

**TEXT FLY
WITHIN THE
BOOK ONLY**

UNIVERSAL
LIBRARY

OU_154679

UNIVERSAL
LIBRARY

OSMANIA UNIVERSITY LIBRARY

Call No. 526.9

Accession No. 13003

Author C 46 E
Chapman, R. W.

Title Elements of astronomy for survey

This book should be returned on or before the date last marked below.

THE ELEMENTS OF
ASTRONOMY
FOR
SURVEYORS

OTHER BOOKS OF INTEREST

<i>Textbook of surveying and levelling</i>	H. Threlfall
<i>Traverse tables</i>	R. L. Gurden
<i>Textbook of geology</i>	J. Park
<i>Geology of gold</i>	E. J. Dunn
<i>An introduction to metalliferous mining</i>	S. Higham
<i>Manual of meteorology</i>	R. M. Deeley

THE ELEMENTS OF
ASTRONOMY
FOR
SURVEYORS

by

THE LATE R. W. CHAPMAN, M.A., B.C.E.

Fellow of the Royal Astronomical Society

Formerly Professor of Engineering in the

University of Adelaide

FIFTH EDITION

Revised and Enlarged

by

J. B. MACKIE

M.Sc., B.E., A.O.S.M., A.M.I.M.M., A.M.N.Z.I.E., F.G.S.

Senior Lecturer in Surveying, School of Mines,

Otago University, New Zealand



CHARLES GRIFFIN & COMPANY LIMITED
LONDON

Copyright

CHARLES GRIFFIN & CO. LTD
42 DRURY LANE, LONDON, WC2

All rights reserved

<i>First Edition</i>	.	.	.	.	1918
<i>Second Edition</i>	.	.	.	.	1922
<i>Third Edition</i>	.	.	.	.	1925
<i>Fourth Edition</i>	.	.	.	.	1933
<i>Second Impression</i>	.	.	.	.	1942
<i>Third Impression</i>	.	.	.	.	1944
<i>Fourth Impression</i>	.	.	.	.	1946
<i>Fifth Impression</i>	.	.	.	.	1948
<i>Fifth Edition</i>	.	.	.	.	1953

*Printed in Great Britain by
Bell & Bain, Limited, Glasgow*

PREFACE TO FIRST EDITION

ALTHOUGH there are several excellent books on Surveying that deal more or less thoroughly with astronomical observation, it appeared to the writer, as the result of his experience in teaching the subject, that there is a distinct need of an elementary work suitable for the student and for the surveyor who is taking up astronomical observation for the first time. Most of the purely surveying books are content to quote practical formulæ for the reduction of the observations, with little or no attempt to expound the principles by which the formulæ are derived. On the other hand, the theoretical works on astronomy in which the mathematical theory is developed are generally too recondite for the beginner, and deal to a large extent with matters of no special interest to the surveyor. The present work is an attempt to provide an elementary exposition, not only of the practical methods of observation and computation, but of the main principles that must be thoroughly understood if the surveyor is to be master of his profession. Throughout the work the methods of observation are illustrated with numerous fully worked-out actual observations, and a prominent feature of the book is the attention that is given to the effects of observational and instrumental errors of different kinds. A large proportion of the examples set for working have been taken from the papers set for candidates at the examinations for Licensed Surveyors in Australia.

R. W. C.

Adelaide,
September, 1918.

PREFACE TO FIFTH EDITION

THIS little volume has proved so useful in teaching practical astronomy, even though changes in the *Nautical Almanac* have rendered parts of previous editions obsolete, that I have very willingly undertaken the task of bringing it up to date. The very recent publication by the Admiralty of a new set of astronomical tables, in the form of *The Star Almanac for Land Surveyors*, has also made the time for a revision most opportune. This new *Star Almanac* will provide data sufficiently accurate for all ordinary survey work, but if greater precision is required, reference to the *Nautical Almanac* will still be necessary in the case of the Sun. With the stars, very precise work will require the use of another H.M. Stationery Office publication, viz., *Apparent Places of the Fundamental Stars*, since, beginning with the 1952 edition, the positions of the stars will be omitted from the *Nautical Almanac*.

The departures from the last edition of *The Elements of Astronomy for Surveyors* lie mostly in the chapters on Time and Longitude, and in the addition of a new chapter on Position Lines. Many new examples have been provided to replace those that have become obsolete, and the problems at the ends of the chapters have been revised.

As far as the text is concerned, I have endeavoured to retain the late author's own explanations, because they are generally so clearly put. There can be little doubt, in fact, that it is this very clarity which has made the book so popular in the past.

A few minor changes have been necessary to bring the notation into line with that now used in the *Nautical Almanac*.

J. B. M.

Dunedin,
New Zealand.
October, 1952.

CONTENTS

	<i>Page</i>
1. THE SOLUTION OF SPHERICAL TRIANGLES	1
2. THE CELESTIAL SPHERE AND ASTRONOMICAL COORDINATES	8
3. THE EARTH	19
4. THE SUN	31
5. TIME	39
6. THE LOCATION OF OBJECTS ON THE CELESTIAL SPHERE	80
7. ASTRONOMICAL AND INSTRUMENTAL CORRECTIONS TO OBSERVATIONS OF ALTITUDE AND AZIMUTH	89
8. THE DETERMINATION OF TRUE MERIDIAN	104
9. THE DETERMINATION OF LATITUDE	151
10. THE DETERMINATION OF TIME BY OBSERVATION	181
11. THE DETERMINATION OF LONGITUDE	214
12. SIMULTANEOUS DETERMINATION OF LATITUDE AND LONGITUDE FROM POSITION LINES	222
13. THE CONVERGENCE OF MERIDIANS	231
INDEX	241

CHAPTER I

THE SOLUTION OF SPHERICAL TRIANGLES

IN this chapter the principal formulæ of spherical trigonometry, such as will be afterwards applied to calculations on the celestial sphere, are brought together for convenient reference. No attempt will be made to establish the formulæ, which are demonstrated in any of the ordinary books on spherical trigonometry, but a brief synopsis will be given of the usual methods for the solution of spherical triangles under different conditions.

Great circles

The line of intersection made with the surface of a sphere by a plane passing through the centre of the sphere is known as a *great circle*. If this circle passes through two points A and B on the surface of the sphere, then the shortest distance between A and B, measured along the sphere's surface, is that measured along the arc of the great circle joining them. Only one great circle can be drawn to pass through two given points on the surface of a sphere, unless they happen to be at the opposite extremities of a diameter, and the length of the shorter arc of this great circle between the two points is the shortest distance between them. Meridians of longitude on the earth's surface are great circles.

In spherical trigonometry it is always assumed that the arcs representing the sides of the triangles considered are arcs of great circles.

Small circles

The line of intersection made with the surface of a sphere by a plane that does not pass through the centre is known as a *small circle*. The ordinary formulæ of spherical trigonometry do not apply to triangles having sides that are arcs of small circles. A parallel of latitude on the earth's surface is a small circle. It follows that the shortest distance between two points in the same latitude is not that measured along the parallel of latitude, but is that measured along the arc of the great circle joining them.

Spherical triangles

Denoting the angles of a spherical triangle by A , B , and C , and the sides opposite to these angles by a , b , and c respectively, the sides being as usual measured by the angles which they subtend at the centre of the sphere, then we have the following fundamental relations :—

(a) The sines of the angles are proportional to the sines of the opposite sides :—

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad . \quad . \quad . \quad (1)$$

(b) One side of a triangle is expressed in terms of the two other sides, and the angle included between them by one of the three formulæ :—

$$\left. \begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C \end{aligned} \right\} \quad . \quad (2)$$

(c) From these may be derived another set of six useful relationships of which the following two are types :—

$$\left. \begin{aligned} \cot a \sin b &= \cot A \sin C + \cos b \cos C \\ \cot b \sin a &= \cot B \sin C + \cos a \cos C \end{aligned} \right\} \quad . \quad (3)$$

(d) Again the cosine of the side of a triangle may be expressed in terms of the sines and cosines of the angles by one of the formulæ :—

$$\left. \begin{aligned} \cos A &= -\cos B \cos C + \sin B \sin C \cos a \\ \cos B &= -\cos C \cos A + \sin C \sin A \cos b \\ \cos C &= -\cos A \cos B + \sin A \sin B \cos c \end{aligned} \right\} \quad . \quad (4)$$

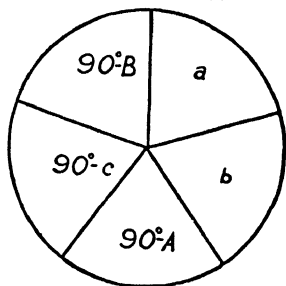
Whilst the formulæ (2) and (3) and (4) are extremely useful in all sorts of investigations into the properties of spherical triangles, they are not adapted to logarithmic computation, and are consequently not suitable for use in the numerical solution of triangles. For this purpose other formulæ are commonly used, derived from these fundamental relationships but expressed in a form more suitable for use with logarithms.

The solution of right-angled spherical triangles

The relationships between the sides and angles of a right-angled spherical triangle are conveniently summarised by the mnemonic

rules due to Napier, the inventor of logarithms, and known as Napier's Rules of Circular Parts.

Denoting the right angle by C, Napier defines five "circular" parts (i.e. a , b , $90^\circ - A$, $90^\circ - c$, $90^\circ - B$) and these are supposed, as in the figure, to be ranged round a circle in the order in which they stand in the triangle. Then, if any one of these five parts is selected and called the *middle* part, the two parts on each side of it are called the *adjacent* parts, and the remaining two are called the *opposite* parts. For instance, if a is chosen as the middle part, $90^\circ - B$ and b are the adjacent parts, and $90^\circ - c$ and $90^\circ - A$ are the opposite parts. Then Napier's Rules are :—



Sine of middle part = product of tangents of adjacent parts

Sine of middle part = product of cosines of opposite parts

Thus

$$\sin a = \cot B \tan b$$

$$\text{and } \sin a = \sin c \sin A.$$

As an aid to memory, it may be noticed that the vowels in the words *sine* and *middle* are the same, so with *tangent* and *adjacent*, *cosine* and *opposite*.

By choosing different parts in turn as the middle parts, we obtain all the possible relationships between the sides and angles, and with a little practice it is easy to choose the particular ones wanted. If we want a relationship between a , b , and c , for example, $90^\circ - c$ must be taken as the middle part, and we have

$$\cos c = \cos a \cos b.$$

If a relationship between a , A , and B is required, take $90^\circ - A$ as the middle part, whence

$$\cos A = \sin B \cos a$$

and so on.

There are six cases to consider in the solution of right-angled triangles, and the formulæ required, readily obtained from Napier's rules, are as follows :—

(1) Given the hypotenuse c and an angle A .

$$\tan b = \tan c \cos A,$$

$$\cot B = \cos c \tan A,$$

$$\sin a = \sin c \sin A.$$

- (2) Given a side b and the adjacent angle A .

$$\tan c = \frac{\tan b}{\cos A},$$

$$\tan a = \tan A \sin b,$$

$$\cos B = \cos b \sin A.$$

- (3) Given the two sides a and b .

$$\cos c = \cos a \cos b,$$

$$\cot A = \cot a \sin b,$$

$$\cot B = \cot b \sin a.$$

- (4) Given the hypotenuse c and side a ,

$$\cos b = \frac{\cos c}{\cos a},$$

$$\cos B = \frac{\tan a}{\tan c},$$

$$\sin A = \frac{\sin a}{\sin c}.$$

- (5) Given the two angles A and B ,

$$\cos c = \cot A \cot B,$$

$$\cos a = \frac{\cos A}{\sin B},$$

$$\cos b = \frac{\cos B}{\sin A}.$$

- (6) Given a side a and opposite angle A ,

$$\sin c = \frac{\sin a}{\sin A},$$

$$\sin b = \tan a \cot A,$$

$$\sin B = \frac{\cos A}{\cos a}.$$

The solution of oblique spherical triangles

- (1) Given the three sides, a , b , and c .

$$\text{Let } s = \frac{1}{2} (a + b + c).$$

Then the angle A may be computed from any one of the following three formulæ:—

$$\sin \frac{A}{2} = \sqrt{\frac{\sin(s-b) \cdot \sin(s-c)}{\sin b \cdot \sin c}},$$

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \cdot \sin(s-a)}{\sin b \cdot \sin c}},$$

$$\tan \frac{A}{2} = \sqrt{\frac{\sin(s-b) \cdot \sin(s-c)}{\sin s \cdot \sin(s-a)}}.$$

Similar formulæ apply, of course, to the other two angles.

(2) Given two sides a and b , and the included angle C ,

$$\tan \frac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{1}{2}C,$$

$$\tan \frac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \frac{1}{2}C.$$

These determine $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$, and hence, by addition and subtraction, A and B .

The side c may be found from either

$$\sin c = \frac{\sin a \sin C}{\sin A},$$

$$\text{or } \tan \frac{1}{2}c = \frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} \tan \frac{1}{2}(a+b).$$

The former of the two alternative formulæ for c is the simpler, but as the value of c is here found from its sine, it is sometimes difficult to determine which of two values is to be given to it. This difficulty does not arise with the second formula.

(3) Given one side c and two adjacent angles A and B .

$$\tan \frac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \frac{1}{2}c.$$

$$\tan \frac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \frac{1}{2}c.$$

These determine $\frac{1}{2}(a+b)$ and $\frac{1}{2}(a-b)$, and hence, by addition and subtraction, a and b .

C may be found from either

$$\sin C = \frac{\sin A \sin c}{\sin a}.$$

$$\text{or } \tan \frac{1}{2} C = \frac{\sin \frac{1}{2} (a - b)}{\sin \frac{1}{2} (a + b)} \cot \frac{1}{2} (A - B).$$

Similar remarks applying to the two formulæ as in case (2).

(4) Given two sides a and b , and the angle opposite one of them A . This is generally known as the ambiguous case.

B may be found from

$$\sin B = \frac{\sin b}{\sin a} \sin A,$$

which will usually determine two possible values of B . If the value of $\sin B$ obtained is greater than unity there will be no solution at all.

Having determined B , C and c may be found from the formulæ:—

$$\tan \frac{1}{2} C = \frac{\cos \frac{1}{2} (a - b)}{\cos \frac{1}{2} (a + b)} \cot \frac{1}{2} (A + B),$$

$$\tan \frac{1}{2} c = \frac{\cos \frac{1}{2} (A + B)}{\cos \frac{1}{2} (A - B)} \tan \frac{1}{2} (a + b).$$

(5) Given two angles A and B , and the side opposite one of them, a .

The solution in this case is similar to case (4), and two solutions are often possible:—

$$\sin b = \frac{\sin B \sin a}{\sin A},$$

after which the same two formulæ as in case (4) determine $\tan \frac{1}{2} C$ and $\tan \frac{1}{2} c$.

(6) Given the three angles, A , B , and C ,

$$\text{let } 2S = A + B + C.$$

Then a may be computed by any one of the following three formulæ:—

$$\sin \frac{a}{2} = \sqrt{\left\{ -\frac{\cos S \cos (S - A)}{\sin B \sin C} \right\}},$$

$$\cos \frac{a}{2} = \sqrt{\left\{ \frac{\cos (S - B) \cos (S - C)}{\sin B \sin C} \right\}},$$

$$\tan \frac{a}{2} = \sqrt{\left\{ -\frac{\cos S \cos (S - A)}{\cos (S - B) \cos (S - C)} \right\}},$$

and similar formulæ may be used for the other two sides.

Spherical excess

The sum of the three angles of a spherical triangle is always greater than 180° , the difference $A + B + C - \pi$ being known as the *spherical excess*.

If this is denoted by E , the area of any spherical triangle $= Er^2$, the spherical excess being in circular measure, and r denoting the radius of the sphere.

CHAPTER 2

THE CELESTIAL SPHERE AND ASTRONOMICAL COORDINATES

The celestial sphere

WE may easily imagine, looking up to the heavens on a cloudless night, that the stars are distributed over the surface of the spherical vault of sky above us. It is not really so, because refined measurements have proved that the distances of the stars differ tremendously. For practical purposes we are never concerned with the distances of the stars, but only with their directions, and in order to record these it is exceedingly convenient to picture the stars as distributed over the surface of an imaginary spherical sky having its centre at the position of the observer. Thus has arisen the conception of the *celestial sphere*, which we may consider as a geometrical device to enable us to record and measure the directions of the stars.

In Fig. 1, suppose that O represents the position of the observer. With O as centre, imagine a spherical surface described with a radius of any length we please; we may make it a few feet or a few thousand miles, it makes no difference. Now, let A, B, and C

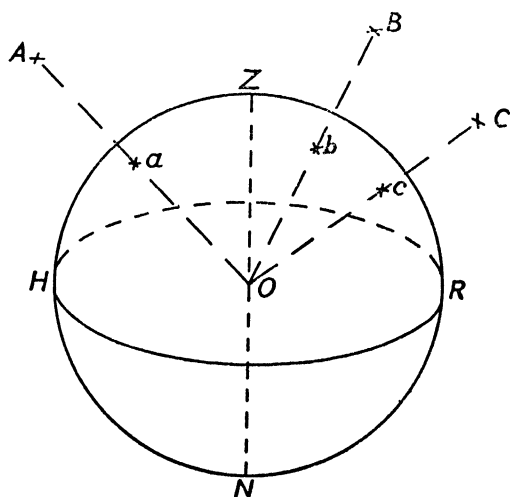


Fig. 1

be three of these immensely distant stars, and let the lines OA , OB , and OC cut our imaginary sphere in a , b , and c respectively. Then, if we are only concerned with the directions of the stars, we may just as well picture them as occupying the positions a , b , and c as their actual places A , B , and C . In fact, to the observer at O their appearance would be unaltered. So, proceeding in this way, we may picture all the stars in the sky as occupying places on this imaginary surface, which is then known as the *celestial sphere*. It may be considered as the spherical surface upon which the stars *appear* to lie, but, of course, in reality they are not all equally distant from us, and they are only represented in this way in order conveniently to measure their directions.

If through the point O a vertical line be drawn to intersect the celestial sphere over the observer's head in Z , and to cut it vertically below his feet at N , the point Z is called the *zenith* and the point N the *nadir*. The zenith is thus the point in the celestial sphere directly over the observer.

If a horizontal plane HR be drawn through O —a plane, that is to say, at right angles to the vertical OZ , the direction in which gravity acts—it will cut the celestial sphere in a great circle, which is called the *celestial horizon*. To an observer whose eye was close to the surface of a calm ocean, the celestial horizon would form the boundary of the visible part of the celestial sphere.

The apparent motion of the stars

Continued observation shows that, leaving the few planets out of account, the stars always maintain the same relative positions, and hence they are commonly referred to as the *fixed stars*. Whilst however, there is no motion relative to one another, they all appear to revolve from east to west in a period slightly less than twenty-four hours round a point in the sky that is known as the *celestial pole*. The motion is just as though the whole celestial sphere, carrying the stars, revolved about an axis passing through this point and its own centre. The ancients thought that this was really what occurred, but we know now that this motion is apparent only, and is due to the fact that we view the stars from a revolving earth. Thus, referring to Fig. 2, the whole of the stars appear slowly to describe circles about a point P in the celestial sphere just as though the whole sphere revolved about the axis OP , so that every star completes its circle in the same time. Some stars, such as A , which are comparatively near to the point P , describe only a small circle, which never takes them below the horizon, so that

such stars are always visible. Thus the Southern Cross in the latitude of Southern Australia can be seen at all times, and never sets. Other stars, such as B and C, which are further away from P, describe much larger circles, which take them, as is shown in the figure, below the horizon for a portion of their revolution, so that such stars rise in the east and set in the west. This diurnal motion of the stars may be prettily demonstrated by exposing a fixed camera containing a highly sensitised plate directed towards the celestial pole on a clear night, leaving the plate exposed for an hour or two. The images of the brightest stars will leave trails upon the plate which are all seen to be arcs of circles having a common centre at the celestial pole.

Now, the stars are so distant that their apparent direction in space is absolutely unaltered by any movement of the observer over the earth's surface. The direction of any particular star is precisely the same, even when determined by our most refined instruments, whether viewed from Melbourne, London, or Perth. More than

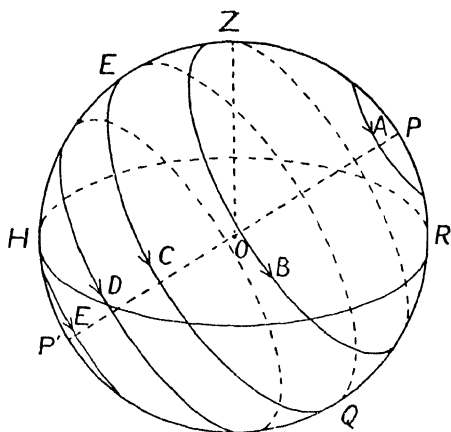


Fig. 2

this, we know that the earth, in the course of a year, describes a path round the sun that is approximately a circle whose diameter is about 186 millions of miles, yet even this great shift of the point of observation produces no appreciable change in the directions of the fixed stars. At intervals of six months apart, when the points of observation, that is to say, are distant something like 186 millions of miles, a slight difference in direction amounting in no case to more than 1.5 seconds of arc, may be detected in a few stars with

the refined observations possible at fixed observatories. But even this cannot be found with the great majority of the stars, so that we may regard the position of the observer on the earth's surface as of absolutely no importance when measuring the direction of the stars in space. Looking at Fig. 2, we may regard the earth as a tiny speck at O, the centre of the great celestial sphere, and no matter where we take the point O on this tiny speck, the direction of the line O P remains the same within the possibilities of our means of measurement, so that the lines joining any one of the fixed stars to different points on the earth's surface may all be considered as parallel.

It follows from this that the portion of the sky visible to an observer at any point on the earth's surface presents exactly the same appearance as it would do if it were possible for him to view it from the earth's centre. This statement refers only to the fixed stars.

Therefore, if we imagine an observer anywhere on a small spherical earth at the centre of a great celestial sphere of dimensions infinitely great compared to the earth, and suppose the earth to rotate about an axis through its centre, the successive pictures of the sky presented to the observer during a revolution will be precisely the same as they would be if the earth remained stationary and the great celestial sphere itself were to rotate about the same axis.

Thus, looking at Fig. 2, if we produce the line P O backwards to cut the celestial sphere below the plane of the horizon in P', the fixed stars appear to the observer at O to revolve on the celestial sphere about the axis P P'. In reality it is the earth that is revolving, and it is the earth's axis that lies in the direction P P', so that the celestial poles P and P' are the points in which the axis of the earth, if infinitely produced, would cut the celestial sphere. If the observer is in the southern hemisphere, the pole P visible to him will be that to which the earth's south pole is directed. If he is in the northern hemisphere the visible celestial pole is that towards which the earth's north pole points.

Celestial equator

If we take a plane through O perpendicular to the line P P', it will cut the celestial sphere in a great circle E Q, which is known as the *celestial equator*. Its plane clearly is coincident with the plane of the equator of the earth. Since two great circles of a sphere always intersect at opposite extremities of a diameter, it follows that a star revolving in the celestial equator has its path divided into two equal

parts by the circle of the celestial horizon $H R$, so that the time during which it is visible above the horizon will be equal to the time it is out of sight below.

Thus, to an observer in southern latitudes, the celestial pole P lies to the south and, since the line $P P'$ (Fig. 2) marks also the direction of the earth's axis, the celestial pole will be in the direction of the true geographical south. Any star, such as B , lying to the south of the celestial equator, will trace the greater part of its circular path above the plane of the horizon. On the other hand, a star, such as D , to the north of the celestial equator, will trace out the smaller portion of its path only above the horizon, so that it will be visible for less than half of its time of revolution. Stars such as E , sufficiently far to the north, will not be visible at all to a person in this latitude, but will complete the whole of their revolution below the plane of the horizon, as shown in the figure.

Astronomical coordinates

If we wish to mark the position of a point on a plane, we may do so by measuring its distances from two fixed straight lines at right angles. A knowledge of these two distances is sufficient to enable us to fix the position of the point, but one distance only would not be enough. Measured in this way, these two distances are spoken of as the "coordinates" of the point. Now, in astronomical observation, we commonly require to determine the position of a star on the celestial sphere, and so it is necessary to have some system of coordinate measurement applicable to the purpose. Either one of two sets of coordinates is commonly employed. In the first set, the coordinates are *altitude and azimuth*; in the second they are *right ascension and declination*. We will consider the two sets in turn.

Altitude and azimuth

In Fig. 3, let O be the position of the observer, Z the zenith, P the celestial pole. Then the plane $Z O P$ will cut the plane of the horizon through O in the north and south points N and S . $S Z N$ is known as the plane of the *meridian*.

Suppose that B is a star describing its circular path $A B C$ round the pole P .

The plane $Z B O$ cuts the plane of the horizon in the line $D O$.

Then it is clear that if we know the angle $D O N$, which is the angle that the plane $Z O D$ makes with the plane of the meridian, our knowledge is sufficient to fix the position of the plane $Z O D$.

If in addition we know the angle B O D , the position of the star B may be fixed on the celestial sphere.

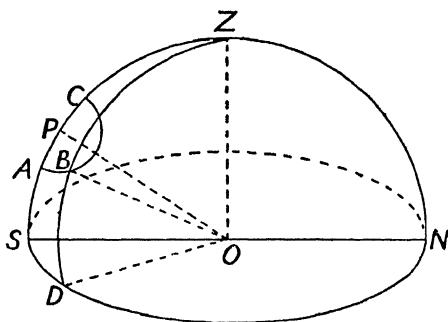


Fig. 3

The angle DON , which the plane passing through the zenith and the star makes with the meridian, measures what is known as the *azimuth* of the star. It is generally measured from the north towards the right.

The angle BOD , measuring the angular altitude of the star in a vertical plane above the horizon, is spoken of as the *altitude* of the star.

Instead of the altitude we may measure the angle ZOB , which is known as the *zenith distance*, and is clearly the complement of the altitude.

If we know both the altitude and azimuth of a star at any time we can mark its position on the celestial sphere. The ordinary theodolite is adapted for measurement in this system of coordinates.

Right ascension and declination

In Fig. 4, let O be the position of the observer, Z the zenith, P the celestial pole, and S P Z N the plane of the meridian.

Suppose that B is a star travelling round the pole in the direction of the arrow in a circle of which only half is shown.

QDQ' is the plane of the celestial equator drawn through O at right angles to OP .

P B D is the arc of a *great circle* of the celestial sphere intersecting the celestial equator in D. The plane of this great circle must pass through O, and the angle P O D is a right angle.

Then clearly if we know the position of the point D on the celestial equator, and also know either the angle P O B or the

complementary angle BOD , we shall be able to fix the position of the star B on the celestial sphere.

The position of the point D on the equator may be determined if we know its angular distance from some known fixed point also on the equator. The fixed point selected for the purpose is known as the *first point of Aries*. It is usually indicated by the symbol γ , denoting a pair of ram's horns. The exact nature of this point we shall discuss a little later on, but for the present all that we want to know is that it is a point whose position can always be accurately determined.

If we know, then, the angular measure of the arc γD —that is to say, the angle which the arc subtends at the centre O , and also the direction in which it is measured from γ —that is sufficient to determine D .

To avoid any confusion as to the direction in which the arc γD should be measured, it is always measured from γ towards the east—that is to say, in the opposite direction to that in which

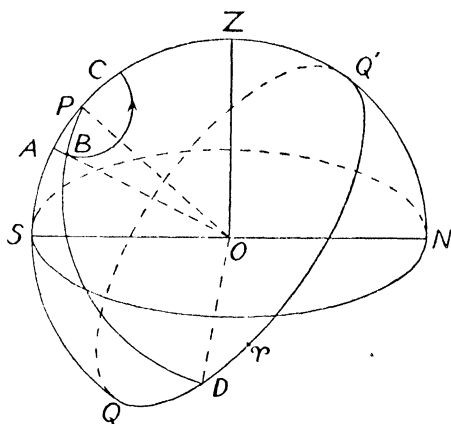


Fig. 4

γ travels round the celestial equator QQ' —because γ moves round with the rest of the fixed stars from east to west.

Measured in this way, the angular measure of the arc γD is known as the *right ascension* of the star B . It may have any value from 0° to 360° . It is commonly denoted by the letters R.A.

The right ascension of the star being known, its position may be fixed if we know either the angle POB , the angular measure of the arc PB , or the angle DOB , the angular measure of the arc DB .

The angular measure of the arc P B is known as the *polar distance* of the star B. It is generally denoted by the letters N.P.D. or S.P.D., according as it is measured from the north or the south pole.

The angular measure of the arc D B is called the *declination* of the star B, and the circle P B D is known as the *declination circle* of the star. In conformity with the practice of the *Nautical Almanac* we shall adopt the convention of regarding declinations north of the celestial equator as positive (+) and declinations south of the celestial equator as negative (-). In this connection, however, it should be noted that in the *Star Almanac for Land Surveyors* northerly declinations are given as "N." and southerly declinations as "S."

Polar distance and declination are always complementary to each other, their sum being 90° , so that if one is known the other is found by simple subtraction from 90° .

Comparative advantages of the two coordinate systems

The altitude and azimuth of a star are readily measured with a theodolite, and serve to fix the position of a star at any particular instant, but owing to the diurnal motion of the stars these co-ordinates are continually changing.

On the other hand, the right ascension and declination of a star are constant, for the reference point, the first point of Aries, partakes of the diurnal motion of the stars. These coordinates are in consequence the most convenient for recording the relative positions of the stars on the celestial sphere. Thus in the *Nautical Almanac* the stars are catalogued and tabulated by their right ascensions and declinations.

The sidereal day and sidereal time

As the revolution of the whole system of stars about the polar axis takes place with absolute uniformity from east to west, the period of revolution serves as a convenient unit of time for astronomical purposes. All the stars complete their circles of revolution in the same period, which is known as the *sidereal day*. This day is about 4 minutes shorter than the ordinary day. Sidereal clocks, adjusted to keep sidereal time, the sidereal day being divided into 24 hours, are used in fixed observatories. Such clocks are arranged to mark $0^h 0^m 0^s$ when the first point of Aries, the point on the celestial equator from which right ascensions are measured, crosses the meridian of the observer. Thus the *sidereal time* at any instant is the interval that has elapsed, measured in sidereal hours, minutes,

and seconds, since the last meridian transit of the first point of Aries.

Looking at Fig. 4, it is clear that all stars on the same declination circle, such as P B D—that is to say, all stars having the same right ascension—will cross the meridian at the same instant. A star whose right ascension is 180° will cross the meridian 12 sidereal hours after the first point of Aries, and one whose right ascension is 15° will cross the meridian at 1^h sidereal time. Thus we deduce the important result that *the right ascension of a star, when reduced to time at the rate of 24 hours for 360° or 1 hour for 15° , gives the sidereal time at the moment when it crosses the meridian.*

Hour angle

In Fig. 4 the angle B P Z, which is the angle that the plane of the declination circle P B D makes with the plane of the meridian, is known as the *hour angle* of the star B. If we know the hour angle of a star, and also its polar distance, we can clearly mark the position of the star on the celestial globe, so that these two may be used as another system of coordinates. The hour angle of a star is continually changing, but owing to the uniform character of the star's motion, it varies at a constant rate. If the hour angle is 90° measured towards the east, then the star will take 6 sidereal hours to reach the meridian. Thus a knowledge of the hour angle at once gives us the time the star will take to reach the meridian, if it be on the eastern side of it, or the time that has elapsed since the star crossed the meridian, if it be on the western side.

From the definitions of right ascension, hour angle and sidereal time, it follows that

$$\text{sidereal time} = \text{hour angle} + \text{right ascension}.$$

Further, it should be obvious that the sidereal time can be called the right ascension of the meridian.

Prime vertical

The plane through the zenith at right angles to the meridian—that is, the vertical plane running east and west—is known as the *prime vertical*. The east and west line, which is the line of intersection of the prime vertical with the plane of the horizon, is also the line of intersection of the plane of the celestial equator with the horizon, as will be evident from Fig. 2.

Synopsis of astronomical terms

For purposes of reference, the principal quantities dealt with in this chapter are illustrated in one figure (Fig. 5).

Fig. 5*a* is drawn for an observer in the southern hemisphere, and Fig. 5*b* for the northern hemisphere.

EXERCISES

1. The R.A. of a star being $35^{\circ} 20'$, what is the local sidereal time when the star is in the meridian ?

Ans. $2^{\text{h}} 21^{\text{m}} 20^{\text{s}}$

2. If the R.A. of a star is 295° and the sidereal time is 15 hours, is the star to the east or west of the meridian ?

Ans. To the east

3. What is the declination of a star that rises exactly in the east ?

Ans. 0°

4. What is the hour angle of the star in Question 2 ?

Ans. 70°

5. The declination of a star is -35° . Determine its S.P.D. and its N.P.D.

Ans. 55° and 125°

6. If the first point of Aries crosses the meridian exactly two hours, as measured by a sidereal clock, after a certain star, what is the R.A. of the star ?

Ans. 330°

7. The declination of the Pole Star is $+89^{\circ} 02'$. What is the difference between its greatest and least zenith distances ?

Ans. $1^{\circ} 56'$

8. At the time of the year when the R.A. of the sun is zero, determine approximately the time of rising of a star with declination 0° and R.A. 150° .

Ans. 4 p.m.

9. What is the point whose declination is equal to the latitude of the observer, and whose hour angle is zero ?

Ans. The zenith

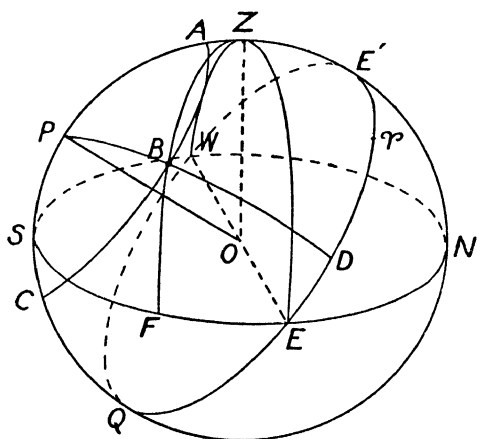


Fig. 5a

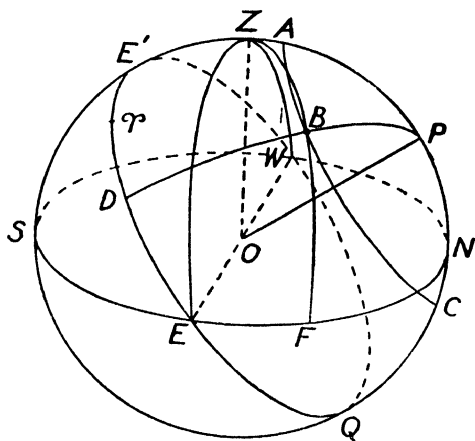


Fig. 5b

O is the observer.

S W N E, the plane of the horizon.

Z, the zenith.

P, the celestial pole; O P, the polar axis.

S P Z N, the plane of the meridian,

E' W Q E, the celestial equator.

W Z E, the prime vertical.

N, S, W, E, the North, South, West, and East points.

B, any star.

Z P B, the hour angle of B.

P B D, the declination circle of B.

P B, the polar distance of B.

B D, the declination of B.

ED, the right ascension of B.

Z B F, the vertical through B.

B F, the altitude of B.

B Z, the zenith distance of B.

N F, the azimuth of B.

CHAPTER 3

THE EARTH

The earth a globe

That the earth is a globe is no longer a matter for dispute. It has been circumnavigated and mapped and measured, and no other supposition will fit the facts. We see its round shadow as cast upon the moon during a partial eclipse. We see the planets as great balls of comparable dimensions revolving at different distances round the great central sun. The law of gravitation explains the form of their orbits and enables their movements to be predicted with the greatest exactness. That our earth is a globe like these, revolving in a similar way round the sun, is the only satisfactory hypothesis that will account for their apparently involved movements in the heavens. The whole of the apparent movements of the heavenly bodies are readily accounted for on the supposition that the earth is a globe, and no explanation even plausibly satisfactory has been advanced on any other supposition.

In the case of some of the planets we can actually observe that they are in rotation in a manner similar to that in which we assume our own earth must rotate to account for the phenomena of night and day and of the diurnal rotation of the stars. In the planet Mars we see the poles or extremities of the axis of rotation surrounded by white caps analogous to the caps of ice and snow that surround the poles of our own earth.

Terrestrial latitude and longitude

The extremities of the axis of rotation of the earth are called the *poles*, and are distinguished as the *north* and *south* poles.

A plane through the earth's centre at right-angles to the axis cuts the earth's surface in a circle known as the *equator*. Every point on the terrestrial equator is thus equidistant from the north and south poles.

In order to mark the position of a point on the earth's surface, it is necessary to have a system of coordinates similar to those we have already discussed in connection with the celestial sphere.

Suppose that P (Fig. 6) is a point on the earth's surface, the position of which it is desired to locate. A plane passing through P and the earth's axis will cut the earth's surface in a great circle

$N P M S$, which is known as a *meridian*. Suppose this meridian cuts the equator at the point M . Then clearly, if we know the position of the point M on the equator, and also the length of the arc $P M$ or the angle which it subtends at the earth's centre, we shall be able to fix the point P .

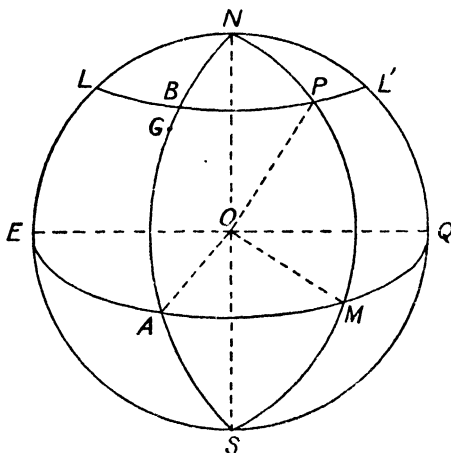


Fig. 6

The position of M on the equator is determined by the *longitude* of P .

To measure this, some arbitrary place A must be selected on the equator as a starting point. The point actually chosen is the point of intersection of the meridian passing through Greenwich, shown as $N G A S$ in the figure, and the equator. The angular measure of the arc $A M$ —that is to say, the angle $A O M$ —is known as the *longitude* of P . Thus, all points on the meridian passing through P have the same longitude. All points on the meridian $N G A S$, passing through Greenwich, have zero longitude. The longitude of other places is reckoned as so many degrees east or west of Greenwich until we come to 180° , which is the longitude of the meridian exactly opposite to the Greenwich meridian.

The angle $P O M$, which is the angle between the direction of the vertical at P and the vertical at M , measures what is known as the *latitude* of P . If we draw a plane through P at right angles to the earth's axis, it will intersect the earth in a small circle $L P L'$ parallel to the equator. Such a circle is called a *parallel of latitude*, and all points on the same parallel clearly have the same latitude.

Latitude is measured as so many degrees north or south of the equator. The latitude of the north pole is 90° N.

Thus, if we know the position of the meridian of zero longitude, the latitude and longitude of a place are sufficient to enable us to mark its position on the globe.

The length of a degree of longitude

If the parallel of latitude through P intersects the meridian through Greenwich in B, it is clear that the arc B P will be much smaller than the arc A M. It will have the same angular measurement on a much smaller circle. If P were very near to the north pole, the arc B P would be very small indeed. Thus two places in the same latitude but differing by, say, ten degrees of longitude, will be very much closer together if they are in a "high" latitude—that is to say, a latitude approaching 90° —than they will be if both are on or near the equator. Thus a degree of longitude has its greatest value, when measured in distance along the earth's surface, at the equator, its value becoming less and less as we approach the poles. At the equator a degree of longitude is equivalent to a distance of about 69 miles.

A degree of latitude, on the other hand, is always of approximately the same value, about 69 miles, whether it is measured near the poles or near the equator, because it is measured along meridians which are all great circles of the same diameter.

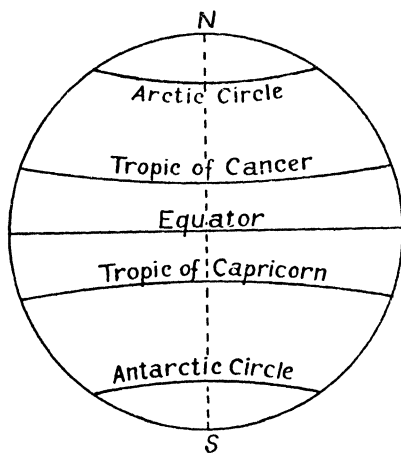


Fig. 7

The zones of the earth

Certain parallels of latitude divide the earth's surface into five belts or divisions, termed *zones*. These mark in a general way a

natural division of the earth's surface according to climate. The parallel of latitude $23^{\circ} 27\frac{1}{2}'$ north of the equator is termed *the tropic of Cancer*, and the corresponding parallel south of the equator is termed *the tropic of Capricorn*. As we shall presently see, at all places between these parallels at some part of the year the sun shines directly overhead at mid-day. As a consequence, the belt included between these is the hottest portion of the earth's surface, and it is known as the *torrid zone*.

The parallel of latitude $66^{\circ} 32\frac{1}{2}'$ north of the equator is called the *Arctic circle*, and the corresponding parallel south of the equator the *Antarctic circle*. The belt between the Arctic circle and the tropic of Cancer is known as the *north temperate zone*, and that between the Antarctic circle and the tropic of Capricorn as the *south temperate zone*. The regions around the two poles bounded by the Arctic and Antarctic circles respectively are termed the *frigid zones*. At all places within the frigid zones the sun is below the horizon at mid-day for some portion of the year.

The altitude of the celestial pole is equal to the latitude of the place of observation

In Fig. 8, let O be the position of the observer and C the earth's centre. Then the direction of the pull of gravity at O is in the direction OC. This, then, will mark the direction of the vertical at O, and the zenith, Z, of the observer will be in CO produced.

HR, at right angles to OZ, marks the plane of his horizon.

If CP, the earth's axis, be produced to cut the celestial sphere in P_1 , then P_1 will be the celestial pole.

Draw OP_2 parallel to CP_1 .

Then the celestial pole being, as we have seen, at a distance from the earth that is practically infinite in comparison with the earth's radius, OP_2 will mark the direction in which the celestial pole is seen by the observer at O.

Draw the plane of the equator ECQ at right angles to the earth's axis.

Then, from our definition, the latitude of O is measured by the angle ECO.

Now the angle $ZOP_2 =$ the angle $OC P_1$, and the complements of these angles are equal.

Therefore, the angle $P_2OR =$ the angle ECO—i.e., the altitude of the pole = the latitude of the observer.

It follows from this that if the observer travels equal distances north and south from O, since his latitude will change by equal

amounts, the altitude of the celestial pole will also be increased or decreased by equal amounts. As this is actually the case from observation, the fact forms a strong proof of the sphericity of the earth.

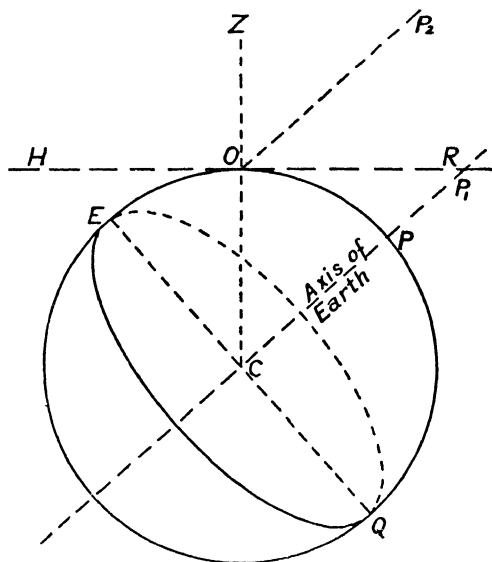


Fig. 8

To find the shortest distance, measured along the earth's surface, between two places whose latitude and longitude are given, assuming the earth to be a true sphere

In Fig. 9, let P and R be two places whose latitudes and longitudes are known.

The shortest distance between P and R, measured along the earth's surface, will be the length of the arc of the great circle joining them.

Draw the meridians passing through P and R.

Then if we know the latitudes, we know the angular measure of the meridian arcs NP and NR, N being the north pole.

If P is in north latitude, the arc NP is the complement of the latitude. If R is south latitude, the arc NR is $90^\circ +$ the latitude.

The angle P N R is the difference of the longitudes of P and R if both are measured in the same direction, or the sum of the longitudes, if one is east and the other west.

Thus in the spherical triangle NPR , we know the sides NP and NR and the included angle PNR .

Then by the ordinary methods of spherical trigonometry we can compute the angular measurement of the great circle arc $P R$, and consequently its lineal measurement, if we know the radius of the earth.

The radius of the earth is approximately 3,960 miles.

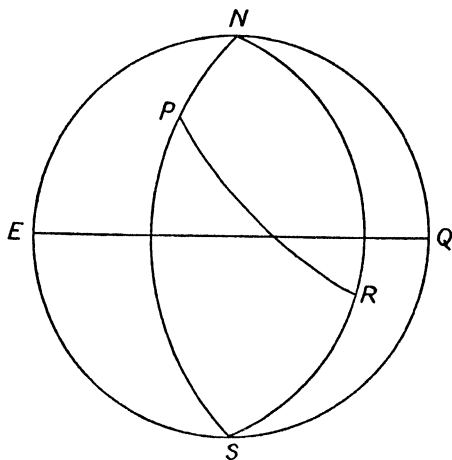


Fig. 9

EXAMPLE.—Find the shortest distance measured along the earth's surface between Perth (long. $115^{\circ} 50' E.$, lat. $31^{\circ} 57' S.$) and Brisbane (long. $153^{\circ} 1' E.$, lat. $27^{\circ} 28' S.$), assuming that the earth is a sphere of radius 3,960 miles.

In this case, both places being in the southern hemisphere, it will be preferable to solve the triangle $S P R$ (Fig. 9) rather than $N P R$.

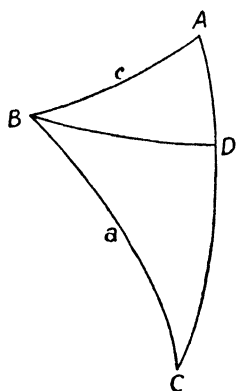
If A denotes the position of Brisbane, B of Perth, and C the south pole, we shall have in the spherical triangle $A B C$ (Fig. 9a)

$$C A = b = 90^{\circ} - 27^{\circ} 28' = 62^{\circ} 32'$$

$$C B = a = 90^{\circ} - 31^{\circ} 57' = 58^{\circ} 03'$$

$$C = 153^{\circ} 1' - 115^{\circ} 50' = 37^{\circ} 11'$$

Since we only want to find c , the simplest way to solve this triangle is to divide it into two right-angled triangles by drawing a great circle arc BD to cut CA at right angles.



Then we have from the right-angled triangle BDC

$$\tan CD = \cos C \tan a.$$

$$\log \tan a = \log \tan 58^\circ 3' \quad . \quad 10.2050545$$

$$\log \cos C = \log \cos 37^\circ 11' \quad . \quad 9.9012980$$

$$\log \tan CD \quad . \quad . \quad . \quad 10.1063525$$

$$\therefore CD = 51^\circ 56' 47'',$$

$$\text{and } \cos c = \cos AD \cdot \cos BD$$

$$= \cos (b - CD) \cdot \frac{\cos a}{\cos CD}.$$

$$\log \cos a = \log \cos 58^\circ 3' \quad . \quad . \quad . \quad 9.7236026$$

$$\log \cos (b - CD) = \log \cos 10^\circ 35' 13'' \quad 9.9925435$$

$$9.7161461$$

$$\log \cos CD = \log \cos 51^\circ 56' 47'' \quad . \quad 9.7898616$$

$$\log \cos c \quad . \quad . \quad . \quad . \quad 9.9262845$$

$$\therefore c = 32^\circ 26' 49''$$

The circular measure of this angle is $\cdot 5663$ radian.

$\therefore$ The distance required $= \cdot 5663 \times 3,960 = 2,242.5$ miles.

The more usual method of solving the triangle ABC , having given the two sides a , b , and the included angle C , would be to find first the angles A and B by means of the formulae

$$\tan \frac{1}{2} (A + B) = \frac{\cos \frac{1}{2} (a - b)}{\cos \frac{1}{2} (a + b)} \cot \frac{1}{2} C,$$

$$\tan \frac{1}{2} (A - B) = \frac{\sin \frac{1}{2} (a - b)}{\sin \frac{1}{2} (a + b)} \cot \frac{1}{2} C,$$

and then find c from the formula

$$\sin c = \frac{\sin C \cdot \sin a}{\sin A}.$$

If this method is adopted to find c , it must be remembered that when $\sin c$ is found there are always two possible solutions, since the sine of an angle $=$ the sine of its supplement. Some care is, therefore, necessary in selecting the appropriate value from the two values determined by the tables.

This ambiguity may be avoided by using the formula

$$\tan \frac{c}{2} = \frac{\cos \frac{A+B}{2}}{\cos \frac{A-B}{2}} \cdot \tan \frac{a+b}{2}$$

in order to find c . This has the further practical advantage that the tables have already been opened at

$$\frac{A+B}{2}, \frac{A-B}{2} \text{ and } \frac{a+b}{2}.$$

Seven figure logarithms have been used in the example worked above, but five would have been ample to give the result to the tenth of a minute.

EXERCISES

In all of these examples the earth is to be taken as a sphere of radius 3,960 miles.

1. Find the shortest distance measured along the earth's surface between Mount Gambier (longitude $140^{\circ} 45'$ E., latitude $37^{\circ} 50'$ S.) and Palmerston (longitude $130^{\circ} 50'$ E., latitude $12^{\circ} 28'$ S.).

Ans. 1,856·8 miles

2. Find the shortest distance measured along the earth's surface between Baltimore (lat. $39^{\circ} 17'$ N., long. $76^{\circ} 37'$ W.) and Cape Town (lat. $33^{\circ} 56'$ S., long. $18^{\circ} 26'$ E.).

Ans. 7,893 miles

3. How far would a place be due south from the equator if the altitude of the S. celestial pole was exactly 20° ?

Ans. 1,382·3 miles

4. Two places are in S. latitude 30° , one longitude 115° E., and the other 35° E. Find the difference in the paths of the two ships sailing from one port to the other, one along the parallel of latitude and the other along the arc of the great circle joining the places.

Ans. 112·7 miles

5. What is the declination of a star that passes through the zenith at a place in latitude 35° N. ?

Ans. $+ 35^{\circ}$.

6. A ship sails along the great circle joining two places, each of latitude 45° N., the difference between their longitudes being $2a$. Show that the highest latitude φ reached during the passage is given by the formula $\cot \varphi = \cos a$.

7. A ship from latitude $8^{\circ} 25' \text{ N.}$ sails south for 600 nautical miles. What latitude is she in?

Ans. $1^{\circ} 35' \text{ S}$

8. At a place in latitude φ North, a star with declination δ rises 60° E. of North. Show that $\cos \varphi = 2 \sin \delta$.

The figure of the earth

If, as in Fig. 10, F and G are two points on the same meridian, their difference of latitude will be measured by the angle FOG . If we know this angle, and also the length of the arc FG, we shall

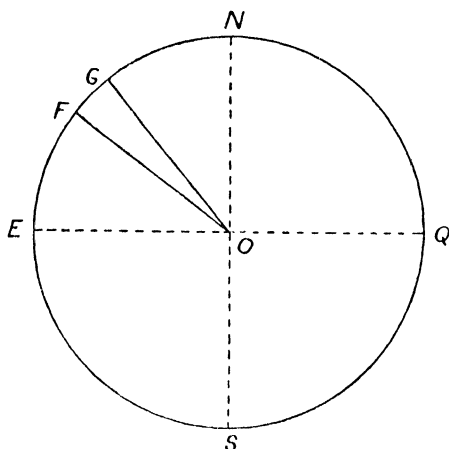


Fig. 10

then be able to calculate the length of the earth's radius FO. The difference of latitude between F and G may be determined by astronomical observation, measuring the altitude of the celestial pole at each place. The length of the arc FG may be either directly measured or it may be computed by means of a triangulation survey from a measured base line on some suitable adjacent part of the earth's surface. Determinations of the radius of the earth on these simple principles were made by the Greeks 2,000 years ago.

If the earth were a true sphere, measurements of the radius of the earth made in this way at different parts of its surface would be all the same. But when it became possible to make the necessary observations with sufficient precision it was found that such was not the case. When Newton discovered and investigated the

results of the law of gravitation in the seventeenth century, he proved that one consequence was that if the earth is a plastic body, revolving on an axis and acted on by its own attraction, it must take the form of a slightly flattened sphere with its polar diameter less than its equatorial diameter. Measurements of two arcs made by the Cassinis in France seemed, on the other hand, to indicate that the length of a degree of latitude decreased towards the north, which would imply that the shape of the earth was such that its polar diameter was greater than its equatorial diameter, contrary to Newton's gravitational theory. The French Academy equipped two expeditions in order to settle the problem. One of these measured an arc in the equatorial regions of Peru (1735-1741), and the other an arc in the polar regions of Lapland (1736-1737). The results showed that a degree of latitude was longer in the polar regions than in parts near the equator, and corroborated Newton's theory. Since then many arcs have been measured in different parts of the world, and the observations have conclusively established the fact that the shape of the earth is not a true sphere, but is very approximately an *oblate spheroid*, the figure formed by revolving an ellipse about its minor axis.

The shape of the earth is thus like that of a sphere slightly flattened at the poles. The amount of flattening is not, however, very great. The length of the earth's polar axis may be taken as 7,900 miles, and its equatorial diameter as 7,927 miles. Thus if a model were made 20 feet in diameter the polar diameter would be shorter than the equatorial by a trifle over three-quarters of an inch.

More exactly still, it is found that the change in the length of a degree of latitude which takes place as we proceed along a meridian is not the same along all meridians. It seems that the equatorial section of the earth is not exactly circular, but is very slightly elliptical. The exact shape would thus appear to be more nearly an *ellipsoid*. For practical purposes, however, all computations in geodetic work are based upon the assumption that the figure of the earth is an oblate spheroid.

Geographical and geocentric latitude

If in Fig. 11, P represents some point on the meridian N Q S, N and S being the north and south poles, then, making allowance for the fact that the section N Q S E is not a circle but an ellipse, the direction of the horizontal at P will not be at right angles to P O, O being the earth's centre, but will be in the direction of the tangent to the ellipse at P. This is the direction taken by the

surface of still water at that point. Consequently the direction of the vertical there is not OP but GP , where GP is the normal at P —that is to say, it is at right angles to the tangent. Thus, if we measure the latitude of P by astronomical methods, observing the

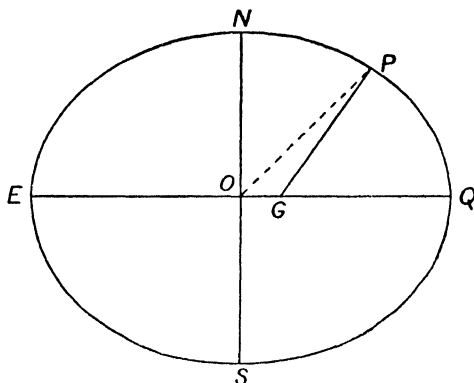


Fig. 11

altitude of the celestial pole above the horizon at P , we shall measure the angle PGQ and not the angle POQ . The angle PGQ thus measures what is called the *geographical* or *geodetic latitude*. This is the ordinary latitude that is used for astronomical and geodetic purposes.

It is clear, however, that the value of the angle POQ , if it can be readily determined, might be equally well used in order to fix the position of P on the meridian. This angle measures what is termed the *geocentric latitude*. The difference between the geocentric and geographical latitude of a place is never very great. There is no difference at all, either at the poles or at the Equator, and the maximum difference is in latitude 45° , where it amounts to about $11' 44''$ of arc. The geocentric latitude cannot be directly observed. It is computed from the geodetic latitude by the formula:—

$$\tan POQ = \frac{ON^2}{OQ^2} \tan PGQ.$$

When speaking of latitude in this book, it will always be the geodetic latitude that is meant unless otherwise specified.

EXERCISES

1. At a place in lat. 42° S. a line is run from a point A on a bearing of 220° for a distance of 2,400 chains to a point B .

Assuming the earth a sphere of 3,957 miles radius, find the bearing from B to A.

Ans. $40^{\circ} 15' 12''$

2. Given that latitude of London is $51^{\circ} 32' N.$, latitude of Jerusalem $32^{\circ} 44' N.$, bearing of Jerusalem from London, $110^{\circ} 04'$. Find the longitude of Jerusalem, its distance from London, and the bearing of London from Jerusalem.

Ans. Longitude, $37^{\circ} 25' 12'' E.$

Distance, 2,278 miles.

Bearing of London from
Jerusalem, $316^{\circ} 00' 16''$

3. The latitude of a Trig. Station A is $33^{\circ} 51' S.$, and its longitude is $151^{\circ} 12' 42'' E.$ The bearing and distance to another Trig. Station B is $284^{\circ} 08' 44''$, 105,600 feet.

Compute the latitude and longitude of B, and the bearing of B to A, on the assumption that the earth is a sphere with radius 20,890,790 feet.

Ans. Longitude, $150^{\circ} 52' 27'' E.$

Bearing, $104^{\circ} 20' 01''$

4. Find the great-circle distance in English statute miles from Wellington, N.Z., to Panama, treating the earth as a sphere, and one degree as equal to $69\frac{1}{2}$ statute miles.

Wellington, - - Lat. $41^{\circ} 17' S.$, Long. $174^{\circ} 47' E.$

Panama, - - Lat. $9^{\circ} 00' N.$, Long. $70^{\circ} 31' W.$

Ans. 7,900.4 miles

5. Two places are each in latitude $50^{\circ} N.$, and their difference of longitude is $47^{\circ} 36'$. Find their distance apart.

Ans. 2,078 miles

6. On a level plain three poles of equal height are set up in a straight line at distances of 660 yards, and a man looks from the top of the first pole at the top of the third. Show that, owing to the rotundity of the earth, the top of the second pole will be above his line of sight, and calculate the difference approximately, taking the radius of the earth as 3,960 miles.

Ans. $1\frac{1}{8}$ inches

CHAPTER 4

THE SUN

The sun's apparent motion among the stars

Like the fixed stars, the sun shares in the apparent general daily rotation of the heavens, but unlike them it does not always maintain the same position relative to other objects on the celestial sphere. In addition to its daily circling of the sky, it appears to shift its position gradually with respect to the stars. Neither its declination nor its right ascension remains constant. Very little consideration will show that its declination must alter during the year, for, if it did not, the sun would always describe the same circle in the heavens. If this were the case, then, like the fixed stars, it would always rise and set at the same points on the horizon, and it would always attain the same altitude when on the meridian. Since it does not do this, it is clear that the declination of the sun must change during the year. That the sun has also a movement in right ascension among the stars is not quite so obvious, but the fact may be readily inferred if we watch the stars that are visible in the east on succeeding mornings just before sunrise or in the west just after sunset. Stars in the east that rise just before the sun, so that in a very short time after rising they are masked by the sun's rays, will on each succeeding morning be seen for a longer time. Similarly stars in the west, setting just after the sun, will be visible for shorter and shorter periods as we watch them on successive evenings until finally they are lost altogether in the strong sunlight, other stars further east taking their places. Hence we infer that the sun has a progressive movement among the stars from west to east.

The problem of determining the sun's place on the celestial sphere with regard to the fixed stars was a difficult one to early astronomers, because as soon as the sun becomes visible its strong light prevents the stars from being observed at the same time. Some used the moon, and Tycho Brahe (1546-1601) used the bright planet Venus in order to get the connection, observing the relative positions of the sun and moon or of the sun and Venus when both were visible, and afterwards measuring the position of the moon or Venus with regard to the stars when the sun had set. But as both the moon and Venus also move amongst the stars, the movement that had taken place in the interval had to be allowed for,

and the method was thus not particularly simple. The sun's position is nowadays determined by much more accurate methods.

The earth's orbit round the sun

All of these movements of the sun are apparent only and not real. Just as its apparent daily rotation in the heavens is due to the rotation of the earth on its axis, so the sun's apparent movements in right ascension and declination are really due to the fact that the earth moves in a great orbit round the sun once a year.

Actually the earth moves round the sun in a path that is very nearly a huge circle with a radius of about 93 millions of miles. More accurately, the path is described as an ellipse, one focus of the ellipse being occupied by the sun. The curve traced out by the centre of the earth lies in a plane that passes through the centre of the sun ; for the practical purposes of surveyors the plane may be regarded as fixed. The earth traces out its complete orbit once a year, and all the time it is spinning on its own axis once a day, the direction of the spin on its axis being the same as that in which it moves round the sun. The earth's axis is not at right angles to

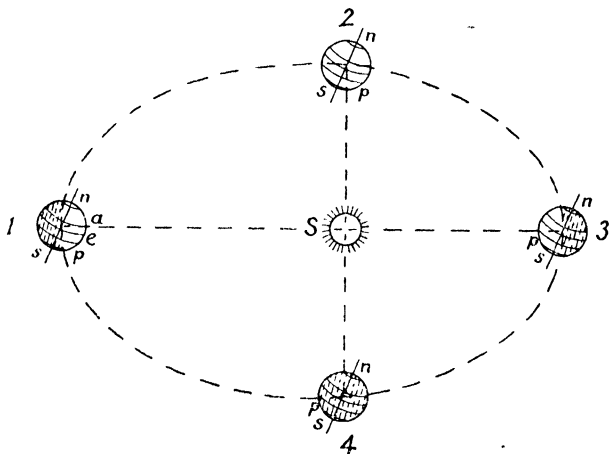


Fig. 12

the plane of its orbit, but it makes with the plane an angle of approximately $66^{\circ} 32\frac{1}{2}'$. That the direction of the earth's axis is practically constant we know from the fact that the position of the celestial pole amongst the fixed stars shows no appreciable shift throughout the year, except to refined observation. Thus, as is illustrated in Fig. 12, the earth moves round the sun, spinning on

its axis, which is inclined to the plane of the orbit, and the axis always remains parallel to itself, pointing in the same direction amongst the fixed stars whose distances, it must be remembered, are practically infinitely great even in comparison with the immense distance of the earth from the sun.

When the earth is in the position marked 1, the sun will be shining directly overhead in a place such as *a* north of the equator. If *e* is a point on the earth's equator on the same meridian of longitude as *a*, *O* being the earth's centre, the angle *a O e* will be the complement of $66^{\circ} 32\frac{1}{2}'$ or $23^{\circ} 27\frac{1}{2}'$ —that is to say, *a* will be a point on the tropic of Cancer. In this position, then, the sun at mid-day will be vertically overhead at all points on the tropic of Cancer. This statement is not quite accurate, because the earth does not remain in the one position in its orbit while it makes a complete revolution on its axis; it is moving forward in its orbit all the time, but as it takes a whole year to go round the sun, its relative movement is not very great in one day.

As the earth moves from position 1 to position 2, its axis always remaining parallel to its original direction, it will be seen that the sun will appear to shine directly overhead at points successively nearer and nearer to the equator, until in position 2 the sun's rays fall vertically at the equator.

Similarly, as the earth moves on to position 3, the sun's rays will fall vertically at points further and further south of the equator, until at position 3 the sun will appear at mid-day to be overhead at a point on the tropic of Capricorn. From there on to position 4 the sun will shine vertically at points successively nearer to the equator, until at 4 the sun is once more overhead at the equator.

The earth is in the position marked 1 on June 22, in that marked 2 on September 23, at 3 on December 22, and at 4 on March 21. Thus, if we consider the appearance of the sun to an observer at some point *p* to the south of the tropic of Capricorn, on June 22 the sun will appear to be further from the zenith and lower down in the sky than at any other period of the year. On December 22, when the earth is in position 3, the sun at mid-day will be nearer the zenith than at any other time of the year.

The earth's orbit being an ellipse, its distance from the sun is not constant. It is furthest from the sun when about in the position 1, and nearest to the sun about in the position 3.

The equinoxes

On March 21 and September 23, the sun, being vertically overhead at the equator, will appear to an observer at any part

of the earth to be in the celestial equator. Now, we have seen that when any heavenly body is in the celestial equator its path is bisected by the horizon, so that the time during which it can be seen in the sky is equal to the time during which it is invisible. Thus, when the earth is in either of these positions the days and nights are of equal length all over the world. These points are consequently called the *equinoxes*.

Motion in right ascension and declination

It thus appears that on March 21 and September 23 the sun's declination is zero, as it lies on the celestial equator. From March 21 to September 23 it will appear in the sky to the north of the equator, so that its declination will be north with a maximum value of $23^{\circ} 27\frac{1}{2}'$ on June 22. From September 23 to March 21 its declination will be south with a similar maximum value on December 22.

It is also evident that the sun's right ascension changes throughout the year, because as the earth revolves round it the apparent position of the sun among the fixed stars must obviously change. The stars that would be seen by an observer on the earth when in position 1, looking in the direction of the sun, would be seen by an observer at 3 when looking in the direction opposite to that of the sun. Clearly, in the course of the year the sun will trace out a complete circle among the fixed stars.

The declination and right ascension of the sun are given in the *Nautical Almanac* for 0^h G.M.T. and for Greenwich Apparent Noon on every day of the year. The values at intermediate instants may be found by interpolation. Illustrations of such calculations are given in Chapter 9 when dealing with sun observations.

The sun's semi-diameter

The disc of the sun subtends at the eye of an observer an angle of about half a degree. By accurately measuring the angle subtended by diameters taken in different directions, we find that these are all equal, so that the disc is circular in form. In order to mark the position that the sun occupies on the celestial sphere at any time, we require to determine the position of the centre of the circular disc. But there is no mark at the centre that we can recognise, and so in practice we must observe a point on the edge of the sun and then make an allowance for the distance of this point from the sun's centre.

From what we have just seen of the nature of the earth's motion round the sun, it is clear that the sun is not at all times of the year at the same distance from us, and consequently we should not expect its angular diameter to remain constant. As the earth completes its orbit round the sun in a year and then goes over the same path again, we might anticipate that the variations in the value of the sun's apparent diameter would follow a yearly cycle. This is found to be the case, a slow decrease taking place from about January 3 to about July 4, and a slow increase during the second half of the year.

As the semi-diameter is frequently required in reducing sun observations, the values are chronicled for every day in the year in the *Nautical Almanac* at both 0^h G.M.T. and Greenwich Apparent Noon. In the almanac for 1951 the maximum value of the semi-diameter is given on January 3 as 16' 17.51'', and the minimum on July 4 as 15' 45.33''.

To plot the position of the sun's centre on the celestial sphere

Supposing that we know the direction of the true north and south, and also the latitude of the place of observation, we may readily measure the declination of the sun at mid-day. With a telescope pointed in the direction of the meridian we may observe the altitude of the sun's upper or lower edges (*limbs*, as they are usually called) at the moment when it crosses the meridian.

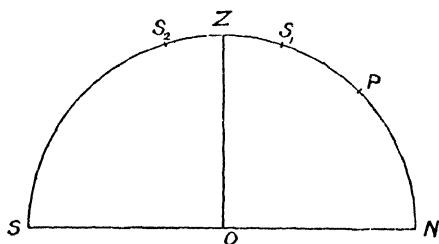


Fig. 13

Making due allowance for the sun's semi-diameter, we shall thus obtain the meridian altitude of the sun's centre. Thus, as in Fig. 13, where P represents the pole, Z the zenith, we measure either $S_1 N$ or $S_2 S$, according as the sun is in a position such as S_1 or as S_2 . Now, we have previously shown that the altitude of the celestial pole, P N, is equal to the latitude of the place. Thus, if the sun is situated as at S_1 , on the same side of the zenith as the pole, the difference between the observed altitude $S_1 N$ and the latitude P N

gives the sun's polar distance PS_1 . If the sun is at S_2 , on the opposite side of the zenith to the pole, then the arc S_2N is equal to 180° —the observed altitude SS_2 . The difference between S_2N and the latitude PN gives the sun's polar distance as before. The declination of the sun is the complement of its polar distance.

Having measured the declination of the sun in this way, in order to fix its position on the celestial sphere, it only remains to determine the difference between its right ascension and that of some star whose coordinates are known. But we have seen that the difference of right ascension of any two stars is measured by the interval in time between their transits across the meridian, as given by the sidereal clock. If, with the sidereal clock, the times be measured when the first and second limbs of the sun cross the meridian, the mean of the two times will give the instant when the centre crosses the meridian. If, therefore, the time of passage across the meridian of some selected known star is also observed, the interval between the two times, reduced to degrees, will give the difference between the right ascension of the sun and the star.

These observations give us the elements necessary to plot the position of the sun.

The sun's apparent path on the celestial sphere

In Fig. 14, let A represent the position of the selected fixed reference star as plotted on a globe representing the celestial sphere,

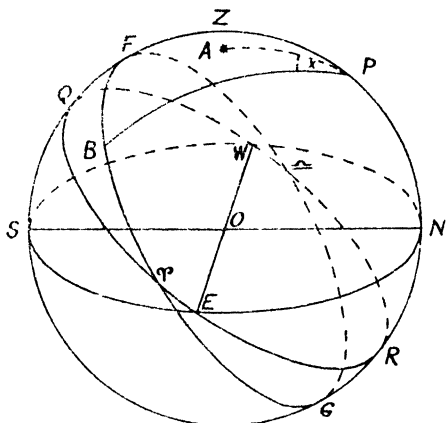


Fig. 14

P being the pole, QR the great circle of the equator, and $SWNE$ the horizon. Then, if we set out the angle APB equal to the observed

difference of right ascension and measure off the arc PB equal to the observed polar distance of the sun, the point B will represent the position of the sun's centre on the star globe.

When observations similar to those just described are made day after day, and the corresponding positions of the sun plotted on the globe, those positions are all found to lie on a great circle, which cuts the equator at two opposite points Υ and $\simeq$ in the figure, and is inclined to it at an angle of about $23^\circ 27'$.

The great circle, the plane of which contains the sun's yearly path, is called the *ecliptic*, and the angle this makes with the equator is spoken of as the *obliquity of the ecliptic*.

Its points of intersection with the equator are called the *equinoctial points*, one (Υ) is known as the *first point of Aries*, and the other ($\simeq$) as the *first point of Libra*.

The sun is at the first of these points on or about March 21 (the *vernal equinox*), and at the second about September 23 (the *autumnal equinox*), its declination being then 0° and its polar distance 90° .

As we have already seen, Υ is the point selected on the equator as that from which right ascensions are measured, so that the right ascension of Υ is 0° and that of $\simeq$ 180° .

At the two points on the ecliptic whose right ascensions are respectively 90° and 270° , the sun will have its greatest declination north and south of the equator. These are known as the *solstitial points*. The sun reaches them on or about June 22 and December 22. On June 22 the sun has its greatest declination of about $23^\circ 27'$ north of the equator, and on December 22 its greatest declination south.

In this and the foregoing two chapters, we have assumed that the direction of the earth's axis of rotation is fixed in space and that the so-called fixed stars always have exactly the same positions on the celestial sphere relative to one another. While we shall retain these ideas for the purposes of explanation, it is as well to note that they are not exactly true. A study of the *Star Almanac*, for example, will show that the right ascensions and declinations of the 'fixed' stars are changing slightly all the time.

Because the earth is not quite a true sphere, the mutual attractions between it and the sun, moon and planets cause a slow secular movement of the line of the equinoxes (*i.e.*, the line joining Υ and $\simeq$) around the plane of the ecliptic. The direction of this motion is opposite to the movement of the earth in its orbit, and amounts to about $50.3''$ per year. The phenomenon is called the *precession of the equinoxes*, or simply *precession*. Further, because of

this precession and the obliquity of the ecliptic, the pole of the equator (the celestial pole) will, in about 25,000 years, describe a circle around the pole of the ecliptic, if the plane of the ecliptic be accepted as the plane of reference. Thus, through the centuries, γ will migrate away from its present position in the constellation of Pisces, and, before long, the celestial pole will begin to move away from the present Pole Star.

The attractive forces causing precession do not remain constant because of the changing distances between earth, sun and moon, and more particularly, because the plane of the moon's orbit does not lie in the ecliptic. As a result, a small periodic oscillation of the earth's axis, called *nutation*, is produced.

Since the earth travels in its orbit at a speed which is an appreciable fraction of the velocity of light, there results a displacement of each star from its mean position by a constant amount of about $10.4''$. This displacement is known as *annual aberration*. A similar smaller effect, called *diurnal aberration*, results from the earth's diurnal motion ; it is nil at the poles where the velocity of rotation is nil, and a maximum at the equator.

Both the solar system and the 'fixed' stars are in motion, and are therefore gradually changing their absolute positions in space. The amount of this *proper motion* is different for every star but is so small that it does not perceptibly affect their *relative* positions as far as we are concerned.

It is mainly, then, the phenomena of precession, nutation and aberration which produce these slight variations in the right ascensions and declinations of the stars.

EXERCISES

1. Determine the meridian altitude of the sun at a place in latitude 30° , (a) at the equinoxes, (b) during the summer solstice.

Ans. 60° and $83^\circ 27'$

2. Find the latitude of the place where the greatest altitude of the sun in midsummer is 60° .

Ans. $53^\circ 27'$

3. At a place in lat. 80° N., on a certain day the sun at mid-day just appears above the horizon. Find the sun's declination. Find also the altitude of the sun at mid-day when its declination is $+20^\circ$.

Ans. -10° and 30°

TIME

Notation

Greenwich Mean Time	.	.	G.M.T. (U.T.)
Greenwich Apparent Time	.	.	G.A.T.
Greenwich Sidereal Time	.	.	G.S.T.
Greenwich Mean Noon	.	.	G.M.N.
Greenwich Apparent Noon	.	.	G.A.N.
Local Mean Time	.	.	L.M.T.
Local Apparent Time	.	.	L.A.T.
Local Apparent Noon	.	.	L.A.N.
Local Sidereal Time	.	.	L.S.T.
Local Standard Time	.	.	L.Std. T.
Right Ascension	.	.	R.A.
Hour Angle	.	.	H.A.
<i>Nautical Almanac</i>	.	.	<i>N.A.</i>
<i>Star Almanac</i>	.	.	<i>S.A.</i>

(Greenwich Mean Time is sometimes called Universal Time with the abbreviation U.T.)

Sidereal time

To measure time we require some form of perfectly uniform motion, and the most perfect motion of this kind in the heavens is provided by the apparent revolutions of the fixed stars. We may consider the earth to turn on its axis with absolutely regular speed (although there are practically negligible departures from perfect uniformity detected by refined observations) and, as the stars are so distant that the movement of the earth in its orbit round the sun produces no apparent effect upon their relative positions, the consequence is that the stars complete a revolution round the celestial pole at a perfectly regular rate in a fixed and constant time. To the astronomer then, this presents the simplest way of measuring time. The period of a complete revolution of the stars round the pole is known as the sidereal day, and the time measured in this way is termed sidereal time.

Apparent solar time

Convenient as the above method of measuring time is to the astronomer, it is obviously unsuited to ordinary purposes of life. It is the day as determined by the sun that controls our habits and rules our lives. The *apparent solar day*, or period of time between successive transits of the sun across the meridian, is, however, variable in length, and it is impossible to regulate a clock so that it shall indicate exactly 12 o'clock just when the sun is on the meridian. The reason for this may be seen from Fig. 15, which shows in an exaggerated way the movement of the earth in its orbital revolution round the sun. Suppose that, when the earth is in the position marked 1, the sun is directly overhead to an observer at A, and that, if it could be seen, the star F would appear in the same direction. As the earth revolves on its axis it also travels forward in its orbit, so that at the end of a sidereal day it is in the position marked 2. If the observer has been carried round to the point B, so that the same star F appears vertically overhead, the star being at practically an infinite distance, B F will be parallel to A F. The interval between these two positions marks a sidereal day. But to bring the sun overhead, to the same observer, he must wait till he is carried round the extra distance B C. The solar day will then be longer

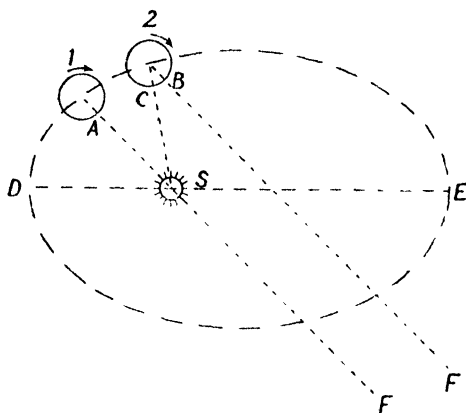


Fig. 15

than the sidereal day by the length of time required to traverse this extra distance. Whilst the sidereal day is the time taken by the earth to make a complete revolution on its axis, the apparent solar day is the time taken to make a little more than a revolution.

Now, the earth does not move in a circular but in an elliptic

orbit round the sun, so that sometimes it is nearer to the sun than at others. When it is nearer to the sun it is a deduction from the law of gravitation that it must travel faster in its path than when it is further away. The result is that the little extra bit, B C, through which the earth has to turn in the interval of time that has to be added on to the sidereal day to give the apparent solar day, is not always the same, and the apparent solar day is thus not of constant length.

We have seen that the right ascensions of the fixed stars are practically constant. This is not quite correct, as there is a very small and slow variation which we may disregard for the present. But if a celestial body were to move in right ascension its period of revolution about the pole would still be constant, although not the same as that of the stars, provided the movement was a uniform one. The difficulty with the sun as a time-keeper is that its motion in right ascension is variable.

Mean time

The right ascension of the real sun changes by 360° in the course of a year, but the rate of change is not always the same. We might conceive of an imaginary body travelling with the sun, so that its right ascension changes by the same amount in the course of the whole year, but having its motion in right ascension perfectly uniform. Such an imaginary sun would form a perfect time-keeper, we could regulate our clocks to mark noon when it should be on the meridian, and it would have the great practical advantage that the time so indicated would never be very far different from that of the actual sun. This imaginary sun is called the *mean sun*, and the time indicated by it is called *mean solar time*. The mean sun is pictured as moving along the equator with uniform speed, so that its motion is the average of that of the actual sun in right ascension. A *mean solar day* is the interval between two successive transits of the mean sun across the meridian.

The three systems of time measurement

There are thus three kinds of time to be considered.

1. *Sidereal*, as determined by the revolution of the stars.
2. *Apparent solar*, as measured by the actual sun or a sundial.
3. *Mean solar*, as measured by the mean sun, which is the ordinary time kept by our clocks.

The hour angle (Chapter 2) of the real sun + 12 hours gives the apparent time or time indicated by a sundial ; e.g., if the hour

angle of the real sun is 30° (2 hours), the apparent time is $12 + 2$, or 14 hours, which is the same thing as 2 o'clock apparent time in the afternoon ; if the hour angle of the real sun is 210° (14 hours), the apparent time is $12 + 14$, or 26 hours, but as there are only 24 hours in one day, this must represent an apparent time of 2 a.m. the next day. If the hour angle is given as an *east* hour angle, it must be treated as negative ; hence, if an east hour angle of 4 hours is given for the real sun, the apparent time is $12 - 4$, or 8 o'clock in the morning.

Similarly, the hour angle of the mean sun $+ 12$ hours gives the mean time at that instant.

Local apparent noon is the instant when the real sun is on the meridian ; local mean noon is the instant when the mean sun is on the meridian.

Both the civil and astronomical days begin at midnight and end the following midnight. In civil practice we divide the 24 hours into two periods of 12 hours each with noon half-way between ; times between midnight and noon are ante-meridian (a.m.), and between noon and midnight they are post-meridian (p.m.) Astronomers, however, do not adopt this division, but reckon the day from 0 hours to 24 hours. It should be noted that 0 hours on, say, September 16, is the same instant as 24 hours on September 15.

Equation of time

The difference between the mean and the apparent solar time at any particular instant is known as *the equation of time*. It will thus be seen to be the difference between the hour angles of the true and the mean suns. Until the advent of wireless, it was the practice of navigators to determine apparent time from observations of the sun, and to deduce the mean time from the apparent time so obtained by application of the equation of time. The equation of time was thus the amount that had to be added to apparent time to obtain mean time.

Nowadays, however, mean time can be obtained more easily by astronomers from stellar observations, or by navigators through the medium of wireless signals, so that it is more convenient to regard the equation of time as the correction that must be applied to mean time to obtain apparent time. It is in this sense, then (apparent—mean) that the *Nautical Almanac* tabulates the values of the equation of time for every day in the year (*N.A.* pages 6 to 21) and it is in this sense that it will always be given in this book.

For example—

At 0^h G.M.T. on 1951 October 12, the equation of time is given in the *N.A.* (p. 18) as + 13^m 11.14^s. This means that 13^m 11.14^s must be added to the mean time to give the apparent time. Thus, at 0^h G.M.T. on 1951 October 12, the apparent solar time (G.A.T.) is 0^h 13^m 11.14^s. In other words, the true sun is 13^m 11.14^s *ahead* of the mean sun.

At 0^h G.M.T. on 1951 January 20, the equation of time (p. 6, *N.A.*) is — 10^m 46.88^s. Therefore, at 0^h G.M.T. on 1951 January 20, the G.A.T. is 23^h 49^m 13.12^s on January 19. At this time of the year the true sun is *behind* the mean sun.

Values of the equation of time for mean times lying between successive midnights may be taken out by interpolation.

Pages 22 to 29 of the *Nautical Almanac* give the Greenwich mean times of Greenwich apparent noon for every day of the year. The difference between 12 hours and the given Greenwich mean time immediately gives the value of the equation of time at Greenwich apparent noon on the date concerned. In all problems, then, requiring the equation of time for a known apparent time, use is made of the G.M.T. of transit of the sun at Greenwich, with interpolation where necessary; the sense of application is obvious from the table.

The value of the equation of time at 6-hourly intervals may be obtained from the *Star Almanac for Land Surveyors* by subtracting 12 hours from the quantity *E* given in the Sun Tables. The sign, in this case, will again be in the sense Apparent — Mean. A useful interpolation table for *E* is provided at the end of the *Almanac*.

The difference between mean and apparent solar time is due to two causes:—

1. The earth does not move in a circle round the sun, but in an ellipse, as already explained and shown in Fig. 15. The result is that the motion of the earth in its orbit is not uniform, but varies with its distance from the sun, being fastest when it is nearest.

2. The imaginary mean sun is assumed to move always in the equator, whereas the actual sun moves along the ecliptic. It will be seen from a study of Fig. 14 that uniform motion along the ecliptic does not represent uniform motion in right ascension, and therefore does not correspond to uniform motion of the mean sun along the equator.

The second of these causes produces somewhat the greater effect, and the actual equation of time is due to the combined effect of the two.

The equation of time vanishes four times a year, on or about April 16, June 14, September 2 and December 25. From December 25 till April 16 it is negative (in the sense Apparent — Mean) with a maximum value of about $14^m 20^s$ on February 12. From April 16 to June 14 it is positive, having its greatest value of about $3^m 44^s$ on May 15. From June 14 to September 2 it is again negative with a maximum value of $6^m 24^s$ on July 27. Between September 2 and December 25 it is positive once more, attaining its greatest positive value for the year, about $16^m 23^s$, on November 4. These dates are approximate only, since they are taken from the 1951 *Nautical Almanac* and are not always precisely the same in different years.

Following are examples which involve the use of the tables in the *Nautical Almanac*:

(a) Find the value of the equation of time at 16^h G.M.T. on 1951 July 1.

From *N.A.*, 1951, p. 14—

Value of equation of time at 0^h G.M.T.

July 1 = $- 3^m \quad 28.41^s$

At the right hand side of the equation of time column on p. 14 are given the changes between successive values of the equation of time. The change between the value for 0^h , July 1, and that for 0^h , July 2, is $- 11.82^s$.

Now 16^h G.M.T. on July 1 is 16 hours after 0^h G.M.T. on July 1, and 16 hours is equal to 0.6666 of a day.

Thus, $- 11.82^s \times 0.6666$. . . = $- 0^m \quad 07.88^s$

Adding algebraically, the value of the equation of time at 16^h on 1951 July 1 = $- 3^m \quad 36.29^s$

Note.—The above result is not, strictly speaking, correct. In order to obtain a precise answer, we must interpolate by using second and third differences*, which gives us $- 3^m 36.32^s$. The discrepancy of 0.03^s in the first answer will hardly affect the surveyor, however, unless he is engaged in highly refined work.

* An explanation of interpolation by successive differences and use of appropriate tables will be found in the *Nautical Almanac* for 1941; p. 745 ff. Notes on interpolation tables, with examples, are given in the *Nautical Almanac* for 1951 on p. 612. A very full explanation is given in *Plane and Geodetic Surveying*, vol. 2, chap. 1 (Clark: Constable & Co. Ltd., London, 1951).

(b) Find the value of the equation of time at 16^h Greenwich apparent time on 1951 October 3:—

From *N.A.*, 1951, p. 28—

G.M.T. of G.A.N., October 3 . . . = 11^h 49^m 15.61^s

Subtracting this from 12^h, we get equation
of time = + 10^m 44.39^s

G.M.T. of G.A.N., October 4 . . . = 11^h 48^m 56.99^s

Subtracting this from 12^h, we get equation
of time = + 11^m 03.01^s

Since the apparent time given lies between apparent noon on October 3 and apparent noon on October 4, the value of the equation of time must lie between + 10^m 44.39^s and + 11^m 03.01^s. The difference between these two values is 18.62^s (*i.e.* the same as the difference between the G.M.Times of G.A.N. on October 3 and October 4, given at the right hand side of the G.M.T. column).

The given G.A.T. (16^h) is 4^h after G.A.N. on October 3 ; 4^h = 0.1666 of a day.

$$18.62 \times 0.1666 = 3.10^s.$$

The equation of time is increasing, thus 3.10^s must be added to + 10^m 44.39^s to give the required value, *viz.*, + 10^m 47.49^s.

Note.—Here, again, interpolation by successive differences would be necessary if a precise answer were sought.

EXAMPLE.—Find the equation of time at 8^h G.M.T. on 1951 February 16, using the *Nautical Almanac*.

$$\text{Ans.} - 14^m \ 13.0^s$$

Local mean time

The *local mean time* at any place is reckoned by counting as 12 hours the instant when the mean sun last crossed the meridian of the place. As the earth rotates uniformly on its axis from west to east, it follows that the further east a place is situated the sooner will the sun cross the meridian, and, therefore, the *later* will be the local time. All places on the same meridian of longitude have their noons at the same instant, and, as the earth turns, one meridian after another is brought opposite to the sun. Thus, the interval of time between the local noons at two different places will depend upon their difference of longitude.

As the earth turns through 360° in 24 hours, it follows that a difference of 15° of longitude corresponds to a difference of 1 hour in time, $15'$ of arc corresponds to a difference of 1 minute of time, and $15''$ of arc to a difference of 1 second of time.

Thus, if we know the longitude and the local time at one place A, we can readily compute the time at any other place B whose longitude is given. We have only to convert the difference of longitude into time, at the rate of 15° per hour, and add this to the time at A if B is to the east, or subtract it if B is to the west from A.

EXAMPLE.—If the longitude of A is $36^\circ 03' 37''$ E., and the local mean time is September 5, $1^h 31^m 17^s$, find the time at B in longitude $3^\circ 27' 13''$ E.

The difference of longitude = $32^\circ 36' 24''$.

To convert this into time, we simply divide by 15, giving us, as the difference in time between the two places, $2^h 10^m 25.6^s$.

As B is to the west from A, this has to be subtracted from $1^h 31^m 17^s$, giving us as the time at B, September 4, $23^h 20^m 51.4^s$.

Should one longitude be east from Greenwich and the other west, we must add them instead of subtracting. in order to get the angle between the meridians.

EXAMPLE.—A ship sails from London on January 2 at 1 p.m., and arrives in Melbourne (longitude 145° E.) at 6 p.m. on February 8. Find the time occupied by the voyage.

Ans. $36^d 19^h 20^m$

Local sidereal time

The local sidereal time at any place is reckoned by counting as 0^h the instant when the first point of Aries last crossed the meridian of the place. Therefore, in precisely the same way, if we know the longitudes of two places A and B and the *local sidereal time* at A, we can compute the corresponding sidereal time at B. For the earth turns on its axis through 360° relative to the fixed stars in 24 sidereal hours, and, therefore, a difference of longitude of 15° corresponds to a difference of 1 hour in the sidereal times. The method to be used for finding the sidereal time at B is thus exactly the same as that just illustrated.

EXAMPLE.—If the sidereal time at A, long. 35° E., is $12^h 30^m$, find the sidereal time at the same instant at B, long. 27° W.

Ans. $8^h 22^m$

Apparent solar times at the same instant at places in different longitudes

The equation of time or difference between apparent and mean times is the same all over the world at the same instant. Consequently the difference between the apparent solar times at two places A and B is precisely the same as the difference between the local mean times. The same method again then can be used to determine the apparent time at B, having given the apparent time at A.

EXAMPLE.—If the apparent solar time at A, long. 45° W., is $1^{\text{h}} 30^{\text{m}}$, and the equation of time is $6^{\text{m}} 10^{\text{s}}$ to be subtracted from mean time, find the corresponding mean time at B in long. 10° W.

Ans. $3^{\text{h}} 56^{\text{m}} 10^{\text{s}}$

Standard time

To avoid the confusion arising from the use of different local times in each town, most countries now adopt the system of using the time on a particular meridian through, or near, the country, that lies an exact number of hours from Greenwich. For example, until recently the standard time for New Zealand was taken as the time of the meridian $172\frac{1}{2}^{\circ}$ E, viz., $11\frac{1}{2}$ hours ahead of Greenwich mean time. For a good many years there has been a $\frac{1}{2}$ -hour of daylight saving, throughout the whole of the year, making the clock time exactly 12 hours ahead of G.M.T. Since daylight saving has become permanently incorporated in the country's time it was decided to make the standard time for New Zealand the time on the meridian 180° E., viz., 12 hours ahead of G.M.T.

To change standard time to local mean time

This problem has really been discussed already, for the difference between standard time and local mean time at any place is that due to the difference of longitude between the given place and the standard time meridian used. For places east of the standard meridian local mean time is later than standard time, and for places to the west the local time is earlier.

EXERCISES

1. The standard time meridian for New Zealand being 180° E. find the local mean time at Dunedin (long. $170^{\circ} 30' 54''$ E.) when the standard time is $8^{\text{h}} 25^{\text{m}} 10^{\text{s}}$ p.m.

Ans. $19^{\text{h}} 47^{\text{m}} 13.6^{\text{s}}$

2. The Eastern Standard Time meridian for the United States of America is 75° W. If the local mean time is $10^{\text{h}} 17^{\text{m}} 18^{\text{s}}$ at a place in the State of New York, the longitude of which is $73^{\circ} 58' \text{ W.}$, find the Eastern Standard Time.

Ans. $10^{\text{h}} 13^{\text{m}} 10^{\text{s}}$

Note.—A complete list of standard times for various countries is given in the *Nautical Almanac* for 1941, with amendments in subsequent issues.

To reduce a given interval of mean time to sidereal time and vice versa

It will be seen from the consideration of Fig. 15 that in the course of its complete orbital revolution round the sun the earth will make exactly one turn less with respect to the sun than it does with respect to the fixed stars. There are approximately $365\frac{1}{4}$ mean solar days in the year, and, therefore, in the same period there are $366\frac{1}{4}$ sidereal days. More exactly, according to Bessel, the year contains 365.24222 solar days, and hence 365.24222 solar days = 366.24222 sidereal days.

Therefore, if m be the measure of any interval in mean time and s the corresponding measure in sidereal time,

$$\frac{m}{s} = \frac{365.24222}{366.24222}.$$

Thus, if m be given, s can be found, or *vice versa*.

Tables to facilitate the reduction are given in the *Nautical Almanac* (Tables III and IV), and less elaborate ones in *Chambers's Mathematical Tables*.

When tables are not used, the simplest way to make the computation is as follows:—

To convert an interval of mean solar time to sidereal time, add 9.8565 seconds for each mean solar hour. Dividing by 60, this gives us 0.1642 second to be added for each minute and 0.0027 second for each second of mean time.

Thus, to convert an interval of $6^{\text{h}} 33^{\text{m}} 17^{\text{s}}$ of solar time into the equivalent interval of sidereal time, we have—

$$\begin{aligned} 6 \times 9.8565 &= 59.139 \\ 33 \times 0.1642 &= 5.418 \\ 17 \times 0.0027 &= .046 \end{aligned}$$

$$\underline{\underline{64.603 \text{ seconds} = 1^{\text{m}} 4.6^{\text{s}}.}}$$

The addition of this to the given solar time gives us $6^h 34^m 21.6^s$ as the equivalent sidereal interval.

For the ordinary work of surveyors only one decimal place is required and the working should not extend beyond two places.

To convert an interval of sidereal time to the equivalent interval of mean solar time, *subtract* 9.8296 seconds for each sidereal hour. Dividing by 60 we get 0.1638 seconds to be subtracted for each sidereal minute, or 0.0027 second for each second.

Thus, to find the interval of solar time equivalent to an interval of $6^h 33^m 17^s$ of sidereal time, we have—

$$\begin{array}{rcl}
 6 \times 9.8296 & = & 58.978 \\
 33 \times 0.1638 & = & 5.405 \\
 17 \times 0.0027 & = & 0.046 \\
 \hline
 & & 64.429 \text{ seconds} = 1^m 4.43^s. \\
 \hline
 \end{array}$$

Subtracting this from the given interval of sidereal time gives $6^h 32^m 12.57^s$ as the equivalent mean time interval.

Given the sidereal time at 0^h G.M.T. on any given date to find the local sidereal time at 0^h L.M.T. at any other place on the same date.

From pages 6 to 21 in the *Nautical Almanac* the Greenwich Sidereal Times are tabulated (under the heading Apparent Sidereal Time) for 0^h G.M.T. on each day of the year. From these it is necessary, in most work in which the time has to be brought into the calculations, that we should be able to deduce the L.S.T. at 0^h L.M.T. on the corresponding day at the place of observation.

From what we have already done, it will be evident that if we have two clocks, one set to keep sidereal time and the other to keep mean time, the sidereal clock will complete its day in a shorter period than the other, and consequently will be continually gaining. According to the last article, it will gain at the rate of 9.8565 seconds for each mean solar hour.

Now, at a place in west longitude, midnight (0^h) occurs a certain number of hours *after* midnight (0^h) at Greenwich, the interval depending upon the longitude. But the tabulated sidereal time at 0^h G.M.T. is the difference between the readings of the sidereal and mean time clocks at that instant. Consequently, by

the time it becomes midnight at the place in question, the sidereal time will have gained still further on the mean time clock at the rate of 9.8565 seconds for each hour of longitude. Thus the L.S.T. at 0^h L.M.T. will be *greater* than the G.S.T. at 0^h G.M.T. by an amount computed at the rate of 9.8565 seconds for each hour of west longitude.

Similarly, at a place in east longitude, midnight occurs before the corresponding midnight at Greenwich, and in this case the L.S.T. at 0^h L.M.T. will be *less* than the G.S.T. at 0^h G.M.T. by an amount computed in the same way according to the longitude.

EXAMPLE.—On 1951 October 1, the G.S.T. at 0^h G.M.T. is given in the *N.A.* as 0^h 35^m 40.798^s. Determine the L.S.T. at 0^h L.M.T., (a) at a place in longitude 57° 33' 28'' W., (b) at a place in the same longitude E.

(a) 57° 33' 28'' is equivalent to 3^h 50^m 13.87^s.

$$3 \times 9.8565 = 29.569$$

$$50 \times 0.1642 = 8.210$$

$$13.87 \times 0.0027 = .037$$

$$37.816 \text{ seconds.}$$

Therefore, for a place in west longitude we must *add* this on to the 0^h 35^m 40.798^s, giving 0^h 36^m 18.61^s as the L.S.T. at 0^h L.M.T.

(b) If the place is in east longitude, we must *subtract* the 37.816 seconds, giving 0^h 35^m 02.98^s as the L.S.T. at 0^h L.M.T. in that case.

EXAMPLE.—On 1951 December 9, the G.S.T. at 0^h G.M.T. is 5^h 07^m 43.166^s. Compute (a) the G.S.T. at 0^h G.M.T. on December 10, (b) the L.S.T. at a place in longitude 43° 35' West at 0^h L.M.T. on December 9.

$$\text{Ans. (a) } 5^{\text{h}} \quad 11^{\text{m}} \quad 39.72^{\text{s}}$$

$$(b) \quad 5^{\text{h}} \quad 08^{\text{m}} \quad 11.80^{\text{s}}$$

Given the local mean time at any instant, to determine the local sidereal time

The local mean time gives us the interval measured in solar hours, minutes and seconds, that has elapsed since local mean midnight (0^h L.M.T.). We may readily turn this interval into sidereal

hours, and so obtain the number of sidereal hours, minutes and seconds that have elapsed since 0^h L.M.T. But in the preceding article we have seen how the L.S.T. at 0^h L.M.T. may be determined on any given date at a place in any longitude. Consequently we have only to add to this the number of sidereal hours, minutes and seconds that have since elapsed, to determine the sidereal time at the instant. We therefore proceed as follows:—

1. From the tabulated G.S.T. at 0^h G.M.T. on the date in question, compute the L.S.T. at 0^h L.M.T. by allowing for difference in longitude.

2. Turn the given L.M.T. into sidereal hours, minutes and seconds, and add this to the L.S.T. at 0^h L.M.T.

EXAMPLE.—*Find the sidereal time at Mount Hamilton (longitude 121° 38' 44.40'' W.) on 1951 October 17, the L.M.T. being 21^h 17^m 32^s.*

Dividing the longitude by 15, we get the difference in local times between Mount Hamilton and Greenwich to be 8^h 06^m 34.96^s.

The gain of the sidereal over the mean time clock in this interval, at the rate of 9.8565 seconds per hour is 1^m 19.93^s.

From the <i>N.A.</i> , we get G.S.T. at			
0 ^h G.M.T. on 1951 October 17	.	.	1 ^h 38 ^m 45.66 ^s
Add	.	.	01 ^m 19.93 ^s
L.S.T. at 0 ^h L.M.T.	.	.	1 ^h 40 ^m 05.59 ^s
But 21 ^h 17 ^m 32 ^s of mean time, when			
turned into sidereal time	.	.	21 ^h 21 ^m 01.87 ^s
Add, to get required L.S.T.	.	.	23 ^h 01 ^m 07.46 ^s

EXAMPLE.—*Find the sidereal time at Dunedin, New Zealand (long. 170° 30' 54'' E.), on 1951, October 18, the local standard time being 9^h 17^m 32^s a.m.*

The standard time for New Zealand is that of the meridian, 180°, or 12^h, E.

Difference in local times between Dunedin and Greenwich is 11^h 22^m 03.60^s.

The gain of the sidereal over the mean time clock in this interval at the rate of 9.8565 seconds per hour is 1^m 52.04^s.

G.S.T. at 0 ^h G.M.T. on 1951, October 18	1 ^h	42 ^m	42.21 ^s
Subtract		01 ^m	52.04 ^s
L.S.T. at 0 ^h L.M.T.	1 ^h	40 ^m	50.17 ^s
L.Std.T. given, October 18	9 ^h	17 ^m	32.00 ^s
Standard meridian fast on local meridian		37 ^m	56.40 ^s
∴ Local mean time, October 18	8 ^h	39 ^m	35.60 ^s
Turning this interval into sidereal time we get	8 ^h	41 ^m	00.96 ^s
But, from above, L.S.T. at 0 ^h L.M.T.	1 ^h	40 ^m	50.17 ^s
Therefore, L.S.T. required	10 ^h	21 ^m	51.13 ^s

Given the sidereal time at a place whose longitude is known, to determine the corresponding local mean time

If we can find the L.S.T. at 0^h L.M.T., then by subtracting this from the given L.S.T. we find the number of sidereal hours, minutes and seconds that have elapsed since 0^h L.M.T. Turning this interval of time into solar time will give us the number of mean time hours, minutes and seconds since 0^h L.M.T.—that is to say, the L.M.T. required. The rules of procedure are thus:—

1. From the tabulated G.S.T. at 0^h G.M.T. on the date in question, compute the L.S.T. at 0^h L.M.T. by allowing for difference in longitude.

2. Subtract the L.S.T. of 0^h L.M.T. from the given sidereal time. Turn the difference into mean solar time, and the result will be the mean time required.

EXAMPLE.—Given that the sidereal time at Mount Hamilton is 23^h 01^m 07.46^s on 1951 October 17, the longitude of the place being 121° 38' 44.40" W., find the corresponding local mean time.

As in the first example of the preceding section, we obtain L.S.T. at 0^h L.M.T. .

Given sidereal time	1 ^h	40 ^m	05.59 ^s
Difference	23 ^h	01 ^m	07.46 ^s
	21 ^h	21 ^m	01.87 ^s

Turning this interval into mean solar time, by the aid of the tables, we get the L.M.T. required as

21 ^h	17 ^m	32.00 ^s
-----------------	-----------------	--------------------

EXAMPLE.—Given that the sidereal time at Dunedin (longitude 170° 30' 54" E.) is 10^h 21^m 51.13^s on 1951 October 18, find the corresponding local standard time.

As in the second example of the preceding section, we obtain the L.S.T. at 0^h L.M.T.

Given sidereal time	1 ^h	40 ^m	50.17 ^s
Difference	10 ^h	21 ^m	51.13 ^s
	8 ^h	41 ^m	00.96 ^s

Turning this interval of sidereal time into mean solar time we obtain the L.M.T. as

But standard time is fast on L.M.T. by	8 ^h	39 ^m	35.60 ^s
Add, to obtain the L.Std.T. required		37 ^m	56.40 ^s
	9 ^h	17 ^m	32.00 ^s

When the given sidereal time is less than the L.S.T. at 0^h L.M.T., we must add 24 hours to the former, in order to obtain the correct sidereal time interval, as in the following example:—

EXAMPLE.—Given that the sidereal time at Dunedin (longitude 170° 30' 54" E.) is 9^h 18^m 46.27^s on 1951 May 1, find the corresponding local standard time.

From N.A., G.S.T. at 0 ^h G.M.T. May 1	14 ^h	32 ^m	27.67 ^s
Difference due to longitude		01 ^m	52.04 ^s
Therefore, L.S.T. at 0 ^h L.M.T.	14 ^h	30 ^m	35.63 ^s
Given sidereal time with 24 ^h added	33 ^h	18 ^m	46.27 ^s
Difference	18 ^h	48 ^m	10.64 ^s
Convert this interval to mean solar time, giving the L.M.T. as	18 ^h	45 ^m	05.82 ^s
Standard meridian fast on local meridian		37 ^m	56.40 ^s
Add, to get the L.Std.T. required	19 ^h	23 ^m	02.22 ^s

In the above example, the sidereal clock stands at 14^h 30^m 35.63^s at 0^h L.M.T. on 1951 May 1. In order to register the given sidereal time, it must first turn through 9^h 29^m 24.37^s to reach 24^h, and then through 9^h 18^m 46.27^s. These two added together give the sidereal interval (18^h 48^m 10.64^s) after 0^h L.M.T. on May 1.

Alternative method for determining the L.S.T., having given the L.M.T.

In the preceding methods for computing the L.S.T. from L.M.T. or *vice versa*, it is necessary first of all to compute the L.S.T. of 0^h L.M.T., and then to transform another interval of time from solar to sidereal or from sidereal to solar. In the methods about to be described the theory is perhaps a little more complex, but there is only one transformation of a time interval necessary, so that the actual computation is a little shorter.

From the given L.M.T., allowing for the difference of longitude, we readily compute the corresponding mean time at Greenwich.

This gives us the interval in mean solar time that has elapsed since the last Greenwich midnight (0^h G.M.T.). Turn this interval into sidereal time, and we get the number of sidereal hours, minutes and seconds that have elapsed since the last 0^h G.M.T.

But from the *Nautical Almanac* we get the G.S.T. at the last 0^h G.M.T. By adding to this the sidereal interval just found, we get the G.S.T. at the instant with which we are concerned. All that now remains to be done is to apply the difference of longitude to this last G.S.T. in order to find the required L.S.T.

We thus get the following rules of procedure:—

1. Allowing for the difference of longitude, compute the G.M.T. of the instant in question; turn this time (which is an interval of mean time since 0^h G.M.T.) into an interval of sidereal time.

2. From the *Nautical Almanac* obtain the G.S.T. at the previous 0^h G.M.T., and add to it the sidereal interval found in 1. This gives the G.S.T. corresponding to the instant in question.

3. Apply the difference of longitude to the G.S.T. found in 2, to yield the L.S.T. of the instant in question.

As illustrations, for purposes of comparison, we shall take the same examples as those already worked.

EXAMPLE.—Find the sidereal time at Mount Hamilton (longitude $121^\circ 38' 44.40''$ W.) on 1951 October 17, the L.M.T. being $21^h 17^m 32^s$.

L.M.T. at Mount Hamilton, October 17	21^h	17^m	32.00^s
Add west latitude	8^h	06^m	34.96^s
Corresponding G.M.T., October 18 .	5^h	24^m	06.96^s
Transform into sidereal time (<i>N.A.</i> , Table III)	5^h	00^m	49.282^s
	24^m	24^m	03.943^s
			06.016^s
			0.963^s
Sidereal interval since 0^h G.M.T. October 18	5^h	25^m	00.204^s
G.S.T. at 0^h G.M.T. October 18, (from <i>N.A.</i>)	1^h	42^m	42.214^s
G.S.T. at instant	7^h	07^m	42.42^s
Subtract west longitude	8^h	06^m	34.96^s
L.S.T. required	23^h	01^m	07.46^s

With this arrangement of the computation, *west* longitude is first added and then subtracted, *east* longitude is first subtracted and then added.

EXAMPLE.—Find the sidereal time at Dunedin, New Zealand (longitude $170^{\circ} 30' 54''$ E.), on 1951 October 18, the local standard time being $9^h 17^m 32^s$ a.m.

The standard time for New Zealand is that of the meridian 180° or 12^h E.

L.Std.T. at Dunedin, October 18	.	9^h	17^m	32.00^s
Longitude of standard meridian				
(subtract)	.	12^h	00^m	00.00^s
Corresponding G.M.T., October 17		21^h	17^m	32.00^s
Transform into sidereal time (<i>N.A.</i> , Table III)	21^h	21^h	03^m	26.986^s
	17^m		17^m	02.793^s
	32^s			32.088^s
Sidereal interval since 0^h G.M.T.				
October 17	.	21^h	21^m	01.867^s
G.S.T. at 0^h G.M.T., October 17		1^h	38^m	45.658^s
G.S.T. at instant	.	22^h	59^m	47.53^s
Add east longitude	.	11^h	22^m	03.60^s
		34^h	21^m	51.13^s
Subtract 24^h	.	24^h	00^m	00.00^s
L.S.T. required	.	10^h	21^m	51.13^s

NOTE.—Subtracting 24^h as above does not involve any change in the date; the sidereal clock cannot register $34^h 21^m 51.13^s$ since it is graduated up to 24^h only. The date is governed by the mean time instant given in the problem.

Alternative method for determining the L.M.T. having given the L.S.T.

Knowing the longitude of the place, we can compute the sidereal time at Greenwich at the same instant. From the *Nautical Almanac*, as before, we get the G.S.T. at the previous 0^h G.M.T. Subtracting these two results gives us the interval in sidereal time that has elapsed since 0^h G.M.T.

If we turn this interval into mean solar time, we therefore get the interval of mean time that has elapsed since 0^h G.M.T. But the L.M.T. corresponding to 0^h G.M.T. is readily determined by allowing for the difference of longitude. Adding to this, therefore, the interval of mean time that has since elapsed, we obtain the L.M.T. required.

The principal difficulty arises in places with east longitude, where it may happen that the instant under consideration really precedes 0^h on the same day at Greenwich. This cannot happen with places having west longitude. If this is the case, it will be at once noticed from the fact that the sidereal time at 0^h G.M.T. on the day in question, as found from the *Nautical Almanac*, will be *less* than the computed G.S.T. at the instant.

We thus get the following rules for determining the L.M.T., having given the L.S.T.:—

1. Allowing for the difference of longitude, compute the G.S.T. at the instant in question.

2. From the *Nautical Almanac* find the G.S.T. at the previous 0^h G.M.T. and then, by subtraction, the number of sidereal hours that have elapsed since. Turn this interval of sidereal time into mean time.

3. Add this interval of mean time on to the L.M.T. corresponding to 0^h G.M.T., and the result is the L.M.T. required.

EXAMPLE.—Given that the sidereal time at Mount Hamilton is $23^h 01^m 07.46^s$ on 1951 October 17, the longitude of the place being $121^\circ 38' 44.40''$ W., find the corresponding L.M.T.

L.S.T. at Mount Hamilton	.	23 ^h	01 ^m	07.46 ^s
Difference due to longitude (W.)	.	8 ^h	06 ^m	34.96 ^s
Corresponding G.S.T.	.	31 ^h	07 ^m	42.42 ^s
G.S.T. at 0^h G.M.T., October 17	.	1 ^h	38 ^m	45.66 ^s
Interval of sidereal time since 0^h G.M.T.,				
October 17	.	29 ^h	28 ^m	56.76 ^s
Equivalent interval of mean time	.	29 ^h	24 ^m	06.96 ^s
L.M.T. corresponding to 0^h G.M.T.,				
October 17,	October 16	15 ^h	53 ^m	25.04 ^s
Add, to get L.M.T. required, October				
16	.	45 ^h	17 ^m	32.00 ^s
Subtract 24 ^h	.	24 ^h	00 ^m	00.00 ^s
L.M.T. required, October 17	.	21 ^h	17 ^m	32.00 ^s

Note.—In this case subtracting 24^h does involve a change in date, because we are dealing with *mean time* which governs the date.

EXAMPLE.—Given that the sidereal time at Dunedin (longitude $170^\circ 30' 54''$ E.) is $10^h 21^m 51.13^s$ on 1951 October 18, find the corresponding local standard time.

L.S.T. at Dunedin	10 ^h	21 ^m	51.13 ^s
Difference due to east longitude	11 ^h	22 ^m	03.60 ^s
Corresponding G.S.T.	22 ^h	59 ^m	47.530 ^s
G.S.T. at 0 ^h G.M.T., October 18	1 ^h	42 ^m	42.214 ^s
Instant <i>precedes</i> 0 ^h G.M.T., October 18, by	2 ^h	42 ^m	54.684 ^s
Equivalent interval of mean time	2 ^h	42 ^m	28.00 ^s
L.Std.T. corresponding to 0 ^h G.M.T., October 18	12 ^h	00 ^m	00.00 ^s
L.Std.T. required, October 18	9 ^h	17 ^m	32.00 ^s

(2^h 42^m 28^s is subtracted from 12^h 00^m 00^s because instant precedes 0^h G.M.T. October 18.)

It should be noted that, if we had subtracted the G.S.T. at 0^h G.M.T. October 18 (1^h 42^m 42.214^s) from the G.S.T. corresponding to the given L.S.T. (22^h 59^m 47.53^s) we would have obtained a figure of 21^h 17^m 05.316^s sidereal interval after 0^h G.M.T. on October 18. This would have given us a mean time interval after 0^h G.M.T. October 18 of 21^h 13^m 36.10^s. Upon adding this to the L.Std.T. of 12^h 00^m 00^s corresponding to 0^h G.M.T. on October 18, we would have obtained a L.Std.T. of 33^h 13^m 36.1^s October 18, or 9^h 13^m 36.1^s on the October 19, whereas we were asked to find a L.Std.T. on October 18.

Comparison of the preceding methods

As it is a most important thing that the student should thoroughly grasp the principles involved in the transference of time from one system of time measurement to the other, it is a good exercise for him to master both the first method given and the alternative method in each of the preceding cases. The first method, however, involves less thinking and is more mechanical than the other, so that it is the method generally adopted and the one probably most suited for ordinary computations. It has an obvious advantage where a number of times are to be reduced for the same place, as the correction for longitude is the same for all.

Determination of the L.M.T. of transit of a known star across the meridian

One very important application of the preceding work is the calculation of the time of transit of a known star across the meridian, or, as it is commonly termed, the time of culmination.

The *Nautical Almanac*, for years up to and including 1951, supplies us with a table of the right ascensions and declinations of the principal stars in the sky. From 1952 onwards the table will be omitted and will appear only in the *Star Almanac* and the *Apparent Places of Fundamental Stars*. It has been shown in Chapter 2 that the R.A. of a star, expressed in time, is the sidereal time at the moment when the star is on the meridian. Thus the problem is simply that of determining the L.M.T. corresponding to the local sidereal time measured by the R.A. of the star. This we may do by one of the methods we have been considering.

EXAMPLE.—Find the time of culmination of α Triang. Aust. on the evening of 1951 August 17, at a place in New Zealand whose longitude is $170^{\circ} 20' E.$, the time to be measured in the standard time of the meridian $12^h E.$

G.S.T. of 0^h G.M.T., August 17	21 ^h	38 ^m	15.886 ^s
L.S.T. of 0^h L.M.T. at a place in long. $170^{\circ} 20' E.$ computed as in previous work	21 ^h	36 ^m	23.962 ^s
R.A. of α Triang. Aust. = L.S.T. at time of culmination	16 ^h	43 ^m	32.470 ^s
Therefore, interval of sidereal time elapsed since 0^h L.M.T.	19 ^h	07 ^m	08.508 ^s
Equivalent interval of mean time	19 ^h	04 ^m	00.576 ^s
This, therefore, would be the L.M.T. at time of culmination.			
Difference between L.M.T. and time of the standard meridian		38 ^m	40.000 ^s
Therefore, L.Std.T. of culmination	19 ^h	42 ^m	40.58 ^s
(i.e. 7 ^h 42 ^m 40.6 ^s p.m.)			

Time of transit of the first point of Aries

In the preceding work we have adopted the usual practice of effecting the change from sidereal to mean time or *vice versa* by means of the column in the *Nautical Almanac* giving the G.S.T. at 0^h G.M.T. But there is another column among the data for the Sun at 0^h G.M.T. on each day; this is headed "Transit of First Point of Aries," and gives the G.M.T. at the instant when the first point of Aries is on the meridian at Greenwich, for every day of the year. Each time in this column, then, is the G.M.T. corresponding to 0^h G.S.T. on each day of the year. These times may thus also be used to effect the change from sidereal to mean time and *vice versa*.

Given the G.M.T. of transit of the first point of Aries, to determine the L.M.T. of transit at a place in any other longitude

The sidereal clock, as we have seen, is always gaining on the clock keeping mean solar time, at the rate of 9.8565 seconds per mean solar hour, or at the rate of 9.8296 seconds for each sidereal hour. Now the G.M.T. of transit of the first point of Aries is the reading of the mean time clock when the sidereal clock reads 0^h. It is the difference between the readings of the two clocks at this instant. As the sidereal clock is gaining on the other, this difference will get less as the time increases. Now, at a place in west longitude the transit of the first point of Aries will take place after an interval of time measured in sidereal hours, minutes and seconds obtained by dividing the longitude by 15. Thus, when this transit occurs the mean time clock will not be as far ahead of the sidereal clock as it was at Greenwich, and the Greenwich reading of the mean time clock will have to be diminished by subtracting 9.8296 seconds for each hour of longitude.

This reasoning assumes that, whilst different clocks at various places on the earth's surface will have different readings according to the longitude, the difference between the readings of the sidereal and mean time clocks at any place is the same all over the world at the same instant. This must be so according to the reasoning by which we have established the rules for determining the local mean and sidereal times at a place A, having given those at a place B. For we should alter both the sidereal and mean times at B by the same amount, depending upon the difference of longitude between B and A, in order to find the corresponding times at A.

Accordingly we get the rule for finding from the *Nautical Almanac* tables the time of transit of the first point of Aries at any place. If the place of observation be not on the meridian of Greenwich, the mean time must be corrected by the *subtraction* of 9.8296 seconds for each hour (and proportional parts for the minutes and seconds) of longitude, if the place be to the west of Greenwich; but by its *addition*, if to the east.

EXAMPLE.—On 1951 February 1, the G.M.T. of transit of the first point of Aries is 15^h 15^m 55.25^s. Compute the local mean time of transit on the same day (a) at a place in longitude 57° 33' 28" West, (b) at a place in the same longitude East.

(a) 57° 33' 28" is equivalent in time to 3^h 50^m 13.87^s.

$$3 \times 9.8296 = 29.488$$

$$50 \times 0.1638 = 8.190$$

$$13.87 \times 0.0027 = 0.037$$

37.715, say 37.72 seconds.

Otherwise we might more conveniently obtain this result direct from Chambers's Tables.

Therefore, for a place in west longitude, we must subtract this from the $15^h 15^m 55.25^s$, giving $15^h 15^m 17.53^s$ as the L.M.T. of transit of the first point of Aries.

(b) For a place in east longitude we must add the $37.72'$, giving $15^h 16^m 32.97^s$ as the L.M.T. of transit in this case.

EXAMPLE.—Given that the G.M.T. of transit of the first point of Aries on March 1 is $13^h 25^m 49.78^s$. Find the G.M.T. of transit on March 2. Find also the L.M.T. of transit at a place in longitude $45^\circ W$.

Ans. $13^h 21^m 53.87^s$

and $13^h 25^m 20.29^s$

Given the L.S.T. at any place and the G.M.T. of transit of the first point of Aries on the same day, to determine the L.M.T.

The local sidereal time at any place measures the interval in sidereal hours since the transit of the first point of Aries over the meridian of that place. By turning this, therefore, into mean time hours we get the interval since the transit in mean time hours. But we have just seen how we may calculate the L.M.T. of transit of the first point of Aries from the information in the *Nautical Almanac*. The addition of the two results will give us the L.M.T. required. The rule of procedure, therefore, may be expressed: Turn the given sidereal time into mean time and add it to the computed L.M.T. of transit of the first point of Aries (Υ).

As the transit of Υ may take place at any time of the day, some care is necessary in selecting the right transit, as is illustrated in the following example:—

EXAMPLE.—Given that the L.S.T. at Mount Hamilton is $23^h 01^m 07.46^s$ on 1951 October 17, the longitude of the place being $121^\circ 38' 44.4'' W$, find the corresponding L.M.T.

Looking up in the *N.A.* the G.M.T. of transit of the first point of Aries on October 17 we find it is $22^h 17^m 34.61^s$. This is very near midnight, and the L.M.T. of transit will not be very different. If we were to add 23^h on to this it would very clearly carry us over into the next day, October 18, so that the transit we must select to work from is that on October 16.

G.M.T. of transit of Υ October 16	22^h	21^m	30.520^s
Allowance for long., to be subtracted		1^m	19.714^s
L.M.T. of transit of Υ October 16	22^h	20^m	10.806^s
Mean time equivalent of $23^h 01^m 07.46^s$			
(add)	22^h	57^m	21.196^s
Therefore, L.M.T. required, October 17	21^h	17^m	32.00^s

Given the sidereal time at 0^h G.M.T. to compute the G.M.T. at the next transit of the first point of Aries

The figures in the two columns of pp. 6-21 of the *N.A.* which give the sidereal time at 0^h G.M.T. and the G.M.T. of the transit of the first point of Aries, may readily be deduced one from the other.

Thus, suppose the sidereal time at 0^h mean time is denoted by s . Then at 0^h mean time, s sidereal hours have elapsed since γ was on the meridian, and, therefore, in $24-s$ sidereal hours γ will again be on the meridian.

If we express $24-s$ sidereal hours in mean solar time, the result will clearly represent the number of mean solar hours that have elapsed since 0^h mean time, and will consequently represent the mean time at the next transit of γ .

For example, on 1951 May 1, the sidereal time at 0^h G.M.T. is 14^h 32^m 27.67^s. Subtracting this from 24^h, we get 9^h 27^m 32.33^s. Turning this into mean solar time, the result is 9^h 25^m 59.35^s, which, therefore, represents the mean time at the next transit of γ .

The converse problem may be dealt with in a similar way.

EXAMPLE.—On April 15 the G.S.T. at 0^h G.M.T. is 13^h 29^m 22.78^s. Find the G.M.T. of the next transit of γ .

Ans. 10^h 28^m 53.91^s

Given the G.M.T. of G.A.N. on a certain date, to find the L.M.T. of L.A.N. on the same date

Here we are given the G.M.T. of transit of the real sun at Greenwich, and we are required to find the L.M.T. of transit of the real sun at some other place whose longitude is known.

If the place is east of Greenwich, L.A.N. will occur before G.A.N., but if the place is west, L.A.N. will occur after G.A.N. Thus, if we are dealing with a place in east longitude, and we are given the G.M.T. of G.A.N. on the date in question, we must know the difference between the given G.M.T. of G.A.N. and the G.M.T. of G.A.N. on the day *before*, in order to interpolate for the difference of longitude. This is because apparent noon at the place occurs *before* apparent noon at Greenwich on the day concerned.

On the other hand, if we are dealing with a place in west longitude, we must know the difference between the given G.M.T. of G.A.N. and the G.M.T. of G.A.N. on the day *after*, in order to

interpolate for the difference of longitude, because apparent noon at the place occurs *after* apparent noon at Greenwich on the day in question.

The L.M.T. of L.A.N. obtained in this way is used in making observations of the sun when it is at, or near, transit (*e.g.*, observations for latitude).

(NOTE.—During the interval of time between apparent noon at the place and at Greenwich—*i.e.*, the interval due to the difference of longitude—the mean sun will gain a little or lose a little on the real sun, depending upon the time of the year. There will thus be a slight change in the equation of time during the interval, and it is really this which we are computing in finding the L.M.T. of L.A.N. from the G.M.T. of G.A.N.)

EXAMPLE.—Given the following data from the *N.A.* for 1951:—

Sun at Transit at Greenwich

Date	G.M.T.			
	h	m	s	
June 30	12	03	22.44	
July 1	12	03	34.38	+ 11.94
July 2	12	03	46.09	+ 11.71
July 3	12	03	57.54	+ 11.45
July 4	12	04	08.70	+ 11.16

Find the L.M.T. of L.A.N. on July 2 at a place (a) in longitude 135° E., (b) in longitude 60° W.

(a) The place is 135° E., therefore its L.M.T. is 9^h ahead of G.M.T. $9^h = 0.375$ day (Table V, *N.A.*). From the given data in the table above, the difference in seconds between G.M.T. of G.A.N. on July 1 and July 2 is 11.71^s .

Thus, $11.71^s \times 0.375 = 4.39^s$.

Since the values of the G.M.T. in the given table are decreasing as we go back from July 2, this difference of 4.39^s must be subtracted from the G.M.T. of G.A.N. on July 2 to yield the L.M.T. of L.A.N. on the same date:—

July 2, G.M.T. of G.A.N.	12 ^h	03 ^m	46.09 ^s
Difference due to longitude E.			– 04.39 ^s
July 2, L.M.T. of L.A.N.	12 ^h	03 ^m	41.70 ^s

(b) The place is 60° W., therefore its L.M.T. is 4^h behind G.M.T. $4^h = 0.1666$ day (Table V, *N.A.*). From the given data, the difference in seconds between G.M.T. of G.A.N. on July 2 and July 3 is 11.45^s .

Thus, $11.45^s \times 0.1666 = 1.91^s$.

Since the values of the G.M.T. in the table are increasing as the dates increase, this difference of 1.91^s must be added to the G.M.T. of G.A.N. on July 2 to give the L.M.T. of L.A.N. on the same date:—

July 2, G.M.T. of G.A.N.	12 ^h	03 ^m	46.09 ^s
Difference due to longitude, W.			+ 01.91 ^s
July 2, L.M.T. of L.A.N.	12 ^h	03 ^m	48.00 ^s

NOTE.—In the above examples, the differences were obtained by linear interpolation which will give an answer correct enough for most purposes. If, however, an exact answer is required, interpolation using second, and, if necessary higher differences must be employed.

For example, to obtain a more exact answer in each of the cases above, using Table XX in the *N.A.*—

(a)	$f_0 =$	12 ^h	03 ^m	34.38 ^s
	$\Delta' =$	+	11.71 ^s	
	$\Delta_0'' + \Delta_1'' =$	—	0.49 ^s	
	$n =$		0.625	
	$f_0 =$	12 ^h	03 ^m	34.38 ^s
	$11.71^s \times 0.625 =$			+ 07.32 ^s
	Table XX			+ 00.03 ^s
	L.M.T. required =	12 ^h	03 ^m	41.73 ^s
(b)	$f_0 =$	12 ^h	03 ^m	46.09 ^s
	$\Delta' =$	+	11.45 ^s	
	$\Delta_0'' + \Delta_1'' =$	—	0.55 ^s	
	$n =$		0.1666	
	$f_0 =$	12 ^h	03 ^m	46.09 ^s
	$11.45^s \times 0.1666 =$			+ 01.91 ^s
	Table XX			+ 00.02 ^s
	L.M.T. required =	12 ^h	03 ^m	48.02 ^s

Given the L.M.T. at any place, to find the corresponding L.A.T.

From the given L.M.T., by applying the longitude expressed in time, we can obtain the corresponding G.M.T. The *N.A.* (pp. 6-21) gives the value of the equation of time for 0^h G.M.T. on every day of the year. Hence, by interpolation to the instant of G.M.T. already found we can find the equation of time at the given L.M.T. (See previous article on equation of time.) Application of this to the L.M.T. will give the required L.A.T.

EXAMPLE.—Given that the *L.M.T.* at a place in longitude 65° 43' 30'' E. is 20^h 22^m 54^s on 1951 July 1, find the corresponding *L.A.T.*

Given L.M.T.	.	.	.	.	.	20 ^h	22 ^m	54 ^s
Longitude, E.	.	.	.	.	.	4 ^h	22 ^m	54 ^s
Corresponding G.M.T.	.	.	.	.	.	16 ^h	00 ^m	00 ^s

Entering the *N.A.* table of values of equation of time at 0^h G.M.T. with this G.M.T., we find by interpolation, as in example in previous article on equation of time, that the value of the equation of time at G.M.T. 16^h on July 1 is

-- 03^m 36.29

Applying this value of the equation of time to the given L.M.T. of 20^h 22^m 54^s gives us the required L.A.T.

20^h 19^m 17.71^s

Given the L.A.T., to find the corresponding L.M.T.

From the given L.A.T., by applying the longitude expressed in time, we can obtain the corresponding G.A.T. The *N.A.* (pp. 22-29) gives the G.M.T. corresponding to 12^h G.A.T. for every day of the year, from which, by inspection, we can find the value of the equation of time at each 12^h G.A.T. (See previous article on equation of time.) Interpolation to the instant of G.A.T. found from the given L.A.T. will enable us to compute the correct value of the equation of time to apply to the L.A.T. to yield the required L.M.T.

EXAMPLES.—

(a) Find the *L.M.T.* corresponding to *L.A.T.* 3^h 33^m 42^s on 1951 October 4, at a place in longitude 173° 25' 30'' E.

Given L.A.T., October 4	.	.	.	3 ^h	33 ^m	42 ^s
Longitude, E.	.	.	.	11 ^h	33 ^m	42 ^s
Corresponding G.A.T., October 3	.	.	.	16 ^h	00 ^m	00 ^s

Entering the Transit Table of the Sun with this G.A.T., we find by interpolation, as in the example in previous article on equation of time, that the value of the equation of time at G.A.T. 16^h October 3 is .

10^m 47·49^s

From the G.M.T. at G.A.N. on October 3, we see that the apparent time is ahead of the mean time. Therefore, we must subtract this 10^m 47·49^s from the given L.A.T. of 3^h 33^m 42^s in order to obtain the required L.M.T.

3^h 22^m 54·51^s

(b) Find the L.M.T. corresponding to L.A.T. 14^h 25^m 18^s on 1951 March 15, at a place in longitude 8^h 03^m 21^s W.

Given L.A.T., March 15 14^h 25^m 18^s

Longitude, W. 8^h 03^m 21^s

Corresponding G.A.T., March 15 22^h 28^m 39^s

From N.A., p. 23—

Equation of time at
G.A.N., March 15 . . . = 9^m 11·67^s

Equation of time at
G.A.N., March 16 . . . = 8^m 54·74^s

Difference . . . = 16·93^s

10^h 28^m 39^s = 0·43656 day.

Difference for 0·43656 day
= 16·93^s × 0·43656 = 7·39^s

Since value of equation of time is decreasing, subtract this 7·39^s from 9^m 11·67^s in order to get equation of time at 22^h 28^m 39^s G.A.T. on March 15. This gives the equation of time at the instant of L.A.T. with which we are concerned as 9^m 04·28^s

From the G.M.T. of G.A.N. March 15, we see that mean time is ahead of apparent time at this time of the year. Therefore, we must add the 9^m 04·28^s found as above to the given L.A.T. of 14^h 25^m 18^s in order to get the required L.M.T.

14^h 34^m 22·28^s

An alternative method, which does not involve taking out the separate values of the equation of time for each G.A.N. is to interpolate, for the difference in time after the previous G.A.N., directly into the table giving G.M.T. of G.A.N. This will give us (12^{h} —equation time) for the instant with which we are concerned, from which, by inspection, we can obtain the equation of time at the instant. Application of this to the given L.A.T. will yield the required L.M.T. The examples given above are re-worked by this method:—

(a) Given L.A.T., October 4	3^{h}	33^{m}	42^{s}
Longitude, E.	11^{h}	33^{m}	42^{s}
Corresponding G.A.T., October 3	16^{h}	00^{m}	00^{s}
G.M.T. at G.A.N., October 3	11^{h}	49^{m}	15.61^{s}
(This is equal to 12^{h} —equation of time at G.A.N.)			
G.A.T. 16^{h} October 3 is 4^{h} , or 0.16667 day, after G.A.N. October 3. The difference between G.M.T. of G.A.N. on October 3 and October 4, from the Table, is -18.62^{s} . Hence $-18.62^{\text{s}} \times 0.16667$			
			-03.10^{s}
$\therefore$ (12^{h} —equation of time) at instant of given L.A.T.	11^{h}	49^{m}	12.51^{s}
From which we deduce by inspection, the value of the equation of time at this instant		10^{m}	47.49^{s}
Given L.A.T., October 4	3^{h}	33^{m}	42.00^{s}
Since mean time is behind apparent time, value of equation of time must be subtracted from apparent time to give required L.M.T.			
	3^{h}	22^{m}	54.51^{s}
(b) Given L.A.T., March 15	14^{h}	25^{m}	18^{s}
Longitude, W.	8^{h}	03^{m}	21^{s}
Corresponding G.A.T., March 15	22^{h}	27^{m}	39^{s}
G.M.T. at G.A.N., March 15	12^{h}	09^{m}	11.67^{s}
22^{h} 28^{m} 39^{s} G.A.T. March 15 is 10^{h} 28^{m} 39^{s} after G.A.N. March 15, or 0.4365 day after.			
Difference between G.A.N. March 15 and March 16 = -16.93^{s} .			
Thus, $-16.93^{\text{s}} \times 0.43656$			-07.39^{s}

∴ (12 ^h — equation of time) at instant of given L.A.T.	12 ^h	09 ^m	04·28 ^s
∴ Equation of time at instant		09 ^m	04·29 ^s
Given L.A.T.	14 ^h	25 ^m	18·00 ^s
Add equation of time to get required L.M.T. (because L.M.T. is ahead of L.A.T.)	14 ^h	34 ^m	22·28 ^s

In all the above examples, interpolation using second and, if necessary, higher differences, would be essential for very refined work.

Nautical Almanac data with regard to time

In the *Nautical Almanac*, on pages 6-29, various data are given that are useful in time computations. The sidereal time at 0^h G.M.T., the G.M.T. of transit of the first point of Aries, the equation of time at 0^h G.M.T., and the G.M.T. of apparent noon are given in each case for every day in the year. In addition, the sun's right ascension is given for 0^h G.M.T. and for G.A.N. These tabulated values are not all independent, and it is good practice for the student to take a *Nautical Almanac* and deduce certain of the tabulated values from others that are given. Here are a few of the exercises that may be practised in this way:

1. From the sidereal time at 0^h G.M.T. on one day compute its value for the next day.
2. From the sidereal time at 0^h G.M.T. find the G.M.T. of the next transit of the first point of Aries.
3. From the G.M.T. of transit of the first point of Aries determine the sidereal time at 0^h G.M.T. on the same day.
4. From the right ascension of the sun at 0^h G.M.T., and the equation of time, deduce the sidereal time at 0^h G.M.T., and the right ascension of the sun at G.A.N.
5. From the sidereal time and the sun's right ascension at 0^h G.M.T., deduce the equation of time.

Time data in the Star Almanac for Land Surveyors

The *Star Almanac for Land Surveyors* is a new publication, of which the first issue was for 1951. It is a much smaller volume than the *Nautical Almanac*, but it contains all the astronomical information required by surveyors in the normal course of their work ; further, the data concerning the sun are given for every six hours throughout

the year. Convenient tables for interpolation are given at the end of the *Almanac*. The introduction gives an explanation of the terms and quantities used, as well as illustrations of the use of the *Almanac*. It is to be noted that the term Universal Time (U.T.) is used throughout the volume instead of G.M.T.

The quantity E with 12^h subtracted from it gives the equation of time with the sign in the sense (Apparent—Mean).

U.T. + E = G.H.A. of the Sun (Greenwich Hour Angle of the True Sun).

This should, of course, be diminished by 24^h if U.T. + E is greater than 24^h .

G.H.A. of the Sun $- 12^h$ = G.A.T.

Similarly,

L.M.T. + E = L.H.A. of the Sun (Local Hour Angle of the True Sun).

L.H.A. of the Sun $- 12^h$ = L.A.T.

The quantity R with the U.T. added to it gives the G.S.T. Thus, at 0^h U.T., the G.S.T. is equal to R . Similarly R + L.M.T. = L.S.T.

As illustrations of the use of the *Star Almanac*, some of the examples already given in this chapter, and worked out by means of the *Nautical Almanac*, are re-calculated below:—

Given the L.M.T. at any instant, to determine the L.S.T.

EXAMPLE.—Find the sidereal time at Mount Hamilton (longitude $121^\circ 38' 44.4''$ W.), on 1951 October 17, the L.M.T. being $21^h 17^m 32^s$.

L.M.T. Mount Hamilton, October 17	.	.	21^h	17^m	32^s
Longitude Mount Hamilton, W.	.	.	8^h	06^m	35^s
∴ Corresponding U.T., October 18	.	.	5^h	24^m	07^s
R at 0^h U.T., October 18	.	.	.	.	.
18	.	.	1^h	42^m	42.2^s
Change in R for $5^h 24^m$	.	.	.	.	.
07^s (Table on p. 64 of S.A.)	.	.	.	.	.
∴ R at U.T. $5^h 24^m$	.	.	.	.	.
07^s , October 18	.	.	1^h	43^m	35.4^s
Thus, U.T. + R = G.S.T.	.	.	7^h	07^m	42.4^s
Longitude Mount Hamilton, W.	.	.	8^h	06^m	35.0^s
Subtract, and L.S.T. required	.	.	23^h	01^m	07.4^s

EXAMPLE.—Find the sidereal time at Dunedin, New Zealand (long. $170^{\circ} 30' 54''$ E.), on 1951 October 18, the local standard time being $9^{\text{h}} 17^{\text{m}} 32^{\text{s}}$ a.m. (The standard time for N.Z. is that of the meridian 180° or 12^{h} E.)

Local standard time, Dunedin, Oct. 18	$9^{\text{h}} 17^{\text{m}} 32^{\text{s}}$	
Longitude of standard meridian, E.	$12^{\text{h}} 00^{\text{m}} 00^{\text{s}}$	
Corresponding U.T., October 17	$21^{\text{h}} 17^{\text{m}} 32^{\text{s}}$	
R at U.T. 18^{h} , October 17		
$17 \quad . \quad . \quad = \quad 1^{\text{h}} 41^{\text{m}} 43.1^{\text{s}}$		
Change in R for $3^{\text{h}} 17^{\text{m}}$		
32^{s} (Table on p. 64 of S.A.)		
$. \quad . \quad . \quad = \quad + 32.4^{\text{s}}$		
$\therefore R$ at U.T. $21^{\text{h}} 17^{\text{m}}$		
32^{s} October 17	$1^{\text{h}} 42^{\text{m}} 15.5^{\text{s}}$	$1^{\text{h}} 42^{\text{m}} 15.5^{\text{s}}$
Thus, U.T. + R = G.S.T.	$22^{\text{h}} 59^{\text{m}} 47.5^{\text{s}}$	
Longitude of Dunedin, E.	$11^{\text{h}} 22^{\text{m}} 03.6^{\text{s}}$	
Add, and L.S.T. required	$10^{\text{h}} 21^{\text{m}} 51.1^{\text{s}}$	

Given the sidereal time at a place whose longitude is known, to determine the corresponding local mean time

EXAMPLE.—Given that the sidereal time at Mount Hamilton is $23^{\text{h}} 01^{\text{m}} 07.46^{\text{s}}$ on 1951 October 17, the longitude of the place being $121^{\circ} 38' 44.4''$ W., find the corresponding local mean time.

L.S.T. Mount Hamilton, October 17	$23^{\text{h}} 01^{\text{m}} 07.5^{\text{s}}$	
Longitude, W.	$8^{\text{h}} 06^{\text{m}} 35.0^{\text{s}}$	
$\therefore$ Corresponding G.S.T. (= U.T. + R)	$31^{\text{h}} 07^{\text{m}} 42.5^{\text{s}}$	
But, from S.A., G.S.T. (= U.T. + R) at 30^{h} U.T. October 17 (i.e. at U.T. 6^{h} , October 18)	$31^{\text{h}} 43^{\text{m}} 41.4^{\text{s}}$	
$\therefore$ instant is <i>prior</i> to U.T. 30^{h} , October 17 by (sidereal time)	$35^{\text{m}} 58.9^{\text{s}}$	
Convert this interval to mean solar time	$35^{\text{m}} 53.0^{\text{s}}$	
Subtract $35^{\text{m}} 53.0^{\text{s}}$ from U.T. 30^{h} to get U.T. of instant on October 17	$29^{\text{h}} 24^{\text{m}} 07^{\text{s}}$	
Longitude of Mount Hamilton, W.	$8^{\text{h}} 06^{\text{m}} 35^{\text{s}}$	
$\therefore$ L.M.T., October 17	$21^{\text{h}} 17^{\text{m}} 32^{\text{s}}$	

EXAMPLE.—Given that the sidereal time at Dunedin (longitude $170^{\circ} 30' 54''$ E.) is $10^{\text{h}} 21^{\text{m}} 51.1^{\text{s}}$ on 1951 October 18, find the corresponding local standard time.

L.S.T. Dunedin October 18	.	.	.	10^{h}	21^{m}	51.1^{s}
Longitude Dunedin, E.	.	.	.	11^{h}	22^{m}	03.6^{s}
∴ Corresponding G.S.T. ($= \text{U.T.} + R$)				22^{h}	59^{m}	47.5^{s}
G.S.T. ($= \text{U.T.} + R$) at U.T. 18 ^h October 17	.	.	.	19^{h}	41^{m}	43.1^{s}
∴ instant is after U.T. 18 ^h October 17, by				3^{h}	18^{m}	04.4^{s}
Convert this into a mean solar time interval	.	.	.	3^{h}	17^{m}	32.0^{s}
Add to U.T. 18 ^h on October 17 to give U.T. of instant	.	.	.	21^{h}	17^{m}	32.0^{s}
Longitude of standard meridian, E.	.			12^{h}	00^{m}	00.0^{s}
∴ L.Std. T. required, October 18	.			9^{h}	17^{m}	32.0^{s}

NOTE.—It was necessary in the above example to take the G.S.T. at U.T. 18^h on October 17, and not the G.S.T. at U.T. 18^h on the 18th, because the latter would have given us an answer for October 19.

To find the L.M.T. of L.A.N.

The sun transits across the meridian when $\text{L.M.T.} = 24^{\text{h}} - E$.

Since the L.M.T. of L.A.N. is often only roughly required in order to ensure commencing, with a reasonable margin of time, a series of observations of the sun as it approaches and leaves the meridian, it is usually sufficient to take out E for U.T. 12^h on the day in question, and subtract this from 24^{h} to give the required approximate L.M.T. of L.A.N.

EXAMPLE.—Find the approximate L.M.T. of L.A.N. at a place in longitude 135° E., on 1951 July 2.

E at U.T. 12 ^h , July 2	.	.	.	11^{h}	56^{m}	14^{s}
$24^{\text{h}} - E =$ approximate L.M.T. required	.	.	.	12^{h}	03^{m}	46^{s}

If it were necessary to find the L.M.T. of L.A.N. more accurately, we could proceed as follows:—

At local mean noon, L.M.T. July 2	.	12 ^h	00 ^m	00 ^s
Longitude, E.	.	9 ^h	00 ^m	00 ^s
∴ Corresponding U.T. July 2	.	3 ^h	00 ^m	00 ^s
E at U.T. 0 ^h July 2	=	11 ^h	56 ^m	19.8 ^s
Change in E for 3 ^h , using 6 hourly difference of 2.9 ^s , (Table on p. 63 of <i>S.A.</i>) =				— 01.5 ^s
∴ E at U.T. 3 ^h , July				
2	=	11 ^h	56 ^m	18.3 ^s
∴ 24 ^h — (E. at U.T. 3 ^h , July 2) = L.M.T.				
of transit, i.e. 24 ^h — 11 ^h 56 ^m 18.3 ^s = L.M.T.				
of L.A.N.		12 ^h	03 ^m	41.7 ^s

Even this is not absolutely accurate, since we have worked from local mean noon instead of local apparent noon. However, since the six-hourly change in E is never more than about 7.5 seconds, an answer by the above method could not be in error by more than about 0.3 second.

Given the L.M.T. at any place, to find the corresponding L.A.T.

From the given L.M.T. find the corresponding U.T. by applying the longitude expressed in time. From the *Star Almanac*, find E for the nearest given U.T. below the U.T. just deduced. Interpolate, using the six-hourly difference given in the E table, and the interpolation table for the sun at the end of the *Almanac*, to find the value of E at the U.T. deduced from the given L.M.T. Then U.T. + E is equal to the Greenwich hour angle (G.H.A.) of the sun. (If the sum U.T. + E is greater than 24^h, we must subtract 24^h from it.) By adding 12^h to the G.H.A. of the sun we obtain the G.A.T. (again subtracting 24^h if the sum G.H.A. + 12^h is greater than 24^h). Then, by applying the longitude we can find the L.A.T.

EXAMPLE.—Given L.M.T. at a place in longitude 65° 43' 30'' E. is 20^h 22^m 54^s on 1951 July 1, find the corresponding L.A.T.

Given L.M.T., July 1	.	20 ^h	22 ^m	54 ^s
Longitude, E.	.	4 ^h	22 ^m	54 ^s
∴ Corresponding U.T., July 1	.	16 ^h	00 ^m	00 ^s

From *S.A.* :—

E at U.T. 12^h, July 1 . 11^h 56^m 25.7^s

Difference between *E* at

U.T. 12^h and *E* at U.T.

18^h, July 1 = — 3.0^s.

Entering table on p. 63 of

S.A. with difference of 3.0,

and interval of 4^h, we get

the difference for 4^h = — 02.0^s

∴ *E* at U.T. 16^h,

July 1 . . . = 11^h 56^m 23.7^s

Thus, G.H.A. sun (= U.T. + *E*) .

Subtract 24^h, G.H.A. . . .

Add 12^h to get G.A.T. . . .

Longitude, *E*

∴ Required L.A.T.

11^h 56^m 23.7^s

27^h 56^m 23.7^s

3^h 56^m 23.7^s

15^h 56^m 23.7^s

4^h 22^m 54.0^s

20^h 19^m 17.7^s

Given the L.A.T. at any place, to find the corresponding L.M.T.

From the given L.M.T. find the corresponding G.A.T. by applying the longitude expressed in time. From the G.A.T. (increased by 24^h if necessary) subtract 12^h to give the G.H.A. of the sun. Now, we know that this G.H.A. is equal to U.T. + *E*. If, therefore, we can find *E* we shall be able to deduce the U.T. and hence, by application of the longitude, the L.M.T.

Since *E* is always about 12^h in value, if the G.H.A. is less than *E* we must add 24^h to it. Next we look in the table for *E* on the date concerned for a value of *E* which, when added to the U.T. opposite it in the table, yields a sum under six hours less than the G.H.A. already found. This sum is then subtracted from the G.H.A. to give us a time interval, the tabular difference for *E* is found from the table, and with these two values the interpolation table for the sun is entered to obtain a correction for *E*. The *E* which we previously found from the table is now corrected by this amount, and the new value is subtracted from the G.H.A. to yield the U.T. Application of the longitude to this U.T. will give the required L.M.T.

NOTE.—The L.M.T. thus found is not, strictly speaking, quite correct, because we have been using an apparent time interval for interpolation in a table designed for U.T. However, the error will not be large enough to affect anything but the most refined work.

EXAMPLE.—Find the *L.M.T.* corresponding to *L.A.T.* $3^{\text{h}} 33^{\text{m}} 42^{\text{s}}$ on 1951 October 4, at a place in longitude $173^{\circ} 25' 30''$ E.

Given <i>L.A.T.</i> , October 4	.	.	.	3^{h}	33^{m}	42^{s}
Longitude, East	.	.	.	11^{h}	33^{m}	42^{s}
∴ Corresponding <i>G.A.T.</i> , October 3				16^{h}	00^{m}	00^{s}
Subtract 12^{h} to give <i>G.H.A.</i> sun	.	.	.	4^{h}	00^{m}	00^{s}
Increase by 24^{h} (because <i>G.H.A.</i> is less than <i>E</i>), and <i>U.T.</i> + <i>E</i>	.	.	.	28^{h}	00^{m}	00^{s}
At <i>U.T.</i> 12^{h} October 3,						
<i>U.T.</i> + <i>E</i>	.	.	=	24^{h}	10^{m}	44.5^{s}
Interval until 28^{h} (<i>U.T.</i> + <i>E</i> above)	.	.	=	3^{h}	49^{m}	15.5^{s}
Tabular difference for <i>E</i> = + 4.7^{s} . From table on p. 63 in <i>S.A.</i> , with difference = + 4.7^{s} and interval of $3^{\text{h}} 49^{\text{m}} 15.5^{\text{s}}$, correction for <i>E</i>	.	.	=			+ 03.1^{s}
But, from <i>S.A.</i> , <i>E.</i> at 12^{h} <i>U.T.</i>	.	.	=	12^{h}	10^{m}	44.5^{s}
∴ <i>E</i> corrected	.	.	=	12^{h}	10^{m}	47.6^{s}
Subtract from <i>U.T.</i> + <i>E</i> above, to give						
<i>U.T.</i> October 3	.	.	.	15^{h}	49^{m}	12.4^{s}
Longitude, East	.	.	.	11^{h}	33^{m}	42.0^{s}
Add, to get <i>L.M.T.</i> , October 4	.	.	.	3^{h}	22^{m}	54.4^{s}

NOTE.—The above answer differs by about 0.1 second from the answer obtained for the same problem worked previously by the more correct method.

EXAMPLE.—Find the *L.M.T.* corresponding to *L.A.T.* $14^{\text{h}} 25^{\text{m}} 18^{\text{s}}$ on 1951 March 15, at a place in longitude $8^{\text{h}} 03^{\text{m}} 21^{\text{s}}$ W.

Given <i>L.A.T.</i> , March 15	.	.	.	14^{h}	25^{m}	18^{s}
Longitude, west	.	.	.	8^{h}	03^{m}	21^{s}
∴ Corresponding <i>G.A.T.</i> , March 15	.	.	.	22^{h}	28^{m}	39^{s}
Subtract 12^{h} to obtain the <i>G.H.A.</i>	.	.	.	10^{h}	28^{m}	39^{s}

Add 24 ^h (because G.H.A. less than E),			
U.T. + E	.	.	34 ^h 28 ^m 39 ^s
At U.T. 18 ^h March 15,			
U.T. + E	.	=	29 ^h 50 ^m 52.4 ^s
Interval until 34 ^h 28 ^m			
39 ^s	.	=	4 ^h 37 ^m 46.6 ^s
Tabular difference for			
$E = + 4.3^s$.			
From table, <i>S.A.</i> p. 63,			
correction	.	=	03.4 ^s
But, from <i>S.A.</i> , E at			
U.T. 18 ^h	.	=	11 ^h 50 ^m 52.4 ^s
∴ E corrected	.	=	11 ^h 50 ^m 55.8 ^s
Subtract from U.T. + E above, and			
U.T.	.	=	22 ^h 37 ^m 43.2 ^s
Longitude, west	.	.	8 ^h 03 ^m 21.0 ^s
Subtract, to get L.M.T., March 15			
			14 ^h 34 ^m 22.2 ^s

With the aid of the *Star Almanac* the student should practise working out further problems similar to those above (they are easy enough to make up), checking the answers by re-working from the data supplied in the *Nautical Almanac*.

NOTE.—There are nautical almanacs other than that published by the Admiralty. The information about the sun in most of these is presented in a way similar to that in the *Star Almanac*, i.e. in terms of G.M.T. (U.T.), R and E . This type of presentation is very suitable when G.M.T. is always available, as on a ship, because the tables can be entered directly with the G.M.T. of any particular observation. It is not quite so convenient, however, as the student will have seen from the above examples, when the tables have to be entered with a G.A.T. The table for the sun at transit at Greenwich, in the *Nautical Almanac* (pp. 22-29), caters specially for this type of problem.

EXERCISES

1. Express in sidereal time the following intervals of mean solar time:—(1) 16^h 15^m 23^s, (2) 9^h 17^m 18.4^s and (3) 17^h 52^m 33.5^s.

Ans. (1) 16^h 18^m 03.2^s
 (2) 9^h 18^m 49.95^s
 (3) 17^h 55^m 29.69^s

2. Express in mean solar time the following intervals of sidereal time:—(1) $13^{\text{h}} 22^{\text{m}} 17^{\text{s}}$, (2) $21^{\text{h}} 35^{\text{m}} 15.5^{\text{s}}$, and (3) $8^{\text{h}} 55^{\text{m}} 39.7^{\text{s}}$.

Ans. (1) $13^{\text{h}} 20^{\text{m}} 05.56^{\text{s}}$

(2) $21^{\text{h}} 31^{\text{m}} 43.3^{\text{s}}$

(3) $8^{\text{h}} 54^{\text{m}} 11.94^{\text{s}}$

3. In longitude $148^{\circ} 15' \text{ E.}$, what is the L.M.T. corresponding to September 22, $4^{\text{h}} 30^{\text{m}}$ p.m. standard time of the 150th meridian east of Greenwich? Find also the corresponding G.M.T.

Ans. (1) $16^{\text{h}} 23^{\text{m}}$, or $4^{\text{h}} 23^{\text{m}}$ p.m.

(2) $6^{\text{h}} 30^{\text{m}}$

4. Convert Perth apparent time, December 3, $4^{\text{h}} 15^{\text{m}} 20.3^{\text{s}}$, to sidereal time; also Perth sidereal time, December 3, $20^{\text{h}} 26^{\text{m}} 15.7^{\text{s}}$, to Western Australian standard time (time of the 120th meridian East).

Given: Longitude of Perth . . . $7^{\text{h}} 43^{\text{m}} 21.7^{\text{s}}$ E.

G.S.T. at 0^{h} G.M.T. December 3 . . . $4^{\text{h}} 44^{\text{m}} 03.8^{\text{s}}$

G.S.T. at 0^{h} G.M.T. December 2 . . . $4^{\text{h}} 40^{\text{m}} 07.3^{\text{s}}$

Equation of time 0^{h} G.M.T. December 3, $10^{\text{m}} 35.4^{\text{s}}$ to be added to mean time.

Equation of time 0^{h} G.M.T. December 2, $10^{\text{m}} 58.3^{\text{s}}$ to be added to mean time.

Ans. Sidereal time— $8^{\text{h}} 48^{\text{m}} 09.3^{\text{s}}$

L.Std.Time— $15^{\text{h}} 57^{\text{m}} 31.8^{\text{s}}$

5. Given that the G.S.T. at 0^{h} G.M.T. is $14^{\text{h}} 40^{\text{m}} 40.14^{\text{s}}$, find the G.M.T. of the next transit of the first point of Aries.

Ans. $9^{\text{h}} 17^{\text{m}} 48.23^{\text{s}}$

6. Given that the G.M.T. of transit of the first point of Aries at Greenwich is $11^{\text{h}} 19^{\text{m}} 41.39^{\text{s}}$, compute the G.S.T. at 0^{h} G.M.T. on the same day.

Ans. $12^{\text{h}} 38^{\text{m}} 26.96^{\text{s}}$

7. The right ascension of a star being $20^{\text{h}} 24^{\text{m}} 13.72^{\text{s}}$, compute the L.M.T. of its culmination at Madras (longitude $80^{\circ} 14' 19.5'' \text{ E.}$) on September 6, the G.S.T. at 0^{h} G.M.T. on that date being $22^{\text{h}} 57^{\text{m}} 06.95^{\text{s}}$.

Ans. $21^{\text{h}} 24^{\text{m}} 28.48^{\text{s}}$

8. Convert $22^{\text{h}} 22^{\text{m}} 44.58^{\text{s}}$ sidereal time at Greenwich, 1951 January 20, into mean time, given that the mean time of transit of the first point of Aries on January 20 is $16^{\text{h}} 03^{\text{m}} 06.18^{\text{s}}$.

Ans. $14^{\text{h}} 26^{\text{m}} 06.69^{\text{s}}$

9. Find the L.M.T. corresponding to $5^{\text{h}} 17^{\text{m}} 32^{\text{s}}$ sidereal time at Moscow (longitude $37^{\circ} 34' 15''$ E.) given that the G.S.T. at 0^{h} G.M.T. on the same day was $23^{\text{h}} 54^{\text{m}} 52^{\text{s}}$.

Ans. $5^{\text{h}} 22^{\text{m}} 11.76^{\text{s}}$

10. Find the standard time of culmination of α Centauri at Adelaide on 1951 June 1. R.A. = $14^{\text{h}} 36^{\text{m}} 18.68^{\text{s}}$; longitude = $9^{\text{h}} 14^{\text{m}} 20.3^{\text{s}}$ E. Standard meridian is $9^{\text{h}} 30^{\text{m}}$ E. G.S.T. at 0^{h} G.M.T. on the same date = $16^{\text{h}} 34^{\text{m}} 40.92^{\text{s}}$.

Ans. $22^{\text{h}} 15^{\text{m}} 11.76^{\text{s}}$, or $10^{\text{h}} 15^{\text{m}} 11.76^{\text{s}}$ p.m.

11. Find the L.M.T. of transit of δ Crucis over the meridian at a place in longitude $11^{\text{h}} 30^{\text{m}}$ E. on 1951 May 10. Transit of the first point of Aries at Greenwich, $8^{\text{h}} 54^{\text{m}} 32.09^{\text{s}}$ G.M.T. May 9; star's R.A. $12^{\text{h}} 12^{\text{m}} 34^{\text{s}}$.

Ans. $21^{\text{h}} 03^{\text{m}} 03.21^{\text{s}}$

12. The G.M.T. of transit of the first point of Aries for 1951 January 21, is given in the *Nautical Almanac* as $15^{\text{h}} 59^{\text{m}} 10.26^{\text{s}}$. For the same date the R.A. of α Leonis is $10^{\text{h}} 05^{\text{m}} 47.51^{\text{s}}$. Find the exact L.M.T. when α Leonis passed the meridian of a place in longitude 135° E.

Ans. $2^{\text{h}} 04^{\text{m}} 47^{\text{s}}$, January 22

13. Compute the L.S.T. at noon by standard time at Dunedin, New Zealand, on 1951 October 24, given—

Longitude of Dunedin	.	.	.	$11^{\text{h}} 22^{\text{m}} 03.6^{\text{s}}$ E.
Longitude of standard meridian	.	.	.	$12^{\text{h}} 00^{\text{m}} 00^{\text{s}}$ E.
G.S.T. at 0^{h} G.M.T. October 23	.	.	.	$2^{\text{h}} 02^{\text{m}} 25.01^{\text{s}}$

Ans. $13^{\text{h}} 28^{\text{m}} 25.17^{\text{s}}$

14. In the forenoon of 1951 August 1, at Melbourne, longitude $9^{\text{h}} 39^{\text{m}} 54^{\text{s}}$ E., a mean time chronometer was compared with a sidereal clock known to be 14.6 seconds fast on true local sidereal time. It was found—

Time by sidereal clock	.	.	.	$8^{\text{h}} 18^{\text{m}} 09.00^{\text{s}}$
Time by chronometer	.	.	.	$11^{\text{h}} 41^{\text{m}} 34.32^{\text{s}}$

The data in the appended table are taken from the *N.A.*:—

Sun at 0^h G.M.T.

Date (1951)	Apparent R.A. of Sun h m s	Equation of time (Apparent—Mean) m s s
July 31	8 37 34.35 233.99	— 6 19.93 +2.56
Aug. 1	8 41 28.34 233.40	— 6 17.37 +3.16
Aug. 2	8 45 21.74	— 6 14.21

Determine (a) the sidereal time at 0^h G.M.T., August 1.

(b) the R.A. of the sun at Greenwich Apparent Noon, August 1.

(c) the error of the mean time chronometer on Victorian standard time (meridian 10^h E.).Ans. (a) 20^h 35^m 10.97^s(b) 8^h 43^m 26.1^s(c) 0^h 20^m 54.99^s slow

15. Find the L.S.T. corresponding to 8^h 35^m 42^s a.m. local standard time at Sydney (longitude 10^h 04^m 49.2^s E.), on 1951, September 3, given the following information from the *Star Almanac*:—

Sun—September 1951

U.T.	h	R	h	m	s
2	0	22	41	20.7	
Sun.	6	22	42	19.9	
	12	22	43	19.0	
	18	22	44	18.2	

From the “Interpolation Table for *R*”, p. 64, *S.A.*, the difference for 4^h 35^m 42^s = 45.3^s.

Standard meridian is 10^h E. of Greenwich.Ans. 7^h 25^m 34.7^s

16. Given that R at U.T. 0^h on 1951 June 15, is $17^h 29^m 52.7^s$, find the U.T. of the next transit of the first point of Aries.

Ans. $6^h 29^m 03.4^s$

17. Given that the L.S.T. at Santiago, Chile (longitude $70^\circ 41' 16.5''$ W.), on 1951 April 22, is $16^h 12^m 16.8^s$, find the L.M.T. The following information is available from the *Star Almanac* :—

Sun—April 1951

U.T. h		R h m s		
22 Sun.	0	13	56	58.6
	6	13	57	57.8
	12	13	58	56.9
	18	13	59	56.1

Ans. $2^h 14^m 09.7^s$

18. The local standard time at a place in longitude $64^\circ 22'$ W. on 1951 August 25, was $10^h 04^m 03^s$. Find the corresponding L.A.T. The standard meridian of the place is $4^h 30^m$ W.

From the *Star Almanac* :

Sun—August 1951

U.T. h		E h m s		
25 Sat.	0	11	57	39.0
	6	11	57	43.0
	12	11	57	47.0
	18	11	57	51.1

Ans. $10^h 14^m 23.7^s$

19. The L.A.T. at a place in longitude $2^h 16^m 48^s$ E. on 1951 November 16, was $8^h 41^m 16.3^s$. Find the L.M.T.

From the *Star Almanac* :

Sun—November 1951

U.T.	h	E		
		h	m	s
16 Fri.	0	12	15	23·1
	6	12	15	20·6
	12	12	15	18·0
	18	12	15	15·3
	24	12	15	12·7

Ans. $8^{\text{h}} 25^{\text{m}} 55\cdot8^{\text{s}}$

THE LOCATION OF OBJECTS ON THE CELESTIAL SPHERE

IN order that the surveyor may pick out and observe a particular star with a theodolite, it is frequently necessary, more especially when he wishes to make the observation in daylight or evening twilight, that he should know the altitude and azimuth of the star at the given time. From the *Nautical Almanac* he obtains its right ascension and declination, and from these data he has to compute altitude and azimuth. In this chapter we will deal with this problem and show how, given the position of a star in one system of coordinates we may determine its coordinates in another.

A. Knowing the latitude, longitude and time at the place of observation and the right ascension and declination of a particular star, it is required to determine its altitude and azimuth

In Fig. 16, let P be the pole, S the star, Z the zenith, $A Z P B$ the plane of the meridian.

Draw the great circle through Z and S to intersect the horizon in H .

If we know the local mean time we can compute the corresponding sidereal time by the methods of the last chapter. But we have seen that the right ascension of the star is the same thing as the sidereal time at the moment of the star's transit across the meridian. Consequently the difference between the sidereal time at the instant of observation and the right ascension of the star gives the interval in sidereal time between the moments of the star's transit across the meridian and of observation—that is to say, it gives, when turned into degrees, minutes and seconds, the hour angle of the star $S P Z$. If the sidereal time at the moment of observation is less than the right ascension of the star, the difference measures the angle $S P Z$ towards the east of the meridian, if the right ascension is the less, the angle is measured toward the west.

Thus, in the spherical triangle $Z S P$, we know $Z P$, the complement of the latitude, and $S P$, the polar distance of the star which is the complement of the declination, and the included angle $Z P S$.

From these data we can compute the third side $Z S$, which is

Thus, sidereal interval after 0 ^h G.M.T.			
July 19	=	7 ^h 31 ^m 13.923 ^s	
But, from <i>N.A.</i> , G.S.T. at 0 ^h G.M.T.			
July 19	=	19 ^h 43 ^m 55.741 ^s	
Thus, G.S.T. at given instant of			
L.Std.T.	=	27 ^h 15 ^m 09.664 ^s	
Reduce by 24 ^h	=	3 ^h 15 ^m 09.7 ^s	
Longitude of place, E.	=	11 ^h 22 ^m 00.0 ^s	
Thus, L.S.T. required			
	=	14 ^h 37 ^m 09.7 ^s	
Using the <i>Star Almanac</i> —			
Given L.Std.T., July 19		19 ^h 30 ^m 00.0 ^s	
Longitude of standard meridian, E.		12 ^h 00 ^m 00.0 ^s	
Corresponding U.T., July 19			
		7 ^h 30 ^m 00.0 ^s	
R at U.T. 6 ^h July 19,			
from <i>S.A.</i>	=	19 ^h 44 ^m 54.9 ^s	
Increase in <i>R</i> in 1 ^h 30 ^m			
(p. 64, <i>S.A.</i>)	=	14.8 ^s	
Thus, <i>R</i> at U.T. 7 ^h 30 ^m ,			
July 19		19 ^h 45 ^m 09.7 ^s	
∴ G.S.T. (= G.H.A. Aries, = U.T. + <i>R</i>)			
		27 ^h 15 ^m 09.7 ^s	
Reduce by 24 ^h		3 ^h 15 ^m 09.7 ^s	
Longitude, E		11 ^h 22 ^m 00.0 ^s	
∴ Local sidereal time required			
		14 ^h 37 ^m 09.7 ^s	

We have already seen that the R.A. of a celestial body added to its hour angle gives the local sidereal time at the instant under consideration. If, therefore, we subtract the R.A. of Antares from the L.S.T. we have just computed, we shall obtain the hour angle of the star at 7.30 p.m. L.Std.T.

Computed L.S.T.	14 ^h 37 ^m 09.7 ^s	
R.A. of Antares	16 ^h 26 ^m 27.3 ^s	

Hour angle of Antares	22 ^h 10 ^m 42.4 ^s	
---------------------------------	---	--

This means that, at 7.30 p.m. L.Std.T. July 19, 22^h 10^m 42.4^s sidereal time has elapsed since Antares was last on the local meridian at upper transit. Expressing this another way, we can say that, at 7.30 p.m. L.Std.T., it will be (24^h - 22^h 10^m 42.4^s) = 1^h 49^m

17.6° sidereal interval before Antares comes on to the meridian at upper transit. In other words, the *east* hour angle of Antares at 7.30 p.m. L.Std.T. is 1^h 49^m 17.5^s, or converting to arc by multiplying by 15, the east hour angle = 27° 19' 24".

Referring now to Fig. 16, we have

$$\begin{aligned} \text{ZP} &= \text{co-latitude} = 44^\circ 08' 00'' \\ \text{PS} &= \text{south polar distance } (90^\circ - \text{declination}) \\ &= 63^\circ 40' 13.7'' \\ \text{P} &= \text{east hour angle} = 27^\circ 19' 24'' \\ \cos \frac{1}{2} (\text{SP} - \text{ZP}) &= \cos 9^\circ 46' 06.8''; \log = 9.9936572 \\ \cot \frac{1}{2} \text{P} &= \cot 13^\circ 39' 42''; \log = 10.6142775 \end{aligned}$$

$$\begin{aligned} &\log \text{product} = 10.6079347 \\ \cos \frac{1}{2} (\text{SP} + \text{ZP}) &= \cos 53^\circ 54' 06.8''; \log = 9.7702404 \end{aligned}$$

$$\begin{aligned} &\log \tan \frac{1}{2} (\text{Z} + \text{S}) = 10.8376943 \\ &\frac{1}{2} (\text{Z} + \text{S}) = 81^\circ 43' 55.3'' \\ \sin \frac{1}{2} (\text{SP} - \text{ZP}) &= \sin 9^\circ 46' 06.8''; \log = 9.2296017 \\ \cot \frac{1}{2} \text{P} &= \cot 13^\circ 39' 42''; \log = 10.6142775 \end{aligned}$$

$$\begin{aligned} &\log \text{product} = 9.8438792 \\ \sin \frac{1}{2} (\text{SP} + \text{ZP}) &= \sin 53^\circ 54' 06.8''; \log = 9.9074163 \end{aligned}$$

$$\begin{aligned} &\log \tan \frac{1}{2} (\text{Z} - \text{S}) = 9.9364629 \\ &\frac{1}{2} (\text{Z} - \text{S}) = 40^\circ 49' 25.3'' \\ \text{Thus } \frac{1}{2} (\text{Z} + \text{S}) &= 81^\circ 43' 55.3'' \\ \frac{1}{2} (\text{Z} - \text{S}) &= 40^\circ 49' 25.3'' \end{aligned}$$

$$\begin{aligned} \text{Sum} = \text{Z} &= 122^\circ 33' 20.6'' \\ (\text{Difference} = \text{S} &= 40^\circ 54' 30.0'') \end{aligned}$$

Thus the star lies in the direction 122° 33' 20.6" east of south or on a true bearing of 57° 26' 39.4".

To find its altitude,

$$\begin{aligned} \sin \text{P} &= \sin 27^\circ 19' 24''; \log = 9.6618235 \\ \sin \text{SP} &= \sin 63^\circ 40' 13.7''; \log = 9.9524330 \end{aligned}$$

$$\begin{aligned} &\log \text{product} = 9.6142565 \\ \sin \text{Z} &= \sin 122^\circ 33' 20.6''; \log = 9.9257598 \end{aligned}$$

$$\begin{aligned} &\log \sin \text{SZ} = 9.6884967 \\ \text{Thus SZ (zenith distance)} &= 29^\circ 12' 53.6'' \end{aligned}$$

Therefore, the *true* altitude of the star is the complement of this, or $60^{\circ} 47' 06.4''$.

Very commonly for such calculations it is sufficient to compute the position to the nearest minute, and in that case five-figure logarithms are sufficient, unless the cosines or secants of angles less than 5° are involved, or the sines or cosecants of angles within 5° of 90° . With this limitation five figure logarithms give all the accuracy required for most work with ordinary theodolites, and considerable time is saved by their use.

B. Having observed the altitude and azimuth of a star, the time of observation being noted, it is required to determine its right ascension and declination

The latitude and longitude of the place of observation are supposed known.

Then in the figure, Z being the zenith point, P the pole, and S the star, as before :—

In the spherical triangle ZSP, ZP is known, being the co-latitude ; ZS, the zenith distance, is also known, and the angle SZP, which the vertical plane passing through the star makes with the meridian.

Thus we know two sides and the included angle, and the triangle may be solved to find SP and the angle SPZ.

The formulæ to be used are those of the preceding problem.

$$\tan \frac{1}{2} (S + P) = \frac{\cos \frac{1}{2} (Z P - Z S)}{\cos \frac{1}{2} (Z P + Z S)} \cot \frac{1}{2} Z,$$

$$\tan \frac{1}{2} (S + P) = \frac{\sin \frac{1}{2} (Z P - Z S)}{\sin \frac{1}{2} (Z P + Z S)} \cot \frac{1}{2} Z,$$

$$\sin S P = \frac{\sin Z \cdot \sin Z S}{\sin P}.$$

The angle S P Z, being turned into hours, minutes, and seconds, at the rate of 15° for one hour, measures the sidereal time that will elapse before S comes to the meridian if S is to the east, or the interval of sidereal time since S was on the meridian if it is to the west.

But the right ascension of the star is the sidereal time when it is on the meridian.

Therefore, to obtain the right ascension of the star, add the time value of the angle S P Z to the local sidereal time at the moment of

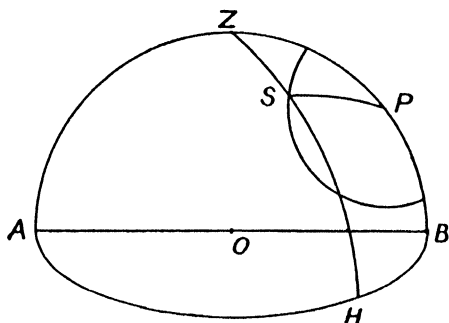


Fig. 17

observation if the star is to the East of the meridian, and subtract it if the star is to the West.

The declination of the star is, of course, the complement of the computed polar distance SP .

C. Having computed the altitude and azimuth of a star for a given time of observation, it is required to determine its approximate position at some short interval of time afterwards

When a surveyor is preparing for daylight observations of a star, it will be generally necessary for him to take at least two readings of its position. To give him time to read the verniers and reverse the instrument before taking the second observation, he requires to know the altitude and azimuth of the star at an interval of five or ten minutes after the first reading.

The computation for the second position may, of course, be made by precisely the same method that we have already used for the first, in which case several of the logarithms already taken out will be useful for the calculation.

But it is rather shorter to make use of the following formulæ :—

If x denotes the slight *increase* in the hour angle SPZ (to be reckoned negative if the angle is decreasing), y and z the corresponding small increases in the zenith distance ZS , and the azimuth angle PZS respectively. Then

$$y = \sin PS \sin PSZ \cdot x \quad . \quad . \quad . \quad (1)$$

$$z = - \frac{\cot PSZ \cdot y}{\sin ZS} \quad . \quad . \quad . \quad (2)$$

The values of $P S Z$ and $Z S$ to be used in the equations are those found in the first calculation.

To establish the formulæ, let $A B C$ (Fig. 18) be a spherical triangle. Then if b and c remain unchanged, we require to find the small changes y and z in a and B respectively if the angle A is increased by a small amount x .

By the ordinary formulæ for spherical triangles we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\text{and } \cos (a + y) = \cos b \cos c + \sin b \sin c \cos (A + x)$$

Subtracting gives

$$\begin{aligned} \cos a \cos y - \sin a \sin y - \cos a \\ = \sin b \sin c (\cos A \cos x - \sin A \sin x - \cos A). \end{aligned}$$

Now, if x and y are very small, we can, if they are measured in circular measure, replace $\sin x$ and $\sin y$ by x and y respectively, and put $\cos x$, $\cos y$ each equal to unity. Doing this, we get

$$-y \sin a = -\sin b \sin c \sin A \cdot x.$$

Putting $\sin c \sin A = \sin C \sin a$, this becomes

$$y = \sin b \sin C \cdot x,$$

which is the first formula given.

Since we have here simply the ratio of y to x , the result will hold good in whatever system of measurement y and x are expressed, provided they are both measured in the same system, both in degrees or both in circular measure.

Further, by the law of sines,

$$\frac{\sin (B + z)}{\sin b} = \frac{\sin (A + x)}{\sin (a + y)}.$$

Expanding and substituting as before, we get

$$\begin{aligned} (\sin B + z \cos B) (\sin a + y \cos a) &= \sin b (\sin A + x \cos A) \\ \text{and } \sin B \cdot \sin a &= \sin b \cdot \sin A. \end{aligned}$$

$\therefore$ subtracting, and neglecting the product of two small quantities y and z ,

$$z \sin a \cos B + y \cos a \sin B = x \sin b \cos A.$$

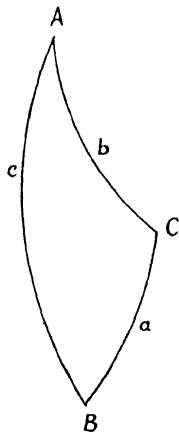


Fig. 18

Putting

$$x = \frac{y}{\sin b \sin C}$$

$$z \sin a \cos B = y \left(\frac{\cos A}{\sin C} - \sin B \cos a \right) = -y \frac{\cos B \cos C}{\sin C}.$$

$$\therefore z = -y \cdot \frac{\cot C}{\sin a}, \text{ which is the second formula.}$$

To illustrate the application of the formulæ we will extend the scope of the example already worked out in Section A of this chapter, and compute the position of Antares 5 sidereal minutes after 7.30 p.m. L.Std.T.

From the previous work, the angle $PSZ = 40^\circ 54' 30''$.
 $PS = 63^\circ 40'$, $ZS = 29^\circ 13'$.

$$\log \sin PS = 9.95242$$

$$\log \sin PSZ = 9.81614$$

$$\log \text{product} = 9.76856$$

$$\text{Product} = 0.58690$$

In this example the hour angle of the star is measured to the east; therefore, x is negative, and -5 minutes of time $= -1^\circ 15'$ of arc.

$$\therefore y = -0.58690 \times 75' = -44'$$

$\therefore$ The new zenith distance $= 29^\circ 13' - 44' = 28^\circ 29'$ and the new altitude $= 90^\circ - 28^\circ 29' = 61^\circ 31'$.

$$\log \cot PSZ = 10.06224$$

$$\log \sin ZS = 9.68852$$

$$\log \text{quotient} = 10.37372$$

$$\text{quotient} = 2.3644$$

and $\cot PSZ$ is positive, $\therefore z = +2.3644 \times \{ -(-44') \}$

$$\therefore z = +104'$$

$\therefore$ The new azimuth is $122^\circ 33' + 104' = 124^\circ 17'$ east of south, or on a true bearing of $55^\circ 43'$.

If results are required only to the nearest minute, the above method is quite sufficient, provided the small differences are not much more than 2 degrees of arc.

With such observations much better results may be obtained if, instead of taking two observations only, the surveyor arranges to make several determinations of the star's position, perhaps eight,

lasting over a period of twenty minutes, half being taken with the instrument reversed in altitude and azimuth. In this case the simplest procedure is to make separate calculations for the altitude and azimuth for the first and last observations at predetermined times, and then the positions for the intermediate observations may be obtained with sufficient precision by simple linear interpolation.

CHAPTER 7

ASTRONOMICAL AND INSTRUMENTAL CORRECTIONS TO OBSERVATIONS OF ALTITUDE AND AZIMUTH

Parallax

THE fixed stars are so distant from us that their directions always appear to be the same, no matter from what point upon the earth's surface they are observed. Even with our most refined instruments no difference can be detected, because their distance is practically infinitely great in comparison with the diameter of the earth. But with the members of our own system, the sun, the moon, and the planets, we are dealing with bodies incomparably nearer to us, and their relative positions amongst the fixed stars of the sky are not precisely the same when viewed from different places. It is, therefore, essential that their registered right ascensions and declinations should be referred to some definite point upon the earth, in order that they may be available to all observers. The point selected is the earth's centre, because, having observed the direction of a planet from any station on the earth's surface, it is an easy matter to deduce its position as it would appear at the earth's centre, and conversely if the position of the object is tabulated as it would be seen from the centre of the earth we may readily find its position as seen from any place on the earth's surface. The selection of the earth's centre as the imaginary place of observation greatly simplifies the computations, and consequently most astronomical observations of bodies in our own solar system are reduced so as to show what the result would be if the observation could have been at the centre of the earth. The registered right ascensions and declinations of the *Nautical Almanac* are those the different bodies would have if viewed from the earth's centre.

The difference between the direction of a heavenly body as seen from the earth's centre and as seen from the place of observation is known as its *Parallax*.

Thus, as in Fig. 19, if S is the sun or planet observed, A the point of observation, and O the earth's centre, the parallax of the body is the angle A S O, the difference in the directions of A S and O S.

If AH_1 is the direction of the horizontal at A , the altitude of S is the angle SAH_1 . If OH_2 is drawn parallel to AH_1 , then the difference of the angles SOH_2 and SAH_1 = the difference of the

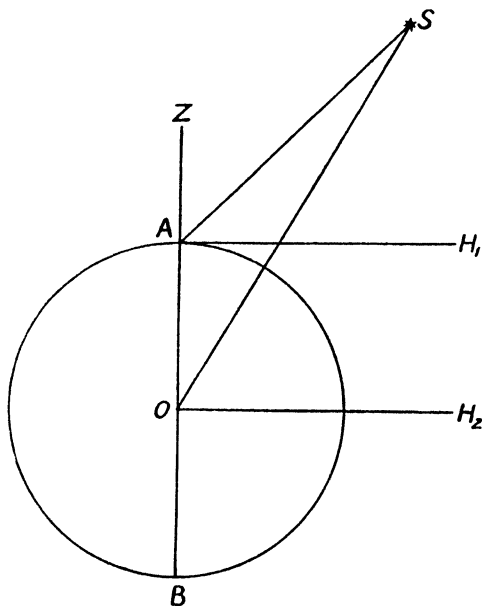


Fig. 19

angles SOB and SAB which = the angle ASO . Thus, if we call p the parallax, $p = \text{angle } ASO = SOH_2 - SAH_1$. Clearly the angle SOH_2 is always greater than the angle SAH_1 .

If z denotes the zenith distance of S as observed from A , r the earth's radius OA , and d the distance OS , then,

$$\text{From the triangle } AOS, \frac{\sin p}{\sin z} = \frac{r}{d}.$$

If the body is observed on the horizon—that is to say, if $z = 90^\circ$ —the corresponding value of p is called the *horizontal parallax*. Call this P .

$$\text{Then} \quad \sin P = \frac{r}{d}.$$

$$\text{Therefore,} \quad \sin p = \sin P \cdot \sin z.$$

Since p and P are very small; except in the case of the moon, whose parallax sometimes exceeds 1° , we may substitute the angles for their sines and write

$$p = P \sin z.$$

The horizontal parallax of the moon and principal planets is given in the *Nautical Almanac* for every day in the year, and that of the sun at intervals of 10 days. The parallax for any other altitude is given by the above simple formula.

Parallax is greatest when the body is on the horizon, and diminishes with the altitude until it becomes nothing when the body is in the zenith.

We see from Fig. 19 that the effect of the parallax upon a celestial object is to make its altitude appear less when observed from A than it would be if seen from O. Consequently, when reducing observations to the earth's centre, we must *add* the correction for parallax to the observed altitude, or

True altitude = observed altitude + parallax.

Parallax has no effect upon the azimuth of an object in the sky ; the correction is made to altitude only.

This statement is strictly correct only when the earth is regarded as a perfect sphere. If the spheroidal form of the earth is taken into account there will be parallax in azimuth as well as in altitude. Even then, however, the correction in azimuth is too small to be worth considering except in the case of certain special lunar observations.

The horizontal parallax of the sun ranges between 8.65 and 8.95 seconds. At an altitude of 60° its parallax is reduced to half of this.

A table giving the sun's parallax for various altitudes appears in *Chambers's Mathematical Tables*.

ATMOSPHERIC REFRACTION

When a ray of light passes from one medium into a denser medium, as from air into water or from air into glass, it is bent or *refracted* towards the normal to the bounding surface. Thus, as in Fig. 20, if a ray of light passes from the medium A to a denser medium B, traversing the path P Q R, the refracted ray Q R will always make a smaller angle with the normal to the separating surface than the incident ray P Q. The direction of bending is always such that the bent or refracted ray lies in the same plane as that passing through the incident ray P Q and the normal Q N. The

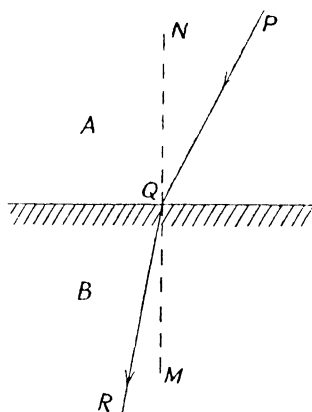


Fig. 20

law governing the amount of bending is that the ratio between the sines of the angles PQN and RQM is constant for these particular media and the value of this ratio is known as the *coefficient of refraction*.

Similarly, when a ray of light from a celestial body reaches the atmosphere surrounding the earth, it is bent slightly out of its original path. If the atmosphere were a uniform homogeneous medium with a definite upper surface it would be comparatively

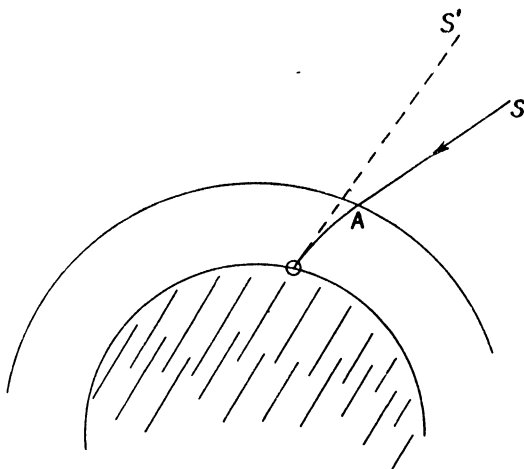


Fig. 21

easy to determine the precise amount of bending of the ray. But the density of the atmospheric air diminishes with the height above the earth's surface. Consequently a ray from a star S (Fig. 21), when it reaches the upper limit of the earth's atmosphere at A, is only very slightly bent, but the amount of bending gradually increases as it passes into the lower and denser layers of air. Its path from A to an observer on the earth's surface at O is thus a curve, and the ray ultimately reaches the observer so that it appears to him to come in the direction of OS' . Thus, the observer sees a star apparently at S' in the celestial sphere, whereas in reality the star is at S. The effect is that the star is apparently raised above its true position, and its apparent altitude is greater than the true altitude if it could be observed from O with no intervening atmosphere. The observed altitude of a celestial body must, therefore, be corrected in order to deduce its true altitude, the correction being always *subtracted* from the observed altitude. The amount of

bending of the ray varies somewhat with the pressure and temperature of the air, but it is greatest for stars on the horizon, and gradually decreases to nothing for a star in the zenith. For a body on the horizon the mean value of the correction is 33'—that is to say, a star will be just visible on the horizon when it is really 33' below it. Thus the sun, whose diameter is about 32', is visible just above the horizon when it is in reality just below it.

It will be seen from the figure, since the refracted ray always lies in the plane containing the incident ray of S A, and the normal to the spherical bounding surface at A, that S and S' will lie in the same plane as the vertical at O. This means that refraction produces its effect entirely in altitude, and has no influence upon the apparent azimuth of a heavenly body. Thus no correction in azimuth is necessary on account of refraction.

As we do not know the exact laws which govern the pressure and temperature of the earth's atmosphere at different heights, nor even the distance to which it extends around the earth, no satisfactory computation of the amount of refraction at different altitudes can be made from theoretical considerations alone. By making different assumptions as to the character of the earth's atmosphere various formulæ have been derived, but as their demonstration generally requires mathematics of a rather advanced character, we shall not attempt the problem here. In any case, as we cannot be sure of the correctness of the assumptions that have to be made in order to derive the formula, the values of the constants used have to be obtained and checked from actual observations. There are various ways by which the amount of refraction at different altitudes may be actually measured, and for practical purposes that formula is selected which best fits the results of such measurements.

The formula that has found most favour, and which has been most used by astronomers for this purpose, is that of Bessel,

$$r = A (B t)^M T^N \cot a,$$

where a = the apparent altitude,

r = the amount of refraction in seconds of arc,

B , a factor depending on the height of the barometer,

t , a factor depending on the reading of the thermometer attached to the barometer,

T , a factor depending on the reading of a thermometer so exposed as to give the temperature of the external air.

A, M, and N are factors depending on the altitude of the celestial body.

When suitable values are given to the different factors, this formula can be made to fit in with the results of actual observations on refraction with great precision, and where great accuracy is required this is the formula that is most generally adopted. To use the formula it is, of course, put into the logarithmic form—

$$\log r = \log A + M (\log B + \log t) + N \log T + \log \cot a,$$

and the values of M, N, $\log A$, $\log B$, $\log t$, and $\log T$ are obtained from appropriate tables. Such a table is published in *Chambers's Mathematical Tables*.

The constants M and N in the above formula do not differ sensibly from unity if the altitude is considerable. If these are taken each = 1, the formula may be put into a form which makes the application of tables much simpler. For the values of B, t , and T are each unity for certain particular values of the barometric height, and for certain special temperatures of the attached and unattached thermometers. Consequently for this particular condition of the atmosphere, which we may take as the standard condition, we have $r = A \cot a$.

If now we denote by r_1 the amount of the refraction for any other temperature and pressure, we have—

$$\begin{aligned} r_1 &= A \cdot B \cdot t \cdot T \cot a, \\ \therefore r_1 &= B \times t \times T \times r, \end{aligned}$$

or refraction = the refraction for altitude a under the standard or mean conditions multiplied by the factors B, t , and T, depending on the height of the barometer and the temperatures recorded by the attached and unattached thermometers.

A table of refractions constructed for standard conditions of the atmosphere is commonly termed a table of *mean* refraction. With the aid of such a table and subsidiary tables for B, t , and T, we may first of all find the value of the “mean refraction” for the measured altitude, then select the values of B, t , and T for the particular conditions of the atmosphere, and the true refraction = the mean refraction multiplied by $B \times t \times T$.

This is the method of determining the refraction most commonly adopted for ordinary purposes, and it gives accurate enough results unless the altitude is very small. The necessary tables are in *Chambers's Mathematical Tables*.

For many purposes, and more especially for high altitudes, it is quite sufficiently accurate to use the value of the refraction as given

in the mean refraction table. The refraction is always less than $1'$ if the altitude is greater than 45° , and for zenith distances up to 20° the refraction is practically $1''$ per 1° .

A very useful critical table of refractions is given in the *Star Almanac for Land Surveyors*; an explanation of how it is used is given in the introduction to the *Almanac*.

Corrections to Observations on Account of Residual Instrumental Errors

It forms no part of the purpose of this book to enter upon a discussion of the construction of the ordinary instruments of the surveyor and the methods of adjustment. These are matters dealt with in text-books on surveying. It will be assumed that the reader is acquainted with the construction of the surveyor's transit theodolite and with the usual methods of securing its accurate adjustment. But even when the adjustments have been made with great care, there commonly remain certain residual errors which affect the accuracy of the celestial observations, and these must be taken into account if the best results are to be obtained. The two most important are, (1) an error due to the fact that the line of collimation of the telescope is not accurately at right-angles to the transverse axis about which the telescope turns, and (2) an error produced if this transverse axis is not absolutely horizontal. We will consider the effect of each of these in turn.

The effect of an error of collimation

Let us suppose that the line of collimation of the telescope, instead of being accurately at right-angles to the axis about which the telescope turns, is in error by a small angle ϵ ; that is to say, the telescope makes an angle $90^\circ - \epsilon$ on one side and $90^\circ + \epsilon$ on the other side with the axis. On turning the telescope about the transverse axis, which is adjusted so as to be horizontal, the line of collimation would, if in accurate adjustment, trace out a vertical plane passing through the zenith. But if in error, and the line of collimation is not at right-angles to the axis, then, as it is plunged up and down, it will trace out a conical surface and on the celestial sphere it will trace out a circle parallel to a vertical circle through the zenith. Thus, as in Fig. 22, if there were no collimation error the collimation line of the telescope would trace out the great circle $ZS'N$, but if in error it will sweep out the parallel small circle LSM . Now, suppose that the star S is observed in such a telescope, and let SS' be an arc of a great circle drawn at right-angles to ZN . $SS' = NM = \epsilon$ the collimation error.

If we draw the great circle arc ZS , then ZS is the true zenith distance of the star. But the observed zenith distance is ZS' . Similarly the correct azimuth is measured by the angle HZS , whereas the azimuth as read on the instruments is HZS' .

In the right-angled triangle $SS'Z$, SS' being denoted by c , we have

$$\cos SZ = \cos S'Z \cos c.$$

If c is very small, as should be the case if the instrument is in good adjustment, we may take $\cos c = 1$, and, therefore, practically $S'Z = SZ$, or no correction will usually be necessary to the observed zenith distance or altitude.

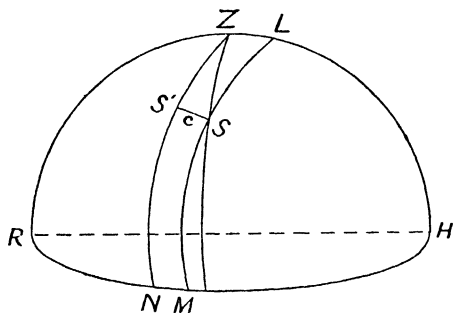


Fig. 22

This will not hold good if the observation is made on a star so close to the zenith that SZ and $S'Z$ are small and comparable with c . In that case $SZ^2 = S'Z^2 + c^2$.

Also, denoting by Z the angle $SS'Z$, the error in azimuth, we have

$$\sin c = \sin SZ \cdot \sin Z,$$

and since c and Z are both small, we may write

$$Z = c \cdot \operatorname{cosec} SZ,$$

or the error in azimuth = the collimation error multiplied by the cosecant of the zenith distance.

The error in azimuth thus becomes very great if the star is near the zenith, but is $= c$ for a star on the horizon. If an observation be made to measure the horizontal angle between a star and some referring object, pointing the telescope to each in succession, then this error occurs at each observation. If both star and referring object are at the same vertical altitude, the measure of the horizontal angle between the two is not affected by the error. But if, as will be

usual, the two are at different altitudes, then the error at each observation must be taken into account.

The following table shows the way in which the error varies with the altitude of the star :—

*Error in Azimuth corresponding to a Collimation
Error c for Various Altitudes of Object*

Altitude of Star . . .	0°	30°	60°	70°	80°	85°	89°
Error in azimuth . . .	c	$1.15c$	$2c$	$2.92c$	$5.76c$	$11.47c$	$57.3c$

The elimination of instrumental errors by changing face

Although we have in the preceding paragraph investigated the effect of a given collimation error, it is very seldom that the surveyor will need to take this error into account, because in all important work the observations are taken in such a way as to eliminate its effects. This is done by observing each angle twice, with the vertical circle or face alternately to the left and to the right. After the angle has been read once the telescope is reversed in direction by turning about its horizontal axis, and the whole of the upper part of the theodolite is turned through 180° until the first object is again sighted, and the angle is again read with the instrument in this reversed position. The operation is commonly referred to as “changing face,” and should be adopted in all theodolite observations, as it gives a means both effectual and simple of eliminating the chief instrumental errors. An error in collimation will not affect the horizontal angle between two objects if both are at the same altitude, but if the altitudes are different, then if the collimation error makes the measured angle a little too great when the vertical circle is facing the left it will make it just as much too small when the vertical circle faces the right, and thus the mean of the two readings gives the correct result.

Now, when measuring the azimuth of a star, we have to sight the telescope to a *moving* object, and it is not possible, therefore, exactly to repeat the measurement because in the interval of time taken in changing face the position of the star is slightly changed. But it is characteristic of all the more accurate methods of astronomical measurement suitable for the surveyor, that reliance is never placed upon one observation, but the methods are so arranged that a series of observations can be made at short intervals, the face of the instrument being alternately changed from right to left, so that a mean may be obtained from which instrumental errors are largely eliminated.

The error made if the transverse axis of the telescope is not truly horizontal

This error, just as that due to collimation with which we have dealt, may also be largely eliminated by the method of changing

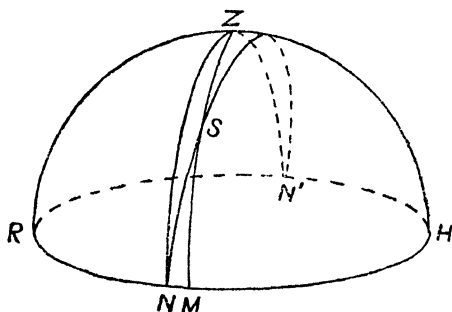


Fig. 23

face. But in this case the elimination is not so perfect, and as it is an easy matter by means of a striding level actually to measure the departure of the axis from the horizontal at each observation, it is frequently desirable to observe the error and allow for it in the computation.

If the axis of the telescope is not truly horizontal, the line of collimation, when the telescope is turned about the axis, will not trace out a great circle in the sky passing through the zenith, as it should do, but will trace out a great circle inclined to the vertical. Thus in Fig. 23, if NZN' denotes the great circle that would be traced out in the celestial sphere if the axis were horizontal, NSS' denotes the circle actually traced out if the axis is inclined at a small angle a . Let S be a star observed with this telescope, and draw the great circle ZSM passing through the zenith and the star.

The angle $ZNS = a$.

The actual observed altitude of the star is measured by the arc NS , whereas the true altitude is given by the arc MS .

Again, the azimuth of the star is actually measured on the circle of the horizon from the point N , whereas it should be measured from the point M . So that the error in azimuth is the angular measure of the arc MN .

In the right-angled triangle N S M, the angle S N M = $90^\circ - a$. Therefore, by Napier's rules, we have

$$\sin N M = \tan a \cdot \tan M S,$$

or, since both N M and a are small,

$$N M = a \cdot \tan M S.$$

That is to say, the error in azimuth = the error in level multiplied by the tangent of the altitude of the star.

Again, by Napier's rules,

$$\sin M S = \sin N S \cdot \cos a,$$

and since a is small and $\cos a$ may be taken = 1, it follows that we may take $M S = N S$, which means that no appreciable correction has to be made to altitude. The error produced is practically in azimuth only.

The error in azimuth increases with the altitude of the star. It is zero on the horizon, becomes = a for an altitude of 45° , and is very great for stars near the zenith.

Error in Azimuth corresponding to a Level Error 'a' in the Axis for Various Altitudes of Object.

Altitude of star	0°	30°	45°	60°	70°	80°	85°	89°
Error in azimuth	0	$0.58a$	a	$1.73a$	$2.75a$	$5.67a$	$11.43a$	$57.3a$

Determination of the level error of the axis by means of the striding level

In order to make practical application of the correction just investigated, it is necessary actually to measure the level error of the transverse axis of the theodolite for each observation. This is readily done by means of the striding level, a very sensitive spirit level supported by two legs with V bearings at the bottom, which can rest upon each end of the transverse axis of the theodolite. The tube of the level is marked off in divisions, the values of which are known or may be readily determined by test. The graduations read outwards from the centre towards both ends. To eliminate errors of construction the readings should be taken in pairs, the striding level being read first in one position and then reversed on its bearings with each observation. Both ends of the bubble are read on each occasion, the observer standing so as to face the direction in which the instrument is pointed. He reads first the left-hand end, then the right, then reverses the level and reads again.

Suppose, as in Fig. 24, the bubble extends from A to B, O being the centre of the graduations and C the middle point of the bubble.



Fig. 24

Then $CB =$ half the length of the bubble.

$$\begin{aligned}
 &= \frac{OA + OB}{2} \\
 \therefore OC &= OB - CB = OB - \frac{OA + OB}{2} \\
 &= \frac{OB - OA}{2}.
 \end{aligned}$$

This, therefore, measures the deflection of the centre of the bubble from its normal position, and, when multiplied by the value of 1 division of the level, gives the angular measure of the deflection from the horizontal.

Suppose that the readings of the left-hand and right-hand ends of the bubble are l_1 and r_1 respectively before reversal and l_2 and r_2 after reversal of the level on its bearings. Then, according to the first reading, the error of the axis is $\frac{l_1 - r_1}{2}$, and according to the second reading $\frac{l_2 - r_2}{2}$. Thus the mean determination is

$$\frac{l_1 + l_2 - (r_1 + r_2)}{4}.$$

We thus get the following rule for finding the error in level of the horizontal axis, after a series of striding level readings taken in this way. Add up the left-hand readings. Add up the right-hand readings. Subtract the two sums and divide by the total number of readings. The result is to be multiplied by the value of the level graduation in seconds of arc.

If the striding level were perfect in construction, then the reading obtained on reversal should be the same as that given previously : $\frac{l_1 - r_1}{2}$ should $= \frac{l_2 - r_2}{2}$. Any difference is due to an

error in the striding level, and is equal to twice the striding level error. Thus the error of the striding level itself

$$= \frac{l_1 - r_1 - (l_2 - r_2)}{4}.$$

For example, if the left-hand readings of the bubble of the striding level are 6.3 and 4.8, the corresponding right-hand readings being 5.2 and 6.8, we proceed as follows :—

L.	R.
6.3	5.2
4.8	6.8
11.1	12.0
	11.1
	4) 0.9
	0.22

Therefore, if one division on the level corresponds to 14'' inclination, the angle the axis makes with the horizontal is $0.22 \times 14 = 3.1''$.

In this case the sum of the readings to the right is greater than the sum of the readings to the left, and, therefore, the right-hand end of the axis is the higher. This would mean that the azimuth of a star (measured from the north towards the right) would appear to be greater than it really is, and the correction to be made would consequently have to be subtracted. If the left-hand end of the axis were the higher the correction to azimuth would have to be added.

If the preceding readings were taken with the striding level on the transverse axis of a theodolite when a star was being observed at an elevation of $42^\circ 33'$ and the azimuth reading was $127^\circ 33' 10''$, the correction to be made to azimuth would be $3.1 \times \tan 42^\circ 33' = 3.1 \times .918 = 2.8''$, and the corrected azimuth would be $127^\circ 33' 7''$.

We have assumed up to the present that the striding level readings are taken with the theodolite in one fixed position, and what this test really measures is *the angle made with the horizontal by the line joining the points of contact of the two ends of the axis with the wyes of the striding level*. But it may be that the two ends are not precisely of the same diameter, and if that be so the method so described would be insufficient to determine the error of the axis. Thus, if the diameters of the two ends of the axis differ by $\frac{1}{1000}$ inch in a length

of 4 inches the central line of the axis will make an angle of about 26 seconds with the line joining the points of contact of the wyes of the striding level. The real error of the central line of the axis therefore can only be completely determined by reversing the face of the instrument so as to get other readings with the axis reversed in direction.

In any important observation where it is possible, this is usually done in any case to eliminate other instrumental errors. The full test is then made as follows :—Exhaustively level to set vertical axis plumb. Take *four* level readings, two with axis in one direction as described above, and two with the axis reversed in direction, the instrument being reversed in altitude and azimuth. Any difference between the resulting two indicates horizontal axis inequality, and the final mean gives the true level error in the axis.

Allowance for error of alidade level

In most modern theodolites intended for astronomical observations, no level is attached to the telescope itself, but instead a delicate level, known as the alidade level, is attached to the vernier or microscope arms of the vertical circle, and the circle turns with the telescope so that when the telescope is horizontal the verniers are at zero.

With this form of instrument, when reading vertical angles, each reading should be repeated by changing the face of the instrument, and to allow for any slight departure from true horizontality in the setting of the theodolite, the alidade level should be read on each occasion. In this case the readings of the two ends of the bubble are commonly referred to as O and E, according as they are at the object or eye end of the telescope.

The principle involved is exactly the same as that of the striding level just described. The error in level will be found by dividing the difference between the sums of the readings of the object end and eye end by the total number of readings, and then multiplying the result by the angular value of one division of the scale of the spirit level. If the readings of the object end are greater than those of the eye end, then the zero line is pointing slightly upwards, and the correction must be added on to the observed altitude. If the readings of the eye end are the greater, then the correction is to be subtracted. So that

$$\text{Correction to altitude} = + \frac{O - E}{(\text{number of readings})} \times (\text{value of 1 division})$$

Thus, suppose that two observations are taken, one with the face of the instrument to the left and the other with face right, as follows :—

	O	E
F. L.	5	9
F. R.	7	7
	<u>12</u>	<u>16</u>
		12
		4) <u>4</u>
		1

Thus, if the angular value of one division on the level is $14''$, it will follow that the altitude measured must be reduced by this amount.

Clearly this correction applies to vertical angles only, and does not affect the measurement of horizontal angles.

The latest types of microptic theodolite are usually equipped with a built-in alidade level which is arranged with prisms to show the bubble split longitudinally ; opposite half-ends of the bubble are brought into the field of view, and, by turning the alidade screw, these can be brought together so that they appear as one single end of the full bubble when the latter is level. There are no divisions marked on this type of bubble tube, but it is a simple matter to set the bubble quickly to the centre of its run with the alidade screw just before reading the vertical circle micrometer, thus eliminating O and E readings.

CHAPTER 8

THE DETERMINATION OF TRUE MERIDIAN

THE determination by observation of a true north and south line is a very important and common operation for the surveyor, and there are many ways in which it may be done. In practice, however, preference is given to such methods as will allow a set of observations to be taken so that instrumental errors may be eliminated, some readings being taken with F.R. (face right) and others with F.L. (face left), and also to such methods as do not require too great an interval of time between the observations. There is an objection to methods which require stars to be sighted at an interval of several hours, not only on the score of practical convenience, but because the atmospheric refraction may have changed considerably in the time that has elapsed. We shall confine our attention to the principal methods in actual use.

Referring mark

When determining the azimuth of a star or other celestial object, it is necessary to have a referring mark whose azimuth may be measured with respect to that of the star, so that the true direction may be found of a fixed reference object. It is commonly indicated in field notes by the letters R.M. (or sometimes by the letters R.O. which stand for referring object). It is highly desirable that there should be no need to refocus the telescope after pointing to a heavenly body and then directing it to the referring mark, and this requires that the referring mark should be where practicable about a mile away. When stellar observations are being taken the referring mark should be made to imitate the light of a star as nearly as possible. This may be done with an electric lantern placed in a box or behind a screen, through which a small circular hole is cut to admit the light to the observer. The face of the screen may be painted with stripes, so that it may be readily observable in the day time. If the referring mark is not to appear larger than a star in the field of view of the telescope, the diameter of the hole must

not be more than about a third of an inch at a distance of one mile. Some observers prefer a narrow vertical slit on the screen, and others use a larger hole with two cross wires at right angles to each other.

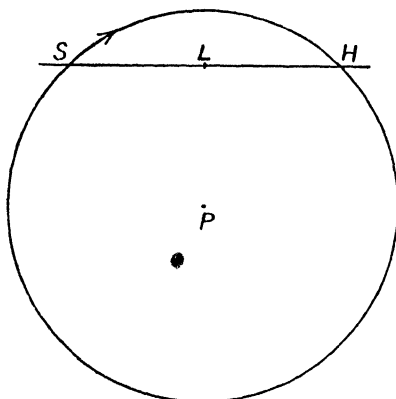


Fig. 25

First method : By equal altitudes

To mark out a true north and south line, we have to determine the direction of the celestial pole, and the simplest method is probably that of observing a star at equal altitudes. No calculations are necessary, and no knowledge of the latitude, longitude, or local time is required by the observer.

If the circle in Fig. 25 represents the circular path of a star round the pole, the problem is to determine the direction of the centre P of this circle. Suppose that the star is observed at S , and then, keeping the angle of elevation of the telescope unchanged, the observer waits until he sees the star again at H at the same altitude. Clearly the point L , midway between S and H , will be vertically above the pole P , and all that the observer has to do to get his true meridian is to bisect the angle between S and H . Nothing could be simpler in principle, but certain precautions are necessary to get accurate results.

In the first place, when fixing either the points S or H , we are really marking the point of intersection of the horizontal line with the circle. Now, we can fix the intersecting point of two lines most accurately when the two lines are at right angles, and so the best position for the line SH is when it passes somewhere near P . As the star takes 24 sidereal hours to complete its circle round the pole,

this would mean that the second observation would be made about 12 hours after the first. This would be often impossible and generally inconvenient. If, on the other hand, the line SH is taken too near the top of the circle, the star is moving so rapidly in a horizontal direction that it is not possible to secure good intersections.

Two simple observations at S and H , such as we have just described, would not be sufficient to enable instrumental errors to be eliminated, and so in practice at least four observations are made, as illustrated in Fig. 26. They will be made somewhat as follows :—Set the instrument to zero and point to the R.M. Point to the star in the position S_1 , measuring the horizontal angle between S_1 and the R.M., and noting also the altitude of S_1 . Then change the face of the instrument and point again to the star, which will by this time be at S_2 . Again note horizontal angle and altitude. Keeping the telescope clamped at the same vertical angle, unclamp

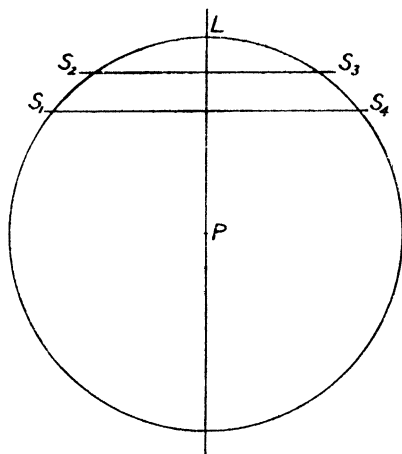


Fig. 26

the upper plate and move the telescope round, waiting until the star is again seen in the position S_3 . When the star is got into the field of view of the telescope, the upper plate is clamped and the star followed by means of the tangent screw until it again coincides with the centre of the cross wires. Having read the horizontal angle, the face of the instrument is again changed, the altitude of the telescope is set to the reading at S_1 , and the star is again followed until at S_4 it once more is in the centre of the field. Finally, the telescope is pointed to the R.M.

The direction midway between S_2 and S_3 should, of course, if there are no errors, coincide with that midway between S_1 and S_4 . This will not usually be the case, but the mean of the two results is taken and instrumental errors are largely eliminated.

If a , b , c , and d be the angles which S_1 , S_2 , S_3 , and S_4 make with the R.M., then if the R.M. be outside the angle subtended by $S_1 S_4$ at the observer's eye, the angle that the R.M. makes with the true meridian will be

$$\frac{a + b + c + d}{4}.$$

If, on the other hand, the direction of the R.M. lies between S_1 and S_4 , the angle will be

$$\frac{a + b - (c + d)}{4}.$$

The reason for this difference will be seen from Fig. 27, where OP represents the true meridian bisecting the angle between

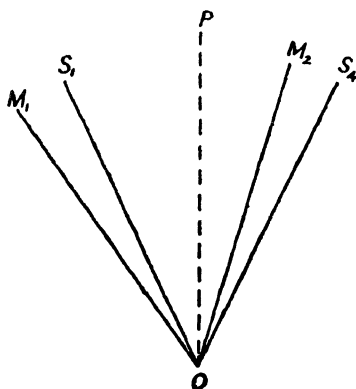


Fig. 27

$O S_1$ and $O S_4$. If the referring mark is in such a position as M_1 , outside the angle $S_1 O S_4$, the *sum* of the angles $M_1 O S_1$ and $M_1 O S_4$ is double the angle $M_1 O P$. But if the referring mark is in such a position as M_2 , within the angle $S_1 O S_4$, the *difference* of the angles $M_2 O S_1$ and $M_2 O S_4$ is double the angle $M_2 O P$.

The accuracy of the method may be further increased by taking a number of settings at say 10' or 20' intervals in altitude, the instrument being reversed for half the readings in *each* side to eliminate collimation and horizontal axis errors. The alidade bubble should be read to ensure that the altitudes are precisely the same for corresponding readings on the two sides.

The polar distances of the stars are not absolutely constant, as the theory of the method assumes, but undergo very slight changes during the year, which are tabulated in the *Nautical Almanac*. In the course of 24 hours, however, the alteration never amounts to more than a small fraction of a second of arc, and, therefore, need not be considered.

An unknown error may be introduced by changes in the atmospheric refraction during the considerable interval of time that must separate the first and second sets of observations. The method will give results quite sufficiently accurate, however, for the ordinary purposes of the surveyor, it may be carried out without the use of mathematical tables or *Nautical Almanac*, and it involves no knowledge of the position of the observer. Its great practical disadvantage is the length of time over which the observations must extend, and to carry them out the surveyor must be up for the greater part of the night. Consequently other more convenient methods are usually favoured by surveyors.

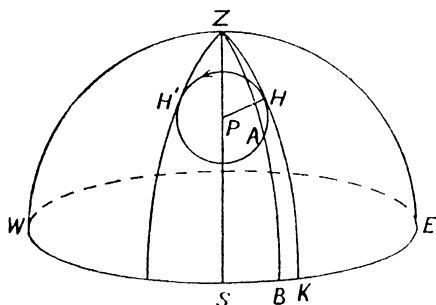


Fig. 28

Second method : By a circumpolar star at elongation

In Fig. 28, let P be the celestial pole, Z the zenith of the observer, W, S, and E the west, south, and east points respectively on the circle of the horizon, or the west, north, and east points according as the observer is in the southern or northern hemisphere. The small circle with P as centre represents the path of a circumpolar star A. The vertical plane passing through the zenith of the observer and the star traces out the circle ZAB on the celestial sphere. This will be the circle swept out by the telescope of a theodolite when the telescope, after being directed to the star, is turned in a vertical plane about its transverse axis. As the star moves from the position shown in the figure this vertical plane will

make a greater and greater angle with the plane of the meridian ZPS until the star arrives at the position H , where the vertical circle ZHK , swept out by the telescope, is a tangent to the circular path of the star. This is the point where the vertical plane containing the star makes its greatest angle with the plane of the meridian. At this point the star is said to be *at elongation*, and, clearly, its motion being vertical, it is in a favourable position for observations upon its azimuth, because its horizontal movement is so slight for some time before and after it arrives at H . There will be a corresponding point H' in the path of the star to the west of the celestial pole, and the points H and H' are referred to as the points of eastern and western elongation respectively.

It is clear from the figure that the points H and H' will always be at a greater altitude than the celestial pole P , but the smaller the circle of the star's path or the greater the declination of the star, the more nearly will the altitude of H and H' approach that of P .

Now, if an *Almanac* star is selected for observation, we shall know its declination, and the polar distance PH is the complement of the declination. If, in addition, we know the latitude of the place of observation, then, in the right-angled spherical triangle ZPH , we shall know PH and ZP , which is the complement of the latitude. Hence, by Napier's rules, we can compute the angle PZH . We have

$$\sin PH = \sin ZP \sin PZH$$

$$\text{or} \quad \sin PZH = \cos \text{declination} \times \sec \text{latitude}.$$

This calculation gives us the angle that the star at H makes with the meridian. Hence, if we measure the angle that the star at H makes with some referring mark, the azimuth of the R.M. is determined.

The method so far indicated would require the direction of the star to be measured at the exact moment of elongation. But we have set it down as a general principle that at least two observations should be made, one with F.L. and the other with F.R. and it becomes important to enquire what error in azimuth will be made if sufficient time is taken to obtain two readings.

On making the necessary calculations, it will be found that, for a place in latitude 30° , the azimuths of stars at different polar distances will not alter by $5''$ after the moment of elongation until the following times have elapsed :—

Polar distance of star	Time after moment of elongation before azimuth changes by 5"
10°, ...	3 min. 33 sec.
15°, ...	3 min. 7 sec.
20°, ...	2 min. 35 sec.
30°, ...	2 min. 11 sec.

As there will a corresponding and nearly equal period before elongation, it follows that for a star whose polar distance is 10° there will be a total time of about 7 minutes during which its motion is so nearly vertical that the total change of azimuth in that period is not more than $5''$. For a star whose polar distance is 30° , the corresponding period is $4\frac{1}{3}$ minutes.

If, then, the surveyor, as will commonly be the case in ordinary work, is not seeking to determine the true meridian nearer than within $20''$, it will be quite sufficiently accurate to take two observations of the star, one with F.L. and the other with F.R., not at the exact moment of elongation, but one just before and the other probably just after elongation. The time required to read both verniers, reverse face, and set the telescope again on the star should not be more than three or four minutes, so that there should be time to get both observations within the period we have just calculated during which the azimuth of the star does not alter by $5''$. The nearer the star is to the pole the greater the length of time available for the observations.

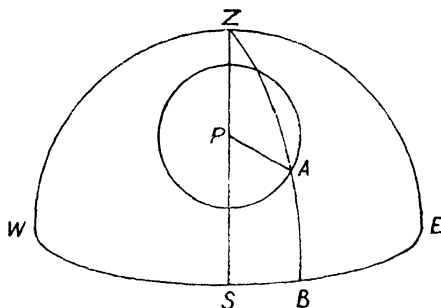


Fig. 29

The average value of the angle that the star makes with the meridian, as determined by two observations in this way, is clearly always a little less than the angle at elongation. In order to get the most accurate results with this method, it is better not to use the formula for the star at elongation at all, but to get a careful set of

four observations of the star *near* elongation, observing the altitude of the star at each measurement. In Fig. 29, let A represent the star moving in its circular path round the pole P, Z the zenith, Z A B the vertical circle passing through the zenith and the star. Then, in the triangle Z P A, if the altitude of the star is measured, the values of Z A ($90^\circ -$ the altitude) and Z P (the co-latitude) and P A (the polar distance of the star) are known. If

P A = p = polar distance of star,

P Z = c = co-latitude,

Z A = z = zenith distance,

$$s = \frac{1}{2} (p + c + z),$$

$$\sin \frac{1}{2} P Z A = \sqrt{\frac{\sin (s - z) \sin (s - c)}{\sin z \sin c}},$$

$$\text{or } \log \sin \frac{1}{2} P Z A = \frac{1}{2} \{ \log \sin (s - z) + \log \sin (s - c) \\ + \log \operatorname{cosec} z + \log \operatorname{cosec} c \}.$$

Such a set of observations should be made in the following order :—Point to R.M., point to star, reading altitude and horizontal angle, reverse face, and point to star again. Turn back to R.M., and read angle. Then another pair of observations is made in the same way. The mean of the first two observations and the mean of the second two are then used as the data for two separate computations of the azimuth of the R.M. by means of the formula we have just given. The average of the two results, if the work is carefully done, will give a very accurate determination. An example is given a little further on.

A more convenient method for reducing any number of observations taken near to elongation is given at the end of this chapter.

Calculation of the time of elongation

In order to prepare for these observations, it will generally be necessary for the surveyor to work out beforehand the time at which the star will elongate. In Fig. 28 the angle Z P H measures, when turned into time, the sidereal time that must elapse before the star at H comes on to the meridian. But when the star is on the meridian the sidereal time is given by the R.A. of the star.

Thus the sidereal time when the star is at H is = the R.A. of the star — the hour angle of the star Z P H. This sidereal time has then to be turned into mean time by the methods we have previously discussed.

EXAMPLE.—*To find the time of eastern elongation of Canopus on 1951 January 10, at a place in latitude $45^{\circ} 51' 50''$ South, longitude $170^{\circ} 30' 20''$ East.*

<i>R.A. Canopus (α Carinae)</i>	.	.	.	6 ^h	22 ^m	54 ^s
<i>Declination Canopus</i>	.	.	.	-- 52 ^o	40'	02.1''

We first of all find the time of culmination, the L.S.T. at that instant being given by the R.A. of the star. Thus, at culmination—

L.S.T.	.	.	.	.	6 ^h	22 ^m	54.0 ^s
Longitude, E.	.	.	.	.	11 ^h	22 ^m	01.3 ^s
Corresponding G.S.T.	.	.	.	.	19 ^h	00 ^m	52.7 ^s
G.S.T. at 0 ^h G.M.T. January 10	.	.	.	.	7 ^h	14 ^m	50.0 ^s

Sidereal time interval after 0 ^h G.M.T.	.	.	.	.	.	.	.
January 10	.	.	.	.	11 ^h	46 ^m	02.7 ^s
Convert to mean solar time	.	.	.	.	11 ^h	44 ^m	07.0 ^s

<i>i.e.</i> G.M.T. of culmination at place concerned, January 10	.	.	.	.	11 ^h	44 ^m	07.0 ^s
--	---	---	---	---	-----------------	-----------------	-------------------

Longitude, E.	.	.	.	.	11 ^h	22 ^m	01.3 ^s
---------------	---	---	---	---	-----------------	-----------------	-------------------

$\therefore$ L.M.T. of culmination, January 10	.	.	.	.	23 ^h	06 ^m	08.3 ^s
--	---	---	---	---	-----------------	-----------------	-------------------

(Using the *Star Almanac*)—

L.S.T.	.	.	.	.	6 ^h	22 ^m	54.0 ^s
Longitude, E.	.	.	.	.	11 ^h	22 ^m	01.3 ^s

Corresponding G.S.T. = U.T. + R	.	.	.	.	19 ^h	00 ^m	52.7 ^s
---------------------------------	---	---	---	---	-----------------	-----------------	-------------------

From S.A., U.T. + R at 6 ^h U.T., January 10	.	.	.	.	13 ^h	15 ^m	49.2 ^s
--	---	---	---	---	-----------------	-----------------	-------------------

Sidereal interval after 6 ^h U.T.	.	.	.	.	5 ^h	45 ^m	03.5 ^s
---	---	---	---	---	----------------	-----------------	-------------------

Convert to mean time interval	.	.	.	.	5 ^h	44 ^m	07.0 ^s
-------------------------------	---	---	---	---	----------------	-----------------	-------------------

Add to U.T. 6 ^h to give U.T. of instant of culmination	.	.	.	.	11 ^h	44 ^m	07.0 ^s
---	---	---	---	---	-----------------	-----------------	-------------------

Longitude, E.	.	.	.	.	11 ^h	22 ^m	01.3 ^s
---------------	---	---	---	---	-----------------	-----------------	-------------------

$\therefore$ L.M.T. of culmination January 10	.	.	.	.	23 ^h	06 ^m	08.3 ^s
---	---	---	---	---	-----------------	-----------------	-------------------

We have the time from elongation to culmination, now to find which will be measured (Fig. 28) by the angle ZPH . From the right-angled triangle ZPH in that figure we have

$$\cos ZPH = \frac{\cos \text{dist}}{\sin PH} = \cot ZP = \cot \text{declination} \times \tan \text{latitude}$$

$$\cot \text{ declination} = \cot 52^{\circ} 40' 02.1'' ; \log = 9.8823535$$

$$\tan \text{latitude} = \tan 45^{\circ} 51' 50''; \quad \log = 10.0130983$$

$\log \cos 38^{\circ} 10' 54.6''$ --- 9.8954518

$\therefore$ angle ZPH = $2^{\text{h}} 32^{\text{m}} 43.6^{\text{s}}$ sidereal time = $2^{\text{h}} 32^{\text{m}} 18.6^{\text{s}}$ mean solar time. $\therefore$ L.M.T. of eastern elongation = $23^{\text{h}} 06^{\text{m}} 08.3^{\text{s}} - 2^{\text{h}} 32^{\text{m}} 18.6^{\text{s}} = 20^{\text{h}} 33^{\text{m}} 49.7^{\text{s}}$. (L.M.T. of western elongation = $23^{\text{h}} 06^{\text{m}} 08.3^{\text{s}} + 2^{\text{h}} 32^{\text{m}} 18.6^{\text{s}} = 1^{\text{h}} 38^{\text{m}} 26.9^{\text{s}}$ January 11.)

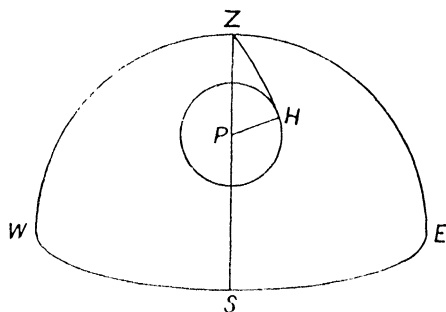


Fig. 30

Azimuth, altitude, and hour-angle at elongation

In Fig. 30, P denotes the celestial pole, Z the zenith of the observer, and H a star at elongation. In the right-angled triangle ZPH we have ZP = the co-latitude, PH = the star's polar distance or the complement of the declination, the angle ZPH = the hour angle of the star, PZH = the azimuth of the star if P is the north celestial pole or the supplement of the azimuth if P is the south celestial pole, ZH = the star's zenith distance or the complement of the altitude. Hence we have the following relations :—

$$\cos ZPH = \cot \text{dec.} \times \tan \text{lat.}$$

$$\sin P Z H = \cos \text{dec.} \times \sec \text{lat.}$$

$$\sin \text{ altitude} = \operatorname{cosec} \text{ dec.} \times \sin \text{ lat.}$$

EXAMPLE OF OBSERVATION OF STAR AT ELONGATION FOR AZIMUTH

Date—1951 January 11.

Star—Canopus.

Place—Dunedin, N.Z.

R.A.—6^h 22^m 54^s.

Latitude—45° 51' 50" S.

Declination— — 52° 40' 02.1". Longitude—170° 30' 20" E.

Computed L.M.T. of western elongation

(see previous article) 1^h 38^m 27^sStandard time fast on L.M.T. 37^m 59^s∴ L.Std.T. of western elongation 2^h 16^m 26^s

OBSERVATIONS

Object	Face	Horizontal circle		
		A	B	Mean
R.M.	R	118° 09'	298° 09'	118° 09' 00"
Star	R	227° 44'	47° 44'	227° 44' 00"
Star	L	47° 44'	227° 43' 30"	227° 43' 45"
R.M.	L	298° 09'	118° 09'	118° 09' 00"

Mean angle between star and R.M., 109° 34' 52". R.M. to East
of star.

CALCULATION

Formula.— $\sin A = \cos \text{declination} \times \sec \text{latitude.}$

log cos dec. = 9.7827900

log sec lat. = 10.1571629

log sin A = 9.9399529

A from south = 60° 33' 35"

Azimuth of star = 240° 33' 35"

Angle between R.M. and star = 109° 34' 52"

Bearing of R.M. = 130° 58' 43"

EXAMPLE OF OBSERVATION OF SIAR NEAR ELONGATION FOR AZIMUTH

Date, . . . 1950 May 21. Barometer, . . . 29.6 inches.
 Star, . . . α Trianguli Australis. Thermometer, . . . 47° F.
 R.A., . . . 16^h 43^m 26.4^s. Place, . . . Station B, O.S.M. Survey
 Declination, . . . -68° 56' 28.9". Camp, Tirohanga, Taieri, N.Z.
 R.M., . . . Lamp on azimuth station. Latitude, . . . 45° 50' 42" S.
 Longitude, . . . 170° 17' 57" E.
 Computed Standard Time of eastern elongation, 21^h 01^m 57.2^s.

Object	Face	Standard Time of observation	Horizontal circle			Vertical circle		
			A	B	Mean	A	B	Mean
R.M.	R	h m s	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
R.M.	L		335 14 16	155 14 28	335 14 22			
Star	L	20 58 55.5	155 13 52	335 14 08	155 14 00			
Star	R	21 01 23	148 57 43	328 57 51	148 57 47	49 59 48	49 59 32	49 59 40
Star	R	21 03 09	328 58 14	148 58 26	328 58 20	50 12 57	50 12 47	50 12 52
Star	L	21 05 38	328 58 15	148 58 29	328 58 22	50 22 52	50 22 36	50 22 44
R.M.	L		148 57 54	328 58 04	148 57 59	50 35 52	50 35 40	50 35 46
R.M.	R		155 13 55	335 14 09	155 14 02			
R.M.	R		335 14 17	155 14 27	335 14 22			

Mean angle between R.M. and star—

From first pair, . . . 6° 16' 07.5"

From second pair, . . . 6° 16' 01.5"

COMPUTATION FOR AZIMUTH

	First Pair	Second Pair
Observed altitude, . . .	50° 06' 16"	50° 29' 15"
Refraction, ...	48·5"	47·5"
Corrected altitude, . . .	50° 05' 27·5"	50° 28' 27·5"
Zenith distance z , . . .	39° 54' 32·5"	39° 31' 32·5"
Co-latitude c , ...	44° 09' 18·0"	44° 09' 18·0"
Polar distance p , ...	21° 03' 31·1"	21° 03' 31·1"
2 s , ...	105° 07' 21·6"	104° 44' 21·6"
s , ...	52° 33' 40·8"	52° 22' 10·8"
$s - c$, ...	8° 24' 22·8"	8° 12' 52·8"
$s - z$, ...	12° 39' 08·3"	12° 50' 38·3"
$\log \sin (s - z)$, ...	9·3405117	9·3469333
$\log \sin (s - c)$, ...	9·1649247	9·1549784
$\log \operatorname{cosec} z$, ...	10·1927556	10·1962533
$\log \operatorname{cosec} c$, ...	10·1570154	10·1570154
$\log \sin^2 \frac{1}{2} Z$, ...	18·8552074	18·8551804
$\log \sin \frac{1}{2} Z$, ...	9·4276037	9·4275902
$\frac{1}{2} Z$, ...	15° 31' 33"	15° 31' 31·1"
Z , ...	31° 03' 06"	31° 03' 02·2"
Angle to R.M., ...	6° 16' 07·5"	6° 16' 01·5"
Azimuth of R.M. (from South),	24° 46' 58·5"	24° 47' 00·7"
Bearing of R.M., ...	155° 13' 01·5"	155° 12' 59·3"

Mean bearing of R.M., ... 155° 13' 00·4"

CALCULATION OF TIME OF ELONGATION

L.S.T.	16 ^h 43 ^m 26·4 ^s
Longitude, E.	11 ^h 21 ^m 11·8 ^s
Corresponding G.S.T.	5 ^h 22 ^m 14·6 ^s
G.S.T. at 0 ^h G.M.T. May 21	15 ^h 52 ^m 15·7 ^s
Sidereal interval after 0 ^h G.M.T. May 21	13 ^h 29 ^m 58·9 ^s
Convert to mean solar interval	13 ^h 27 ^m 46·2 ^s
<i>i.e.</i> , G.M.T. of culmination May 21 . .	13 ^h 27 ^m 46·2 ^s
Longitude of standard meridian	12 ^h 00 ^m 00·0 ^s
∴ L.Std.T. of culmination, May 22 . .	1 ^h 27 ^m 46·2 ^s
$\log \cot \text{dec. } (68^\circ 56' 28\cdot9'')$	9·5855044
$\log \tan \text{lat. } (45^\circ 50' 42'')$	10·0128118
$\log \cos \text{hour angle at elongation}$. . .	9·5983162
Hour angle at elongation	66° 38' 10·6"
Hour angle in sidereal time	4 ^h 26 ^m 32·7 ^s
Hour angle in mean solar time	4 ^h 25 ^m 49·0 ^s
L.Std.T. of culmination, May 22 . . .	1 ^h 27 ^m 46·2 ^s
Hour angle at elongation	4 ^h 25 ^m 49·0 ^s
∴ L.Std.T. of eastern elongation . . .	21 ^h 01 ^m 57·2 ^s

The effect of an error in the latitude

In the preceding calculations we require to know the declination of the star and the latitude of the place. The declination of the star is given by the *Nautical Almanac*, but it is possible that the latitude may not be known with the same degree of precision. In Fig. 30, we have

$$\sin Z \cos \varphi = \cos \delta \quad (1)$$

where φ = latitude, δ = declination, Z = angle P Z H.

Suppose that a small change y in the latitude produces an alteration x in the azimuth Z , δ remaining unaltered.

$$\text{Then } \sin (Z + x) \cos (\varphi + y) = \cos \delta.$$

Expand each of these terms, remembering that x and y are small, so that $\sin x$, $\sin y$ may be replaced by x and y respectively, and $\cos x$, $\cos y$ by unity. We then get

$$(\sin Z + x \cos Z) (\cos \varphi - y \sin \varphi) = \cos \delta.$$

Subtracting (1) from this equation and neglecting the term involving the product of x and y

$$x \cos Z \cos \varphi - y \sin Z \sin \varphi = 0$$

or

$$x = y \tan \varphi \tan Z$$

$$= y \tan \varphi \frac{\sin Z}{\sqrt{(1 - \sin^2 Z)}} \\ = y \tan \varphi \frac{\cos \delta}{\sqrt{(\cos^2 \varphi - \cos^2 \delta)}} \quad \text{from (1)}$$

Thus $x = 0$ if $\varphi = 0$, and $x = \infty$ if $\varphi = \delta$. φ cannot be greater than δ , because if so Z P is less than P H (Fig. 30), and the formulæ would not apply. For such a star there is no position of greatest elongation, as the azimuth of the star during its revolution completes the circle of the compass. If $\varphi = \delta$, the path of the star passes through the zenith.

The following table gives the values of the error in azimuth compared with the error in latitude, as calculated by the preceding formula, for various values of φ and δ .

RATIO OF ERROR IN AZIMUTH TO SMALL ERROR IN LATITUDE

Declination of star observed	In Latitude 20°	In Latitude 30°	In Latitude 40°
60°	.22	.4	.7
70°	.14	.24	.4
80°	.06	.1	.19

In the cases tabulated an error in latitude of, say, $5''$ will produce an error in azimuth of less than $5''$, the tabulated ratios being all less than 1. The error in azimuth may, however, be much greater than the error in latitude, if the star observed has a declination approaching the value of the latitude.

In any given latitude, the error is least when the star selected is nearest to the pole. From the formula, $x = 0$ if $\delta = 90^\circ$. This and other considerations, as we have seen, all point to the desirability of selecting a star for observation as near to the celestial pole as possible.

Star observations in daylight

It is often a very great convenience to the surveyor to be able to make his observations for meridian in the day time. The method that we have just described of taking observations on a star at or near elongation may be used perfectly well in daylight, provided that a sufficiently bright star is selected. Such work is done most easily in the late afternoon.

The following are suitable stars for such daylight observations in the southern hemisphere :—

α Carinae (Canopus), α Eridani (Achernar), α^2 Centauri, β Centauri, α Crucis.

In the northern hemisphere suitable stars are :—

α Lyrae (Vega), α Aurigae (Capella), α Cygni (Deneb).

As these stars cannot be seen with the naked eye in daylight it is necessary to compute the position of the one selected for observation before directing the telescope to it. The time of elongation may be computed by the method already discussed, and the azimuth and altitude of the star at elongation determined by the formulæ given. When these calculations are made the star may be readily picked up.

To select the most suitable star, compute roughly the sidereal time when it is desired to make the observations. A star must be selected which culminates some 4 or 5 hours before or after this. That is to say, the star chosen must have a right ascension some 4 or 5 hours greater or less than the computed sidereal time.

Third method : extra-meridian observations on sun or star

Suppose, in Fig. 31, that S denotes any heavenly body which moves in a circle round the celestial pole P. Let Z be the zenith of the observer. Then if the altitude of S is observed at any instant, and if in addition we know the latitude of the place and the declina-

tion of the celestial body, then in the spherical triangle PZS we know the three sides $SZ = z = 90^\circ - \text{altitude}$, $PZ = c = 90^\circ - \text{latitude}$, $PS = p = 90^\circ - \text{declination}$. Consequently, we can determine the angle Z which the vertical plane through S makes with the true meridian.

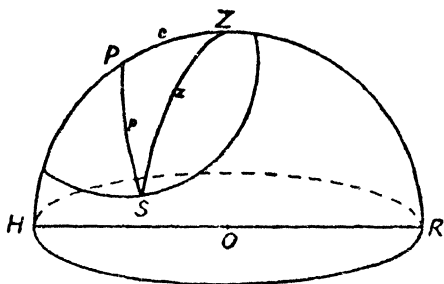


Fig. 31

If we write $s = \frac{1}{2} (p + c + z)$, then

$$\sin \frac{1}{2} Z = \sqrt{\frac{\sin (s - z) \cdot \sin (s - c)}{\sin z \cdot \sin c}},$$

or

$$\cos \frac{1}{2} Z = \sqrt{\frac{\sin s \cdot \sin (s - p)}{\sin z \cdot \sin c}},$$

from which

$$\log \sin \frac{1}{2} Z = \frac{1}{2} \{ \log \sin (s - z) + \log \sin (s - c) + \log \operatorname{cosec} z + \log \operatorname{cosec} c \},$$

and similarly for the second formula.

A more detailed discussion of these formulæ is given in the account of extra-meridian observations for time.

The method may be applied either to a star or to the sun, but for the sun we require a little more information than in the case of a star. To solve the spherical triangle we must know both the latitude of the place and the declination of the celestial object. In the case of a star we can get the declination from the *Nautical Almanac*, and as the declination changes very slightly throughout the year, we only require to know the approximate date of the observation in order to get the declination as accurately as is necessary. But, with the sun, the declination changes very rapidly, and in the *Nautical Almanac* its value is given at 0^h G.M.T. for every day in the year. In order to obtain the declination at any other instant, we must know the Greenwich time at the moment in question, and this means that we must know both the local mean time and the longitude. An

error of one minute in the time may produce an error of $1''$ in the sun's declination. With the sun, therefore, the time of observation must be noted as well as the altitude.

The method is also well suited for daylight observations upon stars, as the very brightest stars are available for this class of observation. Sirius (magnitude -1.4) is a very suitable star.

If the observation is made in the northern hemisphere and S is to the east of the meridian, the angle P Z S is the azimuth of the celestial body. If S is to the west of the meridian, the azimuth $= 360^\circ - \text{P Z S}$.

If the observer is in the southern hemisphere, then the azimuth $= 180^\circ - \text{P Z S}$ or $180^\circ + \text{P Z S}$, according as S is to the east or to the west of the meridian.

Extra-meridian observations of a star

At least two measurements of the altitude and the horizontal angle made with the R.M. should be taken, one with the F.L. and the other with F.R. Since the mean refraction for objects at an altitude of 45° is $57''$, it is necessary to correct for refraction in the measurement of the altitude. As the proper correction for refraction is somewhat uncertain for stars anywhere near the horizon, the star selected for observation should have an altitude of at least 30° . The order of procedure should be as follows :—

Point the telescope to the R.M.

Turn the upper part of the instrument round so as to direct the telescope to the star, reading both verniers on the horizontal circle. Measure also the altitude of the star.

Reverse the face of the instrument.

Again point telescope to star, measuring horizontal angle and altitude.

Turn the upper part of the instrument, this time in the reverse direction, until the telescope points to the R.M.

In the interval between the two pointings to the star it will have moved considerably in altitude. If we average the two altitudes and with the value so obtained solve for Z by the formula given, the result will give us the azimuth corresponding to this mean altitude, but that is not exactly the same thing as the mean of the azimuths in the two observed positions. Provided, however, that the difference in altitude of the star at the two observations is not more than one or two degrees, the error thus made is so slight that it is not worth considering.

When the observations are to be made upon a bright star in the day time, it will be necessary, first of all, to compute the azimuth and altitude of the star for the time of the first observation in the manner explained and illustrated in Chapter 6. The azimuth and altitude 5 or 10 minutes later may then be deduced, as shown in the same chapter.

Extra-meridian observations upon the sun

The sun, being an object of large size in the field of view of the telescope, cannot be observed in the same way as the stars. The observer must sight to its limb, and in this case, where both horizontal angle and altitude are to be measured, it may be sighted in any of the four quadrants formed by the cross wires of the telescope. The four different positions in which it may be observed are shown in Fig. 32, the two cross wires at right angles being brought by means of the tangent screws so as just to touch the sun's limb in each case. The centre of the sun's disc is the point considered in all our computations, and this then is the point whose position we seek

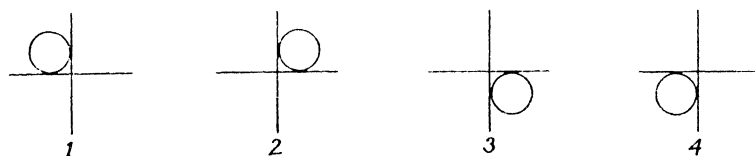


Fig. 32

to determine. Clearly, the centre of the cross wires is midway between the centres of the sun discs in positions 1 and 3, so that the mean of the readings in these two positions should give us the altitude and azimuth of the sun's centre. Similarly the mean of the readings in positions 2 and 4 will give the position of the sun's centre. Two observations of the sun in opposite quadrants are sufficient, but it is better still to make four observations in the four positions sketched. They should be made as follows :—

Take the reading of the R.M.

Turn to the sun and observe altitude and horizontal angle with the sun in quadrant 1 of the cross-wire system.

Then, as quickly as possible, by means of the two tangent screws, bring the sun into quadrant 3 of the cross wires, and again read horizontal angle and altitude.

Turn back to the R.M.

Reverse the face of the instrument and take another reading on the R.M. followed by two more observations of the sun in precisely the same way as before, but this time with the sun in quadrants 2 and 4. Finally, return to the R.M.

Be careful to note the time of each observation of the sun.

During the whole time occupied by the four observations the sun's position will have changed too much for accurate results to be obtained by averaging the measured altitudes and times of the four observations. There should, however, be very little time lost between the first two readings, with the sun in quadrants 1 and 3, and the measured altitudes and times of these two may be averaged together and a computation made for the corresponding azimuth of the referring mark. Similarly, another computation is made, by averaging the readings with the sun in quadrants 2 and 4, from which the azimuth of the referring mark is again determined. Thus we obtain two computed azimuths, one with each face of the instrument, and the average of the two is taken.

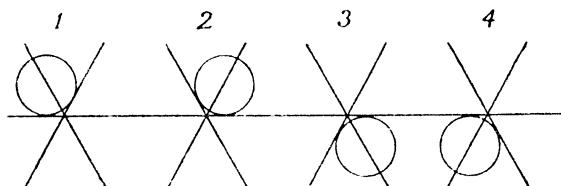


Fig. 32a

The two succeeding observations made without change of face in quadrants 1 and 3 or quadrants 2 and 4 are sometimes a little simplified by what is known as the "run through" method. In this method the observer, after making the first observation, leaves the telescope clamped in vertical arc, and makes the second observation when the sun has just crossed the horizontal wire by moving the vertical wire to the correct position, with the aid of the tangent screw attached to the horizontal circle. The necessity of recording a second set of vertical angles is thus avoided. The objection to this method is that the two observations cannot be made in such quick succession as is possible by the method outlined above, and consequently the error made by taking the average of the two observations is greater.

Very commonly only two observations of the sun are made, and in that case the best procedure is as follows :—

1. Observe the R.M., say, with face L. 2. Observe the sun in, say, quadrant 1 with face L. 3. Reverse face and observe the sun again as quickly as possible with face R. in quadrant 3. 4. Observe R.M. again with face R. The average of the two observations is then taken as the basis of a single computation.

Should the telescope have its cross wires of the form shown in Fig. 32*a*, the observations will be precisely the same, but the various positions of the sun's image will be as illustrated.

For good work the altitude readings should always be corrected by means of the alidade level, reading the E and O ends at each observation

Computation of sun's declination from Nautical Almanac data

In the *Nautical Almanac* the sun's declination is given for both 0^h G.M.T. and apparent noon at Greenwich for every day of the year, and also the 24-hourly difference. For approximate work it is sufficient to find values of the declination at intermediate times by direct interpolation, but for more accurate results, second differences should be used, since the rate of change is not constant. The second difference correction can be obtained very conveniently from Table XX in the *Nautical Almanac*.

EXAMPLE.—Required the value of the sun's declination at 4^h 15^m p.m. on 1951 January 6, the time being South Australian Standard, that of the meridian 9^h 30^m East.

Given L.Std. T., January 6	.	.	.	16 ^h	15 ^m	00 ^s
Longitude of standard meridian, E.	.	.	.	9 ^h	30 ^m	00 ^s
Corresponding G.M. T., January 6	.	.	.	6 ^h	45 ^m	00 ^s
6 ^h 45 ^m = 0.28125 day						
Sun's dec. at 0 ^h G.M.T., January 6			—	22°	37'	14.7''
First difference = + 419.0'' × 0.28125		+			01'	57.8''
Second difference, from Table XX, N.A.		—				02.7''
Sun's declination at given time	.	.	.	—	22°	35' 19.6''

Computation of sun's declination from Star Almanac data

The *Star Almanac for Land Surveyors* gives the sun's declination, to an accuracy of 0.1' (or 6''), for every 6 hours throughout the year ; the tabular 6-hourly differences are also given. The value of the declination at intermediate times is obtained by use of an interpolation table at the end of the *Almanac* (p. 63 in the 1951 issue).

The example given in the last article will be re-worked using the *Star Almanac* :—

The U.T. corresponding to the given L.Std.T. is 6^h 45^m on January 6th.

Sun's dec. at U.T. 6^h, January 6 — 22° 35·5'

Change, in 45^m (using table on p. 63 and
tabular difference of — 17) — 0·2'

Sun's dec. at U.T. 6^h 45^m, January 6 — 22° 35·3'

Corrections to sun observations

We have already seen in Chapter 7 that the sun is one of those bodies the observed altitude of which must be corrected for parallax. It must also be corrected for refraction, as shown in the same chapter.

Either from want of time or through the intervention of clouds the surveyor may be unable to complete the series of four observations, but any single observation will enable him to determine the position of the sun's centre by making proper allowance for the sun's semi-diameter, the value of which is tabulated in the *Nautical Almanac*.

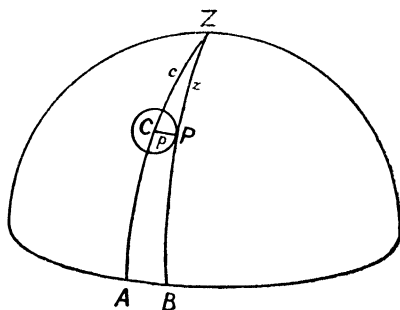


Fig. 33

There is no difficulty with regard to the determination of the altitude of the sun's centre from one observation, as the semi-diameter has simply to be added on or subtracted as the case may be. If, for instance, with a reversing telescope the sun is observed in quadrant 1, it will mean that we are actually sighting the upper limb of the sun, and the measured altitude will have to be reduced by the value of the semi-diameter given in the *Nautical Almanac*.

But with the observations for azimuth the matter is not quite so simple. Thus, in Fig. 33, if C denotes the centre of the sun's disc, Z the zenith, Z C A the vertical trace on the celestial sphere passing through C and the zenith, Z P B the vertical plane just touching the edge of the sun's disc, then the error in azimuth made by sighting the limb instead of the centre of the sun's disc is the angle C Z P. But in the right-angled triangle Z C P we have

$$\sin C P = \sin C Z \cdot \sin C Z P.$$

Now C P is an angle of about 15 minutes, and its circular measure differs from its sine by 1 only in the seventh place of decimals. Consequently, we may write

$$C P = \sin C Z \times C Z P$$

$$\therefore C Z P = C P \times \operatorname{cosec} C Z,$$

or correction in azimuth

$$= \text{semi-dia.} \times \sec. \text{altitude sun's centre.}$$

The effect of an error in latitude upon the calculated azimuth

Referring to Fig. 31, we shall determine the effect of an error in latitude if, in the spherical triangle P Z S, we investigate the effect upon the angle Z of a small change in c , the sides p and z remaining constant.

Let x be the change produced in Z by a small alteration y in c . Then

$$\begin{aligned} \cos p &= \cos c \cos z + \sin c \sin z \cos Z \quad (\text{formula (2) Chap. I.}) \\ \text{and } \cos p &= \cos (c + y) \cos z + \sin (c + y) \sin z \cos (Z + x). \end{aligned}$$

Subtracting these two equations, writing x and y in place of $\sin x$ and $\sin y$, and unity in place of $\cos x$ and $\cos y$, we get

$$\begin{aligned} 0 &= \cos z \cdot y \sin c + \sin z \sin c \cos Z \\ &\quad - \sin z (\sin c + y \cos c) (\cos Z - x \cdot \sin Z) \\ &= \cos z \cdot y \sin c - y \sin c \cos c \cos Z \\ &\quad + x \sin z \sin c \sin Z, \end{aligned}$$

neglecting the term involving the product of x and y .

$$\begin{aligned} \therefore x &= \frac{-\cos z \sin c + \sin z \cos c \cos Z}{\sin z \sin c \sin Z} y \\ &= \frac{-\cot P \sin Z}{\sin c \sin Z} \cdot y \quad (\text{by formula (3) of Chap. I.}) \\ &= \frac{-\cot P}{\sin c} \cdot y. \end{aligned}$$

P is, of course, the hour angle, and we thus have a simple formula for computing the error in azimuth produced by a given error in latitude at any given time of the day. Clearly, when P is very small—that is to say, at times near to noon—cot P is very great, and the error produced by a defective knowledge of the latitude is much increased.

In Fig. 34 a curve is drawn showing the error in azimuth produced by an error of one second in the latitude, at different hours of the day in latitude 40° . It is really a curve of tangents, and it will be seen that the error is very much greater at or near noon than at any other time. The error is least at 6 a.m. or 6 p.m. With increase

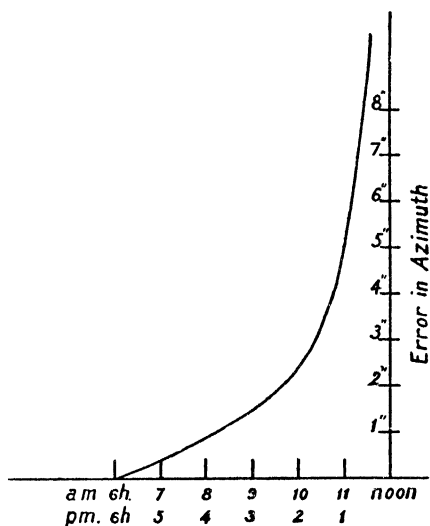


Fig. 34.—Error in azimuth for extra-meridian observation of the sun, corresponding to error of one second in latitude, at different hours of the day in latitude 40° .

in the latitude of the place of observation the error would be greater still, becoming very great for latitudes near the pole.

The effect of an error in the sun's declination upon the calculated azimuth

If a slight alteration y is made in the value of p (Fig. 31), c and z remaining constant, then it may be shown in a manner similar to that of the work just preceding that

$$x = \operatorname{cosec} c \operatorname{cosec} P \cdot y,$$

where x is the corresponding change made in the azimuth Z . The establishment of this formula we will leave as an exercise for the student.

In Fig. 35 a curve is drawn showing the error in azimuth produced by an error of 1 second in the declination at different hours of the day at a place in latitude 40° . Again the error is very great near

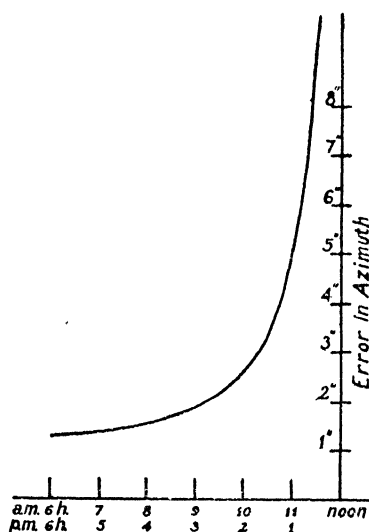


Fig. 35.—Error in azimuth for extra-meridian observation of the sun, corresponding to error of one second in declination, at different hours of the day in latitude 40° .

mid-day, and is least at 6 a.m. and at 6 p.m. As in the previous case, with increase in latitude of the place of observation the error also increases, becoming so great in latitudes near the pole that the method would be quite unreliable in arctic or antarctic regions.

It will be noticed that the two errors we have just discussed are of opposite signs, so that, if the declination and latitude are both too large, the errors tend to neutralise each other.

The effect of an error in the longitude of the place of observation

An error in longitude will produce an error in the computed Greenwich time at the instant of observation, and this in turn will

produce an error in the calculated declination. An error of 1° in longitude will produce an error of 4 minutes in time. Now the rate of change of the sun's declination varies at different seasons of the year, but its maximum rate of change is less than 1 minute of arc per hour. Thus an error of 1° in longitude, or 4 minutes in time, will produce an error in the declination that is always less than 4 seconds. As we have just seen, the resulting error in azimuth is never less than the error in declination, but if the observation is not made within two hours on either side of noon, the azimuth error is not much more than the declination error. It will thus seldom happen that the longitude is not known approximately enough for the surveyor.

The effect of an error in the measured altitude

Referring again to Fig. 31, we have in the spherical triangle S P Z

$$\cos p = \cos c \cos z + \sin c \sin z \cos Z.$$

Let x denote the small change in Z produced by a small change y in z , p and c remaining constant. Then

$$\cos p = \cos c \cos (y + z) + \sin c \sin (y + z) \cos (x + Z).$$

Subtracting and simplifying these equations, regarding z and y as small quantities, we finally arrive at the result

$$x = -\cot S \operatorname{cosec} z \cdot y$$

where S is angle P S Z.

Thus x will be infinitely great when $S = 0^\circ$ or 180° , which is the case when the sun is on the meridian. And, again, we arrive at the result that the resulting error in azimuth is very great if the observation is made near noon, but is small if S is anywhere near 90° .

The best time for extra-meridian observations

The preceding discussions all point to the desirability of making the observations upon the sun as far away from noon as possible. But if we observe it when too low down in the heavens, the refraction becomes a very uncertain quantity, and consequently it is impossible to measure the altitude with precision. For this reason it is generally considered inadvisable to make the observation with the sun at an altitude lower than about 15° . With this limitation it is desirable, in order to minimise the effects of errors in altitude, latitude, and declination, to make the observation as far from noon as possible. So that if the readings are made in the morning, they

EXAMPLE OF EXTRA-MERIDIAN OBSERVATIONS OF THE SUN FOR AZIMUTH

Date, . . . 1951 June 18. Barometer, . . . 27.5 inches.
 Place, . . . Dunedin, N.Z. Thermometer, . . . 51.5° F.
 Latitude, . . . 45° 51' 29.4" S. Longitude, . . . 170° 30' 36.1" E

Object	Face	Standard Time of Observation	Horizontal Circle	Vertical Circle
R.M.	L		0° 00' 03"	
+	L	14 ^h 34 ^m 05 ^s	297° 02' 38"	73° 41' 23"
+	L	14 ^h 35 ^m 28 ^s	296° 09' 03"	74° 20' 21"
R.M.	L		359° 59' 56"	

Mean angle between R.M. and sun = 63° 24' 10". Mean altitude of sun = 15° 59' 08".

R.M.	R		180° 00' 19"	
+	R	14 ^h 42 ^m 02 ^s	114° 42' 21"	15° 39' 01"
+	R	14 ^h 43 ^m 17 ^s	114° 58' 23"	15° 00' 57"
R.M.	R		180° 00' 15"	

Mean angle between R.M. and sun = 65° 09' 55". Mean altitude of sun = 15° 19' 59".

COMPUTATION

	Left face	Right face
L.Std.T. of observation, June 18	14 ^h 34 ^m 46 ^s	14 ^h 42 ^m 40 ^s
Longitude of Standard Meridian, ...	12 ^h 00 ^m 00 ^s	12 ^h 00 ^m 00 ^s
Corresponding G.M.T., ...	2 ^h 34 ^m 46 ^s	2 ^h 42 ^m 40 ^s
Sun's dec. at 0 ^h G.M.T., ...	+23° 23' 10.9''	+23° 23' 10.9''
June 18, ...	+	+
Variation since 0 ^h G.M.T., ...	+	+
Sun's dec. when observed, ...	+23° 23' 20.8''	+23° 23' 21.3''
Observed altitude	15° 59' 08''	15° 19' 59''
Refraction, ...	-	--
Altitude corrected for refraction, ...	15° 56' 05''	15° 16' 47''
Parallax, ...	+	+
Corrected observed altitude, ...	15° 56' 14''	15° 16' 56''
Zenith distance= z , ...	74° 03' 46.0''	74° 43' 04.0''
Sun's polar distance= p , ...	113° 23' 20.8''	113° 23' 21.3''
Co-latitude= c , ...	44° 08' 30.6''	44° 08' 30.6''
2 s , ...	231° 35' 37.4''	232° 14' 55.9''
s , ...	115° 47' 48.7''	116° 07' 27.9''
($s-p$), ...	2° 24' 27.9''	2° 44' 06.6''
log sin s , ...	9.9544075	9.9531989
log sin ($s-p$), ...	8.6233609	8.6786959
log cosec z , ...	10.0170223	10.0156351
log cosec c , ...	10.1571173	10.1571173
log cos ² $\frac{1}{2}Z$, ...	18.7519080	18.8046472
log cos $\frac{1}{2}Z$, ...	9.3759540	9.4023236
$\therefore \frac{1}{2}Z$, ...	76° 15' 06''	75° 22' 21''
$\therefore Z$ (from south), ...	152° 30' 12''	150° 44' 42''
Bearing of sun, ...	332° 30' 12''	330° 44' 42''
Mean angle between R.M. and sun, ...	63° 24' 10''	65° 09' 55''
$\therefore$ Bearing of R.M., ...	35° 54' 22''	35° 54' 37''
Mean, ...	35° 54' 29.5''	

should be made as soon as possible after the sun has reached an altitude of 15° . Similar remarks will apply to the stars, which should be observed as far away from the meridian as possible, so long as they are at an altitude of at least 15° above the horizon.

Fourth method : Time observations upon a close circumpolar star

The method about to be described is the one chiefly adopted on geodetic surveys where the highest attainable degree of accuracy is desired. The observations consist in measuring a series of angles between a close circumpolar star and the R.M., noting the time at which each pointing is made to the star. No altitudes need be measured, and as the time may be measured with sufficient precision by means of a chronometer, the method is simple, as well as being capable of great accuracy. In the northern hemisphere the star α Ursæ Minoris (Polaris) is a very convenient one for the purpose. Being a star of the second magnitude, it can be readily found, and it is within about 1° of the N. Pole. λ Ursæ is within 1° of the Pole, but is a much fainter star, being of magnitude 6.6. Other suitable northern circumpolar stars are 15 H Cephei (Mag. 5.2) and δ Ursæ Minoris (Mag. 4.4). In the southern hemisphere, unfortunately, there are no stars near the pole sufficiently bright to be readily found without first of all calculating their position. The best star for the purpose is σ Octantis, which is within $52'$ of the S. Pole. It is, however, of magnitude 5.5, and in order to find the star it is necessary to know beforehand the approximate bearing of the R.M. This may be found from a daylight observation by one of the methods previously described.

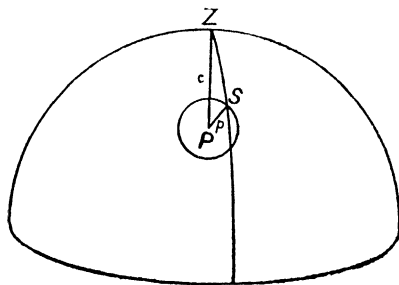


Fig. 36

In Fig. 36, let P be the celestial pole around which moves in a small circle the circumpolar star S. Let Z be the zenith. Then in the spherical triangle ZPS, $ZP = c = \text{co-latitude}$, $PS = p$

= polar distance of star, $ZPS = t$ = hour angle of star. PZS
 = Z = azimuth angle of star.

From formula (3) in Chapter 1.

$$\cot p \sin c = \cot Z \sin t + \cos c \cos t,$$

$$\therefore \cot Z = \frac{\cot p \sin c - \cos c \cos t}{\sin t}$$

$$= \frac{\sin (c - x) \cot t}{\sin x}, \quad . \quad . \quad . \quad (1)$$

where $\tan x = \tan p \cos t$.

The hour angle t , in time, is found by taking the difference between the R.A. of the star, which is the sidereal time when the star is on the meridian, and the sidereal time at the moment of observation. To determine this we must know both the local mean time and the longitude of the place. Thus, we require to know the R.A. and declination of the star and also the latitude and longitude of the place of observation.

For the most accurate work the striding level should be used to determine the error in the measured azimuth of the star owing to any defect in the levelling of the transverse axis of the telescope. This will produce an appreciable effect upon the azimuth of the star owing to its altitude, but as the R.M. will usually be near the horizon, it will not as a rule be necessary to apply any correction on this account to the reading taken to it. If, however, the R.M. should be at a considerable altitude, it would be necessary to read the striding level both when the telescope is pointed to the star and when it is directed to the R.M.

The series of observations necessary may be arranged in several different ways. The following is a recommended programme :—

1. Point twice upon the R.M. and read the verniers of the horizontal circle at each pointing, the instrument being F.L.

2. Read twice on the star with F.L., noting at each pointing the exact time, the reading of each end of the striding level, and the readings of the horizontal circle.

3. Read twice on the star with F.R., the instrument being reversed, noting the time and bubble readings as before.

4. Read twice upon the R.M. with F.R.

According to this programme the striding level is left with the same ends on the same pivots throughout the observations.

Another suggested programme is :—

1. Set the instrument to zero, point to R.M., and read the circle.
2. Intersect star and take the time.
3. Read the striding level and reverse it.
4. Read the circle.
5. Intersect star again and take the time.
6. Read the striding level.
7. Read the circle.
8. Point to R.M. and read the circle.

In turning back to R.M. the instrument is moved in the opposite direction. The instrument is now reversed, the setting on the R.M. increased by $22^{\circ} 30'$, and the operation repeated until angles have been read all round the circle.

A series of observations having been taken by one of these systems, the hour angle of the star and the corresponding azimuth will, of course, be different for each pointing. Each separate observation will give us the azimuth of the R.M., and we wish to get the mean or average of these determinations. We may compute the azimuth of the star for each pointing separately by means of formula (1), deducing from each computation the azimuth of the R.M., and then take the average of the different results. This is the simplest procedure, involving no mathematical difficulties, and when only a few observations have been taken this is the best plan to adopt. But when there are a number of observations the calculations may be lessened by computing the azimuth corresponding to the mean of the several hour angles. This would not be the same as the mean of the different azimuths, but the latter may be derived from the former by applying a correction known as the "curvature correction." In the case of a close circumpolar star, and a series of observations not extending over about half an hour, the curvature correction is given by the formula

$$\text{Correction} = \tan A \frac{1}{n} \sum \frac{2 \sin^2 \frac{1}{2} (T - T_0)}{\sin 1''},$$

where A = computed azimuth of star at the mean hour angle of n pointings,

T_0 = mean of the n hour angles.

T = any one of the separate hour angles.

The establishment of this correction is rather beyond the mathematical scope of this work.

The true mean azimuth always lies nearer the meridian than the azimuth corresponding to the mean hour angle.

The expression $\frac{2 \sin^2 \frac{1}{2} (T - T_0)}{\sin 1''}$, which is usually denoted by m , has to be evaluated for each observation, and as the same form also enters into the computation of circum-meridian observations for latitude, tables have been computed, available in various works, such as Chauvenet's *Astronomy* and *Trigonometrical Surveying* by Close in which the values are tabulated for different values of $T - T_0$. The use of such tables greatly facilitates the computation, as the curvature correction is then found by adding the different values taken from the tables, dividing the sum by n and multiplying by $\tan A$. A table giving the value of m at intervals of 10 seconds of time up to 19 minutes is given at the end of Chapter 9.

CIRCUM-ELONGATION OBSERVATION FOR AZIMUTH

The following account is extracted from a paper by the author published in the *Transactions of the Royal Society of South Australia*, vol. xxxix, 1915. The mathematics involved is rather more advanced than that in the rest of this work, but the method is of sufficient importance to make it desirable to insert it :—

On account of its convenience and comparative simplicity, the observation of a circumpolar star at elongation is, amongst surveyors, the favourite star observation for the determination of a true azimuth. The great disadvantage of the method is that only one observation can be made with the star actually at elongation, and there is thus no opportunity to eliminate instrumental errors in the same way as may be done when a series of observations of the same star are made by taking half the readings with the instrument reversed. As a rule the motion of the star in azimuth is so slow, when near to elongation, that with an ordinary transit theodolite two observations can be made and treated as though the star were actually at elongation without introducing an error sufficient to be measured by the instrument. But a much higher degree of accuracy is possible with the method if a series of half a dozen observations are made on each side of elongation, and the object of the present paper is to discuss the convenient reduction of such a series of observations. For the reduction of a similar set of observations made upon a *close* circumpolar star there is a well-known method that is particularly applicable to the Pole star of the

northern hemisphere. Unfortunately in the southern hemisphere the close circumpolar stars are very faint and not easy to work with. σ Octantis has a polar distance between $51'$ and $52'$, but its magnitude is $5\frac{1}{2}$, so that it is not readily distinguished by the surveyor. The bright southern stars that are most convenient for the determination have commonly a polar distance of about 30° , and to these the formula for close circumpolar stars cannot always be applied without introducing appreciable error.

Two methods are possible for a series of observations made before and after elongation. We may read the verniers of the horizontal circle and note the time at each observation, or we may read the horizontal circle and also the altitude of the star at each observation. The former method is preferable, provided that the surveyor has the correct local time, as errors due to a defective knowledge of atmospheric refraction are not then introduced. The latter method however, involves no knowledge of the time, and is much more convenient when the observations have to be carried out single-handed. In both cases the azimuth of the star at each observation is corrected by the appropriate formula to give the azimuth of the star at elongation, so that practically we obtain a series of observations at elongation instead of only one.

Notation

The following abbreviations will be used throughout :—

z denotes the zenith distance of the star in any position.

p „ polar distance of the star.

A „ horizontal angle between star and pole.

φ „ latitude of place of observation.

c „ co-latitude of place of observation.

h „ hour angle of the star in angular measure.

t „ value of hour angle expressed in sidereal time.

z_0 , A_0 , h_0 , and t_0 denote the values of z , A , h , and t respectively when the star is at elongation.

First method : Horizontal angle and time being noted at each observation

In the spherical triangle having the star, the celestial pole, and the zenith as its angular points we have the following fundamental relations :—

$$\cos A \sin z = \cos p \sin c - \cos c \sin p \cos h, \quad (1)$$

$$\sin A \sin z = \sin p \sin h, \quad (2)$$

and from the corresponding right-angled triangle when the star is at elongation

$$\sin A_0 = \frac{\sin p}{\sin c} = \frac{\cos p \cos h_0}{\cos c}, \quad (3)$$

$$\cos A_0 = \cos p \sin h_0 \quad (4)$$

(1) $\times$ (3) - (2) $\times$ (4) gives

$$\sin z \sin (A_0 - A) = \cos p \sin p 2 \sin^2 \frac{1}{2} (h_0 - h) \quad (5)$$

This is an exact equation, but is unsuitable as it stands for use in reduction of observations.

Putting $\frac{\sin p}{\sin z} = \frac{\sin A}{\sin h}$, (5) may be written

$$\frac{\sin (A_0 - A)}{\sin A} = \cos p \frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h}$$

or, writing $y = \cos p \frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h}$

$$\sin A_0 \cot A - \cos A_0 = y.$$

A_0 is constant, and, therefore, A may be regarded as a function of y .

Differentiating, we have

$$-\sin A_0 \frac{1}{\sin^2 A} \frac{dA}{dy} = 1,$$

and $-\sin A_0 \frac{d^2 A}{dy^2} = 2 \sin A \cos A \frac{dA}{dy}.$

Therefore, when $y = 0$

$$\frac{dA}{dy} = -\sin A_0 \text{ and } \frac{d^2 A}{dy^2} = \sin 2 A_0,$$

and consequently, by Taylor's Theorem

$$\begin{aligned} A = A_0 - \sin A_0 \frac{\cos p \cdot 2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h \sin 1''} \\ + \sin 2 A_0 \frac{\cos^2 p \cdot 2 \sin^4 \frac{1}{2} (h_0 - h)}{\sin^2 h \sin 1''} - \dots \end{aligned}$$

provided that $A_0 - A$ is measured in seconds of arc.

This is a convenient converging series for the determination of the difference between A and A_0 , in which the terms diminish so rapidly that in all ordinary work it is not necessary to take into

account any term except the first. Thus, if the observations are made at a place in latitude 30° , on a star with a polar distance of 30° , and are continued for fifteen minutes of time on each side of elongation, the extreme value of $h - h_0 = 3^\circ 45'$. The corresponding value of the first term in the series then works out at $229''$, or $3' 49''$, and that of the second term at less than $\frac{1}{2}''$. If $t - t_0 = 30$ minutes, or $h - h_0 = 7^\circ 30'$, then under the same conditions the first term $= 902''$ and the second term only $5\frac{1}{2}''$. With the same polar distance and in the same latitude, the limiting value for $t - t_0$, in order that the second term may not be greater than $1''$, is about 19 minutes. On repeating the calculations for a place in latitude 20° , and again for a place in latitude 40° , it is found that in neither case does the limiting value of $t - t_0$ differ by more than a minute from the value previously found if the second term in the series is to be less than $1''$.

It thus appears that, even if the mathematical reduction of each single observation is to be correct within $1''$ of arc, it is sufficient to use only the first term of the series if the observations extend over a period of about 19 minutes on each side of the elongation. The average of the whole series may be correct within this limit, even if the time extends over a considerably longer period, because the error in reduction will exceed $1''$ only in the case of the extreme observations.

A further considerable simplification would be made in the reduction if it were possible to treat the denominator as constant and write $\sin h_0$ instead of $\sin h$. With any single observation the error made, if this is done, may be considerable. For instance, at a place in latitude 30° , if $p = 30^\circ$, for an observation made 15 minutes before elongation, the difference made in the value of the second term, when $\sin h_0$ is written in the denominator instead of $\sin h$, is about $5''$, whilst for an observation made 30 minutes before elongation the difference is about $35''$. But, if we have a series of fairly well-balanced observations made both before and after elongation, the values of h range fairly evenly on each side of h_0 , and on averaging up the set there will be very little difference whether we use h or h_0 , the difference being generally of the order of $1''$. So that in such a case it is usually quite sufficient for the surveyor to use h_0 instead of h . We may then make a further slight simplification by putting

$$\frac{\sin A_0 \cos p}{\sin h_0} = \tan A_0 \cos^2 p.$$

Practical computation

We therefore conclude that, for the ordinary work of the surveyor, a series of well-balanced observations extending to about half an hour on each side of elongation on any circumpolar star may be reduced to a series of observations at elongation by the formula

$$A_0 - A = \tan A_0 \cos^2 p \, 2 \frac{\sin^2 \frac{1}{2} (h_0 - h)}{\sin 1''}, \quad (6)$$

in which $A_0 - A$ is given in seconds of arc.

If, however, only one or two observations are to be reduced, as may be the case if the star at elongation has been obscured by clouds, or the observations are badly balanced and have been made mostly on one side of elongation, or if the greatest possible degree of accuracy is required in the computations, the formula used should be

$$A_0 - A = \sin A_0 \frac{\cos p \, 2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h \sin 1''}, \quad (7)$$

This form may be obtained directly from (5) by considering $A_0 - A$ as a small angle so that the sine may be written equal to its circular measure.

If it is required to make the computation within $1''$, then, for observations more than 18 minutes from elongation, the value of $A_0 - A$ given by formula (7) should be corrected by being decreased by the amount

$$\sin 2 A_0 \frac{\cos^2 p \, 2 \sin^4 \frac{1}{2} (h_0 - h)}{\sin^2 h \sin 1''}. \quad (8)$$

As the expression $\frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin 1''}$ has to be evaluated in the reduction of circum-meridian observations for latitude, tables of the value of the expression and its logarithm have been prepared, and are available in Chauvenet's *Astronomy*, Close's *Astronomical Surveying*, and other works. An abbreviated table is given at the end of Chapter 9. Similar tables for $\frac{2 \sin^4 \frac{1}{2} (h_0 - h)}{\sin 1''}$ are also available. The computation by any one of these formulæ is much facilitated by the use of these tables. Five-figure logs are sufficient.

$$\text{Writing } \tan A_0 \cos^2 p = B, \, m = \frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin 1''},$$

(6) becomes

$$A_0 - A = B m, \text{ where } B \text{ is a constant.}$$

Thus for each observation we get $A_0 = A + B m$, and, averaging the whole series,

Mean value of A_0 = mean value of $A + B \times$ mean value of m .
Therefore, mean angle between R.M. and star at elongation = mean observed angle between R.M. and star $\pm B \times$ mean value of m .

EXAMPLE.—In the following example the method is applied to the reduction of a series of observations taken upon α Trianguli Australis near western elongation :—

Star observed.— α Trianguli Australis.

Place.—Maori Hill, Dunedin.

Right Ascension.— $16^{\text{h}} 43^{\text{m}} 31.25^{\text{s}}$.

Latitude.— $45^{\circ} 51' 36''$ South.

Declination.— $-68^{\circ} 56' 54.4''$.

Longitude.— $11^{\text{h}} 22^{\text{m}} 02^{\text{s}}$ East.

Date.—1951 September 21.

Standard Meridian.— 12^{h} East.

COMPUTED VALUES

Standard time at elongation — $21^{\text{h}} 50^{\text{m}} 08.8^{\text{s}}$

$A_0 = 31^{\circ} 03' 02.5''$

$h_0 = 66^{\circ} 37' 56.8''$

Face	Object	Mean readings on horizontal circle	Standard time of observation	Interval of mean time between ob- servation and elongation	Correspond- ing interval in sidereal time
		° ' "	h m s	min. sec.	min. sec.
L	R.M.	139 58 05			
L	Star	211 01 48	21 41 43.5	8 25.3	8 26.7
R	Star	211 03 45	21 43 38.0	6 30.8	6 31.9
R	R.M.	139 58 30			
R	Star	211 04 12	21 46 56.4	3 12.4	3 12.9
L	Star	211 02 46	21 48 49.4	1 19.4	1 19.6
L	R.M.	139 58 03			
L	Star	211 02 59	21 51 45.6	1 36.8	1 37.1
R	Star	211 04 13	21 53 43.2	3 34.4	3 35.0
R	R.M.	139 58 30			
R	Star	211 03 46	21 56 35.4	6 26.6	6 27.7
L	Star	211 01 40	21 58 26.0	8 17.2	8 18.6
L	R.M.	139 58 00			
L	Star	211 00 53	22 01 11.8	11 03.0	11 04.8
R	Star	211 01 36	22 02 54.9	12 46.1	12 48.2
R	R.M.	139 58 30			

Mean observed angle between star and R.M., $71^{\circ} 04' 29.5''$.

Solving by means of (6), we obtain from the tables :—

$t_0 - t$		m
min.	sec.	
8	26.7	140.0
6	31.9	83.8
3	12.9	20.3
1	19.6	3.5
1	37.1	5.1
3	35.0	25.2
6	27.7	82.0
8	18.6	135.6
11	04.8	241.0
12	48.2	321.8
		10) 1,058.3
		Mean value of m , 105.8

$$\log \tan A_0 = 9.77964$$

$$\log \cos^2 p = 9.94000$$

$$\log 105.8 = 2.02449$$

$$\log Bm = 1.74413$$

$$\therefore Bm = 55.5''$$

$\therefore$ Mean value of angle between star and R.M. at elongation

$$= 71^\circ 04' 29.5'' + 55.5'' = 71^\circ 05' 25.0''$$

$$\text{Bearing to star at W. elongation} = 211^\circ 03' 02.5''$$

$$\therefore \text{Bearing to R.M.} = 139^\circ 57' 37.5''$$

The computation by means of the more accurate formula (7) is rather longer. In this case we write

$$B = \sin A_0 \cos p \text{ and } m = \frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h \sin 1''},$$

and work on the same lines as before. To illustrate the method the computation in this case is also worked out as follows :—

$t_0 - t$	$h_0 - h$	h	$\log \frac{2 \sin^2 \frac{1}{2}(h_0 - h)}{\sin 1''}$	$\log \sin h$	$\log m =$ difference of two preced- ing columns	m
min. sec.	° ' "	° ' "				
8 26.7	2 06 40	64 31 17	12.14612	9.95556	2.19056	155.1
6 31.9	1 37 58	64 59 59	11.92324	9.95727	1.96597	92.5
3 12.9	48 14	65 49 43	11.30749	9.96014	1.34735	22.3
1 19.6	19 54	66 18 03	10.54406	9.96173	0.58233	3.8
1 37.1	24 16	67 02 13	10.70757	9.96414	0.74343	5.5
3 35.0	53 45	67 31 42	11.40140	9.96570	1.43570	27.3
6 27.7	1 36 56	68 14 53	11.91381	9.96792	1.94589	88.3
8 18.6	2 04 39	68 42 36	12.13225	9.96930	2.16295	145.5
11 04.8	2 46 12	69 24 09	12.38201	9.97131	2.41070	257.5
12 48.2	3 12 03	69 50 00	12.50758	9.97252	2.53506	342.8
					10) 1140.6	
Mean value of m ,						114

$$\log \cos p = 9.9700015$$

$$\log \sin A_0 = 9.7124782$$

$$\log 114 = 12.0569049$$

$$\log Bm = 1.7393846$$

$$\therefore Bm = 54.9''$$

This differs by only 0.6'' from the figure obtained by the previous method.

[Note.—In the above table $(h_0 - h)$ is $(t_0 - t)$ expressed in angular measure. The figures in the column headed $\log \frac{2 \sin^2 \frac{1}{2}(h_0 - h)}{\sin 1''}$ are obtained by taking the logs of the values of m found in the previous method : *e.g.*, $(t_0 - t) = 8^m 26.7^s$ gives, from the table at the end of the chapter on latitude, a value of m equal to 140.0. $\log 140.0 = 12.14612$; and so for the other values of $(t_0 - t)$. The logs obtained in this way are not absolutely correct, particularly for small values of $(t_0 - t)$, but the final result is affected in only a very small way. For example, using the computed logs of $\frac{2 \sin^2 (h_0 - h)}{\sin 1''}$ in the above table made the sum of the values of m equal to 1140.7 instead of 1140.6.]

The difference between the results of the two calculations is so small that clearly the more simple approximate method is quite sufficient for the surveyor. If the computation be made for the last four observations only, the difference between the results of the two methods amounts to $2''$, and for the last observation alone the difference is $4''$. For the surveyor it is only necessary to use the more accurate method of calculation for unbalanced observations at a considerable time for elongation.

It may be proved that, *provided the observations extend evenly over an equal time on each side of elongation*, there is no need for the surveyor to know the local time with great precision, an error of 1 minute in the time producing an error of only about $1''$ in the azimuth.

But if the observations do not extend on each side of elongation the case is different, and a more accurate knowledge of the time is essential.

Second method : Horizontal angle and altitude being noted at each observation

With the same notation as before, the star being in any position, we have

$$\cos p = \cos c \cos z + \sin c \sin z \cos A.$$

Writing $x = z - z_0$, this becomes

$$\cos p = \cos c \cos (z_0 + x) + \sin c \sin (z_0 + x) \cos A.$$

The quantities p , c , and z_0 being constants, this equation gives A as an implicit function of x .

Differentiating the equation three times in succession, the work being rather long but quite straightforward, we find that when $x=0$

$$\begin{aligned}\frac{dA}{dx} &= 0, \\ \frac{d^2 A}{dx^2} &= -\frac{\cot p}{\sin z_0} \\ \frac{d^3 A}{dx^3} &= \frac{3 \cot p \cos z_0}{\sin^2 z_0}.\end{aligned}$$

Therefore, by Taylor's Theorem

$$\begin{aligned}A &= A_0 - \frac{\cot p}{\sin z_0} \frac{(z - z_0)^2}{2} \sin 1'' \\ &+ \frac{\cot p \cos z_0}{\sin^2 z_0} \frac{(z - z_0)^3}{2} \sin^2 1'' \quad . \quad . \quad (9)\end{aligned}$$

provided that $A_0 - A$ and $z - z_0$ are expressed in seconds of arc.

To get some idea of the relative values of the terms in this series, we find, if the star observed has a polar distance of 30° and the latitude is also 30° , then $z_0 = 54^\circ 44' 09''$, and if $z - z_0 = 1^\circ$, the second term works out at $66''$ and the last term to $0.8''$. If $z - z_0 = 2^\circ$ the values become $264''$ and $6''$ respectively.

The last term in (9) is equal to

$$\frac{\cos^2 p \cos c}{\sin p (\cos^2 p - \cos^2 c)} \times \frac{(z - z_0)^3}{2} \sin^2 1'',$$

and has, therefore, an infinite value if $p = c$, in which case the star passes through the zenith. This is clearly of no practical importance.

The following are the values of the last terms in different latitudes for a star 30° distant from the celestial pole, if $z - z_0 = 1^\circ$:—

Latitude					Value of Last Term in 9
50° ,	.	.	.	.	$3.5''$
40° ,	.	.	.	.	$1.5''$
30° ,	.	.	.	.	$0.8''$
20° ,	.	.	.	.	$0.4''$
10° ,	.	.	.	.	$0.2''$
0° ,	.	.	.	.	$0.0''$

If $z - z_0 = 2^\circ$ the preceding values should be multiplied by 8.

It follows, therefore, that for the ordinary work of the surveyor the correction involved in the last term of the series is quite negligible for observations extending over a range of altitude of 2° , or 1° on each side of elongation, provided that the star does not pass within 10° of the zenith. At places near the equator the observations may clearly extend over a very much greater range of altitude with the same degree of precision.

To determine over what range of time the observations may extend, we find on differentiating the equation

$$\cos z = \cos c \cos p + \sin c \sin p \cos h$$

$$\text{that } \frac{d z}{d h} = \frac{\sin c \sin p \sin h}{\sin z} = \sin p \text{ for a star at elongation.}$$

This = $\frac{1}{2}$, if $p = 30^\circ$.

Thus, the rate of change of altitude at elongation does not depend on the latitude, but simply on the polar distance of the star, and for a star distant 30° from the pole we have

$$d h = 2 d z.$$

Therefore, if $d z = 1^\circ$, $d h = 120'$ of arc, or 8 minutes of time, the altitude of the star near elongation thus changes by 1° in about 8 minutes. For stars closer to the pole the time taken for the same change of altitude will be greater.

Practical computation

We conclude that for a set of observations extending over a range of altitude of about 2° , or 1° on each side of elongation, occupying in the case of a star with a polar distance of 30° , about 16 minutes of time, it is amply sufficient to use the formula

$$A_0 - A = \frac{\cot p}{\sin z_0} \frac{(z - z_0)^2}{2} \sin 1'' \quad . \quad . \quad . \quad (10)$$

It should be noticed that the error made by the use of this formula in the final reduction of a set of observations will be very much less than the error made in the reduction of the single observation furthest from elongation. We have based the stated limitations upon the error made in the reduction of the single observation, so that for a complete set of observations the time occupied may be extended somewhat beyond the limits given above. In low latitudes the observations may extend over a greater range than in high latitudes. In latitude 10° , for instance, the observations may extend over half an hour, and formula (10) will still give the average result of the set of readings correct within less than $1''$.

If the range of altitude is too great, or it is desirable to compute $A_0 - A$ with the greatest precision possible, then this value must be *reduced* if $z > z_0$, or *increased* if $z < z_0$, by the amount

$$\frac{\cot p \cos z_0}{\sin^2 z_0} \frac{(z - z_0)^3}{2} \sin^2 1'' \quad . \quad . \quad . \quad (11)$$

The computation by means of (10) is somewhat facilitated by making use of the same tables for circum-meridian calculations as have been shown to be suitable for the reduction by the first method. For since $z - z_0$ is a small angle, we have, within the degree of accuracy to which the tables are computed.

$$\frac{(z - z_0)^2}{2} \sin 1'' = \frac{2 \sin^2 \frac{1}{2} (z - z_0)}{\sin 1''}$$

and consequently we can take the value of $\frac{(z - z_0)^2}{2} \sin 1''$ straight from the tables.

Then, writing

$$B = \frac{\cot p}{\sin z_0}, m = \frac{(z - z_0)^2}{2} \sin 1''$$

we get for each observation, just as in the previous method,

$$A_0 = A + B m;$$

or, angle between R.M. and star at elongation

$$= \text{observed angle between R.M. and star} \pm B m.$$

Since B is a constant, we therefore get, on averaging the whole set of observations :—

Mean angle between R.M. and star at elongation

$$= \text{mean observed angle between R.M. and star}$$

$$\pm B \times \text{mean value of } m.$$

Whether the + or - sign is to be used depends upon the position of the R.M. and upon which angle between the star and R.M. is measured. It will be obvious in any particular case which sign should be taken.

If the tables for m are not available, then it is better to write

$$B = \frac{\cot p}{\sin z_0} \frac{\sin 1''}{2}, m = (z - z_0)^2$$

and proceed as before, this time computing m for each observation. Thus the use of the tables does not really make very much difference.

A defective knowledge of refraction does not seriously affect the accuracy of the work. For even if the altitude is in error by $15''$, the resulting error in azimuth is only about three-quarters of a second of arc.

The following example illustrates the method of reduction. It will be seen that the calculations are simple, and the method is undoubtedly capable of much greater accuracy than the ordinary methods of making elongation observations :—

Star observed—Achernar.

Right Ascension— $1^h 35^m 58.3^s$

Declination— $-57^\circ 28' 35.8''$.

Date—1951 September 21.

Place—Maori Hill, Dunedin, N.Z.

Latitude— $45^\circ 51' 36''$ South.

Longitude— $11^h 22^m 02^s$ East.

Standard Meridian— 12^h East.

COMPUTED VALUES

Standard time at Eastern elongation—23^h 00^m 11.2^s.

$$A_0 = 50^\circ 32' 07.5''.$$

$$z_0 = 31^\circ 39' 58.8''.$$

Face	Object	Mean readings on horizontal circle	Observed zenith distance	z = Observed zenith distance corrected for refraction	$z - z^\circ$	m
		° ' "	° ' "	° ' "		
R	R.M.	139 58 29				
R	Star	129 29 27	32 27 02	32 27 37	47' 38.2"	19.80"
R	Star	129 28 52	32 10 36	32 11 11	31' 12.2"	8.50"
R	Star	129 28 38	31 58 33	31 59 07	19' 08.2"	3.15"
L	Star	129 26 42	31 37 07	31 37 41	2' 17.8"	0.00"
L	Star	129 26 58	31 21 46	31 22 19	17' 39.8"	2.70"
L	Star	129 27 22	31 06 26	31 06 59	32' 59.8"	9.50"
L	R.M.	139 58 02				

$$6 \quad 43.65''$$

Mean value of m

$$7.28''$$

Mean observed angle between star and R.M. = $10^\circ 30' 15.7''$

$$B = \frac{\cot p}{\sin z_0} = \frac{\tan 57^\circ 28' 35.8''}{\sin 31^\circ 39' 58.8''} = 2.987.$$

Therefore, mean value of angle between R.M. and star at elongation = $10^\circ 30' 15.7'' - (7.28 \times 2.987)$

$$= 10^\circ 30' 15.7'' - 21.7''$$

$$= 10^\circ 29' 54.0''.$$

Bearing to star at eastern elongation = $129^\circ 27' 52.5''$

$$\therefore \text{Bearing of R.M.} = 129^\circ 27' 52.5'' + 10^\circ 29' 54.0''$$

$$= 139^\circ 57' 46.5''.$$

(Note.—The surveyor will realise that azimuth as determined by astronomical observation will be in terms of the meridian through the station at which the instrument is located. On the other hand, the bearings in a traverse which is tied to local minor triangulation points, or to standard survey points, will be in terms of the meridian through the origin of the meridional circuit (the "initial") within which those points lie. Therefore, if traverse bearings and those obtained by astronomical observation are to be compared, a correction will have to be made for convergence of the meridians through the initial and the station used for observing. See Chapter 13.)

EXERCISES

1. At a place in latitude 30° N., prove that the azimuth of a circumpolar star having a declination of $+80^\circ$ when at eastern elongation is $11^\circ 34' 00.8''$, and that the hour angle of the star is then $84^\circ 9' 25.3''$.

Find the time taken for the azimuth to decrease by $5''$.

Ans. 3 min. 34 sec.

2. At a place in latitude 30° S., prove that the azimuth of a circumpolar star having a declination of -60° when at western elongation is $215^\circ 15' 51.8''$, and that the hour angle of the star is then $70^\circ 31' 43.6''$.

Find the time that elapses before the azimuth is diminished by $5''$.

Ans. 2 min. 11 sec.

3. In latitude 37° S., the sun's declination being -14° , show that at 9 a.m. the sun's azimuth is $72^\circ 14' 39''$.

4. Compute the azimuth of a star having a declination of -75° when at eastern elongation, at a place in latitude 30° S.

Ans. $162^\circ 36' 39.4''$.

5. Demonstrate that if two circumpolar stars A and B are in the same vertical at some instant on the east of the meridian, A being above B, they will later be simultaneously on the vertical making the same angle on the west of the meridian, B being then above A.

6. Given the following information from the *Nautical Almanac*, find the sun's declination when the L.M.T. at a place in longitude 50° W. is 1941 June 1st $16^h 00^m 00^s$.

Sun at 0^h G.M.T.

Date	Declination
May 30	$+21^\circ 40' 37.2''$ "
	$+537.9$
May 31	$+21^\circ 49' 35.1''$
	$+515.2$
June 1	$+21^\circ 58' 10.3''$
	$+492.2$
June 2	$+22^\circ 06' 22.5''$
	$+469.1$
June 3	$+22^\circ 14' 11.6''$

Ans. By linear interpolation :

$+22^\circ 06' 08.3''$

Using second differences :

$+22^\circ 06' 09.14''$

7. The corrected observed zenith distance of the sun on the afternoon of March 17 at a place in latitude $34^{\circ} 56' S.$ is $62^{\circ} 19'$. If the sun's declination is $-1^{\circ} 28'$, compute its azimuth, to the nearest minute of arc, at the time of observation.

Ans. $289^{\circ} 20'$:

8. At a place in latitude $41^{\circ} 12' 40'' S.$ and longitude $11^h 39^m 34^s E.$ on the evening of January 15 (with the object of checking a traverse bearing), the altitude and bearing of a second magnitude star were obtained through a break in the clouds. It was necessary to compute the approximate R.A. and declination of the star to identify it, in order to obtain the precise elements for the calculation. From the following data, find the star's R.A. and declination.

Star's true altitude	$43^{\circ} 52' 34''$
Bearing corrected for convergence	$131^{\circ} 03' 14''$
Sidereal time at 0^h G.M.T. Jan. 15	$7^h 35^m 21^s$
N.Z. Std. time ($12^h E.$)	$20^h 50^m 51^s$

Ans. R.A. $8^h 42^m 54^s$
Dec. $-54^{\circ} 22' 11''$

9. Determine the difference of azimuth of the sun at its rising in mid-winter and mid-summer, also the difference (expressed in mean solar time) in the lengths of the days at these two times. Assume the latitude of the place to be $30^{\circ} N.$, and the greatest declination of the sun $\pm 23^{\circ} 27'$. Disregard corrections for refraction and parallax.

Ans. Difference of azimuth
 $= 54^{\circ} 42'$.

Difference of lengths of days
 $= 3 \text{ hrs. } 52 \text{ min.}$

10. In latitude $30^{\circ} 18' S.$, longitude $123^{\circ} 40' E.$, the following sun observation was taken at $4^h 45^m$ p.m. L.M.T. :—

Altitude	Object (as seen)	Bearing to sun	Bearing to R.M.
$22^{\circ} 28' 30''$		$258^{\circ} 43' 30''$	$357^{\circ} 46'$
$22^{\circ} 04' 30''$		$258^{\circ} 51' 30''$	$357^{\circ} 46'$

The sun's declination for the day, at 0^h G.M.T. was $-20^{\circ} 12' 39''$, and for the next day $-20^{\circ} 25' 25''$, the semi-diameter being $16' 14''$. Find the true bearing of the R.M. (Allow for refraction and parallax.)

Ans. $357^{\circ} 44'$.

11. Find the bearing and altitude of a star at its eastern elongation, also the L.M.T. of elongation, on the night of October 29–30. The latitude of the place is 31° S., the longitude 8 hours West, the R.A. of the star is $6^{\text{h}} 21^{\text{m}} 30^{\text{s}}$, its declination $-52^{\circ} 37'$, and sidereal time at 0^h G.M.T. on October 30 $2^{\text{h}} 29^{\text{m}} 58^{\text{s}}$.

Ans. Bearing $134^{\circ} 54'$.

Altitude $40^{\circ} 25'$

L.M.T. $23^{\text{h}} 39^{\text{m}} 36^{\text{s}}$ October 29

12. In latitude $25^{\circ} 58'$ N. Polaris was observed at its eastern elongation, its declination for the date being $+88^{\circ} 44' 20''$. Compute the azimuth of the star.

Ans. $1^{\circ} 24' 10''$

13. At Adelaide (latitude $34^{\circ} 55' 38''$ S., longitude $9^{\text{h}} 14^{\text{m}} 20^{\text{s}}$ E.) a forenoon observation was made of the sun on 1951 June 24.

From two observations taken with F.R. the mean angle between R.M. and sun was $85^{\circ} 27' 25''$, the mean altitude $24^{\circ} 00' 40''$. The mean time was $10^{\text{h}} 07^{\text{m}} 30^{\text{s}}$ (standard time of meridian $9^{\text{h}} 30^{\text{m}}$ E.). With F.L. the mean angle between R.M. and sun was $87^{\circ} 15' 20''$, the mean altitude $24^{\circ} 53' 25''$, the mean standard time $10^{\text{h}} 15^{\text{m}} 30^{\text{s}}$. The sun's declination at 0^h G.M.T. on 24th June was $+23^{\circ} 26' 13.6''$ and at 0^h G.M.T. on 25th June, $+23^{\circ} 25' 17.4''$. The angle between the sun and R.M. was measured from the sun to the right. Determine the true bearing of the R.M. (Allow for refraction and parallax).

Ans. $118^{\circ} 08' 58''$

14. During the evening of July 28, several bearings of α Centauri were observed when it was near elongation. Find the true bearing of the referring lamp, which was assumed to be $179^{\circ} 00' 00''$.

Longitude, $9^{\text{h}} 39^{\text{m}} 54^{\text{s}}$ E., latitude, $37^{\circ} 49' 53''$ S.

R.A. of star, $14^{\text{h}} 33^{\text{m}} 48^{\text{s}}$, declination, $-60^{\circ} 29' 16''$

Sidereal time, 0^h G.M.T. July 28, $20^{\text{h}} 23^{\text{m}} 12.21^{\text{s}}$

OBSERVATIONS

Standard Time, 10 ^h East of Greenwich h m s	Bearing of α Centauri ° ' "	Weight of observation
22 32 15	217 29 10	2
22 37 20	217 32 44	3
22 42 06	217 34 48	3
22 47 12	217 36 02	2
22 52 05	217 35 25	4

Ans. 180° 00' 14"

THE DETERMINATION OF LATITUDE

THERE are many possible ways by which the surveyor may determine the latitude of the place of observation, but, as in the previous chapter, we shall here confine our attention to the most practicable and most generally used methods.

First method : By meridian altitudes of sun or star

This is a very convenient and simple way of finding latitude, where the greatest possible precision is not required, and depends upon the fact we have already discussed in Chapter 3 that the altitude of the celestial pole is equal to the latitude of the place of observation. It follows that the latitude may be at once obtained by observing the meridian altitude of a body whose declination or polar distance is known. This is the method commonly used by the sailor at sea, the altitude of the sun at apparent noon being observed with a sextant. In Fig. 37, if O denote the position of the observer, Z the zenith point, P the celestial pole, then if an object be observed at S_1 , we have $AP = AS_1 - PS_1$, or latitude = meridian altitude - polar distance. This might represent the position of a circumpolar star at its upper culmination. If it were observed at lower culmination it would be in the position S_2 , and in that case $AP = AS_2 + PS_2$, or latitude = meridian altitude + polar distance.

In other cases the object observed may be on the opposite side of the zenith to P. If E denotes the point where the celestial equator intersects the meridian, the body may be at S_3 or S_4 . Since $BE + PA = 90^\circ$, it follows that BE = the co-latitude. Then at S_3 we have $BE = BS_3 - ES_3$, or co-latitude = meridian altitude - declination. When the body is in the position S_4 its declination will be south if the observation is made in northern latitudes or north if the place is in south latitude. In that case we have $BE = BS_4 + ES_4$, or co-latitude = meridian altitude + declination.

Thus in all cases the latitude can be very simply obtained provided that we know the declination of the celestial body.

The observed altitude must be corrected for refraction as discussed in Chapter 7, and as the amount of this correction depends upon the pressure and temperature of the air, it is necessary, if the correction is to be made as accurately as possible, that thermometer and barometer readings should be taken at the time of observation. Usually it will be sufficient to take the refraction correction straight from the table of mean refractions, without troubling to allow for the difference between the actual temperature and pressure from that for which the table of mean refractions is made out, because the maximum change in the refraction due to an alteration of temperature only amounts to about 3'' per 10° F, and for a change of pressure to about 5'' per inch of barometer.

In the case of the sun, since it is the altitude of the upper or lower limbs that must be observed, and it is the altitude of the sun's centre that is required, a correction must be made for its semi-diameter. Another correction also must be made to allow for parallax. Both of these are found from the *Nautical Almanac*. With observations upon the fixed stars neither of these corrections is needed.

If the altitude of the sun is observed with a sextant on land an artificial horizon must be used, in which case the double altitude is measured. The following is an illustration of such an observation, made in south latitude :—

Double altitude sun's lower limb,	.	64°	13'	10''
Index error	.		+ 4'	5''
		2)64°	17'	15''
			32°	08' 37.5''
Refraction	.		— 1'	55''
			32°	06' 42.5''
Parallax	.			7''
			32°	06' 49.5''
Semi-diameter	.		+ 15'	50''
Altitude of sun's centre,	.	32°	22'	39.5''
Declination N,	.	19°	47'	53''
Co-latitude,	.	52°	10'	32.5''
∴ latitude =	.	37°	49'	27.5''

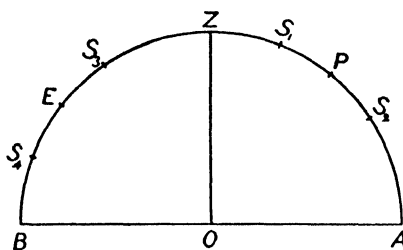


Fig. 37

With observations upon the sun, if the local mean time is known, the time of apparent noon may be found as already described, by using the data in the *Nautical Almanac*. The altitude of the sun's lower or upper limb may then be observed at the proper instant as measured by the watch. The effect of an error in time will depend upon the latitude of the observer and the declination of the sun. In latitude 45° , with the sun on the celestial equator, an error of 1 minute in time will produce an error of only $2''$ in the measured altitude. Under the same conditions if the time is wrong by as much as 10 minutes, the altitude measured will be too small by $49''$. So that for the ordinary purposes of the surveyor, when the observation is made in this way, it is not necessary to know the local time with great exactness. If the approximately correct time is not known the sun is followed by the observer and the altitude measured when it attains its greatest value.

With observations upon the stars the same general principles will apply. With close circumpolar stars it is possible to take two observations for altitude upon the same star, the face of the instrument being reversed after the first reading is taken. When this can be done the accuracy of the determination is increased, but as a rule the altitude changes too rapidly for this to be possible. In latitude 30° , for instance, the altitude, 5 minutes before and after its culmination, of a star having a polar distance of 30° is $43''$ less than when on the meridian.

Zenith pair observation of stars

A great improvement upon the accuracy of simple meridian observations may be effected by making observations upon two stars which culminate at approximately equal altitudes on opposite sides of the observer's zenith. The altitude of one star having been observed at culmination, the instrument is reversed in azimuth and

the meridian altitude of the second star is then measured. The two stars must, of course, be chosen so that the second culminates at a convenient interval after the first. The method is commonly referred to as that of latitude determination by "zenith pair observations." No attempt is made to take two observations on the one star, and the combination of the two results largely eliminates errors of refraction and errors due to the graduation of the vertical arc. Thus, in Fig. 37, if S_1 and S_3 denote the two observed stars, we obtain from the observation upon S_1

$$\text{lat} = A S_1 - P S_1 = A S_1 - p_1, \quad (1)$$

and from the observations upon S_3

$$\text{co-lat.} = B S_3 - E S_3 = B S_3 - (90^\circ - p_3),$$

$$\therefore \text{lat.} = 180^\circ - B S_3 - p_3 \quad (2)$$

Taking the average of the determinations (1) and (2), we obtain

$$\text{lat.} = 90^\circ + \frac{A S_1 - B S_3}{2} - \frac{p_3 + p_1}{2}.$$

Thus, in the final determination it is the *difference* of the measured altitudes $A S_1$ and $B S_3$ that is required, and as any error in the allowance made for refraction will affect both the altitudes alike, the error will practically disappear when we subtract them. Consequently, the method almost eliminates errors due to an uncertain knowledge of the refraction, and also enables instrumental errors to be largely eliminated because these will be practically the same with each observation and will have no effect on the *difference* of the two altitudes.

The face of the instrument is *not* reversed when reading the altitude of the second star, and the alidade bubble is indispensable to ensure that the pointing is the same in altitude for the same reading in the two positions.

If the local time and consequently the sidereal time is known with fair accuracy, the best way is to intersect each star at the instant when the sidereal time is equal to the star's right ascension. This is found from the *Nautical Almanac*, and the two stars will be selected for convenience, if possible, so that their right ascensions differ by from 10 to 30 minutes. If the time is not known accurately, then the telescope must be directed to the true meridian, and the altitude measured when the star intersects the vertical wire. Readings must be taken also of the barometer, thermometer, and alidade level.

EXAMPLE.—Date, 1951 September 24.

α Tucanae, observed altitude (South) = $75^{\circ} 21' 23''$.

Alidade level : O = 5.8, E = 3.2. 1 division of level = $15''$.

Barometer = 28.6 inches. External thermometer = 53° .

Attached thermometer = 51° .

Note.—O means objective end of telescope ; E means eyepiece end.

Compute refraction from “Bessel’s Refractions” as given in Chambers’ Log Tables, from the formula :—

$$\text{True ref.} = \text{mean ref.} \times B \times t \times T.$$

Mean refraction alt. $75^{\circ} 21'$	=	$15.1''$	.	1.17898
B for 28.6 inches	=	0.966	.	$\bar{1}.98497$
t for 51°	=	0.998	.	$\bar{1}.99913$
T for 53°	=	0.992	.	$\bar{1}.99651$

True refraction	=	$14.4''$	.	1.15959
-----------------	---	----------	---	---------

Observed altitude	=	$75^{\circ} 21' 23''$
-------------------	---	-----------------------

Refraction	=	$- 14.4''$
------------	---	------------

	=	$75^{\circ} 21' 08.6''$
Level	=	$+ 19.5''$

True altitude	=	$75^{\circ} 21' 28.1''$
---------------	---	-------------------------

Polar distance	=	$29^{\circ} 29' 51.4''$
----------------	---	-------------------------

Latitude	=	$45^{\circ} 51' 36.7''$ South.
----------	---	--------------------------------

Fomalhaut, observed altitude (North) = $74^{\circ} 01' 30''$.

Alidade level : O = 3.5, E = 5.5.

Barometer = 28.6 inches. External thermometer = 52° .

Attached thermometer = 51° . Date, 1951 September 24.

Mean refraction alt. $74^{\circ} 01'$	=	$16.6''$	.	1.22011
B for 28.6 inches	=	0.966	.	$\bar{1}.98497$
t for 51°	=	0.998	.	$\bar{1}.99913$
T for 52°	=	0.994	.	$\bar{1}.99738$

True refraction	=	$15.9''$	.	1.20159
-----------------	---	----------	---	---------

Observed altitude	=	$74^{\circ} 01' 30''$
-------------------	---	-----------------------

Refraction	=	$- 15.9''$
------------	---	------------

	=	$74^{\circ} 01' 14.1''$
--	---	-------------------------

Level	=	— 15·0''
True altitude	=	74° 00' 59·1''
Zenith distance	=	15° 59' 00·9''
Declination	=	29° 52' 38·5''
Latitude, Fomalhaut (N)	=	45° 51' 39·4''
α Tucanae (S)	=	45° 51' 36·7''
Mean	=	45° 51' 38'' South.

(Because of the high altitudes, the above observations had to be made with the aid of a diagonal eyepiece.)

Meridian altitudes of a star at both lower and upper culminations

If the meridian altitudes of a star be observed at both lower and upper culminations, then, if these be separately corrected for refraction, the mean of the two altitudes will give the altitude of the celestial pole, which is equal to the latitude of the place. The method does not require a knowledge of the declination of the star, but as this information is always to be obtained in the *Nautical Almanac*, there is no practical advantage to the surveyor, save perhaps in very exceptional cases. On the other hand, the long interval necessary between the two observations is a very practical inconvenience. Consequently, the method is not one in practical use amongst surveyors, although it is employed by astronomers at fixed observatories.

Second method : By circum-meridian observations

Observations of stars or the sun taken near to the meridian are commonly spoken of as circum-meridian observations. By taking a series of altitudes of a star or the sun for some few minutes both before and after it crosses the meridian, instrumental errors may be largely eliminated, and by proper methods of reduction the results may be used to give a very accurate determination of latitude. It is necessary to have the means of accurately noting the time of each observation, and then each altitude may be corrected or reduced so as to give us the corresponding altitude on the meridian itself. Thus a series of "circum-meridian" altitudes becomes equivalent to a series of measurements taken on the meridian itself, and in the taking of such a set of observations the instrument may be reversed

and its errors eliminated in a way that is not possible with a single meridian observation. Still greater precision may be attained by taking such observations upon equal numbers of stars north and south of the zenith, at approximately equal altitudes.

In Fig. 38, let Z be the zenith, P the celestial pole, and S the observed star. As this is to be near the meridian, the angle SPZ will be small.

Let $z = SZ$, the zenith distance,

$p = SP$, the polar distance of the star,

$c = PZ$, the co-latitude,

$t =$ the hour angle SPZ .

Then, from the triangle SPZ ,

$$\cos z = \cos c \cos p + \sin c \sin p \cos t. \quad (1)$$

Let x be the correction that has to be applied to the observed zenith distance, z , in order to deduce the zenith distance when the star is on the meridian.

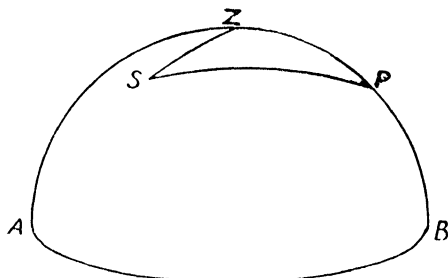


Fig. 38

Then meridian zenith distance $= z - x = p - c$.

If, now, in equation (1) we write $\cos t = 1 - 2 \sin^2 \frac{t}{2}$, we get

$$\cos z = \cos (c - p) - 2 \sin c \sin p \sin^2 \frac{t}{2}.$$

$$\therefore \cos z - \cos (z - x) = -2 \sin c \sin p \sin^2 \frac{t}{2}.$$

$$\therefore 2 \sin \frac{x}{2} \sin \left(z - \frac{x}{2} \right) = 2 \sin c \sin p \sin^2 \frac{t}{2}.$$

$$\therefore \sin \frac{x}{2} = \frac{\sin c \sin p \sin^2 \frac{t}{2}}{\sin \left(z - \frac{x}{2} \right)}.$$

If x is small, we may now replace $\sin \frac{x}{2}$ by the circular measure of $\frac{1}{2}x$, which is $\frac{1}{2}x \sin 1''$, provided that x is measured in seconds of arc. Also, we shall make very little difference to the result if, instead of the $\sin \left(z - \frac{x}{2} \right)$ of the denominator we write $\sin (z - x) = \sin (p - c)$. Thus, if x is the correction to be applied in seconds of arc, we obtain

$$x = \frac{\sin c \sin p}{\sin (p - c)} \cdot \frac{2 \sin^2 \frac{t}{2}}{\sin 1''},$$

or, in the form in which it is more usually written, if a denotes the observed altitude, A the altitude on the meridian, φ the latitude, and δ the declination,

$$A - a = \frac{\cos \varphi \cos \delta}{\cos A} \cdot \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}.$$

It will be noticed that if a series of observations is taken upon the same star, the first factor in this expression, i.e. $\frac{\cos \varphi \cos \delta}{\cos A}$, is the same for each observation. We will denote this by B . If we

write $m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$, we have

$$A = a + B m.$$

The value of m in seconds may be computed, knowing the value of t , or more conveniently it may be taken from tables such as are given in Chauvenet's *Astronomy* or the Army manual, *Field Astronomy*, or from the abbreviated table given at the end of this chapter.

Thus, if a_1, a_2, a_3 , etc., denote a series of observed circum-meridian altitudes of the same star, and m_1, m_2, m_3 , etc., are the corresponding values of m , we obtain a series of values for the corresponding meridian altitudes given by the equations

$$A_1 = a_1 + B m_1$$

$$A_2 = a_2 + B m_2$$

$$A_3 = a_3 + B m_3, \text{ etc.}$$

Therefore, if we denote by A_0 the mean of the deduced meridian altitudes, by a_0 the mean of the actual observed altitudes, and by m_0 the mean of the computed factors m , we have

$$A_0 = a_0 + B m_0.$$

With the aid of tables for m , the reduction of the observations thus becomes extremely simple. We take the mean of the values of m , multiply by B , and add the product to the mean of the observed altitudes.

The deduced mean meridian altitude is then corrected for refraction and the latitude is computed as an ordinary meridian altitude observation.

The value of B involves both the latitude and the meridian altitude, since

$$B = \frac{\cos \varphi \cos \delta}{\cos A},$$

but the value of φ used in this is the approximate latitude as deduced either from the map or from a simple meridian observation, or as calculated from the maximum observed altitude. The value of A used is the meridian altitude computed from the approximate latitude and the known declination of the star. The approximate value of B thus deduced is quite sufficiently accurate, when multiplied by m , to give the correction required. A still higher degree of accuracy may be obtained by repeating the calculation, using for B the value of the latitude as first computed.

Before starting the actual observations, it is necessary to calculate the time of the star's meridian transit. The observations should then be made within about ten minutes on each side of this. The t in the formula is the hour angle, and for stars is the interval of sidereal time between the instant of actual observation and the instant of meridian transit, expressed in angular measure at the rate of 15° per hour. For the sun it is the corresponding interval of solar time similarly expressed in angular measure.

The method involves an accurate knowledge of the local time, and is then capable of a high degree of precision. To get the best results the errors should be balanced by taking an equal number of observations on stars both north and south of the zenith. An equal number should be selected on each side at approximately equal altitudes. The errors are likely to be greatest for stars observed near to the zenith, especially when the place of observation is near to the equator. The range of observed altitudes should, if possible, lie between 40° and 75° above the horizon, and the closer the stars are observed to the meridian the better will be the results.

More exact methods of reduction of circum-meridian observations

The approximate formula that we have given is the one usually adopted for the reduction of circum-meridian observations. A still closer approximation may be obtained by using the more elaborate formula

$$A = a + B m + C m',$$

$$\text{where } C = B^2 \tan A \text{ and } m' = \frac{2 \sin^4 \frac{1}{2} t}{\sin 1''},$$

A and B having the same significance as before.

The correction introduced by the third term in the formula is usually very small when the observations are made close to the meridian. If the value of t in minutes does not exceed two-fifths of the zenith distance of the star in degrees, then it can be shown that the correction introduced by the term $C m'$ is never more than $1''$, so that the more exact formula is only required where the highest precision is sought.

This may be obtained in a manner similar to that employed in Chapter 8 for the corresponding formula for circum-elongation observations for azimuth.

The limits of time for the observations

According to what we have just seen, the greatest interval of time in minutes between any observation and the instant of meridian transit should not exceed two-fifths of the zenith distance of the star in degrees if the error in reducing the observation to the meridian is to be limited to $1''$. It is not possible to work so precisely as this with the instruments commonly used, and the time may be extended somewhat beyond this limit. In general, it seems a good rule to say that the greatest value of t in minutes of time should not exceed one-half of the zenith distance in degrees. Thus, if the altitude of the star is 50° , the observations may be made within 20 minutes on each side of the meridian transit. In that particular case the maximum error would still amount to only $1''$, but in other cases the error may be somewhat greater if this rule is followed, but never so much as $3''$, provided that the star is not within 10° of the zenith.

Circum-meridian observations of the sun

As a general rule, it is more convenient for the surveyor to make observations upon the sun than upon the stars, and exactly the

same method as we have described may be followed for circum-meridian observations of the sun. Obviously the sources of error cannot be balanced in the same way as with stars by taking observations both north and south of the zenith, so that such precise work is not possible. There is another difficulty arising from the fact that the sun's declination is not constant and, if the observations extend over 30 minutes, it may vary by as much as 30". If, however, a similar number of observations are made both before and after apparent noon, the errors will very nearly balance in the mean, provided that in the computations the value of the declination used is the value at apparent noon. This is not exact, but sufficiently so for all but the most precise work.

An even number of observations should be made, usually eight.

The first observation will be to the sun's upper limb with F.R. Then two in succession to the lower limb with F.L., next two in succession to the upper limb, the instrument being reversed, once more with F.R. Two more to the lower limb with F.L., and finally one to the upper limb with F.R. With this order the sun's diameter is eliminated in the mean. The alidade level should be read and recorded at each observation. The method of recording and the calculation are shown in the accompanying example :—

EXAMPLE OF CIRCUM-MERIDIAN OBSERVATION OF SUN FOR LATITUDE

Place, ... Maori Hill, Dunedin, N.Z.
 Longitude, ... 11^h 22^m 02^s East.
 Date, ... 1951 July 20

Sun's limb observed	Face of instrument	Standard time	Vertical circle		Mean
			A	B	
		h m s	° ' "	° ' "	° ' "
U	R	12 38 38	23 02 27	23 02 41	23 02 34
L	L	12 40 37	23 34 24	23 34 34	23 34 29
U	L	12 42 34.5	23 02 44	23 03 00	23 02 52
L	R	12 44 00	23 34 38	23 34 46	23 34 42
U	R	12 45 50	23 03 00	23 03 06	23 03 03
L	L	12 47 34.5	23 34 33	23 34 45	23 34 39
U	L	12 49 33	23 02 10	23 02 20	23 02 15
L	R	12 51 13.5	23 33 31	23 33 43	23 33 37

Mean observed altitude = 23° 18' 31.4"

PRELIMINARY COMPUTATIONS

For approximate latitude		For time of apparent noon		For B	
Highest observed altitude, . . .	23° 34' 42"	G.M.T. of G.A.N. 1951 July 20	12 ^h 06 ^m 11.7 ^s		
R and P, . . .	02' 01.9"	Correction for longitude .	-01.9 ^s	log cos latitude, .	9.8429
	23° 32' 40.1"	L.M.T. of L.A.N. 1951 July 20 . . .	12 ^h 06 ^m 09.8 ^s	log cos declination .	9.9706
-Semi-diam., .	15' 45.9"	Standard time fast on L.M.T.	37 ^m 58.0 ^s	log sec altitude .	10.0369
Corrected altitude, .	23° 16' 54.2"	L.Std.T. of L.A.N. . . .	12 ^h 44 ^m 07.8 ^s		
Declination, . . .	20° 51' 29.9"			log B, . . .	9.8504
Co-latitude, . . .	44° 08' 24.1"			B, . . .	0.708
Latitude, . . .	45° 51' 35.9"				

COMPUTATION FOR LATITUDE

<i>t</i>		<i>m</i>
min.	sec.	"
5	29.8	59.3
3	30.8	24.2
1	33.3	4.7
0	07.8	0.0
1	42.2	5.7
3	26.7	23.3
5	25.2	57.7
7	05.7	98.8
		8) 273.7
m_0 , ...		34.2''
m_0 B, ...		24.2''
Mean observed altitude, ...		23° 18' 31.4''
A_0 , ...		23° 18' 55.6''
Refraction		— 2' 10.0''
		23° 16' 45.6''
Parallax, ...		08.1''
Corrected altitude, ...		23° 16' 53.7''
Declination, ...		20° 51' 29.9''
Co-latitude, ...		44° 08' 23.6''
Latitude, ...		45° 51' 36.4''

Third method : Latitude by prime vertical transits

The prime vertical has been already defined as the vertical plane at right angles to the meridian, running truly east and west. Stars with polar distances less than 90° and greater than the distance of the pole from the zenith—*i.e.*, greater than the co-latitude of the place—will cross the prime vertical twice in a sidereal day. If the interval of time between the east and west transits of a star be measured, and the declination of the star be known, then the latitude can be readily computed. Thus, in Fig. 39, let E'Z W represent the prime vertical of the observer, Z being the zenith. Let P be the celestial pole and A C B the portion of the star's path, described on the same side of the prime vertical as the pole. A and B are the points where the star's path intersects the prime vertical. If A and B respectively are joined to the pole P by arcs of great circles, P A and P B will each be equal to the star's polar distance or to the complement of its declination. Then, in the spherical triangle A Z P, the angle at Z is a right-angle, P Z = the co-latitude, P A = the star's polar distance, and the angle A P Z, if turned into time at the rate of 15° per hour, will represent half the interval between the transits at A and B measured in sidereal time.

From Napier's Rules we have

$$\cos APZ = \tan PZ \times \cot AP,$$

whence $\tan \text{latitude} = \tan \text{declination} \times \sec t,$

where t = half the interval of sidereal time between the transits expressed in angular measure.

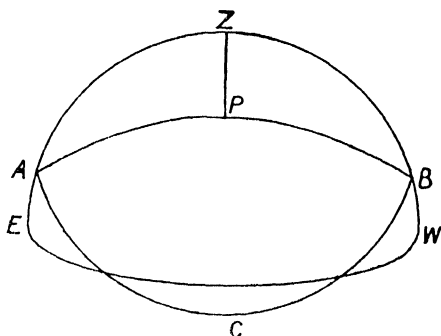


Fig. 39

By this method the errors due to uncertainty with regard to refraction are largely eliminated, because the times of transit are observed instead of altitudes. The method does not require a knowledge of the exact local time, as it is an interval of time that has to be measured, consequently it is sufficient for the surveyor to have a watch or clock whose rate is known.

It will be obvious that in places where the elevation of the celestial pole is small—that is to say, in places near the equator—the paths of such stars as move across the prime vertical will intersect it very obliquely, and it will not be possible to secure a good determination of the exact time of intersection. A precise measurement will be more easy in places of higher latitude.

The effect upon the determination of an error in the measurement of the time interval

To make a determination of latitude by the method just described, the surveyor has to set out the direction of the prime vertical, and also to measure the time interval between the east and west transits. In order to judge therefore of the degree of precision of which the method is capable we require to investigate the effect of small errors in each of these measurements.

If we denote the latitude by φ and the declination of the observed star by δ , we have

$$\tan \varphi = \tan \delta \cdot \sec t \quad (1)$$

If a small error y is made in the measurement of t , and x is the corresponding error made in the latitude,

$$\tan (\varphi + x) = \tan \delta \cdot \sec (t + y).$$

Expanding and writing $\tan x = x$, $\cos y = 1$, $\sin y = y$ since x and y are small, we have

$$\frac{x + \tan \varphi}{1 - x \tan \varphi} = \frac{\tan \delta}{\cos t - y \sin t}.$$

$\therefore$ neglecting the product of the small quantities xy , we get

$$x \cos t + \tan \varphi \cos t - y \tan \varphi \sin t = \tan \delta - x \tan \delta \tan \varphi.$$

Making use of (1), this becomes

$$x \left(\frac{\tan \delta}{\tan \varphi} + \tan \delta \tan \varphi \right) = y \tan \varphi \sqrt{1 - \frac{\tan^2 \delta}{\tan^2 \varphi}},$$

$$\therefore x \frac{\tan \delta \sec^2 \varphi}{\tan \varphi} = y \tan \varphi \sqrt{1 - \frac{\tan^2 \delta}{\tan^2 \varphi}},$$

$$\therefore x = y \frac{\sin 2 \varphi}{2} \sqrt{\left(\frac{\tan^2 \varphi}{\tan^2 \delta} - 1 \right)}.$$

The student who understands differential calculus can obtain this result at once by differentiating equation (1), keeping δ constant.

From this equation we get the important practical deductions that, if δ is nearly $= \varphi$, x will be very small, and that if δ is nearly $= 0$, x will be very large. So that it would seem that the stars most suitable for observation are those whose declinations are nearly equal to the latitude. A star having a declination the same as the latitude would pass through the zenith point, and the declination must be somewhat less than the latitude for the method to be possible. On the other hand, a star with zero declination would pass through the E. and W. points on the horizon at the prime vertical for all latitudes, the interval of time between its transits would be exactly twelve hours no matter what the position of the observer, and no determination of latitude could be made. It would apparently follow, that the best stars to select are those that cross the prime vertical near the zenith. But a star crossing the prime vertical very near to the zenith intersects it so obliquely that it is not possible to make an accurate determination of the time of transit. The distance from the zenith, at which the path of the

star will make a sufficiently large angle with the prime vertical to enable a good measurement of the transit to be made, will depend upon the latitude of the observer. The practical conclusion is that the stars observed should be as high up on the prime vertical as is consistent with an exact determination of the time of transit. Stars which cross it low down must be avoided, as they lie near the celestial equator, and the error in latitude produced by a slight error in time is then very large.

A definite calculation will give a better idea of the effect of a defective measurement of the time interval. If we take a place in latitude 30° S., and suppose the observation to be made on a star with a declination of -10° , then $x = 1.3 y$. Now t in our formula is half the total time interval between the transits, so, if this whole interval is in error to the extent of one second of time, $y =$ half a second. But half a second of time is equivalent to 7.5 seconds of arc, and this multiplied by 1.3 gives 9.7 seconds of arc as the error in latitude caused by an error of one second in the time interval.

If in the same latitude the star observed has a declination of 20° , then, from the same formula, $x = 0.52 y$. In this case a mistake of 1 second in the total time interval will cause an error of 3.9 seconds in the latitude. If the declination is 25° , $x = 0.32 y$, and the corresponding error in latitude is 2.4 seconds. In higher latitudes the errors are still greater.

Clearly, even if the surveyor is to be content with a determination of latitude to the nearest minute of arc, he must be able to rely upon his measurement of the time interval within a few seconds.

The effect of an error in the direction of the prime vertical

The error arising from a defective setting out of the prime vertical is not nearly so serious, because, if this is marked out so that the time of the eastern transit of the star is earlier than it should be, then the time of the western transit will be correspondingly hastened, so that the interval between the transits will be very little different from that when the prime vertical is correctly located. Thus, in latitude 30° , the measurements being made on a star with a declination of 20° , even if the prime vertical is set out as much as 1° out of its true position, the resulting error in the latitude determination is less than 1 minute of arc. So that a comparatively rough determination of the prime vertical is sufficient for the surveyor's purpose. It is, of course, most important that the instrument shall be in accurate adjustment, so that it will sweep out a truly vertical circle. But

instrumental errors may be largely eliminated by taking observations on alternate nights with the instrument reversed.

Although the method is capable of giving results of great precision, the practical inconvenience caused by the long interval between transits and the necessity for exact time measurements rather put it out of court as a suitable method for ordinary surveyors in the field.

The same method may be applied, with some modification of formulæ, to any vertical circle whatever. But the prime vertical circle is the most suitable for accurate work.

Striding level correction to prime vertical observations

The striding level should always be used with prime vertical

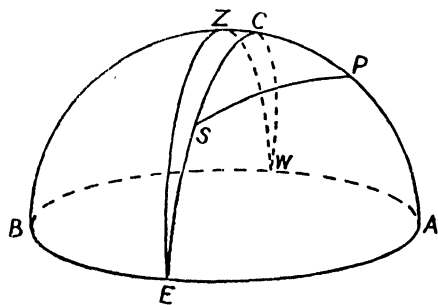


Fig. 39a

observations as the resulting determination of the latitude is in error by an amount equal to the angle which the transverse axis of the telescope makes with the horizontal. Thus, in Fig. 39a, if P denotes the celestial pole, Z the zenith, and E W the east and west points on the horizon, then, if the striding level shows an error in the horizontality of the transverse axis of the telescope, the circle upon which the observations are actually made will be E C W instead of the true prime vertical E Z W. The star is observed to transit at the point S, the angle S P C = t , and the angle S C P is a right angle.

Thus, we shall get

$$\cot CP = \tan \text{declination} \times \sec t.$$

The true co-latitude is then $CP \pm ZC$, the + sign being taken if, as in the figure, C is on the same side of Z as P, and the - sign being used if C and P are on opposite sides of Z. This is determined by the direction of the level error, and ZC = the angular measure of the level error.

Thus, to make the correction, the computation for latitude is made in the ordinary way, and then we add or subtract the striding level error.

EXAMPLE.—*At a place in S. latitude the interval between the passage of Sirius across the prime vertical is 6 hrs. 09 min. $19\frac{1}{3}$ sec. mean time. The mean readings of the bubble on striding level were 10 N. and 14 S., each division being = $20''$. The declination of the star = $16^\circ 35' 33''$. Determine the latitude.*

$$\begin{aligned}
 & 6 \text{ hrs. } 09 \text{ min. } 19\frac{1}{3} \text{ sec. of mean time} \\
 & \quad = 6 \text{ hrs. } 10 \text{ min. } 20 \text{ sec. of sidereal time} \\
 & \quad = 92^\circ 35' 00'' \text{ of arc} \\
 \tan \text{ lat.} &= \tan \text{ dec.} \times \sec. 46^\circ 17' 30''. \\
 \tan \text{ dec.} & \quad \quad \quad \quad \quad \quad \quad \quad 9.4741732 \\
 \cos 46^\circ 17' 30'' & \quad \quad \quad \quad \quad \quad \quad 9.8394702 \\
 & \quad \quad \quad \quad \quad \quad \quad \quad \hline
 & \quad \quad \quad \quad \quad \quad \quad 9.6347030
 \end{aligned}$$

$$\therefore \text{ lat.} = 23^\circ 19' 37''.$$

But the striding level error necessitates a correction

$$= \frac{14 - 10}{2} \times 20 = 40''.$$

As the south end of the transverse axis is the higher, the derived latitude is too small.

$$\therefore \text{ corrected latitude} = 23^\circ 20' 17'' \text{ S.}$$

Fourth method : By the altitude of the pole star at any time

Provided that the exact local time and the approximate longitude are known, the latitude may be found from an altitude observation of a close circumpolar star at any time. In the northern hemisphere the pole star is commonly selected for this purpose, and special tables are given in the *Nautical Almanac* and in the *Star Almanac* for reducing the observations. In the southern hemisphere unfortunately there is no bright star sufficiently near to the pole to make the method a convenient one for the surveyor.*

In Fig. 40, let S be the circumpolar star, Z the zenith, and P the pole as before. Then, with the previous notation, if

$$\begin{aligned}
 z &= S Z, \text{ the zenith distance,} \\
 p &= S P, \text{ the polar distance of the star,} \\
 c &= P Z, \text{ the co-latitude,} \\
 t &= \text{the hour angle } S P Z.
 \end{aligned}$$

* σ Octanis may be used, but its approximate position will have to be computed in order that it may be located.

From the triangle $S P Z$ we have

$$\cos z = \cos c \cos p + \sin c \sin p \cos t,$$

or, if a is the observed altitude, and φ the latitude

$$\sin a = \sin \varphi \cos p + \cos \varphi \sin p \cos t.$$

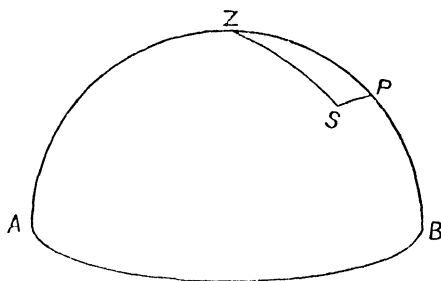


Fig. 40

Now a will differ from φ by a small quantity, which is always less than p . In the case of the pole star p is also small, being about $58'$ at present. Let

$$a = \varphi + x,$$

where x is a small correction.

$$\therefore \sin \varphi \cos x + \cos \varphi \sin x = \sin \varphi \cos p + \cos \varphi \sin p \cos t.$$

$$\therefore \sin \varphi \left(1 - \frac{x^2}{2} + \dots\right) + \cos \varphi \left(x - \frac{x^3}{6} + \dots\right)$$

$$= \sin \varphi \left(1 - \frac{p^2}{2} + \dots\right) + \cos \varphi \cos t \left(p - \frac{p^3}{6} + \dots\right).$$

Neglecting the square and higher powers of x and p in this equation, we get $x = p \cos t$, which is the value of x to a first approximation.

Next, retaining the squares of x and p , but neglecting the higher powers, we get

$$x \cos \varphi = p \cos \varphi \cos t - \frac{p^2}{2} \sin \varphi + \frac{x^2}{2} \sin \varphi.$$

Substituting for x^2 the value $p^2 \cos^2 t$, we obtain then as a second approximation

$$x = p \cos t - \frac{1}{2} \tan \varphi \sin^2 t \cdot p^2.$$

The second term in this expression is very small, and as $\tan \varphi$ differs from $\tan a$ by only a small quantity, the difference when multiplied by p^2 will be too small to take into account, so that we may write

$$x = p \cos t - \frac{1}{2} \tan a \sin^2 t \cdot p^2.$$

In this formula x and p are in circular measure, but if x and p are measured in seconds we may write

$$x = p \cos t - \frac{1}{2} p^2 \tan a \sin^2 t \sin 1'',$$

so that we have for the latitude

$$\varphi = a - p \cos t + \frac{1}{2} p^2 \tan a \sin^2 t \sin 1''$$

The formula is, of course, an approximation only, but it can be shown that it is sufficiently accurate to give the result within $1''$ of the truth.

To determine t , the sidereal time must be known accurately at the moment of observation, and t is then the difference between the sidereal time and the right ascension of the star turned into angular measure.

Four altitudes should be taken in as quick succession as possible, one with F.R., two with F.L., and then again one with F.R., the alidade level being read at each observation, and the chronometer times noted. The mean of the altitudes and the mean of the chronometer times are then taken as the basis for the reduction as a single observation.

(Note.—A convenient table (" Pole Star Table ") for determination of latitude from observations of Polaris is given in the *Star Almanac for Land Surveyors*. The same table provides data for azimuth determinations, using Polaris.)

A rough method for the determination of latitude by noting the rate at which altitude of sun or star changes near the prime vertical

This is only a very rough and approximate method at best, but it is interesting because of its simplicity, and because it requires no knowledge of either the local time or the declination of the body observed. But it is not to be classed along with the previous methods.

In Fig. 41, let Z be the zenith point, P the celestial pole, and RS two consecutive positions of the sun or star. The change of altitude will be measured by the difference between the arcs ZR and ZS , and the interval of time between the two positions will be measured in angular measure by the angle SPR .

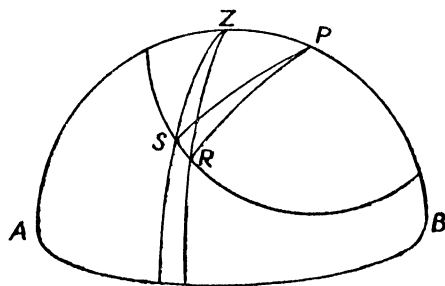


Fig. 41

In the triangle ZPR , ZR = zenith distance = z , PZ = co-latitude = c , PR = polar distance = p , RZP = azimuth measured from elevated pole = A , ZPR = hour angle = B .

In the triangle ZPS , suppose that z has become changed to $z - y$, and B to $B - x$, c and p remaining unaltered.

Then from the formulæ of spherical triangles, we have

$$\cos z = \cos p \cos c + \sin p \sin c \cos B,$$

and $\cos (z - y) = \cos p \cos c + \sin p \sin c \cos (B - x).$

Subtracting these expressions, and regarding x and y as small quantities, so that we may write $\cos x = 1$, $\sin x = x$, etc., we obtain

$$y \sin z = \sin p \sin c \sin B \cdot x.$$

But
$$\frac{\sin z}{\sin B} = \frac{\sin p}{\sin A},$$

$\therefore y = x \cdot \sin c \sin A,$

or
$$\cos \text{latitude} = \frac{y}{x} \operatorname{cosec} \text{azimuth}.$$

Thus, in order to determine the latitude, all we have to do is to measure the change of altitude y that takes place in a given time whose angular measure is x . If t = the interval of time in seconds, $x = 15 t$ seconds of arc.

As the ratio $\frac{y}{x}$ is to be multiplied by cosec A , and the observation is made near the prime vertical, an error in the azimuth A will have but a small effect upon the result.

A convenient way of making the observations is to take the time required by the sun, in the afternoon or early morning, to cross the horizontal wire of the telescope, observing at the same time the

sun's approximate bearing. For an afternoon observation, bring the sun's lower limb into contact with the wire and start the stop watch. When the sun is about bisected by the wire, read the approximate azimuth of its centre. Stop the watch at the instant that the upper limb becomes tangent to the wire.

EXAMPLE.—*At a place in south latitude on March 17, the sun took 2 min. 46.4 secs. to transit the horizontal wire of a theodolite the bearing of its centre being 289° 20'.*

$$\text{Diameter of sun} = 32' 11.3'' = 1,931.3''$$

$$15 \times 2' 46.4'' = 15 \times 166.4 = 2,496$$

$$\cos \text{ lat.} = \frac{1,931.3}{2,496} \times \operatorname{cosec} 109^\circ 20'.$$

$$\log 1,931.3 \quad . \quad . \quad . \quad . \quad 3.2858497$$

$$\log \operatorname{cosec} 109^\circ 20' = \sec 19^\circ 20' \quad 10.0252082$$

$$13.3110579$$

$$\log 2,496 \quad . \quad . \quad . \quad . \quad 3.3972446$$

$$\log \cos 34^\circ 55', \quad . \quad . \quad . \quad . \quad 9.9138133$$

Therefore, the latitude is $34^\circ 55'$ S.

It will be found on trial in this example that if the azimuth is 1° out, the computed latitude is about $30'$ in error, and we must know the azimuth of the sun within $2'$ if we wish to find the latitude to the nearest minute. If the observation had been made with the sun nearer to the prime vertical, however, an error in azimuth would not produce anything like so serious an effect.

To get anything like accurate results, the time must be measured with great precision. In the above example an error of one whole second in the time causes an error of nearly three-quarters of a degree in the latitude. With a stop-watch the time may be estimated to the tenth of a second, but it is evident that only approximate determinations of latitude are possible by this method with the instruments at the disposal of the surveyor.

The method is of interest, because it may be practised upon a star without the use of any *Nautical Almanac* tables. It will give best results in high latitudes with observations made as near to the prime vertical as possible.

There are many other methods by which latitude may be determined, but for the most part they are not so convenient nor do they allow of the same elimination of instrumental errors as the four

standard methods described. The following is an illustration of a method in which horizontal angles only have to be measured :—

Determination of latitude by the measurement of the horizontal angle between two circumpolar stars at their greatest elongations one on each side of the meridian

Let c be the co-latitude, p_1 and p_2 the respective polar distances of the two stars, A_1 and A_2 the azimuths at elongation, one being measured to the east and the other to the west.

The measured angle = $A_1 + A_2$.

$$\text{Then} \quad \sin p_1 = \sin c \sin A_1 \quad . \quad . \quad . \quad (1)$$

$$\text{and} \quad \sin p_2 = \sin c \sin A_2 \quad . \quad . \quad . \quad (2)$$

(1) + (2) gives

$$2 \sin \frac{p_1 + p_2}{2} \cos \frac{p_1 - p_2}{2} = 2 \sin c \sin \frac{A_1 + A_2}{2} \cos \frac{A_1 - A_2}{2}.$$

(1) - (2) gives

$$2 \cos \frac{p_1 + p_2}{2} \sin \frac{p_1 - p_2}{2} = 2 \sin c \cos \frac{A_1 + A_2}{2} \sin \frac{A_1 - A_2}{2}.$$

Dividing one equation by the other gives

$$\tan \frac{p_1 + p_2}{2} \cot \frac{p_1 - p_2}{2} = \tan \frac{A_1 + A_2}{2} \cot \frac{A_1 - A_2}{2}.$$

Since $A_1 + A_2$ is known, this enables $A_1 - A_2$ to be computed. Hence A_1 is found.

$$\text{Then} \quad \sin c = \frac{\sin p_1}{\sin A_1}.$$

EXAMPLE (taken from *Handbook of Instructions to South Australian Surveyors*).—Observed horizontal angle $77^\circ 45'$ between Canopus and β Tri. Aus. at opposite elongations, polar distances $37^\circ 22'$ and $26^\circ 57'$.

$$\tan \frac{p_1 - p_2}{2} = \tan 5^\circ 12' 30'', \quad . \quad 8.9597747$$

$$\tan \frac{A_1 + A_2}{2} = \tan 38^\circ 52' 30'', \quad . \quad 9.9064310$$

$$18.8662057$$

$$\tan \frac{p_1 + p_2}{2} = \tan 32^\circ 09' 30'', \quad . \quad 9.7984562$$

$$\tan \frac{A_1 - A_2}{2} \quad . \quad . \quad . \quad 9.0677495$$

$$\therefore \frac{A_1 - A_2}{2} = 6^\circ 40'$$

$$\text{and} \quad \frac{A_1 + A_2}{2} = 38^\circ 52' 30''$$

$$\therefore A_1 = 45^\circ 32' 30''$$

$$\sin p_1 = \sin 37^\circ 22', \quad . \quad . \quad . \quad 9.7831268$$

$$\sin A_1 = \sin 45^\circ 32' 30'', \quad . \quad . \quad . \quad 9.8535522$$

$$\cos \text{lat.}, \quad . \quad . \quad . \quad . \quad 9.9295746$$

$$\therefore \text{latitude} = 31^\circ 45' 20''.$$

TABLE GIVING VALUES OF m FOR REDUCTION OF CIRCUM-MERIDIAN OBSERVATIONS

$$m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}$$

The values of m are given in seconds of arc

Value of t in Minutes of Time	Additional Seconds of Time					
	0	10	20	30	40	50
0	0.0	0.1	0.2	0.5	0.9	1.4
1	2.0	2.7	3.5	4.4	5.4	6.6
2	7.8	9.2	10.7	12.3	14.0	15.8
3	17.7	19.7	21.8	24.0	26.4	28.8
4	31.4	34.1	36.9	39.8	42.8	45.9
5	49.1	52.4	55.8	59.4	63.0	66.8
6	70.7	74.7	78.8	83.0	87.3	91.7
7	96.2	100.8	105.6	110.4	115.4	120.5
8	125.7	130.9	136.3	141.8	147.5	153.2
9	159.0	165.0	171.0	177.2	183.5	189.8
10	196.3	202.9	209.6	216.4	223.4	230.4
11	237.5	244.8	252.2	259.6	267.2	274.9
12	282.7	290.6	298.6	306.7	315.0	323.3
13	331.7	340.3	349.0	357.7	366.6	375.6
14	384.7	393.9	403.3	412.7	422.2	431.9
15	441.6	451.5	461.5	471.5	481.7	492.0
16	502.5	513.0	523.6	534.3	545.2	556.1
17	567.2	578.4	589.6	601.0	612.5	624.1
18	635.9	647.7	659.6	671.6	683.8	696.0

For intermediate values of t the corresponding values of m may be found by simple interpolation.

EXERCISES

1. At a place in latitude North, the meridian zenith distance of α Cephei (declination $+ 61^{\circ} 58' 21.1''$) is determined as $26^{\circ} 54' 28.3''$ N. The meridian zenith distance of α Aquilæ (declination $+ 8^{\circ} 29' 22.7''$) is found as $26^{\circ} 34' 27.5''$ S.

Find the latitude of the place.

Ans. $35^{\circ} 03' 51.5''$.

2. In latitude 30° S. the times of transit of a star whose declination is $- 20^{\circ}$ are observed across the prime vertical. If the direction of the prime vertical is in error by 1° , show that the measured interval of time will be too great by about 14 seconds.

3. An observation made in Antarctica on 1912 November 19, gave the altitude of the sun's centre as $42^{\circ} 07.8'$, the temperature being 17° F., and the barometer reading 27.2 inches. Correct for refraction and parallax, and compute the latitude of the place, given that the sun's declination is $- 19^{\circ} 21.6'$.

Ans. $67^{\circ} 14.7'$ S.

4. The declination of the sun being $- 20^{\circ} 39.9'$, its meridian altitude is observed as $43^{\circ} 17'$. The correction for refraction and parallax being $- 00.9'$, determine the latitude of the place.

Ans. $67^{\circ} 23.8'$ S.

5. The sun is observed on the prime vertical, morning and afternoon, the times by watch being $7^{\text{h}} 30^{\text{m}}$ and $4^{\text{h}} 14^{\text{m}}$. The sun's declination is $17^{\circ} 31' 30''$. Compute the latitude.

Ans. $37^{\circ} 17' 18''$

6. At a place in S. latitude the interval between the passages of Sirius across the prime vertical is $6^{\text{h}} 9^{\text{m}} 19.33^{\text{s}}$ mean time. The mean readings of the bubble on striding level were 10 N. and 14 S., each division being $= 20''$. The declination of the star is $- 16^{\circ} 35' 33''$. What was the latitude of the place of observation ?

Ans. $23^{\circ} 20' 17''$ S.

7. The hour angle of Aldebaran (dec. $+ 16^{\circ} 20' 15''$) when on the prime vertical was found to be $4^{\text{h}} 35^{\text{m}} 19.5^{\text{s}}$. What was the latitude of the place of observation ?

Ans. $39^{\circ} 04' 03''$ N.

8. At a place in the southern hemisphere γ^2 Ceti (dec. $+ 2^\circ 51' 22''$) was observed at equal altitudes of $48^\circ 02' 20''$, and the interval in mean solar time between the two occurrences was $16^h 12^m$. Required the latitude of the place.

Ans. $39^\circ 05'$

9. Antares crossed the prime vertical at $13^h 52^m$ sidereal time. Find the latitude of the place of observation.

R.A. of Antares $16^h 23^m$

Dec. ,, ... $-26^\circ 13'$

Ans. $31^\circ 54' 49''$ S.

10. The altitudes of a star when it crosses the meridian and prime vertical are respectively 65° and 10° (corrected). Find the star's declination and latitude of the place.

Ans. Lat., $29^\circ 58' 39''$.

Dec., $4^\circ 58' 39''$ ($-$ in S. lat.,
or $+$ in N. lat.)

11. The altitude of Sirius on the prime vertical is found to read $39^\circ 48'$. The declination of Sirius is $- 16^\circ 35' 20''$. Find the latitude of the observing station. Allow for refraction.

Ans. $26^\circ 30'$ S.

12. At a place in south latitude the altitude of a star was observed at its upper and at its lower culminations, the altitude corrected for refraction at upper culmination being $60^\circ 45' 15''$, and at lower culmination $10^\circ 16' 15''$. Find the latitude of the place of observation and the declination of the star.

Ans. Lat., $35^\circ 30' 45''$ S.

Dec., $- 64^\circ 45' 30''$.

13. At a place in south latitude, the magnetic bearing of β Hydri at its western elongation was $185^\circ 47' 35''$, and that of θ Carinae at its eastern elongation was $137^\circ 24' 42''$.

Declination of β Hydri, ... $- 77^\circ 44' 29''$

,, θ Carinae, ... $- 63^\circ 56' 36''$

Determine the latitude of the place and the magnetic variation.

Ans. Latitude, $36^\circ 24' 56''$

Variation, $9^\circ 30' 20''$ E.

14. The altitude of Regulus at $10^{\text{h}} 08^{\text{m}}$ sidereal time was $46^{\circ} 52' 32''$ (fully corrected) in a place of S. latitude.

R.A. of Regulus $10^{\text{h}} 03^{\text{m}} 17^{\text{s}}$

Dec. „ $+12^{\circ} 26'$

What was the correct altitude when on the meridian ?

Ans. $46^{\circ} 53' 26''$

15. On the 9th March, at a place south of equator in 140° E. longitude the following altitudes of α Virginis (Spica) were observed near its meridian passage and their times taken with a chronometer keeping local mean time :—

Observed altitudes	Local mean times
$57^{\circ} 40' 36''$	$2^{\text{h}} 02^{\text{m}} 18^{\text{s}}$ a.m.
$44' 34''$	$05^{\text{m}} 54^{\text{s}}$ „
$48' 40''$	$10^{\text{m}} 50^{\text{s}}$ „
$50' 10''$	$15^{\text{m}} 58^{\text{s}}$ „
$49' 30''$	$22^{\text{m}} 10^{\text{s}}$ „
$46' 40''$	$27^{\text{m}} 00^{\text{s}}$ „
$42' 35''$	$31^{\text{m}} 02^{\text{s}}$ „

The sidereal time at 0^{h} G.M.T. on March 9th was $11^{\text{h}} 03^{\text{m}} 21.19^{\text{s}}$.

R.A. of Spica = $13^{\text{h}} 20^{\text{m}} 41.4^{\text{s}}$.

Dec. „ = $-10^{\circ} 43' 00''$

Find the latitude of the place.

Ans. $42^{\circ} 52' 51''$ S.

16. The declination of a star being -40° , what are the latitudes of the places where its meridian altitude will be 80° ?

Ans. 50° or 30° S.

17. In south latitude two stars are observed on the meridian, one north and the other south of the zenith, the difference of zenith distances being found to be $13' 03.45''$ the declinations of the stars being $-42^{\circ} 44' 04.63''$ and $-45^{\circ} 38' 37.48''$ respectively. Find the latitude.

Ans. $44^{\circ} 17' 52.8''$ S.

18. A south circumpolar star was observed at equal intervals shortly before and after its elongation, when it was found to change its altitude from $44^{\circ} 35'$ to $47^{\circ} 35'$, during an interval of $19^{\text{m}} 47^{\text{s}}$, by a watch keeping correct mean time. Find the polar distance of the star and the latitude of the place of observation.

Ans. $37^{\circ} 13' 53''$

Lat. $33^{\circ} 35' 00''$ S.

19. At 6.10 p.m., local mean time, by watch on September 15, in longitude $151^{\circ} 06' 30''$ East, the magnetic bearing of σ Octantis was $170^{\circ} 37' 30''$, the bearing of the referring mark being $72^{\circ} 50' 45''$, and the corrected altitude of the star was $34^{\circ} 36'$.

R.A. of σ Octantis	.	.	19 ^h 12 ^m 48 ^s
Dec.	„	.	—89° 14' 49"
Sidereal time at 0 ^h G.M.T. Sept. 15,	23 ^h 31 ^m 13.7 ^s		
„ „ „ Sept. 14,	23 ^h 27 ^m 16.7 ^s		

Find the latitude of the observer and the true bearing of the referring mark.

Ans. Lat. $33^{\circ} 54' 18''$ S.
Bearing, $81^{\circ} 52' 12''$

20. On March 6, the altitude of Polaris, when corrected for instrumental errors and refraction, is found to be $46^{\circ} 17' 28''$, the mean time of observation being 7^h 43^m 35^s p.m. and the longitude of the place 37° W.

Sidereal time at 0 ^h G.M.T., March 6,	10 ^h 51 ^m 31.5 ^s		
R.A. of Polaris	.	.	1 ^h 27 ^m 37.7 ^s
Dec.	„	.	+88° 51' 08"

Find the latitude.

Ans. $46^{\circ} 03' 36''$ N.

21. The observatory at Stockholm is in latitude $59^{\circ} 20' 33''$ N., and that at the Cape of Good Hope in latitude $33^{\circ} 56' 03.5''$ S. The declination of Sirius is $-16^{\circ} 35' 22''$. Find the altitudes of Sirius when on the meridian at Stockholm and at the Cape of Good Hope respectively.

Ans. $14^{\circ} 04' 05''$ and
 $72^{\circ} 39' 18.5''$

22. The upper transit of a south circumpolar star was observed to occur at 7^h 05^m 28^s p.m. local time, and to reach its greatest western elongation at 11^h 44^m 30^s p.m., when its observed azimuth was $33^{\circ} 48'$.

Find the latitude of the place of observation and the declination of the star.

Ans. Lat., $31^{\circ} 02' 52''$ S.
Dec., $-61^{\circ} 32' 11''$

23. On 1951 March 13, at a place south of the equator, in longitude $9\frac{1}{4}$ hrs. E., at 6 minutes before apparent noon the altitude of the sun's lower limb was found to be $58^{\circ} 04' 20''$, at which time clouds prevented further observation. The sun's declination at G.A.N. March 13 is $-3^{\circ} 07' 51.7''$, and on March 12, $-3^{\circ} 31' 28.1''$. Find the latitude of the place by reduction to the meridian, the sun's semi-diameter being $16' 07''$, its parallax $5''$ and refraction $37''$.

Ans. $94^{\circ} 55' 12''$ S.

24. The altitudes of a star when it crosses the meridian and the prime vertical of a place are a and b . If φ is the latitude of the place, show that

$$\cot \varphi = \tan a - \sec a \sin b.$$

25. The meridian altitude of Altair is $51^{\circ} 55' 45''$, its declination being $+8^{\circ} 34' 34''$, and the meridian altitude of β Pavonis is $52^{\circ} 54' 32''$, its North polar distance being $156^{\circ} 36' 18''$. Find the latitude of the place of observation.

Ans. $29^{\circ} 30' 15.5''$ S.

26. At a place, south of the equator, the meridian zenith distances of the two stars γ^2 Norma and σ Scorpii were observed, the former to the south, the latter towards the north. The observed difference of the zenith distances was found to be $19^{\circ} 21''$. Find the latitude of the place of observation.

Declination of γ^2 Norma . . . $-49^{\circ} 57' 08.3''$

„ σ Scorpii . . . $-25^{\circ} 23' 31.2''$

Another observer, stationed some distance to the north, found the difference of the zenith distances of these stars to be exactly the same. Determine his latitude also.

Ans. $37^{\circ} 50' 00.25''$ and
 $37^{\circ} 30' 39.25''$

27. The mean altitude reading from four observations of Polaris was $51^{\circ} 39' 34.25''$, the mean readings of the alidade level E., 5.5, O. 6.5, one division of level = $15''$, mean chronometer time $19^h 09^m 54.8^s$, the chronometer keeping L.M.T. and being $3^m 24^s$ fast. The longitude of station was $0^h 02^m 09^s$ E. G.S.T. at 0^h

G.M.T. on the day of observation was $1^{\text{h}} 03^{\text{m}} 35.8^{\text{s}}$. Declination of Polaris, $+88^{\circ} 45' 50.8''$; R.A. of Polaris, $1^{\text{h}} 22^{\text{m}} 26^{\text{s}}$. Barometer, 30.27in. Thermometer, 42° F. Compute the latitude of the place.

Ans. $51^{\circ} 23' 34''$ N.

THE DETERMINATION OF TIME BY OBSERVATION

IN this chapter it is proposed to consider the principal methods available to the surveyor for the practical determination of the local mean or sidereal time by observation. Other methods have been devised, but the methods about to be described are those that have proved in practice to be the most convenient and satisfactory. Nearly all the ordinary time determinations of the surveyor are made by the second of the following methods, a convenient observation that may be carried out in the daylight, and by which the time may be readily found with ordinary instruments with an error of not more than one or two seconds. One second of time will, of course, correspond to $15''$ of hour angle.

First method : By meridian transits

We know that the local sidereal time at the instant that a star is on the meridian is measured by the R.A. of the star. Consequently, if we make the observation upon a star whose R.A. is known, by setting a theodolite up in the meridian and noting the time of transit of the star across the vertical wire, we have clearly a very simple way of finding the sidereal time at that instant and thus of determining the error of a watch or chronometer.

A similar observation may be made upon the sun, by noting the times of transit of the E. and W. limbs. The mean of these times will be the time of transit of the sun's centre, which takes place at *apparent* noon. From the *Nautical Almanac* we can find the G.M.T. of G.A.N. for the given date, from which the L.M.T. of L.A.N. may be found. If only one limb be observed, then allowance must be made for the time occupied by the sun's semi-diameter in crossing the meridian.

EXAMPLE.— On 1951 December 1, at a place in longitude $9^{\text{h}} 45^{\text{m}}$ E., the meridian times of transit of the E. and W. limbs of the sun across the vertical wire of a theodolite were taken with a watch supposed to keep the standard time of the meridian $9^{\text{h}} 30^{\text{m}}$ E. The observed times of transit being $11^{\text{h}} 32^{\text{m}} 32.5^{\text{s}}$ and $11^{\text{h}} 34^{\text{m}} 52.5^{\text{s}}$, determine the error of the watch.

From *N.A.*, G.M.T. of G.A.N., December 1 11^h 48^m 50.4^s
 Correction for longitude ($-21.8^{\circ} \times .40625$) — 08.9^s

L.M.T. of L.A.N., December 1 . 11^h 48^m 41.5^s
 Standard time slow on L.M.T. . . 15^m 00.0^s

L.Std.T. of L.A.N. . . . 11^h 33^m 41.5^s

But time of transit of sun's centre, *i.e.*, the
 mean of the observed times, . . . 11^h 33^m 42.5^s

$\therefore$ the watch is fast by . . . 01.0^s

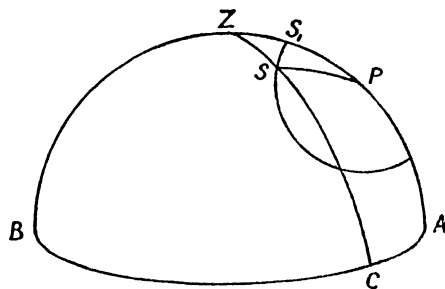


Fig. 42

The effect of an error in the direction of the meridian

If the instrument be in accurate adjustment, but the directions of the meridian be in error, then the meridian set out will pass through the zenith of the observer, but not through the celestial pole. In Fig. 42, let ZC denote the erroneous meridian, making an angle that we will call e with the true meridian ZPA . Then a star will intersect the apparent meridian at S , and the time noted will be either too soon or too late, according as the meridian is wrongly marked out to the east or west of the true direction, the error being measured by the hour angle SPZ , which we will call h .

$$PZ = c = \text{co-latitude}$$

$$PS = p = \text{polar distance of star.}$$

Then, in the triangle PZS ,

$$\cot p \sin c = \cot e \sin h + \cos c \cos h.$$

Since e and h are both small, we may write, without appreciable error, h and e instead of $\sin h$ and $\sin e$ respectively, and may put $\cos h$ and $\cos e$ each $= 1$.

$$\therefore e (\cot p \sin c - \cos c) = h.$$

$$\therefore h = e \frac{\sin (c - p)}{\sin p}. \quad (1)$$

Thus h will have its smallest value when p is nearly $= c$; that is to say, when the observed star makes its meridian transit near the zenith.

If in equation (1) $c = 60^\circ$, or the latitude of the place is 30° , and $p = 40^\circ$, then, if $e = 01'$ of arc, $h = 32''$ of arc or 2 seconds of time. Thus, in this case, an error of 1 minute of arc in the direction of the meridian will make the time of transit wrong by two seconds.

It is clear, therefore, that the method requires the meridian to be very accurately set out, and the instrument must be in perfect adjustment, if good results are to be obtained by this method.

In Fig. 42 we have illustrated the case where the star transits above the celestial pole. If the lower transit had been observed, then the angle h would be the supplement of the angle $S P Z$, and in this case the formula would become

$$h = e \frac{\sin (c + p)}{\sin p}.$$

Both are included in the general formula,

$$h = e \frac{\sin \text{zenith distance}}{\sin p}, \text{ or } e \frac{\cos \text{alt.}}{\cos \text{dec.}},$$

which applies to all cases.

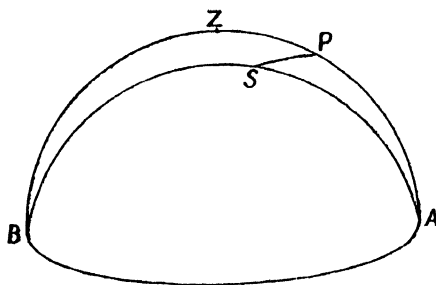


Fig. 42a

The error is thus very great if the polar distance of the star is small, and is least for those stars that transit near the zenith.

The effect of an error in the horizontality of the transverse axis

The direction of the meridian may be accurately set out with the telescope horizontal or nearly so, and yet, if the transverse axis is not horizontal, the line of sight may depart considerably from the meridian at high altitudes. If the angle made by the transverse axis with the horizontal be determined by means of the striding level, the necessary correction to the time of transit may be made as follows :—

In Fig. 42a, the meridian actually swept out by a telescope with the transverse axis slightly tilted is represented by A S B, A and B being the north and south points, and Z the zenith. The transit of the star is observed in consequence at a point S on this circle, and the error in time is measured by the angle S P Z.

In the triangle B P S

S P = p = polar distance of star,

B P = $180^\circ - \varphi$ = supplement of latitude,

Angle P B S = e = error measured by striding level,

Angle B P S = x = required error in time of transit.

$$\therefore \cot S P \sin B P = \cot e \sin x + \cos B P \cos x.$$

$\therefore$ treating x and e as small quantities,

$$\cot p \sin \varphi = \frac{x}{e} - \cos \varphi$$

$$\therefore x = e \frac{\sin (\varphi + p)}{\sin p}, \text{ or } e \frac{\sin \text{altitude}}{\cos \text{dec.}}$$

This formula gives us the hour angle of the star at the moment of observation. Usually e and, therefore, x will be in seconds of arc, and x must then be divided by 15 to determine the error of the observed time of transit in seconds of time. Clearly the transit will be observed either too soon or too late according to the direction of tilt of the transverse axis.

If the star transits below the pole, x will be the supplement of the angle B P S, and we get

$$x = e \frac{\sin (\varphi - p)}{\sin p}, \text{ which again } = e \frac{\sin \text{alt.}}{\cos \text{dec.}}$$

The error in time in this case increases with the altitude.

EXAMPLE.—At a place in latitude 30° S. the sidereal time of transit of a star across the meridian is observed to be $12^h 30^m 17.5^s$, the declination of the star being $-58^\circ 30'$. The readings of the striding level, one division of which = $13''$, are:—

L.	R.
6.0	5.0
3.6	7.2
9.6	12.2
	9.6
	4) 2.6
	0.65

$$0.65 \times 13 = 8.45''.$$

$$\therefore \text{error in hour angle} = 8.45 \times \frac{\sin 61^\circ 30'}{\sin 31^\circ 30'} = 14.21''.$$

This is equivalent to 0.95 second of time.

As the right-hand side of the axis is the higher, and the telescope is directed towards the south, the transit is, therefore, observed too soon by this amount, and the corrected time of transit across the meridian is $12^h 30^m 18.45^s$.

Meridian transits on both sides of the zenith

A considerable improvement may be made in the accuracy of the method by taking observations of the times of transit of two stars, one on each side of the observer's zenith.

In Fig. 43, let Z denote the zenith, P the celestial pole, A Z P B the direction of the true meridian, and C Z D the direction of the meridian actually set out, the figure being drawn as though the celestial sphere were viewed from above. Suppose that the times

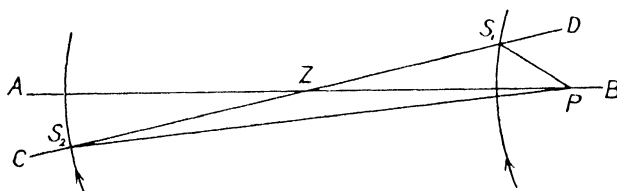


Fig. 43

of transit of two stars are observed, one at S_1 and the other on the opposite side of the zenith as at S_2 . Then, since both stars move in the same direction, as shown by the arrows, if the observed time of transit of S_1 is later than it should be, owing to the faulty determination of the meridian, the time of transit of S_2 will be correspondingly earlier. If the stars are well selected, it may be that the time errors of the two observations are equal and opposite, so that the mean of the two results will give a correct time determination

in spite of the error in the setting out of the meridian. This will be the case if the hour angle $S_1 P Z$ is = the angle $S_2 P Z$, for then one observation will be just as much too soon as the other one is too late.

The conditions that this may be the case are readily obtained as follows :—

Let angle $B Z D = e$ = meridian error, and suppose that the hour angle $S_1 P Z = S_2 P Z = h$.

c = co-latitude $P Z$.

Then, from the triangles $S_1 P Z$, $S_2 P Z$,

$$\frac{\sin h}{\sin e} = \frac{\sin Z S_1}{\sin P S_1} = \frac{\sin Z S_2}{\sin P S_2}.$$

But, since the error e is small, we may write very approximately $P S_1 = c - Z S_1$ and $P S_2 = c + Z S_2$.

$$\therefore \frac{\sin (c - Z S_1)}{\sin Z S_1} = \frac{\sin (c + Z S_2)}{\sin Z S_2}.$$

$$\therefore \sin c \cdot \cot Z S_1 - \cos c = \sin c \cot Z S_2 + \cos c.$$

$$\therefore \cot Z S_1 - \cot Z S_2 = 2 \cot c.$$

This, then, is the condition that has to be satisfied by the zenith distance of the two stars if the observations are to be so balanced that by taking the mean of the two we eliminate, or nearly so, the error due to a faulty setting out of the meridian.

The following table, based upon the above formula, gives the proper zenith distance of the star on the opposite side of the zenith to the pole, corresponding to different zenith distances of the other observed star, for different latitudes :—

Zenith distance of star on same side as pole	Zenith distance of star on opposite side of zenith to the pole							
	Lat.10°	Lat.20°	Lat.30°	Lat.40°	Lat.50°	Lat.60°	Lat.70°	Lat.80°
5°	5° 09'	5° 20'	5° 34'	5° 51'	6° 18'	7° 09'	9° 34'	85° 0'
10°	10° 39'	11° 26'	12° 29'	14° 30'	16° 55'	24° 22'	80° 0'	..
20°	22° 40'	26° 21'	32° 08'	43° 05'	70° 0'	..	..	..
30°	35° 56'	44° 53'	60° 0'	86° 55'	..	..	..	..
40°	50° 0'	65° 07'	87° 53'	..	..	..	..	..
50°	64° 04'	83° 39'	..	..	..	..	..	..
60°	77° 20'	..	..	..	..	..	..	..
70°	89° 21'	..	..	..	..	..	..	..

The advantage of selecting the two stars in this way may be illustrated by a computed example. Suppose that the place of observation is in latitude 30° , and that the polar distance of the star observed on the same side of the zenith as the pole is 40° , so that its zenith distance is about 20° . Suppose, further, that the marked meridian is as much as 1° in error.

Computing with these data the spherical triangle SPZ of Fig. 42, it may be shown that the hour angle SPZ is $2^m 04.8^s$. In other words, the observed transit will take place too soon by this amount.

Now, according to the table, the star observed on the opposite side of the zenith should have a zenith distance of $32^\circ 08'$. Suppose it actually has a zenith distance of 32° , equivalent to a polar distance of 92° . Then, computing in the same way the hour angle of this star when on the faulty meridian, we find that its observed transit will be too late by $2^m 04^s$.

Thus from one observation the chronometer would be set too fast by $2^m 04^s$, and from the other it would be set too slow by about the same amount, and the mean of the two observations would give the time correct to the nearest second in spite of the fact that the direction of the meridian is 1° in error.

If, however, the zenith distances of the two stars are not balanced in the way indicated, the accuracy of the mean result is nothing like so great. If, for example, the two zenith distances were the same, the star observed on the opposite side of the zenith to the pole having a zenith distance of 20° , or a polar distance of 80° . Then, on computing the spherical triangle, it will be found that the observed transit of this star is too late by $1^m 24^s$, so that the mean of the two observations is then in error to the extent of about 20 seconds.

Second method : By extra-meridian observations of sun or star

This is, as a rule, the most convenient and suitable method for the determination of time by the surveyor. It consists in the measurement of the altitude of sun or star when out of the meridian, at the same instant noting the chronometer time. Then, from a knowledge of the latitude of the place and the declination of the body observed we may compute the proper local time at the instant of observation, and so determine the error of the chronometer.

The most favourable time for making such an observation will be when the altitude of the celestial body is changing most rapidly,

and this will be the case when it is near the prime vertical. This position has also other advantages, as we shall see in the course of the discussion.

As an altitude has to be measured, refraction must be allowed for, and as there is considerable uncertainty about this at low altitudes, the star observed should have an altitude of at least 15° .

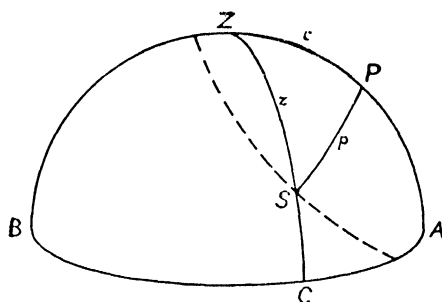


Fig. 44

The method involves the solution of the same spherical triangle that we have discussed in connection with extra-meridian observations for azimuth. Thus, in Fig. 44, if S is the star observed, then in the spherical triangle ZPS we know the three sides :—

$ZP = c = \text{co-latitude,}$

$SP = p = \text{polar distance of star,}$

$ZS = z = \text{zenith distance, or the complement of the observed altitude.}$

Therefore, we can compute the hour angle SPZ, from which we can find the local sidereal time if we know the R.A. of the star, or this at once gives us the local apparent time in the case of the sun.

Let the angle SPZ = h .

Then, we have three available formulæ adapted to logarithmic computation, any one of which may be used for computing h . They are—

$$\begin{aligned} \text{if } s &= \frac{1}{2} (z + c + p) \\ \sin \frac{h}{2} &= \sqrt{\frac{\sin (s - c) \cdot \sin (s - p)}{\sin c \cdot \sin p}}, \\ \cos \frac{h}{2} &= \sqrt{\frac{\sin s \cdot \sin (s - z)}{\sin c \cdot \sin p}}, \\ \tan \frac{h}{2} &= \sqrt{\frac{\sin (s - c) \cdot \sin (s - p)}{\sin s \cdot \sin (s - z)}}. \end{aligned}$$

The choice of a formula

Of the three formulæ, that for $\cos \frac{h}{2}$ is somewhat the simplest, as we must find s in any case, and we have then only to find $s - z$ in addition. With the sine formula we have one more subtraction to make, but there is the advantage that only tables of log sines are used, and there is less risk of mistake in taking out the logarithms.

If, however, we are utilising the same observation, as may be done, for the determination of azimuth in addition, then we shall require to compute also the angle $S Z P$. In this case it is a decided advantage to select the tangent formula for the computation of both angles, for we shall then need only to look up four logarithms, as the same expressions $\sin s$, $\sin (s - c)$, $\sin (s - p)$, and $\sin (s - z)$ will occur in the tangent formulæ for both angles. If, on the other hand, we use the sine or cosine formula for the two angles, it will be necessary to look up six logarithms.

Another important point in the selection of a formula is this : The variation in value of the tangent of an angle, as the angle increases from 0° to 90° , is very much greater than in the case of a sine or cosine. Consequently a table of tangents will enable us to determine the value of an angle with greater precision than a table of sines or cosines. This is of practical importance when the angle under consideration is near to 0° or 90° . Thus there is very little variation in the value of the cosine of an angle up to 2° or 3° , and, if we wish to determine the values of such small angles to seconds, a table of cosines is not nearly so good as a table of tangents. Similarly, there is very little variation in the sine of an angle near to 90° , and it becomes difficult to compute such angles with precision from a sine table. It follows, therefore, that if h is near 0° or near to 90° , the tangent formula is the best one to adopt.

Data necessary for computation

In addition to the measured altitude, we require a knowledge of the latitude of the place and the declination of the body observed. The declination for a star is taken straight from the *Nautical Almanac*, but the declination of the sun has to be found by using approximate values for the longitude and local time. If the result obtained shows that the assumed local time is very much out, the calculation should be repeated by using the corrected value of the local time found from the first computation.

Arrangement of the computation

It is worth some trouble to make a neat form for the computation. A good arrangement reduces the work, and is an aid to accuracy. The following, for instance, is one method adopted :—

$p = 59^\circ 34' 48''$	log sin	= 9.9356770
$c = 76^\circ 05'$	log sin	= 9.9870611
$z = 66^\circ 34' 19''$		<u>19.9227381</u>
$2 \) \ 202^\circ 14' 07''$	subtract from	20.
$s = 101^\circ 07' 03.5''$	log $\frac{1}{\sin p \sin c}$	= 0.0772619
$s - p = 41^\circ 32' 15.5''$	log sin	= 9.8215856
$s - c = 25^\circ 02' 03.5''$	log sin	= 9.6265032
		<u>2 \) \ 19.5253507</u>
$\therefore \frac{1}{2} h = 35^\circ 22' 48''$	log sin	= 9.7626753

Where the same observation is to be utilised for both time and azimuth, a neat device is to proceed as follows :—

log sin $(s - c)$	= say	9.949960
log sin $(s - p)$		9.046045
log sin $(s - z)$		<u>9.875721</u>
		28.871726
subtract log sin s		<u>9.945558</u>
	$2 \)$	<u>18.926168</u>
		9.463084

From this we have simply to subtract log sin $(s - z)$ and log sin $(s - p)$ in order to get $\tan \frac{h}{2}$ and $\tan \frac{Z}{2}$ respectively.

	9.463084	9.463084
log sin $(s - z) =$	<u>9.875721</u>	<u>9.046045</u>
log tan $\frac{h}{2} =$	9.587363	log tan $\frac{Z}{2} =$
		10.417039

Having computed the hour angle to find the time of the observation

In the case of a star the angle S P Z, turned into time by dividing by 15, measures the interval of sidereal time after or before the time of culmination, according as the star is observed on the west or east of the meridian. But the R.A. of the star is equal to the

sidereal time at the instant of culmination. Therefore, the sidereal time at the moment of observation is obtained by adding (or subtracting) the value of h to the R.A. of the star. This may be turned into mean time in the way already discussed.

Thus, if the R.A. of the star is $7^h 30^m$, and the angle h is 35° , the star being observed in the west, then the local sidereal time at the moment of observation is $7^h 30^m + 2^h 20^m = 9^h 50^m$.

If the sun has been observed, the value of the angle h at once gives the interval of solar time before or after the meridian transit of the sun—that is to say, it gives us the local apparent time. We have already seen in Chapter 5 how the local mean time corresponding to an instant of local apparent time may be found.

Suppose, for example, that the angle h , for a sun observation, is $48^\circ 20'$, the observation being made at a place in longitude 60° E. on 1951 May 23 in the afternoon. We have, therefore,

Local apparent time, May 23	.	.	.	15 ^h	13 ^m	20 ^s
Longitude,	.	.	.	4 ^h	00 ^m	00 ^s
∴ Corresponding G.A.T. May 23	.	.	.	11 ^h	13 ^m	20 ^s
G.M.T. at G.A.N. May 23	.	.	.	11 ^h	56 ^m	34.31 ^s

The instant we are concerned with is $46^m 40^s$ before G.A.N. May 23, *i.e.*, 0.0324 day before. 24-hourly difference for period preceding G.A.N. May 23 = 4.24^s .

∴ difference for $46^m 40^s = 4.24 \times 0.0324^*$
 (to be subtracted) = 00.14^s

∴ (12 ^h —equation of time) at given	
L.A.T.	11 ^h 56 ^m 34.17 ^s
∴ equation of time at given L.A.T.	03 ^m 25.83 ^s

It is obvious from the G.M.T. of G.A.N. on May 23 that mean time is behind apparent time on this date. Therefore, to obtain the required L.M.T., we must subtract $03^m 25.83^s$ from $15^h 13^m 20^s$, giving $15^h 09^m 54.17^s$.

* Interpolate using second and higher differences if greater accuracy is required.

Averaging several observations of the same star

In practice it is usual to take at least two, and commonly four, observations in as quick succession as possible, half being taken with F.L. and half with F.R. The computation is then made as though one observation only had been taken, the mean of the altitudes being assumed to be the true altitude at the mean of the noted chronometer times.

The object of this procedure is to eliminate instrumental errors, but this is done at the expense of introducing another error due to the fact that the assumption made is not mathematically exact. The investigation of the magnitude of the error thus introduced into the work is too complex for insertion here, but it may be stated that the surveyor is quite safe in thus averaging altitude observations extending over a range of 2° in altitude under ordinary conditions. The error thus made in an extra-meridian time determination is then generally only a small fraction of a second of time, its exact magnitude depending upon the latitude of the observer, the declination, and hour angle of the heavenly body. It is least when the hour angle is nearly 90° .

Observations on both east and west stars

It is a great improvement in accuracy to take one set of observations upon a star in the east and another corresponding set, under conditions as similar as possible, upon a star in the west. The averaging of two such sets of observations tends to eliminate certain classes of errors, and this should always be done where the highest accuracy is sought. If, for example, the refraction assumed is too great, the corrected altitude will be too low, and the computed time will be too early for a star in the east, while it will be correspondingly too late for a star in the west. If the two errors are about equal, as will be the case if the E. and W. stars make about the same horizontal angle with the meridian, and are observed at about the same altitude, then the average of the two sets of results will be correct. Similarly, the effects of any systematic error in the measurement of altitude are eliminated by pairing sets of observations in this way. The same applies to extra-meridian observations for azimuth.

DETERMINATION OF TIME BY OBSERVATION 193

EXAMPLE OF EXTRA-MERIDIAN OBSERVATION ON SUN FOR TIME

Forenoon Observations

Place—Maori Hill, Dunedin. Thermometer—61° F.
 Longitude—11^h 22^m 01.4^s E. Barometer—27.4 inches.
 Latitude—45° 51' 36.5" S. Date—1951 July 22.
 Value of 1 division of bubble Standard Meridian—12^h E.
 —10".

Chronometer keeping approximately standard time.

OBSERVATIONS

Observed limb	Face	Vertical angles			Level		Chronometer time
		A	B	Mean	E	O	
		° ' "	° ' "	° ' "			
L	L	15 28 00	15 28 08	15 28 04	10	10	10 16 38.8
U	R	16 08 13	16 08 25	16 08 19	11	9	10 18 02.7
U	R	16 16 56	16 17 06	16 17 01	11.5	8.5	10 19 30.3
L	L	15 56 13	15 56 27	15 56 20	10	10	10 21 20.5
			Means,	15° 57' 26"	10.6	9.4	10 18 53.1

Computation for sun's declination at assumed approximate time of observation.

Approximate L.Std.T. of observation,
 1951 July 22 10^h 18^m 53.1^s
 Longitude of standard meridian, 12^h 00^m 00.0^s

Corresponding approximate G.M.T.,
 1951 July 21 22^h 18^m 53.1^s
 This is 22^h 18^m 53.1^s after 0^h G.M.T.,
 or 0.92978 day.

Sun's declination at 0^h G.M.T., +20° 40' 47.5"
 1st difference (−683.5" × 0.92978) − 10' 35.5"
 2nd difference (−41.6" × −0.01628) + 00.7"

∴ Sun's declination when observed +20° 30' 12.7"
i.e., Sun's south polar distance when
 observed 110° 30' 12.7"

Formula— $\tan \frac{h}{2} = \sqrt{\frac{\sin(s-c) \cdot \sin(s-p)}{\sin s \cdot \sin(s-z)}}$

CALCULATION

Mean of observed altitudes,	15° 57' 26''
Level correction,	06''
	15° 57' 20''
Refraction and parallax,	02' 50.4''
Corrected altitude,	15° 54' 29.6''
Zenith distance = z ,	74° 05' 30.4''
Co-latitude = c ,	44° 08' 23.5''
Sun's south polar distance = p ,	110° 30' 12.7''
$2s$,	228° 44' 06.6''
s ,	114° 22' 03.3''
$s - c$,	70° 13' 39.8''
$s - p$,	3° 51' 50.6''
$s - z$,	40° 16' 32.9''
$\log \sin (s - c)$,	9.9736102
$\log \sin (s - p)$,	8.8285915
$\log \operatorname{cosec} s$,	10.0405210
$\log \operatorname{cosec} (s - z)$,	10.1894532
$\log \tan^2 \frac{h}{2}$,	19.0321759
$\log \tan \frac{h}{2} = \log \tan 18^\circ 10' 04.3''$	9.5160879
h ,	36° 20' 08.6''
h (in time)	2 ^h 25 ^m 20.6 ^s
Local apparent time = $12^h - h$,	9 ^h 34 ^m 39.4 ^s

For equation of time—

In computing the sun's declination, we saw that the approximate G.M.T. of the observation was 22^h 18^m 53.1^s on July 21, or 0.92978 day after 0^h G.M.T. on 1951 July 21.

Equation of time at 0^h G.M.T. 1951
July 21 — 06^m 13.48^s

1st difference ($-3.18^s \times 0.92978$), ... — 02.96^s

2nd difference ($+1.09^s \times -0.01628$), ... — 00.02^s

$\therefore$ Equation of time at instant of observation, ... — 06^m 16.46^s

From above, L.A.T. of observation,	9 ^h	34 ^m	39.4 ^s
Equation of time (with sign reversed, because given sign is in the sense Apparent minus Mean),		+ 06 ^m	16.5 ^s
∴ L.M.T. of observation,	9 ^h	40 ^m	55.9 ^s
Difference for standard meridian,		37 ^m	58.6 ^s
∴ Local standard time,	10 ^h	18 ^m	54.5 ^s
Chronometer time,	10 ^h	18 ^m	53.1 ^s
Error of chronometer,			01.4 ^s slow

EXAMPLE FOR REDUCTION

With the same instrument as that used in the above observation a similar set of four sun observations had been taken on the same morning at the same place. The mean altitude obtained was $13^{\circ} 38' 02.3''$, the average alidade level readings being *E.* 10.5, *O.* 9.5. The mean of the chronometer times was 9^h 56^m 57.9^s. Barometer, 27.4 inches ; thermometer, 57° F.

Nautical Almanac data required will be found in the example worked above, since the dates are the same. Latitude, longitude, and standard time are also as above. The chronometer being supposed to keep standard time, determine its error.

Ans. 01.1^s slow.

(Note.—The example for reduction above was an observation done by way of a “trial run,” while waiting for the sun’s altitude to become greater than 15° .)

The effect of an error in latitude

It is important that we should know to what degree of precision the latitude must be known in order that the time may be determined. This may be readily investigated in a manner similar to that adopted with corresponding problems previously.

From the spherical triangle *S P Z* of Fig. 44,

$$\cos z = \cos c \cos p + \sin c \sin p \cos h.$$

If *c* is too *large* by a small amount *y*, then, for the same measured zenith distance *z*, *h* will be too *small* by an amount *x*, and we shall have

$$\cos z = \cos (c + y) \cos p + \sin (c + y) \sin p \cos (h - x).$$

Subtracting these two equations, and treating x and h as small quantities, we readily get

$$x = y \frac{-\cos c \cos h \sin p + \sin c \cos p}{\sin c \sin p \sin h}$$

$$= y \frac{\cot Z}{\sin c},$$

where Z denotes the azimuth angle $S Z P$.

This shows that x will be very large compared with y , if Z is nearly equal to 0 , or if c is nearly 0 . That is to say, a small error in the latitude will produce a very large error in the time if the body is observed near to the meridian, or if the observation is made in high latitudes near to either terrestrial pole.

On the other hand, if Z is 90° —i.e., if the observation is made on the prime vertical— x is 0 , and an error in latitude makes no difference. In this case the angle $S Z P$ is a right-angled triangle, and we can get a relation between p , z , and h that does not involve c at all, so that a knowledge of the latitude is unnecessary. If the observation is made near to the prime vertical, therefore, an error in latitude will produce very little effect on the time determination.

The following table, based upon the above formula, gives the error in time corresponding to an error of $1'$ in the latitude for different azimuth angles :—

ERROR IN TIME CORRESPONDING TO $1'$ ERROR IN LATITUDE *

Azimuth of observed body	Latitude of place				
	0°	30°	40°	50°	60°
	Seconds	Seconds	Seconds	Seconds	Seconds
45°	4.0	4.6	5.2	6.2	8.0
60°	2.3	2.6	3.0	3.5	4.5
80°	0.7	0.8	0.9	1.1	1.4
90°	0.0	0.0	0.0	0.0	0.0

* If the word *declination* be substituted for *latitude*, the same table will give the error in time due to an error of $1'$ in the declination, the first column representing, not the azimuth, but the angle $Z S P$.

This all points to the desirability of making the observation as near to the prime vertical as possible.

The effect of an error in the measured altitude

By a method similar to that adopted in the last paragraph it may be readily shown, if x is the error in the hour angle corresponding to an error y in the observed altitude, that

$$x = y \operatorname{cosec} Z \operatorname{cosec} c$$

x clearly becomes very great if either Z or c is small, and it has its least value when Z and c are each 90° . Thus, again, an error of observation has the least effect when the observation is made on a celestial body near the prime vertical, and the most favourable place for making the observation is at the equator.

TABLE SHOWING ERROR IN TIME DETERMINATION OWING TO AN ERROR OF 1' IN THE MEASURED ALTITUDE, WITH DIFFERENT AZIMUTHS OF THE OBSERVED BODY.

Azimuth of observed body	Latitude of place				
	0°	30°	40°	50°	60°
	Seconds				
45°	5.6	6.4	7.3	8.7	11.3
60°	4.6	5.3	6.0	7.1	9.2
80°	4.1	4.7	5.3	6.3	8.1
90°	4.0	4.6	5.2	6.2	8.0

This table deserves a little careful consideration, as it shows the degree of precision with which altitudes must be measured if the time is to be determined within one second. Under the most favourable possible conditions an error of $\frac{1}{4}$ minute of arc will cause an error of one second in the time, and it may produce an error of two seconds or even more.

EXAMPLE.—*In extra-meridian observation for time set out at length in the paragraph just preceding show that an error of 1' in the measured altitude will produce an error of 9 seconds in the time.*

The effect of an error in the declination of the sun caused by a defective knowledge of longitude or local time

With star observations the *Nautical Almanac* gives us the declination of the star with all the precision that is required, but with sun observations the surveyor has first of all to compute the declination. To do this he requires to know both his longitude and the approximate local mean time.

From the formula

$$\cos z = \cos c \cos p + \sin c \sin p \cos h$$

it appears that the relation between an error in p and an error in h will be of precisely the same nature as the relation between an error in c and an error in h . So that if x denotes the error in the hour angle corresponding to an error y in the declination,

$$x = \frac{\cot Z S P}{\sin p} \cdot y.$$

Thus the table already given, showing the error in time caused by 1' error in latitude, also gives the error in time caused by 1' error in declination, provided that the first column is taken as representing the angle $Z S P$ instead of the azimuth.

We have already seen that the maximum rate of variation of the declination of the sun is a little less than 1' per hour. So that to get the declination of the sun to the nearest minute it is sufficient to know the time to the nearest hour. But one hour of time corresponds to 15° of longitude, so that it is seldom that the surveyor will not know his longitude sufficiently well for this purpose.

It will be seen from the table that, in order to determine the time to the nearest second, it will be necessary to know the declination within only about one-fifth of a minute of arc under almost the worst conditions of observation considered in the table. For this it will be usually sufficient to know the local time within a quarter of an hour.

If the local time is not known with sufficient accuracy, its value must be assumed for the purpose of finding the approximate declination. This is then used in a preliminary calculation made to determine the time. The calculation is then made over again, using the approximate local time so found in order to get a more accurate value of the sun's declination, which in turn is used in the computation to obtain a more accurate determination of the local mean time.

Third method : By equal altitudes

If a star be observed at the same altitude on opposite sides of the meridian, the two observations must clearly be made at equal intervals of time before and after the star's meridian transit. Thus, in Fig. 45, if the star be observed in the two positions, S_1 and S_2 , so that the zenith distances $Z S_1$ and $Z S_2$ are equal, then, if P is the celestial pole, the two hour angles $Z P S_1$ and $Z P S_2$ must be equal.

It follows that the mean of these two observed times is the time of the star's meridian transit. But the local sidereal time at the instant of the star's meridian transit is determined by the star's R.A., which is given by the *Nautical Almanac*. This local sidereal time may be reduced to mean time, and a comparison of this with the average of the two observed chronometer times determines the error of the chronometer.

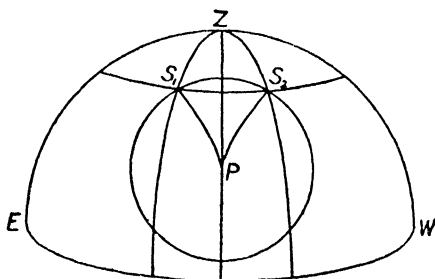


Fig. 45

With stars the method is capable of giving very accurate results, and it has the great advantage that no knowledge is required of latitude, declination, or even azimuth, and errors of graduation of the instrument have no effect upon the result. But to the surveyor it has the obvious drawback that a considerable interval of time must elapse between the observations.

As the accuracy of the determination depends upon the altitude's being the same at the two observations, the star should have an altitude of something more than 45° , in order to eliminate the uncertainties of refraction near the horizon.

EXAMPLE.—On 1914 September 1, β Crucis was observed east of the meridian at $10^h 42^m 30.5^s$ by a chronometer keeping sidereal time. It was again at the same altitude west of the meridian at $14^h 51^m 20.7^s$. Find the error of the clock.

East,	.	.	.	.	.	10^h	42^m	30.5^s
West,	.	.	.	.	.	14^h	51^m	20.7^s
					2)	25^h	33^m	51.2^s
Meridian transit by chronometer,	.	.	.	.	.	12^h	46^m	55.6^s
R.A. of star,	.	.	.	.	.	12^h	44^m	50.3^s
Chronometer correction,	.	.	.	.	.		02^m	05.3^s

As the chronometer is fast, the correction is to be subtracted from the chronometer reading.

If, as is more usual, the chronometer keeps local mean time, the sidereal time at the meridian transit of the star must be reduced to local mean time in order to compare with the chronometer time. This cannot be done without a knowledge of the longitude.

EXAMPLE.—At a place in longitude $8^{\text{h}} 35^{\text{m}} 27^{\text{s}}$ E., on the evening of 1951 September 1, the star α Pavonis is observed east of the meridian at $19^{\text{h}} 09^{\text{m}} 20.5^{\text{s}}$, with a watch keeping L.M.T. It is again observed at the same altitude to the west of the meridian at $9^{\text{m}} 30.2^{\text{s}}$ after midnight. Find the error of the watch, having given—

G.S.T. at 0^{h} G.M.T., September 1, ... $22^{\text{h}} 37^{\text{m}} 24.2^{\text{s}}$

R.A. of α Pavonis, ... $20^{\text{h}} 21^{\text{m}} 53.3^{\text{s}}$

Ans. $2^{\text{m}} 54.5^{\text{s}}$ slow

It is desirable, in order to make the determination as precise as possible, that a series of observations should be made upon the star on each side of the meridian, instead of one observation only. A few times should be taken when the star is on the east of the meridian at altitudes differing by 20 or 30 minutes of arc. A corresponding series of times should then be taken when the star is on the west of the meridian at the same altitudes. Since all that we want to ensure is that the altitude is the same at corresponding observations east and west of the meridian, there is no particular object in reversing the face of the instrument. The whole set of observations may be taken with the one face.

The error due to a slight inequality in the altitudes of two corresponding observations

If in Fig. 45 ZS_1 = zenith distance of the first observation = z ,

ZP = co-latitude = c

PS_1 = polar distance = p

h = hour angle ZPS_1

Z = angle S_1ZP = azimuth of star

$$\cos z = \cos c \cos p + \sin c \sin p \cos h, \quad . \quad . \quad (1)$$

Suppose now that at the second observation the zenith distance, instead of being z , is $z + y$, being in error by a small amount y . Then the hour angle ZPS_2 will be in error by a corresponding amount x , so that instead of being h , it will be $h + x$. Then, from the spherical triangle ZPS_2 ,

$$\cos (z + y) = \cos c \cos p + \sin c \sin p \cos (h + x). \quad . \quad (2)$$

Subtracting (2) from (1), treating x and y as small quantities, we get

$$y \sin z = x \sin c \sin p \sin h.$$

But
$$\frac{\sin z}{\sin h} = \frac{\sin p}{\sin Z},$$

$\therefore x = \frac{y}{\sin c \sin Z}.$

We see thus that the error x in the hour angle, corresponding to an error y in the second altitude, will be least when $Z = 90^\circ$, and will be greater the smaller the value of Z . We draw, therefore, the practical conclusion that the observations are best made on stars near the prime vertical.

If the declination of a star is slightly less than the latitude, it will cross the prime vertical near the zenith and the interval between the times of transit will be small. This, therefore, is a convenient observation to make, and the conditions are favourable to accuracy.

The determination of time by equal altitudes of the sun

The above method is an extremely simple one as applied to the stars, because the declination of a star remains constant during the period over which the observations extend. But in the case of the sun the declination changes so rapidly that it cannot be considered as constant, and the theory becomes complicated by the fact that allowance must be made for the alteration of declination in the interval between the observations. Referring again to Fig. 45, if p denotes the polar distance of the sun when it is on the meridian, then at the first sight, when the sun is at S_1 , the polar distance will be $p \pm y$, and at the second sight, when the sun is at S_2 , the polar distance will be $p \mp y$. The $+$ or $-$ sign is to be taken in the first of these expressions according as the sun is approaching or leaving the elevated pole.

If p were constant, we should have

$$\cos z = \cos p \cos c + \sin p \sin c \cos h.$$

But if at the first observation, S_1 , the polar distance is $p + y$, the hour angle will be $h + x$, and we have

$$\cos z = \cos (p + y) \cos c + \sin (p + y) \sin c \cos (h + x).$$

Subtracting these two equations, and treating x and y as small quantities, we get,

$$0 = y \sin p \cos c - y \cos p \sin c \cos h + x \sin h \sin c \sin p.$$

$\therefore -x = y (\cot c \operatorname{cosec} h - \cot p \cot h).$

Under these conditions the first observation will be made when the sun is at an hour angle $h + x$ *before* apparent noon, where x is given by the preceding expression, and it may be positive or negative according as $\cot c \operatorname{cosec} h$ is $<$ or $>$ $\cot p \cot h$.

Similarly the second observation will be made with the sun at an hour angle $h - x$ *after* apparent noon, and it may be shown in the same way as before that the value of x is given in this case also by the same mathematical expression.

The mean of these two observed times will therefore be when the sun is at an hour angle x before apparent noon.

When the sun is leaving the elevated pole, instead of approaching it, the mean of the two observed times will be when the sun is at an hour angle x after apparent noon.

Thus, the true time of transit—*i.e.*, the time of apparent noon—is given by

Mean of observed times $\pm \frac{1}{15} y (\cot c \operatorname{cosec} h - \cot p \cot h)$.

The quantity y is the alteration in the sun's declination in half the time interval between the two observations.

The quantity h is half the time interval between the two observations reduced to angular measure.

The $+$ sign is to be taken if the sun is *leaving* the elevated pole, and the $-$ sign when it is *approaching* the elevated pole.

Just as with star observations, it is necessary, in order to obtain the best results, that a series, say four or six, of observations should be taken to the sun in the forenoon and a corresponding set in the afternoon, the sights in each case being taken alternately to the upper and lower limbs.

EXAMPLE.—At Adelaide, longitude $9^{\text{h}} 14^{\text{m}} 20^{\text{s}}$ E., latitude $34^{\circ} 55' 38''$ S., on 1951 July 21, equal altitude observations of the sun were taken in the forenoon and afternoon. The means of the noted times were $9^{\text{h}} 35^{\text{m}} 03^{\text{s}}$ and $14^{\text{h}} 37^{\text{m}} 15^{\text{s}}$ by a watch keeping mean time.

	12 ^h 00 ^m 00 ^s	
subtract	9 ^h 35 ^m 03 ^s	
	2 ^h 24 ^m 57 ^s	
add	2 ^h 37 ^m 15 ^s	
	2) 5 ^h 02 ^m 12 ^s = time between observa-	
		tions
	2 ^h 31 ^m 06 ^s	$h = 37^{\circ} 46' 30''$

subtract from $2^h 37^m 15^s$

$0^h 06^m 09^s$

i.e., $12^h 06^m 09^s =$ time by watch at L.A.N.

c , ... $55^\circ 04' 22''$

Declination of sun at G.A.N.

1951 July 21 $+20^\circ 35' 05.4''$

Correction for longitude	{	1st diff.	
		$-673.2 \times .38495$	$+ 04' 19.1''$
		2nd diff.	
		$-41.8 \times -.05919$	$+ 02.5''$

Declination of sun at L.A.N. $+20^\circ 39' 27.0''$

$\therefore p$, ... $110^\circ 39' 27.0''$

$\cot c \cdot \operatorname{cosec} h$, ... $= 1.140$

$\cot p \cdot \cot h$, ... $= -.486$

$\cot c \operatorname{cosec} h - \cot p \cot h$, ... $= 1.626$

Change in declination in

$2^h 31^m 06^s$, ... $= 71.73''$,

and sun is *approaching* elevated pole,

$\therefore$ time of apparent noon $= 12^h 06^m 09^s - \frac{1.626 \times 71.73}{15}$ secs.

$= 12^h 06^m 09^s - 7.8^s$

$= 12^h 06^m 01.2^s$

But, from the *N.A.*, the L.M.T. of L.A.N. is $12^h 06^m 13.9^s$.

Thus the watch is 12.7^s slow.

Fourth method : Almucantar method for time observations

In 1884, S. C. Chandler, at the Harvard College Observatory, U.S.A., devised a form of instrument in which the telescope was fixed at a constant angle with the vertical, so that the line of sight traced out a horizontal circle on the celestial sphere, and observations for the determination of latitude and other purposes were made by noting the times of transit of stars across the fixed horizontal

circle. The instrument was named an "almucantar," and it proved to be capable of very remarkable work. The same principle may be readily applied with an ordinary theodolite, and experience has shown that extremely accurate determinations of time are possible in this way.

Any horizontal circle may be used for the observations, but the most convenient is the one that passes through the pole of the observer. This has been named the "co-latitude circle," its zenith distance being everywhere equal to the co-latitude. The formulæ for reduction then become very simple. The method consists in observing the times of transit of a series of east and west stars, somewhere near the prime vertical, across the horizontal wire of a

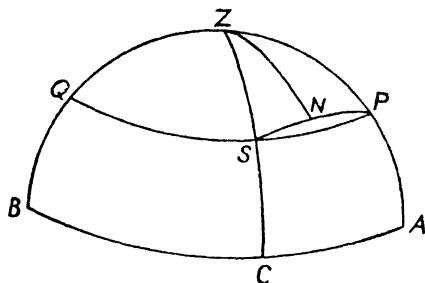


Fig. 45a

telescope that is set to an altitude equal to that of the pole. Allowance must be made for refraction, and, therefore, the telescope is actually set so that its altitude as read off on the vertical circle is equal to the latitude of the place plus refraction.

In Fig. 45a, Z denotes the zenith, P the celestial pole, A and B the north and south points, P S Q the co-latitude circle. Let S denote the position of a star, somewhere near the prime vertical, as it crosses the co-latitude circle.

Let $ZP = c = \text{co-latitude}$.

$PS = p = \text{star's polar distance}$, measured, of course, along the great circle arc P N S and not along the small circle P S Q.

Angle $SPZ = h = \text{hour angle of star}$.

Angle $SZP = Z = \text{azimuth of star measured from elevated pole}$.

Then, since $ZS = c$, ZSP is an isosceles triangle, and, if ZN be drawn perpendicular to the great circle arc joining S and P, it will divide SZP into two equal right-angled triangles.

From the triangle Z N P

$$\cos N P Z = \tan P N \cot Z P$$

$$\therefore \cos h = \tan \frac{p}{2} \cot c = \tan \frac{p}{2} \tan \varphi \quad (1)$$

if φ is the latitude of the place.

To determine the azimuth at which a star will cross the co-latitude circle, from the same triangle

$$\cos Z P = \cot N Z P \cot N P Z.$$

$$\therefore \cos c = \cot h \cot \frac{Z}{2},$$

$$\text{or } \cot \frac{Z}{2} = \sin \varphi \tan h. \quad (2)$$

Formula (1) enables the time of transit to be computed, and formula (2) gives the azimuth if required.

If an observation on one star in the east is balanced by a corresponding observation on a star in the west of somewhere about the same declination, then the mean of the two time observations will give a correct result even if the co-latitude circle is considerably out. If, for instance, the co-latitude circle is set out too low, the observed time of transit in the east will be too soon, but that in the west will be too late, and if there is not much difference in the declinations of the stars the time of transit will be just as much too soon in the one case as it is too late in the other. Thus by averaging the two results any small error in the setting out of the co-latitude circle is practically eliminated, and it is not necessary, therefore, in order to apply the method that the latitude of the place should be known with precision. An approximate latitude will suffice.

For precisely the same reasons as have been investigated when dealing with extra-meridian observations for time, slight errors in latitude, declination, and altitude will have least effect upon the result when the stars observed are near the prime vertical. The stars should be selected from a zone of about 20° on each side of the prime vertical.

EXAMPLE.—On 1951 May 3, in Lat. $31^\circ 56' 45''$ S., the transit of β Orionis was observed in the west across the co-latitude circle at $8^{\text{h}} 55^{\text{m}} 01.5^{\text{s}}$ by a watch keeping sidereal time. The transit of α Virginis was similarly observed in the east at $9^{\text{h}} 20^{\text{m}} 08.4^{\text{s}}$. Determine the error of the watch.

	β Orionis	α Virginis
Declination, ...	— 8° 15' 21.6"	— 10° 54' 40.1"
p , ...	81° 44' 38.4"	79° 05' 19.9"
$\frac{1}{2}p$, ...	40° 52' 19.2"	39° 32' 39.9"
$\log \tan \frac{1}{2}p$, ...	9.9372030	9.9167903
$\log \tan \varphi$, ...	9.7948752	9.7948752
$\log \cos h$, ...	9.7320782	9.7116655
h , ...	57° 20' 35"	59° 00' 50"
h , in time, ...	3 ^h 49 ^m 22.3 ^s	3 ^h 56 ^m 03.3 ^s
R.A. of star, ...	5 ^h 12 ^m 10.9 ^s	13 ^h 22 ^m 38.8 ^s
Computed time, ...	9 ^h 01 ^m 33.2 ^s	9 ^h 26 ^m 35.5 ^s
Observed time, ...	8 ^h 55 ^m 01.5 ^s	9 ^h 20 ^m 08.4 ^s
Error of watch (slow), ...	06 ^m 31.7 ^s	06 ^m 27.1 ^s
Mean determination of watch error—6 ^m 29.4 ^s slow.		

Adjustment of telescope during observation

It is the most essential thing for accurate work, in observations of this kind, that the telescope should throughout make exactly the same angle with the horizontal. It is not of such importance that the altitude should be exactly equal to the latitude, as it is that the altitude should remain the same throughout the observations. Now, no matter how carefully a transit theodolite is adjusted, the bubble attached to the vertical circle will not remain precisely in the centre of its run as the telescope is turned from star to star. It is, therefore, essential to accurate work that this bubble should be adjusted to the centre of its run just before the star crosses the horizontal wire in each case. This must be done, of course, by the adjusting screw on every transit theodolite that moves both telescope and vertical circle together without affecting the altitude reading. After the reading on the vertical circle has been set for the first star so that the altitude is equal to the latitude plus refraction, the altitude screw which would alter this reading must on no account be touched. But at each observation the horizontal line of the vertical circle must be adjusted without altering the reading of the vernier.

To get the most accurate results observations must be made upon a number of stars, at least six in the east and six in the west, and the mean of all the determinations is taken. The east and west stars should be selected so that the angles in azimuth that one set make to the east are as nearly as possible equal to the angles that the other set make to the west.

SUNDIALS

Whilst the sundial does not provide the surveyor with a means of determining local time with anything like the precision obtainable by the methods that have been described, it enables the time to be fixed accurately enough for the ordinary purposes of life. Sundials were commonly used for checking clocks in country places before the advent of domestic radio, and although they are now somewhat rare, descriptions of the main types are included here as a matter of interest, and also as an aid to any surveyor who may be asked to design a dial for ornamental purposes.

When a sundial is illuminated by the direct light of the sun the shadow of a straight line or sharp straight edge is thrown upon a plane containing a graduated circle so marked that the apparent solar time is indicated by the reading at the place where the shadow intersects the circle. The plane containing the graduated circle may be either horizontal, vertical, or inclined. The straight edge, the shadow of which is thrown upon the circle, is always set up so as to be parallel to the earth's axis. It is called the *stile*, or *gnomon* of the dial. When the graduated circle or "plane of the dial" is horizontal we have what is known as a *horizontal dial*, and as this is the most common form we will consider it first.

The horizontal dial

In Fig. 46, let $M B L A$ represent the plane of the dial, which we may suppose to be extended indefinitely so that $M B L A$ is the circle in which it intersects the celestial sphere. $C P$ is the direction

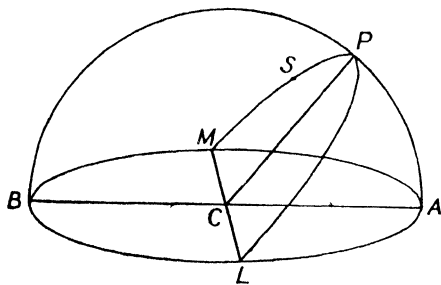


Fig. 46

of the gnomon, which again we may suppose to be produced to intersect the celestial sphere in the celestial pole P . $B P A$ is the plane of the meridian.

If now S denotes the position of the sun, the line of intersection of the shadow of the gnomon $C P$ with the plane of the dial will be the line of intersection of the plane containing $C P$ and S with the plane $M B L A$. $M P L$ represents in the figure the plane passing through S and $C P$, and $M C L$ is the line of intersection of this plane with the plane of the dial, or $C L$ is the direction of the shadow of the gnomon.

Neglecting the slight alteration in the declination of the sun during the hours of daylight, S will describe a circle uniformly on the celestial sphere about P as centre. The angle $S P B$ is the hour angle of the sun, decreasing or increasing uniformly with the time according as the observation is made in the morning or in the afternoon.

Then in the right-angled triangle $L P A$

$A P = \varphi =$ latitude of place.

Angle $A P L = h =$ hour angle of sun.

$A L = x =$ required division along the dial corresponding to hour angle h .

$\therefore \sin \varphi = \cot h \tan x$, or $\tan x = \sin \varphi \tan h$.

Thus, to graduate the dial for the hourly intervals before and after noon, we must put $h = 15^\circ, 30^\circ, 45^\circ$, etc., in succession and compute the corresponding values of x , knowing, of course, the value of φ .

Thus, if the latitude of the place is 30° , the first hourly division on each side of noon will be marked out at an angle with $C A$ given by

$$\log \tan x = \log \sin 30^\circ + \log \tan 15^\circ,$$

from which $x = 7^\circ 38'$.

The next hourly division, indicating either 10 a.m. or 2 p.m. will make an angle with $C A$ given by

$$\log \tan x = \log \sin 30^\circ + \log \tan 30^\circ,$$

from which $x = 16^\circ 6'$, and so on.

The reading of the shadow of the gnomon gives the local apparent time which must be corrected by the equation of time, as given by the *Nautical Almanac*, in order to obtain the mean time. A table of corrections may easily be drawn out for different times of the year.

The prime vertical dial

In this case the plane of the dial lies in the prime vertical. In Fig: 47 let $A L B M$ be the plane of the dial, which we will again

TIME OF RISING OR SETTING OF A CELESTIAL BODY

This is not of much value for the determination of time, because of the uncertainty of refraction on the horizon. In Fig. 48, if $A S B$ be the plane of the horizon, Z the zenith, P the celestial pole, and S the body, which is exactly on the celestial horizon, then the spherical triangle $P S A$ is right-angled at A , and

$$\cos S P A = \cot S P \tan P A.$$

$$\therefore \cos (\text{hour angle } S P Z) = -\tan \text{dec.} \times \tan \text{lat.}$$

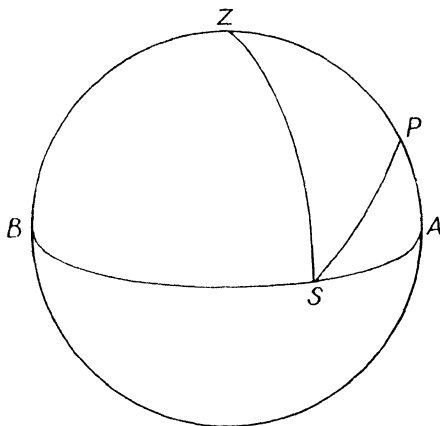


Fig. 48

From this the hour angle of the body at rising or setting may be computed, and this will determine the apparent solar time in the case of the sun or the sidereal time if a star is observed.

We have here neglected the effect of refraction, which, amounting as it does to about $36'$ on the horizon, will cause stars to be just visible when they are really $36'$ below the horizon.

To find the azimuth of the body, we have

$$\cos S P = \cos S A \cos P A,$$

$$\text{or} \quad \cos S A = \frac{\sin \text{dec.}}{\cos \text{lat.}}.$$

EXERCISES

1. At a place in lat. 35° S., the bearing of a wall is 110° . Find the apparent time at the equinox when it casts no shadow.

Ans. $3^h 50^m 24.5^s$ p.m.

2. Find the true bearing and apparent time of sunrise in lat. 32° S. when the sun's declination is -20° . (Take the sun's centre and neglect refraction and parallax.)

Ans. Bearing, $113^{\circ} 47'$
Time, $5^h 07^m 25^s$

3. Rigel was observed east of the meridian on the horizontal wire of a theodolite at $7^h 05^m 20^s$ p.m. by a watch which is supposed to keep West Australian standard time (120th meridian). It was also observed at the same altitude west to cross the horizontal wire at $1^h 25^m 30^s$ a.m. Neglecting the rate of the watch, find its error.

Date of first observation,	.	.	.	January 5
Longitude of locality,	.	.	.	$115^{\circ} 50' 26''$ E.
Sidereal time at 0^h G.M.T. Jan. 5	.	.	.	$6^h 52^m 47.55^s$
Sidereal time at 0^h G.M.T. Jan. 6,	.	.	.	$6^h 56^m 44.11^s$
R.A. of Rigel,	.	.	.	$5^h 10^m 07.29^s$

Ans. $16^m 9.8^s$ slow.

4. On 1950 July 16, in lat. $33^{\circ} 15' 13''$ S. and longitude $10^h 04^m 50^s$ E., the observed altitude of the sun's centre (corrected for refraction and parallax) was $31^{\circ} 54' 45''$, bearing $10^{\circ} 35' 15''$ magnetic, the R.M. bearing $86^{\circ} 54' 15''$ magnetic, local mean time by watch being $10^h 48^m 00^s$.

Sun's declination at 0^h G.M.T., July	
16th,	$+21^{\circ} 30' 13.1''$
Sun's declination at 0^h G.M.T., July	
17th,	$+21^{\circ} 20' 30.3''$

Equation of time (to be added to apparent time) at 0^h G.M.T., July 16th	$5^m 50.60^s$.
--	-----------------

Equation of time (to be added to apparent time) at 0^h G.M.T., July 17th,	$5^m 56.52^s$.
---	-----------------

Find the true bearing of the R.M., the magnetic variation and the error of the watch.

Ans. Bearing, $98^{\circ} 42' 14''$
Variation, $11^{\circ} 47' 59''$ E.
Watch, $3^m 29^s$ fast.

5. At a place $40^{\circ} 51' 20''$ S., $140^{\circ} 20' 30''$ E., at $9^h 10^m 20^s$ a.m. L.M.T. by a watch on 1951 September 2, the sun's preceding limb was found by compass bearing to be $58^{\circ} 14' 20''$, and the observed altitude of the upper limb $27^{\circ} 11' 15''$.

Declination at 0^h G.M.T., September 1, +8° 39' 48·6''

Declination at 0^h G.M.T., September 2, +8° 18' 06·3''

Sun's semi-diameter at 0^h G.M.T., September 1, ... 15' 52·32''

Sun's semi-diameter at 0^h G.M.T., September 2, ... 15' 52·55''

Equation of time (to be added to apparent time) at 0^h G.M.T., September 1, ... 0^m 19·90^s

Equation of time (to be added to apparent time) at 0^h G.M.T., September 2, ... 0^m 01·24^s

What was the declination of the compass and the correct L.M.T. of observation ?

Ans. Declination, 9° 17' 18'' W.

L.M.T., 9^h 07^m 47^s

6. At a place in latitude 32° S. a vertical rod 6 feet high casts a shadow 15 feet long in a direction bearing 75° 12'. What is the apparent time and the approximate time of the year ?

Ans. 5^h 12^m p.m.

December.

7. If the time be found by a single altitude, show that a small error in the latitude will have no effect on the time when the body is in the prime vertical.

8. At a place in south latitude and in longitude 151° 22' E., on September 10, the sun's western limb crossed the meridian at 11^h 58^m 22^s by a chronometer, the observed altitude of the upper limb being 56° 33' 20''.

Find the error of the chronometer on standard mean time (of 150° E.).

The following data are for Greenwich apparent noon :—

	9th September	10th September
Sun's R.A., ...	11 ^h 07 ^m 31·89 ^s	11 ^h 11 ^m 07·69 ^s
„ Declination, ...	+5° 37' 19·2''	+5° 14' 41·6''
„ Semi-diameter, ...	15' 54·47''	15' 54·72''
Equation of time to be subtracted from apparent time, ...	2 ^m 24·09 ^s	2 ^m 44·78 ^s

Ans. Chronometer is fast 7^m 29·9^s

9. On 1950 January 3, at a place latitude $30^{\circ} 15' S.$, longitude 148° East, the following sun observation was taken :—

Observed altitude	Alidade bubble		Approximate L.M.T. by watch	Angle from R.M.
	E.	O.		
$38^{\circ} 07' 15''$	3	7	$8^h 06^m$ a.m.	$112^{\circ} 14' 40''$
$39^{\circ} 18' 37''$	2	8	$8^h 10^m$ a.m.	$114^{\circ} 51' 20''$

Magnetic bearing of R.M., $200^{\circ} 10' 20''$

Bubble divisions on alidade = $20''$

Required : Magnetic variation and error of watch.

Data from *Nautical Almanac* :—

Sun's Declination.

0^h G.M.T., January 2nd, $-22^{\circ} 59' 25.4''$

0^h G.M.T., January 3rd, $-22^{\circ} 54' 08.4''$

Equation of time (to be added to apparent time)

0^h G.M.T., January 2, $3^m 42.64^s$.

0^h G.M.T., January 3, $4^m 10.67^s$.

Ans. Magnetic Variation, $9^{\circ} 41' 52''$ E.

Watch slow, $6^m 45^s$.

CHAPTER 11

DETERMINATION OF LONGITUDE

THE difference of longitude between any two places on the earth's surface, as we have already seen, is measured by the difference between either their local sidereal times or their local mean times at the same instant. The problem, then, of the determination of the difference in longitude between A and B amounts to that of the determination of the difference in the local times at A and B. By the methods we have considered in the last chapter we may by astronomical observation determine the local time at A at some instant, and a means must be found of determining what is the local time at B at the same instant, if we are to ascertain the difference of longitude.

The problem presented is usually that of the determination of the *difference* of longitude between two places rather than the fixing of the absolute longitude of a place as measured from the now universal standard meridian, that of Greenwich. Usually we seek to find the difference in longitude between a point on a survey and some fixed observatory in the country or some other point on the survey, the longitude of which has been previously determined.

In all cases the local time at some instant must be determined at the place whose longitude is required by one of the astronomical methods of the last chapter. The corresponding local time at the reference station is then usually found by reception of wireless signals. However, if wireless equipment is not available, the local time at the reference station may be found by use of portable chronometers, or if the distance is not too great, by flashlight signals. It should be borne in mind that if precision in longitude determination is required, the time differences must be accurately measured ; an error of one second in time will mean an error of 15'' in arc.

By reception of wireless signals

(a) *Where moderate accuracy is required.*---Most civilised countries broadcast wireless time signals several times a day on various frequencies. These signals are usually in terms of the local standard time, and can be used to check a chronometer or watch-chronometer

employed for astronomical work. In most cases the signals take the form of six " pips " marking the 55th, 56th, 57th, 58th, 59th and 60th seconds of the last minute of selected hours. Comparison with the chronometer can usually be made to an accuracy of within one second. Regular checks by this method will reveal the gain rate of the chronometer, which can be plotted as a graph and used for finding the correct standard time of an observation.

If short wave wireless equipment is available, time signals can be obtained directly from the B.B.C., London (G.M.T.), or from any of the other powerful transmitters situated throughout the world, such as WWV, the station of the Central Radio Propagation Laboratory, National Bureau of Standards, Washington, D.C. Station WWV is particularly useful, because continuous, high-powered transmissions on 5, 10, 15 and 20 mega-cycles per second are provided, day and night. Modulations of 1, and 440 cycles per second are impressed on the 5 mega-cycle carrier, and modulations of 1, 440, and 4000 cycles per second on the other carriers mentioned. The 1 cycle per second modulation is stated to be a 0.005-second pulse, the beginning of which marks the beginning of each second to an accuracy of one part in 1,000,000. The pulse is omitted on the 59th second of every minute. The audio frequencies of 440 and 4000 c.p.s. are interrupted at precisely one minute before each hour and each five minutes thereafter (59th minute, 4 minutes past the hour, etc.) ; they are resumed in precisely one minute. During each silent interval Eastern Standard Time (5 hours behind G.M.T.) is given in telegraphic code. A station announcement is given in voice on the hour and half hour.

It is often not necessary to know the actual local standard time of emission of the time signals, provided, of course, the pattern is known. Most countries have, for their standard times, zone times which differ from G.M.T. by exact multiples of half-an-hour. Thus the time signals received will usually fall either on the hours or half-hours of local standard times, and it is a simple matter to use them as a check (provided that the correct local standard time is known to within a few minutes).

(b) *Where greater accuracy is required.*—This method is, in essence, very similar to the last one, except that the method of checking the chronometer permits of greater accuracy. The observer must be equipped with either a sidereal or a mean-time chronometer, and a wireless set powerful enough to receive a standard rhythmic signal from one of a number of stations situated in various parts of the

world. A list of such stations, together with times of emissions, frequencies and call signs is given in *The Admiralty List of Radio Signals, Vol. II*. London, published annually. The standard rhythmic signal consists of a series of regularly spaced "pips" sent out so that they divide each minute into 61 equal intervals; the signals last for five minutes and are broadcast at intervals during the day. The "pips" are each about $\frac{1}{10}$ th second in duration. At the beginning of the series, and at the end of each minute a dash, about $\frac{4}{10}$ ths second long, is substituted for the "pips".

Assume first of all that the observer has a mean time chronometer beating half-seconds. Observations of the chronometer are usually made on the full-second beats and not on the half-second beats, since the latter may not be quite accurate. The observer, then, watches the chronometer and at the same time concentrates on listening to the full-second beats. When the rhythmic wireless signal commences, he will hear the "pips" gradually overtaking the chronometer beats, since there are only 60 of the latter to 61 of the former in one minute. As soon as the signal comes through, he should start counting: "zero" (dash), "one" (pip), "two" (pip), ... and so on. The signal "pips" will overtake the chronometer beats, and coincidence will usually appear to take place for about five signal "pips." The observer should note the middle "pip" of the coincident series and record both the number of this "pip" and the full second of the chronometer on which it occurred. For example, suppose that, at "pip" number 21 coincidence occurred and appeared to continue until "pip" number 25, and that the full-second beat of the chronometer corresponding to "pip" number 21 was 43. The observer records "pip" number 23 as corresponding to the 45th second. He then waits until the next dash is heard, when he starts counting as before, until the second coincident series occurs, whereupon he records the mean of the "pips" and the seconds again. The procedure is repeated for the third, fourth and fifth minutes of the duration of the time signal.

EXAMPLE, using the rhythmic time signal from Rugby, station GIC, on 81.64 Mc/s (34.72 metres) from 9^h 35^m to 10^h 00^m G.M.T. :—

Note.—The 2nd dash of the signal corresponds to the 61st "pip", the 3rd dash to the 122nd "pip," the 4th dash to the 183rd "pip", the 5th dash to the 244th "pip" and the 6th dash (end of signal) to the 305th "pip."

Rugby Time Signal			Mean Time Chronometer		
h	m	"pip"	h	m	s
9	55	23	9	56	45 (mean of 43, 47)
		61+23	9	57	45
		122+24	9	58	46
		183+24	9	59	46
		244+23	10	00	45
Means	9 55	122+23.4	9	58	45.4
or	9 57	23.4	9	58	45.4

61 intervals between the signal "pips" are equal to 60 seconds, so that one interval is equal to $\frac{60}{61}$ of one second.

Thus, in the above results, 23.4 signal intervals will be equal to $\frac{23.4}{61} \times 60$ seconds, or 23.02 seconds.

Thus, at 9^h 57^m 23.02^s G.M.T., the chronometer read 9^h 58^m 45.4^s. The chronometer is therefore 1^m 22.38^s fast on G.M.T.

(The chronometer beats and the signal "pips" at slightly different frequencies, when compared in the above way, are sometimes referred to as a "time vernier".)

If a sidereal chronometer is used, the coincidence will occur at longer intervals (about 72 sidereal seconds) than with the mean time chronometer; this is because the frequency of the sidereal chronometer beats is closer to the frequency of the wireless "pips," hence the difference between the frequencies (or the beat frequency) will be less, *i.e.*, there will be a longer period between coincidences. For this reason it will usually be possible to obtain only four coincidences during the five-minute period of the signal. Another point to be noted is that coincidence will appear to take place for about 8 or 9 "pips."

EXAMPLE.—Date of Observation, 1951 May 22.

Rugby Time Signal			Sidereal Chronometer		
h	m	"pip"	h	m	s
9	55	31	01	51	16 (mean of 12, 20)
		61+44	01	52	28
		122+58	01	53	41
		244+10	01	54	53
Means	9 55	142.5	01	53	04.5

To get the mean time of the mean of the Rugby signals :—

$$\frac{142.5}{61} \times 60 = 140.16 \text{ seconds.}$$

$$\therefore \text{Meantime of mean of Rugby signals} = 9^{\text{h}} 55^{\text{m}} + 2^{\text{m}} 20 \cdot 16^{\text{s}} \\ = 9^{\text{h}} 57^{\text{m}} 20 \cdot 16^{\text{s}}$$

$$\text{Converted to a sidereal interval,} \\ \text{this} = 9^{\text{h}} 58^{\text{m}} 58 \cdot 29^{\text{s}}$$

$$\text{But sidereal time at } 0^{\circ} \text{ G.M.T.,} \\ \text{May 22} = 15^{\text{h}} 55^{\text{m}} 15 \cdot 33^{\text{s}}$$

$$\therefore \text{Sidereal time corresponding to} \\ 9^{\text{h}} 57^{\text{m}} 20 \cdot 16^{\text{s}} = 01^{\text{h}} 54^{\text{m}} 13 \cdot 62^{\text{s}}$$

$$\text{But mean of chronometer times} = 01^{\text{h}} 53^{\text{m}} 04 \cdot 50^{\text{s}}$$

$$\text{Chronometer slow by} \quad \quad \quad 01^{\text{m}} 09 \cdot 12^{\text{s}}$$

The above methods are subject to the personal equation of the observer, and, for refined work, chronographs and other devices have been devised to obviate this. The chronograph incorporates a moving paper tape which has a printed line running down its centre. The second-beats of a chronometer are arranged to operate electrically a pen which makes short marks along one side of the line, at right angles to it, as the tape moves evenly forward. The signal "pips" operate another pen on the opposite side of the line to make a second series of marks. When the tape is removed, the exact points where the marks coincide can be seen. The cathode ray oscillograph also provides a means of comparing chronometer beats and time signals visually to find coincidence. Another method of obtaining aurally a better comparison between chronometer and time signal, is to use a microphone operated from an electric contact on the escapement wheel to feed the full-second chronometer beats into one earphone of a head-set, while the signal "pips" are fed into the other.

By portable chronometers

Since the time when chronometers that will retain a fairly uniform rate have been generally available, this has been the general method for the determination of longitude at sea. Every ship carries a chronometer, which keeps either Greenwich time or the local time at some known port, and from an astronomical observation the Captain is thus able to ascertain the difference between his local time and that of the chronometer. The method is very simple and convenient and nowadays wireless signals can be used to keep an accurate check on the chronometer. If wireless checks are not possible it is essential that the chronometer should keep a constant rate, and the conditions on board a ship are much more favourable for this than is usually the case when chronometers are

carried about from place to place on land. So that for land work the box chronometers used at sea are commonly replaced by chronometer watches which are more easily carried and are found to be more satisfactory.

Suppose now that it is required to determine the difference in longitude between A and B. The watch or chronometer must first be regulated at station A. Its error on the local time at that place must be determined and its "rate"—*i.e.*, the amount that it gains or loses in 24 hours—must be found. On the assumption that the rate remains constant this will enable the local time at A to be found from a reading of the chronometer at any time afterwards. If then the chronometer be transported to B and an astronomical observation be made there for the determination of local time, it will be possible to find from the chronometer the local time at A at the same instant.

EXAMPLE.—*At A, September 8, the chronometer at 8 p.m. was found to be 2^m 6.5^s fast, and it was gaining at the rate of 2.58^s in 24 chronometer hours.*

At B, September 9, from an astronomical observation which gave the local time as 9^h 12^m 35^s p.m., the reading of the chronometer was 9^h 12^m 30.6^s.

What is the difference of longitude?

The interval of time, as measured on the chronometer, between the two readings is $25^h 10^m 24.1^s = 1.049$ days.

Therefore, in this interval the chronometer has gained $1.049 \times 2.58^s = 2.7^s$.

Thus, at B the chronometer was fast by $2^m 9.2^s$, and the local time at A was $9^h 10^m 21.4^s$, corresponding to the local time of $9^h 12^m 35^s$ at B.

Thus, the time at B is in advance of that at A by $2^m 13.6^s$, or B is to the east of A by $0^\circ 33' 24''$.

The accuracy of the method is affected by the fact that the rates of chronometers are not perfectly constant, and particularly by the fact that the rate whilst being carried is not the same as when at rest. The best way to minimise the error is to use several chronometers, from each of which a longitude determination is obtained, and the average of the results is taken. If possible, after the observations have been made at B, the chronometers should be carried back again to A and another comparison made with the local time there.

This method is now never used by surveyors except where wireless or telegraphic communications are not available.

By flashlight signals

When two stations are visible one from the other, flashlight signals may be sent from one at ten second intervals as determined by the tick of a sidereal chronometer and recorded at the other by means of a chronometer keeping mean time. Or the signals may be sent from an intermediate station that is visible from both. The observers at each station must of course have obtained their local time by proper observation, and the difference between their local times at the instant of the signal gives at once the difference of longitude. The signal may be made by the flash of a heliograph by day or the eclipse of a bright light at night.

The following example gives the results of observations made in this way in Western Australia to determine the difference of longitude between the Perth Observatory and Mount Maxwell, about 17 miles away to the east. The signals were made by means of an acetylene lamp placed in a box, the light shining through a hole over which a photographic snap-shutter was fixed. The shutter was released at the proper second and the time of the flash noted as it was seen through a theodolite at the other station. The example is taken from the *Western Australian Handbook for Surveyors* :—

DIFFERENCE OF LONGITUDE

1909	Mount Maxwell to Observatory	Observatory to Mount Maxwell	Mean result
Nov. 6 . .	1' 7.96"	1' 8.64"	1' 8.30"
Nov. 7 . .	1' 7.87"	1' 8.56"	1' 8.21"
Nov. 8 . .	1' 7.82"	1' 8.54"	1' 8.18"
Nov. 9 . .	1' 7.81"	1' 8.61"	1' 8.21"
Nov. 13 . .	1' 7.93"	1' 8.53"	1' 8.23"
Mean, . . .			1' 8.25"
Personal equation, . .			+0' 0.06"
Difference of time, . .			1' 8.31"

Other methods

Before wireless time signals came into common use for *relative* longitude determinations, observations of the moon were often used in *absolute* determinations. The most reliable of methods using the moon (although not by any means as accurate as wireless

methods) was by observation of occultations of stars. Since a surveyor would be unlikely to use this method nowadays, no description is given in this volume. However, if any reader is interested in lunar observations for longitude, he is referred to earlier editions of this work which have accounts of determinations by lunar distances, by lunar culminations and by lunar occultations.

SIMULTANEOUS DETERMINATION OF LATITUDE AND LONGITUDE FROM POSITION LINES

A graphical method of obtaining latitude and longitude from ex-meridian observations of two stars whose azimuths differ by approximately 90°

ALTHOUGH this method is commonly used by navigators it has not, until recently, received much attention from surveyors. It is based on the fact that, at any instant, there is an infinite number of points on the earth's surface from which the zenith distance of a particular star, or the sun, is the same. These points, if plotted on the earth's surface, would form a *small* circle with centre the geographic position of the star (*i.e.*, the point where the line joining the star to the centre of the earth intersects the earth's surface) and angular radius equal to the zenith distance of the star. If two suitable stars were observed, and their zenith distance circles plotted on the earth's surface, these circles would intersect at two points, at either of which the observation station could lie. The selection of the correct point would be simple if the approximate latitude and longitude of the station were known. In practice it is, of course, not possible to plot the entire zenith distance circles, but the portions in the vicinity of their intersection may be regarded as straight lines (called "position lines") which are plotted as such. The directions of these lines will be at right-angles to the respective radii of the zenith distance circles joining the observation point to the geographic positions of the stars. The radii are arcs of great circles which give the azimuths of the stars in terms of the observer's meridian. Hence, the directions of the two position lines can be found if these azimuths are calculated.

In Fig. 49 $PO P'$ is the axis of the earth, and EQ the equator. S_1 and S_2 are two stars whose geographic positions are, at the instant under consideration, S_1' and S_2' . Consider an observer at Z_1 : for him the zenith distance of S_1 will be measured by the arc $Z_1 S_1'$ of a great circle the plane of which passes through O , Z_1 , S_1'

and S_1 . The direction $Z_1 S_1'$ makes with the meridian $P Z_1 P'$ is the azimuth of S_1 . There is an infinite number of points on the earth's surface from which the zenith distance of S_1 will be equal to $Z_1 S_1'$. These will lie on the zenith distance circle Z_1, Z_2, Z_3, Z_4, Z_5 . The other star, S_2 , observed from Z_1 , will have zenith distance equal to $Z_1 S_2'$ and azimuth equal to $P Z_1 S_2'$. The zenith distance circle is shown by Z_1, Z_5, Z_6, Z_7 .

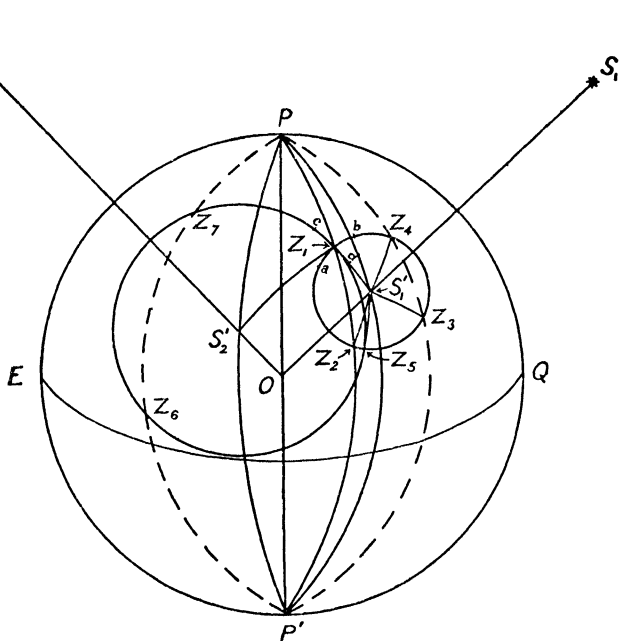


Fig. 49

Now the two zenith distance circles in the diagram intersect at Z_1 and Z_5 , so that there are two points on the earth's surface at which the observer could be located. In this case we know he is at Z_1 , but if we did not know this, the great difference in latitude between the two possible points would rule out Z_5 which is practically within the tropics. The arcs $aZ_1 b$ and $cZ_1 d$ of the zenith distance circles which intersect at Z_1 are, near Z_1 , practically straight lines whose directions are at right angles to $Z_1 S_1'$ and $Z_1 S_2'$ respectively.

From the practical point of view, the observer must first of all assume a latitude and longitude which he may obtain roughly from a map of his area, or from approximate observations. *He must*

know the time—either *Local Standard Time* or *G.M.T.*—accurately. Two suitable stars whose azimuths differ by about 90° are then selected and their altitudes together with the times of observation are measured as precisely as possible. (See Chapter 10, “Extra Meridian Observations of Sun or Star”.)

Using the assumed position and the observed time, the altitude and azimuth of each star are computed. The *observed* altitudes are then corrected and compared with the computed ones. If the observed altitude is greater than the computed one, it means that the true position of the observer is nearer the geographic position

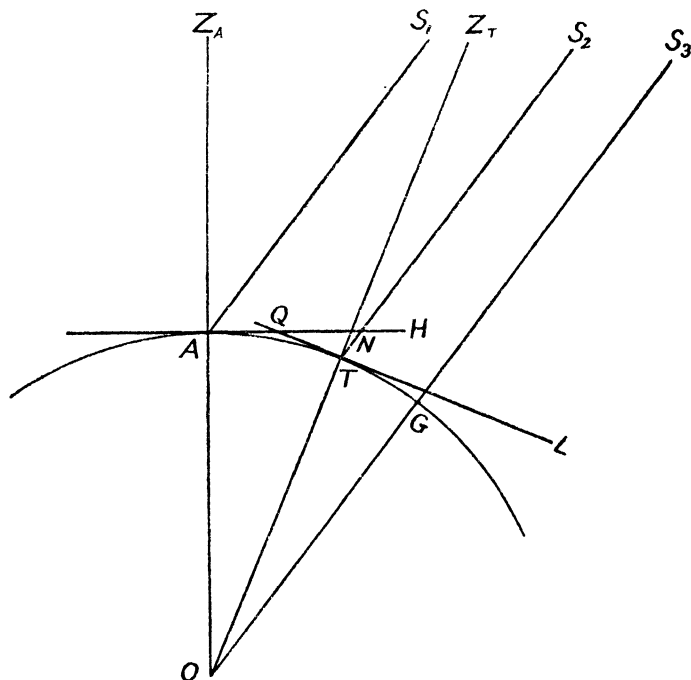


Fig. 50

of the star than the assumed one ; if less, he is further away from the geographic position of the star. The difference between observed and computed altitudes measures the arc of the earth's surface between the true and assumed positions.

In Fig. 50 :

A is assumed position with zenith Z_A .

T is true position with zenith Z_T .

$S_1 A$, $S_2 T$ and $S_3 G$ are parallel rays from a star.

G is the geographical position of the star, and O the earth's centre.

AH is the direction of the horizon at A (at right-angles to $Z_A A$); TL is the direction of the horizon at T (at right-angles to $Z T$). AH and TL intersect at Q .

$HA S_1$ is the altitude of the star at A , and $S_2 T L$ its altitude at T .

AH intersects TS_2 at N .

The quadrilateral $AQTO$ is cyclic, therefore the supplement of angle AQT , *i.e.*, angle NQT , is equal to angle AOT , which is a measure of the arc AT .

But in the triangle NQT , the exterior angle $LT S_2 =$ angle $QNT +$ angle NQT . Also angle $QNT =$ angle $S_1 AH$.

Therefore angle $LT S_2$ (altitude at T) = angle $S_1 AH$ (altitude at A) + angle AOT .

That is, the difference between the altitudes at A and T is a measure of the arc AT .

The arc AT is known as the "intercept," and there are two mnemonics relating this intercept and the observed altitude :

GOAT : Greater Observed Altitude, intercept Towards star's geographic position.

LOAF : Lesser Observed Altitude, intercept From star's geographic position.

As a result of his observations and computations, the observer has two observed and two computed altitudes, and two computed azimuths. On a sheet of paper he plots the line NS (Fig. 51) to represent his meridian, selecting any point Z on it to mark his position. Next, he lays off the lines $Za S_1$ and $c Z S_2$ through Z according to the calculated azimuths of stars S_1 and S_2 . Assuming that the observed altitude of S_1 was greater than the computed one, he would measure off Za in the direction of the star and equal to the difference in altitudes, to some suitable scale. Then he would plot the position line $a Z_1 b$ at right-angles to $Za S_1$ through a .

Again, assuming the observed altitude of S_2 was less than the computed one, he would measure off Zc equal to the difference in altitudes, away from the star (to the same scale as before) and plot the second position line $c Z_1 d$ at right-angles to $c Z S_2$ and intersecting the first at Z_1 which is his true position. ($a Z_1 b$ and $c Z_1 d$, although plotted as straight lines, are in reality arcs of the zenith

distance circles, but they may be treated as straight lines, since the radii of the circles are hundreds, if not thousands, of miles long.)

If now a perpendicular be dropped from Z_1 to meet NZS in X , ZX may be scaled off directly to yield the difference of latitude between assumed and true positions. XZ_1 , the difference of longitudes, obviously cannot be scaled in the same way because

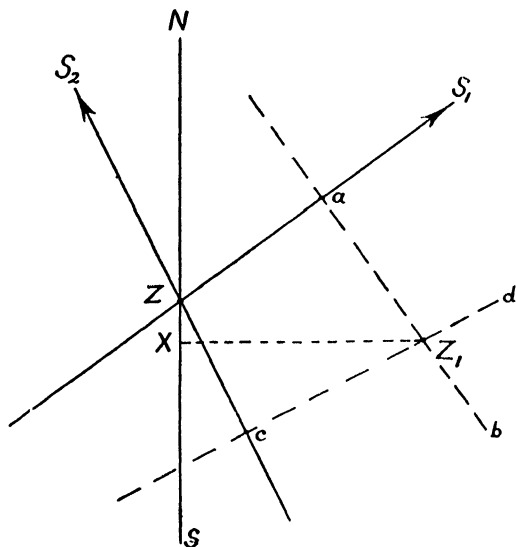


Fig. 51

degrees of latitude and longitude are approximately equal only at the Equator. It can easily be proved, however, that $XZ_1 \times \sec$ latitude is equal to the true difference in longitude. Thus, if XZ_1 is scaled from the plot and multiplied by the secant of the latitude of Z_1 , the difference of longitude is obtained. The differences of latitude and longitude obtained as above are now applied to the assumed latitude and longitude (in the sense indicated by the plot) to give the observer's true position.

EXAMPLE.—On 1951 October 9, at a place in approximate latitude $45^\circ 47' S.$ and approximate longitude $170^\circ 34' E.$, the following observations were made :

Star	L.Std.T. (12 ^h E.)	Altitude (Observed)
Achernar	20 ^h 03 ^m 20	44° 20' 30''
α Crucis	20 ^h 08 ^m 30 ^s	27° 30' 54''

From the *Star Almanac* :

Achernar.—R.A. 1^h 35^m 59^s, Dec.—57° 28' 41''

α Crucis.—R.A. 12^h 23^m 51^s, Dec.—62° 49' 52''

Find the true latitude and longitude by position line method.

For Hour Angles (using *Star Almanac*)—

	Achernar	α Crucis
L.Std.T. 1951 October 9 .	20 ^h 03 ^m 20 ^s	20 ^h 08 ^m 30 ^s
Longitude Std. meridian E.	12 ^h 00 ^m 00 ^s	12 ^h 00 ^m 00 ^s
U.T. 1951 October 9 . .	8 ^h 03 ^m 20 ^s	8 ^h 08 ^m 30 ^s
<i>R</i> at U.T. 6 ^h , 1951 October 9 .	1 ^h 08 ^m 12.4 ^s	1 ^h 08 ^m 12.4 ^s
Change in <i>R</i> , ...	+20.3 ^s	+21.1 ^s
<i>R</i> at U.T. of observation, .	1 ^h 08 ^m 32.7 ^s	1 ^h 08 ^m 33.5 ^s
But U.T. of observation, .	8 ^h 03 ^m 20.0 ^s	8 ^h 08 ^m 30.0 ^s
∴ U.T. + <i>R</i> (= G.H.A. ∩) = G.S.T.	9 ^h 11 ^m 52.7 ^s	9 ^h 17 ^m 03.5 ^s
Longitude E. (assumed), .	11 ^h 22 ^m 16.0 ^s	11 ^h 22 ^m 16.0 ^s
∴ L.S.T., ...	20 ^h 34 ^m 08.7 ^s	20 ^h 39 ^m 19.5 ^s
R.A., ...	1 ^h 35 ^m 59.0 ^s	12 ^h 23 ^m 51.0 ^s
Hour angle of Star . . .	18 ^h 58 ^m 09.7 ^s	8 ^h 15 ^m 28.5 ^s
(<i>East</i> hour angle of Achernar) 5 ^h 01 ^m 50.3 ^s		
Hour angle in arc, . . .	75° 27' 34.5''	123° 52' 07.5''
½ hour angle, ...	37° 43' 47.2''	61° 56' 03.7''

For Azimuths—

Co-latitude (assumed),	.	44° 13' 00"	44° 13' 00"
Polar distance,	.	32° 31' 19"	27° 10' 08"
$\frac{1}{2}$ sum (A),	.	38° 22' 09.5"	35° 41' 34"
$\frac{1}{2}$ difference (B),	.	5° 50' 50.5"	8° 31' 26"
log cos B,	.	9.9977344	9.9951762
log cot $\frac{1}{2}$ hour angle	.	10.1114171	9.7268737
log product,	.	10.1091515	9.7220499
log cos A,	.	9.8943306	9.9096400
log tan 58° 37' 32.3"	.	10.2148209	32° 59' 36.7"
log sin B,	.	9.0080832	9.1709120
log cot $\frac{1}{2}$ hour angle,	.	10.1114171	9.7268737
log product,	.	9.1195003	8.8977857
log sin A,	.	9.7929012	9.7659953
log tan 11° 58' 35.5"	.	9.3265991	7° 42' 50.3"

Azimuth of Achernar (E. of

$$\text{South}) = 58^\circ 37' 32.3'' - 11^\circ 58' 35.5'' \\ = 46^\circ 38' 57''$$

Azimuth of α Crucis (W. of

$$\text{South}) = 32^\circ 59' 36.7'' - 7^\circ 42' 50.3'' \\ = 25^\circ 16' 46''$$

For Altitudes—

	Achernar		α Crucis
log cos 58° 37' 32.3"	9.7165272	log cos 32° 59' 36.7"	9.9236232
log tan 38° 22' 09.5"	9.8985707	log tan 35° 41' 34"	9.8563553
log product	9.6150979	log product	9.7799785
log cos 11° 58' 35.5"	9.9904421	log cos 7° 42' 50.3"	9.9960519
log tan $\frac{1}{2}$ zenith dist.	9.6246558	log tan $\frac{1}{2}$ zenith dist.	9.7839266
$\frac{1}{2}$ zenith distance	22° 50' 55.4"	$\frac{1}{2}$ zenith distance	31° 18' 03.4"
zenith distance	45° 41' 51"	zenith distance	62° 36' 07"
Altitude	44° 18' 09"	Altitude	27° 23' 53"

DETERMINATION OF LATITUDE AND LONGITUDE 229

Observed altitudes :	44° 20' 30"	27° 30' 54"
Refraction	0' 58"	01' 49"
Corrected altitudes	44° 19' 32"	27° 29' 05"
Computed altitudes	44° 18' 09"	27° 23' 53"
Intercept (difference)	01' 23"	05' 12"
or	<u>83" towards Star</u>	<u>312" towards Star</u>

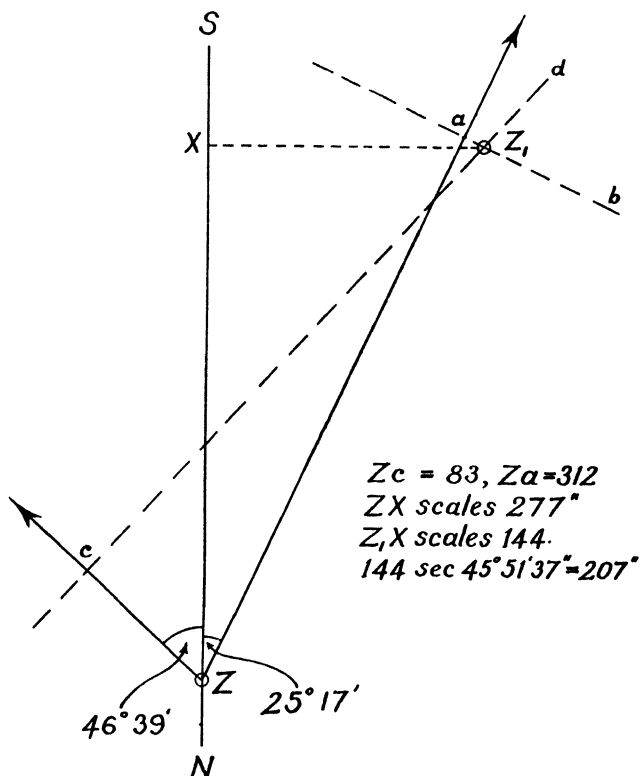


Fig. 52

The plot is shown in Fig. 52 in which the notation is the same as in Fig. 51. The true position (Z_1) is south-west of the assumed position Z. The difference of latitude X Z, scaled directly from the plot, is $277''$, or $04' 37''$; the difference of longitude X Z_1 scales 144, which, when multiplied by the secant of the true latitude, gives the correct difference of longitude as $207''$, or $03' 27''$.

Thus :

$$\text{True latitude} = 45^\circ 47' + 04' 37'' = \underline{\underline{45^\circ 51' 37''}}$$

$$\text{True longitude} = 170^\circ 34' - 03' 27'' = \underline{\underline{170^\circ 30' 33''}}$$

Position lines from sun observations

From the foregoing it will be readily appreciated that a morning and an afternoon observation of the sun will yield two position lines which may be used in exactly the same way as those obtained from star observations. The only disadvantage is that the observations cannot be completed within a short space of time, as is the case with two stars.

THE CONVERGENCE OF MERIDIANS

THE line of sight of the telescope of a theodolite in accurate adjustment, as the telescope is turned about its horizontal axis, traces out a vertical plane. This, if we regard the earth as spherical, we may consider to be a plane passing through the centre of the earth. Therefore, the straight line that is set out by a theodolite is in reality always the arc of a great circle on the earth's surface. Now, unless it happens to coincide with the equator or with a meridian of longitude, any great circle will cut different meridians at different angles. In other words, its bearing will vary from point to point. Thus as we proceed along a straight line set out by a theodolite on the earth's surface, the bearing of the line will not remain constant but will gradually alter. A line the bearing of which was everywhere the same would not be a straight line. A parallel of latitude for instance is such a line, but if the telescope of a theodolite is set out truly east and west at any place its direction would not mark out the parallel of latitude, which is a small circle, but a great circle that would ultimately intersect the equator.

This alteration in the bearing of a straight line is an important matter in surveys of any magnitude, as in latitudes in the neighbourhood of 60° it amounts to considerably over a minute of arc in a line one mile long, and in higher altitudes the alteration is still greater.

In Fig. 53, let N and S denote the north and south terrestrial poles, E L M Q is the equator, and A and B any two points between which the great circle arc A B has been set out.

Let N A M S and N B L S be the meridians through A and B. Then the bearing of the line B A at B is the angle N B A, and the bearing of the same line at A is $180^\circ - \text{N A B}$.

The difference between the bearings of the line A B at the points A and B is known as the *convergence* of the meridians between A and B.

If AB is plotted as a straight line on a plane, then the meridians through A and B will not be drawn as parallel lines, but as lines making an angle with each other equal to the convergence.

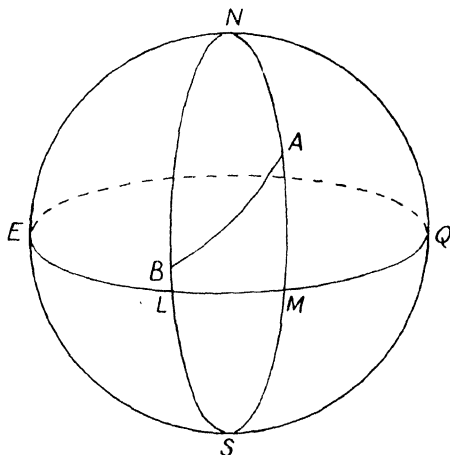


Fig. 53

Denote the convergence by c .

Then $c = 180^\circ - NAB - NBA$.

Let φ = latitude of A and φ' = latitude of B .

$NA = 90^\circ - \varphi'$, $NB = 90^\circ - \varphi$.

Denote the difference of longitude between A and B by m , so that $m = \text{angle } BNA$.

Then in the spherical triangle NBA , having given two sides and the included angle,

$$\begin{aligned} \tan \frac{1}{2} (NBA + NAB) \\ &= \frac{\cos \frac{1}{2} (NB - NA) \cot \frac{1}{2} m}{\cos \frac{1}{2} (NB + NA)} \end{aligned}$$

$$\begin{aligned} \therefore \cot \frac{1}{2} (180^\circ - NBA - NAB) \\ &= \frac{\cos \frac{1}{2} (\varphi - \varphi') \cot \frac{1}{2} m}{\cos \frac{1}{2} (180^\circ - \varphi - \varphi')} \end{aligned}$$

$$\therefore \cot \frac{1}{2} c = \frac{\cos \frac{1}{2} (\varphi - \varphi')}{\sin \frac{1}{2} (\varphi + \varphi')} \cot \frac{1}{2} m,$$

or, inverting

$$\tan \frac{1}{2} c = \frac{\sin \frac{1}{2} (\varphi + \varphi')}{\cos \frac{1}{2} (\varphi - \varphi')} \tan \frac{1}{2} m.$$

In any ordinary survey, the length of the line A B will be very small compared to the earth's radius, and the angles c and m will be so small that $\tan \frac{1}{2} c$ and $\tan \frac{1}{2} m$ may be replaced by $\frac{1}{2} c$ and $\frac{1}{2} m$ respectively without appreciable error.

$\therefore c$ (in circular measure)

$$= \frac{\sin \frac{1}{2} (\varphi + \varphi')}{\cos \frac{1}{2} (\varphi - \varphi')} m \text{ (in circular measure),}$$

and c (in seconds of arc)

$$= \frac{\sin \frac{1}{2} (\varphi + \varphi')}{\cos \frac{1}{2} (\varphi - \varphi')} m \text{ (in seconds of arc).}$$

Again, unless the line A B is a very long one, $\cos \frac{1}{2} (\varphi - \varphi')$ differs from unity by but a very small quantity, so that for ordinary purposes

Convergence in seconds = $\sin$ mid. lat. $\times$ diff. of long. in seconds.

Another convenient form of the result expresses the convergence in terms of the "departure" between A and B; that is to say, their distance apart measured in an east and west direction.

The parallel of middle latitude is a circle of radius

$$r \cos \frac{1}{2} (\varphi + \varphi'),$$

where r is the radius of the earth in miles, and, therefore, if d denotes the departure in miles,

$$\frac{d}{r \cos \frac{1}{2} (\varphi + \varphi')} = \text{the circular measure of } m.$$

$\therefore$ convergence in seconds

$$\begin{aligned} &= \sin \frac{1}{2} (\varphi + \varphi') \frac{d}{r \cos \frac{1}{2} (\varphi + \varphi') \sin 1''} \\ &= \frac{d \tan \frac{1}{2} (\varphi + \varphi')}{r \sin 1''}. \end{aligned}$$

Taking r as 3,958 miles we obtain, therefore, the following rule :—

To the constant log,	.	.	.	.	.	1.7169
Add log tan mid. lat.,	.	.	.	.	.	
Add log departure in miles,	.	.	.	.	.	

The sum is log of the approximate number of seconds in the convergence, ...

Thus for a departure of 1 mile in latitude 20° , the convergence is $19''$ only, but in latitude 40° it is $44''$, and in latitude 60° it is as much as $90''$.

It thus appears that the convergence increases very rapidly in high latitudes, and that in latitude 60° the bearing of a straight line one mile long and running approximately E. and W. will at one extremity be different by 1.5 minutes from what it is at the other.

The amount of convergence is such that when a straight line is run several miles in length the bearing of the line as determined by astronomical observation will differ appreciably at each end. The nearer the place is to the equator, the longer the line will have to be before the difference is sufficient to observe directly. In latitude 40° it is readily observable at the end of an east and west line two miles long, in latitude 60° the line need be only one mile long for the difference to be just as readily detected. There is no such effect in lines running directly N. and S., as such lines form a part of a meridian of longitude, and the convergence is greatest at the extremities of lines of given length, when the direction is E. and W.

The investigation we have given for convergence is of course an approximate one only, and the formulæ obtained are not exact, because the earth is not in reality a true sphere as has been assumed. The results obtained, however, are quite sufficiently accurate for all but the most refined geodetic work.

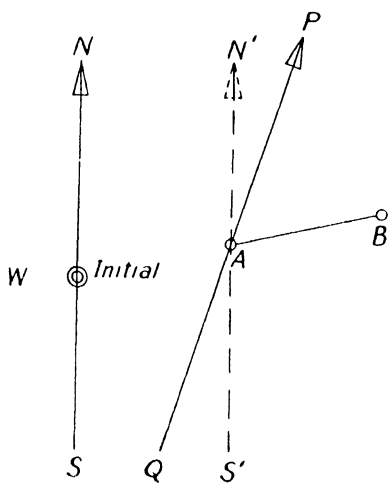


Fig. 54 (a)

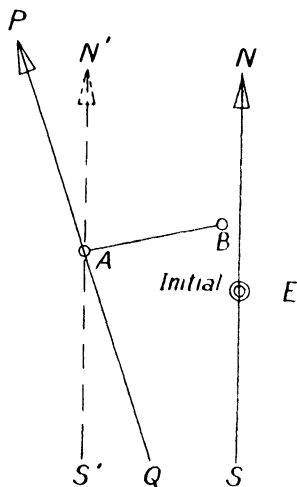


Fig. 54 (b)

In most work carried on by the land surveyor, traverses for control are tied to minor triangulation points or to standard survey points which lie within a particular meridional circuit. All

the bearings between these points in one circuit are in terms of the initial station of the circuit. Thus, if a surveyor wishes to check a traverse bearing by means of an astronomical observation, he must make a correction for convergence between the meridian through the initial and that through his observing station. The amount of the correction is obtained as described in the foregoing part of this chapter. The *sense* in which it must be applied is found as follows :—

A. For places in the southern hemisphere—

- (a) Where the observing station lies to the east of the initial.
Refer to Fig. 54 (a).

N S is the true meridian through the initial.

P Q is the true meridian through observing station, A.

N' S' is the direction of the true meridian through the initial transferred to station A (*i.e.* N' S' is parallel to N S).

The traverse bearing of A B, obtained by the surveyor, will be in terms of N' S', and will be measured by the angle N' A B.

The true bearing of A B, obtained astronomically, will be in terms of P Q, and will be measured by the angle P A B.

The convergence between the meridians is measured by the angle N' A P.

Thus, to convert the true bearing of A B into the bearing in terms of the initial, the convergence (N' A P) must be added.

Rule.—When the station lies to the *east* of the initial, *add* the convergence to the bearing obtained by astronomical observation, to bring it into terms of the meridian through the initial.

- (b) Where the observing station lies to the west of the initial.
From Fig. 54 (b) it will be obvious that the convergence must be subtracted from the bearing obtained astronomically.

Rule.—When the station lies to the west of the initial, *subtract* the convergence from the bearing obtained by astronomical observation, to bring it into terms of the meridian through the initial.

B. For places in the northern hemisphere.—

- (a) Where the observing station lies to the east of the initial.
Fig. 55 (a) is self-explanatory.

Rule.—Where the station lies to the *east* of the initial, *subtract* the convergence from the bearing obtained by astronomical observation, to bring it into terms of the meridian through the initial.

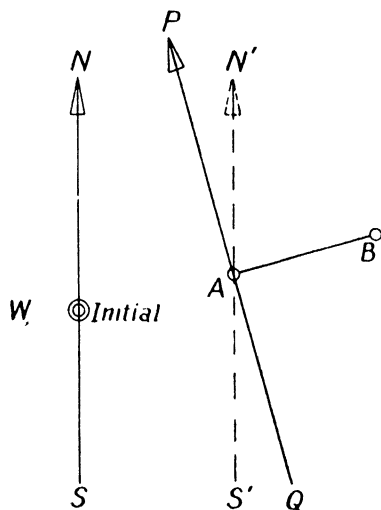


Fig. 55 (a)

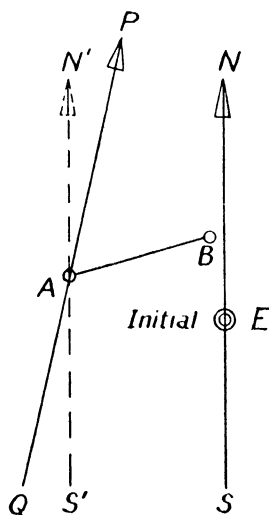


Fig. 55 (b)

- (b) Where the observing station lies to the west of the initial.
See Fig. 55 (b).

Rule.—Where the station lies to the *west* of the initial, *add* the convergence to the bearing obtained by astronomical observation, to bring it into terms of the meridian through the initial.

As an example, we shall find the bearing of the R.M. in the example on page 168 in terms of the initial (Trig. A, Taieri, New Zealand).

Initial : Latitude, $45^{\circ} 51' 39.9''$ S., Longitude, $170^{\circ} 16' 57.2''$ E.

Observing station : Latitude, $45^{\circ} 51' 29.4''$ S., Longitude, $170^{\circ} 30' 36.1''$ E.

Convergence in seconds = $\sin$ mid-latitude $\times$ difference of longitude in seconds.

log sin $46^{\circ} 51' 34.6''$	.	.	.	9.8559039
log 818.9''	.	.	.	2.9132309
log convergence	.	.	.	2.7691348
Convergence	.	.	.	587.7''
or	.	.	.	09' 47.7''.
Mean bearing of R.M., as obtained by sun observation (p. 169)	.	.	.	$35^{\circ} 54' 29.5''$
Convergence (<i>add</i> , since station is <i>east</i> of initial)	.	.	.	09' 47.7''
Bearing of R.M. in terms of initial	.	.	.	$36^{\circ} 04' 17.2''$
(The traverse bearing was $36^{\circ} 04' 10''$)				

Note.—If the co-ordinates of a traverse station are known in terms of the initial, as well as the latitude and longitude of the initial, the latitude and longitude of the traverse station can be found by the use of tables such as those on p. 382 in Park's *Text Book of Theodolite Surveying and Levelling* (Griffin, London, 1922).

MISCELLANEOUS EXERCISES

1. At what height would a signal need to be erected at station B to be visible from the instrument at A, so that the line of sight would be 10 feet clear of the summit of an intervening hill at C ?

Height of instrument at above sea level at A, 488 feet. Station B, 20 miles distant from A, 500 feet. The summit of the intervening hill, 12 miles from A, 442 feet. Neglect refraction.

Ans. 35 feet

2. A man on a height near Pietermaritzburg, 42 miles from Durban, owing to the clearness of the air can see a ship 6 miles out at sea. Looking in the other direction he can see the heights of Drakenburg, which he knows are 110 miles from him. Find the height of the Drakenburg above the sea, taking the radius of the earth as 3,960 miles. (*Educational Times*.)

Ans. Half a mile nearly

3. From a point in latitude 30° South, longitude 120° East, a line at right-angles to the initial meridian is run easterly for a distance of 18 miles. Find the true bearing of the line at its easterly

end, its longitude, and the bearing and distance to a point in that longitude in the same latitude as the starting point. Assume the radius of the earth to be 3,960 miles.

- Ans. (a) $269^{\circ} 50' 59''$
 (b) Longitude, $120^{\circ} 18' 02''$
 (c) Due south, 0.024 mile

4. On the evening of April 12, the altitude at meridian transit of the star α Hydræ, north of the zenith, was observed from two hills, A and B, a considerable distance apart. Altitudes of α Virginis, in the eastern sky, were observed simultaneously from both hills by aid of prearranged signals. Several sets were taken, which, reduced to a mean and cleared of corrections for refraction and level errors, gave the following results :—

At station A the meridian altitude of α Hydræ was $63^{\circ} 22' 40''$ and the altitude of α Virginis was $12^{\circ} 14' 18''$.

At station B the meridian altitude of α Hydræ was $63^{\circ} 44' 40''$ and the altitude of α Virginis was $12^{\circ} 44' 18''$.

The declination of α Hydræ was $-8^{\circ} 16' 26''$, and the declination of α Virginis was $-10^{\circ} 42' 0''$ taken from the *Nautical Almanac*.

Find the distance between the two hills A and B in miles and decimals, and the true bearing of each station, treating the earth as a sphere having a radius of 3,968 miles.

- Ans. Distance = 44.64 miles
 Bearing of B from A,
 $55^{\circ} 31' 04''$
 Bearing of A from B,
 $235^{\circ} 09' 02''$

5. What is, approximately, the spherical excess in a triangle on the earth's surface, two sides of the triangle being 163,421 feet and 154,599 feet respectively, and the observed included angle being $60^{\circ} 05' 12.32''$. What factors do you require for an exact evaluation ?

6. In latitude 45° N. an observer sees a certain star rise in the N.E. If the observer travels to another place with a slightly different latitude, show that the change in direction of the same star at rising will be equal to the change in latitude.

7. Show that all the stars observable from any one place have the same rate of change in azimuth at rising.

8. Prove that the rate of change in altitude of a star is always greatest, when the star is in the prime vertical.

9. From an initial station A in latitude $36^{\circ} 52' 57''$ S., longitude 145° E., traverse lines are run to a point B, which is situated 147,523 links south of, and 347,724 links east of, the initial station A. At B, β Centauri is observed at eastern elongation on March 1, as under :—

		Referring object	Star
F.L.,	. . . (a)	$255^{\circ} 57' 15''$	$141^{\circ} 25' 30''$
F.L.,	. . . (b)	$255^{\circ} 57' 10''$	$141^{\circ} 25' 45''$
F.R.,	. . . (a)	$255^{\circ} 57' 10''$	$141^{\circ} 25' 40''$
F.R.,	. . . (b)	$255^{\circ} 57' 15''$	$141^{\circ} 25' 20''$

From the *Nautical Almanac* :—Sidereal time at 0^h G.M.T. March 1, $10^h 37^m 02^s$. R.A. β Centauri, $13^h 56^m 13^s$. Declination — $59^{\circ} 51'$.

Find the bearing of the R.O. in terms of the initial meridian, and the standard time (of the 150th meridian) of the observation.

Ans. Bearing of R.O., $255^{\circ} 56' 26''$
Std. Time, $11^h 18^m 18^s$ p.m.

INDEX

Abbreviations, 39, 135.

Aberration, annual, 38.

— diurnal, 38.

Alidade level, 102.

— —, allowance for error of, 102.

Almucantar method for determination of time, 203-6.

Alternative method for determining local sidereal time, 53.

— — for determining local mean time, 55.

Altitude, 12.

— and azimuth, determination of, at short interval, 85-8.

— — — determination of, having given R.A., declination, latitude and time, 80-4

— — — of star at elongation, 108

— after star is in known position, 80

— of celestial pole, 22.

— of pole star, for latitude determinations, 168.

Altitudes, equal, method of correction, for sun observations, 201.

— — — of, for azimuth, 105.

— — — of, for time, 198.

—, meridian, method of, at both culminations, 156

— — — of, for latitude, 151-6.

Annual aberration, 38

Antarctic circle, 22.

Apparent motion of the stars, 9-11

— noon, to find local mean time of, 64, 70.

— solar time, 40-2.

— — —, conversion to local mean time, 64, 72.

— — times at same instant in places of different longitude, 47.

Arctic circle, 22.

Aries, first point of, 14, 37.

— — — of, time of transit, 58-61.

Astronomical coordinates, 12-5.

— terms, synopsis of, 18.

Atmospheric refraction, 91-5

— — tables in *Star Almanac*, 95.

Autumnal equinox, 37.

Averaging observations, 192.

Azimuth, 13.

— altitude and hour angle at elongation, 113-6.

—, method of determination of, 80-8.

Bearing, traverse, checking by astronomical observation, 235.

Bessel's formula for refraction, 93.

Calculation of the time of elongation, 111.

Cancer, tropic of, 22, 33.

Capricorn, tropic of, 22, 33.

Celestial equator, 11, 34

— sphere, 8.

— —, to plot position of sun's centre on, 35.

Changing face, elimination of errors by, 97.

Chronograph, 218.

Chronometers, for longitude determination, 214.

—, checking by rhythmic time signals, 215-8.

—, comparison of, 219

Circles, great, 1

—, small, 1

Circular parts, Napier's rules of, 2-4.

Circum-elongation observations for azimuth, 134-46.

Circum-meridian observations for latitude, 156-63.

— —, limits of time for, 160.

Coefficient of refraction, 92.

Collimation, error of, 195.

Comparative advantages of co-ordinate systems, 15.

Convergence of meridians, 231-7.

— — —, application of, to astronomical bearing to check traverse bearing, 235-7.

Coordinates, astronomical, 12-5.

Corrections, instrumental, 95

— to observations of altitude and azimuth, 89-103.

— to sun observations, 124.

Culmination, computation of time of, 57, 153, 159, 162.

—, lower, of star, 156.

—, upper, of star, 156.

Culminations, lunar, for longitude determination, 221.

Data necessary for computation of time, 189.

Daylight, star observations in, 118.

- Declination, 13, 15.
 — circle, 15.
 —, computation of sun's from
 Nautical Almanac data, 34, 123.
 Degree of longitude, the length of, 21.
 Determination of level error of axis
 by means of the striding level,
 99.
 — of true meridian, 104-50.
 Distance between places whose lati-
 tudes and longitudes are known,
 23-5.
 Distances, lunar, for longitude deter-
 minations, 221.
 — of stars, 8.
 Diurnal aberration, 38.
- Earth**, the figure of, 27-9.
 —, its shape, 19-31.
 —, its orbit round the sun, 32
 —, zones of, 21.
 Ecliptic, 37.
 —, obliquity of the, 37.
 Effect of an error in collimation, 95.
 — of an error in direction of prime
 vertical, 166.
 — of an error in latitude, 117, 125.
 — of an error in the longitude of place
 of observation, 127.
 — of an error in the measured
 altitude, 128.
 — of an error in the sun's declination
 upon the calculated azimuth,
 126.
 Elimination of instrumental errors
 by changing face, 97.
 Ellipsoid, the earth an, 28.
 Elongation, altitude, azimuth and
 hour angle at, 113.
 —, calculation of time at, 111.
 —, observation of star at, 108-18.
 —, observation of star near, 111.
 Equal altitudes, method of, for
 azimuth, 105.
 — —, — of, for time, 198.
 Equation of time, 42.
 —, personal, 218.
 Equator, celestial, 11, 34
 Equinoctial points, 37.
 Equinoxes, 33, 37.
 —, precession of, 37.
 Error of transverse axis, measure-
 ment of, 98.
 Errors, instrumental, 97-103.
 —, —, elimination of, by changing
 face, 97.
 Excess, spherical, 7.
- Extra-meridian observations, best
 time for, 128.
 — — for azimuth, 118-31.
 — — for time, 187-98.
 — — on sun or star, 118-31, 187-98.
- Figure of earth, 27.
 First point of Aries, 14, 37.
 — — — —, time of transit of, 58-61.
 — — of Libra, 37.
 Flash-light signals for longitude
 determinations, 220.
 Formula, choice of, for extra-meridian
 observations, 189.
 Frigid zone, 22.
- Geocentric latitude, 28.
 Geographical latitude, 28.
 Geographical position of star, 222.
 Gnomon, 207.
 Great circles, 1.
- Horizon, celestial, 9.
 Horizontal dial, 207.
 — parallax, 90.
 Hour angle, 16.
 — — being known, to determine
 time, 16, 190.
 — — of star at elongation, 108.
- Inclination of earth's axis to plane of
 orbit, 32.
 Initial, conversion of astronomical
 bearings into terms of, 146, 235-7.
 Intercept, in position line computa-
 tions, 225.
- Latitude, by altitude of pole star, 168.
 —, by circum-meridian observations,
 156.
 —, by horizontal angle between two
 circumpolar stars at elongation,
 173.
 —, by meridian altitudes, 151-6.
 —, by prime vertical transits, 163.
 —, by rate of change of altitude near
 prime vertical, 170.
 —, effect of an error in, 117, 195.
 —, geocentric, 28.
 —, geographical, 28.
 —, methods of determination of,
 151-80.
 —, terrestrial, 19.
 Length of a degree of longitude, 21.
 Libra, first point of, 37.
 Lines, position, 222.

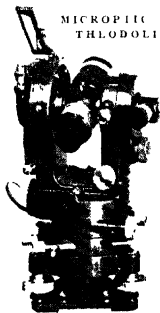
Local mean time, 45.
 — sidereal time, 46.
 — apparent time, 40, 41.
 — noon, to find local mean time of, 64, 72.
 Longitude, by flash-light signals, 220.
 —, by lunar culminations, 221.
 —, by lunar distances, 221.
 —, by lunar occultations, 221.
 —, by portable chronometers, 218.
 —, by reception of wireless time signals, 214-8.
 —, length of a degree of, 21.
 —, methods of determination of, 214-8.
 —, terrestrial, 19.
 Lunar observations for longitude, 221.
 Mean noon, 42.
 — solar time, 41, 42.
 — time, reduction to sidereal, 48.
 — —, conversion to apparent time, 64, 71.
 Meridian, 12
 —, altitudes for determination of latitude, 151-6.
 —, — of a star at both lower and upper culminations, 156.
 —, determination of, by circum-elongation observations, 134-46.
 —, — of, by close circumpolar star, 131.
 —, — of, by equal altitudes, 105.
 —, — of, by extra-meridian observations, 118-31.
 —, — of, by star at elongation, 108.
 —, — of true, 104-50.
 —, transit, computation of time of, 57, 153, 159, 162.
 —, transits on both sides of zenith, 185.
 Meridians, convergence of, 146, 231-7.
 Methods of determining latitude, 151-80.
 — — — longitude, 214-21
 — — — time, 181-210.
 — — — true meridian, 104-50.
 Motion, apparent, of stars, 9-11
 —, — of sun, 32, 34.
 —, in right ascension and declination, 34.
 —, proper, 38.
 Nadir, 9.
 Napier's rules of circular parts, 2-4.

Nautical Almanac, 34, 35, 39, 42, 43, 44.
 — —, data with regard to time, 67.
 Noon, apparent, 39, 42.
 —, mean, 39, 42.
 North temperate zone, 22.
 Notation, 39, 135.
 Nutation, 38.
 Oblate spheroid, the earth an, 28.
 Oblique-angled spherical triangles, solution of, 4-6.
 Oblique dial, 209.
 Obliquity of ecliptic, 37.
 Observations on both east and west stars, 192.
 Occultations, lunar, for longitude determination, 221.
 Parallax, 89-91.
 —, horizontal, 90.
 Parallel of latitude, 20.
 Personal equation, 218.
 Plot position of sun's centre on celestial sphere, 35.
 Polar distance, 15.
 Pole, celestial, 9, 22.
 — star, latitude by altitude of, 168
 — — tables in *Star Almanac*, 170.
 Portable chronometers, 214, 218.
 Position, geographic, of star, 222.
 — lines, 222.
 — —, latitude and longitude from, 222-30.
 Precession of the equinoxes, 37.
 Prime vertical, 16, 163-8.
 — — dial, 208.
 — — transits, for determination of latitude, 163-8.
 Proper motion, 38.
 Recording and receiving wireless signals for longitude, 214-8.
 Referring mark for azimuth observations, 104.
 Refraction, atmospheric, correction for, 91.
 —, —, tables in *Star Almanac*, 95.
 Residual instrumental errors, 95.
 Rhythmic time signals, wireless, 215
 — — —, comparing chronometers with, 215-8.
 Right-angled spherical triangles, solution of, 2-4.

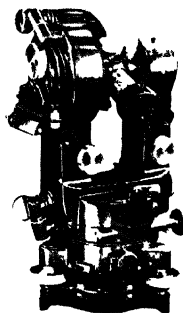
- Right ascension, 12, 13.
 — — and declination, motion in, 34.
 — — of star, to determine, 13, 84.
 Rising of a celestial body, time of, 210.
 Rules of circular parts, 3-4.
- Setting of a celestial body, time of, 210.
 Sidereal day, 15.
 — time, 15, 39.
 — — at 0^h G.M.T., 49.
 — —, reduction to mean time, 52, 55.
 Signals, wireless time, 214-8.
 Small circles, 1.
 Solar time, apparent, 40.
 Solstitial points, 37.
 Solution of spherical triangles, 2-6.
 South temperate zone, 22.
 Sphere, celestial, 8.
 Spherical excess, 7.
 — triangles, oblique, 4-6.
 — —, right-angled, 2-4.
 Standard time, 47.
 — —, to change to local mean time, 47.
 Star, averaging several observations of the same, 192.
 — observations in daylight, 118.
 —, geographic position of, 222.
Star Almanac data with regard to time, 67.
 Stars, apparent motion of, 9-11.
 —, observations on both East and West, for time, 192.
 Stile of sundial, 207.
 Striding level, use of, in azimuth determinations, 132.
 — —, correction to prime vertical observations, 167.
 — —, use of, 99.
 Sun, apparent motion of, 31.
 — dials, 207-9.
 — —, horizontal, 207.
 — —, oblique, 209.
 — —, vertical, 208.
 —, earth's orbit round, 32.
 —, motion in R.A. and declination of, 34.
 —, observations, 121-5.
 —, semi-diameter, 34.
 Sun's apparent annual path, 36-8.
 — — motion among the stars, 31, 34.
 — centre on celestial sphere, to plot position of, 35.
 — declination, computation of, 34, 123-4.
 Synopsis of astronomical terms, 18.
- Tables for astronomical refraction in *Star Almanac*, 95.
 — for reduction of circum-meridian observations, 134, 158.
 Temperate zone, 22.
 Terrestrial latitude and longitude, 19.
 Three systems of time measurement, 41.
 Time, apparent solar, 40.
 —, — —, conversion to local mean time, 64, 72.
 —, determination by almucantar method, 203-6.
 —, — by equal altitude observations, 201.
 —, — by extra-meridian observations, 187-98.
 —, — by meridian transits, 57, 181.
 —, equation of, 42.
 —, local mean, 45.
 —, — —, reduction to sidereal, 48.
 —, — sidereal, 46.
 —, — —, reduction to mean, 48, 52, 55.
 —, mean, 41.
 —, —, conversion to local apparent time, 64, 71.
 — measurement, three systems of, 41.
 —, *Nautical Almanac* data with regard to, 67.
 — observations on a close circum-polar star, for true meridian, 131.
 — of meridian transit, 57, 153, 159, 162.
 — of rising or setting of celestial body, 210.
 — of transit of first point of Aries, 58-61.
 — signals, wireless, 214.
 — —, rhythmic, 215.
 — standard, of different countries, 48.
 —, *Star Almanac* data with regard to, 67.
 Torrid zone, 22.
 Transit, meridian time of, 57, 153, 159, 162.
 — of first point of Aries, 58-61.
 Transits, meridian, on both sides of zenith, 185.
 Transverse axis, error due to want of horizontality, 184.
 Traverse bearing, checking by astronomical observation, 146, 235.
 Triangles, spherical, solution of, 2-6.
 Tropic of Cancer, 22, 33.
 — of Capricorn, 22, 33.

- | | |
|---|---|
| <p>True meridian, determination of, 104-50.</p> <p>Tycho Brahe, and sun's position among the stars, 31</p> <p>Vernal equinox, 37</p> <p>Wireless time signals, 214-18</p> | <p>Zenith, 9.</p> <p>— pair observations of stars for latitude, 153-6.</p> <p>Zone, north temperate, 22</p> <p>—, south temperate, 22.</p> <p>Zones of the earth, 21</p> |
|---|---|

MICROPTIC NO. 1
THEODOLITE



MICROPTIC NO. 2
THEODOLITE



WATTS *Microptic*

SURVEYING INSTRUMENTS

incorporate the efficient "Microptic" reading system, which employs enclosed glass circles and optical micrometers. It gives high accuracy with very simple methods of reading.

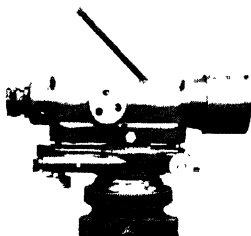
These instruments — Theodolites, Simple Transits, Levels, and Alidades, covering all normal requirements—have been designed to facilitate rapid setting up and use, even in awkward positions ; and, in general, to give the surveyor the instrumental qualities he requires in a convenient and compact form.

Write for catalogues AC/14-16 from

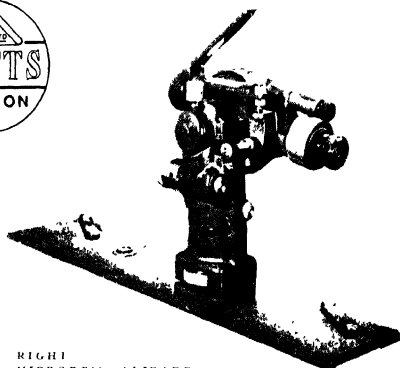
HILGER & WATTS LIMITED
WATTS DIVISION

48 Addington Square, London, S.E.5

Members of the Scientific Instrument Manufacturers Association and of SCIEX



ABOVE
MICROPTIC LEVEL



RIGHT
MICROPTIC ALIDADE

