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NEW RESEARCH TECHNIQUES IN PHYSICS

SYMPOSIUM ORGANIZED BY THE

ACADEMIA BRASILEIRA DE CIENCIAS

(M)

**CENTRO DE COOPERACION CIENTIFICA PARA AMERICA LATINA
UNESCO**

UNDER THE AUSPICES OF THE

CONSELHO NACIONAL DE PESQUISAS DO BRASIL

O " SIMPOSIO sobre Novas Tecnicas de Pesquisa em Fisica" foi promovido pela [Academia](#) Brasileira de Cientias em colaboracao com o " Centro de Cooperation Cientifica para America Latina" da UNESCO e sob os auspicios do Conselho National de Pesquisas do Brasil. Foi seu objetivo promover uma oportunidade para que fisicos do Brasil e de varios paises da America Latina entrassem em contato pessoal com alguns colegas da Europa e dos Estados Unidos da America do Norte. As reunioes scientificas tiraram lugar no Rio de Janeiro e em Sao Paulo durante as duas semanas de 15 a 29 de julho de 1952. Foram apresentadas e discutidas as memorias encaminhadas no presente volume. A esta Academia coube a tarefa de convidar os fisicos norte-americanos e europeus, organizar e supervisionar o programa geral das reunioes scientificas e das visitas a laboratorios e institutos e dirigir o trabalho de impressao do presente volume. Coube ao "Centro de Cooperation Cientifica para America Latina" da UNESCO, convidar os fisicos latino-americanos, supervisionar o trabalho de registro das discussões e contribuir financeiramente para a realizacao do Simposio. Os membros do Simposio foram hospedes da Universidade de Sao Paulo, que coordenou a parte das reunioes scientificas realizadas em suas dependencias naquela cidade. O Conselho National de Pesquisas do Brasil concedeu uma substancial ajuda financeira, sem a qual teria sido impossivel dar a essas reunioes a amplitude de que se revestiram.

Desejamos expressar nesta oportunidade os nossos profundos agradecimentos a todos quantos cooperaram com a Academia para o exito desta reuniao cientifica internacional.

ARTHUR MOSES

J. COSTA RIBEIRO

J. LETTE LOPES

ENTRE las diversas actividades del Centro de Cooperación Científica para América Latina de la UNESCO se encuentra la de organizar Reuniones Científicas internacionales que contribuyan a poner a punto el estudio de ciertos problemas de actualidad.

Durante el período de los 4 años últimos, el Centro ha organizado una serie de Symposia, sobre temas muy distintos.

El primero sobre "Biología Marina" se celebró en Valparaíso (Chile) y fue seguido del que sobre "Biología Andina" tuvo lugar en Lima en 1949. En noviembre de 1950 se celebró en Montevideo el destinado a estudiar los "Problemas fundamentales de estructura y fisiología celular". Un año más tarde los más destacados especialistas latinoamericanos en "Farmacología" fueron congregados por el Centro de Montevideo en Lima y en diciembre de 1951 se reunieron en Punta del Este (Uruguay) un grupo de matemáticos destacados de Ibero-América para discutir acerca de "Algunos problemas matemáticos que se están estudiando en América Latina".

Todos estos Symposia han sido organizados por el Centro de Cooperación Científica para América Latina de la UNESCO en íntima colaboración con Organismos o Instituciones del País donde se celebró la Reunión.

En este volumen se reúnen los trabajos y discusiones del Symposium sobre "Nuevas Técnicas de Investigación en Física" celebrado en Río de Janeiro y São Paulo en julio de 1952.

No hubiese sido posible su Organización sin contar con la eficiente colaboración de la Academia Brasileira de Ciências y del Conselho Nacional de Pesquisas. El apoyo moral y material de estos dos grandes Organismos brasileños, hizo posible sin duda el éxito científico de la Reunión.

El Centro de Cooperación Científica para América Latina de la UNESCO quiere dejar expresa constancia de su agradecimiento a esos Organismos así como a la Universidad de São Paulo que contribuyó a realzar las reuniones del Symposium aportándole su ayuda moral y material.

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NEW RESEARCH TECHNIQUES IN PHYSICS

Meson Research with the Chicago Synchrocyclotron*

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The 170" synchrocyclotron at the University of Chicago accelerates protons to an energy of 450 Mev. When this internal beam of protons strikes a beryllium target, pions (π — mesons) are emitted in large numbers. Negative pions emitted in the forward direction are deflected outward by the magnetic field of the cyclotron and emerge from the vacuum chamber through a thin aluminum window. The cyclotron field separates the pions according to their energy by deflecting those of lower energy more; those of higher energy less. A certain amount of focusing takes place in the fringing field of the cyclotron such that a sizeable fraction of the mesons of given momentum leave the cyclotron in a fairly parallel beam. Channels were cut in the 12 foot thick steel and concrete shield which separates the cyclotron from the experimental room so that a number of these beams having different energies are available in the experiment room.

Since much stray radiation, mostly fast neutrons, come through the channels from the cyclotron, a sector magnet is used to deflect the pions through an angle of about 45° , thereby purifying the beam from the unwanted radiation. Muons and electrons having the same momentum and sign as the pions remain in the deflected beam. The muons amount to about 8%. The electrons are present in negligible numbers for the beams above 100 Mev. Below this energy the electron contamination increases rapidly. For this reason low energy

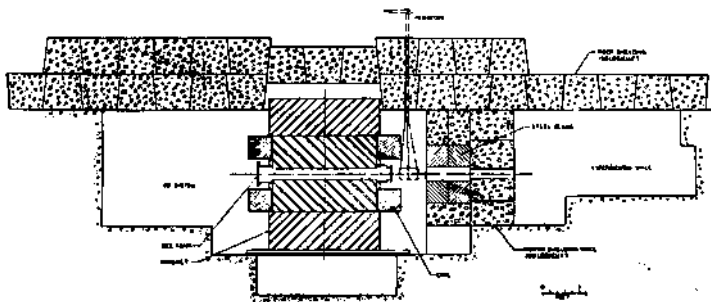


Fig. 1. Cross-sectional view of cyclotron pit showing shielding arrangement.

* Research supported by a joint program of the ONR and AEC.

measurements have been taken by reducing, with a beryllium absorber, the energy of the 122 Mev beam.

Positive pions are obtained by reversing the direction of both the cyclotron and the deflection magnet field. The proton beam in the cyclotron then circulates counterclockwise. The positive pions which are emitted backward from this direction follow the same trajectories and have the same energy as their negative counterparts. The backward emission is unfavorable and the intensity of the positive pions which is obtained in this way is much smaller than that of the negatives.

The intensities are strongly energy dependent. This is due not only to the distribution in energy of the pions which are produced in the target, but also to the peculiar properties of the fringing field of the cyclotron as a spectrometer. The most efficient focusing occurs for pion energies near 120 Mev, and since this energy is close to the maximum for the emission of π in the forward direction, the most intense π beam is that which emerges through the 122 Mev channel. The normal intensity of this beam is about 1000 per cm^2 per second. A 50 cm^2 area (2" X 4") is available in which the energy spread is about $\pm 3 \text{ Mev}$. The π^+ beam in the same channel has an intensity which is about 1/30 of this. Beams of π mesons of usable intensity have been obtained

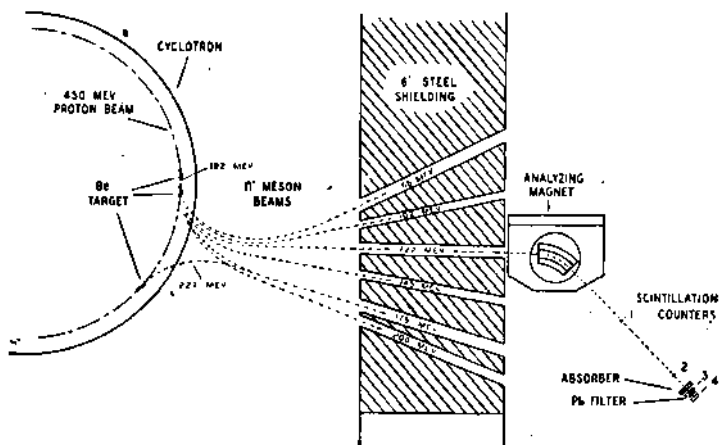


Fig. 2. Experimental arrangement.

from 60 to 227 Mev. The π^+ beams used so far have energies from 60 to 175 Mev. The mesons are usually obtained using a beryllium target 2" long in the beam direction $\frac{1}{4}$ " thick and $\frac{1}{2}$ " high. The target is provided with a thermocouple arrangement so that the energy dissipated by the beam traversing it is recorded. Normal operation is with 15 watts developed in the target. From this it can be deduced that the proton current has a time average value of $1 \mu\text{a}$ and that about 4×10^{10} mesons are produced per second in the target.

Scintillation counters are used to detect the mesons. In particular, liquid scintillators in lucite cells have proven most satisfactory. These can

be made with large areas of quite uniform sensitivity. The speed is quite high ($\sim 6 \times 10^{-9}$ seconds). The use of low density and low Z materials makes the absorption and scattering of pions in them tolerably small. The design of a 2" scintillation cell is shown in Figure 3. The cell is made of clear Incite 3/4" thick overall with 1/16" thick Incite windows. The liquid is phenylcyclohexane with 3 grams per liter of terphenyl and 10 mg per liter of diphenylhexatriene according to the prescription. (KALLMAN & FURST, 1951). The end of the lucite cell is shaped to fit the photocathode of a 5819 tube and good optical contact is assured by a thin layer of clear silicone grease. The whole cell is wrapped in thin aluminum foil which enhances the light collecting efficiency. Phototube and scintillation cell are light proofed with black Scotch electric tape. A μ metal shield is necessary to protect the photomultiplier from the stray magnetic field of the cyclotron.

The 4" diameter scintillators are shown in Figure 3. They are 1" thick overall with 1/16" thick lucite windows. The ends of the lucite cell are shaped to fit the large cylindrical photocathode of the RCA C-7157 photomultiplier. The lucite cell is wrapped in thin aluminum foil and the whole assembly mounted inside a steel box for magnetic shielding.

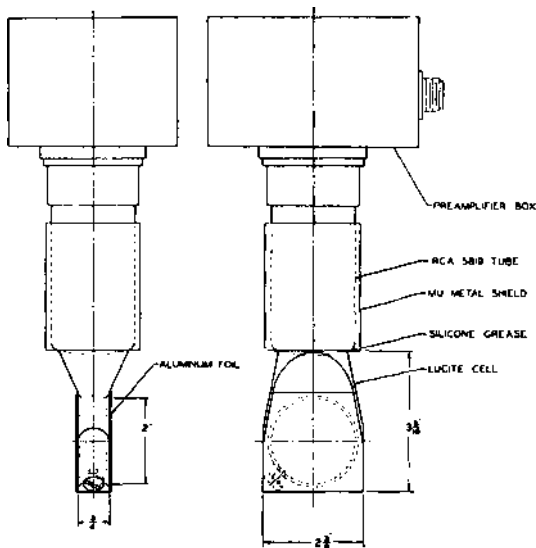


Fig. 3. Two-inch diameter scintillation counters.

For transmission measurements four scintillators are arranged in line as in Figure 2. Coincidence circuits are arranged to record the quadruple coincidences produced when an ionizing particle penetrates all four scintillators and also the double coincidence when the particle traversed the first pair. Thus, the ratio Q/D of quadruples to doubles is the fraction of particles which penetrate the last pair of scintillators after having passed through the first pair.

An absorber placed in front of the second pair of scintillators removes pions from the beam with the result that the ratio, quadruples to doubles, is reduced. Comparison of this ratio with and without absorber is expressed as a cross-section by means of the formula

$$\frac{Q/D \text{ with absorber}}{Q/D \text{ without absorber}} = \exp(-N\sigma)$$

where N is the number of atoms per cm^2 in the absorber and σ is the cross-section in cm^2 .

The last pair of scintillators are made purposefully large, and the absorbers placed as close to them as possible so as to minimize the effects of multiple coulomb scattering and small angle diffraction scattering.

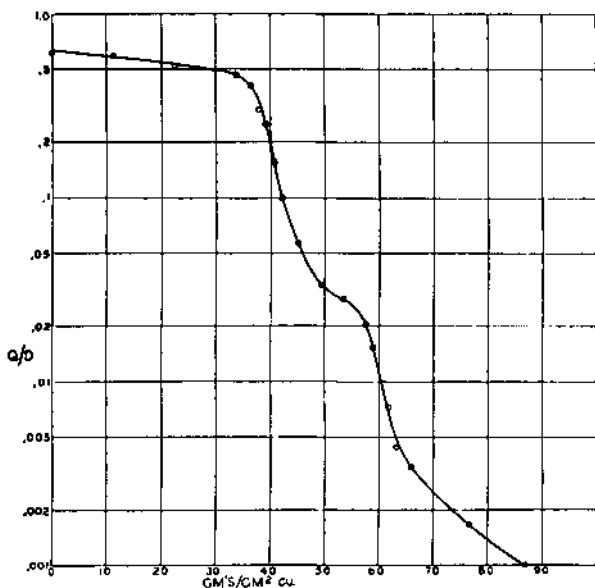


Fig. 4. Range curve of π^+ in copper.

With such an arrangement the range curve shown in Figure 1 was obtained. Copper absorbers were placed in front of counters # 3 and # 4 and the resulting absorption curve plotted logarithmically. This curve taken with positive pions from the 145 Mev channel shows two sharp drops. The first is at the range of the positive pions and the second is at the range of the positive muons that contaminate the beam. The energy of the mesons obtained from this range curve, taking into account energy losses in the scintillators and windows was 123 Mev for the pions and 145 Mev for the muons.

The slow muons which remain when sufficient copper is in place to absorb all the pions have been used by FRY (1952), to study μ produced stars in photographic emulsions.

Of 10,000 μ^- endings studied, 3900 showed the electron typical of the $\mu^- \rightarrow e$ decay. Most of the remaining were stops without tracks, presumably due to $\mu^- + p \rightarrow n + \nu$ in the high Z elements of the emulsion. There were 240 tracks with 1 prong (proton emission) and about 7 in which a proton or deuteron and two alphas were emitted consistent with $\mu^- + C^{12} \rightarrow 2 He^4 + d + 2 n$.

Transmission measurements were carried out (MARTIN, ANDERSON & YODH, 1952) to obtain the nuclear cross-sections for negative pions of energy 109 and 133 Mev. It was found that the cross-sections, absorption + inelastic scattering, have the geometric value $A^{2/3} (\hbar/\mu c)^2$ independent of energy in this range.

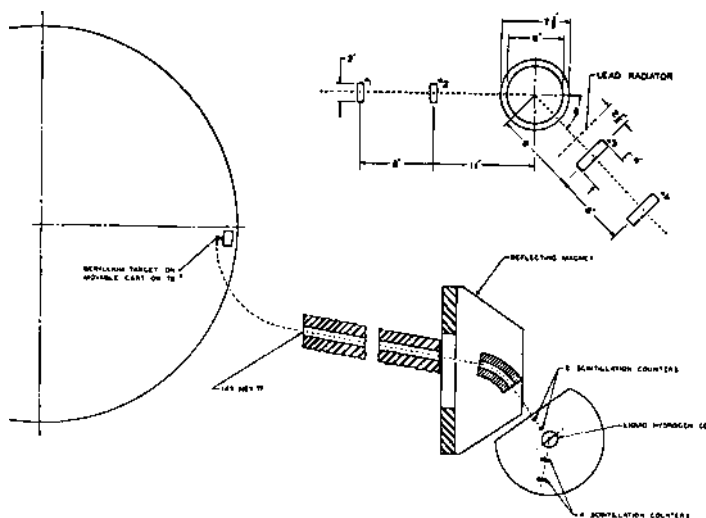


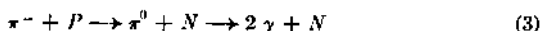
Fig. 5 General experimental arrangement. In the inset the details of the scattering geometry are given.

Transmission measurements with liquid hydrogen were carried out using negative pions (ANDERSON, FERMI, LONG, MARTIN & NAGLE, 1952) of energies from 89 to 217 Mev and also with positive pions (ANDERSON, FERMI, LONG & NAGLE, 1952) over the energy range from 50 to 130 Mev. The results are shown in Figure 0. For negative pions above 135 Mev the cross-section of hydrogen is close to the geometric value $\pi (\hbar/\mu c)^2$. Below this energy, however, the cross-section falls rapidly. This indicates the importance of the p state in the pion-proton interaction. The cross-section with positive pions was found to be several times that with the negatives. This surprising result finds an explanation if the idea that the isotopic spin is a good quantum number is taken seriously. Thus, if the dominant interaction were through states of the pion-proton system of isotopic spin $3/2$, the ratio of the positive to the negative cross-section would be 3.

A more detailed study was made by examining the angular distribution of the scattering from liquid hydrogen using both positive and negative pions, (FERMI, ANDERSON & NAGLE, 1952). The arrangement used is shown in Figure 5. For positive pions only the elastic scattering represented by



was considered possible. With negative pions three processes are believed possible



Equation (2) represents the elastic scattering of negative pions and protons. Equation (3) represents scattering with charge exchange, in which the proton is converted to a neutron and the negative pion is converted

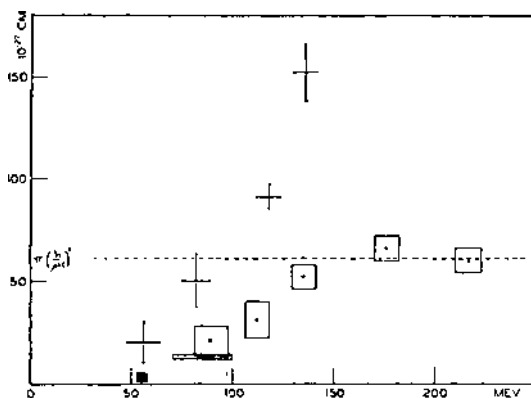


Fig. 6 Total cross-sections of negative pions in hydrogen (sides of the rectangle represent the error) and positive pions in hydrogen (arms of the cross represent the error). The cross-hatched rectangle is the Columbia result. The black square is the Brookhaven result and does not include the charge exchange contribution.

to a neutral pion which almost immediately disintegrates into two photons. Equation (4) is the inverse reaction of the photo-effect in which a photon produces a negative pion by striking a neutron. The cross-section of this last process was estimated by detailed balancing from the inverse process (BISHOP, STEINBERGER & COOK, 1951; FELD, FRISCH, LEBOW, OSBORNE & CLARK, 1952) to be only about $0.8 \times 10^{-27} \text{ cm}^2$.

The sensitivity of the pair of counters #3 and #4 to photons produced in the scattering process is normally rather low. However, it can be increased very appreciably by interposing in front of counter #3 a lead sheet to convert the gamma rays into electron-positron pairs which the counters detect. For this reason, with π^- , four measurements were taken at each angle. Measurements of the ratio Q/D with and without liquid hydrogen in the scattering chamber,

and with and without a $\frac{1}{4}$ " thick lead radiator in front of counter # 3, were made. The difference with and without liquid hydrogen gave the net effect due to the hydrogen.

The differential cross-sections in the center of gravity system are given in Figure 7. These are for pions of energy (in the laboratory system) of 135 Mev. The predominance of the charge exchange scattering over elastic scattering for negative pions is striking. There is a pronounced backward scattering of the π^+ and also for the π^- in charge exchange. However, the elastic scattering of π^- is fairly isotropic.

A phase shift analysis was carried out on the assumption that the scattering is mainly due to states of isotopic spins $3/2$ and $1/2$ and angular momenta $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$. The experimental results are represented quite well at 135 Mev (solid curves) by the phase angles $\pm 26^\circ$, $\mp 11^\circ$ and $\mp 37^\circ$ for isotopic spin $3/2$; $\pm 1^\circ$, $\pm 20^\circ$ and $\pm 2^\circ$ for isotopic spin $1/2$.

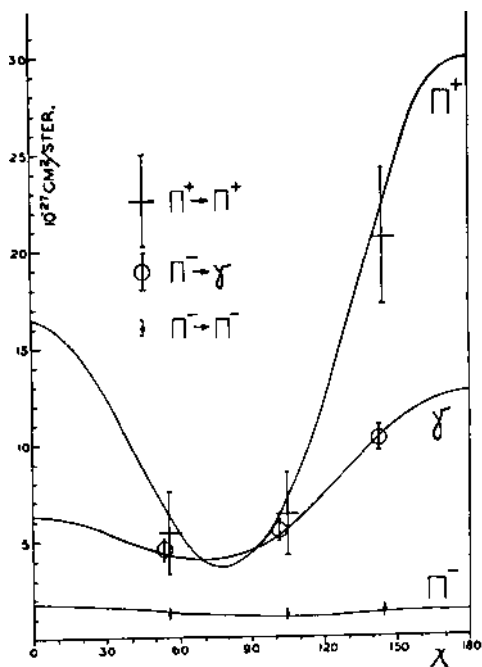


Fig. 7. Cross-sections in the center of mass system at 135 Mev. The three curves have been computed from six phase shift angles.

The success of the attempt to represent the nine experimental data with six phase shifts is a demonstration of the fruitfulness of regarding the isotopic spin as a good quantum number. Such a demonstration would be more meaningful if the experimental error were smaller.

The dominance of the p-state in the pion-nucleon interaction is shown especially for isotopic spins $3/2$. However, the existence of an important interaction for isotopic spin $1/2$ is also exhibited. The interaction in s-states seems to be responsible for the prominence of the backward scattering.

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DISCUSSION

WIGNER: The cross section for the scattering of negative pions seems to be remarkably high. I am wondering whether it is necessary to assume a phase shift in high angular momentum states. Have you made a phase shift analysis?

ANDERSON: NO. The angular distribution measurement has so far been made at 135 and to some extent at 110 Mev. Now, it is true that even that energy may be considered a little bit high to exclude the D states which might enter. But we have hopes for the sake of simplicity that this would not be the case.

Sobre la Relation entre la Radiacion Electromagnetica y la Radiacdn Cosmica Emitidas por el Sol

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Es bien sabido, a consecuencia de las observaciones de Forbush confirmadas por otros ffsicos, que, por una parte, en ciertas raras ocasiones y asociada con las grandes erupciones solares, la intensidad de la radiacdn cosmica aumenta hasta a el doble de su valor normal. Este aumento se observa en la Tierra a latitudes geomagneticas moderadas y altas pero no cerca del ecuador. Por otra parte, siempre o casi siempre que aparecen manchas en la superficie del Sol hay un aumento muy considerable en la intensidad de la radiacdn electromagnetica que viene directamente del astro. Este aumento se percibe principalmente en la region de longitudes de onda de algunos centimetros y de algunos metros. Nos proponemos dar aqui una teoria de estos dos fenomenos.

El primer elemento que interviene en la teoria es el campo magnetico permanente del Sol. Hay gran incertidumbre sobre si, a semejanza de la Tierra, nuestra estrella mas vecina posee un campo magnetico permanente y las observaciones de que se dispone son en gran parte contradictorias. Estas observaciones son de dos clases: las que se refieren a la radiacdn cosmica y las que tienen que ver con el corrimiento Zeeman de las rayas espectrales emitidas en la superficie del Sol. En efecto, los numerosos trabajos teóricos realizados hasta hoy por Janossy, Epstein, Vallarta y Godart y mas recientemente por Dwight indican que la existencia de un campo magnetico permanente del Sol tiene numerosas consecuencias, casi todas de magnitud observable, sobre la intensidad de la radiacion cosmica. El mas notable, en primer lugar, es que las particulas cargadas de baja energia se ven imposibilitadas para llegar a la Tierra. Como resultado, la intensidad de la radiacion cosmica tiene que permanecer constante a partir de una cierta latitud geomagnetica. En segundo lugar, tienen que aparecer variaciones periódicas de la intensidad, de las cuales la mas notable es la variacdn diurna, y ademas variaciones mensuales, semestrales y anuales que deberian ser observables sobre todo a gran altura sobre el nivel del mar y a altas latitudes. Los experimentos realizados hasta ahora son en parte contradictorios pero en general arrojan un resultado negativo. Lo mismo pasa con el corrimiento Zeeman de las rayas espectrales. Sin embargo, hay que tener en cuenta que estos ultimos experimentos son de indole muy delicada, ya que se trata de medir un efecto muy pequeno al que estan superpuestos otros efectos espureos mayores o iguales al buscado.

Los experimentos anteriores a la segunda guerra mundial colocaban la "rodilla" del efecto de latitud de la radiacdn cosmica a 52° de latitud geomagne-

tica, lo cual conduce a un valor del momento del dipolo magnetico solar de 10^{34} gauss-cm³ y a un campo magnetico en el ecuador del Sol de aproximadamente 30 gauss. Este valor esta de acuerdo con el encontrado por Hale y sus colaboradores y basado en el corrimiento Zeeman de las rayas espectrales en el Sol. Con este valor del dipolo magnetico solar se puede calcular que a 55° de latitud geomagnética debería haber un efecto diurno de mas de 5%. Los experimentos recientes estan en desacuerdo sobre la existencia real de la "rodilla" y, si existe, sobre su latitud y no han revelado con seguridad ningün efecto diurno. Por otra parte, los experimentos recientes de Thyssen y Babcock sobre corrimiento Zeeman de las rayas solares arrojan como resultado que si hay en realidad un campo magnetico tiene que ser inferior a 2 gauss. En resumen no hay seguridad sobre el valor del campo magnetico general permanente ni siquiera sobre su existencia real. Las consideraciones que exponemos a continuation, sin embargo, son independientes del valor de dicho campo. Podria ser 100 veces inferior al citado mas arriba sin afectarlas.

El segundo elemento que interviene en la teorfa es el campo magnetico local de las manchas solares. Sobre este se pueden hacer afirmaciones mucho mas seguras que sobre el campo general. Se sabe con seguridad, primero, que las manchas van acompanadas de un campo magnetico cuyo valor maxirno es del orden de magnitud de varies millares de gauss; segundo, que aumenta gradualmente al principio y disminuye poco a poco al fin de la vida de la mancha, conservando un valor constants durante mas de la mitad de dicha vida; tercero, que las manchas siempre aparecen a latitudes solares moderadas y en pares de polaridad opuesta. No se conoce con precision la ley de variation del campo en funcion del tiempo ni se sabe nada sobre como varfa en el espacio en funcion de la distancia al eje de la mancha. A distancias suficientemente grandes del par de manchas y como consecuencia de teoremas generales de la teorfa del potential el campo magnetico de dicho par es el de un dipolo.

El tercer elemento que interviene en la teoria es la existencia de una alta densidad de carga libre en la atmosfera solar. Esta carga libre, formada por electrones, protones y iones en diversos grados de ionization, es de densidad muy elevada, posiblemente hasta 10^{23} cargas elementales por centimetro cubico y su existencia esta asegurada por numerosas observaciones sobre las que no insistiremos aqui.

Pasamos ahora a exponer a grandes rasgos la teoria. Puesto que el campo magnetico de una mancha varfa con el tiempo, hay una fuerza electromotriz dada por la ley de Faraday que actua sobre toda partícula cargada que se encuentre en el campo y la pone en movimiento. Suponiendo que la variacion con el tiempo se nge por la curva de Cowling y que el campo varfa en el espacio inversamente con la distancia del eje de la mancha, hemos estudiado por integration numerica las cSrbitas que describe una partícula cargada durante el tiempo de la aceleracion. Como los exponentes caracterfsticos de Poincare son positivos, el movimiento es inestable tanto en la direction radial como en la axial y la trayectoria se separa rapidamente tanto del piano como del eje de la mancha. Con los valores numericos correspondientes a manchas observadas resulta que un proton, por ejemplo, puede adquirir energfas hasta de 6 Bev. Naturalmente que este processo de aceleracion se aplica a todas las partículas cargadas que esten presentes en el campo magnetico. electrones, protones, iones o nucleos desnudos.

Esto nos lleva a discutir brevemente una objeción hecha por Alfven, y repetida después por Kiepenheuer y otros astrofísicos. Consiste en afirmar que, debido a la alta densidad de carga, el generador mencionado arriba, que nosotros hemos bautizado con el nombre de "cignatron", está de hecho en corto circuito y el proceso no puede en realidad seguir adelante. Esta aseveración parece fundarse en un error. Un generador electromagnético, en efecto, está en corto circuito y muere siempre y cuando no hay suficiente energía disponible en el sistema para mantener el campo magnético variable. Pero esto no es el caso para las manchas solares. Un cálculo sencillo revela, en efecto, que la energía disponible supera por un factor muy grande que puede llegar hasta 10^5 — 10^6 a la energía necesaria para acelerar todas las partículas libres que se encuentran allí. La objeción de Alfven, en consecuencia, carece de validez.

Hemos mencionado que las manchas solares se presentan a latitudes solares moderadas. Si el Sol tiene un campo magnético general permanente, cualquiera que sea su valor dentro de ciertos límites no excluidos todavía por el experimento, entonces precisamente en dichas latitudes se encuentran las regiones prohibidas de Stormer, donde no puede moverse ninguna partícula con energía cinética como la ya apuntada. Se sigue de aquí que las partículas aceleradas por la variación del campo magnético de las manchas solares normalmente no pueden escaparse del Sol. Solo en circunstancias excepcionales pueden hacerlo, y esta posibilidad tiene que ver con la existencia del campo local de las manchas y la circunstancia, ya apuntada arriba, de que a distancias suficientemente grandes el campo de un par de manchas es el de un dipolo.

Si la relación entre las magnitudes de los dos dipolos, es decir, del dipolo correspondiente al campo magnético general permanente, que llamaremos "dipolo permanente", y el que corresponde al que pertenece al par de manchas solares, que llamaremos "dipolo transitorio" es la adecuada, si la orientación relativa de los dos dipolos en el espacio es también la adecuada y si el campo excede de una cierta constante en toda la región afectada, entonces, como lo indicó Godart, se perfora un túnel a través de la región prohibida y por ese túnel pueden escaparse las partículas ya aceleradas. Es claro que estas tres condiciones no se cumplen sino en casos excepcionales. Así se explica por qué solo raramente hay un aumento de la radiación cósmica cuando aparecen las manchas solares y esto generalmente acompañando a las grandes erupciones. Normalmente el túnel permanece cerrado en su desembocadura, o bien ni siquiera existe. El problema de determinar los túneles ofrece algunas dificultades matemáticas y no ha sido resuelto sino para el caso más sencillo de dos dipolos.

Una vez salidas del túnel las partículas cargadas se mueven a través del campo magnético solar y luego, una vez llegadas a la esfera de influencia del campo magnético terrestre, se mueven a través de este último. Para poder llegar a un punto dado de la Tierra es preciso que tengan una energía superior a la energía mínima de llegada a ese punto en la dirección de llegada. Como ya se ha dicho, la energía que adquiere un protón por el proceso de aceleración ya descrito arriba es cuando más de 6 Bev. A esto se debe que no puedan llegar sino a latitudes medias y altas, pero no a latitudes cerca del ecuador. A esto se debe que los aumentos de intensidad de la radiación cósmica no se observan en el ecuador o cerca de él. El problema de la integración de las ecuaciones diferenciales del movimiento-

en el campo de dos dipolos cruzados tambien presenta graves dificultades analiticas y todavia no ha podido ser completamente resuelto.

Las aceleraciones que experimentan las particulas pesadas (protones y iones) son relativamente bajas. No ocurre asl con las ligeras (electrones). Como consecuencia los electrones pierden por radiacion electromagnetica la mayor parte de su energia durante el periodo de la aceleracion. Adaptando las formulas de Schwinger y Pomeranchuk, desarrolladas para la radiacitfn de electrones acelerados por un betatron, al caso presente, se ve que la mayor parte de la energia radiada se concentra en las regiones de longitud de onda de vinos cuantos centlmetros y unos cuantos metros, de acuerdo con lo observado. Tambien se puede prever, y determinar experimentalmente, el estado de polarizacion de la radiacion emitida. Los calculos de polarizacion todavia no han sido hechos.

La radiacion electromagnetica emitida por los electrones pasa libremente a traves del campo magnetico tanto solar como terrestre y en consecuencia siempre que hay aceleracu'm de electrones por el campo magnetico de las manchas solares tiene que aumentar la intensidad de la radiacion electromagnetica que viene del Sol. En consecuencia, casi siempre que aparecen manchas solares hay aumento de radiacic'm electromagnetica, tambien de acuerdo con los resultados experimentales. No es necesario para ello que este" abierto al exterior el tunel; bast a que exista. Solamcnte en el caso excepcional en que no hay espacio disponible para que puedan moverse los electrones durante el periodo de aceleracion cs cuando no hay emision de radiacion electromagnetica.

DISCUSSION

KERST: IS it possible that large sun spots on bigger stars will give much more than 6 billion volts?

VALLARTA: It is a hypothetical question, because we do not know anything about stellar spots. But if the star is of the same type of the sun, and we have no reason to believe that these stars will not behave more or less in the same way as the sun, then indirectly we can believe that there are stellar spots in this type of stars at least. If you consider some of these stars, the larger stars of the same type as the sun, and you make some estimate of the? size of the stellar spots and the maximum magnitude of the magnetic field and the way it. rises to the maximum, then you have the possibility of having much larger energies being generated there. I made some very wild guesses as to what could be expected as maximum energy from these various big stars. It came out to be of the order of 100 Bevs. One wild guess gave as much as 200 Bevs, but perhaps that was too wild a guess.

KKKST: But you still need a mechanism to go to higher energies.

VALLARTA: That is to say to 10^{23} . For instance, I learned at Denver that there is now some evidence, one photographic plate showing one case of a heavy nueleon with an energy of 1.000 Bevs per nucleon hitting something else. I do not think that a star would give that energy. It must have been something else that occurred to the acceleration to come up to that energy.

HABI: Can you expect a smooth average law of energy variation of cosmic rays from such a mechanism?

VALLARTA: We do not know why it is so at the moment, but we do know that apparently the proton component which is the most abundant, and the heavier components, that is, helium, lithium, beryllium and boron, etc. up to iron have the same energy spectrum. That is all we know. Why it is so, I don't think anybody can tell at the moment. That was a question that was rather extensively discussed at Denver some two weeks ago by Professor Fermi. But I do not think anybody can quite understand it. Perhaps one could say this much,—that whatever

processes are responsible for the acquisition of this very high energy, they are the same for protons and other nuclei, but further than this I do not think anyone can go.

ANDERSON: Could you clarify the point about the mean free path? If the density is 10^{23} per cu.cm., the mean free path would be of the order of one meter or so. Would you not expect a rather large number of scatterings?

VALLARTA: The loss of energy by impact with other molecules should be of the magnitude of 1 Bev. which is, let us say, 15% or more of the total acquired energy, which is not too much,

ANDERSON: Would you not expect rather large deflections in the scattering?

VALLARTA: Yes, indeed. There is a large number of deflections in the scattering, but that does not matter. From one point of view they are not so long that they do not stay within the tunnel that I spoke about. They cannot get out of the tunnel, so no matter what happened to these particles they always have to stay within the tunnel. They wander around and eventually they shoot out from the mouth of the tunnel. I do not think that this matters too much in the scattering.

BOHM: I would like a clarification on the question why there is no short circuit in this generator in the sun spots. You state that there was plenty of energy available, and this is what I do not quite understand. You certainly do not accelerate all of them, because there could not be enough energy for this acceleration of a very large mass of particles to 6 billion volts.

VALLARTA: The fact is that unless our calculations are wrong, we have enough energy in the magnetic field of the sun spots. You must remember that the sun spot is very large, in the first place. Very large means dimensions of the magnitude of 50 thousand kilometers diameter, and then the intensity of the field may be of the order of several thousands gauss, and if you figure out the amount of energy that is contained, you find that there is plenty left over after you have accelerated all the particles. That is why I think the short circuit argument is not tenable.

DEUTSCH: Then; is evidence of radio frequency sources other than the sun. Now, can one estimate from the amount of radio frequency that comes from other sources what fraction of these corpuscular radiations we receive might be due to a comparable mechanism? I mean the fraction of the electromagnetic radiation and the fraction of corpuscular radiation.

VALLARTA: That is in reality what Professor Kerst asked a moment ago. Probably the origin of this radio noise comes from other sources or something like what I have described, but I am afraid that nobody has attempted the kind of estimate that you have asked about. I do not know of any such estimate and it would not be so easy to make. Because, for instance, you take one of these regions of the sky, where there are large emitters of radio noise like, for example, the Crab Nebula. Now, the Crab Nebula consists of a great many stars that emit a certain amount of radio noise. It would be very difficult to estimate what corresponds to each of the stars in the Crab Nebula, which has perhaps a 100 million stars. Of course, one may make a wild guess — I would be willing to make one myself.

GROSS: You mentioned the knee of the latitude effect. I wonder if there is sufficient evidence for the existence of a definite knee at sea level. That means, so far, an increase of radiation has only been found at higher altitudes and higher latitudes. Summing up, would it not be just to conclude that there is more evidence for the existence of the knee than for the absence of the knee?

VALLARTA: Unfortunately, this is not the case. If there was a real general field of the sun the knee should be the same no matter what the altitude. It would have nothing to do whatever with atmospheric absorption. The evidence which we have is something like this: If you take the experiments of Pomerantz, he went to latitude of the order of magnitude of 82 geomagnetic degrees, almost to the geomagnetic pole. Intensity seems to be increasing all the way there, although there is a slight discontinuity around 55 degrees, but beyond 55 it continues increasing.

MOSHINSKY: I would like to put a question in regard to Prof. Deutsch's remark. Is there any correlation between the intensity of the cosmic radiation and the intensity of the electromagnetic radiation of the sun spots?

VALLARTA: The electromagnetic radiation appears almost always when you have sun spots. The electromagnetic radiation does not need to have a tunnel. It can come out, or leak out, across or through the forbidden region. The cosmic radiation cannot come out of the sun unless there is a tunnel, and, as I said before, the existence of a tunnel is a phenomenon of very rare occurrence. The electromagnetic radiation is produced whenever electrons have a region in which to move, and that means that the tunnel may be open part of the way or closed at the mouth. That is why there is always electromagnetic radiation emitted but cosmic radiation seldom occurs.

MOSHINSKY: Is there any quantitative relation between the emitted cosmic radiation and the emitted electromagnetic radiation?

VALLARTA: In cases where cosmic radiation appears, there is. This seems to be in fair agreement with experiments. The total electromagnetic radiation, if you compare the energies of the two, seems to be correct within a factor of 1/2.

G. WATAGHIN: Is there some estimate on the upper limit of the average magnetic field in our galaxy?

VALLARTA: Some three months ago, it would have been very easy to answer. The answer would be that it would be 10^{-5} . But lately, questions have been asked about the interpretation of this polarization, mainly by research workers at Harvard. Apparently they do not feel that the estimate is correct, so I do not know at the moment what the answer to that question would be. Apparently, the old estimate is not correct.

On the East-West Asymmetry of the Electronic Component of the Cosmic Radiation

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SUMMARY

The data seem to indicate that there is a small negative asymmetry for the electronic component of the cosmic radiation. The experiment should be repeated under better experimental conditions.

INTRODUCTION

The azimuthal asymmetry of the cosmic radiation has been measured at different latitudes and different altitudes by many investigators. For these measurements a counter telescope pointed at a fixed zenith angle is oriented alternately in the directions east and west and a counting rate is obtained for these two orientations. The asymmetry is usually defined as follows:

$$\sigma = \frac{2 [J(h, \lambda, \omega, \xi)_W - J(h, \lambda, \omega, \xi)_E]}{J(h, \lambda, \omega, \xi)_W + J(h, \lambda, \omega, \xi)_E}$$

where $J(h, \lambda, \omega, \xi)$ is the intensity of the radiation at a place h units above sea level, λ geomagnetic latitude, ω longitude and ξ zenith angle. The subscripts W and E indicate whether the zenith angle is towards the east or towards the west. Usually the counter telescope consists of three counters aligned along an axis and pointed at the desired zenith angle. With such a telescope one can measure the azimuthal asymmetry of the total radiation. To measure the asymmetry of the hard component a lead slab 10 cm thick is introduced between the counters eliminating in this way the soft component. Most of the azimuthal asymmetry measurements that have been done till now have been measurements of the asymmetry of the hard component and of the total radiation. To measure the asymmetry of the soft component the most common procedure has been to take directional counts with and without the 10 cm lead slab and to consider the difference between these two counts as the soft component intensity. Another method of measuring the asymmetry of the soft component is to use three counters located at the vertices of an isosceles triangle with a 2-3 cm lead slab over the topmost counter. With this arrangement, a soft photon or a soft electron that hits the lead slab from above and in the direction of the perpendicular bisector of the base of the triangle will have a high probability of producing a

burst in the lead slab that will trigger the lower counters. This array will have a resolving power which will depend on the dimensions of the isosceles triangle. Any other cosmic ray particle that hits the lead slab will have a much smaller probability of producing a burst and thus the telescope will register soft photons and soft electrons preferentially. A thickness of from two to three centimeters is used because this is the thickness for the maximum of the Rossi (1933) transition curve.

In JOHNSON'S early experiments (1933d, 1934, 1935 a, b) he measured the asymmetry of the total radiation and of the shower producing radiation at different latitudes and altitudes and obtained a positive asymmetry of about 10% for the total radiation and a slight positive asymmetry (of the order of 3%) for the shower producing radiation. From these experiments JOHNSON concluded that the primary particles of the hard component were positive (presumably protons) and that the primary particles of the soft component were a mixture of positive and negative particles. JOHNSON & BARRY'S (1939b) high altitude experiments brought new evidence in favor of this theory when at atmospheric pressures of less than 5 cm Hg the asymmetry of the total radiation was found to be 7% whereas an effect of 60% was expected if primaries were exclusively positive. SCHEIN, JESSE & WOLLAN (1941) balloon experiments indicated that the primary radiation contains almost no shower producing particles such as electrons. This evidence appeared to require in the primary radiation the presence of penetrating particles which do not excite mesotrons but go into the production of the soft component.

More recent experiments (SCHEIN, YNGVE & KRAYBILL, 1947, 1948; WINCKLER, STROUD & SCHENCK, 1948; WINCKLER, 1948; BARBER, 1949; BIEHL, NEHER & ROESCH, 1949 a, b; WINKLER & STROUD, 1949; WINKLER, STIX, DWIGHT & SABIN, 1950; VAN ALLEN & GANGNES, 1950) have been performed, but still the results of the soft component experiments are somewhat confusing. SCHEIN, YNGVE & KRAYBILL (1948) asymmetry measurements in a B-29 airplane brought convincing evidence that the hard component arises from primary rays which are mostly positively charged. In BARBER'S (1949) experiment a comparison of the observed east-west asymmetry with the values predicted from the latitude effect under the assumption that all primaries are positive showed that the observed values are larger than the predicted which seems to preclude the possibility of negative primaries. In this experiment they obtained a value of 3.5% for the east-west asymmetry of the shower producing radiation and an asymmetry of 11% for the total soft component. BIEHL, NEHER & ROESCH (1949a) measured the east-west asymmetry at Peru (geomagnetic latitude 0°) at 30,000 ft with no lead absorber and with lead absorbers 10 and 20 cm thick. The west excess obtained from this experiment was $22.3\% \pm 1.1\%$ for no lead, $27.0\% \pm 1.3\%$ for 10 cm lead absorber, and $30.4\% \pm 1.5\%$ for 20 cm lead absorber, all these results at 3.1 meters of water equivalent. These results indicate that the asymmetry of the total radiation is nearly equal to that of the penetrating radiation. This would mean that there is approximately the same asymmetry for the hard as for the soft component. BIEHL, NEHER & ROESCH (1949a) also found a very high correlation between the latitude and east-west effects assuming all the primary particles to be positive. To make this calculation they used VALLARTA'S (1948) curves and found that at zenith angle 45° east the minimum momentum for positive particles at 25° north geomagnetic latitude is the same as at the vertical on the geomagnetic

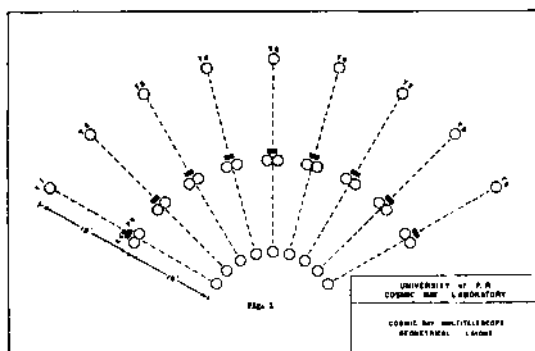
equator. In a similar way for 45° west the minimum momentum at 25° north geomagnetic latitude is the same as at the vertical at 29° north geomagnetic latitude. Thus if all particles are positive, one would expect the same ratio of east to west at 25° north as the ratio in vertical intensities between 0° and 29° north. BIEHL *et al* (1949a) calculated these ratios and they obtained 0.82 and 0.78. While these ratios are very close, BIEHL *et al* (1949a) point out that there is an uncertainty in just how the correct curves in VALLARTA'S (1948) figures should be drawn. From this experiment BIEHL *et al* (1949a) concluded that it was not necessary to assume a different primary particle to account for the penetrating and soft components at the equator and that it is quite likely that one kind of incident positively charged particle will suffice. WINCKLER & STROUD (1949) measured the zenithal distribution of particles at high altitudes with counter telescopes and made simple calculations of the multiplicity and angular divergence of secondary particles. The observations indicate a sharply collimated forward type of production event. The azimuthal, and zenithal distributions indicate a nearly isotropic primary flux.

YEATER (1945) has made a study of the time variations of the cosmic ray directional intensity distribution and has found evidence of some time variations. These variations, occurring within periods of 12 to 28 hours seem most pronounced near the zenith angles 20° and 70° in the east-west plane, and are not found in the north-south plane. These observations would appear to be explained by the effect of the fluctuations of the magnetic field of the earth. These fluctuations modify the allowed cones of certain primary cosmic ray particles of energies near 0.2 and 0.6 stromer. YEATER shows that the east-west positional symmetry of these variations is evidence that there exist positive and negative particles with the same e/m in the primary cosmic radiation.

We wish to point out that according to HEITLER (1937) the soft-component could be easily explained by the cascade theory if the primaries were electrons. According to CACCIAPUOTI (1939) the soft component is not in equilibrium with the hard component. Yet according to FERRETTI (1939) and BHARHA (1938) and EULER & HEISENBERG, (1938) the electrons should be in equilibrium with the hard component. According to Rossi & GREISEN (1942) primary electrons are probably the origin of the soft component. According to HAMILTON, HEITLER & PENG (1943) the primary radiation could be solely protons. The protons produce pseudoscalar mesons and transverse mesons, the former being responsible for the penetrating component, whereas the latter give rise to the major part of the soft component through beta decay. GOLIAN, KRAUSE & PERLOW (1946) rocket experiments also seem to indicate that at least 70% of the incoming primaries are protons. The fraction of primaries which may be electrons is likely to be less than 30%. The small fraction of the primaries alone can not explain the intensity of the soft component and for this reason some other origin is at least partly necessary. According to BERNARDINI, CACCIAPUOTI & QUERZOLI (1948) the measurements concerned with the electronic component of cosmic rays near the earth's surface are not well understood in terms of the usual theories of their origin. According to the theoretical work of HEITLER (1937), EULER & HEISENBERG (1938), FERRETTI (1939), BHARHA (1938), only electrons in equilibrium with the hard component

should be expected between sea level and an altitude of 4-5 km. On the other hand, experiments performed by CACCIAPUOTI (1939), and by BERNARDINI, CACCIAPUOTI, FERRETI, PICCIONI & WICK (1940) seem to indicate that the soft component is not in equilibrium with the hard one, in fact, the increase in intensity of the soft component with height is somewhat faster than is theoretically predicted. These results were confirmed by Rossi & GREISEN (1942). Many theoretical and experimental investigations have since been performed to establish the equilibrium conditions between the electronic and meson component but the conclusions are not as yet in satisfactory agreement.

According to ARLEY (1945, 1940) the proton hypothesis of the primary cosmic rays leads to difficulties in the interpretation of other experiments. Among other things, Arley says that it should give a high positive E - W asymmetry at the top of the atmosphere for both the hard and the soft component, a result which is contrary to what high altitude experiments have shown. This smallness of the E - W asymmetry of the total radiation at high altitudes Arley says could be interpreted as being the result of a high positive E - W asymmetry for the hard component assumed to be produced mainly from primary positive protons, and a



high negative E - W asymmetry for the soft component assumed to be produced mainly from primary negative protons.

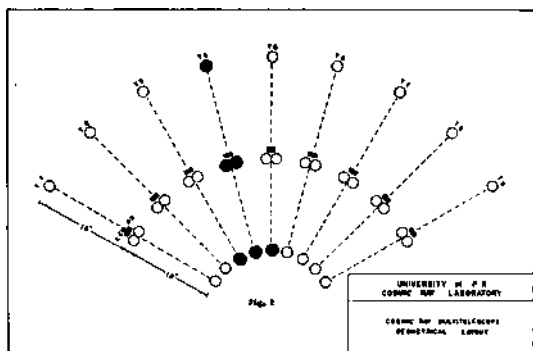
We have measured the symmetry of the electronic component of the cosmic radiation at Rio Piedras, Puerto Rico, geomagnetic latitude $29^\circ N$, longitude $66^\circ W$ and 5 meters above sea level. This is a preliminary report on the results we have obtained so far.

APPARATUS

For this experiment we have built a cosmic ray multitelescope made up of nine telescopes, each of which consists of four counters in four-fold coincidence arranged along 15° intervals of zenith angle as shown in figure 1. This instrument is housed in a wooden building where the roof has a thickness of approximately 6 gm/cm^2 . Above the two middle counters a 2 cm thick lead slab is placed, the length of this slab being equal to the length of the counters. With this arrangement an electron coming in such a direction that it will go thru both the upper counter and the lead slab will probably produce a burst at the lead slab which

will trigger the middle and lower counters and produce a four-fold coincidence. In a 2 cm thick lead slab the probability of an electron producing a burst is very high while the probability of a meson producing a burst is practically zero. In this way we can differentiate between mesons and electrons. A low energy photon can also produce a burst in the 2 cm lead slab but in our arrangement it will not trigger the upper counter. Thus with the geometrical arrangement of figure 1 practically all the four fold coincidences will be due to the electronic component the cosmic radiation. A shower will also trigger the four counters and thus register as an electron but a hodoscope unit attached to the multitelescope provides the means for correcting this. Each telescope operates an impulse counter which registers the count.

The counters we are using are Anton Laboratory type 350. These counters are 5 cm in diameter and 50 cm long and are filled with a halogen mixture. This mixture insures constant operating characteristics over long time intervals. The instrument is operated 24 hours a day and is rotated around an angle of 180° every once a week. Each telescope sustains an angle of approximately 7° in latitude and 60° in azimuth.



The hodoscope unit consists of 30 neon bulbs arranged so that each lights when triggered by a coincidence between a single counter and a master pulse. The neon bulbs are arranged in nine columns of four, each column corresponding to one telescope. The master pulse is produced whenever the four counters of one of the nine telescopes and one or more counters from any of the other eight telescopes fire simultaneously. Thus whenever a master pulse is produced, at least five neon bulbs light, i.e., the four bulbs corresponding to the four counters of the telescope that fired and the bulb or bulbs corresponding to the counter or counters that fired simultaneously. Whenever a shower occurs the bulbs corresponding to all those counters that are triggered by the shower will light unless the rare situation occurs in which a shower fires various counters but not a complete telescope. Our instrument will not register these events where no complete telescope fires but these events occur so rarely that they can be neglected.

The instrument yields the data necessary to measure the asymmetry of the electronic component of the cosmic radiation in two ways: with the aid

of the impulse counter registers and with the pictures obtained from the hodoscope unit. We wish to report the results obtained from the impulse counter registers for a period of approximately 1500 hours. The corresponding hodoscope pictures are being analyzed and a report of such measurement will be made in due time.

As the data obtained from the impulse counters do not differentiate between true counts and counts due to showers we ran the experiment for a period of two weeks and obtained an approximate value for the shower counting rate. This shower counting rate is to be deducted from the actual counting rate of each telescope.

RESULTS AND DISCUSSION

Table 1 shows the results we have obtained until now from the readings of the impulse counters. It is obvious that the uncertainty of our measurements is so high that we can not arrive at a reliable conclusion about the value of the asymmetry we are measuring. Nevertheless we wish to point out that the fact that all telescopes registered a negative asymmetry except telescope 2 and 1, which registered no asymmetry, seems to indicate that there is a negative asymmetry for the electronic component. The experiment should be run for a longer period until the precision measurement becomes reasonably acceptable. Column number 4 of Table 1 shows the approximate shower counting rate which was obtained running the experiment without any lead slabs.

Table 2 shows the results of this measurement where again the uncertainty of our measurement is very high. From this table we wish to point out the fact that with the exception of telescopes 1 and 9 the uncertainty of the measurement is higher than the measurement itself. This would seem to indicate that no asymmetry will be registered by telescopes 2, 3, 4, 6, 7 and 8, while telescopes 1 and 9 might register a negative asymmetry if the measurement were to be made during a long enough period. This result seems to us reasonable because telescopes 1 and 9 are oriented at 60° zenith angle and at such angle any large amount of matter in the room next to the laboratory would affect the measurement. There are a few tons of mimeograph paper stored in the room next door and piled in stacks high enough to affect the lowermost telescopes. For this reason we should neglect the results obtained with telescopes 1 and 9. In Table 1 telescope 1 gave an asymmetry of $-50\% \pm 10\%$ which is entirely unreasonable unless explained by the amount of matter next door.

Table 3 shows a measurement of the asymmetry of the total radiation which was obtained by using three counters in line for each telescope and no lead. Here again the uncertainty is very high due to the shortness of our measurement, but again all telescopes except 2 and 4 give a positive asymmetry as they should. Thus one would be inclined to believe that the instrument can measure asymmetry if operated for a long enough time.

From the analysis of the hodoscope pictures we believe that the uncertainty in our measurements will be less. We have approximately 30,000 pictures and this will take a few months to analyze. We shall make another report as soon as this work is finished. In doing this work we are transferring the data from the film to separate diagrams of figure 1, in such a way that to each frame corresponds one diagram. Figure 2 shows an example of this transferring

procedure where it can be seen that on the frame examined telescope 4 fired and also the lower counters of telescopes 3 and 5. This seems to indicate that the burst produced was wide enough to fire the lower counters. In doing this we will have the data from the film in a more easily examinable way. At a later time this data can be transferred to IBM cards so that any statistical measurement can be performed quickly and efficiently.

If there is a negative asymmetry of the electronic component of the cosmic radiation we will have to look into the problem of the primary particles which give origin to these electrons. It should be emphasized at this moment that the measurements at or below sea level or at any point within the atmosphere give the east-west asymmetry of the secondary particles and not of the primary particles. As we can not assume that in the primary radiation the positive particles and the negative particles will undergo identical multiplication processes, we can only determine the charge composition of the primary radiation at altitudes high enough that the multiplication can be considered to be negligible.

We plan to perform this high altitude measurement but before that, the present experiment should be repeated under better experimental conditions. To this effect funds have been assigned by the University of Puerto Rico to erect a wooden structure isolated from the rest of the University buildings to house the new instrument. The uncertainty in our measurements is due mainly to the inefficiency of the counters. These counters were sold to us under the understanding that their efficiency was about 85% but they turned out to have an efficiency of about 50%. With a four-fold coincidence circuit an efficiency of 85%; is barely acceptable if the experiment is to run for a period of a few months but when efficiency is 50% the experiment is practically impossible unless one *can* run it for a period of years.

ACKNOWLEDGMENTS

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ELECTRONIC COMPONENT

TELESCOPE	CORNTS	TIME IN HOURS	COUNTING RATE IN COUNTS/HR.	SHOWER COUNTING RATE IN COUNTS HR.	CORRECTED COUNT IN COUNTS HR.	INTENSITY'	INTENSITY J_w	ASYMMETRY
1East	3465	1486	2.33	2.08	0.25	0.25 $\pm$ 0.01		
1West	3312	1488	2.22	2.08	0.14		0.14 $\pm$ 0.01	—56% $\pm$ 10%
2East	1799	1696	1.06	0.65	0.41	0.41 $\pm$ 0.02		
2Wed.	1804	1698	1.06	0.65	0.41		0.41 $\pm$ 0.02	0
3East.	2536	1796	1.41	0.90	0.51	0.51 $\pm$ 0.02		
3west	2211	1596	1.38	0.90	0.48		0.48 $\pm$ 0.02	- 6% $\pm$ 8%
4East	1677	1393	1.20	0.71	0.49	0.49 $\pm$ 0.02		
4West	1390	1173	1.19	0.71	0.49		0.48 $\pm$ 0.02	0
6East.	2274	1637	1.39	0.74	0.65	0.65 $\pm$ 0.02		
6West.	2321	1797	1.29	0.74	0.55		0.55 $\pm$ 0.02	—16% $\pm$ 7%
7East.	1794	1728	1.04	0.72	0.32	0.32 $\pm$ 0.01		
7west.	1469	1509	0.97	0.72	0.25		0.25 $\pm$ 0.01	—25%, $\pm$ 7%
8Eatt.	1663	1461	1.18	0.73	0.45	0.45 $\pm$ 0.02		
SWut.	1953	1796	1.09	0.73	0.36		0.36 $\pm$ 0.02	—22%, $\pm$ 10%
9Ea*t	1599	1640	0.97	0.59	0.38	0.38 $\pm$ 0.02		
9Wea.	1625	1796	0.90	0.59	0.31		0.31 $\pm$ 0.02	—21% $\pm$ 12%

REMARKS

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TABLE II. SHOWERS

ASYMMETRY	REMARKS
$-12\% \pm 10\%$	
$-4.6\% \pm 18\%$	
$\pm 12.7\% \pm 15.5\%$	
0	
$\pm 5.4\% \pm 18.4\%$	
0	
$\pm 9.6\% \pm 16.6\%$	
$-20\% \pm 99\%$	

TABLE III. TOTAL RADIATION

TELESCOPE	COUNTS	TIME IN HOURS	COUNTING RATE IN COUNTS/HR.	INTENSITY J_E	INTENSITY J_W	ASYMMETRY	REMARKS
1 East	1110	219	5.07	5.07			UNCERTAINTY TOO HIGH
1 West	1192	216	5.52		5.52	+ 8% $\pm$ 5.8%	
2 East	2056	219	9.39	9.39			
2 West	2033	216	9.41		9.41	0	
3 East	3670	219	16.76	16.76			
3 West	3700	216	17.13		17.13	+2.2% $\pm$ 3.2%	
4 East	3357	219	16.24	16.24			
4 West	3405	216	16.18		16.18	0	
6 East	3899	216	18.05	18.05			
6 West	4080	219	18.63		18.63	+3.1% $\pm$ 3.1%	
7 East	3257	216	15.08	15.08			
7 West	3408	219	15.56		15.56	+3.1% $\pm$ 3.0%	
8 East	2011	216	9.68	9.68			
8 West	2188	219	9.99		9.99	+3.1% $\pm$ 4.2%	
9 East	1119	216	5.18	5.18			
9 West	1220	219	5.57		5.67	+7.1% $\pm$ 6.1%	

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Asimetria de los Mesones en el Ecuador Geomagnetico

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De este estudio se dara cuenta oportunamente, por los autores, en union del Dr. Robert Williams; todos ellos actuando bajo la direccion del Prof. Bruno Rossi.

Nos vamos a limitar aqui a hacer una descripcion, muy somera, del equipo que se viene utilizando y de los problemas que el mismo plantea, tanto en su parte puramente fisica, como en los de la interpretation de sus resultados.

El equipo que se esta utilizando en este experimento es, en esencia, un selector de mesones μ , positivos y negativos, dentro de un campo magnetico de 18.000 gauss, habiendo sido dibujado integralmente por sus autores y construido en los propios talleres del M. T. T., bajo la direccion personal de los mismos.

Con el fin de dar una idea del equipo lo separaremos en 4 partes fundamentales:

- a) Organos sensibles.
- b) Organos selectores.
- c) Organos registradores.
- d) Fuentes de poder.

a) Como organos sensibles, se utilizan contadores Geiger-Muller de 1/2 y 1 pulgadas de diametro, y 56 centimetros de longitud, igualmente fabricados en el M. T. T., y llenos de argon y alcohol a 12 cms. y 20 cms. de presion respectivamente, colocados en telescopio giratorio, en la forma que indica el grabado.

Este telescopio es doble y puede apuntarse en cuatro posiciones diferentes: 0° , $22^\circ 1/2$, 45° y $67^\circ 1/2$, en forma simple con un pasador movable a travs de una plancha perforada, en los angulos arriba indicados.

b) Los organos selectores, estan constituidos por dos tipos diferentes:

- 1) Campo magnetico de 18.000 gauss.
- 2) Selectores electronicos.

El campo magnetico esta formado por dos piezas dobles, para cada telescopio, de hierro dulce, con un limite de saturacion de 21.000 gauss, alrededor de la cual se han efectuado 5 capas de vueltas con alambre de cobre de n.º 19 de espesor y un recorrido de 1.000 metros, para cada parte lo que hace un total de 12.800 metros para el conjunto.

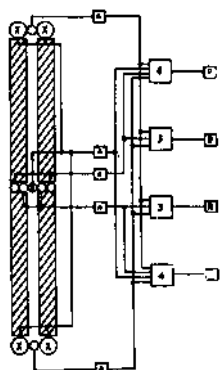


Fig. 1

A = Amplificador
 3 = Coincidencia triple
 4 = » cuadruple
 R = Registrador
 X = Anticoincidencias.

$$dN = F(p, \theta, \phi) dS dp d\omega,$$

donde:

p = momento del meson

θ = angulo zenital

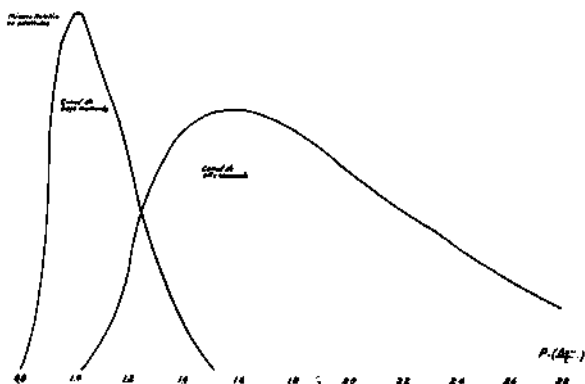
ϕ = angulo azimutal

P = espectro diferencial del momento en funcitin de los angulos de la direction de incidencia del meson.

El problema es el do calcular de los datos arriba citados el numero do particulas que de diferentes momentos atraviesan tres contadores G. M. colocados en la forma que se se sefiala en la fig. 5.

Los tres contadores so enueuntan dentro de un eampo magnetico B , paralelo a los ejes de los mismos. La lfnea L que Se muestra en la figura, tiene en el espaeio las direoeiones θ_0 y ϕ_0 .

Para facilitar el calculo cada tubo sera representado por un area de un piano normal a dicha hnea. Consideremos primero una partícula que pasa por los tres puntos, representados en la fig. n.º 6, del piano del papel.



El meson pierde energia al atravesar el hierro que forma la magneto y por lo tan to pierde momento, sea a una eantidad k unidades por metro, que practieamente es una constante.

Como la trayeetoria es difieil de calcular directamente debemos llegar a ella a traves de ciertas aproximaciones:

1.º --- Dado que $\xi \ll h$ suponer que la distancia recorrida por la partícula entre dos contadores es siempre h .

2.º --- Suponemos tambidn que entre los contadores la partícula sigue una trayeetoria circular cuyo radio esta determinado por el momento de ella en su punto medio.

En un campo magnetico, una partícula cargada (meson en nuestro caso) se mueve en un circulo cuyo radio $R = p/eB$, en donde: e = carga electronica.

En la figura 1, tenemos entonces, que para la parte superior de la trayectoria:

$$R_1 = \frac{p - 1/2 kh}{eB}$$

y en la parte inferior de la figura:

$$R_2 = \frac{p + 3/2 kh}{eB},$$

donde p es el momento incidente, o sea

$$R_1 = \frac{p_1}{eB} \quad y \quad R_2 = \frac{p_2}{eB}.$$

Ello da lugar a que las ecuaciones de los dos círculos se expresan así:

$$(x - x_1)^2 + (y - y_1)^2 = \left(\frac{p_1}{eB}\right)^2 \quad y$$

$$(x - x_2)^2 + (y - y_2)^2 = \left(\frac{p_2}{eB}\right)^2,$$

siendo x_1, y_1 las coordenadas del centro del primer círculo y x_2, y_2 las del segundo.

El primer círculo pasa a través de los puntos $(0, h)$ y $(\xi, 0)$, teniéndose por ello que:

$$x_1^2 + (h - y_1)^2 = \left(\frac{p_1}{eB}\right)^2 \quad o = R_1^2$$

$$(\xi - x_1)^2 + y_1^2 = \left(\frac{p_1}{eB}\right)^2 \quad o = R_1^2.$$

Como anteriormente se indicó $\xi \ll x_1$ y $y_1 \ll h$ y por lo tanto $y_1^2 \ll x_1^2$.

Eliminando los términos en ξ^2 y y_1^2 y solucionando las ecuaciones para x_1, y_1 , se llega a

$$y_1 = \frac{1}{2h} (h^2 - 2\xi R_1) \quad y \quad x_1 = \xi - R_1 \sqrt{1 + (\xi/R_1)^2}$$

Puesto que $\xi \ll R_1, (\xi/R_1)^2 \ll 1$

llegando a que $\sqrt{1 + (\xi/R_1)^2} \approx 1 + 1/2 (\xi/R_1)^2$,

o sea $x_1 \approx -R_1 + \xi$.

El segundo círculo pasa a través de los puntos $(0, -h)$ y $(\xi, 0)$; siguiendo un procedimiento similar se llega a:

$$x_2 = -R_2 + \xi \quad y \quad y_2 = \frac{1}{2h} (2\xi R_2 - h^2).$$

Las derivadas en las dos curvas deben ser iguales en $(\xi, 0)$.

$$\left(\frac{dy}{dx}\right)_1 = \frac{x - x_1}{y - y_1}$$

$$\left(\frac{dy}{dx}\right)_1 \text{ en } (\xi, 0) = \frac{\xi - x_1}{y_1} = \frac{R_1}{(1/2h)(h^2 - 2\xi R_1)}$$

Igualmente $\left(\frac{dy}{dx}\right)_2 \text{ en } (\xi, 0) = \frac{\xi - x_2}{y_2} = \frac{R_2}{(1/2h)(2\xi R_2 - h^2)}$

Ponemos por tanto $\frac{R_1}{h^2 - 2\xi R_1} = \frac{R_2}{2\xi R_2 - h^2}$

Substituyendo los valores de R_1 y R_2 en función del momento p , y resolviendo para éste la ecuación correspondiente, resulta

$$p = kh + (h^2 eB/4\xi) [1 \pm \sqrt{1 + (2k\xi/heB)^2}]$$

Finalmente esta ecuación sólo tiene sentido con signo positivo ya que una partícula debe tener $p \geq 2kh$ para llegar al tercer contador.



Fig. 3

El factor kh que es la pérdida de momento entre dos contadores puede ser representado por p_0 , y $(hc B/4k)$, que tiene las dimensiones de una longitud, por X , resultando portanto:

$$p = p_0 + p_0 (X/\xi) [1 + \sqrt{1 + 1/4 (\xi/X)^2}] \quad (1)$$

Una consecuencia interesante es que se puede deducir un límite para ξ en base de que el mínimo posible para p es $2kh$, o $2p_0$.

Para una parte cualquiera del contador del centro, de área dS , los límites de admisión de partículas que efectúen una coincidencia triple (es decir que atraviesen los tres contadores) pueden observarse gráficamente en la fig. 7.

Representamos en esta figura por α el ángulo de desviación de lo normal de una partícula. El campo magnético existente entre los contadores no tiene efecto alguno sobre la componente de momento paralelo al mismo, por lo tanto solo consideraremos las componentes normales que en la figura se representarían por una línea vertical. Teniendo en cuenta estos conceptos, la ecuación (1) se transforma en

$$p \cos \alpha = p_0 + p_0 (X/\xi) [1 + \sqrt{1 + 1/X^2 (\xi/X)^2}] \dots \dots (2)$$

Los límites de admisión de partículas en la figura 14 se pueden representar matemáticamente por

$$-\frac{(l/2) - u}{h} \leq \lg \alpha \quad ; \quad \frac{(l/2) - u}{h} \geq \lg \alpha ,$$

o dicho de otro modo

$$-\lg^{-1} \frac{(l/2) - u}{h} \leq \alpha \leq \lg^{-1} \frac{(l/2) - u}{h} ,$$



Fig. 4 — Una vista de la magneto.

El ángulo sólido $d\omega$ para nuestra área dS es $n \cos \alpha$, donde n es el límite de la variación en movimiento transversal del trazo de las partículas que, llegando a tocar los dos contadores de los extremos, poseen el momento que consideramos; límite de variación que se halla a una distancia h del eje de las x .

Puesto que η no es una función continua la representamos:

$$\begin{aligned}\eta &= \eta_1 = 0 && \text{cuando } x \leq \xi - r \\ \eta &= \eta_2 = 2r \left(1 - \frac{\xi - x}{r} \right) && \text{cuando } \xi - r \leq x \leq \xi \\ \eta &= \eta_3 = 2r \left(1 - \frac{x - \xi}{r} \right) && \text{cuando } \xi \leq x \leq \xi + r \\ \eta &= \eta_4 = 0 && \text{cuando } x \geq \xi + r\end{aligned}$$

donde r es el radio del contador G. M. El numero de particulas registradas por el aparato en la region de variacion de momento entre p y $p + dp$, por segundo es

$$\frac{dN}{dp} = 2 \int_{\alpha = -\lg^{-1} \frac{(l/2) - u}{h}}^{\lg^{-1} \frac{(l/2) + u}{h}} \int_{u=0}^{l/2} \int_{x=d-r}^{d+r} F(p, \theta, \phi) dx du \frac{\eta \cos \alpha}{h} d\alpha \dots \dots \dots (3)$$

El factor 2 que precede al signo de integration nace por el hecho de que el problema es simetrico con respecto al eje de la figura 14 y la integration para u se toma desde 0 hasta $l/2$ en lugar de $-l/2$ a $l/2$ porque para cada punto u hay otro punto — ?* con el mismo angulo solido y las mismas características; d es la distancia de separation del centro del contador situado en el medio del eje de las y .

Aqui se puede hacer otra aproximacion; supondremos que dentro de una region alrededor de los angulos θ_0 y ϕ_0 la funcion F puede ser representada por

$$F(p, \theta, \phi) = f(p) \frac{\cos^n \theta}{\cos^n \theta_0} g(\phi)$$

Ademas, y puesto que estamos trabajando en el ecuador geomagnetico y el equipo esta siempre dirigido o hacia el Este magnetico o hacia el Oeste magnetico, la funcion $g(\phi)$ debe ser simetrica a nuestro ϕ_0 . Por eso

$$\frac{dN}{dp} = \frac{4 f(p)}{h \cos^n \theta_0} \int_{\alpha=0}^{\lg^{-1} \frac{(l/2) + u}{h}} \int_{u=0}^{l/2} \int_{x=d-r}^{d+r} \eta \cos^n \theta g(\phi) \cos \alpha dx du d\alpha \dots \dots (4)$$

Puesto que se conoce que g varia muy lentamente con respecto a ϕ , podemos escoger arbitrariamente una representation de primer orden para g . Tambien ponemos ϕ_0 arbitrariamente igual a cero. Esto lleva aparejado que g sera diferente para el Este que para el Oeste. En este caso escribiremos

$$g(\phi) = 1 + a \lg \phi = 1 + a \frac{\lg \alpha}{\operatorname{sen} \theta_0} \dots \dots \dots (5)$$

siendo a la constants elegida a base de los datos reales y pudiendo tener un signo doble, ya positivo, ya negativo.

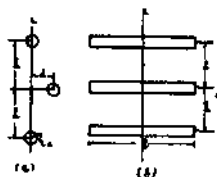


Fig. 5

Si llamamos β_1 al ángulo que efectúa el meson incidente con el plano de la figura 5 (b) y β es su proyección sobre el plano de la figura 6 puede demostrarse que:

$$\cos \theta = \cos \beta_1 \cos \alpha \cos \theta_0 + \sin \beta_1 \sin \theta_0$$

igualmente que: $\sin \beta_1 = \cos \beta_1 \tan \beta \cos \alpha$,

donde θ es el ángulo zenital de la partícula.

Luego: $\cos \theta = \cos \beta_1 \cos \alpha (\cos \theta_0 + \tan \beta \sin \theta_0)$

y por tanto: $\cos \theta = \frac{\cos \beta_1 \cos \alpha}{\cos \beta} \cos (\theta_0 - \beta)$.

Pero considerando que β y β_1 son ángulos pequeños, se puede hacer la aproximación siguiente:

$$\begin{aligned} \beta &= \sin \beta = \tan \beta \\ \cos \beta &= 1 - 1/2 \beta^2 \end{aligned}$$

Debe hacerse la misma aproximación para β_1 .

Después de algunas transformaciones trigonométricas y aproximaciones admisibles matemáticamente puede escribirse que

$$\cos \theta \approx \frac{1 - 1/2 \beta^2 \cos^2 \alpha}{1 - 1/2 \beta^2} \cos \alpha \cos (\theta_0 - \beta), \text{ o } \sin$$

$$\cos \theta \approx (1 + 1/2 \beta^2) (1 - 1/2 \beta^2 \cos^2 \alpha) \cos \alpha \cos (\theta_0 - \beta)$$

y por tanto: $\cos \theta \approx (1 + 1/2 \beta^2 \sin^2 \alpha) \cos \alpha \cos (\theta_0 - \beta)$,

despreciando el término resultante β^4 .

Volviendo sobre la ecuación (4), hacemos

$$\frac{dN}{dp} = \frac{4f(p)}{h \cos^n \theta_0} J_1 \quad \dots \dots \dots (6)$$

donde J_1 es la expresión siguiente:

$$J_1 = \int \int \int \eta (1 + 1/2 \beta^2 \sin^2 \alpha)^n \cos^{n+1} \alpha \cos^n (\theta_0 - \beta) g(\phi) dx du da$$

en donde para simplificar lo escrito hemos suprimido los límites de integración.

Recordando que β es un ángulo pequeño, se puede hacer

$$(1 + 1/2 \beta^2 \sin^2 \alpha)^n = 1 + n/2 \beta^2 \sin^2 \alpha.$$

Expandiendo el término $\cos^n (\theta - \beta)$ en una serie de Maclaurin en β se llega a

$$\cos^n (\theta_0 - \beta) = \cos^n \theta_0 \left[1 + \beta n \tan \theta_0 + \frac{\beta^2}{2} \left\{ n(n-1) \tan^2 \theta_0 - n \right\} \dots \dots \dots \right]$$

o

$$\cos^n (\theta_0 - \beta) = \cos^n \theta_0 \left(1 + \beta A + \frac{\beta^2 C}{2} \right).$$

Puede observarse que los términos $\cos^n \theta_0$ y $\cos^n \theta_0$ del denominador de la ecuación (6) se suprimen mutuamente.

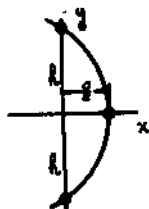


Fig. 6

Debemos señalar que estamos realmente tratando de calcular la suma de las coincidencias producidas por mesones que siguen los dos tipos de trayectorias, que pueden verse en la fig. 8.

Por ello tenemos dos integrales de la clase J_1 , en uno de ellos el ángulo será β y en el otro $-\beta$. Por eso se suprimen las potencias impares de β y se suman las potencias pares de ella.

Escribiendo $\left(\frac{dN}{dp}\right)' = \frac{8f(p)}{h} J_2$, en donde J_2 será

$$J_2 = \int \int \int \eta g(\phi) \left(1 + \frac{\beta^2 n}{2} \sec^2 \alpha\right) \left(1 + \frac{\beta^2 n D}{2}\right) \cos^{n+1} \alpha \, dx \, du \, d\alpha$$

siendo

$$D = C/n = (n-1) \lg^2 \theta_0 - 1.$$

Por tanto: $J_2 = \int \int \int \eta g(\phi) \left[1 + \frac{\beta^2 n}{2} (D + \sec^2 \alpha)\right] \cos^{n+1} \alpha \, dx \, du \, d\alpha. \quad (8)$

Podrá notarse que J_2 es muy difícil de calcular, incluso en forma aproximada, debido a la complejidad de los límites de la integración y la misma naturaleza de la función η .

Resolviendo la ecuación (2) para ξ , tenemos

$$\xi = \frac{2X (P \cos \alpha - 1)}{(P \cos \alpha - 1)^2 - 1/4} \quad \dots\dots\dots (9)$$

y llegamos a la aproximación

$$\xi = \frac{2X}{P \cos \alpha - 1} \left[1 + \frac{1}{4(P \cos \alpha - 1)^2}\right] \quad \dots\dots\dots (10)$$

siendo

$$P = p/p_0.$$

Para una aproximación de primer orden de J_2 se puede considerar:

$$\xi \approx \frac{2X}{P \cos \alpha - 1} \quad ; \quad \beta \approx 0 \quad ; \quad g(\phi) \approx 1 \quad \dots\dots\dots (11)$$

llegando a: $J_2 = J_3 = \int_{\alpha=0}^{1/2} \frac{(1/2) - \alpha}{h} \int_{u=0}^{1/2} \int_{x=d-r}^{d+r} \cos^{n+1} \alpha \, dx \, du \, d\alpha.$

Consideraremos solamente el caso para el cual $d > 2r$, lo que es cierto en nuestro equipo, pero aun antes de calcular la integral se pueden efectuar algunas deducciones relacionadas con el momento de las particular.

El momento mínimo admitido es tal que

$$\xi = d + 2r; \alpha = 0, \quad \text{y por tanto}$$

$$d + 2r = \frac{2X}{P_{\min} - 1}, \text{ o lo que es igual: } P_{\min} = \frac{2X}{d + 2r} + 1.$$

El momento máximo corresponderá a

$$\xi = d - 2r \quad ; \quad \alpha = \lg^{-1} (l/2h) .$$

Consecuentemente:

$$d - 2r = \frac{2X}{\left(\frac{P_{max}}{\sqrt{1 + (l/2h)^2}} - 1 \right)} , \quad \text{a: } P_{max} = \left(\frac{2X}{d - 2r} + 1 \right) \sqrt{1 + (l/2h)^2} .$$

Nuestro primer paso será el de determinar las zonas del plano matematico $x - \cos a$ en las cuales y tendrá los valores n_1, n_2, n_3 y n_4 . Las curvas que separan las zonas estarán expresadas por

$$x = \xi - r \quad ; \quad x = \xi \quad y \quad x = \xi + r \quad \text{respectivamente.}$$

Substituyendo ε por su valor en la ecuacion (11) y resolviendo las ecuaciones para $\cos a$, se obtiene las tres ecuaciones siguientes:

$$\cos \alpha = \frac{1}{P} \left(\frac{2X}{x + r} + 1 \right) = z_1(x)$$

$$\cos \alpha = \frac{1}{P} \left(\frac{2X}{x} + 1 \right) = z_2(x)$$

$$\cos \alpha = \frac{1}{P} \left(\frac{2X}{x - r} + 1 \right) = z_3(x)$$

Estas curvas se muestran en la figura 6 para un caso dado. Obsérvese que sólo nos hallamos interesados en la zona en que $d - r \leq x \leq d + r$, que es la zona que ocupa el contador (i. M. situado en el centro.

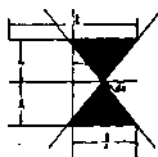


Fig. 7

La gráfica de la figura 16 es general para el caso

$$d < \xi_0 < d + r \quad , \quad \text{donde} \quad \xi_0 = \frac{2X}{P - 1} .$$

El gráfico mencionado contiene también líneas representativas de los valores límites de α correspondientes a diversos valores de u .

Es observable que los límites de la integración estarán determinados en algunos casos por las curvas z y en otros por los valores de u , dependiendo de cuál dará el más bajo valor de α .

Es necesario señalar el punto en el cual tiene lugar este cambio, por eso colocamos $z_m(x) = q(u)$, siendo

$$q(u) = \cos \left[\lg^{-1} \frac{(l/2) - u}{h} \right] = \left[1 + \left\{ \frac{(l/2) - u}{h} \right\}^2 \right]^{-1/2} .$$

Para u estas ecuaciones pueden resolverse así:

$$u = (l/2) - w_m(x) , \quad \text{siendo}$$

$$w_m(x) = \frac{h}{2X + v_m(x)} \sqrt{(P^2 - 1) [v_m(x)]^2 - 4Xv_m(x) - 4X^2} , \quad \text{donde}$$

$$v_1(x) = x + r \quad ; \quad v_2(x) = x \quad ; \quad v_3(x) = x - r ,$$

Existen tres regiones para x , con diferentes propiedades:

$$d - r \leq x \leq \xi_0 - r$$

$$\xi_0 - r \leq x \leq \xi_0$$

$$\xi_0 \leq x \leq d + r ;$$

puediendo escribirse por tanto

$$J_3 = \int_{d-r}^{\xi_0-r} J_4 dx + \int_{\xi_0-r}^{\xi_0} J_5 dx + \int_{\xi_0}^{d+r} J_6 dx .$$

J_4 es 16gicamente cero, por el hecho en esa zona que $n = n_l = 0$. J_5 presenta dos zonas con diferente comportamiento, fijadas por u :

$$0 \leq u \leq (l/2) - w_1(x)$$

$$(l/2) - w_1(x) \leq u \leq l/2$$

Por tanto:

$$J_5 = \int_{u=0}^{(l/2)-w_1(x)} \int_{\alpha=0}^{\cos^{-1} z_1} \eta_2 \cos^{\alpha+1} \alpha d\alpha du + \int_{u=(l/2)-w_1(x)}^{l/2} \int_{\alpha=0}^{\cos^{-1} q} \eta_2 \cos^{\alpha+1} \alpha d\alpha du .$$

Ahora

$$\eta_2 = 2r [1 + \rho(x, \cos \alpha)] , \quad \text{en donde } \rho \text{ queda}$$

definida así:

$$\rho(x, \cos \alpha) = \frac{x - \xi}{r} = \frac{1}{r} \left(x - \frac{2X}{1 + \cos \alpha - 1} \right)$$

Consideremos ahora las integrales indefinidas:

$$\int \cos^{\alpha+1} \alpha d\alpha = \sigma_n(\cos \alpha)$$

y

$$\int \rho(x, \cos \alpha) \cos^{\alpha+1} \alpha d\alpha = \tau_n(x, \cos \alpha)$$

definiciones que nos sirvan para simplificar y abreviar los calculos mas adelante.

Tendremos entonces que

$$\begin{aligned} J_5 &= 2r \int_0^{(l/2)-w_1(x)} [\sigma_n(z_1) - \sigma_n(1) + \tau_n(x, z_1) - \tau_n(x, 1)] du \\ &+ 2r \int_{(l/2)-w_1(x)}^{l/2} [\sigma_n(q) - \sigma_n(1) + \tau_n(x, q) - \tau_n(x, 1)] du \\ &= 2r \left[\frac{l}{2} - w_1(x) \right] [\sigma_n(z_1) + \tau_n(x, z_1)] + 2r \int_{(l/2)-w_1(x)}^{l/2} [\sigma_n(q) + \tau_n(x, q)] du \\ &- r l [\sigma_n(1) + \tau_n(x, 1)] \end{aligned}$$

J_6 tiene tres zonas de diferente comportamiento, dependiendo igualmente de u :

$$0 \leq u \leq \frac{l}{2} - w_1(x)$$

$$\frac{l}{2} - w_1(x) \leq u \leq \frac{l}{2} - w_2(x)$$

$$\frac{l}{2} - w_2(x) \leq u \leq \frac{l}{2}$$

y por tanto,

$$\begin{aligned} J_6 = & \int_{u=0}^{\frac{l}{2}-w_1(x)} \left[\int_{\alpha=0}^{\cos^{-1} z_2} \eta_3 \cos^{n+1} \alpha d\alpha + \int_{\alpha=\cos^{-1} z_2}^{\cos^{-1} z_1} \eta_2 \cos^{n+1} \alpha d\alpha \right] du \\ & + \int_{u=\frac{l}{2}-w_1(x)}^{\frac{l}{2}-w_2(x)} \left[\int_{\alpha=0}^{\cos^{-1} z_2} \eta_3 \cos^{n+1} \alpha d\alpha + \int_{\alpha=\cos^{-1} z_2}^{\cos^{-1} q} \eta_2 \cos^{n+1} \alpha d\alpha \right] du \\ & + \int_{u=\frac{l}{2}-w_2(x)}^{\frac{l}{2}} \int_{\alpha=0}^{\cos^{-1} q} \eta_2 \cos^{n+1} \alpha d\alpha du \end{aligned}$$

Ahora:

$$\eta_3 = 2r [1 - \rho(x, \cos \alpha)]$$

$$\begin{aligned} J_6 = & 2r \int_0^{\frac{l}{2}-w_1(x)} [\sigma_n(z_1) - \sigma_n(z_2) + \tau_n(x, z_1) - \tau_n(x, z_2)] du \\ & + 2r \int_{\frac{l}{2}-w_1(x)}^{\frac{l}{2}-w_2(x)} [\sigma_n(q) - \sigma_n(z_2) + \tau_n(x, q) - \tau_n(x, z_2)] du \\ & + 2r \int_0^{\frac{l}{2}-w_2(x)} [\sigma_n(z_2) - \sigma_n(1) - \tau_n(x, z_2) + \tau_n(x, 1)] du \\ & + 2r \int_{\frac{l}{2}-w_2(x)}^{\frac{l}{2}} [\sigma_n(q) - \sigma_n(1) - \tau_n(x, q) + \tau_n(x, 1)] du \\ J_6 = & 2r \left[\frac{l}{2} - w_1(x) \right] [\sigma_n(z_1) + \tau_n(x, z_1)] + 2r \int_{\frac{l}{2}-w_1(x)}^{\frac{l}{2}-w_2(x)} [\sigma_n(q) + \tau_n(x, q)] du \\ & - 2r \left[\frac{l}{2} - w_2(x) \right] [\sigma_n(z_2) + \tau_n(x, z_2)] + 2r \left[\frac{l}{2} - w_2(x) \right] [\sigma_n(z_2) - \tau_n(x, z_2)] \\ & + 2r \int_{\frac{l}{2}-w_2(x)}^{\frac{l}{2}} [\sigma_n(q) - \tau_n(x, q)] du - r l [\sigma_n(1) - \tau_n(x, 1)] \\ = & 2r \int_{\frac{l}{2}-w_1(x)}^{\frac{l}{2}} [\sigma_n(q) + \tau_n(x, q)] du - 4r \int_{\frac{l}{2}-w_2(x)}^{\frac{l}{2}} \tau_n(x, q) du \\ & - 4r \left[\frac{l}{2} - w_2(x) \right] \tau_n(x, z_2) - r l [\sigma_n(1) + \tau_n(x, 1)] + 2r l \tau_n(x, 1) . \end{aligned}$$

Si

$$\int \sigma_n(q) du = \gamma_n(u)$$

$$\int \tau_n(x, q) du = \delta_n(x, u) ,$$

substituyendo se obtiene:

$$\begin{aligned} \frac{J_3}{2r} = & \int_{\xi_0-r}^{d+r} \left[\frac{l}{2} - w_1(x) \right] \left[\sigma_n(z_1) + \tau_n(x, z_1) \right] dx \\ & + \int_{\xi_0-r}^{d+r} \left[\gamma_n\left(\frac{l}{2}\right) - \gamma_n\left(\frac{l}{2} - w_1(x)\right) + \delta_n\left(x, \frac{l}{2}\right) \right. \\ & \left. - \delta_n\left(x, \frac{l}{2} - w_1(x)\right) \right] dx - \frac{l}{2} \int_{\xi_0-r}^{d+r} \left[\sigma_n(l) + \tau_n(x, l) \right] dx \\ & - 2 \int_{\xi_0}^{d+r} \left[\delta_n\left(x, \frac{l}{2}\right) - \delta_n\left(x, \frac{l}{2} - w_2(x)\right) \right] dx \\ & - 2 \int_{\xi_0}^{d+r} \left[\frac{l}{2} - w_2(x) \right] \tau_n(x, z_2) dx + l \int_{\xi_0}^{d+r} \tau_n(x, l) dx . \end{aligned}$$

Las funciones σ_n , τ_n , γ_n , y δ_n que mencionamos arriba pueden calcularse a través de metodos de aproximacion, como las series de Taylor, MacLaurin, etc. y la integracion de x puede efectuarse entonces numericamente.

Ligeras modificaciones en 1.º expuesto anteriormente se requieren para la determinacin de otros valores de ξ_0 , es decir, en las zonas siguientes:

$$\begin{aligned} \xi_0 &\leq d - 2r \\ d - 2r &\leq \xi_0 \leq d - r \\ d - r &\leq \xi_0 \leq d \\ d + r &\leq \xi_0 \leq d + 2r \\ \xi_0 &\geq d + 2r \end{aligned}$$

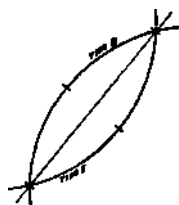


Fig. 8

Si

$$\frac{J_3}{2r} = \epsilon(p), \text{ se llega a: } \left(\frac{dN}{dp} \right)' = \frac{16r}{h} \epsilon \left(\frac{p}{p_0} \right) f(p)$$

lo que ademas permite aislar el efecto debido puramente al equipo, del espectro $f(p)$ de la radiacion incidente.

Efectuaremos ahora un somero analisis del problema fisico sobre las particulas que llegan a nuestros contadores. Ya senalamos que la intensidad

de la radiación que llega a la superficie de la tierra es diferente para cada punto y para poder determinarla se tendrá que conocer:

- 1) El valor del campo magnético de la tierra,
- 2) La composición y distribución de la energía de la radiación cósmica, fuera del campo terrestre.
- 3) La dirección de la radiación en el espacio.

El valor del campo magnético terrestre se conoce con bastante exactitud, desde Gauss, pudiendo estimarse su momento en:

$$M = 8.1 \times 10^{25} \text{ gauss, cm}^3.$$

También se conoce (A. Schmidt) que el denominado centro magnético no se halla en el centro de la tierra sino a 340 kms. de este centro en dirección a los 7° de latitud geográfica Norte y 160° de longitud W.

Para acercarnos a dar valores aproximados a la radiación cósmica en el espacio, solo podemos valernos de la composición y distribución que llegamos a medir en la superficie terrestre (Rossi, 1949). En cuanto a la distribución de la radiación fuera del campo magnético terrestre, la debemos suponer isotrópica.

En base a las consideraciones anteriores puede ahora establecerse un problema matemático, que se soluciona utilizando algunas aproximaciones, que solo vamos aquí a citar, pudiendo el lector consultar la bibliografía que en cada caso señalamos.

Cuando una partícula con masa m y carga c penetra en un campo electromagnético, de componente E y H , las ecuaciones de movimiento de la misma pueden representarse en forma:

$$\frac{d}{dt}(m \vec{v}) = c \left(\vec{E} + \frac{1}{c} \vec{v} \wedge \vec{H} \right)$$

en donde el primer miembro, representa la variación relativista del momento,

desde que $\vec{v} = \frac{d\vec{r}}{dt}$, siendo $\vec{r}$ el radio vector con componentes x , y , z , y el segundo, es igual a la fuerza de Lorentz.

Una solución del planteamiento matemático en el caso de los rayos cósmicos, de varias partículas cargadas, conduce al teorema de Liouville.

La aplicación de este teorema ha sido realizado casi al mismo tiempo, pero en forma independiente, por FERMI & ROSSI (1933), LEMAITRE & VALLARTA (1933), VALLARTA (1933) y SWANN (1933) casi todos partiendo de los primeros

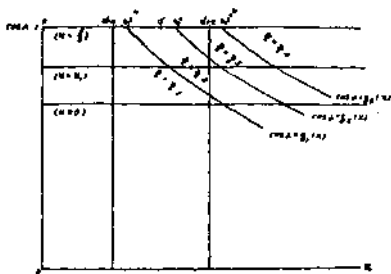


Fig. 9

trabajos sobre el movimiento de las partículas cargadas en el campo magnético terrestre, debidos a STORMER (1930) y suponiendo que la radiación cósmica tiene una distribución isotrópica, cuando alcanza al campo magnético terrestre.

Es conveniente el señalar que nos referimos a la radiación primaria, antes de que esta llegue incluso a penetrar en nuestra atmósfera, ya que para los efectos del campo magnético, esta es insignificante, mientras que para la radiación secundaria juega el papel primordial.

Sin entrar a detallar los complejos cálculos que requirieron a sus autores, la determinación de las órbitas de estas partículas dentro del campo magnético, es necesario señalar que existen direcciones en las cuales ninguna partícula de momento dado procedente del infinito podrá llegar a la tierra, y por otro lado que, la intensidad con que llegan las partículas es la misma, como si en realidad no existiera campo magnético con excepción de su acción en las denominadas zonas prohibidas para cada momento.

De todas maneras uno de los primeros puntos para estudiar el efecto del campo magnético terrestre será el definir en forma total, si la radiación cósmica llega en forma isotrópica como sostienen la mayor parte de los autores o puede estar influenciada por otros factores como el efecto Doppler y la rotación galáctica (JANOSY, 1937) así como el campo magnético del sol en cuyo caso la distribución sería anisotrópica. No mencionamos los posibles efectos de absorción producidos por nebulosas, ni una posible anisotropía en su mismo origen en el Universo.

Para un estudio completo de la teoría de Stormer así como de las direcciones de penetración de las partículas nos referimos a las citas antes expuestas, señalando únicamente aquí que las fuerzas geomagnéticas originan, en las mediciones de la radiación cósmica, de los denominados efectos de latitud y longitud,

así como sus asimetrías Este-Oeste y Norte-Sur, una de las cuales es el motivo de nuestro trabajo.

Las primeras observaciones sobre los efectos de asimetría fueron efectuadas por Rossi (1934) y JOHNSON (1933) que observaron que la intensidad de la radiación cósmica que penetraba en un determinado ángulo con relación a la vertical, era diferente cuando se apuntaba sea, al Este o al Oeste, encontrándose por los investigadores antes mencionados

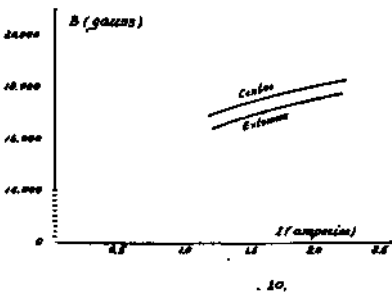


Fig. 10

mayor número de partículas procedentes del Oeste. Además, trabajos posteriores han comprobado que esta asimetría, es función de la altura, de la latitud y lógicamente del ángulo de inclinación con que se mide la misma.

Las observaciones efectuadas por JOHNSON & BARRY (1939) en 1938 con un registro automático llevado hasta una altura de 17.000 mts. y una latitud geomagnética de 20 grados, con un ángulo de 60 grados, dieron como resultado una asimetría de un 7% de partículas procedentes del Oeste y desde luego parecía excluir la posibilidad de una cifra mayor de este número, pero posteriores trabajos

realizados por BIEHL, NEHER & ROESCH (1949) en 1948 hallaron una asimetría sumamente pronunciada.

Igualmente BARBER (1949) obtuvo una fuerte asimetría en alturas aproximadamente de 12 kms, llegando incluso a la conclusión debida a esta fuerte asimetría de que todas las partículas primarias estaban cargadas positivamente. Ello, no obstante, no implicaba que debido a las dificultades de las observaciones y otros factores no pudiese haber un 25% de otras partículas primarias con carga negativa.

So debe ella al hecho de que en las bajas latitudes geomagnéticas se puede suponer que la radiación cósmica tiene un exceso de partículas de un signo determinado, positivo por ejemplo, y que por lo tanto los conos prohibidos con relación a estas partículas positivas primarias apunta hacia el Este, de ahí que tendra que haber forzosamente un número mayor de partículas procedentes del Oeste. Igual consideración pero en sentido contrario, podremos apuntar para las negativas, en el que el mayor número de ellas por la misma teoría de los conos prohibidos exija que proceda del Este.

De acuerdo con ello, la asimetría total Este-Oeste, se halla determinada por la diferencia de las intensidades de la radiación primaria, sea esta positiva o negativa.

El campo magnético da lugar a un fuerte efecto de latitud, encontrado primeramente por CLAY (1927).

A medida que nos alejamos del ecuador geomagnético va aumentando la intensidad de la radiación, hasta alrededor de los 50°; desde esta latitud, prácticamente deviene constante.

Por encima de este cono del efecto de latitud, es decir de los 40 grados, los experimentos de J. C. STEARS, D. C. FROMAN, así con JOHNSON (1939) hallaron asimetrías de un 20% para ángulos entre 20 y 45° mientras que SCHEIN, JESSE & WOLLAN (1941) y SCHEIN, YNGVE & KRAYBILL (1948) en un aeroplano volando entre 50 a 54° de latitud geomagnética y a 9.500 mts. de altura con un absorbente de 22 cms. de plomo, hallaron una asimetría de 13% con un ángulo vertical de 25°.

De acuerdo con la teoría expuesta por JOHNSON (1941), la diferencia en las asimetrías entre las bajas y altas latitudes, y particularmente el menor valor de la asimetría total, que se observa en las altas latitudes se debe al hecho de la desviación magnética de los mesones, después de que estos han sido conducidos y distribuidos simétricamente por la radiación primaria, asimetría que no debería existir si no hubiese un exceso de partículas positivas.

En efecto, la asimetría Este-Oeste en la componente dura de la radiación cósmica se halla interpretada actualmente por casi todos los tratadistas (Johnson, Blackett, Rossi) como producidos por la desviación de los primarios, en mayor abundancia positivos, en este campo magnético terrestre.

Suponemos la conservación de la dirección con que penetra la radiación primaria, para las interacciones que originan las familias secundarias y terciarias, de mesones π y $\bar{\pi}$ respectivamente, lo cual es prácticamente cierto debido a las energías considerablemente grandes a que tienen lugar la creación de estas familias. La medición que se efectúa en la asimetría de las partículas de esta naturaleza, representara, como antes señalabamos el comportamiento de la radiación que la

ha originado. Estas conclusiones se refieren solamente para los efectos geomagnéticos de la radiación penetrante para alturas normales.

La asimetría para un momento dado con relación a una dirección inclinada (zeta) de la vertical se representa por:

$$A(\theta) = 2 \frac{I_0(\theta) - I_E(\theta)}{I_0(\theta) + I_E(\theta)}$$

donde $I_0(0)$ es la intensidad en una dirección y sentido, mientras que $I_E(\theta)$ corresponde a la dirección opuesta, Oeste y Este, respectivamente, para el ángulo θ (zeta) de la inclinación del equipo con relación a la vertical.

Quizá quepa aquí la discusión de uno de los muchos problemas pequeños, pero no por eso menos importantes, con los que se tropieza en un experimento de esta naturaleza. Vamos a considerar, en efecto, el error que se introduce en la asimetría total, como consecuencia de una pequeña desnivelación en los electroimanes del equipo; desnivelación, sea dirigida hacia el Este o al Oeste del mismo.

Supondremos que, para el caso de pequeñas variaciones, este efecto será el mismo que el obtenido por una variación, también pequeña, de las medidas obtenidas en las direcciones θ_0 , ϕ_0 del equipo.

Como antes se ha indicado, la asimetría Este-Oeste se mide por la cantidad:

$$A(\theta) = 2 \frac{I_0(\theta) - I_E(\theta)}{I_0(\theta) + I_E(\theta)}.$$

Si el equipo está apuntado en la posición V, en la que el brazo derecho correspondiente a la dirección Este y el izquierdo a la Oeste, y representando por θ_{01} y θ_{02} los ángulos verdaderos zenitales, que tienen respectivamente los lados que apuntan al Oeste y Este, y por θ_{00} el ángulo deseado, que es igual para ambos lados, se observa que:

$$I_0(\theta_{01}) \text{ es proporcional a } f_1(p) \frac{\cos^{n_1} \theta_{01}}{\cos^{n_1} \theta_{00}}$$

$$I_E(\theta_{02}) \text{ es proporcional a } f_2(p) \frac{\cos^{n_2} \theta_{02}}{\cos^{n_2} \theta_{00}}$$

donde $f_1(p)$ y $f_2(p)$ son los valores de la antes mencionada función $j(p)$, en las dos direcciones, despreciando la diferencia en θ , puesto que esta no nos interesa.

En una región alrededor de $\theta_{01} = \theta_{00}$ se puede escribir aproximadamente:

$$\frac{\cos^{n_1} \theta_{01}}{\cos^{n_1} \theta_{00}} \approx \cos^{-n_1} \theta_{00} \left\{ \cos^{n_1} \theta_{00} + \left[\frac{d}{d\theta_{01}} (\cos^{n_1} \theta_{01}) \right] \theta_{01} - \theta_{00} \Delta \theta_{01} \right\} =$$

$$\cos^{-n_1} \theta_{00} \{ \cos^{n_1} \theta_{00} - n_1 \cos^{n_1-1} \theta_{00} \sin \theta_{00} \Delta \theta_{01} \} = 1 - n_1 \operatorname{tg} \theta_{00} \Delta \theta_{01},$$

donde $\Delta \theta_{01}$ es una pequeña variación en θ_{01} ; es decir,

$$\theta_{01} = \theta_{00} + \Delta \theta_{01}$$

Igualmente:

$$\frac{\cos^{n_2} \theta_{02}}{\cos^{n_2} \theta_{00}} \approx 1 - n_2 \operatorname{tg} \theta_{00} \Delta \theta_{02}.$$

Ahora bien, si la base del aparato se inclina a un ángulo ϵ hacia el Este, tenemos:

$$\Delta\theta_{e1} = -\epsilon, \quad \Delta\theta_{e2} = +\epsilon.$$

Con una substitución sencilla, se llega a:

$$\begin{aligned} A(\theta_{00}) &= 2 \frac{f_1(p)(1 + n_1 \epsilon \operatorname{tg} \theta_{00}) - f_2(p)(1 - n_2 \epsilon \operatorname{tg} \theta_{00})}{f_1(p)(1 + n_1 \epsilon \operatorname{tg} \theta_{00}) + f_2(p)(1 - n_2 \epsilon \operatorname{tg} \theta_{00})} \\ &= 2 \frac{f_1(p) - f_2(p) + [n_1 f_1(p) + n_2 f_2(p)] \epsilon \operatorname{tg} \theta_{00}}{f_1(p) + f_2(p) + [n_1 f_1(p) - n_2 f_2(p)] \epsilon \operatorname{tg} \theta_{00}}. \end{aligned}$$

Teniéndose en cuenta que $f_1(p)$ y $f_2(p)$ son del mismo orden, o igualmente para n_1 y n_2 , y por eso que $n_1 f_1(p) - n_2 f_2(p) \ll f_1(p) + f_2(p)$, la ecuación arriba puede ponerse en la forma siguiente:

$$\begin{aligned} A(\theta_{00}) &= 2 \frac{f_1(p) - f_2(p) + \left\{ [n_1 f_1(p) + n_2 f_2(p)] - \frac{f_1(p) - f_2(p)}{f_1(p) + f_2(p)} [n_1 f_1(p) - n_2 f_2(p)] \right\} \epsilon \operatorname{tg} \theta_{00}}{f_1(p) + f_2(p)} \\ &= 2 \frac{f_1(p) - f_2(p)}{f_1(p) + f_2(p)} + (n_1 + n_2) \epsilon \operatorname{tg} \theta_{00} \left[1 - \frac{(f_1(p) - f_2(p))^2}{(f_1(p) + f_2(p))^2} \right]. \end{aligned}$$

El primer término es la asimetría verdadera al ángulo θ_{00} , es decir, la que se mediría si θ_{e1} y θ_{e2} fueran exactamente iguales a θ_{00} ; y por eso lo llamaremos $A_0(\theta_{00})$. Tenemos entonces:

$$A(\theta_{00}) = A_0(\theta_{00}) + (n_1 + n_2) \epsilon \operatorname{tg} \theta_{00} \left\{ 1 - \frac{[A_0(\theta_{00})]^2}{4} \right\},$$

donde el segundo término es el error en la medida, debido a la desnivelación ϵ .

El error relativo es:

$$R = \frac{A(\theta_{00}) - A_0(\theta_{00})}{A_0(\theta_{00})} = \frac{(n_1 + n_2) \epsilon \operatorname{tg} \theta_{00}}{A_0(\theta_{00})} \left\{ 1 - \frac{[A_0(\theta_{00})]^2}{4} \right\}$$

En casi todos los casos interesantes, $A_0(\theta_{00}) < 0.5$, y por eso:

$\frac{[A_0(\theta_{00})]^2}{4} \ll 1$. Una buena aproximación es, entonces:

$$R \approx \frac{(n_1 + n_2) \epsilon \operatorname{tg} \theta_{00}}{A_0(\theta_{00})}$$

Supongamos, por ejemplo, que

$$A_0(\theta_{00}) = 0.20$$

$$\theta_{00} = 45^\circ \quad \text{u} \quad \operatorname{tg} \theta_{00} = 1$$

$$n_1 + n_2 = 4$$

$$\epsilon = 1^\circ \approx \frac{1}{57} \text{ rad.}$$

En este caso:

$$R = \frac{4 \times 1}{57 \times 0.2} = 0.35$$

El error absoluto es de $0.35 \times 0.20 = 0.07$; o sea que con una desviación de un solo grado, el aparato nos da una cifra de 0.27 en vez de 0.20; lo cual para esta clase de mediciones es considerable.

Otro problema que también es necesario por lo menos mencionar es el debido a la estadística en sí.

El error estadístico en un número arbitrario de conteje N , es $\sqrt{N}$, siendo el error relativo $\sqrt{N}/N \approx 1/\sqrt{N}$.

Representando por R y r los errores relativos y por E y e los errores absolutos y considerando que los errores aritméticos obedecen a leyes conocidas tendremos:

$$E^2 = e_1^2 + e_2^2 \text{ para sumas y diferencias}$$

$$R^2 = r_1^2 + r_2^2 \text{ para productos y cocientes.}$$

Los cálculos para la asimetría se efectúan como sigue:

$$A = 2 \frac{N_0 - N_E}{N_0 + N_E}$$

donde N_0 y N_E representan el número de partículas medidas en dirección Oeste y en dirección Este.

$$\text{En el numerador: } E_n = \sqrt{e_0^2 + e_E^2}$$

$$\text{En el denominador: } E_d = \sqrt{e_0^2 + e_E^2} = E_n = E'$$

Para la cantidad total:

$$R = r_n^2 + r_d^2$$

Ahora:

$$r_n = \frac{E_n}{N_0 - N_E} = \frac{E'}{N_0 - N_E} \text{ y } r_d = \frac{E'}{N_0 + N_E}$$

Por ello:

$$R = E' \sqrt{\frac{1}{(N_0 - N_E)^2} + \frac{1}{(N_0 + N_E)^2}}$$

Obsérvese que:

$$e_0 = \sqrt{N_0} \text{ y } e_E = \sqrt{N_E}$$

$$e_0^2 = N_0 \text{ y } e_E^2 = N_E$$

Al substituir encontramos que:

$$E' = \sqrt{N_0 + N_E} \text{ , obteniendo por tanto:}$$

$$R = \frac{\sqrt{N_0 + N_E}}{N_0 - N_E} \sqrt{1 + \left(\frac{N_0 - N_E}{N_0 + N_E}\right)^2} = \frac{2}{A \sqrt{N_0 + N_E}} \sqrt{1 + \frac{A^2}{4}} = \frac{2}{A \sqrt{N_0 + N_E}}$$

y por tanto:

$$E = RA = \frac{2}{\sqrt{N_0 + N_E}} .$$

Si deseamos, por ejemplo que nuestros datos para E tengan mi valor de 0.01, la suma $N_0 + N_E$ sera de 40.000. En Chacaltaya, colocando el telescopio con 45° de inclinacion, para el canal de alto momento, el numero de particulas que llegan, $N_0 + N_E$, es aproximadamente de 30 por hora. En otras palabras, se requieren unas 1300 horas de datos pra cada tipo de particula, sea positiva o negativa o un total de 2600 horas para completar el experimento. Suponiendo y dadas las condiciones de trabajo, com motores Diesel, es bastante problematic, que se tomen 20 horas de datos por dia se requieran algo asi como 130 dias, o sea mas de cuatro meses.

Despues de las consideraciones anteriores, daremos ahora cuenta de los resultados obtenidos hasta la fecha con nuestro equipo; separando la fase de Echo Lake, de la que se viene haciendo actualmente en Chacaltaya; de esta ultima apenas si hablaremos, pues como se puede deducir de los parrafos anteriores el numero de horas de trabajo es insuficiente para sacar ninguna conclusion, incluso de caracter provisional.

Con unas 1300 horas de datos, tornados en Echo Lake (Estado de Colorado — E.E.U.U.) a una altura de 3240 metros y a una latitud geomagnetica de 49 grados Norte (nritese que nos hallamos sobre la llamada "rodilla" del efecto de latitud) se halla una asimetria aproximadamente igual para los mesones positivos y negativos, siendo distintas, solamente en su signo; es decir, la asimetria de los positivos favorece al Oeste, mientras que, la de los negativos, lo es para el Este.

Estas asimetrias se deben al efecto del campo magnetico terrestre sobre los mesones mismos, despues de su produccion por los rayos primarios y sin que por lo tan to, nos senalen asimetrias en estos ultimos.

Las cifras obtenidas fueron aproximadamente:

12% para el canal de alto momento.

11% * * * bajo momento.

De estas, la segunda representa un numero muy reducido de datos y por ello contiene un error estadistico bastante grande.

Tambien es del caso seiialar que estos resultados no toman en cuenta los efectos de la dispersion (scattering) de los mesones en el hierro del selector magne*tico. Las cifras que mencionamos estan en concordancia con los resultados de GROETZINGER & MCCLURE (1950) obtenidos con un equipo similar en el mismo punto.

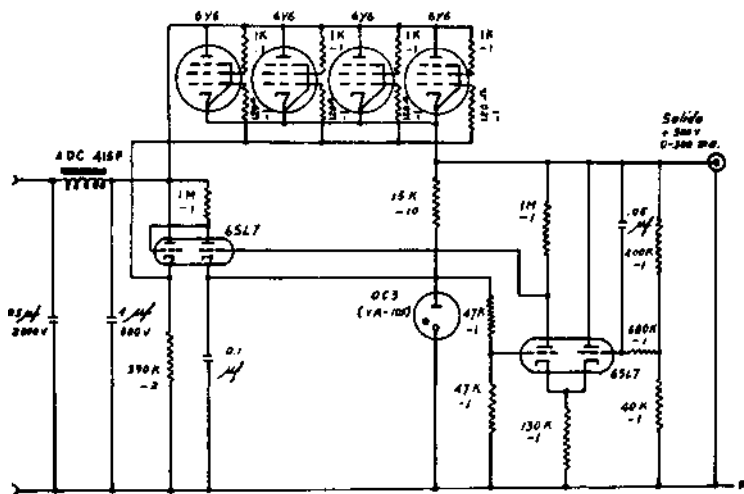
Teoricamente, cuando no hay campo no puede pasar ninguna partfcula que haga contar el equipo; pero, no obstante, se observa que, sin campo hay un numero considerable de datos (alrededor de la tercera parte del numero de los registrados con campo). El efecto, se debe precisamente a la dispersion de los mesones y ello nos indica que tambien cuando esta funcionando el campo, tendrmos un numero de pulsaciones debidas a este mismo fentfmeno de la dispersion el cual se origina en las desviaciones que experimentan los mesones al pasar muy cerca de los nucleos de los atomos de la materia. Hasta la fecha, no hemos llevado a su termino los calculos debidos a este fenomeno y su correccion, serd expuesta oportunamente.

En Chacaltaya y hasta la fecha se tienen datos de unas 300 horas, sin campo magnetico en su mayor parte, y destinados precisamente a efectuar la correccion por dispersion que arriba se senala. Para concluir esta primera parte de nuestro trabajo y con 61, el presente escrito, solo diremos que, estos datos estan de conformidad en cuanto al numero, de contajes, por hora, con los que se podia esperar, dada la altura y ubicacdn geomagndtica del Laboratorio.

APENDICE N.º 1

Los grdficos que se presentan a continuacion, representan los circuitos mas sobresalientes del equipo electronico en funcionamiento. Con el fin de simplificarlos estan incompletos y solo hacemos referencia a las partes que consideramos mas interesantes en cada uno de ellos:

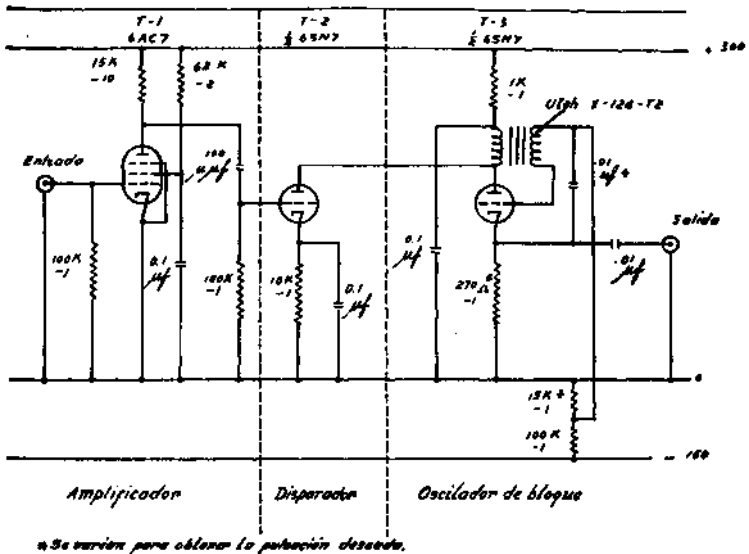
FUENTE DE PODER REGULADA



A.A: del rectificador - dos 3A4-GY en paralelo onda completa - Transformador 500-0-500 (Total 1000V)

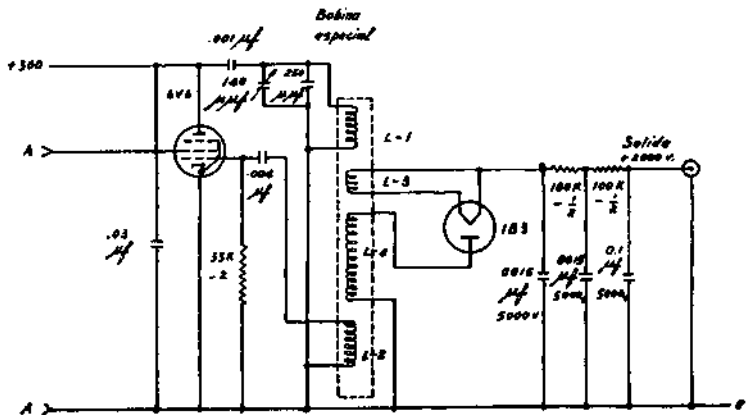
Esta fuente proporciona un voltaje constante de 300 voltios, con una carga variable entre 0 a 300 miliamperios.

ACIPLIFICADoa Y FORMA DOR DE PULSOS



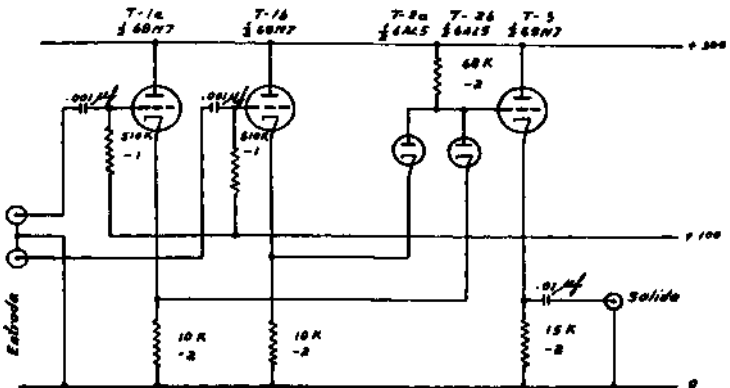
El amplificador mismo no requiere explicación adicional al KUtia. La siguiente etapa, llamada disparador, sirve para iniciar la acción del oscilador del bloque pasando, una corriente relativamente pequeña por el primario del transformador de pulsos. A partir de este punto la acción del oscilador es un fenómeno complejo que se discute en su discusión fuera de este marco; lo que interesa es que ocurre una acción regenerativa entre la placa de la válvula, produciéndose al cátodo una pulsación cuadrada de unos 90 voltios, con duración de 2 microsegundos.

FUENTE DE ALTA TENSION REGULADA, DE RADIOFRECUENCIA



Se utiliza para evitar el empleo de transformadores pesados con gran blindaje aislante, condensadores e inductancias elevadas en el osciloscopio. La lámpara 6 V 6 actúa como oscilador de radio-frecuencia, en virtud de las partes L-1 y L-2 de la bobina; con las partes L-3 y L-4 respectivamente se forman los voltajes para el filamento y la placa del rectificador 1 B 3. La frecuencia empleada es del orden de 300 Kc.

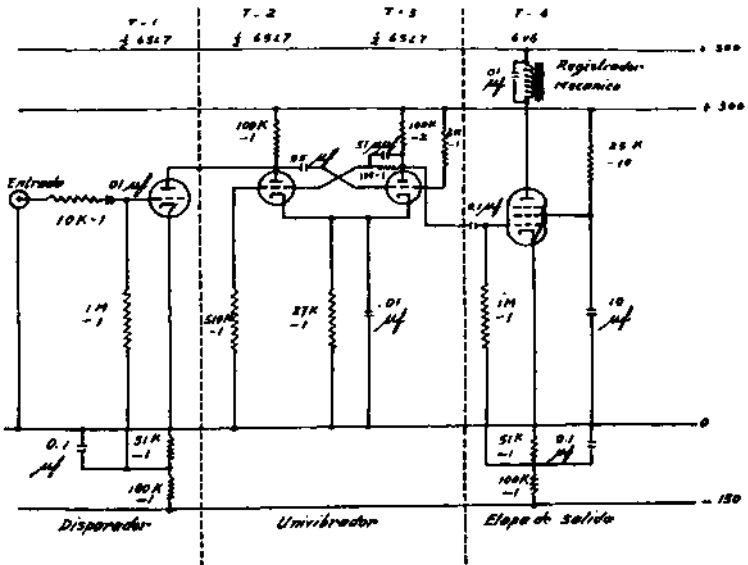
CIRCUITO DE COINCIDENCIAS



Para su funcionamiento, se basa en los siguientes hechos:

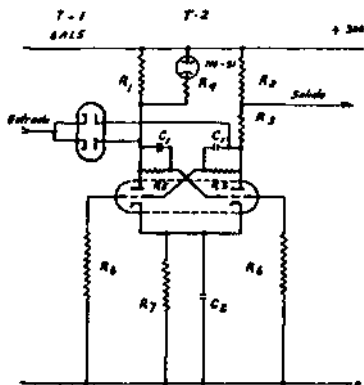
- 1) La impedancia de un diodo electrónico, es unos 200 ohmios en un sentido y prácticamente infinita en el otro.
- 2) La impedancia de salida de un "etodo follower" es muy baja, del orden $1/g_m$ de la valvula.

Cuando el circuito recibe una sola pulsación, de entrada, no actúa en forma ninguna en su etapa de salida. En nuestro diagrama, para que esta última etapa actúe deberá recibir en forma simultánea una pulsación doble en la entrada; en el equipo real se emplearían tripletes y cuádruples coincidencias, que podríamos poner en el diagrama con válvulas T-1c; T-2a; T-2b; T-3 y T-2d, ya que este circuito puede funcionar con cualquier número de diodos en la grilla de la etapa de salida.



Sirve para transformar una pulsación cuadrada positiva de unos 90 V. y 2 microsegundos en otra de mayor amplitud y duración que a su vez mueve el registrador mecánico.

El disparador se utiliza solo para iniciar la acción del univibrador, siendo este posteriormente independiente de aquel. El univibrador tiene una posición estable y otra metaestable; cuando se inicia la acción el mismo, desde su posición estable hasta la metaestable y allí permanece un tiempo determinado establecido, por los valores de las resistencias del circuito, volviendo después a su período inicial. Así se produce una pulsación positiva de la longitud que deseamos esta; dirige la grilla de la etapa de salida, que normalmente no conduce.



T - 2	6 S N 7	6 S L 7
R ₁	22K - 2	100K - 1
R ₂	5.6K - 1	27K - 1
R ₃	15K - 2	68K - 1
R ₄	1M - 1	1M - 1
R ₅	220K - 1	1M - 1
R ₆	100K - 1	510K - 1
R ₇	10K - 2	59K - 1
C ₁	51 μf	59 μf
C ₂	.01 μf	.01 μf

Se usa indistintamente con lamparas 6 S N 7, 6 S L 7, u otras segun se desean resist rar pulsaeiones de alta o baja frecueneia. Solo empleamos nosotros las dos dases antes citado con las rcisteneias que se mencionan en cada caso. Estos circuitos tienen dos posiciones eatables en cada una do las cuales una do las valvulas conduce y la otm no. El diodo doble, sirve para scleccionar pulsos negativos, no respondienddo a los positivos. Se pueden conectar estas etapas en cadena, dando cada una a su salida, la mi tad del numero de pulsaeiones que entran; con ello sc puede hacer que, en base a 5 etapas, se consigna una disminucion con un factor de 32 sobre el numero total de las pulsaciones y la salida de la ultima etapa sirve generalmente para accionar un registrador mecanico. La lampara de neon atua coino indicador para la interpolacion.

LEYENDA DE ABREVIACIONES

μf = microfaradio
 $\mu \mu f$ = micro-microfaradio
 Ω = ohmio
 K = 10³
 M = 10⁶

El numero que aparece despuc's del valor de la resisteneia significa, su disipacion nominal en vatios.

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Properties of Mixed Showers

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Measurements with counter arrangements were made in Peru (4500 m altitude) and in São Paulo (700 m altitude).

The principal problems investigated were:

- a) the density spectrum of atmospheric showers.
- b) the percentage of penetrating particles present in the showers, in several density regions.
- c) altitude effect.

The master arrangement consisted of four trays of counters located along the sides of a square and shielded by 20 cm *Pb*. The length of the sides of the square was 1,0 m. Fourfold coincidences among these trays were the master counting rate. The resolving power of the coincidence set was 5 microseconds. Fivefold coincidences C_5^H and C_5 between the master arrangement and additional trays, one shielded by 20 cm *Pb* and the other unshielded, located in the center of square, were also registered.

We shall first briefly describe what methods were used.

The counting rate C_4 of 1 non-screened trays of counter in coincidence can be written as

$$C_4(S) = \int_0^{\infty} F(\delta) (1 - e^{-\delta S})^4 d\delta$$

where S is the area of each tray, δ is the density and $F(\delta) d\delta$ is the number of incident showers with density between δ and $\delta + d\delta$

The coincidence counting rate C_4^H of 4 trays, each of area S_H shielded by 20 cm of lead will be

$$C_4^H(S_H) = \int_0^{\infty} F(\delta) (1 - e^{-k\delta S_H})^4 d\delta$$

If a surface S_H is found by trial such that $C_4^H(S_H) = C_4(S)$ we shall have $\delta_H S_H = \delta S$ and therefore $\delta_H / \delta = S / S_H$. The ratio δ_H / δ , which we call k , is the percentage of penetrating particles present in the showers. This we shall call briefly method of equal counting rates. This method was employed by ISE & FREETER (1949) who worked at 3000 m and by us in this work. They found 2,5% and we found 2% at both altitudes investigated. These results are in agreement

if we take into account statistical errors and differences in the geometry of the experimental arrangement. The hypothesis that the density spectrum $F(\delta)$ is described by a power law of the type $\delta^{-\gamma}$ has been already checked, for electron showers by COCCONI ET AL (1949), ISE & FRETTER (1949) and others, and for penetrating showers by ISE & FRETTER (1949) and by ourselves in this work. The method used for this check is to vary simultaneously the areas of all counter trays. The value of γ we obtained for penetrating showers by this method was $1,46 \pm 0,03$ in Morococha and $1,50 \pm 0,11$ in Sao Paulo. FRETTER found at 3000 m 1,6 but this arrangement was geometrically somewhat different.

A second method of determining k is to have an extra tray of counters not screened in coincidence with the screened master. The ratio C_5/C_1^H and γ allow us to calculate k in the hypothesis that k does not depend on the density of the showers. We shall call this method for brevity the fifth counter method. This method has the utility of showing whether penetrating atmospheric showers exist with much greater k than 2%. This second method was employed by us in the experiment now reported. MCCUSKER & MILLAR (1951) used recently a similar arrangement but the analysis was made by them on a somewhat different basis. A third method, similar to the one just described, but which depends less on the form of the density spectrum, is to measure also the five-fold coincidences C'' between the master and a screened extra tray. The comparison of C'' and C_5 allows us to calculate k . This method, which we employed simultaneously with the method of the fifth counter confirmed its results. Our measurements in different density regions did not show any appreciable variation in the percentage of penetrating particles. This percentage was found to be $(3,6 \pm 0,4)$ in S. Paulo and $(2,05 \pm 0,14)\%$ in Peru.

We call the attention to the fact that at low altitude (S. Paulo) there is a difference outside statistical errors between the values obtained by the methods of equal counting rates and of the fifth counter.

At mountains altitude, Morococha, no such difference is found.

This indicates that at low altitude, one of the hypothesis needed for the application of the methods given above to penetrating showers, namely the constancy of k , is not valid.

In order to explain the discrepancy one has to suppose that besides the showers having $k = 2\%$ there is at low altitude, a certain number of penetrating showers with very few electrons. This conclusion is in agreement with the results obtained by MCCUSKER & MILLAR (1951) on somewhat different grounds.

These extra showers present at low altitude, could be, as suggested by MCCUSKER & MILLAR, tail ends of normal extensive showers, having the electrons been absorbed during the development of the showers in the atmosphere.

We want to notice that from the registered density of electrons, which varied from 1000 to 6000 part. m^{-2} one can estimate roughly that the mean energy of the showers was very great, of the order of 10^{15} e.v.

Another result of the experiments is that the apparent mean range obtained from the altitude variation of the penetrating showers, is $103,1 \pm 2,0$ gr. cm''^2 .

But the real mean range must be much less, perhaps of the order of the geometrical one, because the showers must be produced at high altitude above the arrangement.

We thank Unesco and Instituto de Biología Andina de la Universidad Mayor de S. Marcos de Lima for providing the laboratory and many facilities during the experiments in Peru.

We thank Cerro de Pasco Corporation for providing part of the lead used in the experiments in Peru, and for many facilities. We thank Conselho Nacional de Pesquisas for having provided the transportation to Peru.

We acknowledge our gratitude to IT. A. Meyer who participated in the preparation of the experiments.

We thank Professors J. Tiomno and D. 13 ohm for stimulating discussions.

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DISCUSSION

PIKRONI: Gostaria de saber o tipo do contadores que os scnhores usaram.

A. WATAOHIN: Contadores de eatodo o externo, tipo Maze.

PIERONI: Que tipos do ouida lo tinhamv para roduriz a influencia da temperatura nas indieaeoes dosses contadores? O Laboratrtrio era aqueeido? Faco a pergunta porque esses contadores, com uma variacao de 5 grans ni temperatura, caem eompletamente fora do "plateau".

A. WATAOHIN: Era aqueeido, mas era mantida permanentemente a temperatura de 15, 20 graus; tinhamos terniometro.

PIKRONI: Uma variacao de temperatura nos contadores expliea muitas variacoes aparentemente devidas a altura.

A. WATAGHIN: Isso foi excluido.

VALLARTA: Hay quo tomar precauciones muy espeeias con eso tipo de contadores para asegurar quo no haya errores do temperatura. Por ejemplo, en la ciudad de Mexico, donde la variation diurna de temperatura es do alrodedor de 15° os absolutamento imposible roalizar esta clase de temperatura si no se toman precauciones muy ospciales para asegurar la constancia do la temperatura dontro del laboratorio, pero entiendo por la explicacion del prof. Wataghin que estas precauciones fuoron tomadas en este experimento y de manera que no hay que sospochar la oxistencia do errores de temperatura aprociablos.

A. WATAGHIN: Nas ocaoes cm que nos tivomos tomoporaturas extornas baixas fo¹ aqueeido. Isto 6 o quo posso dizer.

ESCOBAR: Por ojempro, nosotros tenemos en Chaealtaya tcinperaturas sumamento bajas, al exterior tenemos — 17° pero para todos los contadores tomamos siompre la precaution do quo los contadores ostein siompre a tomoporaturas "standards", Bueno, nuestro caso es muy fdcil, pues ei mismo campo magnetico esta" gonerando la temperatura mas o menos constante y antes do unas 4 o 5 horas do que pongamos a funcionar las altas tonsionos para los contadores tonomos la precaution do quo este funcionando ol campo, do manera quo siompre esta" a temperatura constante, pero en este caso por ojempro tambien imagino yo que on el Peru, Uds. estan trabajando on condiciones similares y los contadores ostarfa na temperaturas do la habitation, nunca a tomoporaturas inferiores,

El Espectro de Energia de la Radiation Cdsmica Primaria Determinada por las Intensidades de Neutrones

M. 8. VALLARTA

Instituto Nariional de la I nvestigacida Cientijica e Universidad de Mexico, Mexico, D.F., Mexico.

Usando las medidas de la intensidad do neutrones de J. A. SIMPSON y suponiendo que hay proporeionalidad entre el nuniero de neutrones medido y el niimero de protones primario, el espeetro de energia de la componente de protones de radiaeiion eosmica primaria puede determinarse por me'todos bien coneoidos. Si el espeetro se escribe en la forma $n = KE^\gamma$ (n , numero de protones primario y E , energia), el analisis da como resultado que γ aumenta del valor 1.18 en la region de baja energia (1.3 a 4.3 Bev) a 1.82 en la region de energia alta (4.3 a 14 Bev). Esto esta de acuerdo con los result-ados obtenidos con medidas de particulas secundarias cargadas a alturas que se pueden aleanzar con globos sonda y cohetes. Esto indica que la componente neutronica se debe efectivamente a la componente protonica primaria.

DISCUSSION

G. WATAGHIN: What is the altitude at which the data on the intensity of neutrons wore taken?

VALLARTA: These are most ly airplane altitude experiments made at about 8.000 meters.

G. WATAGHIN: Some calculations on this neutron intensity made in Turin show that in higher altitudes there is a continuous production of neutrons derived from high energy protons. Probably one should take into account, also this point.

VALLARTA: I think you are quite right, if you take the results of the recording stations. At present there are three stations — there is one in Huncayo (Peril), one in Mexico and one in Chicago. If one takes the results of these three stations there are only three points from which one obtains values of the exponent gamma. These stations are between 3.000 and 2.200 or 2.300 meters, so that it does not touch your point at all.

MOLIERE: In my talk I win describe calculations on the development of cosmic radiation in the atmosphere where we have taken into account the much lower values of Y in comparison with values formerly assumed. Only at higher energies we took the same value for 7, 1,8 for the integral or 2,8 for the differential spectrum. We have a law which is not of the form of a power law, but that asymptotically goes into a power law. At large energies, higher than 14 Bev, the asymptotic law is reached. At about 30 Bev or higher, the power law of 1,8 for the integial spectrum is approximately valid and as a whole we get good results with our assumption of an asymptotic value of the exponent in agreement with that formerly assumed. I would like to ask you whether you have any objections against the assumption of a power at such large energies?

VALLARTA: What seems to be sure is that the energy spectrum at low energy is flatter than it is supposed to be. At very high energies it seems that very high energy particles should be scarcer if the exponent was 2,8.

MOLIERE: According to our results, which have not yet been published, I have not that impression.

WIGNER: I think it is clear that the exponent 7 must increase with increasing energy because otherwise either the total number of cosmic ray particles would be infinite or the total energy would be infinite. In fact, it is clear that it must increase altogether by more than one. The question is whether the increase of the exponent which is observed here is connected with this necessary increase of this exponent. Or do you think that the decrease and increase take place at lower and higher energies?

VALLARTA: We cannot answer this question in final form but the evidence is that what you have said is true, that is, the exponent goes up as the energy increases. The curve, the spectrum, becomes flatter as it goes to higher and higher energies. As regards the question of infinite number of particles of low energy, of course, one never goes to zero energy, that is E_0 is always a cut off except at the pole where one gets all kinds of energies. There one must expect only that the spectrum must be very much flatter than it is elsewhere, it is bent back. But of course that is not known. That is all surmise.

Concord of Cosmic Ray Components in the Atmosphere*

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Qualitatively, the main features of the bulk of Cosmic Radiation, i.e., the nature of the particles constituting the different components and the mechanism of their transmutation and development in the atmosphere, are at present well known. Moreover, quantitatively, we know with more or less accuracy, the energy spectra of the different components including that of primary Cosmic Radiation (i.e. the spectrum of primary protons and of the nucleons contained in primary heavier nuclei), which has been explored by recent experiments with rockets, balloon flights, etc. So it seems a worthwhile task to give a deductive quantitative mathematical description, as complete as possible, of the development of Cosmic Ray components in the atmosphere. This is what my friend Budini and i have tried to do.

The starting point of our theory is the spectrum of primary Cosmic Radiation, which Professor Vallarta spoke about in his preceding lecture. As he pointed out, this spectrum, at least at not too high energies up to about 15 Bev, shows a much slower variation than had formerly been assumed. This behavior of the primary spectrum at lower energies has been taken into account in our theory and turns out to be of great importance in the interpretation of experimental facts. At higher energies we have much support in assuming that the more rapid variation according to a power law of an exponent of ~ 1.8 (for the integral spectrum), which formerly had been assumed to hold throughout the whole energy range, remains valid as an asymptotic law. As a representation of the integral primary spectrum (i.e., the number of primary particles whose energy exceeds a certain value E_0), we used the formula:

$$A \cdot (E + B)^{-1.8} [\text{sterad}^{-1} \text{ sec}^{-1} \text{ cm}^{-2}]$$

where the constants A and B are chosen to fit with experimental data. By intervention of the constant B , which is of the order of 5 Bev, the flattening-out of the spectrum at lower energies is described. A graphic representation of our law for the primary spectrum is given by Figure 1, in double logarithmic scale, where the dotted straight line corresponds to the asymptotic 1.8 power law, while the curved part of the graph describes the slower variation of the spectrum at lower energies.

* The paper reported here will be published in detail in W. Heisenberg, "Cosmic Radiation", 2nd ed., Springer-Verlag, in the press. Parts of the paper will also appear in Nuovo Cimento.

The particles (mainly protons), which constitute the primary component and which on their way from outside the earth, penetrate into the atmosphere, give rise there to the various processes of transmutation by which the different components finally are created, which we observe, for instance at sea level, as "Cosmic Radiation".

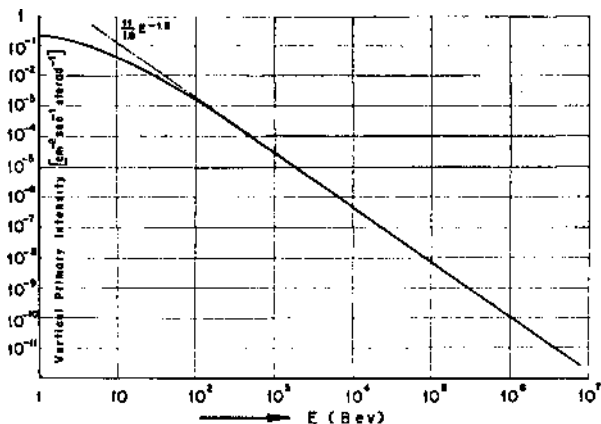


Fig. 1. The integral spectrum of primary nucleons (= protons + nucleons contained in nuclei) — according to Winckler, Peters and others, — power spectrum, after Hilberry.

To give a survey of these processes let us consider a single primary proton of energy E_p which on its way through the air after traversing a certain path length, will undergo a collision with an air nucleus. The average value of this path length according to the geometrical cross section of the air nucleus, corresponds to a layer of about 60 gr/cm^2 . In this process a shower of particles is created which are, on the one hand, nucleons pushed out of the air nucleus, and on the other hand, π — mesons, which in the strict sense of the word, are "created" by this collision. The average energy distribution of these two kinds of particles created by a single collision, may be described by the "creation spectra" $f_p(E_p, E) dE$ and $f_\pi(E_p, E) dE$, respectively.

The nucleons, arising from this first collision will make further collisions by which once more nucleons and π — mesons are created, and so forth, so that the whole process develops through the atmosphere in a kind of cascade.

Also the charged π — mesons are able to make collisions of the same kind, with production of further nucleons and x — mesons. Besides this, as a competing process, they can undergo the $\pi - \mu$ — decay which, at lower energies, is the predominant one, while at higher energies (beginning at about 100 Bev, because of the relativistic increase of life-time), most of the π will collide with air nuclei and so will participate in the above described "nucleonic cascade". For the particles or component playing a role in it (i.e., nucleons plus those π — mesons which collide before decaying), the common name of "W-particles" or "N-component" is used.

This N-component, developing through the atmosphere by the mechanism of nucleonic cascade, may be considered the "skeleton" of Cosmic Radiation in the atmosphere, in so far as all other components are derived from it as secondary. So the μ component is originated by $\pi - \mu$ - decay of the charged π - mesons (viz. those which don't collide before decaying), which are created by collisions of the N-particles. Similarly, the electronic-photonic component in the higher atmosphere is initiated by the decay photons of neutral π - mesons, the latter being created in the same way as those charged.

So the first thing we have to do, according to the mechanism of the nucleonic cascade, is to calculate the energy spectrum of the nucleonic component in its dependence on the atmospheric depth, starting from the primary spectrum. The resultant nucleonic spectrum will serve us as the source from which the spectra of the other components have to be derived.

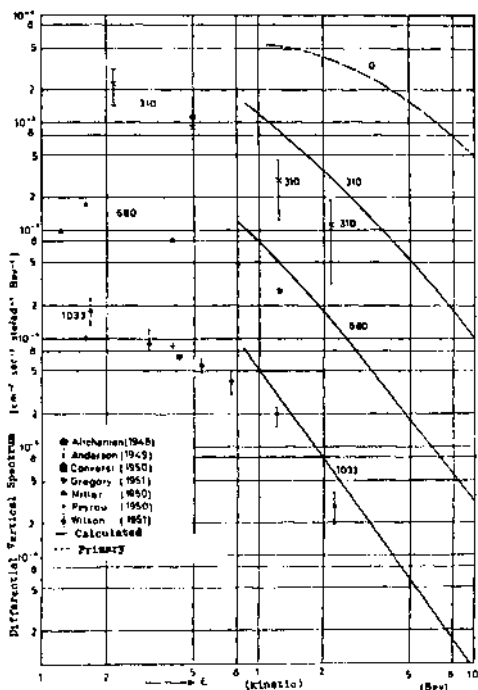


Fig. 2. Differential spectrum of protons in the atmosphere at different altitudes. (The attached figures denote depth in g/cm^2 .)

In performing these calculations a general procedure, based on the extensive use of a folding theorem was adopted which is equivalent to, but much simpler than the usual method of solving diffusion equations.

Two simplifying assumptions (which are common to most of the previous work on the nucleonic cascade) are involved in our method: the first is the neglect

of all angular deviations of the secondary particles from the direction of the primaries, which is good for sufficiently high energies. The other, which is most essential, is the assumption that the above mentioned "creation spectra" are "homogeneous" (or what is sometimes called in literature, "Bremsstrahlung" — type), meaning that these spectra may be written in the form $f_p(E/E_0)dE/E_0$ and $f_\pi(E/E_0)dE/E_0$, depending only on the fraction of the energies of the created and colliding particles. We have support for believing that, for sufficiently high energies, (compared with the rest energy of the particles in consideration), this assumption of homogeneous creation spectra is at least a good approximation.

For the special form of these creation spectra (about which not much is known as yet, theoretically as well as experimentally), we used simple analytical "Ansätze", depending on some parameters which are determined numerically later on by comparison of the results of our calculations with experimental data. It may be noted that our result, so obtained, for π — meson creation spectrum agrees well with Heisenberg's new theory of meson production, which, particularly, leads also to a homogeneous creation spectrum.

The assumption of homogeneous creation spectra has mathematical consequences, well known from cascade theory, which are of importance for the interpretation of certain experimental facts. Suppose a primary component, the spectrum of which is of power form, produces a secondary component by some process which is described by a homogeneous creation spectrum. It follows that the secondary spectrum is of the same power form. Furthermore, if the process under consideration is a complete cascade process, in some material, it follows that the spectrum is absorbed exponentially.

Now, these two facts: reproduction of a power spectrum and exponential absorption, are just what is found experimentally, not over the whole energy range, but asymptotically for large energies and great depths in the atmosphere. Here we have the justification for two of our fundamental assumptions: the asymptotic power form of the primary spectrum and the homogeneousness of the creation spectra.

Our statement about the reproduction of a power spectrum by a homogeneous creation mechanism remains valid, with a slight modification for a spectrum which is only asymptotically of power form at high energies and varies more slowly at low energies, as our primary spectrum does. In this case the shape of the whole spectrum, including its slowly varying low energy part, is approximately reproduced with the modification that this part is shifted towards lower energies in the secondary spectrum. This behaviour can be understood from the degradation of energy by the creation process. In the case of a cascade, the shifting of the flatter part of the spectrum increases with increasing depth.

An illustration of this behaviour of a cascade spectrum is given by Fig. 2, where the results of our calculation for the spectra of the nucleonic component at different atmospheric depths are plotted in double logarithmic scale. (The figures attached to the curves refer to the depth in gr/cm^2).

In Fig. 2 are also included all available experimental data for the nucleonic spectrum. The latter were obtained from the data for the protonic spectrum, (PUPPI A DALLAPORTA, 1952) directly given by experiment, by adding a correcting factor 2, corresponding to a 1:1 ratio of neutrons: protons. It should

be kept in mind, however, that this ratio, which is one at higher energies, has a larger value at lower energies ($\lesssim 2 \text{ Bev kinetic}$), because of the intervention of ionization loss for protons. The experimental points in Fig. 2, therefore, are too low at low energies. On the other hand, our theory is only valid at larger energies ($\gtrsim 2 \text{ Bev kinetic}$), where ionization loss and other complications intervening at low energies, may be neglected.

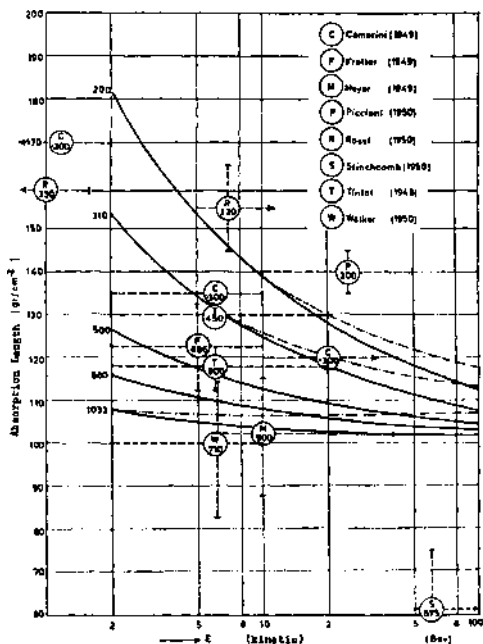


Fig. 3. Absorption length of the vertical nucleonic component as function of energy at different atmospheric depths (figures denoting depth in g/cm^2)

Theoretical results: — without taking into account the participation of π -mesons in the nucleonic cascade. --- with taking into account the participation of π -mesons in the nucleonic cascade. Circles: experimental data.

Good agreement between theory and experiment, therefore, can only be expected for energies starting from about 2 Bev . Unfortunately, experimental data at those large energies are rare. But, from a look at Fig. 2, one has the impression that, with increasing energies, the experimental data (except those at the greatest heights, which are very rough and uncertain), tend asymptotically to our theoretical curves, and this is the best fit to be expected.

This fit was obtained by special choice of the above-mentioned parameters describing the creation spectrum for nucleons. Together with the requirement to give the right asymptotic absorption length these parameters could be fixed rather definitely. The information so obtained, with regard to the nucleon-creation-spectrum, may be described as follows: In a single collision with an

air nucleus, 80% of the energy of the colliding nucleon is transmitted to the nucleonic component. The shape of this creation spectrum is of the low multiplicity type, compatible with the picture of "plural production" for the nucleons emerging from such a collision.

It is of interest to discuss our results on the nucleonic component under the aspect of absorption lengths, since this quantity has frequently been determined by experiment.

As already pointed out, at large energies the absorption is exponential, which means that there the absorption length (i.e., the inverse of the absorption coefficient), is constant. The same is no longer true at smaller energies. As a consequence of the said shifting of the slowly varying part of the spectrum toward lower energies, which shifting increases with increasing depth, the absorption length of the nucleonic component will increase with decreasing energy as well as with decreasing depth in the atmosphere. This behaviour is in general agreement with the experimental findings, which are plotted together with our theoretical curves in Fig. 3. (Cf. for instance the three results of Camerini and co-workers, denoted by "0" in Fig. 3, which agree well with our curve for 310 gr/cm².)

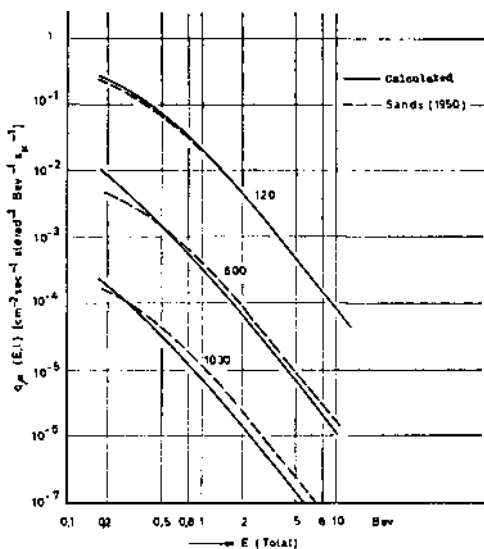


Fig. 4. The source spectrum of μ -mesons at different depths (figures: gr/cm²).

Experimentally much better known than the nucleon spectrum is the spectrum of μ -mesons. Starting from measurements of the spectrum of the μ -component at different heights (partly performed by himself, and partly by other authors), Sands has derived the so-called source spectrum of the μ -mesons (i.e., the number of them created in a certain layer of the atmosphere). In Fig. 4, Sands' results and ours for the μ -source spectrum are compared. Theoretically,

as a consequence of a homogeneous creation spectrum for the π -mesons, the μ source spectrum is a direct reproduction (shifted toward lower energies), of the nucleonic spectrum. According to our calculations, therefore, the μ -source spectrum is of the same asymptotic power form at large energies as the primary spectrum and has, again, a more slowly varying part which is increasingly shifted toward lower energies with increasing depth. The latter feature is not correctly described by the curves of Sands, who, as a simplification in his calculations, assumed a constant absorption length, so that his spectrum appears exactly reproduced, without increasing shifting, in different depths. Nevertheless, it was possible, as Fig. 4 shows, to get an excellent fit between Sands' and our curves in the higher atmosphere where most of the μ -mesons are produced, while in the deeper atmosphere the agreement is as good as can be expected.

This fit, as well for the asymptotic power part at high energies as for the more slowly varying low energy part of the μ -source spectrum, was obtained by determining numerical values for the two parameters describing the above mentioned creation spectrum for π -mesons, the result for the latter being as follows:

- (i) The shape of the π -creation spectrum, as already mentioned, is in good agreement with that given by Heisenberg's new theory.
- (ii) The average energy fraction, transmitted by a single collision of a nucleoli with an air nucleus to the created π -mesons, is about 20%.

The latter result is of interest with respect to the question of energy balance in cosmic radiation. Together with the corresponding result for the fraction of energy of a colliding nucleon transferred to nucleons, viz. 80%, it shows that our picture, according to which only two kinds of particles (viz. nucleons and π -mesons), are created by nucleonic collision, duly accounts for the conservation of energy.

This conclusion, of course is restricted to the range of energy (extending for nucleons from about 2 to 100 Bev), where it is valid. If there exists a lack of energy balance, as found by investigations of Rossi, it can only occur at low energies where our theory is no longer valid. On the other hand, at very high energies we have to expect

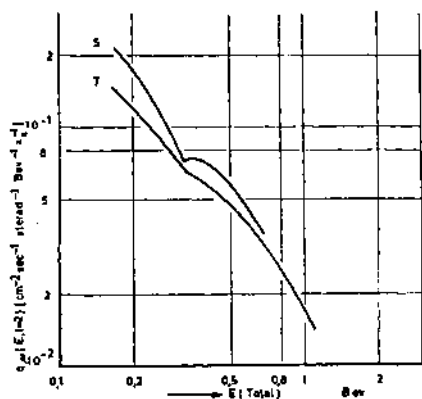


Fig. 5. Sands' anomaly of the source spectrum of μ -mesons. T: theoretical for a depth of 120 gr./cm²; S: after Sands.

that other particles, such as k -mesons and (perhaps), nucleon-pairs, will come into play. The question of k -mesons will be discussed later on in this paper.

Sands' curves for the μ source spectrum, as represented in Fig. 4, are somewhat smoothed. Really, Sands found an anomaly consisting of a sudden change in slope at a certain energy as shown by the curve "S" in Fig. 5. Our explanation for this anomaly is based on the fact that the spectrum of primary protons is cut-off at a certain energy by the geo-magnetic field, while the following generations of the nucleon cascade extend continuously toward smaller energies. As a consequence of these facts, the one part of the source spectrum of μ -mesons

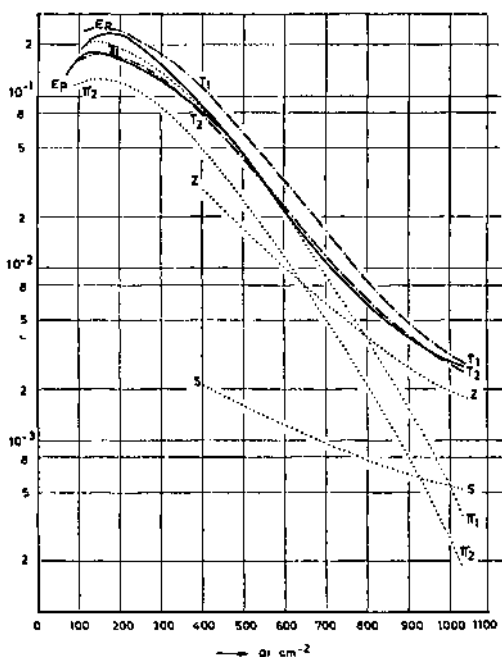
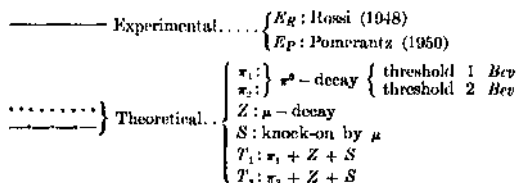


Fig. 6. Vertical intensity of the electronic component ($E > 10^7$ ev) in the air (geomagnetical latitude $> 50^\circ$)



which is produced by collisions of primary protons will fall off rapidly with decreasing energy, while the other part of it, which is due to the following cascade generations of the nucleonic component will, on the contrary, increase with decreasing energy. At small atmospheric depths, where most of the μ -mesons are produced, the contribution of the first part will surpass that of the other

at higher energies, while at lower energies the latter part will preponderate. The anomaly in form of a sudden change in slope will occur approximately at that energy where the contributions of both parts are equal.

On the basis of this idea we calculated the μ -source spectrum for an atmospheric depth of 120 gr/cm², the result being represented by curve *T* in Fig. 5, which, in fact, shows a behaviour similar to Sands' curve. The similarity would have been still more pronounced at a lesser depth of about 00 gr/cm², where the spectrum of primary protons is still more predominant.

It should be noted that a similar anomaly has been found by Camerini and co-workers, for the π -source spectrum.

The correctness of our explanation of Sands' anomaly could be checked by measurements at different geo-magnetic latitudes.

Our above described results concerning the components of nucleons and of μ -mesons are much confirmed by the application of our theory to the electronic component in the atmosphere. P. Budini calculated the total electronic intensity as a function of the atmospheric depth, the results being given by Fig 0. The experimental results are represented in Fig. (>) by the full curve with the two branches "*En*" and "*E_p*", denoting the data of Rossi and Pomerantz respectively, at great heights. The two dotted curves " π_1 " and " π_2 " give the theoretical result for that part of the electronic intensity which is due to the decay of neutral π -mesons, the difference between them consisting of two different assumptions, viz. 1 *Ben* and 2 *Bev*, for the threshold energy for π -creation. The dotted curves "*Z*" and "*S*" respectively, denote the contributions of decay and knock-on electrons of μ -mesons. The sum of the different contributions is given by the dash-dot curves "*T₁*" and "*T₂*", differing by the two assumptions about the threshold. As Fig. 6 shows, there is excellent agreement between the theoretical curve *T₂* and the experimental results at great heights, especially those of Pomerantz.

It should be emphasized that the calculations of that part of the electronic intensity which is due to the decay of neutral π -mesons, assuming that the numbers of neutral to charged π -mesons are 1:2, was based on the same creation spectra as used before with the parameters determined by comparison with experimental data on nucleons and μ -mesons only.

To investigate the role played by *k*-mesons in the production of high energy μ -mesons, we performed some calculations on the μ -component up to energies as high as $\sim 10^4$ *Bev*, which shall finally be reported here:

The first of these calculations concerns the spectrum of the vertical μ -component at sea level, the results being shown in Fig. 7. The two curves (dotted and dash-dot), represent our theoretical results, obtained under alternative assumptions: the dotted curve corresponds to the assumption that all μ -mesons are created by decay of π -mesons — i.e., by the same mechanism responsible for μ -production at lower energies. This curve falls off rapidly at high energies because of the fact that high energy π -mesons (as first pointed out by Greisen), collide with air nuclei and therefore contribute less to μ -production. The other curve, dash-dot, was obtained on the basis of the alternative assumption that μ -mesons are exclusively created by decay of *k*-mesons. As a consequence of their shorter life-time, the latter curve falls off more slowly.

By the comparison of these two results, obtained under extreme assumptions, with experimental data, we hope to get information about the mechanism of μ -production at high energies.

Since direct experimental results on the μ -spectrum at these extremely high energies are not available, we calculated the spectrum of μ -mesons at sea-level from underground measurements of the μ -intensity by means of the range-energy-relation, as given by GEORGE (1925). The result is represented by the shaded band in Fig. 7.

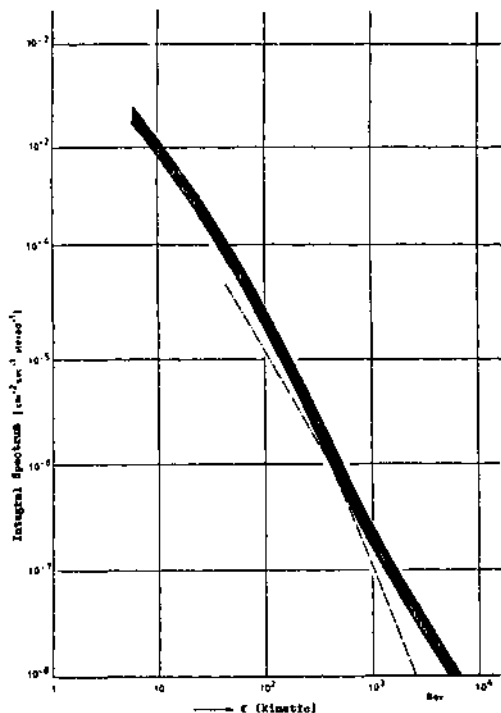


Fig. 7. Integral spectrum of the μ -spectrum at sea level:

Experimental
 Calculated for μ -production $\left\{ \begin{array}{l} \text{from } \pi \text{ alone} \\ \text{from } K \text{ alone} \end{array} \right.$

The comparison of this "experimental band" with our theoretical curves seems to indicate that up to about 500 BeV, most of the μ -mesons are created by decay of π -mesons, while at higher energies almost all of them are due to K-decay.

It should be emphasized, however, that this conclusion is not free of uncertainties, one of which, e.g. being introduced by uncertainties in the range-

-energy-relation at high energies. Direct measurements of the μ -spectrum at high energies, therefore, would be of great importance.

Another possible source of information about the role of k -mesons is given by the angular distribution of high energy p -mesons.

This angular distribution is usually described by a $\cos^p$ - law, where θ is the angle of inclination with respect to the vertical, and p is an exponent which slowly varies with energy. This exponent p , as a function of energy, is represented by Fig. 8, according to our theory. The available values involved in the figure agree well with the theory within the range of its validity. The lower values of p at low energies are due to the angular spread occurring at low energy creation processes, neglected in the theory.

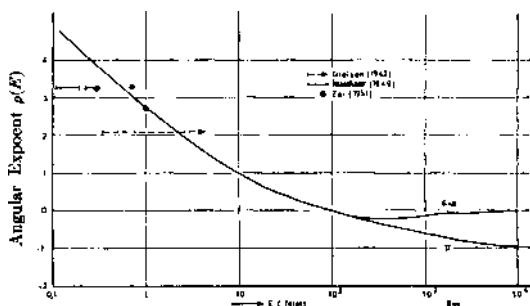


Fig. 8 Exponent p of the cosine-angular-distribution-law of μ -mesons at sea level as a function of energy.

π : calculated for μ -production by π -mesons only

Exp: experimental and calculated for participation of k -mesons.

The two branches of our curve in Fig. 8 correspond to those in Fig. 7: the branch " π " belongs to the assumption of μ -mesons created exclusively by π -mesons, while the branch "exp" corresponds to the "experimental band" in Fig. 7 or to p -mesons of highest energies created by k -decay.

The latter branch tends with increasing energy to the limiting value of $p = 0$, corresponding to an isotropic angular distribution of μ -mesons. The branch " π ", instead, tends to the limiting value of -1 , which means that the number of highest energy π -mesons, provided they are created by w -decay, will increase with increasing inclination. This behaviour, strange as it seems at the first sight, is readily understood by the realization that π -mesons, created in the high atmosphere, travelling in a more inclined direction, have a longer path through low density air and therefore, a greater chance to decay instead of colliding.

Measurements of the angular distribution of high energy p -mesons, therefore, would be of great importance with regard to the question of k -mesons.

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DISCUSSION

G. WATAGHIN: Your results about the mesons floated by k-meson - theoretical results - are calculated on the basis of a production of plural type or multiple type? What is the multiplicity that you admit?

MOLIERE: Multiplicity is not so important for cascade problems because the tail of the creation spectra is the most important part. To calculate multiplicity, one should take into account the deviations from the Bremsstrahlung-type behavior of the spectra. In our calculations, A_1 -mesons are the same as π -mesons with a larger mass and with a smaller life time and for π -mesons we got our law with a power a as it is determined in comparison with experiments: 2.2. This corresponds to multiple production. I could also say that the multiplicity as calculated, is slowly varying with energy and is in good agreement with Heisenberg's new theory of multiple production of π -mesons.

BECK: I would like to ask how far down in the energies we can make these assumptions, in order to know whether one can hope to get a junction with the data we can get in the laboratory.

MOLIERE: I think the energy must not be too low in comparison with the rest mass.

BECK: IS this the only restriction?

MOLIERE: I hope so. It comes out from our results that it is a good assumption that for nucleons starting with about 2 Bev, the theory and the assumption of Bremsstrahlung-type spectrum is good, and for π -mesons, for about 300 Mev or so.

VALLARTA: About the question that was raised a little earlier by Professor Moliere, I think I can see now the point. I think it is this: that the geomagnetic analysis can be extended to energies lower than 60 Bev, that is about the top limit, and that cannot be reached for practical reasons; that is, the geomagnetic analysis cannot be applied to energies which are, let us say, higher than about 20 or 25 Bev, i.e., about half of the theoretical top limit. Now, this, according to what has just been said, applies to very high energies. Therefore, I do not think that we can say anything about this region of the spectrum,

MOLIERE: To this point I could add a remark more: My friend Budini performed new calculations on large air showers and from comparison with certain experiments he found values for exponents 7 which are in good agreement with the value of 1.8 according to our assumption.

New Type of Transition Effects of Nuclear Disintegrations from Cosmic Rays in Solid Absorbers

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In order to analyze the contribution of the different components of cosmic radiation to the production of nuclear disintegrations, usually attributed in short to the so called iV-component, the frequency of stars in photographic emulsions has been measured under various depths of solid absorbers of carbon, iron and lead (SCHOPPER & ROSSLE, 1951).

A set of electrosensitive nuclear plates was vertically placed along the axis of a hill-formed absorber and simultaneously exposed, at 3000 m (Zugspitze observatory) and 420 m above sea level (Weissenau laboratory, Germany). At 3000 m transition curves of the star frequency were found which led to the following analysis: About 90% of the stars at the top of the absorbers or in the free atmosphere are produced by neutrons, protons and fast pions, with — as well known — a mean absorption length according to an $A^{2/3}$ law for the nuclear cross section, eg. 138 g cm^{-2} in air (to be called a-component).

Transition maxima at 1,2 cms in lead and at 5 cms in iron are due to the nuclear interaction of photons (to be called 6-component): their height in iron and lead and their absence in carbon agrees well with the cascade theory, if one states that 10% of the stars in the free atmosphere are generated by photons. This means a cross section of star production by photons in the emulsion of $5 \cdot 10^{-27} \text{ cm}^2$, in good agreement with KIKUCHI'S results (1951).

Subtracting the stars resulting from those a-and-fo-components one finds, in the carbon absorber, a sharply marked maximum at 18 cms, and by an analogous way of analyzing, such maxima at about 20 cms in iron and lead, which one finds shall be called c-maxima.

These maxima represent a new type of transition effects and are characterized:

a) by the fact that they appear, practically independent of the mass and the atomic number of the absorbers at nearly the same depth.

b) by a sharp concave increase beginning at about 15 cms and an equally sudden decrease, which seems to be analogous to the behaviour of the second and third transition maximum of the Rossi-curve, published by THURN & BOTHE (1950, 1951).

With different carbon absorbers: plane, wide and slender forms it could be shown that there exists a defined "maximum length", insofar as only such

forms of the absorbers give raise to a sharply shaped c-maximum which do not contain lateral depths of the magnitude of 1 at vertical depths smaller than 1.

The altitude dependence of the incoming particle enhancing the c-maxima corresponds to an absorption length of $\sim 120 \text{ g cm}^{-2}$ in air. It is not produced at a marked rate within the absorber, as follows from the decrease of the c-maxima.

It seems not possible to describe the mechanism of those maxima by known types of interactions, perhaps by means of consecutive particles, one of which is unstable because of the transition effect in carbon.

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Cerenkov Counters

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Cerenkov radiation particle counters have been in use at the University of Chicago for more than a year. They have been particularly useful in connection with the 450 Mm. proton synchrocyclotron, and they might well be useful for cosmic ray studies. I shall first describe the general properties of the Cerenkov radiation, and later I shall describe the manner in which it is used in particle counters (MARSHALL, 1952).

The Cerenkov radiation is an electromagnetic shock wave, and is analogous to the pressure shock wave produced when a bullet travels at supersonic speed through air. Like the pressure shock wave, the Cerenkov wave is conical with the apex of the cone coincident with the particle producing the disturbance. The radiation is produced when any charged particle traverses a dielectric medium at a speed greater than that of light in the medium. The radiation is directed forward at an angle θ from the direction of the particle, where θ is determined by the relation

$$\cos \theta = \frac{1}{\beta n}$$

The intensity of the Cerenkov radiation is given by the formula

$$I = \frac{4\pi^2 e^2}{h c^2} \Delta \nu \left(1 - \frac{1}{\beta^2 n^2}\right) = \frac{2\pi \Delta \nu}{137 c} \sin^2 \theta \cong 450 \sin^2 \theta \text{ (for } \Delta \nu = 3 \times 10^{11} \text{ cycles per second)}$$

The intensity, I , is expressed in photons emitted per centimeter of particle path. β is v/c , the ratio of the speed of the particle to that of light in vacuo, n is the index of refraction of the radiator material. $\Delta \nu$ is the frequency range over which the radiation is measured. The visible spectrum is roughly of 3×10^{11} cycles per second frequency range.

As an example consider high energy electrons or any other particle of unit charge moving approximately at the speed of light. In Lucite or glass of refractive index 1.50, the intensity from such a particle is approximately 250 photons in the visible per centimeter of path. In water (index 1.31) the intensity would be 175 photons per centimeter.

The Cerenkov radiation from ordinary optical materials includes the visible spectrum. It may be observed visually by immersing a very strong beta radioactive source in water and has a bluish-white appearance to the eye.

In principle a Oerenkov counter can be made simply by exposing the cathode of a photomultiplier to the light produced when a particle traverses a transparent dielectric material. One might expect then to find a pulse in the output circuit of the photomultiplier every time a particle passes through the radiator. These pulses then could be registered electrically. In practice the problem is somewhat more difficult than this because of the small intensity of the Oerenkov radiation and the difficulty of efficient light collection. The intensity is small enough so that, if the light collection efficiency is not particularly large, there may be no photoelectrons at all produced at the cathode when a particle passes. When only small pulses are produced, it may be difficult to distinguish them from thermionic noise pulses which are always present in photomultipliers at normal temperatures. The best method of discriminating against noise is the use of fast coincidence techniques.

A non-focussing Cerenkov counter consists primarily of a body, which may conveniently be cylindrical in shape, of transparent radiator material with a photomultiplier tube so placed that the Cerenkov radiation produced in the radiator is collected with high efficiency by the photocathode of the multiplier. The Cerenkov light produced in a long cylindrical radiator can be transmitted with high efficiency to the end of the radiator toward which the particle is moving by multiple internal reflection. For highly relativistic particles the collection efficiency for a long radiator using total internal reflection is almost always better than 50%. Some loss of light can be expected because the Cerenkov angle for a relativistic particle is such that the light strikes the surface of the radiator just at the angle of total internal reflection if the particle is traveling along the axis of the radiator. Some of the light from a particle not parallel to the axis can be lost through the side surfaces of the radiator. For this type of counter it is essential to use a photomultiplier having its photocathode on the inside surface of the glass envelope, such as the RCA 5819. Multipliers with internal cathodes are inefficient because of the small solid angle subtended by the sensitive area of the cathode at the surface of the envelope. The cathode of a 5819 or of an EMI 5311 can be placed in optical contact with the radiator by the use of transparent oil or grease between the two surfaces. In the case of a liquid radiator the liquid can be in direct contact with the multiplier envelope.

A 5819 with a sensitive cathode area of about 3.8 cm diameter can collect the light from a 3.8 cm Lucite rod with nearly 100% efficiency. With a cathode efficiency of .06 electron per incident photon, the approximate efficiency of a 5819 photocathode, relativistic particles can produce pulses of approximately 16 electrons from the cathode per centimeter of particle path. A ten centimeter long radiator of Lucite then can produce pulses at the cathode up to 160 electrons in size. A water radiator of the same size would yield pulses of up to 110 electrons. While such pulses are not large compared to the pulses available from scintillation crystals, they are larger than is really necessary for efficient counting. Thus one can afford to increase the cross sectional area of the radiator by a factor 2 to 4 over the area of the cathode without loss of counting efficiency. An increase of radiator area by a factor 2 reduces the collection efficiency, and consequently the pulse height, to one half by spreading the light out over a larger area.

In some cases it is possible to have a radiator area larger than the cathode area with no loss of collection efficiency. This is the case for relatively slow

particles with small Cerenkov angle so that the light may be concentrated to some extent by tapering down the end of the radiator. Such particles must of course be heavy ones so that they can have low velocity without having too short a range.

A non focussing Cerenkov counter was demonstrated during the lecture. It consisted of a fast coincidence arrangement using a water radiator Cerenkov counter and a liquid scintillation counter. The water radiator was 5 centimeters in diameter and 4 centimeters long. One end of the Lucite water container was stoppered with the photocathode end of a 5819 multiplier tube. The other end of the radiator was painted black so that light moving in the reverse direction would be absorbed and would not reach the photocathode. Light could reach the cathode of this tube only from a particle moving more or less in the direction of the cathode. The Cerenkov radiation from a particle moving in the opposite direction, i.e. entering the radiator at the cathode end, would be almost completely absorbed by the black paint. A liquid scintillator container of equal diameter was placed in a position coaxial with the Cerenkov radiator, and was in optical contact with the cathode of another 5819. The scintillator material was three grams per liter of terphenyl dissolved in phenylcyclohexane.

The outputs of the two photomultipliers were connected separately to fast amplifiers of 180 ohms impedance and better than 10^{-8} second rise time. The outputs of the two amplifiers were mixed in a fast coincidence circuit using the 6BN6 gated beam tube, and the output of the coincidence circuit was registered directly with a thyratron and a mechanical register. Thus a coincidence count would be registered if a cosmic ray particle of relativistic velocity passed through the scintillator and also through the Cerenkov radiator in coincidence and in the right direction. Since very few cosmic ray particles move upward at the earth's surface, the apparatus would be expected to count only when the 5819 tube of the Cerenkov counter was below the radiator, and not when the apparatus was inverted.

The apparatus performed satisfactorily during the lecture, giving approximately 4 counts per minute in the upright position and about 0.1 count per minute inverted.

Focussing counters have also been constructed. \ shall not go into great detail here regarding these counters since the various designs have recently been published. In a focussing counter an optical system is used which concentrates into a small area light of one Cerenkov angle, and which directs elsewhere light of other Cerenkov angles. A focussing counter then can be used to count particles of one small range of velocities in the presence of and with the exclusion of particles of other velocities. It is possible, for instance, to count 120 *Men* π mesons and not to register electrons or other highly relativistic particles which appear in the same beam. Several optical systems are possible, but the one most successfully applied so far has consisted of a cylindrical Lucite radiator which leads light to the center of a hemispherical Lucite lens by total internal reflection. The lens would focus light of any one Cerenkov angle from particles parallel to the axis into a sharp ring coaxial with the system if it were not for a cylindrical mirror. The mirror is of approximately half the radius of the focal ring and is mounted coaxial with the system. It is constructed by aluminizing the inside surface of a glass tube of approximate size. The light is reflected by

it to form a focal spot on the axis. Light of different Cerenkov angles is focussed at different positions along the axis.

That it is impossible to focus the light to a sharp point can be shown by arguments involving the angular momentum of photons in the system. The actual size of the focal spot depends on the design of the optical system and on the degree of collimation of the particles counted. It is usually somewhat larger than the size of the radiator. If one places at the focal position a photomultiplier cathode, one can obtain pulses the amplitude of which is a maximum for particles of the velocity for which the system is adjusted. In general it is better to use two photomultipliers in coincidence with some arrangement for dividing the light more or less equally between them. Such counters are best applied to rather highly collimated beams of particles since they cannot have a very large sensitive area, and since a small deviation in the direction of the incident particles is sufficient to stop counting.

It is advantageous with Cerenkov counters to use fast electronics, and it is almost completely necessary to use some sort of coincidence arrangement to register counts. The reasons are that the pulses are very short in time, and that they are not particularly large compared with noise pulses from the usual photomultiplier. If slow electronics are used without coincidence, it is quite likely that two or more small noise pulses will add together in such a way as to produce an effect larger than a Cerenkov pulse. Even with fast electronics, single noise pulses may sometimes be too large to be rejected. A coincidence arrangement may be used either between pulses from two multipliers exposed to light from the same radiator, or between one exposed to light from a Cerenkov radiator and another exposed to light from a scintillator through which the same particle passes.

In general we find it necessary to amplify pulses from the photomultipliers with wide band pulse amplifiers by a factor of from ten to one hundred in voltage, and to apply the two pulses to a coincidence circuit with resolving time of the order of 10^{-8} second or better. We use at present coincidence circuits employing the 6BN6 gated beam tube biased beyond cutoff. Positive pulses from both channels must arrive at the grids in coincidence to allow plate current to flow and thus to produce an output pulse.

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DISCUSSION

G. WATAGHIN: What would the photon emission be for the neutron moving with relativistic velocity? How many photons would be emitted for one centimeter of path length?

MARSHALL: I do not know. I imagine that there would be some from the interaction of the magnetic moment of the neutron. I do not know how large it would be. (*)

G. WATAGHIN: Have you made any experiments with neutrons?

(*) A subsequent discussion indicated that the Cerenkov radiation from the magnetic dipole of neutron has a factor $\left(\frac{m}{M}\right)^2 \left(\frac{h\nu}{mc^2}\right)^4$ where ν is the frequency of the electromagnetic radiation, the velocity of which in the medium is equal to the velocity of the particle.

MARSHALL: NO, but I believe that- the intensity would be too small to measure with our background radiation.

VALLARTA: I have heard not so long ago that Phyllis Freier and Ney in Minnesota were going to use this type of counter for cosmic ray work, The idea was to find how many particles were coming straight up. We call it the splash Albedo in cosmic radiation. I don't remember the formula you wrote for the intensity, but does it depend in any way on the mass or the charge of the particle?

MARSHALL: It does not depend at all on the mass. It varies as the square of the charge.

VALLARTA: Could you say anything about the mass of the particles?

MARSHALL: The only effect of the mass is to modify the relations among velocity, energy and range. Any electron that can be counted must be moving at approximately the velocity of light. For electrons of appreciably smaller velocity the range is too small for them to be counted.

RABI: Has any attempt been made to study particles of higher charges?

MARSHALL: Not that I know of.

The Geometric Efficiency of Cylindrical Counters

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ABSTRACT

Counting experiments were made with cylindrical counters fitted with geometry "definers" in order to verify if it is possible to find an area of constant sensitivity in these counters. These measurements are similar to those made by Gleason *et al* with end-window counters. We find that it is indeed possible to achieve these results by means of a careful choice of the dimensions of these definers. The definers also allow a great simplification of the calculation of counter efficiency for a point source. We present a formula for sources placed in any position with respect to the counter. This formula is compared with experiment and the agreement is found to be good. A comparison of absolute beta counting between one of our cylindrical counters (with definer) and an end-window counter (with definer) is made. The agreement obtained is satisfactory.

INTRODUCTION

The accurate determination of the geometric efficiency of a Geiger-Muller counter is one of the most difficult problems that arise when precise measurements are made with counters. It is a well known fact that the largest error that arises in the measurement of the total number of beta particles emitted by a radioactive source is due to the difficulties involved in the determination of the geometry of the arrangement used.

The geometric efficiency of a counter can be: a) calculated; b) measured experimentally with the help of a source that emits a known number of particles per unit time.

a) For point sources the computation of the geometric efficiency can be made easily with help of very well known formulas; when extensive sources of circular shape are subject to a measurement one can make use of the formula (1) due to Blachman.

It is interesting to compare the results of the geometrical efficiency as obtained with the above formulas with the values obtained with the help of a radioactive source: BURTT (1949) has made a comparison between the results

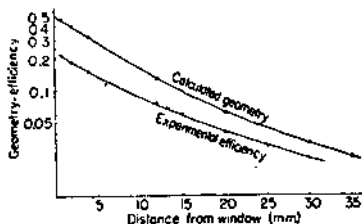


Fig. 1 — Burt's comparison between calculated and experimental geometries.

obtained with a standard source and an end-window counter with the results computed with the formula

$$G = 0,5 \left[1 - \frac{h}{\sqrt{h^2 + r^2}} \right] \quad (1)$$

where h is the distance from the counter window to the source and r is the radius of the window of the counter; the radioactive point source is on the axis of the counter. The results obtained by Burt are in fig. 1; the difference of the values obtained by the use of the methods above can be explained by assuming that the sensitive region of the counter should be smaller than that assumed in the calculations; under this hypothesis, it can be easily shown that both curves do coincide when one assumes that the sensitive region of the counter begins at a distance of about 5 mm from the cathode.

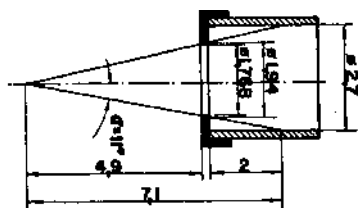


Fig. 2 -- A drawing of the geometry definer used for end-window counters. Distances in centimeters.

and on the energy of the particles that are being observed, others difficulties arise. The above circumstances make this method troublesome and subject to errors (ZUMWALT, 1947).

The difficulties connected with the determination of the sensitive area of an end-window counter can be eliminated by means of a geometry "definer", as suggested by GLEASON, TAYLOR & TABERN (1951), who have shown that an accurate determination of the geometry can be made by means of a plate (of a suitable thickness), adjacent to the counter window, provided with a central circular hole, concentric with the counter cathode (fig. 2). Experiments were carried out in order to determine what the diameter of the definer should be, in order that only the sensitive volume of the counter should be traversed by the incoming particles.

In this paper an attempt is made to utilize geometry definers when cylindrical counters are used. It is a well known fact that the efficiency of a cylindrical counter decreases somewhat in the neighbourhood of the counter ends: if the geometry definer is suitably chosen, it is possible to use those counters for accurate measurements.

b) The determination of the geometric efficiency with the help of a standard radioactive source involves many troublesome corrections due to phenomena such as backscattering, absorption in the air column (between the source and the counter), scattering of the particles by the air, scattering by the walls of the counter housing, etc. Besides, as the sensitive volume of the counter depends on the nature

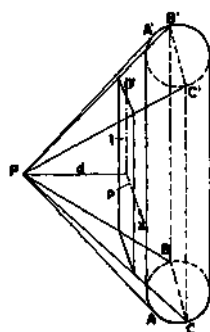


Fig. 3 — Solid angles subtended by a point source in front of a cylindrical counter. xy is the plane of the definer.

The calculation of the geometrical efficiency of cylindrical counter is a difficult problem. As one can see in fig. 3 the solid angle subtended by a source in P is the sum of the solid angle θ , corresponding to $BCB'C$ and θ' which is the sum of the solid angles corresponding to ABC and $A'B'C$. The angle θ can be calculated exactly whereas the angle $\Delta\phi$ can only be calculated under approximations. NORLING'S (1941) calculations of the angle θ , it is interesting to note, refer only to a source that is equidistant from the counter ends and sides.

With the help of a geometry definer (in this case a brass plate provided with a rectangular hole) the angle $\phi\Delta$ is ruled out; we then calculate the geometric efficiency of a point source placed in any position in the neighbourhood of the counter. The comparison of the calculations with the experimental data has provided a good agreement.

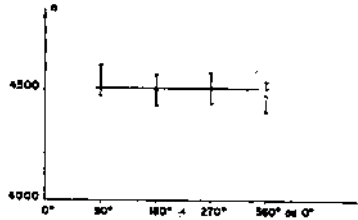


Fig. 4 — Test of uniformity of the counter walls. Abscissa in degrees of rotation of the counter about its axis, from an arbitrary position.

THE EXPERIMENTAL ARRANGEMENT

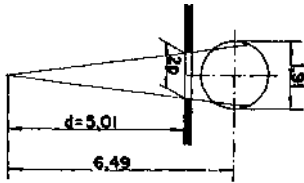


Fig. 5 — A drawing of the geometry definer used for cylindrical counters. Distances in centimeters.

In our measurements an Eck and Krebs counter was used; its walls were 30 mg/cm^2 thick (aluminum equivalent) whereas its diameter was of 1.90 cm and its length 10 cm. In order to decrease the counter background, all measurements reported in this paper were made with the counter placed inside a lead box whose walls were 1" thick and whose internal faces were lined with 5 mm of lucite.

UNIFORMITY OF THE COUNTER WALLS

Some measurements were carried out, in order to test whether or not the walls of the counter used in our experiments had walls of uniform thickness. In order to find out an effect of a possible lack of uniformity of the walls a $RaDEF$ point source was placed in a fixed position in relation to the counter and measurements were made rotating the counter through known angles; the results obtained are illustrated in fig. 4. One can conclude that the average thicknesses of the walls were constant, since the deviations found in our measurements are within the experimental errors.

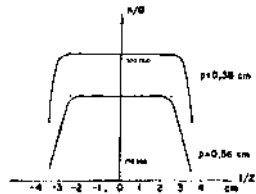


Fig. 6 — Values of n/G as a function of the length of the major side (t) of the definer. n is the actual number of counters and G is the calculated geometry.

EXPERIMENTAL DETERMINATION OF THE DIMENSIONS OF THE DEFINERS

Several measurements were made with different sizes of the definers: keeping a point radioactive source at equal distances from the edges of the definer, several countings were made with different definers (major side — l ; minor side — $2p$); those definers were made of brass and the thickness used were such that no beta ray could go through them.

The observed activities are given by

$$n = N.G$$

where AT is the number of particles emitted by the source by unit time (a *RaDEF* source was used) and G , the geometric efficiency corresponding to the different values of the definers used. G is given by

$$G = \frac{1}{4\pi} \int_{-p}^p \int_{-l/2}^{l/2} \frac{d}{(x^2 + y^2 + d^2)^{3/2}} \cdot dx dy \quad (2)$$

or explicitly

$$G = \frac{1}{\pi} \arctg \frac{pl}{2d(l^2/4 + p^2 + d^2)^{1/2}} \quad (3)$$

where d is the perpendicular distance between the source and the definer (fig.5).

TABLE 1
Values of n/G for different dimensions of the definers

	$l = 4$ cm.	$l = 5$ cm.	$l = 6$ cm.	$l = 7$ cm.
$p = 0.5$ cm.		282,200	264,300	244,100
$p = 0.45$ cm.		283,500	260,100	244,500
$p = 0.38$ cm.	303,200	303,500	299,400	286,300
$p = 0.36$ cm.		302,00	299,200	270,000

If the counter was equally sensitive for all the definers used n/G would be a constant. As one might expect, from the reasons discussed already, this

is not the case as can be inferred from the experimental values of n/G represented in table I, and figs. 6 and 7.

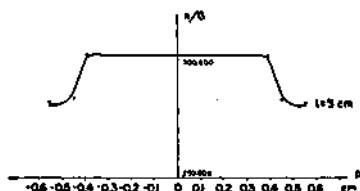


Fig. 7 — Values of n/G as a function of the length of the minor side (p) of the definer.

An analysis of these curves shows that the area of the counter limited by a definer whose major side (l is 5 cm and whose minor side ($2p$) is 0,76 cm. has a constant sensitivity (for $d = 5,01$ cm.).

CALCULATION OF G FOR ANY POSITION OF THE SOURCE

When the source is not placed symmetrically in relation to the definer, there is no known formula, that we know of, allowing a computation of the value of G . We have calculated the case in which the source is displaced by a distance h from the point P , (symmetrical position), remaining, nevertheless, equidistant from the major sides of the definer, fig. 8. The generalization from this case to any other is obvious.

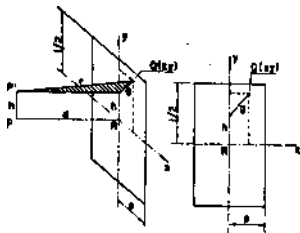


Fig. 8 — Position of the point source (P') whose geometrical efficiency is calculated in Section 4.

The geometric efficiency in this case is given by

$$G = \frac{d}{4\pi} \int_{-p}^p \int_{-l/2}^{l/2} \frac{dx dy}{[x^2 + (y - h)^2 + d^2]^{3/2}} =$$

$$= \frac{d}{4\pi} \int_{-p}^p dx \left[\frac{(l/2 - h)}{(x^2 + d^2)[x^2 + d^2 + (l/2 - h)^2]^{1/2}} + \right.$$

$$\left. + \frac{(l/2 + h)}{(x^2 + d^2)[x^2 + d^2 + (l/2 + h)^2]^{1/2}} \right]$$

or

$$G = G_1 + G_2$$

where

$$G_1 = \frac{1}{2\pi} \operatorname{arctg} \frac{(l/2 + h)p}{d[p^2 + d^2 + (l/2 - h)^2]^{1/2}};$$

$$G_2 = \frac{1}{2\pi} \operatorname{arctg} \frac{(l/2 + h)p}{d[p^2 + d^2 + (l/2 + h)^2]^{1/2}}.$$

A graph of G as a function of h is plotted in fig. 9 where we have also represented the experimental points obtained with a source of $UX_1 - UX_2$ in equilibrium; the curves were normalized for $h = 0$.

INDEPENDENT CHECK OF THE VALUE OF G FOR $h = 0$

In order to check the value of G obtained with formula (3) we have measured the absolute intensity of the source used in the experiments referred above with a mica end-window counter mounted inside a standard counting arrangement. The window of the counter was covered with a geometry definer (fig. 2) and a thickness of aluminum, 30 mg/cm^2 in order to have a situation similar to that of our cylindrical counter. As the geometry of this arrangement was precisely known, the number of disintegrations of the source per minute was found to be $387,000 \pm 5,900$. If the absolute number of disintegrations is calculated from the measurements with the cylindrical counter provided with a definer, as described above, with $G = 0.0123$, one obtains $392,000 \pm 5,610$ disintegrations

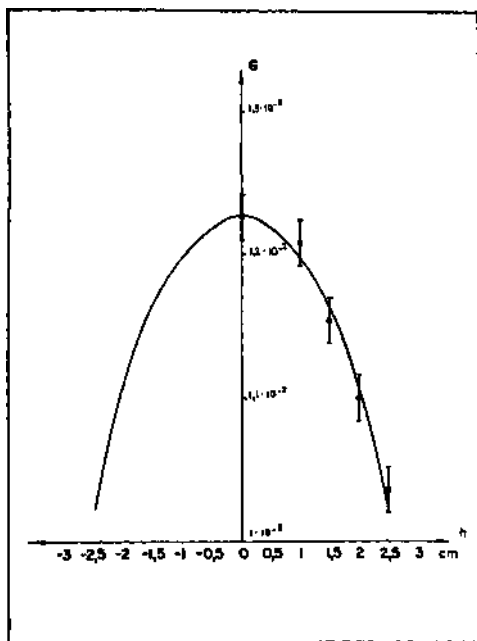


Fig. 9—A comparison between calculated geometry (solid line) and experimental efficiency (points) for a cylindrical counter as the distance h (see above section 4) is varied.

per minute. The agreement is very good, and it shows that the method proposed allows a precise determination even of absolute disintegration rates when the counter wall thickness is known.

SUMMARY

A method of limiting the useful area of a cylindrical counter by means of a geometry definer is proposed and formulas are given for the computation of the geometrical efficiency of the system for point sources in any position with respect to the counter. A comparison between calculated values and the experimental determinations for the geometry of the system is presented. The agreement is strikingly good, which leads the author to believe that cylindrical counters, provided with proper definers may be used quite satisfactory for absolute beta counting.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Prof. M. D. Souza Santos and E. Silva for helpful discussions, to Miss Otavia A. Borello for the help in carrying on the measurements and to Mr. Francisco B. Guidolin for the careful technical assistance during the experiments.

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DISCUSSION

GROSS: Referindo-me aquela figura do contador de particulas α , nao sera" quo so o contador se torna sensível ou insensível, depende da tensao? Aumentando a tensao serd talvez possivel eliminar praticamente a zona insensível? Eu nao sei naturalmente se fazendo isso nao se passa jd da zona de funcionamento do contador. De qualquer modo, fiearia grato com uma informacao justamente sobre a influencia da tensao sobre a zona insensível.

GOLDEMBERG: Em geral tem-se urn contador cujo "treshhold" estd definido, nao se tem alternativa; deve-se trabalhar com aquele "treshhold" e com os contadores, com os quais nos trabalhamos, foi necessdrio, qualquer que seja a tensao dentro do plateau, utilizar esses definidores. Caso nao se faya isso, a razao n/G varia quando se variam as dimensoes desse definidor,

VALLARTA: Me parece recordar que hace ya muchos anos Tuve presente calculos muy extensos sobre eficiencia geometrica de contadores y sistemas de contadores. Estos caleutos fueron publicados en SitzungsberLhte der Preussischen Akademie der Wissenschaften. No ha habido ocasim de volver sobre esos caleutos muy antiguos de Tuve?

GOLDEMBERG: Nao tive oportunidade de consultar os calculos de Tuve; a minim fonte de informacoes foi o trabalho de Norling no Archiv fur Mathematik und Physik, que e uma tese muito complela sobre esse assunto. E o unico eadculo a que Norhing faz referencia, e o de uma fonte pontual colocada simetricamente em relacilo ao contador,

DEUTSCH: I think the situation is, as you probably know, even more; complicated with end window counters because the plateau not only depends on the angle, but also on the counting rate, because the insulating window charge? up and this affects the counting volume.

GOLDEMBERG: A situacao e realmente niais complicada, mas com o "counting rate" que n6s utilizamos, a existencia de uma regio sensível era bem acentuada.

A New Method of Obtaining High Tension for Geiger-Tubes in Light-Weight Equipment

G. HEPP

Centro Brasileiro de Pesquisas Fisicas, Rio de Janeiro, D.F., Brasil.

A circuit was described by SIMPSON (1944), in which the dead-time of a Geiger-Muller-counter is reduced by inverting the tension of the tube immediately after the beginning of a discharge. As a result, the ions which have been formed during the discharge are returned to the central wire and, if the voltage is inverted quickly enough, practically none of them will reach the cathode. The electrons, however, which during the time that the tension is inverted are liberated at the anode by ion-impact and photo-effect, travel to the cathode, which is now momentarily positive, and their number is increased by gas-multiplication. The mean current through the tube is therefore negative; the tube delivers charge to the high-tension source. The energy for this process comes from the amplifier, which provides the high-tension pulse used for inverting the voltage on the tube (fig. 1).

As the source now has to provide voltage and not energy, even a Zamboni-pile, which can be considered as a battery with an extremely high internal resistance, can be used.

Also a charged condenser might be used as high-tension source. The negative current through the Geiger-tube increases the charge of the condenser until equilibrium is reached with the leakage-current of condenser and tube.

Another circuit, which also has the property of delivering charge to a condenser, can be made by putting a diode in parallel with the Geiger-tube. (fig. 2a). As a source of high-tension pulses (in the circuit indicated by a circle surrounding a pulse), a "blocking-oscillator" is used, in which the pulse-transformer has a high-tension winding and which is triggered by pulses from the Geiger-tube. As in this case the voltage on the condenser is equal to the amplitude of the pulse, no reduction of the dead-time can be expected and the circuit of fig. 2b may be used as well, in which the voltage is inverted on the diode only.

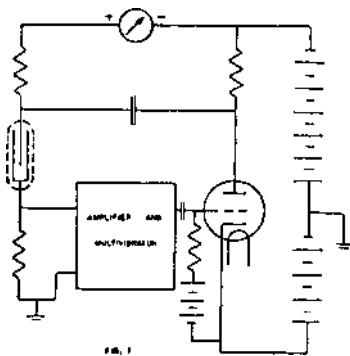


Fig.

The time-constant of the condenser, which is the product of the capacity C and the leakage-resistance R must be sufficiently big, so that the tension does not fall below some critical value E_n when the interval between two consecutive

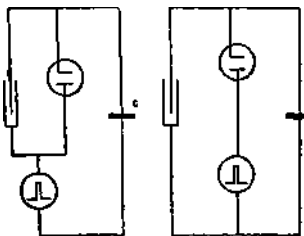


Fig. 2

heur, $RC = (300$ and $E_c/E_n = 0.9$ the chance for the voltage to come down to E_c is in the order of 10^{-6}

If, due to limitation of the current in the blocking-oscillator one pulse is not sufficient to bring the voltage of the condenser back to E_n , it can be shown that for the numerical values given in the example, the result will be substantially the same, as long as only a few pulses are sufficient to rise the tension of the condenser to E_n , even when it came down to E_c . If this is not the case and also for lower values of aRC , calculation becomes complicated. STUMPERS (1050) has studied this problem in the case that there is no limitation of the voltage of the condenser by the pulse-amplitude, assuming an equal charge delivered after every pulse.

A simple arrangement of Geiger-tube and blocking-oscillator is shown in fig. 3. A switch has been provided, with which the negative tension on the grid of the blocking-oscillator can be removed. Without this tension the oscillator is free to oscillate without being triggered by the Geiger-tube-pulses. During this oscillation the condenser is charged and afterwards the charge is maintained by the pulses of the Geiger-tube.

As the requirements for isolation of the connection between Geiger-tube-cathode, diode and condenser C are rather strict, one might consider for some applications to combine these components to one single unit.

pulses happens to be very large.

The probability, that in the time T the voltage of the condenser comes down from its normal value E_n to E_c , is, in the case that the internal resistance of pulse-oscillator and diode can be neglected, approximately

$$aT \left(\frac{E_c}{E_n} \right)^{aRC}$$

in which a is the mean number of pulses per second. For $a = 1/3$, $T = 1$

the chance for the voltage to come down

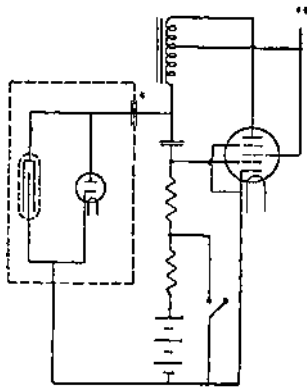


Fig. 3

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HIGH TENSION FOR GEIGER TUBES

DISCUSSION

DEUTSCH: HOW strict are the requirements for isolation?

HEPP: This depends on the current which the blocking-oscillator can give during the pulse. If the mean number of background-pulses per second is a_0 , the tube-current is I , the duration of the multivibrator-pulse is T , the voltage step-up ratio of the transformer is n , the mean charge given to the condenser per second is

$$\frac{a_0 I T}{n}$$

This must be equal to the charge lost by leakage, as the consumption of the Geiger-tube in this case can be neglected. The charge leaking away per second is:

$$\frac{E_n}{R}$$

This gives

$$R = \frac{n E_n}{a_0 I T}$$

For $a_0 = 1/3$, $I = 2 \text{ mA}$, $E_n = 360 \text{ Volts}$, $T = 10^{-4} \text{ sec.}$ and $n = 10$, which are practical values for a very small instrument, R becomes 5×10^{10} .

Although this is high value it is certainly not prohibitive. Modern instruments with ion-chambers and electrometer-tubes are built, in which a much higher isolation is required.

Of course, especially here in Rio, we must take measures to prevent moisture from coming in. But this can easily be done.

RAB: Have you made such an apparatus?

HEPP: Yes and I can show it here to you. It is a portable radiation detector, which works with a filament battery of 1.5 Volt, which delivers 13 HI A during operation, and a 60 Volts battery, from which only a negligible current is taken, when no intense radiation is being measured. The battery has therefore a useful life which is practically equal to shelf-life.

A more detailed description of this apparatus will shortly appear. (Philips Technical Review).

Techniques of Construction of Geiger-Muller Counters in the Centro Brasileiro de Pesquisas Fisicas

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In this communication I will simply describe how we constructed Geiger-Muller Counters in groups of 10 counters each for our own purposes in cosmic ray research and training our people. At the end I will tell some observations on the properties of our counters and what we think could be the explanation of the effects we observed.

As we could not start as soon as we wanted because of many technical difficulties and as on the other hand we wanted to have soon a great number of counters, we could not realize much development work in the beginning.

So we started with the external cathode type described by MAZIC (194(i) and already well investigated from the point of view of behaviour and operation by AHON & GROSS (1950).

As anode we had at the center of the cylindrical tube a tungsten wire of 0,1 mm of diameter.

The cathode was formed by an aquadag layer on the outside wall of the glass tube. These counters generally had a filling of 10 mm of ethyl alcohol vapor and 90 mm of spectroscopically pure argon. In some counters we used as the cathode an internal layer of silver deposited by the well known chemical process of Rochelle salt. In others we produced a pure copper layer as cathode by evaporating copper from the central wire.

In contradiction to the general opinion that it is not necessary to apply extremely clean vacuum treatment, we found as I will point out later that specially for avoiding the photoelectric effects it was necessary to pay attention to all precautions generally employed in high vacuum technique.

The vacuum system we used is drawn schematically in figure n.º I. In the system we have six stopcocks greased with high vacuum grease, $T_1 \dots T_6$. An oil diffusion pump $B.D.$, of 67 liters per second is well protected against sudden air leaks.

Two vacuum gauges of the resistance type $V.P.$ I and $V.P.$ II were constructed in this laboratory using common 40 w 110 volt lamps as gauges. One of them, $V.P.$ I measured the fore pressure that is maintained by the 0 liters flask even when rotary pump is disconnected by stopcock T_1 ; the other gauge $V.P.$ II was intended to measure the pressure on the high vacuum side to be sure that the ionization gauge $V.I.$ would only be put into operation at a sufficient low pressure.

The system was constructed for a daily production of 20 to 30 counters and by connecting a second degassing furnace it would give an output of 40 to 50 a day, with 4 glassblowers, 4 auxiliary technicians and one man at the vacuum system, working 8 hours a day.

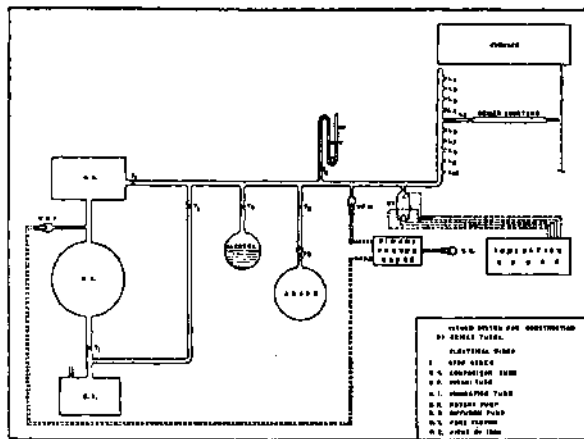


Fig. 1

Everything which does not have to be touched normally is well protected under the table; only the stopcocks and the mercury manometer are on the table.

We could keep the oil diffusion pump in operation even when connecting other groups of counters to the system, because the counters could be roughly pumped through a by-pass by the rotary pump.

To degas the central wire of the counters, we heated it by a number of short current flashes; the time intervals between flashes were adjusted in accordance with the decrease of the degassing pressure observed by our resistance type vacuum gauge. As we shall see later this heating treatment of the tungsten wire is one of the necessary actions to diminish the photoelectric effect and to

lengthen the life time of the counter.



Fig. 2 Photo of the Vacuum System for the construction of counters.

Heating in a hydrogen atmosphere would be even better, but because of technical difficulties we were not able to obtain it. At this point we measured a dynamic final vacuum of between 3 and 5×10^{-5} Torr (mm Hg) with an ionization gauge. Then we baked out the glass

envelopes of the counters by heating them, for about two hours at $\sim 300^{\circ}\text{C}$ with a new type of degassing furnace developed in this laboratory. Instead of the common resistance wire furnace that gives heat by conduction and convection to the tubes to be degassed, we used infra-red lamps of 250 Watts each. On figure 2 one can see this infra-red furnace with the indicating lamps at the right that indicate any lamp which has burned out.

This system of indicators is useful because of the 24 lamps in the bright furnace. This infra-red degassing furnace has all the advantages of the infra-red types. After switching on, the resistance type needs a long time to reach final temperature, sometimes one hour for this size, while the infra-red radiation delivers heat almost immediately, so that tubes reach the desired temperature after some few minutes.

To repair a resistance furnace it is often necessary to dismount it in order to find the burned out part of the wire, but here diagnosis and repair are easy.

Furthermore the infra-red type is more economical, because it is not necessary to heat up material that we don't want to heat. Another advantage is that it is easily constructed.

Of course the temperature cannot be so uniform as that reached by the closed resistance wire furnace. Figure 3 shows a graph of the temperature distribution measured by a thermocouple in close touch with the inner wall of glass tubes that were of the same diameter as the counters. Of course we made sure that we measured the real temperature of the glass and not some other temperature.

The abscissa in the figure gives the distance from the left end of the furnace and the ordinate the temperature. The temperature varies from 280 to 320°C . To reach a higher temperature we have to blacken the tubes outside which will cause an increase of about 50°C .

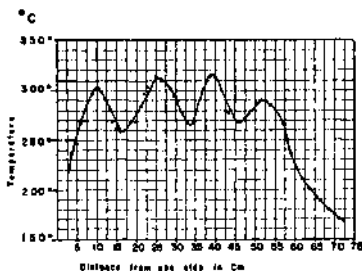


FIG. 3 TEMPERATURE DISTRIBUTION IN THE INFRA-RED DEGASSING FURNACE

But 300°C is just the right temperature for the alkali-lime glass; exceeding 350°C the glass begins to decompose at the surface and delivers gas so that the degassing process practically never comes to an end. On the other hand lower temperatures will not be sufficient for a quick degassing. After two hours' degassing the 10 Geiger tubes, we once more heated the wire electrically for one minute; then we heated the spots where the counters were to be sealed off with the gas torch up to the temperature where the glass begins to become soft, i.e. to suck in. Then we measured after cooling down to room temperature with the ionization gauge a dynamic final pressure of about 5 to 8×10^{-6} Torr which must be the vapor pressure of the stopcock grease. We could not use the very low vapor pressure grease in our stopcocks because of its low viscosity in our climate.

The pumping system was disconnected by the main stopcock, 2\, and the tubes filled with 10 Torr of alcohol vapor and 90 Torr of argon. More than one hour was allowed for diffusion equilibrium to be established before sealing off.

The external surface was cleaned with alcohol and dried with a stream of hot air: the counters were then coated with a conducting layer of aquadag, the ends were cleaned and dried once more with hot air; a cap of copper was stuck to one end by a specially prepared cement (lead oxide and glycerinc)and soft soldered to the wire; the other end of the wire was cut. Both ends were cleaned and hot air dried once again and dipped in insulating black varnish.

As the electrical conditions in counters with internal cathodes are less complicated than those with external cathodes, we tried to construct some with an inner layer, some by silvering the cathode by the well known chemical reduction

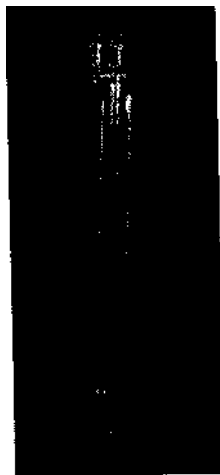


Fig. 4. Photo of counter during construction: copper V's at the central wire.

process of Rochelle salts, and some by evaporating copper. On the other hand, it is more difficult and often impossible to degass a massive metal cylinder, so we produced the copper layer by evaporating thin copper strips under vacuum. Under these conditions we could get a practically gas free metal cathode. Fig. 4 shows a counter of this type before evaporation. The strips were cut into small pieces and formed to a F-shape and then clamped onto the central wire. After the vacuum treatment already described the wire was heated sufficiently hot to evaporate the copper Vs. The copper vapor condensed on the walls and formed a uniform layer for the inner cathode. As it is a bit toilsome to clamp the copper V's onto the thin tungsten wire we tried another method using a small gastorch flame, with which we melted the copper strips while they were in contact with the wire, parts of the strips stuck to the wire and were cut by the flame and then melted to form a small drop stuck tightly on the wire. Because of this treatment the copper drops were oxidized, by heating them in a hydrogen atmosphere they can be reduced to clean copper before evaporation.

BEHAVIOUR AND OPERATION OF THE COUNTERS

As to be expected the length of the plateau shown by the graphs of voltage (potential difference between the wire and the external cathode) against number N of pulses per unit time recorded by the counter depends on the number of particles reaching the counter, from a radioactive source; in fact the plateau becomes longer the nearer the source is placed to the counter. In fig. 5 we see two curves, one of them with the source at a distance of 10 meters, i.e. practically background, the other at a distance of 0,50 meters from the same counter. This

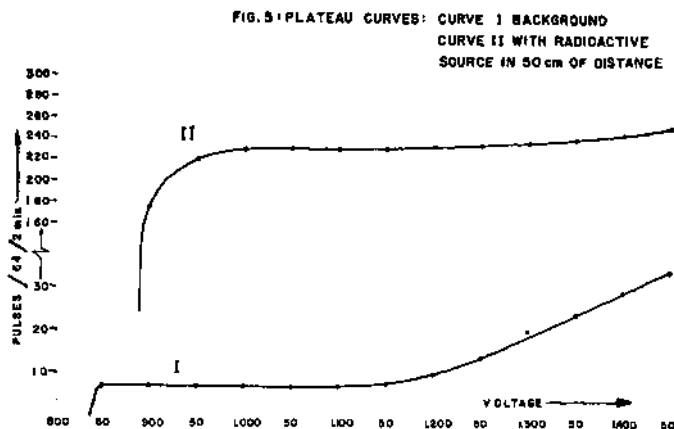


Fig. 5

fact was already pointed out by AROK & GROSS (1950) and was confirmed by our experiments.

In our graphs we ought not to plot the number of pulses against the voltage at the external cathode, but we should consider the voltage drop U within the glass wall, which is roughly proportional to the resistance and to the number of pulses, and is caused by accumulation of charge at the walls:

$$U \sim R.N$$

The voltage that should really be plotted should be the difference $V-U$. The pulse height in general decreases with high counting rates. Using hard glass, one could get "marvelous" counters with very long plateaux, as was recently published by YASIN and collaborators (1951); they were speaking of plateaux extending from 1.200 to more than 3.000 volts with a slope less than 0.4%/100 volts, but they don't explain anything in regard to the dependence of these large plateaux on the counting rate. The counters with internal cathode did not show this effect; in general their plateaux were shorter. But an appreciable advantage of the external type counters is that there will hardly occur continuous discharge at higher over-voltage as might happen with the other cathode type and which might destroy them by the discharge, because the effective overvoltage will be lowered by the accumulation of positive charge on the inside surface of the glass wall.

SOME OBSERVATIONS ON THE PHOTOSENSITIVITY OF THE COUNTERS

It is a well known fact that almost all Geiger Muller counters are more or less sensitive to light. The only way to suppress this is to protect them against light, for example by painting them with a black opaque paint, but sometimes even this will not be sufficient.

From the 600 counters that were constructed in the Centro Brasileiro de Pesquisas Físicas during the last eight months we can already conclude that

only those had little photosensitivity or were not photosensitive which were treated in the very cautious way already described.

All counters that were considered bad, i.e. that had plateaux less than 150 volts and slopes steeper than 5% 100 volts were photosensitive.

From this we must conclude that the photoelectric effect must have something to do with the gases that remained in the counters perhaps in the gas volume or in the solid material, glass or metal parts.

The question remains: is it the gas (oxygen, water vapor or other gases or vapors) in the gas volume or the absorbed gas or both?

We must distinguish between two forms of counters which are photosensitive:

1. those which show the effect from the very beginning of their operation and
2. those which become photosensitive only after some time.

Counters of the first group were not so well vacuum treated as those of the second group.

As NEUKERT & GEERK (1950) found, and later MACKNIGHT & CHASSON (1951), the "time" for the counters to become photosensitive is about 10^6 to 10^8 pulses.

It is easy to calculate the number of alcohol molecules decomposed by one pulse. The pulse height, may be 100 volts, and it is equal to

$$dV = \frac{Q_0}{C} = \frac{n \cdot e}{C} = \frac{n \times 1.6 \times 10^{-19}}{C}$$

Assuming $C = 10^{-11}$ F which corresponds to our counter size we get for the number of electrons arriving at the cathode

$$n \sim 10^{10}$$

The discharge being quenched by the alcohol vapor, an equal number of alcohol molecules must be decomposed, this means that after 10^7 pulses, 10^{17} oxygen or hydrogen or other diatomic gases must have been created; but this number is equal to 1% of impurity in the counters of the first group with photosensitivity right from the beginning or approximately it is the number of molecules that occupy a monolayer on the wall.

The question whether the photoelectric effect occurs at the surface of the solid parts of the counters or in the gas volume has no sense in our opinion, because both the superficial and the volume effects happen. In the counters of the first group there will be the volume effect in majority at the beginning and the surfaces of the counters will not yet have become "activated" by adsorbed

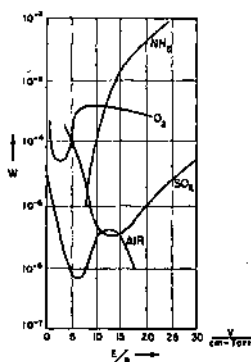


FIG. 6: PROBABILITY FOR ELECTRON ATTACHMENT TO NEUTRAL MOLECULES.

molecules. So we must distinguish between counters that are preferentially volume photosensitive and those that are preferentially surface photosensitive. All counters become after some time more surface photosensitive. This is proved by NEUERT & GEERK (1950), ARON A GROSS (1950) and by our own experiments. NEUERT & GEERK observed photosensitivity of their pure NH_3 counters after 10^6 to 10^8 pulses, then they pumped the gases out and filled the counters with pure argon and alcohol vapor; the photoelectric effect continued. Immediately after the first 10.000 pulses we did the same with our counters of the first group, i.e. counters that showed photosensitivity right from the beginning. These counters after having been refilled with the new argon-alcohol mixture became much less photosensitive (only 10% of that before); but those of the second group that became photosensitive after some time kept almost unchanged their photosensitivity after renewing the argon-alcohol filling. Counters of the second group being degassed during pumping lost their photosensitivity after being refilled with pure argon-alcohol.

ATTEMPT OF EXPLANATION OF THE PHOTOSENSITIVITY

What is the mechanism of the volume and the surface photoelectric effect in the counters?

Obviously the reason for the volume⁴ effect is that the gas or vapor filling was not pure enough, mainly oxygen or water vapor might be the contaminations. And there is no doubt that the surface effect is involved with the adsorbed products of the decomposition process of the quenching vapor and the alkali-lime glass components.

Counters that were not well baked out showed surface photosensitivity from the very beginning; but these we will not consider.

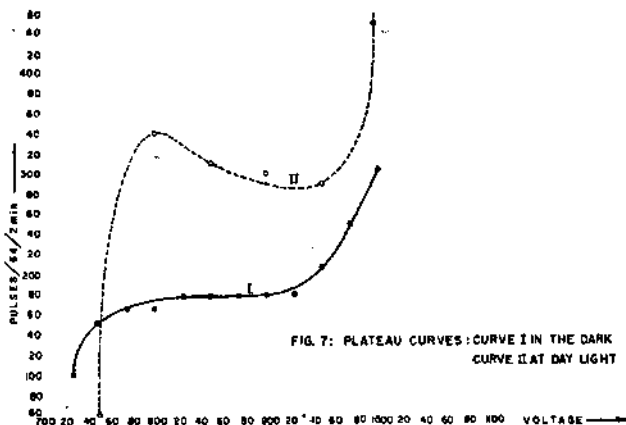
VOLUME PHOTOSENSITIVITY

The light that can pass through the glass must have a wave length longer than 3500 Å. The photons entering the counter from outside can have an energy only smaller than 3.5 eV. But there is not any gas atom that can be ionized by such a small energy, and collisions of second kind, i.e. by gas atoms excited to a metastable state, are unlikely. The most likely explanation of the volume effect seems to us that which takes into account the formation of negative ions promoted by photons. It is known that the electron attachment to oxygen is much higher than that to other gases.

The probability for the formation of negative oxygen ions lies between 10^{-4} and 10^{-3} , as figure n.° 6 (EBERT, 1951) shows, but this is sufficiently high in the case of our counters, because the mean path is of the order of 10^{-3} cm for electrons, so that an electron passing the counter will have more than 10 collisions with the gas and vapor molecules. As FRANCK & GROTIAN (1921) pointed out, the probability for electron attachment to oxygen molecules lies some ten powers higher for molecules that are already excited than for molecules in the ground state.

As it is known on the other hand oxygen can get into an excited state by absorbing an energy of only 1.6 eV (= 7.600 Å) in passing an electron of the

ground state $^3\Sigma$ to $^1\Sigma$ (FRERICHS, 1930), such an excitation might occur in the counters. The oxygen molecule excited to this state has a life time of 7 seconds (CHILDS & MCKE, 1931) as it is a metastable state. And thus the probability of the formation of negative oxygen ions seems to be rather great. Some counters with volume photoeffect show a maximum in the counting rate-voltage-



curve before reaching the plateau, as is to be seen in figure 7. Perhaps this might be explained as well by observing the probability curve for the formation of negative ions (fig. 6). Here is known to be a slight maximum of the curve for oxygen and air and this might give reason for suspecting the existence of resonance-phenomena. Whether there might be relation between the maximum of this curve and the maximum of our curves in fig. 7, or not, cannot be decided from our experiments. But the maximum in the probability curve makes it seem likely that the formation of negative ions may occur after exceeding the voltage maximum for the probability of electron attachment. At this point I should like to emphasize how necessary it is to degas the tungsten wire, as it is well known that tungsten easily adsorbs oxygen; electrons arriving at the central wire with comparatively high energy may release oxygen and this will give more possibilities for the photoeffect as described. Experiments bore this out.

SURFACE PHOTOSENSITIVITY

The surface photoelectric effect is somewhat easier to understand. By the approximately 10^{10} electron collisions during each pulse the alcohol vapor decomposes into hydrogen or oxygen and hydrocarbons. These latter will deposit on the walls and, thus, form a good adsorption surface for the hydrogen and oxygen, so that the work-function of the wall will considerably be decreased. As ABENDROTH (1933) pointed out, the photosensitivity of any material will be increased by several powers of ten, when water vapor had been adsorbed by the material. The maximum of this photoelectricity happens when we reach a monolayer of molecules, i.e. after about 10^7 pulses.

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DISCUSSION

GROSS: I would like to ask whether this figure of 10^9 or 10^8 pulses for the counter to become photo-sensitive refers to counters with internal or external electrodes.

II. SCHWARZ: It refers to counters of the external type.

VALLARTA: Have you made neutron counters?

H. SCHWARZ: Not yet.

RABI: DO you regenerate the counters after they become photosensitive? Can you do something later on to decrease the photosensitivity?

H. SCHWARZ: Yes, we can.

SANTOS: In self-quenching counters, the life of the counter depends on the destruction of the organic vapor which is put inside; now, as the tube life is shortened by impurities which are always present in the quenching vapor and by impurities produced during the discharge, we have found, in our laboratory, even with inside cylinder counters, that it is not very useful to try to make a very high degree of vacuum and to de-gas the tube. We have found that if we have a vacuum of, e.g., 0.01 mm. and then flush the counter with some argon to get rid of oxygen and other electro-negative atoms which might be left inside, the counter behaves as well as it would when a very careful out-gasing is made. I would like to know what, is your experience about that.

II. SCHWARZ: I think there are two answers: first, if one does not treat the counters very well, as it is common in high vacuum techniques, one will shorten the lifetime and then one has, as we observed, always very strong photo-sensitive counters. And if you do not mind the photosensitivity I think it will not be necessary to do such good vacuum techniques. But, on the other hand, you will not have such a long lifetime as with very well prepared counters.

SANTOS: Thank you very much for the explanation; actually, we paint the counters in order to get rid of photosensitivity. Another question I wanted to ask is this: have you ever measured the absolute efficiency of these counters and their dead time?

H. SCHWARZ: We did this in our laboratory and we found that the efficiency of our counters is better than 98% for cosmic rays. The measurement were made by Dr. Salmeron at the Centro.

SANTOS: Dr. Pieroni in our laboratory has found that these counters with an external cathode are very sensitive to change of temperature; actually if you put a standard source at a given distance, in order to have a constant counting rate, the counting rate falls off appreciably for temperature changes of a few degrees. Do you have any experience about that?

II. SCHWARZ: We do not find any dependence on the temperature as far as the ohmic resistance depending on temperature will not change the ends of the plateau, but I suppose the reason is the one I told at the beginning, that to explore something we must have very clean and defined properties of the counters, because if you heat the; counters it will then change the superficial condition of the inside of the counters. That might be the influence of the temperature at your counters.

GROSS: I would like to make some remarks about the temperature effect. I believe that it is entirely due to the ohmic resistance of the glass and I hope to be able to show this in my communication. The second point refers to a remark by Maze who did not heat his counters. He apparently thought that heating was prejudicial because he found that if the

counter is heated, it becomes more photosensitive than when it is not heated. I have no experience in this respect, since we heat the counters. I wonder if you could give some idea about that.

II. SCHWAHZ: I did not have time enough to go into details. I distinguish between two types of photosensitivity: volume photosensitivity and superficial photosensitivity. I think the volume photosensitivity has something to do with the oxygen that remained in the counter or that came out of the internal wire, because it is well known that tungsten wire absorbs extremely oxygen. I found that the probability of electron attachment is stimulated by photons as already found by Franck and Ootian in 1921. So I think that there might be some dependency.

GROSS: But do you think the cross-section of the absorption of photons in the gas with a pressure of 8-10 cm is really sufficient to give photoeffect of the magnitude sometimes observed?

II. SCHWAKZ: Ideal photo effect not because the energy of the light that will come through the glass must be smaller than 3.500 Å, that is, equal to 3.5 eV, and there is no gas atom that could be ionised by such a low energy. The photons must give reason for an excited state. There is an excited state for oxygen only with 1.6 volts, and I suppose that it has something to do with this excited state. Franck and Grotian found that the probability of electron attachment increases more than several powers of ten. At an oxygen molecule the probability for electron attachment is about 10^{-10} at about the field strength of our case and we have a free path of electrons in our counters about 10^{-4} cm. Then; are therefore 10^6 collisions. If Franck and Grotian are right, then when being in an excited state with an increase of the probability of electron attachment by several powers of 10, we will have much more possibility of electron attachment, let us assume an increase of 3 powers of ten, then we already come to one electron that will be caught by each oxygen molecule that was collided. As to the other question, that heating the counters might be prejudicial in regard to photosensitivity, I would like to refer to an article by Abendroth in which he points out that there is a maximum of photosensitivity of platinum covered with a monolayer of H_2O -vapour and that the photosensitivity decreases with larger adsorbed layers, of glass sometimes may have 30-50 molecular layers.

SALMERON: Concerning the influence of temperature in our counters, we have some of these counters working in La Paz and supporting a variation of temperature of more than 20° during the day and working with the cosmic rays, not with radioactive sources, the rate is constant. I could not observe any variations with a rate of about 100 per hour.

SANTOS: With the internal cathodes filled with alcohol and argon we have found that not only the threshold changes with the temperature, but also the background and the sizes of the plateau as well; this is known to be true for all good counters.

PIKRONI: Experimentei esses contadores, baixando a temperatura para não elevá-la a, um valor a que Maze acha que danificaria o contador. Baixando a temperatura, o que se observa é exatamente esse fato: abaixamento do threshold, encurtamento do plateau, que ao mesmo tempo se torna mais inclinado, isto é, deixa de ser um plateau. Se estamos trabalhando com uma tensão dada no contador, variações de temperatura alteram completamente os "counting rates". Além, com diferentes tipos de contadores observamos esse fato. Ao mesmo tempo, observamos também o número de espúrias que os contadores Maze são em percentagem muito grande. O Dr. Guimaraes mesmo, observou esse fato: há cerca de 20 por cento de espúrias nesse tipo de contador ou mais; além disso tem um tempo morto muito grande.

Counters and Ionisation Chambers with External Electrodes*

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In the Maze counter the metal cathode of the ordinary selfquenching Geiger-Mueller counter is substituted by a layer of Aquadag applied externally to the glass envelope. Alkaline glass of low specific resistance ($7,5 \times 10^8$ Megohm/cm) is used. Such counters have been used extensively for cosmic

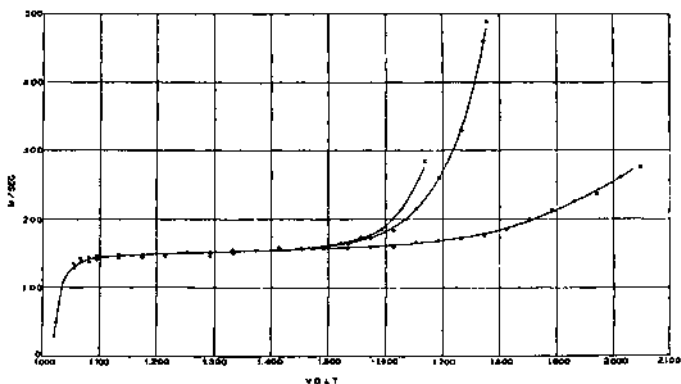


Fig. 1 - Intensity dependence of characteristics. Curve a - background. Curve b - $11,5 \times$ background. Curve c - internal characteristic corresponding to zero flux. (All curves are made to coincidence at low voltages)

ray work in this country and in France. The present paper relates a systematic study of their properties. The main results are:

a) New counters have plateau of about 300 — 500 V with a slope less than 5%. After 2×10^8 counts under a voltage of 200 V above threshold the plateau is still half this value.

b) The capacitance of the glass wall is approximately 10^{-11} F/cm^2 . This is sufficiently high for single pulses to pass through undistorted.

c) The glass acts like a leaky capacitor of time constant 0,5 sec. The charge which it acquires during functioning can be thought of as producing a counter e.m.f. that increases with the mean pulse rate and the value of the applied

voltage. This makes the characteristic intensity dependent. Pulse rates like those encountered with cosmic rays are so low, that no trouble is experienced by this fact.

d) The counter e.m.f. is given by $n \times q(U_i) \times R$ where n is the number of pulses/sec, $q(U_i)$ the pulse size at voltage U_i and R the resistance of the glass. Thus, when U_a is the applied voltage, the internal voltage U_i is given by the equation

$$U_i = U_a - n q(U_i) R$$

This equation explains the intensity dependence of the characteristic and the temperature effect. Indeed, since R is strongly temperature dependent, an increase of temperature reduces and a decrease of temperature increases the counter e.m.f. If the operation voltage of the counter was not sufficiently high above threshold, a strong temperature decrease may produce erratic results.

e) Behaviour and characteristics of the Maze counter can be precisely duplicated by a circuit constituted of a counter with internal electrodes connected in series with a capacitor shunted by a resistor.

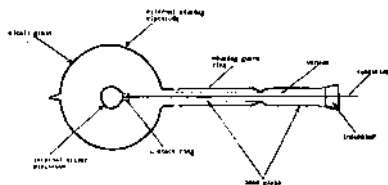


Fig. 2 — Ionization chamber

f) The counter withstands without damage application of high excess voltages, no arc discharge being able to form in the gas.

g) Quenching properties are excellent. The counter can be shown to quench normally in a shorted condition.

h) It is possible to substitute the wire anode by a glass capillary filled with a conducting liquid. Thus an "all glass" counter is made.

The same principle can be applied to the construction of an "all glass" ionization chamber. Such a chamber is constituted by two concentric glass bulbs of alkali glass. The internal sphere carries a silver coating on its inside¹, the external sphere an aquadag coating on its outside. In consequence of the smallness of the currents to be measured, special care must be taken, to avoid parasitic surface currents along the glass. For this purpose a) a guard-ring system is used; b) the glass parts connecting the two bulbs are made of high resistive pyrex or lead glass, and c) the construction of the connecting parts is such as to exclude the possibility of formation of a humidity layer at the free glass surfaces.

With a half-liter filled with argon under atmospheric pressure currents of 10^{-14} A and higher can be measured. This gives a lower limit approximately 4×10^6 roentgen/sec.

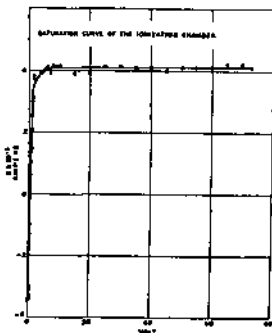


Fig. 3 — Saturation curve of ionization chamber. By γ rays giving 3,000 pairs of ions/cm²/sec in air of N.T.P.

DISCUSSION

VALLARTA: I do not know if it is worthwhile to go back to the question of temperature effects on these counters which was discussed a moment ago, but just the same I would like to ask you what is your experience regarding temperature effects.

GROSS: WE have studied the temperature upwards and not downwards. We have made measurements at about 20° and Dr. Aron has studied the influence of an increase in temperature. Nothing happened, and the characteristics did not deteriorate. Now, if you reduce the temperature, you increase the resistance, and then you will find that the threshold will go up and at the same time you will reach probably sooner, a zone where the characteristics are intensity-dependent. Now, it depends, of course, on the resistance of the glass, whether you will find a sufficiently wide zone, where you can work. I would like to point out that since the plateau is about 400 volts one has a wide choice.

COSTA RIBEIRO: I would like to remark that the difference of behavior of those counters according to temperature could be explained by the fact that the resistance of the glass and the temperature coefficient of this resistance, depends strongly on the nature of the glass employed. This would explain the difference of behavior of the counters of Maze typó that have been employed by the group of Sao Paulo and those that have been built here.

GROSS: I have an experience with these counters here, the resistance of which is shown on the table. The behavior depends also on the voltage you work with. Some people prefer not to work too high, just above the threshold, because that increases the life of the counter. If you do not work high enough the threshold, for instance in certain experiments where you need a very high life, then you will find that there is a very strong temperature dependence. But, if you are free to choose your working point high enough you probably would not.

Photoeffect of Geiger-Counters with External Electrode*

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"Geiger" counters, especially counters with an external cathode, consisting of a layer of graphite applied to the glass envelope ("Maze counters") show a great sensitivity to visible light. This sensitivity is due to photoelectrons produced by photons entering into the space between cathode and anode. We observed the same sensitivity with the all-glass-counters, constructed by Dr.

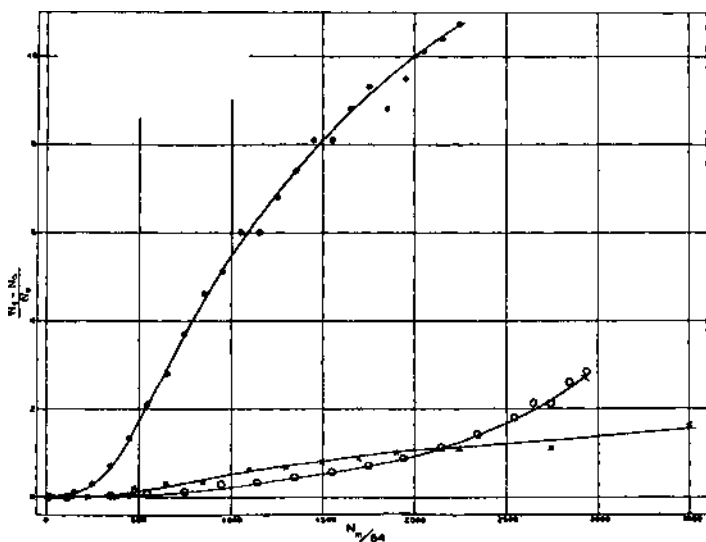


Fig. 1 Increase of photosensitivity with number of pulses for 3 counters

Gross, which have no internal metal-parts. This justifies the conclusion, that the wire anode of the "Maze" counters is not the source of the photoelectrons.

We have made a series of experiments with "Maze" counters, filled with 90 mm Argon and 10 mm vapor of ethyl-alcohol, mounted in a distance of

* Supported by the Conselho Nacional de Pesquisas.

9 cm from a 20 W fluorescent lamp parallel to this lamp in a closed wooden box.

Our researches are not yet finished. The main results are:

- 1) No new counter is photoelectric.
- 2) The counters become photoelectric only after a certain minimum number of discharges has occurred.
- 3) The photosensitivity increases with the number of discharges, which have passed through the counter. In many counters the increase is stronger than linear. Eventually the pulse rate may increase more than 100 fold.
- 4) The minimum number of discharges necessary for the photoeffect to appear and the increase of the sensitivity are very different for different counters constructed in the same manner. Fig. 1 shows the increase of photosensitivity with the total number N_m of pulses which are recorded, giving $(N_1 - N_0)/N_0$ where N_0 is the pulse rate without and N_1 the pulse rate with illumination in function of N_m . The starting point and the inclination are very different for different counters. All counters however are at first not photoelectric.

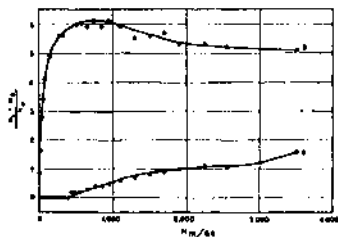
TABLE I

TEMPERATURE °C	TIME OF HEATING	$\frac{N_1 - N_0}{N_0}$	EXTENSION OF THE PLATEAU Volt	INCLINATION OF THE PLATEAU % p. 100 V
—	—	134	350	4,3
100.....	1 hour	134	380	3,2
150.....	5 min.	29	350	3,2
150.....	1 hour	29	400	3,7
150.....	2 hours	0,6	270	7,3

5) With a counter of high photosensitivity and a voltage not sufficiently high above the threshold voltage it may happen, that the recorded pulse rate under illumination decreases and the discharges finally cease completely. One could be led to believe, that the pulse rate is diminished. But one must consider, that the glass-wall acts as a resistor as well as a capacitor; with the increase of the pulse rate produced by the light, the difference of potential across the glass may increase so much, that the voltage of the internal surface (the real cathode) falls below the threshold voltage. A sufficient increase of the external voltage shows the real cause of this phenomenon.

6) When the voltage is removed, the photosensitivity decreases slowly. A counter, which has not been used for some weeks or months, when put into operation again, shows at first a much smaller sensitivity than before. But already after quite a small number of discharges it regains the former photosensitivity that still increases by more discharges. A new counter would have needed at least some thousand discharges to become photoelectric. Fig. 2 shows

this effect. It gives the photoeffect of a new counter (curve *a*) and that of the same counter obtained some time later and after severe electrical treatment (curve *b*). Curve *b* rises much faster than curve *a*. (The maximum is caused by a secondary effect).



[Fig. 2. Photoeffect—*a*) of a new counter ;
b) of the same counter after severe
 electrical treatment.

8) When the gas filling is removed and the counter is degassed some days under vacuum without heating and then filled again, the photosensitivity always disappears completely and the counter behaves like a new one. This is shown in fig. 4.

9) The photosensitivity can also be diminished by heating. Sufficiently high and long lasting heating eventually reduces the photosensitivity to 0. Together with the reduction of the photosensitivity by heat we observed a deterioration of the characteristic of the counter, probably as a consequence of release of gases from the glass surface by the heat. These gases form negative ions and so cause double pulses. Table I shows the behaviour of a counter which has become very photoelectric by the passage of 1.500.000 discharges and then was heated to different temperatures and for different times. A heating to 100°C during 1 hour had not yet any influence neither on the photosensitivity nor on the characteristic. A heating to 150°C one time during 5 minutes and the other time during 1 hour reduced the photosensitivity respectively to $3/5$ and $1/5$, also without any essential modification of the characteristic. A heating to 150°C during 2 hours reduced the photosensitivity almost to 0, but modified the characteristic. Heating to higher temperatures can reduce the photosensitivity still further but modifies the characteristic so much, that a plateau cannot be observed any more. The reduction of the photosensitivity obtained by heat does not last long. The photosensitivity increases to a multiple of its reduced value within hours after cooling without electrical treatment. It is interesting to note, that the dark characteristic of a counter which has deteriorated in consequence of heating, becomes normal again after a sufficiently great number of pulses has passed through. It looks as if the gases are adsorbed again.

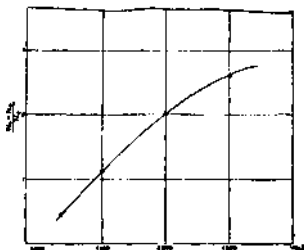


Fig. 3. Photoeffect in function of
 Voltage.

These are the facts known to us. Our researches are chiefly a collection of material and as mentioned before, are not yet finished. The explanations we can give so far are not very satisfactory. But it will be better not to give too many explanations before knowing all facts. Even with metastable atoms an interpretation is difficult on account of their short

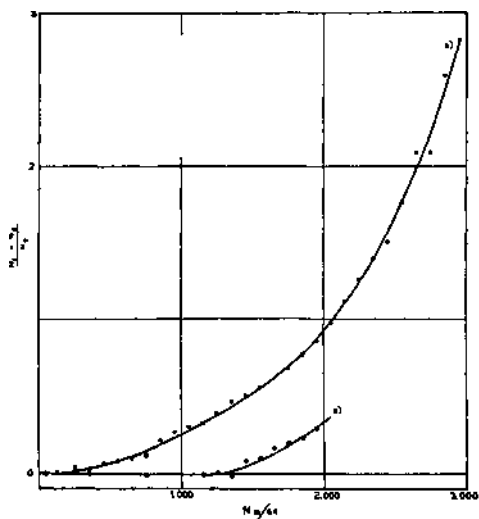


Fig. 4. Effect of new filling. Curve *a* shows photoeffect of a new counter. Curve *b* shows photoeffect of the same counter after new filling.

photoelectric, disappears completely with a careful substitution of the gas filling. It seems therefore, that gas atoms modified by the discharge are the sources of the photoelectrons.

b) Since the photosensitivity disappears also with a sufficient heating of the glass, we assume, that only a thin layer of gas atoms adsorbed by the glass-wall is effective and that only for these work-function is small enough to allow photoemission under visible light. The return of the sensitivity some hours after the cooling can be explained by a new adsorption of the released modified gas atoms after the cooling. (With a careful new gas filling a return of the sensitivity is not observed).

c) The discharge-and voltage-dependence of the effect can be understood in this way.

d) The change of the characteristic, which is observed, when the counter is heated, can be considered as an independent phenomenon, caused by negative ions of adsorbed gases, which together with the adsorbed argon atoms are released from the glass-wall.

life. The expression: "gas atoms modified by the discharge" means gas atoms whose work-function is diminished sufficiently by the discharges to allow photo-emission under visible light after their adsorption by the glass wall, but with a much longer life, than the known metastable atoms. Of course the existence of such atoms is hypothetical.

In conclusion:

a) The photosensitivity of a new counter is 0. That of a counter, which has already become

Discussion of Experimental Results on the Scattering of Protons by Protons

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Previous results on the scattering of protons by protons have satisfactorily established the *S*-wave phase shift over the energy range available from electrostatic generators. These results gave no conclusive evidence for higher phase shifts but did not rule out *P* or *D*-wave shifts which might be expected from reasonable assumptions in regard to the nature of the proton-proton force. Results obtained with conventional cyclotrons did not alter this picture. During the past few years extensive measurements have been made up to energies of a few hundred Mev. These data have proved difficult to explain theoretically and the nature and detailed characteristics of the interacting force are still poorly understood.

Data of higher accuracy in the energy region of electrostatic generators appeared to offer promise for observing contributions of partial waves of higher momenta. For the past few years measurements by H. R. WORTHINGTON, J. N. MCGRUER, and D. E. FINDLEY have been in progress at Wisconsin up to proton energies of 4.2 Mev. Results are now complete except for some details of analysis. In this work, sufficiently stringent requirements were put on all apparatus and all measurements so as to give an overall accuracy of a few tenths of one percent.

The absolute energy of protons from the Wisconsin electrostatic generator is known to .1 % and in this work the energy spread of the protons was held to .1%. Before entering the scattering chamber, which contained hydrogen gas at a pressure of 8 mm mercury, the proton beam was retracted by two diaphragms one meter apart, each with a circular aperture of 1.5-mm diameter. Thus the half-angle of the beam was held to .1°. Through this collimator a beam current of .2 to .4 microamperes was maintained. By means of a series of capillaries in the collimator with pumps between to give differential pumping, the region containing hydrogen was separated from the evacuated region from which the protons entered. Hydrogen was admitted to the chamber through palladium tubing.

Each of the many absolute measurements required in this work presented a problem of substantial proportions. Much time and effort went into the design and construction of an oil manometer for the measurement of hydrogen pressure, and measurements with a probable error of .025% were achieved. Temperatures were stabilized by stabilizing room temperature. They were uniform over the scattering chamber to within .1°C and were measured to .05°C.

The effective scattering volume, the solid angle subtended by the detector, and the angular region under observation were defined by a set of analyzer slits through which scattered protons passed to a proportional counter. These slits were⁴ made from stellite by the Gaertner Scientific Company. Because of the rapid variation of yield with angle, three sets of slits were used to keep counting rates constant to within a factor of ten over the angular region observed. These slits could be changed quickly without opening the scattering chamber. The slits determined a geometric factor and errors in measurement of their dimensions were such that the probable error in this geometric factor was .04% for the largest slits and .14% for the smallest.

Pressure in the proportional counter could be changed without opening the chamber. The overall resolution of the counter-amplifier system was approximately .15 microseconds. The analyzer slit and counter assembly were mounted on a wheel which had been the meridian wheel of a telescope. It was equipped with an angle scale which was accurate to one second of arc and, as used in this experiment, angular settings could be dependably made to .001°.

Contamination was continuously monitored by a counter with a very wide solid angle which viewed the beam at an angle of 90° and thus was sensitive only to substances heavier than hydrogen.

Measurements were made at six different energies from 1.8 Mev to 4.2 Mev and at each of these energies measurements were made at thirteen different angular settings from 6° to 45° in the laboratory system. At each angle and energy setting over 200,000 counts were obtained, and at many of the lower angular positions over one million counts were obtained.

Some of the more important sources of error were as follows:

- 1) Some protons were multiply scattered in the gas and reached the counter after two or three scatterings. This effect was appreciable, but by suitable arrangement of slits and baffles multiple scattering into the beam defined by the analyzer slits was compensated by multiple scattering out of this angular region.
- 2) The proton beam as it passed through the scattering volume contained some protons which were low in energy because of penetration of collimator apertures. The energy spectrum of the incident protons was determined by counting oversized pulses as a function of counter pressure and proton energy.
- 3) Some protons entered the proportional counter after penetration and scattering at the edges of the analyzer slits. This effect led to an increase in the effective size of the analyzer apertures. The magnitude of the effect was studied by pulse-size analysis and by comparison of results from the three sets of slits. The results checked reasonably well with computations. Corrections were appreciable only at very small angles. The sum of this contribution and the contribution described in paragraph three is known to higher accuracy than the separate effects. In the most critical low angle region the sum of these corrections is about .3%.
- 4) The scattering volume contained some contaminant gases which contributed to the scattering yield. Yields were monitored continuously at 90°. Measurements of the angular dependence of contaminant scattering were made. Corrections were appreciable only at small angles and here these corrections were mostly between .4% and .6%.

In an attempt to locate all sources of important errors, extensive data were taken at laboratory angles above 45° . Different pressures were used in the scattering chamber. Pressure in the proportional counter was varied. Different gases were used in this counter, and nickel foil of different thicknesses was used to separate the counter from the scattering chamber. For all of the data, pulsus from the amplifier were fed through two different channels with the discriminator of one channel set at .2 of the overall pulse height and the discriminator of the other set at .4 of the pulse height.

Computation of the corrections was a laborious problem involving much cross-checking for consistency. Analysis of the slit edge corrections and some work on determination of probable errors is still in progress. For the past six-months values computed for cross section have been changed very little by the analysis of errors and further corrections are expected to be insignificant. The set of corrections now appears to be unique and as accurate as can be determined from the data.

In the angular region above about 15° the probable error in cross-section values appears to be about 0.3%. Assessment of probable errors at low angles has not been completed. It will probably be about .6%.

Data were analyzed by H. H. HALL and J. L. POWELL, using at first only an S-wave interaction. Regular departures beyond the estimated probable errors were observed at small angles. A least squares fit to the data was then made assuming a combination of S- and P- wave phase shifts with a weight assignment to each data point according to the experimental uncertainty of the point. Values determined for K_0 and K_1 in this analysis are given in Table I. Values of K_1 appear to be dependable to about $\pm .05^\circ$ and values of K_1 appear to be determined to about $\pm 30\%$.

TABLE I

MEV	K_0 (degrees)	K_1 (degrees)
1.855	44.230	- 0.045
1.858	44.281	-- 0.054
2.425	48.313	-- .075
3.037	50.060	-- .100
3.527	52.414	-- .097
3.899	53.277	-- .104
4.203	53.829	-- .059

The values of $K_{\pm}$ have an energy dependence which agrees reasonably well with ordinary assumptions about the range of interaction, although the value at 4.2 Mev departs considerably from this trend. The anomalous behaviour of the 4.2 - Mev value may be taken to indicate the presence of some; unknown experimental error which could invalidate conclusions relative to the presence of P -waves. Another interpretation might be that the low value of K_1 at 4.2 Mev is genuine and the variation of K_1 with energy behaves in an anomalous manner.

The interpretation which appears most probable is that the absolute value of K_1 continues to increase smoothly with energy above 3.9 Mev and that the

experimental value at 4.2 is low in absolute value by approximately twice the probable error. A plot of the theoretical curves determined by the least squares fit appears to show small systematic departures from the experimental data. These departures are in 90% of the cases less than the probable errors. The systematic departures have a shape which bears a resemblance to the contribution expected from a D -wave with a negative phase shift of the order of magnitude of $.01^\circ$. The improvement in fit by inclusion of the D -wave contribution is too small to be considered good evidence for its existence.

The S -wave phase shifts determined by this analysis agrees well with previous values and confirms the current interpretation of the S interaction in this energy region. The magnitude of the P -wave phase shifts observed can be fitted by assuming charge independence and a conventional two-nucleon interaction in which WIGNER and MAJORANA forces enter with the proportions of .4 and .6, respectively. These conclusions in regard to the interaction are neither unique nor conclusive.

DISCUSSION

WIGNER: One of the interesting things which perhaps one time it will be possible to see on the basis of similar measurements are the three phase shifts of the three different P waves. One might ascertain to what extent they can be explained or described by means of tensor or spin orbit interactions. In principle these measurements should permit one to distinguish between these two alternatives.

HERB: In regard to this problem the P wave has an angular momentum of one unit. The two protons have spins in the same direction so the total spin is one and J may have a value of zero, one or two. The simplest assumption would be that the interactions are the same for $J = 0, 1$ or 2 , but this may not be true. It would, therefore, be very interesting to see whether the three phase shifts could be separately determined. It looks as though the data are not sufficiently accurate to make this possible.

MOSHINSKY: I would like to ask if these data have been interpreted using the boundary condition formulation proposed by Brcit.

HERB: NO, no such analysis has been carried out.

RABI: I would like to ask what your experimental limits are. For example, what was your counting rate on these very sensitive points. My questions are tending to the point whether you believe it is possible to do these experiments much better at these energies. I mean, it seems that you were pretty near the limit and it would take one hundred times as much work to do a little better.

HERB: We appear to be near the practical limits of experimental accuracy. At the lower angles the data are dependable to about five tenths of one percent, and at higher angles about three tenths. To improve this greatly would be extremely difficult.

UABI: Are we about at the end of where we could get?

HERB: It seems to me that the measurement would be more valuable if carried a few million volts higher in energy.

WIGNER: Does this phase shift analysis become more difficult at higher energies?

HERB: Actually, I think it does not, if measurements are made carefully as a function of energy so that we can follow separate phase shifts as they enter and increase in value.

ANDERSON: Could you say something by way of reassurance that the anomaly which you find at small angles might not have; some experimental cause, some; misalignment or some scattering of particles. Do you have some independent check?

HERB: Alignment of all parts was sufficiently accurate so that alignment errors are negligible. Errors in measurements of containments could perhaps be greater than we claim.

ANDERSON: DO you know of some attempt to do the type of measurement in which one detects not only the scattered but the scattering particle as well?

HERB: I should have said more about other experiments; many other measurements have been made recently. At Princeton some very fine work was done at 18 million volts. These results indicate a positive P wave phase shift at 18 Mev. Recent results at Illinois, at about 5 million volts indicate a negative P wave phase shift of about the value; to be expected from extrapolation of the Wisconsin results.

ANDERSON: How big a correction is introduced by scattering of the protons as they pass through the gas from the scattering center to detector?

HERB: Some of the protons do not go into the collector cup but get scattered out. At our lowest voltage this correction is about 0.03%. It is fairly easy to measure and quite easy to correct for. Then we have protons that come from the proper scattering region and start toward the analyzer but are then scattered through a small angle and miss the counter. This scattering out of the beam must be compensated by scattering into the beam. If adequate space is provided around the beam, compensation is good.

The Determination of the (Y, N) Cross-Section for Cu^{63} and Zn^{64}

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INTRODUCTION

Several measurements were carried out on the photodisintegration of Cu^{63} both with monochromatic high energy γ rays (BOTHE & GENTNER, 1937 & 1939; HIRZEL & WAFFLER, 1947; WAFFLER & HIRZEL, 1948) and with the continuous Bremsstrahlung spectrum available from Betatrons (BALDWIN & KOCH, 1945; BALDWIN & KLAIBER, 1940; MCELHINNEY, HANSON, DECKER, DUFFIELD & DIVEN, 1949; JOHNS, KATZ, DOUGLAS & HASLAM, 1950).

The experiments carried out with "monochromatic" γ rays have been made with the 170 γ rays available when lithium and boron are bombarded with protons: those experiments have shown that a (Y, n) reaction takes place leading to the formation of Cu^{62} and the cross-section for the process has been worked out (BOTHE & GENTNER, 1937 & 1939; HIRZEL & WAFFLER, 1947; WAFFLER & HIRZEL, 1948). As the spectrum of the γ rays emitted in these reactions show that there is more than one line energetic enough to photodisintegrate the nucleus of Cu^{63} , the cross-sections give the order of magnitude only, since the behaviour of the reactions at different energies is not known.

Recently, however, it has been shown that the knowledge of the yield of one of the disintegration products of a reaction produced by γ -rays from a Betatron or Synchrotron (as a function of the maximum energy of the Bremsstrahlung spectrum which produced them) and the dose, allow a computation of the photonuclear cross-section involved in the process, provided that the shape of the Bremsstrahlung spectrum is known as a function of the energy (JOHNS, KATZ, DOUGLAS & HASLAM, 1950; BALDWIN & KLAIBER, 1948; PERLMAN & FRIEDLANDER, 1918; ALMY & DIVEN, 1950; KATZ & CAMERON, 1951).

The experiments carried out with Cu^{63} , with Betatrons, show that the cross-section against energy curves have shape similar to resonance curves, that is, they are peaked: in the case of Cu^{63} experiments have shown that there is only one peak over the energy range from 10 to 100 Mev.

It was found that the behaviour referred to above is a general trend of photonuclear processes and several theories were put forward to explain the observed facts since according to the compound nucleus theory, the drop-off of the cross-section should be due to the competition to the (γ, n) reaction afforded by the $(\gamma, 2n)$, etc. and the experiments in a few instances show that the cross-section for these competitive processes is much too small to explain the observed

facts, although several experiments are in good agreement with the compound nucleus theory (EYGES, 1952).

As the process $\text{Cu}^{63}(y,n)\text{Cu}^{62}$ has been measured by several investigators and the process is believed to be well known the induced activity in copper was proposed as a comparison monitor for a beam of X-rays (MCELHINNEY & SIEWERS, 1951).

It is the purpose of this paper to describe some experiments on the (y,n) process in Cu^{63} and Zn^{64} and to discuss the behaviour of these reactions up to 28 Mev.

EXPERIMENTAL ARRANGEMENT

Electrolytic copper plates of 550 mg/cm^2 were irradiated by the beam of the Betatron inside a lucite block (JOHNS, KATZ, DOUGLAS & HASLAM, 1950) in conditions of good geometry. The experimental arrangement was such (fig. J) that the angle subtended by the plates, in the forward direction, was of 15° ; just behind these plates, inside the lucite block, a Victoreen thimble chamber (100 R) was used to measure the energy flux of the incoming radiation.

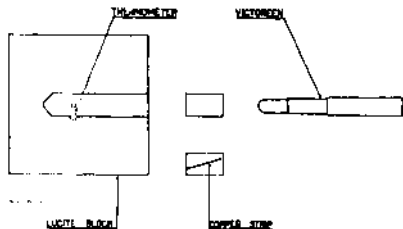


Fig. 1 — Experimental arrangement

Both the thimble and the Condenser R-meter used during the experiments were periodically checked through the use of radium standard and known potential differences, respectively;

the errors in the measurements of ionization are believed to be smaller than 2%.

At several energies measurements were taken with and without, a cadmium shield (3 mm. thick) which surrounded the copper plates inside the lucite block: by that procedure it was possible to evaluate the contribution due to the (n,y) process in Cu^{63} and in Cu^{65} and it was found that even with the highest energy attained in our experiments its influence on the whole activity was smaller than 1%.

In order to avoid the unwanted activity of the $\text{Cu}^{65}(y,n)\text{Cu}^{64}$ reaction, the samples were bombarded during time intervals between 120 and 150 seconds; in the high energy range this circumstance obliged us to decrease the yield of the machine in order that the leadings of the thimble could be within its range.

The instantaneous intensity of the X ray beam was monitored by means of an aluminum ionization chamber, filled with air; this chamber placed behind the lucite block had the purpose of giving information on the yield fluctuations during the bombardment. In the excitation function represented in fig. 2 only measurements in which the instantaneous intensity did not fluctuate by more than 5% were considered; this stability was usual since the Betatron was excited by means of an alternator provided with an electronically controlled magnetic drive.

The induced activities were measured by means of a 1.9 mg/cm^2 mica end window counter (Tracerlab, TGC 3) connected to a Berkeley model 1000 scaler; the efficiency of the counter and of the counting system were accurately known and the geometric efficiency of the measuring arrangement was determined by a method described elsewhere.

DETERMINATION OF THE X-RAY ENERGY

The energy control of the University of S. Paulo Betatron is rather conventional in its main principles (GOLDKMBERG, SANTOS & SILVA, 1951): as the energy acquired by the electrons during the acceleration cycle is proportional to the magnetic flux linking the equilibrium orbit, which is, in turn, proportional to the magnetic field at the orbit, the energy can be controlled by producing an expansion of the orbit at a predetermined field strength. A coil wound across one of the yoke legs developed a potential difference of about 8.000 volts. A resistor and a condenser in series were connected to this coil and the time constant of the RC circuit was large with respect to the period of the alternating voltage. The voltage developed across the condenser, being proportional to the integral of the applied voltage, was, therefore, proportional to the magnetic field at the orbit and hence to the momentum of the electrons.

The kinetic energy of the electrons was adjusted by means of the bias control on a trigger circuit whose output fired a thyatron and a GL 415 ignitron.

It was therefore possible to expand the orbit, at a predetermined value; of the magnetic field. A precision potentiometer was used for this bias control and a helipot was used for the fine adjustment of the bias voltage?

The stability of the machine, its temperature effects, and a comparison of our voltage integrating scheme with the usual one are described elsewhere.

DISCUSSION OF THE EXPERIMENTS

a) $\text{Cu}^{63}(\gamma, n) \text{Cu}^{62}$

Samples of electrolytic copper were bombarded using the technique described in §3. The corresponding excitation function is represented in fig. 2, where the points represent the results of several independent measurements carried out with different copper plates at different beam intensities; this procedure was followed throughout the energy range from 10.9 to 27.8 Mev, in order to have accurate results and eliminate any systematic errors that might arise during a particular bombardment. The obtained values agreed with each other within the inherent accuracy of the Victoreen Condenser /2-meter which was used as a beam intensity integrator in all the measurements.

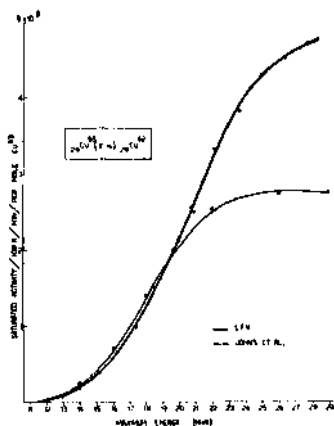


Fig. 2. Excitation function obtained for the $\text{Cu}^{63}(\gamma, n) \text{Cu}^{62}$ reaction.

As the Bremsstrahlung spectral composition changes with the angle about the forward direction (GOLDEMBERG, PIERONI, SANTOS & SILVA, 1952), we have decided to avoid the use of the conventional large monitor chamber

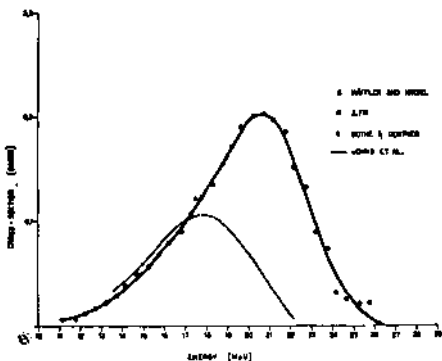


Fig. 3. Cross-section for the $Cu^{63}(\gamma, n)Cu^{62}$ reaction.

and the corresponding beam integrating circuit in order to get rid of any error that might arise from its use; a thimble ionization chamber was therefore used in all our experiments. As the thimble is activated during the exposures, care was taken to avoid its use whilst this "leakage" current might contribute to errors of the order of the inherent error of the instrument in the range of intensities that were measured; the drift produced by this activation is quite

serious at energies around 18 Mev and up and compelled us to wait for the decay of the induced activities since only one 100 R thimble was available during the measurements.

The integrator energy scale was calibrated by means of the threshold of the $Cu^{64}(\gamma, n)Cu^{62}$ reaction whose value 10.9 Mev is well known (MCELHINNEY, HANSON, BECKER, DUFFIELD & DIVEN, 1949); use was made of the parabolic rise of the excitation function near the threshold.

The linearity of the scale was checked with the thresholds of the reactions $C^{12}(\gamma, n)C^{11}$ and $Zn^{64}(\gamma, n)Zn^{63}$ which were found to agree with the accepted values (KATZ & CAMERON, 1951). We have found that the scale calibration was constant in all our experiments; the activities due to the Cu and Zn reactions were measured several times at a given integrator setting and the thresholds checked by the method described above; no discrepancies larger than the experimental errors were observed with the excitation functions measured for the Cu reaction by us several months ago (GOLDEMBERG, SANTOS & SILVA, 1951).

All the ionization measurements made with the thimble chamber were corrected for both pressure and temperature; as the chamber was kept always inside the lucite block (except when readings were taken), its temperature was assumed to be that of the block whose temperature was read with a thermometer imbedded in it.

In fig. 2 the excitation function for the same reaction as obtained by JOHNS, KATZ *et al.*, is plotted in the same scale for comparison: it may be observed that our measurements do not show the rapid fall-off observed by the Saskatchewan group and by several others; it has, however, the same shape observed by MCELHINNEY, HANSON, BECKER, DUFFIELD & DIVEN (1949) who have measured this excitation function up to about 22 Mev.

In fig. 3 the cross section for the (γ, n) process in Cu^{63} as calculated by the "photon difference method" is represented together with the results due to

JOHNS, KATZ, [DOUGLAS & HASLAM 1950]; it may be observed that although the cross-section obtained by us shows a "resonance" peak the energy at the maximum cross-section is of the order of 21 [Mev; its half width is of 6 Mev. In the same figure are plotted the values obtained by BOTHE & GENTNER (1937), (HIRZEL & WAFFLER, 1947) and (WAFFLER & HIRZEL, 1948); the agreement between our results and WAFFLER & HIRZEL is rather good although uncertainties of the Bremsstrahlung spectrum composition as well as in the absolute disintegration rate might well show that the agreement is fortuitous although no large discrepancies are to be expected.

b) $\text{Zn}^{64}(\gamma,n)\text{Zn}^{63}$

Zinc plates of the same dimensions employed in our experiments with copper were bombarded in our standard arrangement. The purity of the Zn sample⁵ was tested by means of the induced activity: the decay curve shows the presence of a single activity with 38,2 min half life due to the formation of Zn^{63} through a reaction (y,n).

During the experiments carried out with Zn, several different irradiation times were employed in order to have high intensities near the threshold; as the energy went up, the irradiation time was decreased so that a two minute irradiation was found to give rise to a strong activity around 20 Mev and up.

The Zn plates are found to be quite uniform and of about 240 mg/cm².

The excitation function (fig. 4) for this reaction rises slowly from its threshold and increases steadily up to 28 Mev although from about 23 Mev onwards a decrease of the slope is noticeable.

The calculated cross section for this reaction is represented in fig. 5; this cross section calculated by the "photon difference method" shows the resonance peak characteristic of the photonuclear processes; this peak is situated at an energy of 20,7 Mev, whereas its value, as found by the Saskatchewan observers (KATZ & CAMERON, 1951) lies at 18.5 Mev. The half-width found in our measurements is of about 11.3 Mev whereas the measurements reported by Johns, Katz et al., give 7.1 Mev.

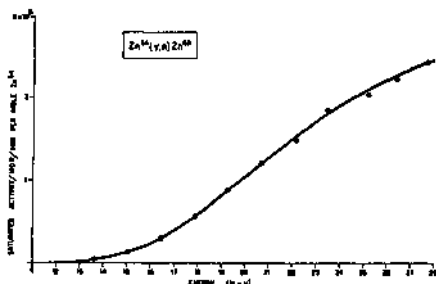


Fig. 4. Excitation function for the $\text{Zn}^{64}(\gamma,n)\text{Zn}^{63}$ reaction.

DISCUSSION OF THE EXPERIMENTS

a) The measured activities were corrected for the experimental influences on their activity such as bombardment time, time delay between the end of the irradiation and the beginning of the activity measurement and the time interval during which the activity was measured; as those measurements involve a precise knowledge of the half-life (see *The Science and Engineering of Nuclear Power*) a precise determination of the half-lives of Cu^{62} was carried out

by utilizing the method discussed by PEIERLS (1935): the value of 9.80 ± 0.07 min was obtained whereas for Zn the value obtained was 38,2 min in accordance with the accepted value.

b) As the calculations of the geometric efficiency of a rectangular source is very tedious, use was made of the comparison of the counting rates of a circular copper source 2 mm. in diameter with a rectangular copper plate of the same thickness, both irradiated simultaneously at 20 Mev in order to have a high specific activity; the measurements were repeated with several sets of plates at counting rates where the loss of the scaler for coincidences was negligible.

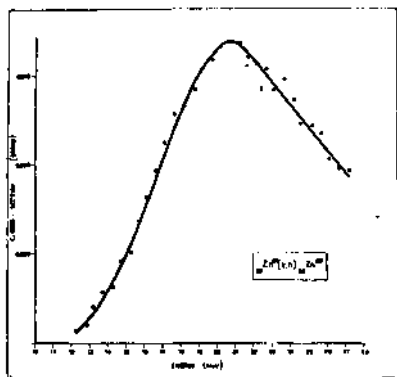


Fig. 5 Cross-section for the $Zn^{64}(\gamma, n) Zn^{63}$ reaction.

The results obtained by both methods showed a remarkable consistency- No correction for "self-scattering" was made.

d) As the element Zn^{62} has a rather complex decay scheme (SKAHOHCJ & PERLMAN, 1948; HUBER, MEDICUS, PREISWERCK & SCHERRER, 1947; BRADT, GUGELOT, HUBER, MEDICI'S, PREISWERK, SCHERRER & STEPHENS, 1946) a correction was made in the computation of the saturated activity.

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By this procedure? it was possible to determine the geometry factor for the rectangular samples used in our experiments.

e) In order to evaluate precisely the self-absorption of the radiation two different methods were employed:

1) the measurement of the absorption coefficient of the radiation emitted by the isotope in the target material (ATTEN, Jr., 1950).

2) the determination of the saturation curve of the specific activity as a function of the target thickness (FRIEDLANDER & KENNEDY, 1939).

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DISCUSSION

KERST: I wonder if you have seen any breaks in the excitation curves, such as those by Katz and Johns? What is the lightest element you have used so far?

SANTOS: SO far we have not been able to locate any sharp discontinuity in our excitation functions. We have bombarded carbon, oxygen, zinc and copper, and we have not found any sharp break which could be ascribed to an excitation level or something else. It may be that our Betatron is not as stable as Katz's. We were not able to confirm the existence of levels.

KERST: In regard to your other measurements with zinc and copper, I think you brought up an important discrepancy that seems to be jumping around from laboratory to laboratory, and it appears as though I were going to make some inter-laboratory check to see what is wrong with some of the determinations of peaks and cross-sections. I think that there are innumerable possibilities for errors, and I know you are getting rid of these successfully. But I have, always been curious since the Canadians mentioned these breaks in the curve of oxygen which are attributed to excited levels in oxygen, since they are examining the activity of the resulting nucleus. I have wondered whether or not they would be apparent in carbon, for example, and I see that you have not found them yet.

BAWITSOUER: Nao se pode sugerir alguma explicacao para a diferenca entre as curvas obtidas em Sao Paulo e as de Katz, por exemplo?

SILVA: A primeira vista supunhamos evidentemente, que talvez estivessem errados em qualquer uma das medidas. Entretanto, todos os checks que podiamos ter feito na funcao de excitacao, foram feitos. Boderia haver, por exemplo, um erro na calibracao da energia. Como disse, calibramos com 4 reacoes, isto e, em 4 pontos diferentes. A calibracao da energia estava correta. Boderia haver erros de medida de intensidade. Isso tambem nao, porque alem de trabalharmos numa dada tensao do contador, tambem tinhamos um "standard", com o qual verificavamos o contador, em cada uma das medidas. De forma que nao posso encontrar de nossa parte, nenhum erro. O fato de a nossa curva subir em altas energias enquanto que a de Katz satura, isto e, depois de uma certa energia ela nao sobe mais, pode-se explicar, supondo-se que talvez haja no betatron de Katz, saturacao do "wafer".

SMITH: Sobre este ponto desejava lembrar que Katz apresenta curvas de secao de choque, com o maximo, na regio de 25 milhoes para outros elementos. Tem assim 2 acima do 22, e 1 com 25,6. Nao me lembro qual foi o elemento.

SILVA: De fato, ele tem aqui uma relacao a qual ele da, para enxofre, 25,9 milhoes, por isso nao posso afirmar que haja saturacao, apenas sugerir.

SMITH: Entao, e muito facil de explicar que ele tenha saturacao na regio de 18.

SILVA: Eu nao tinha pensado nisso, mas, para essa reacao, nao conheco a curva respectiva.

SMITH: Nao e muito diferente das outras. Naturalmente, na regio alta nao pode ser muito boa, porque a energia onde a seccao de cheque esta caindo atinge ate 30 milhoes.

SILVA: EU nao conheco. A unica coisa que posso afirmar e que nao encontro nenhuma explicacao para esta discrepancia dos nossos dados com os deles. Temos que notar que as curvas de excitacao e seccao de choque, descem aproximadamente 20% em relacao a apresentada, como ja havia dito.

Determinamos o coeficiente de "sell" absorcao, com os metodos usualmente indicados, extrapolando para zero a exponencial. Entretanto, como na regio de pequenas espessuras temos competicao dos dois processos citados, o que foi verificado por Burt, a extrapolacao a zero devia ser feita, nao na exponencial, mas na regio mostrada na figura em que a curva desce. Temos de fazer uma correcao de mais ou menos 20% dos nossos valores.

SMITH: A diferenca mais importante e a da energia?

SILVA: Mia fica bem mais alta. A nossa, por exemplo, da 0,203 barns e, para o grupo de Saskatchewan 0,104. E o pico de ressonancia para nos e em 21 Mev, e para eles, 18,1 -- isto, para o cobre; para o zinco, 20,75 para nos, e 18,5 para eles.

O metodo utilizado na determinacao da seccao de choque e tal que o pico depende muito do comportamento dos outros pontos da funcao de excitacao. Ora, ve-se que, na funcao de excitacao de Katz, e na regio de 22 para 24 milhoes, que so inicia, digamos assim, essa saturacao da curva. Eu precisaria ver o comportamento, da curva que possui pico em 25 Mev para ter idia exata da saturacao da curva.

SANTOS: There is one point to which I would like to call the attention in this work. In the beginning we thought that one of the reasons for the displacement of the resonance peak towards high energies might be due to something wrong with the a.c. wave by which we integrated the flux; we decided then to use the same flux integrator as Katz was using and we found exactly the same results. In the meantime, we have devised a new integrator by means of which we integrated the energy by means of a pair of coils sitting right on the top of the equilibrium orbit. This kind of integrator should be completely independent of non-linearities in the iron or things like that. We have found the same results. I might add also that we have observed that the Victoreon chamber which is used to monitor the intensity of the beam gets highly activated due to copper, carbon and oxygen activities and this produces a leakage which might be responsible for the flattening of the curve: one would read more Roentgen than actually the apparatus has emitted. I do not suppose that Katz has made this mistake but I want to say that we knew that this thing might, occur and I we had taken every care in order that this would not give rise to an error in our measurements. A point which should be discussed also is the following: At the University of Illinois, Hanson, Duffield, Mac Elhinney and others have measured the copper excitation functions up to 22 million volts and one of those excitation functions published in their paper agrees with ours up to 22 million volts which was the highest energy they used. Quite recently a paper has been published by Newkirk on the reaction of copper studied with the synchrotron of Iowa University. It has the same behaviour as ours. In that excitation function they came up to about 70 million volts without flattening down; I would think that, our curve is the correct one because we could not find any reason for a systematical error which would be responsible for the observed behaviour.

SMITH: The curves of cross sections of Newkirk are in quite good agreement with those of Katz, that is to say, the curve of cross section given by Newkirk has a peak at slightly less than 8 Mev. They certainly do not agree with Katz', that have a lower energy.

SANTOS: That is true as far as the published cross section is concerned, although I do not know what was the process by means of which this cross section was calculated, and what were the corrections for specific activities. What I can say is this: if you take Newkirk's curves up to 28 million volts, which agree with ours, we cannot use the method of calculation of cross-sections as we know, and make the peak of the energy fall down to about 18 million volts. They go always up to 21,5 Mev.

DEUTSCH: How do you know that your spectrum is the same as on somebody else's machine?

SANTOS: We have used the same donut and same kind of target, and the same expanding devices; we believe therefore that, as both of us have measured this photo disintegration in the forward direction, that the beam has the same spectrum composition, although we are not sure that either of us had the same spectrum composition as described by Schiff.

KERST: After Prof. Santos' remark on monitory, I am reminded of a pitfall that one can fall into when he runs his machine at past 20 million volts. If you have a lucite block with a ionization chamber in, electrons of 20 or 25 million volts can pass through and give you anomalously high ionization readings. If you do not clean the gamma readings perfectly by obstacle, the primordial electrons accelerated in the machine can get off the target and influence the monitor. This should tend to move the maximum energy to lower the maximum cross section, to lower the energy, or give you too many Roentgen. I have seen this happen and if has been necessary in such cases to clean up the beam.

SANTOS: I would like to give some information about this observation of Prof. Kerst. We thought also from another experiment that we carried out that we might have some electrons hitting our ionization chamber (or soft gamma rays due to the Bremsstrahlung of those electrons in the poles of our machine), so we decided to test the behaviour of these high energy points in our excitation functions at different distances in order to check the inverse square law, (this measurement was useful also in order to check the methods of calculating the flux in Roentgens inside the Lucite block because one then supposes that only the forward rays contribute to the ionization). By using the inverse square law experiments, we have found that the yield per Roentgen remains the same up to a distance of the order of two meters from the target.

Polieletrólitos

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Polieletrólitos (Fuoss & CATHKRS, 1947; Fuoss, 1918 a, 1951) são polímeros que contêm átomos carregados, eletricamente. Combinam portanto as propriedades dos polímeros neutros com as dos eletrólitos em uma única molécula; como era de se esperar, estão presentes tanto as similaridades como as diferenças das substâncias originárias. Para apresentar uma analogia matemática, quando se converte um polímero em um polieletrólito, não há somente soma das propriedades dos eletrólitos e dos polímeros, mas também aparece, nas propriedades observadas, o equivalente dos produtos mistos. Viscosidade e condutibilidade são as propriedades físicas características que melhor merecem polímeros e eletrólitos, respectivamente. O objetivo deste trabalho é apresentar revisão de estudo recente em Yale sobre algumas propriedades hidrodinâmicas e elétricas dos polieletrólitos, a fim de ilustrar as similaridades e diferenças mencionadas acima, e desenvolver um modelo que explique as propriedades dos polieletrólitos. Como veremos imediatamente, estes resultados têm importância em muitos problemas que surgem nos campos da bioquímica e da fisiologia.

Os polieletrólitos que surgem em estado livre, incluem proteínas, ácidos nucleicos e alguns polissacarídeos. Em todos eles, temos uma grande molécula que transporta uma carga elétrica múltipla. As proteínas são geralmente anfotéricas; normalmente o *pI*, a molécula ou é um cátion ou em solução ácida ou um ânion em solução alcalina. Os ácidos nucleicos são, do ponto de vista dos polieletrólitos, poliésteres de ácido fosfórico em cadeia longa, e em cujas unidades repetidas, um dos grupos do resíduo de ácido fosfórico é livre para formar sais. Os polissacarídeos podem ser descritos como ácidos poliacarboxílicos em cadeia longa. A estrutura detalhada destes produtos naturais nem sempre é conhecida com certeza, e além disso, diversos deles não são muito estáveis. Geralmente são solúveis apenas em água. O pesquisador que trabalha com tais materiais está portanto sujeito a certas desvantagens, uma das quais, e que não é pequena, é a sua limitada possibilidade de variar a estrutura dos compostos: ele tem que trabalhar com as substâncias fornecidas pela natureza. A viscosidade é um exemplo de polieletrólito obtido por tratamento químico de um polímero que aparece em estado natural; como se sabe, é xantato de celulose. Materiais puramente sintéticos oferecem um campo proveitoso para pesquisa no terreno de polímeros eletrólitos, pois a estrutura e o tamanho das moléculas correspondentes são conhecidos e controláveis. Além disso, são inteiramente estáveis e solúveis em

grande variedade de solventes. Estudando suas propriedades, esperamos aleargar uma correlacao quantitativa entre suas propriedades e sua estrutura. Entao, por analogia, poderemos chegar a melhor compreensao do comportamento de produtos naturais mais complicados.

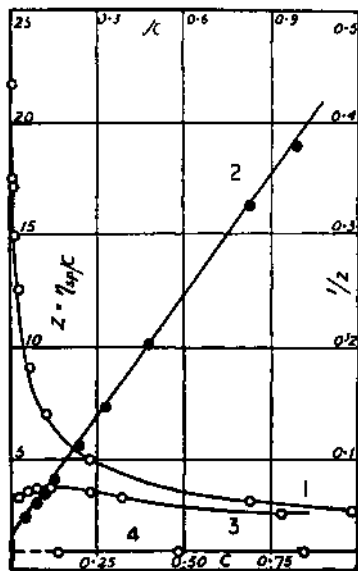
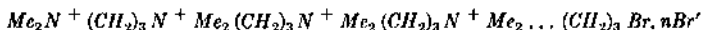
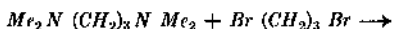


Fig. 1. Viscosidade reduzida de soluções do brometo de poli-vinilpiridina. Curva 1, água como solvente. Curva 3, 0.001 N K Br. Curva 4, 0.0335 N K Br. Coordenadas 1, 3, 4; à esquerda e abaixo. Curva 2, verificação da equação (5); coordenadas à direita e acima.

regados negativamente; em ácido poliácrico completamente neutralizado, os íons contrários são íons de sódio. Sulfonacão de poliestireno produz também um polianião.

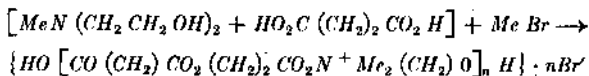
Policationtes também podem ser formados de muitos modos. A polimerização de vinilpiridina (FITZGERALD & Fuoss, 1950), $CH_2 = CH.C_5H_4N$, produz polibase fraca; a quaternização desta por adição de haleto de um polieletrólito forte, no qual os átomos de carbono da cadeia carregam, alternadamente, um íon de piridina. Os íons contrários são, aqui, os íons de haleto. A autoadição (com auto-quaternização simultânea (CHIT & Fuoss, 1951)), de $\alpha - \omega$ diaminas conduz a polieletrólitos fortes nos quais os átomos carregados são ao mesmo tempo átomos da cadeia polimérica; por exemplo,



Compostos similares são formados pela autoadição (LEHMAN, THOMPSON & MARVEL, 1935) de compostos de $\alpha -$ halogênio, $\omega -$ amina. A policondensação

Os polieletrólitos podem ser sintetizados de muitas maneiras, tanto por reações de polimerização como de policondensação. O ácido acrílico, $CH_2 = CH.CO_2H$, contém um grupo vinílico ativo; sol) a reação de radicais iniciadores livres, ele se polimeriza em longas cadeias, cujos átomos de carbono carregam alternadamente um grupo de carboxila. Em solução o anionto e uma macro-molécula, consistindo numa cadeia a qual está ligada um certo número de íons de carboxila. Estes resultam da dissociação de alguns dos grupos carboxílicos. A neutralidade elétrica é mantida pelos íons de hidrogênio que se encontram, parte; no solvente que é preso no volume do polímero enrolado, e parte no solvente entre os íons poliméricos. Os íons menores que contrabalançam a carga dos macro-íons chamam-se íons contrários, ou "Gegenionen". Se se adiciona hidróxido de sódio a uma solução de ácido poliácrico, a carga dos poli-íons é aumentada, pois uma quantidade crescente de grupos neutros de carboxila é convertida em íons de carboxila carregados negativamente; em ácido poliácrico completamente neutralizado, os íons contrários são íons de sódio. Sulfonacão de poliestireno produz também um polianião.

entre ácidos di-básicos e álcoois di-hídricos como metildietanol-amina (EDELSON & Fuoss, 1948), seguida por quaternização, resulta em poliésteres que são polieletrólitos contendo na cadeia ítomos positivamente carregados de nitrogênio; por exemplo.



Variando o número de grupos de metileno no álcool e no ácido, é possível formar polieletrólitos com uma distância entre cargas conhecida com precisão. Outro método de fazer polieletrólitos de densidade de carga variável e co-polimerizar monômeros neutros com monômeros ionogênicos, por exemplo, estireno com vinilpiridina ou anidrido maleico. A solubilidade pode também ser controlada por escolha adequada de copolímeros. Poliantólitos sintéticos (ALFREY JR. *et al.*, 1950) são obtidos por copolimerização de dois monômeros, um ácido e outro básico, como vinilpiridina e ácido acrílico. Em todas as reações acima o peso molecular (ou comprimento médio da cadeia) é controlável pelos métodos convencionais da química polimérica. Vemos assim que grande variedade de materiais sintéticos pode ser obtida para pesquisas: na verdade, podemos desenhá-los uma molécula para um fim específico e então proceder à síntese, com razoável garantia de que o produto terá as propriedades desejadas.

A fim de esclarecer a discussão da viscosidade dos polieletrólitos, recapitularemos primeiro a estrutura dos polímeros neutros e sua correlação com a viscosidade. Um polímero do tipo da borracha (poli-cis-isopreno) consiste em uma longa cadeia de átomos de carbono, aos quais se prendem por ligações covalentes átomos de hidrogênio ou outros átomos, ou ainda grupos de ítomos. Devido à liberdade de

rotação das ligações carbono-carbono, a cadeia é perfeitamente flexível e pode assumir em solução um número tremendo de configurações geométricas. Os dois extremos são fáceis de imaginar; a cadeia completamente estendida, e a mais compacta, completamente enroscada. Devido ao movimento térmico, ambas as configurações são altamente improváveis no sentido estatístico: a primeira se contrairia ou enroscaria e a última se abriria. Entre esses dois extremos, entretanto, há uma série de configurações muito mais prováveis: o cálculo (KUHN, 1931) mostra que o comprimento mais provável $(R^2)^{1/2}$ é proporcional à raiz quadrada do número das ligações na cadeia. O cálculo é exatamente semelhante à média dos quadrados das distâncias atravessadas por uma partícula em movimento Browniano. Se se dissolve um polímero em um solvente de viscosidade η_0 a viscosidade η da solução é mais alta, sendo a diferença

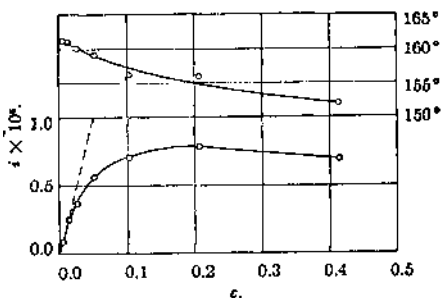


Fig. 2. Orientação e intensidade de birrefringência no gradiente de velocidade 0.655 sec^{-1} .

$$\Delta \eta = \eta - \eta_0$$

A razão $\Delta \eta/\eta_0$ é chamada viscosidade específica; a razão entre a mesma e a concentração, $\eta_{sp}/c = z$, é chamada viscosidade reduzida. O limite de z extrapolado até a concentração zero é $[\eta]$, a viscosidade intrínseca. Em partículas esféricas compactas de densidade 1,0, EINSTEIN (1906, 1910) demonstrou que $[\eta] = 0.025$ (sendo a concentração dada em gramas de solúvel por 100 cm³ de solução). Nos polímeros, os valores experimentais da viscosidade intrínseca são sempre muito maiores do que 0.025, como seria de esperar se os polímeros fossem estruturas enroladas, abertas e difusas. Cálculos teóricos (DENYZ, 1946) mostram que a viscosidade intrínseca é a medida do volume que a molécula do polímero ocupa na solução. A melhor forma desta relação é a proposta por FLOUZY & FOX (1951):

$$[\eta] = \phi (R^2)^{3/2} / M \quad (2)$$

em que ϕ é uma constante universal para todos os sistemas de solventes e polímeros, R^2 é a média dos quadrados das distâncias entre as extremidades da molécula e M o peso molecular. Experimentalmente, medimos a viscosidade em uma série de concentrações e em seguida extrapolamos num gráfico de z em função de c até à concentração zero. Para poliestireno em tolueno e sistemas semelhantes, os gráficos $z - c$ são lineares, com uma inclinação positiva:

$$z = [\eta] + k' [\eta]^2 c \quad (3)$$

So, entretanto, são medidas as viscosidades, o o peso molecular é conhecido, o tamanho da molécula de polímero pode ser determinado. Testes experimentais tem demonstrado que dimensões assim obtidas correspondem perfeitamente as determinações por outros métodos, por exemplo, pela dispersão de luz.

Na figura 1, a Curva 1 mostra a viscosidade reduzida em função da concentração em um polieletrólito forte típico (Fuoss & STRAUSS, 1919), o produto da adição de brometo de butilo e poli-1-vinilpiridina. (O polímero originário tinha um peso molecular de 207.000, isto é, a molécula contém cerca de 2.000

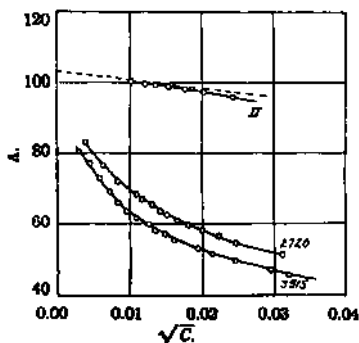


Fig. 3. Condutibilidade em metanol; II, brometo de dimetil-etanolamônio; as curvas mais abaixo, polieletrólitos.

anéis de piridina presos a cadeia de carbono. Em etanol, sua curva de viscosidade era da forma linear dada pela equação (3); o valor extrapolado de (η) era 1.18). Fica desde logo claro que os dados não estão em conformidade com a equação (3): em vez de uma linha reta com inclinação positiva, temos uma curva com tangente negativa, cuja inclinação aumenta rapidamente à medida que a solução se torna mais diluída. A curva mostra que a contribuição do solúvel polieletrólítico para a viscosidade da solução torna-se relativamente maior quando a concentração diminui. Esta simples experiência demonstra a diferença marcante entre

polímeros neutros e carregados, e também serve para sugerir um modelo para a molécula do polieletrólito. Se aceitamos a equação (2), isto é, aceitamos a hipótese de que a viscosidade é o volume da molécula polimérica, então o resul-

tado experimental mostrado na figura 1 indica claramente que o polieletrólito se *expande* quando a solução se dilui.

Esta conclusão pode também ser deduzida de argumentos qualitativos baseados nas forças eletrostáticas intramoleculares. Suponha-se que consideramos a solução a uma concentração particular, e estudamos a distribuição dos íons de brometo. Cada núcleo de piridina no polímero contém um átomo de nitrogênio carregado positivamente; o enorme campo devido a cargas de 2.000 unidades atrairá necessariamente muitos dos íons de brometo negativos e prendê-los-a dentro do volume da molécula. O polícatião se assemelha, sob muitos aspectos, a uma gotícula de solução concentrada, suspensa no solvente. Nem todos os íons contrários, entretanto, estarão dentro do volume dos polícatíons: a probabilidade $W(r)$ de um dado anião estar a uma distância r do centro de um dado cátion é 6

$$W(r) = 4 \pi r^2 \exp(-u/kT) \quad (4)$$

em que u é a energia potencial do anião. Para valores do r maiores do que R , $W(r)$ ainda será finito; em outras palavras, não desaparece a probabilidade de alguns dos íons opostos se localizarem no solvente entre os poli-íons. Suponha-se agora que diluimos a solução que temos considerado: geometricamente, estamos aumentando o volume da solução entre os polícatíons. Em consequência, a fração de íons contrários em relação a qual $r > R$ aumentará, devido ao aumento do termo de volume $4 \pi r^2$ na probabilidade $W(r)$. Alimenta portanto a carga líquida do polímero, isto é, a diferença entre o número dos átomos do nitrogênio positivos numa molécula de polímero e o número de íons contrários presos ou associados. Em consequência da repulsão eletrostática entre cátions não compensados dentro da própria molécula, esta se dilatara. De acordo com a equação (2), esperamos assim um aumento de viscosidade na diluição, exatamente como foi encontrado na experiência.

Embora o raciocínio acima conduza a uma descrição qualitativa dos resultados experimentais, deve-se mencionar que o mesmo não pode constituir mais do que uma explicação parcial, já que pelo menos dois outros fatores que contribuem para a viscosidade foram desprezados. Primeiramente, há o efeito puramente geométrico da mudança de forma do polieletrólito: depois de um certo ponto de dilatação, a molécula não pode mais ser representada como um polímero estatisticamente enrolado e o fator hidrodinâmico na equação (2) se converterá no de um bastão. Em segundo lugar, deve ser considerada a interação eletrostática intermolecular a longa distância. A interação entre moléculas poliméricas do tipo que aumenta o termo positivo na equação (3)

provavelmente será desprezível aqui devido à repulsão entre as macro-moléculas que possuem todas o mesmo sinal de carga; haverá porém uma atração eletrostática entre os poli-íons e suas atmosferas de íons opostos carregados contrariamente.

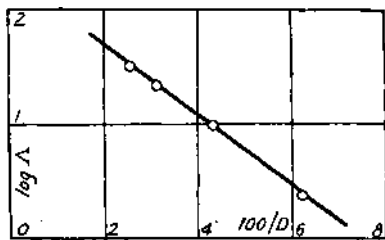


Fig. 4. Condutibilidade em função da constante dielétrica.

Pod-se realizar um teste experimental do modelo eletrostático medindo-se as viscosidades de polieletrólitos, e usando-se soluções de eletrólitos simples como solventes (Fuoss & STRAUSS, 1949) em vez de água. A curva 3 da figura 1 é a curva de viscosidade para o brometo de polivinilbutilpiridínio em solução de brometo de potássio 0,001 N. Em concentrações de poli-sal mais elevadas, onde a concentração iônica do brometo do polieletrólito é igual à de brometo de potássio, a viscosidade é aproximadamente a mesma que na água pura. Mas quando a concentração do poli-sal diminui (enquanto que a do brometo de potássio permanece constante a 0,001 N), a curva passa por um máximo a cerca de um décimo por cento em vez de ascender a valores altos. Abaixo do máximo, os pontos contrários acrescentados sob a forma de brometo de potássio excedem os adicionados como polieletrólito. Se se utilizam concentrações mais altas de brometo de potássio, toda a curva desce o máximo se abaixa e se desloca para concentrações mais elevadas de polieletrólito, e finalmente a curva se torna do tipo linear característica de um polímero neutro. A curva 4 da figura 1 é a curva de viscosidade do polieletrólito em solução de brometo de potássio 0,0335 N. Nosso modelo explica facilmente tal resultado. Na presença de excesso de eletrólito simples, os polímeros possuem grande quantidade de "Gegenionen" e seus campos positivos podem então atrair mais íons negativos. Mas cada anion preso pela molécula diminui a carga líquida, e portanto diminui a repulsão intramolecular; conseqüentemente, a molécula se contrai no sentido da configuração típica de um polímero neutro, e a viscosidade diminui porque seu volume se torna menor. Na verdade, na presença de um excesso muito grande de eletrólito simples, surge outro efeito eletrostático (Fuoss & STRAUSS, 1948) a atração intramolecular entre as cargas positivas nos segmentos do polímero e os anions associados. Esta atração faz a molécula se contrair a um volume muito menor do que o do polímero inicial, como quando a atração inter-iônica estabiliza um cristal iônico, p. ex. cloreto de sódio, em estrutura mais densa. No limite hipotético de um excesso infinito de brometo de potássio, a viscosidade intrínseca do polieletrólito da figura 1 se extrapola a 0,17, aproximadamente um décimo da viscosidade da polivinilpiridina da qual foi formado. Este limite corresponde a gotículas submicroscópicas compactas de solução de polieletrólito neutro altamente concentrado, suspensas em uma solução neutra de brometo de potássio. Como era de esperar, o tamanho limite, medido pela viscosidade intrínseca, depende da natureza do anion no eletrólito simples acrescentado, mas é completamente independente do cátion, pois o último é sempre repelido pelo polímero positivo.

Consideremos agora outra vez a curva 1 da figura 1, a viscosidade do polieletrólito em água pura. Apenas pelo gráfico não se pode prever que haja limite superior para a viscosidade reduzida quando a concentração se aproxima de zero. Intuitivamente, no entanto, sente-se que existe um limite finito a $c = 0$. Se considerarmos outra vez nosso modelo, veremos que a dilatação na diluição não pode exceder um limite superior óbvio, o da cadeia completamente aberta. À medida que a solução vai sendo diluída, aumenta a probabilidade de fuga dos íons contrários do polímero, e quando a carga líquida no polímero se aproxima do limite (que é igual ao grau de polimerização), a repulsão intramolecular vai distorcendo constantemente a molécula. Mas este processo não pode continuar indefinidamente; os átomos do polímero, incluindo, e claro, os carregados, estão

presos entre si por ligacoes covalentes e nao podem se espalhar aldm dos limites estabelecidos por tais lacos. No limite de diluicao infinita, quando todos os fontes contrarios escaparam, a repulsao intramolecular atinge o maximo, e obriga a cadeia polimeica a distorcer-se tanto quanto possivel. Em ontras palavras, a mole'cula se abre e em concentraes muito baixas assemelha-se a uma enfigurae,ao cm forma de vara. Isto porque a blindagem ou cortina eletrostatica formada pelos "Gegenionen" associados diminui, devido ii perda dos liltimos por difusao no corpo do solvente. A ideia de que se trata d'uma cortina eletrostatica sugere que existe algo como uma atmosfera de iontes dentro de cada mole'cula polim&ica, diferindo da atmosfera de Debye-Hluckel pelo fato de possuir uma carga liquida. Sabe-sc que o termo principal na cortina iontica e proporcional a raiz quadrada da concentracao, e somos assim lev ados a suspcitar da presenca de um termo em raiz quadrada na viscosidade dos polieletrolitos. Encontramos empiricamente (Fuoss, 1948b, 1949) que a equacao seguinte representa os resultados experimentais:

$$z = A / (1 + B \sqrt{c}) \quad (5)$$

de modo que, no limite da concentraçao zero

$$[\eta] = A \quad (6)$$

Uma vorifieacao desta equacao e mostrada pola curva 2 da figura 1 que constitui o grafieo da reeiproea de z para nosso polieletrolito naaguam funeao da raiz quadrada da concentracao. Como vemos na figura, os ponton, formam uma linha rota, de acordo com a equayao (5), o agora e facil a extrapolacao para a concentracao zero. Se a forma da equac. au (5) esta certa, a curva 1 (na escala $z - c$) possui a concentracao zero, uma tangente vertical e e" evidente a impossibilidade pratiea de cextrapolac.ao pela curva I.

Voltando a nosso modelo, foi previsto que, em concent ra'des baixas, o polieletrolito se abria e alongaria no sentido de um bastao. A seria entao o coeficiente hidrodinamico para um eilindro longo; tedricamente, este coeficiente e proporcional ao quadrado do comprimento da vara, e portanto, em nosso modelo, proporcional ao quadrado do peso molecular. No polieletrolito da figura I, $A = 50$; em um outro proveniente da polivinilpiridina, de peso molecular 77.000, $A = 7.55$. Os valores de A estao na verdade mais ou menos na razao dos quadrados dos pesos molculares dos dois polimeros. Resultados semelhantes tem sido obtidos tambem em soluctfes de viscose (TAIT *et al*, 1951) e em polimetacrilato de sodio (OTH & DOTY, 1952), que indica uma estrutura em forma de bastao para as moleculas destes polieletrolitos em concentraes baixas.

A birrefringencia (Fuoss & SIGNER, 1951) tambem da resultados que est& de acordo com o modelo deduzido das medidas de viscosklade. Em muitas

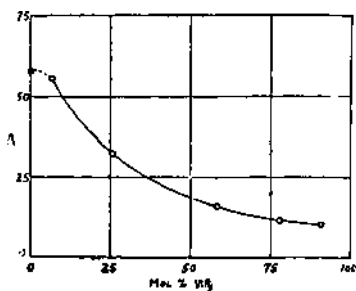


Fig. 5. Condutibilidade em função da fração de vinilpiridina no co-polímero.

substancias birrefringentes, a intensidade da birrefringencia aumenta com a concentracao; em um polieletrolito tipico, tal quantidade passa por um maximo, como mostra a figura 2. Em concentrates baixas, a curva e linear, com grande inelivacao da tangente, correspondendo a pequenissimos coeficientes da difusao rotatoria. Sabe-se que moleculas cilindricas possuem pequenos coeficientes. Quando a concentracao aumenta, cresce cada vez mais o numero de iotes opostos atraidos para OS poli-iones, reduzindo assim a repulsao intramolecular. Isto permite o enroscamento da molecula em configurates de maiores coeficientes de difusao, relativamente mais dificeis de orientar; desto modo, a percentagem de aumento de intensidade sofre um decrdeimo com o aumento de concentracao. O angulo de orientacao tambeYn diminui para concentraoes crescentes. Como teste final, mediu-se a birefringeneia na presenca de grande excesso de brometo de potassio; nao foi observada qualquer orientacao. E isto precisamente o que seria de se esperar se a molecula do polieletrolito tivesse assumido uma pequena e compacta estrutura esferica na presenca de excesso do eletrolito simples, conclusao que ja foi deduzida das experiencias de viscosidade. Conclusoes semelhantes podem tambem ser tiradas do fato de a viscosidade depender do gradiente da velocidade no viscosimetro para solucoes de polieletrolitos (STRAUSS & Fuoss, 1952).

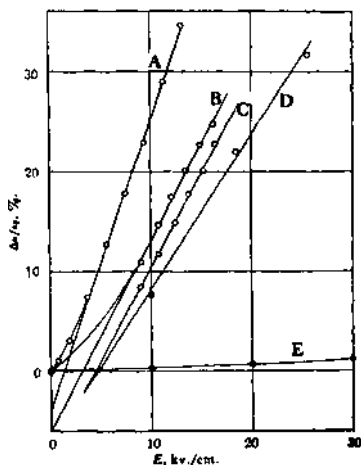


Fig. 4. Efeito de Wien em soluções de polieletrólitos.

Resumindo o comportamento hidrodinamico, concluímos que a viscosidade dos polieletrólitos constitui a medida do volume da molecula do polieletrólito em solucao. Ao contrario de polimeros neutros, entretanto, o tamanho e a forma da molecula dependem da concentracao do polieletrólito (e da de qualquer eletrólito acrescentado). Atribuímos esta dependencia ao efeito das forcas eletrostaticas entre as cargas do polímero e as dos iotes opostos, e em particular, suporemos que uma certa fracao dos ultimos foi presa no polieletrólito retorcido por atração eletrostatica. O teste experimental mais direto da hipotese do associacao e efetuado quando se passa a considerar o aspecto dos polieletrólitos; ate agora, tratamos os mesmos essencialmente como polimeros, cujo comporta-

mento era modificado pela presenca de cargas eletrostaticas nas cadeias. Tratemo-los agora como eletrólitos, cujas propriedades estao modificadas pela restrigao de que uma especie iontica e polimerica.

Em um eletrólito simples comum, como cloreto de sodio, as duas especies de iotes sao semelhantes hidrodinamica e eletrostaticamente; tanto o fonte de sodio como o de cloreto sao entidades de livre locomoeao, mais ou menos do mesmo tamanho. Quando se dissolve em agua um cristal de cloreto de sodio, os iotes do cristal se hidratam e fogem uns dos outros, de modo que se produz uma distri-

buicao aproximadamente uniforme de fontes através da solução. Por "uniforme" queremos dizer que quaisquer elementos de volume contem igual numero de anions e cationes, e mais ainda, que tal numero e igual a media aproximada obtida dividindo-se o numero *total* de anions ou cationes pelo volume *total*. Consideracao mais cuidadosa mostra que, em solventes de alta constante dieletrica, a densidade media dos cationes e levemente maior perto de anions do que a concentracao media, e vice-versa para anions. Para termos precisos, se n_0 e a media aproximada e n a media local,

$$n = n_0 \exp(-e\psi/kT) \quad (7)$$

em que e é a carga do ion e ψ o potencial eletrostático.

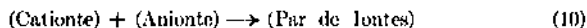
Como se sabe, Debye e Hückel avaliaram o potencial combinando (7) com a equação de Poisson, e assim resolveram, pelo menos em princípio, todos os problemas de soluções diluídas de eletrólitos fortes. Neste caso

$$e\psi/kT \ll 1 \quad (8)$$

e esta condição permitiu a aproximação da função exponencial em (7) pelos dois primeiros termos de sua série, ficando a equação Poisson-Boltzmann reduzida a uma equação linear simples. Um dos resultados da teoria foi a previsão de que a mobilidade dos fontes diminuiria com o aumento de concentração, sendo o decréscimo proporcional à raíz quadrada da concentração. Consequentemente, a teoria confirmou o resultado empírico de Kohlrausch

$$\Lambda = \Lambda_0 - \alpha \sqrt{c} \quad (9)$$

em que Λ é a condutibilidade equivalente e Λ_0 a condutibilidade limite na concentração zero. Finalmente, a teoria prevê o valor numérico da constante. Ela concorda completamente com os dados experimentais que cobrem grande variedade de eletrólitos fortes em água. Para solventes de constante dielétrica mais baixa do que a da água, é necessário modificar-se a teoria original: se D , a constante dielétrica é pequena, a energia potencial $e\psi$ de um fonte pode tornar-se grande comparada com kT (que mede a energia cinética) quando o fonte se aproxima de um fonte de carga oposta, e assim a aproximação (8) não é mais admissível. Se supomos que dois desses fontes formam, ao entrar em contato, uma estrutura estável ao impacto térmico médio, isto é, um par de fontes associado, podemos ainda aplicar os métodos de Debye-Hückel aos outros fontes que estão, em média, a grandes distâncias mútuas em comparação com seus diâmetros (os fontes "livres"). Baseada nesta hipótese, foi desenvolvida uma teoria (Fross, 1935) que corresponde perfeitamente à condutibilidade dos eletrólitos em todos os solventes, para quaisquer valores da constante dielétrica. Resumindo, a condutibilidade geralmente diminui com o aumento de concentração por dois motivos: primeiro, o número de fontes livres diminui, devido ao aumento do número de fontes associados de acordo com a reação:



e segundo, a mobilidade dos fontes livres diminui, devido à interação eletrostática a grande distância, de acordo com (9). Os resultados são sintetizados na equação

$$\Lambda = \gamma (\Lambda_0 - \alpha \sqrt{c\gamma}) \quad (11)$$

em que y e a fragao de eletrolito total presente como iones livres. Dados abrangendo grande variedade de sistemas estao de conformidade com (1.1).

E evidente *a priori* que as consideracoes acima nao podem ser aplicadas a polieletrólitos, porque a distribuicao das cargas em uma solucao polieletrólítica e necessariamente nao-uniforme, *em consequencia da estrutura polimerica*. Em nosso sal de polipiridinio, por exemplo, aproximadamente 2.000 cationes estao presos por ligacoes covalentes, e, nao importa qual a diluicao da solucao, em gramas por litro, as 2.000 cargas de urn dado policatione *nunca* poderao espalhar-se aldo do limite estabelecido pela cadeia completamente estendida. Como ja mencionamos, o campo positivo forte atraira certo numero de iones contrarios e os prendera no volume ocupado pelo polimero enrolado. Os iones contrarios remanescentes se distribuira pelo volume da solucao entre as moleculas polimericas. Por conseguinte, se exploramos a solucao com urn pequeno elemento de volume, achamos uma distribuicao totalmente diversa da observada em solucao de cloreto de sodio. A despeito da situacao do elemento de volume, podemos agora encontrar em vez de numeros aproximadamente iguais de aniones e cat iones: (a), a alta concentracao tan to dos aniones como dos cationes, dentro

dentro dos polimeros ou (b), a concentracao baixa de aniones sem qualquer catione entre as moleculas grandes.

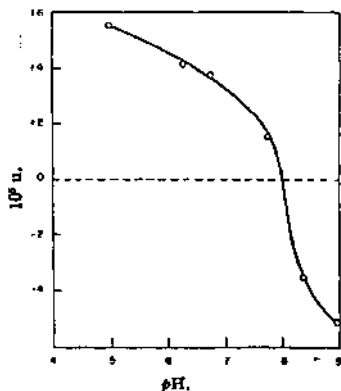


Fig. 7. Mobilidade eletroforética de polianfólicas.

O efeito desta distribuicao sobre a condutibilidade e descrito melhor com urn exemplo (EDELSON & Fuoss, 1918). A figura 3 mostra a condutibilidade de dois polieletrólitos (pesos moleculares 27 20 e 3515) em metanol; foram obtidos por adicao de brometo de metil aos poliesteres obtidos pela condensacao de anidrido succinico e metil-dietanolamina. Em contraste, a curva II apresenta a condutibilidade de urn eletrólito quaternario simples (brometo de dimetil-dietanol-amonio). A tangente teorica do ultimo e" representada pela linha pontilhada; e claro que a curva experimental se aproxima da

tangente limite em concentracoes baixas, enquanto que os pequenos desvios negativos em concentracoes mais elevadas indicam uma formacao moderada de pares de fontes em conformidade com a equacao (10). As curvas dos polieletrólitos sao concavas em vez de convexas, e nelas nao e" evidente nenhuma aproximacao de um valor limite.

Consideremos agora as ultimas curvas do ponto de vista do nosso modelo. Supomos para uma dada concentracao escolhida, que uma fracao definida de iones opostos esteja associada aos policationes, e que os iones remanescentes sejam essencialmente livres. Portanto, a condutibilidade consistira de dois termos: a corrente de iones negativos e a de positivos. A corrente i^* proporcional ao produto $n_j z_j u_j$, em que n_j e" o numero de iones por unidade de volume, z_j e" a sua carga e u_j a mobilidade. Nos iones de brometo, n_1 e" o numero medio de iones

do brometo livres, z_1 e igual a um, e u_1 e a mobilidade dos iões de brometo livres, diminuída por efeitos eletrostáticos a grande distância. Nos polícatíones, n_2 e um número muito menor do que n_1 , ao passo que z_2 , a carga líquida do polícatíone, é muito maior do que z_1 . A mobilidade (HUIZENGA *et al.*, 1950; FRRZGERALD, 1950) é da mesma ordem de grandeza que u_1 em polieletrólitos fortes. Suponha-se agora que diluimos a solução. De acordo com nosso quadro, alguns dos iões opostos associados aos polícatíones se espalharão dentro do volume principal da solução. Isto terá dois resultados: primeiro, a condutibilidade aniónica aumentará, porque o número relativo de iões livres aumenta; e segundo, a mobilidade do polícatíone aumentará, porque a carga líquida e aumentada de uma unidade, cada vez que um aníon associado foge. (O aumento devido a carga mais elevada, naturalmente, será compensado parcialmente pelo acréscimo no coeficiente de fricção que acompanha a expansão da molécula, ocorrida quando a carga líquida aumenta, como consequência da maior repulsão intramolecular). Estes dois fatores, que aumentam ambos a condutibilidade, produzem as curvas acentuadas mostradas na figura 3. Ainda não conseguimos obter um método satisfatório para extrapolar até a concentração zero, embora seja possível calcular (Fuoss & EDELSON, 1951) a condutibilidade limite das moléculas em forma de bastão.

Dev-se acentuar que o equilíbrio entre os iões contrários livres e os associados aos polícatíones e dinâmico. "Gegenionen" estão constantemente se espalhando dentro e fora do volume ocupado pelo polímero enrolado, de tal maneira que o número médio de iões opostos, associados a um poli-fonte, é constante, se calculado para um grande intervalo de tempo. Em princípio, poderíamos computar este número, a partir da função de probabilidade (4), mas a dificuldade está em se obter um valor para a energia potencial U . Quando se trata de polieletrólitos,

$$U/kT \gg 1 \quad (12)$$

no caso de muitas cargas, e as aproximações matemáticas simplificadoras feitas no caso de eletrólitos simples não são mais possíveis. Uma teoria definitiva não foi ainda estabelecida, embora muitos trabalhos já apresentem um começo animador. Destes, o mais satisfatório é o de KATCHALSKY (1951), que pode ser aplicado aos polieletrólitos de baixa densidade de carga, onde o emprego do potencial de Debye-Hückel dentro do polímero enrolado é provavelmente justificado.

A despeito de dificuldades matemáticas, porém, é certo que a energia potencial u é inversamente proporcional à constante dielétrica. Este fato sugere outro teste experimental do modelo. Numa primeira aproximação a condutibilidade é proporcional ao número de iões de brometo livres, e este número é relacionado com a constante dielétrica pela função de probabilidade exponencial (4). Esperaríamos portanto encontrar uma relação linear entre o logaritmo da condutibilidade equivalente em uma dada concentração e a recíproca da constante dielétrica do solvente. GATHERS & Fuoss (1949) mediram a condutibilidade do brometo de polivinilbutilpiridínio em uma série de misturas de dióxano e nitrometano que abrangeu todo o conjunto de constantes dielétricas de 16 a 39. A figura 4 apresenta um gráfico dos seus dados a concentração de 0,001 N em iõe de brometo; $\log A$ e a ordenada e $1/D$ a abscissa. Como mostra a figura,

os dados conduzem a uma linha reta, de acordo com a relação funcional que previmos acima, a partir da teoria geral fundamental. O resultado estabelece claramente a existência de um equilíbrio de difusão entre os "Gegenionen" livres e os associados com as moléculas poliméricas, sendo o equilíbrio determinado por forças eletrostáticas.

Vimos como a densidade da carga e a carga líquida de um íon macromolecular dependem da concentração do eletrólito e da constante dielétrica do solvente. Estas duas características do eletrólito podem variar de outra maneira: por copolimerização do monômero iogênico, e do monômero que permanece eletricamente neutro. Por exemplo, se estireno e vinilpiridina são copolimerizados, e o produto tratado com brometo de butil, obtemos um polieletrólito cuja carga máxima é determinada pela quantidade de vinilpiridina no polímero. As curvas de condutibilidade (GATHERS & Fuoss, 1949) destes sais em nitrometano assemelham-se às da figura 3, e qualquer uma delas pode ser explicada qualitativamente pelos raciocínios já apresentados. Surge, porém, um detalhe que não se compreende à primeira vista: em uma dada concentração de íons de brometo, a condutibilidade é menor, quanto maior for a porcentagem de vinilpiridina no copolímero. A figura 5 e o gráfico da condutibilidade equivalentes destes sais copolímeros a 0,001 N, em função da fração molar de vinilpiridina no copolímero. O estudo cuidadoso do modelo sugere a explicação dos resultados experimentais. Suponha-se que começamos por um copolímero que contém somente um número muito pequeno de grupos de vinilpiridínio. Haverá pequena tendência para associação de íons, porque em nenhuma parte existe elevada densidade¹ de carga, (em toda a parte $kT \ll 1$) e os íons de brometo mostrarão sua condutibilidade costumeira. A mobilidade dos cátions, entretanto, será muito pequena, pois a parte carregada do polímero deve arrastar atrás de si uma volumosa ancora de hidrocabonato inerte. Aumentemos agora vagarosamente a fração de grupos eletrolíticos no copolímero. A mobilidade do policationte tende naturalmente a aumentar à medida em que sua carga aumenta. Às vezes quando esta aumenta, aumenta o potencial, e íons de brometo antes livres ficam agora presos pelos policationtes. Esta ação tem dois efeitos sobre a condutibilidade; primeiro, o número de íons livres de brometo diminui, e com ele a corrente de íons negativos e, por conseguinte, a condutibilidade. Em segundo lugar, a associação de "Gegenionen" com os policationtes diminui a carga líquida dos mesmos, e portanto compensa parcialmente o efeito da carga bruta crescente. Em outras palavras, a mobilidade dos policationtes considerada como função da fração de vinilpiridina principia em zero como uma função linear, mas logo se torna côncava, enquanto o número de íons livres de brometo é uma função monotona decrescente do conteúdo de piridina. A soma das condutibilidades correspondentes dá uma curva da forma mostrada na figura 5. (.) ponto correspondente a nenhuma piridina, até o qual extrapolamos a curva, não é mais do que a condutibilidade do íon de brometo.

A extrapolação parece ser justificada, porque a mobilidade limite de um íon macromolecular com carga sempre decrescente só pode ser zero.

A natureza dinâmica do equilíbrio entre íons livres e associados também pode ser demonstrada diretamente medindo-se a condutibilidade dos polieletrólitos em um campo de alta tensão. Temos usado muitas vezes a expressão

"o volume do polímero estatisticamente enrolado", sem dar-lhe uma definição precisa. Na verdade, não existe definição rigorosa: lembramos que o polímero retorcido é uma estrutura flexível que está constantemente em movimento térmico. Em consequência, ele não pode ter mais do que um diâmetro *medio* R ; os valores instantâneos do R não têm significação. Suponha-se que tracemos uma esfera imaginária de diâmetro R ao redor do centro da gravidade de um dado polieletrólito, e observamos esta molécula. Segmentos da cadeia polimérica atravessarão ocasionalmente a superfície da esfera e se retrairão; ao mesmo tempo, aníons estarão constantemente entrando e saindo da esfera. Ora, um ânion, para o qual $r \ll R$, tem poucas probabilidades de fuga, o outro, para o qual $r \gg R$, tem poucas probabilidades de captura. Mas aqueles cujas distâncias do centro são da mesma ordem de grandeza que R estarão geralmente em um campo eletrostático tal que seu potencial é da ordem de kT ; portanto, bastaria um pouco de energia adicional para libertá-los. Os campos elétricos usados habitualmente em medidas de condutibilidade são demasiadamente fracos para perturbar os equilíbrios iônicos, mas, em campos elevados, Wien mostrou que a corrente aumenta mais rapidamente do que a voltagem. Onsager provou que isso resulta do fato de a associação diminuir nos campos elevados. A figura 6 ilustra o efeito dos campos de potencial elevado na condutibilidade do brometo de polivinilpiridínio em água; campos da ordem de 10-20 kV/cm foram aplicados durante alguns microssegundos, e a condutibilidade foi medida em um ponto especial (BAILEY ET al., 1952). Como vemos, o efeito é muitíssimo maior do que para o sulfato de magnésio (curva E) em água, e é quase tão grande como para um eletrólito típico em tolueno (curva .1). Calculamos deste resultado que vários por cento dos "Gegenionen" estão na região periférica perto do $r = R$, onde o campo forte pode deslocá-los ou adsorvê-los aos íons livres, aumentando assim a condutibilidade.

Como exemplo final de condutibilidade, apresentamos alguns resultados experimentais (ALFHEY JR. ET al., 1952) sobre um copolímero do ácido metacrílico e o N,N-diethylammonium metacrilato, contendo partes aproximadamente iguais (55/45) dos dois monômeros. O primeiro monômero é um ácido fraco, o segundo uma base fraca. Em consequência, o polímero deveria portar-se como um polianião em solução básica e como um policatione em solução ácida. Este resultado é obtido experimentalmente: a figura 7 mostra a mobilidade eletroforética do polímero como função do pH da solução. O polímero é evidentemente um anfólito com um ponto isoelétrico de 8,0. A analogia com proteínas é inoquívoca. Além disso, a viscosidade tem um mínimo pronunciado no ponto isoelétrico, a partir do qual cresce com a adição tanto do ácido quanto do alcali, devido à expansão da molécula, que resulta do aumento da carga.

Temos visto que tanto as propriedades elétricas quanto as hidrodinâmicas dos polieletrólitos conduzem ao mesmo modelo nestas substâncias: um polímero enrolado flexível, cuja configuração e carga líquida são determinadas pela concentração total. Diversas outras propriedades têm sido medidas especialmente pressão osmótica, dispersão de luz e comportamento potenciométrico; o espaço limitado impede discussão de tais resultados.

Para resumir brevemente basta declarar que podem todos ser explicados, pelo menos qualitativamente, pelos mesmos raciocínios que foram feitos aqui para a viscosidade e a condutibilidade.

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DISCUSSION

COSTA RIBEIRO: Desejava perguntar ao prof. Fuoss qual a ordem de grandeza da condutibilidade eletrica desse tipo de materiais.

Fuoss: The order of magnitude of the equivalent conductance of the polysalts is of the order of that of ordinary salts. I did not have time to go into details during the lecture. The current for a given ion is proportional to the product

$$nzu$$

where n is the number of ions of that species, z is the valence or net charge, u is the unit charge and u is the mobility.

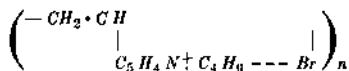
For example, in our polyelectrolyte, n is rather small because it refers to a polyion, z is rather large because we have many charges per molecule, and the mobility is about the same as that of an ordinary ion, while for the counter-ions or the Gegenions, z (the number of charges per ion) is only one, and n is very large, so the product nz is of the same order of magnitude as for an ordinary bromide ion. The reason for approximately equal mobilities is simply this. Each time we add one more charged group to a chain, we increase the force which the external field exerts on the molecule, but each time we also have to insert more atoms which increases the resistance to motion. So, as the charge builds up, hydrodynamical resistance goes up also and the mobility therefore does not change very much.

COSTA RIBEIRO: I would also like to know if there is a possibility of knowing the conductivity in solid and liquid states of the same poly-electrolyte.

Fuoss: These materials, whose molecule weight is of the order of 100 thousand, half a million, a million and so on, are all solids. They do not exist as liquids, unless you go to quite low molecular weights; hence they can be investigated only in solution, or else they can be studied as solids. As solids, they are rather interesting electrolytic conductors. In solution, both the Gegenion and the poly-ion contribute to the current because they are swimming in the solution, but in the solid form the transference number of the poly-ion is zero because it is so large that it cannot move in the medium of high viscosity. But the Gegenions can work their way through the coils of the polymer and then they produce the conductance. It is a typical electrolytic conductance, with an exponential temperature coefficient, which we would expect as conductance, with an exponential temperature coefficient, which we would expect as a consequence of the change of viscosity with the temperature.

COSTA RIBKRO: But is their conductivity in the solid state very low?

Fuoss: It depends on the temperature which, of course, controls viscosity and also, on any possible plasticizer (that is, solvent of low molecular weight such as dibutyl tartrate) which is added, and on the extent of quaternization. As in the case of ordinary compounds, it depends again on the same things: the number of ions and how fast they can move, and how fast they can move depends on the temperature through viscosity. We have not deliberately tried to get very high conductance; as I remember it, our most conductive materials were of the structure:



Polyvinylpyridine quaternized with a little bromide, and plasticized with dibutyl tartrate gives a specific conductance of the order of 10^{-5} : a pretty good conductance, but nowhere near the conductance we get with the same amount of material in water. But I think that, by proper choice of nature and amount of plasticizer, we could get to conductances of perhaps 10^{-3} or 10^{-2} mho,

MEDINA: I wonder if you calculated the activities. You established the equations and these equations should give you the activities and the free energies. How did they turn about?

Fuoss: It is not very practical to calculate activities from conductance data because the latter represent an irreversible process. However, we have made measurements of osmotic pressures. People in other laboratories likewise have measured osmotic pressures and have also made potentiometric titrations: from these data (which represent thermodynamic equilibria) it is possible to calculate the activity coefficients. This turns out to be roughly 0.1 to 0.8, depending on the concentration. That is, at higher concentrations the activity coefficient is quite low, and at lower concentrations it eventually must approach unity. I do not agree; with the way some of these calculations have been made, because no corrections for the association of ions under the influence of intramolecular electrostatic fields were made. The osmotic pressure data, for example, were calculated as if the entire change in osmotic pressure were due to all of the solute. Now, obviously, if some of the Gegenions are trapped in the polymeric coils, they no longer count as independent kinetic units, and therefore could not contribute to the osmotic pressure.

MEDINA: Did you make any attempt to calculate this activity theoretically from potential distribution and concentration distribution?

Fuoss: There have been several theoretical treatments of the poly-electrolyte problem. The whole problem goes back, of course, to a distribution problem, because if we know the distribution, we could determine the potential of a given charge as a function of location; we then could calculate the electrostatic potential energy; from that, the free energy; and finally all the properties of the thermodynamic system. The difficulty is this: if we use the Poisson-Boltzmann equation, the density involves the potential, which is in certain locations very high, and hence the Debye-Huckel approximation cannot be made. The result is a badly non-linear equation. Eventually, we may get an explicit solution of this problem. So far, however, it does not exist. The following approximative cases have been studied: First, the simplest, is Kimball's model where he assumes a central force to account for the statistical coiling of the poly-molecule, and then a uniform distribution of electrostatic charges within that sphere.

Another model which has been used is Debye's: a uniform charge density but with a statistical calculation of the curling effect of the polymer coil. Then Hermans made a calculation in which he assumed essentially that the molecule was already extended but was flexible, and assumed a continuous distribution of charge. They made some mathematical approximations which perhaps are a little hazardous. Kuhn, Kilmle and Katchalsky have made probably the best statistical treatment of the charged coil molecule. They allowed charges located in different spots along a molecule, the model of which is the Kuhn flexible coil. They get into some mathematical difficulties and again approximate their way out. One of the approximations they made was that the electrostatic potential within the coil is that given by Debye's potential. This is justified when we have a low density of charge within the polymer. For instance, let us take polyacrylic acid, which is a very weak electrolyte, and neutralize partially with sodium hydroxide, so that it is five or perhaps ten percent ionized. Then the Katchalsky approximation seems to be justifiable both physically and mathematically. In this range, the activity coefficient calculated from the theory agrees beautifully with his theoretical predictions, but the theory does break down at higher concentrations as expected, because the basic approximation begins to fail. Then, finally, we have obtained an explicit solution of the Poisson-Boltzmann equation for the case of a polymer molecule which has reached the limit of dissociation, so that it is extended to a rigid rod; that is, we have a solution of this problem for the stiff rod-like molecule, with no flexibility at all.

BONHOFFER: We are interested at the moment in getting the concentration and the activity of sodium and potassium ions in metaphosphate solutions. We know then; are changes of polymeric phosphoric acids so your ideas may be applied to this problem. We tried to measure the sodium activity and the potassium activity by using the following method. I would be interested to know what you think about it. We made membranes of ion exchange materials just as Marshall did with his plates of clay. These membranes do not give exact values, but do permit the preparation of membranes which are very last. One can measure the sodium activities; for example, we found in this case that it depends on the concentration of the sodium where the activity coefficient is about 0.1 or 0.2. Do you think it is a very convenient method?

Fuoss: As you know, the method is perfectly sound, so the only possibility of trouble that I can see, would be purely experimental, in the way of leaks or contamination; I know you are a careful experimenter, so those hardly come into the question.

BONHOFFER: I think it is quite easy. The conductivity of these membranes is purely ionic, and I think not with plasticizer. It is also very high.

Fuoss: A plasticizer is present: water. Then there is another method, that Kern has used in measuring the activity coefficient of sodium polyacrylate; again it is a thermodynamic method. He uses a sodium amalgam electrode and measures the electrode potentials, which of course, is thermodynamically found.

Recent Contributions to the Study of the Thermo-Dielectric Effect

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INTRODUCTION

The observation that strong electric charges are developed in samples of dielectrics such as carnauba wax, parafin, colophonium, naphtalene, etc. which, previously melted, were solidified in the absence of any external electric field (COSTA RIBEIRO, 1943), led us to the hypothesis, confirmed by subsequent experiments; that those charges were due to the development of electricity as a consequence of the phase change of the dielectric.

In the course of 1944 we carried out a systematic experimental investigation on naphtalene, and have confirmed *the production of an electric current when a phase change takes place at the inter-face of the solid-liquid dielectric system*, a phenomenon which we proposed to call: *Thermo-dielectric effect* (COSTA RIBEIRO, 1944).

A description of these experiments with the results of the measurements and the conclusions drawn from such observations, have been summarily reported in the "Anais da Academia Brasileira de Ciencias", and extensively discussed and developed in a "thesis" presented to the "Faculdade Nacional de Filosofia, Universidade do Brasil", for the granting of the Professorial and Doctor Degree in Physics (COSTA RIBEIRO, 1945 a, b).

THE THERMO-DIELECTRIC EFFECT

This pioneering work can be summarized in the following way:

In a capacitor whose dielectric is partly in the solid and partly in the liquid state, one of the plates being in contact exclusively with the solid, and the other with the liquid, if a phase change takes place at the inter-face, an electric current, is produced, as a consequence of the phase change (fig. 1).

The current produced (*Thermo-dielectric current*) in such a system (*Thermo dielectric cell*) can be detected and measured by the time rate of charging of an electrometer connected to the plates, if one knows the total electrostatic capacity of the system.

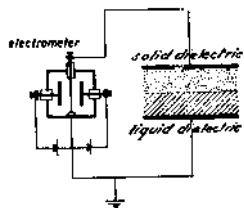


Fig. 1 --- Schematic representation of the thermo-dielectric effect.

The direction of the current produced is related to the direction of the phase change and to the direction of the displacement of the interface; its intensity is related to the time rate of the phase change.

The measurement of the charges developed respectively during the solidification or the melting of the same mass of the dielectric has shown that those charges had opposite signs but the same magnitude, which induced us to think that the effect is a reversible one, so far as the direction of the phase change is concerned (COSTA RIBEIRO, 1945b).

METHODS FOR THE EXPERIMENTAL STUDY OF THE EFFECT

According to the order of magnitude of the thermo-dielectric currents (around 10^{-11} Amperes, for the cells we have worked with) electrometric methods are the most suitable for its measurement, but careful precautions must be taken in order to avoid leakage (COSTA RIBEIRO, 1945b).

Charges developed can be measured by the time integral of the current produced (COSTA RIBEIRO, 1915a, b), or by collecting the total charge in a large and well insulated air capacitor, that is afterwards discharged through a ballistic galvanometer (COSTA RIBEIRO, 1950) (fig. 2).

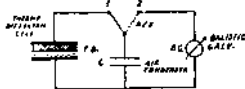


Fig. 2 — Diagram of the circuit employed for the measurement of the total charge developed in a thermo-dielectric cell.

Other methods have been described (COSTA RIBEIRO, 1945a, b, 1950) employing many types of cells, several shapes of electrodes and different ways of measuring the current and the charges. These methods include melting and solidification of dielectrics, that are solid at room temperature or freezing of dielectrics that are liquid at room temperature.

For the quantitative study of the effect we have employed a thermo-dielectric cell whose internal electrode was an aluminium vertical cylinder cooled by a controllable air flow inside the electrode (we used filtered air with low speed in order to avoid completely charges eventually produced by friction).

The external electrode was an helicoidal strip of aluminium involving the internal one. The two electrodes were immersed in the liquid naphthalene, contained in a vessel placed in a thermostatic chamber at a temperature slightly above its melting point. Cooling of the internal electrode by the passage of the air, caused the formation of a coating of solid naphthalene on its surface (fig. 3).

The internal electrode was permanently suspended on the beam of a laboratory balance, with a special device in order to compensate the reaction force of the air escaping the electrode (figs. 4 & 5).

With this system we could measure continuously the variations of mass of the solid deposit of the dielectric. The continuous measurement of the current produced was carried out with a monofiber Wulff electrometer, by the time rate of charging method.

In this way we could draw simultaneously the current-time and the mass time curves, during the processes of melting or solidification (figs. 6, 7 & 8).

LAWS OF THE THERMO-DIELECTRIC EFFECT

From the measurements above reported we could derive the two following laws of the thermo-dielectric effect:

Law of intensities: "In isothermal melting or solidification, occurring at a constant rate, in a thermo-dielectric cell, the intensity of the current produced is proportional to the time rate of the phase change".

$$i = k_1 \, dm/dt$$

k_1 being a constant for a given cell and a given dielectric (fig. 9).

Law of charges: "If a diphasic dielectric system passes from one state of equilibrium to another state; of equilibrium, the total electric charge associated with the change of mass of one of the phases is proportional to this change of mass".

$$Q = \int_1^2 i \, dt = k_2 (m_2 - m_1) = k_2 \, m$$

k_2 , being a constant for a given cell and a given dielectric (fig. 10).

A careful analysis of the results of measurements so far carried out (COSTA RIBEIRO, 1950) shows that, for the same type of cell and the same dielectric, the constants k_1 and k_2 have practically the same value, which can be represented by A .

The discussion of the data so far reported shows that, if measurements are performed with the same type of cell but with different dielectrics, the values of k differ greatly for each kind of dielectric, not only in magnitude but also in sign. This strongly suggests the existence of a *new specific physical constant*, characteristic for each kind of dielectric and which we will call the *thermo-dielectric constant*.

In the case of naphthalene this constant would have a value around 3×10^{-9} coulomb/gram.

THEORETICAL TREATMENT

Comparison of the curves of variation of the thermo-dielectric current with the corresponding variation of the velocity of the phase change (fig. 8) led us (COSTA RIBEIRO, 1945 b) to the conclusion that the current is an hereditary function of this velocity and that the instantaneous value of the current can be expressed by an integro-differential equation of the same form as that established by GROSS (1940) for the absorption current, in the theory of the anomalous behaviour of solid dielectrics.

In the case of the thermo-dielectric effect the current produced, as a consequence of the phase change, can be considered as an hereditary function of the time rate of this phase change ($dm(\tau)/d\tau$) in which $m(\tau)$ is the instantaneous value of the mass of one of the phases, function of the time variable T .

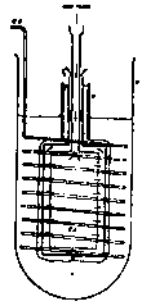


Fig. 3 — Thermo-dielectric cell with immersion electrode. *E.I.* — Immersion electrode; *E.E.* — External electrode (grounded); *L* — Liquid dielectric; *S* — Solid dielectric; *V* — Pyrex insulator.

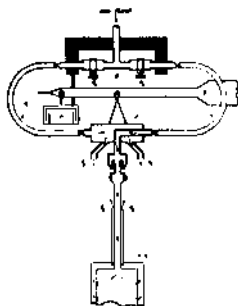


Fig. 4 -- Special suspension system for compensation of the reaction force due to the escaping air. *B* — Beam of the balance; *S* — Air distribution system; *T* — Rubber tubing; *R*₁ and *R*₂ — Mohr's nippers for the control of the air distribution; *A* — Air damper.

The instantaneous value of the thermoelectric current at a given instant (*t*) is the resultant of the remaining contributions, at this instant, of the currents produced by all the variations of the mass, previous to that instant, and can be expressed by the equation:

$$i(t) = \int_{-\infty}^t \frac{dm(\tau)}{d\tau} \varphi(t - \tau) d\tau$$

In which φ is an hereditary function, whose form will be discussed later.

From this equation, and without any particular hypothesis on the internal mechanism of the effect, can be derived the two fundamental laws previously established experimentally.

CINTRA DO PRADO (1946a) has shown that to the hereditary function in this equation can be attributed the exponential form and, CINTRA DO PRADO & SARAIVA DE TOLEDO (1917) employed the simplified equation so obtained, for numerical verifications, calculating a curve that fits very closely with the curve drawn from our experiments (fig. 11).

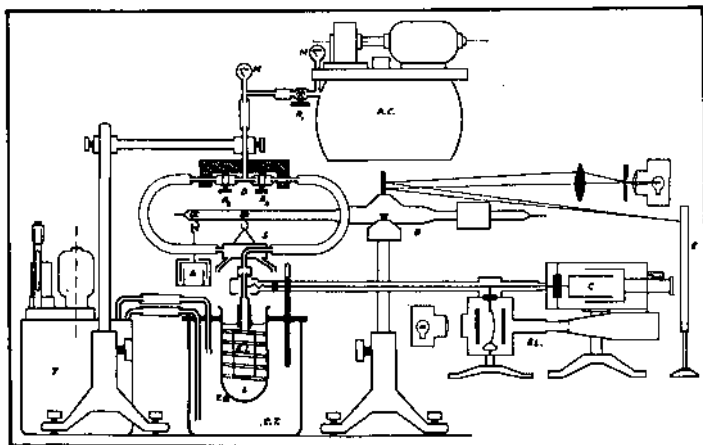


Fig. 5 — Schematic representation of the equipment employed for the quantitative study of the thermoelectric effect. *T.D.* — Thermo-dielectric cell; *E.I.* — Immersion electrode; *C.T.* — Thermostatic chamber; *A.C.* — Air compressor; *B* — Balance; *El.* — Electrometer; *C* — Air expander; *T* — Hoeppler's thermostat.

The assumption of the exponential form for the hereditary function has also led GROSS (1946) to the suggestion of very simple and interesting methods for the local analysis of the mass-time and the current-time curves, and for the

determination of the thermo-dielectric constant and of the time-constant of the system.

SARAIVA DE TOLEDO (1940) studied also, theoretically, some implications derived from the law of charges in the case of cyclic transformations, establishing new ways of calculating the value of the thermo-dielectric constant by the consideration of corresponding values in the above mentioned curves.

HYPOTHESIS ON THE INTERNAL MECHANISM OF THE EFFECT

In order to interpret the origin of the thermo-dielectric effect one can admit the existence of an electric double-layer at the solid-liquid interface. The displacement of the double layer, as a consequence of the phase change, and the capture of part of its charges by the solidified dielectric would account for the observed effect.

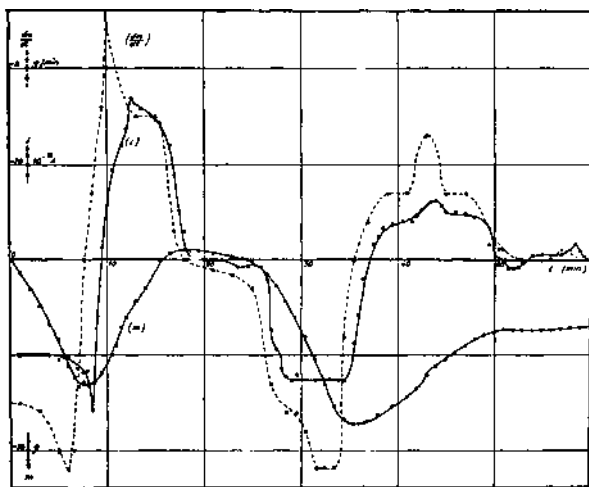


Fig. 6—Results of measurements with naphthalene. m —Variation of the mass of the solid phase; dm/dt —Time rate of the phase change; i —Thermo-dielectric current.

Different hypotheses can be put forward as to the origin of such a double layer. A brief discussion of these hypotheses has been given in our thesis (COSTA RIBEIRO, 1945b).

An electronic mechanism based on the hypothesis of different densities of loosely bound electrons in the solid and liquid phases of the same substance, has been proposed by TIOMNO (COSTA RIBEIRO, 1945a), and the nature of the elementary processes involved has been discussed by CINTRA DO PRADO (1910b) who calculated that, in the case of naphthalene, the number of molecules that pass from one phase to the other corresponding to the liberation of one elementary charge, is of the order of magnitude of 10^6 .

BROAD RANGE OF OBSERVATION OF THE THERMO-DIELECTRIC EFFECT

The observation of the phenomenon has been reported in a large variety of dielectrics, including: different kinds of carnauba wax, ouricuri wax, paraffin, colophonium, naphthalene, naphtol, stearic acid, fluorene, acenaphten, palmitic acid, sacharose, sulphur, etc.

More recently it has also been reported in the solidification of benzene, and in the freezing of water (COSTA RIBEIRO, 1917).

The experimental evidence so far given seems to indicate that the thermo-dielectric effect is not peculiar to a limited number of dielectrics but, on the contrary, is a very general phenomenon. All the dielectrics that we have tested presented the effect in a larger or minor degree.

It is more easily observed in substances of high electrical resistivity, because in such substances, the leakage of the developed charges is very small and these charges can be easily detected and measured by the potential difference that they produce in a sensitive electrometer of low electrostatic capacity.

It is not improbable however, that similar effects should also be produced in the phase changes of substances with lower resistivity, but its observation, in such cases would be much more difficult.

OBSERVATION OF SIMILAR EFFECTS IN DIFFERENT TYPES OF PHASE CHANGES

It is not only in the case of melting or solidification that such phenomena occur; it has also been observed (COSTA RIBEIRO, 1945b & 1950) that charges are developed in crystals of naphthalene precipitated from a concentrated solution of naphthalene in alcohol, benzene, toluene or ether.

A careful investigation is being carried out presently, at our laboratory, by A. DIAS TAVARES, S. MASCARENHAS and E. RODRIGUES (with fellowships granted by the Conselho Nacional de Pesquisas), on the thermo-dielectric properties of crystals of naphthalene, precipitated from a solution of toluene, and of the charges liberated from such crystals, by its dissolution in different solvents.

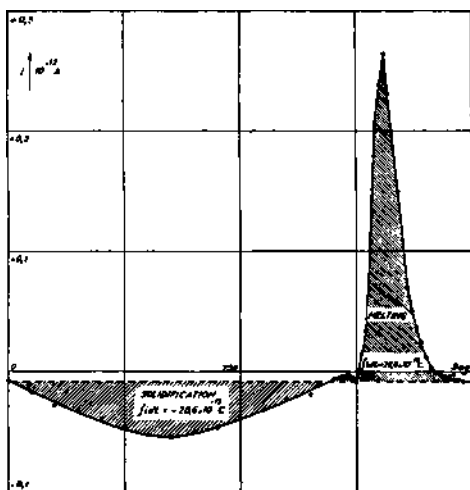


Fig. 7 - Reversibility of the thermo-dielectric effect (measurements done with Paraffin, employing a cell with plane electrodes).

Preliminary results (not yet published) of these experiments seem to indicate that effects very similar to those observed in solidification or melting of naphthalene can be reported in the precipitation or in the dissolution of naphthalene in different solvents.

Residual charges have already been observed in crystals of naphthalene resulting from sublimation of that substance, (COSTA RTBKIRO, 1945b & 1950), and also in solid carbon dioxide (COSTA RIBEIRO, 1947).

The facts above reported seem to suggest that thermo-dielectric effects are observable in all phase-changes in which one of the phases is solid.

It would be interesting to investigate if a similar effect could be observed in transitions of two solid phases of the same substance, as, for instance, in the transition from α to β allotropic varieties of sulphur or in polymorphic transitions of other substances.

DISTRIBUTION OF THE RESIDUAL CHARGES RETAINED IN THE SOLIDIFIED DIELECTRIC

A very careful investigation of these charges has been carried out recently by A. DIAS TAVARES (1952). As a consequence of this study we have strong evidence to believe that, during solidification of naphthalene negative charges are expelled from the solid phase into the liquid one; that the positive charges are spatially distributed in the mass of the solid deposit, and that a thin external film of negative charges is adherent to the surface of the solid deposit, that was in contact with the liquid during the process.

These results are very important because they contribute to explain many particular features of the phenomena observed, and will certainly permit to relate the value of these residual charges, to the value of the thermo-dielectric constant as determined by other methods.

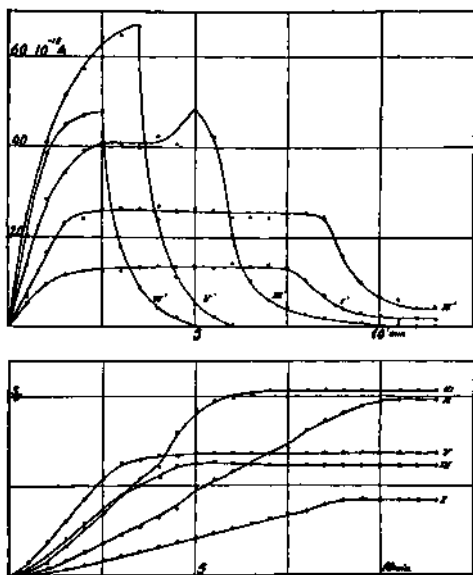


Fig. 8 - Current-time and mass-time curves for different rates of solidification of naphthalene. I, II, III, IV and V - Mass curves corresponding to different transition velocities. I, II, III, IV and V - Respective current curves

ELECTRIC CHARGES DEVELOPED IN THE FREEZING OF WATER. METEOROLOGICAL IMPLICATIONS

Recently we have had knowledge, by a paper of CLAY & KRAMER (1947) that a group of works had been published during the last war by meteorologists in Germany, dealing with the interpretation of atmospheric electricity developed in thunderstorms (FINDEISEN, 1940 & 1943; LANGE, 1940).

Especially remarkable is the paper by FINDEISEN (1940). This author admits that electricity developed in thunderclouds is related to the presence of ice particles.

This hypothesis was suggested to him by the characteristic noises produced in the wireless receiving sets of airplanes, approaching clouds containing ice particles.

With the aim of finding confirmation of this hypothesis, FINDEISEN made the following experiment: A metallic cylinder well insulated, and connected to an electrometer is placed inside a canalisation in which flows a current of air with a high degree of relative humidity or even charged with fog.

If the internal cylinder is previously cooled to a temperature attaining -60°C , the current of wet air is accompanied by the formation of rime at the surface of the cylinder, and the electrometer indicates a positive potential against the external canalisation.

FINDEISEN'S experiment has been interpreted, shortly afterwards, by LANGE (1940) in terms of contact or Volta-potentials between the solid and liquid phases of water; LANGE tried to confirm this point of view by the results of his own experiments on closely associated phenomena to those dealt with by FINDEISEN.

CLAY & KRAMER (1947) considering that there were some discrepancies between the results presented by FINDEISEN and LANGE, carried out new experiments along similar lines.

In many aspects the results of CLAY & KRAMER have confirmed those of FINDEISEN and LANGE, but there was not a complete agreement in some other aspects.

CLAY & KRAMER, for instance, could never observe the negative charge mentioned by FINDEISEN, during the evaporation period of the rime.

CLAY & KRAMER (1947) measured also with a ionium probe the contact potentials of water, ice and rime to air, finding a lower potential for rime covered surfaces than that found by LANGE.

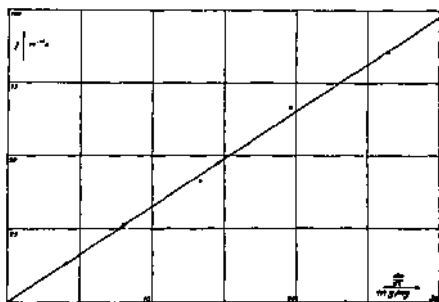


Fig. 9 — Proportionality between the intensity of the thermo-dielectric current and the transition velocity in phase changes occurring at a constant rate (bt. law of the T.D. effect).

These results of CLAY & KRAMER seem more in agreement with ancient determinations of such contact potentials as reported by SOHNCKE (1880) and by FAIRBROTHER & WORMWELL (1928).

One of the experiments of CLAY & KRAMER, employing an apparatus similar to that of FINDEISEN, has shown that the charging of the cylinder failed to occur when the air was completely cleared of all charged nuclei by means of a cotton filter.

This result inclined those authors to the view that the charging of the internal cylinder was conditioned by the contact potentials of the ice or rime to air, but was really due to the capture of charged nuclei in the air flowing around the cylinder.

ELECTRICAL PHENOMENA OCCURRING IN THE FREEZING OF AQUEOUS SOLUTIONS

A very important step in the study of the charges observed during freezing of aqueous solutions has been given recently by WORKMANN & REYNOLDS (1950). These authors have disclosed the important part played by the presence in the liquid, as contaminants, of very small concentrations of different ions in the electrical phenomena occurring during the freezing of such solutions.

Amonium and fluoride ions have proved to be the more effective in producing large potential differences during the freezing. At a freezing rate of 1 cubic centimeter per minute these authors have measured currents whose order of magnitude was 1 microampere.

The authors have pointed out that the character of the effect observed is a function of the nature and concentration of the contaminants in solution with the water.

Furthermore, these authors think that the observed effect is due to the capture of those foreign ions in the ice structure.

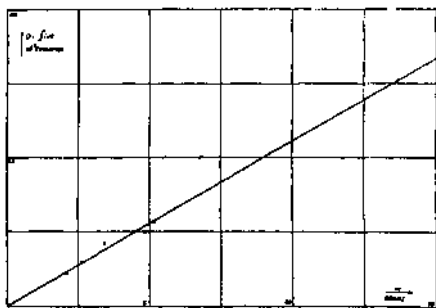


Fig. 10 — Proportionality between the charge developed and the mass of the dielectric which passes from one phase to the other, between two states of equilibrium of the diphasic system (2nd. law of the T.D. effect).

The important part played by ionic contaminants in the electrical effects observed during the freezing of water, as reported by WORKMANN & REYNOLDS, and confirmed by SCHAFFER (1950), suggests at once the idea of interpreting the phenomenon we have observed in naphthalene and other dielectrics and which we called *thermo-dielectric effect*, in terms also of ionic contaminants or impurities present in the material.

It is not however an easy task to decide, at the present moment, if, in a general way, ionic contaminants play an essential part in the thermo-dielectric effect.

The fact that we have observed and measured the currents produced in the solidification and melting of very pure naphtalene (many times recrystallised from alcoholic solutions and finally sublimated), together with the fact that this substance has a non-polar character, is a strong argument for the existence of the effect as a simple consequence of the changing of phase of the pure substance, leading to a thermo-dielectric constant, specific for each kind of dielectric.

This is not inconsistent with the idea that the presence of contaminants can produce an increase or a decrease of the observed effect, as a consequence or the capture, in the solid phase, of ions present in the liquid.

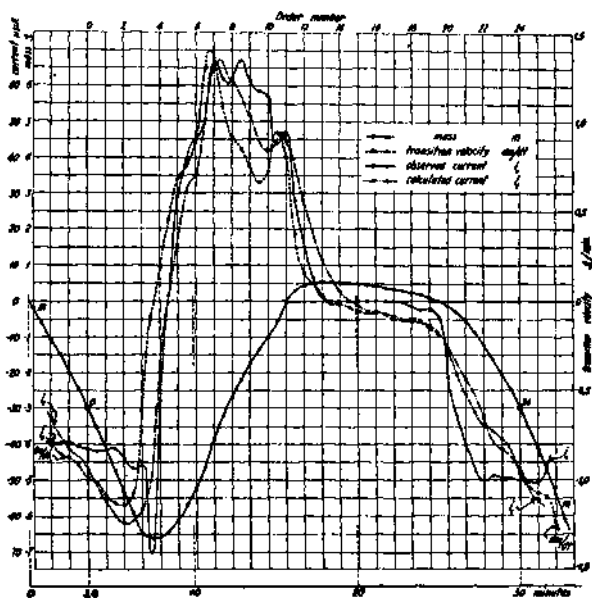


Fig. 11 Comparison between the curves of the observed thermo-dielectric current (i_0) and the calculated curve (i_c) assuming the exponential form the hereditary function (Courtesy of L. Cintra do Prado and P. Saraiva de Toledo).

According to this view, one should consider in the electrical effects accompanying the phase change, two different components: the *thermo-dielectric effect*, related to the nature of the dielectric, considered as a pure substance, and an *effect of Ion capture* by the solid phase, related to the nature and concentration of the contaminant ions.

S. MASCARENHAS is presently investigating carefully at our laboratory, the influence of contaminants on the value of the thermo-dielectric constant of naphtalene, and we hope that the results of such experiments will throw a new light on this problem.

Before this question is clearly settled we don't think that one should simply identify the thermo-dielectric effect, and the effect disclosed by WORKMANN & REYNOLDS, as was done recently by HOFFMANN (1950).

RELATIONSHIP BETWEEN THE THERMO-DIELECTRIC EFFECT AND THE CRYSTALLINE STRUCTURE OF THE DIELECTRIC

It has already been pointed out that the effects observed are strongly influenced by the way in which the solid phase is formed. The solid deposit formed in a thermo-dielectric cell, when the temperature gradient is small and the solidification proceeds very slowly, has the characteristics of a single* crystal, or at least, of an ordered and uniformly orientated deposit of single crystals (COSTA RIBEIRO, 1945 b). If the solidification is rapid, a disordered solid deposit is formed and a much smaller effect is observed, with frequently erratic results.

Careful investigations are being carried out presently by 10. RODRIGUES, in cooperation with Prof. E. TAVOKA, who is studying by X-ray diffraction methods the position of the crystal axis in different types of solid deposits of naphthalene, in order to establish what are the real connections between the observed electrical effects and the orientation of the crystals in the solid phase.

The first results of these studies seem to show that the electrical effects observed in the melting or in the solution of solid naphthalene are strongly influenced by the direction along which the interface solid liquid is displaced as related to the orientation of the axis of the crystals.

CONCLUSIONS

It seems that the disclosing of the very general character of the thermo-dielectric effect and the establishment of the general laws governing it throws a new light on many previous observations of electrical effects accompanying natural phenomena.

We hope that the knowledge of these laws disclosed in the well-controlled phase change of a pure substance like naphthalene, will help to interpret the observations reported in the complex conditions prevailing in the previous experiments on ice and rime by FINDEISEN, LANGE, CLAY & KRAMER. We hope also that new experiments will settle clearly the questions raised by confronting the observations on the thermo-dielectric effect with those recently reported by WORKMANN & REYNOLDS on aqueous solutions, whose important meaning for the understanding of electricity production in the atmosphere must be duly emphasised.

We are sure that the new field of research that has been opened by these correlations of electrical phenomena to the phenomena of phase changes, will lead in the future to interesting and important discoveries, for the benefit of mankind.

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DISCUSSION

GALLONI: Quería saber si se ha observado alguna orientación privilegiada en los cristales en la fase sólida obtenida después de una experiencia de medición del efecto termo-dieléctrico?

COSTA RIBEIRO: Este es un punto importante porque hemos observado que cuando la solidificación se hace muy rápidamente, los cristales que se forman son desorientados y se observa un efecto nulo. En compensación, si uno toma las precauciones necesarias para obtener un monocristal, o por los menos cristales uniformemente orientados, entonces se tienen efectos mucho más concordantes y de valor más grande. Es interesante observar que se puede verificar sencillamente el carácter monocristalino del depósito sólido formado, por medio de luz polarizada y se observa que la célula termodielectrica con que hemos hecho el trabajo, la célula con electrodos de inmersión, realiza prácticamente las condiciones ideales para formación de monocristales porque se tiene un gradiente de temperatura muy reducido entre la pared interna enfriada de la célula y la pared externa en contacto con el líquido termoequivalente y, cambiando la velocidad del aire insuflado, se puede realizar un gradiente de temperatura tan pequeño como se quiera, de modo a poder obtener monocristales en una superficie relativamente considerable. Pero la cuestión, planteada específicamente, de la orientación de los cristales después de la solidificación está siendo investigada por el Prof. E. Távora, junto con mi colaborador Sr. E. Rodrigues, y se ha observado realmente que en las películas sólidas obtenidas parece que la orientación de los cristales de naftaleno es siempre la misma en relación con los ejes cristalográficos. Hay también otra observación interesante en cristales obtenidos por precipitación de naftaleno en solución de toluol, que tienen la forma de agujas. Si estos cristales son fundidos en la dirección longitudinal presentan un efecto mucho menos acentuado que si son fundidos en la dirección perpendicular. Eso podría ser una consecuencia de la influencia de la orientación cristalográfica en relación al sentido en que se procesa el desplazamiento de la interfase, pero puede también ser debido al hecho que la distribución interna de las cargas en el cristal sea de tal orden que en la fusión longitudinal se tenga una neutralización progresiva de las cargas, flujos semejantes han sido observados en cristales tabulares obtenidos en solución de alcohol o toluol; cuando se funden o se disuelven en una dirección paralela a las caras más anchas del cristal se observan efectos mucho más apreciables que cuando la disolución se hace en dirección perpendicular a esas caras.

GOLDEMBERG: Poderia informar se observou alguma relação particular entre a viscosidade de fase líquida da célula e a corrente termodielectrica?

COSTA RIBEIRO: Por enquanto não fizamos nenhuma determinação da viscosidade mas acreditamos que deve haver uma relação, porquanto a viscosidade do líquido deve estar relacionada com a sua condutibilidade elétrica. E no caso do naftaleno, como já tive ocasião de dizer, observa-se facilmente que a condutibilidade é maior na fase líquida do que na fase sólida.

ESCOBAR: Quisiera saber si se ha medido la constante termodielectrica en el caso del agua, y se se ha medido la carga liberada por cm^3 .

COSTA RIBEIRO: En el caso del agua hemos hecho sólo experimentos cualitativos. Es uno de los trabajos que estaban planeados para el laboratorio, en el corriente año, la determinación cuidadosa de la constante termodinámica del agua. Pero la publicación del trabajo de Workmann y Reynolds determine una modificación del plan de trabajo primitivo. Sería ahora necesario conducir estos experimentos teniendo en cuenta no solamente el punto de vista en que nos habíamos colocado, de una constante dependiente de la naturaleza del solvente o del dieléctrico que se solidifica, pero también la evidencia aportada por aquellos autores en el caso del agua, de la acción de contaminantes iónicos. Creemos que el más probable es la existencia de un efecto sin contaminantes que es superpuesto al otro, pero de todo eso se concluye que es realmente muy interesante hacer determinaciones precisas en el caso del agua.

ESCOBAR: Ha llegado a la fecha a alguna conclusión con relación al agua? Ha llegado a tomar algunas mediciones?

COSTA RIBEIRO: Prácticamente no. Hay también una cuestión en el caso del agua que es muy delicada, es que el agua en la fase sólida es un dieléctrico, pero en la fase líquida es un conductor y como la temperatura ambiente es relativamente elevada, los depósitos de hielo que se forman son prácticamente recubiertos de una película líquida conductora. Para evitar la influencia de esa película conductora que anula prácticamente la posibilidad de observación de las cargas, es necesario hacer los experimentos en temperaturas extremadamente bajas lo que, en el clima de Río de Janeiro, infortunadamente, no es muy fácil.

ESCOBAR: Sería muy interesante hacer una cosa similar e incluso a presiones medias o presiones relativamente bajas que es donde tiene lugar la condensación y la formación higroscópicas en el caso del hielo.

COSTA RIBEIRO: Es sumamente importante verificar incluso la posibilidad de aplicación de este efecto para interpretar las diferencias de potencial observadas en las tempestades. Yo pienso que Chacaltaya sería el lugar ideal para experimentos de este tipo porque allí, no tienen los mismos problemas de temperatura ambiente que tenemos en Río.

ARRUDA: Desejava saber, se não seria interessante fazer experiências também com enxofre em estado de fusão, porque tenho a impressão de que o enxofre, logo após a solidificação descende cargas elétricas consideráveis. É uma observação que tive oportunidade de fazer,

COSTA RIBEIRO: Realmente também observamos essas cargas residuais no enxofre solidificado. Não fizemos ainda até agora uma experiência durante o processo de solidificação do enxofre, o que evidentemente deverá ser feito logo. Um dos meus colaboradores, S. Mascarenhas, tem mesmo o grande desejo de fazer experiências, para verificar se se observa algum efeito similar na passagem das variedades alotrópicas α e β do enxofre¹, porque, se se trata de um efeito ligado a estrutura cristalina, é possível que nessa transição de α para β se possa observar alguma coisa de interessante, mas tal experiência ainda não foi feita.

MEDINA: La parte de la corriente¹ que U_d demostro ser proporcional a la velocidad de cambio de fase, probablemente indica que hay una relación entre la energía de activación del cambio de fase y la producción de la corriente. Si esta relación existiera debería manifestarse probablemente como una alteración de la temperatura del cambio de fase en relación con el potencial aplicado a la célula. Se ha hecho alguna determinación de esta naturaleza, se ha tratado de correlacionar el trabajo eléctrico total con la energía de activación del cambio?

COSTA RIBEIRO: NO, ese es un aspecto nuevo que no ha sido todavía considerado. Yo le agradezco de haber llamado la atención sobre eso. En uno de mis trabajos anterior yo hice una observación sobre la posibilidad de hacerse una comparación entre los resultados teóricos de la fórmula de Clapeyron para el cambio de fase y los resultados experimentales de medidas directas de calor de cambio de fase, pero yo pienso que probablemente el orden de magnitud de la energía puesta en juego con el efecto termodinámico debe ser tan pequeña comparada con la otra que corresponde al cambio de fase que solo con mediciones muy delicadas podrían ser observadas las discrepancias.

MEDINA: YO no me refería tanto a la energía total del cambio de fase como aparecería en la ecuación de Clapeyron por ser en efecto mucho mayor que la parte que sería observable, sino me refería más bien a la energía de activación del cambio de fase, tal como aparecería, por ejemplo, en una ecuación de Arrhenius.

COSTA RIBEIRO: Tiene razón, eso no ha sido hecho. Yo le agradezco la sugerencia; es muy interesante.

H. SCHWARZ: Gostaria de saber se o senhor achou alguma dependência do fenômeno termo-dielétrico, com outras const antes como por exemplo o momento de dipolo dos líquidos, o que poderia talvez indicar o caminho para uma explicação do efeito.

COSTA RIBEIRO: Naturalmente e de se supor que todos os processos que envolvam a produção de íons ou a liberação de cargas, possam ter influência sobre a corrente observada. Entretanto, infelizmente não pudemos ainda realizar a exploração dos diversos caminhos que se apresentam no estudo desse novo fenômeno de modo que esses pontos não foram ainda investigados.

KIBELKA: O senhor já tem nas suas referências exemplos de substâncias sem dipolos e com dipolos; quais os valores da constante do seu efeito nos casos de ácido clorídrico e do beta-naftol?

COSTA RIBEIRO: No caso do ácido clorídrico, fizemos apenas uma verificação qualitativa da existência do fenômeno, mas não fizemos ainda a determinação precisa da constante. Entretanto observações feitas por meu assistente, Sr. Dias Tavares, parecem indicar um fato muito interessante: e que as 2 variedades alfa-naftol e beta-naftol apresentam constantes termo-dielétricas dos sinais contrários. Não é esta ainda uma conclusão definitiva, mas o resultado preliminar de experiências recentes e a contaminação do naftaleno comum com essas impurezas seria talvez o motivo pelo qual temos encontrado dificuldades em obter resultados muito coerentes com o naftaleno comum, os resultados ainda divergem em torno de 10 e até 20% em certos casos. De modo que a sugestão é muito interessante, é possível que essa diferença de sentido ou de sinal no valor da constante, esteja relacionada com a estrutura molecular,

The Electret

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Since its discovery the electret (GROSS & DENARD, 1945; GROSS, 1949 & 1950), frequently has been taken as a freak rather than an object worth while of serious consideration. Ad-hoc invented theories increased the distrust it met with. The present exposition shows that the behavior of the earauba wax electret can be accounted for satisfactorily by well known and generally accepted dielectric and conduction effects: dielectric absorption, dependence of dielectric polarization on temperature, and disruptive conduction currents. The unusual feature is the coexistence of all these mechanisms which under normal conditions are encountered separately.

Dielectric absorption, observed in piratically all solid dielectrics, is caused by delayed orientation of dipoles or molecular arrangements, and/or by ionic polarization. In consequence of absorption, a capacitor containing a solid dielectric can neither be charged nor discharged instantaneously.

Temperature exerts a strong influence upon this effect. Although leaving practically unchanged the total amount of charge absorbed, it affects the charging and discharging rates. Both charge and discharge are fast at high temperature and slow at low temperature. When use is made of this effect, a considerable amount of charge can be stored in a shorted capacitor for a very long time. For this purpose the dielectric is charged at a high temperature. Then the temperature is reduced without the applied voltage being withdrawn; no discharge can occur in this stage. Finally the capacitor is shorted at a sufficiently low temperature. Now the rate of discharge will have become so slow that only a fraction of the total charge dissipates within a "reasonably" short time. The rest remains frozen-in; it can however be liberated by reheating.

A dielectric containing a frozen charge is necessarily the seat of electric fields which induce surface charges on the plates covering the dielectric. By lifting the plates, these charges become free and can be measured and discharged. Upon reassembling the system, they appear again like in an electrophor. Thus dielectric absorption combined with an adequate heat treatment will produce the electret effect. The polarity of the absorption charges is such that the surface of the dielectric adjacent to the anode carries a negative charge, that adjacent to the cathode a positive one.

In all devices which have (and can) be used for such measurements, the electrodes make not molecular contact with the dielectric. An interface exists where the dielectric constant is less and the dielectric strength lower than inside

the dielectric. The low value of the d.c. favors field concentration in the interface, the low strength favors breakdown. Consequently even a relatively low mean value of the applied field can give disruptive phenomena in the interphase. Such phenomena have been observed independently in voids and at the surfaces of cables and technical insulators; in this connection they were already studied extensively due to their decisive influence upon the performance of the insulator. They result in transfer of charge from the electrodes to the surfaces of the dielectric, and formation of a ionic surface layer. This charge of course has the polarity of the electrode it comes from. Thus at the anode the dielectric becomes charged positively, at the cathode negatively. After the short circuit, the ionic charge remains trapped on the dielectric: Inside the dielectric, no field is left that could allow for conduction current and "internal" discharge; in the interface, the field has fallen below breakdown threshold and this excludes "external" discharge.

Superposition of the charges due to dielectric absorption and to interface breakdown cause the peculiar behavior and polarity reversal of the global charge as it is observed in experiment. An adequate experimental technique and method of analysis provides one with sufficient evidence for a quantitative interpretation and leaves not much room for extravagant, theories.

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DISCUSSION

FLOSS: I would like to mention some experiments made at the General Electric Co. several years ago, which are on entirely different systems, but show some correlation with your observations and may perhaps resolve the question whether the charge is ionic or dipolar. The system studied was polyvinyl chloride, plasticized with diphenyl and containing tetrabutylammonium bromide as a soluble electrolyte. Here we have a dielectric which contains ions which we know are present, because we deliberately added them. Now, if we measure, just as Dr. Gross did, current as a function of time; at fixed voltage, we find a decrease of current which very much resembles his curves. If we reverse the voltage, the curve immediately goes back to the original value; that is, the charge built up is destroyed at once and short-circuited. If, on the other hand, we at this point turn off the voltage and then simply measure in a bridge; circuit the effective resistance, then we find a very slow discharge current. Charging voltage was 500 volts across a 2 mm. sample; bridge voltage was only ten. Now, here our time scale was of the order of minutes whereas yours were of the order of hours: the difference is that we have very low viscosities in contrast to the very high viscosities of Carnauba wax, but I think the parallel between the two experiments is suggestive.

GROSS: SO, you conclude that there are many ionic charges.

FLOSS: The shape of the curves and the general behavior are quite similar. The only difference is in the time scale, and that is directly traceable, I think, to the viscosity difference.

COSTA RIBEIRO: Não sei se o Dr. Gross teria tido conhecimento de um trabalho cujo resumo foi publicado nas "Letters to the Editor" da Physical Review, de autoria de Franklin, a propósito da produção de hetero-cargas, sem homo-cargas, utilizando-se discos de enxofre para evitar o contacto dos electrodos com a superficie do dieletrico, Tenho a impressao de que e mais uma confirmacao experimental do ponto de vista que Gross sempre tem defendido, em relacao a formacao das homocargas.

GROSS: I think that it is entirely in agreement with the theory that the homocharge is exclusively due to the transfer of ions of charge from the plate to the dielectric. If you separate

the dielectric and the plate by an intermediate layer, then the homocharge would accumulate in the intermediate layer. If the system is disassembled and the intermediate layer removed, then the homocharge is taken away, too.

COSTA HIBEIRO: Gostaria tambem de assinalar uma coisa interessante. Nas experiencias que temos feito com dissolucao de dieteticos verificamos, que e possivel medir as cargas a medida que vai sendo dissolvido o cristal de naftaleno. Seria talvez tentar um metodo semelhante para o estudo da distribuicao interna nas cargas nos eletretos, utilizando-se um solvente que pudesse dissolver completamente a substancia de que e feito o eletreto e examinar como se distribui a carga no interior do eletreto. Qual a opiniao do Dr. Gross sobre essa sugestao?

GROSS: I think it would be interesting, and as a matter of fact we have started some measurements on this line. My assistant Miss Leda Laeerdas some time ago tried to make measurements. We took several discs, in this case ebonite, and they were treated in exactly the same way, that is, charged and the charge was frozen in. Then we took one of the discs and observed the total amount of absorbed charge. The second disc we put in a lathe and shaved off a certain amount at both surfaces and in this way just reduced the thickness of the disc and observed again the charge, and so on. In this way we got a curve which gives in function of the thickness of the discs the frozen-in charge. Actually we got a straight line with a finite inclination. Extrapolation to zero gives the dipole component. The difference between the maximum and the minimum charge gives the ionic component. But I must say that these measurements are not yet finished, so I could not come to a definite conclusion.

COSTA HIBEIRO: AS cargas de atrito desenvolvidas no processo de tratamento mecanico da superficie nao afetam apreciavelmente os resultados das medidas?

GROSS: In this case we did not measure the surface charge with a dissectable condenser. We heat the system and we measure the liberation of the frozen-in charge. Your observation is very important because some people tried to measure charges after the dielectric had been mechanically treated. I do not think these measurements are correct.

Baristors

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ABSTRACT

The author has under development a new class of semiconductors characterized by a notable ohmic variation under compression, traction or torsion together with the properties of symmetrical baristors and thermistors. The baristors, as they have been tentatively named, chemically are mixtures of an oxide and the metal thereof or even different metals and oxides. Nevertheless their electrical properties depend principally upon the process of manufacture or special treatment.

In this paper in addition to a brief survey of the present status of the research the author suggests an elementary theory to explain the effect of mechanical efforts or temperature on the ohmic resistance of baristors. This theory is based on analogies experimentally verified between diffusion phenomena observed in ionized gases and the behavior of baristors in identical circumstances.

Discussion is limited to the properties of baristors, as the manufacturing process still is under laboratory stage and will be covered by a further paper. Samples of baristors are available on request to interested persons.

INTRODUCTION

It is known that there is as yet no completely satisfactory explanation of the behavior of semiconductors, although a great deal of rather disconnected work has been published on their properties. In view of this lack of uniformity it seems desirable to give a brief resume of certain fairly well established principles of the nature of semiconductors which are invoked in the remainder of the discussion.

In the lattice structure of a metal the atoms are so closely spaced that the valence electrons are as much associated with one atom as with another, so that they are relatively free to travel around from one to another. They are therefore called "free electrons" and can move easily in response to electric fields. This means, of course, that the region where they move is not completely filled and also there are not inside the metal, levels of higher energies. Conduction will result from the introduction of the metal into electric circuits and hence the name of *conduction band* given to this energy band. In an insulator lattice the internuclear distance is greater than that at which the energy band overlaps and the valence electrons completely fill one band. In this case, between an empty band of allowed energy level and the filled band there is a region of higher energy level sometimes called the *forbidden region*. Diagrammatically the energy bands of an insulator are

usually represented as in figure 1, where a is the filled band, b is the forbidden band and c is the conduction band. The height ΔE of the forbidden region may be several electron volts high. At room temperatures an applied electric field cannot remove an electron from the filled band to the conduction band unless its strength is of the order of a million volts per centimeter.



Fig. 1

A semiconductor may be regarded as an insulator in which the width of the forbidden region is relatively small. This reduction of the forbidden region may occur a) — because the gap in energy between the filled band and the conduction band is small (of the order of 1 ev) and the semiconductor is said to be of the intrinsic type; b) — because of the presence of extra energy levels as a result, for example, of defects in the crystal lattices and the semiconductor is of the extrinsic type. Intrinsic and extrinsic conduction may occur in the same semiconductor. Chemically pure germanium or silicon or stoichiometric cuprous oxide are semiconductors of the intrinsic type. Nevertheless, stoichiometrically imperfect metallic oxide semiconductors are of the extrinsic type. Extrinsic semiconductors in which the conduction is principally due to the presence of electrons near the conduction band are said to be of the "n—type" (negative charge carrier); the process is called "excess" conduction and the extra levels act as *donators*. Extrinsic semiconductors like Cu_2O are of the "p—type" (positive charge carrier) because of the presence of empty levels ("holes") in the filled (nearly) band; the process is called *deject* or *deficit* conduction, because some of their oxygen atoms have single instead of double negative charges and the extra energy bands act as *acceptors*. Figure 2 shows the energy bands in cuprous oxide. Added foreign material or *impurities* cause extrinsic conductivity in the same way as defects in the crystal. If the impurity has more electrons in its outer shell than the lattice ion there will be free electrons in the crystal and hence an "n—type" of conduction; holes or a p—type conduction will result of impurities with less electrons in its outer shell than the lattice ion.

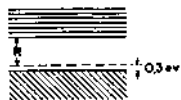


Fig. 2

The resistivity of semiconductors is very susceptible to temperature variations, the explanation being that electrons with added thermal energies may be raised easier from the filled band. This means too that the resistivity decreases with temperature; theoretically it should decrease exponentially and experimental investigations over a wide range of temperatures show that it does so. The ease with which electrons or holes are moved in semiconductors by an applied electric field of one volt per centimeter is called "mobility" and is measured by the drift velocity of the particle in centimeter per second.

NATURE OF THE MATERIAL

The author presents and demonstrates a new class of semiconductors he has under development, characterized by a notable sensitivity to mechanical efforts (compression, traction, torsion) together with the properties of symmetrical baristors and thermistors. The marked property of variation of resistance with compression was the first observed and hence the name of baristors, meaning

pressure sensitive resistors, tentatively coined in the earlier days of the research. This name is not sufficiently broad in meaning to cover all the properties of this class of components, but at the present stage of development it seems too early to decide about a new one.

Discussion here will be limited to the general properties of baristors, as the manufacturing techniques are yet under the laboratory stage and will be covered by a future paper. An account of the earlier steps of the development has already been made on a communication to the Academia Brasileira de Ciencias. This communication was accompanied by a collection of baristors of different compositions. Afterwards, when sufficient experimental data is available, a quantitative treatment covering the different types of baristors will be presented.

This class of materials consists of fused mixtures of oxides, such as ferric oxide (Fe_2O_3), nickel oxide (NiO), cuprous oxide (Cu_2O) with the respective metals or even with the addition of different metals or oxides. The *copper baristor* for instance, is a mixture of cuprous oxide and metallic copper. Up to the present we have prepared copper baristors, iron baristors (magnetic), nickel baristors and many complex baristors, like iron-copper baristors, tungsten-copper baristors, etc. They have resistivities in accordance with the composition and special treatments. The resistance of different kinds of baristors at room temperature ranges from a few ohms to the order of several megohms. The process of manufacture of the copper baristors favors an increased content of CuO or an excess of O in the cuprous oxide. This means more copper ions missing from the Cu_2O crystals and an increased number of positive holes per unit volume. On the other hand, it seems probable that the copper baristor contains interstitial solid solutions where the smaller copper lattice ($a_0 = 3.6$) fits into the larger Cu_2O lattice ($a_0 = 4.25$) as the atomic diameter of copper is very much smaller than that of the oxygen one. The validity of this assumption is open to question and is to be verified by X-ray and further investigations.

ELECTRICAL PROPERTIES

For the sake of brevity we shall illustrate the behavior of baristors by taking the copper baristor as example. Other baristors behave in similar way, though they may differ entirely in chemical composition. In Cu_2O the highest filled band is the Cu^+ band and the hole is provided by a Cu ion converted to a cupric Cu^{++} ion. The introduction of metallic copper into the cuprous oxide lattice provides an additional bridge to the conduction band and hence the extreme sensitivity of the device as a semiconductor. It can easily be seen why the resistance of baristors is so sensitive to variations of temperatures and indirectly to variations of current or voltage. The conductivity of a semiconductor is expressed by

$$\sigma = n e \mu_0$$

and

$$n = (2) n_0^{\frac{1}{2}} \cdot 5 \cdot 10^7 T^{\frac{3}{4}} e^{-\frac{K}{2KT}} \quad (1)$$

where

n is the number of electrons in the conduction band when the temperature is low and n small compared with n_0 .

e is the charge of the electron.

u_0 is the mobility.

n_0 is the density of centres of impurity or lattice vacancies.

T is the absolute temperature in °K.

R is the depth of the centers of impurity or lattice vacancies below the conduction band.

k is Boltzmann's constant.

The factor 2 in brackets prevails only when *every* band is occupied by two electrons of opposite spins (WRIGHT, 1950). In case of copper baristors, where we have both extra energy levels and metallic copper, R is very much reduced and n_0 increased. This may account for the extreme sensitivity of baristors to temperature variations. By connecting an ohmimeter to a baristor it is possible, at room temperature, to detect temperature variations of 10°C or less. On the other hand, at relatively high critical values of T (or of the current through the baristors) they practically short circuit the generator across them. This seems to indicate that n has reached a very high value and the electron gas become degenerate. At these temperatures (very below the melting point) the baristors behave like semimetals. This phenomenon suggests the use of baristors as thermal relays, since below the melting point of the material the resistance resumes its former value when the temperature is lowered.



Fig. 3



Fig. 4

Fig. 3

Fig. 4

Figures 3 and 4 are I/V characteristic curves of a copper baristor and of a complex baristor (iron-copper) respectively. Voltages along the x -axis.

STRESS-SENSITIVE DEVICES

It is well known that the resistivity of semiconductors decreases, in general, at high pressures. In particular the resistivity of tellurium decreases by a factor of more than 000 at a pressure of 30,000 kg/cm². Tellurium is a semiconductor of the intrinsic* type with an energy gap between the filled band and the conduction band of about 0.38 ev. The decrease of the resistivity of tellurium is explained by J. BARDEEN as a result of a decrease in the energy gap, the gap

becoming very small (0.015 ev) at 30,000 kg/cm² (BARDEEN, 1949). Nevertheless, the resistance of a button of baristor under compression applied by the operator's fingers may vary considerably, say from 5 megohms to 30,000 ohms; after the removal of the pressure the resistance resumes its former value. Further investigations confirmed that the resistance variations of baristors were in dependence on deformations or changes of dimensions produced by the efforts. We shall shortly see that other experiments on the behavior of baristors as low pressure electron gas containers lead to the same conclusion. There are some striking analogies between the behavior of baristors and ionized gases. For brevity, in the present communication let us

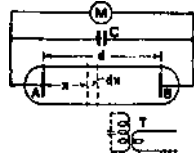


Fig. 5

be restricted to an analogy someway correlated to the explanation of the stress-sensitivity of baristors. To the gas at low pressure enclosed between the two electrodes A and B of a glow tube (for instance, an ordinary fluorescent lamp) is applied a high frequency field by means of a small Tesla coil T, figure 5. The capacitance C is high enough to provide a 11. F. short circuit, so as to avoid any external difference of potential between A and B. By ionization of the gas a D. C. current will flow through the low internal resistance meter M. The direction of this current is reversed when the coil is moved from one electrode to the other and no current is produced if the coil is held in a position midway between A and B. Keeping the coil near one of the electrodes the position of the glow tube may be reversed (if provided with identical electrodes) without reversing the direction of the current, because this is strictly a diffusion phenomenon. The diffusion in case of ionized gases may be treated exactly as in neutral gases. "Any conclusion deducible from a study of neutral gases can be immediately extended to the motion of ions in a gas, for ions will behave very much like one neutral gas of very low concentration mixed with another gas of much higher concentration (MAXFIELD & BENEDICT, 1941). Such being the case and if we assume that the diffusion process has been established long enough for a steady state to be reached, the number of particles crossing a given unit area in unit time will be

$$-D \frac{d\rho}{dx} = \rho v$$

where D is the diffusion coefficient, ρ the concentration of ions and

$$v = -\frac{D}{\rho} \cdot \frac{d\rho}{dx}$$

the mean velocity of the diffusing ions. Supposing all ions of the same charge (q) and sign, the transport of an elementary charge $dQ = q\rho dx$ from a unit cross-sectional area of A to B will be made in the time $t = d/v$ and the current density read by the meter should be

$$J = \int_A^B \frac{dQ}{t} = -q \frac{D}{d} \int_A^B d\rho$$

Or

$$J = q \frac{D}{d} (\rho_B - \rho_A)$$

This result is in accordance with our preceding experiments and shows that "the circuit current is dependent only on the ion concentration immediately in front of the electrodes and not on the ion distribution in the intervening space (COBINE, 1941). The effects of reflexion and ion neutralization at the electrodes were neglected. They may cause a reduction in the current value given by the formula above.

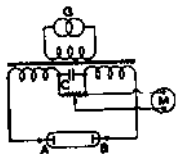


Fig. 6

Now if a baristor is put in place of the glow lamp, very similar effects are observed, but the operation is rather awkward due to its small dimensions. To operate in better condition the electric field may be obtained by means of a low frequency voltage, as represented in figure 6. In this circuit any type of glow lamp or baristors can be experimented with. As in the previous experiments, by provoking different ion concentration in the electrodes of the glow tubes we can obtain D. C. currents and the direction of these currents also depends on p_A and p_B . Here again exactly the same phenomenon occurs with baristors put in place of the glow lamps as if there were different charge concentrations in the vicinities of the inserted leads A and B and the current through the meter seems to have direction and value depending on n_A and n_B . Furthermore, mechanical stresses, particularly compression, applied to locations A and B of dissimilar areas modify the current as if the values of n_A and n_B were changed.

This lends support to the assumption that the deformations or strains existing around the point of insertion of the leads, or provoked by pressures applied to the leads, are sufficient to locally decrease the energy gaps. The special lattice structure of the baristors may explain why relatively low pressures can modify the energy gaps.

We are now in a position to understand the behavior of baristors under mechanical efforts. When submitted to relatively low uniform (hydraulic) pressures there are practically no changes of dimension and no changes in energy gap occur.

If the applied efforts produce elastic deformations in some locations, there will be variation of energy gap causing the resistance to vary; in this case as one would expect the resistance resumes its former value after the removal of the efforts. Permanent deformation is accompanied by a permanent modified resistance.

In brief, the variation of the resistance of baristors under mechanical efforts is here explained as variation of R , equation 1, as the result of elastic deformations.

Temperature differences at A and B also modify the direction and value of the current in the external circuit. Furthermore, if this difference of temperature is high enough a current is obtained even without application of any external voltage. Baristors are indeed very sensitive thermo-electric devices.

CONCLUSION

It is perhaps too early to discuss the possible uses of baristors although some applications appear to be obvious. A considerable amount of measurement and engineering work has yet to be done. The main difficulty to overcome is concerning the precision of measurements. As it became apparent from the fore-

going theory all contacts must be well defined and stable, otherwise reproducible results are difficult to obtain.

The production of baristors of small dimensions is no more a problem and they may be obtained at very low cost. At present, they are made in the form of buttons, beads (with 1, 2, 3 or 4 inserted leads) and tablets. In a near future we hope to be able to obtain the baristors in any size or shape.

The author acknowledges the valuable assistance of Mr. C. J. Rezende in the design and construction of many apparatus used in connection with this research.

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DISCUSSION

COSTA RIBEIRO: Qual a maneira mais conveniente de se fazer a verificação experimental desta variação da resistência com a pressão aplicada, nesse tipo do material?

BARBOSA: Temos empregado vários processos, inclusive bombas hidráulicas; mas, quando usamos bombas hidráulicas com o material totalmente imerso no líquido (óleo por exemplo), e com pressões não muito altas, pois não dispomos em nosso laboratório de pressão superior a 250 quilos por cm^2 , não se podem verificar variações da resistência. Então é preciso deixar uma das faces exposta. Por exemplo, o material é cortado sob a forma do cubo e uma das faces fica exposta à pressão atmosférica, por meio de uma membrana de borracha ou qualquer outro recurso. Nessas condições, é fácil fazer as medidas, tendo-se um manômetro graduado, do manômetro que as leituras se apresentam diretamente a compressão exercida sobre o material. Quanto aos aparelhos da medida, um simples ohmômetro geralmente dá resultados satisfatórios porquanto as variações são muito nitidas. As resistências dos baristores em alguns casos, como o exemplar que tenho aqui, podem baixar de 5 megohms ou mais, até a ordem de 150 ou 200 mil ohms. Os baristores do ferro, por exemplo, têm resistências muito baixas, 1 ohm ou menos as quais são também facilmente verificáveis em simples ohmômetros. Para resistências muito baixas sem dúvida, é aconselhável uma ponte.

COSTA RIBEIRO: Desejo saber principalmente qual a ordem da grandeza da variação da pressão necessária para produzir essas variações da resistência.

BARBOSA: Essa ordem da grandeza é uma incógnita, porquanto, em geral, as verificações são feitas com osciloscópios e as pressões são obtidas por meio de vibradores. Entretanto, nas medidas com a bomba hidráulica temos verificado que pressões de 1 e 20 quilos por cm^2 permitem variações de milhares de ohms. Há dificuldades também quando se exerce compressão com a mão ou com dispositivos tais como balanças, porquanto a pressão unitária vai depender da área de contacto, que é de difícil determinação. São essas pequenas dificuldades que ainda estamos procurando resolver, sendo de notar, também, que no caso dos semicondutores, o especialmente dos baristores, cuja sensibilidade é muito elevada, não se deve esperar a obtenção das medidas com a precisão que se obtém em metais.

COSTA RIBEIRO: Esses baristores em forma de pastilhas de sua autoria devem prestar-se melhor a isso, não?

BARBOSA: Sem dúvida. Entretanto, há apenas 15 dias que conseguimos pela primeira vez obter baristores em pastilhas.

COSTA RIBEIRO: O tratamento mecânico da superfície como o polimento, por exemplo, não influi sobre as propriedades do material?

BARBOSA: Provavelmente não influi, mas o tratamento térmico influi acentuadamente. Eu gostaria até de dizer, porque não vive realmente tempo de falar sobre tudo o que desejava,

que o baristor apresenta uma propriedade muito importante: a partir de certo valor da temperatura a resistência cai bruscamente a quase zero. Cai tão to que promove curto circuito violento no gerador a que está ligado. Como se vê, há sérias dificuldades na realização das medidas. Esta sendo tentado o emprego do baristor como "relay" térmico, porque a temperatura em que ocorre o fenômeno é crítica; pela elevação da temperatura, em dado momento, parece que o gás eletrônico se torna degenerescente e o baristor se aproxima de um metal, promovendo o curto circuito a que nos referimos.

POMPEIA: Quando o baristor atinge a temperatura em que deixa de apresentar as propriedades características, voltando a temperatura normal ambiente, ele se torna inteiramente as condições anteriores?

BARBOSA: Sendo atingido o ponto de fusão, o material se decompõe dando lugar a cobre e Cu_2O . Não sendo atingido o ponto de fusão, a resistência volta novamente ao valor primitivo; e por isso que poderemos empregá-lo como "relay" térmico.

POMPEIA: Quer dizer que não se alteram as propriedades de baristores?

BARBOSA: A experiência poderá ser repetida tantas vezes quantas se queira. Pelo menos assim aconteceu todas as vezes em que a realizamos.

GROSS: Gostaria de saber 1.º: a ordem de grandeza da condutividade até a temperatura normal e 2.º: se o material é policristalino ou que tipo de estrutura possui?

BARBOSA: Quanto a primeira pergunta, devo dizer que as constantes físicas não foram ainda determinadas, exatamente pelas dificuldades decorrentes da grande sensibilidade do material, principalmente nos pontos de engastamento das hastes. Qualquer variação de pressão, o próprio fato de se colocar a mão sobre o baristor, variações de temperatura, modificam imediatamente as condições de experiência. Não conhecemos realmente a condutividade até agora só temos medido resistências, pois não conseguimos determinar o volume exato das peças e as áreas de contato, dada a maneira pela qual são feitos os engastamentos das hastes. Com os baristores em forma de pastilha, ficara resolvido o problema, porquanto as formas regulares facilitarão a determinação da condutividade. Quanto a segunda parte, devo declarar que, por enquanto dispomos apenas de micro-fotografias do material e por essas micro-fotografias não se pode avaliar exatamente qual a textura. Contudo essas micro-fotografias parecem indicar que o material contém soluções sólidas intersticiais de cobre e Cu_2O . A estrutura definitiva do material somente será conhecida mediante o exame pelos Raios-X. O comportamento elétrico dos baristores levou-nos a arquitetar a teoria que apresentamos.

GROSS: No caso de haver uma estrutura polieristalina poder-se-ia pelo menos pensar na possibilidade da grande sensibilidade ser provocada pelas resistências entre os grãos?

BARBOSA: Nilo creio (me seja cste o caso, porquanto a resistência volta muito normalmente ao valor primitivo, desde que se tomem cuidados especiais. Além disso, há diversas correlações com outros semi-condutores (variação da resistência com temperatura e com as correntes elétricas, etc.) mostrando-nos que a estrutura deveria ser mais ou menos como procuramos estabelecer. Em todo caso, tudo isto são hipóteses, pois o material não está ainda completamente estudado. As pesquisas são relativamente recentes e é possível que ainda cheguemos a conclusões diferentes. Mas, ainda a respeito das suas observações, devo declarar que, não me foi possível, por falta de tempo, desenvolver completamente o assunto. Peço-lhe portanto a gentileza de reportar-se ao trabalho completo onde apresentamos uma confirmação dessa teoria, obtida por meios indiretos.

ARRUOA: Em relação a primeira fotografia do oscilograma apresentado, eu queria chamar especialmente a atenção para o fato, que se verifica de não se iniciar a descarga para a qual a corrente cresce sem controle, na condição de uma tensão mínima bem definida, como no caso da ionização de um gás. Acontece que, na descarga de gases, conforme a condição de ionização, uma curva exatamente idêntica aquela mostrada foi verificada. Normalmente a descarga do gás se inicia a um certo potencial de ionização, cresce sem controle e depois volta, mas não com uma curva exatamente do tipo mostrado. Quando, entretanto, as correntes são relativamente grandes, a curva não parte mais do mesmo ponto da abscissa e a ionização apresenta curvas do tipo mostrado pelo Prof. Barbosa para o gás neon ou outro gás rarefeito. A característica de disparar a corrente sem controle, apresenta curvas exatamente dessa mesma natureza.

GALLONI: Los ingenieros elasticos utilizan para medir tensiones en las construcciones o en los modelos la variación de resistencia de un pequeño alambrecito. Ahora, quisiera saber que tamaño tienen estas pastillas que podrían ser muy útiles para medir esas tensiones.

BARBOSA: Perfeitamente. As pastilhas podem ser construídas, no momento, até meio centímetro de diâmetro, mas contamos no futuro obtê-las em dimensões quaisquer. Para medidas dos esforços nos materiais, talvez seja possível formar o baristor na própria peça a ser estudada, ou então ligá-lo como V. S. se refere.

COSTA RIBEIRO: Poderia nos dar um esclarecimento sobre o processo de obtenção desses materiais?

BARBOSA: Temos obtido esse material em chamas de alta frequência. Quando se usam frequências e tensões muito elevadas, a descarga se processa de modo diferente do usual, no sentido de que o arco formado apresenta somente a chama e não o dardo comum às descargas intensivas. Esta chama parece ser altamente redutora, e pode ser dirigida sobre o material a ser fundido, colocado por exemplo, dentro de um cadinho. Esta chama tem temperatura muito elevada, temperatura que também não posso dizer qual seja porquanto é superior ao limite dos instrumentos de que dispomos. Funde-se facilmente o tungstênio, tudo nos levando a crer que a elevação da temperatura resulte de bombardeio eletrônico.

SALA: Queria saber se existe algum atraso ou se é imediatamente que volta o baristor, a situação normal, vamos dizer, quando se retira a pressão.

BARBOSA: Parece que volta imediatamente, porquanto temos construído "pick-up" fonográficos que reproduzem a música razoavelmente. É claro que a volta da resistência ao valor primitivo, sonicate ocorre quando não seja excedido o limite de elasticidade do material.

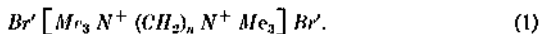
Eletrolitos Bolaformes

RAYMOND M. FUOSS

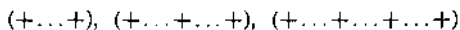
*Contribuido n.º 1110 da Sterling Chemistry Laboratory, Yale University,
New Haven, Conn., U.S.A.*

A teoria moderna (HARNED & OWEN, 1950) dos eletrolitos fortes baseia-se na hipótese de que sua interação é devida somente aos campos eletrostáticos dos íons, que se apresentam como esferas carregadas, rígidas, em um meio contínuo. A teoria corresponde satisfatoriamente aos dados experimentais para grande variedade de eletrolitos do tipo 1-1, mas é inadequada nos casos de grandes íons multiplamente carregados; em relação a polieletrólitos (Fuoss, 1951) e a colóides (VERWEY & OVERBEEK, 1948), por exemplo, um modelo muito mais complexo deve ser evidentemente usado, e mesmo para eletrólitos inorgânicos não simétricos, tal teoria só pode prever, no máximo, as tangentes limites.

O objetivo deste trabalho é rever estudo recente sobre eletrólitos bolaformes (Fuoss & EDELSON, 1951); são estes eletrólitos nos quais pelo menos dois pontos carregados estão ligados por átomos neutros, como nos catiões de sais bi-quaternários do tipo:



Sais de ácidos dicarboxílicos contêm aniões bolaformes. Esperamos que este trabalho seja útil no desenvolvimento de uma teoria eletrolítica mais geral: íons carregados multiplamente, como cálcio, lantânio e tório, podem ser considerados como o limite de eletrólitos bolaformes dos tipos 2-1, 3-1 e 4-1 ao tender para zero o comprimento da cadeia em estruturas do tipo:



Polieletrólitos em cadeia constituem um limite dos eletrólitos bolaformes, quando o comprimento da cadeia e o número de pontos carregados aumentam; eletrólitos coloidais compactos representam outro limite superior quando carga, densidade e tamanho aumentam simultaneamente. Alguns dados de condutibilidade e viscosidade para diversos eletrólitos bolaformes dos tipos 2-1, 1-2 e 2-2 serão apresentados aqui, em conjunto com alguns comentários sobre sua possível interpretação. O plano geral do trabalho de pesquisa é sintetizar-se e estudar-se eletrólitos bolaformes de estrutura conhecida, a fim de se tentar as extrapolações acima propostas.

Um sal quaternário típico é formado pela adição de brometo de metil a metil-dietanol-amina. Sua curva de condutibilidade é representada na figura 1 em círculos semicirculares; a linha reta é a tangente térfica de Onsager. Observa-se

que a curva se aproxima da tangente por baixo, indicando a formacao de pares de ions (associacao) (Fuoss, 1935).

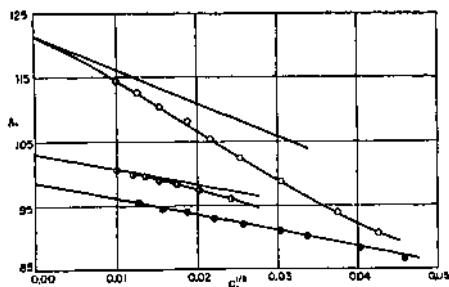
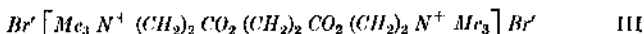
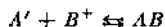


Fig. 1. Condutibilidade em metanol; O, I, $[HO(CH_2)_2]_2 N Me_2 Br$; O, II, $[CH_3 CO_2 (CH_2)_2]_2 N Me_2 Br$; O, III, $Br [Me_3 N (CH_2)_2 CO_2 (CH_2)_2 CO_2 (CH_2)_2 N Me_2] Br$. (J. Am. Chem. Soc. **73**, 271 (1951), Fig. 2.)

hidroxilos no cationte de I por acetilacao, forma-se um eletrolito II consideravelmente mais forte, como se verifica pelo fato de a curva de condutibilidade se aproximar muito mais da tangente limite. Evidentemente, a atracao entre os dipolos (positivos na parte externa) do cationte do sal I e o ionte brometo ajuda a estabilizar o par de iontes de I. No sal II, com um cationte; fortemente protegido, a associacao e^* praticamente insignificante em metanol. Considremos a seguir um eletrolito bolaforme do tipo 2-1



em cujo cationte os dois atomos de nitrogenio carregados positivamente sao ligados por dez atomos neutros. A interacao eletrostatica entre um ionte brometo e qualquer das extremidades do cationte nao pode diferir muito da interacao entre o mesmo ionte e qualquer dos dois cationtes nos sais I e II; todavia, observamos na figura I uma associacao muito maior no caso do sal III. Duas razoes podem ser apresentadas para esse fato: primeiramente, um anionte sente, a uma distancia regular do cationte III, a forza de duas cargas positivas; segundo, uma vez que ha dois possiveis aniontes e cada um deles pode assumir duas posicoes no cationte, seria de se esperar, cm bases puramente estatisticas, que a primeira constante de associacao do sal III fôsse



quando a concentracao aumenta. (Deve-se acentuar que o simbolo AB nao representa aqui um composto co-valente; os iontes componentes A' e B^+ conservam suas identidades em AB , isto e, nao ha transferencia de electrons). A estrutura 6 considerada estavel a impactos termicos somente quando o potencial de Coulomb e grande comparado a kT . Se cobrimos os grupos

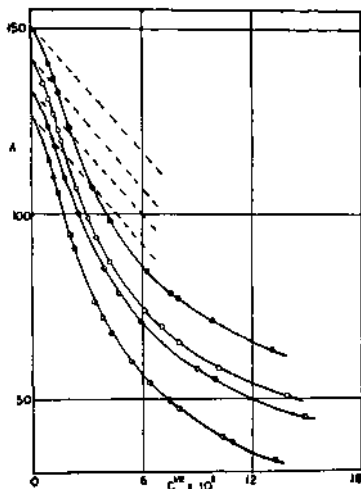
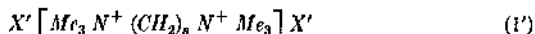


Fig. 2. Condutibilidade em metanol; O, $C_3 I_2$; Br ; O, $(\Lambda - 15)$ para $C_3 I_2$; O, $C_4 I_2$; O, $(\Lambda + 10)$ para $C_5 I_1$. (J. Am. Chem. Soc. **73**, 950 (1951), Fig. 2.)

quatro vezes maior do que a constante de um eletrólito do tipo 1-1 do mesmo tamanho.

Na figura 2 são apresentadas curvas da condutibilidade de vários sais (Fuoss & CHU, 1951) da fórmula geral



em metanol a 25°. As tangentes limites são representadas pelas linhas interrompidas. Assinalamos ainda uma vez que todas as curvas só acham muito abaixo das tangentes limites nas concentrações que estamos empregando, e concluimos que está ocorrendo considerável associação. (Para abreviar a anotação, designaremos os sais de estrutura (1) por 03, etc.).

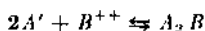
Ovemos mencionar aqui que tais curvas de condutibilidade (e outras semelhantes) podem enganar a primeira vista, pois é possível aproximar-se os dados nas concentrações mais baixas por linhas retas na figura 2. A teoria pede uma relação linear entre Λ e $\sqrt{c}$ em concentrações baixas, e alguns pesquisadores têm usado então, cegamente, as curvas pseudolineares para extrapolação para Λ_0 , a despeito de as tangentes serem muito mais inclinadas do que as exigidas pela teoria; valores de Λ_0 obtidos por tal processo são evidentemente altos demais. Na verdade, a curva de condutibilidade coincide com a tangente teórica a concentração zero. Em concentrações mais elevadas, a associação dos íons começa a tornar-se significativa, íons em pares não contribuem mais para a condutibilidade. A curva teórica de associação é côncava no lado de concentrações ainda mais elevadas e a região pseudo-linear é meramente devida à presença de um ponto de inflexão na curva teórica completa (Fuoss, 1935).

A fim de se fazer uma comparação quantitativa dos eletrólitos representados nas figuras 1 e 2, é necessário desenvolver-se um método para se obter os parâmetros descritivos dos dados.

Dois equilíbrios de associação são possíveis:



Um ânion pode associar-se a qualquer extremidade do cátion bolaforme (2); em seguida em concentrações mais elevadas, outro ânion pode ligar-se a outra extremidade da cadeia (3). (Incidentalmente, cálculos baseados na hipótese de associação simultânea de dois ânions



não puderam reproduzir os dados. Em outras palavras, necessitamos da condutibilidade da espécie catiônica AB^+ , carregada por uma única carga, a fim de reproduzir os dados experimentais). Se escrevermos as equações da ação da massa para os equilíbrios de associação, teremos:

$$[A'] [B^{++}] f_a f_b / [AB^+] f_{ab} = k_2 \quad (4)$$

$$[A'] [AB^+] f_a f_{ab} / [A_2 B] f_{a_2 b} = k_1 \quad (5)$$

As grandezas entre colchetes referem-se as concentrates molares, os f sao os coeficientes do atividade correspondentes, e os k sao as constant es de associacao reciprocas. A condutibilidade κ dada por

$$\Lambda = \sum \gamma_j (\lambda_j^0 - \Lambda_j) \quad (6)$$

onde γ_j a fracao do solvcl presente na espfoie j (aqui, A' , AB^+ e B^{++}), λ_j^0 a condutibilidade limite do ionte j isolado, Λ_j e o decrescimo cm mobilidadocvido a interacao eletrostatica a grande distancia ontre fontes livres. Em principio, substituindo-se (4) e (5) em (6) obteremos

$$\Lambda = F(\Lambda_0, k_1, k_2, c) \quad (7)$$

Entao a substituicao em (7) de valores Λ e c medidos possibilita o calculo das constantes. Na pratica, o processo nao e tao simples, devido a forma complexa das equates. E possivel, entretanto, obter-se solugao das equacoes por meios graficos (Fuoss & EDELSON, 1951; Fuoss & CHU, 1951; SACKS & Fuoss, em impr.). Para os eletrólitos das figuras 1 e 2, o calculo mostra que o equilibrio (3) e desprezível (isto é, nao ha cachos neutros A_2B nas concentracoes consideradas; aproximando, portanto, teremos $k_1 = 0$). Os valores de Λ_0 e k_2 sao apresentados na Tabela I para nossos eletrólitos bolaformes, em conjunto com os de varios outros eletrólitos comuns, (MEAD, HUGHES & HARTLEY, 1933; DAWSON & KEELEY, 1951). (No sal I-1, a última coluna dá naturalmente a constante para o equilibrio $A' + C^+ \rightleftharpoons AC$).

TABELA I

Constantes para eletrólitos em metanol.

SAL.	TIPO	Λ_0	k_2
$Me_4N^+ Br^-$	1-1	125,3	0,033
$C_3H_7N^+ Br^-$	2-1	133,1	0,0010 ₁
$C_3H_7N^+ I^-$	2-1	141,3	0,0008 ₁
$C_4H_9N^+ I^-$	2-1	142,1	0,0010 ₂
$C_5H_{11}N^+ I^-$	2-1	140,1	0,0013 ₄
$Sal III$	2-1	121,3	0,0045
$MgCl_2$	2-1	97,5	0,0005

O brometo de tetrametilamonio (MEAD, HUGHES & HARTEY, 1933) e urn elctrolito normal e pode ser tornado como substantia de referencia; todos os eletrólitos bolaformes da Tabela I tem grupos terminals Mc_3N^+ , e assim a menor distancia a que pode chegar um anionte e, com toda a probabilidade, quase a mesma tanto para eles como para o cationte Me_4N^+ . Seria de se esperar entao, em bases estatisticas, que nos eletrólitos bolaformes k_2 fosse a quarta parte de 0,033 ou seja 0,008. No sal III e no sal C3, k_2 e, respectivamente, 0,0045 e 0,0010; em outras palavras, a presenca da segunda carga aumenta a associacao (k_2 menor) e quanto menor se torna a cadeia, maior e a associacao. Tal resultado e esperado com base na atracao eletrostatica da carga dupla (BJERRUM, 1923). Uraa comparacao entre os sais de tri-, tetra- e penta-metileno mostra que para pequenos valores de n , o valor de k_2 aumenta menos rapidamente do que n , numero dos

dtomos entre as cargas; dste resultado sugere que o cacho $(-)(+...+)$ tende a enroscar-se quando n e menor, devido a atração entre as duas cargas e o anionte. Para valores maiores de n (p. ex. Sal III), a perda de entropia quando se fecha a cadeia excede o ganho de energia na energia livre ($\Delta F = \Delta H - T\Delta S$), e configurates mais alongadas tornam-se muito provaveis (k_2 maior). Em cadeias muito longas com grupos terminais Mg_3N^+ , esperamos que k_2 seja quase igual a 0,008, o limite estatístico.

Em cloreto de magn&io (DAWSON & KEELEY, 1951), duas cargas estao concentradas em urn linico atomo; encontramos (SACKS & Fuoss, em impr.), igualmente, que k_2 atinge apenas 0,0005. Este valor parece constituir urn limite razoavel, se extrapolarmos do Sal I II (onde $n = 10$) atraves do sal C3 ($n = 3$)

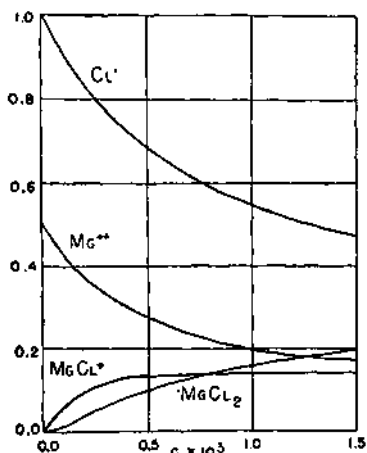


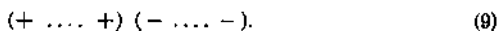
Fig. 3. Distribuição de cloreto de magn&io entre várias espécies em solução de metanol a 20°.

muito depressa quando a concentrayao se torna urn pouco maior do que zero. Conforme ja mencionamos, nos eletrolitos bolaformes, grupos ABA sao insignificantes acima destas concentrates.

Quando a concentracao (ou mais precisamente, a densidade das cargas) vai aumentando, torna-se mais provavel a associayao em cachos maiores (Fuoss & KRAUSS, 1933) sob a influencia de forcas eletrostitticas. Da mesma maneira, aumentando a carga, aumenta a associacao. Se consideramos eletrolitos bolaformes do tipo 2-2, como succinates e adipatos de sais bi-quaternarios, a associacao inicial pode conduzir a duas configurates possiveis: uma forma ciclica



e uma estrutura em cadeia



Pode-se conceber facilmente o desenvolvimento desta ultima mima cadeia com muitas ligayoes. Seria dificil decidir-se entre as estruturas (8) e (9) com base somente

até $n = 0$. (Havcra" naturalmente alguma correção em virtude da diferença entre cloreto e brometo como fonte contrario). Uma diferença marcante entre $MgCl_2$ e os outros sais do tipo 2-1 da Tabela 1 deve ser mencionada. Podemos supor nestes últimos, que $k_2 = \infty$, isto é, formação insignificante de cachos reutros A_2H . No cloreto de magn&io, entretanto, esta aproximação não pode de modo algum explicar a condutibilidade observada. Pelo método de aproximações sucessivas achamos $K_1 = 0,0004$ para o mencionado sal. As concentrates relativas das várias espécies presentes nas soluções de cloreto de magn&io em metanol estão representadas na figura 3. Observa-se que grupos $MgCl_2$ começam a formar-se

cm dados de condutibilidade, mas as medidas de viscosidade (GOLDBERG & Fuoss, em impr.) dao alguma informacao independente.

Em geral, a presenca do urn eletrolito cm solvente dc viscosidade η_0 da uma solucao de maior viscosidade n . O aumento fraeionario on viscosidade especifica

$$\eta_{sp} = (\eta - \eta_0) / \eta_0$$

e constituido por diversos termos. Um deles e devido a forcas atuando entre iotes muito separados (HARNED & OWEN, 1950; FALKENHAGEN & DOLE, 1920; FALKENHAGEN & VERNON, 1932; ONSAGER & Fuoss, 1932); 6 proporcional a raiz quadrada da concentracao. Na figura 4 temos a quantidade $\eta_{sp}/\sqrt{\gamma}$ como funcao de $\sqrt{\gamma}$ para uma variedade dc eletrohtos em metanol; y e aqui a concentracao molar. Ver-se-a que os dados podem ser representados por linhas retas, resultando que a quantidade $(\eta_{sp} - A\sqrt{\gamma})/\gamma$ e uma constante. O termo $A\sqrt{\gamma}$ e o termo eletrostatico no aumento da viscosidade. Se mudarmos para concentrações de massa, portanto,

$$z = (\eta_{sp} - A\sqrt{\gamma}) / c$$

e* uma eonstante para um dado sal e represents a contribuicao do volume das particulas do soluvel para a viscosidade. Quanto a particulas esfeVicas, EINSTEIN (1906, 1911) demonstrou que $z = 2.5 v$, em que v e* o volume especifico do soluvel. Na Tabela II sao dados os valores de z para os sais da figura 4, tanto em metanol como em etanol.

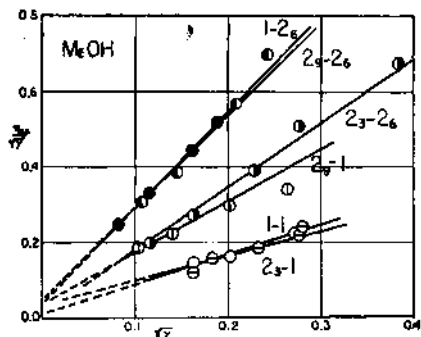


Fig. 4. Viscosidade dos eletrólitos em metanol: 1-2, adipato de tetrabutilamônio; 2-2, adipato de di-(trimetilamônio)-nonano; 2-3, adipato de di-(trimetilamônio)-propano; 2-1, dibrometo de di-(trimetilamônio)-nonano; 1-1, brometo de tetrabutilamônio; 2-1, dibrometo de di-(trimetilamônio)-propano.

TABELA II

Coefficientes z para os sais

	METANOL		
	Bu ₄ N	C3	C9
Br	2,3	2,0	3,2
Ad	4,0	5,0	6,3
	ETANOL		
	Bu ₄ N	C3	C9
Br	3,7	3,4	4,2
Ad	5,7	6,7	7,3

Exceto para os dois sais, brometo de tottrabutildmndio e dibrometo de 1,3-di-(trimetilamônio)-propano em metanol, todos os valores do z são superiores a 2,5. Conhecem-se dois casos em que o coeficiente hidrodinâmico em $z = fte$ superior a 2,5: elipsoides (SIMHA, 1940) e cadeias flexíveis (FLORY & FOX JR., 1951). Em um elipsoide de rotação, com rotação axial igual a 5, o valor de fte^* 5,8; em um polímero em cadeia de peso molecular 10.000, fte^* aproximadamente 10. (Estamos supondo que o volume específico v^* igual a aproximadamente unidade). Por conseguinte, a associação em cadeia dos eletrólitos bi-bolaformes não é excluída pelos valores dos coeficientes da Tabela II. Mas os sais dos tipos 1-2 e 2-1 (por exemplo, adipato de di-(tetrabutilamônio) e os sais C3 e C9) não são suscetíveis de formar cadeias já que um íon é monofuncional no sentido de polimerização, e ambos os sais 2-2 e os 1-2 e 2-1 têm coeficientes mais ou menos do mesmo valor. Os modelos dos últimos $\{+\} \{- \dots -\} \{+\}$ e $\{-\} \{+ \dots +\} \{-\}$ são elipsoidais; concluimos, portanto, em favor da associação cíclica (8) nos compostos bi-bolaformes. O padrão uniforme de aditividade nos coeficientes da Tabela II também requer a estrutura (8).

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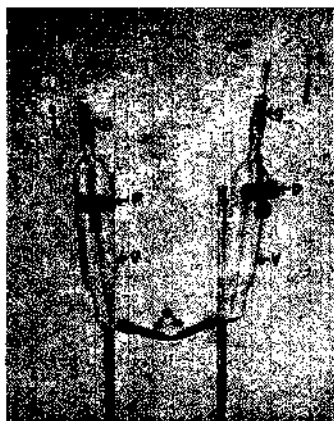


Fig. 2

T al que se ajusta la varilla de vidrio que se fija al soporte de electrodos. Cuando se emplee este dispositivo, no debe esperarse un funcionamiento muy regular del arco de llama, pues el liquido crepita.

Las figuras 4 y 5, son cortes longitudinales de los terminales metalieos usados. El de la figura 4, se emplea cuando se dispone de suficiente canlidad de solucion; el otro se utiliza en las mieropipas.

El porta electrodo *B*, tormina en cono para poderlo ajustar al vaso *V*, en los dos modelos. VA electrodo *A*, de la figura 4 es de 5 mm. de diametro y de unos 60 mm. de longitud, y ha sido rebajado y roscado con paso fino, para poderlo fijai- al soporte *B*, de manera que el liquido que se desliza por el espacio *E*, moje el extremo del electrodo y caiga gota a gota. Al tornillo *H*, se le ha limado una superficie plana que permite el paso del liquido; la tuerca *T* fija sdlidamente el conjunto y es indispensable cuando se desea mantener constants la distancia entre los electrodos, pues los continuos lavados con soluciones acidas, desgastnn el tornillo *R*, y el paso de rosea del porta electrodo.

Kn el modelo de la figura 5, se fija el electrodo a la camisa *B*, por medio de la curia *R* y puede ser utilizado el mismo electrodo *A*, gastado por uso, haciendo nuevos agujeros en el. Como conviene que el espacio nocivo *N*, sea lo menor posible por la hurbuja de aire que algunas voces queda al llenar la micropipa, se reduce estc tamano prolongando el electrodo como se indica en la figura, o llenando el espacio *N* con algodn de vidrio. Se puede tambien sustituir la cuna *R*, por un

B, a cuyo extremo inferior esta soldada la valvula quo ajusta sobre el asiento esmerilado en la pipa; lo que permite cargar la solution impidiendo su derrame, ante de iniciar la exposieion. Usando nu juego de micropipas usf construido, so puede trabajar con un volumen de liquido de unos 5 cm. cubicos. Si el volumen de solucion disponible es menor, puede sustituirse una de las pipas, por el electrodo seco de la figura 3, construido de la siguiente manera: a una masa *M* de cob re, se fija a cono el electrodo 0; esta masa *M* puede ser sustituida por un recipiente del mismo metal, con entrada y salida de agua para enfriamiento. Al extremo del cuello metalico va fijo el conductor *C*, y tambien el tubo de cobre

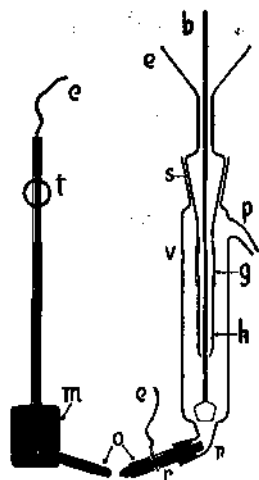


Fig. 3

tornillo roscado, a la camisa B, para fijar el electrodo A, y hacer el contacto sobre otro tornillo, o sobre un collarín del mismo material que ajuste elásticamente sobre la camisa B; en el primer caso rápidamente se gasta el paso de rosca de los tornillos sobre la camisa y en el segundo caso el contacto eléctrico es defectuoso, sobre todo



Fig. 4

cuando se trabaja con agua destilada o soluciones muy diluidas y por ese motivo, el arco de llama no funciona regularmente. El espacio E. es

de más o menos 0,5 mm., pero su tamaño no tiene mayor importancia por poderse regular la salida del líquido, modificando la posición del embudo.

Cuando no interesa mayormente la regularidad del foco luminoso y por lo tanto la distancia entre los electrodos, se puede emplear el modelo de la figura 1, donde la construcción del electrodo se simplifica; el cilindro metálico A, se desliza por el interior de la camisa B, y se fija su posición doblando el conductor C, sobre el borde de la pipa o del embudo. Regulando convenientemente la distancia entre electrodos, la que puede variar entre 0,5 y 2 mm. según la concentración de la solución que se quiere estudiar, y la tensión conectada a los electrodos; y regulando la velocidad de salida del líquido, se puede conseguir que el arco de llama funcione muy regularmente y evitar que el líquido crepite y se pierda en parte por este motivo. En estas condiciones, el dispositivo puede funcionar durante un tiempo muy largo, salvo los pequeños intervalos necesarios para recargar las pipas. El número de gotas que caen por minuto en cada electrodo varía entre 80 y 120, y el volumen total de solución que se derrama por hora es de 1 litro aproximadamente. Cuando se utilice por primera vez un juego de electrodos, conviene para obtener rápidamente constancia en el arco de llama, hacer un pequeño plano en la parte inferior de la punta de cada electrodo, y esperar unos minutos antes de iniciar la exposición, para que el paso de la corriente produzca sobre cada electrodo el cráter. Conviene hacer presente que es indispensable para que el arco funcione regularmente y para conseguir la máxima sensibilidad, que la gota se deslice por la parte inferior del electrodo y llegue al borde del cráter, inclinando los electrodos de manera que esta condición se cumpla.

ELECTRODOS UTILIZADOS

Los electrodos utilizados fueron de *Fe*, *Ag*, *Cu*, *Pt*, o *Ni*, según conviniera, dada la longitud de onda de la línea última del elemento disuelto que se deseaba determinar. Los electrodos de hierro se utilizaron para obtener las placas patrón y de los otros, los más empleados fueron los de *Cu*, por ser fácil conseguir cobre relativamente puro, y tener este elemento un espectro poco complicado. En casos especiales hemos utilizado electrodos de carbono o grafito, y camisa porta electrodo de platino. La dificultad en este caso, es el gran desgaste de los electrodos en el arco de llama, el gran espectro de fondo y en muchos casos, sus impurezas son dificultades insalvables.

Los espectros propios de los electrodos los hemos obtenido siempre, con arco de llama, circulando por las pipas agua tridestiladas; y nos han servido en todos los casos como espectros de comparacion.

En la figura G se reproduce fotograficamente el soporte de electrodos disenado para ajustar facilmente el foco luminoso.

Sobre la cremallera C, esta solidamente fija la chapa metalica M, a la que se ha fresado, paralelamente a su borde anterior, una ranura, por donde pueden correr las piezas D, que llevan soldados los soportes tubulares B, que pueden ser ajustados a la chapa M, con tornillos de mariposa. La distancia entre electrodos se hace moviendo las piezas D y el ajuste final de esta separation se hace con el tornillo O, de paso fino. Los brazos P, tienen en su extremo un collarin elastico donde se fijan los tubos normalmente soldados a los cuerpos de las pipas, lo que permite inclinar los electrodos metalicos, para que la soluci6n llegue al borde del crater de los electrodos.

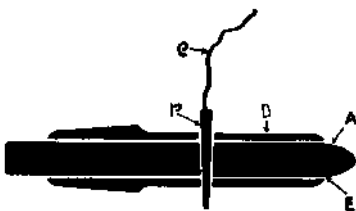


Fig. 5

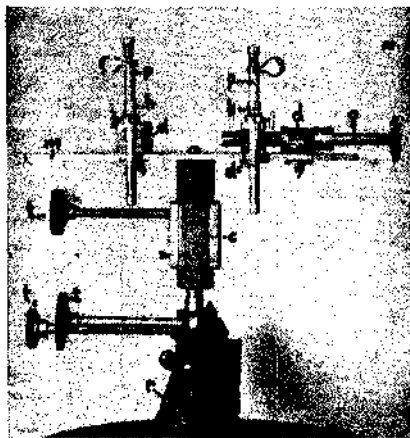


Fig. 6

Para poder regular la altura de los electrodos y disponerlos en un mismo plano, los brazos P corren y giran en los soportes tubulares B. El tornillo T, que actúa sobre el pinón de la cremallera, ajusta finamente en altura el arco de llama, y los otros tornillos inferiores del soporte, centran la imagen, haciendo girar sobre su eje todo el soporte. Para poder lavar facilmente las pipas con solution diluida de HCl, entre dos espectrogramas hemos construido un soporte montado sobre el riel del espectrografo,

girable y con cremallera, sobre el que se coloca el recipiente con la solution caliente de lavado, que permite al levantarse sumergir los electrodos metalicos, y recoger la solution de lavado y el agua destilada al onjugarse las pipas.

LAS SOLUCIONES

Las soluciones fueron preparadas, casi siempre con elementos al estado de cloruros en agua tridestilada, y los aparatos destiladores se construyeron totalmente con vidrio pyrex, a los que se sometio previamente, a continuados lavados con soluciones acidas. Los reactivos quimicos utilizados, fueron estudiados por via espectral previamente.

Para preparar las soluciones de muestras animales o vegetales, hemos destruido la materia orgánica, ya sea utilizando el método químico, o reduciendo la muestra inicial a ceniza. En el primer caso se trató la muestra con ácido nítrico y perclórico; evaporando a sequedad, y disolviendo luego en agua; cuando se utiliza este método debe procederse con mucho cuidado, por el riesgo que significa el empleo del ácido perclórico.

LOS ESPECTROGRAMAS Y EL MÉTODO DE EXCITACIÓN

Los espectrogramas que se publican han sido logrados utilizando tres espectrografos. El primero construido por la casa Zeiss, con tres prismas de vidrio y cámara de 84 cm. de distancia focal; los otros dos construidos por la casa Hilger, ambos con óptica de cuarzo, el primero con cámara de 60 cm. de distancia focal, y el segundo, el gran modelo auto-colimador; iluminando siempre con una lupa condensadora y otra cilíndrica.

El método de excitación, para muestras líquidas fue siempre el de arco de llama, y cuando se estudiaron muestras orgánicas reducidas a cenizas, se utilizó el arco cortado de corriente continua.

El arco de llama lo obtuvimos conectando directamente los terminales del secundario del transformador a los electrodos, por cuyo motivo, al producirse este, la tensión cae enormemente.

Los dos transformadores utilizados, fueron donados por la Compañía CADE al Instituto de Física. El primero es de 1,5 KVA con tensión de entrada de 220 V, y tensión de salida de 4.000 V. El otro de 20 KVA, tensión de entrada de 220 V, y construido con bobinas secundarias múltiples, de manera que, según como se la conecte, puede dar, por etapas iguales de 1.500 V, tensiones entre este valor y 12.000 V.

En adelante, para definir las características del arco de llama, daremos la tensión, en el secundario del transformador en circuito abierto y la intensidad que circula por los electrodos, funcionando el arco de llama. Las placas utilizadas fueron generalmente, las que se emplean para uso fotográfico común, tipo IIP3, fabricadas por Ilford; para las zonas infrarrojas y en alguna oportunidad para las zonas ultravioletas, placas especiales de Eastman Kodak.

Los negativos logrados en todos los casos son sumamente nitidos, y en general con nada o con muy poco espectro de fondo.

En la figura 7, el espectrograma superior, corresponde a una solución en agua, de concentración 1 gramo por litro de ceniza de sangre humana; arco de llama 12.000 V; $I = 3$ Amp; $T = 5$ m; placa 1L de Kodak; electrodos de Cu. El espectrograma inferior se registró, utilizando arco cortado de corriente continua de 8 Amp.; utilizando 10 miligramos de la misma ceniza y exponiendo hasta su consumo total, en un electrodo tubular fabricado con el mismo cobre, que el empleado en el arco de llama.

En los dos espectrogramas aparecen nitidamente las líneas últimas de: Na, Li, K, Rb y otras de menor intensidad de Ca y Fe.

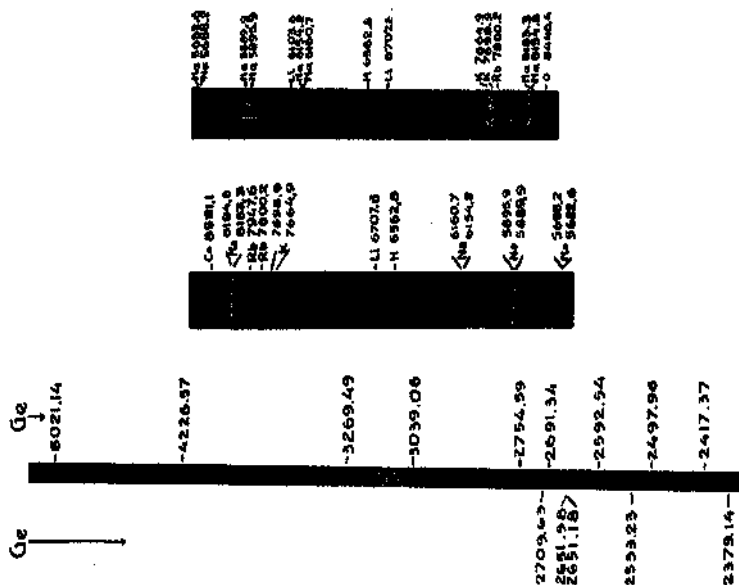


Fig. 7, 8 y 9

Comparando los dos espectrogramas obtenidos, se puede observar que algunas líneas, que por cierto pueden ser reforzadas mucho, aumentando la concentración de la solución, son menos intensas en el espectrograma obtenido con el arco de llama que en otro, pero el espectro de fondo que no puede ser evitado utilizando el arco cortado, prácticamente no existe en el otro espectrograma.

Por otra parte, conviene destacar que para obtener el espectrograma con arco cortado, se gastaron totalmente los 10 miligramos de ceniza, que es la quinta parte de la cantidad contenida en los 50 cm. cubios de solución, que se utilizaron en el arco de llama, y que con este volumen se pueden obtener un número grande de registros, sin variar sensiblemente la intensidad de las líneas; con solo completar, después de cada registro, el volumen residual con agua tridestilada, lo que no produce sensible modificación de la concentración, por ser muy pequeño el consumo espectral como veremos más adelante.

La figura 8 corresponde a un registro de dos espectrogramas, obtenidos con ranura de contacto, y para ambos; arco de llama 0.000 V; $I = 6$ Amp.; $T = 5$ m.; placa 1/4 de Lodak; electrodo de Cu; espectrografo Zeiss.

El espectrograma superior corresponde a una solución de CINA purísimo, de concentración 1 gramo por litro. El espectrograma inferior corresponde a la misma solución, a la que se agregó, 0,2 miligramos de cloruros de Li, Rb y Cs. Las líneas últimas de estos tres elementos, aparecen entonces nitidamente y, por comparación, se verifica que el CINA purísimo no tiene Cs, pero sí vestigios de Li y Rb, que no se indican en la tabla de certificación de impurezas del CINA.

Se estudiaron en las mismas condiciones, 6 muestras de *CINa* purísimo de las cuales, en 4 aparecieron como impurezas *Li* y *Kb* y en ninguna de ellas *Cs*.

La figura 9 corresponde a un espectrograma de una solución en agua de *Ge*, de una concentración de 25 miligramos por litro: arco de llama 12.000 V; 1 = 3 Amp.; $T = 10\text{ m}$; placa *HP3* de Ilford electrodo de *Cu*; espectrografo mediano de Hilger.

La nitidez de este espectrograma es sorprendente: se destacan nítidamente las líneas de *Cu* y *Ge*, y tiene muy poco espectro de fondo.

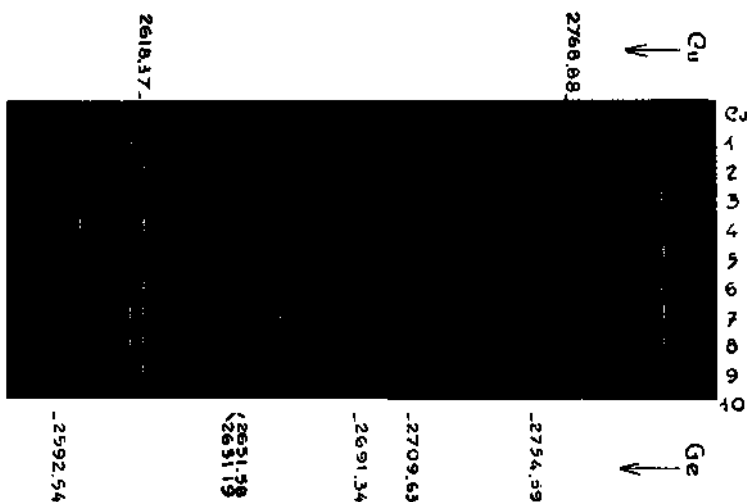


Fig. 10

La figura 10 corresponde a los espectrogramas de 10 muestras de minerales argentinos, estudiado con el propósito de investigar *Ge* — arco de llama 12.000 V; 7 = 3 Amp.; $T = 10\text{ m}$; placa *HP3* de Ilford electrodo de *Cu*; espectrografo gran modelo autocolimador de Hilger.

El espectrograma superior corresponde a los electrodos de *Cu*, corriendo por las pipas agua tridestilada. Los otros espectrogramas, son los de las 10 muestras estudiadas, y por estos se comprueba la presencia de este elemento en las número 10 — 6 — 5 — 3 y 2. El espectrograma es muy nítido y no tiene ningún espectro de fondo.

El cuadro de valores siguientes, da las concentraciones mínimas (en cierto modo la sensibilidad del método *S*) que hemos podido registrar para los elementos que en el figuran, disueltos al estado de cloruro en agua tridestilada, utilizando el espectrografo Zeiss; arco de llama de 12.000 V; 7 = 3 Amp.

ELEMENTO	LONGITUD DE ONDA EN Å	TIEMPO t EN MINUTOS	PLACA	SENSIBILIDAD
Ba.....	4554,0	5	HP3	10^{-9}
Ca.....	3933,7	3	HP3	10^{-11}
Cr.....	4254,4	8	HP3	10^{-8}
Cs.....	8521,1	10	IL	10^{-8}
K.....	7664,0	10	IL	10^{-10}
Li.....	6707,8	10	HP3	10^{-12}
Mn.....	4030,7	8	HP3	10^{-8}
Rb.....	7800,2	10	IL	10^{-8}
Sr.....	4607,3	5	HP3	10^{-9}
Tl.....	5350,5	5	HP3	10^{-8}
V.....	4379,2	8	HP3	10^{-7}

CONSUMO ESPECTRAL

Definimos como consumo espectral, la cantidad de elcmonto, que estando en una muestra dada, con eierta concentracion, se pierde para obtonc el registro fotografico do su linea ultima, utilizando cierta placa, eon un espectrografo dado, y con determinado metodo de exeitaaien.

Hecha la experiencia con *Li*, en una solucion de concentracion, 2 miligramos por litro, para la lfnea 6707 A, utilizando placa HP3, espectrografo Zeiss; arco de llama 12.000 V, $I = 3$ Amp.; el consumo espectral determinado, expresado en miligramos es inferior a 10^{-6} (ISNARDI, 1950).

EXPERIENCES CUANTITATIVAS PRELIMINARES

Al realizar una analisis espectral, interesa la sensibilidad del metodo elegido; pero no es menos importante, el facil manejo del dispositivo experimental. Se justifica asi, la construccion esmerada del soporte porta electrodos ilustrado en la figura 6, que permite comodainente, centrar, levantar, girar, inclinar, variar la velocidad de derranie, regular las distancias entre ekn'trodos etc.; y tambien se justifica la rigida construccion de los terminales metalicos para asegurar la distancia entre olios.

El estudio espectral cuantitativo, de una muestra cualquiera se simplifica, cuando se pueden obtncr de esa muestra, registros fotograficos iguales para tiempos iguales.

Con este proposito hemos realizado algunas experiencias previas, para estudiar el comportamiento de nuestro metodo, modilicando las condiciones en que se produce el arco dc llama, variando la concentracion de la solucirin, la tension conectada, la distancia entre electrodos y la velocidad de escurrimiento del liquido.

Se puede en general por simple observacion del arco de llama, decidir la tension que se debe conectar, para que no se produzca un aumento grande do la temperatura de los electrodos metalicos; que hace disminuir el espacio do escurrimiento del liquido, el que llega a hervir en la parte inferior de las pipas produciéndose entonces depositos salinos en el espacio *E*, sobretodo cuando se trabaja con soluciones concentradas, que hacen variar la velocidad de caida del liquido, por lo eual deja de funcionar regularmente el arco de llama, y al fin se obtura por completo la salida de liquido y se funden los electrodos si no se interrumpe de inmediato el transformador.

De la velocidad de escurrimiento, de la inclinación de los electrodos, y de la distancia entre estos, depende mucho la constancia del foco luminoso. Se puede regular la velocidad de salida del líquido, y la inclinación de los electrodos de manera que la gota moje el borde de su crater y la solución no crepita y regulando luego la distancia entre puntas, se estabiliza la posición del foco luminoso; este último ajuste, el más difícil de lograr, se hace cómodamente por medio del tornillo 0 sin interrumpir la tensión.

Hemos elegido cloruro de litio, para preparar nuestras soluciones, ya que la longitud de onda de su dos líneas últimas (6707, 8 y 0103, 6 Å) hacen posible utilizar el espectrógrafo Zeiss, placas fotográficas comunes HP3; usando una o otra de estas líneas, para estudiar las placas en el microscopio Zeiss, a 6100 voltios.

Para poder medir directamente los tiempos de exposición sin gran error, hemos preparado soluciones de concentración, 0,5 - 1 - 2 y 5 miligramos de cloruro de litio por litro, y para las cuatro soluciones utilizamos arco de llama de 12.000 V., circulando entre electrodos aproximadamente 3 Amp.

Cargadas las pipas y estando todo dispuesto, dejamos correr la solución hasta llegar a derramarse constante; conectamos entonces el transformador exponiendo la placa 30 segundos, y, sin interrumpir el arco de llama, por otros 30 segundos tapamos la ranura del espectrógrafo, para hacer luego el segundo registro con el mismo tiempo, y así sucesivamente hasta llegar a obtener 11 registros.

Analizados los espectros para las cuatro concentraciones, se pudo comprobar que en los 10 últimos, estos no difieren en más del 3%, diferencia que se justifica por la forma en que hicimos la experiencia. El primero de los registros difiere de los otros, en un 15% más o menos, variación que se justifica, por no ser inicialmente estable el arco de llama y por estar fríos los electrodos.

En la misma forma hemos hecho la experiencia para concentraciones, hasta de 10 gramos por litro, y en general podemos decir que la constancia de los registros varía más, a medida que la concentración aumenta, por la acumulación del elemento disuelto en el extremo de los electrodos. Claro está, que fácilmente se puede por dilución reconducir este caso al primero.

Estos primeros resultados, que solo tienen el valor de tales, inducen a suponer, que para soluciones de pequeña concentración, el arco de llama funciona regularmente; que no hay acumulación apreciable sobre los electrodos del elemento disuelto, y que, el consumo de los electrodos es tan pequeño, que no llega a modificar la intensidad del foco luminoso entre una y otra exposición.

Cuando se trate de estudiar soluciones complejas de gran concentración; se puede precipitar por vía química los elementos que no interesen y determinar en el líquido residual, aquellos que interesen y que están en pequeña concentración en la muestra inicial.

Estas experiencias fueron hechas en el Instituto de Física de La Plata, siendo su director el dr. Ricardo Claus, a quien agradezco su continuado apoyo y su constante inquietud por conocer los primeros resultados.

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Proof that Probability Density Approaches $|\psi|^2$ in Causal Interpretation of the Quantum Theory

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In two recent papers (BOHM, 1952a,6) the author has proposed a reinterpretation of the quantum theory, based on the following hypotheses:

(a) A quantum-mechanical system, such as an electron, consists basically of a particle having a precisely defined position, which varies continuously as a function of the time.

(b) This particle is acted on not only by the classical potential, $V(x,t)$ but also by an additional quantum-potential, $U(x,t)$ that is important at the atomic level, but negligible at the macroscopic level.

(c) If we write $\psi = Re^{iS/\hbar}$, where ψ is the wave function, and R and S are real, then the quantum-potential is given by:

$$U = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} \quad (1)$$

The equation of motion of the particle then takes the form

$$m \frac{d^2 x}{dt^2} = -\nabla \{U + V\} \quad (2)$$

With the aid of the above hypotheses, one can obtain the same predictions for all experimental results as are obtained from the usual interpretation of the quantum theory, but with the important difference that processes at the atomic level are described as continuous and causally determined by the equation of motion (2), whereas the usual interpretation assumes that a causal and continuous description of such processes is as a matter of principle unattainable (BOHR, 1934). The causal interpretation provides, however, a conceptual framework that is broader than that provided by the usual interpretation, in the sense that the general class of mathematical theories consistent with the usual interpretation is only a special case of the mathematical theories permitted by the causal interpretation. In fact, the causal interpretation leads to experimental results completely equivalent to those of the usual interpretation, only when the following additional special assumptions are made:

(1) The ψ field satisfies Schrodinger's equation. This assumption is not essentially different in character from the classical assumption that the

electric and magnetic fields satisfy Maxwell's equations, or that the gravitational potentials, $\phi^{\mu\nu}$ satisfy differential equations obtained from the theory of general relativity.

(2) The second order character of the differential equations (2) for the particle motion would suggest that the initial position and the initial velocity of the particle can be chosen arbitrarily. However, to obtain the same results as are obtained from the usual interpretation, we must restrict the initial velocity to $v = \nabla S(x)/m$, where $v(x)$ is the velocity of a particle which happens to be at the point, x . This choice of $v(x)$ could certainly be made at any particular time, say $t = 0$ and as shown by BOHM (1952a), the equations of motion (2) then guarantee that the relation, $v = \nabla S(x)/m$ would be maintained for all time (BOHM, 1952c). Thus we have what is essentially a consistent subsidiary condition.

(3) We assume that we cannot at present predict or control the precise location of an individual particle, but that quantum-mechanical systems now available in practice constitute a statistical ensemble. The probability density, $P(x, t)$ of systems in this ensemble is in principle an arbitrary function satisfying the conservation equation

$$\frac{\partial P}{\partial t} + \nabla \cdot (P, v) = 0 \quad (3)$$

However, in order to obtain the experimental results predicted by the usual interpretation, we must assume that $P = |\psi|^2$. This restriction is consistent in the sense that if it holds initially, it will hold for all time, because as shown in paper I, sec 4, $|\psi|^2$ satisfies the same conservation equation satisfied by P (provided that $v = \nabla S(x)/m$ and that ψ satisfies Schrödinger's equation):

$$\frac{\partial |\psi|^2}{\partial t} + \nabla \cdot \left(|\psi|^2 \frac{\nabla S}{m} \right) = \frac{\partial |\psi|^2}{\partial t} + \nabla \cdot (|\psi|^2 v) = 0 \quad (4)$$

It was suggested in paper I, secs 4 and 9, that the broader conceptual framework provided by the causal interpretation opens up possibilities of making new kinds of theories in connection with the domain of 10^{-13} cm., where present theories seem to be inadequate. One of these possibilities is to give up assumption (1); for in the causal interpretation, one can for example, assume that the equation governing ψ is non-linear and contains inhomogeneous terms depending on the actual location of the particle, assumptions which could not even be consistently formulated in terms of the usual interpretation (BOHM, 1952a). These modifications could easily be made in such a way that the usual Schrödinger's equation would be a good approximation at the atomic level, where the present theory is known to be applicable, while at the level of 10^{-13} cm. totally new results would be obtained. Similarly, we could modify assumption (2) and permit v to differ from $\nabla S/m$, by introducing new terms in the quantum potential. With a proper choice of these new terms the difference between v and $\nabla S/m$ could be made negligible at the atomic level, but of crucial importance in the domain of 10^{-13} cm. (BOHM, 1952a).

It is clear, however, that under conditions in which assumptions (1) and (2) fail, (3) will in general also fail; for the true probability density, $P(x, t)$ will still be conserved, while $|\psi(x, t)|^2$ in general will not, since the conservation

of $|\psi|^2$ follows from (1) and (2). But if deviations from (1) and (2) really exist at the level of 10^{-13} cm. then we might expect the cumulative effects of the resulting differences between P and $|\psi|^2$ to be felt ultimately even at the atomic level, unless there exist opposing processes, which tend continually to re-establish the equality of P and $|\psi|^2$. In this paper we shall prove that the randomly distributed collisions between atoms furnish just such opposing processes, which in the absence of perturbations from the level of 10^{-13} cm., would cause an arbitrary probability distribution, $P(x, t)$ to decay into $|\psi(x, t)|^2$ with the passage of time. Clearly, the demonstration of this result constitutes an important part of the causal interpretation, because it completes the proof that the causal interpretation could have an experimental content different from that of the usual interpretation at the level of 10^{-13} cm., and still lead to agreement with all experimental data that can now be understood in terms of the usual interpretation.

The general lines of our proof that $P(x, t)$ tends to approach $|\psi(x, t)|^2$ were already suggested by BOHM (1952b). We first point out that all experiments in quantum mechanics to date have been concerned with statistical ensembles of systems (such as hydrogen atoms) drawn from large aggregates of matter, where they have been interacting with other systems for a very long time. When we do an experiment now, we have no choice but to draw our samples from such an ensemble, whose members have undergone this lengthy process of collision with other atoms, electromagnetic waves, sound waves, and other disturbances which can alter the physical condition of the members of our ensemble. Now, each collision of, for example, a hydrogen atom with another atom, can change the positions of the precisely defined particles located in each atom in a way that depends on the collision parameter and the initial velocity of approach. Since then¹ is a statistical ensemble of different kinds of collision, we conclude that even if all the particles in our ensembles initially had the same position, they would have some kind of distribution of positions after collision.

To study how this probability distribution changes in a collision, we shall find it convenient to define a function $f(x, t)$ through the equation

$$P(x, t) = |\psi(x, t)|^2 f(x, t) \quad (5)$$

Since $P(x, t)$ satisfies the conservation equation (3), while $|\psi(x, t)|^2$ satisfies the similar equ. (4), we readily find by subtracting (4) from (3) that

$$\frac{\partial f}{\partial t} + v \cdot \nabla f = 0 \quad (6)$$

But the above is just df/dt , the rate of change of f which results from following a particle orbit. We therefore obtain $df/dt = 0$ and

$$f(x, t) = f(x', t') \quad (7)$$

where x' is the position of a particle at the time t' which arrives at the position x at the time t .

Equation (7) is analogous to Liouville's theorem in classical statistical mechanics, with the important difference that in quantum theory, the ratio, $f = P/|\psi|^2$ is what remains constant when we follow a moving particle, while in classical statistical mechanics, it is the density $P(x, p, t)$ of points in phase space

that remains constant. However, by methods that are very similar to those that can be used in classical statistical mechanics to show that $P(x, p, t)$ approaches a constant ⁽¹⁾ along any surface of constant energy, we have shown that $f(x, t)$ approaches a constant, so that P approaches $|\psi|^2$. Moreover, once P has become equal to $|\psi|^2$ no disturbance satisfying conditions (1) and (2) can possibly produce a difference between these two quantities because the relation, $P = |\psi|^2$, is then maintained for all time by the equation of motion of the particles. We conclude then, that in the causal point of view, the statistical distributions of quantum theory have the same origin as those of classical theory; namely, the random collision which cause the various members of an ensemble to distribute themselves more or less chaotically over the range of physical parameters accessible to them.

The above results are deduced with the aid of assumptions (1) and (2), which certainly hold in the atomic domain. If, in this domain, perturbations arising at the level of 10^{-13} cm. should create discrepancies between P and $|\psi|^2$ we should then expect that the discrepancies would die out in some mean time, T , which is determined by the solutions of the equations of motion of the various particles involved and by the rates of collisions of these particles. The difference, $P - |\psi|^2$ in the steady state will be determined by the balance between the mean rate, R , at which perturbations surge up from the level of 10^{-13} cm., and the mean rate, $(P - |\psi|^2)/T$ at which a perturbation already in existence dies out. This yields

$$P - |\psi|^2 = RT \quad (8)$$

Now in the absence of any specific hypotheses as to what is happening at 10^{-13} cm., it is impossible for us to know anything about R , except that with the limitless number of conceivable hypotheses available, any value at all is possible. Thus, if we assume the causal interpretation, we can always regard any experiments which show that P is equal to $|\psi|^2$ as "a posteriori" evidence that R is so small that discrepancies between P and $|\psi|^2$ have not yet been detected. This means that no experiment can possibly show that the usual interpretation must be chosen in preference to the causal interpretation. On the other hand, an experimentally observed discrepancy between P and $|\psi|^2$ would clearly indicate that the usual interpretation was untenable and that a causal interpretation was probably needed ⁽²⁾.

An actual experimental test of the relation, $P = |\psi|^2$ would be impractically difficult at present, but as shown in paper II, the usual formulae for transition probabilities are, in the causal interpretation, consequences of

⁽¹⁾ Note that this result is not to be confused with the statement that $p(x, p, t)$ remains constant when $x(t)$ and $p(t)$ are solutions of the equations of motion. For in this case, even though p depends on x , p and t , the dependence is such that there is no net change in p when we follow a trajectory of a part in phase space. However, after many collisions, the functional form of $p(x, p, t)$ itself changes and approaches a constant along any surface of constant energy. Similarly, from equ. (7), we see that $f(x, t)$ is a constant along any particle trajectory, regardless of the form of f , but we now assert that after many collisions, the functional form of f changes in such a way that f approaches a constant.

⁽²⁾ In fact, we can go further, and say that after we reach the stage of postulating a particular example of a causal theory, experiments are conceivable (BOHM, 1952), which would permit us to infer the necessity of each element in the hypotheses underlying that particular theory; or in other words, to "observe" or detect these elements.

the assumption that $P = |\psi|^2$. Hence, if this assumption is not entirely true, we should expect to find discrepancies, probably rather small, between the observed mean rates of transitions and the rates predicted by present theories. We must study these discrepancies under conditions in which ψ is known to a high degree of accuracy. From this point of view, the best transition processes to study would probably be those involving radiation from atoms of hydrogen. However, to reduce the rate at which discrepancies between P and $|\psi|^2$ die out, we should avoid collisions by keeping the atoms under conditions of extreme isolation, and avoid the effects of thermal radiation by keeping them at very low temperatures. To increase the rate, R , at which discrepancies between P and $|\psi|^2$ come up from the level of 10^{-13} cm., we should use atoms whose nuclei are very highly excited. These requirements appear to be very difficult to satisfy simultaneously at present, but it is possible that a more careful study would disclose other systems which would be more suited for demonstrating an experimentally observable difference between P and $|\psi|^2$. In this connection, it would now be of interest to make plausible speculative hypotheses concerning the level of 10^{-13} cm., with a view to estimating the magnitude of the expected differences between P and $|\psi|^2$. Experimental evidence testing the accuracy of the prediction of the usual interpretation concerning rates of transition is at present rather rough and limited in quantity, and of course, no experiments have as yet been done with a view to finding conditions that are likely to lead to a maximum discrepancy between P and $|\psi|^2$.

It is perhaps interesting to note at this point that as a result of the fact that P approaches $|\psi|^2$, the causal interpretation now involves exactly the same number of hypotheses as does the usual interpretation. In both points of view, one postulates that ψ satisfies Schrödinger's equation; and the very excellent agreement of the observed energy levels with the predicted values provides a convincing proof of the validity of this equation, at least for the stationary states. In the usual interpretation, one makes the additional hypothesis, however, that in the measurement of any observable, A , the probability that we shall obtain the eigenvalue, a , is $|C_a|^2$, where $\psi(x) = \sum C_a \psi_a(x)$ and $A \psi_a(x) = a \psi_a(x)$. In the causal interpretation, one makes instead the hypothesis that there is a particle with a precisely defined position, and with a velocity, $v = \nabla S/m$. From this hypothesis, one deduces that $P = |\psi|^2$; and as shown in paper II, this result is sufficient to prove that the probability of obtaining the result, a , in the measurement of the observable, A , is $|C_a|^2$ (under conditions in which there are enough collisions to establish the validity of $P = |\psi|^2$).

As has been shown by BOHM (1952), the existence of the postulated precisely defined particle variables could in principle be verified by a direct measurement of the particle positions, if the hypotheses (1), (2), and (3) need to be given up in any domain, for example at the level of 10^{-13} cm. The basic hypothesis of the usual interpretation that the probability of obtaining the eigen-value, a , of the observable, A , is $|C_a|^2$ could in principle also be subjected to a systematic experimental test, but as we have seen, this is in practice very

difficult to do at present, especially with the requisite degree of precision ⁽³⁾. Thus, as the situation stands now, neither hypothesis has yet been verified experimentally. It should be repeated, however, that whereas no experiment supporting the usual interpretation could possibly disagree with the causal interpretation, a failure to obtain experimental results in agreement with the basic hypothesis of the usual interpretation would provide strong evidence in favor of the broader conceptual framework consistent with the causal point of view.

We shall summarize the results of this paper in terms of the simplest example of the disintegration of two uranium nuclei having the same wave function. Now one of these nuclei may decay tomorrow and the other in two billion years. The usual interpretation of the quantum theory states, however, that *today*, there **can** be no physically describable difference between these nuclei, since they both have the same wave function, and since the wave function is said to determine all physically significant properties of a system. But the most elementary scientific procedure would suggest that if two objects are observed to act differently, this should be regarded as "a posteriori" evidence that there must in fact be some physical difference between them. Indeed, in the causal interpretation, the difference in the two uranium nuclei is explained very simply in terms of the assumption that each nucleus has in it a set of particles with precisely defined positions, which determine in principle exactly when that nucleus is going to decay (BOHM, 1952a). The difference in times of disintegration is then ascribed to the differences in positions of the various particles in the two nuclei. In fact, if we consider a statistical ensemble of such nuclei, all having the same wave function, then as has been shown, the particles in different nuclei will be in different positions as a result of collisions suffered by these nuclei over the past few billion years; and these collisions will have produced a probability density of $|\psi|^2$ that the particles in a particular nucleus take a given set of positions. Thus, we are able (BOHM, 1952 a), to explain the statistical ensemble in decay times of a large number of nuclei.

In the usual interpretation, however, the role of statistics is very different, for it is said that the precise future behavior of an *individual* system is *completely arbitrary*, in the sense that it cannot be related to any physically definable properties of that system, or of anything else existing in the world today. Only the *probability* of decay is a *statistical ensemble* of systems is said to be determined by physical factors now in existence. Nevertheless, it is admitted that the behavior of an *individual* system such as a uranium nucleus can be physically significant, since, for example, if this nucleus disintegrates, the resulting particle can activate a Geiger counter, which can initiate a large scale process, such as the setting off of a bomb. In this situation, the question of when a particular nucleus

⁽³⁾ The best test of this hypothesis to date probably occurs in connection with Coulomb scattering, where the asymptotic wave function is known exactly through solution of Schrodinger's equation, and where probability of arrival of a particle at a given angle, θ , is observed experimentally. However, this procedure only tests hypothesis, $P(x,t) = |\psi(x,t)|^2$, for the asymptotic part of the wave function, and then only to a very rough order of precision. It is necessary to check this hypothesis close to the scattering center itself. Moreover since the slightest failure of the hypothesis that the probability of obtaining the eigenvalue, a , of the observable, A , is $|\langle \psi | A | \psi \rangle|^2$ would make usual interpretation untenable, we would need experiments of a very high order of precision, before we could obtain a reasonable experimental basis for supposing that it might perhaps be possible to obtain a theory dealing with the level of 10^{-13} cm., without utilizing the new possibilities opened up by causal interpretation.

decays clearly has physical significance, since it determines, for example, whether the bomb will explode tomorrow or in two billion years. In accordance with the postulate that only the behavior of a statistical ensemble of uranium nuclei can be determined by physical factors now in existence, however, the usual interpretation states that the time at which a particular bomb goes off is completely arbitrary, from a physical point of view. What the assumption of the at present "hidden" precise definable particle variable does is to remove this element of arbitrariness from the theory.

Now, it has always been the common practice in science to recognize that a hypothesis that is able to remove arbitrariness by relating previously unrelated facts, events, conditions, or domains of phenomena has a high probability of being at least partially correct and therefore fruitful, even if at the particular time when the hypothesis is introduced, all of its elements cannot yet be verified. For example, the atomic hypothesis was introduced long before individual atoms could be detected, simply on the basis of the fact that it connected the various rules of chemical combination and the perfect gas laws, which had previously been supposed to have no underlying relations connecting them. This hypothesis not only proved fruitful, but ultimately led to experiments which permitted the direct observation of the postulated atomic particles, experiments which would probably never have been done if the atomic hypothesis had not been regarded as having a high probability of being at least partially correct. If the early proponents of the atomic hypothesis had worked in the spirit of the usual interpretation of the quantum theory, however, they would have said that the atomic hypothesis was "unnecessarily complicated" because one could avoid the postulation of atoms by the "simpler" assumption that the facts of chemistry and the perfect gas laws, were unrelated. In order to satisfy the principle of "economy of concepts" they would then have given up the atomic hypothesis, or at least would not have taken it seriously enough to continue the search for experimental evidence of the existence of individual atoms. Indeed, it was just on the basis of such arguments that nineteenth century positivists like Mach contended that the concept of atoms was "metaphysical" and should therefore not be accepted. It should be clear, however, that the "simplification" obtained by assuming that there is no underlying causal relation between facts of chemistry and the perfect gas laws is a trivial advantage, in comparison to the new possibilities opened up by the atomic hypothesis as a result of its unification of hitherto unrelated fields through the postulation of the as yet undetected atoms. Similarly, it seems to the author that the "simplification" resulting from the hypothesis that there is no underlying causal connection between the future behavior of an individual system and the present physical state of the world is likewise a trivial advantage in comparison to the new possibilities opened up through the unification of these things by postulating the precisely defined "hidden" particle variables. These new possibilities are seen to be particularly important at this time, if we consider the fact that physicists using the usual interpretation do not seem to have made appreciable progress during the past twenty years in explaining the properties of elementary particles, particularly their finite self energy, the large number of kinds of such particles, and the various processes by which they are created, destroyed, and transformed into each other.

The parallel between the atomic hypothesis and the hypothesis of the at present "hidden" precisely defined particle variable is very close. Thus, when the atomic theory was first proposed it did not seem to have any new experimental content, because it led to macroscopic conditions (such as the gas laws and the law of chemical combination) which were already treated in terms of purely macroscopic theories. In experiments which were possible at that time, the atoms manifested themselves only statistically, and it was not possible to deduce the size of an individual atom. Moreover, in any domain in which the macroscopic laws (such as the gas laws) were applicable, it was not even possible to obtain experimental evidence for the existence of individual atoms. Such evidence could only come from experiments involving new phenomena which could not adequately be treated by the extrapolation of macroscopic laws to the microscopic domain. The first new phenomenon of this kind was the Brownian motion, a perpetual fluctuation which contradicted the macroscopic assumption that the behavior of a system in thermal equilibrium was specified completely by its equation of state. Brownian motion was not discovered, however, until 1827, a full 24 years after Dalton first proposed the atomic hypothesis to explain the laws of chemical combination, and the first estimate of the size of an individual atom on the basis of measurements of the effects of Brownian motion was made only in 1865.

Similarly, we note that in the causal interpretation of the quantum theory we can now predict only the results of experiments which can already be predicted adequately by the usual interpretation, and that experimental evidence in favor of the postulated precisely defined particle variables could come only from some domain in which the predictions of the present form of the quantum theory break down. The causal interpretation is in a sense, however, in a better position than the atomic hypothesis was when first proposed, because at that time, there was no indication that macroscopic laws were inadequate at small distances, whereas, as we have already pointed out, the extrapolation of the present quantum theory to the level of 10^{-13} cm, leads to unsatisfactory results. The proponents of the usual interpretation, who feel that it is better to assume an arbitrary relation between past and future of an individual system than to relate these things causally by the postulation of the as yet "hidden" precisely definable particle variables, therefore seem to be in a position very analogous to that of the physicists who in the nineteenth century wanted to abandon the atomic hypothesis on similar grounds. In fact those latter physicists had a better basis for their arguments than that of those who now uphold the usual interpretation; for at least they wished to replace the postulation of atoms by empirical laws already proved experimentally; i. e., the gas laws and the laws of chemical combination. On the other hand, as we have seen in Sec. 1, there does not yet exist a satisfactory experimental proof of one key corresponding "empirical laws" underlying the usual interpretation; namely, that in the measurement of an arbitrary observable, A , the probability of obtaining the eigenvalue, a is $|\langle C_a | \psi \rangle|^2$ where $\psi = \sum_{\alpha} C_{\alpha} \psi_{\alpha}$; nor is a sufficiently precise and systematic experimental proof of this law likely to be available in the near future.

In conclusion, it would seem that the causal interpretation is considerably more promising than the usual interpretation; first because on the basis of the

same number of hypotheses, it explains all the facts explained by the usual interpretation in a way that unites previously unrelated things, and secondly because the very step needed to achieve this unity opens up a wide new domain of theories, one of which may be needed to replace the present inadequate theory at the level of $1(T^{13}\text{cm})$. Moreover, the usual interpretation, suffers from the disadvantage that the necessity of adopting it cannot be proved experimentally; for as we have seen in Sec. I, all experimental results consistent with the usual interpretation are also consistent with the causal interpretation, whereas: experimental results are conceivable which are consistent with the causal; interpretation, and not with the usual interpretation.

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DISCUSSION

ANDERSON: It is known that, in ordinary quantum mechanics (here is a whole class of phenomena that have a very beautiful explanation as a consequence of the idea that particles can be identical. In causal theory, it would seem that two particles like two electrons would be tagged and therefore there could be no such thing as complete identity,

BOHM: The causal point of view leads to exactly the same results as the usual point of view in this problem because; of the quantum potential. That is, we have the ψ function, the two particles are not completely described by this, but the motion is determined by the ψ field. Now, one can by more detailed consideration find that as a result of the force due to the ψ field, for example, two particles will never occupy the same position. This is one of the consequences of the anti-symmetry of the wave function, and all of the other consequences can be shown in a similar way. In other words, as long as the laws of the quantum mechanics take their present form, those two particles will behave identically. If the laws change at another level, then it might be possible to tag them but we are simply not able to do it right now.

ANDERSON: It is implicit in your theory that in principle they are taggable and therefore the consequences which result from the exact identity of particles must be lost?

BOHM: All I wish to do is to obtain the same experimental results from this theory as are obtained from the usual theories, that is, it is not necessary for me to reproduce every statement of the usual interpretation. Whether they are in principle taggable or not is not at present verifiable; but if I can get the same results, say that at present they cannot be tagged and that they will behave in such a way and that you will get certain energy levels and certain non-overlapping particles, then that is all really that you can ask to be done. No experiment can tag them at present, but an experiment is conceivable. The difference in the two points of view is that in the causal point of view an experiment is conceivable which might tag them, but we do not yet know how to do this. As a matter of fact, phenomena can be conceived of which might permit us to do this. On the other hand, in the usual point of view it is assumed that it will never be possible.

ANDERSON: But there is a point of principle then;

BOHM: This principle has not been proved experimentally. That is, nobody has shown by any experiment that it is in principle impossible to tag these particles. All that has been shown by experiment is that then; exist certain energy levels and that they are such and such. You may take the exclusion principle as a principle to explain these experiments. But another principle would also explain them. Therefore, we do not have yet enough evidence to decide between these two principles. But evidence is conceivable which will permit a decision.

ANDERSON: One would like to see the threshold at which we could decide once and for all whether there exist some tests of the theory. There are a whole class of experiments being carried out which conceivably could be used for such a test. I can think, for instance, of the experiments that Deutseh has been carrying out on positronium where you have a substance, the positronium atom if you wish, that is made after a short time, lives only a very short time and has a well defined energy level.

BOHM: The energy levels will be the same in both points of view regardless of what happens. The only thing that would be different would be the rate of transitions and the difference? might be rather small. It is important to notice that there might be a very small difference at the level of the atomic dimensions which will be difficult to observe. The situation is perhaps analogous again — then; are two analogies that you can draw — one is with the atomic theory in the early stages when it was not known how small you would have to go to observe something like the Brownian motion. So at the moment you do not know how small you will have to go to observe these particles. Now, careful measurements of the rates of transition might show a difference. But if it did not show a difference it would not prove that the usual interpretation is correct. But if it did show a difference, it would show that it was wrong.

ANDERSON: Why not? The defenders of quantum mechanics would say as they did for instance when the Lamb shift was first discovered that they were merely missing some renormalization of the mass but that the basic ideas of quantum mechanics were not violated.

BOHM: It depends on the situation, and how far you have to get. Within a certain degree of accuracy, the quantum theory is pretty well established. I was going to mention the Lamb shift analogy. As I can remember, I heard a lecture in 1940 on a minute discrepancy on the spectrum of hydrogen which was hardly observable, but which later turned out to be the Lamb shift. Of course, this sort of thing is what I had in mind—that even a very minute discrepancy in the probability of transition would prove that, the usual interpretation is untenable, provided it would not get into the domain where one would not be sure of the calculation of the wave function.

KERST: Would the determination of the absolute cross-section for the production of the Bremsstrahlung at exceedingly short wave length be a test?

BOHM: Yes, it could, but only after the theory is developed further. I did not state clearly enough perhaps that at very short distances we would expect a big discrepancy, but that it would not prove the causal interpretation unfortunately, because there is the point Anderson just made that perhaps some further change in the usual interpretation like the introduction of 8 matrices, non-local theory or something else, might do it. I think that experiments of that kind might ultimately permit such a test when you will have a system of data which could be fitted by theory.

KERST: The wave length would be in the region that you want it to be in, at least beyond the region where we know how things are,

BOHM: The fact that the usual theory fails is not enough to prove that the causal theory is necessary because there are many other ways in which the usual theory might fail. On the other hand, the probability must be given by this rule: $\psi^* \psi$ and we can get a clear test at this point.

MOSHINSKY: Is there no reaction of the motion of the particle on the wave field?

BOHM: One could say that at the level of the usual Schrodinger equation there is none, but there might be. One might argue again that the usual Schrodinger equation is an approximation which neglects the reaction of the particle on the wave field. At the level of 10^{-13} centimeters, this would be important. That is, in electrostatics a similar thing is quite often done. Suppose, for example, that you take the state of the radio wave; in the first approximation you could first consider the electron pushed by the radio wave, but when it comes to more accurate description you would consider a modification of the radio wave by the electron. Perhaps one of the modifications needed is to consider a similar reaction on the wave function; but to obtain the same results as are obtained by the usual interpretation, which is an adequate approximation, certainly at an atomic level, you do not need to consider such a reaction.

MEDINA: I wonder if you can introduce the spin in your theory? Is it possible to predict the existence of a spin of a particle as in field theory? Can we obtain the same statistical results and effects?

BOHM: It does not predict the spin of the particle but describes it. I think one could give a similar causal interpretation to the Pauli equation and the Dime equation and it seems enough to show that it works. In the Pauli equation one assumes that one has a particle with a finite size, very small, which is capable of rotating as well as moving in space. The Pauli spinors are then connected directly with the description of the rate of rotation of this particle; that is, in a stationary state, this particle is described as spinning at a certain speed, so the result is that one interprets the Pauli equation as the equation of motion of a particle spinning under the action of certain quantum mechanical potentials.

RABI: I would like to know just what you mean in your theory by a particle. What physical idea exists there? Is it a classical mass particle such as is assumed in the early Newtonian mechanics? Just what sort of object are we talking about, when we speak of a particle and causal interpretation and what physical evidence can you bring out to make us believe that such a thing exists in nature?

BOHM: At the moment we merely assume that the particle has a certain mass and in a non-relativistic theory that it follows an equation: this is Newton's equation of motion; and the particles may have a spin. We think that the particles may well have many other properties which have not as yet manifested themselves. I keep on referring to the atomic theory -- when the particles were first proposed, very little was known about them except that they had a mass and a position and a momentum. The rest was not relevant because it did not affect the statistical behavior of the large scale level. For example, these particles could have been doing hand springs and this would not have an effect on the pressure in perfect gas. So the same question could have been asked, and was asked, in the nineteenth century. What did these atomic particles mean? Nobody knew exactly what they meant at that time, time, but nevertheless there was a certain advantage to be able to relate causally certain things that previously had been assumed to have no particular relation between them, such as being able to explain the perfect gas law $PV = RT$ instead of deriving it empirically and being able to show a connection between chemistry and the gas law.

RABI: You have described, you say, a particle which may have a mass, it may have a charge and it may have a spin. The spin part of course is something which comes directly from quantum mechanics. In other words I do not see in any of your statements any ideas other than those which came from quantum mechanics except that you hint at infinite possibilities, and I am sure then* you are right. The history of physics has shown that we cannot predict the future; of physics; it always turns out very differently, and there I agree with you. But I do not see how the causal interpretation gives us any line to work on other than the use of the concepts of quantum theory. Every time a concept of quantum theory comes along, you can say yes, it would do the same thing as this in the causal interpretation. But I would like to see a situation where the thing turns around, when you predict something and we say, yes, the quantum theory can do it too.

BOHM: Let me say what I think about the situation. First of all, I say that there is no particular reason, if you take the quantum theory in the present, to adopt either interpretation. The question is, why choose one or choose the other? They both give the same results. I think one should choose the causal point of view, the one that at the moment gives a united, simple and direct explanation of certain things which could not be imagined in the usual point of view. For example, it makes it much easier to teach the subject and to think of it because, for example, usually one says that the particles in a certain orbit jump from one to the other, but one is not supposed to think of the orbit and this is difficult to teach. So there will be a certain advantage there. But, another point is that exactly the same criticism that you are making was made against the atomic theory—that nobody had seen the atoms, nobody knew what they were like, and the deduction about them was gotten from the perfect gas law, which was already known. The third point is that, as I said, it opens up many new possibilities and I will describe a few. I did not describe it in this talk because these possibilities are largely speculative. But one possibility is that one should explain this quantum mechanical potential that I wrote down. In other words, you may say that if you assume a certain quantum mechanical potential, you can explain all the effects causally. Now the question is how to explain this potential in terms of a theory which would cover a still broader range of phenomena. One possibility of doing it is to assume, as several people are already doing, such as notably Dime, that the space is filled with an ether or substratum. In fact, the existence of a sea of negative

energy electrons already seems to indicate that point of view might be taken seriously. If you assume such a substratum, whose particles are interacting in some simple way, you have something very similar to an ion gas in classical physics because the substratum consists of protons and electrons — negative energy of course. If you look at this theory more carefully, you discover that when you try to solve this problem of how these particles will interact, then you get certain relations that are very reminiscent of the causal interpretation of the quantum theory. For example, the relation $V = S$ follows as a very good approximation in terms of this substratum. Another thing that follows naturally is that you get potentials that look very much-like the quantum mechanical potentials. They are not exactly right but there are certain characteristics that are very reminiscent of it. This suggests therefore that what we call a particle, a free particle, is not a particle, but that space is filled with the matter at some other level, that particles are structures most of which are localized in a certain small space but a certain part of which extends out and which we call a field, say, a $\wedge$ field, an electric field, a magnetic field, a gravitation field and so on. This point of view is essentially the sort of idea which Einstein has been proposing for a long time, very similar to it any way, that is, that you have a certain field which covers all space and the particles are highly localized, the disturbance in this field, a part of which extends out a long way, is responsible for the interaction. On the other hand, the transformation of one particle into the other can be very easily understood in this way because you might say that there are many possible stable structures of this type and when you disturb it, for example, by colliding with another one at very high energy, then this particle begins to oscillate and be disturbed in a violent way, may break up into so many more particles. That is one of the lines of possibilities which may be investigated, I think it would take some time to know whether this would have any possibility of success.

LKITE LOPES: There is at present no indication of the experimental results which points to a special new theory. Do you have an idea for a precise scheme below this region of 10^{-13} cm. that you pointed out?

BOHM: Yes, however, I do not know whether I should repeat it again. I think that at the moment there is no particular reason to choose; one interpretation over the other except the one which I gave and which I think is important, that is provides a more unified description of the phenomena. Whenever you obtain a more unified description, you have a feeling that you are on the right track, that is, a description that leaves no disunited disjointed things. I also think that what I have discussed shows a considerable lot of evidence that within this line; you would be able to attack the problems which are facing physics at the 10^{-13} cm. such as the large number of elementary particles, their stability and their transformation into each other.

On Kinematic and Dynamic Laws of Symmetry

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The classical conservation laws of mechanics originate in the symmetry of space time. They may be called therefore kinematic conservation laws. The energy and momentum principles are examples herefor.

Recently, much interest has centered on conservation laws which originate in the symmetry of the properties of particles rather than of space time. The law of the conservation for electric* charges is such a law but others have become recognized lately.

An attempt will be made to enumerate the principles of symmetry in the properties of particles which one has reason to surmise at present.

DISCUSSION

TIOMNO: Could the conservation law for electric charge; be expressed in terms of isotopic spin?

WIGNER: It may be possible to do this. I would like to point out, however, that, at least in the case of proton and neutron, the electric charge and the Z component of the isotopic spin are not equal. The former are, of course, 1 and 0; the latter, however, $\pm 1/2$.

TIOMNO: Is there a general conservation law for the total isotopic spin as one might expect from the symmetrical meson theory?

WIGNER: It does not seem to me that, there is. Thus, for instance, the transition from the virtual state of the deuteron to the normal state involves a change of total isotopic spin from 1 to 0. The symmetry of the meson theory is a symmetry of this particular interaction. Since, there are other interactions, notably the electromagnetic interaction which do not exhibit this symmetry, no conservation law follows from it,

LEITE LOPES: There is so far no experimental evidence; for the existence of the¹ negative proton. On the other hand, it is not possible at present to make a theory which describes only positive particles, let us say. Do you have* any simple explanation for this fact? Do you believe that it is possible to have a theory which would describe only positively charged particles?

WIGNER: I am afraid not. It is surprising, as you say, that all this time and in spite of the high energy processes that we have seen, no negative protons have been observed. Calculations seem to show that it is barely possible to understand this fact. But considering how little we know, it is a real cause of wonderment that no negative protons have been observed. It would be possible of course, to assume that both negative proton and positive proton would have the same neutronic charge. That would explain the absence of proton pair formation. However, I do not want to adopt this explanation. In fact, I fear that I cannot contribute anything to the resolution of this problem.

MEDINA: I only want to make a remark in connection with dr. Tiomno's question. It seems that the elementary charge* symmetry you are referring to, is the; symmetry due to the

invariance of the field, or gauge invariance of charge fields under rotation of the complex field functions. Every charged field has a complex wave function and is symmetric under changes in phase. The total charge operator of the field is proportional to the infinitesimal rotations of charge space. Is that what you referred to? In regard to the isotopic spin space, there is another symmetry of the same type, Is that what you were talking about?

WIGNER: Yes, it is.

BOHM: I would just like to ask prof. Wigner if he has any speculative ideas as to whether there is some geometric or mechanical transformation which corresponds to this symmetry?

WIGNER: I think none beyond the point which dr. Medina just so well described. I think, that we should admit that we do not have an understanding of the deeper causes of any dynamic symmetry. There seems to be an analogy suggested by experimental facts. However, an explanation in the same sense as we have an explanation, for instance, for the hydrogen spectrum, is entirely absent. We see only connections but not more than that.

Diffraction in Time

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SUMMARY

In a previous note a dynamical description of resonance scattering was given, and transient terms appeared in the wave function describing the process. To understand the physical significance; of these terms, the transient effects that appear when a shutter is opened are discussed in this paper. For a non-relativistic beam of particles, the transient current has a close mathematical resemblance with the intensity of light in the Fresnel diffraction by a straight edge. This is the reason for calling the transient phenomena by the name of diffraction in time.

The shutter problem is discussed for particles whose wave functions satisfy the Schrodinger equation, the ordinary wave equation, and the Klein-Gordon equation. Only for the Schrodinger time-dependent equation do the transient wave functions resemble the solutions that appear in Sommerfeld's theory of diffraction. The connection of transient phenomena with the time-energy uncertainty relation, and the interpretation of the transient current in a scattering process, are briefly discussed. The relativistic wave functions for the shutter problem may play an important role in the dynamical description of a relativistic scattering process.

INTRODUCTION

In a previous paper (MOSHINSKY, 1951)* we analyzed the dynamical behavior of a resonance scattering process, when the scatterer (represented by an appropriate boundary condition) was introduced into the beam of incident particles at a definite time. As a result of the introduction of the scatterer, transient terms appeared in the wave function representing the process, and with the help of them we could make the transition from the initial state (plane wave) to the final state (plane plus scattered waves).

Transient terms are to be expected in a dynamical description of resonance scattering, from the analogy that this description has with the theory of resonant electric circuits (WIGNER, 1919). As is well known, in circuit theory the appearance of resonances in the stationary current is closely related with the transients of the circuit, as the same parameters (resonant frequencies and damping factors) appear in both (JAEGER, 1919).

The transient terms in a scattering process contain, besides those that could be considered the analogue of electric circuit theory, terms that are related to the time-energy uncertainty relation (MOSHINSKY, 1952), as quantum mechanics is used in the description of the scattering.

* To be referred to as (A).

To understand the physical meaning of the transient terms in the resonance scattering process, it seemed of interest to analyze first the transient effects that appear when the propagation of a beam of particles is interrupted. The more complicated phenomenon where an actual scatterer (represented by a potential or by boundary conditions (WIGNER, 1940; FESHBACH, PEASLEE A WEISSKOPF, 1947)) is introduced into the beam of incident particles, will be briefly discussed at the conclusion of this paper.

We deal here with the transient terms in the wave function that appear when a shutter is opened. It will be shown in the next section, that when the state of the beam of particles is represented by a wave function satisfying the time-dependent Schrodinger equation, the transient current has a remarkable mathematical similarity with the intensity of light in the Fresnel diffraction by a straight edge (BORN, 1933). The transient phenomena have therefore been given the name of diffraction in time.

The form of the transient terms of ψ that appear when the shutter is opened, is strongly dependent on the type of wave equation satisfied by ψ . In the present paper we analyze the transient terms that appear when the ψ satisfy the Schrodinger equation, the ordinary wave equation, and the Klein-Gordon equation. Only for the Schrodinger equation is there an analogy with the phenomena of optical diffraction, which has to do with the resemblance that the solutions have with those that appear in SOMMERFELD'S (1935) theory of diffraction.

II. THE SHUTTER PROBLEM

The problem we shall discuss in this section is the following: a monochromatic beam of particles of mass m , energy $(\hbar^2 k^2/2m)$, moving parallel to the x -axis, is interrupted at $x = 0$ by a shutter perpendicular to the beam, as illustrated in Fig. 1. If at $t = 0$ the shutter is opened, what will be the transient particle current observed at a distance x from the shutter?

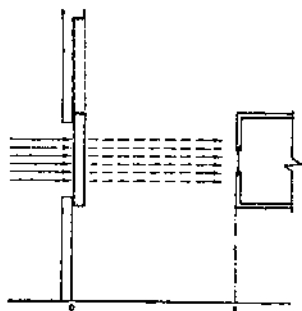


Fig. 1. The shutter problem

To set the mathematical problem, we must first give the behavior of the shutter, i. e. if it acts as a perfect absorber (no reflected wave), or a perfect reflector (an ∞ potential barrier), or something between the two. For simplicity we will assume that the shutter acts as a perfect absorber, though it can be easily shown that for $x \gg \lambda$ (where λ is the wave length $\lambda = (2\pi/k)$), the transient current obtained below holds for any type of shutter, so long

as it acts as a device that, when closed, keeps the beam of particles on only one side of it.

For non-relativistic particles, the wave function $\psi(x,t)$ that represents the state of the beam of particles for $t > 0$, satisfies the time dependent Schrödinger equation:

$$-i(\partial\psi/\partial t) = (\hbar^2/2m)(\partial^2\psi/\partial x^2), \quad (1)$$

and the initial conditions:

$$\psi(x, 0) = \begin{cases} \exp(ikx) & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases} \quad (2)$$

The solution of (1,2) can be immediately given with the help of the functions $\chi(x, k, t)$, introduced in (A) and defined by:

$$\chi(x, k, t) = \exp(imx^2/2\hbar t) \exp(y^2) \operatorname{erfc}(y), \quad (3a)$$

where:

$$\operatorname{erfc}(y) = 2\pi^{-1/2} \int_y^\infty \exp(-y^2) dy, \quad (3b)$$

$$y = \exp(-i\pi/4) (2\hbar t/m)^{1/2} (x - vt), \quad v = (\hbar k/m). \quad (3c)$$

The function $\chi(x, k, t)$ satisfies (1) as:

$$\begin{aligned} & (\hbar/2m) (\partial^2 \chi / \partial x^2) + i (\partial \chi / \partial t) = \\ & (4it)^{-1} \exp(imx^2/2\hbar t) \left[\frac{d^2}{dy^2} - 2y \frac{d}{dy} - 2 \right] \exp(y^2) \operatorname{erfc}(y), \end{aligned} \quad (4)$$

and from the definition (3b) of the error integral function, we see that the right hand side of (4) vanishes. Furthermore, when $t \rightarrow 0$, $|y| \rightarrow \infty$, and $\arg y$ is either $-\pi/4$ if $x > 0$ or $(3\pi/4)$ if $x < 0$. From the asymptotic properties of the error integral function when $|y| \rightarrow \infty$, summarized in appendix 2 of (A), we have:

$$\exp(y^2) \operatorname{erfc}(y) \rightarrow \begin{cases} 0 & \text{if } -(\pi/2) \leq \arg y \leq (\pi/2) \\ 2 \exp(y^2) & \text{if } (\pi/2) \leq \arg y \leq (3\pi/2) \end{cases} \quad (5)$$

We conclude from the above that:

$$\psi(x, t) = \frac{1}{2} \chi(x, k, t) \quad (6)$$

satisfies the equation (1) and the initial condition (2).

When $t \rightarrow \infty$, $|y| \rightarrow \infty$, and $\arg y = (3\pi/4)$, so using (5) again we obtain:

$$\psi(x, t) \rightarrow \exp i[kx - (\hbar k^2/2m)t], \quad (7)$$

which would be the expected stationary form of the wave function.

The transient current $J(x, t)$ takes from (3,6) the form:

$$\begin{aligned} J(x, t) &= (\hbar/2im) [\psi^* (\partial \psi / \partial x) - \psi (\partial \psi^* / \partial x)] \\ &= (v/4) [|\chi(x, k, t)|^2 + \left(\frac{1}{2} \pi \hbar v t\right)^{-1/2} \\ &\quad Im. \{ \exp i[(\pi/4) - (m x^2/2\hbar t)] \chi(x, k, t) \}], \end{aligned} \quad (8)$$

where Im stands for the imaginary part of the curly bracket. The expression (8) simplifies considerably if we assume that $x \gg \lambda$. As we shall show below $|\chi(x, k, t)|$ differs appreciably from zero only when t is of the order or larger than

the time of flight $T = (x/v)$. For $t \geq T$ we have that $(kxt)^{-1/2} \leq (kx)^{-1/2}$ is very small. Therefore, the second term in (8) is ≈ 0 for all $t > 0$ if $x \gg \lambda$, and in this case the current becomes;

$$J(x, t) = (v/4) \left| \operatorname{erfc} \left[-(\pi/2i)^{1/2} u \right] \right|^2 \quad (9)$$

where;

$$u = (kx)^{1/2} (\pi t/T)^{-1/2} [(t/T) - 1]. \quad (10)$$

From the asymptotic behavior (7) of ψ when $t \rightarrow \infty$ we see that the stationary current is $J_0 = V$. Using a simple relation (JAHNKE & EMDE, 1945) between the error integral function and the Fresnel integrals, the ratio of the transient to the stationary current becomes;

$$[J(x, t)/J_0] = \frac{1}{2} \left\{ \left[\frac{1}{2} + C(u) \right]^2 + \left[\frac{1}{2} + S(u) \right]^2 \right\}, \quad (11)$$

where;

$$C(u) = \int_0^u \cos\left(\frac{1}{2} \pi u^2\right) du, \quad S(u) = \int_0^u \sin\left(\frac{1}{2} \pi u^2\right) du. \quad (12)$$

The right hand side in (11) is identical to the expression (BORN, 1933) for the intensity of light in the Fresnel diffraction by a straight edge.

The variable u has though a different meaning from the optical problem, as it is now the function of time given by (10).

In the Cornu Spiral diagram of Fig. 2 the right hand side of (11) is one half of the square of the radius vector from the point $(-1/2, -1/2)$ to the point on the spiral whose distance from the origin, along the curve, is u . From (10) we see that when $0 < t \leq 0$ then $-\infty < u \leq 0$ while when $T \leq t < \infty$ then $0 \leq u < \infty$. With the help of the Cornu Spiral we see that when t goes from 0 to T , the ratio (J/J_0) increases monotonically from 0

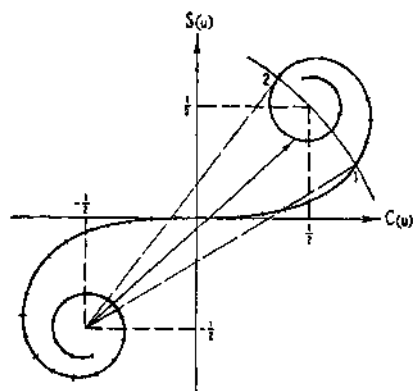


Fig. 2. Cornu spiral

to $1/4$, while when t is larger than T , then (J/J_0) behaves as a damped oscillation around the value $(J/J_0) = 1$, tending to this value when $t \rightarrow \infty$. This behavior is illustrated in Fig. 3.

From the standpoint of classical mechanics, it is clear that $(J/J_0) = 0$ if t is less than the time of flight T , while $(J/J_0) = 1$ if $t \geq T$. This behavior is also illustrated in Fig. 3 by the straight line.

A good measure of the "width" of this diffraction effect in time, can be obtained from the difference τ between the first two times at which (J/J_0) takes the classical value, i.e. $\tau = t_2 - t_1$, as shown in Fig. 3. The times at which the curve in Fig. 3 intersects with the straight line $(J/J_0) = 1$, correspond to

the values of u obtained from the intersection of the Cornu Spiral with the circle of radius $\sqrt{2}$ and center $(-\frac{1}{2}, -\frac{1}{2})$. The values u_1, u_2 that correspond to t_1, t_2 are the lengths along the Cornu Spiral from the origin to the point 1, 2 in Fig. 2, so that we have: $u_2 - u_1 = 0.85$. As $x \gg \lambda$ we see from (10) that t_1, t_2 are very close to T , so we may write:

$$\tau \simeq (u_2 - u_1) (\pi/kx)^{1/2} T = 0.85 (\pi x h/m V^3)^{1/2} \quad (13)$$

As an example (of possible interest in the operation of neutron velocity selectors), we have that for 300° K neutrons at a distance $x = 1$ m, the diffraction width would be:

$$\tau = 0.27 \times 10^{-8} \text{ sec.}$$

Borrowing from the terminology of optics, we could say that in classical mechanics a shutter casts a sharp shadow in time, i.e. the beam current jumps suddenly from zero to the stationary value at $t = T$, while in quantum mechanics there is a diffraction effect in time as illustrated in Fig. 3. When we make Planck's, constant h tend to zero, we see from (13) that the diffraction width τ tends also to zero, and the quantum mechanical current reduces to the classical current, as one should expect.

The analogy between the transient current (11) and the diffraction phenomena in optics, raises the question whether the transient currents for other types of wave equations show this analogy. We shall see in the following sections that this is not the case, and that only when ψ obeys the Schrödinger equation, does it resemble the two wave functions that appear in Sommerfeld's, theory of diffraction.

The transient current (11) increases monotonically from the very moment in which we open the shutter, and therefore, in principle, an observer at a distance x from the shutter could detect particles before a time (x/c) , where c is the velocity of light. This would imply that some of the particles travel with velocities larger than c , and the error is due of course, to employing the non-relativistic Schrödinger equation in the analysis.

We shall discuss in the following sections the transient wave functions associated with two relativistic equations, the ordinary wave equation and the Klein-Gordon equation. It will be shown in this case, that at a distance from the shutter, the wave function $\psi(x, t)$ is zero for $t < (x/c)$, thus safeguarding the relativity principle. When in the solution for the Klein-Gordon equation we make $c \rightarrow \infty$ it will reduce essentially to (6), thus giving the transient current (11) in the non-relativistic approximation.

III. THE ORDINARY WAVE EQUATION

We shall now deal with the same problem of the last section, but assume that ψ satisfies the ordinary wave equation:

$$c^2 (\partial^2 \psi / \partial x^2) = (\partial^2 \psi / \partial t^2) \quad (14)$$

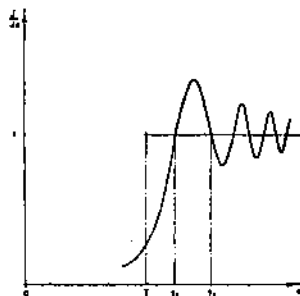


Fig. 3. Diffraction in time

If initially ψ and its time derivative are given by

$$\psi(x, 0) = F(x), \quad (\partial \psi / \partial t)_{t=0} = G(x) \quad (15)$$

then as is well known (WEBSTER, 1933), $\psi(x, t)$ has the form:

$$\psi(x, t) = \frac{1}{2} \left[F(x + ct) + F(x - ct) + c^{-1} \int_{x-ct}^{x+ct} G(x') dx' \right]. \quad (16)$$

For $t < 0$ when the shutter was closed, we had on the left side of the shutter:

$$\psi(x, t) = \exp[ik(x - ct)], \text{ for } x < 0$$

This implies that at $t = 0$ we have:

$$F(x) = \exp(ikx), G(x) = -ikc \exp(ikx), \text{ if } x < 0, \quad (17)$$

while $F(x) = G(x) = 0$ if $x > 0$.

Introducing these initial conditions in (16) we obtain:

$$\psi(x, t) = \begin{cases} 0 & \text{if } t < (x/c) \\ \exp[ik(x - ct)] - \frac{1}{2} & \text{if } t > (x/c). \end{cases} \quad (18)$$

The absolute value of the wave function jumps suddenly to the value $1/2$ at $t = (x/c)$, and it oscillates periodically thereafter. This behavior has certainly no resemblance to the diffraction in time effect obtained in the previous section.

IV. THE RELATIVISTIC SHUTTER PROBLEM

If we wish to extend the discussion of the transient effects of section IF, to a beam of electrons at relativistic energies, we would have, of course, to replace equation (1) by Dirac's equation. A fundamental change creeps in though by the fact that the wave function will involve the negative, as well as the positive energy eigenfunctions. A complete description of the phenomena would necessitate then the introduction of hole theory, and the single particle picture would have to be abandoned.

As our main interest concerns the transients associated with the different wave equations, we shall not attempt to discuss here the problem when hole theory is involved, but rather will restrict ourselves to the single particle picture in which states of both positive and negative energy are available. Furthermore, the features of spin and statistics are irrelevant to our analysis, so that for simplicity, instead of Dirac's equation we shall use the Klein-Gordon equation. We shall briefly indicate though, at the end of this section, the relation of the solution for the Dirac equation with the solution to be obtained for the Klein-Gordon equation.

The wave function $\psi(x, t)$ satisfies now the equation:

$$(\partial^2 \psi / \partial x^2) = c^{-2} (\partial^2 \psi / \partial t^2) + \mu^2 \psi, \quad (19)$$

where: $\mu = (mc/\hbar)$.

If initially ψ and its time derivative are given by:

$$\psi(x, 0) = F(x), \quad (\partial \psi / \partial t)_{t=0} = G(x), \quad (20)$$

then using the Fourier integral theorem, we obtain:

$$\psi(x, t) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} [f(\kappa) \cos(\epsilon ct) + (\epsilon c)^{-1} g(\kappa) \sin(\epsilon ct)] \exp(i\kappa x) d\kappa, \quad (21)$$

where $f(\kappa), g(\kappa)$ are the ordinary Fourier transforms of $F(x), G(x)$ and $\epsilon = (\kappa^2 + \mu^2)^{1/2}$.

For $t < 0$, when the shutter was closed, we assume as in the previous section, that:

$$\psi(x, t) = \exp[i(kx - Ect)], \text{ for } x < 0$$

where E is the energy in units of reciprocal length, i.e.

$$E = (k^2 + \mu^2)^{1/2}. \text{ This implies that at } t = 0 \text{ we have:}$$

$$F(x) = \exp(ikx), \quad G(x) = -iEc \exp(ikx) \text{ for } x < 0, \quad (22)$$

while $F(x) = G(x) = 0$ for $x > 0$.

The Fourier transforms $f(\kappa), g(\kappa)$ become then (HEISENBERG, 1943):

$$f(\kappa) = (2\pi)^{-1/2} \int_{-\infty}^0 \exp[i(k - \kappa)x] dx = (2\pi)^{1/2} \delta_+(\kappa - k) \quad (23)$$

$$= (\pi/2)^{1/2} \{ \delta(\kappa - k) + i[\pi(\kappa - k)]^{-1} \}$$

$$g(\kappa) = -iEc f(\kappa). \quad (24)$$

Introducing (23,24) in (21) we obtain:

$$\begin{aligned} \psi(x, t) &= \frac{1}{2} \exp[i(kx - Ect)] \\ &- (4\pi i)^{-1} P \int_{-\infty}^{\infty} (\kappa - k)^{-1} (\epsilon + E) \exp[i(\kappa x - \epsilon ct)] \epsilon^{-1} d\kappa \\ &- (4\pi i)^{-1} P \int_{-\infty}^{\infty} (\kappa - k)^{-1} (\epsilon - E) \exp[i(\kappa x + \epsilon ct)] \epsilon^{-1} d\kappa. \end{aligned} \quad (25)$$

As usual, when $\delta_{\pm}$ functions are employed, the integrals must be interpreted in the sense of Cauchy's principal value (WHITTAKER & WATSON, 1943) as indicated by the P . The initial conditions at $t = 0$ can be obtained from (25) by closing the contour in the κ -plane, from above if $x > 0$ and from below if $x < 0$. The first and second integrals in (25) give the contribution from the positive and negative energies respectively.

To eliminate the branch points at $\kappa = \pm i\mu$ in (25), we introduce in the first integral the change of variable:

$$\xi = \mu^{-1}(\kappa + \epsilon) \quad (26)$$

from which we have: $\kappa = (\mu/2)(\xi - \xi^{-1})$, $\epsilon = (\mu/2)(\xi + \xi^{-1})$. The limits of integration $[-\infty, \infty]$ for the κ transform into $[0, \infty]$ for the ξ .

In the second integral we introduce a ξ corresponding to negative energies, i.e. $\xi = \mu^{-1}(\kappa - \epsilon)$. It is clear that with this ξ , the integrand in the second integral becomes the same function of ξ that we have in the first integrand. Furthermore, for this ξ the limits of integration are $[-\infty, 0]$.

Combining the two integrals we obtain:

$$\psi(x, t) = \frac{1}{2} \exp[i(kx - Ect)] - (2\pi i)^{-1} \int_{-\infty}^{\infty} (\xi + z)(\xi - z)^{-1} (\xi \xi^*)^{-1} \exp\left\{\frac{1}{2} i \mu [(x - ct)\xi - (x + ct)\xi^*]\right\} d\xi \quad (27)$$

where: $z = \mu^{-1}(k + E)$.

The integrand of (27) has a pole at $\xi = z$ and an essential singularity at $\xi = 0$, but the integral is convergent, and in the appendix it is evaluated by a suitable deformation of the contour.

From the analysis given in the appendix, it follows that:

$$\psi(x, t) = 0 \quad \text{if} \quad t < (x/c), \quad (28a)$$

$$\psi(x, t) = \exp i(kx - Ect) + \frac{1}{2} J_0(\eta) - \sum_{n=0}^{\infty} (\xi/iz)^n J_n(\eta), \quad \text{if} \quad t > (x/c), \quad (28b)$$

$$\text{where:} \quad \xi = [(ct + x)/(ct - x)]^{1/2}, \quad \eta = \mu(c^2 t^2 - x^2)^{1/2}, \quad (29)$$

and the J_n are the Bessel functions of order n .

The wave function $\psi(x, t)$ of (28) satisfies the Klein-Gordon equation, as can be easily seen when we express this equation in terms of the variables ξ, η of (29), so that (19) takes the form:

$$\left(\frac{\partial^2}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial}{\partial \eta} - \frac{\xi^2}{\eta^2} \frac{\partial^2}{\partial \xi^2} - \frac{\xi}{\eta^2} \frac{\partial}{\partial \xi} + 1 \right) \psi(\xi, \eta) = 0 \quad (30)$$

As $\xi^\alpha J_\alpha(\eta)$ is a solution of (30) for any α , we see that each term of the series in (28b) satisfies the Klein-Gordon equation.

For $n=0$ the solution $J_0[\mu(c^2 t^2 - x^2)^{1/2}]$ of the Klein-Gordon equation is well known in quantum electrodynamics (DIRAC, 1934; SCHWINGER, 1949). For $n \neq 0$ the solutions $\xi^n J_n(\eta)$ do not seem to have had a wide use in quantum field theory, possibly because they are not in themselves Lorentz-invariant, but become so, only after multiplication by z^{-n} .

When the rest mass of the particle vanishes, then $\eta \rightarrow 0$ and $z \rightarrow \infty$, so that all the terms in the series of (28b) vanish except $J_0(\eta)$ which becomes 1. The solution (28) reduces in this case, to the solution (18) of the ordinary wave equation as we should expect.

We are now interested in the asymptotic values of $\psi(x, t)$ when $t \rightarrow (x/c)$ and when $t \rightarrow \infty$. In the first case, we see from (29) that $\eta \rightarrow 0$ and we can replace the Bessel functions by their asymptotic values:

$$J_n(\eta) \rightarrow (2^n n!)^{-1} \eta^n \quad (31a)$$

The series in (28b) becomes then:

$$\sum_{n=0}^{\infty} (\xi/iz)^n J_n(\eta) \rightarrow \sum_{n=0}^{\infty} (n!)^{-1} (\xi \eta / 2iz)^n = \exp(\xi \eta / 2iz) \quad (31b)$$

For $t = (x/c)$ the series takes the value $\exp[i(k - E)x]$ and the asymptotic form of $\psi(x, t)$ is (1):

$$\lim_{t \rightarrow (x/c)} [\psi(x, t)] = \frac{1}{2} \quad (32)$$

The wave function $\psi(x, t)$ jumps suddenly from zero to the value $1/2$ at $t = (x/c)$, just as in the case of the ordinary wave equation, and it does not build up continuously from $t = 0$ as in the case of the Schrödinger equation.

To obtain the asymptotic form of $\psi(x, t)$ when $t \rightarrow \infty$, we notice first that in this case, $\xi \rightarrow 1$ and $\eta \rightarrow \infty$. Replacing in the series of (28b) the $J_n(\eta)$ by their asymptotic values:

$$J_n(\eta) \rightarrow \left(\frac{1}{2} \pi \eta\right)^{-1/2} \cos\left[\eta - \frac{1}{4}(2n + 1)\pi\right], \quad (33a)$$

we get:

$$\begin{aligned} \left| \sum_{n=0}^{\infty} (\xi/iz)^n J_n(\eta) \right| &\rightarrow \left| \left(\frac{1}{2} \pi \eta\right)^{-1/2} \sum_{n=0}^{\infty} (iz)^{-n} \cos\left[\eta - \frac{1}{4}(2n + 1)\pi\right] \right| \\ &\leq \left(\frac{1}{2} \pi \eta\right)^{-1/2} \sum_{n=0}^{\infty} z^{-n} \end{aligned} \quad (33b)$$

From its definition $z = \mu^{-1}(k + E) > 1$ for any $k > 0$ so that the series in (33b) converges, and therefore, when $\eta \rightarrow \infty$ the series in (28b) tends to zero. The asymptotic value of $\psi(x, t)$ when $t \rightarrow \infty$, becomes then:

$$\psi(x, t) \rightarrow \exp i[(kx - Ect)], \quad (34)$$

which is the stationary form of the wave function that we expect.

To pass from the relativistic wave function (28b) to the non-relativistic wave function (6), we have to take $c \rightarrow \infty$. The simplest way to make this transition is through the expression (25) for $\psi(x, t)$. When $c \rightarrow \infty$ we see that $\epsilon ct = (x^2 + \mu^2)^{1/2} ct$ tends to:

$$\epsilon ct \rightarrow \mu ct + (\hbar k^2/2m)t + \dots, \quad (35)$$

where all the other terms of the development vanish for $c \rightarrow \infty$. A similar result holds for Ect . Furthermore, when $c \rightarrow \infty$ we see that $(E/\epsilon) \rightarrow 1$, and therefore, the negative energy part of (25) vanishes, while the rest takes the asymptotic form:

$$\begin{aligned} \psi(x, t) &\rightarrow \exp(-i\mu ct) \left\{ \frac{1}{2} \exp i[kx - (\hbar k^2/2m)t] \right. \\ &\quad \left. - (2\pi i)^{-1} P \int_{-\infty}^{\infty} (x - k)^{-1} \exp i[kx - (\hbar k^2/2m)t] dk \right\} \\ &= \exp(-i\mu ct) \frac{1}{2} \chi(x, k, t). \end{aligned} \quad (36)$$

(1) The asymptotic behavior of $\psi(x, t)$ can also be derived, in a simple and rigorous fashion, from the integral representation of the wave function that is given in the appendix.

The last result follows from the evaluation of the integral in (36), that was carried in appendix 1 of (4).

It follows from (36) that when $c \rightarrow \infty$, the transient current obtained with the relativistic wave functions reduces precisely to (11).

Finally, we would like to remark that an analysis similar to the one carried above, can be given for the Dirac wave equation. Choosing a definite direction of spin, the Dirac wave function reduces to two components, each of which can be expressed, for $t > (xk)$, in terms of linear combinations of the wave function (28b) and $J_n(n)$.

V. ANALOGIES WITH SOMMERFELDS DIFFRACTION THEORY

In the previous sections, the shutter problem for different types of wave equations was analyzed, and we obtained the corresponding wave functions. Rather than to compare the related transient currents with the intensity of light in optical diffraction (as suggested by the form of the non-relativistic current (11)), we shall look into the analogies between the wave functions themselves, and a family of solutions in Sommerfeld's theory of diffraction.

For electromagnetic diffraction problems in the plane we need appropriate solutions of the two dimensional Kirchhoff equation, which in polar coordinates has the form:

$$(\nabla^2 + k^2) \phi \equiv (\partial^2 \phi / \partial r^2) + r^{-1} (\partial \phi / \partial r) + r^{-2} (\partial^2 \phi / \partial \theta^2) + k^2 \phi = 0, \quad (37)$$

where $k = (2\pi/\lambda)$ and λ is the wave length of the radiation.

The well known family of solutions of (37) that is obtained with the help of the Riemann surface (BAKER & COPSON, 1939) analysis of Sommerfeld, can be written in the form:

$$\chi'(r, \theta, \theta_0) = \exp(ikr) \exp(y'^2) \operatorname{erfc}(y'), \quad (38a)$$

where:

$$y' = -\exp(-i\pi/4) (2kr)^{1/2} \sin \frac{1}{2} (\theta - \theta_0), \quad (38b)$$

and θ_0 is an arbitrary parameter.

The χ' satisfy (37) as can be seen from the fact that:

$$\begin{aligned} [\nabla^2 + k^2] \chi'(r, \theta, \theta_0) &\equiv k^2 (2ikr)^{-1} \exp(ikr) \\ &\times \left[\frac{d^2}{dy'^2} - 2y' \frac{d}{dy'} - 2 \right] \exp(y'^2) \operatorname{erfc}(y'), \end{aligned} \quad (39)$$

and as shown in (4), the right hand side of (39) vanishes.

Despite the fact that the time dependent Schrodinger equation is of the parabolic type, while the Kirchhoff equation is of the elliptic type, they have solutions of the form (3) and (38) which are remarkably similar. No such similarity exists between the two functions (18) and (28) of the hyperbolic

relativistic wave equations, and the family of solutions (38), so that we do not expect in this case, a resemblance between the transient current and optical diffraction effects.

When a semi-infinite perfectly reflecting plane is introduced at $\theta = -(\pi/2)$, in the path of an electromagnetic wave polarized in the plane and propagating in the direction $\theta = 0$, the wave function $\phi(r, \theta)$ that satisfies the boundary conditions at $\theta = (-\pi/2), (3\pi/2)$ and the asymptotic behavior at $r \rightarrow \infty$, becomes: (BAKER & COPSON, 1939),

$$\phi(r, \theta) = \frac{1}{2} [\chi'(r, \theta, 0) - \chi'(r, \theta, \pi)] . \quad (40)$$

A corresponding problem for the transient current appears when the shutter is represented as a perfect reflector, i.e. an ∞ potential barrier. In this case, we must replace in the initial conditions (2), $\exp(ikx)$ by $\exp(ikx) - \exp(-ikx)$, and with the help of (5) we immediately obtain that:

$$\psi(x, t) = \frac{1}{2} [\chi(x, k, t) - \chi(x, -k, t)] . \quad (41)$$

From (40), and assuming that $r \gg \lambda$, we obtain the characteristic Fresnel diffraction effect (BORN, 1939) for the intensity of the electromagnetic wave in the vicinity of $\theta = 0$. From (41), and assuming $r \gg \lambda$, we obtain the transient current (11), which also shows a Fresnel diffraction effect in the vicinity of $t = T$, where T is the time of flight $T = (x/v)$.

VI. CONCLUSION

It is well known that ordinary diffraction phenomena for beams of particles are closely associated with the position-momentum uncertainty relations (HKISENBKKG, 1930). The appearance of diffraction in time effects for the transient current, when an obstacle is introduced or removed from the incident beam, suggests that these effects are closely connected with the time-energy uncertainty relation. This is shown to be the case by analyzing the transient wave function when an impenetrable spherical potential is introduced into the beam (MOSHINSKY, 1952). The ratio of the transient to the stationary scattered current has in this case, also the form (11). If the wave function is transformed to the energy-angular momentum representation (MOSHINSKY, 1952), it shows that when a scattered particle is detected at a time Δt after the introduction of the scatterer, there is an indetermination ΔE in its energy, such that:

$$\Delta t \Delta E \simeq h . \quad (42)$$

The χ functions that are used in the non-relativistic shutter problem, appear also (in the form $\chi(r, \pm k'', t)$ where r is the distance from the scatterer and k'' the momentum of the incident particles) in the dynamical description of the scattering by an impenetrable spherical potential (MOSHINSKY, 1952), and in the resonance scattering processes (A). These functions account then for

the transient effects due to the time-energy uncertainty relations. When a scatterer has a structure, such as in the single level scattering process (A), there are other terms in the transient wave functions that depend on the poles of the scattering matrix. The transient effects that these terms give rise to in the current, are then no longer general quantum mechanical effects, but are related to the specific nature of the scatterer.

The important role that the x functions play in the dynamical description of non-relativistic scattering processes, suggests that their relativistic generalization, given by the wave function $\psi(x, t)$ of (28), may play a similar role in any dynamical description of relativistic scattering processes.

The author is indebted to Professors M. S. Vallarta and J. Lifshitz for helpful discussions.

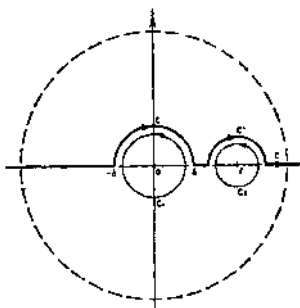


Fig. 4. Integration contour for the relativistic wave function

APPENDIX

In this appendix, we shall evaluate the integral in (27), so as to determine the explicit form of the relativistic wave function.

Instead of taking the principal value of the integral in (27), we shall make the integration along the contour C of Fig. 4, where the singular points $\xi = 0, z$ of the integrand, are bypassed from above by means of the semicircles C' , C'' . From Fig. 4 we obtain:

$$P \int_{-\infty}^{\infty} - \int_C = \int_{-\delta}^{\delta} - \int_{C'} - \int_{C''} \quad (43)$$

The integral around C'' must be interpreted as the limiting value when the radius of the semi-circle tends to zero. As the integrand has a simple pole at $\xi = z$, we obtain:

$$- \int_{C''} = \pi i \operatorname{Res} (\xi = z) = \pi i \exp i [(kx - Ect)] \quad (44)$$

From the form of the integrand in (27), we see that the first two integrals on the right hand side of (43) vanish when $\delta \rightarrow 0$, assuming that $x, t > 0$. As the integrand has an essential singularity at $\xi = 0$, but is regular outside this point (except for the pole at $\xi = z$), we conclude that the first two integrals of (44) add to zero for any $\delta < z$.

From (27) and (43) we obtain then:

$$\begin{aligned} \psi(x, t) = & - (2\pi i)^{-1} \int_C (\xi + z) (\xi - z)^{-1} (2\xi)^{-1} \exp i (\mu/2) [(x - ct) \xi \\ & - (x + ct) \xi^{-1}] d\xi \end{aligned} \quad (45)$$

When $x > ct$ we can close the contour from above, as the contribution from the dotted semi-circle is clearly zero, and the integrand is analytic in that part of the ξ plane above C , we conclude that:

$$\psi(x, t) = 0 \quad \text{if } x > ct. \quad (46)$$

When $x < ct$ we close the contour from below. Again the contribution of the dotted semi-circle is zero, but now we have the singular points $\xi = 0, z$ inside the contour. As the singular points are not branch points, we obtain that:

$$\psi(x, t) = (2\pi i)^{-1} \left[\int_{C_1} + \int_{C_2} \right], \text{ for } x < ct, \quad (47)$$

where the contours C_1, C_2 are now the closed circles surrounding the points 0 and z respectively.

The second integral in (47) is given by twice the value of (44). For the first integral we introduce the variables ξ, η defined by (29), and changing to the variable of integration given by:

$$\xi' = -i\xi^{-1}\xi, \quad (48a)$$

we obtain:

$$(2\pi i)^{-1} \int_{C_1} = (2\pi i)^{-1} \int_{C_1'} \left[(\xi' - z')^{-1} - (2\xi')^{-1} \right] \exp \left[\frac{1}{2} \eta (\xi' - \xi'^{-1}) \right] d\xi', \quad (48b)$$

where $z' = -i\xi^{-1}z$, and the radius of C_1' is less than $|z'| = \xi^{-1}|z|$.

Using the well known formula (WHITTAKER & WATSON, 1943):

$$\begin{aligned} \exp \left[\frac{1}{2} \eta (\xi' - \xi'^{-1}) \right] &= \sum_{n=0}^{\infty} \xi'^n J_n(\eta) \\ &+ \sum_{n=1}^{\infty} (-1)^n \xi'^{-n} J_n(\eta), \end{aligned} \quad (49a)$$

as well as the fact that for $|\xi'| < |z'|$ we have:

$$(z' - \xi')^{-1} = z'^{-1} \sum_{n=0}^{\infty} (\xi'/z')^n, \quad (49b)$$

we obtain straightforwardly that:

$$\begin{aligned} (2\pi i)^{-1} \int_{C_1} &= \text{Res}(\xi' = 0) \\ &= - \sum_{n=1}^{\infty} (\xi/iz)^n J_n(\eta) - \frac{1}{2} J_0(\eta). \end{aligned} \quad (50)$$

Introducing this result into (47), we obtain finally the explicit form of $\psi(x, t)$ for $x < ct$ as given by (28b).

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DISCUSSION

BECK: I would like to ask about the possibility of observing such a phenomenon. Would it be possible, for instance, to find deviations from the stationary treatment of the scattering by a molecule, the nuclear molecules?

MOSHINSKY: What would be the experimental arrangement that you have; in mind?

BECK: I have a molecule, or a crystal. In the stationary state we get the usual interference phenomenon. If now we let incide the wave; packet for a fraction of time, then this could give you deviations which would be observable. I do not know whether it is possible, but this would be very useful to be tested.

MOSHINSKY: First of all, then; is a diffraction of time, so to say pure, in the sense that it has nothing to do with the structure of the scatterer but just on the fact that a wave packet is limited to one half of the space. Now, we can see that there is a definite width for the diffraction effect $r = 0.85 (\pi \times \hbar/m v)^{1/2}$. This width would be larger the lower the energy we could achieve. If one could go to very low energies, for example, when one is working with thermal neutrons at very low temperatures, the diffraction width could become large enough to be observable.

MEDINA: I just want to have something clear. I think you made the remark that the result obtained from the Klein-Gordon equation is similar to Dirac's equation. That seems very surprising.

MOSHINSKY: The similarity between the wave functions for the Klein-Gordon and the Dirac equations that I mentioned, concerned the mathematical functions that appeared in both.

VALLARTA: Prof. Bohm mentioned in his lecture a while ago that there was no known way of checking the transient solutions of the Schroedinger equation and that all the known checks have to do with the stationary solutions of the Schrodinger equation. In this analysis that Moshinsky has made, there is an instance of a transient solution of the Schrodinger equation and of the corresponding Gordon-Klein equation. In principle it should be possible to check this experimentally because the times that are involved here are not entirely beyond present experimental techniques. So, I think it has a bearing on the previously discussed question.

MOSHINSKY: I would like to point out that the last problem discussed corresponded to the scattering of a particle by rigid sphere. This could be applied to Bohm's equation of motion for a particle that is scattered by a rigid sphere.

BOHM: In regard to the remarks by Professor Vallarta, I believe that what he says is completely correct. I would like to add that if you did this test at a very low temperature so as to get down the rate of collision and simultaneously used some material that had been subjected recently to very intense nuclear activity, so as to encourage the discrepancy between the probability and the Psi square, this might provide a test if the experiment was done very precisely.

G. WATAGIIN: If you consider an ideal experiment which works for such a sort time that you have a great uncertainty in the energy, then what kind of new phenomena are you expecting to obtain from Dime's equation, for instance, when there is a polarization of vacuum.

MOSHINSKY: The analysis which has been made here concerns the single particle picture in both the Klein-Gordon and Dime's equation. Now, of course, I have to use both the negative and positive energies, and if one interprets the negative energies in connection with the creation of pairs, then one will have them during a very short time. When the stationary state is established, we have a plane wave of positive energy throughout space and the transient effects that, involve both positive and negative energies, will have disappeared.

G. WATAUHIN: Have you considered this case of Dime-equations?

MOSHINSKY: Yes, I have considered the case when a Dime equation is involved.

Relationships Between Particle and Field in Covariant Field Theories

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We have been able to develop a general approach to the very special problems that one faces in classical field theories that are invariant with respect to transformation groups that contain arbitrary functions. We have been especially interested in applying our method to the coordinate transformation group of covariant field theories, but we have found that the group of gauge transformations of electromagnetic theory can serve us as a useful guide.

In each case we have shown that the invariance properties of the theory cause the field equations to determine some aspect of the motion of particles. These particles are treated as regions of space where the field equations are not satisfied. The equations that determine the motion of the particles are in the form of two dimensional spatial surface integrals whose values do not depend upon the surface of integration chosen. The electromagnetic field equations give rise to conservation of charge, while the field equations of covariant theories lead to the four conservation laws of energy and momentum for the particles. We have derived general expressions for these conservation laws. They are the generalizations of the approximate equations of motion found by KINSTEIN, INFELD and HOFFMANN in the general theory of relativity.

These⁴ restrictions on the motion of the particles arise because the field equations satisfy a set of differential identities. In covariant field theories they are four in number and they are called the generalized BIANCHI identities because their origin stems from the same invariance properties as the well known BIANCHI identities of the general theory of relativity. Electromagnetic theory possesses a single gauge identity. In the LAGRANGIAN formalism these identities can be written in the form of strong conservation laws, and the equations of motion can then be shown to be the integral form of these.

By making use of the invariance properties of the action integral we have been able to free ourselves from the LAGRANGIAN formalism and to set up a consistent HAMILTONIAN scheme. We have found general expressions for the primary and secondary constraints and the equations of motion of particles. The constraints are vanishing relations between the field variables and their canonically conjugate momenta. These constraints, together with the equations of motion, are associated with a generating functional that arises from the application of an invariant variation to the action integral. This generating

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functional is the generator of the infinitesimal transformations under which the theory is invariant. The functional and its total time derivative are shown to vanish, and since it depends upon arbitrary functions, the coefficients of these arbitrary functions must be the constraints. We have also been able to show that the constraints and the HAMILTONIAN form a function group, by noting that the POISSON brackets between two different generating functionals is a generator of the same type. When we make the transition to LOHENTZ covariant theories it is possible to show that the new generating functional defines the conservation laws of linear and angular momentum for the fields.

Causal Explanation of Probability Distributions in Classical Statistical Mechanics

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Classical Statistical Mechanics is usually based on the hypothesis that thermodynamic systems constitute a statistical ensemble with a uniform probability on any surface of constant energy in phase space. This hypothesis has not, however, been verified experimentally. For macroscopic averages are so insensitive to the assumed distribution that the agreement between thermodynamics and statistical mechanics provides almost no information about the actual distribution of systems in phase space. Moreover, it is clearly impossible by direct measurements of the system of atoms to verify that the probability distribution on a surface of constant energy is really constant. Although the precise form of the distribution is of no importance for equilibrium problems, it could be important in systems far from equilibrium.

We wish to discuss here a program for using the information supplied by the laws of motion to make possible an improved basis for statistical mechanics. The basic property of the orbits that interests us here is the extreme instability arising from the fact that the angle of deflection in collision is very sensitive to the initial positions and velocities of the incident particles. We see then that systems originally contained in a small rectangular element in phase space will tend to separate from each other after many collisions. Although according to Liouville's theorem, the total volume occupied by the systems remains constant, the shape of the volume element will alter radically, taking the form of a chaotically tangled thread-like structure, which ultimately covers a very wide region of phase space. Although the probability distribution at this time actually fluctuates chaotically and violently as a function of position in phase space, these fluctuations take place on such a small scale that macroscopic averages are essentially the same as those obtained with a continuous distribution. Moreover, if we consider all those systems arriving at a given time t , in a given rectangular element d , the reversibility of the equations of motion permits us to deduce that they came from a similar chaotically tangled thread-like element in the distant past dW . Thus, although the density following a given point remains constant, the mean density in a small element dQ (which is all that is significant for macroscopic averages) is the average of densities coming from the chaotically tangled thread-like element dQ' described above. On the basis of this fact, we shall show that this mean density approaches a constant after many collisions. The process is analogous to what happens when a droplet of insoluble fluid ink is stirred up

into a glass of water. Ultimately we obtain what to the unaided eye seems to be a continuously black fluid, whereas a microscopic examination would disclose a chaotically tangled thread of black ink suspended in a transparent background.

To sum up, we have shown that an ensemble of systems starting in arbitrarily small domain of phase space d will continually be transformed into an ensemble that is uniform for all macroscopic purposes. But we know on the basis of common experience that no matter how our ensemble of systems starts out, there will always be small uncontrollable fluctuations or errors in the initial conditions which cause the ensemble initially to occupy a small volume element dt . Thus the hypothesis of probability distribution can now be introduced in an experimentally verifiable way. Possible methods of replacing the concept of probability in statistical mechanics by a concept of chaos will be discussed with details to be given at the meeting.

Some Experiments on the Annihilation of Positrons

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It is known that the rest mass of a pair of positive and negative electrons can be transformed into electromagnetic radiation. If the two particles collide without interacting with any other body, the emission of at least two quanta is necessary for conservation of momentum. When, in particular, the electron and positron annihilate at rest, the two quanta will be emitted in opposite directions, each of them carrying an energy me^2 . This simplified view, however, is not a complete description of the phenomenon of annihilation, which does not necessarily occur at rest, and which can occur with the emission of an arbitrary number of γ -rays, provided that the sum of their energy, momenta, and angular momenta equals the energy, momentum and angular momentum of the initial electron-positron-system.

Here, I will report on a series of investigations on the details of the annihilation process; these have been carried on in the last few years at Washington University and at the Carnegie Institute of Technology, and are presently continued at this latter Institution.

ANGULAR DISTRIBUTION OF TWO QUANTUM ANNIHILATION RADIATION

The deviations from co-linearity of the two annihilation quanta, and thus the momenta of the annihilating pairs, were studied with a coincidence method (DE BENEDETTI *et al*, 1950). Two anthracene scintillation counters were located at a distance of 240 cm from each other, with the source of positrons half way between them. The source consisted of a thin sample of Cu^{64} , surrounded by gold foil, in such a way that most of the positrons annihilated in gold. One of the counters was displaced perpendicularly to the line connecting it to the source, and the number of coincidences was recorded for different positions, at intervals of a few mm. It was found that the coincidences did not vanish immediately when the two counters were out of line with respect to the source, and this indicates that the annihilation γ -rays are not always emitted exactly in opposite directions. The counting rate decreased exponentially with the angular distance from alignment, the mean angle being 4.5×10^{-3} radians. This result is in agreement with DUMOND'S observations on the width of the annihilation line (DIT MOND *et al.*, 1949), and has since been confirmed and extended to other positron absorbers by WARREN and GRIFFITHS (1951).

It is natural to associate the angular deviation to the motion of the electron and positron of the annihilating pair. In fact, it is immediately seen

that the angular deviation, expressed in radians, is equal to p/me , where p is the momentum of the center of mass of the annihilating pair. Thus, it follows from this experiment that the momentum of the annihilating pair is of the order of magnitude of the momenta of the valence atomic electrons, or of the free electrons in a metallic conductor. One is therefore led to the following conclusions:

- a) The positrons rarely annihilate against the electrons of the deeper atomic shells;
- b) The momentum of the annihilating positron is not larger than that of the conduction electrons in a metal.

A theoretical analysis shows that one can expect in fact that the positrons survive long enough after the end of their ionized path to practically reach thermal equilibrium with the lattice of the solid substance in which they are stopped. The positrons are then so effectively repelled by the electrostatic field within the atoms that they are sure to annihilate only with the outside (or with the free) electrons.

HALF-LIFE OF POSITRONS IN CONDENSED MATERIALS

The theoretical value of the annihilation cross section is $\pi r_0^2 c/v$ (where r_0 is the classical electron radius) if one assumes that the relative velocity v of the electron and positron is small compared to the velocity of light c , and if one neglects the effect of electrostatic attraction between the two members of the pair. Using this value for the cross section, and taking into account the fact that only the valence electrons contribute to the annihilation, one is led to predict positron lives which could easily be measured with present techniques: for *Li*, *Na*, *K*, one obtains respectively 2, 3.6 and 7×10^{-9} sec. However, it is not legitimate to neglect the effect of Coulomb attraction, and this reduces considerably the positron life. Though it is difficult to compute this effect accurately, one can estimate its importance by using a model which should be approximately valid at least in the case of the alkali metals; the positron is considered at rest, the presence of the lattice is disregarded, and the free electrons are treated as a FERMI gas. With this model one obtains half-lives of 1.2, 1.8 and 2.9×10^{-10} sec for *Li*, *Na* and *K* respectively.

After developing suitably fast coincidence equipment, an experiment (DE BENEDETTI & KICHINAS, 1952) was performed in order to find if differences in positron half-lives could be detected in various absorbers. The method of delayed coincidences between the nuclear 7-rays of Na^{22} and the annihilation radiation from the positrons of the same source, was used.

Two stilbene counters were pointed at the source, at about 90° from each other. Their biases were so adjusted that a coincidence was recorded only if a nuclear 7-ray was detected in one of the counters, and an annihilation γ by the other. In the disposition of the absorbers special care was taken in order to avoid differences in the amount of energy degradation of the 7-rays, which could have produced spurious delays. It was found that in all metals tested, including *Li*, *K*, *Al*, *Cu* and *Pb*, the life of the positrons was the same within the experimental errors which were about 5×10^{-11} sec. Sulphur, graphite, *NaCl* behaved in the same manner, while other substances, like quartz, paraffin and water exhibited a measurably longer half-life, the apparent differences being

1 or 2×10^{-11} sec. These results have been confirmed and extended by BELL & GRAHAM (1952) who, following the decay curves further than we did, have shown that certain substances (for instance fused quartz and polystyrene) display two decay components, one of the order of 10^{-10} sec, and another of the order of 10^{-9} sec.

It is premature to attempt an interpretation of these results, and, waiting for more data, one can only put forward some conjectures. It seems improbable that the equality of half-lives in metals, which contradicts the expectations from the model previously discussed, could be explained by the annihilation with inner electrons, which were neglected. It might become legitimate to ask whether the positron and the electron form some kind of state, which could be of the same nature in all metals, and account for the observations with its fast decay. It must be kept in mind, however, that such state could not be the positronium atom, for which there is no room in the metal lattice and which would be revealed by an anomalous amount of three-quantum annihilation. The longer half lives of other substances are probably connected with their chemical and crystalline properties. It is possible that this work, which was started as a study of the properties of elementary particles, might result in providing a useful tool for the investigations of the nature of the chemical bond and of the structure of solids.

THREE QUANTUM ANNIHILATION

The phenomenon of three quantum annihilation is theoretically predicted to happen when positrons and electrons annihilate from the triplet state. However, this phenomenon has a small cross section and the expected probability of its occurrence is, for random collisions of positrons and electrons, only 1/370 of the probability of 2-quantum annihilation (ORE & POWKLL, 1949). Furthermore, it is more difficult to detect than 2-quantum annihilation, since the radiations emitted are not co-linear, but only co-planar.

The experiment (DE BENEDETTI & SIEGEL, 1952) was undertaken with a source of No^{12} (around 10^6 dis/sec) whose positrons were allowed to stop in a solid. In order to increase counting efficiency NaI(Tl) scintillators were used. Three of these were connected in coincidence; in order to reduce as much as possible the effects of the nuclear γ -rays of Na^{22} (which only contribute random counts), each counter was also connected to a differential pulse height selector, which admitted pulses only in the desired region of pulse height. The pulse height selectors were also used for the study of the energy of the three coincidence γ -rays. The effect was sought with the three counters coplanar with the source, and the background was measured by rotating one of the counters out of the plane, though maintaining it at the same distance from the source.

The counting rate with the counters coplanar was about 2c/min, with a background of 1 c/m with the counters not coplanar. After a study of counter efficiency and an analysis of the geometry experiment, it was found that the difference was consistent with the value of 1/370 for the ratio $3\gamma/2\gamma$. The experiment agrees with the theory within the errors (largely due to counter calibration) of 20%.

DEUTSCH'S discovery of the formation of stable positronium atoms in certain gases, (DEUTSCH, 1951) made it possible for us to obtain stronger sources

of three quantum annihilation and to proceed to a more detailed investigation of this phenomenon. This is because each triplet positronium atom behaves as an isolated stable system which must annihilate with the emission of three gamma rays; hence, the ratio $3\gamma/2\gamma$ could be as high as 3, instead of $1/370$, if all positrons led to the formation of positronium.

To test this prediction a A/a" source of a strenght comparable to the one previously used, was deposited on thin ZAPON support and enclosed in the center of an aluminum container 2.5 cm in diameter, In a typical experiment the container was filled with SF⁶ vapor at various pressures. The triple coincidence counting rate was found to increase about linearly with the density of the vapor, until it reached 33 c/'min at 0.060 g/cm². With this source, and using the differential pulse height selectors, we were able to study the energy of the coincident 7-rays for different angular arrangements. It was pleasing, if not surprising, to verify in an unambiguous manner the prediction of the laws of conservation of energy and momentum. The data obtained will also yield the angular distribution of the three quanta, but have not yet been analyzed in this respect.

We plan to continue our study of the physics of three quantum annihilation with an experiment on the polarization of the 7-rays; and we consider the possibility of investigating the chemical properties of positronium by observing the influence of different gases on its formation and stability.

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DISCUSSION

GOMIDE: 1 would like to ask you a question about the scintillation counters. Why did you use sodium iodide crystal in the second experiment instead of anthracene crystal as in the first experiment?

DE BENEDETTI: In the first experiment, we used anthracene crystal because stilbene and sodium iodine had not been found at that time. In the second experiment, we used stilbene or liquids, because we needed speed in the half life determination. In the third experiment we used sodium iodide counters because of their high efficiency to 7 rays and because of their energy resolution.

OCCHIALINI: What do you anticipate that would happen if in your experiment for the measurement of delays you would compare water and then ice, or sulphur, in the liquid state, and in the crystalline one?

DE BENEDETTI: I did not try ice personally but I know that it was tried by tho Canadians. I think that they got a measurable life as we did in water. However I do not trust comparing results of two different laboratories in such delicate tilings,

OCCHIALINI: I am trying to hint that the effect can be a crystalline one, and not a molecular one.

DE BENEDETTI: In effect the Canadian group have found that there is a difference between crystalline quartz and fused quartz,

Recent Experiments on Positronium

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RUARK (1945) first called attention to the possibility that a positron and an electron may form a bound structure analogous to the hydrogen atom. He proposed the name positronium for this quasi-atom. During the past 18 months the author and coworkers have made an experimental study of positronium (DEUTSCH 1951a, b; DEUTSCH & DULIT, 1951; DEUTSCH & BROWN, 1952). It is the purpose of this paper to present the methods and results of this study. Theoretical predictions concerning the properties of positronium have been published by WHEELER (1946), PIRENNE (1947), BERESTETSKII (1949), ORE (1949), FERRELL (1951), and others. The main results of the theory are the following:

The gross structure of the energy states is identical with that of hydrogen except for the effect of the smaller reduced mass. Thus, disregarding all fine structure the positronium energy terms have the values $E_n = -m_0 c^2 \alpha^2 / 4n^2$, just half of the corresponding hydrogen terms. In particular the ionization energy is 6.8 ev. Fundamental differences between the fine structure of positronium and that of hydrogen arise from three factors: 1. The symmetry in the motion of the two particles causes shifts due to orbital and spin-orbit interactions. These are mainly of interest for excited states which we have not investigated experimentally. 2. The large magnetic moment of the positron compared with that of the proton makes the magnetic spin-spin interaction of the order of the fine structure, i.e. $m_0 c^2 \alpha^4$ while the corresponding interaction in hydrogen only produces hyperfine structure. 3. The possibility of annihilation of the pair. This gives rise to a new short-range interaction characteristic of the particle-antiparticle combination. This "virtual annihilation" term raises the energies of triplet states but does not affect the singlet levels. In particular, the ground state is split by both the magnetic and the annihilation interaction, the ^3S state called ortho-positronium lying higher than the ^1S para-state. The splitting, to the order considered here is given by $\Delta W = m_0 c^2 7 \alpha^4 / 12 = 2.044 \times 10^5 \text{ Mc/sec.}$

The behaviour of these two states with respect to real annihilation is of special interest. The singlet undergoes annihilation at the rate $\lambda = 8 \times 10^9 \text{ sec}^{-1}$ with the emission of two quanta of energy $m_0 c^2$ each, in opposite directions. Two-quantum annihilation is forbidden in the ortho state. It decays ORE & POWELL (1949) at the rate $\lambda = 7 \times 10^8 \text{ sec}^{-1}$ with the emission of three quanta having a continuous energy spectrum. It is this "forbidden" decay which is used to detect and investigate positronium.

The author has demonstrated (DEUTSCH, 1951a) that positronium is formed abundantly and is stable against collisions in many gases. It is observed when positrons from a radioactive source are stopped in the gas. When positronium is formed, one quarter of the positrons decay by two-quantum emission with a mean life of 1.25×10^{-10} sec, too short to be observed by us. Three quarters decay with a mean life of 1.4×10^{-7} sec, which is conveniently measurable. This mean life is independent of the density in gases in which positronium is stable. The gamma-ray spectrum is continuous and this feature makes it readily distinguishable from the two-quantum process. On the other hand, when positrons are annihilated without forming positronium this occurs overwhelmingly by the two-quantum process with the emission of a discrete gamma ray "line" of 0.51 Mev. The decay rate under these conditions must always be proportional to the density of available electrons. In a gas this means proportionality with pressure.

The life-time was measured by the method of delayed coincidences between nuclear gamma rays emitted from the source and annihilation quanta arising in the gas. The apparatus will be described. In most gases the decay curves indicated a mixture of decay processes. In one gas, freon, the life-times were sufficiently different over a convenient range of pressures to permit a good measurement of the pressure-independent decay constant of ortho-positronium. The result (DEUTSCH, 1951b) was $\lambda_0 = 6.8 \pm 0.7 \times 10^6 \text{ sec}^{-1}$. At the time of these experiments three different theoretical results had been published for this life-time. The experimental result is in excellent agreement with one of these (ORE & POWELL, 1949) which has since been shown to be also theoretically the correct one.

The gamma-ray spectrum was observed by means of a scintillation spectrometer. Appropriate lead collimation assured that only annihilation radiation arising in the gas was recorded. A quantitative measure of the relative amounts of two- and three-quantum annihilation was made possible by the fact that in certain gases ortho-positronium is destroyed or "quenched" by collisions. In these gases, at sufficient pressure, the decay constant proportional to gas density and the gamma ray spectrum is a pure line spectrum. By observing the change of the gamma ray spectrum as a function of partial pressure of the quenching gas we found two groups of gases whose quenching cross section per molecule is of the order of 10^{-16} cm^2 . These are the odd-electron gases NO and VO and the halogens. The former probably convert ortho- to para-positronium by electron exchange while the latter may form positronium "compounds". Oxygen has a quenching cross section several hundred times smaller than the gases mentioned but still large enough to quench effectively at all triplet annihilation at partial pressures above about 1/2 atmosphere. Of numerous gases studied these are the only ones with an appreciable tendency to destroy positronium.

In most gases about 15 percent of the positrons form positronium. In order to capture an electron from a gas molecule a positron must have kinetic energy of at least $(V_i - 6.8) \text{ eV}$ where V_i is the ionization potential of the gas. For kinetic energies greatly in excess of V_i positronium formation should again be improbable. If the statistical fluctuations in the slowing-down process cause a positron to "miss" the critical energy region it can never form positronium

unless an external energy source is provided from which it can regain the necessary kinetic energy. It was observed that the application of a strong radiofrequency field to argon more than doubled the probability of positronium formation. In argon the lowest kinetic energy required for inelastic collisions is above that for positronium formation. In most other gases the reverse is true. Thus the addition of a small amount of a polyatomic gas like CO_2 was found to suppress the effect of the electric field in argon.

The most interesting subject of our experiments has been the ZEEMAN effect of the ground state. Positronium has no permanent magnetic moment and therefore no linear ZEEMAN effect in any of its states, since both the charges and the magnetic moments of the two particles are equal and opposite in sign. There is, however, a second order effect which, in the ground state, lowers the energy of the singlet level and raises that of the $m = 0$ component of the triplet. The components with $m = \pm 1$ are not affected by the field. In the magnetic field the $m = 0$ components represent mixtures of singlet and triplet wave functions. In particular the singlet admixture to the triplet level amounts to $x^2/4$, where $x = 4\mu_0 H/\Delta W$. This singlet admixture makes two-quantum annihilation possible for positronium atoms for which it is forbidden in the absence of a magnetic field. Without a field, $3/4$ of the positronium decays by the three-quantum process; in a very strong field this fraction is $1/2$, and in intermediate fields it is $f(H) = 1/2 + \lambda_0(1 - x^2)/4(\lambda_0(1 - x^2) + \lambda_p x^2)$, to a sufficient approximation. $f(H)$ was measured by observing the change of the gamma-ray spectrum as a function of H . The result (DEUTSCH & DULLE, 1951) is in good agreement with theoretical expectation and may be interpreted as measurement of ΔW through x . It clearly shows the existence of the "annihilation force" term in the ground state splitting.

A much more precise measurement was possible by means of radio-frequency methods. The separation between the $m = 0$ and $m = \pm 1$ components in a magnetic field is given by $h\nu = (\Delta W/2) \{(1 + x^2)^{1/2} - 1\}$. It is possible to induce transitions between these components by applying an alternating magnetic field H_p of the frequency indicated, at right angles to the steady field H . Since the $m = 0$ component decays almost entirely by two-quantum emission if $x^2 \gg \lambda_0/\lambda_p$ the transition can be detected by the change in the gamma ray spectrum. In the range $\lambda_0/\lambda_p \ll x^2 \ll 1$ the resonance frequency ν , the line width $\Delta\nu$, and the square of the matrix element for the transition are all proportional to x^2 . Thus the fractional line width is independent of the magnetic field chosen and is $\Delta\nu/\nu \approx 6 \times 10^{-3}$. Similarly the strength of H_p required is independent of H . H_p must be about 5 gauss to induce transitions in ten per cent of the $m = \pm 1$ component before annihilation. Radiofrequency fields of this magnitude can be obtained conveniently in certain frequency ranges. We selected a value $\nu = 3000$ Mc/sec because a resonant cavity at this frequency has convenient dimensions, electronic equipment is readily available and the magnetic field $H = 9000$ gauss at resonance can be easily produced. The apparatus used will be described. The frequency was kept constant and the magnetic field was varied near resonance. The change in the gamma spectrum observed in a scintillation spectrometer was used to indicate resonance. A series of measurements by this method gave results slightly at variance with above formula for $h\nu$. The observed ν exceeded the calculated value by 0.7 ± 0.15 percent. This discrepancy has now been removed

by the theoretical calculation of KARPLUS & KLEIN (in press) which takes into account a number of corrections of order $\alpha^5 m_0 c^2$, among them the anomalous magnetic moment of the electron, virtual two-quantum annihilation in the singlet state, vacuum polarization and some others. If the experimental results, with the proper corrections, are interpreted as a measurement of ΔW , the present experimental value is $2.035 \pm 0.003 \times 10^5$ Mc/sec, in excellent agreement with the corrected theoretical value of 2.0337×10^5 Mc/sec.

Experiments are in progress to improve the accuracy of the measurements, to study the effect of the nature and pressure of the gas on the resonance and to determine the width of the line.

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DISCUSSION

BECK: Since the experiments fit so well the theory, I would like to ask if there is no possibility to look from another side, e.g. at the annihilation life time which may be very hard to be measured. But then we may have more chances to find deviations from the theory.

DEUTSCH: The present measurement of the life-time of the triplet state, is only good to, something like 10%. The ratio of the life-times can in principle be measured by these magnetic quenching experiments. The difficulties are of course collision effects. I think the greatest hope would be to make a beam of slow positronium and then study the width of the line. We are studying the width of the line. I am afraid we will find that it is greater than the theoretical width because there are small collision effects. I do not want to discuss it. If you are interested, we can perhaps discuss it afterwards. There are collision effects which are not quite obvious, in principle. I think this is certainly possible and we will continue to try.

Efeitos do * 'Scattering' em Absorventes, em Curvas de Absorção de Espectros p Contínuos

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RESUMO

Com um arranjo para contagem de raios β construído especialmente e com um contador de janela de mica foram feitas diversas medidas das propriedades dos raios β emitidos por uma fonte de ^{90}Sr - ^{90}Y . O fenômeno bem conhecido da "focalização" dos raios β no contador por meio de absorventes foi observado e por meio de um "definidor de focalização" pudemos confirmar a interpretação usual deste fenômeno. Experiências foram feitas variando-se a distância dos absorventes à fonte; pudemos verificar assim qualitativamente que a lei de "scattering" por absorventes torna-se isotrópica para grandes espessuras do absorvente. Verificamos também que o aumento da radiação focalizada com a espessura do absorvente tem a forma de uma curva de "growth" de uma substância radioativa; associando isto ao fato de que a absorção de um espectro β contínuo é aproximadamente exponencial, sugerida uma interpretação do fenômeno do "self-scattering".

INTRODUÇÃO

O método mais comumente usado para a determinação da energia máxima dos raios β emitidos por camadas finas de substâncias radioativas é o de se medir a curva de absorção dessa radiação em folhas de alumínio. A extrapolação visual dessa curva de absorção dá a máxima espessura de alumínio que os raios β podem atravessar; esta espessura ou "range" extrapolado é ligada por relação simples à energia máxima da radiação emitida (relações "range"-energia) (KATZ & PENFOLD, 1952).

Este método de extrapolação tem, porém, certos defeitos; o mais flagrante consiste no fato de que as curvas de absorção medidas são modificadas notavelmente por efeitos de "scattering" dos raios β nas diferentes partes do arranjo experimental empregado e nos próprios absorventes. Uma extrapolação feita nestas condições não pode fornecer dados muito precisos, considerando-se principalmente que os pontos próximos da máxima espessura correspondem a baixas intensidades e portanto a grandes erros estatísticos. Em particular, se a substância emitir raios γ a interpretação dos resultados torna-se ainda mais difícil.

Outros métodos foram sugeridos para a análise das curvas de absorção sendo mais comumente usado o de FEATHER (1938). Este método tem seu ponto

de partida na [hipotese](#) de que as curvas de absorcao de diferentes emissores β tenham a mesma forma a menos de uma transformacao da escala de abcissas (espessura dos absorventes). Usando-se urn emissor β (*RaE*, sugrido por FEATHER) corao padrao, cujo "range" extrapolado e^* conhecido por outros me'todos, pode-se analisar as curvas de absorcao; a analise consiste em dividir-se o "range" do *RaE* cm 10 partes iguais e calcular-se que reducao de intensidade produzem 1/10, 2/10, . . . , n/10 do range maximo; em seguida, observa-se na curva a estudar que espessuras de absorventes produzem reducoes de intensidade equivalentes; multiplicando-so estas espessuras por 10/n, obtem-se o que se chama "range" aparente. He a [hipote.se](#) feita acima sobre a forma das curvas de absorcao fosse verificada, esses "ranges" aparentes seriam todos iguais e dariam, usando-se uma formula de "range"-energia, a energia maxima do espectro β medido. Na realidade porem, a forma das curvas de absorcao de quase todos os emissores e^* muito diferente da curva de abson;ao de *RaE* (e^* urn fato sabido que o espectro de energia do *RaE* e anormalmente deslocado para a regio de baixas energias (MARINELLI, BRINKERHOFF & HINE, 1947)); os "ranges" aparentes dao, pois, uma sucessao de valores diferentes; com eles pode-se construir urn grafico em que se coloca nas abcissas frames de "range" e nas ordenadas os "ranges" aparentes; uma extrapola(ao da o "range" extrapolado. Dovido a escolha do *RaE* como padrao, essa extrapolacao e as vezes dificil ou mesmo duvidosa.

Foi sugerida por BLEULER & ZUNTI (1946) e GLENDENIN (1948) que se usasse a curva de absorcao dos raios β do UX_2 como curva de absorcao padrao, uma vez que o espectro desse emissor e mais "normal" que o do *RaE*; este unico fato deve permitir uma extrapolacao mais pccisa no metodo acima.

A importancia que o UX_2 tem como padrao para o metodo de Feather justifiea urn estudo acurado das peculiaridades que sua curva de absorcao pode ter. Parece-nos sobretudo interessante efetuar-se urn estudo da influencia dos absorventes nas curvas de absorcao do UX_2 devido as complicacoes introduzidas nelas pelo UX_1 , com o qual o UX_2 csta em equilibrio transitorio, nas fontes usadas. Mostraremos que o procedimento de se colocar um absorvente sobre a fontc a fim de se cortar a radiacao do UX_1 nao e aconselhavel. Aproveitaremos as curvas de absorcao medidas para analisar os efeitos que o "scattring" multiplo dos raios β introduz nos absorventes; com isso pretendemos aprofundar um pouco os estudos feitos para o *RaE* por ELLIOT & SHAPIRO (1951) e NOVEY & ELLIOT (1951). Isto nos permitira uma interpretacao mais detalhada do fenomeno do "self-scattering".

ARRANJO EXPERIMENTAL E TECNICA DE CONTAGEM

Utilizamos em nossas medidas um contador Geiger-Muller Tracerlab tipo *TGCI* com uma janela de mica de $2,5 \text{ mg/cm}^2$ de espessura; o patamar do contador era de 300 volts com um coeficiente angular de 0,3%. O contador foi montado no interior de uma caixa de *Pb* de paredes de uma polegada de espessura, cujo interior era revestido com 5 mm de lucite, cujas vantagens para este fim sao bvias devido ao seu baixo numero atomico medio (YAFFE, 1950). Abaixo do contador existiam prateleiras que permitiam colocar a fonte e os absorventes a

distancias diferentes do contador (fig. 1). O "background" era de 25 impulsos por minuto. A fim de evitar efeitos perturbadores, que o fundo da camara de contagem poderia introduzir, a prateleira de suporte mais baixa distava 4,5 cm do fundo.

O "scaler" utilizava nas medidas era decimal (Berkeley, modelo 1.000). As contagens foram feitas durante tempos fixos. Antes de iniciarmos as medidas, aplicamos o "test" do χ (chi) de Pearson (JARRET) a fim de verificar se o comportamento do "scaler" era normal; o resultado obtido foi satisfatorio.

A FONTE

A fonte utilizada foi feita com a fase aquosa de nitrato de uranila $UO_2(NO_3)_2 \cdot 6H_2O$ dissolvido em eter etilico; est a operacao separa o nitrato de uranila, que permanece na fase eterea. Uma pequena gota da fase aquosa foi depositada a em um filme de acetato de celuloze preparado pela tecnica usual e evaporada por uma lampada de infravermelho; o deposito restante, que ocupava uma area de $0,03 \text{ cm}^2$ era coberto por outro filme de acetato de celuloze de 10 microgramas por cm^2 a fim de proteger a fonte. O "sandwich" de filmes de acetato de celuloze era montado num anel de suporte de aluminio (').

Na fonte preparada o uranio nao estd presente de modo que o UX_2 (meia vida - 24,5 dias) se encontra em ecpilíbrio transitorio com o UX_1 (meia vida - 1,14 minutos); a fonte decai com a meia vida do UX_2 . A energia mdxima da radiao UX_2 6 2,32 Mev e a do UX_1 , 0,13 Mev.

Como prova da pureza da fonte, medimos o decrescimo da atividade da fonte durante 10 dias; a exponencial obtida dava a meia vida correta do UX_2 ; portanto a fonte era pura. O fato de a curva de decrescimo da atividade ser rigorosamente exponencial, ou seja, uma reta num grafico semi-logarltmico, constitui uma indicacao satisfatoria da estabilidade do equipamento de contagem.

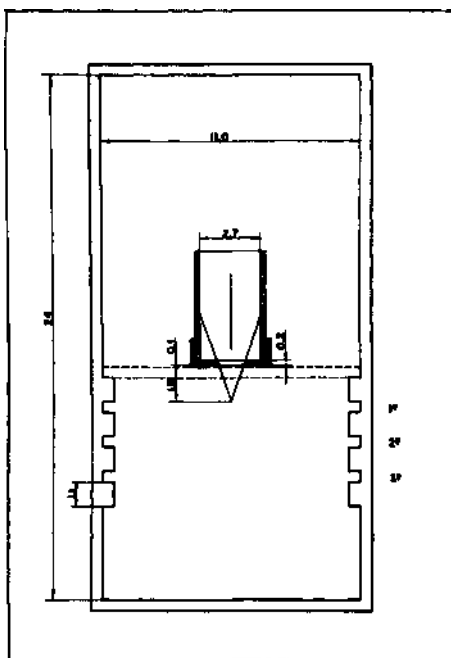


Fig. 1 — O arranjo experimental; distâncias em centímetros.

(') Agradecemos ao dr. A. Cervellini pela preparação da fonte.

O FENOMENO DA "FOCALIZACAO" DE RAIOS β POR ABSORVENTES

Uma das alternativas possiveis ao se medir uma curva de absorcao de um espectro β e colocar-se todos os absorventes sobre a fonte. O resultado que se obtem no caso de uma fonte de $UX_1 - UX_2$ e o da fig. 2 - curva B; para comparacao representamos na mesma figura uma curva obtida colocando todos os absorventes junto a janela do contador — curva A. Fonte a 3,5 cm da janela do contador.

O fenomeno da "focalizacao" de raios β explica este resultado como o observaram JOHSON & WILLARD (1949): parte da radiacao que nao atingiria o contador se os absorventes estivessem todos junto a janela do contador, sofre "scattering" multiplo no absorvente colocado sobre a fonte e e langado no contador (fig. 3). BUKIT (1949) esclareceu a razao pela qual sao "focalizadas" mais parti-

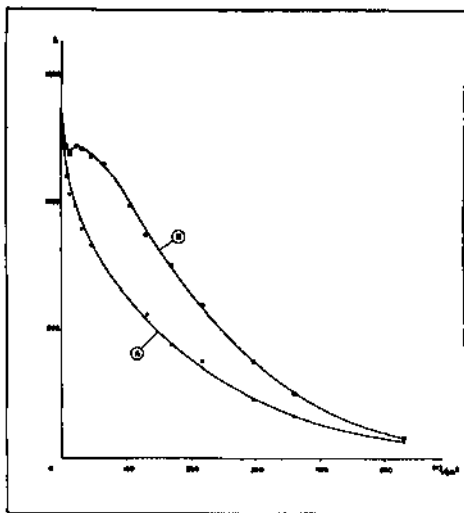


Fig. 2 — Curvas de absorção da radiação β do $UX_1 - UX_2$ em coordenadas comuns. Curva A - absorventes junto à janela do contador. Curva B - absorventes sobre a fonte.

culas no contador do que retiradas do feixe que incidiria no contador se o absorvnte nao existisse ("desfoealizadas"); o angulo solido segundo o qual a fonte ve o contador e pequeno; a medida que aumenta a espessura dos absorventes que cobrem a fonte, a "desfocalizacao" dos raios β aumenta, porem, a fracao da radiacao emitida pela fonte e que nao incidiria no contador se o absorvente nao existisse, e" muito maior do que a fracao que incidiria nele; portanto, mesmo que o "scattering" retire particulas do feixe, outras que tambem sofreram "scattering", sao focalizadas para o contador e compensam largamente esta perda; a intensidade medida aumenta a medida que a espessura do absorvente aumenta; ate" que esse aumento seja compensado pela radiacao, que e absorvida, ou que devido ao "scattering", deixa de atingir o contador.

Evidencia experimental da ideia de Burt foi obtida por nos, medindo uma curva de absorcao completa com a fonte na 2.^a prateleira do nosso arranjo e absorventes na 1.^a prateleira (fig. 4 — curva C); em seguida repetimos a experiencia cobrindo os absorventes com um "definidor de focalizacao" (fig. 5), cuja finalidade era de evitar que radiacao inicialmente fora do cone segundo o qual a fonte ve" o contador o atingisse. Dessa forma a radiacao "desfocalizada" nao pode ser compensada e a curva de absorcao deve revelar este fenomeno. Com efeito, o resultado obtido esta na figura 4 — curva D, assim como uma curva em que todos os absorventes estao sobre a fonte (curva B) e outra em que os absorventes estao junto a

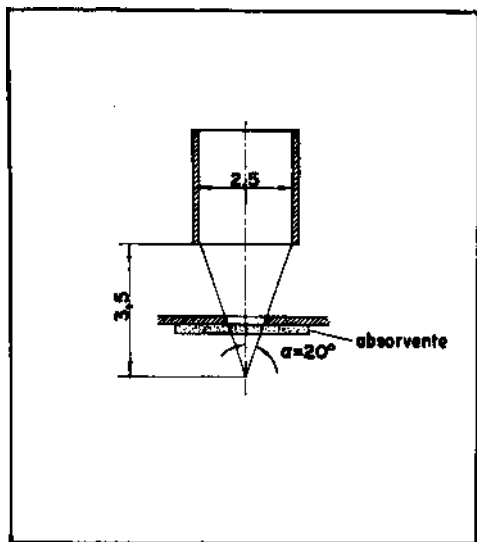


Fig. 5 - Arranjo em que foi medida a curva C da fig. 4. Cobrindo o absorvente eslii o "definidor do foalizacao".

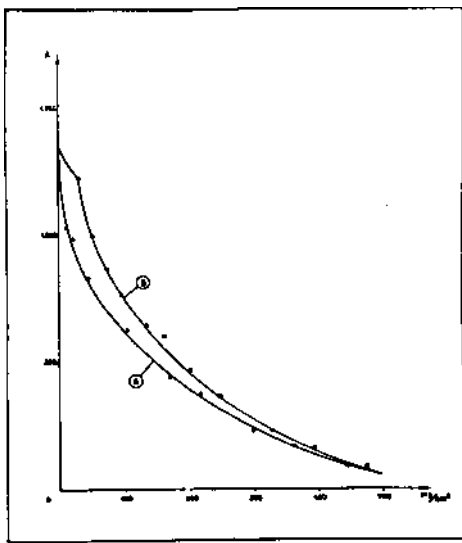


Fig. 6 - Curvas de absorcao da radiagao β do $UX_1 - UX_2$, em coordenadas comuns. Curva A - Absorvontes junto a janela do contador. Curva B - 29,9 mg/cm^2 s6bre a fonte e os absorventes restantes junto a janela do contador.

•colocados junto a janela do contador. A curva de absorcao obtida e* a da fig. 6 — curva B, em coordenadas comuns. A curva A foi obtida com todos os absorventes junto a janela do contador.

VARIACAO DA INTENSIDADE DA RADIACAO "FOCALIZADA" COM A ESPESSURA DOS ABSORVENTES

ELLIOT & SHAPIRO (1951) estudaram tcdricamente o que se poderia esperar quando se variava a posicao dos absorventes entre a fonte e o contador; em seu trabalho o calculo e* feito admitindo-se uma lei simples para a distribuicao angular da radiacao emitida por fontes e absorventes; admite-se ainda, que a fonte e* puntual, que o absorvente e* piano e infinito e que o contador e* pontual. ELLIOT & SHAPIRO obtiveram dois resultados muito interessantes no caso de fontes que emitem isotrdpicamente (e o caso de fontes finas, como a nossa)

a) o "scattering" pelo absorvente e istropico. Se I_0 e a intensidade emitida pela fonte, a intensidade que o contador registra e* dada por

$$I = \frac{I_0}{8 l^2}$$

l — distancia da fonte ao contador; como se ve', neste caso a intensidade medida nao depende da posicao do absorvente.

b) o "scattering" pelo absorvente obedece a uma lei de coseno; o resultado obtido esta na figura 7, que da a lei de variacao da intensidade com a distancia do absorvente a fonte.

Experimentalmente ELLIOT & SHAPIRO SO estudaram a variacao da intensidade medida com a distancia do absorvente a fonte, utilizando uma dada espessura de absorvente (151 mg/cm^2 de aluminio). lrealizamos medidas com absorventes de diferentes espessuras colocando-os em 3 posicfies diferentes entre o contador e a fonte, a 3,5, 1,9 e 0 cm da fonte.

Supondo-se que quando o absorvente esta junto a janela, isto e, que a 3,5 cm da fonte nao haja

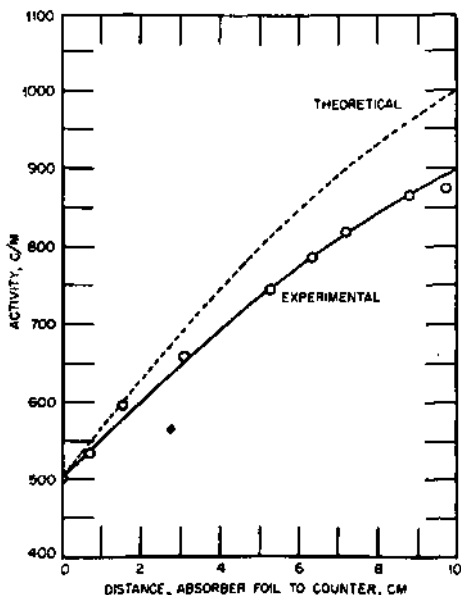


Fig. 7 — Resultados obtidos por ELLIOT & SHAPIRO (1951) admitindo que a fonte é isotrópica e que o "scattering" pelo absorvente obedece a uma lei de coseno. Os pontos correspondem a medidas efetuadas com o RaE.

contribuicao para a intensidade medida devida ao "scattering", calculamos o aumento relativo de intensidade que surge quando o absorvente e^* colocado nas outras posicoes. Os resultados obtidos sao os seguintes:

ESPESURA DO ABSORVENTE (mg/cm ²)	AUMENTO RELATIVO DE INTENSIDADE	
	$d = 1,9$ cm	$d = 0$ cm
7,03	1,06	1,094
12,7	1,061	1,148
21,9	1,053	1,235
29,9	1,104	1,35
45,2	1,146	1,38
106	1,18	1,58
132	1,25	1,57
171	1,28	1,6
218	1,34	1,58
298	1,44	1,64
381	1,4	1,54
445	1,46	1,5

d — distancia do absorvente a fonte.

Os graficos de variacao de intensidade correspondentes a algumas linhas da tabela acima estao na figura 8. O andamento geral destas curvas confirma as id6ias que se tem acerca do "scattering" mtiplo que sofrem os raios β nos absor-

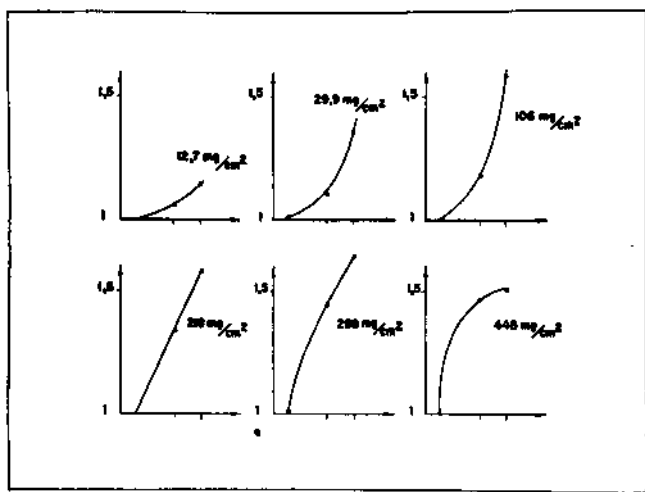


Fig. 8 — Variacao do aumento relativo de intensidade com a posicao do absorvente, para diversos absorventes. Nas abcissas distancias do absorvente a janela do contador.

ventes; para grandes espessuras, a lei de "scattering" se torna isotropa, isto e, conforme o caso a) tratado por ELLIOT & SHAPIRO, a intensidade medida deixa de depender da posicao do absorvente.

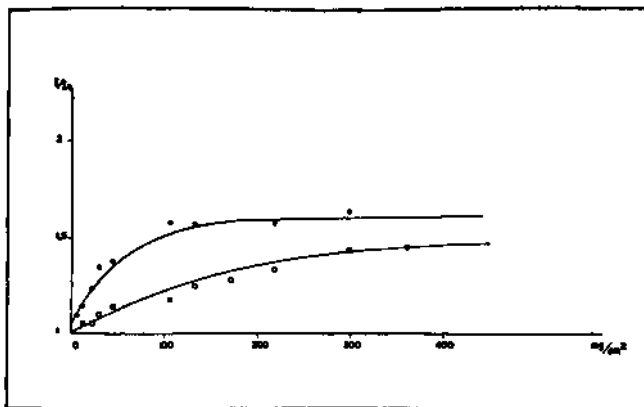


Fig. 9 — Aumento relativo da intensidade medida com os absorventes em posições determinadas em função da espessura do absorvente. A curva superior corresponde a absorventes sobre a fonte.

Pode-se observar na tabela acima, que os valores colocados em uma vertical dão o aumento relativo da intensidade para diversas espessuras de absorventes, isto é, a lei de variação da intensidade de radiação "focalizada" com a espessura dos absorventes. Gráficos dessa lei estão na figura 9.

Para espessuras de absorventes para os quais a lei de "scattering" é isotrópica, as 2 curvas obtidas deviam coincidir (no caso a), tratado por ELLIOT & SHAPIRO); atribuímos as discrepâncias obtidas ao fato de que as aproximações feitas no cálculo afastam-se muito da realidade.

Pode-se verificar que a espessura de absorvente para a qual a lei de "scattering" é isotrópica quando os elétrons incidentes são monocromáticos e de energia máxima de 2 Mev, é de ordem de grandeza do valor obtido nos gráficos da fig. 9 (FOWLER, LAURITSEN & LAURITSEN, 1918).

INTERPRETAÇÃO DO FENÔMENO DO "SELF-SCATTERING"

As curvas de acréscimo relativo da radiação "focalizada" com a espessura do absorvente têm a forma de curvas de "growth" de uma substância de família radioativa; no nosso caso a energia máxima da radiação medida e o arranjo experimental determinam a forma dessa curva. Como a absorção da radiação β é aproximadamente exponencial, parece-nos plausível supor-se que o fenômeno do "self-scattering" seja devido a uma combinação de dois processos que têm lugar em uma fonte espessa:

- absorção da radiação das camadas inferiores da fonte pelas camadas superiores, com um coeficiente de absorção u ;
- "growth" de radiação "focalizada" no contador pelas camadas superiores da fonte, com coeficiente de absorção u' ;

É fácil ver-se que I_0 é a atividade específica verdadeira da fonte e x a sua espessura, I , a atividade medida é dada por

$$I = I_0 \frac{u'}{u - u'} (e^{-u'x} - e^{-ux}) \quad (1)$$

Experiências preliminares foram realizadas com a radiação do Cu^{62} , preparado neste laboratório pela reação $Cu^{63}(\gamma, n)Cu^{62}$. No Cu^{62} o fenômeno do "self-scattering" é* muito acentuado, devido ao fato de o seu número atômico ser baixo e a energia máxima da radiação emitida ser alta. Ao que parece, uma fórmula como a (1) pode descrever bem os resultados obtidos. O elemento utilizado não, é* muito apropriado para estas experiências, por ser um emissor de pósitrons. Serão realizadas experiências com um emissor de elétrons.

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Gamma Rays and Low Energy Alpha Particles from Po^2

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Though the disintegration of Po^{210} is one of the first to have been studied in the history of classical radioactivity, some of its aspects remained an open problem which has received the attention of several investigators in recent years. Most of the α particles of Po have an energy of 5.3 Mcv, but a weak (branching ratio $\sim 10^{-5}$) γ radiation of 800 Kev was observed. Attempts to detect the corresponding short range α group not only had failed, but in some cases had led to results of difficult interpretation.

This problem was attacked with a coincidence method. The pulse height distribution of the α -particles in a fast liquid scintillator was measured directly, and in coincidence with the γ -rays from the same source. The ratio between the single* and the coincidence counting rates was about 10, and it was necessary to use a coincidence circuit of resolving time of around 10^{-8} sec. in order to minimize the random counts.

It was found that the α -particles in coincidence with the γ -rays had a lower energy than those of the main group, the difference corresponding to the γ energy.

After having established in this manner the genetic relationship between α and γ radiations, we proceeded to the study of their angular correlation. Despite the low counting rate the experimental results were quite reproducible. The number of coincidence was 8 ± 1.5 c/hour at angles of $(2n-1)\pi/4$ (data taken at 45° , 135° and 225°), and 1 ± 1 c/hour at angles of $2n\pi/4$ (data taken at 90° and 180°).

If one assumes that the spins of Po^{210} and Pb^{206} are zero, and that the parity of these two nuclei is the same, this result lead unambiguously to the conclusion that the α particles of Po are emitted in a d state, and that the γ -rays following them are electric quadrupole.

El Modelo de Capas en Relación a la Teoría Mesónica de las Fuerzas Nucleares

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Admitiendo la intervención en las fuerzas nucleares de un campo mesónico escalar neutro de mesones pesados, es posible explicar las desviaciones de los momentos magnéticos nucleares de las líneas de SCHMIDT y otros aspectos del modelo nuclear de capas propuesto por JENSEN *et al.* (1918) y por MAYER (1950).

Un nucleón con momento magnético anómalo es polarizable en un campo mesónico exterior. Debido a esto, el nucleón suelto del modelo de capas (odd nucleon) en el campo mesónico del resto del núcleo presenta una modificación de su momento magnético intrínseco. Desviaciones de las líneas de SCHMIDT en el sentido del observado pueden ser producidas solamente por un campo mesónico escalar neutro con acoplamiento escalar. Un campo simétrico producirá desviaciones de uno y otro lado de estas líneas. Los campos pseudoescalar y pseudo-vector, originados por un sistema de nucleones apareados no conducen a la polarización del nucleón suelto. Un campo vectorial neutro produciría apartamientos de las líneas de SCHMIDT en el sentido opuesto al observado.

Identificando los mesones escalares neutros con las partículas V^0 de masa $m \sim 800m_e$ (ARMENTEROS *et al.*, 1951), se obtienen desviaciones de los momentos magnéticos del orden de los observados atribuyendo a la constante de acoplamiento del campo escalar el valor $g \sim 2.8$.

Si además, se admite acoplamiento vectorial para el campo escalar es posible explicar los momentos eléctricos cuadrupolares que presentan los núcleos de neutrón suelto.

El hecho que el campo escalar supuesto es de muy corto alcance, produce un potencial medio que tiene exactamente la forma del potencial requerido por el modelo de capas. Además, ARAKI (1951) ha mostrado que cuando se tiene en cuenta el retroceso del núcleo en el proceso de emisión virtual de mesones escalares aparece en la energía de interacción un término que tiene la forma de acoplamiento spin-orbital. Su aplicación al modelo de capas, con los valores de masa y constante de acoplamiento conduce a un fuerte acoplamiento spin-orbital del orden del requerido en el esquema de MAYER (1950).

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Magic Numbers and Pseudoscalar Mesons

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As has been pointed out by MARIA GOEPPERT — MAYER and by HAXEL, JENSEN and SUESS, the empirical magic numbers of nuclei can be understood by the shell model, if one assumes a strong coupling between spin and orbital angular momentum of a single nucleon moving in the mean potential produced by the other nucleons. There exist also attempts to obtain the correct magic numbers from special assumptions about the form of the potential, but the introduction of spin-orbit coupling seems to be physically more reasonable. The question remains, how to explain the strong spin-orbit coupling, it could be easily understood (GAUSS, 1952; HEISENBERG, 1950) if the mesons producing the interaction between nucleons would have non-zero spin. But from recent experiments on IT-mesons it can be shown, using only general conservation principles, that these particles have very probably spin zero and odd proper parity (if the relative parity of proton and neutron is even). Since it is also probable, that the T - mesons constitute the main part of nuclear forces, the question arises how to explain a strong spin-orbit coupling by a pseudoscalar field.

In the case of pseudoscalar coupling, the interaction HAMILTONIAN contains, in the nonrelativistic limit, the term $\frac{g}{2M} \vec{\sigma} \cdot \vec{\nabla} \Phi$ (M = mass of the nucleon) which commutes with total angular momentum, but not with orbital angular momentum alone. It leads, therefore, to a splitting of usually degenerated energy levels of equal l but different j ($j = l \pm 1/2$). This means that the term proportional to $\vec{\sigma} \cdot \vec{\nabla} \Phi$ has, in general, an analogous effect as the usual spin-orbit coupling which is proportional to $\vec{\sigma} \cdot \vec{\nabla} \Phi_0 \times \vec{p}$. Calculating the splitting $\Delta \epsilon$ caused by $\frac{g}{2M} \vec{\sigma} \cdot \vec{\nabla} \Phi$ the perturbation of first order vanishes, because the operator changes the parity of a state. In second order, we get a finite $\Delta \epsilon$ which has also the correct sign. Terms of higher order are factorised by the corresponding powers of $(m/2M)g$. Even if g is of the order one, as it seems to be probable, they lead in this non-relativistic limit to smaller contributions. One can expect, that the splitting is large enough to explain magic numbers, because in the case of pseudoscalar coupling also the potential terms are of the order $m/2M$. Calculations on the magnitude of the splitting are in progress.

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DISCUSSION

MOSHINSKY: I would like only to ask if there is any kind of field interaction that would lead to the 'Thomas' term that you wrote down before

$$\vec{\sigma} \cdot (\text{grad } V \times \vec{p})$$

OEHME: With a vector-meson-field it is possible to get a term of such a form because you can couple the field directly with the spin-density-tensor. This was shown for example by Gauss in Göttingen.

WIGNER: I am wondering whether you could tell us how the origin of your coordinate system is defined. This is important for the $\vec{\sigma} \cdot \vec{L}$ term in which $\vec{L}$ must be defined as the angular momentum about a definite point,

OEHME: $\vec{L}$ is the angular momentum of the extra shell nucleon about the center of mass of the shell. This center of mass enters because; the virtual meson field of the shell is centered at this point. However, the structure of the virtual meson field has not yet been worked out.

Elimination of Divergencies in Field Theory

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An analysis of the divergencies appearing in Field Theory shows that all divergent integrals which have to be eliminated by renormalization arise from the use of singular solutions of field equations, and even from improper solutions such as the causal functions appearing in the S matrix and the related field commutators. These singularities, some of them already encountered in classical theory, originate, in principle, in our notion of an elementary point particle. It has been suggested several times that a consideration of a "finite size" of the particle should eliminate all singularities from the theory. However, besides the difficulty of introducing a particle of finite size in a relativists invariant manner, as soon as one considers a finite extension for a particle, several questions arise as to form, distribution of mass, charge, stability of the particle, etc., which questions should obviously be meaningless if the particle is to be considered as elementary. On the other hand, the introduction of non-local fields which seems to avoid the above mentioned difficulties has had no apparent success up to the present.

These considerations have suggested that an analysis of the problem under a different point of view might be justified. It seems that all the trouble comes from the fact that we are using in the background of our calculations, ideal entities endowed of finite, well determined physical properties such as mass, charge and spin, and yet nothing that would correspond to physical extension. The introduction of a property of the particle somehow connected to coordinate space can obviously not be done if we assume, as it has been done, that the kind of measure encountered in Physics is the Lebesgue measure. However, as it is well known, there are other types of measure, some of them allowing to assign a finite measure to a single point. One may assume that in the same manner that a physical point may possess mass, charge and spin, it may also be endowed of a certain finite measure although no physical extension in the ordinary sense. As in General Relativity, the distribution of matter determines the metric of space, one may assume that the measure is also determined by the presence and distribution of particles. Thus, in the space usually encountered in Field Theory, neglecting gravitational effects, the metric can be assumed to be flat, but as soon as particles appear, the measure is not of the Lebesgue type. Each point occupied by a particle will possess a certain finite measure and ultimately, each particle will also be characterized by a constant "a" having the dimensions of length. The introduction of these ideas into the mathematical formalism of the theory is performed by substituting ordinary

Lebesgue integrals by Stieltjes integrals and ordinary differentiation by a more general differentiation with respect to a measure. In this manner, singular solutions of the field equations can be avoided. For example, the electrostatic potential of a point charge turns out to be

$$\phi = \frac{e}{r+a}$$

and the mesostatic potential of a point charge becomes

$$\phi = g \frac{e^{-kr}}{r+a}$$

In the quantized theory these ideas can be introduced by assuming that each commutator (or anti-commutator, according to the case) is associated to a certain measure. Functions such as $\delta(x)$, $D(x)$, $\Delta(x)$, etc., become proper, with finite discontinuities, but have a meaning only when used in the proper, Stieltjes integrals. A simple calculation in the case of a scalar real meson field ϕ coupled to a scalar complex meson field ψ with a coupling of the form $g \psi^* \phi \psi$ shows that the self-energies of both particles are finite and so are the vacuum polarization effects. In fact, it can be shown that the contributions of primitive divergent graphs which in ordinary theory would appear, become finite.

DISCUSSION

WATAGHIN: Generally from such a theory one should expect that for low energies and momenta it should give the usual results. I do not know if one can make in your theory a passage to the limit of the parameter "a" approaching to zero. But the problem for instance in the non-local theories is to distinguish high energy interactions and low energy interactions and have the correspondence with the old usual theories for slow moments. Could you tell us something about this question?

MEDINA: As a matter of fact, this turned out to be very simple. In some cases, that is before you calculate the Stieltjes integral, if you take the limit as "a" goes to zero you always fall into the usual result. I gave no logical proof of this, but at least in all the cases I have studied I fall to the same results that ordinary theory would give. This can be seen very simply. For instance, Feynman made the observation that if instead of using Dime's $\delta(x^2)$ one uses a function $f(x^2)$ whose value is very large on the light cone, very small everywhere else, but remains always finite and has a finite width, one obtains convergent results. One can show that in the limit as the width of $f(x^2)$ goes to zero and the height to infinity, one obtains the usual results. What I am doing here is, roughly speaking, in a way similar to this, except that in Feynman's case the condition was arbitrary and besides if one uses his $f(x^2)$ the field amplitudes do not fulfill Maxwell's equation or Klein-Gordon's equation, Dime's equation or whatever equations one started with. But, if one assumes that the metric has a jump "a" and at the same time one uses a delta function

$$\delta(x) = \begin{cases} \frac{1}{a} & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases},$$

actually this behaves as far as the final result is concerned, as if one were using a function of rectangular form of height $1/a$ and width a so that if "a" is small, suppose of the order of 10^{-13} cms., for macroscopic purposes it would behave as a Dime's δ . Further if "a" goes to zero, then the

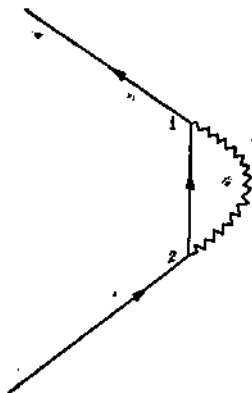
height goes up indefinitely and the width approaches zero so that the limit of this expression is Dirac's δ , and the same relations occur as in usual theory. However, after calculating the Stieltjes integrals, there is no way of taking the limit.

WATAGHIN: AS far as I can see, your method could be made equivalent to a non-local theory.

MEDINA: I do not think it is.

WATAGHIN: Feynman in one of his attempts is cutting off in the graphs the propagation laws, whereas you are cutting off the interactions on the vertices. Is it so or am I wrong in this point?

MEDINA: NO, I think I understand your question, but the alteration that brings about the theory is not in the vertices but in the propagation factors. For instance, in the case of a self-energy graph:



a virtual particle is emitted at 2 and absorbed at 1. We have some interaction factor at 1 and 2, which depends on the kind of fields and couplings, and propagation factors for the two virtual particles appearing in the process. I do not alter the interactions since the differential equations and types of fields are the same in my theory as in ordinary. However, the propagation factors $S(x_2 \cdots x_1)$ and $\Delta(x_1 \cdots x_2)$ are altered. They become proper functions which have to be used in connection with adequate Stieltjes integrals. As the passage to momentum space is made, convergence factors appear, my $S(k)$ and $D(k)$ being different from the ordinary ones. Alterations are thus introduced though not in the elementary interactions but rather on the propagation of virtual quanta.

WATAGHIN: Christian Moeller, Kristensen, Claude Bloch and myself in forthcoming papers discuss many aspects of non-local theories. Perhaps the present discussion could be continued after my communication where I will recall their attempts.

MEDINA: I would be glad to. But from the start I could say this. This theory is not equivalent to non-local theory. That is easy to see. In non-local theories your operators, from the start, do not commute with the coordinates. The change here is in the measure of the space, not in the field.

LEITE LOPES: Your theory is not equivalent to a regularization of the singular delta functions, or is it?

MEDINA: I do not think so. I do not have any trouble of this sort. I have not compared my integrals with the integrals appearing in Pauli's regularization procedure but I do not think they are the same. Any comparison between both things, probably is hard to be done. One uses Riemann's integral and the other uses the Stieltjes integral. It is hard to compare two things that are so different. It is as if you wanted to compare the distance in a flat space with the distance in the Riemannian space.

LEITE LOPES: Is the new Δ function still vanishing outside of the light cone?

MEDINA: Yes, of course. It satisfies the same conditions, namely

$$i) (\square - k^2) \Delta(x) = 0$$

$$ii) \Delta(x) = 0 \text{ if } x^2 > 0$$

$$iii) \int_{\sigma} \frac{\partial \Delta(x)}{\partial x_{\mu}} d\sigma_{\mu} = 1$$

It is defined formally in the same way, except that the measure is different.

BECK: May I ask how the self-energies come out in this method?

MEDINA: For a quantized field I have carried a detailed calculation only for the case of two meson fields $\phi(x)$ and $\psi(r)$ with a coupling of the form $g \psi^* \phi \psi$ and they turned out to be finite, as well as the polarization of vacuum. For a Dirac field ψ coupled to an electromagnetic field A , I have only carried the calculation for classical, not quantized, theory and the electron self-energy turned out to be finite. I am sure, however, that this finiteness will also be found for the quantized fields.

BECK: In what sense? Can it be for the electrons something of the order of mc^2 ?

MEDINA: As I said, no quantum mechanical calculations have been made in the electromagnetic case. But for the non-quantized theory, the " a " in the final results appears in expressions which resemble very closely those that one could obtain considering an electron of radius " a ", so that " a " turns out to be of order of 10^{-13} cm. and the self-energy of the order of mc^2 .

WIGNER: There is one question I would like to ask, and it is a very trivial question, but I think it would clarify matters for some of us. In what sense is $1/(r+a)$ a solution of the Laplace equation?

MEDINA: There are two answers to your question; as far as the classical picture is concerned, the program is to substitute ordinary differentiation by differentiation with respect to a measure. Here one does not take the limit as the coordinate difference goes to zero, but rather as the measure difference goes to zero. This means, as a simple calculation shows, that in the case of

$$\Delta \phi + 4 \pi \rho = 0$$

the equation would be of the form

$$\frac{1}{R^2} \frac{d}{dR} \left(R^2 \frac{d\phi}{dR} \right) + 4 \pi \frac{e}{a} \delta(r) = 0$$

R is the measured distance to the particle. With a particle at the origin of coordinates, the particle having an intrinsic measure " a ", the measured distance from any point P , to the particle is $R = r + a$ and the solution of the foregoing equation is

$$\phi = \frac{e}{r+a}$$

From the point of view of quantized theory, this potential can be obtained as the result of the exchange of two virtual quanta, by using the modified A and S function or the corresponding Fourier transforms in the momentum representation.

WIGNER: Does not that mean that the field equations depend on a different measure depending on where the origin of the field is? Ordinarily the field equations use a differentiation which is independent of the origin of the field. Apparently, you use a differentiation which depends on the point from which the particle originates.

MEDINA: Yes, the space becomes non-isotropic, as far as measure is concerned.

WIGNER: Not only non-isotropic but at the same point you have a different measure depending on the origin of the field. Alternately, you may want to give up the principle of superposition.

IVfemNA: Yes and no. I mean the following: if you take this intuitive picture in classical theory, you will be right. As a matter of fact, the whole theory should have been abandoned if it were not possible to translate it into quantum terms. Fortunately, this difficulty does not arise when one considers the field quantized, that is, if one abandons the idea of a point particle, because the trouble arises from the fact that we are talking now of a point particle located at the origin. In field theory one has a function defined in the whole four-space. What the theory tells, is what to do with the integrals that appear in the expansion of the S matrix. Perhaps it would seem arbitrary to some extent to say that the potentials depend not on the coordinates but on the measured distances. However, there is probably an argument in favor of this view, and this is the following: if one considers for instance, the set of points $(x) = (x_0, x_1, x_2, x_3)$ of Minkowsky space, then one can introduce a set of kets $|x'\rangle$. Assuming that these variables form a perfect set, that is, a complete set of operators which commute with each other, the kets $|x'\rangle$ will form a basic set. Now, what quantum theory tells, is that if one makes an observation of any coordinate of the system, no matter what the state might be, one measures a value x_μ' such that

$$x_\mu |x'\rangle = x_\mu' |x'\rangle$$

Now, what is this x_μ' ? According to the postulates of quantum theory, x_μ' is the measure of the coordinate, or the observed value of the coordinate. There is a question arising here as to whether x_μ' is the coordinate of the point in the mathematical sense or a physically measured value of the distance between the point and some plane of reference. One will arrive to a different theory depending on the interpretation and the assumption made on x_μ' . Because a field ϕ can be considered as an operator satisfying $[x_\mu, \phi] = 0$ and therefore as a function $\phi(x_\mu)$ of the operator x_μ . If this operator acts on some of the basic kets, one obtains

$$\phi(x) |x'\rangle = \phi(x') |x'\rangle$$

and the x_μ' appearing in $\phi(x')$ are exactly the same as the x_μ' we had before. Now the quantities $\phi(x')$ are subjected to a second quantization. The important thing however is to realize that the x_μ' are exactly the same in both cases, for in this manner one sees that before the second quantization is performed one has plenty of room to introduce measure in ordinary quantum theory.

L. SCHWARTZ: I do not see exactly the relation between this theory and my theory of distributions. I cannot speak exactly, but I have the following impression: here you consider some basic measures and some new measures having some density with respect to this basic measure. For instance, you have a basic measure at the origin and you define 5 as having the density equal to zero and one at the origin, so that it represents exactly the measure. In my system you cannot consider for measures the differential equations and you cannot generalize measures in the distribution satisfying differential equations. So that you may for instance have for Poisson's equation a solution which is a measure or a distribution. In this case, as said by Prof. Wigner, the differential equation does not depend on any particular choice. I think in your method you consider some basic measure which you choose after considering the differential equations of some particular point and with respect to this basic measure you consider a density and you look for a differential equation which is satisfied by this density and since the basic measure can be arbitrary it can be changed. You have a differential equation, the consequence of which depends on the basic measure, as you show in this example.

MEDINA: I regret to say that I do not know your method. It seems, however, that I work precisely in the opposite sense, that is to say, I do not establish the differential equation after I have defined the measure. I assume that it is the field that defines the measure, and not the measure that defines the field. Perhaps an example of a more familiar nature would be appropriate here. The situation is similar to that arising in general relativity. In this case one assumes that the metric is determined by the presence and distribution of mass in the sense that if matter is absent, the metric is flat, and if matter is present the tensor $R_{\mu\nu}$ instead of being equal to zero is proportional to the matter-energy tensor T^μ_ν . In this case one has a system of differential equations which allows the determination of the metric when one knows the distribution of matter. In the case of measure, we do not have any differential equations of this

sort. However, one may assume that the measure, in presence of fields, departs from the ordinary, and one may try to look for such departures. I agree with you when you say that there are many possibilities. As a matter of fact, there is an infinite number of possibilities, and this is just one of them. Of course, when I tried to look for different measures, what I did was to select the simplest of all, the one that would depart the least from Lebesgue's measure in our space. However, I insist upon the fact that the measure is determined by the fields, and that I am not speaking of a universal measure. Each field coupled with another determines a different measure. This is an assumption.

I., SCHWARTZ: After some preliminaries of the problem you choose some measures and when you consider μ , μ^+ , μ^- and so on, there are densities with respect to this same measure.

MEDINA: Yes. That is right. All the remaining functions have to be defined with respect to that measure. If not, the integrals would have no meaning at all.

WATAGHIN: Should you have different measures for different particles?

MEDTNY: I do not know, it is probably so. There is no way of telling that a priori. Probably each particle does have its own measure. One says that a particle has a certain charge e , a certain mass m , a certain spin s . Now, in the same sense, it may possess a certain measure a . Once this is assumed, one can carry on calculations and after comparing with experiment it would be possible to tell whether this a is universal or not. It is possible that all the particles have the same measure, but it is possible that different particles have different measures. I do not think that this makes much difference as far as the foundations of the theory are concerned.

FIALHO: I should like to know whether the discontinuity of the measure can be made Lorentz-invariant.

MEDINA: Yes. We have introduced and have defined a measure in Lorentz space, $\mu(s)$ defined for all sets of points S in Lorentz space, belonging to an algebra of sets. The measure I have used was selected to be invariant under the orthochronous Lorentz group $\mathcal{L}^+$ in such a way that the constant a has the same value for all physically admissible Lorentz frames of reference. The theory however, as it is now, could not be extended to the group of continuous transformations of general relativity.

On the Pseudoscalar Meson Theory of the Deuteron

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We report on an attempt to find a mathematically valid consequence of the YUKAWA pseudoscalar meson theory which disagrees with experiment. As is well known, the symmetrical theory with gradient coupling predicts in first order a nuclear potential:

$$V = g^2 \mu (\tau_1, \tau_2) \left\{ \frac{1}{3} (\sigma_1, \sigma_2) + S_{12} \left(\frac{1}{3} + \frac{1}{x} + \frac{1}{x^2} \right) \right\} \frac{e^{-x}}{x} \quad (1)$$

where $x = Ur$ and U is the reciprocal COMPTON wave-length of the T-meson; S_{12} is the tensor force. But this is not a true mathematical consequence of the theory because (a) the nucleons are treated non-relativistically, and (b) only the term of first order in g has been included. Nevertheless, at first sight we might expect that a complete mathematical theory would not essentially alter the potential (1) outside some small value of x . The relativistic corrections ought to come at the Compton nuclear wave length $x \sim 0.15$ and should be small for x large relative to this. The corrections of order g^4 come from the exchange of two mesons. They represent intermediate states of energy 2μ and produce exponentials like e^{-2x} . Hence they might be expected to have effects at $x \sim 0.5$ and here only of order g^2 or 20% of the first order term (1) (however, see below).

We have tried to see whether the present data on $n - p$ forces (the deuteron properties in particular) could be consistent with potential (1) outside some small $x = x_0$. What the potential is inside of this rough limit x_0 we do not know. The procedure is illustrated by imagining for a moment we were dealing only with a central potential, which is the case of (1) for the singlet state:

$$V_s = -g^2 \mu \frac{e^{-x}}{x} \quad (2)$$

We solve the wave equation for $u = x\psi$ for S -wave scattering for all x in this potential, but do impose the usual boundary condition that u vanishes at the origin. Starting from the outside and proceeding inward this ought to give the correct wave function at least until x_0 — that is over the greater part of the interaction range. At x_0 it should be fitted in slope and value to a completely unknown function coming from inside x_0 . But the solution from the inside is probably energy insensitive until momenta of order x_0 are involved. Assuming the slope to value unknown but constant at x_0 would permit thereby a determination of the scattering cross-section for much higher energies than the present approximations

of the theory of effective range and scattering length would permit. Instead of choosing an x_0 and giving the slope there, since there is really only one unknown constant, we have for convenience carried the solution all the way to $x = 0$. The behavior of the solution extrapolated in this manner to the origin is a new effective constant to describe the inadequacy of the potential (2) for short distances. One expects such things as the scattering at energies below momenta XQ^1 and properties depending on the wave function not too close to the origin to be correct¹ described by (2) with its new boundary condition. Actually, we have two accurate data on the singlet interaction, the scattering length and the effective range. The first determines the asymptotic behavior $1 - x/a_0$ of the solution as $x \rightarrow \infty$. We used it, $a_0^{-1} = -0.06$, to start the numerical solution of the radial wave equation from the outside. The effective range r_0 depends on an integral, $\int_0^\infty dx [(1 - x/a_0)^2 - u_0^2]$ where u_0 is the zero energy u , but we needed the experimental value $r_0 = 2.76 \times 10^{-13}$ cm. to determine the as yet unknown g^2 . By trial and error we determined $g^2 = 0.18$. The wave function at the origin behaved like a series involving BESSEL functions. We did not go on to calculate phase shifts from potential (2) and boundary conditions for higher energies but turned instead to the triplet state potential. If χ and φ represent the S and D parts of the wave function

$$x\psi = \chi + \frac{\sqrt{2}}{4} S_{12} \varphi$$

the SCHRÖDINGER equation becomes for the deuteron ground state:

$$\ddot{\chi} = A\chi - \sqrt{2} B\varphi; \quad \ddot{\varphi} = \left(A + B + \frac{6}{x^2}\right)\varphi - \sqrt{2} B\chi \quad (3)$$

where $A = \epsilon^2 - C \frac{e^{-x}}{x}$, $\epsilon^2 = -\frac{ME}{\mu^2}$, $C = \frac{M}{\mu} g^2$;

$$B = 6Ce^{-x} \left(\frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{3x} \right)$$

using the g^2 value obtained from the singlet state. Of course these equations have a singularity, but although we shall carry the solution to the origin, we do not imagine (3) to be valid below some x_0 . The equations have an explosive exponential solution (of the form $k \exp(\alpha/\sqrt{x})$) near the origin. This exponential rises on decreasing x so soon and so rapidly that for appreciable k it does not seem possible that any modification of the potential for small x could bring the solution within a reasonable size again. Therefore, we have assumed that our solution has as x tends to zero an asymptotic form with $k = 0$. It varies then as $\cos(\alpha/\sqrt{x} + \lambda)$. This boundary condition has been suggested by VON NEUMANN and PAULI and discussed by CASE (1950). We consider λ as another unknown constant whose value depends on the true potential. We used binding energy of the deuteron as the experimental quantity determining this constant. With the solutions obtained in this way numerically we have three quantities which we can compare to experiment. (Table I).

TABLE I

QUANTITY		EXPERIMENT	THEORY
Quadrupole moment	$\int_0^\infty (x\varphi - 0.353 \varphi^2) x^2 dx$	$2.73 \times 10^{-27} \text{ cm}^2$	$3.53 \times 10^{-27} \text{ cm}^2$
Effective range	$\frac{1}{2\epsilon} - \int_0^\infty (x^2 + \varphi^2) dx$	$1.65 \times 10^{-13} \text{ cm}$	$2.32 \times 10^{-13} \text{ cm}$
% D state	$\int_0^\infty \varphi^2 dx / \int_0^\infty (\varphi^2 + x^2) dx$	$4 \pm 2 \%$	10.2 %

The second column tells how these quantities depend on the properties of the ground state wave function. The last two show a serious disagreement between theory and experiment. There is too much tensor force. Estimates show that the potential must be wrong by its own order of magnitude at least as far out as $x = 0.7$.

If the argument about the validity of (1) for x large given above were valid, we would have here evidence that the YUKAWA pseudoscalar theory with gradient coupling would not give correct results even if the potential were calculated accurately. Unfortunately the argument about the g^4 corrections is not valid. A direct calculation of the g^4 static correction by TAKETANI, MACHIDA & ONUMA (1951) (and confirmed independently by us) shows that large combinatorial coefficients come in so that the g^4 order corrections are larger than the g^2 potential (V) for $g^2 = 0.2$ even as far out as $x = 1$. Therefore, the potential (1) has neither theoretical nor experimental justification. This conclusion has also been obtained much earlier by TAKETANI, NAKAMURA & SASAKI (1951) in much the same way.

It is unlikely that any calculation by a perturbation expansion (for example, using the diagrams and methods of electrodynamics) has value for nucleon- π -meson problems. At least such a calculation should be accompanied with a good argument establishing its validity.

Incidentally, the g^4 potential was carried out to first order in v/c as well. It shows a spin-orbit force too large but of the right sign for the nuclear shell model of HAXEL, JENSEN and SUESS and of MAYEY. It arises from the fact that the non-relativistic nucleon-meson coupling (r, q) — where q is the meson momentum — must, for Galilean invariance, be replaced by $(a, q-p E/M)$ where E is the meson energy, p the average momentum before and after interaction, and M the proton mass. A velocity v increases q by Ev and p by Mv so the term is now Galilean invariant. Hence, we apparently cannot conclude from the existence of the spin-orbit force that the π -meson theory of nuclear forces is incorrect.

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DISCUSSION

WIQNER: Have you studied the relation of your work to the theory of Levy?

LEITE LOPES: I just saw the June 1 issue of the Physical Review which has a Letter to the Editor by Levy. He considers the second and fourth order non-relativistic potential as given by the pseudoscalar theory down to a distance of the order 0.4×10^{-13} cm. Inside this region, he cuts the above potential and replaces it by an infinitely repulsive potential. One can then understand that such a strongly repulsive force compensates the strong tensor potential and the theory can be adjusted to the deuteron quantities and low energy nucleon-nucleon scattering. The same investigation has been carried out, I think, by a group of Japanese physicists. Of course, we do not know yet what this repulsive potential is; it may be connected with particles heavier than pions.

SCHUTZER: Does this strong repulsive potential substitute the constant you wanted to introduce?

LEITE LOPES: I think so. However, our idea was to make no statement about the nuclear interaction below a certain distance, as we do not know it. Our constant was meant to replace such an interaction, in the present state of our knowledge.

Wigner pointed out that according to Levy's recent work, perturbation theory breaks down for distances smaller than 0.3×10^{-13} cm., and for these distances he then assumes a strong repulsior.

Nuclear Models and Manybody Forces

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A description of the structure of atomic nuclei is given in terms of a model which starts from the one-particle-model, as suggested by the existence of the nuclear magic numbers. According to its assumptions, the experiences resumed by the liquid drop model permit to arrive at conclusions concerning the forces acting between nucleons. After it has been shown that a quantitative representation of these forces by ordinary two-body forces and by exchange forces is impossible, a more phenomenological description can be adopted which can be interpreted as a set of manybody forces, which come into play only at high nuclear densities and then account for the saturation of nuclear forces. A quantitative check of our assumptions is given by computing the surface part of the binding energy. In terms of our description, the forces between two particles turn out to depend not only on their distance but also on the nuclear densities within the range of interaction and to be much stronger at the surface than in the interior of the nucleus. This fact justifies, a posteriori, the assumption of almost free particles in the interior of the nucleus. On the other hand, Bohr's manybody model, — according to which every arriving particle, due to strong interaction, gives off at once its energy to all degrees of freedom heating the whole nucleus, — is to be understood by the interaction which takes place on the surface.

On the Origin and the Effects of Spin-Orbit Coupling in Nuclei

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INTRODUCTION

The nuclear shell model has gone through many modifications since it has been first proposed by BECK (1928 *a* & *b*, 1930). Its latest form, proposed by MAYER (1949, 1950) and by HAXEL, JENSEN & SUESS (1948, 1950), brought us a long step nearer to a detailed understanding of nuclear structure. One might say that nuclear theory has now reached the stage which atomic spectroscopy reached when HUND has established his famous coupling rules. The difference is, of course, that atomic theory had, in HUND'S time, a sound knowledge of electronic interaction but lacked the knowledge of the principles of the proper mechanics to evaluate the consequences of this interaction. We, on the other hand, believe to possess the principles of mechanics but lack the knowledge of the laws of nuclear interaction.

Let me review first briefly the shell model of MAYER and of HAXEL, JENSEN & SUESS. The particles, in our case the nucleons, move, in every shell model, in first approximation under the influence of a common potential. This must be a short range potential in the nuclear case; that is, a potential which decreases rapidly to zero at large distances. A comparison of the order of the levels in several short range potentials (WIGNER, 1951) leads to the result that this order depends but little on the detailed behaviour of the potential and is, in general, as follows:

1s(2)	1p(6)	1d(10) 2s(2)	1f(14) 2p(6)	1g(18) 2d(10) 3s(2)	1h(22) 2f(14) 3p(6)	1i(26) 2g(18)
2	8	20	40	70	112	

The numbers below the columns give the total number of nucleons which can be accommodated in the orbits of the column in question, and in all previous columns. They should represent particularly stable proton and neutron numbers. This may be true for the first three numbers: 2, 8 and 20. It is surely not true of the later numbers of the series, i.e. of 40, 70, 112. Instead, the particularly stable neutron or proton numbers are 50, 82, 126.

One may try to explain this discrepancy by assuming that the order of the levels is not that given in the preceding table and this possibility has been

explored by FEENBERG & HAMMACK (1949) and by NORDHEIM (1949). The j - j -coupling shell model, which we are discussing, assumes instead that at least the levels of the first row are split by a spin-orbit interaction into two levels, the first of which is lowered by the spin-orbit interaction to such an extent that it joins the levels of the preceding column. Thus, for instance, the $1g$ level is split into a $1g_{02}$ and a $1g_{72}$ level and the ten orbits of the former, which form a half shell, are lowered so much that they belong into the preceding column. This column thus acquires ten more orbits and, together with the preceding columns, has place for 50 particles. The other magic numbers, 82 and 126, are explained in a similar fashion by the splitting of the $1h$ and $1i$ levels. In the former case, the twelve orbits of the $1h_{11/2}$ levels give, with the 70 orbits of the first five columns, the magic number 82. In the latter case, the fourteen $1i_{13/2}$ orbits join the 112 orbits of the first six columns to give the magic number 126.

The idea of the spin-orbit interaction and of the splitting of levels into sublevels has proved a most fruitful one. It has helped to correlate nuclear spin values, magnetic moments, nuclear isomerism and plays a significant role also in beta decay. It gives, instead of the above, the following somewhat more elaborate but also more detailed scheme.

$1s_{1/2}(2)$	$1p_{3/2}(4)$	$1d_{5/2}(6)$	$1f_{7/2}(8)$	$1g_{7/2}(8)$	$1h_{9/2}(10)$
	$1p_{1/2}(2)$	$2s_{1/2}(2)$	$1f_{5/2}(6)$	$2d_{5/2}(6)$	$2f_{7/2}(8)$
		$1d_{3/2}(4)$	$2p_{3/2}(4)$	$2d_{3/2}(4)$	$2f_{5/2}(6)$
			$2p_{1/2}(2)$	$3s_{1/2}(2)$	$2p_{3/2}(4)$
			$1g_{9/2}(10)$	$1h_{11/2}(12)$	$2p_{1/2}(2)$
					$1i_{13/2}(14)$
2	8	20	50	82	126

It also leads to the conclusion that in the proper wave functions for nuclei the spins of the nucleons are firmly coupled to their orbits to give individual total angular momenta j for all nucleons. These individual j are coupled to each other in the last step of the coupling scheme. This is the origin of the name " j - j coupling shell model".

Many propositions have been made to explain the apparently strong spin-orbit coupling. The oldest of these is due to the German physicist G.vus (1949) who derived a direct spin orbit force by assuming a vector meson to transmit nuclear interaction. Instead, I shall try to evaluate the consequence of the following assumptions.

1. The nuclear interaction can be described as a sum of interactions between pairs of nucleons. This assumption is certainly true only approximately and we have heard, even today, of a different type of approach.

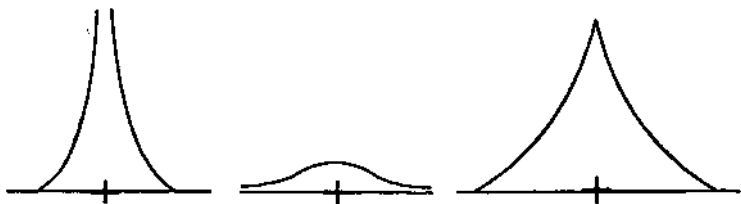
2. In addition to the usual central, spin independent interactions there is a tensor interaction as proved by the quadrupole moment of the deuteron. However, there is no direct spin-orbit or vector interaction.

This second assumption is surely the simplest possible. Again, we have heard even today about calculations based on different assumptions. It is well, perhaps, if I remark at once that I consider the second assumption far from being proved by anything that I shall say or can say. On the other hand,

the consequences of the above two assumptions are surely well worth a thorough exploration even if they should prove false ultimately. In fact, they are worth a very much more detailed exploration than I am able to give at this occasion.

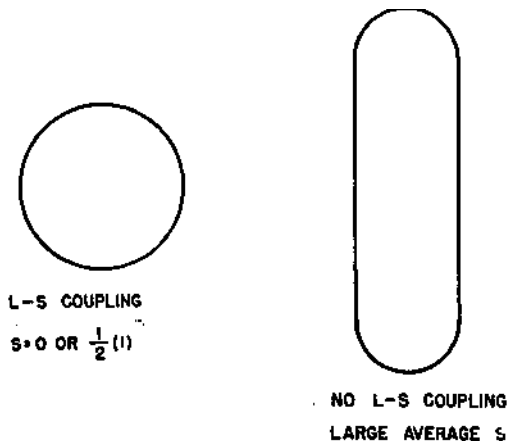
THE EFFECT OF THE TENSOR FORCES

It can well be said that the calculations of ordinary quantum mechanics consist of a reconciliation of, and a compromise between, two opposing tendencies: the tendency of the potential energy which seeks to concentrate the wave function to a very small region where the potential is very small, and the tendency of the kinetic energy which opposes any steep gradient in the wave function and tries to make it as flat as possible. Thus in the case of the hydrogen atom, the



wave function would look like the first picture if the potential energy would have full sway, it would look like the second one if the kinetic energy could have its way. Actually, it looks like the third one.

In the same way, the calculations which we are aiming at should reconcile three tendencies: that of the central forces, in particular the MAJORANA exchange



force, that of the tensor forces, and finally that of the kinetic energy. If only the first two were present the nucleus would again be very small and concentrated to the immediate neighborhood of a point. The compromise to which the existence of the central and the kinetic energy would lead have been worked out in some

detail before the war. It leads to a spherical nucleus with RUSSEL-SAUNDERS (L - S) coupling, just as we learn it in ordinary spectroscopy. However, in contrast to this, it leads to a very low total spin momentum ($S = 0$ for even nuclei, $S = 1/2$ for even-odd or odd-even nuclei and $S = 1$ for odd-odd nuclei).

The compromise between tensor forces and kinetic energy is more difficult to work out. It would lead to an elongated nucleus in which no known coupling scheme is valid. Neither L nor S are good quantum numbers but the average value of S is quite high. Its value will depend on the relative strengths of the kinetic energy, which favors low S , and of the tensor forces which favor large S .

What we have to consider is, therefore, the compromise between the tendencies which manifest themselves in the above two pictures. It is a great help that we can use, as a starting point, as simple a picture for the internal structure of the nucleus as the shell model in which the particles move, in first approximation, independently of each other. In spite of this, we have not yet acquired a full picture of even the qualitative features of the results to which our assumptions lead and I can give therefore only a very fragmentary description of the properties of nuclei which would follow from our assumptions. It may be well to anticipate, even at this point, that the calculations do not lead to a simple j - j -coupling scheme to which a strong spin-orbit force would lead. Neither does the accurate validity of the single particle picture follow. Therefore, if these features of the present shell picture remain valid, the basic assumptions which I have outlined above will have to be modified.

RESULTS OF CALCULATIONS

There are three questions which the shell model poses:

1. The interaction of the particles within closed shells.
2. The interaction of the particles outside of closed shells with the particles in closed shells.
3. The interaction of the particles outside of closed shells with each other. In addition, in the j - j -coupling scheme, there are two types of closed shells: a shell, such as the $1g$ shell, may be fully closed, *i.e.* all its 18 orbits occupied. On the other hand, only the $1h_{9/2}$ shell may be closed, and the $1g_{7/2}$ shell empty. In this case the $1g$ shell contains only 10 particles. This is a particularly interesting case since, as we have seen before, it leads to a particularly stable, "magic" configuration.

1. There is no conflict between the wave functions stipulated in the j - j and the L - S -model if one has a fully closed shell. The wave function is, in both cases, a single determinant which contains all the orbits of the two shells. The two theories would write this determinant in different forms, the two expressions are, however, identical. This may be surprising at first hearing and its real interpretation is, unquestionably, that the shell model is not adapted in this case to the description of the tensor forces. For the wave function thus obtained, the total value of the tensor forces is zero, that of the central forces quite considerable and the compromise forced upon one by the shell model is

all in favor of the central forces. Actually, it is questionable whether, for instance, the $1g$ shell is ever entirely full without other shells being partially filled: the filling of the $1g_{7/2}$ and of the other shells of the 5^{th} column probably proceed simultaneously.

The situation is radically different if only the $1f_{9/2}$ shell is filled. This gives a wave function which is again a single determinant but not the determinant which the central forces would favor. On the long range force approximation, *i.e.* assuming that the range of the forces is large as compared with the nuclear dimension, the effect of the central and of the tensor forces can be easily calculated. The effect of the central forces is decreased as compared with the wave function of the L-S-model, the effect of the tensor forces, which would be zero in the A-S-model, is increased. This is, of course, what one would expect from a compromise solution.

The corresponding numbers are, per particle

$$\frac{l}{2l+1} L_M \quad \text{and} \quad -\frac{7}{24} \frac{l(2l+3)}{2l+1} L_T$$

in which L_M and L_T are the MAJORANA and tensor matrix elements. The coefficient of the latter is larger but the matrix element itself is smaller. Nevertheless, the latter expression is, particularly for high l , larger than the former and will modify the L-S wave function very considerably in the direction of the $j-j$ coupling. As for order of magnitude, L_M is about $30/A$ where A is the atomic number and the whole energy gain is of the order of $1/4$ to $1/2$ Mcv per particle.

It is interesting to compare the above figures with those which one would obtain if one assumed the $j = I - 1/2$ shell to be lower. Contrary to what one may have expected, the situation is very much the same. The decrease of the part of the binding energy which is due to central forces is again overcompensated by the positive binding due to tensor forces. The difference between the internal energies of the $j = I + 1/2$ and $j = I - 1/2$ half shells is quite small. It favors the former shell but so little that one is surely unable to account in this way for the rather substantial energy difference between the two half shells which amounts to about 2 Mcv. This energy difference must derive, therefore, from another source which we shall try to trace in the following. Before doing this, let us finish the present subject of interaction between closed shells by remarking that a fully closed shell has no average tensor interaction either with a fully closed shell or a closed half shell. However, the two closed half shells mutual interaction is large and repulsive. This is evident since it must compensate for the attractive interactions within the half shells. This does lead to a self-stabilization, and a very substantial self-stabilization of the half-shells, which I consider to be significant and to which reference will be made again later. Thus the calculations to which I have referred so far do explain a substantial separation of the two shells; they do not give, however, a sufficiently clear decision which should be filled first.

2. The interaction of a particle outside a closed shell, or closed half shell, with the particles in the closed shell, or half shell, is the same as the average interaction of all the particles in the shell to which this particle belongs, with the closed shell or half shell. Hence the interaction of a particle outside a shell

with a full shell vanishes in the shell model, that of a particle outside a half shell is repulsive. Thus when, for instance, the $1g_{9/2}$ shell is filled, it will exert a repulsion on a single particle in the shell, of a rather substantial amount. This may have one of two consequences: either there will be a break in the binding energy curve, giving the appearance of great stability to the $1g_{9/2}$ shell, or it will lead to the filling of other shells of our fifth column before the $1g_{7/2}$ shell is filled. Actually both events seem to take place.

The interaction of an extra particle with those of a full closed shell vanishes only as long as one uses the wave functions of the shell model without modification, i.e. it vanishes only in first approximation. The next approximation leads to a modification of the wave function of the shell model and to a substantial attractive interaction. This has to be so since otherwise nuclei with only one particle outside of a closed shell would not be stable. As far as tensor forces are concerned, this modification of the wave function, and the corresponding energy gain, has first been calculated by S. DANCOFF. However, his work has been largely superseded, as far as numerical results are concerned, by the work of FEINGOLD (1952).

FEINGOLD'S calculations are confined to the $1p$ shell in which the $j-j$ -model is not yet expected to unfold itself fully. He used a variational method and found an energy gain, due to tensor forces, of the order of 7 Mev per particle. This goes far to correct the defect in binding energy which earlier calculations showed, under neglect of the tensor forces. At the same time, the wave function is modified only to the extent of 4. 1/2%, i.e. the shell model gives in this case, for only a single particle outside the closed shell, about 95% of the wave function correctly. (In this case, again, the $j-j$ and L-S—models give the same wave function). At the same time, FEINGOLD obtains a difference of 0.8 Mev between the $j = l + 1/2$ and the $j = l - 1/2$ levels. This, as well as the whole effect of the tensor forces, results from a polarization of the closed shell, in FEINGOLD'S case the He^4 nucleus. This has to be so since we saw that the unmodified closed full shell does not interact with an outside particle. The splitting of the level, i.e. the energy difference between the $j = l + 1/2$ and the $j = l - 1/2$ levels is due to a rather involved operator

$$V = 9 f(r_{12}) f(r_{13}) (r_{12} \cdot r_{13}) (r_{12} \times r_{13} \cdot \sigma_2 \times \sigma_3).$$

This operator should give, in first approximation, the same result which the ordinary tensor operator gives in second approximation. Those familiar with group theory will note that this V is a vector operator and this causes it to give a different value for the $j = l + 1/2$ and $j = l - 1/2$ levels; $j(r)$ gives the dependence of the tensor forces on the distance.

It is satisfactory that we have obtained, at least in second approximation, a splitting between the $j = l + 1/2$ and $j = l - 1/2$ levels. It is also satisfactory that the $j = l - 1/2$ level is below the $j = l + 1/2$ level and this may be the real explanation that the former shell is filled first, the latter only afterwards. We noted before that the first approximation gives no appreciable difference between these levels. It is not satisfactory that the calculated energy difference, 0.8 Mev, is considerably below the observed difference of about 3 Mev. The calculation is inaccurate, as all variational calculations, and there is some reason

to believe that one would obtain a larger energy difference with an improved calculation. However, it remains questionable whether the whole difference can be bridged this way.

We can further note that the interaction is not of a simple 18 type; it is much more complicated than the prototype of such interactions

$$(\sigma_1 + \sigma_2 \cdot p_1 - p_2 \times r_{12}) f(r_{12}) .$$

In fact V is a three-body force. Hence, its effect is more difficult to assess under more general conditions. This is only another expression for the fact that the tensor forces do not lead to the simple l - l -coupling scheme. The relatively involved nature of V has, on the other hand, the consequence that it is similar to a dispersion force, i.e. it is, like the LONDON-WANG effect with a rather high degree of accuracy additive for the particles outside the closed shells.

3. The above should complete our discussion of the interaction of a single particle outside of closed shells or half shells with these shells or half shells. There are only fragmentary data for the interaction of particles within unfilled shells. Most of the results are due, here, to MAYER and to KURATH, both at Chicago. Most of their results are only qualitative but reproduce some of the empirical facts in a striking fashion. However, they are based on a complete neglect of the tensor forces or of any other forces which might cause a spin-orbit coupling.

The only calculations which take the tensor forces into consideration and with which I am familiar are those of FEINGOLD. They are confined to the j shell, i.e. to very light nuclei and are summarized in the following

Nucleus	E_T	%	ΔE_{calc}	$\Delta E_{exp.}$
Li^6	10 Mev	$5\frac{1}{2}\%$	1.4 Mev	3.5 Mev
Li^7	9.5 "	4 %	0.3 "	0.48 "

The E_T is the total increase in binding energy, due to tensor forces, the percentage modification of the wave function is given in the third column. The fourth and fifth columns give the calculated and experimental energy difference between normal state and an excited state. We see that, again, the calculated energy differences have the same sign but are smaller than the experimental ones. Similar remarks apply to those made after the He^5 calculation. It should be noted though that the percentage deviations of the wave function, given in the third column, are from the wave function of the L - S -coupling scheme, not from the wave function of the l - l -scheme. This seems to me to be more unsatisfactory from the point of view of the j - j scheme than from that of the calculations here presented: a rather thorough survey of the experimental data which I have undertaken about a year ago (WIONER, 1951) indicates that the L - S -coupling wave functions give a more fair picture of the properties of these nuclei than the j - j -coupling wave functions. The situation may well be different for heavier nuclei.

CONCLUSION

The assumption of central and tensor forces, acting between pairs of nucleons, does not lead to a simple j - j -coupling scheme. It does lead, however, to a picture which shows at least a qualitative similarity to observations and

reproduces many of the qualitative features which one derives, ordinarily, from the j - j -coupling scheme. It is not possible to determine from the calculations which have been carried out so far whether the tensor forces give a full substitute for the phenomenological j - j -scheme. The feature of the calculations which are presented here and which is perhaps most attractive is the repulsion between half shells which may be the reason for the rather marked discontinuity of the binding energy at the magic numbers. The usual $j-j$ scheme does not give a full explanation for the larger magnitude of the discontinuities at these than at all other completed shells.

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DISCUSSION

ANDERSON: Could you make clear where the distinction between the neutron-proton interaction and the neutron-neutron interaction, occurs in your work?

WIGNER: Most of the time I was speaking about either neutron-neutron or proton-proton interaction and not about neutron-proton interaction. Clearly the shells must be due to interactions between like particles. Otherwise, the proton numbers which show great stability would depend on the number of neutrons which are present and vice versa. However, the quantitative work of Feingold, and many of the earlier calculations, do take both interactions into account. However, the considerations concerning half shells, which were more general and qualitative, only the proton-proton or neutron-neutron interaction were taken into account.

ANDERSON: YOU would say then that this is evidence for a tensor interaction between neutron and neutron. Do you know of any other, any independent experimental evidence?

WIGNER: I do not know of a very direct evidence, but it now seems that the quantum number T , the isotopic spin quantum number is a good quantum number. It can be a good quantum number only if the neutron-neutron interaction is substantially equal to the proton-neutron interaction. Hence the neutron-neutron interaction must contain a tensor part because we know from Rabi's work on quadrupole momentum of the deuteron that the proton-neutron interaction does.

RABI: Do you have an estimate of the order of magnitude of this repulsion between half shells?

WIGNER: I would rather like to give the repulsion of a single particle by the closed half shell. This is close to the experimental value, which is probably quite accidental because most of the other results are very far from it.

RABI: In that case, I do not quite understand what you meant when you said that, when you began to fill the next half shell after the $g_{9/2}$, the $g_{7/2}$, the particles are distributed in quite a number of states, let us say, you add the first particle and it definitely goes to the $g_{7/2}$, you would then know where the next one went. Is that it?

WIGNER: When the $g_{9/2}$ shell is filled then the $g_{7/2}$ orbits are raised. As a result, the next particle may not even go into the $g_{7/2}$ shell but into another shell.

OEHME: Are any parts not filled very well in making this assumption which you wrote under number one. This means the neglect of the many body forces in the nucleus, and I know that if one wants to introduce manybody forces it is nearly impossible to make mathematics like that, but can one say anything more or less intuitive of how manybody forces will change for example the interaction of extra core nucleoli with core?

WIGNER: I do not know. It is very likely that manybody forces play a role. In fact, I think it is hard to believe that if there are no manybody forces any independent particle model would be very accurate. In other words, probably the fact that the independent particle model is at all approximately correct is probably due to some degree to the joined effect of several mesons or to something that can be described with manybody forces. But in the calculations which have been carried out by Feingold and others, the manybody forces were not taken into account.

ANDERSON: If I may get back to the point that you raised about the isotopic spin, I am not willing to accept so readily your explanation because neutron-neutron and proton-proton belong to an isotopic spin triplet, whereas the neutron-proton is mixed in isotopic spin singlet and isotopic spin triplet, and the interaction might be different. The existence of a tensor interaction exhibited in the deuteron does not necessarily show that such an interaction will also be present in neutron-neutron or proton-proton interactions.

WIGNER: Unquestionably, there might be a difference between neutron-neutron and proton-neutron interaction but the weight of evidence, and some of it was presented by you, is that they are nearly alike. A piece of evidence which is not often quoted, comes from mirror nuclei. The transition probability for β - decay between mirror nuclei would be smaller if there was a substantial difference between like particle and neutro-proton interactions.

On a Statistical Theory of Non-Local Fields and on the Production of Fundamental Particles

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I. INTRODUCTION

The limitations in the choice of a mathematical formalism adequate to the description of physical laws were always introduced in the theory: in the prerelativistic physics such limitations were assumed in the choice of Euclidian space and absolute time and the use of a causal Newtonian mechanics of point — particles, and in the present day physics many other limitations are introduced in the relativity and in the quantum theory.

It seems that new limitations must be assumed in order to obtain a consistent theory of fundamental particles and of nuclear forces. The first step is, in our opinion, the assumption of a non — local theory of fields, satisfying the postulates of relativity and of the quantum theory.

The definition of *non local interactions* and of *non local fields* has been given many times. Speaking of interactions we shall distinguish the *direct interaction* between two particles or two fields (of the type one has in the Compton effect, or in the high energy proton-proton collision giving rise to meson production) from the *indirect interaction* due to the exchange between two particles of some other type of particles (created and annihilated during the interaction), as in the example of Coulomb interaction interpreted by means of the Dirac-Fock-Podolsky formalism.

We assume all direct interactions to be short-range interactions and thus we exclude "actions at distance". MARKOV and YUKAWA have pointed out that the representation of the fields in the usual quantum theory of fields should depend explicitly on the pairs of non commutable operators x_μ, p_μ ($\mu = 0, 1, 2, 3$)

$$[x_\mu, p_\nu] = i\hbar g_{\mu\nu}; x_\mu = x'_\mu \delta(x'_\mu - x''_\mu); p_\nu = i\hbar \delta'(x'_\nu - x''_\nu) g_{\nu\mu}$$

and thus the field components ψ_K are in general non diagonal in the x_μ representation (e.g. in the representation where x_μ are diagonal $\langle x' | \psi_K | x'' \rangle$ is not of the type $f(x') \delta(x' - x'')$). Recently BLOCH, KRISTENSEN & M LLER and RAYSKY have shown how such a non local theory of fields can be built, and how the Lagrangian operator, the commutation relations and the S-matrix formalism can be established. Such theories are called non local, because the field components ψ_K are operators, which cannot be defined unambiguously in one point of the space time.

We shall introduce for every field component new parameters:

$$\xi_\mu = \frac{1}{2} (x'_\mu + x''_\mu) \quad \eta_\mu = \frac{1}{2} (x'_\mu - x''_\mu)$$

and assume that the existing fields belong to the general type of "separable" fields:

$$\langle x' | \psi | x'' \rangle = \sum_r \chi_r(\xi_\mu) \varphi_r(\eta_\mu) \quad (1)$$

We shall call ξ_μ external coordinates and η_μ internal coordinates, making some assumptions on the behaviour of $\varphi_r(\eta_\mu)$. According to BLOCH and others, we can normalise $\varphi(\eta_\mu)$ in the following way:

$$\int \varphi_r(\eta_\mu) d^4\eta = 1 \quad (2)$$

and obtain for every field component ψ an "amplitude":

$$\psi(\xi_\mu) = \sum_r \varphi_r(\xi_\mu)$$

In all proposed non local theories this "amplitude" is assumed to correspond approximately to the usual type of representation of a local field in the cases of "slowly varying fields" or for small values of momenta (where obviously the definition of "small" or "slow" is related to special choice of the Lorentz frame of reference and to the value of a universal constant D). Let $\psi_r(\xi'_\mu)$ and $A_s(\xi''_\nu)$ be two such representations of amplitudes of two fields. Then the usual bilinear local interaction is described by linear combinations of

$$\psi_r(\xi'_\mu) A_s(\xi''_\nu) = \int \psi_r(\xi'_\mu) \delta(\xi'_\mu - \xi''_\nu) \dots A_s(\xi''_\nu) d^4\xi''.$$

Whereas the substitution of $\delta(\xi' - \xi'')$ by a form-factor $D\left(\frac{1}{2}(\xi' - \xi'')\right)$ gives an example of a non-local interaction:

$$1 = \int \psi_r(\xi'_\mu) D\left(\frac{\xi'_\mu - \xi''_\mu}{2}\right) A_s(\xi''_\mu) d^4\xi'' \quad (3)$$

In order to fulfil the covariance requirement, the universal form factor D must be an invariant with respect to the full Lorentz-group.

It is well known (MARKOV, PAIS & UHLENBECK; MANUS & PETERLS; FEYNMAN.), that such an interaction is equivalent to the formalism, proposed years ago, of *relativistic cut-off factors* in the momentum representation.

Indeed, putting $\xi = \frac{1}{2}(\xi' + \xi'')$; $\eta = \frac{1}{2}(\xi' - \xi'')$, (3) can be written:

$$1 = \int \psi_r(\xi + \eta) D(\eta) d^4\eta A_s(\xi - \eta) \quad (3')$$

and, indicating by $G(K)$, $f(p')$, $g(p'')$ the Fourier transforms of $D(\eta_\nu)$, ψ_r , A_s :

$$D(\eta_\nu) = \int G(K_\mu) d^4K e^{iK\lambda \eta_\lambda}; \quad \psi_r(\xi_\mu + \eta_\mu) = \int f(p') d^4p' e^{ip'_\mu(\xi^\mu + \eta^\mu)}$$

$$A_s(\xi_\nu - \eta_\nu) = \int g(p'') d^4p'' e^{ip''_\nu(\xi^\nu - \eta^\nu)}$$

we obtain

$$\begin{aligned} I &= \int d^4 p' d^4 p'' f(p') g(p'') e^{i(p'_\mu + p''_\mu) \xi^\mu} \int e^{i(p'_\mu - p''_\mu + K_\mu) \eta^\mu} d^4 \eta \Gamma(K) d^4 K = \\ &= \int d^4 p' d^4 p'' f(p') g(p'') e^{i(p'_\mu + p''_\mu) \xi^\mu} G(p'_\mu - p''_\mu) \end{aligned} \quad (3'')$$

where the cutoff factor $G(p'_\mu - p''_\mu)$ is a relativistic invariant depending on the 4-vector $p'_\mu - p''_\mu$. The inverse theorem is also true. Putting $G(p' - p'') = 1$, one obtains usual local interactions. We want to point out that the simple claim of invariance of $D(\eta)$ or of $G(p' - p'')$ is by no means sufficient to obtain consistent and convergent results in the theory of interactions. As was mentioned in some previous paper, the definition of D and G rests essentially on the choice of this operator in some special Lorentz frame of reference, depending on the momenta of the interacting particles. It is the purpose of the following remarks to discuss this problem and the solution of it suggested in previous work.

II. THE C-SYSTEM OF REFERENCE

We begin with consideration of group of n relativistic particles having coordinates $x_\mu^{(K)}$ and momenta $p_\mu^{(K)}$ ($\mu = 0, 1, 2, 3$; $K = 1, 2, \dots, n$) and having no interactions, and we shall make use of the elegant method suggested recently by SHIROKOV, modifying it slightly.

Let us put: $c = 1$, $\hbar = 1$ and for the metric-tensor components: $g^{00} = -g^{11} = -g^{22} = -g^{33} = 1$.

Let:

$$P_\mu = \sum_{K=1}^n p_\mu^{(K)} \quad (4)$$

and let us consider the case:

$$P_\mu P^\mu = P_0^2 - P_1^2 - P_2^2 - P_3^2 = m_0^2 > 0 \quad (4')$$

Introducing the unit-vector velocity:

$$u_\mu = \frac{1}{m_0} P_\mu \quad u_\mu u^\mu = 1 \quad P_\mu u^\mu = m_0 > 0$$

we shall find useful to consider separately the "components" of 4-vectors and tensors parallel to u_μ and "normal" to this velocity.

Putting

$$\begin{aligned} \tau^{(K)} &= x_\mu^{(K)} u^\mu & \rho^{(K)} &= p_\mu^{(K)} u^\mu \\ y_\mu^{(K)} &= x_\mu^{(K)} - \tau^{(K)} u_\mu & \pi_\mu^{(K)} &= p_\mu^{(K)} - \rho^{(K)} u_\mu \end{aligned}$$

we have:

$$y_\mu^{(K)} u^\mu = 0 \quad \pi_\mu^{(K)} u^\mu = 0$$

Introducing the angular momentum tensor $M_{\mu\nu}$ and its dual tensor

$M'_{\sigma\lambda} = \frac{1}{2} \Sigma_{\sigma\lambda\mu\nu} M^{\mu\nu}$ (where $\Sigma_{\sigma\lambda\mu\nu}$ is the totally antisymmetrical pseudotensor), and putting

$$g_{\mu} = M'_{\mu\sigma} u^{\sigma} \quad \Gamma_{\sigma} = M'_{\sigma\lambda} u^{\lambda} \quad (6)$$

we have:

$$g_{\mu} u^{\mu} = 0 \quad \Gamma_{\sigma} u^{\sigma} = 0$$

The center of mass frame of reference, or the C -system, is defined by the condition: $\vec{P} = \Sigma \vec{p}^{(K)} = 0$

or $P_1 = P_2 = P_3 = 0 \quad P_0 = m_0 = \Sigma p_0^{(K)}$

$$u_0 = 1 \quad u_1 = u_2 = u_3 = 0$$

Thus the components of g_{μ} are $(0, M_{10}, M_{20}, M_{30})$

and of Γ_{σ} are $(0, M_{23}, M_{31}, M_{12})$

An arbitrary displacement of the origin and a rotation in space can still be applied to the C -system.

(One has also: $\tau^{(K)} = x_0^{(K)} \quad \rho^{(K)} = p_0^{(K)} \quad \overset{\rightarrow}{\pi}^{(K)} = \vec{p}^{(K)} \quad \pi_0^{(K)} = 0$)

In order to define conveniently the world line of the center of mass, let us consider, in the C -system, the simultaneous position of n particles corresponding to: $\tau^{(K)} = x_0^{(K)} = t \quad (K = 0, 1, \dots, n)$.

One has then: $\vec{x}^{(K)} = \vec{y}^{(K)} \quad x_0^{(K)} = t \quad y_0^{(K)} = 0$

The space-time coordinates Y_{μ} of the center of mass can be defined by:

$$m_0 Y_{\mu} = \Sigma \rho^{(K)} x_{\mu}^{(K)}$$

The space coordinates are: $y_1 = g_1 \quad ; \quad y_2 = g_2 \quad ; \quad y_3 = g_3$;

By a suitable displacement of the origin one obtains:

$$Y_0 = t = x_0^{(K)} \quad y_1 = y_2 = y_3 = 0$$

In the quantum theory the definition of the world line of the center of mass Y_{μ} cannot be given because of the uncertainty relations. Indeed the commutation rules give $Y_{\mu} P^{\nu} - P^{\nu} Y_{\mu} = i \hbar \delta_{\mu}^{\nu}$ and $\Delta y_{\mu} \Delta P^{\mu} \gtrsim \frac{\hbar}{2}$.

Thus if a group of n non interacting particles is characterized by their momenta $p_{\mu}^{(K)}$ (sharp values) and by the corresponding plane waves, it is obviously impossible to define the space - time coordinates of the center of mass. Considering the case where there is an interaction between particles, one has to take into account the variations of $p_{\mu}^{(K)}$ and the creation and annihilation of particles. The introduction of a non local quantum theory of fields seems to be unavoidable.

III. THE D_1 -DOMAIN OF INTERACTION

Let us consider a representation of a field $\langle x' | \psi | x'' \rangle$ supposing the x to be diagonal, and let us assume that the internal coordinates $\eta_{\mu}^{(i)} = \frac{1}{2} (x'_{\mu} - x''_{\mu})$ are unobservable. The function $\varphi(\eta^{(i)})$ normalized as in (2), will be defined later

and will vanish for every value of η_μ satisfying $|\eta_\mu| > l$, where l is a universal length. In other words we assume that there are no means to distinguish between the coordinates x_μ , and the external coordinates ξ'_μ of a particle.

Let us consider two "classical" particles and let x'_μ and x''_ν be their coordinates, and p'_μ p''_ν their momenta. We shall define a D_1 -domain of interaction assuming that the interaction takes place if, and only if, the relative coordinates

$\eta_\mu = \frac{x'_\mu - x''_\mu}{2}$ satisfy the following condition:

$$2(\eta_\mu u^\mu)^2 - \eta_\mu \eta^\mu \leq l^2 \quad (8)$$

where $u^\mu = \frac{1}{m_0} P^\mu$ and $P_\mu = p'_\mu + p''_\mu$

$P_\mu P^\mu = m_0^2$ (P^μ and η_μ are commutable in the quantum theory). In the case of two colliding particles: $m_0 > 0$

In the C -system of two particles the condition (8) becomes:

$$\eta_0^2 + \eta_1^2 + \eta_2^2 + \eta_3^2 \leq l^2 \quad (8')$$

(8) restricts the 8-dimensional space of events (x'_μ, x''_μ) to a special subspace D of couples of events, and this subspace is defined in an invariant way by (8). Obviously in the quantum theory the complementarity of the measurements of x_μ and p_μ restricts our possibilities to measure the coordinates Y_μ of the center of mass in the case when p'_μ p''_μ and u_μ are known with given uncertainties Δp , Δu .

In a similar way one can formulate the conditions of interaction between n particles. In the case of 3 particles one defines the D_1 -domain by the requirement that two conditions must be satisfied by the relative coordinates of the couples (2,3) and (3,1):

$$\begin{aligned} 2(u^\mu \eta_{\mu}^{(2,3)})^2 - \eta_{\mu}^{(2,3)} \eta^{\mu(2,3)} &< \left(\frac{l}{2}\right)^2 \\ 2(u^\mu \eta_{\mu}^{(3,1)})^2 - \eta_{\mu}^{(3,1)} \eta^{\mu(3,1)} &< \left(\frac{l}{2}\right)^2 \end{aligned} \quad (9)$$

It follows that also (8) is satisfied by the relative coordinates of the couple (1,2). We say that (9) defines the D -domain of interaction of the group of 3 particles.

The special choice (8) or (8') is suggested by the symmetry considerations and by the consideration of a classical model of short range interaction in which the time interval of interaction decreases when the spatial collision parameter of the trajectories of two incident particles increases.

IV. THE G-OPERATOR

C. Bloch, Kristensen and Moller and Haysky have shown how an S -matrix can be calculated by the method of Yang and Feldman, and have pointed out the non hamiltonian form of a non-local theory. The hamiltonian formalism is obtained in the limit of slowly varying fields.

We shall also postulate that the non local theory must give the results of the conventional local quantum theory in all cases in which the direct interaction,

considered in the corresponding C - system, involves only momenta of created and annihilated particles, which are small compared with the critical momentum $p_c = \hbar/l = l^{-1}$.

It induces us to assume that in this limit (all $|p_\mu| \ll p_c$) the form-factor $D_l(\eta)$ coincides with $\delta(\eta)$, and the cutoff factors $G(p' - p'')$ or $G(p)$ are $= 1$.

In order to illustrate our method, let us examine the Compton effect and the multiple meson production. In the Feynman graphs of Compton effect, we shall proceed by introducing the G - factors at the vertex-points [and shall not alter the propagation laws of free particles (plane waves)]. As it was pointed out by Ascoli, the C - system for the first graph in which the photon is absorbed first, is the system of the two incident particles, whereas for the second graph, in which the photon is emitted first, the C system is the rest system of the incident electron.

In the case of a high energy collision of two protons, giving rise to the production of one or more mesons, the (7) -system of two incident protons is individualised as the one in which the total kinetic energy of the particles before the interaction is a minimum. The experimental data concerning primary high energy protons in cosmic rays show (see the discussion in a forthcoming paper) that in most probable cases the protons lose a considerable fraction ΔE of their total energy $E = p_0^{(1)} + p_0^{(2)}$ and that $\Delta E/E \rightarrow 1$ as E increases.

We postulate that the statistical competition between many modes of release and distribution between created particles of this great amount of energy ΔE is determined by the statistical weight of each mode and this statistical weight is altered by the cutoff G - factors for each created particle. So we find for n created particles that the element of the S matrix will be altered by a factor $G(p^{(1)}) G(p^{(2)}) \dots G(p^{(n)})$ which, in the local theory, is $= 1$.

From previous discussion it follows that a special choice of the D_l -domain or of the G - factors can be justified only by the comparison with the experiments.

A discussion of the suggested choice will be given in a forthcoming *Part 2* of this attempt of a non-local theory (in the *Nuovo Cimento*).

DISCUSSION

BOHM: Could you explain in more detail just how the multiple production would be related to the non-local field theory and how it could furnish evidence?

WATAGHIN: Let us consider two nucleon collisions. Then, the first question arises how much energy can be transferred to the created mesons. What is the degree of inelasticity -- how much of the kinetic energy (measured in the centre of mass system) of the two protons is transformed in meson field? As far as we know there is some evidence in favor of very high inelasticity for high energy collisions which is of the order of 20 to 25% for collisions in which the energy of incident protons, measured in the C -system, is of the order of 10 ev , and goes in some high-energy cases to $\sim 90\%$ of inelasticity. In an example which was discussed by C. F. Powell in Copenhagen, practically all of the incident kinetic energy was transformed. Let us consider the center of mass frame of reference. Let us indicate then by E_1 and E_2 the relativistic energies of two incident protons and by E_1 , E_2 the final energies,

$$E_1 + E_2 = E'_1 + E'_2 + \sum_{\mu=1}^n \epsilon_\mu = E'_1 + E'_2 + n \bar{\epsilon}$$

where ϵ_μ are the total energies of the created particles. We have the final energies of the two nucleons after the collision negligible in comparison with these initial ones and we want to know the probabilities of various possible values of n , $\bar{\epsilon}$ and of various spectral distributions of ϵ_μ .

Let us consider an example — (the total energy transformed $E_1 + E_2 \rightarrow E_1 + E'$ is of the order of 10 (Section volts. The problem of spectral distribution and of the number n of created particles is connected with the statistical factors in the momentum space. From the form factor $D(\mu)$ or from its Fourier transform $G(p)$ one can deduce these statistical factors and one finds that the probability to obtain a created particle of a very high momentum is strongly reduced by the cut-off operator $G(p)$. Evidently if we cut-off high energies then the probability to obtain in the collision one meson of very high energy becomes less than the probability to obtain many mesons of low energy. In the C -system we find that in the above example¹ the most probable value of n is of the order of 10^{10} ev, and this event corresponds to $n \sim 10^5$.

BOHM: Is it still possible to obtain multiple production, for example, if you have a strong coupling as Lewis and Oppenheimer among others have studied but not finished?. They have considered scalar forces that are so strong at short distance that you might still produce many mesons and not have a cut-off and not have a non-local theory.

WATAGHIN: I think that if you consider pseudo-scalar mesons with pseudo-vector coupling, then for high momenta you obtain a very strong divergence, so that you must cut off.

MOSHINSKY: In what way will the theory alter the angular distribution of the produced particles?

WATAGHIN: The choice of the form-factor proposed here gives an isotropic angular distribution in the center of mass system.

MEDINA: Perhaps I am now in a better position to answer your question in Rio de Janeiro. I must say that I noticed that the similarity between your results and mine is much closer than I thought in the beginning. As a matter of fact I do obtain a cut-off factor very similar to yours except that there appears as a Haenkel function. The origin of the factor, however, is entirely different. In my theory it arises from the fact that if one introduces a measure in configuration space, in order to have an adequate unitary transformation (Fourier transform) allowing to pass to momentum space, one has to introduce? a measure in the latter, which measure is determined by the one previously used in configuration space. This measure determines the cut-off. As regards your question as to the universal character of a , I think that you are exactly in the same case where I am since your fundamental length is not necessarily a universal length. You could change your length for different couplings, as I can change my a . Would not that be so?

WATAGHIN: To this last question I would like to state that in the present attempt of a non-local theory I introduce just one universal length, and try to see if one can deduce different mass-values from such a theory.

LEITE LOPES: The usual quantum theory of fields gives a precise prescription for associating particles with fields. How do you associate particles with non-local fields?

WATAGHIN: C. Moller and Kristensen show in a forthcoming paper that it is possible to apply the usual quantisation method to the non-local fields.

LEITE LOPES: What are the commutation rules?

WATAGHIN: I assume the validity of the usual commutation rules for free fields outside of the interaction regions. Now, what happens in these regions I cannot explain in a simple way. The interaction is different for different Fourier components, so one should use the commutation rules in momentum space. For those in space-time one must take into account the fact that different Fourier components obtain different relativistic cut-off factors.

MOSHINSKY: I would like to point out that it is possible to construct relativistic interactions in which no cut-off procedures are involved by considering interactions that are restricted to a 4-dimensional sub-configuration space of the eight-dimensional configuration space-time of two particles. It is also possible by this procedure to construct compound particles that can disintegrate into two or more elementary particles.

WATAGHIN: Is it in your opinion excluded that such a result could be obtained by a kind of non-local theory?

MOSHINSKY: In the case of pure scattering the interactions were of a local character.

BOHM: As regards the question of gauge invariance, have you any possibility of a solution to this kind of a problem?

WATAGHIN: I shall refer to you some talks I had with Pauli and with Moeller. Kristensen and Moeller in a forthcoming paper — in the Danish Academy of Sciences — show that the gauge invariance can be fulfilled in the S-matrix theory when you integrate in space and

time. They think that it is impossible to fulfill the gauge invariance exactly in a non-local theory and it seems to be the correct answer.

MEDINA: I would like to make a remark in connection with Dr. Moshinsky's question. It seems to me that in ordinary field theory without the necessity of going to measure or to non-local interaction we already had that sort of compound particle that he mentions. This is what we call a virtual particle. It seems to me that Dr. Wigner once pointed out the fact that virtual particles going from one vertex to another of a graph do not possess the ordinary properties of a particle. This is an indication that these particles should be considered as some sort of compound particles.

MOSHINSKY: The procedure of introducing boundary conditions in a subspace of the configuration space-time of a number of particles, leads to compound particles with structure which may have a real or virtual character, depending on the position of the poles of the scattering matrix.

On the Low Energy μ -Mesons from π -Meson Decay*

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It has been recently proposed (PRIMAKOFF, 1951; NAKANO *et al.*, 1951, EGUCHI, 1952; FIALHO & TIOMNO, *in press*) that the short M-mesons track observed in the decay of π -mesons (FRY, 1952) should be explained as radiative decay:

$$\pi \rightarrow \mu + \nu + h\nu \quad (1)$$

in competition with radiationless decay

$$\pi \rightarrow \mu + \nu \quad (2)$$

The agreement of the theoretical value for the frequency of μ -mesons with energy smaller than 3.5 Mev., computed under the assumption that the μ -meson has spin 1/2 and no anomalous magnetic moment, with the experimental

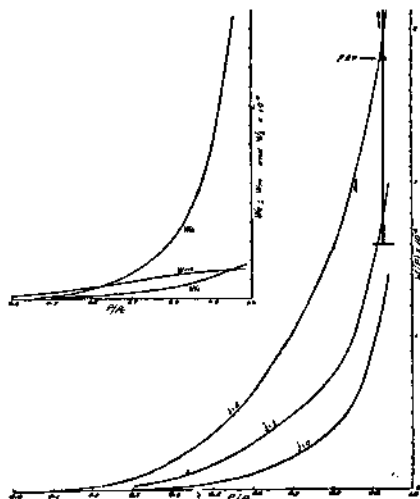


Fig. 1

result is satisfactory. The fact that the theoretical result is somewhat smaller than the experimental one leads us to examine the possibility that the μ -meson has an anomalous magnetic moment ($j_e h / 2 \mu c$) as suggested in the paper by FIALHO

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& TIOMNO (*in press*) (here referred as I). We hope that these results will be helpful to decide on the properties of the μ -mesons when more precise results will be known about the frequency and energy distribution of "anomalous" π -decays.

We make the same assumptions as in Paper I, except that we take for the Hamiltonian of interaction of the μ -meson field with the electromagnetic field A_μ :

$$c \bar{\psi} \gamma^\mu \psi A_\mu + (j e \hbar/2 \mu c) \cdot \bar{\psi} \gamma^\mu \gamma^\nu \psi F_{\mu\nu} \quad (3)$$

The probability per unit time per decay of the radiative process (1), is readily found by FEYNMAN's method (we take $\hbar=c=1$):

$$dW = (\pi/8 \rho \mu^4 E_\nu E \omega) \sum |M|^2 D \quad (4)$$

where μ is the mass of the μ -meson, $\mu\rho$ is the mass of the π -meson, assumed at rest, μE_ν , μE and $\mu\omega$ are the energies of the neutrino, μ -meson and photon, respectively; the summation is to be extended to the polarizations of the photon and spins of the μ -meson and neutrino. Also

$$D dp d(\cos \theta) = (E_\nu \omega^3 p^2 / 8 \pi^4 K \cdot p_\nu) dp d(\cos \theta) \quad (5)$$

is the number of states where the μ -meson has a momentum between μp and $\mu(p + dp)$ and an angle between θ and $\theta + d\theta$ with the momentum μK of the photon.

Finally

$$M = (1/\mu) \cdot \left(\psi_\nu \left| \gamma_5 \sqrt{4\pi} G \frac{p_\lambda \gamma^\lambda + k_\lambda \gamma^\lambda + 1}{(p+k)^2 - 1} \sqrt{4\pi} e \right. \right. \\ \left. \left. \left[j/2 \gamma^\lambda \gamma^\eta (e_\lambda k_\eta - e_\eta k_\lambda) + \gamma^\lambda e_\lambda \right] \right| \psi_\mu \right) \quad (6)$$

is the matrix element for the transition expressed in the momentum space.

We find:

$$W(P, \theta) dp d(\cos \theta) = G^2 e^2 P / \pi E \rho \cdot \{f_c + (1+j)^2 f_m + j^2 f_1\} dp d(\cos \theta) \quad (7)$$

where f_c and f_m are the same as in expressions (6a), (6b) of I, and

$$f_1 = (E_0 - E) \rho^3 p^2 \sin^2 \theta / \{(\rho - E + P \cos \theta)^3 (E - P \cos \theta)\} \quad (7a)$$

Here $\mu P_0 = (\rho^2 - 1) \mu / 2 \rho$ and $\mu E_0 = (\rho^2 + 1) \mu / 2 \rho$ are respectively the maximum momentum and energy of the emitted μ -meson.

Integrating over the angle θ we obtain for the probability per unit range of P/P_0 :

$$w(P/P_0) = w_c + (1+j)^2 w_m + j^2 w_1 \quad (8)$$

with

$$(w_c; w_m; w_1) = W_0 e^2 P^2 / 2 \pi E P_0 \cdot (F_c; F_m; F_1) \quad (9)$$

where

$$W_0 = G^2 / 2 \cdot \mu (\rho^2 - 1)^2 / \rho^3 \quad (10)$$

is the probability per unit time of process (2); F_c and F_m are the same as in expressions 7a, 7b, of I and

$$F_1 = (\rho E - 1) - (E_0 - E) / P \cdot \lg \{ (E + P - 1/\rho) / (E - P - 1/\rho) \} \quad (11)$$

The probability that the π -meson will have a momentum less than P was found by graphical integration:

$$W(P) = W_e + (1+j)^2 W_m + j^2 W_1 \quad (12)$$

with

$$(W_e; W_m; W_1) = \int_0^{P/P_0} (w_e; w_m; w_1) dP/P_0 \quad (13)$$

In Fig. 1 (upper left corner), we give the values of W_e , W_m and W_1 plotted against P/P_0 . In the same figure (lower part), the curves for $W(P)$ are given for $j = 0, 1, 2$. The experimental point obtained by FRY is also indicated. As we can see, the magnitude of the experimental error does not yet permit us to decide which is the value of the magnetic moment of the μ -meson. Other experimental points will be helpful to decide which curve corresponds to the actual case.

The author is indebted to Prof. J. TIOMNO for the suggestion and orientation of the present work, and to Prof. R. P. FEYNMAN for helpful discussions.

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DISCUSSION

BUCK: I would like to ask about one point in the assumptions which intervenes in this calculation: that is essentially the Coulomb field of the π -meson. The radiation is due to the Coulomb field of the μ -meson which is moved and to the magnetic spin field of the μ -meson which may be normal or anomalous. But now the question is whether we can suppose, just for symmetry reasons, that the π -meson being a pseudo-scalar particle has really only a Coulomb field or whether some anomaly may arise there. The only pseudo-scalar particle which we know well is the positronium. For its behaviour it is certainly essential that it has not only a Coulomb field but a field that has a rather complicated structure. I should like to ask whether then¹ are any reasons which could induce us to believe that if there is some electromagnetic field of the π -meson of more complicated symmetry its effects must be so small that they can be neglected with respect to an anomalous magnetic moment of the μ -meson.

FIALHO: I have taken for the π -meson the usual pseudo-scalar description which does not consider any more complex structure of the field. I don't know how the results could be modified in the formulation that you suggested.

DEUTSCH: I should like to add a question to that of Prof. Beck. Isn't it so that in the case of positronium the electromagnetic properties of the singlet S state which is the state to consider, are essentially due to the possibility of internal excitation that is due to the possibility of interference with the triplet state. If you would assign similar properties to the π -meson, would you not tacitly assume a similar possibility of internal excitation or structure for the π -meson? Was that what you meant to imply?

BECK: Yes, it would mean essentially that there are higher states which may be excited meson states.

WIGNER: I understand that you consider the consequences of the possibilities that the μ -meson has an anomalous magnetic moment. Such an anomalous moment is most probably due to some interaction of μ -meson with another field. There was a good deal of discussion whether one can describe this situation adequately by a phenomenological term as you use it. Can you give us your opinion about this?

FIALHO: If the μ -meson has an anomalous magnetic moment of the magnitude here considered it is surely due to interactions with known fields. Therefore it can be only of Pauli's phenomenological kind.

TIOMNO: I would like to add an observation about the main results of this work. At the present moment, experimental errors on the radiative decay spectrum are still very large and it is possible that, in the end, the experimental results will be consistent with the presence of no anomalous magnetic moment. So, the main conclusion about this work may be stated as follows: If the experimental results will turn out larger than in the case of no anomalous magnetic moment, then this result will exclude the possibility of spin zero for the μ -meson. The experimental result on the energy spectrum of radiative decay probably will not allow a decision among the two possibilities of spin $1/2$ and spin $3/2$ because in both cases one may have an anomalous magnetic moment of the type considered by Pauli.

The Interpretation of Racah's Coefficients

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The Racah coefficients are ordinarily used to obtain the proper wave functions for atomic or nuclear configurations. They can be interpreted also in connection with matrix elements of scalar operators which are irreducible tensors of a certain rank a as far as the spin variables are concerned, and tensors of equal rank as far as positional coordinates are concerned. The two states of the matrix elements in question must belong to the same representation J as far as rotation of all the coordinates is concerned but they may belong to different representations L and L' with respect to the rotation of the positional coordinates. The same applies to the representations S and S' which relate to the rotation of the spin coordinates. The Racah coefficient $W(J, L, L', S, S')$ gives the ratio of the matrix element in question to the matrix elements of the product of the irreducible tensor operators between states which are characterized with L, S and L', S' .

The Racah coefficients have also a geometrical significance in the product space of the $2J + 1$ dimensional spaces of the irreducible representations of the rotation group. This point arose during a very stimulating discussion with Dr. Herring.

livery rotation gives a transformation of the vectors of this space by means of the irreducible representations. The quantum theoretical operation of time inversion commutes with all rotations; it provides an invariant metric in the space which permits the raising and lowering of indices. The coefficients of the vector addition model are the invariant vectors of this space.

Every representation can be subjected to unitary transformations. These do not leave the vector addition coefficients invariant. The Racah coefficients are those combinations of the former coefficients which are invariant also under the latter transformations.

On the Connection of the Scattering Matrix With Causality

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The scattering matrix (S - matrix) formalism (WHEELER, 1937; HEI-SKNBERG, 1943) is very helpful for the study of the scattering of particles and also of nuclear reactions (WIGNER & FISENBUD, 1947). The derivation of the scattering matrix from the potential interaction is straightforward, but this potential is not always known; besides this, theories with well defined scattering matrices which do not correspond to a potential interaction are possible. It is also known that the poles of the S - matrix on the positive imaginary axis of the momentum complex plane determine the bound states of the particle plus scatterer. These facts make it important to know the restrictions that should be imposed on the S - matrix in order to obtain a theory with physical meaning.

The property that the S - matrix should be unitary was recognized since the beginning as one of such conditions:

$$S^\dagger S = 1 \quad (1)$$

This is a consequence of the condition for conservation of probability. Also the property that

$$S(-k) S(k) = 1 \quad (2)$$

is usually imposed in the one-particle theory. This is a consequence of the principle of microscopic reversibility.

KRONIG (1946) made the suggestion that the imposition of the causality condition on the scattering matrix formalism for the scattering of particles would result in a supplementary condition for the S - matrix. "Causality condition" in this connection means that there can be no scattered wave before the incident wave reaches the scatterer.

In a previous paper we have shown that the condition for causality, defined in a somewhat different way, however, implies that the poles of the analytical function $S(k)$ which describes the elastic scattering of non relativistic spinless particles with zero angular momentum are either on the imaginary axis or in the lower half plane of the complex k - variable (SCHUTZER & TIOMNO, 1951; MOSHINSKY, 1951). This property is the analogue of the one discovered by FOSTER, CAMPBELL & ZOBEL (1922 - 1924) for radio amplifiers and by KRONIG (1942) and KRAMERS (1927) for the scattering of light on atoms. For these cases the above authors have shown that the poles of the scattering function S are all in the lower half plane of the complex energy variable. It is known that the fact that the

poles of the scattering function are not in the upper half of the k - plane (except the imaginary axis) is useful because it permits one to understand the behavior of cross sections within not too wide ranges of the energy near resonance (KAPUR & PEIERLS, 1938; SIEGERT, 1939).

In the present paper the result of our previous paper is extended to the case of arbitrary orbital angular momentum and the meaning of the time dependent scattering matrix a ($/ - /$, r , r') is analysed. It is shown that the condition of "causality", defined as in the previous paper, imposes that there can be no poles of $S_i(E)$ on the upper half of the energy plane (MOSHINSKY, 1953) (first Riemannian surface). At first we restrict ourselves to a scatterer with finite radius " a ".

Let us consider a packet of incident waves:

$$\psi_i(\vec{r}, t) = \frac{1}{r} \sum_{l=0}^{\infty} P_l(\cos \theta) \varphi_l^i(r, t) \quad (3)$$

with

$$\varphi_l^i(r, t) = \sqrt{r} \int_0^{\infty} g_l(E) H_{l+\frac{1}{2}}^{(2)}(kr) e^{-iEt} dE \quad (4)$$

k being equal to $\sqrt{2mE}$. We use natural units: $c = \hbar = 1$.

The corresponding scattered wave is given by

$$\varphi_s^i(r, t) = \sqrt{r} \int_0^{\infty} g_l(E) [S_l(E) - 1] H_{l+\frac{1}{2}}^{(1)}(kr) dE. \quad (5)$$

In the preceding formulae $H^{(1)}$ and $H^{(2)}$ denote the Hankel functions of the first and second kind, respectively. We relate the first and second sheets of the E - plane with the two halves of the k - plane in the way used in reference 4.

We consider now the functions $S_l(E)$ in the first sheet and define the quantities

$$\sigma_l(r; r', r') = \frac{1}{2\pi} \sqrt{\frac{r}{r'}} \int_{-\infty}^{+\infty} [S_l(E') - 1] \frac{H_{l+\frac{1}{2}}^{(1)}(kr)}{H_{l+\frac{1}{2}}^{(2)}(k'r')} e^{-iE'r} dE'. \quad (6)$$

This integral should be taken along an "indented" path, the half circles around the poles of $H^{(1)}/H^{(2)}$ and the eventual poles of $S_l(E)$ on the negative E - axis being on the upper part of the E - plane. The scattered wave can be expressed as

$$\varphi_s^i(r, t) = \int_{-\infty}^{+\infty} \sigma_l(t - t'; r, r') \varphi_l^i(r', t') dt' \quad (7)$$

for $r, r' > a$.

If we take $r = a$ we see that the scattered waves at a time t are determined by the corresponding incident waves on the scatterer surface at times t' .

In our previous paper we assumed as condition for causality the following one.

"There can be no contribution to the scattered wave at a time t from the incident wave on the scatterer surface at times prior to t' ".

This was only satisfactory for s - scattering in the case of finite scatterer; in general we should impose such condition only for very large radii r . In the case of higher angular momenta, even if the scattering potential is zero outside a sphere of radius a , such a condition cannot be imposed because the centrifugal force

potentials* are not limited to a finite region. So we assume a more general condition for causality, which is valid *even for long range potentials*:

"There can be no contribution to the scattered wave at a time t from the incident wave on a surface with very large radius ($r, r' \rightarrow \infty$) at times prior to t'' ."

Or, in view of the independence of the partial waves we impose on the functions σ_l :

$$\sigma_l(\tau; r, r') = 0, \text{ for } \tau < 0; r, r' \rightarrow \infty \quad (8)$$

In this case we take:

$$\sigma_l(\tau; r, r') = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (S_l - 1) e^{ik'(r+r')} e^{-iE'\tau} dE', \quad (6a)$$

the "indented" path having the half circles around the poles of $S_l(E)$ (on the negative E - axis) in the upper part of the E - plane.

From this we obtain the result that there can be no poles of $S_l(E)$ on the upper half of the E -plane.

The following analysis is helpful for understanding the meaning of the functions $\sigma_l(\tau; r, r')$. We restrict ourselves, for the sake of simplicity, to the case of $l = 0$ and of finite scatterer but the extension of the procedure to cases $l \geq 1$ is immediate, except for a few convenient modifications.

Let us consider the functions

$$\varphi_k(r) = \frac{1}{\pi \sqrt{2}} \sin(kr + \delta) \quad (9a)$$

and

$$\varphi_n(r) = \frac{e^{i\epsilon_n}}{\pi \sqrt{8}} e^{-K_n r} \quad (9b)$$

where $k_n = i K_n$ are the poles of the $S_0(k)$ functions

$$S_0(k) = e^{2i\delta(k)} \quad (10)$$

on the positive imaginary k - axis.

If we impose that these functions satisfy a completeness condition (we should observe that these are exact wave functions outside the scatterer as we assume that the scatterer has a finite radius):

$$\int_0^\infty \varphi_k^*(r') \varphi_k(r) dk + \sum_n \varphi_n^*(r') \varphi_n(r) = \frac{1}{4\pi} \delta(r - r') \quad (11)$$

for $r, r' \geq a$.

This is surely the case if S_0 is derived from a potential interaction. Then, as HEISENBERG (1946) has shown, we have the relation:

$$|C_n|^2 = 2\pi i \operatorname{Res}_{k=iK_n} S(k) > 0. \quad (12)$$

Conversely, if we assume

$$i \operatorname{Res}_{k=iK_n} S = i \operatorname{Res}_{k=iK_n} S(k) > 0, \quad (13)$$

the completeness condition (11) is satisfied if we take

$$|C_n|^2 = 2i\pi \operatorname{Res}_n S.$$

* For higher angular momenta difficulties arise from the zeros of $H_{l+\frac{1}{2}}^{(n)}(kr)$ in the first quadrant of the k -plane. We are thankful to Professor M. Moshinsky who called our attention to an essential mistake in the original form of the present paper.

It seems that such completeness condition should be imposed on the S - matrix. So we impose this restriction in the form of condition (13) for the poles on the positive imaginary axis.

Now let us consider the "completeness function":

$$G(r, r'; t, t') = \int_0^\infty dk \varphi_k^*(r') \varphi_k(r) e^{-iE(t-t')} + \frac{1}{8\pi^2} \sum_n |\epsilon_n^+|^2 e^{-K_n(r+r')} e^{i|E_n^+|(t-t')}, \quad (14)$$

$$G(r, r'; t, t') = \frac{1}{4\pi} \delta(r - r'), \quad (14a)$$

with $|\epsilon_n^+|^2$ given by (12).

Using the properties of the S function, we find:

$$G = G_0 - \frac{1}{8\pi^2} \int_{-\infty}^{+\infty} dk (S - 1) e^{ik(r+r')} e^{-iE(t-t')} + \frac{i}{4\pi} \sum_n \text{Res}_n S \cdot e^{-K_n(r+r')} e^{iE_n^+(t-t')}, \quad (15)$$

where G_0 is the completeness function for free waves ($l = 0$):

$$G_0(r, r'; t, t') = \frac{1}{2\pi^2} \int_0^\infty dk \sin kr' \sin kr e^{-iE(t-t')} dk. \quad (16)$$

Finally one obtains, by changing the contour of integration to the indented path used in the definition of $\sigma(r, r')$ (see expression (6)):

$$G - G_0 = -\frac{m}{8\pi^2} \int_{-\infty}^{+\infty} (S - 1) e^{ik(r+r')} e^{-iE(t-t')} \frac{dE}{k}. \quad (17)$$

Comparing (17) with expression (6) we find that

$$\sigma_0(t - t'; r, r') = \frac{4\pi i}{m} \frac{\partial}{\partial r} (G - G_0). \quad (18)$$

Expression (18) gives the connection between the time dependent scattering function σ_0 and the completeness function G (for $l = 0$). For higher values of l a similar expression is valid only asymptotically.

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On a Dynamical Theory of Scattering

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I. INTRODUCTION

In a recent paper, SCHUTZER & TIOMNO (1951) have shown that a causal description of scattering can lead to restrictions on the S function (the single component of the S matrix) of the problem. In this note we shall introduce a dynamical description of the scattering that will give us, in an explicit form, the causal description of the process. With the help of this dynamical description we shall attempt to obtain restrictions on the S function, when arbitrary angular momenta and relativistic energies are involved.

The causal behaviour of scattering means for SCHUTZER & TIOMNO (1951), the absence of a scattered wave before the incident one reaches the scatterer. In a formulation of single level scattering in Fock space, we (MOSHINSKY, 1951) introduced an alternative criterion of causality in which the time development from the initial state (plane wave) to the final state (plane plus scattered waves), was given explicitly. The time dependent wave function that gave this development (MOSHINSKY, 1951) contained the poles of the S function as parameters. It was possible to show that this wave function gave an appropriate description of the scattering process, only when the poles of the S function were restricted to the lower half or the imaginary axis of the K (wave number) plane. This restriction on the position of the poles is the same as the one obtained from the causal description of SCHUTZER & TIOMNO.

The interactions by means of boundary conditions in Fock space that we introduced (MOSHINSKY, 1951) were restricted to non-relativistic states of zero relative angular momentum. It has been shown by LOZANO & MEDINA (1952), that the time dependent wave function for the system of two particle (in the center of mass frame) interacting through boundary conditions, can be given in terms of the S function of the problem by the expression:

$$\psi_0(r, k, t) = \frac{1}{2} P \int_0^\infty [\delta_-(\epsilon - E) + S_0(\kappa) \delta_+(\epsilon - E)] \\ \exp(-i\epsilon t) [\hat{h}_0^-(\kappa r) + S_0(\kappa) \hat{h}_0^+(\kappa r)]^* d\epsilon, \quad (13)$$

where the index o indicates that we are considering interaction for zero angular momentum, and we have:

$$\delta_{\pm}(\epsilon - E) = \frac{1}{2} [\delta(\epsilon - E) \pm i\pi^{-1}(\epsilon - E)^{-1}], \quad \epsilon = \frac{1}{2}k^2, E = \frac{1}{2}k'^2 \quad (1b)$$

$$h_0^{\pm}(kr) = (\pm ikr)^{-1} \exp(\pm ikr) \quad (1c)$$

In what follows we shall use natural units in which $\hbar = m = c = 1$.

Rather than try to generalize the description of interactions in Fock space to arbitrary angular momentum and relativistic energies, we shall consider the generalization of the time dependent wave function $\psi_0(r, k, t)$ of (1a), which is given explicitly in terms of the S function of the scattering problem. Restricting ourselves to scattering processes in which the angular momentum is a constant of motion, the stationary state representing the plane plus scattered waves can be written as (MORR & MASSEY, 1949):

$$\begin{aligned} \psi_s(r, k, t) = & \sum_{l=0}^{\infty} (2l+1) i^l P_l(\cos \theta) \{ j_l(kr) \\ & - \frac{1}{2} [1 - S_l(k)] h_l^+(kr) \} \exp(-iEt), \end{aligned} \quad (2)$$

where the index s of the wave function indicates its stationary character, $S_l(k)$ is the S function associated with each angular momentum, and (SCHAFFRON, 1941):

$$h_l^{\pm}(\rho) = (\pi/2\rho)^{1/2} H_{l+1/2}^{(2)}(\rho) = (\pm i\rho)^{-1} (\mp i)^l \exp(\pm i\rho) Z_l[(-i\rho)^{-1}], \quad (3a)$$

where Z_l is a real polynomial of order l in $(-i\rho)^{-1}$, of the form:

$$Z_l = \sum_{p=0}^l a_p (-i\rho)^{-p} \text{ where } a_p \text{ real } > 0. \quad (3b)$$

The time dependent wave function $\psi(r, k, t)$ that takes us from the initial plane wave to the final plane plus scattered waves (2) can be written in the form:

$$\psi(r, k, t) = \sum_{l=0}^{\infty} (2l+1) i^l P_l(\cos \theta) \psi_l(r, k, t), \quad (4)$$

where the ψ_l must satisfy the following restrictions:

$$\psi_l(r, k, 0) = j_l(kr) \equiv \frac{1}{2} [h_l^-(kr) + h_l^+(kr)], \quad (5a)$$

$$\lim_{t \rightarrow \infty} [\psi_l(r, k, t)] \rightarrow \frac{1}{2} [h_l^-(kr) + S_l(k) h_l^+(kr)] \exp(-iEt). \quad (5b)$$

Outside the region of interaction $\psi_l(r, k, t) P_l(\cos \theta)$ must satisfy a free particle wave equation, i.e. the Schrödinger time dependent equation for non-relativistic particles $E = k^2/2$ or the Klein-Gordon equation for spinless relativistic particles ($E = (k^2 + 1)^{1/2}$).

We know that (1a) satisfies restrictions (5) and the Schrödinger time dependent equation when $r \neq 0$ (LOZANO & MEDINA, 1952). We propose now

a generalization of (1a) in the form given below, and we shall show in the following sections that it satisfies (5) and the appropriate wave equations. The $\psi_i(r, k, t)$ is now given by:

$$\psi_i(r, k, t) = \frac{1}{2} P \int [\delta_-(\epsilon - E) + S_i(\kappa) \delta_+(\epsilon - E)] \exp(-i\epsilon t) \times [h_i^-(\kappa r) + S_i(\kappa) h_i^+(\kappa r)]^* d\epsilon, \quad (6)$$

where $\delta_{\pm}(\epsilon - E)$ is defined by (1b) and $h_i^{\pm}(\kappa r)$ by (3). The P stands for principal value of the integral (WHITTAKER & WATSON, 1943).

The definition (6) for $\psi_i(r, k, t)$ applies only (as is also [LOZANO & MEDINA, 1952] the case for (1a)) when there are no bound states, i.e. when $S_i(\kappa)$ has no poles on the positive imaginary axis (MØLLER, 1945) of the κ -plane. When there are bound states, we must add to (6) the residues of the integrand at the positive imaginary poles, but as the explicit form of $\psi_i(r, k, t)$ to be obtained below will remain unaffected (MOSHINSKY, 1951) we can restrict our analysis to the relation (6).

The definition (6) of $\psi_i(r, k, t)$ is not as yet complete, as the integration limits with respect to the energy are not indicated. It is at this point that the non-relativistic and relativistic problems differ. In the first $\epsilon = \kappa^2/2$, $E = k^2/2$ and the integration with respect to ϵ takes place from 0 to ∞ . In the relativistic case $\epsilon = (\kappa^2 + 1)^{1/2}$, $E = (k^2 + 1)^{1/2}$ and ϵ is a two valued function of κ . Following NING HU (1948) we now carry out the integration over both sheets of the Riemann surface corresponding to positive and negative energies, and on each of them the absolute value of ϵ goes from 1 to ∞ .

It is important to notice that the only information we have on the scattering process concerns the $S_i(\kappa)$ function and not the Hamiltonian, and that therefore, the time-dependent description of the scattering process that satisfies the conditions (5), is not given in a unique way by (6). Yet the importance of the present developments lies in the fact that for an $S_i(\kappa)$ which is a meromorphic function of κ , the $\psi_i(r, k, t)$ of (6) can be expressed as a sum of basic time dependent wave functions (designated by $\chi_i(r, \kappa, t)$ in the non-relativistic and $\omega(r, \xi, t)$ in the relativistic case) associated with each pole of the $S_i(\kappa)$ function. These basic wave functions have a very fundamental significance in connection with the time-energy uncertainty relation (MOSHINSKY, 1952 a) and the phenomenon of diffraction in time (MOSHINSKY, 1952 c). It is felt therefore, that these basic wave functions would make their appearance in any scattering problem in which the Hamiltonian equation could be integrated, as was the case for the single level scattering (MOSHINSKY, 1951) in Fock space and the scattering by a short range potential (MOSHINSKY, 1952 b). As the restrictions to be obtained for the poles of the $S_i(\kappa)$ function are derived only from the behaviour of these basic wave functions, it is felt that they must have general validity.

In Sections II and III of this note we obtain explicitly the time-dependent wave functions $\psi_i(r, k, t)$ given by (6) for the non-relativistic and relativistic case. In section IV the restrictions on the position of the poles of $S_i(\kappa)$ that result from this dynamical description, are discussed.

II. THE DYNAMICAL DESCRIPTION OF NON-RELATIVISTIC SCATTERING

We shall now derive from (6) the explicit form of $\psi_l(r, k, t)$ in the non-relativistic case, i.e. when $\epsilon = \kappa^2/2$, $E = k^2/2$. We first list some properties of $S_l(\kappa)$ and $h_l^\pm(\rho)$ which will be needed in the following analysis:

$$S_l^*(\kappa) S_l(\kappa) = 1, \quad (7a)$$

$$S_l(-\kappa) S_l(\kappa) = 1, \quad (7b)$$

$$h_l^{\pm*}(\rho) = h_l^\mp(\rho); h_l^\pm(-\rho) = (-1)^l h_l^\mp(\rho). \quad (7c,d)$$

The relations (7a,b) (NING HU, 1948; WIGNER, 1952) are valid for real k and from them we have:

$$S_l^*(-\kappa) = S_l(\kappa). \quad (8)$$

The identities (7c,d) follow from the definition (STRATTON, 1941) of $h_l^\pm(\rho)$.

With the help of (7a) we can write (6) in the form:

$$\begin{aligned} \psi_l(r, k, t) = & \frac{1}{2} P \int_0^\infty [\delta_-(\epsilon - E) + S_l(\kappa) \delta_+(\epsilon - E)] h_l^+(\kappa r) \exp(-i\epsilon t) d\epsilon \\ & + \frac{1}{2} P \int_0^\infty [\delta_+(\epsilon - E) + S_l^*(\kappa) \delta_-(\epsilon - E)] h_l^-(\kappa r) \exp(-i\epsilon t) d\epsilon \quad (9) \end{aligned}$$

We express (9) in terms of the variable κ using $\epsilon = \kappa^2/2$, and in the second integral we make the change of variable $\kappa \rightarrow -\kappa$. Integrating out the ordinary δ function and using (8), we obtain for even l that:

$$\begin{aligned} \psi_l(r, k, t) = & \frac{1}{4} [1 + S_l(k)] h_l^+(kr) \exp(-iEt) \\ & + \frac{1}{4} [1 + S_l(-k)] h_l^+(-kr) \exp(-iEt) \\ & + \frac{1}{2\pi i} P \int_{-\infty}^\infty \frac{1 - S_l(\kappa)}{\kappa^2 - k^2} h_l^+(\kappa r) \exp(-i\epsilon t) \kappa d\kappa. \quad (10a) \end{aligned}$$

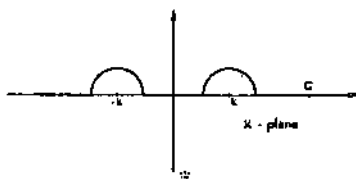


Fig. 1.

Instead of taking principal value of the integral (10), we carry out the integration around the contour C indicated in fig. 1 and we see that:

$$P \int_{-\infty}^\infty - \int_C = \pi i \operatorname{Res}(\kappa = k) + \pi i \operatorname{Res}(\kappa = -k) \quad (10b)$$

From (10a, b) we obtain:

$$\psi_l(r, k, t) = j_l(kr) \exp(-iEt) + (2\pi i)^{-1} \int_C (\kappa^2 - k^2)^{-1} [1 - S_l(\kappa)] h_l^+(kr) \exp(-i\kappa t) \kappa d\kappa. \quad (11)$$

In the absence of poles of $S_l(\kappa)$ in the upper part of the κ plane, the initial conditions are clearly satisfied as at $t = 0$ we may close the contour C from above. Furthermore, as $h_l^+(kr)$ has (STRATTON, 1941) a pole of order $l + 1$ at $\kappa = 0$, we must require that $1 - S_l(\kappa)$ have a zero of at least order l at $\kappa = 0$, for the integral (11) to exist.

We shall now determine (11) explicitly under the assumption that $S_l(\kappa)$ is a meromorphic function which is regular at ∞ , i.e. that $S_l(\kappa)$ is a rational function of κ . When $S_l(\kappa)$ has an essential singularity at ∞ , the solution obtained below still holds (as can be seen by application of the Faltung theorem to Fourier integrals) but instead of a finite sum it becomes an ∞ series. If $S_l(\kappa)$ is a rational function of κ , then $Q_l(\kappa)$ defined by:

$$Q_l(\kappa) \equiv \frac{i}{2} \frac{1 - S_l(\kappa)}{\kappa^2 - k^2} = \sum_{\alpha} A_{\alpha} (\kappa - \kappa_{\alpha})^{-1}, \quad (12)$$

is also a rational function which furthermore, goes to zero when $\kappa \rightarrow \infty$ as $S_l(\kappa)$ is regular at ∞ . If the poles of $S_l(\kappa)$ are simple, $Q_l(\kappa)$ can be decomposed in partial fractions as is also indicated in (12), where $\kappa_1 = k$, $\kappa_2 = -k$, and the rest of the κ_{α} 's are poles of $S_l(\kappa)$. The A_{α} 's are residues of Q_l at these poles.

As $1 - S_l(\kappa)$ must have a zero of at least order l at $\kappa = 0$, we see that $Q_l(\kappa) \kappa^{-p}$ where $0 \leq p \leq l$, is finite at the origin, and that therefore:

$$(-i\kappa r)^{-p} Q_l(\kappa) = \sum_{\alpha} (-i\kappa_{\alpha} r)^{-p} A_{\alpha} (\kappa - \kappa_{\alpha})^{-1} \quad (13)$$

Using (13) and the definition (3) of $h_l^+(\kappa r)$, we obtain:

$$\begin{aligned} \kappa Q_l(\kappa) h_l^+(\kappa r) &= (ir)^{-1} (-i)^l Q_l(\kappa) Z_l[(-i\kappa r)^{-1}] \exp(i\kappa r) \\ &= (-i)^l (ir)^{-1} \left\{ \sum_{\alpha} Z_l[(-i\kappa_{\alpha} r)^{-1}] A_{\alpha} (\kappa - \kappa_{\alpha})^{-1} \right\} \exp(i\kappa r) \end{aligned} \quad (14)$$

With the help of (14) we can write (11) in the form:

$$\psi_l(r, k, t) = j_l(kr) \exp(-iEt) - (-i)^l r^{-1} \sum_{\alpha} \left\{ A_{\alpha} Z_l[(-i\kappa_{\alpha} r)^{-1}] \frac{1}{\pi i} \int_C (\kappa - \kappa_{\alpha})^{-1} \exp\left(\kappa r - \frac{1}{2} \kappa^2 t\right) d\kappa \right\} \quad (15)$$

The integral in (15) has been evaluated before (MOSHINSKY, 1951) and for a κ_{α} that is below the contour C , we have:

$$- (\pi i)^{-1} \int_C (\kappa - \kappa_{\alpha})^{-1} \exp\left(\kappa r - \frac{1}{2} \kappa^2 t\right) d\kappa = \exp\left(\kappa_{\alpha} r - \frac{1}{2} \kappa_{\alpha}^2 t\right) \operatorname{erfc}(y), \quad (16)$$

where:

$$\operatorname{erfc}(y) = 2\pi^{-1/2} \int_y^{\infty} \exp(-y^2) dy, \quad (17a)$$

$$y = \exp(-i\pi/4) (2t)^{-1/2} (r - \kappa_{\alpha} t). \quad (17b)$$

Defining $x_l(r, k, t)$ by:

$$\begin{aligned} x_l(r, \kappa, t) &= \left| i \kappa r h_l^+(\kappa r) \exp \left(-i \frac{1}{2} \kappa^2 t \right) \operatorname{erfc}(y) \right| \\ &= (-i)^l Z_l [(-i \kappa r)^{-1}] \exp i \left(\kappa r - \frac{1}{2} \kappa^2 t \right) \operatorname{erfc}(y), \end{aligned} \quad (18)$$

we have that the $\psi_l(r, k, t)$ of (15) takes the form:

$$\begin{aligned} \psi_l(r, k, t) &= j_l(kr) \exp(-iEt) \\ &+ r^{-1} \sum_{\alpha} A_{\alpha} x_l(r, \kappa_{\alpha}, t). \end{aligned} \quad (19)$$

The wave function (19) gives a dynamical description of non-relativistic scattering for any angular momentum l . For $t = 0$, $\psi_l(r, k, t) = j_l(kr)$ as $x_l(r, \kappa_{\alpha}, t)$ vanishes when $t \rightarrow 0$, because $|y| \rightarrow \infty$ and $\arg y = -(\pi/2)$ so that $\operatorname{erfc}(y) \rightarrow 0$ independently of the value of κ_{α} . It can be easily seen that $\psi_l(r, k, t) P_l(\cos \theta)$ satisfies the time dependent Schrödinger equation for any $r \neq 0$, in case:

$$\sum_{\alpha} A_{\alpha} \kappa_{\alpha}^{-p} = 0, \text{ for } 0 \leq p \leq l. \quad (20)$$

From (13) we realize that (20) holds because $1 - S_l(\kappa)$ has a zero of at least order l at $\kappa = 0$. The only further restriction we must impose on (19) is that at $t \rightarrow \infty$ it shall take asymptotic form (5b). In section IV we shall see that this imposes restrictions on the position of the poles of the $S_l(\kappa)$ function. For $l = 0$ the solution (19) reduces to the solution of the Hamiltonian equation in Fock space for many level scattering (MOSHINSKY, 1951).

III. THE DYNAMICAL DESCRIPTION OF RELATIVISTIC SCATTERING

In this section we shall analyze the dynamical description of scattering of relativistic spinless particle, and for the sake of simplicity, we shall restrict ourselves to the case of zero angular momentum $l = 0$. The generalization to arbitrary angular momentum is straightforward.

For relativistic scattering the S_0 function is no longer a meromorphic function (WIGNER, 1952), of κ , but a function $S_0(\kappa, \epsilon)$ of both κ and $\epsilon = (\kappa^2 + 1)^{1/2}$. In general, it has therefore branch points at $\kappa = \pm i$. We shall demand for $S_0(\kappa, \epsilon)$ that it satisfies the relations (7) for both (NING HU, 1948), positive and negative values of the energy, i.e.:

$$S_0^*(\kappa, \epsilon) S_0(\kappa, \epsilon) = 1 \quad (21a)$$

$$S_0(-\kappa, \epsilon) S_0(\kappa, \epsilon) = 1 \quad (21b)$$

From (6) we can now construct the relativistic $\psi_0(r, k, t)$ taking into account that the integration takes place over both values of the energy, so that we have:

$$\begin{aligned}\psi_0(r, k, t) = & \frac{1}{2} P \int_1^\infty [\delta_-(\epsilon - E) + S_0(\kappa, \epsilon) \delta_+(\epsilon - E)] \exp(-i\epsilon t) \\ & \times [h_0^-(\kappa r) + S_0(\kappa, \epsilon) h_0^+(\kappa r)]^* d\epsilon \\ & - \frac{1}{2} P \int_1^\infty [\delta_-(-\epsilon - E) + S_0(\kappa, -\epsilon) \delta_+(-\epsilon - E)] \exp(i\epsilon t) \\ & \times [h_0^-(\kappa r) + S_0(\kappa, -\epsilon) h_0^+(\kappa r)]^* d\epsilon, \quad (22)\end{aligned}$$

where $\epsilon = +(\kappa^2 + 1)^{1/2}$, $E = (k^2 + 1)^{1/2}$ and the $h_0^\pm(\kappa r)$ are defined as in (1c).

From the definition of the $\delta_\pm$, and taking into account the expansion (DIRAC, 1947) of $\delta(x^2 - a^2)$ and the relation $x \delta(x) = 0$, we obtain:

$$\delta_\pm(\epsilon - E) = (\epsilon + E) \delta_\pm(\epsilon^2 - E^2) \quad (23a)$$

$$\delta_\pm(-\epsilon - E) = \delta_\mp(\epsilon + E) = -(\epsilon - E) \delta_\pm(\epsilon^2 - E^2) \quad (23b)$$

Substituting (23) in (22) and changing from ϵ to κ in the integration, we have:

$$\begin{aligned}\psi_0(r, k, t) = & \frac{1}{2} P \int_0^\infty [\delta_-(\kappa^2 - k^2) + S_0(\kappa, \epsilon) \delta_+(\kappa^2 - k^2)] [1 + (E/\epsilon)] \\ & \times \exp(-i\epsilon t) [h_0^-(\kappa r) + S_0(\kappa, \epsilon) h_0^+(\kappa r)]^* \kappa d\kappa \\ & + \frac{1}{2} P \int_0^\infty [\delta_-(-\kappa^2 - k^2) + S_0(\kappa, -\epsilon) \delta_+(\kappa^2 - k^2)] [1 - (E/\epsilon)] \\ & \times \exp(i\epsilon t) [h_0^-(\kappa r) + S_0(\kappa, -\epsilon) h_0^+(\kappa r)]^* \kappa d\kappa. \quad (24)\end{aligned}$$

The integrands in the two integrals are functions of κ defined in a κ plane in which cuts are made from $\pm i$ to $\pm i\infty$ along the imaginary axis. The two integrals in (24) differ now from those that appear for the non-relativistic problem by the factor $\frac{1}{2} [1 \pm (E/\epsilon)]$ which is invariant to a change $\kappa \rightarrow -\kappa$ as $\epsilon = (\kappa^2 + 1)^{1/2}$. Furthermore, we have from (21) that $S_0^*(\kappa, -\epsilon) = S_0(\kappa, \epsilon)$. Using these facts, we can now follow a procedure very similar to the one of the previous section, to obtain for $\psi_0(r, k, t)$ the form:

$$\begin{aligned}\psi_0(r, k, t) = & j_0(kr) \exp(-iEt) \\ & + (2\pi i)^{-1} \int_C (\kappa^2 - k^2)^{-1} [1 - S_0(\kappa, \epsilon)] h_0^+(\kappa r) \frac{1}{2} [1 + (E/\epsilon)] \exp(-i\epsilon t) \kappa d\kappa \\ & + (2\pi i)^{-1} \int_{-\infty}^\infty (\kappa^2 - k^2)^{-1} [1 - S_0(\kappa, -\epsilon)] h_0^+(\kappa r) \frac{1}{2} [1 - (E/\epsilon)] \exp(i\epsilon t) \kappa d\kappa. \quad (25)\end{aligned}$$

The first two terms of (25) correspond to the first integral of (24), i.e. to positive energies, and we have introduced the contour C of Fig. 1 to avoid using the principal value (6) of the integral. The last integral in (25) corresponds to the negative energy integral in (24), and because of the factor $[1 - (E/\epsilon)]$ there are no poles in the integrand, so there is no need to use principal values or change to the contour C . Also because of the factor $[1 - (E/\epsilon)]$ there is no term of negative energy to correspond to the first term of (25).

The integrals in (25) have branch points at $\kappa = \pm i$. To eliminate them we follow a procedure similar to the one used in the analysis of the phenomenon of diffraction in the time (MOSHINSKY, 1952 c). In the first integral we introduce the change of variables:

$$\xi = \kappa + \epsilon, \quad (26a)$$

so that

$$\kappa = \frac{1}{2} (\xi - \xi^{-1}), \quad \epsilon = \frac{1}{2} (\xi + \xi^{-1}). \quad (26b)$$

The limits of integration $[-\infty, \infty]$ for the κ transform into $[0, \infty]$ for the ξ . Furthermore, as:

$$Im \kappa = Im \xi \frac{1}{2} [1 + |\xi|^{-2}], \quad (27)$$

we see that the signs of the imaginary parts of κ and ξ are equal, so that the contour C which bypasses $\kappa = \pm k$ from above, transforms into a contour C' in the ξ plane that bypasses $\xi = z, z^{-1}$, where $z = k + E$, also from above.

In the second integral of (25) we introduce a ξ corresponding to negative energies $\xi = \kappa - \epsilon$. It is clear that with this ξ the integrand in the second integral becomes the same function of ξ as in the first integrand. Furthermore, for this ξ the limits of integration become $[-\infty, 0]$.

Combining the two integrals in (25), and substituting $h_a^+(kr)$ by its value $(i\kappa r)^{-1} \exp(i\kappa r)$, we obtain:

$$\psi_0(r, k, t) = j_0(kr) \exp(-iEt) - r^{-1} (2\pi i)^{-1} \int_{C'} i \frac{1 - S_0(\xi)}{(\xi - z)(\xi - z^{-1})} \exp \left\{ i \frac{1}{2} \left[\xi(r-t) - \xi^{-1}(r+t) \right] \right\} d\xi, \quad (28)$$

where $S_0(\kappa, \epsilon)$ is now $S_0(\xi)$ a meromorphic function of ξ , and the contour C' goes from $-\infty$ to ∞ in the ξ plane, bypassing z^{-1} , $z = k + E$ from above.

The integrand of (28) has an essential singularity at $\xi = 0$, but the integral converges, and as $r, t > 0$ it can be shown (MOSHINSKY, 1952 c) that the contour can be deformed so as to bypass $\xi = 0$ from above. The contour C' will now go from $-\infty$ to ∞ bypassing $\xi = 0, z^{-1}, z$ from above as shown in fig. 2.

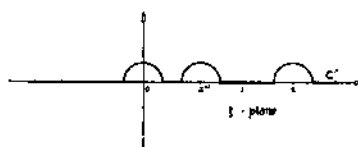


Fig. 2.

To obtain the explicit form of (28) we shall assume that the meromorphic function $S_0(\xi)$ has no essential singularity at ∞ (i.e. it is a rational function), and in case its poles are simple, we can make the expansion:

$$i \frac{1 - S_0(\xi)}{(\xi - z)(\xi - z^{-1})} = \sum_{\beta} \frac{B_{\beta}}{\xi - \xi_{\beta}}, \quad (29)$$

where $\xi_1 = z$, $\xi_2 = z^{-1}$ and the other ξ_β are the poles of $S_0(\xi)$ in the ξ plane. From (28) and (29) we see that $\psi_0(r, k, t)$ takes the form:

$$\psi_0(r, k, t) j_0(kr) \exp(-iEt) + r^{-1} \sum_{\beta} B_{\beta} \omega(r, \xi_{\beta}, t), \quad (30)$$

where the function $\omega(r, \xi_{\beta}, t)$ is given by:

$$\omega(r, \xi_{\beta}, t) = -(2\pi i)^{-1} \int_{C''} (\xi - \xi_{\beta})^{-1} \exp\left\{i \frac{1}{2} \left[\xi(r-t) - \xi^{-1}(r+t) \right]\right\} d\xi. \quad (31)$$

When the poles ξ_{β} are below the contour C'' , we can evaluate (31) following the procedure outlined in the appendix of MOSHINSKY (1952c) and we obtain:

$$\omega(r, \xi_{\beta}, t) = 0, \quad \text{if } t \leq r, \quad (32a)$$

$$\begin{aligned} \omega(r, \xi_{\beta}, t) = \exp\left\{i \frac{1}{2} \left[\xi_{\beta}(r-t) - \xi_{\beta}^{-1}(r+t) \right]\right\} \\ - \sum_{n=1}^{\infty} (\xi/i \xi_{\beta})^n J_n(\eta), \quad \text{if } t > r, \end{aligned} \quad (32b)$$

where:

$$\xi = [(t+r)/(t-r)]^{1/2}, \quad \eta = (t^2 - r^2)^{1/2}, \quad (33)$$

and $J_n(\eta)$ is a Bessel function of order n .

The function $r^{-1} \omega(r, \xi_{\beta}, t)$ given by (32) satisfies the Klein-Gordon equation, as it has been shown (13) that $r^{-1} \xi^n J_n(\eta)$ satisfies the equation for any n , and the exponential in (32) can be written as $\exp i(\kappa_{\beta} r - \epsilon_{\beta} t)$ (where $\kappa_{\beta}, \epsilon_{\beta}$ are given in terms of ξ_{β} by (26b)), so that it also satisfies the equation. Furthermore, it has been shown (MOSHINSKY, 1952c) that when $t \rightarrow r$ from inside the light cone, $\lim_{t \rightarrow r} [\omega(r, \xi_{\beta}, t)] \rightarrow 1$ for any ξ_{β} , so that the summation $\sum_{\beta} B_{\beta} \omega(r, \xi_{\beta}, t) \rightarrow \sum_{\beta} B_{\beta}$, and this last summation is zero, as we can see from (29).

If now $\psi_0(r, k, t)$ is given by (30), in which $\omega(r, \xi_{\beta}, t)$ is defined by (32), we see that not only condition (5a) is satisfied, but no scattered wave reaches the observer before a time $t = r$ (the velocity of light has been taken as 1). From the previous paragraph we also see that $\psi_0(r, k, t)$ would satisfy the Klein-Gordon equation for $r \neq 0$, and that it is continuous across the light cone. The only further restriction we need to impose on $\psi_0(r, k, t)$ is that when $t \rightarrow \infty$, it shall tend to (5b). In the following section we shall see that this imposes restrictions on the poles of the $S_0(\xi)$ function.

IV. THE POLES OF THE S FUNCTION

In the previous sections we have given the explicit time-dependent wave functions for the description of non-relativistic and relativistic scattering, which satisfied the initial conditions (5a) and the appropriate wave equations. The only parameters in these wave functions were poles of the S functions.

We shall now see what restrictions are to be imposed on the poles of the S function, so that the wave functions (19,30) take the asymptotic form

(5b) when $t \rightarrow \infty$. As the scattered part of the wave functions is given in terms of $\chi_l(r, \kappa, t)$ functions in the non-relativistic case, and $\omega(r, \xi, t)$ functions in the relativistic case, we shall investigate their asymptotic behavior when $t \rightarrow \infty$ as function of the complex variables κ, ξ .

From (18) we have that:

$$\chi_l(r, \kappa, t) = (-i)^l Z_l [(-i\kappa r)^{-1}] \exp(i r^2/2t) \exp(y)^2 \operatorname{erfc}(y), \quad (34)$$

where y is given in (17b). The asymptotic behavior of (34) depends on that of $\exp(y)^2 \operatorname{erfc}(y)$ when $|y| \rightarrow \infty$, so that using the results of the appendix 2 of reference 2, we see that for $t \rightarrow \infty$ we obtain:

$$\begin{aligned} \chi_l(r, \kappa, t) &\rightarrow i\kappa r h_l^+(\kappa r) \exp\left(-i\frac{1}{2}\kappa^2 t\right) \text{ if } -\pi/4 \leq \arg \kappa \leq \frac{3\pi}{4} \\ \chi_l(r, \kappa, t) &\rightarrow 0 \quad \text{if } \frac{3\pi}{4} \leq \arg \kappa \leq \frac{7\pi}{4} \end{aligned} \quad (35)$$

Outside the first quadrant in the κ -plane $\chi_l(r, \kappa, t) \rightarrow 0$ when $t \rightarrow \infty$ as $\operatorname{Im} \kappa^2 < 0$. Inside the first quadrant $\operatorname{Im} \kappa^2 > 0$ and $\chi_l \rightarrow \infty$ when $t \rightarrow \infty$. It is clear that no physical solution can be obtained when the poles of the $S_l(\kappa)$ function are inside the first quadrant, i.e. if $\operatorname{Im} \kappa_\alpha > 0$, $\operatorname{Re} \kappa_\alpha > 0$. Because of the symmetry of the poles of the $S_l(\kappa)$ function with respect to the imaginary axis (NINO 1948) implied by the conditions (7), it follows that there can be no poles of $S_l(\kappa)$ on the upper half of the κ -plane outside of the imaginary axis. This condition guarantees that (5b) is satisfied for any r in case there are no bound states (i.e. no poles on the positive imaginary axis), as from (35) χ_l vanishes when $t \rightarrow \infty$ for any poles below the real axis as well as for $\kappa = -k$. The only term in (19) that survives is $\chi_l(r, k, t)$, and as the corresponding residue is $A_l = i(2k)^{-1} [1 - S_l(k)]$ we see from (35) and (19) that $\psi_l(r, k, t)$ takes the asymptotic form (5b). When there are bound states we have $\kappa = i|\kappa|$ and the real exponential in $h_l^+(\kappa r)$ has a negative sign so it vanishes for large r . The condition (5b) is then satisfied for values of r large compared to the radius $|\kappa|^{-1}$ of the bound state. The restriction we obtain for the poles of the scattering function $S_l(\kappa)$ for arbitrary angular momentum, are then the same (SCHÜTZER & TIOMMO, 1953) as those obtained by Schützer and Tiommo i.e. no poles in the upper κ plane outside the imaginary axis.

For the relativistic case discussed in the previous section, we shall carry the analysis in the ξ plane defined by (26a), as $S_0(\kappa, \epsilon)$ is a meromorphic function $S_0(\xi)$ of ξ . We need now the asymptotic properties of $\omega(r, \xi, t)$, and from the discussion given in the appendix of this paper, we see that for $t \rightarrow \infty$ we have:

$$\begin{aligned} \omega(r, \xi, t) &\rightarrow 0, \quad \text{if } |\xi| < 1, \\ \omega(r, \xi, t) &\rightarrow \exp i \left[\frac{1}{2}(\xi - \xi^{-1})r - \frac{1}{2}(\xi + \xi^{-1})t \right], \quad \text{if } |\xi| > 1. \end{aligned} \quad (36)$$

As $\epsilon = \frac{1}{2}(\xi + \xi^{-1})$ we see that $\operatorname{Im} \epsilon = \frac{1}{2} \operatorname{Im} \xi [1 - |\xi|^{-2}]$. If $|\xi| > 1$ and $\operatorname{Im} \xi < 0$, we have $\operatorname{Im} \epsilon < 0$ and for $t \rightarrow \infty$ the ω function vanishes. When

$|\xi| > 1$ and $\text{Im } \xi > 0$ then $\text{Im } \epsilon > 0$, and for $t \rightarrow \infty$ we have $\omega(r, \xi, t) \rightarrow \infty$. It is clear therefore, that $S_0(\xi)$ can have no poles in the region $\text{Im } \xi > 0$, $|\xi| > 1$ above curve C'' of fig. 3. Furthermore, when ξ is real, the corresponding κ is real, and because of the unitary condition (21a), $S_0(\xi)$ cannot have real poles. The poles of $S_0(\xi)$ are then either on the semicircle of radius 1 above the real axis, or at points below the curve C'' excluding the real axis. On the semicircle $\xi = \exp i \gamma$, with $0 \leq \gamma \leq \pi$, so that $\kappa = i \sin \gamma$, $\epsilon = \cos \gamma$, and the poles correspond to bound states of the system. From (36) we see that $\omega(r, \xi, t)$

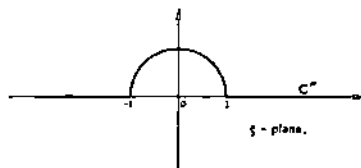


Fig. 3

vanishes when $t \rightarrow \infty$ for all poles below C'' as well as for $\xi = z^{-1}$ (because $z = [k + (k^2 + 1)^{1/2}] > 1$). In the absence of bound states, the only term that survives is $\omega(r, z, t)$, and as the corresponding residue is $B_1 = i(2k)^{-1} [1 - S_0(z)]$, we see from (36) and (30) that $\psi_0(r, k, t)$ takes the asymptotic form (5b) for $t = 0$.

The restriction on the position of the poles of $S_0(\xi)$ that we obtain from the asymptotic condition (5b), is that there are no poles above the curve C'' of Fig. 3, i.e. in the region where $|\xi| > 1$ and $\text{Im } \xi > 0$. From (26) we see that:

$$\text{Im } \kappa = \frac{1}{2} \text{Im } \xi [1 + |\xi|^{-2}] ; \text{Im } \epsilon = \frac{1}{2} \text{Im } \xi [1 - |\xi|^{-2}] , \quad (37)$$

so that the forbidden region in the ξ plane corresponds to points in the κ and ϵ planes whose imaginary parts are positive. Inversely, if $\text{Im } \kappa > 0$, $\text{Im } \epsilon > 0$ we see from (37) that $\text{Im } \xi > 0$ and $|\xi| > 1$.

In the particular case that the relativistic S_0 function is a meromorphic function of κ , i.e. when it does not depend explicitly on ϵ and therefore has no branch points, the restrictions of the previous paragraph can be extended to say that $S_0(\kappa)$ can have no poles in the upper part of the κ -plane, except for poles that correspond to bound states for which $\kappa = i|\kappa|$ and $|\kappa| < 1$. This can be seen from the fact that for each pole κ_β in κ plane there correspond two poles $\xi_\beta = \kappa_\beta + (\kappa_\beta^2 + 1)^{1/2}$ and $-\xi_\beta^{-1}$ in the ξ plane. If $\text{Im } \kappa_\beta > 0$ then from (37) we see that both, the imaginary parts of ξ_β and $-\xi_\beta^{-1}$ are positive, and one of them has absolute magnitude larger than 1, so it will be in the forbidden region. The only exception to this occurs when $|\xi| = 1$, i.e. for bound states.

Finally, it can be easily shown that the restrictions on the position of the poles of the relativistic S functions given in the previous paragraphs, will also hold for angular momentum larger than zero.

APPENDIX

In this appendix we shall derive the asymptotic behavior of the function $\omega(r, \xi, t)$ defined by (32), when $t \rightarrow \infty$. We shall first analyze the asymptotic behavior of ω when $|\xi| < 1$. Using the well known relation (WHITTAKER & WATSON, 1943)

$$\begin{aligned} \exp i \frac{1}{2} [\xi(r-t) - \xi^{-1}(r+t)] &= \exp \frac{1}{2} \eta [(k/i\xi) - (k/i\xi)^{-1}] \\ &= \sum_{n=0}^{\infty} (\xi/i\xi)^n J_n(\eta) + \sum_{n=1}^{\infty} (-1)^n (\xi/i\xi)^{-n} J_n(\eta), \end{aligned} \quad (37)$$

(where ξ, η are given by (33)), and the definition (32), we can write for $t > r$, $\omega(r, \xi, t)$ in the form:

$$\omega(r, \xi, t) = J_0(\eta) - \sum_{n=1}^{\infty} (-1)^n (\xi/i\xi)^{-n} J_n(\eta). \quad (38)$$

When $t \rightarrow \infty$ we see from (33) that $\eta \rightarrow \infty$ and $\xi \rightarrow 1$, and $J_n(\eta)$ takes the asymptotic form:

$$J_n(\eta) \rightarrow \left(\frac{1}{2} \pi \eta\right)^{-1/2} \cos\left[\eta - \frac{1}{4}(2n+1)\pi\right]. \quad (39)$$

Substituting in (38) we see that if $|\xi| < 1$, we can write:

$$\begin{aligned} \left| \omega(r, \xi, t) \right| &\rightarrow \left(\frac{1}{2} \pi \eta\right)^{-1/2} \left| \cos\left(\eta - \frac{1}{2}\pi\right) + \right. \\ &+ \sum_{n=1}^{\infty} (-i\xi)^n \cos\left[\eta - \frac{1}{4}(2n+1)\pi\right] \left| \leq \left(\frac{1}{2} \pi \eta\right)^{-1/2} \sum_{n=0}^{\infty} |\xi|^n. \end{aligned} \quad (40)$$

The last series in (40) converges as $|\xi| < 1$, so if $\eta \rightarrow \infty$, we obtain:

$$\omega(r, \xi, t) \rightarrow 0, \text{ when } |\xi| < 1 \text{ and } t \rightarrow \infty. \quad (41)$$

When $|\xi| > 1$ we consider the form (32b) of $\omega(r, \xi, t)$ and from an analysis similar to the above, we have that:

$$\lim_{t \rightarrow \infty} \left| \sum_{n=1}^{\infty} (\xi/i\xi)^n J_n(\eta) \right| \leq \left(\frac{1}{2} \pi \eta\right)^{-1/2} \sum_{n=1}^{\infty} |\xi|^{-n}. \quad (42)$$

As now $|\xi| > 1$, the series converges again, and we see that:

$$\omega(r, \xi, t) \rightarrow \exp i \frac{1}{2} [\xi(r-t) - \xi^{-1}(r+t)], \text{ when } |\xi| > 1 \text{ and } t \rightarrow \infty \quad (43)$$

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DISCUSSION

MEDINA: I would only ask a question of general principles. In your generalization to the relativistic case, you take a description of the scattering by means of an S formalism which is essentially not covariant.

MOSHIINSKY: I consider the scattering of a particle of relativistic energy by a fixed scatterer, and not the interaction between two particles.

MEDINA: What I want to ask is this: Do you think that you can obtain a reliable description of scattering by using a mechanism which is essentially not covariant?

MOSHIINSKY: Out; can extend the present analysis to interactions between pairs of particles, and put the solutions on a covariant basis. One only needs to replace relative distances and times by the space-time coordinates of the two particles, with the help of the center of mass' momentum four vector. The analysis is somewhat similar to what Wataghin indicated in a previous paper; it is also given in a doctoral dissertation (Princeton, 1949).

WIUNKK: Is the theory that you are presenting restricted in any way to definite types of particles, such as Klein-Gordon, or Dirac particles?

MOSHIINSKY: The particle is assumed to be of the Klein-Gordon type. It is possible, though, to extend the analysis to Dirac particles.

WIGNER: Did you make any assumption about the positive or negative nature of the energy?

MOSHIINSKY: I used the single particle picture with both positive and negative energies available.

WIGNER: Could you say more about the result concerning the analytic nature of S ?

MOSHIINSKY: The restriction of the S functions is that there are no poles in that region of the complex plane where $\text{Im} z > 0$ and $|z| > 1$. The z is defined by the equations $k = (z - z^{-1})/2$, $E = (z + z^{-1})/2$. Hence we obtain that in the forbidden region both $\text{Im} k > 0$, $\text{Im} E > 0$.

WIGNER: You imply that there can be no poles for positive energy. Does this mean that there is a fundamental difference between relativistic and non-relativistic theories? In the non-relativistic case there can be poles in the upper half plane on the imaginary axis.

MOSHIINSKY: The imaginary axis of the k plane would correspond to $|z| = 1$ and it would fall on the boundary of the forbidden region. Poles on the imaginary axis of the k -plane are therefore allowed.

TROMBO: You made the assumption that the S function would depend on k but not on the energy, but do you mean that $S(k) = S(-k)$?

MOSHIINSKY: No, the S function is not the same for k and $-k$. What I meant is that $S(k)$ is a meromorphic function of k and has no branch points. This implies that it could not depend explicitly on the energy $E = (k^2 + 1)^{1/2}$.

ANDERSON: Could you say something about the following question. For the description of what kind of experiments does your theory offer some advantage?

MOSHIINSKY: The theory developed does not concern any specific experimental case. What it tries to do is to find a time dependent wave function, that can be associated with the scattering function of any given problem. As the usual experiments deal with stationary states

the present theory does not concern them. If we could determine the transient scattered current when a scatterer is suddenly introduced in the beam, then we would have an experimental situation that my theory would deal with. The width of the diffraction in time effect is of the order of 10^{-8} sec. for thermal neutrons.

ANDERSON: These days we measure times of the order of 10^{-10} sec. So such things are a practical possibility in experiments.

MOSHINSKY: If times up to the order of 10^{-10} sec. can be measured, the diffraction in time effect could, in principle, be analyzed.

SMITH: YOU can't move a shutter very far in 10^{-8} sec.

RABI: Yes, you certainly can move the shutter in the time 10^{-8} sec. sufficiently far to produce an effect.

SMITH: YOU can form your neutron pulses by means of charged particles?

MOSHINSKY: It is quite possible that neutron pulses formed by the incidence of charged particle pulses, will show a diffraction in time effect similar to the one discussed here.

In reply to a question by ANDERSON, MOSHINSKY said that it is possible to obtain from the transient wave function the transient scattered current. In the non-relativistic case this transient scattered current has peaks at the times $t = r/V_n$ where r is the distance of observer from scatterer, and the velocity V_n corresponds to the energy E_n of the n th resonance level. The width of this transient peaks is of the order of ΔE_n when; T_n is the width of the energy level. In the most favorable case for neutrons, the width would be of the order of 10^{-12} sec, and therefore difficult to observe.

In case the transient scattered current could be detected, it could provide a useful tool in the determination of nuclear structure.

The transient phenomena associated with the scattering of two protons could also be described by the present theory.

MEDINA: In connection with those effects found by Moshinsky, I think that you don't have to shut neutrons, because probably Moshinsky's effects can be observed in cross section and those transients probably could have some effects, for instance, in a very fast nuclear reactor, and that would be a way of checking Moshinsky's theory.

MOSHINSKY: If in any physical phenomenon transient cross sections are needed, then the present theory could provide a useful tool in the determination of these transient effects.

Recent Work at the University of Illinois with Gamma Rays and Electrons of 22 Mev to 320 Mev

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It is a privilege to be here to discuss directly with you some of the recent experiments with high energy X-rays and electrons which we are doing at Illinois. I am glad to have the opportunity to visit your laboratories and to see the instruments with which you are pursuing similar researches. Some of you have seen the X-ray equipment of the University of Illinois; but I believe there have been numerous changes since the last visit, so I will briefly describe the facilities before proceeding to discuss the experiments.

Our laboratory, which was being built in 1940 and 1947, has proved to be a very important aid to the successful pursuit of research. It brought together under one roof numerous people interested in the experimental work and allowed a rapid exchange of ideas and an ease of operation completely impossible in the older surroundings in which some of you saw our instruments. The first illustration (fig. 1) shows you the new Physics Research Laboratory building which houses our betatrons. In the large space at one end of the building the 320 Mev betatron was erected. This instrument began operation in February of 1950.



Fig. 1. New Betatron Laboratory.

At the other end of the building are two betatrons — one is a 22 Mev betatron which has been rebuilt a couple of times and which is very similar to the instrument which you use at Sao Paulo. In the room next to the 22 Mev betatron is an 80 Mev betatron which was built as a model for the 320 Mev betatron. By that I mean it is an exact scale model which actually operates; but because the scaling laws reduce the current carrying capacity of the coils, it is not possible to run this machine more than one pulse

every three seconds. However, it is a good instrument for cloud chamber experiments and it has also been used where sources of high energy electron are needed because it is possible to use the electrons which scatter out of the

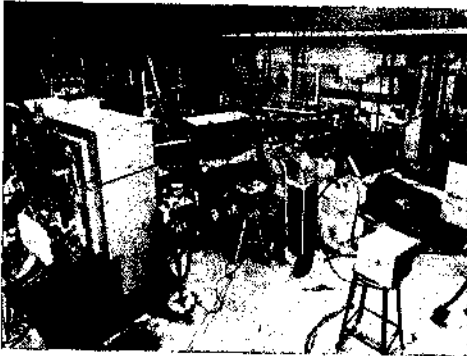
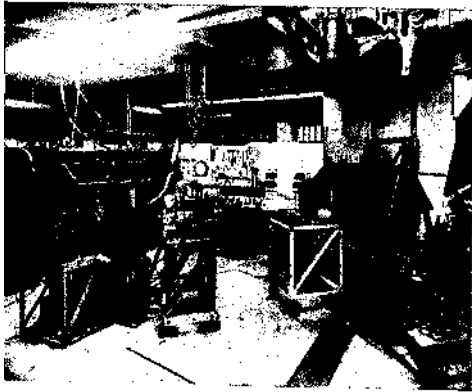


Fig. 2. 22 Mev Betatron with External Electron Beam Experiment.

instrument. In our laboratory building are numerous other facilities such as nuclear emulsion microscopes and the dark rooms for handling nuclear emulsions. There are beta-ray spectrographs. There is an electronic shop, a machine shop, and a glass shop, a library, seminar facilities, and a limited amount of office space. Our stock rooms and supplies are such that we can operate efficiently and quickly. The laboratory is

organized with a certain amount of convenience similar to that which one has in industrial research laboratories and in the larger federal laboratories which have grown up during and since the war. That is, a physicist does not have to occupy

himself with procurement of supplies, with personnel problems of the shop and service people and with numerous maintenance problems connected with the several instruments and of the building. The work we do is not programmatic but has with it the academic liberty of the individual research worker who does the experiments he pleases, in the way he pleases, and who must be a social



creature merely when he employs betatrons or equipment in common with other people. We seem to be perfectly capable of scheduling ourselves in the various botatrons.

The second illustration (fig. 2) shows the 22 Mev betatron which is very similar to the Sao Paulo instrument. In this picture you see apparatus for removing the electron beam to the distance of approximately 10 feet from the Betatron where it enters the scattering chamber, or a chamber for experiments with the electron beam. There is a magnetic focussing lens which pinches the beam

down to a small spot and there are magnetic deflection coils for locating the beam at the proper point inside the experimental apparatus.

The next illustration (fig. 3) shows a view of the 80 Mev betatron. It has approximately the same size as the 22 Mev betatron, but because of unidirectional pulsing and flux biasing the instrument is capable of going to much higher energies. A cloud chamber is also shown in the room with this instrument.

The 320 Mev machine with a liquid deuterium target in its X-ray beam is shown in the next illustration (fig. 4). This instrument gives six pulses of X-rays per second and the energy is easily controllable from a low energy up to 340 Mev. Some of the characteristics of this instrument will be described in another discussion at the conference.

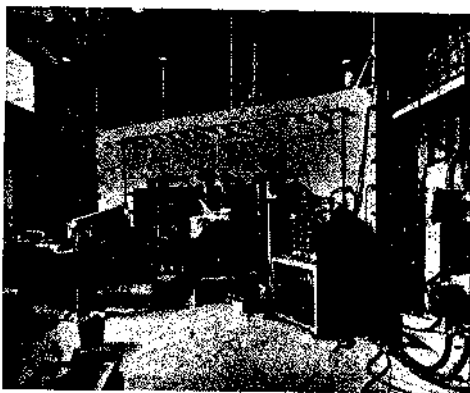


Fig. 4. 340 Mev Betatron with liquid hydrogen target in place.

I. EXPERIMENTAL WORK WITH THE 22 MEV BETATRON

There are several experiments at approximately 20 Mev which have not yet appeared in print. One of these is a recent measurement of the energy

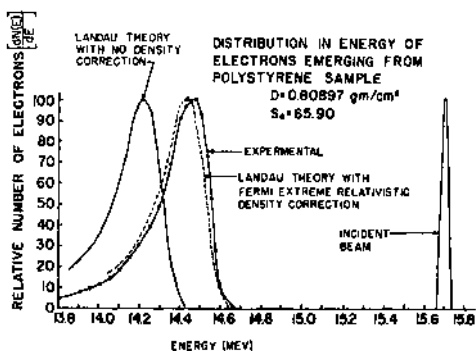


Fig. 5

loss of electrons passing through condensed matter. This work was carried out by Dr. Hanson, Dr. Goldwasser, and Mr. Mills. The electron beam was used for measurements of electron scattering and electron nuclear scattering a year ago, but the recent experiment was a test of the energy loss of electrons passing through condensed material. As you know, the polarization effect on condensed material

decreases the energy loss and it was hoped to test this effect. Actually further measurements are under way this summer by Goldwasser with a sample of condensed material and a sample of the identical material in a gaseous form to determine the polarization effect on energy loss more accurately. The electron

beam passed through thin foils of polystyrene, aluminum or gold and the energy spectrum of the outgoing electrons was analyzed by a magnet. The first slide (fig. 5) shows the experimental result for carbon plus hydrogen — that is the polystyrene foil. The incident beam has an energy shown at the right side of the picture and the experimental curve is shown at the left side. LANDAU'S theory accounting for the straggling of electron energy loss in material is seen displaced from the experimental curve, but when the density correction is applied to the LANDAU curve, the agreement with experiment is extremely good.

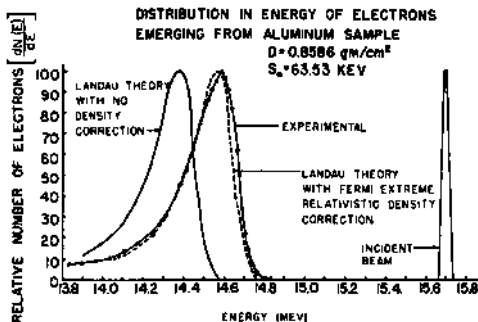


Fig. 6

ionization loss. The radiation straggling distorts the shape of LANDAU'S curve quite a bit, but the correction has been made by YANG and KENNEDY, of the Institute for Advanced Study, to take account of the radiation loss including K and L excitation of gold atoms during the passage of the electrons. The shape of the curve determined by YANG and KENNEDY agrees with experimental curve but the density correction is not sufficient to bring about agreement with experiment at present. The reason for this is unknown.

Some time ago Price and I made studies of the yield of photoneutrons from 51 elements bombarded by X-rays. As a preliminary to this study, we examined the angular distribution of the intensity of photoneutrons for lead, iron, beryllium, and heavy

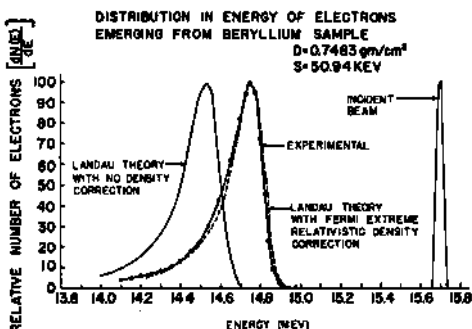


Fig. 7

water. We observed the expected angular asymmetry from deuterium and from beryllium, but iron and lead gave isotropic emission of photo neutrons. Apparently we accidentally chose the wrong heavy elements because many other elements do show angular asymmetries, and this has been the subject of

The next figures (figs. 6 & 7), show the same results for an aluminum and a beryllium foil. Again the LANDAU straggling theory plus the FERMI polarization correction gives excellent agreement with experiment. However, the case of gold (fig. 8) does not result in agreement. Here the 15.7 Mev electron beam suffers a great amount of radiation straggling in addition to

recent work by G. A. PRICE. Of numerous detectors which Price used for the determination of the angular distribution of neutrons of different energies, there are two which give quite interesting results. One is Si^{28} which undergoes an(NT) reaction giving 22.4 minute Al^{28} . The threshold for this reaction is 2.69 Mev neutron energy. This silicon detector is most sensitive to neutrons of more than 5.5 Mev. The result is that with this threshold detector PRICE was able to measure the high energy component of neutrons emitted in the photo-disintegration process. Most of these emitted neutrons have an energy of about 1 Mev. These are the neutrons that are boiled out of a nucleus, but the silicon detects only the high energy group. Fig. 9 shows the angular distribution of fast photo-neutrons from bismuth samples put in the X-ray beam and from copper and beryllium samples put in the x-ray beam. It is evident that the bismuth shows a very strong angular asymmetry.

Experiments with an aluminum detector gave greater accuracy. Aluminum has a threshold of 1.95 Mev and Al^{27} gives Mg^{27} which has a 10.2 minute half-life. Angular distributions resulting from experiments with this threshold detector are shown in the next illustration. It is evident that bismuth, lead, and gold all show strong angular asymmetries while the lighter elements show much less asymmetry. The effect is large in the case of bismuth at 18 Mev as well at 22 Mev.

The next illustrations (figs. 10 & 11) show how the angular asymmetry varies as the function of atomic number. According to WEISSKOPF and EWING, the neutrons which boil out of a nucleus have an energy distribution which contains a relatively larger proportion of high energy neutrons in the case of the light elements than in the case of heavy elements. Therefore, these high energy directional photoneutrons are masked by high energy isotropically emitted neutrons evaporated from the light elements. PRICE used other detectors which catch all the neutrons and degrade their energies so they can be detected in rhodium foils and he also made experiments with an epithermal detector which has some interesting properties.

II. EXPERIMENTS WITH THE 80 MEV BETATRON

An experiment which had partial description in the literature was performed by CURTIS with the 80 Mev betatron for the determination of

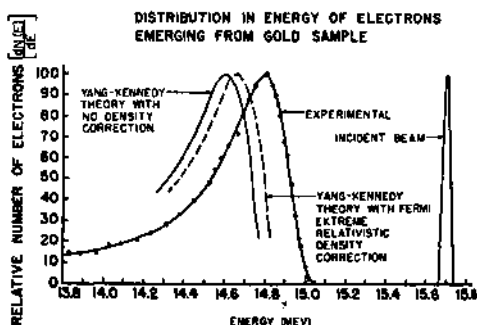


Fig. 8

the absolute cross-section for Bremsstrahlung production by electrons and for the determination of the shape of the Bremstrahlung spectrum. CURTIS used a 00 Mev beam of electrons from the 80 Mev machine. These electrons come out of the machine naturally by scattering of the target. They travel once around the betatron and emerge

from one of the ports. The intensity was sufficient so that CURTIS could put a cloud chamber in this beam and observe radiation straggling of the electrons as they passed through thin foils of lead. By counting the number of transits and the number of radiation straggling the absolute cross-section could be easily determined. Normally this is a very difficult physical measurement, but it comes out simply with this experiment. The illustration (fig. 12) shows the cross section as a function of energy, that is the spectral shape, which CURTIS obtained. This spectral shape shows agreement with theory but it has a magnitude which was about 7% less than the theoretical production. I will have more to say about this anomalously low cross-section after mentioning a few more experiments.

HI. 300 MEV BETATRON

An experiment similar to Curtis's has been carried out by MR. FISHER on the 320 Mev betatron. However, MR. FISHER did not use electrons scattered out of the betatron, but rather he converted the X-ray beam into beams of electrons and positrons. Electron-positron pairs were made by a small lead radiator placed

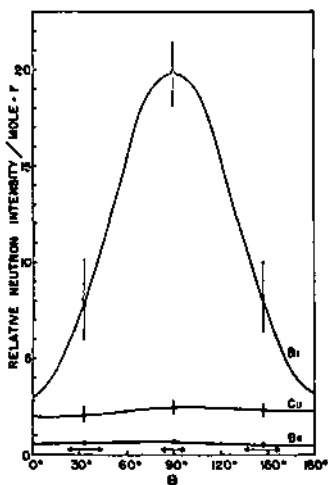


Fig. 9

in a magnetic field, and exposed to X-rays. Out of this magnetic field emerged electrons toward one side of the room and positrons toward the other side of the room. The energy of the pair member was determined by the position in the room where one placed the cloud chamber. By reversing the polarity of the magnet positrons or electrons could be directed to the cloud chamber. In the cloud chamber FISHER had thin foils of gold. Radiation straggling of electrons occurred in these foils. It is very easy to detect radiation straggling particularly if a high energy photon has been emitted because the curvature of the track changes very abruptly crossing the foil. Briefly one could say that FISHER, who is now analyzing his cloud chamber data, found that the positron Bremsstrahlung spectrum has exactly the same shape as the electron Bremsstrahlung spectrum; however,

FISHER'S determination is not yet complete and we do not know if the cross-section is anomalously low as CURTIS found.

Another experiment with the 320 Mev betatron which is about to be published is the determination of the relative cross-section of various elements for pair production in the photon energy range between 50 and 300 Mev's. This experiment was carried out by EMIGH. He used a cloud chamber and foils of two different atomic numbers were placed in the same photon beam. They were spaced so that the photons first went through one foil and subsequently through the other foil behind it in the cloud chamber. The relative pair production from

the two foils could be precisely determined. Assuming aluminum has the correct pair production cross-section, EMIGH found for silver and for lead anomalously lower cross-sections for pair production. The illustration (fig. 13), shows EMIGH'S result together with the result of several other observers demonstrating the discrepancy between the theory which appears in HEITLER'S book *Quantum Theory of Radiation* and the experiment. All experiments uniformly agree on the discrepancy. However, EMIGH'S experiment was on pure pair production. The other experiments have all been on absorption which includes COMPTON absorption as well as pair production, and they require the corrections for the COMPTON effect. The by-products of EMIGH'S experiment was the energy sharing distribution between the pair members. The next illustration (fig. 14) shows how this appears for aluminum, silver and gold. The solid curves are theoretical. The COMPTON event, namely electrons when unaccompanied by positrons, all appear at one end of this spectrum and must be subtracted off. The assumption is made that the number of unmated positions is equal to the number of unmated electrons at the other end of the diagram. Hence, the excess must be COMPTON'S. If this assumption is followed through, the result is shown in the next illustration (fig. 15), namely, the spectrum of COMPTON electrons. The COMPTON electron spectrum shown in this illustration agrees very closely with theory.

I would like to return to the discrepancy mentioned earlier between the bremsstrahlung cross-section and theory and the discrepancy between pair production cross-section and theory. These two differences are of the same nature since the calculations are very similar for each of the processes. The discrepancies usually attributed to the BORN approximation not holding for heavy elements. Certainly heavy elements are outside the range of validity of the BORN approximation, and it is surprising that the BORN approximation has given results so close to the experimental result. One might think that it is only when a very low energy electron or a very low energy positron leaves the vicinity of the target nucleus in the case of pair production or when a very high energy photon and a very low energy electron leave the target nucleus in the case of Bremsstrahlung that the BORN approximation would fail. However, the cross-section as you have seen from the Bremsstrahlung curve is depressed throughout the whole upper end of the spectrum. Consequently, it is not wholly an effect present when one of the out-going charged particles has an extremely low velocity which is usually one of the troublesome limits for the BORN approximation. Likewise, the low pair production section means that even when high energy pair members are created the cross-section must be anomalously low to have a gross effect on the absorption

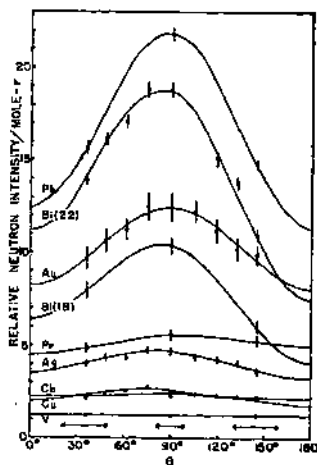


Fig. 10

coefficients. It has taken a few years to get the theoretical work for this done satisfactorily and recently MAXMON, DAVIES and BETHE have been able to correct the BORN approximation and to account for the low experimental pair and Bremsstrahlung cross-sections observed in these experiments. Their result is now in press.

Some of the findings of the radio chemists, PROF. DUFFIELD, DR. HSIAO, and MR. GINDLER have brought to light new activities. Vanadium 53

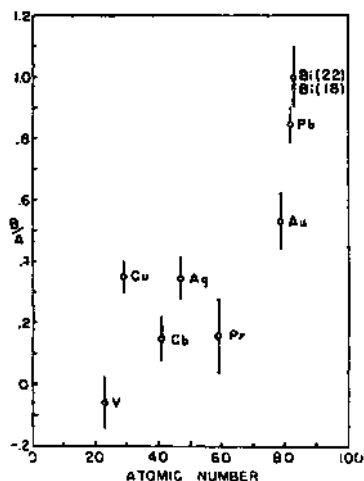


Fig. 11. B/A for Yield = $A + B \sin^2 \theta$ as a function of atomic number.

and GINDLER for these experiments. They have observed photo fission on both bismuth and gold by use of a photomultiplier tube with a zinc sulphide screen on it placed out of the X-ray beam. A thin foil of the element to undergo photo fission is placed in the beam and the fission fragments cross the vacuum to the zinc sulphide screen. The zinc sulphide screen being very thin does not respond brilliantly to electrons but gives a large flash for fission fragments. These experiments are now being extended in the 320 Mev betatron.

A continually interesting-problem is the photodisintegration of the deuteron. Work has been done by FULLER and HANSON using nuclear emulsions with the 22 Mev betatron, and HANSON and others are extending this work to 320 Mev. There are indications, as you know,

It has a 21 hour half-life and emits a negative electron of .65 Mev. One of the interesting things the radio chemists have been exploring is multiple particle emission from high energy photon bombardment. They feel quite certain now that in numerous nuclear events one or several alpha particles are emitted. One can be sure of this because the threshold for photodisintegration involving alpha particle emission is generally lower threshold for photodisintegration involving all the separate particles omitted.

With the high energy available in the 300 Mev. betatron it has been possible to extend some of the photo fissions studies to heavy elements such as gold and bismuth. I would like to describe a system used by DUFFIELD

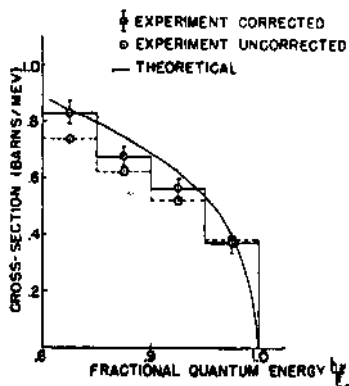


Fig. 12

from some counter experiments that the cross section for photodisintegration of the deuteron rises as one exceeds the meson threshold with the photon energy. However, there is great need of better statistics and better detailed information which we hope will result from exposures which HANSON has just been making at the University of Illinois a couple of weeks ago.

The illustration of the 320 Mev betatron shown earlier contained the liquid deuterium target placed in the beam. Around the liquid deuterium at various angles are placed the nuclear emulsions. The energies of the photo protons produced in the nuclear disintegrations are determined by grain count. We have achieved a rather clean collimation of the X-ray beam and since the betatron has only six open portals from which radiation can issue it was merely necessary to plug up three of these portholes on the side of the machine where the detection apparatus is placed to suppress the room background to a point where the electrons can not bother the emulsions, since electron sensitive emulsions have been used.

An auxiliary experiment which is being carried out by KOESTER, PENFOLD, SMITH and KEUST is one which requires the highest intensity we can get from the betatron because we detect the neutrons emitted from the photodisintegration of deuterium. This experiment is being run on the 300 Mev betatron right now. The emitted neutrons pass through a thick lead shield and produce recoil protons in a long paraffin converter. These protons then go through a proton telescope which selects an energy greater than about 60 Mev. While we find it difficult to obtain a lot of data with this type of apparatus because of the low counting rate, it should give an integral sort of a picture including the smearing effect of a Bremsstrahlung spectrum. The prediction of the results was given by

LEVINGER some time ago in the Physical Review. We have exercised this equipment with carbon targets a few months ago before we had liquid deuterium. The results were described at the Washington meeting of the American Physical Society and I have here an illustration (fig. 16) showing the apparatus with the thick walled lead house which can be moved around to different angles and the proton telescope within the thick walled lead house. The angular distribution which resulted from carbon is shown on the next illustration (fig. 17). The peak at 45° is very definite and the counting rate falls off at smaller angles. LEVINGER made predictions on the angular distribution which would result from a continuous X-ray spectrum directed at carbon and our experimental result differs from these predictions made by LEVINGER. It appears that the angular distribution of the protons and the neutrons from carbon bombarded by 320 Mve

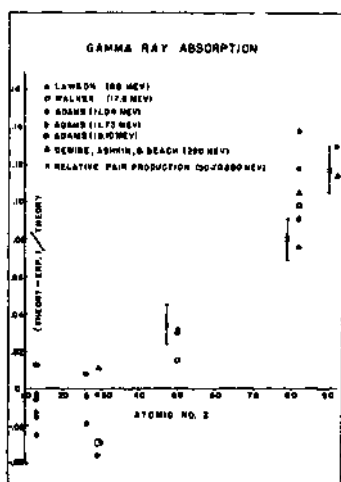


Fig. 13

X-rays is about the same. While the existence of pseudo-deuterons within the carbon nucleus would, according to LEVINGER, give neutrons emitted preferentially at a larger angle than the protons. Since very high energy neutrons of the order of 100 Mev are emitted, they must be accompanied by protons of a similar energy. There is a pseudo-deuteron unit within the carbon nucleus. At high photon energies disintegrating pseudo-deuterons the charged particles go in the direction of the photon and the neutral particle would go slightly backward. If the angular distributions of proton and neutron are identical one is tempted to imagine a rapid exchange of charge and identity between the particles within the nucleus.

An interesting experiment has been carried out by BERNAKDINI, REITZ and SEGRE on the fission of bismuth as a function of photon energy. Bismuth loaded Type D Ilford Plates were exposed directly on the photon beam at 100 Mev,

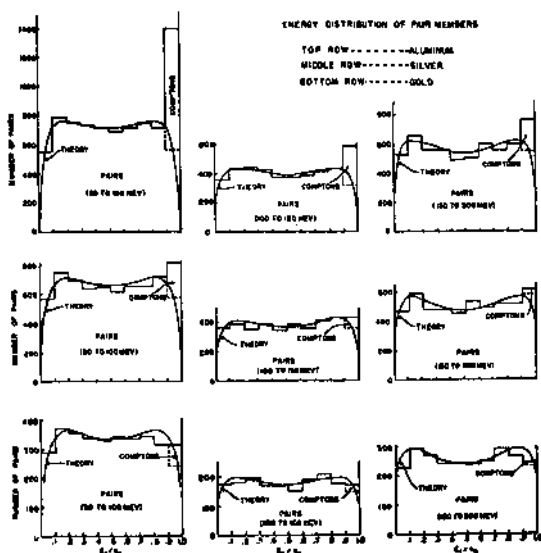


Fig. 14

150, 200, 250, 290, and 320 Mev. Then the plates were carefully processed so that the bismuth fissions were evident and electron backgrounds were small. About 1000 bismuth fissions were produced per plate and the emulsions were exposed to a photon flux sufficient to make 2000 R in an ionization chamber such as the Victoreen X-ray meter placed behind 1/8" of lead. The emulsions were quite clear and free of electron background but the fissions were easy to observe. The relative yields of fissions normalized to the same number of R at various energies were 1, 4, 10, 14, 20, and 28 at each of the above energies. The total number of 500 fissions have so far been recorded on the plate. Estimates of the cross-section from the data so far indicate a rapid rise occurring at the meson threshold, and the suggestion is that mesons are produced and reabsorbed within the bismuth nucleus leading to fission. Apparently about 50% of the meson production is reabsorbed resulting in fission.

Bismuth is an especially suitable element for this experiment because the threshold for photofission of bismuth is rather high. It is something like 50 Mev and since photon absorption generally is most strong around 15 Mev the usual photo-absorption would not provide sufficiently great cross-section at 50 Mev region to lead to a lot of photofission of bismuth by the usual process. Consequently, most of these processes are due to meson effects. Four times $10''$ sq. centimeters is the cross-section of the photofission of bismuth per equivalent photon at 320 Mev, and particle emission from carbon does not show the high energy mesonic effect — it seems to occur mainly with low energy photons.

Another experiment for which BERNARDINI and GOLDWASSER have made exposures is the angular distribution and yield of positive π mesons near the threshold for their production on a liquid hydrogen target similar to the target that HANSON used for liquid deuterium. The deuterium was exchanged for hydrogen and the meson measurements were made.

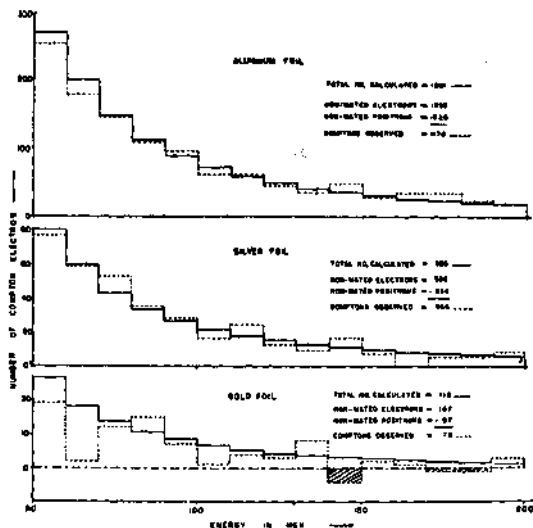


Fig. 15

Another experiment which has been going on for some time is that of BERNARDINI, REITZ, and SEGRE on the possible production of V particles by photons on aluminum. The choice of aluminum was made from considerations of photon absorption and a number of nucleons in the nucleus. I will sketch on the blackboard the experimental arrangement for observing the possible V particles which might be emitted from the aluminum. The data to date show that for every meson produced in the aluminum there are not more than 2 times 10^4 V particles. There are only two doubtful cases. One looks like a real V particle except that the origin of the V is so close to the glass that it may be merely an inelastic scattering of the negative π meson coming from the glass. The outgoing proton has 30 Mev's and the outgoing meson has 4 Mev's, which is just about the correct energy for the V particle disintegration.

Another one of the several counting experiments which are going on using the 320 Mev betatron is the experiment of DR. ROBINSON and MR. LEISS. Their problem is to determine the production of positive π mesons from hydrogen at various angles and at various photon energies. There are several methods which they have been exploring. One of these is to make use of the very narrow burst of X-rays from the betatron. It is possible to make the beam about .0 of a microsecond long and subsequently to detect the positrons from the μ decay resulting from π decay. There are some difficulties with this method because although the X-ray background may be practically gone there are capture gamma-rays from neutrons which are travelling around in the room long after the X-ray burst has produced them. The counting equipment that ROBINSON and LEISS

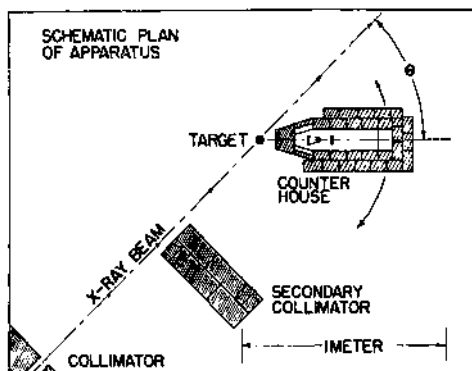


Fig. 16. Apparatus for angular distribution of 100 Mev neutrons from carbon irradiated by 300 Mev gamma rays.

have been using has been undergoing development for a couple of years and with it numerous tests on electrons and positron backgrounds have been made and focusing magnets have been tested for collecting mesons. Counters used by ROBINSON and LEISS have resulting powers of about 2 or 3×10^{-9} seconds while those used by KOESTER, SMITH, KEKST and PENFOLD for the neutrons from the photodisintegration of

deuterium have a resulting power of 10^{-8} seconds. In the latter case coincidence and pulse height discrimination is achieved by displaying the pulses on our oscilloscope and masking the face of the oscilloscope so that if large enough heights are produced, the spot shows from behind the mask it can be counted photoelectrically. The photo multiplier counter for gold and bismuth fission which was mentioned earlier along the work of a much slower counter DUFFIELD and GINDLER is because it used a zinc sulphide screen. However, it is perfectly possible to eliminate electron pile-up during the low resolving

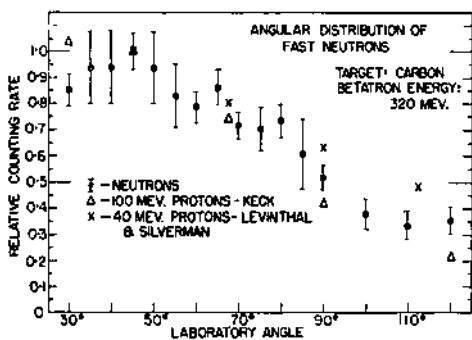


Fig. 17

time of this device so that fission fragments can be counted easily. More will be said in another lecture of this conference about some of the pulse length problems which are important in the use of counting techniques for the detection of nuclear events with our 300 Mev betatron and our 22 Mev betatron. In all the measurements that have been made with the 300 Mev betatron or with the 22 Mev betatron calorimetric determinations of the photon energy flux have been the basis of the photon flux determinations. The calorimetric method was first used with our 300 Mev betatron since the output was rather large. We have something like 50 to 100 $\times 10^{10}$ equivalent photons per minute in our whole X-ray beam. The final determinations have been made by P. D. EDWARDS, lie used a lead cylinder 8 centimeters long and a little more than 4 centimeters in diameter. The betatron was run from 150 Mev to 300 Mev. The temperature rise on the block which occurred on a 5 - minute run was measured by thermistors and bridge circuit. The corrections for leakage of energy from the lead block are a total of 10%. These corrections were determined by a second calibrated monitor which detected the magnitude of the leakage. A similar determination was made with the 22 Mev betatron by DR. LAUGHLIN at the University of Illinois Medical School in Chicago. His determination has been published, although he is refining it, and will have a determination at lower energies and with more accuracy. For use for the 300 Mev betatron we have a flat ionization chamber through which the whole collimated beam can pass. The reading on this ionization chamber in *esu* per centimeter of air path is the measure of the photon flux in *ergs* and is a function of the peak of Bremsstrahlung spectrum energy.

DISCUSSION

ANDERSON: What was the result of the measurement which gave the number of γ rays coming from the large; Betatron? How many do you get?

KKUST: 3 watts. This is about 100 $\times 10^{10}$ equivalent photons per minute in the whole beam.

ANDERSON: If I caught correctly the numbers on the slides which you showed concerning the energy loss of electrons passing through foils, the loss was about 1 Mev per gm per cm^2 , I don't know whether that is correct but, anyway, that is lower than the minimum ionization loss for which one gets normally 1.7 Mev. per gm per cm^2 , in aluminium.

KERST: For water it is around 2.1 though for things that were heavy like lead it was down a little.

ANDERSON: Anyway, there is no reason to believe the experiment shows a bad anomaly in the ionization loss.

KKRST: The only thing that seems to be puzzling is the case of gold where the correction that you apply for polarization is not enough. I notice when looking at the slides that when we apply the polarization correction a theoretical line moves all the way to an experimental line here but in the case of gold it moves only a little way. I think the gold foil of Hanson and Goldwasser must have had a much thinner foil.

ANDERSON: Is then; some reason which could be explained in a simple way about the 10% correction that Bethe found? And what was the physical basis for the correction? Why was the previous theory off?

KERST: The explanations I have heard usually had to do with statements like this: If a pair is formed near its positively charged nucleus the positrons are repelled away and the electron whips around close to the nucleus and this existence of a slow negative electron close to the nucleus for a long time makes more interaction and hence in real experience the cross section is large but one ignores the effect when one makes the calculations. This is one of these

arguments which works only at the end of the pair energy sharing distribution because it would take a slow electron coming out to experience such a distortion at the orbit of the nucleus and I showed the sharing curve just to demonstrate that the whole cross section is suppressed and it is not just the ends that are chopped off.

ANDERSON: I would like to ask a question which refers to the experiment on the V particles. How in the experiment does one eliminate the possibility that the neutron makes the collision with one of the protons of the emulsion producing recoil-proton and a meson that looks like a V . This is always the crucial point in such experiments.

KERST: Yes, that is right, and Bernardini has looked into this.

ANDERSON: And he is one of those who criticize everybody else on just this point.

KERST: Bernardini has estimated how bad this would be in this particular case and he says it is a negligible effect.

ANDERSON: But in the case which is considered doubtful, but nevertheless is shown, there is no way of indicating that a neutron is not involved,

KERST: The disturbing thing about this one case is that it looks so genuine, the proton and the meson have about the right energy and the meson makes a star just as the negative it would.

ANDERSON: Wouldn't it be hard to find a neutron that would do the same thing?

KERST: I think it would, except, Mr. Bernardini would say that the probability of that is less than 2×10^{-4} .

L. MARSHALL: Can you say a little more about the possible interpretation of the break in Bernardini's fission cross section above the threshold for pion production?

KERST: Bernardini found the cross section for fission at 320 Mev is about $4 \times 10^{-27} \text{ cm}^2$, and he feels that about 50% there are just about as many mesons that would be produced by these photons in bismuth as there are fissions resulting. Consequently, he is obliged to say that if you get hold of a bismuth nucleus by way of mesons and shake it, half of the time a meson comes out and half time a meson is reabsorbed and makes fission. He would like to say the cross section is composed of two components, namely something which comes up like this with $E^{1/2}$ for π charged production, and a component which goes up more rapidly for π_0 production, something like E^2 . He would like to be able to determine whether or not this is really virtual mesons effect.

A. BERMAN: I noted that the Curtis' experimental Bremsstrahlung cross section was 7% below the theory; that was the old theory I imagine?

KERST: Yes, that's right.

A. BERMAN: HOW does the new Bremsstrahlung theory compare with the experimental cross section?

KERST: The 10% that Bethe estimated for lead at 50 to 300 Mev. is a figure which applies also to the Bremsstrahlung cross section as well as the pair production cross section.

A. BERMAN: The Curtis value of 7% is then very close to this 10%,?

KERST: Yes, I don't believe one can get any more out of Curtis, experiment than to say that it appears that the cross section is lower than the 1935 calculations by 7% plus or minus 3 or 4%. It's no more accurate than that.

A. BERMAN: In connection with that theory of Maximon, Davies and Bethe, I believe it was calculated on the assumption that the energy of the photon is still much greater than the rest-energy of the electron despite the fact that the theory does improve on the Born approximation. At 20 Mev they do get for lead a correction of 20%, instead of 10%; as you pointed out but the experiment still shows a correction of about 10%, and this might be due to the fact that at 20 Mev the new theory does fail.

KERST: Yes, Bethe was not at all sure about his calculations below 20 Mev and apparently even at 20 it does not agree with the experiment. Your work lands right in among the others, as I understand it.

A. BERMAN: Yes. The correction seems to be about 10% for the entire high energy region including 20 Mev.

KERST: There were some data shown at about 13 Mev and the only high energy work was some total absorption work done at Cornell and also the relative pair cross section function Z between 50 and 300 Mev, done at Illinois.

Photonuclear Reactions in the Intermediate Energy Region

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energy gamma radiation emitted by the reaction, ${}^7\text{Li}(p,n){}^7\text{Be}^{\dagger}$, is higher than the high photonuclear reactions in several substances.

Several known activities, which could be obtained by a $(n,2n)$ reaction were observed and a number of new radioactive isotopes were discovered (Zn^{63} , Ga^{68} , Br^{80} , Mo (17 min.) Ag^{108} , In (1.0 min.) Sn^{120} Te (60 min.)). The nuclear isomerism in Br^{80} was clearly demonstrated and for some radioactive bodies of known half-lives and chemical properties, mass numbers could be assigned.

Little however, can be concluded from experiments with gamma-rays of a given energy since it was not known how the cross-section for the processes investigated depend on the gamma ray energy; the cross-sections for photonuclear reactions cannot be sorted out by means of such experiments since it was shown by DELSASSO, FOWLER & LAURITSEN (1937) that the ${}^8\text{Be}$ gamma radiation is a mixture of 17.5 and 14.5 Mev gamma-rays.

Photonuclear reactions were also produced by Bothe and Gentner in several substances by means of the gamma radiation emitted in the reaction

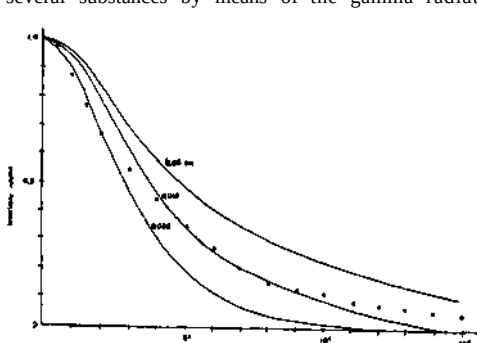


Fig. 1

${}^8\text{B}^{11}(p,\gamma){}^8\text{C}^{12}$, ${}^8\text{C}^{12*}$; in these experiments the gamma radiation was shown to consist of a main line at 11.8 Mev and of a second line of about 1/7 the intensity at 10.0 Mev (FOWLER, GAERTNER & LAURITSEN, 1938).

As generators of monochromatic gamma rays with a variable energy are not available at present time, the behaviour of a photonuclear process as a

function of the gamma ray energy can be studied only with the continuous Bremsstrahlung radiation obtained when electrons, accelerated either by a Betatron or by a Synchrotron, strike a target.

Excitation functions for a given reaction can be measured if the yield of one of the products of the disintegration (such as protons, neutrons etc. or induced activities) is determined experimentally as a function of the electron beam energy and as a function of the X-ray energy flux. The cross-sections, as a function of the energy, can be computed if the absolute yield and the shape of the Bremsstrahlung energy spectrum are known; thresholds can be measured accurately since it is known that for most reactions it is found that the excitation function can be represented by a parabola in an energy interval of a few Mev around the threshold.

The precise measurements of photonuclear yields are subject to various experimental difficulties; some of these difficulties will be discussed in this paper.

In order that the energy spectrum in the forward direction may be calculated with accuracy, it is necessary that the electron beam (responsible for the emission of the Bremsstrahlung radiation) be a monokinetic one before it strikes the target.

Although the data on Synchrotrons are rather scarce, (here are several measurements that show that the spread in energy of a Betatron electron beam is small: the external electron beam of the University of Illinois Betatron (22 Mev) has a half width of about 15 Kev, whereas at the University of Saskatchewan the beam is homogeneous within 30 Kev, allowing the measurements of excitation levels of several light nuclei.

If a monokinetic electron beam of electrons strikes a target, it will give rise to a well defined energy spectrum of the Bremsstrahlung radiation provided that the following conditions are fulfilled:

- a) the target must be thin, or otherwise multiple radiative processes may take place; on the other hand, a thick target gives rise to a large ionization energy loss which spreads the energy distribution of the original electron beam.
- b) the thin target must be positioned in the radially decreasing magnetic field in such a position that a scattered electron, which has suffered a high energy loss by ionization or radiation, will not be captured again by the field and made to strike the target.

By means of a suitable location of the target and by providing a fast expansion pulse, it is possible to avoid multiple traversals of the target.

As those conditions are not usually fulfilled by the standard target utilized in Betatrons, and since it is usual to employ a wire as a synchrotron target, one should not expect the shape of the Bremsstrahlung spectrum to be known accurately enough, in these machines, for a computation of cross-sections.

The targets most commonly used are thick and since, due to the finite cross-section of the electron beam and to the shape of the standard targets used, the beam traverses regions of different

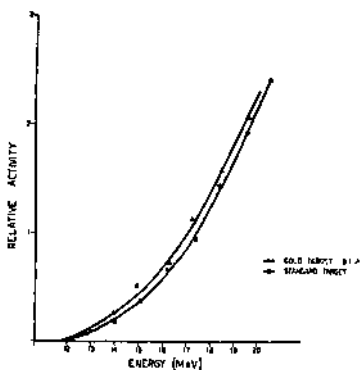


Fig. 2

thicknesses, one should expect the measured excitation functions to be dependent both on the particular target employed and on the geometrical arrangement used (GOLDEMBERG, SANTOS & SILVA, 1951); experiments carried out in our laboratory have shown that this is the case (GOLDEMBERG, SANTOS & SILVA, 1953) and that special precautions must be taken if one wants to measure excitation functions precisely enough to calculate the corresponding cross-sections.

It is well known that the angular distribution of the Bremsstrahlung radiation emitted by a parallel beam of fast electrons in a moderately thick target is much broader than the intrinsic half-width for a single collision (which is of the order of mc^2/E) since the beam is spread out by multiple scattering.

SCHIFF (1940) has calculated the angular distribution of the X radiation under the hypothesis that the target is thin enough so that excessive straggling of the electrons does not take place. Assuming that the electrons undergo a single traversal of the target (of thickness x) and assuming the multiple scattering as described by Williams' formula, the ratio of the intensity at an angle θ (somewhat larger than mc^2/E) to the intensity in the forward direction is given by

$$\frac{I_{\theta}}{I_0} = \frac{E^2 \left(\frac{\theta^2}{2\beta x} \right)}{2\beta x E^2/m^2 c^4 - 0.5772} \quad (1)$$

The representative curves of this function, for three different target thicknesses are illustrated in fig. 1; the experimental points, obtained with the radiation from a standard Betatron target at 20 Mev are represented in the same figure. It can be observed that the actual distribution does not obey eq. (1); it was shown (GOLDEMBERG, SANTOS & SILVA, 1951) that the behaviour of the experimental curve is due to the finite cross-section of the electron beam which, during the expansion process, traverses different thicknesses of the target (as it is known that the above equation gives a reasonable approximation of the actual distribution as measured with a target of a uniform thickness.)

A confirmation of the fact that the spectral composition of the radiation is dependent on the particular target employed in the experiments (GOLDEMBERG,

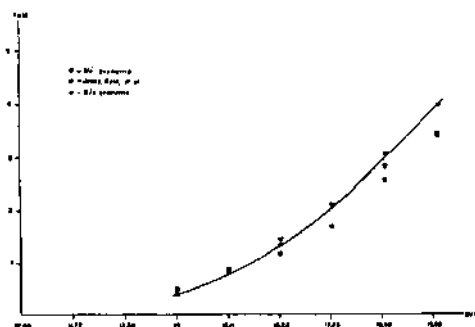


Fig. 3

SANTOS & SILVA, 1953) was obtained by a comparison of the excitation functions obtained for the $\text{Cu}^{62}(\gamma, n)$ reaction with a standard target and with a gold target 0.1 radiation length thick (of uniform thickness) under the same geometrical condition (fig. 2). The observed shape of the excitation functions differ because the high energy tail of the Bremsstrahlung spectrum

is sensitive to the target thickness, a thick target giving rise to a richer yield of soft radiation.

The geometric arrangement plays also an important part in the shape of the measured excitation functions, as one should expect from the considerations briefly outlined above on the angular distribution of the Bremsstrahlung radiation. In fig. 3 are illustrated some excitation functions obtained in our laboratory (GOLDEMBERG, SANTOS & SILVA (1951) for the $Cu^{63}(\gamma, n) Cu^{62}$ radiation, for three different geometrical arrangements about the forward direction.

The geometric arrangements used correspond to: a) the arrangement used by JOHNS *et al.* (1950); b) our own standard arrangement with copper plates subtending an angle of 15 minutes about the forward direction and c) a bad geometry arrangement for which the subtended angle by the copper plates is of 5°.

One can observe that as the angle subtended by the target decreases, the saturated specific activities increase for a given energy, the rate of increase depending on the electron beam energy.

The effects illustrated by those measurements are not due only to the decrease of the radiation intensity with the angle (as one should expect from the angular distribution), but they are also due to the angular dependence of the spectral composition of the radiation; such an effect was observed in our laboratory (GOLDEMBERG, SANTOS & SILVA, 1951) both for the standard and for a gold target of uniform thickness. Those experiments have shown that the relative amount of soft radiation present in the spectrum [which, in this case should be interpreted as the radiation lying below the $Cu^{63}(\gamma, n)$ threshold], increases with the angle about the forward direction, (fig. 4)

JOHNS *et al.* (1950) have investigated the influence of the calculated cross-section shapes upon the spectral composition of the Bremsstrahlung radiation; they have shown that in the case of the $Cu^{63}(\gamma, n) Cu^{62}$ reaction, if one assumes a distribution $(E^{-1} - E_0^{-1})$ with zero photons at the maximum energy, the maximum of the "resonance" is displaced by about 1 Mev towards low energies, as compared with the values computed under a Schiff's distribution, whereas the peak cross-section changes by a factor smaller than two, the former distribution giving rise to a higher value of the cross-section at the resonance peak.

It seems, therefore, that the actual knowledge on photonuclear reactions indicates only the general features of the cross-sections and that a rigorous comparison with the proposed theories (GOLDHABER & TELLER, 1948; LEVINGER & BETHE, 1950) for the photonuclear reactions seems very difficult to be made with the existing experimental data.

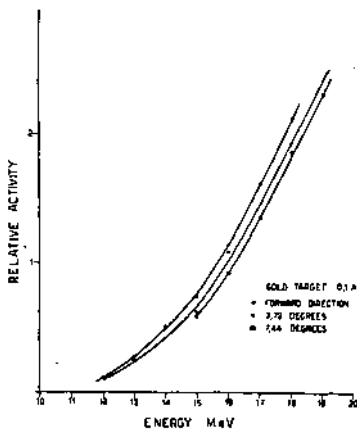


Fig. 4

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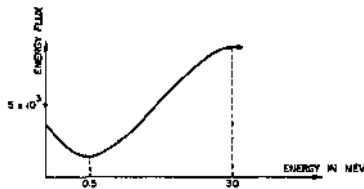
DISCUSSION

ANDERSON: Referring to the hist point, would it not be possible that the reason that you get a higher response to the thick target is that the monitor which you use to measure; the energy fluxes has higher response for the thick target than that of the detector which you use?

SANTOS: We have; used the same monitor for both the standard and the gold target; we have made something like 15 different runs on each cross section so we know the response; with a precision which is better than 1% for both targets. As far as the standard target is concerned up to energies of the order of 22 Mev our excitation functions in the forward direction hit all the data that have been published so far.

ANDERSON: The point is that the sensitivity of the monitor might be energy dependent, it might depend on the shape of the spectrum and since; here you are inve)lve)l in a difference; in the energy spectrum, one has to know quite; well how the; monitor response varies with that spectrum as compared to the eiector response.

SANTOS: The same monitor was used in both cases, and if the spectra were the same we should expect the same response, The spectrum is different and we think that this has to do with soft energy rays because the response of the; Vietoreen insiele an infinite lucite block looks like this:



This is the energy flux required to procedure one roentgen at those energies, therefore one sees that one roentgen at a low energy corresponds to a much smaller energy flux than at higher energies. Now up in this excitation function we have activity divideel by roentgen and one has a smaller ratio for the stanelarel target; than can be infe;preteel as due to a richer soft component in the spectrum.

KERST: There is one experiment that I can mention which checks the calculated sensitivity of a ionization chamber. As has been pointed out by Prof. Santos one has to calculate from the kind of photons what the total ionization will be in the ionization chamber. Laughlin has maele a calorimetric calibration of an ionization chamber and we know that Almy and Diven's calculation of the response of an ionization chamber gives 4,800 ergs per R per cm* agreeing within 10% with calorimetric check.

SANTOS: I might add also that this difference in behavior was observed in our laboratory using two different elements, which had a different photo-neutron threshold, and measuring the ratio for both the standard and the gold target and it turns out that this ratio depends on the target and actually it increases in favor of the element that has a higher threshold: this means that the standard target gives more soft radiation than the gold target. This is a measurement which is quite independent of the ionization.

KERST: I wonder if you think it is worth while to try experiments with thinner targets? It seems to me that a tenth in radiation might still be rather large. A tenth of a radiation length means essentially that the electron loses about a tenth of its energy by radiation going through it, so the loss in your 20 Mev. beam will be of the order of 2 Mev. So on the average you just have very few double radiation events.

SANTOS: We did not want to use a much thinner target because then we might have multiple transversals and that would complicate the spectrum still more.

The Angular Distribution of the Bremsstrahlung Radiation Emitted by a Betatron

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I. INTRODUÇION

In this paper a report is presented of some measurements of the angular distribution of the Bremsstrahlung radiation emitted by a Betatron at small angles about the direction of the incident electron beam.

Although the way in which the electron beam hit the target during the expansion process is very complicated, and a detailed comparison with the theory is difficult due to the change of the spectral composition with the angle (GOLDEMBERG ET AL., 1953) a few interesting results were obtained which throw some light on the nature of the phenomena involved.

Measurements of the angular distribution emitted by Betatrons and Synchrotrons have been reported by other workers (ALLEN-WILLIAMS & ARPLEYARD, 1919a & b; LEES & METCALFE, 1950; LAWSON, 1950; LAXZL & HANSON, 1951) in the same energy range that our experiments were carried out.

Aside from the intrinsic interest of those measurements the knowledge of the angular distribution is of paramount importance for activation experiments made with a Betatron: those measurements are made in the forward direction where the energy spectrum of the radiation is assumed to obey Schiff's spectral distribution (SCHIFF, 1946 & 1951; KATZ & CAMERON, 1951); the knowledge of the angular distribution has been used by several investigators to evaluate the effective target thickness from the decrease of the total intensity as measured at a small angle about the direction of the incident electron beam (SCHIFF, 1946; KOCH & CARTER, 1950).

As the excitation functions for photonuclear processes usually measured with the Bremsstrahlung radiation originated in an internal target of a conventional shape, and since the spectral composition of this radiation is assumed to obey Schiff's formula for a thin target radiation, it is interesting to compare the angular distribution of the radiation emitted by the internal target of a Betatron with that produced by an external electron beam. In this way it is possible to check whether multiple traversals of the target do occur and, therefore, to decide whether Schiff's formula gives a good approximation for the energy distribution from a conventional target. We think that this point is an important one if one wants to attach any meaning at all to cross section calculations.

II. EXPERIMENTAL ARRANGEMENT

As a source of electrons, use was made of the Betatron of the Universidade de Sao Paulo; the measurements reported in this paper were carried out with the radiation originated inside the donut as the electron beam was allowed to hit a target (placed on the injector structure, at a radius of 21,8 cm from the center of the orbit), by means of an expanding circuit.

The experiments were carried out with

- a) A conventional platinum target
- b) A gold target, 0.1 radiation length thick
- c) The injector structure used as a target

As the Betatron was provided with a device by means of which the expansion current could be changed at will, no difficulty was found in adjusting the machine so that the electron beam would hit the targets used.

X-ray pinhole pictures were taken in order to assure that the radiation originated from a single point source.

The measurements of the angular distribution were realized by two different experimental set-ups. As the intensity changes rapidly with the angle about the direction of the incoming electron beam, it is of importance that the detectors subtend an angle as small as possible; otherwise the measured intensity will represent an average value. In our experiments the reaction ${}^{63}\text{Cu}(\gamma, n){}^{62}\text{Cu}$ was used as a detector; the angle subtended by the detectors was of 15'.

It is important to realize that, under the above conditions, the angular distribution measured refers only to a given band of the Bremsstrahlung spectrum, viz. between the threshold for the reaction which was assumed 10.9 Mev (BALDWIN & KOCH, 1945; MCELHINNEY ET AL., 1919), and the maximum energy available from the electron beam.

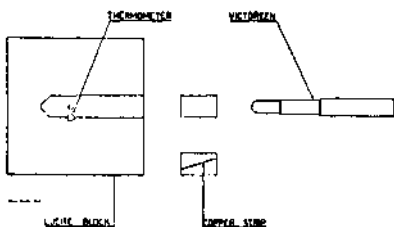


Fig. 1 — Experimental arrangement (1)

In the first experimental arrangement used in our experiments a lucite block which held in its center both the copper plate and a 100 Ii thimble ionization chamber was placed at various angles about the forward direction; this arrangement was used in our measurements of the angular distribution with the conventional target (fig. 1).

In the second arrangement (fig. 2) copper plates of the same thickness and size were submitted to a single irradiation; those plates were placed inside a lucite holder; paraffin and cadmium foils were used in order to eliminate the contribution to the measured activity due to the neutron background of the machine.

This arrangement was afterwards used in all our experiments since by its use relative activities could be measured accurately and small errors from the ionization measurements were eliminated.

It is noteworthy that, as the spectral composition of the Bremsstrahlung radiation changes with the angle about the forward direction (GOLDEMBERG ET AL., 1953), the measurements made with the conventional arrangement are meaningless until a precise knowledge of the spectral energy dependence with angle is available.

This objection applies, of course, to measurements of angular distribution where ionization chambers or photographic films are employed; In these cases, it is very difficult to know what the experiments really mean.

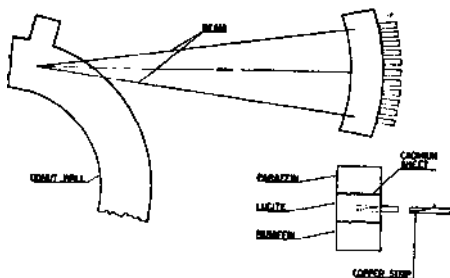


Fig. 2 — Experimental arrangement (II)

in. EXPERIMENTS WITH THE CONVENTIONAL TARGET

A pumped donut provided with a conventional platinum target was used in our betatron, operated at energies of 16,24, 18,47 and 20,4 Mev; the angular distributions were measured with the experimental arrangement described as a § 2 both in the vertical and in the horizontal planes.

The angular distributions in the horizontal plane are represented in fig. 3, where Schiff's theoretical curves for different target thicknesses are plotted (SCHIFF, 1940).

A comparison between our results and Schiff's theoretical curves show if the conventional target behaves as correct, that the radiation emitted by the conventional target behaves as a radiation coming from targets whose thicknesses are a function of the angle about the forward direction.

It can be verified that the obtained results are inconsistent with the hypothesis that this variable thickness might be due to the penetration of the electrons at different depths of the target, due to multiple scattering, before they radiate. Indeed, due to the peculiar shape of the conventional target, one would expect, by assuming this hypothesis, a marked asymmetry in the distribution about the direction of the incoming electron beam; as we have observed a rigorous symmetric distribution, the multiple scattering cannot be responsible for the observed phenomena.

An explanation of the observed distribution might be found in the oscillations about the equilibrium orbit that the electron beam is submitted to during the fast rise of the expansion pulse: radial oscillations do occur and when the beam strikes the target it behaves as though it had a larger average cross-section than the true cross-section at the equilibrium orbit; this cross-section depends, of course, on the target radius but is well known that oscillation phenomena do occur during the expansion at the normal radius of the target of conventional betatrons.

As the conventional target has a curvature radius of about 2 mm in the horizontal plane, those oscillations, which are responsible for an effective increase

of the Brensstrahlung cross-section, may be responsible for the phenomena observed. It is important to point out that this behaviour is to be expected even when the expanding circuit is adjusted for an outward spiraling of the orbit with a pitch small as compared to its assumed cross-section.

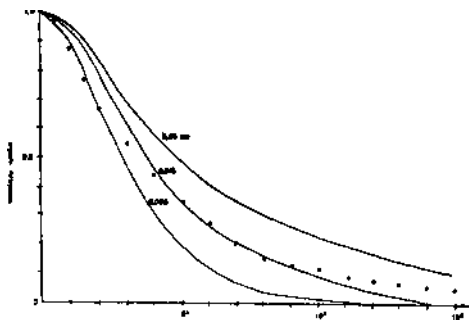


Fig. 3

It is known, on the other hand, that a small pitch of the expanding orbit may give rise to multiple traversals of the target (LAWSON, 1950) in this condition, although a point source radiation is secured, the Brensstrahlung spectrum is enriched by soft and medium energy quanta and any measurement of activities for cross-section computation loses its

meaning, unless the spectrum is measured under the experimental conditions.

Therefore, it seems advisable to use as fast an expansion as possible, although this would give rise to a Brensstrahlung spectrum (from a conventional target) due to a superposition effect of targets of different thicknesses.

This inconvenience may be ruled out by substituting the conventional target by a target of uniform thickness.

The analysis of the experimental points plotted in fig. 3 show also that a knowledge of the angular distribution is not a sufficient criterion for the determination of the effective target thickness unless the target is of a uniform thickness and sufficiently thin (to avoid multiple radiation) and unless the expanding device is such that multiple traversals do not occur in an appreciable amount; the effective target thickness, as measured by this method, is a function of the angle and the estimated values may be wrong by a factor of about 3 for an angle of 5° .

IV. EXPERIMENTS WITH A GOLD TARGET

In order to check the contribution of the gamma rays originating from the injector structure, an angular distribution was taken without a flag (fig. 4). It is seen that, as the half-width for the experimental points is smaller than that corresponding to the standard target, the effective thickness of the injector structure (Mb) is smaller than that of the conventional target.

In order to get a beam originated in a target of uniform thickness, a gold target, 0.1 radiation length was mounted on the injector; the target was highly polished and had sharp edges were highly polished too. A special care was taken in mounting the target on the injector so that its plane was at 90° with the orbit; this care was necessary in order that during the spiralling out of the electron beam, the electrons would have to go through a definite thickness of the target (0.1λ).

This target was a large flag, 5 mm high and 6 mm outside the injector structure; its height was of course larger than necessary, but it was chosen in

order to clear, by Bremsstrahlung, any electrons that might have a large amplitude of vertical oscillation so as to avoid their hitting the injector structure.

This thickness of the target was considered as a good compromise between a low energy loss by ionization and a fairly high probability for radiation; also the probability for multiple radiation is small (see appendix) and the "gain" approaches its saturating value (LAWSON 1950).

The yields obtained with this target were greater than those obtained with a conventional target; besides, as its thickness was uniform, it was observed that, at a given energy, the number of high energy quanta per roentgen as defined, in our experiments, by those having an energy above the threshold for $Cu^3(\gamma, n)Cu^{62}$ was larger than those produced by the conventional target.

KOCH & CARTER (1950) have measured the energy distribution of Bremsstrahlung by determining in a cloud chamber, the energies of positron-electron pairs obtained from 19,5 electrons produced in a platinum target 0,005 inch thick; their experimental results were compared with the theoretically predicted spectrum in the forward direction, for a thin target, by Schiff, using the Bethe-Heitler theory. In the energy range between 13 and 20 Mev, the theoretical intensity was found to be smaller by 8% than the observed value.

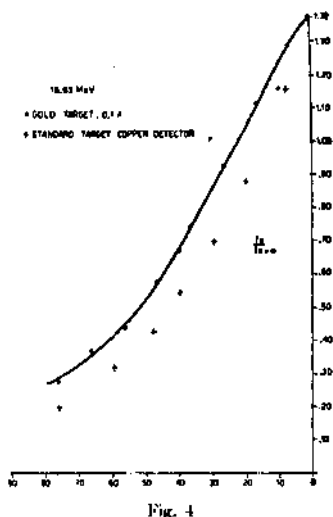
In the computation of photo-nuclear cross-section by the photon difference method (KATZ & CAMERON, 1951), Schiff's distribution of energy is employed in order to calculate the number of photons, in different energy intervals, corresponding to a given maximum energy (that is, the electron energy); the constant C is chosen as 191, instead of the proposed value $C = 111$ for a thin target, as being a good compromise between the thin target and the total spectrum.

It seems, for the reasons already discussed, that a gold target such as was used in our experiments, is a better approximation than the conventional target.

Experimental angular distribution with the gold target.

As we have pointed out, one of the important consequences of an analysis of the angular distribution of the Bremsstrahlung radiation omitted by the internal target of a betatron is that one can compare the experimental results with the data obtained by LANZL & HANSON (1951) who have measured, for targets of various substances and thicknesses, the angular distribution of the Bremsstrahlung radiation obtained by means of an external beam of monokinetic electrons.

By this comparison one can get information on whether multiple passages do occur during the expansion process, and one can decide then whether experiments



on activation and cross-section measurements have a physical meaning when the method of calculation assumed is that of KATZ & CAMERON, (1951).

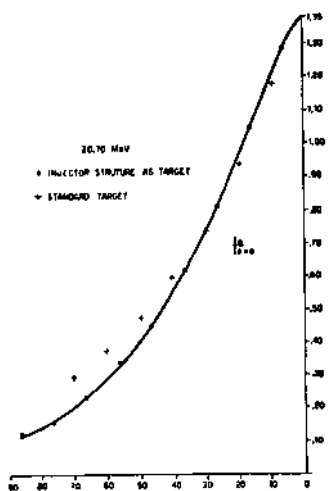


Fig. 5

curve obtained by these authors; one can observe as well that for angles of about $6-8^\circ$ our points (597 mg/cm^2) coincide with theirs as obtained with 247 mg/cm^2 , whereas for smaller angles their points are closer to their theoretical curve and quite apart from our own; this behavior is due to the fact that the relative amount at soft radiation increases with the angle and copper is a high energy detector.

The disagreement between their results and the theory, as well as the behaviour of our points and theirs, when compared with each other, might be explained on the basis of their use of an ion chamber to measure the intensity of the radiation.

An ionisation chamber is very sensitive to the quality of the radiation that hits it: roughly speaking, the energy flux — ergs/cm^2 — required to produce one "roentgen", as measured in an air cavity in a lucite block increases almost linearly with the maximum energy in the energy from 2 up to 20 Mev; it is, therefore, more sensitive for soft than to hard radiation.

We have shown (GOLDEMBERG *et al.*, 1953) that the spectral composition of the Bremsstrahlung radiation changes with the angle about the forward direction and that this change in composition is due to the enrichment of the soft component of the spectrum at the expense of the hard rays.

We think, therefore, that, if a detector sensitive to hard radiation were used, a better agreement with the theory would be found.

On the other hand, the present theories should be modified in order to take into account the angular change of the spectral composition.

In order to check this point and to decide on the questions already discussed in our measurements with the standard target, measurements were made with a gold target 0.1 radiation length thick, using the same arrangement of the previous experiment (fig. 5).

Measurements were made at energies of 16.93 Mev and 20.7 Mev , in order to compare with the data of LANZL & HANSON (1951) and with our own as obtained with the standard target.

The results of our measurements are represented in fig. 6 where the results of LANZL & HANSON are represented as well as the theoretical curves from their equation 0.

One can observe that for gold thicknesses of the order that we have employed, there is no agreement between the experimental points and the theoretical

If there were multiple passages in the internal target of our betatron, one should expect that our points should lie close to the theoretical points at **small** angles and quite wide at large angles, since in the theory of the angular distribution, the spectrum of energy is supposed to remain unchanged.

One can observe from the behaviour of the curves of fig. 6 that just the opposite behaviour is found; although it is rather unfortunate that the experimental points measured with our method cannot be compared with either an experimental measurement based on the use of an ion chamber, or with a theory that does not take into account the variation of the spectral composition with the angle, one can conclude that multiple passages are not likely to take place in our betatron.

One can infer also that the observed points do not lie on a curve of the family described by LANZL & HANSON'S equation

6; it was also calculated that Lawson's distribution does not fit our data.

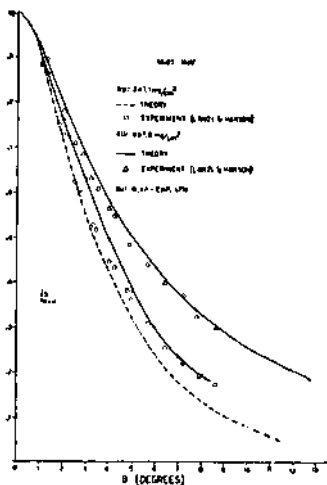


Fig. 6

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Factors Influencing Capture of Electrons Injected into a Betatron

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In addition to discussing factors influencing capture of electrons injected into a betatron, I should like to take this opportunity to discuss numerous other matters pertaining to the characteristics of betatrons and the lore which goes with the performance of the betatron. In addition I would like to describe some of the features of the new 340 Mev betatron which has been in operation since February of 1950 at the University of Illinois. The illustrations (figs. 1, 2, & 3) show that it is composed of a ring of six magnets placed around the central leg of a large transformer. Since the 80 Mev model was an exact scale model of this instrument, the material published in the Review of Scientific Instruments, Vol. 21, pages 402-180, 1950 describes accurately the structure, the scale factor used to enlarge the instrument to the size of the 340 Mev betatron is 4.03 for linear dimensions. This makes the large machine weigh something like 350 tons. The field magnets alone which look very much like a synchrotron weigh about 65 tons. Gathering the radially directed laminations together in six bundles give six crotches from which radiation can be taken, and through which the vacuum can be pumped, and the injector installed and etc. The machine itself is shielded largely by the 1/4 inch iron laminations in the field magnet, however, we do add a little concrete and lead in the crotches facing the detecting apparatus to cut down electron spray of X-rays which do not come out along the axis of the main beam.

Some important characteristics of a machine like this are its yield of X-rays, its maximum energy and how easily it is adjusted, its duty cycle, that is, what fraction of the time is the X-ray pulse on, and how reliable and free from maintenance is the instrument. These data for our 300 Mev machine are that the output is about 3 watts of Bremsstrahlung at 320 Mev in the whole beam. The energy density at the center of the X-ray beam is about 0.13 watts per square centimeter at one meter's distance from the target. This amounts to about 10,000 r per minute at a meter when the r are measured in a Victoreen thimble placed behind an eighth inch sheet of lead. Since we have only six pulses per second the energy in each burst is large—for example, in every pulse there are about 10-20 r at one meter. The machine can be rather easily run down and up in energy as all betatrons can by adjusting the time at which the beam is removed from the betatron. The most energetic runs we have made with it are at 343 Mev. An important feature is the pulse length or the duty cycle. Our natural

pulse length is only about 50 microseconds and since we only get 6 pulses every second we have only 300 microseconds of radiation every second. This is the duty cycle of about 1 in one thousand. This is not very favorable, but it is possible to count several nuclear events in 50 microsecond intervals provided with this instrument. Some counting experiments were rather easy to do with this duty cycle and even with full intensity from the machine. However, if one has an experiment in which he is sensitive to electron background and for which he cannot employ a great deal of shielding then it is very desirable to stretch the pulse out long in time so that electron pile up will not occur in the sensitive time of the detector. For certain purposes it is not desirable to lengthen the pulse because this would destroy the energy resolution of the instrument. The synchrotrons have a nice way of stretching their pulse mainly by slumping the plate voltage of the RF oscillator so that the particles undergoing phase oscillation successively come into the target. In our case since we have an outside target this means that we would have to expand the orbit very gently so that it gradually

is shaved off against the target.

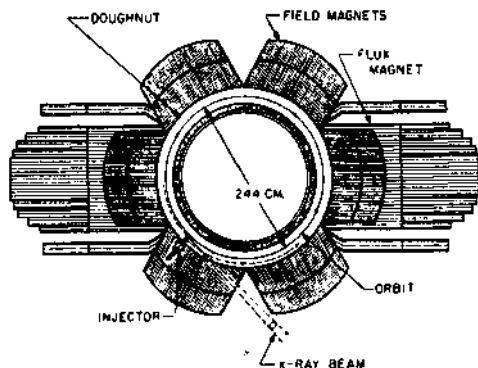


Fig. 1. 340 Mev Betatron

It has been possible with the 22 Mev betatron to lengthen the duration of the X-ray pulse from about 0.2 of a microsecond up to 75 microseconds merely by very slow orbit expansion. This is an important technique for the 20 Mev betatron because it allows counting of charged particles during the X-ray pulse very nicely. The duty cycle with the 22

Mev is then 1.3%. The same trick has been tried with the 300 Mev betatron with some success. It has been easy to lengthen the pulse out to 300 microseconds, and we are attempting to lengthen it still more. The technique being tried to lengthen it still more is to shape the expansion pulse so that the beam very gradually moves out toward the target. To make this simpler the beam is scrambled, so to speak, in the following way: A weak radio frequency bump is put on the magnetic field by small coils outside of the vacuum tube underneath and above the orbit. While the orbit is expanding in radius toward the target, it passes the point where n equals $3/4$. At this point the radial oscillation executes half full oscillation for every revolution of the electron. If the RF magnetic field has a frequency equal of half the rotational frequency of the electron then some electrons will pass over the radiofrequency bump when the amplitude of the radiofrequency bump is zero. These electrons will travel undisturbed in a circle. However, the electrons which pass over the magnetic bump when its strength is at maximum will be shaken in a resonance manner and the amplitude of

oscillation of some electrons in the beam is thus built up. The effect of this is to make a wider beam and to make it easier to move a few electrons at a time out to the target. All properties of this pulse lengthener are not yet tried.

Generally speaking our betatron is trouble free. We had practically no vacuum trouble and our injectors last about five months on the average. There is however a great deal of maintenance to an instrument this large, and a great deal of routine inspection must be carried on regularly. From time to time we have irritating but not fundamental troubles.

I should like now to go on to some of the things we have learned and that other people have learned about the process of injection and the capture of electrons in a stable orbit. For an electron to be captured, it must not return and strike the injector which it has left. The shift of the equilibrium orbit away from the injector is of the order of $\Delta A/A = \Delta E/E$ where ΔA is the change in radio position and A is the displacement of the electron from the equilibrium circle. ΔE is the voltage gained per revolution once around the betatron and E is the voltage at which the particle was injected. On machines of the 22 Mev size one gains about 150 volts per revolution and one injection at 60,000 volts. Consequently $\Delta A/A$ is about 1/400, that is, if A is the centimeter to the equilibrium orbit then ΔA , the shift of the electron per revolution, is only 1/400th of a centimeter. This small shift could not possibly be enough to account for the successful capture of electrons by a betatron although it is true that the crisis does not occur on the first revolution. Theoretically on the order of 10 revolutions are made before the electron strikes the injector structure. Even so, the displacement is not sufficient to explain the operation that has been observed with existing betatrons.

The answer to this puzzle which has existed since the operation of the first betatron is that electrons are shot into existing machines in numbers sufficiently large to have very important space charge and space current effects. These

effects are so great as to dominate the transient behavior of injected electrons and the damping or orbit shifting which I just mentioned previously is one or two orders of magnitude less than the effects which can be produced by space charge and space current. There are several ways in which these currents act. The electron beam, once it is shot into the betatron, circulates around the machine and acts as a resistive loss on the betatron. This decreases the flux within the orbit and as successive electrons are poured into the orbit the first electrons which



Fig. 2 The "C" Magnets of the 300 Mev Betatron.

were shot in find their radii of circulation decreased by the order of a millimeter or two depending on how far out near the point where n equals 1 they were when they left the injector. In the 22 Mev betatron one injects from one-half to 3 amperes and one can collect something like five spirals of electron stream before the acceptance time is over. This current is large enough to produce serious space charge effect and a contraction effect of the order of magnitude that I mentioned. We know that something like 0.2 to 0.3 of these electrons remain in the machine so many of the initial electrons are lost subsequently. It turns out that some of the electrons circulate as many as a hundred times and more before they are lost to the walls or injector of our 22 Mev betatron. Consequently, one does not have a crisis for the order of one hundred revolutions and the ordinary damping first mentioned without regard to space charge effect can take a hold and help shift the orbit away from the injector.

In instruments with different tube properties such a large circulating current may not be achieved. Consequently it is more difficult to get electrons started in machines of smaller proportions and it is also of much greater help to put artificial orbit contraction or to put transient bumps on the magnetic field somewhere to shift the electron beam away from the injector after it has been injected. The transient effects are not very important on the 22 Mev betatron like that used at Sao Paulo and at the University of Illinois. There is another effect that occurs while the circulating beam is being dissipated to the wall soon after injection. While the space charge is being lost to the wall, the electrons last injected observe that the potential valley in which they are executing radial oscillations is becoming deeper and hence the amplitude of their oscillation decreases. This effect for the circulating current of a size used in the 22 Mev betatron is large enough to shift the orbit over approximately 2 millimeters if n is three-fourths. However, if the injector is a little beyond where n equals 1, the shift can be many more millimeters. These several effects do not calculate to give a large absolute displacement if one uses n equals $3/4$ but in practice one finds that betatrons work better with the injector out near where n equals 1, and at such a radius all these transient space charge effects produce an orbit shift far larger than the shift when one calculates for n equals $3/4$. One can go on and elaborate numerous effects. There is an expansion effect on the electrons which are last injected while the circulating beam dies down to a lower level by loss to the wall, but the point I want to emphasize is that these are large effects compared to a single particle ideal orbit. The combinations of these effects of space currents and space charges are complicated and it is not always clear which effect is important in which machine. However, some things have been observed to be true for the 22 Mev betatron. I would like to warn that different instruments behave in different ways. For example, some instruments seem to accept electrons successfully only on the back side of the injection voltage pulse, that is, while the current that is being injected is being shut off. This is definitely not the case with either our 80 Mev or our 300 Mev machine. In these cases it is best to shape the injection voltage wave so that it fits the demanded voltage and pours electrons into the orbit a longer time on the rising side of the voltage pulse. A sufficient reason for some other instrument to behave oppositely would be the existence of a fairly high resistance tube coating. In

which case the very presence of a beam striking the wall of a tube makes an electrostatic bump due to the Hi drop of the current returning to the wall of the tube. This electrostatic bump can be large in size and can deflect the electrons; therefore, it may be necessary to cut the electron beam off suddenly so that this bump does not persist. Our tubes do not have such a high resistance. We are very careful to keep our resistance of the order of 20 ohms to ground so that we are dealing with no spurious potentials and our only perturbations are then magnetic bumps and space charges and space currents of the electron stream itself. It is known that small potentials of the order of 200 volts between the injector and the coating on the tube produce serious changes in the beam intensity. Such an electrostatic bump in the tube may easily create a 60 kilovolt potential if a beam of one ampere strikes a tube coating having 200 ohms to ground.

Another interesting possibility was suggested by HARDEN in England. He pointed out that an electron stream injected from the gun is most intense on the side of the betatron in front of the injector. The existence of a high current changes the value of n and hence one has a different value of n on one side of the machine than he has on the other side of the machine. Also when the beam is cut off or strikes the wall the alternation in n disappears largely. The effect of a different value of n on one side than on the other is an effect exploited by JONES, REAGAN and LAWSON on a synchrotron in which n equals $1/4$

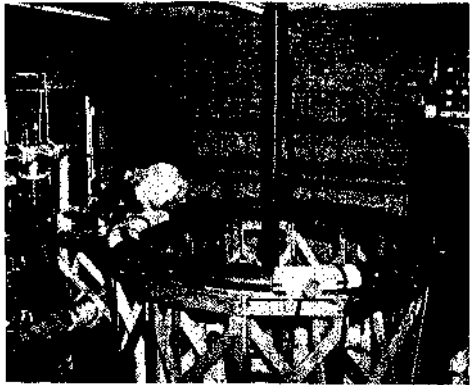


Fig. 3 Vacuum tube for 300 Mev Betatron with Professor Adams.

or with n equals $3/4$ one could get vertical or radial transient focusing in a very rapid way. The easy way to understand the effect is to imagine that in front of the injector one has the diverging lens where n is higher whereas behind the injector one has a converging lens and n is lower than the average n . The result is that straying electrons are brought into less and less amplitude of oscillation as long as they pass through this sequence of diverging and converging lenses. To have this effect operate properly the average value of n must be very near to three-fourths for radial oscillations or one-fourth for axial oscillation. If the lenses are not turned off quickly enough the amplitude of oscillation will grow again after having decreased. A very undesirable property of this type of successive lenses is that a bundle of rays leaving the injector increases in divergence instead of gathering together. Consequently, one can only use guns for this which have a well focussed beam to begin with, because this system of lenses is incapable of collecting much spray from the injector. It is perfectly possible that BARDEN'S

effect works in some particular instance; however, if the value of n is not fairly close to the correct value, the system will not work. All of these space charges or space current effects may act to differing extent. I think there are so many big effects that have been suggested that there are numerous combinations of them which may actually work in the natural situation.

I should like to add some remarks about the importance of having the internal wall of an acceleration tube adequately coated with a semi-conducting layer. As is pointed out a high resistance to ground will give electrostatic potentials which can seriously disturb the path of electrons. It is worse to have bare spots of extremely high resistance because these spots can assume the injection potential. Early in the days of porcelain donuts, it was difficult to coat them completely and the observation was made that a small area of about 2 square centimeters of bare porcelain destroyed the yield of a betatron completely for 15 minutes after which the charge leaked off and it was again possible to get a burst of radiation for a short time. There have been frequent comments about how bare spots are not serious inside of acceleration tubes. This comment needs explanation. Synchrotron acceleration gaps are sometimes bare when exposed to an electron beam. If they become uniformly charged, they form an electrostatic lens which has a focusing property and which would be satisfactory if it would be steady and stable. People who have operated synchrotrons point out that they have numerous magnetic corrective coils on their machines. It is perfectly possible with these corrective* coils to compensate for electrostatic bumps caused where there are spots on the wall. In fact, we have at Illinois some very conclusive evidence on this matter. In 1950 our instrument worked successfully with no magnetic correcting bumps at any point. Subsequently we tried currents in correction coils to see what would happen. At first there was no improvement in the yield. However, as a year or two went by we suddenly realized that magnetic bumps were absolutely essential. In fact, under the old conditions without bumps the yield was only 2% of the yield which we had soon after we turned the machine on in 1950. Naturally the suspicion was there were bare spots on the tube and that we were compensating for electrostatic potentials by adding magnetic field bumps. The tube was opened for the first time and examined and the bare spots were discovered. When the bare spots were covered with a conducting coat, the yield came up to full value without any magnetic bumps and these magnetic bumps or correction coil did no good and if anything did harm, which is the proper way for an instrument to behave if its magnetic field is smooth. I think this resolves in my mind the rather mysterious statement about how magnetic bumps improve the operation of an instrument which is supposed to already have a smooth magnetic field. The reason is that the magnetic field may be smooth, but the presence of electrostatic bumps requires the addition of magnetic bumps for compensation. Spurious bumps are generally large when the main magnetic field is small because they are caused by eddy currents which are 90° out of phase of the main accelerating field. Consequently, as the magnetic field increases, the fractional error in the magnetic field decreases and at high energy the effect of the magnetic bumps would therefore be much less. For this reason it is sometimes worth while to try injecting at the highest possible voltage. Another reason for injecting at high voltages is that if the bumps are

less noticeable, it may be possible to decrease the cross section of the vacuum tube through which the particle must pass because the orbits will be less perturbed at high energy injection. The value of this has been illustrated in the Brookhaven proton synchrotron. Here a prefocused beam of proton is injected and the prefocusing is so good that betatron oscillations were not required for gathering the particles into a bundle. Consequently it will be possible in the future to decrease the gap of such a machine to very small proportions.

Since the main cost of a machine is determined by the volume of gap between the poles of the magnet which is available for the charged particles, the cost of the machine can be brought to an extremely small value by prefocusing of injected particles. It is rather ridiculous that betatrons have such a large tube cross section because the only time the large tube cross section functions is when the particles are injected and when betatron oscillations are being used to collect a beam. When the beam reaches high energy, it occupies a very small space and it is precisely at this time that an immense amount of energy is stored in the large space between the poles of the machine, very little of which is used. Consequently, one should strive in the future for the injection of a prefocused beam from outside the instrument.

I would like to report some experience about special targets. With the Sao Paulo 20 Mev betatron you probably use targets of the kind we have in the Illinois 22 Mev betatron. At times BECKER has used probes in the 22 Mev betatron to get high specific activity of beta rays from samples placed inside near the electron beam. We have tried a similar arrangement on the 300 Mev betatron. A very tiny rod of platinum, one-half a millimeter by one-half a millimeter and about $4\frac{1}{2}$ millimeters long is placed with its long dimension pointing circumferentially around the electron orbit. This small platinum object is held by two fine brass wires. It is supported on a probe 180° around the betatron from the injector, so it appears as a small point of platinum which forms a source of X-rays if the probe is at a smaller radius than the injector case. We have found that this probe does not obstruct the injected electrons appreciably, and it has given us for the first time in the big betatron an extremely fine focal spot with radiation capable of good collimation. We did not have this convenience when the target was mounted on the back of the injector. We once tried a probe that was made of 20 mil tungsten wire bent in the shape of a V with the electron beam hitting the tip of the V. This gave us a small source of X-rays, but the target proved to be rather thin. The beam width was only about $1/2$ of a degree all the way across at half maximum intensity whereas our usual target gives about 1.1° all the way across at half maximum intensity. It is not exactly clear that one can control the thickness of the target which one puts in the betatron because subsequent revolutions can make multiple passages through the target and scattering on the first passage of the target can take an electron out through the side of the structure without passage through the complete thickness of the target.

The advantages of the probe type of target are that we can change injectors without disturbing the target and collimation. We apparently can control target thickness to a certain extent, and we can control the source size.

DISCUSSION

SANTOS: I was very interested in that quotation you made of the use of this field inhomogeneity in order to focus a diverging beam coming from the injector. Do you know whether they have made some experiments or whether it is just a theoretical guess?

KERST: It is just a theoretical guess by Barden of Metropolitan Vickers in England. At the Case Institute they have a small Betatron and they believe they see the effect. They say that they trap electrons on the back side of their injection voltage wave. They believe that they trap electrons at this time because these lenses turn off when the voltage disappears and that is an essential feature of the lenses, one must turn them off. This is their evidence for the effect. They operate with tubes of very high wall resistance and one must turn the beam off anyway or electrostatic potentials would be created on the wall.

Stability Conditions of the Betatron of the Laboratório de Física Nuclear

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1. INTRODUCTION

Between the different experiments that can be carried out with a Betatron, particular importance deserve those that can give precise information on the behaviour of a phenomenon under study as a function of the energy of the electron beam (or the maximum energy of the Bremsstrahlung emitted as the electrons strike a target).

It has been shown (HASLAM *et al.*, 1952; KATZ, 1952) that by means of an accurate control of the energy it is possible to use the betatron radiation in order to measure the energy of the excitation levels of several light nuclei.

In this paper, the stability conditions of the USP betatron are described and two new types of energy integrators are discussed.

II. INFLUENCES DUE TO VARIATIONS OF THE EXCITATION CURRENT

In most Betatrons the energy of the electrons can be adjusted by means of an integrating circuit (BALDWIN & KOCH, 1945; MCELHINEY *et al.*, 1949; KATZ *et al.*, 1950) whose purpose is to allow the electron beam to strike an internal target when a pre-determined energy is reached, irrespective of small changes in the "magnet amplitude".

It is known, however, (MCELHINEY *et al.*, 1949), that a variation of the magnet amplitude gives rise to a small error in the energy control. This error (which depends on a phase shift between the guiding and the acceleration fields and on non-linearities in the iron) is sensitive also to the type of expansion circuitry used.

In fig. 1, curve 1 represents the variation of the saturated activity induced in a copper sample by the reaction $^{63}\text{Cu}(\gamma, n)^{62}\text{Cu}$; the points were obtained by setting the integrator bias at 20 Mev, the energy setting chosen corresponding to the maximum slope of the excitation function for this reaction. The energy integration was obtained by means of an RC integrating network (KATZ, 1952) connected between ground and the high voltage from the magnet excitation coils.

This curve shows how important it is to maintain the magnet amplitude constant. Since the stability of the magnet amplitude is due to the stability

of the frequency stability of the main supply, (which always presents small variations), a stabilizing network should be incorporated in the machine (KERST, 1952).

By means of this circuit, it is possible to keep the amplitude within 2% for a frequency variation of 0,4 cps.

In the Betatron of the Laboratório de Física Nuclear, it was necessary to use an electronic controlled magnetic driven alternator, (Electric Machinery, Minn., Minnesota), owing to the large variations of the main supply frequency. By use of this system it is possible to maintain the frequency of the alternator within 0,1 cps for frequency variations of the input between -8 and 2 cps.

A high degree of stability of the magnet amplitude was obtained by means of this form of excitation; in the betatron excitation circuit, stabilizing resistors were also used (KERST, 1952) so that the magnet amplitude is stable within 2 volts.

For high energy research, the neutralizing capacitors of the frequency tripling transformer are slightly increased, and the magnet volts changed by means of the input voltage of the tripler: by means of a manual control of the alternator excitation it is possible then to change the amplitude between 90 and 130 volts, with instantaneous variations not larger than 2 volts.

In order to decrease the "amplitude effect", a different energy integrator was used. The e.m.f. generating across a coil wound around one of the magnet legs was used with an RC integrating device (GOLDEMBERG *et al.*, 1951). The potential difference across the condenser armatures, which was in phase with the magnetic field at the orbit, and whose magnitude was proportional to the field

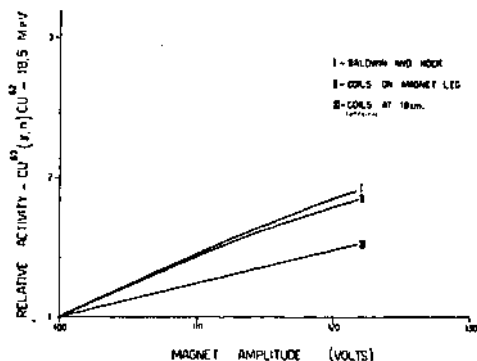


Fig. 1

strength (and hence, to the kinetic energy of the electrons), was used to trip the expander circuit.

This expander circuit (which was used also in our experiments of fig. 1, curve 1), was constituted by a DC amplifier which was used to fire negative feedback followed by a high gain AC amplifier which was used to fire a blocking oscillator (GOLDEMBERG *et al.*, 1951). The sharp pulse (2 microseconds) generated

by the blocking oscillator fired a KU 676 thyratron which, in its turn, was used to excite the starter of a GL 415 ignitron.

The current pulse from this ignitron was fed to the expanding coils by means of a pulse transformer; between the expanding coils and the pulse transformer a network was used to damp critically the expanding pulse and to keep the 180 cps induced e.m.f. out of the transformer secondary⁽¹⁾.

0) The circuit used between the ignitron and the expanding coils is the same utilised at the Universities of Illinois and Saskatchewan (KATZ ET AL., 1950).

No trouble was experienced with jitter from the thyatron and the ignitron since by overvolting the thyatron grid and by applying a large current for a brief time to the ignitron (SLKPIAN & LUDWIG, 1933); it was observed that it was not larger than the observed values with a 5C22 Sylvania Hydrogen thyatron.

The results obtained with this integrating device are illustrated in fig. 1, curve 2. It is seen that this method of controlling the expansion is less sensitive to amplitude variations. As the amplitude of the machine is kept within 2%, the incertitude in the energy, due to this cause, is of about 90 kev, for an energy of 20 Me v.

As there was still an amplitude effect, a new integrating device was used which yielded exceptionally good results.

The ideal method for the determination of the energy of the electrons accelerated by a betatron consists of measuring the magnetic flux through the equilibrium orbit, between the moments of injection and expansion of the beam.

A set of two coils was glued with red glyptal on the external surface of the donut, one above and the other immediately below the equilibrium orbit whose radius was known accurately; as the radius of the two coils were made equal to the equilibrium orbit radius, the induced e.m.f. across its terminals is given by:

$$\oint_{\gamma} \vec{E} \times \vec{ds} = - \int \frac{\partial B}{\partial t} \times \vec{ds} = - \frac{\partial \phi}{\partial t}$$

where ϕ is the accelerating flux.

The output of those coils was connected to the RC integrating network by means of a transformer which supplied 8.000 volts across its secondary.

By using the same expanding circuit described above, measurements of stability against amplitude variation were made; the same detector and the same energy as used in the previous experiments were utilized.

The obtained results are illustrated in curve 3, fig. 1. One can observe that there is a remarkable improvement in stability as far as amplitude variations are considered; the error in the energy setting, due to magnet amplitude variation of 2 volts is of 40 Kev for an energy of 20 Mev.

It is important to point out that this limit of accuracy is due mainly to our expander circuitry and it is hoped that a much higher degree of stability will be reached by means of a more precise amplitude comparison circuit.

III. INFLUENCES DUE TO THE MAGNET TEMPERATURE

It has been observed recently (HASLAM *et al.*, 1952) that there is a large temperature effect in the energy of the machine, which gives rise to variations of the yield for the same energy setting of the integrator.

As such a temperature effect should depend on the nature of the flux integrating system utilized, experiments were made to evaluate how large an error due to this cause might be.

In our measurements, the machine was operated at a constant (within 2%) amplitude and activations of Cu strips at 20 Mev were taken at different times during which the excitation was on.

It was found that our machine reached a stable condition after 20 minutes of heating; this temperature effect was found to vary with the magnet amplitude, being of the order of 11.5% for an amplitude of 100 volts and of 8% for an amplitude of 120 volts.

After a warm-up period of 20 minutes, in either amplitude, the effect disappeared, the activation measurements made being reproducible within the accuracy of 40 Kev for an energy of 20 Mev.

This accuracy should be interpreted as the shift in energy necessary to give rise to activation measurements made with the same energy control setting.

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DISCUSSION

GANS: Uds. han tratado las distintas influencias sobre la estabilidad del betatron. Si todas estas causas quedan constantes, serf a interesante saber del Betatron de Uds., que grado do estabilidad tione el electron mismo en su orbita? Han medido Uds. el campo en funcin del espacio, es decir, la topograffa del campo?

SANTOS: Our Betatron is actually a standard machine which is built in series by Allis Chalmers; the poles have been designed by Prof. Kerst and they should give a field that decreases with the power 3/4 of the distance. When we received our machine, as it has taken a long time in the transport the whole surface was covered with rust and we had to clean it, which we did, but then were not sure that then; were not laminations in short circuit. This would be a big draw-back because then you would have quite a big magnetic field when the excitation current goes to zero and that would disturb the injection. So we made a careful investigation of the field by means of search coils and it turned out that the field was as it should be. We did not bother very much to follow the measurement of the field from zero to the maximum around 360 degrees. We thought a good test of this would be to see whether we could get an electron beam; we could get it and, more than that, we have found that contraction instead of helping did disturb our yield. Therefore I think we can conclude that the field is correct. As far as the change with the distance goes, we have measured that accurately and it is what it was built for.

GANS: PUES hay un criterio muy sencillo y medible sobre el grado de estabilidad de un electron que se mueve en el campo, es decir, si la figura representa esquematicamente el iman y el electron se encuentra en $P(r, z)$

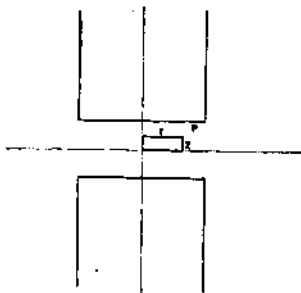


Fig. 2

entonces la aceleración radial puede escribirse y expresarse por una función U y

$$\ddot{r} = - \frac{\partial U}{\partial r}$$

y la aceleración axial en manera semejante $\ddot{z} = - \frac{\partial U}{\partial z}$, es decir existe una especie de potencial y este potencial se expresa de la siguiente manera:

$$U = \frac{e^2}{2mc^2} \left(\frac{\phi(r, z)}{r} \right)^2$$

Si ϕ es el flujo total hasta el punto estudiado, es decir, función de r , de manera que si se dibuja U en una sección meridional en función de r , entonces será una curva quizá de esta forma, y si se hace girar esta figura



Fig. 3

alrededor del eje $r = 0$ resulta una montaña ariular. A describe el equilibrio de equilibrio. Estabilidad hay en la parte B A C, fuera de ella inestabilidad, de manera que la altura de los montes nos da una idea del grado de estabilidad y si en un caso concreto la forma de la montaña resulta no satisfactoria pueden retocarse los polos del imán de tal manera que se mejora el resultado.

SANTOS: Este es el mismo tipo de polo. Tenemos una gran estabilidad de la órbita de equilibrio, tanto no que diz respeito às oscilações radiais, como não que diz respeito às oscilações verticais. A injeção do campo magnético dá efetivamente uma curva desse tipo, e a órbita de equilíbrio nos sabemos que é localizada na posição ideal não que diz respeito às relações ideais, pois ela foi determinada procurando-se o valor mínimo do campo elétrico tangencial. Esse foi modulado com toda a precisão e isso é o valor que nos dá o raio da órbita.

GANS: Y en la dirección axial puede observarse si las líneas de fuerza convergen y en qué grado. Da un criterio de estabilidad en la dirección axial.

SANTOS: Experiments made in the laboratory of Prof. Kerst by Becker using an internal probe have shown that when one expands this orbit and one activates, let us say, copper, and then one makes a radio autography of it, one can see that the beam looks something like this: it may be 1 mm. high and about 1/2 mm. wide and this asymmetry is of course due to small oscillations which depend on the shape of the field. But it is very stable and when one expands the beam against a thin target such as the one that is used for X-rays, one can see that the whole beam comes from a point source. So, I think that the equilibrium orbit is very stable.

Cross Section for Neutral Pion Production by p - p Collisions

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The group at Berkeley measured the relative production of neutral pions by 340 *Mev* protons striking various nuclei but did not observe any production within the limits of experimental error from $p - p$ collisions. Instead their results seemed to indicate that there was neutral pion production practically only from $p - n$ collisions at this energy. For example the pion production cross section for B^n was equal to that for C^{12} . These 2 nuclei differ by 1 proton. Therefore the Berkeley results indicate that the neutrons contribute but the protons apparently do not in a nucleus.

This fact was interpreted as a result of conservation of energy, parity, and angular momentum. At 340 *Mev* kinetic energy of the colliding nucleon there is approximately 170 *Mev* in the bari-central system of the two nucleons. To make a pion requires 140 *Mev*. There remains 30 *Mev* for the kinetic energies of the 3 resulting particles in the bari-central system. In general the meson takes a large part of this energy, consequently the two nucleons are left in an S state with respect to each other.

It is a well known characteristic of nucleon collisions that pions are created mainly in P states. Let us apply conservation of total angular momentum and parity. In the case of $p - p$ collisions there is also the further limitation of the Pauli principle. If the final nucleons are constrained to remain in an Σ state, the $p - n$ collisions can produce mesons in a P state but the $p - p$ collisions can not. Furthermore there are more initial states of the $p - n$ system which can create mesons than is the case for the $p - p$ system. Consequently close to the pion threshold it is reasonable to find a relatively high cross section for neutral pion production from $p - n$ collisions and a low or zero cross section for $p - p$ collisions. These arguments are no longer valid at higher energies when higher angular momentum states of the final nucleons contribute.

It was therefore immediately of interest to find whether the production of neutral pions from $p - p$ collisions was observable at higher energies and if so, to measure it.

Because there is practically no external beam of protons at Chicago this experiment had to be made inside the cyclotron. It was necessary to use liquid hydrogen as the target because in any material containing neutrons the neutral pion yield from the $p - n$ collisions might mask that from the $p - p$ collisions. A target containing liquid hydrogen was placed at a radius of 76 inches in the

cyclotron. Protons of 450 *Mev* maximum energy were scattered into the liquid hydrogen, producing neutral pions which then decayed into photons in the target. Photons emerging in the forward direction, i.e. tangential to the proton beam, were observed in a detecting apparatus placed outside the cyclotron in the experimental area.

As they accelerate in the cyclotron, the protons move out a distance of less than 2 mils per turn. The liquid hydrogen had to be held in a target container inside the cyclotron but the walls of the container had to be more than 2 mils thick. Consequently the direct proton beam would scrape off almost entirely on the walls of the target if it were placed directly in the beam and would never get into the liquid hydrogen. For this reason it was necessary to scatter protons into the target.

The feature of the cyclotron which makes this possible is the property of beam oscillation. At 7(3" radius the radial frequency is approximately equal to the cyclotron frequency, and the frequency of *Z* oscillation is about one fourth the cyclotron frequency. After one revolution from a scattering target the beam has been refocused radially but has reached maximum vertical deflection. Exploratory measurements were made of the focusing of the scattered beam which showed that a 20 mil platinum foil scattered 10% of the beam into 1 square cm. approximately 1 inch from the median plane. Accordingly the hydrogen target was placed above the median plane.

A collimator inside the cyclotron but outside of the beam defined the target area seen by the detecting apparatus.

A second *Pb* collimator defined the solid angle of detecting apparatus seen by the target. The beam consisted of an approximately equal flux of neutrons and gamma rays. The high energy neutrons are an unpleasant feature because they knock on high energy protons. Consequently we could not easily use scintillation counters alone. To measure the photons from π^0 from hydrogen with a pair spectrometer was found to be quite difficult because of the low intensity. We found that the water CERENKOV counters were almost perfectly adapted to this experiment, because the protons in this beam had too low a velocity to make CERENKOV radiation in water, but only electrons and mesons could do it. The flux of mesons was so exceedingly small as to be completely negligible. Therefore a telescope was made consisting of a liquid scintillator and a water CURENKOV counter, which could be placed directly in the neutron photon beam, and which was specific for electrons.

The proton flux was measured by the activity of Na^{24} which it induced in aluminium foils. The cross section for this process has been measured for protons of this energy by Luiz MARQUEZ. The foils were able to be placed directly over the end of the hydrogen target in such a way that one foil integrated the protons which passed through the empty target, and another foil measured the same quantity when the target was filled with liquid hydrogen.

With these various elements of the experiment the differential cross section for photon production was measured to be

$$\left(\frac{d\sigma_\gamma}{d\Omega} \right)_{\text{zero degrees}} = 0.2 \times 10^{-27} \text{ cm}^2/\text{sterad.}$$

A straightforward calculation gives the value of $\sigma_{\gamma}/[d\sigma_{\gamma}/d\Omega]_0$, assuming isotropic pion emission as 5.0 and, assuming P wave pion emission, as 4.0. Accordingly we used a mean value of 4.5 for this quantity, with which we calculate the total neutral pion production cross section for $p-p$ collisions to be

$$\sigma_{\pi^0}(p-p) = 0.2 \times 10^{-27} \times \frac{1}{2} \times 4.5 = 4.5 \times 10^{-27} \text{ cm}^2/\text{sterad}$$

DISCUSSION

HERB: YOU spoke originally of the Berkeley results and you spoke of the expected ratio of $p-p$ and $p-n$ cross-sections. What do your values give for this ratio?

L. MARSHALL: This ratio has apparently decreased a lot from 340 Mev to 450 Mev. It can be understood because now higher angular momentum states can enter which allow the pions — the neutral π^0 — to be created into a P state even in the case of $p-p$ reaction. At 340 Mev the argument was that the pions can not be created into a P -state because there is not energy to let the resultant nucleons go into a P -state and conservation in parity and in angular momentum prevents the creation of the $\pi\pi^0$ meson into a P -state with the nucleons in an S -state.

KERST: The Be test was made with the scattered beam, was it not?

L. MARSHALL: NO, it was made with the direct beam. May be I should say that we did a third experiment on Beryllium using a pair spectrometer. In fact this was the first experiment that we did. We could not however use the pair spectrometer on hydrogen. We did it in the usual way, putting a block of Beryllium in the beam and using a pair spectrometer as detector and we got a cross-section. The differential cross-section as measured on Be for the Cerenkov effect was 4.0 and with the pair spectrometer was 3.5. We had a round-pole pair spectrometer and the efficiency of a round-pole pair spectrometer varies rapidly against energy. We calculated this efficiency: it depends not only on the energy at which one looks, but on the thickness of the converter foils that one uses, also. We calculated three different efficiency curves for three different thicknesses of foils in the pair spectrometer, and used them to obtain the value of 3.5. I believe that the check of these is perhaps valid.

KERST: Both of those were with beryllium in the beam.

L. MARSHALL: Yes.

On the Energy Spectrum of the Bremsstrahlung Radiation Emitted by a Betatron

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INTRODUCTION

Cross-sections for photonuclear processes cannot be calculated precisely by the usual methods (JOHNS *et al*, 1950; KATZ & CAMERON, 1951) unless the spectral composition of the Bremsstrahlung radiation is known accurately.

It is generally assumed that the spectrum in the forward direction from betatrons and synchrotrons are well described by the formula due to SCHIFF (1946 & 1951)

$$I = 2(1-z) \{ \ln \alpha - 1 \} + z^2 \{ \ln \alpha - 1/2 \}$$

where

$$\alpha^2 = \alpha_1^2 \alpha_2^2 / (\alpha_1^2 + \alpha_2^2)$$

$$\alpha_1 = 2W(1-z)/\mu z$$

$$\alpha_2 = C/z^{1/2}$$

This "old Schiff relation with $(7=191$ is actually a compromise in shape between the total Bremsstrahlung spectrum and that in the forward direction calculated with $C = 111$ which is expected to hold only for very thin targets". (KATZ & CAMERON, 1951).

It is assumed that KOCH & CARTER'S (1950) measurement on the Bremsstrahlung spectrum with 19.5 Mev electrons is a good confirmation of the theoretical spectrum, although there are about 10% more quanta in the middle energy range than is predicted by the theory.

It is important however, to point out, that in KOCH & CARTER'S experiments, a platinum target of uniform thickness was used (0.005" Pt) whereas in the betatrons that

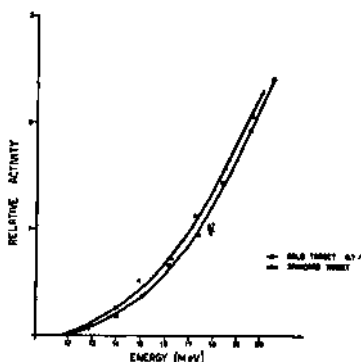


Fig. 1.

have been used for cross-section measurements of photonuclear processes, the target used for Bremsstrahlung production cannot be considered as a target of uniform thickness, its profile being, on a first approximation, that of a Pt disc 2 mm radius and 0.5 mm thick.

JOHNS, KATZ, DOUGLAS & HASLAM (1950) have shown that the shape of the cross-section curves, as well as the "resonance" peak are sensitive to the spectral composition, although the integrated cross section is less so.

As the spectral composition of the Bremsstrahlung radiation originated from a "conventional target" (disc shape) is sensitive to its relative position to the equilibrium orbit and to the shape of the expanding pulse, it seems that a comparison between results obtained in different laboratories is difficult; it would be advisable, therefore, to use a target of a uniform thickness at a known radius and with a standard expansion circuitry; the spectrum should be measured and compared with the theory.

GOLDEMBERG, SANTOS & SILVA (1951) have shown that a conventional target does not give rise to a spectrum similar to that from a target of uniform thickness; by measurement of the angular distribution about the forward direction, the emitted radiation behaved as coming, at different angles, from targets of different thicknesses and it was shown that this behaviour was not due to multiple scattering.

In order to investigate these conclusions, several measurements of the reaction $\text{Cu}^{63}(\gamma, n)\text{Cu}^{62}$ were carried out with a standard target and with a gold target, of a thickness of one tenth of a radiation length.

The results obtained in these experiments are discussed in this paper, together with some preliminary investigations of the angular dependence of the energy spectrum; the latter are important if one wants to attach a meaning to angular distribution measurements.

EXPERIMENTAL ARRANGEMENTS

A gold target, 0.1 radiation length was used instead of a conventional one; this target was placed on the injector structure at a radius of 21.8 cm from

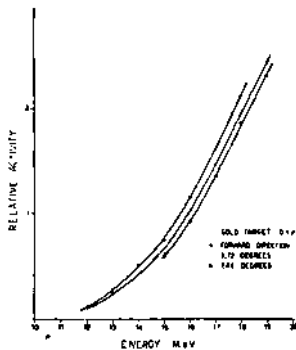


Fig. 2

the orbit center. The target was given an optical flat at its edge in order that it could be located at 90° with the expanding electron beam; this care was necessary in order that the electrons would traverse the same thickness of material. The expansion circuit was adjusted to give fast pulses; the position of the orbit, during the expansion, is not likely to give rise to multiple traversals of the target.

The thickness of the gold foil was chosen so as to have a small probability for multiple radiation; its ionization energy loss is also small (0.3 Mev for 19.5 Mev electrons); on the other hand the target is sufficiently thick

to have enough multiple scattering and the X-ray beam is of an intensity high enough to make activation measurements.

In our experiments with the conventional target, the same circuitry was employed in the expanding circuit and the position of the target was maintained the same as for the gold target.

RESULTS

All the measurements were carried out in the forward direction; the irradiation technique was almost the same described by JOHNS *et al.* (1950), although the copper samples subtended an angle of 15' of the beam and a Victoreen thimble was used in all measurements to measure the beam intensity.

The energy of the electron beam was measured by assuming a value of 10.9 Mev for the $Cu^{63}(\gamma, n)Cu^{62}$ threshold; the energy integrator was calibrated again when the target was changed.

The obtained results for the excitation functions of the $Cu^{63}(\gamma, n)Cu^{62}$ are illustrated in fig. 1; it is seen that the curves do not superpose on each other, the excitation function as obtained with the gold target lying above the one obtained with the conventional target.

As the differences for the activities, for a given energy, increase with the energy and since these differences are much higher than the experimental errors which, incidentally, are of a statistical nature, the conclusion is that the conventional target produces a larger amount of soft radiation than does the gold target (by soft radiation we understand, in this case, radiation of an energy below the threshold of the $Cu^{63}(7, n)Cu^{62}$ radiation).

The cross-sections for this action, as calculated by both the excitation functions, show a slight displacement of the maximum towards higher energies (for the gold target), although the shape of the cross-section curves is very similar, with an increase of about 12% for the cross-section as measured with the gold target Bremsstrahlung radiation.

According to SCHIFF (1940 & 1951), "to a good approximation the energy distribution at all angles is that usually associated with the total radiation".

The experiments referred to above, seem to indicate that, even in the forward direction, this does not seem to be the case.

THE ENERGY ANGLE DEPENDENCE

Excitation functions for the $Cu^{63}(\gamma, n)Cu^{62}$ reactions were measured at different angles about the forward direction in order to test whether the spectral composition remains the same.

The measurements were carried out with our standard irradiation technique at angles of 3.77° and 8.44° about the forward direction: the target-sample distance was kept constant.

The obtained results are illustrated in fig. 2, for the gold target whereas the results for the standard target are given in fig. 3.

During the measurements, the behaviour of the machine was tested periodically by means of the induced activity at a given integrator setting, in a copper plate maintained at the forward direction; by this means it was possible

to conclude that the betatron as well as the Victoreen thimble and electrometer system behaved as expected.

The only interpretation of the curves of figs. 2 and 3 is that the spectral composition changes with the angle about the forward direction, in disagreement with Schiff's expectations; one can see that the conventional target gives rise to a larger amount of soft radiation than does the gold target.

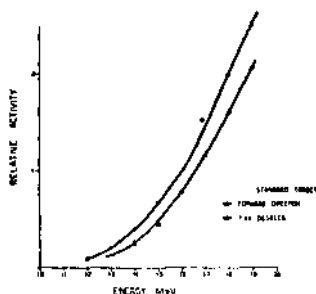


Fig. 3

As the Incite block was displaced sideways, some measurements were carried out in order to check whether Compton electrons and X-rays (produced on the edge of the block struck by the forward radiation) could be responsible for the increase of the soft radiation; absorption measurements with lead and inverse square law measurements (fig. 4) were made leading to the conclusion that the observed effect, viz. the angular dependence of the spectral composition, could not be ascribed either to

Compton effect or to an influence of the diverging electron beam (or its secondaries produced in the poles) from the donut.

It seems, therefore, that the spectral composition of the radiation changes with the angle about the forward direction the relative contribution of the soft photons increasing with the angle.

This conclusion received a further confirmation by means of experiments which were carried out in order to eliminate the ionization measurements of the beam intensity, by means of the activation induced in thin samples of Zn and Ta placed simultaneously at different angles: two sets of these foils were placed in the forward direction and at an angle of $8^{\circ},44$ about the forward direction; they were irradiated simultaneously during 30 minutes at an energy of 27,2 Mev.

The activities measured were due to the reactions $Zn^{64}(\gamma, n)Zn^{63}$ and $Ta^{181}(\gamma, n)Ta^{180}$. As the excitations functions of these reactions (MCELHINNEY *et al.*, 1949; GOLDRMBERG *et al.*, 1953) are sensitive to different regions of the spectrum, one should expect a change in the ratio of the activities if the spectral composition is angular dependent.

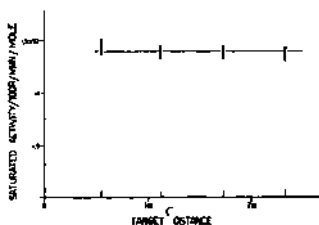


Fig. 4

The obtained results were the following:

Forward direction: Zn/Ta: $6.99 \pm 0,00$

$8^{\circ},44$ about the forward direction: Zn/Ta: 5.92 ± 0.00 .

Those measurements confirm our measurements with the excitation functions at different angles and show that the relative amount of soft radiation increases with the angle about the forward direction.

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DISCUSSION

BOHM: Willi regard to the discrepancy in the angular distribution you should not predict that the energy curve be entirely independent of angle, but find that it depends somewhat on angle. Of course, Schiff's curve must be some kind of an approximation, and this approximation must fail when you get to angles that are about as wide as your angular distribution. How does the angle at which you measure compare with the width of the angular distribution? You have a certain angular distribution, say 10° or 20° and you measure at 4° or 5° or 10° ; and now what is the ratio, what was your angular distribution, how wide was it?

SANTOS: We went up to about 10° . This if you work at 20 million volts is a much higher angle than the intrinsic half-width for a single collision.

BOHM: How wide is the multiple scattering distribution that you measure and how wide is the distribution?

SANTOS: That was something like 4° . That depends on the energy, it was about 4° for 15 million volts and I think it was something like 3° for 20 million volts.

BOHM: You made measurements at 4° ; then Schiff's theory would not be expected to hold at that angle because Schiff's theory is essentially this, that if you have a lot of multiple scattering then if the radiation is only $1/2^\circ$ wide and multiple scattering is 5° wide then in the first 5° you would not expect to find any appreciable change in the spectrum. But if you get to an angle as wide as the width of your beam, then you would expect that Schiff's theory would not apply.

SANTOS: As far as the composition of the radiation is concerned, we went up to angles of $8^\circ 44'$ which are quite large. I think that Schiff's approximation is not a correct one because angular distribution is one thing and spectrum composition is another. When he calculated his angular distribution he took for the intrinsic radiation half-width for a single collision an expression which is energy independent and, therefore, we should not expect to find an energy dependence.

Replying to a question by Herb, SANTOS pointed out that with a very thick target one can have a lot of multiple scattering and one should not expect to find an energy dependence. He said that this can be explained by an observation which was made by Powell, Hartsough and Hill: if one has a collimated beam of electrons hitting a thick target, due to the multiple scattering it is only $1/10$ of the target thickness or so that will contribute to the radiation in the forward direction whereas (due to the multiple scattering) all the other electrons go away and, therefore, in the forward direction one should have something which looks like a thin target spectrum. The energy dependence of the spectrum on the angle makes it very difficult to understand, for instance, what a measurement of an angular distribution means, because if the detector is sensitive to the quality of the radiation, and this is the case with a ionization chamber or with a copper detector operated above the threshold, and if the composition changes with the angle then one does not know what one is measuring and it is very difficult to compare with the theory.

KERST: I would like to make a comment, to defend Schiff because, as far as I understand Schiff's claim, he observed that the spectrum in the forward direction looked very much like the integrated spectrum which appears in the literature, since practically all the energy goes in the forward direction, it cannot help have the same spectral shapes that the whole integrated spectrum would have. I think that the important point is not necessarily whether or not Schiff was right but rather that much work has been done without regard to precisely what are the spectral effects and perhaps the forward spectrum that people have been using is itself not actually known.

SANTOS: We think that this angular dependence that we have found shows that in the forward direction one should have something which is much closer to, let us say, a very thin target spectrum, than to the total spectrum.

Investigations on a Magnetic Ion Source for the Sao Paulo Electrostatic Accelerator

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We very early decided to put our efforts on a magnetic ion source. There are several reasons for this choice. In the first place it appears that to obtain high beam and high efficiency, the capillary arc source, in any of its numerous variations, must be rejected. To my knowledge the only other types which have been found to be practical are the high-frequency source, with or without magnetic field, and the *dc* source with axial magnetic field.

A study of the literature leaves one in considerable doubt as to which of these types of source is the better, since various claims are somewhat conflicting. We decided on the *dc* type of magnetic source because it seemed to offer the best possibilities of obtaining a fool-proof source requiring absolutely no special techniques, no circuitry except standard laboratory supplies, and the minimum of such hard to obtain materials as insulating seals, porcelain supporting members, etc. In brief, this type of source consists of a cylindrical discharge chamber in which a magnetic field is maintained parallel to the axis. A filament is located on or near the axis of the discharge chamber. The anode is cylindrical and the ends of the chamber are closed by plates at a potential slightly lower than the filament. Electrons drawn from the filament cannot go to the anode directly because of the magnetic field. If they lose even a small fraction of their energy they can no longer return to the filament. The result is oscillatory motion until finally they are captured by the anode. Thus each electron becomes much more effective as an ionizing agent than in a non-oscillatory source. This in turn results in sufficient ionization at low pressures so that the exit-hole of the source can be simply a hole drilled in one end-plate. Thus ion extraction is more efficient than in a source with a capillary withdrawing tube. The literature contains several examples of remarkably efficient sources constructed on this principle.

It is clearly not at all difficult, judging from such work as that of KISTEMAKER and DEKKER, to construct a source capable of giving several milliamperes of current. A little thought shows that even a perfect Van de Graaff, returning all of its charge to ground by means of ion current, could hardly be expected to handle even 1 mA. Hence we see that the upper current limit is not a serious cause for concern. What we must be sure of is that the source is capable of delivering an extremely well-collimated beam of protons at good gas efficiency, and preferably at good power efficiency. The experimental source which we have constructed

fulfills these two requirements at least reasonably well. The opening angle of the beam is only 2 degrees, regardless of the current, providing that the source is space-charge limited. I will return to the subject of space-charge limitation later. First I will give a more detailed picture of conditions of operation of our source. I should mention at the start that the source that we now have was designed strictly for experimental work. No arrangement was made for either conduction or forced-air cooling, and so it is quite incapable of operating continuously. In the designs of the source for the Van de Graaff, such measures have been taken.

The first subject to be considered is the location of the hot filament. KISTEMAKEK and DEKKER found an increase in efficiency upon locating the filament in proximity to the exit-hole. We have rejected this type of construction, although we did not really give it a fair trial. We originally placed the filament directly over the exit-hole. The source actually did not work very well, it being impossible to get appreciable beam unless the pressure was raised until a real arc breakdown occurred. When we moved the filament to the back of the source the operation improved tremendously. We probably should have carried on some investigations to determine the cause of this change. However, we were so struck by the tremendous simplification in filament mounting, as well as the increased possibilities of cooling the parts of the source in proximity to the filament, that we left the filament at the back of the source, and did not track down the cause of the improvement in operation.

The magnetic field is obtained by a solenoid coil. In the experimental source the coil was sealed inside the vacuum chamber by means of an O-Ring bounded cylinder. This type of construction is not to be recommended unless certain conditions of rigidity and temperature, not achievable in our ion source, can be maintained. In the source which we have designed for the machine, the coil form is open to the high pressure atmosphere, and finned so that forced air cooling will be easy and effective. The solenoid coil can hardly be said to live up to the requirement that its diameter be negligible with respect to its length. We have made measurements on the field with a moving one-layer coil. The signal was integrated numerically after being placed on a scope. These measurements showed that the field at the ends was only around 75% of the field at the center. Under most conditions of operation the value of the field is not at all critical, so apparently this variation in field is not too serious a defect.

A detailed discussion of the various effects obtained by varying the magnetic field is not in order in a paper of this length. It suffices to say that under the conditions usually encountered, the output current of ions rises very steeply with magnetic field. This striking rise is clearly due to the increase of the ionization efficiency of the electrons, due to oscillation. The total anode current does not increase when magnetic field is applied, but in fact decreases somewhat, due to the increase in the negative space charge of the oscillating electrons. The rise in output becomes less marked above 400 gauss, and shows signs of saturation.

Probably the subject of widest interest, and that causing the widest controversy, is the effect of gas pressure. First I will say that we have found our best operation to be with the filament hot enough and the gas pressure low enough so that the electron current is negative space-charge limited at the filament.

Besides other advantages, this type of operation should substantially increase filament life, due to the shielding effect of the electrons against positive ion bombardment. The current is not exactly the same under operation, as with a high-vacuum in the source, but the presence of the plasma does not increase the electron current by more than 50% over the value in vacuum. We prefer to run the filament at a temperature such that the temperature-limited value of current is three to five times the value of current actually flowing from the filament at any given time. This results in very stable operation, with the output controlled entirely by the anode voltage. Thus we see that we are very far from true arc conditions, and closer to vacuum-tube operation.

We have by no means, however, been able to duplicate the figures given by KISTEMAKER and DEKKER. AS stated in the abstract, we operate at between 4 and 9 times 10^{-4} mm. of H_g . In the paper of Kistemaker and Dekker they state that their source operates at currents up to 13 mA. with an internal pressure of 10^{-5} mm. of H_g . I am not able to advance any mechanism which could explain such a tremendous difference between their results and ours. Our gas efficiency is then considerably lower than theirs. In our source the ratio of the numbers of ions to the number of gas molecules leaving the source approaches 1:1 at high currents, and is ordinarily between 1:2 and 1:3. While these figures are not exceptional, they are quite satisfactory, and considerably higher than those for the older types of source.

The power required by our source is reasonable, being about 0.1 watt/ μ A. This is almost equally distributed between the filament, the magnetic field, and the anode.

Under our conditions of operation the voltage in the anode can take on any value, since a true arc never occurs. Increase of the anode voltage brings about a corresponding output increase, both due to the greater electron current, and in their greater ionization efficiency. We have used with success anode voltage up to 250 volts. From the point of view of beam homogeneity, as well as ionization efficiency, a further increase would probably not bring much improvement. After a great deal of what can only be called fooling around, we came to the conclusion that to obtain good operation we needed heavy electron currents, relatively speaking, without the aid of the gas in the source. This, we believe, is because if the pressure is high so that the ions can aid substantially in electron extraction from the filament, this almost automatically means that the mean free path of the electrons is too small for them to be effective in oscillation. Thus their efficiency falls tremendously, and the whole advantage of this type of source is lost. At first we thought to increase the electron current by reducing the cathode-anode spacing. The current did, of course, increase tremendously when we moved the filament much closer to the anode. But the ion current actually decreased, and the electron efficiency became very low. This effect can be simply explained by the hypothesis that since the filament is on the axis of the source, and the anode is cylindrical, the electrons leaving the filament were subjected to a radial field, and could not oscillate effectively. Finally the question was resolved by placing a, positive grid in front of the filament. This gave the desired increase in electron current, without a corresponding decrease in efficiency. Naturally many electrons are lost on this grid, but the increased number more than compensates for this loss.

We now wish to discuss the results of our investigations on beam-forming, to which we referred earlier. Since this type of source results in the formation of a rich plasma directly in the exit-hole, calculations and measurements as made by Pierce for electron beams are directly applicable to the source. The exit hole is simply an opening in a thin disc of aluminum. This opening, under proper conditions may be thought of as a space-charge limited emitter of ions. Assuming then that we shape our electrodes to compensate for space charge repulsion, the current flowing in the rectilinear beam can be theoretically calculated. The formula is

$$I = 5.4 V^{3/2} / A^{1/2} k^2 \text{ mA.},$$

where V is the extracting potential, expressed in kilo-volts, A is the atomic mass of the ions, and k is the ratio of exit-hole-probe spacing to beam radius. Note that the absolute size of the extracting space does not enter the formula.

We have verified the applicability of this formula to our source, and have found, as expected, that as long as the discharge conditions are such that the source is capable of giving somewhat more current than is actually being drawn, the formula is followed very well. A close check is not feasible unless the beam has been analysed and the distance from the source to the probe checked with higher precision than we were able to do in our present arrangement. Likewise the divergence of the beam is at least approximately correct under these conditions. This was checked by a group of concentric pick up disks which enabled us to measure the beam profile. No precision can be obtained from this sort of measurement, since with the beam striking more than one disc, the actual value of the secondary emission coefficient enters into the calculations. The results do indicate, however, that under space-charge limited conditions, the opening angle is only about 2 degrees.

At first sight, then, all would appear to be well with our source. The only serious difficulty will come when we attempt to focus this beam into our accelerating tube. Mr. NUSSENZWEIG will report on calculations we have made on this problem. We don't wish to anticipate what he has to say, but in order to complete the discussion of the source it will be necessary to quote the following result of these calculations. That is, that an arbitrarily small extracting voltage does not lead to a focusable beam. There exists a certain minimum extracting voltage below which focus is impossible for a given mechanical arrangement, and a given accelerating tube gradient. Our formula for current then shows us that if we are to maintain a space-charge limited condition, there is a certain minimum focusable current. We can, of course, reduce this current quite a bit by increasing k , but physical limitations prevent k from being very large. If we simply reduce the intensity of the discharge a certain part of the beam will be lost due to defocusing. This occurs because the electrodes are shaped to give rectilinear flow in the extracting region, only for a space-charge-limited beam. Part of the beam will still be collimated, and if the unfocused part does not cause too much trouble in the accelerating tube, then we will have solved our problem.

DISCUSSION

HERB: I would like to see a drawing of your ion source.

SMITH: The figure here shows the ion source, not the one we now have in operation, but the one we designed for the Van de Graaff generator. The anode is a thin tube mounted on a piece of pyrex. The exit-hole is shaped according to Pierce's calculations. (Pointing to probe). This piece is going to be separated and probably mounted by insulators here. This is another subject which we cannot discuss at this meeting, but it is a subject which comes under the heading of ion source pulsing. We hope to be able to pulse this probe and thereby get a very fast changing output beam. As regards the cooling everything you see here, practically is finned, so that you can blow air in here. It follows the arrows around. This cools the coil and all the region where the anode is. The head of the source, containing the filament is finned into a depth almost all the way, so it will be quite effectively cooled and I think our cooling problem is pretty well solved by this.

KERST: If you want to decrease your current can't you merely make the hole smaller?

SMITH: Up to a certain point. But if you are going to keep space-charge limitation these surfaces here have to be shaped precisely. Now already in the new design the hole is 0.8mm. and the machinist is wondering how he is ever going to shape the electrode to get the space charge compensated shape on the right when we can barely see it and I suspect that much smaller than that would really be impossible,

KERST: It is a unique situation when you have too much current.

SMITH: I think that most ion sources give an awful lot more current on the test-bench than they ever do in the machine, so I am not at all surprised at this.

Some Problems in the Design and Utilization of Electrostatic Generators

It. G. HERB

University of Wisconsin, Madison, Wis., U.S.A.

The Wisconsin electrostatic generator has been in operation since 1940 when in test runs it gave a beam of protons at a maximum energy of 4.5 Mev. Although a few components such as the charging belt, V-belts, and pulley bearings have been frequently replaced, most of the original parts of the machine are still in good condition. The generator is still operable at 4.5 million volts if it is clean and in good adjustment.

During its twelve years of use, a series of developments were made for improved energy control of the ion beam from this machine. It now operates in the following manner. The ion beam from the generator first passes through a low resolution magnetic analyzer which focuses ions with a particular ratio of charge to mass onto the object slit of a 90° cylindrical electrostatic analyzer which has a maximum resolution of about 5000. The exit or image slits of this analyzer are coupled through amplifiers to a corona point which drains charge from the high voltage electrode of the generator. The voltage of the electrostatic generator is automatically held to a value determined by the electrostatic analyzer.

Voltages supplied to the deflecting plates of the cylindrical electrostatic analyzer are provided by a supply regulated to .01%, which is controlled by the settings of a potentiometer and to vary the energy of the ion beam requires the twist of only one knob.

Because of the ease with which energy of the ion beam may be varied, the machine is well suited for determination of cross section values as a function of bombarding energy. A few examples of excitation functions will be given.

A new electrostatic generator is now being constructed in which insulating support members are made of porcelain cylinders bonded to metal with silver solder. The evacuated accelerating tube of this machine is also to be fabricated in this manner. It will be fabricated throughout so that it can be baked at about 500°C. This tube construction and processing is expected to improve the tube performance, but test results are as yet inconclusive. Assembly and test of the machine should be under way in the fall of 1952.

DISCUSSION

ALBA: What kind of belt do you use?

HERB: We used all sorts of belts in the past, starting with silk ribbons. I don't think we ever used paper, although some people did. We finally settled on ordinary woven cotton.

These belts work pretty well, so for the past ten years we have continued to use them. I think at the moment - a new belt of the same woven cotton, but impregnated with rubber, is being used.

WIGNER: When you spoke of the development of this machine, I was struck by the fact that there are two important general trends that govern this development: improvement of the insulation by increased pressure and subdivision of the voltage. I am wondering what are the limiting factors in these directions. Starting from the oldfashioned Van do Graaff Generator of low pressure, of ordinary gas, no subdivision of voltage and no successive layers of insulators and conductors, you went over to a high pressure gas insulation and threefold subdivision of the voltage. Why did you stop at the point where you stopped and did not use solid insulators, a manifold subdivision of the voltage, etc.

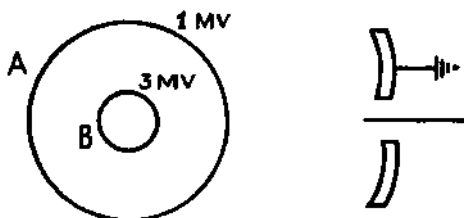
HERB: The gas insulator can be made extremely good. With high pressure sulphur hexafluoride, 1,000,000 volts per inch can be maintained. So, let us say, that is no problem. Solid insulators, if properly designed are no problem up to close to 2,000,000 volts per foot. At both M.I.T. and Los Alamos tiny machines were built in recent years giving four million volts across less than two feet. It turns out that the accelerating tube is so far behind that other problems are just insignificant.

WIGNER: How does the tube fail?

HERB: We don't know. It is probably best to talk about the situation where we don't have an ion beam coming down the tube. When voltage is put across the tube, an X-ray monitor may show the presence of an electron current which increases as voltage is increased. At Wisconsin we happened to pick dimensions such that this effect did not occur. Several machines having tubes of larger diameters have been limited in voltage by this effect. We must somehow have coupling from the top of the tube to the bottom; something must generate those electrons. The fields are far too low to pull many electrons out. We have electrons going up and we now know that there are positive ions going down. But we don't know how electron impinging on surfaces can generate enough positive ions to cause regeneration. There are a number of other effects such as the Malter effect where an insulating layer on a metallic surface picks up a charge and gives yields so high that large numbers of electrons are pulled from the metal. Hydrogen negative ions may play some role in the breakdown. Cranberg at Los Alamos has postulated that loose particles of metal are pulled from the electrodes by the fields. These particles carry sufficient charge so that when they hit an electrode at the other end of the tube, a small mass of metal around the place where the particle penetrates may be raised to a very high temperature. Droplets of metal could be ejected or electrons could be emitted. Attempts to test the validity of these assumptions have not been very successful. Perhaps all of these effects enter to some extent.

SMITH: I don't understand this corona system, because you have three; let us say, plates in your triode and how can the corona current go to anyone of them but the outside one?

HERB: Yes, it will go to the outside one. In recent years we have not used the largest of the 3 electrodes. We have used only the first two. Changing the potential of the outer electrode is satisfactory. If we drain current from it, it drops in potential and everything inside of it follows.



Suppose (A) is the outer one and (S) is the inner one. Suppose the potential of the inner electrode is 1 million volts. Suppose I lower the potential at A by 1 kilovolt, then the potential of electrode B will be lowered by one kilovolt.

Calibration del Generador de Van de Graaff de la Universidad de Mexico

FERNANDO ALBA ANDRADE*

Universidad Nacional de Mexico, Mexico, D.F., Mexico

El generador electrostatico del Instituto de Fisica de la Universidad Nacional de Mexico, fue construido en la High Voltage Engineering Corporation.

Con este generador pueden acelerarse protones, deuterones y electrones, hasta alcanzar una energia de 2 Mev.

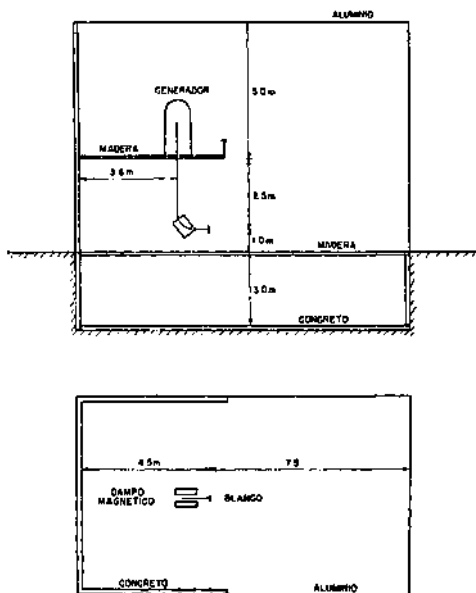


Fig. 1

El aparato consiste de un mecanismo acelerador, un campo magnetico selector y la camara del blanco.

* Becario del Instituto Nacional de La Investigacion Cientffica.

a) *Mecanismo acelerador*

Este mecanismo se encuentra dentro de un tanque vertical, de 88 cm de diametro interior por 2 m de altura que contiene una mezcla de bioxido de carbon y nitrogeno a presion.

Para medir el potencial a que se encuentra la terminal de alto voltaje, se utiliza un voltimetro generador.

El tubo acelerador, esta formado por anillos alternados de metal y pyrex. El vacio en la parte interior del tubo es del orden de 10^{-5} mm. de Hg.

b) *Campo magnetico selector*

El campo magnetico separa a los iones de una misma energia, de acuerdo con sus masas. Las particulas que van a ser usadas como proyectiles, salen por un tubo horizontal a la camara del blanco, y los iones con masa doble, salen por un tubo inclinado, utilizandose para mantener constants el potencial de la terminal de alto voltaje.

La corriente maxima en el tubo acelerador es de 25 micro-amperes, y en el blanco de 1.5 micro-amperes.

c) *Camara del blanco*

En la camara del blanco pueden colocarse 3 diferentes isotopos por estudiar, pudiendo exponerse cualquiera de ellos con un movimiento de rotacion.

LOCALIZACION DEL APARATO

Ya que el aparato va a ser utilizado en el estudio de las reacciones deuterón-neutrón, se construyo un laboratorio en el que el flujo de neutrones dispersados no sea muy grande. El mezzanine y el piso son de madera, soportadas por viguetas de fierro, el techo y la mayor parte de las paredes son de aluminio (Figura 1).

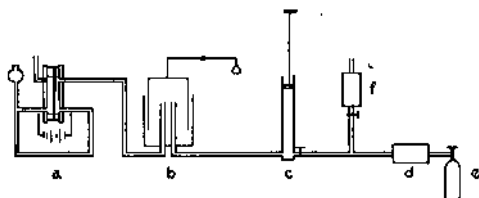


Fig. 2

- a) Aparato de electrolisis.
- b) Gasómetro.
- c) Bomba de mano.
- d) Trompa de aire líquido.
- e) Tanque.
- f) Bomba de vacío.

La calibration del voltimetro generador se hizo utilizando la reaccion ${}^7_3\text{Li}^+ (p,n) {}^7_4\text{Be}^+$, que tiene una energia umbral de 1.88 Mev, los neutrones emitidos se hicieron pasar a traves de parafina, incidiendo despues sobre una placa de plata.

Los electrones emitidos por la plata radioactiva, fueron registrados con contadores Geiger.

La medición del campo magnético entre 500 y 6500 gauss, en función de la corriente, se efectuó con un aparato de bobina rotatoria y con un aparato de inducción nuclear.

Para comprobar la estabilidad de la corriente eléctrica que produce el campo magnético, se dejó trabajando solo al aparato, observándose que en la región de 5000-6000 gauss, $\Delta H \sim 1$ gauss en intervalos de 1 hora, que corresponde a $\Delta E \sim 1.000$ e.v.

Como no fue posible comprar directamente deuterio, se construyó un aparato metálico de electrolisis que produce 1 litro de deuterio en 10 minutos, (Figura 2).

El aparato consiste de 2 cuadros de acero inoxidable de 20 cm. por lado, separados por 2 hojas de hule del mismo tamaño con orificios de 1 dm^2 . Estas hojas de hule van separadas por una tela de asbesto y la mitad superior del orificio con una tela de hule.

Con este aparato de Van de Graaff, van a estudiarse inicialmente las reacciones (d , n) por medio de emulsiones nucleares y contadores de neutrones.

The Problem of Electrostatic Focusing of the Ion Beam in the Van de Graaff Accelerator

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The problem to be discussed in the present communication is that of obtaining a conveniently focused ion beam inside the accelerating tube of the Electrostatic Accelerator of the Universidade de Sao Paulo. The projected ion source has been described in another paper. It was shown in that paper (SMITH & SAIA, 1953) that the type of ion source to be used in the Accelerator requires two important conditions to be fulfilled, namely:

- 1) The source is to be operated under space-charge-limited conditions;
- 2) The initial accelerating field used to extract the ions may vary from its smallest value, which corresponds to minimum beam intensity, to some much higher value.

In the focusing process most commonly followed, this "extracting field" acts in the region between the exit hole of the ion source and an electrode kept at a lower potential, called the "probe". In the region between the probe and the entrance hole of the accelerating tube, the beam is focused by a "focusing field" and injected into the accelerating tube, where it undergoes the action of the "accelerating field". Therefore there are three electrodes to be considered: the "anode" at the exit hole of the ion source, the "probe", and the "cathode" at the entrance to the accelerating tube. We do not think that this arrangement can be further simplified by lessening the number of electrodes.

The parameters which have to be adjusted in order to give a conveniently focused beam are the distances and the potential differences between these electrodes. Ordinarily, this adjustment is made in an empirical way and does not always yield satisfactory results. The only theoretical discussion of the focusing conditions which is known to the authors is that given by MCKIBBEN, in which he studies the focusing effect of the converging lens formed by the aperture at the entrance to the accelerating tube, assuming the beam to have a known fixed source. However, as the probe will in general be at a different potential from that of the other two electrodes, there will be another lens formed at the probe. Since the effect of this lens will be exactly to change the position of the source of the beam, it seems to us that it is an unnecessary sacrifice of precision to neglect its existence.

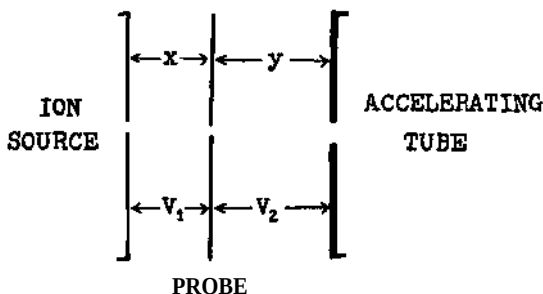
In this discussion we are going to take into account both the "first lens" formed at the probe aperture and the "second lens" at the entrance to the accelerating tube.



Parallel flow of the ions between the exit hole of the ion source and the probe may be obtained in the usual manner, by shaping the electrodes according to PIERCE (1940), in order to overcome space charge repulsion in the region where the energy of the ion beam is still very low. After this region we neglect space charge effects, although they will certainly have an effect at high currents.

The parallel ion beam is rendered converging or diverging by the first lens, giving a real or virtual image at its second focal point. This may then be considered as a point object for the second lens. If we want to have a parallel beam inside the accelerating tube, this point must coincide with the first focal point of the second lens.

The focal lengths can be calculated by means of the usual formula for circular aperture lenses: $f = 4\varphi/(E_1 - E_2)$. Let x and y be the distances and V_1 and V_2 the potential differences, respectively, between the anode and the probe, and between the probe and the cathode, so that all of these parameters are positive. Let b be the uniform field inside the accelerating tube.



The condition giving parallel beam inside the accelerating tube may then be found by equating the focal length of the first lens referred to the coordinate system of the second lens to the focal length of the second lens:

Equation (1):

$$Y + \frac{4 V_1}{\frac{V_1}{x} - \frac{V_2}{y}} = \frac{4 (V_1 + V_2)}{g - \frac{V_2}{y}}$$

In order to simplify the discussion of this equation, we define the dimensionless quantities:

Equation (2):

$$\beta = \frac{x}{y}; \alpha = \frac{g}{V_1/x} = \frac{\text{Accelerating field}}{\text{Extracting field}}; \gamma = \frac{V_2/y}{V_1/x} = \frac{\text{Focusing field}}{\text{Extracting field}}$$

The following relation may then be derived from equation (1):

Equation (3):

$$\gamma = \frac{\alpha + 5}{10} \left\{ 1 \pm \sqrt{1 - \frac{20}{(\alpha + 5)^2} [\alpha(1 + 4\beta) - 4\beta]} \right\}$$

This equation represents a family of hyperbolas depending on the parameter β .

The equation of the asymptotes to these hyperbolas are:

Equation (4):

$$\gamma = 1 + 4\beta \quad \text{Parallel to the } \alpha - \text{Axis}$$

Equation (5):

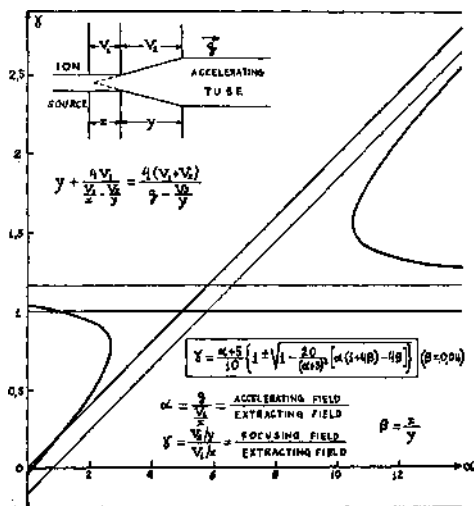
$$\gamma = \frac{\alpha}{5} - 4\beta$$

As β decreases, the left branch of these hyperbolas moves further to the right, and for the limiting case $\beta = 0$ we have the equations of two straight lines:

Equation (6):

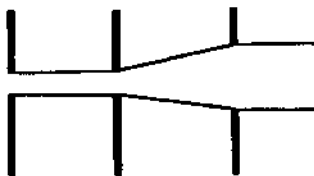
$$\gamma = 1; \gamma = \frac{\alpha}{5}$$

A graph giving one of the hyperbolas of the family, drawn for the value: $\beta = 0.04$, together with its asymptotes and the two straight lines, of equation (6), will now be shown.



We shall divide each hyperbola into three distinct regions, according to the type of solution which they correspond to. The left branch gives two of these regions, one of them being that part of the branch which is under the straight line $\gamma = 1$ and the other being that part which is above. The third region is the whole of the right branch.

The first region, which comprehends the greater part of the left branch, is that for which $\gamma < 1$, so $V_2/y < V_1/x$ and therefore the first lens is *diverging*. It is easily seen that for the whole of this region $\gamma < \alpha$, so $V_2/y < y$ and therefore the second lens is *converging*, as it must necessarily be in this case in order to give a parallel beam. The trajectory of the ions is as follows:



These solutions may be used for normal operating conditions of the Accelerator. For the value of 0 we have chosen and in the region of high beam intensity, they lead to values of the potentials which do not greatly differ from those given by MCKIBBEN. This is only what should be expected, since it is under these conditions that MCKIBBEN'S assumptions are more nearly approached. The difference is principally due to the fact that in MCKIBBEN'S work the source is considered as being fixed, whereas according to our method its positions depend on the values of the applied potentials.

As it is this type of solution that is probably going to be used for all the work to be done with the Accelerator, we shall return to its consideration after examining the remaining types of solution.

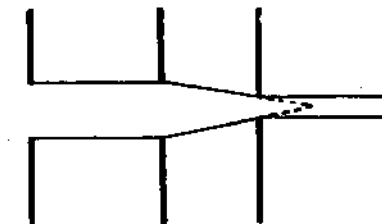
The second region on the left branch is that for which $\gamma > 1$, so that it makes the first lens *converging*. As in this region $\gamma < \alpha$, we have $\alpha > 1$, so the second lens is *diverging*. It is physically evident that this case must correspond to a virtual image formed by the first lens after the entrance of the accelerating tube. This may also be seen analytically in the following way. The focal length of the first lens is given by:

Equation (7):

$$f_1 = \frac{4x}{1 - \gamma}$$

For the mentioned position of the image to be obtained, we must have: $\gamma > 1; |f_1| > y$. This gives the condition: $\gamma < 1 + 4\beta$, which is satisfied by points under the horizontal asymptote, so that the result previously obtained is verified.

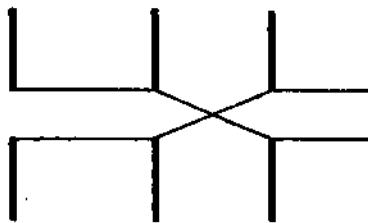
The trajectory of the ions in this case is the following:



In general, this type of solution calls for too high a value of the initial accelerating potential $V_1 + V_2$, but it might conceivably find use under exceptional conditions (such as extremely low gradient in the accelerating tube).

The right branch of each hyperbola gives $\gamma > 1$, and it may be seen at once that on this branch $\gamma < \alpha$, so that these solutions make both the first and the second lens *converging*. Therefore they must correspond to a crossover of the beam. Precisely in the same way as in the preceding discussion, we may show that this in fact is the case, the crossover condition $\gamma > 1 + 4\beta$ being satisfied by all the points of this region.

The trajectory of the ions is therefore:



In general the crossover is to be avoided, but it is possible that these solutions be useful for focusing low intensity beams.

We shall now examine two problems which are of great importance for the design of the actual focusing system. In the whole of the following discussion, we shall restrict our consideration to solutions which belong to the *first region* of the hyperbolas.

The first of these problems is to determine the "magnification", that is to say, the ratio of the final radius of the beam to its initial radius, which is given by the radius of the exit hole of the ion source. The maximum value of the magnification has to be known in order to decide how large the radius of the aperture at the entrance to the accelerating tube must be.

The final radius of the beam may be found by integrating the trajectory of the ions from the probe to the cathode. In cylindrical coordinates, it is given by:

$$M = \frac{2 \left(\frac{dM}{dz} \right)_0 V_1}{E_2} \left[\left(1 + \frac{E_2}{V_1} y \right)^{\frac{1}{2}} - 1 \right] + M_0$$

where M_0 is the initial radius, $\left(\frac{dM}{dz} \right)_0$ is the initial slope and $E_2 = \frac{V_2}{y}$ is the focusing field. (PIERCE, 1949).

The magnification $M = \frac{M}{M_0}$ is then found to be:

Equation (8):

$$M = \frac{1 - \gamma}{2\gamma} \left[\left(1 + \frac{\gamma}{\beta} \right)^{\frac{1}{2}} - 1 \right] + 1$$

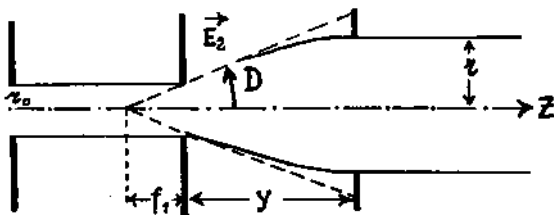
For small values of γ , this reduces to the value we should have found by assuming a rectilinear trajectory of the ions:

Equation (9):

$$M = \frac{1 - \gamma}{4\beta} + 1$$

The magnification has its maximum value when $a = 0$ (the corresponding value of γ being given by equation (3) and decreases when β increases. For $\beta = 0,04$ the maximum value is approximately 10. As we intend to make the exit hole of the ion source 0,8 mm in diameter, this would give a diameter of 8 mm to the entrance hole of the accelerating tube.

The other important problem for the design of the focusing system is the choice of the value of f .



The first restriction that we find to this choice is that, if the thin lens conditions we have assumed are to be obeyed, all the ions must have *paraxial* paths upon leaving the probe, which means that the "angle of divergence" D they make with the axis ought to be smaller than about 10 degrees. At first sight, it appears that paraxial conditions can exist only for large values of θ , since it is for these values that the magnification is smallest and therefore the beam is least

diverging. However, for small values of β , y is very large as compared to x , and, as the focal length of the second lens is always bigger than y , the increase of y compensates for the increase of the magnification, so that we may still have paraxial conditions for small values of β . For $\beta = 0,04$ and $x = 0,6$ cm, if we make the diameter of the exit hole of the ion source 0,8 mm, the divergence of the beam will always be smaller than 2 degrees.

On the other hand, we have good reasons for choosing as small a value for f_1 as possible. In fact, we have already said that, as β decreases, the left branch of the hyperbolas moves further to the right, so that the maximum value of a for which a solution still exists increases. Let α_{max} be this value of a , which corresponds to the point farthest to the right on the left branch. α_{max} is easily found to be given by:

Equation (10):

$$\alpha_{max} = 5 + 40\beta - 4\sqrt{20\beta(1+5\beta)}$$

The corresponding value of γ , which we shall call γ' , is:

Equation (11):

$$\gamma' = \frac{\alpha_{max} + 5}{10}$$

If we take g at its maximum (which for us will be about 13 KV/cm), the point α_{max} corresponds to the minimum value that the extracting field V_1/x may take if a solution is always to exist for all the values of the accelerating field. If β decreases, α_{max} increases, so, if we keep g constantly at its maximum, the minimum of V_1/x decreases. We want this minimum to be as small as possible, for two reasons: 1) because this makes it possible to focus the beam with lower extracting potentials, and therefore with lower beam intensity, than otherwise; 2) because the smaller the minimum of the extracting field is, the larger range of values it may take, and this makes the values of all the parameters less critical, therefore permitting us to have a better and more accurate control over the whole focusing system. For all these reasons we see that we ought to choose the smallest possible value for β .

But can we make β as small as we like? To begin with, this depends on how large we can make the total potential difference $V_1 + V_2$. In our case, this potential difference will be given by a power supply, the maximum rating of which is 60 kV. Clearly this restricts the values that the focusing field V_2/y may take to a well-defined range. There is a definite maximum value for this field and the bigger we make the distance y the smaller this maximum is. Now, for mechanical reasons, as well as the danger of arcing we cannot make the distance x smaller than about 0,6 cm. So, if β is very small, y must be very large. This means that for very small values of β the maximum value that the focusing field V_1/y may take is also very small. Then, even if we take V_2/y at this maximum, for the maximum value 13 KV/cm of the accelerating field the converging effect of the second lens will be very strong. As a consequence of this fact, it follows that the extracting field V_1/x cannot be made smaller than a certain value, otherwise it will make the diverging effect of the first lens so weak that the

second lens is too strong for it and gives a converging beam instead of a parallel one inside the accelerating tube. For practical reasons, then, as we make β smaller and smaller, the minimum value that V_1/x may take increases. Let $(V_1/x)_{\min}$ be this value. Then, $\frac{G_{\max}}{(V_1/x)_{\min}} = \frac{13 \text{ kV/cm}}{(V_1/x)_{\min}}$ gives the greatest value of α which we may use on a given curve, and this value decreases with β .

So we see that there are two entirely different reasons why V_1/x has a minimum, for a given value of θ . One of them is a theoretical reason: the fact that there are no solutions on the left branch of each hyperbola for $\alpha > \alpha_{\max}$; the other is the practical reason we have just seen, which is due to the limited range of our power supply. For very small θ , the practical limitation predominates over the theoretical limitation and drastically restricts the range of variation of V_1/x , so that we again have the same inconveniences which we found for the case of large values of β , where the theoretical limitation predominates over the practical one.

If we do not want this limitation and at the same time want to choose β as small as possible, then there is a definite value of β that we must choose. This is the value for which the theoretical minimum of V_1/x , coincides with the practical minimum. In other words, for this value of β , we can only attain the point $a_{\max}$ on the corresponding curve by making $V_1 + V_2$ equal to the maximum voltage given by our power supply. It is the smallest value of β for which we may use all the solutions given by the curve, from the point corresponding to the maximum intensity of the beam that the machine can handle to the point corresponding to the minimum intensity of the beam for which a solution still exists.

In order to calculate what this value of β is for our case, we take the most unfavorable condition for the accelerating field, that is, $g = 13 \text{ KV/cm}$, and as we want to find the smallest possible β , we take the smallest possible value for x , which as we said is 0,6 cm. The condition that $V_1 + V_2$ shall be equal to 60 KV at the point $\alpha_{\max}$ then gives the following equation:

Equation (12):

$$\gamma' = \frac{\left(60 - \frac{13 \times 0,6}{\alpha_{\max}}\right) \beta}{\frac{130 \times \beta}{\alpha_{\max}}}$$

where γ' is the value of γ at the point $\alpha_{\max}$.

Using equations (10) and (11) to eliminate $\alpha_{\max}$ and γ' , we find an equation in β which gives the value we are seeking. This is found to be approximately 0,039. That is the reason for our choice of $\beta = 0,04$.

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DISCUSSION

J. MARSHALL: Will you be able to vary the potentials on these focusing devices while the machine is in operation and if so, how.

SALA: We can vary the whole range from zero to 60 kilo-volts.

J. MARSHALL: What do you use to adjust it?

SALA: We have a variac and then with strings or with a light beam we can adjust voltages from outside the tank.

NUSSENZVEIG: Of course, the choice of Beta which we made gives a very wide range of variation for the first extracting field and this as I pointed out has both the advantage that we can focus lower intensity beams than otherwise and that all our measurements are less critical than otherwise.

MEDINA: This is a theoretical question, considering that the value of Beta, this ratio of x over y you have considered, is quite small and the distance from the focusing system to the image is probably quite large. As far as your calculation is concerned it seems to me that you have used what is called in optics a *zeroth* order calculation. Don't you think that a consideration of second order effects could alter quite considerably your conclusions? In other words, have you made an estimation of the second order effects for this particular system you are proposing?

NUSSENZWEIG: No, I have not made it, because the whole of this work was an approximation. Generally, people do not give much attention to the focusing system in Van de Graaff generators and sometimes it is quite difficult to focus, so that even a very simple theory as this one would be quite helpful, I think, considering that there are already several approximations involved; as, for instance, the fact that the formula for the focal length is an approximation: the conditions are not exactly the same since we have a space-charge-limited beam here. We had not thought it worthwhile to consider further corrections. We think that we shall see empirically what this will give before that.

VALLARTA: It does not seem very safe under the circumstances in which you are working to neglect the higher effects of the second order and beyond — because of the limitations that you have already placed on your design. It might be that with such severe limitations as you have here the higher order effects would be important because of the values that you have there.

MEDINA: I do not think it should be very hard to make an estimation of the order of magnitude of the second order effects. I dare suggest that it probably should be done, before taking any practical steps. Of course the final adjustment has to be made experimentally.

SMITH: I would like to ask Or. Medina a question. Do you mean by second order effects, the effects arising from the fact that the rays are not truly paraxial?

MEDINA: No, what I mean by second order effects is the following. If you take an eiconal for the whole system, and you can compute that very simply, in this particular case, I think, and you expand the eiconal in powers of the parameters, then all terms coming from the quadratic terms in the eiconal give you second order corrections. All the effects you have considered come only from the first term of the eiconal expansion. Now, the second term should not be very hard to find in this case. The correction thus found might be serious. There are at least 8 effects that may come into consideration.

SMITH: I am afraid space charge effects will be much more important than any optical corrections.

The Circuit Function-a New Concept in Electrical Network Theory (Preliminary Note)

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A network can be represented in different ways. One representation makes use of functions which give the response of the system to a particular type of forcing, another gives the fundamental parameters, e.g. the structure of the network.

The response may be expressed by the steady state admittance or the indicial admittance. The structure of the system, for circuits with lumped elements, may be given by the parameters of the L - C admittance network of fig. 1 (which is one of the 4 fundamental types of equivalent circuits). Not all admittances, however, correspond to systems with lumped parameters when represented by such a network. An infinite ladder network with discrete elements shows d.e. conductance and absorption. When represented in the way shown in fig. 1 it will lead to a structure* with continuously distributed parameters. This suggests a generalization of such a representation. A distribution function of parameters will be introduced, the knowledge of which gives directly the structure of a circuit that corresponds to the given admittance.

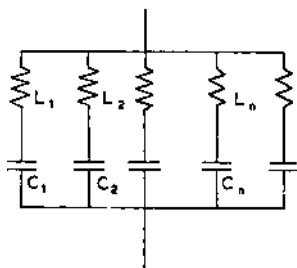


Fig. 1

We call this the circuit function. In consequence of its relation with the structure of the circuit it is a structural function.

I. DEFINITION OF THE CIRCUIT FUNCTION

Consider fig. 1. For the lumped system, the current produced by application of a unit step voltage is given by

$$\sum a_n \sin w_n t$$

with an amplitude

$$a_n = 1/w_n L_n$$

and a frequency

$$w_n = 1/\sqrt{L_n C_n}.$$

Admitting the possibility of continuous systems we shall have an amplitude distribution function such that the amplitude corresponding to systems with frequencies lying between w and $w + dw$ is given by

$$(1a) \quad \rho(w) = \frac{d}{dw} [1/w L(w)]$$

which for discontinuous systems degenerate into

$$(1b) \quad \rho(w) = \sum \frac{1}{w_n L_n} \delta(w - w_n)$$

where δ is Dirac's Delta function.

The indicial admittance $A(t)$ is now given as

$$(2) \quad A(t) = \int_0^\infty \rho(w) \sin wt \, dw,$$

that is as the Fourier transform of the circuit function. Conversely $\rho(w)$ may be obtained from $A(t)$ as an inverse Fourier transform.

II. STEADY STATE ADMITTANCE FROM INDICIAL ADMITTANCE

The current $J(t)$ produced by an arbitrary voltage $U(t)$ is given by the principle of superposition as

$$(3) \quad J(t) = \int_{-\infty}^t \frac{dU}{d\tau} A(t - \tau) \, d\tau.$$

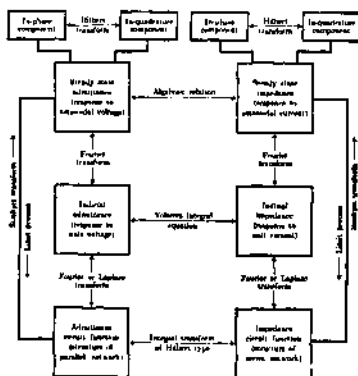


Fig. 2

Considering in particular a voltage

$$U(t) = e^{st}$$

applied at $t = -\infty$, one has

$$(4) \quad J(t) = \left\{ z \int_0^\infty e^{-st} A(\tau) \, d\tau \right\} U(t).$$

The current is proportional to the voltage. The factor of proportionality

$$(5) \quad F(z) = z \int_0^{\infty} e^{-z\tau} A(\tau) d\tau$$

is a generalized steady state admittance. For

$$z = iw + \alpha$$

one gets

$$(6) \quad \lim_{\alpha \rightarrow 0} F(iw + \alpha) = iF(iw) + [\text{Real part}]$$

where $F(iw)$ is the ordinary steady state admittance corresponding to application of a sinusoidal voltage.

III. STEADY STATE ADMITTANCE FROM CIRCUIT FUNCTION

Substituting for $A(\tau)$ in (5) its value (2) one gets

$$(7) \quad F(z) = z \int_0^{\infty} e^{-z\tau} d\tau \int_0^{\infty} \rho(s) \sin s\tau ds,$$

and changing the order of integration

$$(8) \quad F(z) = z \int_0^{\infty} \rho(s) \frac{s}{s^2 + z^2} ds.$$

Introducing $z = iw + \alpha$ this gives

$$(9) \quad F(iw + \alpha) = (iw + \alpha) \int_0^{\infty} \rho(s) ds \frac{s(s^2 - w^2 + \alpha^2) - is 2w\alpha}{(s^2 - w^2 + \alpha^2)^2 + 4w^2\alpha^2}.$$

For $\lim_{\alpha \rightarrow 0}$, the first term gives a principal value, the second term contains a Delta function. It can be shown that

$$(10) \quad \lim_{\alpha \rightarrow 0} \frac{2w\alpha}{(s^2 - w^2 + \alpha^2)^2 + 4w^2\alpha^2} = \pi \delta(s^2 - w^2) = \frac{\pi}{2s} \delta(s - w).$$

And therefore finally

$$(11) \quad \lim_{\alpha \rightarrow 0} F(iw + \alpha) = iw \int_0^{\infty} \rho(s) \frac{s}{s^2 - w^2} ds + \frac{\pi}{2} w \rho(w).$$

The in-quadrature component gives the ordinary steady state admittance

$$(12) \quad F(iw) = iw \int_0^{\infty} \rho(s) \frac{s}{s^2 - w^2} ds$$

and the real component is equal to $\pi w/2$ times the circuit function. $F(iw)$ here appears as an integral over the resonance denominators of the elementary series systems. For lumped systems the behaviour of these functions is shown in fig. 2.

IV. CIRCUIT FUNCTION FROM STEADY STATE ADMITTANCE

It follows now from (11) that

$$(13) \quad \frac{\pi}{2} w_p(w) = \operatorname{Re} \left[\lim_{\alpha \rightarrow 0} F(iw + \alpha) \right].$$

This equation shows that for the sake of obtaining the circuit function, one has to take the expression for the steady state admittance (Fz), substitute z by $iw + \alpha$, and calculate the limit of the resulting expression for $\alpha \rightarrow 0$.

V. APPLICATION: FINITE TRANSMISSION LINE

The steady state admittance of a lossless transmission line of length l , shorted at the far-off end is given by

$$(14) \quad F(z) = \sqrt{C/L} \operatorname{Clgh}(\lambda)$$

where

$$(15) \quad \lambda = \sqrt{LC} z.$$

Introducing $z = iw + \alpha$ and using a known theorem of the theory of hyperbolic functions gives

$$(16) \quad \operatorname{Clgh}(\lambda) = \frac{\sinh 2l \sqrt{LC} \alpha - i \sin 2lw \sqrt{LC}}{\cosh 2l \sqrt{LC} \alpha - \cos 2lw \sqrt{LC}}.$$

Expansion for small values of α gives

$$(17) \quad F(iw + \alpha) = \sqrt{C/L} \frac{l\sqrt{LC} \alpha}{[\sin lw \sqrt{LC}]^2 + [l\sqrt{LC} \alpha]^2} - l\sqrt{C/L} \frac{\sin 2lw \sqrt{LC}}{1 - \cos 2lw \sqrt{LC}}.$$

Thus one has for the steady state admittance

$$(18) \quad F(iw) = -i \sqrt{\frac{C}{L}} \frac{\sin 2lw \sqrt{LC}}{1 - \cos 2lw \sqrt{LC}}.$$

But

$$(19) \quad \lim_{\alpha \rightarrow 0} \sqrt{C/L} \frac{l\sqrt{LC} \alpha}{[\sin wl \sqrt{LC}]^2 + [l\sqrt{LC} \alpha]^2} = \pi \sqrt{C/L} \delta[\sin wl \sqrt{LC}] \\ = \pi \sqrt{C/L} \frac{\delta(w - w_n)}{\left. \frac{d}{dw} \sin wl \sqrt{LC} \right|_{w_n}}$$

where the w_n are the roots of the equation

$$(20) \quad \sin wl \sqrt{LC} = 0.$$

Thus

$$(21) \quad w_n = \frac{n\pi}{l\sqrt{LC}}.$$

Since

$$(22) \quad \left. \frac{d}{dw} \sin w t \sqrt{LC} \right|_{w_n} = t \sqrt{LC} \cos w_n t \sqrt{LC} = t \sqrt{LC},$$

one has finally

$$(23) \quad \rho(w) = \sum \frac{1}{2} \frac{1}{w_n L} \delta(w - w_n).$$

The line is therefore equivalent to a system with lumped parameters. The inductance admittance is

$$(24) \quad A(t) = \frac{1}{2} \sum \frac{1}{w_n L} \sin w_n t.$$

VI. GENERALIZATIONS

These formulae can be generalized:

a) The calculations were made with admittances. An entirely analogous procedure is of course possible on the impedance side, giving a circuit function that corresponds to the structure of another fundamental type of equivalent circuit.

b) In the present paper purely reactive networks were considered. The method however applies also to R-L and R-C circuits.

c) An important case is that of reactive networks with lossy elements. Provided all coils have the same factor of merit, and all condensers the same loss factor, this case can also be treated without much difficulty.

c) Use can be made of the relations that exist between impedances and admittances. It can then be shown that a very general relation holds between the 2 circuit functions, that for admittance system and that for impedance system. With the help of this relation a series circuit can be transformed directly into its equivalent parallel circuit, and conversely. Fig. 2 shows the scheme of relations to which one is led in this way.

DISCUSSION

K. FRANZ: I should like to mention that a function similar to but not identical with Dr. Gross' interesting circuit function has been applied to circuit theory by Caner in Germany and Gonzalez Dominguez in the Argentine. The relation between the two functions is that Caner's function is the integral of Dr. Gross' function and instead of the formula which gives a relation between the impedance $F(Z)$ and the circuit function $P(S)$ there appears a Stieltjes integral; the mathematical basis of it is a general formula demonstrated for instance by Herglotz which allows the representation of any positive function by a Stieltjes integral of this type. Since impedances are positive functions, that is, functions regular in the right half plane and with positive real part in it, this formula is valid for the general impedance, be it lumped or distributed.

GROSS: I should perhaps mention that I have found it more easy to work with 5-functions than with Stieltjes integrals. Actually in the present case both treatments probably would be more or less equivalent.

FUOSS: I hope that you are planning to continue the analysis, especially for the RC circuits, on account of their great value in studying dielectric systems. I imagine that is also on your program.

GROSS: Actually this is one of the points which is on my program. We tried to make similar treatments for general damped systems that would apply to dielectrics and to visco-elastic bodies. The treatment of visco-elasticity is more or less analogous.

KEGEL: You said that it seems to be possible to work with two parameters, let us say LC , ItC and LR . Is there a possibility to apply this method to, let us say, lossy lines, where we have very specified loss for each coil and each condenser?

GROSS: Of course, it would be very interesting to obtain a generalization of the method to systems in which each branch has at least three elements, R , C and L . But as far as I can see it doesn't seem possible to treat this completely general case. I would not even be prepared to assure that there is a unique answer in this general case. However, a very important particular case can be treated perfectly well and that is the case in which one has lossy elements, that is losses in coils and losses in condensers, but in such a way that all coils have the same factor of merit and all condensers have the same loss factor. This is the case I am working on now. I may just point out how it can be treated. In consequence of the fact that all systems have the same damping, the indicial admittance has now the form

$$A(t) = e^{-kt} \int_0^{\infty} p(w) \sin wt \, dw$$

that is we get simply an additional factor e^{-kt} . What we have now to do is try an inversion of this equation. That, of course, is somewhat more complicated but it can also be solved. I could give equations very similar to those given to-day, but it would be to go too far to mention them here. In any event they can be treated and the relations between the circuit functions can be obtained.

FRANZ: I am quite convinced that the principal formulae of this theory are absolutely general and by no means restricted to reactances. May be that the evaluations are more complicated, but the formulae are still valid.

GROSS: I should be very glad to obtain the general relations.

Multiplicador Electrónico en Base a Impulsos Modulados en Duración y en Amplitud

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Nos hemos planteado el problema de desarrollar y de estudiar circuitos electronicos de funcionamiento equivalentes a las operaciones matematicas fundamentales, con fines de obtener datos para el proyecto y la construcción de maquinas de calculo del tipo analogia. Mientras que las operaciones lineales como la adición o la integración, la sustracción o la diferenciación admiten realizaciones por medio de dispositivos que contienen unas pocas valvulas, las operaciones no lineales como la multiplicación exigen circuitos algo mas complicados. En esta comunicación presentamos un multiplicador con 15 valvulas que efectúa mil multiplicaciones por segundo con una precisión del uno por ciento. Los dos factores del producto a evaluar, representados por dos tensiones independientes de entrada, pueden variar entre un pequeño valor positivo y una eota superior del mismo signo. Conviene combinar un multiplicador de este tipo sobre todo con aquellos equipos como los radares, que suministran los datos de entrada del calculo ya en forma de impulsos, aim que no sea necesario utilizar el dispositivo en tal conjunto.

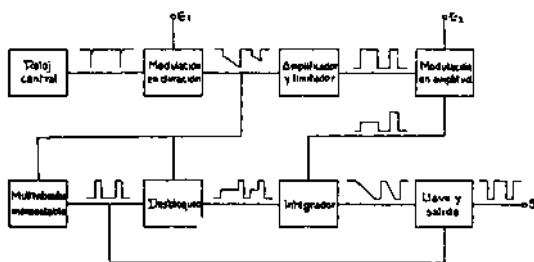


Fig. 1

El circuito estudiado genera mil impulsos rectangulares por segundo, cuya duración es proporcional a una de dos tensiones independientes de entrada y cuya amplitud es proporcional a la otra tensión de entrada. Por lo tanto la carga correspondiente a tal impulso es proporcional al producto de las dos tensiones de entrada. Acumulando la carga en un capacitor, se obtiene en sus bornes una tensión, cuyo valor al final de cada impulso es proporcional al producto de las dos tensiones de entrada. Durante un breve intervalo de tiempo mante-

nemos esto valor de la tensi^om, representative del resultado del calculo, y lo reproducimos en la salida del equipo. Antes de generar un impulso nuevo, borramos el resultado, descargando el capacitor. Queda asi el circuito en condiciones de efectuar otra multiplicacion independiente.

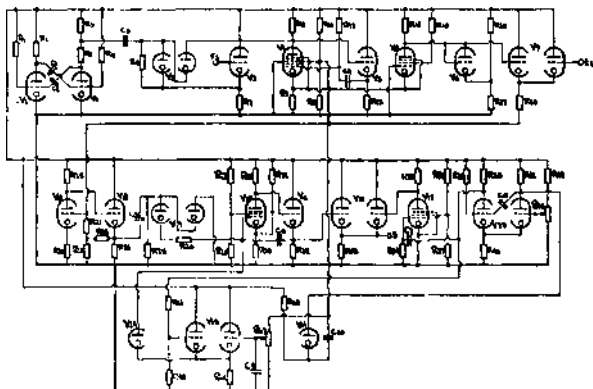


Fig. 2

En la bibliografia acesible a los autores se ha publicado el circuito de un multiplicador con 8 valvulas que rectifica impulsos modulados en duracion y en amplitud, formando asⁱ un promedio sob re muchos impulsos, mientras que nuestro dispositivo realiza una multiplicacion independiente por impulso (GREENWOOD, HOLDAM & MACRAE). Recien aparecio otra publication sobre un multiplicador en base a impulsos, el que electua 400 multiplicaciones por segundo con la precision del uno por ciento por medio de 12 valvulas (*Proc. Inst. Had. Kng.*, 1952). Piste dispositivo utiliza la descarga exponencial en circuitos RC, Haves eletronicas y eomparadores de amplitudes. Mencionamos finalmente la multiplicacion por medio de coincidencia de series de impulsos (CHANCE *et al*).

A continuation exponemos la estructura general de nuestro circuito. ilustrada por el grafico 1. Se llama reloj central a un dispositivo cuya funcion es determinar el ritmo de las operaciones de una maquina de calculo. El nuestro genera una serie de impulsos breves negativos con la frecuencia de mil por segundo, los llamads triggeres, que sirven para arrancar un circuito fantastron. El fantastron monoestable reacciona al trigger produciendo otro impulso, cuya duracion es proporcional a la tension aplicada al borne de entrada E_1 . Realizamos asi la requerida modulation de duracion. Siendo norectangular el impulso del fantastron, se lo amplifica y lo corta en un nivel, dado por la segunda tensintin de entrada aplicada al borne E_2 . Resultan puds impulsos rectangulares modulados en duraeitfn y en amplitud; falta integrarlos. Al impulso rectangular aplicado al integrador, corresponde en la salida del mismo un descenso lineal de tension cuyo valor al final del impulso es proporcional a su area. Este valor final de la tension representa el resultado del calculo y se mantiene durante cierto tiempo. La conservacion de la tension la efectua el circuito de desbloqueo, permitiendo-

al integrador trabajar como tal, mientras que existe o el impulso del fantastron o el impulso generado por un multivibrador monoestable, al cual arranca el salto de tension al final del impulso del fantastron. En ausencia de estos dos impulsos el integrador se bloquea, restableciendose rapidamente su tensiön inieial, que representa el resultado cero. El circuito queda así en oondiciones de efectuar otra multiplicaeion independiente. El resultado del ealculo se conserva en la salida del integrador mientras que existe el impulso de duracirfn fija, y en el mismo intervalo se conecta al integrador la salida definitiva S del equipo por medio de una Have electronica. El resultado esta puds disponible en forma de urn impulso rectangular de tension durante cierta parte fiia del ciclo total de una operacion de la máquina.

Reloj Central
(Multivibrador astable).

1. — Rreja

2. — Placa correspondiente

3. — Salida

4. — Trigger



El esquema completo de nuestro multiplicador electronico esta reproducido en el grafico 2. Nos contentamos con una exposicüm breve de su funcionamiento, refiriendonos para informacirin mas detallada a la bibliografia excelente (CHANCE *et al.*).

El reloj central se compone de un multivibrador astable con dobletriodo V_1 , del circuito diferenciador R_6, C_3 y del dobletriodo V_2 . El multivibrador astable genera en las placas de V_1 una onda cuadrada de mil ciclos con saltos negativos de tension bien definidos, cuya diferenciación en R_6, C_3 da picos positivos y negativos de tension. El dobletriodo V_2 elimina los picos positivos, llegan así al fantastron triggeres negativos con frecuencia igual a la de la onda cuadrada. En las fotografias 3 y 4 se ven la onda cuadrada en la placa de V_1 y los triggeres en un borne de C_3 .

El modulador de duracirn, un fantastron monoestable con acoplamiento catodico, comprende la valvula mezcladora (pentagrid-converter) V_4 y el dobletriodo V_3 . En ausencia de los triggeres el fantastron toma su equilibrio estable con corriente en la primera reja, mucha corriente de pantalla y potencial suficiente de catodo para bloquear la corriente de placa. Al aplicar el trigger negativo a la reja de V_4 a travel de la segunda mitad de V_3 y del capacitor C_4 , se desbloquea

la corriente de placa y V_4 empieza a funcionar como integrador Miller, o sea como amplificador lineal con realimentación negativa por C_4 ; aumenta el potencial de rejá y disminuye el de la placa de V_4 en un descenso lineal hasta que se invierte la fase entre la tensión de rejá y la corriente de placa y se transforma con eso la realimentación negativa en positiva. En el mismo instante la válvula V_4 vuelve a su equilibrio con un salto, posibilitado por la recarga rápida de C_4 a través de V_3 , V_4 y R_9 . El descenso lineal de la tensión de la placa V_4 empieza con un valor dado por la tensión del borne E_1 y proporcional a ella. La tensión mínima y la velocidad del descenso no dependen de la tensión E_1 . La duración del impulso, generado por el fantastron, es proporcional a la tensión de entrada E_1 . En las fotografías 5, 6, 7 y 8 se ven las tensiones de placa, rejá, cátodo y pantalla de V_4 .

Círculo fantastrón.

1. — Placa

← Fig. 5

2. — Reja

← Fig. 6

3. — Cátodo

← Fig. 7

4. — Pantalla

← Fig. 8

En la placa del pentodo V_5 aparecen los impulsos negativos del cátodo de V_4 fuertemente amplificados, de valor máximo limitado por el primer diodo V_6 . En el seguidor catódico con V_7 se limita también el valor mínimo de los impulsos en un nivel proporcional a la tensión del borne E_2 . Resultan así los impulsos rectangulares modulados en duración y en amplitud que se ven en la fotografía 9. Después de una inversión de fase en el amplificador lineal con dobletriodo V_8 y realimentación negativa por el resistor R_{23} se aplican los impulsos a un integrador Miller.

El integrador junto con el circuito de desbloqueo está compuesto del pentodo V_{10} , de la segunda mitad de V_6 , del dobletriodo V_9 y de la primera mitad de V_{14} . Los impulsos rectangulares de signo positivo reaparecen en presentación fiel en el borne del capacitor C_5 conectado al dobletriodo V_9 . La función del dobletriodo V_9 en ausencia es mantener el potencial de C_5 igual a cero y desconectar C_5 del resistor R_{26} , conduciendo la segunda mitad de V_9 y bloqueándose la primera. Al llegar un impulso positivo, la situación se invierte, y se carga el capacitor C_6 a través del resistor R_{26} . Antes de que llegue un impulso en C_5 , la placa de V_{10} toma un potencial elevado y la rejá un potencial bajo, por conducir la primera mitad de V_{14} . Al llegar un impulso en C_5 , se bloquea simultáneamente V_{14} , como veremos más adelante, y el integrador puede trabajar como tab. La tensión de F_{10} disminuye en descenso lineal. La función de V_{10} es mantener casi constante el potencial del borne de C_6 conectado a su rejá, mientras que el otro borne

varia $.4 = 200$ veces mas, siendo A la amplification del pentodo. Se obtiene asi una corriente constante de carga, mientras que existe el impulso rectangular de entrada, a pesar de la variacion fuerte de la tension entre los terminales de C_6 . Las tensiones de placa y de rejilla de V_{10} se ven en las fotografias 10 y 11.

Circuito integrador.

1. --- Impulsos de entrada modulados en duracion y amplitud,

← Fig. 9

2. --- Placa del integrador Miller.

← Fig. 10

3. --- Reja del integrador Miller.

← Fig. 11

4. --- Cátodo del diodo borrador

← Fig. 12

5. --- Salida del equipo.

← Fig. 13

Al terminal* el impulso de entrada, la carga de C_6 queda constante por quedar bloqueada V_{14} durante un intervalo determinado por el multivibrador monoestable con doble triodo V_{13} . Dicha valvula genera un impulso de duracion fija, arrancado por el salto negativo al final del impulso positivo que se obtiene en la pantalla del fantastron F_4 . Es la funcion de C_{10} , R_{48} y de la segunda mitad de V_{14} producir el correspondiente trigger negativo. El impulso positivo de la placa de V_{13} (resistor R_{39}) se ve en la fotografia 14. Al terminar este impulso, vuelve a conducir la primera mitad de V_{14} y se borra el resultado de la integracion a causa de la descarga rapida del capacitor C_6 por V_6 , V_{14} y R_{46} . Todas las variaciones de la tension de placa y de rejilla de V_{10} que acabamos de exponer, se ven en las fotografias 10 y 11, como tambien en la fotografia 12 la tension de desbloqueo en el resistor R_{46} . Es la funcion del seguidor catodico con dobletriode V_{15} reunir los dos impulsos de desbloqueo en R_{46} sin cargar y sin mutuamente acoplar las dos valvulas V_4 y V_{13} .

La parte electronica con dobletriode V_{11} reproduce la tension del integrador en el borne de salida S mientras que existe el impulso de duracion fija del multivibrador monoestable V_{13} , aplicado a una rejilla de V_{11} despues de una amplificacion y inversion de fase en el pentodo V_{12} . Resultan pues en S los impulsos

Conviene agregar algunas observaciones sobre modificaciones posibles del circuito. Para convertirlo en un circuito divisor, podemos generar un impulso rectangular con amplitud proporcional a una tension de entrada. Terminamos el impulso en el instante en el cual su integral sobrepasa un valor dado por otra tension de entrada. La duracion del impulso representa pues el resultado de la division. Es facil obtener una tension de salida proporcional a la duracion del impulso. Este dispositivo tiene la ventaja de no trabajar con realimentacion negativa en regimen lineal; podemos pues prescindir de precauciones contra autooscilaciones.

Para calcular raices cuadradas, podemos generar un impulso de amplitud fija que integramos dos veces. Terminando el impulso en el instante en el cual el resultado de la integracion repetida sobrepasa un valor dado por una tension de entrada, la duracion del impulso es proporcional a la raiz cuadrada de la tension de entrada.

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Applications of the "Polyanalyser"

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The apparatus named "Polyanalyser" by the author is a mechanical synchronous switch, and a selector and combinator circuit, for use with normal cathode ray oscilloscope for the simultaneous study of three electrical phenomena.

The range of frequencies used with this apparatus is 50 to 10000 cycles/seg, tensions between 1 to 500 volts, and currents between 0.01 to 5 amperes.

The selector and combinator circuits permit the use of the apparatus to obtain Lissajous figures or cyclograms, with the respective axis. By this way any desired bipole, in the range of tensions, currents and frequencies indicated, may be studied by the normal oscilogram $V = f(I)$ and $I = f(t)$, or by the cyclogram $I = f(V)$, which gives the dynamic characteristic of the bipole.

Cyclograms may be obtained with a sinusoidal and a direct *d.p.* applied to a system like a vacuum tube (between grid and cathode) and the resultant plate current, giving the $I_p = f(V_p)$ characteristic of the vacuum tube. Applying a rectified tension between plate and cathode and simultaneously and the corresponding plate current, registering the characteristic $I_p = f(V_p)$ may be obtained.

By changing the variables of the system, all the characteristics and the axis may be taken by the polyanalyser and cathode ray oscilloscope.

The possibilities of the apparatus will be demonstrated in various types of bipoles and vacuum tubes characteristics.

DISCUSSION

POMPEIA: Professor Arruda, e possivel registrar maior niimero de fenomenos com sen sistema?

ARRUDA: Quando e necessario registrar-se maior numero de fenomenos, podem-se empregar dois polianalisadores e uma ehave de comutacao de um para outro. Assim podem ser vistos facilmente e fenomenos e ja temos tido oportunidade do trabalhar nessas condicdes.

A menor frequencia e o ndmero de 3 fenomenos foi determinado pelas frequencias industriais 50 ou 60 ciclos/segundo. A 60 ciclos/seg, cada fenomeno e ligado durante 1/50 segundo, com um tempo de interrupyao do 1/450 segundo. Nessas condicdes, pode ser registrado pouco mais de um ciclo na frequencia de 60 ciclos/scg., cm cada tempo de contato

An Electrolytic Tank for the Study of Circuits and Fields

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SUMMARY

A bi-dimensional electrolytic tank is described; a double-sheet tank has been employed, avoiding corrections due to boundary effects, as it has been suggested by Boothroyd, Cherry and Makar. The tank serves a double purpose: bi-dimensional field mapping and analogical circuit computation. Special attention has been given to the techniques of location of polynomial roots, e.g., roots of determinantal equations of networks.

I. INTRODUCTION

The electrolytic tank has been widely employed to map bi-dimensional fields. Attention has been called, more recently (HANSEN & LUNDSTROM, 1945; BOOTHROYD, CHERRY & MAKAR, 1949), to the utilization of the electrolytic tank by means of bi-dimensional field analogies in both the analysis and synthesis of networks.

BOOTHROYD, CHERRY & MAKAR (1949) have studied extensively this particular application of electrolytic tanks. The same authors introduced the double-sheet tank, having thus greatly reduced boundary-



Fig. 1

-effect corrections; they have also determined the best kind of electrolyte for increasing the precision of potential differences measurements between points of the solution. With all these improvements the electrolytic tank has proved a valuable tool to the study of network responses, both in transient and steady state.

We have constructed an electrolytic tank of the mentioned type, which will be described further. As its application to the study of circuits has already been described in detail, we shall discuss here only on the methods for locating the roots of polynomials.

II. DESCRIPTION OF THE TANK AND AUXILIARY CIRCUITS

1. *The Electrolytic Tank.* The bottom of the tank is made of a circular glass plate, with 50,5 cm diameter. Over the periphery of this disc a plywood ring is fixed, impervious-made by immersion in paraffin.

The ring's internal diameter is about 50 cm and its height 3 cm. In order to simplify the local ion of points in the tank, a sheet of graph paper is placed under its bottom.

The electrolytic solution is made of copper sulphate in distilled water (about 6 g/l). The solution is divided into two parts by a circular plate 49 cm diameter and 3 mm wide. This plate is supported 3 mm over the bottom by three insulated blocks (fig. 1). The two sheets of electrolyte have the same width. On the bottom's center there is a cylindrical electrode intended to represent the point at infinity (BOOTHROYD, CHERRY MAKAR, 1949).



Fig. 2

The tank is disposed on a table with a pantographic system. In fig. 2 the tank and its auxiliary equipment are depicted.

2. *The Auxiliary Circuits.* The auxiliary circuits are composed of: a) a generator of oscillations (550 c/s) which furnishes the currents to feed the electrodes, and a switching panel which allows any electrode being connected to any side of the generator's push-pull output transformer; b) a differential voltmeter intended to measure potential differences between points of the field.

a) *Current Generator.* In studying networks or locating roots of polynomials, determined currents must enter or leave the electrolyte sheet by thin cylindrical electrodes placed at convenient points. Furthermore, to avoid polarization and electrolysis effects the currents must be alternative. 550 c/s alternating current has been employed and seems to be a good compromise.

The oscillation is generated by a phase-shift oscillator and then amplified by a power amplifier, both fed by a stabilized power supply (fig. 3). The push-pull output transformer of the power amplifier feeds into a switching board where ten electrodes can be connected on any side of the output transformer by means of switches. The center tap of this transformer is permanently connected to the ground and to the central electrode in the tank's lower sheet.

The current in each electrode can be measured inserting a miliammeter by double-pole, double-throw switches. When the miliammeter is disconnected from a particular electrode, an offset resistance is switched on (fig. 3).

The currents can be regulated by 50,000 ohms potentiometers in series with each electrode.

fr) *Differential Amplifier.* The measuring circuit must perform two functions: zero detection for the plotting of equipotentials and measurement of potential differences between two points of the field, none of which is at the ground potential. In this case a differential voltmeter is needed. The circuit employed is shown in fig. 4.

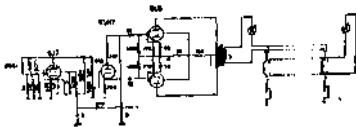


Fig. 3

In the first stage there is a differential amplifier with a constant current pentode inserted in the cathode of the triode amplifier ensuring better insensibility against common potential variations (VALLEY & WALLMAN, 1948). This stage is followed by two frequency-selective amplifiers symmetrically disposed. The output of the selective amplifiers is measured by a high internal resistance multiplescale A. C. voltmeter. When relatively high potential differences are to be measured this voltmeter can be switched on the output of the first stage of amplification. With it, potential differences ranging from 1 millivolt to about 1 volt can be measured.

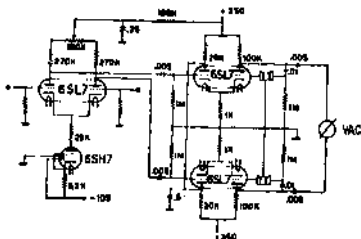


Fig. 4



Fig. 5

The selective amplifiers employ feedback through a twin T network; if the twin T is conveniently unbalanced the band-pass and gain of the amplifier can be adjusted (ORSINI, 1949).

In order to measure the potential gradient in points of the field a double probe is employed, coupled to the input of the voltmeter through a high impedance transformer (fig. 5).

III. LOCATION OF THE ROOTS OF POLYNOMIALS

The determination of polynomial zeroes with the electrolytic tank is based on an analogy due to LUCAS (1888). This analogy is established as follows:

Suppose the polynomial

$$f(p) = a_0 p^n + a_1 p^{n-1} + \dots + a_{n-1} p + a_n = 0 \quad (1)$$

be given, with a_i constant, real coefficients.

Now, forming an arbitrary polynomial of degree $n + 1$ with only single real roots, distinct from those of $f(p)$:

$$g(p) = (p - \alpha_1)(p - \alpha_2) \dots (p - \alpha_{n+1}) \quad (2)$$

the rational function

$$F(p) = \frac{f(p)}{g(p)} \quad (3)$$

which has the same zeroes as $f(p)$ can be decomposed in partial fractions

$$F(p) = \sum_{i=1}^{n+1} \frac{p_i}{p - \alpha_i} \quad (4)$$

Now, as BOOTHROYD, CHERRY & MAKAR (1949) have shown, the double-sheet electrolytic tank can represent all the complex plane, the origin being coincident with the center of the upper sheet. Suppose currents v_i are introduced in this upper sheet through thin cylindrical electrodes disposed on the real axis at the points $(a_i, 0)$. The modulus of the field intensity at any point $p = a + jw$ of the plane is given by

$$|\vec{E}| = k \sum_{i=1}^{n+1} \frac{v_i}{p - a_i} \quad (5)$$

The analogy between (4) and (5) is obvious. Evidently the zeroes of $|\vec{E}|$ will be zeroes of $F(p)$ and also of $f(p)$. So, in order to locate the roots of a polynomial, once formed the corresponding field, we must only search in the tank for the points p_i for which $|\vec{E}| = 0$; the coordinates of these points will be the roots of $f(p)$.

The double probe, already referred to, could be employed in the search for the zero-field points, as BOOTHROYD, CHERRY & MAKAR (1949) have suggested. But it was found more convenient to plot the equipotentials of the field; the zero-field points can then be located by the intersection of equipotentials (JEANS, 1927).

In the practice, before establishing the field (5) in the tank, a convenient scale has to be found so that all zeroes of $f(p)$ are located in the upper sheet of the electrolyte. This corresponds to find an upper restriction to the root moduli of (1). Two simple criteria can be employed at this first stage (WILLERS, 1948; BREVES F°, 1940).

a) no roots can be located outside a circle of radius

$$R = \frac{(a_1 + a_2 + \dots + a_n)}{a_0} \quad (a_0 \neq 0)$$

b) the roots of $f(p)$ are not greater than the greatest of the numbers;

$$\left| \frac{a_1}{a_0} \right| + 1, \left| \frac{a_2}{a_0} \right| + 1, \dots, \left| \frac{a_n}{a_0} \right| + 1$$

Now the numbers fixed by the criteria above are usually very high, so the scale may be too small. But from this first representation of the field we can find a better restriction. In fact, it can easily be seen that

c) no zero-field points can lie outside an equipotential that involves all electrodes without intersecting itself.

Therefore, when such an equipotential has been plotted a new scale can be chosen; the field is then plotted in detail and the points corresponding to the roots can be found. Sometimes a third scale must be chosen, for instance to discriminate two roots located one close to the other. In this case of course only some parts of the field have to be plotted again.

Example — Consider the equation

$$\begin{aligned} f(p) &= p^4 - 3p^3 - 3p^2 + 61p - 156 = \\ &= (p - 3)(p - 4)[p - (2 + j3)][p - (2 - j3)] \end{aligned}$$

Choosing

$$g(p) = p(p - 10)(p - 5)(p + 10)(p + 5)$$

it follows(*)

$$v_1 = -0,0625$$

$$v_2 = -0,0865$$

$$v_3 = -0,1240$$

$$v_4 = +0,4770 \text{ and } v_5 = +0,7910$$

For representing this field five electrodes are placed at the points a , introducing the currents v_i ; the map of equipotentials obtained is shown at fig. 6 from which we get the approximate values of the roots:

$$p_1 \simeq 3,2 \quad p_3 \simeq 2 + j_3$$

$$p_2 \simeq -3,9 \quad p_4 \simeq 2 - j_3$$

These values can now be used as a start point for the determination of more accurate ones by the usual methods in Numerical Calculus.

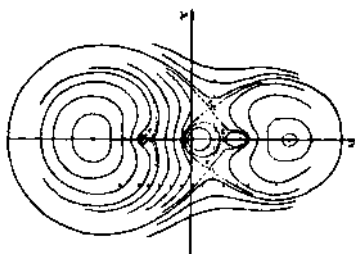


Fig. 6

IV. APPLICATION TO THE DETERMINATION OF NETWORK STABILITY

It is known that the stability of a network can be verified by means of its determinantal equation if the roots of this equation are in the left hand of the complex plane the network is stable.

The determinantal equation can be represented in the electrolytic tank by the analogy indicated above and its roots can easily be located. Moreover, by this procedure we can ascertain without great difficulty the effects that changes in some parameters of the network may have upon its stability.

ACKNOWLEDGEMENTS

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(*) For details to this calculation see VIEIRA (in print).

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DISCUSSION

HEPP: With the small depth of the liquid does not the capillar elevation of the liquid at the electrodes introduce considerable errors?

CAMARGO: We have found the depth of the liquid is sufficient in our case, for there are no sensible modifications in the measurements from a certain point down.

HEPP: Would it not be better instead of using a stabilized tension and a calibrated voltmeter only to use a potentiometer and a zero-detector? In this case the absolute voltage does not come in and the error in the calibration of a potentiometer can be made extremely small.

ORSINI: When locating the roots of the polynomial the variations in currents are all proportional and do not introduce errors in the determination of the roots. The use of potentiometers would complicate the circuitry.

HEPP: Whenever a calibrated voltmeter is used the absolute value of the tension is of course important. The use of a potentiometer short-circuits all the difficulties in stabilization and calibration. With the small depth of the liquid I assume that it is very important to maintain a perfect horizontal position of the tank?

ORSINI: Yes, of course. But we checked horizontally plotting a known field, for instance, an electrode in the center and checking the symmetry.

GOLDBERG: Qual o diametro dos eletrodos usados? Observei que, sendo grande o diametro dos eletrodos, Mes perturbam o campo que se deseja estudar.

CAMARGO: Conheco o fato, mas os eletrodos que usamos eram suficientemente finos para nao perturbarem sensivelmente o campo. Eram do cobre e tinham diametro de 0,5 mm.

WIGNER: What choice of the second polynomial gives the best results? Do you try to choose them close or far from the roots of the first one?

CAMARGO: Capilarity thickness of electrodes are limiting factors for this.

ORSINI: There are yet mechanical difficulties to plotting the field very near the electrodes.

FRANZ: If some or, were roots of the equation the corresponding v_i would vanish: if some roots were nearly coincident the Y_i too would be small, the corresponding electrode would be excited by a very small tension. Answering Dr. Wigner's question I would think it is preferable not to make coincide the roots for better evaluation of the field.

ORSINI: The most favorable case is that in which the real roots are between the electrodes, Mr. Camargo has been charged to organize tables to compute the V_i for chosen ai . If these α_i were not convenient we could change them and obtain the V_i with a little more computation.

KIMBARK: An alternative to an electrolytic tank is a sheet of material of high resistivity, such as a sheet of typewriter carbon paper. This is free from polarization effects and permits the use of a direct current source and indicator. Have the authors considered the use of such material?

ORSINI: In first place, it is not easy to find here a very homogeneous material, in second place, probably for each field one sheet would be needed because it would be difficult to work without scratching the paper.

On the Representation of Orthogonal Polynomials by Transmission Lines (*) (Preliminary note)

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In the present communication we want to show the possibility of representing families of orthogonal polynomials by discrete element transmission lines. The methods developed might be useful for the construction of certain electric calculating machines.

I. A THEOREM ON POLYNOMIALS

We frequently need the following theorem: If $\{U_n(x)\}$ is a set of functions satisfying the equations

$$U_n(x) = (-a_n x + b_n) U_{n-1}(x) - c_n U_{n-2}(x) \quad \text{for } k+2 \geq n \geq k$$

then $U_n(x)$ might always be written as

$$U_n(x) = f(x) p_n(x) + g(x) q_n(x)$$

where $p_n(x)$ and $q_n(x)$ are defined by the equations:

$$p_0(x) = 1$$

$$p_1(x) = -a_1 x + b_1$$

$$p_n(x) = (-a_n x + b_n) p_{n-1}(x) - c_n p_{n-2}(x) \quad \text{for } n \geq 2$$

$$q_0(x) = 0$$

$$q_1(x) = 1$$

$$q_n(x) = (-a_n x + b_n) q_{n-1}(x) - c_n q_{n-2}(x) \quad \text{for } n \geq 2$$

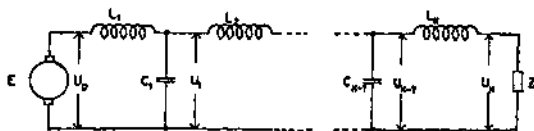


Fig. 1. The structure of the line.

(*) Supported by the Conselho Nacional de Pesquisas.

The proof of this theorem will be omitted. Evidently the $p_n(x)$ and $q_n(x)$ are polynomials of precise order n and $n - 1$ respectively; they play an important role in the theory of continued fractions, for the approximants of certain continued fractions are given by $p_n(x)/q_n(x)$

II. THE TRANSMISSION LINE WITH LUMPED ELEMENTS

Let us now consider a transmission line formed by discrete elements which has the structure shown in fig. 1. We have the following relation:

$$U_0(\omega) = E(\omega)$$

$$U_n(\omega) = \left(1 - \omega^2 L_n C_{n-1} \frac{L_n}{L_{n-1}}\right) U_{n-1}(\omega) - \frac{L_n}{L_{n-1}} U_{n-2}(\omega) \quad \text{for } 2 \leq n \leq k$$

$$U_K(\omega) = \frac{Z(\omega)}{i\omega L_K + Z(\omega)} U_{K-1}(\omega)$$

These equations are obtained applying the mesh- and nodal-equations and are valid but for the steady-state. If we now call

$$a_n^* = L_n C_{n-1}$$

$$b_n = 1 + \frac{L_n}{L_{n-1}}$$

$$c_n = \frac{L_n}{L_{n-1}}$$

$$a_n = \frac{L_n C_{n-1}}{L_1 C_0}$$

with arbitrary C_0 and

$$x = L_1 C_0 \omega^2$$

we get

$$U_n(\omega) = (-a_n^* \omega^2 + b_n) U_{n-1}(\omega) - c_n U_{n-2}(\omega) \quad \text{for } 2 \leq n \leq k$$

and the solution is given by

$$u_n(\omega) = f(\omega) p_n(\omega^2) + g(\omega) q_n(\omega^2)$$

where

$$\varphi(\omega) = E(\omega) \quad g(\omega) = -E(\omega) \frac{\left(1 + \frac{i\omega L_K}{Z(\omega)}\right) p_K(\omega^2) - p_{K-1}(\omega^2)}{\left(1 + \frac{i\omega L_K}{Z(\omega)}\right) q_K(\omega^2) - q_{K-1}(\omega^2)}$$

(We wrote $p_n(\omega^2)$ and $q_n(\omega^2)$ rather than $p_n(L_1 C_0 \omega^2)$ and $q_n(L_1 C_0 \omega^2)$)

III. REPRESENTATION OF POLYNOMIALS BY MEANS OF THE TRANSMISSION LINE

The result obtained shows that the line can be used for generating polynomials related by three member recurrence formulae of the type discussed. In order to show this more precisely let us suppose that we are interested in getting a line furnishing the polynomials defined by:

$$P_0(x) = A_0$$

$$P_1(x) = (A_1 x + B_1) P_0(x)$$

$$P_n(x) = (A_n x + B_n) P_{n-1}(x) - C_n P_{n-2}(x)$$

for $n \geq 2$

In general we must carry out a transformation substituting x by $z - a$ such that all zeroes of the $P_n(z)$ occur for $z > 0$. Also it will be necessary to introduce the polynomials $p_n(z)$ which are given by

$$p_n(z) = \frac{P_n(z)}{P_n(0)} ;$$

so we get

$$p_0(z) = 1$$

$$p_1(z) = -a_1 z + b_1$$

$$p_n(z) = (-a_n z + b_n) p_{n-1}(z) - c_n p_{n-2}(z)$$

for $2 \leq n$

and in this equation the a_n, b_n, c_n are positive numbers. It is now an easy task to calculate the values of the coils and of the condensers. We get from the equations given above:

$$L_n = L_1 \prod_{i=2}^n c_i$$

$$C_{n-1} = \frac{a_n C_0}{\prod_{i=2}^n c_i}$$

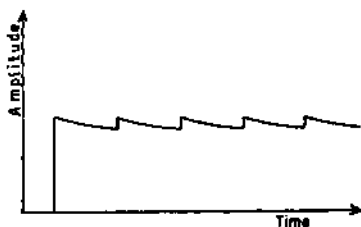


Fig. 2. The amplitude of the tension applied to the line.

the tensions of the nodal points of the line are

$$U_n(\omega) = f(\omega) p_n(\omega^2) + g(\omega) q_n(\omega^2)$$

Since we are interested in the $p_n(\omega^2)$ only one could try to remove the second term. This can be done making $g(\omega)$ equal to zero which calls for a $Z(\omega)$ given by:

$$Z(\omega) = \frac{i \omega L_K p_K(\omega^2)}{p_{K-1}(\omega^2) - p_K(\omega^2)}$$

This expression, however, is very complicated and for this reason the case $g(\omega) = 0$ is of no practical interest.

Another possibility is to make $g(w)$ an imaginary quantity, so that the two terms composing the tension $U_n(w)$ are in quadrature. This might be done by choosing a convenient Z . In this case we can separate the p_n (of) from the $q_n(w^2)$ rectifying the tension $U_n(w)$ with a suitable phase- sensible rectifier.

IV. EVALUATION OF ERROR

In practical application it is important to know the error produced by imperfect condensers and coils. If, for example, the elements composing the line have not exactly the prescribed value we would not get the desired tensions. The following calculation gives in first approximation the error on the tensions as a function of the deviation of the line elements from the required value.

We will first assume that all elements have the right dimensions but that one of the condensers, say C_j , has an admittance $i\omega C_j + \Delta R_j$ instead of $i\omega C_j$. As a consequence of this fact we would expect that all tensions are changed by ΔU_n and we want to calculate these changes. Assuming that products of deltas might be neglected we get:

$$\Delta U_n = - \frac{i\omega L_1 L_{j+1}}{E(\omega) L_j} U_j^2(\omega) q_n(\omega^2) \Delta R_j \quad \text{if } n \leq j$$

$$\Delta U_n = - \frac{i\omega L_1 L_{j+1}}{E(\omega) L_j} U_j(\omega) U_n(\omega) q_j(\omega^2) \Delta R_j \quad \text{if } n \geq j$$

In the same way one can calculate the effect of a coil having an impedance of $i\omega L_j + \Delta S_j$ rather than $i\omega L_j$, in this case we get:

$$\Delta U_n = \frac{L_1}{i\omega L_j L_{j-1} E(\omega)} \{U_j(\omega) - U_{j-1}(\omega)\}^2 q_n(\omega^2) \Delta S_j \quad \text{if } n \leq j$$

$$\Delta U_n = \frac{L_1}{i\omega L_j L_{j-1} E(\omega)} U_n(\omega) \{q_j(\omega^2) - q_{j-1}(\omega^2)\} \{U_j(\omega) - U_{j-1}(\omega)\} \Delta S_j \quad \text{if } n \geq j$$

We see that ΔU_n is always limited.

It is clear that from the expressions given above one can calculate the total variation of the tensions U_n if all elements show deviation from the prescribed values. This enables us to predict the error if we know the tolerances of the elements composing the line. We call attention for the fact that AR_i and AS_j might be complex quantities thus taking into account losses of coils or condensers.

V. LINE WITH LOSSY ELEMENTS

The case of lossy elements constituting the line offers a serious difficulty, for in general it is not possible to reduce the losses to extremely low values. If however the losses are such that $L_n(\alpha_1 + i\omega)$ and $C_n(\alpha_2 + i\omega)$ represent in good approximation the impedances of the coils and the admittances of the condensers

respectively and if α_1 and α_2 are the same for all coils and condensers then there is a possibility to render these losses ineffective

Let us suppose that the determination of the elements of a certain line has been carried out, assuming lossless elements. Let us construct the same line using lossy coils and condensers, whose losses are of the type mentioned above. Finally let the line be determined by an impedance $\bar{Z}(\omega)$ whose characteristic is such that

$$\bar{Z}(\bar{\omega}) = Z(\omega)$$

where $Z(\omega)$ represents the terminating impedance of the perfect line and

$$i\bar{\omega} = \sqrt{(\alpha_1 + i\omega)(\alpha_2 + i\omega)} = \alpha' + i\omega'$$

In this case we would find the following tensions on the line

$$U_n(\omega) = E p_n(\bar{\omega}^2) + g q_n(\bar{\omega}^2)$$

This suggests the question whether an input tension of the complex frequency $(2\omega - \omega') - \alpha'$ would give a response of the form:

$$U_n(\omega) = E p_n(\bar{\omega}^2) = g q_n(\bar{\omega}^2)$$

Really this occurs as one sees calculating the response of the circuit to complex frequencies by means of the Fourier transform. The tensions of complex we are dealing with here are of the form

$$E = E_0 e^{-\alpha' t} e^{i\omega' t}$$

where the time constant α is positive so that it corresponds to a damped oscillation. As the damped oscillation decreases with time it will be necessary to apply at the input a wave of the amplitude shown in figure II.

At the instances where the jump occurs a transient travelling down the line is produced. This transient may be calculated by the Fourier transform, and care must be taken not to use the outputs of the line before the transient has reduced to sufficiently low values. This might be done by an amplifier switched to the output of the line and which is blocked during a certain time after the transient occurred.

VI. EXAMPLE

We mentioned at the beginning that the line might be used for the construction of certain electric calculating machines. As an example we show the possibility of calculating series of the form

$$S = \sum_{i=0}^K \lambda_i p_i(x)$$

by means of the line. We need in this case a line furnishing the polynomials $p_n(x)$. Then we take from each nodal point a certain fraction of the tension U_i getting $\lambda_i U_i$. These tensions must be summed up, which might be done, f.i., by

an electronic circuit like the Rossi-circuit. Finally we must rectify this sum of tensions in order to get rid of the term which contains the $q_n(\omega)$. This rectified tension is then proportional to S , and the variation of the input frequency corresponds to the variation of x , on which depends S .

DISCUSSION

GROSS: IS it possible to find the coefficients of a Fourier series development?

KEGEL: Yes, it is. One can construct a line which furnishes Tchebycheff's polynomials and Tchebycheff's functions of the second kind. One gets these polynomials and functions substituting $x = \cos \theta$ getting

$$U_n = \cos (n \arccos x)$$

$$T_n = \sin (n \arccos x)$$

Now we can make the input tension of the line equal to $f(x)\omega(x)$ where $f(x)$ is the function to be developed and $\omega(x)$ is the weight function. The tensions at the nodal points are then

$$P_n(x) f(x) \omega(x)$$

and it is now necessary to integrate this expression over the interval of orthogonality; this is easier with the use of the line.

Application of the Routh-Hurwitz Criterion to Self-Oscillators

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SUMMARY

The application of Routh-Hurwitz's criterion to the study of the stability of networks is well known; nevertheless, this criterion has been only scarcely employed. In this paper is considered its application to negative-grid self-oscillators.

I. INTRODUCTION

Many self oscillating circuits can be represented by the general scheme of fig. 1. If we suppose that the network is linear, what is possible since here we are concerned only with the stability problem, the scheme of fig. 1 can be replaced by the equivalent circuit of fig. 2.

To determine the conditions of oscillation of this network we have only to consider its response to transients; transient currents will be given by the addition of terms $i_1 e^{p_j t}$, where $p_j = \alpha_j + j\beta_j$ is one natural mode of the network (GUILLEMINN, 1931). If any of the α , is positive the network is *unstable*, i. e., the circuit oscillates; if any of the α , is zero, but none is negative, the circuit is in the *limit of oscillation*.

The equations of the equivalent circuit (fig. 2) can be written at once:

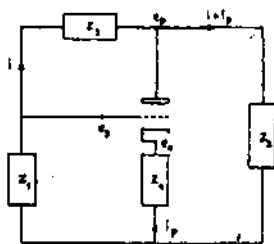


Fig. 2

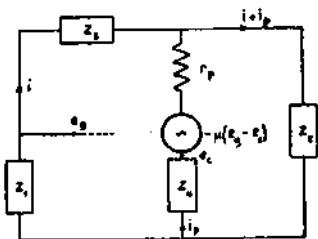


Fig. 1

$$(1) \begin{cases} z_3 i + z_1 i + z_2 (i + i_p) = 0 \\ z_4 i_p + r_p i_p + z_2 (i + i_p) = \mu (e_0 - e_c) \end{cases}$$

where the z 's, functions of the complex variable p , are the generalized impedances of the circuit.

We have also the relations $e_0 = -z_1 i$ $e_c = -z_4 i_p$; re-arranging the equations (1) we get

$$(2) \begin{cases} z_2 i_p + (z_1 + z_2 + z_3) i = 0 \\ [r_p + (1 + \mu) z_4 + z_2] i_p + (z_2 - \mu z_1) i = 0 \end{cases}$$

If this system has a proper solution, then

$$\Delta(p) = \begin{vmatrix} z_3 & z_1 + z_2 + z_3 \\ r_p + (1 + \mu) z_4 + z_2 & z_2 - \mu z_1 \end{vmatrix} = 0$$

or

$$(3) \quad \Delta(p) = f(p)/g(p) = (z_1 + z_2 + z_3)[r_p + (1 + \mu)z_4] + z_2[(1 + \mu)z_1 + z_3] = 0$$

$A(p) = 0$ is the *determinantal equation* of the circuit; by a well known rule (GUILLEMIN, 1931), we can verify that this equation will be, at most, of the fourth degree:

$$(4) \quad a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4 = 0$$

The roots p_j of this equation are the natural modes of the network. So, if the network oscillates, the real part at least of one root must be positive. But, by Routh-Hurwitz criterion (v. MISES, 1930; ROCARD, 1949) the real parts of all the roots will be negative if (supposed $a_0 > 0$) all the following determinants

$$\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}, \quad \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix}, \quad \begin{vmatrix} a_1 & a_0 & 0 & 0 \\ a_3 & a_2 & a_1 & a_0 \\ 0 & a_4 & a_3 & a_2 \\ 0 & 0 & 0 & a_4 \end{vmatrix}$$

are positive. So, to have an unstable network, at least one of these determinants must be negative. From where we arrive to the set of conditions

$$(5) \quad a_1 a_2 - a_0 a_3 < 0$$

or

$$(6) \quad (a_1 a_2 - a_0 a_3) a_3 - a_1^2 a_4 < 0$$

since the condition deduced from the third determinant reduces immediately to (6).

In order to find the conditions of oscillations of a network that can be represented by the general scheme of fig. 1, the following rules must be followed, according to the results above:

a) the impedances $z_i(p)$ are calculated, in the usual way;

b) these values are introduced in equation (3);

c) this equation is reduced to the form (4);

d) inequalities (5) and (6) are formed; these inequalities give the conditions of oscillation.

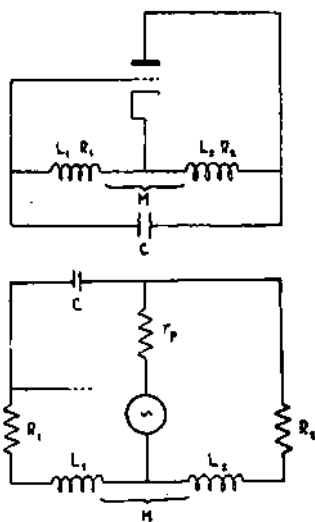


Fig. 3

The application of the Routh-Hurwitz criterion will now be exemplified.

II. HARTLEY OSCILLATOR

The R. F. equivalent circuits of a Hartley oscillator are represented in fig. 3. The generalized impedances are

$$z_1 = (L_1 + M)p + R_1$$

$$z_2 = (L_2 + M)p + R_2$$

$$z_3 = 1/Cp$$

It follows

$$\Delta(p) = [(L_1 + L_2 + 2M)p + (R_1 + R_2) + 1/Cp]r_p + [(L_2 + M)p + R_2] \times \\ \times \{(1 + \mu)[(L_1 + M)p + R_1] + 1/Cp\} = 0$$

Usually we have $R_1, R_2 \ll r_p$; introducing this approximation and re-arranging the equation above it follows:

$$(1 + \mu)(L_1 + M)(L_2 + M)Cp^3 + r_p(L_1 + L_2 + 2M)Cp^2 + \\ + [(R_1 + R_2)r_p C + (L_2 + M)]p + r_p = 0$$

By the condition (5) the circuit will be able to generate oscillations if

$$(L_1 + L_2 + 2M)[(R_1 + R_2)r_p C + (L_2 + M)] - (1 + \mu)(L_1 + M)(L_2 + M) < 0$$

or

$$(L_1 + L_2 + 2M)[(R_1 + R_2)r_p C + (L_2 + M)] < (1 + \mu)(L_1 + M)(L_2 + M)$$

III. ULTRA-AUDION OSCILLATOR

The R. F. equivalent circuit of the ultra-audion oscillator (M. I. T. RADAR SCHOOL, 1946) is shown in fig. 4. We have now

$$z_1 = 1/pC_1$$

$$z_2 = 1/pC_2$$

$$z_3 = (L_3 p + R_3)/(L_3 C_3 p^2 + R_3 C_3 p + 1)$$

Introducing these values in the determinantal equation we obtain:

$$r_p [g_3^2 (C_1 + C_2) + L_3 C_1 C_2] p^3 + [(1 + \mu) g_3^2 + R_3 (C_1 C_2 + \\ + C_3 C_2 + C_3 C_1) r_p + L_3 C_1] p^2 + \{R_3 [(1 + \mu) C_3 + C_1] + \\ + r_p (C_1 + C_2)\} p + (1 + \mu) = 0$$

where $g_3^2 = \sqrt{L_3 C_3}$.

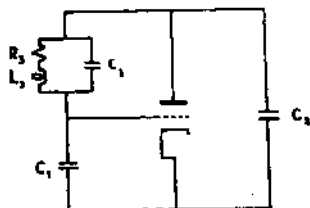


Fig. 4

Since $a_4 = 0$ and $a_3 > 0$, the conditions of oscillation become

$$\left[(1 + \mu) g_2^2 + R_3 (C_1 C_2 + C_2 C_3 + C_3 C_1) r_p + L_3 C_1 \right] \{ R_3 [(1 + \mu) C_3 + C_1] + r_p (C_1 + C_2) \} < r_p [g_3^2 (C_1 + C_2) + L_3 C_1 C_2] (1 + \mu)$$

or, if we introduce the damping constant of the resonant circuit, $\alpha_3 = R_3/2 L_3$, we get

$$\left[\frac{1}{2} (\mu C_3 + C_1) + \alpha_3 (C_1 C_2 + C_3 C_2 + C_1 C_3) r_p \right] \times \left[\alpha_3 (C_3 + C_1) + \frac{r_p}{2} \frac{C_2 + C_1}{L_3} \right] < \frac{r_p}{4 L_3} [C_3 (C_1 + C_2) + C_1 C_2]$$

supposing yet $\mu \gg 1$.

With these two examples the application of Routh-Hurwitz criterion to self-oscillators is sufficiently explained. But we believe that this criterion is specially valuable when numerical values of the parameters are known. The conditions of oscillation can then be easily applied. In particular, the influence of any critical element of the network can be studied without great difficulty.

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Methods of Obtaining High Vacuum by Ionization "Electronic High Vacuum Pump"

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I. INTRODUCTION

The idea to obtain high vacuum by ionization is quite old. Already PLUCKER (1858) points out this possibility. I will give his words in translation;

"Certain gases (oxygen, vapours of chlorine, bromine and iodine) combine more or less slowly with the platinum of the negative electrode, and the resulting combinations deposit on the surrounding glass wall. By this we approach an absolute vacuum".

Of course his conclusions cannot be accepted as a whole; but he seems to be the first to observe electrical clean-up, i.e. adsorption of gases or vapours in an electrical gas discharge. The mechanism of clean-up is explained in the following way: Gas molecules will be ionized and after this drawn to collectors of a negative potential where they will be adsorbed or "shot" in. A simple device of electrical clean-up would operate as a vacuum pump, only for a very short time, because the adsorbing parts will soon become saturated. The positive ions must be drawn to a fore vacuum side. But then the number of ions has to be greater than the number of gas molecules that will diffuse back. This is a necessary but not a sufficient condition for getting a pumping action.

II. ABOUT THE POSSIBILITY OF PUMPING ACTION

We will see now that it is not possible to construct a high vacuum pump with reasonable dimensions in a normal d.e. gas discharge. We assume the following model, see (fig. n.º 1).

We have a tube of x cm of length and a diameter of $2r$ cm. At A may be the cathode and at B the anode. We claim a difference of pressure between the ends of the tube of $p_1 - p_2 > 0$. To the pressure p_1 corresponds the density n_1 and to p_2 the density n_2 molecules per cm^3 .

The necessary assumption for setting up a pressure difference between the ends of the tube is:

number of ions/second greater than gas flow ΔN

$$(1) \quad n_i > \Delta N$$

This assumption is not sufficient from many points of view as we shall see later. To get an idea of the dimensions of the diameter $2r$ and the length x of the tube in relation to the electrical values, we calculate both sides of the inequality (1). The number of ions depends on the number n_e of the electrons coming from the cathode and on the potential difference U between A and B, furthermore on the so called differential ionization s_0 (17), the pressure p and the length of the electron path that may be equal to that of the tube.

$$(2) \quad n_i = n_e \int_0^x s_0(U\xi) p(\xi) d\xi$$

But the concept of the differential ionization means that each electron collides with a gas molecule at least once on its path x . To get more accurate values we have to replace n_e by a value:

$$n_e \left(1 - e^{-\frac{x}{\bar{\lambda}_e}} \right)$$

TABLE 1

Minimum Length $x_{\min}$ for "Electronic Vacuum Pumps"

as zeros of the function: $f(x; J_e; r) = x^2 \left(1 - e^{-\frac{x}{260}} \right) - 508 \frac{J_e}{r^3}$

A) $r = 1,0 \text{ cm}$		$J_e \text{ (Amp)}$ ion current	$S \text{ (liters/sec)}$ at 10^{-4} Torr
$J_e \text{ (Amp)}$	$x_{\min} \text{ (cm)}$		
0.001	735	$1,4 \times 10^{-4}$	0.13
0.01	278	$3,6 \times 10^{-4}$	0.35
0.1	118	$8,5 \times 10^{-4}$	0.8
1,0	52,5	$1,9 \times 10^{-3}$	1,8
10,0	21	$4,2 \times 10^{-3}$	4,0
B) $r = 0.5 \text{ cm}$			
0.001	304	$4,1 \times 10^{-5}$	2,5
0.01	128	$9,8 \times 10^{-5}$	6,0
0.1	56,8	$2,2 \times 10^{-4}$	13,6
1,0	25,8	$4,8 \times 10^{-4}$	30
10,0	12,0	$1,0 \times 10^{-3}$	64

which gives the number of all electrons that collide at least once on their path, $\bar{\lambda}_e$ the mean free path of the electrons in the gas.

$$(3) \quad n_i = n_e \left(1 - e^{-\frac{x}{\bar{\lambda}_e}} \right) \bar{p} \int_0^x s_0(U\xi) d\xi$$

$\bar{p}$ the average pressure between A and B.

Such an electronic pump is only of interest if it acts at pressures lower than 10^{-4} Torr (*mmHg*). To compute the other side of inequality (1), i. e. the number of the back diffusing molecules, we may apply the Knudsen law for the case when the mean free path is longer than the diameter of the tube:

$$(4) \quad \Delta N = \frac{2}{3} r \pi \bar{c} (n_1 \sim n_2) \frac{1}{x}$$

$\bar{c}$ the average thermal velocity of the gas molecules.

Furthermore we are not too pretentious if we claim that our pump reduces the pressure at least down to 1%, i.e. we can neglect n_2 against n_1 . Our inequality (1) gets now the following form:

$$(5) \quad n_e \left(1 - e^{-\frac{x}{\bar{\lambda}_e}} \right) \bar{p} \int_0^x s_e(U_\xi) d\xi > \frac{2}{3} r^3 \pi \bar{c} n_1 \frac{1}{x}$$

As we may derive from measured values (KNOLL, OLLENDORF & ROMPE, 1935) of s_e see fig. No. 2, we can write approximately for the integral $\int_0^x s_e(U_\xi) d\xi = x$ what would be favorable to a shorter tube x .

Substituting n_e by the current I_e , we get the following inequality (6) for the length x of the tube:

$$(6) \quad x^2 \left(1 - e^{-\frac{x}{\bar{\lambda}_e}} \right) > A \frac{r^3}{I_e}$$

$$A = \frac{2 \pi \bar{c} n_1 \bar{e}}{3 \bar{p}}$$

$\bar{e}$ = elementary charge

We regard now the difference of both sides of this inequality (6) as a function of x , I_e , and r :

$$(7) \quad f(x; I_e; r) = x^2 \left(1 - e^{-\frac{x}{\bar{\lambda}_e}} \right) - A \frac{r^3}{I_e}$$

Our electronic pump would operate for all values of x , I_e , and r that give positive values for the function (7). Introducing the numerical values for air at an average pressure in the tube of $\bar{p} = 10^{-4}$ Torr, we get $\bar{\lambda}_e = 4 \cdot \sqrt{2} \bar{\lambda}_{air} = 260$ cm

$$A = 508 \quad \text{Amp. cm}^{-1}$$

$$(7a) \quad f(x; I_e; r) = x^2 \left(1 - e^{-\frac{x}{260}} \right) - 508 \frac{r^3}{I_e}$$

(x and r in cm; I_e in Amps)

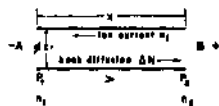


FIG. 1. SCHEME OF PUMP ACTION

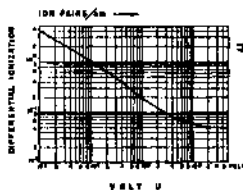


FIG. 2. DIFFERENTIAL IONIZATION OF AIR

Figs. 1 and 2

Computing the zeros of this function (7a) for electron currents between 0.001 and 10 amps and reasonable values for the radius of the tube $r = 0.5$ and 1.0 cm, we get the minimum length x_{min} of the tube. In table n.º 1 we may see the computed values. In the first column we find the electron current I_e , in the second the minimum length x_{min} of the tube, in the third the ion current I_i , for $2 I_e$ and in the fourth column finally the pumping speed at 10^{-4} Torr forcing $2 I_e$ amps through the tube. The ion current has been computed in using the formula (3) for the number of ions, with the values of our case we obtain the formula

$$(8) \quad I_i = 10 \cdot \frac{r^3}{x} \text{ (amps)}$$

The pumping speed at 10^{-4} Torr can be calculated with the formula derived from (4)

$$(9) \quad S = 96,4 \cdot \frac{1}{r^3 \cdot x} \text{ (liters/sec)}$$

if we make pass the double values of the electron current I_e calculated for the minimum length.

Reasonable values for the minimum length are only obtainable for large electron currents. There are no technical difficulties in getting sufficient electrons emitted from hot cathodes but their space charge will be enormous and very high voltage must be applied to get the electrons through the long tube; electrons will attach to the glass walls and charge it negatively and by this sensibly change the electric field. Further calculations on the necessary voltage point out that the voltage.

$$U_s = \left(\frac{I_s}{K} \right)^{\frac{2}{3}}, \quad \text{roughly } K = 10^{-6} \frac{r^2}{x^2}$$

must be of the order of 10^5 volts, even also for the lower currents, as with these we calculate much longer minimum lengths.

From this we can conclude that the construction of an electronic pump in a simple gas discharge tube is not possible.

We must emphasize that we assumed favourable conditions in calculating pumping action of a simple gas discharge. We did not consider re-combination of ions, the high electron bombarding energy nor the technical problems of the incandescent cathode intensely bombarded by ions and other problems. Furthermore we know that s_0 is much smaller at higher voltages and some electrons will hit the walls and not arrive at the anode at all.

We have to find a way to increase the path of the electrons without increasing the length x of the tube. There are two different ways:

1. that of making the electrons oscillate in the axis of the discharge tube (SCHWARZ, 1940 a & b);
2. that of forcing the electrons in spiral paths by a magnetic field within a cylindrical electrode system as for expl. in a magnetron valve or cyclotron.

Instead of our inequality (5) we have now:

$$(10) \quad n_e \left(1 - e^{-\frac{\nu x}{\lambda_e}} \right) \bar{p} \nu \int_0^x s_0(U_\xi) d\xi > \frac{2}{3} r^3 \pi \bar{c} n_1 \frac{1}{x}$$

where ν denotes the increasing factor of the electron path. Substituting the numerical values in (10) as we did before and neglecting the exponential part for sufficiently high values of ν , we obtain the following numerical inequality:

$$(11) \quad x > 22,6 r \cdot \sqrt{\frac{r}{I_e \cdot \nu}}$$

A simple example shows that the conditions for the construction are feasible.

$$r = 1 \text{ cm} ; \nu = 20; I_e = 0.1 \text{ A} ; x_{\min} = 16 \text{ cm}$$

$$S = 6 \text{ liters/sec at } I_e = 0.2 \text{ amps.}$$

Since the ion current in this example would be of the order of 5 mA, here will be practically no negative space charge, consequently the voltage can be much lower.

It is easy to mistake a clean up for a pumping action, i.e. adsorption of the ions on the wall in small volumes. All experimental work on this field must be considered very critically. And it seems to me that the experiments recently done by CHAMPEIX (1950) on this field in a gas discharge tube without a special arrangement for prolonging the ionizing electron path did not show a real pumping action but clean-up.

III. CONSTRUCTION OF AN "ELECTRONIC PUMP"

In our laboratory we constructed an electronic pump (SCHWAUZ, 1943) using experiences obtained from our earlier work already done on this field (SCHWARZ, 1940a & b). The fundamental arrangement of that former type is to be seen in figure No. 3. The tube consists of an incandescent cathode, an anode of ring shape and a third annular electrode that was on the same potential as the cathode. An electron current of about 10 mA was emitted by the cathode

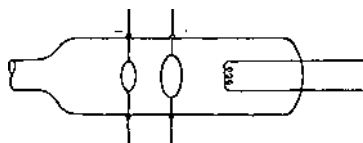


FIG. 3: EXPERIMENTAL TUBE FOR STUDYING CLEAN-UP
(SCHWARZ 1940)

and kept oscillating through the annular anode at about 2000 volts or more; a small magnetic field parallel to the axis of the tube kept the electrons in spiral paths in the centre of the tube away from the anode. In such a tube a very intensive clean-up was formerly observed and the adsorbed gas was found at the glass wall which had become

negatively charged. In our new electronic pump model, fig. n.º 4, we opened the glass wall at four spots between the two rings, but nearer to the anode. We connected four tubes, and at the entrance of each we put in a ring electrode at slightly negative potential. These tubes were joined all together

and connected at F to a common rotatory oil pump as a fore pump. The volume to be brought to high vacuum was connected to the opposite side of the hot cathode at H , and at the entrance of the connection tube another annular electrode B was sealed in coaxial position. It was maintained at positive potential to prevent ions from running to the high vacuum. The pressure at the high vacuum side H was measured by a Pirani gauge and an ionization gauge.

A volume of about 500 cm^3 connected at H was pumped out from a pressure of $5 \cdot 10^{-3}$ to $1,0 \cdot 10^{-5}$ Torr-sometimes $5 \cdot 10^{-6}$ Torr in less than one second.

The pumping was repeated more than 30 times; we did not observe any saturation; but as simple calculations prove that already 3 times would give a mono-molecular layer on the inner wall of the pumping tube, this pressure decrease could not have been caused by clean-up, the adsorbed layer cannot be thicker than 1.7 times that required to form a monolayer, as we already found in our previous work. Then the observed decrease of pressure must have been a real pumping action.

I would like to acknowledge my indebtedness to prof. Cesar Lattes, for granting the possibility to do the final experiments of this work in the Centro Brasileiro de Pesquisas Fisicas.

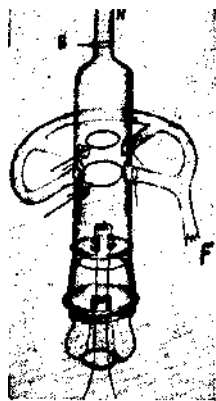


Fig. 4. Electronic High Vacuum Pump.

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DISCUSSION

SALMERON: What is the influence of the nature of the gas on the power of this pump?

H. SCHWARZ: The influence is expressed in the differential ionization s_0 , as this depends on the kind of gas. But considering all possible s_0 in formulae (5) and (10) we can calculate a minimum length x_{min} that will give pumping action for all kinds of gases. The speed is proportional to s_0 .

SALMERON: Was there a way of increasing the ionization by an alternating field inside the tube?

H. SCHWARZ: NO.

HERB: Did you have a fore pump connected to your tube during the tests you described?

H. SCHWARZ: We connected one normal rotary pump to the combined four glass tubes. The rotary pump gave a final pressure that we measured as being 10^{-3} mm Hg .

KERST: Can you start this pump at $2 \cdot 10^{-2} \text{ mm}$ pressure?

H. SCHWARZ: Yes, we can; but then we get difficult conditions caused by the spluttering of tungsten that will deposit on the walls and the strong gas discharge that will need good stabilizer.

On the Lower Limit of the Present Day High Vacuum Gauges

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INTRODUCTION

There is no doubt that in all kind of modern experimental physics where one wants to get rid of air, the lowest vacuum is the most desirable one. But it is unsatisfactory that the present day vacuum gauges cannot measure the low residual pressure that modern technique is able to produce. The lower limit of the best gauges lies at 10^{-8} Torr (mm Hg), but with an error of a factor greater than 2, while on the other hand the lowest gas pressure obtained with modern vacuum techniques is certainly smaller than 10^{-10} Torr. That this is true we found for expl. at adsorption curves of barium getters, fig. 1. Plotting the decrease of pressure with time we should expect an e-function shape; but in very clean vacuum systems this shape did not appear as we see in the figure. The pressure was measured by a radiometer type manometer developed by KLUMB & SCHWARTZ (1944) that is independent of the kind of gas. It is easily derived that the final pressure we obtained in this system was lower than 10^{-9} to 10^{-10} Torr. Already P. ANDERSON (1935) and NOTTINGHAM (1937) concluded from electron emission of pure metals that the pressure they obtained was lower than they could measure, and estimated pressures of the order of 10^{-10} to 10^{-10} Torr. Recently SEDDIG & HAASE (1952) took up the investigations of ANDERSON and developed a method to infer low pressure values from the change of the work function of some metals with the degree of gas adsorption. The velocity of decrease of work function is a measure of the vacuum.

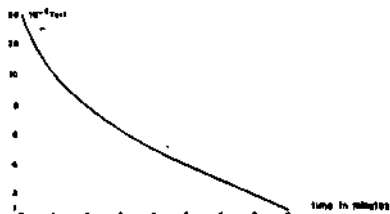


Fig. 1 — Pressure decrease in Ba-Getter Pressure measured by Klumb-Schwarz-Manometer

But why is it so difficult to measure low pressures? The answer is ridiculously simple: because the forces are so small. For a "bad" vacuum of 10^{-4} Torr = 0.1μ , we have only a force of 0.14 mg per square centimeter. We have to do with the small pressure the same as we do with small electric

currents: we have to amplify itself or measurable values that depend on it. Therefore I distinguish between:

1. vacuum gauges without amplifier

and

2. vacuum gauges with amplifier

As vacuum gauges of the first group are able to indicate pressures practically only down to 10 to 100 μ , we will not deal with them here. Among the gauges with amplifier we distinguish the following principles:

1. Compression manometers
2. Heat conductivity manometers
3. Viscosity manometers
4. Radiometer type gauges
5. Ionization gauges.

In an amplifier we always have to put energy. In the compression type we have to use compression work, in the heat conductivity manometers and the radiometer type gauges heat, mostly electric heating; in the viscosity manometers kinetic energy often with help of electrical energy; and lastly in the ionization gauges we have to apply electrical energy.

1. COMPRESSION MANOMETERS

Such as the McLeod-gauge which amplifies directly the pressure by compressing a known volume V can hardly be improved. Its lower limit can be derived from its gauge equation:

$$(1) \quad p = k \cdot h^2, \quad h. — \text{difference in mm of mercury level}$$

where A : the manometer constant gives the degree of sensitivity

$$k = \frac{q}{10 \cdot V},$$

p is the cross section of the capillary wherein part of the gas to be measured has to be compressed from the fixed volume V . The lower limit is a technical limit, how small the cross section q and how large the volume V can be made. ROSENBERG (1939) seems to have reached with $k = 2,4 \times 10^{-7}$ Torr. mm^{-2} a practical lower limit because of the high specific weight of the mercury and capillary forces. To use oil instead of mercury gives many difficulties due to degassing and capillary adherence. The manometer constant $k = 2,4 \times 10^{-7}$ Torr mm^{-2} means that one millimeter difference of mercury level in the two capillaries indicates a pressure of about 10^{-7} to 5×10^{-7} Torr; but only the partial pressure of the permanent gases can be measured; the total pressure cannot because more than 1 μ of mercury vapour is present. As the McLeod will never become a vacuum gauge of quick recording, it will only keep its standard position as an absolute vacuum gauge, esp. for calibrating other types.

2. HEAT CONDUCTIVITY MANOMETERS

2 — *Heat Conductivity Manometers* — From the kinetic theory of gases we know that the heat conductivity K through a gas is proportional to the mean free path X and the number N of the molecules per volume unit:

$$(2) \quad \kappa = \text{const. } \bar{\lambda} N$$

As X is inversely proportional to the pressure and N directly proportional to the pressure, K becomes independent of pressure. But this is only true if we have more collisions between gas molecules than collisions of gas molecules with the walls. At lower pressures (under 100 μ) when the value of the mean free path is of the order of magnitude of the dimensions of the vessels, the heat conductivity becomes dependent on the pressure. The well known Pirani gauge indicates the change of electric resistance of a tungsten wire in the vacuum to be measured, as the resistance depends on the temperature, the temperature on heat conductivity, and this on the pressure, so that we have a function:

$$p = f(R)$$

Another way is to measure the temperature directly by a thermocouple

$$p = f(T)$$

The lower limit of this type is given by the fact that towards lower pressures loss of heat by conductivity of the gas becomes much less than the transport of heat by radiation and conductivity through the solid parts, e.g. metal lead in wires for the current.

The lower limit will result from the heat balance:

$$(3) \quad Q_{\text{total}} = Q_{\text{gas}} + Q_{\text{rad}} + Q_{\text{metal}}$$

From the kinetic theory of gases we can calculate

$$(4) \quad Q_{\text{gas}} = \gamma p \sqrt{\frac{2 \cdot R}{\pi M \cdot T_1}} (T_2 - T_1) 2 \pi r_1 l$$

T_1 = external temperature, T_2 = internal temperature

r_1 = radius of the wire, l = its length, γ = 1 accommodation coefficient. Let us have a practical example:

$T_1 = 293^\circ K$, $T_2 = 423^\circ K$; $M = 29$ (air), $p = 10^{-3}$ Torr, $2 \pi r_1 l = 1 \text{ cm}^2$, then we get

$$Q_{\text{gas}} = 3,3 \times 10^{-4} \text{ cal. sec}^{-1}$$

To have a comparison we calculate Q_{rad} from Stefan-Boltzmann law:

$$(5) \quad Q_{\text{rad}} = \sigma (T_2^4 - T_1^4) 2 \pi r_1 l$$

For our example we get

$$Q_{\text{rad}} = 3,6 \times 10^{-2} \text{ cal. sec}^{-1}$$

This means that by radiation already 100 times more heat will be transported than by conductivity through the gas at that pressure of 10^{-3} Torr. To get an

idea about Q_{metal} we make the simplifying assumption that T_2 decreases from the middle of the wire to both its sides to the temperature T_1 , then we have

$$(6) \quad Q_{metal} = 2 \lambda r_1^2 \pi \frac{T_2 - T_1}{l/2}$$

λ coefficient of heat conductivity through the metal. In our example let us assume $\lambda = 0,1 \text{ cal grad}^{-1} \text{ cm}^{-1} \text{ sec}^{-1}$, then we get with $r_1 = 0,2 \text{ mm}$ and $l = 8 \text{ cm}$

$$Q_{metal} = 8,2 \times 10^{-3} \text{ cal. sec}^{-1}$$

that means as well by conductivity through the metal we have 25 times more heat transported than through the gas volume.

To reach a lower limit we could think of diminishing Q_{metal} by prolonging the wire, but then Q_{rad} would become greater. Practically the lower limit lies at 10^{-4} Torr and all attempts to come to even lower limits are very uncertain in precision.

3. VISCOSITY MANOMETERS

As the viscosity η of a gas is proportional to the heat conductivity it can also be used to measure the pressure of the gas.

The most simple kind of viscosity manometer is a rigid thread of quartz oscillating in the gas to be measured. The damping that depends on the viscosity will be measured by the half-time in regard to the amplitude of the oscillation. The solution of the differential equation of the system delivers for the half-time:

$$(7) \quad t_h = \frac{D \cdot \text{const.}}{p \sqrt{M} + z}$$

where D is the diameter of the quartz thread (cm), p the pressure (Torr), M molecular weight, z a value for the self-damping of the system itself, $\text{const.} = 122$. The lower limit is given by the fact that t_h gets practically independent on the pressure, if the self-damping will become much greater than $p \sqrt{M}$

$$z \gg p \sqrt{M}$$

That happens in the order of 10^{-6} to 10^{-7} Torr.

A great disadvantage of all kinds of gauges of the viscosity type is that they are sensitive against outside vibrations and that they require the use of a stop watch.

We developed together with Klumb a viscosity vacuum gauge which we call resonance-manometer.

This manometer is schematically drawn in fig. n°. 2. It is more rigid and needs no watch. The main parts are the F-shaped loop *Sch* in the vacuum to be measured and the magnet *M*. Through this loop passes a current coming from the photocell *Ph* and amplified by the amplifier, *Ve*. The photocell *Ph* gets its light from the mirror *Sp* fixed on the loop and reflecting the light coming from the slit shaped source *L*. Once the loop *Sch* excited to oscillate in its own frequency (self-frequency) by any means, it keeps on oscillating with

constant amplitudes, because the mirror Sp will receive periodically light in the rhythm of the oscillating mirror. That means that an alternating current coming amplified from the photocell passes through the loop Sch in the constant magnetic field, so that oscillations will continue undamped. The amplitude depends on the viscosity, and this on the pressure. As the frequency is larger than 10 Hz we see the scale Sk illuminated in a whole. The pressure will be indicated by the length of the light stripe on the scale Sk . The lower the pressure the longer is the stripe. Of course the indication of the amplitude could be done by other electric means.

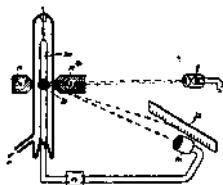


Fig. 2 — Resonance-Manometer of Klumb-Schwarz

4. RADIOMETER TYPE GAUGES

This type is the only gauge of the modern ones that is independent of the kind of gas.

The principle can be derived from the simple figure N.° 3.

We have three plates A, B and C, in vacuum. A is at a higher temperature T_2 than B and C.

$$T_2 > T_1$$

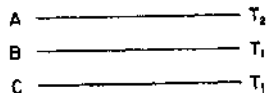


FIG. 3—SCHEME OF THERMAL MOLECULAR FORCES

The molecules that come from A arrive at B with a higher energy than those coming from C, if the pressure is sufficiently low, so that the molecules will arrive at B without colliding with one another. The plate B will be moved by the thermal molecular pressure; the acting force K can be calculated

$$(8) \quad K = \frac{p}{2} \left(\sqrt{\frac{T_2}{T_1}} - 1 \right),$$

where p is the pressure in the system.

I will not describe here the different technical types already known. I will only mention a construction radically different from all the other radiometer types. KLUMB A SCHWARZ (1944) developed a cylindrical arrangement of vanes instead of the common vane in a plane parallel to that of the heated surface, as fig. 4 points out (see also DUSHMAN, Vacuum Technique, 1949 pp. 305-9). The system is suspended at a thin tungsten or quartz thread in the annular space between the outer glass G and inner tube H ; a heater is placed in H . The outer glass can be cooled at a constant temperature T_1 (liquid air for the most sensitive one). The rotational moment that depends linearly on the pressure in the annular space can be observed by the deflection of the mirror attached to the suspension or compensated by an electromagnetic arrangement; the a.c. current necessary for this compensation is then a function of the pressure. This system can mathematically be treated in the same way as a mirror

galvanometer, defining the "reduction factor" C_0 and "gauge sensitivity" E_0

$$(9) \quad C_0 = \frac{1}{E_0} ; \quad C_0 \cdot \Theta = p \\ \Theta = E_0 \cdot p$$

where B denotes the angular deflection.

For the lower limit of this vacuum gauge we have the same conditions as those for a mirror galvanometer. The sensitivity is limited by the mechanical constants of the torsion system.

Practically the lower limit lies at a reduction factor $C_0 = 10^{-8} \text{ Torr} \cdot \text{m}^{-1} \text{mm}^{-1}$, i.e. 10^{-8} Torr for 1 mm of deflection on the scale in 1000 mm of distance from the mirror, using a tungsten wire of 0,01 mm diameter, cooling with liquid air and heating to $T_2 = 350^\circ \text{C}$.

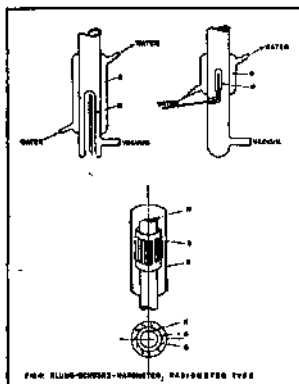


Fig. 4 — Klumb-Schwarz-Manometer; radiometer type

5. IONIZATION GAUGES

The principle of the ionization gauge is the following. In a simple triode with a directly heated filament the electrons accelerated, let us say by 150 volts, will ionize the residual gas molecules, see fig. 5. The positive ions are collected by the third electrode that is at a slightly negative potential. At lower pressures we have then the following simple relation

$$(10) \quad p = C \frac{I_i}{I_e}$$

I_i the ion current to the collector, I_e the electron current.

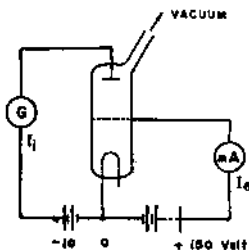


Fig. 5 — Schematic ionization gauge circuit

The constant C is of the order 0,05 to 0,1 if the pressure p is measured in Torr. The electron current I_e normally is of the order 1 mA. Under the best geometrical and electrical conditions of the present day values it is not possible to obtain a lower current for I_i than about 10^{-11} amp., which corresponds to a pressure of $2 \times 10^{-8} \text{ Torr}$, although, as we already pointed out in the beginning, the obtained pressure must be much lower; this means that in the common types the current reaching the negative collector becomes at lower pressures less dependent of the the pressure and that we have to

change our formula (10) into:

$$(11) \quad p = C \frac{I_i + I_0}{I_e}$$

The question is now: To what is the current I_0 due? It is independent of the pressure, or let us say more cautiously, less dependent on the pressure. NOTTINGHAM (see S. Dushman, *Vacuum Technique* 1949, pp. 358/9) has ascribed this zero-current I_0 to photoelectrons emitted from the ion collector and liberated by X-rays which are produced on the "grid" by the incidence of the about 150 volt electrons. This hypothesis seems to us a bit unlikely, when estimating the efficiency of the supposed effects. The minimum wave length of the X-rays produced by the 150 eV electrons is

$$\lambda_{\min} = 82 \text{ \AA}$$

The X-rays can only be Bremsstrahlung and not fluorescence radiation, because the lowest level of tungsten lies higher than this energy. Let us make a rough estimation of the photoelectric yield of the X-rays:

We have an electron current of $I_e = 10^{-3}$ amps. The energy E_x of the X-rays produced by this electron current is only (HANDBUCH DER EXPERIMENTAL PHYSIK, vol. 7, p. 2 (1928)):

$$E_x = 10^{-10} E_e \cdot Z \cdot V$$

in our case $z = 74$; $V = 150$ volts; $E_e = I_e \cdot V$

The number of X-ray photons is:

$$n = \frac{E_x}{h \cdot \nu}$$

If we had 100% yield of photoelectrons, we would obtain from the collector a photoelectric current of:

$$I_{ph} = n \cdot e = \frac{V^2 Z e}{h \nu} \cdot 10^{-3} I_e \quad \begin{matrix} I_e = 10^{-3} \text{ Amps} \\ = 10^{-6} I_e \end{matrix}$$

But not all these X-rays will hit the collector, let us assume a solid angle of 10%.

$$= 10^{-1}$$

Furthermore we cannot assume that every $h \cdot \nu$ quantum will release one electron, because a great part of the weak X-ray energy will be lost within the interior of the metal by heat; let us assume a 10% yield.

From all this we must conclude that only $\frac{10^{-1}}{10^{-11}}$ Amps photoelectrons can be produced by the process supposed by NOTTINGHAM, i.e. 100 times less than the zero-current.

But we must confess that this result does not convince us completely that the hypothesis of NOTTINGHAM is wrong, because we know too little about the weak X-rays and their yield of photoelectrons on solid surfaces. But we think that we need not apply such a complicate mechanism for explaining the lower limit of the ionization gauge, i.e. the zero current I_0 . Tungsten gets vapour pressures in the range of 10^{-6} Torr, if heating to 2600°/°C which is the temperature normally reached by the incandescent wire of the ionization gauge. We will have always a cloud of tungsten vapour near the surface of the tungsten filament, the density of which rapidly decreases towards the colder collector of the gauge.

But in any way the electrons will ionize these tungsten atoms, the number of which at the very low gas pressure is independent of the pressure and only depends on the temperature of the filament. Rough calculations give still a pressure of 10^{-9} Torr within the vapour stream near the surface of the collector electrode. As we already know from our former experiments (SCHWARZ, 1944) there will be always a gas exchange effect at the collector electrode: the ions arriving at the collector will be shot onto it and replace other gas or vapour atoms that will come out. Such a vapor-gas cloud might maintain this exchange process, so that we will always have in the gauge within the ionization space between the hot cathode and the collector a pressure that hardly depends on the pressure in the vacuum system itself. All experimental work recently done on this field and based on Nottingham's theory (LANDER, 1950; METSOV, 1951) cannot undoubtedly prove any of the theories pointed out.

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DISCUSSION

- ORRINI: I would like to know the range of measurements of your viscosity manometer*
 H. SCHWARZ: From 100 to 10^{-6} mm.
 ORSINI: I have seen in your scheme that you employed A. C. current to heat the lamp. Doesn't this introduce perturbations?
 H. SCHWARZ: D.C. current must be used, to have a constant flux of light.
 GROSS: Could you tell me the half time of the oscillations (extrapolated to 0 pressure) for the quartz string gauge?
 H. SCHWARZ: With a thread of quartz of 1 micron diameter I got a half time in the order of hours at 10^{-8} Torr and the extrapolated half time at pressure 0 will not be greater than a factor 2.
 RABI: Have you considered the sensitivity of the Stern-Pirani gauge as modified by Zacharias for the detection of molecular beams? I would expect that the sensitivity is better than 10^{-8} .
 H. SCHWARZ: You may hardly succeed in getting a relative sensitivity of that order in the absolute pressure region between 10^{-2} and 10^{-1} mm Hg, but measuring absolute pressures of 10^{-8} mm Hg is practically impossible with any type of heat conductivity vacuumeters.

Diffusion Cloud Chamber

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The original development of the diffusion cloud chamber was carried out at the M.I.T. and published by LANGSDORF JR. (1938).

In that chamber the control of roof and bottom temperatures was independent; the diffusion was downward and the window was placed in the underside.

The gas used was CO_2 at atmospheric pressure, and the control of temperatures was made by liquid circulation.

About two years ago the diffusion cloud chamber has been improved and the construction was simplified in connection with the ways to obtain low temperature (COWAN, 1950; NEEDLES & NIELSEN, 1950; NIELSEN, NEEDLES & WEDDLE, 1951; BERNARD & ATKINSON, 1952). More elaborated diffusion chambers have been projected for high pressures (MILLER, FOWLER & SHUTT, 1951; SHUTT, FOWLER, MILLER & FOWLER, 1951) and a theory of diffusion chambers was presented (SHUTT, 1951).

Roof heating is usually obtained by electrical means and bottom cooling by direct contact with a dry ice block, or liquid nitrogen (CHOYKE & NIELSEN, 1952).

In this paper is described an atmospheric pressure diffusion chamber, with independent control of roof and bottom temperatures and means for easy control of the temperature gradient in the chamber, in order to maintain the best operating conditions.

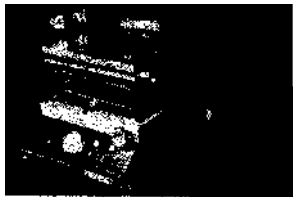
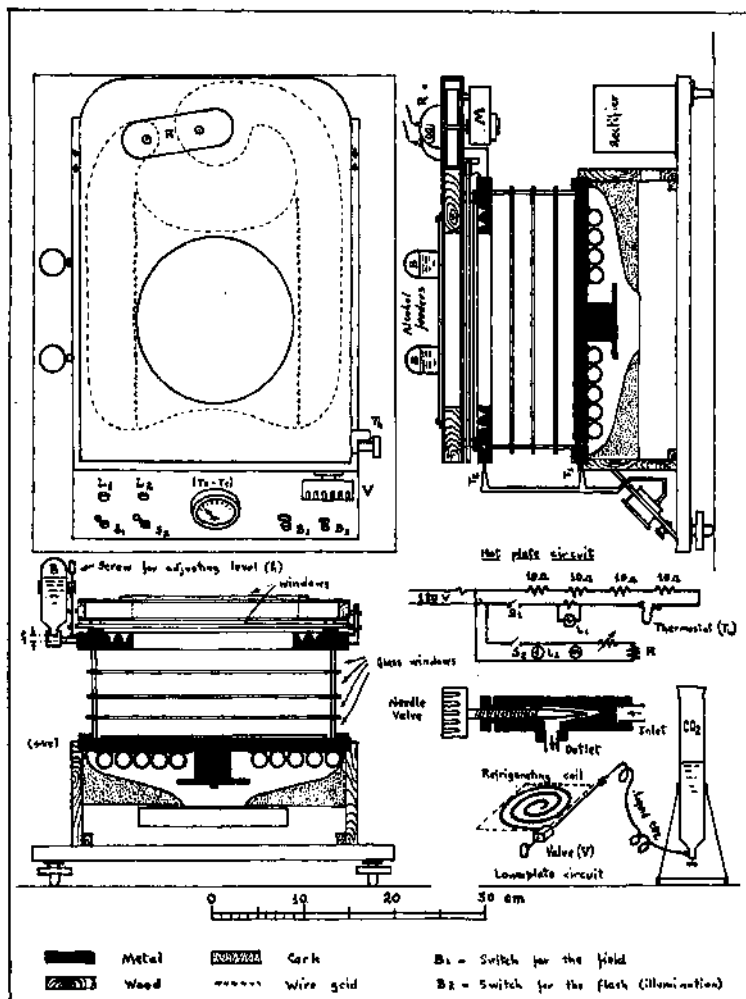


Fig. 1

Fig. 1 shows the diffusion cloud chamber and Fig. 2 the constructional details and circuits.

The principal points to be mentioned are the following (Fig. 2):

- a.) roof heating is made by four resistances, 300 watts total power, and temperature maintained by means of a thermostat (Th);
- b.) windows are cleaned of fog by means of a hot air stream supplied by a closed air circuit, heating resistance (R) and ventilator (motor M);
- c.) bottom cooling is made by expansion of CO_2 liquid into a refrigerating coil, soldered to the bottom plate. The rate of expansion is controlled by a needle



valve (V), and by this means the low temperature is maintained at the optimum condition for the better performance of the chamber;

d.) The temperature gradient is measured by a thermoelectric pair ($T_2 - T_1$) and milliammeter, referred to the roof temperature, this one measured by a mercury thermometer;

e.) Alcohol is fed from a storage (B) through two circular concentric channels. The section of the channels is trapezoidal. The height of the alcohol in the channels can be controlled by adjusting the level of the alcohol container. This is made by two adjusting screws (level h);



Fig. 3



Fig. 4

f.) The windows can be lifted, and the interior of the chamber easily cleaned;

g.) The sides of the chamber are made of glass plates, about 2.5 cm width, separated by metal strips in order to maintain a better temperature gradient. The columns in the corners (Fig. 1) are made of wood, treated with paraffine. The seals in the joints are made with a rubber cement;

h.) A electrical field of about 50 volts/cm, is obtained by conventional rectifying and filtered circuit. A high resistance, about 1 megohm is put in series with the plates.



Fig. 5

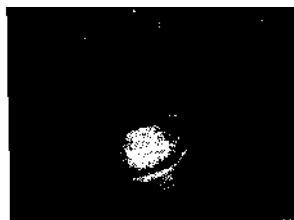


Fig. 6

The following construction details of the diffusion chamber are shown in the figs. 3, 4, 5 and 6:

Fig. 3: hot air circulating chamber, without the glass windows;

Fig. 4: complete roof plate, with alcohol containers, feeding system, concentric channels and imbedded heating resistances;

Fig. 5: bottom plate with refrigerating copper coil, needle valve (V) liquid CO_2 feeding line and baffle;

Fig. 6: complete bottom plate.

The plates are made of brass, the bottom plate is mounted in a wood and cork box shown in figs. 1 and 2. The roof plate is put directly upon the glass sides of the chamber. The first glass window is separated from the roof plate by a cork gasket.

OPERATION

The following steps are followed for good operation of the diffusion cloud chamber: Alcohol is poured in the bottom plate, about 2 mm. This is possible by means of a rubber square cemented in the sides of the plate.

A glass plate blacked with carbon black is sunked into the alcohol pool. It is important that the plate lie uniformly wet. This method gives an excellent background.

Methanol is put in the alcohol containers, with the adjusting screws at the minimum feeding conditions.

The roof heating should be started at this moment, and the temperature adjusted to stable condition by means of the thermostat (*Th*). For methanol the temperature about 55°C is good. The stable condition is obtained in about fifteen minutes.

The bottom cooling should be started after the adjustment of the roof temperature, by opening, gradually the expansion valve (V), and measuring the temperature difference by means of the thermoelectric pair and milliammeter.

hooking into the chamber, the appearance of the cloud must be uniform, like a rain of liquid drops. Stratification is a signal of non-uniform gradient in the chamber, and if this condition is continued, it is impossible to get tracks of particles.

In this case rather than to look for the causes, it is better to start again the operation.

If the cloud is normal, about fifteen minutes after applying the low temperature, and at 55°C, the first tracks of alpha particles may be observed. For testing condition in our work we use a source of alpha and beta particles.

In the subsequent half hour the chamber conditions stabilize, and adjusting the rate of expansion of the CO_2 the more sensible conditions are attained and the indication of the milliammeter is maintained for a control of the temperature gradient.

In that condition the chamber can be used during hours, and the operating conditions may be easily maintained by means of the control of valve (V) and thermostat (*Th*).

The field is applied in order to clean the chamber of old ions by pushing the bottom (B_1) in fig. 2.

RESULTS OBTAINED WITH THE CHAMBER

In order to observe the behavior of the diffusion chamber, the alpha and beta particle source is enclosed in a tube. In front of the tube a removable shield is used for alpha particles; a hole in another shield, filled with berilium oxide, is a source of neutrons in the chamber.

The results are shown in figures 8 to 10. The film used was Kodak Super XX, and the camera, was a Contax.



Fig. 7



Fig. 8

The exposition time is about $1/2$ to 1 second.

The illumination source is a CC carbon arc (20 amperes), and optical condenser system. The area of illumination is about 25 cm width by 2.5 cm height, and about 2 mm over the background.

A more efficient system with gas flash tubes is now under design for better definition of the tracks.



Fig. 9



Fig. 10

Fig. 7: shows alpha and beta tracks originating in the source (bright spot in the photograph).

Fig. 8: shows beta tracks (alpha particles shielded by a thin aluminium foil).

Fig. 9: is the same as fig. 7, but the exposure time is longer.

Fig. 10: shows heavy tracks in the chamber bombarded by neutrons. Exposition is about 1 second, and three V tracks can be seen in the photographs.

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DISCUSSION

SALMERON: Como influi a temperatura da parte inferior? Diminui a espessura da zona de super-saturacao ou diminui a sensibilidade da camara nessa regioao?

ARRUDA: Nao posso dizer exatamente na base das experiencias feitas ate agora, mas parece-me que a sensibilidade da camara se modifica e em certos casos, dentro de 5 graus para a placa fria. Por outro lado, empregando-se uma fonte que dispersa as particular num certo angulo solido, a variacao da temperatura deveria mostrar a variacao da sensibilidade nas camadas. Entretanto o que se nota e uma diminuicao geral da sensibilidade. O assunto, entretanto, deve ser mais bem investigado para uma resposta decisiva.

SALMERON: Teve problemas serios de limpeza ou isso nao causou dificuldades?

ARRUDA: NO que diz respeito a limpeza ha dois aspectos a se considerar: foi observado ser da maior conveniencia limpar-se rigorosamente a camara depois do ser usada, pois os hquidos empregados atacam o metal, tal como aconteceu em camaras anteriormente construidas em alumfnio. Um segundo aspecto, e aquele de operacao isto e, limpeza da poeira em suspensao, que constitui um problema importante na camara de Wilson. Nao trabalhei com camara de Wilson, mas sabe-se que este e um problema serio de limpeza. Sob tal aspeto devo dizer que justamente a camara de difusao apresenta csa enorme vantagem, nao apresentando problemas desse tipo. Esse fato ja e citado na literatura a respeito e o constatamos tambem nas camaras construidas. A camara descrita entra em funcionamento logo que as temperaturas se estabilizam, o que se verifica em tempo relativamente curto — cerca de meia hora.

KERST: Does the chamber always make electron tracks?

ARRUDA: Yes, the chamber is sensitive to electron tracks.

SALMERON: Observou se e necessario mudar periodicamente a posicao da agulha que controla a expansao do CO_2 na parte inferior depois de algumas horas de funcionamento?

ARRUDA: Normalmente o ajuste da agulha e simples, o controle da temperatura sendo imediatamente acompanhado por meio do miliamperímetro. A inércia térmica das placas, entretanto torna muito simples este controle, e se há alguma variação periódica, deve-se em parte a formação de gelo seco na tubulação da serpentina.

ESCOBAR: Desearia una información sobre los dos aspectos siguientes: 1) si se han efectuado experiencias cargando la cámara con argón o otro gas de esta misma naturaleza y 2) si al colocar placas de plomo o aluminio en el interior de la cámara no se alteraron las condiciones de estabilidad de la misma.

ARRUDA: Quanto a primeira pergunta devo dizer que ainda não foi feita essa experiência. Quanto a segunda pergunta posso dizer que, para pequenas lâminas há uma perturbação, mas pode funcionar bem. Para lâminas de alumínio ou chumbo maiores, entretanto, penso que deve aparecer perturbação pois o gradiente de temperatura fica modificado.

ESCOBAR: Igualmente me interesaría conocer la opinión del autor sobre la posibilidad de usar en la parte inferior de la cámara un sistema de vacío para evitar pérdidas de temperatura del CO_2 y conseguir una constante lo más baja posible en la misma.

ARRUDA: Parece-nos que qualquer medida que se tome no sentido de reduzir perdas na placa inferior permitindo manter constante a temperatura, somente trará benefícios ao bom funcionamento da câmara.

GOLDEMBERG: Parece-me que introduzindo-se corpos sólidos no interior da câmara, como chumbo, etc., será muito difícil impedir-se a condensação de vapor nesses corpos.

ARRUDA: De pleno acordo. Observamos que, quando utilizamos uma grade de fios finos como a dos eletrodos da câmara, neles ficavam condensadas as gotas do líquido. A câmara de alta pressão de hidrogênio emprega entretanto tais fios. Outras câmaras os evitam empregando uma corda condutora na região conveniente.

VALLARTA: La experiencia que tenemos adquirida con las cámaras de Wilson construidas en México, de diseño parecido a la del prof. Arruda, es muy semejante a la suya. Solo rífero en el tiempo necesario para sensibilizar la cámara, que en nuestro caso es de dos horas o más, en vez de quince minutos. ¿A qué se debe esta diferencia?

ARRUDA: Já me têm sido referido esse fato em relação a outras câmaras de difusão. Como puderam ver ontem na Escola Politécnica, a câmara por nós descrita entra em funcionamento em cerca de meia hora. É evidente, entretanto, que ela tende a melhorar com maior tempo de operação, principalmente quanto à altura sensível.

Contribution a la Teoria de la Antena Rectilinea

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Se trata de estudiar la distribution de la corriente en antenas rectilineas mediante la teoria de Maxwell. Del mismo problema se ocupó, como se sabe, HALLEN (1918) pero será útil recapitular brevemente sus razonamientos ya sea para aclarar una contradicción que se encuentra en su ecuación integral sobre la que está basada toda su teoría si bien vemos que el error cometido por el no da lugar a consecuencias serias. Luego pasaremos a otro método de resolver la ecuación integral que es equivalente a la resolución de un sistema de ecuaciones lineales y que permite alcanzar cualquier grado de aproximación sin dificultad alguna. Además, tiene la forma de nuestra solución la ventaja que con ella pueden calcularse sencillamente otras propiedades de la antena, como por ejemplo su área de absorción.

I. EL METODO DE HALLEN

La longitud de la antena sea $2L$ y sus extremos sean $z = \pm L$. El punto de alimentación o el lugar del aparato de recepción, $z = 0$. Como la corriente tiene la dirección del eje z vale para el potencial vectorial $A_x = A_y = 0$,

$$(1) \quad A_z = A = \frac{\mu}{4\pi} \int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi.$$

En esta fórmula hemos suprimido el factor $e^{i\omega t}$ y, admitiendo que el material de la antena es de conductibilidad perfecta de suerte que la corriente corre solamente en la superficie, hemos integrado sobre la sección.

En el interior de la antena, A depende solamente de z , porque allí por ser $\vec{\mu} \vec{H} = \text{rot } \vec{A}$ resulta $\mu H_x = \frac{\partial A}{\partial y} = 0$ y $-\mu H_z = \frac{\partial A}{\partial x} = 0$. Por esta razón podemos suponer, para calcular A en el interior, por razones de sencillez el punto campo en el eje del alambre, valiendo entonces

$$(2) \quad R^2 = d^2 + (z - \xi)^2$$

siendo d el radio del alambre.

Teorema auxiliar. Si $I(z)$ es una función par (impar) de z , la integral de la (1) también es par (impar). Esto se comprueba decomponiendo la integral

en los dos sumandos $\int_{-L}^0$ y $\int_0^L$, haciendo en la primera la substitución $\xi' = -\xi$ y

suprimiendo luego el tilde en la letra de integracion. El resultado asi obtenido queda invariado (cambia de signo) al cambiar z por $-z$ segun $I(z)$ sea par o impar. La paridad de $I(z)$ depende del modo de excitacion. En el caso de antenas de transmisitor y de recepcion nos interesaran unicamente las $I(z)$ pares.

Las condiciones de contorno. Primero vale

$$(3) \quad I(\pm L) = 0$$

Luego se calcula $\vec{E}$ mediante los potenciales retardados por

$$\vec{E} = -\text{grad } \phi - i\omega \vec{A}.$$

Aplicando esto a la componente z resulta

$$(4) \quad E_z = -\frac{\partial \phi}{\partial z} - i\omega A.$$

Teniendo en cuenta la relación entre ϕ y A

$$\text{div } \vec{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0$$

que en nuestro caso toma la forma

$$(5) \quad \frac{\partial A}{\partial z} + \frac{i\omega}{c^2} \phi = 0,$$

podemos eliminar ϕ de las (4) y (5) y obtenemos

$$(6) \quad \frac{ik}{c} E_z = \frac{\partial^2 A}{\partial z^2} + k^2 A$$

donde

$$(7) \quad k = \frac{\omega}{c}.$$

La segunda condicion de contorno es distinta segun se trate de oscilaciones libres, de una antena receptora o de una transmisora. En el caso de oscilaciones libres los bornes estan conectados a corto circuito y como en el interior de la antena la componente tangencial

$$(8a) \quad E_x = 0$$

resulta por integracion de la (6)

$$(9a) \quad A = a \cos kz + b \sin kz$$

donde a y b son constantes arbitrarias y no funciones de x e y , porque como ya dijimos A en el interior depende sólo de z .

Igualando la (1) con la (9a) obtenemos para *oscilaciones libres*, suprimiendo en el primer miembro $\frac{\mu}{4\pi}$ y dando a a y b otros valores

$$(10a) \quad \int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi = a \cos kz + b \sin kz$$

Con $b = 0$ resultan por el mencionado teorema auxihar las oscilaciones pares, con $a = 0$, las impares con un nodo en $z = 0$.

Aquí se trata de encontrar los autovalores k que resultan por las condiciones de contorno (3) y calcular I en función de z . $k = w/c$ (vease la (7) resulta complejo con la parte imaginaria positiva, lo que garantiza el amortiguamiento por radiación. No entramos en mayores detalles referentes a este caso tratado por Hallén.

Consideremos ahora el caso de la receptora suponiendo que los bornes, como en el caso anterior, estén conectados a corto circuito y que por razones de sencillez, la onda de amplitud E_0 incida normalmente a la antena. Para que el campo en el interior se anule, el E_z en la (6), que es el campo producido por la corriente en la antena, debe compensar al campo incidente E_0 , vale decir la (6) se convierte en

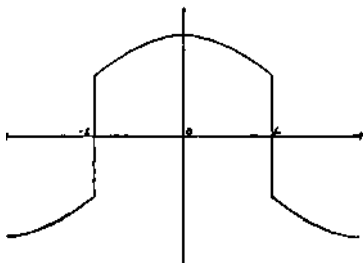


Fig. 1

$$\frac{\partial^2 A}{\partial z^2} + k^2 A = -\frac{ik}{c} E_0$$

o, por integración,

$$(9b) \quad A = -\frac{iE_0}{kc} + a \cos kz + b \operatorname{sen} kz$$

Por combinación de (9b) con (1) resulta

$$\int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi = D + a \cos kz + b \operatorname{sen} kz$$

teniendo la constante dada el valor

$$(11) \quad D = -\frac{4\pi i E_0}{\mu k c} = -i \frac{E_0 L}{\beta \rho}$$

pues

$$c = \frac{1}{\sqrt{\epsilon \mu}}; \frac{1}{4\pi} \sqrt{\frac{\mu}{\epsilon}} = \rho = 30 \text{ Ohm y}$$

$$(12) \quad \beta = k L = \frac{2\pi L}{\lambda}$$

En este caso, en contraposición con el de oscilaciones libres, $k = w/c$ es real y w significa la frecuencia cíclica de la onda incidente.

Puesto que D , como constante, es una función par, podemos decomponer la corriente en I e I' , valiendo

$$\int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi = D + a \cos kz$$

$$\int_{-L}^{+L} I'(\xi) \frac{e^{-ikR}}{R} d\xi = b \sin kz$$

La segunda de estas dos ecuaciones representa una oscilación libre impar que no existe siendo k real. Por esta razón es $b = 0$ y obtenemos para la *antena de recepción a corto circuito*:

$$(10b) \quad \int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi = D + a \cos kz.$$

Para transmisoras seguimos en primer lugar los razonamientos de Hallén. El integra la ecuación (6) sobre el segmento infinitesimal entre los bornes. Si llamamos U el voltaje de alimentación, de manera que

$$\int_{-0}^{+0} E_z dz = -U$$

resulta de la (6)

$$(13) \quad -\frac{ik}{c} U = \frac{\partial A}{\partial z} \Big|_{-0}^{+0}$$

porque A queda finito. Como por otra parte según la (6) en ambas mitades de la antena vale $\frac{\partial^2 A}{\partial z^2} + k^2 A = 0$ y como la integral $\int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi$ debe ser una función par de z , Hallén concluye que

$$\frac{\mu}{4\pi} \int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi = a \cos kz + b \sin k|z|.$$

resultando por la (13) que

$$b = -i \frac{U}{2v}$$

Así obtiene, dividiendo por $\frac{\mu}{4\pi}$ y significando la constante a de otra manera

$$(10c) \quad \int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi = a \cos kz - \frac{iU}{2\rho} \sin k|z|$$

($\rho = 30$ ohm como antes).

Se ve de inmediato que esta ecuación no puede ser correcta, porque la derivada del segundo miembro es discontinua, mientras que la derivada del primer miembro es continua para $z = 0$, pues si se lo diferencia en la integral respecto de z y se integra por partes se anula la parte integrada por ser $I(\pm L) = 0$ y $I(+0) = I(-0)$, quedando

$$\int_{-L}^{+L} \frac{\partial I}{\partial z} \cdot \frac{e^{-ikR}}{R} dz$$

A esta integral, la región entorno a $z = 0$ no contribuye nada porque el núcleo queda finito e I es continua.

El error consiste en el hecho de que la ecuación a derivadas parciales

$$\Delta A + k^2 A = -\mu j$$

con la condición de contorno (13) en el interior de la antena, no tiene la solución (1) sino que vale según el teorema de Green

$$A = \frac{\mu}{4\pi} \int_{+L}^{-L} I(z) \frac{e^{-ikR}}{R} dz - \frac{1}{4\pi} \int \frac{\partial A}{\partial z} \bigg|_{-0}^{+0} \cdot \frac{e^{-ikR}}{R} d\sigma.$$

La integral de superficie se refiere a la sección de la antena en $z = 0$ y como

$\frac{\partial A}{\partial z} \bigg|_{-0}^{+0}$ como constante puede sacarse de la integral obtenemos la integral

$$\int \frac{e^{-ikR}}{R} d\sigma.$$

Si llamamos u a la distancia entre un punto fuente de esa sección y el eje, tenemos que $R^2 = z^2 + u^2$; $d\sigma = 2\pi u du$ y $R dR = u du$, de manera que

$$2\pi \int_{|z|}^{\sqrt{z^2 + a^2}} e^{-ikR} dR = -\frac{2\pi}{ik} \left[e^{-ik\sqrt{a^2 + z^2}} - e^{-ik|z|} \right]$$

y

$$A = \frac{\mu}{4\pi} \int_{-L}^{+L} I(z) \frac{e^{-ikR}}{R} dz - \frac{U}{2C} \left[e^{-ik\sqrt{a^2 + z^2}} - e^{-ik|z|} \right]$$

Por otra parte vale según la (6), como en la antena es $E_z = 0$, $\frac{\partial^2 A}{\partial z^2} + k^2 A = 0$ y teniendo en cuenta la (13) obtenemos la integral

$$A = a \cos kz - \frac{iU}{2c} \sin k|z|.$$

Igualando estas dos expresiones, dividiendo por $\frac{\mu}{4\pi}$ y dando a a otro significado, obtenemos

$$(14) \quad \int_{+L}^{-L} I(z) \frac{e^{-ikR}}{R} dz = a \cos kz + \frac{U}{2\rho} \left[e^{-ik\sqrt{a^2 + z^2}} - e^{-ik|z|} \right]$$

o con otro valor para la constante arbitraria a resulta

$$(15) \int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi = a \cos kz + \frac{U}{2\rho} \cos k\sqrt{d^2 + z^2} - \frac{iU}{2\rho} \operatorname{sen} k\sqrt{d^2 + z^2}$$

Esta es la ecuación integral rigurosa para la corriente de la transmisora. En ella ha desaparecido la molesta discontinuidad de la derivada del segundo miembro.

Para $|z| \gg d$, vale decir con excepción de la región de los bornes podemos reemplazar $(d^2 + z^2)^{1/2}$ por $|z|$ y obtenemos aproximadamente, con otro significado de a , la ecuación de Hallén (10c).

Ahora dos palabras de cómo Hallén resuelve su ecuación integral.

Como el integrando de $\int_{-L}^{+L} I(\xi) \frac{e^{-ikR}}{R} d\xi$ tiene para $\xi = z$ un máximo pronunciado ($R^2 = d^2 + (\xi - z)^2$), él separa de esta integral la parte

$$I(z) \int_{-L}^{+L} \frac{d\xi}{R} = I(z) \cdot \Omega^\times(z)$$

y escribe, por consiguiente, el primer miembro

$$I(z) \Omega^\times(z) + \int_{-L}^{+L} \frac{I(\xi) e^{-ik|\xi-z|} - I(z)}{R} d\xi$$

En esta última integral numerador y denominador se anulan, si se desprecia el radio d , pero el cociente queda finito, de manera que podemos reemplazar R por $|\xi - z|$ y obtenemos

$$I(z) \cdot \Omega^\times(z) + \int_{-L}^{+L} \frac{I(\xi) e^{-ik|\xi-z|} - I(z)}{|\xi - z|} d\xi$$

siendo

$$\Omega^\times(z) = \int_{-L}^{+L} \frac{d\xi}{\sqrt{d^2 + (\xi - z)^2}} = \int_{-L}^z \frac{d\xi}{\sqrt{d^2 + (z - \xi)^2}} + \int_z^{+L} \frac{d\xi}{\sqrt{d^2 + (\xi - z)^2}}$$

o, por la sustitución $z - \xi = u$ en la primera y $\xi - z = u$ en la segunda de estas dos integrales

$$\begin{aligned} \Omega^\times(z) = \int_0^{L+z} \frac{du}{\sqrt{d^2 + u^2}} + \int_0^{L-z} \frac{du}{\sqrt{d^2 + u^2}} &= l n \frac{l+z+\sqrt{d^2+(l+z)^2}}{d} + \\ &+ l n \frac{l-z+\sqrt{d^2+(l-z)^2}}{d} \end{aligned}$$

Esta función tiene el valor $\Omega^\times(0) = 2 l n \frac{2L}{d}$, y varía lentamente con las z crecientes si $L/d \gg 1$ y se aproxima recién cerca de los extremos $z = \pm L$ el valor mínimo

$\Omega^\times (\pm L) = \ln (\pm L/d)$. Resulta, pues, una aproximación si se reemplaza $\Omega^\times (z)$ por la constante

$$\Omega^\times = 2 \ln \frac{2L}{d}.$$

Si $L/d \gg 1$ puede escribirse la (10 c) después de dividir por $\Omega^\times$ y con otro significado de a

$$I(z) = a \cos kz - \frac{iU}{2\rho\Omega^\times} \sin k|z| - \frac{1}{\Omega^\times} \int_{-L}^{+L} \frac{I(\xi) e^{-ik\xi - \alpha\xi} - I(z)}{|\xi - z|} d\xi$$

Gracias al gran valor de $\Omega^\times$ (por cierto que $\Omega^\times$, para $L/d = 100$ tiene el valor 10,60) puede despreciarse primero la integral en el segundo miembro. Así

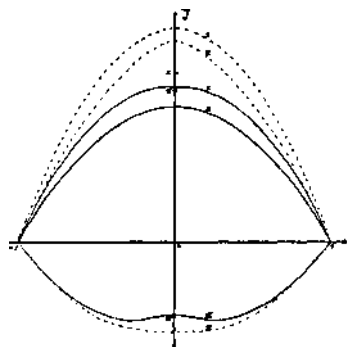


Fig. 2

resulta un valor aproximado de I que corresponde a la teoría elemental y que puedo sustituirse, por el gran valor de $\Omega^\times$, en la integral. Finalmente a debe determinarse de tal manera que $I(\pm L) = 0$. De este modo resulta $I(z)$ proporcional a $\frac{U}{2\rho\Omega^\times}$ y además es función de z y depende de los parámetros $\Omega^\times$ y β (véase la (12)). Esta función se expresa por medio de seno-integrales y coseno-integrales.

Con esta aproximación HALLÉN (1938) ha calculado otra más en la que figuran también términos en $1/\Omega^{\times 2}$.

Para esto tuvo que calcular tablas de seno-integrales y coseno-integrales iteradas, procedimiento bastante complicado cuya continuación es prácticamente imposible.

II. EL METODO NUEVO

Por esta razón hemos desarrollado un nuevo método de solución que vamos a presentar ahora.

Desarrollamos $I(z)$ en una serie de Fourier, y como I debe ser una función para que se anula para $z = \pm L$, tendrá la forma

$$(16) \quad I(z) = \sum_{n=1}^{\infty} C_n \cos \frac{2n-1}{2} \frac{\pi z}{L}$$

Esta serie la substituímos en el primer miembro de las ecuaciones (10). Así obtenemos una función de z que también desarrollamos en serie de Fourier de la misma forma, multiplicando por $\frac{1}{L} \cos \frac{2m-1}{2} \frac{\pi z}{L} dz$ e integrando de 0 a L .

se tienen en cuenta. En la proximidad inmediata de $z = L$ la convergencia es, por esta razón, mala. Mejoramos, pues, nuestro resultado eligiendo a de tal manera que la ecuación integral vale rigurosamente para $z = \pm L$ en la aproximación decaída por nosotros. Esto quiere decir que es para transmisoras:

$$(18) \quad \int_{-L}^{+L} I(\xi) e^{ik\sqrt{d^2+(L-\xi)^2}} \frac{d\xi}{\sqrt{d^2+(L-\xi)^2}} = a \cos \beta - \frac{iU}{2\rho} \sin \beta$$

y para receptoras:

$$= a \cos \beta + D$$

Si sustituimos en estas ecuaciones el desarrollo en serie (16) resulta una ecuación de la forma

$$(19) \quad h_1 C_1 + h_2 C_2 + \dots + h_n C_n + \dots = a \cos \beta + \sin \beta$$

y

$$(19') \quad = a \cos \beta + 1$$

respectivamente, habiéndose suprimido los factores $-\frac{iU}{2\rho}$ y $D = \frac{-E_0 L}{\beta \rho}$ respectivamente (véase la (11)). Las h_n se encuentran calculadas en el apéndice.

El sistema (17) juntamente con las (19) da entonces, cortando después del n -ésimo sumando, $n + 1$ ecuaciones para las $n + 1$ incógnitas $C_1, C_2, \dots, C_n$ y d . El problema se ha reducido así a la resolución de un sistema de $n + 1$ ecuaciones lineales e inhomogéneas para la transmisora y receptora, mientras que para oscilaciones libres el sistema es homogéneo y se presta para la determinación de $C_1 : C_2 : \dots : C_n : d$ y de los autovalores k .

III. LA ANTENA RECEPTORA CON CARGA

Si se trata de determinar la distribución de la corriente en una receptora cargada, puede calcularse por superposición de la corriente $I_1(z)$ de una receptora a corto circuito, vale decir

$$(20) \quad \int_{-L}^{+L} I_1(\xi) \frac{e^{-ikR}}{R} d\xi = D + a_1 \cos kz$$

en la que para $z = 0$ vale $\frac{\partial A}{\partial z} = 0$ y de la corriente $I_2(z)$ de una transmisora en la que el voltaje de alimentación U es reemplazado por $-(I_1 + I_2)_0 Z_c$, siendo Z_c la impedancia del aparato de recepción, según

$$(21) \quad \int_{-L}^{+L} I_2(\xi) \frac{e^{-ikR}}{R} d\xi = \frac{i}{2\rho} (I_1 + I_2)_0 Z_c + d_2 \cos kz.$$

Después de haber resuelto la (20) por nuestro procedimiento, se substituye I_1 en la (21), para lo cual es particularmente ventajoso que I_1 resulte de la (20) ya en serie de Fourier, y luego se resuelve la (21) análogamente a los casos anteriores.

Estos números tienen que multiplicarse por $-iU/2\rho = -iU/60$ para dar la corriente en amperios si se mide U en voltios.

De los coeficientes C determinados puede calcularse la impedancia en primeras cuatro aproximaciones, que viene dada por la formula:

$$Z = \frac{2\rho i}{\sum C_n}$$

TABLA II

	Z en ohm	Z según Hallén
1.ª Aproximación	73,13 + i42,54	0 + i0
2.ª "	85,00 + i45,33	64,8 + i30,3
3.ª "	83,80 + i45,64	80,5 + i40
4.ª "	80,76 + i46,58	—

La figura 2 da I en función de z/L para $L/d = 100$, $\beta = \pi/2$ para la transmisora. Las curvas llenas I, II y III representan la corriente total, la parte real y la parte imaginaria respectivamente, así como resultan de los C de la cuarta aproximación (Tabla I). Las curvas punteadas son las correspondientes a la solución de Hallén, pero en la segunda aproximación, pues debido a la complicación del cálculo, nadie ha calculado la tercera aproximación. Únicamente para $z/L = 0$ existe un valor de Hallén que, correspondiente a las tres curvas, está indicado en el gráfico. Se ve que estos últimos valores se aproximan más a nuestros resultados y corresponden a los valores de la impedancia de la última columna de la segunda tabla.

Para juzgar el grado de exactitud de los valores experimentales de la impedancia Z , que tienen bastante dispersión, reproducimos en la tabla III los resultados de algunos autores.

TABLA III

Z en ohm	AUTOR
83 + i50	KING (1946) (*)
78 + i55	BROWN & WOODWARD (1945)
82 + i50	COCHRANE (SMITH, 1949)
77 + i40	ROBBEER, VILBIN & VOGT (1939)

Se ve por la tabla II que nuestro método es más convergente y, además, más sencillo que el de Hallén. Pero no podemos eallar que en resonancia aparente (por ejemplo $\beta = \pi/2$) el cálculo es más convergente que para otros valores de β . En particular para anti-resonancia ($\beta = \pi$) es necesaria una modificación del procedimiento, principalmente para valores pequeños de z/L .

Para el problema de recepción resultan las C de las ecuaciones (17) y (19'), con los T_m correspondientes (vease apéndice fórmula (m)), las demás constantes teniendo el mismo valor que para el caso de transmisión. La tabla IV nos da los valores de los C hasta la cuarta aproximación, mientras que la tabla V da los valores de la corriente $I(0)$ para la antena en corto circuito.

(*) El valor medido para un L/d un poco distinto fue interpolado por nosotros para $L/d = 100$.

TABLA IV (RECEPTORA)

1.ª APROXIMACIÓN	2.ª APROXIMACIÓN	3.ª APROXIMACIÓN	4.ª APROXIMACIÓN
$C_1 = 0,3906 - i0,2267$	$C_1 = 0,3756 - i0,2314$ $C_2 = -0,0123 + i0,0066$	$C_1 = 0,3663 - i0,2319$ $C_2 = -0,0125 + i0,0069$ $C_3 = 0,0055 - i0,0034$	$C_1 = 0,3652 - i0,2354$ $C_2 = -0,0126 + i0,0072$ $C_3 = 0,0057 - i0,0035$ $C_4 = -0,0034 + i0,0020$

Estos valores deben multiplicarse por $E_0 L/30$ para obtener la corriente en amperios.

TABLA V

APROXIMACIÓN	$I(t)$ EN AMPERIOS
1.ª.....	$(0,3906 - i0,2267)E_0 L/30$
2.ª.....	$(0,3663 - i0,2248)$
3.ª.....	$(0,3652 - i0,2284)$
4.ª.....	$(0,3549 - i0,2217)$

V. LA RESONANCIA VERDADERA

El punto de resonancia (verdadera) se define como el valor de β para el que la parte imaginaria de la impedancia $Z = R + iX$ se anula. Este β_r es prácticamente igual al β para el que $|I(0)|$ tiene su máximo, y se encuentra cerca de $\beta = \pi/2$, de manera que podemos proceder del siguiente modo para obtener un valor aproximado:

En la serie de Fourier para I nos limitaremos al primer término, así que de las (17) y (19) resulta

$$(25) \quad \begin{aligned} g_{11} C_1 &= g_1 a + \Gamma_1 \\ h_1 C_1 &= \cos \beta \cdot a + \sin \beta \end{aligned}$$

Por eliminación de a obtenemos

$$(26) \quad C_1 = \frac{\Gamma_1 \cos \beta - g_1 \sin \beta}{g_{11} \cos \beta - g_1 h_1}$$

y por

$$(27) \quad \begin{aligned} I(0) &= -\frac{iU}{2p} C_1 \\ Z &= \frac{2pi(g_{11} \cos \beta - g_1 h_1)}{\Gamma_1 \cos \beta - g_1 \sin \beta} \end{aligned}$$

Como $2p = 60$ y el denominador de (27) son reales (veanse las formulas (k) y (l) del apéndice) se anula la parte imaginaria de Z si

$$(28) \quad \operatorname{Re}(g_{11} \cos \beta - g_1 h_1) = 0$$

Como esto acontece cerca de $\beta = \pi/2$ resulta por la "regula falsi"

$$(29) \quad \Delta\beta = \beta_r - \pi/2 = -Rc \left(\frac{g_1 h_1}{g_{11} + g_1 \frac{dh_1}{d\beta} + h_1 \frac{dg_1}{d\beta}} \right) \quad \beta = \pi/2$$

o, numericamente

$$(30) \quad \beta_r = \frac{\pi}{2} - \frac{0,354}{\Omega 2,218}.$$

En particular para $L/d = 100$, vale decir para $\Omega = 5,991$, resulta

$$\beta_r = 1,477$$

mientras que KING (1946) observó

$$\beta_r = 1,480$$

La impedancia para $\beta = 1,48$ da en 1a. y 2a. aproximación, respectivamente

$$Z_1 = 61,76 - i0,03$$

$$Z_2 = 67,76 - i6,48$$

Interpolando entre $X = -0,48$ y la 2a. aproximación de la tabla II $X = 45,39$, resulta un valor mejor de la resonancia verdadera ($X = 0$)

$$\frac{2\pi L}{\lambda} = \beta_r = 1,49,$$

es decir

$$L = 0,237 \lambda$$

en lugar de

$$L = \frac{\lambda}{4}$$

según la teoría elemental.

Para $\beta = 1,49$ resulta en 2a. aproximación la resistencia de radiación R por interpolación entre

$$\beta = \pi/2 = 1,5708 \quad R = 85,09$$

$$\text{y} \quad \beta = 1,48 \quad R = 67,74$$

el valor para

$$\beta = 1,49 \quad R = 69,86$$

Pero como en 4a. aproximación segun la tabla II, R es mayor en la proportion 86,76/85,09 a la 2a. aproximación, obtenemos

$$R = 71,2 \text{ ohm}$$

para $L/d = 100$ y el punto de resonancia verdadera $\beta_r = 1,49$.

VI. ALGUNAS APLICACIONES

a) *Area de absorcion de una antena.* RUDENBERG (1908) ha mostrado que la potencia absorbida por una antena receptora no puede ser mayor que

$$(31) \quad P = \frac{3}{8\pi} S \lambda^2$$

Aquí S significa la intensidad de la radiación incidente y λ la longitud de onda. Pero el idealizo la receptora considerandola como un dipolo de Hertz, vale decir como una antena corta frente a λ y provista de una capacidad de techo tan grande que la corriente a lo largo de la antena es constante⁽³⁾. Es fácil examinar si esta relación vale también para una antena rectilínea de longitud comparable con λ .

Para hacerlo pueden deducirse las propiedades de recepción de las de transmisión, utilizando el teorema de reciprocidad (GANS, 1947). Una de las dos antenas a que se refiere el teorema es la que nos interesa, mientras que la otra podemos considerarla como un dipolo de Hertz. Así resulta que renunciamos a deducir aquí la fórmula

$$(32) \quad P = \frac{\left| \int_0^L I(\xi) d\xi \right|^2}{\frac{3}{2} \int_0^1 ds (1-s^2) \left| \int_0^L I(\xi) \cos k s \xi \cdot d\xi \right|^2} \cdot \frac{3}{8\pi} S \lambda^2 = p \cdot \frac{3}{8\pi} S \lambda^2$$

Aquí I significa la corriente en la antena de longitud $2L$ utilizada como transmisora y S la radiación incidente.

Para $kL \ll 1$, el coseno en el denominador toma el valor 1 y resulta al (31), es decir $p = 1$. Esto demuestra que (31) vale más generalmente de lo que podía preverse según la deducción de Rudenberg, es decir para cualquier antena corta o independientemente de la distribución de la corriente, por lo tanto también si $I(\pm L) = 0$, mientras que Rudenberg demostró su relación solamente para I independiente de z .

Para $\beta = \pi/2$, es decir, $2L = \lambda/2$ resulta, si substituímos I por la (16) y nos servimos de los C_n válidos para la transmisora y dados por la 4a. columna de la tabla I:

$$p = 1,104$$

Como nos hemos limitado a $\beta = \pi/2$ este p nos da un límite inferior del área de absorción de una antena rectilínea.

Si hubiésemos despreciado las armónicas de la distribución de corriente, habría resultado

$$p = 1,094,$$

es decir un valor inferior en sólo 1%.

b) *La corriente de reception de una antena con carga.* Otra aplicación, cuyo resultado damos también sin deducción, y basado en el teorema de reciprocidad

⁽³⁾ La fórmula (31) vale también para una antena de cuadro cuyas dimensiones lineales son pequeñas frente a λ .

es la determinación de la corriente de recepción de una antena cargada. Se trata de expresarla por los coeficientes de Fourier del problema de transmisión. Resulta

$$(33) \quad j = \frac{4 E_0 L}{\pi} \cdot \frac{\sum (-1)^{n-1} \frac{C_n}{2n-1}}{2 \rho i + Z_c \cdot \sum C_n}$$

donde E_0 es la amplitud del vector eléctrico de la onda incidente, Z_c la impedancia de carga $\rho = 30$ ohm y los C_n son los coeficientes de la tabla I que se refieren al problema de transmisión.

Para corto circuito ($Z_c = 0$) obtenemos, en particular

$$(34) \quad j = \frac{2 E_0 L}{\pi \rho i} \sum (-1)^{n-1} \frac{C_n}{2n-1}$$

Si, en cambio $Z_c \rightarrow \infty$ resulta de la (33) el voltaje en los bornes de la antena no cargada

$$(35) \quad U = j Z_c = \frac{4 E_0 L}{\pi} \cdot \frac{\sum (-1)^{n-1} \frac{C_n}{2n-1}}{\sum C_n}$$

No es necesario, pues, resolver el problema de recepción como lo hicimos en IV, si se trata solamente de la corriente de recepción y no de la distribución a lo largo de la antena. Si, a pesar de esto, lo hemos hecho para la antena de recepción a corto circuito (IV), obtenemos para la antena cargada sencillamente

$$I(0) = I_0(0) \frac{Z}{Z + Z_c}$$

significando I e I_0 las corrientes de recepción con carga y a corto circuito respectivamente, Z y Z_c las impedancias de la antena y la de carga.

APENDICE

En este apéndice damos los valores de los coeficientes que aparecen en el texto de este trabajo.

1.^a

$$g_{mn} = \frac{1}{L} \int_0^L \cos \gamma z \, dz \int_{-L}^{+L} \cos \alpha \xi \, e^{-ik \sqrt{d^2 + (\xi-z)^2}} \frac{d\xi}{\sqrt{d^2 + (\xi-z)^2}} \quad m \neq z$$

con las abreviaciones

$$\alpha = \frac{2n-1}{2} \frac{\pi}{L} ; \quad \gamma = \frac{2m-1}{2} \frac{\pi}{L}$$

Resulta

$$(a) \quad g_{mn} = \frac{(-1)^{m+n}}{(\alpha L)^2 - (\gamma L)^2} (\alpha L \Sigma_\gamma - \gamma L \Sigma_\alpha)$$

siendo

$$(b) \quad \Sigma_\alpha = \int_0^{2L} \sin \alpha U e^{-iku} \frac{du}{u}$$

y Σ_γ el valor que resulta si cambiamos α por γ .

Vemos que

$$g_{nn} = g_{nn},$$

2.º

$$(c) \quad g_{nn} = K_{\alpha} + \frac{\Sigma_{\alpha}}{2\alpha L} - \frac{\beta [\operatorname{sen} 2\beta + i(1 + \cos 2\beta)]}{2(\alpha^2 L^2 - \beta^2)}$$

siendo

$$(d) \quad K_{\alpha} = \int_0^{2L} \cos \alpha u e^{-ik u} \frac{du}{\sqrt{d^2 + u^2}}$$

y β , como en el texto,

$$\beta = kL = \frac{2\pi L}{\lambda}$$

Si, en particular, $\beta = \alpha L = \frac{2n-1}{2} \pi$ (resonancia aparente), g_{nn} pasa a ser

$$(e') \quad g_{nn} = K_k + \frac{\Sigma_k}{2\alpha L} - \frac{1}{2}$$

siendo

$$\Sigma_k = \int_0^{2L} \operatorname{sen} k u e^{-ik u} ; \quad K_k = \int_0^{2L} \cos k u e^{-ik u} \frac{du}{\sqrt{d^2 + u^2}}$$

Σ_{α} y K_{α} pueden reducirse a funciones conocidas y tabuladas. Por la (b) resulta

$$(c) \quad \begin{aligned} \Sigma_{\alpha} = & \frac{1}{2} S(2\alpha L + 2\beta) + \frac{1}{2} S(2\alpha L - 2\beta) \\ & - \frac{i}{2} C(2\alpha L + 2\beta) + \frac{i}{2} C(2\alpha L - 2\beta) \end{aligned}$$

$$(f) \quad \begin{aligned} K_{\alpha} = & \Omega - \frac{1}{2} [C(2\alpha L + 2\beta) + C(2\alpha L - 2\beta)] \\ & - \frac{i}{2} [S(2\alpha L + 2\beta) - S(2\alpha L - 2\beta)] \end{aligned}$$

con la abreviación

$$(g) \quad \Omega = \ln \frac{4L}{d}$$

Es

$$(h) \quad S(x) = \int_0^x \frac{\operatorname{sen} x}{x} dx$$

el seno-integral, representado en general por $\text{Si}(x)$, y

$$(i) \quad C(x) = \int_0^x \frac{1 - \cos x}{x} dx$$

que está relacionada con el coseno-integral $\text{Ci}(x)$ por

$$C(x) = \ln \gamma x - \text{Ci}(x)$$

($\ln \gamma = 0,5772$, constante de Euler-Mascheroni).

3.º Luego es

$$g_m = \frac{1}{L} \int_0^L \cos k z \cos \gamma z dz$$

$$(k) \quad g_m = (-1)^{m-1} \frac{\gamma L \cdot \cos \beta}{\gamma^2 L^2 - \beta^2}$$

y en el caso de resonancia $\beta = \gamma L$

$$(k') \quad g_m = \frac{1}{2}$$

Para *transmisoras* vale

4.º

$$(l) \quad \Gamma_m = \frac{1}{L} \int_0^L \sin k z \cdot \cos \gamma z \cdot dz = \frac{1}{\beta^2 - \gamma^2 L^2} \left[\beta + (-1)^m \gamma L \sin \beta \right]$$

y en resonancia ($\beta = \gamma L$)

$$(l') \quad \Gamma_m = \frac{1}{2\beta} = \frac{1}{(2m-1)\pi}$$

En cambio para *receptoras*

$$(m) \quad \Gamma_m = \frac{1}{L} \int_0^L \cos \gamma z \cdot dz = \frac{\sin \gamma L}{\gamma L} = \frac{(-1)^{m-1}}{2m-1} \cdot \frac{2}{\pi}$$

5.º Además

$$h_n = \int_{-L}^{+L} \cos \alpha \xi \frac{e^{-ik(L-\xi)}}{\sqrt{d^2 + (L-\xi)^2}} d\xi$$

$$(n) \quad h_n = (-1)^{n-1} \Sigma_n$$

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DISCUSI6N

FRANZ: SO VE que la convergencia numerica del calculo en el caso de resonancia es muy buena. Es posible decir algo sobre la convergencia en el sentido matcmatico?

GANS: En el sentido matematico hay criterios para la convergencia de un sistema de un numero infinito de ecuaciones lineales. Los criterios son, si no estoy equivocado, suficientes y necesarios. En el caso practico, el problema de la convergencia es algo academico puesto que si nos limitamos a 4 terminos por ejemplo, no importa si el sistema es convergente o no convergente. Recuerdo el caso de series semi-convergentes que tambien pueden aplicarse practicamente. Sin estudiar la convergencia o no convergencia hemos calculado hasta el maximo de 4 terminos y nos hemos convencido de que esta aproximacion es suficiente, es decir, hemos substituido el resultado en la ecuacion integral y hemos constatado que practicamente el primer miembro y el segundo miembro dan el mismo resultado. Esto quiere decir que esta cuarta aproximacion es suficiente

VALLARTA: Para el caso de la antena de cuarto de longitud de onda que presents el prof. Gans, las medidas de impedancia estdn en excelente acuerdo con la teoria del prof. Gans. Recientemente tuve la oportunidad de conocer algunos resultados de Collins y estos resultados experimentales son casi identicos a la cuarta aproximacion del prof. Gans. De manera que hay perfecto acuerdo entre la teoria del prof. Gans y el experimento, pero quisiera preguntar si ha ensayado esta misma teoria para otros casos, digamos para el caso en que la antena es de media onda.

GANS: Esto es sumamente diffcil en el caso de la anti-resonancia, la convergencia es muy mala, es decir, se puede, por un artificio particular, calcular la distribution de la corriente que esta de acuerdo casi a lo largo de toda la antena, pero en el centro donde es mas interesante fracasa el metodo. Yo creo que hay que buscar otro melodo para calcular en anti-resonancia.

GROSS: Cuando se sustituye la expansion de Fourier, se obtiene una integral que a mi me parece muy complicada, como se puede obtener la solution de la integral?

GANS: Se sustituye la serie en la integral y luego se desarrolla la integral segun una serie analoga a la de I, es decir se desarrolla A en la misma forma que I; entonces resultan estas constantes por cuadraturas relativamente sencillas.

Nine Channel Coincidence and Anticoincidence Apparatus(*)

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We want to describe an apparatus for cosmic ray work which differs from the apparatus generally used. The difference is that the apparatus to be described allows for taking seven measurements simultaneously.

DESCRIPTION OF THE CIRCUIT

The general structure of the apparatus is given in fig. 1. There are nine input posts for the pulses coming from the counters. The inputs are arranged in groups. The inputs 1 and 2 form the group I, the inputs 3, 4 and 5 form the group II and the inputs 6, 7, 8 and 9 form the group III. The pulses feed to the

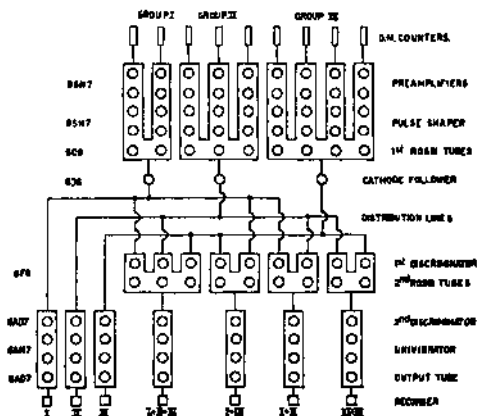


Fig. 1. Block-diagram of the apparatus without power supply.

posts are amplified and equalized by preamplifiers, each of which is working with two 6SN7 tubes in RC-coupling. The first 6SN7 works like an ordinary amplifier without pulse shortening, while the second is used in order to get the desired pulse shape. This is done by sharp differentiation. Positive feed back

(*) Supported by the Conselho Nacional de Pesquisas.

is used in order to reduce the rise-time of the pulses. The preamplifiers may produce either negative pulses of about 3μ sec. duration or positive pulses of about 8μ sec. duration. The amplitude of the pulses is approximately 12 volts. The preamplifiers are followed by the first Rossi shape. Here the pulses coming from the preamplifiers are mixed to give either coincidences or in anticoincidence, according to whether positive or negative pulses are used. A switch is used to obtain coincidences, anticoincidence, or to block the Rossi-tube when necessary. With the switch connected for coincidences, a positive tension of 6 volt is applied

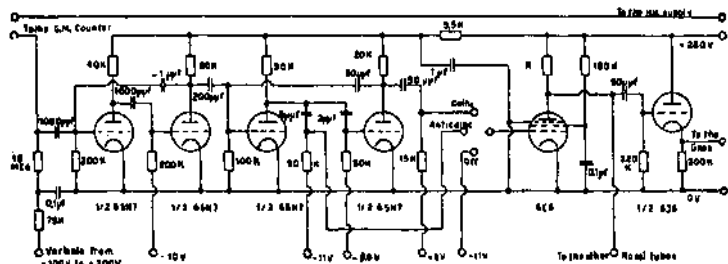


Fig. 2. Diagram of the preamplifier; pulse-shaper, 1st Rossi-tube and cathode-follower. The value of the resistance R is given in text.

to the grid of the 6C6 Rossi tube. Thus the grid wire requires no shielding, because the pulse is insensitive for small pulses induced on the grid. A shielded wire would increase the pulse duration and reduce the pulse height. With the switch connected for anticoincidence a negative tension is used and positive pulses are applied to the grid.

The load resistances of the Rossi stages are $70KU$ for the group I, $60KU$ for the group II, and $50KU$ for the group III. These values are chosen so as to give pulses of the same height in all three groups. The tendency is to keep the resistances low in order to minimize the effects of distributed capacities. The

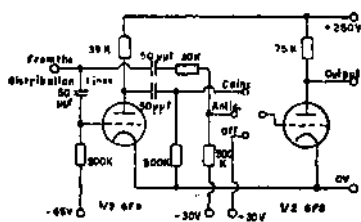


Fig. 3: The first discriminator and the second Rossi-tube. The pulses coming to the output are taken to the anodes of the other second Rossi-tubes and also to the second discriminator.

output pulses from the Rossi tubes are fed into a cathode followed and then to the distribution lines. It has been found necessary to use cathode followed in order to avoid shunting of the load resistances of the Rossi tubes by the capacity of the lines.

The distribution lines take the pulses to the first discriminator tubes. The pulses from the different groups are mixed in the second Rossi stage. In this case too we can choose coincidences or anticoincidence, using negative or positive pulses. For anticoincidence the Rossi tubes are biased far beyond the cut-off and they reline pulses directly from the distribution line without discrimination. The discrimination of the pulses is done by the Rossi-tubes themselves properly biased for this purpose.

High Intensity Accelerated Particle Beams Emerging Through Dynamically Reduced Pressure Stages

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The absorption of corpuscular beams has been found a convenient method for measuring non-stationary states of the temperature and density of gases, especially at low pressures (SCHOPPER & SCHUMACHER, 1951). An apparatus supplying high intensity beams of accelerated monokinetic particles was developed in the following manner: The beam of electrons or protons, accelerated and focused in well known manner in a vacuum tube, emerges into the gas room through a set of diaphragms, detaching chambers of gradually reduced dynamically maintained pressure, by connecting each of them to an air pump. By means of suitable pumps a difference of 2 atm. against vacuum can be reduced by 2 stages, by using diaphragms of 0,2 mm of diameter. Computations and diagrams are shown.

Opposed to the usual method of foilcovered diaphragms, which is not applicable for high intensities of the beam from thermal points of view, the described method allows to issue e.g. a current of 300 μ A of 50 keV electrons into a gas of 2 atm with the mentioned design of 0,2 mm of diameter. In the same way monokinetic proton or deuteron beams from 100 keV to 1,2 MeV were shot into gases of normal pressure.

Applications in spectroscopy, nuclear physics and electron-microscopy are described.

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