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MODERN MAGNETICS

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BY

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AUTHOR'S PREFACE

In the comprehensive treatise on electricity and magnetism edited by Graetz, I have dealt with the whole subject of magnetism, with the exception of magneto-optics and a few minor special questions, in a comprehensive manner corresponding to the character of that work. While doing so it occurred to me that it might be desirable to offer to readers who are not in a position to study so detailed and difficult a work a general account of magnetism, and this idea was cordially received by the editor. In the literature of the subject there is room for such a book; the few books that might be mentioned are above all intended for specialists in the narrowest sense of the term, and moreover are somewhat out of date. These were the considerations which led to the preparation of the present volume. In some passages it makes use of mathematical methods, but for the most part it is so written that any intelligent non-mathematical reader will be in a position to master it. While primarily intended for the teacher and the technologist it should also appeal to the lay reader with a thirst for knowledge. In this book I have of course added chapters on the subjects mentioned above which were undertaken by other writers in the larger work. Thus there is presented within a narrow compass a comprehensive picture of the present position of the theory of magnetism. The text is supplemented by numerous figures, some of them graphic representations of the laws and phenomena concerned; some of them geometrical constructions, and others schematic or detailed representations of apparatus or the arrangement of experiments designed to elucidate the subject. The more important numerical data have also been introduced. In the English edition a select bibliography has been added.

F. AUERBACH

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MODERN MAGNETICS

I

PRELIMINARY

1. **Introduction**—It is usual to say that science unveils the mysteries of nature. But if we look at the matter carefully we shall see that it cannot thus be so summarily dismissed, and that in certain respects it is rather the contrary that is true. To the eye of simplicity there is no secret, for everything seems natural and self-explanatory, and science, therefore, is superfluous. The experienced inquirer makes the noteworthy discovery that the deeper he penetrates into the nature of things, the mystery, far from disappearing, grows ever more profound; the veil may indeed become finer, but it becomes more abundant and more involved.

Magnetism is one among the great mysteries in the field of natural phenomena. From the human standpoint we can divide the things of nature into two classes: those of which we are immediately conscious through the sense organs, and those which we can only indirectly infer through other perceptions of the senses. To the first class of things Magnetism does not belong. Human beings have no magnetic sense. In order to remove any doubt that may exist on this point it has been made the subject of special investigation. People have been placed between the poles of a powerful electromagnet, and under circumstances that excluded any possibility of their giving any but a genuine answer, have been asked to say whether according to their senses the magnet was excited or not. Such experiments have always had a negative result, and it has been shown that it is quite impossible to decide by any direct evidence of the senses whether a body held in the hand is or is not magnetic, or whether one is standing in a neutral or a magnetic field. Now, as will be shown later, there can be no doubt that all substances, the human body not excepted, are in some degree magnetizable, and therefore affected through external magnetic influences which will manifest themselves as a mechanical attraction or repulsion; but these effects as a matter of fact lie quite below the ordinary level of direct perception, and are therefore quite imperceptible. Even if one is carrying several pounds of iron in one's hand one has to stand in an external magnetic field of considerable intensity in order to become aware of the forces experienced by the iron mass, and this would then be regarded not as something mysterious but as something quite natural. A human being, as far as direct sense perception goes, is quite "amagnetic." This needs to be specially emphasized on account of the difficulty that is caused by those who use the word "Magnetism" when they speak of the animal or peculiar human magnetism in connection with spiritualism, occultism, hypnotism, and so forth. The reader will find nothing here concerning things of that sort, and whoever is seeking for information on such matters is advised to lay down this book at once. What we shall consider here are those purely physical phenomena which we can approach indirectly only through their effects in other ways, and particularly through their electrical or mechanical effects, with which we shall deal and into the details of which we shall enter partly on account of their scientific interest, and partly on account of their important modern technical applications.

2. **The Magnetic Field of the Earth**—In that stratum in which we live, immediately surrounding the surface of the earth, various forces are active of which we are directly conscious. We need only mention one as an example here: the force of gravitation. Of this force we are directly conscious inasmuch as we find it easy to go downhill and difficult to go uphill. In the first case we yield to the force of gravitation, urging us downward; in the other we are working against it. Now the region in which a force is operative we call a field; and the surface of the earth is, therefore, a gravitational field. But it is a field in many other senses besides, and with one of them we are concerned here; the surface of the earth is also a magnetic field. That it is so we cannot, as we have already indicated, directly perceive; we cannot say whether we are going with or against the magnetic force; the "uphill" and the "downhill" do not manifest themselves directly to our senses as they do in the case of the gravitational field. We can only infer the existence of this field indirectly through phenomena of which we are conscious;

and which, because we could not otherwise account for them, we find ourselves compelled to ascribe to a new sort of natural force, in short the force of magnetism. The most important among these phenomena are the following:

When we act upon a steel needle, for example a knitting needle, with a certain naturally occurring stone, the lodestone, in a particular way of which more will be said later, and if we allow the needle thus treated to move freely in a horizontal plane or place it carefully, so that it floats, on the surface of still water, we find that it will take up a definite position, so that one end points approximately north and the other south. And we find moreover that it is always the same end that points to the north or the south as the case may be. A rather more careful observation will make it clear that the position of the needle diverges slightly from the north and south direction, and this divergence is called the declination and is found to be of different amount at different parts of the earth's surface. If on the other hand we arrange that the needle can only turn about a horizontal axis (which is best placed so that it is at right-angles to the plane in which the needle places itself when it is free to move about a vertical axis) it will then be found that the needle does not place itself in a horizontal position but that the north end will be inclined more or less downwards (at any rate in our part of the earth) and the angle which it makes with the horizontal is called the inclination. From these two observations taken together it follows that the needle will take up a certain direction in space which is obviously the direction in which the field of force acts at the place under consideration.

In the case imagined it is the magnetic needle which plays the passive part in regard to the earth's field which is supposed to be active. But it can easily be shown that the needle could also take the active part (for in nature, always and everywhere, an interchange of parts of this sort is possible). To demonstrate this the needle itself is placed in a fixed position, and is chosen to be as highly magnetic as possible and preferably in the form of a magnetized rod. We must suppose the magnetism of the rod to be so strong that in its immediate neighbourhood its own magnetic field is predominant and that of the earth is by comparison negligible. If a light iron needle that can turn in a horizontal plane is placed at one end of the bar it will turn in its direction and behave as if this end were attracted by one end of the bar and will not come to rest until the distance between the two is a minimum. If the light needle is hung by a thread it will obey this inclination not merely by turning but by tending to move bodily towards the bar, although in doing so the thread by which it hangs is pulled into an oblique position against the force of gravity. This overcoming of the force of gravity is still more clearly shown when the rod is held vertical and the light needle is brought near the lower end. If free to move, and if the distance be sufficiently small, the needle will fly upwards and remain hanging from the end of the bar.

A difference, however, is to be noticed in regard to the details of the phenomenon according as the needle used in the experiment is in its natural condition, is or is not "bewitched" as one might express it, through previous treatment with the lodestone. In the former case it will be attracted by either end indifferently; but when it is "bewitched," and when therefore the phenomenon indicated is not, as it were, played between an initiate and an outsider, but between two initiates, even though one be large and the other small, a distinct difference is to be observed in their behaviour; one particular end of the needle will always be attracted by a particular end of the rod, and the other will be repelled even if to obey this tendency the needle has to turn round or tilt over. And further that end of the needle which is attracted by one end of the rod will be repelled by the other, and vice versa. A steel bar that has been stroked with the lodestone has therefore, even if its form is quite symmetrical, two different ends which behave in absolutely contrary fashion, and this applies to a small needle as well as to a large bar.

And there is one further point: If, turning once more to the magnetized needle suspended in the earth's magnetic field, it is moved from the north or south position which it assumes and then left to itself, it resumes exactly like a pendulum its original position after a series of oscillations. The frequency of the oscillations is obviously a measure of the strength of the force that is exerted by the field. And it is also found that there is a considerable difference in the force in different parts of the earth's field. And the force is much greater when, instead of the earth's field, that due to an artificial magnet is substituted. A needle that is free to turn when brought close to one end of this magnet executes a series of short, rapid oscillations and very quickly comes to rest in the position imposed upon it by the magnet.

If a magnetized needle is in this way dependent on the earth's magnetic field the question arises whether an arrangement of magnets might not be devised that is less or even not at all subject to its influence. A solution is easy. It is only necessary to fix together two oppositely orientated needles and then hang them from the same thread. Thus a combination is obtained which is termed "astatic." In the ideal case, that is when the two needles are of equally magnetic strength, the arrangement is neutral in its behaviour; it will remain in any position in which it may be placed without tending to change its direction; but if the strength of the two needles is slightly different the combination will take up a definite position (and that in the sense of the stronger needle), but it will do so sluggishly and the oscillations about the meridian position will be slower, the smaller the difference in the strength of the two needles. Such astatic arrangements are very considerably used both in the form described and in many other modifications.

3. **The Magnetic Condition**—These phenomena and many others, considered as a whole, lead to the following conclusion. The condition which a piece of iron assumes as a result of treatment with a piece of iron ore is a quite peculiar one and has no relation to any other, and this we call the magnetic condition. The piece of iron has itself become a magnet and in its turn it can be applied correspondingly to magnetize other bodies. A magnet attracts non-magnetic iron and in the simpler cases the action is most strongly manifest at two points which are called the poles; the line joining these two poles being called the magnetic axis. On the other hand, however, the action of the magnet on another magnet is such that one pole attracts one pole of the second magnet and repels the other, and the other pole of the first magnet acts in a converse sense. The poles which repel each other are called like poles, and those which attract, unlike or opposite poles. Further the pole which by its own free motion turns to the north, and places itself in the direction of the line of declination, is called the north pole and the other the south pole. Thus two north poles repel each other as do also two south poles; but conversely there is attraction between a north and a south pole.

4. **Temporary and Permanent Magnets**—There are many different ways of magnetizing a piece of iron or other magnetic substance, and we shall examine these in detail

a little later. But in the first place we must recognize a very decisive and striking difference which presents itself in this subject. To illustrate this we will take two examples which represent the two extremes and between which all actual specimens of iron can be arranged in order. For the sake of brevity we will call the two bodies (A) and (B). The body (A) has the property, after being treated with the lodestone or an artificial magnet, or in any of the other ways to be considered later, of itself becoming a magnet, but only for so long as the magnetic force acts on it. If the treatment is discontinued and, therefore, the forces are no longer applied, it relapses into its natural, non-magnetic, condition. It behaves in short like one who submits to the spell of the master, but immediately becomes free again when the master retires; out of sight, out of mind. The behaviour of the rod (B) is entirely different. Once magnetized it remains fully magnetized long after the treatment, and when therefore the magnetic force has ceased to act on it. It makes no use of its freedom to relapse into its natural condition. In the one case it may be said that a certain temporary condition is imposed on the body, but in the other there are permanent after-effects. Numerous examples of the same sort are to be found in nature, outside the field of magnetic phenomena. When a piece of elastic is pulled it stretches, but returns to its original condition when the tension is relaxed (Case A). But on the other hand a piece of butter, when it is pressed into a certain shape, retains the shape impressed upon it (Case B). The letter S, let us say, is formed by means of electric glow lamps and stands out in relief as soon as the current is switched on, but vanishes again immediately the current is switched off (Case A) (the after-effects in the eye of the spectator being left out of consideration). If the same letter is written with ink upon paper it remains there permanently and can only be removed by forcible erasure (Case B). In the case before us, we have to distinguish between temporary and permanent magnets. But these are only the two extremes. Various grades of iron lie in between, and retain in greater or less degree the magnetism that has been impressed upon them. We shall examine these important effects (remanence) and all that is connected with them later, and for the present shall only remark that pure iron is very near to the limiting Case (A), and hard steel to the limiting Case (B), and that as material for bodies which are required to be frequently but only temporarily magnetized, soft iron must be chosen, but hard steel for permanent magnets.

5. **Magnetization**—And now as regards the methods whereby the body may be magnetized. For the sake of appearance we must use a definite shape, usually a bar. The simplest procedure is to bring this bar into the earth's field, or rather, since it is already there, to place it in a north-and-south direction, or, more exactly speaking, in the direction of the declination. Not much of a result is obtained in this way since the field is too weak. (Magnetism of position.) It is better therefore to bring the bar into the field of an artificial magnet, placing it in such a position that it lies in the same line or so that it touches it throughout its whole length. It has been proved that the effect is increased when the bar is tapped during this procedure—a fact which already suggests ideas concerning the internal nature of a magnetized body. It is still better not merely to lay the bar in question alongside the magnet, but to stroke it with the magnet, carrying the pole of the magnet along the whole length of the bar to be magnetized. There are various ways of doing this, the single stroke and the double stroke, but it is scarcely worth while to go into the matter more closely here, since there is a much more effective and at the same time much more easily controlled method which depends upon the magnetizing effect of the electric current. And in this connection we must go back a little.

One phenomenon which is as mysterious as magnetism is that of electricity, and it surpasses at the same time all other phenomena with which we are acquainted in its many-sidedness. The special form of its manifestation which concerns us here is the so-called electric current which occurs for example when the two poles of an accumulator are joined by a wire. Nothing of this " current " can be seen, its existence can be only indirectly inferred through its effects. These effects manifest themselves partly inside the body of the current, that is within the wire (for example the heating effect), and partly in surrounding space. In the present connection the most important of these effects is the deflection experienced by a magnetic needle suspended so that it can easily turn as a result of an electric current flowing in its neighbourhood; and this deflection, which will be considered in greater detail later, is greater the stronger the current, the stronger the magnetism of the needle, and the smaller its distance from the wire carrying the current. We cannot describe the phenomenon in general terms except by saying that the electric current makes the space surrounding it a magnetic field.

The effect which we have here considered is of the ponderomotive kind, that is a body, in this case a magnetic needle, is set in motion just as in the case of the purely magnetic effect between two magnets. But there is still another kind of effect which we have now to discuss: the magneto-motive force—the excitation of magnetism in a bar hitherto non-magnetic, and here we have found the method of magnetization which is by far the most generally employed. The iron bar is brought into the field of an electric current so arranged that the effect may be as strong as possible. For this purpose, as will be shown and described later, the circuit of the current must be so arranged that its plane is perpendicular to the axis of the bar and so that the current encircles the bar not once but many times, and thus we arrive at the well-known arrangement: the wire carrying the current is wound into a spiral of many turns and the iron bar is placed inside it. If now the current is switched on, the bar becomes magnetized, and the more strongly the greater the number of the windings, the more closely they surround the bar, and the stronger the current passing through it.

Bodies of Class (A) and Class (B) can be equally well subjected to this procedure, but the result in the two cases will be very different. In Case (B) when a steel bar is used a permanent magnet will be created when the current is switched on, and it can be withdrawn from the coil and henceforth used as a magnet. In Case (A), on the contrary, when a soft iron bar is used, when the current is switched on a magnet is obtained just as before, but it is not a permanent but only a temporary magnet, and when the current is switched off or the bar withdrawn from the coil, it again becomes non-magnetic. We have, therefore, a means of making magnets out of soft iron which remain magnetic as long as they are left in the coil out of which one or both the ends may be allowed to project a little, which for many purposes is very convenient. An arrangement of this sort is called an electro-magnet; and since in this way an enormously stronger degree of magnetism can be produced than in any other way, the electro-magnet in its various forms plays an especially important part in general practice and above all in electro-technology.

6. **Pan-Magnetism**—There is still one point to be added in concluding our first survey of the subject. So far we have spoken of the lodestone as a natural, and of iron and steel as artificial magnets. It might, therefore, seem as if magnetism were a quite special phenomenon and limited to the chemical element iron. But this is not the case. It rather appears that all substances known, whether metallic or not, whether solid, liquid or gaseous, are capable of becoming magnetic, and that magnetism is a general property of matter. Most substances, of course, even when subjected to very powerful treatment, become only very slightly magnetic, and remain magnetic only so long as the magnetic force is continued, and therefore belong to Class (A). Only a few substances, iron and some of its relatives, can be powerfully and permanently magnetized. But we will not go into the matter more closely now, but conclude these preliminary observations, the object of which is rather to introduce the subject in order to proceed to a systematic investigation. Systematic—but we shall take care that the system does not deprive us of the freedom of taking the subject in that living and varied order in which circumstances present it to us. We shall in the first place speak of magnets already made and of their effects and properties, and only after that proceed to direct our attention to the way in which magnetism arises. We shall begin with iron and its allies, although this is only a special part of the subject, and by no means the simplest ; and only then proceed to magnetizable matter in general. We shall bring together facts of experience and theoretical conceptions as we find suitable, so that they may lend each other mutual support. By so doing we shall follow a natural sequence, and the whole system of things will in the end build itself up before our eyes almost without compulsion.

II

MAGNETIC FORCES

7. **General**—If we do not speak very exactly we may say that the space surrounding a magnet is a magnetic field, but this is not in every case correct. It is correct to say so if the magnet is in the form of a bar or sphere or disc, the bar may be straight or bent ; but the more bent it is and the closer its ends are brought together the weaker does the field become, and there is one form of magnet which has no external field at all. (Of the internal field we will not speak here.) That form is the ring. If we imagine the bar with poles, its two ends being bent round into a ring and then welded together, these two poles which, as we know, are of opposite nature, are united and cancel each other out and the ring has no poles. But where there are no poles there can be no external field : a matter we will look into more closely immediately. But first of all comes a question closely associated with this : is there under these circumstances any justification for regarding the ring as a magnet at all ? It is, so to speak, dead and has no outward effect. This is a point which we can and must answer. Its inner life is all the more active ; the inner field is extraordinarily strong and has besides very favourable characteristics which are often, and indeed in the majority of cases, much more important than the outward effect. In a dynamo, for example, it is the inward field alone that comes into consideration : the external field is quite superfluous. It must in fact be eliminated, in the first place because it represents a dissipation of energy, and in the second place because it would cause harm. Where it presents itself it is called a stray field, and in the early years of electrotechnics these stray fields were considerable and spoilt many valuable watches, the iron parts of which became magnetized, to the grief of their possessors. But this difficulty has now been overcome. The modern dynamo is a perfectly enclosed whole, and there is no longer any need to be afraid of its effect upon one's watch.

Let us return once more to the bar which we have bent more and more nearly into a ring shape but just stopping before the two opposing ends touch, so that there is still a gap between them. We have then a further very interesting object before us, the discontinuous ring. It is not so far dissimilar from the closed ring and there is at least one place where relations with the outside world are still maintained, viz. in the gap between the two ends. Here a sharp contrast arises : everywhere else the field is zero, but in the gap and in its immediate neighbourhood it is not only present but is even especially strong. The two opposite poles are so to speak in a condition of the highest tension and discharge themselves outwardly since union is denied them. The discontinuous ring, therefore, represents the most perfect type of an active magnet, especially of an electro-magnet.

8. **The Law of Polarity**—Without poles, no field ! This statement is easily written, but we must now look at it rather more carefully, for in explanation of the effects, not merely of magnetism but of any other outwardly acting force, two theories have been put forward, at the bottom of which a very simple and manifest idea lies, but which are, both as regards the theory of knowledge and in their consequences, substantially different. One of them is the theory of action at a distance. This proceeds from the idea of a centre of force, designated the pole, and which, therefore, can be characterized as the pole theory. The "force" has its "seat" in the pole and operates outwardly, directly and without the agency of any intervening medium. All these conceptions and ideas have for us something mystical about them. But it must be remembered that when we come to analyse any theory there is something enigmatic in it which proceeds from the limitations of our faculties.

On the other hand the theory of action at a distance is distinguished by supreme simplicity and elegance. This can be best seen in the example of gravitation, which through the one simple Newtonian law brings the motions of all bodies in the universe under its sway, and which rules alike the heavenly bodies and the rain-drops falling from the cloud. It has indeed been shown that exactly the same law, which in honour of its discoverer has been called the law of Coulomb —with two modifications relating to the sign and the total strength of the effect—also holds both for electricity and magnetism. But while in regard to gravitation the theory of action at a distance embraces everything without exception and only in the most recent times has a breach been made in it in one particular place, matters in regard to electricity and magnetism are substantially different. There are three experiments which can be tried, each of which compels us of itself to abandon the theory of action at a distance. In the first place the experiment which proves that the strength of the action between the pole and the point affected depends upon the nature of the intervening medium ; in the second place the converse fact that the intervening medium is itself influenced by the action of the forces and that it is thereby changed from its natural condition to another ; and in the third place the fact that the force needs time for its manifestations to pass from the pole to the point affected, so that the greater the distance the greater the length of time required. Obviously each of these facts is at variance with the idea of action at a distance. We must rather assume that the action passes progressively from layer to layer like mechanical pressure or the effects of a blow. It is customary to say that the indication of this in regard to electrical and magnetic forces has been obtained. It must be confessed, however, that this formularization is a little premature. For magnetism, only the first two facts can be directly observed, the third can only be inferred indirectly, and even in the case of electricity, substantial differences are to be observed between its various outward manifestations (charge, current, radiation) ; but for all that, it is correct to say that the whole of the effects taken together compels us to relinquish the theory of action at a distance, and thus leads us to the second of the theories proposed—the theory of the field.

9. **Theory of the Field**—This theory assumes the force, so to speak, as omnipresent ; its causes proceed indeed from a certain place, that is to say every point in the field is its seat as well. To borrow an example from the life of the State, it is a sort of decentralization of power. The essential point is that the force does not spring magically over from the pole to the point considered, but that it flows just in the same manner as matter—water, for example. In this sense one may speak of a flow of force or stream of force. The whole field is filled with lines of force. The poles acquire a new significance and meaning ; they are the points from which the lines of force proceed or towards which they tend, and between which they therefore form a bridge. They are in

short the points of concentration of the forces. It will, therefore, be understood that there are two sorts of poles: sources and sinks. The lines of force flow out of the first and into the second: at the one the force wells up into the field from some place or other and at the other it disappears. It is, therefore, quite natural to speak of a positive or negative pole, and in the case of magnetism the sign is definitely related to the orientation of the needle in the magnetic field of the earth.

10. **The Topography of the Field**—We have already made a start with the topography of the field. We will now proceed further with this art or science—for it is both at the same time. Force we must regard as something that streams in all directions and acts in all parts. As regards the behaviour of this flux two considerations must be kept in view. First we can think of force as streaming out at a given moment from a certain point, and this gives us one element of a line of force: we add to this the element that comes next in time and so proceed. We then have an outward-going line of force, and if we continue to the end, one that reaches from the source to the sink. The line so obtained is the topographical picture of something that takes place in time. We will now in the second place think of a definite moment and consider the position of things at that moment. We will begin with the same line element as previously and at the end of it place the line element which represents the flux of force at that point at the same time that the first line element represents the flux of force at the starting-point. Continuing in this way we obtain a line which represents by its direction the force flux for all points of the line at one and the same instant. In the first case we had a succession, but in the second a simultaneity of line elements, and this latter line we designate a line of force. Moreover it is clear that both sets of lines may happen to coincide; this occurs when the flux of force does not change with time, in other words when it is stationary, and with this condition we shall have much to do. But in the general case when the force may change with time the two sets of lines are different. They have, as Fig. 1 indicates, a common first element, but from that point they diverge and cross over each other like the lines at a railway junction.

The lines of force represent at each point the direction in which the force acts; but of course in all other directions there are components of this force with the exception of one direction-complex which occupies one geometrical surface and which has the property that in any of these directions no flux of force takes place. In accordance with the general laws of geometry this surface is obviously at right-angles to the lines of force, and by imagining a succession of such surfaces we obtain the complete topography of the field. It is filled with lines of force and these are crossed by surfaces which have the property that there is no force action between any two points on one and the same surface: over any given surface there is equilibrium. This can be best expressed by the introduction of a new term which not only here but throughout the whole of exact science is of great utility: the idea of Potential. This is a quantity which is of quite different character from force, for while force is a directed quantity, a so-called vector

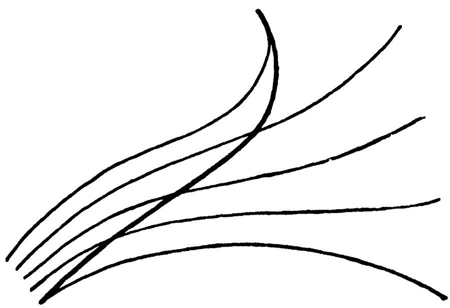


FIG. 1.

possessing a magnitude and a direction, potential is a purely scalar quantity, and at each point of the field has a purely numerical value, from which the force relations in the field can be directly read off and derived, just as from a map with contour lines the gradient of the country represented may be inferred, the lines of maximum gradient at all places being at right-angles to the contour lines. Where the latter lie close together the gradient is steep; elsewhere the slope is less. In the same way the potential is that magnitude whose fall in any direction gives the force or the components of the force acting in that direction. If that direction be found in which the fall of potential is a maximum we at the same time obtain the direction in which the force acts and a measure of its strength.

The direction in which the force acts can now be easily represented: we need only connect all those points together at which the potential has the same value ; such a surface is a surface of equal potential, or if we retain the idea of a map, a contour surface. For another value of the potential we obtain another contour surface, and in short a whole series of them, and at right-angles to them are the lines of force. It is possible to have an infinite number of such surfaces, but it is usual to select them in such a way that in going from any one to the next there is a unit change of potential (or some arbitrary amount depending upon circumstances). Such an arrangement of the contact surfaces gives us a very graphic picture of the field. Where they are close together the force is great (as the gradient in the case of the contour

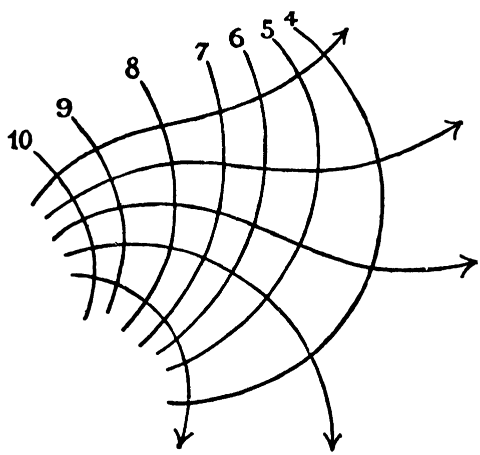


FIG. 2.

line map), where they are widely separated the force is small. On paper a section only of such a field can be represented, and instead of contour surfaces only contour lines, as is shown in Fig. 2, together with the lines of force projected on the plane of the paper.

Also from the infinite possible number of lines of force we must of course make a selection ; but in this case the matter is not so simple as when we were dealing with the contour surfaces of equal potential and calls for more circumstantial treatment.

11. **Flux**—The old theory of action at a distance regarded it, and quite rightly, as its first duty to discover the laws according to which the force at a distance works, and in doing so naturally

set out from the simplest assumption possible, viz. from the assumption that the pole of forces was a point. At a distance r from the pole, what was the effect of the force? From this standpoint the question could only be answered in accordance with experience and observation, or experiments must be made, and this led to the laws of Newton or Coulomb as the case might be. The force decreases as the square of the distance increases: if the distance be doubled, the force is only a quarter as strong; and if it be increased threefold, the force is only a ninth; and when the distance becomes great, it is practically nothing.[1] This law, as has been said, is in the first place a fact of experience. But from the standpoint of the flux of force theory it is a logical necessity, for if the force is supposed to spread out from the point source in all directions, then the stream of force, that is the sum of all the forces passing through a sphere supposed to enclose the point source, is constant, but since it has to spread itself over a greater surface then this specific stream of force, that is the stream of force which passes through unit cross section, becomes more and more diffused. This specific stream of force is what is to be understood by the term force. The force becomes weaker the greater its distance from the source, and the law which here operates is easily intelligible, for we have to do with spherical surfaces which are increasing in size and the area of which will increase in proportion to the square of the radius so that the force must be inversely proportional to the square of the distance from the source.

The simplest way of obtaining the measure of the force is to take the number of lines which pass through unit surface; that is, in the absolute system, the number of lines which pass through a square centimetre, and in order that this number may also be absolute and not merely proportional to the magnitude of the force in any given part of the field expressed in dynes, it is only necessary to presuppose a suitable choice of the lines of force as they flow outwards from the source.

If for simplicity we consider in the first place the source

[1] That is, if F is the force, m_1 and m_2 the attracting masses and r the distance apart,

$$F = \frac{m_1 m_2}{r^2}$$

whose amount is unity at unit distance the force is equal to $1/1^2$, equal that is to say to unity, then care must be taken that just one line of force passes through unit surface of a sphere of unit radius circumscribed about the source. The total area of such a spherical surface is 4π and therefore 4π lines of force must be supposed to come out from the source. (This is not practicable on account of the fraction contained in the number 4π, and how these 4π lines of force could be distributed equally in all directions we do not need at the present moment to ask.) If the total flux from the source is i then the force in the whole field is i times as great as before, so that now $4\pi i$ lines of force must be supposed to flow out from the source. (If i is a quantity of some magnitude the fraction will cause no further trouble.)

12. **Pairs of Poles**—So far we have dealt with the idea of the single or unipolar field. A point now arises which specially concerns us: in other subjects, in gravitation and in electrostatics, this unipolar field presupposes a physical finality; here we must go a step further and transform the field to correspond to the special characteristics of magnetism.

When we are dealing with gravitation we have as an actual picture of the pole the centre of attraction, as for example the sun when planetary space is in question, or the earth when we are dealing with terrestrial space. The sun, and for the matter of that the earth, are of enormous dimensions, but so far as their outward effect is concerned the pole can be regarded as a point situated at the centre of gravity. Therefore the law of inverse squares applies, the distance being always measured from the middle point, the centre of gravity, that is, of the earth or the sun as the case may be. Exactly corresponding to this is the simple pole of electrical force action. But on the contrary a magnetic pole is something that has never existed. Here a *pair* of poles always present themselves, for we have seen that a magnetized bar has a north pole at one end and a south pole at the other. A bar of iron can be brought into a magnetized condition in a variety of ways, either by rubbing it with another magnet or by means of the electric current, but in every case two or four poles or some multiple of two are obtained, or if we are dealing with a ring, no poles at all, but never a single pole; that is to say magnetic poles are always produced in pairs. The sceptic may try in every sort of way to justify himself. He may stroke the bar in such a way, and the procedure is quite an easy one, that he gets a north pole in the middle and a south pole at each end, and then it would seem as if only a south pole were present. But the truth is that the north pole in the middle has twice the strength of each of the two south poles and each of the south poles is bound to it with equal strength. Or the bar may be broken through at the middle in the hope of isolating the two poles, but like Hydras the missing poles spring up again and it is found that each half has again a north pole and a south pole.

Under these circumstances the inverse square law only affords a basis from which to deal with the actual facts. A typical elementary case here is a pair of poles, that is to say two poles which lie near each other and are of equal strength but contrary sign, and it is required to find how such a pair of poles manifests itself in the field which it creates. From the first a decisive difference has to be recognized: while the effect of one pole is equal in all directions so that the equipotential surface is a sphere and the lines of force radial lines, there are here—compare Fig. 3—obviously differences in the various directions. For in the first place there is the direction at right-angles to the polar axis where the effect of the two poles cancels out because there the two poles are at equal distance from the point P_1. Then there is the direction in the line of prolongation of the polar axis represented by the point P_2 where the force is indeed stronger than in any other direction but is still very weak in comparison with the effect of a single pole. For while one pole produces its full effect here the contrary effect of two poles has to be reckoned with, and what counts is the small difference between them, the difference in effect when this deduction has been made being favourable to the nearer pole. Similarly as regards the remaining directions in space which lie obliquely in respect to the polar axis. Here, therefore, we have to deal with a differential effect, and this follows a different law from the principal effect. If the law of an inverse square of the distance holds for the last, then for the first, as the differential

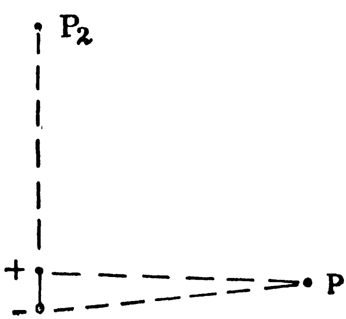


Fig. 3.

calculus shows, and as can be readily seen, the law of the inverse cube holds: the force diminishes in proportion as the cube of the distance increases. Therefore for an equal strength of source it becomes negligible at a much smaller distance, so that one might consider the sphere of influence of a pair of poles to be much less than that of a single pole of the same strength. And moreover the equi-potential surfaces are no longer spherical, they are on the contrary surfaces of extraordinary complexity, as the effects of the poles depend in a very high degree upon the direction. It may be mentioned that, as will be seen from Fig. 4, the equal potential surface in this case consists of two spheres of equal radius which touch each other. The other equal potential surfaces have a

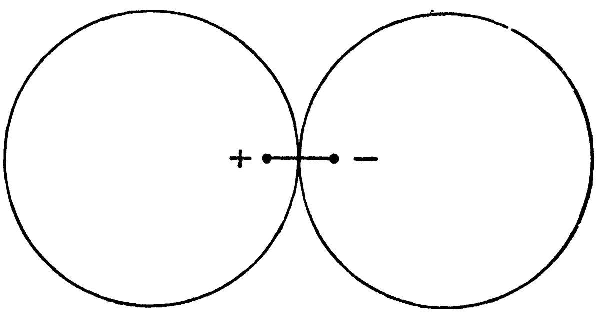


FIG. 4.

similar form and position, the radius of the spheres however being different.

13. **Central Force and Turning Force**—But the cubic law is still not the final one. We have a further step to take in order to be able to consider a specially important case, for we have so far always made a fictitious supposition which does not correspond to reality. We have considered as the active agent a pair of poles, and as the passive object on which the action takes place, so far only a point—the point of application. But as soon as we leave the realm of thought and enter upon that of reality, that is, as soon as we proceed to actual experiment, we are concerned with a pair of poles, i.e. a small needle and not a single pole. How therefore does a fixed bar magnet act upon a movable magnetic needle—it always being presupposed that both objects, rod and the needle, are short in comparison with the distance apart of their centres? It is possible here to adopt the previous method and say that

here also is a case of differential action, because action on the south pole of the needle is in the contrary sense to that on its north pole and therefore only the difference remains which is due to the more favourable position of one pole in comparison with that of the other. And here also the effect will be zero when the needle is perpendicular to the axial line and strongest when it lies along it. Thus we arrive at the conclusion that the force of a magnet on a needle decreases in the ratio of the fourth power of the distance. This force will, therefore, even at the most moderate distances, be scarcely perceptible. But even this way of looking at the matter is not complete : the force arrived at in this way needs a more exact distinguishing mark and then it becomes evident that there is an additional effect in operation. This effect is the displacement force of attraction or repulsion, the consequence of which is that the needle as a whole tends to move towards or away from the magnet. In addition to this if the needle is free a rotation is produced as the effect of a turning moment. And if the needle is balanced on a fixed point there is no displacement of the needle as a whole : the rotation is, therefore, the only effect that occurs. And in regard to this the position is substantially different ; the north pole of the bar repels the north pole of the needle and attracts its south pole and these two effects do not weaken but accentuate each other, for the rotation produced is due not to the difference but to the sum of the two quantities, and the same applies to the effect of the other pole of the bar on the two poles of the needle. Fig. 5 shows this schematically. Here therefore the effect of a bar on one pole is fully exerted, and even indeed in twofold measure, and we obtain the proposition that the turning moment of a bar on a needle decreases as the cube of the distance between their centre points. The turning moment is therefore of a higher order than the displacement force, and at some distance only the first will be appreciable.

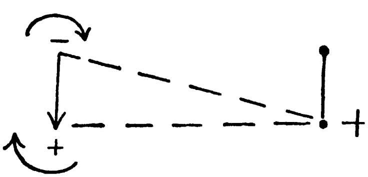


FIG. 5.

14. **The More Simple Formulæ**—When once the fundamental laws have been settled regarding the problem of the two pondero-motive effects of magnets certain systematic difficulties no longer present themselves. But the working

out is generally very laborious and the formulæ finally reached are extremely complicated even when the phenomena are restricted to two dimensions instead of three. For let us consider what the determining factors are (Fig. 6): let 2L be the length of the magnet and $2l$ that of the needle; the pole strength of the bar $\mathfrak{M}$ and the pole strength of the needle $\mathfrak{m}$, the distance apart of their middle points r, Φ and ϕ the angles which the bar and the needle make with the lines joining their centres. The chief difficulty arises

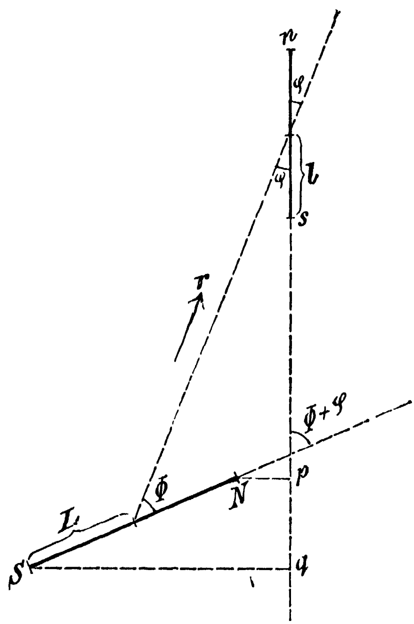


FIG 6.

from the fact that in all the trigonometrical formulæ quadratics arise and that in considering the turning moment the cube is involved. Therefore an exact solution of the problem cannot be obtained, but methods of successive approximation must be employed which give a more and more exact result the further the development is carried. Such an approximation was given by the distinguished physicist Gauss, by whom this theory was chiefly established. Later Lamont, Weihrauch, Chwolson among others have carried the matter further. Here it is sufficient to quote the first approximation which obtains when the bar and the needle are short in comparison with the distance r. Each of the two bodies magnetically considered are then characterized by one simple quantity the product $2\mathfrak{M}L$ and $2ml$, for which we shall write M and m respectively for the sake of brevity. These quantities are called the magnetic moments of the bar and the needle, and exactly correspond to what in mechanics is called the turning moment of a lever, viz. the product of a force and the length of the arm of the lever, and in this case is the product of the pole strength into half the length of the bar taken as the distance between the two poles.

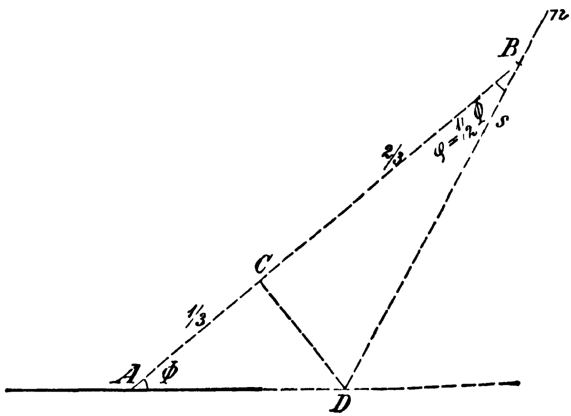


FIG. 7.

The formula for the turning moment exerted by the bar on the needle is therefore

$$D = \frac{Mm}{r^3}(2 \cos \Phi \sin \phi - \sin \Phi \cos \phi) \quad . \quad . \quad (1)$$

An interesting question which arises in this connection is how does the needle place itself under the action of the bar, and the answer given by Gauss is both striking and simple : It places itself in such a way (see Fig. 7) that $\tan \phi$ is equal to half $\tan \Phi$. It will be readily seen that the turning moment is then zero and the needle is then of course in a condition of equilibrium.

15. **Combined Effect of the Magnet and the Earth**—For the rest we shall not linger very long over this problem in the present stage, for it is not a very real one, since usually (unless special precautions are taken) the mutual effect of the two

magnets is disturbed by the presence of a third force, the magnetism of the earth. The needle is therefore subject to two different influences, that of the magnetism of the earth and that due to the fixed bar. It will therefore take up such a position that the two turning moments to which it is exposed are equal. This position is best defined by the angle which the needle makes with the direction of the magnetic meridian, the direction that is of the lines of force due to the earth. If the horizontal intensity of the earth's magnetic field is called $\mathfrak{H}$, then $\mathfrak{H} \sin \alpha = D$, and by giving to D the value already found, and instead of Φ and ϕ the two new angles σ and τ as indicated in Fig. 8, σ being the angle which the bar makes with the

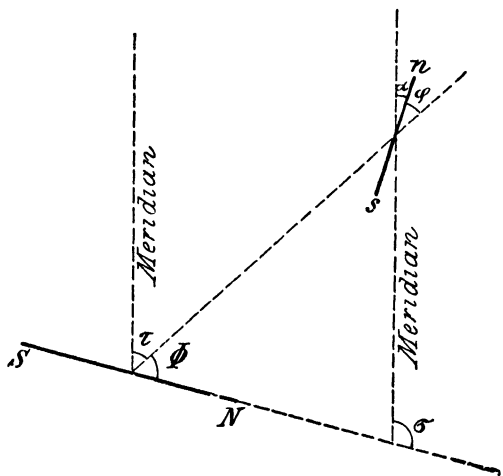


FIG. 8.

meridian, and τ the angle which the line joining the two centres makes with the meridian, by solving the equation for α we then obtain to a first approximation

$$\tan \alpha = \frac{M}{r^3 \mathfrak{H}}\{2 \cos(\sigma - \tau) \sin \tau - \sin(\sigma - \tau) \cos \tau\} \quad (2)$$

16. **Principal Positions**—Special cases are obtained by choosing suitable values for the two quantities σ and τ. If we take $\sigma = 0$ so that the deflecting magnet is parallel with the meridian, then

$$\tan \alpha = \frac{M}{r^3 \mathfrak{H}} \cdot \frac{3}{2} \sin 2\tau \quad . \quad . \quad . \quad . \quad . \quad (3)$$

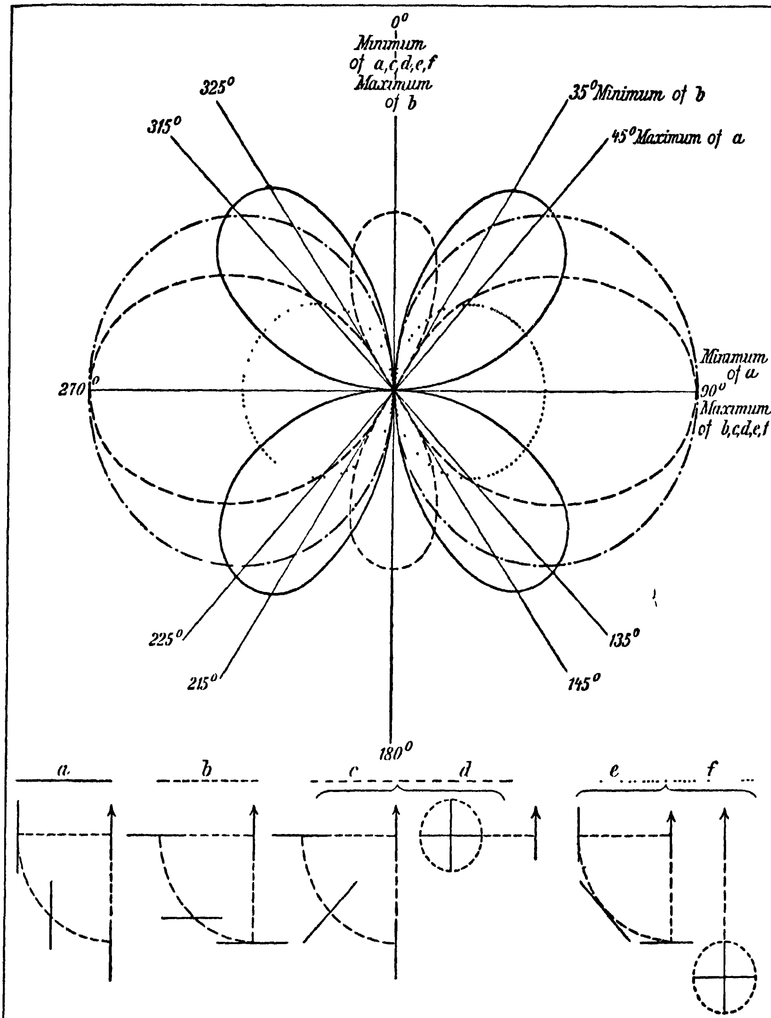
that is to say when a magnet is carried around a magnetic needle swinging in the magnetic meridian in such a way that the bar magnet is at all times parallel to the meridian, then it exerts no turning moment on the needle when it lies exactly left or right, or above or below it; but a maximum effect occurs when it is placed in one of the diagonals (at 45°). Compare curve A (Fig. 9), which consists of the four branches corresponding to the procedure schematically represented in (*a*). If in the second place $\sigma = \pi/2$ then the deflected magnet is perpendicular to the meridian and therefore parallel to the lower edge of the paper, and we now have

$$\tan \alpha = \frac{M}{r^3 \mathfrak{H}}(2 \sin^2 \tau - \cos^2 \tau) \quad . \quad . \quad . \quad (4)$$

By procedure (*b*) if the magnet is moved in a circle around the needle the deflection is greatest, leaving the sign out of account, when it lies to the left or the right; half as great when it lies above or below; and least, that is to say is zero, when it lies about 55° above or below. The dotted curve corresponding consists again of four loops, but they are not as in case (*a*) equal loops, but consist of two large loops lying to the left and right, and two smaller loops lying above and below. In the third and the fourth case, for the special cases 0 and $\pi/2$ may be taken, and we may then determine how the effect varies if the magnetized rod without any displacement be turned about its middle point (procedure (*d*) for position on the left or the right: procedure (*f*) for position above or below). In the first case it will be found that

$$\tan \alpha = \frac{2M}{r^3 \mathfrak{H}} \sin \sigma. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and in the other it is half this value. In both cases the effect is greatest in the horizontal and zero in a perpendicular position. To correspond with this the curve has only two loops, consisting of two equal circles to the right and left, and these circles in case (*d*) have half the radius that they have in case (*f*). Finally in the fifth and sixth case it is of importance to know that the effect varies when the magnet is carried in a circle about the needle in such a way that it continually points towards it or always lies in a position tangential to its path (procedure (*c*) and (*e*)). Here we must return to the original angle and put it equal to 0 or $\pi/2$.



(Deflecting Force of a Magnet on a needle at various positions (*a*, *b*, *c*, *e* in the circle about the needle *a* in the longitudinal position; *b* in the transverse, *c* in the radial; *e* in the tengential position; *d*, *f* in a circle about the needle—*d*, the West (East) position, *f*, the South (North) The algebraic sign of the force is disregarded)

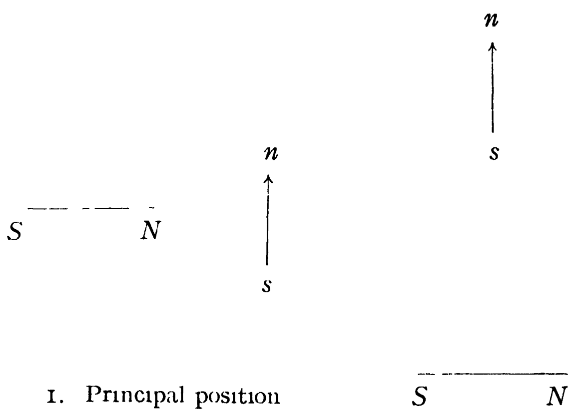
FIG. 9.

In the one case we obtain

$$\tan \alpha = \frac{2M}{r^3 \mathfrak{H}} \sin \tau \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and in the other half this value. The result is exactly the same as in cases (*d*) and (*f*) and the curves given there apply here also.

The four positions of the bar in which the quantities σ and τ have the values 0 and $\pi/2$ are called principal positions; but only two of them are of special interest, those in which $\sigma = \pi/2$. For $\tau = \pi/2$ we obtain the first, and for $\tau = 0$ the second principal positions, and from what has been said we are led to the important proposition: the deflection effect



1. Principal position

2. Principal position

FIG. 10.

is in the first principal position twice as great as in the second. This proposition, however, only holds for the first approximation, a remark which applies indeed to all our statements. The two principal positions are shown in Fig. 10.

The fact that the effect under circumstances otherwise equal is in the first principal position twice as great as in the second furnishes us, as Gauss has shown, with the most exact proof of the square law. And it was Gauss who derived from the formula that he so obtained for the first time an exact method of measurement, and constructed a practical magnetometer, laying the foundation thereby of the whole of modern technical magnetism. We shall return to this later.

17. **Uniform and Unipolar Field**—We are now so far advanced that we are in a position to plot out the map of the magnetic field in accordance with definite presuppositions and from a purely deductive and theoretical point of view with pencil and paper only. But we shall, of course, have to confine ourselves to a plane section of the space surrounding the field, but in many cases a single section of this kind will give with sufficient accuracy the characteristics of the field, and particularly in those cases in which the field in space is either uniform in one of the three dimensions or is symmetrical about one axis. But it is just these cases that play a particularly important rôle. If this is not sufficient a second section of the field must be considered, usually one that is at right-angles to the first. We will not content ourselves, however, with a purely theoretical demonstration, but will endeavour to obtain a confirmation of what has been said through a natural process. Let us imagine a field that is certainly the simplest of all and in that respect even surpasses the field due to a single pole: a uniform field, that is the field in which the force is everywhere equally strong and everywhere acting in the same direction. With such a field we are already acquainted since it is presented to us by nature in the magnetic field of the earth, if a moderately restricted space, an ordinary room for example, is assumed, and also if the space does not lie immediately above the magnetic pole of the earth (a not very likely possibility !). We suppose ourselves then to be at such a distance from the pole that the difference in position of the different parts of the space considered have no effect. The lines of force are everywhere parallel to and at an equal distance from each other. This we can establish by moving a small needle over different parts of the space and observing that both its direction and the time of its swing are at all places the same. A direct picture of the field cannot be experimentally produced because in this instance it is too weak. Uniform fields of very much greater intensity can be produced by artificial means, but this is not so simple, and therefore at this point we cannot go into the matter.

Apart from this the simplest field is that of a single pole, the unipolar field. This case we can settle very briefly. The equipotential lines are concentric circles. They must be so selected that by going from one to the other the potential always changes by an equal amount. At the source it is infinite, at infinity it is nothing, and in between it diminishes in the same proportion as the distance increases. For only by assuming such a law for the fall of potential can we conform to the law of inverse squares. In consequence the concentric circles, which near the source are very close together, become more widely separated as we recede from it. The lines of force are the radii of the circle, and we have already seen how the number of them should be chosen. Naturally this field is not uniform; on the contrary, it is one of the least uniform that can be imagined, and only at a very great distance from the source do we reach a place where anything like uniformity is obtained.

In magnetism (since a single pole is impossible) this case can only be partially realized by plotting the field due to one of the poles of a very long magnet on a plain surface perpendicular to the bar, whereby the effect of the other pole, on account of its distance, can be more or less neglected.

There exists a very elegant method of getting a magnetic field, or rather a section of one, to effect its own delineation and then to compare this experimental result with the theoretical. A piece of cardboard or a glass plate is taken and sprinkled with iron filings, which must be neither too fine nor too coarse, and the whole is then exposed to the magnetic field, and on being assisted by a little gentle tapping it will be found that the iron filings assume a regular grouping. In the present case a long magnetized rod is set up in a vertical position and the plate laid on the surface of the upper pole. We then obtain a sort of picture in iron filings of the unipolar field, which is in effect a system of radial forces represented by a chain of iron particles, generally straight, but of course subject to some slight irregularity in consequence of the imperfections of the experiment, and which, as the distance from the pole increases, show a less pronounced radial tendency.

18. The case of the unipolar field is, however, of special significance as a foundation on which all that follows can be built up. For from the unipolar field the bipolar field can easily be deduced by superposing the two fields one upon the other. The most important case of this kind is obviously that of two mutually opposing poles of equal strength, for this is the case of an ideal bar magnet, and the ideal is more closely approached the longer and thinner the rod. In order

to obtain the field corresponding to this case we draw in the first place each of the two fields taken alone; that is, we construct the circular lines of equal potential, in one case with positive, and in the other with negative values; and in the same way we draw the radial lines of force outwards from

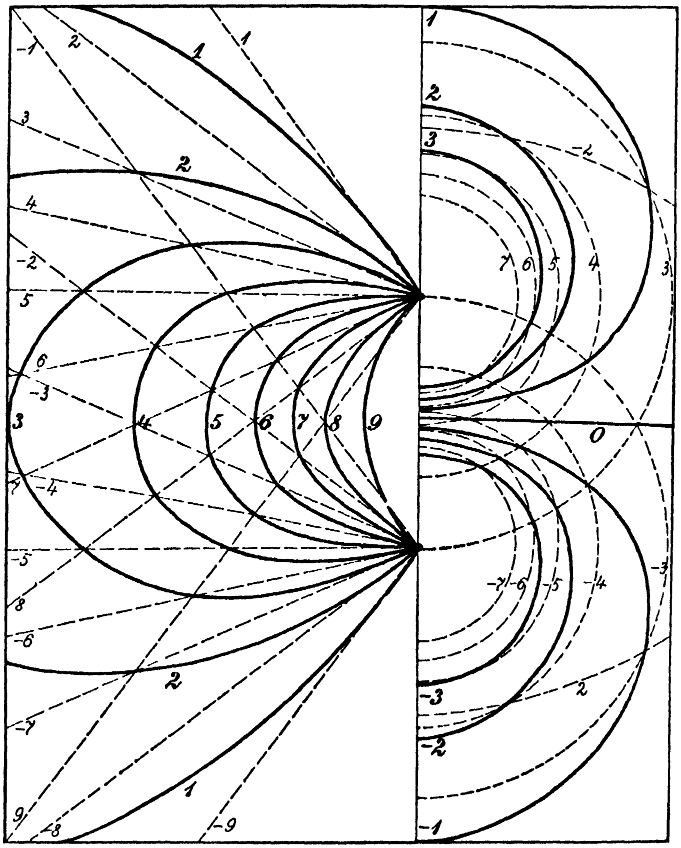


FIG. 11.

each of the two poles and add to them in proper order their numerical values, whether positive or negative. In order not to confuse the picture in Fig. 11, the lines of equal potential, after the example of Maxwell, are drawn only on the right and the lines of force only on the left half of the field. They are indicated by broken lines and the half that has been omitted in each picture can easily be added as a reflected image. If we then take the points of intersection of the two systems of potential lines and add the two together, to each one of them adding the corresponding numerical value of the other, since some of these numbers are positive and the others negative we shall sometimes obtain zero, sometimes positive and sometimes negative values. If we now connect together all the points whose separate values give the same difference by means of a curve and proceed in the same way for all the other series of points, we shall obtain the system of curves which is shown marked by thick lines on the right-hand half of the figure. If we suppose these curves to be continued on to the left half it will be seen that these curves are not circles but curves which run together in the neighbourhood of the middle line. And in the same way it will be seen that

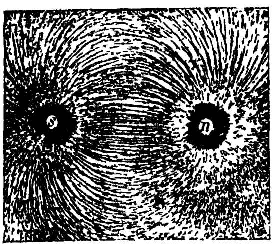


FIG 12

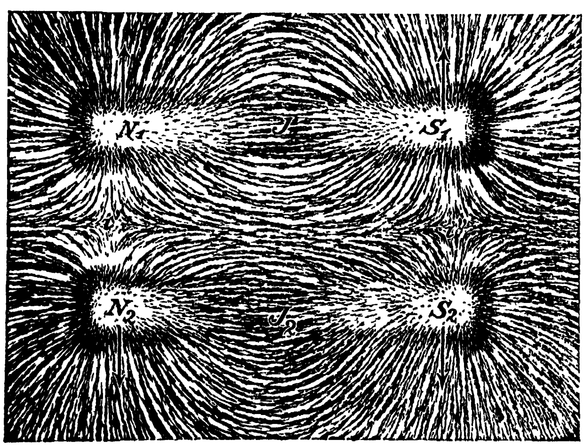


FIG. 13.

the lines of force of the total field, which are represented by thick lines on the left-hand side of the figure, are no longer straight lines but curves connecting the two poles together. In order to compare this result with the natural field a picture can be obtained by means of the iron filings. We can either arrange that two long magnetized bars are placed vertically at the same distance from one another with opposite poles upwards and with the glass plate and the iron filings placed on the top of them (Fig. 12), or a single long bar magnet can be laid horizontally and the plate placed over this. But in this case the picture obtained will be somewhat different because a magnet is not represented by the action of its end poles exclusively, since bridges are formed, though of course they are weaker ones, between the inner points of the bar.

19. Of the great number of cases of special fields which may exist it must be sufficient to select two for illustration

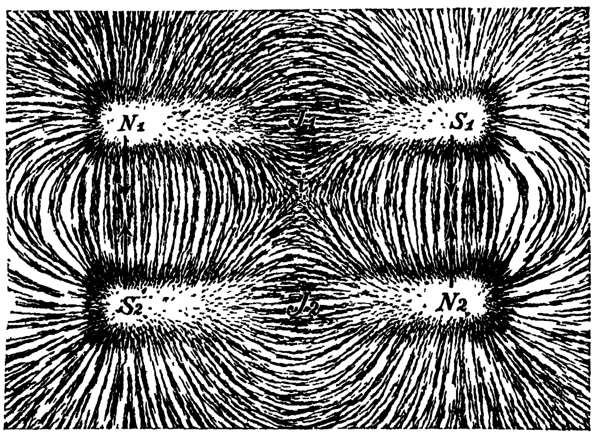


FIG. 14.

here. First we can take the case of two parallel magnets laid close to each other, in the one case with similar poles, and in the other with opposite poles together. The picture in iron filings obtained by means of them is represented in Figs. 13 and 14. In the first case an "open" field is obtained with lines of force which radiate away from the system towards the right and the left, and in the other a "closed" field, the lines of force of which form a series of bridges between the poles. The practical consideration suggested by this is that magnets should always be put away in pairs with their opposite poles together: in this way their magnetism is best preserved. In the figures the point of indifference will be observed of the separate magnets, and in the second case there is for the total field a point of indifference of a kind which presents special characteristics which can only be indicated here.

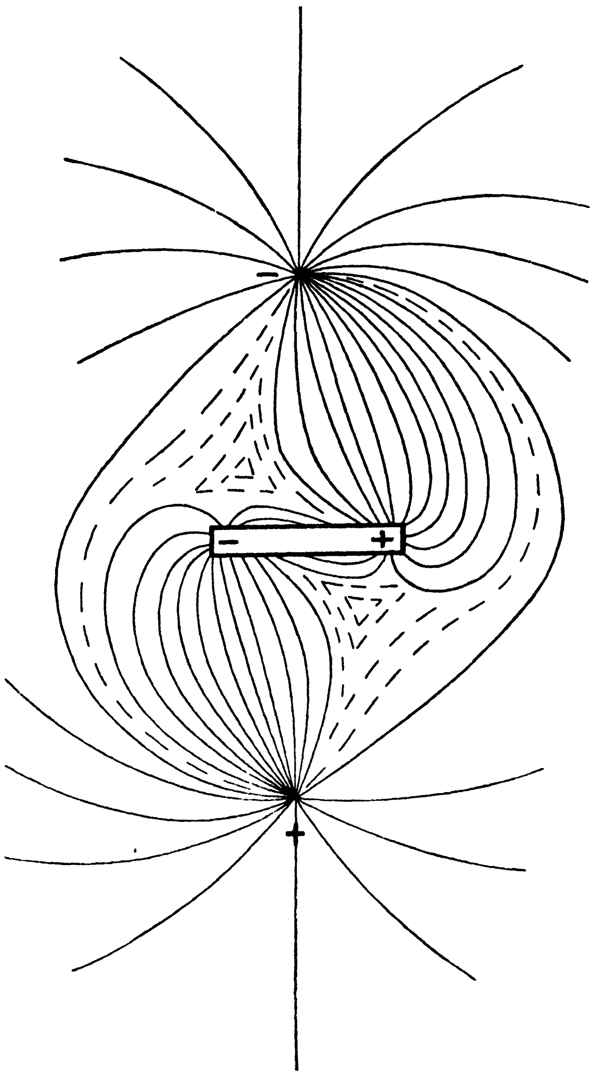


FIG. 15.

The other case is that of a magnetized bar which is placed between two mutually opposing poles of equal strength and at right-angles to the lines joining them. Fig. 15 gives the lines of force as deduced from theory, and Fig. 16 the image in iron filings for this case. It furnishes an excellent opportunity for observing how far theory and experiment agree. It is left to the reader to notice especially the curious whirl in opposite directions of the lines which encircle the two poles of the bar.

It remains to add that there are various other methods for the demonstration of the magnetic field: for example, by means of small magnets balanced on a fine point or hanging by threads, or by means of needles or small spheres floating

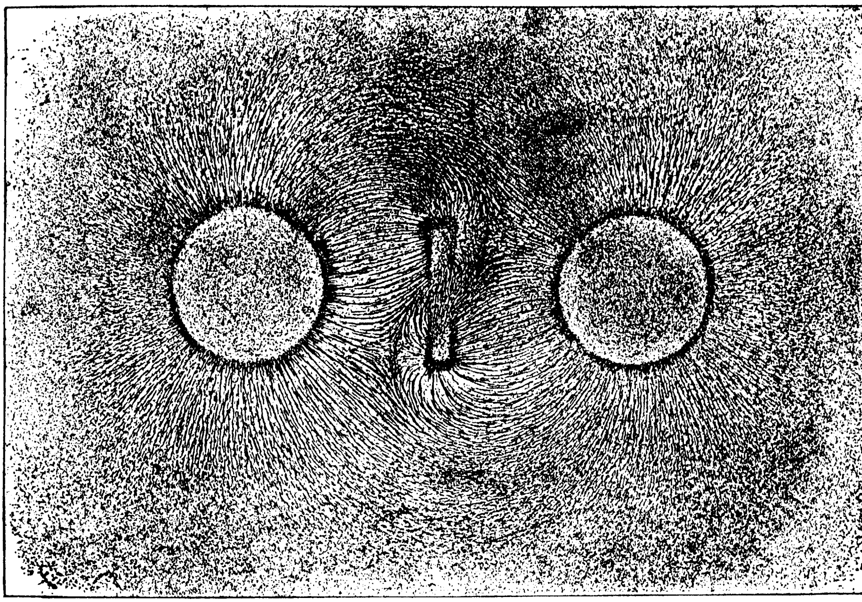


FIG. 16.

on the surface of some liquid. The latter place themselves in definite regular figures (triangles, squares, pentagons, and so forth, with or without a centre point) from which further conclusions can be drawn (Fig. 17).

20. **Molecular Magnetics**—In the beginning we have presupposed that a magnetized bar has at each end two equally strong and contrary poles and that these characterize the magnetic condition. It was, however, stated immediately afterwards that this was only a first rough approximation which is only tolerably true if the bar is long and thin,

but as was mentioned when we were discussing the images obtained with iron filings, even this was only approximately true. That the whole idea was a very imperfect one, and even at times quite false, can be shown in a variety of ways. If such a bar is dipped into a heap of iron filings and drawn out again, it is seen that the iron filings stick most thickly to the ends, the other parts of the bar, however, getting their share. Only the central portion remains fairly free, representing the neutral or indifferent zones already mentioned. Also Figs. 13 and 14 clearly show how bridges are formed between the inner points of the bar. The most decisive fact, however, is the experiment already mentioned, in which a magnetized needle was broken at the middle in the hope of laying bare in this way the neutral point. Experiment showed that this was a vain attempt, and that each half still possesses its two opposite poles, and that this process of division can be carried on indefinitely.

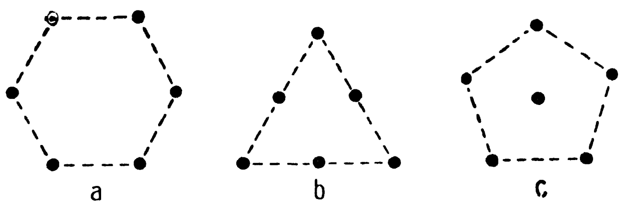


FIG. 17

From this and many other things we have evidence that magnetism is not something which sticks to a number of points but rather is a property belonging to the smallest parts. Therefore, in order to give it its proper place in physical science it must be regarded as a property of the molecules themselves. These little magnets are called molecular magnets. (Recently the term "magneton" has been proposed, of which we shall have to speak later.) Each of these molecular magnets already possesses a north and a south pole, which are of equal strength. For whatever assumption may be made, whether the magnet be small or large, it can always be shown that the two sorts of magnetism are present in equal strength. Of the many proofs of this, just one example may be given. A magnet placed in a uniform field, that of the earth for instance, experiences a turning force (and therefore places itself in a definite direction), but there is no tendency to displacement of the magnet as a whole, and the most striking demonstration of all is supplied by the fact that an inclination needle in a uniform vertical field experiences no change in weight, which would necessarily happen if it had more magnetism of the one kind than of the other, and on this account was attracted more strongly upwards or downwards.

21. **Constitution of Magnets**—On the basis of these facts a picture of the constitution of a magnet can be provisionally stated as follows: It consists simply of a great number of indefinitely small molecular magnets, each of which possesses a definite moment *ml* (the product of the polar strength and the polar distance). That the magnetism of the body is not everywhere equal can be accounted for by supposing that there are different values for *ml*; but it is simpler to regard all molecular magnets as having equal strength and suppose that the density with which they fill different parts of space is variable. Just as in mechanics we have to deal with density of mass, so here we have density of magnetism, that is the magnetic moment contained in a unit of volume. This quantity, which here is obviously of the nature of a directed quantity, that is a vector, is customarily represented by $\mathfrak{J}$ and is called the intensity of magnetization, or simply the magnetization. Then if *dv* is an element of volume we have the equation

$$\mathfrak{J} = \frac{ml}{dv} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If the vector $\mathfrak{J}$ makes with the axes of the co-ordinate system angles whose cosines are λ, μ, ν, the components of $\mathfrak{J}$ are given by the formulæ

$$A = \mathfrak{J}\lambda, \ B = \mathfrak{J}\mu, \ C = \mathfrak{J}\nu \quad . \quad . \quad . \quad . \quad (8)$$

and conversely

$$\mathfrak{J} = \sqrt{A^2 + B^2 + C^2} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It should, however, be noticed that in many investigations the magnetic moment is referred, not to the volume, but to the unit mass of the body, and this quantity is then called the specific magnetism of the substance. Finally, a third magnitude of this sort is the molecular magnetism, or the atomic magnetism, and is proportional to the magnetism of the molecule or the atom respectively. Provisionally (and in general henceforth) we shall have to deal with the magnitude $\mathfrak{J}$.

22. Magnetic Filaments—Thus prepared let us turn to the constitution of the magnet and begin with a simple case with which we are already familiar, but which we shall now look at in a new light. Let us consider a chain of molecules lying in a straight line as being the simplest case and represent them in the first place as in Fig. 18, that is, so that the molecular magnets are small in comparison with their distance from one another. Then in each magnet two poles balance each other, and we obtain zero as the result. This mode of representation is therefore of no value. But it is quite different if we consider the picture in Fig. 19, in which

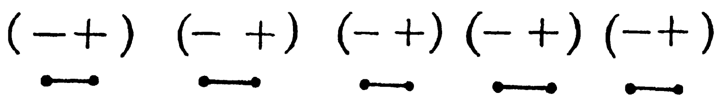


FIG. 18.

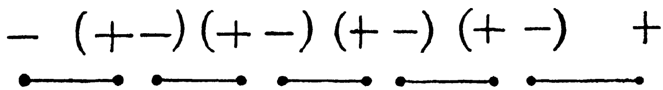


FIG. 19.

the distances are small in comparison with the size of the molecular magnets. Here naturally the neighbouring poles annul each other and only at the ends is there a pole left over, and, as we have shown it, a south pole at the left and a north pole at the right. A model of this sort we call a simple or a uniform filament, its total outward magnetic effect is concentrated in the end points, which therefore can quite properly be regarded as its poles. That this type is approximately realized in the case of a magnetized knitting needle we have already seen.

Instead of a straight line the filament may also be a curved one, but no substantial change in the constitution results so long as the curve remains open. But if it be closed the two end points neutralize each other, and we then obtain the poleless ring to which we have already referred.

A simple filament is characterized by two quantities, the strength of the poles and their distance apart; the last quantity in this instance, not indefinitely small, as in the case of a molecular magnet, but of a finite amount. If one or the other of these quantities be altered we obtain a filament which both intensively and extensively is different, and among the cases then possible that of two filaments which have each a different length and a different pole strength but an equal moment because the product of the two factors is the same. But what is of very great importance is that we can

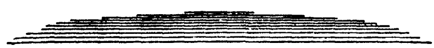


FIG. 20.

lay the simple filaments over one another and in this way obtain the compound non-uniform or complex filament. The possibilities which here arise are very numerous. That which is best adapted to represent the actual facts is obtained by taking a series of filaments of graduated length, as in Fig. 20, and combining them not one above another, but so that each interpenetrates the others, the filaments being such that the pole strength is smaller the shorter the filament. In Fig. 21 it is schematically represented by means of circles which represent the degree of magnetic effect by their size. It will be seen that in this way we obtain a fairly true picture of the appearance presented by a thin magnetic rod that has been dipped into iron filings and then drawn out again. There are poles all over (except quite at the centre), but their strength decreases from the ends to the middle part.

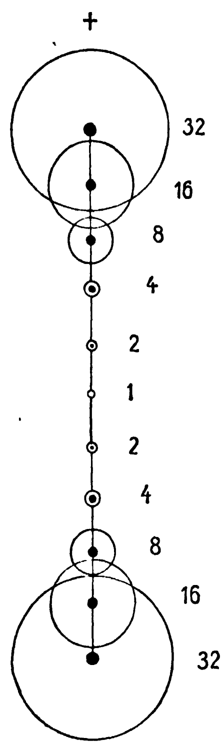


FIG. 21.

23. **Free and Total Magnetism**—What we have said at the conclusion of our description of the method of combining poles together has a direct bearing on the free magnetism which is left over after the annulment of the mutually compensating elements. In contrast to this is the total magnetism characterized by the quantity $\mathfrak{J}$ (multiplied by the volume of the body). While the first in the case of the simple filament is present only at the ends and in the complex case increases from the middle point where it is zero to the ends (at an ever-increasing rate), the total magnetism in the simple filament is everywhere equal and in the complex case it

decreases from the middle outwards because here, where the elementary filaments lie thickest over one another, it is at a maximum and declines towards each end at first slowly and then at ever-increasing rate. Thus we obtain Fig. 22, in which the full line shows the free and the dotted line the

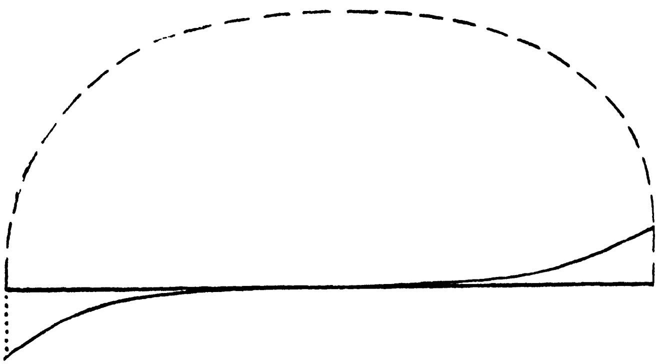


FIG. 22.

total magnetism. It must, however, not be overlooked that the whole consideration is a somewhat arbitrary one and that the idea put forward must be adopted with caution, after it has been carefully considered how far it is applicable in the case under consideration.

24. **Magnetic Shells**—The filament, however, is not the only fundamental type of magnet. There is another of an exactly opposite character in which the individual molecular magnets do not form a chain but lie parallel to one another. Thus a double line is obtained or, if two dimensions are considered, a double surface.

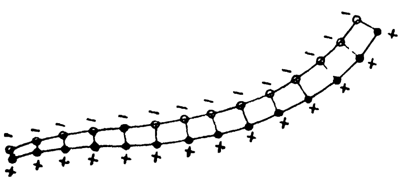


FIG. 23.

Fig. 23 can be regarded either as a double line or as a cross-section of a double surface by the plane of the paper. The upper side of the surface contains only north poles, the under side only south poles. There is also here a large possibility of variety, for the surface may be either plain or curved. In general these double surfaces are spoken of as magnetic shells; filaments and shells are, therefore, the two fundamental magnetic types. And the shell also may be symmetrically constructed, and is then called a simple shell, or it may be built up out of different shells, in which case it is a compound or complex shell. Of special interest is the case in which the shell is closed in on itself, and contains a hollow space of such a kind that all the poles of one kind point inwards and all the others point outwards.

From filaments and shells, or as they are also called solenoids and lamellæ, more general forms of magnets can again be built up. If filaments are laid in bundles alongside each other a solenoidal magnet is obtained and according as the filaments are of equal or unequal length the magnet may be cylindrical, conical or spherical, etc. It is characteristic of a solenoidal magnet that it possesses free magnetism only at the ends of the filaments and therefore only at its surfaces, and indeed only at those parts of the surface which do not run parallel with the direction of the filament; in the case of cylindrical magnets, for example, the free magnetism is present only at the ends, not on the curved surface, but in the case of the conical magnet on the curved surface as well.

If, on the other hand, one considers a number of simple shells laid in layers one over another in such a way that the contours at least form a regularly varying surface, then a lamellar magnet is obtained, and according to the size of the elemental shells, and whether they are open or enclosed, forms of extraordinary multiplicity. A magnet may at the same time have both a solenoidal and lamellar character and then has specially remarkable properties.

Finally, to conclude the whole series of possible magnets, is one which is neither of solenoidal nor lamellar character, because not simple but complex filaments and shells have been employed in its construction. But the constitution is then of such complexity that we cannot here consider it in detail.

25. **Potential of Filaments and Shells**—We now proceed to set forth the essential and outstanding laws according to which the various types of magnets affect surrounding space. In this it is simplest to keep to the idea of potential, from which moreover by differentiation (that is, by finding what is the rate of fall of potential in any direction) the force, or the individual component of the force, can be obtained For a simple filament to which, however, in order to approximate to physical reality we ascribe a certain small cross-

section q, the potential at a point distant r_1 and r_2 from the poles respectively is simply

$$V = \mathfrak{m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = q\mathfrak{J}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad . \quad . \quad . \quad (10)$$

In the case of the complex filament, on the other hand, the effect of the auxiliary poles distributed over the whole length is such that we obtain

$$V = \frac{\mathfrak{m}_1}{r_1} + \int_1^2 \frac{d\mathfrak{m}}{r} - \frac{\mathfrak{m}_2}{r_2} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where r in the middle term relates to the distance of the point considered from any auxiliary pole.

The effect of the shell, on the other hand, is compounded of its elements, and for such an element it follows that if it has a surface ds the potential at a point whose distance is r and in a direction which makes an angle u with the normal to the element of surface, in consideration of the fact that the effect of the two surfaces, the upper and the under, nearly annul each other, and that only the differential coefficient in the direction of the point considered remains, is therefore

$$dV = \mathfrak{m} . \frac{ds}{r^2} \cos u \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

but the right-hand side of the equation (apart from the factor) represents something of the most general application, especially in relation to the laws of perspective: it is the apparent magnitude of the surface ds viewed from the point under consideration; as the distance increases it diminishes as the square of the distance, and where it lies obliquely to the line of sight it diminishes with the cosine. If the apparent magnitude is called dS then the very simple relation is obtained

$$dV = \mathfrak{m}dS = \mathfrak{J}dS \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and over the whole shell when it is of simple type we obtain by integration

$$V = \mathfrak{J} . S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

or, expressed in words: the potential of a simple shell is the product of its magnetization and of its apparent magnitude as seen from the point considered. In this law many other important special laws are involved, of which a few may be briefly quoted: (1) The effect of a magnetic shell apart from the strength of its magnetization depends only on the contour which it presents and is quite independent of the shape of the surface filling this contour (compare in this connection the remarks made in connection with the following number). (2) The effect of a shell on any point lying in a plane passing through its edge is zero. In the case of a plain shell this is obvious without any further demonstration. In the case of a curved shell (Fig. 24) it might be doubted, since from the point under consideration, even when it lies in the plane of the contour which bounds the shell, a piece of the shell is seen, and therefore the apparent magnitude is not zero. But this difficulty only arises if the meaning of the procedure has not been properly understood. We must regard the shell as being made of a transparent material; then not only is the part AB seen but also the part BC: if the first be called positive the latter is the negative side, and since both portions are of apparently equal magnitude they cancel each other out and their effect is zero (3) The potential of an enclosed shell is also zero throughout the whole of the outer space, because for every point outside the two parts of the surface, the inner and the outer, if both are supposed to be visible, exactly annul each other. In the enclosed space the potential is, of course, not zero: here indeed it has the greatest value which the apparent magnitude of a surface can assume, viz. 4π (and this, moreover, must be multiplied by $\mathfrak{J}$). But since this value applies to all the points there is no fall of potential, and in consequence the force effect in the enclosed space, as in the outer space, is zero. From this we shall later draw still further conclusions.

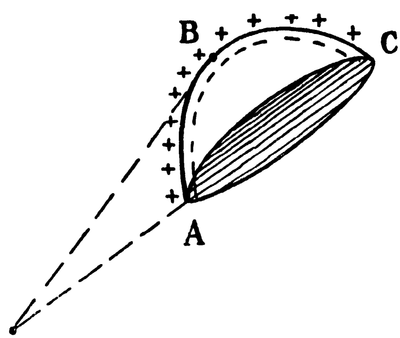


FIG. 24.

26. **Bodies of any Shape**--Finally, as regards bodies of any shape, the potential, as Poisson and Gauss have shown, may be expressed in a very graphic form; it may be resolved into a surface potential and a space potential, the first originating in the magnetism distributed on the outer surface,

the latter on the magnetism distributed on the inner surface. If ds is an element of the outer surface, dv an element of volume, σ the outer surface density, ρ the space density of the magnetism, then

$$V = \int\int \frac{\sigma ds}{r} + \int\int\int \rho\frac{dv}{r} \quad . \quad . \quad . \quad . \quad (15)$$

It is now immediately evident that in a body that has been uniformly magnetized throughout, the space density of the free magnetism is zero, and therefore only the potential due to the outer surface remains. Or, expressed in terms of the total magnetism: the whole magnetism $\mathfrak{J}$, is constant inside the body. To express the idea in a somewhat artificial form as it may at first sight seem, it is at every point inside the body just as great as, on the average, in the neighbourhood of this point. But this artificial method of expression proves to be a very useful one. The difference exhibited by any vector quantity at any point of the field, in comparison with its average value at all points in the neighbourhood of this point, is called its divergence. In sources and all those points out from which, on the whole, a fall occurs, the divergence is positive; in sinks, and points corresponding to them, it is negative. But in the case under consideration, and as can be shown in particular for solenoidally magnetized bodies, it is zero. It can further be proved that also in the case of lamellarly magnetized bodies a certain quantity, but here quite a different one, also becomes zero: the so-called curl of the vector. This is the quantity which, if $\mathfrak{J}$ be regarded as a vector of mechanical displacement, would represent the appropriate rotation about the displacement taken as an axis. We therefore obtain the elegant and valuable proposition: solenoidal magnetism is free from divergence (but has a curl) and lamellar magnetism has no curl (but has a divergence).

Let us now turn once more to uniformly magnetized bodies. Here the magnetic potential, since $\mathfrak{J}$ is constant and the magnetization has everywhere the same direction, can be expressed in the simple form

$$V = \mathfrak{J}\int \frac{\partial\left(\frac{1}{r}\right)}{\partial x}dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

But this can be more briefly expressed if a new magnitude is introduced, namely the Newtonian or gravitational potential P of a body of unit mass. This potential, as we know, is inversely proportional to the distance, and since it relates to an attraction and therefore to a diminution of the distance we must regard it as negative, and accordingly we obtain

$$V = -\mathfrak{J}\frac{\partial P}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

or expressed in words: The potential of a uniformly magnetized magnet is the product of its magnetization into the

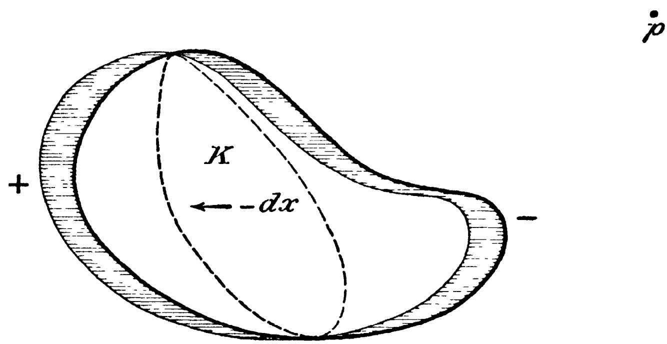


FIG. 25.

fall of its gravitational potential taken in the direction of the magnetization which it would experience if it were filled with a mass of unit density. And geometrically expressed, let it be imagined that the body in Fig. 25 is displaced a little in the negative x direction, then two zones are obtained (shown by the shading), one on the left on the outside, and one on the right on the inside of the figure, and only these zones are concerned in determining the magnetic effect. We are here brought back again to the idea of the outer surface being covered with magnetism, negative on the right-hand side, positive on the left.

III

MAGNETIC INDUCTION

27. **General**—We have so far principally concerned ourselves with the properties of magnets, their constitution and their effect on the surrounding field. We shall now take a step backwards in order to examine more closely a subject which has as yet been only touched upon, namely the origin of magnetism, and try to reduce the question to exact form. This origin of magnets and the production of magnetism in them may be simply called magnetization; but since this expression has already a definite quantitative meaning (see page 35) for the process of magnetization another expression is used (which also will be found to admit of a quantitative meaning), namely magnetic induction. (At times the expression magnetic influence is also used.) The choice of this name is not quite a happy one, partly for the reason already alluded to, partly because a very similar expression, magneto-induction, expresses something quite different—the creation of electric currents through magnetism.

The question therefore is—a question which always arises in connection with every problem in physical science—what relation exists between what we introduce as the "cause" and what we name the "effect"? We do not use these expressions as though relating to the theory of knowledge or as if they had to do with final truth or the primal cause of the accomplished effect, but only as implying that we wish to arrive at a clear understanding and are endeavouring to discover the simplest form that is compatible with completeness.

The two magnitudes between which we have here to discover the relation are the magnetizing force on the one hand and the intensity of magnetization on the other. Of the magnetizing force we can in most cases only form a rough impression, as, for example, in the stroking of an iron bar with a magnet. In such a case the magnetization of this magnet must be exactly known; its distribution; the distance through which it moves over the iron; the intimacy of the contact, and so forth. Since we are usually dealing in nature and the laboratory with energy relations, we must determine those involved in the experiment, and to do this in any procedure in which the human element comes in is not easy. The magnetic force is only exactly measurable when the field is perfectly under control: and this requirement is only fulfilled in a few cases as, for example, when the magnetic field of the earth is used. But this for most purposes is too weak: the most satisfactory means of magnetization is afforded by the electric current. The question how an exact knowledge of the magnetic field of a current is arrived at will not be considered here. We shall take this knowledge for granted, and designate the strength of the field (that is, the magnetizing force acting on a point in which there is a unit quantity of magnetism) generally by the letter $\mathfrak{H}$, which is chiefly used, however, for the horizontal component of the magnetism of the earth. This quantity is a vector and, therefore, according to the usual convention, is indicated by a black letter, and the magnetization $\mathfrak{J}$, the other quantity which comes into consideration here, is also a vector. We have to deal then with the relation between two vectors. The first question then is what is the direction of the two vectors (for each vector has a definite direction, and, moreover, direction in a definite sense like an arrow). The simplest answer would be to take it for granted that the two vectors would agree with each other both in direction and sense: a body would, therefore, be magnetized in such a way that its magnetic axis would lie in the direction of the magnetic field, that is, in the direction of the field with the arrows both pointing the same way. But this assumption would not be generally correct. For first considering the direction, here the assumption is correct in regard to all isotropic bodies, that is, bodies which, whatever the treatment they are subjected to, behave similarly whatever the direction. But it is not correct in the case of anisotropic bodies, of which the most remarkable example are the crystals. Here the direction of the magnetization may very well not agree with the direction of the field, and this indeed is generally the case. This is particularly important because lately we are being more and more led to the conclusion that actually all hard bodies in a certain sense are crystalline, the crystals proper being crystalline on a large scale, and the so-called anisotropic bodies being crystalline on a small scale, that is, in their constituent elements only, so that by summation of the effects of the individual elements throughout the whole body, anisotropism is the result. But for the moment let us leave this antithesis aside and consider what the sense of the direction should be. Here isotropic bodies at once come into question, and if, as would seem reasonable, we said that here there could be no doubt it would be only logical to suppose that the two directions agree, we should find as in many other instances how careful we should be in forming any preconceived ideas regarding nature. For actually it has been shown that there are two classes of matter: those in which the sense of the direction of magnetization is the same as, and those in which it is opposite to, the magnetizing force; and this antithesis is expressed in various other ways. An iron rod places itself between the poles of an electric magnet; a bar of bismuth, on the contrary—and here we come to something unexpected—not axially in the contrary sense but at right-angles to the line joining the two poles. This must obviously be so because the attraction between the poles is satisfied in this case when the bar lies at right-angles. Matter of the first sort is called paramagnetic, matter of the latter sort diamagnetic. Let us leave the consideration of this antithesis for the moment and turn our attention to the numerical values on which the laws of the subject are based.

28. **Magnetization Curves**—Here also a particularly simple assumption might be made, the assumption, namely, that as the strength of the magnetizing field beginning at zero was gradually increased the resulting magnetization would increase at the same rate, so that $\mathfrak{J}$ will be directly proportional to $\mathfrak{H}$; twice the value of $\mathfrak{J}$ for twice the value of $\mathfrak{H}$, ten times the value when $\mathfrak{J}$ was ten times as great, and so on. This assumption is at least partially confirmed in actual experiment, in the case, that is, of almost all substances that have been tested as regards their mode of becoming magnetized. Graphically represented, an ascending straight line is obtained the steepness of which varies with the magnetizability of the substance. This is shown in Fig. 26, in which the strength of the field is set out in a horizontal direction and the resulting magnetization along the vertical axis. The slope of the curve, which for clearness has not been expressed, is in all cases very small, and in the case of diamagnetic bodies is actually downwards. The great majority of substances

are, as we know already, only very slightly magnetizable. But it is just those few exceptions which are particularly important, for they are the substances which on account of their strong magnetizability are naturally the most interesting

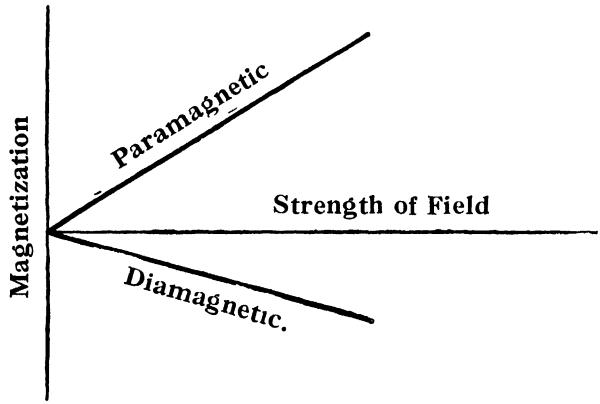


FIG. 26.

of all. We have to deal with iron and the closely related substances to which the general name of ferro-magnetic has been given; and even for these bodies over a certain region this proportionality holds, from the initial value viz., up to a certain limiting value of the magnetizing force and the corresponding magnetization. But outside this region the curve behaves quite differently: three other distinct periods follow the first, and in each of them the ferro-magnetic substances behave in a specifically different way; but naturally in such a way that there is a regular transition between them.

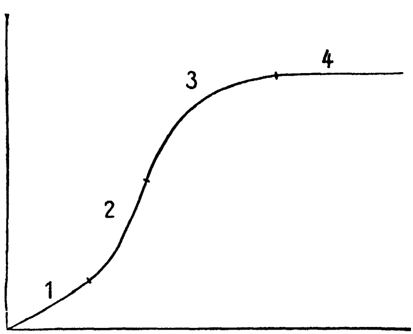


FIG. 27.

The first period is that already described in which the magnetization is proportional to the force, but it must be added the line rises much more steeply than in weakly magnetizable substances, but only to a moderate amount. This corresponds to Section 1 of the curve of Fig. 27. But now when the force has reached this critical value the magnetism begins to increase much more rapidly and we come to Section 2 of the curve. At the end of this period the slope is a maximum, and this point, therefore, is a point of inflection of the curve, that is, a point up to which the curvature is towards one side and after which the curvature is towards the opposite side (in this case to the left above and to the right below), and at the same time a point where the curve, though only for an infinitesimal length, becomes straight. As the force is increased after this point the magnetization still increases, but more slowly, and we get Section 3 of the curve. In the last, Section 4, the magnetism remains constant despite the increase in the magnetizing force. The body in question has obviously become "saturated" with magnetism. A source so obtained is called the magnetization curve.

29. **Remanence and Hysteresis.**—It would be a mistake to suppose that this has settled the problem. We should praise no bridge until we are safely over it. Let us now take, in addition to the ascending curve, the falling curve which is obtained by allowing the magnetizing force, after it has been brought up to its maximum point, to fall back gradually to zero. It is to be remarked that by means of the electric current all this can be effected very easily and very exactly. A variable resistance can be put into the circuit, and this during the first part of the process can be gradually cut out, and when the second part of the curve is to be taken gradually put in again.

What shall we expect as a result of this operation? It might be taken for granted that the process would repeat itself in the backward direction exactly as in the forward, and that the second curve would coincide with the first only that it would run downward from right to left. It is sometimes supposed without further question that the process of nature and technics are reversible and that they will go backwards as well as forwards. In many cases indeed this is approximately true, but it is never exactly true, and in many cases there exists a specially wide gulf between the forward and the backward process. Strictly speaking we may distinguish three cases: in the first the process may be more or less exactly reversible so that the same path is followed in both directions (example, the lifting of a fallen weight); in the second case the process is reversible, but only by following a new course and by the application of other methods (example, the separation from water and sulphuric acid which have been mixed together, and which can be separated by boiling the mixture, leading the steam away, and again condensing it) ; finally in the third case there are processes quite irreversible (example, the burning away of coal to ashes).

In the case of magnetism reversibility is almost exactly observed in the weakly magnetic substances. Here the proverb, easily won and lightly lost, completely applies. If the magnetizing force be diminished the magnetism decreases and completely vanishes at the same time as the force. In the case of ferro-magnetic substances the same remark applies, but only when the magnetization is not carried beyond the end of the second part of the curve, and if after this it is

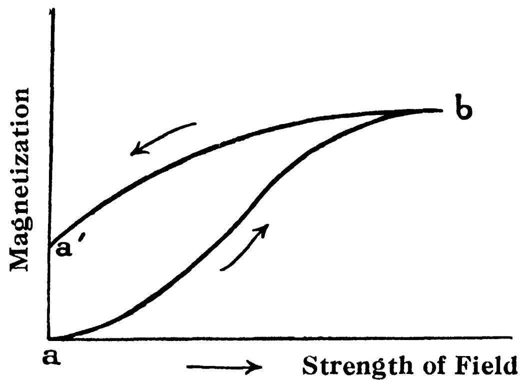


FIG. 28.

then diminished again. But with stronger magnetization irreversibility occurs and it has indeed needed much experiment and theory to bring it artificially under one general rule, and therefore to attain the reversible magnetization of iron of which we still have to speak.

But what does actually happen? Fig. 28 makes the matter clear. When we subject an iron body to a diminishing force it is found that the magnetization indeed diminishes, but for each value of the magnetizing force this magnetization is stronger than it was for the same force on the upward part of the curve. The curve *ba′* lies everywhere above the curve *ab*, and this even occurs when the force is completely reduced to zero (for example, by switching off the magnetizing current). The iron will still show that it possesses a considerable amount of magnetism. This is called the remanent magnetism or more briefly the remanence. The extreme cases we already know. With ideally soft iron, which actually does not occur in nature, the remanence is nothing. The magnetization is then a quite transitory phenomenon. With ideally hard steel, on the other hand, the magnetization once acquired is permanent, and between these two extremes every intermediate stage is represented. But in every case the remanence has a definite value characterized by the length *aa'*.

The phenomenon here described is of great interest which goes beyond the special case under consideration. For it shows that the effect, in this case the magnetization, does not depend on the force to which it is subject at any moment, but also on the forces to which it has been previously subjected. If the latter on the latter part of the curve are smaller than the force actually prevailing at the moment, the effect, in this case the magnetism, is diminished; it is smaller than it actually should be, but if we are on the return part of the curve a higher value is left than would be normally induced. It is as if a sort of memory had to be ascribed to matter. Or, if this way of expressing the facts does not commend itself, there appears to be as it were a sort of inertia which prevents its immediately adapting itself to the new conditions.

The phenomenon is called the magnetic after-effect, or by the Greek name hysteresis (delay). Somewhat varied conceptions have been brought together by means of these two expressions, but that does not belong to this part of the story.

At the final point where the force is zero the body, as we have said, is still magnetic, but it is possible nevertheless to make it non-magnetic again. For this purpose it must be exposed to a negative force, which is achieved when for example the current is reversed and is then again allowed to increase by taking out the resistance. This is indicated in the left-hand side of Fig. 29. The magnetism then declines and for a definite negative force *ac* it is zero: this force, or rather the force acting in an opposite sense to that supposed to reside in the body and exposed to this effect, and which has to be overcome, is called the coercivity of the body. It may here at once be noted that this quantity is much more important and also much simpler than the remanence. The last depends not merely upon the material nature of the body but also upon its shape and the extent to which the magnetization has been carried, and so forth. The coercivity on the contrary is almost exclusively a characteristic constant of the material.

If the magnetizing force is now allowed still further to increase in the negative direction the body becomes henceforth negatively magnetized, with corresponding position of the lines of force and the poles. The curve *cb′* is again obtained running towards the left downwards (which is exactly analogous to that towards the right running upwards). If the force is now gradually diminished the portion of the curve *b′a″* is traced out and then *a″c′* and at last the closing portion

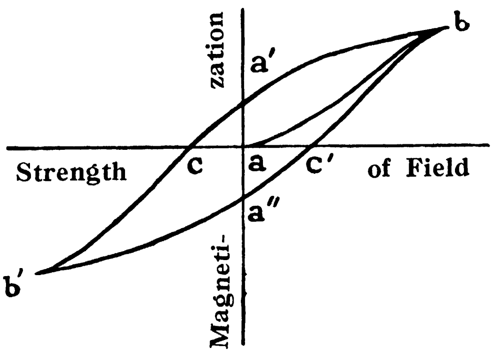


FIG. 29.

c′b. It must be mentioned that the curve will not become exactly closed after the first repetition of this sort of cycle. For this a series of cycles are needed, during which the body gradually adapts itself to the process, but very soon we obtain the same closed curve *b′a″c′ba′cb′*. On the contrary the initial portion *ab* is not repeated. It only plays a transitory rôle, and even so only when the experiment is made with a body which is, so to speak, not yet "experienced," and which has not yet been magnetized. On this account it is sometimes spoken of as the null curve.

30. **Demagnetization** — To the foregoing consideration there are still several others of considerable importance to be added. Two only will be discussed here, both of them equally important from a scientific no less than from a practical point of view. The first relates to the question of the natural condition of the body. When and how does this arise in the cycle under consideration (the null curve naturally being left

out of account)? It might perhaps be supposed at the points c and c', for here the body is certainly non-magnetic. But is it not somewhat suspicious that there should be two such points, while it would be only reasonable to expect one, and that the point standing at the right-hand side of the figure? And this doubt is strengthened by the consideration that neither of these points is a "point of symmetry," in the sense that when one moves from it in either direction a positive effect is obtained. For if the body is magnetized in the positive direction starting from c', it becomes magnetized according to the law of the inflection curve $c'b$; but if, on the contrary, it is magnetized in a negative sense according to the law of the quite differently constituted curve $c'a''b'$, which has no points of inflection (and this quite apart from the fact that this curve runs downward to the left instead of upwards to the right, which is, of course, only natural), and the same remarks apply in a converse sense to the point c. The body is only apparently unmagnetic, but is not in a natural condition, since it assumes one sort of magnetization more easily than the other. And if the force corresponding to c or c' is annulled, this helps nothing, for then the body again becomes magnetic and assumes a condition corresponding to the points a' or a''. From this it might be supposed that the body was for ever "corrupted," that having called up the spirit it is henceforth never free from its influence. But a bar can still be redeemed though by a somewhat complicated procedure, a procedure which is spoken of as demagnetization. It consists in subjecting the body to forces which are alternatively opposite in direction, and which are gradually reduced from their maximum value with a consequent corresponding reduction in the negative and positive values of the magnetization. The procedure is indicated in Fig. 30. At the close of the experiment when the point o is reached the force is then zero and the magnetism zero, and the body is restored to its initial non-magnetic condition.

But it is not permissible to leave the subject without noticing that there is still another difficulty. It has been shown more and more clearly in recent times that magnetism once impressed upon a body cannot again be got rid of. Even when the body is demagnetized by the most refined procedure, for which other and more efficacious methods than that described have been proposed, under certain circumstances it again becomes magnetic—an interesting case of what in medicine is called a recidive. In our subject we speak of spontaneous magnetization, and we shall see that this phenomenon plays a part of some importance in modern theory.

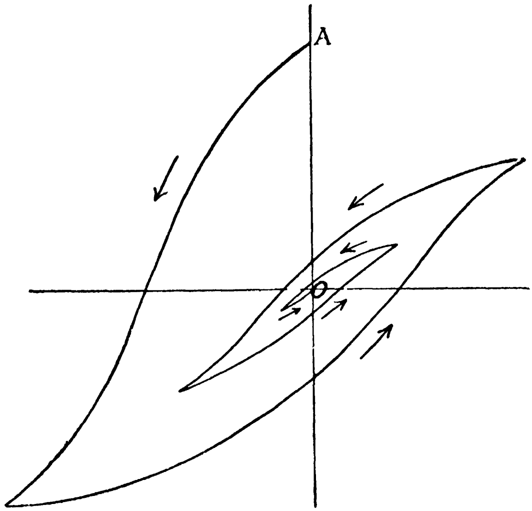


FIG. 30.

31. **Work and Heat**—The following consideration is of distinct significance. When a force is applied in order to produce an effect, work is accomplished and energy consumed. This work is measured as a product of the force and the resistance through which it acts. In mechanics, for example, the lifting of a weight of so many pounds through so many feet is a product of pounds and feet expressed as foot-pounds. Here in the case of work done in magnetizing a body, the magnetizing force and the degree of magnetization attained, which exactly corresponds, have to be multiplied together, or if the body is already in a magnetic condition the magnetizing

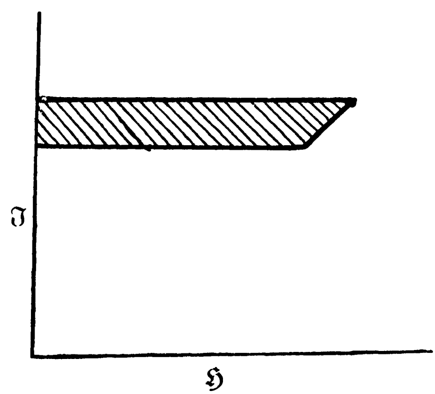


FIG. 31.

force into the increase of magnetization produced. First of all let a very small increase of this sort take place, somewhat as in Fig. 31, where the increase of the force from $\mathfrak{H}$ to $\mathfrak{H} + d\mathfrak{H}$ corresponds to an increase of the magnetization from $\mathfrak{J}$ to $\mathfrak{J} + d\mathfrak{J}$. Then we have as one of the factors of the product to be obtained the quantity $\mathfrak{H}$, or still more exactly, by taking the mean of the initial and the final value, $\mathfrak{H} + \frac{1}{2}d\mathfrak{H}$; but as the other factor $d\mathfrak{J}$ (for the increase of this is the result with which we are concerned); and consequently as a product we have the rectangle with the two factors as sides. Or again, as a surface having exactly the same size, that which is enclosed by a portion of the vertical axis and on the other side by a portion of the oblique line, which, therefore, in the figure has the shape of a small trapezium indicated by cross-hatching. If one goes now through the complete process from the null point to the end point, we obtain as a clear picture of the work done the surface enclosed by the magnetization curve (Fig. 32), the horizontal line *bd* and the perpendicular axis *da*. On the return energy is given out again, but since the return curve, as we have seen, runs higher, this energy is not equal to the total amount of work spent, but only to that represented by the surface *a'bd*. The work done, therefore, is represented by the cross-hatched surface *aba'*. In this there is nothing extraordinary, for the body was originally non-magnetic and is left in a magnetized condition at the end. But the case is different if we consider the complete cycle, one that is in which the null curve has been omitted as in Fig. 33. If we investigate the work that is here spent and regenerated, we find that the difference exactly amounts to the surface which is enclosed by the hysteresis loops, and which is shown by cross-hatching. So much work has ultimately been done, and yet the result of the procedure when the body has been taken through a complete cycle is equal to zero, for the body is once more in exactly the same magnetic condition as at first; and that

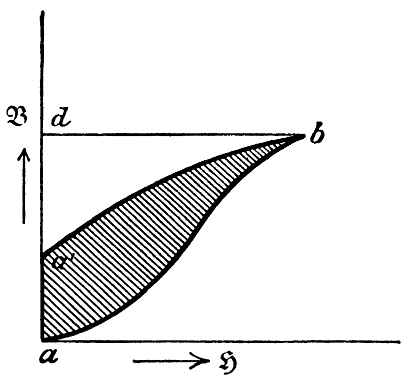


FIG. 32.

equally so whether the cycle begins and ends at *g* or anywhere else. Since according to the principle of the conservation of energy, energy can never be lost, it must reappear in some form other than the magnetic, and observation shows that it reappears as heat. Iron bodies are warmed by a cyclical magnetization, and with each new cycle the heating is increased by an equal amount. The process of magnetization is, therefore, bound up with the dissipation of energy, or as it might also be expressed if one regards the end of the procedure as magnetization, we see that energy has been dissipated.

32. **Dissipation of Energy**.—Therefore, in practice the

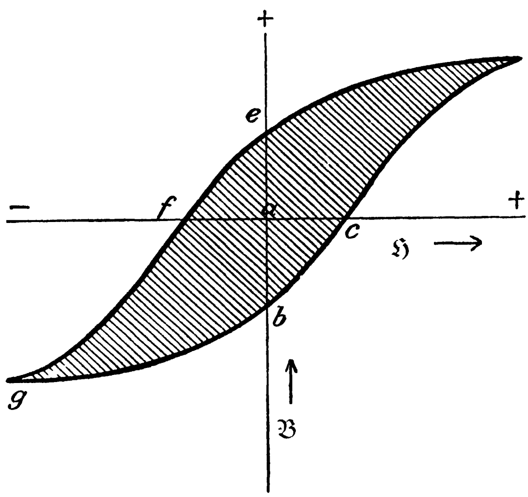


FIG. 33.

problem is to reduce the loss of energy through hysteresis to as small an amount as possible. (In the process other losses occur, viz., those occasioned by eddy currents in the body of the iron, with which we are not here concerned.) For this purpose we have in the first place to investigate on what factors it depends. There are obviously two determining conditions. In the first place the choice of the limits between which magnetization is carried on, and in regard to which we confine ourselves to a symmetrical cycle, one, that is, that has negative and positive values of the same amount. That magnetization must not be carried on to the saturation point is immediately obvious, for in doing so little or no advantage is gained in respect of the magnetization produced, but the heating which is indicated by the broadening out of the hysteresis loop is very considerably increased. On the other hand a high degree of magnetization must be reached, because otherwise the object of the procedure is not attained. At what point the optimum condition occurs depends upon the special circumstances of the case and cannot generally be defined; usually, however, it will not lie higher than the point of inflection of the magnetization curve. Therefore, the cyclic process in the case of a dynamo, for example, should be measured, as it works by means of this sort of alternating magnetization. A more practical consequence is that proper provision must be made to dissipate the heat which in any case will be produced. In addition to this the speed at which the cycle has been gone through is a matter of some importance. In this connection we must consider two extreme cases: on the one hand slow magnetization and demagnetization, in which case it is usual to speak of static hysteresis, and on the other hand the rapid kind which takes place in an alternating-current machine. There is still a third case, that of hysteresis of rotation which occurs during magnetization by the so-called rotational field, in which case the magnetization is not simply positive and negative, changing directly from one state to the other, but passes uniformly through all directions. It has now been finally shown, only, however, after many mutually contradictory or indecisive experiments, that for the same degree of magnetization alternating hysteresis (and apparently also rotating hysteresis) is always greater than static, and is greater, the greater the number of alternations per second. In quick-running machines it always plays a considerable part, and for this reason the question has been much studied in order if possible to eliminate the effects of hysteresis. In this connection an experiment by Gans is of fundamental importance in which a reversible magnetization is obtained. Briefly stated, the experiment consists in measuring backwards and forwards through a portion of the curve. In practice the experiment would be an extremely inconvenient one, and, moreover, Gumlich has shown that even if carried out the final object, freedom from hysteresis, would not be completely attained. On the other hand the superposition of the demagnetization process by means of the alternating field of adequate strength, afterwards progressively diminished to zero, does actually yield the desired result, so that this procedure and the condition obtained by means of it can be regarded as an ideal process of magnetization. In Fig. 34 the ideal curve of a special metal (soft steel) is shown together with the null curve and the two branches of the hysteresis curve.

33. **Iron and Steel**—We are much more interested here, however, in the other determining factor in hysteresis, viz. the nature of the material under experiment. We have already seen that we have to deal with the contrast presented by two extreme cases; quite soft iron—material A—of the introductory remarks, in which there is no remanence, and quite hard steel—material B—in which it is complete. The

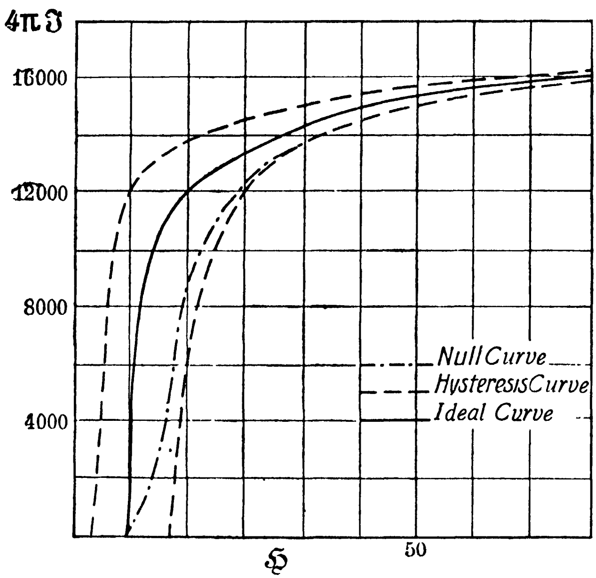


FIG 34.

former is a magnet in a transitory sense, and the latter permanently, and there are all possible gradations between these two extremes. However, it has been shown, as we have already pointed out, that the true characteristic is not the actual remanence but the coercivity. In certain cases the remanence in iron can be greater than in steel; but coercivity is always small in iron and great in steel. This is clearly shown in Fig. 35, in which the null curve and the final loop for three different types of material are represented. The full line represents soft iron; the broken line hard-drawn iron, and the dotted line hard steel. The coercivity, that is the distance from the null point at which the hysteresis curve cuts the horizontal axis, is smallest in soft iron, has an intermediate value for hard iron, and is greatest in hard steel. And this is the decisive fact, even although, as here, the remanence is greater in iron than in steel. It is known apart from

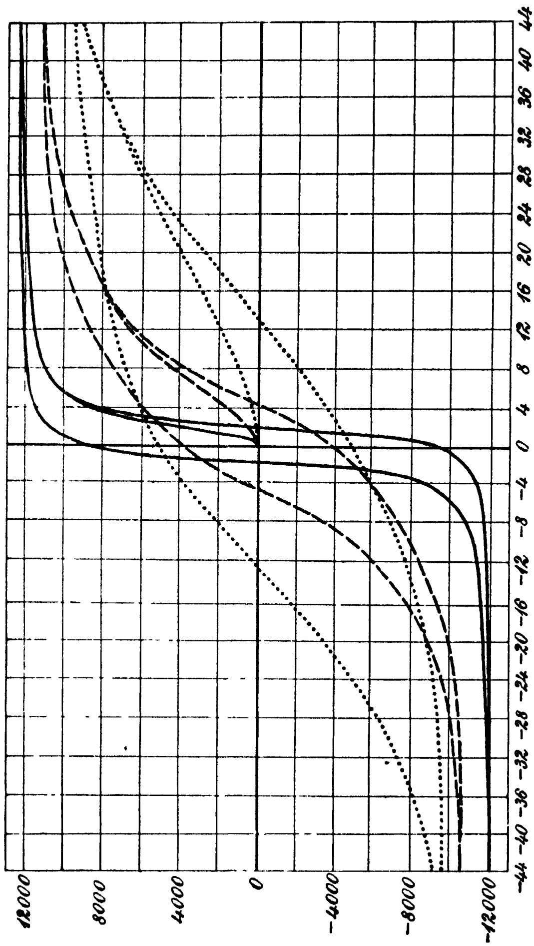


FIG. 35.

this that what it depends on is the magnitude of the area enclosed by the two branches of the magnetization curve, and this, as is here clearly shown, is by far the smallest in soft iron and by far the largest in hard steel. Here one would almost inevitably be led to suppose that the hysteresis can always be further reduced by endeavouring to obtain the iron in as pure a condition as possible, i.e., iron which is as far as possible free from carbon, which is the second principal constituent of steel, and from phosphorus and so forth, and subjecting pure iron of this sort to magnetization. Experiments of this kind have been carried out with ever-increasing success, notably in the Physikalisch-technische Reichsanstalt. A curve has been obtained which it is true still cuts the vertical axis at a considerable distance from the null point, but the horizontal axis close to it ; that is the remanence in this very pure iron is still considerable (which is no disadvantage), but the coercivity is practically zero (which is a fact of decided importance).

34. **The Effect of the Shape of the Body**.—In so far as remanence is concerned it does not depend merely on the nature of the material (nor, as we have seen, to any particular extent on the degree of strength of the magnetization), but also on another factor, the influence of which might at first sight be overlooked, and that is the shape of the body under experiment. The magnetization is of course largely dependent on this, the "temporary" in contradistinction to the "remanent" ; that is, there are shapes suitably or unsuitably adapted in various degrees to magnetization. Let us in the first instance consider a body of unlimited extent. Here we have a case which even if it cannot be realized can very well be imagined, in which the magnetization can so to speak live to the full, and attain its greatest possible value. It is different when the body is of a limited size, for the outside surface then offers an obstacle to the magnetism which is as it were reflected back like a sound wave at the end of an organ pipe, and this backward effect weakens the main effect. A distinction has of course to be made in regard to the direction of the surface. Parts of the outside surface, which run parallel to the direction of magnetization, according to our previous discussion are not prejudicial in their effect on the magnetization. But it is different in the case of surfaces which are at right-angles to the axis of magnetization. There is therefore one definitely limited and therefore practically realizable case which behaves in conformity with the ideal case, i.e., that of the closed ring, for here the only surface runs everywhere parallel to the axis of magnetization. If there is a gap in the ring a disturbance is introduced, but if the two end surfaces are brought close together the weakening effect will be

insignificant. All other bodies, and the cylindrical rod in particular, have disturbing end surfaces, and the disturbance will be stronger the nearer to each other the two end surfaces are, and therefore in a thick short bar it will be greater than in a long thin one, and strongest of all in the case of a flat plate. In this sense one speaks of the demagnetizing force of the surface. The consequence of this is that every shape with the exception of the closed ring suffers a reduction of the magnetization otherwise possible.

It is not easy, however, to free one's mind from the apparent contradiction that arises from the fact that the pole or end surfaces on the one hand are the positive representatives of he magnet, and on the other are places of negative reactive orce. The former applies to the outward effect, the latter o the internal magnetization.

So much for the influence of the shape on the temporary nagnetization. It has a corresponding effect on the remanence, and indeed there are various circumstances which cannot be gone into here that have as a consequence that the effect on the remanence is even still more pronounced. The smaller the demagnetizing force the greater the remanence, and in the case of the flat plate it is therefore a minimum, and in the ring shape a maximum. But this is only on the supposition that there are no other circumstances present that mask the effect.

35. **Distribution**. The form of the body has still another influence on the magnetization, namely, on the distribution in the body itself; and here we have to think not merely of the free magnetism at the poles but of the whole of the magnetism represented by $\mathfrak{J}$. The distribution of the magnetism naturally depends on the properties of the field. If the force which acts on the various parts of the body is itself varied, as in the case of magnetization in a non-uniform field, then the distribution of the magnetism is also non-uniform. But the converse proposition which it might be supposed would naturally follow is not correct: i.e., that in a uniform field a body would become uniformly magnetized. This is true only for bodies of a particular shape, namely, in the closed ring (the proof of this must be taken for granted), in infinitely long circular cylinders, in the sphere and in the ellipsoid: in these cases a constant $\mathfrak{H}$ corresponds to an everywhere equal $\mathfrak{J}$. All other bodies even in a uniform field will become non-uniformly magnetized, one part, that is, more

intensely than another. It depends, as can be seen in a general way, on the mutual support or the contrary which one part of the body exercises on another, and only in particular instances as in the bodies specified above are the effects of this mutual influence everywhere the same.

From all this it might be supposed that in the course of time a very great number of shapes would have been investigated from a magnetic point of view so that in each of them the characteristics of the phenomena and the peculiarities that they might offer would have been determined. But in this respect a noteworthy change has taken place: in earlier times bars were usually chosen for purposes of investigation, because they were easy to produce, and because the bar was regarded as the simplest type of magnet. But now we know that the contrary is true, and that the bar with its end poles, its non-uniform magnetization and so forth, is really a peculiarly complicated case, and rings, spheres, and ellipsoids are usually preferred. The last, particularly when they are elongated, can easily be converted into a cylindrical bar by turning down. We cannot enter into all these investigations, but in the sequel we will return to some specially important individual cases.

36. **Fundamental Formulæ**—In regard to the general subject of magnetic induction we have still to express the facts that we have elicited in an exact quantitative form, and therefore the representation of the ideas that we have arrived at by suitable formulæ

So far we have introduced two vector quantities, the external field $\mathfrak{H}$ and the magnetization $\mathfrak{J}$. If we consider the ratio of the two we obtain a specific characteristic κ. The so-called susceptibility or magnetizability of the material and the equations defining it are

$$\mathfrak{J} = \kappa\mathfrak{H}\,; \quad \kappa = \frac{\mathfrak{J}}{\mathfrak{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

In this form the formula is only general when one regards $\mathfrak{H}$ as being the actual strength of the field at the place where the magnetization $\mathfrak{J}$ is being considered. But according to what has been said earlier, this is only the force of the external field, that is the field which was present before the body was introduced, and when the body is not of a shape that has demagnetizing properties, therefore only when it is a ring or an indefinitely long bar. In other cases, ε the coefficient of shape must be introduced, and then we have the following relation between the true field $\mathfrak{H}$ and the value $\mathfrak{H}_0$ originally present:

$$\mathfrak{H} = \mathfrak{H}_0 - \varepsilon\mathfrak{J} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and from this we obtain by reference to equation (18)

$$\mathfrak{J} = \frac{\kappa}{1 + \varepsilon\kappa}\mathfrak{H}_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

It has to be noticed that for the sphere, κ is equal to $4\pi/3$ and that it has its greatest possible value, namely 4π, in the case of a flat plate placed perpendicular to the field. Of general

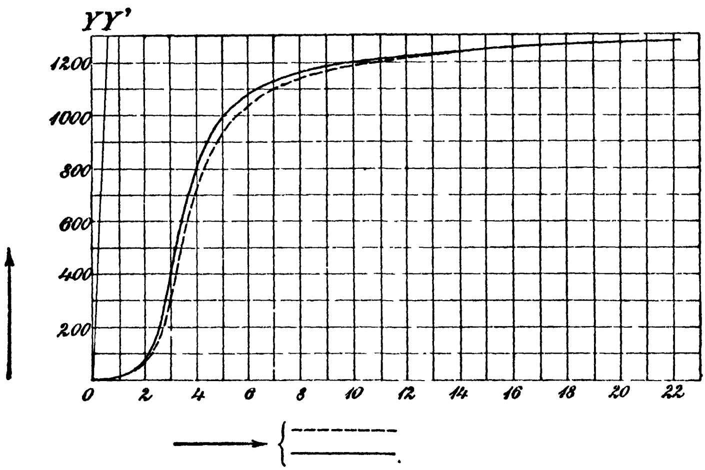


FIG. 36.

application and of great use in practice is the procedure illustrated in Fig. 36, which enables us to go from the $\mathfrak{H}_0$ curve to the $\mathfrak{H}$ curve; the former is shown by the dotted line, the latter by the full line, and is obtained from the first by moving it sideways to the left through an amount which is proportional to the demagnetization term $\varepsilon\mathfrak{J}$, which is represented by the distance between the axis OY and the oblique line OY′. In the figure $\varepsilon = 0{\cdot}00045$.

For most materials κ is a constant and therefore $\mathfrak{J}$, graphically speaking, is proportional to $\mathfrak{H}$, and $\mathfrak{J}$ expressed as a function of $\mathfrak{H}$ is a straight line sloping upwards. In the ferro-magnetic materials this however, as we have seen, is not the case. Here $\mathfrak{J}$ at first indeed increases proportionately, then more quickly, then again proportionately, then more slowly, and finally undergoes no further increase. Similarly in regard to the susceptibility: at first it is constant, then it increases, in the third phase it remains constant, in the fourth it slowly, and in the fifth and last phase it rapidly declines. Therefore in place of the magnetization curve of Fig. 27 we can substitute the κ curve of Fig. 37, which expresses the same facts in another form. The scale used presupposes that $\mathfrak{H}$ and $\mathfrak{J}$ are measured in absolute values. The unit so chosen for the strength of field is called the gauss, and this determines the measure of $\mathfrak{J}$, which only differs from $\mathfrak{H}$ by the numerical factor κ. In the

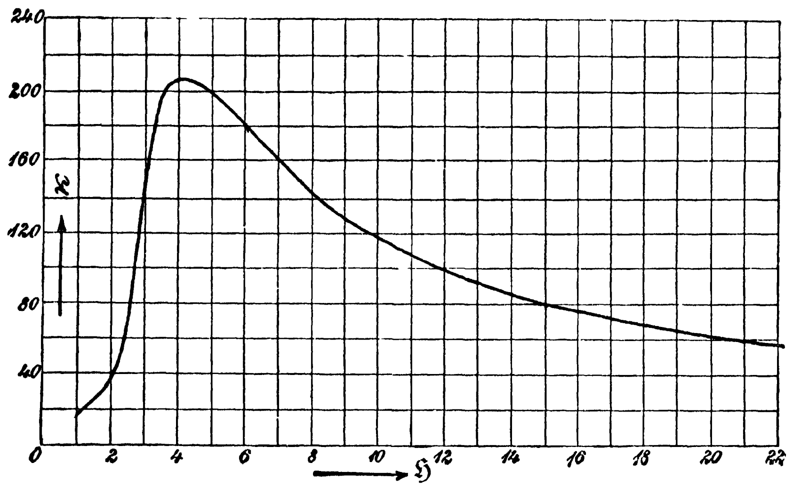
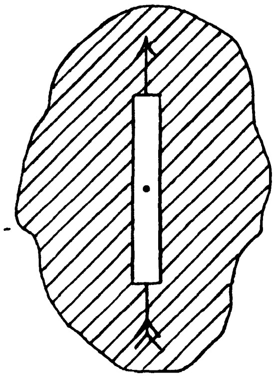


FIG. 37.

example given κ reaches its maximum value for $\mathfrak{H} = 4{\cdot}2$ gauss, but this of course will vary from case to case.

37. "**Induction**"—We might be satisfied as it would seem with this method of representing the phenomena by means of the quantities $\mathfrak{H}$, κ and $\mathfrak{J}$. Nevertheless we should not exhaust the actual facts of the case, and we should lose sight of some very important and illuminating ideas.

We shall therefore now proceed to raise an entirely new question: what is the force inside a body that is subjected to the effect of a field? It is clear that it is not simply the force $\mathfrak{H}$, for the body itself has here reacted upon the field. In the case of weakly magnetizable substances it is true that it has only modified it by a small amount, but the change produced in the ferro-magnetic bodies is very considerable. Indeed in the latter case we must rather consider that the true field has merely given an impulse towards magnetization, and that the principal fact was the mutual excitation of the parts by one another. The inner field has to be thought of as being superadded to the external field. The question is, how strong is this internal field, or, more precisely expressed, how strongly is it excited by the external field? To answer this question we must look at the subject from what at first sight will seem a somewhat peculiar point of view, but which however is justified by the conditions of the case. The elementary magneto-motive force is, as we have shown, a function of the distance in the inverse sense, that is, as the distance increases the force becomes smaller, and as the distance is diminished it becomes greater, and



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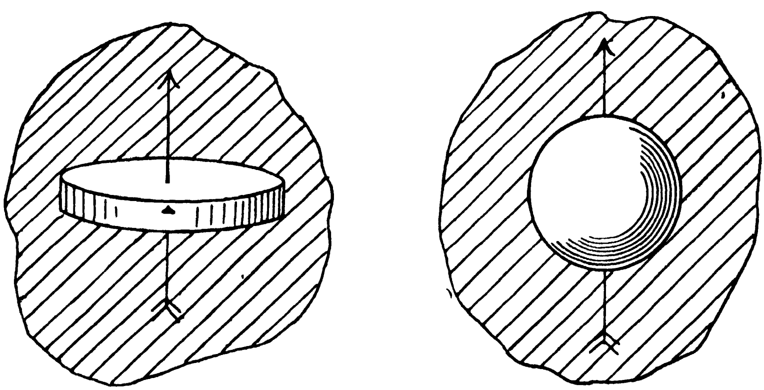

2) 3)

FIG. 38.

therefore at an infinitely small distance it becomes infinitely great. If we consider a point within the body of the iron, we have, since we are immediately surrounded by the iron, an infinitely great force to think of, with which we can do nothing, and we have to find a way of surmounting this difficulty. Thus we can imagine that the point with which we have to deal is enclosed in a hollow space, and thus having got free from the immediate neighbourhood of the iron we can calculate the force. And here it must be confessed the result depends entirely upon the way in which we choose the hollow space (see Fig. 38). There are two extreme cases; the first is a long narrow channel in the direction of the field, and the second a wide but narrow slit at right-angles to it. There is also the possibility of a spherical cavity with the point as centre which represents the mean between these two cases. When we suppose a long narrow channel, then, as a simple construction shows, all internal effect ceases (for the side surfaces of the channel are without influence, and the end surfaces are small and can therefore be neglected). The field therefore has become equal to $\mathfrak{H}$. (Of course only when the body is not limited outwardly; otherwise, as we have seen, there is a weakening effect, which however does not concern us here.) If, on the other hand, we suppose a narrow cleft, then the point under consideration experiences the effect of two mutually opposed and oppositely magnetized plane surfaces; it lies, so to speak, in the middle of a magnetic shell, and in consequence it experiences (see page 41) a force which is 4π times the magnetic intensity, so that the total field becomes equal to $\mathfrak{H} + 4\pi\mathfrak{J}$. For this force there is a special name; it is called the "magnetic induction," which expression has now a definite quantitative sense, and being a vector is represented by the black letter symbol $\mathfrak{B}$. Therefore

$$\mathfrak{B} = \mathfrak{H} + 4\pi\mathfrak{J} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

or since $\mathfrak{J} = \kappa\mathfrak{H}$ (Equation 18)

$$\mathfrak{B} = (1 + 4\pi\kappa)\mathfrak{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

which by the introduction of a new sign

$$\mu = 1 + 4\pi\kappa \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

can be more briefly written

$$\mathfrak{B} = \mu\mathfrak{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

It may be incidentally remarked that also here the demagnetizing force of the upper surfaces must be taken account of, and then if ε is again the coefficient of shape, and $\mathfrak{H}_0$ the original strength of the field, the formula

$$\mathfrak{B} = \frac{4\pi\mu}{4\pi + (\mu - 1)\varepsilon}\mathfrak{H}_0 \quad . \quad . \quad . \quad . \quad . \quad (25)$$

is obtained, exactly corresponding to the formula (20).

The importance of this conception of magnetic induction lies in the fact that it is possible to realize it practically in a very simple manner. This is done by taking an iron body, best of all a closed ring, cutting it across and then bending it out a little so that at the place of section there is a narrow gap. We then have the already mentioned type of the solid ring and in the gap the induction presents itself as a determinate quantity. But more than this, the induction, so far as the inner part is concerned, gives us a generalized conception of the force, and indeed of its highest value. It clearly shows how the iron body raises the field to its highest value through its presence. To explain this we will take two examples. In

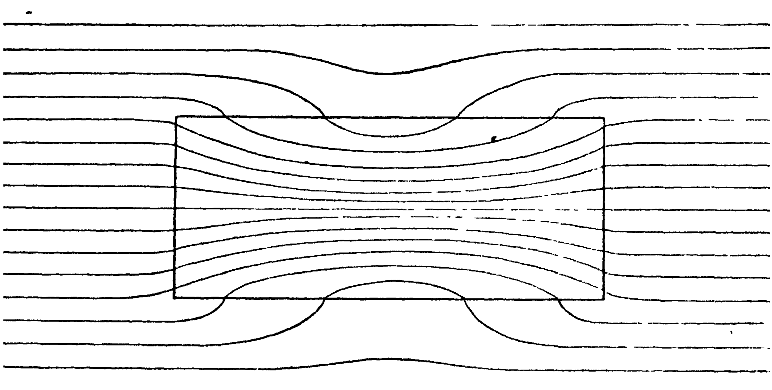

FIG. 39.

Fig. 39 we have an iron rod brought into a field previously uniform ; the lines of force all run from left to right and at an equal distance from one another. But now they are drawn in towards the iron and crowd together there. The lines of force pass more readily through iron than through air, and the extent to which this occurs is represented by the quantity μ, which, on this account, is also called the magnetic permeability. Corresponding to this is the case shown in Fig. 40, where we have two poles N and S with hollow faces standing opposite to each other. The lines of force between would run fairly uniformly as straight lines at an equal distance from one another from the upper to the lower pole. But in consequence of a ring of soft iron being placed between them the lines of force are drawn in together, and are crowded together in the iron. The induced strength of the field on the left and the right is very great, while in the air space it is almost nothing. This case is one which in the early history of the dynamo played an important part as the Gramme Ring.

If we now look back at the conclusions that we have come to, we get the impression that the conceptions introduced have been actually somewhat too abundant: there is the original field $\mathfrak{H}_0$, the field modified by the presence of the body introduced into it $\mathfrak{H}$, the magnetization $\mathfrak{J}$, the induction $\mathfrak{B}$, and in addition the two specific coefficients, κ the susceptibility and μ the permeability. In practice we do not need all these conceptions and in many cases there is some embarrassment in deciding which of them should be chosen to represent the phenomenon. Thus it is largely a matter of taste whether, taking a particular instance, the magnetic condition of a body is characterized by $\mathfrak{J}$ and κ, or by $\mathfrak{B}$ and μ. One mode of representation implies the other, and the relation between κ and μ is exceedingly simple. If the body is only weakly magnetized, κ is only a small proper fraction and it follows from formula (23) that it is only slightly different from unity. If, on the contrary, the body is ferro-magnetic, κ is usually very large, and since it has moreover to be multiplied by 4π we can write to a very close approximation $\mu = 4\pi\kappa$ (neglecting 1). Therefore μ is roughly $12\frac{1}{2}$ times κ.

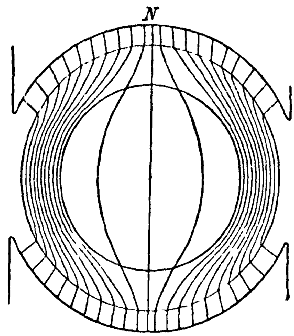

FIG. 40.

38. **The Magnetization again**.—The principal matter is and remains to discover the laws according to which magnetization or induction depends upon the strength of the field, and to evolve a mathematical formula which will represent the magnetization or induction curves. Such a formula it might be thought we already possess in (18) or (24), but here the difficulty and the complexities of the subject have been evaded by means of the coefficients κ or μ, which are not true constants but are themselves dependent on $\mathfrak{H}$. Our object now is the derivation of formulæ connecting $\mathfrak{J}$ (or $\mathfrak{B}$) and $\mathfrak{H}$ in which the other quantities are constants.

Nearly all the physicists who have so far occupied themselves with the study of magnetism have addressed themselves to the solution of this problem, but nothing finally of practical use has resulted from their labours. All the formulæ put forward represent the phenomena only in some but not in all its principal features. Thus, for example, the approach to saturation may be very well represented by Fröhlich's formula,

$$\mathfrak{J} = \frac{\mathfrak{H}}{a + b\mathfrak{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

in which a and b are constants characteristic of the material. As $\mathfrak{H}$ increases $\mathfrak{J}$ increases more and more slowly and approaches a limiting value $1/b$. But the formula breaks down completely in regard to the more rapid increase of $\mathfrak{J}$ in the intermediate portion of the curve. Other formulæ, on the other hand, will represent this very well, but not the condition when saturation is being approached. Those who are acquainted with geometry will see without further proof that the curve of the magnetization must be a curve of the third order since a straight line can be drawn from the zero point which cuts it in two other points, that is in three points altogether, and in addition there are further complications. We must be satisfied with the curves as themselves representing the law, but there are a few other observations to be made.

First comes the question whether the initial portion of the curve is really straight; or, to express it otherwise, does κ (and in the induction curves does μ) at first and so long as the field is weak remain a constant, or does it increase from the very beginning? According to the latest experiments it would seem as if the latter supposition were true. Then comes the question, where does the point of inflexion of the magnetization curve occur, or where is the highest point of the susceptibility curve? The answer to this is that so long as by " where ? " " at what strength of the field ? " is meant, it all depends on the material and the shape of the body. But if we ask, At what degree of magnetization does it occur ? the matter is considerably simplified. In Fig. 41, for example, are shown the induction curves for the same wire after progressive reductions of its length (in consequence of which the demagnetizing effects of the ends become greater and greater). The length is shown by the attached numerics. The point of inflexion moves continually over to the right : that is, the most favourable degree of magnetization occurs the shorter the wire and the stronger the field. On the other hand the points of inflexion always occur at about the same height, that is they correspond to one and the same induction of about 5,000 units, or what is the same thing, to a magnetization of about 400. In this sense, then, the position of the point of inflexion is independent of the length of the rod, and is typical for the material of which it is made. The third question refers to the saturation. Here it is to be emphasized—and this again is a point in which $\mathfrak{J}$ and $\mathfrak{B}$ show fundamental difference of behaviour—that there is a limiting value only for $\mathfrak{J}$ and that on the contrary $\mathfrak{B}$ always goes on increasing,

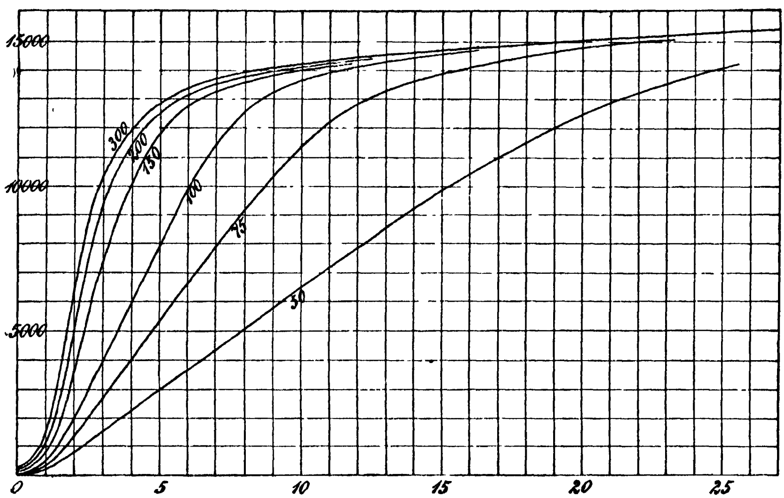

FIG. 41.

for it contains $\mathfrak{H}$ as a constituent, as formula (22) shows. The $\mathfrak{J}$ curve, however, becomes gradually horizontal; the $\mathfrak{B}$ curve continues to rise. The highest value which $\mathfrak{J}$ can reach appears to be about 1,800, and to reach this the field must be increased to about 24,000 gauss. $\mathfrak{B}$ has then, according to formula (21), a value of about 45,000. The quantities κ and μ in this saturated condition have then, however, greatly declined from their maximum value (compare Fig. 37), and while they were 750 and 9,000 respectively at their maximum at the point of inflexion, they now amount to only about 0·1 or 1·8 respectively.

39. **Hysteresis Loss**—It is moreover to be added that $\mathfrak{J}$ and $\mathfrak{B}$ and therefore also κ and μ are not single but double functions of $\mathfrak{H}$, that is, they have different values according

as the point to which they refer was undergoing a process of increasing or diminishing magnetization. Single values are obtained only in processes in which hysteresis does not occur, and with the ideal curve obtained in the manner already indicated.

But as regards the hysteresis itself, and in particular the dissipation of energy which is thereby brought about, we know that this is represented by the energy enclosed by the loop. By integration we obtain for this energy the equation

$$A = \int \mathfrak{J} d\mathfrak{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

corresponding to which others may be obtained in which $\mathfrak{H}$ may be substituted for $\mathfrak{J}$ or $\mathfrak{J}$ for $\mathfrak{B}$. In practice this is called the total loss per cycle, and dividing it by the volume of the iron we obtain the specific value of the total hysteresis loss per cycle and per cubic centimetre of iron. The question now arises what is the law connecting the total loss with the quantities on which it depends, and here it will be appropriate to choose not $\mathfrak{H}$ but $\mathfrak{J}$ or $\mathfrak{B}$ (which of these be chosen makes no substantial difference). It is usual to choose $\mathfrak{B}$ and to understand by it the maximum value attained, whether positive or negative. It can then be shown that the loss V increases more quickly than $\mathfrak{B}$, so that we obtain, according to Steinmetz,

$$V = \eta \mathfrak{B}^{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

where η and β are two numerical constants of which the last is of interest to us. This term β is certainly greater than 1 and smaller than 2, on the average about 1 : 6. Nevertheless the latest investigations have shown that even in the same series of experiments it is not really a constant, and thus the simplicity of the formula is lost. Generally a binomial formula agrees far better :

$$V = a\mathfrak{B} + b\mathfrak{B}^2 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and since it is more convenient in calculation than the exponential formula it will in time completely replace it. Finally as regards the factor η, or in the second formula the absolute value of the coefficients a and b (not their relation to each other) : these in practice are found usually to lie between 0·001 and 0·02. We shall have to return to all this later ; here we shall only remark that the total loss per cycle and per cubic centimetre in soft iron amounts to about 10,000 ergs ; in steel it may be as much as several hundred thousand, and on the other hand in the case of special iron it may sink to a few thousand.

IV

PAN-MAGNETISM

40. **Introduction**—Magnetism is a general property of matter. But this generalization admits of special cases and indeed of contrasts such as are scarcely to be found in any other department of physical phenomena. The contrasts here are such as are only to be found in the very worst forms of plutocracy, in which alongside a little group of the rich there exist the innumerable hordes of the poor. But looking into the matter more closely we find that there are three conditions ; for among the poor are those who do possess something, even though it be a pittance, and those who have not only nothing to call their own, but who even have debts as well. Luckily, or as we may also say, unfortunately (for these would be interesting cases !) the fourth class of those who are very much in debt is missing. The very rich are the ferro-magnetic substances ; the poor are the weakly magnetic, and among the latter are those which experience normal magnetism when they are brought into a magnetic field, and others whose magnetization is in the opposite direction to the normal. The first are called paramagnetic and the latter diamagnetic, and the discovery and investigation of the latter after the preliminary work done by Brugmans (1778) and Becquerel (1827) represents one of the most beautiful and important achievements of Faraday (from 1845 onwards). The name diamagnetism was introduced by him, and he demonstrated that all forms of matter, including fluids and gases, are susceptible to magnetization either in a positive or a negative sense. We will now review the properties of these different classes of matter and consider some of their chief representatives.

(*a*) FERRO-MAGNETISM

41. **Iron and Steel**—If we begin with the ferro-magnetic substances, then in the first place we have to deal with iron and some (though by no means all) of its compounds ; and

in addition with the closely allied elements nickel and cobalt; and, finally, with a series of recently discovered substances, the Heusler alloys.

As every one knows, iron plays a leading part in the arts; we have been living, we may certainly say, for some centuries in the iron age. But under the term "iron" something very varied in its properties is to be understood. Not only does chemically pure iron come under the term (and that in fact least of all, because, as we have already said, it is an extremely difficult substance to produce), but also iron in combination with other elements, and that in more or less loose forms of combination, till we get to the actual "chemical combinations." One substance above all others plays an important rôle here: the element carbon. There are many others, and in particular silicon, manganese, tungsten, and nickel, which still play an important part. A further diversity results from the method of preparing the material, whether the process be one of casting, rolling, or forging, and so on. Connected with this is the still further diversity arising out of what has been called the thermal preliminary treatment of the material; this preliminary treatment usually is one of thermal and mechanical methods combined. The consequence is that the final product we have to deal with exhibits an extraordinary diversity of properties, first from a mechanical point of view (density, elasticity, hardness, brittleness, and so on), and then in its magnetic qualities, with which we are chiefly concerned here.

As regards the carbon content in particular, it is customary to draw up the following scale: wrought iron contains up to $\frac{1}{2}$ per cent. carbon; in the various sorts of steel the content gradually rises to $1\frac{1}{2}$ per cent.; in cast iron it amounts to 2 or 3 per cent., and finally in specular cast iron to as much as 5 per cent. From this it follows that we have not merely to distinguish between iron in the narrower sense and steel. Still less is this possible because we have to consider thermal treatment and particularly the degree and length of the heating process and the rapidity of the cooling, which all play an important part—processes which are known as annealing, quenching, chilling, hardening, and so forth. Only in recent times has a scientific characterization of the various final products been introduced, generally based upon a microscopic examination. The chief results of this investigation are as follows: below 0·9 per cent. of carbon we have a mixture of ferrite, that is pure iron with a material, perlite, embedded in it, in thin layers, which has a mother-of-pearl-like appearance, and corresponds more or less to the formula Fe_3C. If a carbon content of 0·9 is reached the material is all perlite, we have a eutectic mixture; that is, on solidification there is no separation of the various components. Above that there is again separation, but now no longer of ferrite but of cementite, that is of carbide of iron. These relations are shown graphically in Fig. 42; but it must be emphasized that this picture is much too simple, and that we shall later on have to supplement it from a thermal point of view.

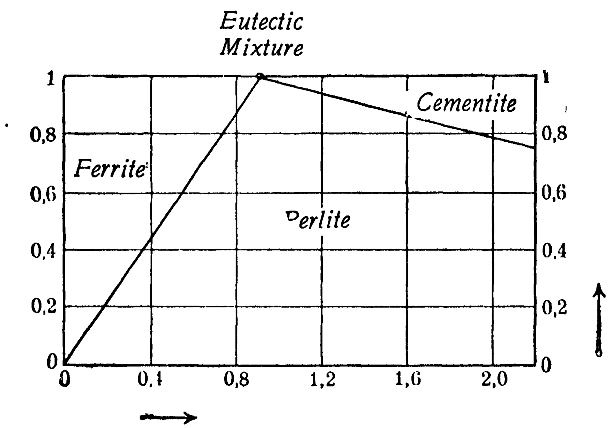

FIG. 42.

An exhaustive characterization, even in any limited sense, of the various sorts of iron and steel according to their magnetic qualities, is not possible here. The greatest contrasts, it may generally be said, relate not to the magnetizability, but to the remanence, coercivity and hysteresis. A characteristic figure (Fig. 35) has already been given. The following table is based on the results of the experiments of Gumlich and Schmidt with ellipsoids of various materials whose technical description is designated in accordance with the following list:

1, Kohlswa 52, five times annealed; 2, Remscheider dynamo steel; 3, Gelsenkirschner cast steel, twice annealed; 4, Kohlswa 52; 5, Kohlswa 50; 6, Cast iron; 7, Böhler tungsten steel annealed; 8, Remscheider tungsten steel unhardened.

No	Length	Thickness.	$\mathfrak{H}$ (max).	$\mathfrak{B}$ (max).	$\mathfrak{B}'$.	$\mathfrak{K}$	μ (max)	U (ergs).
1	33	0·8	151	18,500	7,100	1·0	3,700	11,700
2	26	0·8	152	18,400	8,840	1·3	3,280	12,800
3	33	0·6	165	18,660	8,500	1·6	2,630	13,300
4	18	0·6	149	18,310	8,520	1·7	2,390	16,200
5	26	0·6	156	18,320	9,000	2·1	2,100	20,400
6	33	0·6	155	9,900	4,230	11·9	184	34,300
7	10	1·0	235	17,000	12,900	16·7	433	92,000
8	33	0·8	505	18,720	9,880	27·5	233	116,000

The magnetizability, it will be seen, is in many cases approximately the same, though in the case of cast iron it is about half as great. On the other hand, the remanence varies in the proportion of 1 : 3 and the coercivity in the ratio of 1 : 28, the maximum permeability as 1 : 20 and the dissipation of energy as 1 : 10.

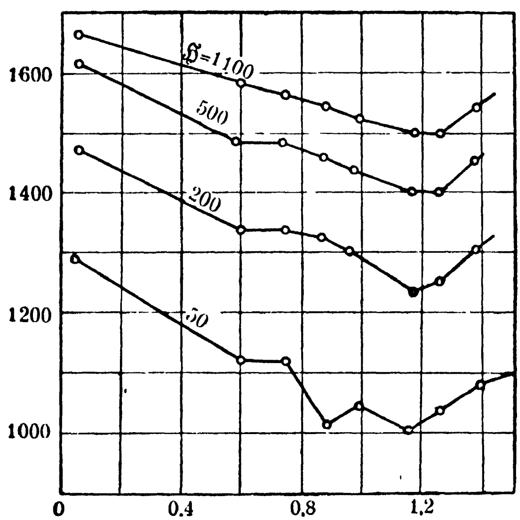

FIG. 43.

In what way the magnetization on the one hand and the Steinmetz coefficient μ on the other depend on the carbon content will be seen from Figs. 43 and 44, the former based on measurements by Holborn, and the other by Benedicks on one hand and Waggoner on the other. Steinmetz and Benedicks have shown on the ground of their own and other experiments that in a very varied series of specimens of steel the hysteresis coefficient η is simply proportional to the coercivity. And this has recently suggested many interesting relations and new ideas.

42. **Alloys**-The most important alloys, as we have remarked, are those of silicon and tungsten. The effects of the addition of silicon are very complicated, but among them is the reduction of the magnetization of saturation, and the coercivity, and the hysteresis, the last to a tenth of its normal value, the permeability on the contrary being increased. Silicon iron is therefore specially well adapted to cyclical magnetization. Tungsten has exactly the contrary effect; here the coercivity becomes enormously increased up to as much as twenty times its normal value. Chromium behaves similarly. Tungsten and chromium are therefore specially suitable for permanent magnets.

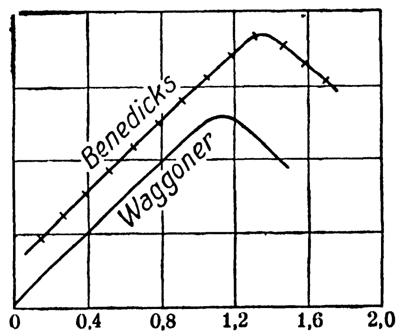

FIG. 44

The effect of manganese is altogether peculiar. In even the smallest quantities it diminishes the magnetizability of the iron; with 4 per cent. it is only a quarter, with 8 per cent. only a twentieth of the normal, and with 12 per cent. practically zero. Manganese is therefore calculated to prevent the process of magnetization. The coercivity and the hysteresis are extraordinarily great.

In conclusion it should be mentioned, in connection with the natural combinations of iron, especially magnetic iron ore (with which the phenomenon was first observed in ancient times), that they can be only a third or a fourth as strongly magnetized as pure iron, and that in detail the effect, especially that on the remanence, and so forth, depends on the chemical combination. This is not constant but varies, for example, in magnetic iron ore between Fe_2O_3 and FeO; and recently Hilpert has shown that both these chemical combinations, taken separately, are not ferro-magnetic, and the magnetizability of magnetic iron ore and such substances is solely a consequence of the combination of the two substances in alligation.

43. **Nickel and Cobalt**—Nickel and cobalt, as is well

known, are closely related to iron chemically and they have also magnetic affinities. The laws of magnetization are the same, only the values attained and the shape of the curves are more or less different. In the first place the initial rise of the curve is much slower than in iron, so that in a weak field the magnetization is much less than that of iron. This is partly made up for later on, and finally nickel for $\mathfrak{J} = 540$ reaches almost a third, and cobalt for $\mathfrak{J} = 1{,}400$ three-quarters, of the value of iron. In remanence and coercivity nickel and cobalt are even superior to iron, at least for moderate degrees of magnetization; then the curves intersect and at the limiting values are much smaller. But it must be borne in mind how difficult it is to obtain these substances free from iron; in a really pure condition they would probably show still greater divergences.

In order not to anticipate matters, we have in dealing with the alloys of iron omitted those formed with nickel and cobalt; but nickel iron and nickel steel have peculiar claims upon our interest. These alloys, according to their nickel content, exhibit weak or strong magnetizability; it declines till a nickel content of 25 per cent. is reached, and then increases again. For the mixture named the permeability amounts only to a few units. The optimum condition occurs with a 5 per cent. nickel content, and at the beginning the induction is indeed below that of pure iron but afterwards it is greater, and in somewhat powerful fields the alloy is decidedly superior. Accordingly 5 per cent. nickel iron is specially suitable for technical purposes, and that too in the positive sense; on the other hand, the use of 25 per cent. nickel steel is to be recommended in a negative sense, that is where a material is needed which it is desired should not become magnetic, as for example in the building of iron ships, where otherwise the use of the compass becomes impossible (in its ordinary form at any rate, but compare what comes later). An understanding of the remarkable behaviour of these alloys is obtained through physico-chemical investigation; it appears that nickel steel up to 25 per cent. nickel is "irreversible," that is, after heating it does not return to its original condition, while the higher percentage mixtures are reversible. But from a general point of view the 25 per cent. nickel iron offers an excellent example of that sort of natural phenomenon which may be designated, irrespective of the particular field of natural science in which it presents itself, interference. Here two effects give when in combination not a stronger but a weaker total effect, and under some circumstances, indeed, none at all, as occurs for instance in the case of the interference of waves (sound, light, or electric waves), where it is easily intelligible as the superposition of a wave hollow on a wave crest. Here, on the contrary, where nothing of this sort occurs, other considerations are required; those, namely, which refer to the internal disposition of the particles and the thermal behaviour.

44. **Heusler Alloys**—We have next to discuss the opposite, so to speak, of what we have just been considering; and the contrast is still more notable because it contradicts the principle that the combination of nothing and nothing is still nothing. We must of course somewhat modify the statement in order to adapt it to the actual case, and instead of saying "nothing," say "very little." If we take any two metals whatever, for example silver and gold, each of them is only very slightly magnetizable, and if we make mixtures of these two metals in varied proportions we expect, and the expectation is justified, that in general the alloys will be slightly magnetizable also. This is true for the greater number of metals that we know, and for the still greater number of their combinations with one another. But twenty years ago a very wonderful exception presented itself—the alloys discovered by Heusler in 1898, and named after him. The decisive constituent is manganin, the others are usually copper and aluminium (but zinc and other metals also come into question). None of these metals is ferro-magnetic, but the manganese bronzes (as the Heusler alloys may be called) are quite distinctly so, and their magnetizability compares very well with that of the other ferro-magnetic substances. The effect appeared at first sight so surprising that people were mistrustful and thought that it was due to the presence of some iron impurity; but the most careful elimination of iron did not alter the fact in the slightest degree. We have therefore here to deal with a quite startling case of a physical property which is not "additive" but "constitutive"; which brings about, that is, a new constitution which is not present in the elements from which it is derived. A manganese bronze has its own curve, quite of the form already known to us, but in each individual case substantially modified by the nature, that is, by the percentage proportions of the alloy. Moreover, in the course of years it has been shown, thanks to the united labour of many physicists, that the phenomenon is peculiarly many-sided and involved, and that the finer features

of the inner constitution and the effects of the thermal treatment, particularly the so-called artificial ageing, all enter into the question—matters to which we shall return later. It is to be noticed here that the maximum of magnetizability presents itself for a given proportion of the constituents, but whilst in certain cases we have to deal with a distinct percentage (in the case of aluminium about 12 per cent.), in others again the chemical equivalent relation is the determining factor; for example, the realization of the chemical

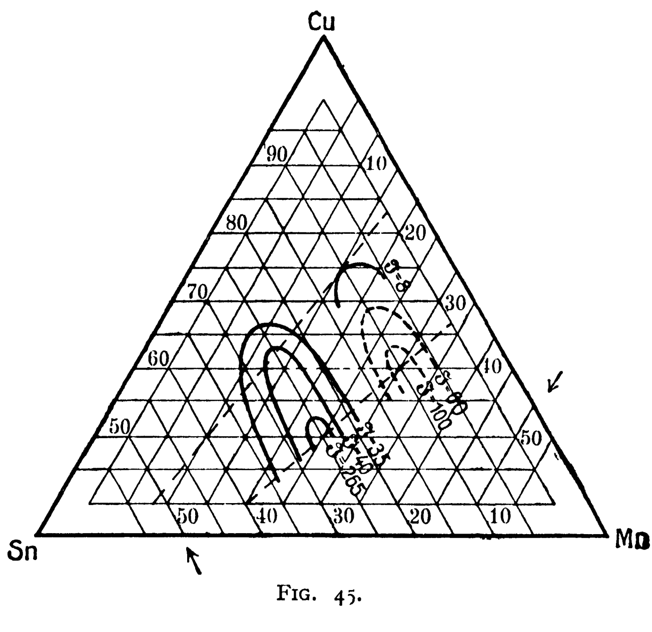

FIG. 45.

formula, $SnMn_3$, $AlCu_3$, and $SnCu_3$. In order to select one out of a crowd of interesting results, we give here (Fig. 45) the van't Hoff triangle in its application to the magnetism of the Heusler alloys. We have here a ternary mixture: tin (Sn), manganese (Mn), and copper (Cu); the proportions of these three substances are represented alongside the sides of the triangle in atomic percentages; each alloy then corresponds to a point within the area of the triangle. The strength of the magnetization of saturation can then be imagined as a vertical line, and the mountain range so built up as two lines of ridges—the principal one is the line of 25 per cent. tin, and a subsidiary line. For the rest the figure speaks for itself.

As a conclusion to this section a comparative representation of the curves of magnetization of the most important ferromagnetic substances is given in Fig. 46, with 5 per cent. nickel iron at the top, and manganese steel at the bottom; these curves have naturally only an average significance.

(*b*) PARA- AND DIAMAGNETISM

45. We now come to substances only weakly magnetic, to which class most substances belong. They offer only a slight

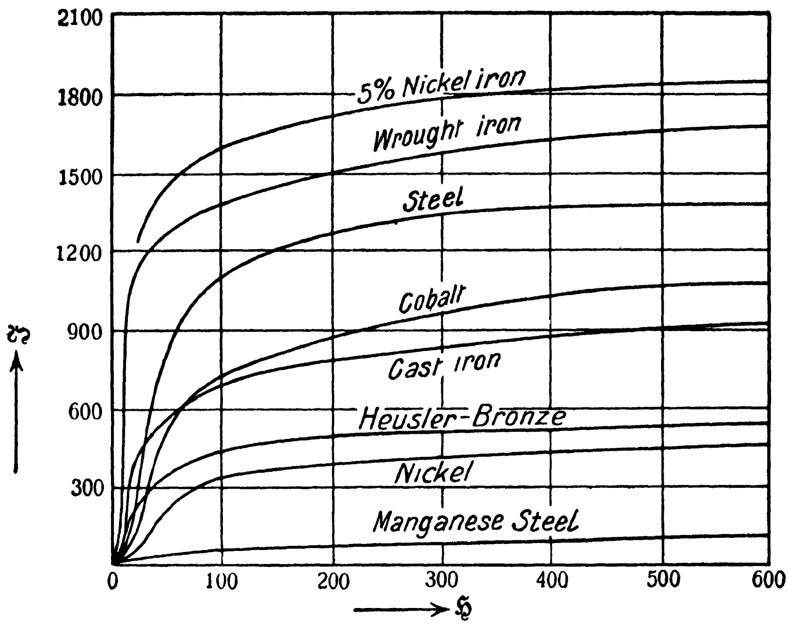

FIG. 46.

practical interest, but a much greater from a purely scientific point of view. The facts relating to this class are of a quite bewildering kind. There are substances which behave like iron but in a much slighter degree; and there are others whose behaviour is the reverse. The latter are called the paramagnetics, and the former the diamagnetics. A ball of bismuth is not attracted by a magnet but repelled and a bar of bismuth between the poles of a horse-shoe magnet places itself not in an axial but in a transverse direction, that is at right-angles to the direction of the lines of the field; but here very pointed pole-pieces must be used, because otherwise complications arise which may lead to confusion. These two experiments are very special ones and they can be varied in a number of ways. But in general terms the contrast of which we are speaking can be expressed according to Gauss in the following way: in fields which possess an axis of symmetry of rotation and a symmetric plane at right-angles to it (and such are those with which we generally have to deal) a para-(dia-)magnetic bar places itself parallel (perpendicular) to the lines of force when the strength of the field from the middle point of the field increases as we move out along the axis, but diminishes perpendicular to it; on the other hand it places itself perpendicular (parallel) to the lines of force when the strength of the field along the axis diminishes and increases perpendicularly to it. Usually we have to deal with fields of the first sort, for example in an electro-magnet with cone-shaped poles. The second case can be demonstrated thus: on the poles of an electro-magnet with plane pole faces a hollow iron cylinder is placed with walls 3 mm. thick, 2 cm. in diameter, and 2 cm. long, and the other pole brought to within 4 mm. of it. In the space inside the hollow cylinder a little bar of platinum, which is paramagnetic, places itself perpendicularly; a bar of bismuth, which is diamagnetic, on the contrary, parallel to the lines of force.

In fluids which have to be investigated in containing vessels, either closed bottles or open trays, the solid substance of which the containing vessel is made must be tested before it is filled with the fluid, for only in this way can the behaviour of the latter be properly deduced. Of various modes of experimenting with fluids the following may be mentioned. A drop of the fluid T (Fig. 47) is brought in a horizontal tube RR′ between the poles PP′. Diamagnetic substances, when the magnet is excited, move in the direction towards R or R′ (according to the direction of the field): paramagnetic substances, on the other hand, are drawn in between the poles. If the fluid under investigation is poured into a watch-glass placed on the pole shoes PP′ of an electro-magnet, a paramagnetic fluid, which was originally bounded by a circular line *abcd*, is drawn down and takes the form *a′b′c′d′* so that it piles itself up slightly over the edges of the pole pieces (Fig. 48). A diamagnetic fluid, on the contrary, forms a hollow over the edges of the pole pieces and piles itself up in the intervening space (Fig. 49).

For gases a soap-bubble method can be used. The bubble is filled with the gas under examination and when placed between the poles rises or sinks according to circumstances. The coloration method evolved by Faraday is very striking. The gas which is allowed to stream between the poles, if it is not naturally coloured, is artificially coloured. For this purpose a number of little tubes which contain paper soaked in ammonia are placed between the poles and extend down to the plane between them; in the pipe on the other side which leads the gas from below is a piece of paper steeped in hydrochloric acid. In the little tubes into which the gas streams a white cloud is formed and in the case of paramagnetic substances in the axially discharging tubes, and in the diamagnetic in the equatorial. The very considerable diamagnetism which a flame possesses, and which Bancalari first observed, can quite easily be recognized by its change of form when placed between the poles of a magnet. In Fig. 50 some of these forms are schematically represented; *a* shows the axial, *b* the equatorial cross-section of the flame; *c* the last when the flame is placed somewhat higher; *d* the picture of a flame placed somewhat at the side; and *e* a very sooty flame of turpentine which, as will be seen, divides into two parts.

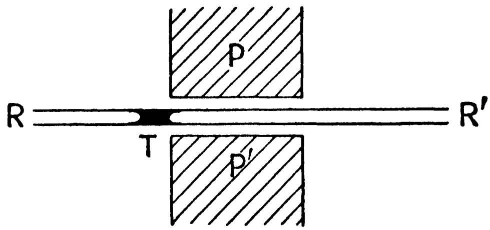

FIG. 47.

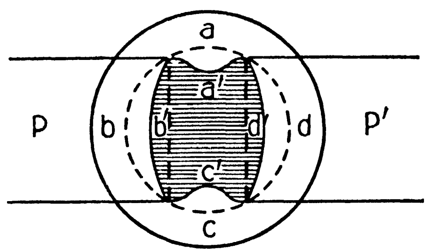

FIG. 48.

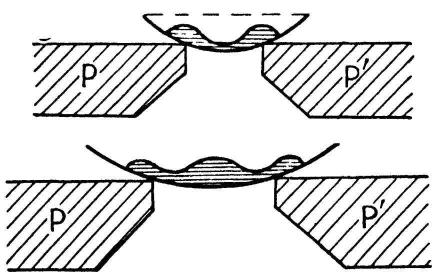

FIG. 49.

But in all bodies, it must now be emphasized, we only observe the differential effect against the surrounding material, therefore against the material of the vessel, or (in all cases) against the air (which is paramagnetic ; see below). Only when the observation is made in a vacuum is the true magnetism observed. This differential effect in regard to magnetic force can be compared with the exactly analogous case of weight, and the principle of Archimedes can be transferred. In the one case as in the other we can either observe the various

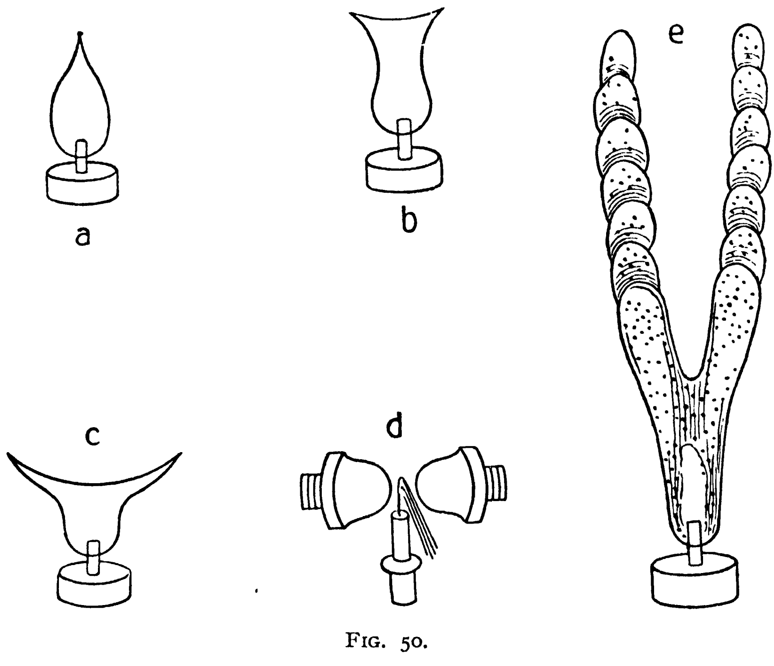

FIG. 50.

bodies which are to be investigated in the same medium ; or one and the same body, which then serves as an auxiliary body, can be observed in various media, which latter are then the object of investigation. Both methods are in use, and it will be readily seen in what cases the first or the second offers the greater advantage. For the rest it is shown that even in a vacuum numerous substances appear diamagnetic, and these we must bear in mind because they are of great significance in connection with the theory of the subject.

46. **Quantitative Relations**—At first, after becoming

aware how widely spread magnetism was, it seemed to be sufficient to determine which bodies are paramagnetic and which are diamagnetic, and in what degree, in which connection it of course makes a difference whether the magnetism is referred to unit volume, unit weight, or atomic equivalent weight (see below). In this way we come as in other subjects to a magnetic series of substances, in which however the majority of substances are not susceptible of being grouped because their differences are too small or too uncertain. If we confine ourselves, therefore, to the substances which are specially adapted to this scheme of classification and consider only the ferro-magnetic substances, we obtain on the atomic basis the following series, which begins with the most strongly magnetic metal and goes on to the most strongly diamagnetic.

(+) Iron — Cobalt — Nickel — Manganese — Chromium — Cerium — Neodymium — Prascodymium — Lanthanum (?) — Palladium — Platinum — Tungsten — Molybdenum — Aluminium — Silicon — (somewhere about here lies the indifference point) — Carbon — Potassium — Sodium — Calcium — Barium — Phosphorus — Arsenic — Copper — Zinc—Cadmium — Silver — Sulphur — Lead — Gold — Bromium — Iodine — Mercury — Antimony — Tantalum — Bismuth (—).

Many of the substances on the magnetic side were at first regarded as magnetic because they contained iron, and it is by no means impossible that some further modification of the list may not be called for. On the whole, however, it shows a certain relation to the periodic classification of the elements ; for those elements in the third column which it is known have the smallest atomic volumes, are also most strongly paramagnetic. To the left and the right are arranged the elements with increasing atomic volume and decreasing atomic magnetism (in fact becoming negative) and the substances on the right are the most strongly diamagnetic.

With such a scale we cannot of course rest satisfied. We must make many more quantitative determinations, and that has been done after numerous and more recently perfected methods. From the rich material thus supplied we can only here select a few samples of the solid substances. The values given refer first to the susceptibility κ, and secondly to the specific magnetism, that is (see above) to the magnetization of 1 gm. (instead of 1 c.c.) of the substance produced by unit field strength χ ; obviously we shall obtain χ from κ if we simply divide by the density of the substance. The values in the following table are only mean values derived from individual values which at times differ very considerably among themselves, partly on account of the dissimilarity of the material and partly because of the imperfection of the method of observation. All the values are to be divided by a million.

Paramagnetic			Diamagnetic		
Material.	κ	χ	Material.	κ	χ
Oxygen (solid)	422	—	Wood (mean)	−0·3	
Manganese	78	10·3	Copper	−0·88	−0·091
Palladium	61	5·4	Sulphur	−0·89	−0·41
Uranium	60	—	Glass (mean)	−0·9	—
Iron oxide	50	—	Zinc	−1·04	−0·146
Platinum	25	1·1	Quartz	−1·2	—
Chromium	24	3·5	Lead	−1·33	−0·104
Tantalum	14	0·84	Fluorspar	−1·35	—
Aluminium	1·7	0·7	Phosphorus (white)	−1·7	−0·93
Sodium	0·52	0·51	Silver	−1·76	−0·167
Potassium	0·5	0·5	Tellurium	−2·0	—
			Gold	−2·8	−0·144
			Bismuth	−14·2	−1·43

As will be seen, three substances, apart from oxygen, stand out from the general scheme: manganese, palladium and uranium. After these, and again apart from iron oxide, platinum and chromium, and on the negative side, bismuth. It is remarkable that manganese, the bearer of the ferro-magnetic property of the Heusler alloy, here stands at the top of the list where it is, of course, as remote as possible from the ferro-magnetic substances. In addition to this, manganese, when specially treated, behaves quite peculiarly. Thus Seckelson has found that manganese electrolytically deposited in the direction of the field is susceptible of very strong magnetism, amounting to a thirtieth of that of iron. And Weiss and Kamerlingh-Onnes found that chemically pure manganese formed under hydrogen is indeed paramagnetic so long as it is in powder form, but when it is melted together it becomes ferro-magnetic to an extent about a hundredth of that of iron.

For liquids and gases some values are here given. They must be multiplied by 10^{-6} in order to give the true value.

FLUIDS

Oxygen (liquid)	+254	Ether	−0·64
Air (liquid)	+180	Alcohol	−0·69
Manganese chloride	+58	Water	−0·73
Chloride of iron	+52	Bisulphide of carbon	−0·8
Chloride of cobalt	+21	Sulphuric acid	−0·83
Sulphate of iron	+16	Mercury	−2·3
Nickel sulphate	+7		
Copper sulphate	+1		

GASES

Oxygen	+0·142	Hydrogen	−0·0005
Air	+0·030	Helium	−0·002
Nitrogen	+0·001	Argon	−0·011

Some interesting points are to be noted here: (1) of liquids, the liquefied gases are by far the most strongly magnetic, which in any case has some connection with the low temperature at which the investigation takes place; compare what is said later. (2) For the rest, naturally the salts of the ferro-magnetic substances are relatively strongly magnetic, being only surpassed by the salts of manganese. (3) No liquid is to any extent strongly diamagnetic. (4) The gases as a whole have exceptionally small values, and in the case of some of them the sign of the value is doubtful (carbon dioxide, hydrogen). It is to be added that the susceptibility κ is proportional to the pressure, from which it follows that the specific magnetism χ is independent of the pressure and therefore is the true characteristic constant. If the above numbers are divided by the density of the gas and therefore multiplied by their specific volume, numbers are obtained ranging from some tens up to over a hundred units.

47. **Physical and Chemical Relations**—And now comes the task of showing how this great mass of quantitative results stands in some sort of relation to the chemical nature and constitution of the substances under consideration! With this task numerous physicists have occupied themselves during recent years, but the results obtained are somewhat conflicting. The relation to the periodic system of the elements, which has already been touched on (page 83), has been set out by Honda and Owen on the basis of very com-

plete material. It is illustrated in Fig. 51. On the axis of abscissæ, about half-way up are the atomic weights, and at the bottom of the figure the so-called "series" according to Mendelejew-Brenner, separated from each other by short cross-hatched lines. The curves fall, as will be seen, into three parts separated by the two specially strong magnetic groups; namely, in the first place, the group Ti, V, Cr, Mn, Fe, Co, Ni; and, in the second place, into the group of the rare-earth elements. These two summits, which, of course, are too high to be shown in the figure, are indicated by $\mathfrak{A}$ and $\mathfrak{A}_2$. The three parts of the curve follow a more or less

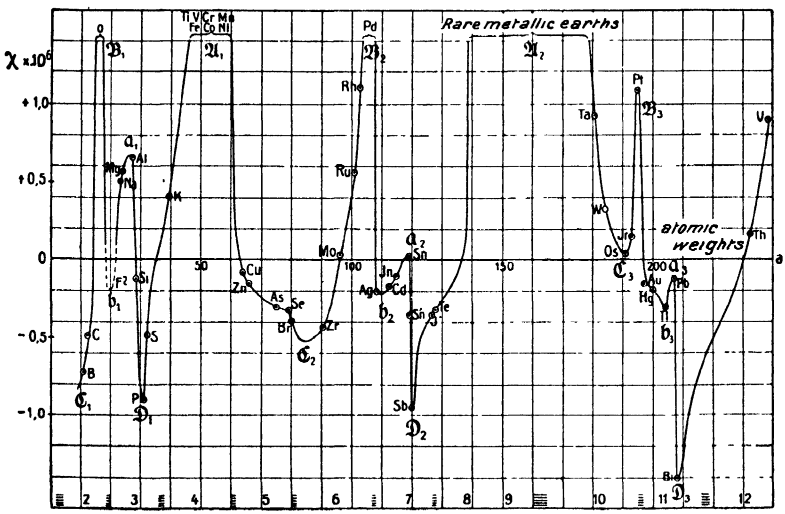

FIG. 51.

similar course, namely, a subsidiary maximum ($\mathfrak{B}_1$, $\mathfrak{B}_2$, $\mathfrak{B}_3$), a flat minimum ($\mathfrak{C}_1$, $\mathfrak{C}_2$, $\mathfrak{C}_3$), a pointed minimum $\mathfrak{D}_1$, $\mathfrak{D}_2$, $\mathfrak{D}_3$), and a secondary minimum (b_1, b_2, b_3). According to Owen there are still two singular points to be added. Elements of the same group and of similar chemical properties often lie at corresponding places, which is a fact from which many inferences may be made.

From the simple substances we may proceed to the compounds, and in turn to the alloys, solutions and combinations. Here indeed are to be found numerous special relations and laws which are valid for quite definite and limited groups of substances; but no very much better understanding of the general nature of the laws of magnetizability has thus been obtained. A few brief indications may therefore be sufficient.

With alloys the additive law seems to hold in special cases only: that is, the magnetizability of the alloy can be determined from the ratio of the parts making up the mixture. In some cases great anomalies present themselves, and indeed very pronounced maxima (and minima) may occur. As an example the graphic representation in Fig. 52 may serve. It relates to the manganese antimony alloys and shows that a pronounced maximum occurs with 32 per cent. manganese, which percentage corresponds to the formation of unsaturated mixed crystals.

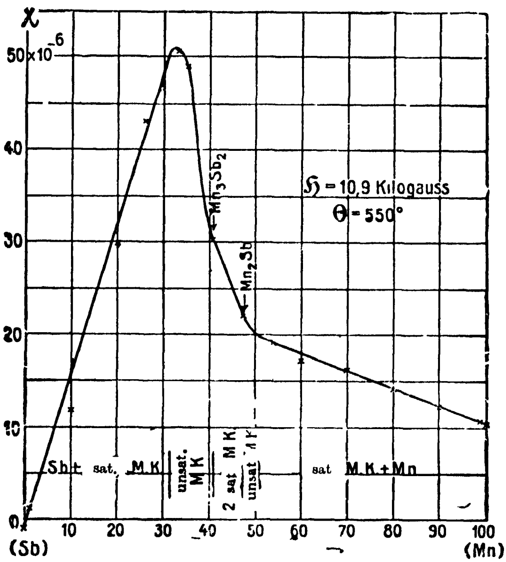

FIG. 52.

Secondly, in regard to solutions: here the question of concentration is one of paramount importance. After long self-contradictory researches designed to clear up the point, G. Wiedemann has proposed the following rule: The specific magnetism of a solution is proportional to the amount by weight of the dissolved magnetic salt. For various solutions this holds where there is no dissociation, alteration of volume, or other special phenomenon, and the specific magnetism is then independent of the solvent. But this rule is still too specialized. Koenigsberger has just shown how it may be generalized and placed on a solid foundation. He arrives at the following equation for the specific magnetism χ ($= \kappa/s$, where s is the specific gravity):

$$\chi = m_0\chi_0 + m_1\chi_1.$$

Here m_0 and m_1 are the parts of the solvent and the dissolved substance in the whole ($m_0 + m_1 = 1$); χ_0 is the specific magnetism of the solvent, and χ_1 a constant whose value is nearly that of the specific magnetism of the solid salt, but is not necessarily equal to it. It will be seen that the first part of the Wiedemann rule is strictly correct only for very strongly magnetic salts, but that for weakly magnetic or diamagnetic materials there will be divergences.

In certain parts of this subject very interesting facts have been established of which at least one example may be given here.

Since water is in itself diamagnetic it must be possible to produce solutions of magnetic salts which are non-magnetic or magnetically inactive, an idea which du Bois has developed for the two following cases:

Chloride of Manganese.		Chloride of Cerium	
	$\kappa 10^6$.	s.	$\kappa 10^6$
0·9992	−0·837	0·9992	−0·837
1·0010	−0·418	1·0529	−0·215
1·0028	−0·127	1·0748	0·000
1·0040	0·000	1·1565	+0·950
1·0054	+0·182	1·2165	+1·596
1·0087	+0·578	1·2697	+2·175
1·0445	+6·819	1·5229	+4·877

In the case of chloride of manganese the indifference point occurs at $s = 1{\cdot}0040$, and in the case of chloride of cerium at $s = 1{\cdot}0748$. Finally for the characteristic comparison of various substances it is of interest in place of κ or χ to introduce two new quantities: the molecular susceptibility κ_m and the atomic susceptibility κ_a, which are defined by the following simple relations (where s = specific gravity, m = the molecular weight, a = the atomic weight):

$$\chi = \frac{\kappa}{s}, \ \kappa_m = \frac{m}{s}\kappa = m\chi, \ \kappa_a = \frac{a}{s}\kappa = a\chi \quad . \quad . \quad (30)$$

One would expect that the atomic susceptibility would be a constant, and this is generally the case, but in certain instances there are divergences, and from this, interesting conclusions can again be drawn in regard to the constitution of the solution under consideration.

The relations in the case of combinations are very complicated. Here also three typical cases are again possible; the value for the combination lies between the single values, or it agrees substantially with that of one of the constituents (this ingredient is therefore decisive), or it lies quite outside; all three cases present themselves. Thus in the halogen combinations of the metals Li, Na, K, Ca, Sr, Ba, it is almost indifferent whether the other element is chlorine, bromine, iodine, or fluorine. Quite generally the influence of the metals, that is (electrolytically expressed), of the kations, is much greater than that of the anions, so that to a first approximation it is only necessary to consider the atomic magnetism of the metal under consideration, but for a closer approximation a correction must be taken into account because of the anions. Weiss has reckoned out the values for a great number of anions and these have been found to agree very closely with experiment. For the metals themselves the following table gives the mean values of the atomic magnetism:

Aluminium	+ 19	Oxygen	+ 1,340
Antimony	− 91	Palladium	+ 565
Lead	− 18	Platinum	+ 268
Cadmium	− 16	Sulphur	− 13
Gold	− 31	Selenium	− 24
Graphite	− 46	Silver	− 11
Copper	− 5·7	Bismuth	− 32
Magnesium	+ 56	Zinc	− 7
Bromium	+ 30	Tin	− 6 5
Mercury	+ 31		

Finally, it is to be noted that the atomicity is of influence on the magnetizability; thus, for example, ferri- and ferro-salts have by no means the same atomic magnetizability.

48. **Remanence and Hysteresis**—The further question now arises, and must be carefully studied, whether in the case of weakly magnetic bodies all these phenomena, which make the magnetization of the ferro-magnetic materials so interesting and diversified, are completely absent — the departure from proportionality between strength of field and

7

magnetization, remanence and hysteresis. It now appears that on the whole these phenomena are actually absent, but it is suggestive that in certain cases they present themselves in those materials which are capable of stronger magnetization. If some physicists have regarded the materials where this is the case as ferro-magnetic (remanence and hysteresis being regarded as indications of ferro-magnetism), it is desirable in the interests of clearness to demonstrate it immediately. Between the strongest of the weakly magnetic substances and the ferro-magnetic substances there is such an immense gap that in this way the nomenclature retains a fixed significance. It must, however, be added that even in weakly magnetizable substances traces of remanence and hysteresis present themselves.

(*c*) MAGNETISM OF CRYSTALS

49. **Crystals**--Crystals also deserve special consideration on account of their magnetic qualities, and that indeed for two reasons, which, if we wished to be paradoxical, we could express by saying that in the first place they behave in a more complicated, and in the second, in a simpler fashion than isotropic bodies. More precisely stated, this means that a sphere of isotropic material behaves in all directions in the same way, in a uniform field it is therefore at rest in all directions. A sphere of crystal, on the contrary, behaves differently in different directions; in a uniform field it will quite definitely place itself so that the axis of greatest susceptibility (or it may be of the smallest negative susceptibility) lies along the direction of the field. It may also place itself with the axis of least susceptibility in this direction, but it is then unstable and at the least disturbance the first condition again presents itself. A sphere of crystal has a different susceptibility in different directions, and all these (just like the corresponding moduli of elasticity) can be graphically represented as radial vectors all drawn from a common centre, the end-points of which will then define an ellipsoid—the ellipsoid of magnetic induction. Its axes are called the axes of principal magnetization and the values of κ corresponding to them are called the principal susceptibilities. In crystals having one axis the surface is an ellipsoid of rotation, and there are only two principal susceptibilities. In regular crystals the surface may even be a sphere and these substances then behave

as regular bodies. But in the majority of crystals the ellipsoid is more or less spherical, and the heterotropy in a magnetic sense is therefore not considerable; thus, for example, spar and quartz have only two very slightly different values of κ, and in topaz (rhombic) the three values are almost equal to one another.

But there are also some ferro-magnetic crystals, and in particular magnetite (magnetic iron ore) and pyrrhotite. The first is only peculiar in the fact that although it belongs to the regular system, nevertheless it does not behave as an isotrope: in the cubic faces the direction of the magnetization only coincides with that of the field for the sides and diagonals of the squares; in all other intermediate directions

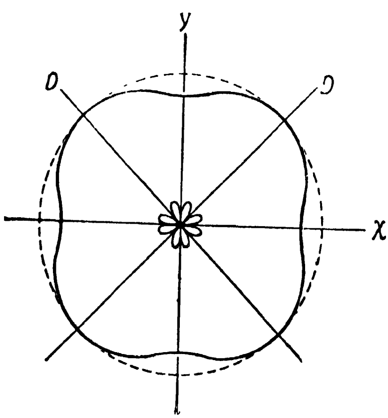

FIG. 53.

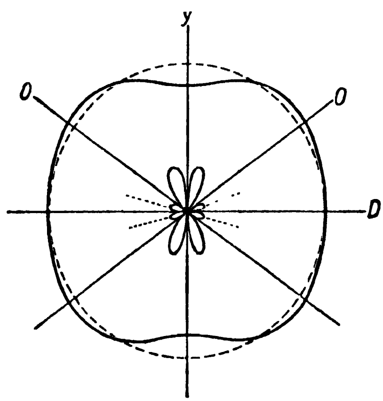

FIG. 54.

the two make an angle with each other so that one can distinguish between two components of the magnetization, one parallel and the other normal to the field. The last form two perfect waves from 0° to 180° with four null points (two axes and two diagonals) and change their sign at each of the null points. In the dodecahedrals the waves of 75°, corresponding with the axes which are at an oblique angle to each other, alternate with others of 105°. In Figs. 53 and 54 the two components are represented, the parallel on the outside, the normal within. In the octohedrals the normal component of the magnetization is so small and runs so irregularly that Weiss came to the conclusion that with an exact orientation of the face there would be complete isotropism, a conclusion which has been substantiated. For the rest it is clear that we can distinguish between a normal and an irregular type and that for each of these types different relations apply. For these and many other discoveries connected therewith we have to thank the indefatigable labours of Pierre Weiss and his scholars. All possible explanations but this proved unworkable: the crystal is built up in a symmetry which is perfectly regular in itself (a simple cubical lattice-work in space) but traversed by an immense number of fissures parallel to the octohedral surfaces, fissures which are microscopically small but large in comparison with the elements of the cubical lattice-work. The four systems of fissures or cleavages produce a demagnetizing force, and everything depends on whether all the four are equally developed or different. Consider Fig. 55: at the left a regular, and in

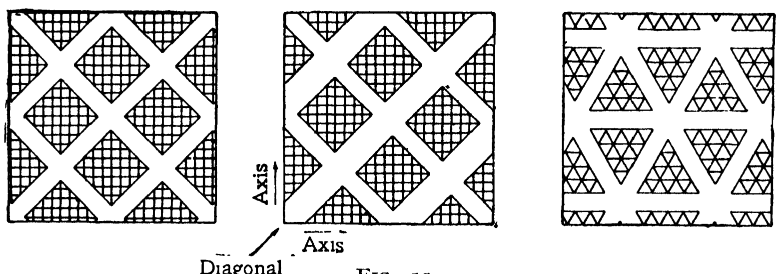

FIG. 55.

the middle, an irregular cubic, and on the right an irregular octohedral has been shown. In pyrrhotite, on the other hand, all observations may be understood by assuming that the substance is made up of rhombic elements which are parallel to the principal axes and are orientated with the subsidiary axes in three positions lying at 120° to one another.

In short, as will have been seen from this brief survey, we have here to deal with extremely complicated relations. Where then, on the other hand, does the great simplicity that has been mentioned reside? That crystals are subject to a simpler law than amorphous hard bodies corresponds perfectly with our present ideas (and the microscopic discoveries which support them), according to which every hard, simple body is a crystal, the so-called isotropic bodies being only quasi-isotropic and in truth consisting of crystalline particles held together at random. This should, however, manifest itself in their magnetic behaviour, and to a certain extent this does happen. But we can only take up this argument when we have considered modern theories in their totality. It should, however, be mentioned here that magnetic iron ore is distinguished from artificial iron by the fact that it possesses natural or " voluntary " magnetism.

V

MECHANICAL AND THERMAL RELATIONS

50. **Introductory**—Nothing is more helpful towards the understanding of a complex of phenomena than the study of the mutual relations, wherever they exist (and they nearly always do exist) between this particular complex and others. In regard to magnetism, such relations are specially abundant, and it may indeed be said that there is scarcely any department of chemistry or physics into which magnetism does not enter. We will consider these various relations in order; first the mechanical, then the thermal, then the optical, and lastly the most important of all, i.e. those relating to electricity.

It is appropriate to begin with the mechanical, and here again with the first and most actual twofold characteristic of matter, its inertia and heaviness, which is manifested by its mass and weight. In this we have one of the few exceptions to what has just been said: the relations of magnetism to inertia and gravitation in spite of all the efforts that have been made to establish them have not been discovered. A bar of iron is not made heavier as a consequence of magnetization; and if the American physicist Bauer found some slight traces of difference of weight in magnetized iron bars according to their orientation in the gravitational field of the earth the matter still stands very much in need of confirmation. Gravitation and magnetism must for the time being be regarded as separate, and only through the application of the principle of relativity to the electro-magnetic theory of gravitation is it permissible to anticipate that the gulf will be bridged.

51. **Relation to Longitudinal Tension**—Extremely interesting, on the contrary, are the relations between magnetism and elasticity, and that in both ways, for elastic changes are accompanied by magnetic, and magnetic changes are accompanied by elastic effects. This is true in various connections;

but here it will be sufficient to consider the two principal cases, longitudinal tension and torsion.

As regards the effects of longitudinal tension on magnetism, the phenomenon is simplest in the case of nickel. Here magnetism is diminished by longitudinal tension and that in proportion to the tension applied. This is true both of temporary and also of permanent magnetism, and of the last in a still higher degree. Fig. 56, in which the three quantities $\mathfrak{J}$, $\mathfrak{J}'$ (Remanence) and κ are shown as functions of the strength of the field $\mathfrak{H}$ for three cases—that in which there is no longitudinal tension and those in which there is a tension of 2 kg. and 12 kg. respectively— shows this better than any series of figures, and it also shows that the departure of κ from constancy and therefore the upward bending of the curve becomes progressively smaller and smaller, and that as the load is increased the point of inflexion moves progressively to the right ; that is, the maximum value of κ is attained at a progressively greater value of the force. Conversely, longitudinal pressure on nickel has the effect of increasing the magnetism, the maximum value of κ is reached sooner and the rate of increase and decrease of κ is greater. As the strength of the field

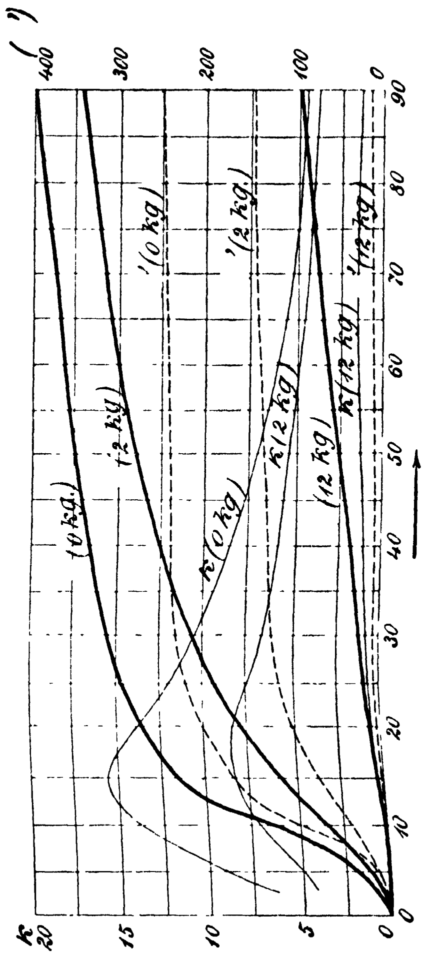

Fig. 56

is increased it is still more clearly shown that the curves draw together; and it is also evident that at saturation the magnetism cannot be further increased by longitudinal pressure. The curves for high loads rise steeply, and quickly bend over to a horizontal direction. The condition of saturation therefore occurs very suddenly. Instead of the curves of magnetization which we have already considered, curves can also be obtained, by a slight change in the method of experiment, which show the magnetism (the ordinate) as a function of the load (the abscissæ) for various values of the strength of the field. With longitudinal tension they fall from left above to right below; in the case of longitudinal pressure they rise from the left below to the right above.

In the case of iron the behaviour is more complicated. Whether longitudinal tension increases or diminishes the magnetism depends upon the strength of the original magnetization: weak magnetism is increased, strong magnetism is diminished, but this only applies up to a certain limit in the case of tensional loads. When the load is still further increased, even if the original magnetism is weak, it is diminished. The above reversal of the phenomenon at the change-over from weak to strong magnetization is called, after its discoverer, the "Villari effect," and the point where it takes place the "Villari critical point." It can be defined in various ways according as it is regarded as a critical tension or as a critical magnetization; in the first case it is a function of the magnetization, and in the second a function of the pressure. This is more evident from the two parts of Fig. 57. For the Villari effect has the result that the magnetization curves cross for various values of the tensional load in the case of iron, as is shown by the figure to the left. In the lower part of the figure is the curve for the heavier loading; next above it, that for the lighter; and finally that for the unloaded wire, highest of all. The crossing points represent the critical magnetization as a function of the load. In the right-hand half of the figure this relation is illustrated differently for the above-mentioned curves which for various strengths of field represent the magnetization as a function of the load. The lowest of these curves, which corresponds to the least strength of field, rises all the time, the uppermost curve falls all the time; the curve lying between first rises and then falls. For each strength of field there is in the first place a definite load for which the magnetization is a maximum, and this value of the load falls more and more as the strength of the field increases. When the curves sink again they will, under certain circumstances, regain their original value; this is the critical load as a function of the magnetization. There are still two other points to be noticed. In the first place there is perhaps even in nickel (and in cobalt as well) a critical point (Heydweiller and others), but the matter has not yet

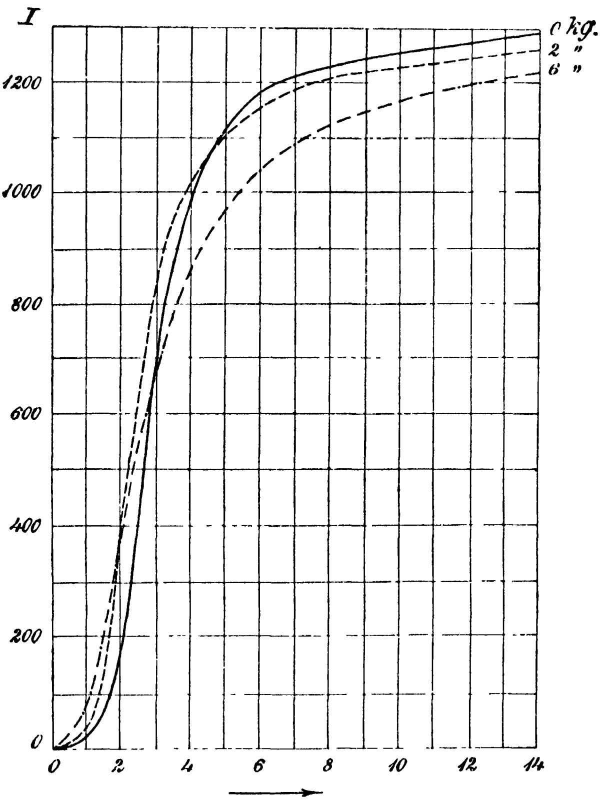

FIG. 57.

been completely cleared up. In the second place, the above phenomena are not inconsiderably modified by the first experiment to which the body is subjected, and even in its permanent condition, in virtue of something that may be called magnetic elastic hysteresis; that is, it depends on whether in the cyclical loading and unloading we are on the outward or the backward part of the curve. But this kind of hysteresis is more complicated than the purely magnetic, for the loops cross each other as will be seen from Fig. 58.

Conversely, by magnetization of a bar a change is brought about in its length. Here again the phenomenon is simplest in the case of nickel, which under all circumstances is shortened, while iron in weak fields is lengthened, and shortened only in stronger fields. Cobalt behaves in various ways according to its method of preparation. Nickel-iron alloys are always lengthened; these facts are set out in Fig. 59. Of special, even if only of historical, interest, is the periodical lengthening and shortening of an iron bar through cyclical magnetization. On this phenomenon Philipp Reis of Frankfurt based the first telephone, which of course was not adapted to practical use. With the alteration of length there is also an alteration of volume that goes along with it, and the whole of these interrelated phenomena constitute magneto-striction. A formal but elegant theory has been worked out (completely in parallel with the sister phenomena of electro-striction) and this has given an impulse to further research.

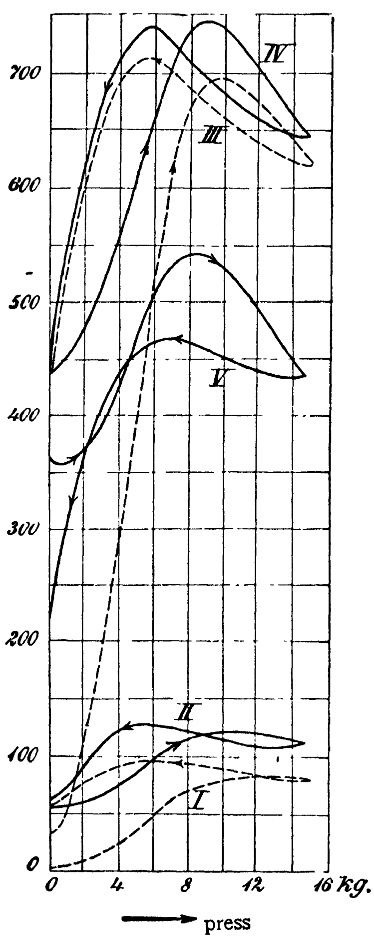

Fig. 58.

52. **Relations to Torsion**. The second case is that of torsion. If we apply, in the first case, a small amount of torsion, well within the elastic limit, we find that in iron the magnetism is at first increased but afterwards diminished to such an extent that in the cyclical condition the twisted bar is less magnetic than one free from torsion. The phenomenon

is most pronounced in soft iron. In hard iron it is weaker, and in steel least of all. In nickel, on the contrary, torsion brings about an increase of the magnetism, at least in moderate fields; in stronger fields an inversion of the phenomenon takes place, indeed—according to Zehnder—an inversion of the magnetism itself. The behaviour of cobalt is again different.

Hysteresis also occurs as a result of the effect of the torsion on the magnetism, that is, the curves of the magnetism with increasing torsion and diminishing torsion are moved

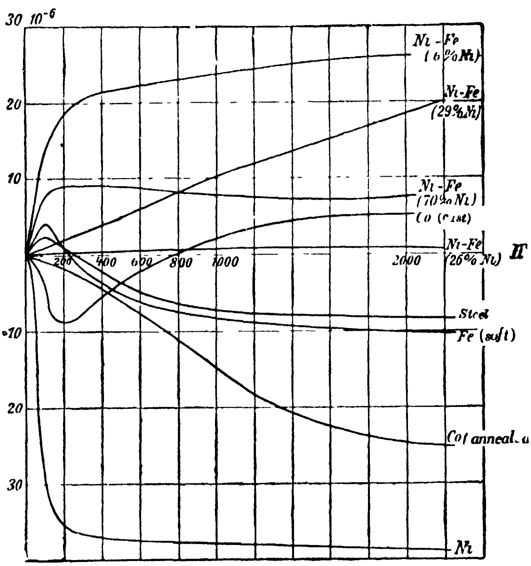

FIG. 59.

sideways with regard to one another, as will be seen in Fig. 60 (where *a* relates to iron and *b* to nickel). The base-line does not correspond to the value $\mathfrak{J} = 0$, but to a very considerable value of $\mathfrak{J}$.

The effect of torsion on magnetism is also shown when the latter is generated not by a coil surrounding the bar but by a current flowing through the bar itself, so that the magnetism is of the circular type. Here the effect is specially interesting; Wiedemann has shown that such a bar when it is twisted, after or during the passage of the current, becomes magnetic. This means that the bar which previously was non-magnetic has become so as a result of torsion ; in other words, the magnetism has changed of itself from the purely circular form to the longitudinal. The bar thus obtains at the end where the current enters a south pole if it has been twisted to the right, and conversely. In nickel the magnetism set up is in every case the contrary.

But how as regards the production of torsion through magnetization ? The following facts have been made out. As a result of purely longitudinal, purely transversal, or purely circular magnetization, a previously untwisted cylinder undergoes no torsion, and it is easy to see why this is so. Where, on the contrary, there is a longitudinal and a circular

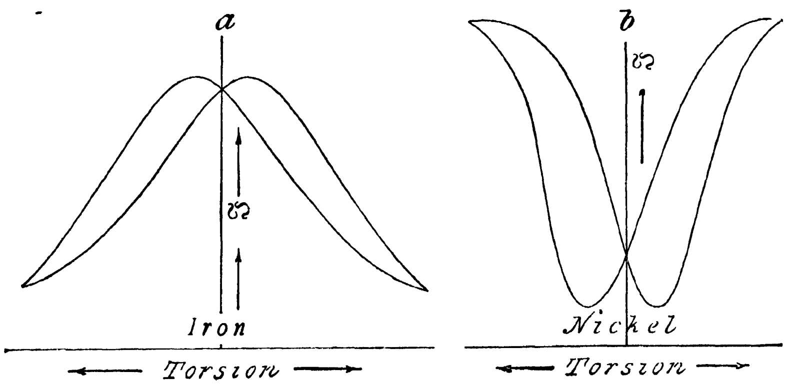

FIG. 60.

field combined, torsion must be set up. The simplest case is that in which the magnetization represents a helix about the cylinder with an angle of 45° and in a right-handed screw direction. The sense of the torsion produced is obtained by considering that in weak fields dilation takes place in iron in the direction of magnetization (in nickel it is contraction that occurs). If, therefore, the field is produced in such a way that through the body when it is hanging in the magnetizing coil a second current is sent from above downwards, then nickel is twisted in the same direction as (but iron in the opposite direction to) that in which the current flows through the magnetizing coil. If the magnetized bar is already twisted, then a longitudinal or a circular field alone will bring about a change of the angle of torsion.

53. **Other Mechanical Relations**—The other mechanical

relations can only be touched on briefly: they relate to the moduli of extension and torsion and to internal friction and capillarity and so forth. Thus, for example, the dropping of a paramagnetic fluid is accelerated in a magnetic field, that of a diamagnetic fluid retarded, and under some circumstances completely stopped. Further, if the narrow upright tube in the apparatus, shown diagrammatically in Fig. 61, is brought into a cross magnetic field the level of the liquid falls (or rises) from z to z', and the change of pressure corresponding to this difference of level is proportional to the susceptibility of the liquid and the square of the strength of the field. A strong field must of course be used in order to obtain any well-marked change of level: water, for instance, in the strongest field that can be produced falls only about a millimetre. This effect is not one of capillarity but of magnetic hydrostatic pressure.

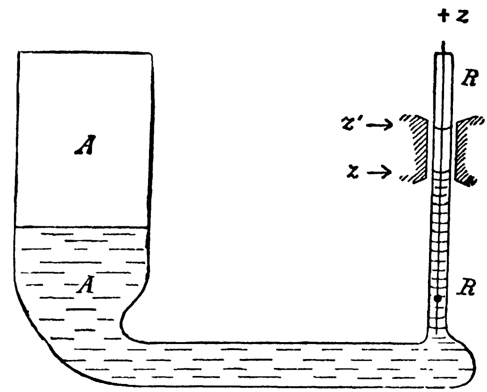

FIG. 61.

Shocks, blows and, in a quite special degree, regular periodic effects such as vibrations, have an extremely interesting and manifold influence on magnetic behaviour. The most important effects are the following: the temporary magnetism (especially in weak fields) is favourably, and the remanent magnetism unfavourably, influenced; the hysteresis is greatly reduced and in some cases by suitable procedure can even be totally eliminated. Compare Fig. 62 (from Maurain), in which S is the ordinary loop; Σ, however, is the new curve with the two branches coinciding in the case in which the vibrations are started when the material is in the condition A_1 or A_2 and is strongly maintained.

Finally, with reference to the processes of crystallization, it is to be added that in a magnetic field crystallization itself often takes place differently from what it otherwise would do: thus needles may be formed instead of plates or leaves.

54. **Heat of Magnetization**—We now come to the thermal relations. Like the mechanical, these also present two different aspects: magnetization generates heat, and heat, on the other hand, influences magnetization.

As for the first phenomenon we need only go back to what has already been said and look at it from the thermal side. The heat which here comes under consideration is called the heat of magnetization; it represents a dissipation of energy (except where the production of heat is the object to be achieved, a case which does not often arise in practice) and is of the same character as all the other manifestations of heat energy, such as the heat of mixture, solution, chemical combination, heat due to the passage of the electric current, and so forth.

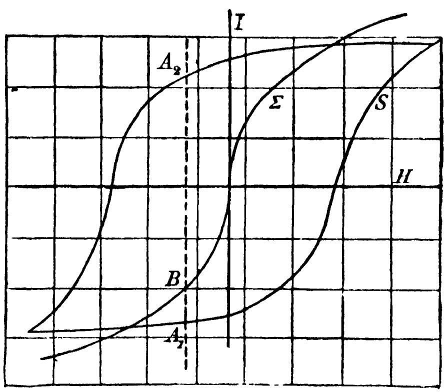

FIG. 62.

The first experiments in connection with the heat of magnetization were undertaken by Joule as part of his great work on the determination of the mechanical equivalent of heat, and for the purpose he used a bundle of iron lamellæ which were rotated over the poles of a magnet. He found, as was expected, that the heat generated was proportional to the square of the magnetism. Later, numerous investigators have devoted themselves to the problem, and two fundamentally different methods have been employed: the method of work with slow cycles, and the calorific method with quick cycles. Specially valuable are the measurements of Warburg and Hönig by both methods (A and B). For the purposes of their experiments bundles of quite thin wires, bundles of thin iron sheet, bars of various size were employed. The cycles were sometimes unilateral, sometimes bilateral (index 1 and 2); they were carried to the point of inflexion and the corresponding susceptibilities were:

Bundle I	Bundle II	Bar I	Bar II	Bundle III
21·7	20·8	12·9	7·2	20·1

In the following table the amount of heat developed for one cycle is given in millionths of a gramme-calorie, and in addition the ratios of these values:

Material	A_1	B_1	A_2	B_2
Bundle I .	9·2	5·3	28·0	17·6
Bundle II .	4·9	5·2	17·6	17·4
Bar I . . .	5·1	13·3	18·9	46·0
Bar II . .	0·4	5·6	1·6	10·1
Bundle III	3·7	2·4	12·0	7·8

Material	A_2 A_1	B_2 B_1	B_1 A_1	B_2 A_2
Bundle I	3·0	3·3	0·6	0·6
Bundle II	3·6	3·4	1·1	1·0
Bar I . .	3·7	3·5	2·6	2·4
Bar II . .	4·0	1·8	14·0	6·3
Bundle III .	3·2	3·3	0·7	0·7

Unfortunately these numbers do not give a clear idea of the phenomena because many other effects are involved (remanence, heating due to eddy currents, heating due to thermal changes in the susceptibility and so forth). In order to obtain an idea of the rise of temperature brought about by the heating due to magnetization, the energy consumed needs to be divided by the density of the iron (7·7) and its specific heat (0·11), and then we have

$$\delta t = 2{\cdot}81 \times 10^{-8} \int \mathfrak{H} d\mathfrak{J}. \quad . \quad . \quad . \quad . \quad (31)$$

The value of the integral in the case of soft annealed iron which has been taken through a complete cycle of the process, that is, when the iron has nearly reached both its positive and its negative condition of saturation, is of the order of 10,000 ergs, and the temperature for 4,000 revolutions of the machine would rise about 1° C. and therefore, if there were about 1,000 turns a minute and the heat generated were not dissipated, it would rise about 15° C. per hour. Actually, the generation of heat on account of eddy currents in the armature is very much greater, or at least was so in the older machines. In modern ones the heat due to the eddy currents produced by electro-magnetic action is reduced to a moderate amount by subdividing the iron of the armature as far as possible. In order to minimize the real magnetic heating the cyclic process is not carried to its full extreme, that is, the iron is only brought to a moderate degree of magnetization. That this is a more advantageous condition will be already sufficiently clear from the form of the magnetization curve, but at the same time it follows from the shape of these curves that the dissipation of energy increases not only absolutely but also relatively if the magnetization is carried between wide limits. The following table will make this clear. $\mathfrak{H}$ is the force, $\mathfrak{B}$ the magnetic induction, and V the energy dissipation. The energy dissipation divided by the induction and the temperature increase per cycle are also given :

$\mathfrak{H}$.	$\mathfrak{B}$.	V.	V : $\mathfrak{B}$.	δT.
1·50	1,974	410	0·21	0·000012°
1·95	3,830	1,160	0·30	0·000033°
2·56	5,950	2,190	0·37	0·000062°
3·01	7,180	2,940	0·41	0·000083°
3·76	8,790	3,990	0·45	0·000112°
4·96	10,590	5,560	0·53	0·000156°
6·62	11,480	6,160	0·54	0·000173°
7·04	11,960	6,590	0·55	0·000185°
26·5	13,720	8,690	0·63	0·000244°
75·2	15,560	10,040	0·65	0·000282°

In Fig. 63 the temperature rises according to Tanakadaté are shown, which occur during one cycle of the process in iron and steel when carried out between the limits indicated by the abscissæ. That the curve for steel at the beginning is lower than the curve for iron is obviously a consequence of the smaller susceptibility of steel. The steel curve would lie above the iron curve throughout its entire length if $\mathfrak{J}$ instead of $\mathfrak{H}$ had been chosen for the abscissæ.

55. **Influence of Temperature on Weakly Magnetic Substances**.—And now let us consider the converse case—the influence of temperature on magnetism. It is a matter of common knowledge that temperature influences all the physical properties of a body, sometimes only slightly, but frequently in a very considerable degree. It influences the volume, the elasticity, the specific heat, the conductibility for heat or electricity, the refractibility of light, and also the magnetic qualities. But its effects in this case are specially interesting in the weakly magnetic substances with which we

will begin. Here a very striking contrast between para- and diamagnetic substances presents itself. In diamagnetic substances the specific magnetism is approximately constant; in paramagnetic substances it is inversely proportional to the absolute temperature. Therefore we have

$$\text{diamagnetic} \quad \chi = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad (32)$$
$$\text{paramagnetic} \quad \chi \times T = \text{constant or } \chi = C/T \quad . \quad (33)$$

These formulæ are commonly known as Curie's laws and the magnitude C as Curie's constant. Nevertheless, it has been shown that there are many divergences from these laws, and that numerous substances do not even approximately conform to them. Thus the magnetism of bismuth, although this substance, as we know, is very strongly diamagnetic, as the temperature rises diminishes to about two-thirds its normal value, and as the temperature falls increases by about one-sixth of its original value at – 182° C. Above all, there are substances whose susceptibility remains constant with change of temperature, some in which it diminishes, and some in which it increases. The same substance, moreover, will in some instances behave differently in different temperature ranges, and it is difficult to lay down any definite rule. In order to give the reader some idea of these varied relations, we reproduce in Fig. 64 a series of curves from the results of Honda and Owen's exhaustive work on the subject. As will be seen the curves run sometimes horizontal, sometimes upwards, sometimes downwards, and some have even definite bends at a particular place. Therefore when, as was formerly usually done, a temperature coefficient of the specific magnetism was given (relative change per degree), this had only a limited significance; mostly it is negative and amounts

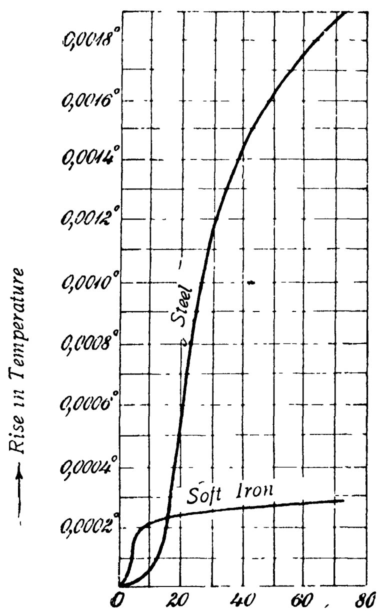

FIG. 63.

to a few thousandths. In conclusion it may be said that on the whole a rise of temperature is unfavourable to the magnetism of weakly magnetic substances.

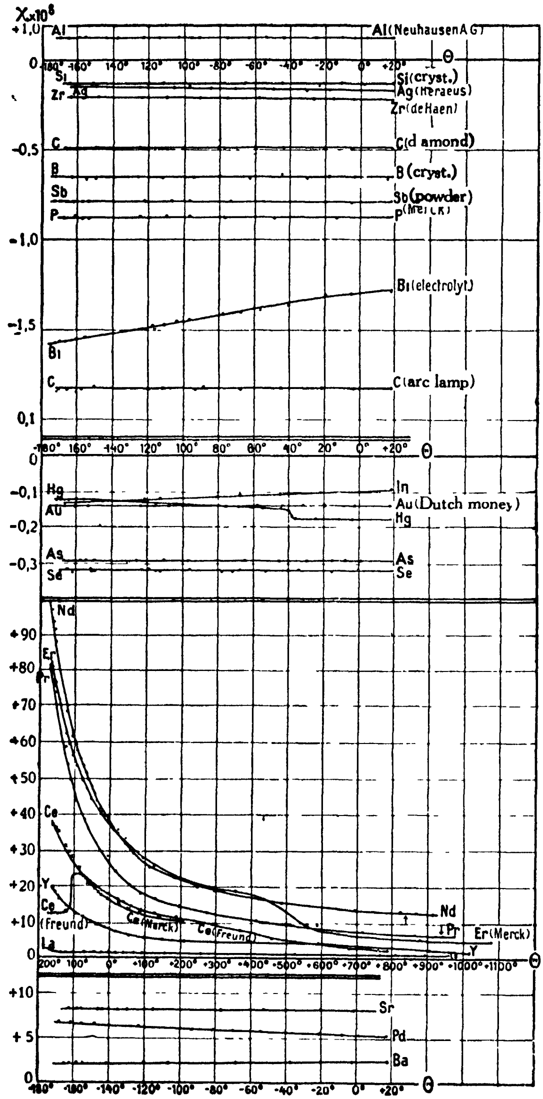

FIG. 64.

56. **Influence of Temperature on Ferromagnetic Substances**—On the other hand, as regards ferromagnetic substances we have in the first place to deal with the influence of temperature on temporary magnetism. In this two different methods may be followed for the purpose of showing the behaviour of the function; we can either for a given strength of field represent the magnetization as a function of the temperature, and repeat the process for all other different strengths of field; or we can represent the magnetization, as we have previously done, as a function of the strength of the field and obtain the magnetization curves for a definite temperature and repeat the process for all

other temperatures. In the first case we obtain the following result : in a weak field the magnetization at first increases to a maximum and then declines ; in a strong field it declines from the beginning. In weak fields the resulting increase is more considerable and the decline which follows the more sudden the weaker the field ; the stronger the field the more rounded does the form of the curve become, and with a given strength of field the ascending branch disappears altogether and it then consists of a branch which at first declines slowly and then more quickly and at last descends very abruptly. The strength of field for which the ascending branch of the temperature curve vanishes can be regarded as the critical strength of field. It is therefore that strength of field below which

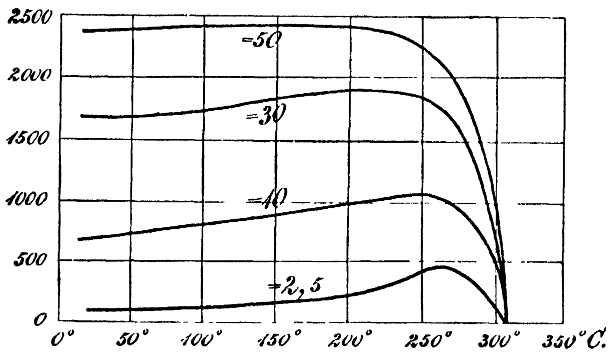

FIG. 65.

the magnetism for higher temperatures is greater and above which for higher temperatures it is smaller than for normal temperatures. Nevertheless, this contrast only holds up to a definite temperature, that, in fact, for which the temperature has its maximum, for beyond that temperature the magnetism for any field strength is smaller for a higher temperature than for the normal. This temperature can correspondingly be called the critical temperature of magnetization. Contrasted with it is another temperature : that at which the curve as it sinks ever more and more steeply cuts the zero axis, and where, therefore, the magnetism vanishes altogether and the material changes from a ferro-magnetic to a non-magnetic or, more correctly speaking, to a paramagnetic condition. This point is called the magnetic conversion-point. The critical point and the conversion-point lie, moreover, in spite of the contrast which they represent, often very close to one another on account of the rapid falling off of the curve.

So far as regards temperature curves, of which Fig. 65 gives an example in the case of nickel. In the other method of representation we get a series of magnetization curves of a shape with which we are already familiar, each of them relating to one particular temperature, and from what has been said already it will be understood that they generally cross one another in such a way that a curve which at first lies below another, later rises above it, as will be seen from Fig. 66, where the abscissæ represent the strength of the field, the ordinates the magnetic induction, and which all relate to forged iron at four different temperatures, namely 10° C., 670° C., 742° C., and 771° C. It is also seen from the figures that the conversion temperature of nickel is about 310° C., but that for iron it certainly lies on the other side of 710° C., for at that temperature a relatively considerable amount of magnetism is still present.

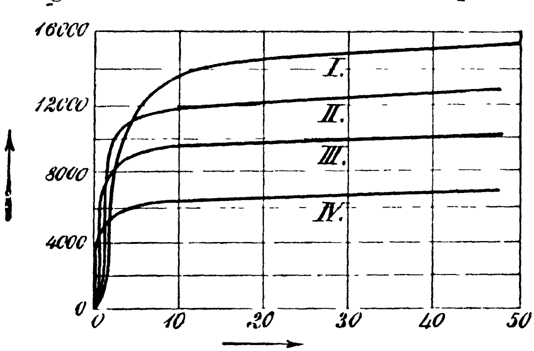

FIG. 66

What has been said does not fully describe the phenomenon. The magnetism of the ferro-magnetic substances at the conversion-point is not dead figuratively speaking, but only apparently dead; a certain weak but latent life is still present and at a certain still higher temperature it suddenly wakes up again. Compare Fig. 67, which is reproduced from the researches of Curie and which shows the temperature curves of iron for six various strengths of field for a temperature of 600° C. and upwards, but which for the sake of clearness gives only the smallest and the greatest values and represents the larger values of the field on a modified scale in which the ordinates are multiplied 1, 10, 100, 1,000 and 5,000-fold respectively. It will be seen that at 800° the magnetism has almost completely vanished. After that point it becomes still weaker, but at 1,270° C. it suddenly springs up again and then dies down anew. Therefore there is in any

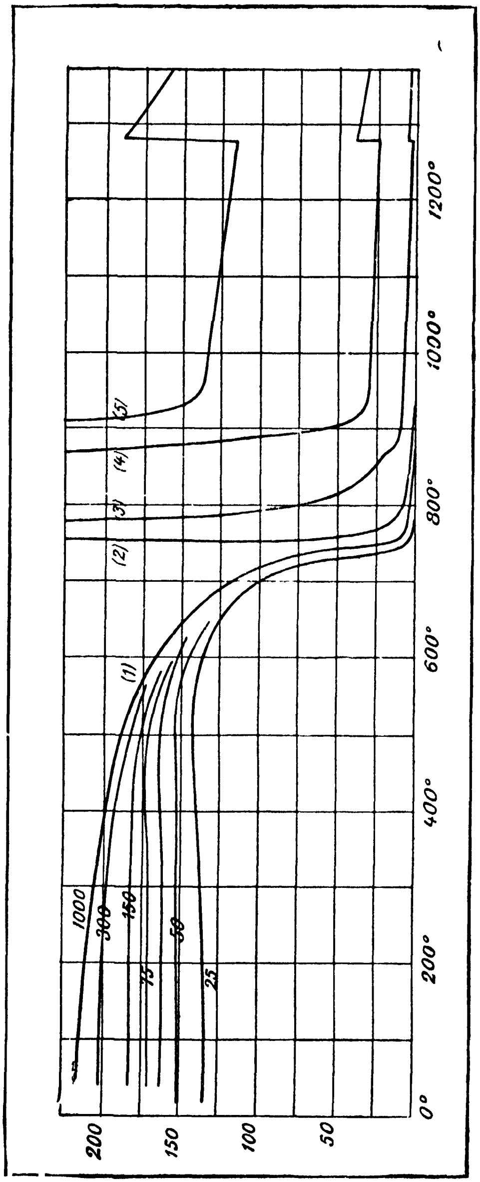

Fig. 67.

case a second inversion-point and a behaviour which is suggestive of complicated internal relations both of structure and the modifications which it may undergo. As we have seen in the case of iron and steel, we have to do not with simple bodies but with a more or less intimate mixture of various components. An illustration of this has already been given in Fig. 42, and all that is now required is to extend this method of representation to a larger temperature range. What we then obtain is schematically shown in Fig. 68. Above the line GOSE lies γ-iron, which at 1,105 C°. holds about 1·7 per cent. of carbon in solution; but at 900° C. only about 0·9 per cent. (eutectoidal alloy); it is magnetizable to only a very slight extent. In the triangle GOM β-iron exists, poor in carbon and non-magnetic. At 760° C. it changes into the highly magnetizable α-iron, which as a structural ingredient is designated ferrite. At 680° C. finally it breaks up into α-iron and iron carbide (Fe_3C) alternating in thin layers. The three points named are the three conversion-points for cooling: Ar_3, Ar_2, Ar_1; for heating the three points Ac_1, Ac_2, Ac_3 correspond to them. Their position for different carbon contents is different. In the case of 0·9 per cent. there is only one such point (S) and above it again three, so that we finally obtain three regions, that of austenite above, cementite (again with Fe_3C, but in this case granular or compact) in the middle, and perlite below, the last of course being ferro-magnetic again.

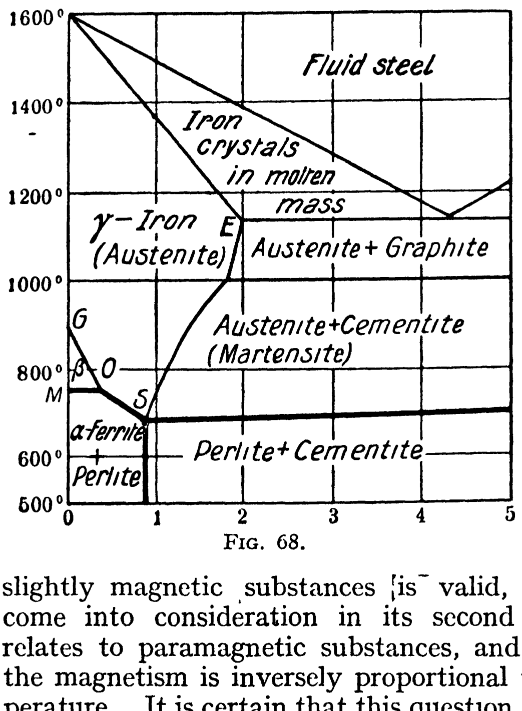

FIG. 68.

57. **Further on the Same Subject**—Leaving this aside we now naturally ask what are the laws which, in the case of ferro-magnetic substances, determine the relation between magnetism and temperature, and especially whether Curie's law for slightly magnetic substances is valid, where it can only come into consideration in its second form in which it relates to paramagnetic substances, and according to which the magnetism is inversely proportional to the absolute temperature. It is certain that this question can now be answered in the negative, for the relation graphically represented would then result in the temperature curve being one branch of an hyperbola, while the actual curves are quite different. These curves do not fall rapidly at the beginning and then more slowly, but slower at the beginning (if indeed they do not even rise slightly for a short distance) and then more and more rapidly. As in the magnetization curves, so also here in the temperature curves, the idea has had to be abandoned of arriving at any simple formula, at least for the whole of the curve up to the conversion-point. Then, however, and when

that point is past, a formula has been found, which again is due to Curie. This indeed is not so strange as it might seem, since only weakly magnetizable substances are under consideration; on the other side of the conversion-point, even ferro-magnetic substances are only slightly magnetizable. One is even a little inclined to go further and to assume that the simple law is valid from that point, and therefore to regard the product χT as constant. But a very slight consideration of numerous experimental results shows that this is not correct, and indeed that it cannot be correct. In other words, the law must be modified, for the starting-point in this region is not the absolute zero of temperature, but the conversion-point itself, and the proper way to express temperature is not from the absolute zero, but as temperature excess above the conversion-point Θ. We thus arrive at the new form of the Curie law for ferro-magnetic substances above the conversion-point:

$$\chi = \frac{C}{T - \Theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

and here, again, C is the Curie constant. This law, however, is valid only up to a certain temperature, somewhere about 930° C.; above that further changes take place into which we cannot enter here.

That in the case of the ferro-magnetic substances still less than in the case of the weakly magnetizable substances is it possible simply to speak of a temperature coefficient of magnetism will be clear from what has been said. It is, however, desirable for practical purposes to have some idea of its value at least within certain definite limits. Between 0° and 100° C.,

Baur (Iron).		Ewing (Iron).			du Bois.	
$\mathfrak{H}$.	ϵ.	$\mathfrak{H}$.	ϵ (Soft).	ϵ (Hardened)	$\mathfrak{H}$.	ϵ. Iron.
0·81	+ 0·0019	2	+ 0·0006	+ 0·0025	500	− 0·00010
1·61	0·0028	4	0·0004	0·0018	1,000	− 0·00015
2·02	0·0018	6	0·0003	0·0017	Steel	
4·85	0·0024	8	0·0002	0·0014	500	− 0·00010
8·07	0·0010	10	0·0001	0·0013	1,000	− 0·00020
16·11	0·0008	12	+ 0·0000	0·0012	3,750	− 0·00025
24·11	0·0002	14	− 0·0000	0·0010	Cobalt.	
32·02	+ 0·0000	20	− 0·0001	0·0005	8,000	− 0·00035
39·84	− 0·0000	30	− 0·0002	0·0002	Nickel	
62·47	− 0·0001	40	—	0·0001		
		50	—	0·0000	12,000	− 0·00135

for example, the values shown in the preceding page may be quoted ($\mathfrak{H}$ is the strength of the field and ε the temperature coefficient):

The temperature coefficient is therefore positive for small values of the strength of the field, and diminishes as the field increases, and is negative in intense fields. In soft iron it is zero for $\mathfrak{H} = 10$ to 15, in harder irons $\mathfrak{H} = 40$ to 50. For weak fields in the neighbourhood of $\mathfrak{H} = 1$, and between 0° C. and 100° C., ε can be taken as about + 0·001. In hardened iron ε = about + 0·002 to 0·003.

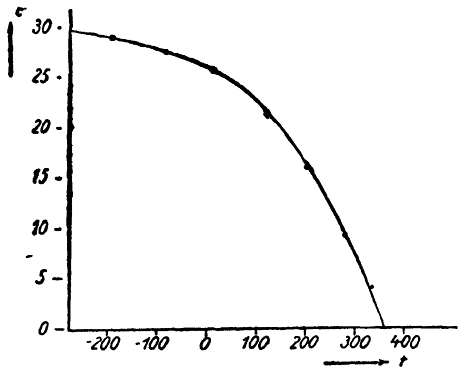

FIG 69

58. **Temperature and Hysteresis** The behaviour of remanence, coercivity, and hysteresis at various temperatures is also very interesting, and in general it may be said that they decrease as the temperature increases, so that they have their maximum values for low temperatures and their minimum for high. Fig. 69, which relates to nickel, and Fig. 70, which relates to iron, may serve as an illustration. The slope of the former is quite regular, with the usual tendency to fall more and more quickly, and according to Gauss can be represented by a comparatively simple formula. The curves for iron, on the contrary, show an irregularity be-

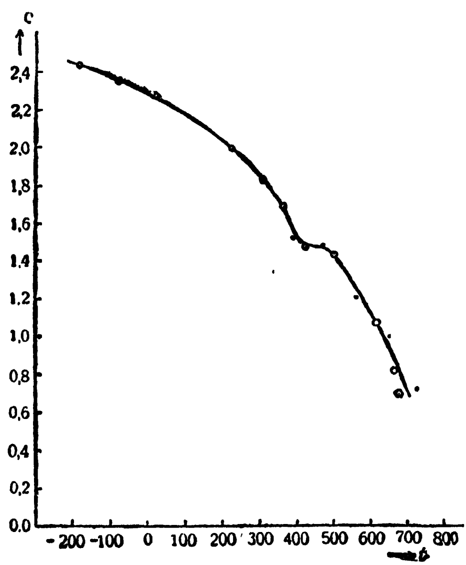

FIG 70.

tween 400° C. and 500° C., and two formulæ have to be combined.

In practice it is of especial importance to be able to specify what may be called the temperature coefficient of permanent steel magnets. It is obviously always negative, varies between the limits 0·0007 and 0·0023 and for hard steel can be taken as approximately equal to one-thousandth. In order to get its normal value it must be carried a few times through a cycle of heating and cooling, until a constant condition is reached. Further, in the case of bars the temperature coefficient is greater the shorter the bar, and the product of

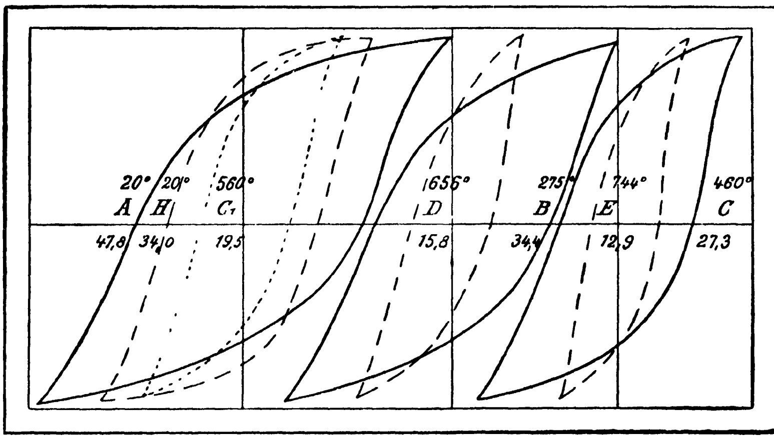

FIG. 71.

the two magnitudes, according to the measurements both of Klemenčič and Loomis, is constant.

The hysteresis loops are finally repeated in Fig. 71 according to the measurements of Kunz, and in order to save space are given as if they were represented about the same initial point. For each of the seven loops the area in square centimetres and the temperature is added, from which it will be seen that A (20°) is the greatest, E (744°) the least, and further that H, although it corresponds to the same temperature as A, is substantially smaller and narrower. The original value, therefore, is not regained after recooling. These curves must, however, be properly understood in connection with the experiments on which they are based. As will be seen, it was provided that the magnetization should in all cases be taken within the same limits, which, in consequence of the falling-off at high temperatures, was only attained by means of an ever greater and greater strength of field. If, on the other hand, the same strength of field is always used, for example, ± 50, we obtain as in Fig. 72, which is after Harrison, not only smaller and smaller, but also flatter and flatter loops.

Curve H in Fig. 71 is, as has been said, substantially narrower than curve A, although they both correspond to the same temperature, but it has been obtained not before but

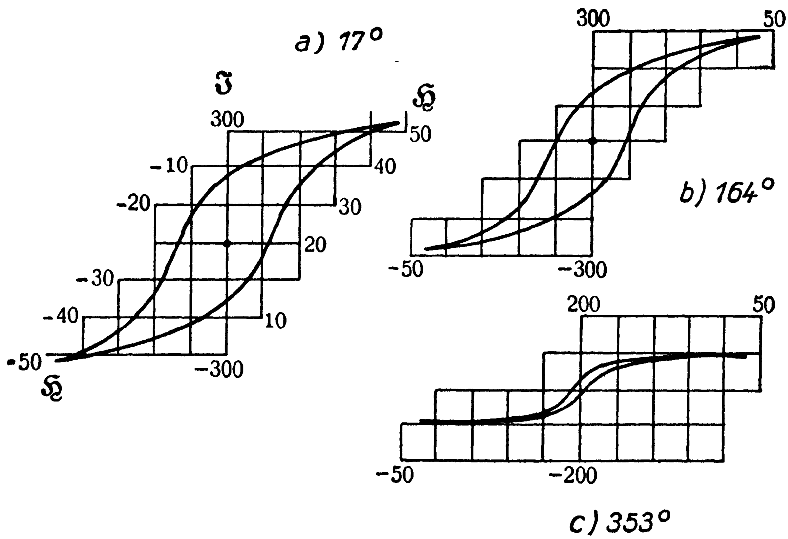

FIG. 72.

after the heating treatment. These phenomena can be regarded as "temperature hysteresis," and more recently they have been studied very extensively and with noteworthy results by various investigators. Here also loops are obtained, but they are usually not simple but of complicated character; for 24 per cent. nickel-iron, for example, Hilpert obtained the loop shown in Fig. 73, that is, a curve which cuts itself. As will be seen, the magnetizability ℑ on cooling down to – 180° C. greatly increases, but on being reheated does not diminish but slowly increases, and then at 400° C. rapidly diminishes; at 700° C. becomes zero, then rises again and remains a constant during the cooling.

Connected with the temperature hysteresis, but having a significance of its own, is the question of preliminary thermal treatment. It has also been mentioned several times incidentally how significant it is to know the nature of the material under consideration. For iron and steel are the products of a manufacturing process; they have already undergone a preliminary thermal treatment, and this treatment is, under certain circumstances, continued in the laboratory. Therefore we have to deal with the heating of the material to a definite temperature, in certain cases to red heat, and also with the manner in which the material is then cooled down again, which may either be sudden (chilling, quenching) or gradually, and in the latter case in all degrees up to a regulated fine-cooling. Then, again, we have to consider not only the temperature to which it has been raised and from which it has been chilled, but also the duration of the process, and the media in which the operation has taken place (a vacuum, oxygen and so forth), and further whether the procedure has taken place once or has been several times repeated. And indeed even without anything being done changes arise which are attributed to the effects of ageing, and which, as was mentioned in connection with the Heusler alloys, may require to be brought about artificially in order to expedite the process.

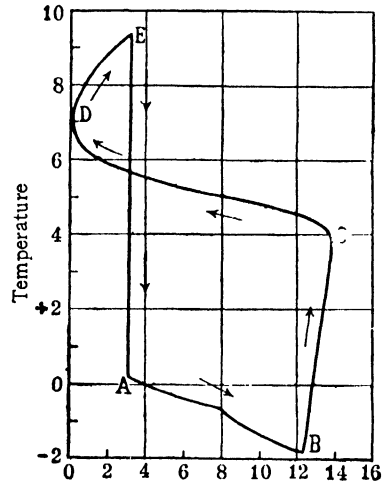

FIG. 73.

VI

MAGNETO-OPTICS

59. **Faraday Effect**—Interesting though the relations of magnetism to mechanical and thermal influences may be, their significance is not to be compared with that of the effects of magnetism on light. For light is a phenomenon perceptible to every one and for which we possess a special sense organ, the eye; while magnetism is perceptible only through its special effects, such as attraction and repulsion. At first sight it might appear that there could be no relation between these two phenomena; and it is obvious that the idea of looking for anything of the kind was not one that would easily present itself. This only happened when a fresh mind, one unhampered by the teaching of any school or authority, Michael Faraday, appeared, who was filled with the thought that all the forces of nature are of the same kind, and that optical phenomena in particular might stand in some sort of relationship, even though at first it were of a mysterious nature, to those of electricity and magnetism. Accordingly he set to work as a genuine empiricist to explore the matter in all directions in order to find some connection between magnetism and light, and after many failures his efforts were finally crowned with success.

It was already known that there is one class of substances, through which polarized light, that is, light in which the ether particles are supposed to move to and fro in a direction perpendicular to the path of the beam, does not pass without being influenced. The direction of the plane in which the light vibrations take place is turned through a certain angle when the light passes through the substance: that is, there is a natural rotation of the plane of polarized light. Such substances are crystals with unilateral symmetry as, for example, quartz, and solutions of corresponding constitution, as, for example, a solution of sugar. With really symmetrical bodies rotation does not take place, but only in those like quartz and sugar, which exist in two varieties differing from each other in the same way as the right hand does from the left. One kind turns the plane of polarization to the right, the other to the left. Bilateral crystals, however, and isotropic bodies turn it in neither direction.

Faraday therefore reasoned that when a body is magnetized it ceases to be isotropic; it acquires in virtue of the attraction of the field a unilateral character, and this he argued should naturally bring about a rotation of the plane of polarization. Through a happy chance he was able in 1845 to confirm this prediction experimentally. A transparent substance has to be employed, and such substances are usually only slightly magnetizable, especially in fields of the strength that were then available. As it happened, a short time previously a new sort of glass had been discovered, composed of boric-acid-flint and lead oxide, which proved to be especially suitable. In this substance the effect that was being sought for was found to occur. It was subsequently found to occur in many others also, and in the course of time was confirmed in regard to all substances. This optical method provided Faraday with an excellent means of confirming his favourite idea that all substances are magnetizable, for in sensitivity the eye surpasses all scientific instruments.

In more recent times the phenomenon has been quite rightly designated the Faraday effect. It occurs in the case of solid, liquid and gaseous substances. In the latter it was first discovered by Kundt and Röntgen in 1879; and it was Kundt who succeeded in 1884 in showing its existence in ferro-magnetic substances, thanks to the circumstance that the effect is here so powerful that it is recognizable and indeed measurable even when we are dealing with so thin a layer of iron that light can pass through it.

60. **Law of the Faraday Effect**—The phenomenon naturally presents itself most simply in the case of plane-polarized light and in investigating the subject this is used almost without exception. The effect must be the same for elliptically polarized light, and here it is the major axis that undergoes rotation. That ordinary light is almost influenced similarly has been shown by Sohncke.

The effect is produced in whatever way the substance is magnetized. It can be brought either into the neighbourhood of a permanent magnet or of an electro-magnet; it only needs indeed to be subjected to the action of the earth's magnetic field; or instead of the field proceeding from a magnet it can be produced by a coil through which a current is passing; and static electricity as, for example, from a Leyden jar, if passed through the coil, has the same effect. But the most marked effects are, of course, obtained with the electro-magnet. The most suitable form is that designed by Ruhmkorff (see below), which for this purpose is modified by having a hole bored through the two iron cores which are surrounded by the magnetizing coils, so that a beam of light can be passed through the whole length of the magnet, as is shown in Fig. 131. The substance to be investigated is placed between the two pole faces. The polarizer is connected with the outer end of one limb and the analyser and its rotating circle with the other. Fluids and gases are enclosed in tubes and these latter have glass plates at the ends. Care must be taken that these plates do not themselves cause rotation. In order to double the effect and to eliminate certain sources of error it is usual to compare the position of the analyser not before and after the production of the magnetic field, but when the field is excited in opposite directions. In order to intensify the phenomenon when it is very weak a multiplication of the procedure may be adopted, a method due to Faraday, in which the light is admitted at the edge of the front surface of the substance, and is only permitted to pass out at the opposite edge of the back surface, so that it is compelled to pass backwards and forwards through the substance repeatedly (some ten or twenty times). The process of deducing exact results from this method is shown later.

The first substances to be investigated all showed rotation in the same sense, namely, in that sense in which the current that is producing the field (or the current which would produce the same field) flows round the substance. It was later found, however, that many substances produce rotation in the opposite direction. Rotation in the first sense was therefore called positive, and that in the opposite sense negative, and it was shown that diamagnetic substances produce positive, and paramagnetic substances negative rotation. But this rule is not generally confirmed, and therefore we have to distinguish four classes according as the susceptibility κ of the substance is positive or negative, and according as the rotation ω is positive or negative. The most important representatives of these four classes are shown in the following table, which is taken from a work by du Bois:

κ Negative.		κ Positive.	
ω Positive (I).	ω Negative (II).	ω Positive (III).	ω Negative (IV).
Potass. ferrocyanide	Titanium chloride	Cobalt	Ferro salts
Borate of lead, etc.		Nickel	Ferri salts
Water, etc.		Iron	Potassium ferricyanide
Hydrogen, etc.		Oxygen	Chromic acid anhydrite
Most solid, liquid and gaseous substances		Nitric oxide	Pot bichromate
		Cobalt salts	Pot chromate
		Nickel salts	Cerium salts
		Manganese salts	Lanthanum salts
		Cupric salts	Didymum salts

Most substances belong, it will be seen, to Group I, while in Group II titanium chloride is so far the only known representative. The ferro-magnetic substances and their salts offer a very varied picture, the three metals themselves belong to Group III, and also the salts of nickel and cobalt ; all the combinations of iron, on the contrary, seem to belong to Group IV or to Group I. In this connection compare what has already been said concerning the para- or diamagnetism of these salts. Reference should also be made to the conjecture of Kundt that elementary substances apparently produce positive rotation.

But in regard to the sense of the rotation an important difference is to be noted between natural and magnetic rotation. Natural rotation in any given substance takes place regularly in the same direction looking from the observer, and therefore so far as absolute space is concerned in either direction according to the direction of the light. Magnetic rotation, on the contrary, is independent of the direction of the light beam, and is always in the same direction, depending only on the direction of the field. A beam that passes through a substance and then returns undergoes no natural but, on the contrary, a double magnetic rotation. From this it follows that when a substance capable of producing rotation is placed in the magnetic field the magnetic rotation adds to or subtracts itself from the natural rotation according to the direction of the beam and the sense of the magnetic field.

61. **Formulæ**—The rotation produced is proportional to

the distance traversed by the light beam, as would be expected; but of course it must be assumed that the direction of the beam coincides with the direction of the field, otherwise we have to multiply by the cosine of the angle between them; and if the light passes perpendicularly to that direction there is no rotation.

Further, at least for weakly magnetizable substances, the rotation is proportional to the strength of the field. This is the law of Verdet; and taken in conjunction with the preceding statement it gives for the rotation Ω the equation

$$\Omega = \omega l \mathfrak{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where $\mathfrak{H}$ is the strength of the field, l the distance traversed by the light, ω the "Verdet constant." Nevertheless, it might be anticipated that proportionality only exists in so far as $\mathfrak{H}$ is itself proportional to the magnetization $\mathfrak{J}$; and that the true inner connection is not between Ω and $\mathfrak{H}$ but between Ω and $\mathfrak{J}$. This must become evident when we turn to the ferro-magnetic substances; and here, in fact, as Kundt has shown, the proportionality is no longer in regard to $\mathfrak{H}$ but rather in regard to $\mathfrak{J}$, and instead of the above formula we can write

$$\Omega = \psi l \mathfrak{J} \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where ψ is the "Kundt constant" or the "coefficient of rotation." If we represent graphically the rotation as a function of the strength of the field, however, it is obvious that we shall not obtain a rising straight line but a curve following the behaviour of the magnetization curve for ferro-magnetic substances: compare the upper curve of Fig. 74 (in which indeed the steeper rise in the middle of the curve does not stand out). For the rest the Verdet and the Kundt constant are obviously connected by the equation

$$\psi = \frac{\omega}{\kappa} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Further, instead of taking as reference the unit of length (or, what is the same thing when the cross-section is unity, the unit of volume), the effect may be expressed in terms of the unit of mass, that is, by dividing by the density. The "specific rotation" is then obtained

$$S = \frac{\omega}{d} \text{ or } \frac{\psi}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

and from this by multiplying by the molecular weight m the molecular rotation

$$M = mS \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

Finally, in the case of solutions, we have to distinguish between the three magnitudes S, S_0 and s (rotation of the solution, rotation of the dissolved substance, and rotation of the medium itself). We then have the relation

$$S_0 = pS_0 + (d - p)\,s, \text{ therefore } S_0 = \frac{1}{p}\,[S - (d - p)s] \quad (40)$$

in which d is the density and p the concentration (grammes

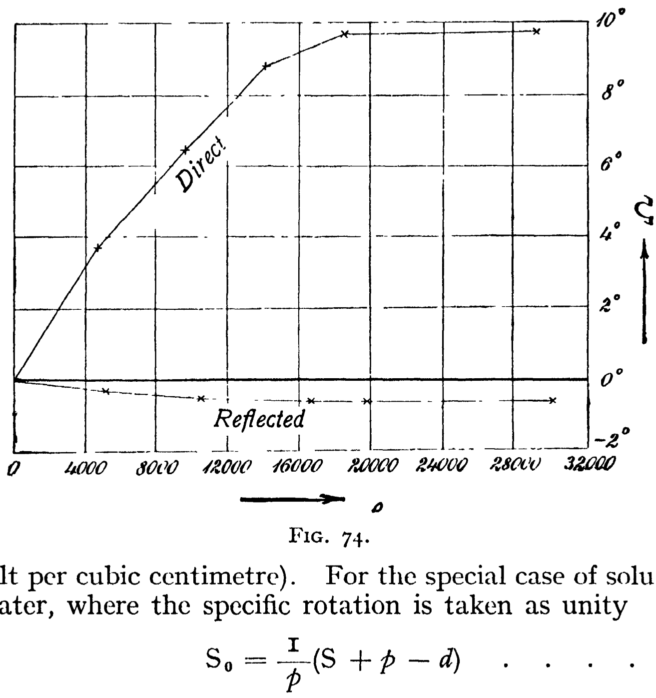

FIG. 74.

of salt per cubic centimetre). For the special case of solutions in water, where the specific rotation is taken as unity

$$S_0 = \frac{1}{p}(S + p - d) \quad . \quad . \quad . \quad . \quad . \quad (41)$$

62. **Dispersion of Rotation**—Our account of the phenomenon is not yet complete, for there are two more influences to be considered. The first, that of temperature, is soon disposed of: in most substances the rotation diminishes with increase of temperature; in the ferro-magnetic substances only does it appear to be independent. Very interesting, on the other hand, is the influence of the wave-length of the light.

As is known, light composed of various wave lengths, on passing from one medium to another undergoes differential refraction, and this phenomenon is called dispersion. Something corresponding takes place here, and accordingly we can speak of the dispersion of rotation, and in this particular connection of the "magnetic dispersion of rotation." For the most important Fraunhofer lines in the spectrum, Verdet has obtained the following relative values which are referred to the line E as unity; the values of the square of the reciprocal of the wave-length, expressed in the same measure, have been added at the top of each column and it will be seen that there is a general proportionality to λ^{-2} and that Ω varies still more, especially in some liquids.

	C	D	E	F	G
λ^{-2} =	0·64	0·80	1·00	1·18	1·50
Distilled water	0·63	0·79	1	1·19	1·56
Pot chloride solution	0·61	0·80	1	1·19	1·54
Zinc chloride solution	0·61	0·78	1	1·19	1·61
Tin chloride solution	—	0·78	1	1·20	1·59
Oil of bitter almonds	0·61	0·78	1	1·21	—
Oil of aniseed	0·58	0·75	1	1·25	—
Bisulphide of carbon	0·60	0·77	1	1·22	1·65
Creosote	0·60	0·76	1	1·23	1·70

Numerous formulæ have been proposed to express the connection of Ω with λ and with the numerical quotient n, but they are valid only over a limited range.

There is, moreover, both in regard to the diffraction and the natural rotation of the plane of polarization out of the normal, the so-called anomalous dispersion. The diffraction or the rotation ordinarily increases with increase of wave-length, but in some substances anomalies present themselves at one or more places in the spectrum. In tartaric acid, for example, the rotation increases between the lines C and F; from F to G, on the contrary, it diminishes by almost the same amount. In the ferro-magnetic substances, according to Kundt in iron, according to Lobach in nickel and cobalt, the anomaly is of a more general character, for here the dispersion regularly increases in passing from the blue to the red rays, as is shown in Fig. 75. Anomalies of dispersion are closely connected, however, with absorption, so that they always present themselves at any place, that is at any wave-length, where an absorption band occurs in the spectrum of the substance. Corresponding observations have been made by Schmauss, which confirm what has been said in the case of cyanin, fuchsin, eosin and similar solutions, as well as in didymium glass. And the lively discussions that these results have provoked have lead to numerous repetitions and developments of his experiments which have fully confirmed them. The dispersion of rotation and absorption do not however always run closely

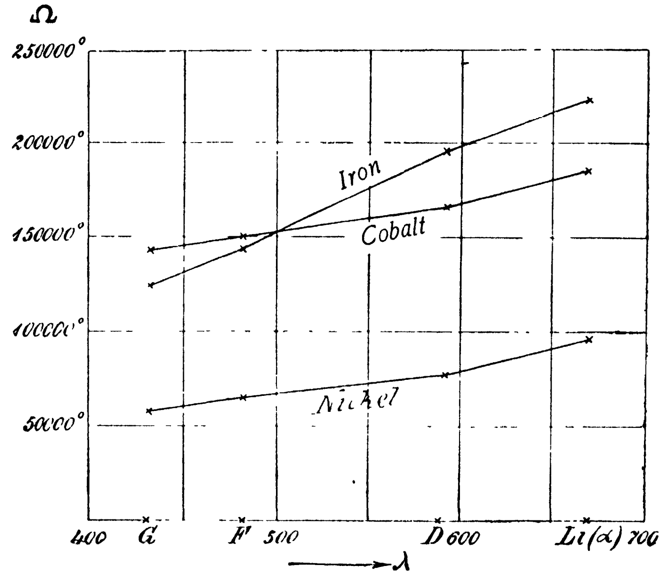

FIG. 75

parallel. In Fig. 76 is an example of the observations of Schmauss for various concentrated solutions of cyanin (the curve for alcohol is added for purpose of comparison), and in Fig. 77 a series of measurements by Elias on a solution of præsodidymium is reproduced (above are the curves of the rotation, below the curves of the absorption, the abscissæ showing the wave-lengths in $\mu\mu$, that is in the millionth parts of a millimetre). The first absorption maximum scarcely makes itself felt; the second, on the contrary (at $\lambda = 482\mu\mu$), has a very pronounced effect.

63. **Numerical Data**—It is necessary to give some of the

numerical results from the especially rich material which has been accumulated in the course of time. The values are expressed in very various ways, either as ω for the unit of

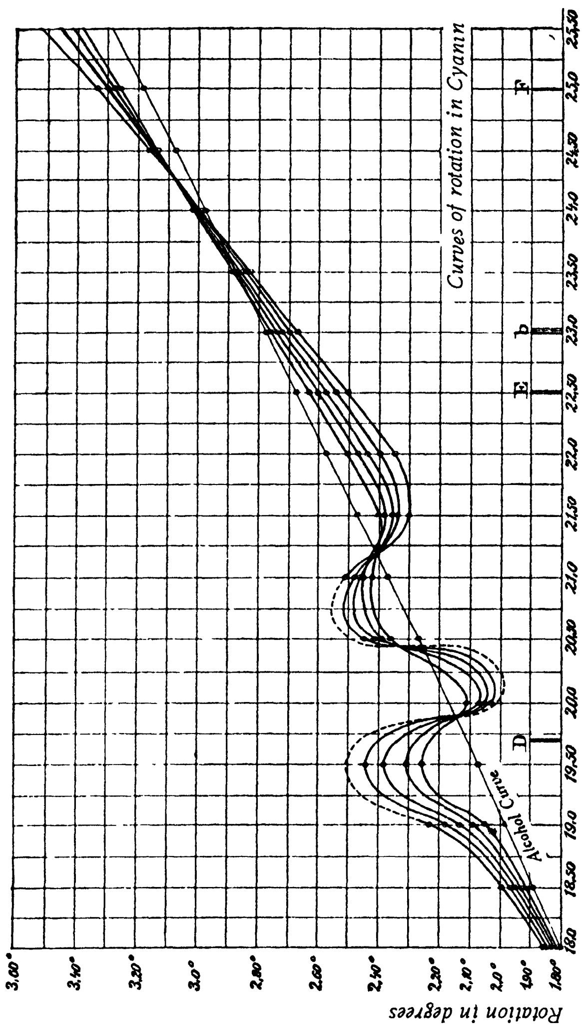

Fig. 76.

field-strength, or as ψ for the unit of magnetization, and in both cases the angle is expressed in minutes of arc or in radian measure, i.e. taking the angle $180/\pi$ as the unit.

In the first place we give a table after du Bois of values of ω and ψ expressed in radian measure at ordinary room temperature for a number of important substances:

ABSOLUTE VALUES OF ω AND ψ

Substance	$\lambda 10^5$	$\omega 10^6$.	ψ.
Cobalt	6·44	—	+ 3·99
Nickel	6·44	—	+ 3·15
Iron	6·56	—	+ 2·63
Oxygen (1 Atm.)	5·80	+ 0·000179	+ 0·014
Sulphuric acid	5·80	+ 0·302	− 4·0
Water	5·80	+ 0·377	− 5·4
Nitric acid	5·80	+ 0·356	− 5·6
Alcohol	5·80	+ 0·330	− 5·8
Ether	5·80	+ 0·315	− 5·8
Chloride of arsenic	5·80	+ 1·222	− 14·9
Bisulphide of carbon	5·80	+ 1·222	− 17·1
Faraday's glass (melted)	5·80	+ 1·738	− 17·7

The table is set out according to the algebraic values of ψ. As will be seen it is not among the ferro-magnetic substances that the maximum occurs. In this respect these substances are surpassed by most solids and liquids, by bisulphide of carbon and by Faraday's glass, which is nearly five times as great. Among the ferro-magnetic substances themselves, cobalt occupies the first, and iron the lowest place. But the ferro-magnetic substances would show an extraordinarily large Verdet constant ω on account of their high susceptibility. How enormous the rotation of the plane of polarization is in the case of iron is made clearly evident by the results of Kundt, which show that for a condition of magnetic saturation the rotation per centimetre is about 200,000°, and therefore that the direction of vibration of the ether particles has been turned completely round in so small a distance as 0·02 mm. Exact values for the maximum rotation are:

Iron:	209,000°	(Kundt);	216,000°	(Lobach).
Cobalt:	198,000°	(du Bois);	180,000°	
Nickel:	89,000°		90,000°	

For the two substances most investigated, water and bisulphide of carbon, the average values derived from a large number of measurements are:

Water at 0° C. and with sodium light, 0·01304′ and 0·000003792.
Bisulphide of carbon at 18° C. and with sodium light, 0·0421′ and 0·0000122.

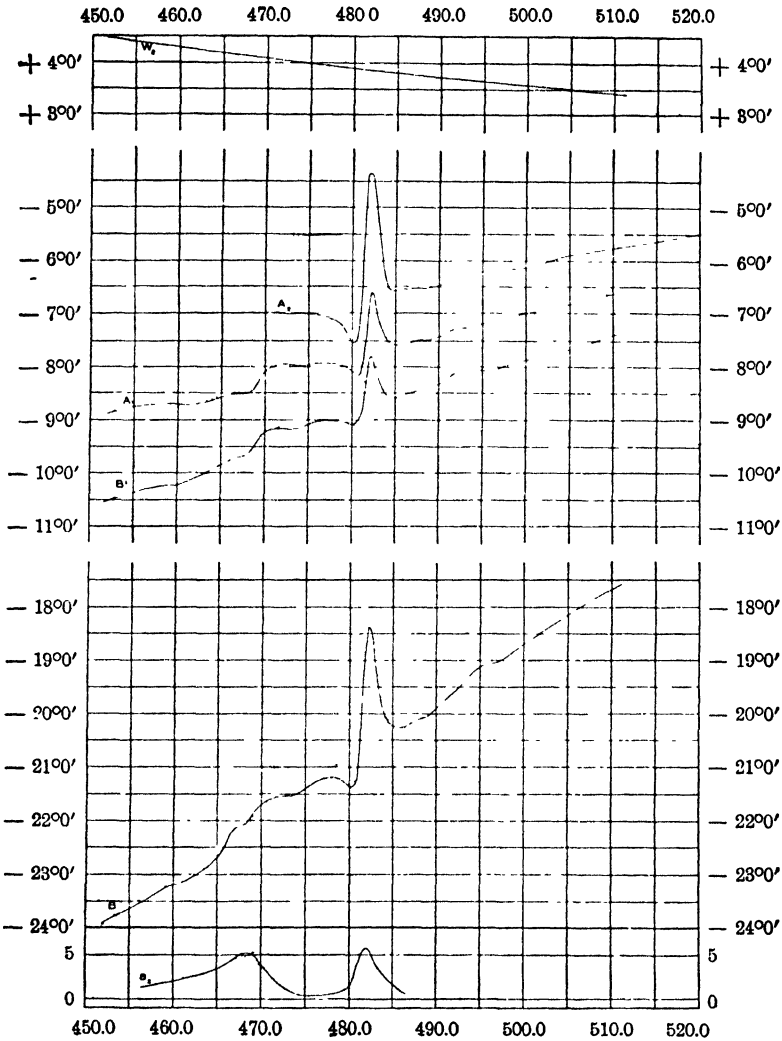

FIG. 77.

How greatly other substances of similar character differ among themselves is shown by the following table of Jena glasses according to du Bois:

JENA GLASSES ACCORDING TO H. DU BOIS

ω in radian measure and for sodium light at 18° C.

Substance	Description	n.	ω.
Boron crown	S—204	1·51013	0·0163
Light boron silica crown	O—1092	1 51660	0·0190
Strongly disperse silica crown . . .	O—1151	1·52017	0 0234
Medium phosphatic crown	S—179	1·56201	0·0161
Heavy baryta silica crown	O—1143	1·57412	0 0220
Ordinary light flint	O—451	1·57522	0·0317
Heavy silica flint	O—469	1·64996	0 0442
Heavy silica flint	O—500	1·75096	0·0608
Heaviest silica flint	S—163	1·89042	0·0888

The heaviest sort of flint glass produces therefore five times the rotation of the lightest crown glass.

Similarly for some solutions according to Wachsmuth the values for Ω, S and M with sodium light are as follows (σ is the density):

Substance	σ.	Ω	S	M
H_2O	1·0000	1·0000	1·0000	1·0000
$CoSO_4$	1·1378	0·9993	0 0029	0 0247
$CoCl_2$	1·1250	1·0991	0 8224	5 9215
$Co(NO_3)_2$	1·1321	0·9620	−0·0328	−0·3325
$Co(C_2H_3O_2)_2$	1·0886	1·0172	0·4770	4·6795
$NiSO_4$	1·1454	1 0730	0 4913	4·2256
$NiCl_2$	1·1058	1·1631	1·5333	11·0569
$Ni(NO_3)_2$	1·1285	1·0444	0·4617	4·6889
$Ni(C_2H_3O_2)_2$	1·0633	1·0490	0·8526	8·3742
$MnSO_4$	1·1607	1·0290	0·2317	1·9435
$MnCl_2$	1·1107	1·1166	1·0434	7·3037
$Mn(NO_3)_2$	1·1135	0·9915	0 1931	1 9205
$Mn(C_2H_3O_2)_2$	1·0864	1·0239	0·5099	5·4777
H_2SO_4	1·8282	0·8652	0 3915	2·1317
HCl	1·1247	1·3541	1 8436	7·4766
HNO_3	1·1898	0·9287	0·2702	1·8916
C_2H_4O	1·0602	0·8576	0·7961	5·3073
$Ni(CO)_4$	1·31	4·244	—	—

In conclusion, for some gases at different pressures, temperatures and wave-lengths according to Siertsema the following formula holds:

$$\Omega \,.\, 10^{6} = \frac{a}{\lambda}\left(1 + \frac{b}{\lambda^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (42)$$

the values for a and b being:

Gas	a	b
Air (100 kg 13·2° C)	190·6	0·242
Oxygen (100 kg 7·0° C.)	271·7	0·0704
Nitrogen (100 kg 14·0° C)	169·9	0·311
Carbonic acid (1 Atm 6·5° C)	269·5	0·307
Nitrogen protoxide (30·5 Atm, 10·9° C)	75·5	0·306
Hydrogen (85 kg 9·5° C)	138·6	0·325

64. **Kerr Effect** We know from the science of optics that when light reaches the boundary of two media it divides into two parts, one of which penetrates the second medium while the other is reflected back into the first. Seeing that the magnetic field exercises an influence on the first of these phenomena we may very well expect that it will do so in regard to the other. The only doubt is whether the phenomenon, which in any case will be very slight in character, will be sufficient to be observed. But in regard to this we have some hope when we consider that we are not dependent on reflection from transparent bodies but that strongly magnetizable substances can be employed, and in particular iron. This may have been the train of thought which led the English physicist Kerr to investigate the reflection of light from a magnetized iron mirror, and that at a time (1876) when the experiments of Kundt on the transmission of light through iron were still to be made. A very positive result was obtained, for it was immediately shown that the phenomenon in this case is more complicated than in the other, and indeed it must be so, for the reflection of light from metals not subjected to a magnetic field is in itself one presenting special features. The effect is consequently of a twofold character, and consists (1) in rotation of the plane of polarization, and (2) in the transforming of plane-polarized light into elliptically polarized light, or of polarized light into another condition of ellipticity, or under certain circumstances into plane-polarized light. The first effect puts the reflection of the light at a magnet into parallel with the transmission through it, the second effect is a property of the reflection itself. It may be characterized by saying that the reflection at the magnetic surface changes the intensity relation and the phase relation of the two components of the light beam, and therefore when the impinging beam is plane-polarized, sets up in it a magnetic component at right-angles to the original.

Since the phenomenon in the case of transmission is already very complicated in consequence of the great number of determinative factors, it is clear that in the case of reflection it will be still more complicated. For here variations may take place magnetically in regard to the strength of the field, the magnetized material and the intensity of the magnetization, the temperature, and the direction in which the reflecting surface of the mirror is cut—that is the angle which it makes with the lines of force. The two limiting cases corresponding to reflection from the end surfaces (angle 0°), and at the curved or equatorial surfaces (angle 90°) are specially important and have been studied in detail. On the other hand a diversity of optical factors has to be considered: the angle of incidence; the angle which the direction of vibration of the incident light makes with the incident plane; the colour; the ellipticity; and, in the case of reflection at the curved surface, the angle which the incident surface makes with the lines of force; and here again two limiting cases stand out according as the plane of incidence is parallel to, or at right-angles to, the lines of force.

The simplest case occurs when the light makes right-angled incidence because here the direction of the plane of vibration plays no part: the plane of polarization of plane-polarized light is simply turned through a certain angle and in the opposite direction to that of the magnetizing current, so that in terms of the previous convention it is to be regarded as a negative rotation; only in the case of magnetite is it opposite. Let us first consider polar reflection. In the first experiments of Kundt the rotation for iron lay between 45′ and 66′, for cobalt between 50′ and 67′, for nickel between 20′ and 23′. In the second series of experiments the strength of the field was measured and for iron the following series was found:

$\mathfrak{H}$	4,990	10,800	16,600	19,800	30,300
Ω	− 0·27°	− 0·55°	− 0·62°	− 0·66°	− 0·67°

These numbers are represented in the lower curve of Fig. 74. While the transmission goes upwards, the curve of reflection goes downwards. Their course for the rest is quite analogous, and one might also expect here that the rotation would not be proportional to the strength of the field but to the intensity of the magnetization, and this supposition was soon confirmed experimentally by du Bois. Nickel was found to have the smallest effect. When the reflecting surface is not polar, and the normal to the surface therefore makes an angle a with the direction of magnetization, the rotation is less; and here also, according to du Bois, the simple sine law is followed. In the case of reflection at an equatorial surface, the rotation therefore becomes zero. The rotation is to be regarded as proportional to the intensity of the normal component of the magnetization in the formula:

$$\Omega = K \, . \, \mathfrak{J}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

The constant K, which is exactly analogous to the Kundt constant, du Bois has proposed to call the Kerr constant; it may be expressed either in angular or radian measure.

Further the rotation is dependent on the wave-length of the light and according to du Bois the relation is as follows: in iron the dispersion is anomalous, that is the rotation diminishes regularly from the red to the violet; in cobalt there is a minimum in the green, in nickel a minimum in the yellow, and finally in magnetite a maximum in the yellow.

In the following table the values of the Kerr constant according to du Bois are given in minutes of arc for the four ferro-magnetic substances, and for the four most important wave-lengths.

Colour	Line	$10^6 \lambda$	Cobalt	Nickel	Iron	Magnetite.
Red . .	Li α	67·1	− 0·0208	− 0·0173	− 0·0154	+ 0·0096
Red . .	- -	62	− 0·0198	− 0·0160	− 0·0138	+ 0·0120
Yellow . .	D	58·9	− 0·0193	− 0·0154	− 0·0130	+ 0·0133
Green . .	b	51·7	− 0·0179	− 0·0159	− 0·0111	+ 0·0072
Blue . . .	F	48·8	− 0·0181	− 0·0163	− 0·0100	+ 0·0026
Violet . .	G	43·1	− 0·0182	− 0·0175	− 0·0089	—

When the incidence is acute we have the more interesting case of the reflection from the side surface, and two cases are here possible, namely, when the lines of magnetic force which run along the surface of the magnet are either perpendicular to the plane of incidence or parallel to it. In the first case no rotation of the plane of polarization takes place. If the plane of incidence is parallel to the lines of magnetization we have to distinguish between the rays which are polarized in the plane of incidence, or are at right-angles to it. The former, for all values of the angle of incidence, experiences a negative rotation; the latter, on the contrary, only for large values of the angle of incidence; for certain values of the angle it is zero, and for smaller values it is positive. For iron the following values by Kundt (|| and ⊥ refer to the direction of polarization), together with the curves of Fig. 78, which are due both to Kundt and to Righi, show this in detail.

Iron i	Iron ||	Iron ⊥	Nickel i	Nickel ||	Nickel ⊥
19·0°	− 4·8′	+ 2·7′	20·0°	—	+ 0·0′
29·9°	− 4·5′	+ 7 3′	30·1°	− 1·7′	+ 1 8′
39·5°	− 6·6′	+ 7 7′	40·0°	− 2·7′	+ 1 4′
50·1°	− 7·7′	+ 6·9′	50 0°	− 4·7′	+ 0 3′
61·3°	− 8·0′	+ 7·5′	61·5°	− 4·2′	− 0 7′
65·0°	− 9·4′	+ 8·7′	65 3°	− 3·8′	− 2·2′
70 0°	− 7 1′	+ 8 1′	75·0°	− 1·1′	− 1 9′
75 0°	− 6 0′	+ 6 8′			
80 3°	—	+ 2·6′			
82·0°	− 4·3′	− 2·3′			
85·3°	− 3·9′	− 1·9′			

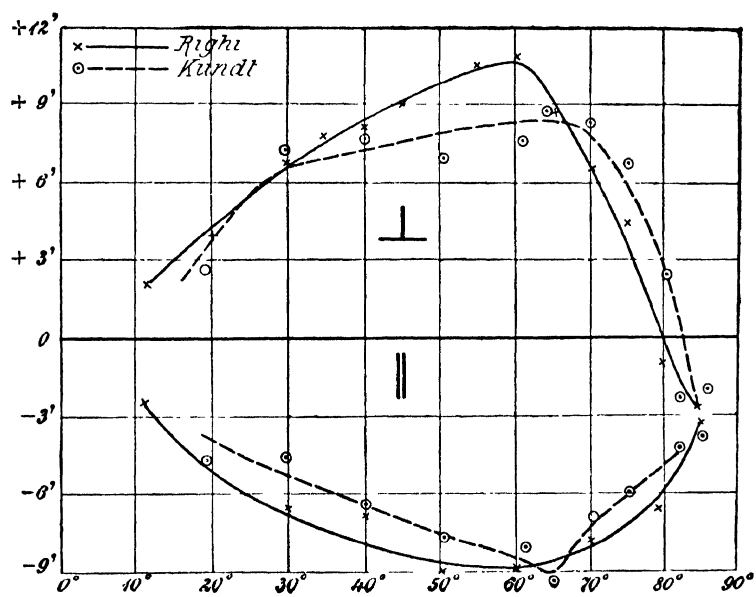

Fig. 78.

The angle of incidence at which this change over of the rotation takes place may be called the critical angle of incidence. It does not depend on the strength of the magnetization ; on the other hand, it is different for different metals, and even for the same metal the values found do not always perfectly agree. Thus for iron it varies from 75° to 81°; for cobalt from 70° to 78°; for nickel from 50° to 60°; and these are apparently only the limits of the true values. In nickel and cobalt the curves, according to Micheli, run somewhat differently, but here it seems to depend very much upon the method of preparation and the purity of the material.

So far we have spoken only of the rotation of the plane of polarization. Actually, however, as was pointed out at the beginning, this is only one aspect of the phenomenon; there is another which consists in the fact that plane-polarization is changed into elliptical, and more generally that elliptical polarization is modified in regard to the relations of the principal axes and the position of the axis of the ellipse. And this may be expressed by saying that by reflection a new magneto-optical component is introduced. But in this connection it must be clearly understood that even in the case of reflection from non-metallic mirrors generally, a new component is introduced which is a property of metallic reflection itself and with which the magneto-optical is now in addition associated. The difference is that in ordinary reflection from mirrors the polarization remains plane if the plane of polarization of the incident beam lies in or is perpendicular to the plane of incidence; in this case the ellipticity of the reflected beam is solely due to the magnetic forces.

We cannot unfortunately go into the interesting experiments by which the Dutch physicists Kaz, Sissingh and Zeeman have substantially widened our knowledge of the Kerr effect. We must also pass over a whole series of magneto-optical phenomena (the Righi effect, the Majarana effect, the Corbino effect) and proceed at once to the last and by far the most interesting of the phenomena relating to this subject, in order that we may go on to the consideration not only of reflection from the medium but also to propagation through the medium.

65. **Zeeman Effect**—In the whole of physical science there has been scarcely any department that has given rise to so many epoch-making discoveries as the remarkable phenomenon that presents itself when diffracted or refracted light of any kind, in particular that sort of combined light that appears to our senses as white light, is caught on a screen, in other words the spectrum. We have only to think of the doctrine of colour and the arguments that arose over the theories of

Newton and Goethe, Helmholtz and Hering right down to the newest phase presented by Ostwald. We have to think of the spectrum analysis of Kirchoff and Bunsen, of the famous Fraunhofer lines on the one hand, and of the relation between the emission and absorption of the beam on the other, which both to physics and chemistry opened up unsuspected vistas and yielded such important results. Finally we have to think of the Doppler effect, which, first observed in the case of sound waves, only found its most important and far-reaching applications in relation to light waves. With all this is associated, as one of the most important links in the chain, the Zeeman effect.

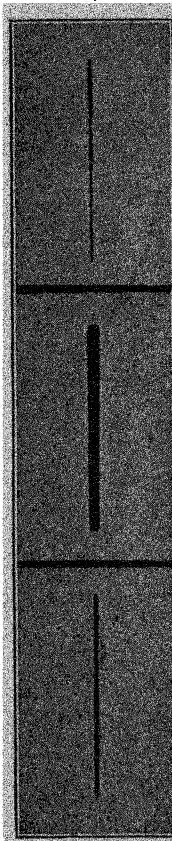

FIG. 79.

Zeeman in Leyden had been for many years occupied with the Kerr effect and the theories connected with it. The question continually arose what effect magnetism had on the propagation of light. Is it not possible and even probable, Zeeman argued, that magnetism has some effect on the emission of light and therefore that the active forces in the light source are subject to the influence of the magnetic field? This question led him in 1896 to resolve the light of a sodium flame between the poles of an electro-magnet into its component parts by means of a Rowland mirror grating. The observation was made in the direction normal to the line of the magnetic axis, and it was found that the yellow sodium lines were broadened out when the magnet was excited (Fig. 79 *a* and *b*). This indicates that under the influence of magnetism the flame emits vibrations some of higher and some of lower frequency than the original. The broadening out is also to be observed in the direction of the lines of force of the magnetic field. It was also shown that to the "direct" effect there corresponded an "inverse" effect, that is the absorption lines that present themselves when white light passes through glowing sodium vapour are likewise broadened out if the vapour is magnetized. But the change observed in the frequency of the vibrations is extremely small: in a field of 10,000 gauss it corresponds to about a thirtieth part of the distance between the two well-known yellow sodium lines. But here a distinction has to be made: in the first place between the direct and the inverse Zeeman effect; and secondly, in each of these two cases themselves between the longitudinal and the transverse Zeeman effect; and finally, we may again as a third case distinguish the negative and the positive Zeeman effect, according as the diminished or increased wave-length (in the longitudinal observation) shows certain properties.

And further, the condition of the light in the central part of the broadened line and that at the edges can be shown to be different; for if a Nicol prism be placed in front of the slot of the spectroscope so that only the horizontal vibrations are allowed to pass through, we obtain the reduction in breadth shown in Fig. 79. That is, the central portions are normal light, the edges are plane-polarized. In the upper halves of Fig. 80 both sodium lines are reproduced for the case in which light circularly polarized in the right-hand sense only is allowed to pass through. If now the current in the electro-magnet is reversed, the pictured half is obtained. The lines are moved sideways, which indicates that the lines are circularly polarized in the opposite sense.

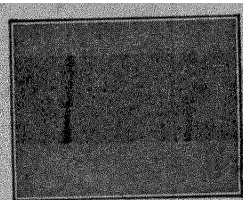

FIG. 80.

Now in consequence of certain theoretical considerations which will be discussed later the probability would seem to be that the broadening out of the spectral lines is not the final but only the incidental result of the observation, and it was therefore necessary to increase the effect by intensifying the conditions of the experiment. For this purpose three methods were adopted: the application of stronger magnetic fields; the use of a more strongly dispersive spectroscope; and the choice of the finer spectrum lines. The stronger fields were furnished by the electro-magnets of du Bois and Weiss; the Rowland grating made possible a higher power of dispersion —the echelon spectroscope of Michelson and the plate spectroscope of Fabry and Perrot on the one hand, and of Lummer and Gehrcke on the other. Specially important in this connection was the Michelson diffraction grating which made possible the separation of lines whose distance apart is only the four-hundredth part of that between the two D lines. As fine spectrum lines we have those which are given by vacuum tubes charged with mercury, thallium, cadmium and other metals. The blue-green cadmium line of 4,678 Angström units, that is of 467·8$\mu\mu$ wave-length, shows itself specially suitable.

We have again to distinguish between observations in the direction of the lines of force of the magnetic field, and those at right-angles to it. In both cases the simplest normal observation is the following: in the direction of the field a subdivision into two parts takes place, the simple spectral lines become a "duplet" and the two lines of which it is composed each lie symmetrically disposed on the two sides of the normal position of the line and are found to be circularly polarized, one in the left, the other in the right-hand direction.

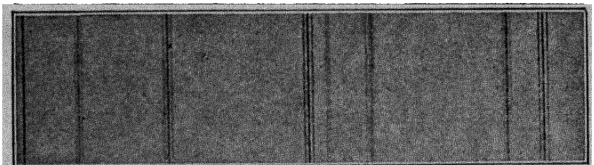

FIG. 81.

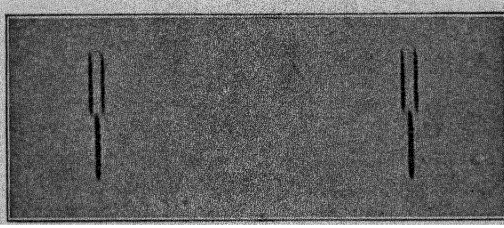

FIG. 82.

But at right-angles to the field there is a subdivision into three parts, and all the three lines of the "triplet" are plane-polarized; the vibrations in the central line, which remains in its normal position, are parallel to the field, and those of the two outer lines are perpendicular to it. In Fig. 81 a group of lines from the spectrum of iron is reproduced, and in Fig. 82 we have the two triplets of the yellow mercury lines. By means of a rhombohedral calc-spar crystal two images were produced, one exactly above the other, of which one consisted only of horizontal and the other only of vertical vibrations. Exactly the same effect occurs in the case of the absorption lines. In very strong fields the effect can be so far intensified that the broadening out of the two sodium lines is half as great as the distance between them.

It is of especial interest that the Zeeman effect can be observed not merely in the laboratory but also in nature. Thus the American astronomer Hale found in 1909 that some absorption lines of the solar spectrum which are usually simple are disintegrated at certain parts of the sun's surface, a fact from which it might well be concluded that in those places the atmosphere of the sun is exposed to a magnetic field. Since the disintegration in question is into duplets or triplets, according as the place lies near the middle of the sun or is approaching its edge, we must suppose that the magnetic lines of force stream out radially from the sun. And this idea in a certain sense is in accordance with another with which we shall later become acquainted.

66. **Zeeman Effect** (*continued*)—But what has been said so far relates only to the simple, normal type of effect. In most cases a further disintegration follows, in which, however, the symmetry in regard to the original position of the spectrum lines is always preserved. If the middle line only is disintegrated, and only into two components, we get for the transversal position a quadruplet, first of the type

| | | |
s   *p* *p*   *s*

in which the letter *s* signifies at right-angles to the field, and *p* parallel to it. Nevertheless, the two middle components can be further separated even to the other side of the position hitherto occupied by the two outer ones. The outer components are always disintegrated so that sextuplets present themselves of the order

| | | | | |
s *s*   *p* *p*   *s* *s*

with still further possibilities of overlapping. Very great multiplicity then arises from the varied intensity relations of the individual lines. Through the continued disintegration of the components of the triplets into 3, 4, 5 . . . lines new complications arise and as many as eighteen components have so far been obtained from a single spectrum line. In the case of longitudinal observations, where normally therefore we have the duplet as the starting-point, the relations are in some respects simpler; but on the other hand, on account of the circular polarization they are still more complicated for new disintegrations, and as a result further types may present themselves. In Fig. 83 the photographic reproductions of some complicated types are given. They relate to the spectrum of a gaseous element existing in the atmosphere which was only discovered in the present century by Rayleigh and Ramsay, the gas neon. In obtaining these photographs an échelon grating was used, and it must be mentioned that its characteristics have to be understood in order to understand the figures properly in all their details. Here the following explanation must suffice : the uppermost division shows the disintegration of the line $\lambda = 660{\cdot}0\mu\mu$ into a sextet at 10,800 gauss. The two inner strong components correspond to the *p* vibrations, the outer four to the *s* vibrations. The two weak duplets just visible on each side belong to the spectra of the adjacent orders. The second division corresponds to the line 671·8 at 5,700 gauss, the disintegration represented possesses five *p* and four *s* components, two of which coincide so that only seven components are visibly separated. The extreme line on the right belongs to the neighbouring spectrum. The third division which relates to the line 650·7 at 9,950 gauss gives a nonett of three *p* and six *s* components. The components are here all separated so that the outermost weak *s* components fall into the region of low intensity and on that account are not noticeable. The fourth division refers to the line 614·3 at 9,300 gauss. The disintegration is into four *p* and eight *s* components, which are all separated. Some of the lines to the left belong to the neighbouring spec-

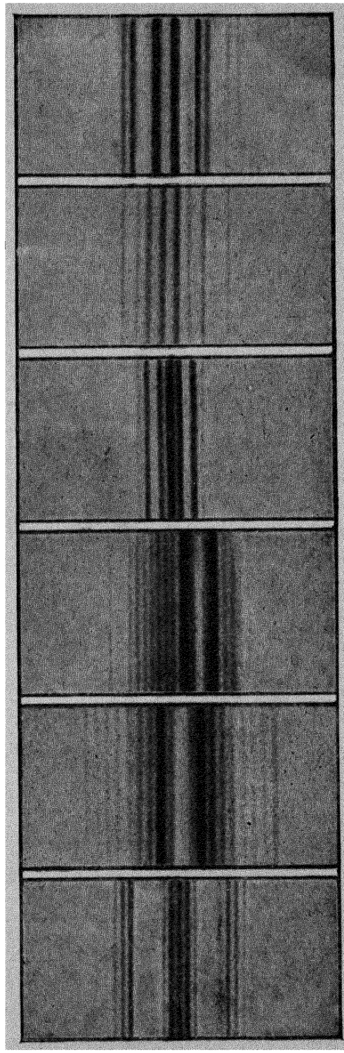

FIG. 83.

trum. The fifth division for the line 633·5 at 8,420 gauss gives an analogous disintegration in which the outer *p* coincides with the inner *s* components. The line spectra broadening out to the right and the left belong for the most part to the neighbouring spectra. The last division represents the disintegration into fifteen components of the line 640·2 at 15,350 gauss. The two pairs of outer and weaker *s* components again fall into the region of low intensity and therefore are not distinct.

What makes these various types specially interesting is the series of laws which they indicate, two of the most significant of which must be mentioned here. In the first place (and having regard to Kirchoff's law we can expect nothing different) emission and absorption are in this respect completely parallel in their behaviour ; i.e., the same sort of disintegration which takes place in any given case of emission also takes place in the corresponding case of absorption. A beautiful example of this, and at the same time of the fact that lines that are apparently identical behave differently in regard to the Zeeman effect, is offered by the two yellow sodium lines. As is known, they are distinguished from each other by the fact that their wave-length is slightly different ($D_1 = 589·6$, $D_2 = 589·0$), and moreover by the fact that D_1 is somewhat brighter than D_2. Nevertheless they give rise to different Zeeman types, for D_1 gives rise to a quadruplet, and D_2 to a sextuplet ; compare Fig. 84. Exactly the same types are presented by the dark double D lines in the solar spectrum.

D_2
D_1

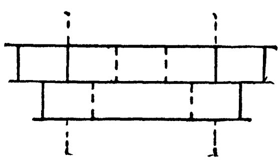

FIG 84.

67. **Zeeman Effect and Series Theory.**—In the second place there is a very noteworthy connection of the types with the series theory of spectrum lines. To understand it we must go back a little. A glowing body gives an emission spectrum, and here all extremes may present themselves, from complete homogeneity, in which therefore only a single wave is sent out so that the whole spectrum consists of one single fine line, to the most extreme universality, as in the solar spectrum which gives all the colours in regular sequence from red to violet. The most usual cases, however, are the intermediate ones, it may be one of partial continuity, or it may be one consisting of separate lines—not as a single line, however, but several. The spectrum of hydrogen, for example, in its visible portion consists of five lines corresponding to definite

Fraunhofer lines in the sun's spectrum, and characterized as follows:

C	F	G′	h	H
656	486	434	410	397

In addition there are eight further sharp lines in the ultra-violet region (the last of them with a wave-length of $317\mu\mu$) as well as numerous other weaker and less sharp lines. Naturally at a very early stage the question was asked whether the wave-lengths of these lines, which are due to the activity of one and the same luminous substance, do not stand in some sort of regular connection one with another. Balmer was the first to succeed, at first on purely empirical grounds, in putting forward a formula which corresponds to the lines of hydrogen. If n is the reciprocal of the wave-length, that is of the number of light waves in a centimetre,

$$n = N\left(\frac{1}{2^2} - \frac{1}{m^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

where N is in all cases equal to 109,675, but m on the contrary must be taken equal to 3, 4, 5, and so forth, in order to obtain the sequence of hydrogen lines. (In order to obtain the numbers given above we must multiply by 10^7 because these numbers relate to $\mu\mu$ instead of centimetres.) For other elements, similar formulæ have been proposed, but they are complicated. The spectrum lines taken as a whole which may be derived from such a formula is called a series; and it may happen that the lines of a substance either all belong to one series or to several, or, on the other hand, cannot be reduced to any regular law.

But now, coming more closely to the point, it has become clear that there is some internal connection between the circumstances which condition the way in which the series presents itself, and those which give rise to the complicated Zeeman types. In particular it is to be noted that the lines of a series have this in common, that they give rise to the same Zeeman type. Preston was the first to observe this connection, but Runge and Paschen have made systematic observations, and numerous investigators have proceeded further on the same line of research. Let us take, for example, the spectrum of mercury. It possesses at least six different series, and in each of them at least two, and in others three lines may be investigated. Each of these series gives rise to a different Zeeman type, but the lines of one and the same series always to the same type. For three series the types are schematically shown in Fig. 85. Very remarkable is the behaviour of helium, the lines of which may be arranged in six series. As Lohmann has shown, the whole of the lines admit of being decomposed into similar triplets with the same difference in the frequency of their components, here therefore we have perfect uniformity. Quite peculiar also is the behaviour of the so-called satellites, that is the weaker lines which are often found in the neighbourhood of certain lines. Thus the helium line $\lambda = 587{\cdot}59$ has a satellite at 587·62. In weak fields it gives rise to an abnormal triplet of nearly normal separation, whose middle components contain *s* and *p* vibrations, but whose outer components are almost exclusively *p* vibrations. If we reckon the Zeeman triplet as the normal form, then we must regard this form as at least a quintet and as arising through the threefold division of the middle and crowding together of the outer components. As the field increases, the middle *p* components and those lying near the red gradually fade away; those lying towards the violet vanish in the middle component of the principal line; the *s* component remains, apparently repelled by the principal line, and as it were pushes back the *s* components of the principal line that lie towards the red, so that their separation becomes asymmetrical. Fig. 86, which is due to Paschen and Back, gives an idea of what happens. Preston has proposed another rule according to which corresponding lines of various elements behave similarly; and this rule, if not quite general, has repeatedly been substantiated.

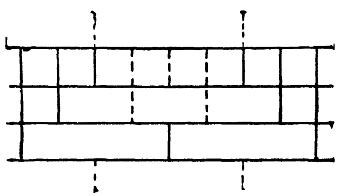

FIG. 85.

68. **Band Spectra**—So far we have only spoken of such spectra as might be called line spectra. But as already mentioned there are other classes of spectra, and the so-called band spectra are of interest here, and specially the decomposed band spectra, that is those in which the band consists of a dense cluster of fine lines in regular sequence. (Spectra with continuous bands do not admit of the Zeeman effect in the usual sense.) The lines are here so thick and the number of them so great that the spectra on the application of moderately strong decomposing forces appear to be continuous and in the

form which might be described as fluted, that is with individual maxima of intensity, which are sharp on the one side and shade off on the other (compare the figures given later). Under stronger decomposing forces the maxima with the shading off on one side are disintegrated more or less extensively into a system of lines which here and there are crowded more thickly together according to the intensity-maxima in the same way as the series lines at the margin of the series, but according to other numerical laws. We must therefore regard the constitution of a perfect band spectra as the superposition of a great number of regular line systems which are crowded at one of their margins; but like the lines in the single bands

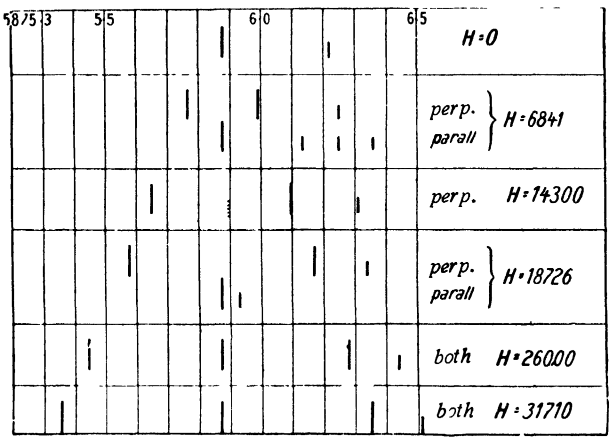

FIG 86

the single bands of a spectrum are themselves usually much more numerous than the series in the same spectrum (Voigt).

A. Dufour (1908) was the first to establish the Zeeman effect in this connection, and for this he used the haloid salts of the alkaline earths. The band spectra of the combinations named, which outside the field were specially investigated by Fabry, have a very complicated structure, and consist of a great number of superposed single bands with flutings at a relatively small distance apart. Flutings occur, some of which form the boundary of the bands on the red, and some on the violet side. For their frequency the approximate law applies $v = A \pm (B \pm Cm)^2$ in which A, B and C are constants and m may have any integral value.

The Zeeman effect was only observed at the borders of the spectrum bands, since the individual lines are crowded too close together to permit of any special investigation. The disintegration is mostly into quartets in which in the longitudinal observation the two *s* components remain visible. In the last case abnormal polarization now and then occurs; i.e., the direction of rotation is contrary to that which follows from the elementary theory of moving charges. At times also the two components appear to be imperfectly circular-polarized, which seems to indicate that the doublet in question only appears to consist of two lines but that really four are present, two of which, being of opposite rotation and of very different intensity, apparently coincide.

The calcium fluoride spectrum may serve as an example. Here the bands designated B, B′, C, D, D′, D″, D‴, were investigated by Fabry. The first two, which correspond to the law $v = A + (B + Cm)^2$, give a weaker effect; the last four, which correspond to the law $v = A + (B - Cm)^2$, a stronger effect. Reproductions from an original photograph by Dufours of the three systems of bands D, D′, D″, are given in Fig. 87. They relate to the longitudinal effect; the red end of the spectrum lies to the right. Above and below are shown photographs taken without the field for purposes of comparison. The flutings, designated D, D′, D″, correspond to wave-lengths of 603·70, 605·08, 606·45 $\mu\mu$ respectively. Of the two photographs taken with the field the upper one represents the behaviour of the waves whose rotation is in the same direction as the magnetizing current; the under one relates to waves rotating in the contrary direction. It will be seen that the disintegration of D″ is normal and that of D′ and D is abnormal. In the short interval that has elapsed since Dufour's discovery many observations have been accumulated showing numerous peculiarities and interesting relations to other phenomena.

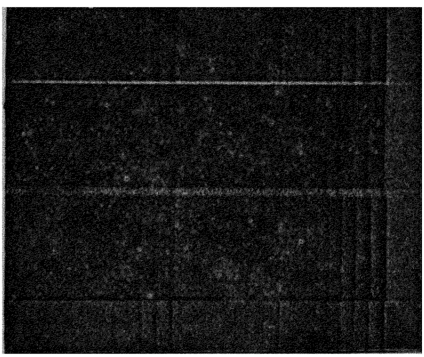

FIG. 87.

69. **Zeeman Effect and Absorption**—We are still far from having exhausted the significance of these phenomena,

and there are at least two other points to be mentioned. The first relates to magnetic double refraction in the case of absorption, and to understand this we must say something about the nature of simple refraction. In a perfectly trans-

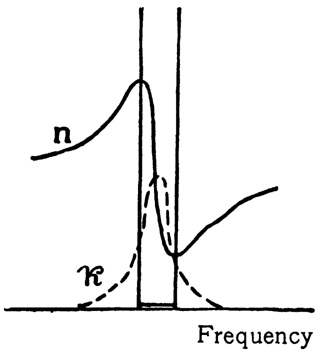

FIG. 88.

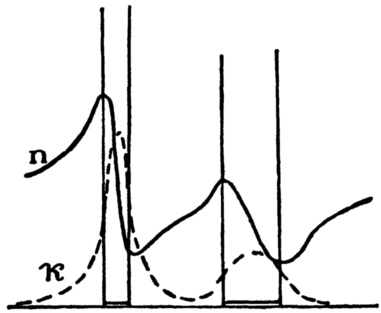

FIG. 89.

parent substance the refractive index regularly increases as the frequency of the vibrations increases; its curve, the dispersion curve, therefore continually rises. But if at any particular place there is a zone of absorption, an anomaly in the absorption presents itself, which has been already mentioned in describing the Faraday effect. The refractive index *n* falls off sharply just before the absorption effect *k* sets in, but rises again afterwards. In Fig. 88 the curve of refraction and the absorption are shown together. Two absorption bands may even be present, and then the two effects are superposed and the effect shown schematically in Fig. 89 is obtained. If however the action takes place in the presence of a magnetic field, then on account of the difference in the velocity of propagation of the rays brought about by the disintegration,

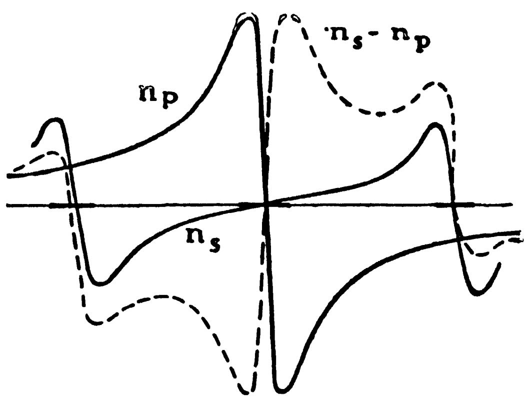

FIG 90.

double refraction occurs, which, as a more exact analysis shows in the case of transversal observations, is of the ordinary kind, but which in longitudinal observations is circular. For the former the scheme shown in Fig. 90 applies ; for the latter, that in Fig. 91. The curves of the components are in one case indicated by n_p and n_s, and in the other by $n-$ and $n+$, and the difference curves are shown by a dotted line. The thickened parts of the horizontal line show the region of observation. Voigt (1898) was the first, followed by Cotton, Zeeman and Geest, and finally Voigt and Hansen (1912), to observe and even to photograph these typical cases of magnetic double refraction.

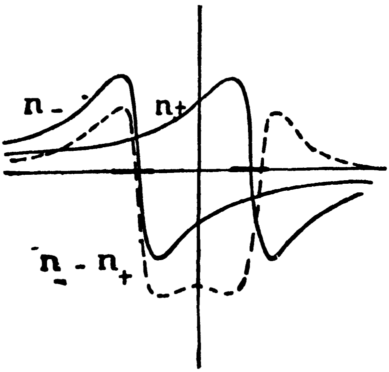

FIG. 91.

An example is given in Fig. 92 which represents the double refraction in sodium vapour (in the neighbourhood of the D line) in the direction at right-angles to the field.

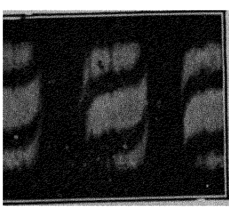

FIG. 92.

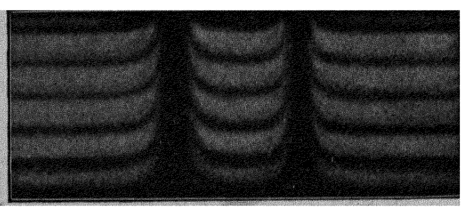

FIG. 93.

Something corresponding to the Faraday effect occurs in the neighbourhood of an absorption band ; almost simultaneously Voigt, Macaluso and Corbino demonstrated its existence experimentally. But in regard to this effect the illustration given in Fig. 93 must suffice.

In the foregoing we have confined ourselves to the consideration of the actual phenomena despite the temptation to bring forward the theoretical side of the subject. This we have done because these theories will be discussed later in their general connection, and it will then be shown that magneto-optics presents one of the richest fields in the whole realm of magnetic theory.

VII

ELECTRO-MAGNETISM

70. **General**—The last relation of magnetism we have still to deal with and it is the most important of all. There is a host of so-called "married words," words, that is, which very often occur in pairs. In physics the words "electricity and magnetism" are such a pair. It is obvious that electricity and magnetism are very closely connected, and the question now arises, What is the nature of this connection? A simple answer cannot of course be given; in the nature of things it must rather be a very complicated one. In the first place, electricity has magnetic effects; and in the second, magnetism has electrical effects (the last follows automatically from the first in virtue of the principle of reciprocal action, which is universally valid throughout nature); and the third consists in a certain parallelism between electrical and magnetic ideas, so that in a quite definite sense we can often substitute one for the other; and finally magnetism can be ultimately reduced to electricity, and therefore only one uniform class of magneto-electric phenomena remains. Indeed the theory of this class of phenomena touches still wider circles; it also includes the phenomenon of light, and lately, physics on the large scale, that is mechanics, has been shown to be, if one may so express it, no longer safe.

These phenomena and these considerations, taken as a whole, may be designated electro-magnetism, and the theory of it as electro-magnetics. But it is convenient to make a distinction so that the magnetic effects of electricity are called magnetism in the narrower sense in contradistinction to magneto-electricity, the electric effects, taken in their totality, of magnetism. In each of the two subjects a further division is then to be considered according as the phenomena in question are of the pondero-motive kind, those, that is, in which masses are set in motion and which are therefore mechanical in character (only that we attribute the specific cause to electricity or magnetism); or, on the other hand, such phenomena as those in which only the magnetic or electric condition undergoes a change (the two extremes, emergence or disappearance of the effect, being included), and which in one case may be characterized as magneto-motive and in the other as electro-motive. In the first case we are principally concerned with the magnetization of iron bodies by the electric current; or, otherwise expressed, with the fact that an electric current creates a magnetic field about itself. In the other we have to deal with the electric current created, or induced, as it is usually expressed, through magnetic action, that is with so-called magneto-induction (not to be confounded with magnetic induction, which we have fully considered at an earlier stage). A special rôle is played by a class of phenomena which are comprehended under the name of the "Hall group," and which become decisive for the question, how far, in the sense above indicated, the pondero-motive effects occur in the field that we are considering; or whether such effects only appear to occur. If we include on the one side all those questions which relate to the equivalence of electric and magnetic ideas, and on the other all applications to practical requirements, we see how rich is the field with which we are at present concerned, and how difficult our task will be to confine ourselves to what is of the greatest actual interest.

71. **Electro-magnetic Effect**—In the first place we have to make sure that static electricity, such as is produced by friction, that is an electric charge at rest, produces no sort of magnetic effect, that a magnetic needle, for example, can be brought into the immediate neighbourhood of highly electrified masses without a trace of any pondero-motive or other sort of effect being observed.. For this to occur electricity in motion is necessary; and therefore, using the word in its widest sense, an electric current.

The fundamental phenomenon we have touched upon already in the first chapter, but we shall now take up its systematic discussion. In the year 1820 Oersted discovered the deflection of the magnetic needle through the electric current. Such a deflection always takes place when an electric current flows in the neighbourhood of a magnet that is free to move. The latter assumes a particular position depending upon the current acting upon it in so far as it is free to move and having regard to any other forces which may be acting. The more powerful the current and the greater the magnetization of the needle the greater will be the deflection. In the beginning Oersted naturally used a very powerful current, so powerful that the wire through which it passed began to glow, and he at first ascribed the effect to the heated condition of the wire; but he soon found that deflection also occurred when the current was so weak that the heating was insignificant.

Immediately after Oersted's discovery was made known it was pointed out that he had been anticipated, and efforts were made, particularly in Italy, to confer the honour of the discovery upon Romagnosi, who had already in 1802 made certain observations relating to the matter. But Romagnosi in no way recognized the significance of his experiments, or he would soon have been led to the explanation of what he observed. We must therefore continue to regard Oersted as the true discoverer.

The deflection takes place equally well whether the conducting substance is in a solid or a fluid condition; that is, whether the current is passing through an electrolyte from the elements in which it is produced, and similarly whatever the electromotive force to which the current is due; it can therefore also be caused by the discharge current in static electricity, or by Hertzian waves or by earth currents. These last are responsible for the perturbations which the magnetic needle sometimes experiences and which upset magnetic and electric observations, especially when there are disturbances in the magnetism of the earth, or when observations have to be made in the neighbourhood of electric power stations. Finally the effect under consideration also manifests itself in connection with convection currents, of which we still have to speak.

The effect, it is to be emphasized, is a deflection, and is perpendicular to the plane in which the needle and the conductor lie. We have to determine what is the direction of this deflection. This is most simply expressed by Ampère's swimming rule: imagine yourself to be lying in the stream of the current in such a way that it flows from the feet to the head, and that the index finger of the right hand is placed in the direction of the current, and the middle finger in the direction towards the magnetic pole, then the thumb will indicate the direction of the force. The advantage of this rule is that the three fingers define the three directions in space and each of them one of the three vectors: current, relative position and force.

Ampère indicated suitable ways of demonstrating these forces by methods which required somewhat powerful currents (of the order of 10 amperes). More recently Kolbe has constructed apparatus which gives quite definite results with 0·5 amperes, and shows the effects very well with 1·0 ampere. In order to observe the effect of the current apart from anything else, the effect of other forces must be eliminated, and therefore in particular that of the magnetism of the earth. This is achieved either by placing the needle so that it is capable of turning in a plane perpendicular to the direction of the inclination, and the conductor carrying the current in a plane perpendicular to this, or by setting up a compensating magnet in the neighbourhood, or finally by using, instead of the simple needle, an astatic combination (see page 5). But in order that the action on the two needles of the combination may not be in a contrary sense it must be arranged that they are exposed to the effect on opposite sides and therefore that the current must be led past between the upper and the lower needle. It is then found that the needle places itself at right-angles to the line drawn from the centre of the needle to the conductor and therefore perpendicular to the plane in which this line and the current lie. If the magnetism of the earth is also acting at the same time the position of the needle will depend upon the direction and the distance of the current, and also on its strength in comparison with the strength of the magnetism of the earth. In all these and the following experiments the Ampère stands have been used, which have been perfected in various ways.

We must once more emphasize the fundamental fact that in dealing with electro-magnetic effects we have not, as in static electricity and magnetism, to deal with forces of attraction or repulsion, but with deflection and therefore lateral forces, that is with forces which are always perpendicular to the line of separation. But since in consequence of the deflection the lines of separation change their direction (Fig. 94), the forces also change their direction, they remain always perpendicular to the latter, and therefore if the latter are radii of circles, they will coincide with the arc of the circle. The result therefore is a turning of the magnetic pole about the line of current, and therefore one may consider the electro-magnetic force as a turning force. Since further the magnetic lines of force are at right-angles to the electrical, they coincide in a plane section of the field of force with the lines of magnetic potential: their respective rôles are simply reversed.

If the electro-magnetic force is one of deflection it may happen that the deviation effects proceed from various parts of the conductor, have various directions of such a kind that the deflectional components to left and right are eliminated, and that the force of repulsion or attraction is left.

72. **The Law of Biot-Savart**—So far we have only spoken of the character and direction of the effect. The complete law was given soon after Oerstedt's discovery by Biot and Savart and since that time has been called the law of Biot and Savart. It is at times also used in a more general sense for the fundamental law of electro-magnetism as well as for the electro-magnetic theory and for the general fundamental law of the field in particular. In its narrower sense the law states that the effect of an infinitely long straight line current on a magnetic pole is inversely proportional to the perpendicular distance between them and can be confirmed either by observation of the length of time of swing at various distances from the current—care being taken to eliminate or allow for the effects of the earth's field—when it is found that the time of swing is proportional to the square root of the distance and therefore the square of the time of swing is proportional to the distance, and hence the force is inversely proportional to the distance. The law may also be proved by the fact that a magnet, which for this purpose may be placed on a wooden ring hanging concentrically about the axis of the conductor and therefore capable of turning in a horizontal plane about the current which is flowing along the vertical axis, experiences no tendency to turn about this axis, from which it follows that the turning moment on the nearer and the further pole is equal and opposite, which can only happen if the force is inversely proportional to the distance. The law holds for the whole of the magnet as well as for the individual poles which are abstractions. That the force is moreover proportional to the strength of the poles and to the strength of the current can be shown by varying these two quantities. We therefore obtain, if $2c$ is the factor of proportionality, the law of Biot-Savart for

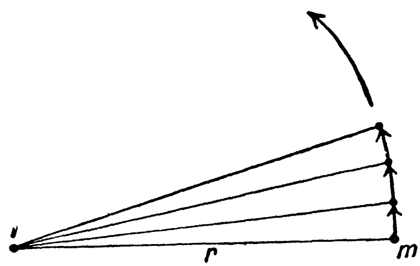

FIG. 94.

the force K of an infinitely long current on a magnetic pole in the form

$$K = \frac{2cim}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

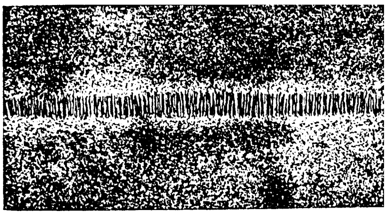

FIG. 95.

The case of a current flowing in an infinitely long straight line cannot of course be altogether realized in practice, but a very close approximation can be made by making it sufficiently long and arranging the connections so that they form a closed circuit, which of course contains the source of electro-motive force, at a sufficient distance from the needle. How far the distant parts must be in order that they do not sensibly affect the needle will depend on circumstances.

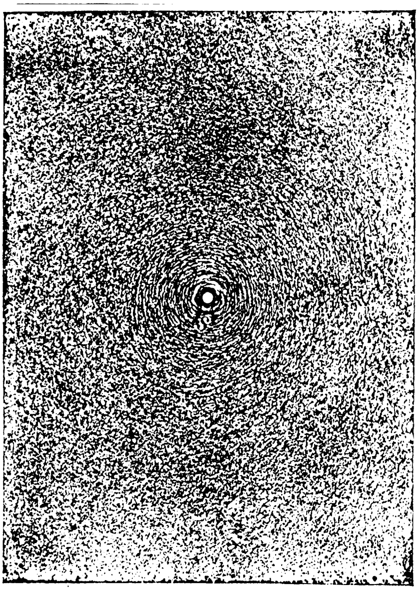

FIG. 96

Since the magnet experiences a side-ways acting force from a current flowing in a straight path, and this side-ways acting force is continuous, it follows, as has already been remarked, that we obtain for the complete line of force a circle with the path of the current as its axis ; and as the system of lines of force, a series of circles lying in planes at right-angles to the path of the current. The contour surfaces perpendicular to

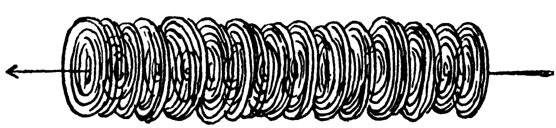

FIG. 97.

these in consequence all contain the current path. The hydrodynamic analogy of this, the axis of a vortex with the stream lines surrounding it, will be recognized, so that we may think of a current flowing in a straight line, with the lines of force surrounding it, as resembling a vortex.

We must here remark that the magnetic field of a current, or rather a plane section of it, can of course be made manifest in exactly the same way as the field of a magnet, by means of iron filings which can be photographed or otherwise fixed. But for this purpose a very powerful current must be used in order to reduce the simultaneous effect of the earth's magnetic field

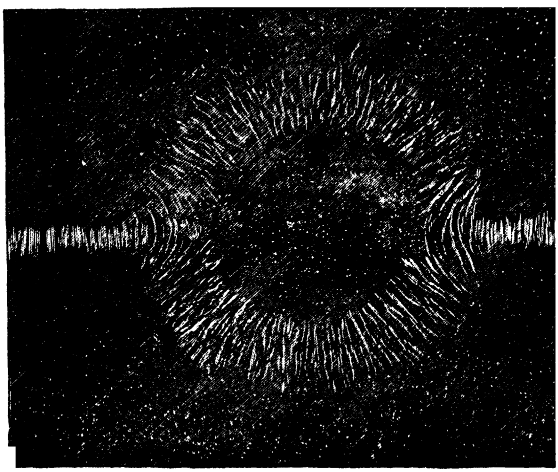

FIG 98

so that it becomes negligible in comparison. For a current in a straight line the filings on a plane surface parallel to the conductor arrange themselves in short lengths parallel to one another (Fig. 95) ; in a plane perpendicular to it they form concentric circles (Fig. 96) constituting a system of concentric cylindrical vortices (schematically represented in Fig. 97). On a flat plate through which a current is flowing the particles place themselves everywhere perpendicular to the stream lines, they therefore represent at the same time the electrical contour lines. Kirchoff in 1847 had already taken advantage of this for the experimental determination of the system of lines of force produced by the passage of a current through a plate. Since then the method has been generally used for purposes of demonstration. Later Lommel has produced pictures of this sort of which Fig. 98 (a ring-shaped sheath) is one example, and Fig. 99 (current paths forming two crossed diagonals) is another.

73. **Elementary Laws**—The abstraction made use of above, by means of which, instead of the whole magnet, its elements the single poles are thought of, can be carried still further. The current also can be resolved into its component parts, and the effect of a single element of current, which of course cannot exist alone, on one pole may be considered. Arrangements can

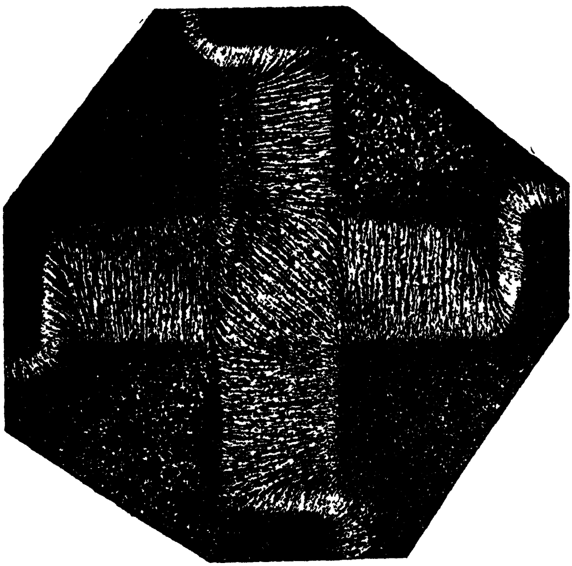

FIG. 99.

be devised by means of which this effect is more or less closely realized. We can also proceed theoretically on the assumption that these elements may be represented by a differential the integral of which will give the Biot-Savart law. In place of the reciprocal r, as will be at once understood, we have the reciprocal r^2 and then, for a current element of length dl making an angle of ε with the line from the element to the pole, we have the expression $dl \sin \varepsilon$, and for the law in question we have

$$dK = \frac{cmidl}{r^2} \sin \varepsilon.$$

But it is to be emphasized that this is not the elementary law, but only one, and that the simplest, of many possible ones. It cannot however be tested by experiment because the effect of one element of current in isolation cannot be realized, at least not in any way that is entirely free from objection.

We have now reached a point when we must say something about the constant c. In the first place it is a dimensional quantity, as a comparison of the terms on the left and right hand side of the equation makes clear; it is in fact the reciprocal of a velocity and can therefore be written in the form $1/v$. Its value may be deduced if K is measured in dynes, dl and r in cm., m in magnetic units, that is in gausses, but i in electrostatic measure. We then obtain

$$v = 3 \times 10^{10},$$

and the elementary law now runs

$$dK = \frac{midl}{vr^2} \sin \varepsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

It is to be noticed that in practice the factor v in the numerator is dropped, which can be done if i be measured in units to correspond. This new system is called the electro-magnetic and leads to the international system of practical units in which the unit value of i is called an ampere.

74. **Circular Currents and Coils** The simplest and most important case of a closed circuit is that in which the conductor forms a circular path and the pole lies upon the axis, that is upon the line which passes through the centre of the circular surfaces. The effect of each element of current can then be resolved into a component perpendicular to the plane of the circle (compare Fig. 100) and one at right-angles to it. If we then integrate the first as well as the last over all the elements of current in the circle, we obtain zero for the former, since the effect of any two elements lying on opposite sides of the circle is to annul each other so that the total force acts along the direction of the axis and assumes the character of an attraction or a repulsion; and for this force, if a is the radius of the circle and x the distance of the pole from the centre of the circle (since $q : r = a : \sqrt{a^2 + x^2}$), we have

$$X = 2\pi cim \frac{a^2}{(x^2 + a^2)^{3/2}} \quad . \quad . \quad . \quad . \quad . \quad (47)$$

and in the special case where the pole lies in the centre of the circle itself

$$X = \frac{2\pi cim}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

In words: the force is directly proportional to the current and the strength of the pole, and inversely proportional to the radius of the circle. Whether, further, it is an attraction or repulsion depends on the direction of the current and the sign of the pole, and can be deduced from the original rule. Here, however, there is a still simp'er rule: the direction in which the pole will move is related to the direction of the current round the circle in the same way as the axial rotation of a screw, i.e., if we have a north pole the relation is that of a right-handed screw (that is, a normal screw); and in the case

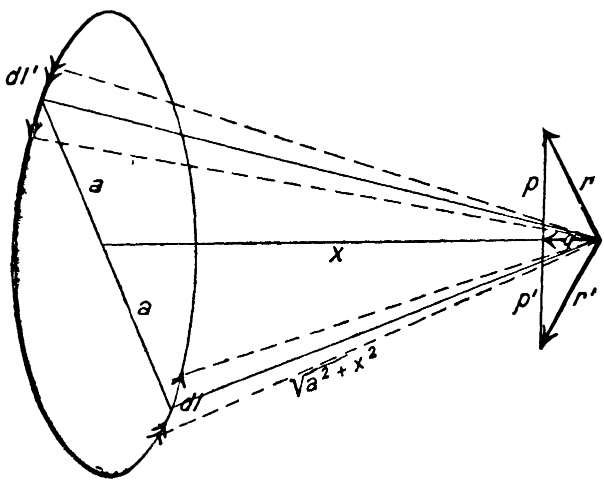

FIG. 100.

of a south pole, is that of a left-handed screw. The relation is not so simple if the pole lies outside the plane of the circle, but is still on the axis. In that case the above general formula (47) must be applied. Here the effect is directly proportional to the square of the radius, but inversely proportional to the third power of the distance of the pole from any point on the circle. If x is small, then, as will be seen, the force is greater the smaller the circle, but if x is large it is greater the greater the circle. And the limiting condition between these two cases is given when $x = \sqrt{2} \times a = 0{\cdot}7a$, that is a position of the pole for which the radius of the circle subtends an angle of about 64°.

The relation becomes more complicated when the pole does not lie on the axis but to one side of it, but in this case—and quite generally—the required result can be obtained by the use of a beautiful proposition of great simplicity which will be set forth here without proof. It relates to the potential of a circular current on a pole, that is it relates to the conception of a scalar magnitude which is applicable to the whole of physical science, namely, that the fall of potential in any direction is a measure of the force in that direction. In the present instance the proposition would run: the potential of a circular current (this expression is to be taken in the quite general sense of a current in a closed circuit) on a pole is proportional to the apparent magnitude of the surface enclosed by the current path as seen from the pole under consideration. The reader will recognize this proposition as something with which he is already familiar, since he will remember that he became acquainted with an exactly similar one in the study of pure magnetism when considering the effect of a magnetic shell—a subject to which we shall have to return. Here it may be remarked in further explanation that the apparent magnitude of a surface is equal to its actual magnitude divided by the square of its distance from the place where the eye is supposed to be placed, and multiplied by the cosine of the angle of its inclination to a plane which is at right-angles to the line of vision. By the help of this proposition the potential can easily be worked out for all cases. It is greatest for points that lie on the axis perpendicular to the plane in which the current is flowing, and zero for points in the plane of the surface. Of special interest is the case of a current which flows through coils that lie on a straight axis, that is where several circular currents are placed close together (actually, of course, the wire runs in a continuous spiral). Here only the two end surfaces come into consideration and the result is very simple: the potential of such a spiral is proportional to the difference of the apparent magnitude of the nearer and the further circular end surfaces as seen from the pole. For the force we then obtain the simple and obvious formula

$$X = \frac{\pi cimn}{l}(\cos\phi_1 - \cos\phi_2) \qquad . \quad . \quad . \quad 49$$

where c is a constant, the so-called factor of reduction, i the strength of the current, m the strength of the pole, n the number of the turns of wire, $2l$ the length of the spiral, and

ϕ_1 and ϕ_2 the angles subtended at the pole by the radii of the nearest and furthest end surfaces. If the pole is outside, the difference must be taken, but if the pole is inside the spiral, the sum, because in this case the end surfaces are seen on opposite sides. In Fig. 101 a curve is given of the internal and external field of such a spiral. It will be seen that inside the spiral over a certain portion of its length the field is moderately uniform but that it falls off rapidly at the ends. In Fig. 102 the behaviour along the axis, for the special case in which the ratio of the diameter to the length of the axis is as 1 : 4, is

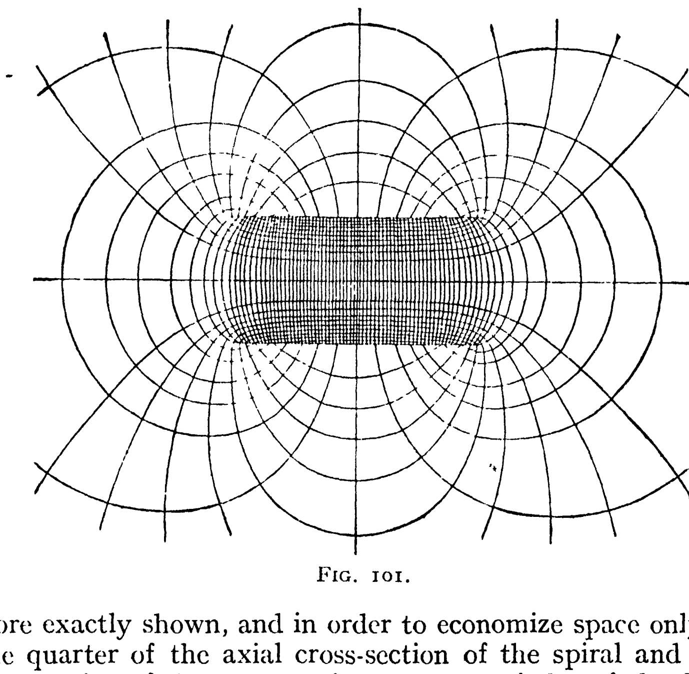

FIG. 101.

more exactly shown, and in order to economize space only for one quarter of the axial cross-section of the spiral and only for one side of the cross-section; the remainder of the figure can easily be added by imagining the part given to be reflected on the other side of the axes. The angle subtended and the value of the sum or difference of the cosines are also added and the last are also graphically represented by means of the curve on the left-hand side. As will be seen the field from the middle point outwards remains constant for a considerable space, and only as the ends are approached does it begin to diminish so that at the end surfaces it is approximately half its value at the middle. But outside the spiral it falls off very rapidly. For a spiral whose length is forty times its diameter the strength of the field for seven-eighths of the total length does not vary more than a hundredth part, and for two-thirds of the length it is constant to one part in a thousand.

An interesting way of showing the electro-magnetic spiral effect is as follows. If a bar magnet is brought near the spiral by being moved along the axial line the nearer pole is more strongly influenced than the further one. According to the position of the north and south pole of the magnet, and according to the direction of the current, the bar magnet is repelled from or attracted into the spiral, in the last case until it occupies a central position inside the coil. The first case occurs when the bar with its north (south) pole is presented to that end of the spiral in which the current looked at from the outside appears to move in the same direction as the hands of a clock

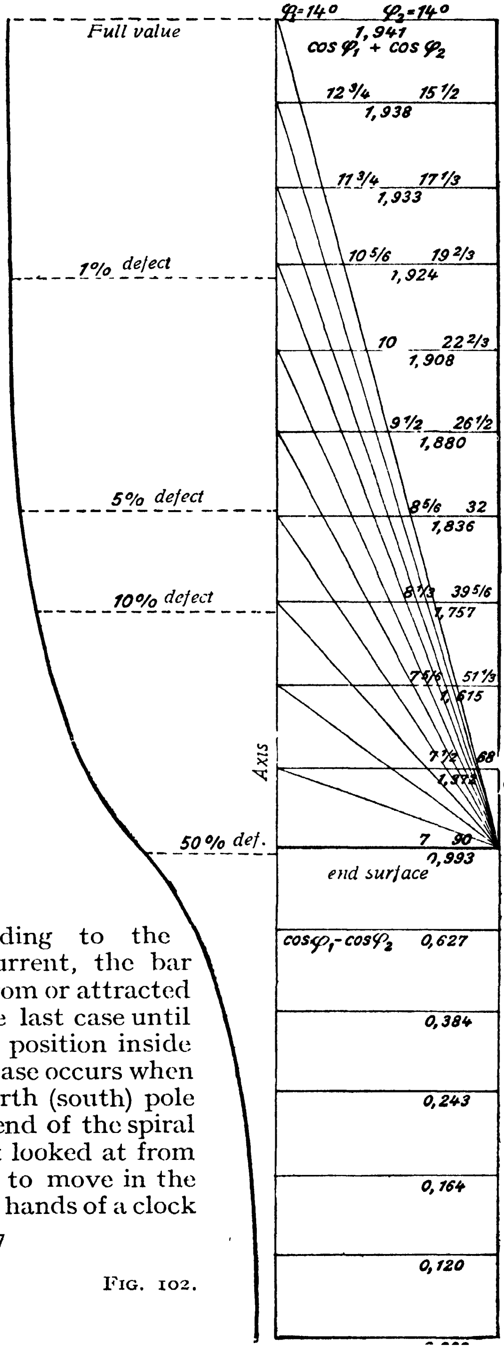

FIG. 102.

(or in the opposite direction in the case of the south pole). The experiment in which the bar is drawn into the spiral is best performed in the vertical position in order to demonstrate visually that the electro-magnetic effect, when the strength of the current and the magnetism of the bar are sufficiently strong, is able to overcome the action of gravity. The bar will then float inside the spiral, but as soon as the current is switched off it will drop out again. If the bar is so long that the two ends protrude above and below it can be pressed downwards with the finger and will then be seen to make vibrations about the central position. If the current is reversed while the bar is in the central position it should theoretically still be in equilibrium. It is however unstable and on the slightest displacement it falls out. Similar experiments are also possible with a bar of soft iron that is initially unmagnetized, but in this case the position of the induced poles is always such that attraction ensues. In addition the introduction of a bar of soft iron, or a bundle of soft iron wires, has the effect of considerably increasing the strength of the field, in the extreme case by as much as μ times the original value, where μ is the permeability of the iron used. The above phenomenon may be described as the magnetic suction effect.

Of other forms of spiral windings, that of the closed ring is of especial interest as it represents the principal part of the oldest form of electric dynamo—the Gramme ring. In this case we have the simplification that the magnetic circuit is closed and therefore there are no poles. The strength of the field is thus the same all round the axis of the ring. On the contrary it is not the same at different points of the same cross-section because the number of windings per unit length is greater on the inner side of the ring; and in fact the maximum and minimum strength are simply in the ratio of the external and internal diameters of the ring. The smaller the cross-section in comparison with the mean diameter of the ring, the less significant does this difference become, and if the ring is very thin the field can be regarded as uniform. It is only necessary however that the section should be narrow in the plane of the ring; in the direction at right-angles to this the thickness does not matter. In this way we arrive at the flat ring type.

75. **Applications**.—The electro-magnetic effect permits of a great number of important applications, some of them scientific, especially in connection with measurement, some

of them purely technical. Only a very brief survey can be given of these applications.

In regard to measurement there is in the first place the tangent galvanometer, which consists of one or more circular windings of copper wire through which the current is passed, and a small magnet needle hanging in the plane of the windings, which is itself placed in the magnetic meridian. The deflection of this needle produced by the current is observed, and from the angle of deflection ϕ the intensity of the current i can be calculated, if $\mathfrak{H}$ the horizontal component of the earth's magnetism, and the constant c (see below), the so-called reduction factor of the instrument, is known. (It is most easily obtained from the electro-chemical effect of the same current.) The needle takes up a position of equilibrium in which the deflectional force of the current is just equal to that of the magnetism of the earth tending to turn it back to its natural position. The first effect is proportional to the sine of the angle of deflection, the second to the cosine; their ratio, therefore, to the tangent. We thus easily obtain for the calculation of i the formula

$$i = \frac{\mathfrak{H}}{c} \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

or if the reduction factor is eliminated by means of a simple calculation and i expressed in amperes (the C.G.S. unit, being ten times greater than the ampere i, would be ten times too small), and if the radius of the copper windings is a, then

$$i = \frac{10}{2\pi} \mathfrak{H} a \tan \phi = 1{\cdot}592\, \mathfrak{H} a \tan \phi \quad . \quad . \quad . \quad (51)$$

This formula is of course only approximately correct, the approximation being closer the shorter the length of the needle $2l$ in comparison with the diameter of the circle $2a$ and the smaller the angle ϕ. A more exact formula runs

$$i = 1{\cdot}592 \mathfrak{H} a \tan \phi \left[1 - \frac{3}{16}\left(\frac{2l}{a}\right)^2 (1 - 5 \sin^2 \phi) \right] \quad . \quad (52)$$

The angle ϕ can either be read off direct by means of a divided circle (but in that case the needle, which must be as short as possible in order to obtain accuracy, has to be extended by means of a thin light rod in order that the circle on which the readings are taken may be sufficiently large) or readings are taken by means of a small mirror attached to the needle.

For particular purposes, as for example the measurement of a very strong or very weak current, the tangent galvanometer may be modified by arranging that the needle does not hang in the plane of the circle, or by placing the ring of wire obliquely to the vertical plane, or by taking two windings of different diameter and sending the current in opposite directions through them so that a differential effect is produced; or, on the other hand, instead of one winding, using several through all of which the current flows in the same direction so that the effect is intensified.

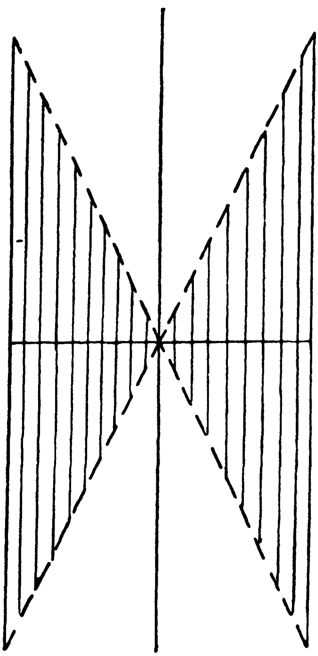

FIG 103

Both as regards theory and practice the most important example is that of the tangent galvanometer of Gaugain and Helmholtz in which the needle hangs at the apex of two cones formed by the two sides of the windings. (Fig. 103 gives the side view diagrammatically.) If the angle of the cone is made equal to 127°, then the distance of the needle from the plane of each circle is equal to the half-radius of the circle, and in the general formula, which is of course valid for this case, for the turning moment.

$$D = 2\pi a^2 i \frac{2l \cos\phi}{u^3}\left[1 + \frac{3}{4}\frac{a^2 - 4x^2}{u^2}\left(1 - 5\sin^2\phi\frac{1^2}{u^2}\right)\right]. \quad (53)$$

where x is the distance of the needle from the plane of the circle, u that of a point from the circumference, the correction term becomes zero and the field in the neighbourhood of the needle extraordinarily uniform. In Fig. 104 this is seen from the disposition and spacing of the lines of force and the equipotential lines.

Galvanometers have been the subject of the most varied development with coils of many windings, in the middle of which, or between which, the magnet hangs. It may be safely stated that there are few departments of the art of measurement which have at their disposal so many exact methods. We have the adjustable type of galvanometer, in which the spools can be placed at a variable distance from one another; galvanometers enclosed in iron shields which protect the instrument from magneto-inductive effects due to external disturbances such as electric tramways; the universal galvanometer, which is provided with means for modifying the connections in a great variety of ways, and so on.

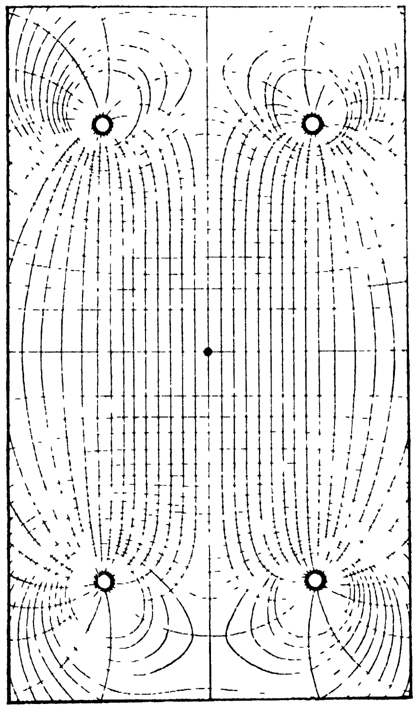

Fig. 104.

Instead of the deflectional effect, the above-mentioned suction effect may be utilized in instruments designed for the measurement of electric current; and this idea in the form of the current balance for technical purposes has undergone prolific developments. The instrument consists of a spool over which hangs a light iron bar (or bar magnet), and through which the current to be measured is passed, which has the effect of drawing the bar down into the coil to a certain amount. The amount of motion can be read off by means of a scale and this can be reduced to the corresponding value of the current in case the scale is not itself divided in such a way as to give the value of the current directly. The counter-force is provided by an elastic spiral spring from which the bar hangs, or a crank lever with a suitable counter-weight at the other end.

Another application of the suction effect is found in the electro-magnetic regulators for arc lamps. When in consequence of the burning away of the moving carbon its distance from the fixed carbon becomes too great the weakening of the current, which is led through a coil, causes an iron core to rise, and this lowers the carbon which is fixed at the other end;

the regulation is most perfectly achieved in the case of the differential lamp, which has two coils with a common iron core between them. Another application is that of the magnetic brake. As regards rotation apparatus compare what is said later.

76. **Magnetic Effect of Convection**– It has already been mentioned that the magnetic effect only occurs when electricity is in motion. But it is indifferent how this motion comes about. Not merely actual electric currents, which we may call currents through conductors, but also convection streams create a magnetic field in their vicinity. Convection occurs when a charged body is set in motion and carries its charge along with it. There would be an exception to this effect if it should happen that the charge is attached to the space itself and therefore slips back relatively to the body carrying it. It is therefore of the very highest fundamental interest to make experiments on this point, and in this connection the effect on magnetic deflection is a very suitable one. The experiment, however, is very difficult to carry out, and it has been shown that a great complication and diversity of phenomena present themselves. Generally there are two kinds of phenomena to be distinguished, according to which the moving body is a conductor or a non-conductor (dielectric). The former phenomenon is called after its discoverer, the Rowland effect, and the latter the Röntgen effect. For the motion, a rotation is of course preferred, and for the moving body a disk; and in all cases, whatever the method of performing the experiment, it is found that the results obtained are of quite normal character and the magnetic needle experiences the deflection anticipated. From this it must be inferred that the electricity is carried along with the moving body and that this convection is on a par with the ordinary conduction current passing through a conductor at rest.

77. **Magneto-electric Effect**—From the principle of reciprocal action it necessarily follows as already indicated that if an electric current exerts a pondero-motive effect on a magnet, the converse must also happen, and that the same laws with the necessary modifications must hold in the one case as in the other. Most of the experiments relating to this subject were originated by Ampère. He used for the purpose of inducing the current a double support that terminated in two small cups filled with mercury. In these cups floated the conductor used in the experiment on two points, and thus it was

very easily set in motion. It is also possible to secure buoyancy by means of a cork. For delicate measurements the conductor must be suspended from one or still better two long wires through which the current is introduced.

The effect of a pole on an element of current is given by equation (46) on page 153. If the constant v be put equal to unity and the electro-magnetic system of units be employed:

$$dK = \frac{im}{r^2} dl \sin \varepsilon \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Here the factor $\frac{m}{r^2}$ is the field of the pole; for a field in any given magnetic measure the strength of which at the place of the element of current is $\mathfrak{H}$, we have then the general equation

$$dK = \mathfrak{H} i dl \sin \varepsilon \quad . \quad . \quad . \quad . \quad . \quad (55)$$

which can be expressed in words as follows: the force which an element of current experiences in a magnetic field is equal to the product of the strength of the current into the area of a parallelogram the two adjacent sides of which represent the field and the element in magnitude and direction. The force is at right-angles to the plane of this parallelogram and its direction is given by the Ampère rule; i.e., suppose yourself to be swimming in the direction of the current and looking in the direction of the field, then the direction of the force is to the left-hand side. Or the left-hand rule may be used: let the index finger of the left hand point in the direction of the field, the middle finger in the direction of the current, then the outstretched thumb will show the direction of the force. Finally, the following rule can also be used in such a case as this. Imagine a corkscrew of the ordinary kind to be placed upright on the plane formed by the element and the direction of the field, then the upward or the downward motion of the axis of the corkscrew will indicate the direction of the force. The force is greatest when the element of current is at right-angles to the direction of the field, and is nothing when the direction of the two coincide. In any case except the last-named the current will move through the field across the lines of force. The same rule holds whether the current be flowing in a finite or an infinitely long straight line, and from what has been said above it will be easy to determine the direction of the motion. For example, in the earth's field a downward-flowing current will experience a force tending to move it to the east; an upward-flowing current a force towards the west. Generally the direction of motion is reversed when either the direction of the current or the field is reversed; if they are both reversed the direction of motion remains the same.

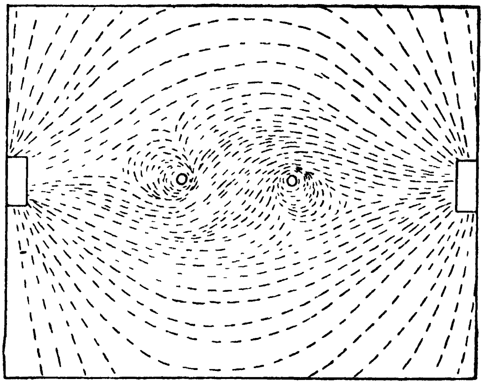

FIG 105.

If a circuit has a particular shape, as, for example, a circle, it is clear that this will not move bodily through a uniform field but will only tend to rotate, and will continue to do so until a position is reached in which the opposite effects on the elements traversed by opposing currents neutralize each other. This obviously occurs when the position is symmetrical; a

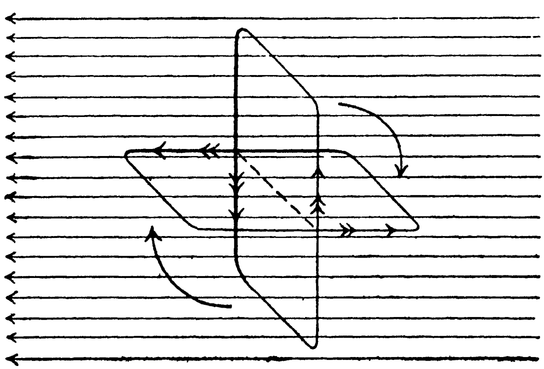

FIG. 106.

circular current will place itself with its plane at right-angles to the field. In Fig. 105 the lines are shown at the moment when the circle lies with its plane in the direction of the field and therefore before it has turned into its final position. Moreover, it is clear that there are two positions at right-angles to the field, and that of these two positions only one is the right one, corresponding, that is, to the position of stable equilibrium, while the other is unstable. This may be demonstrated by a pretty experiment. If a coil carrying a current be laid on the pole of an electro-magnet and the direction of the current through the coil and that of the magnetizing current round the magnet is such that there is stable equilibrium, and if then we reverse one of the two currents, then if there is the slightest asymmetry of position the coil will jump up into the air and turn itself about before falling back on the pole. With a U-shaped magnet it can be caused to move from one pole to the other.

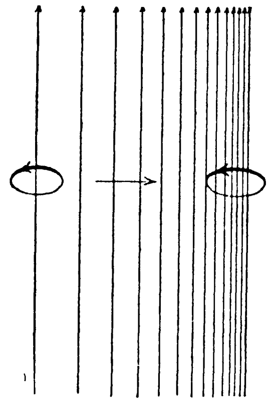

FIG. 107.

The position taken up by a closed current is most clearly expressed by the following proposition which was originated by Gauss but further developed by Maxwell: an electric circuit so places itself in the magnetic field that the number of lines of force passing through it is a maximum. This is valid not only for uniform but also for non-uniform fields, and in such cases not

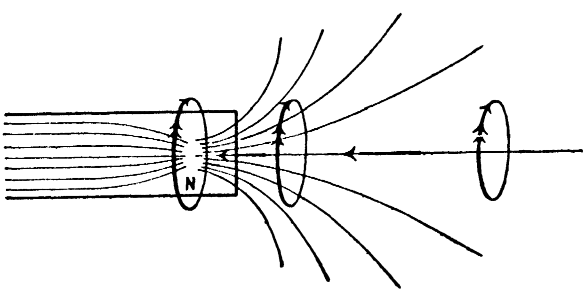

FIG. 108.

only rotation but also lateral displacement takes place and therefore attraction to, or repulsion from, one part of the field to another. This can be followed out in numerous experiments in which, for example, a moving circuit is drawn towards or repelled from a magnetic pole or drawn to the axis of a bar magnet, or in certain circumstances repelled from it. Figs. 106 to 109 will make this clear and call for no special explanation. A coil will, of course, behave like a closed circuit, that is, it will place its axis parallel to the field exactly like a magnet. The motion will be more decided and energetic the stronger the current, the stronger the field, and the greater the number of windings.

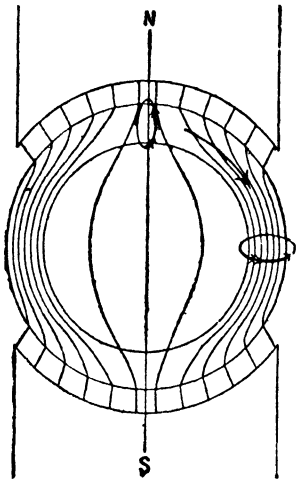

Fig. 109.

In the above considerations artificial magnets were assumed; but the same remarks apply equally well to the magnetism of the earth. For in the field due to the earth a coil suspended bifilarly or floating with its ends dipping into mercury cups will place itself in the axial direction. With such a coil all the experiments can be performed that can ordinarily be performed with a magnet and corresponding results will be obtained. In particular such a coil under the influence of the magnetism of the earth and of an artificial field at right-angles to it will take up a certain angle.

The magneto-electric effect is capable of application to measuring instruments which are spoken of as galvanometers; but in order to express the fact that in contradistinction to the earlier instruments, the magnetic system is fixed and the coil with the current flowing through it is movable, it is therefore specially designated the moving coil galvanometer (Deprez-Darsonval). Of course the magnitudes and relations of the masses are reversed; a massive magneto system is required with a very light moving coil. If the plane of the conductor is originally in the plane of the magnetic medium, and if, when the current is passing, there is equilibrium between the magneto-electric force and that due to torsion in the opposite, then for an angle of deflection ϕ

$$i = \frac{C}{\mathfrak{H}F} \cdot \frac{\phi}{\cos\phi} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (56)$$

where C is the constant of torsion and F the winding surface of the coil. These instruments are well adapted for many purposes because they are far less affected than others by external electrical disturbances.

The effect of the magnetic field is not confined to solid conductors, but also presents itself in the case of fluids and gases, and this leads to especially interesting phenomena where the discharge of electricity in any way is in question. The deflection of the electric arc between carbon or platinum points by the magnetic field was discovered by Davy. And as regards the electric spark of the induction coil, this itself is not deflected from its path but only the cover of light with which it is surrounded. At right-angles to the field simple deflection takes place, but with discharges parallel to the field the light is limited by an S-shaped surface since the two halves are deflected in opposite directions; the middle point remains undeflected. Of very great interest is the influence of the field on the various sorts of rays which have been discovered in the last two or three decades, and which give so attractive a form to the total picture of electric phenomena. That all these rays experience deflection in the magnetic field is indeed scarcely to be doubted; but there are, nevertheless, great distinctions to be made. The most striking relates to the effect of the field on the cathode rays, and the beta rays of radio-active substances; it is less in the case of the canal, anode and alpha rays; least of all in the case of the Röntgen and gamma rays. There is here still another decisive contrast; the direction of the deflection in the canal and alpha rays is opposite to what it is in the cathode and beta rays. All this has in the course of time led to the assumption that in all these rays we have to deal with convection rays, that is, with a stream of thrown-off particles of matter which are very varied in their nature: the canal rays, the alpha rays and the ions being positively charged, while the cathode and beta rays, on the contrary, are very much smaller and therefore much more easily deflected particles, and together with the electrons are negatively charged. These contrasts present themselves in the most interesting fashion in the case of radio-active substances since they throw off at the same time alpha, beta and gamma rays. If this discharge takes place in a magnetic field we have the effect which is shown schematically in Fig. 110. Further, as regards the quantitative laws of the deflection, this depends, to take the cathode rays as an example, apart from the strength of the field to which it is proportional, on the density of the gas in the discharge tube, on the chemical nature of the gas, on the dimensions of the tube and so forth. The greater the potential difference between the electrodes the more drawn out and smaller this is, and the more curved the path of the electrons (just as in the case of a projectile it depends upon the charge); the curvature and the shape of the path can both be calculated. It is of importance as a determinative magnitude to establish the velocity of the electrons. If this is v and the mass m, the electric charge e and the strength of the field $\mathfrak{H}$, we obtain for the radius of curvature the formula

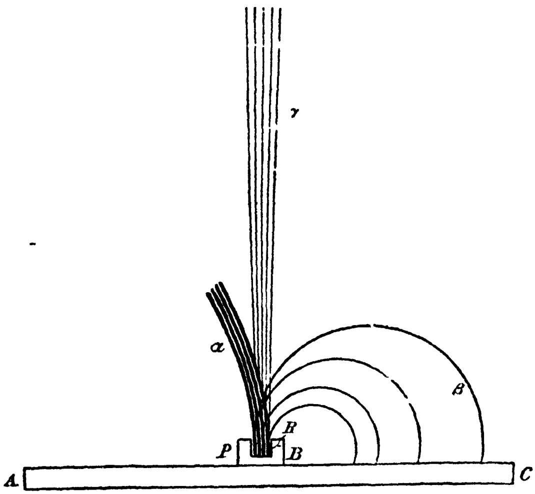

FIG. 110

$$r = \frac{m}{e} \cdot \frac{v}{\mathfrak{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

in words: the curvature of the cathode rays in the magnetic field is directly proportional to the strength of the field, inversely proportional to the velocity, and the proportionality is the relation of charge to mass. If v can be determined independently, e/m can be calculated, and inversely.

78. **Equivalence of Currents and Magnets**—From the pondero-motive effect considered as a whole, one conclusion can be drawn which is decisive for the theory of magnetism. Any given arrangement of magnets is always, so far at least as its outward effects are concerned, equivalent to a given arrangement of currents. We shall consider here only the most important cases of this equivalence:

(1) An infinitely small closed circuit is equivalent to a single pair of poles whose axis is perpendicular to the plane in which the current flows.

(2) A finite closed circuit is equivalent to a shell of the

same shape which, whatever its shape, is supposed to have the line of current at its edge; a circular closed circuit is therefore equivalent to a circular shell; an unbounded straight current equivalent to a shell which, from the line of current as a margin, stretches away into infinity.

(3) A system of concentric circular currents in the same plane and in the same direction, in practice, therefore, a flat coil, is equivalent to a non-uniform shell.

(4) A system of infinitely small circular currents of equal strength, size and in the same direction, and at the same distance from one another, and therefore forming a cylinder or solenoid, is equivalent to a simple magnet filament, that is, to a pair of opposite poles at the ends.

(5) A solenoid of this sort unbounded on the one side is equivalent to a single magnetic pole.

(6) A cylindric system of finite circular currents, therefore in practice a coil of insulated wire with a single layer of windings, is equivalent to a solenoidal magnet, that is, to two opposite disks at the two ends.

(7) A cylindric coil of insulated wire of many layers is equivalent to two opposite non-uniform disks at the two ends.

(8) A coil of wire of any shape and arrangement is equivalent to the magnet of general type which it determines.

(9) The solenoid closed upon itself is equivalent to a closed magnetic filament, therefore a ring-shaped coil of wire, to a ring magnet; neither of these arrangements exercises any outward effect.

To complete the equivalence it is only necessary to find out which are the corresponding sides. For this we merely require to apply Ampère's rule to the various forms mentioned above and the following formula is obtained: if one thinks of oneself as swimming in a closed circuit or coil and looking towards the inner side of it, that side or end of the coil which then lies on the left-hand side corresponds to the inner side of a shell, and the right side or end to the south pole. Or: that side of a closed circuit in which the current appears to flow in an anti-clockwise direction is the north end or side; the other is the equivalent to the south side of a magnetic shell. The former can also be designated the north side of the current and the latter the south side. The same remark applies to the north and south end of a coil of wire. Or conversely: if one stands on the north surface of a shell and looks at the equivalent current flowing at its edge, then this current flows from right to left.

Internally of course the relations are quite different: a bar magnet is a solid thing; the current coil is hollow and consequently in making any comparison of the internal fields one must proceed very cautiously. It would lead us too far to follow out these considerations here. The most fruitful application of this idea of equivalence now under discussion relates indeed to what goes on inside the magnet, namely, to the electrical theory of its magnetic condition, and this we will discuss later in a chapter on the theory.

79. **Rotation Apparatus**—If the electro-magnetic and the magneto-electric effects are rotational in character it must obviously be possible through the application of magnetic and electrical energy to produce rotation, in the first place of magnets about fixed conductors, and in the second place the converse. The difficulty in the realization of this idea is as follows: a magnetic pole does not exist, there exist only magnets, that is, pairs of poles, and such a magnet cannot be set in rotation, for if its north pole tends to go round to the left, its south pole would tend to go round to the right, and these two tendencies would annul each other. At least they would do so if the magnet is rigid; if we could think of it as flexible (like a thin elastic band), then it would wind itself into a spiral round the conductor, but even here the rotation would soon come to an end in consequence of the condition of entanglement that would ensue. If we take the other case, the fixed magnet and the movable conductor, the difficulty which then arises is that this movable conductor must still form part of a closed circuit, one part of which must be at rest if any effect is to result, while the other is movable. And if the latter rotates this again must lead to a frightful complication of contacts which would soon bring the rotation to an end. This can be avoided either through the use of sliding surfaces, or slipping contacts, liquid connections or through the periodical breaking and reversing of the current, effected, of course, by means of some automatic device.

It is on this principle that the numerous apparatus for producing rotation are based, that have been built or described during the last century, especially in the earlier part of that period. They are still instructive to-day on account both of the features that they all possess in common, and also on account of the peculiarities that some of them present. But simple as is the general principle, so complicated does the theoretical and critical examination of each individual case become that even in quite recent times very animated discussions have taken place in regard to them. It must suffice here to represent a few types graphically. Fig. 111 shows the apparatus of Pohl. Here two parallel magnets with like poles together rotate about their common axis, the current from the source passing from *c* to *a* to the mercury cup *b*,

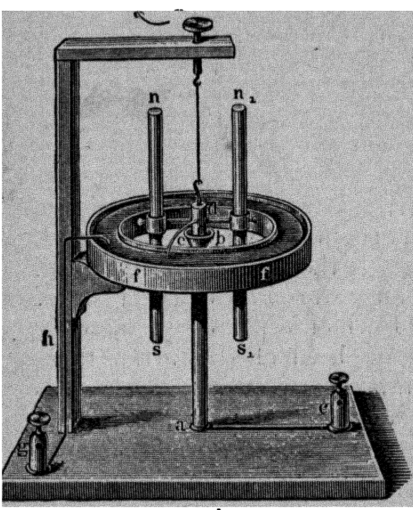

FIG. 111.

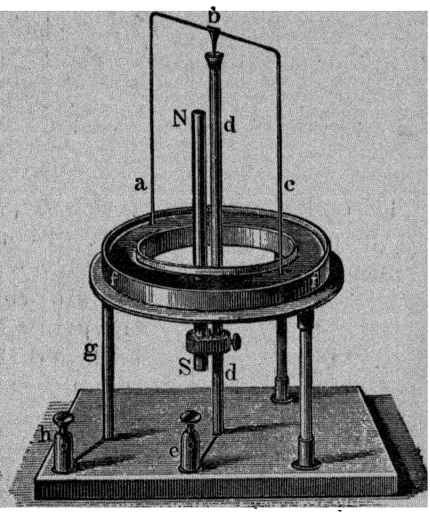

FIG. 112.

through the moving bent piece *e* to the mercury ring *f*, and from here returning by means of the wire *h* to the terminal *g* and thence to the source. The electro-magnetic effect of the current causes a motion of the two connected magnets which, when looked at from above, is in the clockwise direction, but which is reversed if either the direction of the current or the polarity of the pair of magnets is reversed. The motion would not take place if the pair had two unlike poles together or if the current were led in at the top of the axis on a level with the upper poles instead of at the middle. Instead of the pair of magnets a whole cluster of them may be used or a tubular magnet concentric about the axis of rotation. In the apparatus in Fig. 112 the magnet is fixed and the conductor is movable. The current goes from *e* through *d* to *b*, here divides between the two branches *a* and *c*, and then goes through the mercury and the lug *g* to the terminal *h*. The rotation looked at from above is clockwise.

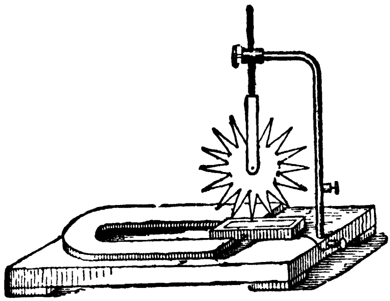

FIG. 113.

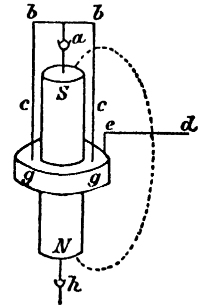

FIG. 114.

That the position of the magnets is somewhat out of the centre is only a necessity of construction. Of historical and, just lately, of practical interest is the Barlow wheel (Fig. 113), in which a star-shaped wheel, one of the points of which dips into a mercury bath and through which the current flows radially, rotates between the poles of a U-magnet. More recently two other types have come to the front. The first is the design of J. Weber (Fig. 114), with axial magnets which at the same time serve as conductors for the current, the return circuit being provided for by the double loop *cbbc* which dips into the annular arm, surrounding but insulated from the magnet. The second is the apparatus due to Nikolajew (Fig. 115). NS is the turning magnet. One end of its winding Z dips into the fixed cup A, the other, ML, into the annular cup LE, which moves with the magnet. If a current flows through ABCDELMZA the magnet rotates. On the contrary, it remains motionless if M is connected to the fixed annular cup PQ instead of to LE. PQ is connected through TR and EL. Finally, it again rotates if the current be carried through FK instead of through DE.

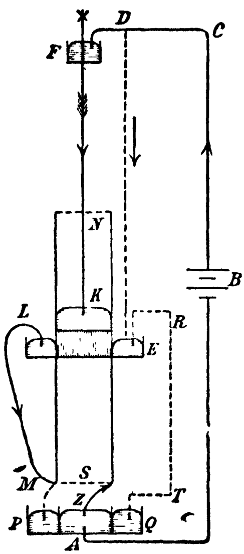

FIG. 115.

80. **Electric Motors**—So far we have been concerned only with rotation apparatus which were of theoretical interest or only useful for purposes of demonstration, but now we come to apparatus of the same kind which are of technical importance. In dealing with electric motors we shall have to be very brief because the details of this subject belong rather to electro-technology. Moreover, there is another point to be remembered: the electro-motor is the converse of another piece of apparatus, the generator or machine for producing current (dynamo). The first serves for the conversion of electric energy into mechanical energy; the latter for the conversion of mechanical energy into electric. Since in these machines the process is perfectly reversible in principle (and also to a large extent in practice) the same model will serve both as a generatoı and a motor. Divergences only arise in the methods of starting up or shutting down of two other sorts of energy, the kinetic energy of the moving parts and the magnetic energy of the field which supplies the opposing electro-motive force. A further difference arises from the service which the model is intended to perform. In practice the creation of electrical energy is almost entirely confined to high-power machines, but in motors the smallest fraction of a horse-power may be required. In small machines the details are therefore substantially different and will necessarily depend on the special circumstances of the case. Apart from this the theory of motors which is based on electro-magnetism coincides with the theory of generators, which on their side come under the head of the induction of currents, and will be dealt with in that connection.

The electro-motor consists of two parts: the field and the armature. The former is fixed, the latter is set in motion by the currents which are sent through it. While in the earliest forms of the machine, now for the most part only of historical interest, the field is energized by one current and the armature coils by another, since the discovery of the principle of the dynamo the energizing of the two parts is derived from a common source (though in particular cases and for practical reasons a separation may be made). Three types are to be distinguished according as the current passes through the two parts one after another, or at the same time, or follows a mixed system. We have therefore series, shunt-wound, and compound motors. The last-named system, which is of especial importance in the case of generators, is of no particular significance in so far as motors are concerned. We are therefore left with the two first-named types, which are shown diagrammatically in Figs. 116 and 117. A glance at the figures and a very little consideration will show that a series machine when used as a motor will turn in the opposite direction to that in which it must turn as a generator for the same direction of the current through the machine, but that a shuntwound motor will turn in the same direction in both cases. Further differences arise in regard to the demagnetizing effect of the armature currents on the field; the neutral zone, over which sparking is reduced to a minimum, in the dynamo is displaced in the direction of rotation, but in motors in the opposite direction.

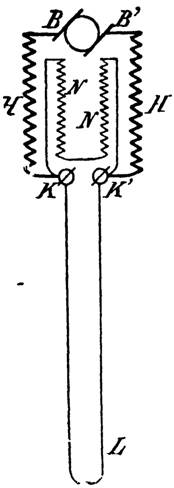

FIG. 116.

The efficiency of an electro-motor is given by the formula

$$\eta = \frac{Ei^{a} - v}{P \,.\, i} \quad . \quad . \quad . \qquad (58)$$

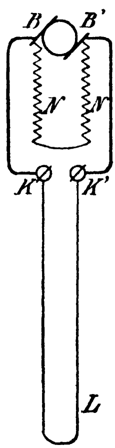

FIG 117.

where E is the back electro-motive force of the armature, P the potential difference at the terminals, i_a the current through the armature, i the external current, and v the sum of all the losses (through mechanical friction, eddy current and hysteresis). Since v can be greatly diminished by sub-division of the iron and proper selection of the material, and since i_a can be made very nearly equal to i and E nearly equal to P, the efficiency can be made very nearly unity. As a matter of fact, except in the smallest models it usually amounts to 80 per cent. or 90 per cent. and sometimes to as much as 95 per cent. Therefore from an economic point of view the electric motor surpasses every other kind of motor and particularly the steam engine, and is therefore more and more superseding it.

The method of working of the direct-current motor is to be deduced from the following equations:

(1) The induced electro-motive force

$$E = N\Phi \frac{U}{60} \cdot \frac{p}{a} \cdot 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad (59)$$

(2) The potential difference at the terminals

$$P = E + i_a w \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

(3) The turning moment

$$D = 1{\cdot}02 \frac{N\Phi i_a}{2\pi} \cdot \frac{p}{a} \cdot 10^{-7} \text{ kg.} \quad . \quad . \quad . \quad . \quad (61)$$

Here Φ denotes the flux of lines of force per pole; to a first approximation it is a function of the magnetizing current i_f, to a second of the armature current as well. In series motors $i_f = i_a$, in shunt motors $i_f = \frac{P}{w_f}$ (w is the resistance, w_f the

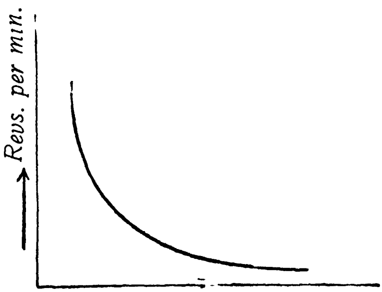

FIG. 118.

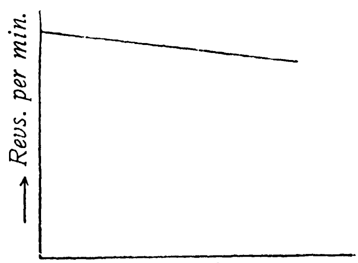

FIG. 119.

resistance of the field windings). N is the number of armature windings, U the number of rotations per minute, p the number of pairs of poles, and a the number of parallel circuits through the armature.

The behaviour of the motor can be represented by its various characteristics, i.e. by curves which show some one important quantity as a function of one of the determinative quantities. In motors, of course, as opposed to generators, the most important characteristic is the speed of rotation. That of a series motor is given in Fig. 118. The turning moment, so far as the field is proportional to the current, increases with the square of the current. The rotation characteristic of the shunt motor is typically represented in Fig. 119. Here the speed of rotation decreases only very slightly between no load and full load (by some 4 or 8 per cent.), and it is easy to keep it quite constant. Here the turning moment is proportional to the current through the armature. Where a large turning moment is required on starting up (as in trains or lifts), the series motor is to be preferred. Where a constant speed is required (pumps, driving of machine tools, etc.), the shunt motor is the better adapted.

The most important practical distinction (apart from that between direct, alternating and polyphase current motors) is that between synchronous and asynchronous motors, each of which two types has certain advantages in regard to the nature of the supply, constancy and certainty of performance, etc.

81. **Interrupters and Vibration Apparatus**—The series

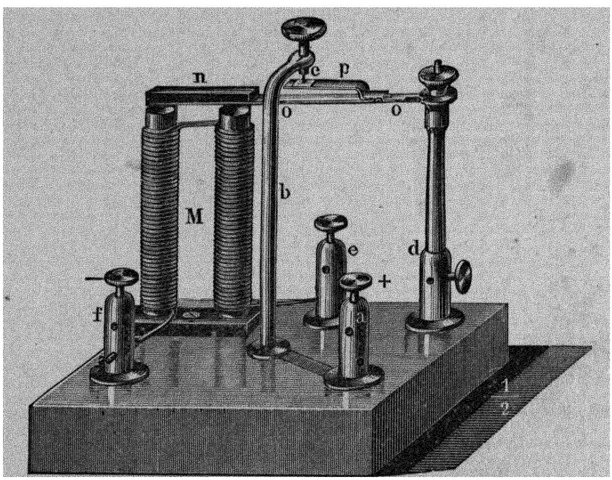

FIG. 120.

of practical applications of electro-magnetism has still to be amplified by mention of apparatus which are concerned not with rotation but with some other sort of periodic motion. In the first place, we have the interrupters, that indispensable component of a great variety of apparatus, as, for example, the induction coil. They are contrivances which break and again close an electric circuit at regular intervals which can be controlled within very wide limits of frequency as required. For the most part a quick alternation is required so that the current must be intercepted several times in the course of a second, a procedure which is usually rendered audible to the ear as a humming or singing note of the same frequency. The most important application of a current interrupted in this way is in the creation of induced alternating currents. Another great field of application is in telegraphy with all its special requirements, especially relays.

The best-known type of interrupter is the Wagner hummer, which was first described however not by Wagner but by Neef. Its most usual form, which it owes to Poggendorf, is shown in Fig. 120. A steel spring, *opn*, rests with its contact piece *n*, which constitutes the armature, a short distance above the electro-magnet M. As soon as it is drawn down by the magnet the current energizing the magnet is interrupted by the circuit being broken at the point *c*. The spring thereupon flies back to its original position and is then again drawn down and so on. The battery is connected to the terminals *d* and *f*, and to *a* and *e*, the wire in which the current is to be interrupted. The periodicity of the arrangement depends upon the form, the distance, and the elastic qualities of the spring *ab*. A second piece of apparatus of the same kind is the string interrupter in its various forms, of which that due to Max Wien is the most carefully thought out and most generally in use. In the third place we have the tuning-fork interrupter: as an electro-magnetic device for obtaining a continuous uniform tone it owes much to Helmholtz and R. Koenig. A typical example is shown in Fig. 121, which is one of the newer forms of the apparatus, which are distinguished from the older forms by greater simplicity and perfection of construction. On the base S a tuning fork is placed so that the plane in which its prongs vibrate is vertical, the axis of the fork being horizontal. The upper prong carries a bunch of platinum wires *δ* attached to a piece of brass *m*, the under

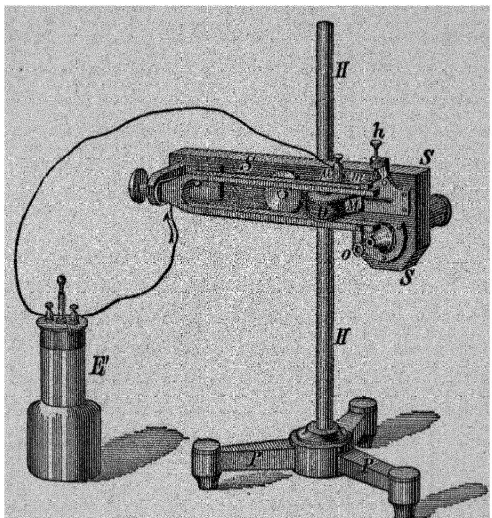

FIG. 121.

prong an object glass with which we are not concerned here. Through a slot in the front part of the base another piece of brass can be moved in or out and fixed in the proper position. This piece of brass carries a coil of wire surrounding a core of soft iron not visible in the figure. A current from a battery, or from the general supply flowing in the direction of the arrow, first enters the stem of the fork and, passing through the upper prong to *m*, then enters the platinum wires *δ*, which are in light contact with a small platinum plate which lies beneath the screw *h*. From here the current runs through the brass piece M and the coil D, then into the brass pillar which is fixed to the piece M but insulated from it; thence it passes back to the battery. There is thus a closed circuit; but the current excites the electro-magnet and this draws the upper prong down a little and the under prong upwards. This causes the contact between the platinum plate and the bundle of platinum wires to be broken, so that the current is interrupted, the prongs then swing freely back again, contact is restored and the fork is thus set in regular vibration, which can be maintained as long as desired and a current of a definite frequency of interruptions is obtained. By choice of the tuning fork used this frequency can be modified. Moreover, modification of the frequency over a small range can be obtained with the same fork by adjusting the position of a sliding weight placed on the prongs. In conclusion, we have also the receiving telephone (the transmitting telephone has in practice long been replaced by the more sensitive microphone).

82. **Magnetization through Electric Currents**—All that we have dealt with so far has belonged to the one sort of electro-magnetic mutual effect, the pondero-motive. Now we shall come to another class, that is, to the magneto-motive effect of the electric current. For practical reasons this particular phenomenon has already been touched upon in the first chapter.

An electric current, as we know, creates around itself a magnetic field. If soft iron is brought into this field it will, under its influence, itself become magnetic. And of all methods of magnetizing iron that in which the electric current is employed is the most important. For in the first place electric currents can be produced of any strength, and therefore any degree of magnetism can be obtained (up to the condition of saturation of the iron under consideration). And in the second place the whole process is of such a kind that any quantitative condition can be produced as desired, and the whole procedure is in every respect completely under control.

The question how the strength of the magnetization is related to the strength of the field has already been considered. We have only further to investigate how the strength of the field, when it is produced by an electric current, is dependent on the characteristics of the current and in the first place upon its strength. The connection is one of the simplest: the strength of the field is exactly and directly proportional to the strength of the current. Our curves of magnetization, those, for example, in Fig. 27, represent equally well the way in which the magnetization (instead of the strength of the field) depends upon the strength of the current, and only the scale and the definition of the abscissæ would need to be altered. Another question relates to the uniformity of the field and the magnetization. In accordance with what has already been explained, the following general rules will hold: in order that a coil should produce a uniform field, it must be wound with parallel equidistant windings on a sphere or ellipsoid; and in order, further, that this field should evoke a uniform condition in an iron body, the latter must likewise have the form of a sphere or ellipsoid. The same condition will also be approximately realized if the spherical or ellipsoidal iron bodies are placed in the middle of a cylindrical coil of sufficient length. It is less completely realized if into the last kind of coil a long bar-shaped iron body is introduced. Uniformity of the field and the magnetization are, on the contrary, very closely realized if the coil has the shape of a ring, and if the body to be magnetized is also introduced into the coil in the form of an iron ring.

Apart from the strength of the current, the strength of the field also depends on the number of windings n_1 in unit length of coil; and in the formula to be evolved a numerical factor is required which depends upon the shape of the coil. From the strength of the field the intensity of the magnetization can immediately be deduced. Thus, for example, in the case of an iron sphere on which the current windings have been regularly placed, the strength of the field produced by the current is

$$\mathfrak{H} = \frac{8\pi}{3} n_1 i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

the intensity of the magnetization

$$\mathfrak{I} = \frac{\kappa}{1 + \frac{4}{3}\pi\kappa}\mathfrak{H} = \frac{8\pi\kappa}{3 + 4\pi\kappa}n_1 i, \quad . \quad . \quad . \quad (63)$$

and the magnetic induction

$$\mathfrak{B} = \frac{3\mu}{\mu + 2}\mathfrak{H} = \frac{8\pi\mu}{\mu + 2}n_1 i. \quad . \quad . \quad . \quad (64)$$

Since κ or μ are usually very large, we have approximately

$$\mathfrak{I} = 2n_1 i, \qquad \mathfrak{B} = 8\pi n_1 \mathfrak{I} \quad . \quad . \quad . \quad (65)$$

For a long cylindrical coil

$$\mathfrak{H} = 4\pi n_1 i, \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

and for a long bar placed inside it,

$$\mathfrak{I} = \kappa\mathfrak{H} = 4\pi\kappa n_1 i. \quad . \quad . \quad . \quad . \quad . \quad (67)$$

So far it has been tacitly assumed that the principal direction of the body to be magnetized is at right-angles to the direction of the current and therefore lies in the direction of the field, as is realized in the case of a bar lying axially inside a cylindrical coil and in other similar cases. Longitudinal magnetism, or magnetism in the direction of the length, is then produced. But what if the principal direction of the body lies in the direction of the current and therefore at right-angles to the field? It is obvious that something quite different, namely, cross-magnetism, must occur, and a specially interesting case arises when a current is passed along the axis of a cylindrical bar of iron itself. Cross-magnetism is produced in this case, but in all directions in the same degree, a condition which may be designated circular magnetism. Instead of the bar, a tube may be taken and the current passed through a wire running along the axis. Such circular magnetism has, of course, no outward effect, but inwardly a condition is set up which has been the subject of much study.

83. **Magneto-induced Currents**—Just as there are two reciprocal effects in the pondero-motive phenomenon, the electro-magnetic and the magneto-electric, so we might reasonably expect a counterpart to magnetization through the electric current, and in this expectation we are not disappointed. Only we must be clear what the expected effect will be. For the magnetizing effect does not proceed from

the presence of the electricity (electricity at rest has no effect), it is rather the effect of an electric stream, or electricity in motion. The exact counterpart in the case of magnetism, it is true, does not exist, for there are no magnetic currents, or magnetism in motion (a subject which we have still to discuss). And therefore we conclude that the corresponding phenomenon does not occur. But nevertheless there is one means of bringing it about. There is no current of magnetism, it is true, but there is one very rough and obvious means of setting magnetism in motion, and that is by moving the substance that carries the magnetism—the iron substance in which it resides. We may therefore make the experiment of moving the magnet and observing what happens in its neighbourhood. Naturally we must give the phenomenon a chance of realizing itself, and therefore we set up a closed copper wire in the neighbourhood, and in order to be able to observe what goes on in this circuit we include a galvanometer in it. When the magnet is brought near to, or moved away from, the wire, the needle of the galvanometer is deflected, but only so long as the motion of the magnet continues. Afterwards the needle returns to its original position even although the magnet has now a different position from what it had at the beginning. Such a current is called an induced current, and the phenomenon itself, induction, and thus we have magnetic induction in contradistinction to electric induction, which takes place when in the neighbourhood of a closed circuit not a magnet, but an electric current, is moved. But this account is still very far from being an exhaustive account of magnetic induction. For instead of bringing the magnet near the circuit we can proceed in a different way; we can leave the magnet where it is but increase its magnetism. While this increase is taking place, a current flows in the circuit; that is, the process of increasing the magnetism has the same effect as bringing the magnet nearer; and of course weakening the magnet has the same effect as drawing it away. Finally, as the extreme case: a quite non-magnetic piece of iron which we magnetize in the neighbourhood of a circuit has exactly the same effect as if we brought the magnet ready made from an infinite distance up to the same position; and the complete demagnetizing of a magnet has exactly the same effect as if it were withdrawn to an infinite distance. All these cases may be brought together by the use of a very obvious quantity, the potential between the two things, namely, the magnet and the circuit. In each we must imagine that a unit current is flowing through the latter (though this, of course, actually is not the case). The phenomenon can then be made to depend on this potential in a very simple fashion, only we must not try to express the strength of the induced current (which would be unnecessarily complicated, since this is again dependent on the resistance of the circuit), but the strength of the induced electro-motive force, and this is simply equal to the decrease of the potential in unit time:

$$e = -\frac{\partial V}{\partial t}. \qquad (68)$$

Whether the change in V takes place on account of change in position or change in intensity of the magnet, is now a matter of indifference.

For the demonstration of the phenomenon it is customary to use not a simple circuit, but a coil of wire of many turns. In regard to the effect produced by motion it is, moreover, obvious that we are only concerned with relative motion, and therefore it is the same thing whether the magnet is thrust into the coil, or the coil put over the magnet. If the magnet is not merely inserted in the coil but is drawn right through it, then, of course, the induced current flows first in one direction and then in the other. And if the bar is caused to move to and fro in this fashion continually we get an alternating current. A somewhat different arrangement was made use of in the old transmitting telephone (which has long been discarded in practice). Here it is not a bar but a thin plate which vibrates in front of a coil of wire. The other case, induction through change in intensity, is best demonstrated by winding two coils over an iron bar, an inner and an outer. The first serves to magnetize the bar and the latter to observe the induction effect. The first coil is connected to a source of current and the latter to a galvanometer. The further procedure will be obvious. Frequently the two methods, change of place and change of intensity, are used simultaneously so as to accentuate the effect. Thus an iron core is placed in the coil of the transmitter of the telephone mentioned above, so that its change in magnetic intensity may contribute to the effect; and the same sort of procedure is resorted to in many ways, particularly in the case of those machines to which we must make a brief reference since they are the most important application of magnetic induction, namely, dynamos or electric current generators.

The electric generator is the converse of the electro-motor already mentioned. Just as in the former the electric current produces rotation in the magnetic field, so here rotation in the magnetic field produces electric current. The original type, the magneto-electric machine, has been displaced by a new form—the dynamo machine, and it is this that has been one of the first causes in bringing electro-technics to its present high state of development. Nevertheless, a detailed consideration of the nature and construction of these machines cannot enter into the scheme of the present book.

84. **Eddy Currents**—Induction currents also occur in the body of the conductor and in certain circumstances completely fill it. They are called eddy currents, or Foucault currents. They in turn react upon the system producing them and frequently surprising effects are produced. For instance, we have the damping of a moving system—to take a concrete case, the damping of the vibrations of a magnetic needle through eddy currents which it sets up in any mass of metal in its vicinity—an effect which is often made use of in galvanometers and magnetometers in order to stop the troublesome swinging of the needle about its proper position of rest. Among the apparatus for the demonstration of the phenomenon the Waltenhofen pendulum may be mentioned. This consists of a ring-shaped copper disk which swings as a pendulum between two opposite poles of an electro-magnet. On the field being excited the pendulum, which would otherwise go on swinging to and fro for a very long time, comes as suddenly to rest as if it were moving through some viscous fluid. The experiments of Thompson are also of extraordinary interest: metal rings or cylinders placed over an electro-magnet are tossed upwards when the field is excited. That this is the consequence of the repulsion effect between the magnet and the eddy currents is shown by the fact that the effect does not occur when the ring is severed at some point so that the circuit is broken. In practice eddy currents, since they represent dissipation of energy, play an important part and everything must be done to reduce them to a minimum. This is best accomplished by subdividing the metallic parts, and it is therefore the almost universal practice, as, for example, in dynamo machines, to build up metallic masses wherever possible in the form of thin sheets or bundles of rods.

85. **Hall Effect** -In the pondero-motive magneto-electric effect the question arises whether the effect is on the conductor itself which carries the current, or only on the electricity which is passing through it. This point cannot be decided by the help of linear conductors, but only by means of flat conductors in which the lines of current are free to move about. After much fruitless experiment the American physicist Hall succeeded in obtaining an indication that the lines of current flow are actually displaced, and this phenomenon has been named the Hall effect. A very thin rectangular plate of metal is placed between the two poles of a

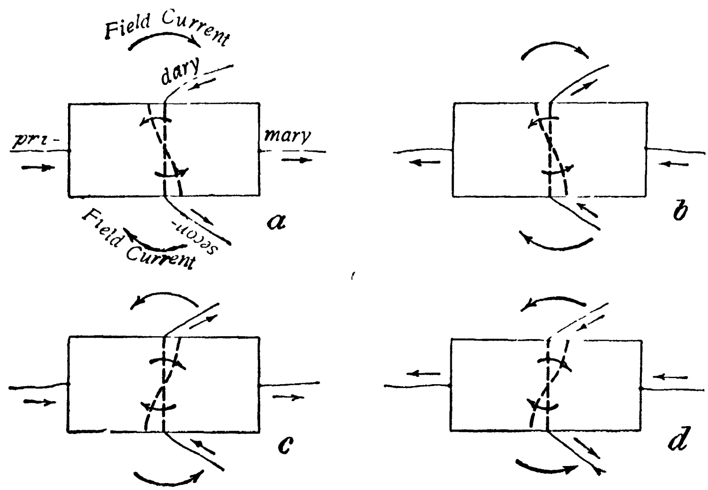

FIG. 122.

magnetic field (Fig. 122), which must be regarded as lying above and below the plane of the paper in close proximity to each other. An electric current is led in and carried away at two middle points at opposite sides of the plate, and at the middle points of the other two sides connection is made to an ammeter. Since these points because of the symmetry of the arrangement are at equal potential, the ammeter indicates no current. But if the magnetic field is excited the contact points of the connections to the ammeter must be moved in opposite directions in order that the ammeter should again indicate zero current. The line of equal potential NN has therefore been turned into the position N'N". If now the current or the field is reversed the deflection of the equal potential line is in the opposite direction. If both are reversed together the deflection remains in the same direction. This indicates that the magneto-electric effect is on the electric stream itself. Rotation in the sense of the current representing the magnetic field is called positive (iron, tin, antimony, etc.), in the contrary direction negative (nickel, gold, silver, copper, bismuth, etc.). The potential difference is

$$e = \frac{\mathrm{R}i\mathfrak{H}}{\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

where i is the primary current, $\mathfrak{H}$ the strength of the field, δ the thickness of the plate, and R a constant, the constant of rotation, or the Hall constant. By a simple conversion we obtain a formula for the angle ϕ through which the equal potential lines are turned when s is the specific conductivity of the material

$$\tan \phi = \mathrm{R}\mathfrak{H}s \quad . \quad . \quad . \quad . \quad . \quad (70)$$

The following are some of the values of R:

Te	+	530	Au	−	0·00071
Bi	−	10·1	Cu	−	0·00052
Sb	+	0·192	Zn	+	0·00041
Ni	−	0·0242	Al	−	0·00038
Fe	+	0·0113	Pt	−	0·00024
Co	+	0·00459	Pb	+	0·00009
Ag	−	0·00083	Sn	−	0·00004

As will be seen there are colossal contrasts; only a few materials show the effect in a very marked degree, the others only slightly. In the ferro-magnetic substances, moreover, R is not a constant but varies with the magnetization. There is, however, no obvious connection with the magnetization and iron and nickel have even opposite signs. In bismuth R diminishes as the strength of the field increases, but for a falling temperature it increases. The influences of heterotropy (crystalline structure) also manifest themselves. There is great variety in the behaviour of the alloys.

Copper-zinc alloys have already been investigated by Hall himself and the following results have been found:

Copper in %	100	81	73	67	6	0
$\mathrm{R} \times 10^6$	− 520	− 404	− 250	− 166	+ 496	+ 820

The behaviour of the two components of this combination are therefore substantially additive. But this is not usually the case; thus the negative coefficient of bismuth through the addition of antimony, although this has a positive coefficient, is made greater. For an alloy of bismuth with 8·35 per cent. of antimony van Aubel found that the value of R was 3·4 times as great as in Bi (which, however, was not quite a pure sample). The effect of adding sulphide of bismuth is still more pronounced, here pure bismuth sulphide has already a value twice as great as pure Bi. On the other hand, the behaviour of bismuth through the addition of lead,

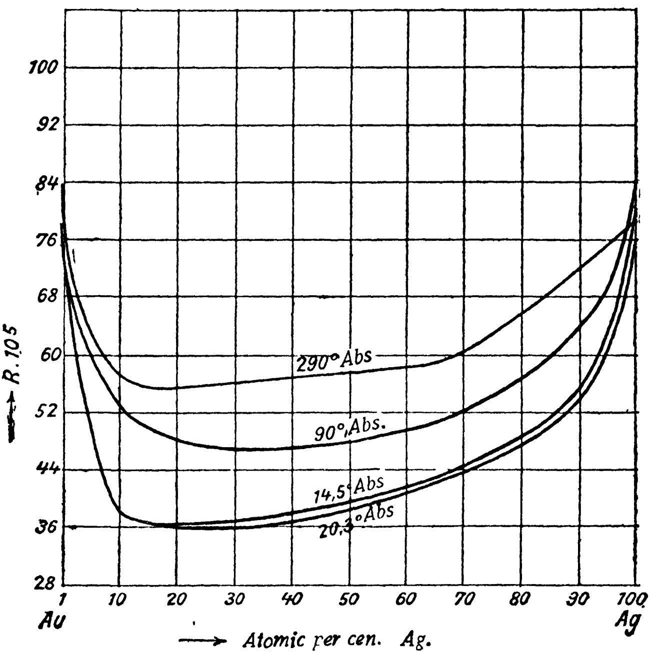

FIG. 123.

according to Leduc, is only slightly affected. Beckmann with gold-silver alloys obtained the following results: If silver is slowly added to pure gold, then although the silver has a greater coefficient, R at first declines to a minimum which occurs approximately at the point where the quantity of the two metals is equal. From there onwards, however, a considerable change in the composition has only a slight effect on R and it is only at the end that the curve rises steeply again. This is shown in Fig. 123, the curves of which all relate to different absolute temperatures. As will be seen, the Hall effect is greatest at the room temperature (the uppermost curve), and diminishes very markedly for the lower temperatures. This, however, does not always occur; in many substances temperature has little or no regular effect. For antimony-zinc alloys we have the effect shown in Fig. 124. In these and many other alloys there are outstanding singular points, corresponding to those mixtures in which the components are present in the ratio of their atomic weights (chemical mixture). There is, as will be seen, a considerable degree of parallelism with the dia- and para-magnetic effects. The Hall effect has been observed and measured in gases; the question of its existence has not, however, been settled in regard to liquids.

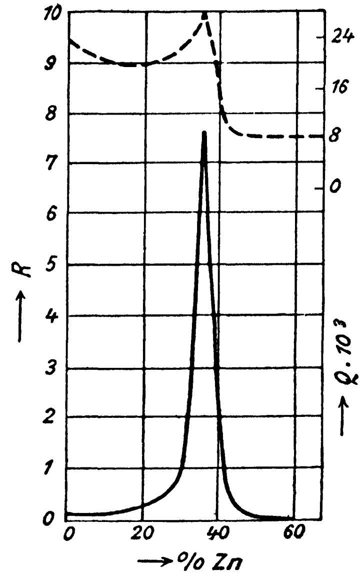

FIG. 124.

The Hall effect is, moreover, only one of a number of examples of interdependent thermal-electro-magnetic effects which are shown in the following scheme.

	GALVANO-MAGNETIC		THERMO-MAGNETIC	
	Potential effect	Temperature effect	Potential effect	Temperature effect
Transverse effect	Hall effect (R) [1	Ettingshausen effect (P) [4	Nernst effect (Q) [7	Leduc effect (S) [10
Longitudinal effect in transverse field	Resistance change in transverse field (G) [2	Peltier effect in cross magnetic and non-magnetic material (M) [5	Thermal force in cross-magnetic and non-magnetic material (L) [8	Change in thermal conductivity in a cross field (F) [11
Longitudinal effect in longitudinal field	Electromotive force of magnetization (G′) [3	Peltier effect in longitudinal magnetic and non-magnetic material (M′) [6	Thermal force in longitudinal magnetic and non-magnetic material (L′) [9	Change in the thermal conductivity in a longitudinal field (F′) [12

There are, as will be seen, transverse and longitudinal effects, and they arise partly from electric and partly from thermal causes, and their effects again are partly electric and partly thermal. The nearest to the Hall effect is the change of resistance in a magnetic field, which may be regarded as a longitudinal Hall effect. It has been carefully investigated in the case of bismuth and has finally led to a practical application in the measurement of the strength of a magnetic field by means of a bismuth spiral. An induction of 1,000 gauss corresponds approximately to an increase of resistance of about 5 per cent. The Hall effect can also be used for measuring the strength of the field.

The thermo-magnetic transverse effect occurs in plates if a primary current of heat is sent through them by maintaining a fixed temperature difference between the two opposite shorter sides. The electro-motive transverse force produced is

$$Q = \left(\frac{q}{k}\right) \cdot \left(\frac{\mathfrak{M}\mathfrak{H}}{\delta}\right) \quad . \quad . \quad . \quad . \quad . \quad (71)$$

where k is the thermal conductivity, δ the thickness of the plate, $\mathfrak{M}$ the flux of heat, $\mathfrak{H}$ the strength of the field, and q a specific constant. Only in the case of bismuth has the latter any considerable value. Here also there is both a longitudinal and a transverse effect. The two together are called the Ettingshausen effect. The converse of the transverse effect, the turning of the isothermals in the magnetic field, is called the Leduc effect. To the same class as the longitudinal effect belongs also the phenomenon that presents itself in the fact that a current flows between two electrodes of the same metal if one of them is in a magnetic field. This current is to be regarded as a special electro-motive force of the magnetization.

VIII

SOME SPECIAL BRANCHES OF THE SUBJECT

86. **The Magnetic Circuit**—The parallelism which in so many respects is to be observed between electric and magnetic phenomena naturally excites the wish to follow it up wherever it is possible so as to get at a similar conception of the subject to that which has proved so extraordinarily fruitful in the field of electric phenomena. We speak of electric currents in reference to the analogy with a stream of liquid. This idea has in the course of time undergone many profound modifications and at times has even been discarded, but finally, as the result of some development or other, it has been resumed again And in the simple case of a constant current in linear conductors the idea may be used to formulate a law which when it has once been enunciated becomes independent of any special conception of this kind, that is Ohm's Law, which states that the strength of the current is equal to the electromotive force divided by the resistance, or, expressed as a formula,

$$i = \frac{e}{w}.$$

Here i may be defined by the electrolytic effect of the current, that is, it may be measured in amperes; e is the potential difference between the poles from which the supply of current is derived, and is measured in volts; and w is a constant determined once for all for the wire which completes the circuit, and is equal to its length divided by its cross-section and divided by an electric constant, the conductivity, and is expressed in ohms.

We have now the task of adapting this idea and the formula in which it is expressed to the case of magnetism, and therefore to speak of a magnetic circuit, of a magnetic current, of a magneto-motive force, of a magnetic resistance and of a magnetic Ohm's Law. The circuit will consist of some form of a horse-shoe magnet with the armature connecting its two ends, or theoretically, and still more simply, of an iron ring or the like. Of a " current of magnetism " it is not permissible to think, because all our experience indicates that magnetism is a residual property of the place where it occurs. But this need not interfere with the idea that underlies this rough-and-ready picture, and therefore the lines of force of the field, or, since we have to deal with the modified (and intensified) conditions inside the body of the iron, the lines of induction must be taken into account, and their density estimated in order to arrive at what may be called the flow of magnetic induction $\mathfrak{F}$. This is simply the induction $\mathfrak{B}$ that we have already discussed, but multiplied by the cross-section Q of the iron. (This must be supposed to be equal through the entire circuit, otherwise we must take its mean value, or still better the integral round the complete circle.)

According to this definition we have

$$\mathfrak{F} = \mathfrak{B}Q \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

The magneto-motive force M must here be regarded as the potential difference for the whole circuit; that is, we must take the strength of the field $\mathfrak{H}$ and integrate it for the whole length L of the circuit, or if we can take the mean value as sufficiently near, then simply

$$M = \mathfrak{H}L \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Finally we imagine the third determinative magnitude, the magnetic resistance, as in exact accordance with the electric, i.e., we regard it as being directly proportional to the length of the circuit, inversely proportional to the cross-section Q, and divided (as in the previous instance by the specific electrical conductivity), so here by the magnetic permeability μ, so that for the resistance we have

$$W = \frac{L}{\mu Q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

And following from this definition we obtain what corresponds to Ohm's Law for the magnetic circuit:

$$\mathfrak{F} = \frac{M}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

If the cross-section is everywhere the same since $\mathfrak{F}$ contains the factor Q it disappears from both sides of the equation;

if for M we use its mean value as defined by the formula above, L also cancels out, and if instead of μ we use the reciprocal quantity $r = \frac{1}{\mu}$, and therefore instead of the magnetic conductivity the "specific magnetic resistance," we obtain the simple formula

$$\mathfrak{B} = \frac{\mathfrak{H}}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

By many writers W is referred to as the "reluctance" and r as the specific reluctance; but lately so many new "ances" have been introduced (resistance, impedance, inductance, reactance, etc.) that it becomes a considerable strain upon the mind to grapple with the whole series successfully. If the circuit consists of parts of varying cross-section and varying permeability, we must proceed exactly as in the case of the electric circuit, consider the sum of the separate terms, or if the variation be of a regular character the integral

$$\mathrm{W} = \int \frac{dl}{\mu q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

In practice the field is usually created by an electric current, then if i is the strength of the current in amperes, and n is the number of turns, our formula takes the form

$$\mathfrak{F} = \frac{1{\cdot}257ni}{\mathrm{L}/\mu\mathrm{Q}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

So much for the theory, let us now consider its significance and justification.

That we should be able in other branches of natural science and also in this particular one to make use of a formula on the lines of Ohm's Law is by no means strange; it merely amounts to the definition of a new magnitude, the resistance. The formula could only be regarded as having anything more than a purely formal significance if the new magnitude could be regarded as a simple characteristic of configuration. This is the case in the fundamental law of the electric current, because the resistance is a constant of the conductor along which it travels, and this can be regarded as the law of the existence of a constant conception of resistance. But this, as we know, does not exactly apply in the case of the magnetic circuit; here the resistance, since it depends upon the permeability, is a function of the force; that is the numerator in the fundamental law is a function of the denominator, and the law really tells us nothing at all. Other considerations have to be taken into account: Ohm's Law in this form is valid only for approximately linear conductors, that is for small cross-sections which are only realized very approximately in the case of the magnetic circuit. Further the part played by the air in the conduction of the force is small indeed, but is substantially greater than in an electric conductor, and of course for high magnetization, when the permeability of the iron becomes small in comparison with that of the air, even the latter plays an important part. In other words the leakage in the wider sense (see above) has to be taken into account, and therefore we must regard the circuit as comparable with that of a high-tension circuit, or with the current in a cable, for which it is well known Ohm's law is no longer exactly applicable. In the case of the current, indeed, there is nothing analogous to magnetic saturation; in the end we should need to have recourse to an analogy with the alternating current with its impedance and so forth. The Kirchoff equations for electric networks when the conductibility is variable become insoluble, as Hanauer, for example, has shown, and a graphic method must be adopted, which again will give reliable results only under certain conditions.

But the unsatisfactory nature of the analogy is best shown when it is applied to permanent magnets. The question was a short time ago keenly debated what it is that remains constant in a permanent magnet when it is subjected to certain operations, when, for example, it is introduced into a magnetic circuit and its resistance changes, then either i or e, to use the terms of Ohm's law in their magnetic application, either the flux of lines of force, or the magneto-motive force, must vary if the other of these quantities is to remain constant. Sometimes the one and sometimes the other answer is given, and only Gans has introduced a certain clarity into the discussion by stating that over such small alterations of the resistance and therefore of the field strength we can afford to leave out of account the hysteresis and in addition regard the permeability as constant. The constancy of the magnetization also conditions the constancy of the magneto-motive force. But it is necessary, in order that one should be able to postulate this latter constancy, that we should be able to regard the quotient of these two magnitudes $\mathfrak{J}/\mu$ as constant (which in fact would seem to be permissible). We see that through this necessary limitation in the interests of clearness the significance of Ohm's law for magnetization is considerably diminished.

Moreover, it becomes clear that the so-called fundamental Ohm's law of the magnetic circuit has no real scientific significance, and that when it is required to apply it to practical purposes one has to proceed very carefully and be guided by the results of experience. But under these conditions it may be and has been of considerable value.

87. **The Ring with an Air Gap** - The ideal case of the magnetic circuit is that in which the total flux of induction remains in the magnet. This case is only completely realized in a ring which is covered with uniform windings. If the windings are more thickly crowded together at one place than at another, or if the cross-section of the ring gradually or suddenly changes between one part and another, lines of force leak out into the surrounding air. As will be readily understood a quite special case arises when the circuit is broken at some point, corresponding to the case of the spark gap in an electric circuit. A typical case of this sort is the slotted ring. Here, as Fig. 125 shows, the lines of force for the most part run parallel with the axis, and those near the axis maintain their parallelism across the gap; but the outer ones bend outwards as they approach the end surfaces so that they escape out of the iron into the surrounding space—a phenomenon which was referred to in Chapter II (page 10) as the dispersion of the stream lines. It will be readily understood that this

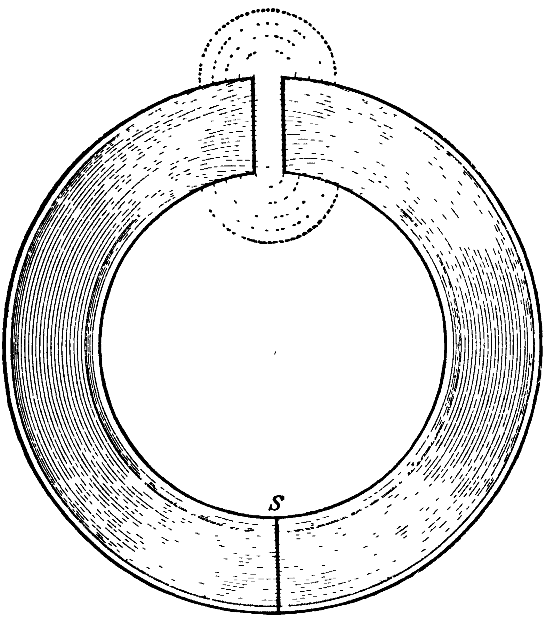

FIG 125.

dispersion is greater, the greater the width of the gap, and for a somewhat extreme case, namely, that of a horse-shoe magnet with an armature, Fig. 126 will show how the dispersion

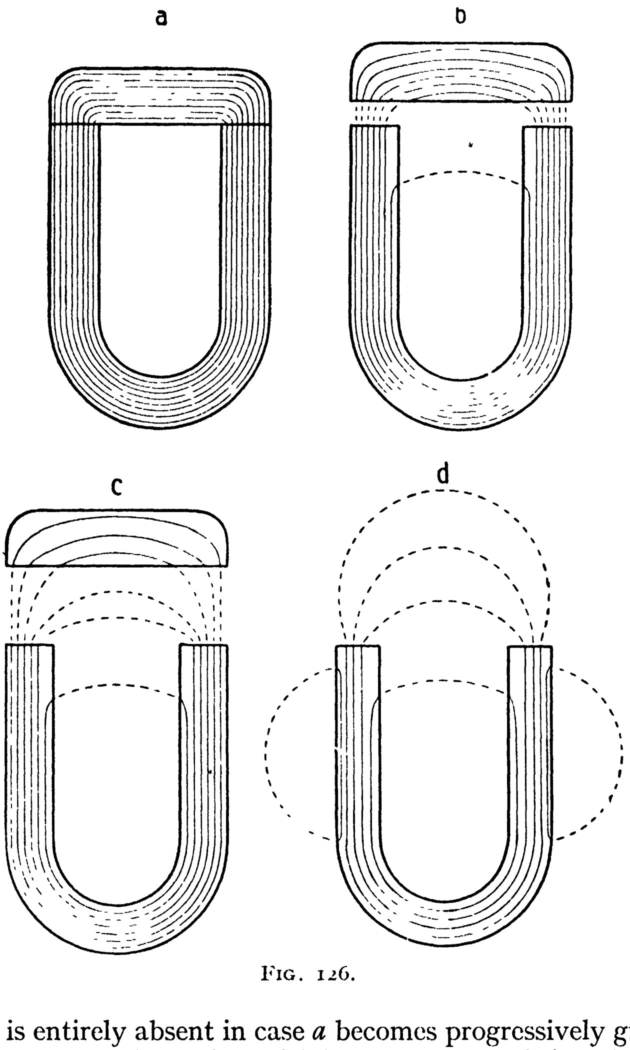

FIG. 126.

which is entirely absent in case *a* becomes progressively greater as the armature is further withdrawn in *b*, *c* and *d*, and in the last case it plays the predominant part. In dynamo machines one of the principal actors of progress has resulted from the fact that we have learnt better how to diminish this dispersion, and in that way to increase the useful effect, and at the same time to eliminate the harmful subsidiary effects. To make clear the effect of a gap in a ring a graphic illustration is given in Fig. 127, which relates to an iron ring which is surrounded at one part by a magnetizing coil (at 0° or 360°), and in consequence the flux of induction is non-uniform; but in the closed ring to which the principal curve refers, the want of uniformity is not so marked. It is very considerable, however, in the lower curve at the point corresponding to 180° in the solid ring, even if the slot be a very narrow one.

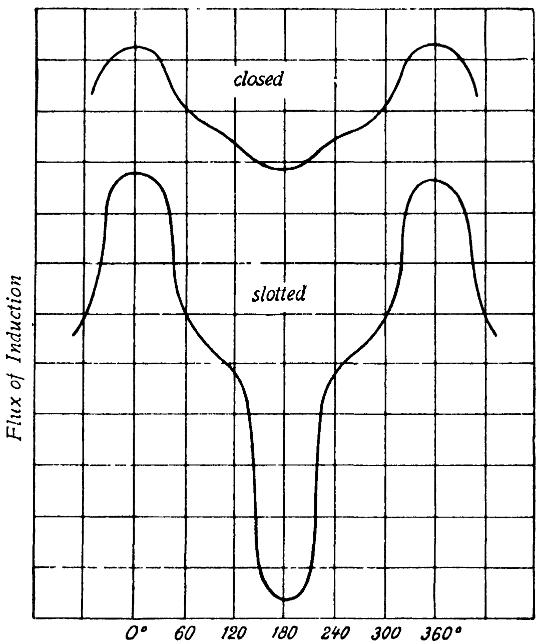

FIG 127

88. **Shielding Effects**—In the closed ring, as also in the horse-shoe magnet with the gap completely closed by the armature, the lines run, as has been said, entirely in the iron. The air space within and without the ring is sheltered from the penetration of lines of force. This leads us to another specially interesting consideration—the magnetic shielding effect. As a more satisfactory example of this we will take not the narrow ring in which the air space is on all sides in contact with external space, but a hollow body, for example, a hollow sphere. If such a sphere of soft iron is brought into the magnetic field it of course itself becomes magnetic, but very little magnetism penetrates to the inner part; and if the walls are sufficiently thick there is almost no effect there whatever. A small iron body enclosed within this hollow space is protected from the external field. This effect can be made use of in an obvious

fashion when it is desired that a certain space shall be confined to the effect of certain forces without other magnetizing forces that might be present exerting a disturbing influence, as in ships' compasses and measuring instruments and the protected type of galvanometer already mentioned (page 161). Moreover this internal shielding effect is the counterpart of an external one by which the external air space is protected by the hollow body from forces which may be active inside it. There are further numerous theoretically interesting and

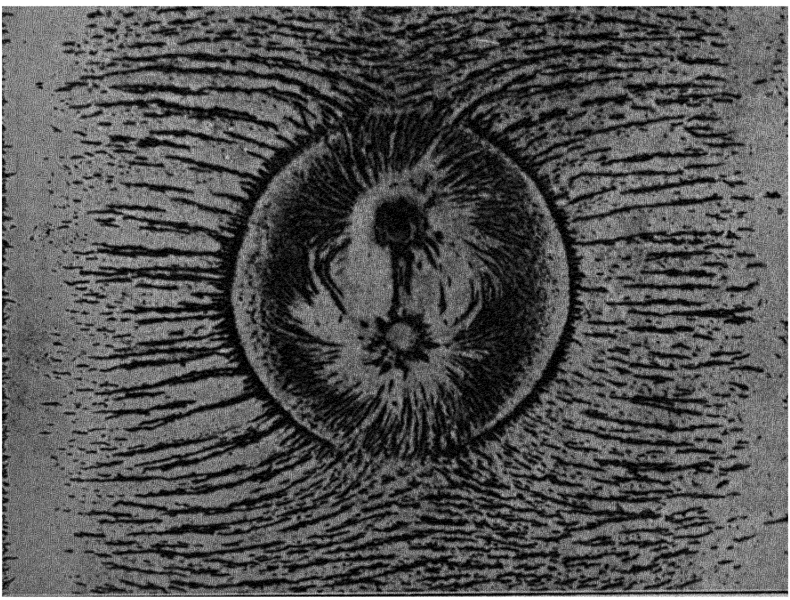

FIG. 128.

practically important special problems directly or indirectly connected with the shielding effect, for example the following: What magnetization is experienced by a hollow cylinder in comparison with a solid, or a hollow sphere in comparison with a solid sphere? (in a solid body the inner parts are certainly in some measure protected by the outer parts). How does the magnetization of tubes depend upon the thickness of the walls? What is the behaviour of a series of tubes placed one inside the other?

The problem of the shielding effect was first worked out on scientific lines by Stefan, but the most exhaustive investigation is due to the Dutch physicist du Bois, who succeeded in resolving the many contradictions which existed between the discoveries of the earlier workers in the subject. It must here be sufficient to illustrate the shielding effect by the reproduction of a photograph which is due to du Bois. Fig. 128 shows the protection of external space from the effects of an inner pair of poles by means of a thin cylindrical shell; as will be seen, the external field is almost completely unaffected.

89. **Fields. Electro-magnets**—We have already several times discussed magnetic fields and characterized them according to their special properties; we will now consider the subject as a whole.

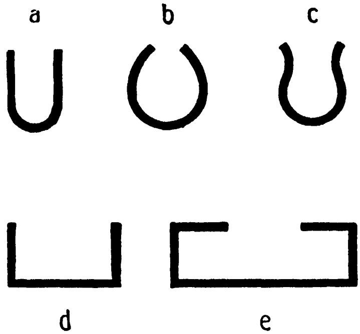

FIG. 129

A field may be either uniform or non-uniform; that is, it may have at all places the same strength and direction, or may be different in strength and direction at different places. There is the intermediate case, that the direction may everywhere remain the same, but that the strength may vary (or conversely). Among the requirements of practical magnetics two cases in particular come to the front: in the first place, to obtain the most uniform field possible; and secondly, to obtain the most powerful field possible. A third may be suggested: to obtain the most extended field possible. But the attainment of this by artificial means is possible only in a very limited degree, and it is a happy circumstance that in the magnetic field of the earth we are already provided with what we require. Of course the earth's field is relatively weak, but "two good things rarely come together," says the proverb, and such is the case here. And there is still another case which can only be partially realized: an extended field which is everywhere uniform. For the most part we must be content to clearly define the problem, and then it may be so to adapt the method that the whole of the requirements are more or less satisfactorily met.

But nevertheless we have still left out of consideration a fourth problem: that the field shall be capable of regulation as required; that it shall be possible to make it strong, medium or weak. The oldest method, the use of the steel magnet, is at once excluded, for the strength of this is fixed. In its place we have the electro-magnet in its various forms, which in the course of the last hundred years represent an ever higher

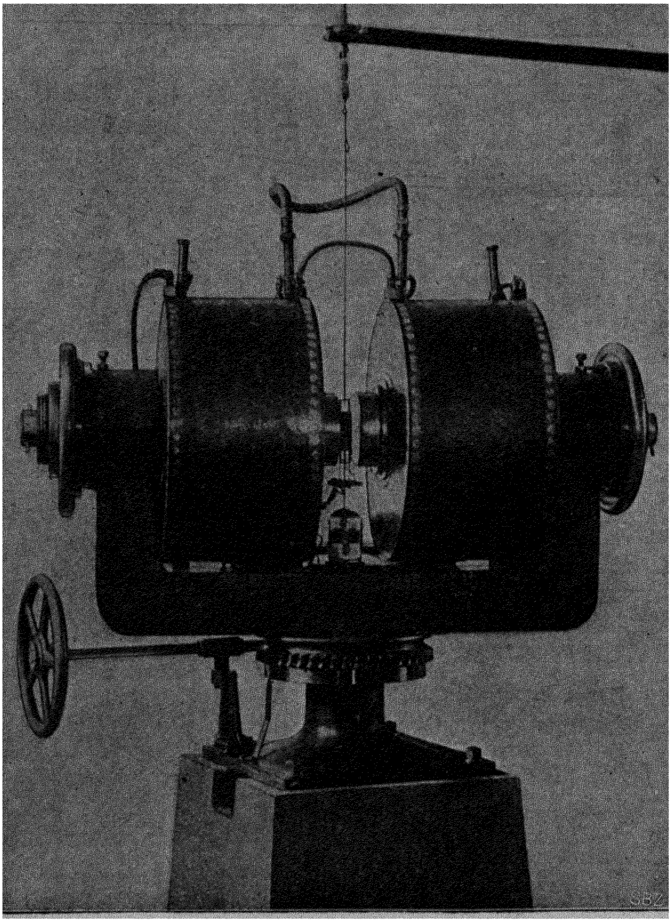

FIG. 130.

development in the direction of perfection. The later forms, as in the U-shaped magnets, the horse-shoe magnets, the Faraday and Ruhmkorff electro-magnets, need only be mentioned here in passing, in order that we may at once turn to the newest forms due to Pierre Weiss and du Bois. All the forms reduce to one or other of the types represented in Fig. 129. The electro-magnet of Weiss (Fig. 130) is related, as will be seen, to type *e*; the two upper limbs, however, are developed and may be moved nearer to or further from each other by means of a screw. The limbs, however, are only half as thick as the yoke in order that magnetic saturation may be obtained in all parts. It is one of the most carefully thought-out forms in regard to rigidity, cooling and adjustment. Twice 1,680 windings are employed, and on account of economy not wire but bands. The limiting current is 60 amperes, and therefore the maximum ampere turns are about 200,000. Between the truncated conical pole pieces, 3 mm. in diameter at the base, and 2 mm. apart, a field of 46,000 gauss can be obtained. Du Bois, on the other hand, first constructed his long electro-magnet, and then, as being less costly, the almost equally efficient half-ring electro-magnet, which is schematically represented in Fig. 131. The adjustment of the distance between the two poles is made along A_1A_2, the distortion of the field by means of K_1K_2. S_1S_2 are the cores, P_1P_2 the conical pole pieces. In the latest, slightly modified model, the intensity of the field can be raised to 50,000 gauss, but only in a restricted space of 1 mm. length and 3·6 mm. breadth.

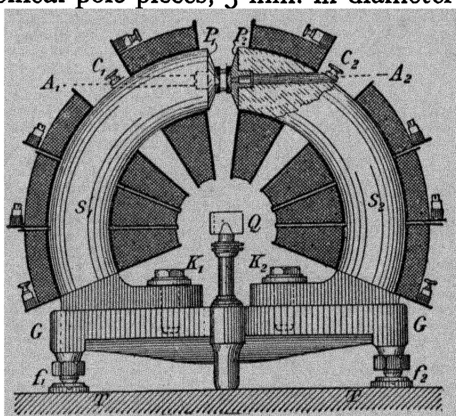

FIG. 131.

90. **Distribution of the Field**—The formulæ for the strength of the field in the space between the poles have in the course of time been more and more perfectly developed and experimentally tested. Between plane-parallel, cylindrical poles of diameter $2r$ and distance apart $2d$ the strength of the field in the middle point on saturation

$$\mathfrak{H} = 4\pi\mathfrak{J}\left(1 - \frac{a}{\sqrt{a^2 + r^2}}\right) \quad . \quad . \quad . \quad . \quad (79)$$

and in the special case when r is great in comparison with a

$$\mathfrak{H} = 4\pi\mathfrak{J}.$$

In addition we must remember that there is the field due to the magnetizing coils themselves, which may under certain circumstances amount to 10 per cent. Since $\mathfrak{J}$ cannot usually be increased over 1,700 the strength of the field cannot exceed 22,000. A better result is achieved when the lines of force are concentrated by giving to the poles the form of truncated cones, and the maximum is reached when the generating lines of the cone make an angle, the tangent of which equals $\sqrt{2}$, that is an angle of 54° 44′. Then we have

$$\mathfrak{H} = 4\pi\mathfrak{J}\left(1 + \frac{2}{3\sqrt{3}}\log\frac{r}{a\sqrt{2}} - \frac{1}{\sqrt{3}}\right) =$$

$$4\pi\mathfrak{J}\left(0{\cdot}2893 + 0{\cdot}8863\log\frac{r}{a}\right) \quad . \quad . \quad (80)$$

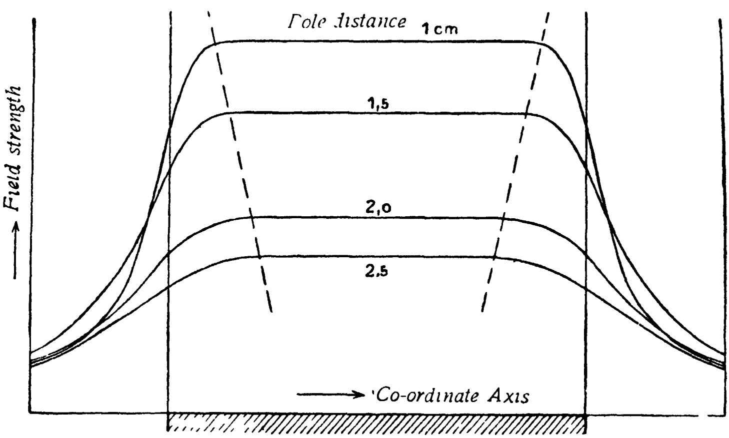

FIG. 132

Therefore, for example, for $a = r/20$ it equals 1·442 ($4\pi\mathfrak{J}$) and with an increased value of r/a the value can be made as great as is desired. The field formed by the cone is however not so homogeneous as that obtained with the cylindrical pole pieces.

The strength of the field between flat pole pieces has been represented by Siegbahn for various amounts of separation of the poles, and these results are given in Fig. 132 for solid poles, and in Fig. 133 for hollow poles (which are used when it is necessary to look through the field in the axial direction), the lower pole piece being shown. As will be seen, over a very large part of the field the strength is constant, but there is a depression in the middle which is greatest when the distance between the poles is small.

A very valuable collection of formulæ has been given by du Bois, together with their working, and relate to cases of practical importance; it is unfortunately not possible to give them here.

Finally with regard to the uniformity of the field the most important example has already been given on page 160; it is the field in which the needle of the Helmholtz tangent galvanometer swings. The best way of creating a strong and in some degree fairly extensive field would be by the use of an elliptically shaped coil, which by suitable arrangement of the

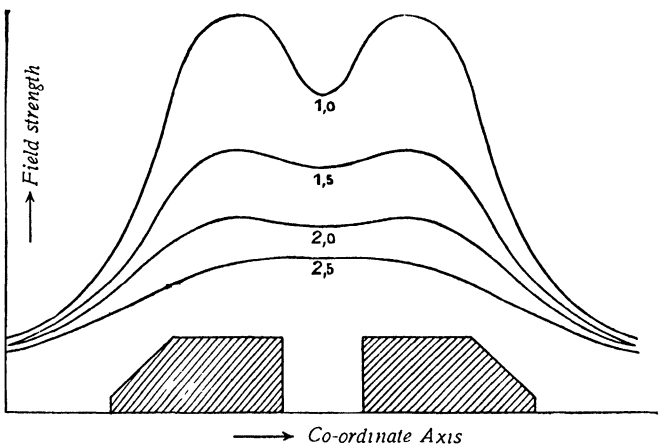

FIG 133

windings produces a perfectly homogeneous field (see the chapter on electro-magnetism). Unfortunately the construction of such coils is so difficult that they can only be regarded as practical in quite special cases. In view of this drawback Bestelmeyer has worked out a compromise method which has shown itself very successful. A cylindrical coil is made whose length is 2·4 times its diameter. The want of homogeneity at the ends amounts to 1 or 2 per cent., and this is corrected for by means of two complementary coils which can be moved in or out of the principal coil. The most suitable values for the diameter, length, and position of the supplementary coils varies somewhat according as one lays special stress on the value of homogeneity along the axis or in the median plane The normal values finally arrived at are as follows:

	Principal Coil	Supplementary Coil
Length	68·72	11·092
Internal radius	13·30	9·39
External radius	15·75	11·73

91. **Magnetic Attractive Force**—Concerning few problems does there exist so extensive a literature as concerning magnetic attractive force; and in few problems is the value of the literature, for the most part so slight and unreliable, at least in the earlier period, as in this. Very rarely does it go beyond the narrowest limits of the problem and the experiments made in connection with it. The reason is that these experiments were made as a result of their obvious interest, at a very early stage of the science, and at a time when the fundamental principles of magnetism were almost unknown. No distinction or no sufficient distinction was made between strength of current and strength of magnetization; between magnetic force and induction; between induction and flux of induction. No attention was paid to the nature of the magnetization, whether, for example, it was uniform or non-uniform, and so forth. The most contradictory results were therefore obtained, which, as we now know, were almost without any general significance.

Only in the most recent times have investigators learnt to arrange their experiments scientifically, and only since then have results of real significance been obtained which are of use in technical applications, in regard to magnets and particularly electro-technics, where such information is required for the most varied purposes and developments, as for example in levers and brakes, and also (a very remote subject!) in ophthalmology, where powerful magnets are used for extracting iron particles from the eye.

The method which consists in determining the weight necessary to pull the armature away from the magnet by means of a suitable balance is still used, only now we are in a better position to take account of various sources of error that may arise. We have to specify whether the process of pulling the armature away is sudden or gradual, side forces must be carefully excluded, remanent magnetism under certain circumstances plays an important part, and there is also the more general question whether the series of experiments is made with an increasing or decreasing magnetism and so forth. The fundamental difference, however, lies in the fact that account is also taken of the character of the magnet in regard to the field in which the attractive forces are being exerted. For if the field and the intensity of the magnetization and so forth are not stated, any mere statement about the attractive force has little value. For this reason one of two methods must be chosen: either very simple magnet forms are taken for which the field of magnetization is known from theory to be as far as possible uniform; or magnets of any form are taken, and then the field and the induction within and without are studied in detail. The first method is to be recommended for experiments with a scientific object, the latter for practical purposes. For in practice the form of the magnet is very often completely defined in accordance with other considerations, and is of course very different from the theoretically simple forms.

Theory under certain simplifying assumptions gives the following formula for the attractive force $\mathfrak{Z}$, per unit area of surface. It is commonly ascribed to Maxwell, but is only indirectly due to him.

$$\mathfrak{Z} = \frac{1}{8\pi}\mathfrak{B}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

where $\mathfrak{B}$ is the induction; therefore, for the whole surface F, the attractive force is

$$T = \frac{F}{8\pi}\mathfrak{B}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

or, if expressed in kilograms and written somewhat differently,

$$T = F\left(\frac{\mathfrak{B}}{5{,}000}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

The attraction is therefore proportional to the surface and to the square of the induction. The stronger the magnetization, the more favourable the conditions in regard to the energy consumed. But the formula is only valid for a homogeneous field, and one in which the lines of force emerge perpendicular to the pole face; in the general case it is much more complicated. The attractive force bears no simple relation to the weight of the magnet itself; but for the smaller forms it may be said that small magnets are more efficient than large ones. The most extreme case is represented by a magnet constructed by S. P. Thompson, the weight of which was 0·1 of a gram, and which would support 250 grams, or 2,500 times its own weight. The attractive force per unit of surface area and therefore the carrying power is here given for some values of $\mathfrak{B}/1{,}000$:

$\mathfrak{B}/1{,}000$.	$\mathfrak{Z}$.	$\mathfrak{B}/1{,}000$.	$\mathfrak{Z}$.
1	0·04	14	7·6
2	0·16	16	10·4
3	0·37	20	16·2
4	0·65	25	25 4
5	1·01	30	36 6
6	1·46	40	64·9
8	2·6	50	101·5
10	4·1	60	146
12	5·8	75	229

When a determination of this sort is made a very interesting phenomenon is to be noticed ; on switching off the current by which the electro-magnet is excited the hanging weight does not drop immediately but only after the lapse of a certain interval of time. The effect can be particularly well demonstrated to a large audience by making the two actions audible. If the two bodies, the magnet and the hanging weight, stick together well enough, this time-lag may very well amount to several seconds.

The other method of approaching the problem alluded to above consists in the complete experimental investigation of the general characteristics of a magnet as it is used in technical applications. This problem in the case of a brake magnet, for example, has been very elegantly solved by Euler and others, and has been given in an outline which allows of the course and the concentration of the lines of force over the whole of the gap between the two bodies to be traced out.

91A. **Time Phenomena**—In all the phenomena hitherto described time as such plays no part. It has rather been taken for granted that the magnetic condition follows simultaneously on the appearance of the force to which it was due. But now the question arises whether there is a time interval required between the appearance of the magnetism and its propagation through space, and whether this time is measurable —a question which, as we know, has been answered in a positive sense in regard to sound, light and electricity. It has become evident that there is here something similar to be determined both in connection with the local appearance of magnetism, and its propagation through space. The first phenomenon can be regarded as a change due to an after-effect, and it is then found that with a possible transition between them there are three types: the quick, the gradual and the slow change after-effect; the first requiring a small fraction of a second, the second requiring seconds or even minutes, and the last hours or days or even years. The phenomenon is so many-sided and involved that we cannot go into it more closely, especially as the circumstances in many respects are not clear. On the other hand, as regards the propagation of magnetism we have gradually come to a clearer understanding of the phenomenon, and it has finally become possible to speak of magnetic waves exactly as of other waves, both of the stationary and the progressive kind, and to give their length and speed of propagation. The problem has been most exhaustively studied by Lyle and Baldwin. By the help of a very suggestive method of the characteristics of the progressive waves, the amplitude and phase of the fundamental vibration and harmonics of bars and wires can be obtained and also the damping coefficient determined, so that it can be not only expressed in words but also graphically represented.

IX

METHODS OF MEASUREMENT

92. **General Survey**—It might very well be laid down as a general proposition that it is impossible to write a textbook that is good in every respect, and to prove such a proposition in the following way. A textbook must actually concern itself with three things that taken together make up the totality of the subject: methods, facts and theories. The question arises in what order should these three aspects of the subject be discussed. The order just indicated would at first sight seem to be the most natural one. But a very little consideration suggests that the reader could only understand the methods, and in particular the methods of measurement, when he already knew something of the facts; indeed the very use of the apparatus presupposes a knowledge of the quantitative laws. And as regards theory it is theory alone that can teach us out of an infinity of possible experiments to choose that one from which we can venture to expect something of intrinsic value. We therefore see that each of the three things in some degree presupposes the two others and at first we do not know what to do. But it will be said, knowledge has gone forward on its course for all that, and therefore it must surely be possible for a textbook which merely gives an account of it to follow that course effectively. But knowledge does not proceed in a straight line, but rather in a series of zigzags, and it makes detours and enters byways that have no exit. In a textbook written on a historical plan all this can easily be taken account of. But of a systematic treatise more is required: it is required that matters should be thoroughly cleared up and that there should be a clear and simple presentation of the subject.

So we have proceeded in the present case. And now we come to the consideration of methods which should really have been presupposed; but in their many-sidedness they touch upon the equally great many-sidedness of the phenomena themselves, and they are therefore only intelligible when we have learnt something about the latter. A paradoxical method of treating the subject is therefore excusable; and also as regards the theory to which we intend to devote ourselves in the next chapter, we may say in anticipation that the same remark applies equally well.

Our subject is rich in content because two different sorts of diversity are embraced in it: that of the quantities to be observed, and the method by which they are to be determined. As regards the quantities to be determined they may be divided into two classes, those relating to the field, whose seat is the air or similar substance; and those relating to the induction, whose seat is in the ferro-magnetic material. In the case of the field we have to deal with its strength and direction; the variations of these in different parts of the field and, it may be. changes dependent upon time. And a very substantial difference arises according as we are dealing with a weak field like that of the earth, or with a medium or a strong field. For each of these cases particular methods are to be preferred. Very much the same sort of remark applies in the consideration of induction as regards the magnetic moment in its magnitude and direction, its distribution throughout the different parts of the ferro-magnetic substance, and as regards any possible changes dependent upon time. The last point however may present itself in various ways since it may relate to the whole course of the process of magnetization, the way in which it arises or dies away, the remanence and so forth, and further as regards phenomena relating to time and place, such as the propagation of the magnetism from place to place or point to point, and its penetration into the deeper parts of the material. The most important quantity to be determined for a ferro-magnetic substance is finally its specific magnetism referred to some sort of unit: thus, if the unit of volume be chosen, the susceptibility as a characteristic of the magnetization; and the permeability as a characteristic of the "induction." Further, we shall have to deal with the total flux of induction, the magneto-motive force and resistance, and so on. The problem will obviously be simplest when considered in connection with permanent magnets, as we shall then be independent of time effects; and we shall have principally to deal with the magnetic moment, its distribution through the material, the poles, their distance apart, the polar axis and its direction, and so forth. Finally the pondero-motive force and the lifting power will come under consideration. Moreover the above matters cannot be very systematically treated since it will happen that several of the quantities named are to be obtained simultaneously or by parallel methods.

As regards the methods themselves they run very closely parallel in the two chief divisions of the subject, whatever their development may be, according as we have to deal with the strength of the field or the intensity of the magnetization. In principle of course any property of the magnetic condition is equally well suited to become the basis of a method of measurement. The chief question is, what is the degree of exactitude of which these various methods admit, and to what circumstances are they best adapted? The answer may be very different in different cases. Here let it suffice for the present to make a short comparison of the most important methods. (1) The magnetometric methods are specially suitable for moderate fields, and give on the one hand the strength of the field and on the other the magnetic moment in relation to the strength of the magnetization when the other of these two quantities is given; of these methods there are a great many variants. (2) The electro-dynamical methods which are specially suitable for rapidly changing conditions of magnetization, for hysteresis and so forth. (3) Methods dealing with the flux of induction, or ballistic methods with their variations; closing of the yoke and isthmus methods, etc. In rings and the like, which have no outward field, they are the only methods applicable. (4) Methods dependent upon the pulling force, the more exact modifications of the old lifting-force methods. (5) Hydrostatic methods. (6) Damping methods. (7) Bismuth methods, specially suitable on account of their convenience. (8) Hall-effect methods. (9) Methods dependent on the ascent of liquids in tubes. (10) Optical methods, based on the electro-magnetic rotation of the plane of polarization of light, in which, instead of passing the beam through the field in the case of ferro-magnetic substances, the refractive (Kerr) effect may be used.

The measurements relate sometimes to single quantities, as for example the magnetic moment of a permanent magnet, or the distance of the poles, or sometimes a whole series of quantities in regard to their dependence as functions of some given factor, as for example the relation of the magnetization or the permeability to the strength of the field or the lines of induction within the substance; in such cases a graphical method may be applied, and curves or systems of curves may be obtained, and indeed an attempt has been made, and not without success, to get an automatic record of such curves.

In conclusion we have to distinguish between the measurements of purely numerical quantities, such, for example, as permeability, and the dimensional magnitudes; and in the last case again between the comparatively simple task of giving relative values, or the much more difficult one of giving absolute values referred to the C.G.S. system or to the electromagnetic system of measurements, whether it be directly or through the medium of certain standards of reference obtained once for all.

It is of course out of the question to treat all these in detail here; we must limit ourselves to a few specially important or interesting cases

(*a*) MOMENT AND DIRECTION

93. **Magnetometer**—We begin with the magnetometers in the narrower sense. The classic representative of this class of apparatus is Gauss and Weber's magnetometer, and the character of the measurements to be obtained with it has indeed become something of a pattern for exact measurements throughout physical science—so there is no wonder if it plays an outstanding part in the business of teaching. It is shown in Fig. 134, the outer parts, in order to make the construction clearer, having been partly removed and shown separately. The essential features are the two magnets, one of which hangs by a thread from a torsion head and naturally in the magnetic meridian, the other, lying on a bar placed at right-angles to it and therefore approximately in an east-and-west direction, is capable of being moved to and fro and from one side to the other, but of course is fixed in position during observation. In addition there is a stand, a cover, a damping arrangement to control the swing, and a mirror fixed to the suspension which turns with the magnet and therefore permits its position to be measured by means of a telescope and scale (mirror reading). Two series of observations are taken—one of the deflections, and one of the oscillations. The deflections obviously give the relation of the magnetism of the fixed bar to the horizontal intensity of the earth's magnetism, for the deflection is directly proportional to the former, and inversely proportional to the latter, whence we have in accordance with the magnetic law

of distance (page 18) the factor $1/r^3$, where r is the distance between the centre point of the two magnets, and in addition, because the torsion of the suspension also comes in, the factor $1 + \theta$ where θ is the so-called torsion relation, that is the ratio of the angle through which the magnets turn to the angle through which the upper torsion head has to be turned (usually

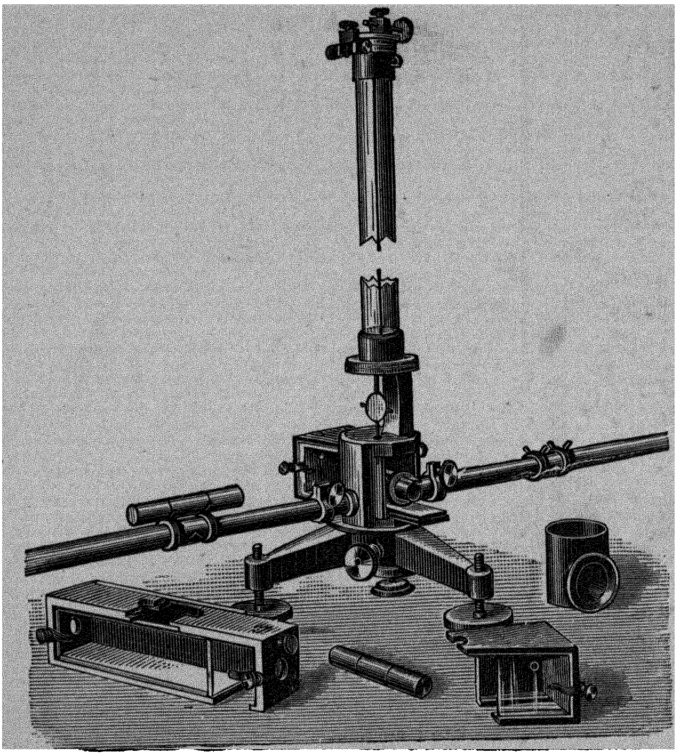

FIG. 134.

only a very small fraction of a complete rotation). A fairly simple formula would then be obtained for the angle of deflection if it were not that a serious cause of error were present, the size of the deflecting magnet, the various parts of which are not all at the same distance r from the suspended magnet, some of them being nearer and some farther than r. It would be necessary to make a minute investigation into the bar and take account of this, but that the whole difficulty can be surmounted by making two observations at two distances,

r_1 and r_2, and observing the two corresponding angles ϕ_1 and ϕ_2, and combining the two observations in a suitable way. A simple calculation then gives for the required relation the formula

$$\frac{M}{\mathfrak{H}} = \frac{1}{2}(1 + \theta)\frac{r_1^5 \tan \phi_1 - r_2^5 \tan \phi_2}{r_1^2 - r_2^2} \quad . \quad . \quad (84)$$

It is to be noted that the fixed magnet, instead of being allowed to act from an east-and-west position (first principal position), may be placed in a north-and-south position (second principal position); the formula, in accordance with what was said on a previous page (page 24), is only changed to the extent that the factor $\frac{1}{2}$ is to be left out. Of course it is not usual to rely on a single observation but to make several in such a way as to eliminate any want of symmetry that may be present, so that in the first principal position the observations are taken both from the east and the west with both the north and the south pole forwards. In this way ten observations are obtained in accordance with the scheme in Fig. 135:

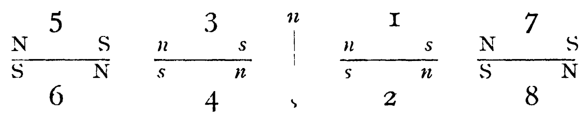

FIG 135

where the various observations are distinguished by numbers. A definite time sequence is also followed so that variations with time are also very largely eliminated. The ten observations are arrived at by taking, before and after the eight observations of the deflection, the position of the hanging magnet when not exposed to the fixed magnet. If these two null positions are divergent then by interpolation the null value of each of the eight observations is obtained and thence the difference between this and the value of the deflection. From the four approximately equal values of r_1 and the four approximately equal values of r_2 the mean is taken in order to obtain the values of ϕ_1 and ϕ_2. Kohlrausch has given a still more exact formula, but that given above is usually sufficient.

Now as regards the observation of the oscillations. The magnet hitherto fixed is put in the place of the hanging one; it is displaced a little and the time of swing t is observed. This obviously gives the product of M and $\mathfrak{H}$, for the time of oscillation is shorter the greater M, and the greater the value of

$\mathfrak{H}$, since both forces here work in the same sense. According to the laws of mechanics if K is the moment of inertia of the bar, and θ' now the torsion ratio, then

$$M\mathfrak{H} = \frac{4\pi^2 K}{(1 + \theta)t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

If the bar is a regular cylinder the moment of inertia K can be calculated by means of the formula

$$K = m\left[\frac{\lambda^2}{12} + \frac{\varrho^2}{4}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

where m is the mass, λ the length and ϱ the radius. If the bar is of more complicated shape then the moment is increased by attaching to it a block or slipping a ring over it whose moment of inertia k is calculated in the same way. The time of oscillation t' is observed anew and we have

$$K = k\frac{t^2}{t'^2 - t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

The observation of the time of oscillation must be very carefully carried out by taking the time of at least 30, 50 or even 100 swings, and is best done by means of a loud-ticking pendulum clock.

Finally, if we put

$$\frac{M}{\mathfrak{H}} = A\,; \quad M\mathfrak{H} = B \quad . \quad . \quad . \quad . \quad . \quad (88)$$

we can combine the two observations and obtain either the magnetic moment of the (first fixed and then hanging) bar, or the horizontal intensity of the earth's magnetic field:

$$M = \sqrt{AB}\,; \quad \mathfrak{H} = \sqrt{\frac{B}{A}} \quad . \quad . \quad . \quad . \quad . \quad (89)$$

Other forms of magnetometer are the sine magnetometer, the torsion magnetometer, the bifilar magnetometer, the astatic magnetometer, the compensation magnetometer and the galvano-magnetometer. If instead of the vertical axis of rotation (suspension) we chose a horizontal one we have the magnetic balances, on the one hand the Helmholtz-Kopsel balance for bar magnetism, in which the procedure is the same as with the Gauss magnetometer, and on the other the Toepler balance for the measurement of the magnetism of the earth. These balances in comparison with others soon to be mentioned do not appear to have been very widely adopted.

94. **Inclination and Declination** Since we have been discussing the determination of the earth's horizontal magnetic intensity, we must now go on to the other, the vertical component $\mathfrak{Z}$. There are many methods, but few of them are much used, $\mathfrak{Z}$ being much more usually determined indirectly, as by determining the inclination i about the horizontal axis of a needle that is free to turn in a vertical plane. $\mathfrak{Z}$ can then be calculated from the formula $\mathfrak{Z} = \mathfrak{H} \tan i$. The total intensity of the earth's magnetism can then be obtained by means of the equations $\mathfrak{F} = \sqrt{\mathfrak{H}^2 + \mathfrak{Z}^2}$ or $\mathfrak{F} = \mathfrak{H}/\cos i$, which will be self-explanatory on reference to Fig. 136. Everything therefore turns on the measurement of i. For this there is either the dip circle instrument in which the movement of the needle must be quite free from friction, and which does not give any very great accuracy, or the earth inductor of Wilhelm Weber. This consists of a wire coil of a very great number of windings. The coil is mounted so that it can be turned about either a horizontal or a vertical axis, and in either position through an angle of 180°, so that its plane before and after the rotation in the one case lies exactly horizontal and in the other exactly vertical and exactly perpendicular to the meridian. That this condition has been fulfilled can be determined either by means of a small magnetic needle in a rectangular frame and a spirit level, or by making some supplementary observations with the help of additional apparatus devised for the purpose. In both cases the axis of rotation must be correctly orientated, that is it must be exactly in the horizontal plane or exactly in the vertical meridian plane. The effect of these two rotations is observed by means of a galvanometer. The necessity of any complicated and uncertain calculations is avoided by comparing the throw produced in each case, a_1 and a_2, so that we have

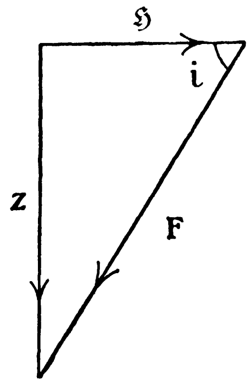

FIG 136.

$$I = \tan^{-1}\left(\frac{a_1}{a_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

If carefully performed the method is capable of giving very exact results. The apparatus by Hartmann and Braun (Fig.

137) represents a type that is capable of observing the effect of the two rotations; that by Leonard Weber and others has been still further perfected.

There is yet another angle involved in the declination, that is the angle which a needle capable of turning about a vertical axis makes with the geographical meridian; it is called the declination, and the symbol for it is usually D. Its measurement is divided naturally into two parts, namely, the determination of the geographical (astronomical) meridian, and

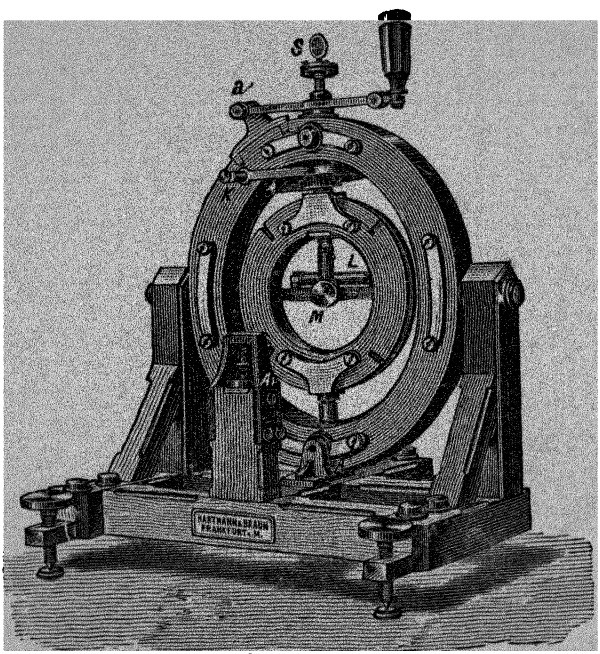

FIG. 137.

that of the magnetic. For the determination of the first, either terrestrial landmarks are used—for example, a church tower, the position of which is known—or astronomical objects, in particular the limb of the sun, or stars at different times; for the other, the dip of the needle. As typical examples of three of the most usual classes of instruments those of Gambey and of Gauss and Lamont may serve. Briefly described, the declinometer of Gauss consists of the usual magnetometer together with a theodolite placed at some distance from it. Care is taken that the magnet hangs free from all torsion and consequently takes up its position exactly in the magnetic meridian plane, which can be easily ensured by hanging an unmagnetized bar in place of the magnet and noting the position and then progressively adjusting the position of the torsion head until the unmagnetized bar and the magnet both take up the same position, so that if a telescope reading is employed the same deviation is read on the scale or (since a scale is not really needed) the same mark appears in the telescope. Or the angle ϕ between the position of the magnet and that of a magnetized bar of equal weight is approximately determined and the factor $\phi\theta$ (θ being the torsion relation) is introduced as a correction. Then by means of the adjusting screws the mirror is placed at right-angles to the magnetic axis of the bar, which is attained when, after turning the bar about so that its under-side comes uppermost, the same mark appears in the telescope. The theodolite is now directed towards the astronomical object, and then turned so that an object placed at the middle of the object glass falls on the cross-wires, and thus one obtains the declination. It is most convenient to bring the astronomical object, through the creation of an image by means of a line, to the same visual distance as the object mark—a hollow magnet may also be used so that by means of a lens at the front end a mark at the back end can be observed. Modern magnetometers are thus made by Brunner and Carpentier in Paris and by Bamberg in Berlin.

95. **The Compass**—The present is a good opportunity to say a few words about the converse problem, that is the determination of the geographical or magnetic meridian by means of the magnet. The apparatus used for this purpose is the compass. Apart from doubtful Chinese information the oldest knowledge of the compass dates back to the eleventh century; but only in the thirteenth does it appear to have been actually adopted, and Flavio Gioja enjoys the distinction of having been the first to make it really serviceable. The ship's compass in its modern form consists of a horizontal turning plane, usually with a sharp pivot which works in a little cup in the magnetic needle, to which is attached a card either subdivided into angles or showing the cardinal points; and a fixed ring surrounding its periphery on which the fore-and-aft line of the ship is marked. The fixed part of the apparatus is suspended on universal bearings so as not to be influenced by the pitching or rolling of the ship. Since the declination is not a fixed amount (see the chapter on the

magnetism of the earth) the astronomical meridian cannot be directly observed but only the magnetic direction, and to each reading the proper correction must be applied.

To ensure that the movement of the compass should be as free as possible the moving part has to be made very light. It has also been shown that a system of several magnets works better than a single needle ; and in the third place the compass must be shielded as much as possible from concussion. The first two conditions have been successfully realized by Lord Kelvin. In his form of the apparatus the compass card consists of an aluminium ring which is connected by means of silk cords to the pivot cup and carries a system of eight fine magnet needles suspended by silk threads. The combination has a powerful magnetic moment, but weighs only 14 grams. Protection against concussion is obtained in the so-called fluid compass in which the case is filled with glycerine or spirit. We cannot enter here into a description of the various forms of ship's compass—the azimuth, the steering and the boat's compass, and so forth. The self-registering compass which continuously records the course of the ship deserves to be noticed, however.

Simple as the theory of the compass may be it becomes very complicated when the disturbances or deviations of the needle through the magnetism of the ship itself is taken into account, which is now quite unavoidable in consequence of the partial or predominant use of iron in ship construction. The magnetism of the ship in the longitudinal direction may be resolved into a horizontal and a vertical component ; and on the other hand, according to its character, into permanent, sub-permanent, or temporary magnetism. It depends partly on the position, in respect to the magnetism of the earth, of the iron masses during construction ; partly on the position of the ship during its journey with respect to the magnetism of the earth. The former is of constant or of slowly diminishing character ; the latter is continually varying, and is particularly affected by the direction of the ship, its oscillations and the geographical longitude in which it happens to be ; but the constant part also exerts an effect on the compass needle which is dependent on the geographical longitude, because this effect is in its turn dependent upon the earth's magnetism, the horizontal component of which varies with the longitude. Further this effect, according to the nature and direction of the magnetic parts from which it proceeds, is either of a semicircular or a quadrantal character; that is to say there are two or four positions of the ship for which it is zero, and two or four others for which it is a maximum or minimum. From all this it becomes clear that the idea of drawing up a table of deviations for the ship is scarcely practicable; and also that the attempt to compensate for these deviations by placing magnets and masses of iron of some particular form in particular positions is a problem which permits of no easy solution. Nevertheless the labours of Flinders, Airy, Kelvin and others, which have been continued down to the most recent times, have succeeded in finding an arrangement which generally speaking satisfies all requirements. In Airy's arrangement, longitudinal and transverse magnets are placed in the neighbourhood of the compass, as well as bars of soft iron. In the Kelvin arrangement these bodies are placed inside the case of the compass itself, and in order to avoid the dangerous influences that these might introduce the compass needle (or system of needles) is made as light as possible. The semicircular error is compensated for by one transverse and two longitudinal magnets which are placed symmetrically with regard to the vertical, underneath the compass; and for the variations in this error due to geographical longitude a vertical soft iron bar is added. The quadrantal error is done away with by two spheres of soft iron which are placed symmetrically on either side of the needle point. Finally a vertical magnet compensates for the errors arising through the oscillations of the ship. By preliminary investigations it is then possible to secure compensation under all circumstances. There are of course plenty of difficulties and on that account the magnetic compass has in many cases been abandoned in favour of one based on a quite different principle, the gyro-compass which depends on the turning force of a rapidly rotating body.

96. **Local Apparatus and Variometers**—When it is a question not of taking absolute measurements but only of comparing the same magnetic magnitudes at different places or at different times, much simpler forms of apparatus are of course sufficient, and attention can then be principally directed towards the design of apparatus which are handier and more convenient in use. Thus if we want to compare the horizontal intensity at different places we can either use the deviation formula

$$\frac{\mathfrak{H}_1}{\mathfrak{H}_2} = \frac{\tan \phi_1}{\tan \phi_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

or the oscillation formula

$$\frac{\mathfrak{H}_1}{\mathfrak{H}_2} = \frac{t_2^2}{t_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

but corrections have still to be applied in either case on account of the temperature and the induced magnetism. If, instead of steel bars, bars of quite soft iron are used, then as a result of the magnetic induction the oscillation formula is completely transformed and now runs

$$\frac{\mathfrak{H}_1}{\mathfrak{H}_2} = \frac{t_2}{t_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

Apparatus for the continuous observation of the quantities under consideration are called variometers, and there are intensity, inclination and declination variometers. The first have various formulæ for the relative changes recorded, that is for the quantity $\delta\mathfrak{H}/\mathfrak{H}$ according to the nature of the suspension, the choice of the principal position, and so forth. For example, by constraining the needle to take up a cross-position through the so-called deflectors, if u is the angle of deflection between the ends of the thread

$$\frac{\delta\mathfrak{H}}{\mathfrak{H}} = \frac{\delta u}{u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

For such purposes magnetometers with quartz threads are commonly used, the elastic behaviour of which is very regular, but in that case the very lightest possible magnetic systems must be chosen. In the instruments of Eschenhagen, for example, they weigh only about 1·5 grams, and the quartz threads only need to be about 0·05 to 0·1 mm. thick, and the sensitivity becomes extraordinarily great so that the very slightest variations in the earth's magnetism can be recorded.

Finally those which function the most perfectly are of course the registering instruments, the magnetographs, usually provided with photographic recording apparatus, such as are in operation in the chief observatories, their construction owes much to Eschenhagen.

(*b*) STRENGTH OF THE FIELD

97. **Electrodynamical Methods**.—From the special cases so far considered we shall now turn to the more general task, the measurement of any sort of field and any degree of magnetization. The two tasks are in many respects similar, still it is

advisable to take each separately, and we shall begin with the measurement of the field. In this connection we can pass over the first of the numerous methods applicable in this case, the magneto-motive, because it has already been discussed.

The most closely related method is the electrodynamical. We can either observe the displacement effect of the field on a linear, or the turning moment on a circular conductor. The last method can be made the more accurate since a coil of many turns can be employed. As an example of the first we have the Kelvin method, the principle of which is shown in Fig. 138. The field is horizontal and perpendicular to the plane of the paper; f is a wire which hangs down between the pole shoes F. The current i is led into it through the mercury cup C. The force acting towards the left which is exerted on it,

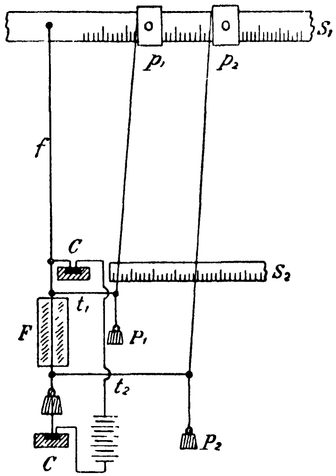

FIG 138

$K = il\mathfrak{H}$ (l being the actual length), is balanced by the pull exerted by the two pendulums p_1P_1 and p_2P_2, from the dimensions of which, the scale readings S_1S_2 and the pendulum weights P_1 and P_2, K in the above formula, and thus $\mathfrak{H}$ in absolute measure, can be determined. As regards the other method the turning moment on a coil with bifilar suspension may be determined (Stenger), or the torsion at the support according to the principle of the turning coil (page 166), Siegbahn's method or the weighing method may be adopted (Angström, Cotton and others). As an example the perfected electrodynamical balance of Weiss and Cotton is shown schematically in Fig. 139. At one end of the balance beam is the conductor circuit ABCD, and this is so formed that AD and BC, being arcs of a circle, experience no effect, nor does CD,

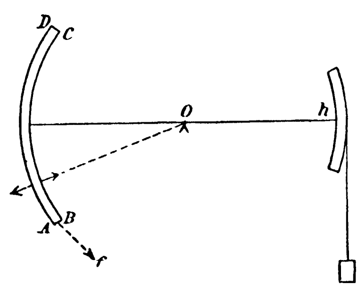

FIG 139

because it lies too far from the field. Only the current element AB is left, and this, because it is on a radius of the circle, experiences the full effect. At the other end of the beam is the pan and weights, and here also a second conductor circuit of other dimensions can be introduced. The measurement can either be made with constant current and variable weights, or conversely with constant weights, and such an adjustment of the current that a balance is obtained. Each method allows of the possibility of using the current by which the field has been created so that only one ammeter is needed. This method has the great advantage that there is no need to make an actual weighing and that the current i can be read very much more quickly and exactly than can p. According to the theory the current element in the field behaves as though its weight had been increased by $p = \frac{\mathfrak{H}li}{9{,}810g}$; conversely therefore we have $\mathfrak{H} = \frac{9{,}810p}{li}$. If we take $\mathfrak{H} = 10{,}000$, $l = 1$, and $i = 1$, p becomes approximately equal to one gram, and this with a delicate balance can be measured to a very high degree of accuracy. For weaker fields l and i must be increased; for this purpose a plane spiral is used built up of many turns and so arranged that the field only affects one side of it.

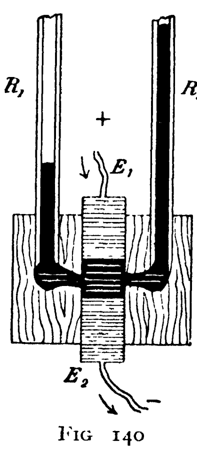

FIG 140

In Kelvin's method if the solid conductors are replaced by a liquid, namely, mercury, we then come to the method of hydrostatic pressure. The mercury is enclosed in a flat insulated chamber (Fig. 140), which is traversed downwards by the current E_1E_2, and which in the direction of the field, that is from front to back, has only a very small thickness d. In consequence of the electrodynamic effect a difference of level is set up through the communicating tubes R_1 and R_2 corresponding to a difference of pressure P. We therefore have $\mathfrak{H} = Pd/i$. Leduc and du Bois have worked out two types which are specially well adapted to the measurement of medium and strong fields. As a method of absolute measurement it suffers from the fact that it is very difficult to determine the small magnitude d with any great accuracy.

98. **Induction Methods**.—The most important and most frequently used method is the induction method. It depends upon the induction of magnetism in soft iron, or the currents which are set up in wire coils as soon as the test piece is moved relatively to the lines of force. Wire coils are almost exclusively used and are placed with their planes perpendicular to the lines of force. They are then pulled outward into another plane or are turned through a definite angle. According to the angle another factor ε enters into the expression for the strength of the field. The method obviously corresponds to the application of the earth inductor. The relative strength of the field is, *ceteris paribus*, simply proportional to the throw of the galvanometer. In order to obtain the strength of the field in absolute measure, the formula

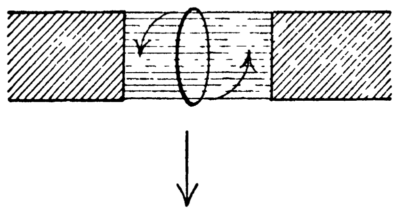

FIG 141

$$F = \varepsilon \frac{w}{n} e \quad . \quad . \quad (95)$$

must be used, in which w is the resistance, n the number of windings in the coil, and e the quantity of electricity flowing through the galvanometer. By the withdrawal of the coil or by turning it through 90°, as will be seen from Fig. 141,

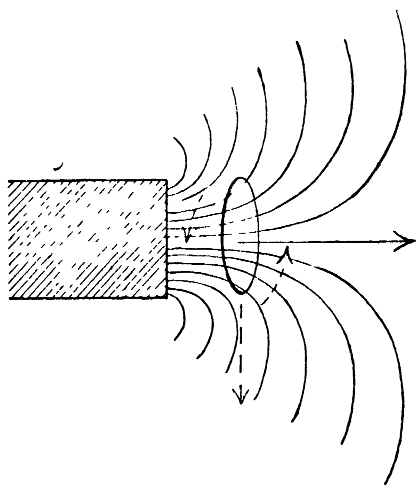

FIG 142

the same effect is obtained because in the one case the whole coil cuts half the lines of force, and in the other its upper half cuts one half and its lower the other, and these two halves produce an additive effect. The factor here is equal to unity. If the coil be turned through 180° (the most convenient procedure) the effect is twice as great, and the factor becomes $\frac{1}{2}$. If the field is not homogeneous, as we have so far assumed, the motion of the coils must be adjusted accordingly. In the case shown in Fig. 142, for example, the coil could not properly be withdrawn either upwards or downwards because obvious errors would be introduced, nor could it be simply rotated, but it must be withdrawn by a motion towards the right, keeping it parallel to itself in doing so. In fields which are varying the galvanometer must be replaced by an alternating-current instrument, best of all an electrometer, or an electrostatic voltmeter, and the difference between the maximum and the effective strength of the current, and in some cases the form of the alternating-current curve, must be taken into account in order to connect e with the throw of the galvanometer in the above formula The effective area of the coil, the total resistance of the current circuit, the time of swing, the reduction factor and the damping coefficient of the galvanometer must be known or determined. The determination of these quantities, with the exception of the first, can however be avoided if in the circuit of the galvanometer and the coil an earth inductor is introduced and the throw b and B, for the coil and the earth respectively, are observed in turn (the throw with the earth inductor is to be obtained by means of a rotation round a vertical axis). If $\mathfrak{H}_0$ is the field of the earth, and if f and f' are the effective areas of the coil and the earth inductor respectively, then the strength of the field

$$\mathfrak{H} = \frac{f'}{f} \cdot \frac{b}{B} \cdot \mathfrak{H}_0$$

Recently the methodical plotting of the strength of the field by means of the so-called test coils has undergone considerable development, and a short account of it must be given. A single turn of wire of known effective area is mostly used as the coil; for the measurement of the current a ballistic galvanometer is employed. The strength of the field is obtained by means of the formula $\mathfrak{H} = cu/f$, where u is the throw of the galvanometer needle, f the effective area of the coil, and c is a factor which has been obtained by means of a special standard (auxiliary coil or steel magnet). Lord Kelvin, Hibbert and others have constructed standards of this sort which have proved very successful. Quite recently we may mention Gans as having gone very carefully into the question of the rapid and convenient reproduction of a determinate field by means of a standard. The most suitable form has proved to be a slotted ring of an iron, the permeability of which over a certain range is approximately constant, the ring being thickly wound with many turns of the current windings.

Finally we come to the class of differential apparatus for the measurement of field strengths. As an example we will give a short description of the method due to Paschen. It consists in the comparison of the field with the field inside a coil through which a known current is passing. On one and the same axis are two inductors J and J′ resembling two drum armatures. They differ however in having no iron core, J rotates in the field to be measured, J′ in the coil. The induced electromotive forces are arranged to oppose each other, and the current i through the coil is so regulated that a galvanometer in the current is not affected. Under these two conditions $ic = \mathfrak{H}$ where c is a constant of the apparatus. The effect of any stray lines of the field on the coil and conversely are eliminated by preliminary experiments of a similar kind, and they can be made so small, at least the latter, that they may be neglected. If w, w', and f are the effective areas of the two inductors and the coil,

$$\mathfrak{H} = \frac{w'}{w} fi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

The three areas are obtained by means of special experiments which depend on the comparison of the resistances of the three coils The procedure is applicable up to about three amperes, corresponding to 650 w'/w gauss, and by means of a slight modification, still further. The accuracy obtainable is of the order of $\frac{1}{2}$ per cent. and in certain cases is still better.

The damping method can also be included along with the induction method. The damping of the oscillations which a conductor experiences in a powerful magnetic field is considerable. Its logarithmic decrement (difference of the logarithms of successive oscillations), which can be very exactly measured, is proportional to the square of the strength of the field, a fact which is in itself sufficient for relative measurements. If absolute measurements are required a special case is made use of and the procedure is as follows. A coil, the moment of inertia of which is K, and the effective area of which is F, is hung by a unifilar or bifilar suspension, and its period of oscillation T is obtained when the circuit is open. This is then closed by means of rheostat (without self-induction) and the rheostat resistance is altered until the oscillations are aperiodic, which can be definitely determined. Let this resistance be W. We then have the simple formula

$$\mathfrak{H}F = \sqrt{\frac{4\pi WK}{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

99. **Hall-effect and Bismuth Methods** — These two methods are closely related to each other, and we shall study each in turn.

(*a*) Hall-effect method. If a current is sent lengthwise through a thin piece of metal sheet, and if it be placed at the same time in a magnetic field the direction of which is perpendicular to its plane, there arises, as has been shown (page 185), a potential difference between opposite points on the edge which can be determined by means of a galvanometer. Within certain limits the deflection will be proportional to the strength of the field. For gold and silver this holds for fields up to about 25,000 units, and in addition it is proportional to the

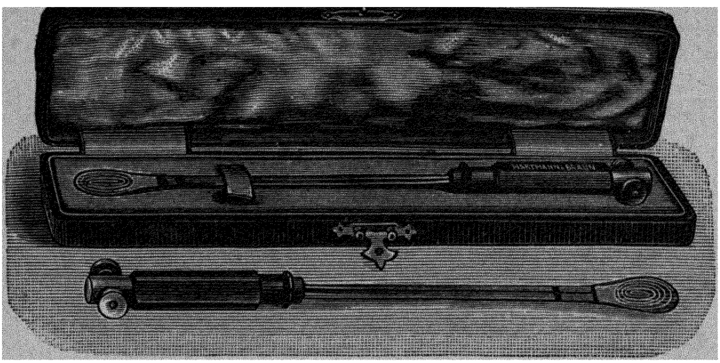

FIG. 143.

primary current, so that the sensitivity can easily be regulated. The method of course presents some difficulties, since sources of error arise and can only be taken account of by methods which necessitate great complications. The reader should consult the work of Peukert and the comments of Zahn.

(*b*) Bismuth method. The electric resistance of metals (see page 188) undergoes a change when placed in a magnetic field. Whilst in most metals this change is very insignificant, in bismuth it is quite considerable, so that the resistance in a powerful field may be double its normal value. Leduc was the first to propose that strength of field might therefore be measured by means of thin plates of bismuth or of tubes filled with melted bismuth, and developed the necessary formulæ for the method. It was Lenard, partly in conjunction with Howard, who succeeded in working out a serviceable method. Pure thin bi muth wire is formed into a flat double spiral and is cemented between thin plates of mica. The thickness of the coil and its cover amounts to scarcely more than a millimeter, and its resistance is about ten ohms. Fig. 143 shows the apparatus in its case. The spirals are placed perpendicular to the direction of the field. Since there is no simple relation between resistance and strength of field, the spiral must be standardized beforehand. To enable direct readings of the strength of the field to be taken Hartmann and Braun have constructed a bridge for use with the apparatus. For the plotting out of a constant field the bismuth spiral has proved of great value. But it cannot without further investigations be applied to the determination of the momentary value of rapidly fluctuating fields on account of the hysteresis effects with which the phenomenon is associated.

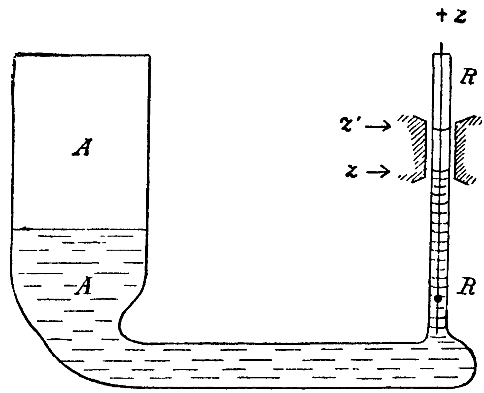

FIG 144A.

100. **Hydrostatic Method**.—This method has been developed by Quincke and du Bois and will be easily understood from the diagram in Fig. 144A (compare page 101). It is essentially a **U**-shaped tube consisting of a wide reservoir AA and a narrow ascending tube RR. If the latter is brought into a field at right-angles to it, the level of the liquid rises (or falls) from Z to Z′. For slightly magnetizable liquids, if p is the pressure corresponding to the difference of level produced and κ the susceptibility, then $\mathfrak{H} = \sqrt{2p/\kappa}$. The effect is of course considerable only in very strong fields; water in the strongest field that can be produced (50,000 gauss) would only fall about 0·6 cm. Iron chloride (concentrated solution) would however rise about 60 cm., but in this case the simple formula would not be quite exact. H. du Bois therefore in his apparatus, which is shown in Fig. 144B, has adopted the artifice of inclining the ascending tube in such a way that it makes an angle u with the horizontal, whereby the method is obviously rendered more sensitive in the ratio 1 : sin u. The **U**-tube is here replaced by the glass vessel AARRGS and

the microscope M turns along with it for the observation of the meniscus E. For the liquid, a half-concentrated solution of the green rhombic nickel sulphate is usually recommended. If g is the specific gravity, d the density of the liquid, and b the oblique rise, then we have

$$\mathfrak{H} = \left(\sqrt{\frac{2gd}{\kappa}}\right)\sqrt{b \sin u} \quad . \quad . \quad . \quad . \quad (98)$$

The quantity in brackets, by suitably selecting the strength of concentration, can be easily adjusted to have a round number for its numerical value—for example, 10,000—and a suitable value can be given to u according to the strength of the field to be measured.

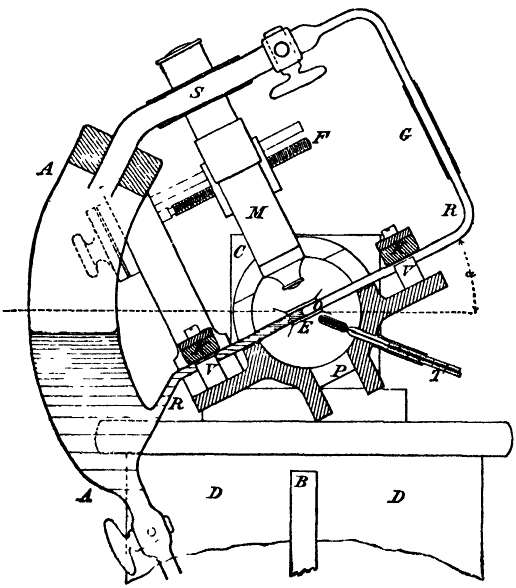

FIG. 141B.

101. **Optical Methods**—Finally we come to the optical methods. These are based upon the Faraday effect (see page 116) in transparent materials. For fields which are uniform in the direction of the light beam the effect is simply proportional to the strength of the field and the thickness of the layer of the material. The proportionality factor which is needed for absolute measurements, that is the rotation which is produced by unit strength of field on a beam passing through unit thickness of material—the Verdet constant—is very exactly known for certain substances (water, carbon disulphide, Jena glass. see page 127). From a good stable glass various standards can easily be made, which are moreover very conveniently intercomparable. H. du Bois has constructed such standards out of the heaviest sorts of flint glass, such as have the highest Verdet constant, and has given them a

slight cotter shape so that the parallel reflections that occur with parallel surfaces are obviated. With a thickness of 1 mm. these standards are suitable for fields of a magnitude of about 1,000 gauss. It is to be emphasized that the direction of rotation depends entirely upon the direction of the field and not on that of the light (see page 119). If therefore a beam is allowed to fall on the plate or the layer of material in a nearly perpendicular direction and is then reflected back again from a deposit of silver on the back surface, double rotation is obtained, and thence if d is the thickness and c the Verdet constant

$$\mathfrak{H} = \frac{\omega}{2cd}$$

(c) MAGNETIZATION AND PERMEABILITY

102. **General. Magnetometric Methods**.—In parallel with the measurement of the field, that is, with the measurement of magnetic force in free space, we have the further and specially significant task of determining the condition within the body itself; and here, since the para- and diamagnetic bodies offer no special points of interest, we limit ourselves to ferro-magnetic substances. In this determination it is substantially a matter of indifference whether we measure the intensity of the magnetization $\mathfrak{J}$ or, if it is more convenient, the induction $\mathfrak{B}$. Both quantities are interconnected by a simple formula and the one can easily be deduced from the other. For the rest the task here is a much wider one, since we are not usually concerned with the determination of a single value, but with a whole series of values of $\mathfrak{J}$ considered as functions of the strength of the field $\mathfrak{F}$, therefore, speaking graphically, with the construction of the whole curve of magnetization, and even, going somewhat further, with that of the hysteresis loops.

The problem presented varies according as we are concerned with the investigation of a definite body or merely with the properties of the material—for example, those of a particular quality of iron. In the last case the material can be given the most suitable shape, the shape, that is, which according to theory permits of certain conclusions from the whole moment of the body regarding the "magnetization" Such forms are the sphere, the ellipsoid, and in particular the elongated ellipsoid of rotation; the long, thin wire shape; and finally the ring. For other shapes we are compelled to rely on more or less uncertain calculations as, for instance, for what is the most important case in practice, that of the cylindrical bar with end surfaces, bundles of thin wires and thin sheets. And also between the first-mentioned forms there is a difference, as the same forms are not equally suitable to the demonstration of theory (spheres, short ellipsoids) as for measurement (elongated ellipsoids, rings). The field causing the magnetization may be that of the earth, or may be derived from steel magnets or electro-magnets or from currents flowing through coils. Since the field exercises an effect of the same kind as that of the body under examination, this effect must be taken into account or allowed for. Both are most easily achieved by means of coils since the effect can then be easily calculated, and any disturbing subsidiary effect on the measuring apparatus can be easily corrected for by means of a compensation coil, and in the event of this compensation not being complete any effect still remaining can be dealt with in other ways. The material to be investigated must be as pure as possible, homogeneous and, at the start, in a non-magnetic condition.

In the magnetometric methods the body is allowed to influence the needle of a magnetometer and the effect is compared with that of the magnetism of the earth or that of some other known force. These methods have already been described; but whereas we were then concerned with the magnetic moment of the body, here $\mathfrak{J}$ will be obtained, and from it κ, and so forth, can be deduced; and not, as in that case, for bars only, but for other shapes. The formulæ, therefore, will usually be different. Since rings have no external effect, we shall in this method be mostly concerned with ellipsoids, and on account of their simplicity with ellipsoids of rotation. If a and c are the semi-axes in the equatorial and polar directions, then the volume is $(4/3)\pi a^2 c$, and therefore the total moment M must be divided by this amount in the case of uniform magnetization, in order to obtain $\mathfrak{J}$. Further, if $\mathfrak{H}$ is the horizontal component of the earth's magnetism, and r the distance—which should not be too small—of the body from the needle, and θ its deflection, then for the first and second principal positions respectively we have

$$\mathfrak{J} = \frac{3r^3\mathfrak{H}\tan\theta}{8\pi a^2 c} \text{ or } \mathfrak{J} = \frac{3r^3\mathfrak{H}\tan\theta}{4\pi a^2 c} \quad . \quad . \quad (99)$$

The actual arrangement of the measurement is shown schematically in Fig. 145. A is the bar and is completely enclosed in the principal magnetizing coil, and a second bar serves to compensate the vertical component of the earth's magnetism. (The latter coil is supplied from C with a constant current which is regulated by D.) B is the magnetometer, E is a coil for the compensation of the direct effect of the principal coil, K is the battery for supplying the principal coil, F is a commutator, G is the galvanometer, and H is a liquid resistance for the steady increase of the magnetizing current. Of the two magnitudes present in the formulæ, $\mathfrak{H}$ is either known or can be referred to the normal value by

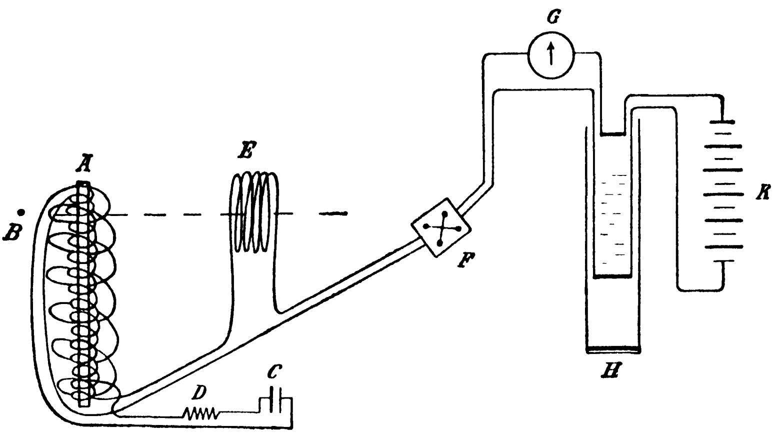

FIG 145

means of a local variometer; and for very exact determinations corrections are to be applied to *r*, the distance on account of the position of the pole.

An improved form of magnetometer has been constructed by Gray and Ross. It was designed to satisfy the following conditions: exact and rapid adjustment, solid construction, any inaccuracy in the setting of the coil in the exact coaxial position shall not introduce errors, the procedure shall be equally well adapted to both weak and strong magnetization, and shall permit of observations being made at any temperature. For this purpose two instead of one compensation coils are used in the east-and-west direction, and a third is added in the north-and-south direction. Of these one serves for coarse and the other for fine adjustment. The whole is mounted on a cross-bar of wood 350 cm. long with arms 135 cm wide, the planks themselves being 24 cm. in width. For high temperatures electric heating is used and for low temperatures immersion in liquid gases. In Fig. 146, A is the solenoid which serves for the reception of the specimens, which are 20 cm. long, E, F, G are the compensation coils, B is the needle and mirror, H is the damping coil, C the lamp provided with a fine cross-wire the image of which appears on the scale at D, and finally J is a compensation magnet.

In conclusion it is obvious that a differential method may be adopted, that is, the magnetization of an unknown may be compared with that of a known magnet. In this connection an analogy to the Wheatstone bridge may use-

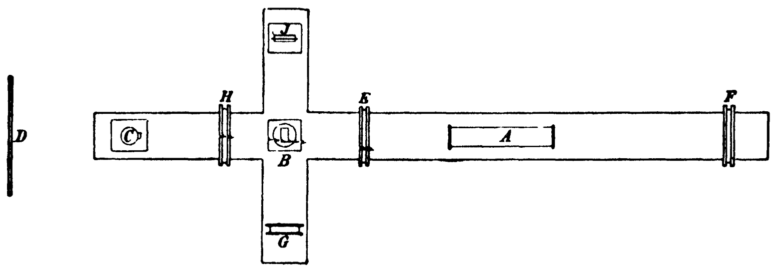

FIG 146

fully be adopted. In Ewing's apparatus, for example, the two bars to be compared are placed in two parallel magnetizing spirals, and are connected magnetically at their ends with two yokes of soft iron Between two projections from the yokes the needle floats. The number of ampere turns surrounding the test-piece can be varied until the deflection of the needle is reduced to zero. In a later form the comparison has been dispensed with and the bar under test compared with a length of air. Thus, in the apparatus of Baily the test-piece is brought into a magnetic circle which contains a known air-gap. Over the air-gap a pair of needles is pivoted which, on the one hand, is deflected by the stray lines of the gap, and on the other by a coil capable of rotation lying behind the magnetizing coil. The former gives a measure of the flux of induction and the latter of the force. The coil is so placed that the needle gives no deflection and then a very simple relation gives the permeability directly. The curve tracer of Searle is also very ingenious. It operates by means of two auxiliary bars which are perpendicular to each other so that the strength of the field is photographically recorded in one direction and the magnetization in another at right-angles to it. Thus, we can obtain directly the magnetization curve or the hysteresis loops.

103. **Electrodynamical Methods**—In the electrodynamical methods the effect on a coil is used in order to determine the magnetism. It is therefore the converse case to that of the moving coil galvanometer (see page 166). The best representative of this type of apparatus is that devised by Koepsel and afterwards repeatedly improved. It is constructed by Siemens and Halske, and, as Fig. 147 shows, consists of a semicircular yoke which connects the ends of the bar, and in a cavity in the middle of this yoke is a coil consisting of a few turns of fine wire, and capable of rotation. The details are so arranged that a reliable result is obtained in the most direct way. The electrodynamical method is specially well adapted for the determination of the momentary value of the magnetization when the force is rapidly changing. Kaufmann has devised an apparatus that is at the same time an hysteresis measurer. Other types have been built by Ewing.

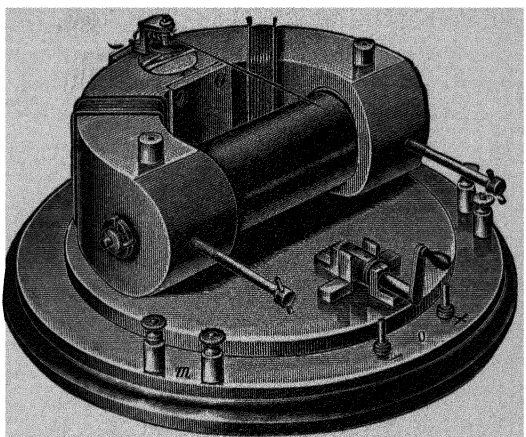

FIG. 147.

104. **Induced-current Methods**—We now come to the methods depending on induced currents, which may also be called the ballistic methods. In addition to the coil which serves to magnetize it, the test-piece is also surrounded, either completely or at some particular place, by another coil, called for the sake of distinction the secondary coil, and this last is put in circuit with a galvanometer with a long period of swing. If now the test-piece is put into the coil or withdrawn from it, or if the current is switched on or switched

off or reversed in the magnetizing coil, or if the position of the body or the strength of the magnetizing current be altered, in short, for any alteration in the test-piece an induced current is set up in the secondary coil which causes a throw of the galvanometer needle which is proportional to the magnetism or the change of magnetism. It is necessary in making the observations to take certain precautions, especially as regards the sudden switching on or off of the current in the magnetizing coil or the setting up of induced currents in the iron. It is also well to take care that all changes of place or current occur gradually. In order to obtain absolute instead of relative measurements, the galvanometer must be calibrated; that is, we must find what throw is produced by a known magnetic force, and for this purpose the magnetism of the earth may be employed by including in the circuit of the secondary coil an earth inductor and turning it quickly about a vertical axis (compare page 213 above). If the throws produced in the principal experiment and by the rotation of the earth-inductor respectively are δ and δ_0, the number of turns of the secondary coil and of the earth inductor, n and n_0, $\mathfrak{H}$ the component of the earth's magnetism if this is used, f the effective area of the earth inductor, then we have the equation

$$\Phi = \frac{2n_0 f \mathfrak{H} \delta}{n \delta_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

where Φ is the flux of induction, that is, the magnetic induction for the whole cross-section (for 1 cm. of the length of the body). In order to obtain the magnetic induction $\mathfrak{B}$ per cubic centimetre, it is only necessary, if the secondary coil is in immediate contact with the body so that there is no air-gap between them, to divide by its cross-section q, otherwise before dividing by q a correction which is usually small must be applied to Φ. From $\mathfrak{B}$ we can then obtain μ, and working backwards, $\mathfrak{J}$ and κ.

Instead of the earth's magnetism the effect of an electric current may also be used as a standard of comparison. Then we have

$$\Phi = 4\pi i n' q' \cdot \frac{\delta}{\delta'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

in which i is the current in a coil which, on being suddenly switched on or off, produces a throw δ', q' the cross-section and n' the number of windings per square centimetre. In comparison with the method which makes use of the magnetism of the earth this method has the advantage in the first place that we do not need to know the value of the earth's magnetism at the particular time and place under consideration. But in addition to this it is simpler and makes one special experiment unnecessary, if, instead of the coil to be used in the comparison experiment, the magnetizing coil is used with the test-piece removed. The effect of the current necessary to produce the magnetization must be deduced, however. For this it is only necessary to refer the reader back to what has already been said. It must be noticed, however, that the ring method cannot here be used because it is not possible to withdraw the iron body. Therefore here the comparison has to be made with the magnetism of the earth.

The induction method, because of its applicability to the most varied cases, is the one most usually preferred. It is specially important in the case of rings because here both the magnetometric and the electrodynamic methods break down. Kirchhoff was the first to refer to it, and Stoletow and Rowland the first to apply it.

In the application of the induction method to bars and similar shapes, the demagnetizing force introduces disturbing effects, and in such cases various arrangements are to be met with, of which the most important are the following.

Closed-yoke Methods. These were first indicated by J. Hopkinson. The bar is put into a single or double closed yoke, that is, its ends are enclosed in a frame of very soft and therefore permeable iron of large cross-section; well annealed wrought iron is the most suitable. This yoke takes up the lines of force that radiate out from the ends, so that the simple formula without regard to any end-effect can be used, and that the more exactly, the longer that portion of the bar on which the magnetizing coil is wound. The method is, of course, the more sensitive the less the permeability of the test-piece in comparison with that of the yoke. Fixed induction coils can be used surrounding the middle of the bar; the effect of varying the current is then observed. Or the induction coil may be drawn off and the resulting induction impulse measured, and for this purpose automatic arrangements may be employed. The Ewing modification is still more convenient. Here two bars of the material to be tested are used, and their ends are connected together in pairs by an armature. The secondary coil is here again placed at the middle of the bar. One of the armatures is so arranged that when it is pulled off one of the secondary coils is pulled off along with it automatically. In Fig. 148 the arrangement due to Hopkinson is schematically represented.

As a special case and a particularly important one the Isthmus method must be here included. It is always suitable where it is possible to give the material to be tested a definitely complicated form. Stefan has shown (see page 200) that the force existing between the poles of a ⊏⊐-shaped Ruhmkorff electro-magnet (compare page 197, Fig. 129*e*) may be considerably intensified, when instead of flat end surfaces, truncated cones are used, which are most effective when the generating angle is 54° 44′ ($\tan^{-1}\sqrt{2}$). As will

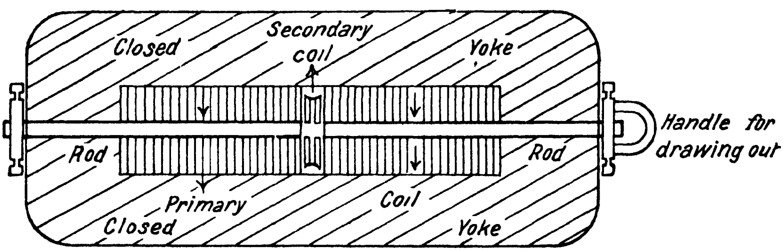

FIG 148

be easily seen, these cones have an effect similar to that of lenses in optics. In the middle of the field, when a is the distance and r the radius of the end surfaces in the first case,

$$\mathfrak{H} = 4\pi\mu\left(1 - \frac{a}{\sqrt{a^2 + r^2}}\right), \quad . \quad . \quad . \quad (102)$$

therefore as a maximum $4\pi\mu$; in the second case, however,

$$\mathfrak{H} = 4\pi\mu\left(1 - \frac{1}{\sqrt{3}} + \frac{2}{3\sqrt{3}}\log\frac{r}{a\sqrt{2}}\right) \quad . \quad . \quad (103)$$

and therefore if $\frac{a}{r}$ be sufficiently small the value may be made as large as desired: for example, if $a = \frac{r}{20}$ it would be $1{\cdot}442 \times 4\pi\mu$.

On this is based the practical adaptation of the Isthmus method by Ewing and Low. In order to achieve the twofold object of obtaining in the middle of the field a powerful but nevertheless uniform force, conical pieces are fixed to the two wide pole faces, and to the end surfaces of these, which are reduced to the smallest size possible, the little rod to be investigated is connected. A still better method is to give to the body to be investigated the form of a bobbin, that is, of a double cone with as slender and short a middle piece as possible. In Fig. 149 is shown such an arrangement constructed by Ewing, the scale being two-thirds the natural size, the true dimensions being added in millimetres. The Kapp apparatus for obtaining $\mathfrak{B}$-$\mathfrak{H}$ curves is also based on the isthmus method.

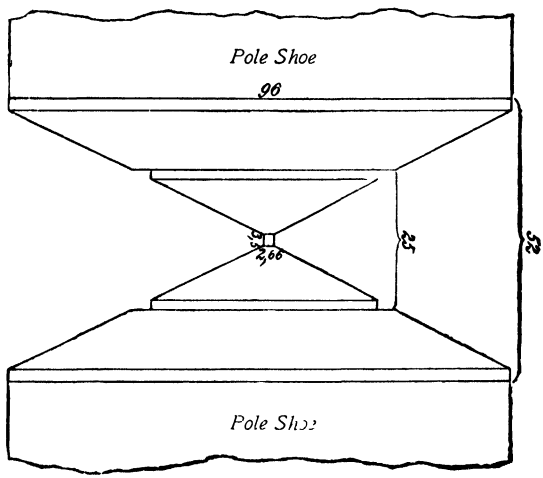

FIG 149

The isthmus has the special advantage that it enables the highest field-strengths to be obtained and at the same time a field that is very nearly uniform. By means of suitable arrangements the strength of the field can also be determined at the same time as the induction, so that the ordinates and abscissæ of the curve are obtained.

105. **Tractive Force Methods**.—We have considered in a previous chapter the tractive force of a magnet. On this, or rather on its scientific development, is based the tractive-force method of determining magnetic induction. Its foundation represents in very simple fashion the Maxwell formula (see page 203, Equation 81), $\mathfrak{Z} = \mathfrak{B}^2/8\pi$. Certain difficulties of course arise in carrying out the idea, which nevertheless du Bois, as the result of many years' study, has been able to overcome. His magnetic precision balance, which is shown in Fig. 150, is equally suitable both for scientific and for practical purposes. It consists essentially of the test bar (in the figure hidden by the magnetizing coils), which is clamped

in position over the base by means of end cheeks, and the yoke, which rests on knife-edges like the beam of a balance and floats freely over the cheeks. Since the knife-edge is not in the centre but somewhat to one side, the two arms of the balance are therefore unequal and the yoke is pulled

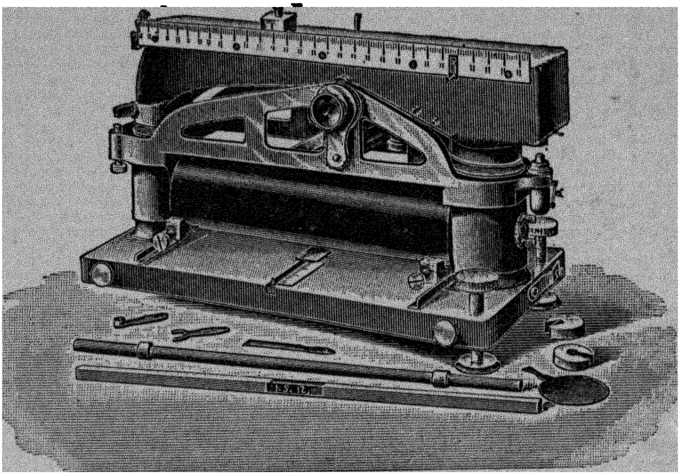

FIG. 150.

down by the stronger attractive force on the left-hand side until it comes up against the cheek. A sliding weight, seen at the top, is now adjusted until the yoke is just pulled away

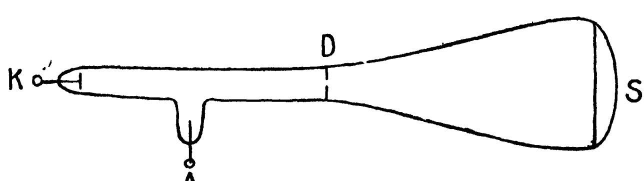

FIG. 151.

again from the under position (which is determined by an adjusting screw). Since the dimensions are suitably chosen and the scale graduated according to the square law, the magnetization can be read off direct, while the field strength is given by the current.

106. **Optical Methods**—Of the optical methods we shall only mention here the deflection of the cathode rays (page

167) by use of the Braun tubes. They are especially suitable for demonstration purposes. The Braun tube is a vacuum tube, in which, as shown in Fig. 151, the cathode rays are compelled to pass through a small hole in the diaphragm D, so as to produce a patch of light on the phosphorescent screen S. This patch of light is displaced when a magnetic pole is brought near the diaphragm. Braun pointed out its applications and Angström and others have worked out the method and have made it serviceable in many branches of inquiry.

107. **Modern Methods of Measurement**—In recent years from about the beginning of the century a system of magnetic measurements has been developed in which, as a matter of fact, it has not been so much a question of developing fundamentally new methods as of perfecting those already known in regard to practical convenience and the assurance that they can be regarded as reliable. Investigation has in particular been directed to determining the principal quantities occurring in magnetic practice, the properties of iron and steel and their alloys which are used in the construction of motors, generators and transformers. A specially important part in this question is played by two classes of quantities: those, on the one hand, which relate to magnetization, induction, flux of induction, susceptibility and permeability; and on the other those relating to the dissipation of energy through hysteresis and eddy currents.

The first things to settle are the standards for the testing of iron sheet as was undertaken at the annual meeting of the Verband deutscher Elektrotechniker in 1910 (similar standardization for wires, bars, and other forms did not seem to call for discussion).

1. The total loss in iron is to be determined by the actual testing of a sample of at least 10 kg. taken from four sheets at $\mathfrak{B}_{max} = 10{,}000$ and $15{,}000$ in C.G.S. units at a frequency of 50 cycles, and to be expressed in watts per kilogram at 20° C. These values relate to an impressed voltage of sine-wave shape, and are to be called the total loss constants (V_{10} and V_{15}).

2. The ageing coefficient shall be the percentage change in the total loss constant for $\mathfrak{B}_{max} = 10{,}000$, 600 hours after it has been heated up to 100° C.

3. The magnetizability of the iron shall be taken as being the density of the lines in C.G.S. units at 300 ampere turns

per cm. (AW/cm.), for the values of AW/cm. equal to 100, 50, 15.

4. The density of iron shall be assumed in ordinary dynamo sheet to be 7·7, and in alloyed sheet as 7·5.

5. For the measurement of the total loss constant a magnetic circuit shall be employed which contains only iron of the quality to be tested and which is assembled under proper conditions.

6. The normal thicknesses of sheet taken shall be 0·3, 0·5, and 0·8 mm.; and variations of 10 per cent. are to be neglected.

7. In cases of doubt the investigation is to be referred to the Physikalisch-technische Reichsanstalt in Charlottenburg.

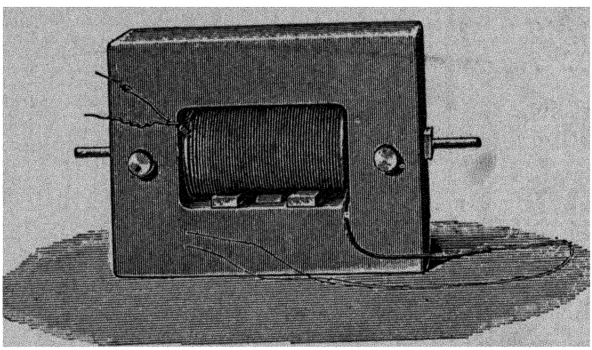

FIG. 152.

Of the method itself only a brief indication can be given here by means of a few examples summarily treated.

1. Ballistic method of the Physikalisch-technische Reichsanstalt.

2. Yoke method of the Reichsanstalt. The apparatus is shown in Fig. 152: the yoke, the primary winding, and the projecting ends of the test bar can be seen; the secondary winding, being inside, is not visible. The yoke is made out of Krupp steel sheet, it is 33 cm. long and has an open internal space of 25 × 8 × 8 cubic cm., a cross-section of 2·32 sq. cm., and therefore, on the average, about 200 times that of the bar to be tested, which is usually turned down to have a diameter of 0·6 cm., and must be at least 35 cm. long. Similar dimensions hold for bunches of iron strip. To make a good contact, cheeks of soft iron are used which are pressed together by means of screws. The primary winding consists of

about 2,000 turns of thick wire and completely fills the internal space. The secondary coil is 1 cm. long.

3. Epstein's method for the testing of iron sheet. Here the leading idea is to exclude all foreign iron, and to distribute the magnetizing coils as uniformly as possible over the whole of the magnetic circuit. Therefore a ring form is chosen, or, as being more convenient to construct, put together and take to pieces, a square as shown in Fig. 153. The circuit therefore consists of four squares of the material to be

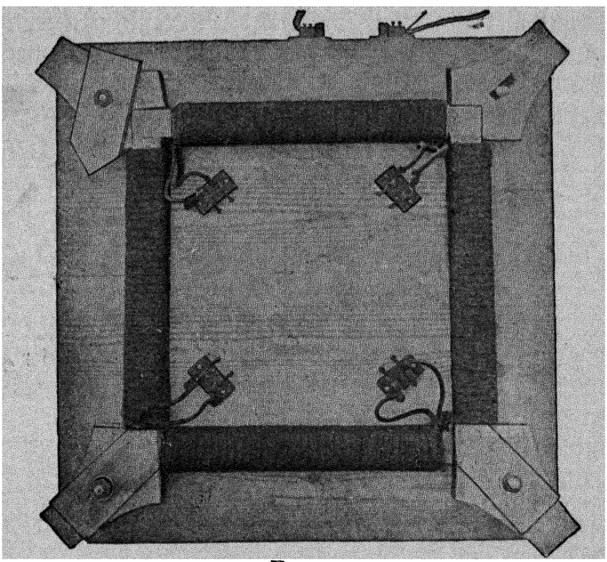

FIG. 153.

investigated, and with the exception of the four corners it is covered with a uniform winding. The measure of the total loss is obtained in watts, and can be separated into that due to hysteresis and that due to eddy currents. For this purpose a special arrangement and method is needed of working the two machines: i.e. the continuous-current generator and the alternating-current machine which feeds the coils. Since the individual iron sheets often vary a good deal in quality they must be taken in such quantity that a good average value may be expected. On this account four bundles of some 60 to 70 strips each are used. The cross-section of the iron q is deduced from the weight and density, and from this the flux of induction $\mathfrak{F} = \mathfrak{B}q$. In order to take account of the dependence of the Steinmetz coefficient (see page 70, formula 28) on the induction, the test is carried out with three values of $\mathfrak{B}$ (6,000, 10,000, and 15,000). The mean electromotive force E in volts for 400 windings and n the number of revolutions (account being taken of a correction factor) is $E = 0{\cdot}0000176\ \mathfrak{B}qn$. The following procedure is adopted. First the generator is adjusted to give the required number of revolutions and voltage. Then the reading of the ammeter and the wattmeter is taken. If the energy lost in the leads is subtracted we then get the true iron loss, and this has then to be expressed as per 100 kg., as well as for per 100 kg. per period. The last calculation is required in order to be able to separate the hysteresis and the eddy-current loss, for if f is the eddy-current loss factor

$$v = \eta\mathfrak{B}^{1{\cdot}6}\, q + f\mathfrak{B}^2 q^2 \quad . \quad . \quad . \quad . \quad (104)$$

therefore

$$\frac{v}{q} = \eta\mathfrak{B}^{1{\cdot}6} + f\mathfrak{B}^2 q \quad . \quad . \quad . \quad . \quad . \quad (105)$$

the curve for v/q is plotted and the point determined where it cuts the axis of ordinates (6,000 : 1·98W; 10,000 : 4·38W; 15,000 : 8·82W). These are therefore the hysteresis losses, and from this the Steinmetz coefficient is obtained, $\eta = 0{\cdot}0013$, in fair agreement for all three values of the induction. Finally by subtraction from the total loss we obtain the eddy-current loss, which is usually very small in comparison with hysteresis loss. The preliminary preparations for the measurement take about 1½ hours, the measurement itself takes about ½ hour, the calculation about ¾ hour, and three persons are required to take the observations. In order to be able to measure the magnetization and the permeability as well, a secondary winding is added, which does not run continuously but is divided up into seven coils on each of the four limbs, so that at each place a separate determination may be made and the leakage field also obtained; then, in order to improve the magnetization, the primary windings at the corners are coned off, and the primary as a whole suitably spaced out (the secondary naturally lies inside the primary).

4. The null method of Lonkhuyzen (Siemens & Halske).

5. The method of Gumlich and Rogowski. The basic principle is as follows: to measure both the induction and the field strength at places that are as far as possible free

from stray field; this is done by means of secondary coils, of which there are four each surrounding the Epstein sheet-iron bundles and which are joined up in series with ballistic galvanometer. Advantage is taken of the fact (on which the Ewing isthmus method is based) that the tangential component of the magnetic force at the point of transition from iron to air does not undergo any sudden change. Therefore it can be measured in the immediate neighbourhood of the outside surface of the iron. Of course, on account of the other coils the field-measuring coils cannot be brought right up to the surface of the iron and the measurement must be taken at some distance away. But it is easily possible to obtain experimentally a curve by means of a small plate that can rise and fall, which represents the percentage decrease of the field strength as we pass outwards, and these curves with necessary precautions can be extrapolated for the opposite direction. It is then shown that the curve for small values of the field strength depends upon special qualities of the iron. But from 10 AW/cm. it is practically uniform. The corners of the apparatus have no practical influence except when the strength of the field is small, as is proved by the effect of the addition of corner-pieces to improve the magnetic circuit. Therefore for normal field strengths of 25 and upwards any special precautions on this account may be dispensed with. In any case, according to the opinion of the author, the errors are much smaller than with the Epstein apparatus. The clamping screws are arranged to suit any disposition of the coils that may be required and specially provide for rapid transposition. To obtain the effective area of the coils for measuring the field and the induction, the magnetizing coils themselves are used. The dispersion of the field expressed as a percentage of the induction is smallest for $B = 5{,}000$ (about 1 per cent. at the corners), greatest for $B = 18{,}000$ (6 to 7 per cent.). The apparatus can also be used for the Epstein method, and the two results so obtained can be compared. The Gumlich and Rogowski apparatus is manufactured by the Allgemeine Elektrizitäts-gesellschaft.

6. The weak point in even the best of the above methods is the measurement of the strength of the field. In the one case we have to rely on the tangential component of the strength of the field being equal on both sides of the boundary surface between iron and air, and then we must be very careful to

exclude any normal component from the measurement, because, if they are considerable, the slightest divergence of the coil from its position parallel to the iron surface will add substantially to the result. It is of great value in all cases where there is great dispersion and the lines of induction in consequence emerge almost perpendicular to the iron. It is, however, almost impossible to adjust the coil to exact parallelism. Or, in the other case, the attempt to obtain individual values of the field strength at a particular place is abandoned and only the line integral of the field strength round a closed circuit is measured. For this purpose the magnetization current i is measured and the Maxwell equation is relied on, according to which the total magnetomotive force is determined by the number of ampere turns in the magnetizing coils. This still requires that we should infer the field strength at the place where the induction is being measured from the total magnetomotive force. The method is best in the case of the ring; in the case of the ellipsoid, as we have already seen, it is difficult to obtain the required shape, in all other forms the matter is extremely uncertain.

These considerations have led Rogowski and Steinhaus to work out a new method in which the magnetomotive force is measured directly. Starting with the equation which expresses the element of the flux in regard to the induction, or the field strength, we have

$$\Phi = qn\int_1^2 \mathfrak{B}_x dx = 0{\cdot}4\int_1^2 \mathfrak{H}_x dx$$

where q is the cross-section of the coil, n the number of turns in unit length, both regarded as constant over the whole coil The flux in a uniformly wound coil of uniform cross-section is therefore proportional to the magnetomotive force between the beginning and the end of the coil. The path along which this force acts is the axis of the coil. According to this the establishment of a measure of the magnetomotive force is in principle clear; the special condition is that there should be a cross-section everywhere equal, and perfect uniformity in the windings. The cross-section will also be chosen as small as possible in order to reduce the effect of any errors that may be present. The magnetomotive force measurer of Rogowski and Steinhaus is constructed in the form of a strip. It is prepared from presspan 60 cm. long, 2·5 cm. wide, 0·1 cm. thick, and carries a double layer of wire of 0·02 diameter.

The cross-section varies by about $1\frac{1}{2}$ per cent.; there are two soldered places present in the wire where it has been broken in the process of winding, the specimen was therefore very well adapted to show the sort of results that can be obtained without any special perfection of construction. It was covered with a rubber band, and small wooden blocks served to protect the ends.

It was now tested whether actually, as theory requires, the total magnetomotive force for the same ampere turns is the same whatever the path; whether it becomes zero when no current encircles the closed circuit; and whether in spite of the introduction of iron into the field, and the distortion thereby produced, the field remains unaffected. The field was produced by means of a four-cornered coil of 458 turns, the ends of the windings for the pressure measurer were connected with a ballistic galvanometer, the throw of which for the reversal of 1 ampere in the magnetizing current was measured. Then a pressure measurer bent into a closed curve, and held together by a clamp, was brought into first one and then another part of the magnetizing coil—a quadrilateral enclosing it more or less closely and supporting it at one corner, into which the iron bundle was brought. An example is given in Fig. 154. The experimental results in all cases confirmed the theory. It only remained, therefore, to standardize the pressure measurer. This was done by observing the throw of the galvanometer on reversal of a current i in the field of a magnetizing coil of N windings. A magnetic tension of Ni/u therefore corresponds to one division of the scale. In the foregoing example the throw on the average was 90·6 divisions of the scale. One scale division therefore corresponds to a magnetic tension of 5·06. If, further, a change of b in the flux of induction corresponds to one division

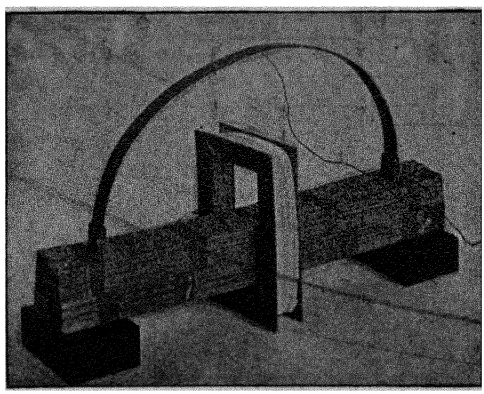

FIG. 154.

on the scale, then the unit of flux corresponds to a magnetic tension of $2Ni/ub$.

7. Finally there is the oscillographic method to be mentioned which really belongs to the electrodynamic method. The oscillograph itself belongs to another subject since it serves primarily for the delineation of oscillating current. It suffices to mention that an exceedingly fine wire loop, which has therefore a very short period of vibration and high damping, is brought between the field poles, which lie very close together. When the current is flowing the loop turns backwards and forwards and its vibrations are projected by means of a small mirror and a suitable optical system on a rotating drum or are photographed. The curves obtained are true so long as the period of the current is short compared with that of the loop. A simple form of oscillograph suitable for magnetic demonstrations is described by Wehnelt and is diagrammatically represented in Fig. 155. F is the bar to be investigated, which is enclosed in the yoke J, which is constructed of soft wrought iron and is of very large cross-section interrupted by a narrow air-gap at O. K_1 and K_2 are screws which fix F and provide for a good magnetic contact. The magnetization coil M is fixed to J. In the gap O is the oscillograph loop of bronze wire in a very weak state of tension. It carries a small mirror which is so constructed that it can fit between the ends of the yoke pieces. This mirror turns about a horizontal axis, the moving-coil galvanometer in the magnetizing circuit about a vertical one. A beam of light, therefore, after reflection from both mirrors will give the required curve. It can easily be arranged that this is so large that it can be seen from a distance, and so clear that it can be traced out with chalk on the usual lecture-room blackboard.

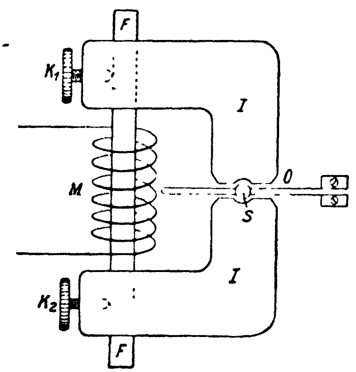

FIG 155.

Of the many methods belonging to the above we can only refer to that of Hausrath, the first purpose of which was an instrument for demonstration but which was also so designed that it could be used for measurement. In one of the cases of the electro-magnet which contain the current loop a turning coil is substituted for the latter. The whole system is shown in Fig. 156 with its cover, which is exactly like the loop case and which is capable of being turned in the cylindrical bored hole in the pole piece. The coil is provided with two ligaments which can be regulated, and the period is capable of variation within wide limits. It carries a small mirror and convenient and effective regulation and damping is provided for. The arrangement is shown in Fig. 157. As far as the loop O the path of the beam shown by the dotted line is the same as in the ordinary oscillograph. From there on it is turned through a right-angle by the three small mirrors 1, 2, 3, so that the required curve is produced on the screen. On one

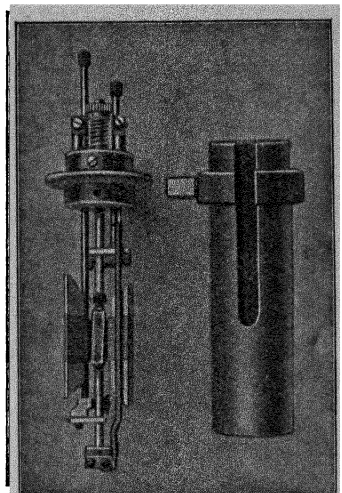

FIG. 156.

side of the terminals p lies the moving-coil instrument with the shunt r_n which can be connected at choice either through the mica condenser C or through an inductionless resistance r_v. On the other side is the pressure across the oscillograph again with a shunt that can be connected through the apparatus for testing the iron E or an inductionless resistance R. The ap-

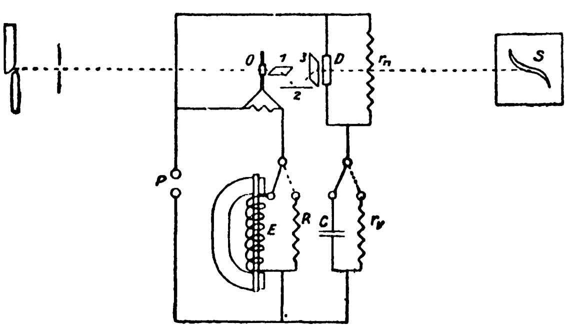

FIG. 157.

parent resistance of C with r_v and E with R are approximately the same. By regulating r_v and R the turning coil is brought into phase with the current. Then of course the curves for the forward motion and the return must fall together, and this curve at the same time gives a measure of the divergence of the pressure from the sine-wave form. The theory is not altogether simple. It is simplest when the ohmic resistance is neglected and the pressure can be regarded as having a sine-wave form. We then obtain the dynamical hysteresis directly. If the ohmic resistance is considerable the curve must be reconstructed, otherwise its area will represent not merely the hysteresis but the total energy loss.

X

MAGNETIC THEORY

108. **Introduction** – From the first and repeatedly in the course of our treatment of the subject, we have been tempted to give to the observations, facts and laws with which we became acquainted a theoretical signification in order to show why the behaviour was such as it was, and not different. This temptation we have to some extent resisted, but now at last we take up the discussion of the subject as a whole and again deal briefly with some of the points already anticipated, though thus to accentuate their significance will do no harm.

There is, in the first place, a fundamental distinction to be made between formal theories on the one side and graphic theories on the other. The first can also (for it comes to the same thing substantially) be called the continuity theory and the latter the molecular theory. A formal theory has always the advantage of introducing nothing in the way of special or doubtful assumptions, and in virtue of the continuity presupposed it lends itself to simple and elegant mathematical formulation, and has in other respects many advantages. But in all sorts of odd places it comes to the limits of its powers; there are many individual facts to which it cannot be applied, and in the last place it is not finally satisfactory for the thinking mind that wishes to probe as deep as possible into the secrets of the matter under investigation. Whether indeed such satisfaction is to be obtained through the pictorial, atomistic theory is another question, the answer to which the critical reader will not require immediately.

109. **The Maxwell Theory**—The formal theory can be developed from various starting-points. At an earlier place the magnetic induction in a narrower sense was chosen as the starting-point; but here, to save repetition and at the same time to aim at something more general, we shall at once begin with the Maxwell general theory. This is a theory of the electro-magnetic field which equally well embraces elec-

tric as well as magnetic phenomena, and is based on the conception, principally due to Faraday, of a space that is filled with lines of force.

But here we must go back a little and, in the first place, put the pondero-motive effects in the foreground. In the second chapter we have based this on certain facile assumptions of action at a distance, and in so doing have obtained quite satisfactory results Action at a distance in the case of magnetism is perfectly parallel with the electrical and the gravitational case, only that we have to deal not with single poles but with pairs of poles; and the consequence of this is that in addition to the attractions and repulsions, there are also rotational effects which may even be predominant since these effects are proportional to the third, while the others are proportional to the fourth power of the distance. One reason for giving up the action-at-a-distance theory arises not from the fact that one rejects such a theory as being something mystical in the ultimately vain hope that by a continuity theory we may arrive at complete knowledge; but, as sometimes happens in human affairs, we kill two birds with one stone. For in regard to electrical phenomena the theory of action at a distance has shown itself untenable and has had to be given up because it has been shown that certain (of course, only certain) phenomena, namely, electrical oscillations, need time to propagate themselves from layer to layer of the medium, and at the same time both affect the medium and are affected by the medium; thence the victory of the field theory. But magnetic phenomena are so closely connected with the electrical that we cannot accept one theory in one subject and reject it in the other. Magnetic phenomena must also be brought into the field theory, although (and this is for the most part passed over in silence) nothing is known in regard to the purely magnetic effects that really disprove the action-at-a-distance theory.

The magnetic field $\mathfrak{M}$ (hitherto $\mathfrak{H}$) of a straight conductor, according to the law of Savart-Biot (page 149), is $\mathfrak{M} = 2i/vr$, or, if we multiply both sides by $2r\pi$, $\mathfrak{M}2r\pi = 4\pi i/v$. If we think of the current as being the axial line of a circle at a distance r, we can look at this last equation as being the result of an integration all round the first and thus arrive at the equation for any closed curve

$$\int \mathfrak{M}_s ds = \frac{4\pi i}{v} \quad . \quad . \quad . \quad . \quad . \quad (106)$$

in which s is an element of the curve and $\mathfrak{M}_s$ the component of $\mathfrak{M}$ along the curve. In the electro-magnetic system of measurements v disappears, and in the practical system with i in amperes there is a factor of 10 in the numerator, so that we obtain

$$\int \mathfrak{M}_s ds = 1{\cdot}258i \quad . \quad . \quad . \quad . \quad (107)$$

In order to get rid of the integral a new idea is introduced, the " curl " or " rotor " of $\mathfrak{M}$, i.e. that quantity which, on integration, leads to the above and is represented by $q(\mathfrak{M})$. If, further, on the right-hand side we substitute the current intensity $\mathfrak{S}$ (the strength of current per unit of cross-section) instead of the strength of the current, we now obtain

$$q(\mathfrak{M}) = \frac{4\pi\mathfrak{S}}{v} \quad . \quad . \quad . \quad . \quad . \quad (108)$$

Finally, we can replace $\mathfrak{S}$ by the formula which connects it with the electric field strength $\mathfrak{E}$:

$$4\pi\mathfrak{S} = \varepsilon\frac{\partial\mathfrak{E}}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad (109)$$

in which ε is the electric constant characteristic of the material, the so-called dielectric constant; and then we obtain

$$\frac{\partial\mathfrak{E}}{\partial t} = \frac{v}{\varepsilon}q(\mathfrak{M}) \quad . \quad . \quad . \quad . \quad . \quad (110)$$

which is the first Maxwell equation. The second can be deduced in a similar way from the facts of magneto-induction, and we then obtain in integral form (line integral of the electric field strength and surface integral of the magnetic induction)

$$\int \mathfrak{E}_s ds = -\frac{d}{dt}\int \mathfrak{B} df \quad . \quad . \quad . \quad . \quad (111)$$

The new equation can be at once expressed in a different way if account be taken of the principle of reciprocal action according to which every effect is equal to its opposite characterized by its opposing curl q':

$$\frac{\partial\mathfrak{M}}{\partial t} = \frac{v}{\mu}q'(\mathfrak{E}) \quad . \quad . \quad . \quad . \quad . \quad (112)$$

where now μ is the corresponding magnetic constant, which we already know as the permeability. This is the second

Maxwell equation. We can also introduce the magnetic induction and so obtain as the counterpart of equation (108)

$$q'(\mathfrak{E}) = \frac{1}{v}\frac{\partial \mathfrak{B}}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

The two Maxwell equations have now been put into the vector form, that is, the quantities occurring in them, $\mathfrak{E}$ and $\mathfrak{M}$, $\mathfrak{H}$ and $\mathfrak{B}$, are vectors or directional quantities and are to be treated as such. If we keep to the usual method of calculation, then instead of dealing with these magnitudes as they stand we must consider their components in the direction of the axes of a rectangular system of co-ordinates E_x, E_y, E_z, and corresponding M_x, M_y, M_z; and then we obtain, if we write the curl components in the form usual in mechanics, the two triple series of equations:

$$\left.\begin{aligned}
\frac{\partial E_x}{\partial t} &= \frac{v}{\varepsilon}\left(\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right) & \frac{\partial M_x}{\partial t} &= \frac{v}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right)\\
\frac{\partial E_y}{\partial t} &= \frac{v}{\varepsilon}\left(\frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x}\right) & \frac{\partial M_y}{\partial t} &= \frac{v}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right)\\
\frac{\partial E_z}{\partial t} &= \frac{v}{\varepsilon}\left(\frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}\right) & \frac{\partial M_z}{\partial t} &= \frac{v}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z}\right)
\end{aligned}\right\} \quad (114)$$

The Maxwell equations can be expressed in words in the following way: the rate of change in respect to time of the electric force is, apart from one universal and one individual constant (the velocity of light, and the dielectric constant respectively), equal to the curl of the magnetic force; and conversely the rate of change in respect to time of the magnetic force is equal to the opposing curl of the electric force multiplied by the ratio of the velocity of light to the magnetic permeability.

The Maxwell equations belong to the most wonderful products of the spirit of discovery in the whole realm of exact science. If beauty can consist of an ingenious and complete simplicity, then they show that even in a subject like ours beauty may present itself, and that it is a factor in the general system of knowledge. The two triple systems, as we see, are linked with one another, the E being expressed in terms of M and conversely. For certain purposes with which we are not concerned here, this interlinkage must be solved. (For example, in the electro-magnetic theory of light.) It must, however, be noticed at the same time that the great simplicity of these equations is purchased by leaving out of account a whole series of phenomena which belongs to our subject. But even if these are taken account of, the equations, which then must include certain additional terms, are not impossibly complicated. The special phenomena relating to electrical conductors, ferro-magnetic materials, crystals, and many other things are referred to here. But with this indication the matter must be left.

110. **Application to Magnetism** -We can, however, derive immediately from our simple equations certain correspondingly simple conclusions. If we apply the second integral equation (111) to two different surfaces limited by s, we obtain for the closed surfaces thereby resulting (B_n is the component of $\mathfrak{B}$ perpendicular to df):

$$\frac{d}{dt}\int B_n df = 0 \quad . \quad . \quad . \quad . \quad . \quad (115)$$

that is, the number of the lines of induction emerging from a closed surface is a constant with respect to time. If no permanent magnet is operating then we have

$$\int \mathfrak{B}_n df = 0 \quad . \quad . \quad . \quad . \quad . \quad (116)$$

From which it follows that all $\mathfrak{B}$ lines are closed on themselves. Further, as a counterpart to the curl, we can also introduce the " divergence " (page 42), that is, the magnitude

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}; \quad . \quad . \quad . \quad . \quad (117)$$

through a suitable combination of the equations of the second triplet this can be easily calculated, and we find

$$\frac{\partial}{\partial t}\,\mathrm{div}\,(\mathfrak{B}) = 0 \quad . \quad . \quad . \quad . \quad . \quad (118)$$

Therefore for the case where no permanent magnets are operating

$$\mathrm{div}\,(\mathfrak{B}) = 0 \quad . \quad . \quad . \quad . \quad . \quad (119)$$

The two equations div $(\mathfrak{B}) = 0$ and $q(\mathfrak{M}) = 4\pi\mathfrak{S}$ may frequently be most conveniently employed as fundamental equations to the theory.

For the theory of pure magnetism the electric currents represent only a means of creating an exact basis; we will therefore now leave them aside and ask what formulæ are valid. In the above case a great simplification was obtained for all parts of the field where no current is flowing, for here $\mathfrak{S}$ becomes $= 0$ and in consequence

$$q(\mathfrak{H}) = 0 \qquad (120)$$

and therefore if V is the magnetic potential

$$H_x = -\frac{\partial V}{\partial x}, \quad H_y = -\frac{\partial V}{\partial y}, \quad H_z = -\frac{\partial V}{\partial z}$$

or more briefly (grad being an abbreviation for gradient),

$$\mathfrak{H} = -\operatorname{grad} V, \qquad (121)$$

and thus we get back to the earlier formula. But here it is clearly shown that if there is no current something else must be introduced, and this can only be done for permanent magnets, which compels us to take hysteresis and "true magnetism" into consideration. Therefore if its density is ϱ_w and the true magnetization $\mathfrak{J}_w$

$$\operatorname{div} \mathfrak{J}_w = \varrho_w \qquad (122)$$

or in V

$$\operatorname{div} \mu \operatorname{grad} V = -\varrho_w \qquad (123)$$

and the energy

$$U = \int \frac{\mu}{2} \operatorname{grad}^2 V d\tau \qquad (124)$$

Nevertheless, if we wish to go further, we must introduce still further modifications on account of the variability of μ.

111. **Special Developments**—Here we must refrain from considering the application of the formal theory to individual phenomena such as initial magnetism, saturation, remanence and hysteresis as well as the behaviour of bodies of various shapes, and only by way of an indication make a few remarks on a few significant points. Hysteresis makes magnetization appear as an irreversible process. The question now arises (it has been touched upon already) whether it would not be possible to bring about forcibly some sort of ideal, i.e. reversible process, and this according to Gans can actually be done. He has developed a beautiful theory of reversible magnetization, which finds its expression in a new constant, the reversible susceptibility.

Further, as regards the phenomenon of diamagnetism, the fact, therefore, that the susceptibility for numerous materials (and indeed for the majority of those that are weakly magnetizable) is negative, is a matter of indifference to the formal theory; it says nothing either for it or against it. On the contrary, Duhem, the great French authority on thermodynamics, has applied to the case of magnetism the two famous principles of thermodynamics, and has shown that a negative susceptibility contradicts the second law. There is, however, a very obvious expedient: diamagnetism is thought of as being only apparent, and it is explained that a body appears diamagnetic when it is more weakly magnetizable than the surrounding medium (compare page 82) —an idea which corresponds completely to the case of a body of low specific gravity in a heavy medium: one may even speak of an "Archimedean principle of magnetism." One difficulty in the way of this explanation lies in the fact that even in a vacuum many bodies appear to be diamagnetic, so that we are left with the further conclusion that the vacuum itself is magnetic, and indeed more strongly magnetic than all those bodies which in a vacuum appear to be diamagnetic. This assumption loses its paradoxical character when we remember that the so-called vacuum is not merely empty space but is filled with the ether, and that the ether, indeed, is infinitely light, but nevertheless in other respects must possess infinite properties in order properly to fill the rôle that has been assigned to it. Still we may say that these differential theories of diamagnetism are not completely satisfactory, which makes us disposed to regard with interest other theories that may present themselves.

It is further to be remarked that the group of Hall effects has been dealt with from the point of view of the field theory, and that in a very striking fashion (Boltzmann, Goldhammer, Heurlinger, among others). The thermodynamic theory has here yielded gratifying results; it has been especially indicated on account of the connection of the Hall effects with thermal phenomena. We cannot, it is true, penetrate very deeply into the inner nature of these phenomena and the facts can only be partially explained.

In a still higher degree does this apply to the application of the field theory to magneto-optical phenomena as it has been developed, for example, by Drude. Here to the Maxwell equations certain terms must be added and the corresponding transformations made. For the Faraday and the Kerr effect we obtain much that is satisfactory; the Zeeman effect remains for the most part completely enigmatical in its attractive many-sidedness. It must suffice here to allude to the most interesting of the purely phenomenological theories, that due to Kolaček.

112. **Analytical Hypothesis**—Let us turn now to the molecular theories of magnetism. The oldest of them is that due to Poisson. According to him a body consists of molecular magnets, that is, of little parts each of which possesses both negative and positive magnetism and at the same time infinite magnetic conductivity, while the medium in which the little parts are embedded is uniform and continuous. Magnetization consists in the process of separating out the particles that in the non-magnetic condition are mixed with opposing magnetisms. How strongly the body can be magnetized depends upon the proportion of the particles k in the total volume (the Poisson constant) and the magnetic conductivity of the body as a whole, as can be easily shown if μ_1 and μ_2 are the values for the medium and the particles, is generally

$$\mu = \mu_1 \frac{2\mu_1 + \mu_2 + 2k(\mu_2 - \mu_1)}{2\mu_1 + \mu_2 - k(\mu_2 - \mu_1)}$$

therefore, since Poisson puts $\mu_1 = 1$ and $\mu_2 = \infty$,

$$\mu = \frac{1 + 2k}{1 - k}, \quad k = \frac{\mu - 1}{\mu + 2} \quad . \quad . \quad . \quad (125)$$

Corresponding to this relation between the coefficients k and μ are those between k and the susceptibility κ:

$$\kappa = \frac{3k}{4\pi(1 - k)}, \quad k = \frac{4\pi\kappa}{4\pi\kappa + 3} \quad . \quad . \quad . \quad (126)$$

Since for iron κ is a fairly large number k becomes approximately equal to unity, that is, the whole space must be regarded as filled with molecules. For spherical-shaped particles such as are suggested by Poisson, this is geometrically impossible, and for other shapes it is very improbable.

113. **Rotation Hypothesis**—The advantages of the analytical hypothesis also present themselves in connection with the direction or rotation hypothesis which is principally due to W. Weber, according to which the particles in the non-magnetic condition of the body are already magnets but with their axes lying in different directions, and which by magnetization are turned into the same direction. This turning into the same direction would, in contradiction to all experience, be effected by the application of the smallest force, if it were not at the same time assumed that there is a resistance

to this turning of the particles, and that such resistance arises from the forces exerted on the particle considered by the surrounding particles which also have imposed upon it its prescribed position. Weber, however, does not pursue this line of thought, he simply assumes that there is an inherent directional force D which, in opposition to the external force X, tends to restore the particle to its original position. On this assumption it is possible, if m is the magnetic moment of the molecule and n the number of them in unit volume, to obtain a relation between the forces X and D, on the one side, and the magnetization on the other:

X =	o	$<D$	D	$>D$	∞
$\mathfrak{I}$ =	o	$\frac{2}{3}\frac{mn}{D}X$	$\frac{2}{3}mn$	$mn\left(1-\frac{1}{2}\frac{D^2}{X^2}\right)$	mn

This gives the uniform increase at the beginning, the gradual slackening down, and the final saturation fairly well, but not

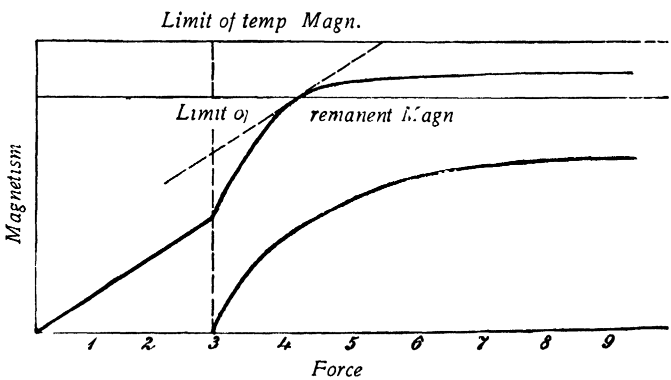

FIG. 158

the acceleration in the middle part of the curve, as we see it, for example, in Fig. 27; moreover, it takes no account whatever of the remanence. These two defects Maxwell has tried to remedy by assuming that a molecular magnet returns indeed after the first small displacement to its original position, but after a large displacement retains a part of it permanently. We then obtain the condition of things illustrated in Fig. 158 with a sudden bend instead of a gradual transition

—a drawback which can only be got over by supposing that the critical point for the various molecules is somewhat different.

A substantial advance was made by Ewing, who took into consideration the fact that each individual magnet was influenced by the neighbouring ones. According to Ewing we

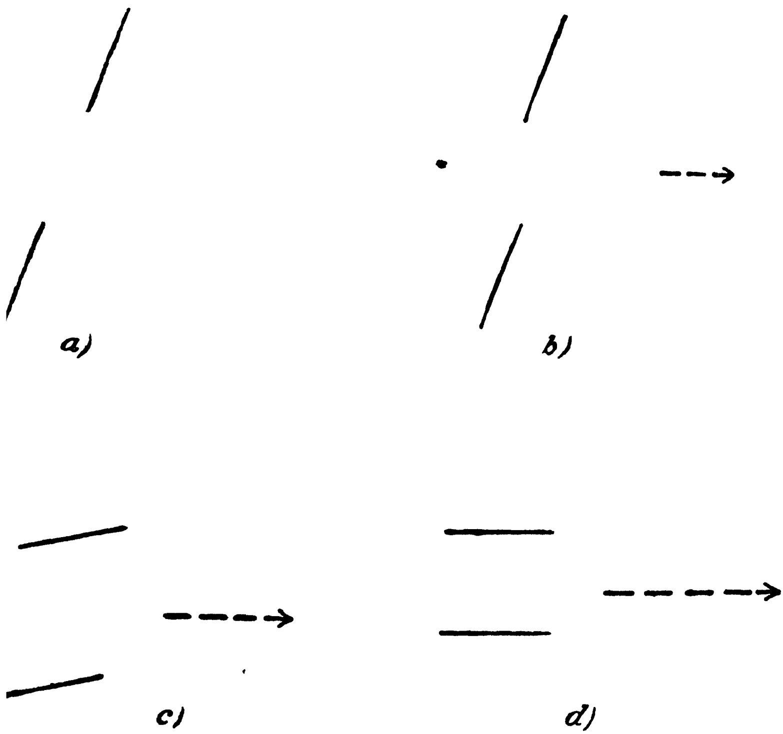

FIG. 159.

must consider the behaviour of a group of two molecular magnets which lie in the same line (Fig. 159): a small force deflects them only slightly (*b*); then there follows a fairly sudden change-over into a position in which, the forces having become stronger, they lie parallel to one another and to the force (*c*); and finally, a further increase of the force has the result that the parallelism becomes more and more perfect. By this very simple idea one may take account of the three phases of the magnetization curve; its first gradual rise, which afterwards becomes accelerated, and finally becomes gradual again. If a group of four or more molecules be taken the representation of the behaviour becomes even better. Ewing has also followed out this thought numerically, and he has also tested it experimentally by means of a large number of small magnetic needles each capable of free rotation. It is very instructive to prepare such a model, but the significance of such rough material theories should not be exaggerated.

114. **Ampère's Theory**.—The consideration of the equivalence between magnets and electric currents (page 168) has already suggested some ingenious theoretical applications; we will now proceed to redeem the promise that was then given. Soon after his discovery (1821), Ampère put forward the hypothesis that magnetism might not be a phenomenon peculiar to itself, but might result from infinitely small currents flowing round the particles of bodies exhibiting it. It would indeed have been simpler to suppose the body as a whole encircled by currents, but then the equivalence for the internal parts would not have been provided for, and, moreover, other difficulties would have presented themselves. As there are two different magnetic theories of magnetism, so also two electric theories are imaginable corresponding to the analytical and the rotational hypothesis respectively. For, in the first place, the direction of the molecular currents will be more or less affected by the magneto-electric ponderomotive action, and in the second place induction effects must also arise, currents must be induced about the molecules or new currents be added to those already existing, and these new currents will not, like other induction currents, die away quickly if it is assumed that the path in which they flow has no resistance, as we must assume regarding the currents which were already present and whose direction only was modified when they were brought into the magnetic field. Only when one brings into operation a new oppositely directed induction by withdrawing the body again from the field will the induction currents again be stopped. In paramagnetic bodies we must suppose the already existing currents to be strong so that they are only weakened to a small extent by the oppositely directed induction currents. In diamagnetic bodies, on the contrary, we must suppose that they do not exist, or are so weak that they are exceeded by the induction currents and in consequence a result of opposite character arises. Why,

however, in many materials, both in the ferro-magnetic and also in the paramagnetic, strong molecular currents are present from the outset but not in the diamagnetic, we cannot say; the theory also leads to some peculiar consequences which, so far, have not been confirmed by experiment.

The question further arises how we are to think of the ampere currents circulating about the molecule. The assumption of currents passing through conductors is rendered difficult by the absence of any resistance. This is well met by the hypothesis, first put forward by Richarz, of rotating electrons. Here also there are many difficulties, and one of the most considerable is that unless a resistance can be supposed we can only in this way represent the facts of diamagnetism; on the contrary, to represent paramagnetism, resistance and an impelling motion must be assumed; but it is the ferro-magnetic substances that present the most formidable difficulties. Before we go into this matter further, we should like to make at this point a small digression.

However great the possibilities of the ampere theory, so far any direct indication of the nature of the molecular currents has been wanting—naturally not merely in the sense that the currents cannot be directly observed, but also in the sense that their mode of action has not been definitely indicated. One suggestion in this direction has already been put forward by Maxwell, but neither he nor Haas, nor Lorentz later, could obtain any indication of the effect experimentally, since it was much too weak. This gap has lately been filled by the work of Einstein and de Haas. According to theory to each of the circling electrons there is given an impulse, which is in the same direction as the vector of its magnetic moment and which stands in fixed relation to it independently of its geometrical relations and the frequency of its rotation about the molecule. The magnetic molecule behaves itself mechanically like a top whose axis constantly coincides with the magnetic axis. If the magnetic condition of a body changes, then the orientation of the top changes and with it the impulse momentum. The change of the impulse momentum must, however, in accordance with the principle of the conservation of energy, correspond to a compensating impulse momentum of a mechanical character, that is, the body by the change of its magnetization will be set in rotation (movement of precession). If the magnetism is evoked by circling electrons, then the magnetization $\mathfrak{J}$ of the body and the impulse momentum $\mathfrak{M}$ of the circling electrons are connected by the vectorial relation

$$\mathfrak{M} = 2\left(\frac{m}{e}\right) \times \mathfrak{J}, \quad . \quad . \quad . \quad . \quad (127)$$

therefore, if we insert the best values at present available for the relation of the charge e to the mass m,

$$\mathfrak{M} = -1{\cdot}13 \times 10^{-7}\mathfrak{J} \quad . \quad . \quad . \quad . \quad (128)$$

This gives the mechanical turning moment $\mathfrak{B} = d\mathfrak{M}/dt$. It is shown, therefore, that under easily attainable conditions with a single demagnetization of a thin iron bar an angular velocity of about 1/100 must arise about the longitudinal axis. In order to increase the effect a resonance method is adopted. A cylindrical iron bar of about 2 mm. diameter hanging in the axis of a magnet coil is brought to the frequency of an alternating current passing through the coil. When the condition of resonance is approached, a rotatory oscillation of the bar about its longitudinal axis must be set up whose amplitude is determined by the constants of the self-oscillation of the bar and the change of its magnetic moment. By calculation we have

$$A = \left(\frac{2}{\pi}\right)1{\cdot}13 \times 10^{-7}\frac{\mathfrak{J}_s}{\kappa \mathfrak{Q}},$$

where $\mathfrak{J}_s$ is the saturation magnetization, κ the damping constant, and $\mathfrak{Q}$ the moment of inertia. By means of a mirror reading, a deflection is found to occur which qualitatively fulfils the conditions; but the agreement quantitatively is at first only very moderate, and could only finally be made satisfactory by various improvements in the method. These improvements include the following in particular: the current coil must be wound directly about the bar, the time of swing must be increased to about two seconds, and instead of an alternating current a direct current commutated by means of a pendulum contact must be used, which renders the determination of the phase and so forth easier. But even so the numerical value found for e/m differs by about 15 per cent. from that otherwise determined.

115. **Electron Theory**—The modern molecular theory of magnetism is associated chiefly with three names: Gans, Langevin and Weiss. Let us first consider the theory of Gans. The useful part of the Weber-Ewing theory is retained, and it is therefore assumed that the position of each

molecular magnet is determined through the magnetic effect of all the others and of the outward forces, and that hysteresis is to be explained through multiple positions of equilibrium of the molecular systems. The theory is so far extended that we are not limited to any special arrangement of the elements, but may assume as general a distribution as possible. Therefore nothing is left but to apply the laws of probability, and to use statistical methods so that we are no longer substantially concerned with the position and direction of individual elements. Further, the magnetic fluid is dispensed with and replaced by rotating or circulating electrons. It is next shown that the constrained rotation of electrically charged particles are equivalent to actual molecular magnets, both actively and passively. In order to represent the direction of the force and the constancy a spherical shape must not be ascribed to the particles, but they must be presented as figures of rotation, which rotate very rapidly about their axis of figure; this sets itself constantly in the direction of the field and retains, therefore, approximately the same momentum, while rotation about any other axis is damped down through its own radiation and cannot therefore be continuously retained.

In many respects the working out of the theory follows the same lines as that of Lorentz for dielectrics. We must therefore seek the law of its distribution, discover, that is, how often each direction is assumed from the neighbouring elements. We must even suppose that the number of degrees of freedom in consequence of the fundamental conditions of magnetism is smaller, and that in consequence the total effect can only be determined through a change in the distribution function. To compute it exactly is very difficult. It is probable that it is nearly the same whatever the direction (in the electrical case it is even zero), which gives the Weber standpoint a more secure foundation.

In the first place the elements may be regarded as at rest, apart, that is, from the rotations of the electrons. We then obtain a very remarkable curve which is shown in Fig. 160*a* and 160*b* by the line D′EC′OCE′D. They contain, however, one part, that, namely, which is shown dotted in the figure, which can never be observed because it corresponds to a diminution of magnetization (Fig. 160*a*) or of induction (Fig. 160*b*) with an increase of field strength, and therefore to an unstable condition. If we exclude the unstable part, we obtain as the magnetization or hysteresis curve respectively in the first rough approximation a rectangle; in the second it has the form shown in the figure, therefore, if we leave aside the continuation, the form EC′E′CE, in which the straight-line portions C′E′ and CE obviously represent merely an obligatory substitute for the unstable portion of the curve. Moreover, a curve of this sort is only actually obtained if

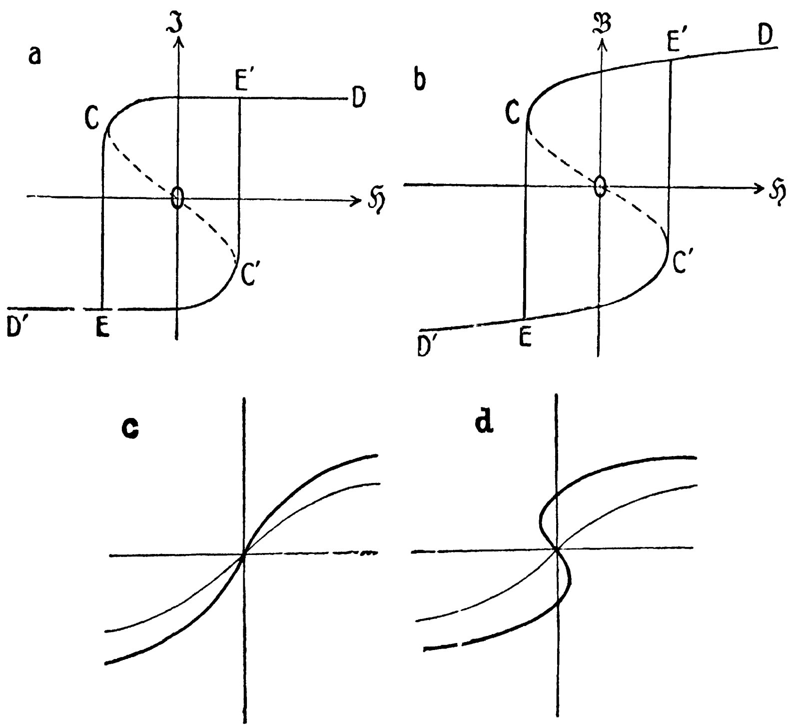

FIG. 160.

out of the isotropic medium that is here assumed of absolutely identical bodies elementary complexes be formed, and out of these a magnetic crystal be built up, in which the elementary complexes are arranged on a three-dimensional frame corresponding to the crystalline structure. In so-called amorphous iron the matter becomes more complicated, and in consequence the hysteresis loop in some respects departs not inconsiderably from the above form.

But the other circumstance comes into consideration, that what we have said so far applies only to molecular magnets at rest. If we now assume a temperature effect and that the molecules move about inside the elementary complex and experience collisions and deflections, we obtain a more general conception of the relations than that applying in the first instance which referred to $T = 0$. We have to distinguish between the outer field, the outer zone, and the near zone; the first two directed, the last statically indeterminate. We can, in the first place, determine the magnetization as a function of the first two fields, and convert the corresponding curve afterwards through a graphical shearing experiment (see later) into the final curve. According to the shearing angle a completely stable curve (Fig. 160*c*) or a curve that is unstable in the middle portion is obtained, the last because in ferro-magnetic materials the effect of the near zone is considerable and therefore the shearing angle becomes large.

The kinetic theories of Langevin refer first to weakly magnetic bodies, and among these, again, diamagnetism and paramagnetism may be ascribed to various causes. Diamagnetism consists in the deformation of the paths of the electrons through the magnetic field. Since these phenomena take place inside the atom it is not influenced by its motion or that of the molecule, it is therefore independent of temperature, which, in fact, is fairly often correct. In order to understand paramagnetism it is assumed that each molecule contains a great number of electrons which describe closed paths each of which supplies a certain momentum. If the resulting momentum on account of the disposition of the paths is zero, the polarization is purely diamagnetic. If it is different from zero, then the orientation of the molecules depends at once upon the field and the thermal agitation; its determination is a problem of statistics and rests upon the Maxwell-Boltzman laws which determine the distribution of the molecules in positions which correspond to the various values of the potential energy.

If m is the molecular magnetization (magnetic moment of the gram-molecule), M its maximum value for $T = 0$ (not including the reaction of the agitation), rT the double value of the kinetic energy for one degree of freedom of the molecule and for brevity,

$$a = \frac{MH}{rT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129)$$

(r is at the same time the molecular gas constant, that is, 83·135 × 10^6 ergs per degree), then for the magnetization formula we obtain

$$\frac{m}{M} = \mathfrak{Ctg}\, a - \frac{1}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (130)$$

which is shown graphically by the curve in Fig. 161. By series development we obtain as a first approximation

$$\frac{m}{M} = \frac{a}{3} = \frac{MH}{3rT} \quad . \quad . \quad . \quad . \quad . \quad (131)$$

and in consequence for the molecular susceptibility the expression

$$\kappa_m = \frac{M^2}{3rT} \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

that is, if M is assumed to be constant, the susceptibility is inversely proportional to the absolute temperature (Curie's law).

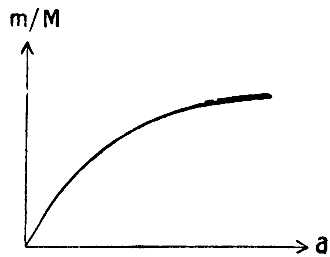

Fig 161

The theory of Langevin is immediately connected with that of Weiss, and indeed stands in the same relation to it as the Van der Waal kinetic theory does to that of Bernouilli (or Krönig). A "molecular" field is introduced which, when added to the external field, with the help of the laws of paramagnetism, makes the strong magnetization of the ferromagnetic substances intelligible. Weiss assumes that the effect of the molecules taken as a whole upon any one of them is equivalent to a homogeneous magnetic field, that is, is proportional to the intensity of the magnetization and has the same direction. Moreover, the forces concerned are only to be formally introduced as magnetic, for, in fact, they are probably of a quite different nature. The molecular field is defined by

$$H_m = nJ = n\frac{d}{p}m, \quad . \quad . \quad . \quad . \quad . \quad (133)$$

where p is the molecular weight, d the density and n is a constant. Apart from this the molecules in a magnetic metal are to be considered as free to rotate as in an ideal gas. If no external field but only the molecular field is present, then

$$a = \frac{ndM}{prT} m \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

And now comes the most remarkable feature of the theory. The system for the values $m = a = H = 0$, as is easily seen, is not stable. While, therefore, experiment makes us acquainted with magnetization only as the consequence of a present or a previous magnetic field, here we must familiarize ourselves with the idea of " spontaneous " magnetization. And there is nothing so strange about this as might at first sight appear, for we know in the case of crystals that they may possess natural magnetism; and an ordinary piece of iron or steel only appears unmagnetic because the crystal-complexes of which they are composed are orientated in various directions so that their outward effect is annulled. The rôle played by the external field is therefore not to call magnetism into existence, but merely to make its observation practicable. Indeed we may go still further: the spontaneous magnetization is that corresponding to the saturation intensity for the temperature concerned; in this respect there is no difference between an unmagnetic (if it be strongly magnetizable) and a magnetized body: the difference lies rather in the parallelization. Of course, the spontaneous magnetization will fall with increasing temperature, and when this has reached a definite value $T = \Theta$, will become zero: the critical point of the magnetization. Whence we immediately obtain

$$\Theta = \frac{ndM^2}{3pr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

or if, in accordance with the formula

$$C_m = \kappa_m T = \frac{M^2}{3r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

the " Curie constant " be introduced (referred to the molecule or the mass):

$$\Theta = \frac{nd}{p} C_m = ndC \quad . \quad . \quad . \quad . \quad . \quad (137)$$

Since here C can be experimentally determined, we obtain information regarding the constant n.

If to the spontaneous field an external field be added, H_a, then let the two temperatures be considered corresponding to a given magnetization, one without and the other with an external field, T and T′ respectively; then we have

$$T' - T = \frac{MH_a}{ar} \quad . \quad . \quad . \quad . \quad (138)$$

The temperature is therefore to be corrected by an amount which is directly proportional to the external field and inversely proportional to a. When T is approximately equal to Θ, and therefore a to the value $\frac{3m}{M}$, and the magnetization to zero, we have

$$\frac{m}{H_a}(T' - \Theta) = \frac{M^2}{3r} = C_m \quad . \quad . \quad . \quad (139)$$

that is, it now becomes

$$C_m = \kappa_m(T' - \Theta) \quad . \quad . \quad . \quad . \quad (140)$$

The Curie constant is no longer simply the product of the susceptibility and the critical temperature, but (compare page 111, Eq. 34) the product of a susceptibility measured at a higher temperature and the excess of this temperature over the critical. This magnetism set up over the critical point may be called the "conditioned magnetism" in contradistinction to the spontaneous.

The theory that has just been sketched is on one side too simple to describe the totality of the phenomena, especially those in the neighbourhood of the conversion point; on the other hand it leaves something to be desired inasmuch as it provides no connecting link between the various magnetizable substances. Both criticisms have been met by Weiss—and that is the second remarkable feature of his theory—and that at one stroke, by carrying on the theory in the direction of a further analysis of the quantity M, the molecular moment of saturation. In the first place one can obviously go back from this to the atomic moment, which is then the characteristic quantity for the atom in question. The further question may be raised, which naturally can only be answered by experimental measurement, whether there is any relation between the various atomic moments. It is very satisfactory to find that the answer is in the affirmative: the atomic moments of nickel, cobalt and iron (we do not need to go into the question here regarding other substances) are related almost exactly as 3 : 9 : 11. The common factor of these numbers for the gram-atom, the only unit directly accessible to experiment, is equal to 1,123. If this is divided by the Avogadro-Loschmidt constant (according to Perrin's latest determination, $68\cdot5 \times 10^{22}$) we then obtain for the atom itself the value

$$16\cdot4 \times 10^{-22}.$$

This common measure of all atomic moments is called, according to Weiss, the magneton. It is a universal natural constant, and the foundation stone from which the magnetic atoms, with these the molecules, and with these the magnets, build themselves up. Along with this simplification however there goes the complication necessary for the representation of phenomena. one and the same material has not one single characteristic multiple, but several: those given above relate to spontaneous magnetism. For conditioned magnetism, iron for example, gives instead of 11 according to the temperature interval the numbers 10, 12 and 20; nickel 8 or 9 respectively instead of 3, and cobalt 15 instead of 9. For the various chemical combinations other numbers are obtained, so that we may say that the ferro-magnetic substances exist in many very different conditions and in each of them the atom contains a different number of magnetons. The essential fact remains that a whole number remains and therefore the idea of the magneton is preserved.

Finally the question arises how the magneton is related to the other universal natural constants recently introduced, to the electron and to the quantum hypothesis: this question has only just been raised and its answer must be deferred.

116. **Applications**—We must now apply our theory to individual problems, especially in relation to mechanics, electricity, temperature and light; unfortunately the space at our disposal only allows us to say a little about these interesting subjects. However it is to be remarked (which may be a matter of some comfort to the reader), that here the theorists have employed all the arts of mathematical treatment, with which we can become acquainted only through tedious and laborious preparation.

The Hall effect and those related to it have been examined in connection with the electron theory by numerous inquirers (Riecke, Drude and Lorentz). If we consider only one sort of electron, that namely of negative electricity which moves with a velocity v, we obtain without anything further a transversal fall of potential in the section of the field in accordance with the formula:

$$E = vH \quad . \quad . \quad . \quad . \quad . \quad . \quad (141)$$

that is, E is proportional to v and H. If, following Baedeker and Steinberg, we take as the conducting substance copper iodide, the concentration of electrons n through substitution of iodide can be shaded off, and then in order to obtain a definite current density $j = n \times e \times v$ the velocity of the electrons must be kept in inverse ratio to n. For a given current density, the Hall effect within certain limits must be inversely proportional to n and therefore to the conductivity, and only when the free path of the electrons is influenced by the iodide content will divergencies occur; both of which deductions are confirmed by observation. But what the theory does not account for is the sign determination of the Hall effect; under all circumstances it gives a negative R, whereas for many substances it is positive. The same remark applies to the longitudinal effect, that is to the change of resistance in the field According to J. J. Thomson an increase must always occur, which indeed is frequently, but not invariably, the case. Everything points therefore to the necessity of modifying the theory in some direction, and this has been done by Lorentz, Riecke and Drude.

Lorentz assumes as before only one kind of freely moving electrons, the negative ones, while the positive remain fixed; but since this does not give us the different signs, he proposes either to ascribe to the positive electrons a limited possibility of motion or to submit the neutral union of positive and negative electricity to a more detailed consideration. The other authors assume from the outset that both sorts of electrons are freely moving. As his final formula for the Hall constant Drude gives the following:

$$R = \frac{ec}{\lambda} \cdot \frac{v_1 \frac{d \log N_2}{dT} - v_2 \frac{d \log N_1}{dT}}{\frac{d \log (N_1 N_2)}{dT}} \quad . \quad . \quad (142)$$

Here c is the velocity of light, e the elementary charge, v_1 and v_2 are the mobilities of the two sorts of electrons, N_1 and N_2 their number per cubic centimetre, T is the absolute temperature and λ the conductivity of the material. Similar expressions are given for the characteristic constants of the other effects. It is thereby shown that R and Q according to circumstances may have different signs, but that on the contrary the signs of P and R must always agree, which indeed is predominantly but not always the case, iron and steel belonging to the exceptions. For the longitudinal effect on the other hand, it follows that it is small compared with the transverse effect and must increase in proportion to $\mathfrak{H}^2$, both of which statements are in agreement with experience. Going still further, the ideas and laws of the kinetic theory of gases have been applied, especially the Maxwell law of distribution, and new results obtained. A very simple formula, for example

$$R = \frac{1}{2\lambda} \frac{e}{m} \frac{l}{v} \quad . \quad . \quad . \quad . \quad . \quad (143)$$

has been arrived at by Koenigsberger (e/m the relation of the charge to the mass, l/v the relation of the wave-length to the velocity), and interesting conclusions follow from it. Taking them on the whole it may be said that the theories of this group are still far from having attained finality but nevertheless have already led to very encouraging results.

The Faraday effect according to the field theory may be represented very satisfactorily; especially in the form that has been given by Drude, it affords a sufficient answer to all substantial requirements that may be made of it. Of the two Maxwell vector equations the first (110) remains unchanged; on the other hand the second is modified by the substitution of terms of the combined vortex type. The dielectric constant ε for transparent substances has its customary value. For absorptive substances, however, if n is the coefficient of refraction, a the coefficient of absorption, and $i = \sqrt{-1}$, we must put

$$\varepsilon = n^2(1 - ia)^2 \quad . \quad . \quad . \quad . \quad . \quad (144)$$

As a result of this enlargement of the theory the Kerr effect can also be included, of course only in its main features; some important details, particularly the Sissingh phase difference, necessitating further modifications. Finally Voigt has shown how the dispersion produced by the rotation of the plane of polarization, together with its anomalies, can be accounted for.

On the other hand the last and most interesting of the magneto-optic effects, the Zeeman effect, has made it necessary to abandon the field theory and in its place to substitute the more effective electron theory, the conclusive demonstration and development of which is again due to Voigt. Let us conclude with as brief account as possible of his demonstration.

In the molecules of the ponderable bodies there are found as we imagine elementary electrical parts, the electrons, which are extremely small and tend towards their position of equilibrium within the body. The fact that the same electron when outside a magnetic field in any condition of excitation possesses the same period or frequency $f = 2\pi/T$ and therefore always sends out the same colour, makes the assumption necessary that the force with which the electron tends towards the position of equilibrium is of the nature of an elastic force, that is, it is proportional to the distance r. On this account it is usually regarded as a quasi-elastic force. If K is the force at unit distance it is therefore generally Kr, and if m is the mass of the electron then the general relation holds $f^2 = K/m$. The most general form of the path of an electron held in this fashion is an ellipse; its form, shape, and orientation depend upon the nature of the excitation. The motion of the electron has as a consequence the sending forth of electric oscillations; otherwise expressed there is in the neighbourhood of the electron an electric field produced of periodically changing magnitude and direction which is regularly dependent upon the motion of the electron. In order to give simple expression to this relation consider the strength of the field at any given point q in the neighbourhood of the position of equilibrium of the electron, p, represented by a straight line with the end point at q such that the length and direction of the line correspond to the strength and direction of the field. Then for a periodic field the end point will describe a closed curve. This plane lies constantly in the plane normal to pq, is geometrically similar to the projection of the path of the electron on this plane, and is inversely proportional in magnitude to the distance. In any light source a great number of electrons are working together which are excited independently of one another so that as a result natural light with constant change of the ellipse in form and position is produced. Only through some sort of definite control can a regular emission, that is polarized light, be obtained, which indeed may be either rectilinearly or elliptically or, as a special case, circularly polarized light.

Now when such a source of light is placed in a magnetic field it affects the natural period of oscillation of the electron. To understand the nature of the effect it is necessary to invoke the laws of the resolution of oscillatory motions. The simplest resolution of an elliptical motion is into two straight-line motions at right-angles to each other (Fig. 162), in which two imaginary electrons e_1 and e_2 are supposed to move like the projections of the actual electron e. Still more generally the resolution can be thought of as being into three rectilinear motions mutually at right-angles. Another sort of resolution of an elliptical motion is into two circular ones, as in Fig. 163, in which the supplementary electron rotates in the smaller circle and the other in the larger, and where the numbers show the position at corresponding times.

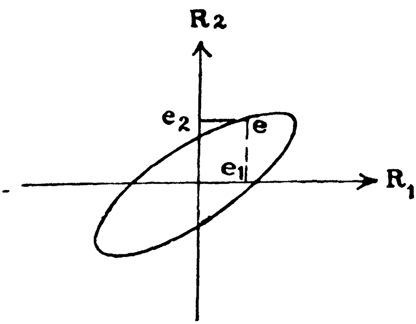

FIG 162

The resolution of an elliptical motion in space would here result in a rectilinear and two circular motions. All these oscillations have the same frequency as the oscillation which they represent.

All this being presupposed we may now apply the fundamental laws deduced by Lorentz for the effect of a magnetic field on a moving electron, this is a law which in regard to the fundamental laws of magneto-electricity (page 163) offers nothing new. The force of the field on the electron is perpendicular to the plane defined by the field and the direction of motion, and is equal to the product of the electric charge e, the strength of the field $\mathfrak{H}$, the velocity v and the sine of the angle between $\mathfrak{H}$ and v. An electron which moves in the direction of the field therefore experiences no effect whatever. If however we are concerned with an electron that under the influence of the force Kr is describing a circular path perpendicular to the direction of the field, then the effect of the

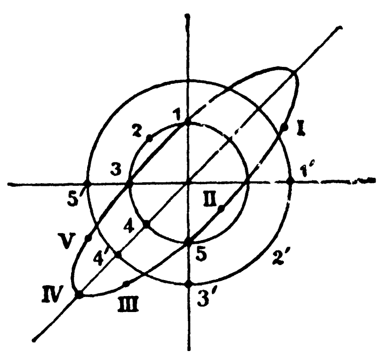

FIG. 163

field is $e\mathfrak{H}v$. If the radius of the circle is at one time r_1, and at the other time r_2, whilst the time of rotation T is in both cases the same, then $v_1 = \frac{2\pi r_1}{T} = r_1 f$ and $v_2 = \frac{2\pi r_2}{T} = r_2 f$ and therefore the effect of the field is $ef\mathfrak{H}r_1$ and $ef\mathfrak{H}r_2$; they increase therefore as the distance and behave exactly like the quasi-elastic forces which hold the electron to its position of rest. This amounts to saying that the field has the same effect as if the quasi-elastic force Kr were altered by an amount $e\mathfrak{H}fr$ and this effect is dependent on the direction of the rotation, the charge of the electron and the direction of the field. Thereby a change of periodicity is brought about, it is either increased or diminished. As regards the change in this quantity it is given by the above-mentioned laws of frequency in the following form:

$$f\pm^2 = \frac{\kappa}{m} \pm \frac{e\mathfrak{H}f+}{m} = f^2 \pm \frac{e\mathfrak{H}f\pm}{m} \quad . \quad (145)$$

or very approximately, since the second term is small in comparison with the first:

$$f\pm = f \pm \frac{e\mathfrak{H}}{2m} \quad . \quad . \quad . \quad . \quad (146)$$

Thus the Zeeman effect both parallel and perpendicular to the field can be quantitatively and qualitatively deduced at least for the simplest fundamental types, and it should be added that comparison with experiment is also qualitatively satisfactory Otherwise expressed, the calculation of the famous relation e/m (charge to mass) from the Zeeman observations leads to values which are in very good agreement with those obtained from an entirely different sort of observations. To represent the more complicated Zeeman types new assumptions regarding the structure of the molecule must be made; but here also hopeful statements have been made in the first place by Ritz and then by other theoretical workers. It is therefore to be assumed that with time the electron theory from this special part of the subject will be extended over the whole theory of electro-magnetism.

XI

MAGNETISM OF THE EARTH

117. **Introductory**—This eleventh and last chapter is to be regarded as in the nature of an appendix. For now we leave the real field of physical magnetism and direct our attention to terrestrial or it may be cosmic phenomena. The magnetism of the earth and what is connected with it belongs therefore not really to physics but to cosmology exactly in the same way as seismology, the investigation of atmospheric electricity, and meteorology. But the picture of magnetic phenomena which we desire to present would be incomplete if we were to shut ourselves in the laboratory and neglected to visit the great field of nature outside. How necessary this is, is best shown by the fact that even for our conception of pure physics important use is to be derived from the observations to be made there, and that the theories connected with the magnetism of the earth, and the experiments which have been devised to substantiate them, will lead us back again to the laboratory. But from what has been said something further follows: that we must be on our guard not to wander too far, and must forego any attempt to exhaust the huge field of cosmic material which here presents itself, and that we have to limit ourselves to what appears to be fruitful and valuable for our principal subject.

We have already learnt something about the fundamental facts at the beginning of our considerations. From the behaviour of a freely turning magnetic needle it followed that on the surface of the earth we are living in a magnetic field, and that its strength and direction vary from place to place. The quantity $\mathfrak{F}$ is a vector and in order to deal with it more conveniently we must resolve it, as we can do, into its horizontal and vertical components $\mathfrak{H}$ and $\mathfrak{Z}$, and $\mathfrak{H}$ it may be still further into $\mathfrak{X}$ and $\mathfrak{Y}$, the northern and eastern components. Of the two angles which then present themselves, that between $\mathfrak{H}$

and $\mathfrak{X}$ is called the declination D and that between $\mathfrak{H}$ and $\mathfrak{F}$ the inclination I. We therefore have the relations:

$$\mathfrak{H} = \mathfrak{F} \cos I ; \quad \mathfrak{Z} = \mathfrak{F} \sin I ; \quad \mathfrak{X} = \mathfrak{H} \cos D ; \quad \mathfrak{Y} = \mathfrak{H} \sin D \quad (147)$$

The gauss has been almost universally adopted as the unit of intensity, and is derived from the absolute system of measurement and is generally written Γ; in cases where a smaller unit is desired the hundred-thousandth part of this unit is chosen and is designated by γ.

118. **Organization**—To each place on the earth's surface there corresponds a definite value of the three elements of terrestrial magnetism, i.e., the declination, the inclination, and the horizontal intensity, which are the three characteristic quantities chosen. If all the points are connected where any one of these magnitudes has the same value a system of curves is obtained which are called the isomagnetic lines. The pictures presented by any of these quantities become still more valuable if from the infinitude of such curves possible a selection is made so that for any two neighbouring curves the difference in value between them is the same; the " density " of the lines, that is their distance from each other, then gives an index to the rate of change of the value. It will be seen that these lines are related to contour lines and isobars, etc. In dealing with terrestrial magnetism a great number of such curves can be distinguished; the most important are the following:

(1) Isogonal lines, i.e. lines of equal declination.

(2) Isoclinic lines, i.e. lines of equal inclination.

(3) Isodynamic lines, i.e. lines of equal total intensity of terrestrial magnetism.

(4) Horizontal isodynamic lines, i.e. lines of equal horizontal components of intensity.

(5) Vertical isodynamic lines, with corresponding significance.

(6) Lines of equal X (north) components.

(7) Lines of equal Y (east) components.

(8) Lines of equal magnetic equilibrium or contour lines, i.e. lines of equal value of the magnetic potential (see below).

(9) Magnetic lines of force or meridian lines, i.e. lines which are obtained when one moves in a direction corresponding to the direction of the force.

(10) Isonomalous lines, i.e. lines in which the deviation

from the true value of any of the above quantities from that deduced by any hypothesis has the same value.

(11) Lines of equal rate of change with time of one of the elements, of which more will be said later.

It is just a hundred years since a beginning was made under the initiative of Gauss (and his fellow-workers Weber and Humboldt) according to a carefully considered and detailed scheme to gather the material required for these isomagnetic lines and therewith for a complete understanding of the magnetic condition of the earth. An ever closer net of magnetic stations has been spread over the surface of the earth, and at the present time there are more than ten thousand of them. In Germany the magnetic observatory on the telegraph hill at Potsdam stands at the head with a branch at the more quietly situated Seddin, and there are corresponding central stations in other countries. They are also points of reference for topographical surveys. An important supplement has always been afforded by the scientific expeditions, especially on the sea, in which the magnetic observations have been either subsidiary to, or the principal object of the expedition. Among the earlier of these that of Ross and Humboldt may be mentioned which led to the discovery of the two magnetic poles of the earth, and the two international polar years, that of 1882–83, devoted to the north polar zone, and that of 1902–3 to the antarctic zone; finally, there were the well-known *Challenger* and *Gazelle* expeditions. It therefore may be said that at the present time, apart from a few outlying oceanic islands where it is still to be desired that there should be stations, there is scarcely any perceptible gap in the net. And for the co-ordination of these efforts the foundation of the international commission on terrestrial magnetism was finally decisive, which met every two years after 1898 until it was interrupted by the world war.

119. **Distribution in Space** –As regards the shape of the isoclines it is here sufficient to give the general features and draw the general conclusions. The isoclines as well as the horizontal isodynamical lines run as a first though very rough approximation parallel to the lines of latitude, except that in America they bend outwards to the south, and in western Europe to the north. The isogonal lines, on the contrary, run entirely between two pairs of points, two of which lie in the north and two in the south; one of each pair is the geographical pole, the others are to be distinguished as the magnetic poles

of the earth. Therefore regarded as a whole the earth is to be thought of as a magnet whose axis makes an angle of about 12° with the axis of rotation, and whose poles (or rather the projection of whose poles on the surface of the earth, for they may themselves lie inside the earth) on the one side lie in the arctic regions of North America and on the other in the South Sea to the south of Australia. The magnetic equator is also of importance (the aclinical line), which differs from the geographical equator in the manner indicated above, as well as a pair of agonic lines, one of which runs through America, and the other from Middle Europe though Persia to Australia, while a third, which is closed upon itself in Eastern Asia, represents a great anomaly. Numerous smaller anomalies present themselves all over the surface of the earth, such districts are called regions of disturbance, and those districts with their own mountain or rock magnetism come specially into consideration in this connection We cannot unfortunately give a series of maps of magnetic lines, but we reproduce the magnetic lines for middle Europe as they run at the present time (Fig. 164). The horizontal isodynamical lines are shown full, the isoclines as broken lines, the isogonical lines as a series of points. The numerical values and a few important places are added.

120. **Variations with Time**--The variations of terrestrial magnetism in regard to time are parallel to its variations in regard to space. The values of the magnetic quantities are not constant, that is, but variable, and the following types of variation are principally of interest:

1. The secular change, greatest for the declination, least for the total intensity (that is, the change is greater in regard to direction than to intensity), and with an average period of 450 years, after which the same values are repeated. In Potsdam, for example, the declination has diminished from $10\frac{1}{2}°$ to the west to $7\frac{1}{2}°$, the inclination in the first half of this period has varied from $66\frac{1}{2}°$ to $66\frac{1}{4}°$, and in the second has risen again to $66\frac{1}{2}$, and the horizontal intensity has first risen from 0·1872 to 0·1888, and then has fallen to 0·1862 Γ. Specially interesting is the question how the position of the pole has altered in the course of the centuries. Only in the most recent times has this question been answered by the careful collation of all the available evidence (in as far as it appears reliable) and curves showing the strange wanderings of the two magnetic poles have been obtained.

An ingenious idea, to be used with caution, has been suggested and worked out by Folgheraiter. Taking account of the fact that certain bodies, for example clay vases with an iron content, on being fired assume the direction of the magnetic field and

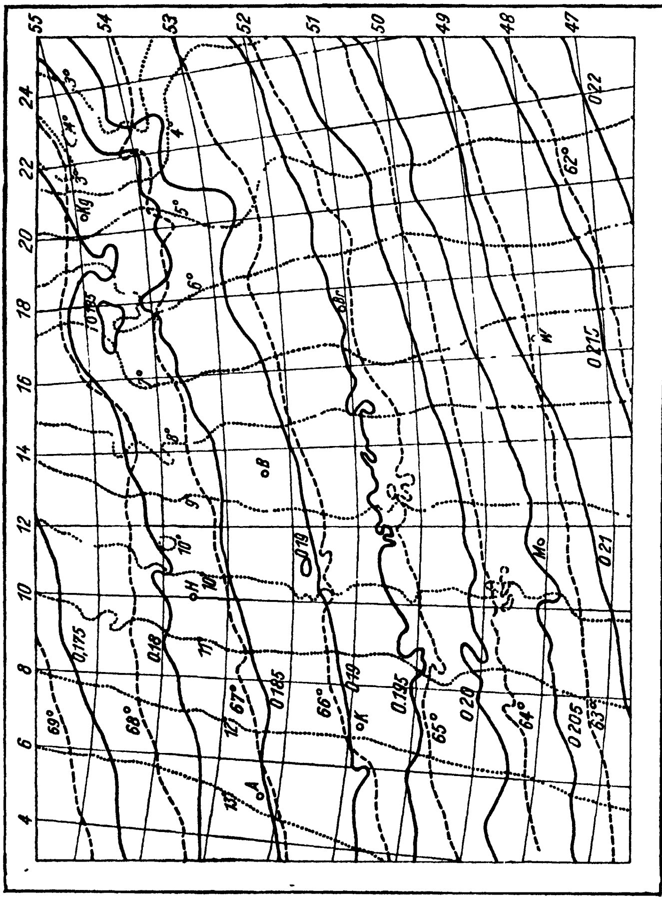

FIG. 164.

then retain it, certain conclusions may be drawn regarding the direction of the earth's magnetic field in ancient times, especially in regard to the inclination. The remarkable result is obtained that in the sixteenth to the eighteenth centuries before Christ the inclination in Italy was partly zero and partly even opposite to what it is at present, and that in

Pompeii in the first century it was as great as it is at present. Brunhes has applied the same idea to the solidification of lava, and among other things has found that at the time when it became hard, the magnetic south pole must have lain somewhere in France.

2. Diurnal variations, and here again those for the declination are the strongest (the greatest westerly value is at two o'clock in the afternoon and least at eight o'clock in the evening), as regards distribution in space are stronger the nearer we approach the poles. The variation moreover is stronger by day than it is by night, and in summer is about twice as great as it is in winter. These variations can be represented in the usual way with the twenty-four hours of the day as abscissæ and the values of the quantity under consideration as ordinates; or still more clearly in the form of a vector diagram. From any point the variations of the north-south components are drawn outward and above or below, and those of the east-west components to the left or right for each hour of the day. The closed curve so obtained is the vector diagram of the daily deflection of the horizontal intensity in magnitude and direction. But now comes something very remarkable: Lamont had already called attention to the fact that the amplitude of the diurnal variation of the terrestrial magnetic elements was not always the same but that these showed periodical variations, the period of which was about eleven years. Since at the same time Sabine demonstrated that there was an equal period for the frequency of the sun spots, it was an obvious thing to infer some connection between the two sets of phenomena. This has been remarkably well justified even in small details, so that, for example, the same variations in the length of the period present themselves at the same time in both sets of phenomena. The connection is such that with the exception of a few (moreover doubtful) cases it is of such a kind that the variations in the terrestrial magnetism run parallel with those of the sun-spot frequency, as is made clear in regard to the declination and the horizontal intensity by the curves in Fig. 165 (the upper curve showing the sun-spot frequency, the middle one the declination, and the lower one $\mathfrak{H}$), and also by the following values which have been determined at Greenwich since 1841. The upper row relates to the period of sun-spot frequency, and the lower row to the strength of the terrestrial magnetic daily variations (in years and fractions of years):

12·55	11·85	11·40	10·45	11·30	13·05	10·90	9·85
12·50	12·00	11·20	10·50	11·80	13·40	11·20	10·00

The mean value of the sun-spot period comes out as 11·42 and

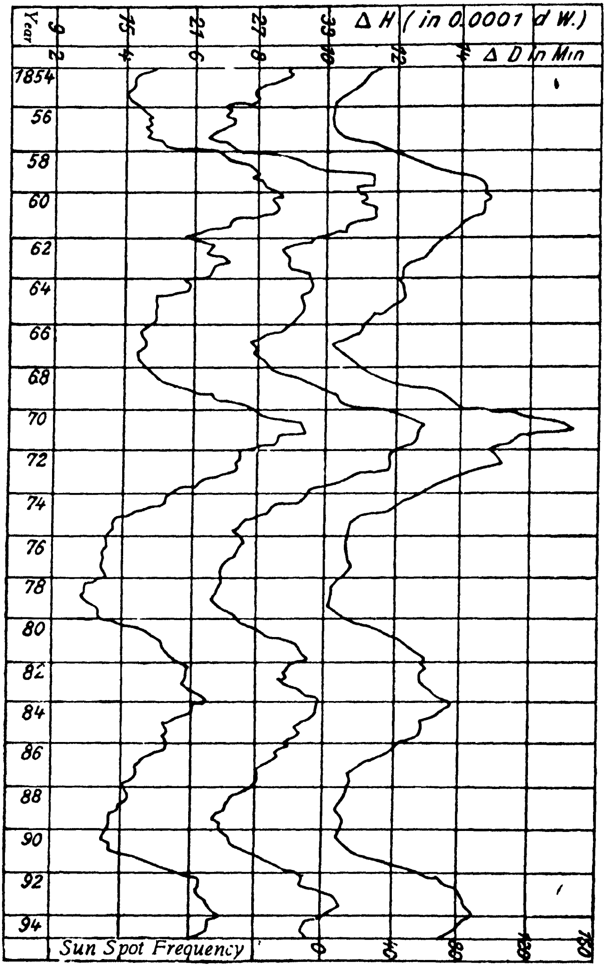

FIG. 165.

that of the daily terrestrial magnetic variation as 11·45, therefore as of almost exactly the same value. The moon also has a certain influence on the variations in terrestrial magnetism.

3. Subsidiary variations. Until quite recently it was assumed that the curves representing terrestrial magnetism, apart from the variations already mentioned or still to be described, were constant and uniform, and the indications of the older recording instruments seemed to confirm this assumption. Later results obtained by means of more sensitive and reliable instruments have shown, however, that it is not so, but that small variations are also present which manifest themselves as countless small ripples superimposed on the main curves, and in these shorter-period variations, still smaller ones can be discovered.

4. Magnetic disturbances These manifest themselves by the needle, hitherto at rest, beginning visibly to quiver and change its direction in a short time through a wide angle; specially violent disturbances such as are frequent at high altitudes were named by Humboldt magnetic storms. In Germany the more recent of the magnetic storms took place one on October 31, 1903, another on September 25, 1909, and another on June 17, 1915 Such disturbances usually occur at all parts of the earth at the same time, though the intensity may be different, and everywhere it follows a parallel course. We are therefore dealing here with a general and not a merely local phenomenon. Again, the connection with the activity of the sun is a fact of the greatest importance. This connection is so close that the outbreak of a magnetic storm coincides exactly with a sun-spot change, or to be more exact with the appearance of a facula.

121. **Earth Currents**—The picture of terrestrial magnetism that we have given would be incomplete if we did not take account of two phenomena which at first sight seem to have nothing to do with it, but which nevertheless have the closest bearing The first takes place within the earth; the other in the heights of the atmosphere. The former are the earth currents to which Barlow was the first to call attention, and which were more closely investigated by Lamont and finally reduced to systematic form by Weinstein. They circulate without intermission in the superficial layers of the earth; generally weak, but sometimes so strong that they interfere with the telegraph service, where, as is well known, the earth is used as a return circuit. The extreme limit of its intensity amounts to a potential difference of about one volt per kilometre. The earth current has a constant part which is of little interest to us, and a variable one. The periodicity of the last has a

striking relation to the variation in the terrestrial magnetic element. The most important period is the daily one; it is stronger for the north-south component than for the east-west; but in both the correspondence is the same. There are maxima on the one side in the morning and the afternoon, and maxima on the other shortly before noon, and the amplitude of the daily variation is greater in summer than in winter. As regards the connection with the terrestrial magnetism the most important question is, which of these two phenomena is to be regarded as primary and which as secondary? Apart from other considerations the matter can be decided by an application of the general laws of induction. If terrestrial magnetism is to be regarded as being the cause of the currents, then these should be strongest at the time when the component of terrestrial magnetism with which it is related is changing most rapidly. If on the contrary the earth current is the primary cause, the magnetic variation must be strongest when the earth currents are strongest and *vice versa*. The latter is actually the case, at least in so far as we are concerned with simple relations and not those dependent upon some special factor.

122. **The Polar Lights**—The other phenomenon to be considered here is the polar light. It is for the most part a soft light, imposing because of its size and manifold character, which presents itself in the northern or the southern sky. According to the form of the light it is classified as follows: (1) Polar light not of a stream-like character, subdivided into (*a*) bows; (*b*) cloud-like masses of light; (*c*) a diffused light. (2) Polar light of a stream-like character subdivided into (*a*) bands; (*b*) curtains; (*c*) corona. The bows are perpendicular to the magnetic meridian; the stream-like forms run parallel to the direction of the inclination, and the rays if produced meet near the point to which the upper end of a free magnetic needle would be directed. The actual connection with terrestrial magnetism seems to occur only in the case of the stream-like variety; and here it is not only qualitative but in a high degree quantitative as well, so that it is almost always safe to conclude if in any locality there is a magnetic storm, but no polar light, that the light will be found to have been visible somewhere else. The zone of greatest frequency surrounds the magnetic pole over a pretty considerable distance (about 20°). Recently very accurate determinations have been made of the height of the polar light. According to

Störmer the northern light on 138 occasions showed the following distribution as regards height (in all cases calculated from the centre of the appearance):

0–50	50–100	100–150	150–200	200–250	250–300	over 300 km.
7	30	64	24	6	5	2

According to this the maximum would seem to occur at a height of 120 km., and therefore in a layer of the atmosphere which consists of two-thirds hydrogen, and the spectrum of the polar light is moreover in agreement with this. As regards frequency of occurrence the polar light is a phenomenon presenting numerous periodicities, of which the eleven-year, the yearly, and the half-yearly are the most important; but the most important feature of all is its coincidence with magnetic disturbances.

123. **The Theory of Gauss**—We are now in a position to turn to the theory of terrestrial magnetism. The first of the still valid theories that are free from any arbitrary assumptions is that due to Gauss. It is in the sense already defined a formal theory and was put forward in one of the most classical and significant of treatises in the whole field of mathematical physics (C. F. Gauss, *General Theory of Terrestrial Magnetism*, 1840). It sets out simply from the assumption that the intensity of the effect produced is inversely as the square of the distance. The field of the earth is intersected by surfaces of equal potential, and the intersection of these surfaces with the surface of the earth represents the contour lines of magnetic equilibrium. The lines of force or magnetic meridians are perpendicular to these. The line of intersection of the surface of zero potential with the surface of the earth is the equator of potential. The point at which the contour surface touches the surface of the earth, and where therefore the potential is a maximum or a minimum and the force is vertical in direction, is a magnetic pole. In theory, of course, many such poles might occur. Actually, apart from local phenomena, there are only two, the north pole and the south pole. The potential is a function of λ, the geographical longitude, and of the polar distance (the complement of the geographical latitude) u. It satisfies the Laplace equation $\Delta V = 0$, and can be represented by the so-called spherical functions. The results of actual observations being given, the coefficients of this series can be determined with a degree of accuracy which depends upon the number of observations available and their suitability

as regards distribution in latitude and longitude. Gauss himself made the calculation for 12 points on 7 parallels of latitude, and the calculation has been repeated later by Ermann and Petersen for 9 points on 10 parallels, and that too for the epoch (1830) to which Gauss's calculation referred. The calculation was then made by Quintus Icilius for 12 points on 10 parallels for the year 1880, and finally by Neumayer for 72 points on 25 parallels (altogether therefore for 1,800 points on the earth's surface) for the year 1885. The magnetic moment of the earth if the radius be called a comes out as $0{\cdot}33092a^3 = 8{\cdot}584 \times 10^{25}$ from Gauss's calculation, and as $0{\cdot}32237a^3 = 8{\cdot}362 \times 10^{25}$ from Neumayer's calculation, and therefore as about 3 per cent. smaller. It is uncertain however whether this difference is due to greater accuracy in the later calculation, or whether some diminution has actually taken place. At present we may state approximately that

$$M = 8{\cdot}33 \times 10^{25}$$

and according to L. A. Bauer this amount diminishes yearly by $0{\cdot}0033 \times 10^{25}$.

If we compare the earth to an artificial magnet then we can say, according to the calculation of Ermann and Petersen, that the earth at a distance of 21,000 metres would exert the same effect as a well-magnetized steel bar of 500 grams weight at a distance of one metre; or, alternatively, the radius of an iron sphere magnetized to saturation, having its axis concentric with the earth, and exerting the same effect as the earth, would be about 243 km., that is about one-twenty-sixth the radius of the earth. According to Gauss we can say that assuming uniform distribution each cubic metre of the earth is as strongly magnetic as eight well-magnetized pound bars would be.

The magnetic axis of the earth, that is, its axis of greatest moment (for this, only the first terms of V are required) according to Neumayer, is the line from 78° 20′ north latitude and 292° 43′ east longitude, to 78° 20′ south latitude, and 112° 43′ east longitude. It is of course a diameter of the earth, which is not the case with the line connecting the actual magnetic poles (see above).

If, in conclusion, the calculated values of D, I and $\mathfrak{H}$ are compared with the observed values for the same places, certain differences of course are found, and in some cases they are considerable (for D and I they amount to 5°, and in the case of

$\mathfrak{H}$ to 8 per cent. of the actual value). According to Neumayer's calculation it is of course smaller, but still not to the extent that might have been expected; there must therefore have been some inexactitude in the assumptions made. This consideration has lately led Adolf Schmidt, the present director of the Potsdam Observatory, to apply improved methods to the Gauss theory in the following directions: (1) Allowance is made for the extent to which the shape of the earth differs from being that of a true sphere; (2) the series are carried further; (3) the assumption is abandoned that the total force should have a potential at the surface of the earth, a part of it is assumed to arise from the effects of currents which pass perpendicularly through the earth's surface—a part which admittedly will be very small; (4) the assumption is abandoned that the whole effect arises from internal causes, it appears that about one-fortieth of the effect is due to external causes. Finally, the conclusion is reached, differing somewhat from Neumayer's result, that

$$M = 8 \cdot 348 \times 10^{25}, \quad \phi = 78^\circ\ 34 \cdot 3', \quad \lambda = 68^\circ\ 30 \cdot 6w.$$

124. **Analysis of the Earth's Magnetism**—We have next to make the attempt to analyse the developed picture that is offered us by what we have learnt of the distribution of magnetism over the surface of the earth, and in the first instance into two parts, one of which corresponds to a uniform and the other to a supplementary part. The former is broadly represented by the magnetic meridians and the parallel circles, and has an axis which is inclined about 11° to the axis of the earth. It is a matter of surprise that this part represents the major component of the total magnetic moment, so that it may be regarded as the normal terrestrial magnetism, and the small remainder as an anomaly Bezold, Bauer and others have gone into this question further and have considerably elucidated the problem.

But another altogether different sort of analysis of the total field is possible, namely into four terms:

$$\mathfrak{F} = \mathfrak{F}_0 + \mathfrak{A} + \mathfrak{B} + \mathfrak{C} \quad . \quad . \quad . \quad . \quad (148)$$

These terms can be distinguished from one another through their behaviour in regard to time: $\mathfrak{F}_0$ is the constant permanent field; $\mathfrak{A}$ is the field of the secular variation; $\mathfrak{B}$ that of the periodic and special daily variation; $\mathfrak{C}$, finally, that of disturbances or magnetic storms. Through a skilful combination of observations and calculations, these terms can be taken in detail and separately examined.

In the first place, as regards the secular variation: this Bauer regards as a curve which is described by a point on the axis (supposed to be prolonged) of a magnetic needle in consequence of secular changes in D and I—obviously an oval curve having a different shape at different places, but generally always described in a clockwise direction. On the other hand a curve is also obtained by taking into account only the permanent term in the earth's magnetism, which carries the needle along a circle of latitude. The interesting result is that these two curves almost coincide. The secular variation arises because the permanent field moves round the earth. If we wish to go more closely into the matter then we must calculate the change with time of the Gauss coefficient. It is then found that the matter is not so simple and that we require to assume a resolution of the magnetic axis into two parts which rotate at different rates. The shorter period amounts to 454 and the longer to 3,147 years. The first corresponds in length and also in phase to the secular change in declination as it is observed in Europe. Why such a rotation takes place is another question. The simplest assumption is that the magnetic axis rotates about the axis of the earth, but so far nothing more definite can be said about it.

We now come to the daily variation. Substantially it depends, as Schuster in particular has shown, upon external forces. Its potential can be calculated and a map of the "lines of equal potential of daily variation" can be drawn. It will be clear that the picture so obtained will be valid only for a particular moment of the day, because this system, as contradistinguished from preceding ones, does not reside on the earth's surface but in surrounding space. Therefore for the ordinates of the map we must use, not the geographical longitude, but the time of the day. The picture actually consists of a system of closed curves, which if we assume the theory of electric currents, may be regarded as a current eddy: about the negative pole the currents circulate in a clockwise direction; about the positive pole in the reverse direction. There are four poles, two positive and two negative, one of each pair being a day pole and the other a night pole. The strongest pole (positive) lies in longitude 30°, and 30° west of the point where it is midday, i.e. at the place where it is ten o'clock in the forenoon.

125. **Physical Theory**—All that we have so far said, from a theoretical point of view, can be brought within the scope of a formal mathematical development. We must now turn to the physical side of the theory, and in the first place consider that part of the terrestrial magnetism which is permanent in character. Here there are obviously three possibilities: the origin of the magnetism of the earth may be permanent magnets; or soft iron masses which are maintained in a magnetic condition by electric currents; or finally they may be electric currents as such. The assumption of permanent magnets, when we come to consider it carefully, presents many difficulties, in the first place on account of the strength of the effect which would compel us to assume that the earth consists largely of iron and indeed of permanently magnetized iron and therefore of steel. This assumption, which at first appears fantastic, recently indeed, on account of the results obtained in two other fields of investigation, gravitation and seismology, has acquired a firmer foundation. For from the theory of gravitation and the corresponding measurements relating thereto, it follows that the mean density of the earth is about 5·5. This value is indeed considerably below that of iron, but we must remember that the crust of the earth has a density of about 2, and that in its interior there must exist considerable hollow cavities and interstices, and we then arrive at a value for the solid interior of the earth which is at least very close to that of iron. Wiechert has come to the conclusion, on account of earthquake observations, that the earth consists of three zones: a crust about 30 kilometres thick; an intermediate layer of some 1,500 kilometres; and a core of about the density of iron. But it must be remembered that at so great a depth there must be an extremely high temperature, higher than the critical temperature for magnetic iron, so that observations made at the surface of the earth would indicate that the iron below could not be magnetic. But the other consideration remains that at these great depths there is also an enormous pressure and that it is possible (there are no experiments dealing with this point) that the critical temperature of the iron is substantially raised by the extreme pressure. Without further knowledge it is not possible altogether to reject this hypothesis. The other question, whether it is a case of permanent magnetism or of magnetism induced in soft iron by

electric currents, has also been much discussed. The latter alternative is supported by the fact that terrestrial magnetism is subject to change both in regard to time and to distribution in space, and this is more intelligible if we suppose that the magnetism is induced by electric currents, whereas, if we suppose it to be due to the presence of masses of permanently magnetized steel, we are compelled to assume changes in the position or to make still more fantastic suppositions. Finally there remains the possibility that the magnetic masses are to be sought, not at the centre, but in the outer crust of the earth. But this is composed of very weakly magnetizable substances, the effect of which would not be sufficient to explain the phenomena concerned.

If therefore we have reason to turn to the purely electrical theory it is to add that this theory in the last few decades has been steadily acquiring firmer foundations. In the first place the phenomenon of earth currents shows that currents may circulate in the earth's crust, and these currents show a remarkable connection with terrestrial magnetism; in the second place a very simple consideration shows that the currents which have to be assumed in order to explain terrestrial magnetism need only to be of moderate and quite possible intensity; and in the third place there is no great difficulty in regard to the direction which must be ascribed to these currents. We have only to assume that in addition to the principal east-west system, there is also a second system of currents that circulates through the land masses along the coast lines in an anti-clockwise direction. If we then assume a suitable ratio between their respective intensities we arrive at the actual relations obtaining in terrestrial magnetism, the inclination of 11° and so forth. But two questions still remain: what is the nature of these currents? Here, in consideration of what has already been discussed, the most favoured hypothesis is that we have to do with the so-called displacement current due to the relative motion of rotation of the earth; in other words, that the earth rotates through the ether, which remains relatively at rest, and that the ether which is relatively left behind induces the phenomenon of the east-west currents as well as the anti-clockwise currents because of the difference of resistance. And secondly, where do these earth currents actually originate? Here we must plainly have recourse to external forces, to the horizontal currents connected with atmospheric electricity, which act inductively on the interior of the earth.

This accords very well with what has been said before, but in regard to details there remains much that is not at all clear.

So much as regards the permanent magnetization. As for the variations, the secular change has already been ascribed to the wandering of the permanent field about the earth (we have here a case where formal theory leads the way to definite physical conceptions). The daily variations, on the other hand, must be ascribed to the effect of electric currents; and from observations on the inclination needle at various geographical latitudes we are led forcibly to the conclusion that these currents have their seat in the atmosphere, and especially in the better-conducting upper layers—an assumption by the help of which these magnetic phenomena are brought into a certain parallelism with the meteorological. It is in good agreement with this that the four poles of the daily variation which have already been referred to, that is the eddy centre of the respective current systems, lies in the so-called "horse latitudes," that is at the border of the circular course of the monsoon, and the polar wind system. Adolf Schmidt has even attempted to refer the magnetic disturbances, the more powerful ones at least, to wandering electric current eddies corresponding to cyclones, which would give the term that has long been used in connection with them—"magnetic storms"—a deeper significance.

Our theory takes, as will be seen, an upward direction; ascendant in the sense that we have looked from the interior of the earth and from the earth's surface upwards towards the atmosphere. But the process is still not at an end; we must take another and a further step; we must go from the earth's atmosphere through space to the sun itself, with its all-dispensing and all-controlling might. For that the sun here also plays a determinating part, follows from the whole sequence of terrestrial magnetic phenomena, the various periodicities (the daily, the 27-day, the yearly, the 11-yearly), and follows in a special sense from the startling parallelism that is to be observed with the sun-spot and sun-facula periods. The next step is to assume that the sun, like the earth, is itself a magnet; and a magnet in fact it is, as is indicated by many astronomical observations, especially those of Hale, who has established some very interesting facts about the magnetism of the sun. Nevertheless the hypothesis of a direct magnetic influence on the earth has to be relinquished because the magnitudes in the ratio of cause to effect are too widely at variance. For if the relation of the effect of the sun S to the effect of the earth E were even in the most favourable circumstances defined by the formula

$$\frac{S}{E} = \frac{1}{4}\frac{\mathfrak{J}_s}{\mathfrak{J}_e}\left(\frac{2r}{d}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (149)$$

where $\mathfrak{J}_s$ and $\mathfrak{J}_e$ are the intensities of the magnetization of the sun and the earth, r the radius of the sun and d its distance from the earth, the calculation shows that even if $\mathfrak{J}_s$ is put $=\mathfrak{J}_e$, S/E works out as about 2×10^{-7}, from which it would follow that the effect on the declination would amount to about one-twentieth of a second of arc, which would be quite a negligible change, whereas it actually amounts to ten minutes and even more. To produce an effect of this magnitude the sun would have to be 12,000 times as strongly magnetized as the earth, specific magnetism referred to unit volume being understood of course, and therefore much more strongly than if it were composed entirely of the best magnetized steel. Thus the assumption of a direct magnetic influence fails altogether. Moreover the heat radiation of the sun does not lead, though this is without prejudice to the previously mentioned parallelism, to any comprehensive theory. Therefore only the theory of electric radiations remains. It was Balfour Stewart, afterwards supplemented by Schuster, who gave the quietus to this theory.

If we give a more detailed examination to the facts of the case especially as regards periodicity and the initiatory phases we are led to conjecture that the influences which set up disturbances are connected with isolated, relatively small, but somewhat long-enduring areas of the sun's surface, and arise approximately at the time of their passage across the apparent middle of the sun's disc. In confirmation of this we have above all the frequent succession of disturbances at intervals which more or less correspond to the synodic time of rotation of the sun.

These and similar considerations have led to the light-pressure theory of Arrhenius, and the Hertzian wave theory of Nordmann, and then to the cathode-ray theory of the sun, first suggested by Birkeland and afterwards extended in all directions and then strikingly confirmed by the theoretical investigations of Störmer. This latter may be called the electron theory of magnetic disturbances and the polar light —for both phenomena are here included.

In the first place, as regards the theory of Störmer, it is a masterpiece of elegance and it is on that account unfortunate that it cannot be given here. It must suffice to mention that the system of equations that is required and which determines the path of a cathode ray particle in the field of the earth cannot be exactly solved and can only be dealt with by means of the so-called mechanical quadrature (a tedious business requiring 4,500 hours of calculation!). According to the value of the arbitrary constant γ chosen, various paths are obtained, and the following classes may be distinguished: there are paths which do not reach the earth; paths which approach the earth and then go off again into infinity; paths

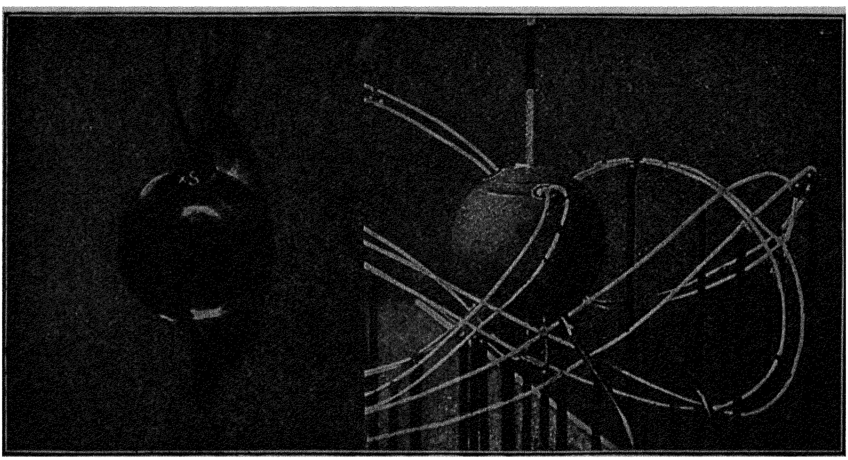

FIG. 166.

which remain in the earth's field; and, finally, paths which penetrate the earth. In Fig. 166 on the right-hand side are shown examples of some paths which strike the earth. But what does the left-hand side of the figure signify?

Here we come to one of the most beautiful results of modern experimentation. It has not been thought sufficient to limit the theory to the explanation of the polar lights and magnetic disturbances (especially in the great Norwegian expeditions); the leading authority on this subject, Birkeland, has rather sought to reproduce them by artificial means in the laboratory, naturally on a very small scale, but under experimental conditions which may be suitably varied, and therefore in the hope of obtaining in this way a closer insight into the actual conditions of the phenomena in question. This hope has been strikingly justified. If an iron sphere covered with a fluorescent substance is introduced into the discharge tube it becomes lighted up on the whole of that side turned towards the cathode-ray stream. But if it is now magnetized the appearance undergoes a fundamental change; a luminous ring makes its appearance on the equatorial plane of the sphere, and as the magnetization is carried further, there are seen to be associated with it two spiral-like girdles in the neighbourhood of the poles which progressively approach them as the magnetization is increased. And these girdles on still stronger magnetization break up into isolated patches of light which float like the polar lights over the miniature earth. Therefore, regarded as a comprehensive demonstration of what takes

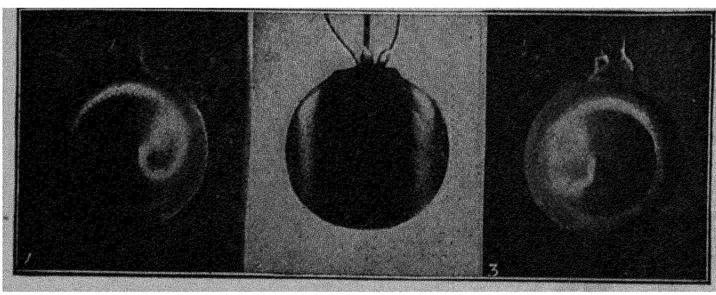

FIG. 167.

place in the actual field of the earth, it is only that part which relates to the light girdle surrounding the equator which finds no place in the terrestrial scheme. Some further pictures of the phenomenon are also given in Fig. 167. It has lately been found that the details of the experiment agree better with the assumption of positive than that of negative rays, and that the distribution of the northern light in particular can be better explained both quantitatively and qualitatively. Since the assumption of canal rays leads to difficulties, we assume as the source of the positive rays, which are the so-called α-rays, radio-active processes going on in the sun, and it can then be understood how it comes about that in the field of the earth, only the positive rays resulting from these processes show themselves active.

The circle of electric and magnetic phenomena, regarded both in its terrestrial and its cosmic aspect, here again leads to a complete and beautiful unity.

A SELECT BIBLIOGRAPHY

[From the enormously extensive literature of the subject only a very small selection can be given here. In all cases only what is of fundamental or most general interest has been quoted.]

I. PRELIMINARY

GILBERT, De Magnete magneticisque Corporibus, etc. London, 1600.

R. NORMAN, New Attractive. London, 1580.

ANASTASIUS KIRCHER, Magnes sive de Arte magnetica, 1634.

MUSCHENBROEK, Diss. phys. de Magnete. Vienna, 1754.

ÆPINUS, Tentamen Theoriæ Electricitatis et Magnetismi. Petersb., 1759.

BRUGMANS, Ueber die magnetische Materie, 1788.

M. FARADAY, Exp. Researches in Electricity and Magnetism. Lond., 1839 ff.

F. NEUMANN, Vorlesungen u. d. Theorie d. Magnetismus. Leipzig, 1881.

G. KIRCHHOFF, Vorles. ü. math. Physik, Bd. 4. Leipzig, 1881.

Cl. MAXWELL, A Treatise on Electricity and Magnetism. Cambr., 1873.

W. THOMSON, Reprint of Papers on Electricity and Magnetism. London, 1872.

MASCART u. JOUBERT, Lehrb. d. Elektrizitat u. d. Magnetismus. Berlin, 1886.

G. WIEDEMANN, D. Lehre v. d. Elektrizitat. Braunschweig.

J. LAMONT, Handbuch d. Magnetismus. Leipzig, 1867.

AUERBACH, Magnetismus in Winkelmann's Handb. d. Physik, Bd. 5. Leipzig, 1905.

AUERBACH, Magnetismus in Gratz Handb. d. Elektrizität, Bd. 4, Leipzig, 1920.

S. SAVARY, Phil. Trans. R. Soc. 36, 295, 1729.

II. MAGNETIC FORCES

TOBIAS MAYER, Gött. Anzeiger, 1760.

J. H. LAMBERT, Hist. de l'Acad. de Berlin, 1765, p. 22.

C. A. COULOMB, Mém. Acad. de Paris, 1785, p 6c6.

C. F. GAUSS, Resultate a. d. Beobachtungen d. magnetischen Vereins, 1836, S. 22, und 1840, S. 26.
C. F. GAUSS, Intensitas vis magneticæ, etc., Comm. Soc. Göttingen, 8, 1841. Werke, Bd. 5, S. 79.
J. LAMONT, Handb. d. Erdmagnetismus.
O. CHWOLSON, Mém. Acad. de St. Petersburg (7), Bd. 31, Nr. 10, 1883.
H. LLOYD, Trans R. Irish Academy, 19, pp. 159 and 249, 1843.
K. WEIHRAUCH, N. Mém. Soc. Nat. Moscou, 14, Heft 4, 1883.
C. H. HANSTEEN, Untersuchungen ü. d. Magnetismus d. Erde. Christiania, 1819.
C. F. GAUSS, Pogg. Ann. 28, S. 241, u. 591.
AUERBACH, Physik in graphischen Darstellungen. Leipzig, 1912 (Tafel 158) (sec. ed. 1924).
H. EBERT, Magnetische Kraftfelder. Leipzig, 1896, 2. Aufl., 1904.
M. FARADAY, Phil. Trans. 1852, 1 ; Exp. Res., Vol 3.
MAXWELL, On Lines of Force, etc. See Collected Papers.
A. M. MAYER, Phil. Mag. (5), 5, p. 397, 1878 , 7, p. 98, 1879.
R. W. WOOD, Phil. Mag. (5), 46, p. 162, 1898.
DE LA RIVE u. GUYE, Compt. rend. 148, p. 161, 1909.

III. MAGNETIC INDUCTION

S. D. POISSON, Mém. de l'acad. 5, p 249 and 488, 1824 ; 6, p. 441, 1827. Ann. chim. phys. 25, p. 113 ; 28, p. 1.
W. THOMSON, Phil. Mag. (4), 1, p. 177, 1851. Ges. Abh. ü. El. u. Magn. 449.
F. NEUMANN, Crelle's Journal 37, S. 44, 1848. Vorlesungen ü. d. Theorie des Magnetismus. Leipzig, 1881.
G. KIRCHHOFF, Crelle's Journal, 48, S. 348, 1854. Pogg. Ann. Erg., Bd. 5, S. 1, 1870. Ges. Abhandlungen, S. 193 u. 223.
J. STEFAN, Wien. Ber. (2), 69, Feb. 1874.
E. BELTRAMI, Mém. di Bologna (4), 5, 1884 ; (5) 1, p. 409, 1891.
C. NEUMANN, Untersuchungen u. d. log. u. Newtonsche Potential. Leipzig, 1877.
A. BEER, Einl. i. d. Elektrostatik usw. Braunschweig, 1865. S. 145 ff.
LEONH. WEBER, Zur Theorie d. magnet. Induktion. Kiel, 1877.
E. RIECKE, Wied. Ann. 13, S. 465, 1881.
A. WASSMUTH, Wied. Ann. 51, S. 367, 1894.
P. DUHEM, De l'aimantation par influence. Paris, 1888. Leçons s. l'él. et le magn., Bd. 2. Paris, 1891.
H. DU BOIS, Wied. Ann. 46, S. 485, 1892.
E. WARBURG, Wied. Ann. 13, S. 141, 1881.
J. A. EWING, Proc. R. Soc., 1882, p. 39; Trans. R. Soc., 1885, p. 549.
J. HOPKINSON, Trans. R. Soc., 1885, p. 446.

LORD RAYLEIGH, Phil. Mag. 6 and 7, 22, p. 175, 1886.
WARBURG u. HÖNIG, Wied. Ann. 20, S. 814, 1883.
J. A. EWING, Proc. R. Soc. 23, p. 22, 1881 ; 24, p. 39, 1882.
R. LIPSCHITZ, In.-Diss., Berlin, 1857.
H. NAGAOKA, Wied. Ann. 57, S. 275, 1896.
G. RÖSSLER, Unt u. d. Magn. d. Eisens. In.-Diss., Zürich, 1892.
G. GREEN, An Essay on the Application, etc. Nottingham, 1828.
O. GROTRIAN, Ann. d. Physik, 23, S. 575, 1907.
J. A. EWING, Magnetic Induction, London, 1892.
H. DU BOIS, Magnetische Kreise, Berlin, 1894.
A. WASSMUTH, Wien. Berichte (2), 102, S. 81, 1893.
J. STEFAN, Wied. Ann. 17, S. 928, 1882.
H. DU BOIS, Wied. Ann. 63, S 348, 1897 ; 65, S. 1, 1898.
A. P. WILLS, Phys. Review, 9, p. 193, 1899.
D. N. MALLICK, Phil. Mag. (6), 15, p. 65, 1908.
M. ASCOLI, Rend. Acc. Linc. 3, p. 190, 1894.
RIBORG MANN, In -Diss., Berlin, 1895 ; Phys. Review, 3, p. 359, 1896.
C. G. LAMB, Phil. Mag. (5), 48, p. 262, 1899.
BENEDICKS, Ann. d. Physik, 6, S. 726, 1901.
THOMPSON and MOSS, Phil. Mag. (6), 17, p. 729, 1909.
O. GROTRIAN, Ann. d. Physik, 34, S. 1, 1911 ; 36, S. 929, 911.
A. OBERBECK, Pogg. Ann. 135, S. 84, 1868.
A. STOLETOW, Pogg. Ann. 146, S. 442, 1872.
H. ROWLAND, Phil. Mag. (4), 46, p. 140, 1878.
H. LEHMANN, Wied. Ann. 40, S. 406, 1893.
T. TAKAMINE, Proc. Math.-phys. Soc. Tokio (2), 5, p. 303, 1910.
P. BARLOW, Gilb. Ann. 73, p. 1, 1823.
J. STEFAN, Wien. Ber. 85 (2), S. 613. Wied. Ann. 17, S. 928, 1882.
F. KIRSTÄDTER, In.-Diss., Leipzig, 1896 ; Wied. Ann. 65, S. 72, 1898.
J. RUSSELL, Trans. Edinb. Soc. 40, p. 631, 1903.
O. FRÖHLICH, Elektr. Zeitschr., 1881, S. 141 u. 170 ; 1882, S. 73.
LORD RAYLEIGH, Phil. Mag. (5), 23, p. 225, 1887. Sc. Papers, 2, p. 579.
GUMLICH u. ROGOWSKI, Ann. d. Physik, 34, S. 235, 1911.
R. GANS, Phys. Zs., 1910, S. 988, u. 1911, S. 1053. Ann. d. Phys. 27, S. 1, 1908 ; 33, S. 1065, 1910.
ST. JOHN, Phil. Mag. (5), 39, p. 297, 1895.
M. WIEN, Wied. Ann. 66, S. 859, 1898. Ann d. Phys. 18, S. 1077, 1905.
MADELUNG, Ann. d. Phys. 17, S. 861, 1905.
J. M. BALDWIN, Phil. Mag. 13, p. 232, 1907.
A. V. WALTENHOFEN, Wien. Berichte 48 (2), S. 564, 1863. Pogg. Ann. 120, S. 650.
F. AUERBACH, Wied. Ann. 14, S. 308, 1881 ; 16, 554, 1882.

J. A. EWING, Trans. Phil. Soc. London, 1885, p. 655.
E. WARBURG, Wied. Ann. 13, S. 141, 1881.
P. Ch. STEINMETZ, Elektr. Zs. 12, S. 62, 1891 ; 13, S. 519, 189: Electrician, 28, p. 384, 408, 425, 1892.
LYLE and BALDWIN, Phil. Mag. (6), 12, p. 433, 1906.

IV. PANMAGNETISM

B. O. PEIRCE, Proc. Amer. Acad. 46, p. 207, 1910.
LYDDALL and POCKLINGTON, Proc. R. Soc. 52, p. 228, 1902.
E. WILSON, Proc. R. Soc. 62, p. 369, 1898.
ST. KREUSLER, Verhandl. d. d phys. Gesellsch., 1908, S. 344.
E. GUMLICH, ibidem, S. 371.
GUMLICH u. SCHMIDT, Elektr. Zs. 22, S. 621, 1901.
BARUS u. STROUHAL, Wied. Ann. 20, S. 621, 1883. Bull. Geo Surv., Nr. 14, 1885.
J. HOPKINSON, Trans. R. Soc., 1885 (2), p. 463.
J. A. EWING, ib. (1), p. 523 ; (2) p. 581.
L. HOLBORN, Zs. f. Instr.-Kunde, 1891, S. 113. Wied. Ann. 61 S. 281, 1897.
C. BENEDICKS, Rech. s. l'acier. Upsala, 1904.
WAGGONER, Phys. Review, 35, p. 58, 1912.
S. HILPERT, Ber. d. chem. Ges. 42, S. 2248, 1909. Verh. d. d phys Ges. 11, 293, 1909. Jahrb. d. Radioakt. 10, S. 91, 1913
GUMLICH, Elektr. Zs. 37, S. 592, 1916.
L. HOLBORN, Zs. f. Instr.-Kunde, 1891, S. 113.
E. WILSON, Electrician, 45, p. 894, 1900.
S. HILPERT, Ber. d. d. chem. Ges. 42, S. 2248, 1909.
H. ROWLAND, Phil. Mag. (4), 48, p. 321, 1874.
J. A. EWING, Phil. Trans. Lond. 179, A, p. 327, 1888.
R. BEATTIE, Phil. Mag. 6, p. 642, 1901.
J. HOPKINSON, Proc. R. Soc., Dec. 1889, Jan. and May 1890.
EWING and PARSHALL, Proc. Inst. Civ. Eng. 126, pp. 50 and 71 1896.
FR. HEUSLER, W. STARK, u. E. HAUPT, Verh. phys. Ges. 1903 S. 219 (with RICHARZ, Sitz.-Ber., Marburg, 13, S. 237, 1904 E. HAUPT, In.-Diss., Marburg, 1904. HEUSLER, Wallach Festschrift, Gött., 1909, S. 467).
HEUSLER u. RICHARZ, Zs. f. anorg. Chemie, 65, S. 110, 1910.
HEUSLER u. TAKE, Phys. Zs., 1912, S. 897.
E. TAKE, Ber. Marb. Ges., 1905, S. 35. Verh. phys. Ges., 1905, S. 133
BRUCE V. HILL, Phys. Review, 21, p. 335, 1905 ; 23, p. 498 1906.
E. TAKE, Ber. Marb. Ges. 1906, S. 299 u. 304. Ann. d. Physik, 20 S. 849, 1906. Verh. phys. Ges., 1910, S. 1059. Habil.-Schrift Marb., 1911.
A. SEMM, In.-Diss., Marb., 1915.

W. THOMSON, Ges. Abh. ü. El. u. Magn. pp. 478, 484, 493, 501.
L. BOLTZMANN, Wien. Ber. (2), 80, 1879 , 83, S. 575, 1881.
M. FARADAY, Exp. Res. 21, § 2418. Trans. R Soc., 1846, p. 41.
BANCALARI, Pogg Ann 73, S 286, 1848.
W. WEBER, Eletr. Maasbestimmungen, insb. u Diamagnetismus, Abh. Sächs. Ges. 1, S. 485, 1852. Pogg. Ann. 87, S. 145, 1852. Werke, Bd 3.
C. MARANGONI, Lum. el. 48, S. 148, 1893.
J. PLÜCKER, Pogg. Ann. 73, S. 559, 1848.
BRUGMANS, Magnetismus. Ludg., 1778.
A. C. BECQUEREL, Pogg. Ann. 10, S. 507, 1827.
E SECKELSON, Wied. Ann. 67, S. 37, 1899.
WEISS u. KAMMERLINGH, Compt rend. 150, S. 687, 1910.
K HONDA, Ann. d Phys. 32, S 1027, 1910.
M. OWEN, Ann. d. Phys 37, S. 657, 1912
G. WIEDEMANN, Pogg. Ann. 126, S. 1, 1865 ; 135, S. 177, 1868. Wied Ann 5, S. 45, 1878 , 32, S 452, 1887.
H. DU BOIS, Wied Ann. 35, S 154, 1888
J. KÖNIGSBERGER, Wied Ann 66, S 707, 1898. Ann d Phys 1, S. 175, 1900.
P WEISS, Arch. de Gen (4), 31, S 401, 1911. Verh. phys. Ges., 1911, S. 718.

S D POISSON, Mém Ac. Paris, 5, pp 247 & 488 ; 6, p. 441, 1823.
J. PLÜCKER, Pogg Ann 72, S 315, 1847 , 76, S 576, 1848 , 77, S. 447, 1849 , 78, S 427, 1849 , 81, S. 115, 1850 , 82, S. 42, 1851.
M FARADAY, Exp Res. 22, 26, 30, 1849-1856.
W. THOMSON, Rep. Br Ass 1850 (2), p 23. Phil. Mag. (4), 1, p. 177, 1851.
W. VOIGT, Lehrbuch d Kristallphysik, Leipzig, 1910, S. 468–534.
F. AUERBACH, Handb. d. Elektr., Bd. 4, S 824–845.
P. WEISS, Thèse, Paris, 1896. J. de Phys. 5, p. 435, 1896 ; 4, p. 469 u. 829, 1905. Arch. de Gen. 19, p. 537 ; 20, p. 213, 1905. C. R. 140, p. 1587 ; 141, p. 245, 1905. Ausserdem V. QUITTNER, In.-Diss., Zurich, 1909 ; KUNZ, N. Jahrb. f. Min., 1907, p. 62.

V. MECH. AND THERM. RELATIONS

C. MATTEUCCI, Ann. chim. Phys. 5, p 416, 1858.
A. WERTHEIM, ib. 50, S 385, 1857.
E. VILLARI, Pogg. Ann. 126, S. 87, 1868.
W. THOMSON, Proc. R. Soc. 23, pp. 445 and 473, 1875 ; 27, p. 439, 1878. Trans. R. Soc. 166 (2), p. 693, 1877 ; Repr., p. 332.
H. TOMLINSON, Phil. Mag. (5) 29, p. 394, 1890.
J. A. EWING, Trans. R. Soc. 1885 (2), p. 523 ; Magn. Ind. p. 182.

A. HEYDWEILLER, Wied. Ann. 52, S. 462, 1894.
G. WIEDEMANN, Pogg. Ann. 103, S. 563, 1858; 106, S. 161, 1859; 117, s. 195, 1862. Wied. Ann. 27, S. 376, 1886; 37, S. 610, 1889.
H. GERDIEN, Ann. d. Phys. 14, S. 51, 1904.
G. QUINCKE, Pogg. Ann. 160, S. 586, 1877. Wied. Ann. 24, S. 375, 1885.
LIEBKNECHT u. WILLS, Ann. d Phys. 1, S. 182, 1900.
CH. MAURAIN, J. de Phys. 6, p. 5, 1907.

J. P. JOULE, Phil. Mag. 23, p. 263, 347, 435, 1843.
WARBURG u. HÖNIG, Wied. Ann. 13, S. 141, 1881; 20, S. 814, 1883.
TANAKADATÉ, Phil. Mag. (5), 28, p. 207, 1889.
J. HOPKINSON, Proc. R. Soc. 45, p. 318. Trans. R. Soc., 1889, A, p. 443.
W. KUNZ, In.-Diss., Tubingen, 1893.
R. GANS, Ann. d. Phys. 42, 1065, 1913; 48, S. 514, 1915.
P. CURIE, C.R. 118, pp. 796, 859, and 1134, 1894 J. de Phys. (3), 4, S. 197, p. 263. C.R. 115, pp 803, 1292; 116, p. 136, 1892; 118, p. 1134, 1894. J. de Phys. 4, S. 197, 1895.
D. K. MORRIS, Phil. Mag. 44, p. 213, 1897.
R. L. WILLS, Phil. Mag. 50, p. 1, 1900.
K. HONDA, Ann. d. Phys. 32, S. 1027, 1910.
M. OWEN, Ann. d. Phys. 37, S. 657, 1912.

VI. MAGNETO-OPTICS

M. FARADAY, Exp. Res. 19, Nr. 2146 ff. Phil. Trans, 1846, p. 1.
KUNDT u. RONTGEN, Wied. Ann. 6, S. 332, 1879, 8, S. 278, 1879; 10, S. 257, 1880.
A. KUNDT, Wied. Ann. 23, S. 228, 1884; 27, S. 191, 1885
E. VERDET, Ann. chim. phys. (3) 41, p. 370, 1854, 43, p. 37, 1854; 44, p. 1209, 1857, 52, p. 129, 1858.
L. SOHNCKE, Wied. Ann. 27, S 203, 1886.
H. DU BOIS, Wied. Ann. 31, S. 941, 1887; 35, S. 160, 1888.
W. LOBACH, Wied. Ann. 39, S. 346, 1890; In.-Diss., Berl., 1890.
A. SCHMAUSS, In.-Diss., München, 1900.; Ann. d. Phys. 2, S. 280, 1900; 8, S. 842, 1902; 10, S. 853, 1903.
FR. BATES, Ann. d. Phys. 12, S. 1080, 1903.
G. J. ELIAS, In.-Diss., Utrecht, 1909; Ann. d. Phys. 35, S. 298, 1911.
R. WACHSMUTH, Wied. Ann. 44, S. 377, 1891.
L. H. SIERTSEMA, Sitt. Ak. Amsterdam, 1895–6, S. 29 u. 317.
J. KERR, Phil. Mag. (5) 3, p. 321, 1877; 5, p. 161, 1878.
H. DU BOIS, Wied. Ann. 39, S. 25, 1890.

A. RIGHI, Ann. chim. phys. (6), 4, p. 433, 1885; 9, p. 65, 1886; 10, p. 200, 1887.
F. J. MICHELI, Ann. d. Phys. 1, S. 542, 1900.
P. C. KAZ, In.-Diss., Amsterdam, 1884.
R. SISSINGH, Wied. Ann. 42, S. 115, 1891; Arch. néerl. 27, S. 173, 1893.
P. ZEEMAN, In. Diss., Leiden, 1893.

P. ZEEMAN, Versl. Ak. Amsterdam, 5, p. 181, 1896. Phil. Mag. (5), 43, p. 226, 316, 1897.
H. A. LORENTZ, Th. électr. de Maxwell, etc. Leiden, 1892.
P. ZEEMAN, Phys. Zeitschr. 11, S. 1, 1910.
A. RIGHI, Phys. Zeitschr. 1 S. 239, 1900.
J. J. BALMER, Wied. Ann. 25, S. 80, 1885.
TH. PRESTON, Trans. Soc. Dubl. (2), p. 7, 1899.
RUNGE u. PASCHEN, Ab. Akad. Berl. 1902, S. 380. Berl. Sitz.-Ber. 32, S. 720, 1902; 34, S. 417, 1904.
W. LOHMANN, Phys. Zeitschr. 9, S. 145, 1908.
A. DUFOUR, C.R. 146, p. 118, 229, 634, 810, 1908; 149, p. 917, 1909.
CH. FABRY, C.R. 138, p. 1581, 1904, 140, p. 578, 1905. J. de phys. (4) 4, p. 245.
W. VOIGT, Handb. d. Elektr., Bd. 4, S. 393–710.
W. VOIGT, Drude Ann. 68, S. 352, 1899. Ann. d. Phys. 24, 193, 1907. Wied. Ann. 67, S. 345, 1899. Ann. d. Phys. 8, S. 877, 1902; 35, S. 101, 1911. Phys. Zeitschr. 13, S. 217, 1912. Ann. d Phys. 45, S. 457, 1914; 42, S. 815, 1913 (with HANSEN).
W. VOIGT, Magneto- u. Elektro-Optik. Leipzig, 1908.
A. COTTON, C R. 127, p. 953, 1256, 1898; 128, p. 294, 1899.
ZEEMAN u. GEEST, Versl Amst. Ak 13, S. 516, 1904. GEEST, Diss., Amst, 1904.
MACALUSO u. CORBINO, C.R. 127, p. 548, 1898; N. Cim. (4), 8, 257 u. 381, 1899.

VII. ELECTROMAGNETISM

H. C. OERSTED, Experimenta ets. Hafniæ, 1820. Gilbert Ann. 66, S. 295.
ZANTEDESCHI, L'elettromagn. rivendicato a Romagnosi. Trento, 1859.
AMPÈRE, Ann. chim. phys. 15, p. 198, 1820.
BIOT u. SAVART, Ann. chim. phys. 15, p. 222, 1820.
B. KOLBE, Zs. f. phys. Unterricht, 23, S. 201, 1910.
G. KIRCHHOFF, Pogg. Ann. 67, 1847. Ges. Abhandlungen, S. 17.
E. LOMMEL, Wied. Ann. 48, S. 462; 49, S. 539; 50, S. 316, 1893.
H. WEBER, Beiträge z. Thd. Potentials, etc. Braunschw., 1901.

G. MINCHIN, Phil. Mag (5), 35, p. 354; 36, p. 201, 1893; 37, p. 204, 1894.
A. GAUGAIN, C.R. 36, p 191, 1853; Pogg. Ann. 88, S. 442.
H. v. HELMHOLTZ (not published), according to WIEDEMANN, Elektr. 3, S. 250
H. HADENKAMP, Pogg. Ann 78, S. 59, 1849
J. STUART, Phil Mag. (4), 45, p. 219, 1873.
J. STEFAN, Wien. Ber. 69 (2), S 165, 1874.
H. v. HELMHOLTZ, Ges. Abh. 1, S. 779 u. 791; Ber. Berl. Akad., 1876, S. 211.
H. A ROWLAND, Amer. J of Science, 1878, p. 30.
H. ROWLAND and C. HUTCHINSON, Phil Mag (5), 27, p. 445, 1889.
W. C. RÖNTGEN, Ber. Berl. Akad, 1885, S. 198. Wied. Ann. 35, S. 264, 1888, 40, S 93, 1890.
A. EICHENWALD, Ann. d Phys. 11, S. 18, 1903.
H. DAVY, Trans. R. Soc. 1821 (2), p. 427
BARLOW, Essay, etc London, 1823.
W. v. NIKOLAJEW, C.R 129, p. 475, 1899.
J. WEBER, Zs. f. Elektrotechnik (Wien), S. 445, 1889.
NEEF, Pogg. Ann. 46, S. 104, 1839.
M. WIEN, Wied. Ann. 42, S. 593, 44, S. 681, 1891.
H. v. HELMHOLTZ, vgl. Tonempfindungen (4. Aufl.), S. 198 u. Beilage 8.
A. v. WALTENHOFEN, Wied. Ann 19, S 928, 1883

E. H. HALL, Amer J of Math. 2, p 287, 1979. Sill. J (3), 20, pp. 52 and 161, 1880; 29, p. 117, 1885, 36, pp 131 and 277, 1888.
E. VAN AUBEL, J. de phys. (3), 1, p 424, 1892
B. BECKMANN, Comm. Leiden, Nr. 132, c 1912
E. VAN EVERDINGEN, Comm. Leiden, Nr. 26 bis 61, In.-Diss., Leiden, 1897.
A. RIGHI, J. de phys. (2), 8, p. 131, 1884.
A v. ETTINGSHAUSEN, Wied Ann. 31, S. 737, 1887
W. NERNST u. ETTINGSHAUSEN, Wied. Ann. 29, S. 343, 1886; 31, 760, 1887.
A. LEDUC, C.R. 104, p. 1783, 1887.

VIII. SOME SPECIAL BRANCHES OF THE SUBJECT

H. DU BOIS, Magnetische Kreise, Berlin, 1894, S. 177 ff.
R. GANS u. R. H. WEBER, Ann. d. Phys. 16, S. 172 u. 178, 1905.
H. LEHMANN, Wied. Ann. 48, S. 406, 1893.
NIETHAMMER, Magnetismus, Stuttg., 1901, S. 20.
J. STEFAN, Wien. Ber. 85 (2), S. 613. Wied. Ann 17, S. 928, 1882.
H. DU BOIS, Wied. Ann. 65, S. 403, 1898.
C. P. WILLS, Phys. Review, 9, p. 193, 1899.

J. RUSSELL, Trans. Edinb. Soc. 40, p. 631, 1903.
W. ESMARCH, Ann. d. Phys. 39, S. 1540 u. 1553, 1912.
J. STEFAN, Wien. Ber. (II A) 97, S. 176, 1888. Wied. Ann. 38, S. 440, 1889.
EWING and LOW, Proc R. Soc 45, p. 40, 1888. Trans. R. Soc. 180, p. 227, 1889.
M. SIEGBAHN, In.-Diss., Lund 1911.
A. BESTELMEYER, Phys. Zs., 1911, S. 1107.
S. P. THOMPSON, Der Elektromagnet. Halle, 1894.
K. EULER, Unt. e. Zugmagneten, etc. In.-Diss., Berl., 1911. El. Zs., 1911, S 269.
LYLE and BALDWIN, Phil. Mag. (6), 12, p. 433, 1906.

IX METHODS OF MEASUREMENT

C. F. GAUSS, Intens. vis. magn., etc., Gott. Abh. 8, 1832. Pogg. Ann 27, S. 241 u. 581. Res. a. d. Beob. d. magn. Vereins, 1, 1837. Werke, Bd. 5.
F. KOHLRAUSCH, Wied. Ann. 31, S. 609, 1887.
W. WEBER, Abh. Gott. Ges. 5 (2), S. 3. Pogg. Ann. 90, S. 209, 1853. Werke, Bd. 2.
L. WEBER, Berl. Ber. 1885, S. 1105.
M. ESCHENHAGEN, Verh. d. phys. Ges., 1899, S. 147. Sitz. Ber. Ak., 1896, S. 965.
A. GRAY, Abs Measures in El. and Magn. London, 1893.
F. STENGER, Wied. Ann. 33, S. 312, 1888.
EDSER and STANSFIELD, Phil. Mag. (5), 34, p. 186, 1892.
KN. ANGSTRÖM, Rep d Phys. 25, S. 383, 1898.
P. WEISS u. A. COTTON, J. de phys. (4), 6, p. 433, 1907.
A. LEDUC, J. de phys. (2), 6, p. 184, 1887.
H. DU BOIS, Wied. Ann. 35, S. 142, 1888.
W. HIBBERT, Phil. Mag (5), 33, p. 307, 1892.
R. GANS, Phys. Zs. 8, S. 523, 1907; GANS u. GMELIN, Ann. d. Phys. 28, S. 925, 1909.
F. PASCHEN, Phys. Zs. 6, S. 371, 1905.
P. LENARD u. HOWARD, El. Zs. 9, S. 340, 1888; LENARD, Wied. Ann. 39, S. 619, 1890.
G. QUINCKE, Wied. Ann. 24, S. 374, 1885.
H. DU BOIS, Wied. Ann. 35, S. 137, 1888; 51, S. 549, 1894.
J. G. GRAY and A. D. ROSS, Phil. Mag. 18, p. 148, 1909.
J. A. EWING, Electrician, 37, p. 41, 1896. Elektr. Zs. 13, S. 516 and 712, 1892; 14, p. 451, 1893.
BAILEY, Electrician, 48, p. 172, 1901.
SEARLE, Proc. Phil. Soc. Cambr. 7, p. 330, 1892.
A. KÖPSEL, El. Zs. 15, S. 214, 1894. Verh. phys. Ges., 1894.
W. KAUFMANN, Verh. phys. Ges. 1899, S. 42.
J. A. EWING, Inst. el. Eng. 24, p. 398, 1895.

STOLETOW, Pogg. Ann. 146, S. 442, 1872.
H. ROWLAND, Phil. Mag. (4), 46, p. 140, 1878.
J. HOPKINSON, Phil. Trans. 176, p. 455, 1885.
J. A. EWING and W. LOW, Trans. R. Soc. 1889 (A), p. 221.
H. DU BOIS, El. Zs. 1892, S. 579. Verh. phys. Ges., 1898, S. 97. Ann. d. Phys. 2, S. 317, 1900. Zs. f. Instr. 20, S. 113 u. 129, 1900.
F. BRAUN, Wied. Ann. 60, S. 552, 1897.
KN. ANGSTRÖM, Oefv. Kgl. Vet. Akad. Förh. 1899, S. 251.
J. EPSTEIN, El. Zs. 1900, S. 303 ; 1911, S. 334 u. 363.
VAN LONKHUYZEN, El. Zs. 1911, S. 1131.
E. GUMLICH u. W. ROGOWSKI, El. Zs. 1912, S. 262.
W. ROGOWSKI u. W. STEINHAUS, Arch. f. Elektrotechnik, 1, S. 141, 1912.
A. WEHNELT, Verh. phys. Ges. 1909, S. 109.
H. HAUSRATH, Phys. Zs. 1909, S. 756.

X. MAGNETIC THEORIES

J. Cl. MAXWELL, A Treatise on Electr and Magn., 2 vol., Oxford, 1873.
R. GANS, Ann. d. Phys. 22, S. 481 ; 23, S. 399, 1907 ; 27, S. 1, 1908 ; 29, S. 301, 1909 ; 33, S 1065, 1910. Phys. Zs. 11, S. 988, 1910 ; 12, S. 1053, 1911.
P. DUHEM, Th nouv. de l'aimantation fondée s. l. Thermodynamique, Paris, 1888. Leçons s. l'él. et le magn., Paris, 1892.
L. BOLTZMANN, Wien. Anz. 1886, S. 77 u. 113 ; Wien. Ber. 94, S. 644, 1886.
D. GOLDHAMMER, Wied. Ann. 31, S. 370, 1887.
HEURLINGER, Ann. d. Phys. 48, S. 84, 1915
F. KOLÁČEK, Elektrina a magnetismus, Prag, 1904, S. 246. Ann d. Phys. 39, S. 1491, 1912.
W. WEBER, Elektrodyn. Maasbestimmungen. Abh. Sächs. Ges. 1, S. 485, 1852. Werke, 3, S. 475.
J. A. EWING, Magnetic Induction, Phil. Mag. (5), 30, p. 205, 1890.
J. M. AMPÈRE, Ann. chim. phys. 15, p. 70 u. 170, 1820 ; Mém. s. l. th. des phén. electrodynamiques, Paris, 1826, p. 323.
A. EINSTEIN u. J. W. de HAAS, Verh. d. phys. Ges. 1915, S. 152 u. 302.
F. RICHARZ, Wied. Ann. 52, S 410, 1894.
J. J. THOMSON, Phil. Mag. (6), 6, p. 673, 1903.
G. A. SCHOTT, Phil. Mag. (6), 15, p. 172, 1908.
R. GANS, Gött. Nachr. 1910, S. 197 ; 1911, S. 118.
P. LANGEVIN, Ann. chim. phys. (8), 5, S. 70 u. 245, 1905.
P. WEISS, J. de phys. (4), 6, p. 661, 1907. Phys. Zs. 9, S. 358, 1908 ; 12, 1911. C. R. Jan. 1911. J. de phys. (4), 10, Nov. and Dec.

E. RIECKE, Wied. Ann. 66, S. 353 u. 545, 1898.
P. DRUDE, Ann. d. Phys. 1, S. 566, 1900; 3, S. 369, 1900.
H. A. LORENTZ, Versl. Akad. Amsterdam, 13, S. 493, 1905. Jahrb. d. Elektronik, 1907, S. 125.
KOENIGSBERGER u. GOTTSTEIN, Phys. Zs. 1913, S. 232. Ann. d. Phys. 46, S. 446; 47, S. 566, 1915.
S R. WILLIAMS, Phys. Review (2), 2, p. 241, 1913.

XI. MAGNETISM OF THE EARTH

W. GILBERT, De magnete . . . et de magno magnete Tellure . . . London, 1600.
EDM HALLEY, Theory of the Variations of the Compass (Trans. R. Soc. 1683); On the Cause, etc. (Trans. R. Soc. 1692).
G. GRAHAM, Observations on the Var. of the Needle. London, 1724.
CHR. HANSTEEN, Unt. ü. d. Magn. d. Erde. Christ., 1819.
GAUSS und WEBER, Resultat a. d. Beob d. magnet. Vereins 1836 bis 1841, 6 Bde., Gott. u. Leipz. C. F. GAUSS, Allg. Th. d. Erdmagnetismus (1840); Ges. Werke, Bd. 5; Intensitas vis magn., ibidem.
J. LAMONT, Handb. d. Erdmagnetismus. Berlin, 1849.
H. LLOYD, A Treat. on Magn. gen. and terrestrial. London, 1874.
E. MASCART, Traité de magn. terrestre. Paris, 1900.
A. NIPPOLDT, Erdmagnetismus. Leipzig, 1903.
AD. SCHMIDT, Enz. d. math. Wiss. (VI (1)). Leipzig, 1917.
G. NEUMAYER, Atlas d. Erdmagnetismus. Gotha, 1891.
H. FRITSCHE, Atlas d. Erdmagnet. f. 1600, 1700, 1780, 1842, u. 1915. Riga, 1903.
G. B. AIRY, On Magnetism (Ueber d. Magn., Berlin, 1874).
F. POCKELS, N Jahrb f Min 1, S. 66, 1897. Wied. Ann. 63, S. 195, 1897; 65, S. 458, 1898.
FOLGHERAITER, zahlreiche Abhandlungen (Rom).
B. WEINSTEIN, Die Erdströme. Braunschw., 1900.
MAUNDER, Astrophys. J. 1905, p. 21.
CHREE, Trans. R. Soc. 212, p. 75, 1912.
K. BIRKELAND, Expéd. norvég. de 1899–1900, Christ., 1901; The Norw. Aurora borealis Expedition, 1902–3, Christ., 1908.
C. STÖRMER, Ber. u. e. Exp. n. Bossekopp. Christ., 1911.
ERMAN u. PETERSEN, D. Grundlagen d. Gaussischen Theorie. Berl., 1874.
AD. SCHMIDT, Arch. d. deutschen Seewarte 1889 u. 1898. Abh. bayr. Akad., 19, S. 1. 1895–96. Meteorolog. Zs., 1911, S. 49.
W. v. BEZOLD, Sitz. Ber. Berl. Akad. 1895 u. 1897.
N. UMOW, Bull. Soc. Moscou, 1902, Nr. 1.
L. A. BAUER, The Phys. Theory of Earth-magn. Terr. Magnetism, 15 to 17, 1910–12.

C. STÖRMER, C.R. 142, p. 1580 ; 143, p. 140 u. 587. Arch. de Gen. 23 u. 24, 1907, u. 32, 1911.
K. BIRKELAND, C.R. 147, p. 539, 1908 ; 150, p. 246, 1910.
P. VILLARD, C.R. 142, p. 1330, 1906 ; 143 ff. 1906 ff. J. de phys. 1908, p. 429.
P. LENARD, Heidelb. Ber. 1910 u 1911.
L. VEGARD, Ann. d. Phys. 50, S. 853 ; 51, S. 495, 1916; Jahrb. d. Elektronik, 1917, S. 383.
G. ANGENHEISTER, Handworterbuch d. Naturwiss. 6, S. 599 ; 7, S. 995.
W. SWANN, Phil. Mag. 24, p. 80, 1912.

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