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The World View of Physics

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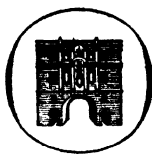
THE HISTORY OF NATURE

THE WORLD VIEW OF PHYSICS

by

C. F. v. WEIZSÄCKER

Translated by
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Preface

to the English Edition

IN re-reading these essays in the English translation I feel a desire directly to address the English-speaking reader. I think Mrs. Grene's translation is excellent, even in the most difficult questions of philosophical terminology. Yet in every text that is not merely technical but a spontaneous expression, trying to convey the author's views and feelings to an audience with a well-defined, particular cultural background, there must be untranslatable elements. Thus, for example, to a German reader a reference to Goethe's views about Nature may serve as a shorthand expression of a more general 'understanding' or 'human' attitude towards Nature which transcends the self-imposed limitations of the scientific approach and which, in principle, can be understood by everybody. But Goethe has not been absorbed into the English speaking cultural tradition to an extent that would make such a reference helpful to an English or American reader. He would rather feel embarrassed by the strangeness of those elements in Goethe's thoughts which are peculiar to the German cultural tradition and which, actually, do not matter too much with respect to the questions I want the reader to consider. These questions, I think, are common problems of the modern world. I did not try, however, to replace such passages, since it is a delicate question whether a spontaneously written exposé can be improved by subsequent alterations that are bound to interrupt the train of thought. But it may be useful to make a few comments.)

These essays have been written during a period of several years. The order in which they are presented here represents a development of the views of the author and of the colleagues and friends with whom he used to discuss such matters. The starting-point is the fact that the scientific outlook that framed most of the typical 'modern' elements in our culture is as unavoidable to us as it is limited. The author feels himself compelled to avoid two misunderstandings of the role of science: the negative one that

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underestimates, neglects, or even rejects the influence of science upon the whole of our lives; and the positive one that considers science to be self-sufficient and that thereby neglects the limitations and even dangers of a merely scientific approach to the fundamental questions of life. The internal changes in science itself, as illustrated by the transition from classical physics to quantum theory, give us some hope that science will by its own developments be able to adapt itself to situations that could not be foreseen by the founders of modern science. Hence the first part of the book is devoted to an outline of the conceptual structure of atomic physics that tries to avoid the more repugnant technicalities such as equations; then the path divides.

One task of the following chapters (especially Chapter Four) is to prepare a fuller understanding of the philosophical consequences of modern physics. Three existing philosophical views are examined: the two 'naive' schools of realism and empiricism, and the philosophy of the *a priori* in the form given to it by Kant.

The insufficiency of 'realism' for the interpretation of atomic physics is an established fact among scientists to-day. Even in the controversy between Einstein and Bohr, Einstein does not deny that fact; he only concludes from it, not that realism but that the actual form of atomic physics should be considered unsatisfactory. In this respect the present essays do not pretend to be more than a commentary on Bohr's views. ¹It is Bohr who has influenced the philosophical ways of thought of all younger atomic physicists in the most profound manner, and I want to use this occasion for expressing once more the admiration and gratitude we all feel for his influence. ²

In the English tradition empiricism was more influential than in German philosophy. Hence in an essay originally written in English I might have spent more space for a discussion of its merits and shortcomings. I think, there is no doubt that empiricism was most valuable in criticizing the naive way in which traditional metaphysics used to translate insights of a particular origin and of a limited scope into statements that were thought to be general and absolute. But by refuting generalizations we do not yet refute the original discovery that had been unduly generalized. Perhaps I may point out a particular fact not mentioned in the book. It has been a dogma of empiricism that there are no 'inborn ideas'. This, however, has certainly never been *empirically* proved. It was rather considered to be self-evident, since the infant does not display any higher intellectual activity. But this argument was well known to *apriorists*, and they used to say that knowledge of which we first become conscious when we

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need it in the field of some particular experiences, therefore does not need to have been acquired by deducing or abstracting it from that experience. It might as well have been the inborn condition of the possibility of such experience—of which, of course, we would not be aware before having had the corresponding experience. Empiricists used to consider this view to be an escape into unobservable complications. But now the study of animal behaviour shows us that at least in the growth of animals precisely the structure exists that was proposed for the growth of human knowledge by the *apriorists*. Exactly as organs grow in the lifetime of growing animals, so grows the ability to use them, and cases in which the 'knowledge' how to use the organ is completed before the organ is complete itself, show clearly that at least in some cases there is 'inborn knowledge'. Nobody who has seen it would deny that the chicken does not 'learn' to run but just knows it. It has been proved that at least some birds in the same way never 'learn' to fly but that they use their growing ability to fly to the extent to which it exists at the respective moments. The 'learning' has been done not by the individual but by the species millions of years ago. This is a poor analogy to the *a priori*—especially since 'inborn' is not identical with '*a priori*'—but does it not show the empirical weakness of the usual empiricist arguments?

We should consider the philosophy of the *a priori* not as a set of dogmas but as the analysis of an important element of our understanding which we ought to know if we want to understand what experience means. This element was probably discovered by Plato, and Kant does not mean more than one important step in a long, and not yet finished, process of analysis. In this respect the present essays cannot be considered to be more than an indication of the problem.

The other task of the present book is a discussion of some problems of our human life that are connected with the growth of science. The atomic bomb has become a symbol of a development by which we scientists ourselves have become aware late enough, of the fact that science may contain dangerous possibilities. This book, except the last chapter, was written before the explosion of the first atomic bomb, but with the knowledge that a bomb might be possible. Yet the book does not propose any specific course of action to be taken in the practical questions of the application of science. I frankly admit that this is a fundamental weakness of the book. My only excuse is that these questions cannot be solved by any general remedy but that they need particular concrete decisions in every single case. Thus

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what can be expressed in a general way will probably not be rules of action, but an attitude of the mind from which the right decisions can possibly spring. It is this attitude I have been seeking throughout the time covered by the writing of these essays. I have not been able to find any convincing solution that avoided Christianity.

If we want to understand the mutual relationship of Science and Christianity we must try to re-interpret both of them. Our time is inclined to take science too much as an absolute truth and Christianity too much as an historical relic. The question of their relationship then usually takes the form: are Science and Christianity reconcilable? This question, I feel, is out of date. Rather we ought to ask whether they need each other, whether they can exist without each other.

In the discussion of this question in Chapters Five and Seven of this book the reader may again feel the difference between the English and American and the Continental, particularly the German, situation. Christianity and Science, it is true, are common to the nations of the West as a whole. But the later developments of Christianity and particularly the ways of the modern departures from it are peculiar to the individual nations. In England and America the moral rules that were historically upheld by Christianity have, in many cases, survived the Christian faith. Morality without religious belief is a common feature of modern Anglo-American life, with an emphasis either on traditional forms or on progress. In Germany, on the other hand, a tendency towards strong irrational religious feeling survived the decay of Christianity. In its high forms this enabled some of the leading figures of German literature and philosophy to have a genuine understanding of pre-Christian pagan religion. On the other hand it made possible pseudo-religious mass movements that got out of moral self-control. The particular form of despair which to-day is called nihilism was well known for decades to those European thinkers who saw the hollowness of much of the civilization of our time; in the last decade it has become a half-conscious feeling of, perhaps, a majority of the people on the European Continent. The reader will find my first chapter tainted by German irrationalism, and the later ones by German or Continental nihilism, and I deny neither of the charges. If there is any advantage in having gone through these states of the mind, it may be the freshness with which the Christian message can be heard in such a vacuum. It may then be easier to dissociate Christianity from an outmoded set of beliefs, habits, and social institutions.

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I see no way, however, of dissociating Christianity from its fundamental experiences. It is, I am afraid, the denial of the experience of sin that threatens to-day, all over the world, to replace considerations of morality by considerations of efficiency. The experience of my own guilt seems to be the only way for me as a human being, to discover that all the apparently efficient measures that forget the needs of the body or of the mind of my fellow-man in the end are not efficient but self-destructive. It seems to me that here is a point where science can only do the next step in its development as one of the fundamental factors in modern life if it accepts to learn the Christian lesson.

C. F. v. WEIZSÄCKER

Gottingen, June 1951

Chapter One

CONTEMPORARY PHYSICS AND THE PHYSICAL WORLD VIEW

THE PROBLEM

A FEW decades ago physical science had a unified picture of the world. This picture offered a frame into which all known physical phenomena fitted. As a model of a scientific image of the world it exercised a decisive influence on all other sciences, and it helped to mould the intellectual face of the age.

To-day this world no longer exists. It was destroyed simultaneously from within and without: physics itself made discoveries which found no place in it, and the neighbouring sciences have again recognized more clearly how their object differs from the object of physics. The scientific value of the facts from which the old physical world picture proceeded was of course not affected by these discoveries. What was recognized as wrong was that in this world view a way of thinking suited to a limited sphere of our experience, had been over-hastily generalized for all spheres.

Our present condition, then, is richer in positive knowledge than the earlier one. But it appears poorer in inner unity. Earlier, the unity of the physical world seemed to make intelligible the unity of reality itself. To-day the physicist finds behind the sphere of immediate experience, which is governed by the so-called classical physics, the

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sphere of the atoms, not directly perceptible, which satisfy new types of order. True, all the inorganic sciences, particularly physics and chemistry, are fundamentally bound together by atomic theory into a higher unity. But the root of this unity, the mechanics of the atoms, does not correspond to our familiar notions of physical objects, and the abstract mathematical form in which it is dressed does not, according to some physicists, satisfy our need for a real understanding of nature. Moreover, a common world view no longer unites the great groups of sciences. Beyond the sciences that are grouped around physics and chemistry the physicist finds an autonomous biology. Beyond that he senses the social and humanistic disciplines whose inner articulation is for the most part already incomprehensible to him. There is no longer any recognizable philosophy which would hold all the disciplines together.

But our mind seeks unity. Nor can science fulfil its task with respect to life, if it loses its orientation in its own domain. In this situation, what is to be done?

Our aim must be to construct a new, more correct and more living picture of the world. True, whether we achieve this aim does not depend on our own volition. But it does depend on us whether we seek it. And this work of seeking is necessary whether or not it be crowned with success to-day or to-morrow. For even if someone came to-day who knew the answer to all unsolved problems, we should not understand him if our own need had not already driven us to put the questions which he answers. No help comes where a need has not even been felt.

We want, therefore, to try to put these questions. It cannot be our aim in this to enumerate unsolved problems. We want rather to initiate a train of thought such that once it has begun, the questions follow of themselves in natural sequence, and some of the answers, even, present themselves after part of the ground is covered. We have herewith no immediate systematic goal. But in order to know what we are talking about, we shall choose as guideposts the much

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discussed concepts of wholeness and perceptibility* and examine their role in the physical mode of thought. The newer biology reproaches the physical way of thinking with its failure to understand wholeness in the processes of life. On the other hand, the objection of many natural scientists to modern theoretical physics is that it has given up the perceptibility of the world of classical physics. We want to test how far both objections are justified, and where we may hope for relief in this crisis. We shall begin with the question of wholeness, which concerns the old physical picture of reality as well as the modern one, and proceed afterwards to the inner transpositions of physical thought in our time, which are characterized by the abandonment of perceptible models of nature.

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AN EXAMPLE OF THE PHYSICAL PICTURE OF REALITY

What does physics know about reality? As an answer we do not want to build up before our eyes the abstract system of physics, but rather look at a concrete object and ask ourselves, what physics teaches about it and what it ignores.

It should be not something made by man but an object such as we actually find in nature. I look about in my home for such objects. All utensils fail to fulfil my condition. But I find a fragment of an Icelandic calcite crystal that I have owned since my school days. I take it in my hand and look at it. What properties does this crystal really have, and what can physics say about these properties? I shall forget the painstaking single steps of measurement and calculation and confine myself to the result which is available in the contemporary theory of the structure of matter.

* The term 'perceptibility' has been chosen to render the German *Anschaulichkeit*. *Anschaulichkeit* is used in the original to describe a quality common to the theories of classical physics: that they describe nature by models formed analogously to things that can be perceived by the senses.

The crystal is heavy, solid, of symmetrical shape, transparent. But I shall begin with an even more elementary property which philosophers regard as characteristic of all matter: the capacity to fill space. The crystal is in fact extended. It is also impenetrable: where the crystal is located, I cannot introduce another body, without pushing it away or demolishing it.

True, I can demolish it. Its parts are then once more calcite crystals: but, as physics teaches, I cannot break it up ad infinitum. The crystal is composed of atoms. True, the entity which chemists have called 'atom' can be further analysed, physically, into electrons and the atomic nucleus, and the latter again into protons and neutrons. But in the first place we suspect, not without reason, that these last units are now really physically indivisible, and secondly, the atom contained in the calcite crystal is itself in any case neither calcite nor crystal, but just a calcium atom or a carbon or oxygen atom. Physics began with the determination of the laws which satisfy the events immediately perceptible by the senses, but now its goal is precisely to investigate the atomic world, which is no longer perceptible by the senses, and to reduce all other types of order to the laws of atomic processes. So the representation of the atom becomes the foundation of the unity of the physical view of the world. We shall recognize in the case of every further property how near physics has already come to this goal. >

Thus, we understand the volume of the crystal as the sum of the volumes of its atoms. That the atoms themselves are practically impenetrable, we can now deduce from wave mechanics; but the question of what that means would lead us here too far afield. We now ask about the other properties.

The crystal has a determinate weight; it is the sum of the weights of its atoms—in fact, essentially, of its atomic nuclei. <

It is a solid body. Its atoms are held together by certain forces. These forces we now know and can calculate quantitatively—not indeed for so complex a thing as this crystal,

but for the basically analogous case of simple molecules. They are electrical in nature: the electric charge is, along with weight, one of the basic properties of the atomic units.

The crystal is chemically analysable according to the formula CaCO_3 . This formula states the ratio in which the three constituent types of atoms occur in the compound. Here we are at the starting point of the whole conception of the atom: the chemical law of constant proportions.)

The crystal exhibits the regular crystalline form. Its atoms are regularly arranged in space, and the form of symmetry of the whole structure is determined by that arrangement which produces the strongest binding of the atoms possible with the available forces.

It is, indeed, transparent, but double refracting: a single line, observed through it, appears double. It was on this property that Huygens in his day founded the decisive demonstration for the wave nature of light. The vibration which is experienced by our eye as light—an electromagnetic vibration, as we now know—also sets the individual atoms of the crystal vibrating. The 'secondary waves' proceeding from the atoms extinguish one another in all directions, except the one direction in which the light waves are propagated. So it is with the normal diffusion of light; on the other hand, the regular atomic arrangement in the crystal, which singles out certain spatial directions, allows a second direction of propagation and thus leads to double refraction. < Experiment shows that the crystal does not conduct electricity. According to atomic physics, this fact is connected with the transparency of the crystal. In good (i.e. metallic) conductors the electrons can move freely and are therefore set by the impact of the electric wave into very strong vibration. Thus the light energy is in a very short time transformed into the kinetic energy of the electrons and the metal does not allow the passage of light. To be sure, the reason why the electrons do not move freely in our crystal is only explained by the more delicate atomic model of quantum theory

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To the optical observations we add the perceptions of the remaining sense-organs, the physical causes of which can all be reduced to atomic processes: sound to vibration of air molecules, smell and taste to chemical changes, and therefore to transposition of atoms, touch to mechanical pressure of impenetrable matter, perceptions of hot and cold to the unordered motion of atoms which we call heat.)

We do not want to lose ourselves in details. After the examples we have given, does not a system like the atomic theory already appear as the adequate expression of the unity of nature as a whole? Nevertheless even with respect to the calcite crystal, physics has already seriously restricted itself to a segment of reality.

WHAT IS LACKING IN THE PHYSICAL PICTURE?

Again, we cannot—nor do we want to—give a complete account of the other side, that is, of those properties of the object which physics has omitted. For the moment, we want to point out through some examples the dangers to which the physical world view easily falls a prey. In the next section we shall then look somewhat more closely at the biological questions as a particular example, in order finally to draw some general conclusions.

When I took the crystal out of its drawer, I thought at first not of any of the properties we have enumerated, but of something quite different, also belonging to it: its personal relation to me. A teacher whom I liked brought it back to me from a trip to Iceland. Through this recollection the crystal appeals to my emotions. Were I not a physicist, perhaps I should forget to ask about anything at all except this memory.

Such emotional values have no place in the physical picture of the world. The pure physicist will say: obviously not, for they are not objective properties of things, but subjective processes 'in me', which no other experimenter can 'reproduce'. That is correct; but who says that these sub-

jective realities are less important for a person who wants to make himself a true picture of the world? What justifies me in considering that what is accessible to all is more real than what is accessible to me alone? Physics itself cannot decide this question, for it is precisely the point of departure of physical method that it inquires only about phenomena accessible to all, and therefore obtains an answer only about such phenomena. What a restriction we have already imposed on ourselves in this point of departure!

There is a rather comfortable way of disposing of the question we have raised here, by saying that there are in fact two equally justifiable 'aspects' under which nature can be regarded: the objective and the subjective. But the problem lies precisely in the fact that the two aspects stand in an inner relationship to one another. How do I really know that my psychological relation to this crystal does not belong to it as an objective property—that is, one perceptible to other people? Is it perhaps only that our sense for the objective but unconscious relations between the physical and the mental is too little developed? Are there perhaps in these relationships objectively existent basic structures to which our conscious, subjective feelings are related as the individual properties of this one fragment of crystal are to its general property of being calcite crystal? Let us demonstrate by an example that this is not just a question of vague fancies.

Nearly all times and peoples, except rationalistic modern Europe, believe in the magic effect of jewels. In this there is an objective relation of the crystal to the soul. One may consider it superstition. Certainly, however, the physical world view is itself incapable of judging the question. For if such effects do not occur in the physical world view, this is only because it has consciously ignored them in advance. But we cannot demonstrate a priori the completeness of the presuppositions on which contemporary physics is constructed. Obviously it is rather a case of an empirical question; and what respectable physicist would deny the reality

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of these effects if they were exhibited to him in practice? Hence we must say that the physical world view, when it is applied without exaggeration, leaves room, in principle, for such effects.

Apart from these examples, which are admittedly problematic for our scientific consciousness, the special structure of physical method appears still more clearly in cases which remain within the framework of the generally accepted physical world view of to-day—for example in the investigation of historical questions.

My crystal comes from Iceland. For the mineralogist its origin is clearly marked in certain properties to which I have not so far paid attention, for instance, its size and its degree of purity. Where do these special properties come from? They are the traces of the geological fate of this piece of matter. Up to now I had not noticed them especially; now they happen to be the object of my interest for they betray objective events of the past, for instance changes of temperature, of pressure and of chemical conditions in the place and at the time of the formation of this crystal. To be sure, the limits of the physical world view are not broken by this consideration, for the history of the earth took place according to the laws of nature; it is, in fact, one of the aims of scientific research to recognize the interplay between the uniqueness of the cosmic process and the universality of natural laws. But it is one thing to ask if the world view leaves space for a certain sector of reality and another to inquire whether it consciously fills that space.

This reminds us, finally, of the general truth, that the physical world view partakes of the incompleteness of experience. Thousands may have held a calcite crystal in their hands, without seeing that it is double refracting. Thousands knew its double refraction and saw in it nothing but one curious fact among others until Huygens recognized it as demonstrating the wave nature of light. How many properties this little crystal may yet have—properties that my senses perhaps already perceive or easily could perceive,

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and whose entrance into my active consciousness might enrich my picture of the world by a new scene!

It is clear, therefore, that the physical view of the world does not embrace all the essential properties of objects. It is not yet equally clear to what extent it necessarily excludes them. These propositions also characterize the present relation of physics to biology.

PHYSICS AND BIOLOGY

It is a justifiable feeling[✓] of many biologists, that the physical way of thinking has, in their science, grasped only a part of the reality, perhaps even the less essential part. Has it not always grasped only those properties of living things which they share with dead things? Nevertheless, the difference is very hard to define. We shall examine critically some suggestions along this line which also shed light on the physical world view itself. Let us take the concept of wholeness as our starting point. For on the one hand it is certain that one cannot understand a living thing if one is unable to understand it as a whole. And yet on the other hand it is not easy to mark off the concept of wholeness we are using here from a concept current in physics also.

Wholeness can be understood as the whole of all properties of an object, or as a special property, namely that of being a whole. In the first sense wholeness is attainable by neither biology nor physics, but both try to approach it. Thus the difference between the two sciences can at any rate not be found in their attitude towards the general truth that the whole is more than the sum of its parts; rather its serious adoption is what separates the good scientist from the bad, both in physics and in biology. True, physics stresses the fact that the crystal consists of atoms. But it is not the 'sum' of its atoms, as little as a wall is the sum of its bricks. In the first place the arrangement of the atoms in space must be determined; only their symmetry makes the crystal a crystal. Moreover, the nature of their interaction must be described.

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In fact, the atoms, which are no longer perceptible by the senses, are even as physical objects characterized only by the nature of this interaction, through which a perceptible entity is produced—i.e., through the specification of the whole whose parts they can be.

Is there then a way of being a whole which is characteristic of living things, but not of non-living objects? In order to deepen the question, let us be guided by the following stately presentation of Carus: 'Suppose we consider the history of the development of any plant or animal or any organic individual whatsoever: one of the first important observations we must make of it is: that a picture of its nature must be conceded in advance of its actual existence. Suppose for example we take the egg of a butterfly, and we perceive a small rounded shell filled with a uniform albuminous fluid. But of the body of the butterfly, of its four wings and its glorious colour, of its thousands of eyes, its antennæ, its feet and its delicately rolled-up tongue, meant for sucking the nectar of flowers, or of any peculiarity of form of the caterpillar, not the slightest indication is present. Nevertheless the picture of this whole many-sided articulation hovers over the formless appearance of the fluid egg, and as it is sketched out in this invisible spiritual picture of the future actuality, the organization shoots forward step by step in crystalline fashion, as it were, and demonstrates irrefutably that a picture of its nature preceded its existence. The same thing holds if we go a step lower to the genuine so-called crystal formations. If we take the purest, most indifferent fluid, water: over it there hovers, or had we better say in it there lives, the picture of actual crystallization according to the law of triangular and hexagonal symmetry, and as the floating drop of water is subjected to the effect of cold, so that the crystallization based on decisive polar attraction can set in, there appears the delicate form of the water crystal as snowflake—i.e., as three or six pointed star. The picture, the type, or idea of this form was therefore present, before the form appeared.'

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These words must strangely move the contemporary physicist. For on the one hand, he must grant their truth, and on the other he does not count the crystal among living things, but has become accustomed to regarding its structure as a physically 'explained' matter. Has this idea of the crystal thus become superfluous in our cosmology, or, on the contrary, does the world of physics itself include such ideas!

The latter conjecture should be correct. Physics constructs the crystal in thought not out of atoms as such, but out of atoms in space. But the idea of the crystal is nothing but its spatial symmetry, and physics shows that whenever certain atoms are regularly arranged in space, this particular symmetry must appear. To be sure, our undeveloped perception of space does not immediately recognize this: but the mathematical discipline of group theory demonstrates in detail that all the actual forms of crystalline symmetry, and only these, follow from the mathematical characteristics of Euclidean space. The ideal element in nature consists in the fact that mathematical laws, which are laws of our thought, really hold in nature. And that deep amazement which we often feel over the inner order of nature is connected above all with the circumstance that, as in the case of crystals, we have already recognized the effects of this 'mathematics of nature' long before our own mathematical knowledge was sufficiently developed to understand its necessity. Is the crystal then really 'explained'? It is, if we take the structure of space as given; it is not, if we see that too as a problem. The former point of view is adequate for the practical physicist. But as long as the latter is possible, it cannot be said that physics explains away the secrets of nature, but rather that it traces them back to deeper secrets.

We have thus received some instruction about physics but have not found any boundary between biology and physics. But perhaps just this is an important addition to our knowledge. When biology takes issue with the physical mode of thought, it is to a great extent a question, not of the opposi-

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tion between biology and physics, but of an opposition which is just as possible within the framework of pure physics: for to those investigators who keep the whole continuously in view, there are always opposed others who confuse the part already explained with the whole. Since the biologist uses physics merely as an auxiliary science, and therefore is more familiar with its results than with its problems, he is especially liable to be seduced into this abuse by the physical mode of thought.

Nevertheless there is a difference between the physical and the biological modes of thought. We understand the mathematics of the crystal, but not—if there is such a thing—that of the butterfly. On the other hand, there is a rich store of biological concepts which to-day at least cannot be reduced to physical concepts, but which describe with precision our daily observations of living things. The attempt has been made to draw a sharp boundary between physics and biology by asserting of a selected number of these concepts that they are, not only to-day but for all time, inaccessible to physical explanation. For example, one could find a specifically biological form of wholeness in the concept of the individual. The crystal can grow without limit, and a part of it is again a crystal; the butterfly lacks both these properties. One might mention, further, processes like growth, metabolism and reproduction, in which new matter continuously becomes the bearer of the same form, the same functional relationship.

However, we can demonstrate every single one of these phenomena with physical models. For example a simple candle flame has the above mentioned characteristics of individuality, as well as metabolism and the possibility of 'reproduction'. The tobacco mosaic virus, which has the properties of living disease-producing agents, and at the same time is a crystallizable chemical substance, may be considered no mere analogy of life, but a genuine borderline case of it. To draw sharp boundaries here would mean, at the very least, to anticipate the unknown results of future

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research. Indeed one should not make any sharp distinction between living and non-living objects, but between the sources of knowledge in pure physics and the sources of knowledge at our disposal in our contact with things that are unquestionably living.

Doubting this distinction, one might ask: besides sense experience and logical thought, which are both applied by physics, what further sources of knowledge can biology have, if it is not to become mysticism? Yet it is a question here of something quite familiar, with the sole difference that it does not fit into the schematic theory of knowledge of the 'physical picture of the world'. All the items of my experience are given to my consciousness, not in mosaic-like separation, but in a context which results from the very fact that I am a living creature which must behave simultaneously as acting and perceiving. In this context something of the meaning of the behaviour of other living things, with whom I am involved in a network of living interactions, is already given me. A stone, to be sure, I know only as object, but a man is necessarily my 'fellow man'. His expressions are carriers of a meaning which I can understand because I am myself a man, and it is both practical necessity and ethical obligation, which stirs spontaneously in me, that I should not take them solely as objects, but do my best to understand them. Now the sphere of biology is shot through with both these modes of interpretation and is unable wholly to dispense with either. An *amœba* is for me scarcely more than an odd physical object. A saddle horse is for me first of all a partner in the game of life, with whom my inner contact is stronger than with many people. Between both extremes there is any number of intermediate stages.

Of course, this 'experience of living contact' is for scientific biology just as much mere raw material as the experience of the realm of objects is for physics. Therefore, it is naturally exposed to the dangers which beset all theory building: we misinterpret and overstep the bounds of our own competence—as for example when, because of the

undeniable analogy between many natural forms and the products of human purposiveness, objective purposes were ascribed to nature in too anthropomorphic a fashion. But here as always, the danger of misuse should be no excuse for giving up altogether a source of experience. On the contrary, it is this source of experience that gives their justification and their characteristic colour to all those biological concepts which were not obtained by a method analogous to physics.

This should also hold, in particular, for the biological concept of wholeness. That concept of the peculiar form of the living creature, a form that continually renews its existence—the concept of the ‘image of being before existence’—would provide the basis for making no more than a distinction of degree between living and physical forms, and would stress the boundary between highly differentiated inanimate matter and living things scarcely more sharply than, say, the distinction between animal and plant. The use Carus makes of it by the concept has already shown us that. Carus himself, however, considers the living form not as a final self-explanatory concept, but as a phenomenal manifestation of the ‘soul’ of the organic life. Thus he repeats in the language of his romantic view of nature that fact which must at least be echoed in every other language adapted to life: the connection of the organic with the human world. My self is a whole for myself in still another, a fundamentally different way from all the objects of external experience, and it would be hard to indicate what enables us fundamentally to distinguish the biological individual from a mere physical thing, unless it be its kinship, however distant and partial, with the human subject, and the consequent possibility of our ‘understanding’ something of its behaviour, whether in direct contact or by analogical inference. As boundary between biology and physics in the contemporary sense, therefore, we should simply designate the place at which the specifically biological concepts proceeding from this understanding are no longer introduced for the interpretation of

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our experience. Whether it is here a question of a boundary in the objects or in our present insight may be left open. On the other hand, to the extent to which biology treats living things simply as objects, as physics does, it is destined to become more and more completely united with physics.

THE METHOD OF PHYSICS

We have seen the physical way of thinking at work in many particular cases and though we have perhaps not clearly designated its limits, we have learned to feel them. Now we shall try to state more exactly the cause of this limitation.

The cause has been thought to lie in 'quantitative' procedure, in the preference of physics for counting and measuring. This, however, is itself only a symptom and not the cause. The goal of physics is in every case to determine exactly what exists. Where several comparable things are present, we can count or measure. Wherever we can do that, we must therefore, as scientists, proceed quantitatively. Accordingly, to make a non-quantitative observation in such a case necessarily remains no more than a first step towards a physical judgment.

We come nearer to the essential point if we look at the attitude of physical thought, and of scientific thought generally, with respect to method. Naïve human thought proceeds from the fact, scientific thought from method. Method (met-odos) is a way. This method has no other goal but just the fact, the factual truth. But I am by no means at the goal from the beginning; rather I must tread the path if I am to come to the goal. We have indeed, in daily life, a rough acquaintance with the partners and objects of our action, and this acquaintance suffices if we only want to move about with some security in our environment. But science begins with the discovery that we know much less than we should like to know or could know—partly because we have forgotten to ask about it, partly because we have contented

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ourselves with false answers to our questions. This inconvenience scientific method is supposed to remedy. In what does this method consist?

It consists, for one thing, in the fact that we proceed from the immediately given. We do not believe what people in general say, but only what we ourselves have seen. We do not form our judgments, on things that are not immediately at hand, through the interplay of instinct and custom as does the man of action but we try to infer them from their effects through carefully multiplied observations. Finally, we no longer accept as general truths the great parables such as the myths and dogmas of religion which summarize in binding form, for a people or an age, the foundations of its attitude to life, and the highest goal of its hope and longing. We accept only those generalizations of manifold items of experience which can in every particular case be objectively confirmed by test, measurement and calculation: the laws of nature.

But while one is methodically taking control of the single 'given', one is always in danger of losing the unmethodical knowledge of the living man; just as the eye that was used to the dark and recognized at least the outlines of its environment, at the lighting of a searchlight sees the objects on which the light is thrown, to be sure, in all sharpness, but only at the price that everything surrounding them sinks into black darkness. If we want to speak of this occurrence not only analogically but with epistemological accuracy, we must recall that there is no absolute 'given'. What experiences are given me depends rather on my situation in life, above all on my attitude and the degree of openness that follows from it. Thus in particular it is possible and, if I want to live at all, indispensable, that at every moment I grasp instinctively something of the whole of the context in which as agent or sufferer I happen to stand. He who can hit the mark when he throws a stone, knows in a half-conscious way as much as he needs to about the trajectory of his stone, although he will usually be unable to integrate the mechani-

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cal equations of projectile motion. The doctor or teacher who understands his art, the good military leader or diplomatic negotiator—each of them knows from direct contact with the other person, from an intercepted glance or a movement of the hand that can be correctly interpreted only by instinct, more than the whole of scientific psychology will teach him. And the orientation of a man to the whole of his life, which is expressed in his cosmology, proceeds from similar powers. It is here that we find the problem of science in general and of physics in particular. It is the practical value of science, that it teaches us to control more and more particular things; the result is technology. It is the danger of science, that it holds us back too readily from the human exertion needed to maintain our living contact with the whole; the result is the disintegration of our world view.

Is there then no knowledge which escapes this dilemma, which combines clarity of awareness with instinctive certainty? We shall not discuss in abstract philosophic fashion whether such knowledge is possible. It seems better to look at a living man who embodies for us an attitude to nature free from the confinement of the physical world view. There have been many people of this sort. Let us choose as an example Goethe, 'the first great European'. What does his example teach us?

Let us begin with the negative side. Goethe advanced a doctrine of colour whose purely physical principles are false. Occasional attempts have been made to revive it. This happened not only out of respect for the poet, but because the view of nature that Goethe teaches us to take impresses receptive minds over and over again. Though no one dared to call Newton wrong as against Goethe, at least it was assumed that there were two equally justifiable theories, one according to Newton and one according to Goethe. This did no service to exact knowledge. Where Goethe contradicts Newton, Newton is right and Goethe wrong. Here even the greatest personality was impotent against the sure course of the physical method. But this necessary criticism is intended

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only to help us recognize clearly where we can learn from Goethe.

The physical view of the world is wrong not in what it asserts, but in what it omits. We recall how Goethe reproaches Newton mainly because he, in order to investigate the nature of light, first shut himself into a dark room, the window of which had only a small open slit, and then investigated the light that had been thus mistreated. And truly, how little is left in this experiment of the living light around us. And this living light is not an unknown for us, since we carry in ourselves the organ that is akin to living light.

*An eye that is not like the sun
Could not behold the sun above.
Lay not in us the God's own power
How could the Godlike win our love?*

This is indeed no blueprint for erecting a teachable system of the natural sciences. The receptivity for those things that are here in question does not belong to every one in the same degree. Every one of us can interpret the simpler actions of a highly developed animal, but few can follow the meaning of the series of all organisms so far as to envisage the 'primal plant' or metamorphosis. This is not a matter of the perfection of the intellect, but of the living force in us.

*For the last time I say again:
Nature has neither husk nor grain.
But you must ask from dawn to dusk
This question: are you grain or husk?*

Surely no special explanation is needed to show that what is being preached here is not subjectivism. The world within us which teaches us to understand the world about us is the same for all who find it; though, to be sure, not everyone finds the way to it. And in the last analysis, it is from the objectivity of that world that the objectivity of physics gets the justification for its existence, more than

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from the external results which are only its perilous reward. True, it is vulgar to hold convulsively to the mathematically verifiable, just to be sure one is proceeding correctly; but then that is not objectivity but weakness. We may recall here a basic law of inner development: that perfection is achieved only insofar as I learn to disregard my own person. This is not only true in the ethical sphere. A Kepler or a Newton, too, strives for the objectivity of mathematics and of experimentation as comprehensible embodiment of that supra-personal truth towards which the only proper attitude is reverence. Precisely pure mathematics, which is well known to be of spiritual significance all the way to the laws of music and religious symbolism, became for Kepler the link between the divine within us and around us. We know his words: 'My book may wait a hundred years for its reader. God too waited six thousand years for the man who would look upon His work.'

PERCEPTIBILITY

THE CONCEPT OF PERCEPTIBILITY*

Our business is not completed with the statement of a general doctrine about the value and limits of the physical view of the world. We have just seen, that, although basically the physical view of the world does not need to deny any realities that have not yet found a place in it, it is nevertheless daily under the temptation to do so. We want to learn really to avoid this error in every particular instance, and to correct it in so far as we have already committed it. We want to do this, not on the side, in our private lives as it were, but precisely in our profession as physicists.

Whoever tries this in earnest soon recognizes his profound impotence. So many good intentions are ruthlessly trampled underfoot by an historically necessary development. He who sees his own path marked out in his own mind

* See note p. 13.

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with the same necessity will not let himself be misguided by this circumstance. But it is for him a duty of prudence to examine where the development itself is co-operating with him. If the method of physics ever leads at all to the knowledge of truth, there must be such points in its development. The very objectivity of the physicist, which occasionally causes him to limit his perspective, can in turn lead him to recognize the limitations.

Where can we expect such a point? Primarily at the place where physics itself leaves the realm of those objects for which its mode of thought was originally made. To-day atomic physics is such a place. Thus we shall now investigate its relation to the older physical world view. As a starting-point we choose the much discussed problem of its non-perceptibility.

When we speak of an attempt to attain perceptibility, we understand two tendencies that are in many respects almost opposite. When we describe something we have perceived, that thing is certainly accessible to perception. Perceptible in the highest sense is Goethe's natural science. As against this, the physical view of the world has always had a tendency towards the non-perceptible. This stems immediately from the endeavour of physics to achieve a unified world view. We do not accept appearances in their many-coloured fullness, but we want to explain them, that is, we want to reduce one fact to another. In this process what is perceptible is often explained by what is not perceptible, and to this extent the atomic theory, the basic concept of which already designates something which is not perceptible to the senses, is from the very start inaccessible to perception.

With the success of the mechanical view of the world, however, a new concept of perceptibility becomes effective: the attempt is made to represent the non-perceptible in the image of the perceptible. True, we cannot see the atoms—but what would they look like if we could see them? Whether it is permissible at all to make this inquiry—that is the question raised by modern atomic physics.

categories of substance and causality, constitute the framework of the experiment.

But we do not immediately perceive the atom itself; it is given us, not as object in space and time, but as the result of a series of inferences from our measurements. Nor can it be described by a model after the pattern of a spatio-temporal object. We characterize it rather through a certain mathematical magnitude, Schrödinger's ψ -function. It takes the place of the mechanical characteristics by which we are accustomed to describe an object, and from which we gain predictions about the outcome of future experiments. But in what way can it do this? An experiment always results in a spatio-temporally defined magnitude. But the ψ -function is not such a magnitude; it is not measured directly. Instead, through a mathematical operation, the type of which is patterned on the type of the experiment in hand, we gain from it a statement of probability about the outcome of the experiment, that is, a statement of the mean value which the magnitude in question will assume in the course of a great many measurements.

Let us pretend that we could predict with certainty the outcome of every possible experiment on an atom, for instance, about its position, its speed, its inner structure. Then we could simply attribute to the atom as properties the measured values of these spatio-temporally defined quantities. Thus we should have achieved the spatio-temporal model of the atom we were looking for. The fact, however, that in most cases only predictions of probability are possible, signifies just this: that we may not naïvely attribute as properties to the atom the results of our quantitative measurements. The same limitation is indicated by the fact that these measured quantities, considered as properties of one and the same objects, are in part logically incompatible. For example, the same atom behaves in some experiments like a spatially concentrated particle, in others like a wave filling the whole of space. Now it is clear that the atom cannot at one and the same time be both particle and wave. The logical

paradox is avoided by the fact that one can never simultaneously make experiments in which the atom reacts in these two different ways. If, say, I have found the atom as particle at a particular place, I can, in an immediate repetition of the experiment, find it at the same place, and can thus say with a certain justice: the atom is there and there. In this case the ψ -function of the atom is of such nature that only predictions of probability are possible about its other magnitudes, say its speed, or about the properties which would characterize it as wave (wave length, phase). On the other hand, if I know the wave properties, I can predict only with probability the particle properties. Hence I may not say: 'The atom is a particle' or 'It is a wave', but 'It is either particle or wave, and I decide by the disposition of my experiments, in which of the two ways it manifests itself.'

Should we say, then, that reality depends on our arbitrary choice? No, not reality, but the picture in terms of which we understand reality. We cannot have any experience of the atom except through experiment; and the experiment is a violation of nature. We force the atom, as it were, to tell us its properties in an inadequate language. Let us try to determine somewhat more precisely the kind of deviation from the older picture.

Actually our spatio-temporal perception does not fail; for we experience of the atom nothing else but spatio-temporal measurements. Nor, as has been supposed because of the occurrence of statements of probability, does the law of causality fail; for we create closed chains of causation through each experiment. But what fails is the integration of these isolated bits of perception and isolated causal chains into an objective model of the event; what fails is the 'objectifiability of nature'. Perhaps we can best speak of a collapse of the category of substance; perhaps we should rather speak of the necessity of adapting our logic, formed by thinking in objects, to the new situation.

Nor is this situation entirely new to human thought. That the event must be observed in order to be known, and yet

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that this very act of observation alters the event essentially, we know from the difficulties of self-observation, in which the subject of knowledge becomes its own object (Bohr). Perhaps something of the necessary logical armour for the working out of these questions is to be found in the philosophy of German idealism, which recognized the limitation of the concept 'object' through the reflection of thought on itself. However, this 'perhaps' applies only to the philosophical elaboration of the questions; what is certain is that in physical terms, the contemporary situation of atomic physics allows no other interpretation than the one given here.

CONSEQUENCES

Many physicists have instinctively put themselves on the defensive against these results, as they did previously against the related but less radical results of the theory of Relativity. Let it be said first of all that this reaction is quite understandable. Whoever takes it too lightly shows only that he himself presumably has not understood how profound a break with the previous programme of physics it is that is demanded of him. The scientific reasons why most physicists now consider the break as final cannot be expounded here. But we can examine the question of what we are to conclude when we have recognized the break as unavoidable.

Ought we to be so unwilling to learn that it was false to subject the ultimate particles of matter to the mechanics of tables, chairs, and billiard balls? Did not this programme contain the same naïveté towards reality that is contained in all hasty generalizations of a physical cosmology? Moreover, what leaves us dissatisfied is at bottom not the fact that the old perspectives have failed, but that nothing new which is immediately intelligible takes their place. When we give up the attempt to describe a living thing as a machine, there still remains our immediate living partnership to compensate us; but nothing like that holds in the case of the atoms. Thus we feel that the task of achieving a genuine understanding of

the atoms is still not accomplished. It may be accomplished; if so, the mistake resides not in the propositions of physics but in our understanding of those propositions. The propositions are formulated in abstract mathematical language. Let us compare them with a musical score. For those who cannot read notes, the musical score is dead; but the man who understands them hears the melody in them, and so recognizes for the first time the inner necessity which held these signs together. It is this inner necessity of the new physics that we have to look for, not by preserving outworn images, but by understanding its function in the great process of thought.

What is this function of the new physics in the development of knowledge? Our considerations, which began with questions, issue here in another question. Neither a single man nor even a single science can give a completely valid answer. To be sure, atomic physics will be further perfected, experimentally and theoretically, in the next decades. It has begun to find technical application, and thus it is earning a place in the practical life of human society similar to that already held by classical physics. But the fundamental question we are raising presupposes at least the co-operation of physics and philosophy. Thus our question belongs in the context of the many questions that arise to-day at the borders between the sciences, and at the place where all the sciences border on life—questions all of which have as their theme the organic unity of knowledge which is itself a part of the greater unity of life. These questions can be attacked only through the active co-operation of a number of people, for the assembling even of the most important knowledge of our time in *one* head has become impossible. We do not want here to anticipate with conjectures the results of this endeavour. It is enough if we have made clear something of the attitude with which some representatives of contemporary physics are ready to undertake such co-operation.

Perhaps, in summing up, we can say this much:

A 'world view' has been destroyed, more thoroughly

than could have been expected. That is no misfortune, but a wholesome lesson. A world view is more than a scientific theory; it is meant at least symbolically to comprehend the whole of reality. Looked at from the point of view of the particular demonstrable items of knowledge, this regard for the whole is always a faith; it is the faith which is the condition of our life. The old physical world view also attempted to represent the whole, but with inadequate means; and it was bound to founder. But physics was the first science of nature to possess a complete system, and so, independently of the 'world view', it played the role, as it were, of a model among the sciences. Perhaps this role now belongs to physics in a new sense. The new physics is the first complete system of natural knowledge beyond the boundaries of the mechanical world view that is expressible with mathematical exactitude. Let us do what we can to make it worthy of this role.

Chapter Two

THE ATOMIC THEORY OF MODERN PHYSICS

A CRITICAL INVESTIGATION OF ITS FOUNDATIONS

THE present investigation is intended to be critical in the sense given to this word by Kant. Thus, my intention is not to criticize particular assertions of modern physics, but to examine what general propositions are basic to it, how these propositions affect the structure of the science and, consequently, what value can be ascribed to them in the framework of the human striving for knowledge as such. To-day at least, it is impossible to carry out a complete investigation of this question. We shall therefore limit ourselves here to one of the most important concepts of modern physics, that of the atom, and to those of its applications which are of interest beyond the limits of pure physics.

THE EXISTENCE OF THE ATOMS

Before me lies a piece of coal. I smash it with a hammer. I make the pieces smaller, I grind the resulting grit to the finest coal dust. A tiny grain of the dust, looked at under the microscope, is still a piece of coal which we ought to be able to divide further and further with finer instruments. Does this go on to infinity, or does the possibility of division definitively stop at some point? Are there parts of matter which have themselves no further parts?

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This question is age old. Leucippus and Democritus are considered the originators of the doctrine of the final, partless parts of matter, the atoms. They attributed to this doctrine a decisive significance for the knowledge of the world and thus made the question of the infinite divisibility of matter a basic question of philosophy. It is very difficult to deduce the exact meaning of that first atomic theory from the extant fragments of the literature. But we shall here suggest at least one train of thought that seems to have been fundamental to the doctrine, because it has remained effective in all later philosophical atomic theories.

The philosophers of the Eleatic school had denied the reality of becoming and of change. For—so they taught—being would be not-being if it contained change. Becoming is the transition from non-being to being, destruction the transition from being to non-being. But non-being is not: there is no non-being and therefore no change. Therefore that which truly is, is to be thought of as an indivisible and unchangeable reality of the most perfect form—the form of a sphere; but our world of multifarious changeable forms is only an illusion. If we strip this train of thought of its antique manner of expression, which we feel to be logically naïve, there remains in it nevertheless at least one motif that has kept its cogency for our thought until to-day: that we should seek and demand the permanent behind the flux of appearances. We believe we understand that a substance remains what it is; a veil of mystery hovers constantly over change.

Now it is this Eleatic problem which the atomic theory tries to solve through an ingenious reinterpretation: that unchangeable being of the Eleatics exists true enough, not however just once, but in ten thousand times ten thousand copies. The atoms, indivisible, smallest bricks of the world structure, are the Eleatic being. They are themselves immutable, but through the constant change of their relative position they produce all phenomenal change. In the question whether the change which the phenomenal world daily

presents to us is real or an illusion, both parties are given their due. For on the one hand change is not a sensory illusion, but exists in the real motion of the atoms. But on the other hand it is not a change of substances, for the atoms do not change, and their movements merely appear to us as generation, transformation, and annihilation of objects. In modern terms: it is not the substance water that is destroyed by boiling as the substance steam arises, but the substance H_2O changes the ordering and state of movement of its atoms. >

The popularity of this doctrine was bound to persist since it promises to explain the fullness of appearances out of seductively simple presuppositions. Thus, for example, it had a large share in the origin of the distinction between the primary and secondary qualities of things. Warmth, colour, taste, and smell are not 'real properties' of things, but only the way the position and motion of certain atoms enter human consciousness; only the extension and motion of the atoms are real. How unified is this cosmic picture—and how poor.

Here we have the reverse side of the engaging simplicity. In all ages there has been raised against the atomic theory the objection which, for us of to-day, has crystallized in the concept of materialism. Is extension really the final essence of matter? Is matter an independent reality, and how is it related to the other realities we know: life, soul, mind? The answer of pure atomic theory to these questions has always been found inadequate. In addition there were conceptual difficulties in the concepts of the atom and empty space.

Instructive though it is, we shall not pursue here the two thousand year old quarrel of the philosophers over our question. As long as no experience comes to the aid of philosophic thought it may indeed illuminate the question from all sides and sharpen our awareness of its profundity, but not resolve it. The turning point in the history of the atomic conception came when natural science had to give it a concrete form in order to explain with its aid, particular

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data of experience. However naïve this form may be, philosophically, contact was made through it with reality as we observe and experience it. And careful collection and interpretation of continually new data led at last to the correction of the original hasty simplifications. Let us follow this road.

The atomic theory of modern science had its beginning in chemistry. An explanation was sought for the fact that, when two chemical elements combine, the proportion between their weights is constant. For this purpose it was assumed that each element consists entirely of uniform and therefore also uniformly heavy atoms, and the 'molecule' of the compound always contains the same number of atoms of each of its elements. Seldom has so simple a scientific concept exercised such all-embracing influence.

The concept of valence and structural formulæ and finally the spatial molecular models of stereochemistry introduced order into the immeasurable realm of chemical experience and explained innumerable particular phenomena. These results do not need to be described here in detail.

The first encroachment of these thoughts on physics took place in thermodynamics. After the law of the conservation of energy had taught us to consider heat a form of energy, it was natural to go further and visualize this form of energy through the assumption: heat is movement of the atoms. The kinetic derivation of the gas laws and the statistical explanation of the second law of thermodynamics were the most important consequences of this assumption. At the turn of this century chemistry and physics already possessed a rich store of knowledge about the atoms. But they knew all this only in the way in which a good criminal investigator can reconstruct his case from circumstantial evidence even before the criminal is caught. There was a general belief in the atoms and it was known that if they existed, they must have these and these properties. Two things however were not known: are the atoms in the last analysis only a useful working hypothesis or do they really exist? and: Are these so-called atoms of natural science, if they exist, also atoms

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in the sense of philosophy, that is, ultimate indivisible units?

Both questions were answered at nearly the same time, but with opposite answers. We know to-day: the atoms of chemistry exist, but they are divisible. This contrast between fulfilled and disappointed expectations goes still deeper. On the one hand the inner consistency of modern atomic physics and the growing richness of its applications have given us a more perfect picture of the unity of nature than could have been hoped for a few centuries ago. On the other hand the root of this unity, the mechanics of the atoms, is surprisingly and at first glance strangely different from the behaviour of the ultimate units of matter as it was conceived by the atomic hypothesis of philosophy as well as science. We shall first describe this new atomic physics, then make a critical examination of its foundations.

THE STRUCTURE OF THE ATOMS

The most convincing demonstration to-day for the existence of the atoms is that the track of a rapidly flying atom in the gas of a cloud chamber can be made visible and photographed. What we observe here is, however, the flight not of complete chemical atoms but of fragments of atoms. To-day, therefore, we can hardly speak any longer of proofs of the existence of the atoms, without at the same time investigating their inner structure.

Before we embark on this investigation, let us look back and ask: with what degree of justification do we still designate these divisible bits of matter we have now discovered as 'atoms'? Our justification comes from chemistry. The 'atoms' of modern physics have all the properties which chemists have ascribed to their atoms. All atoms of an element are alike (except for the distinction in each element of a small number of different types, necessitated by the discovery of isotopes), and a compound arises from the grouping of atoms together to form molecules. Therefore, the

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chemical properties of matter remain constant, in a mechanical division, down to the atoms, but not beyond that point. Every part of a piece of coal, down to the single carbon atom, is still a piece of coal; the parts of a carbon atom, however, are no longer coal but are certain 'elementary particles' out of which all other atoms too are constructed. Moreover the inner cohesion of an atom is firmer, and governed by laws other than that of the atoms with one another.

We shall now subdivide into three subordinate questions the question of the internal structure of the atom. We shall ask about the constituent parts of the atom, about the laws of their interaction, and about the application of this knowledge in theory and practice.

First the constituents of the atom. All atoms consist of very small, solid nuclei (diameter about 10^{-12} cm) and extended outer shells (diameter about 10^{-8} cm). The shells are not compact, but represent simply the space in which are located the very small electrons, the number of which is equal to the 'atomic number' of the element in the periodic table. The existence of the electron as the 'atom of electricity' was first postulated on the basis of the laws of electrolysis; free from atomic nuclei, we observe it as cathode ray. The minuteness of the solid, impenetrable nucleus follows from the high permeability of the atom to rapidly moving electrons or nuclear fragments. The nucleus has a positive electric charge, the (normal) electron a negative charge; their reciprocal attraction binds the outer shell to the nucleus. The nucleus is again a composite. Its parts are two particles of roughly equal weight: the positively charged proton and the uncharged neutron. Both are known as fragments, which can be ejected from a nucleus by bombarding it with other very rapidly moving nuclei; the proton is known also as the nucleus of the lightest element, hydrogen.

Besides the proton, the neutron and the electron, physics to-day recognizes a few further 'elementary particles', which, however, do not occur in atoms as permanent constituents: the 'positron', a positively charged electron;

different sorts of 'mesons', particles recently discovered in cosmic rays which are halfway between the proton and electron in mass; and the 'neutrino', only theoretically postulated thus far, an uncharged, very light particle. In a certain sense the quantum of light energy in the 'photon' must also be counted here.

Are these particles again further divisible? On this point experience has so far offered us no evidence. Experience does show us, however, another property of the elementary particles: they can change into one another. Thus under certain circumstances an electron and a positron are produced from a photon; a proton produces a neutron and a positron; a meson an electron; in the last three cases a neutrino is probably produced at the same time. It is the chief task of contemporary atomic physics to trace out the unitary reality underlying these different phenomenal forms of matter. The considerations in the third section of this essay will perhaps make intelligible the reasons which lead some physicists to the conjecture that the root of this unity is no longer to be found in a further spatial divisibility of the particles.

Now for the laws according to which the atomic units interact; first of all we must say, negatively, they are not the laws of classical mechanics. One cannot regard the elementary particles as points having mass, or small extended bodies which move according to kinetics and dynamics. This is most strikingly demonstrated by the comparison of the atom with a planetary system. The analogy is clear: as nucleus and electrons are small compared with the extension of the atom, so sun and planets are small by comparison with the extension of the planetary system; the space is not permanently filled, but only traversed again and again in constant motion. The Newtonian law of force which has the planets circle around the sun, even sounds mathematically like Coloumb's law of force which binds the electrons to the nucleus. Nevertheless the analogy cannot explain precisely the basic assertion of chemical atomic theory that all atoms

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of the same element are alike, alike in size, alike in their reaction to the surrounding atoms, and because of the existence of sharp spectral lines, responsive only to light vibrations of exactly the same frequency. To be sure, we know only *one* planetary system, now, and therefore we cannot say from experience whether they are all alike; but this is certain, that no law exists in classical mechanics which would prevent their being different. If, for example, another star should for a time come very near the sun, all the planets would be turned out of their orbits by its gravitational effect, and after its passage a wholly altered system of paths would remain. Now the atoms are continuously under such influences from their neighbours. Moreover, they carry in themselves a cause of constant change: the moving charges of the circling electrons according to classical electrodynamics should continuously radiate light. Through the accompanying loss of energy the electronic orbits would necessarily keep shrinking, and finally the electrons would fall into the nucleus.

Actual experience shows that all atoms have a fundamental state, in which they do not radiate at all and constantly maintain a fixed diameter. If a certain finite amount of energy is added to the atom, it passes over into a next higher state from which it can return to the fundamental state, radiating light in which the whole added quantum of energy is at once given off. Smaller amounts of energy the atom, in its fundamental state, simply cannot absorb. On the other hand the next higher state is followed by a series of still higher states. The concept of 'stationary state', to which one is thus led, can no longer be deduced from classical physics. It is a basic concept of the peculiar mechanics of the atom: of quantum theory.

Quantum mechanics is to-day as much a complete system of laws as is classical mechanics. Actually, it does not stand in contradiction to classical mechanics. Rather, it is more inclusive and includes classical mechanics as a limiting case, which is valid for the interaction of a very large number of

atoms. We can only hint here at the structure of quantum mechanics.

One of the most important observations of atomic physics is that all the physical configurations which were described above as 'particles' can under certain experimental conditions also appear as waves. Now between the assertion 'the electron is a particle' and the assertion 'the electron is a wave' there is a relation of contradiction. For when a particle is at a given place, it is by definition not at the same time at another place, but a wave event just as much by definition, happens simultaneously in an extended spatial area. This contradiction is dissolved by the following assertion: An atomic 'particle' is a physical reality which lies beyond the limits of immediate perception, and which we can no longer make accessible to perception by describing it in our spatial and temporal concepts. It is therefore, strictly speaking, neither a particle nor a wave. But in order to acquire any knowledge of it at all, we must bring it into contact with some sort of measuring apparatus; in the simplest case with a microscope or a diffraction grating. The reaction of the measuring apparatus to the atomic configuration we are then driven to interpret in our perceptual concepts: 'a particle was seen here and here' or 'a wave of this and this length was diffracted'. The concepts 'particle' and 'wave' or, more exactly, 'spatially discontinuous event' or 'spatially continuous event' appear therefore as interpretations demanded by the forms of our perception for processes that are no longer immediately perceptible.

In the form of the so-called indeterminacy relations we can indicate the limits up to which each of the two pictures remains physically significant. They can be deduced from the requirement that no contradictions may appear between the consequences drawn from the two pictures. For example, if the place at which a particle occurs has been very exactly determined, then, in terms of the wave picture, the statement must be made: there exists here a 'wave packet' compressed into a very small space. Such a wave packet however is not

stable but at once disintegrates. But in terms of the particle picture, this means that no determinate velocity and direction of further motion can be attributed to the particle. The application of the concept of the velocity of a particle is here to an extent indeterminate, and in fact the more so, the more exactly the position of the particle was known. To this it might be objected: one could take a particle of known velocity and measure its position during its flight: then one knows position and velocity exactly. To do this, however, the particle must be brought into interaction with some other entity, a photon for example, which then serves as measuring device. Through this interaction the observed particle is necessarily deflected out of its course by a certain angle, and this angle increases in proportion as the measurement of position becomes more exact. This would still not be so bad, if one knew the angle of the deflection exactly; in that case, one could then apply it in subsequent calculations as a correction to the originally assumed velocity. But in order to determine this angle one would have first of all to know exactly the position and velocity (or more accurately, momentum) of the photon. But for the photon, too, the relation of indeterminacy holds. Thus one enters into the process of measurement with an initial indeterminacy of certain of the magnitudes defined in the particle picture, an indeterminacy which cannot afterwards be eliminated. In order to transcend the indeterminacy relation by a measurement, we should thus need a measuring device not itself subject to the indeterminacy relation, and that would mean, at the least, a measuring device not composed of matter as we know it.

But the limit of perceptual representation is not yet the limit of scientific knowledge as such. We can make verifiable predictions about the outcome of all experiments which can be set up with atomic entities. For this purpose we assign to them symbolically a mathematical function (the ψ -function) from which by certain operations the probability for the outcome of experiments can be obtained. Thus

we get expressions in the form: 'with this and this probability an electron will be found at this place.' The fact that in this connection we can generally predict not with certainty, but only with probability, is only another expression of the fact that we cannot describe atomic entities completely in one of the two pictures. If, for example, I could predict any time with certainty at what place I should find a given electron, then nothing would prevent my saying: the electron is a particle which is really at this place. In fact, however, I can indeed determine by a measurement the exact place at some instant, but according to the indeterminacy relation, I must do without an exact simultaneous statement of velocity—and thus I do not know where the 'particle' will have moved to in the next instant; and thus, shortly after the measurement I can predict the position again only with probability.

This state of affairs has been called a failure of the law of causation. It would have been better to speak of a failure of the determinism of classical physics. This failure, in its turn, follows from the failure of the objective reification of the processes of nature. By this we mean the following: We can indeed answer experimentally every single question formulated in the concepts of the sensible world (measurement of a place, a velocity, a wave length), we can also point out particular chains of causally connected events, but we can no longer weave together these isolated segments into the web of perceptible and causal relations which alone constitutes the permanent objective nature of the classical scientific world view with its unchangeable 'things'. Every statement about the atom has a clear meaning only in relation to the measurement through which it was obtained. We may choose what segment of reality we want at a given time to comprehend perceptibly; but the whole escapes the grasp of the man who measures.

We shall here leave our basic exposition and turn for a moment to the application of the knowledge we have

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acquired. As a rule, only those who know the applications from the everyday practice of science have a sure feeling for the inevitability of the path we have entered.

The applications occur partly in physics, partly in the adjacent sciences, and have already made their way into practical life. In physics the classical field of application is spectroscopy. Spectral lines give immediate evidence of the inner movement of the atoms, and both of the most important steps in the development of atom mechanics found their first confirmation in the quantitative calculation of the spectra of the two simplest atoms, hydrogen and helium. Among other fields of application let it suffice to name only the explanation of metallic conductivity, of ferromagnetism, of specific heats, and the new creation of whole branches of science, as represented by the transmutation of elements of the 'modern alchemy', nuclear physics, and by the investigation of cosmic rays. In these new discoveries, by the way, it becomes especially clear how experimental and theoretical research work hand in hand. Thus it was the experimental discovery of the neutron that provided the basis for the contemporary theory of the structure of the atomic nucleus, and on the other hand both the other new elementary particles the positron and the meson, for example, were already postulated in theory before their discovery by experiment.

A multitude of possible applications in the adjacent sciences also results automatically from the fact that all objects in which natural science is interested are composed of atoms. Thus the concept of valence and the basic facts of chemical composition have received an explanation in terms of atomic physics, and in the investigation of reaction rates and similar questions of physical chemistry the concepts of atomic physics have proved of fundamental significance. The discovery that such important biological processes as mutation of genes can be produced by the absorption of a simple quantum of energy opens vistas of a field of application whose scope cannot yet be measured. Similarly signi-

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ficant are the applications in astronomy (astrophysics). The knowledge of the laws of atomic physics allows us, among other things, to draw from spectral investigation the important inference that the chemical elements in the stars as well as in the diffuse interstellar matter are not only the same as on the earth, but also occur by and large in the same proportions. And the elaboration of the theory of the atomic nucleus recently made it possible to infer that the transmutations of elements in the inmost depths of the stars are the source of their constantly radiating energy—and thus permitted an approach to the problems of the development of the stars and of the origin of chemical elements.

In recent years atomic physics has become known to the general public to an extent which its exponents would not have dreamt of a short while ago and which they would scarcely have wished for. The reason for this is the practical application of the research, which till then had proceeded at the desk and in the laboratory, in the technical use of atomic energy. I should prefer here not to go into the questions connected with this development. Instead I would cite still another example of the practical application of atomic physics: the use of radioactive materials in chemistry and physiology. Almost all the elements can now be produced in radioactive form, and this gives us an important tool for the investigation of metabolism and also for therapy.

CRITIQUE OF THE CONCEPT OF THE ATOM

There is no doubt that atomic physics gives to natural science a higher measure of inner unity than it has ever before possessed. But is not the unity bought too dear at the price of non-perceptibility? The programme of atomic theory was to reduce the unintelligible multiplicity of phenomena to an intelligible unity. Is the unity we have achieved still intelligible if the way to it leads through the dualism of two 'pictures' and ends in an abstract mathematical function? It is true, indeed, that truth does not

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accommodate itself to our wishes; we must rather accommodate our wishes to the known truth. But is atomic physics a definitive truth? Of its extraordinary success there is no doubt, but is not this success too only an intermediate stage which will at some time be transcended? And does not the hope remain that a later stage will again add to unity the desired perceptibility?

The rest of the present essay can be considered an attempt to contribute something towards answering this question. We shall compare the programme of atomic theory with the fulfilment which it has found to-day, first in respect to the concept of the atom itself, and then under the broader aspect of the perceptibility* of the world view. In so doing we shall find a connection between the objections that shrewd critics of the programme—to name above all Kant and Goethe—have raised long ago, and the difficulties of the present age. To be sure, we cannot build up from such considerations any absolutely certain prophecies about the further development of our knowledge. But they can teach us what we can reasonably hope for, and also on what side in the great unceasing battle of minds we are ranging ourselves when we decide for this or that hope.

Philosophers have long criticized the concept of the spatially extended atom. For example, Leibniz did so in his correspondence with Huygens, and Kant in the antithesis of the second antinomy in the *Critique of Pure Reason*. In modern language, Kant's argument can be summarized as follows: Since there is no smallest part of space, parts of the space filled by an atom can be conceived. Therefore it is also possible to think of parts of the atom which fill these parts of the space. Therefore it is absurd to say the atom has no parts. In short: the concept of the atom and the intuition of space, both taken as strictly as possible, are incompatible.

To this it might be answered that the fact that we can think the parts of the atom does not mean that we can really

* See note p. 13.

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separate them. But Kant does not want to prove by his argument that all matter is infinitely divisible. Rather he is trying in the second antinomy to push to absurdity the thesis of the existence of least parts as well as the antithesis of infinite divisibility, and so to show reason its bounds, which it has overstepped with these two assertions both of which transcend the limits of all possible experience. On the other hand he leaves room for a relative, empirical atomism instead of an absolute one: an atomism which, in application to the later atomic theory of chemistry, one might formulate roughly as follows: The parts of the atom are conceivable, and it is therefore an empirical question whether we can separate them; if we cannot do it to-day, perhaps we may be able to do it later. As long as we cannot divide the atoms, nothing prevents our treating them for the purposes of science as if they were indivisible (not denying their possible divisibility, but ignoring it).

How restrained is this argument, which weighs only the usefulness, and in no way the truth of the atomic hypothesis, when compared with the aims of the original, metaphysical explanations of the ancient atomists and of the modern materialists! So much the better does it correspond, viewed as a whole, to the development and spirit of the modern scientific atomic theory. But it does not exhaust the results of the most recent atomic physics. The latter does actually interpret the concept of the atom in a somewhat more absolute way. Therefore the direction in which it applies to the critique of spatial intuition its knowledge of the incompatibility of the concept of atom with the intuition of space must be, at least in part, contrary to the direction of the Kantian critique.

In the kinetic theory of heat physics has already found itself forced to assert in a very strict sense the indivisibility of the atoms. Heat is there understood as unordered movement of the atoms. The general tendency towards an equalization of temperatures in nature, for example, is explained by saying that the statistically most probable con-

dition is always reached in time; i.e., the condition in which all the atoms (more exactly, all the degrees of freedom) possess, in the mean, the same kinetic energy. Now the question arises whether the parts of the atom cannot move with respect to one another, or, for example, whether the atom is capable of internal vibration. If, for instance, one supposes that the atom is not divisible merely because its parts are held together by very strong forces, then inner vibrations must in any case be possible. But with this an uncontrollable number of degrees of inner freedom would be added to the degrees of outer freedom of the atom, and thereby the ground would be taken out from under the kinetic interpretation of the simplest appearances. If one views the atom strictly as extended matter continuously distributed in space, then the number of degrees of inner freedom is even infinite, and no equilibrium of temperature can ever be established; the atoms of a warm substance must gradually 'get warm' themselves, and finally the whole heat energy would disappear into the infinitely many internal atomic vibrations of very small wave lengths. (This is just the paradox which is known in the theory of radiation as the 'ultraviolet catastrophe' and which first led to quantum theory.) One could try to avoid the difficulty by artificial evasions, such as the assumption of infinitely great forces in the atom. The difficulty impressed itself especially clearly upon the consciousness of physicists, however, when it followed from the observed specific heat of mono-atomic gases that such atoms are normally incapable of even the simplest relative movement of their parts; i.e., rotation, or at least exchange of rotational energy.

But it is just the non-perceptible traits of quantum theory which have eliminated this difficulty. Let us look, for example, at the hydrogen atom in its ground state. Just as it is incapable of giving off energy, so it is also incapable of taking in energy except for an energy quantum sufficient to bring it into one of the higher states. But such large quantities of energy hardly ever occur in normal heat motion,

and therefore play no role in the appearance of equilibrium. Thus it is just the property of the state that it is stationary—a property foreign to classical physics—which obviates the paradox. But this property is connected in a profounder way with the contradiction between the concept of the atom and the intuition of space.

In short we may say: as long as we consider the structure as an atom, we refrain from making more exact statements about the parts of space it fills; when we investigate the spatial details of its inner structure we destroy the atom. Let us explain this in detail: The hydrogen atom in its ground state has a determinate spatial extension. Quantum theory refrains from indicating just where within this space the electron is located. Instead it describes the electron in terms of the wave picture as a charge density spread through the whole space of the atom. The fixed distribution of this charge density is a property of the ground state; to change it, to excite vibrations, means just to raise the atom to a higher energy state. Thus this distribution of charge density characterizes the self-contained configuration of the hydrogen atom which is exactly alike in all individuals. But with the help of a very accurate measuring instrument, we can also try to determine the position of the electron in the atom, and we shall get a definite answer: 'the electron is located here at this place.' Naturally we also know that the whole charge of the electron is located at this place; the continuous distribution of density is destroyed. But in order to achieve this, it was necessary by means of the measuring instrument to add a great deal of energy to the atom, so much that in general the electron will now at once leave the atomic union. Thus we have now demonstrated spatial relations of the parts of the atom, but we have therewith destroyed the atom.

The peculiarity of quantum theory is that it denies neither of the two contradictory modes of consideration—neither the atomic nor the spatial-intuitive. On the one hand we can force the spatial localization of the electron; on the other

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hand, the chemical and thermic phenomena which led to the atomic concept occur only in atoms in which we refrain from localizing the electrons. Mechanics in the classical sense assumes the applicability of spatial intuition, chemistry leads to atomism; one can also say: mechanical and chemical methods of describing minute processes are mutually exclusive. Both are human methods for gaining knowledge about nature, and they retain their correctness for the purpose for which they were devised; but neither of them is valid as a uniquely justified description of an objective nature.

This much we learn from modern physics about the limits of the older attempt to explain nature at once mechanically and atomistically. That both mechanism and atomism were once considered compatible, and even almost identical, was due only to the fact that, as we to-day do consciously, scientists once unconsciously refrained from applying mechanics to the inner structure of the atom—an inconsistency clearly seen by Leibniz. In turn, we learn something about modern physics from the old paradox of the atomic concept: Modern physics must renounce spatial intuition because, and as far as, it wants to be strictly atomistic; and it must renounce the reified objectivity of nature in order to be able to apply—in accordance with the demand of experience—both spatial intuition and the concept of the atom to the same reality.

PERCEPTION AND PERCEPTIBILITY*

Let us turn to the basic question of the world picture. Granted that we have grasped clearly the conceptual relationships of the modern renunciation—does not the perceptibility of the world picture represent an unconditional demand for every science? He who renounces it departs from the ideal of natural knowledge that has become classic for us; but does he not, further, depart from living life?

* See note p. 13.

But what does perceptibility mean? This concept can be applied in two different, almost contradictory senses. It is perceptible, first of all, when I relate what I have perceived. Perceptible is what one calls a description which allows the picture of the perceived object to rise again in the soul of the listener. Perceptible in the highest degree is the *natural science* of Goethe which teaches us to find the primal original phenomena in what we perceive. As against this, the programme of atomic theory is inaccessible to perception from the start. All things are robbed of the essential nature attributed to them by immediate perception and sensation, and reduced to the behaviour of the atoms which no one has ever seen. Light is 'in reality' a hidden motion in dimensions of invisible minuteness, similarly with sound and heat and, through the mediation of chemistry, even smell and taste. The belief in the immediate truth of sensuous perception is sacrificed to the hope of a unified world picture.

To this world picture there is now attached another interpretation of 'perceptibility'. It consists in the hope that the non-perceptible too—that is, the atom—will submit to representation according to the image of the perceptible. According to the doctrine which was dominant from Democritus down to our century, the atoms lack all the properties of sensuously perceptible things except the geometrical ones: extension, shape, position. (To this were added in the newer view the dynamic properties, already less accessible to perception, of mass, force, and energy.) To investigate these properties of the atoms—what is more, to reduce all other natural phenomena to them in so far as possible—was now considered a perceptible explanation of nature, even though it stripped nature of all the immediately perceptible. Without doubt this was a grand project. But shall we regret that it has foundered? Was it not just the gross perceptibility of the geometry of the atoms, which dissolved all the real into a mechanics of small billiard balls and seemed to justify the most extreme materialism? Only after the atom has lost the last sensible quality does its true

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meaning for the physical world view become clear: the unity—real, though remote from our immediate perception—of all that our perception knows only as a multitude of appearances, is systematically held together and symbolically represented in it, but not mechanically explained.

Nevertheless, there is a renunciation expressed in this. If Goethe fought mechanistic thought, he wanted in doing so only to protect the knowledge which was already given to him directly in perception. For us, however, the renunciation of the mechanical model by no means leaves behind an immediate perception of the atom, but rather the abstract mathematics of the ψ -function. This renunciation we must understand as the natural consequence of the undertaking which modern science represents. The world of technical skill and likewise the world of modern experimental art is not the naturally given life sphere of man. Modern natural science makes the most strenuous efforts to produce experience which primeval man could not have, and from this, further, to infer objects and relationships which presumably will not be accessible to any immediate experience. Goethe did not blame Newton for anything so much as for the fact that he shut himself into a dark room and only left a crack in the window open, when he wanted to study the laws of light; on the other hand nothing shows so clearly, how exactly Newton was conscious of the methodological principles of modern science.

Science has not taken this course frivolously. The world immediately given in perception, 'mysterious in the full light of day,' does not contain fully its own explanation. Again and again it forces on us questions which reach the limits of the perceptible. In order to be able to pursue these questions at least a step further, we make ourselves 'with levers and screws', with high tension installations and ultra microscopes, a world of artificial experience formerly unknown. The abstract character of the knowledge which this world affords us is the true reflection of the path by which it is reached. This is clearly seen from the non-objective

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character of the atomic world. Every experiment is an act of violence which we impose on nature. It must react to the violence, and the law of this reaction can be stated in formulæ. But every statement is valid only in relation to the experiment through which it was obtained, and cannot be generalized to a hypothetical, objective, undisturbed state of the particles or waves which we have seen in the experiment; for we must admit that we ourselves have created the perceptible forms of appearance of the real through our experiment.

It may be remarked in conclusion that we have here got from the perspective of physics a glimpse of one of the most mysterious features of the human cognitive power in general. It is in fact only new to the physical mode of thought, but well known to those with some insight into men's spiritual and mental life, that the capacity for knowledge itself possesses a kind of creative power. If the physical world picture dissects the living relationship of I and world—which both, active and suffering at once, become what they are only in the interaction with their opposite poles—into the schematic confrontation of subject and object, then that is itself basically a productive recasting of reality. But while it is generally only a question of leaving out and thinking away parts of experience, modern physics itself produces its own experience as it were by an act of violence. The experiment, which itself brings to birth the state of reality which it shows us, is an especially eloquent material manifestation of the mind which only knows as it creates.

Chapter Three

THE IMPLICATIONS OF THE LAW OF THE CONSERVATION OF ENERGY IN PHYSICS

THE law of the conservation of energy is perhaps being applied in more special fields than any other law of modern natural science. But it has a special relation to physics. It originated in physics and its validity was first demonstrated in physics. Thus its application to other spheres of experience was frequently felt to be the first step towards a radical subjection of these spheres to the physical mode of thought. It became a cornerstone of the point of view, developed in the second half of the nineteenth century, which can perhaps be described as 'the physical world view'.

If we want to discover the exact sense of the law of the conservation of energy, whether out of a special interest or with the aim of examining more exactly the 'physical world view' we must look at its role in physics. What is most remarkable in this connection is the fact that the law survived the development of theoretical physics in the twentieth century—revolutionary as that development was in many respects—not, indeed, without being influenced, but without being shaken. Unlike the larger part of classical physics, it has not been restricted to a definite 'sphere of validity'. On the contrary, it is valid with certainty for all the phenomena of which we now possess any complete theoretical conception. In fact, the hypothesis of a suspen-

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sion of the law has not even been thought useful for the tentative description of any experience known to us.

This fact might suggest the conjecture that it must be possible to grasp the validity of the law of the conservation of energy a priori, independently of any particular experience. Nevertheless modern physics has had so much bad luck with such conjectures that until the establishment of a binding a priori demonstration (towards which not even the first step has been made) it will be prepared for the possibility of an empirically conditioned limitation of the area of validity of the law. But it is interesting to consider what properties of the law have exempted it from all crises up to now, and to add at least some remarks on its possible significance for the future.

The present exposition is therefore directed mainly to the 'constructive' significance of the law, i.e. to the part it has played and by its nature can in the growth of physical systems of thought. This question lies midway between the extremes of a merely philosophical and a merely practical problem. In the face of contemporary physics, however, both the latter questions demand to be heard and should therefore at least be touched on. Nevertheless the exposition of the philosophical significance of the law of the conservation of energy will be made only briefly, and will only touch on the point on which a rather different light is shed by the most recent physics: the relation of Mayer's two principles of the conservation of energy and of trigger action to the principle of causality. The practical side of the law will be handled just as briefly: We shall at least get a general view and provide a couple of examples of the great number of cases in which the application of the law of the conservation of energy to the establishment of energy balances, or for similar purposes, is a decisive aid to contemporary physical research.

THE CONSTRUCTIVE SIGNIFICANCE OF THE
LAW OF THE CONSERVATION OF ENERGY

CLASSICAL PHYSICS

The law of the conservation of energy is the generalization of a theorem of mechanics. Its first important field of application is the theory of heat; its description as 'the first law of thermodynamics' bears witness to this. Finally, it played an essential role in the construction of the only theory of classical physics which cannot be reduced to mechanics: i.e., electrodynamics. The following exposition is therefore divided according to these three theories. Optics does not need to be discussed at length since through the electromagnetic theory of light it can be reduced to electrodynamics.

Mechanics. The historical point of departure for the establishment of the law of the conservation of energy in mechanics is an observation of Leibniz, based on Huygens, about the force contained in a moving body. It is essential for what follows that we recall the contents of this observation. >

Descartes had declared, that the 'force' contained in a moving body is given by the product of its mass and its velocity (mv). Leibniz denied this, having in mind especially bodies which have attained their velocity through a free fall. He considered two different bodies which had been raised to different heights and then dropped, and which therefore reach the ground at different velocities. He then assumed that both bodies would have the same 'force' on reaching the ground if it had taken equal work to raise them to their former height. If, for example, the first body has on reaching the ground twice as great a velocity as the second, one would conjecture, according to Descartes, that the second body must be twice as heavy as the first in order to contain the same 'force'. But Leibniz asserts that it must be, for this purpose, four times as heavy as the first (i.e. 'force' is pro-

portional to the square of the velocity: mv^2). He bases this conclusion on Galileo's law of falling bodies, according to which one must raise a body not twice but four times as high to obtain double velocity of impact. But now, certainly, one must apply the same amount of work to raise one pound one foot four times, and thus four feet in all, as to raise four pounds one foot, once. Therefore, in fact, the same amount of work is employed if one takes the second body four times as heavy as the first.)

This train of thought was the starting point for the long-standing quarrel about the 'true' measure of the 'force' contained in a body. This quarrel is scarcely any longer intelligible to us to-day, since in the last analysis it was only a question of nomenclature. For in fact both quantities, that considered by Descartes as well as that considered by Leibniz, are important in mechanics, and in the construction of the system of mechanics we may choose arbitrarily which of the two quantities we shall call 'force'. Contemporary physics calls neither of the two 'force', but the quantity introduced by Descartes 'momentum', that introduced by Leibniz 'energy'. Yet it was an important conceptual task, which had to be accomplished at some time or other, to recognize that we might take this liberty with our terms. For we are at liberty to designate a given concept with any name we please—not indeed for all concepts of a theory, but only for those which by definition are explicitly based on other concepts already assumed as known. In order to be intelligible at all, a theory must always draw out of everyday speech a stock of concepts which can be considered sufficiently clear for the purpose at hand, and with their help it can define precisely a relatively small number of concepts. In the construction of the theory of mechanics the concept 'force' belonged to this group of concepts offered by everyday speech, and only through discussions of the type mentioned it became evident that this concept was not clear enough.

The approach of Leibniz was fruitful because he already tacitly assumed in his definition that the 'force' must be con-

served in the different phases of the observed event, and so he was led to that magnitude which was later to form the point of departure for the general law of the conservation of energy. This conception of conservation is hidden in the premise that the 'force' is of equal magnitude if equal work was employed to produce it. In this connection, the 'force' is by no means immediately perceptible; it remains hidden at first, as it were, in the body raised by the expenditure of work. To-day we call the 'living force' which is contained in the velocity of the moving body, its kinetic energy, and we call its capacity to acquire kinetic energy by falling (a capacity produced by the expenditure of work) its potential energy. As the body falls, it changes its potential energy to kinetic energy; if it rebounds and reaches again the height from which it fell, then kinetic energy is again being transformed into potential energy. General mechanics demonstrates that the sum of kinetic and potential energy is constant if the acting forces fulfil certain conditions, for instance, absence of friction.

In the formation of this concept we already have the basic principle of the general law of the conservation of energy: although experience shows us a constant change in phenomena, there is something that remains unchanged throughout all changes. This concept meets a deeply rooted need of our thought. We were prepared to introduce a concept as abstract as that of potential energy, which betrays itself by its very name as mere 'possibility', in order to purchase in exchange the invariability of the total amount of energy. But it cannot be taken for granted that nature does us this favour: that is, that we can find a quantity unequivocally determined through the observable properties of the object (in our example through the height of the lifted body), which will always supplement kinetic energy so as to produce a constant. In fact this is no longer possible if we take account of friction: through friction a body loses energy without mechanically determinable compensation. This is where Julius Robert Mayer began.

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✓ *The general law of the conservation of energy.* We do not need here to describe historically the achievement of Mayer and his successors; it is enough to characterize it conceptually.

Through the conviction that in heat, light, chemical affinity, electricity, and magnetism there are hidden so many forms of energy, it was possible to obtain universal validity for the law of conservation. We could then pursue energy through the transformation of its forms; thus the energy lost through friction appeared again as heat, etc. This point of view became a proven theory through the fact that, for every one of the areas of experience mentioned, it proved possible, on the basis of observation, to state a quantitative law which permitted the actual calculation of the energy contained in a state from its observed constituents. In this process it was natural to proceed from mechanics. For example, the work done by a given process was measured by the height to which it could raise a given weight. The calculation of the energy of a physical entity amounted then to calculating the work which this entity could do by a certain change of its state, or which must be applied to bring about such a change. In ascribing such a 'work equivalent' to every change of state, or at least in requiring that such an equivalent exist even for processes which were not yet theoretically understood, lay the real physical content of the law of the conservation of energy. That Mayer's priority was recognized after being at first ignored was probably due less to his general consideration of natural forces than to the fact that he was the first to obtain from experience, by a correct argument, a value for the work equivalent of heat.

The contents of the law of the conservation of energy can be expressed in this form: The work equivalent of a change of state depends only on the initial and final state, but not on the nature of the transition from one to the other. It is a question only of the whole amount of energy released and not of the path through which it is released. We can recognize here the connection with the law, made probable by many observations, of the impossibility of a perpetuum

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mobile—i.e., of a machine working periodically which constantly produces work without altering in other respects the state of the machine or its environment. For if the liberation of energy in the transition from one state of a system to another depended on the manner in which the transition was effected, we could choose for the liberation of energy the more expensive and for the reduction of the system to its original state the more economical path, and thus have retained liberated energy in the whole process without any difference between the final state of the process and its initial state; and if we repeated the process periodically, we should have the machine we wanted.

This consideration reveals what an amazing and bold extrapolation from a few known items of experience to all unknown experience is hidden in a principle like the law of the conservation of energy. For if just one single perpetual mobile were discovered in the whole world, the law of the conservation of energy would be false. To be sure, the law of the conservation of energy shares this boldness with all universal laws of nature, which state a rule according to which *all* phenomena of a given kind must occur. Again and again one is astonished at the thought that among the experimental data observed in a century of the most intensive physical research, all of which were unknown at the time of the establishment of the law, not one plausible example of a violation of the law has been found.

Nevertheless, we can put the question; what would be changed in physics if a perpetual mobile were to be discovered to-day? Our conviction of the universal subjection of nature to law would not be shaken. Rather we should anticipate that this perpetual mobile would need quite definite physical conditions for its operation. We should try to ascertain these conditions empirically and finally on the basis of our new observations to formulate a new general law which would have a content something like this: the law of the conservation of energy is valid when and only when these and these conditions are fulfilled. By this process

the validity of the law of the conservation of energy would be restricted to certain limits, and perhaps we could hope to recognize it ultimately as a special case of a still more general law. But it is of no use for the moment to follow this speculation further; it would be more useful to acquire a feeling for the reasons why just this law should so far have been universally confirmed.

Let us further point out a peculiarity of the general law of the conservation of energy. The work equivalent of a change of state gives only the difference of the energy content of two states. The absolute value of energy, however, that is, the total energy content of a body, remains undetermined. We recognize this already in the potential energy of the free fall: obviously it would be arbitrary to assign to a body, just when it is lying on the earth's surface, the potential energy 'zero', since the definition would depend, for example, on the height above sea level; or the body might fall into a well and so acquire negative potential energy. As long as we do not know the absolutely poorest state with respect to energy at which a body can arrive, energy is determined only up to an additive constant. In this respect the law of the conservation of energy differs essentially from the law of the conservation of mass, to which even at an early date it was considered parallel. The difference was overcome only by the union of both laws in the theory of relativity.

✓ *Application to heat and electricity.* The historical meaning of the law of the conservation of energy for the theory of heat is of a dual nature. First, the law proved decisively the incorrectness of the so-called material theory of heat, which conceived heat as a particular material, and secondly, it furnished the foundation for the construction of a consistent mathematical theory of the phenomena of heat. Although the latter is a particularly fine example of the 'constructive' application of the law, we shall limit ourselves here to a few remarks on the first point.

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The material theory of heat considered heat a kind of matter which could neither be generated nor destroyed. The observation of the unlimited production of heat by friction had already contradicted this assumption, and with the recognition of heat as *one* form of energy among many the break with the old theory was completed. The new doctrine was in general called the 'mechanical theory of heat'. This designation did not at first express the view that heat is really a form of motion, but only that it can be transformed into mechanical work. Mayer himself conceived heat not as motion, but as an independent form of energy. Just as one cannot infer, he said, from the possibility of changing potential energy into motion, that potential energy really is motion, so one cannot infer from the possibility of changing heat into motion, that heat itself is a kind of motion.

Nevertheless, it was precisely the law of the conservation of energy which contributed most to the propagation of the 'mechanical conception of nature': i.e., the conception which held that all the phenomena of nature were produced by purely mechanical action of the more minute parts of matter (and of the hypothetical ether); that, in fact, a physical theory presented no real explanation of the phenomena it dealt with as long as it had not reduced these phenomena to mechanics. On this view, the general law of the conservation of energy can be dispensed with as a particular law of nature. For if we assume that all the forces arising among the most minute particles are determined by an energy dependent on their mutual position (and therefore that there is no friction in the smallest dimensions), then, by virtue of the basic laws of mechanics, the law of the conservation of energy is automatically valid. The empirical fact of the universal validity of the law is then only an indication of the validity of this hidden mechanics, and an indication of the need to reduce the forms of energy arising in experience to mechanical forms of energy in the minute particles. Heat, e.g., then appears as kinetic energy, chemical energy as a certain potential energy of the atoms. Following Helm-

holtz, this view was used as the main argument in favour of the general law of the conservation of energy.

As a matter of fact, a wealth of experience has ultimately demanded that heat should be conceived as unordered movement of the atoms. Thus thermodynamics, which is based on the law of the conservation of energy, has its foundations in a statistical mechanics of the atoms. So much the more important is it therefore, fundamentally, that the law of the conservation of energy is also valid in electrodynamics, which it has not been possible to reduce to mechanics.

We shall, again, only mention here the application of the law of the conservation of energy in the construction of a conceptually complete *electrodynamics*. Thus the law of the conservation of energy, with a few simple additional assumptions, allows us to deduce the mathematical form of the law of induction. But what is the basic position of electrodynamics with relation to the law of the conservation of energy?

Since nothing was known about the way in which electric and magnetic processes might possibly be reduced to mechanics, it was necessary to develop electrodynamics in accordance with observation as a 'phenomenological theory'. It turned out that this theory had to be constructed as a 'field theory'. That is, electromagnetic processes do not act directly at a distance, but they fill the whole of space, and every effect is only transferred from one point to the neighbouring point and so on. Therefore electromagnetic energy must also be spread out in space, and it must be univocally determined by the quantities through which we determine the state of the electromagnetic field, that is, through the two vectors of the electrical and the magnetic field strength. Now how does electromagnetic energy depend on these field quantities? Observations on the work performed by the field can give some indication. But to start with we can also draw some conclusions from the condition that the law of the conservation of energy must hold. For obviously the energy must be such a function of the field quantities

that it does not change no matter how the field changes, as long as no energy is supplied or carried off. If we take as known the laws which the alteration of the field quantities follows (Maxwell's field equations), then by means of this condition we can already limit the possible functions to an extraordinarily small number. It is even impossible by any means to construct a function having the required property out of any and every field equation that one may happen to think of. Thus the law of the conservation of energy presents an incisive and restrictive condition for the possibility of the construction of new theories: only those equations can be considered for which it is possible to define an energy function.

In practical terms, it is necessary to introduce the concept of the spatial density of energy since the energy of the field is extended through space. Energy can stream through space. Thus energy flows from a radio transmitter into the environment; and it is only when energy given off by the transmitter strikes the receiver, that the latter can perceive what has been transmitted. The law of the conservation of energy states in this case that the integral of the energy density over a space to which no energy is added and from which none is taken away, is constant. If we consider a spatial volume through the boundaries of which energy is flowing, then the temporal alteration of the integral over the density of energy in the inner part of the volume must be equal to the integral of the flow of energy through the surface. Thus energy is handled exactly like flowing matter in hydrodynamics.

This holds independently of the question whether we can or cannot assign a mechanical model to electrodynamics. Now all attempts at such a model have failed entirely, and finally Einstein deduced from the Michelson experiment, that the ether, the mechanics of which was thought to be involved in electrodynamics, does not allow even the determination of a velocity which could be attributed to the ether as an objective quality (e.g., a co-ordinate system in which it

rests). The attempt to look further for a mechanical model of electrodynamics was finally abandoned. In fact, the inverse conception even gained ground of founding mechanics on a field theory like electrodynamics. With this, the independence of the law of the conservation of energy from the mechanical view of nature was revealed for the first time. It enters the modern development of theoretical physics as an independent principle.

RECENT PHYSICS

It is characteristic of theoretical physics since 1900 that it has taken seriously the programme we have just sketched. Yielding step by step to the pressure of new data, scientists have given up more and more completely the presupposition that classical mechanics or a theory modelled on it is valid for the whole of nature. Instead the attempt is made to develop an independent theory of the phenomena not apprehended by classical mechanics and, conversely, to understand classical mechanics as a 'limiting case' of the new theory, i.e., as the result of its application to a definite, restricted field of inquiry.

To be sure, this new stand could not be taken, as it were, at one bound. Indeed, it is for reasons of method indissolubly connected with classical physics, and is attainable only on a road that leads through classical physics. For classical physics already embraces the sphere of experience that is easily accessible. All the recent theories are based on experiments which can be carried out only with the most refined experimental technique. But the manner in which the instruments used in these experiments operate must of course be known, theoretically, as exactly as possible, if an unequivocal conclusion is to be drawn from the results of the experiment. But, obviously, this theory of the measuring apparatus must be taken from classical physics. For, in the first place, the interpretation of the experiment, when completed, has yet to furnish the basis of a new theory; there-

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fore, this theory cannot yet be available for the first interpretation. But secondly, even after the setting up of the new theory, the greatest part of the processes in the measuring apparatus will correctly be attributed to classical physics; since the apparatus itself is not—as the effects investigated with its help are—removed from immediate perception, but is merely a specially prepared object of our everyday world and thus belongs to the sphere of validity of classical physics. As an example let it suffice to point to the microscope, which is itself a manipulable visible object, but which furnishes us an image of objects that lie beyond the limits of visibility.

Thus we carve for ourselves, step by step, a path from the known to the unknown, and so we are driven to describe every newly discovered object first of all—tentatively—with the physics we already know, in order, as occasion offers, to let experience teach us a better one. So the development of modern physics is the history of a gradually more and more radical denial of a number of central theses of classical physics along with the conscious preservation of the remainder which is as yet intact. In this development three main phases can be distinguished up to the present: the theory of relativity, the quantum theory, and the theory of elementary particles which is just now being developed. The first two theories have essentially deepened the concept of energy, and have thus broadened the significance of the law of the conservation of energy. It is to be assumed that the third theory too will play a decisive role. Let us now turn to the particular theories.

Theory of Relativity. We shall consider only the Special Theory of Relativity. The General Theory of Relativity does indeed present an important conceptual possibility. But the development introduced by it cannot yet be considered closed, and at the present moment the number of possible empirical proofs of the theory is small. It would be difficult, in this theory, to present the question of the law of the con-

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servation of energy apart from general cosmological considerations, which we do not wish to pursue here.

From our point of view the most important law of relativity theory is the law of the inertia of energy. It permits the two fundamental laws of the conservation of mass and the conservation of energy to be fused in a single law.

That matter can neither come into existence nor be destroyed is an age-old belief of most natural scientists and of the more 'materialistically' inclined philosophers. After mechanics had refined the concept of mass in its double significance as inert mass (measured by the resistance of matter to a change in its motion) and as heavy mass (measured by the weight of matter), the belief in question took the form of a law of the conservation of mass. The law was empirically confirmed—and thus became scientifically fruitful—only when weight was introduced into chemistry (Lavoisier). Since then it has always been supported by experience. The introduction of the law of the conservation of energy occurred from the first as an analogy to it. Energy and mass, force and matter, now seemed to be the two genuine realities of the physical world.

Now relativity theory predicted—and it has been confirmed in this through a profusion of experimental data—that a proportional amount of mass is associated with every amount of energy: energy itself is inert and weighable. A material particle (e.g., an electron), which moves with a velocity close to that of light, resists every further acceleration with a resistance of inertia that increases with the velocity already reached: its mass is increased by an amount which depends on its kinetic energy. If we bring together several separate parts of atomic nuclei (protons and neutrons) for the formation of an atomic nucleus, a sum of energy is liberated which is larger in proportion as the union of the parts in the nucleus is more stable. To the same degree as this energy is dissipated, the mass of the nucleus decreases; it shows a 'defect of mass' compared with the sum of the masses of its parts. Suppose the energy is released

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in the form of electromagnetic radiation: the field of radiation then carries a density of mass proportional to its density of energy—which in certain cases we can even see ‘re-materialize’ in the production of electron pairs.

With this we now have the absolute measure of energy which had been lacking before. For if all energy has mass, then a body cannot in any event give off more energy than corresponds to its total mass (at any rate in so far as we do not need to reckon with the possibility of the occurrence of negative masses). It is another question whether a body can release its total mass in the form of energy—in which case it would at the same time give up its own material existence. For a certain kind of matter, electrons, the question has now been decided, experimentally, in the affirmative. For the atomic nuclei, which furnish the chief contribution to the mass of matter, no such experimental results are yet available.

In any event, the law of the conservation of energy can to-day be maintained only if the weighable mass which does not manifest itself directly as energy, the so-called ‘rest mass’ of bodies, is regarded as a form of energy, and is taken into account in every energy balance. Obviously it can also be said that the law of the conservation of mass can be maintained only if energy is understood as a form of mass (or as a carrier of mass). In short, the two laws are completely fused into one.

The fusion of energy with momentum in the energy-momentum four-component-vector also deserves our attention. A quantity which at the same time has a direction in space, is called a vector; e.g., the velocity of a body, which is only determined if both its amount and its direction are known. The momentum of a body, being the product of its mass and its velocity, has the same direction as its velocity, and thus is also a vector. Energy, on the other hand, is characterized simply by the statement of its numerical value; it has no spatial direction. Such a quantity is called a scalar in contrast to a vector: all scalars can be ordered from

smallest to largest in one scale, while two vectors of equal magnitude can be further distinguished by their direction. A vector is characterized only by the determination of three numbers. It is possible, for example, to set up arbitrarily any three dimensional rectangular system of co-ordinates and to characterize the vector by the three projections of the fixed spatial distance by which it can be represented on the three co-ordinate axes. These projections are called the components of the vector. But the components themselves are not actually physical realities, for the same vector naturally has other components if another system of co-ordinates is laid down. Yet it is sufficient to know the components in any system of co-ordinates, since it is then possible to calculate the components in every other system of co-ordinates.

Now the Special Theory of Relativity compares the expression of physical realities in co-ordinate systems which are not only distinguished from one another by the different position of their axes, but which move with different velocity. The physical properties which we attribute to a body depend on the co-ordinate system in which we observe it. If, for example, the velocity of a railroad train is measured from a stationary point on the earth's surface, then this velocity, and with it likewise the momentum and kinetic energy of the train, have a certain value. If, on the other hand, the velocity is related to a co-ordinate system which is moving with the same velocity as the train, then the velocity of the train naturally has the value zero; in relation to this co-ordinate system the momentum and the energy necessarily also have the value zero. On the other hand, observed from a co-ordinate system outside the earth, say the sun, the train takes part in the very great velocity of the earth in its orbit and thus its momentum and its energy also appear extremely large. That is to say, energy and momentum are relational concepts. In classical physics we might, indeed, be able to ask about the 'true momentum' and the 'true energy', which result from the 'true velocity'

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related to a 'co-ordinate system absolutely at rest'. Yet it has never been possible to establish physically such a co-ordinate system, i.e. one which is 'absolutely at rest', and the theory of relativity asserts the fundamental relativity of uniform rectilinear motion, that is, it asserts that fundamentally all co-ordinate systems with rectilinear and uniform motion are equally valid.

Formally, this is expressed by combining the three components of the momentum with the energy as the four components of a so-called four-vector (the fourth dimension symbolically introduced here is time). Therefore momentum and energy are considered expressions of a reality characteristic of a given body: its dynamic potency as it were. But this expression is different in every system of co-ordinates and we content ourselves with knowing momentum and energy for every system of co-ordinates. To ask about the 'true energy' of a body is as meaningless as to want to know the 'true components' of a vector. The law of the conservation of energy is valid in every system of co-ordinates; only the numerical value which we attribute to energy changes when we choose a new system of co-ordinates, i.e. a new point of view.

Now it is characteristic of relativity theory that energy is united precisely with momentum, for which also a law of conservation is valid. Energy and momentum must be understood as the expression of the dynamic properties of a body, while its position as a function of time is the expression of its geometric-kinematic properties, and there is a single dynamic law of conservation which can briefly be designated as the energy-momentum-law. >

Naturally the mass of a body as well as its energy depends on the system of co-ordinates. But there is a mass characteristic for a body: that is, that value of a mass, which is measured in the co-ordinate system in which the body finds itself at rest. In this system of co-ordinates its momentum and its kinetic energy as well as its velocity have the value zero. The mass which is still present there is called the rest

mass of the body and the energy co-ordinated with it its rest energy.

✍ *Quantum theory.* Quantum theory, too, begins by bringing energy into a close and unexpected relationship with another physical magnitude, i.e. with the frequency of vibration. In the development of the quantum hypothesis, Planck considered the radiation of light which proceeds from a vibrating body (such as an atom). He assumed that in this process the energy of vibration was transferred to the field of radiation, not continuously, but in discrete quanta of energy, whose magnitude, E , is proportional to the frequency of the vibration, ν , and therefore to the number of vibrations per second: $E = h\nu$. The factor of proportionality, h , which can be determined by experiment, is Planck's 'quantum of action'.

This assumption was suggested by experience and has since been admirably confirmed. From the point of view of classical physics, however, it was entirely unintelligible. We know to-day that it is the consequence of a basic fact of all atomic events: the dualism of the wave picture and the particle picture. Every material body (e.g., an atom or an electron) can appear under certain circumstances as a wave spread continuously through space, and every wave event (e.g., light) as an effect of particular localized particles. It depends on the nature of the experiment in hand in which of the two pictures a physical object presents itself to our perception. We must remember here what we mentioned earlier, the peculiar position of classical physics as the path of approach to all fields, even those newly opened. 'Particle' and 'wave' are two visual pictures which come from the conceptual storehouse of classical physics. One might think the dualism could be avoided if both pictures were given up and replaced by the formation of a third concept entirely foreign to classical physics. But both pictures are the simple description of what is seen in experiments: their classical character is already determined by the fact that we can

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receive a sensible perception of the actions of atoms in no other way than through that realm of nature which classical physics is able to describe. Even if we possessed this third concept, in order to compare it with experience, we should have to translate the consequences to be drawn from it into the language of classical physics.

Now this dualism means that in the characterization of the state of an object two entirely different groups of concepts are applied side by side. In the particle picture we speak of the place, the momentum, and the energy of a particle, in the wave picture of the wave length, the frequency, and the speed of propagation of a wave. If the determining factors of a state are given according to one picture, the question arises, whether in these factors something is already said about the determining factors in the other picture. This is indeed the case. Planck's equation is the expression of this relation between the two pictures. An entity which has a given energy in the particle picture, has in the wave picture a frequency proportional to this energy. There is in addition a second functional relation which was discovered by De Broglie: The momentum of a particle, p , is related to the wave number k of the corresponding wave (i.e., the number of wave lengths per cm.) by the equation $p = hk$.

In quantum theory, these relations play the role of axioms which are indeed suggested by experience but can no longer be deduced from other basic laws. Nevertheless, the law of the conservation of energy, in the form in which relativity theory has stated it, allows us to infer that the relation between wave picture and particle picture, if it can be written at all in the form of such simple equations, must have the Planck-De Broglie form. For even before it is known what quantities in the wave picture correspond to the energy and the momentum of a particle, it is certain, first, that these quantities of the wave picture may not change in time (law of conservation) and secondly, that their dependence on the co-ordinate system must be the same as that of energy and

momentum (existence of a four-vector). Now if I observe a wave moving forward uniformly in space, then the two quantities which do not change in this movement, are precisely its frequency and its wave length; and it can be shown that the frequency and the number of wave lengths per centimetre, measured in the direction of propagation do in fact form a four-vector. This consideration furnishes a perfect example for the constructive application of the law of the conservation of energy.

Despite this fundamental role, the law of the conservation of energy did go through a crisis in the evolution of the present form of the quantum theory. It is instructive to consider the causes and the solution of this crisis. It arose from the knowledge that the relation between wave and particle must be partly of a statistical nature. Let us ask what quantity in the wave picture corresponds to the place at which the corresponding particle is located. The wave is located not simply at one place, but in many places at once. Yet, through a proper measuring apparatus (a microscope, a scintillation screen) I can nevertheless measure exactly the place of the particle within the volume filled by the wave. The place which I thus find is not exactly determined by the form of the wave, but varies from case to case even under exactly similar conditions. It has turned out that the *intensity* of the wave at a place determines the *probability* of finding a particle at just this place; more than this probability does not follow from the wave picture.

But now energy is a property of the particles: where we find a particle, we find a considerable amount of energy concentrated in a very small space. But if particles occur in space in a fashion that is not strictly predictable, then we lose sight of the flow of energy. Can the law of the conservation of energy be retained in this situation?

In 1924 Bohr, Kramers, and Slater proposed tentatively a theory according to which the law of the conservation of energy would hold, not in every particular case but only statistically. This theory rests on two assumptions: First the

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intensity of the wave indicates the probability of the presence of a particle; second, the form of the wave at any other place does not change through the appearance of a particle at a given place. If, for example, we look at a wave which originates from the giving off by an atom of a single photon of energy $h\nu$, we now divide the space in which the wave occurs into two equal parts and look for the photon first in one and then in the other part. For each of the two parts, the probability of meeting a photon in it is, according to the first assumption, equal to one-half. But according to the second assumption the form of the wave function in the second spatial division is unaffected by the result of the measurement in the first division, i.e., whether a photon has or has not been found in the first division, the probability of finding a photon in the second division remains one-half. Thus four equally probable results of the whole experiment are possible: a photon occurs in neither of the two divisions, or only in the first or only in the second or one in each. But since the frequency ν of the wave and therewith the energy $h\nu$ of the photon is fixed, the energy content of the whole wave turns out in the first case to be zero, in the second and third $h\nu$, in the fourth $2 h\nu$. So it is only in the second and third case that the energy discovered in the field is equal to the energy introduced into it: in the first case it has disappeared, in the fourth doubled. Therefore, the law of the conservation of energy would hold only in the statistical mean, but not in the particular case.

We cite this theory because it shows how close scientists came, in view of the statistical nature of atomic phenomena, to relinquishing the strict validity of the law of the conservation of energy. Only new data (experiments of Bothe and Geiger and of Compton and Simon) decided against this theory. They showed that the law of the conservation of energy holds even in the particular process. Hence, one of the two presuppositions of the theory had to be dropped; and since the first is the condition for the possibility of uniting the wave and particle pictures, it was necessary to

relinquish the second. In our example this means: we know in advance that the field contains exactly one photon of the energy $h\nu$. If we find a photon in the first spatial division, then it is certainly not in the second. Thus the probability of finding it in the second division is momentarily reduced from one half to zero in the instance in which the photon is found in the first division—that is, the intensity of the light wave in the second division momentarily decreases from a finite value to zero. If, on the contrary, we find that the photon is not in the first spatial division, then the probability of finding it in the second division, increases momentarily from one-half to one; the intensity of the wave in the second division is doubled. This discontinuous change of wave functions through the act of observation (the so-called ‘reduction of wave-parcels’) expresses the fact that, in quantum mechanics, the waves are not physical realities existing in themselves but the expression of a cognition of the experimenting person, acquired through measurement, and can thus in fact be momentarily changed by the acquisition of new knowledge. The transition to the self-existent wave fields of classical physics occurs through the increase in number of the photons in the wave pattern; for if the wave contains many photons, then our learning that a single photon was encountered in the first spatial division scarcely alters the intensity in the second division.

Furthermore, the characteristic properties of the particles—including energy—are, not self-existent given quantities, but the expression of a cognition acquired through measurement. Let us look, for example, at a particle whose position at a given moment has been very exactly measured. In this event the probability of finding it at any other than the observed place is equal to zero. In the wave picture we have therefore to represent it by a ‘wave parcel’, the intensity of which is everywhere equal to zero except at the observed place. Such a wave parcel has no defined frequency and wave length. That means, however, that no defined value of energy and momentum can be said to correspond to it.

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While, therefore, the extended wave pattern of determinate frequency and wave length which we considered earlier allowed only statistical predictions about the position of the particle, we can now, conversely, make no definite prediction of the momentum and energy which would be found in the particle if it were measured; for every possible outcome of the experiment, again only a probability can be stated. (This probability is given in the coefficient of the frequency, or wave length in question in a Fourier-analysis of the wave parcel.)

The quantitative formulation of this relationship in the Heisenberg indeterminacy-relation means that exact knowledge of momentum stands in an exclusive relation to an exact knowledge of position, and exact knowledge of energy, to the exact determination of an instant in time. Let us explain this more exactly in the case of the measurement of energy. The frequency of a wave corresponds to the energy of a particle. In order, however, to be able to ascribe any definite frequency to a wave, we must let it vibrate long enough to let several wave peaks and valleys pass by in the interval; for otherwise it is not a question of a periodic wave, but of a single impact, to which no frequency can be assigned. Therefore, it follows even from the Planck relation alone that the energy of a particle is also defined, not for a very short instant of time, but for a certain span of time only. If energy is measured in a shorter span of time, an indeterminate value is obtained. This has practical consequences. For example, according to quantum mechanics, and in harmony with experience, a moving particle can penetrate an obstacle when the energy of the particle would be, according to classical physics, insufficient to overcome it (Tunnel effect). For this to occur, the only condition is that the obstacle must be so narrow that the short time in which the particle is in the obstacle is not sufficient for an exact definition of the energy which the particle has in this state. In this case there is a certain probability that the particle, if measured, would show a higher energy in this

time-span than before and after the obstacle, and so could traverse the obstacle without contradicting the law of the conservation of energy. The point of departure for this consideration is, therefore, that there is enough time before and after the obstacle to define the energy of the particle as exactly as may be, but not during the actual traversing of the obstacle, and correspondingly the probability of traversing the obstacle decreases the broader the obstacle, and, therefore, the more exactly the energy of the particle during the passage can be determined.

The example shows that the law of the conservation of energy, although it is never false in quantum mechanics, is, as it were, suspended in certain cases. Its validity can be assumed when—but only when—experimental conditions are present which allow us, at least in principle, to test its validity. Comparison with the older conjecture of Bohr, Kramers, and Slater shows that this restriction is not only a consequence of our limited knowledge of nature, but that it is the necessary logical presupposition for the integration of the dualism of waves and particles into the system of classical physics. If we wished to understand the characteristic quantities of one of the pictures as self-existent properties of nature, we should have to admit that laws like the law of the conservation of energy really collapse when empirically tested.

∴ *Theory of elementary particles.* Quantum theory does not explain the characteristic properties of the different sorts of elementary particles which we know to-day. It is, therefore, certain that it must be supplemented by a new, as yet unknown theory. Experience already provides us with some supporting evidence for the properties of this new theory. Since, on the basis of experience, the question whether the law of the conservation of energy will still be valid in the new theory has already been discussed, let us examine these considerations briefly. ∴

↪ Certain atomic nuclei undergoing so-called beta decay

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send out electrons, and in this process change into nuclei of another sort. The energy of the original nucleus and the final nucleus is in each case exactly determined. According to the law of the conservation of energy we should therefore expect all electrons proceeding from an original nucleus of a certain sort to have the same energy, i.e., exactly the difference in the energy of the original and of the final nuclei; for no other observable emission of energy takes place except for the emission of the electron. In fact, however, these electrons have very different energies, all of which lie between the value zero and a certain maximal value. Bohr, therefore, considered the possibility that this process violated the law of the conservation of energy. A closer experimental investigation of the process has, however, rendered it very probable that another conjecture proposed by Pauli is correct: i.e., that an uncharged particle (a so-called neutrino) is given off each time simultaneously with the electron—a particle which has escaped all direct observation up to now because of a very slight interaction with matter. It has in fact been discovered that the difference in energy between original and final nuclei is exactly equal to the maximal energy which an emitted electron can have. Now it seems very reasonable to assume that in every case just this quantity of energy is emitted, but that it is distributed between the two particles in a ratio that varies from case to case; then elements of varying energy will be observed, but with energy always lying between zero and that maximal value. >

Here again experience seems to have decided in favour of the law of the conservation of energy. Especially interesting, however, is a purely theoretical argument by Landau for this solution of the question—an argument cited by Bohr himself. Let us suppose that, on the emission of the electron, a certain quantity of energy disappears entirely at the place occupied by the nucleus. With it a certain mass also disappears. Therefore the gravitational effect of the nucleus on its environment will decrease also. Since, according to the

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Special Theory of Relativity, no effect spreads faster than with the speed of light, the change of the gravitational field which is conditioned by this change in mass, will run off as a wave with (at most) the speed of light. But if we try to describe this wave process according to any sort of differential equation for the gravitational field (e.g., that of the General Theory of Relativity) then it always appears that the wave process carries mass. Just as much mass flows outward in the wave as disappeared inside the atom. Thus the wave equation has automatically corrected the assumption of a loss of energy. Whenever energy disappears anywhere without compensation, it must appear again in the form of a gravitational wave; perhaps the neutrino is the particle coordinated with this gravitational wave. Of course it would be possible to alter the theory of gravitation also, in such a way that it would not contain this transport of mass. Our point was only to show how deeply the assumption of the law of the conservation of energy is anchored in the structure of contemporary physics, so that it is impossible to deny it without reshaping the whole framework of physical theory.

There is so far to-day not much more to say about the role of the law of the conservation of energy in the physics of the future.

THE LAW OF THE CONSERVATION OF ENERGY AND CAUSALITY

J. R. Mayer considered the law of the conservation of energy to be the physically precise formulation of the old proposition 'cause equals effect' and therefore placed it in a close relationship to the principle of causality. He came to this through the explanation of energy (or 'force', in his terminology) as the cause of the movement of matter. Yet the law of the conservation of energy obviously does not embrace the totality of possible causes of material changes.

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The remarkable further group of phenomena, in which cause and effect do not stand in a numerically determined relation to one another, but to which the saying 'small causes, big effects' is more easily applicable, he included under the concept of trigger action.

We shall examine briefly three questions: What role is played in physics by the relation stated by Mayer between the law of the conservation of energy and the causal principle? Is the law of the conservation of energy also affected by the so-called crisis of the causal principle in modern atomic physics? What meaning has the concept of trigger action for physics?

CAUSA AEQUAT EFFECTUM

< It cannot be maintained, that the law of the conservation of energy is a logical consequence of the principle of causality. The principle of causality is not precise enough for that, and the law of the conservation of energy is too narrow. The law of the conservation of energy might rather be called the more exact formulation of a part of the general principle of causality, a formulation, moreover, which was made possible only on the basis of experience.

By causality, physicists as a rule understand simply the assertion that between two states of the same entity at different times, there exists a universal functional relation: 'If the state of a closed system at a moment in time is entirely known, then its state at every other moment is in principle determined.' It is obvious that this principle is logically independent of the law of the conservation of energy. It can be correct even when energy is not conserved (e.g., in pure mechanics, where the forces of friction occur) and it could be wrong, even though energy were conserved (e.g., in quantum mechanics it is at least restricted in its applicability). Nevertheless, this principle, which we can briefly call the principle of determinism, does not embrace the totality of the ideas which we unite by means of

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the pair of concepts 'cause and effect'. We know this much from the asymmetry of the two concepts. While according to determinism the past is just as much determined by the future as the future by the past, we say that a cause produces an effect, but not vice versa. It belongs to the same representational nexus that, at least in everyday life, we do not regard one state simply as the cause of another state, but we imagine a cause to be some kind of self-existent force which possesses the capacity of 'effecting' something. It is clearly this side of the concept of cause which Mayer wanted to reproduce in his interpretation of the 'conservation of force'.

This way of thinking is so foreign to modern physics that many of its prominent representatives regard determinism as absolutely the only confirmable and meaningful formulation of the causal principle. Perhaps it is necessary, therefore, to undertake here a brief justification of the other linguistic usage. The difference between cause and effect, which in a process that follows a determinate course can be reduced to the difference between earlier and later, has a much more pregnant meaning in every interchange between the human being and his environment. The human being can by arbitrary acts influence the future, but not the past. Since we possess no causal description of human acts of will, a one-sided causal chain running into the future begins for us with every such act. Our acts of will occur only as causes and not as effects in our description of reality. Every experiment is such an act; precisely the freedom to choose experimental conditions at will, and to test the predicted effects, constitutes the value of the experiment for the causal analysis of nature. Thus the laws of physics can in general be reduced to the form: 'If the state A is realized, the state B occurs as its consequence.' The analysis of natural phenomena, not produced by us, likewise follows this formula: we regard the state occurring of itself in nature as cause from which we calculate the effect according to the conditional formula of the general law. The difference is only, that in an uninfluenced natural process we can place the beginning wher-

ever we like. That this is not, however, identical with full determinism, is clear in quantum mechanics, which cannot be interpreted at all without taking account of the conditional form of the laws of nature.

Thus the difference of cause and effect is meaningful in the description of nature, because we use the difference of past and future in natural science (even though this usually does not become explicit)—a difference which can perhaps be clothed in the words that the past is factual, but the future, on the other hand, is a field of possibilities. The actual presentation of natural appearances presupposes, never full determinism, but always the possibility of the variation of the conditions, either practically or at least in the pattern of our conceptual formulæ. What was mistaken was the presupposition still maintained in the ‘mechanical view of the world’, whether explicitly or implicitly, that we could grasp *a priori* what effect a given cause must have (the special ‘intelligibility’ of action through pressure and collision, through centripetal forces, through action in contact or the like). Only through experience do we learn to give content to the formula of cause and effect, as well as to understand the limits of its applicability.

Now in what sense can the law of the conservation of energy be said to satisfy this formula? In Mayer’s sense it bridges, as it were, the gap between the concepts of causality and substance. The German ‘*Ursache*’ means etymologically a determinate ‘thing’, the Latin ‘*causa*’ has been reified to French ‘*chose*’. In fact, it is important for the human being to know the thing, the object which he must have at hand in order to exercise certain effects. Energy is certainly such an ‘object’. Thus we could actually take the two laws of conservation of matter and energy as laws of conservation of things and causes.

But with this, on the one hand, causes of the trigger action type have dropped out. And on the other hand, the modern unification of the two laws of conservation has pushed the law of the conservation of energy still farther from the

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category of causality and nearer the category of substance. In so far as these old categories can be applied at all to the present state of research, the law of the conservation of energy will be viewed no longer as corresponding to the category of causality, but to that of substance. Yet its career shows that the very meaning of these categories itself shifts with every forward step of science.

RELATION TO THE ANALYSIS OF THE CAUSAL PROBLEM IN QUANTUM MECHANICS

The second question has actually been answered by the above remarks. Since the law of the conservation of energy has nothing to do with determinism, and has less to do with the general conception of causality than with substance, it is not affected by quantum mechanics, which revises only determinism. Indeed, we have already seen how, on the contrary, it was precisely through the empirical validity of the law of the conservation of energy that the decisive conceptual advance of quantum mechanics, its critique of the concept of thing, was necessitated. Let us here note the logical connection in a few words.

Quantum mechanics retains the principle of determinism in its hypothetical formulation. If a state A is known, the preceding or succeeding state B of the same entity can be calculated from it, and, what is more, according to the laws of classical physics. But it limits the possibility of the fulfilment of the condition introduced by 'if' (we can always know only one of two 'complementary' parts of the determination of a state), and thus removes from the principle part of its sphere of application. We lose herewith the conclusion drawn earlier from the principle of determinism that all events are determined in themselves, and our knowledge only has to implement the nature of that determination. On the other hand, every particular event remains determinable, if we furnish the experimental presuppositions for the implementation of the factors determining it; only the simultaneous

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determination of the corresponding 'complementary' event is thereby rendered impossible.

The object of the critique of quantum mechanics is not, therefore, the concept of the causal nexus, but that of the 'thing or event in itself'. Thus we should rather speak of a critique of the category of substance. Yet the validity of the law of the conservation of energy shows that this way of speaking too is at least ambiguous. We had better say: Quantum mechanics recognizes the relational character of the categories. Substance, causality, etc., designate not realities in themselves, but realities known by men. The limit of their application is thus reached at that point at which the conditions no longer exist which make possible the establishment of a knowable reality. Thus, as we have shown above, the validity of the law of the conservation of energy itself can be presupposed only when the experimental conditions allow, at least in principle, a measurement of energy.

TRIGGER ACTION

The relations of energy alone do not determine the course of events. Whether the weight of a clock falls and its potential energy is thus transformed into heat by way of the kinetic energy of the clockwork, depends on whether or not the pendulum is struck. The small quantity of energy of the striking of the pendulum acts as a trigger. Thus understood, every detailed analysis of a physical process is a study of trigger action and of 'directive' action.

Trigger actions in the specific sense are all the effects of the measuring apparatuses of modern physics. They are all intended, in a controllable way, to cause processes not immediately perceptible to our senses to produce sensuously perceptible effects. Consider the microscope, the telescope, the point discharge counter, the amplifier, etc.

In the theoretical sphere we may just point to one problem of trigger action which has the greatest significance: the physics and chemistry of the processes in the living

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organism. The growth of an organism from an egg and even more the study of gene mutations shows that the determining factors for the process of life are confined to the smallest space are, in fact, of atomic magnitude. >

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The application of the law of the conservation of energy begins with the establishment of work equivalents. We must be able to translate the amounts of energy of the different forms of energy into one another. In this process the physicist uses a fundamental measure of energy the erg, i.e., twice the kinetic energy of one gramme of matter which is moving with the velocity of one cm/sec. This is a very small unit of measurement. A shell of mass 10 kg. and velocity 1000 m/sec. has 5×10^{13} ergs energy. >

Gravitational energy is determined according to the equation, 1 mkg = 0.98×10^8 erg; i.e., with about 10^8 ergs we can lift 1 kg to a height of one metre. >

Expressed in mechanical measurement the energy of heat is very great. It is 1 calorie = 4.19×10^7 erg = 0.427 mkg; i.e., with the energy needed to warm a quantity of water by one degree, the same quantity of water could be lifted 427 metres. This is intelligible according to the kinetic theory of heat. Heat is a hidden, unordered motion of matter—in fact the molecules of air move at normal temperature about 400 m/sec.—i.e., most visible movements are slight, compared with heat motion.

Larger yet are chemical energies. The burning of two grammes of hydrogen with sixteen grammes of oxygen, a process in which eighteen grammes of water are formed, releases 68,000 calories. Speaking in terms of atomic theory this means that the potential energies of the atoms are large compared with their mean kinetic energies. This is understandable since a potential energy of atoms transferred into kinetic energy soon spreads as heat over a large number of atoms. >

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Until recently chemical energies were the largest we could arbitrarily evaluate by technical means. There are, however, two other energies in nature, which far surpass the chemical; they appear as great concentrations of energy in atomic physics and as great total amounts of energy in astrophysics.

The unit of energy in atomic physics is the electron-volt (eV). It is the energy which an electron gains when it traverses a tension of one volt. In the above mentioned hydrogen gas reaction, nearly three electron-volts are liberated for every water molecule formed. One eV is 1.6×10^{12} erg. This unit of energy is very small, since it expresses energy of a single atom. Since 18 grammes of water contain 6×10^{23} molecules of water, the total production of energy is, however, very great.)

All chemical reactions depend on transpositions in the outer part of the atom; with that process energies are set free ranging from the above mentioned value to small fractions of an eV per atom. Much larger are the changes in energy when there are transpositions in the atomic nucleus. They carry roughly 1 to 160 million individual eV per atom. These reactions are the source of energy of the atomic bomb.

Even greater concentrations of energy are found in cosmic rays. These rays consist of single, very rapidly moving particles, which, coming from a source as yet unknown, traverse space, and strike the earth uniformly from all sides. The average kinetic energy of such a particle is about 10 billion (10^{10}) eV, and there exist particles with 10^{14} , perhaps even 10^{16} eV energy.

The stars contain the largest known aggregates of energy. The kinetic energy of the earth in its course around the sun is 2.7×10^{40} erg. The sun radiates in a year 10^{41} erg. According to the evidence of geological data it has maintained this radiation for at least two billion years. There are stars which radiate one thousand times this amount per year.

Through the law of the conservation of energy the immense radiation of the sun has become a physical problem. For the law of the conservation of energy demands, that this

energy was already contained in the sun or steadily introduced into it; thus it raises the question of the mechanics of furnishing this energy. It was with this question that the physics of the sun really began. Mayer himself raised the question and answered it on the assumption that the radiated energy came from transformed gravitational potential energy of meteorites which are constantly colliding with the sun. It has turned out, however, that this energy is not sufficient. Through various other hypotheses we have finally arrived at the present view, theoretically and empirically well supported, that it is energy of the atomic nuclei which are undergoing transmutations in the central region of the sun. Thus the sun would be a great machine set up by nature for the use of nuclear energy. This source of energy is sufficient for more than ten times the minimum age that the sun must have, according to astronomical and geological evidence.

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Chapter Four

THE RELATION OF QUANTUM MECHANICS TO THE PHILOSOPHY OF KANT

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MODERN physics has raised philosophical questions which will scarcely find a complete answer in the framework of the known philosophical systems of the past and present. This essay hopes to serve as preparation for an appropriate answer, by examining some already known philosophical theses which have to do with the questions of physics. It has therefore not a historical but a preparatory-systematic purpose. The physics we shall consider has been limited to quantum mechanics only in order to draw the circle of questions not too broadly in advance; since quantum mechanics is at once the empirically most fully confirmed and the most radical of modern theories, it can also, at the present, teach us most about philosophical problems. The limitation of the philosophical doctrines to be considered, to the system of Kant, is not exclusive. But the inadequacy of the naïve realistic and positivistic views competing with Kant to-day automatically forces the manner of putting the question in the direction in which Kant had started. The answers which Kant gave to his basic questions appear in the light of modern physics neither as true nor false but as ambiguous. In trying to distinguish by means of our present knowledge between a correct and an incorrect interpretation of the Kantian theses, we acquire a principle for a critique of the Kantian philosophy and at the

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same time a point of departure for the further philosophical elaboration of modern physics.)

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It is necessary first of all to summarize briefly the assertions of quantum mechanics. What interests us primarily in this is the question whether quantum mechanics has, as has often been said, given up the perceptibility of the description of nature, and the causal principle. We shall see that both assertions are inexact, and that the decisive point of quantum mechanics consists in the fact that it gives up the 'objectifiability' of natural processes. Finally, we shall consider to what extent final validity can be attributed to the assertions of quantum mechanics from the point of view of physics.)

PERCEPTIBILITY*

The concept of perceptibility is equivocal. In modern physics it receives a limited but exact sense through the equation of 'perceptible' with 'classical'. In what sense and with what right are the two concepts equated?

By 'classical physics' we mean to-day Newtonian mechanics, Maxwell's electrodynamics, and all the disciplines which by any interpretation of their basic phenomena through the use of models can be reduced to mechanics and electrodynamics: such as acoustics (on the basis of the interpretation of sound as wave motion), the theory of heat (on the basis of the kinetic theory of heat), optics (on the basis of the electromagnetic theory of light). Now in what sense is this physics 'perceptible'? To this question we usually get the answer: it describes all physical phenomena as states of entities which occur in three dimensional Euclidean space, and as changes of these states in a

* See note p. 13.

one-dimensional objective time. Yet this is clearly a very abstract conception of 'perceptibility'. Think for example of what Goethe understood by natural science: the perception of the pure phenomena. What characterizes classical physics is precisely that it does not rest with the perception of the original phenomena of light, sound, and heat, but reduces them to imperceptible, only indirectly demonstrable motions. The world view of classical physics, for which these motions were ultimately the only realities, thus denied precisely the physical reality of what we perceive and left it to the unfinished sciences, the physiology of the sense organs and psychophysics, to explain after the fact how those phenomena of perception 'came about'. Classical physics pays this price to purchase the unity of its world view. Thus the perceptually given world is projected, as it were, upon a plane of pure concepts.

Nevertheless, it is precisely the perceptually given world which is pictured in this projection. To be sure, by no means all the propositions of Euclidean geometry are obvious or immediately comprehensible to the imagination of the normal person. But when we wanted to translate our perception of the space in which we discover external objects, into the language of an axiomatic geometry, we seemed to be led unambiguously to Euclidean geometry. To be sure, the immediate experience of light does not contain anything of wave events; but when we wanted to present the phenomena of the propagation of light—down to the subtleties indicated by the words interference and diffraction—by means of coherent concepts, we were forced to speak of a field periodically modulated with respect to space and time. Thus the propositions of classical physics mirror the structure of the world given by perception; the concepts of classical physics constitute the precision-language in which the physicist indicates how an apparatus ought to be built and communicates what he has seen. Δ

In this way classical physics has a peculiar middle position between the world of sensations and the world of things.

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For a theory of sensation and perception it indicates the objective things which produce sensation and appear in perception. For example, it permits us, in the comparison of stimulus and reaction, to define the stimulus objectively. Conversely, in the investigation of the objects of the external world, which to-day more and more runs its course beyond the range of the immediately perceptible, it furnishes us with the language in which every message that comes to us from those objects outside direct perception, must be expressed. Since all that is perceived is described through classical physics, classical physics is in turn the representative of perception in physical research.

At this point we may learn to distinguish between two meanings of classical physics: as a world view, and as a methodological instrument. Statements based on immediate perception and extrapolations to what is non-perceived or to what is even, in many cases, for practical purposes imperceptible, are inseparably interwoven in physics. The world view of classical physics assumes, that all that is not perceived must have, at least basically, the same properties as the objects of perception. This is an hypothesis and, as contemporary physics teaches, to a great extent a false hypothesis. Classical physics as a methodological instrument is untouched by this criticism. For the empirical confirmation of the propositions of physics always demands that, at least indirectly, a connection be established between object and perception (e.g., through devices for enlarging and amplifying, the best known example of which is the microscope). In any event, however, the perceived event in nature or in the measuring apparatus is described by means of the concepts of classical physics. From this situation there follows, in fact, a condition for the correctness of every new theory: it must, in the limiting case in which it is applied to objects of our daily experience, pass over into classical physics.

In what respect does quantum mechanics differ from classical physics? It has discovered that the same physical object appears in two different seemingly mutually exclusive

forms: particle and field (or wave). These two forms, in which all atomic objects appear in experiments, are not selected from a larger number of possibilities of equal value; rather they form a complete disjunction. A particle is a physical entity which, if it occurs at one place, cannot occur simultaneously at a place distant from that one; a field is an entity spread through space. The particle-nature of an entity follows from all the experiments which demonstrate a localization of its effects (e.g., cloud chamber photographs); the field nature of an entity follows from all the experiments which demonstrate an interaction of several places at a distance from one another (interference). Now what meaning can there be in the statement that an electron is both particle and field? To be sure, experience shows us localization as well as interference effects of the electrons. But how is the contradiction apparently contained in this to be avoided? The contradiction does not subsist between the actually observed properties of the electron; it arises only if we assume that these properties would belong to the electron even if we gave up observing it. Let us elucidate this by an example.

If I have just discovered an electron at a given place, then the particle picture says: there is a particle at this place; the wave picture: here is a 'wave parcel' concentrated in the smallest space. Both descriptions are correct, and they do not contradict one another. But after some time the particle must have moved off in some direction or other; the wave parcel, on the other hand, must have spread itself out in all directions. The contradiction apparently contained in this is avoided by the observation that I do not know the momentum (i.e., direction and quantity of motion) of the electron; for the measurement of place presupposes the interaction of the electron with a stationary measuring apparatus, which can by its impact turn the particle aside in any direction. Thus according to the particle picture I can only predict that upon repeating the measurement of place I shall again find the particle somewhere in the vicinity of the first place;

and it is the basic statistical assumption of quantum mechanics, that the probability of discovering the electron at a determinate place is given by the intensity of the corresponding wave at the same place. The contradiction contained in the fact that the particle can traverse only one path and cannot therefore, as the wave picture demands, reach every point of the environment, is avoided through the consideration that the momentum from which the path of the particle could be predicted is unknown and that in consequence I do not know *what* path the particle will traverse. Conversely, wave picture and particle picture can be united only if it is assumed that the momentum of a particle at a given place cannot be known. If we wished to determine the momentum through a second measurement, as we might do without further qualification in classical physics, then, in consequence of the interaction of the electron with the new apparatus, its place—as we can, again, deduce from the simultaneous validity of the wave picture—would be indeterminate to just such an extent, as would make the calculation of the path which would contradict the wave picture impossible because of ignorance of place.

What follows from these laws for the question of the perceptibility in quantum mechanics? The whole of the above discussion is carried through within the framework of perception. We have not assumed that, where experience demands them, particle picture and wave picture are merely approximate presentations. The thesis of quantum mechanics is, rather, the persistence of classical laws; it asserts: when any classically defined quantity is known through measurement, then all the consequences which can be drawn from this knowledge according to classical physics, are exactly correct. That is to say, either picture, where it is demanded by experience, can be applied in all strictness; only we may not assume that the elements not given by the particular experiment in hand, have determinate values. Thus we have not given up the perceptibility of our description of nature, but only the ascription to the perceptible constituents of

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nature of an 'objective' meaning independent of any and every observational context.

This becomes especially clear in the general formalism of quantum mechanics. It describes our knowledge of an object through the specification of an abstract ' ψ -function'. The connection of this function with experience consists in the fact that from it the probability for every possible outcome of every possible experiment can be predicted. And all 'possible experiments' means here only measurement of *classically* defined quantities. Thus in so far as we may equate 'classical' and 'perceptible', quantum mechanics is a fully perceptible, but not objectifiable description of nature.

CAUSALITY

Quantum mechanics makes certain predictions only with probability, for example the prediction of the momentum of an electron to be measured at a known place. Does this mean an abandonment of the principle of causality?

If we call classical physics causal, then in doing so we limit the concept of causality just as above we limited that of perceptibility. Causality in classical physics means nothing more than the existence of a univocal functional relation between states at different times: 'If the state of a closed system at an instant is completely known, then the state of the system in every earlier and later instant can be completely calculated.' Now in quantum mechanics this conditional law becomes not false but inapplicable, because the premise is never realized: The state of a system cannot be fully determined in the classical sense since the acquisition of one item of knowledge (e.g., of the place of a particle) excludes the knowledge 'complementary' to it (e.g., of momentum). However, the conditional law itself actually belonged only to the classical 'world view', and not to its practice in which no state was ever fully known. On the contrary, in practice only so much could be calculated as followed from whatever constituents of the system happened to be known at the

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time: that is, only those consequences could be drawn on which the constituents which happened to be unknown could have no influence. But this still holds in quantum mechanics. In quantum mechanics that conditional law which we designated above as the thesis of the persistence of the classical laws is valid: that is, the proposition which asserts as applied to the present problem: 'If certain constituents of the state of a system are known, then it is possible to calculate all those constituents of earlier or later states which stand according to classical physics in a univocal relation to the known constituents.' This is, however, precisely the causal principle of classical physics. The difference between quantum mechanics and classical physics, therefore, is not to be found in the causal principle, but only in the limits to which the objective determination of a state can be pushed.

The persistence of the classical principle of causality is no accident. For an apparatus is suited to experimentation only in so far as the causal principle holds in it. Only when, in the microscope for example, a chain of univocal relationships leads from the object to the picture, can we make an inference from the position and form of the picture to the position and form of the object. Thus the act of taking cognizance of nature experimentally is tied to the applicability of the causal principle to the instruments of measurement. We can in fact show, by means of examples, that an apparatus ceases to be suitable for measurement when quantum-mechanical indeterminacy makes the inference from the observed process to the process being investigated a statistical one.

OBJECTIFIABILITY

Every observation presupposes a causal chain and affords a perceptible result. There is only one thing we can no longer do: join together single fragmentary perceptions and causal chains in a model of self-existent nature. On the con-

trary: our freely chosen ordering of experiments determines which of the mutually 'complementary' sides of nature comes into our view, and the knowledge of one set of circumstances excludes the knowledge of the set of circumstances complementary to it.

We are now face to face with the cardinal question of quantum mechanics: Is this exclusion a matter of difficulties in our way of knowing? or is it a matter of the concept of objective nature itself? May we presuppose, that the constituents still unknown to us at a given time exist in themselves and are only 'hidden', or may we not presuppose this? The present form of quantum mechanics has decided for the second answer. It denies the existence of hidden parameters; not indeed, when our ignorance rests simply on our decision not to seek some knowledge which is possible from the point of view of quantum mechanics, but when the unknown quantity cannot be known because we have too exact a knowledge of the quantity complementary to it. This is no empty assertion, but a law with definite logical consequences: Suppose for example an unknown observable X has two possible values x and y , i.e. in a measurement of X only these two results would be possible. Further let there be an assertion A , which is false equally when the measurement of X has produced the value x or when it has produced the value y . Then from the assumption that the unknown values exist in themselves we could conclude: ' X has certainly either the value x or the value y ; in both cases A is false; therefore A is certainly false.' However, according to quantum mechanics this inference is false, for A can, for example, be the assertion: the magnitude Z complementary to X has the determinate value z (that is in a measurement of Z we shall certainly find the value z). According to quantum mechanics this assertion is false whenever X has any determinate value, but is true when we have refrained from measuring X and have measured Z and got the value z .

Our example has led us to a fundamental logical property of quantum mechanics. The knowledge that we have of

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nature enters explicitly into it. Thus, in our example, if X is known, the proposition 'Z certainly has the value z ', or, to put it differently, 'I know that Z has the value z ' is false, and on the other hand the mere statement 'Z has the value z ' is neither true nor false, but undetermined, for in a measurement of Z the value z could in fact result. It is the definitive difference of quantum mechanics from classical physics, that it cannot even enunciate its propositions without at the same time expressing the way in which they are known.

The establishment of this fact is just as natural with respect to the practice of classical physics, as it is revolutionary for the world view not only of classical physics, but of most philosophical systems. A complete experimental assertion as it stands in the experimental protocol, sounds in schematic form as follows: 'I have observed this state in this object of experiment under these experimental conditions.' The hypothesis of classical physics states that this proposition can always be replaced by the proposition: 'This state exists in this object of experimentation,' and that propositions of the latter type must necessarily be either true or false, regardless of whether there is a person who knows if they are true or false. This hypothesis was constantly confirmed in the older physics. Over and above that, it corresponds to a basic motif of nearly all science and philosophy: the belief in an objective presence of the objects of our knowledge. Of course we cannot deny that every empirical circumstance is a circumstance known by man. But we do not want to draw from this proposition any consequences touching on the structure of our knowledge itself. The quarrel among the philosophers was at most concerned with the question whether the proposition was significant for our general concept of reality or not. That is, it was assumed, that without changing the conceptual structure of science we could talk about its objects, of which we have knowledge, without explicitly taking account of the fact that we know them; and what was discussed was only what meaning there was in the consequence drawn from this by naïve science, that

objects 'really' exist independently of our knowledge. Quantum mechanics, on the other hand, denies even the premise of this discussion. We shall now try in terms of formal logic and ontology to characterize more exactly the new basis of quantum mechanics.

From the point of view of formal logic, quantum mechanics uses a many-valued truth concept, in which a statement can have along with the values 'true' and 'false' the value 'undetermined, and moreover with such and such a probability of being true.' Let A be a statement about a given concrete state of affairs. Then the full experimental statement has the form, 'I know that A holds.' In place of this, classical physics uses only 'objective' propositions of the form 'A holds.' The full experimental statement permits two sorts of negation: the negation of the objective proposition: 'I know that A does not hold,' and the negation of the knowledge: 'I do not know if A holds.' Of these two negations only the negation of the objective proposition is a real statement about nature for classical physics. The negation of the knowledge, on the other hand, can, according to classical physics, be transformed into the proposition: 'A holds or does not hold; and I do not know, which of the two is so.' For quantum mechanics, on the other hand, the objective proposition 'A holds' can in general be meaningfully stated only when the full experimental statement 'I know that A holds' is true. Moreover, for quantum mechanics the full experimental statement and its two negations are three equally possible statements about nature. The classical transformation of the negation of knowledge is false; in its place as consequence of the negation of knowledge the proposition holds: 'Neither A nor non-A holds.'

This means, ontologically, that the concept of object can no longer be employed without consideration of the subject having the knowledge. To be sure, the empirical subject with his emotions and his personal fate is not introduced into physics, but just two basic functions of consciousness enter into every proposition in the description of nature:

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knowledge and volition. This becomes clearest perhaps in the proposition that the ψ -function specifies the probability for every possible outcome of every possible experiment. The first 'possible' expresses non-knowledge; an event is possible, of which I neither know that it will happen nor know that it will not happen. The second 'possible' on the other hand expresses a power of volition, a capacity; an experiment is possible, which I can either carry out or let alone. It is essential to quantum mechanical 'non-knowledge' that I can at any time know the unknown if I but want to. The only condition imposed by the complementarity of the different ways of putting the question, is, that if I want to know a particular thing, I give up my claim to the knowledge complementary to it. Thus the boundary between the known and the unknown is nothing 'objective', but I can place it arbitrarily where I like; only I cannot reduce it to the vanishing point.

Let us add one explanation. The indeterminacy of measurable quantities has sometimes been correlated with the disturbance of the object by the act of observation. This way of putting it is misleading. For it creates the impression that the object has, before it has been observed, certain properties which are destroyed only by the act of observation. So interpreted however this would mean a retrogression into the way of thinking that preceded quantum mechanics. More correctly we should say: a presupposition for the possibility of ascribing a given property to an object is an arrangement for making measurements which allows us to ascertain this property. If now, through the application of a new measuring device, I switch to the measurement of the quantity complementary to the one previously measured, then the conditions no longer exist under which the previously measured quantity could have any determinate value at all. It is the physical interference with events which is necessary to substitute the new experimental conditions for the old ones that is called, in the aforementioned mode of expres-

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sion, the 'disturbance of the object through observation'. Thus this expression has a clear meaning, if it is not the fictional 'undisturbed' object, but an object already known through observation that is subjected to an observation of a new kind.

THE QUESTION OF FINAL VALIDITY

Now is this change in the concept of object necessary and finally valid? This question must be answered first of all by another question: what can be understood, in general, by the 'final validity' of a physical theory? Strict demonstrations can be constructed within the conceptual system of a complete theory. But that a system is exactly adapted to experience can never be shown with mathematical precision. Even if it could represent correctly all known data of experience, the possibility of future experience contradictory to it would be open. Physics has, however, fashioned for itself a limited concept of final validity in the concept of the sphere of validity of a theory. A theory is said to be correct, not if it embraces all experience that is merely conceivable, but if there is any reproduceable group of phenomena which are correctly represented by it. If a second theory represents a more comprehensive group of phenomena, then it is required that, for the phenomena represented by the first theory, the second theory should fit into the first as a 'limiting case'. Thus classical physics has a sphere of validity, quantum mechanics another, broader one; each is—and this is now really a tautology—final for its sphere of validity.

The question we put at the start can now mean only this: could there not some day be a more comprehensive theory than quantum mechanics, which would enable us after all to answer the question of the objective values of undetermined magnitudes rejected by quantum mechanics? This possibility cannot, in strict logic, be excluded. We can impose on the new theory only one limiting condition: it can be

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shown that this theory can perform the task demanded of it only if it ceases to maintain the persistence of the classical laws. In contrast to quantum mechanics, therefore, it would have to introduce another description of what we perceive immediately instead of that of classical physics—and that in so incisive a fashion, that the distinction between particle and field which we discussed above would be wiped out: the behaviour of a localized particle would have to depend on the state of the whole space in a fashion which would be viewed, classically, as proof of the presence of a field. The difficulty even of imagining something of this sort, and the failure of all attempts undertaken till now in this direction is, for the physicist, perhaps the strongest argument not to expect a further revision of just these traits of quantum mechanics.

Since, however, we are concerned with a question that has not been decided on a strict physical basis, it is necessary to hear the opposing arguments. As advanced to-day these are consciously or unconsciously of a philosophical character. They rest either on a metaphysically conditioned belief in the existence of an absolutely objective external world or on the conviction—based above all on Kant—that the conceptual tools of classical physics are the presuppositions of all possible experience and therefore cannot be criticized on the ground of this experience. Thus as with every great scientific advance, we see ourselves summoned from scientific discussion to philosophizing.

We shall not evade this summons. In the trial now to be undergone, physics no longer functions as judge, but as witness. Even before the question of its conclusiveness has been decided, the existence of quantum mechanics as a conceptually closed system proves to us at any rate the logical possibility of a theory which contradicts the above mentioned philosophical demands. In so doing it puts into our hands the means of discovering gaps in the arguments for the objections, and of showing that the objections are in any event not counterproofs but statements of faith. Finally,

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it incites us to make clear the background of the faith from which the objections stem.

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STATEMENT OF THE PROBLEM

If we now dare to enter philosophical territory, we take warning from many a misadventure of purposeless philosophical debate, and seek for a leading thread, a methodological principle. We take physics as our model and try, as in physics, to stick to the phenomena. We take pains not to accept general laws without due consideration, but to ask: What do we already know, what views follow from this knowledge and what other views transcend it? Experience must teach us whether more is to be attained. In most cases, philosophy cannot absolve the thinking person of the choice of his own point of view; it can only show him what he is doing when he resolves to take this or that point of view.

The object of our investigation is what the physicist calls 'experience'. Physics has a fixed concept of experience and limits itself to examining whether the particular assertion it wants to make really is experience. We wish to investigate wherein this experience really consists and whence it derives its certainty.

We shall choose an example of an elementary proposition drawn from experience: 'Here is a chair.' If we want to designate exactly the source of our knowledge we say, 'I see a chair.' In this proposition we can distinguish at least three sorts of things: the chair, the seeing, the proposition. That is, there are things, there are sensations, there are judgments. But the three are given us as simultaneous and belonging together. Only through sensation do I know of things, and it is only of things that I know through sensation. Moreover my knowledge of things and sensations is at least complete only in the form of judgments, and the judgments with

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which we shall be concerned here are related only to things and sensations.

Now it is probably a drive towards simplification which leads people over and over to separate the three elements. In the following discussion we deal above all with the attempts to see the really 'real' either in the thing or in the sensation. The first attempt we call realism, the second sensualism.

THE REALISM OF CLASSICAL PHYSICS

Physics rests on the possibility of making judgments which go beyond what is given in immediate sensation. Without this fact not even the language we use to communicate a given experience would have meaning. What sensations are designated for example by the expression 'this chair'? There is first of all a series of actual sensations (form, colour of parts, etc.), a small part of which is actually present to my consciousness *as* these sensations, and a greater part of which can be made present to it if I turn my attention to them. Never yet, however, have I been able to call into my consciousness the totality of all the sensations which I have even from just this one chair. In addition there is a series of merely possible sensations, which I do not have for the moment, but which I could produce by free will (the appearance of the back of the chair, bodily feeling when sitting on the chair, exertion when carrying the chair, etc.). Only the confirmed expectation that those possible sensations occur if I do what is required, gives the concept 'chair' the meaning it actually has. In the same way, the expectation that things will remain identical with themselves in the future as in the past transcends what is directly perceived, and so, in general, does the expectation of the reproducibility of the data of experience, which forms the foundation for the construction of natural laws. I may even go so far as to speak scientifically about things that I have never seen (for example an experimental arrangement set up in a foreign laboratory, or a continent I have not yet visited), which no

one has seen (the other side of the moon), which as far as we can tell no one ever will see (the inner parts of the earth).

All these facts are embraced by our everyday consciousness—and by classical physics as well—in the conviction that there are real things which exist, regardless of whether they are perceived or not. In this sense we are all, practically speaking, realists. Even ourselves we know in no other way than as living in a real world. Let us now try, however, to distinguish what in this realism is simple description of the given immediately present to our consciousness, and what in it is theory. The state of affairs we have just pictured is itself responsible for the fact that this boundary is not at all easy to draw. We cannot, for example, give a general criterion for determining in each particular case where this boundary lies, but we can only distinguish two basically different forms of the realistic conviction. These two forms we shall call practical and radical realism.

The distinction proceeds from the fact that all 'things' of which we can speak are basically related to our experience. This is true even of our everyday judgments and becomes clearest where the knowledge of the external world is sought with methodical consciousness in natural science. To be sure, physics does not in fact test every single judgment which transcends experience, but it subjects every judgment of this kind, in principle, to the control of experience: a single indubitable experience to the contrary is enough to exclude it from physics. True, physics does not speak only of things that have entered experience, but it does speak only of things that *can* enter it, directly or indirectly. Now we shall designate as objectifiability this property of our common statements about things: that the content of such statements does not depend on the conditions under which the experience verifying them is obtained. Practical realism, then, declares only that there are objectifiable statements about the external world, and that these fill the sphere of our normal experience. Radical realism on the other hand, says that there are no non-objectifiable statements.

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Only quantum mechanics has made it possible to make this a clear-cut distinction. For it was only the example of quantum mechanics that called our attention to the fact that radical realism is not a logical consequence of practical realism. Practical realism is the attitude of classical as well as modern experimental physics and at the same time the theoretical attitude of quantum mechanics. Radical realism corresponds to the classical 'world picture'. From the empirical point of view there is to-day no longer any reason to adhere to radical realism. The wish to do so, which is nevertheless very widespread, arises from a third position, which we may call metaphysical realism.

Metaphysical realism is distinguished from the theses discussed so far by the fact that it is hard to state it in such a way that it can be tested. It says roughly: 'Things exist in reality.' The only question is what such a proposition may mean. In practical realism it is a pure tautology, for 'things' are precisely what is given by experience and it is therefore not clear what should be called existent if not things. But metaphysical realism usually declares itself dissatisfied with this interpretation. It is persuaded that the world 'in itself', independently of our experience, is a world of things, and infers from this the thesis of radical realism—that is, the independence of *every* true statement about things from the type of experience involved.

◀ It is important to recognize, that metaphysical realism is not a scientific thesis, but a form of world view. It gives its adherent the comforting assurance, that he knows at a decisive point what the world is like. Those who give it up must feel at first like 'Columbus, who had the courage to forsake all the land known till then in the almost mad hope of finding land again beyond the seas'. (Heisenberg.) This is one of the human decisions which cannot be forced by philosophical thought alone. Yet thought can prepare for it. In our case thought can show that the very meaning of the words in which metaphysical realism states its thesis is not clear. This has happened in the philosophy of empiricism.

Let us turn to it and come back once more at the end of our essay, to the peculiar character of personal decision which philosophy has here assumed.

SENSUALISM AND POSITIVISM

Empiricism proceeds from the conviction that we know everything we know through the senses (Locke). We shall set aside the criticism of this view and follow out its consequences. Its first logical extension is the doctrine of Berkeley, that we know nothing at all about a real world. We do not need to go into Berkeley's particular arguments; it is sufficient to point out that every sense impression might be deception, illusion or hallucination. Granted, that it does not follow from this train of thought that there can be no real world corresponding to sense impressions, but only that there need be no real world corresponding to them; yet precisely this is sufficient to invalidate every inference from experience to the real world. The next step, again logically correct, is represented by the doctrine of Hume and Mach, which, as radical sensualism, can be placed opposite radical realism as analogous to it. It argues: Metaphysical realism, which asserts that there is a real world underlying our sensations, and solipsism, which asserts that there is no real world underlying our sensations, are both basically neither demonstrable nor refutable, since they do not allow experience, the only instrument of proof we have, to have validity as such. Thus they are not scientific propositions at all, but only a misuse of words. It is a misuse of words to apply the concept 'real' to a world that lies beyond every possible experience and thus—and only thus—to give to sense perception the pejorative character of 'mere appearance'. If only sensations are given us, then only they and, in a derived sense, 'bundles of sensations' can really be called 'things' or 'natural laws'; everything which makes a claim to reality must be reducible to sensations.

As a critique of metaphysical realism this train of thought

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is one of the finest applications of the methodological principle to which we too wish to subscribe: that we must hold to what is given. If the premise that only sensations are given is correct, then the conclusion drawn from it is also unobjectionable. If the premise is incorrect, then this type of argumentation is still a warning against the overhasty expectation that concepts like 'real' must under all circumstances have a precise meaning. The argument has accomplished enough if it warns us not to employ such concepts without considering what phenomena might possibly correspond to them. Without the feeling for the extent and limits of a concept which only practice in this sort of reflection provides, no one can hope to penetrate into deeper philosophical questions.

As a positive assertion, however, radical sensualism falls into the same error as radical realism. The very criticism which it applies to the concept of 'thing', it fails to apply to the concept of sensation. 'Sensations in themselves' are by no means given to our consciousness. On the contrary, our immediate perception is of 'things', in which only a new concentration of attention discovers particular sensory elements. We do not see spots of colour, but trees, people, even joy in a man's face—which is not a thing, still less a sensation. It is a mere hypothesis—and presumably a false one—that everything given to our consciousness in connection with things can be analysed into elementary sensations. The answer, that we had ourselves acquired the concepts of things as children, in order to tie together sensations which were presented to us at first as disparate, does not meet the methodological sense of our objection. For we do know at least in part our adult consciousness, but the consciousness we had as children in that first period, we do not know at all. To be sure, it is very important to reflect that our consciousness was not always there, but has developed; yet it is not its unknown origin, but only its present condition, that can possibly serve methodologically as a starting point for the whole theory of knowledge. Probably the idea that

experience can be analysed into elementary sensations, is no more than a hypothesis which we read into the development of the child. And this hypothesis itself is at bottom a relic of realism, for our assumption of distinct sensations is to a great extent determined by our knowledge of the sense organs. It is the task of research in the physiology and psychology of sensation, to delimit the 'practical sensualism', which is justified, from the radical variety—just as quantum mechanics has done in the case of realism.

Modern 'positivism' has recognized the error of radical sensualism. But it has not put any positive new position in place of the one rejected: it has simply formulated the critique presented above in conformity with the new view (and has otherwise applied it for the most part in an uncritical fashion). In doing so it assumes a logical-linguistic point of view. That is, it forbids the enunciation of 'meaningless sentences' in science. It supports itself by an argument more or less as follows: Science rests on the fact, that symbols (words, mathematical signs) are correlated with phenomena (the given). It consists in the attempt to establish relations between these symbols which mirror, structurally, the relations existing between the phenomena. There are rules of relation corresponding to the meaning of the symbols. Relations which contradict the rules are not false but meaningless. Such are, for example, grammatically impossible sentences or mathematically impossible assertions like: 'The third focus of this ellipse lies here or here.' Often the meaninglessness of a sentence or a question is a great discovery. For example: "Of what algebraic equation is π the root?" or: 'Where is this electron of given momentum located?' The latter question is meaningful in classical physics, meaningless in quantum mechanics. \}

There is no objection to this train of thought, if it is explained under what conditions we can have any knowledge of the meaninglessness of a sentence. Strictly it is necessary for this purpose, that the sentence belongs to a

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closed system of mathematical propositions in which it can be unambiguously decided what expressions can be demonstrated and even what ones can be formed. It is noteworthy, that this can happen not only in pure mathematics, but also in physics. The condition for it is the existence of a consistent theory of the phenomena in question. Thus before the establishment of such a theory we can at most conjecture the meaninglessness of a statement, but not demonstrate it. Further, because of the openness of experience there is always the reservation that the theory may be revised in the future. That we ever do hazard, in the face of this reservation, the positive conjecture that certain questions are definitively meaningless, rests only on the fact that this conjecture makes room, logically, for the introduction of new affirmative statements: so for example the conjecture of the meaninglessness of the simultaneous determination, to any arbitrary degree of precision, of place and momentum, makes room for the possibility of unifying the wave picture and the particle picture.

If positivism limits its assertions to these cases, then it is correct, but says no more than science already knows. It is then in a sense the null class among philosophical systems with the merit of the most radical self-criticism. It has, further, the merit of having made known some important matters of logic. In fact, however, the representatives of positivism have usually disregarded the narrow limits within which the concept 'meaningless' has a meaning, and have poured out their criticism without distinction, like sun and rain, over bad and good. Nevertheless it would have been possible at least to state the more profound philosophical questions in the language of positivism.

THE PROBLEM OF THE A PRIORI

If we describe knowledge as the correlation of symbols with phenomena, then pure logic is concerned with the rules for relating symbols to one another, while philosophy must

ask, besides, how the correlation of the symbols to phenomena actually takes place. We have observed that, strictly speaking, every word transcends the phenomena. On the other hand we constantly use language to characterize, communicate and distinguish the phenomena. But this means that actually we are not in possession of the phenomena at all. The most indubitably given thing for a philosopher who wants to hold to the phenomena, is language itself. It always takes a special effort to examine the question whether a word or a sentence reproduces the phenomena; and the result of this investigation can always be expressed only in sentences, which use other words without investigating them. To the positivistic confession of faith: 'Only the given is given' we could reply paradoxically: 'The given is not given at all,' or rather more accurately: 'Nothing is definitively or absolutely given.' Our work in knowing is like disentangling a ball of yarn, the ends of which we do not have in our hands, so that we must proceed in both directions from a couple of threads which we have got loosened at the centre.

In questioning first things, then sensations, and finally the concept of the given itself, we have certainly 'left all known land' and find ourselves on the high seas. Are there at least a couple of stars, by which we can find our bearings?

In the last decades, physics, in its special problems, has found itself, in a number of ways, in a similar position. In this situation it has several times proved useful to reverse the approach of the inquiry, and to choose as the starting point of further construction precisely that state of affairs which was the source of all the difficulties. Can we describe accurately the state of affairs which has here led us into difficulties?

We were asking about the nature and source of experience. In the course of this inquiry we ascertained that even the simplest judgment of experience involves assertions, the content of which we neither have experienced nor, in its completeness, ever can experience. The realm of the empirical is not possible without non-empirical elements. The attempt to characterize experience univocally and com-

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pletely through its object (things) or its medium (sensation, the given) has foundered. How would it be if we looked at the non-empirical elements in experience, put their existence at the head of our investigation, and inquired into their efficacy?

Non-empirical is another word for *a priori*. Herewith we enter the circle of problems of Kant's philosophy.

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KANT'S THESES

We may consider here one pillar, so to speak, of the great structure of the Kantian philosophy, in fact the central one: his doctrine of the presuppositions and the object of empirical knowledge. It is suggested by the two pairs of contraries: *a priori*—*a posteriori*, and appearance—thing in itself.

According to Kant all our knowledge does indeed begin *with* experience, but it does not all arise *from* experience. Rather there is *a priori* knowledge, that is, knowledge which does not receive its validation from experience. How can we demonstrate that there actually is such knowledge, and how can we understand that it can exist?

According to Kant all that knowledge is *a priori*, which possesses necessity and universality. Experience does indeed teach us, that something has such and such a character, but not, that it could not be otherwise. If therefore, in the first place, we find a proposition which is conceived simultaneously with the conception of its necessity, then it is an *a priori* judgment. Secondly: experience never gives its judgments true or strict universality, but only acquired and comparative universality (through induction), so that we should really say: As far as we have perceived till now, this or that rule finds no exception. If therefore a judgment is conceived in strict universality, that is, in such a way that no exception whatsoever is allowed to be possible, then it is not derived from experience, but is valid absolutely *a priori*.

Under the compulsion of this argument, the empiricists who opposed Kant found themselves forced to deny the necessity and universality of the a priori knowledge postulated by Kant. We do not want to lose our way in these debates, but to examine at once the reason for this denial. This is: the empiricists' basic belief, that there *could* not be any a priori knowledge, since experience is our only source of knowledge. Once we have obviated this objection, we can always go on to examine the particular items of supposedly a priori knowledge.

That there *can* be a priori knowledge, we grasp, according to Kant, from the fact that there must be such knowledge, if there is to be any knowledge whatsoever from experience. A priori knowledge is, according to Kant, the condition of the possibility of empirical knowledge. It is the element in knowledge which proceeds not from things, but from our own power of knowing. Thus what we normally accept uncritically as empirical knowledge or empirical science, Kant makes the object of investigation—and in so doing he in fact undermines the ground on which the empiricists' counter argument rests. So he enters on the very road which we found it necessary to take earlier on the basis of our analysis of judgments of experience.

We cannot say, however, that he follows this road to its end. For if we ask how he demonstrates the necessity of a priori knowledge for the production of experience, we are brought back to the other train of thought, which simply asserts the existence of a priori knowledge. Kant examines the question, which of the intuitions, concepts and judgments, which constitute our knowledge, are a priori according to his criterion of necessity and universality, and only then does he show, that without this knowledge the rest of our knowledge could not exist. He does not show concretely, how the foundation of empirical on a priori knowledge actually comes about (the problem is only touched on in the section on the 'Schematism of the Concepts of the Understanding', in the *Critique of Pure Reason*), and thus at

the same time he fails to secure his a priori knowledge in its particular details against the empiricists' objection. Indeed he is quite incapable of performing these two tasks, since he uses both the concept of thing and that of sensation more naively than we now dare to do.

But what knowledge is a priori for Kant? A priori, first of all, are the analytic judgments, those whose predicates express nothing but what is already comprehended in the concept of the subject. We must leave aside at the present the various problems involved here, on which modern logic has something to say. A priori, further, are the forms of pure intuition: space and time; the categories of the understanding, among which we meet those concepts of importance for physics: substance and causality; and finally numerous synthetic judgments, especially the axioms of arithmetic and (Euclidean) geometry. It is important to consider, which judgments of physics are a priori. On this question we find a development in Kant's views. In the *Critique of Pure Reason* some principles, e.g., that of the conservation of matter, of inertia, of the equivalence of action and reaction, are said to belong to a 'pure', (i.e., non-empirical) physics. In the *Metaphysical Beginnings of Natural Science* even the law of gravitation is introduced a priori. And in the *Opus Postumum* this tendency goes so far, that we may wonder whether—in our present way of putting it—the entire body of principles of classical physics must not be valid a priori. To be sure, we must see in the uncertainty expressed by this development, the consequence of the above mentioned obscurity about the relation between a priori and the empirical knowledge.

⌞The idea of a priori knowledge has far reaching consequences for the doctrine of the object of knowledge.⌋ Kant proceeds from a purely realistic mode of thought.⌞He talks of things or objects which affect the mind by means of its receptive capacity, called sensibility, and produce sensations.⌋ But that knowledge which we possess a priori is by definition

not a consequence of this operation of things on our minds, even if we become aware of it only in this connection. Since then this knowledge does not come from things, we cannot deduce anything about things from it. Thus for example space is a necessary form of our external perception, for we cannot imagine objects any other way than in space. But just because we know this a priori we cannot have gathered it through experience from the objects and thus we are not justified in attributing spatiality to the objects as they are in themselves. What then do we know in a priori knowledge? It is the presupposition for the possibility of experience, and thus it is knowledge of the necessary properties of things in so far as we have experience of them—of things as appearance. Since, however, all our knowledge rests on a priori knowledge, we know nothing at all about things in themselves, but all our knowledge is concerned with appearance.

However, appearance must not be confused with 'mere appearance' or illusion. True, things cannot be given to us at all except in so far as they 'appear' to us, and every concept about which meaningful positive assertions can be made, even the concept of illusion, can be applied only within the sphere of appearance. If I believe that I see Mr. Brown coming towards me on the street and discover as I come closer that it is not Mr. Brown, then my first impression was mere illusion; but I have exposed the illusion, precisely by examining more carefully the 'appearance' of the person approaching. Kant expressly explains in the section of the *Critique of Pure Reason* on 'phenomena and noumena', that only a negative use may be made of the concept of the 'thing in itself', in order to indicate that about which no positive statement whatsoever can be meaningfully made.

The affinity of these thoughts with the critique of the concept of reality introduced above, is easily recognized. Kant has therefore been accused of inconsistency because he continued to make any use at all of the concept of the thing in itself and did not entirely exclude it from philosophy as meaningless. In fact, even the initial assertion that things

'affect' our sensibility, and are therefore the cause of our sensations, must come under the ban on positive assertions about the thing in itself; for the category of causality which is here employed also has its legitimate sphere of validity only within appearance. In that case we must interpret Kant in such a way that it would be only the appearances of things which affect us; yet by this the whole argument that we know nothing a priori about things would be false, since appearance originates only by means of a priori knowledge, and we can therefore know something about it a priori.

Kant himself decided otherwise. Many passages in the *Critique of Pure Reason* lose their meaning if we strike out the thing in itself. The 'refutation of idealism' of the second edition demonstrates, that Kant did not even consider the possibility of rejecting the question 'whether there are things outside us' as meaningless in this form. Doubtless his system has in this respect a logical inconsistency at a decisive place. But we believe that, precisely through this flaw, facts have found their way into Kant's system which, because its premises were too narrow, would otherwise have been excluded. This very conjecture would make a new interpretation of Kant doubly important.)

For comparison with atomic physics, let us add a remark on Kant's doctrine of antinomies. According to Kant there are questions which force their way unavoidably into our thinking, but to which nevertheless no answer can be given, because in a fundamental way they leave behind them the realm of possible experience. From this antinomies originate: these are pairs of contradictory answers to the same question, both of which can be proved in a way that is conceptually unobjectionable. The solution of this 'dialectical illusion' consists precisely in rejecting the original question as saying nothing about possible experience; and hence reason, if it nevertheless becomes involved with this question, runs, as it were, an empty course, and under the illusion of positive statements is in truth saying nothing.

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As an example let us look at the second antinomy, that of atomism. Here the thesis reads: 'Every composite substance in the world consists of simple parts, and there exists nothing at all except the simple, or what is compounded out of it,' and the antithesis: 'No composite thing in the world consists of simple parts, and there exists nothing at all that is simple.' The proof of the thesis runs in substance: 'For, suppose that compound substance did not consist of simple parts; then if all composition were dissolved in our thoughts, no composite part and (since there are no simple parts) no simple part either, and thus nothing at all would remain, and so there would be no substance at all.' The proof of the antithesis observes, that every simple part of substance (every 'atom') necessarily occupies a certain space, and that as we think of the parts of the occupied space, we are already thinking of parts of the atom.

From the more phenomenalist point of view of to-day, which prefers to say 'divisible' rather than 'compound', these proofs may seem questionable. They are not so from a strictly realistic point of view, which tries not merely to describe but to explain phenomena, through the assumption of real things, the properties of which are not defined (as is 'divisible', for example) with reference to our practical capacities. Thus Kant's argument again proves to be a critique of the concept of thing, in so far as it transcends the limits of possible experience. For it is, as a matter of fact, impossible to decide by direct experiment, whether matter is infinitely divisible or not. Precisely a practical atomism of physics, on the other hand, which treats the smallest particles it has discovered as indivisible, though fully aware of the fiction contained in such a procedure, was considered by Kant an especially fruitful conception.

COMPARISON WITH QUANTUM MECHANICS

We shall now place a concrete model alongside Kant's doctrine, by interpreting it as a description of the cognitive

situation of modern physics. This interpretation presupposes an intentional naïveté. For Kant claims to have founded, with the same concepts, an accurate epistemology of physics and 'every future metaphysics which can ever arise as science'. But when we interpret his concepts with reference to physics in the light of a situation unknown to him, part of the meaning that was aimed at metaphysics will escape us. Yet this temporary omission might be useful even for the metaphysical evaluation of the Kantian philosophy, since it enables us to trace out separately the threads of the various types of problems which Kant has often wound into a single knot.

The physicist investigates the things of the external world. He gains knowledge of them only through sense experience. Thus all knowledge whatsoever which we acquire about things is accessible to perception; and in order that any univocal conclusions at all may be drawn from it, it must occur in a causal context. This we know *a priori*, that is, independently of a particular act of experience. But perceptibility and causality are, so far, still vague expressions. They acquire their exact meaning only in the conceptual system of classical physics. For the contemporary physicist, who knows that he must formulate all his experience in the language of classical physics, the whole of classical physics is in fact *a priori*. That is in no case so clear as in that of the dualism experiments of quantum mechanics; for they lay the groundwork for a non-classical theory exclusively through classically interpreted measurements.

But still there is in things something to be investigated, which is not directly given to our sense experience, for example, the minute particles of which things consist. If we think of the meaning of the atomic theory for chemistry, we are even inclined to admit that we have understood how things are in themselves, only when we have understood the nature of their construction out of their minute parts. Even if this understanding should some day be replaced by an even deeper sort, it is nevertheless deeper than that which is implemented by direct experience.

But how can we experience anything about the atoms? In the last analysis only through sense perception, that is, in the language of classical physics. But precisely because we can know a priori—that is, through the mere consideration of the character of our experimentation—that we can grasp directly only those effects of the atom which can be represented classically—precisely for this reason we have no empirical ground for inferring that the atoms themselves too, in so far as they are not observed, must satisfy the laws of classical physics. Classical physics indicates the only manner in which the atom can enter the sphere of appearance, but for that very reason it states nothing about the atom in itself.

But just as we are engaged in constructing in this fashion the concept of an 'atom in itself', it melts away again in our hands. About the atom in itself physics knows nothing through experience. This vacuum is used by quantum mechanics to remove an apparent contradiction in experience. Particle and wave picture are contradictory if we interpret the observed phenomena as properties of self-existent particles or waves. The contradiction disappears if we consistently apply the imagined concepts of particles and waves only to appearance. The concept of the atom in itself has only the negative significance of showing us what sort of concepts we ought not to introduce into physics. Not even objective physical existence—that is, the capacity of having physically definable predicates, even if we do not know them—can be attributed to the 'atom in itself'. Up to this point we can take over Kant's argument word for word.

Now there is, however, no strict boundary between things of which we have experience and things of which we have no experience. It is precisely through experiment that we force the atom to make an appearance. However, for the logical relation of complementarity—that is, for the fact that the necessary experimental tools for the production of a given phenomenon render impossible the occurrence of certain other phenomena—there is no parallel in the doctrine of

Kant. This is connected with the fact that Kant does not take sufficient account of *will* in the construction of the empirical world. Complementarity permits us to represent the relation between the appearances of the atomic world, not according to the model of the 'atom in itself', but nevertheless in a determinate fashion, namely through the statistical ψ -function. Thus we obtain a non-classical physics of the atoms, which does not indeed deny the a priori character of classical physics, but pre-supposes it.

Especially instructive, perhaps, is the application of complementarity to the atomism antinomy. The 'atoms' of chemistry are known to be further divisible, physically, into nucleus and electrons. But for chemistry their peculiar significance rests precisely on the fact that they are not further divisible by chemical methods, and also that, in so far as they belong to the same isotope of the same elements, they are all exactly similar to one another. It is therefore the first task of atomic physics, to explain these basic facts of chemistry. In the classical particle picture, however, according to which electrons can move around the nucleus on any paths whatsoever, this is impossible. For this purpose we use instead the concept of the stationary state. According to wave mechanics, the concept pertains to the atom as a whole and refrains from specifying the paths of the electrons. But now if any one should ask, where in the atom an electron is located at a given moment, he can at once acquire concrete information by a measurement of place. In the course of this measurement of place, however, the momentum of the electron remains so indeterminate, that in the majority of cases the electron may even break its union with the atom. Thus we have indeed located the electron, but in so doing have destroyed the chemical atom.

It is clear that this 'complementarity of chemistry and mechanics' employs exactly the arguments of the Kantian atomism antinomy. If we want chemistry, that is, the explanation of the properties of matter through the interaction of its minute parts, then these parts must be conceived

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as non-compounded. But then it can have no meaning to talk about what is happening in the parts of the space occupied by them. If, conversely, we want atom mechanics—i.e., if we want to know what happens in the interior of the atom—then we can obtain the answer through an experiment, but in so doing we destroy the atom. We are left with an atom which is compounded and analysable, but which is never in this form the unit of which the matter given us in experience is built.

CONSEQUENCES FOR THE CONCEPT 'A PRIORI'

What, then, have we learned from this comparison?

In the first place, concepts which were thought absolute by Kant have received a peculiar relational character. According to Kant, a priori knowledge was supposed to be not only the presupposition for but also part of every possible science bearing on its object. However, the a prioris of classical physics are a methodological presupposition, to be sure, and not an actual part of the content of quantum mechanics; or, more accurately, they are a part of the content only in so far as classical physics remains a limiting case of quantum mechanics. They do not indeed become false, but—a possibility unforeseen by Kant—limited in their applicability. That is, it is true that no non-spatio-temporal experience—no relation between experienced phenomena which is not, on examination, causal—has been discovered. But it has been shown that an exhaustive interrelation of all of experience in a classical model is not possible, and that we have only to choose where, through experimental investigation, we want to create classical relationships, and where not.

One of these a priori presuppositions is practical realism in the sense defined above, i.e., the fact that not 'sensations in themselves', but perceptions of things are given us, and that we may also speak unhesitatingly about things that are *not* perceived, in the manner of classical physics. This is a relative, justified use of the expression 'thing-in-itself'. Its

limit is the limit of practical realism. Its connection with the absolute concept of thing-in-itself which Kant uses, consists first and foremost only in the formal identity of the Kantian inference of the impossibility of knowing the thing-in-itself with our inference of the possibility of a non-classical behaviour of the atoms.

Our attitude to the *a priori* in modern physics is different from the usual conceptions. Two of them are easily refuted, since they do not see the problem. These are the dogmatic-empiristic thesis which denies altogether the existence of *a priori* elements in knowledge, and the dogmatic—*a priori* thesis which denies the results of modern physics because they contradict the preconceived concepts of *a priori* knowledge. Our differences from the first we have expounded sufficiently; as to the second, the dubious character of its presuppositions should be made apparent merely by the logical possibility of a theory like quantum mechanics. A third possibility, which has to be taken more seriously, is the attempt, not, as we are doing, to reinterpret *a priori* knowledge, but to limit its extent. Even in quantum mechanics a broadened 'causality' is still valid if we admit that mathematically formulated probability relationships fall under this concept. Pure mathematics as a whole (arithmetic, analysis, general forms of geometry) is taken for granted by it as an instrument of thought that is given *a priori*.

Yet this conception seems to us on the one hand too modest, on the other hand too bold. It is too modest, in that it does not observe, that in the limited sense of a procedural premise even the classical causal principle and Euclidean geometry are valid in quantum mechanics; this circumstance must in any event be taken notice of, philosophically. It is probably too bold when it hopes, by limiting Kant's claims, to found a delimited system of *a priori* knowledge which must be part of every future science. On this last question only the future development of science can decide conclusively. But the single instance of failure in Kant's pre-

diction permits us to doubt, whether his criteria allow us to establish any a priori knowledge at all in his absolute sense.

We return herewith to the criticism of Kant's foundation of a priori knowledge as such. A distinction is made between the psychological and transcendental demonstration of an a priori. The former shows only that we inescapably have a certain notion and that it is perhaps genetically the source of certain other notions. The other, on the contrary, proceeds from complete, developed knowledge and investigates, formally, its necessary presuppositions. Only the latter is generally considered valid as a genuine demonstration, since the psychological existence of a notion does not demonstrate its logical necessity. Now although Kant himself conceived the idea of the transcendental demonstration, his actual demonstrative procedure is nevertheless not clearly delimited from the psychological. Of psychological nature, for example, is the foundation of the a priori character of space by means of the proposition: 'We can never form for ourselves a representation of there being no space, although we can at the same time easily think that no objects are encountered in it.' (*Critique of Pure Reason*.) But we meet this form of argument everywhere in the *Critique of Pure Reason*. As a second example let us mention only the demonstration of the synthetic character of the equation 7 plus 5 are 12 (*Critique of Pure Reason*), '... and I may analyse as long as I like my concept of such a possible sum, yet I shall never find twelve in it.' Precisely the overtones in the 'as long as I like' produce the impression of a certain unsureness, which is intelligible to one who knows the problems of axiomatization in modern mathematics. Thus we are finally led to the general question whether from the fact that we do conceive a judgment 'together with its necessity', it actually follows that we are also *justified* in conceiving it with its necessity. The same holds of the universality of judgments.

This question should not be confused with the empiricists' criticism of the concepts of necessity and universality. The empiricist or 'psychologist', who wants to 'explain' our

concepts 'true', 'necessary', and 'universally valid', psychologically, overlooks the fact that he is in the same breath setting up propositions which he regards as true, from which he makes inferences which appear to him necessary, etc. In other words: The concepts of logic designate phenomena of our consciousness, which are immediately given; even before we put the empiricists' question, we know what 'true', 'necessary', etc., mean. Now it may be an instructive inquiry, to track down the psychological genesis of this concept. But as in the attempt to analyse the given 'things' of experience into 'sensations', so here too the logical fact is methodologically, the given, and the psychological interpretation is an hypothesis, which can only justify itself by its performance from case to case.

At the same time, however, that we assume the question of necessity and universal validity of a judgment to be meaningful, we may doubt whether there is an answer to this question which would be absolute, that is, conclusive once and for all. The belief in such an absolute logical value of judgment Kant shares with nearly all the tradition of our philosophy. Of a different opinion are principally the sceptics, who, however, reject the phenomena along with the interpretation and so deny the problems rather than solving them. Plato, too, is of another mind, when through dialectic and myth he saves the tension of the act of knowledge from sinking into a frozen linguistic statement. And does not logical absolutism in fact represent just as much an exaggeration of the given as, for example, the substitution of radical for practical realism?

What has become clear to us about the role of language in philosophy? We use language in order to be able to philosophize at all. On the other hand we know or suspect about each word that it does not exactly designate a given. We have no means of avoiding this situation once and for all, but can only, from time to time, criticize particular words by the use of other words which are themselves uncriticized. We have no reason to surmise that this will be otherwise in the

sphere of thought than in the sphere of perception. We need concepts which do justice to this peculiarly fluctuating character of all knowledge.

It is in this sense that our interpretation of the concept 'a priori' is intended. That every experiment is classically described, we know no more certainly, than that every experience of the external world is spatial; in fact, the first proposition is the more questionable in the extent to which it asserts more. Both propositions have not logical but factual necessity. We ought not to say, 'Every experiment that is even possible *must* be classically described,' but 'Every actual experiment known to us *is* classically described, and we do not know how we could proceed otherwise'. This statement is not sufficient to prove that the proposition is a priori true for all, merely possible future knowledge; nor is this demanded by the concrete scientific situation. It is enough for us to know that it is a priori valid for quantum mechanics. That means that it is valid, independently of every particular one of all the phenomena which are considered at all in quantum mechanics—for the very reason, that it is the condition of the possibility of such experience. This demonstration can be made for the complete system of quantum mechanics and is, in the exact sense of the above definition, a transcendental demonstration only in the more modest framework which our changed conception demands.

When 'a priori' thus becomes a relational concept, knowledge which in relation to some other knowledge is a priori can nevertheless without difficulty be called empirical under another aspect. For example, classical physics itself of course originated by means of experience. Within it there are again relationships which are similar to the relation which it has to quantum mechanics. For example, electrodynamics methodologically presupposes classical mechanics, which is itself neither purely empirical nor purely a priori. To analyse it from our newly won point of view should be one of our first tasks.

We shall not undertake this task at present, but just in-

dicating two closely connected peculiarities of such problems. Mechanics is the procedural *a priori* of electrodynamics, and the whole of classical physics is that of quantum theory. As far as content is concerned, on the other hand, the situation is just the reverse. Quantum theory is a more fundamental branch of natural knowledge than classical physics, and it is only quantum mechanics which explains, for example, the possibility of the existence of solid bodies, which are, of course, necessary in classical physics, if the elementary determinations of spatial measurement are to be realized. Thus in this specific sense it is only the *a posteriori* knowledge which explains afterwards what was already assumed in the *a priori* knowledge. It is an error of many epistemological theories not to have observed this fruitful circle of all knowledge.

This fact can also have as its consequence a peculiar alteration of *a priori* relationships. For example, if we wanted to analyse classical mechanics, then in the face of the question of what is really empirically given we could no longer avoid the concrete analysis of sense perception. In this analysis it is at once apparent that—since we possess classical physics—we know the objects of this perception very much better than the process of perception itself (and that not only as the physical process in the sense organ, but as the phenomenon of sensation). Thus in an altered sense classical physics turns out to be *a priori* for the theory of perception. Indeed, it would probably be advisable, for the characterization of these tangled situations, which Kant could not foresee, to replace the concept '*a priori*', which already has so many claims on it, by more exactly suitable concepts.

LEVELS OF MEDITATION

We have lost ourselves in the application of the Kantian concepts to physics, and have not paid attention to the fact that they were meant to form the foundation of a universal philosophy, indeed, of a metaphysics. But if the philosophy

of physics we have been planning is to prove worthy of confidence on the philosophical side, we must, before we even set up a definite programme for it, return once more to this deepest concern of Kant. Have we any fixed point left at all, from which we can proceed to philosophize? ▴

As physicists we started from realism, which offers metaphysical solace to its adherent through the belief in the objective existence of things. As we recognized the questionable character of metaphysical realism, Kant's concept of the *a priori* seemed at least to furnish us an absolutely sure point in the foundations of our own thought. But now the *a priori* seems itself to have become not a necessary but a merely factual given, which at any rate is capable of and needs further analysis. Let us try once more to master the problems by making the source of the difficulties itself the starting point of a new way of looking at them. The purpose of introducing the concept of the *a priori* was, by studying the conditions under which the knowledge of objects is possible at all, to make intelligible those phenomena which were unintelligible when we tried to understand them from the objects of knowledge alone. Now we have found a fluctuating character in scientific concepts which does not seem to fit with our notions of science; in order to understand it, we step outside science for a moment, and look at the conditions under which it is possible at all as a process of human consciousness.

We have resolved not to say 'Every experiment *must* be classically described' but simply, 'Every experiment *is* classically described.' Thus the factual, we might almost say historical situation of physics is made basic to our propositions. As a matter of fact Aristotle did not yet describe his observations classically; and although we cannot imagine it, we cannot logically exclude the possibility, that a later mankind may again choose a description different from ours; it is not even demonstrated that our capacities for perception and sensuous imagination are constant. In order to speak clearly about what we actually know, we do not need to do

any speculating about what we do not know. Should we not say that the historical situation of human consciousness belongs to the a priori of physics?

After sensualism and psychologism we meet here again, in historicism, the temptation unwittingly to substitute the concept of a genetically explained opinion for the concept of knowledge. But it is only by the very fact that we resist this temptation that we can give the 'historical a priori' its exact meaning. Thus we do not assert that there is only historically conditioned opinion and no knowledge. Not in the form of our assertion, since when we say 'opinion', we already presuppose the concept 'knowledge'. Not in its content, because, for example, it is precisely the relation between the historically conditioned forms of our knowledge which physics postulates. Thus everything which is true at all in the physics of Aristotle, is contained in classical physics, even if it is in part differently expressed; and we should expect the corresponding situation for the relation of our physics to every future one. Nor do we subscribe to the more dangerous formula of historicism, that every epoch has its own truth—at least not if the concept 'truth' is so naïvely understood that it embraces the truths of physics, and if it is allowed that the 'truths' of different epochs contradict one another. For then this proposition is only a more skilful rewriting of the previous one. Men of other times can know something we do not know, and they can also prove that we were wrong; but when we say that something is true, we believe at any rate that the opposite is never true at any time. That forms of expression change and that therefore non-contradictory contents are now and then expressed in contradictory form, is naturally admitted. It is also admitted, that different states of consciousness can, in equally 'true' fashion, document their attitudes to the deepest human questions through contradictory statements; but we wish to take account of this fact precisely in a fashion which does not destroy scientific truth.

But at the same time that we protect in this way the mean-

ing of scientific propositions, and their claim to be true, from succumbing to historical relativism, we must also admit, that, as a matter of fact, truth is always given to us only in the form which the state of mind of our epoch permits, and that there is not even the possibility of speculating beyond the boundaries thus established. If we seek a clear concept for this fact, we must look out for a known process in consciousness, which unites both the full claim to truth and the recognized dependence on the present state of consciousness. We find this in *meditation*.

∟ Meditation is the process by which consciousness takes possession of a truth in such a way that not only the content but also the structure of consciousness is changed. It is connected with the fact that knowledge itself is a living process. Let us compare it with a process of the vital sphere of human life. In learning to ride it is of no use to me that both the riding master and I know what I must do in every situation; the moment must come in which I instinctively do the right thing—in which I ‘can ride’. This ability is only acquired through the attempt to ride, although I doubtless cannot do it at first, and through continued practice. It is just the same if I want to ride the even more unruly horse of knowledge. Continual contemplation, thinking through, calling to mind and practising of truth, sometimes wandering through the area already known and ever and again knocking on doors which are not yet opened, in constant readiness to liken one’s own nature to the truth once it has revealed itself—this is the contribution of the will to meditation. The process which takes place only when this attitude of will is there comes from the powers of the unconscious and consists in a slow but actual transformation in the character of consciousness. It is not different in its essence from every process of maturation. The adult too has a different consciousness from the child; he has different attitudes of will and different drives, and he applies as a matter of course images and concepts which have no intelligible meaning for the child. But all higher knowledge can

be achieved only with a conscious attitude of will to the process of knowledge, and there is knowledge which is accessible only to the strictly meditative road.

If we want to describe the meaning of meditation 'epistemologically', we must go back once again to the concept of the 'given'. Elementary sensations are by no means given us; things and similar sensible unities are. But what is or will be given us depends on our own state. A person who can ride knows instinctively what he previously knew at most theoretically. A scratched up paper is given to the child who cannot read; to the child learning to read, letters; to the adult, whole words and even sentences, and in such a way that the letters are no longer given individually (if this were not so, there would not be so many typographical errors left in proofreading). In immediate personal contact many of the emotions of my fellow men are given me; although through a changed attitude such as suspicion I may lose them again. The significance of the attitude of 'faith' for consciousness does not rest on the fact that faith takes what is unproved for true, but on the fact that through the removal of the 'suspicion' resident in doubt it creates the condition in which certain not easily accessible circumstances can be 'given'. And in this sense we can also say: Meditation changes consciousness in such a way that something is given to it which previously could not be given.

All schools of systematic meditation know the concept of the levels of meditation. It is the essence of meditation that it gradually appropriates truth which cannot be obtained all at once. On this road there are stations which must be run through in series, because one is the presupposition—the 'a priori',—of the next. Obviously this does not mean any rigid schematism; sudden vistas may occur into a depth or height not yet in itself accessible. But it is nevertheless astonishing with what regularity the development of consciousness takes place.

Now the development of science and philosophy can be understood as a grand meditation. The phases of develop-

ment, in which every later one at once presupposes and relativizes the earlier one, correspond to levels of meditation. Thus in the question of the objects of physics the metaphysical realism most closely connected with classical physics signifies a first level. The sceptical and sensualistic critique of realism is an opposed intermediate stage, a transitional state, which has only the negative significance of making clear the need for emergence from the first level. The application of the concept of thing by quantum mechanics to possible observations only—a procedure which makes conscious use of the fact that physics is human knowledge—signifies a new positive level.

Every level can be characterized by what is necessarily and universally valid in it, through its a priori in the psychologically absolute sense. When, through the transition to the next level, this a priori as such is dissolved, then it is not therewith invalidated as knowledge; it becomes a special scientific insight with a definite ground for its validity and definite limits to its validity.

The transition from one level to the next, though evoked by arguments, cannot be logically necessitated. The step from a given level is always in the first instance a step into the dark. For both levels, as different states of consciousness, fail basically to understand one another; above all the earlier does not understand the later. The earlier does not know the mental phenomena which are given for the later and must therefore necessarily misunderstand every attempt to talk about them; in our example, it must interpret every criticism of metaphysical realism as a doubt of the 'reality' of physical things. The later stage on its side contains at any rate the basic laws of the earlier among its 'empirical' judgments; but it loses more and more an understanding of the special metaphysical dignity which was previously attributed to these laws, and in discussions gets into difficulties because it can no longer even formulate in a language meaningful for itself what was meant by those laws over and above their empirical meaning.

Now how are we to interpret Kant? He begins at the level of metaphysical realism. From it he is pressed forward, not like modern physics by experience, but by the discovery of the *a priori*. He recognizes that what metaphysical realism believes is not in the least derivable from the experience given it, and yet that it cannot be omitted, because it represents the presupposition on which it depends for this experience. Through this knowledge the statements of realism acquire a changed meaning; a new level of meditation is achieved. Kant seeks to interpret this level through the distinction between 'appearance' and 'thing in itself'. He uses the concept of 'thing in itself' as a basis, only in order to retain the thing both in its character as 'object' and in its express relation to the person who knows, and thus to protect it from slipping back either into a naïvely realistic or an equally naïve solipsistic interpretation. This danger is particularly acute in the doctrine of Kant, because he has arrived at the new level by a purely philosophical path, and thus does not change the natural science which he finds already existing, but only reinterprets it. From the purely physical point of view, therefore, Kant's 'Copernican revolution' turns out to be a mere change of nomenclature from which nothing follows. It was not until the advent of quantum mechanics that the same level of meditation was achieved by a physical path and the new freedom which this gives to the formation of physical concepts was used for the creation of a physics which no longer *can* be interpreted realistically. Thus it was only modern physics which filled in the empty framework of the Kantian doctrine—to be sure, with the changes and reinterpretations inevitable in such a case.

There remains now the still unsolved task of carrying through a philosophy corresponding to the level of meditation of quantum mechanics. But perhaps it was permissible at this time to submit these tentative considerations in the form of a request for co-operation and criticism.

Chapter Five

THE INFINITY OF THE WORLD

A STUDY OF THE SYMBOLICAL ELEMENT IN NATURAL SCIENCE

NATURAL SCIENCE AND SYMBOLISM

Is the world infinite? Three questions are here contained in one: Does the world stretch without limit into space? Is it of everlasting duration? Are there no limits to the smallness of an object, so that every thing no matter how small would be still further divisible?

These questions impinge on our thought at a certain stage of its development. But how are we to solve them? No matter how much experience we collect, its sum total is always finite. In everything which we know through experience there is a most distant object, an earliest event, a smallest unit. But this very limit of experience disquiets our thought. We ask ourselves, what might lie beyond those limits, and try nevertheless to experience it. To be sure, by this means we always reach only a new limit of experience and never infinity. But in this procedure we nevertheless presuppose that beyond every limit we reach there is something more, be it only empty space or empty time. Thus we involuntarily presuppose the infinity of the world, although we must tell ourselves that no positive experience can ever confirm this presupposition. Our imagination too is prone to this conflict, for as little as we can represent to ourselves

infinity, so little can we really imagine a limit that cannot be passed.

How have people acted when they encountered these questions? Some cautious investigators have refrained from giving an answer. But most thinkers—and with them whole epochs—have formed definite opinions on the question of the infinity of the world. How were they guided in this? The expressed grounds were different, but the result always reflected something of the basic relation to nature of those who held them. Especially in such questions in which no compelling facts determine the answer, the formation of scientific opinions is at the same time symbolical of an attitude of mind, a feeling for life. We shall here pursue this side of the question of infinity through western cultural history up to the present.

How is it that the symbolical element in natural science can move us, especially to-day? To many of us precisely the physical discoveries of our century appear to be the expression of a spiritual change which extends far beyond the limits of physics. Yet we are far from believing that physical discoveries are the cause of this change. Still less may we suppose that the new theories were advanced in order to evoke such a change or even merely to express it. They forced themselves on us as the sober expression of observed facts. If we nevertheless find an exact relationship between them and the thought and experience of our time, then this discovery will make all the deeper impression on us. We can see important sides of our nature in them as in a mirror, and we feel ourselves forced to explain how such a mirroring is possible at all, and what it shows. As preparation for the question of infinity we shall turn for a moment to the basic question of symbolism in natural science.

With the words 'symbol', 'expression', 'mirroring', we have described a fact which in the most general terms can be designated by the name 'meaning'. Scientific propositions are important to us not only for their own sakes, but also because they point to something else, that is, a state of mind

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or a way of being human. They mean something to us. The important point is that we do not regard meaning in this sense as something odd and unexpected, but recognize in it a basic phenomenon of human life. What a multitude of meanings meet us in life!

The carrier of the most inclusive wealth of meaning is language. Every word is a sound which means something other than this sound; thus for example the sound we emit when we say the word 'apple' means something totally different from this sound, namely a real apple. Since all developed knowledge needs language, knowledge always presupposes the existence of meanings. But meaning is not restricted to language. There are the great symbols: the flag, the crown, the cross. In daily life there are the visible, tangible signs, which are noticed not with respect to what they are in themselves, but only with respect to what they mean: letters, coins, traffic signals. There are actions which mean something, whether by convention, like the handshake, or as spontaneous expressive movements like laughter and tears. Whoever knows dream symbolism is aware of the way in which one idea can signify another even in the unconscious. Art, too, presupposes meaning. It is not only the picture which 'means' the object it represents. Music, too, in a mysterious fashion, signifies the movement of the soul. And the example of music reminds us that the capacity of art 'to express the inexpressible' is a special sort of meaning, which is distinct from the representation which the art of painting and sculpture has in common with photography and from the conceptual character which poetry has in common with everyday and scientific speech.

Now what is common to all these sorts of meaning? We might say: 'in all cases one thing stands for another.' Thus we should see meaning as a relation between two things, the 'sign' and the 'signified'; in this context the concept of thing would also embrace the immaterial. But the decisive point is not the mere existence of a relation. It is hidden, rather, in the little word 'for'. We use the sign in place of the thing

signified. But by this process we by no means want to drive away the thing signified, rather we want precisely to keep it before us through the sign. In this sense the sign is there 'for' the thing signified. As long as the sign is used without reflection on its meaning, the duality of sign and thing signified is not even present to consciousness.

When a child cries, we see only the tears and the distorted face; the distress cannot be pointed out directly. But for the child only the distress is present. The tears are not drops of moisture—they are the distress itself. It interprets its environment in this way too. If the older brother says, 'My little sister is crying,' he does not mean by that that she has tears on her face, but that she is distressed. Or more exactly: he means both at once, since the tears are the distress. To be sure, tears too can 'change' in an instant to drops of moisture; as when they fall on the child's hand and there disturb it. Here the 'tear in itself' is discovered. 'Distress in itself' is discovered when we experience the fact that we can be distressed without crying, and cry without being distressed. Despite this discovery the next tears of another person are again naïvely taken as an expression of distress; it needs a wholly new attitude, which we call suspicion, to destroy this immediate givenness of distress in tears. Similarly, our own next distress presses us again to tears, and only the deep change in human attitude which the process of growing older brings, leads in many cases to new signs or to the absence of expression.

This example is valid *mutatis mutandis* for all relations of meaning, and above all for language. When I say 'apple' there does not have to be an apple before me. When I say 'distress', I cannot make the object of my meaning perceptible at all except through signs. But in both instances my attention is taken, not by the sound 'apple' or the sound 'distress', but specifically by what is meant: the real apple, and the real distress. As soon as I reflect on the process of speaking or just on the 'sound of the words', I destroy the obvious relation of the word with the thing. I can be forced

to this by the fact of having exhausted the possibilities of expression, by the inexactness and insufficiency of language. I can re-establish the relationship in a reflective fashion, e.g. through definition, which expressly states what the word is to signify. But to be able to state it, the definition must use other words without reflection; it presupposes other signs in which the thing signified is still immediately present.

A sign which cannot be made superfluous simply by pointing to the object signified, we shall call a symbol. With this usage we include as well those signs which are called symbols in the most serious sense of the word. Just because it is the only visible sign of a reality which is invisible but decisive for our lives, the tattered cloth that is a flag can become an object almost holy. But we also include, intentionally, the seemingly most banal words of everyday speech in the realm of the symbolic. The old conviction of the magic power of the word expresses the knowledge that the symbol stands at the beginning of language. Only he who has the word, has the thing whole. The reasons why this consciousness has faded are worth special consideration. We shall find in the following some phenomena which are related to them. It is philosophically important that we could not reduce the concept of meaning entirely to the concept of pure being. Since the being of the things signified becomes comprehensible to us only through the signs, an exact explanation of the concept of being already presupposes, conversely, the concept of meaning. But we shall not pursue this question further here. Let us return to natural science and its relation to symbolism.

Scientific knowledge can be symbolic in a twofold way: by being itself a symbol, or by encountering symbols and using them. Like every other expression of life, scientific knowledge too can involuntarily describe something of what is in man. It then becomes, in the first instance unconsciously, but sometimes also with conscious recognition, a symbol of his nature or his state.

But, on the other side, the object of science—nature

itself—can be taken by us as symbol. Unsophisticated man has at all times read the face of nature like the face of a fellow man. Nature threatens and can be conciliated. The tumultuous sea is angry at man; the bright sky wishes him well. Gods and spirits are the soul of nature. The stars mark out human fate. The order of the created world sings the praise of the creator. As late as the eighteenth century the variational principles were set up as laws of nature because they were worthy of the divine wisdom. In the course of this development we can see the transition, from the unconscious symbolism of nature which is taken for granted and which is comparable with the unconscious symbolism of human expressive movements, to the conscious act of putting and affirming the question whether natural phenomena are at the same time to be taken in an objective sense as signs of something else. In general, to be sure, modern science has rejected this objective symbolism of nature. ʘ

The question of infinity leads us first to the symbolical value of natural science in the first sense. We shall find again in science the man who made it. The second question, which concerns the objective symbolism of nature, far transcends the limits of our investigation. We shall follow it as far as our subject-matter demands, but shall finally have to let it go without a concrete solution.

ANTIQUITY: THE FINITE COSMOS

The earth is a round disc, surrounded by the stream of Ocean. This proposition from early Greek cosmology is at once a statement about facts and a symbol. It was, without doubt, held to be literally true. Acquaintance with the Atlantic Ocean beyond the Pillars of Hercules and with the Indian Ocean beyond Arabia made it, in fact, a very reasonable geographical hypothesis. But at the same time it expressed in striking fashion something of the Greek cosmic sense. Other geographical information might indeed have altered it in its details, but not basically. ʘ

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The world is finite for it is ordered. Here are our fields and harbours. If we walk across the mountains or sail over the sea, we come to new lands and strange peoples. These peoples too live within set boundaries. Beyond their wilds and woods may dwell still other nations. But sometime we must reach an end. For—in this way we may perhaps give words to the unexpressed feeling—the limitless would be the inconceivable, and the inconceivable does not exist.

For the spatial sense of modern man it is hard to conceive that one does not inquire further beyond each limit one has reached. Ocean may be the boundary of the human world. But is it itself unlimited? Or, what comes beyond Ocean? We may easily conjecture that people in those early times simply forgot to put this question. For later history teaches that mankind was bound at some time to come up against this problem. Nevertheless this conjecture is unjust to the Greek picture. It fails to recognize the foundations of a different state of consciousness on which that picture rests and which it expresses symbolically. It must fail to recognize it, as long as it is not conscious of the peculiarity of its own presuppositions and of the symbolism which is also concealed in our own idea of infinity.

Mythologically, Ocean was one of the Titans. He belonged therefore among those primal powers with whose overthrow the rule of the Olympian Gods begins. Measure and order, image and concept, derived from the world of the Olympians. Therefore this world was in its essence finite. The grey dawn of time, the immeasurable distance did not belong to it. But Ocean was one of the imagined realizations of that measureless world 'beyond the boundaries', in which there is essentially no form, of which there is no concept. The man who would penetrate it undertook not only the impossible, but the impious. For he abandoned the measure which the gods have set for him as a human being—the measure of which the gods themselves are the embodiment. Therefore one could not—must not—ask

how wide the water of Ocean is, or what lies beyond that water.

∟ Greek science soon left behind the doctrine of the world disc, and Greek philosophy occupied itself extensively with the concept of the infinite. But in the mature form of their scientific world view—the astronomical doctrines of which were systematized by Ptolemy, and the philosophical content of which comes essentially from Aristotle—the Greeks have left to posterity what is perhaps the most complete expression of their belief in the world which is finite because it is ordered. √

The earth is a sphere in the centre of the world. About it circle the seven planets. Their orbits can be mathematically represented: they are compounded of circular motions. The motion on these orbits is produced, according to the most widespread view, by seven concentric crystal spheres, on which the planets are located. About them as outermost sphere turns the sphere of the fixed stars.

This picture represents the astronomical observations with all the exactitude one could wish for. It was not revised for fifteen hundred years, because it did not need any revision from the side of experience. Thus the astronomers were not forced to put the question of what lies beyond the sphere of the fixed stars. Nor was there any occasion for the present theory, that the fixed stars are not arranged on a sphere, but are at varying distances from us—since it was not possible to measure the distances of the fixed stars. Only philosophy thought through the question of the limits and what lies beyond the limits, and found a logically satisfactory answer.

According to Aristotle there is no body outside the sphere of fixed stars. But just as little is there 'empty space' there, for there *is* no empty space. To us, to be sure, it seems obvious that space is in itself infinite, even if it should 'contain' only finite bodies. Yet this concept of space as a Something present in itself, independently of matter, is

already a modern notion, by no means necessary to thought. For the Greeks there is no concept of 'space' but only of 'place'. Every body is at a place. Thus its place is a predicate which belongs to the body. But where there is no body, the subject to which we might attribute the predicate of place is lacking. An empty place would be the place of nothing, therefore no place. This inference is by no means sophistical. It is, rather, a necessary consequence of Aristotelian logic which is oriented to being, and according to which a proposition is false, precisely if there is no thing of which it is true.

One might object, however, that the concept of infinite space is prefigured in Euclidean geometry. But Euclid too speaks not of space, but of geometrical figures. These he defines through rules of construction. He does not say: 'there is a straight line through two points,' but 'let a straight line be drawn through two points'. But every figure constructible in practice is finite. This is obvious for closed figures like circle, triangle, etc., but it also holds in a somewhat different sense for the straight line. True, the latter is usually conceived by modern geometry as infinite, but for Euclid it is conceived simply as always capable of further continuation. Accordingly, in a particular postulate he demands the continuation of a given limited straight line in a given direction; this can be accomplished in practice with a ruler, and thus always by finite segments.

This difference of conception can be elucidated by reference to the Aristotelian pair of concepts: act and potency, reality and possibility. In actuality, the straight line is always finite; but it is potentially infinite, for its nature is such that it can always be further extended. Aristotle denies that there is any actual infinite at all, for it would have to have contradictory mathematical properties. Nor, for the same reason, can the world be infinite. Thus Aristotle did not naïvely presuppose the finitude of the world, but derived it from his conviction of the intelligibility of the world. This conviction is not expressed as a particular proposition, but it

enters into all the inferences as an obvious, undiscussed premise. Yet for just this reason it finds in the results of the inferences a symbolic expression of perfect clarity. In the myth of Ocean the infinite and therefore unintelligible becomes visible at least as a grand image beyond the bounds of the finite world. But now, with the conceptual comprehension of the intelligible, the unintelligible has totally disappeared—at least with respect to space: the world is finite, and beyond its limits there is no place.

The question of temporal infinity was also put by the Greeks. In mythology the world is in general limited in time as much as it is in space. Myths of the origin of the world usually start: 'in the beginning there was . . .' In this way a power is placed at the beginning, into whose origins no further inquiry is made. Similarly the myths of the destruction of the world cut off the search for an infinite future. But Greek philosophy takes an infinite duration of the world into account. According to Aristotle the celestial edifice is ungenerated and indestructible. Another theory, which unites in a peculiar way finite and infinite duration, is that of periodically recurring world catastrophes and of an eternal repetition of all events. One might ask whether these theories do not refute our assertion of the symbolical meaning of finitude for Greek thought. But the starting point for Aristotle—as, earlier, for the Eleatics—is not the infinity of duration, but the unalterability of being. And this arises in the last analysis from the demand for the intelligibility of being; somehow to understand change as the transition between being and non-being was in fact one of the most difficult problems for Greek thought. Thus both spatial finitude and temporal permanence express, in the same fashion, the still more deeply rooted belief in the intelligibility of being.

We shall not consider more closely here the problem of the infinitely small. Suffice it to recall that the Greeks are the inventors of the concept of the atom.

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MIDDLE AGES: THE FINITE CREATION

For the Middle Ages too the world is finite. But its finitude has a different meaning from that of antiquity: for over against it stands the infinite God.

The world is finite in space. The Ptolemaic picture is taken over from ancient science. It agrees with astronomical experience, which could not be entirely set aside even in centuries remote from science, since for example the determination of the correct date of Easter depended on it. Further, the Ptolemaic picture is easily harmonized with the text of the Bible. Finally, it furnishes the natural frame for the medieval interpretation of the world. In the middle of the world stands the earth, on which the real purpose of the world's course, the history of man's salvation is acted out. The heavenly spheres likewise belong to the creation but are nearer to God, whose real seat, despite his omnipresence, is thought of primarily in the 'empyreum' beyond the sphere of fixed stars. By this the finitude of space is by no means destroyed. For God as the creator of the world, and thus also of space, is himself non-spatial. Nor, for this reason, can the empyreum have spatial determinations in the strict sense applied to it. Moreover, a soul to whom God grants the vision of divine things in the empyreum will no longer perceive them in a spatial manner. The last cantos of Dante's *Divine Comedy* may furnish us with some conception of this process.

The world is also finite in time. It has a first day—the first day of creation; and a last day—the day of judgment. The first day of creation is dateable historically, almost exactly to the year. When the day of judgment will come no one knows; yet nearly all Christian ages have awaited it in the near future. Before the creation and after the judgment there is no time. For God is supertemporal, and created time when he created the world. This is the logically consistent doctrine; for less philosophical spirits, the information would be sufficient which Luther gave to the question of

what God had been doing in the long time before creation: 'He sat in the woods and cut birch rods for people who ask useless questions.' Here, as in antiquity, the 'impossible questions' are not only logically unanswerable, but it is unseemly for man to put them. To be sure, a new world and eternal life are expected to come after the judgment; yet this does not mean a life of infinite duration, but a state which can no longer be represented in a temporal fashion. For the rest, the doctrine of finite time is not only fixed by the biblical text, but it expresses the only possible relation of medieval man to history. For history is a purposeful process in which God realizes his plan of salvation, and particularly, once Christ has appeared 'in the fullness of time', there remains no meaningful content for a permanently enduring world.

But God is not finite. He is omnipotent, omniscient, omnipresent, and eternal. All this presupposes that his essence has no limits in any respect. Yet the infinity of God is not to be understood primarily as a concept. Like all dogmas it is the symbol of an experience which language can only suggest. Only one who understands this experience can hope to comprehend its symbols. In Christianity a new dimension of the human soul is disclosed. As against the previous world, it must be conceived as infinite in at least three things: in its longing, in its moral demand, and in its religious fulfilment.

The world is finite and therefore intelligible. But he who understands it sees that it is a world of suffering. True enough, space and time are the bearers of its order. But the man who has experienced as deeply as possible what spatial separation, what uncertain future and irretrievable past mean, the man who knows death, knows too the longing for a realm in which the bounds of finitude no longer hold. Now this divine realm is taught and embodied by Christ. It transcends the bounds of finitude in its demand and in its fulfilment. Its demand is unconditioned: 'Ye shall be perfect even as your Father in Heaven.' Only this demand makes

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clear the degree to which the finite world is not only a world of suffering, but a world of guilt. In the knowledge of his own inescapable guilt man perceives his distance from God even more deeply than in the experience of his suffering. But God is also infinite in grace. Salvation begins when man no longer shuts himself off from grace, and it consists in his winning participation in the fullness of God. Every attitude which man can assume of himself limits him; it is God's gift that boundless love can come to power in him.

Christianity has a double relation to the world. The world is God's creation, but has fallen away from God. Dogmatically the possibility of this fall is related precisely to the share in God's perfection which the world received through creation. God made man in his image and therewith gave him freedom of will. In virtue of this freedom man chose the path of alienation from God. The symbol of Lucifer, whose fall preceded, metaphysically, the historical fall of Adam, expresses the fact that not only man but all creation shares the alienation from God. To this doctrine there corresponds a double experience of the world. The original creation, which is to be reinstituted on the Last Day, is the image of God's glory, and the beauty and order of the fallen world, which is all we know, is also by its mere existence a constant hymn of praise to its creator. Consider how St. Francis praises God along with our brother Sun and our sister Moon, our brother Wind and our sister Water! But guilt in the world is the expression of the fall from God and suffering is its consequence. The order of the world is therefore as it were only a substitute solution to bridge the time until the judgment. It is a metaphor, which is to awaken the yearning of the soul for the divine perfection which the creation is really meant to have.

This tension kept men in constant motion. From the entrance of Christianity into history till the end of the high Middle Ages the movement from the world towards God predominates. Then the direction of the inner movement begins to reverse itself, from God back to the world. But it

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was not so much the direction of this movement that determined its fruitfulness, as the relationship into which it brought the two poles God and world. It could, so to speak, bring the one pole over to the other, or it could forget the one because of the other. For early times this meant the decision that either attention to God gives order and life to the world, or it lets the world sink into neglect. We must now turn our attention to the corresponding decision in the movement of modern times.

TRANSITION TO MODERN TIMES: THE INFINITE WORLD AS SYMBOL

Nicolas of Cusa, in the middle of the fifteenth century, was the first to teach the infinity of the world. He proceeded from a consciously symbolic religious view. But with the same argument he led the way to the mathematical natural science of modern times.¹

The infinity of God can be mathematically pictured. In our world of finite things there is no largest thing. A still larger circle can be drawn around every finite circle. But God is the absolutely greatest, the all-embracing. In this respect he is like an infinite circle or an infinite sphere. While our world of finitude is the world of opposites, God as their source is the unity of contradictories (*coincidentia oppositorum*). This unity is incomprehensible to us. But precisely its incomprehensibility is represented by the paradoxical mathematical properties of the infinite. The greater a circle, the less it is curved; and a strictly infinite circle would therefore not be curved at all; the infinite circle is identical with the infinite straight line. If we want to see clearly that this is not just a game with ideas, but the expression of experiences, we must recall that Cusanus is here only transforming a metaphor of medieval mysticism. According to this metaphor, which Master Eckhard also uses, God is an infinite sphere whose centre is everywhere and whose circumference is nowhere. No thing limits God, but he embraces all things;

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and no thing is distant from him, but everything rests at his centre. This holds above all for every human soul. Before salvation we want to escape him and do not know that we cannot; and salvation consists only in becoming aware of this rest in God.

Now according to the doctrine of Cusanus, God infused into the world as much of his perfection as was at all possible if it were still to be different from him. Therefore it also shares his infinity. To be sure, it is not 'absolutely' infinite like him, but it is 'concretely' infinite, that is, it is the spatially unlimited multiplicity of concrete, finite things. The infinity of God transcends every concept, and the infinite sphere too is only an image of him. The world, on the other hand, is in the strict geometric sense an 'infinite sphere'. Thus the world is itself a geometrical image of God.

Two things are noteworthy in this thought: the expressly symbolic mode of thinking and the high valuation of the world.

The older views of the finitude of the world were not stated with the intention of being symbolic. That they were stated in just this form was the natural and unintentional expression of a feeling for life. Symbolic considerations were, at the most, added afterwards when the views already were held as certain. Nicolas of Cusa, on the other hand, consciously uses the symbol as an instrument of knowledge. He proceeds from the conviction that things which are akin to one another in their origin must stand to one another in a relation of similarity. Thus an archetype which is in itself unknowable can be at least approximately understood through its more easily knowable copies. Thus we understand God only through the picture of the world which proceeded from him. And again, among the intelligible things of the world those stand nearest to God which are least material, that is, the concepts of pure mathematics.

Mathematical symbolism is an age-old possession of mankind. On the road from concrete things to their back-

ground, which can no longer be concretely grasped, perfect abstraction is the last level to which thought can still attain. Numbers and figures, as the emptiest and most primary forms of thought, are the simplest and perhaps the purest vessel into which inexpressible experiences can still be poured. To be sure, mathematical symbols are not the truth, but they exhibit as much of it as can be exhibited and hide the rest. The formal laws of art are that remainder of this experience which are still present in the consciousness of the present age. All laws of artistic form have a core of the simplest mathematics. Let us but recall the ratios of musical harmony, the meaning of symmetry and regular sequence in all the arts, the pictorial beauty of mathematically simple figures. And it is the very secret of art that the strictest law of form which has apparently nothing to do with content permits it to express things which escape unrestricted speech.

But for Cusanus—who in this is following a Pythagorean-Platonic tradition—not only does human consciousness use mathematics as an image, but the real things themselves are in their mathematical properties images of their divine origin. Here the way is opened for modern natural science. For what receives attention here is not only the paradoxical mathematics of the infinite which unites with God the universe that is never perceived as a whole, but also the mathematics of the finite which determines the behaviour of finite bodies. Nicolas of Cusa himself was concerned with particular questions of natural science. But if we want to see the full flowering of his mode of thought in mathematical physics and astronomy, we must go on two hundred years further to Johannes Kepler.

Kepler had found his famous laws of planetary motion by looking for a comprehensive representation of the extremely detailed observations of Tycho Brahe. For this representation he set two conditions: undeviating agreement with experience, and mathematical simplicity. These two conditions remain to-day the foundations of exact natural

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science. After we have once set ourselves the task of understanding nature as she presents herself to us, the first condition is only a demand for procedural neatness and thoroughness. The second condition, on the contrary, might appear from an empirical point of view to be an arbitrary addition, for nothing in experience guarantees from the beginning that our observations will in the least satisfy a mathematically simple law. Nevertheless the condition is set again and again—and again and again with success. This success we must feel almost as a miracle, and Kepler's writings breathe everywhere enthusiasm for the constantly renewed confirmation of this miracle. In his '*Harmonice Mundi*' he unites this experience with the world of his faith and his artistic background through the common inclusive concept of harmony. He defines as harmonic certain relations of length which occur in figures that are 'constructible' in the sense of Euclidean geometry. He distinguishes these figures from those which are not strictly constructed (with ruler and compass), because only they in his view 'exist' in the real sense even for a divine mind. Then he tries to demonstrate that musical intervals, astrological configurations (the effect of which on man he explains from a special sensitivity of the unconscious soul to the harmonies occurring in them), and the speeds of revolution of the planets can all alike be reduced to the harmonies thus constructed.

More important for us than these time-bound particulars is the epistemological-metaphysical interpretation which Kepler appends to them. The harmonies like all mathematical concepts, are not derived from experience, but have their origin in the mind itself. Long ago Plato had recognized that mathematics permits inferences of a certainty and exactitude which experience cannot provide. On the other hand we find mathematical laws in nature, in fact mathematics proves to be the most suitable instrument for explaining nature. Thus man, as it were, meets himself again in nature. But since he himself did not make nature, this meeting points to a common origin. Nature is of divine origin, and

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since man is created in God's image, he can understand nature. The man who investigates nature is thinking again God's thought at creation.

It is no accident that just at this point in historical development, the thought of an objective symbol-value of nature was worked out. Let us recall the symbolism of simple expressive phenomena, such as crying. At first the symbol is so obvious that it is not felt to be a symbol at all, but the thing itself. Only after we have learned to distinguish conceptually between sorrow and tears does the proposition 'He who cries is sad' have a meaning. The proposition becomes important because its opposite could be true; it takes distrust to force trust into expressing itself. What the world signifies has to be expressly stated only when the possibility arises that it signifies nothing.

But this possibility arises with the turning back from God to the world, the turning with which the modern age begins. Now the here and now becomes of surpassing importance. The particulars command attention. Symbolic forms through which the Middle Ages had expressed, in a way it took for granted, its faith and its experience, now become empty demands of authority. To the Middle Ages, the truth was at bottom something already given; man had only to appropriate it and form himself according to it. Now on the contrary what assumes importance is the method which guarantees the road to unknown truth and insures success in all undertakings. Alongside the symbolist Kepler stands the methodologist Galileo. Descartes and Bacon want to found philosophy anew by the right method. Even earlier, Machiavelli had treated politics as a methodological problem. In the jesuitical exercises even religious experience is methodically guided. And only in this time could the theological controversy, whether the bread of communion was the body of Christ or only signified it, gain its overriding importance; while the original Christian feeling did not need to put the question because it did not separate being and meaning in the experience. In the framework of this

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reflection, conscious, objective symbolism must be considered an attempt to bind together the great realities of God and world which were already moving apart. Could this attempt succeed?

Let us go back once more to our point of departure in the Middle Ages.

The doctrine of creation and fall is paradoxical. The assumption of a God at once all good and all powerful is logically incompatible with the presence of suffering and guilt in the world. Nevertheless it was impossible to renounce either of the two sides of this doctrine. That the world is filled with suffering and guilt cannot be denied. But neither was the idea of God's omnipotence and benevolence simply laid down externally through dogma; after the experience which is expressed in it was once accessible to men, no lesser idea of God could satisfy them. An image for God like that of the infinite sphere, whose centre is everywhere, means therefore—however near our hearts it lies—at the same time the peak of paradox. In fact, in its interpretation everything depends on recognizing and holding fast to this paradox. This metaphor, like all related propositions of mystics, is not intended to be a 'correct' statement about the world, at least not as long as it is interpreted within the concepts of our everyday life. Only an alteration of the whole being of a man gives him the possibility of entering a new 'level of meditation', in which the concepts exhibit new meanings. Only so far as you let God work in your soul with all power and all goodness will you understand his omnipotence and his benevolence. Then, to be sure, you will know that you are at God's centre, and not only you, but all beings and all events. But in so far as this transformation has not yet taken place in you, you cannot understand the proposition at all and therefore cannot, if you are honest, even state it, unless it be as the expression of yearning and faith. For on the lower level the fact that there is a higher, genuine level can appear only in the form

of paradox; he who draws back from the sting of this contradiction, draws back from the way of solution.

When Nicolas of Cusa lets the world participate in God's infinity, he does not therewith dissolve the paradox. Rather, in his doctrine of the union of contradictories in God and of 'learned ignorance' (*docta ignorantia*), in which alone we can grasp the core of things, he makes it precisely his starting point. Only on that other, genuine level does the closest approach of the world to God acquire its meaning. Only to learned ignorance does its origin stand revealed in every thing. Only love, which God himself gives us, meets the creator in every one of his creatures.

This is a moment of integration in the development of thought which did not last. The very next step, which may have been unavoidable historically seems once and for all to join God and the world by equating them, but it is really the beginning of their irretrievable separation. Even in Nicolas of Cusa the tension of the original experience of the mystics seems at times to slip over into the purely conceptual. The next great proponent of the infinite world, Giordano Bruno—though he still admits conceptually the difference of God and the world—actually speaks only about the world. It is on the world that the glory of infinity now falls—that glory which was unknown to antiquity and, in the Middle Ages, was reserved to God. It is for the sake of this glory that modern man loves and conquers the world. Kepler's feeling is still medieval, and he can guide his thoughts only with dread through the infinite spaces of Giordano Bruno's world. Not much later the shattering of the crystalline celestial spheres appears as a release from an unbearable confinement in which no free mind could breathe.

From this point many roads lead further. Some great exponents of modern 'pantheism' have retained the clear consciousness that this world which we love is a world of paradox, and that therefore the doctrine of the divinity of the world contains a mystery and an infinite task. Where on the other hand the contradiction was avoided and men

lost themselves in the world, there the love of the world reveals a more and more daimonic character. Longing turns to greed, fulfilment to satiety. Admiring contemplation is transformed into that analytical knowledge which is the condition for limitless power. The practical life of modern times contains examples enough of this development. Natural science, especially on its technical side, has taken part in this second road. In its theoretical doctrine it has gone a third road, which is indeed connected with the second, the road of disenchantment.

Kepler perceived the divine harmony immediately in the motion of the planets. Newton derived Kepler's laws from a basic mechanical law which for methodological reasons, he wanted to be kept pure of all metaphysical hypotheses. Only because with this law he could not explain the origin of the planetary system, its regular structure, and its stability over very long periods of time, did he see in these points a demonstration for the existence of an intelligent creator. Laplace believed that he had found the mechanical solution of these questions left open by Newton, and answered the question where in his system there was still room for God: 'I do not need this hypothesis.'

Both steps were unavoidable. Kepler's world picture is the grand phantasy of a single man. In his harmonic speculations deep insights are intertwined with errors. And precisely from the religious point of view we can ask where in his picture that inner tension of the world appears. We probably feel that his love for the world is free from the element of the daimonic, only because it is so naïvely transported by the greatness of God in his creation, and because we know what kind of life stood behind that love. It might perhaps have been conceivable for his successors to go farther within the sense of Kepler's basic position. But here the methodological character of modern science comes into play. Kepler's astronomical knowledge was teachable, his religious experience was not. But once the symbolic relation between God and world is broken, a material relation can-

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not well be salvaged. The step from Kepler to Newton is historically intelligible, that from Newton to Laplace objectively necessary. For Kepler the positive knowledge of science points to God, while for Newton it is just the gaps in this knowledge which leave room for God. But such gaps are usually filled in in further development, and science cannot rest satisfied until they are filled in. Even if the hypotheses of Laplace had been false in some particulars, still every scientist must certainly set himself the goal of making the hypothesis 'God' superfluous in his field. God and the faded, half-religious concepts which have often been substituted for him in recent times, always designate, as scientific hypotheses for the explanation of particular facts, only incomplete points in science, and therefore with the advance of knowledge they find themselves in continuous and dishonourable retreat.

MODERN TIMES: EXTENSION OF THE BOUNDS OF THE KNOWN WORLD

The world now is infinite. But does our imagination suffice to people this world with the infinity of things necessary to it? Let all the stars we see be suns—even then, compared with the infinite, unknown remainder, no more space is filled up by them than by a single grain of sand. The actually infinite world remains an unrealizable demand on our power of imagination. The real use which science has made of the idea of infinity, lies in the potential infinity of the known world. We can always push further the limits of the known world and need not fear that a halt will be called to this advance by anything but the slackening of our powers.

This expansion of the limits was already beginning at a time when the limit of our knowledge was still considered the limit of the world. Copernicus, who still believed in a finite world enclosed in the sphere of fixed stars, had to assume the diameter of this sphere to be more than a thou-

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sand times greater than Ptolemy had thought it. To be sure, even in Ptolemy there is the statement that the earth is only a point in comparison with the sphere of the fixed stars. For otherwise the same star, seen from different, widely separated places, would have to appear in a different position; astronomically speaking, it would have to show a 'parallax'. We all recognize this phenomenon on a train journey, when the telegraph poles seem to hurry quickly by us, houses, which are farther away, seem to recede more slowly, and distant mountains seem almost to stand still. The direction of an object changes perceptibly only when we have traversed a distance which is not too small in comparison with the distance of the object from us. Now the stars stay fixed for our vision, even when we traverse the whole earth. From this we can estimate their enormous distance. Such an estimate, to be sure, is always tied to the limits of accuracy of our observations. A very exact measurement of angle might yet yield a certain change in direction of the star; and thus there follows from the Ptolemaic argument no more than a lower limit for the true extent of the world. Copernicus, now, enlarges this lower limit enormously. For according to him the earth itself is in motion and traverses, in its yearly path around the sun a distance which—as we know to-day—is more than thirty thousand times greater than the diameter of the earth. The fact that the planets appear to lag behind when the earth passes by them is precisely the explanation which Copernicus gives for the occasional retrograde motion of planets. But with the accuracy of measurement then possible, the fixed stars showed no trace of retrograde motion; they remain fixed through the whole year at the same places in the sky. Therefore even the diameter of the earth's orbit must be small to the vanishing point compared with the distance of the fixed stars. The immense, empty world of space which thus was opened was for many astronomers the principal objection against Copernicus. Tycho Brahe thought that God could not have created a space so great by

comparison with the solar system, without any useful purpose; to which Kepler answered, we could then make a similar judgment about the relative size of the elephant and the fly.

When we learned to look on every fixed star as a distant sun and assumed the infinity of the world, the question was no longer: how far is it to the sphere of fixed stars, but: how far is it to the nearest fixed star? With every refinement of the possibilities of measurement we sought anew for the 'annual parallax' of the fixed stars; that is, for their retardation over against the motion of the earth which must allow us to determine their distance. But before this parallax was found, another, purely conceptual question arose, namely the question of the frame of reference for the determination of places and motions in the universe.

For Ptolemy and equally for Copernicus the world has a centre. Every body, at every moment, has a well-defined place in the world; in modern terms: there is a 'natural' co-ordinate system in the world. In the infinite world, on the other hand, every point can be taken as centre. The old concept of the place of a body loses its absolute meaning. The radical consequence of this would be interpreting the concept of place basically as a relative concept. Strictly speaking we could then not say: 'this body is located at this place,' but only, 'seen from that other body, it is located at this place'. So, for example, thought Leibniz. In its stead, physics accepted the doctrine of Newton by which he tried to assure an absolute sense for the determination of place by the introduction of a new physical but immaterial reality, 'absolute space'. A decisive argument for this concept was the purely physical one that only absolute space permitted a simple formulation of the law of inertia and of the concept of acceleration. But the origin of the concept lies admittedly in religious-symbolic thought. In Newton and his predecessors, infinite absolute space appears as the physical image or also as the 'sensorium' of God. Even when physics forgot the origin of the concept, it retained a meaning for

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it which went further than the degree of empirical justification which could be given it. We might even venture to say that in an unconscious way it symbolized the absolute Nature which had taken the place of God.

The first stellar parallax was found in the middle of the nineteenth century. Until then the boundary of the measurable world was advancing as it were in empty space. Then, in the course of one brief century that boundary absorbed into itself all the celestial phenomena visible to the naked eye. In the meantime better telescopes had made it possible to see more and more faintly shining celestial objects, and with them the boundary of the known world still goes on expanding further from decade to decade. What picture of the world has resulted?

The diameter of our earth is 7,900 miles. The moon is separated from us by about thirty times this distance, and the sun by more than ten thousand times as great a distance, that is, about ninety-three million miles. A rifle bullet of constant velocity, would travel about six years from the earth to the sun. Light travels the same distance in eight minutes. About the sun circle the planets, of which the earth is one. Sunlight reaches the farthest known planet, Pluto, in about six hours. All these determinations of distance depend basically on measurements of the direction in which the planets appear to us from different places on the earth. In this connection it is sufficient to make these measurements on one planet that is particularly near; we can then calculate all other distances with the aid of the observed movements of the planets.

The remaining planets are akin to the earth in their structure; in particular they are not self-illuminating. The sun on the other hand is an immense glowing ball of gas. Its diameter is about one million miles. If we put the earth at its centre, the moon could still circle the earth at its normal distance without leaving the sun. According to the evidence of its spectrum the sun consists of the same chemical elements as the earth. The surface temperature of

the sun can be determined from the composition of the light emitted by it to be about 10,000 degrees Fahrenheit. At the centre of the sun, according to a well supported thermodynamic estimate, a temperature of about thirty-six million degrees must prevail. According to our present knowledge, the source of this heat lies in the reactions of the atomic nuclei through which hydrogen is constantly being transformed into helium.

The fixed stars are sun-like bodies. They too are composed of the same chemical elements as sun and earth. The nearest fixed star, Alpha Centauri, is over twenty million million miles distant from us. The light-year is taken as the unit for these great distances—it is the distance which light travels in a year. The distance of Alpha Centauri is four light-years. This distance and that of other rather near stars is determined by the parallax method which we described above. Seen from Alpha Centauri the orbit of the earth around the sun would seem to extend over an angle of not quite two seconds of arc (that is, about one thousandth of the diameter of the full moon), and there is exactly the same difference between the directions of Alpha Centauri seen from the earth, at two opposite points in its orbit. For most more distant stars this deviation is immeasurably small. But several indirect procedures have been developed for the determination of greater distances; among them, the method based on the variable stars goes farthest.>

Certain stars, the Cepheid variables, suffer regular variations of their brightness which presumably are caused by pulsations of the stellar body. Now we have found an empirical law: the greater the mean brightness of a Cepheid, the more slowly its brightness varies about this mean value; the length of the period of variation is a monotonous function of the average brightness of the star known as the 'Period-Luminosity Law'. In this connection we do not of course mean by its 'brightness' the 'apparent' brightness we see, but the total quantity of light emitted by it; for if two equally bright stars are at different distances from us, then

the nearer one will appear brighter. Since we did not at first know the distances of the Cepheid stars, the Period-Luminosity Law was discovered only by comparing Cepheid stars all located in one large group or cloud, which are thus roughly at the same distance from us. We have not yet succeeded in giving a complete explanation of the Period-Luminosity Law; yet it seems clear that to every period of variation there corresponds a determinate physical constitution of the star and therewith a determinate brightness. For the determination of distance the law is applied as follows: If two Cepheids have the same period of variation, then they must be equally bright. If they appear to us as of different brightness, then their distances must be different. If we know the distance of only one such star from direct measurement of its parallax, we can calculate distances of all the others by means of their apparent brightness. Most groups of stars in the sky contain Cepheids, and so our procedure extends to the greatest distances at which the telescope permits any recognition at all of individual stars.

The totality of the visible stars belongs to a great inclusive system, the farther parts of which betray themselves to the naked eye only in the shimmer of the Milky Way. The Milky Way, because it circles the heavens as a narrow band, must have the form of a rather flat disc, and we must be situated near its central plane. Its greatest diameter is about 100,000 light years. According to a rough estimate it contains one hundred billion luminous stars and an equal mass of non-luminous matter which is distributed in interstellar space mostly as dust or gas. Our sun is located in one of its outer regions.

Small nebulous objects in the sky, the so-called spiral nebulae, are distant groups of stars comparable with the system of the Milky Way. In some of the nearest of them we can still perceive some Cepheids. Thus we derive the distance, for example, of the Andromeda nebula—the most famous among the nebulae—of 700,000 light years. Then, with these nearer nebulae, we can empirically ascertain the

average total brightness of a spiral nebula, from which we can then estimate the greater distances of nebulae of the same form which appear much fainter in the sky. The farthest nebula which can be photographed with present day telescopes is thus estimated to be about 500 million light-years away. When the light which we photograph to-day left the nebula, the earth was in its early geological infancy.

The number of spiral nebulae up to the distance to which our best observations reach, can be estimated at about 100 million. The spiral nebulae themselves, are often grouped in loose clusters. These 'nests of nebulae' are the largest known aggregates of matter. They fill the known region of space just about uniformly.

Like the limits of space, the modern era has also cracked open the limits of time which the Middle Ages recognized. When geology found fossils of living things 'before the flood', and when it learned to estimate the age of past geological epochs by counting the strata in sedimentary rock, the calculation of time according to a literal interpretation of the biblical story of creation had to be abandoned. To-day the most certain determination of the time of geological epochs depends on the radio-active minerals. Some elements, above all uranium and thorium, are slowly and spontaneously transformed into lead, with the splitting off of helium. This transformation consists in a decay of the atomic nucleus, and is, like all processes in the nucleus, entirely unaffected by outside interference of mechanical, chemical or thermic nature. It follows from careful laboratory observations of the rate of decay that of a given quantity of uranium exactly half must change into lead in 4.5 billion years. Now if we find a material containing uranium, we need only determine how much lead has already collected in it in order to know its age. In this connection, the age of the mineral is the time since it was put together out of its component parts; for since that moment no lead

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could any longer escape from its interior. Minerals of the oldest known geological age have thus been assigned an age of more than two billion years. This is therefore a lower limit for the time which has elapsed since the origin of the earth. For the age of the sun and the other stars there are estimates which permit us to infer somewhat longer times. We shall treat these estimates in the next section, since they already have to do with the critique of the idea of infinity. There we shall also meet the much more indefinite inferences which are drawn about the future of the universe.

Finally, the modern era has penetrated into the 'infinitely small'. Conceptually, this advance occurred through the establishment of differential calculus and the principle of continuity. True enough the atomic theory of chemistry connected experience first with the ancient idea of indivisible minimal unities. Nevertheless by this means it already teaches us to recognize material entities of a smallness previously unknown. Their diameter is the hundred millionth part of a centimetre (10^{-8} -cm.). Then physics demonstrated that these 'atoms' could be further analysed, into a nucleus 10,000 times smaller and a diffuse electron volume. To-day the nucleus is split further. However, the laws of motion of the parts of atoms give occasion to regard the concept of the infinitely small with reservations. }

THE PRESENT: CRITIQUE OF THE IDEA OF INFINITY

There is a widespread feeling that exact natural science finds itself at present in a crisis. Wherein does this crisis consist? It has arisen not through doubt of the scientific method, but through its rigorous application. Nor has it limited the extent of scientific knowledge, but has widened it and has promoted the inner unity of the whole of natural science to an undreamed of degree. Perhaps we may describe the conceptual content of the crisis as the critique of the inference from the finite to the infinite. }

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All the conclusions described in the preceding section presuppose that the laws of nature known to us also hold good in very distant places and times. For example, we cannot determine the distance of a star by means of a parallax, in the way we have described, if in the universe light rays no longer travel in a straight line; and the determination of the age of the earth collapses if uranium decayed faster or more slowly in earlier ages than it does to-day. These questions must and have been tested. Thus the remains of living things which were similar to those of to-day show that in early geological periods organic chemistry followed the same laws as now; how much more is this then to be expected of the less easily affected nuclear processes. Moreover, we can apply several different methods of age determination, which rest on different sorts of natural laws, and in their agreement the results support one another. To be sure, all such inferences employ probability arguments. Yet this is the one and only method available to empirical science in the establishment of universal laws, to heighten probabilities into practical certainty through the multiplication of evidence.

But with the advance of our knowledge we have come in several directions to a point at which the permanence of natural laws has become either questionable or even refutable from experience. And in fact it was not only the case that changes took place in the special laws which fill in the framework set by the general concepts of infinite space and infinite time, but there is even a question whether these general concepts themselves are still permissible: the idea of the infinite expansibility of the known world as such has been criticized. This criticism was applied in the first instance at very particular points, at which it had almost the character of a technical detail. But to-day we can already see that in many areas of application a general truth is here being established. In our survey, which must necessarily be brief, we shall follow the same road from the particular to the general.

The special theory of relativity criticizes, on the basis of experience, the assumption that a body might have any velocity however great. Instead, the theory sets a finite limit of velocity, that of light. Here velocity is not conceived in relation to an 'absolute space' or 'ether' considered at rest, but it can be measured from the point of view of any other body whatsoever. At first this seems self-contradictory. If, for instance, two particles were fired in opposite directions at nearly the speed of light from a body considered at rest (e.g. two electrons), then according to our usual view the two particles must be moving apart from one another at nearly double the speed of light. But according to relativity theory the velocity of one particle measured from the other, must be less than the speed of light. This apparent contradiction is solved through an exact investigation of how velocities actually are measured. From this we get the famous critique of the concept of simultaneity of distant events and a new rule for the addition of velocities. According to the experimental definition of the concept of velocity, which takes account of the new natural laws, we may not in fact any longer combine very high velocities by simple addition; in particular the velocity of light, added to the velocity of light, again gives the velocity of light.

We now have yet to examine whether we cannot simply accelerate a body further and further until it passes the speed of light. But it turns out that the energy of motion of a body must also have a mass, and that therefore the body keeps getting heavier the faster it moves. Thus a constantly increasing resistance is established to further acceleration. If the body were ever to reach exactly the speed of light, its mass would have become infinite and further acceleration by a finite force would be impossible. This increase of mass with increasing velocity has been directly observed experimentally. Thus we see in the particular case of the concept of velocity how new laws of nature can deprive its application beyond a finite, determinable limit of all meaning. In this it is decisive that we do not employ the concept

naïvely in an area of which we no longer have any immediate perception, but investigate what possibilities for its experimental realization still remain in that area.

Quantum theory states that there can be no action which is smaller than the quantum of action. Thus it criticizes our notion of indefinitely small stretches of space and time. To be sure, it does not forbid us on principle to penetrate indefinitely into the realm of the very small, but it attaches certain dynamic conditions to this penetration. The existence of the bodies of the world surrounding us rests on the stability of the chemical atoms. But this stability is destroyed by the analysis of the inner structure of the atoms. This is valid not only experimentally, but conceptually. For from the properties of the component parts into which we can split the atom, it is not possible to explain the stability of the atoms in a fashion that can be spatio-temporally interpreted. Rather it is the consequence of the new law of the smallest quantum of action. Only stable atoms create the fixed entities which make possible the application of our notions of space and time to bodies 'in themselves'. At less than atomic magnitude the world is no longer spatially articulated, but the transference of further spatial subdivision to this region is a human act of violence which destroys the original constitution of the atoms.

Even this act of violence does not seem to be possible to an unlimited degree. The latest development of atomic physics leads us to conjecture that there is an 'elementary length' below which we cannot proceed. But the consequences of this assumption have not yet been tested in detail.

Limits seem to appear also for the infinitely large, which however have not yet led to complete, empirically confirmed theories. Still it is worthwhile to look at the evidence.

With respect to the temporal infinity of cosmic events the second law of thermodynamics had awakened some doubts even in the last century. This theorem says that there are

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irreversible events in nature. Thus hot water cools of itself, warming its environment, but water at room temperature does not become hot of itself and cool its environment. Mathematically the law asserts that there is a quantity, 'entropy,' which can only increase or remain constant in a system isolated from the environment, but can never diminish. From the point of view of atomic theory entropy is the measure of 'disorder'. For example the concentration of heat energy in a tea kettle signifies an 'order', which is transformed into disorder by the diffusion of the energy over the whole environment. Order is a state which can only be realized in a very special way and which therefore, in practice, never originates of itself. Disorder, on the other hand, is a generic name for the totality of all states in which no definite order is realized; it can thus be realized in a thousand different ways. When therefore any change not precisely determined takes place in nature, it is to be expected with overwhelming probability, that it leads from order to disorder and not vice versa.

Every event in an isolated configuration has, according to the second law, only a finite duration. We begin with an 'ordered' state, e.g., a kettle of hot water in a cold room. Then events run on as long as it takes to reach the 'most probable' state of maximal disorder; in our example: until the heat is evenly distributed through the whole room. When this state is reached, nothing more happens. This consideration has also been applied to the world as a whole, and the claim has been made that it is moving towards an eventless state of evenly distributed heat, the 'heat death'. Correspondingly we must deduce, that it began once with a state of maximal order. Beginning and end need not be sharply marked; the world might have gone, first slowly and then more quickly, from an initial state which had subsisted 'from eternity' into the time of 'events' and it might just as gradually 'freeze up' again. It has been objected that the second law need not hold for the whole of an infinite world, which would have an infinite energy content. But neverthe-

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less systems like our Milky Way and the spiral nebulæ are isolated to a sufficient degree from environmental influences; regardless of what happens to the rest of the world, which is unknown to us, we can expect at any rate that these systems have a beginning and an end. In general we can say that at least every finite part of the world has only a finite 'supply of possible events'.

In very recent times these rather formal considerations have acquired a more tangible content through estimates of the age of the known world. Surprisingly, the same age, of about three billion years, has been established by entirely different methods, for chemical atoms, the stars and the spiral nebulæ. The determination of the age of the chemical atoms makes use of the radio-active elements uranium and thorium. Since these materials gradually decay, they must either be constantly replenished, or have originated a sufficiently 'short' time ago, so that they would not yet have decayed completely. The first assumption has to-day become very improbable; we know no place in the universe which presents the physical conditions necessary for the formation of uranium and thorium. If we decide on the second assumption, then an age of about three to five billion years follows for these (and presumably all complex) elements. The age of the sun can be estimated, since the source of the energy which constantly radiates from it has been found in nuclear reactions, and the speed of exhaustion of this source can be calculated. For example, it follows from the present hydrogen and helium content of the sun that it has been in fact radiating in the present way for several billion years, but not a great deal longer.

Especially remarkable is the determination of the age of the spiral nebulæ. These objects show a spectral shift towards the red of the light emitted by them, and this shift increases in proportion to the distance from us of the particular nebula under consideration. According to the usual interpretation this phenomenon indicates that all the spiral nebulæ are moving away from one another and therewith

also from us. For the red shift of the light consists in the fact that fewer light waves strike per second than in ordinary light, and this is just what is to be expected if the nebulae are moving away from us. If we once assume this interpretation, it follows that the spiral nebulae are moving apart in the same way as the fragments of a great explosion; for after such an explosion the fragments which move fastest must in fact, after the lapse of some time, have moved farthest, just as the dependence of the red shift on the distance shows. Now if we calculate from the speed of motion and the present distance of the spiral nebulae the instant of the 'explosion', we obtain again a point in time lying a few billion years back. This agreement is so striking that we may well ascribe to it a real meaning. That would mean, then, that the parts of the world we know to-day, spiral nebulae, stars and atoms, had been formed at this point in time out of a state essentially different from the present one.

How did the world look before that time? We do not know how to make any inference from its present state to that earlier one. Nor do we know whether we may presuppose for the earlier state the natural laws which hold good to-day. We do not even know whether that state any longer permits us to apply to it the concept of time in a clear sense. For no bridge of direct 'recollection' seems to lead beyond that initial catastrophe; and it is unknown whether the state 'before' that offered any possibility whatsoever for the running of something like a 'clock'. For a clock must 'go', that is, it presupposes that something is happening. But according to the Second Law it is conceivable that for our world the time of events only began with that initial catastrophe. Herewith we return once more to the interpretation of the Second Law: if a natural law shows that, of a time span conceived as infinite, only a finite part can be filled with real events, then we may easily suspect that the notion of an infinite time presupposes a non-physical concept of time, which is merely an ideal construct. We should

then anticipate that a concept of time which takes account of the possibility of the physical realization of temporal determinations, automatically leads to the assumption of a finite time. The objection that, even if nothing is happening, time is passing 'in itself', is just as questionable, philosophically, as for example the assertion that the quantities of state which quantum mechanics leaves undetermined must nevertheless have values determinate 'in themselves'.

To-day the most probable content for filling in the assumption of 'heat death' in the future looks like this: that finally the sources of energy in the stars will dry up and the spiral nebulae scatter entirely in space. Yet we are even less likely to risk a definite prophecy than a definite assertion about the past.

The mathematical means for expressing the thought that the world is perhaps not infinite in space were discovered a hundred years ago in non-Euclidean geometry. According to this geometry, we can conceive an unlimited but nevertheless finite space which can, for example, be made intelligible by comparison with the surface of a sphere. If we travel in a straight line on the earth's surface in any direction we like, we never reach a limit, but we always come back to the place we started from; and the total area of the earth's surface is finite. It would be just as conceivable that, even though the universe might be nowhere 'nailed up with boards', every straight road in it would finally lead back on itself, and the whole spatial content of the world be finite. Such a space we cannot, indeed, conceive as a whole in our imagination. But we should consider that the actual infinity of Newtonian absolute space also escapes our power of imaginative representation. Imaginable are only the finite figures of which the original Euclidean geometry treats.

The General Theory of Relativity introduced the thought of such a finite space as a physical hypothesis. To be sure, still other infinite forms of space are also conceivable; and it is probable that experience will not in the foreseeable

future justify a decision among them. But what is shattered is the way in which the older idea of space was taken for granted. Since non-Euclidean geometry has shown that different structures of space are conceptually possible, it seems plausible to assume with the General Theory of Relativity that space is not a finished 'lodging house' into which matter has been moved, but that it is matter itself which determines the structure of space. Then even the question whether space is finite or infinite, can no longer be treated independently of the question what kind of matter it contains.

Finally, the purely mathematical concept of number has gone through a similar development. In the second half of the nineteenth century, the theory of sets had introduced the concept of the actual infinite into mathematics. Different infinite numbers were compared and distinguished, such as e.g. the aggregate of all whole numbers and the number of all points on a given line. The abstract concept of 'set', which equally embraces finite and infinite totalities, was even chosen as the starting point for the formation of all mathematical concepts. It seemed necessary to do this in order to provide an unobjectionable basis for the fundamental discipline of modern mathematics, differential calculus. It was possible to refute the Aristotelian objections against actual infinity.

But at the turn of the century it became apparent that the theory of sets nevertheless leads to logical contradictions. In the critique which was necessary for the exclusion of the possibility of such contradictions, another conception of number was developed, the clearest version of which was given by so-called intuitionism. According to this view we must say something like this: numbers, as conceptual entities, do not exist 'in themselves' but are the expression of man's ability to count. That there are infinitely many whole numbers thus means only: however far we have counted, we can always count still further. Thus human freedom,

which can always add still another new act to the totality of what has already happened, belongs to the logical presuppositions of this concept of number. Mathematical expressions which begin with the words 'there is' (e.g., 'there is another point between any two points on a given line') pass as meaningful only if we can substitute for the words 'there is' the words: 'it is possible to construct . . .' Thus it is clear that there are no actually infinite aggregates, for although it is always possible to construct more than has already been constructed, every set actually constructed is a finite quantity.

It is true that the intuitionist concepts proved to be too narrow, to serve as a foundation for that portion of mathematics which is, in practice, necessary, in particular for the differential calculus. For this reason the modern 'theory of deduction' tries to justify actually infinite aggregates as technical auxiliary concepts. That is, it does not take for granted the expressions which are meaningless from the intuitionist point of view, but tries to select among them those which do not lead to a logical contradiction when added to intuitionist mathematics; this freedom from contradiction must in each case be expressly demonstrated. From the intuitionist point of view, these additional expressions are, indeed, not genuine mathematics, but a formal game with concepts. In justification of them one can only point out that in practice physics needs differential calculus. If we reflect, however, that concepts like that of a minimal length occur in physics, then it seems to be conceivable that with respect to the behaviour of very small objects, classical differential calculus is just as much only one among several possible theories, as Euclidean geometry is with respect to space on a large scale, and that here too the consideration of the experimental possibilities has the last word.

We are reminded, in conclusion, of Kant's doctrine of antinomies. According to Kant the question of the infinite always leads to contradictory answers, regardless whether it

is a matter of the infinitely large or the infinitely small, of the infinite in space or in time or in the chain of causes. This is because we can ask in two opposite directions, and if we do so, must depart from opposite premises. If we proceed from our own position, then we can always ask about objects lying farther and farther away, and there is no seeing how, in this way, we could come to a limit. If we wish to proceed from an objective reality and explain our own state, then speaking in purely practical terms we can never proceed from infinity, but must choose a point of departure in the finite. But if the explanation is to be definitive, so that there is not still another answer hiding behind it, then we must assume our point of departure as an untraversable limit for the question, in contradiction to our previous assumption.

Modern science has in general taken a 'pragmatic' position. That is, it has given up in advance definitive explanations, and has therefore retained for its questions only the one direction which proceeds from the person to the things; so the contradiction naturally collapses. But the present shows that consistently pragmatic thought destroys both sides of the contradiction. Perhaps we may summarize the above described results and criticisms in the programmatic formula: the world is not in itself finite, but neither is it in itself infinite. Objectively present things of the sort familiar to us, from which we could proceed to explain our own situation, occur only in a finite region surrounding the place at which we ourselves stand. We can leave this region. But then we no longer find objective things, which were already there, but states which we ourselves have made. The laws of nature are transformed from statements about the existent world to statements about what we can do. The two sides of the antinomy are no longer contradictories but alternatives between which we chose. If we want to explain the existent by the existent, we must let it stand in its finitude; if we want to 'get back of it', we must destroy it. How far we can go in both cases, only experience can show us from one instance to another. But what is certain is that

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the concepts which are valid for finite things—whether these things are given us or whether we make them—may not be naïvely applied where nothing is given us any more and where we can no longer make anything.

ON THE INTERPRETATION OF THE PRESENT SITUATION

What do these results mean for us? Do they express something about our nature, about our situation?

Scarcely any one among us has from the first accepted without resistance the new way of thought in science. If we were convinced that, logically, no objection to it could be raised, at least there remained an uneasiness which was in many cases heightened into passionate contradiction. Does not the new way of thinking destroy at their roots our concepts of truth and reality? Does it not destroy all that is absolute and unquestioned?

Where controversy over scientific laws is based on this sort of argument, we may anticipate that they are important not for their own sakes, but because they stand symbolically for human attitudes. What did it mean for the practical life of men at the time of Copernicus whether the earth turns around the sun or rests in the centre of the world? Why need the movement of the spiral nebulae and the structure of the hydrogen atom concern us men of to-day? How can knowledge about the farthest reaches of the outer world disturb the peace of the human soul?

Without doubt the world view of modern science rests on an unconscious symbolism. We have become acquainted with the origin of modern science. The dignity which in the Middle Ages was reserved for God, fell, in modern times, to the lot of the world. Thus our views of the nature of the world had also to assume the task of giving us the inner security which the belief in God had previously given. Convictions like that of absolute space and the reducibility of all natural phenomena to the geometry and mechanics of the atoms owe their power to this role—which was generally

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allotted to them unconsciously, though in the materialistic camp this was also done consciously. The discussion of the question of infinity teaches us the same lesson.

Thus the argument from the second law of thermodynamics to a limited duration of cosmic events produced a will to contradiction which is not explained by the logical gaps that might be contained in the argument. Rather, the feeling of modern man bristles against the suggestion that the world will not last forever. Why? The suffering of every man from time and mutability is in fact so great that he must believe in something exempt from annihilation in order to be able to live. Those who no longer believed in an eternal life needed at any rate to believe in an infinite duration of the material world. The symbolical power of this belief scarcely permits a person to realize that there is little use to him personally in a persistence of his environment after his own death. Finally, the infinity of the world in every dimension is the most striking symbol of the belief in progress. For the medieval Christian, a sufficient goal, worthy of infinite struggle, was set in the return of his soul to its home in God. At the moment in which the goal of our striving is placed outside us, every thing exhibits a limited value only, and our inborn urge towards the absolute can nowhere rest satisfied, but sees every step as justified only as the preparation for the next one. Only the notion of an infinite field of activity provides for this attitude to life at least the symbol of solace; every thought of a limit, however remote, disquiets it deeply.

A deep breach has been made in this symbolism by the discoveries of the present; its very foundations are already destroyed. The new experiments and the methodological reflections aroused by them have destroyed the influence of the symbolism of infinity on the formulation of natural laws. In the question of how natural laws must be formulated, the realistic component of science has again proved victorious over the symbolical one. This victory does not represent any new decision, but is merely the consequence of

the decision which had already been made at the time of Galileo. The world is a realm of facts, and we guide our judgment about these facts by the experience which we can attain about them, and not by the symbolic value which they are supposed to have for us. For could a false or doubtful assertion about facts provide us with a credible symbol? What is new is only the discovery that at a decisive point we ourselves had not yet completed our voluntary restriction to the factual.

To-day, we have reached a degree of scientific disillusionment as extreme as we can imagine, and we look back at the road we have trodden. At all times the world has meant God, but in different ways. Once it was itself divine. Later it pointed back to its creator. In modern times it became a substitute for God. We have not returned herewith to the world of prechristian paganism which was itself divine or, as has been well said, was itself full of gods. Only as a beautiful memory—but for the most part misunderstood—has this world arisen again for modern times. The road back to it was blocked by a stronger drive: the urge to the infinite. The gods of the pagan world are the powers of nature in us and around us. But our natural sphere of life is finite and conditioned. Christianity made the infinite and unconditioned the goal of our striving. The secularized man of modern times has continued to strive in this direction, and has simply sought the fulfilment of his urge on another plane. It is therefore the essence of his relation to nature that he transcends the limits and conditions of his existence, which originates in nature. He presses on, to put it in ancient terms, into a realm where there are no gods, or whose gods are strange to us. He thus acquires a knowledge and a power which would have seemed to all earlier times no less improper than impossible. Knowledge and power were our gods, and they wore the halo of the infinite and the unconditioned.

To-day we begin to discover what we can in truth achieve: not the unconditioned, but an insight into the conditions of

our knowledge and our power. We wanted to press on behind appearances to the things themselves, in order to know them and to possess them; now it appears that precisely beyond our natural perceptual world the very concept of thing can be defined only in relation to the man to whom it appears or who himself makes it. Knowledge of nature and power over nature are in their essence finite. This they are, not like a limited power which could expand further and further in an unlimited space, but their limits lie at the place where they must themselves first produce that with which they are concerned. The symbols which suggested their infinity had to crash, because they themselves were dreams and not real knowledge or real power.

Must we rest content with this? Does the world mean nothing? Is only that man happy—if such a one there be—who can forever evade the question of the meaning and the right with which he uses his knowledge and his power? Does nothing remain for the others, who have run through the movement of modern times, but that in addition to the two ancient aspects of the world: suffering and guilt, meaninglessness has entered as a third?

I do not believe it. I do believe, however, that we must first see the abyss, must bear the emptiness. But I believe that this emptiness means, not the end, but the demand for a decision. Does God no longer speak to us? Our substitute symbolism has broken down, not because it was symbolism, but because it was a substitute. But the silence which has taken its place is eloquent enough, for it puts before us our real situation. We must in fact know only whether we want to hear God at all—not where we wish to hear him, but where he really speaks to us.

It may be right to let this stand here as a question. But in the scientific context there arises a partial question which we wish to discuss in conclusion. Granted that we were ready to hear and would open ourselves therewith to the decisive human experiences which await us, shall we not be

thus led irrevocably away from science? Does not our life then fall into two areas: genuine human existence and the technical-physical world, which is inescapable but in itself meaningless? Can we, without deceiving ourselves, reverse the process of emptying the external world of meaning?

Let us turn our attention once more to the last great union of natural science and objective symbolism. To what degree are we in a different position from Kepler? The purely scientific presuppositions have not changed since his time on the point that is crucial for objective symbolism. The mathematical law of nature was then valid as the medium through which nature represents mind. To-day, to be sure, the mathematical understanding of nature has progressed much farther. It has become so familiar to us that we are perhaps less astonished at its possibility. But we do not have, any more than Kepler did, an empirical-rational explanation for the fact that precisely those laws of nature which maintain themselves in experience, are again and again distinguished from all other conceivable ones by a peculiarly high degree of mathematical simplicity. The often cited principle of economy of thought explains, at the most, why we look for simple laws, but not why we find them. This becomes clearest from the fact that we often look a long time in vain, and that we know very well how to distinguish between a plausible working hypothesis and a well-grounded theory. We penetrate deeper when we think, at Kant's instance, that order in accord with law is the condition of the possibility of experience; that without the existence of natural laws we could not even form the concepts in terms of which we look for them. To be sure, this thought does not explain why there are natural laws and concepts adapted to them. But it does explain why we may not expect to receive a general answer to our question in terms of concepts which are formed in the manner of science. For the mere application of such concepts already presupposes the existence of the kind of law in question.

But it is out of this very situation that Kepler's con-

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sciously metaphysical argument arises. This is expressed rationally in the explicit reduction to God, that is to a reality which can no longer be scientifically described, which is the origin at once of nature and of our minds. But in this there is revealed at the same time an experience of a non-scientific character. The mind which in the objective order of nature meets the mystery of its own origin, experiences how fact becomes translucent, as it were, as bearer of a meaning that is no longer expressible. This experience appeals to another level of personality than the arbitrarily reproducible sense experience which is employed in physical experiment. Apparently the question whether we consider something like an objective symbolism of nature possible comes down to the question how we react to the experiences of this level of our being.

Modern science has excluded that question from the circle of its obligatory premises and has banished it to the private sphere. Thus it determined in advance the result at which it has now arrived. At the same time it gradually removed from that level of experience the means of expression which would make it generally intelligible, and has thus made the way to full consciousness more and more difficult for it. That modern science could not stifle such experience altogether is proved by the very occurrence of the substitute symbolism. What speaks with an even more undisguised voice to those who can hear, is the almost religious relation of many actual scientists to their object. The hesitancy of the modern scientist to speak of this side of his work is for the hearer in many cases as eloquent as the inspired words of Kepler. Not only does it require an attitude of devotion and faith to create the conditions for a discovery; but the awe before the truth as it bursts upon us, before the abyss into which we look when all at once and undeniably we meet the reality we have not made, is close to the awe of man before God.

Seen epistemologically, the separation of symbolism from science is thus only the consequence of the resolve to follow

a certain method. Historically, indeed, this resolve seems to have been unavoidable. It was itself the expression of a certain human attitude. But we may infer that a new change of human attitude can bring this question too into a new light. The results of contemporary science themselves point in this direction. Not the isolated thing, but only the whole of the relation man-thing is valid for modern physics as an intelligible reality. A change in one pole of this relation cannot leave the other untouched. It is idle to speculate on the effects of this change before it has occurred. Certainly it will not turn the clock of history back. It will not destroy the procedurally pure concept of science which the modern age has elaborated. But it will teach us to see something besides that, which we overlooked before because we did not want to see it. Perhaps, one day of which no one to-day can say if it is near or far away, a new man may open his eyes and with astonishment see before him a new nature.

Chapter Six

NATURAL LAW AND THEODICY

PROBABLY no philosopher has penetrated metaphysics and the mathematical sciences with so unified a movement of thought as Leibniz. What he thus achieved in metaphysics with respect to clarity, he certainly did not sacrifice in profundity. But above all he has thus placed the mathematical sciences before the background appropriate to them. To follow his thoughts through is to perceive how, in the choice of our simplest mathematical concepts, we make metaphysical decisions. For us who no longer live in the world of the enlightenment, it may be almost frightening when he moves the plenitude of the real into its very depths with the apparent ease of obvious mathematical inferences—as if he were dealing with a perfect clockwork. But what we have to learn from him is the relationship of all levels of being. As we conceive number and motion, so, undeniably, we conceive God too, and if we wish to conceive God differently from the way Leibniz did, we must be able to conceive number and motion differently too.

For the basic ideas of the *Theodicy* and the *Characteristica Universalis* the relationship which we are here referring to has been recently presented by Heinrich Scholz. He has emphasized, in particular, the way in which the *Theodicy* is constructed according to the variational principles of physics. The following pages are intended to be nothing more than the commentary of a physicist on these ideas.

THE THEODICY

They owe their origin to the wish to give a token of friendship and respect to this man who embodies the unity of metaphysics and mathematics in our time.

THE THEODICY

Leibniz dared to designate the real world as the best of all possible worlds.

Did he not know the world? Did he not know that it is the world of everlasting guiltless suffering, and the world of everlasting inescapable guilt?

He knew of suffering and of guilt. He knew the plaint to which men have let themselves be tempted, ever since they believed in a creator of the world: Why hast Thou not, God, if Thou art Goodness, created a better world? He dared to answer this reproach with the instruments of reason. He dared to say that God had not made a better world because he could not make a better world. But that he could make no better world lay not in his impotence but in the fact that this world is the best possible. If this is understood, then God's creation is justified, the Theodicy has succeeded.

Because of this argument Leibniz is considered a philosophical optimist. But would not such optimism be sophistical self-deception? Granted that we could see that a better world than ours is not possible, would the evil in our world be thus lessened? What does it avail the suffering creature to know that there can be no world in which he need not suffer? Is not, conversely, every hope of a better world cut off by this reasoning? Must not the thesis of Leibniz have as its consequence the most extreme pessimism?

Leibniz might indeed have said in answer to this objection, that the sum total of good in the world outweighs the sum total of evil, and so it is nevertheless better for the world to be than not to be. But what actual sufferer is convinced by this idea? Does not the pessimistic objection reveal at least the daimonic ambiguity of every attempt to solve the

riddle of existence like a problem in arithmetic? The being of God is the judgment on man; can man then even attempt to sit in judgment on God, even if it be to acquit Him?

But perhaps this destruction of the superficial meaning of the *Theodicy* is necessary in order that we may at least have a presentiment of an experience which Leibniz may have had when he dared to think through these ultimate questions—and which is reflected less in the content of his assertions than in the grandiose naïveté of his argument. Knowledge changes man. We bear even the small evils of every day more lightly if we understand their necessity. This does not stem from the wretched comfort contained in the counsel to make the best of them because they are not to be avoided. But consciousness is filled with a new object. The place of the purely factual and the evil which is unintelligible in its facticity is taken by a larger relationship, a necessity, a meaning. Probably the philosopher in Leibniz' sense has a similar experience when he surveys the world as a whole. It is no longer the mass of particular factual evil which is the object of his consciousness, but the meaning of the whole. If he should succeed in seeing not with the eye of his limited human existence, but with the eye of God, then the choice between optimism and pessimism would be resolved. He would see that the world, if it is to be at all, must necessarily be as it is, and thus he would no longer say 'it is good' or 'it is bad', but simply 'it is'. With that all would be said.

Perhaps we may be able to follow Leibniz through this experience. But can we also follow him in thought? Let us turn to the conceptual structure of the idea of the *Theodicy*.

'World' in the strict philosophical sense is a *singulare tantum*. We live 'in the world'. There is not 'a world', there are not 'worlds'. The 'world of the baroque', the 'distant worlds' of astronomy are only segments of 'the world'. Yet Leibniz speaks of 'possible worlds'. In what sense and with what right?

He is standing on the ground of western metaphysics, for which God alone is a necessary being. His being follows

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from the idea of Him. The world on the other hand might or might not be, be thus, or be otherwise; it is contingent.

When God created the world, he had all possible worlds before his eyes to choose from; he selected one of them, the best, and gave it existence. Thus our world became real, it became 'the world'. But every possible world was just as much 'the world'; only the predicate of existence was denied it.

By this argument the uniqueness of the world is made a philosophical theme in a peculiar fashion. The concept of 'possibility', which enables us to conceive things without the predicate of existence, permits us to place the fiction of other, merely imagined worlds, alongside the one world which is the only one we know. But this fiction only serves to distinguish the 'real' world through certain properties from all the possible ones, and thus as it were again to dissolve its contingency. Its properties are conditions of its existence. It is real because it is as it is; and it is unique because its properties are unique, that is, optimal.

It might be said, that this concept achieves the uniqueness of the world by a detour which was indeed necessary for the older metaphysics but is no longer binding on us. 'Being in the world' is said to be a basic determination of being human and therefore a presupposition of all knowledge. Therefore it would also be impossible for the concept of possibility to embrace the concept of world; there are possibles in the world, but there are no possible worlds. This question acquires special weight to-day in view of the attempt of Heinrich Scholz to define logic as the totality of those propositions which are valid in every possible world, and thus to ascribe to it the character of a metaphysics.

We do not dare to discuss this question here in its full breadth. We shall limit ourselves to the reminder that the argument of Leibniz is the philosophical copy of considerations which are at home in physics and are there without doubt legitimate. We wish to bring to mind the content of these physical considerations and to refrain at this point

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from examining explicitly the legitimacy of their transference to philosophy.

VARIATIONAL PRINCIPLES AS NATURAL LAWS

The argument of the Leibnizian *Theodicy* is exactly analogous to that of the variational principles of physics. Let us take as perhaps the most familiar example Fermat's principle of the shortest path of light. This principle declares: A ray of light always chooses that path on which it travels the distance from its initial point to its end point in the shortest possible time. The three basic laws of geometrical optics can be derived from this principle: the law of the rectilinear propagation of light and the laws of reflection and refraction. At the same time the principle expresses the mathematical condition for extending geometrical optics, which works only with the concept of the ray, to a wave optics; and conversely it can be derived from the differential equation which light waves satisfy.

We shall clarify for ourselves the content of the principle, if we derive from it its simplest consequence, the rectilinear propagation of light. Given two points in space, for instance a distant star and an observer on the earth. By what path will light travel from the star to the earth? According to the principle, it will choose the path by which it will arrive in the shortest time. The shortest distance between two points is a straight line. Therefore the light will travel in a straight line.

This inference needs an explanation. We have silently assumed that light travels with equal velocity on all possible paths; only then will the geometrically shortest path also be traversed in the shortest time. This assumption is justified in empty cosmic space. On the other hand the phenomenon of refraction follows precisely from the fact that the velocity of light is different in different substances. If for example a ray of light falls diagonally on a surface of water then it is as we know 'refracted' in such a way that it travels down-

ward more steeply in the water than it did before in the air. The fact is that in water the velocity of light is less than in air. The straight line which united the source of light in the air with the terminus of the ray at the bottom of the vessel containing the water, is indeed once more the geometrically shortest path, but it is not the path which can be travelled in the shortest time. Instead the light gains time if it travels a somewhat longer distance in the air, and thus faster, and can then go through the water, which must be traversed more slowly, on the steeper and therefore shorter path. In the same way also a foot traveller who wants to reach a destination quickly in going across country, does not go as the crow flies if instead he can follow for some time a regularly laid out road.

Now we can make more precise the use of the concept 'possible' which is characteristic of this principle. All geometrically conceivable lines of connection of the source of light with its destination are considered 'possible paths'. These paths are not in the strictly physical sense possible; for the natural law which is formulated in Fermat's principle selects one among them as the real one and therewith stamps the others as really impossible paths. They are possible only for a certain mode of consideration, namely one which does not yet take account of Fermat's principle. On the other hand this mode of consideration does not ignore all natural laws. For example, at every point in space that value must be assigned to the velocity of light which really belongs to it there according to natural laws; that light should travel in water with the speed it has in air is not counted as possible.

We can see the analogy to the argument of Leibniz. For example, a world in which what he considers the logically necessary connection between certain goods and certain evils did not exist would not be designated by him as a possible world. Perhaps we may say that possible worlds in his sense are all those, and only those worlds which have at the same time all the properties which necessarily belong to the real world, with the exception of those properties which

follow only from its special character as best of the possible worlds. Thus they are exactly like the possible paths of light, merely imaginary things, which are constructed for a methodological purpose. At the most they were objectively possible before creation in the same degree as the real world; thus we might for instance say that in the mind of God all paths of light were possible, before he brought it to pass that Fermat's principle should hold in the real world.

This comparison loses its somewhat playful tone, if we consider that the variational principles of physics were for Leibniz not mere analogies, but definitive consequences of the optimal character of the real world. In the best possible world variational principles must be valid, and that such principles are valid in the real world confirms the fact that it is the best. The shortest path of light is the best, and a world in which light did not choose the shortest path could therefore not be the best. This mathematical optimality of the world probably engrossed Leibniz at times much more strongly than the moral; in fact his *Theodicy* is basically an attempt to reduce the moral optimality of the world throughout to a kind of mathematical optimality. That to-day not only optics, but equally mechanics and the theory of electricity, and even the new type of atomic physics, as far as we have yet obtained mathematical mastery of it, can be reduced to variational principles, would be a triumph for him. Only, indeed, if his 'optimistic' interpretation of the variational principles still has a meaning for us. Can we attach some meaning to the assertion that the shortest path of light is the best?

CAUSALITY AND FINALITY

The variational principles have often been understood to be the expression of a finalistic, purposive character of cosmic events. In general the laws of nature are valid for us as the expression of causality reigning throughout nature, according to which the state of a thing and its environment

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at any instant determines the state of the thing in the instant immediately following. The variational principles appear to break through this schema by means of a teleological type of order: the path of light is determined through the end point which the light ray is to reach only after this path has been traversed. The factor determining the event seems here to be not a 'mechanical' cause, but a goal to be reached in the future, a purpose. So it was concluded that there is a plan governing nature, and thus a spiritual principle in nature.

A banal interpretation of this finalism must be at once rejected. When strictly formulated, Fermat's principle does not assert that the path of light must be a minimum, but only that it must be an extreme; it may also be a maximum. As a matter of fact, there are arrangements of optical instruments, in which the actual path of light is not the shortest, but the longest possible; in between lies the limiting case of the 'ideal optical image' in which many paths of light, which are all realized, because they can all be traversed in the same time, lead from the one point to the other, its image. Thus it is not the case that God 'had no time' and therefore developed cosmic events as quickly as possible. But even in the generalized form of the principle, the real path is still distinguished from all possible paths in mathematically transparent fashion. And above all the teleological anticipation of the future is retained; the path of light is still determined by its end point.

Let us explain the opposition of causality and finality through a brief historical consideration. In modern times the concept of cause has become narrower under the influence of natural science. Scholasticism, proceeding from Aristotle, distinguished the famous four kinds of causes: material, formal, efficient and final cause. The investigation of the real meaning of this distinction would lead deep into the problems of Aristotelian philosophy. A simplifying example may remind us of its practical use. Let us look, for example, at a wine glass. Its material cause is the stuff of which it is made, i.e. the chemical substance of the glass.

Its formal cause is (to remain at the simplest level) its shape: that of a chalice. Its efficient cause is what has made the glass: the hand and the breath of the glass blower. Its final cause is its purpose: that wine should be drunk from it. Thus the same thing generally has all four causes at once: they do not compete with one another, but indicate the different points of view according to which the thing can be traced back to a principle ($\delta\rho\chi\acute{\eta}$). >

The modern age, on the contrary, calls something the cause of a thing only when it lies outside the thing. Thus the two first causes, which are only present in the thing itself, at once disappear; according to this way of speaking, matter and form designate the essence, but not the cause of the thing. Corresponding to this changed mode of expression is the polemic of the natural scientists against the scholastic thesis that 'substantial forms' or 'qualities' can be causes—a polemic which entirely misses Aristotle's original meaning. The change in the actual state of affairs corresponding to this change of expression might be said to lie in the fact that the modern age turned to an instrumental conception of science. To the admiring contemplation of things is now joined the wish to master them. If knowledge is power, then it must above all know the means of itself making things and appearances or at least influencing them; for every fact it must know the efficient cause. The criterion for the fact that one really knows the efficient cause, is that one can predict correctly the event produced by it. Thus the concept of cause has been so transformed, that the causal principle in modern natural science has been identified precisely with the complete predictability of natural phenomena. The mathematical expression of this causal concept is the representation of natural processes through differential equations, which express the time differential derivative of the quantities which characterize the state of a thing, through these quantities themselves: the state itself determines from instant to instant its temporal alteration.

Finally, through this emphasis on efficient cause, final

cause is brought into an ambiguous position. On the one side the instrumental thought of modern times fits in very well with the view that natural phenomena are produced according to a plan; the inference from natural phenomena which seem to exhibit a plan back to the purpose of a creator of the world has often been hazarded. That this was already a deviation from the Aristotelian meaning of final cause was for the most part unnoticed. But on the other side the very assumption, that the totality of efficient causes completely determines the course of the future, leaves no room for further, concurring final causes. If the state of a ray of light at an instant already fully determines its further path, then this path—so at least it seems—cannot be influenced, in addition, by the demand that it should reach a given end point as quickly as possible.

It is a decisive cognition of modern mathematics which has penetrated much too little into the general consciousness, that this contradiction between the causal and final determination of events does not in fact exist—at least not as long as we are permitted to give precision to the principle of causality through differential equations and to that of finality through variational principles. The calculus of variations teaches us to express the same mathematical demand either by a variational principle or by a differential equation: the differential equation states the relations which must hold on a small scale (from place to place), in order that the effect demanded by the variational principle should hold on a large scale (Euler). Thus we can calculate, according to the differential equation, what end point will be reached by a light ray which has started to travel in a given direction. This place is determined precisely in such a manner that, in order to reach it in the shortest possible time, the light ray must take exactly the direction which it has in fact taken. Thus the final 'goal' and the causal 'law' are only different ways of expressing the same principles. The goal only states the consequence which must necessarily result according to the law, and the law is set up in precisely such a way that the

effects governed by it must realize the goal. The anticipation of the future, which is expressly stated in the conception of a goal, is also implicitly contained in the expectation that the law will obtain always and everywhere. Strict determinism itself permits the inference from future to past just as much as that from past to future.

We have just been considering the mathematical background of the Leibnizian doctrine of the pre-established harmony. As a matter of fact a clockmaker does judge his clockwork both causally and finally in the same act; he arranges his wheels in precisely such a manner that they spontaneously fulfil the goal set by him in virtue of their mechanical properties. Thus before God there is no difference between the causality and finality of the cosmic process. For us to-day, indeed, this construction too is only an analogy and not the truth itself. Metaphysically we are much farther than Leibniz from regarding God as a distinguished clockmaker. And physically, atomic physics has deprived us of that very determinism which is the foundation of the whole construction. To be sure, atomic physics, too, is governed by differential equations and variational principles, but the quantities which satisfy these mathematical relations are no longer objective properties of existent things, but mere mathematical constructs, from which the probabilities of the results of measurements can be calculated. What is still binding for us in the Leibnizian argument?

Perhaps we may say: it is the reduction of causality and finality to a higher principle which exhibits a certain kinship with the old formal cause. From the outset, we were unable to give a naïvely finalistic interpretation to the variational principles; we should not know how to say what concrete purpose the creator of the world thought to achieve with Fermat's principle. If we had put this question to Leibniz, then he would perhaps have answered: the purposes of God are of a higher kind; God does not need the world for some use—but he wants it to be perfect. And the perfection of a world in which variational principles hold, consists in the

fact that it unites the greatest richness in phenomena by a law as simple, and as transparent for the mind as possible; it consists in the fact that such a world possesses the greatest intellectual beauty. §

Literally the same thing can be said about the mathematical version of the efficient cause. Precisely the mathematical formulation of natural laws has at last conquered the coarsely mechanical tendency of modern physics, the derivation of all things from 'pressure and collision'. For what already held, basically, for the Newtonian axioms of mechanics, became evident in the differential equations of electrodynamics and atomic physics; these equations are not distinguished from other conceivable ones by the fact that they describe mathematically a concept of mechanical action which was already clear beforehand. Rather it is they alone which have given a precise content to our ideas of cause; but they owe their justification to experience and their convincing character to their mathematical simplicity. Mathematical form, in fact a kind of formal cause, remains in physics as the last intelligible content of our old causal concepts. The concept of form is herewith extended to the realm of time. Differential equations and variational principles mean that a physical process represents, temporally, a whole, a configuration; causal analysis appears as the highest stage of morphology, in a fashion which can look paradoxical only when interpreted under false suppositions.

Thus in the mathematical law of nature we find, with Leibniz, mind in matter. If we refrain from every metaphysical interpretation, this means in the first instance, that matter is so constituted that man can think it; through its property of being a possible object of thought, it has a share in mind. A glance at the sphere of problems which concerned Kant shows that this proposition is not trivial. But the mathematical simplicity of natural laws reaches much farther than this. Even the positivistic scientist will be able to follow Leibniz's *Theodicy* in the confession that none of us has ever succeeded in thinking of another possible world

which unites such a plenitude of phenomena with such simple basic laws.

The question of the reality of mind in nature which we have here introduced is united, finally, with our initial question of the right and the limits of the application of the concept of possibility. In the above discussion we have called the possible paths of light of Fermat's principle, as well as the possible worlds of the *Theodicy*, mere ideal things which were constructed for a methodological purpose. Over against the plenitude of the real they have the poverty of the precisely conceived. A real path of light is for a living, observing man a road of wonder; a possible path of light is a geometrical curve and nothing more. But the 'possible things' are on the other hand necessary instruments of thought. For since we do not know in advance the full truth, we can think of the real only by separating it out from the plenitude of the possible. The one marvel, that we can think of the real at all, presupposes the second, that we can meaningfully apprehend the concept of possibility. The attempt to state the conditions under which this can happen we shall no longer undertake here.

Chapter Seven

THE EXPERIMENT

THE MEETING OF SUBJECT AND OBJECT IN THE EXPERIMENT

WHEN Galileo experimented with balls of a definite weight on the inclined plane, when Torricelli caused the air to sustain a weight which he had calculated beforehand to be equal to that of a definite column of water, or when Stahl, at a later period, converted metals into lime, and reconverted lime into metal, by the addition and subtraction of certain elements; a light broke upon all natural philosophers. They learned that reason only perceives that which it produces after its own design; that it must not be content to follow, as it were, in the leading-strings of nature, but must proceed in advance with principles of judgment according to unvarying laws, and compel nature to reply to its questions. For accidental observations, made according to no preconceived plan, cannot be united under a necessary law. But it is this that reason seeks for and requires. It is only the principles of reason which can give to concordant phenomena the validity of laws, and it is only when experiment is directed by these rational principles, that it can have any real utility. Reason must approach nature with the view, indeed, of receiving information from it, not, however, in the character of a pupil, who listens to all that his master chooses to tell him, but in that of a judge, who compels the witnesses to reply to those questions which he himself thinks fit to propose. To this

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single idea must the revolution be ascribed, by which, after groping in the dark for so many centuries, natural science was at length conducted into the path of certain progress.'

According to these true words of Kant, the experiment is a drama between two partners, 'Reason' and 'Nature': Instead of 'Reason', to-day we would prefer to say 'Man'. For, on the one hand, our science does not know a reason other than the human one, and superhuman reason would hardly need to experiment; on the other hand mere reason is not adequate for experimentation, since man needs for that also his hands and his eyes. All this Kant, too, knew, but for us the emphasis lies less exclusively on the rational component.

Accordingly, the experiment would be a give-and-take between man and nature, but not every give-and-take between these two partners is an experiment. The roles of the two partners in this particular process are more clearly defined by the grammatical concepts of subject and object. An experiment exists only where knowledge is sought for. 'To know' is a transitive verb: someone knows something. 'Someone,' the subject, is some man; 'something,' the object, is some thing, process, condition in nature. In the following, I shall use the terms 'subject' and 'object' mostly in relation to the knowledge conveyed by the experiment.

Now of what must the two partners be capable, what must they do or undergo for an experiment to take place? It was just said that man uses, besides reason, his hands and his eyes for the experiment. Only the triad of thinking, acting and perceiving makes the experiment possible. What conditions do the two partners have to meet so that the triad of processes can be active between them?

We cast a first brief glance in the direction of the subject. In our traditional epistemology the role of perception was the most self-evident. The experiment is a kind of sensuous experience, but receptive sensibility does not exhaust the experience gained in the experiment. Indeed, the comparison of naïve perception with the experiment has taught us to see

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how much activity is involved in seemingly pure perception. It is not even sufficient that, in addition to perception, one of the two active dispositions is added to perception: only thinking or only acting. In the first case, the result is philosophy, the second, handicraft. It is fitting that Galileo stands at the beginning of Kant's exposition, because he is probably the first to embody that unity which is no longer either philosophy or handicraft because it encompasses both; each of his manipulations is guided by thought, each of his thoughts by experimental evidence.

THE OBJECT IN PHYSICS

We now turn to the object and confine ourselves at first to the physical sciences. It is not sufficient that man is able and willing to perceive, think and act. An experiment can occur only because and inasmuch as nature itself is capable of being perceived, thought about and acted upon. We do not know, a priori, to what extent nature has these qualities. Only experimental science teaches us to know the conditions under which the experiment is possible inasmuch as these conditions lie on the side of the object. Like every single experiment, the undertaking of experimental science as such requires the courage to undertake something the outcome of which is uncertain. What has the development of science up to date taught us about the conditions which make science itself possible?

Again, from the side of the subject, the most self-evident prerequisite for the experiment seems to be perceptibility of natural phenomena; again the isolated consideration of the latter is deceiving. Certainly experimental science would be impossible if nothing could be perceived, but it would be unnecessary if everything could be perceived. The purpose of the experiment is the exploration of that which has not been perceived hitherto; either because it reveals a process which has not been revealed before or because it reveals effects of a process which continues to remain unrevealed.

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In view of this, Mach's sensualist concept of reality is, to put it mildly, an inexpedient re-naming. 'Esse est percipi' is a phrase which is not in conformity with the spirit of experimental science; first it should not read 'to be perceived', but rather 'being capable of being perceived', and secondly allowance should be made for 'indirect perception', i.e., perception of the effects of something which cannot itself be perceived. Even in the cloud chamber I do not see atoms, but I see drops of water and deduce the existence of atoms. The building and expansion of the cloud chamber contain the element of action, the deducing of the atoms, the element of thinking. Perhaps one may say; immediate sensuous perceptibility of the thing is a sufficient but not a necessary condition of our belief in its reality; immediate sensuous perceptibility of some object connected with a thing is a necessary but not adequate condition for the ability of experimental science to make a statement about this thing.

Thought is a bridge from the perceptible to the non-perceptible part of the real. It connects the perceptions through the concept into a system. It plans the experimental operation and interprets its result. It is the judge who poses the question to the witness, nature. But the inquiry is possible only because and inasmuch as the witness understands the language of the judge. True, as Kant has told us, 'The applicability of the categories of our thought to nature is the prerequisite for the possibility of all experience': only inasmuch as the principle of causality is valid can I understand a phenomenon as the result of a cause which is not apparent and thus deduce the cause; only inasmuch as the laws of nature are valid is there constancy of phenomena, hence meaningful definitions of things, hence scientific communication between men about nature. But we do not know, a priori and generally, that nature will submit to thought. We try to apply conceptual patterns, which appear to us plausible or self evident, to every new realm of reality and we explore experimentally how far we can get with this procedure. The fact that certain laws are prerequisites for the

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possibility of a certain kind of science does not compel nature to obey these laws; where it does not obey them, we have to give up this kind of science. Thus we apply successfully to our measuring instruments Euclidean geometry, the law of causality, and the category of substances; but it is exactly with these instruments that we discover phenomena for which those same principles are no longer adequate. That this occurs to us with objects measuring 10^{-8} cm. or 10^8 light years should perhaps surprise us less than the fact that we still find abstract mathematical, hence thinkable, systems of scientific laws to which such objects are obedient.

The thinking of our science proves itself only in action, in the successful experiment. To experiment means to exert power upon nature. The possession of power is then the ultimate proof of the correctness of scientific thought. The limits of the applicability of certain concepts or laws appear to us only in the form of the impracticability of certain experiments—for instance, the determination of an absolute speed—or in the form of the mutual incompatibility between two sorts of experiments like the measuring of position and impulse in an electron. Concepts like relativity, complementarity, and invariance under transformation groups take account, even in abstract theory, of this operative character of our science. Exponents of an older ideal of science are often surprised at the seeming naïveté with which we ‘confuse’ the non-demonstrability of a situation with its non-existence. In truth, our science has merely adapted its conceptual form by one more degree to the fact, existing since Galileo, in that its ultimate criterion of truth is the experiment. We only have to inquire rigorously whether an experiment is impracticable for us because of inadequate technical means or ideas or for every and any experimenter because it conflicts with a positive law of nature.

But although the operative concepts within a given experimental science will be consistent, they reveal nevertheless that our intellectual point of view is different from that

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of the founders of that science. For them, nature was an existing reality which man had only to learn to know.

The factual situation which Kant's judge wants to discover exists 'in itself' prior to the inquiry. The scientific law indicates what happens in nature objectively without our intervention; it is a law of that which is. For us, it is a law of that which is possible. For our concept of classical physics, it is important that the mathematical form of its scientific laws, the differential equation, yields an unequivocal solution only when initial conditions are given; that this scientific law has the form of the conditional statement: 'If situation A is realized, condition B occurs as its result.' In those realms of nature which cannot be observed immediately, the scientific law becomes more and more only an indication in regard to the possibility and the result of experiments; a law concerning our ability to produce phenomena. This law, too, is objective, because, with certain means, only certain phenomena can be produced. But what has been objectively determined here is no longer a being, but a give-and-take between acting and perceiving. We may speak of objects only inasmuch as they are possible objects of a subject. This sentence, too, is almost a truism since we define knowledge as the knowledge of an object by a subject. But we could not anticipate that the form of our statements about objects would, in terms of pure logic, no longer permit us to pass over in silence the fact that, in each case, they are being pronounced by a subject. The metaphysical hope of classical physicists to gain through science a solid hold on being itself collapses. Contemporary physics compels the physicist to look upon himself as a subject.

THE HISTORICAL PLACE OF THE SUBJECT

We return to the side of the subject. We have summarily designated the subjective conditions of the possibility of the experiment as the human abilities to perceive, to think

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and to act. By a closer inspection of these abilities, we shall perhaps interpret and change our concept of the subject in the same way as we have that of the object—through the contemplation of their objective correlatives. Thus, by a generally anthropological method of inquiry, one might explore the three abilities, the level of conceptualization as defined by their distinction in our thought, and the way in which they are *de facto* interconnected. Here, however, I should like to confine myself to a historical observation.

We probably face the modern polarity of subject and object for the first time in Descartes' antithesis between *res cogitans* and *res extensa*. We recall his way of reasoning. Even if I doubt everything, I know nevertheless that I doubt, hence that I think, hence also that I am. *Cogito ergo sum*. So the most certain in me, the *res cogitans*, is the necessary subject of every cognition. What enabled Descartes to return to faith in the 'external world' is a proof of the existence of God. If the being of God can be deduced from the concept of God, and if I may be convinced that God does not deceive me with the ideas which He has planted in me, and with the ideas which He has granted me, then Nature is real. Now Descartes conceives of Nature as purely spatial, as *res extensa*, and subjects it to the way of thinking of the then beginning classical mechanics. Thus, Nature is, in fact, introduced as the object which can be known to the subject and which exists in itself.

This construction, like the head of Janus, turns one face towards the Christian thought of the Middle Ages and the other towards the scientific thought of the modern era. That Descartes' heart was on the side of the modern era is almost certain. In his triangle—I, God, World—God is only the mediator of the transition from I to World. A later age felt Descartes' God to be a kind of auxiliary quantity introduced to facilitate his calculations and removed from the final equations; and one tried to show that the calculation could be accomplished without the auxiliary quantity. While for centuries, philosophy moved within the problems posed by

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the Cartesian triangle, science chose with determination the 'calculation without the auxiliary quantity', and soon based its self-confidence on the results of this method. Classical science omitted, besides God, the subjective pole, the 'I'. As to-day we again take seriously the duality of subject and object, we find ourselves approaching the place of Descartes; and in this motion we cannot avoid looking in the same direction as that face of his system which was turned towards the past. We shall understand the modern, Cartesian, subject only against the background of Christianity.

The Cartesian *res cogitans* is an impoverishing schematization of that which Christianity called the soul. The rationalism of Descartes is well known. It may be explained through the triad of human abilities which the experiment requires. What Descartes allows as perception is dispassionate statement of fact. His thinking is 'clear and distinct recognition'. His relation to action is of particular significance. His thinking is active and not receptive; hence, his concern is the method of the right use of reason. But physical and affective action has for him no cognitive value; hence, it is difficult for the philosophy which stems from him to do justice to the experiment. Even for Kant, it is spontaneity which places reason above sensuousness. The Cartesian subject is solitary. Its natural ties with reality are severed; love and hatred, pleasure, pain and guilt are theoretically inconsequential emotions. Only an 'I' of this kind can conceive the idea that his own thinking is the only immediate certainty. No wonder that the most difficult problem of this philosophy was the interaction of soul and body. This impoverishment would be unimportant if it were the peculiarity of a single philosopher, but it is historically a probably unavoidable and typical process, which repeats itself in many variants. The difficulties which followed are therefore legitimate problems of the modern era. >

It has been said that the solitude of the Cartesian 'I' is the heritage of the solitude of the Christian, who is a pilgrim in

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this world. That may be; the *cogito sum*, too, may be found in Augustine. But Christian solitude is suffering; and, at least in the communion of saints, it is overcome. Cartesian solitude is serene; it is part of the sovereignty of the perceiving ego. The 'world' in which the Christian is solitary is the totality of his fellow man. But love is always the bridge to his fellow man. The 'world' of the modern scientist is material nature. The Christian is a subject among subjects, the scientist is a subject confronting objects. Therefore, with Descartes, the role of God becomes purely formal. For the Christian, God is indeed the only safe bridge to his fellow man and eventually to the animal and the thing. For God is love. He is not blindly determined, instinctive love, but free-seeing love; He is that understanding which is love. If one omits love from loving knowledge, the Christian God becomes Cartesian, the Christian soul becomes the subject of modern science.

We return to the question of the subjective prerequisites of the experiment. What distinguished both the true Christian and the modern scientist from all people committed to instinct or myth is that the former are free in regard to all earthly things. But freedom is a prerequisite of the experiment. Only where my action and thought are not determined by circumstances, urges or customs but by my own free choice can I make experiments. I do not want to explore here the question of why even mature and late antiquity, which had severed so many ties, experimented little. But why were there no experiments in the Christian world until the end of the Middle Ages? This can hardly have been due to the fact that Christian history was, in fact, for the greater part a history not of freedom but of the authority of word and tradition, community and the higher powers. After all, Christianity knows also the inner freedom against the Church even to the point of martyrdom. I think that the true answer to our question is the transition from loving to purely factual knowledge. Can we experiment where we love? For the Christian, the important thing is his

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fellow man; and he will hardly feel entitled, and certainly not urged to experiment with him. In the voluntary self-limitation of love, the conflict of force and freedom is solved. Free from all voluntary or involuntary restraint, only modern man experiments.

But don't we do an injustice to the scientist if we deny him love of the nature he explores? We do and we don't; here the multitude of meanings of the words in our vocabulary cannot deceive us, as our word, love, encompasses in a bewildering and profound manner the realms of sex, of Eros and of Agape. It also does not distinguish between the relation of 'I' and 'Thou' and the ties of a man to a thing. Who may deny to the scientist love for his object, love of truth, love of beauty? That the brain of a leech, the motion of the earth, the harmony of organic forms or of mathematical relations can distract him from all the rest of life, that is one way of love. But it is not the love of one person for another, of the 'I' for the 'Thou'. It is a love which, in its matter-of-factness, is capable of the highest degree of cruelty. It is the love of one who is lonely and which cannot be reciprocated by its object.

THE HUMANITIES AND BIOLOGY

✓
We have considered the mutual conditions of the meeting between the subject and the object in the experiment. We must now inquire into the limits of the experimental method. There are objections to this method within modern science, and there are objections to modern science as such because it originates in this method. Let us begin by considering the first objections.

The great fields of knowledge, biology and humanities, have experienced an invasion by that mode of thought which they have designated as mechanical, physical, scientific. In varying degrees, they have partly been enriched by it and partly have tried to resist its penetration. As a whole,

scientific thinking has been more successful in biology than in the humanities. Hence, we shall perhaps grasp more clearly the position of outright opposition in the humanities, and the problems raised more clearly in biology.

The objection raised by the humanities against scientific methods I may briefly express by saying that natural science does not know the 'Thou'. That is, the meaning of the postulate that intellectual phenomena should not be explained causally but should be understood. If, as a philologist, I try to understand the meaning an author puts into a text; or, as a historian, to understand the intention of an acting man, I enter as an 'I' into the discourse with a 'Thou'. This kind of meeting with its object, physics does not know, because it does not encounter its object as a subject. This personal understanding is a mode of experience which is accessible to us in regard to our fellow man; but not in regard to the stone, the star and the atom.

A second distinction has to be added which, however, appears to me less profound. Man is a historical being: a large part of the humanities is historical science. By the term, 'historicity,' we mean first the irreversibility of the chronological process, the uniqueness of destiny, the singularity of each situation. The term implies, secondly, the knowledge that this is so; a life conscious of the uniqueness of its own destiny. If we call the first aspect of historicity, 'factual,' the second, 'conscious,' then it is clear that nature is not consciously historical. It is, however, often not clearly realized that nature is factually as historical as man. The second law of thermodynamics and the irreversible development of phenomena, in the organic as well as the inorganic realm, show it. The time-scales of physical 'destinies' are longer, the limits within which we can find an approximately repeatable happening are wider, but this difference is only quantitative.

Must we conclude from this that it is altogether impossible to apply the scientific method to the science of man? The experiment is connected to the belief in law; since I do not

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experiment for the sake of the single case but for the typical event. I experiment in order to be able to predict. Hence, factual historicity sets a limit to the experimental method. I can change nothing of what is passed, and the relevancy of the present experiment for past and future remains problematical. But the position of natural science is no different; not without reason is it forced to name for each law its realm of validity. One cannot deny that the economic necessities of man's daily life or certain psychological processes are likewise realms of validity for certain laws. Therefore, in economics as well as psychology, though within narrower limits, experimentation is scientifically meaningful. But the fact that science does not know the 'Thou' leads us to an entirely different question: not whether one *can* experiment with man, but whether one wants to or may experiment with him. What do I do to my fellow man in treating him in my thoughts or actions as a mere object? In this question seems to focus everything that has rightly been advanced against the application of natural science to man. For, as far as I can see, the personality of my fellow man does not, in pure theory, pose limits to the application of the principle of causality to him, to the carrying out of experiments on him. He who refuses to address his fellow man as 'Thou' deprives himself of the decisive experience about him; but no impossibility keeps me from gaining this experience and from subsequently integrating it, exactly like the experience gained from purely descriptive means, into a pattern of causal processes, indeed from experimenting in the field of that experience as in that of every other. We all know the moments of introspection in which we realize that we, who believe we act freely, have followed a social convention, an economic expediency, an unconscious urge, an illusion; moments in which we become for ourselves the object of causal knowledge. We all know the temptation, in the borderland between hatred, love and indifference, in which modern man moves, to experiment cynically with the most human experiences. Is there any one of us who has not

succumbed to this temptation? This is precisely the experience of our time, that the 'Thou' is a value but a value almost without protection.

This is the place to speak of the biological and medical sciences. For, as a result of the state of science to-day, the thought of systematic experiments on man within the realm of the soul is not yet as familiar to us as within that of the body. We do not have to think necessarily of extreme cases which it is easy to condemn. Every first test of a new medicine, of a new surgical technique, in fact every gathering of personal experience by a young doctor contains elements of the experiment. And contemporary biology is based on the experiment on the living creature from microscopic examination of tissue segments to 'vivisection'. What have we to hope, what to fear, from this kind of science?

The trend to which we owe the extraordinary progress of biology and medicine up to the present day expects everything from the experiment and from causal thinking. Compared with the objects of physics, living creatures are more complicated and more obviously historical; but for natural science this means no more than a technical difficulty, which has resulted in a development of biology which has been slower than that of physics and is even to-day much further removed from a systematic form. The historical factor, which we call accident, requires that we shall probably never be able to understand the possibility, let alone the necessity of roses or elephants in the same way in which we understand even to-day the necessity of acoustic waves or rock-salt crystals. Likewise, it may be that we shall never be able to produce a living organism experimentally, because it is possible that among the necessary requirements for its generation is the natural selection of millions of years. But these are not essential limits to the scientific mode of thinking but technical limits of certain procedures which, within that mode of thinking, can be satisfactorily explained. No one will deny to astronomy its scientific character because we cannot experiment with stars; it is sufficient that in it

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we do not have to use essentially different patterns of thought than in the experimentally accessible realm.

In biology, too, this way of thinking has again and again awakened opposition. It seems to me that what is legitimate in this opposition can again be expressed in the sentence, 'The "Thou" has been forgotten'. This means, theoretically, that the realm of experience has not been fully exploited. To utilize it without anthropomorphism, for instance in animal psychology, demands indeed a very high degree of methodological discipline. The unsolved question of principle which enters here concerns the connection between the 'intelligible' and the 'explicable' manifestations of life—the connection by means of which the body houses, or is, the soul, and by which the soul manifests itself in the body or is the body. But it seems to me an error if the vitalistic attempt, in a narrower or wider sense, is being made to find phenomena which can be verified physically, but which are not essentially to be explained in terms of physics. The organism is theoretically, as little as practically, protected against the experiment—it has to suffer it. What the physicist can observe he will, in all likelihood, eventually be able to explain in his own terms. True, Bohr has weighed the possibility that the life-process might remain inexplicable in terms of physics and that nevertheless the validity of every physical law may be demonstrated by experimental means in the organism. Since a too severe experimental invasion kills the organism, we ourselves would have to choose whether we want to leave life intact or whether we want to observe the physical process. But in physics, too, we can explain many things despite the fact that we destroy them by excessive interference with them in the experiment. The X-ray ionizes the atom while revealing to us its structure. Consequently, Bohr's thought indicates a conceivable but surely not a necessary limit to the theoretical intelligibility of the object, 'life,' in terms of physics. But his thought may also remind us, on the other hand, that the living thing is also a subject, a subject like myself who is part of life. 'In order to

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explore life, one has to take part in life.' (Viktor v. Weizsäcker.) If I gain my knowledge at the price of killing, if the way in which I participate in life is murder, the knowledge gained may be pertinent but is it a blessing?

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The experiment is an exercise of power in the service of knowledge. It makes it possible, conversely, to apply knowledge in the service of power. Its benefits and dangers are the benefits and dangers of power. Power is not only a concern of politics. Politics only reveals for all that which has already existed in all. To experimental science we owe the modern way of life; from the railroad, which made the world small, to hygiene, which multiplied the population, to the questionnaire, which tries to keep in order the large number of people in the small world. We have to accept into the bargain infantile diseases of procedure, changes of living habits. But are we permitted also to accept into the bargain that the procedure is connected with killing for the sake of knowledge and knowledge for the sake of killing? Are these phenomena necessary evils, or avoidable degenerations or proofs of a flaw in our approach which we can escape, if at all, only by complete reversal?

If one is attentive, one can rediscover those questions to-day in every branch of science. One will also find that the scientists themselves are helpless in the face of them. This helplessness is perhaps least serious where it is admitted. I cannot attempt to answer these questions. I can only show, with the help of a personal experience, how complicated they are.

Many years ago, a good friend of mine who works in a biological institute led me through his rooms and showed me, among other things, a large colony of mice which had an inherited disposition to cancer, of which all of them eventually died. In his white laboratory coat, with the devotion of a scientist to his cause, he pointed out to me in the

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scrupulously clean room the accumulated sum of wilfully produced, defenceless suffering. Later, in a circle of collaborators of that institute, I pointed out that contradiction; I encountered astonishment and even doubt as to the seriousness of my question. How can one want science and yet take exception to its means? I must admit that, without explicitly planning to do so and, incidentally, without any weakening of my friendly relations with these colleagues, I have never again entered that institute. That was merely evasion. Things won't change because one individual fails to face them. But what should one alter?

During those conversations, I was told rightly that in my own institute atoms are treated exactly the same way as mice are in theirs. The objection that mice have feeling, while atoms hardly do, was insufficient for me. The chain of beings which connects the atom with man is continuous. Where is the dividing line? Is not the attitude more important than the object? What Goethe missed in Newton's attitude towards light was reverence; and I knew how easily power over atoms could be converted into power over man.

I was reminded, almost convincingly, that such experiments serve medical purposes. Mice suffer so that men may be saved from suffering. But does one realize how terrible this logic is? Could one distinguish it with a clear conscience from the logic of those who distinguish in this way among men, and who persuade themselves that they may cause suffering to certain human beings in order to save suffering from other human beings? Is vicarious suffering possible on any but a voluntary basis? Perhaps so, but he who acknowledges this opinion must be aware of what he is doing.

I will not attempt to answer any of these questions. What matters to-day are not clear distinctions which soothe our conscience and allow us to continue working as before, with the necessary respect for the designated limits. What does matter is for the scientist to understand that even in the most insignificant of his actions he shares, like every other man,

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the responsibility for the whole. He must understand which are the small causes for the great effects for which we suffer. As long as it is not a matter of course for the scientist to consider in his experiment the human element with the same attention as the neatness of his technical execution, I cannot look to science for salvation. What matters is a change of mind.

I want to add explicitly, that as I see it, there is also no generally valid Christian answer to our questions. The Christian is free to do everything which he can do in love. Perhaps we have to join in executing the separation of matter-of-factness and love, but we must also know the demons which this separation unchains in order to become aware of this freedom. The Christian cosmos of the Middle Ages, that fragile order which under the image of a God of love was stamped on a savage world, was a magnificent naïveté which we have outgrown. The modern era, the age of the experiment, has experienced its freedom in the emancipation from that image. It resembles itself a great experiment which cannot be interrupted midway. Perhaps its consequences will teach us some time that what matters is not loveless freedom, nor recognition and reverence of love, but only love itself. ✓

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