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THE IDEALISTIC REACTION AGAINST SCIENCE

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THE IDEALISTIC REACTION AGAINST SCIENCE

BY

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TRANSLATED BY

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PROFESSOR A. E. TAYLOR

IN TOKEN

OF GRATITUDE AND ESTEEM

PREFACE

THIS work must be regarded as a new edition rather than as a mere translation of my book, *La Reazione idealistica contro la scienza*, published in Italy in 1912, since I have subjected the whole of it to a process of revision with a view to improving it and adapting it to the British public. In the concluding chapter I have gathered together the constructive portions formerly scattered through the whole book, so as to give greater prominence to my personal point of view, which is a form of spiritualistic realism, and to make that view clearer. The outlines of a spiritualistic conception of the world sketched therein have already formed the subject-matter of three addresses given by me in the Philosophical Library of Palermo, founded by Dr. Giuseppe Amato Pojero, a true apostle of philosophy, the study of which he strives to further by both precept and practice. The line of thought adopted by me—that of the school of Francesco de Sarlo and his review, *La Cultura filosofica*, defends the rights of the scientific method and of natural reality against the facile denials of the neo-Hegelians. Idealism, which came into vogue in Italy after the decline of positivism, now appears to be on the wane, and the abuse of the dialectic method has resulted in such a confusion of ideas in mental sciences that Croce himself recently lifted his voice in protest against these exaggerations. It is now time to return

to realism, and in England, America, and Germany there are already indications of such a return, which this work of mine would fain hasten in Italy, where, if absolute idealism has attained a large measure of success, other vigorous and original currents of thought, which have disputed the victory with it, are by no means lacking.

This productive trend of thought, which merits attention in other countries as well as in Italy, is not touched upon in the present volume, because it was not included in the general plan of my work, which does not aspire to be a complete history of contemporary philosophy, but merely a study of one aspect of it, *i.e.* of the phenomenon of irrationalism in its relations to criticism of science. Irrationalism, moreover, in spite of the efforts of a few romantic minds, has not taken root in Italy. I shall deal at length with modern Italian philosophy in a separate book, which will also be published in England should the present volume meet with the favourable reception from English readers for which I venture to hope.

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INTRODUCTION

1. *The Reaction from Intellectualism in Contemporary Philosophy.*—One of the essential characteristics of contemporary thought is undoubtedly the reaction from intellectualism in all its forms. The mind of man, which could not rest content with a simple transference of results attained by the methods of the natural sciences to the realm of philosophy, and was reluctant to stay its steps on the threshold of the dim temple of the Unknowable, sought within itself other and deeper activities which should throw open the portals of mystery. Art, moral life, and religious belief were called upon to fill the void left by scientific knowledge; and the reaction went so far as to extend to the human intellect as a whole a distrust which should have been confined to scientific naturalism and its claim to be able to comprehend the infinite riches of mind and nature within a few mechanical formulas. The ruined shrines of the Goddess of Reason, who for so long had tyrannised over the mind, were invaded by the rebel forces of feeling, will, imagination, and every obscure and primitive instinct: thus it came about that Schopenhauer achieved a posthumous triumph over his hated rival Hegel, whose hearers he had in his lifetime vainly endeavoured to entice away, even though he fixed his own lectures for the same hour. Once the blind power of impulse was exalted and the sure guidance of the intellect abandoned, the door was opened to every kind of arbitrary speculation; hence the confusion, Byzantinism, and dabbling in philosophy which during the last twenty years have

obscured thought and masqueraded under the fine-sounding name of idealism. O unhappy Idealism, how many intellectual follies have been committed in thy name! Theosophy, the speculations of the Kabala, occultism, magic, spiritualism, all the mystic ravings of the Neo-Platonists and Neo-Pythagoreans, the most antiquated of theories, débris of every kind, heaped haphazard on the foundation of the speculations of the ages—all these have returned to favour in defiance of the dictates of logic and common sense. Balance and the sense of direction have to a certain extent been lost, the light of intelligence quenched, and man gropes in the gloom of wild inspirations, direct intuitions, and mysterious miracles in the search for some new truth which shall satisfy the inmost needs of the human mind.

2. *Intellectualism and Anti-Intellectualism in the History of Philosophy.*—The reaction from pure intellectualism, which reached its zenith towards the end of the last century, is nothing new in the history of philosophy, but a phenomenon which recurs whenever thought indulges in exaggerated rationalism. In Greece the splendid affirmation of the concept against the subjectivism of the Sophists and the intellectualism which had carried all before it from Socrates to Aristotle was followed by the sceptical dissolution which ended in the ravings of the mystics of Alexandria; while the glow of Christian sentiment came to fill the void left by a cold intellectualism in minds confused by the contradictory formulas of the various systems and the quibbles of destructive dialectic. All through the Middle Ages we see this antithesis of mystic faith and love, which breaks out from time to time with fresh force in protest against the excesses of rationalism: the paradoxical "Credo quia absurdum" of Tertullian stands in opposition to the bold assertions of the gnostics; the "Amo ut intelligam" of S. Bernard and the Victorines marks the reaction of feeling from the intemperate dialectic of Abélard's "Intelligo ut credam";

S. Thomas vainly strives to reconcile these conflicting principles in a higher synthesis, defining clearly the limits of faith and reason. The antithesis of feeling lives on, although in a more moderate form, in the "lumen superius," the "excessus mentalis et mysticus" of S. Bonaventura who counsels his followers to appeal for penetration into the highest truth: to "gratiam, non doctrinam"; "desiderium, non intellectum"; "caliginem, non claritatem"; indeed, another antithesis is added in the voluntarism of Henry of Ghent and John Duns Scotus, which places "Voluntas imperans intellectui est causa superior respectu actus eius" in opposition to the "Simpliciter tamen intellectus est nobilior quam voluntas" of S. Thomas. The exaggerated subtleties of the scholastics and the interminable controversies between the followers of S. Thomas and those of Scotus lead by way of Ockham's scepticism to a re-awakening of the spirit of mysticism in Eckhart and Gerson. The epic struggle still continues in modern philosophy; the first triumphs of mathematical natural science encouraged the boldness of Cartesian rationalism, against which the tormenting doubt of the mystic Pascal struggles in vain. Intellectualism, not content with its theoretical domain, would fain in the teaching of Spinoza invade that of moral life as well, vainly deceiving itself into the belief that it can interpret the action of the passions *more geometrico*, and reaches its extreme in the claim of Wollaston to be able to express the supreme laws of duty as logical relations, a claim calling forth the just reaction of the sentimentalists from Shaftesbury to Smith. The mind of man is once more irresistibly drawn in the opposite direction by the piercing analyses of Berkeley and Hume and the critical genius of Kant, which is at one and the same time the apotheosis of the physico-mathematical method in the order of phenomena and the irrevocable condemnation thereof as an organ of speculation. We see the antithesis once more in the traditional form of feeling regarded as the direct revela-

tion of God in the mystical writings of Jacobi, and in the form of the primacy of practical reason in the work of Kant and Fichte, while in the revolt of the romanticists, the Schlegels, Tieck, Novalis, and Schelling, it takes on the new aspect of poetic intuition which ranks the concreteness of aesthetic vision higher than abstract mathematicism, the individual than the universal, the changeful life of history than the inflexible formulas of mechanical science. In Hegel reason strives to break away from the motionless formulas of the old logic and to comprehend within the triad of a higher dialectic that concrete development which eluded the schemes of mathematical intellectualism, but for all its gigantic efforts it fails to dominate the manifold complexity of experience, or to absorb into the idea the productive wealth of intuition and the vivid glow of feeling; and, while speculative Pan-logism is celebrating the funeral rites of the dead and gone divinities of the romanticists—art and religion, superseded now by thought—we behold the gods arising once more from the tombs to which Hegel and his teaching had consigned them—rising full of the ardour of youth in the mysticism of the later philosophy of Schelling, in the feeling and religious faith with which Schleiermacher and Hamilton sought to supplement our poor intellectual science of the finite and conditioned, in the belligerent will of Schopenhauer who strives to express his deep sense of rhythm in music, beyond the realm of precise concepts. The over-depreciation of scientific intellectualism and of mechanical and abstract mathematicism, which is characteristic of all idealistic speculation, and its claim to take the place of science and to substitute for it a fantastic system of natural philosophy are followed by a fresh glorification of the physical mathematical method, which in its turn, in the exaggerated reaction which set in, laid claim to the place of philosophy, thus invading the realm of the mind. Thus, passing over the criticism of Kant, we return to the

naturalism of the eighteenth century with its crass ignorance of the epistemological problem. Scientific intellectualism, however, after vainly striving to express the highest manifestations of life and consciousness by the aid of its formulas, is forced to stop short at the limits already defined by the genius of Kant. The "Ignorabimus" of Du Bois-Reymond, the Unknowable of the philosopher of *First Principles*, are the most explicit confession of the inability of that method to solve the problems of most vital interest to the mind of man.

3. *Causes of the Reaction from Intellectualism.*— Could thought rest easy in this complacent agnosticism? Could it silence the ever-questioning voice within? There were two ways of escaping this intolerable situation: either to turn to the other functions of the mind for the solution of the problem which had baffled the intellect, or to eliminate the problem altogether, by proving it to be due to faulty perspective and to a false conception of science and of the value of scientific theories. Both ways have been tried; on the one hand, by a return to the moralism of Fichte and the aestheticism of the romanticists, into which the rebellious genius of Nietzsche had breathed new life, the will, as the creative source of all values and of unfettered aesthetic intuition, is exalted above the intelligence; while, on the other, the bases of the mechanical conception and of its chief instruments — geometrical intuition and mathematical calculation—are subjected to a searching examination. This analysis, to which men of science themselves were impelled by the discovery of the new principles of energy, and by meta-geometrical conceptions, resulted in stress being laid upon the active work of the mind in the construction of scientific laws and theories, and has therefore contributed to the triumph of that line of philosophic thought which holds that the fullest revelation of reality is to be found in the aesthetic point of view, and in the practical functions of consciousness. In this way speculative

criticism, determined by imperious demands of the mind which positivism failed to satisfy, came into contact with the new criticism which the new theories called into being in the realm of science itself, thus shaking dogmatic belief in the old geometry and in traditional mechanical science. This valid co-operation of physicists and mathematicians distinguishes the struggle against intellectualism of the closing years of the nineteenth century from the analogous movement of the beginning of the present century; it is also more intense and more extensive, especially as regards its critical aspect. The scientific method which Kant and the idealists had declared to be inadequate in the domain of the absolute had successfully resisted all attacks, entrenching itself within the citadel of the phenomenon which Kant himself had fortified so strongly with his vigorous criticism, but towards the end of the century the reaction spread to this sphere also, and science was not only divested of its speculative office, but its theoretical value was denied as well.

The prevalence of Darwinism and of the theory of evolution in general contributed not a little to this radical change in the concept of science. From this standpoint consciousness too appeared to be a complex of functions, whose meaning could not differ from that of the other organic functions: it was but an additional weapon in the struggle for existence, a means of adaptation. The theoretical function could not be regarded as an exception to this utilitarian value—a value, that is to say, of an essentially practical order—of psychic life as a whole, and the forms of thought, like the other types of the biological world, could not therefore be considered as being immutable and eternal, but rather as being subject to a continuous process of formation by means of successive adaptations to new conditions of life.[1] Science is no longer the standard by which every form of knowledge is gauged, as was the case in the days of the old positivism; it is no longer the eternal mould into which human consciousness must be forced

if it would attain to certainty; it too is an organism capable of that development, renewal, and change of structure which enable it better to fulfil its biological function. That very theory of evolution which had at first sight appeared to prove the mechanical method afresh, and to give it a new weapon wherewith to subdue the rebel world of life, helped rather to depreciate its value and to shake its foundations. Regarded in the light of evolution, was the world what the mechanical theory had held it to be, an eternal persistence of unchangeable substances, an eternal repetition of necessary movements subject to unchangeable laws; or was it rather a perennial becoming, an incessant renewal of forms which cannot be foreseen, and which cannot therefore be subject to the rigid necessity of determinism? Is not variability, that is to say, the possibility of the new, presupposed in all evolution? Can the new be confined within the limits of any mathematical formula? How can mechanics, the science of eternal types, mirror the transient life of the real? It is not to the motionless ideas of reason that we must turn if we would sound the depths of being and grasp it in the productive moment of its generation, but rather to the free creations of imagination and energy. Not *sub specie aeternitatis*, but *sub specie generationis* is the motto of modern logic.[2]

The researches of psycho-physiology and more especially the analyses of perception, which proved the subjective character of those sensory elements which the mechanical theory had raised to the rank of ultimate reality, contributed largely to the change in the conception of science and of knowledge in general. Are not resistance, space, and time presentations no less dependent on the special physiological structure than sounds, colours, tastes, and smells? What right have we to regard the one class as objective and primary, the other as subjective and secondary? Helmholtz considers that the only distinction which can fairly be drawn between these elements is of a practical kind,

in as much as some of them are of more assistance to us than others as guides to reality, awakening in us, as they do, expectations which are habitually verified.

In the first glow of enthusiasm to which these researches gave birth, the possibility of discovering the psychological origin of these presentations and of resolving them into their elements seemed to be a clear proof of the empirical nature of geometrical truths and therefore, also, of mechanics; thus from this point of view also doubt was cast upon the apodeictic value of science *more geometrico demonstrata.*

The reaction from intellectualism, which is, in my opinion, the predominant characteristic of contemporary philosophy, will act as our guide in the study of the prevailing tone of present-day thought touching the theory of knowledge. By the general term "intellectualism," taken in the widest sense of the word, we shall understand those epistemological systems which assign an autonomous value to the cognitive function,[3] and we shall therefore regard as forms of reaction all those currents of thought which make the value of science and of knowledge in general depend upon the ends of other functions of the mind and rank will and imagination above intellect.

Notes to Introduction

[1] Sergi, *L' Origine dei fenomeni psichici e la loro significazione biologica* (Milan, 1885), p. 72.

[2] Dewey, "Does Reality possess Practical Character?" *Essays Philosophical and Psychological in Honour of W. James* (London, 1908).

[3] Intellectualism in the strict sense of the word is the reduction of all the functions of the mind to intellectual processes. The pragmatists and intuitionists, however, in their polemic against the intellectualists, apply the term also to those who, though not going so far, yet look upon the intelligence as a theoretic function of intrinsic value, and do not consider it as identical with or subordinate to practical activity. It would be waste of time to enter upon a discussion of the justifiability of using the word in this sense; the essential thing is that we should clearly understand what concepts we attach to the word.

PART I

THE REACTION FROM INTELLECTUALISM IN THE NEW THEORIES OF KNOWLEDGE

SECTION I

THE BEGINNING OF THE REACTION FROM INTELLECTUALISM

CHAPTER I

AGNOSTIC POSITIVISM

1. *Agnosticism as the Consequence of the Traditional Mathematical Method.*—Agnosticism was the logical outcome of a prejudice which had become more and more deeply rooted in thought from the time of the Renaissance on: a prejudice which affirms that there is no other form of knowledge save that of which we have the perfect model in mathematical physics. The rich results yielded by the quantitative method of studying natural phenomena which modern science had opposed to the fruitless multiplication of hypothetical qualities led to over-estimation of this type of knowledge: everything which could not be comprised in this scheme, everything which from its very nature could not be comprehended within the narrow limits of a precise formula, was for ever banned from the domain of knowledge. Even Kant could not wholly shake off this prejudice; for although the intuition of genius taught him to discern beyond the realm of mathematics and physics that of aesthetics and moral values, he yet considered them as being beyond the pale of true knowledge, and as belonging to the domain of feeling, contemplation, and faith. Positivism with its apotheosis of the scientific method, with its claim to give a comprehensive explanation not merely of natural reality, but also of ethics and aesthetics, by constructing the whole sphere of philosophy on scientific principles, carried this prejudice

to its extreme consequences, declaring those problems for which, from its one-sided, restricted point of view, it could find no adequate solution to be insoluble, and was thus led by faulty perspective to attribute to the nature of human knowledge that inadequacy which was due rather to its own method and system.

2. *The Ignorabimus of Du Bois-Reymond.*—Du Bois-Reymond [1] lays down the dogma that the one and only true exact science is mechanics; all points of view based on teleological, aesthetic, and qualitative principles are but anthropomorphic conceptions, from which we must free ourselves that we may consider nothing in the world but the quantitative aspects of the movement of material masses. What, then, is the essence and source of matter, force, motion, and of their distribution? A mystery which baffles human knowledge! How does the qualitative complexity of sensation and consciousness issue from this world of purely homogeneous magnitudes? Yet another mystery! How about the source of life, the finality of organisms, the highest functions of the mind and free will? These too are inscrutable enigmas, otherwise we might well ask ourselves: Are these bounds really the Pillars of Hercules of human knowledge? Do they not rather mark the limits of your partial and fragmentary conception? Du Bois-Reymond, taking as his starting-point the old prejudice that knowledge is but the power of formulating mechanically, unhesitatingly chooses the first alternative, and cries, "Ignorabimus!" But the mind of man with its higher ideals refused to submit to this "Ignorabimus," and, since science had declared herself unable to satisfy its loftiest moral aspirations and attributed her failure to the congenital defects of our reason, what more natural than that it should seek to meet the requirements of life in some other way? Scientific intellectualism with its sceptical conclusions prepared the soil for the various forms of reaction; indeed it went farther, and sowed the seed, leaving, as did Spencer, the revelation of the Absolute

to religious belief, and to a vague indefinite consciousness incapable of being expressed in precise concepts.

3. *Criticism of Spencer's Agnosticism.*—The philosopher of *First Principles* goes even farther than Du Bois-Reymond, striving as he does to prove that the ultimate essence of things eludes not only scientific knowledge, but also speculative reason, and that because human knowledge can of necessity be but relative. Agnostic positivism, using as its weapons the transcendentalism of Kant, which Hamilton [2] and Mansel [3] had pressed into the service of faith, is forced back on its negative side, on the ancient forms of traditional mysticism, which, though latent, had never really perished, and was ever ready to rise again to do battle with the theological rationalism of the extreme school. In the theory of the Unknowable we see the reappearance of the mystical tendency, finding expression not in the moderate formula "Credo ut intelligam," but rather in the blind aberration involved in "Credo quia absurdum," since the absurd unknowable is in its ultimate analysis but the confession of the powerlessness of that rationalism which is supposed to reconcile the conflicting claims of science and theology. But, we may ask, must thought inevitably lead to such an absurd conclusion? If we examine the Unknowable closely, we shall find that it is simply something which we think or at least vaguely feel to be actual, but which we affirm that we cannot know. We must here make sure that we clearly understand in exactly what sense we use the word "know," since it is just the arbitrary limitation of its meaning which has given rise to certain alleged antinomies.

Spencer admits no other knowledge than that which subjects fact to law, classifying it, resolving it into its abstract relations, determining in what respects it resembles other facts or differs from them; but side by side with this form of mediate knowledge which seeks the intelligible element in the phenomenon brought to its notice, there exists that immediate knowledge which consists in the direct life of conscious reality

as manifested in its individual physiognomy. Any form of consciousness, however embryonic and rudimentary, is already a knowing of the content which is manifested in it. The pain which I feel at a given moment is an actual fact, known by me to be such, though I may not be able to subject it to law, classify it in a system of concepts, or explain it scientifically; real too is the world of colour, sound, and form in its unending variety. The error of abstract rationalism, in its scientific and speculative forms alike, lies in its claim to be able to reduce reality in its entirety to a system of relations, since there exists an individual aspect of things which cannot be expressed in its concreteness by means of abstract relations. It is for this reason that we find ourselves confronted by insoluble antinomies when we attempt to realise this pure system of relations, that we vainly endeavour to find a fixed point in the process of reasoning which leads us from one relation to another, a goal which cannot be in its turn a relation unless we are prepared to continue the process indefinitely. Thought, whose function is the establishment of relations, cannot reach this absolute goal, but our consciousness is not forced to seek it beyond the indefinite series of relations, since it is found within itself as an original possession in immediately experienced facts. Knowledge founded on pure logic is thus doomed to grope in the empty darkness of its own contradictions, unless it will take refuge in the luminous atmosphere of concrete consciousness. If by "knowing" we understand simply the reduction of phenomena to law and their dissolution into abstract elements, then the unknowable will be found, not beyond the bounds of experience, but in the facts themselves in as much as they possess a concrete physiognomy which cannot be translated into abstract relations, and even our own individuality, as presented to us by experience, will be unknown to us! If, on the other hand, we understand by the term "knowledge" not merely logical reflection, but also the immediate

life of the real, nothing is unknowable, since everything which we regard as real becomes a content of our consciousness the very moment we recognise its reality. Try as it may, thought cannot call its own objective value in question, and, while endeavouring to prove its own relativity, posits as the absolute term of reference something made of like substance with itself! This is proved by Spencer's Unknowable, which in the doctrine of transfigured realism is conceived of as the *cause* of phenomena, as being at once *single and the manifold*, in its variations which correspond to empirical changes; as a substance possessed of *persistent modes* connected by an *indissoluble relation* with their conditioned effects—space, time, motion, and force. And yet it is alleged that we know nothing about it! Moreover, we are supposed to have found an absolute model of reality face to face with which thought must perforce own its impotence, as if this model were not just as much a thought! Logical activity will brook no limits, since in the very act of defining these limits it comprehends and transcends them in its universal concepts. A reality absolutely eluding thought is an epistemological absurdity; how can we affirm that it exists without thinking of it in some way?

4. *First Germs of the Reaction from Intellectualism in Spencer.*—If Kant, Hamilton, and Mansel pronounce the Absolute to be unknowable, it is because they wrongly restrict the circle of knowledge to abstract intelligibility; yet at bottom they too grant the possibility of a revelation of this reality in the mind of man. Hamilton writes:

> By virtue of a wonderful revelation we are thus, in the consciousness of our inability to conceive anything but the relative and the finite, inspired to believe in the existence of something unconditioned beyond the sphere of comprehensible reality.

And Spencer explicitly recognises that the so-called Unknowable does not absolutely elude consciousness, but is rather presented thereto in a form differing from precise and determined thought:

Besides that definite consciousness of which logic formulates the laws, there is also an indefinite consciousness which cannot be formulated. Besides complete thoughts, besides the thoughts which, though incomplete, admit of completion, there are thoughts which it is impossible to complete and yet which are still real in the sense that they are normal affections of the intellect. . . . The error fallen into by philosophers intent on demonstrating the limits and conditions of consciousness consists in assuming that consciousness contains nothing but limits, conditions, to the entire neglect of that which is limited and conditioned. It is forgotten that there is something which alike forms the raw material of definite thought and remains after the definiteness which thinking gave to it has been destroyed.[4]

Does not this sound like the voice of Bergson ?

. . . Autour de la pensée conceptuelle subsiste une frange indistincte qui en rappelle l'origine.[5]

The indefinite consciousness of which Spencer speaks becomes the fundamental organ of philosophy in Bergson's intuitive system. If it be this indefinite consciousness surrounding logical thought which presents to us the absolute, the culminating point of every reality, according to the opponents of intellectualism, has it not a cognitive value far beyond the limited, phenomenal consciousness of the intellect ? But Spencer is still too much under the influence of the old mathematical prejudice to draw these bold conclusions from his own premisses ; he therefore persists in designating as unknowable that aspect of reality which cannot be classified and ordered by the scientific method ; he makes a tremendous effort to apply a single mathematical formula to the perennial evolution of mind and nature, to subject the concrete reality of becoming to a law of persistency, to a system of intelligible relations which is outside the limits of time. It is an endeavour which is doomed to failure, and will cause the final crash of the structure of scientific intellectualism, a structure whose foundations are already undermined by its own confession of impotence, by proving its inadequacy in the realm of phenomena as well.

5. *The Evolutionary Method also leads to Reaction.*— The law of preservation from which Spencer is deceived into deducing the necessity of the evolutionary process only applies to the quantitative relations of the forces at work in the system; hence it can give us no information as to the direction the changes will take. The qualitative transformation of forces, on the other hand, is subject to the law of degradation,[6] according to which the imperceptible differences, and more especially the inequalities existing in the redistribution of energy in respect to masses, constantly tend to diminish, so that the natural course taken by physical phenomena makes for the greater homogeneity of the system, though this is diametrically opposed to Spencer's assertion. As far as the principle of conservation is concerned, it is a matter of indifference whether we pass from the homogeneous form of heat to differing forms of energy, or whether the process be reversed, since in either case it remains unchanged in its totality.

As Lalande[7] has well said, this permanency would be equally true even if the progress of the world were suddenly to be reversed, supposing, that is to say, trees were to grow smaller instead of taller, till they returned to the germs from which they had developed, and mankind were to grow towards youth instead of age, reaching the embryonic stage at the end of life instead of at the beginning. Nor is this hypothesis purely fantastical! There are many biological instances of retrogression or involution of organs,[8] yet the law of the persistence of force is in no way affected thereby. For that matter, does not Spencer himself deduce from the law of conservation the necessary dissolution of the system when its cycle has been accomplished? How marvellous is this law, from which we may deduce on the one hand, when it suits us so to do, the necessity of passing to the heterogeneous, and on the other with equal facility the no less necessary return to primitive homogeneity!

The evolutionary process cannot be deduced from a

system of mathematical laws. To the physicist, who would seek in the development of natural phenomena the permanent and universal relations of co-existence and succession, the world is ever the same in its totality and unchangeable in its inexorable mechanical laws. From this point of view the individual aspects of things must be considered as illusions of the senses; we are under the impression that we see an inexhaustible multiplicity of forms where objectively there merely exists a continual repetition of one and the same form, the same mechanism, a uniform play of forces whose action can be calculated and foreseen to a nicety by mathematical means. Since, then, the evolutionary process disappears when we exclude the possibility of the genesis of new forms, of the production of new characteristics, are we not perhaps justified in concluding that the mechanical theory of the universe, interpreted strictly, must also regard the evolutionary transformation of species as an illusory appearance? Mechanism and evolution are two concepts which cannot be derived from one another, since they correspond to two different aspects of nature: one is quantitative permanence and absolute determinism of mathematical law; the other qualitative transformation and fruitful genesis of individual forms, which no set of abstract formulas comprehends in the fulness of its living reality. The evolutionary conception of things could never be made to fit the Procrustean bed of the traditional mathematical method; it was inevitable that it should (if I may so say) insinuate the poison of dissolution into the veins of intellectualism. The living spirit of history, which had animated the idealistic speculation of the beginning of the century, finding its way with Darwinism into the domain of positive research, whilst thus endeavouring to find itself a place in the schemes of science, breaks down their mechanical rigidity, and exposes the tremendous gaps left by empty formulas in the sphere of experience. The scientific method is thus proved to be inadequate not only in the field of specula-

tion, but also in that of phenomena itself. The theory of evolution, whilst thus calling attention to the new forms and new concrete aspects assumed by reality in the process of development, to the irreversible direction of development in time, and to the hierarchical order of the beings which rise little by little to higher forms of life, reveals a world beyond and above abstract mechanism and indifferent to every temporal and hierarchical order—the world of valuation and history, of which Kant caught a glimpse in his *Kritik der Urteilskraft*, and which achieves its triumph over intellectualism in the philosophy of Windelband, Rickert, Münsterberg, and Royce. It is, however, specially in another direction that Spencerian evolution prepares the ground for reaction, *i.e.* in its psychology which maintains that the explanation of all conscious life is to be found in the requirements of biological adaptation. The cognitive function thus becomes but a means for the preservation of the species, consciousness a weapon of defence against natural forces, valuable only for its utility in foreseeing facts, an instrument for the maintenance of organic equilibrium against the influence of perturbing actions in an ever-wider sphere, which is subject, like every other organ, to transformations corresponding to the altered conditions of the environment.[9] Science is not then based on an eternal, universal model, as was asserted by traditional rationalism; scientific theories are born into the world just as are organic species, and like them they perish when they can no longer resist the shock of new experiences. Science, too, has its history, and if we would know the meaning of that history we must seek it, not in the tendency of speculation to grasp the absolute truth of the rational order which is immanent in things, but rather in the needs of life and action. This biological conception of knowledge will pass through the writings of Avenarius and Mach into almost every form of reaction from intellectualism, and will act more especially as the motive power of pragmatism. Spencer's system with its theory

of the Unknowable appealing to a belief, a feeling beyond conception, with its doctrine of the evolutionary intuition of the universe, discrediting, as it does, the traditional mathematical attitude, and putting science at the service of biological adaptation, is not only pregnant with the crisis of scientific intellectualism, but enfolds the first germs of that reaction whose development we shall follow as it strives in various ways to escape from the difficult position in which agnosticism has placed it.

NOTES TO CHAPTER I

[1] *Reden*, two volumes (Leipzig, 1886–87), containing the two famous addresses: "Über die Grenzen des Naturkennens" (1873) and "Die sieben Welträtsel" (1880).

[2] *Discussions on Philosophy and Literature, Education and University Reform* (London, 1853); *Lectures on Metaphysics and Logic* (Edinburgh, 1859–60).

[3] *The Limits of Religious Thought* (London, 1858).

[4] *First Principles* (chap. iv. section 26).

[5] *L'Évolution créatrice* (Paris, 1907), p. 210.

[6] Cp. on this point Chapter III. Part II.

[7] *La Dissolution opposée à l'Evolution* (Paris, 1899), p. 47.

[8] Demoor, Massart et Vandervelde: *L'Évolution régressive en Biologie et en Sociologie* (Paris, 1897).

[9] *Principles of Psychology*, vol. i. pt. iii. chap. xi. p. 383 ff. (Third Edition.)

CHAPTER II

NEO-CRITICISM, VOLUNTARISM, AND THE PRIMACY OF PRACTICAL REASON

1. *The Return to the Critical Method.*—The first indication of the awakening of the mind from the extremely negative attitude of the materialists may be seen in the return to the teaching of Kant; the activity of the subject in the elaboration of science, which had been for long ignored, and had been thrust into the background by the triumphs so easily achieved by the mechanical method, asserts its rights once more and inaugurates the fruitful work of salutary criticism. Scientific intellectualism, having experienced for itself in the failure of its bold attempts to exhaust the totality of things the limitations already defined by Kant, finds itself in the self-same position, face to face with the self-same problems which baffled the thought of the philosophy of Königsberg. It is natural that the solution of these difficulties should be looked upon as the necessary starting-point of the new criticism of value and the limits of human science; but at the same time the need is seen of modifying it to a certain extent in order to bring it into harmony with the results of the theory of evolution and of psycho-physiological research, and it is therefore incumbent upon us to define more clearly the meaning of the *a priori* element, and to assign limits thereto.

2. *Lange.*—Albrecht Lange recognises, as does materialism, the necessity of finding a mechanical ex-

planation for physical phenomena and for the physiological processes of the brain, seeing that we are organically so constituted as to be unable to intuit and conceive the world of phenomena in any other way. There is, however, something which eludes all and every explanation, and this something is the origin of our physiological constitution; thought, no matter what efforts it may make, will always be brought up short by the limiting concept of the thing in itself, a difficulty which it utterly fails to surmount. It cannot even ascertain whether it actually corresponds to anything real or is not rather an illusion born of our special organisation. May not the dualism of phenomenon and noumenon be due to faulty perspective?

This doubt cannot be dispelled by the intellect; poetic imagination alone can guide us beyond the limits of experience. One thing only is certain: that man feels the need of supplementing reality by an ideal world of his own creation, and this creative work brings into play the loftiest and noblest functions of his intelligence.[1] Speculation must not claim to be rational and demonstrative; the more theoretical it is, the more it would compete with science in certainty, the less important will be its part in life; if, on the other hand, it is content to bring the world of actualities into relationship with the world of values, and rises in its conception of phenomena to a moral action, mastering matter by means of form without doing violence to facts, it will raise a temple built up of its ideas meet for the worship of the eternal and divine.[2] Peace will never be attained, the conflict between science and the highest human aspirations will never cease, until the transient character of all that is fictitious in art, religion, and philosophy be recognised.[3]

3. *Criticism of the Physiological Interpretation of the a priori and of the Poetic Intuition of the Absolute.*—To Lange belongs the credit of having asserted loudly the rights of mind as opposed to vulgar materialism; but by thus relegating to the domain of poetry the

solution of those problems which are of greatest interest to the human consciousness, he opens the door to every kind of fantastic and arbitrary speculation, undermining at the same time the most stable foundations of moral life. His physiological interpretation of the *a priori* of Kant deprives science of all essential and universal value, narrowing it down to a sceptical subjectivity, and merely sets the problem without solving it. Our physiological constitution is undoubtedly a part of the world of experience, and therefore the *a priori* conditions of its possibility demand investigation. Lange is not aware that he is moving in a vicious circle: the basis of mechanical explanation is to be found in our organic structure, but this structure in its turn demands a mechanical explanation. It is impossible to conceive of the physiological organism without making use of those intuitive forms and categories which are supposed to be deduced therefrom, and it therefore presumes the laws of thought and the activity of the knowing subject. With Lange begins that confusion between epistemological, psychological, and physiological problems which serves later as the basis of empirio-criticism and pragmatism. Thus, then, the doubt as to the reality of the thing in itself to which it gives expression is but the first step to phenomenalism which calls upon psychology and physiology to explain the illusion of the two opposing terms. On the other hand, if intelligence be unable to discover the sources of our organisation which are also the sources of thought, and if feeling and poetic insight succeed where intelligence failed, is not creative intuition anterior and superior to the intellect? How else, Bergson would ask, can we reach the inmost heart of things? We have seen that the philosophical system of Lange is vitiated by his incurable habit of begging the question; it is nevertheless of great historic value in as much as it strives to overcome sordid materialism, but unfortunately he fails to free himself wholly from its toils, and to place intellectual knowledge on a sure foundation.

4. *The Empirical Prejudice in Helmholtz.*—The same empirical prejudice recurs in Helmholtz,[4] who admits causality as the one and only *a priori* form, but treats it as a species of instinct or impulsive tendency, a purely subjective affair of whose necessity and universality there can in consequence be no guarantee. It is evident that this view of the *a priori* more nearly approaches the mental attitude of the English empiricists than the category of Kant as a condition essential to the intelligibility of the real; it is a law of our nature from which there is no escape, and which for that very reason must ever be to us an obscure and mysterious power for which we can in no wise account. How can a principle which is blind and incomprehensible in itself help us to interpret experience? Above all, what value has the science to which this need gives birth unless it be that of a contingent and subjective construction? The results of the researches of Helmholtz, which seemed to him to contradict Kant's[5] theories on certain points, inclined him to adopt the views of Stuart Mill, and even those expressed earlier by Hume and Berkeley. Intuition *a priori*, the universal necessity of mathematical truths, is excluded by the fact that perception of space is not inborn but acquired in the course of a slow process of experience, and more especially by the fact that it is possible to conceive of a space other than that of which Euclid treats in his geometry.[6] We will leave to the second part of this book the discussion of the value of the new geometrical speculations and the alleged proof that the empiricists have deduced therefrom in support of their teaching; for the present we will content ourselves with pointing out that Helmholtz clearly fails to distinguish between the epistemological and the psychophysiological *a priori*, with the result that his analyses of perception prepare the way for the phenomenalism of Mach and Avenarius.

5. *Liebmann and Schultze.*—The philosophers of the neo-Kantian school were not, however, all thus led astray. Liebmann draws a sharp distinction between

the norms of thought, considered as categorical prescriptions which act as our guides in the search for truth, and the natural laws of psychology, in accordance with which objects are presented to the mind, thoughts change and pass away, and the empirical content of the consciousness is transformed: the *a priori* must not be regarded as an innate idea, but rather as the basis of sensible experience and of the world of speculation.[7] Schultze,[8] another adherent of the neo-Kantian school, observes that the psycho-genetic theory does not in the least advance the critical problem, *i.e.* the analysis of the conditions essential to knowledge at the present stage of our thought. Both Liebmann [9] and Schultze,[10] however, agree with Lange in the doubt he expresses of the existence of the thing in itself, leaning, as they do, towards phenomenalistic idealism, and viewing the noumenon as a mere product of feeling, a poetical creation of the mind.

6. *Criticism of the Neo-Kantian School.*—The theories of the neo-Kantians, whilst thus striving to overcome the difficulties arising from the conception of the thing in itself, and from its relations to phenomena, do but increase those difficulties. We cannot possibly understand how that which is absolutely outside consciousness can stand in any sort of relation to feeling and imagination, the most intimate and subjective functions of the mind of man. The thing in itself, problematic as it may be, is ever present, like some mysterious deity, behind sensible appearances, and our absolute ignorance of its function forbids us to take for granted that the stream of phenomena of which it is the source will always be content to flow in the channel of forms and intellectual categories. Our mental organism may subject phenomena to law, but the Absolute is completely beyond its jurisdiction, and might at any moment reveal an aspect of itself antagonistic to our nature, and by so doing imperil the universal character of science. Are the sensations imparted to us by the unknowable object purely plastic and amorphous? If so, it is

difficult to understand why one form of classification should be more applicable than another, and we end at the same time in reducing knowledge to nothing more than a creation of the subject totally devoid of any objective meaning. Or must we admit data to be possessed of characteristics and a physiognomy of their own which act as a stimulus to the activity of thought in certain directions ? If this be granted, we cannot explain how these stimuli which have their source in the impenetrable heart of things can, by some happy accident, be moulded without any difficulty by the human intellect. This uncertain equivocal position of neo-Kantian philosophy left a painful sense of doubt in the mind which no flight of imagination, even though inspired by genius, and no feeling, however lofty and poetical, could entirely dispel.

7. *Riehl's Monism.*—Riehl, while retaining the unknowable residuum of the thing in itself, has endeavoured to invest science with objective validity by substituting a monistic conception for the subjectivism of the neo-Kantists. The harmony between the activity of thought and the processes of the real, which could not be explained by Kantism pure and simple unless the sensible objects were to be regarded as a creation of the mind, thus returning to the theories of romantic idealism, is, according to Riehl, accounted for by the identity of the unknowable source, in which the streams of thought and objective reality both take their rise, and from which they pursue their course along parallel lines. The unifying activity of the human mind, the one and only true *a priori*, is not purely formal, but has its objective correlative in the unity of nature, the ruling idea of scientific research. The intuitive forms and the categories are not *a priori*, but are constructed by the synthetic activity of thought, which is ever striving to reduce the changeful world of individual perceptions to a reality possessed of social value ; they express necessary conditions, because experience in its manifold forms acquires characteristics of universal

and objective knowledge. Kant considers intuitive forms to be *a priori*, because he confuses mathematical space and time, which are concepts, with sensible space and time; whereas they contain a material element irreducible to pure formal and mathematical relations.[11] This must not be taken to mean that either geometry or mechanics, taking, as they do, sensible data as the basis of their concepts, are of purely empirical and contingent value. The activity of thought elaborates the impressions of the senses in accordance with its own laws, and thus transforms and completes them till they correspond to its need of unity and universality; and this logical reconstruction is neither arbitrary nor violent, but answers to their nature, whose source, as we have already pointed out, is identical with that of thought.

8. *Criticism of Riehl's Philosophy.*—There can be no question that Riehl's conception more nearly succeeds in placing science on a firm basis than does neo-Kantism; but it has the serious drawback of being founded, just as is the transfigured realism of Spencer, upon a metaphysical doctrine against which human thought rebels, *i.e.* on the hypothesis of that unknowable entity which nothing but a miracle could bring within the range of thought. Riehl not only admits its reality, but credits it with being the common cause of the two series, that is to say, with standing in certain functional relations thereto, thus implicitly assuming it not to be beyond the forms of our thought. How indeed could it be so without losing the character of reality? Can we conceive of anything real which cannot be translated more or less definitely into terms of thought? Riehl rightly insists upon the necessity of admitting the existence of a cause producing our sensations, but differing from them, since between the stimulus and the fact of consciousness there is a series of processes which physiology has described to us in all its stages;[12] but the fact that we conceive this cause proves it to be already known to a certain extent. Monism based upon the

unknowable is a castle in the air: how can we affirm that to be one of which we can know nothing? If it be really unknowable, we cannot possibly apply the category of number to it. This obscure basis can only afford an explanation of the harmony of the two worlds if we assume it to be possessed of a certain rationality; otherwise it might be revealed now in one way, now in another, the latter contradicting the former, and the thought-series might consequently develop along lines directly opposed to objective processes. Riehl may apparently succeed in proving the value of scientific concepts, but this is simply because he unconsciously transforms his unknowable into an essentially logical activity, to which he transfers the synthetic unity proper to our consciousness, and which he regards as a kind of potential thought.

9. *Elimination of the Thing in Itself and Transition to Phenomenalistic Monism.*—In its final stage of development critical philosophy, in order to shake off the fetters of absurd agnosticism, has striven to eliminate the thing in itself, and has turned towards that form of monism prevalent in contemporary philosophy. The older form of monism always took as its starting-point the duality of subject and object, of internal and external experience, and of the psychic and physical world, assuming it as an undeniable fact, and then trying to reconcile the opposition of the two regarded as originary empirical data, by resorting to metaphysical speculation; it imagines, that is to say, the existence over and above these two orders of phenomena of a substance sustaining them both, an entity which might be material or spiritual as best suited the exigencies of the individual case, or which might even, should it seem undesirable to define it too sharply, be regarded as a *tertium quid*, neither matter not spirit, unknowable in its essence and hence admirably adapted to run with the hare and hunt with the hounds, or act the part of a buffer state situated between two belligerent powers. The new monism, on the contrary, maintains the unity of the two orders of phenomena

to be a primitive and original datum, the opposition and irreducibility of the two terms to be due to faulty perspective, and to be the outcome of an unconscious metaphysic; it further maintains that as soon as this illusory superstructure is removed, and a return made to the pure sources of immediate experience, which are still unsullied by philosophical reflection, we shall be able to grasp the undivided unity of the real without going beyond the limits of the phenomenal. The unity of the two worlds, the physical and the psychical, is a presentation, not a structure raised by thought; it is the starting-point, not the goal, of philosophical speculation. There are various degrees of this new form of monism—which may be designated empirical as distinguished from the older metaphysical monism—which spread rapidly during the last quarter of the nineteenth century, and is prevalent at the present day. We shall see that in its most advanced form it has the adherence of the critics of the empirical school, of the French intuitionists, and the pragmatists, all of whom agree in denying to original experience the distinction between subject and object, which they regard as a product of posterior reflection, determined by motives of a practical order. A more moderate form of empirical monism is that supported by the philosophers of immanence, who are at one with the radical empiricists in maintaining that subject and object are but two abstract aspects of the phenomenon, or of the content of concrete consciousness, but who assert on the other hand that it is impossible to conceive of conscious reality without making such a distinction, and that the two terms, subject and object, although neither can exist independently of the other, are nevertheless real, in so far as they are distinct though not separate aspects of experience.[13] In short, whilst empirical monism in its earlier form asserts duality to be illusory or at most of purely practical value, the later form views it as the fundamental characteristic of consciousness, and hence also of reality, which without the two terms would be

inconceivable: when the attempt is made to do away with this distinction, the cognitive relation disappears, and with it every form of real life. This last form of monism is undoubtedly of greater value than the older type, which merely moves in a vicious circle while claiming to deduce from a vague neutral experience the opposition of the two terms, which is already presumed by the mere fact that the philosopher thinks this empirical content, that is to say, that he places it before his own *Ego* as something extra-subjective. It is child's play for the philosopher to argue the duality of subject and object from non-differentiated experience, when, try as he may to eliminate all subjectivity, and however sure he may be that he has succeeded in so doing, his *Ego* is still acting as subject, even though he himself may be quite unaware of the fact, and may be under the impression that he is constructing it, whereas it has always been the unseen watcher of his psycho-genetical researches. The critical idealism of Wundt approaches very closely to this first form; the neo-criticism of Renouvier more nearly to the second.

10. *Wundt's Critical Idealism.*—According to Wundt,[14] our presentations are originally identical with the object: they are at the same time thinker and thought. No object exists independently of representative activity; we may admit that objects exist which are not present to us at this moment, and which are possibly not present to any consciousness, but we must attribute to these objects the property of being presentable. If thought and being, the presentation and the thing, be postulated as two distinct realities, the cognitive relation becomes inexplicable, since it is incomprehensible how one can be related to the other. The subject is not anterior to the object—both come into being as the result of a simultaneous process of abstraction from the indivisible object of the primitive presentation the moment thought begins to reflect on the different aspects of the object in question. The distinction between the two orders of facts should be preserved

when it is a question of formulating the data of our experience with the aid of concepts, but we must bear in mind that its value is merely that of a distinction between two stages found in every act of cognition. Subject and object are two logical determinations of one and the same real fact, not two different facts. The starting-point of the cognitive process is neither pure thought nor subjective presentation, but a presentation in which the character of objectivity is immanent. The fact that in common thought, and still more in the scientific elaboration of reality, certain elements break away from the objective presentation and adhere to the subjective nucleus, should not lead us to confound by means of real separation that which is merely a distinction necessary to enable science to reach its goal, and we must above all beware of regarding the concrete presentation as an illusory appearance by substituting for it an abstract conceptual fiction, which, even though it may enable us to order experience in a higher synthesis, can never take its place as true reality.[15] Why is it that thought does not stop short at the immediate presentation, the fact as experienced in the indivisible complex of its three aspects—active, affective, representative—instead of rising gradually to generic conceptions until it attains the most abstract concepts of science and the ultimate notions of metaphysic? The latent contradictions contained in primitive experience act as a stimulus to the thought of man; reason, which is, according to Wundt, the only *a priori*, strives to purge the presentations of the senses in such a way as to fulfil its own most intimate requirements and conform to its supreme laws. We may distinguish three stages in this process of refinement, stages corresponding not to three different forms of cognition, but rather to three gradations of one and the same process of refinement of experience: common or perceptive knowledge, intellectual or scientific knowledge, rational or philosophic knowledge. Thought works spontaneously and unconsciously in the construc-

tion of the practical world, which therefore appears to us to be presented from without, but which is in reality our own work, and bears the stamp of our reason on the complex of its relations. This explains why the every-day world is adapted to the successive logical elaborations of science. In the first stage of knowledge we construct things with their temporal and spatial relations. Time and space are not *a priori* forms, as Kant considered them to be, but elements forming part of the complex of concrete perception, which are only regarded separately to suit the requirements of logic and epistemology. Abstract division is, however, possible in so far as the form permits of variation independently of the matter, and *vice versa* ; hence the sensorial content may be heterogeneous, whilst the spatial properties remain constant and uniform.[16] Thought, in order that it may the more readily eliminate the contradictions inherent in the practical world, separates certain elements from the perceptions, and attributes them to the subject; but, after what has already been said, it is clear that this distinction can only be of a heuretic nature, and that it does not correspond to anything real.[17] Thus it follows that the world, though it may when reduced to purely mathematical terms, as science conceives of it, answer more nearly to the requirements of reason, in as much as it eliminates a large number of contradictions along with the qualitative content of common reality, is nevertheless not possessed of a higher degree of objectivity than is the primitive and still unpurified presentation.

The hypothetical concepts to which science has recourse in its efforts to classify the complex of experience coherently are valueless in the field of real knowledge, although useful in so far as they enable us to comprehend a large number of objects in a single act of thought, and to transfer the same logical considerations from one object to another belonging to the same class. Our cognitive activity has the individual, not the general, as its goal; hence the concepts of real cognitive

value are not general but individual, generic presentations, that is to say, of particular objects. Thus, then, the supreme ideas which satisfy the demand of reason, that the totality of experience which has never been presented in its completeness should be comprised in a coherent whole, cannot be assumed as principles of real knowledge; nevertheless we cannot agree with Kant that the effort to complete the series must confront us with insoluble antinomies or force us to posit a transcendent and unknowable reality beyond the realm of phenomena. The aim of ideas is not to comprehend the thing in itself, but merely to embrace all experience in a harmonious system which shall satisfy the exigencies of reason and sentiment alike.

11. *Wundt's Undecided Position.*—While the older speculative idealism took the subject as its starting-point, deducing the object therefrom by means of dialectic, Wundt reverses the process, starting from the objective presentation and endeavouring to reconstruct the knowing subject by essentially psychological means. In this method lies the fundamental error of his theory. From the psychological point of view it is impossible to deny that self-consciousness is not a primitive fact, but rather something which is slowly acquired by means of an evolutionary process of which the phases can be described, so that the presentation of the object may be existent from the beginning without its being aware thereof. We must not, however, confound the empirical *Ego*, the historic *Ego*, with the epistemological subject which defies every attempt at analysis or deduction, since such analysis and deduction necessarily presuppose its existence. There is a subjective side to every thought, even though we may not be aware of it, and it is this common form which brings within the bounds of possibility the convergence and continuity of individual thoughts, which would otherwise remain extraneous to one another. Neither will it avail to look to memory for an explanation of the synthesis of personality, since memory in its turn presupposes the identity

of the *Ego* from the epistemological point of view. How could we recognise our thoughts of the past if the *Ego* of yesterday had nothing in common with the *Ego* of to-day? If the *a priori* of reason be once admitted, does not that of the epistemological subject necessarily follow? We cannot conceive of reason apart from the identity of the *Ego*. *Ego* = *Ego* is the logical presupposition of every other affirmation of identity whatsoever: the coherency of human personality is the concrete basis of the coherency of thought. If, on the other hand, as Wundt teaches, every thought bear the stamp of objectivity, reason in every stage of its activity, whether conscious or unconscious, must always be ranked as an objective reality. The concepts of science and the ultimate ideas of philosophy must be constituent parts of the real no less than are presentations. Is the distinction between subject and object, then, illusory or not? Is being immanent in every thought or not? Only if we look upon reality as something distinct from thought can we fairly speak of purely formal activity, subjective in the Kantian sense of the word, or of an intellectual or rational elaboration with nothing objective about it; but this, applied to the theory enunciated by Wundt, would be meaningless. In short, either the exigencies of reason are legitimate, in which case they are so not only with regard to the construction of individual concepts, but also with regard to the ulterior processes of thought whose aim it is to eliminate the contradictions latent in earlier stages, and to substitute more coherent systems for the lower forms of reality; or else this claim of reason to do away with all contradictions in the concrete world of experience is unfounded, in which case we must strive to undo this work of falsification as far as possible, in order to return to pure experience and primitive intuition.

Once we start in this direction we must follow it to the end, *i.e.* empirio-criticism and Bergson's doctrine of intuition. Wundt, however, is reluctant to go so far, and he retains the concept of a presentation

which does not bear the imprint of thought, and stops short at the generic image, which for some reason or other is privileged to keep the characteristic of objectivity. We have hardly advanced a step higher, and resolutely banished the concrete presentation, when the ensuing concept ceases to correspond to anything real and becomes but a convenient symbol of a class of objects. Now this uncertain position is not tenable: if the scientific concept does not correspond to something objective, it can, according to Mach, be nothing more than a useful tool; in that case what becomes of the *a priori* of reason? Logical exigency too becomes but a means of saving mental energy, which would otherwise be incapable of mastering the countless successions of individual presentations. It is impossible to conceive of an *a priori* reason which is not also a constituent part of reality. If, as Wundt affirms, the scientific world eliminates the latent contradictions of the world of ideas, it cannot for that reason be looked upon as less real and less objective; unless indeed we are prepared to admit the absurd paradox that the contradictory is more real than that which is devoid of contradictions. Thought and reality are not exhausted by presentation; it is not true that we can postulate nothing as real which cannot be presented; all that can fairly be asserted is that nothing exists which cannot be thought or which, in other words, is inconceivable. Wundt's conclusion would be justifiable from the point of view of nominalism, which denies the existence of the concept apart from the individual image, but he draws a distinction between the concept and the sign which represents and defines it; that is to say, he admits its reality in consciousness; and since, according to him, the being of the object is immanent in thought, the logical conclusion would be that the concept of the universe too is of objective value. Does cognitive activity endeavour to eliminate contradictions because it aims at the individual, or rather because amid the various and transient aspects of things it would fain

grasp the oneness of universal law? There is nothing contradictory about the individual, as presented to us by immediate intuition; the contradictory element is introduced when we attempt to turn a fugitive experience, which is real only at that concrete moment, and will never occur twice in the same way, into a persistent thing, existing independently of ourselves, or, in other words, when we demand a revelation of the universal from immediate presentation. The image must then of necessity seem inadequate, and thought is conscious of the need of transcending it, and constructs scientific and philosophic reality with the help of its concepts. If the aim of knowledge were to grasp the concrete individual there would be no contradiction; the contradiction arises the moment we seek in sensible appearances the characteristics of universality which are proper to thought. Wundt's philosophy, which regards the individual as the end of knowledge, whilst implicitly denying its starting-point, the *a priori* of the reason, must perforce, when carried to its ultimate conclusion, lead to empirio-criticism.

12. *The Return to the Voluntarism of Schopenhauer.*—Wundt shows himself even more plainly as the opponent of intellectualism in psychology and metaphysics than in the theory of knowledge, regarding, as he does, will as the principle, not merely of psychic life, but also of all cosmic development.[18] This return to Schopenhauer, which achieved its first startling success as early as 1869 in Hartmann's philosophy of the Unconscious, is closely connected with the spread of the theory of evolution and of energetic conceptions. The tendency of every organism to self-preservation—the motive power of the struggle for existence—does, as a matter of fact, approximate closely to Schopenhauer's will to live; hence the derivation of intelligence from instinctive life which Darwinism asserted itself able to prove woke an answering chord in his philosophy. On the other hand, the active principle of energy, which had already taken the place of the inert atom in Spencer's *First Principles*, bore

a strong resemblance to the blind unconscious impulse which Schopenhauer regards as the essence of physical reality. Hence it was natural that voluntarism should seem the metaphysical view best suited to explain the processes of evolution and supply that which was lacking in the mechanical theory. The spontaneous and productive activity of will to which consciousness points as the creator of new concrete forms might perhaps have given us the clue to this historical development, which eluded the abstract formulas of scientific intellectualism. As a matter of fact, von Hartmann,[19] Wundt, and Fouillée [20] all call upon the will to perform this task in their systems, of which a new philosophy of evolution forms an essential part.[21]

13. *Von Hartmann's Philosophy of the Unconscious.*— Metaphysical voluntarism, which was thus the outcome of the necessity of unifying scientific intellectualism, and which was called into being by this very doctrine of evolution as its complement, loses that absolute irrationality characterising it in Schopenhauer's philosophy, when it strives to absorb the results of science and to justify its existence therewith. It has ceased to be an unseeing impulse giving birth to the cosmic process, and has become a will, in which reason and idea are present, even though it may be unconscious or but dimly conscious that this is the case. As we have already seen, Wundt regards will as being at the same time rational thought; Fouillée views the "idée-force" as the synthesis of the two principles. Von Hartmann looks upon the unconscious as being both will and idea.[22] We are still far from the exaggerations of empirio-criticism and pragmatism, against which von Hartmann [23] and Fouillée [24] protest; but although these philosophers do not go so far as to agree with Paulsen's assertion that science by reason of its poor practical results must be looked upon as being a thing of the past, and that the future lies open to faith, they adopt a critical attitude towards scientific truth which is very far removed from the

dogmatic faith of the old mechanical theory. Hartmann maintains that physical science will never lead us to certainty, but only to hypotheses of varying degrees of probability. Every scientific concept presupposes an abstraction made from a determinate point of view, a selection of characteristics according to a criterion which is always hypothetical and becomes less and less probable as the distance between concrete experience and ourselves increases, and we approach the ultimate concepts of physical science (energy, potential, entropy, mass, matter, force, etc.). The hypothetical character becomes still more obvious in induction, which goes beyond facts as directly perceived, and strives to determine the objective causes to which they are due. It is but an hypothesis that there is such a thing as real nature, and that our laws are valid in the realm of that real nature—an hypothesis, moreover, which cannot be proved, since we can never go beyond the subjective sphere of our own consciousness, and yet, whatever phenomenalists may say, we must leave it behind if we would construct a system of physical science; he who never goes beyond the domain of sensations and their relations is a psychologist, but he will never succeed in expressing the objective world in scientific terms. Nothing but the realistic presumption that cause, time, and intelligible space of three dimensions have a transcendent validity can enable us to speak with truth of natural laws differing from those of psychology, or of objective movements in conformity with the ideal laws which our thought conceives; but this presumption is an hypothesis and must ever remain so.[25] The progress of physical science may enable us to arrive at an accurate, quantitative determination of the coefficients of probability of each individual law, but it can never transform that probability into certainty.[26]

Thus the whole of von Hartmann's system depends upon an hypothesis, since the very unconsciousness which is supposed to be the cause common to the cognitive

process and the real world is in its turn but probable knowledge similar in nature to all causes transcending the order of phenomena. The efforts of philosophy in this direction are no more successful than those of science; von Hartmann holds, just as does Wundt, that the ultimate principle to which philosophy leads us is, in its absolute transcendence, of purely hypothetical value. Human reason, whatever efforts it may make, must always leave some thing exterior to itself: that element which is beyond logic, and whose dark mysterious depths can never be sounded by thought. Philosophy, Wundt tells us,[27] may prove the necessity of faith in a universal Will, but can never transform that faith into knowledge; hence the assertion of the reality of this supreme principle belongs to the subjective sphere of feeling, which has its rights just as much as has that of reason. Von Hartmann bases philosophy upon a power of divination, partaking of the nature of genius, which identifies us with that unconscious spiritual activity lying at the root of existence more nearly than discursive intelligence could succeed in doing. Every system takes its rise in mystic intuition; hence it is only capable of proof to its originator and to those who by a process of mystical creation can rediscover its essential elements within themselves.[28] This intellectual process, however, which von Hartmann takes from Schelling, must either be conscious in some way or other, in which case the term absolute unconsciousness is out of place, or else utterly transcends our conscious life, in which case it is difficult to see how its existence can be stated as a fact. Von Hartmann in reality falls into the very error which lies at the root of Spencer's agnostic evolution; he ascribes the character of actual reality to something which has no essential connection with consciousness, which existed and could have continued to exist in its absolute unconsciousness even had the idea never happened to be born of the will, and to produce therein by its sudden appearance a feeling of stupefaction, a reaction which is the beginning of

conscious life.[29] Agnosticism is the fatal result of this view of the relation between being and consciousness; it is true that von Hartmann contrives to avoid this sceptical conclusion, but he does so because his Unconscious is only nominally such; in reality his description of it involves a much higher form of consciousness than our own, one capable of grasping in a single act of intuition that which discursive intelligence views as developing in the indefinite and inexhaustible series of time.

14. *Fouillée's Idée-force.*—Fouillée must be credited with having resolutely put aside the absurdity of the epiphenomenal consciousness which is suddenly superimposed upon physical energy or upon the idea emanating from the will in some unknown miraculous way. Unlike those men of science who would fain reduce psychic life to an automatic mechanism, a complex of reflex actions, Fouillée passes from psychology to cosmology, and endeavours thus to prove that physical evolution itself is inexplicable if the factor of consciousness be not taken into account.[30] The starting-point of every process of development is not brute movement, but rather psychical appetitive or reflex process which is at once an obscure presentation, a feeling of a more or less vague nature and an activity.[31]

Mechanical evolution is not a primitive law, but the form and outer sign of the appetitive process which is the true inner essence in ourselves, and in all probability in everything else. Darwinism does not ask us to regard biological and psychological phenomena as a complication of mechanical laws, but would rather have us understand from the struggle for existence that mechanism itself is a form of this struggle for life, which, rightly understood, resolves itself in its turn into a struggle for the minimum of suffering and the maximum of well-being, or, in other words, into a struggle of wills.[32] The world is not a complex of inert atoms, but a vast social organism of wills united by the bond of mutual sympathy. Philosophy in its ultimate

analysis looks upon the idea of the universal association of consciousnesses as the basis of that which was formerly termed nature,[33] and thus replaces scientific abstractions by the full complete synthesis of living reality. Science, which looks upon the relations of things independently of the subject which wills and thinks and of existence in its totality, affords us but an abstract vision, which cannot take in the fulness of reality, the unity of things with the mind which is conscious of them and with the entire universe. Philosophy cannot therefore stop short at the objective synthesis courted by positivism, but must integrate and complete it by re-establishing that intimate relation with the conscious subject which is ignored by scientific research.[34] As the eye of the philosopher learns to range over wider horizons he will see less of the unbending certainty which science attains by limiting its objective; the outlines of ideas will become less sharply defined, until they fade away altogether in that mysterious distance which is the domain of faith, probability, and feeling.[35] Faith and feeling must not, however, rise in revolt against intellectual knowledge, which, limited as it may be, is yet our surest possession, whereas the indefinite character of feeling renders it most liable to fall into error. The heart may have reasons which carry weight, but it is to reason that we must look for recognition of their truth. Love and will may in practice supply that which is lacking in knowledge, but they cannot turn uncertainty into certainty, except by means of an illusion which cannot be taken as a rule of life. The most important of the duties of man is to search for the greatest possible number of assured truths; and there can be no true certainty apart from intellectual certainty. The efficacy of faith is an incontrovertible fact, but its efficacy is no proof of its truth; the creed of the Mohammedan is false, although it has led him to victory.[36]

Fouillée thus raises the standard of rebellion against the excesses of pragmatism and the doctrine of contingency, and resolutely asserts that morality without

reason cannot exist, and that if liberty could only be had at the price of intelligence, it would be better to retain reason even at the cost of liberty than to enjoy freedom without reason, and in defiance of reason itself.[37] Yet, although Fouillée's psychological determinism[38] is the very antipodes of the doctrine of contingency, there can be no doubt that he too has helped to initiate and spread the reaction from intellectualism by means of his evolutionary theory of knowledge, which, while striving to correct and modify the Spencerian doctrine of evolution, is in agreement with its fundamental principle—the biological significance of the intellectual function. He may not fall into the error of deriving consciousness from a physical process, but he persistently looks for the origin of the supreme categories of the intelligence in the need for the preservation of the species, in that will to live which is the motive power of the struggle for existence.

The proposition he endeavours to prove in the *Psychologie des Idées-Forces* is to all intents and purposes identical with that which becomes later the dominant note of pragmatism; knowledge is but a means which we have artificially turned into an end by severing it from the sensitive, emotional, and motor process, the living circle of which it naturally forms part.[39] Neither physical nor logical evolutionism can account for that thought which is deeply rooted in the will, understood, not in the intellectual sense of an ultimate or exemplary cause acting in view of an end which it has before it, of an ideal good ideally conceived, but rather as a tendency to preserve unimpaired the immediate feeling of pleasure which affords satisfaction to the organism.[40]

15. *The Endeavour to reconcile Pragmatism and Intellectualism.*—If these premises be granted, may not the conclusions drawn from them by empirio-criticism and pragmatism be regarded as justifiable? If the theoretical function be really of no intrinsic value by itself; if it be but a tool forged by the will to live in order to ensure the preservation of the organism, then science

and philosophy, which are the offspring of that function, have no end of their own, but owe any importance they may posesss to their relation to practical activity. That belief or theory which enables us to gain the victory in the struggle for existence must be true; that which fails to attain that end, false: such is the one and only criterion of this new instrumental logic. Fouillée, however, shrinks from going to such extremes, though such a conclusion is the logical outcome of his biological theory of knowledge, and takes up a dubious position, a half-way house between intellectualism and pragmatism. By conceiving the idea as possessed of force,[41] he deems it possible to reconcile the theory of the intellectualists, which looks on truth as a harmony, with that of voluntarism, which regards it as an action or active belief, since truth is simultaneously and indivisibly a harmony of actions and ideas, derived from an intelligent will or an active intelligence. Even though the main claim of knowledge to importance may be the work it accomplishes as an instrument of life in the course of evolution, this does not constitute its only value; ideas are undoubtedly of value as forces, but they are also valuable as ideas.[42] They are not merely a residuum of abstraction, but are a manifestation of higher reality, because the process of elaboration to which thought subjects data enables us to penetrate more deeply into the heart of nature. Only by a dogmatic abuse of the transcendent unknowable does it become possible to affirm that thought transforms reality, as if thought were not the same reality at a more advanced stage of development, a higher level of consciousness and action. Reality does not shrink, but rather attains to its full stature in the full consciousness of thought and in the moral will.[43]

Now I quite agree with Fouillée in this vindication of the objective value of thought, and readily subscribe to his criticism of the relativity of knowledge, and of the subjectivism of Kant, whose weak points he exposes more clearly than Wundt, recognising, as he

does, the character of objectivity, not in sensible knowledge of phenomena alone, but also in every stage of scientific and philosophic elaboration;[44] but I fail to see how this declaration of the value of the understanding can be reconciled with the biological view which would trace its origin to the elementary needs of life. If, as Fouillée himself says, it is to the loftiest forms of the mind that we must look for the deepest and fullest revelation of reality, if the participation of this reality in the life of things be in proportion to the higher intellectual, moral, and above all, social determinations it contains; if we approximate more closely to the universal will when we think than when we feel,[45] then neither psychic reflex, obscure appetition, nor the impulsive will to live which aims at immediate satisfaction can afford us an explanation of that intelligent volition which is capable of setting universal ends before itself; we must rather look to the moral will infused with intelligence for the explanation we desire of the rudimentary forms of volition, "How," we would ask in Fouillée's own words, "can the meanest existence be the most faithful transcript of the world?"[46] Whilst Fouillée, the idealistic philosopher, proclaims that man mirrors the universe more faithfully than does the animal, Fouillée, the biologist and psychologist, who is still under the influence of the preconceptions of positivism, looks for the cause of the birth of intelligence in the blind instinct of the animal, and in the vague feeling of the protozoa and its tendency to preserve agreeable stimuli. Thus the way is cut off which leads to that synthesis of volition and idea, whose living expression is found in personality conscious of its ends, not in the rudimentary forms of primitive impulse. The *idée-force* reconciles in itself the opposing systems of voluntarism and intellectualism, if we understand by it active thought or intelligent will; if, on the other hand, we identify it with obscure appetition and the instinctive tendency to preserve life, it degenerates into the absolute irrationalism taught by Schopenhauer. Fouillée halts between the

two meanings, but leans more toward the latter, which, alleging as it does that thought is a product of vital adjustment, certainly cannot be said to recognise its intrinsic value.

16. *Ultimate Consequences of Voluntarism as seen in Paulsen.*—Fouillée's undecided position is not tenable: if we are to look to the will to live for the origin and meaning of the intelligence, we must resolutely put on one side the thesis of the intellectualists according to which the idea is possessed of autonomous value, and we must accept, as do Paulsen and Nietzsche, the extreme conclusions of the voluntarism which regards reason merely as an organ of volition. Paulsen affirms [47] that the whole structure of the mind is a contingent product of the evolution of species: time, space, and the categories have come into being in the course of development, just as the eye, the ear, and the brain have done; hence they represent the variable and secondary part of consciousness, whilst will, which is present in all manifestations of life alike, and may by analogy be assumed to be at the basis of material activity, is the true first and constant principle of both mind and universe. Will, not reason, defines the end of life: intelligence plays but a subordinate part—that of finding the means best suited to the accomplishment of that end; and, since the true task of philosophic research is to determine the meaning of existence, which is manifested only in the relations existing between things and our will, and in that obscure feeling which forms the basis of all our valuations, we must not look to intellectual knowledge with its indifference to every value to bring us into contact with the underlying life of things, but rather to moral and religious belief.[48] The certainty that the world is not an utterly meaningless game of unseeing forces, that the supreme good to which our will is directed is also the end and aim of the cosmic process is not bestowed upon us by science, which is but an apparatus for the registration of the real in the sphere of phenomena, but owes

its origin to our keen sense of duty, and to the religious faith which we place without asking for theoretical proof in a perfect Being, the creator of all values.[49]

17. *Criticism of Voluntarism and the Theory of Faith.* —Paulsen thus returns to the antiquated mysticism of tradition; he exaggerates the agnostic subjectivism which is the weak point of Kant's system, and finds fault with Kant for precisely that which is of the greatest value in his *Kritik der reinen Vernunft*, namely the distinction he draws between the epistemological method and psycho-genetic research, the *pièce de résistance* of the English empiricists. It is indeed strange that Paulsen, who insists so strongly on the inadequacy of the scientific method for the determination of values, should yet call upon the scientific theory of evolution to assist him in his denial of the value of intelligence! As a matter of fact, whatever he may say to the contrary, he is still fettered by the science he would fain reject: is not the analogy which forms the basis of his system, the process of induction from subjective volition to cosmic will manifested in phenomena, an intellectual process? Is not the identification of the end of the universe with good—our loftiest aspiration—a thought rather than a feeling? Are we to form no idea or conception of the end of activity? Paulsen, whilst deluding himself into the belief that he derives the cognitive function from will, really assumes its existence the moment he states that it aims at the preservation of the individual and the species, even though it may do so unconsciously; because it is at bottom the idea of this end which enables the philosopher to deduce the cognitive organ as a means of its realisation; logically then, if not psychologically, this concept is present in the most elementary forms of life. Moreover, in order to explain the biological process of evolution we are continually forced to refer to the conditions present in the environment, to take for granted, that is to say, an external reality having a specified structure to which the organism must adapt itself if it would live; and the concept of this external reality

has already called into action the forms of space and time and all the categories which are supposed to be born in some miraculous way of the will to live, whereas the mind of the philosopher has already taken them for granted the very instant he begins his so-called genetic explanation of the intelligence.

Moreover, the very will to live, which is the starting-point of the whole process of deduction, is (if by it we understand a universal activity) not a feeling, but a concept; it is not pure volition directly experienced, but will reflected by thought, which has thus become the *idea* of a cosmic will embracing all things. The more capable volition is of becoming a principle of philosophic explanation, the more will it become, so to speak, infused with intellect: philosophy cannot be built up on nothing but feeling. Faith, which would fain take the place of knowledge as the basis of the system, though undoubtedly comprising an active and feeling element, also implies an intellectual aspect, no matter in what form it may present itself. The presentation or concept of a thing is essential to belief therein.

Paulsen, like all the apostles of the will, turns to psychological analysis for proofs of the primacy of will; but although the voluntaristic tendency was welcomed so warmly by psychology (it will suffice to quote the names of Höffding, Stout, Ward, and Jodl, in addition to the philosophers already mentioned), it does not appear to me that the results of observation of consciousness afford sufficient proof of such a theory. The strong point of voluntarism was that it combated Herbart's mechanical theory of presentations, by laying stress upon the active and spontaneous element in psychic life; but it went to the other extreme when it treated the will as the principle of all psychological explanation. It is true that if we examine into consciousness we shall find that there is no such thing as an intellectual manifestation in which will does not take part, and that theoretical investigation is impossible without an act of volition

having a definite aim in view, but we must also recognise that there is no stage of practical activity of which sensation (vague and confused as it may be), presentation, and thought do not form constituent parts. Intellect and will always manifest themselves to us as two aspects of a single process, these two aspects being only separable by a process of abstraction. Nor, on the other hand, can the evolutionary genesis of consciousness be adduced in support of the primacy of the will; whatever Paulsen may say to the contrary, practical activity is no less variable than theoretic function, and true will is due to a process of development just as much as intelligence; the gulf between the impulsive tendency of the protozoa and the moral will is no less wide than that between elementary sensibility and the most abstract thought. It is nothing but an abuse of terms to apply the word *will* to such widely differing phenomena, an abuse which has given birth to the illusion of the constancy of volition in contradistinction to the variability of cognitive function in the course of evolution; in reality the two aspects develop on parallel lines, and each is implied by the other.

18. *Nietzsche's Individualistic Voluntarism.*—The reaction from intellectualism reaches its zenith in the works of Friedrich Nietzsche,[50] in which are foreshadowed the aestheticism of Bergson and the pragmatism of Schiller and James, while voluntarism is divested of its universal and determinist character, the last vestige of traditional intellectualism, and assumes the picturesque attitude of the rebel against all and every necessary law, and of the champion of individual genius—the creator of new values—against the universal norms which would reduce everything to the same level. Nietzsche regards knowledge merely as a manifestation by the will towards power which constructs the concept of permanent being and causal necessity, creates space, homogeneous time, and all the scientific categories, classifying events with their help in such a way as to render it possible to foresee them,

and doing so in order that it may assert itself in a given sphere of reality, and establish its supremacy over the perennial flux of phenomena, and the chaotic alternation of variable and fugitive sensations. The significance of truth does not lie in correspondence with an objective order, but rather in the needs and vital tendencies which it satisfies by ensuring the triumph of the individual and the species—an assertion as ingenious as it is paradoxical, into which we shall enter more fully in the more systematic form given to it by empirio-criticists, pragmatists, and the upholders of the doctrine of contingency.

19. *The Philosophy of Freedom: Ravaisson, Secrétan.* —In addition to the causes already enumerated, there are certain other exigencies of an ethical order which helped to bring about the triumph of voluntarism, regarded in its negative aspect as the adversary of scientific knowledge; of these the most important is the necessity of ensuring the freedom of the will which the universal determinism of science threatened to destroy. Kant in his doctrine of the primacy of practical reason had pointed out clearly the way of escape from this difficult position, and it was along these lines, already boldly pursued by Fichte, that a number of philosophic systems were developed, even before the yoke of materialism had been shaken off, which regarded free moral action as the principle of the whole cosmic process, and necessity merely as a more or less illusory presentation of the spontaneity lying at the root of all being. According to Ravaisson,[51] the apparent fatality of nature is the result of a habit formed by the repetition of a free action; hence it is in the spontaneous act that we shall find the true essence of the world: things are not the outcome of brute mechanism, but of the development of a voluntary tendency to perfection and beauty. In like manner Secrétan[52] maintains that necessity owes its origin to free divine creation, that it is not determined by logical reasons, but is the spontaneous outpouring of

love and grace. Knowledge is not the absolute end, but rather a means to the edification of the world whose supreme reason is to be found in the moral order; the will is at the root of everything; consciousness is a means by which the will takes possession of itself; the perfecting of the will is the end of science.[53] "I believe," says Secrétan, "in the primacy of practical reason and cast my vote freely for liberty."

20. *Lotze and the Primacy of Practical Reason.*— Hermann Lotze is the most illustrious representative of this form of ethical or teleological idealism[54] which arose in opposition to the invading forces of materialism. Lotze, though a deliberate and strenuous advocate of the advantages of the mechanical view in the field of empirical science, was not carried away by the current of his day, and thus did not over-rate its importance in the realm of speculation. He was too sensible of ethical and religious needs[55] to rest content with a system of philosophy which robbed the world of all significance, and regarded the soul of man as the product of some unseeing mechanism; hence, he is unwilling to stop short at that which he and Kant alike view as the inevitable conclusion of the theoretic spirit, and turns to the moral life and the world of values in his search for the basis of metaphysics,[56] and to the good, and the ethical norm for the true content of the world, and the substance of all process.[57]

Only by looking at that which ought to be can we understand that which is, since nothing takes place in nature independently of the end and meaning of the whole to which each part owes its existence, and its efficacy.[58] And in the light of the whole and the absolute, we see that the actions of beings which from the finite point of view of science would seem to be determined by mechanical laws, are in reality spontaneous acts; and necessary laws, passively obeyed, are transformed into ideal norms, voluntarily recognised.[59] Value is then the gauge of all reality; each thing exists, and will endure only in proportion to its value; that only will

last for ever which has a part to play in the harmony of the whole.[60] The cognitive function too is subordinated to the end of practical activity, and truth is of value merely as a necessary presupposition of the actualisation of the good.[61] The doctrine of the primacy of practical reason, which, together with the idea of value, is the pivot on which the whole of Lotze's system turns, becomes later on the ruling principle of the philosophies of Windelband, Münsterberg, Royce, and Rickert, and we shall therefore have occasion to discuss it fully in its epistemological and metaphysical developments, when we analyse that form of reaction from intellectualism which constitutes the new philosophy of values. We shall also see a new current of thought spring from the philosophy of liberty, a current which, flowing down from the precipitous peaks of speculation into the field of criticism, and bringing the spontaneity of the transcendent sphere of the absolute into the immanent order of phenomena, will burst asunder that iron mechanism which Lotze, like Kant, recognised as confining liberty within the noumenal principle, which is both eternal and beyond time. The neo-criticism of Renouvier marks a vital stage in this transition, which is of immense importance in the beginning of the reaction from intellectualism, because it leads to the denial of the value of intelligence even in the sphere of phenomena, within which it had hitherto been considered unassailable, since its incapacity was limited to the domain of metaphysical speculation.

21. *Psychological Development of the Theory of the Primacy of Practical Reason in the Phenomenalism of Renouvier.*—Liberty, confined to the absolute, is non-existent as far as we are concerned; that which is of moment to us as an essential condition of moral responsibility is not the liberty of God, but that of man; that is to say, the efficacy of free will in the domain of phenomenal consciousness. If our empirical character be a link in the necessary chain of phenomena, how can the human

being in his concreteness act freely without breaking the bonds of this natural necessity? We have to choose between the law of necessity which lays an iron grasp upon us, stifling our most living aspirations, and the free spontaneity of feeling which knows no limits to the sphere of action of the mind; and of these alternatives, duty commands us to choose the second.[62] The will must not be subordinated to the intellect; on the contrary, the will must conquer the intellectual function. According to Renouvier,[63] all knowledge is belief, and every belief implies a voluntary decision. Thinking involves affirming and judging, every judgment is a voluntary act: thus liberty lies at the very root of intelligence. The problem of certainty is not a logical, but a psychological and moral problem, whose solution depends on that of the problem of liberty. Every assertion which goes beyond the immediately present state of consciousness presupposes, in addition to the reasons for belief, an act of will, a *parti pris* to affirm.

Thought is not something necessary and impersonal —we only believe in proportion as we ardently long and will so to do: reason is practical in all its degrees, in all its forms and in all its functions. The distinction between phenomenon and noumenon must disappear together with that between pure and practical reason: there is but one world, the world of phenomena, the world which we picture to ourselves, the world of space and time, and it is just this world which we must contemplate in the light shed upon it by the idea of moral order. The postulates of practical reason do not carry us beyond the bounds of experience, they rather extend it in accordance with our moral requirements. The categories exist by the same right as do phenomena, and are given with them: they are neither more nor less than universal phenomena, which cannot be deduced *a priori* from a single principle, but which must be extracted from experience. The special characteristic of the categories lies in the fact that, while they are manifested in a particular form in phenomenal data,

they are still presented as something superior to experience, able to act as its guide, and to subject it to laws; so that we expect to see them verified constantly. It is thus that we must understand the *a priori*, not as a subjective form applicable to the data of the outer world. Subject and object are but two aspects of the phenomenon, that which presents and that which is presented. If I apply the term "mine" to presentations, I do so because they are bound up with feelings, acts of volition, and certain material and organic phenomena in such a way as to form a distinct whole, subject to laws of its own; in like manner, when I speak of an object or thing, I understand thereby merely the presentations.

The idea of substance is inadmissible, because it leads to the actual infinite which Renouvier looks upon as a contradictory concept. By the very law of its formation a number must have a successor; we cannot conceive of an actual number without any successor, than which therefore no greater number exists.[64] Let us take the infinite series of numbers as data; let us find the square of each number in the series, we shall then obtain another series which must on the one hand contain the same number of terms as the first series, but which must on the other hand (since it is made up of squares only) contain a smaller number, since the first contains in addition to these all the other numbers; now it is absurd that a number should be at the same time greater and smaller than another.[65] The law of finite number delivers us from the metaphysic of materialism,[66] and enables us to resolve the antinomies by rejecting the antithesis, which violates the law of contradiction, and by forming the concept of finite and interrupted series of phenomena. Renouvier regards the principle of discontinuity or discreteness of phenomena as the supreme law of the real,[67] a law directly opposed to the principle of continuity advocated by Leibnitz. We are thus led to look upon the elements of bodies as being separate from one another, to regard contact

as an appearance, forces as acting between distant points, and acting in time as discontinuous and intermittent, of an essentially periodic and ejaculatory character. All movements, although they appear continuous, are composed of a series of initiatory acts and beginnings, interconnected by laws, *i.e.* mathematical functions which give expression to their reciprocal relations. The idea of cause is thus transformed and purged from the gross images of contact, impulse, and the transference of force from one object to another; the old fetish of causality, and all its attendant images are brought with the help of our reflective knowledge into harmony with phenomena and with our experience of their determinate and invariable concatenation in certain cases by means of certain preceding data. We are enabled to restore to phenomena, under the form of laws, those elements of stability and regularity of which the elimination of the idea of substance apparently deprived them. There is as a matter of fact no phenomenon which does not present itself in a complex of relations: everything is relative; nor can it be asserted that every relation implies terms, that is to say, something which is not relative, since terms are only intelligible in their relations; nor need we say that the relative presupposes the absolute, and is a proof thereof, since the absolute itself is but the correlative of the relative.[68] Even the identity of consciousness or the permanency of personality does not depend upon a spiritual substance, its one condition is the identity and relative permanency resulting from the harmonious diversity and variations of a group of phenomena subject to one law. Consciousness is but an extremely complex function (in the mathematical, not the physiological sense of the word), which implies a large number of laws and subordinate functions (intelligence, sensibility, will). If it be true that everything is, as far as we are concerned, presentation, relation, phenomenon, that that which presents and that which is presented are alike essential constituents of any

and every object of knowledge, and that pure being in itself has no sense, then the concrete unity of the abstract categories will be found in the phenomenon of consciousness. Without consciousness there can be no intelligible presentation; by which I do not mean without *my* consciousness, but without the other functions of the kind which I perceive in the outer world, and, since the world resolves itself into a combination of presentations, reality in its ultimate analysis is but a complex of consciousnesses.[69]

22. *Criticism of Renouvier's Phenomenalism.*— Renouvier's theory of phenomena differs from empiricism in as much as it admits law to be a universal phenomenon comprehending and dominating other phenomena; but, since it introduces discontinuity into the network of things, it puts the existence of stable relations of any kind whatsoever beyond the power of conception. If every phenomenon be a fresh beginning, how can it be dependent on preceding phenomena? If, on the other hand, it be dependent on and conditioned by these preceding phenomena, where is its freedom and spontaneity? If time be discontinuous, if the series of phenomenal, presenting, or presented facts be also discontinuous, then each moment, as a fresh beginning of being, neither can nor should know anything of the moments which have preceded it, but must be regarded as isolated, and as forming a world in itself. If there be any relation between the then and the now, this relation cannot, according to Renouvier, be regarded as other than a phenomenon; we have then to choose between two possibilities: either the relation has continuous duration, and thus affords an explanation of the concrete transition from one term to another, and of their reciprocal interpenetration—which is of course a contradiction of Renouvier's thesis—or it is discontinuous, in which case it must be the outcome of successive elements between which relations exist. Are these relations also discontinuous? If so, we stand committed to that indefinite process which Renouvier sought to avoid.

There is no way of escape short of admitting the continuity of time together with its necessary basis, the continuity of the subject: the discrete terms may be related, but only on condition that there be a subject which, while passing from one to the other, preserves its persistent and continuous identity. The ideas of relativity and discontinuity forming the basis of Renouvier's system exclude each other, since they cannot be united without contradiction. If indefinite retrogression in the order of causes be inconceivable, the transition from nothingness to being and the relation between them are still more so. What would be the value or purpose of the persistence of law and order in such a world of fresh beginnings? Where and how could a law persist? As a general phenomenon it is non-existent in experience, which brings only individual phenomena to our notice; if it be an abstraction, a product of thought, it presupposes a subject which lives continuously in such a way as to be capable of referring one to the other, and of comparing individual instances of the past and present. If consciousness also be merely a general phenomenon, where must we look for the condition essential to the construction of these general relations? A reality made up of nothing but phenomena, and of discontinuous phenomena as well, is merely a series of individual facts, and in such a world we cannot fairly speak of either laws or universal functions, and the categories become but an incoherent collection of phenomena and individual relations.

Renouvier turns for proof of the discontinuity of the real to the alleged contradictions implied in the concept of the infinite, which are supposed to be derived from the impossibility of thinking of a series as ended which is defined as being inexhaustible. The main argument he adduces is and ever has been the stock one of those who would deny the infinite, and may be briefly stated as follows: In order to conceive of an infinite series in its actuality, we must make the synthesis of an infinite number of parts; this

is, however, an impossibility, since to do so would demand infinite time; hence the synthesis will never be completed.[70] Now it is not difficult to see that this argument derives its probative value from the presumption that it is impossible to think of a series or class of objects without passing those objects successively in review, and that the concept results from the synthesis of this enumeration: premises which, if true, would render it impossible to think even of finite numbers. Let us, for example, take a number a thousand times greater than the number of seconds in the average life of man; it is self-evident that, even if we were sufficiently persevering to count all our lives, we could never reach that number: are we, then, to conclude that it is unthinkable? Certainly not: thought does not proceed *per enumerationem simplicem*; the concept is not a series of moving pictures of individuals or individual objects belonging to the same class, but determines simultaneously the whole series with the characteristics defining it. In other words, the presentation of the extension of a concept is not necessary for the formation of that concept, that of its comprehension will suffice: a finite number of indications will enable us to determine an infinite class of individuals. Every concept, every universal law is in itself the positing of an infinite; the denial of the infinite is equivalent to the denial of the universality of thought. The apparent contradiction arises from the alleged exhaustion of this infinity or universality by means of an empirical succession of presented terms, whereas the concept of the infinite, like every thought of universal and necessary law, in as much as it comprehends all time, is external to temporal succession. The absurdity comes in when this same thought is transferred in unvarnished nominalism to the empirical series of moments, since it is then obvious that, as it cannot issue therefrom, it can only realise the concept thereof by passing through the series in its successive order. We conceive of the infinite in the very act of formulating the law which determines it in

its essential characteristics; thus it is not something at which we arrive by enumerating all the terms which succeed each other in time. Those who deny the infinite say: "Let the will strive as it may, it can never reach an ultimate term." We do not, however, define the infinite as the last term of a series; we do not assign it a place in the sequence of finite numbers; on the contrary, we maintain that it is external to that empirical series which develops in time. The neo-criticists have the best of it, because they look upon it from the first-mentioned point of view, that is to say, they regard it as the end of a series, and assert that it must be possessed of the same properties as are finite numbers and that the axiom, "the whole is greater than the part," must hold good of it. This axiom, which, translated into mathematical language, would be more clearly expressed as follows, "The sum of two magnitudes is greater than either of them," holds good of absolute lineal magnitude, but there are other kinds of magnitude, such as vectors, to which it does not apply. In like manner, if it be true of positive numbers, it is not so of negative ones, nor is it valid for complex numbers, the difference between which cannot be defined.[71] What wonder, then, that it should not be applicable to infinite cardinal numbers?

The law of number is at bottom but a dialectic weapon which Renouvier places at the service of the will to believe; nor does he hesitate to distort reality to such a degree as to render it inconceivable, in order to facilitate the introduction of the voluntary act into the series of phenomena.

We can already trace in his phenomenalism the germs of the doctrine of contingency, since it regards the caprice of the will and the ardour of passion as the basis of all judgment and belief, subordinates the demands of the intellect to the exigencies of sentiment and moral aspirations, and denies the necessity of thought which it derives from a voluntary affirmation of the mind. On the other hand, the elimination of the idea of sub-

stance, the reduction of the bond of causality and law to a purely functional relation, and the dissolution of reality, including the world of consciousness, into a conglomeration of phenomena and phenomenal relations are the very concepts which serve later on as the basis of empirio-criticism.

NOTES TO CHAPTER II

[1] *Histoire du matérialisme* (Paris, 1877), vol. ii. p. 171.
[2] *Op. cit.* vol. ii. p. 579.
[3] *Op. cit.* vol. ii. p. 596.
[4] *Handbuch der physiologischen Optik*, p. 455 (1877).
[5] *Die Tatsachen in der Wahrnehmung* (Berlin, 1879), p. 42.
[6] *Über die Tatsachen, die der Geometrie zu Grunde liegen* (Göttingen, 1865).
[7] *Zur Analyse der Wirklichkeit* (Strassburg, 1880), pp. 224 and 251.
[8] *Philosophie der Naturwissenschaft* (Leipzig, 1881–82), vol. ii. p. 36.
[9] *Kant und die Epigonen* (Stuttgart, 1865), p. 27.
[10] *Op. cit.* vol. ii. p. 248.
[11] *Der philosophische Kriticismus* (Leipzig, 1876 87), vol. ii. p. 64.
[12] *Zur Einführung in die Philosophie der Gegenwart* (2nd ed., Leipzig, 1904), p. 59 ff.
[13] Schuppe, *Erkenntnistheoretische Logik* (Bonn, 1878), p. 68; *Grundriss der Erkenntnistheorie und Logik* (Berlin, 1894), p. 7 ff.
[14] *System der Philosophie* (Leipzig, 1889), p. 91 ff.
[15] *Op. cit.* p. 107.
[16] *Op. cit.* p. 116 ff.
[17] *Op. cit.* p. 141.
[18] *System der Philosophie*, p. 387 ff., p. 415 ff., p. 442 ff.
[19] *Philosophie de l'inconscient* (Paris, 1877), vol. ii. p. 237 ff. *Le Darwinisme* (8th ed., Paris, 1905). The first German edition, published under the title *Wahrheit und Irrthum im Darwinismus*, came out in 1875.
[20] *L'Évolution des idées-forces* (Paris, 1890).
[21] Nietzsche's *Wille zur Macht* is an interpretation, though a paradoxical and romantic one, of Darwinism and the principle of selection, which, just as it has raised man above the level of the ape, prepares the way for the advent of the super-man, the *raison d'être* of the existence of the earth.
[22] *Philosophie de l'inconscient* (Paris, 1890).
[23] *Die Weltanschauung der modernen Physik* (Leipzig, 1902), p. 221 ff.
[24] *Le Mouvement idéaliste et la réaction contre la science positive* (Paris, 1896), 2nd ed. p. 63 ff.
[25] Hartmann, *op. cit.* pp. 211-219; *Das Grundproblem der Erkenntnistheorie* (Leipzig, 1889), pp. 104-126; *Kritische Grundlegung der trascendentalen Realismus* (Leipzig, 1885), 3rd ed. pp. 81-90, 107-116; *Kategorienlehre* (Leipzig, 1896), pp. 127-142.
[26] *Die Weltanschauung*, p. 222.
[27] *System*, p. 444.
[28] *Philosophie de l'inconscient*, vol. i. p. 400 ff.
[29] *Ibid.* vol. ii. p. 43.
[30] *L'Évolution des idées-forces*, p. 247.

[31] *Op. cit.* pp. 77-78.
[32] *Op. cit.* Introduction, pp. 53-54.
[33] *Le Mouvement idéaliste*, Introduction, p. 48. This theory is more fully developed in the other book, *Le Mouvement positiviste et la conception sociologique du monde.*
[34] *Op. cit.* Introduction, pp. 37-42.
[35] *Op. cit.* Introduction, p. 56.
[36] *Op. cit.* Introduction, pp. 61-62.
[37] *Op. cit.* p. 227.
[38] *La Liberté et le déterminisme* (Paris, 1872).
[39] *Psychologie des idées-forces* (Paris, 1893), vol. i. p. 274.
[40] *Le Mouvement idéaliste*, pp. 94 and 146.
[41] *Op. cit.* Introduction, p. 66.
[42] *Op. cit.* p. 237.
[43] *Op. cit.* p. 214 ff.
[44] *Op. cit.* pp. 211-220.
[45] *Op. cit.* Introduction, p. 67.
[46] *Ibid.*
[47] *Einleitung in die Philosophie* (17th ed., Stuttgart and Berlin, 1907), p. 432 ff., p. 129 ff.
[48] *Op. cit.* p. 8 ff., p. 131 ff., p. 459 ff.
[49] *Op. cit.* p. 451.
[50] *Werke*, vol. xv. (Leipzig, 1900), p. 265 ff.
[51] *De l'habitude* (Paris, 1838), reprinted in the *Revue de métaphysique*, 1894; *La Philosophie en France au 19e siècle* (Paris, 1868); "Morale et métaphysique" in the *Revue de métaphysique et de morale* (1893); "Testament philosophique," *ibid.*, 1900.
[52] *La Philosophie de la liberté* (2nd ed., Paris, 1866), p. 354 ff., p. 438 ff., p. 437 ff. "Je suis ce que je veux," is in Secrétan's eyes the motto of the absolute (*op. cit.* p. 365).
[53] *Op. cit.* p. 389 ff.
[54] This term is applied by Lotze himself to his system: "In diesen teleologischen Idealismus hat unsere Betrachtung sich aufgelöst, . . . " *Metaphysik* (Leipzig, 1841), p. 329.
[55] "I am old-fashioned enough," he says, "to be sensible of religious needs." *Métaphysique* (Paris, 1883), p. 477.
[56] "Der Anfang der Metaphysik ist nicht in ihr selbst, sondern in der Ethik," *Metaphysik* (Leipzig, 1841), p. 329.
[57] *Op. cit.* vol. i. p. 427.
[58] *Mikrokosmus*, 2nd ed. (Leipzig, 1869), vol. i. p. 442.
[59] *Op. cit.* vol. i. pp. 323, 326.
[60] *Op. cit.* vol. i. p. 439.
[61] *Metaphysik*, p. 328.
[62] Renouvier, *Les Dilemmes de la métaphysique pure* (Paris, 1901), p. 247 ff.
[63] *Essais de critique générale : Deuxième Essai, Traité de psychologie rationnelle* (2nd ed., 1875), vol. ii. p. 135 ff.
[64] *Année philosophique*, 1868, p. 37.
[65] *Troisième Essai, Les Principes de la nature* (2nd ed., 1892,) vol. i. p. 55.
[66] *Premier Essai*, vol. i. p. 66; *Critique philosophique*, 1876, vol. ii. p. 69.
[67] *Troisième Essai*, pp. 20-30, 70-80.
[68] *Premier Essai*, vol. i. p. 110.
[69] *Premier Essai*, vol. iii. p. 207.
[70] Evellin, *La Raison pure et les antinomies* (Paris, 1907), p. 2.
[71] Couturat, *De l'infini mathématique* (Paris, 1896), p. 452.

CHAPTER III

EMPIRIO-CRITICISM

1. *Old and New Positivism.*—We must distinguish two periods in the history of positivism: of these the first is marked by a dogmatic belief in physical science, which is set up as the model of every form of knowledge; the second, dating from about 1870, goes still farther, and subjects science itself to searching criticism in order to eliminate any traces of metaphysics which might be sheltering themselves beneath the cloak of experimental theories; it no longer looks upon science as an unchangeable model, but studies the process of formation through which it passes, and turns to the human organism in its search for the psycho-physiological basis of this evolutionary genesis. This latter period approximates less closely in its methods to the older system of positivism than it does to the empiricism of David Hume and Stuart Mill, of which it is the logical conclusion, and which it completes by the addition of the biological concept of consciousness and the latest researches of physiological psychology. While positivism in its earlier forms endeavoured to eliminate metaphysics by proving all reality to be capable of *scientific* explanation, taking refuge in agnosticism when this synthetic effort failed rather than acknowledge itself beaten by speculation, the new positive philosophy resorted to a purer and more ingenuous form of experience than self-styled scientific experience, and set itself the task of proving the enigmas of the universe to be non-existent. Metaphysical

problems are the result of faulty perspective; if we will but change our point of view, divest our minds of all *a priori* schemes, and look at things as they are immediately brought to our notice instead of through the tangled web of mechanical formulas, the illusion will vanish and problems cease to exist.

2. *Factors determining the Transition from One Form to the Other, and their Influence upon the Thought of Mach.*—This change in the concept of science was to a great extent brought about by those general factors with which we have already dealt in the introductory chapter. When, between 1860 and 1870, Ernst Mach first began to give specimens of his scientific labours, three great events had taken place which could not fail to be of permanent influence on the mind of the young thinker. It was but a short time since the publication of Darwin's *Origin of Species*, a work which Mach described as "initiating new life in every branch of science by means of a revolution in method no less fruitful than that which owed its first great impulse to Galileo." Helmholtz was giving to the world the results of his researches into the analysis of optical and auditory perceptions, which, with the works of Fechner and Hering, threw open a hitherto unexplored world to experimental research. A more comprehensive theory, the outcome of the new principles of energetics, was being put forth by Carnot, Mayer, Joule, Clausius, and Kelvin—a theory far loftier than the mechanical conception of nature, which in its ultimate analysis inevitably ended in the *Ignorabimus* of De Bois-Reymond; whilst in the work of Rankine[1] we have the first proclamation of the reform in physical methods. If we would form a right estimate of the importance of Mach's work, we must bear these three factors in mind. From Darwin he derived the historical evolutionary method, together with the biological valuation of science; from the analyses of Helmholtz and Hering those sensorial elements which he regarded as the ultimate basis of reality, and as neutral ground on which the conflicting

claims of nature and mind might be reconciled; whilst the failure of the mechanical theory impelled him to search for the reasons of that failure and to purge science from the last traces of metaphysics still lurking in it. Mach's first writings treat of the psycho-physiology of the senses, that is to say, of that class of research work which had been so splendidly inaugurated by Helmholtz, Fechner, and Hering, and the fact that he had done experimental work of this kind during the ten years from 1860 to 1870 was certainly not without influence on his philosophic thought; indeed, his enthusiasm for the analysis of the perceptions and for the resultant sensorial elements induced him to place too much reliance upon them: sensation, to which he had devoted so many years of patient research, is ever present in his consciousness like an *idée fixe*, and this thought obsessed him till it gradually developed into a sort of monomania. He sees sensorial elements everywhere! What is perception? a group of sensations! the presentation? another sensation! the concept? a combination of sensations! human reason? a special sense! the *Ego*? a collection of sensations! the will? a series of sensations! Mach has fallen victim to the very malady which his keen eye had noted in physicists, who are so accustomed to making use in the course of their daily work of the concepts of mass, atom, and force, that they end by overrating these means for the classification of phenomena, and confound the means with the end of research, positively raising them to the rank of reality. Thus the mental habit formed by Mach during these first ten years of experimental work in the domain of the sense organs led him to overestimate those sensorial elements which were but the product of artificial analysis, but which he regards as facts immediately given in direct experience, and therefore real apart from that complex known as the mind of man.

3. *Hypostatisation of the Sensorial Elements.*—According to Mach,[2] reality is a combination of sensations stand-

ing in definite relation to one another. It is erroneous to consider them as symbols of things; on the contrary, the thing is a mental symbol of a complex of sensations of merely relative stability. The true elements of the world are colours, sounds, pressures, spaces, and durations, not things (objects or bodies). The exigencies of practical life force us to seek something persistent, some unchanging nucleus, amid the unceasing variations of these elements, and this gives rise to the common distinction between the thing and its quality, whence is derived the metaphysical concept of substance in which the accidents are inherent. As a general rule, the nucleus of the object is considered to be the combination of tactile and spatial qualities which appear to be more stable than colours, sounds, or odours. When the ulterior development proves such a distinction between primary and secondary qualities to be arbitrary, since these qualities are at bottom all sensorial elements and are all more or less variable, the need of finding something persistent leads to the absurd thought of a thing in itself from which sensations emanate. But if we divest the object of all its sensible qualities, what is left of reality? This absurdity is eliminated when we reflect that the idea of permanent substance is an illusory construction of thought, resulting from the practical necessity of finding stable points of orientation amid the continual renewal of the facts of experience; and that things are artificial schemes, stereotyped forms which we have substituted for the manifold variety of sensations in order that we may make use of them more easily and more readily, and that they are on that account of purely economic value. The idea of a substantial *Ego* originates in an illusion of the same kind. We regard the complex of memories, tendencies, and feelings which is apparently bound up with that particular group of sensations composing our bodies as a persistent *quid*, but in reality it too is subject to transformations which are so gradual as to escape our notice. This complex does not indeed change

suddenly or as a whole, but new elements are added to the relatively stable mass of memories by a continuous process of development which has an illusory appearance of permanence. The *Ego*, then, like the objects of the outer world, is a structure serving a purely practical purpose: this complex of elements closely connected with the body and its conservation is of the utmost importance to the will with its shrinking from pain and desire for pleasure; but to the intellect considered apart from economic advantages the line drawn between the *Ego* and the world seems an arbitrary one. There is no reason why only those elements should enter into the content of the *Ego* which are of more direct interest to our organism—pleasure and pain, for example—and the sensations of seeing and hearing be excluded. There are no sharply defined boundaries beyond which the *Ego* may not pass; on the contrary, it may expand until it comprehends and embraces the whole world. The alleged contrast between the mind of man and external reality is non-existent; the *Ego* and material bodies are alike the result of different combinations of the same elements. If the fundamental identity of psychic and physical facts be admitted, it will naturally follow that there is no real difference between psycho-physiological and physical research: it is the aim of both to establish functional relations between the elements; only whereas physical science studies the sensations considered apart from the complex which we call the human organism, physiological psychology determines the ties of dependence uniting these same elements to the group of sensations composing our body and to the complex of facts (memories, feelings, acts of volition) forming the essence of our personality. There is nothing in either the inner or the outer world which is not the outcome of combinations of these elements. Perceptions, presentations, concepts, will, sentiments, in a word the whole of conscious life, are but different combinations of a limited number of elements. Wherein lies the difference between presentation and sensation?

Is it not in the fact that we have placed them in different domains, that is to say, in relation to different elements? The difference between the illusory image and the perception of the real is one of a practical order only: the most fantastic dream is just as much a fact as any other, and if dream images were more coherent, more normal, and more stable, they would be of even greater practical importance to us. The question whether the world be real or merely a dream is one of no scientific importance. If presentation be identical with sensation, it is no difficult matter to reduce perception to a group of sensorial elements with their associative links; Mach therefore eliminated the word "perception" from his vocabulary, substituting for it the term "sensation." Our concepts owe their origin to sensations and the connections existing between them:[3] the sensorial elements are latent in the concepts, just as are chemical elements in bodies. The apparent distance between the sensorial, concrete, and individual presentation and the concept is illusory, there is a continuous process of transition from one to the other. The whole value of the concept really lies in the fact that it sums up in a simple and orderly manner a long series of experiences, and can suggest these experiences at the right moment; it is "potentially intuitive" (*potentiell Anschauliches*). The concept of sodium, for instance, results from the combination of its properties, which are in their turn nothing but as many sensorial experiences, either actually passed through or merely presented.

4. *Science as Mental Economy.*—The definition of the concept, or, when this is obvious, its name, acts as a stimulus to an activity which is exactly determined, critical, comparative, or constructive. This may be a series of physical, chemical, anatomical, or mathematical operations, either actually completed or merely presented, and whose sensible result is usually part of the extension of the concept. The concept is to the scientist what notes are to the pianist, a scheme suggesting corresponding actions. If this be the nature of concepts,

then the knowledge of the outer world gained by their help can be only of practical value: science, the outcome of the needs of life, like every other manifestation of organic life, must be subject to the universal law of evolution; it is not a fixed system of immutable truth, as was maintained by the supporters of the mechanical method, but changes in accordance with the varying requirements of adjustment. The ideas which age-long experience had rendered habitual jostle one another as if struggling for their very existence every time it is a question of grasping a new idea, and form the starting-point of the necessary change. The hypothetical method of accounting for new phenomena is entirely based on this procedure; for instance, when, in order to explain to ourselves the movements of the heavenly bodies, we represent them as phenomena of gravity, we do but adapt the schemes of past experience (with which we are familiar) to a new fact. It is by means of this process of adjustment of thought to facts, and of the reciprocal adjustment of ideas to one another, that scientific life develops; the former process constitutes observation, the latter originates theories.[4] The origin of science clearly proves to us that its work is purely biological, that is to say, its task is to serve as a guide to man in the intricate maze of natural facts. The starting-point of its formative process will be found in the mechanical arts. The birth of science was due to the need of making it possible to communicate the experience gained in the practice of the arts and of extending it beyond the limits of place and the necessity of the moment.[5] We must distinguish three periods in the evolution of science: the first experimental, the second deductive, the third formal. The first is in direct contact with reality. The second inaugurates the substitution of mental images for facts, thus obviating the necessity of having recourse to observation on each occasion: scientific work then becomes essentially subjective, a structure raised by the mind for the mind in which imagination plays an important part. The

traditional mechanical theory belongs to this second stage of science. When we come to the third period all idea of objectivity seems to be entirely eliminated: the attempt is made to arrange scientific results in a synthetic framework, with no other end in view than that of convenience and utility; when science has reached this stage, it no longer imagines itself able to sound the depths of reality, but strives to avoid mental toil and to economise the efforts of thought.[6] The aim of all the works of Mach is to hasten the advance of science from the second to the third period, from the mechanical to the purely formal stage.

5. *Criticism of the Traditional Mechanical Theory.*—Mach regards the view that mechanical science is the basis of all other branches of physics as a prejudice arising out of the fact that the fundamental discoveries of mechanics were the first to be made in point of time; but, although this may afford a psychological explanation of the genesis of the mechanical doctrine, it is no justification thereof.[7] We must first of all observe that there is no such thing as a purely mechanical phenomenon: such phenomena are the result of a process of abstraction which may be either instinctive or intentional and serves to facilitate their study; but every phenomenon must, strictly speaking, belong to all the branches of physical science. The general laws of mechanics are on the same level as the other general laws of physics; therefore they can enjoy no special privileges, and we have no right to say that some are more fundamental than others. That which is of longer historical standing does not necessarily always serve to explain later discoveries; as facts become known and classified, other and wider intuitions will replace it in the process of scientific adjustment. It may be urged that by reducing the concepts and laws of other branches of physical science to those of mechanics, we are enabled to express all physical knowledge in a simpler, more systematic, and more convenient form; as a matter of fact, however, the mechanical hypothesis effects no saving of either

time or trouble. In the majority of cases it is ill adapted to the phenomena it is supposed to explain; hence it involves fresh corrections, which in their turn give rise to additional complications, obscurity, and labour.[8] In our present state of ignorance as to the true constitution of bodies and their ultimate elements, the mechanical method can only afford us approximate presentments; it therefore follows that there will be a difference between the logical deductions of the system and the data of experience; thus in many cases it is impossible to give even an approximate presentation of real phenomena, such as the irreversibility of the transformation of heat in work, according to the principle laid down by Carnot. We are then forced to introduce invisible movements which experience has not revealed to us and never will reveal, and which complicate the theory to such a degree as to divest it of simplicity and economy alike. Even if it be conceded that the mechanical theory can furnish a working hypothesis in one branch of physical science, we do but duplicate the relations of these phenomena by adding a second system of symbolic relations, thus augmenting our mental labours. The mechanical theory undoubtedly served its purpose in the period which gave it birth, in as much as it emancipated human thought from theological prejudices; in reality, however, it did but substitute a mechanical system of mythology for the animism of the ancient religions, and this mechanical mythology is just as much a fantastic exaggeration of partial knowledge as is the conception it supplants, since neither the world nor even a part of it, but merely one of its aspects, is comprehended in mechanical formulas.[9] Space, time, and movement can lay claim to no greater confidence than colour, sounds, and smells, seeing that they too are sensations; mechanical concepts are allowable in the order of mechanical facts, which they present to our notice both simply and economically; but their application elsewhere is arbitrary, and inevitably leads to the *Ignorabimus*.

Du Bois-Reymond failed to see that his riddles arise from confounding certain means of classifying a series of phenomena with the object itself, and from treating those means as if they were absolute reality, and that the problems which he regards as insoluble cease to exist when we substitute the concrete world of our sensations for these abstract entities. If we would remain true to the method which led the greatest scientists to their greatest discoveries, we must confine our physical science to the exact expression of facts without trying to set up more or less fantastic and arbitrary hypotheses beyond the limits of perception and experience. Newton's maxim, "Hypotheses non fingo," should be that of every experimentalist, whose aim it should be to discover the connections between the movements of masses, the variations of temperature, the variations in the value of the functions of the electric potential, and of chemical variations, without thinking anything into these elements which is not the direct result of observation. For instance, in the case of the theory of electricity every hypothesis of a fluid or ethereal medium becomes worthless when we reflect that all electrical facts are given when we know the value of the potential and the electrical constants.[10] If we thus eliminate arbitrary hypotheses, physical science is reduced to its merely formal aspect, to a quantitative and conceptual expression of facts, and all useless presentations will disappear, together with the spurious problems connected therewith. Modern formal physical science has not lost that characteristic of unity which was perhaps the one and only advantage of the atomic theory; it attains the same ends without having recourse to useless complications and arbitrary hypotheses, by establishing permanent quantitative relations between the various electric, calorific, and mechanical processes; it takes note of the correspondence between the concepts of the various branches of physical science (between mass and thermic capacity, the amount of heat and the potential of an electric charge, the velocity of movements, tem-

perature, and the function of the potential) without allowing an exaggerated love of simplicity to betray it into underestimating their fundamental difference, and thus becomes a kind of comparative physics.[11] Though it may still formulate a law of balance between *vis viva*, amount of heat, the potential of an electric charge, etc., on the one hand, and a determined amount of work on the other, it does not understand thereby that mechanical work is the basis of these processes, but merely that a constant quantitative relation exists between mechanical, electric, calorific, and similar processes. Its aim is to establish functional relations between the elements of experience; every other aim, like that commonly attributed to it of investigating the causes of phenomena, is a remnant of the old animistic conception. When we speak of cause and effect, we isolate in an arbitrary manner those circumstances which have most bearing on our practical end, but in nature cause and effect are non-existent. *Die Natur ist nur einmal da.*[12] Nature never repeats herself; repetitions of the same occurrences, in which A and B are always connected, exist only in our abstract imitation of phenomena. The science of the future must eliminate the concepts of cause and effect, which savour of fetishism, and are lacking in clearness of form, and must substitute for them the more exact notion of mathematical function. Even temporal succession may in its ultimate analysis be reduced to a system of relations of dependence, and the word "time" in physical science is used merely in order to save ourselves the labour of a complex series of relations. Physical time is an abstract idea, and should be distinguished from the sensation of time:[13] the one is real, as are all the sensorial elements on the same level; the other formal, like all concepts, and of purely economic value. When we say that the acceleration of a falling body increases at the rate of 9·81 m. per second, it is equivalent to saying that the velocity of the body towards the centre of the earth is 9·81 m. greater when the earth has completed $\frac{1}{86400}$

of its rotation. But, it may be objected, how are we to explain phenomena if the idea of cause be eliminated? Mach's answer is that the so-called causal explanation is, in ultimate analysis, but the description of a fact or real relation; there is no essential difference between the naturalist's system of classification and the physicist's explanatory theory; the only advantage of the latter lies in its greater simplicity and economy, an advantage which it owes to the very nature of physical phenomena and to their quantitative character, which renders it possible to comprehend various and complex categories of phenomena in a small number of differential formulae.[14] There is no difference in the degree of evidence on which the various sciences rest, or in the demonstrative efficiency of their methods; their researches are one and all directed towards the same object, are conducted in the same way, and have the same end: the facts of experience are the only source of their principles and the true reality which they seek to reproduce in thought; their common aim is to effect the greatest possible economy in the operations of the intellect, and this aim is the only justification of their existence. They do not, however, all attain the same measure of success; hence the difference in their economic value. Mathematics realises this ideal of the maximum economy of thought more nearly than any of the others; physics approaches it by reason of its simple functional formulae: herein lies their value, not in the greater degree of objectivity to which they attain, as the mystics of science would fain assert. Scientific knowledge gains economically, but loses objectively as its schemes become more generic and more abstract. Here we have the weak point of science, which is forced to impoverish concrete reality, to look at it from a one-sided point of view, and to reduce it to a mere frame-work of abstract formulae in order to fulfil practical requirements. Our intelligence endeavours in this way to make up for the natural limitations of memory; if memory were able

to keep a faithful record of facts and their individual relations so as to bring them forward at the right moment, science would be useless, since we should have something else as a guide of our actions; but memory being unfortunately very limited, its deficiencies must be supplemented by a scheme of concepts, which can replace a complex series of individual images of facts. Since we cannot, for instance, recall case by case the spaces of descent corresponding to each individual time, we substitute the much more convenient formula $s = \frac{1}{2}gt^2$ for the long table of figures; in the same way, since we cannot bear in mind the individual cases of refraction of light for different angles of incidence and media of refraction, we express this long series of numbers in the brief formula $\frac{\sin i}{\sin r} = \mu$. Finally, the human intellect does but imperfectly supplement the memory of facts; scientific knowledge derived therefrom, though a biological necessity, is really but a matter of practical order, whose value must be gauged by economic standards.

6. *Unconscious Metaphysic and Contradictions in Mach's Phenomenalism.*—Mach's work falls into two divisions; negative criticism and positive construction: the former, which is a profound confutation of the traditional mechanical theory, is undoubtedly of the utmost value, and inaugurates that salutary movement of critical revision of science whose development assumed such large proportions later, more especially in France, thanks to the work of Poincaré, Milhaud, and Duhem. It is in great measure due to the influence of Mach that scientific men no longer take up that dogmatic attitude towards their theories which characterised positivism in its earlier form. When, however, he endeavours to build up a new intuition of the world on the ruins of the mechanical theory, and substitutes the element of sensation for the material atom, he does but replace mechanical by sensorial mythology. The atom was the hypostasis of an abstraction; what else is the

sensorial element? It is not presented immediately to intuition, but is the result of scientific analysis. Reality, which we experience directly, is the concrete world of complex perception, which in its turn does not exist in isolation but is presented to us in the organic context of our conscious personality. Physiological psychology resolves this real organism into abstract elements in order to find something constant in the series of individual intuitions, or, as Mach would put it, in order to arrange them economically in a system of relations; we must not, however, confound the result of reflective analysis, arrived at by a process of scientific abstraction, with reality perceived by intuition. The world, as conceived of by Mach, is not the world of immediate intuition, but the world seen through the analyses of Helmholtz and Hering, seen, that is to say, through a schematic and artificial theory of his own, which would fain reduce the whole complex world of consciousness to a mosaic of sensations. I will not stop to discuss this theory, in which the desire for economy and simplicity is carried too far; I would merely observe that, even if the possibility of such a reduction be granted, elements and relations, when called upon to make up psychic life, would not be facts of immediate experience, but conceptual schemes, products of mental reflection of the same nature as those used in physical science. Hence the words, "*Physician, heal thyself*," might well be addressed to Mach. In like manner his own criticism of the constancy of causal relations might well be turned against his conception of the object of science. If it be true that *die Natur ist nur einmal da*, if it be true that nature never repeats herself, then not only are the persistent ties between cause and effect non-existent, but there can be no stable connection between facts. Mach, on the contrary, affirms that functional relations between elements are not only persistent, but absolutely real.[15] Does he himself not in this statement hypostatise a concept? To sum up, are mathe-

matical relations, as expressed in formulas, intellectual schemes of an economic nature, or do they exist objectively in reality? If nature never repeats herself, how can we speak of *bleibende Gesetze*, or of *beständige Beziehungen*? Bergson, following Mach's premisses to their logical conclusion, might well say that the constancy of laws and the persistency of relations are just as much artificial schemes originating in the need for action and of orientation in reality as is the persistency of things and of the *Ego*; but nothing in nature is ever repeated, each fact is a new revelation. The economic theory of science, when carried to its logical consequences, leads to the intuitive method of the *new philosophy*; Mach has stopped half-way, and his position is therefore both equivocal and contradictory. On the one hand he asserts that the concept does not correspond to anything real by reason of its schematic nature: on the other he affirms the reality of functional relations which are nothing more or less than systems of concepts: the constancy of relations is not something known by intuition, but rather the product of mental reflection. The reality of concepts is the conclusion to which Mach's reasoning should logically lead him; if the concept too be a union of sensorial elements, it is just as real as are all relations between elements.

7. *Petzoldt's Law of Univocal Determination.*—Moreover, functional dependence cannot escape those very criticisms which are applied by empirio-criticism to the concept of cause. Petzoldt, following out Mach's train of thought, maintains that the concept of cause is not applicable, since the conditions determining a phenomenon are infinite, and a selection of one of these conditions as its true cause is an arbitrary proceeding; further, it is possible to insert an infinite number of intermediate terms, having a determinate effect, between one phenomenon and the next; hence an infinite number of causes might be adduced.[16] He therefore proposes to substitute the law of univocal determination (*das*

Gesetz der Eindeutigkeit) for the causal relation; the advantage of this law lies in the fact that it holds good in cases of reciprocal action as well; it may be briefly enunciated as follows: "For every phenomenon, elements may be found upon which the phenomenon depends univocally; no other result could be unique, since it would admit the possibility of at least one other with equal rights to existence." For instance, a ball moving on a horizontal plane under the influence of an impulse which propels it forward in a direction parallel to the horizontal plane will pursue a straight line, since, if it deviated in an oblique direction, there would be another oblique line on the opposite side of the original line and inclined at the same angle to it and might equally well be pursued; hence the course of the ball would not be univocally determined as it would be in pursuing the original direction, which is unique in space.[17] This principle is not deduced from individual experiences, but is the necessary presumption of our every action and our every thought. If nature were indeterminate we should be in the position of a soldier exposed to the fire of the enemy without any means of defence. Indetermination in nature would be chaos; in thought, madness.[18] The law of univocal determination is independent of individual experiences, and therefore *resembles* an *a priori*; but Petzoldt, in his fear of going beyond the bounds of experience, hastens to warn us that this is but a comparison, since this law in ultimate analysis is but a general fact which we must be content to recognise without attempting to explain.[19] But whatever Petzoldt may say, this principle will, if it be enunciated as a universal law, transcend the fact of experience, which never justifies us in affirming anything beyond the limits of past observation: the philosophy of pure experience must, if it would remain coherent, stop short at the fact immediately experienced in the passing moment, since everything else is an addition made by thought, an interpretation of data in accordance with its requirements. When you state

that relations of mathematical dependence exist between elements, that nature is a cosmos determined in all its parts, you unknowingly invest experience with the organisation proper to your own thought; you assume that nature ever remains coherent in itself, and that its elements are linked together just as are your mental concepts, that is to say, you implicitly assert that reality is not a complex of pure data, but an organic thought governed by our own human logic.

Now if we once grant the necessity of going beyond data and of assimilating them to the intelligence, if we would not be condemned to experience them only in their non-communicable immediacy, is it not an arbitrary proceeding to restrict the application of thought to certain special relationships, and to use the convenient pretext of anthropomorphism as a reason for placing under a ban other categories no less necessary to the intelligible explanation of data? The relations of functional dependence do not suffice to formulate all the aspects of things. The functional formula cannot comprehend any kind of activity or any successive development of events in a determinate direction. The terms of mathematical functions can as a matter of fact be inverted; if the variations of x be dependent on the variations of y, it may equally correctly be said that y varies whenever that x does so; whereas the essential characteristic of physical processes and of real phenomena in general is irreversibility, which finds adequate expression in the order of non-invertible succession of cause and effect. Further, if functional relations can in some way record the relations between abstract phenomena which are subject to quantitative relations, they are not applicable to the concrete causality of individual facts. The tie uniting an individual fact (which must not be confounded with the abstract physical phenomenon) to the complex of its antecedents cannot be translated into a functional formula: is this fact then indeterminate? I do not think that Petzoldt would be prepared to make such an admission, thus denying the law of univocal

determination. On the other hand the application of this law in no way eliminates those difficulties which Petzoldt attributes to the use of the concept of cause (which are not, however, as serious as he supposes) when he raises afresh the objections urged by Aenesidemus. It is undoubtedly possible to insert intermediate terms between the antecedent phenomenon *A* and the subsequent phenomenon *B*, but, even when this has been accomplished by the scientist, does it not remain equally true that *A* is a necessary condition of the existence of *B*? Its action may not be a direct one, but this in no way detracts from its efficacy. We are not justified in declaring the concept of cause to be obscure and inapplicable simply because a large number of active factors co-operate more or less directly in the production of a phenomenon. Undoubtedly every effect results from the action of countless circumstances, and if we would really determine it in its totality, we should need to take them all into consideration; but it should be observed that these factors are not all on the same level, or of the same importance, so that we are justified from the scientific as well as from the practical point of view in selecting those which act most directly and most effectively, and in neglecting those whose action is unimportant or remote. Thus, in Petzoldt's example of a building being ruined by a snow-storm, the main and direct cause is the excessive weight of snow on the roof, and it is superfluous to go back to the circumstances to which the formation of the snow and its fall in that special place are due, such as the lowering of the temperature, the variations of atmospheric pressure, the property possessed by water of freezing at zero, etc. This complexity of circumstances which makes it difficult to determine individual events is much simplified by scientific research, which only takes into account certain elements of the fact, which it isolates from the others. This enables us to determine phenomena with precision, by leaving out of consideration all those aspects which are not of interest to us from the point of view from

which we are looking at them, and which may be traced to the action of other factors. If we thus isolate certain aspects of the complex fact by means of a process of abstraction, we are enabled to resolve the combination of antecedents into elementary factors each one of which determines one of the consequent factors whose synthesis gives birth to the fact. Here we have the secret of the experiment leading to the exact determination of phenomena, nor need we alter our conception of the causal relation in order to attain thereto; more especially since the tie of functional dependence would in no way do away with the difficulties which we encounter in the determination of the fact in its concreteness. Petzoldt, in order to score an easy victory in his criticism of the causal relation, has recourse to sophistry, and confounds the abstract phenomenon as studied by physical science with the fact in its historic reality, failing to see that the same difficulties arise in the case of functional dependence, which involves the choice of one or more of the countless elements on which the phenomenon depends, just as much as does the causal relation. In no case, moreover, is it possible to determine the fact without transcending the limits of experience, thus introducing into the datum certain functions proper to thought, and attributing to facts that logical and mathematical structure proper to human intelligence. Does not Mach declare functional relations to be real and objective? Every attempt to divest reality of the forms of classification introduced into it by thought is of necessity doomed to failure; human intelligence cannot get outside itself, it works unconsciously in the consciousness of the philosopher, even when it imagines that it has succeeded in eliminating itself. Of this we have clear proof in the case of Richard Avenarius, who came under the same influence as Mach, and arrived at much the same conception of the value of science, although working independently of him.[20]

8. *The Principle of Minimum Effort, as set forth*

by Avenarius.—The work of Richard Avenarius, the founder of the philosophy of pure experience, aims at re-establishing pure experience by eliminating every arbitrary addition made by thought, and at affording a psychological and physiological explanation of the origin of the metaphysical illusion. Avenarius holds that the whole development of philosophy and knowledge in general may be reduced to the principle of the minimum expenditure of force.[21] Whatever our conception of the mind in relation to the organism may be, we must allow that its part in the preservation of life is of the utmost importance, and it is just this biological utility which gives rise to the principle of the minimum expenditure of force, in order that sufficient may remain for other no less necessary functions.[22] If the mind were endowed with inexhaustible force, it would not matter whether a greater or smaller amount of that energy were expended; but, since this force is limited, it is obvious that the mind must strive to economise it. The eminently biological interpretation which Avenarius attaches to his principle is proof positive of the influence exercised on his thought by the theories of Darwin: in reality, he, like Mach, does but apply the general law of adaptation to the development of knowledge. We see the working of this principle of the minimum expenditure of force in the realm of theoretic function, which Avenarius includes as a whole under the Herbartian term "apperception." To what, if not to the necessity of economising force, is due the logical demand for the elimination of contradictions? When the contradictory element ceases to exist, we have the advantage of being able to reduce two representative masses to one only; whilst in the contrary case, since apperception is constantly compelled to put on one side the representative group which acted as appercipient, in order that it may be free to produce another, there is a useless expenditure of force, which appears to consciousness as a sense of discomfort. Every method of classification is in

itself a great saving of labour, in as much as it organises the representative masses in such a way as to facilitate orientation and the finding of a common solution of problems which would otherwise have to be resolved one by one. The results of habit, which are of the greatest importance in the development of knowledge, are also a special instance of the same principle. Habitual reactions are the easiest, hence the tendency to judge the new by the old. The concept merely represents a saving of energy, since it enables us to comprehend a large number of objects with a minimum effort of consciousness. This principle is put into practice in all the sciences, and more especially in the explanatory sciences, and enables us to condense individual laws and concepts into general laws and concepts, thus effecting an economy of force.[23] Philosophy, which aims at giving us a universal concept of the world, is the goal to which we are led by the need of economising the force which the consciousness has at its disposal.

We advance gradually, rejecting as we go all useless items added to experience by the subject, and endeavour thus to purge it of all superfluous elements. These additions are of three kinds: the mythological, which invest real data with the form of our whole being; the anthropopathic, which attribute our feelings to objects; the intellectual or formal, which add to experience certain forms proper to the human intellect (cause, substance, etc.).[24] In our own day scientific evolution has freed the concept almost entirely from mythological and anthropopathic additions, but the *a priori* element of rational concepts has not yet been eliminated and still persists in the realm of experimental science. The aim of the criticism of pure experience is to purge experience from this last residuum, which, as opposed to the principle of the minimum expenditure of force, places more in data than is required, with the result that its thought involves waste of energy; this aim is the antithesis of Kant's criticism of pure reason,

which asserts that phenomena cannot be explained without these categories.[25] Natural sciences indeed regard material atoms which are set in motion by forces and act upon the other atoms of the system with an intrinsic necessity, as the result of experience; but as a matter of fact these material atoms cannot be considered as data of pure experience. No observation of things in motion, however complete, will enable us to perceive force, and in the only case in which such perception is possible, *i.e.* in the sensation of our own muscular effort, force is not presented to us as moving; the transition from the effort which we feel to the muscular movement eludes us; they are two heterogeneous facts, between which no bond of action has been revealed to us by experience. Not only force, but necessity also must be excluded, since necessity can be but the constraint or violent action of force; experimental data tell us nothing about such violence: we are aware that one fact precedes another, but not that one exercises constraint upon the other. Cause, conceived of as force acting of necessity, must be excluded from pure experience as an anthropopathic conception; both it and the concept of force can only be retained if they are reduced to mere empirical relations of sequences, having a definite degree of probability of verification in the future as well.[26] The same may be said of the concept of substance: experience only gives us groups of sensations, some of which are variable, some relatively stable; but, since the qualities may change without the thing being destroyed, we come to regard the thing as being to a certain extent independent of its properties, hence the illusory belief that a certain substratum will be left, even if the thing be stripped of its qualities. We can only retain the idea of substance if we look upon it as an auxiliary of thought, which is incapable of grasping the idea of change except in relation to something stable; but we must beware of passing from this subjective function of the idea of substance to its objective reality.[27]

9. *Biological Explanation of Scientific and Philosophic*

Knowledge.—The whole complex process of development of psychic life, and more especially of knowledge in its various stages (common, scientific, philosophical), may be reduced to a sequence of three parts, a sequence having a physical parallel in constant vital rhythm: disturbance of the normal organic equilibrium; intermediate processes for the re-establishment thereof; re-establishment of this equilibrium, and of the conditions favourable to its preservation: these are the three essential stages of every vital series.[28] There is no process of knowledge which will not fit into this scheme: it is the psychological and physiological basis of the whole history of both science and philosophy, and affords full proof of the biological value of knowledge. Avenarius, like Mach, recognises the influence exercised in this direction by the prevalent Darwinism, and by psycho-physiological research, and, having once admitted the principle, carries it out to its extreme consequences, instead of stopping short, as does Wundt, at the loftier functions of mind. There is nothing either in the realm of theoretical activity or in that of practical and artistic activity to which the dynamics of the nervous system do not correspond physiologically, and of which they fail to afford an explanation. Psycho-physical parallelism holds good of all psychic processes without exception. These processes develop in accordance with the common type of the vital series, which consists, as we have already observed, of three successive phases. In the initial stage we find psychic values which, in contradistinction to that which up to now has been designated real, true, secure, certain, known, habitual and self-evident, are rather unexpected, unusual, extraordinary, new, marvellous, problematic, and even untenable, but which are not, however, mere nothing, since they could not in that case be of any interest to us. This unpleasant stage of dissatisfaction stimulates our quest for everything which it lacks: existence, security, certainty, knowledge, truth, evidence, order, law, clearness and determination. In the second

stage we pass from dissatisfaction and desire to the quest itself, and there is a tendency to close the series with the final term which is presented afresh as the same, the true, the existing, the rule, the secure, the certain, the known and the self-evident.[29]

This scheme applies to psychic activity in every form, the most rudimentary and the most complex alike: to the processes of consciousness of the child and the savage, and equally to those of the greatest genius in the world of thought, art, or action. The functions of civilised man, endowed with all the theoretic aids of art and science, are but a quantitative intensification of the methods and experience of primitive man. Expressed in terms of physiology, this involves the existence of a fundamental nervous process capable of extraordinary intensification and development, which consequently forms the basis of the most complicated processes of the nervous system. Let us take a very simple example: If you pinch a frog's leg, the animal will draw it back. The whole process falls into three divisions: the disturbance of the normal physiological conditions essential to organic life; a series of movements having the purpose of effecting an escape from the injurious stimulus; the restoration of normal conditions.[30] The mechanism of the higher and more complex functions of the nervous system is in no way different: we merely have a number of reciprocally connected series instead of a single vital series, as in the case of the simple reflex actions; the difference is one of degree, not of kind. The process of loss and regain of equilibrium forms in its entirety a complete oscillation whose special character determines the psychic process depending thereon. The facts of knowledge, even in its higher logical forms, rest upon the same foundation: that which we term constant, secure, or known corresponds in ultimate analysis to habitual oscillations of frequent recurrence. The non-existent, uncertain, and unknown are those things which do not correspond to such oscillations. We regard as true

that to which our thought is adapted: the new, in so far as it is in direct contrast to pre-existing mental systems, gives us at first the impression of being false and impossible, but we gradually grow used to it, adapting ourselves to it until we end by looking upon it as true; the theory of evolution, for instance, had to battle against existing mental habits before it in its turn became habitual. Every epoch has a relatively stable system of cognition, its *logischer Bestand*, to quote Petzoldt's expression,[31] a system corresponding to the equilibrium of the vital oscillations at that stage of development; the whole course of human evolution tends towards the attainment of an absolutely constant system together with perfect adaptation of the organism to its environment.[32] Progress is not indefinite: the various series are subjected to a process of steady reduction to those component parts which are essential to a rapid, simple, and unalterable conclusion, whilst the final term approximates ever more closely to an absolute constant.[33] Concepts which were originally extremely unstable and indeterminate become more and more determinate and limited; the new and unknown will be more and more rapidly reduced to the known, until a concept is attained which can be applied to every possible form of experience by eliminating and leaving out of account all variable elements. This ideal can only be attained when experience has been purged from every arbitrary adjunct of thought, and a return has been made to the natural concept of the world. Before the birth of philosophical speculation, the world might have been described by me (or any other human being) as follows: "I, with all my feelings, am placed in an environment made up of many constant parts, related to each other in various ways. In this environment other human beings with their manifold expressions have their place as well, and that which they say is usually to a certain extent related to and dependent on the environment. Other men speak and act as I do myself; they answer my questions, as I do theirs; they modify certain parts of

the environment or endeavour to keep them unchanged, indicating their actions by means of certain words just as I do myself; I am thus led to suppose that other men are beings like myself, and that I am a being like them."

10. *Introjection.*—This is the natural concept of the world which lasts until its transformation is brought about by the theories of philosophical psychology.[34] Is a change thereof really necessary? Even a superficial analysis will suffice to show that this concept is made up of two parts, whose values differ from a logical point of view: experience and hypothesis. The hypothesis lies in the meaning attached by me to the actions of other men, and more especially to linguistic expression, taking for granted, as I do, that this expression has reference to facts of consciousness, acts of volition, sentiments and thoughts, as in my own case. It might be possible to eliminate this hypothetical part, and to regard human beings as highly complicated pieces of mechanism, but we must bear in mind that if we deny the consciousness of other human beings we leave the realm of experience, since in the only case in which we know the movements of a human being in all their relations (*i.e.* in the case of our own actions) we have experienced these sentiments, thoughts, and volitions, whereas experience has never shown us that these actions are derived from a purely mechanical source. Hence the hypothesis that other men are beings like myself, which Avenarius terms the fundamental empirio-critical hypothesis of the psychological equivalence of man, is admissible.[35] If we let R stand for the whole of our own direct experience of the environment, and E for the expression of the experiences of other human beings, we may then say that the values R and E do not differ as to their nature, but are homogeneous contents of experience, and are mutually comparable. The values of experience are commonly divided into thoughts and facts; but there is no absolute difference even in the nature of these two categories, since thoughts and facts may be compared

with one another: thus a portrait may be compared with the image of the absent person. The absolute division between the inner and outer world is due to faulty perspective, an illusory effect which Avenarius terms "introjection." [36] A person tells me that he sees the same tree as I do; hence I am led to think an image of the tree into the consciousness of that person, and to draw a distinction between the tree as a fact of my experience and his perception thereof. Since it is possible for me to put myself in his place, I end by introducing into myself also an image which I distinguish from the tree. Here the error is due to changing the point of view: the thing is a presentation not for me but for the other person, to whom by means of introjection I attribute the perception of the thing in question; for me it is a presentation of fact, something which I find there (*Vorgefundenes*).

The error becomes more serious when the sensation, as distinct from the thing, is localised in the brain, whence it is supposed to be projected outwards in the act of perception. The brain is not the habitation, seat, instrument, producer, or organ of thought: experience merely tells me that I possess a thought and a brain in the sense that both belong to the group of facts which we call the *Ego*; it merely authorises me to establish a tie of logical dependence between them, but not to locate one within the other.[37] On the other hand, I find the complex of elements (thoughts, sentiments, volitions) which make up the so-called *Ego* in precisely the same way as I find the complex of elements which I call tree; they are both homogeneous contents, and both are found already in existence (*vorgefunden*). If, however, we like to make use of the word "datum," the *Ego* may be considered as coming under that designation in the same sense as the tree; as data, or, to put it better, as things found already existing (*vorgefunden*), the various complexes are all on the same level.[38] As long as I confine myself to describing the content of experience, as found, the *Ego* and the environment differ only in the

relations of the elements, not in their general form or in the fact that one is immediate and the other mediate, one subject and the other object. As I am in my experience, so is the tree in my experience: if I say, I experience the tree, I understand thereby merely that an experience results from the more ample complex of elements called *Ego*, and from the other less ample complex called tree. The *Ego* is distinguished from the environment merely by the greater wealth and complexity of its elements. This co-ordination proper to every experience, in which the complex *Ego* forms the relatively constant member, and part of the environment, whether it be a tree or another human being, the other and relatively variable member, is termed by Avenarius "principal empirio-critical co-ordination." [39] The human individual, as the relatively constant member of this co-ordination, may be designated as the central member (*Centralglied*), and the part of the environment as the opposing member (*Gegenglied*). In accordance with the principle of psychological equality other individuals may act as central members of empirio-critical co-ordination, since by their *Ego* we do not understand a subject of experience, but something found (*vorgefunden*), a datum amongst others of the same kind. Fundamental empirio-critical co-ordination, which is taking the place of illusory introjection, does not change the natural concept of the world, but gives prominence to a general relation included therein.[40] The variations of the natural concept of the world are useless, and must be eliminated as superfluous in accordance with the principle of economy. The philosophical concept of the world must approximate ever more closely to a purely empirical content, which substitutes for the riddles of the universe an idea of the world containing nothing which is not of the nature of something found, of an experimental datum. In conclusion, if we would attain to pure experience, we must not only eliminate all concrete forms and the products of introjection, that is to say, every kind of metaphysic and anthropomorphism, but also intro-

jection itself as an unconscious function of the subject. There will then remain that concept of the world whose content is the totality of all that is found, whether belonging to the group which acts as central member or to the complex of an opposing member. We thus return to the natural concept of the world.[41]

11. *Criticism of the Philosophy of Pure Experience.*— But, we would ask, is a form of philosophic knowledge possible in which thought does not add anything to data ? If we divest reality of all the forms of classification which are introduced into it more or less consciously by the subject, what is left of it ? A chaos of data devoid of any relation of stable dependencies. Philosophical knowledge, or, in other words, intelligence, which must not be confounded with the fact as experienced, begins the moment a wider meaning is assigned thereto, as we ascend to the law, the order of which it forms part, and the concept which comprehends it. To stop short at the given is to deprive science of the conditions essential to its life. A concept of the world, no matter how embryonic, no matter how natural it may be, always goes beyond the fact of experience ; the philosophy of pure experience itself, in so far as it strives to construct a universal concept of the world, cannot avoid adding to data something which they do not contain. Do we not go beyond the limits of the given when we assume the expressive signs made by other human beings to be the revelation of psychic life like our own ? Do we not go beyond it when we, like Avenarius, endeavour to describe the manifold life of experience, and to comprise it in general schemes ? The centres of co-ordination, the elements, the complexes are generic formulas answering, though but imperfectly, to the need of finding in things a certain unity, certain constant characteristics ; but unity is constructed by thought and is not a datum. When we say that the various contents of experience are homogeneous, not heterogeneous, we unconsciously go beyond the data. Facts are neither immediately heterogeneous nor

immediately homogeneous, they are as they are found, each one of them has its own individual character. If, then, we admit that it is impossible to avoid adding something to experience which is not derived therefrom, there arises the critical necessity of selecting that one of the various adjuncts which will render the data easiest of comprehension. To eliminate them all would amount to dooming ourselves to understand nothing. We should dismiss not merely metaphysical problems, but also problems of every kind whatsoever. The coherent conclusion of such a system of philosophy would be to condemn every concept and every form of intelligence, and would involve the identification of knowledge with the immediate intuition of reality. Avenarius, however, whatever he may say on the subject, is not prepared to give up tracing relations of logical dependence between different data, or arranging them in categories, or completing experience with the help of intellectual forms which go beyond it. For the most part his essays in this direction are not specially successful. His schemes of classification savour too strongly of the artificial, and betray an attempt to reduce by force the countless processes of psychic life to a single type, so as to make it possible to connect them with the simple mechanism which he regards as the basis of cerebral life. The same disturbance of equilibrium is called upon to act as the physiological term corresponding to unlike psychic facts, and the reason why in any given case it should produce one process rather than another is not apparent. At best Avenarius affords us an explanation of the action of habit and practice on the various functions of consciousness ; but he does not give us in the case of each psychic value a fact capable of determining that value in the central nervous system. The habitual, the customary, the repetition of the same oscillations are at one time psychologically described as pleasing, at another as beautiful, at another as true, at yet another as good ; why is this ? Let us grant that all positive values of the mind correspond to habitual repetitions, and negative

values to deviations from habit; what is the psychological basis of their diversity? A different form of oscillation! This leaves us where we were before, with the additional drawback of having two facts to explain instead of one. We have not succeeded by this method in determining the psychic fact, but have left it as undetermined as before. So much then for rejecting the help of physiology! The attitude of Avenarius as regards the problem of reality in relation to the subject is extremely vague and undecided; he is under the impression that he has definitely disposed of the idealistic phase by assigning equal importance to the *Ego* and the world as contents of experience, but by regarding, as he does, the *Ego* as the central member of the co-ordination, he as a matter of fact still makes the reality of the opposing member, *i.e.* the outer world, depend upon its existence. Avenarius was reluctant to take the final step of the hypostatisation of sensations, by severing them, as did Mach and his own disciple Petzoldt, from the complex of the *Ego*, and investing them with entirely independent reality. If the outer world can be reduced to a complex of sensorial data, if these data exist only in connection with the central member termed the *Ego*, it follows that the objective world is not possessed of reality except in so far as it is connected with the *Ego*. Thus we come back to idealism, and we, moreover, implicitly admit that the functions of the *Ego* and the environment in empirio-critical co-ordination are not identical. Is the relation between the *Ego* and the outer world the same as the extrinsic relation between two parts of the environment, or is it a relation *sui generis*? Avenarius tries to get round the difficulty by having recourse to the metaphors of the centre of co-ordination, and the opposing member; but these metaphors, which are supposed to describe this special relation, misrepresent its nature, by leading us to think of the relation which may exist between any two complexes of the outer world, as, for instance, between the sun and one of its planets, and has nothing to do with the cognitive

relation. A complex may act as the centre of co-ordination of other complexes without being in the least aware of their existence. The *Ego* is not a content of experience which simply stands in relation to other contents: its special characteristic is that it knows that which stands in relation to itself, and is conscious of its own relations, whereas the opposing member knows nothing of the *Ego*.

The cognitive relation will not result merely from placing two contents of experience in a relation of propinquity; the difference between the *Ego* and the environment is not just one of complexity and fulness; it is the difference between a subject capable of self-knowledge without reference to others and an object which is a content of experience merely in relation to the *Ego*. One is a value of existence in itself, the other becomes so only in relation to the cognitive subject. The unity and continuity of consciousness cannot be derived from a simple extrinsic connection of facts which would remain extraneous to one another were it not for the existence of a subject capable of synthesising the various items of the series in a single act. The succession and co-existence of empirical elements is one thing, and consciousness of the relations of succession and co-existence another. Mach and Avenarius, whilst imagining that they take facts and their ties of dependence as the starting-point from which to deduce the concrete unity of the subject, fail to see that they postulate it the moment they admit the consciousness of relations. The mechanism of the nervous system, to which Avenarius turns for an explanation of the recognition of diversity or identity, thus reducing it to the transition from a familiar cerebral oscillation to one which is less habitual or *vice versa*, presupposes the existence of these logical relations: how, if this were not so, could we speak of the functional dependence of psychic processes upon the vital series of higher order in the nervous system? How could we state either that the *same* oscillations have recurred

or that *new* ones have been originated ? The mechanism of the brain and its relations to psychic life and to the environment do not form a complex of things found (*Vorgefundenes*), but rather systems of relations already bearing the imprint of thought. Thought does not then result from experience except in so far as we ourselves have placed it in experience. That which Avenarius shows us is not pure experience in the strict sense of the word, but experience already formulated into certain mental systems. The purely *found* which is external to every form of thought and every relation to the cognitive subject is an abstract fiction which may be found helpful in symbolising the starting-point of psychic formations, but corresponds to nothing real. Moreover, pure data, being the product of analysis and abstraction, presuppose the work of reflex thought, which discriminates between the world of naïve, primitive experience and those elements unconsciously added thereto by the activity of the subject. Hence empirio-criticism is doomed to move in a vicious circle, from which it can only hope to escape by recognising the *a priori* function of the subject. The truly real is the conscious personality in the concreteness of its content, not scattered facts, isolated from the unity of the subject.

12. *Hodgson's Metaphysic of Experience.* — In Hodgson's *Metaphysic of Experience*,[42] which has many points of contact with German empirio-criticism, we have the same attempt to argue the antithesis of subject and object from a primitive, undifferentiated experience. Hodgson, like Avenarius, would fain purge experience by analysis from the hypotheses, the "assumptions," which have been added to it in the course of psychic evolution, such, for instance, as the ideas of cause, action, mental substance, etc. These mental superstructures, which transcend immediate perceptive experience, do not correspond to any objective reality, but merely represent forms of combination of real facts, points of orientation amid the chaos of immediate

data of sensibility. Eliminate these adjuncts, and we have the continuous flux of the concrete current of consciousness, the process of immediate experience of which every psychic fact is an item separable from the rest only by means of a process of abstraction. Each of these items contains within itself by virtue of the continuity of conscious life all preceding phases; hence each perception mirrors the past. This retrospective reflection (from which Hodgson derives the term "philosophy of reflection" which he applies to his system) gives rise to the distinction between subject and object, since the psychic contents of the past form the object of the present perception which acts as subject. If these premisses be granted, what should be the logical conclusion of the system? If the mental categories be hypotheses of purely practical value, reflecting nothing real, if the concept be but an economy of thought, then true, genuine reality is the immediate life of the transient moment of consciousness. Hodgson ought then to end in something resembling Bergson's intuition, but he too, like Avenarius, finishes by contradicting himself, and, while denying the objective value of the categories, yet makes use of them in order to form a conception of the material organism, the nervous system, as a real active substance, a condition necessary to the existence of psychic life. According to Hodgson, consciousness, though in its nature a primitive and irreducible fact, is yet dependent for its existence on material conditions. This position is equivocal and untenable, since, if consciousness be qualitatively an ultimate fact, it is obvious that the existence of this irreducible quality cannot be derived from matter. Objects are, when all is said and done, constructions of the subjective perceptions, fragments of consciousness, objectified mental functions; hence Hodgson ought really only to speak of psychic facts conditioning other facts of consciousness, and from his point of view nothing justifies him in positing anything differing from phenomena of consciousness, still less in making the existence of psychic

life dependent thereupon. That which I experience immediately is the flux of consciousness in its continuity, and this real continuity is the primal and indubitable fact which, amidst changeable perceptions, constitutes the reality of my concrete *Ego*. If my psychic life be not the outcome of a mere aggregate of elements ranged side by side in an extrinsic relation, as Mach and Avenarius would have us believe, if it be rather the compenetration of these elements in a living unity, as Hodgson rightly maintains, is it not this living unity, a concrete and active substance, of which we have immediate experience, while the material substance with all the mechanism of atoms is but a hypothetical structure ?

13. *Kleinpeter's Subjectivism.*—Kleinpeter saw this plainly, and his teaching on this point is a notable advance on that of Mach and Avenarius.[43] The *Ego* is not a sum of psychic facts, and even though it is possible to distinguish different distinct parts in consciousness, these parts have nothing to do with the parts of a physical body. Sensations are never presented as such apart from everything else, but always as *my* sensations, and the bond between these elements in consciousness differs from the connection which may exist between them outside the *Ego*. The primitive datum is not the element taken by itself, but the totality of consciousness, which we must regard as an ultimate and irreducible fact. We may speak of other beings in as much as we refer them to our own *Ego*, the living model of all reality ; where could we find anything we know better ? The elements of knowledge in all its forms cannot but be of psychic origin, hence they can never leave the sphere of individual life. All knowledge is originally only of value to the individual who has built it up, and the thought-product of one individual is of importance to another only when certain presuppositions can be verified. It is mere matter of chance that this can usually be done. Strictly speaking, I cannot even state with certainty that other individuals exist ; I

ought therefore to confine myself to that which I am experiencing at this moment; hence knowledge would only be of value during the passing moment which gives it birth.[44] The equivalence of all thinking subjects is but an hypothesis: that which is valid in the eyes of one person may not be so in the eyes of another who does not start from the same premisses; nothing can be proved to him who will make no concessions. "Subjective opinion, not objective certainty, is the one and only end to which science can attain."[45] A scientific work is only of value when the reader concedes certain postulates to the author. The function of science is the saving of mental labour; she says to the reader: "You must recognise the truth of certain principles, from which, once they are granted, we derive a long series of other theorems; in the case of the former your immediate experience is required, for the latter you can save yourself this mental labour." Our knowledge is relative and provisional, since its existence depends upon the verification of two fundamental presuppositions, *i.e.* the psychological equivalence of men and the uniformity of nature. Who can guarantee that they will always be verified?[46] Human thought is not endowed with the spirit of prophecy; it must confine itself to the description of that which is and that which may be expected, but it cannot lay down laws for the future.[47] The natural series of facts eludes the activity of our mind, which, when confronted with it, can but act as looker on, and strive to reproduce it as skilfully as possible in thought by means of systems whose validity is entirely dependent upon the probable success of their forecasts. If the work of science be successful, it is a lucky chance; from our point of view this is the utmost which can be said.[48] This candid confession, with which Kleinpeter concludes his critical analyses of the value and basis of the sciences, affords a proof of the dire results of the empirical method when it is carried to its logical consequences. A caprice of the will in the beginning of human thought; a series of

happy accidents which have rendered it possible to apply it to the world of experience; behold in very truth a system of philosophy before which all riddles will flee away! Empirio-criticism alleges that it has explained everything by reducing the intellectual function to a combination of expedients destined to economise mental labour, but it is unable to know or say how or why these expedients succeed. It is indeed strange that facts do not revolt against our economic demands, that they submit to being classified in concepts and logical systems and to being foreseen, that they behave, in fact, for all the world as if they too had a leaning to economy and a turn for mathematics! Empirio-criticism must of necessity, if followed to its ultimate consequences and freed from its implicit contradictions, lead to the doctrines of contingency, intuitionism, and pragmatism, since it reduces the theoretic function to the practical attitude of consciousness, and looks to the fact of immediate experience for the truest revelation of reality. If the fact in its original immediacy be the true real, how can we grasp it without divesting ourselves of every intellectual form? The concept of pure experience, the functional relation, the stable dependence of elements are remnants of the old intellectualism: we must go farther, and deny all and every permanent relation, law, and form of conceptual reflection, if we would attain to that deeper experience which abstract formulae have falsified and impoverished. Mathematical functions too are of merely practical value; the law of univocal determination is but an economic expedient for the mastery of the inexhaustible wealth of experience. Nature never repeats herself: her every moment is a new creation which no intellectual effort can ever grasp, and which can only be experienced by means of intuition. Creation, unfettered action—these alone, Bergson will tell us, can sound the depths of fugitive being.

NOTES TO CHAPTER III

[1] "Outlines of the Science of Energetics," *Proceedings of the Philosophical Society of Glasgow*, 1848–55, vol. iii. p. 382.

[2] *Die Analyse der Empfindungen* (5th ed., Jena, 1906), chap. i.

[3] "Aus den Empfindungen und durch deren Zusammenhang entspringen unsere Begriffe," *Erkenntnis und Irrtum*, 1st ed. p. 142.

[4] *Erkenntnis und Irrtum* (1st ed., Leipzig, 1905), p. 162.

[5] *Die Mechanik in ihrer Entwicklung* (4th ed., Leipzig, 1901), p. 78, etc.

[6] *Op. cit.* p. 80 ff.

[7] *Die Prinzipien der Wärmelehre* (2nd ed., Leipzig, 1900), p. 437 ff. The same thesis is maintained by Andrade, *Revue de mét. et de morale*, March 1899, p. 178.

[8] *Die Mechanik*, etc., p. 529 ff.

[9] *Op. cit.* p. 543.

[10] *Op. cit.* p. 531 ff.

[11] *Op. cit.* p. 531 ff.

[12] *Op. cit.* p. 513.

[13] Mach only looks upon the shortest times as sensorial data; periods of longer duration are constructed by a process of reflection.

[14] *Die ökonomische Natur der physikalischen Forschung* (Vienna, 1882), reprinted in the *Populär wissenschaftliche Vorlesungen*.

[15] "Diese Gleichungen oder Beziehungen sind also das eigentlich Beständige" (*Die Prinzipien der Wärmelehre*, 2nd ed. p. 424). "Jede physikalische Beständigkeit kommt schliesslich immer darauf hinaus, dass eine oder mehrere Gleichungen erfüllt sind, also auf ein bleibendes Gesetz im Wechsel der Vorgänge" (*ibid.* p. 342).

[16] *Einführung in die Philosophie der reinen Erfahrung* (Leipzig, 1900), pp. 27 ff., and 55. Lalande too makes an analogous criticism of the principle of causality, which he regards as a vague and incomplete idea, a rough approximation, for which the principle of continuity and mathematical identity should be substituted ("Remarques sur le principe de causalité," *Revue philosophique*, 1890, vol. ii. p. 225).

[17] *Op. cit.* p. 37.

[18] *Op. cit.* p. 40 ff.

[19] *Op. cit.* p. 44.

[20] Avenarius learnt the strict method of physiological research in Ludwig's school at Leipzig, and was initiated by Drobisch into Herbartian philosophy, whence he derived the notion of the inertia of apperceptive masses being due to the need of conservation of the mind, which, when interpreted physiologically, harmonised with the new conceptions of Darwinism.

[21] *Philosophie als Denken der Welt gemäss dem Prinzip des kleinsten Kraftmasses* (2nd ed., Berlin, 1903), p. 3 ff.

[22] *Op. cit.* p. 12 ff.

[23] *Op. cit.* p. 24.

[24] *Op. cit.* p. 38.

[25] *Op. cit.* p. 46 ff.

[26] *Op. cit.* pp. 52-54.

[27] *Op. cit.* pp. 56-62.

[28] *Kritik der reinen Erfahrung* (Leipzig, 1888–91), vol. ii. p. 238 ff.

[29] *Op. cit.* vol. ii. pp. 218-222.

[30] *Op. cit.* vol. ii. p. 204 ff.

[31] *Einführung in die Philosophie der reinen Erfahrung* (Leipzig, 1900), vol. i. p. 198. Petzoldt has modified the theory of Avenarius on the

subject of logical, ethic, and aesthetic values. Avenarius looked upon these values as derived from the function of language, by which the psychic facts of one individual assume a different character of a secondary or epi-characteristic order when communicated to another person: thus, for instance, immediately experienced pleasure and pain become aesthetic pleasure and pain when communicated to us by others; knowledge, communicated through the medium of language, becomes science, true, if the communication be made by a reliable person, false in the contrary circumstances. Petzoldt, on the other hand, regards these values also as primary and fundamental.

[32] Petzoldt lays special stress on this tendency to stability as a motor force of evolution (1904, vol. ii. p. 72 ff.). The principle of the minimum effort of economy is in its essence a tendency to stability (*ibid.* p. 94).

[33] Avenarius, *op. cit.* vol. ii. p. 302.

[34] *Der menschliche Weltbegriff* (2nd ed., Leipzig, 1905), p. 5.

[35] *Op. cit.* p. 8.

[36] *Op. cit.* p. 26 ff.

[37] *Op. cit.* p. 76 ff.

[38] *Op. cit.* p. 82.

[39] *Op. cit.* p. 84.

[40] *Op. cit.* p. 93 ff.

[41] *Op. cit.* p. 110 ff.

[42] *The Metaphysic of Experience* (London, 1898).

[43] *Die Erkenntnis der Naturforschung der Gegenwart* (Leipzig, 1905), pp. 24-30.

[44] *Op. cit.* p. 43 ff.

[45] *Op. cit.* p. 9.

[46] *Op. cit.* p. 141.

[47] *Op. cit.* p. 124.

[48] *Op. cit.* p. 141.

CHAPTER IV

ENGLISH NEO-HEGELIANISM

1. *Two Attempts at Escape from the Agnostic Position.*—Thinkers have tried to escape from the agnostic position in two different ways: one, whose course we have followed through neo-criticism and empirio-criticism, aims at the critical elimination of the problem by the reduction of all reality to phenomena only, and the dismissal of that Absolute which appeared to baffle knowledge, a proceeding reminding us of a child who imagines that by shutting his eyes to something of which he is afraid he can destroy it; the other is the return to that speculative method which positivism had vainly endeavoured to replace by science. Some of these speculative attempts, which were inspired by post-Kantian idealism, have been already treated in their relation to neo-criticism; this applies more especially to those which are closely connected with the teaching of Fichte, Schelling, and Schopenhauer, and which are more or less deeply tinged with romanticism and irrationalism; we must now sketch the outlines of that movement of thought which arose in England in opposition to traditional empiricism and its ultimate tendency, agnostic positivism, claiming to be able to supply that which was lacking in scientific intellectualism, and reaching in the works of Hegel a higher form of rationalism.

2. *The Eternity of Thought, as Affirmed by Green in Opposition to Empiricism.*—The philosophy of Thomas Hill Green [1] appears to be a reaction from the empiricist

and psycho-genetic method which had for centuries been the predominant feature of English philosophy,[2] a righteous vindication of the eternity of consciousness and thought against those who would fain regard it as a contingent phenomenon, having its origin in time, and doomed to vanish in time. It must be said in his favour that he is not carried away by the facile enthusiasm for the new theory of evolution, and that he clearly saw the *petitio principi* concealed in every alleged biological explanation of consciousness.[3]

The world of nature and experience, in so far as it is a series of inter-connected facts, presupposes the conscious and intelligent principle which is supposed to be derived therefrom;[4] an experience without a subject is an epistemological absurdity, just as would be an eternal system of relations (such as the physicist's conception of the world) without an Eternal Thought to impart reality to that system. The consciousness of change cannot be in its turn a process of change, since it must be present at all stages of that process; experience of a series developing in time presupposes a conscious principle external to time, and hence not of natural origin.

We cannot conceive of any reality external to this Eternal Thought which comprehends within itself the whole system of objective relations: the dualism of Kant, according to which the form of phenomena, *i.e.* their relations, is derived from the intellect, while matter, *i.e.* sensations, takes its rise in some mysterious source beyond all thought, is therefore inadmissible.[5] Kant's error lies in assuming as a possibility the existence of a formless sensation, not qualified by thought; whereas every form of experience implies at least the distinction between the actual fact and the preceding moment, and hence an intellectual reference. If everything be eliminated which can be expressed in terms of relations, no reality will remain. If we divest our knowledge of a thing of every relation, that is to say of every thought, not even simple consciousness is left, since consciousness cannot exist where change and difference cannot be

noted, and where there is no relation of sequence and intensity between the sensation experienced at the moment and those preceding it.[6] In the most elementary act of perception we establish a relation between terms which can only be given in and by virtue of relations, and that which enters into this conscious relation is not sensation as such, but the fact that *the sensation is felt*.[7] If, for instance, I recognise the action and presence of the fire in my vicinity, that which forms an integrant part of my knowledge is not the impression of heat, but merely the idea that I feel warm. This is proved by the fact that if I go farther away in order to make sure that the heat is produced by the fire, the impression of heat diminishes in intensity, whereas the perception of the scorching fire does not become more precise or undergo any change. Further, a too intense sensation does not act as an aid to knowledge, but rather as an impediment in its path, and, whilst the impression is perpetually subjected to a process of transformation, the fact conceived of its existence remains always the same. For instance, the sensation of red conveyed by a lady's sunshade may vary in intensity, but there is no change in my knowledge of the fact that the sunshade is red in this determinate way. Knowledge in its ultimate analysis consists of relations, and experience, when all is said and done, is but a manifold of thought relations. If it be impossible to derive thought from sensation, as the empiricists do, the inverse procedure is equally unjustifiable, because, just as there is no such thing as pure sensation, there is no such thing as pure thought: these two phrases merely stand for abstractions to which there exists no corresponding reality either in the facts of the world or in the consciousness to which these facts stand in relation.[8] Sensation and thought do not exist independently of each other, but are two inseparable aspects of the same living experience.[9] By this we do not mean that all sentient animals must also be capable of thought: the relations from which the reality of their sensorial life is derived do not exist for their con-

sciousness, but for the Absolute Thought, in which the component relations of phenomena exist to all eternity, even when empirical consciousness is not aware of their presence. So long as we feel without thinking, the world of phenomena is non-existent *for us*, yet we possess a certain form of existence, since, even if the relative sensations be not real facts for our consciousness, they yet exist in the consciousness of the Absolute.[10] The action of the mind does not consist in abstracting certain attributes from things as presented to us by experience, thus mutilating experience and rendering it barren; it is rather thought itself which constitutes the attributes and makes them into objects by colligating them with one another. Knowledge does not pass from the concrete to the abstract, from rich and full perception to poor and empty conception; on the contrary, it passes from the universal to the individual. The categories do not stand at the end, but at the beginning; they are not ultimate truth, but rather that which we apprehend in even the simplest perception; they are the most universal and primitive of relations, by means of which we create objects in order of progression, determining them by means of relations which grow more and more numerous and exact, until we attain individual concrete ideas possessed of a greater wealth of synthetical relations. Knowledge goes through two phases, the one spontaneous, the other reflective: in the former we pass from the universal to the individual, and interpret things according to the laws governing our mental activity without being aware that we are doing so; in the reflex phase we retrace our steps from the concrete to the abstract, defining clearly the relations existing between individual objects; these relations are, of course, not derived from experience as such, but rather from that which we ourselves have unwittingly introduced into it in the first stage of knowledge. Empiricists leave this second phase out of their reckoning, and ignore the activity of the mental principle in the spontaneous construction of the world.

The difference between conception and perception, the imaginary and the real, the general abstract idea and the individual concrete presentation, amounts to this: that in the case of perception we have, in addition to the conceived relations which constitute the idea of the object, the thought that this idea is or has been felt; whereas in pure conception relations are considered independently of the impressions which they determine, *i.e.* of the fact that these impressions are or are not present. In the case of the single concrete idea, there is but one actual or possible impression, determined by a network of relations which are extremely numerous, and which have been noted more or less vaguely from the first; whilst simpler and more general relations may be equally well verified by a large number of relations without determining any one of them. A perfectly adequate conception of the conditions of a phenomenon would therefore in no way differ from its reality, since it would of necessity include amongst those conditions the relation that the phenomenon can be and is felt.

If objects exist only by virtue of their relations, relations in their turn are possessed of no consistency apart from the harmonious system of all relations, towards which of its very nature thought must tend. The consciousness of a unique system of relations at once universal and coherent is the criterion of truth and reality to which we unconsciously look even in our most elementary acts of judgment: a relation is real and true when it is in logical agreement with the whole manifold of known and knowable relations, and is false when it contradicts them.[11] Macbeth, when he imagines that he sees a dagger before him, is deceived because he has established false relations between his own actual sensations and other sensations, relations, that is to say, which are out of harmony with the whole system of relations constituting the universe. All our researches into the objective nature of appearances have one and the same aim—the discovery of an unchangeable order

of relations, a complete system having nothing external to itself. The unity of the system, *i.e.* the unity of nature, is presupposed in all knowledge, and is the basis and gauge of its certainty. In mathematics this certainty is undoubtedly more stable, and rests upon a surer foundation, but we are not therefore justified in placing the exact, *a priori*, necessary science in opposition to the *a posteriori* and contingent natural sciences. In reality, mathematics, like other sciences, is the result of experience, in the sense that it consists in the analysis of the unconscious products of primordial mental creation, and that it rediscovers in things the relations unconsciously infused into them by thought. Its one and only claim to superiority lies in the fact that it is based on the simple and general conditions governing the existence of natural objects, that is to say, on quantitative and spatial conditions, of which it is possible to conceive apart from all others. The natural sciences, on the other hand, are not contingent, as is thought by those who place them in opposition to mathematics, since induction is not based on experience, analogy, or custom derived from many repetitions, which could never be sufficient authority for laying down a universal law. We do not pass from the known to the unknown, since such a transition would be unintelligible, nor from like to like, since we should have no authority for such a transition, but from identical to identical. In order to assert that that which has been recognised as true in one case holds good of a whole class, we must know that all the cases in question, whether they have come under observation or not, are identical as regards a certain aspect, that is to say, as regards that relation at all events to which the present induction refers.

The conditions of a natural phenomenon are extremely numerous and are never repeated, hence it follows that at times some of them may escape us; a geometrical problem, on the other hand, depends only upon conditions with which we are thoroughly acquainted, therefore

in the one case we attain unconditioned, in the other conditioned, truths. Our knowledge of nature is constantly being extended by connecting facts which are increasingly coherent in their nature, and co-ordinating relations which tend to become more and more complex, and the proof and criterion of the truth of the simpler relations are to be found in the system which harmonises them in itself. The falsity of a theory can only be demonstrated by proving it to be inexplicable; that is to say, by showing that it cannot be connected with other groups of relations. The uniformity and unity of nature become more and more evident the more closely we enquire into it, but, on the other hand, we cannot investigate it without believing it to be already uniform and one, and without implicitly admitting that nature constitutes in itself a unique system of relations which condition each other *ad infinitum*, presuppose and imply one another *ab aeterno*, of which individual objects are but the ultimate consequences and combinations, that is to say, without presupposing that nature has a significance present in its totality to Absolute Thought.[12] Our consciousness, being subject to the limitations of time, cannot fully grasp this significance or identify itself entirely with the Divine Mind, but all human knowledge presupposes this significance, of which our knowledge is the gradual revelation in time. In the interpretation of the great book of nature, in which the Thought of God is revealed to the soul of man, the same thing occurs which each one of us may observe when reading a sentence or phrase; single words succeed one another by means of a process developing in time, but the thought that the whole sentence or phrase must have a meaning is present with us from the moment we begin to read, and, when we have reached the end, this meaning is present to our consciousness as a simultaneous whole, not as a series of successive elements.[13] Thus, although the psycho-physiological organism may develop empirically in time, our thought in the act of grasping universal

relations places itself outside time, and shares in the Absolute Thought. That which we term our mental history is not the development of this eternal aspect of consciousness by which we are made one with God, and which is not subject to development in time, but is rather a history of the process by which the animal organism becomes the vehicle of this development. The empirical consciousness in its incessant evolution and its interruptions and disturbances should not make us forget its eternal element, that Absolute Thought, which is consciousness of time, but which is not itself in time; which is consciousness of becoming, but escapes all change.[14]

3. *Criticism of Green's Pan-logism.*—Green's philosophy with regard to scientific research differs widely from the empty dialectic of Hegel, which alleged that all the determinations of nature could be constructed out of nothing by means of artificial negations. English Neo-Hegelianism lays no claim to the place of science; its aim is rather to integrate the fragmentary results attained by science, and to find amid the isolated laws and supreme categories of the real the ultimate tie of necessity binding them together in thought. In this direction Green's epistemology makes a notable step in advance on the logic of Hegel, but he does not succeed in shaking off the prejudice of pan-logism, and persists in the assertion that living concrete reality can be reconstructed by means of a system of abstract relations. There is something in the psychic fact as immediately experienced which no effort of dialectic can ever identify with a system of conceptual relations; sensations, feelings, passions, impulses, volitions as they are given in the concreteness of the human personality, are possessed of an individual aspect which cannot be foreseen, and which, as we shall see, plays into the hands of the opponents of intellectualism. Green asserts that there is no difference between conceiving and feeling, that it will suffice to add to the idea of horse, for instance, the relation of being felt, for the concept of horse to be

transformed into the perception thereof; it is, however, one thing to think of feeling and another actually to feel.

Green, however, passes with the greatest ease from the concept of the sensation to the sensation itself, without perceiving that between the two there is an impassable gulf fixed. We may think that all the conditions necessary to the verification of a sensation have been fulfilled, but this will not make us feel. If it were so, the blind could restore their own sight by studying a treatise on optics! Nor can the sophistical argument be adduced that one of the essential conditions is lacking, namely, the normal structure of the eye, since, according to Green, the complete concept of this structure should suffice to transform the idea into a real fact. The intuitionists cannot succeed in accounting for the constant and universal in reality; Green goes to the opposite extreme, and places himself in a position which prevents his understanding that which is individual, concrete, and changeable in the history of the world. From his point of view, indeed, there is no such thing as time, there exists merely the concept of time, which is something external to time; there is no such thing as change, but only the idea of change, which is external to it; there is no such thing as an individual, but only the concept of one which by its very nature must of necessity be universal. If reality then be a network of eternal relations, present in their totality to the Absolute Consciousness; if even a human person be but a fragmentary group of those relations, does it not become impossible to explain the evolutionary motion of things, their incessant transformation, and everything which is most spontaneous, living, and fruitful in the concrete development of our inner life? In the eternal immobility of the idea, time with its efficacious rhythm, and the world as a whole, become, to quote Bradley, merely an illusory appearance. But is not the birth of this illusion too an inexplicable mystery, if human consciousness be a web of unchangeable

relations ? How did it issue from the eternity of thought in order to project itself into time ? Moreover, if the Absolute live in it, must not its every idea perforce be true ? In Green's philosophy there is no room for error and illusion, which can only be understood if we admit a certain degree of independence and spontaneity in the individual subject as against Absolute Consciousness. We can form no conception of objective reality as a system of relations, complete, fixed, and unchangeable from all eternity, since every new form which makes its appearance, every individual in his concrete physiognomy, becomes the centre in cosmic evolution of a fresh network of countless relations which tend to become more widely extended, more interwoven, and more complicated in successive moments. The reduction of the Absolute to the eternal contemplation of ideas eternally present to its consciousness amounts to the same thing as turning it into a *caput mortuum*, like the impassive deities of Epicurus in their blissful ease. We cannot conceive of a consciousness which is not life, development, perennial creation, and fruitful activity, nor of a thought which cannot be enriched by new relations, whilst preserving the coherency and identity of its fundamental laws ; nor of any form of spirituality without qualitative development, or which is not manifested in original actions which cannot be foreseen. Pan-logism aims at the absorption of everything into a system of eternal relations, and must therefore inevitably end in denying the reality of that which is most vital and most concrete in the world of consciousness. If its premisses be granted, Bradley's philosophy is the necessary conclusion.

4. *The Reductio ad Absurdum of Pan-Logism in Bradley's Philosophy.* — Bradley[15] maintains that the world, as given to us by experience and as constructed by science in its concepts, is but an illusory appearance of a deeper reality of which philosophy should strive to sound the depths by speculative methods, after having exposed the contradictions latent in the

world of appearances. The concepts of which physical science makes use lend themselves perfectly to the determination of limited phenomena, but lead to contradictions when we attempt to use them to express the true essence of reality; they are relative concepts, characterising things in relation to and in comparison with other things, but they can tell us nothing of the terms of those relations; they are *working ideas*, which are of no theoretical importance, but have only the value of useful fictions, practical compromises.[16] Bradley starts from the principle of contradiction,[17] which he regards as the supreme criterion of every reality, and, acting on the strength of this, the one and only article of his logical code, considers himself entitled to administer summary justice to all scientific concepts and intellectual categories — substance, quality, relation, space, time, movement, change, causality, force, activity — which from his point of view imply contradictions and hence cannot correspond to anything real. Let us, for example, consider the relation between the thing and its properties: its substance is not identical with any one of them; what is it then? Merely a link connecting its qualities; but what do we mean by the assertion that one quality is related to another? Neither of them is identical with the other or with the relation to the other; thus the number of contradictions is augmented rather than diminished. Quality does not exist apart from relations, but relation in its turn is not conceivable except as existing between qualitative terms: on the one hand, it would appear that quality is the result of relations, since qualitative difference cannot exist apart from a process of distinction; on the other, it would appear that relations in their ultimate analysis are forms of quality. Nor is it of any avail to draw a distinction between two elements in a quality, one pre-existent to the relation, and rendering that relation possible; the other resulting from the relation itself; since we should have to explain the mutual relation of these two elements, both belonging to one

and the same thing; we are, that is to say, confronted with the problem of a new relation no less inexplicable than the first, and so on *ad infinitum*. The relation cannot be identified with the things related, and, taken by itself, is nothing.[18] No less contradictory are space,[19] time,[20] movement,[21] activity,[22] causality,[23] etc., because when these concepts are resolved into various combinations of qualities and relations, the difficulties set forth above will arise afresh. The fundamental concepts of the special sciences are then mere appearances due to faulty perspective, which must be eliminated by rising to a higher experience embracing all possible appearances transfigured to a greater or less degree in an integral harmonious system; these illusions, however, are possessed of a certain degree of reality; there is no such thing as a truth which is entirely true, just as there is no such thing as an error which is entirely false; we can only speak of a greater or lesser degree of truth; error is partial truth, it is false only because it is one-sided and incomplete. All appearances are real in some way or other, and to some extent, and the right modifications and transformations (*supplementation and arrangement, addition, qualification*) may bring them too into the system of the Absolute.[24] In like manner every finite truth, like every fact, must be to a certain extent unreal and false, and the unlimited nature of the unknown renders it impossible to determine with certainty in the last analysis the proportion of error contained therein. If our knowledge were a system, we could determine the position of each thing in the whole, and gauge accurately the proportion of truth and error contained therein, but the nature of our knowledge renders such a system out of the question.[25] Thought originates in the separation of the *what*, the ideal meaning, the predicate, from the *that*, immediately felt existence, the subject; error, falseness, lies in uniting a *what* and a *that* which do not correspond to each other. In the harmony of the whole, each *what* will find its proper *that*, and every illusion

will consequently vanish; the ideal will coincide with the existent, the intelligible with the thing given. This is impossible to our finite consciousness, which develops in time and has its being in the world of appearances derived from the separation of idea from fact; notwithstanding this, we can approximate in some fashion, and with varying degrees of success, to the total harmonic system by striving to eliminate the contradictory element in phenomena, and to render our thought more coherent and more complete.[26] The ideal at which knowledge aims is the re-union of idea and fact—an ideal which it can never fully realise; thus its efforts in this direction imply a latent contradiction, since, on the one hand, knowledge is only possible in virtue of the distinction between the *what* and the *that*, the predicate and its subject, which are elements indispensable to the judicial function; whilst on the other, its development and perfecting should lead to the elimination of this distinction. There can be no clear and full understanding of truth with this distinction between data and their ideal significance; the moment this difference vanishes, truth ceases to exist and knowledge gives place to the true and real life of the Absolute.[27] Truth and knowledge are then but illusory appearances, like everything else which implies the separation of idea from fact, and they tend to transcend the bounds of intellect, and to become fused in a form of intuition and universal life of which we can hardly form an abstract idea, an immediate concrete experience, in which all the elements—sensation, emotion, thought, and will—are fused into one comprehensive feeling.[28] Finite beings cannot enter into the fulness of the life of the Absolute, or have specific experience of its constitution, but human consciousness can form a certain idea of it by retracing its steps to that primitive and diffused feeling to which the distinction between subject and object, and the differentiation of elements was as yet unknown. This intuition, which must embrace and harmonise the various phenomenal aspects

of consciousness, will, intellect, imagination, which, considered separately and postulated as absolutes, give rise to contradictions, although it possesses the immediacy of feeling, is nevertheless not subject to the limitations of every kind of distinction and relation as feeling is, but transcends all distinctions and relations, and therefore contains them in a higher unity within itself.[29] It is a form of psychic or spiritual experience (*sentient experience*),[30] because there can be no reality external to the mind, and the truth of a thing is in proportion to its spirituality;[31] but the modes of conscious experience are too one-sided for any one of them to give us the immediate intuition thereof, hence we must rest content with forming an abstract conception of it by, so to speak, "passing to the limit" of the various appearances.

5. *Criticism of Bradley's Dialectic.* — Pan-logism thus ends in an act of apostasy, and its dialectic leads to its own annihilation in a form of mystical intuitionism, whose static and contemplative character distinguishes it from that of Bergson.[32] It is the conception of the one unchangeable and eternal being of the ancient Eleatic philosophers, as opposed to the perennial flux of Heraclitus, and the inevitable end of those who give themselves over to the hollow dialectic of reason divorced from its vital content and articulated in rigorously identical formulas. What then is left of reality? A principle devoid of life and motion, something which has not even the logical coherence of our thought, since this thought is only valuable and important in so far as it opposes itself to the fluctuations of experience and assures the stability of concepts amid the manifold changes of images. Removed from this environment, its function ceases to be possible, and thought itself is arrested and vanishes into nothingness. The law of identity, if it is to be of any efficacy and value, must be applicable to a multiplicity in which the movement of thought is developed; if it be divested of such a content, it ceases to be conceivable. The Absolute, as a mere identity of

permanence, is something the reality of whose existence is beyond our power of thought. Moreover, even if we admit that it *is* possible to think of it as a limited concept, it will still be incomprehensible how such perfect identity can give birth to the illusion of multiplicity, or an inviolable law of permanence to the phantasmagoria of a world in process of evolution.

It is useless to say that change is illusory, since we still have to explain how the illusion arises, because, even if it be nothing else, it is a psychical fact which we experience directly, and whose existence is consequently undeniable. Our thought refuses to admit that if the law of things is a perfect identity the manifold content of consciousness with its unceasing transformations can be derived therefrom without the imperturbable inflexibility of being undergoing any change. Either it is unrelated to phenomenal occurrences and abstract, lifeless unity remains immovable to all eternity in its ataraxy, in which case it is a *caput mortuum* with which the world of phenomena can readily dispense; or it must be the adequate reason of the constant renewal of consciousness and experience, in which case it cannot preserve its fixity of quietism. But, Bradley would urge, even the change of finite consciousness in time is illusory in so far as it assumes the separation of fact from idea, the *that* from the *what*. What authority have we, then, for forming such an opinion of it? The principle of contradiction is only of value as the law governing the judgment, and implies the distinction between two concepts standing in a definite relation to each other, that is to say, they must be such that one excludes the other; now, according to Bradley, judgment and the distinction between conceptual terms are appearances relative to our finite point of view; hence even the principle of contradiction can be but a law of appearances, an illusory law; how can it, then, be set up as a criterion of absolute reality? Who can guarantee that this law is applicable to absolute reality, and is not rather an error of perspective like

the rest? May not Hegel be right in assigning to contradiction a place in the very heart of the idea, and in looking upon it as the germ from which its development springs? When we assert with Bradley that finite thought is an appearance, we can no longer consistently regard the principle of contradiction as an absolute criterion any more than any other logical axiom. If logical principles be set up as judges of reality, we grant by implication the value of finite thought, judgment, and human reason. On the other hand, if the unconditioned value of the axioms be granted, it is not a necessary inference, as Bradley, following in the footsteps of Herbart and the ancient Eleatics, would have us believe, that movement and change are illusory, because they contradict the laws governing our thoughts. Multiplicity and transformation, as we receive them directly from intuition, are not contradictory in themselves, the contradiction exists only between our one-sided concepts. We may resolve movement as presented to us by intuition into abstract elements, but in that case we must bear in mind that each one of these elements is but a partial view, a limit which we ourselves have laid down in order to facilitate analysis and research, and which does not correspond to any real division. Thus, on the one hand, if we isolate certain persistent and uniform elements from the process of change which we apprehend by means of immediate intuition, we may state that body remains unchanged; if, on the other hand, we look at it from another point of view, and introduce only varying elements into the idea, regarded in abstraction from the complex of intuitive data, we should be led to a diametrically opposite conclusion. Who, however, can fail to see that the opposition in such a case is the artificial creation of our partial view? Change is in no way contradictory to the principles of human intellect; it is true that the birth of the new amid the unceasing flux of experience eludes the grasp of our abstract concepts, which are constrained to sacrifice the wealth of mind and nature

on the altar of their universality and identity, but even if concrete becoming cannot be adequately transcribed into terms of abstract thought, it does not follow that it is illogical. The contradiction vanishes when we substitute intuited reality in its fulness for incomplete abstract terms. In like manner will vanish the other alleged contradictions seen by Bradley in the exercise of thought; if thought appear to be overwhelmed by a flood of absurdities, it is because its supreme categories have been divested of all intuitive content, and are then supposed to fulfil their functions in the resulting void. The activity of judgment may be explained by establishing relations between terms which are not wholly the creation of thought, as is asserted by Green, but always have their source in intuitive data; there must necessarily be a limit to the resolution of terms into relations, since there is always something left which cannot be translated into relations. Thus, to use Bradley's illustration,[33] there is nothing contradictory in the relation between the two properties of whiteness and sweetness in sugar, since the two terms do not exist merely by virtue of the relation which we set up between them, but exist also in as much as they are immediately felt. Their relation is rendered sufficiently consistent by the unity of the subject, even though it be impossible to identify it with either of the two terms. The absurdity arises only when the relation is separated from the subject, and considered as an entity in itself, a thing. In like manner the necessary basis of the relations of succession implied in the concepts of cause, action, force, energy, etc., will be found in the continuity of the epistemological subject, hence it is not surprising that they should give rise to contradictions if we isolate them from that subject. Nothing but the continuous presence of the subject can bridge over the gulf between one term and another, and enable the intellect to grasp the relation between an antecedent which has ceased to exist and a certain consequence which has not yet come into existence. Duration, extension, action, and change

only become contradictory when looked upon apart from the living continuity of consciousness, and, even though we cannot succeed in re-constructing them analytically in their concreteness by means of pure acts of thought without being confronted by insurmountable difficulties, it does not follow that either they or thought are empty appearances, but only that logical relations do not exhaust the whole of reality in every concrete moment of consciousness, and that there exists an individual physiognomy of the world which cannot be reduced to mere systems of relations.

6. *Mystical Degeneration of English Neo-Hegelianism: McTaggart.*—English Neo-Hegelianism, after striving in vain in the teaching of Green to dispose of the agnostic position of intellectualism by absorbing into an eternal system of relations those irreducible elements which are ignored by scientific knowledge, degenerates with Bradley into a form of scepticism and intuitionism. The mystical degeneration of Hegelianism is still more marked in McTaggart, who no longer regards dialectic as the very life of the Absolute, as did Hegel, but considers it to be merely a subjective means for the re-construction of the eternally perfect system of individual minds, whose harmonious synthesis gives birth to the Absolute, by disposing of the abstract appearances of the reality which develops in time.

This ultimate synthesis of reality cannot be attained by discursive thought, which is unable to reconcile perfect and imperfect, temporal and eternal, the *Ego* and the *non-Ego*, experienced immediacy and mediate or rational knowledge, but can be reached only in the state of love in which other beings lose their exteriority and appear to us in the very form of our *Ego*.[34] English Neo-Hegelianism thus ranks sentiment above reason, and aims at the pole of convergence of contemporary philosophy, the denial of the cognitive value of intelligence, and the search for some more direct means of penetration into reality. Thus the many and various currents of thought which spring

from the irresistible longing to burst the bonds of agnosticism which stifle the most living aspirations of the soul, mingle their waters in the wild, rushing torrent of the reaction from intellectualism.

NOTES TO CHAPTER IV

[1] Green published nothing in his life-time but articles in reviews and an Introduction to the works of Hume (1874–75). After his death his *Prolegomena to Ethics* was edited by A. C. Bradley and published in 1883, and his complete works were collected and published in three volumes by Nettleship (Longmans, Green & Co., London, 1885).

[2] Hume, *A Treatise on Human Nature*, edited by T. H. Green and T. H. Grose (new edition, London, 1878). The "Introduction" prefixed by Green to vol. i. is a criticism of the philosophy of Locke (pp. 5-132), Berkeley (pp. 133-151), and Hume (p. 161 to end).

[3] "Mr. Herbert Spencer and Mr. G. H. Lewes: Their Application of the Doctrine of Evolution to Thought," *Contemporary Review*, 1877–78, vol. xxx. pp. 25-53, 745-69; vol. xxxii. pp. 751-772.

[4] ". . . Experience in the sense of a consciousness of events as a related series . . . cannot be explained by any natural history, properly so called. It is not the product of a series of events" (*Prolegomena to Ethics*, Oxford, 1899, 4th ed., p. 25. See also his introduction to Hume, p. 164 ff.).

[5] *Prolegomena to Ethics*, p. 45.

[6] *Op. cit.* p. 54 ff.

[7] *Op. cit.* p. 75 ff.

[8] "We admit that mere thought can no more produce the facts of feeling than mere feeling can generate thought" (*op. cit.* p. 60).

[9] *Op. cit.* p. 58.

[10] *Op. cit.* p. 57.

[11] *Op. cit.* p. 18 ff.

[12] *Prolegomena to Ethics*, p. 34, p. 62 ff.

[13] *Op. cit.* p. 85 ff.

[14] *Op. cit.* p. 80 ff.

[15] *Principles of Logic* (London, 1883); *Appearance and Reality* (London, 1893).

[16] *Appearance and Reality*, p. 284.

[17] "Ultimate reality is such that it does not contradict itself: here is absolute criterion" (*op. cit.* p. 136 ff.).

[18] *Op. cit.* p. 20 ff.

[19] *Op. cit.* p. 35 ff.

[20] *Op. cit.* p. 39 ff.

[21] *Op. cit.* p. 44 ff.

[22] *Op. cit.* p. 62 ff.

[23] *Op. cit.* p. 54 ff.

[24] "Error is truth, it is partial truth that is false only because partial and left incomplete" (*op. cit.* p. 192). "Error is truth when it is supplemented" (*op. cit.* p. 195).

[25] *Op. cit.* p. 54.

[26] *Op. cit.* p. 364 ff. According to Bradley, the appearance which

most nearly approximates to reality and is possessed of the greatest proportion of truth is the one which demands the least *addition* and *rearrangement* for its conversion into the Absolute.

[27] *Op. cit.* pp. 163 ff., 361, 545 ff.

[28] *Op. cit.* p. 227. "For our Absolute was not a mere intellectual system. It was an experience overriding every species of one-sidedness, and it was a living *intuition*, an immediate individuality.

[29] *Op. cit.* p. 242.

[30] *Op. cit.* p. 144.

[31] *Op. cit.* p. 551.

[32] F. C. S. Schiller in an article entitled: "Mysticism *versus* Intellectualism," published in *Mind* (January 1913, p. 87), protests against my interpretation of Bradley's philosophy. He is under the impression that I intended to say that Bradley wilfully and deliberately reduced English Neo-Hegelianism *ad absurdum*; it is, however, obvious that I merely assert that Bradley's dialectic unconsciously reduces pan-logism *ad absurdum*, and on this point Schiller and I are really agreed.

[33] *Appearance and Reality*, p. 20.

[34] *Studies in Hegelian Cosmology* (Cambridge, 1901), pp. 270-292; *Studies in Hegelian Dialectic* (Cambridge, 1896), pp. 214-276.

SECTION II

THE REACTION FROM INTELLECTUALISM

CHAPTER I

THE DOCTRINE OF CONTINGENCY AND INTUITIONISM

1. *The Aesthetic and Moral Conception of the Universe: Secrétan, Ravaisson.*—Modern French philosophy results from the confluence of two great currents of thought: the philosophy of liberty, of which Kant's doctrine of the primacy of pure reason is the source, which vindicates the rights of will and feeling against the claims of the intellect; and the work of critical revision of science, of which Mach was the pioneer. It derives its moral and aesthetic conception of the universe from the philosophy of liberty as developed by Secrétan and Ravaisson, and its arguments against the necessity of law from the new criticism of science, thus striving to eliminate the antitheses existing between the intelligence and free will. It holds, as does Secrétan, that the essence of the world is an act of unfettered expansion, an act of love and infinite benevolence; hence the real tool of philosophy is, as Ravaisson clearly saw, artistic inspiration and feeling for religion, not definition and scientific analyses;[1] it is intuition which enables us to grasp the active substance of the *Ego*, and affords the irresistible evidence of feeling, evidence above all argument and all calculation. This sense of indefinite effusion, to which French philosophers of the new school look for the revelation of the Absolute, is neither Kant's Good Will nor yet the free affirmation of the *Ego*, governed by the imperious law of duty inherent in its very nature, as conceived of by Fichte, but rather

that feeling of untrammelled expansion and abandonment proper to creation and aesthetic contemplation. "Beauty," as Ravaisson had already said,[2] "and more especially beauty in its most divine and perfect form, contains the secret of the world." The cosmic process is not a mechanism of necessary and eternal movements, circumscribed within the inflexible limits of a system of mathematical formulae, as intellect, guided by science, had imagined it to be ; it is rather the perennial creation of a marvellous work of art, which from the rude outlines of the inorganic world has gradually evolved into the higher forms of spiritual life. The significance of cosmic evolution lies not in that to which natural beings have already attained, but in their higher aspirations and in the ideal towards which they more or less consciously tend. True reality is not the necessity of phenomena, as Kant imagined in his *Kritik der reinen Vernunft*, but the world as seen in the light shed upon it by the idea of beauty and liberty, as he beheld it in his *Kritik der Urteilskraft.*

2. *Émile Boutroux and the Contingency of Natural Laws.*—This concept of the moral and aesthetic finality of the universe is the dominant feature of the system of philosophy set forth by Boutroux, in which the two currents of which we have already spoken meet for the first time. According to Boutroux, the supreme principles of things are moral and aesthetic laws regulating the spontaneous activity of beings in their ascent to God. Natural laws are in themselves in no way absolute or eternal, they are merely the expression of a transitory phase which may be superseded or left behind ; they are but habits formed by the creature which, instead of going forward, rests content with forms already realised and tends to persist in those forms, in which it recognises the imprint of the ideal.[3] When the Good and the Beautiful are completely triumphant, these artificially acquired fixed images of an essentially living and flexible model will vanish, and necessary law will give place to the spontaneous effort of the will to

attain perfection and the free hierarchy of minds. The inner heart of things is manifested in the concrete work of imagination and in the spontaneity of will, not in the empty, stereotyped forms of the intellect. When we try to confine them within the bonds of abstract formulae of permanency, that which is most real in them — qualitative multiplicity in the inexhaustible wealth of its creations—eludes our grasp. Science then increases rather than diminishes the distance between ourselves and the inmost nature of reality. How and why, then, have the constructions of science come into being? Boutroux maintains that all the labour of science is but an attempt to adjust things to the law of identity of thought, and to make them carry out our will more readily.[4] We see the beginning of this work of adjustment in the realm of logic itself; concept, judgment, syllogism all contain something more than mere logical principles, *i.e.* the multiple as contained in the one, the relation between the explicit and the implicit, so that, while they are not exactly *a priori*, neither are they *a posteriori*: the mind of man contains within itself the laws of pure logic; but, since the matter presented to its notice does not appear to it to be strictly in conformity with these principles, it endeavours to adapt logic to things, and devises a complex of proceedings and symbols designed to make reality easier of apprehension.[5] The laws of pure logic are necessary, and there can be no doubt as to their objective validity; but their weakness lies in the very thing which constitutes their strength, since they fail to determine the nature of the things to which they apply. The art of syllogism in its turn, being a product of the mind, affords in itself no guarantee of objective validity, but the fact that our reasoning attains its end is a proof that there must be some relation between human intelligence and the nature of things in general. There must be something at the heart of things which, though it may not be intelligence such as that with which man is endowed, is yet possessed of some analogous property

or quality, a tendency towards intelligibility; hence reasoning represents a method of interrogation and interpretation which we may rightly apply to nature. Things tend to order, classification, and the relation of genus, species, and law; but, just as we ourselves are possessed not only of intelligence, but also of a complex of active faculties, so we may attribute a principle of activity and spontaneity to nature.[6] The intelligence is the rule of this activity, but it is impossible to say to what extent it is realised therein; knowledge of the special laws in question can alone afford us any idea of the degree to which logical necessity is active in nature.

As we advance by slow degrees from the abstract sciences to the more concrete, from the indeterminate to the complete determination of facts, we leave necessity and logical evidence farther and farther behind. Mathematical laws, which most nearly approach this evidence, cannot, however, be deduced from pure logical relations, as was thought by Leibnitz. Logic presupposes a given whole, a concept which we propose to analyse, and admits into it elements placed in juxtaposition without determining the connecting link between them; the work of mathematics is on the contrary essentially synthetic: it posits those relations which logic presupposes, creates a connecting link between the parts, generates the composite whole instead of assuming it as a datum. It thus introduces new elements beyond the grasp of thought, inasmuch as it constructs these relations of composition, differentiates the identical by means of intuition, and uses in its generalisations the argument from recurrence, that is to say, from complete induction. Even though mathematical laws be not immediately derived from the nature of the mind, and are not therefore *a priori*, we are not justified in affirming that they are *a posteriori* forms of knowledge, since they refer to limits which are not the result of experience, but represent purely ideal terms towards which tends a quantity which is supposed to increase or decrease

indefinitely. Mathematics must be regarded as an intelligent, voluntary adjustment of thought to things, a form which enables us to eliminate qualitative diversity, a mould into which reality must be poured in order that it may become intelligible, but we must beware of making it objective, if we would not risk falling into the absurdity of infinite actual number: the rock on which every system of mathematical realism has foundered. Idealism in its turn imagines that it can save the reality of mathematical laws by regarding them as the indispensable basis of the world of presentations which the mind projects out of itself; whereas these laws fail on the one hand to attain perfect intelligibility, and on the other, are not the only ones possible. We give them the preference merely because they afford a simpler and more convenient explanation of external phenomena than any others.[7] It is a fact that our mathematical system is applicable to reality, but we cannot say *a priori* to what extent it is so. Man is not an anomaly of nature, hence that which satisfies his intelligence cannot fail to be related to other things; we may therefore conjecture a correspondence between the laws of mathematics and those of nature, but experience alone can show us how far this analogy extends. The laws of reality which most closely resemble mathematical relations are the laws of mechanics, but these laws introduce a new element—force—which cannot be reduced to purely mathematical relations, since it contains the concept of physical causality. Many agree with Kant in his assertion that the notion of causal law is derived from our mental constitution, and is therefore *a priori* and necessary in order to bring phenomena within the scope of thought, and to unify them in consciousness. Now there can be no doubt of the need of unity, but it cannot be maintained that this need is greater than any other and is the motive power of our whole intellectual life; nor, on the other hand, can the categories be regarded as means suited to that end, since they leave things external and

extraneous to one another, joining them together in a purely artificial way like the stones in a building, whereas, if we would conceive of things, we must necessarily apprehend their relations and natural affinities, and see into what groups they fall, and how they are unified.[8]

Mechanical laws are not, however, the result of experience: we cannot observe the uniform and rectilinear movements of a body which is subjected to no extraneous influence, or the persistency in repose of a body to which no impulse has been given. We do not find continuity, precision, isolated relation, and the constancy of law in experience, they are one and all constructions of thought. Mechanical laws are, however, no arbitrary fiction, but represent the character we must ascribe to things if things are to be expressed by help of the symbols at our command, the matter which physical science must place at the service of mathematics in order to effect its union therewith. Moreover, facts prove that certain natural phenomena lend themselves to this need, so that the notion of mechanical law makes itself felt in all scientific research, at all events in the form of a guiding principle.[9] But we cannot think of mechanical laws as such being realised in the nature of things: the concepts from which they result cease to be intelligible when they are turned into objective entities. On the other hand, we cannot accept the idealistic conception of them as projections of the mind, since they bear witness to the existence of something differing from the mind, even though it cannot be absolutely severed from it. Things are possessed of certain characteristics which suggest the invention of mechanical laws and show a certain analogy with that which takes place in conscious life, more particularly in our habitual actions, which, although originally produced by the activity of thought, become detached therefrom and go on independently. Words follow one another without being determined by thought; our states of consciousness persist and act on one another with a certain inertia and mechanical

force analogous to that of things. We isolate the world of atoms and mechanical forces by means of a process of artificial abstraction, and look upon it as sufficient to itself; in reality, not only is it impossible to conceive of atoms and mechanical causality apart from a mind which thinks them, but also to isolate mechanical movement from the physical and organic phenomena existing in nature. Can we tell whether mechanical laws are the cause or the effect of other laws? [10] The mathematical element in mechanical laws cannot be applied strictly to reality, and the nature and cause of the experimental element they contain remain unknown. Physical phenomena cannot all be reduced to abstract mechanics in which all movement is reversible, whilst in concrete mechanics friction stands in the way of reversibility, as for instance in the case of the oscillations of a pendulum which should theoretically go on for ever, but which in experimental reality is retarded and finally actually arrested by the resistance of the air. Thus the principle of degradation of energy is in direct contradiction to the laws of reversible movement. All physical laws laid down by us are but the result of a process of abstraction, hence they cannot be a reflection of facts in their concrete complexity; we isolate certain elements in order to facilitate study, but can there be such a thing as a phase of nature which is sufficient to itself and is not susceptible to the influence of the other phases? Do all the qualities and forms of existence which we have eliminated in order to include phenomena in the system of our equations remain inactive, beyond measurable forms of magnitude, like the impassive gods of Epicurus above this world of ours? What guarantee have we that physical laws are not the result of evolution, just like the species of the animal world, and that their stability may not therefore be contingent and transitory? [11] The complication and degree of contingency are still greater in the case of chemistry, since that science admits elementary bodies differing as to quality. The services rendered

to chemistry by the atomic theory are of undoubted value since that theory affords a ready means of notation, but it can lay no claim to being a metaphysical revelation of the nature of things. The atom is certainly useful as a unit of classification, but we must beware of confounding metaphor and entity.[12] The only reason impelling us to believe in universal mechanism is, as Descartes rightly points out, the confidence we place in the truth of clear ideas and in their relation to reality.[13] The attempt to express everything proper to living organisms, to the processes of consciousness and the forms of social life, in terms of mathematics and mechanics is of necessity doomed to failure, since the qualitative element which mechanics treat as negligible is the dominant element in these phenomena, where spontaneity triumphs over automatism. Thus reality in its entirety cannot be reduced to mathematical elements, still less to the necessity of logical principles; as we descend from the lofty heights of pure reason to living, concrete reality, it will steadily become more and more impossible to comprehend existence in its multifarious aspects within the limits of our intellectual schemes. The necessity of law is rigorously valid only for logical principles; beyond these bounds liberty and contingency reign supreme over the processes of nature, and our efforts to assimilate things to our thought and to introduce logical necessity into them can never meet with a full meed of success. Science, the outcome of these endeavours, is therefore never of objective value, and, while failing on the one hand to satisfy our demand for evidence and logical universality, stands condemned on the other to leave external to itself that which is most real in things—their qualitative aspect, their incessant transformation, the act of perennial creation which is their very essence, just as it is the basis of the mind of man.

3. *Criticism of the Theory of Contingency.* — The philosophy of Boutroux is a legitimate reaction from the excesses of the mathematical spirit, which endeavoured

to reduce reality to a bare skeleton of empty formulas and looked upon everything as illusory which could not be thus classified. The world of sound, colour, and form in its varying moods is just as much an aspect of reality as are the constant and universal relations revealed to us by thought. Every moment of the life of the universe is a fresh revelation, hence the absurdity of the theory of Laplace that the whole future history of the world could be foreseen did we but know all the mechanical conditions at a given moment. Science does not and cannot foresee facts in their historic reality, but merely certain general characteristics of phenomena. If nothing new were ever to happen in nature, no evolutionary movement would be possible, since such a movement presupposes the birth of new forms, which, because new, cannot be foreseen. But although the fact in its concrete individuality cannot be forced to submit to the rule of universal law which results from the abstraction of the variable element and the individual aspect of things, it does not follow that it cannot be comprehended in the formula of law as far as certain of its characteristics are concerned which are subject to constant repetition. It is true that every fact contains a new element, but it always contains as well something which recurs, properties which persist in spite of variations in specific conditions, and can therefore be foreseen. Boutroux himself is forced to acknowledge that science assumes, is indeed constrained to assume, the existence in things of a certain tendency to intelligibility, although he tries to prove this tendency to be but an inveterate habit impelling Nature to repeat herself. The moment, however, habit presupposes repetition it ceases to be able to afford an explanation thereof: an action cannot become habitual and mechanical unless it has been repeated several times. It is nonsense to speak of habits in a world of ever new and original creations. The identity of the ideal after which every being strives affords no explanation of the repetition of the same actions unless a certain structural homogeneity and persistency in these actions be taken

for granted. If there were nothing constant in these beings, if their life differed completely from moment to moment, whilst yet aiming at the same ideal and striving to reproduce their previous work out of mere caprice, as Boutroux's bold imagination led him to suppose, the same actions could never be repeated, since neither their interior tendencies nor the means of actualising them would be the same as before. Habit demands the repetition of the same acts, and this repetition involves in its turn a certain persistency of previous conditions. Nor is this all; it further presupposes the principle of causality, since, should these conditions remain unchanged and yet not give rise to the same actions, habits could not be formed. Boutroux, in order to find some explanation of the success achieved by science, should in the last analysis admit the existence in reality of persistent and universal characteristics corresponding to the laws laid down by thought. Far from so doing, he arbitrarily reduces evidence and necessity to mere logical principles in their empty, abstract forms, and considers himself entitled to deny the characteristic of universal necessity to the formations of thought, since they contain elements which cannot be deduced from these pure principles.

4. *Milhaud and the Limits of Logical Certainty.*— Milhaud, following out Boutroux's line of thought,[14] asserts that the mind must renounce all certainty in the domain of reality, because the recognition of this certainty would involve the reality of an ideal condition which may be approximated but can never be attained, namely, the exclusion of all matter imposed upon the mind in the construction of the elements concerning which it reasons. The more nearly this ideal condition is fulfilled, thus justifying the use of the principle of contradiction, the more subjective, that is to say relative to ourselves and not to things, will be the knowledge resulting therefrom. In the sphere of mathematics, for instance, the more we gain in certainty by forming concepts of ideal elements, divested

of every sensible quality, purely mental structures, the more we lose in objectivity. The subjective character of logical certainty is obvious in the formation of concepts which synthetise certain properties of things but can never exhaust the entire content of reality; hence there will ever remain something which is not comprised in our concepts, and which may be revealed to us by future experience. If then our concepts be always of necessity incomplete, the principle of contradiction which holds good of them can never be a criterion of reality, since it is not impossible that the incompatibility of the thing with a given characteristic may be due to the incompleteness of our concept, as for instance in the case of black and white swans.[15] The principle of contradiction cannot be a criterion of objective reality even in the case of $2+2=4$, since it is not absolutely impossible to conceive of a world in which the union of 2 and 2 may result in the creation of another unit. Milhaud goes even farther than Mill, and asserts that this criterion does not apply even in the case of *A* and *not-A*, because, since the attribute *A* and the attribute *not-A* are sensations or syntheses of sensations which present themselves to us in the record of known phenomena, we cannot decide *a priori* that their union is an impossibility; experience may yet have surprises in store for us. Here it is obvious that Milhaud fails to distinguish between contradictory and contrary; the negation of *A* is not a positive quality to be found in experience, it merely expresses the thought that a certain characteristic is not compatible with a certain concept; but Milhaud insists that if we define *not-A* by means of its property of being absent when *A* is present, the principle of contradiction resolves itself into a mere tautology. This is not the case: we do not define *not-A* as the property excluding *A*, but as the absence of *A* from a concept, and the principle of contradiction teaches us that if *A* be conceived of as incompatible with a given concept, it cannot be thought of at the same time as being amongst the characteristics thereof.

Milhaud is wrong when he agrees with Boutroux's assertion that thought cannot go beyond the affirmation of barren identities, devoid of any content whatsoever, without losing its universality and certainty. Thought in its real activity is synthetic, and gives birth to new concepts and relations, which, while they are suggested to it by experience, yet acquire a universal and necessary character in the course of the process of elaboration to which it subjects them. The principle of identity severed from all concrete activity of thought, from every concept, and from every judgment, as presented to us by Boutroux, is a mere *flatus vocis*, which is, and must be, devoid of all significance. Between what terms can identity be established if not between concepts ? What is the affirmation of identity if it be not an act of judgment ? It is true that multiplicity cannot be deduced from identity ; but can identity exist apart from multiplicity ? Every moment of thought without exception is a search for the one in the manifold, for the identical in the diverse. The advance and development of thought are possible just because thought, whilst amplifying its content in the constructive work of the individual sciences, is continually affirming its identity, coherence, and universality. Herein lies the significance of Kant's *a priori* synthesis which enables thought to escape from the grave dilemma in which Hume had placed it, the choice between barren certainty and uncertain fruitfulness, which appears in another form in the philosophy of Milhaud and Boutroux. The limitation of the certainty of thought to $A = A$ amounts to the same thing as denying that certainty altogether ; even the simplest argument contains a diversity of terms, though Boutroux would regard this as removing it from logical necessity. Would it not be more accurate from your point of view to say that logical certainty is a pure fiction which is non-existent just because no such thing as perfect identity exists ? This is the logical conclusion to which your philosophy should lead you, rather than to the limitation of certainty

to $A = A$, which, apart from the multiplicity of the concept and the judgment, is an absurdity, or rather a combination of meaningless signs. Boutroux appears to prove his case because he compares scientific laws with a non-existent model of certainty, *i.e.* that type of absolute identity which amounts to the denial of thought. He does not venture to deny the value of logical necessity, but he strives to isolate it as far as possible from the world of experience, lest even a suspicion of it should find its way into that world in defiance of the doctrine of contingency. But how is it possible for a world of unceasing creation suddenly to give birth to a thought which proclaims itself identical and demands the identical? Is this tendency to intelligence which Boutroux sees in things original, or is it rather a product of habit? Can the logical tendency and effort made by thought to assimilate the manifold content of experience to the principle of identity be regarded as legitimate, or is it too rather a habit formed by the mind, a kind of inertia, a cessation of creative activity which doubles on its own track instead of going forward? Boutroux does not venture to go as far as this absolute denial of the value of logical thought, and is forced to make a certain concession to intellectualism in the shape of this tendency to intelligibility, which has its mysterious source in the indeterminate flux of the real.

5. *Bergson's Doctrine of Intuition.*—The reaction from intellectualism reaches its zenith in the teaching of Bergson. This reaction would undoubtedly be salutary did it confine itself to affirming the value of the individual and of concrete reality, and if it tended to accentuate once more the fulness of the conscious individual and of the world of which he is the reflection, as opposed to those philosophical systems which impoverish the content of mind by reducing it to a handful of abstract formulae devoid of life and movement. Bergson, on the contrary, opposes exaggeration to exaggeration, impoverishment to impoverishment.

Intellectualism reduced nature and the mind to an inert skeleton, doomed by the working of an inflexible law to reproduce itself in the vacant solitude of time; Bergson's fantastic mysticism reduces the universe to a perennial stream of forms flowing in no definite direction, a shoreless river whose source and mouth are alike unknown, deriving the strength for its perpetual renewal from some mysterious, blind, and unintelligent impulse of nature, akin to the obscure will of Schopenhauer. The new method of knowledge advocated by Bergson consists in drifting with this stream, divesting ourselves as far as possible of all intellectual thought, retracting our false conception of distinct things clearly determined in space and ordered in necessary series in time, retiring from the world which is but the deceptive work of our own needs into the intimacy of our minds in order to identify ourselves with creative activity. The Deity of Plotinus was a lifeless, abstract unity, and mind by identifying itself therewith lost all mobility; Bergson's Deity is continually in process of formation and is perpetually being renewed in the productiveness of unexhausted creation,[16] and our mind, by being reabsorbed into the swift current of Divine life, becomes once more possessed of this evolutionary power of movement, from which the fixed, stereotyped forms of pure intellect generally divert it by turning it towards the material things which owe their existence to the necessity of action. How and why do they divert it? Why does consciousness turn suddenly to the past and solidify its fluidity into distinct objects instead of pursuing its course towards the new and the future? Here we have an enigma which the metaphysic of Bergson would fain conceal beneath the decent veil of poetry, but which the expert, penetrating eye of the philosopher sees in all its nakedness. Can it be for the sake of action? But need and practical necessity are only significant, and can only arise in a world of things and persons distinct from the active subject; or, in other words, in the empirical world of

matter as already constituted with its various divisions. Outside the material order, there is no meaning in the need of action, or, at all events, the meaning is a different one, and the backward return of mind and the formation of the practical world unnecessary. Is not the perennial creation of consciousness the loftiest affirmation of its freedom of action? Why should it not rest content, why should it seek an outlet in the current of its time instead of remaining within the magic circle of its continual self-renewal? The presupposition in the Absolute of a practical need, the source of intelligence and matter alike, amounts to moving in a vicious circle, since practical need in its turn cannot exist apart from intellect and matter.

Bergson maintains that the intellectual function is not original, but owes its being to the practical function, since it is in reality neither more nor less than a more accurate adaptation of consciousness to the conditions of life. For this reason intelligence, which came into being merely in order to ensure that our body should fit perfectly into its environment, moves at its ease amid solid and inert objects, the fulcrum of our action and the tools of our industry, but is unable to grasp the true nature of life or the true significance of the evolutionary movement.[17] No one of the categories of thought, such as unity, multiplicity, mechanical causality, intelligent finality, is exactly applicable to the phenomena of life; our intellectual formulas cease to be adequate when we pass from inert objects to living organisms. Shall we then proclaim the essence of life to be unknowable? Shall we not rather acknowledge that our difficulties and contradictions arise from the desire to apply to it the habitual forms of our thought which are only suited to matter? The intelligence can grasp something of the Absolute so long as it confines itself to the physical objects upon which it is modelled; it becomes powerless the moment it attempts to cross these limits, and to invade the realm of life. We must not, however, be led to conclude that

life transcends the limits of our cognitive faculties, since conceptual logical thought is not the only form of knowledge; around it is a misty cloud of the same substance as the luminous nucleus we call intelligence—intuition, which gives us a direct grasp of the real in that process of perennial creation which is its very life. Intelligence sees nothing in things beyond the aspect of repetition; the irreducible and irreversible element in the successive movements of cosmic evolution eludes it. Mechanical explanations hold good of the systems which our thought has artificially severed from the continuous flux of the universe; but it cannot be admitted *a priori* that the universe in its totality together with the systems which naturally are formed in its image are capable of a mechanical explanation, since in that case time would be useless and devoid of all reality. The essence of the mechanical explanation is to look upon the past and the future as calculable in terms of the present, and to argue from this that everything is given; according to this hypothesis, past, present, and future would all be visible at a glance to a superhuman intelligence capable of making the necessary calculation, and the apparent duration of things, together with its creative process, would be merely the expression of the weakness of a mind which cannot know everything at once. Thus a confusion arises between time—ever fresh multiplicity—and mathematical space, constructed so as to be able to act upon things; the living world of quality gives place to the abstract schematism of pure quantity. If we would grasp reality in its creative essence, we must divest consciousness of the whole of the artificial superstructure which we have raised in order to adapt it to the exigencies of practical life, place ourselves once more in pure duration, and live in the depths of the *Ego*. Time as understood by physical science is not true, real duration, the process of incessant creation, but rather a homogeneous scheme conceived by analogy to mathematical space, which, by solidifying the flowing life of the mind into homo-

geneous moments external to one another, turns it into a necessary mechanism of recurring states. Beneath homogeneous duration, the extensive symbol of true duration, psychological analysis reveals to us a duration whose heterogeneous moments are perpetually penetrating one another; beneath the numerical multiplicity of conscious states it shows us a qualitative multiplicity; and beneath the superficial and symbolic *Ego*, an *Ego* in which succession implies fusion and organisation.[18] Since the symbolic *Ego* is better suited to the requirements of social life, we generally lose sight of the deeper *Ego*, and tend to solidify our impressions so as to express them in language. Mere rude speech, with its clear-cut outlines, giving, as it does, definite form to the stable, common, and therefore impersonal element in the impressions of mankind, is incapable of recording the delicate fugitive impressions of our individual consciousness. If, however, we remove these schemes and eliminate from the *Ego* everything which we have introduced into it in order to render it easier of communication and better suited to practical requirements, every moment of its life will be revealed in its concrete physiognomy, which cannot be foreseen; the bonds of necessity will be burst asunder, and the mind stand forth as a free creation of qualities which are ever new. Science, the outcome of the necessity of action, is fated to leave exterior to itself true reality with its unceasing changes and creative spontaneity; it can only act on time and movement if it first eliminates the qualitative element, which is its very essence. Reality in its fulness eludes the methods of science; if we would grasp it, we must leave intelligence behind and retrace our steps to the springs which the need of action drove the mind to forsake. This undertaking, which Bergson regards as the task of the theory of knowledge, presents many difficulties, and is beyond the power of pure intellect. A mere careful analysis of the categories of thought will not suffice, they must be generated. As regards space, we must make an effort *sui generis* to trace the retro-

gression and degradation of the mind into spatiality.[19] If we place ourselves at the apex of our consciousness, and gradually descend thence, we feel that our *Ego* extends into inert recollections external to one another, instead of tending towards an indivisible and acting will. We thus grasp the principle of this movement of descent towards spatiality, which is continued in the matter of perception and is completed by the physicist, who shows a thorough understanding of his task when he impels matter towards the ideal space of geometry. The philosopher, on the other hand, misconceives the goal which he should strive to attain, when he follows in the footsteps of the physicist in the vain hope of being able to advance still farther in the same direction, since he ought rather to climb the hill which the physicist is descending in order to bring matter back to its source and form a cosmology which is, if I may so express it, inverted psychology. From this new standpoint everything which the physicist and geometrician regard as positive becomes an interruption or inversion of true positivity, which must be defined in terms of psychology.[20] The reluctance shown by philosophers to consider things from such a point of view is to be attributed to the fact that the logical work of the intelligence is in their eyes a positive effort of the mind, but, if by spirituality we are to understand an advance towards ever new creations, towards conclusions which cannot be deduced from the premisses and which are not determined thereby, it must be acknowledged that a representation which is hedged in by relations of necessary determination, through premisses whose conclusion has been inherent in them from the beginning, is moving in the opposite direction to spirituality. The special laws of the physical world are derived from this tendency, which is really a negative one. No one of them, looked at separately, is possessed of objective reality; it is but the work of a man of science who has looked at things from one particular point of view, isolating certain variable elements, and measuring them by certain conventional

units. There is, however, a certain approximately mathematical order immanent in matter, an objective order to which our science approximates as it advances, because, if matter be the relaxation of the inextensive into the extensive, and hence of liberty into necessity, it must tread in the footsteps of geometry, since, though it does not originally coincide with pure homogeneous space, it is derived from the same movement; and if it never fully attain that goal, if mathematical laws be not wholly applicable to it, it is merely because it cannot entirely shake off duration, and thus transform itself into pure space.[21] It is impossible to lay too much stress upon the artificial character of the mathematical form of a physical law, and hence also of our scientific knowledge of things. Our units of measurement are conventional and, so to speak, extraneous to the intentions of nature: how can it be supposed that nature has gauged the related modalities of heat by the dilations of a mass of mercury, or by the varying pressure of a mass of air maintained at the same volume? The act of measuring is a purely human one, implying the real or ideal superposition of two objects a certain number of times. Nature has made no provision for such a process, she neither measures nor counts; yet physical science adopts this plan successfully, how are we to account for this? Its success would be inexplicable were the movement which constitutes materiality not identical with the movement which, when extended by us to its limit, *i.e.* to homogeneous space, teaches us to count, measure, and follow the variations of certain terms, some of which are functions of the rest. The intelligence effects this extension simply by means of a process of self-extension, because, since intellectuality and materiality are of the same nature and are originated in the same way, it naturally tends towards space and mathematics. If mathematical order were something positive, if laws like those of our codes were immanent in matter, the success achieved by science would partake of the miraculous. What chance would there be of our finding

nature's unit of measurement, and isolating just those variables chosen by her in order to determine their reciprocal variations? On the other hand, the success attained by mathematical science would be no less incomprehensible if matter were not possessed of everything requisite for its inclusion in our schemes. There remains, then, but one plausible hypothesis—that there is nothing positive in mathematical order, but that it is the form to which the interruption of the evolutionary movement automatically tends, and that materiality is an interruption of the same kind. It then becomes comprehensible that, while science is contingent, relative to the variables it has chosen and to the order in which its problems have been set, it can yet achieve success. It might have been totally different as a whole and yet have been successful, simply because no definite system of mathematical laws is based upon nature, and mathematics in general merely represents the direction towards which matter tends.[22]

6. *The Fundamental Error of Bergson's System.*—In Bergson's view that the intellectual function is derivative lies the fundamental error of his system; this view gives rise to the attempt—as vain as it is ingenious—to deduce intelligence from the practical attitude of consciousness, which is in its turn confounded with the imaginative or creative function. Bergson's intuition is as a matter of fact neither more nor less than the aesthetic attitude of the human mind, in which we find that experience of unfettered creative spontaneity and unbounded expansion which his poetic imagination regards as the very essence of universal reality, so that he might join with Schelling in acclaiming art as the fullest revelation of the Absolute, and with Froschammer[23] in looking upon imagination as the heart of the cosmic process.[24] But if the essence of the life of things is to be sought in creation and in artistic contemplation, it is difficult to understand how intellect and the practical world of matter can have sprung from such a source, or why the duality of subject and object and the other

factors determining the real came into being. The contemplative mind should indeed remain for ever in the magic realm of dreams and flow on to all eternity in a poetic phantasmagoria, growing ever richer in new creations as it takes its course. What freak of fancy induces it to turn back instead of pursuing the even tenor of its way? Why does it try to dam the stream of its day, double on its own track, and endeavour to return to the past, thus letting matter limit its freedom instead of rejoicing in its liberty? Bergson deceives himself into thinking that in practical need, the mysterious offspring of universal consciousness, we have the explanation of all difficulties. Is the practical function of mind primitive or derivative? If it be primitive, it assumes a distinction of terms as co-existent, since action, as Bergson himself maintains, can find no hold in the unceasing flux of time; matter, that is to say, in Bergson's sense of the word, would in such a case have no beginning, but would be primitive, like the practical function. If practical activity, on the other hand, be of later origin, Bergson should make clear to us how and why the intuitive life of the real and its creative power suddenly give place to the empirical world of action and intelligence. With all his metaphors Bergson fails to convince us that continuous, creative activity can give birth to practical, discontinuous activity, and this activity in its turn to the objective world with all its determinations. The pure act of will is psychological in its nature, and is of itself powerless to leave the sphere of intimate consciousness; even if it be granted that it demands a discontinuity of terms, these terms will never appear external to the consciousness, but will always keep the character of inner experience. The mere fact that discontinuity makes its way into the psychic world cannot give rise to the apprehension of something which is, or at all events appears to be, exterior thereto. Consciousness might be able to distinguish a series of states in its continuous flow, and to find in them a foothold for its actions without, for

that reason, coming out of itself. Do we not experience this when we voluntarily modify our train of thought? It is true that freedom of action implies the distinction between subject and object, between the agent and his act, but the two terms can both exist as part of consciousness. The mind is not compelled, if it would affirm its liberty, to come out of itself and create the illusion of a world completely extraneous to itself, and which it must endure as an extraneous necessity. According to Bergson, the arrest of the vital flux, the turning back of creative activity, will suffice to produce the intellectual process; it is the inversion of the evolutionary movement which has given birth both to the intellectuality of mind and the materiality of things; the intellect is negative in value in comparison to intuition, it is the mind abjuring itself, turning in a moment of weariness to gaze at what it has already accomplished instead of creating new forms, ceasing its march forwards and relaxing into the materiality of space. It would be out of place here to enter into the question whether the work of the intellect does not demand as great (or even greater) intensity of effort as artistic creation: Bergson himself, if he could but divest himself of systematic preconceptions and put the question to his own consciousness, would be forced to admit that it is an error to look upon intellectual work as a relaxation. Another, looking at the matter from another point of view, might rather assert that the logical process involves an excessive mental strain, and that aesthetic contemplation is productive of a sense of calm and repose, almost as if the mind were drifting at the mercy of its own current. These are but subjective impressions on which I will not insist lest I should seem to attach too much importance to Bergson's metaphors. His view of intelligence is too limited and fragmentary, and he altogether fails to see its poetical side. The creative activity of the mind is revealed in concepts no less clearly than in intuition; the abstract logical schemes and the deduc-

tions of an already systematised science are one thing, logical scientific work in its concrete process of discovery quite another. The demonstration of a new relation between phenomena which before appeared to be heterogeneous, the harmonisation of laws already known to us in a new theory with the help of a new principle, the formulation of the equation of a new curve, this is a function of intellect, and a creative function as well; and if progress towards the new and the genesis of new products be, as Bergson states, the sign of a positive movement of the mind, then intelligence, no less than intuition, stands revealed as a positive process, and the world as conceived by it is no less real than that continuous flux of images which Bergson regards as the essence of the universe.

7. *The Two New Rules of Invention propounded by Wilbois.*—The intuitionists do not really deny the creative activity in the realm of science, but endeavour to reduce it to the aesthetic function. Wilbois,[25] in opposition to the methods of Stuart Mill, which postulate determinism, propounds two rules of invention: the aesthetic sense and the sense of progress. The rules of scientific invention should constitute as a whole a kind of *aesthetic of the laboratory*: no strictly scientific rule can enable us to discover truth, which lies concealed amid the complexity of our qualitative perceptions, and aesthetic instruments alone can reveal it to us; the search for beauty is then a condition of science. Aesthetic is a chapter of logic.[26] The history of a scientific work bears a strong resemblance to the history of a work of art; discovery is a creation, a poem.[27] The truth, however, which is based on beauty can be of a provisional order only; the aesthetic sense must therefore be united to the sense of progress, which will spur us on to the criticism and continual renovation of facts and theories. It is, of course, true that there is nothing less bound by rule than the creation of a principle, since genius is always something which cannot be foreseen; there is, however, a certain effort which is

characteristic of all great inventors; they have only reached the goal of discovery by means of a process of self-renewal, preceded by a long dark night. Facts will only reveal themselves to him who seeks them with the temperament of the artist, and whose whole mind strives after the future of science.[28]

8. *The Arbitrary Character of Spiritual Activity in Scientific Construction, as viewed by Le Roy.*—This prevalently aesthetic character of intuitionism leads it to exaggerate and exalt the element of personal activity in scientific construction: not only theory, not only law, but fact itself is a free creation of the man of science. Positivism in its older form went into ecstasies over the fact, which it failed to distinguish from pure data; the new positivism, as the new French philosophy has been termed by one of its most ardent disciples, Le Roy,[29] sees in the fact of common sense an unconscious product of spiritual activity directed towards practical action. The very name of fact, says Le Roy,[30] should put us on our guard against the common belief: *that which has been made* cannot be an immediate datum. There are no such things as isolated facts, but all is diffused in all; nothing of any kind can be defined except by means of the ties binding it to the universe as a whole. All isolation, all fragmentation, is relative to the selected point of view. The relativity of facts is still more obvious in science, in as much as it is not satisfied with any facts whatsoever, but seeks for facts of significance for the law or theory which is to be proved. Of course, everything is not created by men of science; facts contain a mysterious residuum of objectivity, but science, employed as it is in that fragmentation necessary from its point of view, does not take into account this primitive material which philosophic criticism alone can reveal to us in intuition. It is not what is objective but what is artificial in facts which is of interest to science.[31] The character of personal creation and of artificial schematism becomes more marked in the scientific law and theory. The law, whose office it is to fix that which

is constant in the variations of phenomena, is but a general formula, a schematic model, a type of classification summing up under a single heading an inexhaustible multitude of individual events. The only characters of facts persisting in the law are those which are of greatest interest to us, and whose relative permanency gives us a handle for our action. Thus the laws of Galileo are a brief condensation of all that is worth recording amongst the infinite details of all the real or possible falls of bodies. Laws are then but aids to memory, principles of arrangement, which, by simplifying and impoverishing concrete reality, cause us to lose immediate contact with nature. They appear to us less like elements of things than constructions of our minds, products and symbols of our disposition to vary perpetually the angles from which we contemplate the constancy of the world, and much more like true and proper definitions established by an arbitrary decree. The law of fall defines the closed system, the law of definite proportions does the same in the case of the chemical combination as distinct from the mechanical mixture.[32] From such a point of view laws cannot be verified, because they are the instruments with which we effect in the continuity of primitive data that splitting up into fractions which is indispensable to the action of our thought, and because they themselves constitute the method and criterion of which we must make use in order to test them accurately. For instance, in order to verify the law of the reflection of light we should require a plane mirror, and in order to make such a mirror we must make use of this same law. Further, the very project of seeking for laws in the world involves the postulate of universal determinism, which is neither an evident *a priori* principle nor yet capable of *a posteriori* proof, but is a mere decree of our mind which never fails, because each time it is endangered science rushes to the rescue by inventing a new concept and setting up a new convention. Law is then purely arbitrary, in as much as it demands the definition of a unit of measure. The smallest appliance of the laboratory, the minimum

of experimental technique, presupposes—as Duhem and Milhaud in particular have pointed out—in its apparent simplicity a large number of definitions, postulates, conventions, and decrees of the mind. The contingency of laws is further dependent upon the method by which they are obtained: physical science seeks for constant quantities and finds them, because it desires to do so, by doing ingenious violence to nature; but these constant quantities do not reflect anything objective, and express in reality but the weakness of our senses, the approximations we have accepted, the requirements of our practical needs and of our discursive reason. But, it may be asked at this stage, how does it come about that a phenomenon is always produced in determinate circumstances? Is not this a proof of the reality of constants? It is hardly surprising, replies Le Roy, that the law should be verified the moment we decide to exempt from its rule all those cases in which it is not verified; when experience fails we calmly declare that the requisite conditions have not all been fulfilled, or that some as yet unknown cause (which is thus defined) is at work, and has determined a modification of the phenomenon.[33] As regards theory, all agree in recognising its conventional and hypothetical character, which in no way distinguishes it from law, as is erroneously believed, when the variability of theory is placed in opposition to the stability of law. The chief office of theory is not to forestall the known, but to furnish us with a general scheme of representation, capable of adaptation to a category of laws.[34] Theory absolutely eludes control; it cannot be proved *a priori*, since it is based upon contingent definitions, nor *a posteriori*, since in its case an *experimentum crucis* is out of the question: experimental contradiction proves the existence of an error in the system of laws applying to the case in point, but does not tell us exactly where this error is to be found, and it is possible for us to modify some law in such a way as to leave the theory intact.[35] Theory

is not the outcome of experience, but is a system of symbols wholly created by the mind, a conventional language which can be changed as often as we choose to do so; hence the possibility of different theories of the same order of facts. The man of science is at liberty to choose the one for which he has a personal preference, or which is best suited to the end he has in view, conceiving of phenomena by means of differential equations and integrating symbols, having recourse to geometrical and mechanical schemes, or making use of the images of the flow and circulation of energy. Each of these modes of expressions is legitimate from its own point of view; and our choice of one in preference to another is due to the fact that the one selected is better suited to the type of our mentality, corresponds more nearly to our habits, and proves more convenient to us. There is no such thing as a universal limiting theory, which should serve as a unit of measurement by which to judge the degree of truth of the others; there merely exists theories to which we give the preference. Even in the realm of geometry, as has been proved by Poincaré,[36] there do not exist evident principles possessed of universal and objective value in preference to all others, there are merely conventions established by the mind which have become more or less habitual, and whose convenience is the only point capable of discussion. Rational science, the ultimate aim of all scientific work, is a device of the mind for the conquest of the world, constructing with the aid of our resources alone a scheme of the universe which enables discursive thought to reproduce at will the whole development of nature without having recourse to experience. The ideal of science is the attainment of knowledge which is wholly ours, which is entirely our creation, subject to us and contained within us. Its mission is not to attain to some external necessity or other concealed ready-made in the real, but rather to manufacture the truth for which it seeks, the perfect instrument of action, the system of perfectly tractable discursive

symbols.[37] Scientific truth is like moral good; it is not received from without, it makes and creates itself; is a system of fictions, of merely social value, which succeed in the practical world. How is it that they are successful? Here we have a difficult problem which Boutroux and Bergson vainly strove to solve, and with which Le Roy too finds himself face to face. It will not suffice to show that this success is obtained by eliminating everything which cannot be made to fit into our schemes; or that science is not relative to true knowledge, and that its successes belong to the order of discourse and industry: all these observations, says Le Roy,[38] are legitimate, but leave something still unexplained. In order to establish any determinism whatsoever, the data must lend themselves thereto: nature must contain certain elements, must be possessed of certain characteristics which bring our science within the bounds of possibility. Were incoherency the basis of things, the decrees of our legislative action would be continually overturned by the unexpected; whereas we see that our elaboration works well. Is not this a sign that it is true, that it has at all events a partial grasp of reality, that something in the universe corresponds thereto? Knowledge is power and foresight, but inversely, power and foresight are knowledge. Hence Le Roy is forced to admit that laws are in some way possessed of objective value, because they enable us to grasp a real and constant order of phenomena, at least with the approximation of common life. Matter contains something which thrusts itself upon us, opposes our undertakings, and limits our liberty: how are we to conceive of it? If everything be eliminated from matter which is due to the work of the mind, nought but potentiality will be left: nature is only actualised, developed, and made explicit by the work of minds which by means of discrimination introduced number and space into primitive continuity; which contract by means of memory a plurality of moments into differentiated syntheses which become sensible qualities, and

chisel out and solidify by means of action the objects and facts of common sense. As criticism undoes their work, matter becomes less clear and more involved, bodies dissolve, qualities are confused, images vanish away, till nothing is left but a kind of latent tension towards development and actuality.[39] We thus reach pure matter, which can only be conceived of as the virtuality of an order common to all minds, as the capability of laws. Matter at times rather resists the decrees of the reason and the will; hence it seems to us to be endowed with true activity, a tendency, a desire, a straining after determinism. It is no ready-made web, no tissue into whose unyielding meshes our action cannot hope to penetrate; it merely shows that it is impossible for the mind to vary the rhythm of its duration beyond a certain limit, it is a combination of stationary waves in our psychic life, or rather of waves which advance less rapidly. The real activity manifested in matter induces me to believe instinctively in its objective existence; in order to conceive of this activity, we must admit the existence of spiritual monads in those regions where common sense sees but brute matter: consciousness more or less clear, at a lower or higher degree of tension, endowed with a greater or lesser degree of reality and liberty. Matter is illusory in the sense that it is our work, it is like a decree by virtue of which every free action develops reactions; but, on the other hand, it is real in the sense that we are pre-determined to construct it. We feel its weight in two ways: as an institution which is the outcome of human art (actual matter), and as a decree which aims at pre-forming our action (pure matter).[40] If it be not the work of the individual mind, it is that of mind in general; the fact that each of us finds at the dawn of life matter which has already been elaborated must not lead us to conclude that this holds good of the totality of monads. A group of hereditary habits, re-inforced by education, a relatively stationary wave in the flux of my becoming, a knot in my duration

which I have neither strength nor time to unloose: this is a material reality. The world is born and advances by means of the inventions of liberty; it is preserved and assured by the inertia of habits.[41] Pure matter has a certain existence, it is not a meaningless word; but it only becomes a thing through consciousness: in itself it is merely an obscure desire of images and attitudes, an instinct, an appetency, a will to live, a kind of tension towards a final cause which develops it and puts it in motion; it exists *de iure* rather than *de facto*.

9. *The Physical World as an Instrument of Moral Life.*—If we thus conceive of the world as an appeal to the mind, a limit placed on its action, we still have to explain the origin of this obscure tendency and its office like that of a decree; hence we must refer to a Being transcending necessity which subjects mind to matter. Thus we have once more in modern philosophy the old cosmological proof of the existence of God.[42] Why does matter exist? To what end have we this invention, which must be regarded as an unfortunate one from a certain point of view, limiting the liberty of the mind? According to Le Roy, the answer is simple enough: the ballast of matter is necessary in order to enable the mind to descend from the plane of dreams to the plane of effective action. Matter is a fatal result, and at the same time an unavoidable means, of discursive life without which the formation of society would not take place, the individual would not enter into possession of himself, and the world would not exist: such is its nature and its mission. The new philosophy admits a hierarchy in which moral and religious action occupies the highest place, and brings—as means—both practical and discursive action into subjection unto itself. Matter, the creation of mind, is, according to Wilbois,[43] the stadium in which morality is prepared. In the act of discovery the intuition of the beautiful, the dominion of time, the duties of abstinence, humility, and abnegation to which we have striven to submit and which leave their seal

and imprint in the most profound of facts, the appeal to the future to which we feel ourselves to be more subject than we are to the past, the creative power which we feel stirring within us, which raises us above matter, and introduces us into a new world, all these afford us unique moments of experience which we shall only find again in the fullest hours of our moral and religious action. Scientific life is then the first step towards loftier spontaneity; invention is fulfilled in virtue. Matter has a final cause which only permits it to become solidified into determinism in a certain direction, and this final cause must be sought in the activity of the scientist: the formula of inertia is the final cause of the fact of the fall of heavy bodies, the Newtonian formula of the fact of universal attraction: matter tends towards law. Axioms and categories, forms of intellect and sensibility, all become, all are evolved.[44] The mind of man is plastic, and can change its most intimate desires, provided it takes the requisite time to do so. Intelligence and the categories are then no mere inflexible, fixed forms, as Kant would have us believe; one *a priori*, however, remains: the primordial needs and natural tendencies of action, because primordial action to which we turn for an explanation of the genesis of matter and reason, must not be incoherent; it must be regulated in order that it may carry out lasting and coherent work. The true *a priori* is of a religious and moral order. We organise matter and create practical life and social relations, because the moral law rules and directs us: everything is in the last analysis based upon the mystery of duty. The task of philosophy is to illuminate it and subject it to a scrutiny which will cause it to reveal its specific originality and inspire our intuition of it with fresh life.[45] Science, by reason of the very attitude it assumes, and the artifices of which it makes use, increases the distance between itself and living concrete reality, which thus becomes for it a noumenon which it cannot penetrate. Philosophic intuition alone can bring us once more into immediate contact with reality. The work of the new

philosophy is to throw off the bonds of number and space; to break through the lifeless forms of a crude language; to rise above discursive thought in order to define it and pass judgment upon it; to rediscover the living springs of logical mechanism in the mobile depths of spiritual life.[46] This direct appeal to the inner soul of things, the method peculiar to philosophic intuition, will be found also in art, which enables us to penetrate the symbolic meaning and dim soul of appearances, and shows us the dynamic penetration of the living being beneath the rigidity of the outline; but it will be found to a greater degree in the subtle and illogical art, so full of dreams and mystery, of the modern symbolists than in musical and penetrating art.[47] Art is, as it were, the preface to philosophy; but its fictions, though they may reveal the mysterious basis of being, do not penetrate it, as philosophy does, just because they are fictions. The real in its intimate and autonomous value is in no case discursive; it can only be experienced, loved, or accomplished according as it is termed truth, beauty, or good.

10. *Ethical Action as a Means of penetrating Reality: Blondel.*—Blondel,[48] another opponent of intellectualism, states that according to that which we have experienced, loved, and done, we shall know and possess in another way, we shall touch, penetrate, and enjoy things differently. In love and sacrifice will be found the fullest revelation of being: without love there can be no understanding. Charity is the organ of perfect knowledge, because it puts into us that which is in others, whereas selfishness isolates us and makes us impenetrable. The willing acceptance of suffering, voluntary submission to the natural necessity which we find within us, and which imposes limits upon us from without through the extrinsic action of things, these alone will enable us to attain to the full possession of ourselves and of the universe. In this supreme act of abnegation, which is an act of faith, in this ultimate option of the will in favour of true Being, which is not to be found in the ephemeral enjoyment of the passing

moment, but in the life of the Eternal, the true and intimate experience of Absolute Reality will be reached.

11. *Criticism of Intuitionism.* — The intuitionists endeavour with all their might to prove that there is nothing objective in facts, laws, and scientific theories; that the whole of science is an arbitrary and artificial construction, which is entirely created by the mind for the needs of action, and that its success is due to the fact that the matter to which it applies is also an unconscious creation of the mind of the race under the pressure of these same needs. But this merely shifts the problem; Le Roy perceived this and accordingly modified Bergson's theory to a certain extent, a theory which, as we have already pointed out, failed to explain how free creative activity could suddenly give birth to a practical need. He recognises—and in this he is at one with Boutroux—that we must admit the existence, at all events in pure matter,—matter, that is to say, which has not been elaborated by the mind,—of a tendency towards intelligence owing its existence to a moral imperative. Moreover, Le Roy considers intuition to be not only feeling and volition, but more especially reason;[49] it is the mind experienced in the wealth of all its aspects. I consider that this view virtually transcends Bergson's fantastic intuitionism: the mind is no longer an indefinite creative power, but an activity regulated by an end, which is not the product of our action nor yet of the action of the race, but a command emanating from a transcendent Being. How is it possible to conceive of a universal end without presupposing the mind which tends thereto to be possessed of a certain vision of that end, that is to say, of a certain intelligence? If this end exists in us, does it not determine our actions, and guide them in certain directions? What becomes of intuitionism if action be governed by motive? Further, if there be a certain coherence in the actions which result in the construction of matter, the mind which performs those actions must also partake

of the character of coherency proper to logical thought. How can we then say that thought falsifies reality? The moral law, as a universal command, calls for a mind capable of thinking the universal. If, amid the manifold and transient appearances of things, we seek the nature common to them all, if, even adopting Le Roy's point of view, we affirm monads to be spiritual essences having common tendencies and aspirations, we go beyond the moment of intuition, to grasp in our own and in countless other consciousnesses the universal characteristics of spirituality, which cannot be intuited, but only conceived. There can be no philosophy without concepts: even your metaphysic is not the immediate life and intuitive communication of ever new impressions, but conceals beneath its metaphors a system of concepts every whit as abstract as those of intellectualism; the organon of your philosophy is not intuition, which, however far-reaching it may be, cannot give more than the passing moment, but *the concept of intuition*, the *thought* that that immediately experienced activity is not peculiar to your mind alone, but is common to all consciousnesses; the *thought* that your inmost actions, like the actions of all mankind, tend towards a universal end. The concept is not just an expedient for rendering social life possible, but is the one and only way in which the universal can be manifested to consciousness; even that which the new philosophy terms intuition is at bottom but a vague, dim, indistinct concept, which is unconscious of itself. Further, scientific activity as a whole, in as much as its aim is to emphasise that which is universal amid the various and changing aspects of phenomena, is not directed towards action only, but tends to show us a character of reality. In short, either you believe that the various spiritual actions have something in common which is ever being repeated, in which case you must recognise the value of intelligence, which alone is capable of grasping this universal element; or you hold that each action is entirely new, in which case you are not justified in affirming that *all* the appearances of nature are

spiritual actions, that *all* tend equally to the *same* end, have their source in the *same* supreme principle, in *one and the same* moral command. There undoubtedly exist in man forces other than intelligence, and of these philosophy must take account, but it can only do so by means of concepts. Feeling and instinct, as Poincaré rightly observes,[50] may act as its guide, but cannot render it of no avail; they can direct the glance, but not take the place of the eye. Intelligence is a tool with which we cannot dispense, if we would philosophise and not compose music or poetry; even if we conclude with the primacy of action, it is intellect which arrives at this conclusion. Philosophy which does not make use of the intelligence is a contradiction in terms: in order to set up a system of philosophy, any system of philosophy whatsoever, even the *new philosophy* itself, we must leave behind us concrete duration, and the intuitively experienced moment, and look at the world *sub specie aeternitatis*. We must not confound the cognitive function with the aesthetic function. Both contain creation and activity; but the product of intelligence is always something of universal value, whereas the product of imagination gains in value in proportion as it is original, and gives utterance to the individual personality even in its transient impressions. Art may break the bonds of natural necessity and create for itself a world which is ever new; but this freedom of creation is denied to science and philosophy alike. The concept, too, is a production of mind, but it is not of value as a creation or as a flash of fruitful genius, as is asserted by Wilbois, who strangely confounds the beautiful and the true, but only in as far as it corresponds to something objective which is independent of the creative consciousness. Moreover, we cannot set up this correspondence by artificial means: there are always elements beyond the control of our will, which are thrust upon us, not only in actual matter, which Le Roy regards as a construction of the race which has in the course of time become an instinctive mechanism, but also in

pure matter, divested of everything introduced into it by the act of perception.

12. *Theoretical Value of Science.*—The scientific man does not create scientific fact at will, but allows himself to be guided by the suggestions of brute fact. The examples adduced by Le Roy are no proof to the contrary. When the astronomer states that the eclipse took place at nine o'clock, he does not create the hour, but deduces it from a brute fact, from what his watch tells him : undoubtedly he might say that it was eleven, but, by doing so, he would set up another convention. The only truth contained in Le Roy's thesis is that the scientific man plays an active part in the selection of the facts which are worthy of notice : an isolated fact is of no interest to him ; he selects one, if it can aid him to predict others ; but facts are facts, and their conformity to previsions is not dependent upon our free activity.[51] In like manner the choice of the unit of measurement is certainly arbitrary, but this is no proof of the arbitrariness of natural law. Abbé Mariotte estimated the volume of gases in cubic inches, and the column of mercury by which their pressure is measured in feet ; has the law, which was true according to the old units of measurement, ceased to be so since the adoption of the new units ? The only thing which is dependent on the choice of the unit is the number which measures a given magnitude, not the magnitude itself : the principle of homogeneity allows the physicist to pass from one system of fundamental units to another by means of a simple calculation, without being obliged to take fresh measurements. The formulas of physical laws do not change when the unit is altered, because the relations between magnitude which they express are independent of the units selected.[52] To this Le Roy rejoins that Mariotte's law undoubtedly still holds good when the metre instead of the yard is taken as the unit of length ; but that this is due to the fact that the two units stand in a constant relation, so that the transition from the one to the other is really but a change

of script; this, however, would no longer hold good if the distance from the sun to the earth were adopted as the unit; in that case time would intervene, and the formula of the law would be modified, yet it would not cease to be of use in the ordering of phenomena. Moreover, may not the constancy of the unit of measurement be a fiction? The choice of a unit is equivalent to the choice of a point of view from which to contemplate the world; and this point of view depends upon the conditions of our physical activity, our habits of speech, and the feebleness of our senses. In conclusion, the devices used to define these units, and the fact that every measurement is made by means of a more or less complicated experiment, force us to state that the most insignificant quantitative law is dependent upon an immense number of conventions and decrees which make it solidary with science as a whole.[53] But, we may ask, is then the postulate that the unit of measurement, and hence the magnitude, is possessed of constant value absolutely arbitrary? It is true that experience never affords us absolute constancy, but we see that the more nearly the causes of error are eliminated, the more nearly, that is to say, we succeed in isolating the phenomenon, thus realising the ideal conditions of our thought, the more closely does it approximate thereto. We see then that experience yields to our exigencies, and that it may be resolved into abstract elements and translated into our formulas; may we not fairly argue from this that there must be some constants in nature, even if they are not the same as ours? We cannot of course assert that our concepts, our divisions, our points of view, the structure, that is to say, of our science in its minutest details corresponds to the structure of things; our knowledge is always a reconstruction which may be accomplished in various ways, by means of various systems of concepts and various relations, and hence with the help of different formulas; but this variety is no proof that science is arbitrary and unable to afford us anything real; it is rather a proof of the contrary, since it shows us that the

possibility of translating that group of phenomena into mathematical language is not a contingent fact, relative to a point of view, but is in the very nature of things, and is independent of our special way of looking at them. Le Roy himself [54] ends by recognising that, if the choice of the unit of measurement be arbitrary, there is something which is independent of our will, *i.e.* the fact that nature is measurable. We can decompose phenomena into abstract elements in various ways, and take certain functional relations into consideration rather than others, but it does not depend upon us whether the law be verified or not: once we have chosen a point of view, we are no longer free to make facts say what we please: nature sets us a limit, pits her decrees against ours, and puts us in danger of failure if we do not respect them. Le Roy says that our laws are but definitions in disguise, and succeed merely because we choose, and because we eliminate the cases which do not accord with them. When I state that phosphorus melts at a temperature of 44° C., I am under the impression that I am enunciating a law; in reality, however, I am merely defining phosphorus; were another body to be discovered which, while possessed of all the properties of phosphorus, did not melt at 44°, another name would be given to it, and the law would still be true. Thus, again, when I say that heavy bodies, falling freely, traverse spaces proportional to the squares of the times, I do but define free descent; whenever the condition is not fulfilled I shall say that the descent is not free, so that the law can never be contradicted. Now it is obvious that, were it possible to reduce the law to this, it would be of no use either as a means of knowledge or as a principle of action. When I say that phosphorus melts at 44°, I mean thereby that every body possessed of all the properties of phosphorus except the point of fusion will melt at 44°; taken in this sense, the proposition is a law which may be of service to me, since, if I find a body possessed of these properties, I can predict that it will melt at 44°. It is possible that another

body like phosphorus may be discovered which will not melt at 44°, but in that case the other properties will not be absolutely identical, and, should they seem to be so, it will be due to our imperfect means of observation.[55] Thus it would avail me nothing to have given the name of free descents to those descents which take place in conformity with the laws of Galileo, if I did not know under what conditions the descent can be termed free or approximately so; the law establishes precisely that if these conditions be fulfilled, the descent will take place in accordance with the formula of Galileo. The free descent is not defined by the law, but is determined by other criteria, absence of any obstacle, perfect void, etc. If this were not the case, the law would be reduced to a meaningless tautology. Poincaré has endeavoured to correct and moderate the paradoxical conclusions reached by Le Roy by exaggerating the importance of the conventional element in scientific construction. When a law has been sufficiently confirmed by experience, we are at liberty to adopt two attitudes towards it: we may either leave it amongst the other laws which are subject to constant revision, or we may set it up as a principle, by adopting a convention of such a nature that the law must of necessity always remain true. In order to do so, we intercalate between the two brute facts *A* and *B*—the former terms of the primitive law —an abstract concept of a more or less fictitious order, resolving that law into a principle which is a definition, and thus removed from all experimental control, and a part which can still be verified. Thus the law, "the stars follow the law of Newton," may be resolved into these two others: gravitation follows the law of Newton; gravitation is the only force which can act upon the stars; of these the former is a definition which is beyond the control of experience, the latter, on the contrary, must be verified. The latter only can be termed true or false; whereas the principle, which has now been crystallised, is neither true nor false, but merely convenient.[56] If it be possessed of a degree of truth and

certainty which is lacking in the experimental truths from which it has been extracted, this is due to the fact that it may be reduced in the last analysis to a mere convention, which we are entitled to set up, since we are assured that no experience will contradict it. That which is gained in certainty is lost in objectivity. Principles are disguised conventions and definitions; but their adoption is not a purely arbitrary proceeding, it is not a whim of ours, because certain experiences have shown us that these conventions are useful.[57] On the other hand, the term conventional cannot be applied to the whole of science: there is always a portion of the law which cannot be transformed into principle. Although, then, the conditions may vary, something constant will always be left, the relation between the two brute facts. The part of facts which is the creation of the man of science is only the convenient language in which he enunciates them.[58]

13. *Criticism of the Arguments of Duhem against the Objective Value of Science.*—Duhem[59] has adduced new arguments in favour of the thesis of Le Roy, and against the criticisms of Poincaré. Physical observation is not merely the observation of a phenomenon, but is also the theoretical interpretation of this phenomenon, which replaces the concrete data of observation by an abstract and symbolic presentment. Thus, for instance, Regnault has not left us the description of the concrete facts of his experiments on the compressibility of gases, but only a transcription of them into abstract terms, temperature, pressure, volume, etc.; and each of these notions presupposes a theory which must be known in order that the explanation of the experiment may be intelligible. Between phenomena, as actually observed in the course of an experiment, and the result of this experiment as formulated by the physicist, an extremely complicated intellectual elaboration is interpolated, which substitutes an abstract and symbolic judgment for a description of concrete facts. The use of the instruments found in the laboratory would

be impossible, unless the concrete objects composing these instruments were replaced by an abstract and symbolic representation, giving a hold to mathematical reasoning, and if this combination of abstractions were not subjected to deductions and calculations, in which adhesion to the theory is already implied. Before accepting the result of the experiment made by the physicist, we must then see the theory on which it is based. The contradiction which so frequently exists between experimental data does not lie in the facts but in the theories upon which the interpretation of them has been based.[60] The result of an experiment in physics is not as certain as a fact observed in a non-scientific way, by the mere sight or touch of a man of healthy mind and body, because it is always subordinated to the confidence inspired by a whole mass of theories. A law of common sense, being a general judgment, may be true or false ; not so the laws which physical science, at its highest stage of development, enunciates in the form of mathematical propositions. These laws are indeed always symbolic, and a symbol is, strictly speaking, neither true nor false ; it can only be said that it has been more or less well chosen to represent a certain reality, that it expresses this reality in a more or less accurate, and more or less detailed form. Let us consider a series of analogous facts : to the physicist the discovery of the law governing these facts means the discovery of a formula containing the symbolic representation of each of them ; and, since the symbols are indeterminate, the formula uniting them will be so also. An infinite number of different formulas can be made to correspond to one and the same group of facts, seeing that there is an infinite number of values which may be chosen as the approximate results of experimental measurements. These formulas are algebraically incompatible, but they are all equally acceptable to physics, since they determine the phenomenon more nearly than observation can do. The choice of one rather than another of these formulas is made not because

the one selected is truer than the others, but because it is simpler.[61] A physical law is provisional, because it represents the facts to which it applies by means of an approximation which physicists at present consider to be adequate, but which will one day cease to satisfy them; it is relative in the sense that this approximation may suffice for the use which one physicist desires to make of it, but not for another. The same physicist at two different periods of his life may accept or reject a law; this would not be possible could the law be either true or false, since in that case a contradiction would be involved.[62] A physical law is provisional also because it is symbolic, and cases will always be found in which the symbols which enter into it no longer represent reality in a satisfactory manner. The struggle between reality and laws will continue indefinitely: sooner or later reality will confront every law formulated by the physicist with some rude contradictory fact; but physical science will never weary of correcting, modifying, and complicating the law, that it may give place to one which is more comprehensive, containing the rule applying to the exception revealed by experience.

Duhem is right in asserting, in contradiction to Poincaré, that the transition from brute fact to scientific fact, and hence to law, is not a mere change of language, but is fact pervaded by theoretical elements; except that from these premisses it is not allowable to conclude that law is purely conventional, approximate, and symbolic, and cannot be termed either true or false. We must clearly understand what meaning we attach to the idea of scientific truth. We have already seen, when discussing empirio-criticism, that to reduce the true to the useful, convenient, and economical is simply to beg the question, because a law or theory would be of no service in foreseeing, did it contain nothing objective, or did it fail to reflect some real relation. The true criterion of which science makes use is the harmony and logical coherency of the system on the one hand, and agreement with the facts

and relations of experience on the other. The experience of things, however largely spiritual activity may contribute to it, is not entirely the creation of the subject, but contains elements derived from extraneous factors, which to a certain extent guide and regulate the development of the scientific construction: the fact that sensations succeed one another and co-exist in this or that way does not depend upon our will, and is not a convention of our making. Even the most rabid contingentists have been forced to stop short at this insuperable barrier. The two criteria of which we have spoken are not of course a measure of *absolute and eternal truth*, which will ever remain a pure limiting concept, a regulating idea of the cognitive function; they will, however, enable us to estimate our knowledge, and to determine *the degree of truth* contained therein, according to the greater or less degree to which it realises that ideal. The system which is absolutely coherent from the logical point of view, and which affords perfect satisfaction to the intelligence, will never be fully realised, hence the ceaseless efforts made by philosophy and science; this does not, however, amount to saying that all systems are of equal value, or that they are all on the same level: though no one of them be *absolute and eternal truth*, a hierarchy may be established among them, in which those which are most harmonious and comprehensive will rank highest. Thus we shall never arrive at the construction of a theory able to resist all the attacks of future experience, we shall never succeed in exhausting the realm of the unknown, but comparison with the mass of known facts will teach us to decide which is the most adequate formula, and which most nearly approaches the truth. Duhem says that law is not true, but approximate; what meaning, however, are we to attach to the word *approximation*, and how are we to decide its degree if we do not admit the existence of a law which is true in the absolute sense, a law which we are ever striving to approach more nearly by correcting and completing our

formulas? The results of our measurements are not possessed of absolute constancy, hence various values may be obtained which are mutually exclusive from the mathematical point of view, but which are equivalent from the physical point of view, because our appliances are not sensible of those differences; are we then, asks Duhem, to say that they are all true? [63] Is not this a logical absurdity? Assuredly, if we take the word *true* in the absolute sense, because in that case the real value of that magnitude can be but one; but in the relative sense the other values may be more or less true, in proportion as they approach more or less closely to this real value; and, though we do not at present succeed in drawing such a distinction, the future of science will enable us to do so by perfecting our appliances. We shall then be forced to correct our formulas; this does not, however, imply that these formulas were absolutely indifferent to truth and falsehood. The physicist does not accept two different formulas as equivalent because he is indifferent to the search after truth; he is well aware that one of them must be more nearly true than the other, but science does not for the time being afford him the means of selection. If truth be thus understood in the relative sense, Duhem's criticism confirms rather than weakens the truth-value of scientific law: the law is true just in so far as it is approximate; the more approximate it is, the truer will it be. If we put aside this unit of measurement, if we deny that law is of any value as a revealer of one aspect of the real, will not the affirmation that, as science advances, formulas will become more precise and laws *more* approximate become utterly meaningless? Our thought is not Absolute Thought; but neither is it a system of fictions devoid of all objective value. Our knowledge, fragmentary as it is, yet reveals one aspect of reality to us; it can lay no claim to embrace all the relations of things, but it strives to comprehend as many as possible by varying its points of view; it contains something false, but it strives ever more earnestly to eliminate it, by extending the range of its

observations. It is certainly an arbitrary act, a pure convention to isolate a phenomenon, and to take into consideration merely one of its aspects apart from the rest; but the scientific man, when he does this, is well aware that the aim of this separation is only to facilitate study, and, when he wishes to think of the phenomenon in its true reality, he takes into account the other elements which he has neglected. Something will always escape him, his system is of necessity incomplete, but the laws which he formulates, though they may not be exactly the relations of things, and all the relations, approximate ever more nearly thereto, as they become increasingly true. Thus, then, if there is no *absolutely certain* experimental law, there are principles which have stood the test of facts better than others, and have shown us how to render the world of experience intelligible, so that we may regard them as being *relatively more certain than others*; therefore, should an experiment not verify a prevision of ours, we are justified in suspecting this failure to be due to the less solid portion of our construction. If, when we have modified one of these less certain laws, we see that the experiment is successful, we may be relatively sure that the defect originated there. Finally, if Duhem's arguments prove that the value of absolute reality cannot be ascribed to physical law, they fail entirely to show it to be incapable of a greater or lesser degree of truth.

NOTES TO CHAPTER I

[1] *Revue de métaphysique et de morale* (1893), vol. i. pp. 20-22.

[2] *La Philosophie en France dans le 19e siècle*, p. 322.

[3] *De la contingence des lois de la nature* (Paris, 1899), 2nd ed. p. 170. The reduction of physical laws to habits had already been accomplished by Ravaisson (*De l'habitude*, p. 39).

[4] *De l'idée de loi naturelle dans la science et la philosophie contemporaines* (Paris, 1901), 2nd ed. p. 143.

[5] *Op. cit.* pp. 15-16.

[6] *Op. cit.* p. 20.

[7] *Op. cit.* p. 26.

[8] *Op. cit.* p. 35.

[9] *Op. cit.* p. 38.
[10] *Op. cit.* p. 49.
[11] *Op. cit.* p. 60.
[12] *Op. cit.* p. 68.
[13] *Op. cit.* p. 81.
[14] *Essai sur les conditions et les limites de la certitude logique* (Paris, 1894), p. 233.
[15] *Op. cit.* p. 19.
[16] *L'Évolution créatrice* (Paris, 1907), p. 270.
[17] *Op. cit.* p. 216 ff.
[18] Bergson, *Essai sur les données immédiates de la conscience* (Paris, 1889), p. 96 ff.
[19] Plotinus also regards extension, if not as an inversion of the original Being, as a weakening of its essence, one of the last stages of degradation (*Enneads*, IV. iii. 9-10; and III. vi. 17-18).
[20] *Op. cit.* p. 227 ff.
[21] *Op. cit.* p. 237.
[22] *Op. cit.* p. 239.
[23] *Die Phantasie als Grundprinzip der Weltprozess* (Munich, 1898).
[24] Baldwin, too, in his *Thought and Things*, vol. i. preface, p. x. *sq.*, looks to aesthetic experience, which alone is capable of transcending the dualism of subject and object, individual and universal, etc., created by logical and practical activity, for the fullest and most direct revelation of reality: "It is, on the contrary, in a form of contemplation, aesthetic in character, that the immediacy of experience constantly seeks to re-establish itself. In the highest form which comes to itself as genuine and profound aesthetic experience, we find a synthesis of motives, a mode in which the strands of the earlier and diverging dualisms are merged and fused. In this experience . . . consciousness has its completest and most direct and final apprehension of what reality is and means."
[25] "L'Esprit positif," *Revue de métaphysique et de morale* (September 1901), p. 597 ff.
[26] *Op. cit.* p. 598.
[27] *Op. cit.* p. 608.
[28] *Op. cit.* p. 643.
[29] "Un Positivisme nouveau," *Revue de métaphysique et de morale* (March 1901), p. 140 ff.
[30] "Science et philosophie," *Revue de métaphysique et de morale* (September 1899), p. 515 ff.
[31] Le Roy, "Science et Philosophie," *Revue de métaphysique et de morale* (September, 1899), p. 518.
[32] *Un Positivisme nouveau*, p. 144.
[33] *Op. cit.* p. 524.
[34] *Op. cit.* p. 527 ff.
[35] Cp. on this point the analyses of Duhem, *La Théorie physique* (Paris, 1906), p. 308, and of Milhaud, *Le Rationnel*, p. 66.
[36] We will enter on a more detailed discussion of these new theories in the realm of mathematics and physics in the second part of this work.
[37] Le Roy, *op. cit.* p. 559 ff.
[38] "Sur quelques objections adressées à la nouvelle philosophie," *Revue de métaphysique et de morale* (July 1901), p. 409 ff.
[39] *Op. cit.* p. 413.
[40] *Op. cit.* p. 415.
[41] *Op. cit.* p. 417.
[42] *Op. cit.* p. 425.

[43] *L'Esprit positif*, p. 638.
[44] Le Roy, *op. cit.* p. 428.
[45] "Science et philosophie," *Revue de métaphysique et de morale* (January 1900), p. 66.
[46] "Science et philosophie," *Revue de métaphysique et de morale* (November 1899), p. 719.
[47] *Op. cit.* p. 731.
[48] "L'Illusion idéaliste," *Revue de métaphysique et de morale* (November 1898), p. 742; *L'Action* (Paris, 1893), pp. 443 ff., 468, 479.
[49] *Op. cit.* pp. 314-317.
[50] *La Valeur de la science* (Paris, 1906), p. 217.
[51] Poincaré, *op. cit.* p. 233.
[52] Couturat, "Contre le nominalisme de M. Le Roy," *Revue de métaphysique et de morale* (January 1900), p. 88. Cp. also Brunschwigg, "La Philosophie nouvelle et l'intellectualisme," *ibid.* (July 1901), p. 468.
[53] "Réponse à M. Couturat," *Revue de métaphysique et de morale* (March 1900), p. 225 ff.
[54] *Op. cit.* p. 226.
[55] Poincaré, *La Valeur de la science*, p. 236.
[56] Poincaré, *op. cit.* p. 238 ff.
[57] *Science et Hypothèse* (Paris, 1905), p. 163.
[58] *La Valeur de la science*, p. 231.
[59] *La Théorie physique*, p. 235 ff.
[60] *Op. cit.* p. 261.
[61] *Op. cit.* p. 279.
[62] *Op. cit.* p. 283.
[63] Duhem, *op. cit.* p. 246.

CHAPTER II

ANGLO-AMERICAN PRAGMATISM

1. *Pragmatism as Evolutionary Transformation of English Empiricism.* — Anglo-American pragmatism, though at first sight it may appear to be a revolutionary philosophy, is in reality but an evolutionary transformation of the old English empiricism. James himself shows this clearly in the dedication of his last volume, *Pragmatism*, to the memory of Stuart Mill, and also in the sub-title of that work, *A New Name for Some Old Ways of Thinking.*[1] Into what facts can an idea be resolved ? What is its value in terms of special experiences ? This is the problem which James considers that the English school has always more or less consciously striven to solve. Locke, in fact, treats the question of personal identity in this way, reducing its meaning to the series of our detailed recollections, the only part thereof which can be verified in a concrete manner, and eliminating the idea of spiritual substance as useless and unimportant. By means of the same procedure Berkeley finds in sensations, connected and ordered in different ways, the practical value of the idea of matter, and David Hume recognises in the concept of cause merely the habitual tendency to expect like phenomena in like circumstances.[2]

2. *The Pragmatism of Peirce.* — Charles Sanders Peirce,[3] who was the first to make use of the word *pragmatism*, really did nothing but set forth explicitly that principle which had been the instinctive guide of

these philosophers. According to Peirce, the activity of thought tends to attain rest in a belief, because it is only then that we can have a stable and sure guide to our actions upon objects. Beliefs are rules of action, and the function of thought has no other end than the production of active habits. All ideas which fail to determine any difference in the practical results of thought form no true and proper part of its meaning. In order to develop such a content, we need therefore merely specify exactly what line of conduct it is suited to produce: there is no possible distinction in the meaning of our ideas which does not give rise to a difference of a practical order. Hence, if we would attain to complete lucidity of thought relative to an object, we must merely consider what sensations, either immediate or remote, are to be expected from it, and what reactions we should prepare in case the object should prove to be true. The positive significance of ideas lies in these practical consequences, all other more or less subtle distinctions are valueless.

3. *Utilitarianism and Pragmatism.* Anglo-American pragmatism did not, however, stop short at Peirce's position,[4] but went still farther. It does not merely say: "The value of ideas lies in the practical consequences," but speaks of *good* results, and ends by simply identifying the true with the useful and opportune. The utilitarianism of Bentham and Mill, which was at first confined to the sphere of moral life, boldly entered upon a desperate undertaking, rather than confess itself vanquished in the age-long struggle against the loftiest human ideals. The cognitive and aesthetic functions alike, with their disinterested ends, contrasted too forcibly with the thesis of utilitarianism, proving, as they did, the mind to be capable of recognising values external to and above social and individual pleasure: were not the search for truth and the creation of beauty the strongest contradiction of utilitarianism? It was natural that empiricism should muster its forces for the assault on this impregnable citadel of human

disinterestedness. Bacon had already insisted on the practical end of science, stating that, *ipissimae res sunt veritas et utilitas*, but he had also added: *Atque opera ipsa pluris jacienda sunt quatenus sunt veritatis pignora, quam propter vitae commoda*:[5] and had placed the *contemplatio rerum* above the *inventio fructus*. We rejoice in the light because with its help we can work, read, and see one another, but the very sight of the light is of greater value than all its manifold advantages. Locke, in his *De arte medica*, had stated still more forcibly: "Those who apply themselves seriously to finding and combining abstractions take great pains for a thing of little account, and would do well, although they be men, to play with the puppets of their childhood . . . there is no knowledge worthy of the name but that which leads to some new and useful invention, which teaches us to do something better, more quickly, and more easily than formerly. Every other speculation, however singular and ingenious it may be, whatever its appearance of depth, is but vain and idle philosophy, an occupation for those who have nothing to do." "Our business in this world is not to know everything, but to know that which concerns the conduct of our life." David Hume says: "Indulge your passion for science, says she (nature), but let your science be human and such as may have a direct reference to action and society. . . . Be a philosopher, but, amidst all your philosophy, be still a man."[6] Empiricism, however, although containing germs which, when developed and carried to their ultimate consequences, might easily degenerate into pragmatism, was far from denying the value of disinterested research. "Were the generality of mankind contented," says Hume, "to prefer the easy philosophy to the abstract and profound, without throwing any blame or contempt on the latter, it might not be improper, perhaps, to comply with this general opinion and allow every man to enjoy, without opposition, his proper taste and sentiment. But as matter is often carried farther, even to the absolute

rejecting of all profound reasonings, or what is commonly called metaphysics, we shall now proceed to consider what can reasonably be pleaded on their behalf." [7] Treating of the problem of human liberty, he makes an observation which might well have been written by William James: "There is no method of reasoning more common and yet more blamable than, in philosophical debate, to endeavour the refutation of any hypothesis by a pretence of its dangerous consequences to religion and morality. When an opinion leads into absurdities it is certainly false, but it is not certain that an opinion is false because it is of dangerous consequences." [8]

Notwithstanding these reserves, it was, however, but a step from utilitarian empiricism to pragmatism. One of the factors which contributed most effectually to this transformation was undoubtedly the evolutionary theory applied to the development of human consciousness. Had not Spencer said that thought with its logical structure is but a means of adaptation, an organ originating, like all other organs, in vital necessities? Is not the advantage of the species the biological meaning of psychic life? Empirio-criticism, transferring the old English empiricism to Germany, had already applied this principle to the evolution of science and philosophy: [9] what wonder, then, that in the native lands of Spencer, Darwin, and Romanes [10] a logical and epistemological theory was built up on this postulate, which had never been called in question? Why should pragmatism indeed waste time in discussing the theoretic value of that principle when it was so convenient to accept it? What matter if the intellect were not convinced by it? It is opportune, and that is enough: "Human arbitrariness," pronounces James, "has done away with the divine necessity of scientific logic." [11]

4. *Reasons for the Prevalence of Pragmatism: James's Will to Believe.*—Pragmatism would not, however, have taken hold and spread so rapidly had it not found favourable soil in minds weary of the abstract

formulas of scientific naturalism and dissatisfied with the dreary prospects which it held out to souls yearning for faith and hope. The will to believe came at just the right moment to fill the void left in the mind by Spencer's Unknowable, that fatal outcome of the scientific method which had been exalted to the rank of a philosophic method. The door was open to the inspirations of feeling; since intellect, backed up by mathematical science, had done such poor service, and had been constrained to own itself beaten, why not try to find a substitute for it? Is not the human mind endowed with other and greater energies capable of surmounting these obstacles and breaking down those barriers with which agnosticism strove to hamper its free activity? The *raisons de cœur* of the mystic Pascal, the moral and religious arguments which David Hume would fain banish from philosophic disputes, take their part in the fray. We have once again in the contemporary social consciousness that tragic situation which led to Kant's Primacy of the Practical Reason, which is called upon to decide the struggle between intellectual exigencies and the imperious needs of duty. James's essay on the Will to Believe [12] is at bottom but a revised and corrected edition *ad usum Delphini* of the doctrine of the primacy of the practical reason, which had already been taken up again and elucidated with greater acumen and philosophical depth by Lotze and Renouvier, yet James is not ashamed to say: "The mind of Kant is the strangest and most intricate possible of museums of antiquities!" [13]

In this first essay James does not as yet declare himself plainly to be an opponent of intellectualism, but suggests his method to us as a means of deciding those questions only which intellectualism leaves open. Our passional nature not only may but should decide in all cases of true option between two opposite alternatives in which a choice based on *intellectual grounds* is impossible, since to say under such circumstances: "I will not make any decision, but will leave the matter

open," is in itself a passional decision just as much as a decision in the negative or affirmative would be, and is equally exposed to the risk of missing the truth.[14] Even James himself puts us on our guard against carrying anti-intellectualism too far: in the concrete the liberty to believe should only be applied to those practical options which the intellect is unable to decide; and these *passional decisions* must be avoided as much as possible, whenever facts allow of our doing so. Whenever the choice between missing and gaining a truth is not absolutely indispensable to life, we may relinquish the probability of gaining a truth, or in any case avoid the risk of believing in something which is not true, by not deciding until we are in possession of objective evidence. In science more especially we must never be in too great a hurry to choose hypotheses; doubt is preferable to falling into error.[15]

So far, James has not left the "museum of antiquities" of the primacy of the practical reason: will and feeling are called upon merely to supply that which is lacking in the intellect which may and should await conviction when it is a question of beliefs which are not vital. James, although he takes up the standpoint of "*radical empiricism*," [16] still recognises the autonomous existence of truth and the ability of the human mind to attain to it by means of successive approximations, but, while absolutists not only believe themselves to be capable of truth, but affirm that they are already in possession thereof, and can see no salvation apart from their dogmatic philosophy, the empiricist is never sure that he has attained it.[17]

5. *Differences between "la Philosophie nouvelle" and Pragmatism.*—Anglo-American pragmatism soon went beyond this stage of prudent relativism, and, intoxicated by its easy success, did not hesitate to declare war on intellectualism in the realm of science as well. This result was brought about to no small degree by the criticisms of Mach, Ostwald, Pearson, Milhaud, Poincaré, and Duhem, and more especially by those of Bergson

and Le Roy, who had laid special stress on the economic value of scientific theories and on the active, personal, and arbitrary element in the determination of facts and laws. The *Philosophie Nouvelle*, to which indeterministic idealism in France gave birth, yet bears to a certain extent the stamp of its metaphysical origin; whereas pragmatism, which is derived by natural evolution from empiricism, strives as far as possible to remain true to the programme drawn up by Peirce. This imparts a physiognomy of its own to Anglo-American pragmatism, which distinguishes it clearly from the new French philosophy. The action spoken of by Bergson, Le Roy, and Blondel is not practical external action, empirical fertility in the realm of facts, but profound, interior, experienced action, the concentration of the mind on itself in order that it may intuitively grasp its creative activity.[18] The truth of the idea, according to pragmatists, lies in its consequences, in its empiric content; according to intuitionists, on the other hand, its true reality lies in that spiritual action which precedes it. The former descend from ideas to facts, and hence tend to realism; the latter rise from discursive thought to the creating mind, and therefore reach idealism.

6. *The Humanism of Schiller.*—The logical and epistemological development of pragmatism is mainly due to Schiller and Dewey: James has merely spread their doctrines and applied that method more especially to religious problems.[19] He is, however, after Peirce, the first inspirer of the new current of thought, and must be therefore regarded as the spiritual leader of the school,[20] which gathers together in the unity of the method tendencies and applications of different kinds.[21] It is indeed to him, as the most human of philosophers, that Schiller dedicates his chief work, which proclaims itself, with a touch of the solemnity of a *Novum Organum*, to be a reform of customary logic.[22] This has been up to now a pseudo-science of that non-existent and impossible process commonly called pure thought, in whose name

we have undertaken to banish from our mind the most minute trace of interest, desire, and emotion, as we should the most pernicious source of error. The new logic, on the contrary, considers that it is an emotional postulate, which takes the first place in the acquisition of our knowledge because there is no such thing as an argument which is not derived from an internal passion of the mind, and which is not based upon a more or less sentimental belief, and upon a subjective need.[23] The old saying of Protagoras, "Man is the measure of all things," is, when interpreted aright, the greatest discovery of philosophy: it does not lead to scepticism, but impels science to enquire how man can measure, and what expedients will enable him to bring his measures into agreement with those of his fellows. Humanism regards human consciousness as the centre of the universe, and makes use of its guidance alone in the world of experience, rejecting every *a priori* principle in whose name the possibility is claimed of reducing that which is the concrete type of every reality to an illusory appearance of some fantastic Absolute.[24] Knowledge is not impassive contemplation of the Absolute, but a form of practical activity which belongs to the sphere of moral responsibility. Pure reason is a mere fiction: an intellect which is of no value to the ends of life is a monstrosity, a pathological aberration, a failure in adaptation which must sooner or later be eliminated by natural selection.[25] The idea of value is more primitive than that of fact; without valuation there is no knowledge. The ultimate problem of philosophy may be summed up in these questions: What is reality? To what end, for what useful purpose is it real? The reply will naturally vary according to the end. The direction of our effort, which is determined by the desire and the will to know, enters as a necessary factor into the revelation of reality. Reality, considered as unknowable, is nothing; as something unknown it is only potentially real. The nature of things is not determinate but determinable, like that of our fellows; the

nature of the answers given is determined by our questions. The notion of a fact in itself is an anachronism, just as is the idea of a thing in itself. It is quite untrue that we count for nothing in the construction of the world; on the contrary, our action is essential and indispensable, since without us fact would not be *what is made.* Within what limits and in what direction the world is plastic and can be moulded by our action we do not yet know for certain, but we know enough to transfigure the aspect of existence in relation to ourselves. The disparity between our power and exterior forces, great as it may be, is yet not incommensurable, and nature has never yet refused to reply when she has been questioned in the pragmatical method.[26] We make choice of the conditions under which reality is to be manifested to us, and may, if we have chosen badly, provoke by our action a hostile reply, and therewith our destruction, hence the choice is at our own risk. Pragmatism thus imparts fresh vigour to the sense of responsibility by leaving in the world much that is indeterminate.

Every cognition, however theoretic it may be, is of practical value, and is therefore potentially a moral act. Even the so-called eternal principles of mathematics are human constructions, postulates, that is to say, demands which we make of our experience because it is necessary to us that it shall become a cosmos adapted to life; they appear to be obvious and axiomatic because they are so firmly rooted in our mental habits that it occurs to no one to call them into dispute.[27] At bottom, theoretic principles, like practical ones, derive the whole of their meaning and value from their utility to us. That alone is necessarily true which is necessary to our needs. The true is the useful, the useless is the false. The definition of the true as agreement with the object is not tenable, because we should need to have independent knowledge of the thought on the one hand and the reality on the other, which is absurd. The other method of conceiving of truth as

systematic coherency is no better, first of all because not every system is true, and we should therefore need another criterion to distinguish the true from the false systems, and secondly, because the bodies of truth which we acquire in our science are all partial and incomplete systems which are not in harmony with the rest, hence it follows that no actual system is true. It may be urged that actual systems are approximations to an ideal system, which is absolute coherency, but what right have we to suppose that there is one system only and not several different ones ? Reality may be constructed in different ways by varying our efforts, and the exigency of the system is but the need of some harmony producing emotional satisfaction, and not of a purely logical and formal coherency.[28] Pure thought is, as we have already stated, non-existent: logic cannot be divorced from psychology. A truth which is actually present is in the first place a process of consciousness, and, as such, subject to a large number of psychological influences, such as desire, interest, attention, will, etc. The same thing may be said of coherency, which is primarily a psychic fact and cannot therefore be attained by means of argument, but is immediately felt. The movement of thought is initiated, sustained, and guided by interest: knowledge is a form of valuation which does not differ essentially from the rest. There is nothing to guarantee the agreement of the valuations of one man with those of his fellow-men, or with his own made at another time; but the necessities of social life demand the systematic coherence of all truths. Single interests are subordinated to the principal ends of life, therefore certain of them disappear according to the law of natural selection: of the subjective valuations of truth those only survive which are of social utility, and which answer best to the common aspirations of man.[29] Our preference for certain conceptions is due to a mere criterion of convenience: the Copernican hypothesis gained the preference over the Ptolemaic only because it required a smaller number of auxiliary hypotheses,

and because the calculations it involved were simpler. Thus, too, the geometry of Euclid still practically reigns supreme over other systems, because its application to the world of our experience is easier and more convenient; metageometricians have had to confine themselves to imagining other worlds subject to laws other than those of Euclid's geometry. Certainty in the sense of intrinsic coherency, of harmony with the definitions and postulates from which we start, has nothing to do with the question of the objective validity of principles, which depends upon the possibility of systematising our experience by means thereof. Applied geometry is not certain, but useful.[30] Geometrical judgments are universally valid, solely because it is greatly to our interest to keep them so.[31] In like manner, if in the realm of physical sciences we admit the existence of universal and eternal laws, and also that the individuality of things in their special spatial and temporal determinations is negligible, we do not so act because we are convinced of the theoretic validity of that supposition, but rather because we are constrained thereto by its practical convenience; we must, that is to say, make previsions concerning the future existence of things in order to regulate our conduct. The postulate of the persistency of laws does not reveal to us any necessity of nature, but is merely a methodological expedient, answering to the need of finding formulas which will enable us to calculate events without awaiting their verification.[32] The things of common sense, the atoms of the physicist, the Absolute of the philosopher are but schemes for ordering the manifold qualities of phenomena corresponding to certain practical requirements, but these abstractions, in as much as they are instruments which can produce effects upon experience, become possessed in our thought of the value of reality.[33] Immediate experience does not satisfy our needs, and for this reason we construct realities answering our practical requirements better; this must not, however, lead us into the error of regarding

as illusory that sensible world which must always remain a necessary point from which we start and to which we refer. We can make different constructions according to the ends we have in view, and these constructions are frequently of a contradictory nature: even in the one realm of physical phenomena we have the theories of atoms, electrons, and vortex-rings; and explanatory schemes become more numerous when we pass from one science to another; so much so that we might well ask: Is the real world that postulated by physical science, or that of geometry, psychology, or ethics? The philosopher feels the need of eliminating the discords between these worlds, of attaining to an ultimate reality of a more satisfying nature, capable of embracing within itself and harmonising the schemes of the different sciences, and thus putting an end to our uncertainties; but this ultimate reality must preserve its connecting links with the world of appearances which it was called upon to explain. Immediate experience is to a certain extent more real, that is to say more directly real, than those schematic constructions which have been erected upon its foundations: if we destroy this foundation, the whole edifice will collapse.[34] Phenomenal appearances must be preserved: the world is really coloured, sonorous, hard, painful, spatial, and temporal. The units of measurement of the reality introduced will in ultimate analysis be found in these appearances, and the assumption thereof will be vain if they in no way serve to fill the gaps therein and to transform them actively. That which must decide whether an introduced reality be merely a fiction of the imagination or exist effectively is its efficacy, the power imparted to us by its aid. The scientific theory of the transmission of light through the air is preferable to the poetical idea of the fluttering of the wings of hypothetical cherubim, since we may discover how to act upon the air but could never have any means of control over the movements of these invisible cherubs.[35] The more capable reality shows itself of rendering life harmonious, the truer it is; that

knowledge which fails to satisfy and which imparts no power to us is false. The ultimate goal of cognitive activity is not an infinite and complete system of relations, within the limits of which there is room for endless discussion, but the vision or immediate perception of a reality which has absorbed into itself all truths without destroying or denying them, the luminous and transparent contemplation of that perfect harmony which embraces all things and reveals the whole meaning of the cosmic process in the full expression of its supreme goodness and its divine beauty.[36] The ideal of life is not the arrest of motion, but the perfection of motion in the equilibrium of an activity which is self-sustaining: the Ἐνέργεια ἀκινησίας of Aristotle.[37] True being is not immutable substance, but perennial activity; it is not something transcending experience, but that which brings it to perfection. There is no reason to confine this perfection to divinity, as does Aristotle. We can easily conceive of a cosmos of beings whose activity has overcome change and attained to perfect equilibrium. The realisation of themselves, and of the potentiality possessed by each one in its own nature, should take the form, not of a revolting, cruel, and pathological restlessness, not of an unending effort, but rather of an activity which, overcoming change and time, is preserved in a harmonious equilibrium.[38] The harmony of the cosmos is not capable of logical proof, but we must assume it as a postulate which satisfies a profound longing of ours; in the last analysis there is no stronger evidence of the intelligibility of the world which is taken for granted by physical science than there is of ethical and religious beliefs, such as the belief in a moral order and in the immortality of the soul. There is only one method of valuation, *i.e.* the measure of efficacy in life; there is only one foundation, which is always a postulate of an emotional nature.[39]

7. *Dewey's Instrumental Logic.*—John Dewey lays special stress upon this analogy between scientific and moral judgments, which may be said to constitute the

essence of pragmatism.[40] It is usual to draw a distinction between moral judgments, whose aim is the solution of special cases, and which are dependent upon the personality of the judge, and scientific judgments, which give expression to universal and objective relations, independent of individual tendencies. Now, according to Dewey, this is erroneous: it is true that the man of science seeks for general laws, but he, too, in reality sets himself the task of solving special cases; the laws are but the means, not the end, so much so that, if the law proves inadequate to solve the special problem in question, it is changed, whereas the specific case always remains unaltered: science and morals alike aim at the individual. On the other hand, even in scientific judgments, the personality of the judge affects the preliminary classification of possibilities or predicates, the choice of the individual cases to be studied, and the method of verifying the hypotheses, whether by means of experiment or of demonstration. The *character* modifies, guides, and suggests the judgment. If the use of the resources of science, of experimental technique, of systems of classification, etc., which guides the act of judging and therefore determines the content of the judgment, be dependent upon the interest and inclination of the judge, we are forced to recognise that the two forms of judgment are perfectly analogous.[41] There is, however, a certain difference: whilst the intrusion of the personal element is of no consequence in scientific judgments, in moral judgments, on the contrary, this factor qualitatively colours the meaning of the situation; hence, seeing that this element practically amounts to nothing in scientific judgments, it is logically useless to take it into account, whereas it is impossible to ignore it in moral judgments, which are of themselves true and proper acts whose efficacy is felt in the practical conduct of life. In short, whereas in scientific judgments the character, that is to say the complex, of natural tendencies, technique, habits of thought, etc., is a uniform and impartial condition, in the moral sphere

its action is explicit, causing us to prefer one special judgment to another, and it becomes hence the predominant factor.[42] This does not, however, entitle us to regard scientific judgments as differing in nature from others, or to abstract thought arbitrarily from the individual psychological content. Dewey expresses surprise that, in spite of the progress made by the evolutionary method in natural science, certain systems of logic (it would have been more accurate had he said all sensible ones) persist in drawing a sharp distinction between the problem of origins and that of nature, between genesis and analysis, between the history and the validity of thought.[43] The whole meaning of the evolutionary method, which has brought forth so much fruit when applied to biology and sociology, is that each distinct organ, each structure, each formation group of cells and elements must be regarded as a means of adjustment to special situations of the environment: its meaning, character, and value are known only when it is considered as an instrument demanded by a specific situation. Psychology, as the natural history of the various attitudes and various structures through which thought passes in the course of its development, and as the knowledge of the conditions from which it issues and the ways in which it acts by the stimulation or inhibition of other states of consciousness, is indispensable to logical valuation. Thought is but a form of adaptation to its generative conditions, and its validity must therefore be judged in relation to its efficacy in the solution of these problems. Instrumental logic, according to Dewey, knows nothing of the object in itself of a thought in itself, but recognises a series of values varying with the variation of the functions, and which can therefore be determined only by reference to them.[44] Thought is not something pure and absolute, existing for itself, whose office is to reflect and represent a world of independent realities, but a function which has been formed, like the rest, in the course of experience, and which originated in determinate needs. Its every

stage is of value in so far as it corresponds to the exigency of certain conditions; the later stage in which other values arise does not entitle us to pronounce the preceding moment to be false. The stimulus to the function of thought must be sought in a situation in which the elements conflict, in a state of reciprocal tension, which would lead to the dissociation of experience were the re-organising work of reflective thought not to intervene and re-establish the equilibrium of the system, causing distinctions to arise in the heart of the primitive non-differentiated totality.[45] In this work of restoration and re-integration (re-definition, re-relation),[46] lies the whole meaning of the logical function, whose antecedent is therefore always to be found in a conflict between the various parts of the world of physical, social, or intellectual experience. This situation, which constitutes the starting-point, with its tensional elements remains something objective, but is, in as much as it sets thought a problem, suggestive of the subjective phases of another system, that is, which at present appears to us to be the more or less uncertain solution of the conflict. In short, the situation, from being a single principle, inevitably tends to become polarised and to dichotomise itself: there is something which is not affected by the contest of incompatibles, there is something which remains assured and beyond question; on the other hand, there are elements which become doubtful and precarious. This gives rise to the general division of the sphere into facts (the given, the presented) and ideas (the conceived, the thought). That which remains indubitable is the fact of the conflict or tension with that specific colouring, that individual physiognomy which cannot be replaced, but which is immediately felt; the new relations and new positions assumed by the elements in the re-integrated system are, on the other hand, of a problematic nature. The memory of the past gives us other experiences, other contents which teach us to interpret present facts; these contents are distinct from the facts themselves, as pure

possibilities, ideas, thoughts, ways of conceiving; but this division is merely relative: in other words, the given and the thought are but divisions of labour, co-operative instruments of an economic commerce whose object is the preservation of the integrity of experience.[47] The concept is not a formal stamp or seal of the ready-made order, applying to a content, but is a continuous transformation of data in a certain direction. The proof of the validity of an idea is its functional or instrumental efficacy in effecting the transition from an experience which is relatively in conflict to another which is relatively integrated. The difference between ideas and facts does not lie in their content, but in their function: the idea is simply that part of total experience which is regarded as tentative and corresponds to the predicate of the judgment, whilst the situation which has called it forth forms its subject.[48] The idea is not universal in itself, but is a particular content which acquires universality through its re-organising function; it is not of value in as much as it is a copy or sign of other contents, but in so far as it is an instrument of action, an economy in the process of thought.[49] In other words, the idea, being chosen because it fulfils a certain office in the evolution of unified experience, can only be proved by verifying whether it succeeds in the task which it has set itself. The conception of thought as a purely formal activity, which is exercised upon an independent matter, is meaningless; moreover, the success of such an action would be a miracle of a most remarkable order. Were the instrument and the material originally extraneous to one another and to the result, their reciprocal adaptation to the attainment of a valid result would be simply miraculous; it is however easy to explain when we reflect that both of them have been chosen and elaborated with special view to that end, that is to say, to the preservation of harmonious experience. The adaptation of thought is not pre-arranged all at once from the first, but is accomplished little by little, case by case, accord-

ing to the exigency of the particular situations. The problem of the validity of thought in general, as distinct from the value of this or that special process, is only raised when thought is arbitrarily considered apart from its historical position and its material context. The logical function, on the contrary, is an active and fruitful part of the evolution of experience : each cognitive act makes a difference to and in things, and the more it reveals to us the perennial becoming of the word, and teaches us to see the universe not *sub specie aeternitatis*, but *sub specie generationis*, the more adequate it is to its object. This does not, of course, imply that thought creates reality, as is alleged by idealism, since its function is merely to reorganise and reconstruct an empirical situation which already exists.[50] But this existence, on the other hand, must not be conceived of as something transcending that conscious experience, which is non-differentiated matter, and gives birth by a simultaneous process to subject and object.[51] When activity develops without being interrupted or arrested, and no conflict arises between the motor responses adapted to the various parts or aspects of the situation (as, for instance, in aesthetic activity), the distinction between subjective and objective is not apparent to the consciousness of the agent ; but this opposition is formed only when an impulse to reaction in a certain direction meets with an obstacle to its complete manifestation.[52]

8. *Pragmatical Elimination of the Duality of Subject and Object.*—With regard to the problem of the reality of the external world, it will be seen that pragmatism is in agreement with empirio-criticism, in as much as it does not admit the primitive duality of subject and object, but claims to be able to rise to an undifferentiated experience which is their common source. James maintains [53] that sensible reality and our sensation thereof are absolutely identical with each other : our sensations are not miniature internal duplicates of things, but things themselves in so far as they present to us. The content of the physical world does not

differ from that of the psychic world; even when we dream of a mountain of gold, which undoubtedly does not exist apart from our dream, it appears to us in our dream like an object possessed of physical existence. In the same way the content of our memories is not an internal subjective fact, projected outwards by ourselves, but is the distant object itself. The act of thinking this content is not a duplication thereof, but is the association into one group of other mental facts, such as the emotions which it has excited, with the effort of attention and the ideas which have recalled it. The phenomenon, placed in these relations, appears to us to be thought; if, on the other hand, we consider it in relation to the other facts of a physical order, it will appear objective. Consciousness is not an entity, an activity which is added to its content, but is reducible to the complex of these "pure experiences," which can be mutually related in various ways and belong to various groups at one and the same time, so that that which in a certain context of contiguous facts is classified as a physical phenomenon may figure in another group as a fact of consciousness, just as a particle of ink may belong simultaneously to two straight lines, one vertical, the other horizontal, provided it be situated at their point of intersection.[54] Subject and object, thought and thing, are but practical distinctions, of great importance certainly, but of functional order only, not ontological, as classical dualism represents them to be, since they are in the last analysis made of the same stuff, which cannot be defined but must be experienced immediately, namely, the stuff of experience in general.

9. *Plasticity of Experience according to James.*—The starting-point of reality is the flux of our sensations which are thrust upon us, coming to us without our knowing whence: their nature, their order, and their quantity elude our action. They are neither true nor false, they merely *are*; the distinction between true and false only applies to what we say of them, to the name we give them, to our theories touching their nature

and their relations.[55] The second element of reality consists in the relations between our sensations or between their images in our consciousness. Some of these relations are variable and accidental, as, for example, those of time and place; others are fixed and essential, because they are based upon the internal nature of their terms. Both these kinds of relations are immediately perceived, and are "facts," but the latter kind is of greater importance to the theory of knowledge, because it embraces the "eternal" relations, which are apprehended each time their sensible terms enter into relation, and which must be eternally recognised by logical and mathematical thought. The third element of reality is formed by preceding truths, which are taken into account in all new research: this second factor, which is of less resistent quality than the others, always ends by giving way to us, but even the two first are not entirely impervious to our action. It is true that the sensations "*are*": their flux is independent of us, but we make our choice among them, according to what best serves our interests, recording some of them, omitting others, and arranging them in the order most convenient to us. What we say of reality depends upon the point of view from which we regard it: the Englishman looks upon the battle of Waterloo as a victory, the Frenchman as a defeat; to the optimist the universe is a good thing, to the pessimist it is the worst possible evil. That which is proper to things is their indeterminate being — the *that*, — but their determinate nature, the *what*, depends upon the *which*, that is to say, upon our way of regarding it. We are given the block of marble, but *we* have to carve the statue out of it.[56] The same thing may be said of the "eternal" parts of reality: we disturb and arrange at will our perceptions of intrinsic relations, we classify them in one series rather than in another, consider one more fundamental than another, until our beliefs with regard to them constitute those systems of truths which we term logical and geometrical,

whose form and order are obviously the work of man. The two first elements of reality, sensations and relations, are mute elements, and tell us absolutely nothing about themselves: we must speak for them. It is impossible to conceive of reality independently of human thought, unless it be regarded as something in course of arising in the realm of experience, an evanescent and indeterminate flux, plastic matter, to which we must give the finishing touches. The world is not, as rationalists would have us believe, the infinite folio edition, the *édition de luxe* complete to all eternity, which individual consciousnesses fail to decipher in its entirety, and which they reproduce in so many small, finite editions, full of misprints, and more or less mutilated and distorted; it is rather an edition which is not yet perfect, and is in process of gradual completion, more especially through the activity of thinking beings.[57] These beings tend to mould reality in various ways, according to the special ends they have in view, and the flux of sensations passively assumes those forms. The number 27, for instance, can be regarded as the cube of 3, the product of 3×9, as $26 + 1$, as $100 - 73$, and in infinite other ways, all of them equally correct; thus a chessboard may be regarded as made up of black squares on a white ground, or of white squares on a black ground. Reality can be printed in our human editions, which are all equally true, provided they answer the purpose for which they were elaborated. The historian and the moralist regard the individual as a person; the anatomist as an aggregate of tissues; the histologist as a complex of cells; the chemist as an aggregate of molecules. It is for us to condense into things at will the liquid flux of sensible reality, thus creating not only the subjects, but also the predicates of our judgments, which merely express the relations in which things stand to human interests and feelings.[58] In the cognitive function, as in practical life, we are the creators of truth and law.

10. *Ideas as Instruments of Action.*—Ideas are not

true in themselves, but rather in as much as they put us into a satisfactory relation with other portions of our experience, they are abbreviating schemes, which save us the trouble of following the interminable series of special phenomena.[59] Each scientific theory solves a problem of maxima and minima: it is instrumentally true if it demands the minimum of intellectual effort, and if it adapts previous systems to new facts with the minimum amount of alteration. The view expressed by Ramsay, which seeks the origin of radiations in the internal potential energy of the atom, is generally accepted as true, because it extends the old concept of energy with the least possible alteration in the nature thereof. We declare those ideas to be true which serve to connect the stable mass of previous knowledge with the new contents of experience, thus fulfilling a mediatory function (marriage-function, go-between).[60] There is no absolute criterion of truth: each theory is merely possessed of an instrumental value of adaptation to certain special conditions.[61] The idea does not exist, but is made, and becomes true with the facts: its truth is the process of its verification (*veri-fication*); its validity is the process of its convalidation (*valid-ation*). If we follow up the mental image of a thing, we are led actually to see the thing: we thus have the complete verification of it. These guides of thought, simply and completely verified, are undoubtedly the originals and prototypes of the process of truth; the other mediate and abstract forms of knowledge are conceivable as primary verifications, inhibited, multiplied, or substituted for one another.[62] The larger number of the notions of our life are not directly verified, the possibility of verifying them is in practice enough for us, so long as we find nothing contradictory of them, and we generally give credence to our ideas if they harmonise with past experience, even if it be that of other persons. The reality of the past is guaranteed by its possible agreement with the present, with actual facts, which are the final term of reference, because every true process

must in the last analysis lead to sensorial experiences which can be directly verified. In practice the work of verification is much facilitated by the fact that in nature things do not exist singly, but are arranged into species; hence, when we have verified part of the class, we feel ourselves entitled to extend our judgment to the rest. In addition to these truths which are only indirectly or potentially verified, there exist ideal relations (for instance, $2+2=4$; the effect begins when the cause begins to act) which we recognise as being eternal and immediately true, and which therefore seem to be exempt from the common law of verification; but even in this sphere of pure mental relations truth is in reality gauged by the convenient direction it is able to give to our activity in the world of experience. We refer one abstract idea to another, and arrange them in the great system of logical and mathematical truths, into which sensible facts will eventually fit; and it is just this possibility of application, this characteristic with which the logical structure is endowed of being prepared for every kind of imaginable object of experience, which gives our eternal truths their guarantee of reality.[63] Truth is but a collective name serving to indicate certain processes of verification, just as health and strength are names for certain vital processes. The true is merely the convenient in the sphere of our thought, just as the right is merely the opportune in the sphere of our action; and, just as there is no such thing as absolute convenience, so there is no such thing as absolute truth: Ptolemaic astronomy, Euclidean space, the logic of Aristotle, are examples of systems which were convenient for some centuries, and from certain points of view only, and have since been changed by science.[64] Finally, truth is a form of good, not, as is usually supposed, a category distinct from the good and co-ordinate with it.[65]

11. *Criticism of Pragmatism.*—All the efforts made by Anglo-American pragmatism to reduce the cognitive

function to the practical function and knowledge to action cannot fail to appear vain to the unprejudiced man who analyses the distinctive characters of the two functions. Undoubtedly the human mind is an activity in its every moment; even in knowledge it is not a passive receptacle of impressions which it receives from without, but is the reconstruction of reality in accordance with its intimate laws. This must not, however, lead us into the mistake of confusing the various forms of spiritual activity, and neglecting the specific differences which impart to each of them a physiognomy proper to itself, and an independent value in the life of consciousness. Cognitive *doing* is not practical *doing*, just as it is not artistic *doing*: the attitude assumed by us differs widely in the three cases. In the theoretic function it is a fact that we are conscious of reflecting upon something which exists independently of our subjective activity, which puts itself in opposition in various ways to our will, which is, in short, possessed of a nature of its own. The belief that reality has nothing determinate about it, that it is plastic matter which will yield to our every whim, may be useful in so far as it increases the sense of our personal responsibility (a statement which is, however, open to dispute), but is contradictory to human experience, which pragmatists themselves acknowledge to be the fundamental criterion.[66] The object is not an amorphous flux of sensations which can be segmentated and ordered as we please, but is the centre of a system of reactions which, as Schiller himself admits, may sometimes play us false and despatch us into the other world. Our action is not always successful, but at times meets with obstacles in the outer world, which proves that this world is not of an absolutely malleable nature, and that, though we can modify it in part, we cannot do so in every direction. *Facts* do not tell us that which we would have them say, they are not *that which we have made them*, as is alleged by pragmatists, who confer the rank of philosophic arguments on verbal quibbles. Beyond the sphere of

our individual life there are other active beings, some like ourselves, others apparently more or less heterogeneous; and the facts of creation are not merely a creation of our own, but also the result of the co-operation of extraneous activities, even of a non-human order. Even if in some respect they reflect our action, answer to certain exigencies of ours, and bear the stamp of human labour, in others, infinitely numerous, they elude our will and refuse to yield full satisfaction to our desires. In short, our action is one of the factors of reality, but not the only one; and if by analogy thereto we are to conceive of countless others whose combination gives birth to the life of the universe, what possibility is there of establishing dominion over them if they contain nothing determinate? A complete lack of determination in events would deprive our will of all dominion whatsoever, because our caprice would be confronted by its undying foe, the caprice of things, and nothing but a miraculous coincidence could enable our action to divine the right course amid the arbitrary succession of facts. The success of human previsions, upon which pragmatists lay so much stress, presupposes in phenomena a certain constancy of relations which we are unable to modify, and can merely imitate in our mental constructions, and which is, on the other hand, the necessary postulate of life. Does not the development of organic functions, the formation of useful habits and of stable adaptations, demand a certain persistency in the conditions of the environment? Was it vital need which forced the world into certain repetitions of its processes? What meaning has the struggle for existence in a world where there is nothing to fight, where the plastic matter of experience may be traversed in every direction without offering the least resistance to the *Wille zur Macht*, and the assertion of our supremacy? Where nothing determinate exists, the words useful, opportune, convenient, so misused by pragmatists in their pseudo-explanations of the genesis of thought, become devoid

of all value, since no idea, no means of adjustment, would be more likely to succeed than another, and that which chanced to hit upon the right course once would inevitably be doomed to fail in future experience; or, even if the arbitrary hypothesis of absolute plasticity be granted, they would all be of equal value and all equally successful, and every criterion of selection, every possibility of distinguishing between the useful and the useless, the true and the false, would be abolished. There can be no meaning in any category where determinate character does not exist. If thought with its durable and coherent structure were not the reflection of some order or system of stable relations inherent in the nature of things, it would not only be meaningless in itself, as pragmatists are never tired of telling us, but it would be worthless as an organ of life. The evolutionary theory, like all scientific conceptions, in its explanatory principles presupposes an organic structure of the real and situations in the environment to which life must adapt itself and which are therefore not created thereby.[67] Pragmatism, which accepts blindfold and dogmatically the biological origin and meaning of mental life, ends by contradicting its own postulate, when it denies the presupposition of all natural selection, that is to say, the objective physical order. Pragmatists are indeed very enigmatical on this point: they affirm and do not affirm, in order not to clash too violently with common sense. Dewey admits before logical thought a more or less organic situation, which is not, however, the absolute absence of determination; Schiller recognises in the external world resistent factors capable of establishing a limit to action, although he proposes not to take it into account, and to act so long as no obstacle intervenes (as if the most elementary action did not presuppose a more or less explicit knowledge of these factors!); James, on the one hand, states that sensations are thrust upon us, and come from some unknown source, and that we have no control over their *nature, order,*

and quantity,[68] on the contrary, he adds that our beliefs must *obediently* take into account not merely accidental relations, but also those which are essential and eternal, which are based upon the internal nature of terms, and which are not created by us, but perceived as facts; yet a few pages farther on he affirms that the order of sensations, and, in general, of every determination of them, depends upon us.[69] Is then the order of sensations independent of our action, or is it created by us? Do their intrinsic and eternal relations, which are immediately revealed to us, form something "*grounded on the inner natures of their terms*" or not? It is then not true that the task of moulding reality is entirely in our hands: it contains relations which we recognise, not because it suits us so to do, not in so far as they are useful, but because we are forced to do so by the nature of things and of our thought. The world allows us to modify it up to a certain point, provided that we in our turn submit "*obediently*" to it, recognising its universal and eternal relations. Things have a nature of their own, their mode of action is governed by certain rules which are not of our making, but must be sought and recognised by our subjective activity, if we would in any way establish dominion over them. *Natura non nisi parendo vincitur*, wrote Bacon, the great ancestor of modern empiricists. We hold a finger to the fire, and we feel it scorch us: this common experience impels us to formulate the law, "fire burns," which serves to govern our future actions; but does this constant relation between the perception of the finger in contact with the fire and the pain of the burn exist only in so far as it may aid us to avoid the burn in future? Did we create it? Is our law true because it is useful to us, or is it not rather useful to us because it is true, *i.e.* because it records in terms of thought a relation between objective facts? Pragmatists have a holy horror of the theory which regards knowledge as a copy or image of things and relations independent of the act of knowing, but must not the idea in some way reproduce in itself

something objective, if it would be an efficacious instrument of action ?[70] It is true that the term copy or image is not a very appropriate definition of the cognitive function, which is not in the least like a photograph or picture of the world : this does not, however, do away with the fact that the value of thought must be sought in its ability to reconstruct an ideal world, which, while satisfying its requirements, contains in itself the largest possible number of objective relations. In this reconstruction, in which thought may be almost said to remake consciously that which has been made by nature, creative activity is undoubtedly manifested, but the ideal product which results therefrom is not of value to us as an action of the mind, that is to say, as expressing its aptitude to transfigure and direct the content of consciousness, but derives its entire significance from reference to another real process, to which it must correspond in an adequate manner.[71] At this point pragmatists may observe : Is not the psychic content, which cognitive activity modifies in order to attain its ends, part of the general world of experience, that is to say, of reality ? If we transform our consciousness, do we not at the same time actively modify real things ? The sophism here is obvious : even if we are prepared to accept the epistemological absurdity of contents of consciousness, colours, forms, sounds, etc., existing outside that individual and subjective context in which experience presents them to us ; even if we are prepared to adopt the stand-point of James, the relations between the experiences constituting the psychic world still remain distinct from the relations between the experiences of the physical world.[72] Our cognitive activity can modify relations of the former kind, but cannot, and must not, change the exterior relations of facts : herein lies the difference between the theoretical and practical attitudes. I perceive an increase of temperature, and see the mercury rise in the tube of the thermometer : I can in thought transfigure this relation and imagine, for instance, the

same increase of temperature without the rise of the mercury or with a lowering thereof, but my activity, though it may in this way modify the psychic relation between the two facts, can exercise no influence over the external physical relation, which will always remain the same. I may devise as many theories as I like, but only on condition that I recognise that determinate sequence of phenomena which is not created by me, but forced upon me. My ideas are of cognitive value only if I restrict the sphere of my activity to the interior reconstruction of these relations without attempting to alter their objective nature, if my action confines itself to the domain of consciousness, and recognises the independent reality of things as an insuperable barrier. Practical activity, on the contrary, bursts these bounds and invades the sphere of objects, modifying not only ideas and their relations, but things in their physical reality. Human individuality is a disturbing factor which must be eliminated from the theoretic point of view in order to affirm that which exists independently of subjective action; in the moral act, on the contrary, the personality of the individual is an essential factor. In spite of his thesis, Dewey has been forced to recognise this, and whereas he had started with the intention of demonstrating the perfect analogy between theoretic and moral judgments, he ended by owning that the human element, which he calls "character," is of no efficacy in knowledge.[73] Of course, even in the cognitive function it is impossible to prescind from the subject, but this subject is not the psychic changing subject, with its individual needs and feelings, but rather the epistemological subject of a thought which is of universal structure. Knowledge presupposes the need of knowing, logical harmony affords a satisfaction *sui generis*, herein we can up to a certain point fall in with Schiller's views, provided that this need and satisfaction be not taken in a sophistical sense in their individual varieties, and the deduction made therefrom that there is no other logic but that which Ribot terms

the logic of the feelings.[74] Thought does not exist apart from a psychological structure, which imparts concreteness thereto, but this does not imply that its intimate laws do not remain unchanged amid the variations of the concrete content of consciousness in which its forms are actualised: one and the same mathematical demonstration, for instance, may be translated psychologically into auditory, visual, or motor images, according to the type of imagination of the subject, but the meaning and value of the deduction do not for that reason vary in the three cases. When logicians speak of "pure thought," they do not in the least mean to return to the Platonic conception of a world of ideas severed from psychological reality and from all concrete content; they merely wish to affirm that it is possible to study thought apart from this or that special content, not from all content whatsoever, which would be an absurd hypothesis. The norms of truth are not dependent upon the variable and contingent structure of the human subject, although therein alone can they make their efficacy felt and enjoy the light of knowledge. In like manner the fact that the knowledge of things is realised through the action of concrete individuals, and in connection with certain vital needs, does not imply that their reality is created by these subjective actions and exists in that determinate form merely because they have constructed the needs of the organism for their own use and consumption. Things have a nature of their own, and act according to laws which are not forced by thought upon a flux of indeterminate sensations, as is believed by James, who thus unwittingly recrosses the threshold of Kant's "museum of antiquities," but are revealed to us in the relations of experience. Sensible data are not absolutely amorphous, they are not adapted to assume any form and every form, but are possessed of characters and needs of their own, which we are bound to respect. Sensations are not indifferent and mute, they do not say all we would have them say, but speak to us in the language of all

other beings, whether human or non-human, which exist external to our individuality. It is the business of thought to interpret these countless voices of things, and to understand their profound meaning. The publication of the universe, to quote another figure of speech used by James, is not yet completed, and awaits its perfecting from the activity of man; but how could we collaborate in this great work if we did not first strive to decipher the numberless signs which the past has imprinted on its ancient pages? How could we understand that eternal language if things were entirely extraneous to the nature of the intelligence? The logical organism is not an artificial arbitrary mould into which we force a plastic and indifferent matter which submits to all our requirements, but is rather the very structure of reality, whose ideal meaning is revealed in human thought.

NOTES TO CHAPTER II

[1] *Pragmatism, a New Name for Some Old Ways of Thinking: Popular Lectures on Philosophy* (William James, London, 1908). The dedication of this work, which is in itself sufficiently significant, runs as follows: "To the memory of John Stuart Mill, from whom I first learned the pragmatic openness of mind and whom my fancy likes to picture as our leader, were he alive to-day."

[2] *Op. cit.* p. 20.

[3] In his article, "How to make our Ideas clear," in the *Popular Science Monthly* (January 1878), vol. xii. p. 286, translated in the *Revue philosophique* (January 1879), vol. vii.

[4] Peirce, in Baldwin's *Dictionary of Philosophy*, severs his responsibility from that of James, stating that he is unable to follow him in all the conclusions he draws from his principle.

[5] *Novum Organum*, Aph. 24.

[6] Hume, *Essays and Treatises*, vol. iii., *An Enquiry concerning Human Understanding* (London, 1770), p. 8.

[7] *Op. cit.* p. 9.

[8] *Op. cit.* pp. 136-137.

[9] This application has been made not only by Mach and Avenarius, but also by G. Simmel, who regards utility as the essential factor in the survival of our mental elements and the only creator of reality. "Über eine Beziehung der Selectionslehre zur Erkenntnistheorie," *Arch. f. system. Philosophie* (1895), pp. 39 and 41.

[10] It would be superfluous for me to quote the well-known works of these authors. Of special importance is Baldwin's work, *Thought and*

Things, vol. i. ; *Functional Logic or Genetic Theory of Knowledge* (London, 1906), which, however, somewhat modifies the " extravagant first hypotheses of the pragmatic revolutionaries " (Preface, p. viii).

[11] *Op. cit.* p. 57.

[12] *The Will to Believe and other Essays in Popular Philosophy* (New York, 1897). Balfour, in his work *Foundations of Belief*, had already given prominence to the action of non-intellectual factors in the foundation of faith.

[13] " The Pragmatic Method," *Journal of Philosophy*, i. p. 686.

[14] *The Will to Believe*, p. 11.

[15] *Op. cit.* pp. 19-20 and p. 30 ff.

[16] *Op. cit.*, Preface, p. vii.

[17] *Op. cit.* p. 12. The term " Pragmatism " was taken up again by James in a speech made by him at the University of California, from which time its fortune was assured.

[18] Le Roy distinguished three kinds of action, practical, discursive, profound, and regards the last named as the criterion of philosophy. The mind must be severed from practical life, and from the illusions of speech produced thereby, if it would withdraw into its deeper life (*Revue de métaphysique et de morale* (May 1901), p. 325).

[19] *Human Immortality : Two Supposed Objections to the Doctrine* (Boston, 1908). *The Varieties of Religious Experience : a Study in Human Nature* (New York, 1902).

[20] The first work of James in which the pragmatic method appears is the article " Sentiment of Rationality," *Mind* (1879), No. 15. His work *The Principles of Psychology*, published in 1890, also shows a method of analysis of preception which is essentially pragmatistic.

[21] Dewey, in " What does Pragmatism mean by Practical ? " *Journal of Philosophy, Psychology, and Scientific Method*, vol. v. pp. 86-99, distinguishes three applications of the term pragmatism, according to whether it refers to objects, ideas, or beliefs. When applied to objects it means the future replies asked of us by an object ; applied to ideas, it indicates the changes effected by them, as attitudes, in things ; applied to truth or belief, it implies the question of value, of importance. Lovejoy distinguishes no less than thirteen kinds of pragmatism (" The Thirteen Pragmatisms," *ibid.* vol. v. pp. 5-12, 29-39). On this point see Armstrong, " The Evolution of Pragmatism," *ibid.* vol. v. p. 645 ff.

[22] *Studies in Humanism* (London, 1902). As early as 1892 he had unknowingly followed the pragmatic method in his article, " Reality and Idealism," *Philosophical Review* (September). Other works by Schiller are " Riddles of the Sphinx " (London, 1894) and " Axioms as Postulates," published in a collection of essays by various authors entitled *Personal Idealism* (London, 1902).

[23] *Humanism*, p. 10 ff.

[24] The shaft is, of course, aimed at Bradley's *Appearance and Reality*. Pragmatists in England and America have adopted a warlike attitude towards the neo-Hegelians.

[25] *Op. cit.* p. 8 ff.

[26] *Op. cit.* pp. 12-14.

[27] *Op. cit.* p. 33.

[28] *Personal Idealism*, p. 123.

[29] *Humanism*, p. 59.

[30] *Op. cit.* p. 89 ff. Here the influence of Poincaré is plain.

[31] *Op. cit.* p. 92.

[32] *Op. cit.* p. 104.

[33] *Op. cit.* p. 120.

[34] *Op. cit.* p. 195. Here, as is obvious, Schiller is defending the reality of appearances against Bradley.

[35] *Op. cit.* p. 200.

[36] *Op. cit.* p. 203.

[37] *Op. cit.* p. 218. Schiller feels the influence of Ostwald's energetic conception.

[38] *Op. cit.* p. 227.

[39] *Op. cit.* p. 263.

[40] *Logical Conditions of a Scientific Treatment of Morality* (Chicago, 1903). Pragmatists regard Dewey as a disciple of their school of thought. Dewey, in his criticism of the work of James in the *Journal of Philosophy, Psychology, and Scientific Method*, for February 1908, has made reserves on this count; he admits, however, that his point of view is eminently pragmatistic.

[41] *Op. cit.* p. 14.

[42] *Op. cit.* pp. 16-20.

[43] *Studies in Logical Theory*, by James Dewey, with the co-operation of members and friends of the department of philosophy (Chicago, 1903), p. 14. James judges this work to be fundamental for the pragmatistic theory (*Pragmatism*, p. 8).

[44] *Op. cit.* p. 18.

[45] *Op. cit.* p. 39.

[46] "The restoration of a deliberately integrated experience from the inherent conflict into which it has fallen" (*op. cit.* p. 47).

[47] *Op. cit.* p. 52.

[48] Helen Bradfort Thomson, "Bosanquet's Theory of Judgement," in *Studies in Logical Theory*, p. 111.

[49] *Op. cit.* p. 114. Cf. also in the same volume, Simon Fraser, "Typical Stages in the Development of Judgement," p. 128 ff.; Myron Lucius Ashley, "The Nature of Hypotheses," p. 153 ff.; William Clark Gore, "Image and Idea in Logic," p. 193; Dewey's article, "Some Stages of Logical Thought," *Philosophical Review* (September 1900), and that by Rogers in the same review (1898), entitled "Epistemology and Experience."

[50] "Does Reality possess Practical Character?" in *Essays Philosophical and Psychological in Honour of W. James* (London, 1908), p. 80.

[51] "The Psychological Standpoint," *Mind* (1886), p. 4 ff.; "Knowledge as Idealisation," *Mind* (1886), p. 86; "The Reflex Arc Concept," *Psychological Review*, vol. iii. p. 368 ff.

[52] Henry Waldgrave Stuart, "Valuation as a Logical Process," from *Studies in Logical Theory*, p. 225.

[53] "La Notion de conscience," *Records of the Fifth International Congress of Psychology* (Rome, 1905), p. 148 ff.

[54] *Op. cit.* p. 152.

[55] *Pragmatism*, p. 244.

[56] *Op. cit.* p. 247.

[57] *Op. cit.* p. 259. The same conception will be found in Schiller: "The world is essentially ὕλη, it is what we make it. It is fruitless to define it by what it originally was or by what it is apart from us; it is what is made of it. Hence . . . the world *is plastic*" (*Personal Idealism*, p. 60).

[58] *Pragmatism*, pp. 251-255.

[59] *Op. cit.* p. 58. James, like all pragmatists, accepts Mach's economic theory of science.

[60] *Op. cit.* p. 64.

[61] *Op. cit.* pp. 170-194.

[62] Strong has laid special stress upon this process of substitution, defining knowledge as a series of experiences which are substituted for one another in a way satisfactory to the direction of conduct. Cf. the essay "Substitution" in *Essays Philosophical and Psychological in Honour of W. James.*

[63] *Op. cit.* pp. 208-210.

[64] *Op. cit.* pp. 216-223.

[65] *Op. cit.* p. 75: "Truth is one *species of good*, and not, as is usually supposed, a category distinct from good, and co-ordinate with it."

[66] Cf. on the pragmatistic notion of ὕλη the polemic between Kallen and Gifford in the *Journal of Philosophy*, vol. v. (1908), Nos. 4 and 11.

[67] Baldwin, in opposition to pragmatism, insists upon the necessity of recognising the independent reality of the environment. He remarks that even reflex thought is never wholly autonomous: reality, the fact in itself, must be postulated as that to which thought is adjusted in its progressive movement. To deny this would be equivalent to abjuring the pragmatistic method, since it would then be necessary to return to the idealistic position, according to which thought is a teleological system which is sufficient unto itself and develops of itself ("The Limits of Pragmatism," *Psychological Review* (January 1904), p. 30).

[68] "Over their nature, order and quantity we have as good as no control" (*Pragmatism*, p. 244).

[69] "By our order we read it in this direction or in that . . . we shuffle our perceptions of intrinsic relation and arrange them just as freely. We read them in one serial order or another" (*op. cit.* p. 247).

[70] Strong, "Pragmatism and its Definition of Truth," *Journal of Philosophy*, vol. v. (1908), p. 256.

[71] Peirce in his article, which pragmatists regard as the programme of their line of thought, laid down as the fundamental postulate of the scientific method the existence of a reality, "whose characteristics are absolutely independent of the ideas we may have of them" (*Revue philosophique*, vi. p. 566).

[72] James himself, moreover, says that his conception of truth "is realistic and conformable to that dualism which in matter of theory of knowledge constitutes the conception of common sense" ("The Pragmatical Account of Truth and its Mis-Understanders," *Philosophical Review*, vol. xvii., 1908). Cp. also "The Meaning of the Word Truth, remarks at the Meeting of the American Philosophical Association" (Cornell University, December 1907).

[73] Albert Schinz, "Professor Dewey's Pragmatism," *Journal of Philosophy, Psychology, and Scientific Method*, vol. v. (1908), p. 617. Cp. also the later work of Schinz, *Anti-Pragmatism* (Paris, Alcan, 1909), p. 72. Schinz examines pragmatism in relation to the social and religious conditions of the American environment.

[74] The human nature of which Schiller speaks cannot, according to Dewey (*Psychological Bulletin* (September 15, 1904), p. 336), be taken to be a purely subjective being. Each thing possesses an existence-value only in so far as the intelligence, understood *in the wide sense* of the word, concedes this value to it.

CHAPTER III

THE PHILOSOPHY OF VALUES AND THE HISTORIC METHOD

1. *The Philosophy of Values and the Primacy of Practical Reason.*—Of all the forms of reaction from intellectualism, the philosophy of values is the one which is most directly related to Kant's doctrine of the primacy of practical reason, inspired, as it is, by the *Kritik der Urteilskraft.* It, too, like pragmatism, reduces the true to the good ; but, whereas pragmatism looks at the moral law from the point of view of empiricism, and sets up the useful, the convenient, and the opportune as the only criterion of truth, the philosophy of values, true to the concept of the categorical imperative, sees in the law of duty a universal norm, and the affirmation of an objective value in every judgment.

The doctrine of the primacy of practical reason arose in the thought of Kant from the need of reconciling theoretical exigencies and practical interests, and of eliminating the conflict between pure reason, which forbids us to leave the realm of phenomena, and practical reason, which impels us with irresistible force towards the ultimate ideas of metaphysic. According to Kant, it is indeed only by subordinating theoretic thought to practical reason that we can reach that loftier harmony which would be impossible of attainment were the two functions placed on the same level, or were practical interests illegitimately subordinated to speculative exigencies, which, like all forms of interest, are in the last analysis also included in the sphere of practical

life. Kant failed to see that the conflict was the result of his arbitrary mutilation of knowledge, which banished from the realm of true science to that of aesthetic contemplation all those forms of judgment and all those categories of which the physical-mathematical sciences do not make use. He regards these sciences, with traditional rationalism, as the true type of all knowledge; everything which cannot be comprised in their schemes is therefore not considered to be true knowledge; is it not natural that we should find ourselves face to face with insoluble antinomies, when trying to exhaust all reality with inadequate categories, and applying the conceptual schemes created by thought in order to render the physical world intelligible, to totally different phenomena? Side by side with the natural sciences and above them, have we not the historic and human sciences, the sciences of spiritual values? Have we any right to deny them the name of sciences or systems of knowledge, because their methods are not those of the natural sciences, because they do not arrive at mathematical formulas? Is it metaphysical to make use of the idea of end in the human world, in the world of facts which are produced by conscious wills? The necessity of integrating Criticism with a wider and more complete concept of knowledge, comprehending not merely the judgments of the natural sciences, but also the other no less scientific forms of judgment, gave birth to the philosophy of Rickert, which, whilst developing certain fundamental ideas which had already been unfolded by Windelband, endeavours to find an epistemological basis for the doctrine of the primacy of practical reason, placing himself in opposition on the one hand to the metaphysical development of this doctrine attempted by Fichte, Lotze, and Renouvier, and taken up again later on by Royce and Münsterberg, and on the other to the psychological interpretation, which, having been first touched upon by Renouvier, was destined to become so widespread with the advent of pragmatism. Rickert indeed does not attempt to do

away with the antinomy of pure reason and practical reason by synthesising them in a metaphysical principle, conceived as an absolute will or as a supreme Personality, the conscious creator of all values; neither does he justify, like Dewey and Schiller, the identification of theoretic and practical judgments by a psychological analysis which comprehends them in their empirical and contingent reality, and gives prominence to their common passional and utilitarian character; he rather proceeds by an essentially epistemological method, and seeks the ultimate presuppositions of every judgment of truth in the universal norms of the Ought.[1]

2. *Philosophy as the Science of Universal Values: Windelband.*—In this procedure, which starts from the existence of universal affirmations in order to reach the conditions which make them possible, and may be considered a new application of the Kantian method, Rickert develops the programme clearly outlined by Windelband in his *Präludien.* Windelband regards critical philosophy [2] as the science of necessary and universal values (*Die kritische Wissenschaft von den allgemeingiltigen Werten*): universal values are its object, criticism its method. It examines whether there be a science possessing a universal and necessary truth-value; whether there be an art, that is to say an intuition and a sentiment, possessing necessary and universal beauty-value; whether there be a morality, that is to say a volition and an action, possessing universal and necessary goodness-value. A distinction must be drawn between the judgments (*Urteile*) in which the convenience of two representative contents is expressed, and the other kind of judgments (*Beurteilungen*), which express the relation between the judging consciousness and the represented object. There is a fundamental difference between the two judgments, "this thing is white" and "this thing is good," in spite of the identity of grammatical form. In the purely theoretic judgment we problematically establish a connection between two presentations without giving

any opinion as to their value; in the *Beurteilungen*, on the other hand, we ascribe or deny universal validity to that relation. This latter class of judgment, which presupposes a determinate end as a unit of measurement, and is of significance only to him who recognises that end, and is presented to us with the two fundamental alternatives of pleasing and displeasing, approving and disapproving, accepting and refusing, constitutes the special object of philosophy, whose business is not, like the special sciences, to determine the natural necessity of facts, their *Müssen*, but their Ought, the *Sollen*, that which all must recognise to be equally valid, even if it does not exist in practice or is not actually a fact.[3] These valuations must be distinguished from individual feelings of pleasure and pain, since they are not simple attractions or repulsions of a Hedonistic order, determined by physiological conditions, but rather judgments subject to universal norms.[4] Some thinkers attribute a merely relative value to these judgments, affirming them to be a product of the variable conditions of society; but no one has ever been found to have a serious belief in relativism; it is a *fable convenue*. He who is not content with the mere affirmation of relativism, but strives to put it to the test, will deny it, recognising universal norms of thought.[5] Normative law differs widely from natural and causal law, which may correspond to the norms, but is often far removed from them. This does not, however, lessen the value of the normative laws which establish that which ought to be, even if it be never fully effected. The laws of thought, set up by logical consciousness, are not identical with the laws of representative association, but neither are they something entirely different from them and opposed to the mechanism of presentations. True associative relations are determined in consciousness by the same natural laws which determine false associations, and are distinguished from these false associations merely by their conformity to ideal norms. Truth is the one white ball

amongst many black ones.[6] The psychic mechanism leads indifferently to beauty and ugliness, truth and falsehood, good and evil, and is hence incapable of deciding their value. The acceptance of the norm is forced upon the empirical consciousness by immediate evidence of which no causal explanation can be given: genetic psychology can merely tell us how and to what extent that norm is actualised; it can resolve the question of fact, but is incompetent to decide the question of right.[7] The presupposition of the critical method is the belief in universally valid ends, and in the capacity of human consciousness to recognise those ends. The historic importance of Fichte lies in the stress laid by him upon this teleological character of the critical method; but he was wrong in endeavouring to deduce from the determination of the end the means of its realisation as well. Even if the norms be not based upon experience and do not derive their value therefrom, it is nevertheless only by means thereof that they can become possessed of clear consciousness. The one end of the whole activity of thought is the realisation of its norms: that which is commonly termed an object, the copying of which is the task of science, is reduced, when properly analysed, merely to a rule of connections between representations. We do not need to know whether this rule correspond to an absolute and independent reality; it will suffice to observe that some of our representative associations are adjudged to be true, others false, in accordance with a norm which is valid for them all. The concept of truth cannot mean the agreement of presentations and things, which are two mutually incommensurable terms, but merely the reciprocal harmony of the presentations: of secondary and primary, abstract and concrete, hypothetical and sensorial, theory and facts.[8] Immediate certainty lies in two points which are diametrically opposed: the sensations and the general principles or axioms, according to which the relations of the sensations must be apprehended. All the propositions established and proved by the individual sciences

are intermediate products between axioms and sensations.[9] In order that knowledge may exist, we must presuppose the possibility of ordering the sensations according to these principles—we must, that is, start from the postulate that the relations of our sensations can be logically ordered. These two factors of knowledge are both indispensable: there is no such thing as exclusively deductive or exclusively inductive knowledge. The value of axioms is determined by a universal end for each thought, and must be unconditionally recognised, if we would attain that end. It is a hopeless undertaking to base upon empiric theory and genetic research those axioms which are the necessary presupposition thereof.[10] The universal end alone imparts meaning and value to our knowledge; only when thought is regarded as a moral duty can this end be attained. During the activity of thought moral force restrains extraneous impressions, personal interests, and the temptations of the imagination.[11] There is no such thing as knowledge of the world *sub specie aeternitatis*; but though our knowledge be limited to that which is transformed in time, the sentiment of that which is universally valid sheds the light of eternity abroad in our minds. Not in science (*Wissen*), but in moral consciousness (*Gewissen*), does the mind of man partake of the eternal. Eternity will not be known, but experienced.[12]

3. *Reduction of Being to the Ought: Rickert.*—Rickert,[13] following in the footsteps of Windelband, has endeavoured to prove that the transcendent object is reducible to the Ought, the *Sollen*. The opposition between subject and object may be understood in three ways: firstly, as the opposition of the animated body, the psychic-physical subject, to the environment; secondly, as the opposition of the world of consciousness with its whole content to that which is external to itself; of the immanent to the transcendent; thirdly, as the opposition of the conscient subject to the content of consciousness.[14] Correspondingly, the word object assumes three different meanings:

firstly, the spatial world which is external to the body; secondly, the transcendent object; thirdly, the object immanent to consciousness. The reality of the immanent object cannot be called in question; the only reality admitting of doubt is that of the transcendent object, which is not immediately certain, as is thought by Riehl, but is merely an induction of our own. Is this induction legitimate? This is the problem which Rickert would fain solve. When the whole content of consciousness is ascribed to the object (that is to say, in the third method of understanding the opposition of the two terms), nothing is left of the subject but mere conscious being: consciousness without a name, of a generic, impersonal kind, which can never become the object, the *Bewusstsein überhaupt*, which must not be confounded with the psychological individual subject. Everything which is individual in the subject is an immanent object to consciousness, hence it follows that the mind in its individuality cannot be regarded as transcendent. Generic consciousness is neither an immanent reality nor a transcendent reality, but merely a concept; it only means that which is common to immanent objects, that is to say, to all contents of consciousness, that which cannot be more nearly described; it is really another name for the only being which is immediately known to us, the general concept, the form or species of this being, that is to say, of the immanent object, in contradistinction to that form of being proper to the transcendent object. Each immediately presented being is a being in consciousness, an indubitable fact, incapable of further analysis.[15] The meaning of knowledge is certainly based upon the conviction that an order independent of the subject may and should be discovered; but we may yet ask, Must this order be an order of transcendent things, a transcendent reality? If knowledge consists in the presentment, it must be compared with a reality of which it is the sign or copy; but this point of view is untenable, because presentments, like things, belong

to the content of consciousness, that is to say, to the object; hence in presentments there is no cognitive relation between a subject and an object, but only the mutual relation of two objects. Now in order to recognise this agreement, a subject is required; and this knowledge cannot be in its turn a presentment, since in that case we should have to recognise a fresh agreement, and the process would go on *ad infinitum*.[16] True knowledge consists in the judgment, which is not a mere relation of presentments, but always contains an affirmation or negation of reality.[17] Knowledge is an affirmation or a negation by reason of its logical essence, and belongs to the active phenomena of consciousness. Affirmation and negation are but forms of attraction or repulsion, determined by pleasure or pain. This practical nature of all knowledge distinguishes it sharply from pure presentment: judgment is always an acceptance or a refusal, an approval or a disapproval, that is to say, in ultimate analysis the recognition of a value.[18] Cognitive value differs, however, from other values: whereas Hedonistic valuation is valid only for the individual at the determinate point of time and space in which he experiences pleasure or pain; in the logical judgment something is affirmed which is of value to us independently of that moment and of those determinate circumstances.[19] In our judgments we do not feel ourselves at liberty to deny or affirm in an arbitrary manner, but feel ourselves bound by a sentiment of proof, we submit to an extra-individual power which constrains us to make that affirmation or negation. If I hear a sound and wish to pronounce judgment upon it, I am unconditionally forced to judge that I hear the sound. This necessity (*Urteilsnotwendigkeit*), which is not proper to mathematical judgments only, but is common to all judgments in which something is affirmed, even to judgments which refer to reality of experience, must not be confounded with causal necessity, since it is only the logical reason of our affirmation.[20] It is not a *Müssen*, but a *Sollen*, an imperative, whose necessity

in judgment we allow, and which is accepted by our will. The object of our knowledge is not being (*Sein*), but the Ought (*Sollen*), deriving its adhesion from a judgment, and constituting the universal order which we feel ourselves constrained to recognise.[21] Rickert agrees with empirio-criticism in its desire to confine the problem of science to the classification of the contents of consciousness; but this order is not regarded by him merely as a convenient classification, but as an order conforming to a universal norm, and independent of the subject in the sense that it must be valid, even were no one to recognise it.[22] Its unconditioned value cannot be denied with self-contradiction, because every contrary judgment would imply the transcendence of that Ought. Truth does not depend upon individual tastes, as relativists would have us believe: he who affirms that there is no value of absolute truth implicitly contradicts himself when he asserts this fact as a certainty. All human judgments may be mistaken: there is only one which can never be false—the judgment that there exists a value of absolute truth. It is impossible to doubt the transcendent Ought as an object of knowledge, no matter from what point of view it be posited, because it is the necessary condition of all affirmations, even of those of a sceptical order.[23] That alone exists which is judged to do so. Hence not being, but the Ought is the logically original concept.[24] Rickert terms his system *transcendental idealism*: idealism, in as much as it recognises no other immediately presented being than the presentment; transcendental, in as much as it admits a transcendent object beyond the content of consciousness, a *Sollen*, an ideal, to which the conscient subject tends.[25] Even experience, that which is perceived, data, the fact, are such only in so far as they are recognised, and hence presuppose from the gnoseological point of view a judgment and a norm. Every judgment is based on experience as far as its content is concerned; its form is, however, related to the *Sollen*, which cannot be empirically presented; in this way the opposition between

empiricism and rationalism is overcome.[26] In perception the thing is presented as a complex of qualities, of contents of consciousness, but these properties do not exhaust the thing: it contains a network of necessary relations, answering to a transcendent norm; its independence does not consist in being independent of the subject, but in the necessity of the judgments with which we posit these relations. The thing in its objectivity is then reduced to a transcendent norm or rule of the connections between the presentations, which demands recognition.[27] Even the necessary relations of causality derive all their objective value from the recognition of that ideal norm.[28] There can be no opposition between the theoretical and the practical man, between knowledge and volition; even knowledge is based upon a categorical imperative, and logical evidence is but one case of moral certainty.[29] The antinomy of the *Sollen* and the *Müssen* is overcome when we reflect that even the form of natural necessity must be founded on the *Sollen*, if it is to have objective significance.[30] In many systems of philosophy the intellectual values were ranked above all others, and moral, religious, and aesthetic life so debased or intellectualised that it lost all meaning of its own. Contemporary voluntarism, whilst striving to overcome the antinomy between the theoretical and the practical man, goes to the other extreme, depriving knowledge of all foundation and opening the door to arbitrariness. Science naturally sees danger in this intrusion of individual volition and sentiment, and is therefore impelled to combat the new doctrines; on the other hand, sentiment and volition cannot rest content with the conclusions arrived at by science. Thus the theory of the will, instead of eliminating strife, does but add fuel to the flame.[31] Only epistemological subjectivism, which places valuation at the base of science, enables us to do away with the antinomy between the intellect and the other functions of the mind. The recognition of truth also is a categorical imperative which is unconditionally forced upon us.

Every act of cognition hence presupposes an autonomous will capable of conforming to an ideal norm. From such a point of view theoretical and practical activity do not conflict with one another, but we perceive them to be two different manifestations of the same consciousness of duty, in which logical values find their super-logical basis.[32] The consciousness of duty, as practical will, is conceptually antecedent to logical will, that is to say, the will to truth: the tendency to truth presupposes the tendency to do one's own duty, because the cognitive judgment is a special form of practical activity. From this is derived the absolute value of the conscious will to duty, as an unconditioned necessity even for the theoretical man, because the recognition of one particular value presupposes the recognition of value in general. In a reality which is indifferent to duty, in a world which is not adapted to the realisation of truth-values, all judgment would become meaningless. The value of knowledge depends upon the conviction that ethics, not logic, reigns supreme, that the world is guided in such a way as to bring the realisation of the ends of knowledge within the bounds of possibility; it is dependent, that is to say, upon faith in the moral objective order (*objective Weltmacht des Guten*).[33]

4. *Natural and Historical Sciences.*—If human consciousness be thus placed in the centre of the universe, the cosmic process will no longer seem the random product of an obscure necessity, a decoction of mixed atoms,[34] but rather the progressive historic actualisation of the ideal. History, in as much as it enables us to watch the realisation of universal values in the world of concrete consciousness, thus becomes the fundamental organ of philosophy. Historical science alone, interpreted in its widest sense, can fill the gaps left by the formation of scientific concepts, it alone can substitute reality in the fulness of its individual aspects for the empty abstractions of science.[35] Scientific knowledge cannot consist in the imitation or representation of single objects

in their individuality, because, however large the number of known objects may be, an infinite number of others remains still unknown; moreover, each object can be resolved into a complex of relations and elements, which become gradually more numerous as our knowledge becomes greater.[36] Science would not be possible were there no means of overcoming multiplicity in either form, external and internal, extensive and intensive alike. This purpose is served by the concept, which in its extension overcomes extensive multiplicity, in its content intensive multiplicity. The more perfect science becomes, the more completely will the intuitive element be eliminated, and together therewith the individual character of things, that is to say, that physiognomy of theirs which is never repeated twice in the same way.[37] We can live and experience reality directly, but the instant we attempt to explain it by means of science, that which makes it truly real will elude our grasp; the more nearly our concepts approach perfection, the farther we shall go from concrète reality. If a certain residuum of empirical reality still remain in science, it is an element which science has been unable to comprehend, to translate into judgments, or to overcome thoroughly. Neither the atom nor the simple sensation can be regarded as an object of knowledge, but merely as means thereto: it would be absurd to posit as the end of science the discovery of a reality which cannot be experienced.[38] Moreover, it is not the business of science to copy reality; on the contrary, as Rickert paradoxically affirms, the less reality its concepts contain, the more nearly do they approach perfection.[39] If, however, we must be on our guard against the hypostatisation of scientific concepts, we must be equally careful not to go to the opposite extreme, by depriving science of all objective value. The place of being, which concepts are totally unable to represent, must be taken by the value which they *ought* to have; scientific concepts are not true in so far as they copy real things, but in so far as they are valid for reality.[40]

Science then leaves external to itself facts in their individual concreteness, which are of no interest from its point of view; but the individual, though it may not be of scientific importance as such, is of great value to us from the aesthetic and moral point of view. The apprehension of reality, in that aspect thereof which never recurs twice in the same way, is the task of historical knowledge, understood in its widest sense, that is to say, as applying not only to human facts, but to all events in their individual physiognomy.[41] If historical facts be by their very definition such as never recur twice, not only will it be difficult to find historical laws, as is commonly supposed, but the very concept of historical law will be a contradiction in terms. It is commonly asserted that historical personalities are inexplicable by reason of the complexity of their factors; but this applies to all things in their individual aspect, to a piece of sulphur, the leaf of a tree, etc., which are therefore just as inexplicable as Goethe or Kant.[42] This distinction between the natural [43] and the historical sciences is of methodological value only, since in reality the two forms of knowledge are blended with each other. On the one hand, historical elements will always be found in the sciences, because the sciences, with the exception of mathematics and rational mechanics, have not been able to free themselves entirely from the empirical element; on the other hand, history does not always refer to that which is truly individual, but frequently treats of phenomena which, whilst of a generic character, are relatively singular. The naturalistic method may be used in that which is relatively historical, but that which is absolutely historical, as, for instance, a human personality with its individual characteristics, eludes the grasp of the scientific concept, unless the individual be considered as the type or example of a species.

Though the concept of law may not be applicable to historical facts in the absolute sense, it does not follow that these facts are not determined by causes. A distinction must be drawn between the principle of

causality, which demands of a cause for every fact, and the causal law, according to which, given the same causes, the same effects must be verified.[44] The connection existing between an individual cause and an effect which is also individual, as, for instance, between the earthquake of November 1, 1775, and the destruction of Lisbon, may be termed *an historical causal connection*; that which is common to a group of these individual connections constitutes the *causal law*, which embraces that which recurs in concrete historical successions. The opposition between nature and history is not an opposition between necessity and liberty, because if law is excluded from historical knowledge, cause is not. Still less can it be said that accidental facts are the object of history; if by accidental we understand everything which cannot be comprised in a general concept or a scientific law, the whole of concrete reality is a complex of accidents subject to no law whatsoever: it is a chance that Saturn has a ring and that the earth has none, that Frederick the Great won a battle at Leuthen. If, on the other hand, we understand by chance, as opposed to causal necessity, that which has no cause, nothing in the world is contingent, and everything is necessary, Saturn's ring and the victory of Frederick the Great alike.[45] Historical causality differs, however, from scientific causality: in science it may be stated that the same cause will produce the same effect; in history such a statement is impossible, since there are not two equal causes and effects. Further, the principle of the equivalence of cause and effect cannot be applied to historical causality, because the historical event is always something heterogeneous to its cause.[46] History may seek the influences of the environment and of past conditions upon the individual, but only in so far as the environment and the historical moment are considered in their individuality; if we pass from these individual complexes to generic concepts of the mind of the people, the spirit of the age, etc., we forsake the historic point of view and adopt the attitude of naturalistic treatment.[47]

The individual elements of history can be combined into a higher unity only by referring to a universal value. It is useless to try to impart significance to life and to history by taking up the standpoint of science, which regards all individuals, as examples of generic concepts, as being of the same importance and meaning, and which therefore recognises no difference in value.[48] If we would distinguish the essential from the non-essential in the world of experience, in a way which is universally valid, we must have a criterion of selection, an ideal norm which will enable us to eliminate everything which is not of importance to the attainment of that universal end, and to arrange the most important moments of historical development in a hierarchical scale of values, independent of subjective caprice.[49] Historical thought, like moral volition, is bound up with the irrationality of the world, that is to say, with that element individual which cannot be deduced from a system of concepts. All rationalistic systems, in as much as they presuppose that each moment of development is predetermined in an eternal idea beyond time, nullify the meaning of historical individuality. Where everything is logically necessary, as in Hegel's absolute idealism, no distinction of values is possible, because all moments are placed on the same level.[50] Empirical reality is absolutely irrational to us.[51]

If the concrete individual be true reality, and the concept, instead of leading nearer to it, rather increases the distance between us and it, what is the use of science? The ideal of knowledge is a subject capable of embracing the whole of past, present, and future reality in a single intuition; but man, strive as he may to increase his knowledge, by prolonging the series of existential judgments, can never attain that ideal, absolute knowledge of all individuals. He therefore requires a substitute, and science is the most complete compensation imaginable. It thus becomes of absolute value to man, and necessary from the human point of view which has but a limited horizon. He who desires an end must also

desire the means thereto, hence the presupposition that judgments valid for all instances may be deduced from a limited number of facts must be accepted as a necessary and universal principle.[52] Only by reference to an absolute value which ought to be, is it possible to explain the universal validity which we attribute to law; empiricists in their genetic researches are doomed to move in an eternal vicious circle, presupposing the universality of those principles which they imagine themselves to be deriving from experience. When it is stated that repeated succession necessarily produces in each case indissoluble association of ideas, is not the causal law presupposed ? [53]

Science, however, necessary as it is from the human point of view, cannot take the place of history, because the scientific concept, which is but a means of mastering the infinite multiplicity of things, cannot give us anything real. Historical knowledge approaches more nearly to the ideal than does scientific knowledge; but it too is at bottom relative to the limitation of the empirical subject in time. The total intuition of the universe is denied to the finite intellect, therefore human knowledge can only attain its end discursively by means of a series of successive acts, which, seeing that they tend towards an unconditioned and transcendent end, and are the progressive actualisation thereof, in their totality constitute an historical development. The process of realisation of human knowledge cannot be known scientifically, but only historically,[54] this proves the necessity and value of historical knowledge from the point of view of the empiric subject.[55]

5. *Criticism of the Philosophy of Values.*—The philosophy of values makes, like pragmatism, the mistake of trying to reduce the theoretical to the practical function, positing as the object of knowledge not that which is, but that which ought to be. If, in fact, from the metaphysical point of view there is no doubt that we must have recourse to our moral experience in order to understand fully both the meaning of things and their profound

unity; that no philosophy can afford us a satisfactory intuition of the world without reference to an end; that the mind of man with its universal values would be simply an inexplicable monstrosity in a mechanical world wholly extraneous to its ideals; we must, on the other hand, recognise that the Ought, even if it be not derivable from being, yet finds its presupposition and its necessary subsistence therein. An end, a norm, a value which is not such to any consciousness is inconceivable, and consciousness is not merely valid, it *is*. Rickert considers the priority of the *Sollen* over the *Sein* to be proved by the fact that every form of being whatsoever, whether psychic or physical, real or ideal, sensible or super-sensible, given or inferred, presupposes a meaning and an objective value in the judgment affirming such being.[56] But could the judgment be valid, did it not exist in some way? The validity of the judgment implies in its turn the reality thereof, and of the thought which judges. Rickert's argument is based upon the false postulate that *nothing exists unless it be judged to do so*; whereas we have an experienced assurance of the concrete existence of our thought, an assurance preceding any explicit recognition thereof. I feel that I exist, even before I judge myself to be in existence, and my reality does not depend upon the value of the judgment which recognises it; on the contrary, the judgment is valid because it is based upon that primitive and indestructible fact which is the immediate consciousness of the being of my thought. A *Wert an sich* absolutely transcending consciousness is not an intelligible concept, and finds itself confronted by the same difficulties as the old idea of the *Ding an sich*. How can the value which is in itself apart from conscious being be made one therewith in the cognitive act? How can the transcendent become immanent? Who will show us how to overcome this dualism of being and meaning, reality and value? To this Rickert[57] replies that dualism is created by the intelligence, which, in its desire to explain facts, separates conceptually that

which is one in origin: we have immediate experience of the unity of meaning and being, in as much as we in knowing identify ourselves with truth: it might be said to be that which is best known, did not knowledge already imply a division; it is inexplicable, not because it is above all knowledge, but because it is previous to every conceptual distinction. This *inexplicable*, which Rickert's penetrating mind finds at the basis of his conception of knowledge, is not, in my opinion, a necessary product of intelligence, but results from his arbitrary concept of objective value, which he defines as being external to every form of being whatsoever, and hence also to consciousness. If value be non-existent,[58] if it absolutely transcend consciousness, no dialectic effort will enable me to pass from the one term to the other; but this concept of *value in itself* without any reference to a consciousness is not thinkable; still less is it possible to think of a value which *does not exist* either actually or ideally. Our thought refuses to conceive of something *absolutely non-existent*, because every judgment posits being either implicitly or explicitly. The affirmation of being is immanent in every act of judgment, even in acts referring to values; I am not constrained to say "*the value is*" by a mere imperfection of language, the very nature of thought forces me to do so. I cannot think that something is valid, and is of intrinsic worth, without implicitly thinking that it is in some form or other. A value which is non-existent is a meaningless phrase, because nothingness, the absolute negation of being, is the negation of thought. What wonder if, when absolute value is defined in this unintelligible manner, it should be impossible to conceive of the transition from *non-existence* to existence? It is not necessary absolutely to break through the relation between value and the object and consciousness in order to place knowledge and valuation upon a universal and objective foundation; it will suffice for them to be independent of the individual subject, while still in relation to a Universal Consciousness. The significance

of the act of judgment goes beyond the empirical sphere of my consciousness and individual existence, and is therefore *relatively transcendent*, but it does not transcend the sphere of consciousness and existence in *the absolute sense*. Things certainly contain a network of laws and relations which would exist even had man never been born, and had never known them; but this order cannot be thought of as real except in relation to an Absolute Thought capable of comprehending universal reality in the concreteness of its infinite Consciousness. The *Bewusstsein überhaupt*, which is an abstraction of that which is common to human consciousness, does not of course exist apart from them, and cannot therefore act as the subject of real relations, still less of the universal norms which existed previous to the origin of man. The consciousness which we must postulate in order to find a foundation for the objective order and the eternity of values must not then be an abstract concept, a pure logical fiction, but a living Personality with a concrete content.

The identification of knowledge with practical activity, which is the fulcrum of Rickert's theory, does not in the least correspond to that which is revealed to us by our inner experience: the attitude assumed by us in the two cases is very different. In the cognitive function we certainly feel at liberty to affirm or deny the existence of a fact or of a real relation, but we are at the same time conscious that that fact and relation exist, even if we do not recognise them; in the practical attitude, on the other hand, the reality of that which is willed seems to us independent of our subjective action: the end will continue to be of value, it is true, even if I do not actualise it, but its realisation, its transition from the ideal order to the sphere of objective existence, is dependent upon my free will. The relation of the reality to the *Ego* differs, then, considerably in the two functions: our mind is practical only in so far as it modifies, or at all events proposes to modify, that which exists, and feels itself to be the active *cause* of such modifications;

it is theoretical if it recognises objects and relations whose existence does not depend at all upon its volition. I do not hereby intend to maintain that consciousness in the cognitive process reflects things passively, but merely that the mind is not active in the theoretical function in the same sense as it is in the practical attitude. That which I perform and am at liberty to perform in knowledge is the act of judgment: this act, not the existence of the object affirmed, depends upon my will. There is no judgment without the will to judge; but does it therefore follow that to judge and to will are the same thing? When I affirm the reality of a fact, I certainly feel the need of affirming it; it is not, however, this exigency which constitutes the object of my judgment: that which I judge is the existence of the thing, not the norm which governs my judgment. Rickert maintains that the affirmation of reality presupposes the value of the existential judgment; but, I would reply, the *presupposition* is one thing, the *object* another: there must be no confusion between the two. In the affirmation of existence the validity of my judgment is implied, not the value of the object affirmed; the existential judgment is valid just because it recognises a fact of which my consciousness gives me immediate assurance: that is to say, that my individuality does not exhaust the sphere of being, that external to myself there exist other individuals and other real phenomena. That which Rickert terms the immanent object, and which he too recognises to be beyond question, will suffice to impart value to the existential judgment; it is enough that the things and relation affirmed by me be independent of my individual life, it is not at all necessary to have recourse to an object absolutely transcending consciousness. My judgment, "This sheet of paper upon which I am writing exists," is valid in the sense that the sheet continues to exist even when I no longer perceive it, that is to say, in the sense that its being does not depend upon me. The same may be said of the relations between the sensorial contents, whose existence

we affirm, and which we strive to reconstruct in that which is objective about them, that is which is real independently of this or that individual. It is the existence of these relations, which are immanent in the phenomenal order, which constitutes the object of the scientific judgment, which is valid, no less than is the historical judgment, not because it refers to a transcendent norm, but in as much as it affirms a system of real relations. In order to comprise in the scientific concept that which recurs in natural processes, this repetition must exist in some way. Had each individual moment of the cosmic process nothing in common with the preceding moments, were the causal historic connections absolutely different from one another, how could they be abridged into a law? Are constant relations artificially created by us, or do they not rather exist in things? Rickert does not even set himself the problem, and rests content with the dogmatic affirmation that science presents us with nothing real without affording us an explanation of its success.

The absolute distinction between natural and historical sciences, a distinction derived in Rickert's opinion, not from the diversity of their matter which is always the varied and complex content of experience, but from the method followed in its elaboration and from the difference in the proposed end, results in the creation of an artificial dualism in the heart of the cognitive function itself, which aims, on the one hand, at a law, and, on the other, at an individual; whose end in science is the concept, while in history it uses the concept merely as a means. Theoretical activity, on the contrary, has, and can have, but a single end: the comprehension of reality in the fulness of all its aspects, the vision of the fact in the light of the total system of its relations. Its starting-point is the fact experienced in its individual physiognomy, its goal neither is nor can be other than a concept in which the intuitive moment loses nothing of its concrete reality, but gains all those determinations which escape immediate consciousness. Even if we accept Rickert's

definition of historical knowledge, its final goal will yet in the last analysis be a system of concepts: when you affirm that a certain fact is, that it succeeds or precedes another event, that it is co-existent with others, that the cause thereof will be found in a certain anterior situation; when you determine its ideal value as compared with that which you regard as the meaning of the world, the individual, as such, considered in its non-recurring aspect, has ceased to be the end of your knowledge, which is rather the individual *sub specie aeternitatis*, not merely intuited, but above all thought. The historian, like the scientific man, is not satisfied with merely experiencing the fact, but would fain understand it, and, in order to do so, he must rise to the concept.

6. *Historical Knowledge in Münsterberg's Philosophy of Values.*—If the end of history be the same as that of science, may not the matter which is the object of its elaboration be different? This thesis, the very antipodes of that propounded by Rickert, has been recently set forth by Münsterberg,[59] who proposed to give us a complete system of the philosophy of values, carrying it back from epistemological criticism to Fichte's metaphysical idealism, a transformation necessary in the eyes of any philosopher who was not prepared to stop short at the untenable position of the pure norm, the absolute *Ought*, the transcendent *Sollen*, the absurd concept of a value in itself, divorced from every form of existence and consciousness. Münsterberg is at one with Rickert in the battle against relativism and the subjectivism advocated by the new sophists, who, though they must be credited with forcing us to turn from the artificial construction of abstract scientific knowledge to the immediate world of consciousness, yet fall into error when they profess to stop short at this individual immediacy. We owe the vindication of historical reality and practical activity against the mechanical ideal and positivism to the empirio-criticists, intuitionists, and pragmatists;[60] but if, when we have reached this point, we would not be forced to retreat, if concrete life, not

a sterile conceptual scheme, is to be our starting-point, we must go beyond the passing moment by means of affirmations of universal and eternal value, in order to form an intuition of the world.[61] But this value—and on this point Münsterberg differs from Rickert—while independent of single individuals, must not be conceived of as external to all spiritual life whatsoever, but in an absolute will, an original action (*Urtat*), in the Super-*Ego* (*Über-Ich*) which is perennial and incessant effort; not useless and painful, as Schopenhauer deemed it to be, but full of inexhaustible joy in every pulsation of life, which renews and intensifies the eternal will and raises it to a higher power.[62] The principle of the world is no inert matter, no motionless God, but living activity in perpetual flux, which is at once the preservation and augmentation of itself (*Steigerung des Wollens*).[63] This ultimate basis of all values is not comprehended either by thought or moral consciousness or aesthetic sentiment, which will never enable us to go beyond the finite *Ego*, but by the philosophical conviction that there is a value transcending human experience, in which all empirical values find their higher unity. We may express such a conviction in concepts and judgments, but by so doing we merely make use of an auxiliary means, which does not transform it into scientific knowledge; it more nearly approaches religious belief than conceptual thought.[64] We cannot do without this conviction, which is in reality an action of our own, because it is in this way alone that our will can be fulfilled: in willing the unity of all values, that is to say of our will with itself, is concluded the universal action in which every exigency is satisfied, every question answered, every tendency completed. Fidelity to oneself to all eternity (*Sich selber treu sein in Ewigkeit*) is the action ensuring the salvation of all the values of the world.[65]

Münsterberg thus regards value as identical with that which is willed by the pure or super-individual will; from the concept of this ultra-personal will which urges us on beyond the subjective world, and is ever

affirming itself, he endeavours to derive all values by a process of deduction which is a vicious circle from Rickert's point of view,[66] since the existence of absolute will presupposes the validity of the judgment affirming it. Münsterberg rejects this priority of the *Sollen*, which would imply that value is independent of the will, whereas he considers that *to have value* and *to be willed* form a perfect equation. If this equation be assumed, it follows of necessity that the will cannot choose the false, ugly, or immoral; it is for this reason that Münsterberg declines to accept Rickert's view of value as an *Ought*, a norm to which we submit of our own free will, whilst having it within our power to will the contrary. The concept of the *Sollen*, far from elucidating the idea of value, introduces into it a character which is not proper to it, that is to say, freedom of choice between ought and non-ought, without which we cannot speak of *Sollen*.[67] When we desire to judge, we only desire to choose the truth, and there is no question of deciding between two alternatives. Of course error may arise, but only because we think it to be truth, not because we choose the false as such. Even in moral action, although value and the Ought may appear to coincide, the *Sollen* is devoid of meaning, because there is no such thing as a will contrary to the good. Does the highwayman steal because he prefers the action of robbing? Certainly not: at bottom he prefers that which is right, and if he is induced to commit a theft, the force impelling him to do so is the seduction of pleasure, not the will to evil. Were he to prefer the criminal act, and look upon it as something good, he would be a moral lunatic, not an immoral man. Immorality consists not in willing that which should not be preferred, but in acting contrary to that which is and which alone can be willed: the good.[68] The values, in the last analysis, whether they be theoretical, aesthetic, ethic, or religious, are all alike based upon the satisfaction (*Befriedigung*) of the pure will, which must not be confounded with individual hedonistic feeling, since the end of the ultra-personal

will is not pleasure, but its own indefinite and inexhaustible development, the progressive intensification and the preservation of itself.[69]

There is one act of will which has nothing to do with our pleasure or pain: the will that there shall be a world, that the content of our life shall be of value not merely to us as life, but shall be independently affirmed in itself. This original action, which imparts eternal meaning to our existence, and without which life would be an empty dream, a chaos, a cipher, is neither a truth, nor a beauty, nor a duty, nor a sacred good, but is the fundamental conviction from which all these values are derived. Such a conviction is an absolutely free act which cannot be enforced by any proof; it is impossible to adduce any argument to the man who refuses to ascribe value of reality to his experience, and hence to that combination of facts which questions him, since argument and discussion are impossible unless the objective validity of the world be presupposed. Only the man who is prepared to take this first step can receive the proof that in this step all other values are comprised.[70] Münsterberg presents us with a systematic deduction of them, starting from the single principle of the will that an objective world shall exist and thus imparting fresh life to the epic exploits of speculative idealism.[71]

7. *The Attempt to deduce all Values systematically.*— If all values originate in the fact that we separate experience from our individual personality and consider it as existing by itself, the existence-value must be found on the threshold of valuation. Against this it might be urged that the affirmation of the real is not a valuation; that a judgment is valid, not that which is affirmed therewith; this, however, results in the dogmatic establishment of a dualism between the object which is recognised and the act of recognition, whereas they are both merely indivisible aspects of the same volition, whose end and content is in the end itself. Each experience in its concreteness is an active process,

not something passively endured, as the physicist believes, looking at it, as he does, from an abstract point of view. Things are to me originally ends and means, objects of my will and my attention which attract me or repel me; hence there is no such thing as an experience which is not a valuation at the same time.[72] The proof of reality is generally sought in the confirmation of the experienced fact by means of other experiences, either of ourselves or our fellow-men, but the repetition of the same fact in one individual and the agreement with the experiences of various persons might both be a pure accident, a mere coincidence of illusions; hence it is not an adequate foundation for the existence-values, whose ultimate basis is the will that a world shall exist for itself independently of our subjective individuality, the exigency that the content which is present to me at this moment shall not be an object to me alone, but to every other consciousness under the same conditions. Experience, however, though it does not suffice to give us the reality values, yet serves as a limit and guide to that exigency of volition which is their true foundation, weakening or re-enforcing it according as it is opposed to the facts or in harmony therewith. He who would make his dreams and imaginings into the contents of every other subject would meet with such resistance from his fellow-men as to deprive him of all power.[73] If the objective existence of things be derived from the will that the facts experienced by us shall also be experiences possible to other individuals, it is obvious that the consciousness of the reality of our fellow-men must precede the affirmation of things. The general belief is rather that we first perceive the bodies of other men, and then from the resemblance of their external aspect argue to their psychic life by a process of induction. Münsterberg maintains that nothing can be more false than such a conception, which is the outcome of the habit of considering reality naturalistically and abstractly. We grasp the action of our volition and of the will of

others in an intuitive and immediate manner which precedes the knowledge of things; we feel ourselves and others directly, not as objects, but as tendencies of attraction or repulsion.[74] But why do we not attribute to the flower which wills to be gathered, the fruit which wills to be tasted, the line which draws us to follow it, the rhythm which urges us to dance, the same objectivity which we ascribe to our fellow-men and the higher animals? Do they not all alike make us feel their exigencies directly? Undoubtedly the flower, the fruit, the line, and the rhythm all will something, but their volition is exhausted in the exigency experienced by us, whereas animals and other human beings are continually assuming fresh attitudes with regard to objects. The criterion of subjective existence is the rediscovery of the same experience of will as identical with itself under new forms.[75] This intimate relation of volition to itself constitutes what is commonly termed the soul.[76]

Our individual volitions are not merely limited by exterior things and by other subjects, but they also recognise the existence above themselves of acts of will independent of any subjective caprice and possessing reality-value to us in as much as they always recur in the same way. These acts of ultra-personal will are the absolute valuations (*die absolute Bewertungen*) as opposed to which the volitions determined by necessity and subjective tendencies appear to us to be accidental and unreal, as do the products of the imagination in comparison to objective nature: only if we will with the consciousness that we shall always will thus, shall we have an absolute valuation. Even the existence of values, like that of subjects and objects, is based upon the pure satisfaction of the will, which rediscovers the original experience unchanged in new experiences.[77]

The values of existence, which we have thus deduced from a single principle, constitute the world of primitive experience in its triple aspect: external, internal, and social, as it is immediately presented. To each of these

three kinds of values there corresponds a value of culture, which is discovered by reflection when the will becomes conscious of its ends. We then go beyond the bounds of immediate experience by means of connections which are established between facts belonging to the same order of reality; there thus arise the various forms of knowledge, whose objective validity we must prove by keeping the starting-point of our deduction ever in mind, *i.e.* the will which tends to preserve its own identity amid various experiences. Everything which satisfies this super-individual will, each identity which is discovered between the different moments of existence, is of independent and absolute value, and is not a mere act of subjective thought.

8. *The Historic World of Subjective Wills and the Mechanical World of Objects.*—Seeing that every connection is based upon identity, it is obvious that things can only be linked with things, subjects with subjects, valuations with valuations: the first species of connection gives birth to the science of nature, which is the consideration of objects with regard to that in them which remains identical; the second to history, which connects single subjective wills, thinking them in their identity; the third to reason, the system of pure valuations conceived in the unity of their absolute principle.[78] Science and history have then the same end, and only differ as to the matter which they elaborate; one of them seeks the identity of things, the other that of wills, which, as we have already seen, cannot be reduced to objective contents, but are comprehended in their immediacy by a peculiar form of experience. The general law is not the true end of scientific knowledge, which tends rather to connect phenomena reciprocally in such a way that the identical permanence of one affords an explanation of the other; it matters little that the two phenomena thus united are produced but once, and do not recur: the formulator of a law merely exacts that it shall be possible to verify it afresh in a single case. The generic concept is not a con-

stituent of science, but is, like language, merely an auxiliary and extrinsic means, a practical organ rendering the same service to the scientist as the hand does to the painter.[79] The ideal of science is not a system of laws, but a system of things, such that its preservation throughout time carries with it all variations perceptible in the outer world. Were it possible to think the whole natural series of phenomena, pursued *ad infinitum* in both directions, as a series of atoms remaining identical in their movements and accelerations, there would be no need to have recourse to generic laws, but, since it is impossible for each individual to follow the whole series in both directions to the two extremes, in order to solve the problem definitely as nearly as possible, laws are constructed summing up in themselves the results of the preparatory work of our predecessors in such a way as to avoid the necessity of beginning again at the beginning each time.[80] The atom of mechanics is not a mere fiction, but is true and existent, because, when experience is reduced to mathematical properties only and divested of all qualitative content, the end of the ultra-personal will is realised, and that pure satisfaction is attained which Münsterberg regards as the basis of existence-value. The physicist is therefore within his rights as long as he confines himself to affirming the reality of the world conceived of by him ; he falls into error when he makes nature a system sufficient unto himself apart from that eternal act of will which can alone impart existential significance to it.[81]

Individual and subjective wills, which are eliminated by naturalistic treatment in order to construct an independent world, constitute the matter proper to historical knowledge, which studies man not as a psychophysical being, devoid of value, not as a thing amongst other things, but as a subject which wills and values freely. This world of free wills escapes the law of causality, because they are external to time in their immediate life : just as it would be meaningless to ask whether the will of Napoleon measured two or three

inches, whether it was white, red, or green, so it is nonsense to attribute duration to it: his wars lasted for years, the movements of his body a few seconds only, but the attitudes of his will are not for that reason limited by time, which is merely a property of objects. The will is wholly comprehended if we understand its ends: he who asks the cause of an act of will puts naturalistic interest in the place of historical interest. In the immediate communication of two subjects, as, for example, of two friends who are discussing politics, one will penetrates the other and understands its meaning without setting it up to itself as an objective content, without enquiring whether it precede or follow his own, or be co-existent therewith, still less what antecedents have determined it. It would at first sight seem a hopeless task to seek in different individual wills that identity which is necessary to the attainment of a system of historic knowledge; but, when I agree with the friend who is judging in his affirmations or negations, when I share his hopes and antipathies, do I not will his will? (*mit-erlebtes, mitgewolltes Wollen*).[82] The comprehension of a subject in its historical existence involves the discovery of the will of other subjects in the will of the individual, and this is attained by resolving that personality into his single political, judicial, economic, scientific, artistic, religious, and ethic volitions, each one of which receives identically within itself the will of another individual or group of individuals. Thus we shall say that he is a Darwinist, a Wagnerian, a Marxist, etc., if in his partial volitions we find the same theoretic affirmations as in Darwin, the same aesthetic and musical preferences as in Wagner, the same social and political tendencies as in Marx. In historical elaboration it must not be forgotten that every new creation is also partially an imitation: in history, as in the world of nature, nothing absolutely new takes place. He who desires something new desires something old, only he desires it in a different way. The artistic, ethical, religious, political, or scientific genius does but gather within himself the

incitations to will which he feels from a thousand sides; the only thing in him which is really new is the original synthesis, and we shall therefore have succeeded in understanding him historically if, when analysing his will, we find therein the wills of those who have exercised influence upon him.[83] Things are not entirely excluded from the historical world; but they are included there in their original reality, as means and ends of the will, not as parts of nature: the flood or earthquake which sweeps away a civilisation is of interest to the historian only in so far as it is dreaded by individual minds; the historical importance of the boat which carried Caesar lies in the fact that it satisfied his desire to obtain news of the fleet.[84]

The normative sciences (logic, ethics, aesthetics, religious dogma) differ from history because the object of their search is the identity not of the subjective values, but of the ultra-personal valuations, which do not depend upon the free choice of individuals. As historical subjects, we may place ourselves at variance with our valuations; as moral subjects, on the other hand, we will and must will in that determinate way only. Here, too, the fundamental problem is the same: given a value, we must find in the current of life a new super-individual will, which shall, in spite of its diversity, prove identical with the first. Just as chemistry can derive the compound from the elements and the elements from the compound whilst preserving the same things (atoms), so logic in the induction of the single affirmations from the ultra-personal will passes to the general valuation, and *vice versa* in the deductive process derives the single elementary valuations from the complex affirmation, always finding the same fundamental identity in these transitions. The same may be said of all the other values, whether aesthetic, ethical, or religious, in which the ultra-personal attitude always remains identical, while yet being realised and taking concrete form in new situations of life, in new exigencies, which set the will fresh problems.[85]

9. *Criticism of Münsterberg's Philosophy.*—No great amount of acumen is necessary to perceive the artificial element in this systematic deduction of all values, and more especially of the cognitive function, from the ultra-personal will. The will of which we have all had experience is always the will of an individual *Ego*, of a concrete historical subject; volition, even if it be exercised with the consciousness that it will always will in that way, does not therefore cease to be the action of a determinate person. The first artifice will then be found in the starting-point of the deduction, that is to say, in the concept of an extra-subjective will, from which the subject is then supposed to be derived. If the existence of the Super-*Ego* be a free conviction of individuals, their reality is the presupposition of the whole deduction. It is not the pure will which imparts existence-value to the *Ego*, but rather the *Ego* which gives a basis to that as to every other will. We have immediate assurance of our subjective life, an assurance which cannot be shaken by sceptical doubt, whereas the existence of an impersonal will is not something which all of us are prepared to acknowledge. In order that the reasoning of Münsterberg may mean anything, we must postulate that pure will is of greater value than individual and momentary desires; that the will which is always coherent with itself is of greater value than changing and contradictory volitions; this presupposes a criterion of valuation, a consciousness of value which cannot be derived from that pure will itself without committing a vicious circle. Or shall we say that these contingent and individual values do not exist? Münsterberg actually has recourse to this artifice, and, seeing that the volitions of the historic subject, which frequently take a direction contrary to the good, the beautiful, and the true, cannot be reconciled with his theory which identifies value with being willed, he arbitrarily banishes these actions from the sphere of will. The thief who commits a robbery really wills the good; does he who denies that he is guilty of a

crime, while well aware that he has committed it, still prefer the truth, in spite of the fact that he states the false? Here we have the paradox to which we are forced to resort in order to save the equation: *to have value = to be willed*, cost what it may to do so, an equation contradicting our intimate experience, which bears witness that it is possible to be conscious of the good, the true, and the beautiful, whilst at times not willing them intensely enough to give them the preference over Hedonistic and individual ends. The robber who commits a theft of his own free will knows which would be the right course, but pure volition is too feeble in him and is conquered by the direct will to enjoy. If he had really willed the good, and the good only, he would not have stolen; in appropriating the property of another, he has preferred his own pleasure. For him to be termed immoral, it suffices that he is conscious of the good; it is not necessary, as Münsterberg maintains, that he shall will only the good. On the contrary, were this the case, we could not accuse him of being immoral, since he would then be an irresponsible being. He who steals without intending to do so, like the kleptomaniac, is ill, not guilty. It is one thing to be conscious of the value of an object, it is another to will that object: they are two widely different moments which do not always coincide in our minds. He who tells a lie in order to avoid being sent to prison still recognises the value of truth, but at the moment he prefers the falsehood which is of personal use to him. The reality of the crime he has committed remains mercilessly with him, even though he may repent and no longer desire it, but may rather strive to put it out of his mind. The past is no longer willed, but this does not bring about its destruction; we continue to attribute value to the men and the events of the past, even though it may be meaningless to say that we still will them. This is a proof that that specific reaction of our whole consciousness, which enables us to feel the value of things, is not an act of will, still less so

is the affirmation of existence, which, as we have already seen when discussing Rickert's theory, has nothing to do with the apprehending of a value. My subjective existence is not something dependent upon my will; on the contrary, my being appears to me the condition of my will, not merely of fugitive and incoherent volitions, but of that pure will, which, while acting *sub specie aeternitatis*, never ceases to be *my* will. To live in spite of oneself, to feel oneself exist even when one's own individual life is of no value to one, even when the will to exist has been shattered in oneself, is not this the cruel despair of the pessimist ?

Objective reality and the assertion of it are even more obviously independent of the act of will. Münsterberg is correct in maintaining against the empiricists that the evidence of objective being cannot come to us from without; but he is mistaken in attributing this recognition to the satisfaction of the pure will, so that the reality of the world would be in ultimate analysis a conviction which the individual is at liberty to accept or not, and which no proof can force upon him. It is not will which impels us to transcend the fugitive moment and the sphere of individual life, but rather thought, which, in its laws and its function is presented to us as possessed of the characters of necessity and universality, and affords us the immediate and indestructible certainty that there must be countless other experiences beyond the limited sphere of our own experiences. If identity within certain limits be a sign of objective existence, this is not the case in order that the will may be satisfied, but because it is a law of reason, whence we are of necessity led to think of nature in its independent being with that same coherency which is proper to our thought—a coherency which must not be, however, understood as abstract identity, but as concrete unity of the various moments in time, which does not exclude the qualitative multiplicity of real becoming. Münsterberg, on the contrary, by placing objectivity in the absolute permanency of the identical, turns nature into

a rigid and abstract system of immutable things, and thus establishes another arbitrary equation: *nature = world of mathematical physics*, as if independent existence were reducible only to stereotyped objects and not to evolutionary processes as well; as if it were not possible to conceive of individual aspects and heterogeneous qualities in things external to our subjective life. The new in the objective world is no less existent than the identical; on the contrary, it is a necessary condition of phenomenal happening, which quantitative formulas cannot entirely explain. Is change real only in relation to our subjective will? Is the world transformed only in our immediate experience thereof? Certainly not; science itself recognises, side by side with the laws of permanency, the irrevocability of natural processes in time. The mechanical world is not the whole of nature, but merely the skeleton thereof—a skeleton which must be clothed with muscles, nerves, and blood if we are to be able to think of it as a system of real things and events. The equation set up by Münsterberg between objective reality and identity of permanency is then of no greater value than the others we have already discussed: in order to affirm the independent existence of a given concrete situation, I am not in the least bound to recognise that it is identical with another. If we thus restore to objective reality that incessant motion of development of which Münsterberg arbitrarily robs it, petrifying it into immovable things, it will at once be revealed to us in its historical life, and the artificial dualism of the two forms of knowledge will be eliminated. Münsterberg, while on the one hand robbing nature of all its activity, on the other hand divests history of its every content, reducing it to the mere abstract moment of subjective activity external to time. Is it however possible to think of a concrete will aiming at and realising an end without positing any temporal relation therein? If there be an experience of which time is a constituent element, it is undoubtedly the experience of our will; the relation between present

and future is essential to the tendency towards an end which ceases to be conceivable when duration, the distinction between that which is present to us and that which appears to us as the future goal of our efforts, is abolished. It is useless for Münsterberg to tell us that this distinction of successive moments is illusory, and that in eternal action the will is identified with the willed, since we know no will but our own, and must start from our own experience in order to form a concept of the absolute will. Further, according to Münsterberg's theory, does not history treat just of subjective volitions? We must then determine the characters of these volitions, not those of absolute valuations, by derivation from personal experience. A will in which there is no transition from a state of dissatisfaction to one of greater completeness, from an initial moment to a final moment, is an absurdity not merely psychologically, but also logically. Were that which we seek to coincide with our present condition, there would be no such thing as tendency; desire ceases the moment it is gratified. In short, we can speak of will only on condition that the term of the action do not exist in its completeness at the point from which we start; if we annihilate this interval, volition will disappear as a fact of concrete experience, and cease to be thinkable even as a concept, since contradiction would then be implied. That which is fully achieved, that which is actual in its absoluteness, cannot be in course of being done, since to do means to realise, and nothing can be realised but something which is not as yet real. Even if it be admitted that the end of the will is another will, this second will is never perfectly identical with the first; if this were not so, there would be no meaning in the action: the intensification of the will, the raising thereof to a higher power, assumes the existence in the goal to be attained, which imparts meaning to the volition, of something which is lacking in the initial moment. Now, how are we to interpret this actualisation of ever loftier stages in the cosmic process of the will, and the distinction of absolute

volition in the single subjective tendencies, unless we understand them to be a series of successive terms, each of which contains more than its predecessor? If you attempt to suppress this succession, it is meaningless to speak of will, which is process developing in a determinate direction. What will become of the progressive actualisation, of which Münsterberg speaks, the incessant becoming of the world, in which each volition tends towards a more complete will, which is in its turn the starting-point of another act of will, if we place it outside time? That which affords satisfaction to our will is not another will, but a perception, a presentation, a concept—a state of our consciousness, not a new activity. We do not desire another desire, we do not make efforts in order to arrive at other efforts, we do not act in order to continue to do so. Certainly there is no such thing as a will without content; certainly the end and the activity directed thereto are fused in consciousness into a concrete synthesis; it does not, however, follow that the content and the action of willing are absolutely identical; even if we do not succeed in separating them, we must yet distinguish between them, since our experience, the only source from which we can draw, if we would know what will is, always presents them to us as distinct. The statement that consciousness can be wholly reduced to action, that the subjective life is will and nothing but will, merely because every psychic phenomenon contains a moment of activity, is equivalent to reducing the whole of physical reality to the single property of extension, merely because there is no such thing as a body which does not occupy a space. By a similar train of reasoning the intellectualist in his turn might reply that, since will devoid of representative or conceptual content does not exist, vague and confused as that content may be, will is reducible to presentation or concept! This is a theory from which Münsterberg shrinks in holy horror, but which is at bottom the outcome of a sophism of the same nature as that which serves to prove his voluntarism.

Moreover, in the concept of history Münsterberg in the last analysis, speak as he may of ends and wills, has not succeeded in shaking off the prejudices of traditional intellectualism, and of abstract mathematicism. Of what avail is it to substitute the will for atoms if it be then treated as if it were the outcome of a combination of will-particles, each one of which must be acknowledged to be identical with any other? Of what avail is it to vindicate living experience against the schemes of the mechanical theory, if the concrete personality is then to be resolved into a series of atomic wills in the endeavour to discover identity between these elements and the elementary volitions of other subjects? A system of abstract equations, even though its terms consist of volition-atoms instead of extended particles, a system of identities external to time is the very negation of history; since in such a system there is no meaning in speaking of that development in a determinate direction which is the essential characteristic of the historical and teleological process. In relations of identity it is a matter of indifference in what order the terms are placed, and these terms can always be inverted; hence it is impossible to decide which of two elementary volitions which are acknowledged to be identical is the original, and which of them is a mere imitation of the other: if, for instance, certain tendencies be found in the subject Münsterberg which are identical with those found in the subject Fichte, it may equally well be said that Fichte's will lives again in Münsterberg or *vice versa*. If we prescind from time and confine ourselves to positing identity between wills, the irreversible order of development will vanish, and we have not even a criterion by which to establish that hierarchy of wills, that progressive intensification of them, which Münsterberg maintains to be the essence of absolute will. Two identical volitions are absolutely on the same level, and one cannot be regarded as the end of the other in such a way as to make it allowable to place it a degree higher

in the scale of values. The system of identities then does not help us to understand history, but rather destroys it. That which constitutes the life of historic reality, and which we have to reconstruct and comprehend, is the rise of new ends and new volitions in concrete consciousnesses: even when you succeed in resolving an individual personality into certain volitions which have been *previously* willed by others (we may say so, but Münsterberg could not affirm it even from his extra-temporal point of view), you have not understood the individual at all, because that original imprint which imparts to his will a new physiognomy, that cannot be reduced to preceding wills, has escaped your notice. Your formulas contain everything except the consciousness of that subject; everything save his history, which does not consist, as you maintain, in finding a will identical to itself, but in the knowledge of reality in its development, and hence also in those unique aspects which can never be identified with others. In order to affirm the independent existence of a subjective will, it is not at all necessary to acknowledge it to be identical with our own will or with that of another subject; indeed the contrary is the case, since we distinguish the different subjects from ourselves and from one another only in so far as they appear to us possessed of characteristics irreducible to one another. Were we to find the same volitions in them all, were we to experience these wills as we immediately experience our own, were it possible to identify them with our own, all exteriority and independence of the single subjects would disappear. That which makes one person a being independent of others is not the universal identity which makes them the same, but the singularity which makes it impossible to reduce them to one another. I recognise the *alter* external to my *ego*, not because he wills that which I will, but because his will, strive as I may, refuses to be identified with my own will. Even if we agree, relatively speaking, if our hearts beat in unison, it is impossible for me to include his will within my consciousness,

since it always remains extraneous to my individuality. Münsterberg in his theory does not account for the wide difference between the intimate unity in which *my* volitions are bound together and the identity which may exist in the ends willed by me and another individual. If subjective existence consisted in finding a will identical, the relation between my volition of yesterday and my volition of to-day, both being directed to the same end, should seem to me the same as the agreement between my will and that of another person. But what a profound difference there is between the two forms of unity! My volition of this present moment may agree completely with the will of another individual and be absolutely contrary to that which I willed yesterday, yet my two volitions, however contradictory they may be, are far more intimately connected than is my will with that of another. Even in the case of a capricious and hysterical subject, whose mind is constantly changing, the consciousness of his own identity is never absent, because this personal identity is not derived from the unity of the end, to which the single volitions may converge, but from the common origin from which they irradiate; it is an indefinable imprint due to their origin, not to their meaning. Two wills may be identical as to their end, but this is not enough to make them a subjective unity; and, conversely, the individual does not lose the identity of his consciousness, even though he may change the content of his desires every minute; neither do we cease to recognise that he is one subject, even though we may not succeed in finding a constant meaning in his volitions. The coherency of the will is the criterion of judgment of the moral character, not the sign of subjective existence. We always feel ourselves to be the same persons, even though we may not be able to order the volitional acts of our whole lives in such a way as to enable us to see them as the progressive actualisation of the same plan and of an identical end. Inner experience, in which we experience ourselves immediately, must undoubtedly

be distinguished from knowledge of the exterior world: things affirmed in their independent reality are not directly apprehended as is our subjective life. Münsterberg would include in the sphere of immediate experience the knowledge we possess of other subjects, which are hence supposed to be experienced in the same way as we experience our own *Ego*: a theory which, were it true, would annihilate the unique distinctive character of my own will as opposed to that of others; because, if I regard my consciousness as a subjective world rigidly opposed to the consciousness of other individuals, I do so by virtue of that character of immediacy which is found in no other apprehension of reality. If the will of others were revealed to me in the same way, I should be unable to distinguish it from my own will; this is, however, not the case: when I have closed my eyes, stopped my ears, and placed myself at such a distance from others that I cannot be touched, I continue to experience my own will, but know nothing of the will of others. I can only receive intelligence of the consciousness of others by means of the sound of words, the sight of their features and gestures, and by tactile and muscular sensations: that is to say, mediately, not immediately. Only when these contents are interpreted as signs of an external reality do we believe in the existence of another subject; an existence which is not admitted if these contents be regarded as mere subjective images. When I am convinced that the voice of a person which I thought I had heard is a hallucination of my own, the existence of that subject will also become an illusion to my mind. The existence, independent of my will, of the friend who is talking politics with me is a dream of mine if I do not actually see and hear him: the existence of other subjects will vanish along with the objective reality of their contents. The logical presupposition of the latter then will be found in the former, not *vice versa*, as Münsterberg would affirm. That which exists independently of me, no matter whether it be a thing or a consciousness, is never

apprehended with that immediacy with which I experience my subjective facts, but is always reconstructed by thought by means of a more or less conscious process. It will not originate psychologically in a true and exact deduction from outward resemblances, but it undoubtedly implies the more or less clear concept of a *quid* beyond the sphere of my subjective consciousness. I do not apprehend the will of my friend any more directly than I do his face and his words ; it exists for me, not because I will in agreement or disagreement with him (which is not always the case, since it is possible for me to remain indifferent to his political views), but because I think it and reconstruct it conceptually in its meaning from that which he says to me. Even if I will the same thing, I do not on that account experience his will directly, I only experience *my own* will immediately ; that there is in existence the similar will of another individual, having the same end in view, is a more or less explicit addition of thought. We then posit the will in another subject, not because we experience it directly, but on the contrary in as much as we think of it as extraneous to our consciousness, and not identifiable with that which is directly grasped thereby without the help of the senses. Münsterberg's treatment of the subject and object of volition, of that which is desired and the person who desires, gives rise to constant ambiguity.

If it be then admitted that history should treat of subjects only (an arbitrary limitation), its matter will not on that account be presented to us in a form of experience differing from that of objects. History cannot be made up of that which is immediately experienced, because this does not go beyond our own *Ego*. The wills of other subjects, the ends they have had in view, the plans of action which they have realised, are thought and reconstructed by me in their independent reality, just in the same way as I think and reconstruct all the other actions of nature. The will of another is apprehended in a more direct way than physical force, and exists historically, because its being is not dependent

upon my will, since it is real apart from the immediate moment of my consciousness. Spiritual activity is manifested as energy working in the objective world, which is not motionless permanency of inert atoms, as Münsterberg conceives it to be, but concrete historic development. External to history is the world of mathematical and mechanical abstractions, not objective reality in its entirety. The world which our thought recognises to be independent of individual subjects, even if it be severed from the wills of single persons, does not for that reason become a dead, meaningless thing, but preserves all its value as an activity directed towards an end. Nature in its complete organisation and in its evolutionary process is only intelligible as the actualisation of an end in time. Hence not even teleological consideration can constitute a characteristic sufficient to exclude historicity from the objective world in order to confine it to individual wills only. Neither can it be urged in opposition that a difference will always remain, since in the case of nature the end is universal and objective, whereas in history it is personal and subjective, because, as Münsterberg himself maintains, historical reality does not lie in the single will, but in the connections of wills and in their identity. The momentary desire, if it remain an experienced intuition, has no historical existence ; in order that it may acquire such existence it must be put into relation with other volitions in such a way as to give prominence to their universal meaning. The will of Caesar becomes history only when conceived with regard to that which is real about it, not exclusively for me, not for this or that individual who re-experiences it in himself, but for all possible subjects. The historian, no less than the man of science, must eliminate that which is contingent in the way in which a personality appears to different individuals in order to determine it in its true existence. History is altogether impossible unless we sever, so to speak, the individual action from our subjective life in order to transfer it into the objective process of reality.

In this transition things, too, which Münsterberg considers to be matter for history merely as means and ends of a given subject, are of necessity included in the scheme of nature, understood, of course, in its concreteness, not in the schematicism of mechanical science. In what other way can we conceive the independent existence of the processes serving as means to the realisation of certain individual ends, but through the forms and categories necessary to the construction of the objective world? If we return to pure primitive intuition, to immediate experience in which the contents exist merely as an integral part of the subjective will, we cannot speak of Caesar as a real individual, distinct from us who compile history, still less of the boat and the night voyage, which do not exist for us, confined as we are to the experienced immediacy of our consciousness. In order to understand Caesar's wish and the satisfaction thereof by means of a boat, in order to think all this as an historical reality, we must conceive of the voyage and the boat as they were present to Caesar's mind; who is prepared to maintain in all seriousness that, when Caesar was on the point of embarking, the movement which was to take him away and the boat only seemed to him to be real because they satisfied his desires? Caesar might have imagined in his consciousness a more convenient way of going to search for the fleet, but the mere mental presentation would not have satisfied his will; the boat did so just because he recognised it as an objective thing, independent of his will. Moreover, from Münsterberg's point of view there would be no difference between Caesar dreaming of embarking and Caesar embarking in reality. How, indeed, are we to distinguish between the two contents, both of which satisfied his subjective desire, unless we place in opposition the real voyage and boat and the imaginary voyage and boat, unless we think, on the one hand, of a world of things existing independent of the single subject, and, on the other, of a world only possessing the value of reality within that subject? The historian also, when

placing objects in relation to individual wills and considering them as means to the actualisation of their ends, always conceives them as real in the external world, nor is it possible for him to do otherwise. Restore all its concreteness to nature, all its objective content to history, and the gulf fixed by your artificial formulas between the two aspects of existence will disappear. The consideration of values and ends does not preclude the scientific explanation of real processes, but rather completes and comprehends it by raising it to a higher synthesis.

10. *Münsterberg's Super-Ego and Royce's Absolute Consciousness.*—The concept of the Super-*Ego* which supports all the values and imparts a universal basis thereto undoubtedly represents an advance as compared with Rickert's *Ought*, suspended in the void of its absolute transcendence ; but the conscious activity of the ultra-personal will, as conceived of by Münsterberg, external to all duration, the eternal action without any subject is not a principle which is intelligible in itself, still less one which can account for the becoming of the world and our individual life in time. To Josiah Royce [86] must be ascribed the credit of having placed the philosophy of values upon a more solid speculative basis, by grafting it on to the vigorous stem of English neo-Hegelianism. Rickert's *Sollen* is brought down by him from its transcendent sphere in order to be transferred into the ideal order immanent in Green's Absolute Consciousness ; this Consciousness in its turn gains from Rickert's individual reality and his concept of history that mobile life which enables it to escape from the universal system of eternal relations, the pan-logism of Hegel, which is powerless to resolve into itself the concrete development of beings in time. The idea becomes the meaning of a will, and is vivified by the warmth of personal experience. Thus in Royce's system the different streams of reaction from intellectualism, whose course from their common source we have endeavoured to follow, as they wind along, now far apart,

now close together, meet and merge in a harmonious whole: voluntarism and neo-Hegelianism, contingentism, and the historical method, the philosophy of values and pragmatism. Royce, like the pragmatists, believes that our ideas do not consist of pure images, but that they always imply consciousness of the way we propose to act with respect thereto; they are instruments serving special ends, and must therefore be judged in relation to these ends.[87] The idea of a sword implies, for example, the memory of the appropriate act, and the way of using it; the idea of friends differs from that of enemies by reason of the different attitude which we intend to assume towards them. Popular psychology regards understanding as a passive reception of the truth, and defines will as a productive force, thus artificially severing the two functions, which it afterwards vainly strives to unite; but this separation is false, since experience shows us that there is no knowledge without the will to know, that in our intellectual processes we are generally guided by those same interests which are commonly regarded as stimulating the will; on the other hand, our conscious volition implies the immediate knowledge of itself. The intellect and the will are then but two aspects of one and the same process; when I know, I act, hence my theoretical life is also practical.[88] We do not observe any external fact without observing at the same time more or less clearly our attitude with regard thereto, our estimate of its value, our response to its presence, and our intentions with respect to our future relations to that fact.[89] Each thought is a will process; each conscious action an idea visible and tangible in its immediacy.[90] But though Royce agrees with the pragmatists in defining reality in terms of action as that which is capable of satisfying an interest, a desire, or a volition,[91] he differs from them in according a higher place to the end and absolute fulness of the Divine Will, which can never find adequate expression in finite consciousnesses, than to the ends and relative satisfaction of human wills. Incomplete and contingent

truths are thus subordinated to an eternal and absolute truth-value, which is not dependent upon any individual, but is based upon the Universal Consciousness. This affirmation of an objective will, which imparts a common form to, and imposes a categoric exigency on single individual wills in spite of their irreducible variety, connects Royce's philosophy very closely with the teaching of Windelband, Rickert, and Münsterberg.[92] Royce too regards our recognition of facts as voluntary submission to an *Ought* (corresponding to Rickert's *Sollen*), and the *Ought* is not a force constraining from without, but something which, while in opposition to the momentary impulse, is willed by us as the most complete expression of our rational nature. That which determines us to recognise one system of special facts as real rather than another is the *Ought* of recognising those facts which at a certain moment enable us to actualise our will better than we could do were we not to recognise them.[93] We are not constrained thereto by violence from without, but from within, by the very nature of our will which strives after a more complete expression than the present one: the theoretical *Ought* of our judgments about facts, like the practical *Ought* of Ethics, is in the last analysis definable only in terms of what Kant called the Autonomy of the Will.[94] A fact is generally conceived as something external to our power, as necessary, resistant, extraneous to the will; this, however, depends upon the essence of our will which seeks its satisfaction in an end, a form of life, which is in its fulness external to the actual moment, and is never completely attained; hence that sense of limitation and incompleteness which makes the real seem to us to be beyond the realm of our action. If facts seem to us to be extraneous, it is because they imply aspects which must at present appear to us infinitely remote from our fragmentary consciousness; because the end which imparts the meaning of reality to them is not my human end in its transitory aspect, nor that which I can actualise in one moment of my life, but that which *ought to be*,

and which I feel in my will as an absolute exigency, the ultimate goal of my complete satisfaction, even though it may be unattainable in the empirical process of our voluntary actions, which develop in time. It is due to this inexhaustibility that it appears external to our empirical consciousness ; but it is really within us as the end of our will, as the perfect expression of that meaning to which we never succeed in giving complete expression ; it does not absolutely transcend consciousness, but is immanent in every stage of its development, and finds eternal satisfaction in the Divine Will, which embraces and completes the whole infinite series of moments of time and space accordingly. Royce thus puts aside the absurd concept of the absolute transcendence of value, which, as we have already seen, is the bane of Rickert's system, and rightly maintains, as Green does, that no absolute object is thinkable except in relation to an Absolute Consciousness. In order to impart meaning to our cognitive and practical life we must transcend the empirical human world ; but the reality which is the goal after which we strive must also be conceived of as a form of conscious experience.[95]

11. *Reduction of the External to the Internal Meaning of the Idea.*—If the truth-value of the idea be dependent upon the end which it more or less partially actualises, upon that which Royce [96] terms the *internal meaning*, immanent in consciousness (*purpose embodied in the idea*), not transcendent like Rickert's *Sinn*, we must reject the common conception which considers truth as consisting in a correspondence between the idea and an exterior object, that is to say, in its *external meaning*. Against this conception, the thesis of realism, Royce aims the shafts of his vigorous dialectic, in order to prove its innate absurdity, and thus to clear the way for the reduction of the external to the internal meaning of the idea. Realism views the object as that which is absolutely independent of thought, which would exist even were it unknown, and does not therefore stand in any necessary relation to conscious-

ness: the cognitive relation is purely accidental, so that even if it be eliminated, the reality of the object will endure.[97] Realism as a philosophic conception is presented either in the pluralistic form, or in that of monism, but in both cases it gives rise to insuperable difficulties. In the pluralistic form the relations between different individuals, whether they be conceived as material atoms or as spiritual monads, remain inexplicable; hence the efforts made by philosophers to account for their reciprocal actions by means of secondary hypotheses, frequently of a paradoxical order, such as pre-established harmony, the accidental aspect and the like. In reality the manifold beings remain extraneous to one another, and it is not possible to establish any link between them, or to affirm any character common to them all without implicitly denying the hypothesis by admitting a universal embracing them all. In order to conceive a relation between two individuals of the realistic world in time and space, we must, if we would adhere to the hypothesis, think of this connecting link as a third being, independent of the first two, which therefore cannot serve to connect them.[98] The realist is thus forced to take refuge in monism, admitting a single, internally complex being, whose various aspects or moments are so connected that nothing can change or disappear without giving rise to change in the whole. Even this refuge, however, will fail realism, since from its own point of view it is bound to admit two independent beings — the idea and the object; and, if reality be defined as independence, one must be completely extraneous to the other, and cannot stand in any relation thereto, or have any characteristic in common therewith. How can we then maintain that it corresponds with the object, or that it represents it in any way?[99] The realistic theory itself has nothing to do with the world which is completely independent thereof! It thus contradicts itself when, after placing the real object absolutely outside thought, it exacts that the idea shall resemble it and correspond to it. Such an agreement

cannot, moreover, be set up as an absolute criterion of truth; the resemblance borne by the idea to the object does not suffice to make it true, if this object be not the one purposed by the idea, and, conversely, if the idea carry out adequately the plan it had proposed, it will be true even if it have but a vague and schematic resemblance to the external object. Agreement as such does not constitute truth, but agreement which is willed (*intended agreement*) for certain special ends: thus in some cases a concrete representative image may be preferable to an abstract symbol; in others, when we have a different purpose before us, a complex of algebraical symbols may be preferable as a representation of objects to the pictures thereof. Do you desire to reproduce a musical phrase? Your song will be out of tune if it fail to bear a concrete resemblance to the series of sounds heard on other occasions. Do you rather propose to study acoustics? It is then no longer a question of seeking a concrete resemblance, but rather of finding a correspondence between certain abstract relations, which are mathematically formulated in your ideas, and the greater or lesser agreeableness of certain combinations of notes.[100] Just as it is impossible to say in general whether a razor is better than a hammer, without reference to the use we propose to make of them, so these instruments of a higher order—our ideas—must be judged with regard to their specific end and conscious purpose. The idea decides its own meaning, chooses its object and the special form of agreement or resemblance which it desires to have therewith; it assigns itself a task and submits to its plan of its own free will.[101] The external meaning of the idea upon which realism lays so much stress is therefore in ultimate analysis subordinated to the internal meaning, which alone decides its truth.

At the very antipodes of realism we find another conception of being, which Royce regards as equally impossible of acceptance, although he considers it

superior from the epistemological point of view: the conception of mysticism. Whereas the realist considers the external meaning only, the mystic knows the internal meaning alone and condemns all finite ideas, because they fail to attain that absolute satisfaction, that perfect calm in which the will finds its ultimate and adequate expression, the goal of its desires and researches. Realism considers being as independent of knowledge; mysticism, on the contrary, defines it as immediate experience (*immediate feeling*); the former says, Seek the truth outside thyself in the outer world; the latter, Seek the truth within thyself, withdrawing thyself from external and contingent facts, which never satisfy thy inmost will, into the calm solitude of thy own mind.[102] The superiority of mysticism to realism lies in its critical attitude, since it does not dogmatically require that we shall accept its definition of being, but appeals to our experience to determine that which we truly call real, and finds it in full possession and complete satisfaction. It is a thoughtful doctrine, which knows itself and consciously faces its paradoxes without ignoring its own contradictions, as does realism.[103] But if the mystic be right in affirming that there can be no reality wholly independent of our knowledge, that in ourselves alone is to be found the criterion by which to distinguish between the true and the false, the real and the unreal, the eternal and the transient contents, he is wrong in entirely repudiating the world of facts and finite ideas, since the state of Nirvana, which he proposes to attain, taken by itself, is nothing, and derives its whole value from the contrast between it and that incomplete reality which is termed illusion. Now, if being cannot be attributed to the first member of such a relation of contrast, the second will disappear also. Annihilation is something only so long as I seek it, it is a positive ideal only in as much as I strive to attain it; pure immediacy has a content only because it satisfies the imperfect will; hence it derives its whole meaning from contrast and other relation with finite

facts. If our conscious ideas be nothing, the Absolute is also nothing.

Yet another inadequate solution of the problem of reality is critical rationalism, whose spiritual ancestor is Kant, and which, as opposed to the independent individual of realism, identifies being with the universal validity of the idea, with the law, the type, the common form of all possible experience.[104] Physical and mathematical sciences offer us countless examples of these universally valid truths, which always transcend the sphere of our actual experience, and can therefore never be completely verified. In the physical and mathematical sciences alike consciousness is confronted by a fact or empirical process, ideally constructed in the case of mathematics,[105] and presented by experience in that of physical science, but the marvellous thing in both cases is that the observation or the experiment guarantees a universal affirmation with regard to an infinity of objects which neither are nor can be present in their totality to any human being. How are these two aspects to be reconciled: on the one hand, the empirical contingent fact in its transitoriness; on the other, the affirmation of a universal validity? What is a valid experience in the moment in which it is not presented to me, but in which I merely suppose it possible? What is a valid truth when no one verifies its validity? Can completeness of being be attributed to the pure universals which have not as yet been verified? Validity is an ambiguous term: when it is applied to the ideas which we are actually verifying, it means that they are expressed in concrete in experience; when applied to the whole realm of valid truth in general, to the world of nature which is not now under our observation, or to mathematical truths which are not present to us, it means that this realm is in some way possessed of a character which we cannot verify and which is never entirely actual in our human experience.[106] In concrete experience the validity of ideas is presented to us by an individual fact; in the realm of possible being

the same validity appears as a universal, abstract, formal law. Now is it possible to admit two species of beings, both of which are known to be valid, but of which one is individual, the other universal; one empirical, the other purely ideal; one present, the other merely possible; one possessed of concrete life, the other a pure form? Must not the same life of concrete experience throb in all beings? Our will, which is also knowledge, finds adequate expression — its full and perfect determination — in the individual alone. A truth which remains purely valid fails to satisfy it, because it does not enable it to leave the domain of the indeterminate; the judgment which formulates the universal law excludes certain possibilities, but tells us nothing about the positive content of concrete reality: eliminating little by little the various possibilities, we might go on indefinitely without ever reaching the end of the series, that unique experience, which is completely determined, and leaves no other possible alternative external to itself.[107] This defect of pure abstract reasoning in universal terms is supposed to be remedied by the appeal to experience; but experience cannot in a finite time confirm the law in its universality. The detailed judgment thus arrived at, though possessed of a positive content, does not authorise us to exclude all other possibilities; when you define an individual by certain abstract characteristics, you cannot be sure that one individual alone corresponds thereto in reality, neither can human experience give you absolute proof thereof, since it is incapable of exhausting the whole sphere of existence. The presupposition that there is no other person identical with a member of your family is one which you may justify from the metaphysical point of view, but which it is vain to ask experience to confirm.[108] To us the individual, the complete determination may be the object of love and hope, will and desire, faith and action, but never of present discovery; it can never be defined either in terms of generic concepts or as a datum of finite experience, but only as

a limit towards which are directed our active researches and our conscious will which strives after complete satisfaction in a form of experience at once fully determined, unique, and exclusive.[109]

Criticism of the three principal existing solutions of the problem of reality thus leads us on to the fourth conception of being, which Royce regards as alone capable of synthesising the others in itself, eliminating their inherent contradictions and reducing them to a harmonious whole. According to this fourth conception the real is the complete embodiment in individual form and in final fulfilment of the internal meaning of finite ideas.[110] The object, the *other*, is the full actualisation of our end and of our will which is at present imperfectly incorporated in our ideas — the completion of that which we now possess only in part in our finite vision ; it always appears to us to be out of reach of our ideas and finite experiences just because they never succeed in realising it in its exhaustive and individual concreteness.[111] It is true in a certain sense, as is asserted by realism, that the object exercises authority over the finite idea, since the idea can only seek that which it consciously intends to seek, that which demands the determination of its will for singularity and that final expression for which nothing can be substituted, and must therefore be in subjection to its own plan ; but this authority exercised by the object over the idea must not be transformed into an absurd independence. Thus Royce and the mystic alike consider that in that final stage the world and the *Ego* are absolutely identical, but the *nescio, nescio* of the mediaeval mystic merely serves to express the present inability of the transient idea to actualise the end in full, not the essential nature of true being ; the satisfaction of the will, fragmentary though it be, will be found in earth, not in heaven. Critical rationalism gives us the universal essence of the object, and the characteristics which it possesses in common with our present idea, and the experiences of the moment in which we define it, but

it leaves undetermined precisely the being of the object as *other*, as different from the finite idea. The fourth concept of being embraces everything which is true in the three former views and corrects the false elements contained in them: being exercises authority over finite ideas, as is asserted by realism; it is valid, in conformity with the demands of critical rationalism; it is identical with the true internal meaning, as is affirmed by mysticism, but its reality is not external to the idea, as in the realistic conception; and, as opposed to the assertions of the mystic, its imperfections, its incompleteness, its struggles, and its strivings in time are also real, as an integral and constituent part of the Absolute, which is not a mere law of abstract validity, as is affirmed by the rationalistic theory, but rather a will embodied in a concrete life, a life giving full expression to that meaning which each passing moment of human consciousness actualises only partially and which is the object of its ideal aspirations.[112]

A sceptic might say, "I admit the existence of nothing beyond human experience, there is no experience which is more complete and more perfect"; but in making this assertion, he would admit that the present content of his consciousness is perfectly adequate to his idea of being, that is to say, that this experience is not of a transient nature, but is an experience at once absolute, individual, and definitive.[113] Moreover, even the sceptic must admit the existence between different subjects of relations whose reality can only be conceived in a Consciousness which is able to embrace in one inclusive act all single consecutive processes together with their relations. If everything which is real exists merely as a known fact, the actualisation of a conscious end, there can be no reality in the distinction between different known beings and their reciprocal actions except in so far as they satisfy a conscious will. This Supreme Knower of the universe, who is not an empty unit external to time, but includes all temporal processes in their infinite variety, and all the manifold

meaning of our minds in the inexhaustible riches of his content, must be conscious of himself, since, were his own being and his spiritual unity to escape him, he could (according to Royce) be real only to some other consciousness to which his internal meaning was present; but this cognitive relation, like all relations, could in its turn only exist as a fact present to the Absolute Consciousness, which, since it must be aware of its relation to the other consciousness, must know each one of the terms, and hence also itself.[114] Everything then which develops and lives in time exists in God; it is neither absorbed nor destroyed, but is rather preserved in its individual physiognomy; finite consciousness, just as it exists in ourselves, with its strivings and defeats, its mistakes, its temporality and limitations, is all present from the absolute point of view; but it is seen together with the solution of its problems, the attainment of its ends, the overcoming of its defeats, the correction of its mistakes, the final completion of temporal processes, the perfecting of that which is faulty in us.[115] The Absolute knows everything we know, and as we know it; our experience is not transmuted or reduced in some ineffable way in order that it may become one with the Divine Life, but persists in that Life wearing the same concrete aspect which it does in us. Even our pain exists in God, since the full triumph of eternity can only be attained through the sorrows of time; in struggle with pain, conflict with suffering, victory through striving, will be found the loftiest fulfilment of the life of the mind. Were God ignorant of pain, He could not know the sublimity of victory; just as there is no courage without conquered fear, so in the life of suffering there is no conscious heroism without present tribulation.[116] Thus Royce, as opposed to Bradley, resolutely affirms the reality of finite experience and of its becoming, replacing the purely logical and static concept of reality, as a harmonious system by a dynamic interpretation of it in terms of conscious will. The reality of time is from such a standpoint a condition necessary to moral life, and to the

actualisation of the Absolute Will, which would be inconceivable without development.[117] Within the duration of His present God embraces the infinite series of successive moments, which forms an individual whole in its single and definitive meaning. It is this unity of plan that, determining as it does the position of each moment in the irreversible process of time, which has the form of a well-ordered series, defines it, and thus renders it thinkable as a present totality. The error fallen into by critics of the actual infinite lies in the belief that the place of each term in the series must be found by means of empirical counting, whereas in reality the definition of the series predetermines at one stroke the successive order of its elements. When we say that the series is present in its completeness, we do not mean that there is a last term which can be reached by passing its members in review one after another in a kind of roll-call, but that these elements are seen together in a total experience, all at once, with the place pertaining to each in the system, just as the definition determines them in their individuality, their unique and irreplaceable meaning.[118]

12. *Error.*—If our every presentation and our every thought exist in the Absolute Consciousness, if truth be dependent upon the idea itself which selects its object, task, and meaning, does not the existence of error become inexplicable ? If all be real in God, how can we affirm the existence of anything false ? If it be the idea which says, " I will mean this or that," how is it possible for facts to give it the lie ? If the object be that which the idea freely wills it to be, how can it place itself in opposition thereto, and be out of harmony therewith, thus giving rise to that which is commonly called error ? This question is the rock on which all pantheistic systems are doomed to split, and Royce's philosophy shares the fate of the rest. He does not attempt to avoid it by means of dialectical devices, but faces it boldly. Green's panlogism, which absorbs and dissolves the individual into the system of eternal

relations which are present to the Absolute Consciousness, affords no explanation whatsoever of the possibility of error, which it regards as no less real than that which we commonly call truth, though existing in a different order of relations. He certainly distinguishes between variable and arbitrary relations and the eternal, unchangeable system of relations, which acts as our objective criterion, and which we place in opposition to our subjective life,[119] but this individual variability together with everything changeable and contingent in the history of the world finds no adequate justification in his system. Bradley's doctrine of degrees of truth goes more deeply into the problem than does Green, though he too is very far from affording a solution of it. Bradley regards everything in the realm of human thought as appearance; hence there is error in everything, but every error contains a certain amount of truth, just as every truth contains a certain amount of error; it is therefore possible to distinguish various degrees, according as the appearance must be subjected to a greater *supplementation* or *rearrangement* in order that it may be transformed into absolute experience. Bradley does not, however, afford us any explanation of the existence of error, which, partial as it may be, is always a deviation from the absolute system; he does not explain why the severance of the *what* from the *that*, the beginning of so much evil, takes place, and he leaves us in an uncertain sceptical position. If human experience be illusory, why and how does this great illusion exist? Are appearances necessary for the constitution of the reality of the Absolute which, as you yourselves admit, is nothing apart from them, or are they not? Bradley replies that they are not all necessary to the same degree; but if we divest appearances of that which is true in them, that is to say, of that part which is necessary in order to give a concrete content to Universal Consciousness, for whom and in what way does the residue exist? It is not necessary to the life of the Absolute, which could be manifested in its perfection without them; it does

not exist in the experience of the Absolute, which would in that case become contradictory; why and where then does it originate? Does anything exist external to the Absolute Consciousness or not? If there be no subject distinct therefrom, where did and does the separation take place between the *what* and the *that*, the idea and the fact? Bradley stops short at this problem, and proclaims the inadequacy of the human intellect, thus turning us away on the threshold of mystery with our difficulties unsolved. Royce, on the contrary, does not refuse us an explanation of that transitory and imperfect element which marks the distinction between human and divine experience, and, whilst accepting the theory of degrees of truth, does not take up Bradley's purely negative position,[120] but endeavours to make clear to us the reason for the existence of these different degrees, and determines the vague concept of *supplementation* by means of a more exact idea of the relations between finite thought and the infinite Consciousness, explaining to us the nature of that greater completeness which our soul finds in God. That which Green and Bradley treat as an inexplicable accident is to Royce a necessary moment in the life of the Absolute, whose will can only attain to perfect satisfaction through incompleteness. The will, that is to say, the idea bounded by time, never attains this fulness of individual life; it is never able to express its definitive meaning; in the process of actualising the will we can therefore distinguish various stages (corresponding to the different degrees of truth), each one of which is more or less distant from the end and embodies the internal meaning of the idea in a more or less adequate manner. The end is not always clearly present to consciousness; we pass from a vague, indeterminate state of restlessness to a definite state of will and resolution; before arriving at it we ask ourselves, "What do I desire? What is my real end?" and the answer to such a question may make large demands upon our time, and may be erroneous as regards intelligence of our end.[121] The

unique, unambiguous expression capable of fully determining our will may, when our will has not yet reached its goal, differ from that which at the present moment seems to us to be the meaning of the idea. Here we have the explanation of the possibility of error, which lies in the inadequacy of the present stage of the volitional process to express its true end. The object which may conflict with my partial and fragmentary will, that is to say, with my will not as yet fully realised, and may to a certain extent contradict it, is not an external thing, an obstacle to my conscious activity, but rather my will itself in its phase of complete actualisation, my final intention, my total meaning determinately and definitely expressed.[122] In the last analysis that which decides whether an idea be true or false is the object which it has predetermined and the plan which it has proposed to carry out. The idea chooses, so to speak, the game it will play, but, coherently with itself, it cannot change it arbitrarily or alter the rules, thus failing to fulfil its task: thus, for instance, if it proposes to find its complete determination in a group of sensorial experiences (as occurs in the verification of physical theories), these experiences exercise authority over it and demand its full submission; but this is in reality not the exercise of extrinsic force, since it is the idea itself which has freely determined to find the agreement with facts. It is possible that it may fail, but to what else can this failure be ascribed than the end proposed by it, that is to say, to the object and form of correspondence which it has chosen as its term of reference? [123] If it then be contradicted, it is so of its own free will, just as the lover who is rejected by his lady owes this repulse to his desire to possess her, a repulse which he would not have had to endure had he not chosen just this particular woman as the object of his adoration.[124] There can be no doubt that facts can resist our momentary desire, but at bottom it is the will that recognises these limits, which are objective in the sense that they differ

as more complete expressions from that which is now present to us. Even in appearing extraneous to me, nature embodies my will, since I recognise that my conscious activity is limited and controlled only in so far as it can attain the desired completion through these limitations and this control.[125]

13. *The World of Science and the World of Valuation.* —Belief in the reality of the external world is not forced upon us from without and is not justified by the resistance met with by muscular effort, as some philosophers have thought, but is guaranteed from motives of a social order, and makes itself felt by us as the requirement of a duty. The physical world is that part of human experience which is common to all men, which enables them to understand one another, and thus acts as the basis of their co-operation ; failure to recognise it would therefore be equivalent to cutting oneself off from civilised society.[126] The existence of our kind is then the presupposition of the reality of nature, not *vice versa*, as is usually affirmed when our belief in the inner life of other men is explained by an induction by analogy based upon likeness of features and objective physical manifestations. We are social beings primarily by reason of hereditary instincts ; we instinctively love, fear, and watch our fellow-men, who are therefore real to us as immediate objects of desire or repulsion, in so far as they supply us with that which we lack, complete our fragmentary meanings, answer our questions, and render our experience and consciousness of our ends more perfect.[127] In this direct penetration of minds and their internal meanings which causes us to feel ourselves part of a single *Ego* is found the world of valuation, the most profound of truths in its concrete history ; the external mechanical aspect, the world of scientific description, is but an inadequate symbol, necessary to the empirical communication of finite individuals in time and space, a common scheme formed by eliminating the variable subjective element, in order that all men may be able to establish an

understanding amongst themselves in their practical relations.[128]

To civilised man the difference between the mind and material objects is so great not by virtue of any experience conferring upon him the right to assert it positively, but by virtue of the fact that our relations to the physical world are possessed of a social meaning which tends to contrast more and more strongly with our practical relations to living man. Owing to this contrast more and more prominence is given to that aspect of nature which enables us to have dominion over it, and to which the success of practical life is due, the unbending, uniform, mechanical aspect, whereas the increase in the common heritage of civilisation raises the importance of our fellow-men in our eyes.[129] This sharp distinction, which is a mere artifice whose existence is justified by reasons of a practical order, must not, however, lead us into the mistake of making a dualistic separation between the two forms of reality. If we compare with the life of consciousness, not the abstractions with which mechanical science presents the physical world to our notice, but rather physical processes in their concrete complexity, the apparent contrast will vanish and that which is common to both nature and the mind will take the prominent place which is its due.[130] Both are, as a matter of fact, subject to an irreversible becoming, though the process of transformation in nature is considerably slower; in both we see the tendency to form habits, which are not, however, absolutely fixed, but form rhythms of temporary duration which are finally dissolved by the perturbing action of irrevocable change. The slowness of natural processes prevents our consciousness adapting itself to the rhythm of that psychic life which lies at their base; hence the erroneous opinion which classes them as unconscious phenomena. In reality they, too, form part of a finite consciousness, embracing millions and millions of years in the duration of its present. There is nothing against our conceiving other consciousnesses

whose relation to time differs from ours, which have a different *time-span*.[131] Single animals and objects cannot each be possessed of consciousness, but they form part of a conscious process which embraces them in its vast present; just in the same way as human individuals with their inner life are included in other finite experiences, of which the phenomena of racial memory and instincts are indications. The birth and death of a man are not the beginning and end of a conscious life, but a mere change in this consciousness with a longer *time-span*.[132] Royce then does not agree with Berkeley in regarding matter as something illusory, he rather considers it to be real in the same sense as our fellow-men are real: it is a finite consciousness which is identified with the life of the Absolute in the same degree as our individual mind is identified therewith.[133] Each consciousness can, whilst preserving the unity of its plan, form part of a vaster plan of conscious experience, and is not enclosed within itself, and external to all other consciousnesses, as are the monads of Leibnitz. The relations of communication of finite individuals are an indication of their unity in the Absolute, which embraces them all as moments in a conscious process having an ultimate and single meaning.[134] That which characterises and distinguishes different individuals is not the plurality of substances which are inconceivable in their realism, but the unique nature of the plan which each one of them actualises in his consciousness and the originality of the end which he embodies. I place myself in opposition to others, because my life is possessed of a unique meaning which even in the Absolute remains distinct from the ends of other persons.[135] In this originality, this unique imprint preserved by our will in the Divine Consciousness in which each one of us gives perfect expression to his meaning, lies the essence of human liberty. My will, though a moment of the Absolute Personality, is, in as much as it is possessed of a unique physiognomy, determined by nothing but itself: that

which I will is still willed by me even in God.[136] My experience is not absorbed, transmuted or reduced in some ineffable way, but exists in the Eternal Mind just as it does in myself with all its imperfections and all its errors.[137] Eternity does not annihilate the succession of time, but embraces it all within itself in its infinite present, just as we distinguish the various moments of a rhythmical measure with its successive notes, while yet grasping it as a whole in our consciousness.[138]

14. *Criticism of Royce's Philosophy.*—In Royce's philosophy the reaction from intellectualism reaches its speculative acme in a system grandly architectural in its monumental lines. Royce does not attempt to evade the great problems which have baffled the mind of man throughout the ages, but rather faces them with a vigour of thought born of genius. Liberty, immortality, the existence of evil and error all find their metaphysical justification in his system, a justification which, though it may fail to satisfy us wholly, is yet the most momentous attempt ever made to solve these problems by means of pantheistic intuition of an ethical and religious order. As we have already seen, Royce considers that the existence of error is due to the fact that the true meaning of the idea, as an act of will, is never fully actualised: before the complete satisfaction of the will is attained, a series of stages must be passed through in which the end is not present in its full determination. The transition from dissatisfaction to satisfaction is the very essence of the volitional life, and hence also of knowledge, which Royce regards as identical therewith. Truth is that which I will, that which I propose to do, my plan of action; and, should I err, I do so, not because there is something external to myself which sets itself in opposition to my idea, but because, though the meaning of my thought depends upon myself and myself alone, I have no clear consciousness thereof at the initial and intermediary stages of the volitional process. From the first I have no determinate knowledge of my true end; in that case,

however, we would ask Royce how it is possible to state that this end is dependent on myself alone, that I have selected it, if it is not that which was present to my consciousness. If the result attained by the process of my research differ from that of which I am conscious at a certain moment (and it is just this difference to which the possibility of error is due), am I justified in affirming that this ultimate result was the one which I had proposed, and that the contradiction which facts so frequently present to my ideal hypotheses is dependent upon my will ? To take an example : a scientific man might conceive the notion that the motion of falling bodies is uniform, but he is unable to actualise this idea, or to impart to it the intensity and concrete determination of experience ; does this failure depend upon himself ? How can he seriously maintain that this contradiction is due to the internal meaning of the idea, if the experienced fact of the constant increase in velocity were not the end of his researches ? He had neither foreseen nor chosen it : his will was directed towards a totally different end, and he was fully conscious of this internal meaning. That which contradicts it was not willed by him ; hence it cannot be regarded as the internal meaning of the will of that individual *Ego*. Neither can it be said that the end proposed by the student was the constant acceleration of motion, although he was not aware of it, since it is not allowable to consider that as being willed which was not present to the consciousness of the individual, and which is therefore at times the very opposite of the desired end. If the idea be not aware of its true meaning, how can it be maintained that this is the free choice of the idea ? Is it possible to will without knowing what one wills ? In reality, the assumption by our idea of individual concrete form never appears to us to be dependent upon the idea itself, even when the idea is confirmed by experience : the transition from the abstract hypotheses to the determinate form of perception is not due merely to the will to actualise that idea. In the cognitive

function we are conscious that this transition is not due to our will alone, and that it depends upon something external to our will, so much so that it is sometimes, as in the case of error, a hindrance to the realisation of our idea. Metaphysics may even conceive the *other*, which is external to my will, to be the actualisation of a Higher Will, but this Will is not my individual will. Royce's argument is entirely based upon the confusion of these two wills, which he postulates as being identical, thus making the volitionary process in man into a stage of a wider process. Such an inclusion of consciousness in one another, of the mind of man in the mind of the species, and of this mind of the species in the consciousness of matter, and of them all in the Absolute Will is, however, inconceivable. It is an undeniable fact of experience that each finite *Ego* presents an impenetrable front to other individuals, and that, strive as it may, it can never apprehend the phenomena of consciousness in others with that immediacy with which it apprehends the processes of its own consciousness. To this impossibility of fusion, not to singularity of plan, is due the fact that each mind places itself in opposition to the rest, refusing to regard as its own those feelings, thoughts, and acts of will of which it can have no immediate experience within itself, and can only reconstruct indirectly, and by analogy. In our criticism of Münsterberg's system we have already observed that the unity of the *Ego* cannot be reduced to the unity of the end: the consciousness of oneself is one thing, moral character another. The unification of the various stages in the life of an individual in such a way that he always feels himself to be the same is so little the work of a plan developed by that life that there exist individuals whose plans are both incoherent and unstable, but who, nevertheless, do not lose consciousness of their own personal identity. From the ethical point of view it is absolutely impossible to co-ordinate their actions in such a way as to make them appear the expression of a single design, yet we continue to look

upon them as persons who are conscious of themselves; and *vice versa*, different individuals may at times work towards the same end without on that account feeling themselves to be a single *Ego*. It is no argument to say that each of them works towards it in an original way, since, even in the case of a single individual, the various moments of life, though directed towards one and the same end, each give expression thereto in a different way at each period of time, so that each individual would need to be split up into as many persons as there are moments in his life. If the personality be determined by the original method of expressing a meaning, then each single act of will, seeing that it cannot be reduced to the rest, would constitute a mind. In short, from Royce's standpoint it is impossible to distinguish between the processes of consciousness of a single individual and those of different individuals; at bottom the ultimate meaning is the same in them all while the expression of it is singular in each. Experience, on the contrary, shows us that the psychic facts of one consciousness are related in a peculiar way, differing entirely from the extrinsic relation which may connect the phenomena of different consciousnesses. Metaphysical speculation should explain and not destroy the indubitable data of our experience. If we hold these data firmly, what would take place in the Absolute Personality were it obliged to live our conscious life in itself just as we ourselves live it? Each of us, even in the Divine Consciousness, would feel himself to be impenetrable to the rest, hence the various simultaneous and successive moments of that supreme spiritual life would continue to be extraneous to one another, and God would feel Himself to be a different person from one moment to the next. He might indeed regard our single lives as different expressions of one and the same meaning, but this would not cause Him to feel Himself to be the same in each one of us; He would be in the same position, as far as His self-consciousness is concerned, as a victim of hysteria in whom duplication of personality has taken place. If,

on the other hand, we admit that the various self-consciousnesses of single individuals lose this characteristic of incommunicability in the Absolute Mind, we contradict Royce's thesis from another side, since in that case it ceases to be true that our experience remains unchanged in God. Moreover, on this last hypothesis we should cease to feel ourselves to be distinct persons. In short, the inclusion of human consciousness in the divine consciousness only becomes possible if one or the other lose the characteristic of personality—self-consciousness. If we would preserve this characteristic in both of them, we must give up the absolute immanence of the will of man in the Will of God. Every personality in its self-consciousness is in its very nature transcendent with regard to other persons. Seeing then that my *Ego* must be distinguished from other *Ego*'s, and from the Absolute *Ego*, that which in the act of cognition appears to me beyond the bounds of my will may certainly be regarded as the expression of a will, but this will, far from being identifiable with my own will, is a transcendent object as far as I am concerned. That which is experienced by other individuals in the intimacy of their consciousness is a real life transcending the sphere of my will: who would venture seriously to assert that other minds exist only in so far as they express the internal meaning of an idea of mine, and embody a plan of my devising? Reduce all reality to conscious wills, and it will be no less true that each consciousness is a transcendent object with regard to every other consciousness. Is the existence of other *Ego*'s dependent upon me? Assuredly not! Then if I think this existence, may it not be an external meaning with respect to my idea? Are my friends, and those who are dearest to me, real only in so far as they satisfy *my* desire? Or do they rather exist in themselves in the intimacy of their consciousness, an intimacy which I cannot directly penetrate? Does *this existence of theirs in themselves* differ from my thought which takes it as its object or not? The idealist, finding

himself in such a strait, takes refuge in the Universal Mind, a subterfuge which avails him nothing, since, even if the immanence in God of my thought and of the other consciousness thought by me be granted, my thought will still remain something distinct from the person whom I think. For instance, I conceive the reality of the individual named Royce, and even supposing my concept, and the consciousness of Royce to form part of one and the same spiritual life, my idea with its internal meaning on the one hand and the subject Royce on the other will nevertheless remain two distinct things which cannot be fused into one unless the consciousness of the great American philosopher be annihilated.

But if being be thus placed outside the idea, will not an insuperable dualism be the result? No; because we say that the two terms must be kept distinct, but do not assert that one is entirely independent of the other, so that reality would remain unchanged, even if thought be eliminated. Consciousness is not indifferent to being; the bond between reality and thought is not a mere accident which might actually be dispensed with, but an essential relation. Nature and the human mind are moments of one and the same evolutionary process, a process which is the actualisation of an ideal plan, and their organic unity affords sufficient justification of the cognitive relation. Such a teleological unity undoubtedly implies an Absolute Consciousness which thinks it, but does not require that the two terms and their relation shall exist only in that Eternal Thought. The physical world and the human mind are objects of the Divine Mind, but do not exist merely as His ideas. This distinction between finite beings in their spontaneous activity, and the Absolute *Ego*, whilst it is the only way of accounting for the possibility of error, does not isolate them in such a way as to render their relations inconceivable, and, more especially, preserves in every stage of their immanent development that ideal meaning which is the profound reason of their

existence. Each centre of spontaneous life bears within itself the stamp of the common origin, and the close tie uniting it to others in the solidarity of the eternal plan which is carried into effect throughout the infinite vicissitudes of history. Nature, as a necessary moment of this process which embodies an ideal plan in its objectivity, though the result of a combination of actions which are independent of our *Ego*, is nevertheless not extraneous to those ends which constitute the deepest essence of our mind. Thus when we invest it with the forms of consciousness and interpret it in terms of our thought, we do not falsify it, but rather express its true meaning.

15. *Ward on the Realm of Nature and the Realm of Ends.*—Nothing but a spiritualistic view of the world can, without encountering the difficulty of absolute idealism, afford an intelligible explanation of the unity of nature and thought, and the universal teleology of the *ought to be*, which the philosophy of values regards as controlling the evolutionary movement of experience. If the universe be not a brute mechanism, but the realm of ends and of history, the outcome of the interweaving of spontaneous individual activities whose goal is the actualisation of the ethical order, only a theistic conception will enable us to comprehend it. The logical completion of the philosophy of values can only be found in a form of spiritualism, and to James Ward belongs the credit of having frankly recognised this fact. Ward, who in his Gifford Lectures waged a glorious warfare against agnostic naturalism, sees, like Royce, Münsterberg, and Rickert, in the historical and concrete aspect of the world its true reality as opposed to the abstract, mechanical fictions of science. The cognitive attitude which endeavours to describe the world as it exists independently of our subjective activity, presents us merely with an abstract fragment of reality, because it neglects its ties with the subject, whereas in the practical attitude, in which things are conceived as objects of attraction and repulsion, as means and ends of our

spontaneous activity, we have the fulness of reality without any abstraction whatsoever, and the whole of experience in its double aspect, both subjective and objective.[139] Whereas the cognitive attitude entirely fails to take into account the subjective factor of experience, which is therefore subjected to a process of mutilation, the attitude of valuation will not allow us to neglect the object, since our life really consists in an exchange of actions therewith. Science, as has been proved by the criticisms of Mach, Boltzmann, Kirchhoff, and Pearson, gives us but a symbolic description of phenomena, and symbols are abstract by their very nature ; physical science with its mechanical schemes, no less than arithmetic and geometry, "is incompetent to furnish a concrete presentment of a real and living world. Its essentially formal character has become increasingly evident with every improvement in its methods." [140] We are now very far removed from the time in which our psychic experiences were thought to be a vain appearance of the movements of material masses, which were regarded as absolute reality : the terms are now inverted. " What we see and feel, the facts of perception, become the real phenomena. Instead of the states of consciousness supervening upon certain motions of mass-points or some peculiar complex of ethereal vortices, these motions, etc., prove to be but ideal conceptions superimposed upon phenomena by the mind, that seeks to connect them in respect of their quantitative relations." [141] History, not science, can show us reality in its concreteness ; this reality is not an eternal repetition of necessary motions, but rather " the intercourse, the co-operation " of individual subjects which tend freely and spontaneously to their preservation and their perfecting.[142] All beings are animate, though in different degrees : in proportion to the growth of our knowledge will be the disappearance of the artificial barrier between the psychic and the physical, mind and nature, the world of liberty which gives rise to actions which cannot be foreseen and the world of necessity which admits of

mathematical calculation. He who contemplates the world in its historical aspect regards the constant uniformity of physical laws merely as the result of an average in which variations in opposite senses have been eliminated by summation. May not the same thing take place in the case of free human actions, which are sometimes disguised by the apparent uniformity of statistic averages ? [143] The contingency which Ward recognises in the world is not "that of chance, but that of freedom," which consists in conformity to what ought to be. Absolute conformity—the complete actualisation of the realm of ends—is an ideal for us; but little by little in the course of evolution we shall see the harmony of free co-operation, together with the elimination of conflicts, being realised more and more nearly.[144] This order and harmony lead us perforce to the theistic hypothesis, which alone can make us understand how it is possible for the many to co-ordinate their actions in systematic unity. Ward rejects not only the dualism of matter and mind, but also that of subject and object: there is but one living experience: that which we call objective reality is merely experience itself in that part thereof which is common to all subjects. The separation of subject and object is due to the exchange of action amongst different subjects by means of that process which Avenarius termed "introjection." [145] In like manner there exists no absolute distinction between sensibility and reason, since sense, no less than understanding, gives us something objectively real, and the universal world of reason is at bottom but the individual world itself in so far as *it is brought into harmony* with the contents of experience of other conscious beings living in society. Ward therefore looks upon knowledge not as a passive reflection of an external reality by means of a special faculty called intellect, but rather as an active expansion of experience into a larger sphere of life by means of that tendency to preserve and perfect which lies at the root of all being. Theoretical activity is in ultimate analysis but a form of practical life: there is

no cognition the impulse to which will not be found in determinate needs, and the whole scientific construction is at bottom but a means for the extension of the sphere of our power. Ward regards *conation*, not *cognition*, as "the central feature of experience." [146] Thus he conceives everything as subordinate to human ends, and as bearing the imprint of the activity of our mind and our subjective selection, and ultimate reality too as formed in the likeness of our intimate experience. Is this anthropomorphism? Ward replies with truth that "in a sense we are always anthropomorphic," since we can never divest ourselves of our consciousness; hence not only spiritualistic intuition but the very mechanical interpretation of the universe, which in the last analysis derives its concepts from our human experience, is of an anthropomorphic nature.[147]

Whilst we fully agree with Ward in his spiritualistic conclusions, we are not equally ready to accept the concept of the value of science and of the theoretical function in general in its relations to practical life which he derives from empirio-criticism, and which we have already discussed at length. On the other hand, we fail to see how his idealistic theory of knowledge, which would annihilate the dualism of subject and object, of internal and external experience, can be reconciled with pluralism. If other beings do not exist merely in so far as I think them, but exist also in themselves in their spiritual intimacy, my experience of them from without is of a different order from the experience which each one of them enjoys of itself from within. When I think another individual, his being as present to himself and the idea of him formed by me constitute an irreducible duality. The existence of anything outside thought may be denied, but in that case we must also deny the truth of pluralism which regards each individual as existing outside the consciousness of other individuals. The plurality of subjects and their existence even outside the Divine Mind, which must be granted in the theistic hypothesis, leads perforce to realism.[148]

NOTES TO CHAPTER III

[1] In the Preface to the second edition of his book, *Der Gegenstand der Erkenntnis* (Tübingen und Leipzig, 1904, p. vi.), Rickert insists upon the epistemological character of his method: ". . . Meine Schrift will nur Erkenntnistheorie und nicht Psychologie oder Metaphysik geben, d.h. sie will das entwickeln, was auch für den Psychologen und den Metaphysiker Voraussetzung ist und daher nicht gut Object psychologischer oder metaphysischer Untersuchungen sein kann."

[2] *Präludien* (Freiburg, 1884), p. 28 ff.

[3] *Op. cit.* p. 36 ff.

[4] Here the contrast with the teaching of pragmatism is obvious.

[5] *Op. cit.* p. 43.

[6] *Op. cit.* pp. 221-224.

[7] *Op. cit.* p. 227.

[8] *Op. cit.* p. 130.

[9] *Op. cit.* p. 251 ff.

[10] *Op. cit.* pp. 256-261.

[11] *Op. cit.* p. 210.

[12] *Op. cit.* p. 320 ff.

[13] *Der Gegenstand der Erkenntnis. Einführung in die Transzendentalphilosophie* (Tübingen und Leipzig, 1904, second edition, p. vi. The first edition was published in 1892).

[14] *Op. cit.* p. 12 ff.

[15] Rickert here shows the influence of Schuppe's philosophy of immanence: *Erkenntnistheoretische Logik* (Bern, 1878); *Grundriss der Erkenntnistheorie und Logik* (Berlin, 1898). Schuppe regards subject and object as two aspects of the only reality,—consciousness, external to which nothing exists. The *Ego* is not a substance, but merely consciousness of oneself. The different consciousnesses possess one part which differs from other consciousness and which constitutes their individuality, and one which is common to them all ("gattungsmässiges Ich"), which constitutes objective reality. To the latter belong abstract concepts, which are presented to us as elements of perception (*Grundriss*, p. 31 ff., p. 90 ff.).

[16] Rickert, *op. cit.* pp. 78-84.

[17] *Op. cit.* p. 97.

[18] "Erkennen ist anerkennen oder verwerfen. . . . In jeder Erkenntnis wird ein Wert anerkannt" (*op. cit.* pp. 108-110).

[19] *Op. cit.* p. 112. Note also here the difference between the philosophy of values and pragmatism.

[20] *Op. cit.* pp. 111-114.

[21] *Op. cit.* pp. 115-123.

[22] *Op. cit.* p. 124 ff.

[23] *Op. cit.* p. 141.

[24] *Op. cit.* p. 157.

[25] *Op. cit.* p. 165.

[26] *Op. cit.* p. 185.

[27] *Op. cit.* p. 199.

[28] *Op. cit.* p. 197 ff.

[29] *Op. cit.* p. 234.

[30] *Op. cit.* p. 240.

[31] *Die Grenzen der Naturwissenschaftlichen Begriffsbildung* (Tübingen und Leipzig, 1896–1903, p. 695 ff.).

[32] *Op. cit.* p. 697 ff.
[33] *Op. cit.* pp. 737-739.
[34] Windelband, *Präludien*, p. 75.
[35] Rickert, *op. cit.* p. 24.
[36] *Op. cit.* p. 34 ff.
[37] *Op. cit.* pp. 230-237.
[38] *Op. cit.* p. 244.
[39] *Op. cit.* p. 246.
[40] *Op. cit.* p. 247.
[41] *Op. cit.* p. 256.
[42] *Op. cit.* pp. 258-260.
[43] It should be noted that by nature Rickert does not mean merely the physical world, but in general that which recurs in the psychic world as well. Windelband had already replaced the old distinction between natural and moral sciences by that between the sciences of events ("Ereigniswissenschaften") and sciences of laws ("Gesetzeswissenschaften"), applying the term *idiographisch* to the method of the former, and *nomothetisch* to that of the latter (*Geschichte und Naturwissenschaft*, Strassburger Rektoratsrede, 1894). Xénopol too, in his *Les Principes fondamentaux de l'histoire* (Paris, 1899), makes a similar division, distinguishing the *faits de répétition* from the *faits de succession*. The first suggestion of a division of the kind occurs in Humboldt's *Cosmos*, but Cournot was the first to determine it clearly and to extend it to all the sciences (*Considération sur la marche des idées et des événements dans les temps modernes*, Paris, 1872, Preface, p. 4). Hermann Paul, who was not acquainted with Cournot's work, draws a distinction in his *Principien der Sprachgeschichte* (Halle, 1880) between the *Gesetzwissenschaft* and the *Geschichtewissenschaften*.
[44] *Op. cit.* pp. 412-414.
[45] *Op. cit.* p. 416 ff.
[46] *Op. cit.* p. 422.
[47] *Op. cit.* p. 428.
[48] *Op. cit.* p. 622.
[49] *Op. cit.* pp. 641-649.
[50] *Op. cit.* pp. 650-653.
[51] "Die empirische Wirklichkeit für uns absolut irrazional ist" (*op. cit.* p. 511).
[52] *Op. cit.* p. 680.
[53] *Op. cit.* p. 685.
[54] *Op. cit.* p. 687.
[55] *Op. cit.* vol. ii. p. 98.
[56] "Zwei Wege der Erkenntnistheorie" (from *Kantstudien*, vol. xiv. No. 2), published separately in Halle, 1909, p. 37.
[57] *Op. cit.* p. 57 ff.
[58] "Unter Sollen verstehen wir gerade das, was nicht ist oder nicht existiert" (*op. cit.* p. 16).
[59] *Philosophie der Werte*, *Grundzüge einer Weltanschauung* (Leipsig, 1908). Münsterberg had already sketched the first outlines of this conception in his earlier works: *Psychology and Life*, 1899; *Grundzüge der Psychologie*, vol. i. 1900; *Science and Idealism*, 1906.
[60] "Die Geschichte tritt wieder in ihr Recht. Der wollende Mensch wird zum Ausgangspunkt und der psychophysische Mechanismus verschwindet endlich aus der Metaphysik. Dem Positivismus folgt der Voluntarismus" (*Philosophie der Werte*, p. 36).
[61] *Op. cit.* pp. 29-38.
[62] *Op. cit.* p. 39 ff., pp. 449-481.

[63] "Das eine also wissen wir: am Anfang war die Tat" (p. 456); "Die Welt ist lebendige Tat" (p. 476).

[64] *Op. cit.* pp. 438-481. "Diese Bewertung der Ganzheit ist also ein Wert sonderer Art, ein metaphysischer Überzeugungswert, der vom logischen Zusammenhangswert grundsätzlich zu trennen ist und der aufs engste mit dem religiösen Glaubenswert zusammengehört" (*op. cit.* p. 444).

[65] *Op. cit.* p. 481.

[66] "Deshalb lassen sich auch die theoretischen Werte nicht aus einem überindividuellen Willen ableiten. Der Wille ist als etwas Feindes dem Sinn gegenüber immer Sekundär" (*Zwei Wege der Erkenntnistheorie,* p. 50).

[67] Münsterberg, *op. cit.* p. 53 ff.

[68] *Op. cit.* p. 57 ff.

[69] *Op. cit.* p. 60 ff., p. 461 ff.

[70] *Op. cit.* p. 74 ff.

[71] We shall, of course, confine our explanation to the theoretical values, and omit the others which are not comprised in our subject.

[72] *Op. cit.* pp. 85-89.

[73] *Op. cit.* pp. 94-97.

[74] "Der Wille trifft unmittelbar den Willen" (*op. cit.* p. 106).

[75] *Op. cit.* p. 110 ff.

[76] *Op. cit.* pp. 112-114.

[77] *Op. cit.* pp. 115-117.

[78] *Op. cit.* pp. 120-122.

[79] *Op. cit.* p. 131.

[80] *Op. cit.* pp. 134-139.

[81] *Op. cit.* p. 130.

[82] *Op. cit.* pp. 150-165.

[83] *Op. cit.* pp. 167-169.

[84] *Op. cit.* p. 170.

[85] *Op. cit.* pp. 174-181.

[86] Our exposition of Royce's thought is based upon his chief work, *The World and the Individual* (New York, 1901).

[87] ". . . your intelligent ideas of things never consist of mere images of the things, but always involve a consciousness of how you propose to act towards the things of which you have ideas" (*The World and the Individual*, vol. i. p. 22). "Ideas are like tools. They are there for an end" (*ibid.* p. 308). Royce quotes Stout's *Analytic Psychology* (vol. ii. pp. 114, 124), in which ideas are regarded as "plans of action."

[88] "When I know, I am acting. My theoretical life is also practical" (*op. cit.* p. 27).

[89] *Op. cit.* p. 434 ff.

[90] *Op. cit.* p. 153.

[91] "The conscious expression of an interest, of a desire, of a volition" (*op. cit.* p. 41). Royce himself, in a communication made by him to the third International Congress of Philosophy, "The Problem of Truth in the Light of recent Research," has termed his system "absolute pragmatism."

[92] As early as 1881, in an article published in the *Journal of Speculative Philosophy,* entitled "*Kant's Relation to Modern Philosophical Progress,*" Royce insisted upon the active character and ethical meaning of the recognition of facts, to which Rickert and Münsterberg, acting independently of Royce, called attention later. Thus, before reading the works of Rickert and Münsterberg, he arrived at the distinction between

the world of valuation and the world of description, which he compares for the first time in his book, *Spirit of Modern Philosophy* (Boston, 1892). He acknowledges, however, the influence exercised by the works of these writers upon the final form assumed by his system (*op. cit.* vol. ii. Preface, p. 6 ff.). These coincidences are not, however, purely accidental, but are due to a common source: Windelband, with whom Royce was undoubtedly acquainted, and more remotely to Lotze.

[93] *Op. cit.* vol. ii. pp. 32-41.

[94] "And the theoretical Ought of our judgments about facts, like the practical Ought of ethics, is after all only definable in terms of what Kant called the Autonomy of the Will" (*op. cit.* vol. ii. p. 32).

[95] *Op. cit.* vol. ii. p. 24.

[96] *Op. cit.* vol. i. p. 24 ff.

[97] *Op. cit.* vol. i. p. 62 ff.

[98] *Op. cit.* vol. i. p. 128.

[99] *Op. cit.* vol. i. p. 134 ff.

[100] *Op. cit.* vol. i. pp. 306-308.

[101] *Op. cit.* vol. i. pp. 318-319.

[102] *Op. cit.* vol. i. pp. 176-179.

[103] *Op. cit.* vol. i. p. 185 ff.

[104] *Op. cit.* vol. i. p. 204 ff.

[105] Royce, who is a diligent student of mathematical logic, and believes that metaphysic can profit largely by modern researches ("The metaphysic of the future will take fresh account of mathematical research," *op. cit.* vol. i. p. 527), applies it freely in his system. As regards mathematical procedure he follows the theory set forth by Peirce, according to which abstract reasoning consists in a process of experiment upon an artificial object constructed by the mathematician and observed by him in his own consciousness, just in the same way as one observes an external object, *i.e.* in that which Mach has termed "Gedankenexperiment" (*op. cit.* vol. i. p. 225).

[106] *Op. cit.* vol. i. p. 260 ff.

[107] *Op. cit.* vol. i. p. 280 ff.

[108] *Op. cit.* vol. i. p. 293 ff.

[109] *Op. cit.* vol. i. p. 297 ff.

[110] "What is, or what is real, is as such the complete embodiment in individual form and in final fulfilment of the internal meaning of finite ideas" (*op. cit.* vol. i. p. 339).

[111] *Op. cit.* vol. i. p. 346.

[112] *Op. cit.* vol. i. pp. 353-366.

[113] *Op. cit.* vol. i. p. 373.

[114] *Op. cit.* vol. i. p. 391-400.

[115] *Op. cit.* vol. ii. p. 302.

[116] *Op. cit.* vol. ii. pp. 408-410.

[117] *Op. cit.* vol. ii. p. 122 ff., p. 344.

[118] *Op. cit.* vol. i. pp. 581-584.

[119] Green, *Prolegomena to Ethics*, pp. 28-30.

[120] Royce, *op. cit.* vol. i. p. 419.

[121] *Op. cit.* vol. i. p. 327.

[122] *Op. cit.* vol. i. p. 389.

[123] *Op. cit.* vol. i. pp. 334 and 389.

[124] *Op. cit.* vol. ii. p. 31.

[125] *Op. cit.* vol. ii. p. 41 ff.

[126] *Op. cit.* vol. ii. p. 186.

[127] *Op. cit.* vol. ii. pp. 170-174.

[128] *The Spirit of Modern Philosophy*, Part ii. p. 398. Royce refers to Kirchhoff and Mach for this conception of science.
[129] *The World and the Individual*, vol. ii. p. 181 ff.
[130] *Op. cit.* vol. ii. p. 217 ff.
[131] *Op. cit.* vol. ii. p. 228 ff.
[132] *Op. cit.* vol. ii. p. 233.
[133] *Op. cit.* vol. i. p. 236.
[134] *Op. cit.* vol. ii. p. 238.
[135] *Op. cit.* vol. ii. pp. 276-286.
[136] *Op. cit.* vol. ii. p. 330.
[137] *Op. cit.* vol. ii. p. 408.
[138] *Op. cit.* vol. ii. p. 142.
[139] Ward, *The Realm of Ends, or Pluralism and Theism* (Cambridge, 1911), p. 430 ff.
[140] *Naturalism and Agnosticism* (London, 1899), vol. i. p. 82 *sq.*, p. 151 ff., p. 179 ff.
[141] *Op. cit.* vol. ii. p. 103.
[142] *The Realm of Ends*, p. 20 ff., p. 431 ff.
[143] *Ibid.* p. 5.
[144] *Op. cit.* p. 434.
[145] *Naturalism and Agnosticism*, vol. ii. p. 182 ff.
[146] *Op. cit.* vol. ii. p. 134 and p. 232 ff.
[147] *Op. cit.* vol. ii. p. 257.
[148] There are already signs in contemporary philosophy of a return to epistemological realism; it will suffice to quote Külpe's book, *Die Realisierung* (Leipzig, 1913), and the works of Meinong and Russell to which we shall refer later on. Of importance is the work entitled *The New Realism; Co-operative Studies in Philosophy*, by Edwin B. Holt, Walter T. Marvin, William Pepperell Montague, Ralph Barton Perry, Walter B. Pitkin, and Edward Gleason Spaulding (New York, The Macmillan Company, 1912).

PART II

THE NEW THEORIES OF MATHEMATICS AND PHYSICS

CHAPTER I

NON-EUCLIDEAN GEOMETRY [1]

1. *Traditional Geometry and the New Theories of Gauss, Lobatchewsky, and Bolyai.*—Until the beginning of the nineteenth century Euclid's geometry had seemed to be the perfect, unchangeable model of all scientific certainty: Cartesian rationalism, inspired by Kepler's words, *Ubi natura, ibi geometria*, had placed it at the foundation of all knowledge of things idealised in pure extension, and had finally in the teaching of Spinoza claimed to establish an *Ethica, more geometrico demonstrata*. The keen analyses of Berkeley and the bold criticism of Hume were alike powerless to shake this ancient faith, because in Kant's *Transcendental Aesthetic* the ideality of the pure intuition of space achieved a triumph over nominalism and empirical scepticism. Only at the beginning of last century did the construction of a different geometry begin to seem possible; and about fifty years later these new speculations formed the subject of lively discussions and polemics in the world of philosophy by reason of the consequences which were supposed to be deducible from them against the *a-priority* of space.

The failure of the many attempts to deduce the axiom of parallels directly from the other axioms [2] induced Gauss,[3] Lobatchewsky,[4] and Bolyai [5] to make use of a bolder method, a kind of *reductio ad absurdum*.[6] If the axiom of parallels could be logically derived from the others, by denying it while retaining the rest, one should arrive at contradictory results; but the three mathe-

maticians, while denying it, reached a geometry which was logically consequent, and drew the conclusion therefrom that this axiom was logically independent of the rest and essential only to the Euclidean system. Lobatchewsky substitutes the following proposition for Euclid's axiom: "In relation to a straight line, all the other straight lines in the same plane may be divided into two classes: those which intersect the given line, and those which do not intersect it: a line which forms the limit between these two classes is termed parallel to the given line; for each point external to the line there exist two parallels, which are symmetrical in relation to the perpendicular drawn from that point." From these premisses, and by means of the Euclidean synthetical method, he deduces a series of propositions, of which the following is the most important: "The sum of the internal angles of a triangle is either always less than, or always equal to, two right angles: in the latter case the whole system becomes Euclidean."

2. *The Empiricism of Riemann and Helmholtz, and the Dispute with the Neo-Kantians.*—The new geometrical speculations failed at first to excite philosophic interest, but after Riemann and Helmholtz had turned to them for proof of Stuart Mill's empiricism, the dispute began between the meta-geometricians on the one hand and the neo-Kantians on the other. Riemann [7] subordinates the concept of space to the more general concept of magnitudes having manifold extension (*der allgemeine Begriff mehrfach ausgedehnter Grössen*): in order to determine how many kinds of space are logically possible, we must first find out in how many ways magnitude can have manifold extension. When the number of spatial varieties which are conceptually possible has thus been fixed, it is the work of experience to determine which of them is represented by our space, that is to say, by the space in which the world with which we are acquainted is situated. After having determined the general concept of space as a continuous manifold of points, each

of which is dependent upon algebraic values, and having posited as its fundamental property constant curvature [8] which is a necessary condition of the free mobility of figures, Riemann reduces the number of dimensions to three, and proves that in this species of space three varieties are possible, according as the value of the constant of curvature is positive, negative, or null, *i.e.* spherical, pseudo-spherical, and flat or homaloidal space.[9] The last named corresponds to Euclidean space; the second to that of Lobatchewsky; the first is a new variety introduced by Riemann. In spherical three-dimensional space no line passes through a point external to a straight line which does not meet this latter; two points do not always, as in Euclidean geometry, determine one straight line only, and the sum of the angles of a triangle is greater than two right angles. This geometry is termed spherical, because for the case of two dimensions it is identical with the geometry of the surface of the sphere, in which the arc of maximum circumference plays the part of a straight line.

Helmholtz [10] arrives at the same conclusions, but follows a different procedure in the transition from the manifold of n dimensions to Euclidean space, deducing from certain hypotheses on the movement of bodies the analytic function of the space-constant, which Riemann takes as his starting-point. Of n-dimensional manifolds he takes into consideration those containing non-deformable and mobile systems; he further imagines the movement of a perfectly free solid and restricts the concept of the spatial manifold by the condition that if $n+1$ points of a system be fixed, it must return to the initial position. Of the spaces thus defined he then determines Euclidean space, assuming the number of the dimensions to be equal to three, and admitting the dimensions of a point to be capable of indefinite increase.

Riemann and Helmholtz agree in thinking that the possibility of conceiving other systems of geometry proves the empirical origin of axioms. Riemann thinks

it possible that these axioms are not valid in the case of the infinitely small which does not admit of observation; as a matter of fact the empirical concepts upon which spatial measures are based—*i.e.* the concepts of the rigid body and the luminous ray—do lose their validity in the infinitely small; it is therefore perfectly conceivable that the relations of spatial magnitude cease to correspond therein to the presuppositions of geometry; hence we must grant this hypothesis, which enables us to give a simpler explanation of phenomena. Helmholtz also endeavoured to prove that all the axioms are empiric, basing his conclusions on meta-geometry as well as upon the results of psycho-physiological research. To the objections of Land, Krause, Becker, and others who defended Kant's position, affirming that even if it be possible to conceive of other spaces, there can be no intuition but that of Euclidean space, Helmholtz replied [11] that the representation of non-Euclidean spaces is difficult because we are not accustomed to it, but not actually impossible, and that Kant's arguments, even were they formally valid, would not prove the *a-priority* of Euclidean space in particular, but merely that of space in general, which embraces both Euclidean and non-Euclidean spaces. The *a-priority*, in short, does not imply a preference for certain spatial relations over others, but is independent of the relations affirmed in the axioms of geometry which constitute the matter of the idea of space and can therefore only be justified by experience. In order to prove the possibility of rendering non-Euclidean space intuitive, Helmholtz makes use of the work of Beltrami,[12] who had endeavoured to impart an intelligible Euclidean sense to Lobatchewsky's plane geometry, showing all the propositions established by this mathematician in plane geometry to be valid in ordinary Euclidean space on the surfaces of constant negative curvature, which Beltrami terms pseudo-spherical. The dispute between Helmholtz and the pure neo-Kantians turned more especially on the possibility or impossibility of repre-

senting such a surface intuitively. Helmholtz, in order to prove his thesis, defines imaginability as the ability to form a complete representation of the sensible impressions which the object makes upon us according to the known laws of our organs of sense;[13] and endeavours to enumerate, in conformity to this definition, the series of sensible impressions which the phenomena of pseudo-spherical space makes upon us.[14] An observer, having sight and a criterion of judgment like our own, would, on first entering the pseudo-sphere, continue to see luminous rays or lines of vision as straight lines, just as in plane space, and as they are in reality in the spherical representation of pseudo-spherical space. The visual image of the objects of the pseudo-sphere will then make the same impression upon him as if he were in the centre of Beltrami's representative sphere. More remote objects will appear to surround him at a finite distance (about a hundred feet, for instance), but if he approaches them, he will see them expand before him—and that more in depth than superficies—and contract behind his back, and will realise the error of judgment committed by his eyes. If he have seen two straight lines which appear to him to run parallel for a distance of 100 feet, at which point the world comes to an end for him, he will recognise, as he approaches more closely, that this dilation causes objects to become more and more remote the farther he advances; behind him, on the contrary, their distances, which in the former position appeared to intersect one another at one identical point only, will appear in the new position behind him with the same intersection at 100 feet, and, approach as he may, he will never reach the point of intersection.

Land[15] objects that Helmholtz's description is couched in conceptual terms, and is therefore no proof of the possibility of intuiting the new space; moreover, as Stallo truly remarks,[16] it is not enough to ask ourselves what would be the nature of the impressions produced upon us by the objects of that fantastic world;

we must also ask whether they can be imagined together in the spatial order and the required form, in conformity to the known laws of the representative faculty. The perception, as conceived of by Beltrami, of points, lines, and surfaces of pseudo-spherical space in the interior of an ordinary spherical surface, whose points correspond to the infinitely remote points of pseudo-spherical space, does not enable us actually to represent that space, since it is one thing to intuit the projection and another to imagine the projected figure, which will only be present to our intuition if we have already known it in some other way. The projection of a solid on a plane does not in itself enable us to form a mental image of that three-dimensional figure; if we succeed in imagining it, we do so because we already possess intuitive knowledge of the solid in question. In like manner, a figure of plane space will never—let Klein [17] say what he will—enable us to imagine a figure of pseudo-spherical space projected upon it. The vocabulary [18] which brings the elements of the pseudo-sphere into correspondence with those of Euclid is of an artificial nature: nothing but a convention can assimilate Beltrami's geodesics to the straight lines of the plane; and his calculi take us very far from intuition into the realm of pure analysis, in which symbols and equations still keep the names of lines, surfaces, figures situated on the superficies, lengths of lines, displacements of figures, merely by virtue of their literal resemblance to the cases which directly recall these intuitive forms.[19] The geometrician may be driven to reduce the entities of his geometry to certain analytical properties which can be expressed in algebraic language, assuming as their definition the fact of correspondence to certain equations; but he thus renounces the intuitive form of the figures, whose equations merely express certain abstract relations; if these relations be transformed, he neither can nor may assert that his formulas still correspond to intuitive representations.

3. *Intuition and Concept in Geometry.*—The dispute

between the empiricists and the neo-Kantians as to the possibility of representing non-Euclidean space intuitively took its rise in the prejudice that mathematical space and figures can be the object of immediate intuition. It is obvious that there thus arises a confusion between the spatial forms of bodies, as perceived and represented in their concreteness, and the ideal types conceived by the mathematician. No geometrical space or figure, be it Euclidean or non-Euclidean, is representable in the strict sense of the word. The triangle on which the mathematician reasons is not the triangle which is drawn on the paper or mentally imagined, but the triangular entity which he has defined, that is to say, the concept, not the intuition of the triangle. The same may be said of space in general: perfect continuity, unlimited divisibility, absolute homogeneity, the number of dimensions, the degree of curvature, etc., are conceptual determinations which may be formed on the occasion of an experience or the image of one, but were not given therein as such. When Helmholtz affirms that congruence, that is to say, the possibility of displacing a geometrical figure without deformation, is drawn from experience of mechanical rigidity, he fails to perceive that he is moving in a vicious circle, since, in order to prove that a body has not changed in form, we must have recourse to measurement, which presupposes the homogeneity of space. But, if experience fail to give us the figures of mathematical space, may not these figures be revealed to pure intuition? Investigation, however accurate it may be, of the mental functions does not afford proof of the existence of any such mysterious faculty. The representation is subject to the same limitations and the same inaccuracies as the sensorial perception; thought alone with its concepts can complete and perfect it in such a way as to make it the starting-point of rigorous deductions. Intuition, as such, can merely give us something individual and contingent: the represented circle always remains this or that particular circle, and our

observations, or, if you will, our mental experiments, on it will be valueless for all other cases, unless there be behind the intuition the thought that all which we deduce is derived, not from the properties of the single figure which is present to us, but from the characteristics which it possesses in common with others of the same kind. Now a figure considered in the abstract as the type of a class is no longer an intuition, but a concept. The existence of geometry is in itself the inconfutable proof of the falsity of nominalism and empiricism. It is not true, as Stuart Mill asserts, that figures existing in the thought of the mathematician are mere copies of those given to us in experience, and are hence imperfect: unless the ideal type of the figure were present to our mind, how could we judge of the inaccuracy of the sensorial data? [20] It is true that this type *cannot be represented*; this does not, however, imply that it cannot be *thought*: the mistake made by Stuart Mill, and earlier by Berkeley and Hume, lies in the identification of thought and representation.

4. *Tannery on the Contingency of Geometrical Truths.* —If, then, we cannot speak of pure *a priori* intuition, because the exactness of mathematical concepts passes the bounds of imagination, is there nothing of an *a priori* nature about space and mathematics? Is the possibility of constructing different systems of geometry an argument against this *a-priority*? Must we not rather modify the conception of the *a priori*, and define its limits more exactly? Tannery has chosen the former alternative, affirming that geometry contains nothing necessary: Riemann's analyses and later studies have proved that the mathematical concept of space is formed by associating different notions, which are absolutely distinct from one another (magnitude, continuity, dimension, triplicity, measure, identity of the unit of measurement in different dimensions, distance, analytical law relative to the distance of two points, etc.), and that there is nothing subjectively necessary in the association of these notions. Tannery considers

that there is no doubt that the laws of our thought are at work in the construction of these concepts, but they do not help to determine in a necessary way their associative links, which may be inverted, ordered and connected in every possible way, without going beyond the bounds of logic. Every proposition on space is then subjectively contingent, and in no way different in this respect from the other propositions which formulate the laws of external phenomena.[21]

5. *The General Geometry of Calinon and Lechalas.*— Others, while recognising the impossibility of defending Kant's position, hold that though the geometry of Euclidean space may not be *a priori*, the *a-priority* of a more general geometry should be granted.[22] This thesis, which was, as we have seen, propounded by Helmholtz, who had confined the characteristic of *a-priority* to this more general geometry, only leaving to experience the task of selecting the series of axioms which are valid for our space, has been taken up again and developed by Calinon,[23] who has elaborated the essential definitions capable of serving as the basis of a geometry of three-dimensional spaces, of which Euclidean geometry would be but a particular instance. Taking these definitions as a starting-point, we thus create by a series of logically deduced theorems a general geometry depending upon a parameter whose value must be fixed by experience, not by thought. This geometry is a purely rational construction, devoid of postulates, based upon *a priori* definitions and wholly created by thought: experience only intervenes later to act as a practical guide in the choice of the parameter best suited to transcribe the phenomena of our experience. Whereas Helmholtz over-depreciates the formal task of the mind, reducing it to a vague notion devoid of any particular space-relation, and thus departing very far from Kantian idealism, Calinon, while divesting Euclidean geometry of its *a priori* character, leaves the mind not only the power of forming the concept of a space without content, but also that of creating a complete geometry

capable of indefinite development. At first sight it would appear that in this construction the fundamental elements of our geometry are derived from rational notions posited from the first with a series of necessary restrictions; in reality, however, they are not deduced but are the outcome of the habitual elements of our intuition. Calinon presents three-dimensional spaces as a generalisation of the surfaces of ordinary geometry, and derives all his definitions without exception by generalisation or simply by analogy from the properties of the plane, the straight line, and the surface. The concrete materials which have guided thought in the construction of definitions are always present. "There are surfaces," says Calinon, "such as the plane and the sphere, in which the figures can be moved without changing their form; we will call these surfaces identical with themselves; in like manner we will give the name of spaces identical with themselves to those spaces in which figures can be displaced without deformation, as in Euclidean space." Calinon endeavours to prove that general geometry, as defined by him, is well defined and presents the maximum of generalisation possible to the mind of man without leaving the realm of intuition: he does not admit geometries having more than three dimensions, because they take us entirely away from intuition, and can be reduced to the mere analytical aspect of formulas to which no geometrical meaning can be attached: since we have no idea of a figure having more than three dimensions, there is not only correspondence, but absolute identity, between this figure and the equation representing it. This statement does not, however, hold good of the various three-dimensional spaces of general geometry, because these spaces are either but little removed from Euclidean space, or, if the difference from it be considerable, they can always be resolved into infinitely small elements, differing very little from Euclidean elements, so that, though we may not conceive a space of this kind as a whole, we can readily conceive each of its elements.[24]

Milhaud [25] rightly remarks that this intermediate position cannot be accepted; we can only assume two attitudes with regard to intuition: we must either accept it in all its exigencies, or dispense with it altogether. Calinon, like Helmholtz, aims at the ideal creation of an intuition differing from our own, made up of elements taken from Euclidean space; but this artificial construction has nothing to do with true intuition, and cannot endow analytical formulas with geometrical meaning. We may make a figure of Euclidean space correspond to a certain formula, but such a proceeding is a mere arbitrary convention, which might be equally well applied to a space having more than three dimensions, in contradiction of Calinon's assertion. Renouvier and Broglie have also drawn attention to the fact that even general geometry cannot dispense with a number of postulates in its primary propositions, and Lechalas,[26] in order to defend Calinon's thesis against this objection, has proved a series of theorems on identical surfaces and their geodesics without enunciating any postulates; but he constantly has recourse to intuitive data, introducing, for instance, in the course of the deduction the notions of superficies, line, point, figure, displacement, etc., without saying what they are, and arguing thereon as if they had a logical meaning and the methods of association were legitimate. Who can fail to see that a whole series of postulates is thus taken for granted? Lechalas, on the contrary, asserts that general geometry can be based solely on the axioms of magnitude, which he regards as purely analytical, and that all its theorems can be deduced from them with the principle of contradiction as the only criterion.[27] These theorems would thus possess an apodictic value, although the space of our real world is contingent like every other phenomenon. General geometry, being devoid of postulates, has the universal and necessary value of analysis, of which it is an application; whereas Euclidean geometry, which is merely a branch of it, is based upon postulates of an essentially synthetic nature, designed to

define and specify the particular space to which they apply.

6. *Criticism of these Theories : the a priori Properties of Space.*—The fundamental error of the rationalistic construction of Calinon and Lechalas is the same as that lying at the root of Riemann's philosophy of space, *i.e.* the alleged possibility of deriving the determinations peculiar to space from pure relations of magnitude.[28] Were the *a priori* element reducible only to the analytical combination which can be deduced from the general axioms of magnitude, we should merely have an algebra, never a general geometry of universal and necessary *a priori* validity. Calinon and Lechalas apparently succeed in constructing their geometry by purely rational methods, because they derive from experience and spatial intuition the constituent elements of space, namely, those properties which distinguish it from other magnitudes and which make geometry a branch of science distinct from a pure algebraic calculus. If this content be eliminated from space, nothing is left of it, and it is an arbitrary proceeding to give the name of geometry to the science which results therefrom. The diversity of positions, which constitutes the qualitative content of space, and is irreducible to mere relations, is that which is empirical in it; but if this heterogeneity of spatial elements be *a posteriori*, space, as conceived by the mathematician, is *a priori* in the sense that it is a means of making experience intelligible. Homogeneity, continuity, and infinity are *a priori* characteristics of geometrical space, in as much as they represent that which thought has infused of its own into the manifold of positions, filling up the gaps left by experience, and leading it on to its ideal perfection in order to satisfy its demand for rationality. Everything in space which is intelligible is a pure construction of thought. Dimensions are not given, but are a product of the analysis made by thought of indistinct spatial experience in order to translate that experience into intelligible quantitative relations. The number of

dimensions consequently is also *a priori*. It is not true that it is only possible to determine that number by having recourse to experience; it is not, moreover, easy to understand how our hypothesis could be verified by experience when once a dimension ceases to be a datum, and is taken to be a complex of ideal relations constructed by thought. If by the *a priori* we are to understand the condition indispensable to the intelligibility of experience, it is obvious that we must attribute to geometrical space such a number of dimensions as is *necessary and sufficient* for the transcription of our spatial experience into intelligible terms. Now no one will deny that three dimensions answer perfectly to this requirement: one or two would not be enough to exhaust the whole content of our empirical space; four or more would be superfluous, since there are no spatial data which are not comprised within three-dimensional geometry. If we posit four spatial co-ordinates instead of three, the position of a point will be no better defined; four-dimensional space is not more extensive than that having but three, so as to comprise it within itself as the superficies does the line, but coincides with it, however widely the analytical description may differ. Three dimensions exhaust *the whole* of empirical space, nothing is left to be determined by another dimension; a fourth dimension may then be excluded *a priori*, since it does not render more intelligible any aspect of our experience, the only experience of which we can speak, unless we propose to write a scientific romance. We might apply an old aphorism and say: "*Dimensiones non sunt multiplicandae praeter necessitatem.*"

7. *Vain Attempt at an* a priori *Deduction of Three-dimensional Space: Cohen, Natorp.*—We do not mean hereby that the three dimensions of space can be deduced *a priori* from a pure rational exigency: if thought has constructed mathematical space, and has done so in that determinate form, it is due to the fact that experience demanded this construction in order that it might be transcribed in intelligible terms. If we do not take the

special characteristics of the empirical data into account, we shall not see why reason has been forced to construct the category of space with just that number of dimensions. The attempt made by Natorp,[29] following in the footsteps of Cohen,[30] to deduce space with all the other determinations of nature from the pure activity of thought is undoubtedly a failure. In ultimate analysis Natorp's proof does but show that without space and just this form of space we could never attain to the complete univocal determination of the object of thought. But is space necessary to the thought of a completely determined and individualised existence? Do we not conceive the unique reality of our consciousness, and of the consciousness of others in its concrete determination without being obliged to make use of spatial co-ordinates? The characteristic of reversibility, which differentiates spatial from temporal series, is so far from necessary to the requirement of complete individualisation that certain philosophers (Royce amongst others) who, regarding as the end of knowledge complete individuality exclusive of every other alternative (much as does Natorp), consider its principal characteristic to be the irreversible process of time, *i.e.* a series developing in a single order in a determinate sense. The possibility of inverting this order, the abstract homogeneity of space, which allows us to make the two opposite directions issuing from one and the same point coincide by rotating one of them in the plane round this point, would, according to these philosophers, remove us farther from univocal determination. Is not reality sufficiently individualised as a irreversible series of moments? Why add to the single direction of time the infinite multiplicity of directions contained in the concept of space? Do we thus gain in determination? I do not think so. Try as you may, you will never succeed in deriving from the demand for an ultimate determination, which leaves nothing undetermined, the necessity of an infinite number of directions issuing from a point, and of their connection in a continuous and homogeneous

system, which is the fulcrum of your proof of the three-dimensional nature of space. What Natorp regards as necessitating a new dimension is the logical exigency of the continuity of the transition from one direction to the opposite: thus the second dimension of the plane is necessary in order to allow of the continuous transition from one direction issuing from a point to the opposite direction; a third dimension is necessary in order to conceive the continuous rotation of the semiplane on one of its axes in such a way as to come to coincide with the opposite semiplane. We do not require more, because with three dimensions only the rotation of the plane can be inverted and the transition made continuously from one direction to the opposite. Hence Euclidean space is necessary and sufficient for the production of a homogeneous and continuous system of spatial determinations, such as is required by the concept of existence.[31] In this proof of his Natorp postulates two propositions: first, that one direction only developing in an irreversible order, as, for example, the series of instants of time, would suffice for the univocal determination of reality; second, that opposite directions are necessary, and the order of the elements of the series must always be reversible, because, were it not so, there would be a sudden saltus from one direction to the opposite one, whereas thought by the law of continuity exacts a gradual transition from one to the other, that is to say, a series of intermediate directions.[32] Now with regard to the first proposition, we note that from a logical point of view an existence completely determined in an irreversible series is very far from inconceivable; if our thought constructs an opposite direction, and other intermediate ones of all senses, it does so because the series of tactile, visual, and kinaesthetic sensations cannot be ordered in one sense only, and the data of sensible experience could not be exhausted, much less understood, with the temporal direction alone. Three-dimensional space remains unexplained, unless the exigencies peculiar to its data, together with their special

characteristics, be taken into account as well as the exigencies of thought. As to the second proposition, it may be observed that if the continuous transition from one order of elements to the opposite order were really necessary to thought, this exigency should be satisfied not only in the case of the opposite senses of the plane with reference to any one of its straight lines, but also in that of the opposite orders of three-dimensional space with reference to any place dividing it into two parts. In the case of space also it should be possible to conceive a continuous transition between the two opposed senses, a rotation leading by degrees to the inversion of the order of the elements with reference to the plane. Now it is well known that in three-dimensional space rotation of such a nature as to lead to the coincidence of symmetrical solid figures is not always possible. Must we conceive a fourth dimension, as has been argued by some philosophers? Natorp stops short at three; but in that case can opposite orders exist for thought without a continuous transition from one to the other? If this be so, why not stop short at the plane, or even at the straight line? The truth is that it is not the part of pure thought to exact three dimensions and three only for real existence, but that the epistemological justification of such a determinate number of dimensions must be sought in the fact that it is necessary and sufficient to exhaust and make intelligible the series of sensorial data with their characteristics which are immediately experienced by us.

8. *Impossibility of an Experimental Proof of Euclidean Geometry: Stallo, Poincaré, Couturat.*—The limitation of the number of dimensions to three thus appears to us justifiable *a priori* as the only adequate solution of the problem which spatial experience has set to thought. Can the same be said of the postulate of parallels? Most metageometricians are agreed in affirming that experience alone can decide between the Euclidean and non-Euclidean systems, a thesis which has been subjected to special criticism by Stallo, Poincaré, and Couturat.

Supposing, says Stallo,[33] an astronomer turns his telescope on to a star in order to determine its parallax, and that he finds this parallax to be of appreciably higher value than that of less distant stars; supposing, in other terms, the angle of intersection between the straight lines of his sight to differ from that required by the laws and facts known to astronomy and optics: to what conclusion will he come? We may safely say that no one would attempt to explain this anomaly of the parallax by a pseudo-sphericity of space; he would rather try to find the physical cause producing it, like Bradley, who, having discovered during the first half of the eighteenth century that the apparent displacement of the star γ of the constellation of the Dragon, due to the revolution of the earth, differed in direction and amount from that resulting from calculation, was led after many hypotheses to explain this anomaly by the aberration of light. Couturat [34] draws attention to the fact that in astronomical triangles the three vertices are not accessible, so that it is impossible to measure the three angles directly, and, if one of them be calculated by means of the others, the proposition is postulated which ought to be proved. How are we to make sure, then, that the sides of the triangle are straight lines? By means of the definition of the straight line, in virtue of which not more than one such line can pass through two given points; this is, however, an axiom which permits of exceptions in Riemann's spherical space, and still more in Klein's elliptic space, and which Russell regards as being of empirical value only. Those who maintain that Euclidean geometry can be verified by means of measurement implicitly assume that our space has a constant curvature, and that our measuring instruments therefore remain invariable in their displacement. Now Poincaré [35] has proved that hypothetical beings living in a non-isogeneous space would of necessity be led to conceive of it as isogeneous, because their measuring instruments would vary proportionately to the bodies to be measured

and to their own bodies, and the relation of measure would therefore remain constant. Lechalas tried to evade this objection [36] by considering rays issuing from the same centre and falling upon a rectilinear base at different angles: these rays, which will meet in a point in isogeneous space, will not do so in a non-isogeneous space, and this would afford an empirical criterion by which to verify the isogeneity of space. Couturat, however, does not regard this reply as decisive,[37] because Lechalas attributes the form of Euclidean straight lines to the second series of rays, forgetting that in Poincaré's non-isogeneous space the index of refraction is inversely proportionate at every point to the absolute temperature, *i.e.* to the linear dilatation at that point. The luminous rays will not then be rectilinear and can therefore converge until they meet in spite of the deformation of the base, since they in their turn would undergo a deformation. The isogeneity of space cannot be verified, because all measurement presupposes it, by postulating the constancy of the unit. The experimental verification of the Euclidean axioms thus revolves in a vicious circle. Moreover, as was already pointed out by Calinon,[38] we know only the tangents of the luminous rays emitted by the stars at the point where they strike our eye; it is therefore possible that the rays are not rectilinear, but that they are subject to two conditions: each of them must be determined by two points; all of those emanating from the same star must, no matter what their form may be, meet at a point, which is the real position of the star, as if they were rectilinear. Any geometry can be applied to the real world through the medium of a suitable hypothesis as to the path of luminous rays. Klein [39] gives utterance to a similar thought, regarding, as he does, all forms of space as equally compatible with our experience: by far the larger part of the universe is indeed only known to us by means of sight or perspective, that is to say, through its projective properties; hence it can be constructed indifferently in any one

of the metrical geometries which are projectively equivalent.

Poincaré [40] has subjected the alleged experimental proof of Euclidean geometry to further criticism. Experiences merely teach us the relations of bodies to each other ; neither of them either does or can tell us anything about the relations of bodies to space or about the mutual relations of the different parts of space. Russell affirms that dynamics would cease to be possible without Euclidean geometry ; now it is obvious that if we forsake Euclid's geometry for that of Lobatchewsky we should be forced to modify the enunciations of all the laws of dynamics, which could not exist without a language to express them, and which are modified along with this language, but this does not, however, imply that the verification of one of these laws proves the truth of this or that special expression. The human mind frames with the help of its own resources alone a system of signs or symbols suited to represent the system of relations which we term experience ; and, since signs are of their very nature conventional, it can construct an infinite number of equivalent systems, which can be translated into one another, just like different languages. Euclidean geometry is but one of countless others which we might equally well have chosen, and we give it the preference, not because it gives a more faithful reflection of objective reality and is imposed on us by experience, but because we regard it as a more convenient language than the rest.[41] Geometrical axioms are neither synthetic *a priori* judgments nor experimental facts, but conventions. The choice we make among all possible conventions is guided by experimental facts, but it is free and is limited by nothing but the necessity of avoiding all contradiction. Thus the postulates can be rigorously true, though the experimental laws which have determined their adoption are of a merely approximative nature. In other words, the axioms of geometry are but disguised definitions ; hence there is no sense in asking whether Euclidean geometry be true or false

Such a question amounts to the same thing as asking whether the metric system be true and the older measures false: one geometry cannot be possessed of a greater degree of truth than another; it can merely be more convenient, because it is simpler.[42] Euclidean and non-Euclidean geometry rest upon a common basis, *i.e.* a continuum of three dimensions, which is the same for all, and differs only as to the figures delineated therein, and as to the method of measuring them adopted.[43] Either of the two spaces may then issue from this amorphous continuum, just as either a straight line or a circle may be drawn upon a blank sheet of paper at will. We are acquainted in space with rectilinear triangles in which the sum of the angles is equal to two right angles, but we also know curvilinear triangles in which the sum of the angles is less than two right angles; if we call the sides of the first straight lines, we are adopting Euclidean geometry; if we apply that name to those of the second, our geometry is non-Euclidean. Neither experience, nor intuition, which may equally well represent the Euclidean and the non-Euclidean triangle, can decide the question; the Euclidean straight line is merely preferable to the rest, because it differs but little from certain natural objects from which the non-Euclidean straight line differs very considerably. In like manner we confine the number of dimensions to three for reasons of a practical order, which have worked for centuries in the psychological genesis of space in such a way as to constitute a mental habit. We do not know from experience that space has only three dimensions, it is merely more convenient to attribute three to it, because our sensations are thus more simply ordered. The number of dimensions is not then an *a priori* necessity, but is suggested by practical reasons: usually we place the muscular sensations imparted to us by movements of accommodation and convergence in the same series, because the members thereof correspond exactly to one another, so that they can be regarded as depending upon a single variable;

but were we to distinguish between the sensations of convergence and those of adjustment, we should have a four-dimensional space; if in touch we were to distinguish between the various series of sensations to which movements of different kinds give rise, we should construct a space having as many dimensions as there are muscles.[44] It is not beyond the bounds of possibility to conceive of physical phenomena in a space having more than three dimensions: Hertz's system of mechanics gives us an example. Hertz, in order to eliminate the idea of force, supposed visible material points to be subject to certain invisible bonds connecting them with other invisible points, and that which we term force to be the effect of these ties. Suppose a system to be formed of n material points either visible or invisible: we shall then have $3n$ co-ordinates in three-dimensional space; Hertz, however, thinks it would be more convenient to regard them as the co-ordinates of a single point in a space having $3n$ dimensions. The ties of which we have already spoken will force this point to remain on a superficies (having any number of dimensions smaller than $3n$), and in order to pass from one point to another on this surface it will always take the shortest way. The whole of mechanical science can thus be simply and elegantly summed up with the help of this single principle. Be the value of this hypothesis what it may, the fact that Hertz was able to conceive it as more convenient than the habitual hypothesis proves that three-dimensional space is not irresistibly imposed upon mechanics.[45]

Russell,[46] in reply to the criticisms of Couturat and Poincaré, has insisted upon the possibility of an experimental proof of Euclidean geometry. It is commonly stated that the inaccuracy of the measurement of astronomical angles leaves the question undecided; it is not, however, essential to our purpose that the line with which we experiment should be absolutely straight or that the bodies should be absolutely rigid; it will suffice to know the limit within which the lines deviate

from the straight and the bodies from absolute rigidity, the bounds within which the spatial constant must necessarily be found. Even if a formal demonstration be not possible, our proof will suffice from an empirical point of view. We are unable to determine accurately that our space is Euclidean, just as we are unable to give a rigorous proof of the law of gravitation ; on the other hand, however, no theoretical limit can be assigned to the degree of approximation of which such a proof is susceptible. In short, the Euclidean axioms are susceptible of empirical proof in the same sense as other scientific laws, that is to say, it can be shown that they constitute the simplest hypothesis for the explanation of facts, although it is possible to imagine other facts of which a slightly non-Euclidean space would afford a simpler explanation. The empirical nature of these axioms does not, however, imply that they may any moment become false or that the fourth dimension may be discovered in some other country or planet : be the nature of space what it may, we cannot admit that it is capable of varying with time. It is in the last analysis permissible to doubt whether space be three-dimensional, but only in so far as we throw doubt upon truths of fact and the data of the senses. Finally, the Euclidean axioms are not propositions relative to our sensibility, imposed by *a priori* intuition, but are as certain as any truth of fact can be.

9. *Euclidean Geometry more Rationally Complete than the Rest.*—Russell's thesis [47] does not appear to me wholly acceptable because it fails to explain how human thought came to ascribe a greater degree of evidence to the axioms of Euclid than to the laws derived from experience. Is this a mere illusion ? Or is it based upon a characteristic peculiar to the science of space? In my opinion the latter alternative should be chosen, and we must therefore determine wherein lies the distinctive character of geometrical truths. We have already seen that pure intuition cannot act as the basis of the Euclidean axioms, because no such faculty exists among the functions of consciousness. Strive as we

may to extend and correct our representation, we remain very far from the perfection and ideal limits of the mathematician: it is not possible to imagine an infinite straight line or two parallel lines in their geometrical precision. If we are unable to prove the axioms by means of perception, still less can we do so by means of imagination which is subject to the same limits of the threshold and the same errors as perception; because, whereas in external measurements exact instruments afford us the means of reducing the errors of sensorial data, and of transcending limits, and thus approximating to the ideal set up by mathematics, the psychophysiological structure of the organism sets insuperable bounds to our imagination of geometrical forms and dimensions. Thought alone with its concepts is capable not only of transcending the limits of subjective representation, but of being more accurate than any measurement made with objective instruments. The character of necessity and universality pertaining to geometrical concepts is derived from the ideal nature of those concepts. But, we might here observe, does not non-Euclidean geometry contradict Euclidean? Is it not then absurd to pretend that they are both equally necessary and universal? The contradiction is, in my opinion, only apparent, and arises from an abuse of terminology. Euclid's triangle is not the same figure as that of Lobatchewsky or Riemann, hence it is not surprising that that which can be proved of the one should not be applicable to the other. If you call that a straight line which has hitherto been called a curve, it is only natural that you should arrive at different conclusions; just as the anatomy of the dog would cease to be the same if by this name you understand another animal such as the horse. If by a straight line we understand that defined by Euclid and by a triangle the rectilinear triangle, the sum of the internal angles is equal to two right angles; this does not, however, prevent the sum of the angles of a triangle whose sides are not straight, but correspond to what Euclid

calls a curve, being greater or less than two right angles.

Euclidean geometry does not exclude these curvilinear triangles, but affirms with justice that they are not the only ones in existence, and that it is possible to conceive of others the sum of whose angles is equal to two right angles. Such an assumption is not only allowable but necessary, unless we would cause an arbitrary interruption of the continuity of the process of thought. Euclid's rectilinear triangle may be regarded as the limit of the series of the triangles of the so-called spherical spaces and of the series of triangles of the pseudo-sphere, in which the degree of curvature of the sides gradually decreases. There is no sense in asking whether this limit *really* exists, that is to say, whether it exists in empirical space: the essential thing is that it shall exist *ideally*, and that it shall be impossible to avoid conceiving it, if we would not interrupt the continuous transition from a positive to a negative degree of curvature. Little by little, as the degree of curvature decreases, the sum of the angles will approach more and more closely to two right angles and will be identical therewith in the passage to the limit. Though the various systems of geometry may reach different conclusions, they do not contradict one another, because their axioms and theorems refer to different figures. The concept of three-dimensional space in its infinite homogeneity remains unchanged in them all, the only things which undergo change are the figures drawn in space and depending upon the value of the spatial constant. The difference between the three geometries may be stated as follows: whereas Euclidean geometry does not exclude the existence of triangles other than the rectilinear triangle, but merely distinguishes them and calls them by another name, the two other geometries banish true rectilinear triangles and take nothing but curvilinear triangles into consideration. They are not false, but defective in so far as they are partial, they fail, that is to say, to exhaust all the figures and rela-

tions of figures which can be conceived in ideal space.[48] Euclidean geometry hence appears to us to be the most complete, in as much as it can transcribe the figures of the other geometries into terms of its own figures by means of a mere change of language; whereas the other two, excluding as they do from the first by means of an artificial hypothesis the possibility of certain figures, give us but a fragment of three-dimensional geometrical space. Thus if we are asked: Which of the three geometries is true? we have no hesitation in replying that all three are true, in as much as they refer to different figures; at the same time, from the epistemological point of view, we feel justified in stating that Euclidean geometry more nearly fulfils the demand for the complete intelligibility of space than the others do, since it alone is capable of exhausting the whole of its ideal existence.

Though figures and geometrical space exist only ideally in thought, they are not mere fictions or convenient signs upon which no judgment of truth or falsehood can be passed, as Poincaré asserts. Even with reference to applied geometry it is certain that if no figure fulfils perfectly the conditions laid down by mathematicians, an indefinite approximation to this ideal type is possible, and we observe that the more nearly these conditions are verified, the more perfectly do we encounter the properties which are of necessity connected therewith in our thought.[49] We should not find our geometry more convenient than any other were the space of experience not more naturally transcribed into certain conceptual terms than into others: experience is not absolutely indifferent, it is not capable of assuming any spatial order whatsoever, it rather suggests to thought the direction in which the process of idealisation should be accomplished; it shows us, for instance, that the more nearly the degree of curvature of the sides of the triangle approximates to zero, the more closely will the sum of the angles approximate to two right angles, so that, continuing conceptually the process which in experience

and intuition can be pursued only up to a certain point, we conclude by passing to the limit that in the rectilinear triangle the sum of the internal angles is equal to two right angles. This conclusion of ours is legitimate, because the empirical data are arranged in regular series, which point out to us the road we ought to take, and thought, once started on this road, arrives at the ideal limit by a law inherent in its very nature. The concept which results therefrom with all its properties is not a fiction depending upon our caprice, but rather the one and only and necessary conclusion at which it is possible to arrive, given those empirical or intuitive data, and given the nature of our thought; in this way alone is it possible to make the data of experience intelligible, by a transformation of empirical into ideal space. The possibility of arriving within certain limits at various systems of geometry is due to the fact that one and the same order of concepts can be translated into different languages, which rob science of none of its universal value, unless we would by an absurd nominalism identify the concept with the symbol expressing it. Is not the possibility of translating one system of signs into another, and non-Euclidean geometries into Euclidean terms and *vice versa*, as Poincaré himself maintains, a proof that equivalent symbols express one and the same concept? A dictionary is only possible when the different terms correspond to the same idea which is thought universally by all men, though they give expression to it in different ways. In like manner the system of analytical signs and their relations is capable of infinite variations, because it depends upon certain conventions; this does not, however, change the concept of geometrical space, which is universally recognised, because it is the only one which meets the demand for perfect intelligibility.

NOTES TO CHAPTER I

[1] A complete bibliography of the works on these new geometrical speculations has been published by Halstead ("Bibliography of Hyperspace

and non-Euclidean Geometry," *American Journal of Mathematics*, vol. i. p. 261 ff.; vol. ii. p. 65 ff.). For the history of non-Euclidean geometry see Klein, *Vorlesungen über nicht-euclidischen Geometrie* (Göttingen, 1893); Russell, *An Essay on the Foundations of Geometry* (Cambridge and London, 1897).

[2] For the history of these attempts, of which that of Legendre in 1833 was the last, see Stäckel and Engel, *Die Theorie der Parallellinien von Euclid bis auf Gauss* (Leipzig, Teubner, 1895).

[3] "Disquisitiones generales circa seriem infinitam," *Comm. recent. Soc. Gött.* 2 (1811–13); "Theoria residuorum biquadraticorum," *ibid.* 7 (1828–32).

[4] Lobatschewsky first published his results in Russian in the *Proceedings of the University of Kazan* (1829–30), but they remained unknown until he translated them into French: "Géométrie imaginaire," *Journal de Crelle* (1837), vol. xvii.

[5] His results first appeared as an appendix to a work of his father's: *Appendix, scientiam spatii absolute veram exhibens, a veritate aut falsitate axiomatis XI. Euclidei (a priori haud unquam decidenda) independentem; adiecta ad casum falsitatis quadratura circuli geometrica* (translated into French by Houël in 1868, republished by Hermann, Paris, 1896).

[6] The method had been suggested almost a century previously by the Italian Saccheri, whose work, *Euclides ab omni naevo vindicatus* (Mediolani, 1733), remained unknown till Beltrami discovered it.

[7] "Über die Hypothesen, welche der Geometrie zu Grunde liegen." This dissertation, which was read by Riemann before the faculty of philosophy of the University of Göttingen, was published after the death of the author in 1866 (*Abhand. d. Kgl. Gesellschaft der Wissenschaft zu Gottingen*, vol. xiii. p. 133 ff.).

[8] Note that the curvature of an n-dimensional manifold is a purely analytical expression having a merely symbolic affinity with ordinary curvature. Russell, in order to avoid misunderstanding, makes use of the term "space-constant."

[9] Klein calls these three species of space elliptic, parabolic, and hyperbolic ("Über die sogenannte nicht-euclidischen Geometrie," *Mathematische Annalen*, vol. iv. p. 577). Klein's elliptical space is not, however, absolutely identical with spherical space, since while in the latter the geodesic lines may have two points in common, they cannot have more than one point of intersection in elliptic space.

[10] "Über die Thatsachen, die der Geometrie zu Grunde liegen," *Nachrichten der Kgl. Gesellschaft der Wissenschaft zu Gottingen* (3 giugno, 1865).

[11] Cp. his polemic with Land in *Mind*, vols. ii. and iii.

[12] "Teoria fondamentale degli spazi di curvatura costante," *Annali di mathematica*, vol. ii. (1868–69); "Saggio d' interpretazione della geometria non-euclidea," *Giornale di matematica*, vol. vi. (1868).

[13] *Vorträge und Reden*, vol. ii. p. 237.

[14] "Les Axiomes géométriques," *Revue scientifique* (June 1877).

[15] *Mind*, vol. ii. p. 45.

[16] *La Matière et la physique moderne*, p. 191 ff.

[17] *Op. cit.* p. 573.

[18] Klein, Poincaré, and Lie have constructed analytical developments which contain the series of theorems of the three systems of geometry corresponding to the three different hypotheses. A special vocabulary has been formed allowing of the translation of the enunciations of non-Euclidean geometry into Euclidean terms.

[19] Milhaud, *Essai sur les conditions et les limites de la certitude logique*, p. 167.

[20] Pasch (*Neuere Geometrie*, Leipzig, 1882) endeavoured to construct a geometry by strictly empirical methods, maintaining the object of this science to be not types of ideal figures, but things themselves in their material form! Thus he regards a point as a corpuscle which cannot be further divided into parts within the limits of our observation! Even Pasch, however, is forced to abandon the empirical method when, for instance, he assumes the penetrability of solids, which is not a matter of experience, and the motion of them without deformation, as he does in the definition of the congruence of two figures. Of a more moderate order is the empiricism of Freycinet, *De l'expérience en géométrie* (Paris, 1903), who, while maintaining that geometrical axioms are experimental truths, resulting from observation of the outer world, yet introduces a process of purification and idealisation.

[21] *Revue philosophique*, vol. iii. p. 574.

[22] It should be remembered in this connection that Kant himself conceived the idea of a general geometry: "Eine Wissenschaft von allen diesen möglichen Raumsarten wäre unfehlbar die höchste Geometrie, die ein endlicher Verstand unternehmen könnte," *Gedanken von der wahren Schätzung der lebendigen Kräfte* (1747), p. 10.

[23] "Les Espaces géométriques," *Revue philosophique* (1889, 1891), 2; "Sur l'indétermination géométrique de l'univers," *ibid.* (1893), 2.

[24] "Les Espaces géométriques," *Revue philosophique*, vol. xxxii. p. 375.

[25] *Op. cit.* p. 192.

[26] *Introduction à la géométrie générale* (Paris, 1904); *Étude sur l'Espace et le temps*, 2nd ed. (Paris, 1909), p. 68 ff.

[27] Veronese must be credited with having recognised that in order to speak of a geometry with more dimensions it will not suffice to prove that it is possible to find an analytic expression for it, but that it is also necessary to show that we are dealing with real true geometrical beings. He therefore endeavours to construct spaces having more than three dimensions with the help of the ordinary intuitive elements. Given space of three dimensions and a point outside it, he constructs, *e.g.*, space of four dimensions, and the others analogously by the synthetic method. This, of course, presupposes the conceivability of a point external to our space, which is decidedly problematic, since, whatever efforts we make, the point thought of will always remain in three-dimensional space.

[28] The same error will be found in Erdmann (*Axiome der Geometrie*, 1877), who, like Riemann, claims to deduce space from the concept of magnitude. In order to define space as magnitude he starts from its two most obvious properties, continuity and the three dimensions; but, since these two properties are also, according to Erdmann, common to the manifolds of sounds and colours, to distinguish space from these manifolds it is necessary to add that in space, though not in sounds and colours, the dimensions are homogeneous and permutable. In the more general case the manifold is n times determined (n-bestimmt); in space it is n times extended (n-ausgedehnt).

[29] *System der Philosophie*, pt. i., "Logik der neuen Erkenntnis" (Berlin, 1902), p. 162 ff.

[30] *Die logischen Grundlagen der exakten Wissenschaften* (Leipzig, 1910), p. 303 ff.

[31] *Op. cit.* p. 306 ff.

[32] *Op. cit.* p. 326.

[33] *Op. cit.* pp. 179-181.

[34] *Revue de métaphysique et de morale* (May 1898), p. 371 ff.

[35] *Revue générale des sciences pures et appliqués* (January 30, 1892).

[36] *Études sur l'espace et le temps*, 2nd ed. pp. 181-197.

[37] *Revue de métaphysique et de morale*, vol. iv. (1896), p. 648.

[38] " L'Indétermination géométrique de l'univers," *Revue philosophique* (1893), vol. xxxvi. pp. 595-607.

[39] *Zur ersten Verteilung des Lobatchewsky Preises*, p. 20.

[40] " Des Fondaments de la géométrie à propos du livre de M. Russell," *Revue de mét. et de mor.*, p. 251 ff.; " Réponse à M. Russell," *ibid.* p. 79 (January 1900).

[41] *Science et hypothèse*, p. 65 ff.

[42] *Ibid.* p. 90 ff.

[43] *La Valeur de la science*, p. 61.

[44] *Science et hypothèse*, p. 72 ff.; *La Valeur de la science*, p. 103 ff.

[45] *La Valeur de la science*, p. 126 ff.

[46] " Les Axiomes propres à Euclide sont-ils empiriques ? " *Revue de mét. et de mor.* (November 1898), p. 760; " Sur les axiomes de la géométrie, *ibid.* (November 1899), p. 685 ff.

[47] Note that Russell is here speaking of applied geometry; in the next chapter we shall see what view he takes of pure geometry.

[48] The non-Euclidean systems of geometry exclude, for instance, the possibility of similar figures. Delbœuf regards this as the main argument against non-Euclidean geometry, since the impossibility of enlarging the figure without changing the form thereof appears to him to contradict the perfect homogeneity of space (" L'ancienne et les nouvelles géométries," *Revue philosophique*, vol. xxxvii. p. 380 ff.; cp. also *Megamicros, ou les effets sensibles d'une réduction proportionnelle des dimensions de l'univers* (Paris, Alcan, 1893).

[49] Poincaré's objection that it is impossible to separate in the experiment that which is due to physical actions from that which depends upon the special structure of space is valueless, since we can cause the spatial conditions to vary while keeping the physical conditions unchanged, and *vice versa*, thus effecting a relative separation between the effects of the one and those of the other.

CHAPTER II

THE NEW LOGICAL ELABORATION OF PURE MATHEMATICS

1. *Fusion of Logic with Mathematics towards the Middle of the Nineteenth Century.* — Until the middle of the nineteenth century mathematics and logic had remained apart in spite of the attempts made by Jungius and Leibnitz to bring them together,[1] but during the second half of the century mathematicians were seized by logical scruples unknown to their predecessors, and began to analyse all the methods of proof, to verify the concatenation of their theorems, to investigate the hypotheses and postulates which crept surreptitiously into their arguments, and to give prominence to the axioms or principles which were the starting-point of their deductions, and upon which their theories depended. The infinitesimal calculus, which had till then preserved something paradoxical and mysterious in its principles, found its strict basis in the theory of limits ; the theory of functions, over which notions of intuitive origin had long held sway, was refined and deepened ; geometry and mechanics, freed as far as possible from intuition, became hypothetical deductive systems, based upon a certain number of axioms and postulates, from which everything else is logically deduced. Mathematicians, in laying bare the foundations upon which their science rested, were thus led to form two new theories which were destined to serve as the basis of all others : the theory of assemblages and that of groups, that is to say, the science of multiplicity and that of order.

Moreover, the science of number and magnitude began to be regarded merely as a small division of a more far-reaching science based upon notions about which there was nothing quantitative, a view opposed to that of the school of Weierstrass, which claimed to reduce the content of analysis and of pure mathematics as a whole to the single idea of number, in its endeavour to "arithmeticise mathematics," as Klein expresses it.[2] Logic, on the other hand, strove to extend its domain, which had previously been limited to the relations of inclusion of concepts, and to discover new forms of deduction. Formal logic, borrowing the method and symbolism of algebra,[3] established itself under the two-fold form of the calculus of classes and the calculus of propositions, finding surprising analogies between these two branches, and enlarged its borders till it became a general logic of all relations[4]; since, moreover, the simplest and most elementary relations are to be found in mathematical theories, it was only natural that it should apply itself to analysing and verifying the concatenation of the propositions, and to proving mathematical axioms by reducing them to purely logical principles. Thus the gulf between logic and mathematics was bridged over: the calculus of classes presented itself as the most elementary part of the theory of assemblages, and the logic of relations as the indispensable basis of the theory of groups and the theory of functions. This fusion of logic and mathematics, which began during the past century in the bary-centric calculus of Möbius (1827), the calculus of equipollences of Bellavitis (1832), in Grassmann's theory of extension and Hamilton's method of quaternions, became an accomplished fact in the works of Peano, Pieri, Whitehead, and Russell.

2. *Peano's Logistic and its Application to Arithmetic and the Geometrical Calculus.*—Peano was not content merely to apply logical symbolism to arithmetic, which he formulated into a strict deductive system,[5] but also developed a geometrical calculus, in which he makes use of the notation contained in Grassmann's *Aus-*

dehnungslehre, a work which had hitherto remained almost unknown owing to the elevation and abstruseness of its concepts.[6] The geometrical calculus in general consists of a system of operations carried out upon geometrical entities, analogous to those of algebra on numbers, and enables us to express the results of geometrical constructions in formulas, to represent geometrical propositions by means of equations, and to substitute a transformation of equations for an argument. This calculus, while presenting certain analogies with analytical geometry, differs from it in that, whereas in analytical geometry calculations are made on the numbers which determine the geometrical entities, in this new science the calculations are made on the entities themselves. Pieri,[7] in a series of interesting articles, has contributed largely to the new logical elaboration of geometry, whose goal is the establishment of an accurate distinction between primitive and indefinable geometrical entities and those derived from them by definition, between the properties which must be postulated and propositions drawn therefrom by means of a pure deductive calculus.[8]

3. *Geometry as a Hypothetico-Deductive System: Pieri.*—Pieri considers that geometry in all its branches should be increasingly affirmed and consolidated as the study of a certain order of logical relations, shaking off little by little the somewhat feeble bonds between it and intuition, and assuming in consequence the form of an ideal science of a purely deductive and abstract order. From this point of view we are led to acknowledge with Pasch that if geometry is really to be a deductive science, its arguments must not depend upon the intuitive meaning of geometrical concepts, but must be based on those relations only which are imposed on those concepts by postulates and definitions. The primitive entities of any deductive system whatsoever (the geometrical point, for instance) must lend themselves to arbitrary interpretations within certain limits assigned by the primitive propositions, so that each

mathematician is at liberty to attach what meaning he pleases to the words and signs representing those entities, provided that this meaning be compatible with the general properties laid down by the postulates and definitions. Geometry thus becomes a hypothetical science whose object is not the space of experience, but merely an assemblage of abstract entities, created by our mind, upon which an act of our will imposes certain arbitrary properties expressed in the postulates. Nominalism thus achieves a victory over the old realistic conception of geometry.[9]

4. *Whitehead's Universal Algebra.*—Whitehead[10] in his universal algebra, which admits of very extensive application, has realised the idea of a "universal characteristic" conceived by Leibnitz. This universal algebra embraces not merely geometry and kinematics, but also mechanics and mathematical physics, because it lends itself to the representation of matter with its various qualities, and hence of all physical phenomena. The power and fruitfulness of the new algorithm are due to the fact that it shakes off the yoke of arithmetic, and freely defines new combinations having but a formal and partial analogy with arithmetical operations, instead of confining itself, as did analytical geometry, to discovering everywhere the properties of the addition and multiplication of numbers, and admits different rules of calculation suited to every application, instead of confining itself, as did classical algebra, to treating its symbols according to the laws of numerical calculation. This algorithm offers immense advantages in geometry, because, instead of introducing three parallel and simultaneous equations relative to the three axes, which repeat the same geometrical fact, relation, or construction, it expresses in a single formula the one fact, no longer dismembered into its three projections, but with its nature and physiognomy undeformed and without the intervention of extraneous quantitative concepts, thus delivering us from the arbitrary element contained in every system of co-ordinates. While in analytical

geometry the letters always represent numbers and indirectly the magnitudes measured by these numbers, in the calculus of extension the letters are a direct representation of geometrical elements (points, lines, planes), which not only have nothing in common with numbers, but are not magnitudes either (at least not primarily), and the formulas are a direct transcription of geometrical constructions. This allows of the application of the new algebra to the geometry of position and enables it to give us a faithful translation of projective constructions without having recourse to the notions of number and measure. The algebraic symbols, which represent the positions, have a co-efficient expressing the intensity of that point; and this intensive magnitude, from which geometry makes abstraction by assuming it to be always equal to 1, is of great use to mechanics and physics, since it makes it possible to treat by means of universal algebra those physical qualities of matter which are susceptible of various degrees of intensity. Thus mathematics is no longer the science of number and magnitude, but becomes the science of all the types of formal and deductive reasoning. That a certain assemblage of objects of thought should verify the principles of such an algebra will be the sufficient condition that it should also of necessity verify perforce all the propositions derived from them: in short, the starting-point alone is hypothetical, but once it has been verified, everything else follows of deductive necessity.[11]

5. *Russell on the Identity of Logic and Mathematics.*— This work of the logical systematisation of mathematics was completed in Russell's book,[12] the aim of which is to prove the fundamental identity of logic and mathematics, by showing all mathematical propositions to be based upon eight indefinable notions and twenty undemonstrable principles, which are also the primitive notions and principles of logic itself. Pure mathematics thus assumes an exclusively logical character, and its necessity becomes hypothetical, depends, that is to say, upon certain conditions; the theorem is true if

such and such an hypothesis be true. Russell [13] defines pure mathematics as the totality of propositions having the form, p implies q, in which p and q are propositions containing the same variables, and containing none but logical constants; that is to say, it is a class of formal implications which are independent of any and every content, hence the paradox: "Mathematics is a science in which we never know what we are speaking of, or whether what we say is true." [14] In applied mathematics, which endows these formal implications with a material content. the theses of the theorems become true only in the degree and measure in which their hypotheses are verified by these data. Geometry in so far as it refers to space and to the real world is the arithmetic of concrete numbers; that is to say, all its applications to the numeration of objects and the measuring of real magnitudes belong to applied mathematics.

In order to reconstruct pure mathematics Russell makes use of Peano's logistic; he begins by determining the laws of the calculus of propositions, of classes, and of relations, which constitute a logical basis able to support the whole edifice.[15] If we take these elements as our starting-point, we can define all the other notions and prove all the other propositions of pure mathematics. In this particular, Russell [16] modifies the theory of Peano's school,[17] which admits a special group of indefinables for every branch of mathematics; of the three forms of definition considered by Peano, nominal definition, definition by postulates, and definition by abstraction, he recognises the first only. An object x is nominally defined by an equivalence having the form $x = a$, in which a is an expression formed with the elements already known. It is merely the bestowal of a new name, an abbreviation, which is not theoretically necessary, however convenient and practically indispensable to the progress of science it may be. Every defined sign or word may therefore be suppressed provided that its value be substituted for it; should

such a substitution not be successful, the definition is not complete.[18] Such an explicit definition is not always possible; in such a case, if we would define a complex of notions, we must have recourse to an assemblage of relations verified by it; but this group of postulates only defines those entities to a certain degree, and may leave their value undetermined, like a system of equations with several unknowns. The definition by abstraction is also imperfect: this definition consists in stating in which cases a mathematical or logical function is equal to itself for different values of the variable. These forms of definition can be but provisional and must be transformed into nominal definitions in a strictly deductive system; it should, however, be borne in mind that even in this more perfect stage a definition is merely a convention of language and must not therefore be regarded either as a principle or as a source of truth; strictly speaking, it is neither true nor false, because the only thing responsible for the consequences is the existential judgment which must accompany and justify it by a theorem or postulate affirming the compatibility of the concepts constituting the meaning of the new symbol, that is to say, the existence of at least one individual verifying all these properties.[19]

6. *Ordinal and Cardinal, Finite and Infinite Number.* —Arithmetic is the first branch of pure mathematics to be presented to logical elaboration. Weierstrass considers that the whole of analysis can be constructed with the idea of number alone, but Russell, while recognising the utility of the work performed under his influence, since to it is due the exclusion of intuition from the domain of mathematics, regards the reduction of the primitive ideas of mathematics to the concept of number alone as arbitrary. In the theory of groups and substitutions,[20] the essential idea is that of order, hence this idea too must be admitted to be primitive side by side with that of number. Certain mathematicians have maintained that of the two forms of numbers, ordinal and cardinal, the first named alone is irreducible and

a priori; this view, however, is not accepted by Russell and Couturat,[21] who affirm the idea of order to be less primitive and simple than that of number. Number may be defined in a purely logical way, by use of the notion of class alone. Two classes have the same cardinal number when a univocal and reciprocal correspondence can be established between their elements, that is to say, a one-to-one relation, or, to put it briefly, when they are *equivalent*. Each number defined in this way is regarded as the name of a class: the class of the apostles, for instance, is one of the classes whose number is twelve or, in more common parlance, is a dozen. In short the cardinal number of a class u would be the assemblage of the classes equivalent to u. Thus each of the cardinal numbers is defined independently of the others, and no order is assigned to them: in ultimate analysis mathematical equality is reducible to logical equality, that is to say, to the identity of a concept. Thus even arithmetical operations can be defined without the intervention of the idea of order; arithmetical addition, for instance, is defined by means of logical addition: the arithmetical sum of two cardinal numbers a and b (corresponding to two classes a and b) is the cardinal number of the logical sum of the classes a and b, conditionally on these classes being disjuncted, that is to say, on their having no element in common. By means of the logic of relations Whitehead has given a general proof of the associative law for addition and multiplication, and of the distributive law of multiplication in relation to addition; he has defined the powers of a cardinal number, the combinations and permutations of any number of objects whatsoever, and has proved the principal theorems relating to these notions, as, for instance, the generalisation of the binomial formula. Thus Russell and Whitehead, taking purely logical principles as their starting-point, have succeeded in giving formal proof of all the propositions of the theory of assemblages discovered by Cantor, and thus in confirming the logical validity of this

theory by divesting it, as they do, of any appeal to intuition.

The cardinal definition of number, given above, is preferable to that of Dedekind,[22] who begins by defining numbers as simple ordinals, which are supposed to become possessed of their cardinal meaning when they are applied to the enumeration of concrete classes; because, whereas the cardinal definition extends to all numbers, finite and infinite alike, the ordinal definition is applicable only to a part of them, *i.e.* to those which are obtainable, starting from 0 by repeated additions of 1, and for which the principle complete of induction is valid. This principle is not a synthetic *a priori* principle, as was maintained by Poincaré,[23] but merely a property characterising finite numbers and forming part of their definition. It is not valid of the transfinite classes defined by Cantor,[24] as those assemblages which are equivalent to a proper part of themselves. The class of *all* whole numbers (which Cantor indicated by the number ω) is equivalent to that of all even numbers which forms a part of it, because each term of the one can be made to correspond with a term of the other, and *vice versa*.[25] If in comparing the classes we also take the order of elements into account, we shall arrive at the concept of ordinal number. The two well-ordered series M and N belong to the same type, or are *like* one another if they can be made to correspond univocally and reciprocally in such a way that, m and m' being two elements of the one, and n and n' the corresponding elements of the other, the relation of position of m and m' is the same as that of n and n'. An ordinal number is then nothing but the abstract concept of a type of order common to like *classes*, and is thus defined without having recourse to intuition, because the idea of order is in its turn definable by the pure logic of relations. The linear order of an open series, for instance, is reducible in the last analysis to a transitive, asymmetrical relation; that is to say, to a relation between the terms x, y, z of the class, such that if it exist between x, y and y, z, it also

exists between x and z, but cannot be inverted; that is to say, the two relations xRy and yRx exclude one another reciprocally.

7. *The Continuum.*—The continuum may be defined by means of pure relations of order without having recourse to spatial intuition. The ordinal definition of the continuum, which was formulated for the first time by Enriques [26] and Cantor,[27] has been simplified by Whitehead as follows: "A series is continuous, (1) when it has a first and last term, and when all its segments (upper and lower alike) have a limit; (2) when it contains a denumerable, compact series, such that a term of this latter series will be found between any two terms of the former series." [28] The idea of magnitude does not enter into this definition, but only that of order, because the concepts of segment and limit can be defined by means of pure ordinal relations. A segment of a series E is an assemblage S containing some, but not all, of the terms of E, not containing a last term, and containing all the terms of E which precede some term of S. A term x is the limit of a series S, if all the terms of S be anterior to x, and if every term which is posterior to all the terms of S be posterior to x. Cantor [29] has shown that all continua are equivalent, whatever be the number of their dimensions; that is to say, it is possible to establish a one-to-one correspondence between them, so that nothing distinguishes them, from the point of view of the number of their elements. There are as many points in a rectilinear (or curvilinear) segment, as in a square or cube; this paradoxical truth has been illustrated by Peano, who has invented a curve which fills a square, that is to say, which passes through all the points of the square. Couturat [30] regards this a manifest proof of the incompetence of intuition in matters of geometry. Continua of a different number of dimensions are not equivalent and indiscernible except from the point of view of the cardinal number of their elements; but they are distinguished from one another by their ordinal properties. A segment, a

square, a cube, do not differ from one another as classes of points, but only as regards the order and arrangement of these points, that is to say, in ultimate analysis as regards the relations set up between them, since every order consists in a system of relations. That which constitutes the continua having several dimensions, no less than the linear continuum, is not an assemblage of points, but an assemblage of asymmetrical relations; space is not a simple manifold, but an ordered manifold. A one-dimensional assemblage is a simple series whose terms are absolute individuals, *i.e.* not relations; a two-dimensional assemblage is a double series whose terms are themselves simple series, *i.e.* a relation of relations; a three-dimensional assemblage is a triple series whose elements are double series, *i.e.* a relation whose terms are relations of relations, and so on.[31]

8. *Geometry as a Hypothetico-Deductive System.*—Pieri [32] constructs the whole of projective geometry with nothing but the two notions of point and of the connecting line between two points, defining straight lines as relations of a certain type, and points as the problematic terms of these relations. In such a hypothetico-deductive system, there ceases to be any such thing as a primitive, undemonstrable proposition, because all the postulates form part of the definition of projective space and constitute its hypothetical properties. No one of them is categorically affirmed; it is merely stated that if a space be possessed of certain properties enunciated in its definition, it will also be possessed of the others which are enunciated in the theorems. Thus projective geometry is reduced to the form of a vast implication, and is therefore included in pure mathematics, which knows no other principles than those of logic; the same may also be said of descriptive geometry.

Metrical geometry is historically anterior to the geometry of position; it is, however, logically posterior thereto, since it of necessity implies relations of position (for the most part unnoticed or neglected) upon which

relations of magnitude are superposed ; it is not the most elementary, but the most complex part of geometry. The most elementary truth, "two points determine a straight line," which is commonly regarded as the definition of the straight line, implies a large number of postulates. Neither Euclid nor his imitators can give us any demonstration which is logically exact, and does not imply an appeal to intuition. Couturat pronounces the rigid accuracy of Euclidean geometry to be but a legend [33] ; for proof of this statement it will suffice to take into consideration the numerous errors which Russell [34] has pointed out in the first twenty-six propositions of Euclid. It is only in our day that mathematicians have begun to take into account all the postulates implied in the elements of geometry, and, subsequently to the enunciation of these postulates, the whole of geometry has been reconstructed in a purely analytical manner, taking as the starting-point certain primitive notions, which Peano [35] considers to be reducible to the point and distance. Others on the contrary assume the primitiveness of the notion of congruence, which is the presupposition of the possibility of super-position, because two figures are not congruent when they can coincide, but, *vice versa*, in order to coincide, they must first be congruent ; and Pasch postulates congruence between figures of any description ; Hilbert, between segments and angles ; Veronese, between segments only.[36] Pieri,[37] on the other hand, regards motion and the point as primitive concepts ; but this is at bottom merely a mere intuitive way of representing the relation of congruence, since in geometry we do not take into consideration the continuous series of intermediate positions of the moving point, but only the initial and final positions. Metrical geometry, like that of position, is reducible to a purely logical form, if the postulates be transformed into a definition of metrical space : we give the name metrical space (Euclidean or non-Euclidean) to an assemblage endowed with the properties enunciated in the postu-

lates.[38] Metrical geometry, or rather, each of the metrical geometries, then assumes the form of a vast implication : if a certain assemblage of entities verify the fundamental properties enunciated in the postulates, it will verify all the theorems of the corresponding geometry. Thus geometry, or to express it more accurately, the geometries, are no longer based upon primitive, undemonstrable propositions, and are no longer possessed of axioms of their own differing from the general axioms of logic. This transformation is legitimate, since so long as we are constructing pure geometry, we speculate on ideal spaces whose real existence we do not affirm ; therefore we can, and indeed should, divest postulates of their categorical character, in order to reduce them to mere hypotheses problematically posited ; it is necessary because, in order to reason on a space, we must define that space, and we can only define it by enunciating its characteristic properties, from which all the others are logically derived.[39] The definition of a space is the totality of the conditions which are necessary and sufficient for the determination of all its properties. Now a postulate must either belong to this totality of conditions, in which case it ceases to be a postulate, and becomes an integral part of the definition, or it does not belong to it, in which case it is a theorem, which can and therefore must be proved. There is no room in a logically constructed geometry for postulates of any description ; in short, as has been pointed out by Poincaré, postulates are but disguised definitions, or rather, parts of a definition : Euclid's famous postulate, for instance, is a mere complement of the definition of the straight line, and of Euclidean space. Geometry thus ceases to be an autonomous science having its special principles, and based upon synthetic *a priori* judgments, and becomes a series of formal deductions, dependent upon a definition from which logical consequences are derived *ad infinitum*. Geometry no longer has primitive notions of its own, just as it has ceased to possess primitive propositions ; in fact the

primitive notions in all the systems upon which we have touched may be reduced to two: a class-concept, which is termed point, and a notion of relation (order, congruence, motion), which sometimes appears in the guise of a class-concept (straight line, segment, vector), when the analysis is not followed out to its conclusion. Now the notion of point does not enter into the logical structure of geometry: points have ceased to be anything but the elements of certain assemblages, or rather the terms of certain relations: they are objects of which we know one thing only, viz.: that if certain fundamental relations subsist between them (*i.e.* the relations which are enunciated in the axioms) all the theorems logically derived therefrom will be verified.[40] Thus pure geometry, like a system of implications, affirms nothing with regard to points: strictly speaking, it neither knows them nor has need of them. On the other hand, the relations constituting the second primordial datum of the various systems are no longer indefinable, because the logic of relations permits of their definition by means of their formal properties. Mathematics disregards the intrinsic nature of objects in order to take into consideration nothing but their relations, so that if two assemblages of objects, of a totally different nature, verify the same relations, they will be submitted to the same theory; a proceeding of which mathematical physics affords us many examples. But, if mathematics be an abstract science, it is not such because it disregards the properties of the objects of experience, as is believed by empiricists, but because it has ceased to be a science of objects at all, and has become a pure, formal science, dealing only with the form of object and their relations. This explains why mathematical truths are of *a priori* universality and necessity: their objectivity is due to the fact that, while they refer to no object in particular, they are applicable to all possible objects, like the laws of logic, whose characteristics they possess, and of which they are the necessary consequence. Geometry is distinguished from logic and

the other branches of mathematics, because it studies multi-dimensional series, *i.e.* ordered multi-dimensional assemblages, whereas arithmetic studies a series having but one dimension, the natural series of numbers.[41] Pure, wholly *a priori* geometry is an implication of the form, if A be true, B is true: applied geometry, which is an empirical science, says: A is true, therefore B is true, affirming the existence of A and B in actual objective space, whereas pure geometry confines itself to affirming their logical, ideal connection. Intuition, which is absolutely excluded from pure geometry, prevails in applied geometry, because it is indispensable to give a direction and a support to the primitive notions, and for the verification of the primitive propositions (postulates) from which all the rest are derived. Whereas pure geometry, for instance, ignores points, and must ignore them, applied geometry has to determine in real and objective space those elements to which it will give the name of points, and their various relations, straight lines, plane, etc., because it is only when this condition is fulfilled that geometrical propositions acquire a real sense and become capable of verification.

9. *The Analytic Character of Mathematical Truths according to Couturat.*—Russell regards pure mathematics as being identical with logic from the standpoint of form, in as much as it proceeds *by logical principles from logical definitions*, and is distinguished therefrom merely by the special content of the relations it studies; hence its character of necessity, which renders it possible to say with Benjamin Peirce: *Mathematics is the science which draws necessary conclusions.*[42] Mathematics thus assumes an analytical character as opposed to the Kantian thesis which would turn it into a system of synthetic *a priori* judgments. That which distinguishes the analytical and synthetical attributes of a concept is the purely logical fact that they do or do not form part of the definition of that concept: everything which is contained in the definition of a concept

or is logically deduced therefrom is of an analytical character; everything which is added thereto for reasons of extra-logical necessity is of a synthetic nature.[43] It is a mere matter of words whether a definition be termed analytical or synthetical; in reality all mathematical definitions are purely nominal, that is to say, they consist in the determination of the meaning of a new term, supposed to be unknown, as a function of old terms whose sense is already known; it is a convention as to the use of a simple sign which is substituted for a combination of signs; it can therefore in no way influence the analytical or synthetical character of the propositions deduced from, or rather deduced by means of, it.[44] The concept of the sum of 7 and 5 does not, according to Kant, contain anything more than the union of these two numbers, which does not at all imply the thought of the single number: analyse the concept of this sum as much as you will, you will fail to find the number 12, hence you must transcend this concept and have recourse to intuition, by counting on your fingers for instance. Now Couturat[45] regards this as a gratuitous affirmation; the sum of 7 and 5, for the very reason that it implies the union of the two numbers, or, to put it more accurately, that of their units in a single number, contains this very number, in as much as it determines it univocally; there is not merely equality but absolute identity between $7+5$ and 12, as can be proved by recourse to the definition of numbers, according to which each is equal to its predecessor *plus* 1, and to the equation $a+(b+1)=(a+b)+1$, which is derived from the definition of sum. Kant's error is explained by his conception of logic, a conception both too restricted and too simple. We may, he says, turn and turn our concepts as much as we please without ever succeeding in finding the sum by merely resolving our concepts into their component parts. Couturat, however, observes[46] that Kant starts from the false presupposition that all concepts are composed of partial concepts, so that it

will suffice to resolve them into their component parts in order to discover all their properties. Concepts of number cannot be defined *per genus proximum et differentiam specificam*, or be resolved into logical factors; this, however, merely proves the inadequacy of classical logic, and does not give us the right to conclude that arithmetical addition is beyond the grasp of true logic, since it can and must be defined by means of the logical sum of the classes a and b: the sum of 7 and 5 is the number of the collection formed by uniting the two collections represented by 7 and 5. Intuition may even be necessary as a psychological help; this does not, however, alter the fact that the concept of $7+5$ must by its very definition contain that of 12, or rather is identical with it. The synthetic character of judgments does not depend either upon the nature of the concepts or upon their origin and mode of formation; it is possible to form analytic judgments upon empirical concepts which are the product of an intuitive synthesis.[47] Not even geometrical judgments are synthetic; that is to say, the judgment that the straight line is the shortest distance between two points is not an axiom, but a theorem which can be proved by means of the definition of inequality and of the sum of segments and angles. Kant regards the fact that in the course of mathematical demonstrations we are frequently forced to have recourse to auxiliary constructions as affording proof of the synthetic character of geometrical truths. These auxiliary constructions are, however, either merely aids to the imagination, corresponding to elements which have already been analytically determined, or useless, since they are not indispensable in the demonstration.[48] Neither is the existence of symmetrical figures an argument in favour of the synthetic character of geometry, since this merely proves the impossibility of reducing space to its metrical properties only. It is not true that two symmetrical figures are absolutely equal; they are so merely as regards the relations of magnitude, but differ as to the relations of order which

may be defined by means of the logic of relations: in the end two inverse orders correspond to inverse relations, and the conversion of relations is a logical operation.[49]

10. *Intuition in Mathematics.* — The new logical elaboration of pure mathematics may be regarded as a return to the rationalism of Leibnitz as opposed to the intuitionism of Kant. Mathematicians do not all, however, agree in placing an absolute ban upon intuition. Felix Klein,[50] after distinguishing three classes of mathematicians — the logical, the formalist, and the intuitive — vindicates the rights of intuition in the construction of mathematics, and emphasises its usefulness and even necessity in every sphere thereof, even in the realm of abstract number, thus rehabilitating geometrical intuition, which had been over-depreciated and neglected by the excessive rigour and exclusive purism of mathematicians of the formal and logical schools. He shows, for instance, that a very simple geometrical representation serves to interpret the equivalence of binary quadratic forms in the theory of numbers, and to illustrate Kummer's conception of ideal numbers in algebra. It is not possible to base mathematics solely upon logical axioms, and intuition must do its share.[51] Poincaré, too,[52] maintains the legitimacy of intuition, affirming that the two different types of mathematicians, the intuitive (with whom he ranks Klein, Bertrand, Riemann, and Lie) and the logical (such as Meray, L'Hermite, Weierstrass, and Kowalevski), are equally necessary to the advance of science since analysis and synthesis alike have their allotted task to fulfil. Analysis gives us single parts but not a general view; logic and intuition are then both necessary, the former affords us strict certainty and is the instrument of proof, the latter is the organ of discovery. There are discoverers in the ranks of the analysts; they are, however, but few in number: sensible intuition is the more usual tool of discovery.[53] To the accusation of barrenness brought against logistic by Poincaré and

other critics, Couturat has with justice replied that logistic does not claim to be an instrument of discovery.[54] Only a crass *ignoratio elenchi* can place the psychological fact of discovery in opposition to logistic, since it is not its business to inspire or explain discovery, but merely to control and verify it. Would you reproach the science of metre or harmony with failing to create poetical or musical genius ? Poincaré's analysis of the two opposite types of mathematicians is undoubtedly extremely interesting from the psychological point of view, all the more since a similar difference exists amongst men of science in the realm of physics with regard to their concrete procedure of discovery and their mode of thought, and of setting forth their theories—a difference which has given rise, as we shall see later, to two opposite schools. We must, however, beware of confusing the psychological question with the epistemological problem, and must draw a distinction between that which is dependent upon the individual structure of the thinker and upon his type of imagination, and that which is essential to all thought whatsoever, given the nature of his branch of knowledge.

The criterion by which to gauge the necessity of intuition cannot be derived from psychological observation ; in order to be entitled to affirm that a science cannot dispense therewith, we must prove that it is impossible for the pure activity of logical thought to establish its principles, and to draw all the conclusions from those principles. Now as long as we confine ourselves to the realm of abstract relations, as studied by the new mathematical logic, and to that of pure quantity, the object of arithmetic and of algebraic analysis, there is no need to appeal to intuition in order to define the supreme concepts upon which these sciences are based, and to deduce all the theorems from them. The fact that it is possible, within certain limits, for mathematical calculus and logical theory to become fused and to pervade one another, affords proof of the profound affinity between the two orders of knowledge

which are the outcome of the reflections of thought upon its own function, and hence exclude all consideration of intuitive matter. Number is a product of the analytic and synthetic activity exercised by thought upon itself, associating its single acts in complex units and resolving the units which had previously been undivided into elementary acts. The human mind finds it within itself like a law of its nature, an intrinsic necessity, of which it cannot divest itself without denying its own very nature. The existence of cardinal number is not something hypothetical, as the logistics would affirm, but rather an axiomatic truth which thought discovers by reflection upon its own acts, and recognises immediately as the necessary presupposition of its function. There is no concept without the possibility of distinguishing a whole in its parts, and of recombining them into a synthetic unity: the ability to grasp the one in the many, and the many in the one is the very essence of human thought, which is therefore by its own activity implicitly led to presuppose the existence of number. *Dura lex, sed lex.* We may seek to escape from this law by living with Bergson in the fantastic world of creative and indivisible intuition, but if we thus deny number, we deny discursive thought at the same time, and with it the whole science of logic. We are as certain of the existence of number as we are of that of our own logical thought, since thinking is potential numeration.

11. *Irreducibility of Mathematics to Logistic.*—If such be the nature of cardinal number, it will at once be obvious that all attempts to define it with the help of pure logical constants must of necessity move in a vicious circle. We cannot therefore be surprised that Poincaré [55] and various other critics have discovered in the alleged definitions a more or less explicit use of numerical concepts and terms or of a word in the plural, which naturally presupposes that which is to be defined. Thus Couturat defines *zero* as the number of the elements of the *null* class, *i.e.* of the class con-

taining no element, or again as the number of objects fulfilling a condition which is never fulfilled, and 1 as the number of the elements of a class of which any two elements are identical. Couturat[56] replies that the logistical definition of 1 does not in the least imply that of 2, but means, properly speaking, that "1 is the class of the classes u, not being null, such that if x be a u, and y be a u, x is identical with y, whatever x and y may be"; hence, in the place of x and y we may put an indeterminate plurality of terms, provided that they all be identical. But, it may be urged, does not *plurality*, however indeterminate it may be, presuppose the idea of number? When you define the cardinal number of a class u as the totality of the classes which are equivalent to u, and add that two classes have the same cardinal number when a univocal and reciprocal correspondence can be established between their elements, that is to say, when these elements can be made to correspond term for term, or to put it more briefly, when the two classes are similar, you appeal to the logical calculus, which is in its turn impregnated with numerical concepts. Even if we put aside explicit numeral terms (as, for instance, in the definition of the logical product or sum of *two* or *more* propositions) which could be disguised by saying x,y instead of 2, x,y,z instead of 3, and so on, it is an undoubted fact that the simplest logical proposition cannot be formulated without establishing a relation, that is to say, without implicitly positing a multiplicity synthetised into unity: the logical constants, by whose means number is supposed to be defined, already contain number within themselves as their necessary presupposition. How is it possible to conceive the relation of a term to a class of which it is a member, without at the same time positing the concept of multiplicity of terms upon any one of which we fix our thoughts? What meaning can be attached to the word *class*, if you do not presuppose the concept of a plurality, which may be as indeterminate as you will, but which is still a plurality of logical individuals? Does not the

notion of *any and every term* too, imply the thought of a multiplicity of terms, composing one and the same class, belonging, that is to say, to one and the same conceptual synthesis? While we recognise with Russell[57] that the class may be defined *intensionally*, that is to say, by means of the comprehension of the concept determining it, without having recourse to the actual enumeration of its terms, that is to say, to its extension, it is certain, on the other hand, that in the logical calculus we are continually forced to leave the pure consideration of the intension in order to refer to the entities belonging to these classes.[58] Thus when we define the logical product of the classes a and b as the assemblage of the values of the variable x, which are at the same time a and b, we do not by the expression *assemblage of the values of* x mean the intension of the concept x, but rather the extension thereof. Is not this assemblage of values a numerical plurality? Thus when we define the equality of numbers by means of the similarity of classes, and say that one term and one term only of the one must correspond to one term of the other, we are already taking for granted the thought of the class as a collection of terms, that is to say, we regard it as being composed of a certain number of terms.

The attempt to give a logical definition of the idea of linear order will also end in a like *petitio principii*. Russell and Couturat define linear order as an asymmetrical relation, and this is in its turn defined as a relation which can never be inverted; that is to say, that the two relations xRy (*i.e.* the relation between x and y), and yRx (*i.e.* the relation between y and x) are mutually exclusive. Is it, however, possible to define the inverse of a relation without presupposing the concept of order? The intuitive assistance afforded by the succession of terms in the schemes xRy and yRx enables you to avoid the use of the word "*order*" in the definition of inverse relations, and deceives you into thinking that you have succeeded in avoiding a vicious circle, but, were you asked to give an abstract definition of the inverse of a

relation without making use of the signs x and y, you could only say that a relation is converted when the *order* of the two terms is changed. The existence of non-invertible series is not a mere hypothesis, or an arbitrary convention, but rather an axiomatic truth which we derive from reflection upon our acts of thought, together with the primitive and indefinable general concept of order. Analysis of the logical function, which brings to our notice cases in which our thought is capable of passing from the truth or falsity of one proposition to that of another, without, however, being able to make the inverse deduction, affords us immediate and apodictic certainty of the existence of non-invertible relations, and hence of series of acts of thought developing in a determinate order. We cannot, moreover, conceive of this series as limited without at the same time placing a limit to the function of logical thought, which by its very character of universality transcends all limitations. From this point of view the proposition that each number has a successor ceases to appear an arbitrary convention, a mere hypothesis set up by us in the definition of number, as logisticians would have us believe, and becomes a law which our mind recognises in the series of its denumerable acts, refusing to conceive of a last term to its function beyond which no further act of thought is possible. It would be absurd to conceive such a term, since to do so would be tantamount to contradicting that value of universality which thought apprehends immediately in the affirmation of its principles.

In geometry the impossibility of defining all mathematical concepts with the help of logical constants only becomes still more apparent. If we consider the hypothetico-deductive systems attentively, we shall at once see that their definitions refer to conceptual entities, having nothing geometrical about them, and if they recall spatial figures and relations to a certain extent, they do so because the terms commonly used to indicate certain intuitive properties are introduced into

the definitions. Thus when we say: "An assemblage of one dimension is a series whose terms are absolute individuals, that is to say, are not relations; an assemblage having two dimensions is an assemblage whose terms are themselves relations, that is to say, a relation of relations; an assemblage having three dimensions is a relation whose terms are relations of relations," and so on; our definition does not determine anything spatial, and if we think of the straight line, the plane, and ordinary space, we do so because the word *dimension* suggests intuitive extension to our minds. Thus when in projective geometry we enunciate the postulates relative to the point and say that points form a class, that there exists at least one point, and that if a be a point, there exists a point differing from a, the word point is an appeal to intuition and imparts a geometrical meaning to these postulates which otherwise would have none, seeing that they can refer to any individuals composing a class. In like manner, when we say that, given the two points a and b, there is an entity which we call the straight line ab, which is a class of which every individual is a point, etc., if the words straight line and point, which recall geometrical entities to our minds, be removed, these formulations would apply to any class of individuals verifying all those properties. Neither do we determine anything really spatial in the postulates of metrical geometry when we say that the point and motion are generic and class concepts, that every assemblage of points is termed a figure, that two figures are identical when they are composed of the same points, etc.; since, if the words points, figure, etc., be eliminated, propositions will remain which are valid in the case of certain assemblages. In short, hypothetico-deductive systems do not succeed in individualising geometrical entities in their definitions, but treat of generic relations of assemblages and classes, which only apply to the specific case by a mental restriction, due to the terms of common geometry and to the spatial intuition which is associated with them. In ultimate analysis, when we formulate

these systems, we either fail to create a geometry, and merely create a logic of classes, assemblages, and relations, since that which we enunciate does not refer to the point, the straight line, the figure, etc., but to any entity which is a term of those relations, and to any assemblage of like entities; or, if we succeed in individualising these concepts, and in determining them specifically, so as to impart a geometrical sense to them, our success is due to the fact that we recall intuition by the use of corresponding terms, and derive therefrom the properties expressed by the definitions and postulates. The new systems bear the same relation to the science of space as a genus to its species: it is true that they differ from the old analysis, since they treat of relations which cannot be reduced to mere relations of magnitude, but they possess in common with it the characteristic of being reducible to a pure logical calculus, which, though it may be applicable to geometry, is not for that reason geometry, just in the same way as general logic is not the science of space, although the properties of concepts, and the deductive laws formulated thereby, are necessarily valid of geometrical figures also.

12. *Criticism of the Hypothetico-Deductive Systems.*— The new logical elaborations claim to divest mathematical truths of their categorical character, and to transform them into hypothetical judgments, but this interpretation of the value of mathematical knowledge is thoroughly vitiated by the empiristic preconception that geometrical and arithmetical principles and deductions require to be legitimatised by experience. If this prejudice, which contrasts forcibly with the ideal character of mathematical constructions, be put aside, there ceases to be any sense in speaking of hypotheses. Geometrical and arithmetical concepts exist of necessity in our thought, and are of universal value, in as much as experience would not be intelligible without them; they are not mere arbitrary fictions, which may or may not be verified by empirical facts, but are conditions indispensable to the intelligibility of the facts. A non-

denumerable reality, or a reality which, while denumerable, did not conform to the laws which our mind recognises as necessary in the various forms of calculus, would be an absurdity baffling the intelligence. To say: If experience verify our definition of number, it will verify all the theorems which follow therefrom, has as little sense as to say: If experience verify logical axioms, our arguments are applicable to it! The same thing holds good of geometrical concepts, which, whilst they are not a product of the simple reflection of thought upon itself, since they derive their content from intuition, yet, in as much as they represent the only adequate means of translating the empirical data of space into intelligible terms, appear to us to be equally universal and necessary. Even in the case of geometrical truths there is no sense in the hypothesis: "If an entity exist which verifies these relations," since that entity does and must exist ideally, and this is enough, since geometry neither does nor should take empirical existence into consideration. When logisticians define or claim to define a geometrical entity by means of a certain number of properties, they of course assume that a concept corresponds to that complex of symbols, that is to say, that that entity is thought in some way or other. When it is stated that all mathematical definitions are nominal, says Couturat,[59] it is not meant that mathematical concepts are reducible to names, in accordance with the thesis of nominalism, but that they can all be logically and explicitly defined in terms of certain primitive notions, and may therefore be regarded as names given to certain combinations of those notions. Thus even this new sign must correspond to the concept resulting from the synthesis of the primitive notions. Moreover, continues Couturat,[60] in order to reason later on this concept and invoke its properties, it must be possible to affirm that individuals of the class exist, that is to say, that the conditions defining it are not logically incompatible; each definition must therefore be accompanied by a theorem or a postulate, affirming

the *existence* of the object defined. Then, we ask, what becomes of the hypothetical character of mathematics the moment that its concepts have and must have an ideal existence? When such an existence, which is essential to ulterior deductions, has been proved or recognised in some other way, we shall have the right to affirm categorically: "An entity exists in our thought which is endowed with those properties."

The demand that the object defined shall be thought and recognised as existing cannot be satisfied in the realm of geometry without having recourse to intuition which has been idealised and purified by thought. How indeed are we to prove by means of logistic alone that the notions and relations with which we define a geometrical entity are mutually compatible? [61] Only two ways are open to us: we must either prove that none of the possible consequences of those premisses contradict one another; a never-ending process of verification, which could therefore never afford us perfect certainty; or we must show by an example that there exists a class of individuals verifying all those properties. "If," says Pieri,[62] "A, B, C, be propositions belonging to a deductive system Γ, their compatibility may be said to be proved if in some domain Δ an interpretation can be found of the primitive ideas of Γ manifesting all the properties enunciated by the propositions A, B, C, provided that such a domain do not embrace any one of those propositions amongst its premisses, and that the consistency of its principles be already established or granted *a priori*." In other words, the justification of one system is sought from another system, and of course we must, unless we would continue the process *ad infinitum*, stop at one of those systems which posits the ideal compatibility of its primitive propositions by means of an axiom or a postulate. Pieri is forced to acknowledge that, "it will never be possible to afford deductive proof of the truth and consistency of the whole system of logical premisses." [63] For geometry the proof of the ideal existence of an entity verifying all these properties

lies in the concept of the figure in which we find the verification of all these properties and relations. We may amuse ourselves by forming countless combinations of logical constants and relations; but these combinations are not all possessed of geometrical meaning, that is to say, they do not all determine an object existing in ideal space. Logicians choose from all possible combinations those which are endowed with geometrical sense, but in this selection they are in the end guided by the old intuitive notions. Moreover, the ideal existence of these entities, which in formal combinations [64] remains a mere hypothesis which cannot be translated into our intuitive notions of space, becomes an axiomatic truth when these notions are present, because the certain verification of all the postulates afforded us directly by thought in the concept of the figure transforms the hypothetical judgment into a categorical one.

13. *A priori Synthesis in Mathematics.* — Though the truths of mathematics are necessary and *a priori*, this does not imply that they are purely analytical, as Couturat maintains: it is erroneous to believe, as David Hume did, that thought is synthetical only in empiric knowledge, when it derives its materials from experience, and that *a priori* deduction never affords us anything new. Even in abstract calculation mathematical thought is essentially inventive and constructive, and every analysis presupposes the synthesis completed in the act of definition. In so far as you can derive the rules of arithmetical operations from the rules of the logical calculus, your thought has already formed in the eight indefinable notions and twenty undemonstrable principles the conceptual synthesis necessary for all the following analyses. Modern logic differs from the older form in that it condenses and absorbs within itself in a more generic form principles and definitions which were previously scattered throughout the different branches of mathematics; so that, whereas in the old constructions the foundations were laid little by little as the need for them became

apparent in order that the process of deduction might be continued, and the single synthetic acts were carried out case by case, in the new logical edifice the whole basis is constructed from the very first and the particular syntheses are condensed into one wider synthesis. This form of systematisation of pure mathematics may also be preferable to the older one, in as much as it saves useless repetitions of analogous synthetic acts in the individual sciences, and in their different departments, and gives prominence to the logical unity of mathematical knowledge; it does not, however, eliminate the synthesis which is merely shifted, concentrated, and disguised in the definitions and initial principles. Moreover, the work of constructive activity is not wholly excluded even in successive deductions; this activity manifests itself in the combination of the elements originally defined into new products, which are not merely a juxtaposition of entities and principles, admitted from the first, but organic syntheses of them. Each new notion is defined by a complex of properties, and, although these properties may have been known already through the definitions, the thought of the whole is always something new; even if it be merely thought of as possible, at all events we must have a concept of it. Neither can it be said to be but a new name, since in the hypothetical judgment it is implicitly admitted that an objective entity can exist which verifies all the properties at one and the same time; this entity is certainly not the word, but that which is symbolised by the word, that is to say, that which is thought by its means. The Kantian doctrine which would derive the synthetic character of mathematics from its intuitive nature does not hold good of the calculus, to which intuition is not essential, but the activity of thought in its *a priori* deductions is none the less fruitful in new products, which, constructed as they are in accordance with its intimate laws, bear the imprint of universality and necessity.

14. *Russell on the Philosophical Consequences of*

Mathematical Logic.—Russell has considered himself entitled to draw certain important philosophical conclusions from his logico-mathematical theories.[65] The theory which resolves the continuum into an infinite number of distinct elements without any contradiction has robbed of their value the idealistic doctrines of space and time which were based upon the alleged antimonies of the actual infinite. Realism is thus restored to its place of honour. Mathematics further teaches us that there are certain fundamental and irreducible concepts (the logical constants), and certain primitive undemonstrable propositions, which enable us to construct a system of truths, recognised by us, in their deductive relations, without the need of an appeal to experience. Every general proposition transcends the bounds of sensible knowledge, which is limited to the particular case, and never authorises us to formulate a principle having universal extension. Induction is at bottom but a deduction based upon a certain premiss, *i.e.* upon the principle of induction, which cannot be derived from experience without a vicious circle, and must therefore be recognised as an *a priori* truth. Empiricism is then false, but no less false is the idealism which believes that the salvation of the universality of *a priori* truths lies in reducing those truths to subjective forms of the mind. If general truths were merely the expression of psychological facts we could not be sure of their constancy or legitimately make use of them in order to deduce one fact from another, because they would not connect facts, but only our ideas about facts. Mathematical logic then forces us to admit a species of realism in the Platonic and scholastic sense, to conceive, that is to say, a world of universals having a reality of its own external to space, time, and the mind of man. This world must *subsist*, though it may not exist in the same sense as that in which particular facts exist. We have direct *knowledge by acquaintance* of universals, just as we have of sensible data; of material objects on the other hand, existing in

time and space, independently of our subjective perceptions, we can only have indirect *knowledge by description*, by making use of a combination of universal ideas and sensible experience. Such knowledge is, however, always far from perfect, whereas our immediate intuition of the eternal relations constituting in their ideal combinations the system of mathematical logic, is extremely perfect.

15. *The New Realism.*—Closely connected with this Platonising tendency of the Cambridge school, of which the chief representatives are Bertrand, Russell, and George Moore, is the new realism, which has recently been the subject of so much discussion in England and America.[66] This new realism is based upon the mathematical theory of the externality of relations set forth by Russell, according to which terms are in no way altered by the relation established between them, so that one and the same entity may be a constituent of many different complexes. Knowledge itself is an external relation: the object, when known, remains the same object which existed outside the relation with the subject. It therefore follows that everything which we perceive or imagine exists objectively also just as it is presented to our consciousness. The new realism differs from the older realism in that it does not admit that the things external to thought are different from the contents present to our consciousness, that is to say, in its "*epistemological monism*," which identifies the real object with that which is immediately present to us in the act of perception. On this point the new realism disagrees with Russell, who maintains that "if there are to be public neutral objects, which can in some sense be known to many different persons, these objects are not identical with the private and particular sense-data which appear to various people."

In my opinion the fundamental error of the new realism will be found in its starting-point, that is to say, in the theory of the externality of relations applied to the cognitive relation. Relations are external only in abstract mathematics, in which the terms can be

anged side by side, and united by a sign which sym-
olises their relation, without in any way modifying
hem. The number 8, for instance, will always remain
he same number in all the relations in which it can be
laced to other numbers: 8×4, $8+3$, $8-5$, $8>2$, etc.
But if we leave the world of abstractions for the realm
f concrete reality, it is a matter of common experience
hat things are subjected to physical changes by the
ction of other things standing in certain determinate
elations to them. It has been rightly observed that
he new realism fails to afford an adequate explana-
ion of the production of illusions and hallucinations,
ll of which should correspond to objects existing
utside consciousness just in the same way as they are
magined. The new realism cannot turn to the physio-
ogical action of the organism for an explanation of
llusion, since by doing so it would deny its doctrine of
he externality of relations: if the object appears in
lifferent ways according to the different organisms
with which it enters into relations, it is obvious that
hese relations are not external. The illusory appearance
s a real fact, but it is so only in that particular context
f psycho-physiological relations; and every other
ontent of sensation is real in the same sense; but to
dmit that this content remains identical even when
he relations are changed is to contradict our most
ertain experiences.

16. *Meinong's Theory of Objects.*—Russell's mathe-
matical realism has many points of contact with Meinong's
theory of objects.[67] As opposed to empirical sciences
(*Wirklichkeitswissenschaften*) which deal with existing
reality (the *Dasein*), the theory of objects is the science
of the *Sosein*, the rational essence, which can be
elaborated *a priori* independently of any considera-
tion of existence (*Wirklichkeitsfreie oder Daseinsfreie
Wissenschaft*). We have an example thereof in
mathematics, whose object is not the real, but the
ideal, that which subsists (*besteht*), although it may
not exist as do the phenomena studied by the empirical

sciences. In addition to the facts immediately experienced, such as colours, sounds, desires, etc., we must admit objects of a higher order, such as likenesses, equalities, diversities, complexes, etc., upon which we pronounce *a priori* judgments. According to Meinong, the theory of objects embraces not only the study of these relations, but also that of a series of objects which of their very nature are not comprised within the field of any of the sciences hitherto known, and which Meinong terms *homeless objects* (*heimatlose Gegenstände*). Sensorial contents, for instance, cannot be the object of the psycho-physiology of the senses, which does not treat of these contents in themselves and in their relations, but only of the way in which they are subjectively apprehended in relation to the organism ; nor yet that of physics whose task it is to study the objective stimuli of sensations, for instance, vibrations of the air or of ether, not sounds and colours. Another, as yet unexplored field of research, is that of *impossible objects*, as, for instance, the round square, non-extended matter, etc. Russell [68] has remarked that these objects would divest the principle of contradiction of its universal validity, but to this Meinong [69] has replied that this principle is valid of objects which are possible and real, not of those which are impossible, and that the round square, non-extended matter, etc., are not reducible to mere *flatus vocis*, mere complexes of letters, since these complexes exist psychologically, which is not the case with impossible objects.

The delicate subtleties of Meinong's arguments cannot, however, invest this alleged theory of impossible objects with meaning. How can you rationally elaborate that which is contradictory, or apply the laws of thought to that which is constructed in antagonism to those laws ? With regard to sensorial contents which Meinong considers should be studied by the new theory, his assertion that research into them is not within the province either of physics or psychology does not appear to me to be accurate. When Meinong states that the

object of physics is not colours and sounds, but rather the vibrations of ether and air, it is clear that he is confusing the object of science with the explanatory theory; you might equally well say that chemistry does not study metals, salts, acids, etc., but the relations of atoms. If sensorial contents be thus eliminated from the theory of objects, nothing will remain but relations and complexes, which Meinong regards as having no existence, hence the name science of the non-existing, which distinguishes the theory of objects from the empirical sciences. Now it seems to me that an artificial division is thus made between the two forms of knowledge which renders their interpenetration in the concrete life of science impossible of explanation. Were *a priori* deduction merely the instrument of knowledge of the non-existent, it would be an abuse to make use of it in the science of the existent. How, too, could this illegitimate transition attain success? How could mathematical deduction, the organ of knowledge unfettered by existence, enable us to make previsions concerning existence?

17. *Criticism of Russell's Theory.*—Russell's theory partly escapes these criticisms, because it regards the ideal world as subsisting, if not existing; it, too, however, has made the grave mistake of setting up an insuperable barrier between sensible cognition and rational knowledge. This new form of realism raises once more the self-same difficulties which perplexed the mind of Plato, and which were already emphasised by Aristotle. The doctrine of the exteriority of relations accentuates the want of agreement between the two worlds, and turns the process of knowledge into an incomprehensible mystery. On the one side, in fact, we have eternal relations; on the other, things in time and space; and in addition to and external to them both, human consciousness. Each relation is external to every other, each thing external to all others, each consciousness external and impenetrable to the rest. We may now ask, How and by what miraculous means is that union of the ideal and the real brought about which is necessary to the

cognitive act? The subject, reduced to a mere mirror which is to reflect external relations on the one hand and the facts of experience on the other, while itself remaining unaltered, will certainly not be equal to such a task of mediation, because even with regard to it and in it the universal idea and immediate experience will remain outside each other. It is useless to turn to an exterior relation between the idea and immediate experience, first, because in the world of objective things constructed by science the concept and the fact do not remain external to each other, but pervade one another in a synthesis which transfigures both of them; and further, because the external relation between the idea and the phenomenon would not resolve the problem, but would merely add another eternal relation to the others which the mind must receive from without. Of this new relation, in turn, the question might once more be asked, "How is its union with the sensible world possible?"

We are confronted by still more serious difficulties in the conception of the relation of the subject to these real entities. How can the revelation of them to consciousness be possible? Does not consciousness undergo a certain modification in the act of intuiting ideas through this very act? Or are we to grant, in order to save the theory of the externality of relations, that the subject thinking the principle of inclusion, for instance, is the same subject as that to whose consciousness it was not present before? Does not sensible experience modify the subject to a certain extent? Do subject and object remain external to each other in that experience also? Or is that experience a purely subjective, individual fact, not due to the action of external reality? In that case by what right do you make use of it, as well as of the idea, in order to gain indirect knowledge of real things? How do eternal relations make their way into the consciousness? How is a relation set up between thought and that which is thought? If this relation, too, be an external one,

we return to our starting-point, since we must explain how such a relation, which you affirm to be real, can become the object of your mind, that is to say, how it can enter into relation therewith. In the incoherent dust-storm of facts and external relations we shall vainly seek that synthetic principle which shall account for the possibility of knowledge, and that organic unity of thought and reality which alone can render knowledge intelligible.

NOTES TO CHAPTER II

[1] Leibnitz was conscious of the need for the extension of mathematics, and conceived a geometrical calculus which could be directly applied to figures without the mediation of numbers and express *situs* directly, as algebra does *magnitudo* (*Lettre à Huygens*, September 8, 1679). "Datur . . . analysis geometriae sublimior per proprios characteres, qua multa pulchrius breviusque quam per algebram praestantur" (*De scientia universali seu calculo philosophico*, 1684). Cp. Couturat, "L'Algèbre universelle de Whitehead," *Revue de mét. et de mor.*, May 1910; *Les Principes de mathématiques* (Paris, 1905), pp. 1-2.

[2] "Sur l'arithmétisation des mathématiques," *Nouvelles Annales de mathématiques*, March 1807.

[3] The first suggestion of this "scriptura universalis" will be found in Leibnitz, "Logicae inventionis semina," *Math. Schriften*, vol. v. Logistic assumed a specially scientific form through the works of De Morgano, *Formal Logic* (London, 1847); Boole, *An Investigation of the Laws of Thought* (London, 1854); Schroeder, *Der Operationskreis der Logikkalkuls* (Leipzig, 1872); *Algebra der Logik* (Leipzig, 1890-95); and Peano, *Formulaire de mathématique* (Turin, Bocca, 1894–1906).

[4] This extension is mainly due to Russell, "Sur la logique des relations avec des applications à la théorie des séries," *Revista di matematica del Piano*, vol. vii. (Turin, 1902), pp. 115-147.

[5] *Arithmetices principia, nova metodo exposita* (Turin, 1889).

[6] *Calcolo geometrico secondo l' Ausdehnungslehre di H. Grassmann, preceduto dalle operazioni della logica deduttiva* (Turin, 1888), Preface, pp. 5-6.

The following are examples of these geometrical notations: If the volume $ABCD$ indicate a tetrahedron, the formula $ABCD=O$ means that the points $ABCD$ are situated in the same plane; if A be a point and a be a plane, $Aa=O$ (*i.e.* tetrahedron Aa is nothing) indicates that the point A is situated in the plane a; if a, b be two lines, $ab=O$ means that the lines a, b are situated in the same plane, *i.e.* either that they meet or are parallel (*op. cit.* p. 33).

[7] Peano, *I Principii di geometria logicamente esposti* (Torino, 1889), p. 3.

[8] "I Principii della geometria di posizione composti in sistema logico deduttivo," *Memorie della R. Accademia delle Scienze di Torino*, serie xi. vol. xlviii., 1898; "Nuovo Metodo di svolgere deduttivamente la geometria proiettiva," *Rendiconti del R. Istituto Lombardo*, vol. xxxi., 1898; "Della geometria elementare come sistema ipotetico deduttivo. Mono-

grafia del punto e del moto," *Memorie della R. Accademia delle Scienze di Torino*, serie ii. vol. xlix., 1900.

[9] Pieri, "Sur la géométrie envisagée comme un système purement logique," *Bibliothèque du Congrès International de Philosophie* (Paris, 1901), vol. iii. pp. 268-377.

[10] *A Treatise on Universal Algebra* (Cambridge, 1898).

[11] Couturat, Article quoted in *Rev. de mét. et de mor.*, May 1900, p. 259 ff.

[12] *The Principles of Mathematics*, vol. i. (Cambridge, University Press, 1901).

[13] "Pure mathematics is the class of all propositions of the form '*p* implies *q*,' where *p* and *q* are propositions containing one or more variables, the same in the two propositions, and neither *p* nor *q* contains any constants except logical constants." The logical constants according to Russell are: implication, the relation of a term to a class of which it is a member, the notion of *such that*, the concepts of relations, of propositional function and of class, the notion of denoting and of *any or every term* (*op. cit.* p. 3 e p. 106).

[14] Russell, "Recent Work on the Principles of Mathematics" (*International Monthly*, vol. iv. No. 1, p. 84).

[15] Russell, *Principles of Mathematics*, pp. 13-32. We cannot here set forth the whole development of these calculi: we merely give a few examples in order that the reader may be able to form a general idea of them. In the calculus of propositions the logical product of two propositions means that both propositions are true at one time, and the commutative law applies thereto (if, for example, *p* be true together with *q*, *q* is true together with *p*), as do also the associative law (if *p* be true together with the simultaneous affirmation of *q* and *r*, the simultaneous affirmation of *p* and *q* is true together with *r*); the principle of simplification (the simultaneous affirmation of *p* and *q* implies the affirmation of *p*); and the principle of composition (if *p* imply *q* and *p* imply *r*, *p* implies the simultaneous affirmation of *q* and *r*). The logical sum of two propositions is defined as the proposition which is implied in each of them and implies every proposition implied in each of them. In the calculus of the propositions is enunciated amongst others the principle of the hypothetical syllogism; if *p* imply *q* and *q* imply *r*, *p* implies *r*. In the calculus of classes a propositional function is defined as a logical function which becomes a proposition for every value attributed to the variable or variables: the totality of values verifying the propositional function constitutes a class. An axiom of the calculus of classes is that if two propositional functions be equivalent, the corresponding classes are equal. The logical product of the classes *a* and *b* is the totality of the values of the variable *x*, which are at the same time *a* and *b*; the logical sum of the classes *a* and *b* is the totality of the variables *x*, which are either *a* or *b*. The calculus of relations deals with the relations of equivalence, implication, etc., in general. One of the axioms of this calculus is that every relation can be inverted, that is to say, that if the relation R exist between any two terms *x* and *y*, a relation also exists between *y* and *x*, which is termed the inverse of R. Another axiom is that every relation has its negative which is also a relation. It can be proved that the inverse of the negative is identical with the negative of the inverse. If there exist a relation R between the individuals *x* and *y*, and a relation S between *y* and *z*, there will exist between *x* and *z* another relation which is termed the relative product: the relative product of two relations is also a relation: the relation of uncle, for instance, is the

relative product of the relations of brother and father. Relative multiplication in general is not commutative; for instance, my brother's father is not my uncle.

[16] *Op. cit.* p. 112.

[17] "Les Définitions mathématiques," *Bibliothèque du Congrès*, vol. iii. p. 279; Burali-Forti, "Sur les différentes méthodes logiques pour la définition du nombre réel," *ibid.* p. 289; Padoa, "Essai d'une théorie algébrique des nombres entiers, précédé d'une introduction logique à une théorie déductive quelconque," *ibid.* p. 309; Burali-Forti, *Logica matematica* (Milan, 1894), pp. 120-130.

[18] Peano, *op. cit.* p. 288.

[19] Couturat, *Principes des mathématiques*, p. 39 ff.

[20] The theory of substitutions was originated by Galois. The name substitution is applied to the transition from one arrangement of objects, that is to say, from one permutation to another. An assemblage of substitutions of n objects form a group if the result obtained by effecting successively two substitutions of this assemblage can be obtained with a single substitution of the assemblage; in other terms, if the product of two substitutions be equivalent to a single substitution of the assemblage. In particular, all the possible substitutions of n objects form a group. The notion of group is very general and can be applied not only to substitutions, but also to all arithmetical operations and to analytical transformations of all kinds. Thus the labours of Sophus Lie have made the theory of groups of transformations into a method of marvellous extension and fruitfulness.

[21] Russell, *op. cit.* pp. 111-120; Couturat, "Sur une définition logique du nombre," *Revue de mét. et de mor.*, January 1900, p. 21 ff.; "Sur les rapports du nombre et de la grandeur," *ibid.*, July LM'M, p. 422 ff.; Russell, "On the Relations of Number and Quantity," *Mind*, N.S., vol. vi. No. 23.

[22] *Was sind und was sollen die Zahlen?* (Brunswick, 1887).

[23] *Science et hypothèse*, p. 20 ff.

[24] "Grundlagen einer allgemeinen Mannigfaltigkeitslehre" (Leipzig, 1883), *Math. Ann.* vol. xxi.

[25] See what has already been said in Chapter II. Part I. on this apparent paradox *à propos* of Renouvier's criticism of the infinite.

[26] "Sui fondamenti della geometria proiettiva," postulate vii., *Rendiconti del R. Istituto Lombardo*, serie ii. vol. xxix., 1897.

[27] "Beiträge zur Begründung der transfiniten Mengenlehre," *Math. Annalen*, vol. xlvi., 1895, and vol. xlix., 1897.

[28] Russell, *op. cit.* p. 299; Couturat, *op. cit.* p. 95; "Sur la définition du continu," *Rev. de mét. et de mor.* vol. viii. pp. 157-168.

[29] *Math. Annalen*, vol. xxxvi. pp. 157-160.

[30] *Op. cit.* p. 132.

[31] It is superfluous to remark that the relations in question are non-symmetrical and transitive, like those characterising linear order. Russell, *op. cit.* p. 374 ff.; Couturat, *op. cit.* p. 134.

[32] In the two articles already quoted: "I Principii della geometria di posizione, etc.," and "Nuovo modo di svolgere deduttivamente la geometria proiettiva." In the latter, Pieri constructs projective geometry with primitive entities (projective point and homography) and postulates differing from those of the first article. These two hypothetical and parallel developments are an extremely interesting example of equivalent systems, *i.e.* such as permit of the primitive entities of the one being defined by those of the other, and the primitive propositions of the one being deduced from the postulates of the other. This proves the choice of indefinable entities

and of primitive propositions, which are not capable of proof, to be arbitrary (*Rendiconti del R. Istituto Lombardo di Scienze e Lettere*, serie ii. vol. xxxi. p. 797).

[33] *Op. cit.* p. 82.

[34] *Op. cit.* pp. 404-407.

[35] "La Geometria basata sull' idea di punto e di distanza," *Atti della R. Accademia delle Scienze di Torino*, 1902.

[36] Enriques, *Questioni riguardanti la geometria elementare* (Bologna, 1900), Article 3, "Della congruenza e del movimento."

[37] In the Article already quoted, "Della geometria elementare come sistema ipotetico deduttivo."

[38] The following are some of the postulates laid down by Pieri (*Memorie della R. Accademia delle Scienze di Torino*, serie ii. vol. xlix. p. 179 ff.) as the basis of metrical geometry:—(1) The point and motion are generic and class concepts. (2) At least one point exists. (3) If p be a point, there exists a point differing from p. Every assemblage of points shall be termed a figure. Two figures are identical (or coincide) when they are composed of the same points. Two points are identical (or coincide) when every figure containing one of them contains the other also. (4) Every motion is a bi-uniform correspondence of two figures; this means that identical points correspond to identical points, and different points to different points, etc.

[39] Couturat, *op. cit.* p. 204.

[40] *Op. cit.* pp. 205-207.

[41] Russell, *op. cit.* p. 372.

[42] *American Journal of Mathematics*, vol. iv., 1881, p. 97 ff.

[43] Couturat, *op. cit.* pp. 242-246; cp. also Frege, *Grundgesetze der Arithmetik begriffschaftlich abgeleitet* (Jena, 1893–1903), in two volumes. In this work the whole of arithmetic is constructed analytically.

[44] Couturat, *op. cit.* p. 250.

[45] *Op. cit.* p. 253.

[46] *Op. cit.* p. 257.

[47] *Op. cit.* p. 267.

[48] *Op. cit.* pp. 279-285.

[49] *Op. cit.* p. 293.

[50] *Conférences sur les mathématiques* (Paris, 1895), Lecture VIII.

[51] *Op. cit.* p. 45.

[52] *La Valeur de la Science*, p. 11 ff.

[53] Both Borel in "La Logique et l'intuition en mathématique," *Revue de mét. et de mor.*, May 1907, and Lucas de Peslouan in *Les Systèmes logiques et la logistique* (Paris, 1909), p. 13 ff., p. 220, also place the necessity of intuition for the discovery of mathematical truths in opposition to logistic.

[54] "Pour la logistique," *Rev. de mét. et de mor.*, March 1906, p. 214 ff.

[55] "Les Mathématiques et la logique," *Revue de mét. et de mor.*, November 1905, pp. 215-835; January 1906, pp. 17-36; May 1906, pp. 294-317. "À propos de la logistique," *ibid.*, 1906, pp. 866-868. See also the criticisms of Winter, Borel (*ibid.*, May 1907), and Reymond, *Logique et mathématiques* (Paris, 1908), p. 133 *sq.* These criticisms have been replied to by Russell: "Sur la relation des mathématiques et de la logistique," *Revue de mét. et de mor.*, November 1905, pp. 906-917; "Les Paradoxes de la logique," *ibid.*, September 1906, pp. 627-650; Couturat, "Pour la logistique," *ibid.*, March 1906, p. 208 *sq.*; Pieri, "Sur la Compatibilité des axiomes de l'arithmétique," *ibid.*, March 1906, p. 196 ff.

[56] "Pour la logistique," *Revue de mét. et de mor.*, March 1906.

[57] *Op. cit.* p. 69.

[58] Couturat (*op. cit.* p. 51) writes as follows: "It is certainly not forbidden to think of concepts and their relations through their intensions, but they only enter into our formulas by their extension, and their relations in extension are those which form the basis of the logical calculus."

[59] *Op. cit.* p. 36.

[60] *Op. cit.* p. 39.

[61] Poincaré lays special emphasis upon this point in his polemic against the logisticians (Article already quoted).

[62] See Article quoted above.

[63] Pieri in his more recent utterance (*Uno Sguardo al nuovo indirizzo logico-mathematico delle scienze deduttive* (Catania, 1907), p. 51 ff., observes that Russell's logical constants, though sufficient for the construction of numerical and geometrical entities, are insufficient to assure us of their existence. He considers that the following existential judgment should be included among our logical premisses: "There exists at least one infinite class of entities," to deduce the existence of the whole numbers from them. From this it would then be possible to derive the existence of any other mathematical entity we choose.

[64] Besides, Russell too recognises the necessity of having recourse to intuition, "Les Paradoxes de la logique," *Revue de mét. et de mor.*, September 1906, p. 630 ff.

[65] "L'Importance philosophique de la logistique," *Revue de mét. et de mor.*, 1911, p. 283 ff.; *The Problems of Philosophy* (London, 1912); "The Basis of Realism," *Journal of Philosophy, Psychology, and Scientific Methods*, vol. viii. p. 158 ff.

[66] "The Programme and first Platform of six Realists," *Journal of Philosophy, Psychology, and Scientific Method*, vol. vii. p. 393 ff.; *The New Realism; Co-operative Studies in Philosophy*, by Edwin B. Holt, Walter T. Marvin, William Pepperell Montague, Ralph Barton Perry, Walter B. Pitkin, and Edward Gleason Spaulding (New York, 1912). Dewey, "The Short Cut to Realism examined," *Journal of Philosophy, Psychology, and Scientific Method*, vol. vii. p. 553 ff.; Spaulding, "Realism, a Reply to Professor Dewey," *ibid.* vol. viii. p. 63 ff.; Dewey "Rejoinder to Dr. Spaulding," *ibid.* vol. viii. p. 77 ff.; Drake, "The Inadequacy of Natural Realism," *ibid.* vol. viii. p. 365 ff.; Dewey, "Brief Studies in. Realism," *ibid.* vol. viii. p. 393 ff.; p. 546 ff.; Calkins, "The Idealist to the Realist," *ibid.* vol. viii. p. 449 ff.; Lovejoy, "Reflections of a Temporalist on the New Realism," *ibid.* vol. viii. p. 58 ff.; "On some Novelties of the New Realism and Subjectivism," *ibid.* vol. x. p. 43 ff.; Cohen, "The New Realism," *ibid.* vol. x. p. 197 ff.; Perry, "Some disputed Points in neo-Realism," *ibid.* vol. x. p. 449; Pratt, "Professor Perry's Proofs of Realism," *ibid.* vol. ix. p. 573 ff.

[67] "Über die Stellung der Gegenstandstheorie im System der Wissenschaften," *Zeitschrift für Philosophie und philosophische Kritik*, October 1906, p. 48 ff.; January 1907, p. 105 ff.; April 1907, p. 1 ff.; *Untersuchungen zur Gegenstandstheorie und Psychologie* (Leipzig, 1904), which in addition to the first article by Meinong, entitled "Über Gegenstandstheorie," contains papers by his followers, Ameseder, Mally, Frankel, Benussi, Liel, and Saxinger.

[68] *Mind*, vol. xiv., 1905, p. 533.

[69] "Über die Stellung der Gegenstandstheorie," Article 1, p. 63 ff.

CHAPTER III

ENERGETICS

1. *Traditional Mechanism.*—From the Renaissance until the middle of the nineteenth century scientific thought was dominated by the concept that the key to all natural phenomena was to be sought in mechanics. In the reaction from the Scholasticism which saw in every new phenomenon the working of some new occult quality, and had no hesitation in multiplying the number of these qualities *ad absurdum*, the science of the Renaissance was inspired by the ruling idea that the true essence of physical reality is to be found in quantitative relations. If, as Kepler affirms, *mundus participat quantitate*, what but mechanics, which Leonardo da Vinci terms " the paradise of the mathematical sciences," can serve as the foundation upon which all experimental science is to be mathematically built up ? The master mind of Galileo regarded the book of nature as written in mathematical characters, and the geometrical properties of bodies, figure, magnitude, rest, and motion as the *first and real accidents*, which exist objectively, whereas other qualities only exist in relation to the organ of sense.[1] In like manner Descartes, reducing matter to extension only and making all variations of bodies dependent upon motion, absorbs physics into mechanics and mechanics into geometry.[2]

In spite of the criticisms of Berkeley and Hume, the division between primary and secondary qualities, taken up by Hobbes and Locke, is accepted as a dogma by all

those scientists the tendency of whose researches enables them to say with Descartes: *Terram totumque hunc mundum instar machinae descripsi*, and the dream of a universal system of mechanics is realised in Huygen's undulatory theory of light,[3] Bernouilli's kinetic theory of gases, and Mayer's mechanical theory of heat. Just, however, when the great work seemed to be accomplished with the inclusion in the same system of the mechanical laws of the great revolutions of the planets and the imperceptible motions of atoms, and the hope had dawned that it might be possible to comprise even the phenomena of life within these universal formulas, the first doubts as to the value of mechanical explanations began to arise.

In the first part of this book we have studied the general causes of a philosophical order which brought about the critical revision of science and a new epistemological valuation of it towards the end of the nineteenth century; we shall therefore now confine ourselves to pointing out the scientific reasons which led physicists to reconstruct their theories upon new bases.[4]

The supreme principles of energy, undoubtedly the greatest triumph of the last century in the realm of theoretical physics, did much to undermine faith in traditional mechanism. The principle of conservation of energy appeared at first sight to confirm the mechanical theory: Helmholtz, Clausius, and Kelvin, indeed, took Mayer's discovery of the equivalence of heat and work to be a proof of the possibility of reducing all forms of energy to kinetic energy; an interpretation which was not legitimate. Mayer's experiments merely proved that it was possible to transform one form of energy into another, that a determinate quantity of one such form always corresponded to a determinate quantity of another; it did not, however, afford any authority for regarding one form of energy, *i.e.* the kinetic, as the basis of all other forms. We should be equally entitled to conclude that heat, light, motion, and chemical affinity are but different manifestations of the same electrical

energy. As Ostwald correctly observes, the equivalence of all forms of energy, far from authorising us to reduce one of these forms to another, places them all on the same level. In any case the principle of conservation, if it could not be adduced in support of the mechanical theory, since it rather impelled thought to transcend that theory and to comprehend it as a special instance in a wider theory, based upon the general laws of energetic transformations, did not, on the other hand, constitute a decisive argument against it.

2. *Carnot's Principle.*—The heaviest blow to the mechanical theory was dealt by the other principle—that of Carnot. Mechanics, as ordinarily understood, is the study of reversible phenomena: if to the parameter representing time, which has acquired increasing values in the course of the development of the phenomenon, we assign decreasing values which cause its inversion, the whole system must once more go through just the same stages as it has already traversed. Now Carnot [5] and Clausius [6] have shown (the latter even more plainly than the former) that this is not verified in the transition from thermal to kinetic energy: if a determinate quantity of work be expended in order to raise the temperature of a body, we cannot return exactly to the initial stage of the process by inverting the cycle: however perfect the machine may be, we shall never obtain from the lowering of the thermal level the same amount of work as we have employed: there will always be a portion of the thermal energy which is not transformed into work, which is, as it is commonly expressed, lowered or degraded. This irrevocability, which is a general characteristic of the physical world, and this evolution of nature in a determinate direction do not admit of explanation by the mechanical theory. Were physical phenomena exclusively due to the motions of atoms, whose mutual attraction depended upon nothing but distance, they should be reversible: when all the initial velocities are reversed, the atoms subjected to the action of the same forces should pursue their

trajectories in the contrary sense, just in the same way as the earth would describe in a retrograde sense the same elliptic orbit which it describes in a direct sense, were the initial conditions of its motion inverted. The attempts which have been made to reconcile Carnot's principle with traditional mechanics have failed to give satisfactory results: it has almost always been necessary in the course of the deduction to introduce some new hypothesis, independent of the fundamental principles of mechanics, and equivalent in reality to one of the postulates upon which the ordinary enunciation of the second law of thermodynamics is based.[7] Helmholtz has endeavoured to include Carnot's principle in the principle of least action; this does not, however, eliminate the difficulties involved in the mechanical interpretation of the irreversibility of phenomena. Gibbs,[8] Boltzmann,[9] and Planck have advanced extremely interesting ideas on this point: they consider that the principle indicates that a given system tends to the configuration offering the maximum probability: thus two different gaseous masses, placed in two separate receptacles, will, when communication is opened between the two receptacles, become diffused into one another, and it is extremely improbable that in the course of the reciprocal shocks the two kinds of molecule will assume a distribution of velocity bringing them back to the initial state by a spontaneous phenomenon. Carnot's principle would then merely express a law of probability: an extraordinary concurrence of circumstances would be required in order to render phenomena reversible. The return to the primitive state demands a very great interval of time, but is not absolutely impossible. This attempt at reconciliation is undoubtedly sufficiently ingenious: at bottom it does, however, but oppose the possibility of a fact to a phenomenon proved by experience, though this procedure does not answer to the rules of scientific method. Physical theories must formulate the laws of observed phenomena, not of those which

are more or less probable. It is better to seek another theory which affords a more satisfactory explanation of the phenomena which have been hitherto observed than to speculate upon mere possibilities, in the hope of saving the mechanical hypothesis at all costs.

3. *Evolutionary Genesis of Chemical Elements.*—In the realm of chemistry also the new discoveries have led science to abandon the old concept of the indestructible atom and to substitute an energetic conception for the old atomic theory. As early as 1815 Prout, starting from the hypothesis that the atomic weights of all simple bodies were exact multiples of the weight of hydrogen, had maintained the unity of composition of the elements, regarding them as formed by successive condensations of hydrogen. The periodic law, first suggested by Newlands in 1864, and elaborated by Mendelejeff in 1869, according to which the chemical and physical properties of elements are periodic functions of their atomic weights, confirmed the hypothesis of the unity of composition of the elements; so that, when William Crookes, in a discourse on "The Genesis of Elements," [10] given at the Royal Institution on February 18, 1887, and in his presidental address to the Chemical Society in the following year on "Elements and Meta-Elements," [11] proving the falsity of the belief that chemical elements have existed *ab aeterno* as we now observe them, and cannot be subjected to change or decomposition, maintained that these elements are neither simple, nor primordial, nor created all at once, but that they are the result of evolution, he did in reality but formulate an idea which had, if I may so put it, already been for long in the air of science. Crookes, however, must be credited with having placed the evolutionary hypothesis in chemistry upon an experimental basis by analysing the spectra of certain metals belonging to the series of rare earths, such as yttrium, samarium, thorium, etc. If one of these metals, yttrium, for example, be subjected to a long and tedious process of fractionisation, and the parts thus obtained become

phosphorescent by the action of the induction-spark in tubes with the rarefaction of a millionth part of an atmosphere, different spectra will be obtained, revealing the existence of specifically distinct substances. The conclusions thus reached by Crookes have been corroborated by Lockyer's long and painstaking spectral analyses,[12] which prove the erroneousness of the theory that each chemical element has but one spectrum. The application of a higher degree of temperature than had previously been employed led Lockyer to admit the existence of component parts of a still more subtle nature in substances which had been regarded as simple. The same element will present different spectra if it be exposed to different degrees of temperature: certain lines in the spectra of iron, calcium, and magnesium are not visible at a low temperature, but become more intense when the temperature is raised—a proof that the atoms composing these substances are not immutable and indivisible, but are subject to change and dissociation.[13]

The recent discoveries of cathode rays, X-rays, and radio-active bodies such as uranium and radium, and the proof that radio-activity does not belong to certain bodies only, but constitutes a general property of matter, have corroborated from another point of view the hypothesis of the evolutionary formation of elements and of their dissociation; since such radiations, according at all events to Le Bon's theory,[14] which is based upon numerous experiments, are but the product of the dematerialisation of atoms, which thus restore by means of a slow process of dissipation the energy stored up in them during the period of cosmic formation. Energy and matter are at bottom but two forms of one and the same thing; one is a stable, the other an unstable form of intra-atomic energy; when the atoms are dissociated, that is to say when matter is dematerialised, the stable form of energy, which is termed matter, is merely transformed into the unstable form known as electricity, light, heat, etc.[15]

4. *Physical Chemistry.*—The researches of physical

chemistry, and more especially of its most flourishing branch electro-chemistry, which were inspired and promoted mainly by Wilhelm Ostwald,[16] have done much to direct thought towards an energetic conception of material transformations. If Dalton's atomic theory be rejected as a useless and arbitrary hypothesis,[17] chemistry enlarges its borders and looks to a dynamic theory for a more comprehensive basis; chemical combination is regarded as a special instance of physical mixture, and a physical definition thereof is accordingly sought. It is mainly due to the labours of Van 't Hoff that the profound and accurate theory of diluted solutions, which institutes a comparison between solutions and gases from the thermo-dynamic point of view, came into being independently of any mechanical hypothesis; thus the fundamental laws of chemical transformations, *i.e.* the laws of equilibrium and of velocity of reaction, are independent of any mechanical presupposition whatsoever. The law of mass-action, by which bodies are chemically efficacious in ratio of their concentration, can be grasped perfectly well without the help of hypotheses, so much so that Berthollet, an opponent of Dalton's conception, was the first to enunciate it, and the application of it to single chemical processes was made by Guldberg, Waage, and Wilhelmy without having recourse to the attractive forces of atoms. Gibbs' law of phases [18] and Van 't Hoff's principle of mobile equilibrium are infallible guides, enabling us to foresee and calculate many chemical reactions, when the qualitative difference of bodies is disregarded: and the principles of thermo-dynamics thus render it possible to dispense with the old atomic theory, which, however efficacious it may be as a representation of a certain class of experimental facts, cannot afford a general view of all the phenomena.

5. *Energetics in Rankine and Spencer.*—Energetics, which has found its staunchest advocate in Wilhelm Ostwald, would fain replace this narrow one-sided scheme by wider concepts and principles, which are not

borrowed from one special branch of physics, regarded arbitrarily as the basis of all the others, but comprehend in one vast synthesis the characteristics common to the different classes of phenomena. Such a concept is nothing new either in the realm of science or in that of philosophy [19]: if we are not prepared to return to the dynamism of Leibnitz, which, as opposed to the mechanical theory of Descartes and Gassendi, regarded motion and extension as phenomenal manifestations of force, which last it defined as, *ce qu'il y a dans l'état présent, qui porte avec soi un changement pour l'avenir*,[20] we must at all events refer to two of his immediate predecessors, Rankine and Spencer. Some time before Ostwald, Rankine in an article published in 1846,[21] after distinguishing the *abstractive method*, which confines itself to the data of experience and does not theorise about that which lies beyond those data, but contents itself with gathering together the characteristics common to the various orders of phenomena in order that it may rise by induction to general and abstract concepts and laws, from the *hypothetical method*, which rather has recourse to conceptions of a conjectural order with regard to that intimate constitution of objects which is not revealed to us by the senses, advises us to reject the latter method, which is the one adopted by traditional mechanicism, in order to raise the new edifice of energetics with the aid of the former. If we would purge physics of the arbitrary hypotheses of masses, movements, and imperceptible forces, and reduce it to the abstractive form, we must find the properties which are common to the various groups of physical phenomena, forming classes of a more and more extensive order, until we reach the most general concepts and principles. Energy is precisely the most general property of physical facts, in as much as the power of producing an effect and of being a potentiality of changes is a characteristic common to them all; hence its laws must constitute the fundamental principles of the new physical theory, from which the special laws

governing the different groups of phenomena are derived by a process of deduction. Spencer, too, as is well known, looking at the matter from the philosophical and more particularly the psychological point of view, in his *First Principles*, after a not very successful analysis of the presentations of space, time, matter, and motion, arrives at "force," which he regards as the ultimate and irreducible element in the experience of the outer world.

6. *Ostwald's Phenomenalistic Programme.*—Ostwald goes still farther than Spencer: the concept of energy not only shows us how to systematise the experience of the external world, but enables us to penetrate the innermost recesses of human consciousness, and sheds its light on the loftiest spheres of the mind, the noblest manifestations of art and morality. He affirms that matter only exists in thought; the real, that which truly acts upon us, is energy.[22] We must get wholly rid of figurative hypotheses and analogies with mechanics, and construct a science devoid of hypotheses—*eine hypothesenfreie Wissenschaft.*[23] Is such an undertaking possible? Some men of science would absolutely deny such a possibility, maintaining that every mathematical formulation of phenomena is based upon some hypothesis: thus in mechanics we speak of perfectly rigid bodies and of absolutely frictionless fluids, etc., which do not exist in nature and are therefore only admitted by hypothesis. This argument, however, fails to distinguish between two different things. There can be no doubt that the relations expressed by natural laws are never perfectly realised in experience, because they refer to abstractions, that is to say to real phenomena, minus certain aspects thereof which we voluntarily leave out of consideration. This holds good of all the magnitudes which we disregard and look upon as null, not because they are so in reality, but because they are too small for us to measure. We frequently disregard even measurable quantities, because we have not yet found out how to calculate them. Ostwald terms this general procedure which is dependent upon the nature of our

mind *the method of abstraction* (*Abstractionsverfahren*), in order to distinguish it from all other scientific methods, and more particularly from hypotheses properly so called.[24] In hypotheses the procedure is reversed: we do not for convenience' sake disregard certain parts of phenomena, but add to them characteristics which are not given to us by experience, and which can never admit of scientific proof, by forming images or models drawn for the most part from the realm of mechanics, which represent certain aspects of phenomena in an intuitive and communicable form. The images are of course chosen in such a way as to represent the properties of the phenomena by means of corresponding properties of the images. Can an image be found capable of affording a complete presentation of all the characteristics of phenomena? Ostwald considers this to be an impossibility, because when using images in the representation of phenomena it is impossible to avoid adding to them certain essential parts which are found in the model, but not seen in the phenomenon; now between these extraneous parts and the corresponding characteristics of the phenomenon a contradiction will inevitably arise sooner or later, which will make the given model useless. Is it not possible, however, to make such a choice of images that this contradiction will not arise? No, because, did the image and the object coincide in all their parts, they would be the same thing. A phenomenon can only be perfectly represented by means of itself: every representation thereof by means of other phenomena of a more or less analogous order must of necessity contain extraneous elements. But, it may be objected by the advocates of hypotheses, all the mathematical formulas, by means of which we express, for example, the relation between velocity and time of descent, between tension and current, are also merely images of reality and not reality itself, so that science from start to finish is constructed with the help of these images. Ostwald replies to this that we must draw a distinction between formulas and images:

formulas possess no part which is peculiar to them, but merely those elements which we put into them and derive from experience. Every letter is a symbol, standing for a phenomenal multiplicity having the character of a magnitude, and has no further meaning; the relations set up by formulas between such symbols, corresponding to objective magnitudes, can all be verified by means of experience. In models or images, on the contrary, properties are added which are not revealed to us by phenomena, and cannot therefore ever be verified. We must for this reason dismiss these arbitrary hypotheses: science must set itself the problem of representing phenomena in such a way that those elements only shall be comprised in the representation which can be observed in experience and those characteristics which are not capable of verification put aside. This method excludes all intuitive models or physical images and leaves as means of representation nothing but numbers and algebraical signs which stand in their stead. We must, however, bear in mind that hypotheses are frequently concealed in formulas. How are we to recognise them? If every magnitude comprised in the formulas be measurable in itself, we are really dealing with a relation between phenomena, a true natural law: if, on the other hand, the formulas contain magnitudes which are not measurable in experience, we may rest assured that we shall find ourselves confronted by hypotheses in mathematical form. Let us take, for instance, the formula of the kinetic theory of gases, $pv = \frac{1}{3}mnc^2$. If we apply this criterion, we see at once that measurable magnitudes are symbolised in the first side of the equation, namely pressure and volume, whereas the second side, on the contrary, in which m represents the mass of a molecule, n the number of molecules, and c their velocity, includes non-measurable magnitudes.

7. *Criticism of the Traditional Mechanical Theory.*— The traditional mechanical theory, in as much as it is wholly based upon the use of hypothetical images, fails to answer to the requirements of scientific method:

its theories do not give a true reflection of the relations of phenomena; it rather makes arbitrary additions thereto, in the mistaken belief that it is thus enabled to penetrate more deeply into the nature of things. In thermal phenomena, for instance, no mechanical property comes under immediate observation, and, if it be admitted that these phenomena consist in movements, we go beyond facts and construct an imaginary world of invisible particles, describing in space trajectories which are equally invisible. Experience can, of course, never prove the falsity of similar theories, because our imagination, being at liberty to form any image it likes of these movements, can always modify them in such a way as to bring them into agreement with facts. But if this hypothesis cannot be shown to be false by experimental proof, it must be rejected from the methodological point of view, since it does not simply transcribe that which is given by experience, but contains elements which can never be verified. The upholders of the mechanical theory maintain that their procedure is legitimate, because it endeavours to reduce the unknown to the known, as is done in every explanation; but this amounts to the postulate that mechanical phenomena are better known than other phenomena. This is by no means true at the present stage of science, since the very same functional relations between magnitudes to which our knowledge of the laws of motion and masses is in the end reducible have been established between the magnitudes of the other groups of thermal, electrical, and chemical phenomena, without the help of the mechanical hypothesis. Undoubtedly there was a time in which mechanical phenomena were better known than others; it is, however, owing to a mere historical accident that the motions of masses were the first to attract the attention of students of science.[25] Had researches into heat been made first, there would have been a tendency to write books entitled: *Motion considered as a Form of Heat*, instead of those bearing titles such as that of Tyndall's

work: *Heat considered as a Form of Motion.* Hypotheses are the perishable part of science, there is, however, something which lives on as a lasting acquisition; namely, the laws which express the relations between the magnitudes of experience. *Naturgesetze sind dauernd, Hypothesen sind vergänglich.*[26] The laws of stoicheiometry will endure when the atomic theory exists only in the pages of dusty forgotten books on our library shelves, and Ohm's laws will hold good for ever, no matter what representation of the essence of electric energy may be prevalent in the future. But if Ostwald denies the usefulness of hypotheses even as heuretic means, and affirms that we have nothing to do but keep our eyes open in order to make new discoveries, and that working hypotheses are indispensable only to those who are not capable of advancing without them, just as crutches are to those who cannot walk without their aid, he intends this assertion to apply merely to conjectures as to what lies beyond phenomena, not to anticipations of experience, which he terms *prototheses* in order to distinguish them from hypotheses properly so called, and retains as a valid aid to scientific research.[27] We set up a *prototheses* if, on the ground of observations which are not wholly adequate, we establish a mathematical relation between measured magnitudes or a causal relation between variations which have been observed, and then proceed to make researches with the object of proving whether facts confirm these presuppositions or not. Prototheses are legitimate, because they do not leave the realm of experience and do not make arbitrary additions to observed phenomena.

8. *Energy as a Universal Substance.*—Science and philosophy must be constructed of materials which either have been proved or are at least capable of being verified. Ostwald proposes to give us a synthetic concept of nature, based exclusively on experience which he defines as *das, was man erlebt,*[28] that which is immediately experienced. If the name substance is applied to that which is permanent in the external

world and the name accident to that which is variable, at the present stage of science we may make the statement that energy is at one and the same time the substance and the most general of accidents.[29] All other physical concepts (mass, quantity of motion, etc.), whose magnitude is subject to the law of conservation, are applicable only to a determinate field of natural phenomena: energy is the only one which is found in all known phenomena without exception. This concept is therefore better suited than any other to afford a solution of the problem set by the concept of substance. Energy not only exists in all natural phenomena, but also determines them completely: all processes without exception can be accurately represented when it has been determined which energies undergo variations in space and time. Inversely the question: Under what circumstances does a phenomenon occur, or a certain thing take place? can generally be answered by indicating the action of the energies put into play. We can express all we know of the external world in terms of energy, which thus appears to us to be the most general concept hitherto formed by science, because it solves not only the problem of substance but that of causality as well.[30] Energy may be defined as work or everything which is derived from work, and can be transformed into work. We experience it directly in the sensations of the muscular effort which is necessary to overcome a determinate resistance. All other sensations, however, are also due to transformations of energy, to labour accomplished in the organs of sense. That which we hear is the work done by the oscillations of the air in the inner ear, that which we see is but chemical work, the product of luminous energy; if we touch a hard body, we feel the mechanical work expended in the compression of our finger-tips and of the object; while both tastes and smells are due to a consumption of chemical energy. Generally speaking, our knowledge of how the world is ordered and of its properties is always due to energies or amounts of work: from such

a point of view nature in its totality is seen as a distribution in space and time of energies which are variable both with respect to space and to time, with which we are acquainted only in so far as these energies pass into our bodies and particularly into the sensorial organs adapted to their reception.

9. *Reduction of Matter to Energy.*—To what are matter and bodies reducible from the point of view of energetics? The variations in form of a body can be produced by the performance of work: an elastic body can change in form with the employment of a determinate quantity of work, which it gives back when it returns to its original conditions: we here have an *energy of form,*[31] *i.e.* an energy which is dependent upon the form of the body. In the same way the volume of a body can be diminished by compression applied in such a way as to preserve its form, by expending energy which will be restored by the body when it reassumes its original volume, and may be termed *energy of volume* because it depends upon the volume of the body.[32] Unless energy be added from without, every body must preserve its form and volume in accordance with the principle of conservation of energy. In touch we experience only the spatial relations of energy of form and volume. In the last analysis the weight of bodies is reducible to energy: in order to raise a weight a certain amount of energy must be consumed, which is restored during its fall. This energy may be termed *energy of distance,* in as much as it is dependent upon distance,[33] and takes the place of the old *force of attraction.* If a body be dropped, it loses energy of distance and acquires velocity, *i.e. energy of movement.*[34] This energy is dependent upon another circumstance as well as upon acceleration: experience teaches us that the labour required in order to impress the same velocity on two stones of different weights will differ in quantity. This second property, upon which the energy of a body in motion is dependent, is termed *mass.* In scientific phraseology the word mass means nothing but this

relation to an energy of motion : it is senseless to define it as quantity of matter, since we are ignorant as to what matter is, and as to how its quantity can be measured. We say that the mass of two bodies is equal when one of these bodies can be substituted for the other without any variation in the velocity when the same amount of work is expended. The principle of inertia can also be explained by the law of conservation of energy.[35] In the form imparted to it by Newton it would appear that a body will follow its rectilinear direction, if nothing intervenes to disturb it, because it has taken it into its head to do so ! In reality, however, that which is termed inertia is merely the conservation of energy of motion, so long as no other energy is added from without.[36] The laws governing the motion of bodies in space are derived from the reciprocal relations between energy of distance and energy of motion. These two forms of energy are not localised in the extension occupied by the matter of each individual body, but extend through space as a whole. A single point endowed with energy of distance is unthinkable, since this energy is dependent upon its approximation to or recession from another point, and the reciprocal spatial relations of different bodies are therefore essential to it. The enigma of action at a distance thus vanishes.

The fundamental concepts and laws of mechanics are thus comprehended as a special case within the wider theory of energetics ; matter resolves itself into a complex of energies ordered together in space. Exposition in energetic terms offers the great advantage of representing facts without having recourse to any hypothesis whatsoever : the only concepts introduced into this exposition are those of magnitude and intensity, which can be observed and measured ; nothing is admitted into bodies which cannot be proved by experience and measurement. Some objectors say it is preferable in teaching to start from the concept of force in order to derive that of energy from it, since the former is a primitive datum,

whereas the latter can only be defined as the product of force and space. Ostwald replies that a datum is that which we experience by means of our senses, and these react, as we have already seen, not to forces, but to energies. From such a standpoint the concept of energy may be regarded as original with greater right than that of force. We may further observe that the resolution of energy into its two factors can be accomplished in a thousand ways, according to the length of space selected: for the same work there may therefore be a thousand different forces, and the choice of one rather than another is arbitrary. Work is then a more general concept by comparison with the accidental product of the force and space which has been selected.[37] The fact that the concept of force was discovered first in point of time does not imply that it is simpler. The various forms of energy are distinguished qualitatively, and cannot be identified with one another, because each one of them possesses specific characteristics. We may, of course, assume determinate quantities of energy of distance, movement, heat, electricity, etc., to be equal to one another, if in the process of transformation into the same form of energy they produce the same quantity thereof; but this is the only equality which exists, and they differ in every other aspect, just in the same way as two pieces of iron and silver may be equal in weight while differing in every other respect. Thus, for instance, the marked dualism of electric and magnetic energy is not in the least apparent in heat, which is determined by a single number when the unit is given; kinetic energy has a direction in space, whereas energy of volume acts in every place, and in all directions, where a change of volume is possible.

10. *Energetics and Vital Psychic Phenomena.*—The concept of energy not only affords us a means of systematising physical phenomena, but also enables us to order biological and psychic phenomena scientifically. Even vital processes are in the last analysis reducible to transformations of energies, which, unlike

the processes of the inorganic world, possess the property of conserving the system. If it be possible in each single case to determine by what means and by the action of what energies the problem of conservation of the organism has been solved, everything which we can fairly ask to know has been said on the subject of the phenomena of life. Biological facts present no insoluble enigma, and contain nothing which cannot be expressed in terms of energetics. This is the only sense in which vital phenomena need an explanation, since to explain, according to Mayer, is but to know a phenomenon in all its aspects. Against the view of the neo-vitalists, it may be urged that when we know what energies take part in the phenomena of life, and in what the means of self-regulation and conservation of organic individuality consist, we shall have the explanation of the phenomena of life.[38]

Nor do the phenomena of consciousness elude energetics. The researches of physiological psychology have proved that no psychic process ever takes place without a consumption of energy. Consciousness must not be regarded as a phenomenon concomitant with the energetic processes developed in the brain, according to the theory of psycho-physical parallelism, but as an energy due to the transformation of the chemical energy of the brain, which is subject to the same laws as the other forms. The fact that all the processes of the external world can be represented as transformations of energy is explained if it be granted that consciousness is itself energetic and communicates this property to external experience.[39] The relations between mind and body cease to present any difficulty when energy is put in the place of matter. On the old materialistic conception, which affirms that mind is a property of the material atoms of the brain, this property should, according to the law of conservation, also be met with in these atoms outside the organism in the physical world, that is to say, we should logically arrive at animism. This difficulty vanishes in energetics. It is

in fact possible to change a determinate quantity of energy into another without any measurable residuum being left; experience is therefore not against the thought that certain forms of energy require certain relations in order to come into being, and that the quantities of them which are thus obtained can by transformation undergo a complete change into other forms. This is precisely what takes place in the case of the spiritual energy which is born of a transformation of the chemical energy of the brain under determinate conditions, and is then changed back into it again, completely restoring the work consumed.[40] The energy of consciousness is the loftiest and rarest form known to us: it is found in extremely complicated organisms only, and not always in equal quantity and efficacy even in human brains; we must not, however, be surprised that such a form of energy should arise only under determinate circumstances, since this is the case with the other energies as well: for instance, a small number of crystals only are suitable for the production of electrical energy by means of pressure.[41] Thus by putting energy in the place of matter, it is possible to construct a philosophical synthesis, which will afford an adequate explanation of all the phenomena of mind and matter.[42]

11. *Criticism of Energetics: Ostwald the Phenomenalist, and Ostwald the Metaphysician.*—It does not lie within our province to estimate the work done by Wilhelm Ostwald in the domain of chemistry, where his influence has undoubtedly brought forth fruit, and where he has done pioneer work by striving to demolish the artificial barriers between chemistry and physics. It is true that his endeavours to banish the atomic theory in the deduction of the fundamental laws of stoicheiometry (law of definite proportions, multiple proportions, and proportional numbers) have not met with assured success; this does not, however, alter the fact that his ideas, paradoxical as they may be, have inspired fresh researches of the utmost importance. In the realm of philosophy Ostwald is by way of

being a phenomenalist, and is so as long as he is merely engaged in forming the theory of scientific method and confines himself to waging war against the atomic theory; in this direction he is a faithful follower of Mach, in as much as he affirms that science should be reduced to the enunciation of the pure mathematical relations existing between the phenomena of experience; but he forsakes Mach when he replaces the sensations by energy as their universal substance. The metaphysician then triumphs over the empiricist, and—a more serious matter this—Ostwald is unaware of it, and is under the delusion that he has put forward no hypothesis, and has used nothing but the pure data of experience in the construction of his philosophy of nature. But is energy an experimental datum? Ostwald is constantly telling us that the sensations give us nothing but energy, and fails to see that he is thus making the absurd admission that a sensation can impart knowledge of a concept to us. Sensations in themselves afford us no immediate revelation of either matter, force, or energy: it is senseless to discuss whether they give us the one or the other. Sensorial data are the heterogeneous qualities of colours, sounds, smells, etc., not energy. Energy is a construction of our thought which we cannot regard as reality, unless we would fall into the old error of hypostatising a concept. When you say that sensations are energy, you go beyond phenomena, beyond that which is experienced, *das, was man erlebt*, you make an induction about the objective cause of the sensations. A return is thus made to those hypotheses with which we were supposed to be able to dispense, and we prove afresh that it is impossible to philosophise without adding anything to data. Empiricists delude themselves into thinking that they are deriving everything from experience; they fail, however, to perceive that it is no difficult matter to derive from experience that which the mind of man has first unknowingly placed there.

Energy then is a concept, not a datum. Is there

any meaning in assuming it to be an objective and universal reality? At bottom when the content of this idea is thoroughly analysed, it will be seen that it is reducible to a mere potentiality of changes, *i.e.* of sensations, and our thought refuses to conceive of a pure potentiality as real, still less to regard it as a permanent entity, a substance. The physicist, and more especially the phenomenalistic physicist, such as Ostwald claims to be, cannot take the law of conservation of energy as signifying the permanency of a substantial entity, but merely as the expression by means of a simple and abbreviated formula of a system of relations of proportion between electrical, mechanical, thermal, and other quantities: the law is only possessed of scientific value and content when taken in such a sense. When, on the other hand, it is stated that energy is the common source of phenomena, we are giving utterance to a mere tautology which tells us nothing of physical reality: changes take place, because energy, *i.e.* the potentiality of changes, exists! This amounts to the resurrection of the ancient *virtus dormitiva*, *virtus sanativa*, and the like, of blessed memory, which our fathers fondly imagined to be the explanation of all things. According to Ostwald's definition, the concept of energy would comprehend not only these potentialities, but actual work as well; he thus calmly postulates that the causes and effects of work are of the same nature as work itself. Now, is it evident that the cause of a phenomenon need be homogeneous with the phenomenon itself? Is not one of the most serious objections to the mechanical theory that from the fact that heat, electricity, etc., *can cause* motions, it draws the conclusion that heat, electricity, etc., *are* motions? Ostwald falls into the same error when he regards mechanical work as being of the same nature as its electrical, thermal, and other causes. The name is changed, but not the fundamental error, *i.e.* the transposition of effect into cause. It is an error which becomes still more serious when the cause or effect is no

longer a phenomenon of the physical world, but a process of consciousness, because the general principles of energetics cannot be applied to a world whose variations are purely qualitative, and which it is therefore impossible to measure.[43] The world of mind, and, to a certain extent, that of life as well, in as much as it is perennially evolved by the multiplication of its qualities and forms, cannot be explained by the principles of physical energy, which, when left to itself, should in accordance with the second principle of thermo-dynamics sink towards a greater uniformity.

Ostwald, in spite of his protests of phenomenalism and his theoretical affirmations, does but substitute a new metaphysic for the old one of materialism, and is very far removed from the method suggested by Ernst Mach.[44] Empirio-criticism regards nothing as existing but phenomena, and their relations; everything which is force, impulse, activity, potentiality, etc., is a remnant of anthropomorphism, an hypostatisation of the tendencies and desires proper to human consciousness. The concepts of work, energy, force, mass, etc., are in Mach's eyes mere schematic, shorthand signs indicating complex systems of quantitative relations between the different qualities of experience. Ostwald, on the contrary, does not look upon the concept of energy as a functional formula, an analytical expression, but as a real entity, the true substance of the world. Of what avail is it for him to adopt Newton's *Hypotheses non fingo*[45] as the motto of his own scientific method, when in reality he is perpetually transcending the phenomenal, objectivising an abstract construction of thought? The concept of energy, says Helm,[46] approaches so closely to facts as to exclude all danger of hypostatisation; what else is he doing, however, when he affirms that "energy is *the true element* of the world, because everything we know about the world we know also about energy?"[47] What else is Ostwald doing—and doing even more plainly—when he proclaims energy to be the universal substance and accident?

12. *Mechanics as the Necessary Basis of Energetics.*— No sensation, not even that of muscular force, gives us energy, which is an extremely complicated concept, in as much as it contains in itself other elementary concepts, *i.e.* the concept of cause and force, the concept of resistance, the concept of space, through which resistance is overcome: Ostwald's argument to prove that energy is a simpler element than force is a sophism. It is not only physically, but also psychologically possible to resolve work into its component parts: when a workman lifts a kilogramme to the height of a metre, he may represent the weight of the kilogramme (that is to say, the resistance encountered in raising the arm) as an element by itself, and the displacement of a metre as another element. The concept of resistance is derived from momentary muscular sensations before that of energy. Moreover, Ostwald himself, when attempting to define work, cannot avoid having recourse to the more elementary concept of resistance overcome through certain space. It is of no avail to argue that different forces can be made to correspond to one and the same work (by changing the value of the other factor, *i.e.* of space, in such a way that the product will remain constant), and that, therefore, the concept of work is more general as compared with that of force, since this argument can be turned the other way by saying that one and the same force can correspond to different amounts of work if the space be made to vary proportionately! Ostwald also maintains that an integration is saved in the exposition of physics in terms of energy; he fails, however, to see that this saving is due to the fact that in energetics we consider the ultimate effect, the sum total, without taking into account individual addenda, which are excluded from the first with a *sic volo, sic iubeo.* Now if in many cases the final result is the only thing of any interest to us, this does not mean that it is the only one of importance, and that more accurate and more detailed knowledge, such as that afforded us by dynamical physics, is not preferable to the

simpler but incomplete concept of mechanical facts offered us by energetics. It is true, for instance, in a phenomenon of capillarity, that we should give ourselves useless trouble and labour if we undertook to measure the intensity of force at each point of the imperceptible distances of displacement; that which is of greatest interest in this case is the sum of the quantities of work, which is obtained at once by separating two particles of the liquid from one another, because the constant of capillarity offers us a datum which is sufficient for our calculations. But if the energetician would make his knowledge so exact as to assign the augmentations or diminutions of work for every displacement of a micron, and if he would then reach the limit, so as to obtain the forces acting at every point of the field of force, he will be obliged to have recourse to calculations no less complex than those which are necessary in the methods of dynamical physics. Moreover, it is all very well for Ostwald to say that exposition in terms of energetics is simpler, and therefore preferable from the didactic point of view: it is impossible to define and understand what energy is without resorting to mechanical work, that is to say, to the movement and resistance of a material mass; and whatever efforts we may make, the concepts of mechanics will always remain the foundation of the building, the necessary starting-point even for him who would fain attain a higher synthesis. What else is energy but the potentiality of work? Can we form the concept of work if we disregard mechanical work? This form of labour alone is suggested to us by experience: all other forms are constructed by analogy. Electrical, luminous, thermal, and chemical phenomena are not presented to us immediately as work or energy, and if we interpret them in this way our interpretation is based upon an hypothetical induction. If we speak, for instance, of the resistance of an electric current, of energy necessary to overcome this resistance, we do so because we in reality consider ourselves entitled to

transfer by the law of analogy to electrical phenomena those concepts which have enabled us to render the motions of material masses intelligible.

When we feel a rise in the temperature of a body, or see water issue from hydrogen or oxygen in determinate proportions, do we experience any work directly? In that case do we feel energy, or do we not rather transcend the datum, assuming behind it a concept constructed by ourselves from the suggestion given by the muscular force necessary to overcome the resistance offered to us by a material body? Are we not thus setting up a hypothesis? The distinction drawn by Ostwald between hypothetical magnitudes and magnitudes which are directly measurable in experience does not bear criticism, since in the last analysis, if we except spatial magnitudes, all other magnitudes are measurable, not directly, but indirectly by means of hypotheses, which may be legitimate but are none the less hypotheses for that: thus time is measured by starting from the presupposition of a uniform motion; temperature by reference to the expansion of a heated body, and by presupposing that this expansion remains uniform in permanent gases; quantity of heat by the determination of temperature in bodies whose weight is known; weight by means of comparison with a normal weight, granting that the arms of the balance are equal on both sides; electric energy by means of the displacement of the magnetic needle, and so forth. Motion and spatial magnitude will always remain the necessary standard of reference of all other more or less hypothetical magnitudes, and energetics cannot avoid taking mechanics as its model. The age-long effort to absorb the whole of physics into rational mechanics is not a mere historical accident, or a fact which can be explained by the association of ideas: its deepest reason is to be found in the demand for intelligibility, which is wholly satisfied by the laws of pure motion. Mechanics do not propose to describe the motions of nature, as is alleged by Kirchhoff,[48] for the very simple reason that motions, as it studies

them and transcribes them in its equations, have never existed in nature, and no observation can reveal them to us. It would be the same thing to say that the object of geometry is to describe in the simplest way the figures of the bodies given us in experience! It is not merely a question of determining what the phenomena are (*welches die Erscheinungen sind*), putting aside researches into their cause with holy horror, because, were the office of science reducible to this alone, it would be perfectly useless. Does not immediate perception suffice to show us what the phenomenal is? Does not the phenomenal exhaust the whole of reality for you? Is there anything else to investigate? Phenomenalists, however, do not rest content therewith, because the demand for rationality is unconsciously at work in them too, urging them to construct beyond the empirical world an ideal world more easily penetrated by the light of intelligence.

NOTES TO CHAPTER III

[1] *Opere* (Florence, 1842), vol. iv. pp. 332-338.

[2] *Principia philosophiae*, Part II. Article 23: "Omnis materiae variatio, sive omnium eius formarum diversitas pendet a motu."

[3] At the beginning of his *Treatise on Light*, Huygens defines as true philosophy that in which the causes of all natural phenomena are sought in mechanics: " . . . in vera philosophia, in qua omnium effectuum causae concipiuntur per rationes mechanicas; id quod meo iudicio fieri debet, nisi velimus omnem spem abjicere in physicis intelligendi," *Opera reliqua* (Amsterdam, 1728), vol. i. p. 2.

[4] Cp. on this point Abel Rey's work, *La Théorie de la physique chez les physiciens contemporains* (Paris, 1907).

[5] *Réflexions sur la puissance motrice du feu* (1824).

[6] *Théorie mécanique de la chaleur* (Paris, 1868), vol. i. p. 331 ff.

[7] Poincaré, *La Physique moderne, son évolution* (Paris, 1908), p. 83 ff.

[8] *Elementary Principles in Statistical Mechanics* (New York and London, 1902).

[9] *Über die Prinzipien der Mechanik* (Leipzig, 1903).

[10] *Proceedings of the Royal Institution*, vol. xii. Part I. No. 81, p. 37, 1887.

[11] *Transactions of the Chemical Society*, vol. liii. p. 487, 1888.

[12] *Phil. Trans.* vol. clxiv., Part II. p. 491, 1873.

[13] *Proc. Roy. Soc.* vol. xxx. p. 30, 1879.

[14] *L'Évolution de la matière* (Paris, 1906), p. 34 ff.

[15] *Op. cit.* p. 9.

[16] His *Lehrbuch der allgemeinen Chemie*, published in 1885, may be regarded as the programme of the new branch of science.

[17] Ostwald considers that it is possible to deduce the fundamental laws of stoicheiometry, *i.e.* the law of multiple proportions and that of proportional numbers, by taking chemical dynamics as the only basis, and thus making the atomic theory superfluous ("Éléments et combinaisons," *Journal de chémie physique*, 1904, vol. ii. p. 377; "Über die Grundlagen der Stöchiometrie," *Zeitschrift für physikalische Chemie*, vol. xviii. p. 337; vol. xix. p. 607; vol. xxii. p. 252; vol. xxiii. p. 78.

[18] The first attempt to construct a chemistry dispensing with the concept of matter and reducing everything to energy is to be found in Gibbs' work, "On the Equilibrium of Heterogeneous Substances," *Transactions of Connecticut Academy, 1876–1878.*

[19] For the history of energetics see George Helm, *Die Energetik nach ihrer geschichtlichen Entwickelung* (Leipzig, 1898).

[20] *Lettre à Basnage.*

[21] "Outlines of the Science of Energetics," *Proceedings of the Philosophical Society of Glasgow, 1848–1855*, vol. iii. p. 382.

[22] "Die Materie ist ein Gedankending" . . . "Das Wirkliche, d.h. das, was auf uns wirkt, ist nur die Energie . . . Du sollst dir kein Bildnis oder kein Gleichnis machen!" (*Die Überwindung des wissenschaftlichen Materialismus*, a part of which has been translated into French under the title, "La Déroute de l'atomisme contemporain," in the *Revue générale des sciences*, 1895, vol. vi. p. 954).

[23] In the preface to his work, *Vorlesungen über Naturphilosophie, gehalten im Sommer 1901, an der Universität* (Leipzig, 1902), Ostwald tells us that he has endeavoured to write a book without introducing into it or making use of any hypothesis, "Ich habe mich bemüht, ein Buch zu schreiben, in welchem keine Hypothese aufgestellt oder benutzt worden ist." Herein he follows the methodical tendency of Mach, to whom he dedicates his work, and of Mayer, who, with Mach, exercised great influence on the formation of his thought, and the dedication of whose first work, *Bemerkungen über die Kräfte der unbelebten Natur*, published in 1842, runs as follows, "Freunden klarer, hypothesenfreier Naturauffassung." Mayer, in his other work published in 1850, *Bemerkungen über das mechanische Aequivalent der Wärme*, summed up his ideas on the method of science in these words: "The most important, not to say the only rule, for the investigation of nature is to bear in mind that our problem is to learn to know phenomena before being able to seek explanations and inquire into higher causes. If a fact be known in all its aspects, it is thereby explained and the problem of science is at an end."

[24] *Op. cit.* p. 206 ff.

[25] A similar idea has been expounded by Andrade, "Du rôle de l'association des idées dans la formation des concepts métaphysiques du mécanisme," *Revue de mét. et de morale*, March 1899, p. 177 ff. Had Pasteur lived before Galileo, says Andrade, the idea of life, and not the ideas of number, space, and force would at present form the basis of scientific concepts. The mechanical scheme has been imposed on the human mind merely by force of an association of ideas.

[26] *Op. cit.* p. 215, and p. 399 note.

[27] *Op. cit.* p. 14.

[28] *Op. cit.* pp. 91 and 141.

[29] *Op. cit.* p. 153.

[30] *Op. cit.* p. 158.

[31] *Op. cit.* p. 167.

[32] *Op. cit.* p. 169.

[33] *Op. cit.* p. 177.

[34] *Op. cit.* p. 184.
[35] *Op. cit.* p. 187.
[36] *Op. cit.* p. 188.
[37] *Op. cit.* p. 175.
[38] *Op. cit.* p. 317.
[39] *Op. cit.* p. 394.
[40] *Op. cit.* p. 398.
[41] *Op. cit.* p. 397.
[42] Ostwald has no hesitation in including in Energetics both aesthetic and moral feelings.
[43] Cp. on this point my book, *La Misura in psicologia sperimentale* (Florence, 1905), published by the Instituto Reale di Studî Superiori.
[44] Friedrich Wolfgang Adler has criticised Ostwald's philosophy of nature from the point of view of empirio-criticism, *Die Metaphysik in der Ostwald' schen Energetik* (Leipzig, 1905).
[45] In the Scolium Generale with which he concludes his work *Philosophiae naturalis principia mathematica* (London, 1686), Newton says: " So far I have accounted for the phenomena presented to us by the heavens and the sea by means of the force of gravity, but I have as yet assigned no cause to this gravity. . . . I have not been able to deduce from phenomena the *raison d'être* of the properties of gravity, and I have not set up hypotheses. Everything, indeed, which is not derived from phenomena must be termed hypotheses; and hypotheses, whether metaphysical or physical, whether of occult qualities or of mechanics, have no place in experimental philosophy."
[46] *Die Lehre von der Energie* (Leipzig, 1887), p. 16.
[47] " Die Energie ist das wahre Element der Welt, denn alles, was wir von der Welt wissen, wissen wir über die Energie " (*op. cit.* p. 56).
[48] " Vorlesungen über Mechanik, Vorrede," *Vorreden und Einleitungen zu klassischen Werken der Mechanik* (Leipzig, 1899), p. 119.

CHAPTER IV

THE NEW QUALITATIVE PHYSICS

1. *Vain Endeavours to reduce all Physical Qualities to Figure and Movement alone.* — The new qualitative physics, theoretically developed by Duhem, which is also a reaction from the traditional, universal, mechanical theory, remains more faithful to the methods of phenomenalism. We have already seen that the characteristic of modern science as opposed to scholastic philosophy is the attempt made by it to reduce to a minimum the number of qualities in order to transcribe all natural phenomena in terms of pure quantity, and to expound them *more geometrico*. The most notable effort in this direction was made by Descartes, who idealised matter into pure geometrical extension; but he failed to perceive that motion was not thus rendered more intelligible, since if a body be identical with the part of extension occupied by it, how can it be said without absurdity that one and the same part of extension occupies two different places in succession ? [1] As Leibnitz rightly observed, we must recognise that nature contains something more than pure extension and its changes. In mathematics it is a matter of indifference whether we say that a body A moves towards B, or B towards A, because the relations of the distances of the two points are the same in both cases; this cannot, however, be said in mechanics without altering our explanations. The motions of the constellations, for instance, are in agreement with a certain celestial

mechanics when the stars are regarded as fixed; they cease to be in accordance with it if the earth be assumed to be stationary. Motion cannot be reduced to pure quantitative relations, because it is always relative to a given fixed point, *i.e.* to a determinate quality; if the point of reference be changed, the law ceases to hold good. Let us consider, for instance, the law of inertia, according to which a material point, at a great distance from every body, must move with a uniform rectilinear motion, and let us suppose that this law is verified when the motion of the material point is relative to a certain body which is regarded as fixed. If we attribute fixity to a new body which was before animated by a rotatory movement in relation to the other, our isolated material point will cease to move in a straight line, and will describe any trajectory we choose, according to the law we are pleased to impose on it.

The Cartesian attempt to construct all physical reality by means of figure and motion having failed, the attempt was made to reduce as far as possible the number of fundamental qualities. Gassendi and Huygens, adhering to the ancient atomic theory, admitted only space, time, and material mass as irreducible; Newton and Leibnitz thought it necessary to add the idea of force; and these four elements remained in Lagrange's analytical mechanics.[2] The idea of force, however, was too suggestive of the occult qualities of the Middle Ages, and it was therefore natural that on account of its metaphysical colouring the attempt should be made to banish it from the domain of science. In Bernouilli's kinetic theory of gases we return once more to the atomic theory, but even the most convinced adherents of this theory, such as Boltzmann,[3] ascribe to it merely the value of a mechanical model which imitates certain properties of gases and can afford the experimenter certain useful indications, and are very far from believing bodies to be in reality composed of small particles. The mechanical theory of heat is confronted by difficulties which are no less great. Gibbs,[4]

who has made the most notable attempt to reduce the laws of thermo-dynamics to the principles of mechanics, was forced to conceive atoms as small fluid masses, capable of change of form and independent of an indefinite number of variables; this takes us very far from the small rigid solids so dear to the atomists. Do what one will, the idea of force, when turned out of the door, will find its way in again by the window: those who, like St. Venant and Kirchhoff, while retaining Lagrange's mechanics, regard the notion of force as derivative, and choose to see in the product of a material point by its acceleration, not a quantitative symbol capable of representing the various intensities of force and of acting as their measure, but the very definition of force, succeed, it is true, in developing their dynamics with absolute strictness and in starting from equalities which are true by definition, but they remain confined within a circle of empty and sterile identities, and, when they would fain transform them into synthetic principles giving us some knowledge of bodies and their motions, find themselves obliged to break through the analytic rigidity of their formulas and to resume all the experimental intuitions of which they had previously divested the general notion of force.[5] The others who, like Hertz,[6] retain the terms representing forces, but regard these forces as fictitious, reducing them to forces of inertia, generated by imperceptible motions, and to forces of connection or relation, due to the presence of imperceptible masses, tell us nothing of these invisible motions, so that, if on the one hand they can never find themselves in contradiction with experience just because of this indeterminateness, on the other they explain nothing to us, because they fail to reveal to us the structure of those invisible mechanisms which are supposed to take the place of forces.

Kelvin[7] has striven to carry the process of simplification still further, eliminating not only force but mass as well in his theory of the vortex-atom. According to Kelvin the universe is full of a homogeneous

and incompressible matter whose motions are subject to the same laws as those laid down by Euler's equations for perfect fluids. At first certain forces, which are incompatible with the equilibrium of any fluid whatsoever, set this matter in motion and created therein a large number of vortex-rings of all forms and dimensions; these forces then vanished, leaving in the world only apparent forces, capable of explanation by means of the pressures and forces of inertia of the universal fluid. These forces can neither generate a new vortex-atom nor annihilate or divide one of those already in existence. The matter which is brought before our senses is composed of such rings, which have thus become true physical atoms. It has been urged against Kelvin's theory that the fictitious forces, generated by the pressure of this fluid, cannot account for universal gravitation, and that the principles of mechanics cannot be deduced from the properties of vortex-rings: as Maxwell has truly observed, we do not know where in the vortex-atom the invariable element is to be found which can be regarded as its mass.[8]

2. *Duhem's New Mechanics.* — From our summary of the various attempts which have been made, it is clear how chimerical and vain it is to try to reduce all the properties of bodies to figure and motion only, why such a reduction would be obtained at the cost of complications terrifying to the boldest scientific imaginations, and why it would be contradictory to the nature of material things. It is for this reason that Duhem, renouncing the Cartesian ideal, has no hesitation in admitting the qualitative element into physics. This does not, however, involve that, returning to Scholasticism, we are to admit as many occult virtues as there are effects to explain: far from it! The new physics aims at the most economic presentation of natural phenomena; this is one of its reasons for opposing those complex mechanical theories which cause thought to wander in the intricate maze of imperceptible

motions and weary the imagination in the effort to intuit invisible masses. The physicist will therefore strive to reduce the number of ultimate qualities to the minimum; each time a new effect is presented he will endeavour to reduce it to the qualities which have been already defined, and only after recognising the impossibility of such a reduction will he resign himself to admitting a new quality into his theory and introducing a new species of variables into his equations.[9] On the other hand, when he calls qualities ultimate, he will have no intention of affirming that they are by nature irreducible, or of shutting out the possibility that the progress of science may render feasible that reduction which we are at present unable to make. Light, for instance, is now presented as an ultimate quality, but the day on which the electromagnetic theory achieves its final triumph will see it reduced to the periodic changes of another quality, *i.e.* dielectric polarisation. Are we, therefore, to conclude that the number of qualities admitted into our theories will diminish daily, that matter, the object of our researches, will become less rich in essential attributes, tending towards the uniform and geometrical simplicity of Cartesian space? Certainly not! Undoubtedly the very development of theory may from time to time bring about the fusion of two distinct qualities, such, for instance, as light and dielectric polarisation; but, on the other hand, the unceasing progress made by experimental physics will always lead to the discovery of new categories of phenomena, and in order to classify them and arrange them under laws, it will be necessary to endow matter with new properties. It is impossible to foretell with certainty which of these two opposing processes—one of reduction, the other of complication—will succeed in carrying off the victory; all that we can affirm is that, at all events at present, the latter current, which is much stronger than the former, is leading our theories to conceive a matter which is more and more complex and more richly endowed with attributes.[10] The new mechanics does not then hesitate

to admit into its equations terms differing in form and nature: terms of viscosity, attrition, hysteresis, electro-kinetic energy, chemical reactions, etc., thus imparting to the word motion all the extension attributed to it by Aristotle; it does not confine itself to the study of change of place and form, but treats also of the mutations by which the different qualities of a body increase or decrease in intensity, by which a body becomes hot or cold, is magnetised, reacts chemically, etc., so that for the new mechanics a system in equilibrium would not be only that in which there is no change of figure or position, but a system in which the various parts do not become hot or cold, in which the electric and magnetic distribution remains the same, which is not subject either to fusion, freezing, evaporation, or chemical reactions.[11] If, however, it does not attempt to transcribe all the qualities of matter in terms of figure and motion, it has no intention of hereby banishing the mathematical method. In order to make of physics a universal arithmetic, as Descartes desired to do, it is not in the least necessary to imitate the great philosopher and exclude all quality, since the language of algebra allows us to reason on the different intensities of a quality. The purely qualitative character of a notion is no barrier against the use of numbers in order to present its various stages, since one and the same quality can be presented in infinite degrees of intensity, which we may indicate by different numbers. It should, however, be observed that since a magnitude is not simply defined by an abstract number, but by a number in conjunction with concrete knowledge of a certain unit of measurement (for instance, the number *five* is not enough to determine a length, *yards* must be added), the intensity of a quality is not fully represented by a numerical symbol, but a concrete procedure must be added to this symbol giving us the *scale* of the intensity in question; only in this way is it possible for the algebraic propositions enunciated by us relative to the numbers representing the different degrees of the quality

studied to acquire a physical sense. The scale studied is of course constructed upon a quantitative effect produced by the intensive variations of that quality; and the effect is chosen in such a way that its magnitude will gradually increase as the quality becomes more intense. Thus in a glass tube immersed in a hot liquid the quicksilver expands in proportion to the heat of the liquid: here we have a quantitative effect which allows us to construct a scale of temperatures and to indicate the different intensities of heat by means of numbers. In this way the selection of a suitable scale makes it possible for us to substitute the consideration of numbers which are subject to the rules of algebraic calculation for the study of the different intensities of a quality. The advantages sought by physicists of the past in the substitution of a hypothetical quantity for the qualitative property revealed to them by the senses, and then measuring the magnitude of this quantity, is equally obtainable without resorting to the hypothesis of a true and proper magnitude behind the sensible datum, by making choice of a suitable scale.[12]

3. *The Economic and Objective Value of Scientific Theories.*—The new mathematical physics, when substituting a numerical symbol for a quality revealed by experience, does not deceive itself, like the old physics, into the belief that it is penetrating further into natural reality, nor does it believe that the calculations to which we may subject the degrees of temperature, for instance, can tell us more about the intimate essence of the quality represented by these degrees than direct observation can teach us. The aim of physical research is not of a speculative but a practical order; our theories neither can nor should claim to give us an explanation, but merely to represent phenomena in the most economic way, assisting us to master the world of matter and to modify it in accordance with our needs. The logical order in which our knowledge is classified will afford the physicist a convenient and certain working system; it will enable him to find readily without risk of error or

omission all the laws upon which the solution of a given problem depends.[13] The new mechanics is not directed towards speculative and metaphysical contemplation of the essence of things, but answers to the practical necessity of acting upon the bodies of the external world and of modifying them according to our requirements. Its first object is just to know which are the different bodies which can be substituted for our personal activity in order to further or arrest a change of things, what machines can be substituted for workmen in the carrying out of certain kinds of work. The work which we should have been obliged to do ourselves, had we acted upon the system which is being transformed, we regard as accomplished by the body or bodies which we have substituted for ourselves or our fellow-men ; let us then apply the concept thus formed to those cases also in which the modification undergone by the system is of such a nature that our personal actions can neither further nor arrest it, *e.g.* to chemical reactions. When a material system is transformed in the presence of foreign bodies, we regard these bodies as contributing to the transformation, and give the name of work to such a contribution. Of what nature this may be is a metaphysical problem which transcends the limits of science, and is therefore put aside by the physicist, whose more modest aim it is to create a mathematical expression which will serve to symbolise that contribution.[14] The work which bodies foreign to a system accomplish upon it may be represented by a modification of this system, that is to say, by the augmentation undergone by a certain magnitude absolutely independent of the nature of the foreign bodies : this magnitude is the total energy of the system. The part of this energy which is dependent solely upon the state of the system, and not upon local motion, is termed internal or potential energy ; the other part, which is dependent solely upon local motion, and not upon the state of the system, kinetic or actual energy.[15] Let us take two different material elements,

and let us, starting from the state of rest, launch them with the same acceleration; in general we accomplish two different tasks, and are naturally led to think that the relation of these two effects is independent of the common acceleration impressed on the two elements. This relation, which depends solely upon the nature of the two elements, is termed mass, and is proportionate to the work required in order to launch them with a determinate velocity. If we consider an isolated system, no body exists external to it; hence the work of foreign bodies is *nil*. Therefore in every modification of an isolated system the value of the total energy remains constant.[16] The procedure by which we have reached the concept of energy and the principle of its conservation is a revelation of the eminently practical character of physical theory. It proposes nothing else than to give us a system of mathematical propositions, deduced from a small number of principles, whose aim it is to represent as simply, completely, and accurately as possible the whole of experimental laws. Duhem does not, however, stop short at this economic and pragmatistic conception of science, but is conscious of the necessity of transcending it by attributing to physical theories a certain cognitive value also, as revealing the real relations of things.[17] The ease with which each law finds its place in the classification made by the physicist leads us to believe that this whole, which is so admirably arranged, is not entirely artificial, and that its order does not result from the arbitrary grouping of laws by the genius of a single man of science. Without being able to account for our conviction or to shake it off, we see in the exact order of that system the sign by which a natural classification is recognised; and, while making no claim to explain the reality concealed beneath phenomena, we feel that the arrangement of laws made by our theory corresponds to real affinities between things. The physicist who sees an explanation in every theory deceives himself into the belief that in luminous vibration he has apprehended the true and intimate basis of the

quality manifested to us by our senses under the form of light and heat, and believes in the existence of a body, the ether, whose different parts are animated by a rapid vibratory movement. We do not share these illusions, says Duhem. When in the course of an optical theory we continue to speak of luminous vibration, we do not think of a true and proper oscillatory movement of a real body, but merely imagine an abstract magnitude, a pure geometrical expression, whose length, which is periodically variable, enables us to enunciate the hypotheses of optics, and to discover by means of calculation the experimental laws of luminous phenomena. This vibration affords us a *representation*, not an *explanation*. But when, after making various attempts, we succeed by means of this vibration in formulating a system of fundamental hypotheses, when we see how on the plan traced by these hypotheses the immense realm of optics, hitherto chaotic and confused, becomes ordered and organised, it is impossible for us to believe that such an order and organisation do not image forth a real order and organisation; that the phenomena classed together in theory, such as interference fringes and the colours of thin films, are not in reality kindred manifestations of one and the same property of light; and that on the contrary the phenomena which are separated by theory, such as the spectra of diffraction and the spectra of dispersion, differ also in their essence. Thus if, on the one hand, physical theory never affords us an explanation of experimental laws, and never reveals to us the reality hidden behind sensible appearances, on the other, in its progressive development, it gives us reason to suspect that the logical order in which it arranges experimental laws is the reflection of an ontological order, and that the relations established by it between the data of experience correspond to real relations between things. Were the theory a purely artificial system we should expect from the discovery of a new law a refutation rather than a confirmation of it; it would indeed be a truly marvellous accident were this law to find its place

in the theory established on the basis of other laws. It is precisely because we recognise in the principles of our theory a correspondence with the real order of things that we are not surprised to see its consequences anticipate the phenomena of the future and guide us with unerring foresight to the discovery of new laws. The physicist can give no rational justification of this profound faith in the objective value of the order established by his theory, because he is unable to go beyond the data of experience, and to prove the order established by him between experimental laws to be the reflection of an order transcending experience, but, on the other hand, he cannot escape this irresistible belief. Strive as he may to convince himself that his theories cannot grasp reality, and are of service only as affording him an economic representation of experimental laws, he cannot succeed in believing that a system capable of ordering the most unlike laws so simply and so easily only possesses the value of an arbitrary convention; no argument will succeed in destroying his faith in a real order, of which his theories are an image which daily becomes clearer and more faithful.

4. *Criticism of Duhem's Theory.* — Duhem has assuredly made a great advance upon the narrow economic conception of science, endeavouring, as he does, to vindicate its value against the more or less fantastic negations of the *philosophie nouvelle*. To my mind, however, he seems to stop half-way, being unwilling to allow any possibility of physical explanation. The physicist certainly does not explain in the sense of grasping the essence of things with his theory, and Duhem is perfectly right in doing battle against that unconscious metaphysics which drives many physicists to posit certain concepts as true and proper real entities, almost deifying them, and making them the ultimate essence of things; he is right in waging war against the upholders of the mechanical theory who fall down and worship the invisible motions of material atoms or of imponderable ether; but there is a vast difference

between this and saying that the physicist does not explain. We must have a clear understanding as to the meaning of words : *to explain* does not in the least mean to grasp the thing in itself, to penetrate the absolute and transcend the bounds of experience ; it rather means *to make the world of phenomena intelligible* ; now how can it be denied that physical theory with its logical organism presents natural reality to us in an intelligible form ? Are not the logical demands of the principle of sufficient reason—the basis of all explanations—fulfilled when we have succeeded in reducing particular facts to experimental laws, and these laws in their turn to the more general principles of the theory ? This of course presupposes that reality—the object of scientific knowledge—is possessed of logical structure : were it totally devoid of intelligible elements, it would be impossible to understand how the human mind could succeed in weaving it into the web of exact formulas and intellectual categories. The objective existence of a rational order is not a blind act of faith or a mere concession to common sense, as is thought by Duhem ; it is rather the necessary postulate of all knowledge, whose justification lies just in the impossibility of conceiving a knowable reality, which does not contain within itself the conditions requisite for becoming known, *i.e.* for being translated into terms of thought. Neither do we need to transcend the sphere of experience in order to affirm the objective value of this order, or to conceive it, as Duhem does, to be something which is behind phenomena and transcends them, since in the world itself, as revealed to observation and experiment, we find those intelligible elements which afford us the means of applying the categories, that logical skeleton without which the objects of experience would be reduced to a chaotic and incoherent medley of sensations.

Duhem says that we cannot judge whether mechanism be true or false, because it always finds some way of bringing itself into agreement with experience by imagining invisible motions and masses, and because, moreover,

a theory is a pure system of conventions which eludes that judgment; but he affirms that there is another criterion which authorises us to put mechanism aside, namely, the mental economy which is the aim of theory. The new physics affords us a simpler and more natural arrangement, and is therefore preferable to the complications of the mechanical theory. Now this advantage is only apparent, because the new physics is simpler only so long as it remains in the abstract sphere of conventions, but when it desires to give them a meaning, when it would bring its formulas into harmony with facts, it is forced to make corrections and to add new terms in its operations; and since, according to Duhem's theory, it is not necessary for these auxiliary magnitudes to have a physical meaning, there is ample opportunity of complicating formulas, provided they end by agreeing with facts. If the traditional mechanical theory contained an arbitrary element, here the arbitrary is raised to the dignity of law. Moreover, the criterion of simplicity is not sufficient to enable us to judge the value of a theory: nature may be much more complex than we desire and be wholly indifferent to the economising of our mental effort: the real aim of theory is to transform the world of experience into an intelligible world, and its truth will therefore be in proportion to its success in attaining this end. A system of symbols and conventional relations, no one of which can lay claim to a physical sense, is certainly not fitted to satisfy our demand for rationality. A system of signs is not sufficient to give us an intelligible theory; these signs must correspond to concepts having a certain objective content and to thoughts of real relations. Duhem ends by recognising that theory must correspond to an objective order, but from his point of view such a correspondence seems to be an inexplicable mystery, a miracle which is indeed passing strange, since he does not regard the qualities of experience as magnitudes, and, if we transcribe them into numbers, we do so in order to label them by a convenient symbol

which enables us to find them readily, in the same way as we indicate the books in a library by various cardinal and ordinal numbers in order that we may be able to arrange and catalogue them. Yet in theory we then subject our numbers to calculations, that is to say, we end by treating them as magnitudes, establishing equations between these signs and deriving therefrom, in accordance with the laws of mathematical analysis, other relations which are verified in experience. How can this be done, if there is no natural affinity between the qualities of experience and our numbers, but only a conventional link? Is it not inexplicable that our calculations, which are only valid of magnitudes, should be of such use in enabling us to foresee that which is not in its nature a magnitude? How does that which is not quantitative obey the laws of quantity? The application of mathematics to natural phenomena is not intelligible unless the existence of a quantitative element in every fact of experience be assumed; and if it be not given immediately to consciousness as such, we are entitled to transcend the immediate experience, and to conceive the objective cause of our sensations in such a way that it may become possible for us to treat it as a magnitude. We must, not of course, fall into the error of the old mechanical theory by turning the conceived magnitude into an absolute entity; but is it not possible to transcend the immediate datum of sensation and still remain in the realm of phenomena? None of the opponents of the mechanical theory, not even Mach himself, has had the courage to deny the legitimacy of the induction which regards the vibratory movement of particles of air as the stimulus of auditory sensations; no one will seriously maintain that the physicist, who substitutes oscillations for immediately apprehended sounds, merely describes and sums up his sensations of sound in an economical formula.

From the methodological point of view nothing can be urged against the mechanical reconstruction of optical and thermal stimuli; we still have to consider whether

such an interpretation be not in contradiction to experimental data. The greatest difficulty of the mechanical theory is, as we have seen in the preceding chapter, derived from Carnot's principle; and Duhem and Ostwald have as a matter of fact tried to raise the new theoretical edifice on the principles of thermo-dynamics. But both opponents and the physicists who have endeavoured to defend the mechanical theory by trying to reconcile it with the principle of degradation of energy have failed alike to take into account the ideal nature of the principles of mechanics. Mechanics does not consider movements as they are empirically given, but substitutes for them a perfect model which is never completely realised in natural phenomena. According to the laws of pure mechanics, a pendulum should continue its isochronous oscillations to all eternity, whereas it stops after a certain time; a projectile, thrown in a straight line, should pursue the same direction with a uniform motion *ad infinitum*, whereas in reality we see it fall after having described a parabola. Are we to conclude that the mechanical interpretation is not legitimate even in the case of motions? Shall we not rather think that our laws are not exactly verified because the conditions thought by us are not realised with that perfection conceived by us? All the principles of mechanics have the character of ideal types, obtained by passing to the limit in order to satisfy our need of rationality; hence it is not surprising that the distance between them and the phenomena of experience becomes ever greater the more perturbing causes intervene. The very law of conservation of energy which the new physics would substitute for the old principles of mechanics possesses, as they do, merely the value of an ideal limit: a system which is not subject to any action from outside never has existed and never will exist. How can we claim from mechanics that absolute verification which no principle of physics can ever have? The possibility of a reversible cycle is only an extreme case, like a perfectly conservative system, since it is valid of a perfect machine, in which

there would be no dispersion, no loss, and no friction. This machine will always remain a pure ideal, but we can approach it by endeavouring as far as possible to avoid friction and thus to augment the "return" of the machine, that is to say, the quantity of work resulting from the lowering of the thermal level. In every natural phenomenon there is something mechanical, but not the whole of it is mechanical; it is therefore necessary in every branch of science to complete by means of other explanatory concepts those universal principles of mechanics which present only the universal form of physical reality, which can be reconstructed *a priori*—the warp necessary to him who would weave an intelligible world. These complementary elements must, however, be as few as possible, and it is always legitimate to endeavour to resolve the new phenomena revealed to us by experience into these four elements, by transmuting them into an ideal form in which our intelligence recognises itself. What is the aim of the age-long work of science, if it be not to bring to light the thought contained in the inmost nature of things?

NOTES TO CHAPTER IV

[1] Duhem, *L'Évolution de la mécanique* (Paris, 1903), p. 18 ff.
[2] *Op. cit.* pp. 37-71.
[3] *Leçons sur la théorie des gaz.* (Paris, 1902).
[4] *Elementary Principles in Statistical Mechanics* (New York, 1902).
[5] Duhem, *op. cit.* p. 57 ff.
[6] *Die Prinzipien der Mechanik in neuem Zusammenhang dargestellt* (Leipzig, 1894).
[7] "On Vortex-Atoms," *Proceedings of Royal Society of Edinburgh*, vol. vi. p. 94, etc., 1869. [Read February 18, 1867.]
[8] Duhem, *op. cit.* p. 176.
[9] Duhem, *La Théorie physique*, p. 201 ff.
[10] *Op. cit.* p. 211.
[11] *L'Évolution de la mécanique*, pp. 343-345.
[12] *La Théorie physique*, pp. 185-193.
[13] *L'Évolution de la mécanique*, p. 108.
[14] *Op. cit.* p. 222 ff.
[15] *Op. cit.* p. 225.
[16] *Op. cit.* p. 226 ff.
[17] *La Théorie physique*, pp. 34-43.

CHAPTER V

THE THEORY OF MODELS

1. *The Two Types of Ideation in Physicists: the Abstract and the Concrete.*—One of the most controverted points in the discussion of physical theories is undoubtedly the value to be attributed to concrete representative elements in the systematisation of empirical laws. Some thinkers (Rankine, Mach, Ostwald, Duhem) would absolutely banish all concrete images whatsoever in order to reduce theory to a pure system of concepts and mathematical relations; others (Faraday, Kelvin, Lodge, Maxwell) will have none of this abstract formalism, and resort continually in their theories to concrete representations of phenomena. As Duhem [1] has shrewdly observed, such a difference is to a great extent dependent on the type of ideation of the physicist who constructs the theory. In some individuals the faculty of conceiving abstract ideas and of reasoning is more highly developed than the power of imagining concrete objects; others, on the contrary, have to make a great effort in order to conceive in an abstract way, whereas they possess a marvellous power of imagining an extremely complex combination of objects in their sensible form. These visualisers of science, whose thoughts, like those of Stelio Effrena in Gabriele d'Annunzio's *Fuoco*, are all translatable into a clear vision, are specially common in England. In the treatises on physics published by English writers, we find a model, a concrete image at

every turn: abstract notions do not afford them satisfaction. "So long as we adhere to this mode of expression," says Lodge,[2] "we cannot form a complete mental picture of the actually occurring operations." We must then form one of those models so much used by the great English school of mathematical physicists, whose splendid achievements will—Lodge tells us—shine out in the future as the glory of the nineteenth century.[3] The use of models is so necessary to the physicists of this school that they end by confounding the sight of the model with the understanding of the theory.

Kelvin[4] says that his object is to show it is possible to construct a mechanical model satisfying the required conditions in every category of physical phenomena which we have to consider, no matter what these phenomena may be. The real meaning of the question, "Do we understand a certain physical subject or not?" appears to him to be, "Can we construct a corresponding model?" By understanding a phenomenon the physicists of the English school mean forming a concrete representation of it, constructing a model imitating it: to comprehend the nature of material things is to imagine a mechanism whose working represents and simulates the properties of bodies. They give the preference to mechanical explanations, because motion can easily be represented, but they do not resort to it alone, nor disdain to take the elements of their images even from tactile and muscular sensations, provided that they make an efficacious appeal to the imagination. Like S. Thomas, they would fain see and handle. Do they, for instance, purpose to construct a theory of electro-static phenomena? Instead of conceiving in the space which separates the two conductors lines of abstract force, without breadth and real existence, the English physicist materialises these lines until they attain to the dimensions of a vulcanised indiarubber tube; for a group of ideal lines of force, conceivable only by reason, he substitutes a bundle of visible and tangible elastic cords, whose extremities are firmly affixed to the

surfaces of the two conductors, and which are distended and tend at the same time to become thicker and shorter; when the two conductors approach one another more closely he sees these elastic cords pull them, and each become shorter and more inflated, as in the celebrated model of electro-magnetic actions imagined by Faraday, which Maxwell and the whole English school admired as a work of genius. The elements with which the English physicist constructs his models are not abstract conceptions, but concrete bodies, like those which are around us, solid or liquid, flexible or inflexible, fluid or viscous, and their properties are not conceived in the abstract, but imagined by means of sensible examples: rigidity recalls the image of a block of steel; flexibility that of a thread of silk; viscosity that of glycerine. In order to express more forcibly the concrete character of the bodies from which he manufactures his mechanisms, Kelvin has no hesitation in designating them by the most familiar terms, speaking of the re-percussions of a bell, of cords, gelatine, etc. The image can of course be varied according to the taste of the physicist, affording wide scope to his imagination. In order to represent the structure of matter, for instance, Kelvin imagines the most widely different models without any connecting link, which would undoubtedly contradict one another did they claim to afford us a revelation of the *true* constitution of matter; but Kelvin himself is careful to warn us that the mechanical structure of his models must not be regarded as existing objectively in nature.[5] They are rough models, "unnatural mechanically." Their meaning is essentially psychological and subjective; their construction answers to that demand for clear and concrete representation which exists in the individual type of mentality of certain physicists. We must beware of attributing a logical or epistemological significance to them. As long as Kelvin and Lodge say to us, "Our mental construction does not allow us to follow a series of abstract concepts, and we therefore feel the need of the help of

concrete images," we have nothing to find fault with; and as psychologists we note with interest the representations in which their thought takes concrete form. When, however, they generalise their individual peculiarity, and identify comprehension and imagination, and would fain reduce physical theory to a kinematograph of images, their affirmation becomes illegitimate and arbitrary.

2. *The Nominalistic Prejudice of the Theory of Models: Hertz.*—The logical error which lies at the root of all these theoretical constructions is the nominalistic prejudice concealed in the ambiguous sense of the word representation. This term is used on the one hand by Maxwell and Hertz to indicate a species of reproduction or copy made by thought of phenomena (what Mach calls the imitation of facts in thought), on the other, all the characteristics of the concept are attributed to it. According to Maxwell the progress made by exact science is based upon the discovery and development of appropriate *representations*, which in their generality put us in a position to embrace a great number of phenomena, and to make useful previsions about them, and are sufficiently precise to serve as a basis for mathematical reasoning. According to Hertz,[6] the problem of natural science is to enable us to foresee the facts of the future in such a way that we may regulate our practical life accordingly. In the transition from the observations and experiments of the past to the prevision of the future we act as follows: we form in our thought a symbolic image (*Bild*) of the phenomenon, of such a nature that its logical consequences are always the images of the necessary consequences of the phenomenon in the natural order. This is possible just because — as is proved by the experience of many centuries — a certain harmony exists between nature and our minds. In reality we do not know, and have no means of proving, whether there exists a fuller agreement between our images and objects than the one fundamental relation by which

their logical consequences correspond to the natural consequences of phenomena; but this harmony alone is sufficient for the ends of science, hence there is no need to search any further. It is obvious that we may form different images of the same phenomenon, of which we regard as acceptable (*zulässig*) only those which contain no contradiction of the laws of our thought; of these we shall term those exact (*richtig*) which satisfy the fundamental requirement we have laid down, *i.e.* that their logical consequences shall be faithful representation of the natural consequences of objects. Two images of the same external phenomenon, both of which are admissible and exact, may, however, be distinguished by a greater or lesser degree of convenience (*Zweckmässigkeit*); that image will be most convenient which reflects a greater number of essential relations of objects and contains a smaller number of superfluous or free relations. These free relations, relations, that is, which do not correspond to natural relations of phenomena, can never be entirely eliminated, because they are of necessity inherent in images for the sole reason that they are images created by our mind. The tribunal of reason can decide definitively and at once whether an image be admissible or not; the question whether it be exact or not can also be answered with precision, but only with respect to the present stage of our experience, leaving the possibility of an appeal to new and more accurate researches; on the other hand, it is impossible to decide equally easily whether an image be convenient or not, since one image may be advantageous in one respect, another in another; only by testing them is it possible after a certain time to judge which is the more convenient.

Against the theory of Hertz we would observe that the aim of science is to grasp that which is universal and permanent in the inexhaustible multiplicity of facts; and the universal and permanent can only be attained by a concept, never by an image, whose characteristics are just the opposite—individuality and

variability. The image, taken by itself, may be of value in the realm of art, which seizes the fugitive aspects and the individual physiognomy of things; but in the realm of science its office is merely to symbolise a concept and to render it communicable in some way or other. What may legitimately be discussed is whether this form of expression is in practice preferable to the common symbolic language of science and the algebraic signs of mathematical functions. The substitution of the image for the concept would amount to the denial of the universality of science and its objective validity. The worshippers of models frequently confound representation and concept, making use of the former term even when it is a question of properties implying conceptual elements. Hertz speaks of logical consequences of images: the term *Bild* does not stand for a particular representation, but is extended to mean a complex system of ideal relations which copy certain relations existing between natural phenomena. It is, however, well to set this ambiguity aside and distinguish that which is proper to the concrete representation from the characteristics belonging to the concept. The model is not by itself a scientific theory, but it presupposes that theory in the thought of the man of science. He does not imitate the concrete phenomenon by means of another concrete phenomenon; he does not make an individual image correspond to a physical fact, but expresses and embodies, we may almost say, the concepts which he already possesses of phenomena; and what makes the variety of images and the translation into so many different tongues possible is just the unity of the law which becomes concrete in these different forms. The theory is not to be found in the series of models, considered in themselves, but in the system of conceptional relations which they symbolise.

Even in those physicists who state that they cannot understand without forming an image, the concept exists as a psychological reality distinct from the

representation, and confers on the latter a universal meaning, beyond that which is actually seen and felt in the model. It is one thing to say, "I cannot understand without imagining," and another to identify understanding and imagination. The working of the model would have no meaning and would not be understood did the physicist not already possess concepts of certain mechanical properties and of their resemblances to other general properties of electric phenomena, for instance, which he does not imagine in the concrete but conceives in the abstract. Hence—to analyse thoroughly the logical process in which the concepts takes its rise—it will be seen that the tissue of abstractions presupposed thereby is far wider than that which appears explicitly in the formulas of abstract theories. On the one hand, in fact, we have the properties and mechanical relations expressed by the model; on the other, the properties and relations of the phenomena about which the physicist is theorising; and lastly, the relations of analogy between the two orders of properties and relations. Take away this system of ideal relations and nothing will be left of the model but the perception of the moment in its individuality without any scientific meaning.

3. *Value of Concrete Representations in Physical Theory.*—Well and good, the advocates of models will reply, we do not deny the existence of concepts, without which science would be annihilated; we merely wish to affirm the necessity of the concrete representation. Thought, Rey [7] has recently observed, needs the image in order to do battle against abstract theories: a mind capable of comprehending an abstract notion without any concrete support neither does nor can exist from the psychological point of view. Now this is not altogether the case: the possibility of thought without images is proved by careful psychological analysis,[8] and even the most resolute disciples of nominalism may sometimes catch themselves following an abstract train of thought, without being able to find any verbal or concrete image in immediate recollection; even in the cases when

they do find such an image, if they examine them dispassionately, they can convince themselves that the image is but a small fragment of the total thought and cannot therefore claim to exhaust that thought.

There are, of course, different gradations in individual types, but in no case is there a complete absence of abstract thought, although a greater wealth of images may sometimes follow in its train. This is the case with the English physicists. Others, on the contrary, do not require the help of the concrete representation, the system of mathematical signs sufficing to fix their thought-series. But, Rey might object, are not these signs images also? Let us have a clear understanding: the signs used by the mathematician have no content by themselves, but merely serve to recall a concept or a relation, and to fix it objectively. Models, on the contrary, inasmuch as they must of necessity be concrete phenomena, possess determinate and specific properties, which never perfectly coincide with the phenomenon to be represented, there will always be certain properties, certain relations in the model to which we find nothing analogous in the phenomenon to be studied. This was already noticed by Hertz, who applied the term superfluous or free to those relations to which we do not find any correspondence in the phenomenon. Ostwald is therefore perfectly right in affirming that any presentation of one phenomenon by means of another is of its very nature inadequate; this drawback may be reduced to a minimum, but can never be eliminated. An image differing from the phenomenon will always be to a greater or lesser extent a falsification of it. Not so the algebraic sign, which translates the measure of a magnitude given in experience, and serves merely to indicate it and to enable us to communicate it easily. The mathematical sign is a representation which has lost its concreteness and is, therefore, looked at in itself, of no value; it is a pure symbol which has been divested of its changeable aspect in order to reflect the stability of the concept. It is, we may almost say, a dead,

fossilised image, but that which it has lost in concreteness it has gained in universality. The model, on the contrary, is a concrete phenomenon, which exists by itself, as well as in the capacity of an image of another phenomenon, and just for this very reason it can never be a perfect symbol. It is not, like the algebraic sign, a mere construction of our thought, which contains only that which we have placed in it, but possesses as a real phenomenon characteristics of its own which cannot be eliminated. Its functions should be to aid the understanding and render the effort of abstraction less arduous ; but this, if it gives it an individual value, cannot raise it to the rank of a symbol of a universal language. Even those who feel the need of giving their thoughts concrete form do not all make use of the same order of images, so that what is a suggestive model to Kelvin or Lodge suggests nothing to others, because it does not conform to the particular psychological structure of the individual. The usefulness of images depends, therefore, upon the mental constitution of the individual; hence each individual must be left at liberty to represent the phenomenon in his own way. If physical theory be reduced to these images, what becomes of its universality ? The algebraic language of abstract theories is from such a standpoint preferable to the model; all the more so since schematic signs, just because they lack a concrete content of their own, do not prevent physicists who feel the need of doing so constructing an image adapted to their own type of ideation. Formulas supply the universal warp, into which each one is at liberty to weave as he will. If you are unable to think the force of attraction in the abstract, no one will prevent your imagining between the earth and the moon (both more or less reduced in size in order to bring them within the scope of imagination), an elastic thread or cord, drawn from both ends in some way or other ![9] These images, which are sometimes suggested by the different departments of physics or by certain special laws, neither can nor should take the place of theory itself, which, if it would be

worthy of the name, must enable us to understand the physical world by giving greater prominence to its logical structure. We are only satisfied when we have reduced the complex of phenomena to a system of concepts which reflects the logical unity of our thought. How can we rest content with a disconnected series of images which exclude one another, in defiance of the principle of contradiction ?

4. *The Model not an Indispensable Means of Discovery.*—The advocates of models glorify images as means of discovery of the utmost potency. Abstract theory, says Rey,[10] is an excellent descriptive means of summing up what we know, but it does not enable us to foresee and anticipate experience, whereas theory should be of heuretic value. The hypothesis which is fruitful in the physical realm is of necessity an imaginable hypothesis, one constructed in terms of perception and sensation; the concept, which is a mere summary of experience, cannot serve as a means of discovery. Invention is a special property of imagination. Now we are willing to concede to Rey that theory should also impel thought to new discoveries, and that its value is in great measure dependent upon its fruitfulness; we are not, however, inclined to grant that experience can only be anticipated by means of images. Cannot an hypothesis be formulated also in conceptual and mathematical terms ? Before verifying a relation or a mathematical formula, the man of science constructs it in his thought; and even when he has verified it, this formula and in general the system of concepts constructed by him does not remain something definitive, which has no need of completion and modification. Concepts may give place to others of a more comprehensive order, formulas to others which are more complete and correspond to new observations; and the genius of the man of science may be manifested in the construction of these concepts and of these truer and fuller formulas, without his being forced to resort to concrete representations. Why indeed should images have the monopoly

of discoveries? May not the concept, like the image, refer to possible experiences? Is not the mind at liberty to construct new concepts, just as it is at liberty to create new images?

The history of physics proves, moreover, that abstract theories too are fertile in new inventions. From a doctrine which is the type of abstract theories—thermo-dynamics—Maxwell inferred an essential relation between the theoretical and the practical isotherm, and Gibbs the fundamental equations of osmotic pressure. Thermo-dynamics was also the only guide of Van 't Hoff during his first labours along the same lines, while experimental induction provided Raoult with the laws necessary for the progress of the new doctrine, which was already mature and fully constituted when mechanical models came to bring it help of which it did not stand in need, which was of no service to it. In a large number of cases, a model of a theory which was already formed has been constructed either by the originator of the theory himself or by some other physicist; little by little the model has caused the abstract theory, without which it would never have come into being, to fall into oblivion, until finally the model has been presented as the means of the discovery, whereas it was merely a way of explaining it.[11] We must not confound the use of a model with the use of analogy, which is undoubtedly an extremely productive method. If two classes of phenomena possess certain analogies, the physicist will make use of these analogies as a guide to the classification of the phenomena of the group which is not as yet known in a system of the same type of phenomena as the former group. Thus the analogy between luminous and acoustic phenomena led Huygens to the idea of luminous waves; a resemblance between the propagation of heat and that of electricity in a conductor enabled Ohm to transfer to the latter category of phenomena all the equations which Fourier had established for the former. There sometimes exists between two completely heterogeneous orders of phenomena merely an analogy in the equations

which serve to formulate them; this is the case, for instance, between the distribution of stationary temperatures and electro-statics. Then one group of phenomena may illustrate the other, and, when a problem belonging to the former is solved, one of the latter is solved as well. This correspondence is certainly of value, because it is not only a notable mental economy, but may lead to new discoveries. Here we are clearly not dealing with resemblances between images; on the contrary, the phenomena in their concreteness appear wholly heterogeneous, and their analogy is revealed only when they are reduced to an abstract system of conceptual symbols.

5. *Fertility of the Concept.*—What more evident proof of the fertility of the concept? It is useless to say that science cannot advance a step without models and sensible images. Thomson's finest discoveries — the electrical transportation of heat, the properties of variable currents, the laws of oscillating discharges, and many others—have been made by means of the abstract systems of classical thermo-dynamics. When he turns to mechanical models for assistance, he confines himself to setting forth and representing the results he has already obtained. Thus the model of electro-static and electro-magnetic actions described by Maxwell in his article, "On Physical Lines of Forces," does not appear to have helped that physicist to create the electro-magnetic theory of light; he endeavours, it is true, to deduce from this model the two formulas essential to the theory, but the very way in which he directs his attempts proves that the results were already known to him by other means. We do not hereby intend to deny that some physicists may be inspired to new discoveries by these images, but, if we observe closely, the concrete representation as such never acts as the inspiration, but rather the model, in so far as it is a system of concepts symbolised by a mechanism. Even in this direction its value will always be relative to the psychical structure of the individual, and we have no right to lay down a universal law

affirming the impossibility of extending the domain of scientific knowledge without the use of images. Those who do not feel the need of sensible representations in thinking can anticipate experience by means of concepts, expressing their hypotheses in abstract terms and in some cases even in mathematical formulas.

When Rey tries to infer the sterility of the concept from its nature he starts from the false presupposition that it is only a summary of experience, a schematic index of perceptions. He does not see that his argument might be turned *a fortiori* against the image, which is nothing but the residuum of a perception or of several perceptions combined. How, we might ask him, making use of his own logic, can the image, which is the memory of a past experience, anticipate the future? Just because, the reply might be, the mind is not a passive receptacle of impressions, which are combined more or less automatically, as the childish and simple psychology of the positivists teaches us; but is rather creative activity, which evolves from the elements of experience a world of new formations transcending the realm of obscure facts in order to take possession of the vaster domain of possible experience. This creative activity does not manifest its function only in the fantastic play of images, but elaborates the higher syntheses of concepts, revealing new relations which find expression in new laws and new theories. The scientific concept is something more than a mere summary of perceptions; it is not an *abridged experience*, but an *idealised experience*, and its fruitfulness lies in its ideal character. In respect to experience, it is not an impoverishment but a raising of it to a higher power; it is experience purified and carried to its ideal limit in order that it may satisfy the demands of necessity and logical universality. All the truly rational laws and concepts of science possess this character of ideal limits, which experience can approach more and more closely in proportion as the required conditions are verified, but which neither are nor ever

can be completely realised. Every scientific concept is therefore in itself an anticipation of the future; the stamp of universality imprinted on it by thought impels it to transcend past experience and foresee the future. Our thought does not rest content with merely making a more or less economical record of perceptions, but seeks its own ideal nature in those perceptions, creating concepts which correspond more and more nearly to that type of unity which is its supreme law. There is no danger of its becoming stationary or sterile, since, as we have already seen, the ideal of reason is never fully realised; hence the ceaseless effort of science to harmonise known laws in higher syntheses and to establish its sway more firmly over the future. The formulas of our scientific theories, although suggested by experience, always transcend it; they are not only the reflection of the known, but also an effort to divine the unknown. Even in the most abstract theories this hypothetical element, the life of science and the fertile root of its progressive development, is never lacking. It matters little whether the hypothesis take concrete form in images or be expressed in mathematical signs; in either case it is a system of concepts, and only crass nominalism can stop short at the external model, and attribute to it that productiveness which is rather to be found in the thought of him who constructs it and in the idea which it expresses.

NOTES TO CHAPTER V

[1] *La Théorie physique*, p. 87 ff.
[2] *Modern Views of Electricity* (London, 1889), p. 25.
[3] *Ibid.* p. 5.
[4] *Notes of Lectures on Molecular Dynamics* (Baltimore, 1884), p. 131.
[5] *Ibid.* p. 131.
[6] "Die Prinzipien der Mechanik, Einleitung," *Vorreden und Einleitung zu klassischen Werken der Mechanik* (Leipzig, 1899), p. 123 ff.
[7] *L'Énergétique et le mécanisme au point de vue des conditions de la connaissance* (Paris, 1908), p. 92.
[8] Binet, *Étude expérimentale de l'intelligence* (Paris, 1904).

[9] The French physicist Andrade has made a system of this method of representation, by founding *the school of the thread*, as he himself terms it. This school endeavours to reduce everything to the consideration of certain material systems of negligible mass, regarded in the state of tension and as capable of transmitting considerable forces to distant bodies, systems of which the thread is the ideal type.

[10] *Op. cit.* p. 121 ff.

[11] Duhem, *La Théorie physique*, p. 150 ff.

CONCLUSION

OUTLINES OF A SPIRITUALISTIC CONCEPTION OF THE WORLD

1. *Intuitionism, Pragmatism, and Intellectualism as Partial Views.*—The labour of the critical revision of science has not been fruitless, inasmuch as it has served to shake faith in the old dogmas and to purge thought from the unconscious metaphysics which led it to exalt its abstract constructions to the throne of absolute reality. These abstract constructions are certainly not devoid of value, since they express the intelligible element in the physical world, but they neither can nor should claim to exhaust reality in the fulness of its manifold forms. Philosophy alone, soaring as it does to greater heights and returning to the undefiled sources of every kind of knowledge, can reintegrate in its concreteness the world of consciousness of which scientific schemes afford us but a fragmentary and partial view. Apart from the intimate context of the human mind, the true type and gauge of all reality, scientific concepts are meaningless; all the contradictions and absurdities before which agnosticism halts, ascribing them to the congenital weakness of the intellect, are due to this hypostatisation of these fragments of thought, which, when divorced from the rest of consciousness, seem full of yawning gaps and mysterious gloom. Scientific experience is not the only one which can and must serve as the basis of philosophical speculation: our mind possesses other functions which are no less vital and no less original and profound, and have laws and

exigencies of their own which cannot be reduced to the categories of the natural sciences. Art and moral life are not a tissue of illusions, but spheres of conscious reality, whose existence cannot be questioned, and philosophy must take these data also into account if it would give us a total conception of the world. The contemporary speculative currents of thought which have arisen in opposition to intellectualism possess the merit of having asserted in no uncertain voice the rights of feeling and will against the excesses of scientific materialism, and the cold indifference of agnostic positivism; but they have allowed themselves to be carried too far by the reactionary movement, and have ended by going to the opposite extreme and sacrificing understanding to mystical inspirations, wild flights of imagination, and the crude utilitarianism of practice.

Only by restoring the fulness of conscious personality without any arbitrary mutilation and recognising a proper and autonomous value in all three fundamental functions of the mind, seeking their concrete synthesis in the unity of the human subject, not in an alleged supremacy of one over another, can the age-long strife of the spiritual energies be stilled. Every attempt to reduce or subordinate one to another, while doomed to failure, since each function of consciousness is possessed of characteristics and exigencies of its own which the others cannot meet, leads of necessity to an exaggerated reaction in the opposite direction, because the mind tends to assert itself as a whole and will not brook reduction and impoverishment. It is for this reason that in the history of philosophy we see periods of extreme rationalism followed by epochs of raving mysticism, and *vice versa*; and thought, oscillating between these two extreme poles, has seldom succeeded in finding a lasting equilibrium. Will and imagination have their rights, but they cannot take the place of reason, which alone can give us the concept of the universal and is therefore the proper organ of philosophy. We may take the data requisite for the completion of

scientific experience from the aesthetic and practical function and from the valuative attitude of the human mind, but it is the task of reason to deduce a higher synthesis from these data, and to transfigure them into an idea which comprises in its universality all the manifestations of being and contemplates them *sub specie aeternitatis.*

2. *The Reality of Concrete Thought.*—Without thought there can be no philosophy, hence there can be no doubt that all speculation whatsoever must begin with the concrete act of thought. Thought is the primary and incontrovertible basis of all certainty, the living model of every other reality; he who doubts the existence of his own thought cannot affirm that any other object exists, seeing that his own thought is the one and only medium through which all other knowledge reaches him. If my conscious activity were an illusion, would not the world it constructs be a twofold illusion? It is folly to try and place a deeper objective reality in opposition to thought regarded as a simple subjective reality; he who makes such an attempt must inevitably end by contradicting himself, by placing a fragment of consciousness on the throne of the Absolute. What else is the impenetrable extended atom of the materialist? what else is the movement and the system of mechanical laws, *more geometrico demonstratae*, but a projection out of the mind of certain forms and categories and of certain conscious experiences? Time, space, and the relation between them which gives birth to the mechanical notion of movement, substance, causality, a complex of kinaesthetic representations, and all the principles of formal logic: these are in the last analysis the contents and conscious functions whose varied combinations result in the world of the materialist. May it not be possible, moreover, to reduce the energy of modern philosophers of nature to the categories of potentiality, motion, and action, and to the representation of muscular effort? A world which is thus built up of elements derived from thought can only be real if thought

itself be so also. The reality of nature presupposes the reality of consciousness as its essential epistemological basis. Thus we do but move in a vicious circle when we try to derive thought from the world of mechanics or energetics, since there is nothing left of that world when the concepts, principles, and ideas used by conscious activity in its construction have been eliminated. The unknowable itself, the impenetrable and mysterious noumenon which certain philosophers have endeavoured to place beyond the range of thought, is, when reduced to a mere subjective appearance, the vain shadow of a body beyond our grasp, nothing more or less than the projection into the objective of the categories of cause and substance. Thought cannot, while denying its own absolute value, transfer the reality it denies into a mysterious *quid*, because this *quid* is constructed by the activity of thought in the very act which thinks of it as existing. A limit which is thought ceases for that very reason to be a limit to thought. Since being is a mental category, it is impossible to conceive of the existence of that which absolutely eludes thought. It is true that there are infinite aspects of the real which defy the limits of an abstract concept; but by knowing we do not merely understand the act of forming an abstract concept, but also that of affirming the reality of our concrete experience, as passed through by us in the moment which is perpetually being renewed. We may say that we know the pain and joy of the passing moment, and that we know them as true realities, even though we may try in vain to translate them into a generic scheme. Agnosticism is a fatal consequence of the arbitrary mutilation of thought, taken in an abstract sense and forced to act in a void. The idea severed from the intuitive life of consciousness is undoubtedly full of contradictions, but why seek to sever it from that concrete experience from which alone it derives the significance and value of real cognition? Reality only eludes thought when thought is reduced to mathematical formulas; it is not the world of concrete thought

which is contradictory, but rather that fragmentary mechanical system by which you would replace it as being sufficient to itself.

The denial of the reality of conscious life and its reduction to a world of appearances cannot possibly be justified. By what criterion do we pronounce such a sentence upon it? We certainly shall not find it in another type of reality absolutely external to that life, since we can know nothing of this model unless it reveals itself in some way or other to consciousness, without, that is to say, presupposing the truth of this conscious revelation, and thus contradicting ourselves. Neither can we find it in a logical principle (as, for instance, the principle of contradiction, of which some thinkers have made use in order to degrade the world of thought into an illusory appearance), since, if this principle be a law resulting from that consciousness which they regard as a vast illusion, how can it be the principle of truth? He who regards it as an absolute criterion of the real is thus led by that very act to recognise its objective value, and hence to grant by implication the objective reality of the thought which acknowledges it as its highest law.

3. *Concrete Thought as the necessary Organ of Philosophical Enquiry.*—We have stated that without thought there can be no philosophy, and have accordingly excluded from the first the possibility of dispensing with the concept and judgment in philosophic research. Every philosopher indeed assumes the existence of that which he regards as true reality and imparts to his philosophic conception a value which transcends the passing moment of his mental life; now it is impossible to assume the existence of anything without performing an act of judgment, at all events by implication; it is equally impossible to attribute to the experience of consciousness at a given moment a value which transcends the finite moment, without looking at it from a universal point of view, that is to say, without raising the immediate experience to the rank of a concept. As

long as we confine ourselves to experiencing in ourselves the perennial creation of new concrete qualities, to feeling the unfettered energy of our will, and to experiencing a certain feeling of love or faith, we do not see beyond the passing moment, and we drift with the current of life, which does with us as it will; we are not philosophers. A philosopher is one who is not content merely to live, but who is capable of reflecting upon life; who is not the plaything of time, but can establish his dominion over it by placing himself in the infinite moment of which it forms part. The fullest life of the real, and its most intimate possession—the aspiration of every philosopher—are not to be found in immediate experience, in the intuitive data which are but fragments of reality no less abstract in their seeming concreteness than the concepts of the intellectualist, but rather in concrete thought, which does not efface the moment of life but rather raises it to a higher degree of truth, showing it to us in the light of all its relations to the rest of the universe. Intuition severed from this organic whole is not rich in life, but rather poor. If we consider the concept apart from intuition in its abstract schematism, it will undoubtedly seem but a skeleton, inert in its riches, excluding all possibility of change, and powerless in its barren identity to give us so much as a single throb of life. But in concrete thought the concept is not severed from intuition, and makes no claim to be a substitute for it; it merely completes it, raising it to a higher power and revealing it to us in all the fulness of its relations; hence intuition is not impoverished, but enriched. The concept is the conquest of a synthetic unity which presents an intuitive moment in the light of a system of concrete relations with other moments; it is the conquest of a peak from which we can view a vaster horizon without losing sight of the concrete particular. But just as he who has scaled the heights would see nothing but a void around him were he to raise his eyes from the landscape, so he who, having attained to this unity, would fain dispense with the

concrete experience from which he set forth on his way will find nothing in his mind but a formula devoid of all real content. The energetic activity of the will, the ardour of feeling, the perennial and spontaneous renewal of mind and world, all these will elude the concept thus understood in its motionless abstraction; but concrete thought, which neither denies nor excludes the immediacy of direct experience, but comprehends it whilst yet transcending it, reveals the depths of the moment which gives birth to new forms of existence and the fruitful, creative power which alone can supply the clue to the eternal becoming of things. Concrete thought is not a cold abstract conception, but experience seen in the glow of the idea which breathes the warmth of feeling, and is moved by the energetic impulse of spontaneous volition, which inspires it with the enthusiasm of faith. All the demands which abstract intellectualism failed to satisfy, and which gave birth to the philosophy of intuition and action, will find their fulfilment in concrete thought, which is not a mere concept but an intuitive life, at once volition and feeling, and in its full concreteness is no other than the whole mind in action.

4. *The Substantiality of the Ego.*—Thought is, as a matter of fact, always activity developing in an individual consciousness, in which the act of conception does not involve the cessation of volition and feeling. There is no such thing as impersonal thought, *i.e.* thought which is not thought by a conscious subject. I may certainly in the act of thought place myself *sub specie aeternitatis*, and conceive of the truth which is presented to me as possessed of a value beyond the limitations of my judicial personality, but I cannot divest this recognition of that subjective and personal character common to all my acts of thought which causes me to call them " mine." The conscious subject remains an individual subject even when its cognition is of universal validity: the universality attributed to the object and to the content of the judgment must

not be transferred to the subjective moment of the act of thought. It is of course true that thought cannot be severed from the thinker, but the two moments can and should be distinguished in consciousness, and we have no right to ascribe to the second of the two terms the characteristics which immediate experience has shown us to be proper to the first of them. I think the universal, but I do not on that account identify myself with it or divest myself of the contingencies and individual properties of my empirical person, or cease to be that determinate *Ego*. Nor must it be objected that the individual moment of each act of thought in its concreteness differs from that of every other act of thought, even of the same individual, and that there is consequently nothing permanent which could constitute the spiritual substance of the person, since, when we speak of the identity of the *Ego*, we do not mean to reduce the mind to an empty, motionless unity, incapable of variation or development, but merely wish to say that the activity of the *Ego* in the various phases of its evolution first feels directly and then reflectively recognises in itself a characteristic and constant physiognomy which justifies it in regarding itself as a substance. Those who dispute the substantiality of the *Ego* have formed a false concept of substance, one derived from the abstract materialistic conception of the atom, regarded as a thing which is inert and rigid in its absolute identity. If the substantiality of the *Ego* were of this nature, I should have no hesitation in acknowledging it to be a pure fiction, in no way partaking of the character of objective reality, since the identical, strictly understood, cannot be the expression of any concrete reality. In the real life of thought identity does not exist apart from diversity, or unity apart from variety. We may for purposes of study consider the identical apart from the diverse, but we must not say that the identical taken by itself possesses the sufficiency of concrete reality. Absolute identity would be death, the negation of conscious life;

we can, however, conceive it without annihilating thought, in as much as the concrete act of cognition in which one of the terms of the identity is present differs from the successive act in which the second term is presented to us. The formula $A = A$ has meaning, because a certain diversity is understood: thought when it passes from the first to the second A does not remain absolutely identical: if it did, the duplication of the first A and the consequent identification would both be impossible, we should remain stationary at the first A and be unable to form any judgment. Identity is always relative, never absolute. Even in mathematics, the science in which the principle of identity is most strictly applied, there is no such thing as the establishment of relations between absolutely identical terms. There is always a certain element of diversity which renders mathematical thought fruitful and progressive, and without which it would be doomed to an isolated barren existence in each of its concepts and would be unable to establish relations between one concept and another. Mathematical equality is always identity of diversity. Two congruent triangles are not identical in every respect, but differ as to their position in space; in the arithmetical equation $8 + 4 = 12$, the first member is not absolutely identical with the second, but differs from it in as much as the units of which it is composed are differently arranged. Absolute identity is a mere fiction; concrete thought advances and develops by identifying the diverse and diversifying the identical.

Thus the word substance as applied to mind denotes no empty, formal identity, but rather an activity which recognises its own identity amid variations and preserves its characteristic unity throughout its development. Undoubtedly every moment brings forth something new in our consciousness, but do not let us indulge in sophisms: there is a very great difference between admitting this fact and saying that everything is new and nothing permanent. From a logical point of view there is an infinite series of degrees between all and

nothing. Those who proclaim the testimony to the permanency of the *Ego* to be an illusion which they endeavour to explain by the relative persistency of certain contents of consciousness, and more especially of internal, rhythmic sensations and the persistence of memories from past life, fail to realise that they take for granted that very identity of the *Ego* which they would fain deny. I believe myself to be the same *Ego* who existed some years ago (so they say), because, though I am subject to change, this change is so gradual that it escapes my notice: it is an illusion of the same nature as that which makes a person who gazes at the hour and minute hands of a watch think that they are stationary when they are in reality moving. We experience the same thing as mothers whose children are constantly with them, and who consequently do not notice their growth and development and complain that they are not growing, though friends who see them again after an interval of some months are struck by the difference in their height. Thus if some magic spell could enable us to deprive ourselves for some years of the continual presence of our spiritual and corporal life, we should not recognise ourselves as the same beings when we returned. Is not this proved by the phenomena of dual and alternating personality, caused by profound amnesia or by sudden changes of organic sensibility which make the *Ego* lose consciousness of its own identity?

Those who take up this line of argument forget that the identity of the *Ego* which remembers and recognises is the epistemological basis of both memory and recognition. How could I recognise that I am or am not the *Ego* of days gone by if nothing had persisted in me, if the *Ego* which looks at the many-coloured pictures spread before it by consciousness had nothing in common with the *Ego* which beheld other changing scenes in the past? If everything were subject to change, who could perceive that change? If the becoming of the mind were a series of discontinuous acts, each of

them entirely new as regards the preceding, it would be inexplicable how the memory and the recognition of past contents and acts could form part of the present act; were this so, the acts of thought of one and the same person would remain extraneous to one another, and would be incommunicable just as are the acts of thought of two different persons.

5. *The Sophisms of the Idealist.*—Idealism, which claims to embrace the whole of reality in a single eternal act of thought which is ever being renewed in the dialectic rhythm of its life, cannot account for the profound and ineffaceable difference between the form of the intimate relation which links together the successive acts of thought of the same individual and the extrinsic form of relation which may exist between the thoughts of different individuals. The possibility of the simultaneous existence of various acts of thought, which cannot be communicated in their subjectivity until speech or the dumb language of eye or gesture reveals to one soul the secret of another, remains an incomprehensible enigma from the idealistic point of view. My past penetrates and pervades my present; do what I may, I cannot banish it from my act of thought: its shade, like the ghost of Banquo, stands before me, try as I will to close my eyes; its voice echoes in the solitary places of my mind, try as I will to silence it and to stop my ears. Thus the present itself only exists in virtue of the past which lives afresh in it; and the continuity of psychic life makes it impossible to draw a boundary line dividing that past from the actual moment of consciousness. The special character of the *Ego* as an individual subject is to be found in this intimate interpenetration of different acts of thought succeeding one another in the spiritual becoming; and it is just this fusion which distinguishes that becoming from the relation between the consciousnesses of different individuals. These consciousnesses defy all attempts at direct penetration, and remain incommunicable in their intimacy; we can only reconstruct the subjective

life of others by the help of outward signs, whereas I can read my own past without seeing, hearing, or making any induction whatsoever. Now from the idealistic point of view, which denies the substantiality of the individual *Ego* and the plurality of various subjects, such a far-reaching difference is incomprehensible, since, if the various consciousnesses existed merely by virtue of the eternal act of thought which embraces them all, they should penetrate one another as do different moments of the same spiritual life, and it should be possible for me to read the mind and past of another just as I do my own. Every obstacle to the intimate communication and fusion of minds should vanish, and one mind be visible to another without any intervening veil of secret thought or feeling. Who, however, would venture to say that his mind is possessed of unlimited knowledge of the intimate life of other minds? Moreover, if two of these minds remain impenetrable, when indirect communication through the senses is removed, how can it be asserted that they form part of the same act of thought? Are not reciprocal transparency and unity of the various parts of the content essential if we are to speak of a single act of thought and not of a plurality of acts? Can one part of actual thought be ignorant of that which is known to another part? It is clear that we should then not have one thought but two thoughts, absolutely external to each other. The only hope for the idealistic thesis is in the admission that the Absolute is afflicted with hysteria and suffers from dual personality, as hysterical patients do!

But, the idealist may retort, how could I affirm the reality of other acts of thought if they did not exist in my thought? When I state that I understand the thought of others, do I not thus appropriate it as my own? It is not, however, difficult to discover the sophistry of this train of reasoning, the stock argument of idealism, the fortress within which it entrenches itself and declares itself to be invincible. Comprehend-

ing the thought of another individual does not imply identifying oneself with it as a subjective activity, but merely having the same content present to my consciousness as the other person has present to his; the two *Egos* are identified as regards the objective moment, not the subjective aspect. The thing thought is the same, not the act of thought. Hence, whilst I understand that which others think, whilst, that is to say, I possess the content of thought in common with others, I do not cease to feel that I am distinct from them as a conscious activity.

The idealist's argument has a certain appearance of validity simply because it starts from the false assumption that it is the variety of content which differentiates the various individuals, and that the subjective act is always the same. It is indeed obvious that if the distinction were derived solely from the content of the different minds they would be fused the moment they thought the same things; but, as we have already pointed out, the weakness of idealism lies in this very presupposition, since it makes it incomprehensible why direct communication should be impossible. The various conscious unities differ not merely as regards their content, but also as subjective activities; it is precisely as subjective activities that they remain impenetrable by direct means, and distinct even though they may be thinking the same thing. By "*subjective aspect*" I mean that character of the processes of my consciousness in virtue of which I can experience them directly, whereas I can never have direct concrete experience of the thought of others, but can only form an abstract conception of it. Am I to say that the consciousness of another person consists of my abstract conception, or should I not rather affirm that it has a concrete existence differing from my thought of it? In short, should the way in which I form an abstract thought of the consciousness of another be distinguished from the way in which it is concretely present to himself or not? Does his being, as a conscious individual, differ

from the thought I may have formed of him or not? Do not I in my turn in the intimacy of my consciousness differ from his thought of me? The being of my consciousness is one thing, the thought which those around me may have formed of me as a conscious subject is another, and *vice versa*. It is true that I cannot affirm the existence of the *Ego* of others without thinking of it; it is also true that nothing can be known without becoming the immanent object of my concrete act of thought, but to argue from this that everything exists *merely* as the immanent object of that act is pure sophistry. By what right do we exclude the possibility that that which is present in that act may and does exist also independently of it, and in a way differing from that in which it is present to us? Take the case of an individual who never goes out: it is clear that he can only know other people if they come to see him; what should we say if he were to take it into his head to state that other people only existed within the four walls of his house. Such reasoning certainly cannot be termed a logical masterpiece; but, *mutatis mutandis*, is the idealist's argument any more convincing? "The consciousnesses of others," he says, "and all the things which I affirm to be real, cannot be known by me unless they come in and form part of the actuality of my thought; hence they only exist in this concrete act of thought!" But a yet more subtle sophist might ask the sophist: "To what act do you refer? To the present one? Well, it eludes you the very moment you try to grasp it, and that act which you proclaim to be the only one in existence, the creator of existence, has vanished into the non-existence of the past, which can only exist by virtue of another act, which will in its turn, almost as if out of revenge, proclaim itself the only in existence, and so on *ad infinitum*. But the preceding moment, which you deny and relegate to the hell of non-existence, will rebel against its judge and say, "Thou art what I was; and I, since I existed in my full concreteness before thou wast born, cannot owe my

existence to thee who wast not then in the world." Had the past a concrete existence even before the act which affirms it came into being or not? Did that existence which is the object of my present thought owe its being merely to that thought? I do not think idealism will be prepared to go to such absurd lengths: there is then nothing for it but to admit the existence of something, *i.e.* the past, as independent of the act which now recalls it.

6. *Proof of Realism.*—I have said that the argument which excludes the possibility of existence independently of the act of thought is not conclusive, because it is not impossible that what is present in the act of thought exists also outside that act. But, the idealist retorts, how can you prove this independent existence, unless it be possible to get outside your thought? We reply that there is no need to get outside the act of thought in order to assure ourselves of the certainty of such an existence, since we find the incontrovertible proof of realism in that very concrete act of thought which we, like the idealist, regard as the centre from which reality radiates, and not in a blind common sense or an external resistance opposed to us from without. Every act of judgment has a subjective aspect constituting its concrete individual physiognomy, by means of which it presents itself to me as mine; it has also always a significance that in its universality transcends the momentary act of thought which affirms it. It must, moreover, be noted that that which I assert to be universally valid is not the subjective act of thought, the transient moment of consciousness in which that truth is affirmed, but that which I judge to be true. My belief in the value of my thought is due to the certainty that the truth thought by me is not truth by virtue of this fugitive moment which affirms it, but that it also exists independently thereof; this would not be intelligible if the actual moment exhausted the whole domain of reality, just as the distinction drawn by us between subjective and objective would cease to be compre-

hensible. Even if it be granted that such a division is illusory, we should still have to explain how such an antithesis came into being. Why does not the act of thought remain in its eternal uniqueness, merely affirming itself instead of constructing other objects differing from itself? Let us admit that its nature is to annihilate itself perennially in order to rise again in the infinite rhythm of the present which is ever taking flight into the past; at best such an admission can account for the continuous renewal of the life of my thought, but not for the admission of anything independent of it, since the idealist cannot deny that this positing of a real world external to the act of judgment exists at least as an illusion.

The act of thought is not in the least obliged to place itself in opposition to a world external to itself in order to render the process of self-renewal possible: the act of thought could transform its content without leaving its subjective intimacy. Each moment of life would then be equally the negation of its precursor, and the dialectic movement would take place, yet thought would not be conscious of anything differing from itself. Dialectic can find no justification for assuming the existence of a limit external to thought, since if thought be eternal actuality of consciousness, the knowledge that the opposition was the result of its own act could never be absent from it. In short, if the thesis of idealism were correct, each one of us should be aware at every moment of his life that the antithesis of subject and object, the opposition between nature and our minds, owes its existence solely to our present act of consciousness. Now I must frankly own (and in this most of my fellowmen who are not idealists will agree with me) that all my searchings, past and present, into the secrets of my consciousness have failed to reveal the knowledge that this opposition is of my own creation; neither can it be urged that I construct it without being conscious of it, since from the idealistic point of view there is no such thing as unconscious activity. A proceeding of which

I am unconscious would, in fact, be a real activity external to thought, and this would imply a return to realism. Now, seeing that there is no doubt as to the existence in my mind of the recognition of an object which is affirmed as existing in reality even if that special act of thought had not come into being; seeing that being in its universality, like every eternal truth, is recognised in the very act of judgment which affirms it as independent of itself; and seeing, on the other hand, that the testimony of thought cannot be pronounced illusory without postulating unconscious activity, *i.e.* a reality external to thought, it is obvious that all roads alike lead to the thesis of realism.

7. *The Truth of Self-Consciousness.* — We then acknowledge the existence of my consciousness and of other consciousnesses external to it; a conclusion to which the analysis of the concrete act of thought has led us. But what am I, and what is the world of entities which thought posits outside me? Here we have another problem: let us begin by finding the solution of the first part of it. My mind is just as it is revealed to my consciousness; there is no sense in supposing that spiritual substance in itself is other than that of which I am conscious, since I can speak of my being only in so far as I have immediate knowledge of it. Any other mysterious, impenetrable *quid*, hidden behind the scenes of consciousness, which I might assume to be my true self in contradistinction to the one revealed to me by concrete thought, would always in ultimate analysis prove to be made up of elements of my actual consciousness, and thus I should be involved in self-contradiction. If we attempt to eliminate all these elements from thought, nothing real will be left, since reality itself is a form of conscious thought. Moreover, what reason have I to suppose the reality of my mind to be other than that of which I am immediately conscious? When dealing with the *Ego*, there is no sense in speaking of phenomenality, relativity, or subjectivity of cognition. It is indeed comprehensible up to a certain point that

outward things, being revealed through the medium of forms peculiar to our minds, may appear other than they are in themselves, since thought, when it invests objects with itself, may project something into them which is peculiar to itself, and should not be attributed to nature; but it is really incomprehensible that mind should change itself by appearing through the medium of its own forms. These forms are subjective, but is not the mind subjective also? For any falsification to be possible, it would be necessary for the subject to see itself through an extra-subjective function; this is obviously absurd, since a non-subjective cognition is an impossibility. Relativity and phenomenality presuppose two heterogeneous terms, one of which must be relative to and appear to the other; but in the case of self-consciousness subject and object are identical, and mind, being relative to and appearing to itself alone, cannot but possess absolutely accurate cognition of itself. That which is relative only to itself is to all intents and purposes absolute.

8. *The Knowableness of Nature.*—We now pass on to the second part of the problem: what are natural beings in themselves? It would appear as though having once admitted the existence of something real external to thought we must inevitably founder on the rock of agnosticism, which we have from the first pronounced to be absurd by reason of its inherent contradictions. This is, however, not the case: on the contrary, the very path by which we came to posit the reality of nature will of necessity lead us on to its knowableness. If I admit that something exists outside the momentary act of judgment, I do so because the laws and categories which this judgment of mine puts into play are presented to me as possessed of a universal value infinitely transcending the subjective moment. This means that the value of those laws and categories would exist even were I not to recognise it; it is not merely subjective, but is objective as well. It exists not only within but also without me: *to this my*

concrete thought bears witness. Being, then, and all its categorical determinations exist outside the subjective act of knowledge exactly as they do in that act. It is impossible to doubt the objective truth of the categories and supreme principles of reason without contradicting oneself, since any world which can be conceived as real will always bear the stamp of these categories and principles. Everything of which we can think as existing in reality must be rational; the illogical is an absurdity, the death at once of reality and of the thought of it. Those who assert that the categories and norms of reason are relative to our physiological constitution and to the special contingencies of vital adjustment, in the very act of affirming this, think of the organism, the environment, and their relation, thus putting into action those norms and categories, and, while they deceive themselves into thinking that they are conceiving a moment of the evolution of reality in which they are not valid, illogically make use of them to construct the world at that moment of time, thus presupposing their unconditional value in that evolutionary phase of the cosmic system also.

9. *Mind the Truth of Nature.*—There is then no such thing as an unknowable reality: now this would be incomprehensible did no essential relation exist between nature and mind. If there were no connecting link between reality and thought, if their union were purely fortuitous, knowledge would not be possible; why indeed should that which is in thought correspond in any way to that which is entirely extraneous to it? How could thought arrogate to itself the right to invest the world with its forms and to ascribe objective value to itself if nature and mind had nothing in common? Those who regard human consciousness as a mere accident in the cosmic becoming and regard the world of nature as self-sufficient must of necessity arrive at agnostic conclusions, since, if the true reality of things be complete in itself and perfect outside consciousness,

the addition of consciousness and its subjective forms could only change and falsify it. The agnostic thesis is the inevitable outcome of the mechanical theory of evolution, because the subject, if it be but a transitory moment, one of the countless things which may come into being in the cosmic becoming, cannot lay any claim to privileges not accorded to the others, look upon itself as representative of these others, or presume to contain their truth within itself. The naturalistic and mechanical conception is not, however, proof against even the most superficial criticism, since nature and its mechanism cannot be regarded as generating the mind; from the moment that it vanishes into nothingness the concept of it is divested of all those elements which belong to consciousness. It is hopeless to endeavour to conceive of a *quid* anterior to thought and giving birth thereto, since such a *quid*, if regarded as real in any sense, is already invested with the forms of thought, and is, so to speak, a fragment of thought projected into the past. The past itself, moreover, is only constructed by the aid of the category of time, which is a function of thought. Consciousness cannot then be placed materialistically on the same level as other things, since it is the luminous centre from which the reality of the universe radiates. Quench this light, and everything will vanish into the dreary gloom of nothingness. It is undoubtedly true that nature does not exist only in the act of thought, it is not necessary to postulate the absolute identity of the real and consciousness in order to account for the possibility of knowledge, but there is no doubt that this possibility is only intelligible when we admit the existence of an essential relation between nature and the mind. In order to give the mind the right to project its forms into the physical world, it will suffice to suppose human consciousness to be the end towards which the becoming of things tends. Nature, when thus regarded as a means to the advent of the spiritual life, will find therein the profound revelation of its true being. If thought

clothes the world with itself and projects its forms into things, it acts not out of tyranny and caprice, but rather because it recognises in itself the meaning of the world. The law by which it feels itself bound, the idea for the triumph of which it suffers and struggles, which is at once its glory and its pain, is not some special and exclusive end of its own, but the ideal to which all beings tend, the inner meaning of their unconscious activity. Just as a genius from time to time reveals to his fellow-men something which they had vaguely felt, but were unable to intuit and conceive lucidly, so human consciousness acts with regard to things in nature, which only acquire in and through it that knowledge of their own virtuality which they were powerless to realise of themselves; in the mind which embraces them, the act of thought which reconstructs them, they are raised to a higher power of reality, thus becoming truer than they could be of themselves while outside human knowledge. If we would penetrate farther into the heart of things we must not divest ourselves of consciousness, diminish its content and degrade it in order to put ourselves, so to speak, on their level; we must not lower ourselves and stoop to nature, but must rather raise her to our level and rise above ourselves, by intensifying our inner life, rendering our thoughts wider and more coherent, our will more free and conscious, our ideals more definite and lucid. The fulness of absolute reality lies before us, not behind, and we can only approach more nearly to this goal by striving to give expression to that which is as yet only potential in our mind. We must replace the static conception of the object which regards it as complete in itself in an inaccessible sphere by a dynamic view which beholds it in its progressive ascent to a higher form of knowledge and reality. Nature and the complex of beings of which it is composed certainly do not exist only in so far as they are in human thought, but in the transition from external existence to the light of knowledge, far from losing their character of reality, they acquire reality in a higher degree; they are not falsified,

as the agnostic would assert, but rise to higher truth. We too, then, may, in a realistic sense, take as our own Hegel's profound saying, "*Mind is the truth of nature.*"

Knowledge is not the passive reflection of things, a sort of photograph of them, but the active elevation of the object to the life of the subject. All nature is contained in thought, but thought contains more than nature — its profound meaning. Some philosophers maintain that it is a defect of cognition to be unable to grasp things as they are independently of thought, and to know merely what they are for consciousness: I rather regard this as constituting the grandeur and power of mind; its inability, that is, to become the mere passive recipient of outer things, its power of transcending and integrating them, and of completing that evolutionary process left unfinished in the physical world. Mind is the necessary complement of nature; thought is the perfection of reality. If nature be divested of this, its final fulfilment, nothing is left but an undecipherable enigma, giving rise to insoluble antinomies. What wonder if the physical world in its abstract mechanism, not being self-sufficient, should teem with contradictions when the attempt is made to set it up as a complete system? Give nature its value and concrete meaning as an instrument of spiritual life, and the spectre of the unknowable will vanish.

10. *Natural Monads.*—We have said that we must refer to the aims of mind if we would understand nature; we do not, however, intend to imply thereby that nature must be interpreted as an assemblage of spiritual monads, endowed with a greater or less degree of consciousness. The reduction of all things to systems of conscious units conceived by analogy to our own mind is an arbitrary proceeding, since the analogy is only legitimate where there is some outward evidence of psychic life, and affords no help to us from the point of view of intelligibility. If we conceive matter to be an assemblage of elementary consciousnesses, seeing that the character proper to the conscious *Ego* is, as we

have already observed, its inability to communicate directly with other *Egos*, we should arrive at the doorless, windowless monads of Leibnitz, which for purposes of intercommunion are forced to have recourse to the *deus ex machina* of the pre-established harmony. If we would eliminate the inconvenience of incommunicability without resorting to this expedient, there is nothing for it but to endow these monads with the ability to act upon others from without, and to be acted upon themselves in like manner. It is, however, obvious that monads would in that case cease to be merely minds, but would be bodies as well; their activity would not be merely psychical, but would partake of the physical as well. Thus, instead of simplifying the problem of matter, we should clearly have rendered it more complicated, since, while we should still have to explain the nature of matter as a system of bodies, we should also have to ascertain the nature of the psychical life of these monads. The consciousness which we might reasonably ascribe to them would be too rudimentary to afford us an explanation of the intricacy and rational co-ordination of their actions. If everything they do were present to their consciousness, if, that is to say, the rationality which our intellect recognises in the world of nature were wholly comprised within the sphere of consciousness of these monads, they would have no cause to envy Newton on the score of intelligence, devoid of brains as they are! I do not consider that procedure by analogy authorises us to draw such conclusions! In any case, then, the real activity of monads would extend farther than their consciousness; hence here also we find ourselves confronted by the problem of understanding nature as a complex of external actions.

11. *The Sense in which the Contents of Sensation are Real.*—If nature cannot be reduced to an assemblage of conscious monads, neither can it be regarded, on the other hand, as resulting from the various relations of the contents of sensation looked upon simply as realities which persist even when external to our mind. Sensation

is a fact which occurs every time a certain combination of subjective and objective conditions is present, and if some of these conditions be not fulfilled, it is obvious that the fact cannot occur in that way, or be possessed of that determinate quality. A particular shade of red, for instance, cannot exist as such if the subjective, psychological, and physiological conditions necessary to its verification be eliminated. Sensation, with its varied content, is always relative to the subject and its special physiological structure: we all know that the sensation produced by one and the same stimulus will vary according to the different organs upon which it acts, and the different conditions under which it does so. Which of these qualities presented to us by the object is the true one? They cannot all be true at the same time, since this would involve a violation of the principle of contradiction, hence we must choose one of these different appearances as the true property of the object at that moment; but what authorises us to choose one rather than another? In any case our choice must always be arbitrary.

We do not hereby wish to convey that the sensorial content is devoid of all reality. Given these conditions, internal and external to the psycho-physiological organism, and the facts which are felt will really occur; what sensation reveals to me is a real moment in the life of the universe. The sensible aspect of things is not an illusory appearance, but reality itself as it exists at that moment, in that combination of relations to which it owes its special physiognomy. Even those appearances which are commonly known as illusions of the senses are real facts which will be repeated every time those determinate conditions within and without the human organism are fulfilled; we term them illusory because under other circumstances other facts appear to us; as a matter of fact, however, they are just as real as the others, the bent stick in the water is as real as the straight stick out of the water, each one in the order of its special relations. What wonder, then, that if

the relations to which a phenomenon owes its special physiognomy be changed, the phenomenon should also be changed? The transformations of the sensible world are no argument against its reality, unless indeed we are prepared to postulate arbitrarily that that alone is real which remains identical with itself, thus returning to the ancient Eleatic conception of the one unchangeable and eternal Being. As long as we confine ourselves to affirming the reality of the sensorial contents in that determinate context of relations, we do not involve ourselves in any contradiction; the contradiction arises when we attempt to claim the rank of persistent reality for a phenomenon which is true only at that moment and in that determinate situation, and to set up as unconditioned that which has only a conditioned existence. Sensation is real, but it is so only in the passing moment; we cannot say to the instant, with Faust: "*Verweile doch, du bist so schön!*" without laying ourselves open to that which is fatal to all thought—the contradictory and the absurd.

12. *Concept of Nature.*—Neither the procedure by analogy which projects consciousness in a degraded form into material units, nor the hypostatisation of the sensorial contents, torn arbitrarily away from the intimate unity of the subject by which they are experienced, can reveal to us what nature is; thought alone, in as much as it is capable of universal functions, can transcend the subjective sphere to enable us to grasp that which is truly real in the world of things: it alone can extract from sensible concrete experience that which is truly intelligible in it, that which exists not merely in the passing moment, but in the eternity of mind. Nature is as thought constructs it with its universal concepts and universal principles: the truth of nature is the thought of nature, since the rational world built up by science and philosophy out of the scattered fragments of sensible experience forces itself by the necessity of its laws upon the mind, which cannot deny its value without denying itself. It is the business of the individual

sciences to give us detailed knowledge of the various spheres of being; knowledge which cannot be regarded as being final once for all, since reality itself is in a state of perennial evolution, and since new facts and orders of existence may consequently arise in the course of the incessant transformation of things. To philosophic speculation is assigned the task of seeking out that imperishable meaning of being, that eternal warp whose changing woof may indeed be woven by science, but which science may not alter, much less destroy. The philosopher, contemplating things in the interior light diffused by his thought, sees in the world of nature a complex of real centres of spontaneous activities, which, limiting one another externally as they do, appear to be mechanical, inert, and rigid, but are really in their inmost depth—which is free tendency towards the ends of mind—mobile and living, a complex of actions which, just because they are directed towards thought, are possessed of profound rationality, which must not, however, be understood as abstract identity, but rather as the concrete unity of their becoming. Were there no such thing in the world of nature as this creative spontaneity, that continuous evolution of the world, which even the upholders of the mechanical theory have been forced to recognise, would be an impossibility; moreover, such spontaneity cannot be regarded as emanating from one centre only, since in that case the plurality of conscious subjects revealed to us by experience, and the undeniable limitation which contact with other monads sets to their activity would alike be incomprehensible, and it would be impossible to account for the genesis of the external mechanism to which the reciprocal encounter of their actions gives birth. Where these spontaneous centres accidentally meet in their spontaneous actions, without any co-ordination of their activities, there we have the inert matter, the equilibrium, the relative and temporary permanency of physical body arising from the annihilation of activities aiming in opposite directions. Development is possible only

when actions are co-ordinated and have a common end, that is to say, in the organism in which spontaneity is most plainly manifested and comes to light as life. Moreover, the organism is but the tool of the mind, the complex of actions which are necessary in order that consciousness may be produced and may develop from a mere glimmer into the full light of thought. I say "necessary," not "sufficient," since, wherever consciousness makes its appearance, we have a new production whose adequate reason cannot be found in nature, but only in a Higher Consciousness. We have already shown that it is folly to attempt to derive mind from nature, the ideal from the real; yet the physical world in its objectivity presents itself as the necessary organ of spiritual life. The body is not something with which the mind can dispense, it is not the forbidding prison which the Platonists depicted in such gloomy colours, it is no torture-chamber in which mind is doomed to expiate some mysterious crime, but rather the fertile soil in which alone the plant of spirituality can develop and blossom. That which degrades and depresses the body has the like effect on consciousness. Do not let us then unduly exalt nature, as naturalism does, by placing it on the throne of absolute reality, but do not let us, on the other hand, depreciate it unduly, as the Platonists did. It is not a reality which is self-sufficient, but neither is it the absence and the negation of reality: its true meaning lies in its teleological relation to the life of consciousness, as a necessary moment of the process leading to the sublime heights of thought.

13. *The Vicious Circle of Empiricism.* — If the principles and supreme concepts with which thought reconstructs the world of nature be universally valid, because, as has been already proved, all doubt as to their value is contradictory, they cannot be regarded as being derived from changing experiences. These experiences can only testify to that which has occurred at the special moment at which it has occurred, but, taken by themselves, in no wise authorise us to make

any affirmation which transcends the sphere of observed facts. From such a point of view, no scientific law can be more than a summary of the phenomena of the past and of that part only of nature which has been the object of observation: its value will be purely historical, as a more or less schematic representation of a phase of the cosmic process, but it can tell us nothing either of the future or of that part of the world which has not yet come under observation. Even had we been able to verify a certain uniformity in natural phenomena and the repetition of some of their aspects, we should not on that account be justified in asserting that things *must* always happen thus, given the same conditions, but merely that they *usually do so.*

Further, is the derivation of the categories and norms of reason from the facts of experience an undertaking which has ever been successful or is ever likely to be so? Every attempt of the kind invariably ends in a vicious circle, since the empiricist is forced in his genetic explanation of mental functions to make use of these very categories and principles, which are therefore only got out of experience because the philosopher has placed them in it without being aware that he has done so. Were he to reflect upon his concrete procedure, he would see that he has not in reality started from immediate experience or from pure fact, but from thought fact, *i.e.* from fact which has already been clothed with all the forms of his thought, which is ever present, and from which he cannot free himself if he would form any concept of the things of which he treats, and enter into any discussion about them. Do not take the positivist seriously when he proclaims that he will presuppose nothing *a priori*, that he will admit nothing which does not result from facts: can he, when he makes these statements, divest himself of his thought, his mental activity, and of the norms which regulate it; can he put aside his *Ego* which judges, unifies, and distinguishes the contents of experience? Is the fact the starting-point of his philosophy, or is it

not rather his thought of the fact ? Moreover, in this activity of his thought, are not all those functions present and at work which he deludes himself into thinking he has derived from something outside himself ?

Thus he says that being is but a generalisation of the character of objectivity found in all sensorial contents, and he fails to see that, since he starts from the facts of experience as from something real, he has from the very first conceived of them as existing, thus putting into action this very category of being. He goes through the same little performance with the other categories. Identity is due to the fact that two similar phenomena are presented, and the parts common to both abstracted from those which differ ; thus the mind is of course assumed to be capable of identifying the contents in certain of their parts, since the presence of two similar facts is something which differs widely from the consciousness of their partial identity. How often are we not confronted by similar things without being in the least aware of their relation of likeness ? When we say that two similar sensations are presented, have we not already applied the category of identity ? If you succeed in deriving the categorical determinations from concrete experience by means of a process of abstraction, it is due to the fact that experience itself has already been characterised in your thought by means of these determinations, so that in ultimate analysis thought derives from sensorial facts by means of conscious reflection only that which it has previously placed there by its spontaneous and unconscious activity. The presupposition of what we believe we are explaining genetically is the cardinal illusion of empiricism. Thus when David Hume reduces the concept of causality to a habit generated by the action of repeated sequences of phenomena which determine in thought an association by temporal contiguity, he makes use of the very idea of the causal relation between repetition and habit in order to derive therefrom the genesis of the concept of cause.

14. *Irreducibility of Thought to Practical Activity.*— Thought cannot be considered as a function derived from other psychic activities which are not thought. The needs of organic adaptation, vital necessities, the requirements of practical action, to which many philosophers turn for an explanation of the genesis of intelligence, presuppose the physical and social environment as already constructed with all its determination; they presuppose the organism, conceived as already in existence with its organs and functions, and also the concept of life, of adaptation, and of the action of the organised individual upon the things which surround him. In all these various forms of reality, of which the philosopher makes use in his biological interpretation, we find already at work those very ideas and principles which are supposed to be derived from them. How can we derive thought from life if life can only be spoken of in so far as we form a concept thereof?

Intelligence, on the other hand, could not be an instrument of action, a weapon which gives us the victory in the struggle for existence, did its ideas and norms not correspond in some way to objective reality: if the world were essentially irrational, of what value could reason be in it? If everything were renewed every moment *ab imis*, it would be impossible to form even the simplest of our concepts. Could we ever have succeeded in abstracting certain common and persistent characteristics had these characteristics never been found in the transient content of our experience? The concept is useful because it enables us to foresee, but were nature a perennial flux devoid of all determinate character, were nothing constant in the becoming of the world, science would be useless and even injurious to human life. He who submitted to the guidance of its laws of persistency, and acted in conformity with them, would find himself perpetually in conflict with the new and altered conditions of the environment. To act as if something were lasting where nothing is so, to repeat the same acts when no external situation is ever even

partially repeated is assuredly not the best way of preserving life. The pragmatists have fashioned for their own use a nature which is submissive to human will, which treats its requirements with respect, allows itself to be moulded in accordance with its needs, and obeys all the capricious commands of scientific theories. It is matter so fluid that it can be adapted to every line of thought: this they maintain to be the explanation of the success of our previsions. The man of science, like an absolute monarch, issues a decree from his laboratory, and natural forces, like obedient subjects, never resist his will! One writer has gone so far as to speak of spells and magic; may he not have been the most logical of all the pragmatists, for how can he who accepts the premisses of this philosophy explain the verification and success of science unless he be prepared to credit human volition with a mysterious power over physical phenomena? The manipulation to which we subject experience in order to make it fit into the scientific concept is of a purely arbitrary nature, yet (marvellous to say!) this arbitrary proceeding is crowned with success. Do you know why it is successful, say the pragmatists? Because things are real only in so far as they are desired and willed: hence their reality is unable to resist our will simply because it is the product of our will. If, however, the environment were what it is merely in virtue of our will, if there were nothing determinate external to will, all actions would be equally successful. The difference between useful and harmful would then become meaningless: the organism would find no resistance to be overcome in an environment which would submit to its every wish. Meanwhile pragmatists turn to the theory of evolution and the law of adaptation for an explanation of the origin of intelligence and the concept. But to what is the will to be adapted if things do not exist independently of it, if they are only what it has decreed? Will constructs the intelligence as an instrument for adapting itself to the environment, but the

environment, on the other hand, is merely that which we wish it to be! Those of us who persist in treating the old-fashioned path of logic will regard this reasoning as a vicious circle, but some pragmatist or other will probably look on it as an enchanted circle from which some magic charm will show him the way out.

15. *Inadequacy of Nominalism.*—When we asserted that thought cannot be derived from sensations, we implicitly admitted the concept to be something very different from particular, more or less schematic images, and from their associative relations; we have, that is, refuted the nominalistic thesis. The old psychological argument, which has always been the stock weapon of nominalism, that it is impossible to think abstractly without having an image or a word present is in no way borne out by the observation of our consciousness. On the contrary, introspection tells us that we frequently follow a long and complicated train of reasoning without making use of internal speech or having a concrete representation for every concept: the images accompanying thought by no means exhaust the whole of its content. Even when the image does exist, we are perfectly aware that the concept is not the image, but rather that which the image represents; if we take it to be representative of a class of whose individual members we certainly do not form distinct images, if this one individual stands in our mind for all, do we not thereby recognise that thought can refer to objects which are not actually presented? When the mathematician reasons about a given individual triangle, but is at the same time convinced that his proof holds good of all possible triangles, when the physicist thinks of the law of Galileo as true not only of the body whose fall he follows in his imagination, but also of all possible falling objects, it is obvious that his thought extends objectively infinitely farther than the individual case which is present to him; he could not be sure of the universal validity of his theorem or law were he not able to conceive the whole class in his act of judgment.

If the whole class is present to him, and this presence is not in the image which is only an individual member thereof, is it not clear that the concept of the class is something very different from the idea of that individual?

Were the nominalistic thesis true, thought ought never to transcend the limits of the image, and since this image is but the reproduction of a perception in a less vivid and less clear form, it follows that thought should be bound by the law of the sensorial threshold. Could thought not transcend the image, it would not only be impossible to perceive and represent the intensities of stimuli and the magnitudes of objects, which are below the threshold, but even to think of them. The length of a thousandth part of a millimetre, for instance, could not be conceived if we failed in forming a concrete image of it. We should thus be led to the amazing conclusion that nothing exists outside the bounds of the threshold of consciousness; that the weight of a milligramme, for instance, cannot exist, because it eludes our tactile and muscular perception, and consequently cannot be represented. Could that be real which is not thinkable in some way or other? It is of no avail to appeal to the aid of symbols, and say that even if we have not the direct image in this case, we can represent this magnitude by means of symbols, since, if that for which the symbol stands were not somehow present to our thought, the sign would be meaningless, and would represent nothing but itself.

It is at present fashionable among theorists about science to speak of symbols, signs, imitations, and images, but these terms, far from shedding light on the cognitive relation, only give rise to confusion. Is the scientific concept a symbol? Does it only serve to recall the objects and facts of a determinate class, which are supposed to constitute true reality? Is it merely a shorthand sign? Were this the nature of the concept, it should be confined to schematising certain characteristics, to cataloguing them so as to render it easier to find them again; it should be but a register

of the past, which would never tell more than we have experienced, but rather much less owing to the necessity for abbreviation. Now, if we observe any principle, law, or physical concept whatsoever, we shall find that the exact opposite is the case; experience is not simply copied or abbreviated, but is rather completed, its gaps are filled up, and it is extended beyond the bounds of past observation in space and time; it is perfected and idealised in forms which satisfy the need of unity, continuity, and universality in our thought. The importance of the recent criticisms of science of Boutroux, Milhaud, Poincaré, Le Roy, and Wilbois is due to the prominence they have given to this constructive activity of mind which brings the world of experience to its logical perfection. In the scientific concept, then, the phenomena given in perception attain to a greater degree of coherence and intelligibility than in the practical world, and hence to a higher degree of truth, and approach more nearly to that total and wholly intelligible system which is the highest aim of thought. The sign and the symbol are valueless of themselves except in so far as they represent or serve to communicate something else: the scientific concept, on the other hand, is valuable in itself, in so far as it constitutes a step in advance towards that ideal of systematic coherence for which we yearn. The symbol is always worth less than the thing symbolised, and if it be possible for us to have the thing directly present with us, it is better to put the sign on one side and look at the thing directly. The concept, on the other hand, is always of greater value than the series of facts which acted as the starting-point of its formation; it tells us more than facts can do; hence, if the concept be replaced by phenomena in their concreteness, our knowledge must be the loser. Even if we assume, as Mach does, a memory capable of registering every individual fact, such a memory would be less valuable than intelligence which derives an organic world from this incoherent record of facts, and not only teaches us to know the past, but is rich in pre-

visions of the future. Intuitionists lay much stress upon the schematisation, simplification, and impoverishment to which the concept subjects concrete reality; but do we know less when we see in the moment we experience not only its transient aspect, but also the invisible bonds uniting it to the other moments of universal life? Every new relation, every new affinity which science discovers between a phenomenon and the countless other facts of nature shows it to us in a new light, and reveals an aspect of it which had hitherto remained unseen. Can a phenomenon be exhausted by the concrete appearance presented to us in intuition? Are there not rather countless other aspects in it which elude intuition, and which should be reconstructed by us in thought?

16. *Theoretical Value of the Scientific Concept.*—Not only the philosophic idea, but also the scientific concept is, in my opinion, of cognitive value, and is a necessary moment of thought, which cannot be reduced to practical activity. I cannot therefore accept the dualism artificially set up by Croce between philosophy and science as different functions of the mind, one theoretical, the other practical; between the pure concept, at once universal and concrete, and the pseudo-concept in which at least one of these two characteristics is always lacking, either, that is to say, concreteness or universality, so that it is either a universal devoid of representative content, like the concepts of mathematics, or a generic representation devoid of universality, like the concepts of animal and vegetable species. There can be no doubt that the mathematical or physical concept, considered in itself, apart from the presentations which formed its starting-point, ceases to correspond to anything real, and becomes a mere abstract fiction, on which we have no right to confer the rank of reality, by substituting it for the individual fact, and for immediate experience.

The criticisms of the contingentists and empirio-criticists were salutary as against this false interpretation

of scientific concepts, this hypostatisation of them which led to the denial of the reality of concrete world of sounds, colours, and forms, and placed concepts as real true entities in opposition to the direct life of our consciousness, which was declared to be merely illusory. But is such a severance from the world of intuition necessary ? Must consciousness inevitably repudiate intuition if it would rise to the physical and mathematical concept ? Must it of necessity clothe the shadow with a material form ? The fact that this abuse has been committed by some is no valid ground for maintaining that the function of the scientific concept is to take the place of the immediately experienced individual fact, and to banish that fact from consciousness. Its allotted task is not to supplant direct experience, but to complete it, raise it to a higher power, and show it to us, so to speak, in a new light, in a wider context of relations, and from a loftier standpoint: if, then, it would preserve its cognitive importance, it must remain in continuous relation with the experienced fact from which it started ; if it fails to do so, it will stumble in the void and run the risk of falling into the absurd. Vibration cannot take the place of sound, which will continue to be a true and real fact, even when I recognise the truth of acoustic, but if sound which is merely heard be real, sound which is thought in relation to vibration as well as presented in its concreteness is still more real, because by means of this concept it acquires new relations without losing anything of its intuitive reality. It is true that the scientific concept considered abstractly apart from any content whatsoever is not real, nevertheless the synthesis perennially effected in thought of the intuited individual and the universal determinations formulated by science is of theoretical value. Science merely gives us the predicate of the cognitive judgment whose subject is to be found in concrete experience. The error of scientific intellectualism lies in its claim that the predicate is self-sufficient and can take the place of the subject ; whereas, apart

from its relation to the subject, it is but an abstract entity incapable of realisation. This abstract character which mars the purity of knowledge when it is artificially severed from the intuitive atmosphere which can alone impart concrete life to it, is not a defect peculiar to the scientific concept, but one common to all concepts, even to the universal concepts of philosophy, which become mere abstract fictions when we endeavour to divorce them from the individual presentations which endow them with a concrete content.

The possibility of such a separation has been proved by all forms of pan-logism or philosophism, as it is termed by Croce, which consider philosophic concepts in their empty abstractness, and claim for them the place of immediate intuition, thus falling into an error akin to that of scientific intellectualism. Are we to deny the theoretical value of the concept merely because being, becoming, and mind itself degenerate into empty abstractions when severed from concrete consciousness and regarded apart from intuitive and individual life? Are we to disavow the value of the philosophic concept on account of the abuses of dialectic? Assuredly not, then why confine ourselves to considering scientific knowledge as misused by the intellectualists? Why claim that the predicates of science are possessed of the consistency of reality apart from their synthesis with immediate experience, when even the universal predicates of philosophy are not possessed of it? A triangle which is not a triangular object of some kind or other does not possess reality outside thought, neither is there such a thing as indeterminate mind which is not some individual consciousness or other; thus in both cases the synthesis of the idea and the concrete fact alone is real. If the philosophic concept be defined as the *a priori* synthesis of the universal and the individual, the scientific concept must be considered in the same spiritual synthesis; it will then no longer appear empty and abstract, but rather full of that intuitive life which it presupposes and pre-

serves in its concrete reality, and enriches with new determinations, revealing and distinguishing in it aspects and relations which had hitherto escaped notice in the light of principles and laws governing more or less vast realms of existence. Thought, when it has risen to the scientific concept, does not exclude from itself or annihilate intuition, but presupposes it, raised to a higher power, in various orders of relations, in order that its position in the system of reality may appear more and more determinate. Its function may appear to be to impoverish if it be considered in itself and apart, but it will manifest itself to us as an advance towards a greater degree of concreteness when that union with intuitive life is re-established without which knowledge neither does nor can exist. The maximum of concreteness is not to be found in immediate experience, in the passing moment which, taken in itself and severed from its relations to the vast world of intuition, is but a fragmentary and abstract vision, but rather in the individual fully determined by all its relations to the rest of the universe. Moreover, these relations are not constituted by the universal concepts of philosophy alone, but by all those other concepts which, although merely generic in their nature, yet correspond to more or less extensive and persistent systems of real relations.

Are we to deny the theoretical value of certain scientific concepts just because they are not universal and do not apply to all presentations? Such a proceeding does not seem to me justifiable. Supposing that none of what you contemptuously term pseudo-concepts had been formed, that the intuitive world were presented to us only in the tissue of pure concepts, would reality be better known? Would not our vision of it rather be dimmer and less determinate? The categories show you the relations common to all facts, without which nothing is thinkable; they represent the minimum of logical determination essential to the conception of their existence; but there are characteristics and relations which, whilst not applying to the

whole system of being, are constituents of a certain order of reality, and are hence no less necessary than the categories to its complete determination. These characteristics and relations, to which scientific concepts give prominence, teach us more about the fact than the categories alone can reveal, and enrich direct intuition with elements which it could never grasp, confined as it is to the unique, the individual, and the non-recurring aspect. Croce would be right in denying the theoretical significance of the pseudo-concept only if the becoming of the world left nothing unchanged in its course except the universal characteristics of the categories, and if there were not an infinite series of degrees between the eternal and the irrevocable moment. But, besides that which occurs only once and that which is ever being repeated, there exists that which is of greater or less duration; besides the characteristics common to all presentations and that which is a unique occurrence which can never be repeated, experience shows us aspects and relations in reality common to more or less extensive classes of facts. We cannot determine the position which belongs to each moment of becoming in the total system, we cannot truly understand its meaning in its determination, unless we grasp these relations as well; the pure concept teaches you that a given individual animal exists for an end, and precisely for that end which is the universal meaning of existence, that it is a moment in the life of mind, but the pseudo-concept alone can show you what moment and in what specific relations it stands to other moments, to what degree it actualises that end and by what means it tends thereto. The philosophic concept shows me the universal meaning, intuition the meaning special to each individual, but the scientific concept alone can enable me to understand how these individual meanings make up the total meaning. From the standpoint of the pure concept it is impossible to estimate the various degrees of dependence and to establish the various orders of special relations, since each fact is equally united in it with all and equally

distinct from all by reason of its singular character; between two human beings, for instance, there exists the same categorical unity and the same intuitive irreducibility as between a man and a fly! The category, in as much as it is a universal determination, is predicable of the whole sphere of reality; there are then no two facts, however heterogeneous and unlike, which cannot be united in the pure concept: everything is, and everything becomes; everything is possessed of an individual physiognomy, an irreducible quality, and lives in the unity of the universal mind. The pure concept is the synthesis of the determinations which are valid for all the intuitive moments of reality, and which therefore unite them all in the same degree. Only the scientific concept can enable us to determine the various orders of relations, to measure their extension, to distinguish the different degrees of unity and diversity, showing us how the fundamental unity which lives in all things is realised in the various spheres of existence and may be recognised in the unities of temporary processes; how the eternal and infinite idea is determined in syntheses limited in time and space, but not, however, wholly reducible to a single intuition.

17. *History, Science, Philosophy.*—The method of the concrete knowledge of history and philosophy is frequently contrasted with the abstractive method of science: a distinction which is justified if science is understood to mean a system of self-sufficient concepts which are to take the place of immediate experience—the view taken by old-fashioned intellectualism—since, if the stability of the concept is obtained by the elimination of the subjective and individual element from consciousness, it is obvious that such a concept cannot claim to be the whole of reality in the fulness of its life. But if scientific thought be understood in its true sense, not, that is to say, as the negation of the intuitive activity of our mind but as the raising of it to higher syntheses without any loss of concreteness, the arbitrary distinction

drawn between the method and end of science and the method and end of history will vanish, unless indeed we propose to regard the bare assertion of the reality of single events as historical knowledge; this would not, however, amount to recognising a difference in ends, since science, too, starts from the affirmation of the fact. The end of science is not to attain to abstract laws, but rather to comprehend the facts of experience in the light of laws; and history in its turn, if it aspires to be true knowledge, and not merely the successive enumeration of a disconnected series of events, must aim at the concept, the union of those events in an order of relations which grows more and more complex and determinate. Theoretical activity neither has nor can have more than one end: the comprehension of reality in the fulness of all its aspects. Its starting-point is the fact experienced in its individual physiognomy, its goal neither is nor can be other than a concept in which the intuitive moment loses nothing of its concrete reality, but rather gains all those determinations which escape immediate consciousness. Individual reality integrated in the universal system of all its relations is the goal of philosophic thought; to this ideal end tend history and science alike, and they should therefore, when fully actualised and perfected, coincide with one another in the unity of philosophy. The severance commonly made between history and philosophy and history and science is a practical device justified by the limited nature of the human mind, like that between philosophy and special sciences, and between the various individual sciences. To a thought capable of taking in at once all the universal and special determinations in the single fact there would be no such thing as a plurality of sciences, there would be only science, that is to say philosophy, which would also have the concreteness of history; but the limitations of our consciousness render us incapable of this simultaneous view, hence the necessity of considering separately first one and then another determination of the fact, of studying first one order of relations and

then another—spatial relations, for instance, apart from physical and chemical changes, and these changes in turn apart from relations to the world of conscious life, an unavoidable necessity which philosophy itself cannot escape, constrained as it is to treat in one volume of aesthetic intuition, in a second of the pure concept, in a third of practical activity, which is in its turn sub-divided into economic and ethical activity, whereas spiritual reality is intuition, thought, and action in one concrete whole. Unfortunately, even the philosopher, though he knows well that the various moments or aspects of the real do not exist apart from one another, is constrained by the limitations of his thought to treat of them separately, hence the accusation brought against him by some foolish critics of dividing mind into compartments! The essential thing is not to lose sight of the synthesis which has been split up in order to facilitate research, and not to reproach scientific knowledge with the abstraction which is a practical drawback, that cannot be eliminated even in the realm of philosophy, at all events not until some genius shall arise capable of comprehending in a single act of thought all the universal and special determinations of the single event, all its relations to the system of facts which have existed and which exist in the passing moment. The separation of historical research from scientific elaboration is, like all other divisions of theoretical activity, of purely practical value; the distinction between the verification of the existence of facts and their order of succession and co-existence, that physiognomy which is never repeated twice in the same way, and the study of all the other determinations and relations which are repeated, and are therefore comprised in the realm of scientific knowledge, is one of convenience only. Theoretical activity does not face in two opposite directions: towards abstract law on the one hand, towards the individual on the other; it has always the same end: the vision of concrete facts in the light of the conceptual systems which determine them in their relations. There are not two logics, one

of science, the other of philosophy, but logic is one as thought is one.

18. *Impossibility of a Dialectical Deduction of the Categories.*—If it be a hopeless undertaking to search for the empirical genesis of mental categories, the attempt to derive them by the dialectical method from pure rational activity is no less foolish. Thought, starting, as Hegel does, from indeterminate being, could never of its own intrinsic necessity reach all other determinations were they not already in the consciousness of the philosopher and did he not know that which he had to construct. Reality in its intuitive fulness is present to him even when he thinks of being in its abstractness; hence he has no difficulty in deceiving himself into the belief that he is deducing by means of intrinsic necessity that which is known in another way, when he defines each determination artificially by means of the combined play of other concepts arranged symmetrically in such a way as always to reproduce the same rhythm of thesis, antithesis, and synthesis. Thus, to say that the point, according to the dialectic of Hegel, is the negation of space as non-differentiated externality; that the line in its turn is the self-negation of the point, and the surface the negation of this negation, is an arbitrary method of defining these concepts which does not in the least correspond to their true content; how then can it be asserted that the various complications of these abstractions give rise to their determinate characters? The concept of the point, in negating itself, leads to the concept of the line; but why? Because the line has been arbitrarily defined by the philosopher as the negation of the point. In this way dialectic can indeed perform miracles, by defining a concept by a postulate as the negation of some other moment of real thought, and then presenting it to us as the inevitable result of this negation. Moreover, there is no moment of thought which cannot be thus arbitrarily manipulated, because, since each differs from the other, *it is not* that other; hence in Hegelian phrase-

ology it may be termed the negation of that other, and may therefore be placed towards it in the relation of antithesis to thesis; then, if we select a third concept which differs from the second, it can always be presented as its negation, that is to say, as the negation of the negation, or synthesis. Speculative imagination will thus afford a wide field for the most varied combinations; and no philosopher who adopts the dialectical method need be at a loss for opportunities of displaying his inventive genius by ordering the determinations of the real in different ways, arranging them at will in the sacred triads of thesis, antithesis, and synthesis. Let us leave this performance to those to whom it affords intellectual entertainment; for my part I must own that I have not as yet reached such speculative heights as to find any enjoyment in it.

Absolute contradiction, the unity of being and non-being, of thought which everlastingly posits itself and annihilates itself in the very act of self-affirmation, are fictions no less abstract than the absolute identity of being with itself: if the formula $A = A$ is empty, the other which it is proposed to substitute for it, $A = non\text{-}A$, is the concentration of emptiness. The contradictory indeed, the *non-A*, the negation of any concept, does not correspond to any conceptual determination; taken by itself, it does not characterise even abstractly any determinate sphere of reality, but merely indicates everything falling without the extension of the concept which we are taking into consideration. Hence pure negation never suffices to generate in thought the concept of other determinations; it leaves that which is outside the range of the concept which is negated wholly and entirely indeterminate. It merely tells us that there is always something beyond the limits of that which we negate, but it gives us no information as to the nature of this something, *i.e.* of all the other determinations of reality which are outside our concept and must supplement it in order to attain to the concrete fulness of being. If, then, we have no other means of knowing

what these other determinations are, it is useless to turn to dialectical negation for information; moreover, such negation is devoid of logical meaning when applied to the abstract category of being. It is meaningless to say that something does not exist except in the sense that it is excluded from one particular sphere of existence in order that it may be considered as existing in another order: for instance, there is meaning in the assertion that the perfect circle of the mathematician does not exist, because by this we merely mean to state that this figure in its perfection does not belong to the things of the outer world as empirically perceived, but is only an ideal object of thought. Negation then can only refer to a more or less extensive portion of being, but becomes entirely meaningless when attributed to being in its universality. Absolute non-being is not thinkable, since that which is thought exists at all events as an object of thought. Our judgment moves always in the domain of being, because that which is judged must be present to consciousness, that is to say, it must *be* in some way or other. Non-being cannot, then, be admitted as an abstract category: it is a mere verbal expression to which there is no corresponding idea. Negation here can only mean the absence of all content and of all special determination: being (and this is precisely Hegel's own argument), in as much as it is absolute abstraction, absolute indetermination, is identical with nothing; it is, however, obvious that the word *being* is here used in an equivocal sense, since the being which is affirmed in the thesis is being in its abstractness, whereas the being which is negated in the antithesis is determined being. Now Hegel forgets to prove that being in its abstractness is not, taken by itself, a determination of thought; even if it be lacking in all the determinations derived from the other categories, it does not follow that it may not itself be a primitive determination, a primitive attitude of thought, devoid, it is true, of content, but not of form, as Hegel unjustifiably adds. The being of the category is being as a

universal form, which cannot, it is true, be thought in its abstractness devoid of any content and severed therefrom, but can nevertheless be distinguished therefrom in thought as that to which the content is indifferent. It exists in the same sense and to the same degree as all other concepts, all other abstract determinations, exist: that is to say, not as a separate reality, but as a distinct moment in the organism of concrete thought resulting from the synthesis of all those determinations. To understand becoming, then, we do not, and need not, escape from being by an absolute negation. Movement, life, and development are not a transition from non-being to being, but rather a transition from one special determination of being to another. Neither of these determinations is posited as the absolute negation of the preceding, but as something different; and the different, which is always a positive quality, must not be confused with the contradictory, which denies without determining anything new as a concrete reality. If the identical which remains identical does not enable us to comprehend becoming, but holds us fast in a rigid concept, the dialectic which repudiates this identity, whilst it helps us out of this concept by denying it, precipitates us into the gloomy void of infinite indetermination, in which we should be doomed to grope for ever if experience with its changing content did not afford us the new determination we seek. Thus neither the thought which remains absolutely identical nor the thought which is ever dying and ever being born again out of nothing, thanks to its restless mania for self-contradiction, is that concrete thought which we have affirmed to be the necessary organ of philosophical speculation. Dialectic is concrete only in name; in reality it is a play of abstract fictions, a formulary imposed upon and falsifying the true life of our minds. Thought need not deny the past in order to ensure its own existence, but should merely live it over again in itself, enriched by the new contents which the flux of experience has brought with it; total death is not a

condition of self-renewal. The *Ego* may feel itself to be the same, and may really remain so in certain of its aspects and yet be transfigured in others. The new does not wholly exclude the persistence of the old, but is fused with it and organised in the unceasing motion of thought which is ever *aliusque et idem*, like the sun hymned by Horace in his *Carmen Saeculare*.

19. *Ideal Genesis of the Scientific Categories.*— Absolute rationalism and absolute empiricism are alike baffled by the problem of the genesis of the scientific categories: the former taking into account rational exigencies only, the latter pure experience only. If the world, as conceived by science, answers to the supreme norms of reason on the one hand, it possesses on the other certain characteristics which are not the outcome of any logical necessity, and can only be explained by reference to the details of our experience. Why is it that thought does not order the contents of sensation in time alone without arranging them also in a space? Why has this space three dimensions instead of a larger number? From the purely logical point of view there is nothing against the thought of a reality developing in a single series of facts without any spatial plurality of co-existent objects, or against the concept of a space of four dimensions. It is not then reason which is driven of its own intrinsic necessity to construct space and to endow it with three dimensions. If thought does so, it is due to the fact that it finds in its own empirical content certain special properties which are not intelligible unless we construct that category in that particular form. The supreme concepts with which the physical world is interwoven are assuredly not derived from experience; the logical activity of thought constructs them in their ideal purity, but the stimulus to their creation must be sought in the various presentations of experience. When our logical activity is confronted by certain facts and by certain special relations of those facts, there arises the problem how to render them intelligible, that is to say, how to elaborate them in such a way as to satisfy our

eternal demand for rationality. The various scientific categories represent precisely the solution of this problem; they are the means made use of by thought in order to evolve a rational world out of the incoherent chaos of facts. But in this process of elaboration logical activity is bound to respect the exigencies of empirical data, which are not something amorphous, indifferent as to what form they assume, but are possessed of determinate characteristics which thought must take into account in its ideal constructions.

Time, space, motion, causality, force, substance, and quantity, are one and all instruments constructed by our reason in order to find amid the various successive and co-existent appearances of the empirical world that unity and identity which is its supreme law. An experience of irreducible qualities, having nothing in common, each enclosed in itself and independent of the rest, would assuredly fail to satisfy thought which, being by reason of its universal nature posited as the living model of every other reality, is entitled to demand from all other beings and the system of them that concrete identity and that coherency which it recognises in itself. I say concrete unity, because scientific activity tends towards such an end, as I have already explained, even though reasons of practical convenience may constrain it to consider the identical apart from the different, to move, that is to say, in the realm of abstractions. Each new fact is a new problem demanding a new solution which must be understood; the categories are just the means which make the solution of the problem, that is to say, the transformation of the empirical chaos into a rational cosmos, the rise of experience to reason, a possibility.

20. *Cause, Substance, Quantity, Time, Space.*—Thus causality with all its laws is but the means which enables us to unify successive facts and identify the different, thus tracing the action of the old in the new. Substance is a concept constructed in order to grasp the concrete unity of simultaneous appearances amid their multi-

plicity and the persistence of the identical amid changes. Quantity is the expression of the effort to reduce qualitative variety to unity by considering different qualities as manifestations of one and the same quality which remains homogeneous in its augmentation or diminution. The time and space of mechanics in the homogeneity, continuity, and infinity of their structure afford a clearer revelation of the active work of logical thought, which in the successive moments of becoming and in the heterogeneity of simultaneous facts has emphasised an identical element which unites and comprehends them all in its uniformity. Experimental data, however, have not been altogether indifferent to such constructions: immediate experience in its various forms has also helped to guide the activity of thought into determinate channels, and has initiated that process which reason has since completed and brought to its ideal limit. Thus, if thought on the one hand demanded unity and identity, experience too acted as a spur to such research, proving, as it did, that certain aspects of reality recur, that not everything is new in the becoming of the world. If the mind exacted from facts that coherency which finds expression in the law of causality, if it required that nature should not contradict itself, but act identically under identical circumstances, experience did not refuse to comply with this demand, but rather gave its full consent to it, inviting it indeed by the constancy of certain successions of facts to investigate its intimate rationality. Moreover, if space and time, as mathematical categories, cannot be deduced from experience in their pure ideality, concrete experience possesses characteristics suggestive of these rational constructions. The space of the physicist has three dimensions, because three dimensions are necessary and sufficient to make our experiences intelligible: one or two would not suffice, four or more would be superfluous, since no empirical data exist which cannot be explained in space of three dimensions, and whose logical order will not be found therein.

21. *Primitive and Derivative Categories.*—But, it may be observed, if experience be thus called upon to explain the genesis of the scientific categories, shall we not also end in the vicious circle of empiricism? How is it possible to conceive this experience without putting the categories in action? To this objection we would reply that undoubtedly certain categories are presupposed in our ideal reconstruction, but they do not include either cause, substance, quantity, time, or mathematical space, but rather other categories, which are really primitive and fundamental, and are conditions essential to the thinkableness of any form whatsoever of experience. Such categories are identity and diversity, without which no function of thought, and hence no experience, is conceivable, since we cannot speak of any fact which is not thought, that is to say, distinguished from or identified with other facts. And with the unifying and differentiating activity of thought, we have presupposed the category of being, that is to say, the affirmation of facts as existing. Reality, diversity, and identity are not categories which can be placed on the same level as the others whose ideal genesis we have reconstructed, since, even if we dispense with these latter, we can still conceive of a form of psychical experience, whilst no fact, even of a spiritual order, is thinkable which is not more or less explicitly affirmed as existing, and which is not in some way unified or distinguished in the context of concrete thought. In our reconstruction we make no claim, as empiricism does, to start from pure fact and to derive reason from it, but we start from original and immediate forms of thought in order to derive other forms from them, not mechanically or naturalistically, but by means of the activity of thought itself which strives to derive from the empirical world a world more in conformity with its exigencies. We presuppose these norms, together with the primitive categories, as universally valid, not by a mere unreasonable act of faith, but because they constitute the minimum which is indis-

pensable to the existence of thought, and hence to all philosophic research. To attempt to philosophise without them would be the same thing as trying to fly outside the air in an absolute void. Their existence is sufficiently justified by the impossibility of dispensing with them. Epistemology can but emphasise these functions and these elementary principles, thus proving them to be necessary to thought, which would commit an act of suicide were it to attempt to get rid of them. With respect to the other categories which we have considered as being derivative, it is the task of the theory of knowledge to show how they are born of the demand for rationality, by deriving them, that is, from the end to which the activity of thought tends, that of rendering experience intelligible. This is no causal or naturalistic explanation, which would be a vicious circle, but is a teleological explanation which starts from the end of knowledge in order to derive from it the means necessary to its attainment.

22. *Ideal Genesis and Value of the Mechanical Interpretation of Physical Phenomena.*—If the ideal genesis of science be that which we have briefly described, the mechanical interpretation of physical phenomena is not a mere caprice, a result of the need for mental economy, but represents the means by which the chaotic world of sensations is to be transformed into a world more transparent to the light of intelligence. It is not due to a mere chance, an accidental historical precedence, as Ostwald and Andrade would have us believe, that mechanics has become the foundation of the temple of science. When Ostwald tells us that had the discoveries relative to thermic phenomena been made first, Tyndall's work, *Heat considered as a Form of Motion*, would have been replaced by one entitled, *Motion considered as a Form of Heat*, he commits a gross error from the epistemological point of view, since the concept of heat could not have performed the task fulfilled by motion as a means of making physical experience intelligible. The ideal forms of space and

time having been constructed, motion, also in an ideal form, appeared capable of resolution as a whole into their mathematical relations, and consequently perfectly intelligible: mechanics alone approached this type of rationality which geometry presented to thought.

The error of the phenomenalists consists in the belief that motion can be placed on the same level as the other qualities revealed to us by immediate experience, as, for instance, a variation of heat; it is not, however, difficult to see that they confuse pure motion, which is an ideal construction of the same order as space and time, with the qualitative data constituting the starting-point of that conceptual formation. It must not be urged against the mechanical interpretation that, whereas all sensations are equally subjective, it attributes a higher degree of reality to one particular sensorial datum than to other such data without any epistemological justification for so doing, because the motion with which it would reconstruct the world of physical phenomena has nothing to do with sensations, just as geometrical space has nothing to do with the empirical elements which acted as stimulus to its formation. Immediate consciousness receives a series of muscular, tactile, and visual sensations, differing from one another in quality; in other words, what is given is a qualitative change from which thought derives time, space, and pure motion, resolving it into a system of relations and integrating it in accordance with its laws in such a way as to render it intelligible. Motion must not then be put on the same level as the qualitative changes which give birth to the world of phenomena, since it is a condition no less necessary to the intelligibility of experience than are space and time. We are unable to conceive of any physical action which is transmitted from one point in space to another except in terms of motion; the objective stimuli, the sources from which our sensations are derived, are localised by us as at a greater or lesser distance in space; thus we may be able to form some concept of the specific nature of their

action, but never to eliminate the motion necessary to render the transition of these actions from the origin of the stimulus to our organs of sense intelligible. Thus the theory of emission has given place to that of undulation, and of late to the electro-magnetic theory, but the transmission of light in all the theories is only conceivable as motion. Moreover, a legitimate inductive procedure leads us to motion as the cause of some of our sensations. None of the opponents of the mechanical method—not even Mach himself—has ventured to deny the legitimacy of the induction which regards the vibratory movement of the particles of air as the stimulus of the auditory sensations: the phenomenalists have not reached this point, because in order to do so they would have been forced to deny the existence of that which they can see and handle: vibratory motions can, in fact, be registered by the kymograph, and can be felt by touching a vibrating cord. Had the audacity of Mach, Ostwald, and Duhem reached such a pitch, the gramophone would have found in the sinuosities of its discs a high, shrill voice to raise in solemn contradiction of their statements. The old mechanical method, vitiated more or less by materialism, interpreted the results of scientific research upon the stimuli of the auditory sensations by the affirmation that the absolute reality is the movement of masses, of which sound is but an illusory appearance,—an unconscious metaphysic, against which the phenomenalistic reaction had a salutary effect. Sound is not an illusion, but a real phenomenon of consciousness which cannot be reduced to motion; vibration is the cause of sound, not sound as such. There is nothing of a metaphysical nature about the mechanical interpretation of the objective world if it be understood in this sense. When the physicist, under the guidance of the law of analogy, states that our sensations of light or heat are products of vibratory motions, he does not intend to state that heat and light are illusory appearances of a deeper reality, but merely means that the cause of these phenomena will be found

in special vibrations; he does not, in short, identify heat with motion, or claim to reduce it to motion, but, while distinguishing between them, he places them in a relation of cause and effect. Experience authorises him to make this induction, by showing him the close connection existing between that which he perceives directly as motion and that which gives him a sensation of light or heat. It is true that the vibrations cannot in this case be either seen or touched, but are we not entitled to transcend the bounds which the law of the threshold sets to our kinaesthetic sensations? Do we deny the existence of the distance of a thousandth part of a millimetre because it escapes our perception? No sensible physicist would be ready to deny the existence of movements which cannot be perceived on the strength of Newton's much-abused motto, *Hypotheses non fingo*, as though it were possible to set up an intelligible system of physics without going beyond the limits of perception. The alleged ability to create a system of physics without hypotheses has developed into a regular spiritual epidemic.

We do not believe that it is possible for all the branches of physical science to be absorbed by mechanics, because each of them contains something which cannot be reduced to mere laws of motion; but mechanics will ever be their common foundation. The error of Descartes lies in his belief that the whole of physical reality can be reconstructed with the aid of the laws of motion alone: even in the realm of pure mechanics the idea of force is necessary if the transition from rest to motion and the changes in the velocity and direction of motion are to be made intelligible. The concept of force, employed in rational mechanics, cannot be regarded as an empirical datum, and must not be confounded with the sensation of muscular effort which has merely been the stimulus to and occasion of its construction; its very definition proves it to be logically derived from the principle of causality. Force, then, like motion, cannot be placed on the same level as the sensible qualities,

which can be reduced to others, but must be considered as an indispensable means to the understanding of physical experience, like time, space, and motion. The science which studies the relations between these concepts and translates them into equations is of just as great apodeictic and universal value as mathematics: no physical phenomenon can be thought apart from these concepts and their relations. No physical theory can dispense with them, no matter what progress may be made by science, no matter how far the domain of its researches may be extended. The new electrical theory of matter, though it modifies the older concept of material mass and gives us a new and different interpretation of the properties of matter expressed in terms of electro-magnetism, continues to make use of the concepts of force, displacement, velocity, and acceleration, and every other imaginable theory must of necessity make use of these concepts just as the electrical theory does. Pure mechanics are then the basis and general form of all physical science; but though space, time, motion, and force are necessary, they are not sufficient to exhaust the whole content of external experience. In every natural phenomenon there will always be found something mechanical, but not everything is mechanical; it will therefore be found essential in every branch of physical science to integrate with the help of other explanatory concepts the universal principles of mechanics which merely present to us the universal form of physical reality, the warp with which he who would weave an intelligible world cannot dispense. There should, however, be as few as possible of these integrating elements; and it will always be permissible to attempt to resolve into these four concepts the new phenomena revealed to us by experience, transmuting them into an ideal form in which our intelligence may recognise itself. What, indeed, can be the aim of the age-long work of science, if it be not to bring to light the thought contained in the intimate nature of things?

23. *Spiritual Meaning of Science.*—The whole complex structure of the mechanical world is but a vast framework erected by the mind of man in order to raise experience to the unity of reason, and to understand amid the few scattered and fragmentary indications afforded us by nature that Living Thought which finds expression there, as in our own innermost mind. Science, by emphasising the rational unity which gathers together the phenomena of the universe into one vast organism, teaches us to understand them truly, since the word "understand" neither has nor can have any other meaning than to draw out the intelligible meaning of things, and to discover in the unconscious actions of natural beings those same principles which constitute the substance of our reason. Scientific knowledge teaches us to hear the inmost soul of things vibrating in unison with our own souls, and it is just this which enables us to understand them and to absorb into our *Ego* the revelation of their true being. It is commonly said that the man of science studies nature in order to wrest her secrets from her, it would perhaps be equally true to say that the silent voice of things, speaking to us through experience, asks of our mind the revelation of the end to which their unconscious activity tends. Obscure being would fain rise to the light of thought, and the soul of the man of science who listens to that voice is not the *Ego* of the solitary egoist who seeks his own intellectual pleasure, but rather the heroic soul which knows the virtue of sacrifice, and which, filled with yearning love for these lowly beings, strives to impart to them a spark of the flame which burns so brightly within itself: it is the soul which gives itself freely and is itself enriched by its self-sacrifice. The fire of a thought which is communicated burns all the more brightly and expands in the vortex of experience which gives it its unfailing nourishment.

24. *Epistemological Proof of the Existence of God.*—We have said that nature does not exist only in the mind of man, but also outside our thought. The cognitive relation would not, however, be intelligible if there were

no essential connection between nature and consciousness. If the mind of man were a pure accident in the cosmic becoming, a transitory moment which might be dispensed with, what would give our consciousness, which is but a very small part of the universe, the right to shed the rays of its forms and subjective idea over the whole of it? Would it not be an inexplicable miracle that outward things should submit to be moulded into these mental forms and that the data of experience should be thus easily bent to the requirements of reason? It is on this ground that we have asserted the necessity of a teleological view which conceives nature as a system of actions directed towards the mind of man, in which the revelation of their meaning will be found; it is on this ground that we have regarded reason as the final goal at which our experience aims, and to which by its means all the beings in the world tend. In this way only can we succeed in at once comprehending and legitimatising thought's projection of itself into things, since in this way their deepest meaning is found precisely in thought.

The teleological relation which unites nature to the human mind necessarily leads us to the idea of an Absolute Consciousness to which the two terms and their relations are present. An end which is not thought by some consciousness is an epistemological absurdity: it is only in thought that that which is not as yet real can be ideally anticipated. It is thus obvious how the monads of nature can tend towards consciousness without yet being conscious, since the idea of the human mind which they tend to actualise, though not thought by them, is present to that Eternal Thought which is the principle of their existence. The difficulty, on which we have already touched, of the human consciousness which is apparently the offspring of a complex of unconscious actions, will vanish at the same time since the mind is for us a new creation of the Absolute Consciousness in respect to the system of material conditions which are necessary but not sufficient for its generation.

Thus human thought is not derived by an epistemological absurdity from an antecedent unconscious reality, but owes its origin to another thought previous to which nothing existed.

In opposition to this, it might be urged that even if the idealistic thesis be rejected, and the existence of reality external to human thought be admitted, the legitimacy of the projection by consciousness of its forms into all things might still be justified by assuming the existence of a real connection, not a mere teleological and ideal link, between these forms and the mind; for instance, by regarding nature and mind as manifestations of one and the same substance. But this view affords no solution of the problem, and only brings us back to the very point from which we started, since we should still have to explain how thought could place itself in relation with this hypothetical substance, and what right it had to transfer its subjective forms to it. Such a substance must indeed either be unknowable, in which case we should find ourselves once more involved in the contradictions of agnosticism, which have been already exposed; or it is known and conceived in some way or other, in which case it implies that we consider ourselves entitled to reconstruct it with the categories and principles of our mind. Hence we find ourselves face to face with that very problem for the solution of which we turned to that third substance, which is neither material nor spiritual, and was supposed to act as a connecting link. Are we to have recourse to a fourth substance in order to make the relation between the supposed third substance and thought intelligible? It is obvious that such a procedure, which could be continued *ad infinitum* without any result, would never enable us to overcome the difficulty.

If, then, we exclude the thesis of absolute idealism which eliminates the problem of the relations between thought and external reality by simply suppressing this latter term, if we reject all and every form of realistic

monism for the reasons already stated, there is but one way of explaining the essential relation which must exist between nature and the mind of man in order to make knowledge possible and to justify its value: we must posit between the two terms a link of an ideal order in an Absolute Consciousness to which both are present as successive phases of the realisation of an eternal design of that Consciousness. We do not then consider mind as being statically identical with nature, as realistic monism does, since their unity does not lie in a real substance, but rather in a process of which the various moments find their ideal synthesis in the end which unifies them.

25. *Faith in the Value of Science is Faith in God.*—If this ideal synthesis can only be understood if an Absolute Thought be postulated, it is obvious that faith in God is the necessary outcome of faith in the objective value of science. The man who demands that nature shall conform to the normative laws of his mind is assuming a rational order to be immanent in the heart of things, though he may be quite unaware that he is doing so. What else is that belief in the uniformity of natural laws, which is the basis of all scientific prevision, but belief in the coherency of nature, that is to say in the presupposition that cosmic processes are due not to caprice, but to reason? Further, if reason be regarded as real even when external to the human subject, if the *ought* which its norms contain be conceived of as valid even apart from our mind, are we not necessarily led to an Eternal Consciousness, for which these norms may be of value?

This will be evident if we reflect that science considers itself entitled to construct with the forms and principles of our mind not only the actual reality of things, but also the history of the evolution of the world before the birth of man. Thus the celestial mechanics of Laplace, which formulate in terms of mathematics the cosmic processes which took place before human consciousness came into being, imply the belief that the supreme

categories and axioms which form the framework of our intelligence were objectively valid before that consciousness existed ; they imply the idea that the world, before the birth of man, was of such a nature as to make it possible to reconstruct it in terms of consciousness. Were this not the case, what authority would Laplace or his disciples who try to find a mechanical interpretation of the genesis of the world have for projecting into the past the concepts and principles of a thought which had not then come into being ? This means that, even before consciousness existed, the world was adapted to transcription into our mathematical formulas and mental schemes. Now such an adaptation of a reality to something which had not yet been born is inexplicable, unless it be admitted that nature tends towards the ends of the mind, and that the cosmic process is the Manifestation of an Absolute Thought. Only if reality be looked upon as the expression of a mind analogous to our own have we any right to demand that the same ideal norms which form the essence of our reason shall be valid therein, and to presuppose in the order of objective things a mathematical system governed by the same laws as our own. Only that which bears the stamp of the Creative Thought can legitimately be understood and reconstructed in terms of thought.

The rationality which science postulates in nature leads us to Divine Consciousness as its necessary epistemological complement, because a reason, an objective system of concepts and ideal relations, without consciousness, is something which we cannot understand. Reason, in as much as it implies norms, that is to say, reference to ends and values which can only have an ideal existence, cannot be divorced from consciousness. An idea, an *ought*, extending to that which is not as yet actual, to all the possible experiences of the future, can only exist as a moment in a conscious thought. A norm which is not a norm for any consciousness is a logical absurdity. He who believes in the objective value of his science must then also believe in God : if an Absolute Thought

does not exist, nature cannot be rational, and if there is no rationality in things, the reconstruction which we make of them with the categories and principles of our mind is an arbitrary projection of no value whatsoever. He who doubts the existence of God must doubt the objective value of his cognition. We seek the deepest reasons for faith not in some blind feeling, nor yet in an illogical will to believe at all costs, but in those very rational motives which lie at the root of the exigencies of science. The scientific man who sets himself to understand nature manifests his faith in the rationality of the world by the very act of turning to her in the yearning of his soul, and works all unknowingly for the glory of God, even though he may call himself a materialist. The voice of the Eternal speaks to his reluctant mind through sensible appearances; he is the unconscious priest of an undying religion, of that faith whose temple is the universe, and whose inexhaustible revelation will be found in the inmost depths of the mind.

26. *Denial of the Conflict between Pure Reason and Practical Reason.*—God is thus brought before us as the necessary basis of the possibility of knowledge: the criticism of pure reason itself leads to Him even apart from the exigencies of moral life. We therefore take a firm stand against those who, following in the footsteps of Kant, speak of an alleged want of harmony between intellect and feeling, the world of existence and the world of values, between pure reason which is confined to the sphere of hopeless necessity and practical reason which soars to the fruitful liberty of the mind. It is at present fashionable to depict in lurid colours the struggle between the two opposing exigencies of the mind, which is confronted by the painful dilemma of intellectual reasons and *raisons de cœur*, and is asked to make its choice between them, that is to say, between intellect without faith and faith without intelligence. As against this view, we affirm that there are not two reasons, but one reason only, at once pure and practical, a reason which in no way denies feeling, volition, and spontaneous

tendency towards an end, but which holds them to be its vital content just as much as the rest of our concrete spiritual experience. The error of intellectualism does not lie in making use of reason, but in mutilating experience by regarding physical facts as the only ones worthy of consideration, that is to say, in assuming sensible experience as its only basis, whereas our ethical, aesthetic, and religious feelings, of whose norms we are intimately conscious, are facts every whit as real as the fall of a body or an electric discharge. Concrete thought, as we understand it, must comprehend the integral totality of experience, and hence embrace both feeling and volition in the fulness of its higher synthesis. It does not renounce this living portion of its content, but strives to understand it, just as it endeavours to render sensible experiences intelligible.

Undoubtedly, if we restrict pure reason to the comprehension of sensorial data alone, we shall find ourselves confronted by insoluble antinomies; and the rest of our mind will revolt against a reason which fails to take an essential part of consciousness into account, the very part, moreover, which is most truly ours and of which we are most certain. Are we on this ground to say that the intelligence is the root of all evil, and not rather the one-sided use of it, its arbitrary limitation to a single range of facts and the equally arbitrary banishment of the others from the realm of philosophic knowledge? Does thought cease to be theoretical when the data of moral experience form its content? If this content is not intelligible, unless we grant the existence of an Absolute Consciousness only in relation to which there can be any meaning in speaking of an imperative category, and a universal end transcending our subjective life, do we not in this way arrive at a theoretical certainty of the existence of God? To put it briefly, I am consciously constrained to pursue an end whose value is manifested to me as something having also an existence independent of my individual personality, as something which is not of my creation, but which I

merely recognise of my own free will; this experience is a fact no less indubitable than the expansion of a body resulting from a rise in temperature. My theoretical reason is confronted by such a fact, and tries to understand it, and can only succeed in rendering it intelligible by supposing that an Absolute Subject exists which is above human subjects, and to which that end and value are present, since it is obvious that an ideal can only exist objectively in a consciousness.

Does he who takes up this line of argument prove the existence of God theoretically or not? The fact which forms the starting-point is certainly moral, but the intellect which argues from it is the very intellect which makes inductions and deductions in the sphere of physical facts, and the conviction at which it arrives is always of the same order. We go, however, still farther, and maintain that it is not even necessary to start from the moral fact to attain to the certainty of an Absolute Consciousness, because we shall be equally led to that certainty by the consideration of the universal and objective value which we attribute to our thought in the cognition of natural phenomena. Kant in his *Kritik der reinen Vernunft* simply presupposes that the data of experience, though they come to us from an external reality, the thing in itself, submit to be moulded by our intellect, to which they offer no resistance when it arranges them in the subjective forms of space, time, and the categories. He ascribes to reason the right to legislate for the world of experience, but he fails to justify this right, or to explain to us how it comes about that sensorial data, which are derived from a source external to the subject, never rebel against these authoritative decrees of consciousness. If the thing in itself be independent of thought, how is it that its manifestations fit so perfectly into subjective forms? Is not this a miraculous coincidence? Even if it be admitted that such a lucky chance has occurred and may occur sometimes, by what right does our intellect claim that the empirical data will always

obey its decrees? What authorises it to assert that it *must* always be so? This *ought*, which Kant presupposes in experience, implies the concept that the thing in itself is not entirely independent of thought, but stands in an essential relation to it; it implies that nature in its unconsciousness is not extraneous to the ends of mind, but rather answers fully to the exigencies of rationality. The noumenon, though Kant may declare it to be unknowable, is at bottom assumed to be essentially rational: were this not the case it would be impossible to explain how it comes about that its phenomenal apparitions can be ordered according to the principles of our mind. Data not only are, but *must* be intelligible, this is the tacit postulate of the *Kritik der reinen Vernunft*; this means that the noumenon, far from escaping the toils of the intellect, does and must conform to its exigencies. Such conformity will, when we seek to explain it, necessarily lead us to the idea of finality immanent in nature conceived as a means to the advent of mind; an idea which was developed later by the genius of Kant himself in his *Kritik der Urteilskraft*; and, with the concept of end, we are necessarily led to the idea of an Absolute Thought also.

27. *Legitimacy of Other Types of Science, differing from Mechanics.* — If Kant refused to ascribe theoretical value to the finalistic conception of the world, and relegated it to the realm of contemplation and faith, he did so because he regarded causal explanation as exclusively legitimate in the realm of knowledge, and arbitrarily set up mathematical physics as the prototype of cognition. It is, however, clear that there is nothing to justify this apotheosis of the methods of mathematical physics as the sole organ of all scientific knowledge. Are there no phenomena brought to our notice which differ in character from physical phenomena? Can it be assumed *a priori* that the categories which serve to render physical phenomena intelligible must be necessary and sufficient for the comprehension

of other kinds of phenomena? If the office of the category be that which we have pointed out, it is obvious that, if the phenomena be varied, the means serving to emphasise their inner rationality must also be varied: the category, being an instrument for making experience intelligible, is also relative to the order of phenomena to be explained. Assuredly no one will wish to maintain the paradox that physical facts exhaust all reality; no one will wish to erase with an arbitrary stroke of the pen, vital, psychical, social, and human phenomena from the great book of nature, and these phenomena are undoubtedly presented as endowed with characteristics which make it impossible to reduce them to physical facts. If life, consciousness, and society contain something distinguishing them sharply from the physical world, why should we try to force these new orders of facts into mechanical schemes? By what authority do you deny the legitimacy of the use of other categories when those of mechanics are either useless or inadequate? By what authority do you presuppose that the only science is that which is *more geometrico demonstrata*, and exclude from the list of sciences all those which cannot be formulated mathematically?

There are then other types of knowledge whose claim to the name science is no less well founded than that of mechanics, and the categories which they employ in order to render phenomena intelligible cannot be simply banished from the realm of pure reason, just because they are not necessary in what Leonardo termed the paradise of mathematical sciences. "Infinite Goodness hath such ample arms" [1] as not to refuse to receive into the bliss of knowledge ideas of a non-mathematical order, since, if it be true from one point of view as Plato has it that ὁ θεὸς γεωμετρεῖ, when we take the mechanical expression of that Eternal Thought into consideration; if God be, as Dante has depicted Him, "He who a compass turned on the world's outer verge"; [2] if the whole physical universe be ordered *mensura, pondere et numero*, as we read in the Bible,

which inspired Leibnitz to assert that *Dum Deus calculat et cogitationem exercet, fit mundus*; it is, on the other hand, no less true that the Divine Consciousness is manifested in the fertile creation of new concrete forms, and in the free tendency towards the higher ends of the mind. The mechanical aspect of things must not so engross our attention as to prevent us from realising their living foundation and spontaneous activity which, though the equilibrium of opposing forces may at times render it less apparent, is always, even in its hidden energy, the unexhausted source of the cosmic process. The world, as conceived by mechanical science, is motionless persistency of laws and beings, the abstraction of that which endures, and leaves outside itself that living part of the world which is ever being transformed in its evolution.

28. *The Categories of Liberty and Finality.*—This most living part, which is nearest to us, which stirs within us, struggles, suffers, and hopes amid the tumult of our mind, cannot be understood by means of the schemes and formulas of mechanical science, but demands for its comprehension another and higher order of categories and principles. Such are the concepts of liberty and finality with which we can dispense in the world of physical experience, but which are necessary to the comprehension of our spiritual experience, and with it of reality in its true concreteness. Shall we deny the free initiative of our consciousness merely because the mechanical concept of causality leads us to exclude it, merely because in inert bodies the impulse to action must always come from without? It is obvious that by so doing we should fall into the above-mentioned error of trying to apply a category which is only suited to render one order of phenomena intelligible illegitimately to another and entirely heterogeneous class of facts which are not external to one another as are mechanical facts, but pervade one another in their intimate unity; hence it is meaningless to say that the action of one of them sets the other in motion, seeing

that the individual facts cannot be conceived as acting each on its own account. It is meaningless to talk of one psychical fact determining the next, because spiritual activity is found not in the individual moment, apart from the rest, but in the organic whole, in that indivisible context which we call the *Ego*. The spring of every conscious action must then be sought in the whole, that is to say in our mind.

The categories of liberty and finality are of great epistemological value, because only in virtue of them is it possible to form the complex of our experiences into a systematic unity. The causal relation connects one moment of the cosmic process with its successor, thus rendering their unification possible; in this way, however, each link is isolated from the others, and the manifold causal series remain independent of one another and cannot be systematised in a higher unity. Our reason therefore does not rest satisfied with causal explanation, which drags it on from term to term without ever enabling it to attain to that concrete unity of experience which is its supreme goal. It would fain find something identical amid the diversity of the separate causal successions; it seeks, amid the differences of the countless series of cause and effect, whose interweaving gives birth to the life of the universe, that rational unity which it eternally demands. A world in which there could be no link between one relation of causality and the rest, in which no series of causes and effects had anything in common with others, would be a chaotic clash and succession of events which thought would be utterly unable to understand, because, as we have already said more than once, to understand only means to find the unity of reason amid the variety of facts. In such a world thought would seek itself in vain.

The ideas of liberty and finality enable us to attain this higher unification which the category of cause fails to give us. Liberty enables us to transcend the indefinite process by which we are borne in a giddy whirl from one causal term to another, and makes it

possible for thought to embrace in one synthesis that series which would otherwise be left devoid of unity and hence of rationality. It is not a negation of causality in general, but rather the raising of it to a higher principle which renders it intelligible : it is a necessary complement of purely mechanical extrinsic action, which is transmitted from term to term, by an initial action which does not lead us back to anything external, but whose source is in the very subject whence it emanates. Mechanical transmission does not in the least help us to understand action, since it—if I may so put it—sends us from Herod to Pilate, and each of the terms in turn washes its hands of us, and sends us to ask an explanation from a preceding term. Now it is obvious that in this way we shall get ever farther from the true spring of action, without making any progress towards understanding it. In short, mechanical causality gives us the simple transmission, but not the origin of action, much less then does it enable us to understand it. Only in the experience of the creative spontaneity of our mind can we find once more that principle which will help us to understand the fertile genesis of actions. When we have once more entered into the heart of our own substance, in which the light of consciousness sheds its rays upon activity, we no longer feel the need of turning to something external to ourselves for an explanation of the origin of action, because its primitive source is within us, and is revealed to itself in its eternal genesis. In the inner concrete causality which finds its whole *raison d'être* in itself, and not in something extrinsic to itself, is found the logical fulfilment of mechanical causality. Moreover, the mechanical links through which the original action is transmitted will find the concrete unity of their process in that action and the end thereof. If the individual moments, however much they may differ, be considered as phases in the actualisation of an end, they are unified in that end ; the causal series, however heterogeneous they may be, are also reduced to unity by

the common end to which we conceive them to be directed.

The categories of finality and liberty, like the other categories we have investigated, do not owe their origin to a pure exigency of thought, but represent the logical solution of problems raised by experience; they are the only means of understanding it and raising it to reason. Thus the evolution of reality, the production of new elements, new worlds, new organic species, which are undeniable data of experience, requires the possibility of the creation of new concrete forms, because the repetition of an identical mechanism affords no adequate explanation of it; hence the necessity of assuming the existence at the base of things in nature of that same creative spontaneity which is revealed to us by our consciousness. Further, the convergence of various causal series towards a common function, which may be observed in organisms, and which is characteristic of life, can only be understood by means of the category of finality. The simplest of vital functions, for instance the process of assimilation in a cell, demands that various chemical reactions shall be co-ordinated in such a way as to result in the reintegration of the cellular substance. Now, even if every reaction taken by itself be capable of reduction to a constant relation of causality, and can therefore be explained by the ordinary laws of chemistry, chemistry does not afford us any explanation of the succession of the different reactions in such a determinate way as to lead to the conservation of the cell. The same may be said of all other physical and chemical facts whose combination leads to the complex organism; if an animal or plant be resolved into its component parts, scientific analysis will find in the single elements nothing which cannot be explained by physical and chemical laws, but have we thus found any chemical and physical explanation of life? Certainly not, because life is not to be found in each of the single facts, but in their combination in that special order and arrangement which ensures the preservation of the organism.

Why do the various causal series of physical and chemical processes come into contact with one another at those determinate points which correspond to the vital phases of the organised individual? There are only two possible solutions of this problem, fortuitous coincidence and finality. It is quite certain that chance cannot always by some miraculous means bring about such a lucky concurrence of the circumstances which ensure organic integrity. You might just as well expect always to hold all the trumps in your own hand! Finality is then the only category which can afford us an explanation of the constant convergence of these heterogeneous causal series towards the same point, since causality, I would say once more, enables us to understand the single successions of facts, when isolated from one another, but not the way in which they are combined in each case in order to bring about the different vital phenomena.

29. *The Accusation of Anthropomorphism.*—Our teleological interpretation of the process of reality and the theistic view which is its necessary epistemological complement will undoubtedly be accused of being an anthropomorphic illusion. Ever since the day when Xenophanes dared to subject the Greek myths to the cold criticism of thought, and to exclaim, "Every one pictures the gods as being like himself: the negro thinks of them as being black and flat-nosed, the Thracian as fair and blue-eyed, and, were horses and oxen able to paint, they would undoubtedly depict the gods as horses and oxen," the accusation of anthropomorphism has become a commonplace in polemics directed against spiritualistic and idealistic intuitions of the universe. Just as the accusation of being a "metaphysician" is supposed to annihilate a philosopher, so it is persistently held that the *coup de grâce* has been given to a philosophical system by the discovery that it represents the whole or a part of reality as akin in its essence to the human mind.

Is such an accusation valid? Can it detract from the truth of a philosophical conception? Can all and

every form of anthropomorphism be rejected *en bloc*, or should we not rather distinguish between anthropomorphism and anthropomorphism, just as we distinguish between sound and unsound metaphysic ? Is it possible to speculate on the inmost nature of things without drawing more or less consciously and deeply upon our spiritual experience ? A cursory examination of the systems devised from the most remote ages up to our own day will suffice to convince us that all philosophers who have not taken refuge in indifferent scepticism or convenient agnosticism have been unable to avoid deriving from their own consciousness both the elements and the principles essential to the construction of their systems, thus setting up a particle of their spiritual life as absolute reality, although they are frequently unaware of doing so. We need not be possessed of much acumen or wit in order to show the anthropomorphic element in the idea of a personal God : even the positivists and monists of the school of Haeckel were equal to that discovery ! We have been told till we are weary by every more or less shallow critic of religious beliefs that the primary cause of all things is represented by theism in the likeness and similitude of the spiritual personality of man, raised to its highest power ; it is therefore unnecessary to insist upon it again. There is no difficulty in finding the anthropomorphic elements harboured in every idealistic conception. Whether the universe be regarded as an organic process and cosmic evolution as the necessary actualisation of an idea immanent in nature, whether the becoming of reality be reduced to the unfolding of an impulsive tendency or looked upon as springing from a productive instinct of creation, in ultimate analysis we can only posit as the basis of things an aspect or function of human consciousness. Plato who raises the concepts of the human mind to immutable essences, Fichte who sees in the cosmic process the perennial endeavour of the *Ego* to render its freedom efficacious, Hegel who ascribes the value of absolute reality to

the dialectical movement of thought, Schopenhauer who regards will as the basis of things, Schelling, Frohschammer and Bergson who, in the eternal becoming of the world behold the work of an inexhaustible imagination, all derive the principle of their explanations from inner experience, and seek in their own minds the spark which shall lighten the world. What else is done by the most convinced materialists, the implacable foes of all finalistic order, those who would defend universal mechanism to the death ? At first sight it would appear that the world, as conceived of by them, is completely free from infiltrations of spiritualism : what can be more foreign to mind than inert matter, whose motions are subject to inflexible laws ? What can be more contrary to the free spontaneity of consciousness than physical determinism ? The worshippers of the atom and of energy, the dreamers of universal mathematics thus lull themselves to rest under the pleasing delusion that they have banished for ever from the realm of nature those ancient myths which told of spiritual substances, the arbiters of their will, and diffused into all things a breath of the infinite mind and the quickening ray of intelligence; they delude themselves into thinking that they have transcended the bounds of consciousness and penetrated into the very heart of the Absolute, of which the mind of man is but an illusory appearance, a shrouding veil, which must be drawn aside if we would enjoy the vision of the hidden goddess. We regret our inability to share their enthusiasm, recognising, as we do, that these concepts are just as deeply tinged with anthropomorphism as are other philosophical theories. What does the endeavour to translate the multifarious variety of phenomena into precise formulas mean but the tendency of thought to seek itself in nature and to mould her on the model of its own inmost laws ? Intelligence, an essential characteristic of the mind, is thus implicitly extended to all things, and the human mind in defiance of its unconscious detractors conceives of that which remains identical in nature in its own image. From what source do

the causal ordering of phenomena and the postulate of the uniformity of laws derive their consistency and meaning if it be not from the very substance of the mind? So much has been said against the anthropomorphic origin of the idea of end, just as if that of cause had any other. We will put aside the notion of efficient causality which would enable us easily to have the best of the argument by proving that notion to be but an extension of voluntary activity, and turn our attention to that empirical connection of constant succession which positivists, following the example of Stuart Mill, have endeavoured to substitute for the relation of cause and effect. When it is stated that in the physical world, given certain conditions, a determinate phenomenon must necessarily and invariably follow, are we not ascribing to nature that logical coherency which is proper to our thought? When we demand that in the physical world the experience of the future shall not contradict the laws established in the past by a process of induction, are we not really postulating the existence in things of something analogous to our reason? May not this intuitive belief in the rationality of nature be the projection of a characteristic of the human mind into the rest of the universe? It matters little whether such a projection be made consciously or unconsciously. What has been said of the causal nexus may be applied to all the other principles of classification used in scientific work. Fundamental scientific concepts are but projections of the principal kinds of internal experience: from what but the immediate sense of our personal identity amid the various changes in the content of consciousness is the conception of a permanent substance amid the variations of phenomena derived? Let Haeckel say what he will against anthropomorphism, his law of substance is at bottom a conception of the universe constructed by mind in its own image just as much as the idea of a personal God. Is not the concept of the fundamental unity of all things, the very essence of monistic intuition, modelled upon

the living concrete unity which we feel in our consciousness ?

If we eliminate these relations which form, so to speak, the framework of the physical world, we shall have nothing left but sensorial data which are in the last analysis themselves special modifications of consciousness, having no reality apart from consciousness, and from that context which constitutes our inmost human personality. This fact has not prevented a spurious anthropomorphism from detaching these particles of psychic life from the spiritual organism to which alone they owe reality and meaning, in order to endow them with the rank of independent essences: matter with all its attributes is at bottom made of spiritual stuff no less than that God whose place it has been so often called upon to fill, and the sensations and anticipations of sensations to which phenomenalistic empiricism would like to reduce universal reality are mere fragments of consciousness, projected outwards by the most serious of epistemological absurdities. This rapid survey of the principal philosophical intuitions has, I think, afforded sufficient proof of the fact that we must conceive reality in terms of consciousness, if we wish to know; hence, if by anthropomorphism in the widest sense of the word, we understand the representation of the basis of things in the likeness of the human mind in its totality or in some of its aspects, all systems are anthropomorphic. It could not be otherwise, since reality can only be known through the medium of the forms of the mind. Knowing this, we do not hesitate to conceive all the other centres of active spontaneity, and the common principle from which their existence is derived, as modelled on our spiritual substance in its most living and concrete aspects.

30. *Eternity of Creation.*—This principle is for us an Absolute Self-conscious Personality, which is, like our mind, Volition, Thought, and Imagination in one indivisible whole, an *Ego*, which is not motionless and shut up in an abstract identity, but eternally renews

itself in its inexhaustible life. Creative activity is its essence, just as it is in the essence of our mind, which experiences it in itself, and has therefore concrete cognition of it: it is no obscure mystery, no incomprehensible dogma, but rather something which is revealed to us in the continual evolution of universal reality and of our own consciousness. The question, "Why has God created the world?" is then meaningless to us who are incapable of conceiving a Mind which is not fruitful and active creative spontaneity. The work of creation is as eternal as that Consciousness which manifests its abundant life in that work. The lot of the theistic conception is not indissolubly bound up with that of a beginning of the cosmic process in time, since it is possible to reach the Personal God even if we concede the eternity of the world. The work of creation has no end, just as it had no beginning: we behold its accomplishment with our own eyes in everything which lives and is subject to change, in the opening flower, the sprouting seed, and the glowing dawn in the heavens.

Notes to Concluding Chapter

[1] Dante, *La Divina Commedia*, Purgatorio, Canto 3, line 122 (Longfellow's Translation).

[2] Dante, *La Divina Commedia*, Paradiso, Cant 19, lines 40-41 (Longfellow's Translation).

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