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ELECTRICITY AND MAGNETISM

(To Advanced and Scholarship Level)

by

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First printed . . . 1958

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Made and Printed in Great Britain for the English Universities Press, Ltd., London,
by C. Tinling & Co., Ltd., Liverpool, London and Prescot.

GENERAL EDITOR'S FOREWORD

SIR GRAHAM SUTTON, C.B.E., D.Sc., F.R.S.
Director-General, Meteorological Office,
Formerly Dean of the Royal Military College of Science, Shrivenham,
and Bashforth Professor of Mathematical Physics

THE present volume is one of a series on physics and mathematics, for the upper forms at school and the first year at the university. The books have been written by a team of experienced teachers at the Royal Military College of Science, where, among other things, students are prepared for London (External) Degrees in Natural Science and Engineering. The series therefore forms an integrated course of study based on many years' experience in the teaching of physics and mathematics.

In preparing their manuscripts the writers have been mainly guided by the examination syllabuses of London Univeristy, the Joint Board of Oxford and Cambridge and the Northern Univeristies Board, but they have also taken a broad view of their tasks and have endeavoured to produce works which aim to give a student that solid foundation without which it is impossible to proceed to higher studies. The books are suitable either for class teaching or self-study; there are many illustrative examples and large collections of problems for solution taken, in the main, from recent examination papers.

It is a truism too often forgotten in teaching that knowledge is acquired by a student only when his interest is aroused and maintained. The student must not only be shown how a class of problems in mathematics is solved but, within limits, why a particular method works and, in physics, why a technique is especially well adapted for some particular measurement. Throughout the series special emphasis has been laid on illustrations which may be expected to appeal to the experience of the student in matters of daily life, so that his studies are related to what he sees, feels and knows of the world around him. Treated in this way, science ceases to be an arid abstraction and becomes vivid and real to the inquiring mind.

The books have therefore been written, not only to ensure the passing of examinations, but as a preparation for the exciting world which lies ahead of the reader. They incorporate many of the suggestions which have been made in recent years by other teachers

and, it is hoped, will bring some of the new points of view into the classroom and the study. Last, but by no means least, they have been written by a team working together, so that the exchange of ideas has been constant and vigorous. It is to be hoped that the result is a series which is adequate for all examinations at this level and yet broad enough to satisfy the intellectual needs of teachers and students alike.

AUTHOR'S PREFACE

THIS book has been written primarily to meet the need of students taking the advanced and scholarship level examinations in physics of the General Certificate of Education. It also caters for those preparing for the open scholarships of various universities including Oxford and Cambridge. For convenience, those sections included specially for scholarship students are marked with an asterisk. Representative examples taken from past papers of the university examining bodies are worked out in the text and a number are added at the end of each chapter for solution by the student.

The subject has been developed from the fundamental concepts of point charges and point poles. Properties of charge distributions in isolation and on conductors are examined in some detail because it is believed that a firm understanding of fundamental electrostatics is essential in order to appreciate the processes involved in conduction in gases, liquids and solids. A simple theory of magnetism is given in an early chapter in order that magnetic field strength and magnetic induction may be introduced before discussing current electricity. The point pole has been used as the basis of all magnetic effects but it has been stressed that the fundamental magnetic element is the dipole. At this stage of development it is convenient to regard the dipole as a combination of point poles rather than attempt to introduce a more correct but far more complex electronic picture.

The classical approach to the study of magnetism and electricity has the advantage of closer alliance with historical development. It is believed that some correlation with historical background is required in order that the student will not feel overawed by the complexity of modern thought without some inkling of its gradual emergence. In spite of this it was decided to relegate to a later chapter the more difficult experimental techniques associated with electrostatic principles. This chapter includes also the experiments associated with magnetostatics. The latter are not difficult to perform but they are now regarded with disfavour and the extensive ramifications that the experiments allow often lead to gross distortion of the importance of the topic.

Rationalized M.K.S. units have been used throughout the text. The new units have been introduced at the outset and defined relative to the fundamental inverse square laws. Although the latter assume an unfamiliar form when M.K.S. units are used, the new forms are committed to memory easily providing the numerical values of the con-

stants ϵ_0 and μ_0 are not used. This method of introduction has been chosen deliberately in spite of the recommendations of the Science Masters Association published in their report on the Teaching of Electricity and in contrast to the ideas expressed by leading members of the Institution of Electrical Engineers.

Using the inverse square laws as starting points the method of teaching outlined above may be persued. Further, this method has the advantage that it does not require drastic reorientation of ideas by those students who have reached 'O' level G.C.E. using classical concepts with C.G.S. units. It also means that the book may be used by electrical engineers and first year degree students who wish to adopt the M.K.S. system of units without abandoning their earlier ideas.

It is a pleasure to thank the following for reading the manuscript at various stages and making helpful comments: Mr. R. Stone, Senior Science Master, Manchester Grammar School, Mr. G. W. E. Ghey M.B.E., Head of the Science Department, Britannia Royal Naval College, Mr. K. W. Lyon, Senior Physics Master, Bristol Grammar School. I am also indebted to Dr. D. Fender, Reading University, for assistance with the diagrams and to Mr. J. W. Skinner, Mr. J. Stanway and Lt. H. Tomkins R.E.M.E. at Shrivenham, for their criticisms of certain sections of the text. Finally, my thanks are due to Sir Graham Sutton F.R.S., the Editor of this series of text-books, for his very helpful advice and comments.

I am indebted to a number of the examining bodies for permission to reproduce examination questions. These are mainly G.C.E. Advanced and Scholarship Level papers, but also include the Oxford, Cambridge and Manchester Open Scholarship and the London Intermediate B.Sc. papers. The authorities concerned are: The Senate of the University of London, the Joint Matriculation Board of the Universities of Manchester, Liverpool, Leeds, Sheffield and Birmingham, the Oxford and Cambridge Schools Examination Board, the University of Oxford Delegates for Local Examinations, the University of Cambridge Local Examinations Syndicate, the University of Durham School Examinations, the University of Manchester and the Associated Examining Board.

C. G. Wilson, Royal Military College of Science,
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CONTENTS

		Page
	INTRODUCTION	xi
Chapter 1.	ELECTROSTATICS	1
Chapter 2.	CAPACITY AND CONDENSERS	40
Chapter 3.	ELEMENTARY MAGNETISM	66
Chapter 4.	ELECTRIC CURRENT (Fundamental Laws)	94
Chapter 5.	MAGNETIC EFFECT OF CURRENT: METERS	109
Chapter 6.	MEASUREMENT OF RESISTANCE, KIRCHHOFF'S LAWS, POTENTIOMETER	152
Chapter 7.	THERMAL EFFECTS	180
Chapter 8.	ELECTROLYSIS	219
Chapter 9.	ELECTROMAGNETIC INDUCTION	252
Chapter 10.	MAGNETIC PROPERTIES OF MATERIALS	297
Chapter 11.	ALTERNATING CURRENT, GENERATORS AND MOTORS	324
Chapter 12.	CATHODE RAYS, ELECTRONS AND ELECTRON PHYSICS	363
Chapter 13.	X-RAYS, RADIOACTIVITY AND ATOMIC STRUCTURE	407
Chapter 14.	NUCLEAR PHYSICS	427
Chapter 15.	ADDITIONAL MAGNETISM AND ELECTROSTATICS	443
	APPENDIX	487
	ALPHABETICAL LIST OF THE ELEMENTS	505
	THE RADIOACTIVE SERIES	506
	PERIODIC TABLE OF THE ELEMENTS	509
	TABLE OF PHYSICAL CONSTANTS	510
	ANSWERS TO EXERCISES	511
	TABLES	515
	INDEX	534

INTRODUCTION

Electricity and Matter

The earliest recorded evidence of electrical effects is found in Greek writings which record the power of amber to attract light pieces of matter. This is easily demonstrated by rubbing a piece of amber with some dry material such as fur, flannel, human hair, etc., and bringing the amber near some small dry pieces of paper. The pieces of paper tend to rush to the amber and behave as though they were alive. To explain this effect it is supposed that the rubbing process causes the production of electric " charges " on the surface of the amber.

In explaining this simple example of electrical effects the term " electric charge " was introduced. Today, all effects which are recognized as being of electrical origin, such as lightning, the electric lamp, the dynamo, the electric motor, are related to electric charges either at rest on the surfaces of bodies, or moving through the matter composing bodies in the form of a current of charge.

One of the most important properties of a current of charges moving through a body is magnetism, a simple definition of which might be: the ability to attract small pieces of iron. The science of magnetism began when this property was discovered and explanations of magnetic attraction tended to be similar to those for effects between electrified bodies. The association of magnetism with electric current, however, made it plain that such magnetic effects arose primarily from electric charges in motion, and that magnetism was really secondary to electricity.

The theory of electricity and magnetism is based essentially on the existence of electric charge. It is impossible to describe charge in terms of simpler concepts like shape, colour, fluidity, etc. One can only surmise from the results of experiments that charge is always associated with mass and exists in two forms known as positive and negative charge. Forces exist between electric charges and the two kinds are distinguished by the forces they exert on each other. A positive charge attracts a negative charge, whilst a positive charge repels another positive charge, and likewise two negative charges repel each other. The effect of two equal charges of opposite sign acting together on a third charge of either sign is to annul the force each would exert separately on the third charge, and from this point of view the negativeness or positiveness of a particular charge has significance.

Experiments carried out by Sir J. J. Thomson and others, towards the end of the last century (Chapter 11), proved that any negative charge

possessed by a body consists of a definite number of fundamental units of charge. The elementary charges can exist in the free state as single entities, but they are always associated with mass of the same amount. These small indivisible pieces of matter with their definite negative charge are called *electrons*. It was subsequently shown that all positive charges, produced or found in nature, are made up of fundamental units of the same magnitude as that on the electron, but generally associated with a much greater mass. This more massive positively charged particle is called a *proton* and is known to be the core or *nucleus* of the hydrogen atom. Experiments on radioactivity have since shown that particles of the same mass and of the same charge as the electron, but with a positive sign, also exist. They are called *positrons*, but since their presence is never obvious, except when conducting experiments in nuclear physics, they are neglected in this general account of matter and electricity.

Chemistry is the science which deals with the various reactions which are possible between the different kinds of matter. In order to explain chemical reactions the existence of *elements* is assumed. An element is defined as a substance made up of indivisible particles alike and unique in all respects. These particles, called *atoms*, were initially conceived as extremely small spheres of matter, having a definite mass and size depending on the element. Experiment showed that the masses of the atoms of elements relative to the mass of the hydrogen atom were approximately whole numbers. To explain this Prout made the original suggestion that all other types of atoms were built up of hydrogen atoms, but it was not until the early part of the present century that this view, in a modified form, was accepted.

Sir J. J. Thomson showed that electrons were obtained from matter and, therefore, must be contained within atoms. The hydrogen atom was the lightest known and presumably the simplest in structure. Experiments showed that both electrons and protons could be obtained from hydrogen and it was natural to suggest that the hydrogen atom consisted of one proton combined in some way with one electron. Since the electron and proton have charges of equal magnitude but of opposite sign, their combined electrical effect at some distance from the atom, large compared with the size of the atom, is zero. Thus, the neutral nature of the ordinary hydrogen atom is explained.

All other atoms are supposed to have a similar structure, viz. a positively charged nucleus of matter surrounded by a definite number of electrons. The nucleus is supposed to consist of a number of protons equal to the number of electrons, so that the external electrical effect is zero, and to make the mass of the atom the correct value relative to hydrogen, a number of *neutrons* is considered to exist in the nucleus. Neutrons, which were not discovered until 1932, are similar to protons except that they have no charge. The masses of the electrons surrounding the electron are neglected in determining the relative mass

of an atom since they are negligible compared with the mass of the proton or neutron. This conception of the structure of atoms is now generally accepted.

Neglecting any consideration of cosmic ray particles and positrons, which, as mentioned earlier, are not necessary to any simple theory, the only kind of charge thought to exist, is that associated with protons and electrons. This implies that electric charge is a property of these elementary particles and not an entity capable of existing on its own. All electrical and magnetic effects must be explained in terms of these charges. Thus, to explain the attraction of rubbed amber for light objects like pieces of paper, it is supposed that some of the electrons from the surface atoms of the amber are transferred to the rubbing agent in the friction process of rubbing. This removal of negative charge from the amber surface leaves it with a net positive charge which is able to attract the negative charges in the light objects. Since the negative charges in the light objects are firmly attached to the atoms there, the attractive force causes the whole object to be attracted to the amber. The electrons transferred to the rubbing material cause it to be negatively charged.

The magnetic effect associated with the movement of electric charge in the form of a current is detected only when electric charges move in the same direction in substances called *conductors*. These substances are usually metals, but solutions of acids, bases, salts, and such substances in the molten state also permit the flow of electric charges through them, although the ability to conduct the charge is not so great as with metals. Other substances do not normally permit the flow of electric charges in them and are called *insulators*.

The electric charges which flow in a metal to form what is called an *electric current* are electrons belonging to the metal atoms. In the solid state of the metal the atoms form a fixed lattice of points. In this state the atomic forces which normally hold the electrons to the nuclei are altered and some electrons, usually about one per atom, are set free by the atoms. The " free electrons ", as they are called, cannot escape from the metal at ordinary temperatures, because of forces at the surface of the metal which repel any motion outwards. Such electrons, however, move through the spaces between the relatively fixed atoms of the metal in an erratic fashion, colliding with the atoms and gaining and losing kinetic energy but, on the average, always possessing the same amount of energy at a given temperature. The creation of an electric current in a metal simply means causing the electrons to move in a more orderly fashion in a particular direction. When this orderly stream of electrons is created, magnetic and other effects associated with an electric current come into being.

Although all electric and magnetic effects are ultimately to be traced to movements of electrons or protons, it is generally too difficult to consider quantitatively the motion of individual particles. In the

ensuing chapters on electrostatics, i.e. the study of electric charges at rest on substances, the fundamental particle aspect of electric charges may often seem obscure. We may speak of a layer of charge on bodies, as though the charge were conceived as a thin film of jam or something which can be spread over solid matter. This is convenient from the point of view of the mathematical development of the theory of electricity, but the atomic nature of the charges concerned should always be borne in mind.

CHAPTER 1

ELECTROSTATICS

1.1. Detection of Charge

All insulators such as glass, ebonite, etc., attract small pieces of matter when rubbed. Each is electrified by another in a friction process, i.e. each becomes " charged " with positive or negative electricity. If a glass rod rubbed with silk is suspended from its mid-point, bringing up the rubbing silk causes attraction. A similarly charged glass rod repels the mounted one. The glass is said arbitrarily to be positively charged and substances which attract it negatively charged; those which repel it are charged positively. A more sensitive detector of charge is a small pith-ball suspended from a fine silk thread. If the pith-ball is covered with thin metal foil, it becomes a small insulated conductor. The ball is electrified by allowing it to touch a charged rod; in the presence of other charges it is repelled or attracted. On charging the pith-ball electrons pass from the metal foil to a positive charging rod, or, to the metal foil from a negative charging rod.

1.2. Force between charges

Coulomb (1736–1806) investigated the force between charged bodies, with a torsion balance. A small metal sphere A is fixed at the end of an insulated rod (Fig. 1.1). A similar sphere B is fixed at one end of an ebonite arm which is suspended at its mid-point by a fine thread from a torsion-head. When A and B are similarly charged they repel each other and the ebonite arm rotates. The twist in the thread brings B to rest at a definite distance r from A. A light beam reflected from a small mirror M attached to the thread, gives a deflection which is twice the angle of twist θ. The turning moment G producing this twist is given by:

$$G = c \, . \, \theta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where " c " is the torsional constant of the thread. If the distance from the thread to the centre of B is l units of length the turning moment caused by the electrostatic force F is:

$$G = Fl \text{ (for small } \theta) \quad . \quad . \quad . \quad . \quad (2)$$

By equating (1) and (2), F is found.

By using the same charges on each sphere but altering the initial position of the torsion head, different equilibrium positions are obtained corresponding to different values of r. It is found that:

$$F \propto \frac{1}{r^2}$$

To find the variation of force with charge, A and B are charged together from the same source. On separation each sphere possesses a similar quantity of charge. Using the torsion balance, the force is found for a given distance apart. The charge on B is then removed by connection to the earth with the finger and the charge remaining on A is divided equally between A and B when they are brought into

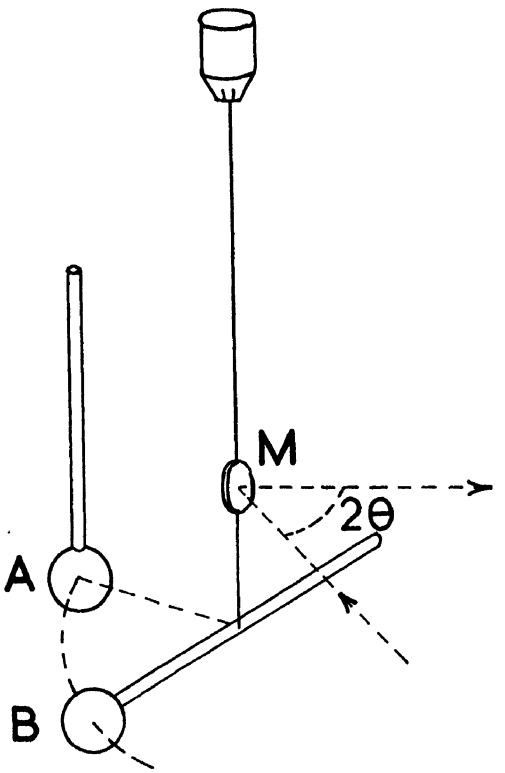


Fig. 1.1

contact again. If the magnitude of the original charge on A is called q_o, the new charges on A and B are each $q_o/2$. By repeating the torsion experiment the force between the two new charges at the same distance away is determined. The process is repeated using charges $q_o/4$, $q_o/8$, etc., at a constant distance apart. The experimental results show that $F \propto q^2$, where q is the charge on either sphere.

The combined results yield the relation: $F \propto \frac{q^2}{r^2}$

i.e. $$F = \frac{q^2}{pr^2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where p is a constant of proportionality.

This equation leads to a system of units for measuring charges, since if F is measured in dynes, r in cm., and p put arbitrarily equal to unity,

In this case the charge acquired by A and L, and hence the divergence of the leaf, depends only on the potential of C relative to earth.

When C is connected to a charged conductor its potential becomes that of the conductor so that the divergence of the leaf is a measure of the potential of the conductor. The electroscope may be calibrated as a voltmeter by noting the angular deflection of the leaf corresponding to known potentials. The angular deflection is approximately proportional to the potential.

In the following experiments all apparatus must be thoroughly dry, since a film of moisture forms a conducting path ultimately leading to earth.

Experiment 1. *The Potential of Conductors.* An insulated metal conductor of any shape such as A in Fig. 1.11, is charged by gently

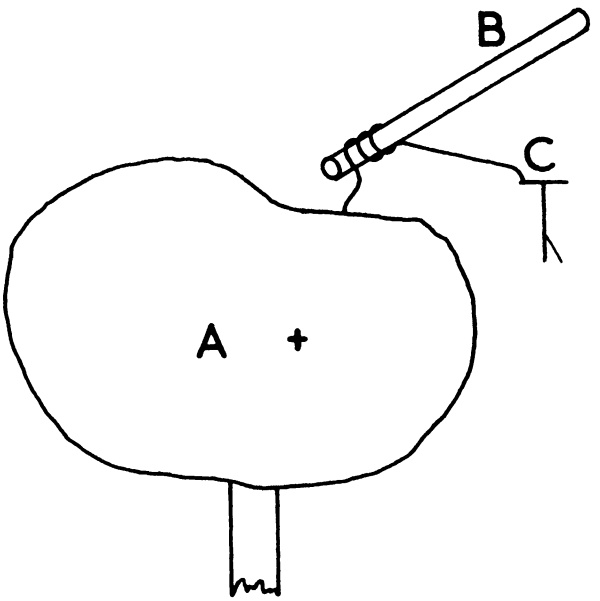


Fig. 1.11

stroking it with a charged rod. Some thin wire is wrapped round an insulating rod B near one end; one end of the wire is connected to the cap C of an electroscope, the other left free. Initially the electroscope is earthed by touching momentarily with the finger. The rod B is now held so that the free end of the wire touches A. The electroscope leaf diverges indicating the potential of A. The divergence of the leaf remains the same as the wire is moved over the surface of A indicating that the surface is an equipotential.

Experiment 2. *Sharing of Charge.* A is charged as before and connected to the electroscope with the wire held on B. The electroscope deflection is a measure of the potential of A. A second insulated

conductor which is uncharged is brought into contact with A. The electroscope leaf falls to a new deflection indicating a reduction in the potential of A. The divergence remains the same when the second conductor is connected to the electroscope indicating that it is at the same potential as A. On separation the potentials still remain the same.

1.23. Electrostatic Induction

An uncharged conductor placed in an electrostatic field shows evidence of being charged although it is given no charge by external means. The phenomenon is called *electrostatic induction*. It is demonstrated by placing an uncharged insulated conductor AB near a positively charged insulated metal sphere C (Fig. 1.12). Small pieces

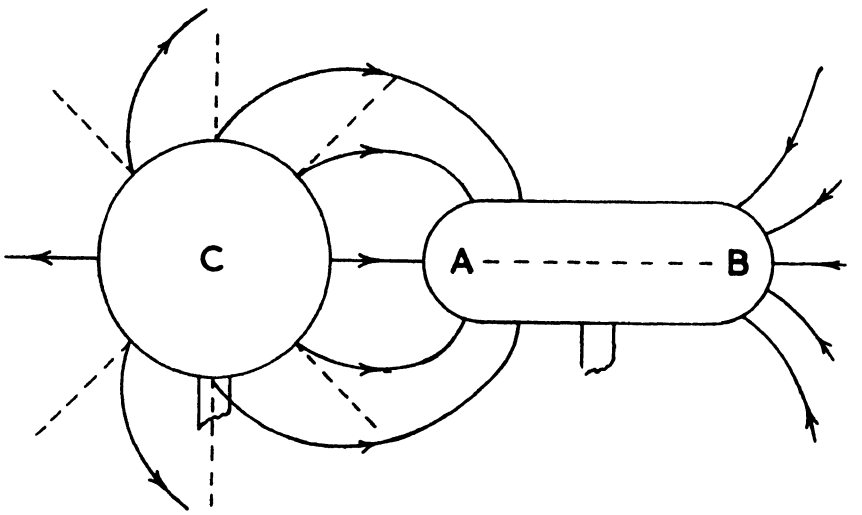


Fig. 1.12

of paper or cork dust are attracted to the ends of AB. No attraction is found near the middle of AB. When AB is connected to an electroscope the leaf diverges showing that the conductor has acquired a potential. When C is removed all evidence of charge and potential on AB disappear.

This phenomenon is explained as follows: when AB is placed in the field of C, the points on the surface near A have a higher potential than those near B. As a result, electrons flow from B to A resulting in a deficiency of electrons at B and an excess near A. The current of electrons lasts for a negligibly short time and ceases when the surface of A is once more an equipotential. Thus A appears to be negatively charged and B positively charged. The original field of C is distorted, and AB has a positive potential less than that of C (Fig. 1.13). It still has zero net charge since it has not lost or gained electrons.

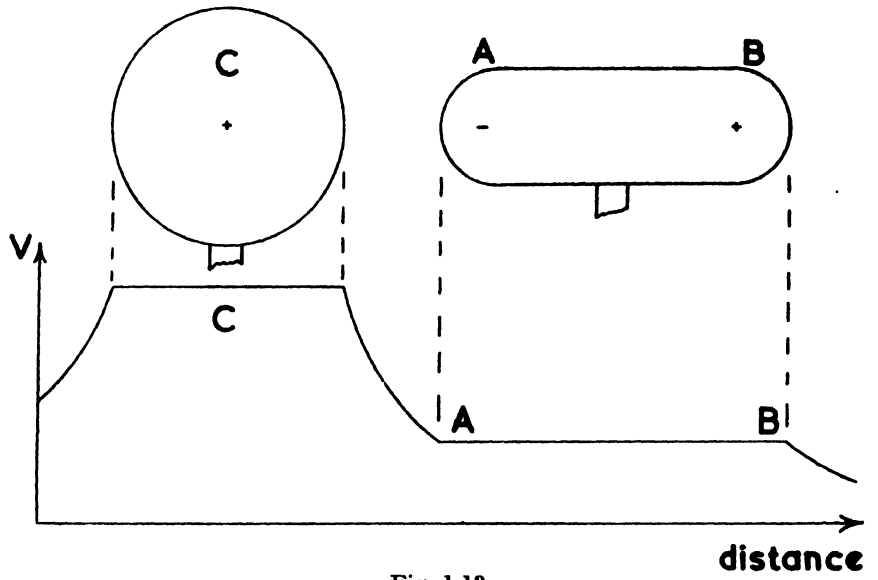


Fig. 1.13

1.24. Charging by Induction

If AB is earthed momentarily with the finger whilst still near C, electrons flow from earth to AB until its potential is zero. A negative charge is left on AB but its potential is zero (Fig. 1.14). When C is

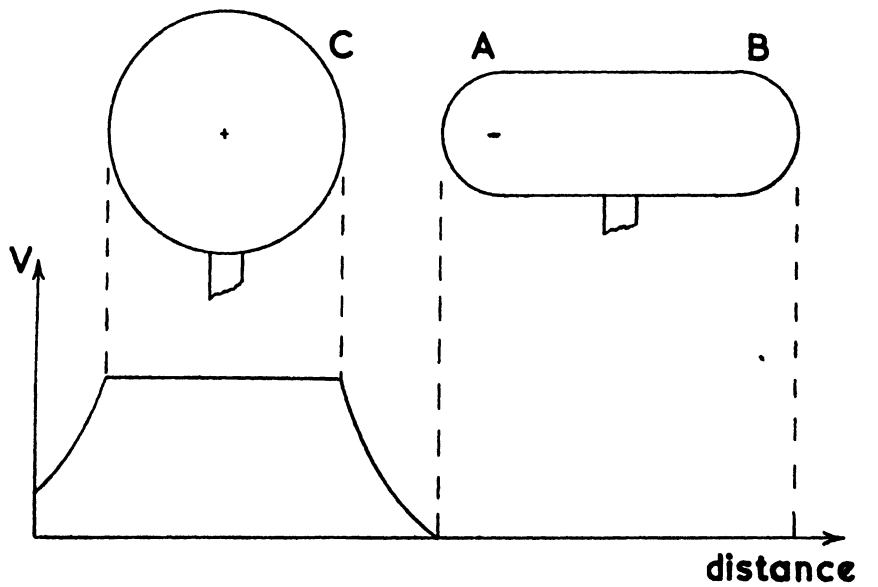


Fig. 1.14

removed from the neighbourhood of AB the latter acquires a negative potential due to its negative charge, the field being as shown in Fig. 1.15. Insulated conductors are conveniently charged by this means.

An electroscope is also charged by induction; a charged ebonite rod (negative charge) is held near the cap of the electroscope which is momentarily touched whilst under the influence of the rod. The rod is then removed and a positive charge remains on the electroscope. A positively charged rod (glass, etc.) is used to charge the instrument negatively.

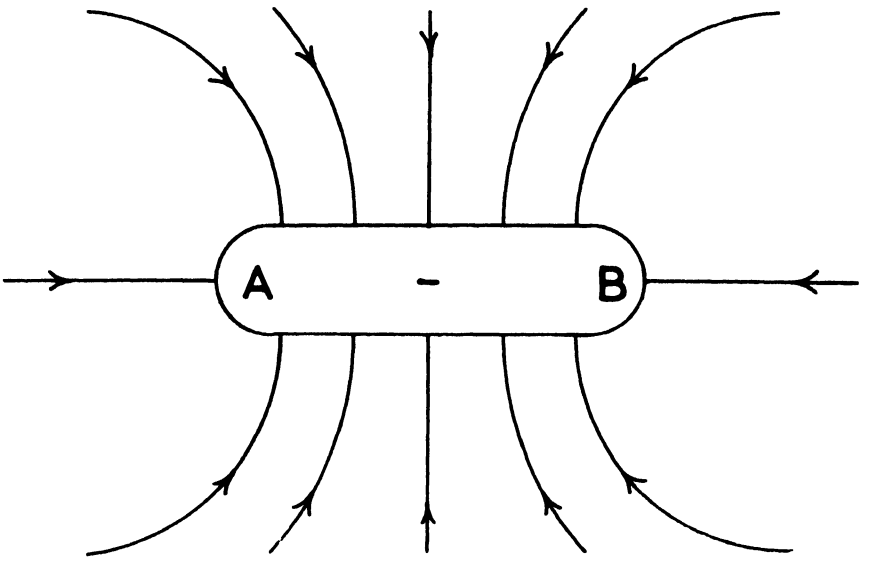


Fig. 1.15

1.25. Hollow Conductors—Faraday's Ice-Pail Experiment

Hollow metal conductors have special properties which were investigated by Faraday (1791–1867) using an ice-pail as an approximate representation of a complete hollow conductor. In a modern version of the *ice-pail experiment* a copper calorimeter is used. The can is placed on the cap of an uncharged gold-leaf electroscope, (Fig. 1.16). A small metal sphere, suspended by means of a thread of silk or nylon is charged by induction, and gently lowered into the can without touching the sides. The electroscope deflection is constant at all points well inside the can. On removing the sphere the deflection falls. The sphere retains its original charge.

The charged sphere is again lowered into the can and the gold leaf diverges. The sphere is allowed to touch the inner surface of the can and the leaf remains divergent. On removal the sphere is found to have lost its charge and no charge can be abstracted from the inside of the can by making contact with an insulated conductor.

These results are interpreted as follows:

(i) When the sphere is inserted in the can a negative charge is induced on the inside of the can because all the lines of force from the sphere end there (neglecting those which escape from the mouth

of the can). Since the can is uncharged an equal and opposite positive charge is induced on the outer surface of the can, (Fig. 1.16). When the sphere is removed the induced charges disappear and the leaf falls. The divergence of the leaf shews no change with position when the sphere is below the mouth of the can because the number of lines ending on the can is then constant, i.e. the potential of the can is independent of the position of the sphere.

(ii) When the sphere touches the inside of the can its charge neutralises the induced negative charge on the inner surface, the can is left with a positive charge and the sphere is discharged. Thus, the induced charge on the inner surface of the can is equal and opposite to the inducing charge on the sphere. The can now has its own field and potential (Fig. 1.17).

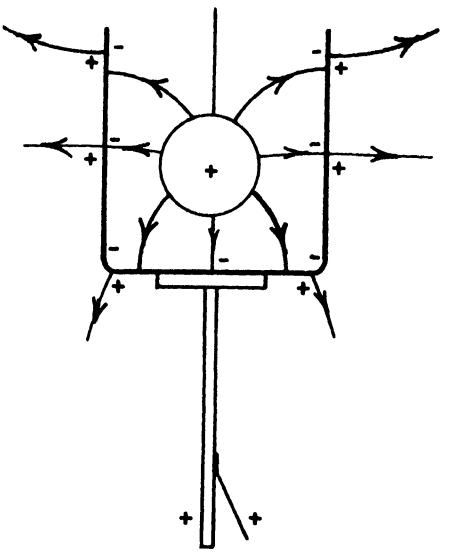


Fig. 1.16

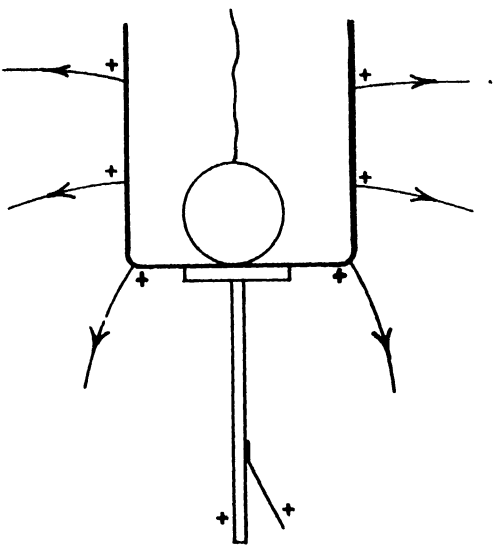


Fig. 1.17

Faraday's ice-pail was sufficiently deep for the " end-effects " near the mouth to be neglected and as a result of this experiment the the following generalizations may be made:

1. There is no *net* charge *inside* a hollow conductor.

2. When charges are maintained on insulated bodies inside a conductor the induced charge is equal and opposite to the inducing charge.
3. All points inside a hollow conductor containing no separate charges must be at the same potential which depends solely on the charge on the outer surface of the conductor. (This follows because there is no charge and hence no field inside the conductor).

*1.26. Hollow Conductors and Gauss' Theorem

Let A and B represent the inner and outer surfaces of a hollow conductor P in Fig. 1.18. Suppose an imaginary surface C is drawn in the conductor between A and B. Since no electric field exists in the

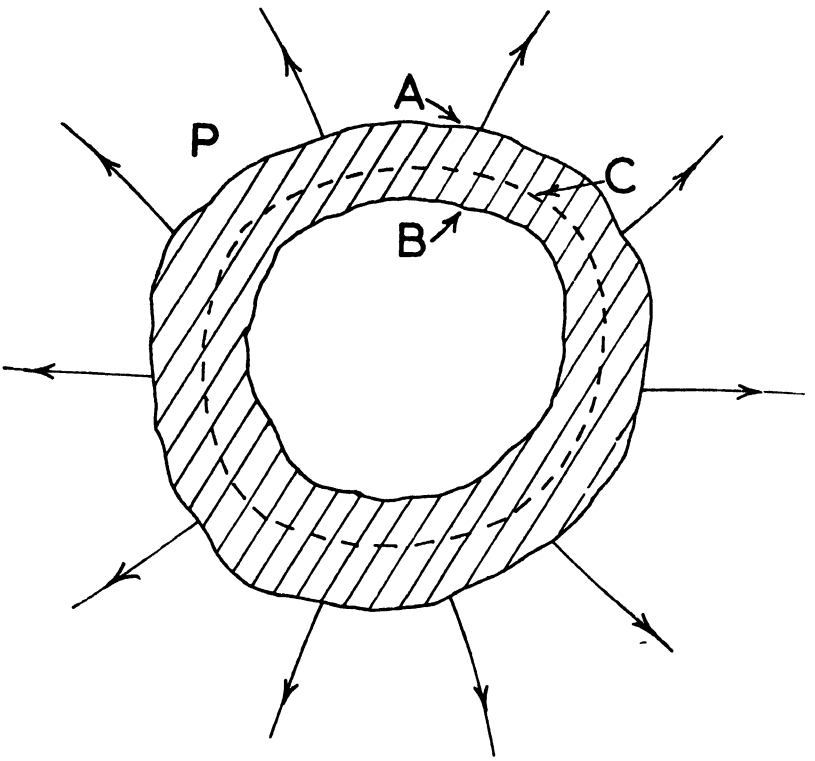


Fig. 1.18

material of a conductor the total flux of induction through C is zero. From Gauss' theorem, therefore, there is no net charge enclosed by C and hence none in the hollow space of the conductor. This result is quite independent of whether P is charged or uncharged.

Suppose a small insulated conductor carrying a charge $+q$ coulombs is placed inside B without touching it. The flux through C is still zero so that an induced negative charge of the same magnitude as q must exist inside C, the only place where it can reside being the surface B.

Now, suppose an imaginary surface C' is drawn outside A completely enclosing P. The total charge inside P is $+q$ coulombs so that the flux through C' is equal to $+q$. Since a charge of $-q$ coulombs exists on B, the flux through C' must arise from a charge $+q$ distributed over the outer surface A; the surfaces of P are thus charged as shown in Fig. 1.19. The conductor P has a positive potential because of $+q$

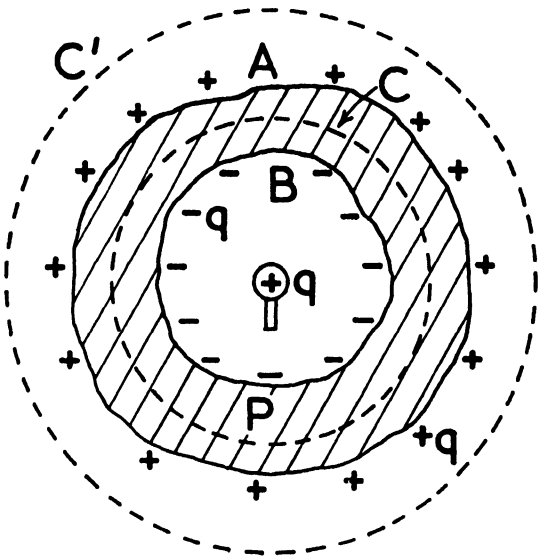


Fig. 1.19

on its outer surface. If the conductor is charged, in addition, by external means, its potential (relative to earth or infinity), will depend on the total charge on its outer surface.

1.27. Distribution of Charge on the Surface of Conductors—Experiment

In this experiment a proof-plane, consisting of a small brass disc D fixed at one end of an insulating rod B (Fig. 1.20), is used to transfer small samples of charge from the surface of a charged insulated conductor such as A. A is charged by induction and D is placed in contact with its surface. The proof-plane is then inserted inside a metal can placed on the cap of an uncharged gold-leaf electroscope. All the charge on D is given to the electroscope and can when D touches the inner surface of the latter. The electroscope measures the potential of the can on acquiring the charge, but since the can potential is dependent solely on this charge, the deflection is a measure of the charge. Since the charge transferred to D depends on the surface concentration on A, the deflections are a measure of the surface density of charge on A.

The proof-plane is now placed in contact with other points on the surface and the charge given up to the electroscope, the latter being earthed after each deflection. The deflections show that the proof-plane acquires a maximum charge from points on the surface having the

greatest curvature. When a sphere is used the deflections are independent of the points of contact on the surface.

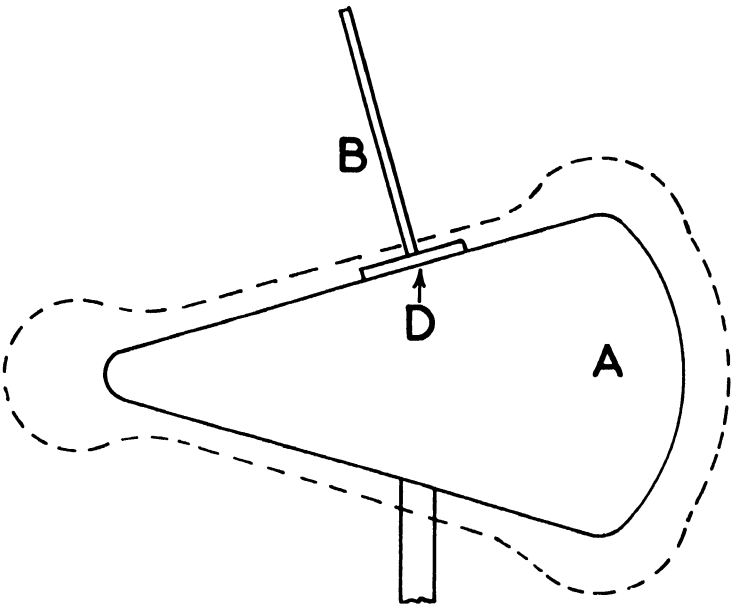


Fig. 1.20

From such experiments follows the general law: the surface density of charge on an isolated and insulated charged conductor increases with the curvature of the surface. The results are illustrated in Fig.

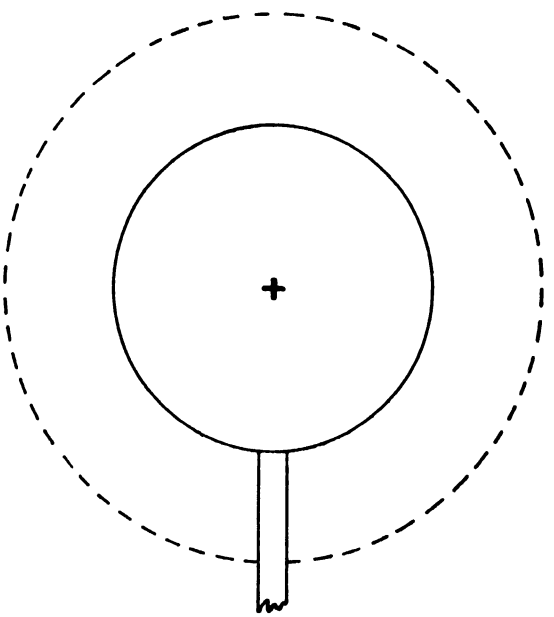


Fig. 1.21

1.20 and 1.21, where the height of the dotted line above the surface represents the surface density.

1.28. Metal Points

A charged and insulated pointed metal object, e.g. a needle, has a curvature which increases strongly at the pointed end. From above, therefore, the charge density increases rapidly at the pointed end. The high density of charge gives a large electric field strength just outside the point. This may be sufficiently high to cause *dielectric breakdown* of the surrounding air. This means that the few free electrons or charged atoms naturally present in the surrounding air are accelerated by the strong field to such an extent that they are able to knock electrons out of neighbouring neutral atoms to form more free electrons and positively charged atoms—a process called *ionization*, (p. 363). The additional electrons are similarly affected and cumulative ionization results. The negative charges travel to a positively charged point and neutralize the charge. With a negatively charged point the positively charged particles effect the discharge. For air at normal temperature and pressure the minimum field strength (*breakdown strength*) required for breakdown is 3×10^6 volt.m.$^{-1}$ (approx.).

The discharge of an insulated charged conductor likewise occurs when an *earthed* metal point is placed nearby. In this case an induced charge tends to be produced on the metal point, the process being similar to that represented by Fig. 1.15. Because of the high curvature of the point the induced charge density is so high that ionization occurs and the charge is neutralised as rapidly as it is produced. This property is utilised in *lightning conductors* to cause leakage of charge from buildings under the influence of charged clouds.

1.29. The Electric Fields of Conductors

(i) *Any Conductor:* On a very small area dS of a conductor surface the charge distribution may be assumed constant, say σ coulomb.m.$^{-2}$ The lines of induction leave the surface normally and since one line leaves unit charge the total number passing through a plane area dS m^2., very close and parallel to the area dS on the surface, is: $\sigma \, . \, dS$. Hence, the number of lines passing through unit area at right angles is σ. By definition, therefore, at points outside but close to any charged conducting surface: $D = \sigma$ *coulomb.m.*$^{-2}$

The electric intensity at such points is given by:

$$E = \frac{D}{\epsilon_0 \epsilon} = \frac{\sigma}{\epsilon_0 \epsilon} \text{ volt.m.}^{-1} \quad . \quad . \quad . \quad (22)$$

(ii) *Sphere:* Let the radius of an insulated and isolated spherical conductor be " a " metres and suppose it carries a charge $+q$ coulombs.

The charge distribution is uniform (σ coulomb.m.$^{-2}$). From above, at points near the surface $D_0 = \sigma$, and since $\sigma = q/(4\pi a^2)$:

$$D_0 = \frac{q}{4\pi a^2} \text{ coulomb.m.}^{-2} \quad . \quad . \quad . \quad (23)$$

All the lines of induction leave the sphere normally and are therefore radial (Fig. 1.22). The number of lines passing through an imaginary sphere of radius r metres is: $4\pi a^2 D_0$, i.e. q.

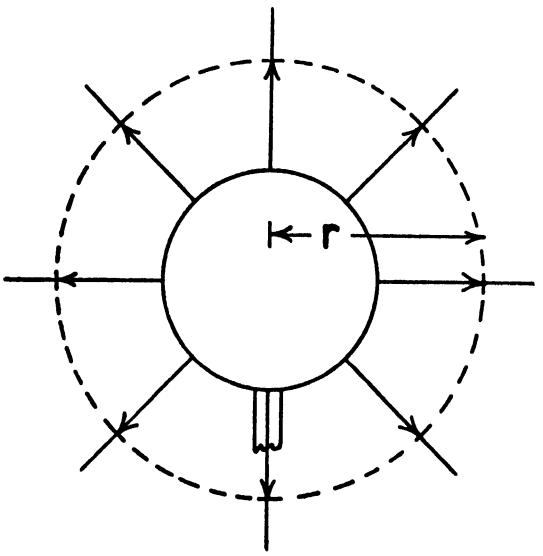


Fig. 1.22

By definition, the total number of lines passing through the same sphere is also given by: $4\pi r^2 D_r$,

Hence: $4\pi r^2 D_r = q$

i.e. $$D_r = \frac{q}{4\pi r^2} \text{ coulomb.m.}^{-2} \quad . \quad . \quad . \quad . \quad (24)$$

From (23): $$E_0 = \frac{D_0}{\epsilon_0 \epsilon} = \frac{q}{4\pi \epsilon_0 \epsilon a^2} \text{ volt.m.}^{-1} \quad . \quad . \quad . \quad (25)$$

and from (24): $$E_r = \frac{D_r}{\epsilon_0 \epsilon} = \frac{q}{4\pi \epsilon_0 \epsilon r^2} \text{ volt.m.}^{-1} \quad . \quad . \quad . \quad (26)$$

These results are identical with those for a point charge, so that the field *outside* a charged sphere is the same as that of a point charge at its centre. This result was assumed in the Coulomb torsion balance experiments (p. 2).

From (26) the electric intensity at a point s metres from the centre of the sphere is:

$$E_S = \frac{q}{4\pi \epsilon_0 \epsilon s^2} \text{ volt.m.}^{-1}$$

By definition, the potential at a point distant r metres from the centre of the sphere is:

$$V_r = -\int_{\infty}^{r} E_s \,.\, ds = \frac{-q}{4\pi\epsilon_0\epsilon} \cdot \int_{\infty}^{r} \frac{ds}{r^2} \text{ volts}$$

i.e. $$V_r = -\frac{q}{4\pi\epsilon_0\epsilon}\left[\frac{-1}{r} + \frac{1}{\infty}\right]$$

i.e. $$V_r = \frac{q}{4\pi\epsilon_0 r\epsilon} \text{ volts (as for point charge)} \quad . \quad (27)$$

Hence, the potential at the surface of the sphere, i.e. the potential *of the sphere* is:

$$V_o = \frac{q}{4\pi\epsilon_0\epsilon a} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

(iii) *Infinitely Large Plane Conductor:* The charge distribution is uniform (σ coulomb. m.$^{-2}$) since the curvature of the plane surface is constant. The lines of induction leave the surface at right angles and travel outwards as parallel lines (Fig. 1.23). Near the surface $D = \sigma$ from (i) above. Because the lines of induction are parallel, D is the same at all points outside the plane and normal to the plane:

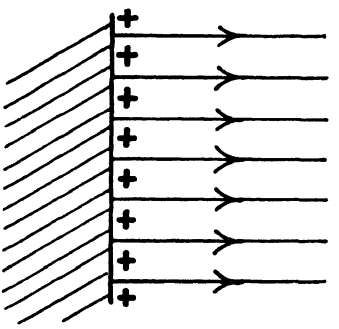


Fig. 1.23

i.e. $$D = \sigma \text{ coulomb.m.}^{-2} \quad . \quad . \quad . \quad . \quad (29)$$

(*N.B.* This result is not the same as (7) for the *plane layer of charge*, where $D = \sigma/2$. This is because the charge on a *conducting* surface can send lines of induction outwards only).

From (29): $$E = \frac{D}{\epsilon_0\epsilon} = \frac{\sigma}{\epsilon_0\epsilon} \text{ volt.m.}^{-1} \quad . \quad . \quad . \quad (30)$$

Since the field is uniform, i.e. E = constant, the p.d. between the plane and a point t m. from the plane is:

$$V = -\int_{t}^{0} E \,.\, ds = Et = \frac{\sigma t}{\epsilon_0\epsilon} \text{ volts} \quad . \quad . \quad (31)$$

It is not practicable to use infinitely large plates for producing a uniform field but the uniformity of field of a small plate is greatly

increased by placing an earthed parallel plate close to the charged plate (Fig. 1.24). This arrangement constitutes a *parallel plate condenser* (Chapter 2). The field lacks uniformity at the edges because it is not completely limited to the volume between the plates (edge effect).

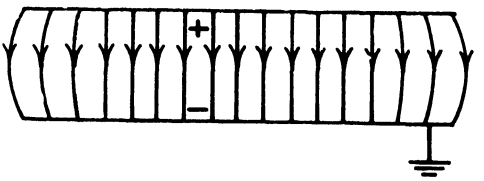


Fig. 1.24

The uniformity is further improved by using discs instead of rectangular plates and placing a plane annular conducting ring around the charged plate and maintaining it at the same potential. Those lines from the edge of the main charged plate show little divergence- -providing that the space between the ring and the conductor is very small. The annular ring is called a *guard ring*.

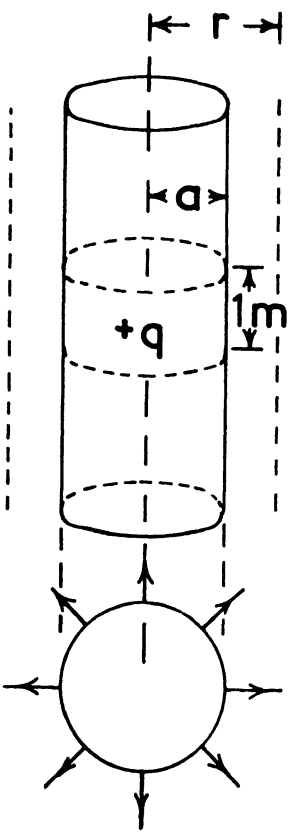


Fig. 1.25

(iv) *Infinitely Long Charged Cylinder:* Let the radius of the cylindrical conductor be "a" metres. Because of its constant curvature the surface density of charge is constant. Let the *charge per unit length* be $+q$ coulomb.m.$^{-1}$. By symmetry, the lines of induction are radial (Fig. 1.25). Very close to the surface therefore:

$$D_0 = \sigma = \frac{q}{2\pi a} \text{ coulomb.m.}^{-2} . \quad (32)$$

The total number of lines leaving unit length of surface is hence q. Thus, q lines pass through unit length of a concentric cylinder of radius r metres, and $q/(2\pi r)$ per unit area of this cylinder,

i.e. $$D_r = \frac{q}{2\pi r} \text{ coulomb.m.}^{-2} . \quad (33)$$

The corresponding values of E are:

$$E_0 = \frac{q}{2\pi\epsilon_0\epsilon a} \text{ and } E_r = \frac{q}{2\pi\epsilon_0\epsilon r} \text{ volt.m.}^{-1} \quad (34)$$

*1.30. Use of Gauss' Theorem in determining D and E in above fields

(i) *Surface of any Conductor:* Assuming uniform charge density over an infinitesimal area of surface dS m.^{2}, so that D is constant

and normal at points just outside it, the flux through the surface of a small "pill-box" of cross-section dS with end faces just inside and outside the surface, is given entirely by the flux through the outer end face, i.e $\Phi = D \,.\, dS$; the charge inside the closed surface is $\sigma \,.\, dS$, so that by Gauss' theorem:

$$D \,.\, dS = \sigma \,.\, dS$$

hence: $D = \sigma$ coulomb.m.$^{-2}$ and $E = \dfrac{\sigma}{\epsilon_0 \epsilon}$ volt.m.$^{-1}$

(ii) *Charged Sphere:* Assuming uniform surface density of charge σ coulomb.m.$^{-2}$ the lines of induction are radial. The total flux of induction through a concentric sphere of radius r metres is hence: $4\pi r^2 D_r$. The total charge inside the sphere is $4\pi a^2 \sigma = q$ coulombs (say), so that:

$$D_r = \frac{q}{4\pi r^2} \text{ coulomb.m.}^{-2} \text{ and } E_r = \frac{q}{4\pi \epsilon_0 \epsilon r^2} \text{ volt.m.}^{-1}$$

(iii) *Infinite Plane:* This case is treated like (i) above using a pill-box surface. The field is uniform and $D = \sigma$.

(iv) *Charged Cylinder:* The "Gaussian" surface chosen is a concentric cylinder of unit length. If the charge per unit length is q coulombs, the charge enclosed is q. The charge density is uniform and D radial so that the flux is: $2\pi r D_r = q$.

$$\therefore D_r = \frac{q}{2\pi r} \text{ coulomb.m.}^{-2} \text{ and } E_r = \frac{q}{2\pi \epsilon_0 \epsilon r} \text{ volt.m.}^{-1}$$

Example

(a) *Describe Faraday's ice-pail experiment, including such details as the method of charging the sphere. Explain the lessons which are to be learnt from it.*

(b) *Explain the action of an earthed metal point in discharging an insulated charged metal sphere placed near to it.*

(c) *Calculate the highest potential to which an isolated conducting sphere of 50 cm. radius can be raised if air begins to conduct electricity when the potential gradient is 30,000 volts per cm.* (North G.C.E. Adv. L.).

Parts (*a*) and (*b*) are discussed in the text.

(*c*) Let the maximum charge carried by the sphere be $+q$ coulombs and the corresponding potential V volts, then:

$$V = \frac{q}{4\pi \epsilon_0 \epsilon a} \text{ volts}$$

The electric intensity at the surface is:

$$E = \frac{q}{4\pi \epsilon_0 \epsilon a^2} \text{ volt m.}^{-1}$$

whence: $V = Ea$

Since, $E = 3 \times 10^6$ volt.m^{-1}; $a = 0{\cdot}5$ m.;

$V = 3 \times 10^6 \times 0{\cdot}5 = 1{\cdot}5 \times 10^6$ volts

Example

If a piece of metal has a charge $+0{\cdot}1$ *micro-coulombs and is placed inside a hollow metal sphere of radius* 20*cm. (without touching it) what is the potential of the sphere? What will the potential of the sphere become if* (a) *the sphere is temporarily earthed and then left insulated,* (b) *if the metal subsequently touches the inside of the sphere?* (Part of Lond. G.C.E. Adv. L.).

From above, induced charges of $-0{\cdot}1$ and $0{\cdot}1$ micro-coulombs are produced on the inner surface and outer surfaces of the sphere (Fig. 1.26). The potential of the

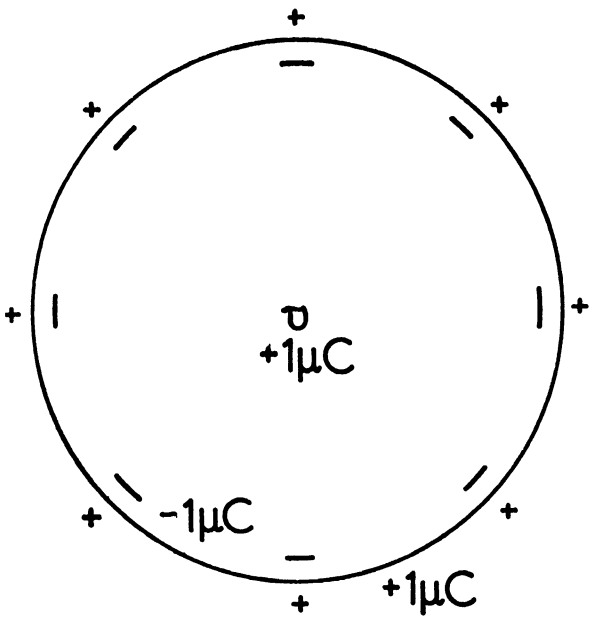


Fig. 1.26

sphere relative to earth is determined solely by the outer surface charge q, hence:

$$V = \frac{q}{4\pi\epsilon_0\epsilon a}$$

where "a" is the radius of the sphere. Since $\epsilon_0 = 1/(36\pi) \times 10^{-9}$ M.K.S.; $q = 10^{-7}$ coulombs; $a = 0{\cdot}2$ m.; and $\epsilon = 1$:

$$V = \frac{10^{-7}}{4\pi \cdot \frac{1}{36\pi} \cdot 10^{-9} \times 0{\cdot}2} = 4{,}500 \text{ volts}$$

(*a*) When the sphere is momentarily earthed its potential is reduced to zero. The positive charge on the outer surface disappears but the induced negative charge inside remains. Thus: *the potential of the sphere is zero.*

N.B. There is a p.d. between the sphere and the metal holding the charge *inside* so that the latter has a positive potential, relative to the sphere).

(*b*) When the metal touches the sphere the induced negative charge is neutralised and no charge remains on the metal or sphere and both are at the same potential, viz. *zero potential.*

1.31. Mechanical Force at the Surface of a Conductor

Consider an infinitesimal area dS m.^{2} at A on the surface of a conductor (Fig. 1.27). Let the surface density of charge on dS be σ

coulomb.m.$^{-2}$ so that the total charge on dS is $\sigma.dS$ coulombs. Let P_1 and P_2 be two points just outside and inside dS, respectively.

From (22) above, the electric intensity at P_1 is:

$$E_1 = \frac{\sigma}{\epsilon_0 \epsilon} \text{ volt.m.}^{-1} \quad . \quad . \quad . \quad . \quad (35)$$

and from the property of conductors, at P_2:

$$E_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

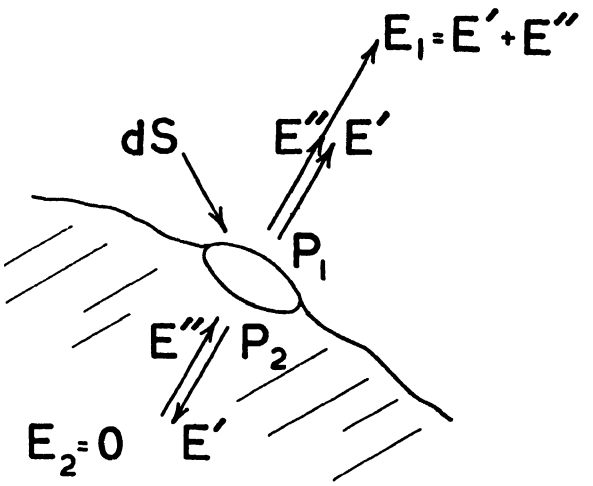


Fig. 1.27

The field on both sides of dS may be derived from two sources: (i) the charge on dS alone and (ii) the charge existing on the rest of the surface. The component of field strength due to (ii) acts on the charge on dS giving rise to the force which is to be determined.

Assuming that dS is equivalent to a charged plane layer, the field strength at P_1 (very close to dS) caused by dS alone is:

$$E_1' = \frac{\sigma}{2\epsilon_0 \epsilon} \text{ volt.m.}^{-1} \text{ (from (7)).} \quad . \quad . \quad (37)$$

The plane layer of charge at dS also gives rise to an equal and opposite field strength (E_2') at P_2, the direction being into the conductor.

From (35) the rest of the surface charge must give rise to a field strength component E_1'' at P_1 such that:

$$E_1 = E_1' + E_1'' = \frac{\sigma}{\epsilon_0 \epsilon} \text{ volt.m.}^{-1} \quad . \quad . \quad (38)$$

Since $E_1' = \sigma/(2\epsilon_0\epsilon)$, $E_1'' = \sigma/(2\epsilon_0\epsilon)$ in the same direction as E_1'.

At P_2 the field of the rest of the charge E_2'' neutralises E_2' to give

$E_2 = 0$, i.e. $E_2' = \sigma/2\epsilon_0\epsilon$. Thus, $E_1' = E_2'$ (same direction), which is expected since P_1 and P_2 are close together.

Since the charges on the conducting surface excluding dS give rise to the field strength $E_2' = \sigma/2\epsilon_0\epsilon$ at points close to the surface (and hence at points *on* the surface) which is directed normally outwards, the force acting on the charge on dS is:

$$f = \frac{\sigma}{2\epsilon_0\epsilon} \times \sigma \, . \, dS = \frac{\sigma^2 dS}{2\epsilon_0\epsilon} \text{ newtons}$$

Hence, the force per unit area:

$$F = \frac{\sigma^2}{2\epsilon_0\epsilon} \text{ newton.m.}^{-2} \quad . \quad . \quad . \quad . \quad . \quad (39)$$

This is a mechanical force tending to push the charge off the surface. A much larger force than that given by (39) is required to remove the positive or negative charge associated with charged conductors since the charge is effectively bound to the metal by strong atomic forces.* As a result the electrostatic force creates a tension within the material similar to that produced by an external mechanical stress.

1.32. Charged Soap Bubble

Suppose a soap-bubble is produced at one end of a thin tube, the radius of the bubble being r metres and the surface tension of the soap solution T newton.m.$^{-1}$.

The bubble is maintained in equilibrium by forces acting normal to the surface inwards caused by surface tension and those acting normally outwards caused by the excess air pressure inside the bubble. The surface tension force is $4T/r$ newtons per square metre, whilst the excess pressure gives rise to a force p newtons per square metre (say). For equilibrium:

$$p = \frac{4T}{r} \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Suppose the tube holding the bubble is charged to a potential V volts. The bubble is charged as a hollow conductor and has a uniform surface density of charge of σ coulomb.m.$^{-2}$ (say). The additional surface force acting outwards is hence: $\sigma^2/(2\epsilon_0\epsilon)$ newton.m.$^{-2}$. This causes the bubble to expand to a radius r' metres (say), so that the excess air pressure is reduced to p' and the new equilibrium condition becomes:

$$p' + \frac{\sigma^2}{2\epsilon_0\epsilon} = \frac{4T}{r'} \quad . \quad . \quad . \quad . \quad (41)$$

* Charge densities giving rise to electric fields of 10^8 volt.m.$^{-1}$ (approx.) are required to cause the emission of electrons from metals (*auto-electronic emission*).

Example

A soap bubble is blown on the end of a fine glass tube inside an exhausted container, the source of air pressure then being cut off by means of a tap. The bubble is then charged to a potential of 6,000 *volts and the radius of the charged bubble is found to be* 1 *cm. If the surface tension of the soap solution is* 3×10^{-2} *newton.m*$^{-1}$*, find the original radius of the bubble.* (Part of question, North H.S.C. Schol.).

Let the original radius of the bubble be r_o metres and the initial gas pressure inside the bubble p_o newton.m^{-2}. Since the bubble is blown in an exhausted container p_o is the *excess* air pressure inside the bubble, hence:

$$p_o = \frac{4T}{r_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

When the bubble is charged it expands to a radius r say, and the gas pressure is reduced to p (say). Assuming the perfect gas law:

$$p_o v_o = pv$$

where v_o and v are the original and final volumes of the bubble. Hence:

$$\tfrac{4}{3}\pi r_o^3 p_o = \tfrac{4}{3}\pi r^3 p$$

i.e.
$$p = \frac{r_o^3}{r^3} \cdot p_o$$

and from (42):
$$p = \frac{4Tr_o^2}{r^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If the charge on the bubble is q coulombs its potential is

$$V = \frac{q}{4\pi\epsilon_o r} \text{ volts.} \quad (\epsilon = 1)$$

If the surface density of charge is σ coulomb.m^{-2}, $q = 4\pi r^2\sigma$, so that

$$V = \frac{4\pi r^2\sigma}{4\pi\epsilon_o r} = \frac{r\sigma}{\epsilon_o} \text{ volts}$$

i.e.
$$\sigma = \frac{V\epsilon_o}{r} \text{ coulomb.m.}^{-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The equilibrium condition for the bubble is

$$p + \frac{\sigma^2}{2\epsilon_o} = \frac{4T}{r}$$

whence on substituting for p and σ from (43) and (44) above:

$$\frac{4Tr_o^2}{r^3} + \frac{V^2\epsilon_o^2}{2\epsilon_o r^2} = \frac{4T}{r}$$

i.e.
$$r_o^2 = r^2 - \frac{V^2\epsilon_o r}{8T}$$

Since $r \approx 10^{-2}$ m.; $V = 6 \times 10^3$ volts; $T = 3 \times 10^{-2}$ newton.m.$^{-1}$:

$$r_o^2 = 10^{-4}\left(1 - \frac{36 \times 10^6}{24 \times 10^{-2}} \times \frac{1}{36\pi} \times 10^{-5} \times 10^{-2}\right)$$

$$= 10^{-4}(1 - 0{\cdot}1326)$$

i.e.
$$r_o = 9{\cdot}31 \times 10^{-3} \text{ metres} \quad 0{\cdot}931 \text{ cm.}$$

1.33. Absolute Electrostatic Voltmeter

This instrument consists of two parallel circular metal plates A and B (Fig. 1.28), the radius of B being smaller than A and surrounded by the annular " guard-ring " B', the external diameter of the guard-ring being the same as that of A. B is earthed and suspended from one arm of a sensitive balance which is counterpoised so that B lies in the plane of B'. The p.d. to be measured is connected between A and B. When A is charged an opposite induced charge is produced on B and the field between them is extremely uniform.

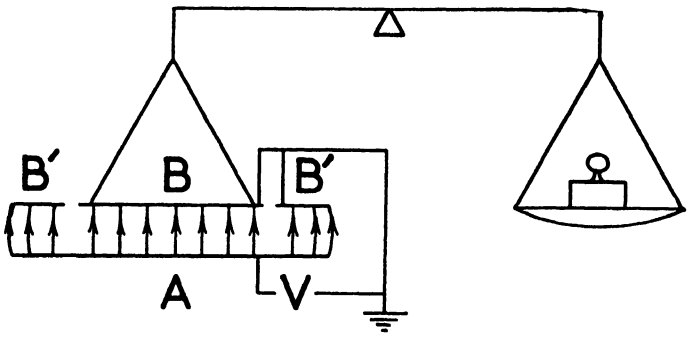


Fig. 1.28

Let the p.d. between A and B be V volts, the distance between the plates t metres, the area of B, a m^2., and the charge per unit area of either plate σ coulomb.m.$^{-2}$.

The mechanical force which acts on B by virtue of its induced charge causes it to be attracted to A. The force of attraction is:

$$f = Fa = \frac{\sigma^2 a}{2\epsilon_0 \epsilon} \quad . \quad . \quad . \quad . \quad . \quad (45)$$

The original position of B is restored by adding a small weight m. kgm. to the opposite pan of the balance. For equilibrium:

$$\frac{\sigma^2 a}{2\epsilon_0 \epsilon} = mg \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Since $$V = -\int E_s \, . \, ds = \frac{\sigma t}{\epsilon_0 \epsilon} \text{ volts, [(31)]:}$$

$$\sigma = \frac{V \epsilon_0 \epsilon}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Substituting for σ in (46):

$$\frac{V^2 \epsilon_0^2 \epsilon^2 a}{2\epsilon_0 \epsilon t^2} = mg$$

whence: $$V = t\sqrt{\frac{2mg}{\epsilon_0 \epsilon a}} \text{ volts} \quad . \quad . \quad . \quad (48)$$

Since $\epsilon = 1$ for air or vacuum, V is measured absolutely. For p.d.'s less than thousands of volts the mass required for balance is small, and the voltmeter is unsuitable for ordinary use.

Example

A parallel plate air condenser is made from two discs, each of radius 8 *cm., placed* 1 *cm. apart. The upper plate is earthed and hung from a balance. The lower plate is initially at earth potential and is then raised to* 15,000 *volts. What is the change in the apparent weight of the upper plate? (Ignore edge effects).* (Part question Camb. Open Schol.).

Ignoring edge effects the field between the plates is uniform and the force acting vertically downwards on the upper plate is its weight plus the total electrostatic force given by $\sigma^2 A/(2\epsilon_0\epsilon)$. The apparent weight of the upper plate is thus increased by the amount (expressed in force units):

$$w = \frac{\sigma^2 A}{2\epsilon_0\epsilon} \text{ newtons}$$

Since: $\sigma = V\epsilon_0\epsilon/t$ coulomb.m.$^{-2}$ from (47) above:

$$w = \frac{V^2\epsilon_0^2\epsilon^2 A}{2\epsilon_0\epsilon t^2} = \frac{V^2\epsilon_0\epsilon A}{2t^2} \text{ newtons}$$

Using: $V = 1{\cdot}5 \times 10^4$ volts; $t = 0{\cdot}01$ m.; $A = \pi a^2 = 64\pi \times 10^{-4}$ m.^{2}, $\epsilon_0 = 1/(36\pi) \times 10^{-9}$ M.K.S.;

$$w = \frac{2{\cdot}25 \times 10^8}{2 \times 10^{-4}} \times \frac{1}{36\pi} \times 10^{-9} \times 64\pi \times 10^{-4} \text{ newtons}$$

i.e. $w = 0{\cdot}2$ newtons.

A body having a mass of 1 kgm. has a gravitational force acting on it equal to 9·81 newtons, so that the mass equivalent to w is:

$$m = \frac{0{\cdot}2}{9{\cdot}81} = 0{\cdot}0204 \text{ kgm.} = 20{\cdot}4 \text{ gm.}$$

Hence, the apparent change in " weight " of the upper plate is 20·4 gm.

EXERCISES

1. Describe one experiment which tests the validity of the law of inverse squares in electrostatics. Point charges of $+10, +20, +30$ and -10 micro-coulombs are placed at the corners A,B,C,D, respectively of a square of side 50cm. Find the magnitude and direction (relative to the diagonal BD) of the force on a positive point charge of 1 micro-coulomb placed at the centre of the square. (Oxford Open Schol. Subsid.).

2. Define the term " electrostatic potential ".
   Three positive charges of 1, 2 and 3 $\times 10^{-9}$ coulombs are arranged at the corners of an equilateral triangle of side 1 m. Find (a) the potential and (b) the electric field strength at the centre of the triangle. (Oxford Open Schol. Subsid.).

3. Give an account of Faraday's ice pail experiment.

If you were supplied with a negatively charged sphere A, and an insulated ice pail, show how it would be possible to give a second sphere B:

(a) A negative charge and zero potential.

(b) Zero charge and a negative potential.

(c) A negative charge and a positive potential.

(Manch. Open Schol.).

4. Explain the meaning of *potential at a point in an electrostatic field* and obtain an expression for its magnitude at a distance r m. from the centre of an isolated spherical conductor carrying a charge of Q coulombs.

How may a conductor be charged inductively when a second charged conductor is available? Illustrate and explain the process by means of potential diagrams, assuming the conductors to be spheres. (Durham G.C.E. Schol.).

5. Define *intensity of electric field, potential gradient at a point* What is the connection between these quantities ?

Explain why there is a tendency for electricity to discharge from points ?

A surface discharges into the air when the potential gradient is 3×10^4 volts per cm. To what potential can a sphere of radius 3 metres be raised before it begins to lose its charge to the air ? (Camb Local G.C.E. Adv.L.).

6. Define the electric intensity (electric field strength) and the electric potential at a point in an electric field, and explain the relation between them.

What factors determine the maximum charge that can be stored on an insulated conductor in the air?

Calculate the mechanical force between the plates of a parallel-plate air condenser, which has plates of area 100 sq.cm. separated by 0·5 cm., when the p.d. between the plate is 600 volts. (Ox. Local G.C.E. Schol.).

7. What is meant by (a) the intensity of an electric field and (b) total normal induction ?

Gauss' theorem states that the total normal induction over a closed surface is numerically equal to the total charge inside the surface. Using this theorem, or otherwise, determine the intensity outside and close to an electrified surface surrounded by air, in terms of the surface-density of charge.

The electric field near the earth's surface is directed inwards and has a value of 300 volts per metre. If the earth is regarded as a spherical conductor of radius $6{\cdot}38 \times 10^6$ metres, deduce the surface density of charge and the corresponding potential which would give rise to the observed field. Show also that a water drop of 10^{-5} cm. radius and carrying a charge of 1 electron will be supported against the force of

gravity anywhere above and near to the earth's surface. (Charge of an electron $= 1{\cdot}6 \times 10^{-19}$ coulombs) ($g = 9{\cdot}81$ m.sec.$^{-2}$). (London G.C.E. Schol.).

8. Show that Gauss's theorem follows from an inverse square law of force, and give examples of its use in electrostatic problems.

A total charge " e " is spread uniformly through the volume of a sphere of radius " a ". Draw graphs to show the variation of electric field and electric potential with distance from the centre, both inside and outside the sphere. (Camb. Open Schol.).

9. Define *unit electrostatic charge* and *dielectric constant.*

Give an account of any theoretical and experimental basis for the inverse square law of force between electrostatic charges. (London G.C.E. Schol.).

10. Give an account of the direct and indirect evidence in favour of the inverse square law of force between electrical charges.

Two similar spherical particles of mass M carry each a charge Q. Initially one is at rest on a smooth plane surface, and the other is projected along the surface directly towards it from a great distance with velocity V. Describe qualitatively the subsequent motion of the particles, and find an expression for the closest distance to which they approach one another. (Camb. Open Schol.).

11. By applying Gauss' theorem, or by any other method, find, for a conductor charged with a surface density of σ coulombs per sq. m. (*a*) the electric intensity near the surface, (*b*) the mechanical force on each sq. m. of its surface.

A pendulum consists of a small conducting sphere of mass 5 gm. suspended by a long insulating fibre Its normal periodic time is 2 sec. If the pendulum bob is charged with $\frac{2}{3} \times 10^{-9}$ coulombs and it is caused to oscillate with small amplitude so that the sphere is moving just above a large horizontal metal plate which is charged with a uniform charge of $\frac{1}{3} \times 10^{-4}$ coulombs per sq. m., find the approximate difference between the time of swing of the pendulum in this case and in that of the normal oscillation. (London G.C.E. Schol.).

12 Obtain an expression for the force per unit area on the plates of a charged parallel plate condenser.

The bob of a simple pendulum of length 100 cm. consists of a vertical metal disc of diameter 5 cm. and mass 50 gm. At 1 cm. on either side of this disc and parallel with it are two large fixed metal plates. The bob of the pendulum is maintained at earth potential and the two fixed plates at potentials of $+1200$ volts and -1200 volts respectively. Calculate the period of the pendulum for very small displacements in a direction at right angles to the plane surfaces. (Camb. Open Schol.).

CHAPTER 2

CAPACITY AND CONDENSERS

2.1. Capacity of a Conductor

The potential acquired by a conductor is always proportional to the charge given to it, i.e.:

$$Q \propto V \text{ and } Q = CV \quad . \quad . \quad . \quad (1)$$

The constant of proportionality C, is called the *capacity* of the conductor.

Definition: *The capacity of a conductor is the amount of charge required to raise its potential by unity.*

The M.K.S. unit of capacity corresponding to coulombs and volts for charge and potential is called the *farad.*

2.2. Factors influencing Capacity

The capacity of a conductor depends on (i) the presence of other conductors (especially earthed conductors), (ii) the dielectric constant of the medium, and (iii) the surface area. These effects are demonstrated by experiment:

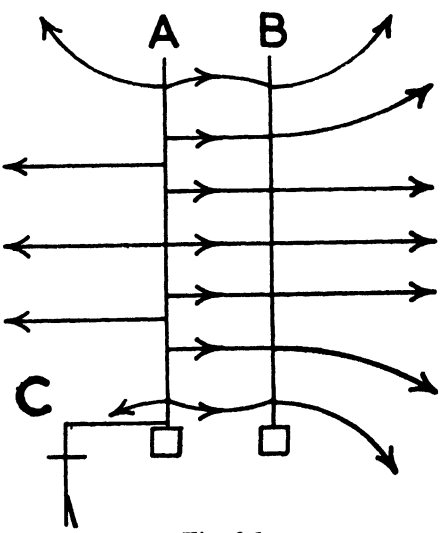


Fig. 2.1

(i) The insulated plane conductor A (Fig. 2.1) is charged by induction. The deflection of the electroscope C, to which A is connected, indicates the potential of A. An insulated second plate B which is uncharged, is slowly brought nearer to A. The electroscope deflection decreases slightly, indicating a reduction in the potential of A. The effect is reversed when B is withdrawn. The reduction in potential of A caused by the presence of B means an increase in the capacity of A since its charge is unaltered. The effect is much greater if B is maintained at earth potential.

The decrease in the potential of A effected by the presence of nearby conductor B, is a consequence of the induced charges produced on B. The latter cause a redistribution of charge on A and a distortion of its

field. More lines of induction leave the side of A nearer to B with the result that less work is done in bringing up a unit charge from infinity to A, i.e. the potential of A is reduced (Figs. 2.1 and 2.2).

(ii) Using the same charged conductor A, a thick slab of dielectric, such as glass or ebonite, is brought slowly near to A. The electroscope deflection falls indicating an increase in the capacity of A. The effect is reversed when the dielectric is withdrawn. The decrease in potential of A is again caused by a redistribution of charge. In this case the displacement of the dielectric molecules (p. 477), is responsible for concentrating the field on the side of A nearer to B (Fig. 2.3). The effect increases with the value of ϵ.

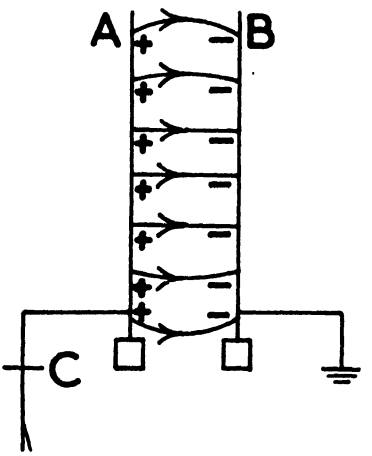


Fig. 2.2

(iii) The effect of the surface area is demonstrated by using a conductor consisting of a sheet of tinfoil, one end of which is attached to a rod capable of rotation in insulated bearings. The free edge is weighted

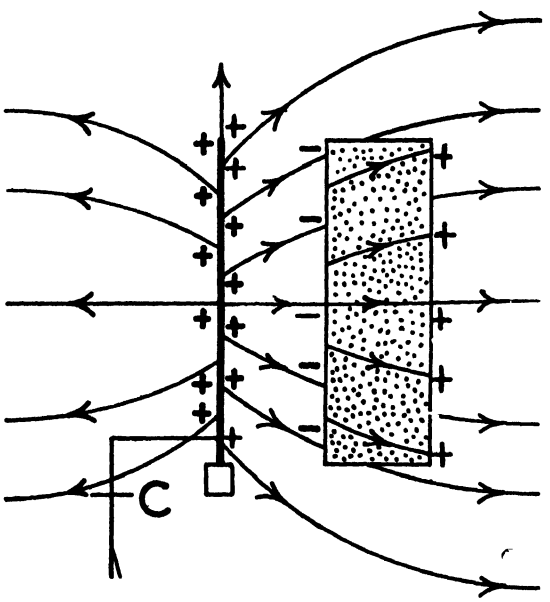


Fig. 2.3

to maintain the foil vertical. The potential of the foil is measured with a gold leaf electroscope (Fig. 2.4). The foil is charged by induction when rolled up. The foil is then slowly unrolled. The electroscope deflection falls indicating an increase in capacity of the foil.

The decrease in potential of the foil, in this case, is caused by a gradual reduction in the surface density of charge as the foil area increases.

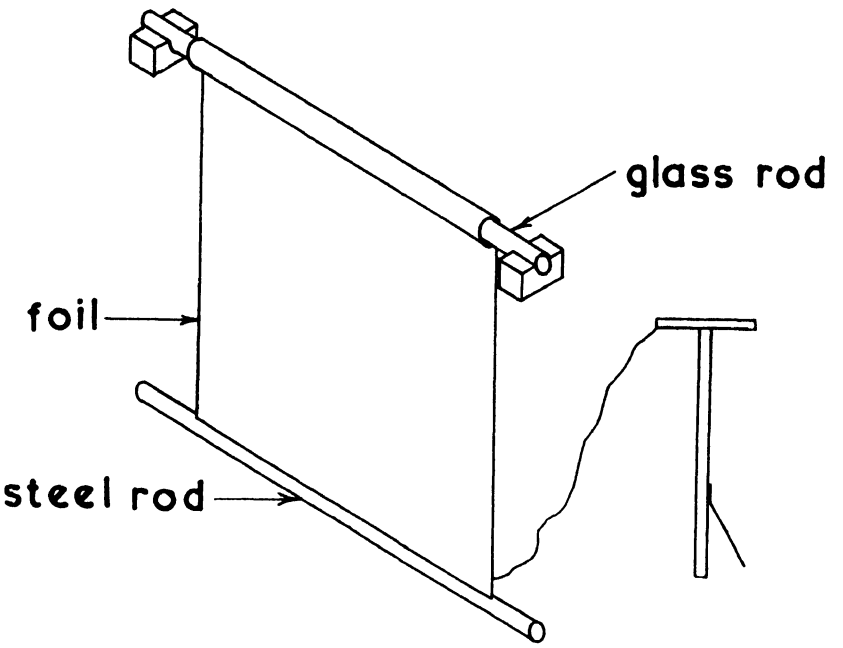


Fig. 2.4

2.3 Condensers

Any two conductors between which an electric field may be maintained form a *condenser.*

The capacity of a condenser is the charge required on one conductor to raise the p.d. between the conductors forming the condenser by unity.

The conductors A and B in the above experiment (i) form a *parallel plate* condenser. By conducting experiments in an exactly similar manner it is shown that the capacity of a condenser depends on (i) the state of electrification of either plate, i.e. whether one plate is earthed, etc., (ii) the plate separation, (iii) the dielectric between the plates, and (iv) the area of the plates. When one plate is earthed the capacity increases when the plate separation decreases, increases as the dielectric constant of the medium increases and increases as the plate area increases.

In practice, condensers are made of symmetrically shaped conductors; the capacity is almost independent of the presence of nearby conductors if one conductor (plate) is earthed. Isolated conductors are condensers whose earthed " plate " is formed by distant earthed conductors on which the lines of induction end. A general term covering any arrangement is *capacitor.*

2.4. Calculation of Capacity

(i) *Isolated metal sphere:* Let the sphere have a radius r m. and possess a charge $+q$ coulombs. The potential of the sphere is given by:

$$V = \frac{q}{4\pi\epsilon_0\epsilon r} \text{ volts.}$$

Hence, the capacity of the sphere is:

$$\underline{\underline{C}} = \frac{q}{V} = \underline{\underline{4\pi\epsilon_0\epsilon r}} \text{ farads} \quad . \quad . \quad . \quad (2)$$

For a sphere of 1 cm. radius in vacuum:

$$C = 4\pi\epsilon_0 \times 10^{-2} = \tfrac{1}{9} \cdot 10^{-11} \ F.$$

Hence, a sphere having a capacity of 1 farad must have a radius of 10^{10} m. The earth is a conducting sphere of radius $6{\cdot}3 \times 10^6$ m. (approx.), so that its capacity is only $1{\cdot}6 \times 10^{-3}$ farads. It follows that a sphere of capacity 1 farad has a radius 600 times greater than that of the earth. The farad is thus an extremely large unit; more common units are micro-farad ($1\mu F = 10^{-6}F$), and the pico-farad ($1pF = 1\mu\mu F = 10^{-12}\ F.$).

(ii) *Spherical Condenser* (Outer sphere earthed): The spherical condenser consists of a metal sphere A of radius " a " m. surrounded by an earthed metal concentric sphere B of radius b m. (Fig. 2.5). If A is

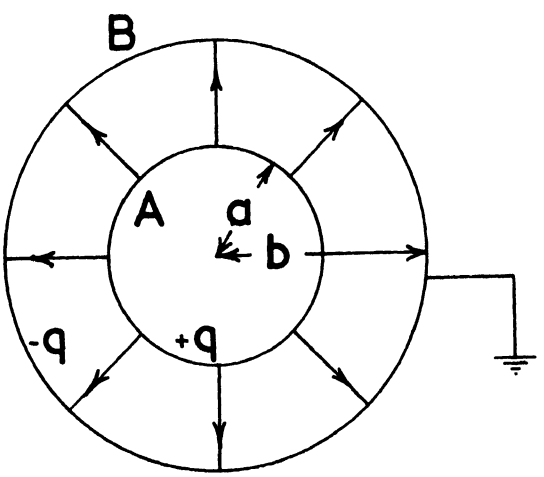


Fig. 2.5

given a charge $+q$ coulombs, an induced charge $-q$ coulombs is produced on the inner surface of B. If A were isolated its potential would be $+q/(4\pi\epsilon_0\epsilon a)$ volts. The effect of the negative charge on B is to

reduce the potential of A by the amount $-q/(4\pi\epsilon_0\epsilon b)$ volts, so that the net potential of A is:

$$V = \frac{q}{4\pi\epsilon_0\epsilon}\left\{\frac{1}{a} - \frac{1}{b}\right\} \text{ volts} \quad . \quad . \quad . \quad (3)$$

Since the charge on A is $+q$ coulombs, the capacity of the condenser is:

$$C = \frac{q}{V} = \frac{4\pi\epsilon_0\epsilon}{\frac{1}{a} - \frac{1}{b}} = \frac{4\pi\epsilon_0\epsilon ab}{b - a} \text{ farads} \quad . \quad . \quad (4)$$

The same result is obtained by noting that the distribution of charge on A is unaffected by the presence of B and hence the field intensity at any point distant r m. from A, between A and B, is unaffected by B (the extent of the field is merely diminished in this case). Thus, the potential at the point considered is given by:

$$V_r = -\int_b^r E \cdot ds = -\int_b^r \frac{q}{4\pi\epsilon_0\epsilon r^2} \cdot ds = \frac{q}{4\pi\epsilon_0\epsilon}\left\{\frac{1}{r} - \frac{1}{b}\right\} \quad (5)$$

Hence, the potential at the surface of A, $(r = a)$, is given by:

$$V_a = \frac{q}{4\pi\epsilon_0\epsilon}\left\{\frac{1}{a} - \frac{1}{b}\right\} \text{ volts}$$

and:

$$C = \frac{q}{V_a} = \frac{4\pi\epsilon_0\epsilon ab}{b - a} \text{ farads.}$$

**Spherical Condenser* (Inner sphere earthed): In this case the sphere B is insulated and A is earthed by a fine wire passing through a small hole in B (Fig. 2.6). If B is charged positively, induced charges appear on

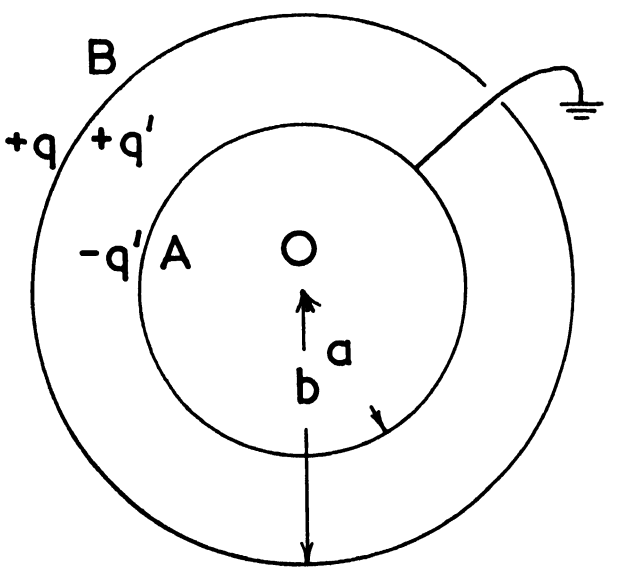


Fig. 2.6

A and the inner surface of B. The induced charges give rise to a field such that the p.d. between A and B is equal and opposite to the potential of B relative to earth (if A were insulated its potential would be that of B). Let the induced charges be $\pm q'$ coulombs and the *total* charge on the *outer* surface of $B + q$ coulombs. The field strength at a point P between A and B, distant r m. from the centre O, is:

$$E_P = \frac{-q'}{4\pi\epsilon_0\epsilon r^2} \text{ volt.m.}^{-1}$$

Hence, the p.d. between A and B is:

$$V = \frac{-q'}{4\pi\epsilon_0\epsilon}\int_a^b \frac{dr}{r^2} = \frac{q'}{4\pi\epsilon_0\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ volts} \quad . \quad (6)$$

The potential of B relative to earth is also given by:

$$V = \frac{q}{4\pi\epsilon_0\epsilon b} \text{ volts} \quad . \quad . \quad . \quad . \quad (7)$$

From (6) and (7):

$$\frac{q'}{4\pi\epsilon_0\epsilon}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\epsilon_0\epsilon b}$$

i.e. $$q' = q \cdot \frac{a}{b-a} \text{ coulombs} \quad . \quad . \quad . \quad (8)$$

Hence the total charge on B, viz. $(q + q') = \frac{qb}{b-a}$ coulombs, and the capacity of the condenser is thus:

$$C = \frac{q+q'}{V} = \frac{qb}{b-a} \cdot \frac{4\pi\epsilon_0\epsilon b}{q} = \underline{\frac{4\pi\epsilon_0\epsilon b^2}{b-a} \text{ farads}} \quad . \quad (9)$$

(*N.B.* In both cases it is assumed that the radii of the inner and outer surfaces of B are the same).

(iii) **Cylindrical Condenser* (very long): This consists of a charged cylinder A of radius "a" m., surrounded by an earthed concentric cylinder B of radius b m. (Fig. 2.7). Let the charge *per metre* of A be $+q$ coulombs. For long cylinders the lines of force between A and B are radial (plan of Fig. 2.7). From (1.29) the field intensity at P, distant r m. from the axis is:

$$E_P = \frac{q}{2\pi\epsilon_0\epsilon r} \text{ volt m.}^{-1}$$

Hence: $$V_P = -\int_b^r E \, . \, ds = -\frac{q}{2\pi\epsilon_0\epsilon r} \, . \, ds = \frac{q}{2\pi\epsilon_0\epsilon}\Big[-\log_e s\Big]_b^r$$

i.e. $$V_P = \frac{q}{2\pi\epsilon_0\epsilon} \cdot \log_e\left(\frac{b}{r}\right) \text{ volts} \quad . \quad . \quad . \quad (10)$$

Hence the potential of A is:

$$V_A = \frac{q}{2\pi\epsilon_0\epsilon} \cdot \log_e\left(\frac{b}{a}\right) \text{ volts} \quad . \quad . \quad . \quad (11)$$

The capacity of the condenser is:

$$c = \frac{q}{V_A} = \frac{2\pi\epsilon_0\epsilon}{\log_e\left(\frac{b}{a}\right)} \text{ farads } \textit{per metre} \quad . \quad . \quad (12)$$

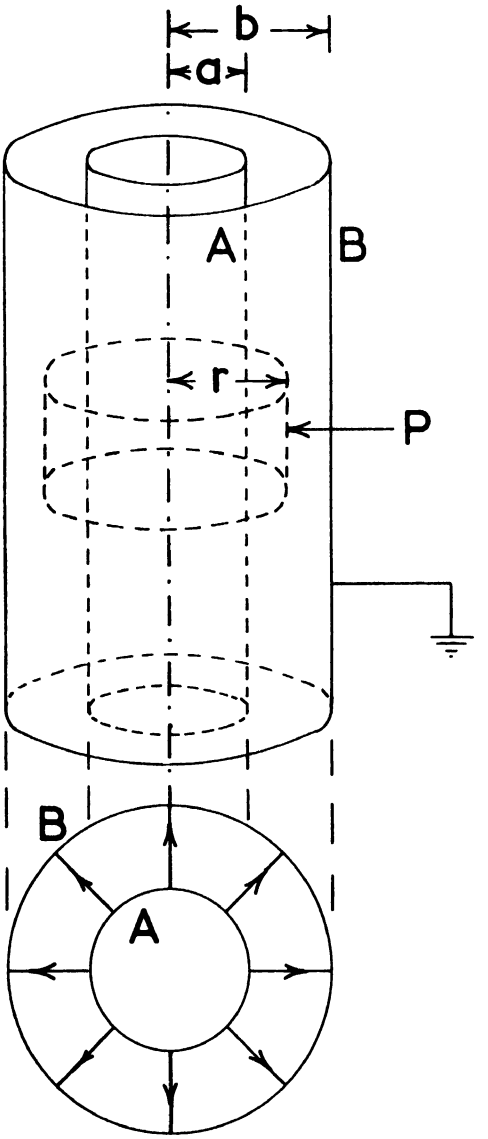


Fig. 2.7

In a cylindrical condenser of finite length, the radial nature of the lines of induction is maintained by using guard-ring cylinders at both ends. These are charged to the same potential as the inner cylinder. Such condensers (Fig. 2.8) are used as standards, since their capacity can be determined from a knowledge of the dimensions. They have small capacity because the factor $1/\log_e(\frac{b}{a})$ cannot be made very large without difficulty.

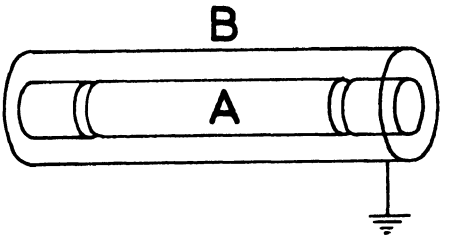


Fig. 2.8

(iv) *Parallel plate Condenser:* This consists of two similar plane parallel plates such as X and Y of Fig. 2.9, each having an area of A m.^{2}. Let the plate separation be t metres and the dielectric constant ϵ. Plate Y is maintained at earth potential.

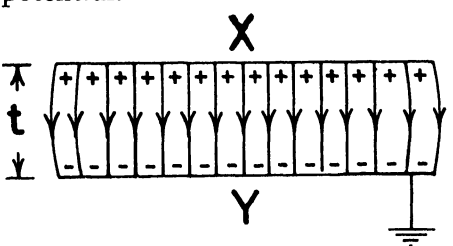


Fig. 2.9

If X has a total charge of $+q$ coulombs its surface density of charge (on the side facing Y) is: $\sigma = q/A$ coulomb.m.$^{-2}$. Assuming the plates are large compared with the separation, all the lines of induction are parallel between the plates and no charge exists on the side of X further from Y. Thus, the field intensity at any point between the plates is: $E = \sigma/(\epsilon_0\epsilon)$ volt.m.$^{-1}$ (p. 29). Hence, the potential of X relative to Y is given by:

$$V = -\int_t^0 E \, . \, ds = \frac{\sigma t}{\epsilon_0 \epsilon} \text{ volts} \quad . \quad . \quad . \quad (13)$$

Hence, the capacity of the condenser is:

$$C = \frac{q}{V} = \frac{A\sigma}{\frac{\sigma t}{\epsilon_0 \epsilon}} = \frac{A\epsilon_0\epsilon}{t} \text{ farads} \quad . \quad . \quad . \quad (14)$$

The capacity of a parallel plate air condenser is generally small, e.g. if $A = 100$ cm.^{2}, $t = 1$ cm., $C = 0{\cdot}926 \times 10^{-11}$ farads (from (14)). Using the same size of plate it would be necessary to reduce the separation to 10^{-5} cm. to effect a capacity of $1\mu F$. This is virtually impossible with air as the dielectric. Thus, to make capacitors of this order of magnitude special methods have to be adopted as below.

2.5. Construction of Large Condensers

One convenient method of manufacturing condensers is illustrated in Fig. 2.10. Two long rectangular metal foils are interleaved between

thin sheets of waxed paper which act as a dielectric. The end L of the sandwich is rolled towards M to form a compact parallel plate system of relatively large area and small separation. Each metal foil is connected to a terminal wire and the capacitor dipped in wax to preserve the insulation.

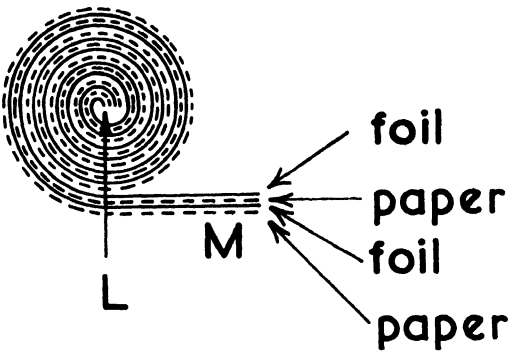


Fig. 2.10

For larger capacitance, electrolytic condensers are manufactured. An electrolytic condenser consists of an axial electrode on which is deposited a very thin film of some metal oxide by means of electrolysis—the electrode acts as the anode in an electrolytic cell whose cathode is a concentric containing can. After the " forming " process the outer can and electrolyte are retained in the condenser as the earthed " plate ". Because of the extreme thinness of the oxide deposit, which forms the dielectric, the capacity of the condenser can be made very large—tens of micro-farads. To prevent damage to the dielectric caused by a current flow in the wrong direction in the electrolyte, one terminal is always marked red; this terminal is the non-earthed terminal.

Example

Define capacitance of a conductor. How is it affected by the presence of a neighbouring conductor which is (a) *uncharged and insulated,* (b) *earthed? How would you demonstrate these effects experimentally?*

Assuming the expression for the potential of an isolated conducting sphere, show that the capacitance of such a sphere will be increased by a factor n *if it is enclosed within an earthed concentric sphere, the ratio of the radii of the spheres being* n/(n–1). (London G.C.E. Adv. L.).

The term capacitance is often used instead of capacity, so that parts (*a*) and (*b*) are answered in the text.

For an isolated conducting sphere: $C = 4\pi\epsilon_0 a$ (in vacuum). When surrounded by an earthed sphere of radius b, the capacity becomes:

$$C' = 4\pi\epsilon_0 \cdot \frac{ab}{b-a}$$

Hence:
$$\frac{C'}{C} = \frac{ab}{a(b-a)} = \frac{b}{b-a}$$

Since: $\frac{b}{a} = \frac{n}{n-1}$, it follows that: $b = \frac{an}{n-1}$ and: $\underline{\frac{C'}{C} = n}$

Example

Define potential, capacity.

Obtain an expression for the potential at a point " x " metres away from a point charge of " q " coulombs.

Hence calculate the capacity of a condenser made up of two concentric thin spherical shells, the inner one being charged and the outer one earthed.

The radii of the two spherical shells which form an air condenser of this kind are 10·0 *cm. and* 10·5 *cm. After the inner shell has been charged to a potential of* 100 *volts, the outer shell is taken apart and removed. What is the final potential of the charged inner shell?* (Oxford Local G.C.E. Adv. L.).

The first three parts of the question are answered in the text.

The capacity of a spherical air condenser is given by:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} \text{ farads}$$

Hence, the charge on the inner sphere is:

$$Q = CV = 4\pi\epsilon_0 \frac{abV}{b-a} \text{ coulombs}$$

Using: $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ M.K.S.; $a = 0{\cdot}1$ m.; $b = 0{\cdot}105$ m.; $V = 100$ volts:

$$Q = 4\pi \times \frac{1}{36\pi} \times 10^{-9} \frac{(0{\cdot}0105)}{(0{\cdot}005)} \times 100 = \frac{2{\cdot}1}{9} \times 10^{-7} \text{ coulombs}$$

When the outer sphere is removed the charge on the inner sphere is unchanged but the capacity of the sphere remaining is:

$$C' = 4\pi\epsilon_0 a \text{ farads}$$

Hence, the new potential of the inner sphere is:

$$\underline{\underline{V'}} = \frac{Q}{C'} = \frac{2{\cdot}1}{9} \times 10^{-7} \times \frac{36\pi \times 10^9}{4\pi \times 0{\cdot}1} = \underline{\underline{2{,}100 \text{ volts}}}$$

Example

Derive from first principles, an expression for the capacitance per unit area of a parallel plate condenser of infinite extent.

A parallel plate condenser is made of two plane circular plates separated by " d " metres of air. When a parallel faced plane sheet of glass 2 *mm. thick is placed between the plates, the capacity of the system is increased by* 50 *per cent of its initial value. What is the distance between the plates, if the dielectric constant of the glass is* 6? (London G.C.E. Adv. L.).

(i) The capacity of a very large parallel plate condenser is derived above (p. 47). Hence, the capacitance per unit area is: $\underline{\underline{c}} = C/A = \underline{\underline{\epsilon_0\epsilon/t}}$.

(ii) Let the area of either plate be A m.^{2} The capacity of the condenser with *air* between the plates is then: $C_0 = \frac{A\epsilon_0}{d}$ farads.

When a sheet of dielectric of thickness h m. is placed between the plates with separations as shown in Fig. 2.11, the electrostatic induction between the plates is unaltered but the field strength at points within the dielectric is reduced. Let the surface density of charge on the upper plate be σ coulomb.m.$^{-2}$. The induced charge density on the lower earthed plate is hence $-\sigma$ coulomb.m.$^{-2}$. The induction at any point between the plates is given by:

$$D = \sigma \text{ coulomb.m.}^{-2}$$

Hence, the electric intensities at points inside the dielectric and in the air spaces are given by:

$$E_d = \frac{D}{\epsilon_0 \epsilon} \text{ and } E_a = \frac{D}{\epsilon_0} \text{ respectively}$$

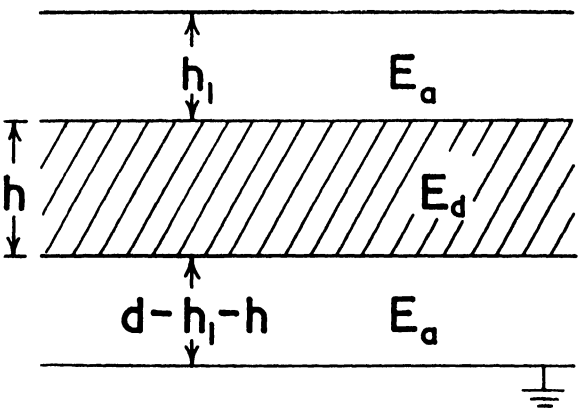


Fig 2.11

Since the fields in the air spaces and the dielectric are uniform the p.d. between the plates is given by:

$$V = E_a h_1 + E_d h + E_a (d - h_1 - h) \text{ volts}$$

i.e. $$V = \frac{\sigma}{\epsilon_0}\left\{h_1 + \frac{h}{\epsilon} + d - h_1 - h\right\} = \frac{\sigma}{\epsilon_0}\left\{d - h + \frac{h}{\epsilon}\right\} \text{ volts}$$

i.e. $$V = \frac{\sigma}{\epsilon_0}\left\{d - h\left(1 - \frac{1}{\epsilon}\right)\right\} \text{ volts}$$

By definition, the new capacity of the condenser is:

$$C = \frac{Q}{V} = \frac{A\sigma}{\frac{\sigma}{\epsilon_0}\left\{d - h\left(1 - \frac{1}{\epsilon}\right)\right\}} = \frac{A\epsilon_0}{d - h\left(1 - \frac{1}{\epsilon}\right)} \text{ farads}$$

Hence: $$\frac{C}{C_0} = \frac{\frac{A\epsilon_0 d}{\left\{d - h\left(1 - \frac{1}{\epsilon}\right)\right\}}}{A\epsilon_0 / d} = \frac{d}{d - h\left(1 - \frac{1}{\epsilon}\right)}$$

Using: $C/C_0 = 1{\cdot}5$; $h = 2 \times 10^{-3}$ m ; $\epsilon = 6$; $\epsilon_0 = 1/(36\pi) \times 10^{-9}$ M.K.S.:

$$1{\cdot}5 = \frac{d}{d - 2 \times 10^{-3}(1 - \frac{1}{6})} = \frac{6d}{6d - 10^{-2}}$$

whence: $$\underline{d = 0{\cdot}005 \text{ m.} = 5 \text{ mm.}}$$

2.6. Condensers in Series

Condensers connected in series are arranged as in Fig. 2.12. Here C_1, C_2 and C_3, are the capacities of three condensers. The outer members of the series are connected to the p.d. V volts. If the outer plate of C_1

acquires a charge $+Q$ coulombs, the opposite plate of this condenser acquires an induced charge of $-Q$ coulombs. The plate of C_2 connected to C_1 must thus acquire a charge $+Q$ coulombs since there is no net charge on the connected plates. The other plates also acquire induced charges as shown. The p.d.'s across the separate condensers are accordingly given by:

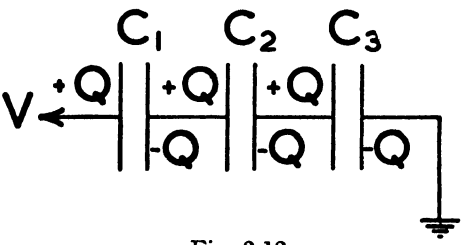


Fig. 2.12

$$V_1 = Q/C_1,\ V_2 = Q/C_2 \text{ and } V_3 = Q/C_3 \text{ volts}$$

Since the p.d. across the series of condensers is V volts:

$$V = V_1 + V_2 + V_3 = Q\left\{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right\} \text{ volts}$$

The equivalent capacity of the series, C, is defined by:

$$C = \frac{Q}{V} = Q \div Q\left\{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right\} \text{ farads}$$

Hence:
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For n condensers, therefore: $\dfrac{1}{C} = \sum_{r=1}^{n} \dfrac{1}{C_r}$ . . . . . (15)

2.7. Condensers in Parallel

Condensers in parallel have one plate of each condenser connected together and the opposite plates connected together, (Fig. 2.13). When

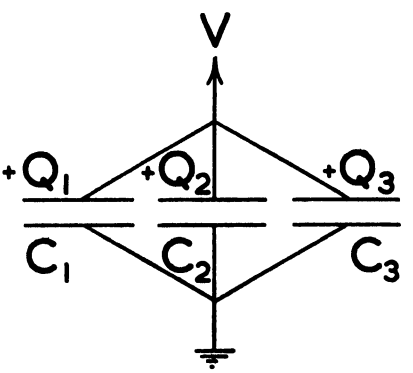


Fig. 2.13

a

the two sets of plates are connected to a p.d. of V volts, the charges on the separate condensers are:

$$Q_1 = C_1V; Q_2 = C_2V; \text{ etc. coulombs.}$$

The total charge on all the similarly charged plates is hence:

$$Q = Q_1 + Q_2 + Q_3 = V(C_1 + C_2 + \ldots) \text{ coulombs.}$$

Hence, the equivalent capacity of the system is given by:

$$C = \frac{Q}{V} = \frac{V}{V}(C_1 + C_2 + \ldots) = C_1 + C_2 + \ldots \text{ farads.}$$

For n condensers, therefore: $C = \sum_{r=1}^{n} C_r$ . . . . . (16)

Both forms of connection are useful. If, for example, a capacitance of $1\mu F$ is required but a $1\mu F$ condenser is not available, two $2\mu F$ condensers may be connected in series or two $0{\cdot}5\mu F$ condensers may be connected in parallel.

2·8 Large Parallel Plate Condensers

Large standard condensers consist of a large number of parallel plate condensers in parallel. Alternate plates are connected together and brought out to two insulated terminals on an earthed box as in Fig. 2.14.

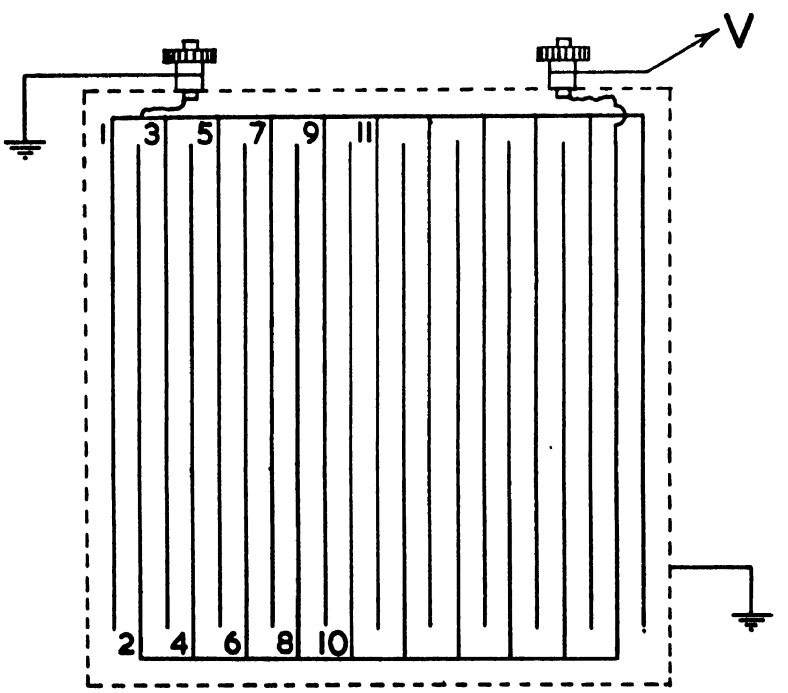


Fig. 2.14

If similar plates numbered 1, 3, 5, ... n, are earthed and those numbered 2, 4, 6, ... $(n-1)$, are connected to a potential V, the capacitor is equivalent to $n-1$ condensers in parallel. If the plate area is A m.^{2}, the dielectric constant ϵ and t the distance between adjacent plates, the capacitance of the condenser is:

$$C = \frac{(n-1) \cdot A \epsilon\epsilon_0}{t} \text{ farads.}$$

For a typical condenser in which $n = 101$, $A = 100$ cm.$^2 = 10^{-2}$ m.^{2}, $\epsilon = 1$ (air dielectric), $t = 2$ mm. $= 2 \times 10^{-3}$m.:

$$C = \frac{100 \times 10^{-2} \times 10^{-9}}{36 \times \pi \times 2 \times 10^{-3}} = \underline{4{\cdot}42 \times 10^{-9} \text{ farads.}}$$

Larger capacitances than this are made by using smaller plate separation with thin mica sheet as the dielectric ($\epsilon = 6$).

Example

Derive expressions for the capacity between two concentric spherical conductors when (a) *the outer, and* (b) *the inner sphere is earthed. Explain carefully the distribution of charges on the two conductors in each case.*

A condenser consists of two concentric metal spheres of radii 5 cm. and 6 cm. The space between the spheres is half filled with a liquid whose dielectric constant is 3·7. Calculate the capacity between the two spheres when the outer is earthed. Express your answer in practical units. (Manchester Open Schol.).

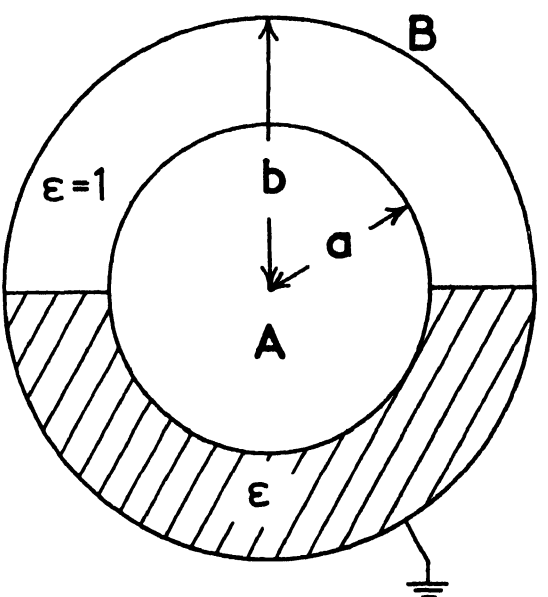


Fig. 2.15

Parts (*a*) and (*b*) are answered in the text.

When the space between the spheres is half-filled with dielectric, as in Fig. 2.15, the condenser is equivalent to two hemispherical condensers in parallel, one con-

taining air and the other liquid. The capacity of each hemispherical condenser is half that of a complete spherical condenser. Hence, for the upper half of the condenser of Fig. 2.15:

$$C_1 = 2\pi\epsilon_o \cdot \frac{ab}{b-a}$$

For the lower half: $$C_2 = 2\pi\epsilon_o\epsilon \cdot \frac{ab}{b-a}$$

The total capacitance is given by:

$$C = C_1 + C_2 = 2\pi\epsilon_o \cdot \frac{ab}{b-a}(1 + \epsilon) \text{ farads}$$

Using: $a = 0{\cdot}05$ m.; $b = 0{\cdot}06$ m.; $\epsilon = 3{\cdot}7$; $\epsilon_o = 1/(36\pi) \times 10^{-9}$ M.K.S.:

$$C = 2\pi \times \frac{1}{36\pi} \times 10^{-9} \times \frac{5 \times 6}{10^{-2}} \times 10^{-4} \times 4{\cdot}7 = 7{\cdot}83 \times 10^{-11} \text{ farads}$$

Example

State Gauss's theorem in electrostatics, and use it to show that the capacity of a parallel plate condenser in a vacuum with a plate separation of d *is* ϵ_o/d *per unit area. What would be the change in this capacity if an insulated uncharged conducting plate of thickness "* x *" were introduced between the plates?*

Five conducting plates A, B, C, D *and* E *of large extent are placed parallel in a vacuum so that adjacent plates are separated by a distance "*d *". The two outer plates A and E are kept earthed. What will be the capacity per unit area between* (a) *plate* B *and earth when the central plate* C *is also earthed,* (b) *plate B and plate* D *when C is insulated, and* (a) B *and* D *when* C *is earthed?* (Oxford Open Schol.).

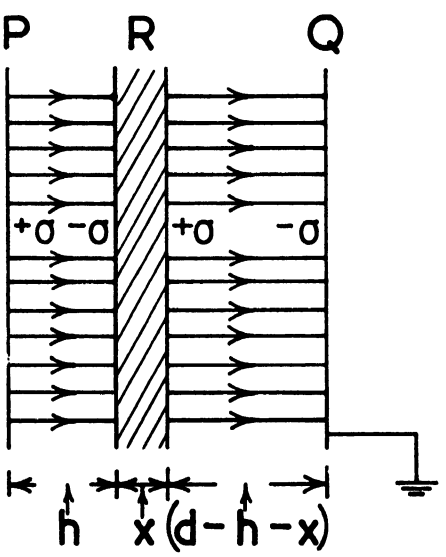


Fig. 2.16

(i) Gauss's theorem (p. 10) is used to shew that the field between the plates is given by: $E = \sigma/\epsilon_o$ volt.m.$^{-1}$ (p. 30). It follows then that the p.d. between the plates is: $V = Ed = \sigma d/\epsilon_o$ volts and the capacity per sq.m.: $c = \sigma/V = \epsilon_o/d$ farad.m.$^{-2}$.

(ii) Let the uncharged plate R be inserted between the plates P and Q so that the separations are as shown in Fig. 2.16. There can be no electric field inside R and lines of force from P end on the nearer side of R, producing an induced charge of opposite sign. Since R is uncharged an induced charge of equal amount appears on the further side from P, producing lines of force as shown. The field strength in the vacuum is thus unaltered and the presence of R merely reduces the extent of the field.

Let the original charge on P be $+\sigma$ coulomb.m.$^{-2}$. The induced charges on R are thus $+\sigma$ and $-\sigma$ coulomb.m.$^{-2}$. The field strength in the vacuous spaces is hence: $E = \sigma/\epsilon_o$ volt.m.$^{-1}$ and the p.d. between the plates becomes:

$$V = Eh + E(d - h - x) = (d - x)\frac{\sigma}{\epsilon_o} \text{ volts.}$$

The capacity is, therefore:

$$c' = \frac{\sigma}{V} = \frac{\epsilon_0}{d-x} \text{ farad.m.}^{-2}$$

The difference in capacity is given by:

$$\Delta c = c' - c = \frac{\epsilon_0}{d} - \frac{\epsilon_0}{d-x} = \frac{\epsilon_0 x}{d(d-x)} \text{ farad.m.}^{-2}$$

(iii.*a*) Let B be raised to V volts relative to earth by connection to a battery one pole of which is earthed (Fig. 2.17). Let the density of charge on B be

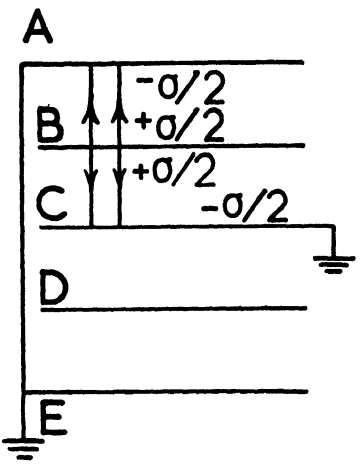


Fig. 2.17

σ coulomb.m.$^{-2}$. By symmetry, the induced charges on A and C are $-\sigma/2$ coulomb.m.$^{-2}$. No charge is produced on D which remains at earth potential since it is virtually surrounded by earthed conductors (C and E).

The field strength at points between B and C (or B and A) is given by:

$$E = \frac{\sigma}{2\epsilon_0} \text{ volt.m.}^{-1}$$

Hence, the p.d. between B and C (i.e. earth) is:

$$V = Ed = \frac{\sigma d}{2\epsilon_0} \text{ volts}$$

Hence, the capacity between B and C is, by definition:

$$c = \frac{\sigma}{V} = \frac{2\epsilon_0}{d} \text{ farad.m.}^{-2}$$

(iii.*b*) Assuming B is again raised to V volts with a charge density of σ coulomb.m.$^{-2}$, the charge densities on the upper and lower faces of B will be different, say σ_1 and σ_2 coulomb.m.$^{-2}$. The induced charge densities on the other plates will be as shown in Fig. 2.18. The field strength at points between A and B is given by: $E_{AB} = \frac{\sigma_1}{\epsilon_0}$ volt.m.$^{-1}$

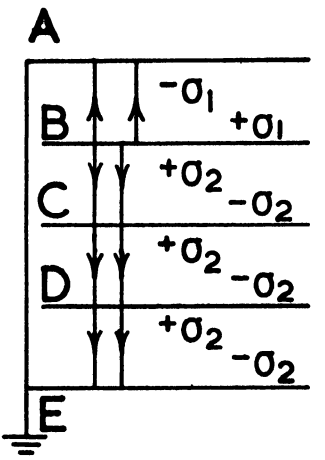


Fig. 2.18

Hence, the p.d. between A and B is:

$$V_{AB} = E_{AB}d = \frac{\sigma_1 d}{\epsilon_0} \text{ volts}$$

The field strengths between B and C, etc. are given by:

$$E_{BC} = E_{CD} = E_{DE} = \frac{\sigma_2}{\epsilon_0} \text{ volt.m.}^{-1}$$

Hence, the potential of B relative to E is given by:

$$V_{BE} = (E_{BC} + E_{CD} + E_{DE})d = \frac{3\sigma_2 d}{\epsilon_0} \text{ volts}$$

Since A and E are at earth potential:

$$V_{AB} = V_{BE}$$

i.e.

$$\sigma_2 = \frac{\sigma_1}{3}$$

The p.d. between D and earth is thus: $V_{DE} = \frac{\sigma_2 d}{\epsilon_0} = \frac{\sigma_1 d}{3\epsilon_0}$ volts

Hence:

$$V_{BD} = V_{BE} - V_{DE} = \frac{3\sigma_2 d}{\epsilon_0} - \frac{\sigma_2 d}{\epsilon_0} = \frac{2\sigma_1 d}{3\epsilon_0}$$

and the capacity between B and D is given by:

$$C = \frac{\sigma_1 + \sigma_2}{V_{BD}} = \frac{4\epsilon_0/(3\sigma_1)}{2d/(3\sigma_1)} = \underline{\frac{2\epsilon_0}{d} \text{ farad.m.}^{-2}}$$

(iii*c*) When C is earthed and B is raised to V volts to give rise to charge densities σ_1 and σ_2 coulomb.m.$^{-2}$ on its faces, induced charge densities appear on A and C as in Fig. 2.19; D remains at earth potential as in (*a*). The field strengths between A and B and between B and C are given by:

$$E_{AB} = \frac{\sigma_1}{\epsilon_0} \text{ and } E_{BC} = \frac{\sigma_2}{\epsilon_0} \text{ volt.m.}^{-1}$$

Whence: $$V_{AB} = V_{BC} = \frac{\sigma_1 d}{\epsilon_0} = \frac{\sigma_2 d}{\epsilon_0} \text{ volts}$$

whence: $$\sigma_1 = \sigma_2 = \sigma/2 \text{ and } V_{BD} = V_{BC} = \frac{\sigma d}{2\epsilon_0} \text{ volts}$$

The capacity between B and D is thus:

$$c = \frac{\sigma}{V_{BD}} = \frac{2\epsilon_0}{d} \text{ farad.m.}^{-1}$$

Thus the capacity in each case is the same, viz. $2\epsilon_0/d$ farad.m.$^{-1}$

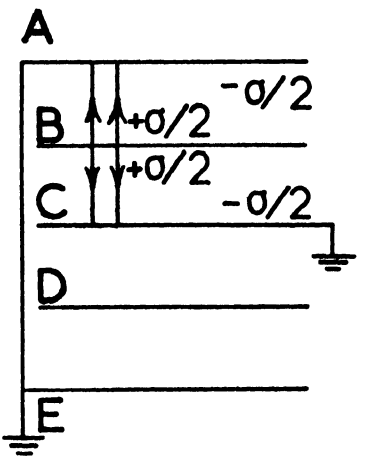


Fig. 2.19

2.9. The Energy of an Electric Field

It is probably difficult to appreciate that an electric field is a source of energy, since the field in vacuo contains nothing of material significance. This situation may be contrasted with the case of a hot body which has energy by virtue of the fact that its molecules are in violent motion. It is easy to abstract energy from the hot body since on contact with other bodies the molecules are able to transfer the energy by direct collision. In the electric field energy is abstracted by free charges which move under the influence of the field—there is no collision transfer mechanism involved. The process is called " action at a distance ".

Early physicists like Faraday and Maxwell could not conceive energy being derived from an electric field without some contact between the charges and the charged bodies producing the field. They gave elastic properties to lines of force, making them realistic links between the charged bodies and the free charges. It was convenient to give substance to these imaginary lines for the sake of understanding the forces involved between charges. Maxwell imagined *tubes of force*, which are imaginary hollow tubes having lines of force forming their bounding surface, starting from a positively charged body and ending on a

negatively charged body. The tubes behave like elastic tubes, being in tension to an extent depending on the strength of the electric field. The tension in the tube described the force of attraction between the positive and negative charges at the ends of the tube.

It can be shown using elastic tubes of force that the substance of the tubes is in equilibrium if a compressive force acts perpendicular to the surface of the tubes. Each tube is thus in tension due to a stress acting along its axis, and in compression due to a lateral pressure acting normally to its surface. The magnitude of each kind of stress is equal to $ED/2$ newton.m.$^{-2}$ where E is the intensity and D the induction at that point.

From a theoretical point of view it is convenient to imagine the whole of an electric field to be filled with an elastic medium. Maxwell imagined such a medium to exist in a vacuum or in the interstices of a dielectric and called it the *aether*. This aether, which has never been detected, had to have extraordinary properties to explain electric forces, and, in particular, any small volume of it was submitted to the two types of stress mentioned above. The aether was strained by these stresses and as a consequence possessed potential energy. Following well-known methods for determining the work done in straining elastic bodies, the potential energy per unit volume, i.e. the *energy density*, was found to be: $\mathscr{E} = \frac{1}{2}ED$ joule.m.$^{-3}$ at any point.

Modern theory accepts the force experienced by a charge in a field as an experimental fact probably incapable of further elucidation in mechanical terms. However, the formula for the energy density deduced by Maxwell is essentially correct as is proved by the correctness of results obtained with its use.

2.10. Proof of Energy Density Formula

It is easy to prove the energy formula in special cases. Consider, for example, a condenser consisting of two parallel plates X and Y (Fig. 2.20); X has a surface.density of charge of σ coulomb.m.$^{-2}$ and Y an

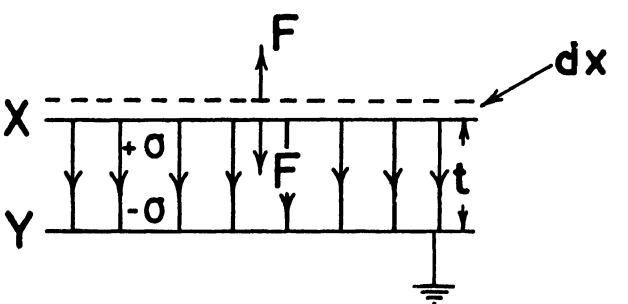


Fig. 2.20

induced density of $-\sigma$ coulomb.m.$^{-2}$. Assuming large plates (uniform field) the intensity between X and Y is: $E = \sigma/\epsilon_0\epsilon$ volt.m.$^{-1}$. The

mechanical force acting on the surface of X is: $F = \sigma^2 A/(2\epsilon_0\epsilon)$ newtons, where A m.^{2} is the area of X. If X is moved a small distance dx m. away from Y, work is done by the agent pulling the plate:

$$dW = F \,.\, dx = \frac{\sigma^2 A}{2\epsilon_0\epsilon} \,.\, dx \text{ joules} \quad . \quad . \quad . \quad (17)$$

By the conservation of energy theorem, this represents energy acquired by the field as potential energy.

The whole of the electric field between X and Y may be imagined as being created by withdrawing X from an initial position very close to Y to its final position t m. away. The total work done, i.e. the energy of the field is:

$$W = \int_0^t dW = \int_0^t \frac{\sigma^2 A}{2\epsilon_0\epsilon} \,.\, dx = \frac{\sigma^2 A t}{2\epsilon_0\epsilon} \text{ joules} \quad . \quad (18)$$

Since the volume of the field is given by: $v = At$ m.^{3}, the energy density is:

$$\mathscr{E} = \frac{W}{v} = \frac{\sigma^2}{2\epsilon_0\epsilon} \text{ joule.m.}^{-3}. \quad . \quad . \quad . \quad . \quad (19)$$

Since $E = \frac{\sigma}{\epsilon_0\epsilon}$ and $D = \sigma$: $\mathscr{E} = \frac{1}{2}ED = \frac{1}{2}E^2\epsilon_0\epsilon$ joule.m.$^{-3}$. (20)

2.11. Energy of a Charged Condenser

A charged condenser stores potential energy in the field between the plates. The energy of a parallel plate condenser is given by (18). Since the capacity of this condenser is given by: $C = A\epsilon_0\epsilon/t$ farads and $\sigma = E\epsilon_0\epsilon = V\epsilon_0\epsilon/t$ coulomb.m.$^{-2}$, it follows that:

$$W = \frac{\sigma^2 A t}{2\epsilon_0\epsilon} = \frac{V^2\epsilon_0{}^2\epsilon^2 A t}{t^2 \,.\, 2\epsilon_0\epsilon} = \frac{AV^2\epsilon_0\epsilon}{2t} = \tfrac{1}{2}CV^2 \text{ joules} \,. \quad (21)$$

Since $C = Q/V$: $W = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{\frac{1}{2}Q^2}{C}$ joules . . . . (22)

Alternative Deduction for all Condensers

A condenser is imagined to be charged by bringing successive increments of charge $+dq$ from infinity to one of the plates, say X of Fig. 2.9. Let the charge on X at some intermediate stage during this process be $+q$ coulombs and its potential v volts. On adding the next increment $+dq$, the work done is:

$$dW = v \,.\, dq \text{ joules} \quad . \quad . \quad . \quad . \quad (23)$$

since the amount of work done in bringing up unit charge is v joules, by definition. (During the addition of $+dq$ the potential of X rises by the

corresponding increment of potential dv; thus, the work done should be: $(v + dv)dq = v.\,dq + dv\,.\,dq$ but both dq and dv are very small and the product $dv\,.\,dq$ is negligible compared with $v\,.\,dq$). The total work done in charging X with a charge $+Q$ coulombs, so that its final potential is V volts, is:

$$W = \int dW = \int_0^Q v\,.\,dq \quad . \quad . \quad . \quad (24)$$

Since the capacity of the condenser is given by: $C = \frac{q}{v} = \frac{Q}{V}$:

$$W = \int_0^Q \frac{q}{C}\,.\,dq = \tfrac{1}{2}Q^2C = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \text{ joules} \quad . \quad (25)$$

2.12 Sharing of Charge

When two conductors (or capacitors) are connected, charge is transferred from one to the other until the potential of each is the same (p. 18). The final charge, potential and energy, of each conductor is determined if any two of the three variables: charge, potential and capacity, are known initially. In this determination the criteria used are: (i) the initial aggregate charge is unaltered on sharing; (ii) the final potential (or p.d.) of each is the same; and (iii) the capacity of the equivalent conductor formed when they are in contact is the sum of the separate capacities (capacitors in parallel).

Consider two conductors A and B, having capacities C_a and C_b farads, respectively. Suppose A has an initial charge of $+Q$ coulombs whilst B is uncharged. The initial potentials are thus:

$$V_a = \frac{Q}{C_a} \text{ and } V_b = 0 \text{ volts} \quad . \quad . \quad . \quad (26)$$

When A and B are connected, positive charge flows from A to B, (i.e. electrons flow from B to A), until both have the same potential: V volts. Let the charge remaining on A be Q_a coulombs.

Using criterion (i) the charge on B is: $Q_b = Q - Q_a$ coulombs. When A and B are in contact the equivalent capacity is: $C = C_a + C_b$.

Hence:
$$V = \frac{Q}{C_a + C_b} = \frac{Q_a}{C_a} = \frac{Q - Q_a}{C_b}$$

whence:
$$Q_a = \frac{QC_a}{C_a + C_b} \text{ and } Q_b = \frac{QC_b}{C_a + C_b} \text{ coulombs} \quad . \quad (27)$$

The initial energies are:

$$W_a = \frac{\tfrac{1}{2}Q^2}{C_a} \text{ joules and } W_b = 0 \quad . \quad . \quad . \quad (28)$$

The final energy of A is given by:

$$W'_a = \frac{\frac{1}{2}Q_a^2}{C_a} = \frac{1}{2} \cdot \frac{Q^2C_a^2}{(C_a + C_b)^2 C_a} = \frac{1}{2} \cdot \frac{Q^2 C_a}{(C_a + C_b)^2} \text{ joules} \quad (29)$$

The final energy of B is:

$$W'_b = \frac{\frac{1}{2}Q_b^2}{C_b} = \frac{1}{2} \cdot \frac{Q^2 C_b}{(C_a + C_b)^2} \text{ joules} \quad . \quad . \quad (30)$$

The total final energy is:

$$W = W'_a + W'_b = \frac{1}{2} \cdot \frac{Q^2}{(Ca + C_b)^2} \cdot (C_a + C_b) = \frac{1}{2} \cdot \frac{Q^2}{(C_a + C_b)} \text{ joules} \quad (31)$$

In the process of sharing charge there is a loss of energy given by:

$$W = W_a - (W'_a + W'_b) = \frac{\frac{1}{2}Q^2}{C_a} - \frac{1}{2} \cdot \frac{Q^2}{C_a + C_b} = \frac{\frac{1}{2}Q^2 C_b}{C_a(C_a + C_b)} \text{ joules} \quad (32)$$

The energy lost is that converted into heat in the connecting wire when the electron current flows between the conductors (p. 286). The time required for the charge transfer depends on the resistance of the connecting wire and the capacities of the conductors. For ordinary size capacitors and normal connecting wire this time is negligibly small (cf. p. 284). When conductors are placed in contact without using connecting wire the same loss in energy occurs. The energy is dissipated at the points of contact. A certain amount of energy may be dissipated in the formation of a spark which is caused by dielectric breakdown as the conductors approach each other.

When a condenser is discharged by connection to the earth, (27) applies with A as the conductor and B the earth. Since, B is much greater than A, $C_b \gg C_a$ and (27) becomes:

$$Q_b = \frac{Q \cdot C_b}{C_b} = Q$$

i.e. all the charge is given to the earth. This also happens when the two plates of the condenser are connected together.

The discharge of a condenser is used for taking flash-photographs. The apparatus consists of a very large capacitor which is charged by connection to a high voltage dry battery. A very bright flash is produced when the condenser is discharged through a special type of discharge tube containing gas at low pressure. The gas glows brightly for a fraction of a second, enabling high speed photographs to be taken.

Example

Describe and explain an experiment to demonstrate the change in the capacity of a parallel plate condenser produced by inserting a sheet of dielectric between the plates.

Derive the expression for the energy stored in a condenser of capacity C *when a potential difference* V *is applied across its plates.*

A parallel plate condenser in air has a capacity of $\frac{1}{3} \times 10^{-9}$ *farads. Calculate the change in energy of the condenser when the space between the plates is filled with an insulator of dielectric constant* 5, (a) *if the potential differences between the plates is kept constant at* 600 *volts,* (b) *if the charge on the plates is kept constant at* 2×10^{-7} *coulombs. State the unit in which the results are expressed.* (Northern G.C.E. Adv. L.).

The first two parts of the question are answered in the text.

When the space between the plates is filled with a dielectric the original capacity C is increased to $C' = C\epsilon$. In this case, therefore, the capacity is increased from $C = \frac{1}{3} \times 10^{-9}$ to $C' = \frac{5}{3} \times 10^{-9}$ farads.

(*a*) The original energy is: $W = \frac{1}{2}CV^2 = \frac{1}{6} \times 10^{-9} \times (600)^2 = 6 \times 10^{-5}$ joules.

The final energy is: $W' = \frac{1}{2}C'V^2 = \frac{5}{6} \times 10^{-9} \times (600)^2 = 30 \times 10^{-5}$ joules.

Hence, the *increase* in energy is: $W' - W = \underline{2{\cdot}4 \times 10^{-4} \text{ joules.}}$

(This energy increase is derived from the work done in providing more charge on the plates: $Q' = C'V$).

(*b*) The original charge on the condenser is:

$$Q = CV = \tfrac{1}{3} \times 10^{-9} \times 600 = 2 \times 10^{-7} \text{ coulombs}$$

Since this is unaltered, the new energy is:

$$W' = \frac{\frac{1}{2}Q^2}{C'} = \frac{2 \times 10^{-14} \times 3}{5 \times 10^{-9}} = 1{\cdot}2 \times 10^{-5} \text{ joules}$$

Hence the *loss* in energy is: $W' - W = \underline{4{\cdot}8 \times 10^{-5} \text{ joules.}}$

(The loss in energy is caused by the reduction in field strength on introducing the dielectric) (cf. p. 478).

EXERCISES

1. Derive an expression for the potential at a point r m. from a point charge of " q " units. Hence show that the capacity of an isolated spherical conductor is given by: $4\pi\epsilon_0$ times the radius in metres.

   Draw diagrams to show the distribution of potential in the neighbourhood of a positively charged spherical conductor (*a*) before, (*b*) after an uncharged cigar-shaped insulated conductor is brought near. Give a brief explanation. (Camb. Local G.C.E. Adv. L.).

2. Describe and explain how you would use an electroscope together with the necessary accessories (*a*) to demonstrate the factors which influence the capacity of a parallel plate condenser, (*b*) to show that there is no electric field inside a hollow charged conductor. Discuss briefly the practical and theoretical importance of the fact stated in (*b*). (Northern H.S.C.).

3. What is meant by the *electric potential* of a conductor ? What factors determine the potential of a conducting plate, and how may their influence be shown experimentally ? Can an uncharged body be at

a potential which is not zero, or a charged body at zero potential (give examples) ?

A plane aluminium sheet of 900 sq. cm. area, and 1 mm. thick, is hung symmetrically between two large vertical earthed plates, 5 mm. apart. What is the capacity to earth of the aluminium sheet ? To what potential would it be raised when given a charge of 1 micro-coulomb ? Will there be any change in its potential if it is moved 1 mm. towards one of the earthed plates? (O. & C.J.B. G.C.E. Adv. L.).

4. What do you understand by "electrical potential" and by "capacity of a conductor"?

Calculate the capacity of an isolated sphere of radius " a ".

A metal sphere of radius 10 cm. is charged to a potential of 600 volts and two hemispheres made of thin metal of radius 11 cm. are brought up from a distance, without touching the first sphere, so as to form a second sphere concentric with the first. What is the potential of the first sphere relative to the earth:

(*a*) if the hemispheres are earthed.
(*b*) if the hemispheres are insulated.
(*c*) if the hemispheres are insulated, but are momentarily connected to the inner sphere by a wire carried on an insulating handle ? (Camb. Open Schol.).

5. Describe the structure and mode of action of a gold-leaf electroscope. How may the instrument be employed to find the distribution of charge over the surface of a charged conductor ?

Assuming the intensity of the electric field between the two plates of a parallel plate air-condenser to be uniform and given by σ/ϵ_0, where σ is the surface density of charge, derive an expression for the capacitance of the condenser.

A multi-plate condenser consists of parallel sheets of tin-foil separated by thin sheets of paraffined paper, alternate sheets of tin-foil being electrically connected. In such a condenser 81 circular sheets of tin-foil are separated by paper 10^{-3} cm. thick and the diameter of each sheet is 10·0 cm. Calculate the capacitance of the condenser in microfarads if the dielectric constant of the paper is 2·4. (London G.C.E. Adv. L.).

6. State the factors which determine the capacitance of a condenser. Describe, briefly, experiments you would perform to verify your statements.

Obtain an expression for the energy of a charged condenser.

Find the energy which may be stored in condensers of 2 and 4 microfarads when taken singly, in series and in parallel, when a potential difference of 100 volts is available. (London G.C.E. Adv. L.).

7. What is meant by (*a*) the electrostatic potential at a point, (*b*) the energy of a charged condenser.

Four particles each of mass m and charge q are held at the corners of a square of side ' a '. What will be the motion if they are released ?

Find an expression for their ultimate velocity. (Oxford Open Schol.).

8. Define the terms "electric intensity", "potential" and "capacity".

Obtain from first principles an expression for the capacity of a parallel-plate condenser.

The plates of a parallel-plate condenser are each of area 20 sq. cm., the separation is 3 mm., and the dielectric is a slab of mica of dielectric constant 6, which completely fills the space between the plates. The condenser is connected across a high-tension supply at 400 volts. Find the charge on the plates; and calculate the change in the energy of the condenser if the mica is slid out from between them, the p.d. being maintained still at 400 volts. (Oxford Local G.C.E. Adv. L.).

9. Show how the capacitance of two condensers (*a*) in series, (*b*) in parallel may be calculated when the individual capacitances are known.

A condenser of capacitance 10^{-10} F. is connected in series with a pair of condensers of capacitances 50×10^{-12} F. and 75×10^{-12} F. which are connected in parallel. If a potential difference of 3,000 volts is applied to the terminals of the combined condenser so formed, calculate the energy stored in the condenser of capacitance 10^{-10} F. (Durham G.C.E. Adv. L.).

10. Define the *electrostatic potential* of an isolated conductor.

Obtain an expression relating the energy of a charged conductor to the charge on it and its capacity.

Two insulated spherical conductors of radii 5·00 cm. and 10·00 cm. are charged to potentials of 600 volts and 300 volts respectively. Calculate the total energy of the system. Also calculate the energy after the spheres have been connected by a fine wire. Comment on the difference between the two results. (Northern G.C.E. Adv. L.).

11. A condenser with a capacity of 4μF. is charged by a battery to a potential difference of 2 volts. How much energy is stored in the condenser?

If the condenser is disconnected from the battery and then connected in parallel with a second condenser of capacity 2μF., initially uncharged, what will be the energy stored in each condenser?

Discuss your answers. (Manch. Open Schol.).

12. Define electrostatic potential, capacitance.

State the factors which determine the magnitude of the capacitance of a parallel plate condenser and describe experiments to support your statement.

A conductor *A*, capacitance 2μF., is charged to a potential difference of 400 volt. When its terminals are joined by very fine short wires to the terminals of another condenser *B*, the potential difference between the terminals of *A* finally becomes 100 volt. Calculate (*a*) the capacitance of *B*, (*b*) the energy in joules dissipated as heat in the wires. (London Inter. B.Sc.).

Since $r \gg l$, the expansion of the denominators gives:

$$F_1 = \frac{pm}{r^n}\left\{\left(1 + \frac{nl}{r}\right) - \left(1 - \frac{nl}{r}\right)\right\} = \frac{2lmnp}{r^{(n+1)}} = \frac{Mnp}{r^{(n+1)}} \quad . \quad . \quad (14)$$

where M is the magnetic moment of the dipole.

Let P_2 be a point distant r metres from O along the perpendicular bisector of NS (Fig. 3.4). The force exerted on unit pole at P_2 by N or S has the same value

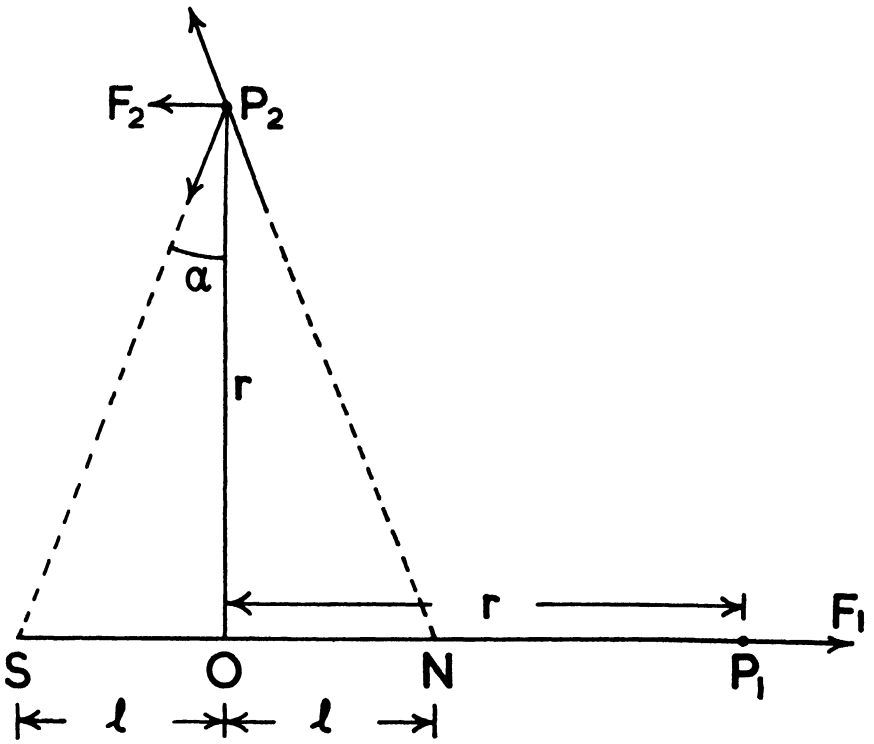


Fig. 3.4

and each force is inclined to P_2O at the same angle a, as shown. The resultant force acting at P_2 is given by:

$$F_2 = 2 \times \frac{mp}{(P_2N)^n} \cdot \sin a = \frac{2mp}{(r^2 + l^2)^{n/2}} \cdot \frac{l}{(r^2 + l^2)^{1/2}} = \frac{pM}{(r^2 + l^2)^{\frac{n+1}{2}}}$$

i.e. $$F_2 = \frac{pM}{r^{n+1}\left(1 + \frac{l^2}{r^2}\right)^{\frac{n+1}{2}}}$$

Again assuming $r \gg l$, the term $(l^2/r^2) \ll 1$, so that it may be neglected by comparison and F_2 becomes:

$$F_2 = \frac{pM}{r^{n+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Dividing (14) by (15):

$$\frac{F_1}{F_2} = n$$

Using a deflection magnetometer (p. 443) it is shown that $(F_1/F_2) = 2$, i.e. $n = 2$, within the limits of accuracy of the experiment. This is known as *Gauss' Proof of the Inverse Square Law.*

For the second part of the question let NS and $N'S'$ represent two short magnets of pole strength m and m' and separations $2l$ and $2l'$, respectively (Fig. 3.5).

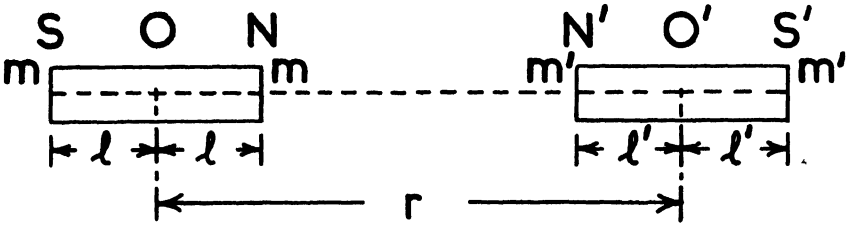


Fig. 3.5

Let the centres of the equivalent dipoles, (O and O'), be separated by r metres.

Using the inverse square law the force exerted between N and N' is given by:

$$F_1 = \frac{mm'}{4\pi\mu_0(r - l - l')^2} \text{ (in air or vacuo)}$$

the force between S and N' is:

$$F_2 = \frac{mm'}{4\pi\mu_0(r + l - l')^2}$$

the force between N and S' is:

$$F_3 = \frac{mm'}{4\pi\mu_0(r - l + l')^2}$$

the force between S and S' is:

$$F_4 = \frac{mm'}{4\pi\mu_0(r + l + l')^2}$$

The net force acting on N' is given by $F_1 - F_2$ and the net force acting on S' is given by $F_3 - F_4$. These net forces act in opposite directions so that the aggregate force acting on $N'S'$ is:

$$F = F_1 - F_2 - F_3 + F_4$$

Hence:

$$F = \frac{mm'}{4\pi\mu_0 r^2}\left\{\frac{1}{\left(1 - \frac{l + l'}{r}\right)^2} - \frac{1}{\left(1 + \frac{l - l'}{r}\right)^2} - \frac{1}{\left(1 - \frac{l - l'}{r}\right)^2} + \frac{1}{\left(1 + \frac{l + l'}{r}\right)^2}\right\} \quad . \quad (16)$$

Since r is large compared with l and l' each term of (16) may be expanded using the binomial theorem and all terms containing inverse powers of r greater than two neglected.

Whence:
$$F = \frac{24ll'mm'}{4\pi\mu_0 r^4}$$

Since $M = 2lm$, and $M' = 2l'm'$:

$$F = \frac{6MM'}{4\pi\mu_0 r^4}$$

Using: $F = 10^{-3}$ newtons, $M = 2 \times 10^{-6}$ weber, $r = 0{\cdot}3$ metres, $\mu_o = 4\pi \times 10^{-7}$ M.K.S.:

$$10^{-3} = \frac{6 \times 2 \times 10^{-6} \times M'}{16\pi^2 \times 10^{-7} \times (0{\cdot}3)^4} = \frac{3 \times 10^5 M'}{324\pi^2}$$

whence: $M' = 1{\cdot}066 \times 10^{-5}$ weber . metre

3.10. Field Strength at any point in the field of a Dipole

The field strength at any point may be calculated providing the distances from each pole are known. If the dipole is small the field strength is calculated more simply in the following manner:

Assume that the magnetic moment of the dipole is a vector quantity whose magnitude is $M = 2lm$. and whose direction is along the axis of the dipole, the positive sense being SN. Let the point P be distant r metres from O and let $PON = \theta$ (Fig. 3.6).

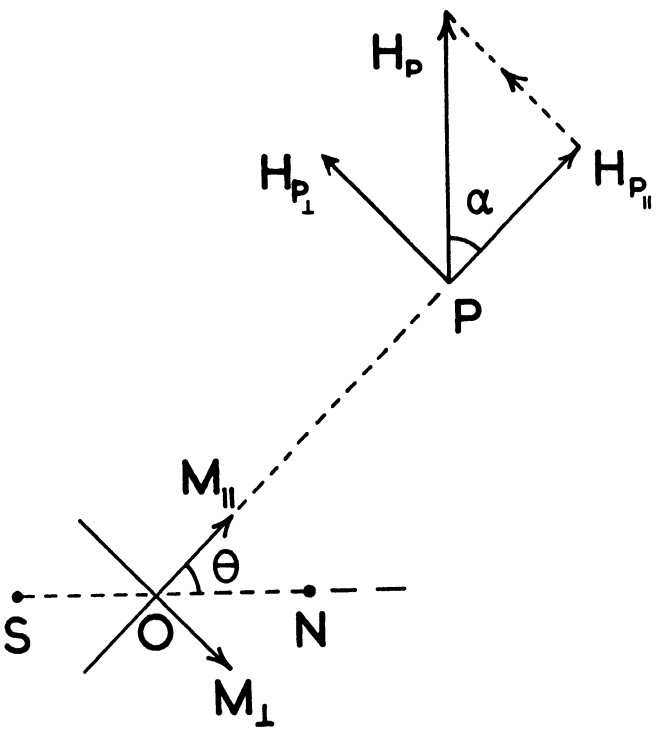


Fig. 3.6

The vector M is resolved into two components:

$$M_\parallel = M \cos\theta \text{ (along } OP)$$

and: $$M_\perp = M \sin\theta \text{ (perpendicular to } OP).$$

Since P lies on the axis of the component dipole $M_\parallel$, the field strength at P created by this dipole is given by (10) as:

$$H_{P\parallel} = \frac{2 . M_\parallel}{4\pi\mu_o r^3} = \frac{2M \cos\theta}{4\pi\mu_o r^3} \text{ (along } OP) \ (r \gg l) \qquad (17)$$

Likewise, since P lies on the perpendicular bisector of $M_\perp$:

$$H_{P\perp} = \frac{M_\perp}{4\pi\mu_o r^3} = \frac{M \sin\theta}{4\pi\mu_o r^3} \ (\perp \text{ to } OP) \quad . \quad . \quad (18)$$

The resultant field strength at P is the vector sum:

$$\text{i.e.} \quad H_P = \sqrt{H_{P\parallel}^2 + H_{P\perp}^2} = \frac{M}{4\pi\mu_o r^3}\sqrt{4\cos^2\theta + \sin^2\theta} \quad . \quad (19)$$

and H_P is directed at an angle α to OP, where:

$$\tan\alpha = \frac{H_{P\perp}}{H_{P\parallel}} = \frac{\sin\theta}{2\cos\theta} = \tfrac{1}{2}\tan\theta \quad . \quad . \quad (20)$$

3.11. The Suspended Bar Magnet

A freely suspended bar magnet behaves like a suspended dipole. If the magnet has poles of strength m. wb. distant $2l$ m. from each other and it is suspended in a uniform field of strength H n.wb.$^{-1}$, the force exerted on each pole is mH newtons. If the axis of the magnet is

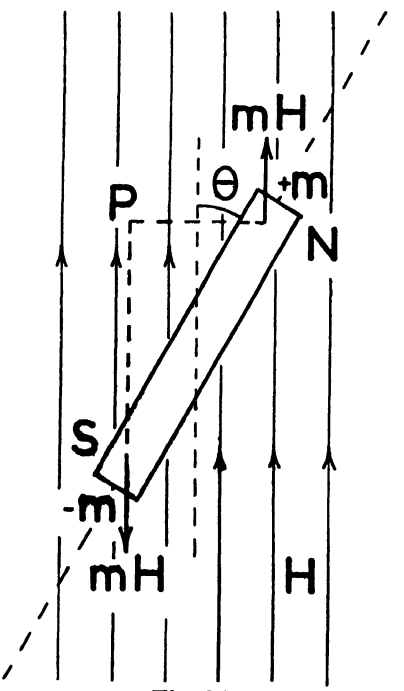


Fig. 3.7

maintained at an angle θ to the direction of the field, as in Fig. 3.7, the couple acting on the magnet is:

$$G = mH \times NP = mH \,.\, 2l \,.\, \sin\theta$$

$$\text{i.e.} \quad G = MH \sin\theta \text{ newton.metres.} \quad . \quad . \quad . \quad (21)$$

When $\theta = \pi/2$:

$$G_{max.} = MH \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Hence: *The moment of a bar magnet is the couple needed to maintain the axis of the magnet at right angles to a uniform field of unit intensity when it is freely suspended.*

3.12. The Earth's Magnetic Field

A freely suspended or pivoted magnet aligns itself so that its axis points roughly N and S (in the southern hemisphere the direction is reversed). This effect is caused by the *magnetic field of the earth.* The earth's magnetism is quite complex (Chapter 15) but its field is sensibly uniform over a wide area and in this country its direction is inclined to the horizontal at an angle of approximately 70°. This inclination is called the *Angle of Dip.* At any point the magnetic field strength vector H_0 can be resolved into two components H_H and H_V, H_H being the horizontal component in the plane containing H_0 and H_V the vertical component. The magnets used in instruments are generally suspended or pivoted so that their axes are horizontal, either by using an axis off-centre or suitable counterpoising. The horizontal component only is then able to influence the deflection of the magnet. The vertical plane containing H_0 points roughly N and S, so that H_H causes the suspended magnet to point in the N–S direction, i.e. along the *magnetic meridian.*

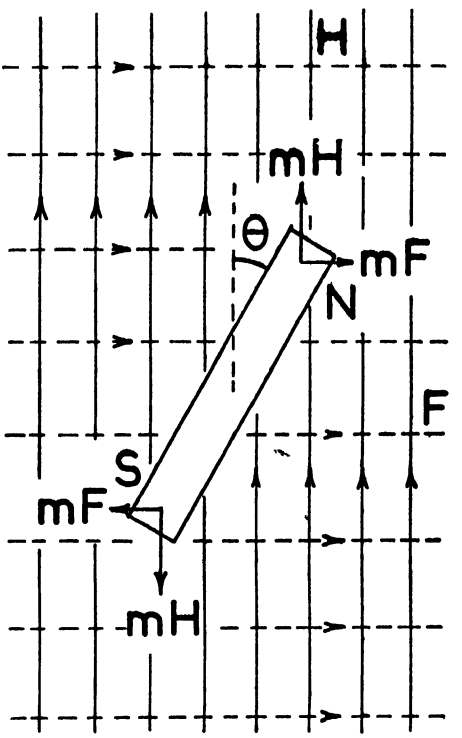


Fig. 3.8

3.13 Suspended Magnet in Crossed Fields

If a uniform magnetic field is created whose direction is at right angles to that of another uniform field the resultant field strength at any point is the vector sum of the two components. A freely suspended or pivoted magnet comes to rest with its axis pointing in the direction of the resultant field.

Suppose the two fields have strengths H and F n.wb.$^{-1}$, and the equilibrium position of the magnet is such that its axis makes the angle θ with H (Fig. 3.8). The couple caused by the field H is anticlockwise and give by:

$$G_H = MH \sin \theta$$

The couple caused by the field F is clockwise and given by:

$$G_F = MF \cos \theta$$

For equilibrium:

$$MF \cos \theta = MH \sin \theta$$

i.e.

$$F = H \,.\, \tan \theta \quad . \quad . \quad . \quad . \quad (23)$$

This expression is required when using a *deflection magnetometer* to measure magnetic field strength. The horizontal component of the earth's magnetic field is used as H and a field F, at right angles to H, may be measured. The deflection magnetometer consists of a small pivoted magnet mounted in a brass draught-proof case provided with a circular scale graduated in degrees (Chapter 15).

Example

Prove that the intensity of the magnetic field due to a short bar magnet at a point on its axis produced is twice the value at a point equally distant in its equatorial plane.

A short bar magnet of magnetic moment $2\pi \times 10^{-7}$ *weber.metre is placed on a horizontal table with its* N *pole pointing* 30° *east of magnetic north. In what direction will a small compass needle set, if it is placed on the table* 20 *cm. magnetic north of the bar magnet.* ($H_H = 14$ *amp.m.*$^{-1}$). (London G.C.E. Adv. L.).

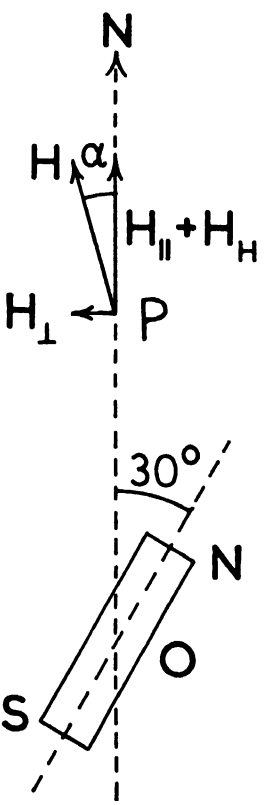


Fig. 3.9

The first part of the question is covered in the text.

Let NS represent the magnet with its axis inclined at 30° to the magnetic meridian (Fig. 3.9). The compass at P, 0·2 metres from the centre O, sets itself along the direction of the magnetic field at P. The component of M in the direction of the meridian is $M_{\parallel} = M\cos 30° = M(\sqrt{3}/2)$ wb.m., whilst the component at right angles is $M_{\perp} = M\sin 30° = M/2$ wb.m. Since P lies on the axis of $M_{\parallel}$:

$$H_{\parallel} = \frac{2M_{\parallel}}{4\pi\mu_o\mu r^3} = \frac{2M(\sqrt{3}/2)}{4\pi\mu_o\mu r^3}$$

Since: $\mu = 1$, $r = 0{\cdot}2$ m., $\mu_o = 4\pi \times 10^{-7}$ M.K.S., $M = 2\pi \times 10^{-7}$ wb.m.:

$$H_{\parallel} = \frac{2 \times 2\pi \times 10^{-7} \times (\sqrt{3}/2)}{16\pi^2 \times 10^{-7} \times (0{\cdot}2)^3} = \frac{\sqrt{3} \times 10^3}{64\pi} = 8{\cdot}61 \text{ n.wb}^{-1}$$

The total field strength in the direction of the meridian is:

$$H = H_{\parallel} + H_H = 8{\cdot}61 + 14 = 22{\cdot}61 \text{ n.wb.}^{-1}$$

Since P lies on the perpendicular bisector of $M_{\perp}$:

$$H_{\perp} = \frac{M_{\perp}}{4\pi\mu_o\mu r^3} = \frac{M/2}{4\pi\mu_o\mu r^3} \text{ n.wb.}^{-1}$$

i.e.
$$H_{\perp} = \frac{\pi \times 10^{-7}}{16\pi^2 \times 8 \times 10^{-10}} = \frac{10^3}{128\pi} = 2{\cdot}49 \text{ n.wb.}^{-1}$$

The direction along which the compass needle sets is given by:

$$\tan\alpha = \frac{H_{\perp}}{H} = \frac{2{\cdot}49}{22{\cdot}61} = 0{\cdot}11$$

Hence:
$$\alpha = 6°17'$$

i.e. the compass sets along a direction 6°17′ west of magnetic north.

Example

Find the magnitude of the magnetic field due to a small magnet of moment " M " (a) *at a point on its axis at a distance* " d " *from its centre;* (b) *at a point on its perpendicular bisector at a distance* " d " *from its centre.*

Three small magnets having their centres at the corners of an equilateral triangle ABC, and being free to rotate about their centres, can rest in equilibrium with the magnet at A parallel to BC and with those at B and C perpendicular to AB and AC respectively. Prove that the magnetic moments of the magnets are in the ratios $\sqrt{3}$:4:4. (Camb. Open Schol.).

The first parts of the question (*a*) and (*b*) are answered in the text.

In the second part the magnets are in equilibrium with their axes arranged as in Fig. 3.10. The condition for equilibrium of any magnet is that the couples exerted on that magnet by the other two are equal and opposite.

Let the distance between adjacent magnets be r metres and let the moments of the magnets be M_A, M_B and M_C, respectively.

Since A, (i.e. the point A), lies on the perpendicular bisector of magnet B, the field strength at A due to B is:

$$H_{AB} = \frac{M_B}{4\pi\mu_o\mu r^3} = \frac{M_B}{c} \text{ (magnet is small)}$$

where:
$$4\pi\mu_o\mu r^3 = c$$

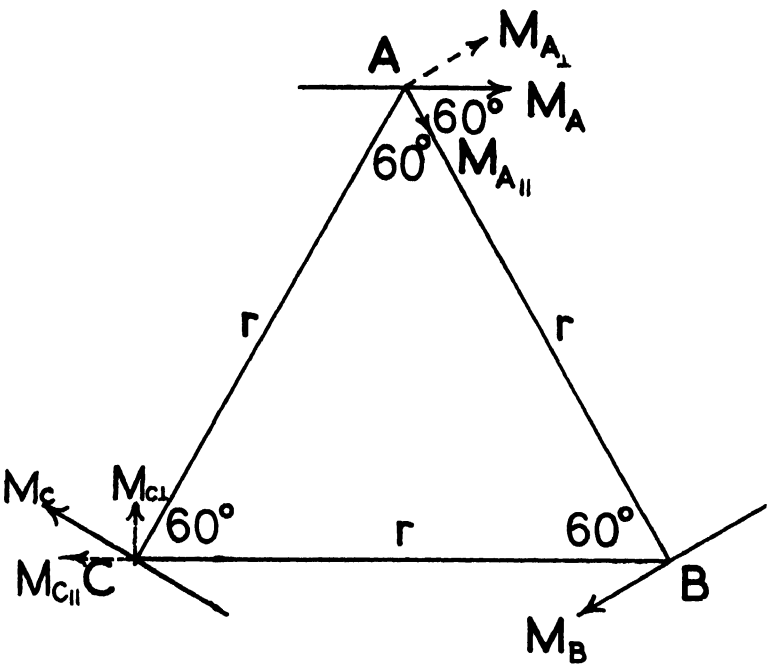


Fig. 3.10

The field may be assumed uniform along magnet A, so that the anticlockwise couple exerted on A is:

$$G_{BA} = M_A H_{AB} \sin 30° = \frac{M_A M_B}{c} \sin 30°$$

Since A lies on the perpendicular bisector of magnet C the field strength at A caused by C is:

$$H_C = \frac{M_C}{4\pi\mu_0\mu r^3} = \frac{M_C}{c}$$

and the clockwise couple exerted on A is:

$$G_C = M_A H_C \sin 30° = \frac{M_A M_C}{c} \sin 30°$$

For equilibrium: $$G_B = G_C$$

so that: $$M_A M_B \sin 30° = M_A M_C \sin 30°$$

i.e. $$\underline{M_B = M_C}$$

The point B lies on a straight line inclined at 60° to the axis of the magnet at A. The field strength at B due to A is hence given by using (19) with $\theta = 60°$; the two components of H_A are:

$$H_{A_\parallel} = \frac{2M_A \cos 60°}{c}$$

and: $$H_{A_\perp} = \frac{M_A \sin 60°}{c}$$

$H_{A\parallel}$ is inclined at 90° to the axis of the magnet at B, whilst $H_{A\perp}$ is parallel to the axis of this magnet. Since the latter gives rise to no couple on B, the resultant anticlockwise couple acting on the magnet is:

$$G_{AB} = \frac{2M_A \cos 60° \, M_B \sin 90°}{c} = \frac{M_A M_B}{c}$$

Since point B lies on a straight line inclined at 30° to the axis of magnet C, the two components of the field of C at B are given by:

$$H_{C\parallel} = \frac{2M_C \cos 30°}{c}$$

and:

$$H_{C\perp} = \frac{M_C \sin 30°}{c}$$

The component $H_{C\parallel}$ is inclined at 30° to the axis of B, whilst $H_{C\perp}$ is inclined at 60° to the axis of B, so that the resultant clockwise couple exerted on B is given by:

$$G_{CB} = \frac{2M_C \cos 30° \, M_B \sin 30°}{c} - \frac{M_C \sin 30° \, M_B \sin 60°}{c}$$

i.e.

$$G_{CB} = \frac{M_C M_B}{c}\left(2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2}\right) = \frac{M_C M_B}{c} \cdot \frac{\sqrt{3}}{4}$$

For equilibrium:

$$G_{AB} = G_{CB}$$

Hence:

$$\frac{M_A M_B}{c} = \frac{M_C M_B}{c} \cdot \frac{\sqrt{3}}{4}$$

i.e.

$$M_A = \frac{\sqrt{3}}{4} \cdot M_B$$

From above, $M_B = M_C$, so that:

$$M_A : M_B : M_C :: \sqrt{3} : 4 : 4 \quad \textit{Q.E.D.}$$

3.14. Oscillations of a Suspended Magnet

If a freely suspended magnet is slightly displaced relative to its axis of rotation and then released it oscillates with a constant frequency.

Let the angle between the axis of the magnet and the magnetic meridian at t seconds after release be θ radians. The couple acting on the magnet caused by the earth's field H_H is:

$$G = MH_H \sin \theta \text{ newton.m.}$$

where M is the moment in wb.m.

Assuming no torsion in the suspending fibre and no frictional effects:

$$G = I \cdot (d^2\theta/dt^2),$$

Where I is the moment of inertia of the magnet about the axis of suspension and $(d^2\theta/dt^2)$ is the angular acceleration.

$$\therefore I \cdot \frac{d^2\theta}{dt^2} = - MH_H \sin \theta \quad . \quad . \quad . \quad (24)$$

The negative sign is inserted because the angular acceleration decreases as θ increases. If the maximum value of θ is small, sin θ is approximately the same as θ in radians, hence:

$$\frac{d^2\theta}{dt^2} = \frac{-MH_H \cdot \theta}{I} \quad . \quad . \quad . \quad (25)$$

i.e. Angular acceleration = — (constant) × angular deflection.

This is the equation of *Simple Harmonic Motion* whose period is:

$$T = 2\pi\sqrt{\frac{1}{\text{constant}}} = 2\pi\sqrt{\frac{I}{MH_H}} \quad . \quad . \quad (26)$$

The oscillations cease when the energy is dissipated in overcoming air friction. Equation (24) may be modified to take this into account but the effect on the period is small. Torsion in the suspending fibre may affect the period appreciably. If the *torsional constant* is τ, the couple opposing motion when the angular deflection is θ (θ increasing) is $\tau\theta$, and (24) becomes:

$$I \cdot \frac{d^2\theta}{dt^2} = -\tau\theta - MH_H\theta = -(\tau + MH_H)\theta \quad . \quad (27)$$

whence:
$$\frac{d^2\theta}{dt^2} = \frac{-(\tau + MH_H)\theta}{I} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

and the period:
$$T = 2\pi\sqrt{\frac{I}{\tau + MH_H}} \quad . \quad . \quad . \quad . \quad (29)$$

3.15. The Vibration Magnetometer

A simple vibration magnetometer consists of a small bar magnet suitably mounted and suspended from its mid-point by means of a fine thread or an unspun silk fibre. The magnetometer is screened from draughts by a cylindrical glass case.

Equation (26) is used for the measurement of the earth's magnetic field strength H_H. If the magnet has a rectangular cross-section and is suspended by means of a non-magnetic wire cradle of negligible mass; $I = \text{Mass}\left\{\frac{(\text{length})^2 + (\text{breadth})^2}{12}\right\}$; if the cross-section is circular and the suspension is similar: $I = \text{Mass}\left\{\frac{(\text{length})^2}{12} + \frac{(\text{radius})^2}{4}\right\}$ (kgm.m.^{2}).

When other magnetic fields are present the *resultant* field strength is

given by adding vectorially H_H and the field strength being measured, say F, i.e. $\vec{H} = \vec{H}_H + \vec{F}$, and hence (26) becomes:

$$T = 2\pi\sqrt{\frac{I}{M(\vec{H}_H + \vec{F})}} \quad . \quad . \quad . \quad (30)$$

Absolute measurements of field strengths are rarely made using this magnetometer but it is used extensively for comparative measurements. (cf. Chapter 15).

Example

What is meant by (a) *the moment of a magnet,* (b) *the horizontal component of the earth's magnetic field?*

Obtain an expression for the period of oscillation of a bar magnet suspended so as to swing freely in a horizontal plane in a uniform magnetic field.

A small magnet A *oscillates with a frequency of* 15 *per minute when in the earth's field alone. A bar magnet* B *is placed horizontally with its centre vertically below that of* A *and with its north-seeking pole pointing north. In its equilibrium position* A *still has its north-seeking pole directed northwards, but when disturbed oscillates with a frequency of* 10 *per minute. What would be the frequency if* B *were turned in the horizontal plane through* (a) 180°, *and* (b) 90°? (Durham G.C.E. Adv. L.).

The first two parts of the question are covered in the text.

Let the moment of inertia of A be I kgm.m.^{2} and its magnetic moment M wb.m. When oscillating in the earth's field (H_H) alone:

$$T = 2\pi\sqrt{\frac{I}{MH_H}}$$

i.e.
$$\frac{1}{15} = 2\pi\sqrt{\frac{I}{MH_H}} \quad . \quad . \quad . \quad . \quad (31)$$

When B is present with its axis along the magnetic meridian the equilibrium position of A is unaltered (Fig. 3.11), so that, since A is small, the field in which it

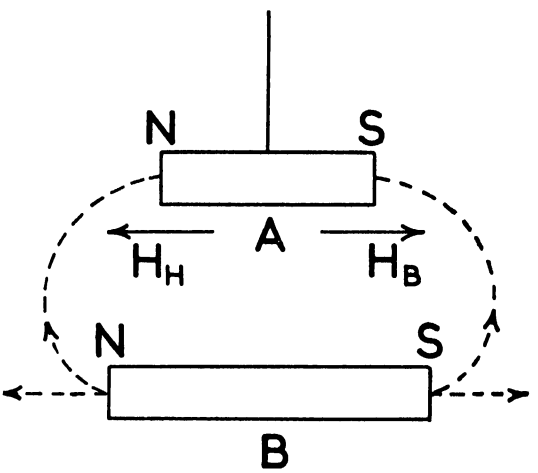


Fig. 3.11

now oscillates is given by $H_H - H_B$, where H_B is the field strength at A caused by B, and:

$$T_1 = \frac{1}{10} = 2\pi\sqrt{\frac{I}{M(H_H - H_B)}} \quad . \quad . \quad . \quad (32)$$

On squaring (31) and (32) and dividing:

$$\frac{100}{225} = \frac{H_H - H_B}{H_H} = 1 - \frac{H_B}{H_H}$$

i.e.
$$H_B = \frac{125}{225} \cdot H_H = \frac{5}{9} \cdot H_H$$

(*a*) When B is turned through 180° the field at A due to B is reversed and the new periodic time is:

$$T_2 = 2\pi\sqrt{\frac{I}{M(H_H + H_B)}} = 2\pi\sqrt{\frac{I}{MH_H(1 + \frac{5}{9})}} = 2\pi\sqrt{\frac{I}{MH_H \cdot \frac{14}{9}}} \quad (33)$$

Dividing (33) by (31):

$$15T_2 = \sqrt{\frac{9}{14}}$$

Hence the number of oscillations per minute is:

$$n_2 = \frac{1}{T_2} = 15 \cdot \sqrt{\frac{14}{9}} = 18{\cdot}71$$

(*b*) When B is rotated through 90° the field at A is made up of two components at right angles so that the resultant field in which it oscillates is given by: $\sqrt{H_H^2 + H_B^2}$, and the periodic time becomes:

$$T_3 = 2\pi\sqrt{\frac{I}{M\sqrt{(H_H^2 + H_B^2)}}} = 2\pi\sqrt{\frac{I}{MH_H\sqrt{1 + (\frac{5}{9})^2}}} \quad . \quad (34)$$

Hence, dividing (34) by (31):

$$15T_3 = \frac{1}{\sqrt[4]{1 + (\frac{5}{9})^2}}$$

so that the number of oscillations per minute is:

$$n_3 = 15\sqrt[4]{1 + (\tfrac{5}{9})^2} = 15\sqrt[4]{106} = 16{\cdot}05$$

Example

A short bar magnet, placed 30 cm. due magnetic E. of a magnetometer needle and pointing towards it, produces a deflection of 45°. The same magnet suspended by a torsionless fibre in a horizontal plane in the place formerly occupied by the magnetometer is found to have a period of oscillation of 2·0 sec. If the moment of inertia of the magnet about the axis of oscillation is 4×10^{-6} kgm.m.², find the horizontal component of the earth's magnetic field. (Northern H.S.C.).

In Fig. 3.12, *NS* represents the magnet and *ns* the magnetometer needle. Since the field of *NS* is perpendicular to the horizontal component of the earth's field at *ns*, its value is given by:

$$F = H_H \cdot \tan \theta$$

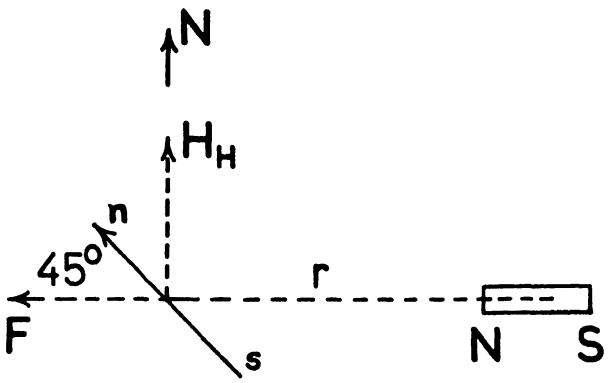


Fig. 3.12

Since *NS* is short:

$$F = \frac{2M}{4\pi\mu_o r^3}$$

Hence:
$$\frac{2M}{4\pi\mu_o r^3} = H_H \cdot \tan \theta$$

and:
$$M = 2\pi\mu_o r^3 H_H \cdot \tan \theta \quad . \quad . \quad . \quad . \quad (35)$$

When *NS* is suspended in the earth's field alone the periodic time is given by:

$$T = 2\pi\sqrt{\frac{I}{MH_H}} \quad . \quad . \quad . \quad . \quad (36)$$

Squaring (36) and substituting for M from (35):

$$T^2 = \frac{4\pi^2 I}{2\pi\mu_o r^3 H_H \tan \theta \cdot H_H} = \frac{2\pi I}{\mu_o r^3 H_H^2 \tan \theta}$$

Hence:
$$H_H^2 = \frac{2\pi I}{T^2 \mu_o r^3 \tan \theta}$$

Since: $I = 4 \times 10^{-6}$ kgm.m.^{2}, $T = 2$ secs., $r = 0{\cdot}3$ m., $\mu_o = 4\pi \times 10^{-7}$ M.K.S., $\theta = 45°$ and $\tan \theta = 1$:

$$H_H^2 = \frac{2\pi \times 4 \times 10^{-6}}{4 \times 4\pi \times 10^{-7} \times 27 \times 10^{-3} \times 1} = \frac{10^4}{54}$$

i.e.
$$\underline{H_H = 13{\cdot}6 \text{ n.wb.}^{-1}}$$

3.16. Magnetic Induction and Dipole Theory of Magnetism

In the elementary theory it is supposed that magnetic materials consist of very small magnetic dipoles. The effect of a magnetic field is to align the dipoles in the direction of the field and so cause induced magnetism. The alignment is opposed by the " heat " motions of the molecules with which the dipoles are associated. Thus, magnetization increases with the strength of the applied field and disappears when the

field is removed. This picture applies only approximately to the strongly magnetizable substances such as iron, nickel, cobalt, etc., (ferromagnetics) and modifications are essential to account for the unique properties possessed by these materials.*

The alignment of the magnetic dipoles is called *magnetic polarization*. In the case of a regular bar specimen placed with its long axis parallel to the uniform field, poles appear at either end and it behaves like a bar magnet. This *induced magnetism* is demonstrated by placing a short piece of soft-iron rod on the axis of a strong bar magnet, as in Fig. 3.13. If some small iron tacks are sprinkled over the ends of the specimen they will adhere firmly whilst it remains near the magnet. When the latter is removed the tacks fall off and the specimen appears to be unmagnetised. Induced magnetism is responsible for the adherence of iron filings to the poles of permanent magnets. Each filing becomes magnetized by induction and the force between induced and inducing pole causes adherence.

Fig. 3.13

In an unmagnetized specimen the dipoles have a random arrangement caused by heat motions preventing any group of dipoles producing a resultant field strength at an external point (Fig. 3.14a). The applied field causes rotation of the dipoles so that each becomes aligned to some extent depending on the field strength. In the case of strong alignment the poles of one kind only appear in the end faces, (Fig. 3.14b). (In general, an *excess* of similar poles is produced in the end faces, but this is not easy to illustrate).

The excess of similar poles in the end faces gives rise to an external magnetic field, which, if it could be dissociated from the original field would be very similar to the field of a large dipole whose poles cover the end faces of the specimen. If the cross-section is small the end faces approximate to point poles and the specimen behaves like a true dipole. Magnetic lines of force pass from the N. pole to the S. pole so that the original field is reduced at points between the poles internally. The alignment of the elementary dipoles is dependent on the resultant of the original field and that of the

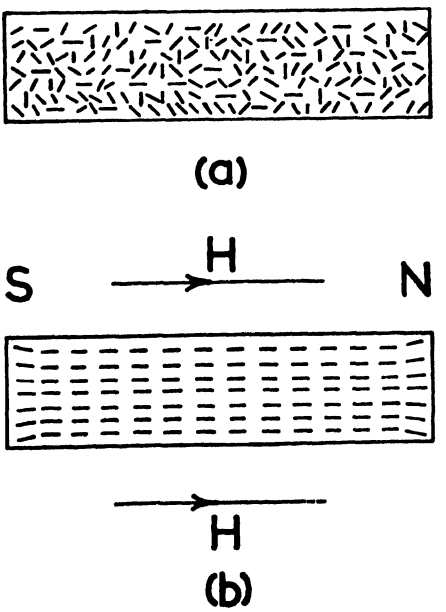


Fig. 3.14

* These include permanent magnetism, hysteresis, etc. (cf. Chap. 10).

specimen itself. The latter field is called a *demagnetizing field* since it reduces the magnetizing effect of the original field. The resultant field is called the *magnetizing field.* If the values of these field strengths at any point in the specimen are H_o, H_d and H_m, where the subscripts refer to original, demagnetizing and magnetizing fields:

$$H_m = H_o - H_d.$$

Near the poles of the specimen (externally) the original field is increased because of the additional lines of force created.

3.17. The Origin of Permeability

Magnetic induction is responsible for the reduction in force between magnetic poles in a medium other than vacuo, e.g. if a point N. pole is imagined inside an extremely large mass of magnetizeable matter the dipoles are aligned radially, S. (Fig. 3.15). (The alignment is not

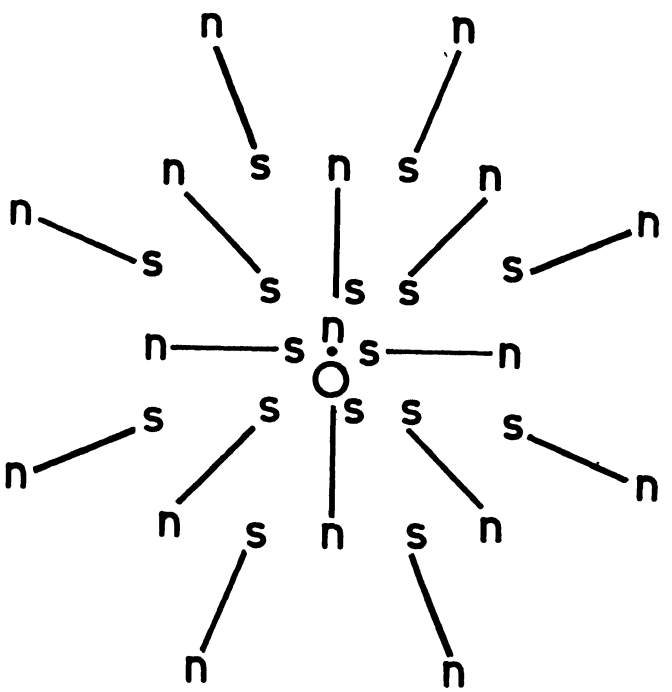


Fig. 3.15

complete and the diagram represents the equivalent effect). The S. poles near the original N. pole effectively reduce its strength so that the effect on any other pole is reduced. This is taken into account by introducing relative permeability μ in the inverse square law:

$$F = \frac{mm'}{4\pi\mu_o\mu r^2}$$

3.18. Magnetic Induction Vector B

Because of the different degrees of alignment of the magnetic dipoles of different substances an excess amount of pole of one sign appears at the interface between two media in the presence of a magnetic field. This effect is called the appearance of " free " poles. Because of this the magnetic field strength changes abruptly at boundary surfaces and any theoretical determination of field strengths becomes very difficult without extensive knowledge of free pole distributions.

To avoid the difficulties the *magnetic induction vector* or *magnetic flux density* (B) is introduced, defined by the relation:

$$B = \mu_0 \mu H \quad . \quad . \quad . \quad . \quad . \quad (37)$$

It follows from (8) that the value of B at a point r m. from a point N. pole is given by:

$$B = \frac{m}{4\pi r^2} \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Similarly, at a point on the axis of a bar magnet (or dipole):

$$B = \frac{2M}{4\pi(r^2 - l^2)^2} \quad . \quad . \quad . \quad (39)$$

In all cases B is independent of the medium, since μ does not appear in the formulae. This property makes B more important than H. From (38) it follows that B is measured in: *webers per square metre* (*wb.m.*$^{-2}$).

The direction of the magnetic induction vector is the same as that of the magnetic field strength vector H at that point (in isotropic media, assumed throughout). It is possible to measure B, as described later, (Chapter 9).

3.19. Flux of Magnetic Induction

For a point N. pole, B is directed radially from the pole and has the same value at all points on a sphere of radius r with the pole at the centre. The *flux of magnetic induction* through the spherical surface is:

$$\Phi = 4\pi r^2 \times B = m \text{ (webers)} \quad . \quad . \quad . \quad (40)$$

From this it follows that the unit of flux is the same as that of pole strength, viz. *weber* (wb.). In general, the flux of magnetic induction through any surface is defined as for electric displacement (p. 10), by the relation:

$$\Phi = \int B_n \, . \, dS \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Lines of induction are drawn in a magnetic field, the tangent at any point on a line representing the direction of B at that point. As with lines of force the number of lines drawn per unit area normal to their

direction represents the value of B. From (40) the number of lines of induction leaving a N. pole of strength m. wb. is m. The number of lines leaving a unit pole is, accordingly, *unity*.

3.20. *Gauss's Theorem in Magnetism

By assuming the separate existence of N. and S. poles it is possible to show, by exact analogy with Gauss' Theorem in electrostatics (p. 10), that:

$$\int B_n dS = \Sigma m \quad . \quad . \quad . \quad . \quad (42)$$

i.e. *the flux of magnetic induction through any imaginary closed surface is equal to the net pole strength inside that surface.* In practice N. poles are found always associated with S. poles, since the fundamental entity is the magnetic dipole, so that Σm is always zero. Gauss' Theorem in magnetism must hence be written:

$$\int B_n \,.\, dS = 0 \,. \quad . \quad . \quad . \quad . \quad (43)$$

3.21. *Boundary Theorems

At the interface between two magnetic media of different permeabilities the magnetic field strength and magnetic induction change in value. The changes are governed by two theorems:

(i) *The normal component of magnetic induction is the same at points close to either side of the boundary.*

(ii) *The tangential component of magnetic field strength is the same at points close to either side of the boundary.*

(i) Consider the points P_1 and P_2 which are close to opposite sides of the boundary between two media having relative permeabilities μ_1 and μ_2 (Fig. 3.16). Construct an imaginary " pill-box " having an infinitesimal cross-section dS m.^{2}, the end faces containing P_1 and P_2.

The normal components of induction at P_1 and P_2 are: $B_1 \cos\theta_1$ and $B_2 \cos\theta_2$. The flux of magnetic induction through the curved surface of the pill-box is negligible compared with that through the end-faces. The total flux is therefore:

$$d\Phi = B_1 \cos\theta \,.\, dS - B_2 \cos\theta_2 \,.\, dS$$

(Flux entering a surface is negative and vice versa).

From Gauss' Theorem: $d\Phi = 0$.

Hence: $B_1 \cos\theta_1 = B_2 \cos\theta_2$.

i.e. $\underline{B_{n1} = B_{n2}}$.

This proves the theorem.

(ii) The tangential components of field strengths at P_1 and P_2 are: $H_1 \sin\theta_1$ and $H_2 \sin\theta_2$, respectively (Fig. 3.16). If a unit N. pole is carried round a closed path passing through P_1 and P_2 and parallel

to the surface, its length being dl, the work done on going across the interface is negligible compared with that done parallel to the interface. The latter is given by:

$$dW = H_1 \sin \theta_1 \, . \, dl - H_2 \sin \theta_2 \, . \, dl.$$

The total work done along a closed path in a magnetic field must be zero, otherwise, energy is lost or gained by the field and this would be contrary to the conservation of energy theorem, hence:

$$H_1 \sin \theta_1 = H_2 \sin \theta_2$$

i.e.

$$\underline{H_{t1} = H_{t2}}$$

This proves the theorem.

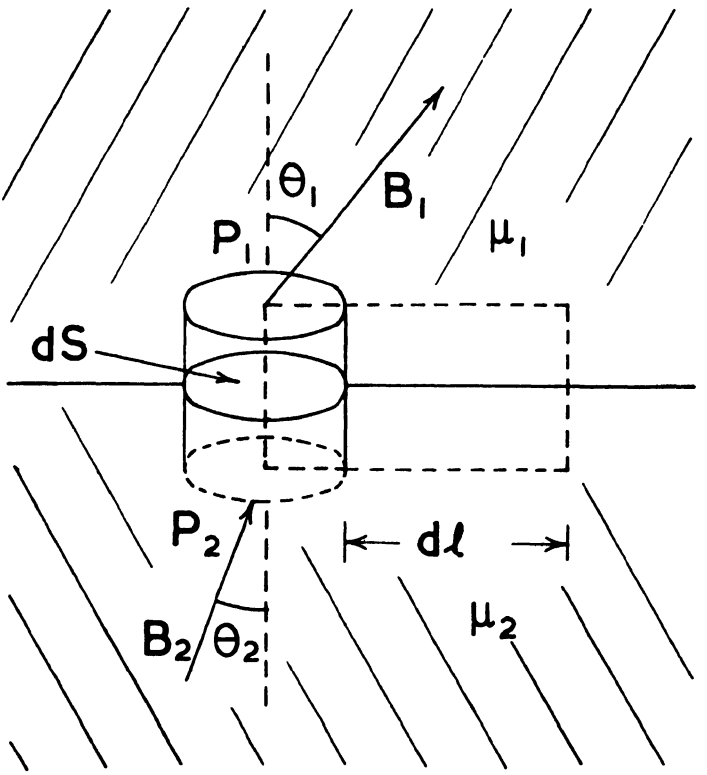


Fig. 3.16

3.22. The Importance of B and H

The value of the magnetic induction vector at any point governs the force exerted on a conductor carrying a current, the couple exerted on a coil carrying a current in a magnetic field, the force on an electron in a magnetic field, etc. Furthermore, whenever the flux of magnetic induction through a closed circuit changes an e.m.f. is produced in that

circuit. Magnetic field strength governs the alignment of magnets, the magnetization of materials, etc., and forms the basis of an extremely important link between magnetism and electricity (Ampère's Theorem).

3.23. Magnetic Potential

By analogy with electrostatic potential it is possible to define a magnetic potential:

The magnetic potential at any point in a field is the work done in bringing up a unit N. pole from infinity to that point.

Magnetic potential is thus measured in joules per weber (j.wb.$^{-1}$). The magnetic potential at any point is given by:

$$\Omega = -\int_{\infty}^{r} H_s \,.\, ds \quad \text{(cp. (18) of Chapter 1)} \,. \qquad (44)$$

where H_s is the component field strength along the element of path ds.

The concept of magnetic potential was extremely useful historically in determining the connection between magnetism and electricity (p. 112). In proving theorem (ii) above the criterion used was effectively: *the change in magnetic potential along a closed path is zero.*

3.24 Magnetic Potential in the Field of a Point Pole

The field strength at a point s metres ($s \gg r$) from a point N. pole of m. wb. is given by:

$$H_s = \frac{m}{4\pi\mu_0\mu s^2}$$

Hence, the potential at a point r m. from the pole is:

$$\Omega = -\int_{\infty}^{r} H_s \,.\, ds = \frac{m}{4\pi\mu_0\mu}\left[\frac{1}{s}\right]_{\infty}^{r} = \frac{m}{4\pi\mu_0\mu r} \text{ joule.wb.}^{-1} \qquad (45)$$

3.25 Magnetic Potential at a point in the field of a Dipole

(i) *Point on axis:* Referring to Fig. 3.2:

Since $NP = r - l$, the potential at P due to the N. pole is:

$$\Omega_N = \frac{+m}{4\pi\mu_0\mu(r-l)}$$

and since $SP = r + l$, the potential at P due to the S. pole is:

$$\Omega_S = \frac{-m}{4\pi\mu_0\mu(r+l)}$$

Hence, the total magnetic potential at P is:

$$\Omega = \Omega_N + \Omega_S = \frac{m}{4\pi\mu_0\mu(r-l)} - \frac{m}{4\pi\mu_0\mu(r+l)} = \frac{m(r+l-r+l)}{4\pi\mu_0\mu(r^2-l^2)} \,. \qquad (46)$$

i.e. $$\Omega = \frac{2lm}{4\pi\mu_0\mu(r^2 - l^2)} = \frac{M}{4\pi\mu_0\mu(r^2 - l^2)} \quad . \quad . \quad (46)$$

If $r \gg l$: $$\Omega = \frac{M}{4\pi\mu_0\mu r^2} \text{ joule.wb.}^{-1} \quad . \quad . \quad . \quad (47)$$

(ii) *Point on Perpendicular Bisector* (Fig. 3.3).

For any point such as B, on the perpendicular bisector of the dipole, the distance from each pole is the same so that the potentials due to each pole are equal and opposite and the *magnetic potential is zero.*

The above results are similar to those obtained for the corresponding electrical systems, except that ϵ_0 is replaced by μ_0 and q is replaced by m.

EXERCISES

1. Define the terms *magnetic moment* of a magnet and *magnetic field strength* at a point in its field.

   Deduce an expression for the field strength due to a bar magnet, of magnetic moment M and magnetic length $2l$, at a point on its axis distant r $(> l)$ from its centre.

   If a magnet is of magnetic length 8 cm., find the percentage error caused by neglecting the magnetic length of the magnet in calculating the magnetic field strength due to the magnet at a point on its axis 32 cm. from its centre. (London G.C.E. Adv. L.).

2. Calculate the magnitude and direction of the magnetic field due to a bar magnet which is 10 cm. long and has a magnetic moment 10^{-6} weber.metre, at a point 20 cm. from its mid-point measured along a line inclined at 45° to its magnetic axis.

   You may assume that the magnetic moment is a vector, but are to deduce from first principles any formula used. (London G.C.E. Adv. L.).

3. The needle of a compass consists of a small magnet of moment of inertia 10^{-6} kgm. metre^2. When the compass needle is placed on the axis of, and 10 cm. from one pole of another magnet, it oscillates twice per second if given a small angular displacement and released. If the poles of a second magnet are 10 cm. apart and of strength $8\pi \times 10^{-6}$ weber, what is the magnetic moment of the compass needle? (The earth's field may be ignored). (Part of Camb. Open Schol.).

4. Define *unit magnetic pole* and *magnetic field strength.*

   A uniform steel needle 10 cm. long has a mass of 1 gm. It is magnetized to a pole strength of $4\pi \times 10^{-6}$ weber and is suspended horizontally in the earth's field. How far from the centre of the needle must the suspension be attached if the vertical component of the earth's field is $109/(4\pi)$ amp.metre^{-1}? (Oxford Open Schol.). (Subsid.).

5. A small magnet is suspended by a quartz fibre attached to a torsion head. The latter is adjusted so that the magnet lies with its north-seeking pole pointing north. In this position the period of small oscillations is found to be 1 sec. The torsion head is then turned through 90° and the equilibrium position of the magnet is found to be at 60° to the magnetic meridian. The torsion head is then turned to 180° from its first position. Show that there is a position of stable equilibrium with the north-seeking pole pointing south and find the period of small oscillations about this position. (Oxford Open Schol.).

6. Explain how the horizontal component of a magnetic field may be determined by timing the oscillations of a bar magnet of known magnetic moment suspended by a fine thread. If the thread were not very fine, what effect would this have and how could allowance be made for it. Deduce any formulae you use.

   A bar magnet of mass 100 gm. and magnetic moment 10^{-4} weber.m. was suspended freely at one end in a vertical magnetic field and allowed to make small oscillations in a vertical plane: the period was found to be 0·414 sec. The magnet was then reversed and the periodic time of the oscillations was then found to be 0·276 sec. If the length of the magnet was 10 cm., find the strength of the field in which the magnet was suspended. (Northern H.S.C. Schol.).

7. Derive an expression for the moment of the couple required to hold a bar magnet with its axis inclined at an angle θ to a uniform magnetic field.

   A magnet supported, with its axis horizontal, by a torsionless fibre is displaced from its position of equilibrium in a uniform magnetic field through a small angle, θ radians, and then released. Write down an expression for the period of vibration of the subsequent horizontal oscillations. Indicate the effect on the periodic time of using a suspension with a finite torsion (e.g. a wire).

   A magnet, suspended by a wire so that its axis is horizontal, comes to rest in the magnetic meridian when the wire is free from torsion. It oscillates with small amplitude in a horizontal plane about the position of rest with a periodic time of 5 sec.

   To estimate the amount of torsional control in the wire, the top of the wire is twisted through a right angle and it is found that the magnet at the lower end comes to rest at 5° from the meridian.

   At a second point it oscillates with a periodic time of 6 sec.

   Assuming that the torsional control in the wire is constant, compare the horizontal magnetic fields at the two points. (London G.C.E. Schol.).

CHAPTER 4

ELECTRIC CURRENT
(Fundamental Laws)

4.1. Introduction

When two insulated conductors at different potentials are connected by a wire an electric current flows consisting of a stream of moving charges in the connecting wire. By convention, the moving charges are assumed positive, flowing from higher to lower potential. In *metals* the current charges are *electrons*, i.e. negative charges, so that current really flows from lower to higher potential. This is of no consequence, since the effects of a current of positive charges are the same as those of an equal current of negative charges in the opposite direction.

A direct current produces three major physical effects, viz. (*a*) a magnetic field surrounding the conductor, (*b*) heat in the conductor and (*c*) the disintegration of special types of conductor called *electrolytes*. These effects form the basis of detection, measurement and application of electric currents, as described later.

The minute currents in electrostatic experiments last for a fraction of a second and are called *transient currents*. In order to maintain a continual *direct current* (D.C.), a *constant* p.d. is required between two points. This necessitates a source of energy called a source of *electromotive force* (e.m.f.), from which the moving charges may abstract energy to overcome *resistance* to flow.

4.2. Identity of Static and Current Electricity

Current derived from static electricity gives the same effects as a steady current so that static and current electricity are the same. To demonstrate this, large condensers are used as the sources of static electricity with a poor conductor for the current flow, (Fig. 4.1 (*a*)).

C_1 and C_2 are condensers having a capacity of several μF. The plates P and X of C_1 and C_2, respectively, are connected to gold leaf electroscopes E_1 and E_2, respectively. These indicate the potentials of P and X relative to the earthed plates Q and Y. P and X are also connected by long damp strings l_1 and l_2 to two stout insulated copper rods M and N. The connection between P and X is completed by placing an insulated conducting rod S across M and N.

C_1 is charged by momentarily connecting P and Q across a battery of several hundred volts: E_1 only shows a deflection. When M and N are connected by S, the leaf of E_1 gradually falls whilst that of E_2 slowly diverges, indicating a fall in p.d. across C_1 and an increase in p.d. across C_2. Eventually both E_1 and E_2 show a steady divergence indicating

Hence the required resistance is made up of two groups of four resistors as shown in Fig. 4·7.

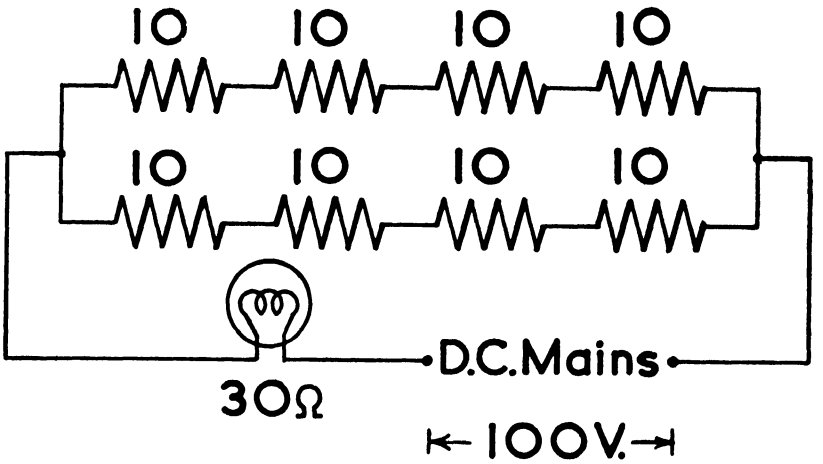


Fig. 4.7

4.12. Internal Resistance of a Cell

The current flow in a cell consists of positive and negative ions going in opposite directions. They gain and lose energy by collisions with the atoms or molecules of the electrolyte. There is hence a resistance to flow in the cell itself. If the e.m.f. is E volts and the p.d. between the plates when a current of strength I amps. flows is V volts, the p.d. in the electrolyte is: $v = (E - V)$ volts. Ohm's Law applies to the current flow in the cell so that:

$$I \propto v, \text{ i.e. } I = \frac{v}{r} \quad . \quad . \quad . \quad . \quad (10)$$

where r is called the *internal resistance* of the cell.

Since $v = Ir = E - V$, and $I = V/R$ for the external circuit:

$$I = \frac{E}{R + r} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This is the form Ohm's Law takes for a complete circuit containing a source of E.M.F.

4.13 Alternative Definition of e.m.f.

By definition, the work done in taking one coulomb through a p.d. of V volts is V joules. Hence, from above, the total work done in taking one coulomb completely round the circuit is: $V + v = E$ joules. Hence, an alternative definition of e.m.f.:

The e.m.f. of a cell is the work done in taking a unit charge (coulomb) completely round the circuit.

Example

The positive terminals A *and* C *of two cells are connected by a resistance of* 5 *ohms and their negative terminals* B *and* D *by a resistance of* 8 *ohms. The e.m.f.s of AB and* CD *are* 1·5 *volts and* 1·1 *volts respectively and their corresponding internal resistances* 1 *ohm and* 2 *ohms. Terminal* D *is earthed. Find the potentials of the mid-points* H *and* K *of the resistances joining* AC *and* BD *respectively.* (Northern G.C.E. Adv. L.).

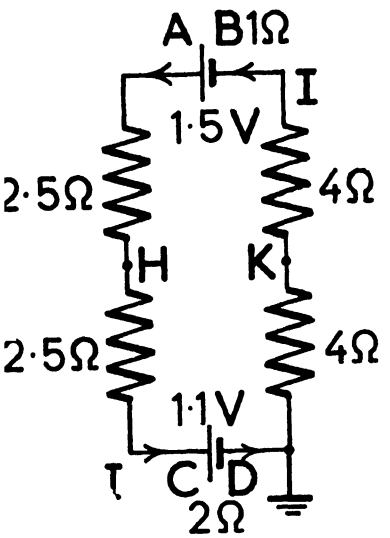


Fig. 4.8

Referring to Fig. 4.8, D is earthed and its potential is zero. All points in the circuit have an "absolute" potential relative to this zero.

If there were no current flowing A would be at $+1{\cdot}5$ volts relative to D, and C would be at $+1{\cdot}1$ volts relative to D. Since A is at the higher potential, conventional current flows from A to C and round the complete circuit. The net e.m.f. responsible for the current is: $E = 1{\cdot}5 - 1{\cdot}1 = 0{\cdot}4$ volts. The total internal resistance is: $r = 1 + 2 = 3$ ohms. The total external resistance is: $R = 5 + 8 = 13$ ohms. Hence, applying Ohm's law:

$$I = \frac{E}{R + r} = \frac{0{\cdot}4}{16} = \frac{1}{40} \text{ amps.}$$

By Ohm's law the p.d. along DK is:

$$V_{DK} = IR_{DK} = \frac{1}{40} \cdot \frac{8}{2} = 0{\cdot}1 \text{ volts}$$

Since current flows in the direction D–K, K is at a *lower* potential than D, its value being: $-0{\cdot}1$ volts.

Because of the p.d. across the electrolyte of CD in the direction $C \rightarrow D$, the potential of C is increased from $+1{\cdot}1$ volts to $1{\cdot}1 + Ir_{CD} = 1{\cdot}1 + (\frac{1}{40} \times 2) = 1{\cdot}15$ volts. The p.d. between C and $H = IR_{CH} = \frac{1}{40} \cdot \frac{5}{2} = 0{\cdot}0625$ volts. Thus, H is at a potential higher than C, its absolute value being:

$$V = 1{\cdot}15 + 0{\cdot}0625 = 1{\cdot}2125 \text{ volts}$$

(*N.B.* The p.d. between the plates of CD is *increased* when current flows because the flow is opposite to that normally taken).

4.14. Rate of doing Work: Power

The potential energy lost by one coulomb flowing through a p.d. of V volts is V joules, so that when Q coulombs flow the energy lost by the charge is: QV joules. If the current strength corresponding to this flow of charge is I amps. and the time of current flow is t secs.: $Q = It$. Hence the energy lost in t secs. is:

$$W = IVt \text{ joules} \quad . \quad . \quad . \quad . \quad (12)$$

This energy ultimately appears as heat in the conductor. From (12) the *rate of loss of energy is*:

$$\underline{P = IV} \text{ joules per second} \quad . \quad . \quad . \quad (13)$$

P is the *rate of working* of the current or the *Power* developed in that part of the circuit. The unit of power: *joule per sec.*, is called the *watt.* Larger units are the kilowatt (1 kw. $= 10^3$ watts) and the megawatt (1 mw. $= 10^6$ watts). The *watt-hour* is the name given to the common unit of consumption of electrical energy. The domestic unit of energy consumption is the *Kilowatt-hour* (1 kw.hour $= 10^3 \times 3600 = 3{\cdot}6 \times 10^6$ joules).

Electric appliances are rated in watts or kilo-watts, e.g. a 1kw. electric fire dissipates 10^3 joules per second. Similarly electric lamps have ratings between 15 and 250 watts depending on the energy they consume.

Since one coulomb loses E joules when it flows round a complete circuit containing an e.m.f. of E volts, the rate of working in a *complete circuit* is given by:

$$\underline{P = EI \text{ watts}} \quad . \quad . \quad . \quad . \quad (14)$$

Both (13) and (14) may be expressed in terms of R and I or V and R instead of V and I:

From (13): $\underline{P = I^2R \quad \text{watts}} = \underline{V^2/R \quad \text{watts}} \quad . \quad (15)$

From (14): $\underline{P = I^2(R + r) \text{ watts}} = \underline{E^2/(R + r) \text{ watts}} \quad . \quad (16)$

It follows from the conservation of energy that a cell delivers energy to a circuit at the rate of: $P = E^2/(R + r)$ joules per second: this is derived from chemical action.

Example

A Daniell cell having an e.m.f. of 1·1 *volts and an internal resistance of* 0·5 *ohms is connected in series with a resistance of* 10 *ohms. Determine the current strength in the circuit, the p.d. across the resistance and the power delivered by the cell.*

The total resistance in the circuit: $R + r = 10{\cdot}5$ ohms.

Hence the current strength is:

$$I = \frac{E}{R + r} = \frac{1{\cdot}1}{10{\cdot}5} = 0{\cdot}105 \text{ amps}$$

The p.d. across the resistor is:

$$V = IR = 0{\cdot}105 \times 10 = \underline{1{\cdot}05 \text{ volts.}}$$

The power delivered by the battery is:

$$P = EI = 1{\cdot}1 \times 0{\cdot}105 = \underline{1{\cdot}155 \text{ watts.}}$$

4.15. Maximum Power delivered by a Cell

In Fig. 4.9, the cell of e.m.f. E volts and internal resistance r ohms delivers a current of strength 1 amps. to the resistance R ohms. Hence:

$$I = \frac{E}{R + r} \text{ amps.}$$

The work done per second in the external circuit is:

$$P = I^2R = \frac{E^2}{(R+r)^2} \cdot R \text{ watts} \quad . \quad . \quad . \quad (17)$$

Since E and r are constant, P depends on R only and has a maximum given by:

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^2} - \frac{2E^2R}{(R+r)^3} = 0$$

i.e. $R + r = 2R$

or: $R = r$

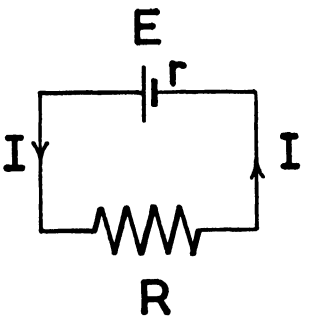


Fig. 4.9

It follows that a cell delivers its maximum power to the external circuit when the external resistance is equal to the internal resistance. The cell is working most efficiently when this applies. Putting $r = R$ in (17):

$$P_{max.} = \frac{E^2R}{4R^2} = \frac{E^2}{4R} \quad . \quad . \quad . \quad . \quad (18)$$

The efficiency of the cell, treated as a machine, is given by:

$$\eta = \frac{\text{useful energy supplied}}{\text{total energy dissipated}} = \frac{I^2R}{EI} = \frac{R}{R+r}$$

At maximum efficiency, therefore: $\eta_{max.} = R/2R = 0{\cdot}5$ (or 50%). Cells are rarely used at maximum efficiency since, generally, $r \ll R$.

Cells in Series and Parallel

Cells are connected in series when the positive terminal of one is connected to the negative terminal of the next cell, as in Fig. 4.10. The connected cells form a "battery" whose total e.m.f. is the sum of the separate e.m.f's. If the cells are identical, each having an e.m.f. of E_o volts, the total e.m.f. is: $E = nE_o$, where n is the number of cells in the battery.

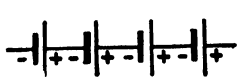


Fig. 4.10

If the internal resistance of each cell is r ohms, the current delivered to an external circuit having a resistance R ohms is:

$$I = \frac{nE_o}{R + nr} \text{ amps.} \quad . \quad . \quad . \quad . \quad (19)$$

When all the positive terminals of a group of cells and all the negative terminals are connected together, as in Fig. 4.11, the cells are connected

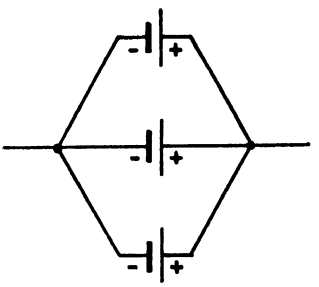


Fig. 4.11

in parallel. The e.m.f. of the battery is the same as that of each cell providing the cells are identical. If the e.m.f. of each cell is E_o volts and the internal resistance of each is r ohms, the current delivered to an external circuit R ohms is:

$$I = \frac{E_o}{R + \frac{r}{n}} = \frac{nE_o}{nR + r} \text{ amps.} \quad . \quad . \quad . \quad (20)$$

(If the cells are not identical, current flows in the circuit formed by any two cells because a net e.m.f. exists).

Batteries are constructed of identical cells such as accumulators, dry cells, etc. The main requirement of a battery other than p.d., is ability to deliver sufficient current strength without a decrease in e.m.f. All cells show a marked decrease in e.m.f. when the current exceeds a given amount. If a lamp is rated at: 6 volts, 24 watts, a current of 4 amps. is required at a p.d. of 6 volts to give optimum illumination. A battery of series accumulators must be capable of delivering this current. If the cells give a maximum of 2 amps. only without any deleterious effects, two series batteries of 3 accumulators each are connected in parallel.

EXERCISES

1. Describe experiments which show the identity of static and current electricity. (London G.C.E. Adv. L.).

2. What do you understand by the *electromotive force* (e.m.f.) of an accumulator?

An accumulator, when directly connected to a constant d.c. supply of 2·5 volts, is being charged with a current of 0·5 amps. If the terminals of the same cell are connected to a resistance of 7·6 ohms, the discharge current is found to be 0·25 amps. Calculate the e.m.f. and the internal resistance of the cell. (Oxford Open Schol. Subsid.)

3. State Ohm's Law. Define *specific resistance.*

Two wires of identical shape have specific resistances s_1 and s_2. They are connected first in series, then in parallel, to form resistances R_s and R_p respectively. Calculate the ratio R_s/R_p. How could this ratio be determined experimentally? (Oxford Open Schol. Subsid.)

4. What conditions are necessary if a battery of cells are to deliver maximum power to a resistance connected across its terminals?

It is desired to heat a wire of resistance 3 ohms, using a battery of six cells, each of e.m.f. 1·1 volts and internal resistance 1 ohm. Show how the cells must be connected in order that the wire will be heated as rapidly as possible. Find the rate in cals. per minute at which heat is developed with this arrangement. (1 cal. = 4·2 Joules). (Manch. Open Schol.).

5. Obtain from first principles an expression for the combined resistance of a number of resistances connected (*a*) in series, and (*b*) in parallel.

A 6-volt battery of negligible internal resistance is connected in series with a 3 ohm and a 5 ohm resistance. A further resistance of 3 ohms is connected in parallel with the 5 ohm resistance. Find the current flowing, and the power dissipated, in each resistance. Check your answer by showing that the sum of these powers is equal to the power supplied by the battery. (Oxford Open Schol. Subsid.).

flows the coil rotates and ultimately comes to rest at an angle θ to its initial position. In this position the forces acting on the horizontal sides AB and CD are in the plane of the coil and do not affect its rotation. The forces acting on the vertical sides BC and AD are at right angles to the magnetic induction B and in a horizontal plane (Fig. 5.24 (b)). They give rise to a couple:

$$G = BI \times b \times a \cos\theta = BIA \cos\theta \text{ newton.m.} \quad . \quad (34)$$

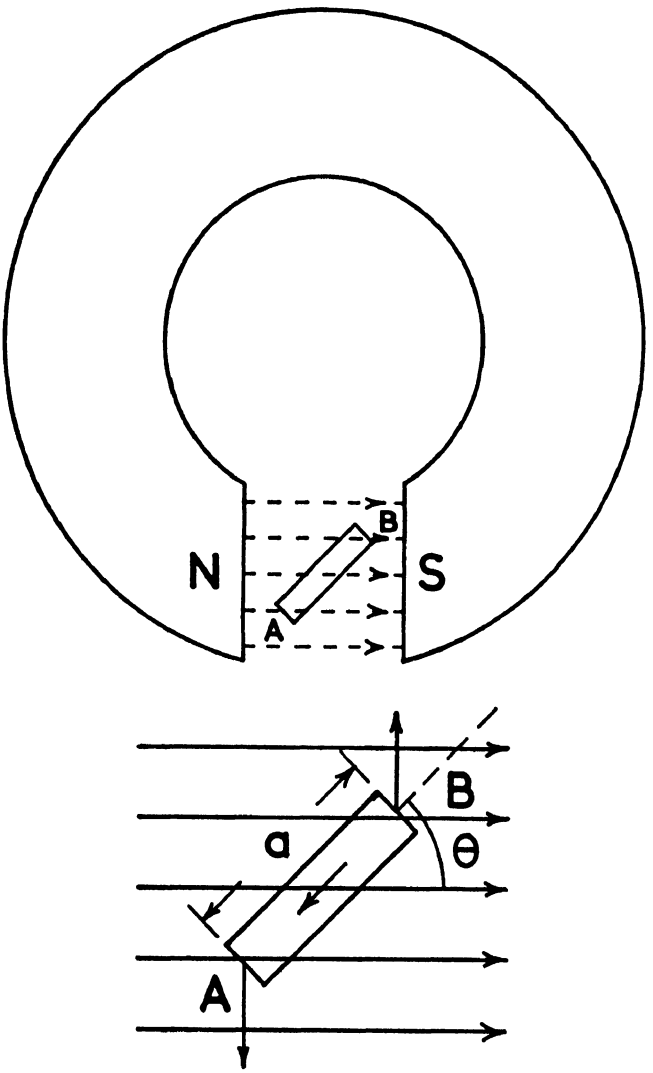


Fig. 5.24(b)

where $AB = CD = a$ m., $BC = AD = b$ m., and $A = ab$ m.^{2} = cross-section of coil.

For N turns the total turning moment is:

$$G = BINA \cos \theta \text{ newton.m.}$$

Equilibrium is maintained by the twist in the suspension, given by:

$$G = \tau\theta \text{ newton.m.}$$

where τ is the torsional constant of the suspending fibre, i.e. torque per radian twist.

Hence: $$I = \frac{\tau\theta}{BAN \cos \theta} \text{ amps.} \quad . \quad . \quad . \quad . \quad (35)$$

For small deflections, $\cos \theta \to 1$, whence:

$$I = \frac{\tau\theta}{BAN} \text{ amps.} \quad . \quad . \quad . \quad . \quad . \quad (36)$$

In practice θ is never measured in radians but in scale divisions on a lamp and scale. Deflections are always small but to ensure that $\cos \theta$ may always be replaced by unity in (35), N and S are provided with concave pole pieces and a soft iron cylinder is placed inside the coil so that lines of induction enter and leave the cylinder radially (Fig. 5.25). The plane of the coil is then always parallel to B and (36) is accurate.

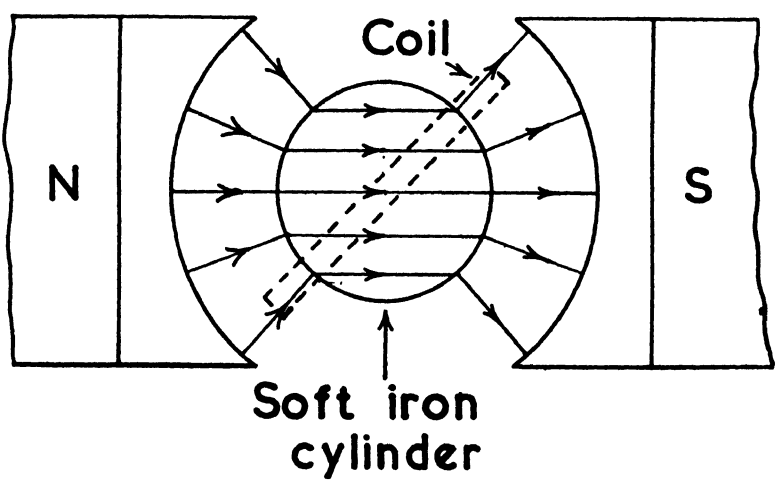


Fig. 5.25

Assuming θ is measured in scale divisions, (36) may be put in the form:

$$I = \lambda\theta \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where λ is a constant which is known as the current sensitivity* of the

* The constant $\frac{1}{\lambda}$ measured in scale divs. per amp. is often used as the current sensitivity.

galvanometer. It is measured in amps. (or micro-amps.) per scale division. Sensitive galvanometers have small values of λ. For maximum sensitivity, therefore, BAN must be as large as possible and τ as small as possible. This necessitates very fine suspensions, a large number of turns and a very strong permanent magnet. By virtue of the latter the galvanometer deflection is virtually independent of the earth's magnetic field.

Damping

On passing a current through the coil a couple acts causing rotation. The coil is accelerated from rest and acquires kinetic energy which carries it beyond its true equilibrium position θ_0. The coil rotates until the excess twist in the suspension brings it to rest ($\theta > \theta_0$). The excess twist causes rotation in the opposite direction which does not cease until the kinetic energy again acquired is converted into potential energy ($\theta < \theta_0$). Thus, the coil oscillates about its equilibrium position, the kinetic energy acquired from its initial deflection being converted into potential energy and vice versa. These oscillations cease when all the energy of the coil is dissipated in overcoming the damping forces present.

Galvanometer damping is caused by (i) air viscosity and (ii) induced *eddy currents* in the coil and former. Air viscosity gives rise to a small opposing couple which cannot be avoided. Eddy currents arise from the e.m.f.'s induced in the coil and former as they rotate in the magnet field. The induced currents give rise to a magnetic field which causes interaction opposing the motion producing the currents (Lenz's Law) (Chapter 9). The damping effect which arises increases with the strength of the eddy currents and can, therefore, be controlled by altering the resistance in the galvanometer circuit* or choosing the former material carefully. For heavy electromagnetic damping, as it is called, the coil former is made of copper so that the eddy-currents have maximum strength. For measuring steady currents this is very desirable and results in the coil coming rapidly to rest after a deflection.

5.23. Ballistic Moving Coil Galvanometer

This is a moving coil galvanometer having very small electromagnetic damping and a long period of oscillation (several seconds). Under these conditions the galvanometer deflection is a measure of the total *charge* which passes through the coil, providing this occurs in a time, short compared with the period of oscillation. Very small electromagnetic damping is achieved by winding the coil on a former made of insulating material such as bakelite, and the long period is achieved by using a very large number of turns (giving a large moment of inertia) and a very fine suspension (small torsional constant). From the theory given in Chapter 9, it is shown that the initial deflection of the galvanometer is

* Galvanometers on " open " circuit ($R = \infty$) have minimum damping whilst galvanometers which are " short " circuited have maximum damping ($R = 0$).

proportional to the total charge which passes through the instrument, i.e.:

$$Q = a\,\theta$$

where a is a constant known as the *ballistic constant* or *ballistic sensitivity*, and measured in coulombs (or micro-coulombs) per scale division.

Example

When a resistance of 10^6 *ohms is connected in series with a galvanometer having a resistance of* 1,000 *ohms and a cell having an e.m.f. of* 1·5 *volts (negligible internal resistance), the galvanometer shows a steady deflection of* 20 *cm. on a scale* 1 *m. away. What is the current sensitivity of the galvanometer?*

Referring to Fig. 5.26:

$$I = \frac{E}{R+G} = \frac{1{\cdot}5}{10^6 + 10^3} = 1{\cdot}495 \times 10^{-6} \text{ amps}$$

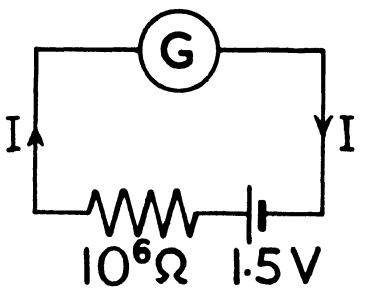


Fig. 5.26

Since $I = \lambda\theta$:

$$\lambda = \frac{I}{\theta} = \frac{1{\cdot}495}{20} \times 10^{-6} = 7{\cdot}475 \times 10^{-7} \text{ amps.cm.}^{-1}$$

5.24. Determination of Sensitivity and Resistance of Galvanometer

In the circuit of Fig. 5.27, P is a known small resistance (1 ohm say) and Q a much larger one (1,000 ohms say), both being connected to an

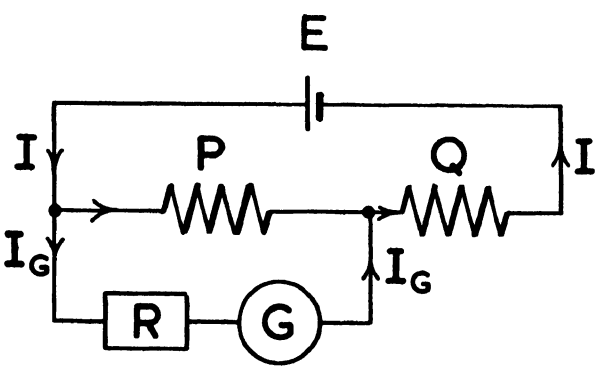


Fig. 5.27

accumulator of e.m.f. E volts. The galvanometer of resistance G ohms is connected in series with a resistance box, whose value R is always greater than P, so that the current in G is negligible compared with that in P and Q. Hence, if the latter is I amps., the p.d. across P is IP volts. Hence, the current in G is:

$$I_G = \frac{IP}{R+G} \text{ amps.}$$

Assuming $R \gg P$:

$$I = \frac{E}{P+Q} \text{ amps.}$$

whence: $$I_G = \frac{EP}{(P+Q)(R+G)} = \lambda\theta \text{ amps.} \quad . \quad . \quad (38)$$

where θ is the deflection in scale divisions.

From (38): $$\lambda\theta = \frac{k}{R+G} \left(k = \text{const.} = \frac{EP}{(P+Q)} \right)$$

Rearranging: $$R = \frac{1}{\theta} \cdot \frac{k}{\lambda} - G \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

In the experiment θ is found for various values of R and the latter is plotted against $1/\theta$ to give a straight line whose gradient is (k/λ). Since k is known, λ can be determined. The intercept on the R—axis gives $-G$, i.e. the galvanometer resistance.

5.25. Ammeters

Moving coil galvanometers are not sufficiently robust for ordinary use. For the measurement of larger currents the moving coil principle is retained but the moving part is pivoted and torsional control is replaced by springs.

A typical ammeter (milliammeter) is sketched in Fig. 5.28. The coil C consists of several turns of insulated wire wound on a circular metal former fitted with pivots resting in agate bearings P_1 and P_2. Filling most of the space inside C is a soft iron sphere F, which is fixed to the case. A pointer is attached to the upper pivot and rotates over a scale graduated in amps. (milliamps). The hair springs S_1 and S_2, control the deflection of the coil.

When current flows in the coil interaction between its field and that of the permanent magnet NS causes rotation which is opposed by S_1 and S_2. The pole faces of NS are shaped so that the lines of induction are radial, and, as for the moving coil galvanometer, the deflection is

proportional to the current strength. Calibration of the scale is obtained by using known currents.

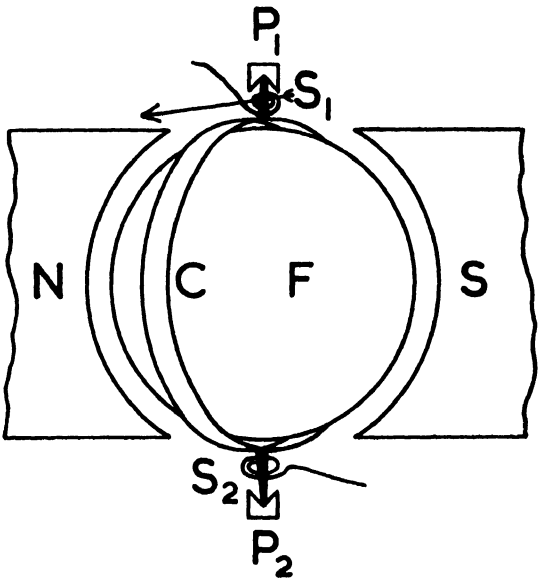


Fig. 5.28

Shunts

The resistance of an ammeter is small compared with any circuit resistance so that the circuit current is not appreciably affected by the presence of the meter. To increase the range of the meter a *shunt*, consisting of a low resistance, is placed across the meter terminals. The value of shunt resistance required to increase a given meter range by a factor n is found as follows: Let the ammeter resistance be R ohms and the required shunt resistance S ohms. If the full scale reading of the meter is I amps., the current which must pass through S when the range is nI amps., is $(nI - I)$ amps. Referring to Fig. 5.29, the p.d. across the meter is:

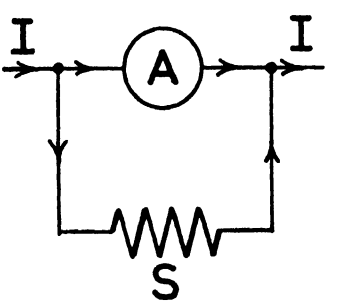


Fig. 5.29

$$RI = S(nI - I) \text{ volts}$$

whence:

$$S = \frac{R}{n-1} \text{ ohms.} \quad . \quad . \quad . \quad . \quad (40)$$

If $n = 10$, $S = R/9$; if $n = 100$, $S = R/99$, etc. In this way a meter designed to read 10 ma. full scale, may be arranged to read 100 ma., 1 amp., etc. The scale cannot be extended indefinitely because the shunt resistance becomes progressively smaller and contact resistance caused by dirt etc., may be greater than the shunt resistance.

5.26. Voltmeters

Moving coil voltmeters are used for D.C. p.d.'s from millivolts to hundreds of volts. They consist, essentially, of a milliammeter in series with a high resistance. The function of the high resistance is to prevent appreciable current flowing through the meter when it is placed across two points in a circuit whose p.d. is being measured (Fig. 5.30).

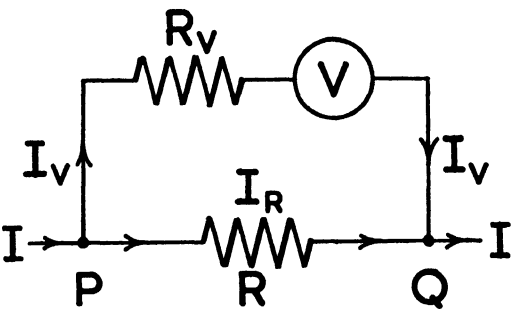


Fig. 5.30

Referring to Fig. 5.30, the current in the voltmeter is I_V amps. and that in the main circuit resistance R is I_R amps. Hence:

$$I_V R_V = I_R R$$

Since:

$$I_V + I_R = I$$

$$I_V R_V = (I - I_V)R$$

i.e.

$$I_V = \frac{R}{R + R_V} \cdot I \quad . \quad . \quad . \quad . \quad (41)$$

Hence the p.d. across R is:

$$V' = I_V R_V = \frac{RR_V}{R + R_V} \cdot I \quad . \quad . \quad . \quad (42)$$

By design $R_V \gg R$, so that $V' = IR$ approx. Further, since I is almost the same as the current in R before the voltmeter was connected, V' is approximately the same as the original p.d. across R. From (42), $I_V \propto V'$, so that the reading of the milliammeter is a measure of V' and hence the p.d. across R.

Good voltmeters have resistances R_V of thousands of ohms. The sensitivity of the instrument depends on the sensitivity of its milliammeter.

Different voltage ranges are obtained by using different series resistances, the full scale reading on any range being determined by the maximum current of the milliammeter. If the latter is I amps. and a full scale reading of V volts corresponds to R_V ohms: $IR_V = V$.

For a new full scale reading of nV volts, the new resistance required R'_V, is given by:

$$nV = IR'_V = \frac{VR'_V}{R_V}$$

whence: $$R'_V = nR_V \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

i.e. the series resistance is increased by the factor n.

The minimum p.d. measured by the voltmeter is limited by the milliammeter to values of the order of 0·1 volts. The maximum range is limited by insulation difficulties to values of the order of hundreds of volts.

5.27. Multi-reading Instruments

Since ammeters can be used as voltmeters, multirange instruments are devised having several ranges of current strength and voltage. A switch on the instrument is used to " switch-in " different current shunts and another switch is used to add different series resistances for voltage ranges.

Example

A milliameter has a full scale reading of 10 *ma. and an internal resistance of* 0·1 *ohms. How would you arrange for the instrument to be used to measure* 1 *amp. and also to give a full scale reading of* 10 *volts?*

To increase the range a shunt is required whose resistance is given by: $S = R/(n - 1)$. Since $R = 0{\cdot}1$ ohm., $n = 100$:

$$S = \frac{0{\cdot}1}{99} = 0{\cdot}00101 \text{ ohms.}$$

To use the meter as a voltmeter a series resistance is added. Since the maximum current in the meter is 10 ma. the p.d. across the meter and series resistance is: $V = I_V\,(R + R_V) = 10 \times 10^{-3}\,(R_V + 0{\cdot}1)$ volts. To read 10 volts, therefore:

$$10^{-2}\,(R_V + 0{\cdot}1) = 10$$

i.e. $$R_V = 999{\cdot}9 \text{ ohms.}$$

Example

Describe the construction and explain the principle of a moving coil voltmeter, illustrating your answer by diagrams.

Current from 200 *volts mains is sent through resistances of* 400 *ohms and* 600 *ohms connected in series. What will be the potential difference between the ends of the* 400 *ohm coil? If an accurate voltmeter of resistance* 1,000 *ohms is connected in parallel with the* 400 *ohm coil, how many volts will it read?* Oxford & Cambridge J. B. G.C.E. Adv. L.).

The construction of a moving coil voltmeter is described on p. 141.

The circuit for the problem is given in Fig. 5.31. When the voltmeter is not connected in the circuit the current strength is given by:

$$I_0 = \frac{E}{R + r} = \frac{200}{400 + 600} = 0{\cdot}2 \text{ amps. (assuming } r = 0).$$

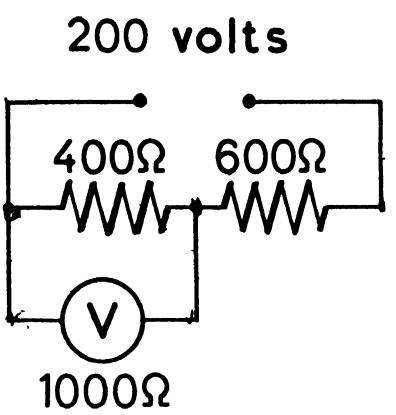


Fig. 5.31

Hence, the p.d. across the 400 ohm coil is:

$$V = I_o \times 400 = 80 \text{ volts}$$

When the voltmeter is connected across the coil the equivalent resistance of resistor and voltmeter in parallel is given by:

$$\frac{1}{R'} = \frac{1}{400} + \frac{1}{1000} = \frac{7}{2} \times 10^{-3}$$

i.e. $$R' = 2/7 \times 10^3 \text{ ohms.}$$

The new current strength in the main circuit is:

$$I'_o = \frac{200}{\frac{2}{7} \times 10^3 + 600} = \frac{7}{31} \text{ amps.}$$

The current passing through the 400 ohm coil is given by:

$$I' = I'_o \cdot \frac{1000}{400 + 1000} = \frac{7}{31} \cdot \frac{5}{7} = \frac{5}{31} \text{ amps.}$$

$\therefore$ the new p.d. across the coil is:

$$V' = I' \times 400 = \frac{5}{31} \times 400 = 64{\cdot}5 \text{ volts.}$$

i.e. the p.d. across the coil is initially 80 volts but when the voltmeter is connected it is reduced to 64·5 volts—this is recorded by the meter.

Example

State Ohm's law and describe how you would test its validity. Why, would an experiment involving the use of a moving coil ammeter and a moving coil voltmeter be unsatisfactory?

In order to calibrate a galvanometer an accumulator of e.m.f. 2·00 *volts and negligible resistance is connected in series with two resistances P and Q. A resistance R and the galvanometer (resistance G) are joined in series and then connected to the ends of* P. *The galvanometer is shunted by a resistance* S. *If* P = 200 *ohms,* Q = 1,880 *ohms,*

R = 291 *ohms*, S = 10 *ohms*, G = 90 *ohms and the deflection is* 20 *divisions, calculate the current sensitivity in microamperes per division.* (Northern H.S.C.).

The first part of the question is discussed in Chapter 5.

The equivalent resistance of S and G in parallel in Fig. 5.32 is given by:

$$\frac{1}{r} = \frac{1}{S} + \frac{1}{G}$$

i.e. $$r = \frac{SG}{S+G} = \frac{10 \times 90}{10 + 90} = 9 \text{ ohms.}$$

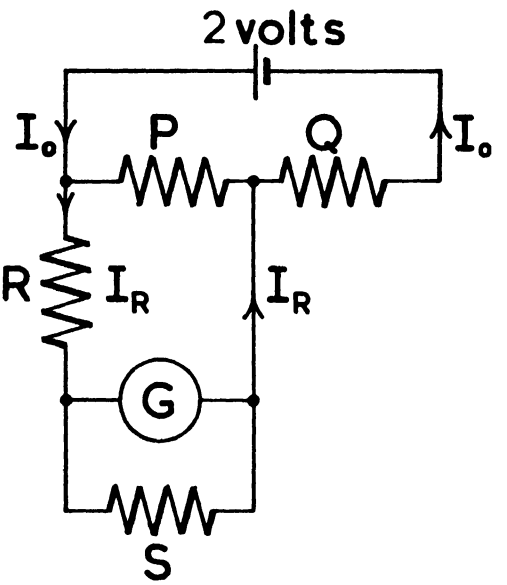


Fig. 5.32

The equivalent resistance between the ends of P is given by:

$$\frac{1}{r'} = \frac{1}{P} + \frac{1}{R+r} = \frac{R+r+P}{P(R+r)} = \frac{291 + 9 + 200}{200\,(291 + 9)} = \frac{500}{6 \times 10^4}$$

i.e. $$r' = 120 \text{ ohms.}$$

The total resistance in the circuit is hence:

$$r_0 = r' + Q = 120 + 1{,}880 = 2{,}000 \text{ ohms.}$$

The current strength in the main circuit is thus:

$$I_0 = \frac{E}{r_0} = \frac{2}{2000} = 10^{-3} \text{ amps.}$$

The current strength in R is:

$$I_R = I_0 \cdot \frac{P}{R + r + P} = 10^{-3} \times \frac{200}{500} = 4 \times 10^{-4} \text{ amps.}$$

Hence, the current strength in G is:

$$I_G = I_R \cdot \frac{S}{S+G} = 4 \times 10^{-4} \times \frac{10}{100} = 4 \times 10^{-5} \text{ amps.}$$

∴ the current strength per division, i.e. sensitivity of the instrument is:

$$2 \times 10^{-6} \text{ amps. div.}^{-1}$$

Example

Describe the structure of a moving coil galvanometer and discuss the factors which determine the sensitivity of the instrument. Explain the practical considerations which limit the sensitivity.

Explain how a moving coil instrument may be used to measure potentials of the order of 100 *volts. What would you do, using such an instrument, to estimate potentials of the order of* 10,000 *volts, assuming that you may use any materials in the laboratory to adapt the instrument for this purpose.*

A moving coil voltmeter has a resistance of 220 *ohms and when applied to a cell of negligible internal resistance it correctly records a potential difference of* 2·2 *volt. What reading will it give when connected to a cell of* 1·8 *volt e.m.f. and* 5 *ohms internal resistance? What steps could be taken to make the readings of the instrument more nearly equal to the true values?* (London G.C.E. Schol.).

(i) The moving coil galvanometer is described in the text. Sensitivity is limited practically by the fineness of the suspension (small τ) and the size of coil and its corresponding mass (large BAN).

(ii) For the measurement of p.d.'s of 100 volts a series resistance is required, as discussed in the text. For p.d.'s of 10,000 volts an extremely large series resistance would be needed if a simple voltmeter arrangement were used. Such an arrangement would be undesirable because of the danger of insulation breakdown across the resistor and subsequent damage to the meter. A better plan is to use the voltmeter to measure a known fraction of the high voltage. This is achieved by using a chain of known resistors across the supply, the resistors being chosen so that they can dissipate the power involved without undue heating, e.g. if 100 resistors of 1,000 ohms each are used to form the chain, the power dissipated in each is 10 watts and the p.d. across each member is 100 volts. The p.d. across one member of the chain is measured and hence the total p.d. estimated. All the resistors of the chain should be immersed in oil to prevent sparking.

(iii) In the problem, the voltmeter records the correct p.d. across the cell of negligible resistance, i.e. it records the e.m.f. of the cell. The current through the meter is:

$$I = \frac{E}{R_V} = \frac{2{\cdot}2}{220} = 0{\cdot}01 \text{ amps.}$$

When the voltmeter is connected to the second cell the current strength is:

$$I' = \frac{E'}{R_V + r} = \frac{1{\cdot}8}{220 + 5} = 0{\cdot}008 \text{ amps.}$$

Since a current of 0·01 amps. corresponds to a reading of 2·2 volts the current of 0·008 amps. corresponds to a reading of:

$$V = \frac{2{\cdot}2}{0{\cdot}01} \times 0{\cdot}008 = 1{\cdot}76 \text{ volts}$$

For more correct readings a higher series resistance is required. The deflection is thereby reduced and recalibration is necessary.

5.28. Power Measurement

(i) *Direct Method.* The measurement of power between any two points may be carried out with a voltmeter measuring p.d. and an

ammeter measuring current strength. Fig. 5.33 shows the connections for a " load " such as a lamp, fire, etc., represented by its resistance R. The Power P is given by: $P = IV$ watts. The resistance of the ammeter must be small compared with R and the resistance of the voltmeter must be large compared with R.

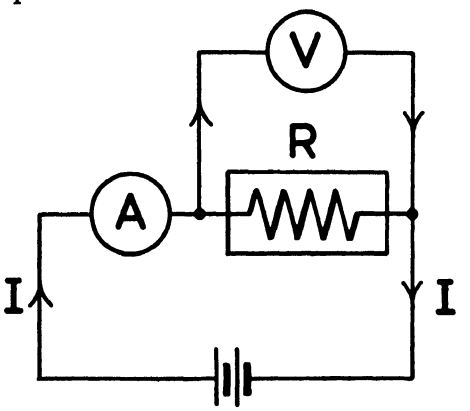


Fig. 5.33

(ii) *Dynamometer Wattmeter.* A dynamometer wattmeter is sketched in Fig. 5.34. A small fraction of the current flowing in the load R, say I_V amps. passes through the two " voltage " coils A and B. This is achieved by using a high series resistance R_V. Almost the whole

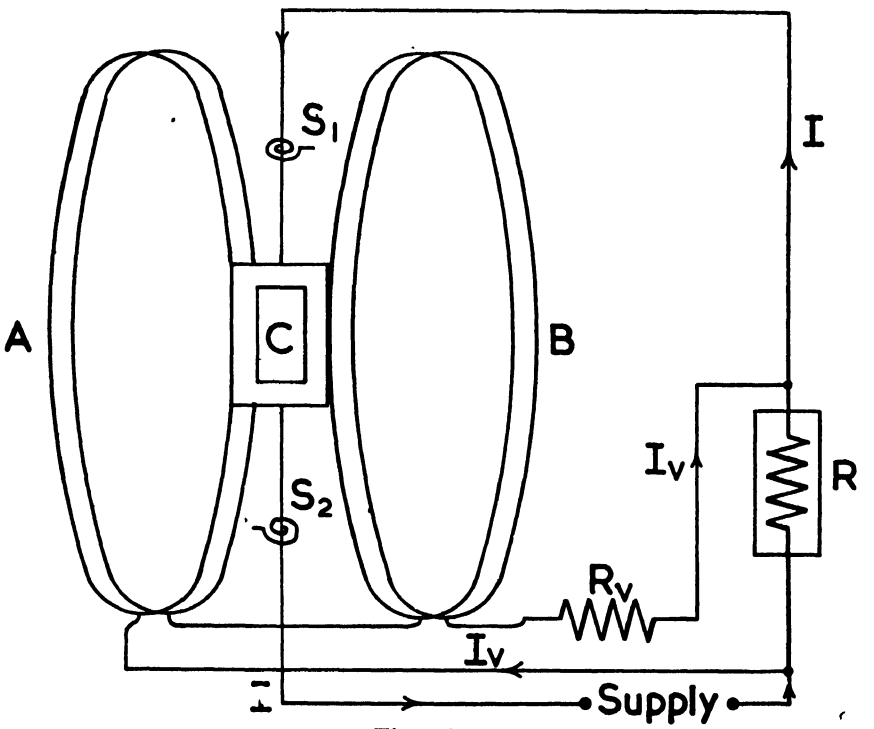


Fig. 5.34

of the load current I amps. passes through the current coil C, which is suspended midway between A and B, and connected in series with R.

Coils A and B provide a uniform magnetic field and C behaves exactly like the suspended coil of a moving coil galvanometer when current flows, i.e. it is deflected. The deflection is controlled by the hair springs S_1 and S_2. The deflection is proportional to the product of the magnetic induction B and the current strength I, (p. 135). Since $B \propto I_V$, it follows that the deflection is proportional to II_V, i.e. deflection $\propto IV \propto P$.

If R_V is removed and A, B and C are connected in series, the same current flows in each coil and the deflection is proportional to I^2. The instrument thus becomes a dynamometer ammeter. The ammeter can be adapted in the usual manner for use as a voltmeter. This type of ammeter and voltmeter is most suitable for A.C. measurements since the deflection is independent of the direction of the current flow.

EXERCISES

1. Define the *absolute electromagnetic unit of current*, and obtain from first principles an expression for the magnetic field at distance R m. from a long straight wire carrying a current of I amperes.

   Describe how you would attempt to verify experimentally that the intensity of the magnetic field for a given value of I is proportional to $1/R$. (Oxford Local G.C.E.).

2. State Laplace's Rule giving the magnetic field at a point at a distance r from a small element ds of a wire carrying a current i, the element being inclined to a line drawn from it to the point at an angle θ. Apply this rule to calculate the field at the centre of a circular coil of n turns and radius r when a current i flows through it. Thence define the electromagnetic unit of current.

   How would you demonstrate experimentally that the magnetic field at a point due to long straight current conductor varies inversely as the distance from the conductor? (Durham G.C.E. Adv. L.).

3. Give the reasons for the various adjustments and readings which are desirable when setting up and using a tangent galvanometer for the measurement of current.

   How would you find (*a*) the resistance, (*b*) the sensitivity of the tangent galvanometer when the deflection is 45°? (London G.C.E. Adv. L.).

4. Two plane coils, each of "n" turns and radius "r", are placed so that the centre of each coil is on the axis of the other. If each coil carries the same current, show that the greatest uniformity of field near the mid-point between the coils is obtained if the distance between the coils is equal to "r".

An absent-minded professor connects the coils so that the currents in them flow in the opposite sense. Draw a rough sketch showing how the field on the axis varies with distance and examine quantitatively how it varies near the mid-point between the coils. (Oxford Open Schol.).

5. Derive an expression for the magnetic field strength on the axis and near the centre of a long solenoid when a current passes through it.

A current of 1·5 amp. is passed through a solenoid containing 900 turns wound on a cylinder 100 cm. long and about 1 cm. in diameter. Calculate the magnetic field (*a*) at the mid-point of the solenoid and (*b*) at the mid-point of one end of the solenoid. (London G.C.E. Schol.).

6. Describe a moving coil galvanometer. Explain how it is possible to use one and the same instrument as an ammeter and as a voltmeter. (Oxford Open Schol. Subsid.).

7. Give a labelled diagram of a sensitive moving coil galvanometer and enumerate the factors which determine its current sensitivity.

An unknown high resistance, a galvanometer of resistance 100 ohms and a cell of negligible resistance are connected in series. The galvanometer deflection is noted. When the galvanometer is shunted with a reistance of 4 ohms and the high resistance is replaced by a resistance of 5,000 ohms the deflection is nine-tenths of the former value. Calculate the high resistance. (Durham G.C.E. Schol.).

8. Describe with the aid of diagrams the structure and principle of action of a moving coil galvanometer. Explain what is meant by its current sensitivity and state how high sensitivity is obtained in practice.

The following table gives the readings on the scale of a moving coil milliammeter when it is joined in series with a variable resistance R and a source of negligible internal resistance and of steady e.m.f. Use the results to deduce the resistance of the milliammeter.

R (ohms)	30	40	60	80	150	200
Meter Reading (scale div.)	95	77	56	43	24	18

(London Inter. B.Sc.).

9. Describe a moving-coil galvanometer, and explain its action. Discuss, in a general way, the factors which affect its sensitivity. (The mathematical theory of the instrument is not required).

A flat circular coil consisting of a single turn of radius 10 cm. carries a current of 5·00 amp. When it is set with its plane in the magnetic meridian, a small magnetic needle suspended at its centre is deflected through 57½°. When the coil is set with its plane perpendicular to the meridian the needle makes 50 oscillations in 25·0 sec. Calculate the possible periods of oscillation when no current is passing through the coil. (Camb. Local G.C.E.).

10. Explain the principle of the moving-coil galvanometer, and describe how you would calibrate the instrument.

Discuss the possible effects of temperature change on the accuracy of its readings. (Oxford Local G.C.E.).

11. Describe an experiment which demonstrates that a mechanical force is exerted on a current-carrying conductor suitably placed in a magnetic field.

Derive an expression for the deflexion of the coil in a moving-coil galvanometer supposing that the coil is rectangular in shape and that the magnetic field in which it moves is radial.

A galvanometer, whose sensitivity is 0·20 divisions per microampere, gives a full scale deflection of 100 divisions when connected in series with an accumulator (e.m.f. = 2·2 volts) and a 4,300 ohm resistance. How can it be adapted to give a full scale deflexion for a current of 1·0 milliamperes? (Northern G.C.E. Adv. I.)

12. Draw labelled diagrams showing the essential parts of a moving coil galvanometer. Indicate the couples acting on the coil and the direction of the field and of the current when the coil is in equilibrium in the field.

What is the effect and purpose of winding the coil (*a*) on a metal frame, (*b*) on an insulating frame? (Northern H.S.C.).

13. Give a brief description of moving-coil and moving-magnet galvanometers indicating how the deflecting forces and controls differ in the two types.

Derive a relation between the steady current passing through a moving-coil galvanometer and the deflection produced. How would the period of this instrument on open-circuit be affected by increasing (*a*) the number of turns in the suspended coil, (*b*) the thickness of the suspension, and (*c*) the strength of the magnetic field in which the coil is suspended? (London G.C.E. Adv. L.).

14. Describe the structure and explain the action of a moving coil galvanometer. Obtain an expression for its steady current sensitivity.

A rectangular coil of 100 turns of mean area 4 cm.^{2} is hung with its plane vertical, in a horizontal uniform magnetic field of 4×10^4 amp.m.$^{-1}$. The plane of the coil is parallel to the field when there is no twist in the suspension. The coil is deflected through 60° when a steady current of 1 milliampere passes through it. Calculate the torsion couple exerted by the suspension per radian twist. (London G.C.E. Adv. L.).

15. Describe with diagram the magnetic field associated with a current flowing in a long straight conductor. Mark clearly the directions of the current and the magnetic field.

Draw a second diagram showing the field associated with two parallel conductors each conveying a steady current. Indicate the direction of the mechanical force which one of them exerts on the other.

Describe an instrument in which this mutual effect between two conductors is utilised to measure *either* current *or* power. (London G.C.E. Adv. L.).

16. Draw a rough diagram of the magnetic field in the vicinity of a circular coil of wire carrying a steady current.

Describe how you would investigate the dependence of the field at the centre of the coil on (*a*) the current, (*b*) the number of turns, and (*c*) the radius of the coil. Write down a formula for the intensity of the field at the centre in terms of these quantities, and explain the derivation of any constant that occurs.

A circular coil of 30 cm. diameter and of 100 turns is fixed in a vertical plane and carries a current of 15 amps. At its centre is hung, in the same vertical plane, a small square coil of 5 mm. sides and of 160 turns. What forces act on this coil when a current of 1 mA. is passed round it? (O. & C.J.B. G.C.E. Adv. L.).

17. Describe with full experimental details an experiment to test the validity of Ohm's law for a metallic conductor.

An accumulator of e.m.f. 2 volts and of negligible resistance is joined in series with a resistance of 500 ohms and an unknown resistance X ohms. The readings of a voltmeter connected successively across the 500 ohm resistance and X are 2/7 and 8/7 volts respectively. Comment on this and calculate the value of X and the resistance of the voltmeter. (Northern G.C.E. Schol.).

18. Describe a moving coil galvanometer.

Compare the sensitivity (*a*) for current and (*b*) for voltage of two galvanometers, one of which has a coil and former "n" times smaller in all linear dimensions than the other. The number of turns in the coil and the magnetic field are the same in both cases and the torsional constant of the suspension is adjusted so that the time of swing is also the same. (Oxford Open Schol.).

19. Calculate the force on an element of a current-carrying wire with its length normal to a uniform magnetic field. You may assume that the magnetic field at a point P due to a small element δs of the wire is $i.\delta s.\sin\theta/(4\pi r^2)$ where θ is the angle between the wire and the line joining P to the element, i is the current in the wire and r is the distance of P from the element. Apply your result to calculate the electromagnetic torque acting on the coil of a moving-coil galvanometer when a current is passing through it, and discuss the factors affecting the sensitivity of such an instrument.

Derive an expression for the tension T in a rigid circular loop of wire of radius R carrying a steady current i when it is situated in a magnetic field H normal to the plane of the loop. (London G.C.E. Schol.).

20. A moving coil galvanometer to give a deflection of about 5 degrees per microampere is to be made from the following materials:

(*a*) a permanent magnet giving a field of 2×10^5 amps. per metre in a gap about 2 cm. cube;

(*b*) fine copper wire, 0·1 mm. diameter and density 9 gm.cm.$^{-3}$;

(*c*) phosphor bronze wire (rigidity modulus 5×10^{10} newton.m.$^{-2}$) which is available in a variety of diameters from 0·5 mm. down to 0·02 mm.

Carry out " rough " calculations to arrive at a possible design, and estimate the period of swing on open circuit.

(The couple "C" necessary to twist a wire of radius "a", length "l" and rigidity modulus "k" through an angle θ is given by $C = (\pi k a^4/2l)\theta$). (Camb. Open Schol.).

21. An electric D.C. source is capable of providing 1,000 volts and a maximum current of 1 milliamp. You are provided with a voltmeter reading a maximum voltage of 100 volts and internal resistance 100,000 ohms. Given any resistances you require, show how you would measure this 1,000 volts accurately, stating clearly any corrections you would apply. (Manch. Open Schol.).

22. Two thin coils each of n turns of wire in the form of a square of side $2b$ m. are set up in parallel planes $2b$ m. apart, having a common axis. Each carries a current "i" amps in the same sense Find the magnetic field half-way between the coils on their common axis. A similar coil of "m" turns, the lengths of whose sides are $\frac{1}{100}$ of those of the large coils, and which carries a current "j" amps, is placed at the half-way point on the axis of the large coils and parallel to them. Calculate the force on each of its sides. (Oxford Open Schol.).

23. Find an expression for the torque on a plane coil of effective area A carrying a current i in a magnetic field H whose direction makes an angle θ with the plane of the coil.

Two plane circular coils of effective area A having a common vertical axis are rigidly fixed to one another so that their planes are perpendicular, and they are free to rotate about their common vertical diameter. The system is placed in a uniform horizontal magnetic field, H and the current i_1 is maintained through one coil. Find the angle through which the system turns when a current i_2 is passed through the other coil. (Oxford Open Schol.).

CHAPTER 6

MEASUREMENT OF RESISTANCE, KIRCHHOFF'S LAWS POTENTIOMETER

6.1. Measurement of Resistance

The simplest method of determining a resistance is to connect it in series with an ammeter A and a suitable source of e.m.f. A voltmeter V is connected across the resistance R (Fig. 6.1). If the readings of A and V are I amps. and V volts, respectively:

$$I = \frac{V}{R}$$

so that:

$$R = \frac{V}{I} \text{ ohms.}$$

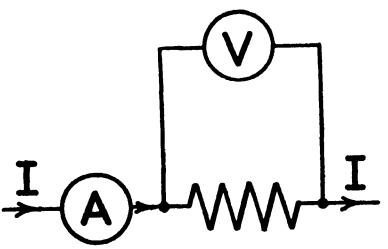


Fig. 6.1

Accuracy is limited because a fraction of I flows through V. This current is negligible if $R_V \gg R$. Some rough idea of the value of R is thus desirable. The method is suitable for approximate values, providing meters with suitable ranges are available.

6.2. Wheatstone Bridge

For accurate measurement in the range 1 to 10^4 ohms. a circuit devised by Wheatstone is used. It is a " null-method " of measurement, since the criterion used is that no current flows through a galvanometer. The Wheatstone network consists of four arms, AB, BC, AD and CD, containing resistances R_1, R_2, R_3 and R_4, ohms respectively, (Fig. 6.2). R_3 is a standard resistance of value similar to the unknown R_4. Current is supplied to the network by a steady source of e.m.f. such as an accumulator. A moving-coil galvanometer G is connected between B and D.

Suppose currents I_1, I_2, I_3 and I_4 amps. flow as shown. If no current

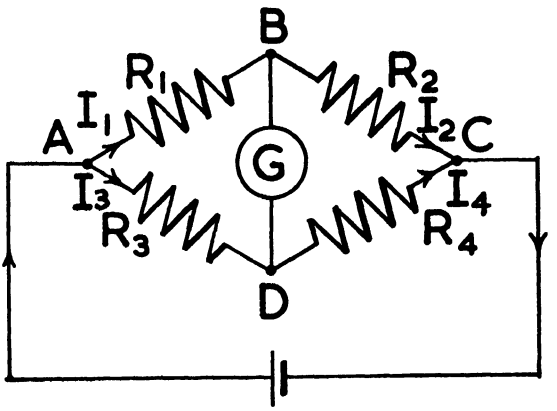


Fig. 6.2

flows through G the potentials of B and D are the same and p.d. along AB = p.d. along AD, i.e.:

$$I_1R_1 = I_3R_3 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly:

$$I_2R_2 = I_4R_4 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Furthermore, $I_1 = I_2$, and: $I_3 = I_4$, so that (2) becomes:

$$I_1R_2 = I_3R_4 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Dividing (1) by (3):

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

and hence:

$$R_4 = R_3 \cdot \frac{R_2}{R_1} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus R_4 may be found, if R_3 is known, by determining the ratio R_2/R_1.

In the practical form of the circuit, called a *Wheatstone Bridge*, R_1 and R_2 are parts of a long uniform wire AC, (Fig. 6.3). Since the wire has a constant diameter its resistance per unit length is constant. The sliding contact B moves along AC, AB and BC corresponding to R_1 and R_2 respectively, ($R_1 \propto AB$ and $R_2 \propto BC$). The galvanometer G is connected between B and the mid-point D of a copper strip of negligible resistance. Connected to the same copper strip are adjacent ends of R_3 and R_4. Strips of copper link the opposite ends of R_3 and R_4 to A and C as shown. A suitable cell is connected between A and C.

To measure R_4, B is adjusted until no current flows through G. Before this " balance-point " is reached a shunt is placed across G to prevent heavy currents damaging the coil. At balance:

$$R_4 = R_3 \cdot \frac{R_2}{R_1} = R_3 \cdot \frac{l_2}{l_1} \quad . \quad . \quad . \quad (5)$$

where l_2 and l_1 are the lengths of AB and BC respectively.

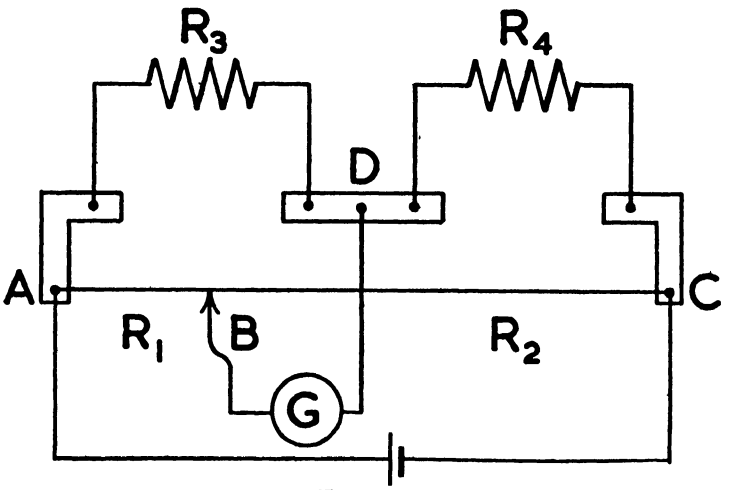


Fig. 6.3

Accuracy of Measurement

The accuracy depends on the measurement of l_1 and l_2, and the certainty of the balance point. The latter depends on (i) the sensitivity of the galvanometer and (ii) the width of the contact B. The distance moved on either side of a balance point before the galvanometer detects a current is a minimum when its resistance is approximately the same as R_1, R_2, R_3 and R_4, i.e. the galvanometer is most sensitive when all the resistances are equal. By using a knife-edge type of contact (ii) becomes unimportant.

A metre rule fixed beneath AC is used to measure l_1 and l_2 to the nearest 0·5 mm., so that if the contact has a negligible width the balance point is estimated to 0·5 mm. Providing l_1 and l_2 are at least 10 cm. the inaccuracy in their measurement is not greater than $0{\cdot}05/10 \times 100$ per cent, i.e. 0·5 per cent. The ratio l_1/l_2 has an inaccuracy not greater than 1·0 per cent, since the separate percentage errors are added on dividing. An uncertainty in balance caused by galvanometer insensitivity of 2 mm., which is probable, is thus higher than the uncertainty in measuring l_1 and l_2.

To minimise the errors, R_3 is chosen to be about the same value as R_4 and the balance point is near the centre of the wire. The error in the balance point is then least, say 1 mm. in 50 cm., i.e. 0·5 per cent. To minimise heating effects, all resistances are made of manganin wire. In laboratory bridges heating is avoided by taking quick readings.

6.3. Post Office Box

This is a convenient form of the Wheatstone network. It consists of a compact series of resistances arranged so that different values may be selected to act as three arms of a Wheatstone net. In Fig. 6.4 (*a*), the terminals A, B, C and D, correspond to the junctions of the Wheatstone net in Fig. 6.2. The resistances R_1, R_2 and R_3, consist of coils

of wire wound non-inductively, (p. 274), with their ends soldered to heavy brass separating pieces of negligible resistance, (Fig. 6.4(*b*)).

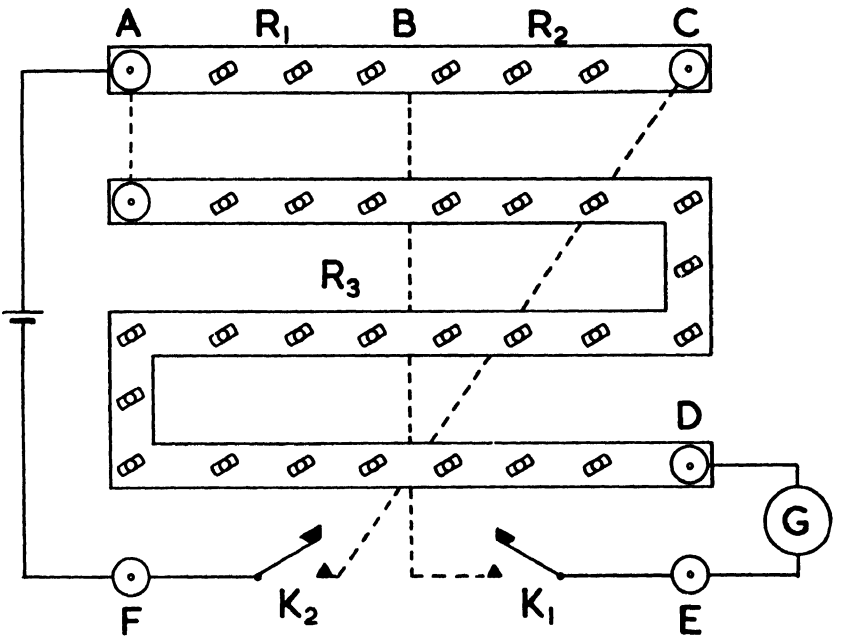


Fig. 6.4 (*a*)

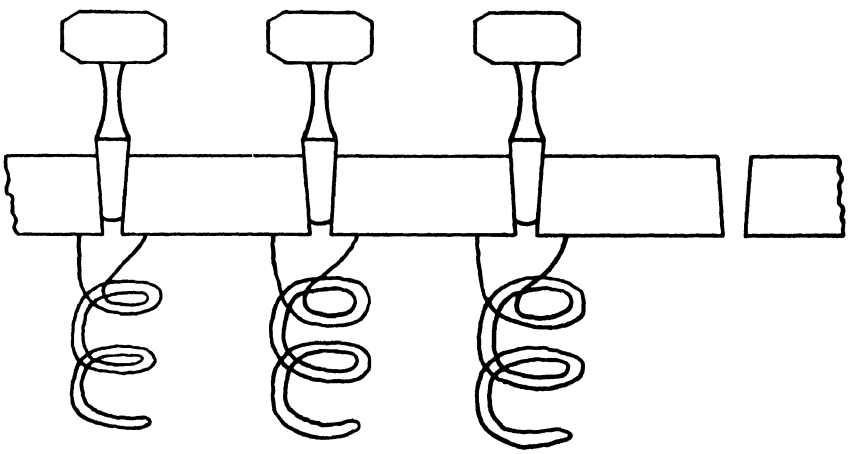


Fig. 6.4 (*b*)

Each coil resistance is an integral number of ohms. To alter the resistance of an arm brass " shorting " plugs are removed or inserted (Fig. 6.4 (*b*)). The arms AB and BC each consist of three coils of value 10, 100 and 1,000 ohms, to give ratios R_2/R_1 of 100: 1, 10: 1, 1: 1, or 1: 10, and 1: 100. For this reason AB and BC are called the *Ratio Arms*. The arm AD consists of a large number of separate coils to give an integral value of ohms between 1 and 10,000 ohms for R_3.

The point B is usually connected internally to a convenient terminal E via a tapping key K_1. Similarly, C is connected internally, via the tapping key K_2, to F. The galvanometer G, linking B and D, is connected to D and E, the unknown R_4 is connected between C and D, whilst the cell supplying the bridge is connected between A and F. In determining R_4 the ratio arms are made equal, their values being nearest to the surmised value of R_4. Key K_2 is then depressed before K_1, so that G does not record the transient current that flows on connecting the battery into the circuit. This current is caused by circuit inductance and if detected (K_1 depressed before K_2), gives rise to a " kick " on the galvanometer (p. 274). R_3 is varied until G shows no deflection, when: $R_3 = R_4$. To prevent too much current flowing through G when the bridge is not balanced it is shunted with a variable low resistance. The shunt is increased as the balance point is approached and finally removed.

Using equal ratio arms an exact balance is not possible, unless the unknown is an integral number of ohms since the smallest variation in R_3 is one ohm. Hence the value of R is found to within one ohm of its correct value. This represents an inaccuracy in the determination of a resistance of 100 ohms of about 1 per cent. For smaller resistances the percentage inaccuracy is greater. For small resistances, therefore, R_2:R_1 is made 1:100, so that for the nearest balance point: $R_4 = (1/100) \times R_3$. This allows the determination of R_4 to the second decimal place, to within one unit. However, the galvanometer is not very sensitive with this arrangement and the uncertainty in the balance point makes the percentage inaccuracy much higher.

6.4. Determination of Specific Resistance

The *specific resistance*, (resistivity) of a given metal, is found by determining the resistance of a long length of uniform wire using a Wheatstone Bridge or P.O. Box. If the length of the wire is l m. and the diameter is d m., the specific resistance of the metal is given by:

$$\rho = \frac{\pi d^2}{4} \cdot \frac{R}{l} \text{ ohm.m.} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The inaccuracy involved in this experiment lies in the measurement of R and d, since the resistance of a metre of wire of 1 mm. diameter is of the order of one ohm only.

6.5. Measurement of Low Resistances

For resistances much less than one ohm the simple Wheatstone Bridge and the P.O. Box are unsuitable. Instead, the circuit of Fig. 6.5 is used. A steady current I amps. passes through the unknown resistance R and a standard resistance S. The value of S is similar to the estimated value of R. A galvanometer G is first connected across

R and then connected across S, the deflections θ_1 and θ_2, being noted.

When G is connected across R the current flowing through it is given by Ohm's Law:

$$I_1 = I \cdot \frac{R}{R+G} = \lambda\theta_1 \quad . \quad . \quad . \quad . \quad (7)$$

where λ is the current sensitivity of the galvanometer and G its resistance. Similarly, the current strength in G when it is connected across S is:

$$I_2 = I \cdot \frac{S}{S+G} = \lambda\theta_2 \quad . \quad . \quad . \quad . \quad (8)$$

Dividing (7) by (8):

$$\frac{R}{S} \cdot \frac{S+G}{R+G} = \frac{\theta_1}{\theta_2} \quad . \quad . \quad . \quad . \quad (9)$$

Since G is much greater than R or S, this is simplified to:

$$\underline{\underline{\frac{R}{S} = \frac{\theta_1}{\theta_2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

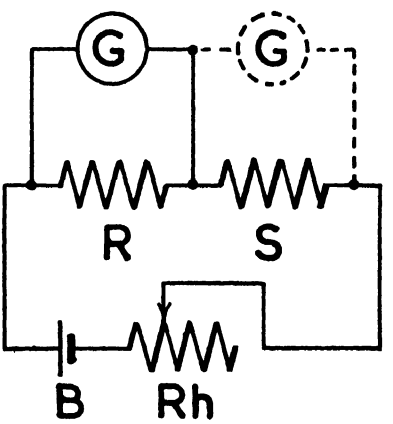


Fig. 6.5

Suppose $R = 0{\cdot}03$ ohms and $S = 0{\cdot}01$ ohm. If B is a 2 volt accumulator, the current flowing through R and S when the value of Rh is zero is: $2/(0{\cdot}04)$, i.e. 50 amps. This current is too heavy for the accumulator and causes serious heating effects. If $\lambda = 10^{-6}$ amp.cm.$^{-1}$, $\theta_2 = 10$ cm. (for reading accuracy) and $G = 100$ ohms ($>>S$):

$$\frac{IS}{G} = 10{\cdot}10^{-6} = 10^{-5} \text{ (from (8))} \quad . \quad . \quad (11)$$

$$\therefore \quad I = 10^{-5}\frac{G}{S} = 10^{-1} \text{ amps.}$$

Since the total resistance of the circuit is roughly that of Rh, the latter

must thus be about 20 ohms. A trial experiment is necessary to obtain optimum values for θ_1 and θ_2.

6.6. Measurement of High Resistance

A high resistance, i.e. greater than 10^4 ohms, may be measured in several ways. One method is to connect the resistance (R) in series with

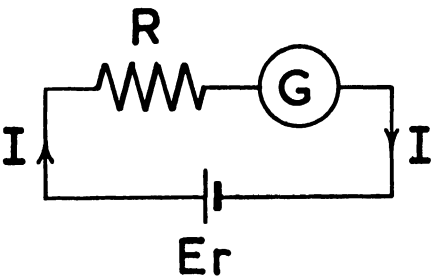


Fig. 6.6

a cell of known e.m.f. (E) and a galvanometer of known sensitivity (λ) as in Fig. 6.6. The current passing through G is given by:

$$I = \frac{E}{R + G + r} = \lambda\theta$$

where G is the galvanometer resistance and r is the internal resistance of the cell. Assuming $R \gg G \gg r$:

$$\lambda\theta = \frac{E}{R}$$

i.e.
$$R = \frac{E}{\lambda\theta} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Hence by measuring θ and knowing E and λ, R may be found.

A common method of measuring resistance greater than 10^6 ohms uses condenser discharge theory (Chapter 9).

6.7. Measurement of Resistance using Multipurpose Meter

Multipurpose meters often have provision for measuring resistances. A basic circuit is illustrated in Fig. 6.7. A dry battery is connected via a switch S, through a milliameter mA and a rheostat Rh, to the terminals T_1 and T_2. When the terminals are short-circuited by connecting a thick wire between them, and S is closed, a current flows through mA of strength:

$$I_o = \frac{E}{Rh + r + R_m} = \frac{E}{R_o} \text{ amps} \quad . \quad . \quad (13)$$

where $R_o = Rh + r + R_m$, E is the e.m.f. of the battery and R_m the resistance of the milliameter. This current should give a full scale

reading on the meter, but if the dry battery has deteriorated this will not be so and *Rh* is adjusted.

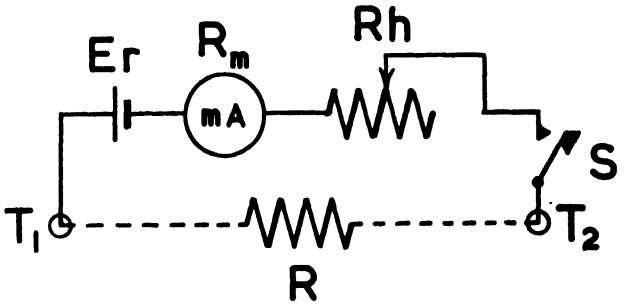


Fig. 6.7

When the resistance to be measured (R) is connected between T_1 and T_2 and S closed, the new current strength is given by:

$$I = \frac{E}{R_o + R} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Since E and R_o are constant, I can be used as a measure of R, the scale of the meter being calibrated in ohms. From (14) the scale is not linear, and for large R the scale is very compressed. The limit of R which can be measured is about 10^5 ohms.

Megohmmeters use a similar principle for high resistances up to about 10^7 ohms. A large e.m.f., supplied by a hand-driven generator, is used.

6.8. Resistance of a Galvanometer

(i) *Direct Method.* A galvanometer resistance is much higher than an ammeter resistance since its coil consists of a large number of turns of fine wire. One method of measurement is to clamp the coil and use it as R_4 in a Wheatstone Bridge or P.O Box. Care must be taken to prevent too much current flowing through it and causing damage.

(ii) *From sensitivity determination*—this method is given in Chapter 5, (5.24).

(iii) *Kelvin's Method.* The circuit is illustrated in Fig. 6.8. It consists of a Wheatstone network in which the galvanometer of resistance G replaces R_4, and the detector is replaced by a tapping key K_1. To find G the resistances R_1 and R_2 are equal and similar to G, say 100 ohms each. The variable resistance *Rh* is adjusted for a suitable deflection of G when K_2 is closed; R_3 is then adjusted until on depressing K_1 there is no change in the steady deflection of G. The p.d. between B and D is then zero, otherwise current flows through K_1 causing a change in the deflection of E. Hence:

$$\frac{R_1}{R_2} = \frac{R_3}{G} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and since $R_1 = R_2$; $R_3 = G$.

Difficulty is experienced in arranging a suitable current flow through *G*. If *G* has a sensitivity of microamps. per cm., 10^{-5} amps. only is required to give a suitable steady deflection. This means that *Rh* is about 10^5 ohms. Small changes in R_3 will not then give appreciable changes in deflection and accuracy is rather low.

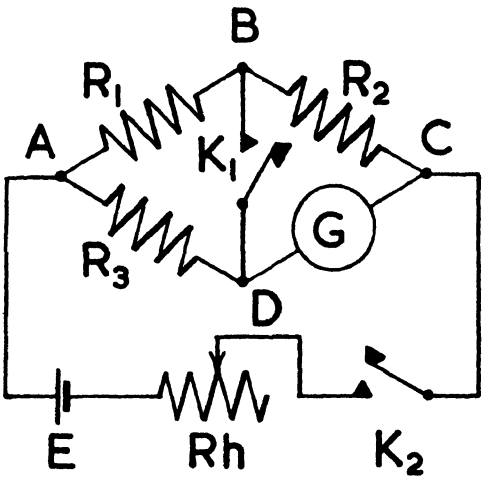


Fig. 6.8

6.9. Resistance of a Battery

(i) *Mance's Method.* The "internal" resistance of a cell may be very low, as with accumulators, or a few ohms as in the Daniell Cell. Mance's method uses the circuit of Fig. 6.9. This is a Wheatstone

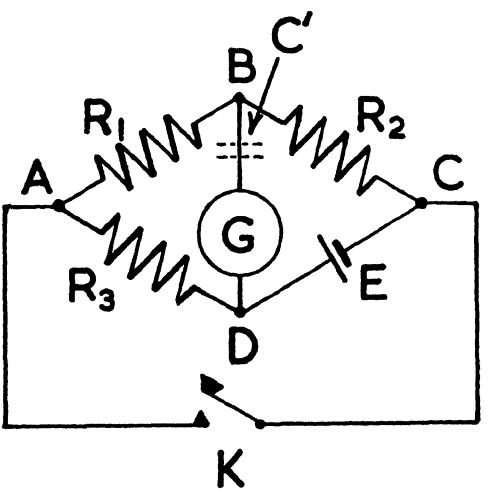


Fig. 6.9

network in which R_4 is replaced by the cell. The ordinary supply battery is replaced by the tapping key *K*; R_1 and R_2 are equal and R_3 variable. When *K* is closed a steady current flows through the galvanometer *G*. R_3 is varied until no *change* is observed in this deflection.

Points A and C are then at the same potential and no current flows through K, otherwise when K is depressed the current in K would affect the current in G. Here G is used as a detector of *change in current.*

The observation of small changes in G is difficult. To avoid its steady deflection, G is connected in series with a condenser of capacity about one microfarad (C' in Fig. 6.9). The condenser does not pass direct current but changes of p.d. across it cause current changes in G. The method is inaccurate because of the insensitivity of the galvanometer in such a circuit.

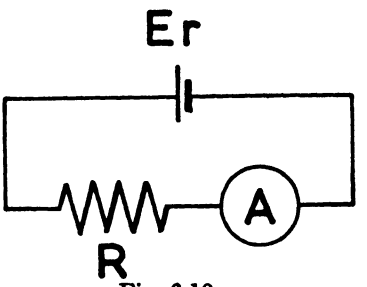


Fig. 6.10

(ii) *Variation of Current Strength with External Resistance.* This method uses the circuit of Fig. 6.10. The current I amps. is measured with an ammeter A, when the cell of e.m.f. E volts and internal resistance r ohms and the known resistance R are connected in series. From Ohm's Law:

$$I = \frac{E}{R + r} \text{ amps.}$$

whence:
$$R = \frac{E}{I} - r \text{ ohms} \quad . \quad . \quad . \quad . \quad (16)$$

Hence, various values of R are used and plotted against $1/I$, (Fig. 6.11); the cell resistance is given as an intercept on the R axis.

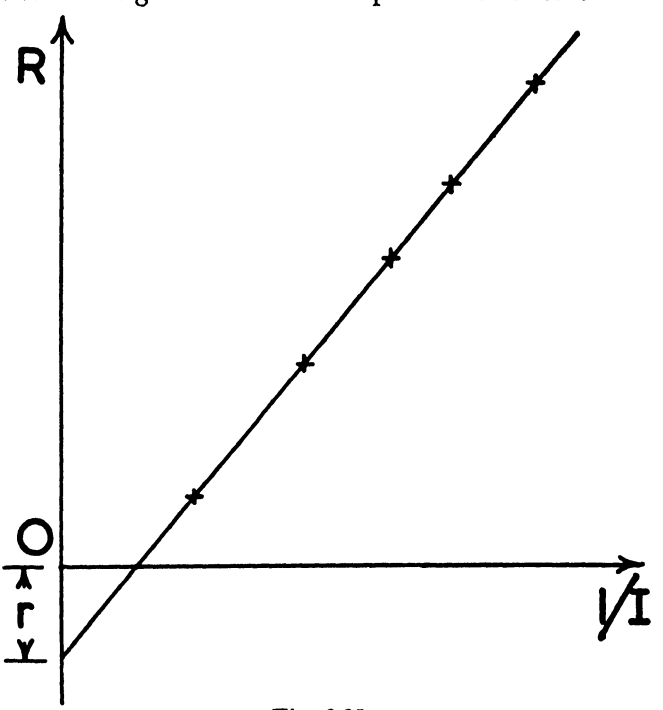


Fig. 6.11

(iii) *Potentiometer Method.* Internal resistance is best determined by using a potentiometer circuit (p. 172).

6.10. Comparison of Resistances

(i) *Carey-Foster Bridge.* This is a modification of the Wheatstone Bridge (Fig. 6.12) the modification being the addition of resistances P and Q to the arms R_1 and R_2 (cf. Fig. 6.3). The latter are formed by the uniform wire ST, which is 100 cm. long with a resistance of ρ ohm.m.$^{-1}$, say.

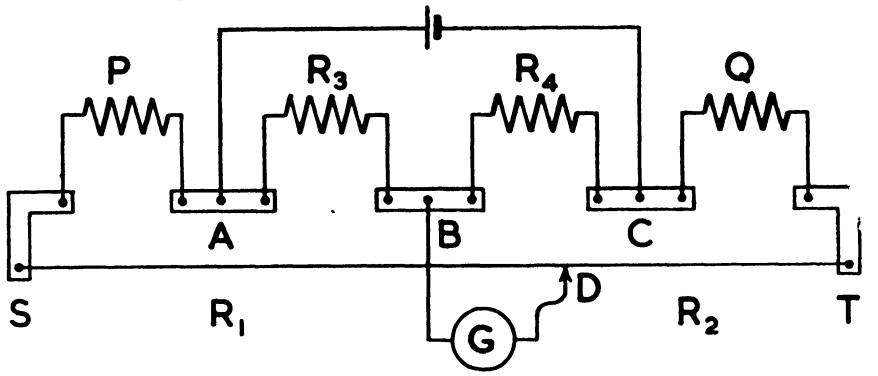


Fig. 6.12

The circuit was originally used to determine the ratio of the "*International Ohm*" to the "*Absolute Ohm*". The International Ohm was somewhat arbitrarily defined in 1861 as the resistance of a column of mercury of definite length (106·3 cm.) and mass (14·45 gm.) at 0°C. This was used as a standard as near one ohm as possible with methods of measurement then available. There is a significant difference between it and the modern standard: the absolute ohm. Each nominal one ohm is connected as P or Q in Fig. 6.12. Resistances R_3 and R_4 are similar to P and Q, but not accurately known. Using approximately equal resistances P, Q, R_3 and R_4, gives optimum accuracy.

A balance point is first found with P and Q connected as shown. this is x_1 m. from B, the condition for balance is:

$$\frac{P + x_1\rho + r_s}{Q + (100 - x_1)\rho + r_t} = \frac{R_3}{R_4} \quad . \quad . \quad . \quad (17)$$

where r_s and r_t represent contact resistances created at the ends of S

Adding unity to both sides of (17) and inverting:

$$\frac{Q + (100 - x_1)\rho + r_t}{P + Q + 100\rho + r_s + r_t} = \frac{R_4}{R_3 + R_4} \quad . \quad . \quad (18)$$

P and Q are now interchanged and a new balance point is found at x_2 m. say, from S.

The standard cell is a specially prepared cell of accurately known e.m.f., say E_o volts (Chapter 8). If the standard cell gives rise to a balance point l_o m. from A, it follows from (37) that:

$$E_o = k \,.\, l_o \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Hence:
$$\frac{E_1}{E_o} = \frac{l_1}{l_o}$$

l:
$$E_1 = \frac{l_1 \,.\, E_o}{l_o} \quad . \quad . \quad . \quad . \quad . \quad (39)$$

rom (38): $k = E_o/l_o$ volts.m.$^{-1}$, hence, if any other p.d. gives a ınce point l_2 m., say, from A, that p.d. is given by:

$$\frac{l_2 \,.\, E_o}{l_o} \text{ volts.}$$

No current flows through the cell when a balance point is found so that the instrument measures e.m.f.'s accurately. Moving-coil voltmeters cannot measure e.m.f.'s accurately since they take current. Electrostatic voltmeters do not pass current but they are insensitive for small p.d.'s. For these reasons a potentiometer is widely used.

6.15. Applications of the Potentiometer

(1) *Comparison of e.m.f.'s.* For the comparison of the e.m.f.'s of any two cells it is not necessary to calibrate the potentiometer. A balance point is found for each cell, as in Fig. 6.18. If the e.m.f.'s are E_1 and E_2 and the balance points are l_1 and l_2 m. from A:

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

ractical Hints

The pole of the cell connected to A must be the same as that of the ıttery E connected to A. As a check C is first moved to A and the rection of the galvanometer deflection noted. The contact is then ved to B and the deflection noted. If the two deflections are in posite directions the connections at A are correct. If the deflections e in the same direction the connections are wrong and the e.m.f. being easured must be reversed. If the deflections are still in the same direcon the e.m.f. being measured is too great, i.e. the p.d. along AB due to e current from E is less than the e.m.f. being measured. In this case E increased. In all experiments excess current flow in the galvanomter avoided by using a low resistance shunt.

The accuracy of the potentiometer measurements are limited by the sensitivity of the galvanometer, the type of contact used and the ability

to read the distances measured. A measure of the accuracy is determined as indicated for the Wheatstone Bridge.

(2) *Internal Resistance of a Cell.* This is measured with the circuit of Fig. 6.19. A balance point is first found (with key K open) at a point l_o m. from A, say. Hence, if the e.m.f. of the cell is E volts:

$$E = k \,.\, l_o \quad . \quad . \quad . \quad . \quad . \quad (40)$$

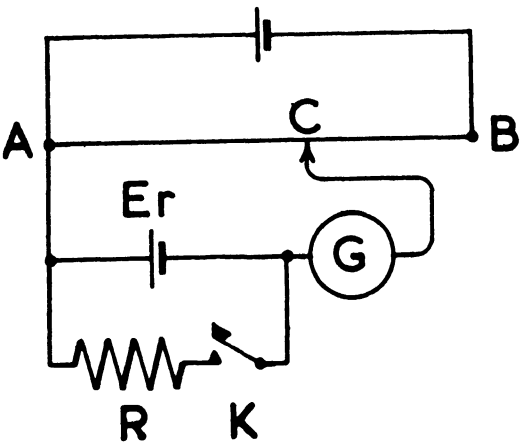


Fig. 6.19

The key is now closed and a known resistance R ohms connected across E. The p.d. across the plates is reduced by Ir volts, where r ohms is the internal resistance. A new balance point is found at l m. from A, so that:

$$E - Ir = k \,.\, l \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Substituting: $I = \dfrac{E}{R + r}$ in (41):

$$E - \frac{Er}{R + r} = k \,.\, l$$

i.e.
$$\frac{ER}{R + r} = k \,.\, \frac{l_o R}{R + r} = kl \quad . \quad . \quad (42)$$

i.e.
$$\frac{R}{R + r} = \frac{l}{l_o}$$

or:
$$r = \frac{R(l_o - l)}{l} \quad . \quad . \quad . \quad . \quad (43)$$

Rewriting (43):
$$\frac{r}{R} = \frac{l_o}{l} - 1$$

i.e.
$$\frac{1}{l} = \frac{1}{R}\left(\frac{r}{l_o}\right) + \frac{1}{l_o} \quad . \quad . \quad . \quad (44)$$

Hence, by using different values of R and plotting $1/l$ against $1/R$, the intercept on the $1/l$ axis gives $1/l_o$, whilst the gradient of the straight line graph gives r/l_o. Thus, r is obtained.

(3) *Comparison of Small Resistances.* Small resistances are compared accurately using the circuit of Fig. 6.20. The potentiometer wire AB is supplied by the battery E_1. The current strength in AB is adjusted using the variable resistor. The specimen resistances R_1 and R_2 are supplied with a steady current by battery E_2. The current strength in this circuit (I amps.), is adjusted by Rh and its value checked by the ammeter A.

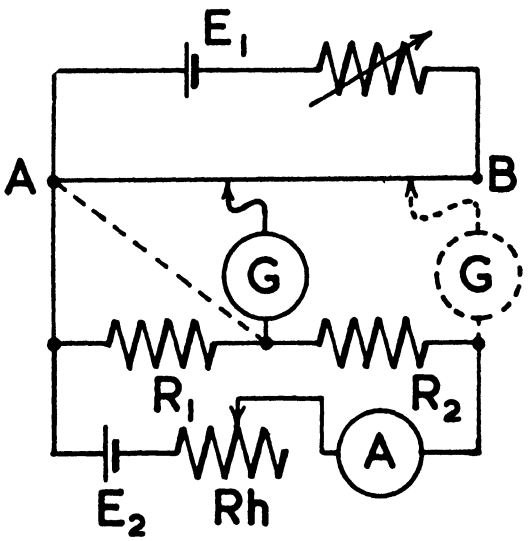


Fig. 6.20

The experiment consists of measuring the p.d.'s across each resistance. The ends of R_1 are first connected to the potentiometer as shown. Suppose a balance is obtained l_1 m. from A. The p.d. along R_1 is given by: IR_1 volts, so that:

$$IR_1 = k \,.\, l_1 \,. \quad . \quad . \quad . \quad . \quad . \quad (45)$$

The ends of R_2 are now connected to A and C, respectively, as shown by dotted lines in Fig. 6.20. A new balance point is found at l_2 m. from A, so that:

$$IR_2 = k \,.\, l_2 \,. \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Dividing (45) by (46):

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

For accuracy l_1 and l_2 must be as large as possible; the indefiniteness of the balance point and the reading error are then small by comparison. To achieve this, I is adjusted carefully so that the p.d. along R_1 or R_2 is comparable with that along AB.

(4) *Measurement of Current Strength (Calibration of Ammeter)—The Rayleigh Potentiometer.* Current strength is measured by passing the current through a standard resistance and measuring the resulting p.d. with the potentiometer. Instead of the simple potentiometer the *Rayleigh Potentiometer* may be used. In this potentiometer the wire AB of Fig. 6.18 is replaced by two resistances boxes P and Q of equal maximum value. A battery supplies the potentiometer current and the galvanometer G is connected as before. In Fig. 6.21 the p.d. across the standard resistance S ohms, is measured.

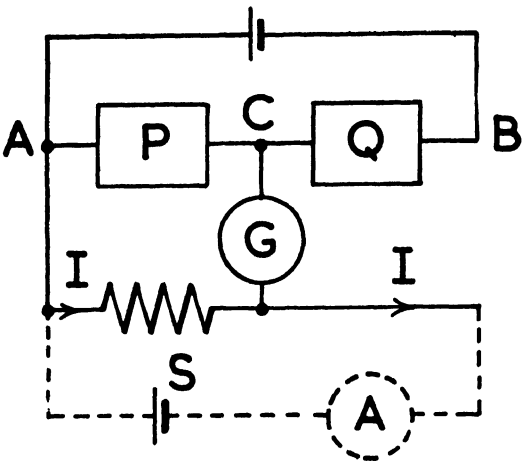


Fig. 6.21

All the plugs in P are initially removed, whilst all the plugs remain in Q. The total resistance between A and B is thus the maximum resistance available in P, say R_0 ohms. To find a balance point, plugs are taken from Q and placed in corresponding positions in P, so that the resistance between A and B is always R_0 ohms and the potential along AB is constant. Zero current flows through G when the p.d. across AC is equal to that across S, i.e. IS.

Suppose a balance point is obtained when the resistance of P is R ohms. If the current passing through P and Q is I' amps., the p.d. along P is $I'R$ volts. Hence:

$$I \,.\, S = I'R$$

i.e.

$$I \propto R.$$

The potentiometer is calibrated for reading current strength directly by using a standard cell (E_0 volts) between A and C. If a balance is obtained with P_0 ohms removed from P:

$$E_0 = I'P_0$$

$$I \,.\, S = \frac{E_0 R}{P_0}$$

and: $$I = \frac{E_o R}{P_o S} \text{ amps.} \quad . \quad . \quad . \quad (47)$$

The circuit is also used for calibrating an ammeter connected as at A.

The advantages of the Rayleigh Potentiometer are (i) the avoidance of a sliding contact and (ii) the use of known resistances instead of an imperfect wire. Sliding contacts cause spurious currents from thermo-e.m.f.'s (p. 200) created by the heat of the hand or by friction when moved.

Example

Explain the principle of the potentiometer. Draw circuit diagrams to show how you would use a potentiometer (a) *to compare the e.m.f.'s of two cells*; (b) *to compare the resistance of two coils;* (c) *to measure an electric current.*

An electric current is passed through an ammeter and a standard coil of resistance 0·100 *ohm, connected in series, and is adjusted until the ammeter reads* 5·00 *amps. The p.d. between the ends of the standard coil is balanced by the p.d. across* 128·8 *cm. of potentiometer wire. A standard Weston cell of e.m.f.* 1·018 *volts is found to require* 254·5 *cm. of the same potentiometer wire for a balance. What is the error in the ammeter reading?* (Oxford and Cambridge Joint Board, G.C.E. Adv. L.).

The first part of the question is covered in the text.

Since the standard cell requires 254·5 cm. of wire for balance, the p.d. per cm. of the potentiometer wire is:

$$k = \frac{1{\cdot}018}{254{\cdot}5} \text{ volts.cm.}^{-1}$$

Hence the p.d. across the standard coil is:

$$V = kl = \frac{128{\cdot}8}{254{\cdot}5} \times 1{\cdot}018 = 128{\cdot}8 \times 0{\cdot}004 \text{ volts.}$$

i.e. $$V = 0{\cdot}5152 \text{ volts.}$$

$\therefore$ the strength of the current passing through the ammeter is:

$$I = \frac{V}{R} = \frac{0{\cdot}5152}{0{\cdot}1} = 5{\cdot}152 \text{ amps.}$$

Hence, the error in the ammeter reading is 0·152 amps. (too low).

Example

Describe and explain how you would use a potentiometer (a) *to compare the electromotive forces of two cells,* (b) *to test the accuracy of the* 1 *ampere reading of an ammeter.*

The e.m.f. of a cell is balanced by the fall of potential along 150 *cm. of a potentiometer wire. When the cell is shunted by a resistance of* 14 *ohms the length required is* 140 *cm. What is the internal resistance of the cell?* (Northern H.S.C.).

(*a*) The comparison of e.m.f's is described on p. 171.

(*b*) For testing the accuracy of the 1 ampere reading of an ammeter the potentiometer is used as described for the measurement of current strength. It is not necessary to use a Rayleigh potentiometer for this purpose. A standard 1 ohm resistance is connected in series with an accumulator (2 volts approx.), a rheostat, and the ammeter. The current strength is arranged to be 1 ampere as measured on the ammeter. The *apparent* p.d. across the resistor is thus 1 volt. This p.d. is measured on a calibrated potentiometer. The difference gives the error of the ammeter.

When the resistance is placed across the cell in the problem, the current which

flows is: $I = \frac{E}{R + r}$, where E is the e.m.f. of the cell and r the internal resistance. Hence, the p.d. across the plates of the cell is now:

$$E = Ir = E - \frac{Er}{R + r} \text{ volts.}$$ From the potentiometer readings:

$$E = kl_0 = 150k$$

and:
$$E - \frac{Er}{14 + r} = kl = 140k$$

Dividing:
$$1 - \frac{r}{14 + r} = \frac{140}{150}$$

i.e.
$$14 = \frac{196}{15} + \frac{14r}{15}$$

hence:
$$r = 1 \text{ ohm.}$$

6.16. The Potentiometer Voltage Divider

A potentiometer may be used to supply a circuit with a variable voltage, providing the resistance of the circuit is greater than that of the potentiometer circuit. In Fig. 6.22 a potentiometer is shown supplying

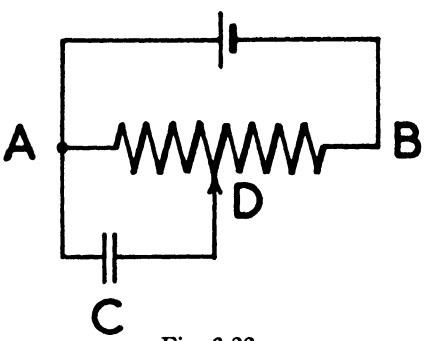


Fig. 6.22

a variable voltage for charging a condenser of capacity C. The usual potentiometer wire is replaced by a rheostat between A and B. The p.d. between A and D is given by:

$$\frac{V_{AD}}{\text{e.m.f. of cell}} = \frac{AD}{AB} \quad . \quad . \quad . \quad . \quad (48)$$

In this form the potentiometer is called a *potential divider* and (48) is true for a load which takes current from the supply, providing this is negligible compared with the potentiometer current.

The above arrangement is used extensively for supplying variable voltages to the electrodes of valves, etc. To economise in the use of power the value of the variable resistance is made as high as possible consistent with the demands of the load. Very convenient types of variable resistance are made for this purpose having total resistances between several hundred ohms and a few megohms.

EXERCISES

1. Calculate the resistivity of mercury if a thread 50 cm. long in a tube 1 mm. internal diameter has a resistance of 0·61 ohms.

   Describe a method suitable for determining the resistivity of mercury supplied in a tube having dimensions of the order of magnitude mentioned. Give reasons for the procedure adopted. (Northern H.S.C.).

2. Describe the Wheatstone net method of comparing two resistances and discuss its accuracy. Outline the method you would adopt when two very low resistances are to be accurately compared.

   An accumulator, of 2·05 volts and negligible internal resistance, is applied to the terminals A and C of a Wheatstone network $ABCD$. Resistances AB and AD are 2,000 ohms each and BC and DC are 2,100 ohms each, at balance. The resistances AB and BC are now transposed; calculate the potential difference between the ends of the galvanometer branch BD when it is on open circuit. What value of resistance will now have to be placed in parallel with the arm DC in order to restore the balance of the network. (London G.C.E. Adv. L.).

3. Discuss the various methods available for the measurement of potential difference.

   A voltmeter has a resistance of 1,000 ohms, and the scale reads up to 10 volts. It is to be used in conjunction with an ammeter of resistance 0·1 ohm reading up to 10 amps. Examine the suitability of this combination for the direct measurement of resistances (*a*) of the order of 1 ohm, (*b*) of the order of 1,000 ohms. (Oxford Local G.C.E. Schol.).

4. Describe how you would measure two of the following: (*a*) the resistance of a very sensitive galvanometer (e.g. full scale deflection = $\frac{1}{10}$ microampere); (*b*) the resistance of an electric lamp bulb when its filament is at 0°C.; (*c*) the specific resistance of distilled water (which may be about 10^6 ohm.m.). (Oxford Open Schol.).

5. How would you compare (*a*) two small resistances of the order of $\frac{1}{10}$ ohm and (*b*) two high resistances of the order of 10^6 ohms? (London G.C.E. Adv. L.).

6. Describe and explain how you would compare two resistances each of the order (*a*) 10^{-1} ohm, (*b*) 10^3 ohm, (*c*) 10^6 ohm. (London G.C.E. Adv. L.).

7. Explain the terms *electromotive force* and *potential difference* as applied to a cell and illustrate by reference to an experiment in which these quantities are measured.

   A circuit $ABCD$ is made up of a cell AB, of e.m.f. 1.08 volts and internal resistance 2 ohms, a resistance BC of 8 ohms, a cell CD of 2·92 volts e.m.f. and 3 ohms internal resistance, and a resistance DA of 5 ohms. The positive terminal A of the cell AB is earthed. Find the potential at C (*a*) when C is the positive terminal of the cell CD and (*b*) when the cell is reversed making C the negative terminal. Show clearly the deduction of your results. (London G.C.E. Schol.).

8. Explain the principle of the potentiometer, and point out its advantages.

How would you use a potentiometer to measure
(*a*) the internal resistance of a cell;
(*b*) a very large current? (Oxford Open Schol. Subsid.)

9. Describe how a potentiometer may be used to measure an unknown potential difference of about 100 volts.

Why is it that a potentiometer is capable of higher accuracy than a moving-coil voltmeter? (Oxford Open Schol. Subsid.).

10. A cell of negligible internal resistance and e.m.f. 2·2 volts is connected by wires of negligible resistance across the ends AB of a potentiometer wire whose resistance is 1·1 ohms and length "l" cm. A second cell of internal resistance B is joined at A and is balanced on the wire, using a galvanometer of appreciable resistance, at a point "x" cm. from A. What is the e.m.f., E, of the cell.

If the value of E were to be increased until the balance length "x" became equal to "l" what would be the value of E under these conditions? Also what current would flow through AB? How much current would flow from each of the cells?

Describe the experiment you would perform to find the value of B if a resistance of known value were also available.

(Draw circuit diagrams of all cases considered). (London G.C.E. Adv. L.).

11. Describe how to use a potentiometer, standard resistance and other equipment to calibrate an ammeter of range 0–1 ampere. (Full credit will be given only for a detailed account including numerical value of the components used).

An ammeter of resistance 0·75 ohms requires a potential difference between its terminals of 75 millivolts to give a full scale deflection. Calculate the resistance of a conductor which, connected to this instrument, would convert it into an ammeter suitable for measuring currents up to 10 amp. Show on a diagram how it would be connected and state a condition which it should satisfy. (Northern G.C.E. Adv. L.).

12. Explain the principle of the potentiometer. Compare its usefulness with that of a Wheatstone bridge.

Describe, giving a clearly labelled circuit diagram, a method of calibrating an ammeter by means of a potentiometer. (Durham G.C.E. Adv. L.).

13. Explain in general terms why the Wheatstone bridge network is unsuitable for the comparison of (*a*) two low resistances, (*b*) two high resistances.

Draw a labelled diagram of the circuit for the comparison of two low resistances by the wire potentiometer method. Explain the following:
(*a*) a balance point is obtained with one resistance and not with the other;
(*b*) a balance point cannot be obtained with either resistance;
(*c*) the value obtained for the ratio of the two resistances changes progressively when the comparison is repeated with progressively smaller currents through them. (Northern G.C.E. Schol.).

14. Explain how a potentiometer may be used to measure (*a*) a current, (*b*) a resistance. *A* and *B* are the end points of a potentiometer wire of length 100 cm. and resistance 10 ohms. The positive pole of a cell of e.m.f. 2 volts and internal resistance 5 ohms is connected to *A* and the other pole to *B*. A second cell of e.m.f. 1·0 volt and internal resistance 5 ohms is connected in series with a galvanometer of resistance 45 ohms and the whole connected to the potentiometer in the usual way. Find where the sliding contact on the potentiometer wire must be placed in order that the current through the galvanometer should be zero. What current will pass through the galvanometer if the sliding contact is moved 5 cm. nearer *B*. Indicate on a circuit diagram the direction of this current. (Durham G.C.E. Schol.).

15. State Kirchhoff's laws for an electrical network and show how they may be used to obtain the balance condition in a Wheatstone bridge.

A Wheatstone bridge is used to measure a resistance and at the balance point the resistances of all four arms are equal. Calculate how the accuracy of the measurement depends upon the resistance of the instrument used in detecting the balance point. (Cambridge Open Schol.).

16. State Kirchhoff's laws for currents in a network, and outline the evidence on which they are based.

Resistances *R* form four edges *AB*, *BC*, *CD*, and *DA* of a tetrahedron, the other two edges *AC*, *DB*, being resistances 2*R*. Find the total resistance between the corners *A*, *B*. (Oxford Open Schol.).

17. Describe with the aid of a circuit diagram, how you would determine the e.m.f. and internal resistance of a given cell. What further experiments are necessary in order to justify the assumption that the e.m.f. and internal resistance are constant?

A variable resistance *R* is connected across a cell of constant internal resistance *r*. Find the value of *R* for which the power dissipated in it is greatest. If *R* is shunted by a fixed resistance *S*, find the new value of *R* for which the power dissipated in it is greatest. (Oxford Open Schol.).

18. State Kirchhoff's laws for an electrical network.

A circular brass ring is supported by a brass rod forming a diameter whose centre is pierced by an axle so that the ring can rotate about an axis through its centre and perpendicular to its plane. The ring and the diametral rod both have the same resistance per unit length, ρ; the radius of the ring is "*a*".

If one terminal of a battery of e.m.f. *V* volts is connected to the axle, and the other to a fixed brush bearing on the outside of the ring, find how the current through the battery varies with time as the ring rotates with uniform angular velocity ω. (Oxford Open Schol.).

CHAPTER 7

THERMAL EFFECTS

7.1. Introduction: Resistance Heating

Work is done by an electric current in overcoming resistance and the energy expended appears as kinetic energy of the positive ions of the conductor, causing them to vibrate more vigorously about their mean positions. This is observed as an increase in the temperature. Current flow then becomes more difficult because of the increased possibility of collisions occurring between the moving charges (electrons) and positive ions. This increases the resistance and reduces the current. Since heat is lost by conduction, convection and radiation, an equilibrium state is eventually produced when all the heat lost is equal to the heat generated by the current. The resistance then has a steady value and the current strength is constant.

If a p.d. V volts, is placed across a resistance R ohms., the initial current strength is: $V/R = I$ amps., and the initial rate of dissipation of energy is: $P = IV = I^2R$ watts. If the conductor loses heat sufficiently rapidly there is no appreciable temperature rise, the resistance does not change appreciably and conditions remain steady. This is assumed in all the circuits discussed so far. Most resistances and rheostats allow rapid cooling to the surrounding air, so that temperature and resistance changes are not appreciable.

7.2. Measurement of the Variation of Resistance with Temperature

In this experiment the circuit of Fig. 7.1, is used. The specimen coil C (iron, copper, etc.) has a resistance of a few ohms at room temperature. It is supported by the ebonite or paxolin lid, in a beaker containing paraffin or similar insulating liquid. The coil is connected as one arm of a Post Office Box. The paraffin is heated externally and its temperature measured with a mercury thermometer.

The resistance of the coil at 0°C. is first obtained by immersing the container in melting ice for some time and balancing the bridge. Resistances at other temperatures are found by heating the container gently, stirring vigorously to maintain uniform temperature and taking quick balance points on the bridge.

When values of resistance are plotted as a function of the temperature T (°C.) they lie on a straight line represented by the equation:

$$R_T = R_0 (1 + \alpha T) \quad . \quad . \quad . \quad . \quad (1)$$

where $R_T =$ resistance of coil at T°C., R_0 is the resistance at 0°C., and

α is a constant known as the *temperature coefficient of resistance.* The slope of the straight line is $R_0\alpha$, whilst the intercept on the R_T axis is R_0, so that α may be determined. Strictly speaking, therefore, it is not necessary to determine R_0 experimentally, since this is obtained from the curve.

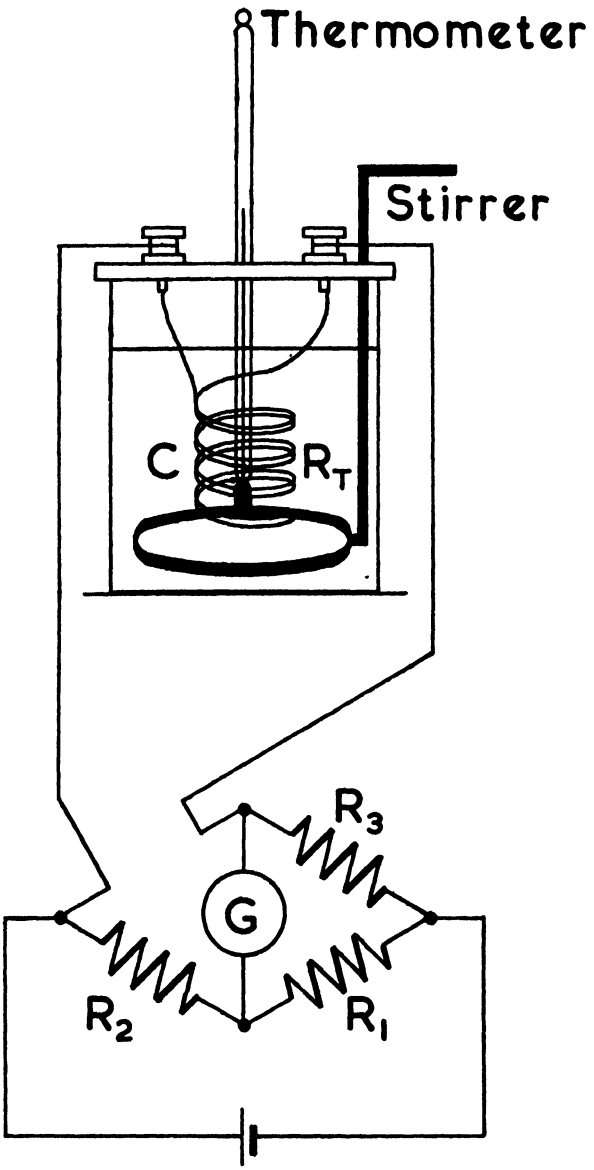


Fig. 7.1

The *coefficient α is the change in resistance per degree C. rise in temperature of a specimen of the material which has a resistance of* 1 *ohm at* 0°C. Its value varies with different conducting materials, as shown in the following table:

Metal	α (*per* °C.)
Copper	0·0040
Mercury	0·00089
Nickel	0·0054
Manganin	0·00001
Platinum	0·0037
Tungsten	0·0057

Since α for manganin is extremely small it is used for making accurate resistances whose variation with temperature must be small (e.g. bridge wires, potentiometer wires, etc.).

Although (1) is true over small ranges of temperature (e.g. 0° to 150°C.), deviations become apparent over large ranges of temperature, which show up as a pronounced curvature on the R_T *vs.* T. graph. The resistivity *vs.* temperature curve is drawn for tungsten in Fig. 7.2. (Tungsten is used for lamp filaments running at temperatures greater than 2,000°C.).

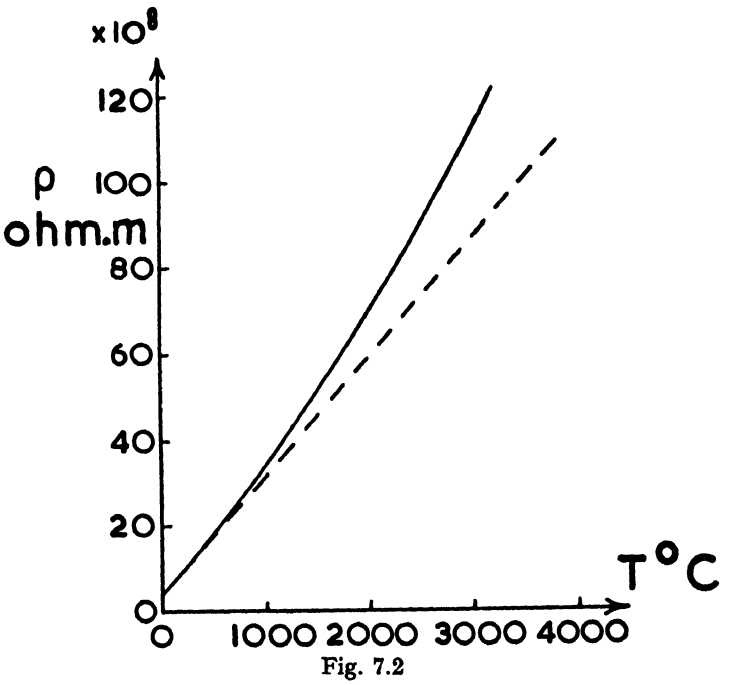


Fig. 7.2

Over such wide ranges of temperature the following formula applies:

$$R_T = R_0(1 + \alpha T + \beta T^2) \quad . \quad . \quad . \quad (2)$$

where β is another characteristic constant much smaller than α.

Example

Describe an experiment for determining the variation of the resistance of a coil of wire with temperature.

An electric fire dissipates 1 kilowatt when connected to a 250 v. supply. Calculate to the nearest whole number the percentage change that must be made in the resistance of the heating element in order that it may dissipate 1 kilowatt on a 200 v. supply. What percentage change in the length of the heating element will produce this change of resistance if the consequent increase in the temperature of the wire causes its resistivity to increase by a factor of 1·05? The cross-sectional area of the wire may be assumed constant throughout the calculation. (Northern G.C.E. Adv. L.).

The experimental determination of resistance variation with temperature is outlined on p. 181.

The energy dissipated per second by the fire is given by:

$$P = 1 \text{ kilowatt} = 1{,}000 \text{ joule.sec.}^{-1}$$

Since the p.d. across the element is given (250 volts) the resistance of the hot element is obtained from the formula:

$$P = \frac{E^2}{R} \text{ (cf. Chapter 4)}$$

i.e. $$R = \frac{E^2}{P} = \frac{250 \times 250}{1000} = 62{\cdot}5 \text{ ohms.}$$

When the p.d. across the element is reduced to $E' = 200$ volts, and the power is maintained at the same value, the resistance of the element must be reduced to:

$$R' = \frac{E'^2}{P} = \frac{200 \times 200}{1000} = 40 \text{ ohms.}$$

$$\therefore \text{ percentage change in resistance} = \frac{R - R'}{R} \times 100 = \frac{(62{\cdot}5 - 40)}{62{\cdot}5} \times 100$$

$$= \underline{\underline{36\%}}$$

Let the length of the original element be l metres, its radius r metres and the resistivity of the wire ρ ohm.metres. Then:

$$R = \frac{\rho l}{\pi r^2} = 62{\cdot}5 \text{ ohms.}$$

Since the diameter of the wire is constant but the resistivity becomes $1{\cdot}05\rho$ Ω.m. the new length of element l', is given by the formula:

$$R' = \frac{1{\cdot}05 l' \rho}{\pi r^2} = 40 \text{ ohms.}$$

Hence: $$\frac{R'}{R} = \frac{40}{62{\cdot}5} = \frac{1{\cdot}05 \cdot l'}{l}$$

whence: $$l' = 0{\cdot}6095 \, . \, l$$

$\therefore$ the percentage change in the length $= \frac{l' - l}{l} \times 100 = (1 - 0{\cdot}6095) \times 100 =$ 39·05 per cent.

Example

A ½-kilowatt electric radiator is made of nichrome wire and used with 240 volt mains. If the resistance of the wire when cold is 100 ohms and the temperature coefficient of resistance of nichrome is: $1{\cdot}7 \times 10^{-4}$ per °C., calculate the rise in temperature of the wire when hot.

The energy dissipated by the radiator per second is: $P = 500$ watts.
The p.d. across the radiator is: $V = 240$ volts.
Let the hot resistance be: R_T ohms.

Then: $$P = \frac{V^2}{R_T}, \text{ and: } R_T = \frac{V^2}{P}$$

whence: $$R_T = \frac{(240)^2}{500} = 115{\cdot}2 \text{ ohms.}$$

Let the cold resistance be: R_t ohms.
If the resistance of the radiator wire at 0°C. is R_o, then:

$$R_T = R_o (1 + \alpha T) \text{ and } R_t = R_o (1 + \alpha T)$$

whence: $$\frac{R_T}{R_t} = \frac{1 + \alpha T}{1 + \alpha t} = (1 + \alpha T)(1 + \alpha T)^{-1}$$

Using the Binomial Theorem:

$$(1 + \alpha t)^{-1} = 1 - \alpha t + \frac{2\alpha^2 t^2}{2!} - \frac{6\alpha^3 t^3}{3!} +$$

Since α is very small and t is not large, all terms involving $\alpha^2 t^2$ and higher powers may be neglected, so that:

$$\frac{R_T}{R_t} = (1 + \alpha T)(1 - \alpha t) = 1 + \alpha T - \alpha t - \alpha^2 Tt$$

neglecting $\alpha^2 Tt$ in this expansion gives:

$$\frac{R_T}{R_t} = 1 + \alpha (T - t)$$

i.e. $$R_T = R_t(1 + \alpha (T - t)) \text{ (approx.)} \quad . \quad . \quad (3)$$

From the data given: $R_t = 100$ and $\alpha = 1{\cdot}7 \times 10^{-4}$, hence:

$$115{\cdot}2 = 100 (1 + 1{\cdot}7 \times 10^{-4}(T - t))$$

i.e. $$(T - t) = \frac{15{\cdot}2}{1{\cdot}7 \times 10^{-2}} = 894°C.$$

i.e. the rise in temperature of the radiator is 894°C. (approx.)

Example

How would you make an accurate determination of the coefficient of increase of resistance with temperature of a metal supplied in the form of a wire?

The resistance of the tungsten wire of an electric lamp at 20°C., is 40 ohms. When the current through the lamp is 0·5 amp. the potential difference between its terminals is 200 volts. If the coefficient of increase of resistance with temperature of tungsten is constant and equal to 5×10^{-3} ohm.deg.$^{-1}$C. find the temperature of the filament. (Durham G.C.E. Adv. L.).

The determination of the temperature coefficient of resistance is best carried out using a P.O. box as described on p. 180.

The resistance of the hot lamp filament is given by:

$$R_T = \frac{V}{I} = \frac{200}{0{\cdot}5} = 400 \text{ ohms.}$$

If the resistance of the filament at 0°C. is R_o ohms:

$$R_T = R_o(1 + \alpha T) = R_o(1 + 0{\cdot}005T)$$

Likewise the resistance of the filament at 20°C. is given by:

$$R_{20} = 40 = R_o(1 + 20 \times 0{\cdot}005) = R_o \times 1{\cdot}1$$

i.e. $$R_o = \frac{40}{1{\cdot}1} \text{ ohms.}$$

Substituting for R_o above:

$$R_T = 400 = \frac{40}{1{\cdot}1}(1 + 0{\cdot}005T)$$

i.e. $$1 + 0{\cdot}005T = 11$$

$$\therefore \underline{T = 2{,}000}$$

i.e. the temperature of the hot filament is 2,000°C.

7.3. Resistance Thermometry

The variation of resistance with temperature is used for measuring temperature. The resistance element of such a thermometer is usually made of platinum, (M.Pt. = 1,700°C.), because it can be obtained pure, does not oxidize readily and is virtually unaffected by corroding agents such as water, carbon dioxide, etc.

A platinum resistance thermometer consists of a coil of fine platinum wire R, non-inductively wound on the end of a cylindrical former made of silica or other ceramic (Fig. 7.3). Copper leads PP connect the coil to end terminals X and Y. Exactly similar leads CC, form a loop which runs parallel to PP and ends in terminals M and N. PP and CC are connected in opposite arms of a bridge so that the resistance of PP is compensated for at all temperatures. CC are thus called the *compensating leads*. The thermometer is enclosed in a protecting ceramic sheath. The resistance of R at 0°C. is several ohms.

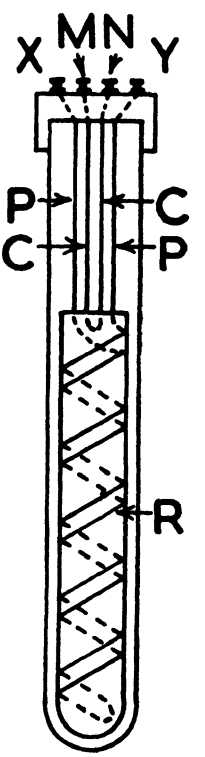


Fig. 7.3

To measure the change of resistance a modified Wheatstone network known as the *Callendar-Griffiths Bridge* is used (Fig. 7.4). The ratio arms AB and BC are equal whilst the third arm AD contains a small variable resistance r, the part ED of a resistance wire EF and leads CC. The fourth arm contains the coil R_T, leads PP and the part DF of EF. Balance points are obtained using the moving-coil galvanometer G. The bridge is initially balanced, by varying r, with the sliding contact at the mid-point of EF. If

$EF = 2l$ m., its resistance per unit length is ρ ohms.m.$^{-1}$ and the resistance of R at this temperature is R_{T1}, when no current flows in G:

$$r + R_C + \rho l = \rho l + R_{T1} + R_C$$

where R_C is the resistance of CC or PP,

$$\text{i.e.} \qquad r = R_{T1} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

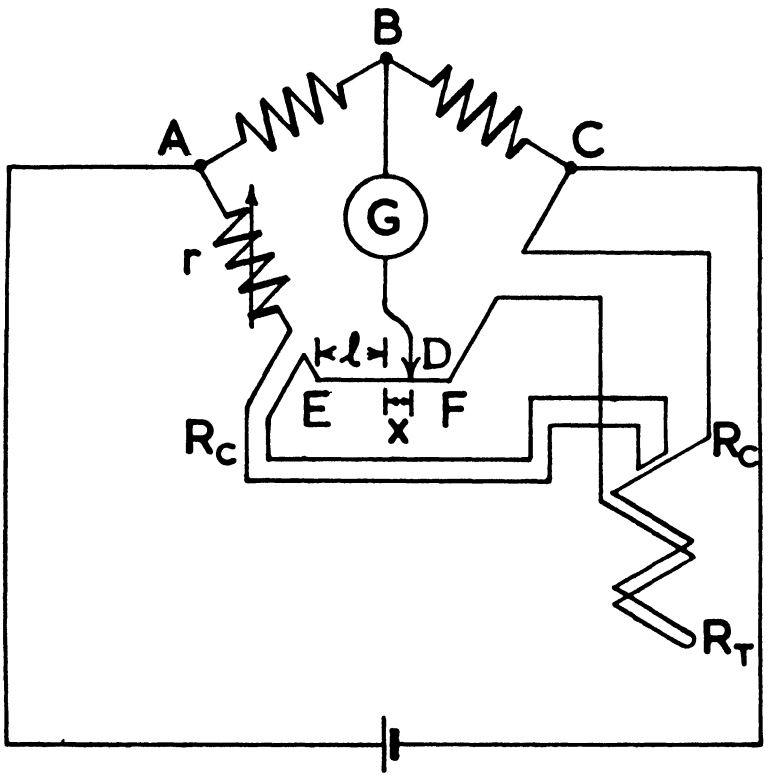


Fig. 7.4

At a higher temperature T_2, balance is obtained by moving the position of D, say x m.

Then: $$r + (l + x)\rho = (l - x)\rho + R_{T2}$$

whence: $$R_{T2} - r = 2x\rho$$

From (4): $$R_{T2} - R_{T1} = 2x\rho \quad . \quad . \quad . \quad . \quad . \quad (5)$$

i.e. the change is resistance is proportional to the shift of D. A linear scale may thus be attached to EF so that the position of D reads temperature difference.

The scale of EF covers a few deg. C. only, so that for an extended temperature range r must be altered.

The variation of resistance with temperature described above is characteristic of metals and alloys. Much interest is now taken in *semi-conductors*. These are substances whose conductivity is intermediate between that of metals and insulators, and are generally oxides or sulphides of metals such as lead, copper, etc. The number of free electrons available for conduction is much less than in metals but increases with the temperature. Their resistance thus decreases with the temperature, the variation being approximately logarithmic. Small beads of special semi-conductors are now used as resistance thermometer elements known as *thermistors*. Because of their large variation of resistance with temperature they can be used with a Wheatstone bridge without modification.

Example

Define temperature coefficient of resistance, and indicate the general variation of the resistance of pure metals over a wide temperature range.

How is a platinum resistance thermometer calibrated and used for the measurement of high temperatures?

A platinum resistance thermometer has a resistance of 24·00 *ohms at* 0°*C. Estimate the temperature of the enclosure in which the measured resistance of the thermometer is* 61·00 *ohms, assuming the temperature coefficient of resistance of platinum to be* 0·00370°C^{-1}. *State, giving your reason, whether you would expect this estimate to be very accurate.* (London G.C.E. Adv. L.).

The first part of the question is answered in the text.

A platinum resistance thermometer is calibrated by using the Callendar & Griffiths bridge at known standard temperatures (" fixed points "). The most common fixed points are the melting point of ice (0°C.), the boiling points of water (100°C.) and sulphur (444·6°C.), the melting points of silver (960·8°C.) and gold (1063°C.). The resistance thermometer is inserted in melting-ice, the vapour of boiling water, molten sulphur, etc., and its resistance at the fixed points determined. A graph is then constructed of temperature against resistance.

For high temperature measurement the resistance is determined at that temperature and from the calibration graph the temperature is noted.

Let the temperature of the enclosure in the problem be T°C. The resistance at this temperature is given by:

$$R_T = R_0(1 + \alpha T)$$

Since: $R_0 = 24$ ohms, $R_T = 61$ ohms, and $\alpha = 0{\cdot}0037°C^{-1}$.

$$61 = 24(1 + 0{\cdot}0037T)$$

$$\therefore T = \frac{61 - 24}{24 \times 0{\cdot}0037} = \frac{10^4}{24} = 416{\cdot}7°C.$$

This determination is not accurate because it cannot be assumed that the resistance of the platinum varies linearly with the temperature over such a wide range.

7.4. Mechanical Equivalent of Heat

The work done per second by a current is given by: $W = IV$ joules, when I and V are measured in amps. and volts, respectively. All this

energy is eventually lost to the surroundings as heat so that by enclosing a conductor in a calorimeter the energy loss may be measured in heat units, viz. calories. If the rate of heat production is H calories, then: $IV = HJ$, where J is the factor converting calories into joules and called the *Mechanical Equivalent of Heat.*

7.5. Electrical Methods of Determining J

1. The simplest method uses a resistance coil R, suspended in water contained in a copper calorimeter C (Fig. 7.5). The calorimeter is provided with a stirrer S and thermometer T. To reduce heat losses C is placed in a larger calorimeter C' which is packed with cotton wool. Current is supplied to R by means of the battery B and the rheostat Rh and its strength is measured by the ammeter A. The p.d. across R is measured with the voltmeter V.

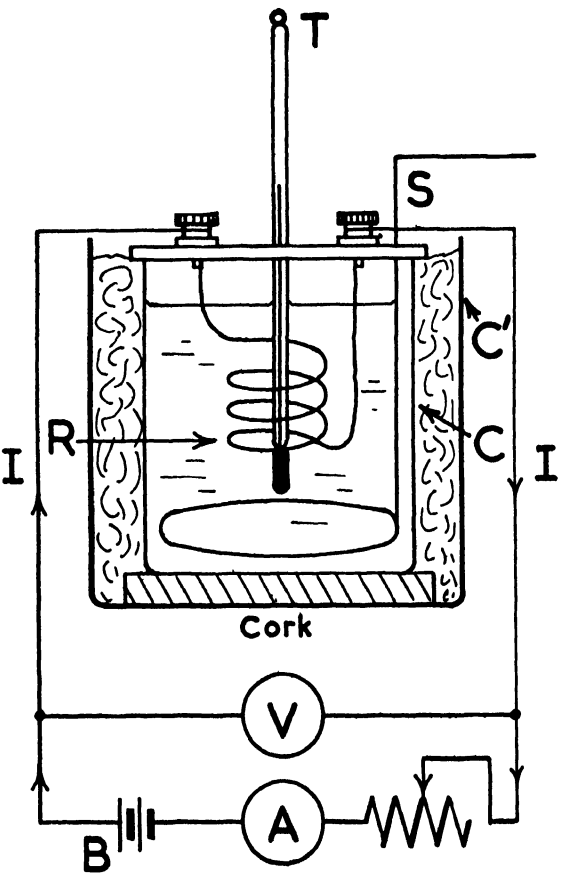


Fig. 7.5

The experiment consists of passing a current I amps. for a suitable time t secs. and noting the temperature change of the water T°C., say. Assuming the resistance of R does not change appreciably the p.d. across it will be constant V volts, say.

If the mass of the water in C is m gm. and the water equivalent of the calorimeter is M gm., the heat gained by the calorimeter and contents is $(M + m)T$ calories. The electrical energy dissipated by the current is: IVt joules, so that:

$$(M + m)TJ = IVt$$

i.e.
$$J = \frac{IVt}{(M + m)T} \text{ joules per calorie.} \qquad (6)$$

The greatest source of inaccuracy is the heat loss from the calorimeter.

2. *Constant Flow Calorimeter*

A better method uses the *constant flow calorimeter* of Callendar and Barnes. In this apparatus a constant current is supplied to a long resistance spiral R, contained in a glass tube AB through which water flows at a constant rate (Fig. 7.6). The current strength (I) and voltage (V) are measured. The water is heated as it flows through AB, the temperature increasing along the length of AB. The vacuum jacket P surrounding AB reduces heat losses caused by conduction and convection.

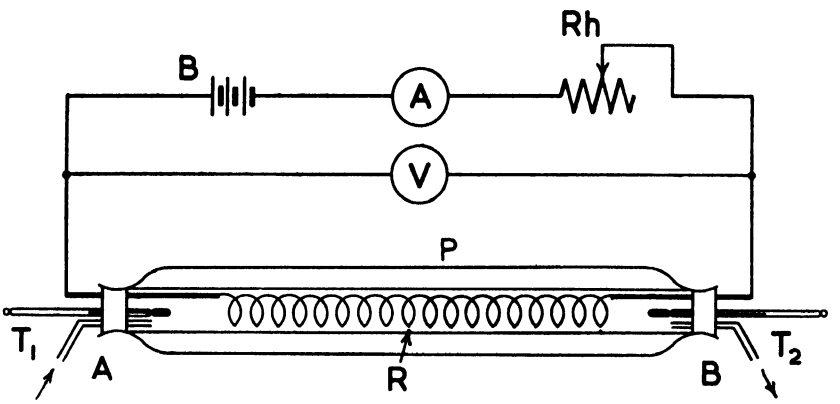


Fig. 7.6

After the water has been flowing for some time, the temperatures at the inlet and outlet, as measured by thermometers T_1 and T_2, become constant. In this state all the heat generated by the current per second is utilised in raising the temperature of the mass of water flowing through AB per second (neglecting heat loss). The rate of water flow is found by collecting a volume of water in a given time. Suppose this is m cm.^{3} per second and the temperatures of inlet and outlet are T_1 and T_2°C. The heat gained by the water in AB per second is: $m(T_2 - T_1)$ cals. The electrical energy lost by the current is IV joules per second.

Hence: $mJ(T_2 - T_1) = IV$

and:
$$J = \frac{IV}{m(T_2 - T_1)} \text{ joules . cal.}^{-1} \quad . \quad (7)$$

This formula neglects heat losses but these may be taken into account by repeating the experiment with different values of I and V and altering the rate of flow of water until $(T_2 - T_1)$ is the same. If the heat loss is H cals. per second, it follows that:

$$mJ(T_2 - T_1) + H = IV$$

and
$$m'J(T_2 - T_1) + H = I'V'$$

where the dashes signify the new values.

Eliminating H:
$$J = \frac{(IV - I'V')}{(m' - m)(T_2 - T_1)} \text{ joules.cal.}^{-1} \quad . \quad . \quad (8)$$

The use of (8) requires careful measurement of temperature since $(T_2 - T_1)$ is only a few degrees. With mercury thermometers capable of reading accurately to $\frac{1}{5}$°C., $(T_2 - T_1)$ is uncertain to $\frac{4}{5}$°C., since the extreme values of T_2 and T_1 are $T_2 \pm 0{\cdot}2$ and $T_1 \pm 0{\cdot}2$°C., respectively, and the worst possible values of $(T_2 - T_1)$ differ, therefore, by 0·8°C. If $(T_2 - T_1) = 4$°C., this represents a maximum error of 20 per cent. Likewise, if IV and $I'V'$ are not very different, any small inaccuracy in measuring current and voltage gives rise to an appreciable uncertainty in: $(IV - I'V')$. Since mass can be measured to 10 mgm. with a balance, the error here is not serious. In the original Callendar and Barnes apparatus platinum resistance thermometers were utilized for temperature measurement and current and voltage were measured by the potentiometer method (Chapter 6).

Example

An insulated wire of length 80 *cm. and cross section* 0·16 *sq.mm. is immersed in* 250 *gm. of water and a current of* 3·0 *amperes is passed through it. If the temperature rises* 17°C. *in* 20 *minutes, find the resistivity of the wire, assuming that all the heat goes into the water.* (London Inter. B.Sc.). ($J = 4{\cdot}2$ Joules. Cal.$^{-1}$).

Since the temperature rise is comparatively small the resistance of the wire may be assumed constant — R ohms; let the resistivity be ρ ohm.m.

The wire cross-section:

$$A = 0{\cdot}16 \text{ sq.mm.} = 1{\cdot}6 \times 10^{-7} \text{ sq.m.}$$

The wire length:

$$L = 80 \text{ cm.} = 0{\cdot}8 \text{ m.}$$

Hence
$$R = \frac{\rho L}{A} = \frac{0{\cdot}8\rho}{1{\cdot}6 \times 10^{-7}} = 5{\cdot}10^{6}\,\rho \text{ ohms.}$$

The energy dissipated by the current:

$$I^2Rt = 9 \times 20 \times 60 \times 5 \times 10^{6}\rho = (5{\cdot}4) \,.\, 10^{10}\rho \text{ joules.}$$

The heat gained by the water:

$$m(T_2 - T_1) = 250 \times 17 = 4250 \text{ cals.}$$

The energy equivalent of this heat:

$$= 4250 \times J = 17850 \text{ joules.}$$

Since the energy lost by the wire is gained by the water:

$$5{\cdot}4 \times 10^{10}\rho = 17850$$

hence: $\rho = 3{\cdot}306 \times 10^{-7}$ ohm.metres.

7.6. Calorimetric Determination of Resistance

Providing the value of J is known, experiment 1 (7.5) may be modified to determine the resistance of the coil. In this experiment the voltmeter is not required. A steady current I amps. is passed through R for a known time t seconds, until the temperature rise of the water is about 10°C., say T°C. Using the same symbols as above, the heat gained by the water and calorimeter is: $(M + m)T$ calories, so that the energy equivalent is:

$$W = (M + m)TJ \text{ joules}$$

The total energy delivered by the current (assuming R constant) is given by:

$$W = I^2Rt \text{ joules}$$

Hence: $$I^2Rt = (M + m)TJ$$

i.e. $$R = \frac{(M + m)TJ}{I^2t} \text{ ohms} \quad . \quad . \quad . \quad (9)$$

Example

How would you investigate experimentally the way in which the rate of production of heat in a wire carrying an electric current depends on the current strength?

A heater coil immersed in a vessel of liquid maintains the whole at a temperature of 20°C. when the current is 2·0 amps. The temperature of the air is 12°C. Assuming that Newton's law of cooling is obeyed, find the current needed to maintain the vessel at 60°C. under the same conditions. (Take the temperature coefficient of resistance of the material of the heater as 0·004 per C. degree). (Oxford Local G.C.E. Adv. L.).

(i) In order to investigate the rate of production of heat in a wire a calorimeter is used as for the determination of J. Providing the thermal capacity of the water and calorimeter is known the amount of heat produced by a measured current strength for a known time is calculated. The experiment is repeated using different current strengths, the time being chosen to give a suitable rise in temperature of 5 to 10°C. From the results a graph is plotted of heat produced per second against the current strength. (Heat losses from the calorimeter should be taken into account in the usual manner).

(ii) Let the resistance of the heater coil at 0°C. be R_0 ohms, then the resistance at 20°C. is given by: $R_{20} = R_0(1 + \alpha . 20) = 1{\cdot}08R_0$ ohms. Likewise the resistance

at 60°C. is: $R_{60} = R_0(1 + \alpha. 60) = 1{\cdot}24R_0$ ohms. The energy dissipated by the current per second at 20°C. $= I^2R_{20} = 4 \times 1{\cdot}08R_0 = 4{\cdot}32R_0$ joules.sec.$^{-1}$ If the current strength required to maintain the liquid at 60°C. is I' amps. the energy dissipated per second at this temperature $= I'^2R_{60} = 1{\cdot}24I'^2R_0$ joules.sec.$^{-1}$ Since the rate of heat loss by the vessel and liquid is proportional to the difference in temperature between the liquid and surroundings (Newton's law), the heat lost at 20°C. when the surroundings are at 12°C. is given by:

$$H = c(20 - 12) = 8c \text{ cals.sec.}^{-1}$$

where c is a constant of proportionality. The energy loss is thus:

$$W = 8cJ \text{ joules.sec.}^{-1}$$

Since this energy loss is the same as that supplied by the current per second:

$$8cJ = 4{\cdot}32R_0$$

whence:
$$\frac{cJ}{R_0} = 0{\cdot}54$$

When the liquid is at 60°C. the rate of energy loss is: $48cJ = 1{\cdot}24I'^2R_0$

whence:
$$I'^2 = \frac{48}{1{\cdot}24} \cdot \frac{cJ}{R_0} = \frac{48 \times 0{\cdot}54}{1{\cdot}24} = \frac{25{\cdot}92}{1{\cdot}24}$$

i.e.
$$\underline{I' = 4{\cdot}57 \text{ amps.}}$$

7.7. Electrical Heating in wires—Temperature Equilibrium

The heating effect of a current is utilised in many ways. In the electric fire and lamp a coil of wire is heated to a high temperature and in the latter case the temperature is so high (>2000 deg.C.), that some electrical energy is converted into light. A third application is the electric fuse which is inserted in a circuit to prevent excess current creating a fire hazard. The fuse consists of a piece of wire of low melting point which melts and causes a break in the circuit when the current strength exceeds a definite value.

Two special cases of a heated wire are considered theoretically, depending on whether the rate of loss of heat by convection or radiation is more important. In both cases the loss of heat by (thermal) conduction to the surroundings is assumed to be negligible:*

(i) Let the length and radius of a given wire, be l metres and r metres, respectively. Let the resistivity of the wire be ρ ohm.m., so that the resistance of the wire is: $R = \dfrac{\rho l}{(\pi r^2)}$ ohms.

Let the strength of the current in the wire be I amps. The rate at which electrical energy is dissipated is then given by:

$$W = I^2R = \frac{I^2\rho l}{\pi r^2} \text{ joules.sec.}^{-1}$$

* Assuming air is the medium since the thermal conductivity is very small.

The heat equivalent of this energy is:

$$H = \frac{I^2\rho l}{\pi r^2 J} \text{ calories.sec.}^{-1} \quad . \quad . \quad . \quad (10)$$

where J is the mechanical equivalent of heat.

Assuming a small temperature change, so that convection is the main source of loss of heat, Newton's law applies, i.e. the rate of loss of heat per unit area of surface of the wire is proportional to the difference in temperature of the wire and its surroundings.

Let the equilibrium temperature of the wire be θ deg.C. above the temperature of the surroundings. The rate of loss of heat is hence:

$$H = 2\pi r \,.\, l \,.\, c\,\theta \text{ cals.sec.}^{-1} \quad . \quad . \quad . \quad (11)$$

where c is a constant of proportionality. From (10) and (11):

$$2\pi r l \theta c = \frac{I^2\rho l}{\pi r^2 J}$$

Hence:

$$\theta = \frac{I^2\rho}{2\pi^2 r^3 c J} \text{ deg.C.} \quad . \quad . \quad . \quad (12)$$

From (12) it follows that θ is independent of the length of wire; for small temperature changes ρ may be considered constant, so that for a constant current strength $\theta \propto r^{-3}$.

(ii) When the temperature of the wire is much higher than that of its surroundings, or when the wire is heated in vacuo (conduction and convection zero), the loss of heat by radiation only need be considered. Assuming a similar wire and current flow, let the equilibrium temperature of the wire be T deg.K. (absolute) and the temperature of the surroundings T_0deg.K.

The rate of loss of heat is given by *Stefan's Law*: the rate of emission of heat per unit area of surface is directly proportional to the fourth power of the absolute temperature, i.e. $H_1 = \sigma\epsilon T^4$ cals.m.$^{-2}$sec.$^{-1}$, where σ is *Stefan's constant* $= 1{\cdot}3 \times 10^{-8}$ cals.m.$^{-3}$sec.$^{-1}$deg.$^{-4}$, and ϵ is a constant of the wire called the *thermal emissivity*. If the wire constitutes a "black" body $\epsilon = 1$, but in general, ϵ is less than unity. The wire also receives radiant heat from the surroundings at the rate given by Stefan's law: $H_2 = \sigma\epsilon T_0^4$ cals.m.$^{-2}$sec.$^{-1}$. Hence, the net rate at which the wire *loses* heat is given by:

$$H = H_1 - H_2 = 2\pi r\sigma\epsilon l(T^4 - T_0^4) \text{ cals.sec.}^{-1}$$

In general $T \gg T_0$ so that $T^4 \gg T_0^4$, whence:

$$H = 2\pi r\sigma\epsilon l T^4 \text{ cals.sec.}^{-1} \quad . \quad . \quad . \quad (13)$$

The electrical energy dissipated per second is given by (10), hence, from (10) and (13):

$$T^4 = \frac{I^2\rho}{2\pi^2 r^3 \epsilon\sigma} \text{ deg.}K. \quad . \quad . \quad . \quad . \quad (14)$$

To a first approximation the resistivity may be assumed to vary directly with the absolute temperature, i.e. $\rho = aT$, where a is a constant of proportionality, whence:

$$T = \left(\frac{I^2 a}{2\pi^2 r^3 \epsilon\sigma}\right)^{\frac{1}{3}} = \left(\frac{I^2 a}{2\pi^2 \epsilon\sigma}\right)^{\frac{1}{3}} \cdot \frac{1}{r} \quad . \quad . \quad (15)$$

From (15), it follows that the ultimate temperature reached by the wire is independent of its length, and, for a constant current strength, is inversely proportional to the radius of the wire.

In both cases a negligible loss of heat by conduction to the connecting wires is assumed. This is not true, but it is too difficult to estimate this loss of heat theoretically.

Example

Describe the method you would use to measure a small resistance (< 1 *ohm*) *and the precautions you would take to obtain an accurate result.*

A fuse wire of diameter 0·5 *mm. fuses at* 15 *amps. What diameter fuse wire of the same material and length must be used to fuse at* 5 *amps., assuming that all the heat is lost by radiation?* (Oxford Open Schol.).

The determination of a low resistance is discussed in Chapter 6.

Assuming all the heat is lost by radiation, the fuse wire may be treated as in (ii) above, the wire melting at equilibrium; the melting point of the wire is then given by (14), viz.:

$$T = \left(\frac{I^2\rho}{2\pi^2 r^3 \epsilon\sigma}\right)^{\frac{1}{4}} = \frac{bI^{\frac{1}{2}}}{r^{3/4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where $b = \{\rho/(2\pi^2\epsilon\sigma)\}^{1/4}$ = constant.

If the fusing currents in the two wires are I_1 and I_2 amps. and their radii are r_1 and r_2 mm., it follows from (16):

$$\frac{bI_1^{\frac{1}{2}}}{r_1^{3/4}} = \frac{bI_2^{\frac{1}{2}}}{r_2^{3/4}}$$

Hence:

$$r_2^3 = \frac{I_2^2 r_1^3}{I_1^2}$$

In the problem: $I_1 = 15$ amps., $r_1 = 0{\cdot}25$ mm., and $I_2 = 5$ amps., therefore:

$$r_2^3 = \frac{25 \times (0{\cdot}25)^3}{225}$$

i.e.

$$r_2 = 0{\cdot}12 \text{ mm.}$$

$\therefore$ the diameter of the fuse wire is 0·24 mm.

Example

A wire of length l *m., radius* r *m., and resistivity* ρ *ohm.m. is heated* in vacuo *by a current* i *amp. which raises it to a steady temperature of* T°K. *Assuming that* ρ *is proportional to* T, *that the rate of radiation of energy per unit surface area of the wire is proportional to* T^4, *and that heat losses by conduction through the supports are negligible, find an expression for* T *in terms of the other quantities involved.*

Compare (a) *the radii,* (b) *the lengths of the filaments of a* 100-*volt* 60-*watt tungsten lamp and a* 230-*volt* 100-*watt tungsten lamp, assuming that the operating temperatures of the two filaments are the same.*

Explain briefly why electric lamp filaments are run at the highest possible temperature consistent with a reasonable life. (O. & C. J.B. Schol.).

The formula required in the first part is deduced in (ii) above. Using the given symbols:

$$T = \left(\frac{i^2 a}{2\pi^2 r^3 b}\right)^{\frac{1}{5}} \text{ deg. K.} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where a is the constant in the relation $\rho = aT$, and b is the constant in the relation $H = bT^4$, [$b = e\sigma$ of (14)].

Let the radii and the lengths of the 60-watt and 100-watt lamp filaments be r_1 and r_2 and l_1 and l_2, respectively. The strength of the 60-watt lamp current is:

$$I_1 = \frac{P_1}{V_1} = \frac{60}{100} = 0{\cdot}6 \text{ amps.}$$

Since the resistance of this lamp is: $R_1 = \frac{\rho l_1}{\pi r_1^2}$ ohms, the power is:

$$P_1 = I_1^2 R_1 = \frac{I_1^2 \rho l_1}{\pi r_1^2} \text{ watts} \quad . \quad . \quad . \quad . \quad (18)$$

For the 100-watt lamp, the strength of the current and power are given by:

$$I_2 = \frac{P_2}{V_2} = \frac{100}{230} = \frac{10}{23} \text{ amps.}$$

and:

$$P_2 = I_2^2 R_2 = \frac{I_2^2 \rho l_2}{\pi r_2^2} \text{ watts} \quad . \quad . \quad . \quad . \quad (19)$$

The lamps have the same operating temperatures so that ρ is constant. Dividing (18) by (19):

$$\frac{P_1}{P_2} = \frac{60}{100} = \frac{(0{\cdot}6)^2}{\left(\frac{10}{23}\right)^2} \cdot \frac{l_1 r_2^2}{l_2 r_1^2} = 1{\cdot}904 \cdot \frac{l_1 r_2^2}{l_2 r_1^2}$$

whence:

$$\frac{r_1^2}{r_2^2} = \frac{19{\cdot}04}{6} \cdot \frac{l_1}{l_2} = 3{\cdot}17 \cdot \frac{l_1}{l_2} \quad . \quad . \quad . \quad . \quad (20)$$

For each lamp the operating temperature is given by (17), so that:

$$\frac{I_1^2 a}{2\pi^2 r_1^3 b} = \frac{I_2^2 a}{2\pi^2 r_2^3 b}$$

whence:

$$\frac{r_1}{r_2} = \left(\frac{I_1^2}{I_2^2}\right)^{\frac{1}{3}} = \left(\frac{0{\cdot}6 \times 23}{10}\right)^{\frac{2}{3}} = \underline{1{\cdot}24}$$

Using (20): $\frac{l_1}{l_2} = \frac{1}{3 \cdot 17} \times \left(\frac{r_1^2}{r_2^2}\right) = \frac{1}{3 \cdot 17} \times (1 \cdot 24)^2 = 0 \cdot 484$

i.e. the ratio of the radii of the 60-watt to the 100-watt lamp is 1·24,
" " " " lengths of filaments of the two lamps is 0·484.

Lamp filaments are run at the highest temperature possible to obtain maximum light output. The percentage of electrical energy converted into visible light increases with the temperature. The limiting temperature is determined by the vapour pressure of the filament metal. If this is too high vapour condenses on the inner surface of the bulb and absorbs light and at the same time the filament is rapidly destroyed.

7.8. Thermoelectricity

The heating effect of an electric current is an *irreversible* effect, i.e. heating does not become a cooling effect when the current direction is reversed. The process is similar to the production of heat by friction—heat is produced independent of the direction of motion. An extremely important *reversible* heat effect that appears in an electric circuit is called the *Thermo-Electric Effect.* This has an entirely different explanation to the resistance heating effect. The effect consists of three related phenomena known as: (i) the *Seebeck Effect*, (ii) the *Peltier Effect* and (iii) the *Thomson Effect.*

(i) *Seebeck Effect*

This was discovered by Seebeck in 1821. He showed that if a strip of antimony and bismuth were joined together at one end and the other ends connected to a galvanometer, as in Fig. 7.7, a deflection is observed

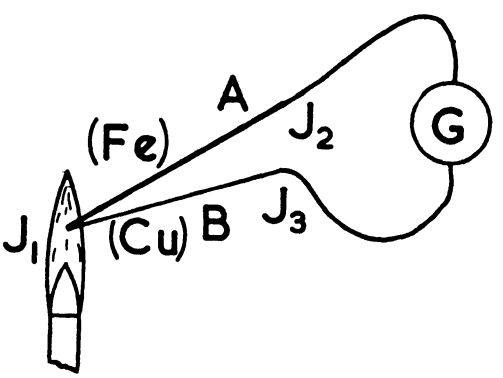


Fig. 7.7

when the junction of the two metals is heated. The deflection increases with the temperature of the junction. The effect is exhibited by any two dissimilar metals. The effect is demonstrated by using a " thermo-junction " made by twisting together (or soldering) one end of an iron and copper wire and connecting the free ends to a galvanometer. On heating the junction gently with a Bunsen burner the galvanometer

shows a steadily increasing deflection. With more intense heating the deflection falls to zero and ultimately increases in the reverse direction (cf. p. 202).

(ii) *Peltier Effect*

Peltier discovered this effect in 1834. He shewed that when a current is passed through a circuit containing two dissimilar metals arranged so that two junctions are formed, heat is evolved at one junction and absorbed at the other. This is demonstrated using the circuit of Fig. 7.8. A current of several amps. is supplied by the battery B to rods of

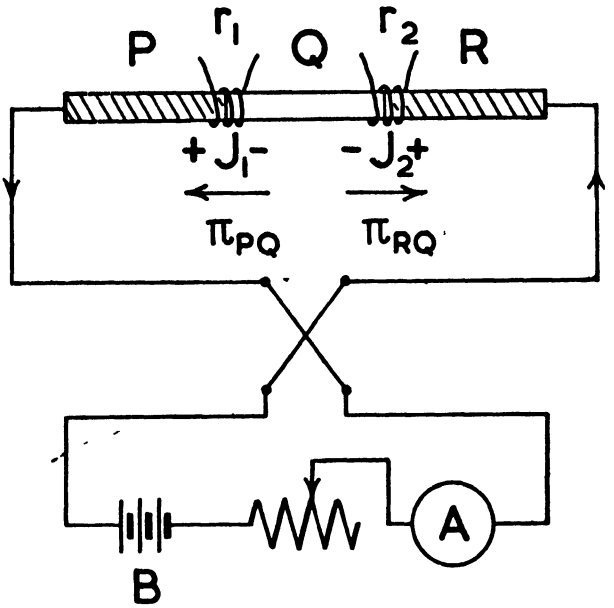


Fig. 7.8

antimony P and R, and a rod of bismuth Q, the junctions being J_1 and J_2. Wrapped round J_1 and J_2 are fine wire resistance coils r_1 and r_2, respectively. The latter act as resistance thermometers and are connected in opposite arms of a Wheatstone bridge. When current flows r_1 increases and r_2 decreases, showing that the temperature of the junctions has changed accordingly. On reversing the current r_1 decreases and r_2 increases, i.e. the effect is reversed. Large resistance heating is avoided by making P, Q and R of large cross-section.

7.9. Peltier Coefficients

Both the Seebeck and Peltier effects are explained by supposing that an e.m.f. exists at the junction of any two dissimilar metals; e.g. in Fig. 7.7 e.m.f.'s exist at the junctions of A and B. These e.m.f.'s are called *Peltier Coefficients* and denoted by π_{AB}, the subscript signifying the junction metals. The values of Peltier coefficients depend on (i)

the junction temperature, (ii) the metals of the junction. If the temperatures of the junctions in Fig. 7.9 are the same, the Peltier coefficients have the same value and both are directed from A to B. The net e.m.f. is thus zero and no current flows. If the temperature of one junction, say J_2, increases, its Peltier coefficient becomes π'_{AB}, say, where π'_{AB} may be greater or less than π_{AB}, depending on the metals. This causes a net e.m.f. given by:

$$E = \pi'_{AB} - \pi_{AB} \quad . \quad . \quad . \quad . \quad (21)$$

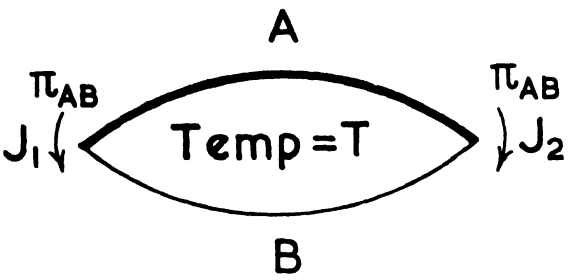


Fig. 7.9

Assuming Peltier coefficients the following experimental laws may be proved:

1. Law of Intermediate Metals: *The thermal e.m.f. produced by two different metals having junctions at different temperatures is independent of a third metal introduced in the circuit, providing that the junctions formed by this metal are maintained at the same temperature.*

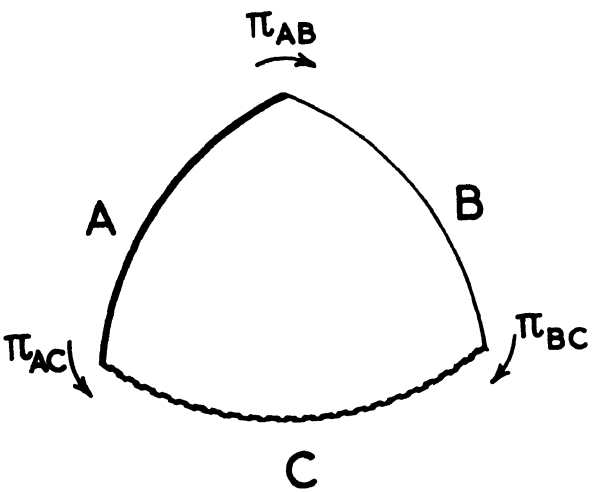


Fig. 7.10

Proof: Suppose the circuit of Fig. 7.10 consists of three metals A, B and C. Peltier coefficients exist as shown. For similar junction temperatures the net thermal e.m.f. is:

$$E_0 = \pi_{AB} + \pi_{BC} - \pi_{AC} = 0 \quad . \quad . \quad . \quad (22)$$

If the junction of A and B is maintained at a temperature higher than the others, π_{AB} becomes π'_{AB} say, and the net e.m.f. is now:

$$E = \pi'_{AB} + \pi_{BC} - \pi_{AC}$$

From (22) however: $\pi_{BC} - \pi_{AC} = -\pi_{AB}$

hence: $$\underline{E = \pi'_{AB} - \pi_{AB}}$$

This is the thermal e.m.f. that exists in the circuit when C is absent. Hence the law of intermediate metals follows.

2. Law of Intermediate Temperatures: *The thermal e.m.f. produced by two different metals with junctions at different temperatures* T_1 *and* T_3, *is given by the sum of the thermal e.m.f.'s which exist between the same junctions at* T_1 *and* T_2 *and* T_2 *and* T_3, *where* T_2 *is some intermediate temperature.*

Proof: Consider a circuit consisting of two different metals A and B

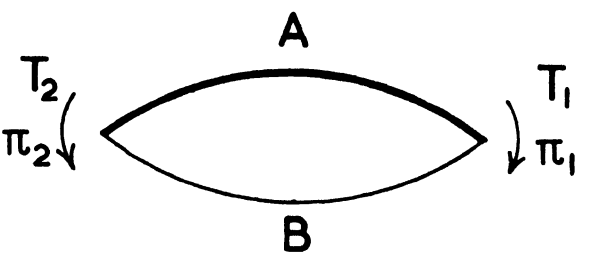


Fig. 7.11 (*a*)

(Fig. 7.11(*a*)), with junction temperatures T_1 and T_2. The net thermal e.m.f. is:

$$E_{12} = \pi_2 - \pi_1 \quad . \quad . \quad . \quad . \quad (23)$$

where the subscripts now refer to temperatures. Suppose T_1 is raised to T_3, where $T_3 > T_2 > T_1$. The new state of affairs is shown in Fig. 7.11 (*b*) and the new thermal e.m.f. is:

$$E_{23} = \pi_3 - \pi_2 \quad . \quad . \quad . \quad . \quad (24)$$

Adding (23) and (24): $E_{12} + E_{23} = \pi_3 - \pi_1$

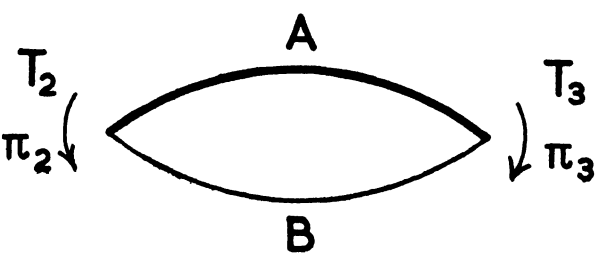


Fig. 7.11(*b*)

The R.H.S. of (24) is the e.m.f. which exists when the junction temperatures are T_1 and T_3, hence:

$$E_{13} = E_{12} + E_{23} \quad . \quad . \quad . \quad . \quad (25)$$

This proves the law. From this law it follows that if the thermal e.m.f. is known for two metals when one junction is maintained at a constant temperature, the e.m.f. for a different constant junction temperature is found by subtracting the e.m.f. for the junctions at the two constant temperatures, (cf. p. 203).

7.10. Explanation of Seebeck Effect

Referring to Fig. 7.7, Peltier e.m.f.'s exist at J_1, J_2 and J_3, assuming the wire of the galvanometer coil and connections is the same. The arrangement is similar to Fig. 7.10. Before the junction $A-B$ is heated the net e.m.f. in the circuit is zero and no current flows. When the junction is heated π_{AB} changes and a net e.m.f. exists given by (21), the galvanometer and connections forming an intermediate metal. This thermal e.m.f. causes the deflection of G. The subsequent changes in deflection depend on the changes of π_{AB} with temperature.

7.11. Explanation of Peltier Effect

Referring to Fig. 7.8, Peltier coefficients exist at J_1 and J_2 as shown. The arrows indicate the direction current would flow if no other e.m.f. existed in the circuit. Each junction is equivalent to a cell whose plates are the dissimilar metals in the immediate vicinity of the junction, one of which is charged positively and the other negatively; in Fig. 7.8, P and R are positive and Q is negative. When current is passed in the direction shown, energy is gained by the charge crossing J_1, since its potential is raised by the Peltier e.m.f. If the Peltier e.m.f. is π volts the energy gained per coulomb is π joules. This energy must be derived from the metals at the junction since no heat is supplied externally; the junction is accordingly cooled. The converse occurs at J_2 the charge losing energy to the junction which is thereby heated.

7.12. Measurement of Thermal e.m.f.'s

Thermal e.m.f.'s are measured with a potentiometer, using the circuit of Fig. 7.12. The two metals, such as copper and iron, form junctions J_1 and J_2. J_1 is immersed in melting ice ($T = 0$°C.) and J_2 is contained in a glycerine bath. The free ends of J_1 and J_2 are connected to the end A of the potentiometer wire AB and the position 3 of the single-pole double-throw switch S. Position 1 of S is connected via a galvanometer G to the sliding contact C. The current supplied to AB by the battery is limited by the resistance boxes R_1 and R_2.

The maximum p.d. developed across the junctions is only a few millivolts, say x mV. and greatest accuracy in measurement is obtained

when the p.d. along AB is similar. The total value of $(R_1 + R_2)$ is hence given by using Ohm's Law:

$$I(R_1 + R_2) = E - x.10^{-3}$$

where I amps. flows in AB and E volts is the e.m.f. of the battery. If the resistance of AB is r ohms and the internal resistance of the battery is negligible:

$$I = \frac{E}{R_1 + R_2 + r} \quad . \quad . \quad . \quad . \quad (26)$$

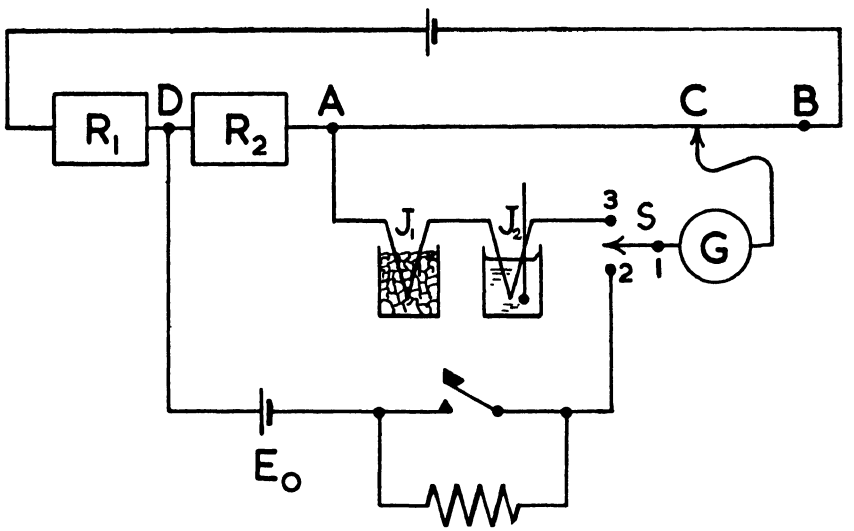


Fig. 7.12

Substituting for I in (26):

$$\frac{E(R_1 + R_2)}{R_1 + R_2 + r} = E - x.10^{-3}$$

whence:
$$R_1 + R_2 = \frac{r(E - x.10^{-3})}{x.10^{-3}} \text{ ohms} \quad . \quad . \quad (27)$$

An approximate value of $(R_1 + R_2)$ is required and since $E \gg x.10^{-3}$:

$$R_1 + R_2 = \frac{Er.10^3}{x} \text{ ohms. (approx.)} \quad . \quad . \quad (28)$$

The separate values of R_1 and R_2 are chosen by calibrating the potentiometer: a standard cell of e.m.f. E_o volts is connected between D and B using positions 2 and 1 of S. The cell cannot be connected across AB directly (cf. p. 170) because the p.d. along AB is a few mV. only, by design. With C on the point B the value of R_1 is increased and that of R_2 decreased by similar amounts (or vice versa), until no

current flows through G. The p.d. between D and B is now E_0 volts, and since $(R_1 + R_2)$ is unaltered, I remains the same and the p.d. per m. of AB is given by:

$$k = \frac{r}{L} \cdot \frac{E_0}{(R_2' + r)} \text{ volts.m.}^{-1} \quad . \quad . \quad . \quad (29)$$

where L m. is the length of AB and R_2' the final value of R_2.

The thermal e.m.f. (E_T) is measured by removing the standard cell, connecting 1 and 3 of S, and adjusting the position of C until no current flows through G. For balance at a point l m. from A:

$$E_T = k \,.\, l = \frac{r}{L} \cdot \frac{E_0}{(R_2' + r)} \,.\, l \text{ volts} \quad . \quad . \quad (30)$$

The variation of E_T with temperature is obtained by heating the glycerine bath.

7.13. Variation of Thermal e.m.f. with Temperature

The variation of thermal e.m.f. with temperature for two wires with one junction at a fixed temperature is shown in Fig. 7.13. The curve is

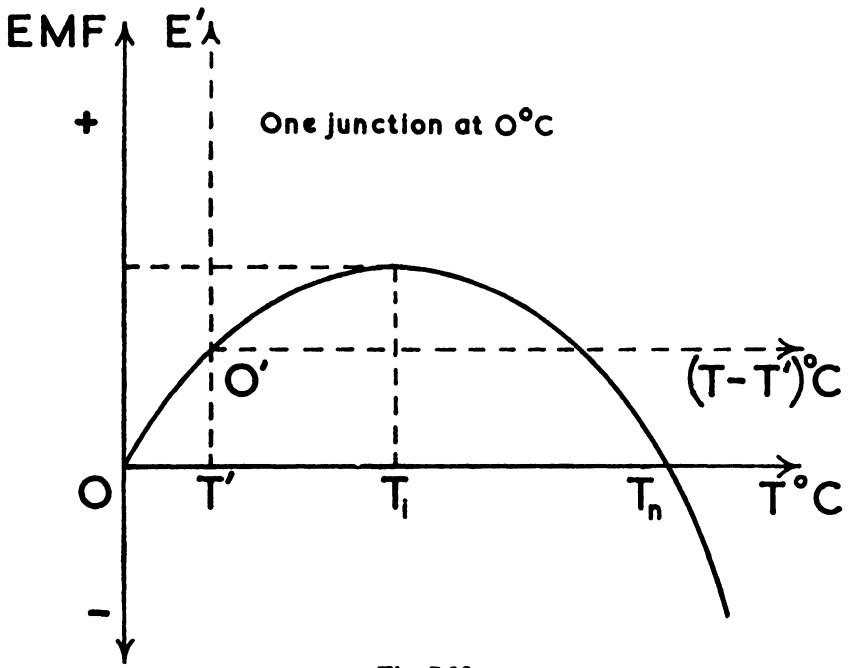


Fig. 7.13

parabolic (approximately), with a maximum value of e.m.f. at a temperature T_i, called the *Inversion Temperature*, (about 280° when Cu and Fe are used). The temperature at which the e.m.f. becomes zero and reversal takes place is called the *Neutral Temperature* (T_n)

(575°C. for Cu-Fe). Both these temperatures are characteristic of a given " couple ". The variation of e.m.f. may be represented (approximately) by:

$$E = aT + bT^2 \quad . \quad . \quad . \quad . \quad (31)$$

where a and b are constants and T is the temperature difference between hot and cold junctions.

N.B. From the law of intermediate temperatures the thermal e.m.f. when the " cold " junction is maintained at any intermediate temperature T' is given by using the new origin O'.

Example

Explain with the aid of circuit diagrams how to set up and standardize a potentiometer arrangement capable of measuring voltages up to 2·000 *volts to an accuracy of* 1 *millivolt. The following apparatus is available: two resistance boxes each containing* 2,000 *ohms in steps of* 1 *ohm; an accumulator charged to just over* 2 *volts; a standard cell of e.m.f.* 1·018 *volts, a variable resistance and, a sensitive galvanometer.*

The e.m.f. of a Leclanché cell is found by this arrangement to be 1·423 *volts. Describe the experimental procedure which would lead to this conclusion.*

A thermocouple gives 900 *microvolts for a temperature difference of* 100°C. *between the hot and cold junctions. Assuming that a uniform wire of length* 1 *metre and resistance exactly* 1 *ohms is available, indicate how the above potentiometer arrangement may be modified to permit the measurement of temperatures up to* 100°C. (Northern H.S.C.).

(i) The circuit for measuring p.d.'s up to 2·00 volts, is shown in Fig. 7.14. The resistance boxes P and Q, accumulator E and variable resistance Rh form a Rayleigh potentiometer, (Chapter 6). All the plugs are initially removed from Q and the total resistance of P and Q is maintained at 2,000 ohms by removing plugs from P and placing them in the corresponding positions in Q.

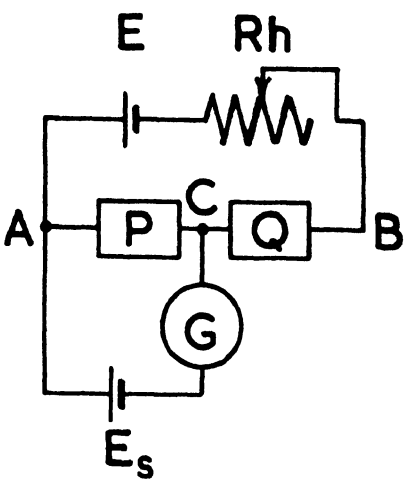


Fig. 7.14

The potentiometer is arranged to read a maximum of 2 volts by adjusting the current passing through P and Q until the p.d. across P and Q, i.e. V_{AB}, is exactly 2 volts. This is achieved by connecting the positive terminal of the standard cell (E_s) to A and the negative terminal, via the galvanometer G, to C. When $V_{AB} = 2$ volts, the p.d. *per ohm* between A and B is 10^{-3} volts, since $R_{AB} =$ 2,000 ohms. Since $E_s = 1{\cdot}018$ volts the resistance required in P for no current in G, is $1{\cdot}018/10^{-3} = 1{,}018$ ohms. To standardize the potentiometer, therefore, the value of P is made 1,018 ohms ($Q = 982$ ohms.), and Rh is varied until no current flows through G.

(ii) To measure any other p.d., connections are made to A and C (via G) and the value of P is altered (maintaining $P + Q = 2{,}000$), until no current flows in G. If the resistance in P required for balance is P ohms, the value of the p.d. is $10^{-3}P$ volts. Since the smallest variation in P is 1 ohm the accuracy of measurement is 10^{-3} volts, i.e. 1 millivolt. In the case of the Leclanché cell a balance point would be inferred from the fact that the galvanometer current flows in opposite directions when P has the values 1424 and 1422, so that the estimated correct value is 1423, i.e. 1·423 volts.

(iii) To measure up to 900 microvolts the uniform wire (DA) is placed in series with P and Q (Fig. 7.15). This makes the total resistance between D and B, 2001 ohms. The maximum p.d. along DA must now be 9×10^{-4} volts, i.e. the p.d. per ohm is 9×10^{-4} volts. To re-standardize the potentiometer use the standard cell, as before, between A and C. The resistance now required in P is $1{\cdot}018 \div (9 \times 10^{-4})$, i.e. 1,131 ohms. Hence, P is given this value and Rh adjusted until G shews no deflection. The p.d. along DA is now 9×10^{-4} volts and the p.d. *per cm.* is 9×10^{-6} volts.

(iv) For the measurement of temperatures, the ends of the thermocouple are connected between D and a sliding contact F, the latter connection containing the galvanometer. One junction is placed in ice and the other in the given temperature enclosure. The thermal e.m.f. is balanced by moving the sliding contact along DA until no current flows in G. If the length required for balance (DF) is x cm. the thermo-e.m.f. is $9x.10^{-6}$ volts. Since 100 cm. of wire are equivalent to a temperature difference of 100°C. and a thermo-e.m.f. of 9×10^{-4} volts, x cm. of wire is equivalent to a temperature of the hot junction of x°C.

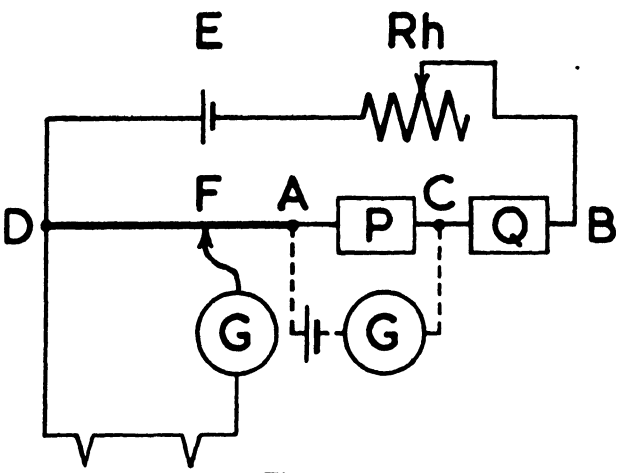


Fig. 5.15

7.14. Origin of Peltier e.m.f.'s

According to modern theory the number of free electrons per unit volume in a metal at a particular temperature is constant; the individual free electrons have different kinetic energies varying from zero to a maximum dependent on the temperature; the number of free electrons having a given kinetic energy is constant. Only those electrons with kinetic energies near the maximum value can continually lose and gain energy by collision with the positive ions of the metal. The rest of the free electrons are said to be *degenerate*; they cannot share their energies by collision. Those electrons which share their energies on collision, i.e. the *non*-degenerate electrons, are responsible for Peltier e.m.f.'s.

The junction of two different metals forms a single conducting path, but on either side of the boundary the numbers of non-degenerate electrons per unit volume having given kinetic energies is different. Electrons diffuse from the metal, say A, with the larger concentration of higher speed electrons to the other metal, say B. As a consequence,

the A side of the junction has an excess positive charge and the B side an excess negative charge. These charges create an electric field across the junction *within* the metals. The diffusion of electrons is a manifestation of the desire to create equal concentrations of equally fast electrons on either side of the boundary. The process ceases when the p.d. created by the electric field is sufficient to stop the electrons crossing.

In the equilibrium state the concentrations of electrons of similar energy on both sides of the boundary are the same. The excess of electrons in B and the deficit in A are confined to regions very close to the boundary so that the electric field exists within a very small region containing the junction. The work done on taking a unit positive charge from B to A across the junction gives the p.d. which exists. This p.d. is the Peltier e.m.f.

When the temperature of a junction changes the concentrations of free electrons in both A and B are altered, the equilibrium at the junction is disturbed and the Peltier e.m.f. changes. The net positive charge on the A side is not necessarily increased, since the concentrations of faster electrons in B might change more than those in A to cause diffusion of electrons from B to A and, hence, a reduction in the Peltier e.m.f.

7.15. Thomson Effect

The concentration of non-degenerate electrons of given energy depends on the temperature of the metal, so that if a temperature difference is maintained between two points in the *same* metal, differences in electron concentrations at the two points causes diffusion, the separation of charge and the creation of an e.m.f. between those points. This is called the *Thomson e.m.f.*; its value is generally much less than that of the Peltier coefficient between two different metals. In copper it is found that electrons diffuse from the hot to the cold part of the metal, so that the Thomson e.m.f. in copper is directed from the cooler to the hotter parts of the metal.

The Thomson e.m.f. existing between two neighbouring points having a temperature difference of dT°C. is proportional to dT, i.e. $dE \propto dT$. Hence: $dE = \sigma.dT$, where σ is a constant called the *Thomson Coefficient* for that metal. The total Thomson e.m.f. between any two points at temperatures T_1° and T_2° C. is given by:

$$E_t = \int dE = \int_{T_1}^{T_2} \sigma.\, dT \quad . \quad . \quad . \quad (32)$$

The Thomson coefficient varies with the temperature so that the expression for E_t cannot be integrated without a knowledge of this variation.

Thomson coefficients are positive or negative. If E_t increases in the direction of increasing T, i.e. both dE and dT are positive, then σ is also

positive, whilst if dE represents an increase in the direction of decreasing T, i.e. dE is positive but dT negative, then σ must be negative; e.g. σ_{Cu} is positive whilst σ_{Fe} is negative; lead is unusual because σ is very small at ordinary temperatures.

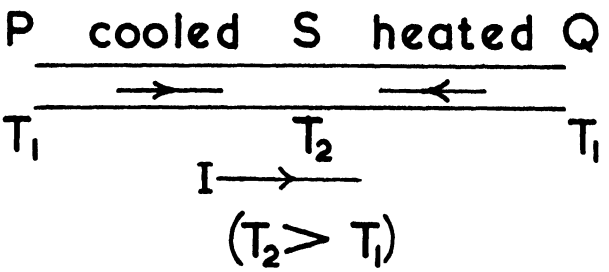


Fig. 7.16

If the wire PQ of Fig. 7.16 has a positive Thomson coefficient and is heated, so that both ends have a temperature T_1 which is less than T_2 at the mid-point S, Thomson e.m.f.'s exist as shown by the arrows which indicate the direction of increasing potential. If a current is passed through the wire from P to Q by means of an external battery, energy is gained from the source of e.m.f. existing between P and S and given up to the source between S and Q. Accordingly, PS is cooled and SQ is warmed up. This is called the *Thomson Effect*. It was deduced theroretically by Lord Kelvin (Thomson).

7.16. Demonstration of Thomson Effect

The extremely small Thomson effect is demonstrated with the apparatus of Fig. 7.17. AB is a uniform copper wire soldered at its ends to a thick copper rod ADB. The mid-points C and D are connected to a sensitive moving-coil galvanometer G. The four sections AC, CB,

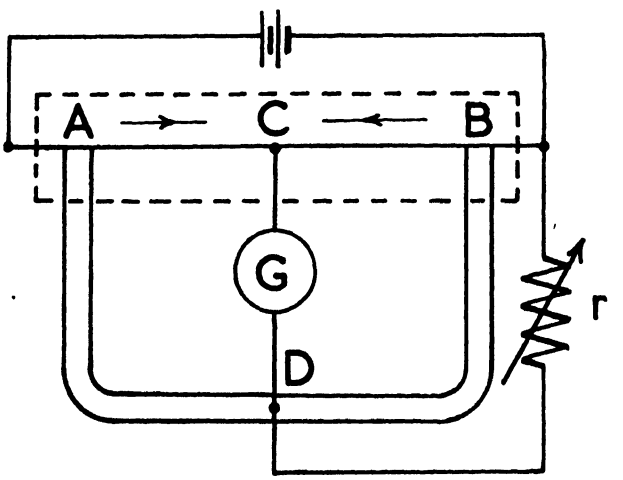


Fig. 7.17

AD and DB, form the arms of a Wheatstone Bridge. AB is buried in plaster of Paris so that its temperature distribution is unaffected by draughts. The large current from the battery E causes resistance heating in AB. When thermal equilibrium has been established the bridge is balanced by adjusting a small variable resistance r between D and B. The latter effects a fine control over the balance point. When the current to the bridge is reversed the bridge can no longer be balanced. This is caused by Thomson e.m.f.'s in the wire AB as explained below.

When current flows large temperature differences exist between C and A, and C and B, owing to thermal conduction along ADB. Thomson e.m.f.'s act as shown to cause a slight additional cooling in AC and a slight heating in CB, so that the thermal gradients on either side of C are not quite the same. When the current is reversed after balancing the bridge, the Thomson e.m.f.'s are reversed and a heating effect appears in AC and a cooling effect in CB. This causes corresponding resistance changes and unbalancing of the bridge.

7.17. Total Thermal e.m.f.

In any simple circuit consisting of two dissimilar metals, such as A and B in Fig. 7.18, the junctions being J_1 and J_2 at different temperatures T_1 and T_2, $(T_2 > T_1)$, the total thermal e.m.f. in the circuit is given by:

$$E_{12} = \pi_2 - \pi_1 + \int \sigma_A \, . \, dT - \int \sigma_B \, . \, dT$$

i.e.
$$E_{12} = \pi_2 - \pi_1 + \int (\sigma_A - \sigma_B) \, . \, dT \quad . \quad . \quad . \quad (33)$$

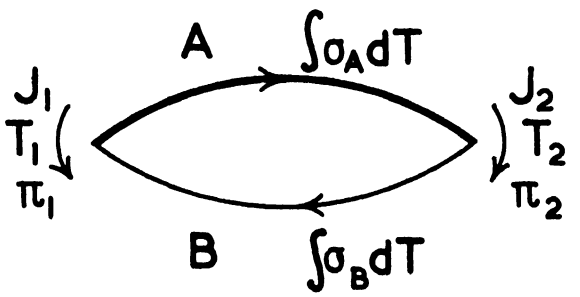


Fig. 7.18

The term $\int (\sigma_A - \sigma_B) \, . \, dT$ is very small compared with $\pi_2 - \pi_1$, and is often neglected. By taking this into account in all cases it can be shown that the laws of intermediate metals and intermediate temperatures are still true.

7.18. Thermo-electric Power and Thermo-electric Diagrams

Assuming that the variation of thermal e.m.f. with temperature difference T is parabolic:

$$E = aT + bT^2$$

where a and b are constants, it follows that:

$$\frac{dE}{dT} = a + 2bT \quad . \quad . \quad . \quad . \quad (34)$$

The constants a and b can be determined by choosing two suitable points on the curve of E *vs* . T and solving simultaneous equations.

From (34), (dE/dT) is a linear function of T, and if plotted against T gives a straight line which is extremely useful in assessing the capabilities of a given thermocouple. Because of its importance (dE/dT) is called the *Thermoelectric Power* and represented by p. Instead of determining p as a function of the temperature for every conceivable thermocouple, it is usual to determine p for a circuit consisting of a given metal and lead. The curves showing the variation of p with temperature for various metals and lead are almost straight lines. They are called *Thermoelectric Lines* and plotted on the same graph to give a *Thermoelectric Diagram*. From this diagram valuable information regarding the thermo-electric properties of any two metals is obtained.

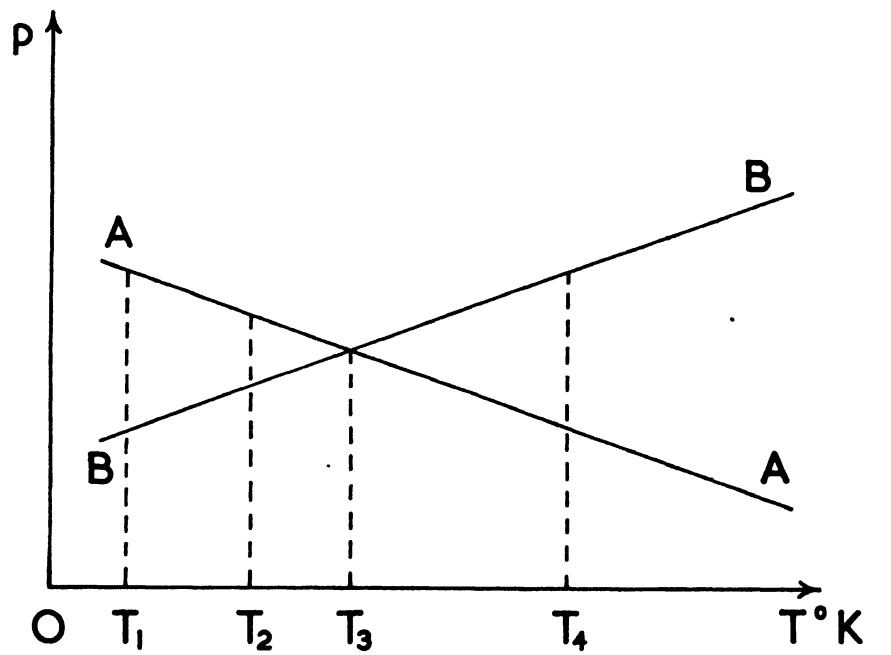


Fig. 7.19

Fig. 7.19 shows the thermoelectric lines for two metals A and B. Since: $p = (dE/dT)$, the narrow strip formed by two ordinates T and $T + dT$ has an area: dE $(= p.dT)$. Hence, the area underneath a thermoelectric line between two temperatures T_1 and T_2 gives $\int_{T_1}^{T_2} dE$, i.e. the thermal e.m.f. for a couple consisting of that metal and lead between those temperatures.

If the e.m.f. for metal A and lead between T_1 and T_2 is represented by E_{APb} and that for metal B and lead between the same temperatures by E_{BPb}, the difference in areas beneath both curves between T_1 and T_2 represents:

$$E_{APb} - E_{BPb}$$

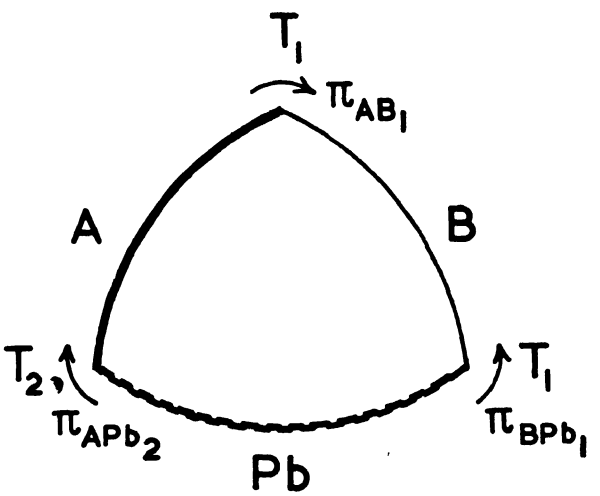


Fig. 7.20

If A and B were connected in series with a lead wire Pb (Fig. 7.20), and the junction temperatures were as shown:

$$E_{APb} = \pi_{AB1} - \pi_{BPb1} + \pi_{APb2} \quad . \quad . \quad . \quad (35)$$

since B is an intermediate metal. If the temperature of the junction $A-B$ is raised to T_2, the ends of A will be at the same temperature and then:

$$E_{BPb} = \pi_{AB2} - \pi_{BPb1} + \pi_{APb2} \quad . \quad . \quad (36)$$

Substracting (36) from (35):

$$E_{APb} - E_{BPb} = -(\pi_{AB2} - \pi_{AB1}) \quad . \quad . \quad (37)$$

The R.H.S. of (37) is the thermal e.m.f. in a circuit consisting of A and B only with junction temperatures T_1 and T_2, i.e. E_{AB}, hence:

$$E_{AB} = E_{APb} - E_{BPb} \quad . \quad . \quad . \quad (38)$$

It follows that *the difference in areas beneath the two lines between* T_1 *and* T_2 *represents the thermal e.m.f. of* A *and* B *between* T_1 *and* T_2. From a comprehensive thermoelectric diagram the thermal e.m.f. between any two temperatures for a couple consisting of any two metals can thus be measured easily.

Since the two lines for A and B intersect, the area between them when T_1 is maintained constant and T_2 increases has a maximum value when $T_2 = T_3$. This temperature is thus the Inversion Temperature for the two metals. Beyond T_3, the area between the lines is negative and, eventually, when $T_2 = T_4$, the negative and positive areas are equal, i.e. $E_{AB} = 0$. The temperature T_4 is thus the Neutral Temperature. It is quite clear that a thermoelectric diagram gives very useful information in a compact form.

7.19. Applications of Thermo-electricity

1. *Power*: Since a net e.m.f. is produced in a circuit containing two dissimilar metals with junctions at different temperatures, the thermoelectric effect acts as a source of power. However, it is a most inefficient one since the total e.m.f. is always small.*

2. *Thermoelectric Pyrometer*: Thermal e.m.f.'s are utilized for the measurement of temperature over a wide range. In practice, a *thermocouple*, consisting of two fine wires of different metals welded together at one end, is mounted in an insulating ceramic tube. The free ends of the wires are led to terminals at the end of the tube (Fig. 7.21). The terminals are connected directly to a moving coil galvanometer calibrated for direct reading of temperature difference. In this form the junctions of the thermocouple wires with the galvanometer leads are maintained at room temperature. The variations in room temperature do not affect the readings providing that the hot junction temperature is fairly high. This pyrometer is used for measuring furnace temperatures. Calibration is achieved by using fixed temperature points.

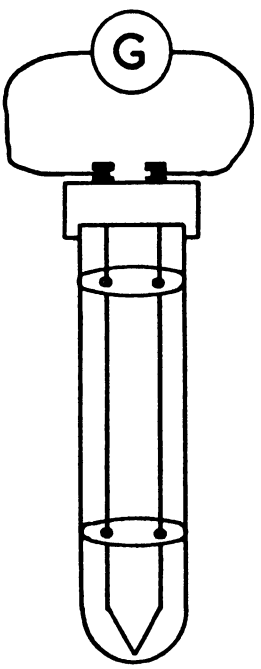


Fig. 7.21

In the laboratory, thermocouples are used for the accurate measurement of temperature, using a potentiometer to measure the thermal e.m.f. (p. 201). The thermal e.m.f.'s for many common pairs of metals are known as a function of the hot junction temperature when the cold junction is at 0°C., and potentiometer readings are calibrated in terms of temperature difference.

Choice of Thermocouple Wires: This depends on the range of temperature required. For high temperatures (say > 1,000°C.), the metals must not melt or show an inversion effect, hence platinum is often coupled with a platinum-rhodium alloy, (up to 1,700°C.). For tempera-

* The thermoelectric power of some semi-conductors is relatively large and efforts are being made to produce a battery utilizing such materials as thermo-junctions which can be heated by the sun.

ture differences of one or two hundred degrees a copper-iron couple is useful. It is limited by its inversion temperature at 285°C. A copper-constantan couple is more useful because it gives a larger e.m.f. for the same temperature difference. The general requirements are always a steady measurable variation of e.m.f. with temperature and avoidance of inversion and neutral points.

3. *Detection of Heat Radiation. Radiation Pyrometer*: Thermojunctions are used for the purpose of detecting and measuring heat radiation. This has led to the method of measuring large furnace temperatures using a *Féry Radiation Pyrometer*. The instrument consists of a cylindrical metal tube of several cm. diameter containing an adjustable concave mirror M (Fig. 7.22). Radiation from a furnace

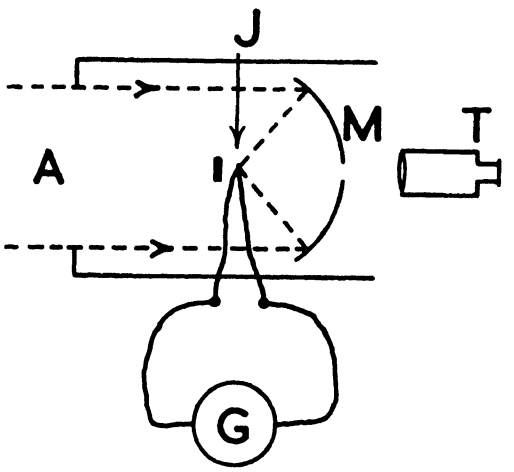


Fig. 7.22

enters the large aperture A and is focused by M on to a small thermojunction J. The latter is suitably shielded from the effects of the direct beam of radiation and is connected externally to a sensitive galvanometer G. A small telescope T is fitted inside the pyrometer so that J may be viewed through a hole in M. This allows the radiation to be accurately focused on J, the visible wavelengths being used for this purpose. The pyrometer is calibrated using known furnace temperatures.

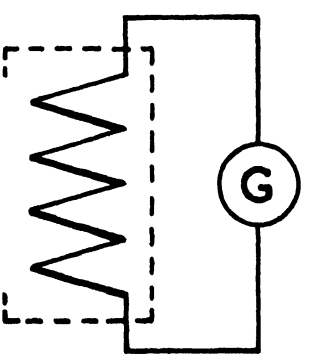


Fig. 7.23

Thermopile: For measuring the intensity of heat radiation, several junctions of dissimilar metals are joined in series as in Fig. 7.23. One set of junctions is exposed to the radiation whilst the others are covered by an insulating lid. The exposed ends are blackened so that most of the radiation falling on them is absorbed. This arrangement is called a *thermopile*; the junctions are usually made with small antimony and

bismuth strips. The radiation raises the temperature of the exposed ends relative to the opposite ends. The thermal e.m.f. depends on the number of junctions used. A galvanometer is connected to the thermopile to measure the current which flows. This is proportional to the radiation intensity.

Boy's Radiomicrometer: This is used for the measurement of very small quantities of heat radiation and consists of a thin blackened copper disc C attached to the junction J_1 of a loop of antimony and bismuth (Fig. 7.24). The loop is suspended by means of a fine metal fibre in the

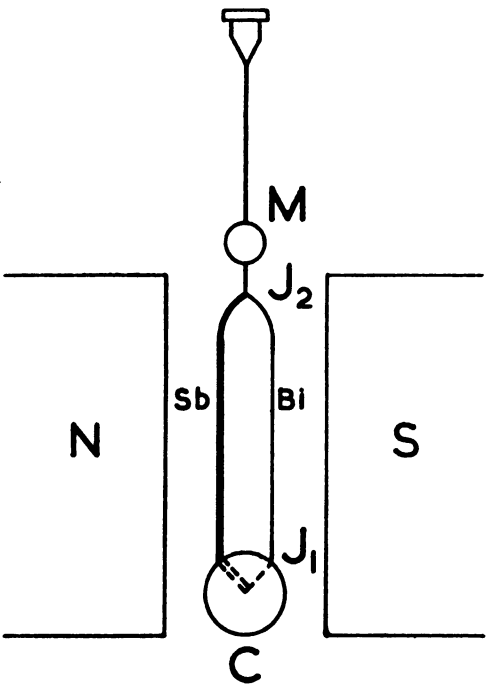


Fig. 7.24

space between the pole pieces of a permanent magnet NS. Radiation falling on C raises its temperature slightly, and the temperature of J_1 is increased relative to the opposite junction J_2. The thermal e.m.f. produced in the loop creates a current whose magnetic field interacts with that of the magnet. The loop rotates, the deflection being controlled by torsion in the suspension. The instrument is extremely sensitive.

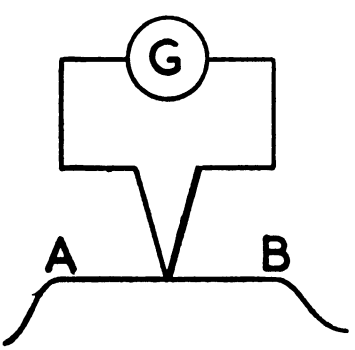


Fig. 7.25

4. *Thermo-Ammeter*: Thermojunctions are used to measure A.C. current strengths. A small thermocouple is welded to a fine platinum strip AB (Fig. 7.25). The A.C. passes through AB and causes resistance

heating which raises the junction temperature. The current which flows through the galvanometer G is a measure of the current in AB. The effect is independent of the direction of the current.

EXERCISES

1. Describe in detail the measurement of a resistance by means of *either* a P.O. box *or* a metre bridge, and explain what experimental precautions should be taken. (The theory of the method is *not* required).

   For what range of resistances is the method you describe most suitable?

   Indicate how the temperature coefficient of resistance of copper can be determined by experiment. (Northern H.S.C.).

2. Describe a moving coil ammeter and give the theory of the use of the shunt with such an instrument. What are the essential features in the design of a shunt for an accurate instrument?

   A badly designed shunt for an ammeter is made of iron whose temperature coefficient of resistance is 0·005 per deg. C. When 20 amp. pass through the circuit in which the ammeter is placed the temperature of the shunt rises by 10°C. What, approximately, is the error on the reading at this graduation, if the resistance of the shunt at room temperature is 0·100 ohm and that of the moving coil is 100 ohms?

   What other error may be introduced by using iron as the material of the shunt? (London G.C.E. Schol.).

3. State Kirchhoff's laws for steady currents in a network of conductors.

   A Wheatstone's bridge is balanced at 20°C., using three standard 1 ohm resistances of negligible temperature coefficient, and a piece of iron wire of temperature coefficient $6{\cdot}2 \times 10^{-3}$ per °C. The galvanometer has a resistance of 5 ohms, and a sensitivity of two divisions per milliampere; the battery has negligible internal resistance, and an e.m.f. of 2 volts. To what temperature must the iron wire be heated in order to cause a deflexion of ten divisions in the galvanometer? (Oxford Local G.C.E. Schol.).

4. Give a critical account of the experiments required to determine the variation of the resistance of a filament lamp (for example, a 240 volt, 60 watt lamp) with the current through it over a range of currents varying from a very small value to the normal working value.

   A Wheatstone network $ABCD$ consists of two ratio arms, AB and BC, each 100 ohms, a variable resistance R and a filament lamp whose resistance, P, varies with the current through it according to the formula $P = 50 + 1000I$ for the range of current under consideration, I being in amperes. A galvanometer is connected across AC and a battery, of e.m.f. 2·0 volts, is connected, in series with a rheostat across BD. The rheostat and variable resistance R are adjusted until the bridge is balanced when a current of 10 milliamperes is flowing

through the lamp. The rheostat is kept fixed at this value and the positions of the battery plus rheostat and the galvanometer are interchanged. Calculate the new value of the lamp resistance at balance, the balance being achieved by adjusting R. (Northern G.C.E. Schol.).

5. Give an account of the variation of resistance of solids with temperature.

A length of wire may be used to define a scale of temperature (resistance scale) by measuring its resistance at the temperatures of melting ice and of steam at S.P. and subdividing the fundamental interval into 100 parts. How will the temperature measured on this scale differ from that on the ideal gas scale?

The resistance of the length of wire is 12·00 ohms in melting ice, and 17·50 ohms in steam at S.P. and has the following values at the temperatures given in degrees on the ideal gas scale:—

Temp.	175°	220°	300°	450°
Resistance	21·2	23·4	27·2	33·0 ohms

What is the temperature of the wire (*a*) in degrees on the ideal gas scale, and (*b*) in degrees on the resistance scale, when its resistance is 22·00 ohms? (London G.C.E. Schol.).

6. Describe *briefly* how you would find the resistance of a coil of copper wire which has a nominal value of 2 ohms at room temperature, by (*a*) a calorimetric method, (*b*) an ammeter-voltmeter method and (*c*) a potentiometer method.

Give a diagram of the electrical connections in each case and the underlying principles of measurement. Could the accuracy of the results be improved, and if so why, if the coil were made of a material of low temperature coefficient like manganin?

The slide wire of a metre bridge is exactly 100 cm. long and has a uniform resistance of 0·0100 ohm.cm.$^{-1}$ The bridge is used with an accumulator of 2·00 volts and negligible internal resistance to compare two resistances of approximately 2·00 ohms each. If the galvanometer has a resistance of 100 ohms calculate approximately the out-of-balance current through the galvanometer when the potential gradient along the wire is unaltered by a small displacement of the contact on the wire from the balance point. (London G.C.E. Adv. L.).

7. Deduce from first principles an expression for the rate at which heat (in calories per sec.) is generated in a circuit of resistance R ohms, in which a known current i amp. passes.

Explain how you would measure *all* the unknown quantities involved in the formula. (London G.C.E. Adv. L.).

8. Define the M.K.S. units of current, potential difference, resistance.

Obtain, from first principles, an expression for the heat in calories dissipated in a conductor of resistance R ohms when a potential difference V volts is maintained across it for a time t seconds.

An electric heating element to dissipate 450 watts on 250 volt mains,

is to be made from nichrome ribbon of width 1 mm. and thickness 0·05 mm. Calculate the length of ribbon required. (The resistivity of nichrome is 110×10^{-8} ohm.m.). (London G.C.E. Adv. L.).

9. What is meant by the mechanical equivalent of heat?

A current of 1·5 amps. is passed through a coil of wire having a resistance of 1·3 ohms immersed in 40 gm. of water contained in a lagged calorimeter which has a water equivalent of 10 gm. The temperature of the water rises 6°C. in 7 minutes. Find the mechanical equivalent of heat. (Oxford Schol. Subsidiary Physics).

10. Describe and explain how you could determine *either* the mechanical equivalent of heat (J) *or* the latent heat of vaporisation of water (L) by an electrical method.

A litre of water at room temperature (20°C.) is put into an electric kettle also at room temperature. How many " units " (kilowatt-hours) of electricity are used

(a) if the current is switched off when the water boils (assuming none has been lost as vapour) and

(b) if the current is switched off after $\frac{1}{8}$ of the water has boiled away?

(You may assume that 20 per cent of the energy supplied is wasted, that the water equivalent of the kettle is 50 gm., that $L = 540$ cal.gm.$^{-1}$ and that $J = 4{\cdot}2$ Joules.cal.$^{-1}$). (Oxford Schol. Subsidiary Physics).

11. Describe an accurate method for measuring the mechanical equivalent of heat.

Using the Callendar and Barnes method, a current of 0·75 amp. passing through the heater gave a temperature rise of 3°C. with a rate of flow of 5 gm. per sec. When the rate of flow was reduced by two-thirds, the same temperature rise was produced by a current of 0·5 amp. What was the potential difference across the heater? ($J = 4{\cdot}2$ Joules.cal.$^{-1}$). (Oxford Schol. Subsidiary Physics).

12. Define the units of *current, potential difference* and *resistance.*

Obtain an expression for the heat produced in a resistance by the passage of a current for a known time.

A battery of e.m.f. of 8 volts and internal resistance 1 ohm is connected in series with coils of resistance 3, 4 and 5 ohms respectively. The 4-ohm coil is immersed in a liquid contained in a calorimeter and the rise in temperature of the liquid is found to be 1 deg. C. per minute. What would be the rate of rise in temperature if the 3 and 5 ohm resistances were connected in parallel in the circuit? (London G.C.E. Adv. L.).

13. Deduce from first principles an expression for the heat developed in a resistance R ohms, when a current of I amperes flows for t seconds.

A wire carrying a current of 3 amperes reaches a steady temperature of 30°C. in a room which is maintained at a steady temperature of 16°C. What will be the steady temperature of the wire when the current is increased to 4 amperes? The emissivity of the surface of the wire may be assumed to remain constant and the coefficient of increase of resistance of the material of the wire with temperature

may be taken as 0·0036 per degree Centigrade. (London G.C.E. Adv. L.).

14. Describe an experiment to show how the heating effect of a given electric current depends on the resistance of the conductor.

Assuming that the rate of heat loss from the surface of a bare wire, heated electrically, obeys Newton's law of cooling, show how the final temperature difference set up between the wire and its surroundings depends on (*a*) the value I of the steady electric current in the wire, (*b*) the resistivity ρ of the material of the wire and (*c*) the diameter d of the wire.

Would you expect the same wire carrying the same current to attain the same temperature if it is covered by insulating material? Give reasons for your answer. (London Inter. B.Sc.).

15. Derive an expression for the rate of generation of heat in a wire of resistance R ohms when carrying a current I amp. Describe how you would verify the expression experimentally.

Calculate the heat radiated/*unit area/unit time* from the surface of a straight tungsten filament lamp in terms of the current (I), radius (a) of the wire and the resistivity (ρ) of the tungsten. Assuming no end effects and the filaments to be running at the same temperature, determine the ratio of the filament radii of a 50-watt, 110-volt lamp and a 100-watt, 220-volt lamp. (London G.C.E. Adv. L.).

16. Describe an experiment to determine the resistance of a wire by a calorimetric method.

It is desired to construct a 5-amp fuse from tin wire which has a melting-point of 230°C. and resistivity 22×10^{-8} ohm.m. at that temperature. Estimate the diameter of the wire required if the emissivity of its surface is 21×10^{-1} cal. per sq m. per sec. per deg. C. excess temperature above the surroundings whose temperature is 20°C. Neglect the heat lost by conduction along the wire. ($J = 4{\cdot}2$ joules.cal.$^{-1}$). (Northern H.S.C.).

17. A straight tungsten wire, 100 cm. long, is heated *in vacuo* to a temperature of 2200°C. by a current from a 200-volt D.C. supply and consumes power at the rate of 100 watts. Assuming the specific resistance of tungsten to be proportional to the absolute temperature, calculate (*a*) the diameter of the wire, (*b*) the emissivity of the tungsten surface, (*c*) the voltage at which the power consumed would be 30 watts.

Assume the emissivity to be independent of temperature and explain clearly what other assumptions you make. Indicate briefly what other considerations would enter into the calculation if an alternating instead of a direct current supply were used.

(Stefan's constant $= 5{\cdot}7 \times 10^{-8}$ joules.m.$^{-2}$, degree^{-4}, sec.$^{-1}$. specific resistance of tungsten at 727°C. is $2{\cdot}5 \times 10^{-7}$ ohm.m.). (Camb. Open Schol.).

18. An electrical circuit is composed of copper wire 3 mm. in diameter and is protected by a fuse wire of different material with a diameter of

1 mm. Calculate for how long a current of 30 amp. may flow through the circuit before the fuse melts and what the temperature of the copper will then be. You may ignore losses of heat in the short time involved.

	Copper	*Fuse material*
Density (kmg.m.$^{-3}$)	$8{\cdot}9 \times 10^3$	$10{\cdot}7 \times 10^3$
Resistivity (ohm/m.cube)	$1{\cdot}7 \times 10^{-8}$	$3{\cdot}5 \times 10^{-7}$
Specific Heat (cal.kgm.$^{-1}$deg.C.$^{-1}$)	90	50
Melting Point (°C.)	—	200
Latent Heat of Fusion (cal.kgm.$^{-1}$)	—	8,000

$J = 4{\cdot}18$ joules.cal.$^{-1}$

(Manchester Open Schol.).

19. What are the essential parts of a common potentiometer? What are its advantages compared with other types of voltmeter?

Give a circuit diagram, showing the value of any resistances or other circuit elements you may include, of a 20-ohm potentiometer, driven by a 2-volt accumulator, used to measure:

(*a*) the e.m.f. of a thermocouple known to produce a maximum e.m.f. of 5 mV.

(*b*) the current flowing in a circuit, which is known to be of the order of 10 amp.

In measuring the e.m.f. of an unknown cell (whose e.m.f. is in fact 1·1 volt and whose internal resistance is 1 ohm), an experimenter taps in at the centre point of a 50-ohm potentiometer wire supplied by a 4-volt accumulator of low internal resistance. If he uses a 14-ohm galvanometer without any protection, what current will flow through it? (Oxf. & Camb. Joint Board G.C.E. Adv. L.).

20. Describe the construction of the standard Western cell, and explain its action.

The e.m.f. of a given thermocouple for a certain temperature difference between the junctions is of the order of 0·004 volt. How would you use a potentiometer to measure this, in terms of the e.m.f. of a standard cell?

On testing the assembled circuit, it appears to be satisfactory so long as the thermocouple is in action; but with the standard cell the galvanometer deflects in one direction only, no matter where contact is made on the potentiometer wire. State possible causes of the trouble, and indicate how you would proceed to put it right. (Oxford Local G.C.E. Adv. L.).

21. Describe an experimental method of producing a thermo-electric e.m.f.

How may a thermojunction be used to measure temperature?

Why is a copper-iron junction not used to measure temperature above 250°C., although a copper-constantan junction is often so employed? (London G.C.E. Adv. L.).

22. Describe experiments, one in each case, to illustrate the (*a*) Seebeck effect, (*b*) Peltier effect.

Give a diagram and explain the method of use of an apparatus,

based on the Seebeck effect, to detect radiation from a furnace or similar source of heat. (London G.C.E. Adv. L.).

23. Give an account of the Seebeck, Peltier and Thomson effects.

Describe how you would use a thermocouple for the measurement of temperature and explain how you would calibrate it.

What are the chief advantages and disadvantages of thermoelectric thermometry? (Oxford Local G.C.E. Adv. L.).

24. What is a thermocouple?

Explain the use of a potentiometer to measure the small electromotive forces developed by a thermocouple.

What are the relative advantages and disadvantages of a thermocouple used as a thermometer as compared with the resistance thermometer? (London Inter. B.Sc.).

25. Give an account of the Seebeck Effect.

Describe the structure and use of two instruments depending on this effect, one designed for the measurement of high temperatures and the other for the detection of infra-red radiation. (Durham G.C.E. Schol.).

26. Give an account of the properties of thermocouples and give *two* examples of their use.

Explain how you would measure the e.m.f. of a thermocouple by means of a potentiometer circuit, assuming that the maximum e.m.f. to be measured is $\frac{1}{50}$ volt. (Camb. Local G.C.E. Adv. L.).

27. Give an account of the chief phenomena of thermoelectricity, and explain the use of thermoelectric lines plotted for various metals against lead.

Discuss the advantages and disadvantages of the thermocouple as an instrument for the measurement of temperatures. (Oxford Local G.C.E. Adv. L.).

28. Write a short account of the variation with temperature of the electrical resistance of pure metals.

Give a brief description of a platinum resistance thermometer, and explain the principle of the electrical circuit in which it is normally used.

The resistance of the tungsten filament of an electric lamp measured by means of a Wheatstone bridge was found to be 44·0 ohms at 0°C. and 65·7 ohms at 100°C. When the lamp was running normally on 230 volt mains it took 100 watts. Calculate the temperature of the hot filament measured on the tungsten resistance scale. (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

CHAPTER 8

ELECTROLYSIS

8.1. Introduction

The conduction of electricity through solutions of various substances and molten salts is known as *electrolysis*. Such conduction always involves chemical action in the solution. The liquids are called *electrolytes*, the common electrolytes being solutions of inorganic salts, acids and bases. To produce electrolysis, two electrodes connected to an external source of e.m.f. are immersed in the electrolyte. Current enters at the *anode* and leaves at the *cathode*.

Electrolysis was investigated by Faraday in the mid-nineteenth century. His findings formed the basis of the *ionic theory* of electrolysis.

Note: *In the theory involved in this chapter all masses are measured in grams and lengths in cm. for convenience.*

8.2. Chemical Effects at the Electrodes

Two main actions occur during electrolysis; (i) the metal atoms of the electrolyte are liberated at the cathode; (ii) the non-metal part of the solute molecules (radicles) are liberated at the anode. Whether chemical action takes place at either electrode depends on the metal used. This is made clearer with examples:

(i) When a solution of copper sulphate ($CuSO_4$) is "electrolysed" using platinum electrodes, the copper atoms are deposited at the anode to form a coating of pure copper. The sulphate radicles (SO_4), which are deposited at the anode, do not remain there since SO_4 cannot exist in the free state. A chemical effect occurs equivalent to the reaction:

$$SO_4 + H_2O \longrightarrow H_2SO_4 + O \quad . \quad . \quad . \quad (1)$$

The oxygen atoms form diatomic molecules and escape as oxygen gas from the surface of the anode.

(ii) When copper electrodes are used with $CuSO_4$, copper is deposited at the cathode but the sulphate radicles react with the copper anode to form copper sulphate which goes into solution. The concentration of the copper sulphate solution is thus maintained, whereas with platinum electrodes the concentration decreases as electrolysis proceeds.

(iii) *The electrolysis of water*: pure distilled water is a dielectric and shows a negligible ability to conduct; the conduction process is made easy by adding some dilute sulphuric acid. The sulphuric acid molecule H_2SO_4 is split up into a metallic-like part: H_2, and a sulphate radicle: SO_4. With platinum electrodes hydrogen atoms are liberated at the

cathode as a gas. The sulphate radicles liberated at the anode produce oxygen according to (1). The sulphuric acid content in the water remains constant and the liberated hydrogen and oxygen thus come from the water molecules.

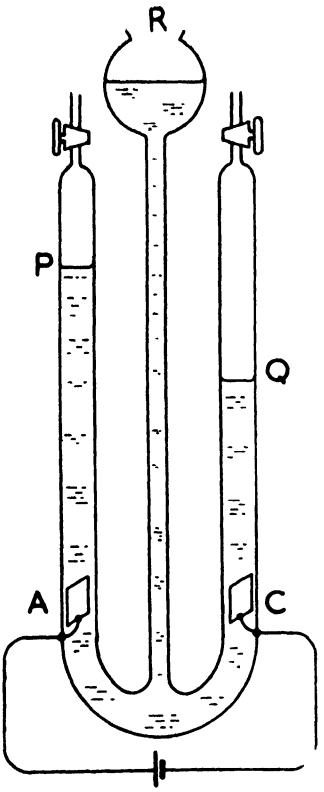


Fig. 8.1

The electrolysis of water is effected with a water *voltameter*. This is shown in Fig. 8.1. The tubes P and Q and reservoir R are initially filled with the acidulated water. The platinum anode A and cathode C are connected to an accumulator. The solution in P and Q is displaced by the gases liberated, the volume of gas in Q being twice that in P. The gas in P is oxygen, and that in Q is hydrogen.

8.3. Products of Electrolysis

It is possible to predict the ultimate products of any electrolysis by using chemical knowledge In general, the liberated metallic (or hydrogen) atoms do not take part in chemical reactions with the cathode. If, however, the metal normally reacts with the solvent, the ultimate products are those expected from such reactions. For example, using sodium chloride in aqueous solution, the liberated sodium produces hydrogen according to:

$$Na + H_2O \longrightarrow NaOH + H \quad . \qquad (2)$$

Incidentally, this makes it impossible to produce metallic sodium by electrolysis from aqueous solution.

The radicles liberated at the anode are often incapable of free existence, and react with the solution or the electrode material. For example, the sulphate radicle reacts with electrodes of *Cu, Zn,* etc. but not with *Pt.*

8.4. Faraday's Laws

From experiments conducted with different electrolytes, Faraday deduced the following laws:

(i) *The mass of an element liberated or evolved at an electrode during electrolysis, is directly proportional to the quantity of charge which flows.*

(ii) *When the same quantity of charge flows through different electrolytes, the mass of any element liberated or evolved is directly proportional to its chemical equivalent.*

8.5. Electrochemical Equivalent

If the mass of substance liberated by a current I amps. flowing for t secs. is m. gm., it follows from (i) that:

$$m \propto It$$

i.e.
$$m = zIt \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where z is a constant known as the *electrochemical equivalent* (e.c.e.) of that element.

The e.c.e. of an element is the mass of that element in gm. liberated or evolved during electrolysis when one coulomb of charge flows.

8.6. Equivalent weight, valency, etc.

The chemical equivalent of an element may be defined as that number of grams of the element which combine with 8 gm. of oxygen. Equivalent weight was formerly defined as the number of grams of an element which combine with one gram of hydrogen, but since elements form compounds with oxygen far more easily than with hydrogen, it is preferable to use the new definition relative to oxygen. For example, sodium forms the oxide Na_2O, in which 46 gm. of sodium combine with 16 gm. of oxygen (atomic weight of sodium = 23, atomic weight of oxygen = 16), hence the equivalent weight of sodium is 23 gm. This also follows from the hydride NaH.

The *valency* of an element represents its *combining power*, and is defined as the number of atoms of hydrogen which combine with one atom of the element. Hence, the valency of any element is obtained from the formula of the compound it forms with hydrogen, e.g. from the formulae HCl and MgH_2 it follows that the valencies of chlorine and magnesium are unity and two, respectively. Since two atoms of hydrogen combine with one atom of oxygen, valency is also determined as half the number of atoms of the element which combine with one atom of oxygen. Thus the valency of sodium is ascertained to be unity from the formula Na_2O. Elements behave as though they had more than one valency, e.g. nitrogen forms the oxides: N_2O, NO, N_2O_3, NO_2 and N_2O_5. Substituting $2H$ for each O-atom in these formulae gives the valency of nitrogen as one, two, three, four and five respectively.

From above, it follows that one gram-atom of an element with valency v combines with v gram-atoms of hydrogen. Hence, $\frac{1}{v}$ gram-atoms of the element combine with 1 gram of hydrogen or 8 grams of oxygen, i.e. *the equivalent weight of an element* ($\mathscr{E}$) *is given by the atomic weight A divided by the valency v*;

i.e,
$$\mathscr{E} = \frac{A}{v} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

8.7. Experimental Proof of Faraday's Laws

1. *First Law.* A copper voltameter is used consisting of a cylindrical glass vessel, containing a solution of copper sulphate. Three parallel copper electrodes A, A' and C, are suspended from the ebonite lid (Fig. 8.2). The outer electrodes A and A' are connected together to form the anode and C forms the cathode. The cell is connected to a Key K, a tangent galvanometer $T.G.$, a battery B and a rheostat Rh, in series. The tangent galvanometer is used because it is an absolute instrument which does not require calibration by comparison with a standard ammeter. The reversing switch is always used with the galvanometer.

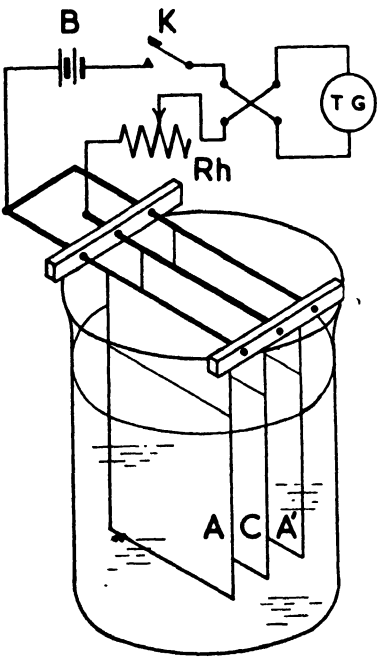


Fig. 8.2

After cleaning with sand-paper and washing with distilled water C is weighed. It is replaced and $T.G.$ adjusted for normal use. K is depressed momentarily and Rh adjusted to give a suitable 45° (approx.) deflection on $T.G.$ It is finally closed at the same time that a stop watch is started. Electrolysis is allowed to proceed for a definite time (t secs.) the reading θ of the galvanometer being noted. After electrolysis C is carefully removed. It is found to be covered with a clean film of metallic copper. The electrode is washed with distilled water and dried very carefully above a Bunsen flame. When dry it is reweighed and the copper deposit noted: m gm.

The experiment is repeated using the same current strength I but different times of electrolysis. It is also repeated using different values of current strength for the same time. The curve of m against t . tan θ is a straight line passing through the origin. It follows that: $m \propto t$. tan θ, and since tan $\theta \propto I$, $m \propto It$, i.e. the mass liberated is directly proportional to the quantity of charge which flows.

E.C.E. Copper

From the tangent galvanometer readings values of current strengths are obtained in amps. For each experiment: $m = zIt$, so that: $z = \frac{m}{It}$. By this means the e.c.e. of copper is found to be $3{\cdot}29 \times 10^{-4}$ gm. coulomb^{-1}.

2. *Second Law.* Several voltameters containing different electrolytes are connected in series, (Fig. 8.3); V_1 . . . V_4, contain copper sulphate

solution, silver nitrate solution, a solution of ferric chloride and dilute sulphuric acid. All have platinum electrodes.

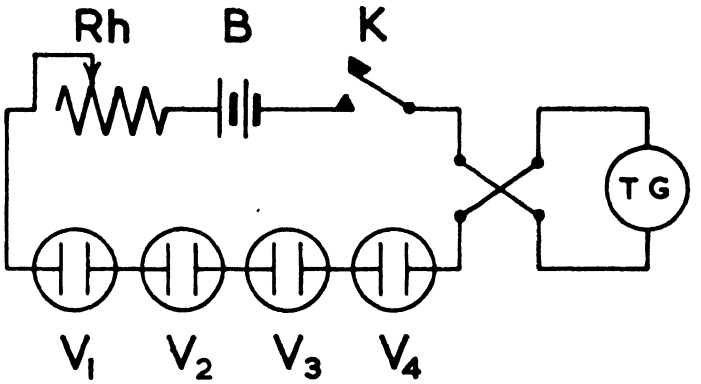


Fig. 8.3

The experimental procedure is similar to 1., the masses liberated at the various cathodes being determined for the same time interval and constant current strength. The elements deposited are copper, silver, iron and hydrogen. The masses of the metal deposits are found by direct weighing, whilst the mass of the hydrogen is found by first converting the volume liberated, at the given temperature and pressure (atmospheric minus the saturation vapour pressure of water), to the equivalent volume at N.T.P., using the formula: $\frac{p_o V_o}{T_o} = \frac{p_1 V_1}{T_1}$. The volume V_o, at N.T.P., is multiplied by the density: $8{\cdot}988 \times 10^{-2}$ gm. per litre at N.T.P., to give the mass.

From the formulae for the solutes used, the valencies and equivalent weights of the elements liberated are found. The valency of copper in copper sulphate is two, so that the equivalent weight of copper is $\frac{63{\cdot}51}{2} = 31{\cdot}75$. The valencies of the silver and iron are one and three, respectively, and the equivalent weights are: 108/1 and 56/3, i.e. 108 and 18·7 respectively.

It is found that the masses liberated are directly proportional to their equivalent weights, i.e.:

$$m_{Cu} : m_{Ag} : m_{Fe} : m_H :: \mathscr{E}_{Cu} : \mathscr{E}_{Ag} : \mathscr{E}_{Fe} : \mathscr{E}_H :: 31{\cdot}75 : 108 : 18{\cdot}7 : 1 \quad (5)$$

thus proving Faraday's law for masses deposited at the cathodes of the four voltameters.

8.8. Conductivity of Electrolytes

Polarization and Back e.m.f. If, a p.d. less than 1·7 volts is applied between electrodes in dilute sulphuric acid, the initial current strength falls until after a few minutes it is negligible. This is demonstrated

with the circuit of Fig. 8.4. The voltameter is connected to the Daniell cell B, (e.m.f. = 1·1 volts), by closing the points a and b of switch S. The current is read on the milliameter $m.A.1$.

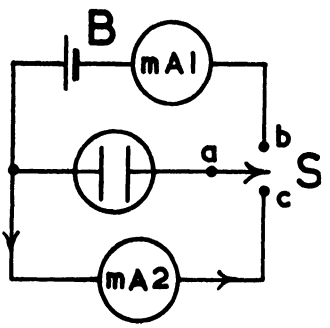


Fig. 8.4

The current strength soon becomes negligible and a and c are then connected. It is found that a small current flows for a short time through the milliammeter $m.A.2$. The source of e.m.f. producing this current is the voltameter itself. Current flows in the direction indicated, so that the potential of the anode is greater than that of the cathode. This p.d. opposes that of the Daniell cell and is built up during electrolysis to prevent current flow. It is known as a *back e.m.f.* When a sufficiently large p.d. is used across the voltameter the maximum back e.m.f. is developed which is 1·7 volts. When current ceases to flow the cell is said to be *polarized.*

All voltameters using electrodes of different material to the metal atom of the solute give a back e.m.f. With electrodes similar to metal atoms in solution, polarization does not occur. Thus, with copper electrodes and copper sulphate solution there is no back e.m.f. In the case of dilute sulphuric acid the polarization is caused by charged hydrogen and oxygen atoms (ions) being adsorbed on the electrodes. These charges set up an electric field which destroys the applied field and prevents current flow. When the applied p.d. is removed, the adsorbed atoms give rise to the back e.m.f.

Experiments on Conductivity. Kohlrausch found that when sufficient voltage was applied to overcome back e.m.f. the current strength was always proportional to the difference in the applied p.d. and the back e.m.f., i.e.:

$$I \propto V - E_b \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where I is the current strength in amps., V the applied voltage and E_b the back e.m.f. Thus Ohm's Law is valid in a slightly modified form, viz. $I = \frac{V_e}{R}$ where R is the resistance of the electrolyte and $V_e = V - E_b$. Kohlrausch also found that Ohm's Law is obeyed in the original form, i.e. $I \propto V$, when polarization is absent.

For plane parallel electrodes, distance L cm. apart and having a cross-section of A cm.^{2}, the specific resistance of the electrolyte whose resistance is R ohms, is: $\rho = \frac{RA}{L}$ ohm.cm. and the *conductivity* of the electrolyte is given by:

$$g = \frac{1}{\rho} = \frac{L}{AR} \text{ ohm}^{-1}\text{cm.}^{-1} \text{ (mho.cm.}^{-1}\text{)} \quad . \quad . \quad (7)$$

8.9. Experimental Determination of "g"

Polarisation is reduced to a minimum if alternating current is used, or if the current is made and broken with a high frequency. Kohlrausch's method of finding conductivity is based on this fact. A simple form of the apparatus is sketched in Fig. 8.5. The electrolytic cell consists of

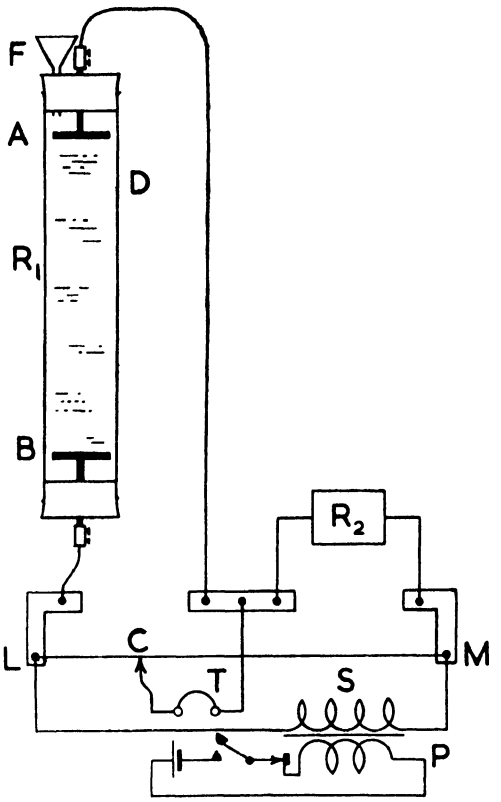


Fig. 8.5

two copper disc electrodes A and B soldered to stout copper wires held in rubber bungs at the ends of the glass tube D. The tube is filled with the electrolyte via F.

The electrodes are connected as one arm R_1 of a Wheatstone bridge, the arm R_2 being a resistance box of several thousands ohms. The bridge is supplied by the secondary S of a small induction coil (buzzer). When the primary key is depressed the e.m.f. across S is reversed many times per second.

The telephone receiver T replaces the usual bridge galvanometer. The receiver diaphragm vibrates with the frequency of the alternating current producing sound of the same frequency. The criterion for bridge balance is hence silence from T.

Assuming no inductance or capacity, the usual formula holds for Wheatstone's Bridge, and at balance:

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where l_1 and l_2 represent LC and CM in cm. For greatest sensitivity R_2 is similar to R_1. Complete silence is never achieved with simple apparatus because of alternating current effects associated with the components of the apparatus. As a result the position of C is usually in doubt to several mm. Assuming A and B are the same diameter as D, so that current flows in parallel lines between them, the cross-section of the electrolyte is found by measuring the diameter (d cm.). Knowing the distance L between A and B, the conductivity of the electrolyte is given by:

$$g = \frac{L}{R_1 \frac{\pi d^2}{4}} = \frac{4Ll_2}{\pi d^2 l_1 R_2} \text{ mho.cm.}^{-1} \quad . \quad . \quad (9)$$

The difficulties associated with non-parallel current flow are avoided by using an electrolytic cell having fixed electrodes. The quantity L/d^2 is then found by using an electrolyte whose conductivity is known. Since L/d^2 is a constant, the cell may be used for subsequent conductivity measurements, by measuring R_2, l_1 and l_2. Potassium chloride is used extensively as a standard electrolyte for this purpose.

8.10. Ionic Theory of Electrolysis

Faraday explained his experimental results by assuming that the current in electrolysis was carried by charged atoms which he called ions. The theory was developed by Arrhenius in 1887. This ionic theory is able to explain the following experimental results:

(i) The conduction process begins immediately after the application of the smallest p.d., providing no polarization exists.

(ii) Ohm's Law is obeyed.

(iii) Faraday's laws of electrolysis are obeyed.

Arrhenius supposed that solute molecules were dissociated into a *positive ion*, consisting of the metal or hydrogen atoms of the molecule carrying a definite positive charge (*cation*), and a negative ion, consisting of the rest of the molecule (radicle) carrying an equal and opposite negative charge (*anion*). The dissociation of the solute molecules is not necessarily complete, i.e. there exists a *degree of dissociation*, depending on the solute and its concentration in the solution.

Ions have properties different to those of uncharged atoms, e.g., the cation formed when sodium chloride is the solute is represented by Na^+, the superscript indicating its positive charge. If this ion had the normal properties of the sodium atom it would react immediately with

water to give sodium hydroxide and hydrogen. That the latter does not occur is sufficient evidence for the difference between the ion and atom.

When current flows, ions give up their charges to the electrodes and are liberated as ordinary atoms which give rise to secondary reactions.

8.11. Solution and Dissociation

The dissociation of solute molecules on solution is explained by supposing that in the solid state the substance is crystalline, consisting of oppositely charged metal and radicle ions held together in a regular structure by electric forces. On immersing the solute in a solvent which, like pure water, is an insulating medium with a large dielectric constant ($\epsilon = 80$ for H_2O) the electric force between neighbouring ions is reduced by the factor ϵ, [$(F = q_1q_2/(4\pi\epsilon\epsilon_0 r^2)$ instead of: $F = q_1q_2/(4\pi\epsilon_0 r^2)$ in vacuum)]. The bonds between the ions are weakened and if ϵ is large enough rupture occurs, the process being assisted by the kinetic energy the ions normally possess. The ions then become free in solution. In molten salts dissociation is caused by excessive heat motions.

The degree of dissociation depends on the ionic binding forces and *recombination*. Recombination is the formation of a neutral molecule from two oppositely charged ions. This happens when ions collide in their random motions through the solution. In the equilibrium state the rate of recombination is equal to the rate of dissociation which is then constant.

Electrolytes are often classified as weak or strong. In weak electrolytes the solute is only slightly dissociated—examples are acetic acid solution and any organic compound in aqueous solution. Strong electrolytes are almost completely dissociated and most inorganic salts, acids and bases (in aqueous solution), belong to this category.

8.12. The Theory of Conduction (polarization absent)

Suppose an electrolyte having n positive and n negative ions per c.c. is contained between two parallel plane electrodes of cross-section S cm.^{2} separated by a distance of d cm. When a p.d. of V volts is applied to the electrodes a uniform electric field is created whose intensity is: $E = \frac{V}{d}$ volt.cm.$^{-1}$. Let the ions of opposite signs have charges of $+q$ and $-q$ coulombs. When the electric field is created each positive ion is submitted to a force $Eq \times 10^{-2}$* newtons urging it towards the cathode, whilst each negative ion experiences a similar force urging it towards the anode.

The force acting on the ions causes superimposed motion in the direction of the field, which modifies the otherwise random motion.

* The factor 10^{-2} is necessary because E is measured in volt.cm.$^{-1}$

The ions still undergo collisions which prevent continual acceleration in the direction of the field and are equivalent in their effect to a resistive force. After a small initial acceleration this resistive force becomes equal to the applied force and the ions acquire a steady *drift velocity* in the direction of the field.

The drift velocity of each type of ion is proportional to E. If the drift velocities are u_+ and u_- cm.sec.$^{-1}$:

$$u_+ = k_+E \text{ and } u_- = k_-E \quad . \quad . \quad . \quad (10)$$

where k_+ and k_- are called the *ionic mobilites* of the positive and negative ions respectively. *Ionic mobility is thus the drift velocity acquired by an ion in a field of unit intensity.* (Unit: cm.sec.$^{-1}$ per volt.cm.$^{-1}$).

The drift of ions commences immediately the field is applied, so that those ions adjacent to an electrode and possessing the appropriate sign of charge are immediately collected there. They give up their charge and a flow of current is detected in the external circuit. The current strength is the amount of charge delivered up to either electrode per second. This is the same as the quantity of charge passing any point in the electrolyte per second.

The number of positive ions passing any point in the electrolyte per second is given by:

$$N_+ = Snu_+ \quad . \quad . \quad . \quad . \quad . \quad (11)$$

since each ion travels u_+ cm. per second and the total number passing the point is contained in a prism of cross-section equal to that of the containing vessel (S cm.^{2}) and length u_+ cm. The total positive charge passing this point per second is hence:

$$I_+ = Snqu_+ \text{ amps.} \quad . \quad . \quad . \quad . \quad (12)$$

Likewise, the number of negative ions passing the same point in the opposite direction is:

$$N_- = Snu_- \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Whence:

$$I_- = Snqu_- \text{ amps.} \quad . \quad . \quad . \quad . \quad (14)$$

A current of negative charges in one direction is exactly equivalent to a current of positive charges in the opposite direction. Hence, the total ion current strength is:

$$I = I_+ + I_- = Snq(u_+ + u_-) \text{ (amps.)} \quad . \quad . \quad (15)$$

Since $u_+ = k_+E$ and $u_- = k_-E$:

$$I = SnqE(k_+ + k_-) \quad . \quad . \quad . \quad . \quad (16)$$

Since $E = \frac{V}{d}$:

$$I = \frac{SnqV}{d}(k_+ + k_-) \quad . \quad . \quad . \quad . \quad (17)$$

The mobilities of the ions are constant for a given concentration so that from (17):

$$I \propto V \quad . \quad . \quad . \quad . \quad . \quad (18)$$

i.e. Ohm's Law is obeyed.

8.13. Faraday's Laws

The numbers of atoms deposited at each electrode per second are the same, otherwise one electrode gains charge relative to the other and the p.d. between the two electrodes changes, which is contrary to experience.

From (15) the number of ions giving up their charges per second at each electrode, is given by:

$$N = \frac{I}{q} = Sn(u_+ + u_-), \quad . \quad . \quad . \quad (19)$$

since the current strength at each electrode is the same as that in the electrolyte.

The total mass of the atoms liberated in t seconds at the cathode is hence:

$$M_+ = Snm_+ (u_+ + u_-)\, t \text{ gm.} \quad . \quad . \quad . \quad (20)$$

where m_+ is the mass of each atom in gm.

From (15): $I = Snq\,(u_+ + u_-)$ (amps.)

Hence: $$M_+ = \frac{m_+}{q} . It. \text{ gm.} \quad . \quad . \quad . \quad . \quad (21)$$

From which it follows that the mass deposited is proportional to the quantity of charge which has passed, which is Faraday's First Law. Furthermore, by definition, the electro-chemical equivalent of the atoms liberated is given by:

$$z_+ = \frac{M_+}{It} = \frac{m_+}{q} \text{ gm. coulomb}^{-1} \quad . \quad . \quad . \quad (22)$$

Similar formulae are obtained for the anode.

Faraday's Second Law follows from these results if it be supposed that *the charge carried by each ion is equal to the valency of the element multiplied by a constant unit of charge.* If the latter is e coulombs, the charge carried by each ion is: $q = ve$, and (22) becomes:

$$M_+ = \frac{m_+}{ve} . It \text{ gm.} \quad . \quad . \quad . \quad . \quad (23)$$

The masses deposited by the same quantity of current at the cathodes of two different electrolytes, is then given by:

$$\frac{M'_+}{M''_+} = \frac{m'_+ \, It \, . \, v''e}{v'e \, . \, It \, . \, m''_+} = \frac{m'_+ v''}{m''_+ v'}$$

Since the mass of an atom is given by Am_H, where A is the atomic weight and m_H the mass of the hydrogen atom:

$$\frac{M'_+}{M''_+} = \frac{A' m_H v''}{A'' m_H v'} = \frac{\mathscr{E}'}{\mathscr{E}''} \quad . \quad . \quad . \quad (24)$$

where $\mathscr{E}'$ and $\mathscr{E}''$ are the equivalent weights of the two elements concerned.

8.14. Electrode Processes

The processes involved at the electrodes during electrolysis are quite complex. In particular, the concept of secondary reactions is not now regarded as being correct. Fundamentally, the process of electrolysis involves the transfer of electrons to positive ions at the cathode and from negative ions to the anode.

An aqueous solution contains ions other than those of the solute, viz. hydroxyl $(OH)^-$ and hydrogen ions H^+, caused by a slight dissociation of the water molecules. The hydrogen ions of the water and the positive ions of the solute are both able to acquire electrons from the cathode. The deposition of a particular ion is dependent on the applied potential between the electrodes, the ion concentration and the nature of the electrodes. If it is easier for the hydrogen, rather than the metal ions, to acquire electrons hydrogen only is liberated at the cathode. This occurs when ferrous sulphate ($FeSO_4$) solution is electrolysed using platinum electrodes. With silver nitrate ($AgNO_3$), however, the silver ion is more easily neutralised than the hydrogen ions. In simple cases the resultant effect is the same as that expected from assuming secondary reactions to occur.

At the anode the negative ions of the solute, or the $(OH)^-$ ions of the water, can give up their electrons or the positive ions of the electrode metal may go into solution. The process which occurs is that which is easiest from the energy point of view. With silver nitrate, using silver electrodes, neither the $(NO_3)^-$ nor the $(OH)^-$ ions give up their electrons, but the silver atoms of the electrode form silver ions *in* the solution, i.e. the anode dissolves. With dilute sulphuric acid and platinum electrodes the $(SO_4)^=$ ions are not liberated and Pt ions do not pass into solution, but $(OH)^-$ ions give up their negative charges and subsequently dissociate to form gaseous oxygen and water ($2OH \rightarrow H_2O + O$). The resultant effect is equivalent to that noted earlier ($SO_4 + H_2O \rightarrow H_2SO_4 + O$), but the incongruity implied in the argument that the sulphate ion forms sulphuric acid, which is immediately dissociated again, is avoided.

Under standard conditions, using platinum electrodes, a minimum p.d. exists for the transfer of charge at the electrodes. This is known as the *decomposition voltage*. It is 0·7 volts for silver nitrate, 2·3 volts for zinc sulphate, 1·7 volts for sulphuric acid, etc. This is of importance when electrolysing a solution of mixed salts. Since each salt solution has its characteristic decomposition voltage, the applied p.d. can be arranged so that only one kind of metal ion is deposited at a time. When this has been completely removed from the solution the p.d. is raised to allow the next metal to be deposited, and so on. The idea is utilized commercially in separating pure metals from solution.

8.15. Atomicity of Electricity

One of the most important features of the ionic theory is the integral nature of ion charges. This is readily understood in the light of the modern conception of the atom. A positive ion is an atom or molecule from which one or more electrons have been removed, and a negative ion is an atom which has acquired electrons. Because of this, ions can be represented by their chemical symbols with super-scripts, indicating electron gain or loss, e.g. Na^+, Cl^-, H^+, Ca^{++}, $(SO_4)^=$, etc.

Prior to the development of atomic theory the possibility of their being a natural unit of charge led to much conjecture on the *atomicity of electricity*. The idea was linked with atomic theory developments in chemistry. To explain the laws of chemical combination Avogadro had put forward a famous hypothesis, which states that equal volumes of all gases contain equal numbers of molecules, at the same temperature and pressure. Assuming the truth of this hypothesis, it is not difficult to show that one gm-molecule of any substance must contain the same number of molecules. This is *Avogadro's Number* ($N = 6{\cdot}023 \times 10^{23}$).

8.16. The Faraday

Since molecules split up into equal numbers of positive and negative ions on solution, it follows that one gram-atom of any element liberated in electrolysis contains N atoms and if each carries a charge ve coulombs, the charge required to liberate a gram-atom of any element must be Nve.

Hence, the charge required to liberate one *gram-equivalent* of an element is: $\frac{Nve}{v} = Ne$ coulombs. This quantity of charge is called the *Faraday* (F). The Faraday is determined using a silver voltameter (p. 235). ($F = Ne = 6{\cdot}203 \times 10^{23} \times 1{\cdot}601 \times 10^{-19} = 96500$ coulombs approx.).

8.17. The Faraday, Electrochemical Equivalent and e/m_H

The charge required to liberate one gm. equivalent of a substance ($\mathscr{E}$) is F coulombs, hence from (23):

$$\mathscr{E} = \frac{m_+}{ve} \cdot F \quad . \quad . \quad . \quad . \quad (25)$$

The mass of the atom is given by: $m_+ = Am_H$, so that:

$$\mathscr{E} = \frac{Am_H}{ve} \cdot F = \frac{\mathscr{E} m_H}{e} \cdot F$$

whence:

$$\underline{F = \frac{e}{m_H}} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Hence, *the value of the Faraday is the same as the ratio of charge to mass of a hydrogen ion* (i.e. a *proton*), which is constant.

From (22) it follows that:

$$z_+ = \frac{m_+}{q} = \frac{Am_H}{ve} = \frac{\mathscr{E} \cdot m_H}{e} = \frac{\mathscr{E}}{F} \quad . \quad . \quad (27)$$

and for different elements:

$$\frac{z_1}{z_2} = \frac{\mathscr{E}_1}{\mathscr{E}_2} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The above relations are found to be true experimentally, thus supporting the atomicity of electricity.

Example

State Faraday's laws of electrolysis.

Find the volume of oxygen at N.T.P. liberated per second in a water voltameter carrying a current of 1 *ampere. Atomic weight of oxygen* = 16. (*Faraday's constant* = 96,500 *coulomb/gm. equivalent. A gm. mol. of gas occupies* 22·4 *litres at N.T.P.*). (Oxford Open Schol. Subsid.).

Faraday's laws of electrolysis are given in the text.

Since oxygen is divalent its equivalent weight is:

$$\frac{A}{v} = \frac{16}{2} = 8 \text{ gm.}$$

Since 96,500 coulombs are required to liberate 1 gm. equivalent of any element the mass of oxygen liberated by one coulomb is given by:

$$m = \frac{8}{96{,}500} \text{ gm.}$$

Oxygen is diatomic so that its molecular weight is: $2 \times 16 = 32$, and 32 gm. of oxygen in the gaseous state occupy 22·4 litres at N.T.P. Hence, the volume of oxygen liberated at N.T.P. when 1 amp. flows for 1 second in a water voltameter is given by:

$$v = \frac{m}{32} \times 22{\cdot}4 \text{ litres}$$

i.e.

$$v = \frac{8}{96{,}500} \times \frac{22{\cdot}4}{32} \times 10^3 \text{ c.c.}$$

i.e.

$$\underline{v = 0{\cdot}058 \text{ c.c.}}$$

Example

Describe an experiment to investigate the way in which the current through a water voltameter varies with the p.d. applied across it. Describe the kind of variation observed, and give a brief explanation of it.

A p.d. of 4·0 *volts gives a current of* 1·0 *amp., and a p.d. of* 6·5 *volts a current of* 2·0 *amps. Find the power which must be furnished to liberate hydrogen from the water voltameter at the rate of* 0·2 *gm. per hour. Find also the proportion of this power which is expended in generating heat in the voltameter. (The electrochemical equivalent of hydrogen is* 0·00001045 *gm. per coulomb.).* (Oxford Local G.C.E. Adv. L.).

(i) The variation of current strength with applied p.d. is investigated by using a water voltameter (p. 220) connected in series with a battery of accumulators, a rheostat and a milliammeter. Across the electrodes is connected a high resistance voltmeter. The rheostat is adjusted so that the p.d. across the voltameter, is a small value, say 0·2 volts, and the current flowing is noted. The rheostat is adjusted to give p.d. variations of 0·2 volts at a time and the current strength noted for each voltmeter reading up to a maximum of about 6 volts.

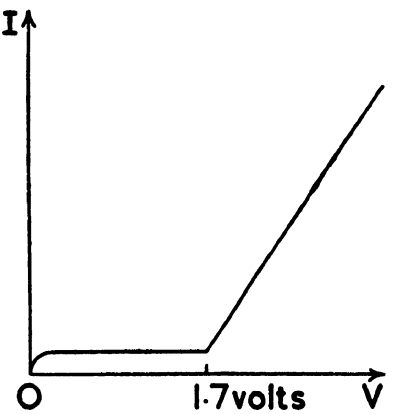


Fig. 8.6

The variation of current strength with applied p.d. is as shown in Fig. 8.6. For p.d.'s less than about 1·7 volts (cf. p. 224), the *steady* current strength is very small. Beyond this p.d. the current strength increases linearly with the applied p.d.

(ii) Assuming the current through the voltameter to be negligible below a minimum p.d. V_0, and proportional to the excess p.d.:

$$I \propto (V - V_0) = k(V - V_0) \text{ where } k \text{ is a constant.}$$

Since $V = 4$ volts when $I = 1$ amp., and $V = 6{\cdot}5$ volts when $I = 2{\cdot}0$ amps.:

$$1 = k(4 - V_0) \text{ and } 2 = k(6{\cdot}5 - V_0)$$

whence on dividing:

$$\frac{1}{2} = \frac{4 - V_0}{6{\cdot}5 - V_0}$$

i.e. $\underline{V_0 = 1{\cdot}5 \text{ volts.}}$

it follows that:

$$1 = k(4 - 1{\cdot}5) = 2{\cdot}5k$$

i.e. $\underline{k = 0{\cdot}4 \text{ amps.volt}^{-1}}$

Hence: $I = 0{\cdot}4(V - 1{\cdot}5)$ (i)

The mass of hydrogen deposited is given by:

$$m = z_H It$$

since $m = 0{\cdot}2$ gm. when $t = 1$ hour $= 3{,}600$ seconds, the current required is:

$$I = \frac{m}{z_H t} = \frac{0{\cdot}2}{1{\cdot}045 \times 10^{-5} \times 3600} = 5{\cdot}316 \text{ amps.}$$

The p.d. required is given by substituting for I in (i):

$$5{\cdot}316 = 0{\cdot}4(V - 1{\cdot}5)$$

i.e. $$V = 14{\cdot}79 \text{ volts}$$

Hence, the power required to liberate the hydrogen at this rate is:

$$P = IV = 5{\cdot}316 \times 14{\cdot}79 = \underline{78{\cdot}6 \text{ watts.}}$$

Since Ohm's law applies to the voltameter in the form:

$$I = \frac{V - V_o}{R}$$

the effective resistance of the voltameter is given by:

$$R = \frac{V - V_o}{I}$$

From above it follows that:

$$R = \frac{1}{k} = \frac{1}{0{\cdot}4} = 2{\cdot}5 \text{ ohms}$$

Hence the power wasted is generating heat in the voltameter

$$= I^2R$$

$$= (5{\cdot}316)^2 \times 2{\cdot}5 \text{ watts.}$$

Therefore, the proportion of power wasted $= \frac{I^2R}{IV} \times 100\%$

$$= \frac{5{\cdot}316}{14{\cdot}79} \times 2{\cdot}5 \times 100\%$$

$$= \underline{89{\cdot}9\%}$$

i.e. the power required to liberate hydrogen at the given rate is 78·6 watts the proportion of power wasted as heat = 89·9%.

Example

The electrochemical equivalent of silver is $1{\cdot}118 \times 10^{-3}$*gm./coulomb. and the atomic weight of silver, which is monovalent, is* 108. *Calculate the ratio of the charge to the mass of a hydrogen ion (proton).* (Oxford Open Schol.). (Part question).

By definition, the e.c.e. of silver z, is the mass of silver liberated by one coulomb. Since silver is monovalent the positive charge carried by each ion is the same as that on the proton: e coulombs.

By Avogadro's hypothesis the number of silver atoms in one gram-atom, i.e. 108 gm., is equal to Avogadro's number N. Hence, the number of silver ions deposited when one coulomb passes is: $zN/108$.

Hence, the charge on each ion $= \frac{1}{zN/108} = \frac{108}{zN}$ coulombs.

1 gram of hydrogen contains N atoms also, so that the mass of one hydrogen atom is: $m_H = 1/N$ gm.

Hence

$$\frac{\text{charge on proton}}{\text{mass of proton}} = \frac{108 \times N}{zN} = \frac{108}{z} = \frac{108}{1{\cdot}118 \times 10^{-3}} \text{ coulomb.gm.}^{-1}$$

i.e. $$\frac{e}{m_H} = 9{\cdot}654 \times 10^4 \text{ coulomb.gm.}^{-1}$$

or: $$e/m_H = 9{\cdot}654 \times 10^7 \text{ coulomb.kgm.}^{-1}$$

8.18. The Silver Voltameter

(1) *Determination of F*

The silver voltameter consists of a platinum bowl C as the cathode, and a solid silver cylinder A as anode, (Fig. 8.7). Connections to these electrodes are made via the terminals T_C and T_A. The electrolyte, which is silver nitrate solution, is contained in C. Beneath A is a porous pot G in which impurities in the silver are collected (" anode sludge ").

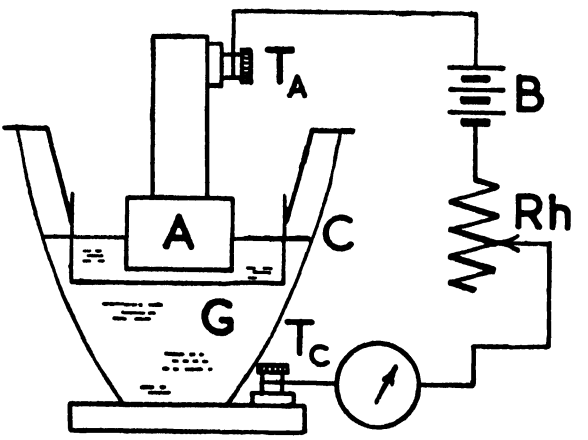


Fig. 8.7

To determine the Faraday, the cathode is first cleaned and weighed. The voltameter is then filled with silver nitrate solution and a steady current (I amps.) is passed for a known time (t seconds) from the accumulator battery B. The current strength must not exceed 0·2 amps. per cm². of immersed anode surface, otherwise the deposit of silver on the cathode does not adhere firmly.

During electrolysis the $(NO_3)^-$ radicle forms a silver nitrate molecule ($AgNO_3$) with a silver atom of the anode, so that for each silver atom deposited one dissolves in the solution and the concentration remains constant.

The mass of silver deposited is found by reweighing C after washing. From Faraday's first law:

$$M = zIt$$

From (27): $$z = \frac{\mathscr{E}}{F}$$

So that: $$M = \frac{\mathscr{E}}{F} . It$$

$$\therefore \quad F = \frac{\mathscr{E}}{M} . It \quad . \quad . \quad . \quad . \quad (29)$$

The Faraday is thus determined knowing the equivalent weight of *Ag*.

(2) *Measurement of Current Strength*

Since the mass of an element deposited in electrolysis is proportional to the current strength, it may be used as a measure of current strength. The procedure takes time, necessitates delicate weighing and is generally inconvenient. However, the idea was used to define the *International ampere* in 1908. At this time, the only absolute instruments were relatively inaccurate tangent and Helmholtz galvanometers. It was decided, therefore, to measure the amount of silver deposited by a current strength measured approximately by meters, and define arbitrarily an exact mass of silver deposited per second as a measure of the ampere which could be accepted as a legal standard. The International ampere was thus defined as *that current strength which when flowing through a specially designed silver voltameter causes the deposition of silver at the rate of 0·00118 grams per second.*

In the laboratory the silver voltameter is used for measuring steady current strengths using (3), knowing z_{Ag}. A most useful function of the voltameter is the calibration of an ammeter which is placed in series with it when measuring current.

8.19. Cells

A cell is a device for producing electrical power which utilizes an electrolyte and two metal electrodes. The latter are called the positive and negative poles of the cell, the former having a higher potential than the latter. There are several different kinds of cells, known as *primary* cells, *secondary* cells, *concentration* cells, *fuel* cells, etc. Primary cells supply electrical energy by direct conversion of chemical energy whilst secondary cells must first be " charged " before they can be used to supply power.

8.20. Primary Cells

(1) *Simple or Voltaic Cell*

This consists of one zinc and one copper plate immersed in dilute sulphuric acid. The cell was first used by Volta in 1800. He used plates of zinc and copper separated by felt moistened in dilute sulphuric acid; by adding successive plates of different metal, using the same acid impregnated material, he constructed a *Voltaic Pile.* This consists of

several simple cells in series. The e.m.f. of the simple cell is just over 1 volt. When the plates are connected externally positive current flows from the copper to the zinc plate, so that the copper is the positive and the zinc the negative pole. As current flows, zinc is dissolved and hydrogen is liberated at the copper plate. If there were no defects the current would continue until all the zinc was dissolved.

The current in the external circuit consists of electrons flowing from the copper to the zinc. In the electrolyte the mobile charges are positive and negative ions, viz. H^+ and $(SO_4)^-$, respectively. The laws of electrolysis apply to this flow of charge in the cell and, as a consequence, hydrogen is liberated at the copper and the $(SO_4)^-$ ions combine with the zinc to form zinc sulphate.

The defects of the cell are (i) steady solution of the zinc, even on open circuit, and (ii) polarization at the positive pole. Commercial zinc is used in simple cells and this contains impurities, such as iron, which form small voltaic cells of their own and cause the solution of the zinc. This process is known as *Local Action*. Pure zinc is barely soluble in sulphuric acid but its use is prohibited by cost. Local action is considerably reduced by rubbing the zinc surface with mercury to form a protective amalgam coating, i.e. a mercury-zinc alloy.

The polarization of the cell is mainly caused by hydrogen bubbles not escaping from the copper surface; they form an adsorbed layer which prevents deposition of more hydrogen and produces a back e.m.f. This defect is remedied to a great extent by (i) removing the hydrogen bubbles mechanically by scraping or brushing, and (ii) using some oxidizing agent (depolarizer) to use up the hydrogen atoms on formation.

(2) *Daniell Cell*

Because of its defects the simple cell is not used as a source of power. A more useful type of primary cell, called the *Daniell Cell*, avoids polarization. The electrodes are copper and zinc, as in the simple cell, but each metal plate is immersed in a different electrolyte. In Fig. 8.8, the copper plate A (positive) is immersed in a saturated solution of copper sulphate contained in a glass jar. The zinc rod (negative) C is immersed in a saturated solution of zinc sulphate contained in the porous pot P. The porous pot prevents diffusion between the two solutions, but allows charged ions to flow freely through the pores when current flows.

The e.m.f. of this cell is approximately 1·1 volts and is fairly constant, providing the copper sulphate solution is kept saturated. When current flows in an external circuit the sulphate ions $(SO_4)^-$ carry their charges to the zinc and form zinc sulphate, so that zinc goes into solution. Zinc ions Zn^{++} and copper ions Cu^{++} travel towards the copper, the former passing from the porous pot into the $CuSO_4$ solution, whilst the latter give up their positive charges at the copper electrode. From the copper sulphate solution, therefore, both copper and sulphate ions are

lost and the concentration of copper sulphate is reduced. No gases are liberated and polarization is absent.

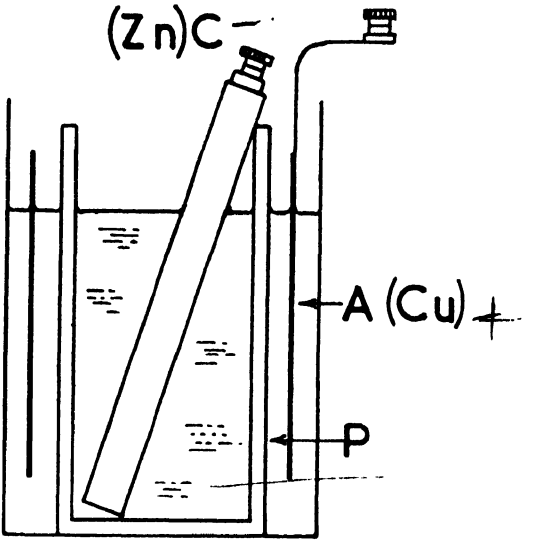


Fig. 8.8

In some forms of Daniell cell the zinc is immersed in a dilute solution of sulphuric acid. The zinc dissolves forming zinc sulphate, whose concentration depends on the amount of acid used. The e.m.f. of the cell is sufficiently constant for it to be used as a rough standard.

(3) *Leclanché Cell*

This cell has a carbon plate as positive and a zinc rod as negative pole. The latter is immersed in a strong aqueous solution of sal-ammoniac (NH_4Cl), contained in a characteristic form of glass vessel, shown in Fig. 8.9. The carbon plate is surrounded with a mixture of powdered carbon and manganese dioxide contained in a porous pot. The latter is an oxidizing agent and prevents polarization.

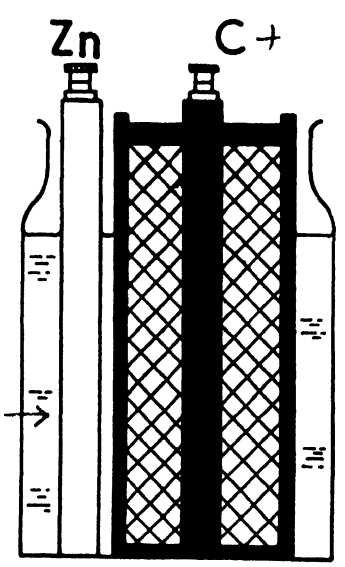


Fig. 8.9

The e.m.f. of this cell on open circuit is about 1·4 volts. When current flows the $(NH_4)^+$ and Cl^- ions of the electrolyte, carry their charges to the carbon and zinc electrodes, respectively. If there were no depolarizer present the $(NH_4)^+$ would cause the formation of ammonia NH_3 and hydrogen at the carbon electrode and polarization would result. The manganese dioxide does not play any part in the conduction mechanism since it is an insulator, but it

effectively oxidizes the hydrogen forming water and the oxide Mn_3O_4, according to the reaction:

$$3MnO_2 + 2H_2 \rightarrow Mn_3O_4 + 2H_2O$$

The admixed carbon provides intimate contact between the released hydrogen and the oxide, and forms conducting paths between the sal-ammoniac and the carbon electrode proper.

In spite of the depolarizing agent the cell is polarized after several minutes, especially if large currents are taken. It is extremely useful for intermittent currents, as in the operation of a door bell, in some telegraphic signalling circuits, etc. The cell needs little attention—adding distilled water to the electrolyte to counter evaporation.

8.21. The Dry Battery

The Leclanché cell has been rendered almost obsolete by the dry cell. This is more convenient because of its compact form and freedom from accidental spillage. The construction is similar to the " wet " Leclanché cell. The positive pole is a carbon rod placed in a thin canvas bag containing powdered carbon and manganese dioxide. This is inserted in a thin zinc cylinder (negative pole) containing a stiff mixture of sal-ammonia solution, flour, gum and zinc chloride, and maintained in position with a surface layer of tar. A small vent in the tar allows release of gases.

The e.m.f. of the dry cell is about 1·5 volts when fresh. Batteries are manufactured consisting of cells in series, with voltages between 1·5 and several hundreds. The batteries are used continuously for relatively small currents (milliamps.); with larger currents the battery e.m.f. is reduced considerably. They deteriorate if kept in the unused state for any considerable length of time. In spite of disadvantages, dry batteries are used extensively in all kinds of devices from common electric torches to portable electronic devices such as radio sets, Geiger " counters ", etc.

8.22. Theory of Primary Cells—Electrode Potential

To explain the origin of the e.m.f. it is supposed that on immersing any metal in an aqueous solution it tends to dissolve to a very limited extent forming positive ions. The free electrons in the metal cannot enter the solution and hence the metal acquires an excess negative charge.

The extent to which a metal dissolves in an aqueous solution depends on the nature of the metal, the concentration of ions already in solution, the temperature, etc., but is so minute as to be undetectable. The controlling factor opposing the continued solution of the metal is the negative charge acquired. This tends to attract any positive ions present in solution. The negative charge on the electrode causes the metal to acquire a definite negative potential relative to the solution.

The solution cannot maintain a p.d. across itself because it is a conductor, hence, it is supposed that an excess number of positive ions remain very close to the electrode under the influence of the attractive negative charge. This arrangement of opposite charges is called an *electrical double layer.* (cf. p. 96).

The absolute negative potential of a metal in a solution cannot be measured, because any attempt to effect such a measurement necessitates a second electrode being inserted in the solution. The latter gives rise to its own *electrode potential* and only the *p.d.* between the two metals is measured.

A *standard electrode* is used to measure the p.d. between itself and a metal in solution. The standard (hydrogen) electrode consists of hydrogen gas bubbling over a platinum electrode at atmospheric pressure, immersed in a solution containing one gm-equivalent of hydrogen (as ions) per litre. The hydrogen gas forms an electrode whose atoms can pass into solution as positive ions, leaving a negative charge. The potential achieved by the gas is the same as that of the adjoining platinum which makes electrical contact with the gas.

The electrode potential of any metal relative to the standard electrode is measured by making up a cell consisting of the metal in a solution of one of its salts and the hydrogen electrode. If the salt solution is *normal*, i.e. contains one gram equivalent of metal per litre in the form of ions, the measured p.d. is called the *Standard Electrode Potential* of that metal. This varies slightly with small differences of ion concentration.

The following table gives the standard electrode potentials $E°$ of various elements:

Metal	*Ion*	$E°$(*volts*)
Li	Li^+	−2·96
Na	Na^+	−2·72
Mg	Mg^{++}	−1·55
Zn	Zn^{++}	−0·76
Pb	Pb^{++}	−0·12
Cu	Cu^{++}	+0·34
Ag	Ag^+	+0·80
Cl	Cl^-	+1·36
Au	Au^+	+1·36

8.23. Theory of Daniell Cell

Assuming each electrode is immersed in a normal solution of its salt, the p.d. between the two is the difference between their standard electrode potentials. From the table, this is: $0·34 - (-0·76) = 1·1$ volts, which is the e.m.f. of the cell.

Origin of Energy

In order that the cell may provide power energy must be supplied and

the origin of this energy must be the processes taking place at each electrode. At the zinc electrode ions go into solution and this causes the release of energy.

At the copper electrode copper ions are neutralised and deposited on the electrode, a process which is the reverse of solution and requires energy. For each zinc ion formed one copper atom is deposited, so that the net energy gained is equal to the difference between the release of energy on solution of a zinc ion and the energy lost on deposition of a copper atom. The net energy gained when one coulomb of ions is created or neutralised is the same as the total amount of work which can be done by one coulomb flowing round the circuit, i.e. the e.m.f. of the cell.

This viewpoint is confirmed by determining the heat of reaction when a zinc rod is placed in a solution of copper sulphate. The zinc displaces copper, atom for atom, according to:

$$Zn + CuSO_4 \rightarrow ZnSO_4 + Cu \quad . \quad . \quad . \quad (30)$$

The net effect occuring in the cell is exactly similar:

$$Cu^{++}: SO_4^{=} + Zn \rightarrow Zn^{++}:SO_4^{=} + Cu \quad . \quad . \quad (31)$$

The result obtained from calorimetric determinations of the heat of reaction in (30) is usually expressed in calories per gm. molecule of copper displaced (or Zn dissolved), but if this is expressed as joules of energy released per coulomb of charge transferred from Cu^{++} to Zn^{++} ions, it is the same as the e.m.f. of the cell.

The detailed theory of the other primary cells discussed above is much more complex and the reader is referred to text-books on electrochemistry for further details.

Example

State Faraday's laws of electrolysis. How would you verify the laws experimentally? Discuss briefly the phenomenon of polarization in electrolysis and how it is overcome in the Daniell cell.

Calculate a value for the e.m.f. of a Daniell cell from energy considerations using the following data: 1 *gm. of zinc dissolved in copper sulphate solution liberates* 796 *calories.* (*e.c.e. of zinc* = 0·000340 *gm./coulomb;* 4·18 *joules* = *one calorie*). (London G.C.E. Adv. L.).

The first part of the question is covered in the text.

The heat liberated when zinc dissolves in copper sulphate solution provides the energy for the passage of the ions between the electrodes of the cell. The charge Q, produced in the form of ions when 1 gm. of zinc dissolves, is given by using Faraday's law, since the charge required to liberate a zinc atom from solution is the same as that acquired when a zinc atom dissolves, hence:

$$m = zIt = zQ$$

Putting: $z = 0{\cdot}000340$ gm.coulomb^{-1} and $m = 1$ gm.:

$$Q = \frac{1}{0{\cdot}000340} \text{ coulombs}$$

The energy liberated per coulomb of charge produced on Zn^{++} ions is able to effect the passage of that charge through the cell and deposit an equivalent amount of copper and is hence equal to the e.m.f. Since 796 calories are produced when one gm. of zinc dissolves, the energy liberated per coulomb is:

$$E = 796 \times J \times 0{\cdot}000340 = 796 \times 4{\cdot}18 \times 0{\cdot}000340 \text{ joule} \cdot \text{coulomb}^{-1}(\text{volts})$$

i.e. $E = 1{\cdot}13$ volts.

8.24. Other Primary Cells

A cell made up of two electrodes in solutions containing their own ions, separated by a porous or other kind of barrier which prevents diffusion, has an e.m.f. whose value is the difference between the electrode potentials. The origin of the energy available from such cells can always be traced to the difference in energy released on solution of the metal of the negative electrode and the deposition of metal on the other electrode. Such cells are *reversible*, since a p.d. applied in the reverse direction causes the converse to take place, deposition occurring where solution occurred formerly and vice versa.

8.25. Secondary Cells—Accumulators

These are devices which store electric charge and give an electric current in an external circuit for a limited time dependent on the amount of charge stored, i.e. the *capacity* of the accumulator. There are two well-known types of accumulator: (i) the Lead-Acid Accumulator, and (ii) the Nickel-Iron Alkaline Accumulator.

(i) *The Lead-Acid Accumulator*

This consists of two lead plates, on one of which is produced a layer of chocolate-coloured lead peroxide PbO_2, and on the other a layer of grey spongy lead, by a "forming" process. The prepared plates are immersed in sulphuric acid and a current passed until the separate plates have a fresh grey or brown colour, respectively. The cell is then charged, i.e. able to act as a source of e.m.f.

Discharge of Cell

When used to produce current the oxide coated plate forms the positive pole and the metal plate the negative pole. In the acid electrolyte sulphate ions migrate to the lead plate and hydrogen ions migrate to the oxide coated plate. On deposition hydrogen ions reduce the lead peroxide to the monoxide which forms lead sulphate, as follows:

$$PbO_2 + H_2 \rightarrow PbO + H_2O \quad . \quad . \quad . \quad (32)$$

$$PbO + H_2SO_4 \rightarrow PbSO_4 + H_2O \quad . \quad . \quad (33)$$

The lead sulphate is not soluble in the electrolyte and it forms a coating

on the plate. At the negative plate the SO_4^- ions also form lead sulphate according to:

$$Pb + SO_4 \rightarrow PbSO_4 \quad . \quad . \quad . \quad . \quad (34)$$

When both plates are coated with $PbSO_4$ the discharge of the accumulator is complete and no more electrical energy may be obtained.

During the discharge process sulphuric acid is effectively removed from the solution to form lead sulphate, so that the specific gravity of the solution is reduced. In the freshly charged state the specific gravity is about 1·21, whilst in the fully discharged state it is about 1·16. The specific gravity of the electrolyte is used as a guide to the state of the cell, a special form of hydrometer being used for testing.

The e.m.f. falls as the discharge continues. In the charged state the e.m.f. is slightly greater than 2·0 volts. This remains fairly steady, but falls to about 1·87 volts near the end of the discharge, (Fig. 8.10).

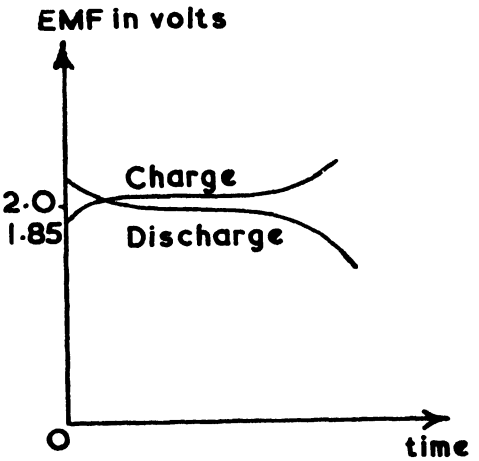


Fig. 8.10

Charging Process

After discharge the accumulator is restored to the charged state by passing current through the cell in the opposite direction. This is achieved by applying a p.d. so that the positive plate becomes the anode and the negative plate the cathode. The voltage required across the plates must always be just greater than the e.m.f. developed by the plates as charging continues. The initial charging p.d. is about 2·04 volts and as electrolysis proceeds this must be increased to about 2·5 volts. The increase in applied p.d. is associated with the energies required to liberate the gases formed at the electrodes. The variation in charging p.d. is shown in Fig. 8.10.

During the charging process H^+ ions migrate to the negative plate and reduce the lead sulphate to metallic lead:

$$PbSO_4 + H_2 \rightarrow H_2SO_4 + Pb \quad . \quad . \quad . \quad (35)$$

At the positive plate the $SO_4^=$ ions produce lead peroxide and sulphuric acid:

$$PbSO_4 + SO_4 + 2H_2O \rightarrow PbO_2 + 2H_2SO_4 \quad . \quad (36)$$

The resultant changes are the production of a layer of lead peroxide on the positive plate, a layer of spongy lead on the negative plate, and an increase in the concentration of sulphuric acid.

Forming Processes

The initial " forming " of the accumulator is carried out in two ways, known as the Planté and Fauré processes. In the former process an oxidising agent is added to a dilute solution of sulphuric acid maintained between two corrugated lead plates. Current is passed through the cell and a layer of lead peroxide is formed. The current is reversed and the lead peroxide is reduced to spongy lead by the products of electrolysis. The process is repeated and, slowly, both plates are " formed ", one coated with PbO_2 and the other pure lead.

In the Fauré process a mixture of red lead Pb_3O_4 and sulphuric acid is pressed into a lead grid and dried. Two similar plates are immersed in weak sulphuric acid and a current passed through the cell. At one plate the Pb_3O_4 is reduced by liberated hydrogen to spongy metallic lead, and at the other plate the red lead is oxidized by the $SO_4^=$ ions to the peroxide PbO_2. This method is much cheaper than the Planté process but the plates are less durable.

Accumulators—Notes

(*a*) It is not wise to short circuit accumulators because of their small internal resistance—the large current causes the plates to buckle and the active deposits to crumble off; (*b*) Accumulators should never be left in the discharged condition because the $PbSO_4$ tends to harden and act as a high resistance insulator (*Sulphation*), instead of being porous and allowing ions to pass through it; (sulphation is avoided by charging before leaving the cell); (*c*) Lead-Acid accumulators are heavy and costly but extremely useful for producing very steady currents of several amperes; (*d*) The capacity of an accumulator, i.e. the total charge the cell is capable of delivering, is expressed in *ampere-hours*, (the product of the current strength and the time which discharges the accumulator): e.g. a 100 ampere-hour cell supplies 2 amps. for 50 hours or 10 amps. for 10 hours; (*e*) The efficiency of lead-acid accumulators is about 75 per cent, i.e. three-quarters of the energy used to charge the accumulator is available for use.

(ii) *The Nickel-Iron Alkaline Cell*

This cell was invented by Edison as a lighter and more robust alternative to the heavy lead-acid accumulator. The two plates are made of a steel framework into the spaces of which are pressed the

active materials required. The positive plate has a mixture of nickel hydroxide $Ni(OH)_3$ and nickel flakes, the latter allowing conduction to the underlying metal. The negative plate initially contains iron oxide mixed with a small amount of mercury. The electrolyte is a 20 per cent potassium hydroxide solution.

Initial Charging Process

On first charging the cell, potassium ions pass to the negative plate and produce hydrogen by the usual chemical reaction with water. The hydrogen reduces the iron oxide to metallic iron:

$$FeO + H_2 \rightarrow Fe + H_2O \quad . \quad . \quad . \quad (37)$$

The OH^- ions liberated at the positive plate cause the formation of a nickel oxide:

$$Ni(OH)_3 + OH \rightarrow NiO_2 + 2H_2O \quad . \quad . \quad . \quad (38)$$

In this *initial* charging process the electrolyte is diluted.

Discharge and Recharging

On discharge, the potassium ions migrate to the positive plate and form the oxide Ni_2O_2:

$$2NiO_2 + 4K + 2H_2O \rightarrow Ni_2O_2 + 4KOH \quad . \quad . \quad (39)$$

At the negative plate the OH^- ions cause the formation of iron oxide:

$$Fe + 2OH \rightarrow FeO + H_2O \quad . \quad . \quad . \quad (40)$$

For each NiO_2 molecule reduced at the positive plate one molecule of water is used up and one molecule of water is formed at the negative plate. The electrolyte is thus unchanged and its specific gravity remains constant. On recharging, the Ni_2O_2 is converted to NiO_2 and the FeO reduced to Fe according to:

$$\text{Positive Plate: } Ni_2O_2 + 4OH \rightarrow 2NiO_2 + H_2O \quad . \quad (41)$$

$$\text{Negative Plate: } FeO + 2K + H_2O \rightarrow Fe + 2KOH \quad (42)$$

Again the caustic potash concentration is unchanged.

The charging potential required for this cell increases gradually from about 1·5 to 1·8 volts, whilst the e.m.f. on discharge varies steadily from about 1·5 to 1·1 volts and then falls rapidly to a low value. The efficiency of the cell is about 60 per cent. The cells can be made with relatively small amounts of electrolyte and active material, and the weight per unit of energy obtained is much less than that of lead accumulators. They possess the advantage that mechanical shock does not affect the plates seriously.

Batteries are made by connecting storage cells in series; $12V$ or $6V$ batteries are used extensively in motor-cars for starter motors, ignition, lighting, etc., (the charged state of a car battery is maintained by the car dynamo). Batteries of large capacity are also used for driving small vans, remote country lighting, etc.; in the laboratory, accumulators provide a convenient source of supply for most D.C. experiments.

Example

Describe and explain the action of one *type of primary cell and* one *type of secondary cell.*

A battery of 30 *accumulators is to be charged at* 4 *amps. from* 240 *volt D.C. mains. How may this be done? What percentage of power will be wasted? (Assume that the mean voltage of each accumulator during charging is* 2·2 *volts and that its internal resistance is* 0·1 *ohm).* (Cambridge Local G.C.E. Adv. L.).

The first part of the question is covered in the text.

The circuit required for charging the accumulators is shown in Fig. 8.11.

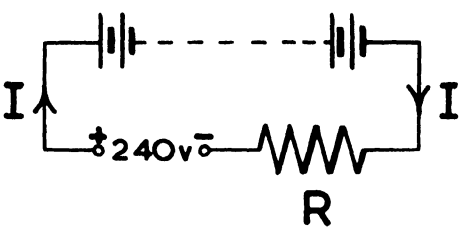


Fig. 8.11

The charging current (I) flows in the direction indicated. The series resistance R ohms limits the value of I to 4 amps. The total resistance of the battery is $r = 30 \times 0{\cdot}1 = 3$ ohms. Since the mean voltage of each accumulator is 2·2 volts the e.m.f. of the battery during charging is $V = 30 \times 2{\cdot}2 = 66$ volts. This e.m.f. opposes the mains supply.

From Ohm's law: $$I = \frac{E - V}{R + r}$$

Since the applied e.m.f. $E = 240$ volts:

$$4 = \frac{240 - 66}{R + 3}$$

hence: $$R = 40{\cdot}5 \text{ ohms.}$$

Power is wasted in the internal and external resistance of the circuit and is given by:

$$P_W = I^2(R + r) = 16(40{\cdot}5 + 3) = 696 \text{ watts}$$

The power supplied by the mains is given by:

$$P = EI = 240 \times 4 = 960 \text{ watts}$$

Therefore, the percentage power wasted $= \frac{P_W}{P} \times 100 = \frac{696}{960} \times 100 = 72{\cdot}5\%$

8.26. Fuel Cells

In an attempt to find a more efficient source of electrical power than the normal type of generator, which converts the energy derived from burning coal or oil with an efficiency which is generally less than 30 per cent, the fuel cell has been proposed. This endeavours to use an electrolyte between two electrodes such as oxygen and carbon, so that oxygen ions are delivered up to the carbon to form carbon dioxide. The energy thus released is the same as that obtained by burning carbon in oxygen and, in the cell, almost all this energy may be used to drive current through an external circuit. For successful operation the electrolyte must allow oxygen ions only to migrate, so that energy is not used up in other reactions. Theoretically, the energy conversion is approximately 100 per cent efficient, but, so far, no practical fuel cell has been developed, although research is being actively pursued in many countries.

8.27. Concentration Cells ✓

These consist of two similar electrodes each immersed in the same solution but with different concentrations. The two solutions are prevented from mixing by keeping them in separate compartments connected by a siphon tube. The electrode potentials depend on solution concentration, and the difference gives the e.m.f. of the cell. When the plates are connected externally the same metal is deposited at the positive plate as is dissolved at the other. Such cells provide useful electrochemical data.

8.28. Applications of Electrolysis

1. *Electroplating*

Metal objects may be coated with a more durable and expensive metal by making them the cathode of a cell whose electrolyte contains cations of the metal to be deposited. The current strength is controlled to ensure a firm coating.

2. *Electrotyping*

Metal engravings, printer's type, etc., are reproduced by first making a wax mould. This is coated with graphite and used as a cathode in a copper voltameter. After a sufficient time a thin copper reproduction of the original is obtained. This can be used for printing, after mounting on a suitable base.

3. *Production of elements*

Some molten salts, e.g. *NaCl*, are electrolytes. The metal may be deposited at a suitable cathode when the melt is electrolysed. This process is used for the preparation of alkali metals. Other metals are refined by using the impure metal as the anode and the pure metal as

cathode in an aqueous solution of a salt of the metal. The applied p.d. is such that only the required ion is deposited at the cathode. The electrolysis of water is used for the production of hydrogen and oxygen, and *deuterium* (p. 439).

4. *Standard Cell*

The Weston standard cell consists of an H-shaped glass vessel, at the bottom of each limb of which is sealed a platinum wire forming the electrodes. One limb (R.H.S. of Fig. 8.12), contains an amalgam of

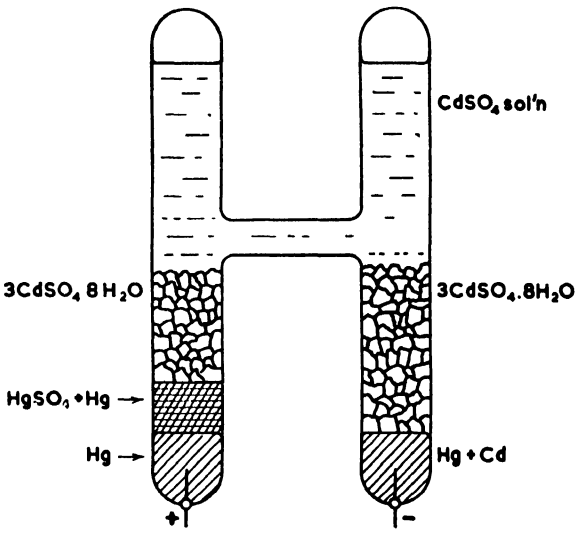


Fig. 8.12

cadmium and mercury, on the top of which is placed a layer of small crystals of cadmium sulphate $3CdSO_4 . 8H_2O$. In the opposite limb is a small amount of mercury covered with a layer of paste consisting of solid mercurous sulphate $HgSO_4$ and saturated cadmium sulphate solution. Cadmium sulphate crystals cover the paste and the cell is filled with a saturated solution of cadmium sulphate. Providing the temperature is constant, the e.m.f. is constant (1·0183 volts at 20°C.); the e.m.f. falls with temperature according to:

$$E = 1{\cdot}0183 - 0{\cdot}0000406\,(T - 20) \text{ volts} . \quad . \quad (43)$$

The Weston standard cell is reversible. When the electrodes are connected externally, cadmium ions go into solution at the cathode and drift towards the opposite electrode. At the anode mercury ions are deposited. If the current is passed in the opposite direction, by connecting a cell of larger e.m.f. across its terminals, mercury ions go into solution at the cathode and cadmium ions are deposited at the cathode. The electrolyte is equivalent to a mixture of saturated

solutions of mercurous and cadmium sulphates and the concentrations of mercury and cadmium ions are unchanged when the cell is in use. By this means the e.m.f. is maintained independent of the direction of current strength.

EXERCISES

1. State Faraday's laws of electrolysis. Briefly indicate how you would verify them experimentally.

   Assuming that the minimum e.m.f. necessary to decompose water is 1.50 volts, and that 34,480 calories are liberated when 1 gm. of hydrogen is burnt to form water, calculate the number of coulombs which will liberate 1 gm. equivalent of any substance during electrolysis. ($J = 4{\cdot}2$ calories). (Cambridge Local G.C.E. Adv. L.).

2. State Faraday's laws of electrolysis, and show how they are explained by ionic theory.

   A potentiometer circuit is set up in the usual way, and a Daniell cell of e.m.f. 1·08 volts just balances the potential fall down 75·6 cm. of wire. In another circuit, a copper voltameter is in series with a battery and a resistance R of 0·5 ohm, and the p d. across R is balanced by the fall down 67·2 cm. of potentiometer wire. Find the mass of copper deposited in one hour. (The electrochemical equivalent of copper is 0·00033 grams per coulomb). (Oxford Local G.C.E. Adv. L.).

3. A tangent galvanometer and a copper voltameter are connected in series with a battery. The coil of the galvanometer has 75 turns of mean radius 9 cms. When a certain current is passed through the circuit, the deflection of the galvanometer is 57° 30′, and 5·35 mgm. of copper are deposited in 5 minutes. Given that the electrochemical equivalent of copper is 0·329 mgm./coulomb, find the horizontal component of the earth's magnetic field. (Oxford Open Schol. Subsid.).

4. State Faraday's laws of electrolysis and describe how you would verify them by experiment.

   A copper refining cell consists of two parallel copper plate electrodes, 6 cm. apart and each 1 metre square, immersed in a copper sulphate solution of resistivity 1·20 ohm.cm. Calculate the potential difference which must be established between the plates to provide a constant current to deposit 480 gm. of copper on the cathode in one hour. (e.c.e. of copper = 0·000329 gm.coulomb^{-1}). (London Inter. B.Sc.).

5. What are Faraday's laws of electrolysis? Describe how you would verify them.

   What evidence do these laws provide for the belief that there is a fundamental unit of electric charge, and how could the results of the experiments you describe be used to estimate its magnitude? (Cambridge Open Schol.).

6. State Faraday's laws of electrolysis and describe experiments to verify them.

   A difference of potential of 60 volts is maintained between two electrodes 12 cm. apart in a solution of common salt. How long will it take a chlorine ion to travel 3 cm. in the solution? (The mobility

of chlorine ions may be taken as 0·00053 cm./sec. per volt/cm.). London G.C.E. Adv. L.).

7. Describe the electrolytic processes which occur in a Daniell cell when its terminals are joined together through a small resistance.

A steady current of 5·0 amp. is passed through a silver voltameter in series with a coil of wire of 10 ohms resistance immersed in 200 gm. of water. What will be the rise of temperature of the latter when 0·10 gm. of silver has been deposited? (Assume that the e.c.e. of silver = 0·001118 gm./coulomb. $J = 4{\cdot}2$ joules/cal. Thermal capacity of the coil and vessel = 10 cal./deg.C.). (London G.C.E. Adv. L.).

8. Define the m.k.s. units of electric current and quantity of electricity. Describe a laboratory method of measuring the total quantity of electricity passing in an electrical circuit containing an unsteady battery.

A charged thunder cloud is producing raindrops of average diameter 3 mm., each carrying a charge of 3×10^{-15} coulomb. How heavy must be the rainfall (in cm. per hour) for the electric current density to the earth to be 5×10^{-11} amp./m.^{2}? (London G.C.E. Adv. L.).

9. Describe the construction of the lead accumulator, and give an account of the chemical actions that take place at each plate during charging and discharging.

A 12-volt 40 ampere-hour accumulator is used to operate the 2 h.p. starter motor of a car. Assuming the arrangement is completely efficient, find the current taken. If the starter is used for an average period of 10 sec. on each occasion, and the accumulator is not recharged in the intervals, how many times can the car be started before the accumulator is discharged? (Take 1 h.p. to be 746 watts). (Oxford Local G.C.E. Adv. L.).

10. Give a short account of the phenomenon of electrolytic polarization.

Describe a lead accumulator, and indicate in general terms the nature of the changes that take place in the plates and the electrolyte during charging and discharging.

A battery of 24 accumulators in series is charged from 100 volt D.C. mains. If the internal resistance of the *battery* is 2 ohms and the e.m.f. of each *cell* during charging is 2·5 volts, what series resistance will be required to give a charging current of 10 amps? What percentage of the energy taken from the mains will be converted into heat? (Oxford & Cambridge Jt.Bd. G.C.E. Adv. L.).

11. What is meant by electrolytic polarization? How does it affect the working of a simple voltaic cell?

Describe an experiment to show that, when a direct current is passed between platinum plates immersed in dilute sulphuric acid, a back e.m.f. is set up by the polarization of the plates.

Give a brief account of the lead accumulator, and indicate in general terms the chemical changes that take place during charging and discharging. Why is a hydrometer useful for indicating the state of charge of an accumulator? (Oxford & Cambridge Jt.Bd. G.C.E. Adv. L.).

12. Describe an experiment to show that electrolytic polarization produces a back electromotive force. Explain how this polarization is minimized (or eliminated) in (*a*) a Daniell cell, (*b*) a dry cell.

Compare the merits of the hydrometer and the voltmeter as an indicator of the extent to which a lead accumulator is discharged. (Northern G.C.E. Adv. L.).

13. Describe *one* type of primary cell with which you are acquainted. Explain the functions of its different parts and indicate the origin of the electrical energy.

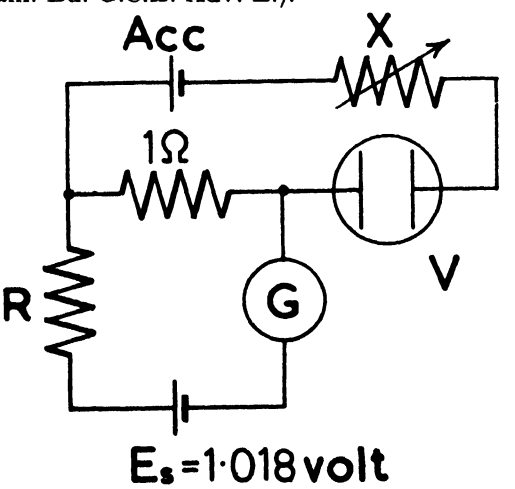
The readings of a high resistance voltmeter connected to the terminals of a Leclanché cell are (*a*) on an otherwise open circuit, 1·40 volts, (*b*) when a 2-ohm coil is connected across the terminals in parallel with the voltmeter, 1·10 volts, (*c*) on removing the coil after it has been connected for a few minutes, 1·30 volts, (*d*) after another half-hour, 1·40 volts. Explain these readings and calculate the internal resistance of the cell. (Northern H.S.C.).

14. Give an account of the lead accumulator, and discuss the changes that take place during charge and discharge.

A battery of accumulators, consisting of 50 cells, each of internal resistance 0·01 ohm and e.m.f. 2·2 volts, joined in series, is being charged through a resistance of 2 ohms by a 120 volt D.C. generator. Calculate the charging current. Explain what is likely to happen if the power supply operating the generator is suddenly cut off. (Oxford Local G.C.E. Adv. L.).

15. Explain the principle of the potentiometer. Describe how you would use a potentiometer to compare two low resistances of the order of 0·01 ohm.

In the accompanying diagram V represents a copper voltameter joined in series with a standard 1 ohm coil, an accumulator and the variable rheostat X. The standard cell E_s in series with the resistance R and galvanometer G is joined as shown to the terminals of the standard 1 ohm coil. The rheostat X is adjusted until G is undeflected. What mass of copper is deposited per hour on the cathode of V? (Electrochemical equivalent of copper = 0·00033 gm. per coulomb.). (Assoc. Exam. Bd. G.C.E. Adv. L.).



CHAPTER 9

ELECTROMAGNETIC INDUCTION

9.1. Induced e.m.f.'s

Faraday discovered that e.m.f.'s are produced in conductors when the magnetic field strength changes in the vicinity. The conditions under which such "induced" e.m.f.'s are produced are illustrated by the following experiments:

The ends of the coil S in Fig. 9.1, are connected to a moving-coil

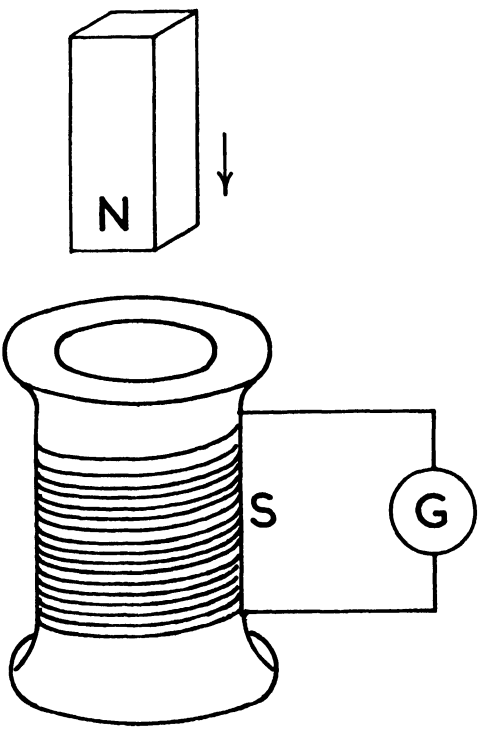


Fig. 9.1

galvanometer G. When a bar magnet is quickly plunged into S, G shews a deflection which increases with the speed of the magnet. If the magnet is rapidly withdrawn a deflection is obtained in the opposite direction. The galvanometer deflection also increases with the strength of the magnet. Similar results are obtained if the coil is moved and the magnet fixed.

The galvanometer deflection is produced by an e.m.f. which occurs only *when relative motion exists between magnet and coil.* The phenomenon is known as *Electromagnetic Induction.*

In Fig. 9.2, LM is a long wire connected to a battery, rheostat Rh, ammeter A and key K', forming a *primary circuit*; PQ is a similar wire connected to two stout copper rails PC and QD. The galvanometer G is connected to stout copper rods which slide on PC and QD and make contact at J and K; $PQKGJ$ forms the *secondary circuit.*

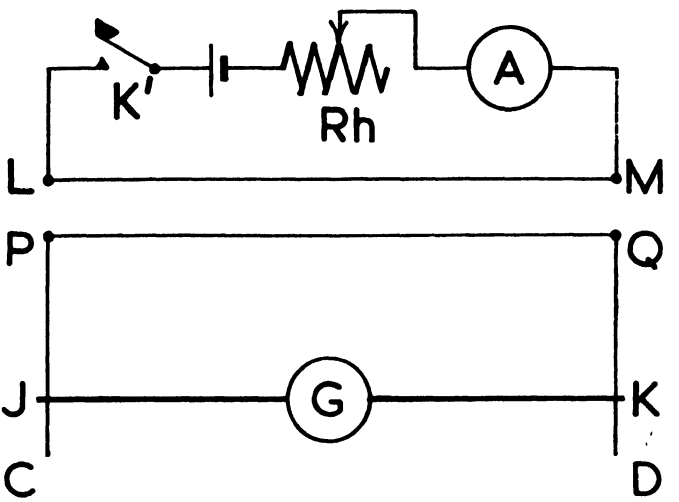


Fig. 9.2

1. (*a*) LM, PQ and JK are arranged to be parallel and K' is closed. A momentary deflection is observed on G, the current flowing from P to Q.
   (*b*) K' is opened; a deflection in G is observed in the opposite direction.
   (*c*) Both (*a*) and (*b*) shew increased deflections when the current strength (I) in LM is increased.

The deflections are caused by induced e.m.f.'s which last *whilst the magnetic field created by a primary is changing.* The e.m.f.'s increase with I, i.e. with field strength.

2. (*a*) K' is closed. After the momentary deflection of G, JK is moved rapidly from PQ by sliding along PC and QK. A deflection is observed due to current passing from P to Q.
   (*b*) With K' still closed the motion of JK is reversed. The deflection is reversed.
3. With K' closed and JK fixed, the secondary circuit is moved bodily away from LM. The galvanometer again shows a deflection which is reversed when the motion is reversed.
4. A block of soft iron is inserted in the secondary and 2 is repeated. The deflections are increased considerably.

From these experiments is deduced: (i) an induced e.m.f. is produced in a circuit which is submitted to changing conditions in a magnetic field or vice versa; (ii) the e.m.f. depends on magnetic field, the circuit area and its rate of change; (iii) the e.m.f. increases with the relative

permeability of the medium; (iv) the direction of the induced current is such that its magnetic field tends to oppose the change producing it.

9.2. Use of Ballistic Galvanometer - Quantitative Demonstrations

To obtain quantitative results a uniform field is provided in place of the primary circuit of Fig. 9.2, and a ballistic galvanometer is used as a detector. The deflection of a ballistic galvanometer is proportional to the total charge which passes through it. If the current strength at any time t during the generation of an induced e.m.f. E volts, is I amps.:

$$I = \frac{dQ}{dt} = \frac{E}{R+G} \quad . \quad . \quad . \quad . \quad (1)$$

where $R + G$ represents the circuit and galvanometer resistance in ohms.

Since: $I \,.\, dt = dQ$, the amount of charge that flows in time dt:

$$dQ = \frac{E}{R+G} \,.\, dt \text{ coulombs}$$

On integrating for the time the e.m.f. exists:

$$Q = \frac{\int E dt}{R+G} \text{ coulombs} \quad . \quad . \quad . \quad (2)$$

Since $Q \propto \theta$, where θ is the ballistic deflection:

$$\underline{\theta \propto \int E \,.\, dt} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

i.e. a ballistic deflection measures the time integral of the induced e.m.f.

The earth's magnetic field replaces the primary circuit and a ballistic galvanometer replaces G. Repeating 2., JK is moved rapidly to different distances along the rails: *θ is proportional to the distance moved*, i.e. *is proportional to the area swept out by JK*. By varying the inclination of the rails (α), it is found that *θ is proportional to sin α*. The deflection is greatest when α is the angle of dip, i.e. H_0 is normal to the plane of the rails. Combining these results: *the ballistic deflection is proportional to the product of the area swept out by JK and the normal component of field strength*, i.e. *θ is proportional to the change in magnetic flux through the circuit $PQJK$*.

Were it possible to repeat 4. using materials of widely different relative permeability (μ), it would be found that $\theta \propto \mu$, when α and the area swept out by JK were constant, hence, *θ is proportional to the change in flux of magnetic induction* ($B = \mu\mu_0 H$),

i.e. $$\theta \propto \triangle(B_n \,.\, A)$$

where B_n is the normal magnetic induction at any point in $PQJK$ and A

base of a retort stand, suspended from the mid-point of its shorter edge by means of thin wire. If one of the vertical edges is lightly tapped to give an impulse to the system, the block rotates slowly, but during the time of the impulse the deflection of the block is negligibly small. The subsequent motion consists of oscillations with decreasing amplitude. The initial maximum deflection of the coil is called the "*kick*" (or throw). This is proportional to the total charge that passed through the coil. If θ_1 is the kick when Q coulombs pass:

$$Q = \alpha \,.\, \theta_1 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where the constant of proportionality α is called the *ballistic constant* or the *ballistic sensitivity* of the galvanometer; it is usually expressed in micro-coulombs per scale division.

*9.7. Theory of Galvanometer

To prove (11), suppose the galvanometer coil is undamped so that after the initial kick it oscillates indefinitely. The effect of damping is taken into account later.

At any time t during the time of current flow, when its strength is I amps., the couple acting on the coil is:

$$G = BANI \text{ newton.m. (cf. (5.22,) Chap. 5, } \theta = 0).$$

The equation of motion of the coil at this time is given by:

$$\text{Couple } (G') = \text{Moment of inertia of coil } (MI) \times \text{angular acceleration } \left(\frac{d^2\theta}{dt^2}\right)$$

i.e. $$G' = (MI) \,.\, \frac{d^2\theta}{dt^2} = (MI) \,.\, \frac{d\omega}{dt} \quad . \quad . \quad . \quad (12)$$

Where θ is the deflection and ω the angular velocity, $[\omega = (d\theta/dt)]$. Since θ is negligible during the time current flows the opposing coupl caused by torsion is also negligible and $G = G'$, whence:

$$BANI = (MI) \,.\, \frac{d\omega}{dt}$$

i.e. $$BAN \,.\, I \,.\, dt = (MI) \,.\, d\omega$$

Since $I \,.\, dt = dQ$: $$BAN \,.\, dQ = (MI) \,.\, d\omega \quad . \quad . \quad . \quad (13)$$

When current ceases to flow Q coulombs have passed through the galvanometer and the angular velocity of the coil reaches a value ω_o, so that on integrating (13):

$$BANQ = (MI) \,.\, \omega_o$$

i.e. $$\omega_0 = \frac{BANQ}{(MI)} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The torsion couple $(\tau\theta)$ is important when θ becomes appreciable. Let the time (t_1) now be measured from the time current flow ceases; the new equation of motion is:

$$(MI) \cdot \frac{d^2\theta}{dt_1^2} = -\tau\theta \ (G = O)$$

i.e. $$\frac{d^2\theta}{dt_1^2} = -\frac{\tau\theta}{(MI)} = \text{const.} \times \theta \quad . \quad . \quad . \quad (15)$$

This is the equation of simple harmonic motion; the period is:

$$T = 2\pi\sqrt{\frac{1}{\text{const.}}} = 2\pi\sqrt{\frac{(MI)}{\tau}}$$

The solution to (15) is:

$$\theta = \theta_0 \cdot \sin\sqrt{\frac{\tau}{(MI)}} \cdot t_1 \quad . \quad . \quad . \quad (16)$$

where θ_0 is the *amplitude of the motion.*
From (16) the angular velocity is given by:

$$\omega = \frac{d\theta}{dt_1} = \sqrt{\frac{\tau}{(MI)}} \cdot \theta_0 \cdot \cos\sqrt{\frac{\tau}{(MI)}} \cdot t_1 \cdot \quad . \quad (17)$$

When $t_1 = 0$, $\omega = \omega_0$, so that:

$$\omega_0 = \sqrt{\frac{\tau}{(MI)}} \cdot \theta_0 \quad . \quad . \quad . \quad . \quad (18)$$

From (14) and (18):

$$\frac{BANQ}{(MI)} = \sqrt{\frac{\tau}{(MI)}} \cdot \theta_0$$

i.e. $$Q = \frac{\sqrt{\tau(MI)}}{BAN} \cdot \theta_0 \quad . \quad . \quad . \quad . \quad (19)$$

whence: $$Q = a\,\theta_0, \text{ where } a = \frac{\sqrt{\tau(MI)}}{BAN} \quad . \quad . \quad (20)$$

The ballistic galvanometer may be used for measuring steady currents and these are related to the steady deflection θ_s, by the formula:

$$I = \lambda\theta_s = \frac{\tau}{BAN} \cdot \theta_s \quad \text{(Chapter 5)}$$

Substituting $BAN = \frac{\tau}{\lambda}$ in (20):

$$Q = \lambda \frac{\sqrt{\tau(MI)}}{\tau} \cdot \theta_0 \quad . \quad . \quad . \quad . \quad (21)$$

and since: $T = 2\pi \sqrt{\frac{(MI)}{\tau}}$, (21) may be written:

$$Q = \frac{\lambda T}{2\pi} \cdot \theta_0 \quad . \quad . \quad . \quad . \quad . \quad (22)$$

so that:

$$\underline{a = \frac{\lambda T}{2\pi}} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The ballistic constant is thus related to the steady current sensitivity and the period.

***Correction for Damping**

In practice the " kick " of the galvanometer is less than θ_0 given by (22), because of damping by: (i) air viscosity and (ii) induced currents (*electromagnetic damping*); (i) is much smaller than (ii) and is a friction effect giving rise to an opposing couple proportional to the angular velocity, say $\beta \cdot (d\theta/dt_1)$, where β is a constant. In electromagnetic damping, currents are induced in the coil caused by the flux change as it rotates. The induced current creates an opposing couple (Lenz's Law), which is proportional to the angular velocity of the coil and may be represented by: $\gamma(d\theta/dt_1)$ (for constant circuit resistance).

The total opposing couple caused by damping is: $(\beta + \gamma)\ (d\theta/dt_1)$, and the true equation of motion after the charge has passed is:

$$(MI) \cdot \frac{d^2\theta}{dt_1^2} = -(\beta + \gamma) \cdot \frac{d\theta}{dt_1} - \tau\theta$$

i.e. $$\frac{d^2\theta}{dt^2} + 2k \cdot \frac{d\theta}{dt} + n^2\theta = O \text{ (neglecting the dashes)} \quad . \quad (24)$$

where: $$2k = \frac{(\beta + \gamma)}{(MI)}, \text{ and } n^2 = \frac{\tau}{(MI)}.$$

Equation (24) is the equation of *damped harmonic motion.* The solution when damping is small, $(2k \ll n^2)$, is:

$$\theta = \theta_0 \cdot e^{-kt} \cdot \sin \sqrt{(n^2 - k^2)} \cdot t \quad . \quad . \quad (25)$$

Fig. 9.9. is a plot of θ against t. The maximum values of θ have a decreasing value on either side of the zero. The time between successive maxima of the same sign, i.e. the *period* of the motion, is:

$$T = \frac{2\pi}{\sqrt{n^2 - k^2}} \text{ (from (25))} \quad . \quad . \quad . \quad (26)$$

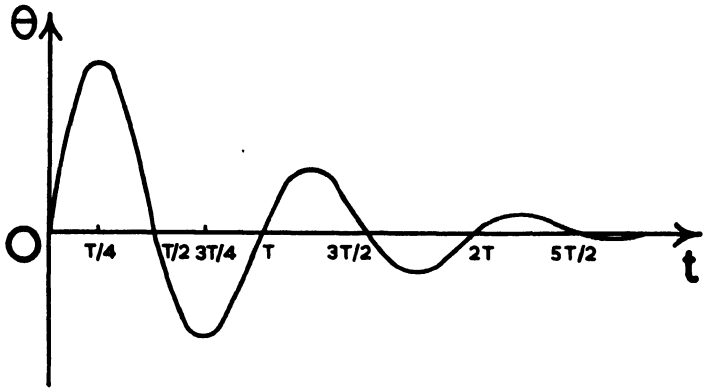


Fig. 9.9

The successive maxima of θ are given when: $t = T/4,\ 3T/4,\ 5T/4$, etc., i.e.:

$$\theta_1 = \theta_o \,.\, e^{-kT/4},\ \theta_2 = \theta_o \,.\, e^{-3kT/4},\ \theta_3 = \theta_o \,.\, e^{-5kT/4}, \text{ etc.}$$

Hence:
$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots = e^{kT/2}$$

taking logs.:
$$\log_e\left(\frac{\theta_1}{\theta_2}\right) = \log_e\left(\frac{\theta_2}{\theta_3}\right) = \ldots = \frac{kT}{2} = \delta \quad . \quad . \quad (27)$$

The constant δ is called the *logarithmic decrement* of the motion. It is a measure of the damping.

From above:
$$\theta_1 = \theta_o \,.\, e^{-\delta/2} \,.$$

i.e.
$$\theta_o = \theta_1 e^{\delta/2} = \theta_1\left(1 + \frac{\delta}{2} + \frac{\delta^2}{4.2!} + \ldots\right)$$

Since damping is always assumed to be small, this becomes:

$$\underline{\theta_o = \theta_1\,(1 + \delta/2)} \quad . \quad . \quad . \quad . \quad (28)$$

Hence the corrected expression for Q in terms of the observed kick is:

$$\underline{Q = \alpha\,\theta_1\,(1 + \delta/2)} \quad . \quad . \quad . \quad . \quad (29)$$

This equation must be used for accurate work, but in the applications which follow damping is neglected for simplicity.

9.8. Determination of α — Calibration of Ballistic Galvanometer

The constant α is determined by discharging a condenser through the galvanometer, using the circuit of Fig. 9.10. The rheostat *Rh*, is used as a potential divider across the accumulator *B*, so that the condenser *C* (several μF.) is provided with a variable p.d.; *C* is charged by connecting points 1 and 2 of the switch *S*. The charging p.d. is measured with the high resistance voltmeter *V*. After charging, which takes a negligible time, *C* is discharged through the galvanometer *BG* by connecting points 1 and 3 of *S*. The kick of the galvanometer θ_1, is noted.

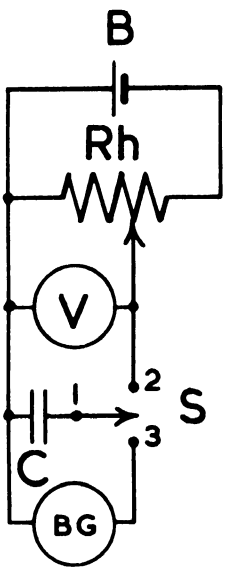


Fig. 9.10

The charge lost by the condenser on discharge is: $Q = CV$ coulombs, (*C* in farads and *V* in volts).

Hence: $Q = \alpha\,\theta_1 = CV$ (neglecting damping)

whence: $\alpha = \dfrac{CV}{\theta_1}$ coulombs per scale division . . (30)

9.9. Applications of the Ballistic Galvanometer

(1) Measurement of Capacity

If α is known *C* may be found from (30). This is the basis of capacitance measurement. For the *comparison* of capacitances α need not be known. In this experiment each condenser is charged to the same p.d. using the circuit of Fig. 9.10. The condensers are discharged separately through the ballistic galvanometer. If the initial swings are θ_1 and θ_2, and the capacitances are C_1 and C_2, farads:

$$Q_1 = \alpha\,\theta_1 = C_1V$$

and: $$Q_2 = a\,\theta_2 = C_2 V$$

whence: $$\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The method is useful for capacitances of the order of microfarads, but for lower capacitances an excessively large charging p.d. is required to give appreciable ballistic kicks.

9.10. (2) Measurement of Magnetic Flux and Field Strength

For this purpose a " flip-coil " (" search-coil ") is used, consisting of a number of turns (N) of fine wire wrapped round a circular non-magnetic former of small thickness and mean radius r m. The coil is provided with a non-magnetic handle. The ends of the coil are connected in series with a ballistic galvanometer (resistance G ohms), and a resistance box R ohms (to limit the deflection). The coil is placed with its plane at right angles to the field producing the flux. It is then removed quickly to another point. From (10) the change in flux through the coil is related to the galvanometer kick θ_1:

$$\frac{N(\Phi_1 - \Phi_2)}{R + G} = Q = a\,\theta_1 \quad . \quad . \quad . \quad (32)$$

The change in flux is thus determined. If the magnetic fields giving rise to Φ_1 and Φ_2 are uniform and the corresponding values of magnetic induction are B_1 and B_2: $\Phi_1 = B_1 \pi r^2$ and $\Phi_2 = B_2 \pi r^2$, so that:

$$B_1 - B_2 = \frac{a\,\theta_1(R + G)}{\pi r^2 N} \text{ wb.m.}^{-2} \quad . \quad . \quad (33)$$

The change in magnetic induction is thus found.

To determine B_1 or Φ_1, the search coil is removed to a point where $\Phi_2 = 0$, $(B_2 = 0)$. In such cases the magnetic field strength H_1 is found, using $B_1 = \mu_o H_1$ (in air).

The field strength between the Pole Faces of an Electromagnet

The electromagnet of Fig. 9.11, consists of coils wrapped round the vertical limbs of the iron yoke. The free ends of the yoke have flat pole faces N and S, the magnetic field between them being uniform. The magnet is energized by a current of several amps. in the coils.

To measure the field strength between the pole faces a search coil C is placed with its plane parallel to the pole faces, so that lines of induction pass through it normally; C is connected to a resistance box R and ballistic galvanometer BG. The initial flux through C is:

$$\Phi_1 = \pi r^2 B = \pi r^2 \mu_o H \text{ (in air) wb.}$$

The coil is suddenly removed to a point where $H = 0$ (arm's distance away). From (32), since $\Phi_2 = 0$:

$$\frac{\pi r^2 \mu_0 HN}{R+G} = \alpha\,\theta_1 \quad . \quad . \quad . \quad . \quad . \quad (34)$$

H is obtained from this expression in amp.m.$^{-1}$

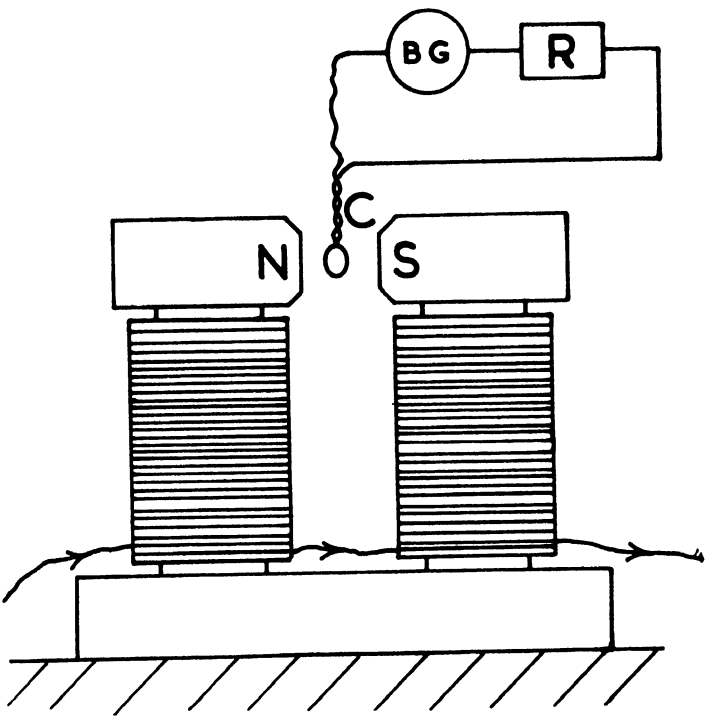


Fig. 9.11

When the field is not uniform the search-coil is made of such dimensions that the field is sensibly uniform over its cross-section. The method is limited to field strengths greater than 10^4 amp.m.$^{-1}$. For smaller field strengths the *earth inductor* may be used.

9.11. (3) The Earth Inductor

This is used for measuring the earth's field strength and the angle of dip. It consists of a flat circular coil C of N turns of radius r m. The coil is mounted inside a wooden frame F, so that it can rotate about a diameter defined by pivots P_1 and P_2 (Fig. 9.12). The frame can rotate about a horizontal axis defined by fixed pivots P_3 and P_4. The ends of C are connected to a ballistic galvanometer BG in series with a resistance box R.

To measure the horizontal component H_H, $P_1 - P_2$ is made vertical. Using a compass, the plane of C is initially arranged to be perpendicular

to the magnetic meridian. Lines of induction corresponding to H_H now pass through C normally and the initial magnetic flux is:

$$\Phi_1 = \pi r^2 B_H \text{ wb.}$$

and the flux-turns is: $N\Phi_1 = \pi r^2 B_H N$. wb. turns
The coil is rapidly rotated through 180° and the flux becomes:

$$\Phi_2 = -\pi r^2 B_H \text{ wb.}$$

The change in flux turns is thus $2\pi r^2 B_H N$; this gives a kick θ_1.

From (32): $$\frac{N(\Phi_1 - \Phi_2)}{R+G} = a\,\theta_1$$

so that: $$\frac{2\pi r^2 B_H N}{R+G} = a\,\theta_1 \quad . \quad . \quad . \quad . \quad (35)$$

B_H and hence H_H may be determined absolutely from this expression.

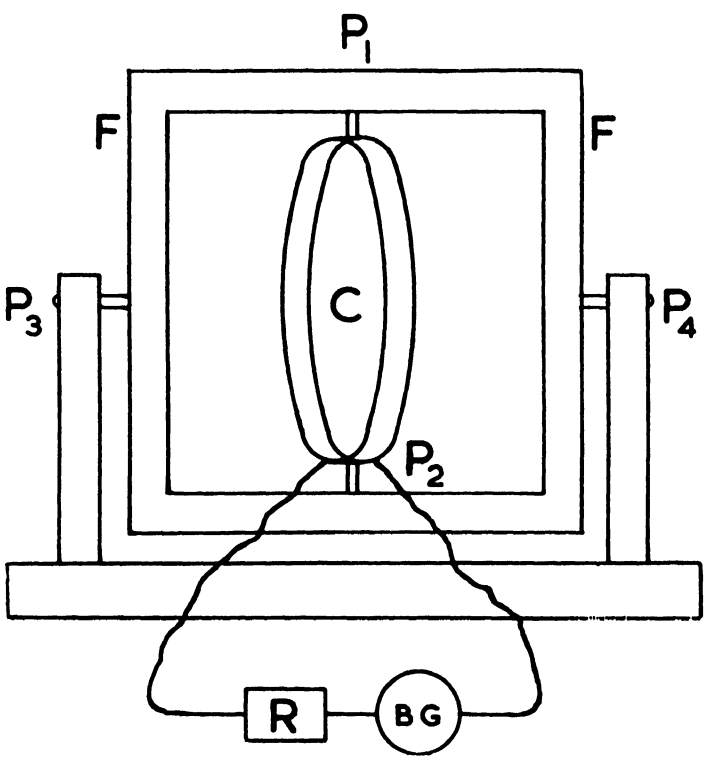


Fig. 9.12

For the vertical component, the axis $P_1 - P_2$ is horizontal. The coil is then rotated through 180° from an initial position in which the plane

of C is horizontal. The change in flux-turns is: $2\pi r^2 B_V N$, and the galvanometer kick θ_2, given by:

$$\frac{2\pi r^2 B_V N}{R+G} = a\,\theta_2 \quad . \quad . \quad . \quad . \quad (36)$$

Dividing (34) by (36):

$$\frac{B_V}{B_H} = \frac{H_V}{H_H} = \frac{\theta_2}{\theta_1} = \tan D \quad . \quad . \quad . \quad (37)$$

where D is the angle of dip.

This method of determining D avoids the tedious corrections of the dip circle (Chapter 15). With a sensitive galvanometer, θ_1 and θ_2 may be several cm. so that θ_2/θ_1 is determined with reasonable accuracy.

Example

What are the distinguishing features of a ballistic galvanometer? Explain their significance.

Explain how a search coil and ballistic galvanometer are used to explore the magnetic field between the poles of a horse-shoe magnet.

In an experiment to determine the strength of the field between the poles of an electromagnet a search coil, consisting of 100 turns of wire closely wound on a cylinder of radius 1 cm., was placed suitably between the poles and connected to a ballistic galvanometer of sensitivity 1 microcoulomb per unit scale deflection. The total resistance of the circuit was 50 ohms. When the coil was rapidly rotated through 180°, a throw of 200 scale division was obtained. Calculate the strength of the magnetic field. (Northern G.C.E. Adv. L.).

(i) The distinguishing features of a ballistic galvanometer are given in the text.

(ii) In order to explore the field between the poles of a horse-shoe magnet a small search coil is used as explained in the text.

(iii) In the problem $a = 10^{-6}$ coulombs per scale division and the total resistance in the circuit is: $R + G = 50$ ohms. For the purpose of field strength measurement the correct placing of the search coil is with its plane at right angles to the lines of induction. The change in flux turns is then given by: $N\Delta\Phi = 2\pi r^2 N \mu_0 H$ weber . turns. Since $N = 100$, $r = 10^{-2}$m., $\mu_0 = 4\pi \times 10^{-7}$ M.K.S. and H is unknown:

$$N.\Delta\Phi = 2\pi \times 10^{-4} \times 100 \times H \times 4\pi \times 10^{-7} = 8\pi^2 \times 10^{-9} H \text{ weber-turns}$$

Hence, using (32):

$$a\,\theta_1 = 10^{-6} \times 200 = \frac{8\pi^2 \times 10^{-9} H}{50}$$

i.e.
$$H = \frac{10^7}{8\pi^2} = 1{\cdot}27 \times 10^5 \text{ amp.m.}^{-1}$$

Example

State the laws of electromagnetic induction.

Explain why the swing of a galvanometer is damped by the presence of a short-circuited turn on the coil former. Why is the iron-core of an A.C. choke laminated, and in what direction must the laminations lie?

When measuring the flux density inside the gap of a large electromagnet, a search coil connected to a ballistic galvanometer (which measures the total charge which flows round a circuit) is placed with its plane normal to the flux, and then removed to a position where the flux density is negligible. Show that the total charge which flows through the galvanometer is independent of the rate of removal of the coil. In the case

of a coil of 0·5 sq.cm. area and of 50 turns, the total resistance of the coil and galvanometer being 20 ohms, the fling of the galvanometer was the same as that obtained when a 1 μF condenser charged to 110 volts was discharged through it. What is the flux density in the gap? (Oxford & Cambridge Jt.Bd. G.C.E. Schol.).

(i) The laws of electromagnetic induction are given on p. 255.

(ii) When a turn on the galvanometer coil is short-circuited a relatively large induced current flows as the coil rotates, because the induced e.m.f. is generated in an extremely low resistance circuit. The induced current produces its own magnetic field, which, by Lenz's law, gives rise to a damping torque opposing the motion.

An A.C. choke consists of a coil wound on an iron former. It possesses a large self-inductance (p. 274) and acts as a device for reducing A.C. current strengths in a manner similar to a resistance in D.C. circuits (p. 335). If the core consisted of a single piece of iron, the flux changes, caused by the alternating current, would create large induced (eddy) currents. Eddy currents give rise to undesirable heating of the iron and they are minimised by inserting thin sheets of high resistance material at right angles to their path. Thus, the core consists of sheets of iron (laminations), separated by thin layers of a dielectric.

By Lenz's law, the magnetic field produced by the eddy currents must oppose or support that of the alternating current in the coil of the choke and hence be parallel to the coil axis. The currents, therefore, flow in circular paths about this axis. Thus, the laminations must be *parallel* to the axis of the coil.

In the problem, the flux change through the coil is $\Delta\Phi = BA$, where B is the flux density in the gap and A is the area of the coil. The change in flux-turns is hence:

$$N(\Delta\Phi) = BAN$$

Since $A = 5 \times 10^{-5}$m.^{2}, and $N = 50$ turns:

$$N(\Delta\Phi) = B \times 5 \times 10^{-5} \times 50 = 2{\cdot}5 \times 10^{-3}B \text{ weber-turns.}$$

The total charge which flows through the galvanometer is given by:

$$Q = \frac{BAN}{R + G} \quad \text{from (32)}$$

This expression is independent of the rate of removal of the coil since it is independent of the time. Providing the flux-change is completed in a time short compared with the period of the galvanometer the ballistic fling is also independent of the time of removal. Hence:

$$Q = \alpha\,\theta_1 = \frac{BAN}{R + G} = \frac{2{\cdot}5 \times 10^{-3}B}{20} \text{ coulombs}$$

Also: $Q = CV = 10^{-6} \times 110 = 11 \times 10^{-5}$ coulombs.

$$\therefore \alpha\,\theta_1 = 11 \times 10^{-5}$$

Hence: $$\frac{2{\cdot}5 \times 10^{-3}B}{20} = 11 \times 10^{-5}$$

i.e. flux-density $$B = 0{\cdot}88 \text{ wb.m.}^{-2}$$

Example

Explain the earth inductor method for the determination of the angle of dip.

The ends of a flat circular coil of 1,000 *turns, each of diameter* 10·0 *cm., are joined together. The coil is placed on a horizontal table and then turned over. If the resistance of the coil is* 25 *ohm what charge, in microcoulombs, will flow round the coil?*

Establish any formula used, (assume $H_H = 45/\pi$ *amp.m.*$^{-1}$*; angle of dip* $= 67°$*).* (London Inter. B.Sc.).

The determination of the angle of dip is described above.

With its ends joined together the coil forms a complete circuit of $N = 1{,}000$ turns. The flux through the coil on the table is: $\Phi = BA$, where B is the magnetic induction corresponding to the earth's vertical field strength H_V, $(B = \mu_o H_V)$. Since $H_V = H_H \tan D$, $B = \mu_o H_H \tan D$. The flux change on turning over the coil is twice the initial flux, so that the change in flux-turns is:

$$N.\Delta\Phi = 2 \,.\, BAN = 2\mu_o H_H \tan D \,.\, AN$$

The charge which flows in the coil as a result of the change in flux-turns is given by (32):

$$Q = \frac{2\mu_o H_H \tan D \,.\, AN}{R}$$

Since the diameter of the coil is 0·1 m., $A = 2{\cdot}5\pi \times 10^{-3}$m.^{2};

$H_H = 45/\pi$ amp.m.$^{-1}$, $D = 67°$, $N = 1000$, $\mu_o = 4\pi \times 10^{-7}$ M.K.S., $R = 25$ ohm:

$$\therefore Q = \frac{2}{25} \times 4\pi \times 10^{-7} \times \frac{45}{\pi} \times 2{\cdot}356 \times 2{\cdot}5\pi \times 10^{-3} \times 1000$$

i.e. $Q = 26{\cdot}65 \times 10^{-6}$ coulombs $=$ 26·65 microcoulombs.

9.12. Self-Inductance

Since a current in a coil gives rise to its own magnetic field, it produces a flux through itself. When the current strength changes the flux changes and an e.m.f. is induced in the coil, called a *self-induced e.m.f.* The phenomenon is known as *self-induction.*

The field strength at any point in the vicinity of a coil is proportional to the current strength so that the flux and the number of flux-turns through the coil is proportional to the current strength.

i.e. $$N\Phi \propto I$$

Hence: $$N\Phi = LI \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The constant of proportionality (L) is called the *self-inductance* of the coil. When the current strength changes, the self-induced e.m.f. is:

$$E = -\frac{Nd\Phi}{dt} = -\frac{LdI}{dt} \text{ volts} \quad . \quad . \quad . \quad (39)$$

From (38) the dimensions of the unit of inductance on the M.K.S. system are: Flux ÷ Current strength, and the unit is: weber.ampere^{-1}, (wb.amp.$^{-1}$). This unit is called the *henry.*

From (39), the henry may be defined as follows:

A self-inductance has a magnitude of one henry if an e.m.f. of one volt is produced in it when the current passing through it changes at the rate of one amp. per second.

Self-inductance, often called inductance, is a constant for any given

coil. It is dependent on the number of turns, the radius, length of coil and permeability of the medium. It is determined theoretically in simple cases.

9.13. Self-Inductance of a Long Solenoid

Suppose the solenoid has n turns *per metre*, radius r m., length l m., and is filled with a medium of relative permeability μ. When I amps. flow through the coil, the magnetic field strength is given by: $H = nI$ amp.m.$^{-1}$ and is directed along the axis. The flux through the coil is hence:

$$\Phi = \pi r^2 \mu \mu_0 H = \mu \mu_0 \pi r^2 n I \text{ wb.}$$

The number of flux turns is: $N\Phi = nl\Phi = \mu\mu_0\pi r^2 n^2 l I$
By definition, therefore:

$$L = \mu \mu_0 \pi r^2 n^2 l \text{ henries} \quad . \quad . \quad . \quad (40)$$

(40) does not apply to short coils because of the uncertainty of the field near the end faces.

*9.14. Growth of Current in an Inductive Circuit

Self-inductances are made of wire which has an associated resistance. It is usual, therefore, to represent an inductance by its coil-like symbol with a series resistance. Any self-induced *e.m.f.* is produced across the coil only; when current flows a *p.d.* exists across the series resistance as well. Pure self-inductance has no effect on steady current but it plays a very important part in its growth or decay.

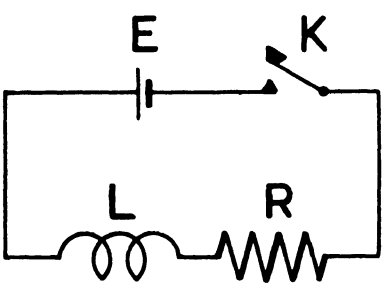


Fig. 9.13

Suppose a circuit containing a self-inductance L henries and total resistance R ohms (including that of the inductance), is supplied with an e.m.f. E volts (Fig. 9.13). When the key K is closed, a current begins to flow which takes a finite time to reach its steady value, given by Ohms Law: $I_0 = E/R$ amps. Whilst the current is increasing the magnetic flux through the coil increases and a self-induced e.m.f. is created which opposes the increase (Lenz's Law). The effect of the inductance is to increase the time taken for the current to build up to its final value I_0. The way in which I varies during this time is obtained by analysis.

Suppose K is closed at time $t = 0$. Let the current strength at time t be I amps. The induced e.m.f. at this time is hence: $E = -L.(dI/dt)$ volts. The p.d. across R is IR volts, so that applying Kirchhoff's Law:

$$IR = E - L \cdot \frac{dI}{dt}$$

i.e. $$\frac{L}{R}\cdot\frac{dI}{dt} = -(I - \frac{E}{R} \quad . \quad . \quad . \quad (41)$$

Since E/R is constant, $d(I - E/R) = dI$, so that:

$$\frac{d(I - E/R)}{(I - E/R)} = -\frac{R}{L}\cdot dt$$

Treating $(I - E/R)$ as a variable and integrating both sides:

$$\log_e (I - E/R) = -\frac{R}{L}\cdot t + \text{constant} \quad . \quad . \quad (42)$$

The constant of integration is determined by using the fact: $I = 0$, $t = 0$:

Hence: $\log_e(-E/R) = \text{constant}$

Therefore: $\log_e(I - E/R) - \log_e(-E/R) = -\frac{R}{L}\cdot t$

i.e. $$\log_e \frac{I - E/R}{-E/R} = -\frac{R}{L}\cdot t$$

Taking antilogs: $$\frac{I - E/R}{-E/R} = e^{-Rt/L}$$

i.e. $$I = \frac{E}{R}(1 - e^{-Rt/L}) = I_o(1 - e^{-Rt/L}) \quad . \quad (43)$$

As t tends to ∞, $e^{-Rt/L}$ tends to zero, so that I tends to I_o. Curves of I against t are plotted in Fig. 9.14. The speed at which the curve

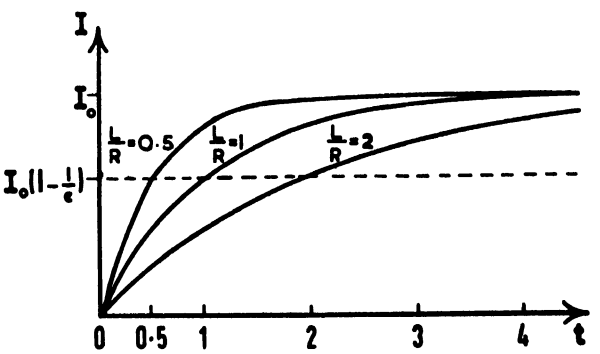


Fig. 9.14

approaches I_o is governed by the quantity: L/R. Theoretically, it takes an infinite time for I to reach I_o, but when $t = 4L/R$, $I = I_o(1 - e^{-4}) = 0{\cdot}982I_o$; L/R is thus a guide to the time taken to

reach steady conditions and is called the *time constant* of the circuit. Putting $t = L/R$ in (43) gives $I = I_0(1 - e^{-1})$, so that the time constant is the time taken for the current strength to approach to within $1/e$ of its steady value.

9.15. Demonstration Experiment

The variation of I with t is demonstrated by using a *choke*, (an iron-cored inductance of several henries), in series with a lamp P, a suitable battery and a key K (Fig. 9.15). The lamp replaces R of Fig. 9.13.

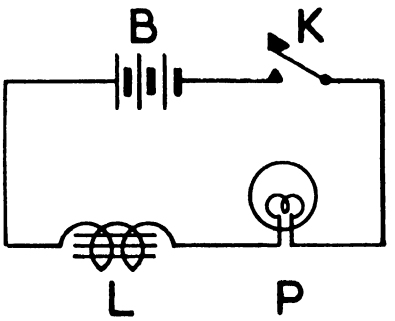


Fig. 9.15

When K is closed the brightness of the lamp gradually increases, taking a few seconds to reach normal intensity. It is interesting to note that when K is opened the light disappears immediately.

Self-induced e.m.f.'s become apparent as " transients " when circuits are " made ". For example in the P.O. Box (p. 156), if K_2 is depressed *after* K_1, the self-induced e.m.f. produced by inductance in the circuit gives a kick on the galvanometer. Coils are wound " non-inductively " to minimise these effects; this is achieved by using a loop of wire and winding so that the current flows in opposite directions in neighbouring turns and their magnetic fields cancel.

9.16. Energy Stored by an Inductance

Potential energy is stored by a magnetic field so that when a current grows to a steady value (I_0), a definite amount of potential energy is stored in the growing magnetic field. The potential energy stored is equal to the work done by the current in the inductance. At the time t the induced e.m.f. is: $E = -L(dI/dt)$ volts and the rate of working is: $EI = -LI(dI/dt)$ watts. The work done in time dt is hence:

$$dW = EI \,.\, dt = -LI(dI/dt)dt = -LI \,.\, dI \text{ joules}$$

In the time taken to establish the field, I changes from 0 to I_0, so that the total work done by the current is:

$$\int dW = -L\int_0^{I_0} I \,.\, dI \text{ joules}$$

i.e. $$\underline{W = \tfrac{1}{2}LI_0^2 \text{ joules} = \text{Potential Energy stored}} \quad . \quad (44)$$

9.17. Decay of Current in an Inductive Circuit

When K is opened in Fig. 9.13, the current strength falls to zero. This means a rapid collapse of the magnetic field and a rapid change of

flux through L, producing a large self-induced e.m.f., which, by Lenz's Law, tends to maintain the original current flow. The induced e.m.f. is much higher than the battery e.m.f., and since the circuit is broken it appears across the key. The e.m.f. may be sufficiently high to cause a "breakdown" of the air, and a spark to cross the key-gap. This is dangerous.

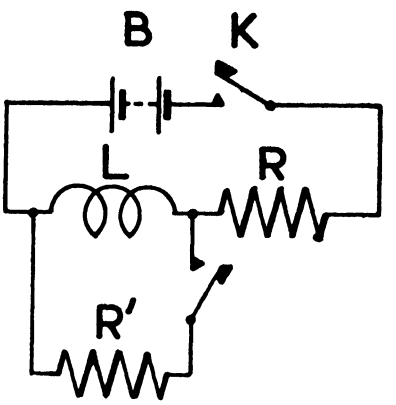


Fig. 9.16

One method of avoiding sparks is to close a subsidiary switch across a resistance in parallel with the inductance (R') (Fig. 9.16), before the main circuit is opened. This allows the self-induced e.m.f. produced across L to create a current in the subsidiary circuit and dissipate the energy of its magnetic-field when the main switch is opened.

Example

State the laws of electromagnetic induction. Explain what is meant by self induction and define a unit of self inductance. Give a general indication of how the current in a circuit, to which an e.m.f. is applied, rises from zero to it final value, indicating the effect of self inductance.

A solenoid is 40 *cm. long and has* 10 *turns per cm. Each turn of the solenoid has an area of* 50 *sq.cm. What is the value of the self inductance of the solenoid (the end effect may be neglected)?* (London G.C.E. Adv. L.).

The first part of the question is discussed in the text.

From (40): $L = \mu_0 \pi r^2 n^2 l^2$ henries.

From the given data: $\pi r^2 = 5 \times 10^{-3}$m.^{2}, $n = 1{,}000$ per metre, $l = 0{\cdot}1$ metres, $\mu_0 = 4\pi \times 10^{-7}$ M.K.S., hence:

$$L = 4\pi \times 10^{-7} \times 5 \times 10^{-3} \times 10^6 \times 0{\cdot}4 \text{ henries}$$

i.e. $L = 2{\cdot}51 \times 10^{-3}$ henries

Example

A coil having an inductance of 0·45 *henries and resistance* 10 *ohms is connected in series with a tapping key, a* 35 *ohm resistor and a battery of e.m.f.* 12 *volts. At what time after pressing the key will the current strength be half its steady value? What is the value of the steady current?*

From (43): $I = I_0(1 - e^{-tR/L})$

When $I = \frac{I_0}{2}$: $\frac{I_0}{2} = I_0(1 - e^{-tR/L})$

Hence: $e^{tR/L} = 2$

taking logs to the base "e":

$$\frac{tR}{L} = \log_e 2 = 0{\cdot}693$$

The same phenomenon occurs in 2 if the current strength in 1 changes. The number of flux-turns in 2 is: $N_2\Phi_2 = MI_1$, and the induced e.m.f. is:

$$E_2 = -M \cdot \frac{dI_1}{dt} \quad . \quad . \quad . \quad . \quad (47)$$

M depends on the cross-sections of the coils, their positions and the relative permeability of the medium.

Whilst the current strength in each coil changes, self-induced e.m.f.'s are also produced, given by: $-L_1(dI_1/dt)$ and $-L_2(dI_2/dt)$, respectively. It can be shown that if the coils are close together, so that all the lines of induction produced by the current in one coil pass through the turns of the other coil:

$$M^2 = L_1L_2 \, . \quad . \quad . \quad . \quad . \quad (48)$$

If the coils are more distant from each other:

$$M^2 = kL_1L_2 \quad . \quad . \quad . \quad . \quad (49)$$

where k is a constant depending on the distance apart of the coils and is less than unity; k is called the *coupling coefficient.*

The unit of mutual inductance is the same as that of self-inductance, viz. the henry.

9.19. Theoretical Calculation of Mutual Inductance—Standard Inductance

A standard mutual inductance is made by winding a coil of m turns round the central part of a long solenoid (Fig. 9.18). Suppose the

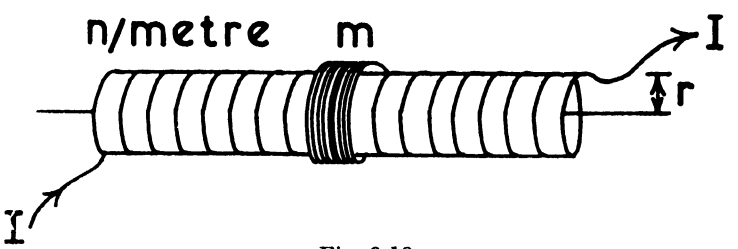


Fig. 9.18

solenoid has n turns per m. of radius r m. When I amps. flows through the solenoid the magnetic field strength inside is: $H = nI$ amp.m.$^{-1}$ The magnetic flux through the secondary is:

$$\Phi_2 = \mu_0\mu\pi r^2H = \mu_0\mu\pi r^2nI \text{ wb.}$$

The number of flux turns in the secondary is:

$$m\Phi_2 = \mu_0\mu\pi r^2mnI$$

From (45): $M = \mu_0 \mu \pi r^2 mn$ henries . . . (50)

9.20. Determination of a using Standard Mutual Inductance

The ballistic constant is found using the circuit of Fig. 9.19. The steady current (I amps.) flowing through the primary of the inductance

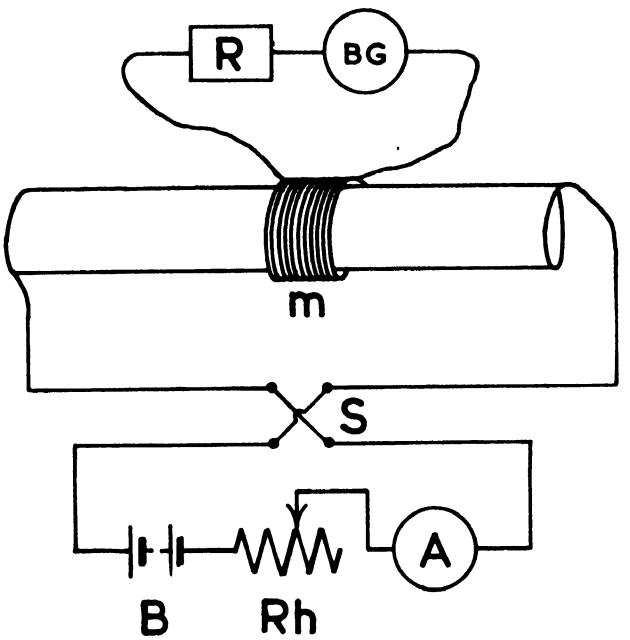


Fig. 9.19

is reversed and the change in flux through the secondary causes a ballistic kick (θ_1). The initial and final fluxes through the secondary are equal and opposite and the change in flux turns is hence:

$$m(\Phi_1 - \Phi_2) = 2MI \text{ wb.}$$

Hence: $$Q = a\,\theta_1 = \frac{2MI}{R+G} \quad . \quad . \quad . \quad . \quad (51)$$

Since M is known from (50), a is determined.

*9.21. Determination of Mutual Inductance

Mutual inductance is rarely calculated because of the difficulty in determining the flux through either coil. To determine M experimentally, a ballistic galvanometer BG is connected as in Fig. 9.20. The two coils L_1 and L_2 belong to the primary and secondary circuits respectively; L_1 is supplied with a known current I amps., measured with the ammeter A, This current is broken by opening the key K;

the change in secondary flux turns is MI. This produces a kick θ_1 on BG. Hence:

$$\alpha\,\theta_1 = \frac{MI}{R+G}$$

and:
$$M = \frac{\alpha\,\theta_1(R+G)}{I} \text{ henries} \quad . \quad . \quad . \quad (52)$$

The resistance R includes the resistance of L_2 and the box R.

Although the transient flow of current produces a self-induced e.m.f. in L_2, it has no effect on BG since the initial and final values of its magnetic flux are zero.

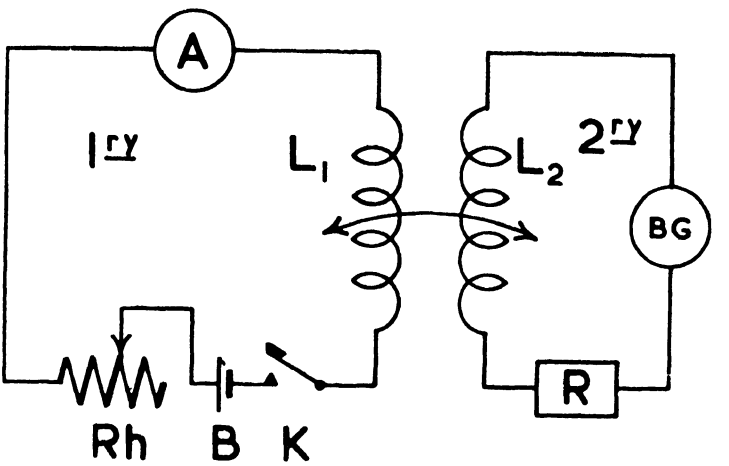


Fig. 9.20

9.22. The Induction Coil

The induction coil was devised by Rhumkorff in 1851 for the production of very high voltages. It consists of two coils wound on the same core. The primary P consists of one or two hundred turns of insulated thick wire wound on the core of soft iron wires F, (Fig. 9.21). Insulated soft iron wires are used, rather than a solid cylinder, to reduce eddy currents.

The ends of P are connected to one pole of the battery and a steel spring B, respectively. Attached to the free end of B is a short piece of soft iron A. When no current flows in P, B makes contact with an adjustable platinum tipped screw D. The primary circuit is completed by connecting D to the opposite pole of the battery. The condenser (C) between B and D has a special function.

The secondary coil S consists of a very large number of turns of fine insulated copper wire around P. It has a high resistance (some ten thousand ohms), and its ends are led to the pointed metal rods G_1 and G_2 mounted on the ebonite casing; G_1 and G_2 form an adjustable spark-gap.

The current which flows when the primary circuit is closed magnetizes F like a bar magnet and A is attracted and the circuit broken at D. The current falls, F becomes demagnetized and the spring restores contact at D. The primary current flows again and the cycle of operations is repeated with a frequency dependent on the stiffness of B and the coil inductances.

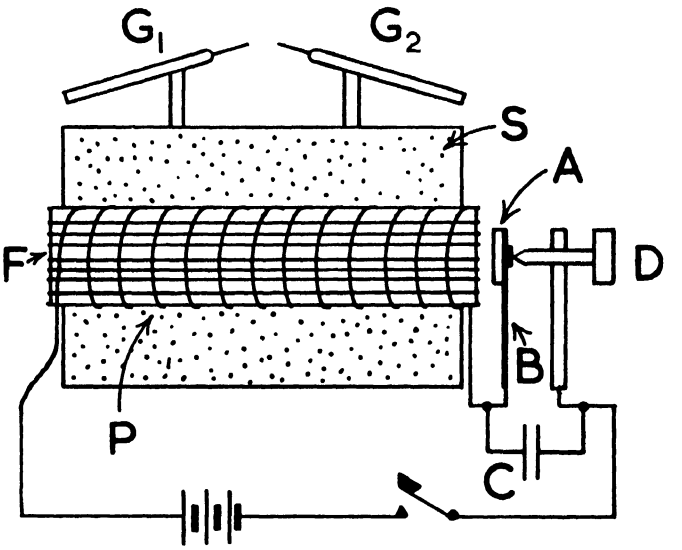


Fig. 9.21

When the current increases in P the increasing flux through S creates an induced e.m.f. When the primary circuit is broken at D, the flux through S is suddenly changed to zero and a larger e.m.f. is induced across it in the opposite direction. For each cycle of operations, therefore, an e.m.f. (E_2) is produced across the secondary which changes its direction when the current is interrupted. The value of E_2 when the primary circuit is broken, i.e. at the " break ", is much higher than at the " make ", because the time of break is much less than that of make. Thus, sparks are produced across G_1G_2 at the " break " only and to a first approximation the secondary voltage may be considered unidirectional.

The time variation of E_2 and primary current are shown in Fig. 9.22. The length of spark across G_1G_2 is a measure of the maximum value of E_2. Sparks of several cm. corresponding to p.d.'s of several kilovolts may be produced.

The Function of the Condenser

At the " break " the self-induced e.m.f. produced across P is very large and it causes the condenser C to be charged as shown in Fig. 9.23. The condenser accepts the energy originally stored in the magnetic field of P and prevents a spark crossing the gap B—D.

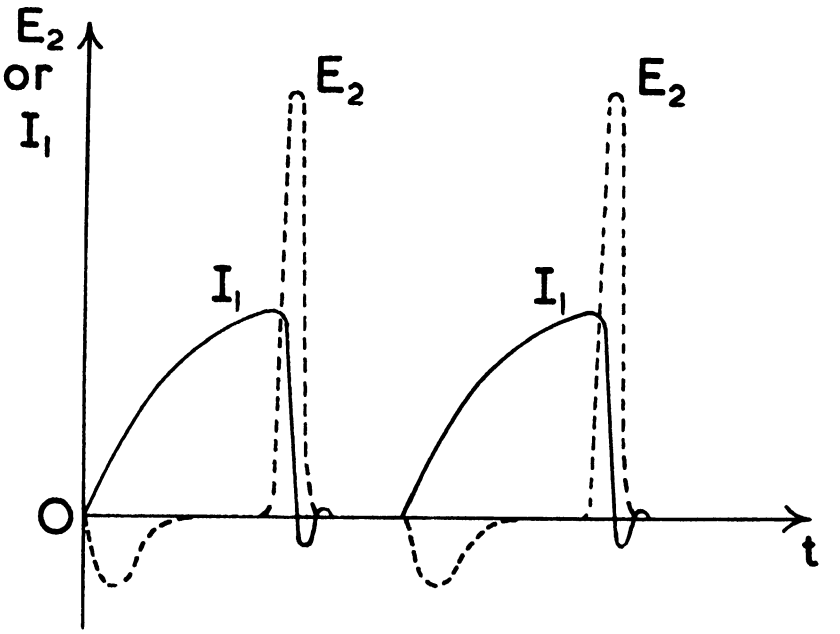


Fig. 9.22

The charged condenser and P form a complete inductance-capacity-resistance circuit. The condenser discharges round the circuit and recharges in the opposite direction, energy being dissipated in the resistance. This is repeated but the strength of the associated oscillatory current decays rapidly. Because of the small period of the oscillations, determined by L and C, and the rapid dissipation of the energy, the

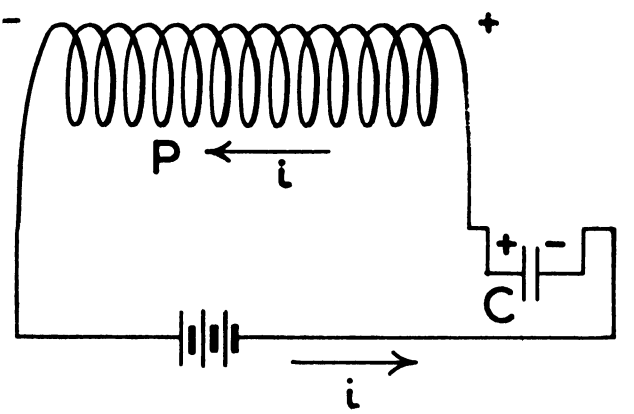


Fig. 9.23

time of break is made extremely short. Thus, the rate of change of magnetic flux and the induced e.m.f. in S is very large. Without the condenser the breakdown across B—D prevents a quick collapse of the field associated with P.

At the " make " C is short-circuited by the contact B—D and the

self-induced e.m.f. in P prevents the rapid growth of the current. Thus, the rate of change of flux and the induced e.m.f. is much smaller.

The condenser thus prevents excessive wear at the contact B—D by avoiding spark formation and provides a large e.m.f. across S at break only.

Effect of Load Resistance

If a resistance is connected across the ends of the secondary, induced *current* flows and unless the resistance is very high (megohms), so that the current strength is small, the large back e.m.f. due to *self*-inductance prevents the e.m.f. growing across G_1G_2. The power dissipated in the secondary can never be greater than that dissipated by the battery in the primary. If, therefore, the secondary current is high the e.m.f. is low. In all the applications of the induction coil, e.g. X-ray tube and discharge tube supply, the current strength supplied is very small.

*9.23. Transient Currents involving Condensers: 1. Charging of Condenser

The charging of a condenser has similar characteristics to the $L-R$ circuit. Suppose the condenser in Fig. 9.24, has a capacity C farads, the resistance R ohms and the cell an e.m.f. E volts. On pressing the key K a current begins to flow so that positive charge collects on the plate A and leaves plate B. In actual fact, electrons leave A and arrive at B at the same rate, so that A becomes positively charged and B negatively charged to the same extent. The movement of charge constitutes the current between A and B which creates a p.d. across R so that the p.d. of C is less than E. The current ceases eventually and the condenser is then charged to E volts. The speed at which the charging takes place depends on R since this limits the charging current. If R is very small the charging is practically instantaneous whilst if R is large, (of the order of megohms), the charging may take several seconds or minutes.

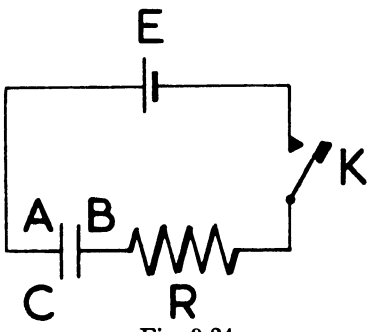


Fig. 9.24

Suppose the current strength at a time t secs. after closing K is I amps. The p.d. across R is: $V_R = IR$ volts, and if the charge on either condenser plate is Q coulombs, the p.d. across C is: $V_C = Q/C$ volts. From Kirchhoff's Law:

$$V_R + V_C = E$$

i.e.

$$IR + \frac{Q}{C} = E$$

By definition: $I = dQ/dt$, so that:

$$R \cdot \frac{dQ}{dt} + \frac{Q}{C} = E \quad . \quad . \quad . \quad (53)$$

whence: $$CR \cdot \frac{dQ}{dt} = -(Q - EC)$$

Since E/C is constant, dQ is the same as $d(Q - EC)$, so that:

$$CR \cdot d(Q - EC) = -(Q - EC)$$

and: $$\frac{d(Q - EC)}{(Q - EC)} = -\frac{1}{RC} \cdot dt$$

Integrating both sides gives:

$$\log_e(Q - EC) = -\frac{t}{RC} + \text{constant} \quad . \quad . \quad (54)$$

The constant of integration is found by using the fact: $t = 0$, $Q = 0$;

hence: $$\text{constant} = \log_e(-EC)$$

substituting in (54) and taking antilogs:

$$\frac{Q - EC}{-EC} = e^{-t/RC}$$

i.e. $$Q = EC(1 - e^{-t/RC}) = Q_0(1 - e^{-t/RC}) \quad . \quad . \quad (55)$$

where $Q_0 = EC$, is the final charge on the condenser.
The variation of Q with t is given in Fig. 9.25. The curves are similar to

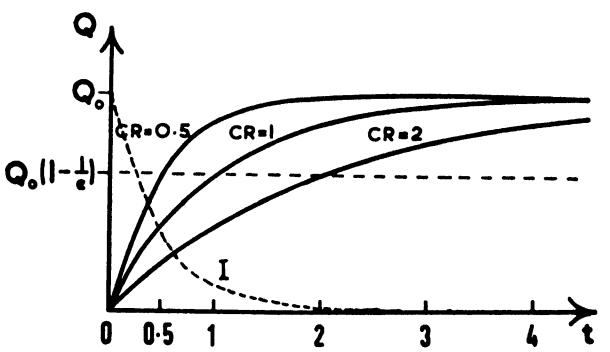


Fig. 9.25

those of Fig. 9.14. The speed at which Q approaches the final value Q_0, depends solely on CR, which is the *time constant* of the $C-R$ circuit.

The condenser may be considered to be fully charged after a time $t = 4RC$ approx. ($e^{-4} = 0{\cdot}0183$).

If $C = 1\mu F = 10^{-6}F$, the time constant is: $T = RC = 10^{-6}R$. When the condenser is connected directly across a battery, R is probably less than one ohm, and $T < 10^{-6}$ secs. which is negligible. If $R = 1$ megohm, $T = 10^{-6} \times 10^{6} = 1$ second, and the time for charging is relatively long.

Variation of Charging Current and Voltage of Condenser

By differentiating (55) with respect to time:

$$I = \frac{dQ}{dt} = \frac{-EC}{RC}(-1) \,.\, e^{-t/RC} = \frac{E}{R} \,.\, e^{-t/RC}$$

Since: $I = E/R$ when $t = 0$, E/R may be written I_0, whence:

$$I = I_0 \,.\, e^{-t/RC} \quad . \quad . \quad . \quad . \quad (56)$$

The variation of I with t is shown as a dotted line in Fig. 9.25.

The charging p.d. across the condenser at any time is:

$$V_C = \frac{Q}{C} = E(1 - e^{-t/RC}) \quad . \quad . \quad . \quad (57)$$

Likewise:
$$V_R = IR = E \,.\, e^{-t/RC} \quad . \quad . \quad . \quad (58)$$

It is seen that: $V_R + V_C = E$ always, as required by Kirchhoff's law.

Energy required to charge C

Energy is dissipated by the current flowing through R and this appears as heat at the rate IV joules per second. The total potential energy acquired by C is $\frac{1}{2}CE^2$ joules, so that the total energy delivered by the battery during the charging process is:

$$W = \int_0^\infty IV_R dt + \tfrac{1}{2}CR^2 \text{ joules}$$

Substituting for I and V_R from above:

$$W = \int_0^\infty \frac{E^2}{R} \,.\, e^{-2t/RC} \,.\, dt + \tfrac{1}{2}CE^2$$

i.e.
$$W = \tfrac{1}{2}CE^2 + \tfrac{1}{2}CE^2 = CE^2 \text{ joules} \quad . \quad . \quad (59)$$

The energy dissipated by the resistance is thus equal to the potential energy acquired by the condenser and is independent of the value of the resistance.

***9.24. Discharge of a Condenser**

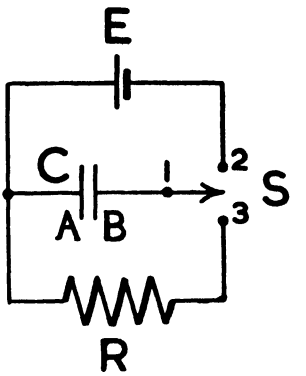


Fig. 9.26

Let the condenser of Fig. 9.26 be initially charged by the cell, using points 1 and 2 of the switch S. The condenser is discharged through the resistance R ohms by connecting points 1 and 3 of S. Plate A is then connected to plate B externally and charge leaves A to go to B, forming a current. This current strength is limited by R, causing the discharge to take time.

Let the current strength at a time t secs. after closing 1 and 3 be I amps., so that the p.d. across R at this time is: $V_R = IR$ volts. If the charge remaining on either condenser plate is Q coulombs, the p.d. across C is: $V_C = Q/C$ volts. There is no source of e.m.f. in the circuit so that Kirchhoff's law gives:

$$IR + \frac{Q}{C} = O$$

Since: $I = dQ/dt$:

$$R \cdot \frac{dQ}{dt} + \frac{Q}{C} = O \quad . \quad . \quad . \quad . \quad (60)$$

i.e.
$$\frac{dQ}{Q} = -\frac{dt}{CR}$$

Integrating:
$$\log_e Q = \frac{-t}{RC} + \text{constant} \quad . \quad . \quad . \quad (61)$$

The constant of integration is given by using: $t = 0$, $Q = EC = Q_o$ (say), so that:

$$\text{constant} = \log_e Q_o$$

Whence, on substituting in (61) and taking antilogs:

$$\frac{Q}{Q_o} = e^{-t/RC}$$

i.e.
$$\underline{Q = Q_o \cdot e^{-t/RC}} \quad . \quad . \quad . \quad . \quad (62)$$

The variation of Q with time is thus exponential as shown in Fig. 9.27. When $t = RC$, Q is reduced to Q_o/e, i.e. $Q = 0.368\ Q_o$, and, as before, it is called the *time constant* of the circuit. For laboratory capacitances (i.e. microfarads), the time constant is small unless R is of the order of megohms.

Using the relation: $I = dQ/dt$:

$$I = \frac{-Q_o}{RC} \cdot e^{-t/RC} = -\frac{E}{R} \cdot e^{-t/RC} \quad . \quad . \quad . \quad (63)$$

Following the procedure of the previous section:

$$V_R = IR = -E \cdot e^{-t/RC} \quad . \quad . \quad . \quad (64)$$

$$V_C = \frac{Q}{C} = \frac{Q_0}{C} \cdot e^{-t/RC} = E \cdot e^{-t/RC} \quad . \quad . \quad (65)$$

The negative sign in (64) indicates that V_R opposes V_C; the two p.d.'s are equal and opposite.

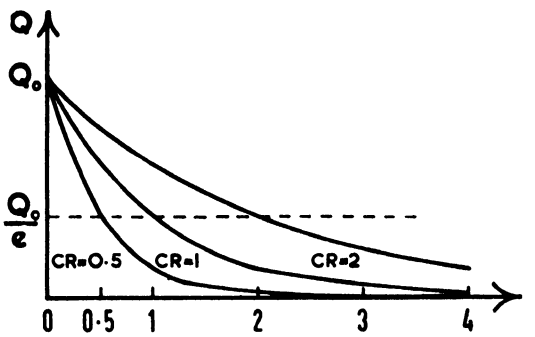


Fig. 9.27

9.25. Dissipation of Condenser Energy

When a condenser discharges into a circuit its potential energy is converted into kinetic energy of the moving charges of the current and thus dissipated as heat in the circuit resistance. This is confirmed as follows:

At any time t: $I = -\frac{E}{R} \cdot e^{-t/RC}$ (from (63)).

The rate of working in R is: $I^2R = \frac{E^2}{R} \cdot e^{-2t/RC}$ joule. sec.$^{-1}$

The total energy dissipated in R is:

$$\int_0^\infty I^2R \cdot dt = \frac{E^2}{R} \cdot \left[-\frac{RC}{2} \cdot e^{-2t/RC} \right]_0^\infty = \frac{E^2C}{2}$$

This is the same as the initial potential energy of the condenser.

*9.26. Loss of Energy on Sharing Charge

When insulated charged conductors are connected by a wire, charge flows from one to the other until their p.d. is zero. A loss of energy occurs caused by " Joule " heating in the connecting wire (p. 61). This is shown by considering the charged conductors as condensers A and B, the earthed " plates " of which are the common walls of the

room (Fig. 9.28). Let the resistance of the connecting wire be R ohms and suppose the connection is made when $t = 0$ secs. If the current flowing from A to B at the time t is I amps., the application of Kirchhoff's law gives:

$$V_A - V_B + IR = 0 \quad . \quad . \quad . \quad . \quad (66)$$

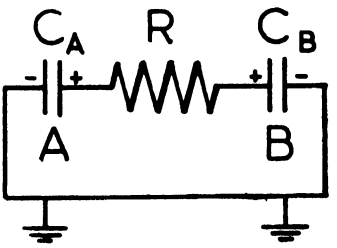


Fig. 9.28

Let the initial charge on A be Q_0 and that at time $t = Q$ coulombs. The charge on B at this time is: $(Q_0 - Q)$ coulombs, since charge is conserved. Hence:

$$V_A = \frac{Q}{C_A}; \text{ and } V_B = \frac{Q_0 - Q}{C_B} \text{ volts}$$

(66) then becomes: $$\frac{Q}{C_A} - \frac{Q_0 - Q}{C_B} + IR = 0$$

Since: $I = dQ/dt$: $$\frac{Q}{C_A} - \frac{Q_0 - Q}{C_B} + R \cdot \frac{dQ}{dt} = 0$$

Rearranging: $$\frac{dQ}{dt} + \frac{Q}{R}\left(\frac{1}{C_A} + \frac{1}{C_B}\right) - \frac{I}{R} \cdot \frac{Q_0}{C_B} = 0 \quad . \quad . \quad (67)$$

This equation may be solved in a manner similar to that of (60), or more simply as follows: differentiate (67) with respect to t and replace (d^2Q/dt^2) by (dI/dt) and (dQ/dt) by I, so that:

$$\frac{dI}{dt} + \frac{I}{R}\left(\frac{1}{C_A} + \frac{1}{C_B}\right) = 0 \quad . \quad . \quad . \quad (68)$$

Equation (68) is similar to (60) with I replacing Q, and the solution is:

$$I = I_0 e^{-t/R\left(\frac{1}{C_A} + \frac{1}{C_B}\right)} = I_0 e^{-t(C_A + C_B)/RC_AC_B} \quad . \quad . \quad . \quad . \quad (69)$$

I_0 is the constant of integration equivalent to Q_0 of (60) and $\therefore I_0 = V_0/R$, where V_0 = initial potential of A.

The rate of dissipation of energy in R is:

$$I_o^2 R = I^2 R \,.\, e^{-2t(C_A + C_B)/RC_AC_B} \text{ joule sec.}^{-1}$$

The total energy dissipated in R is:

$$I_o^2 R \int_o^\infty e^{-2t(C_A + C_B)/RC_AC_B} \,.\, dt \text{ joules.}$$

i.e. $$W = \frac{RI_o^2 C_A C_B R}{2(C_A + C_B)} \text{ joules}$$

Since: $I_o = V_o/R = Q_o/RC_A$:

$$W = \frac{R^2 Q_o^2 \,.\, C_A C_B}{C_A^2 R^2 2(C_A + C_B)} = \frac{\frac{1}{2}Q_o^2 \,.\, C_B}{C_A(C_A + C_B)} \text{ joules} \quad . \quad (70)$$

This expression is identical with (2.12) of p. 61, thus proving that the energy lost in the process disappears as heat in the connecting wire.

*9.27. Determination of High Resistance

If C is known in a $C-R$ circuit, R may be determined by measuring the values of Q which remain on C after allowing it to discharge through R for known intervals of time. From (62):

$$\log_e Q - \log_e Q_o = \frac{-t}{RC}$$

Since: $\log_e y = \log_e(10) \,.\, \log_{10} y = 2.303 \log_{10} y$, this equation may be written:

$$2{\cdot}303 \,.\, \log_{10} Q - 2{\cdot}303 \,.\, \log_{10} Q_o = \frac{-t}{RC} \quad . \quad . \quad (71)$$

The charges (Q) are found by discharging the condenser through a ballistic galvanometer, and if the galvanometer kicks are represented by θ, $Q = a\,\theta$, and (71) becomes:

$$2{\cdot}303 \,.\, \log_{10}(a\,\theta) - 2{\cdot}303 \,.\, \log_{10}(a\,\theta_o) = \frac{-t}{RC}$$

i.e. $$\log_{10}(\theta) = \frac{-t}{2{\cdot}303 \,.\, RC} + \log_{10}(\theta_o) \,. \quad . \quad . \quad (72)$$

This equation is of the form: $y = mx + c$, and if $\log_{10}(\theta)$ is plotted against t, a straight line graph is obtained whose gradient is:

$$m = \frac{-1}{2{\cdot}303 \,.\, RC}$$

By determining m, RC is found and hence R, knowing C.

The circuit is shown in Fig. 9.29; C is charged using points 1 and 2 of the switch S and then discharged for a time t through R, using points 1 and 3 of S. The charge remaining on C, viz. Q, is discharged through the ballistic galvanometer BG, by closing points 1 and 4 of S. Since the time constant of the circuit consisting of C and the galvanometer is negligible a ballistic throw θ is obtained.

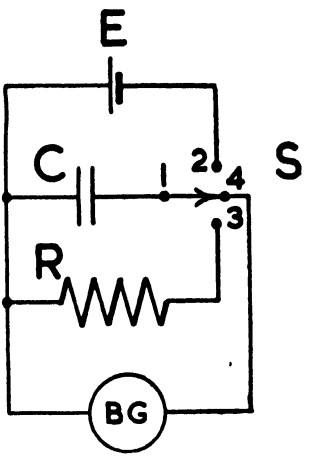


Fig. 9.29

The experiment is repeated for different t, and the graph drawn of $\log_{10}\theta$ against t.

This method of determining resistance is only applicable when CR is large enough to enable appreciable values of t to be used. If $C = 1\mu F$ and $R = 50$ megohms, $T = 50$ seconds and values of t up to one or two minutes may be used. The method is used for resistances of one megohm or more.

*9.28. Leakage Resistance of a Condenser

If a condenser has an imperfect dielectric, it acts like a high resistance (leakage resistance) and allows charge to leak slowly from one plate to the other. Since no condenser is perfect the leakage resistance should be taken into account when measuring a high resistance. The leakage resistance forms a resistance in parallel with R so that the measured value of R is really the equivalent resistance R_e, given by $\frac{1}{R_e} = \frac{1}{R} + \frac{1}{R_l}$, where R_l is the leakage resistance.

9.29. Discharge of Condenser through an Inductance

The combination of a condenser and an inductance forms an extremely important circuit in the field of wireless. Suppose a condenser of capacity C farads is charged to E volts by connecting points 1 and 2 of the switch S in Fig. 9.30. If points 1 and 3 are then connected, the condenser discharges through the inductance L henries, which for the sake of simplicity is assumed to have zero resistance.

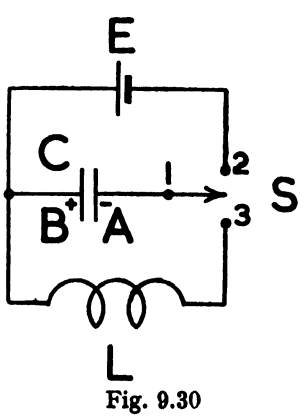


Fig. 9.30

Charge begins to flow from plate A to plate B via L as soon as 1 and 3 are connected, but as the current increases a self-induced e.m.f. is produced across L which opposes this growth of current. The current eventually reaches a maximum value and the induced e.m.f. is then zero. The p.d. across C is then zero also, by Kirchhoff's law. The

magnetic field associated with L is a maximum, but the electric field between the plates of C is zero. No energy is lost ($R = 0$) and the potential energy originally associated with C is all converted into magnetic potential energy of L.

This condition is not stable since current is still flowing from A to B. Thus, B becomes positively and A negatively charged. As the reverse p.d. builds up across C the current strength falls and a self-induced e.m.f. is produced across L which tries to maintain the decaying current. Eventually current ceases and the magnetic energy of L is once more converted into electrical energy of C.

The p.d. across C is now equal and opposite to its original value. The condenser thus starts to discharge in the direction B to A and the sequence of events is repeated, electrical energy being converted into magnetic energy and back again into electrical energy. The complete cycle of discharge and re-charge of C is repeated indefinitely with the *natural frequency* of the circuit $\left(f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}\right)$. The circuit is known as an *oscillatory circuit* since the current strength varies sinusoidally with the time.

9.30. Electromagnetic Waves

The magnetic field strength (H) at any point in the vicinity of the inductance of Fig. 9.30, is proportional to the current strength, hence, H varies sinusoidally at any point. The varying magnetic field produces a varying flux through any area in space so that, by Faraday's Law, an induced e.m.f. is produced round its perimeter. If a circuit coincided with the area chosen a current would flow, but even when no circuit is present the induced e.m.f. makes itself manifest as a varying electric field whose line integral: $\int E_s \,.\, ds$, round the perimeter of the area, is equal to the value of the e.m.f. Thus, at any point near L there is electric field strength vector associated with the magnetic field strength vector, both varying sinusoidally with time.

The energy density of either field (electric or magnetic) varies, because the field strength vectors vary, so that energy must flow to or from the inductance. The velocity of energy flow is finite and because of this, energy which reaches a distant point from the inductance is not able to return sufficiently quickly as the current changes. An infinitely large velocity of energy flow would be required to follow the field strength variations predicted by the current strengths at that time. Energy is thus lost from the system continually. This energy is propagated outwards with a velocity which is found experimentally to be the same as the velocity of light in that medium. Theory shows that the velocity of propagation of the energy is given by:

$$c = \frac{1}{\sqrt{\epsilon\epsilon_0\mu\mu_0}}$$

In vacuo, $\epsilon = \mu = 1$ and the velocity of light $= 3 \times 10^8$ m.sec. (approx.). Since the value of μ_0 is chosen to be: $4\pi \times 10^{-7}$ in the definition of unit pole, the value of ϵ_0 must be $(1/36\pi) \times 10^{-9}$ (approx.).

The variations of electric and magnetic field strengths in time and space in the vicinity of L constitute an *electromagnetic wave*, since the curve representing H (or E) at points along a straight line, appears to move like a wave as H (or E) varies with time.

Similar effects are produced between the condenser plates. There is no actual movement of charge between the plates but magnetic effects appear when the charges *on* the plates vary, corresponding to a current flow between the plates. This imaginary current was called a *displacement current* by Maxwell (p. 477). The complementary electric and magnetic fields existing between the plates give rise to a loss of energy as an electromagnetic radiation also. The rate of loss of energy from C and L is extremely small unless L is made in the form of an *aerial* (p. 395). This is because the fields of L and C are mainly confined to small regions of space, viz. within the coil and between the plates, respectively.

*9.31. Effect of Resistance in an L—C Circuit

If a resistance is included in series with L and C of Fig. 9.30, a p.d. is generated across it when current flows, given by: $V_R = IR$ volts. Accordingly, the equation of the circuit becomes:

$$L \cdot \frac{d^2Q}{dt^2} + R \cdot \frac{dQ}{dt} + \frac{Q}{C} = 0 \quad . \quad . \quad . \quad (73)$$

This equation is exactly similar to that describing the motion of a damped galvanometer coil (p. 263). The solution is similar and given by:

$$Q = Q_0 \cdot e^{-Rt/L} \cdot \sin\left\{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \cdot t + \frac{\pi}{2}\right\} \quad . \quad (74)$$

Both the charge and current oscillations are " damped " so that their amplitudes decrease with time and ultimately become zero. In this case energy is lost continually in the resistance in the form of heat. The frequency of the " damped " oscillations is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad . \quad . \quad . \quad . \quad (75)$$

When the resistance is small, f reduces to f_0, so that the frequency of oscillations is approximately the natural frequency of the circuit.

EXERCISES

1. State the laws of Electromagnetic Induction.

   A straight horizontal wire 1 metre long is held pointing East and West and is allowed to fall from a height of 20 metres. Neglecting air resistance and assuming the wire does not change its orientation, find the e.m.f. between its ends as it strikes the ground. The horizontal component of the earth's field is $50/\pi$ amp.m.$^{-1}$ If the wire were cut into two equal halves, and a small condenser connected between them, would the condenser become charged during the fall? ("g" = 9·81 m.sec.$^{-2}$. (Oxford Open Schol. Subsid. Physics).

2. State and explain the laws of electromagnetic induction.

   A horizontal wire 2 metres long, pointing E. and W. is dropped from rest. What is the potential difference between its ends after it has dropped 50 feet?

   (The horizontal component of the earth's magnetic field = $45/\pi$ amp.m.$^{-1}$. g = 9·81 m.sec.$^{-2}$). (Oxford Schol. Subsid. Physics).

3. How would you demonstrate the laws of electromagnetic induction, using a magnet, a solenoid with windings clearly visible, a cell and a suitable galvanometer? Explain carefully how you identify the directions of the induced currents in each instance.

   An engine travels north at a uniform speed of 60 m.p.h. in a straight horizontal path. Calculate the e.m.f. set up at the ends of a conducting axle 150 cm. long if H_H = 16 amp.m.$^{-1}$ and the angle of dip is 68°. (You may assume that 8 kilometres = 5 miles). (London G.C.E. Adv. L.).

4. What are the special characteristics of a ballistic galvanometer? Explain, using circuit diagrams, how you would use a ballistic galvanometer to compare (*a*) the capacitances of two condensers each of the order 1 μf., (*b*) the magnetic fields between the poles of two electromagnets.

   A coil of 100 turns each of area 400 cm.^{2} is connected in series with a ballistic galvanometer of sensitivity 100 divisions per microcoulomb, the total resistance of the circuit being 2000 ohm. Calculate the galvanometer deflection when the coil is turned quickly from its initial position in a horizontal plane through 180° about a horizontal axis. How does the speed with which the coil is turned affect the result? ($H_H = 45/\pi$ amp.m.$^{-1}$. angle of dip = 67°). (London G.C.E. Adv. L.).

5. State Lenz's law relating to induced currents; describe and explain an experiment to illustrate it.

   A circular coil whose terminals are short circuited lies on a horizontal table. Explain how to calculate the charge which flows round the coil when it is turned over on the table. Calculate this charge in microcoulombs using the following data: number of turns on the coil = 500; mean radius = 20 cm.; resistance of coil = 10 ohms; angle of dip = 60°; $H_H = 45/\pi$ amp.m.$^{-1}$. (Northern G.C.E. Adv. L.).

6. Describe the procedure necessary to measure the magnetic field, of the order of 10^5 amp.m.$^{-1}$, between the poles of an electromagnet, using a

search coil and a ballistic galvanometer whose sensitivity in scale divisions per microcoulomb is known. Show how the field strength may be calculated from the readings obtained.

Explain the modifications that would have to be made in the dimensions of the coil and in the procedure to permit a determination of the horizontal component of the earth's magnetic field. (Northern G.C.E. Adv. L).

7. State and explain Faraday's laws of electromagnetic induction, and describe how you would demonstrate them quantitatively.

A search coil of 10 turns, each of area 2 sq.cm., is connected in series with a ballistic galvanometer and a resistance which makes the total resistance of the circuit 1,000 ohms. When the search coil is suddenly withdrawn from between the poles of a magnet, the galvanometer gives a throw of 10 scale divisions. When a condenser of 1 microfarad capacity charged to a potential of 2 volts is discharged through the galvanometer, a throw of 5 divisions is obtained. What was the strength of the field between the magnet poles? (Assume that any corrections for damping have been included in the figures given for the galvanometer throws). (Cambridge Open Schol.).

8. What measurements would you make to show how the electromotive force induced in a circuit is related to the magnetic flux through the circuit?

Estimate the potential difference between Dover and Calais when the tide is running north at a speed of 2 metre sec.$^{-1}$ How would you attempt to measure this difference of potential? (Assume that Calais is 22 miles east of Dover. Make rough estimates of any other quantities you require). (Cambridge Open Schol.).

9. Describe a ballistic galvanometer and explain how you would use it to compare the capacity of two (approximately 10μF) condensers.

A coil connected to a ballistic galvanometer is inserted suddenly into a magnetic field. Show that the throw of the galvanometer is approximately independent of the rate at which the coil is inserted into the field. (Oxford Open Schol.).

10. What are the chief features in the construction of a moving coil galvanometer that will render it sensitive, with little damping and a long period of swing?

Show that the initial swing of the coil of such a galvanometer is proportional to the quantity of electricity passing through it, and explain how the galvanometer may be calibrated to measure quantities of electricity.

A search coil with 60 turns and resistance 2 ohms is connected in series with a ballistic galvanometer of resistance 10 ohms. When the coil is withdrawn from the field between the poles of an electromagnet the galvanometer gives a first deflexion of 105 divisions. If the sensitivity of the galvanometer is 2 microcoulombs per division, find the flux between the poles of the electromagnet. (Oxford Local G.C.E. Schol.).

11. Describe an experiment in which a search coil and a ballistic galvanometer are used to investigate how the intensity of the magnetic field in the middle of the air gap between the poles of an electromagnet varies with the current in the coil of the magnet.

The rectangular coil of a moving coil mirror galvanometer consists of 50 turns each 5 cm. × 2 cm., the longer sides being vertical. It is hung in a uniform horizontal magnetic field by a suspension which exerts a torsional couple of 2×10^{-6} newton.m. per radian twist. The plane of the coil is parallel to the direction of the field when there is no twist in the suspension. A current of 5×10^{-4} amp. in the coil produces a deflection of 25 cm. from a central zero on a scale placed at a distance of 100 cm. from the galvanometer mirror. Calculate the intensity of the magnetic field. (Northern H.S.C. Schol.).

12. A maker's catalogue gives the following data for a certain moving coil galvanometer: resistance 15 ohms; scale deflection 60 mm. per microamp., scale deflection 60 mm. per microcoulomb, both deflections being noted with the scale 1 metre from the galvanometer mirror.

Describe how you would verify this specification having at your disposal a standard cell, standard resistances, a standard condenser and the usual facilities of a physical laboratory. Give diagrams of the electrical circuits employed. (Northern H.S.C. Schol.).

13. Explain how you would determine *accurately* the value of (*a*) a small magnetic field, e.g. the earth's magnetic field; (*b*) a large magnetic field such as that due to an electro-magnet.

Suggest a method of measuring a large magnetic field which lasts for only 0·01 sec. (Cambridge Open Schol.).

14. Describe and discuss the vibration magnetometer and the search-coil methods of measuring the strength and the direction of magnetic fields (*a*) for weak and (*b*) for strong fields.

A small search-coil, connected to a ballistic galvanometer, is set with its plane perpendicular to the magnetic field at a point A in the field. On removing the coil out of the field a galvanometer throw of 16·0 divisions is obtained. When the coil is replaced in the same position at A and is then removed to a point B in the field so that its plane is perpendicular to the field at B, a throw of 7·0 divisions is observed. Calculate the ratio of the periods of vibration of a suspended magnet when placed at A and B respectively. (London G.C.E. Schol.).

15. When I amperes are flowing in a circuit, the work done in carrying a unit magnetic pole completely round the circuit once by any path is I joules. Define the two quantities underlined, and by using this result calculate the magnetic field strength at a point on the axis of a long solenoid of length l with N turns carrying a current I amperes.

Two solenoids A and B are wound side by side over a soft iron core. A is connected to a galvanometer and B to a battery and suitable switch. Describe and explain what is observed when the current in B is (*a*) switched on, (*b*) reversed, (*c*) switched off. (Oxford Local G.C.E. Adv. L.).

16. Explain what is meant by *Self Inductance* and *Mutual Inductance.* A 10 volt battery is connected in series with a coil having an inductance of 10 henries and a resistance of 10 ohms. What is the current flowing in the circuit 2 seconds after switching on? (Manch. Open Schol.).

17. A standard mutal inductance of 1 millihenry is to be made by winding a secondary coil closely over the centre of a long solenoid of 5 cm. diameter and 20 turns per cm.

How many turns are required on the secondary?

What are the uses of such a standard inductance? (Manch. Open Schol.).

18. Define *capacitance* and *self-inductance.* Derive an expression for the energy required to charge a condenser of capacitance C farads to a potential difference of V volts.

A coil of self-inductance 2·00 henrys and resistance 0·102 ohm is connected in series with an accumulator of 2·04 volts and negligible internal resistance. What is the energy residing in the magnetic field of the inductance if the work done in establishing a current I in a self-inductance L is $\frac{1}{2}LI^2$? (London G.C.E. Adv. L.).

19. Explain how a moving coil galvanometer may be used to measure quantities of electricity by the ballistic method. Indicate any special properties of the galvanometer necessary for this purpose and describe how it may be calibrated.

A condenser of 2 microfarads capacity is charged and then immediately discharged through a ballistic galvanometer. The condenser is recharged to the same potential difference, allowed to discharge through a high resistance for a period of 10 seconds, and is then discharged through the same galvanometer. If the throw in the first discharge is twice that in the second discharge, find the value of the resistance in ohms. Explain all the steps in your calculation. ($\log_e x = 2{\cdot}303\log_{10} x$). (Northern H.S.C. Schol.).

20. A condenser of capacity $1\,\mu$F is connected in series with a 10,000 ohm resistance across the terminals of a 300 volt battery. Calculate the amount of heat generated in the resistance. ($J = 4{\cdot}18$ joule.cal.$^{-1}$). (Manch. Open Schol.).

21. A charged condenser is discharged through a resistance. Show formally how the voltage across the condenser varies with time.

A moving coil galvanometer has a current sensitivity of 200 millimetres per micro-ampere and a resistance of 60 ohms and a time of swing on open circuit of 2 sec. There is also a resistance of 1000 ohms in series. The galvanometer and a $1\,\mu$F condenser and a voltage source of 0·01 volt are connected with a rotary switch in such a way that the condenser is charged to 0·01 volt and then connected to the galvanometer in every switching cycle. Calculate how many times per second the switching must be done to obtain a galvanometer deflection of 10 cm.

What would be the effect of raising the series resistance from 1000 ohms to 1 megohm? (Cambridge Open Schol.).

22. State the laws of electromagnetic induction and illustrate them by reference to the construction and working of an induction coil.

Why, in a properly designed coil, is the potential difference between the secondary terminals much greater at break than at make of the primary circuit? (Northern H.S.C.).

23. Define *magnetic flux* and *coefficient of self-inductance.*

A current of i amps. is passed through a solenoid of N turns and of length l m. Assuming the length is large compared with the diameter, derive an expression for the value of the magnetic field at the centre of the solenoid. (You may assume if necessary that the work done in taking a unit magnetic pole once round a circuit carrying a current i amps. is i joules). Hence deduce an expression for the total flux linking each turn and obtain an expression for the coefficient of self-inductance of the solenoid.

Explain why an iron core is used in an induction coil and why it is in the form of a bundle of iron wires. What is the reason for winding the secondary coil in sections? (London Inter. B.Sc.).

CHAPTER 10

MAGNETIC PROPERTIES OF MATERIALS

10.1. Ferro-, Para- and Diamagnetics

Faraday shewed that magnetism could be induced in all substances. He separated substances into three groups: those easily magnetized by induction—*ferro-magnetics*, and those with extremely weak tendencies to become magnetised—*para- or dia-magnetics*. Paramagnetics are magnetized in the same direction as the magnetic field and diamagnetics in the opposite direction. The induced magnetism of non-ferromagnetics requires extremely delicate apparatus for detection. They do not retain their magnetism when removed from the field (cf. **10.25**).

10.2. Magnetism Induced by a Uniform Field

It is proposed to discuss induced magnetism with particular reference to a ferromagnetic specimen, because this is easily demonstrated experimentally, but the theoretical results apply, strictly speaking, to non-ferromagnetics only.

Suppose a long cylindrical specimen of unmagnetized material, e.g. soft iron, is placed with its axis parallel to a uniform magnetic field, e.g. the earth's field. The lines of force, plotted with a compass needle, are

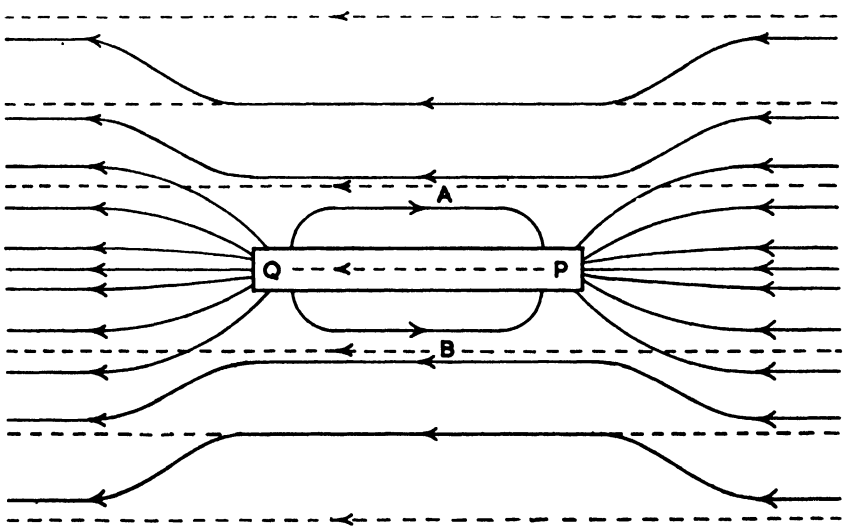


Fig. 10.1

shown in Fig. 10.1. The dotted lines of force represent the initial uniform field.

Lines of force crowd into the end-face P and leave the end-face Q,

both faces becoming origins of new lines of force. By inspection, P has a south polarity and Q a north polarity. The specimen behaves like a bar magnet, and is said to be *magnetized by induction.* The same behaviour is exhibited by a paramagnetic substance, but it is impossible to detect it using a compass needle. A diamagnetic shows an opposite effect, P becoming an N pole and Q a S pole, the number of lines of force leaving P or entering Q being less than the original number.

10.3. Theory of Induced Magnetism

In the simple theory it is supposed that the molecules* of ferro- and para- magnetics behave like permanent dipoles. In the unmagnetized state the molecular magnets are arranged in a chaotic fashion, the resultant magnetic moment being zero. When a field is applied each molecule experiences a couple tending to align it in the direction of the field. The kinetic energy of the molecules prevents perfect alignment unless the field is extremely strong. The induced magnetism depends on the degree of alignment. Complete alignment produces *magnetic saturation.*

In the specimen of Fig. 10.1, the degree of alignment is roughly the same throughout. Excess poles of one kind exist in the end-faces and are called *free poles.* The quantity per unit area of end face is represented by σ; σ is constant over the central regions of P and Q, but smaller near the edges. The free poles give rise to the characteristic field of a bar magnet which is superimposed on the initial uniform field to give the resultant field of Fig. 10.1.†

10.4. Intensity of Magnetization J

This is a measure of the induced magnetism and defined:

The Intensity of Magnetization at a given point in a substance (J) *is the magnetic moment per unit volume at that point.*

J is given approximately by dividing the resultant vector sum of all the molecular moments δM, in a small volume δV surrounding a given point, i.e. $J = \dfrac{\delta M}{\delta V}$ (approx.). In the limit:

$$J = \frac{dM}{dV} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

J is a vector quantity whose direction is that of the magnetic field strength at that point (for isotropic materials).

From (1) the M.K.S. units of J are: $\dfrac{\text{weber.metres}}{(\text{metres})^3}$, i.e. wb.m.$^{-2}$

* The magnetization of ferromagnetics is now referred to " domains " rather than molecules, cf. p. 307.

† At points A and B the resultant field is zero; these points are known as ***neutral points.***

10.5. Uniformly Magnetized Specimen

In a *uniformly magnetized specimen* the alignment of the elementary magnets is the same at all points, i.e. J *is constant.* A long thin specimen parallel to a uniform field is fairly uniformly magnetized. If the free pole per unit area is σ wb.m.$^{-2}$, the total magnetic moment is:

$$M = A \,.\, 2l\sigma \text{ wb.m.}$$

where A is the cross-section in m.^{2} and $2l$ is the length in m.

Hence:
$$J = \frac{Moment}{Volume} = \frac{A2l\sigma}{A2l} = \sigma \text{ wb.m.}^{-2} \quad . \quad . \quad (2)$$

Perfect uniform magnetization is rarely achieved because of the demagnetizing effects of the free poles (p. 300).

10.6. Relation between B, H and J

The magnetic field external to the long uniformly magnetized specimen is the resultant of two fields: (i) the original field and (ii) that of a bar magnet. If the strength of (i) is H_o amp.m.$^{-1}$, $B_o = \mu_o H_o$ wb.m.$^{-2}$ (medium is vacuum). To determine the induction corresponding to (ii) the magnetized specimen is considered as an array of successive layers of dipoles whose axes are parallel to H_o.* The pole strength per unit area of each layer is the same as that at P and Q, viz. σ.

The lines of induction leaving the N poles of one layer pass to the adjacent S face of the next layer (Fig. 10.2). Since m lines leave a pole

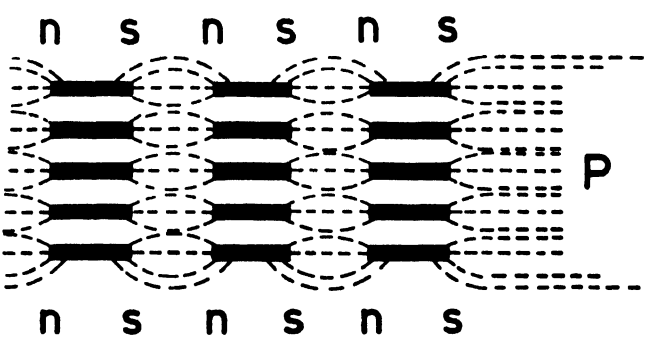


Fig. 10.2

of strength m wb., the number leaving unit area of a layer is σ. Thus, at the end-faces σ lines per m.^{2} enter or leave normally, it being impossible to detect variations which occur within molecular distances. An observer is also unable to detect the discontinuities inside the layers and to all intents therefore, σ lines of induction enter each unit area of P, travel through the specimen and leave again at Q. This result is

* The number of dipoles in each layer is chosen to give the same effect as the imperfectly aligned dipoles of the specimen.

predicted by the boundary theorem (i), (p. 89). By definition, the magnetic induction inside (or just outside) the specimen, caused by magnetization, is:

$$B' = \sigma$$

$\therefore$ inside (or just outside) the specimen:

$$B = B_0 + B' = \mu_0 H_0 + \sigma$$

Since $\sigma = J$:

$$\underline{\underline{B = \mu_0 H_0 + J}} \quad . \quad . \quad . \quad . \quad (3)$$

10.7. The Effect of Demagnetizing Field

In general the dipole alignment above is not uniform and depends on the surrounding dipoles. Inside a specimen the effect of the neighbouring dipoles is a minimum because each dipole is surrounded by neighbours. Near the surface the forces acting on a dipole are not symmetrical and misalignment results (Fig. 10.3). The alignment is thus non-uniform except in the central region.

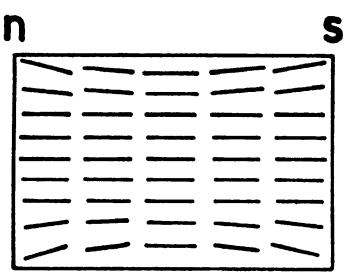


Fig. 10.3

The non-uniformity of magnetization is most pronounced in short thick specimens of relatively large surface area. In long thin specimens the misalignment is small and often neglected.

The misalignment of dipoles is that which would be produced by a magnetic field created by the free N and S poles with vacuum between them. This imaginary field opposes H_0 inside the specimen and is a *demagnetizing* field (H_d). Using this conception, the *resultant* magnetizing field (H_m) inside the specimen is:

$$\vec{H}_m = \vec{H}_0 - \vec{H}_d \quad . \quad . \quad . \quad . \quad (4)$$

The relation between B and J at any point now becomes:

$$B = \mu_0 H_m + J \quad . \quad . \quad . \quad . \quad (5)$$

Since: $H_d \propto J$, i.e. $H_d = NJ$, where N is a constant depending on the dimensions of the specimen:

$$H_m = H_0 - NJ \quad . \quad . \quad . \quad . \quad (6)$$

and:

$$B = \mu_0(H_0 - NJ) + J \quad . \quad . \quad . \quad (7)$$

It can be shown that the relation: $B = \mu_0 H + J$, is true inside or outside any magnetised specimen, where H is the true magnetizing field, i.e., $H = H_m$.

10.8. Magnetic Susceptibility

For a point inside a uniformly magnetized specimen in air (zero demagnetization):

$$B = \mu_0 H_0 + J$$

Also (by definition): $B = \mu_0 \mu H_0$

$$\therefore \quad \mu_0 \mu H_0 = \mu_0 H_0 + J$$

$$\therefore \quad \mu_0 \mu = \mu_0 + \frac{J}{H_0} \quad . \quad . \quad . \quad . \quad (8)$$

i.e. $$\mu_0(\mu - 1) = J/H_0 = \kappa \quad . \quad . \quad . \quad (9)$$

κ is the *magnetic susceptibility* of the specimen and defined as *the intensity of magnetization per unit magnetizing field.* Its unit is: wb.m.$^{-2}$/amp.m.$^{-1}$, i.e. wb.m.amp.$^{-1}$

Ferromagnetics have large susceptibilities, but since μ varies with H, κ is not constant. Paramagnetics and diamagnetics have very small susceptibilities which are positive and negative, respectively, (μ is slightly greater and slightly less than unity, respectively). In the latter, μ and κ are constant.

10.9. Ferromagnetics and J — H curve

Ferromagnetic materials have most important applications. Their permeability, and hence B, varies with H; B and J also depend on whether H is increasing or decreasing and this effect is called *magnetic hysteresis.* Much information about ferromagnetics is obtained by determining a "$J - H$" or "$B - H$" curve, as follows:

The apparatus required is indicated in Fig. 10.4. The long solenoid C_1 provides a uniform magnetizing field H, whilst the coil C_2 acts as a compensating coil. The coils are placed on opposite arms of a deflection magnetometer D with their axes at right angles to the earth's field H_H. The same current passes through C_1 and C_2 but the windings are in opposite directions and their fields oppose each other. For a suitable fixed position of C_1, C_2 is adjusted until the magnetometer shows no deflection for various current strengths controlled by Rh_1 and Rh_2. When the specimen AB is along the axis inside C_1, the induced magnetism caused by the magnetizing field is the sole cause of the deflection of the magnetometer needle ns.

The specimen is a long thin rod, so that the demagnetization is negligible, and the free poles are treated as point poles.

AB is first demagnetized, so that the induced magnetism is independent of its previous history. This is achieved by placing AB inside a large coil connected to an A.C. mains supply in series with several rheostats. Starting with several amps., the rheostats are gradually brought into the circuit until the current is reduced to zero.

The slowly reduced magnetization and demagnetization caused by the A.C. demagnetizes the specimen.

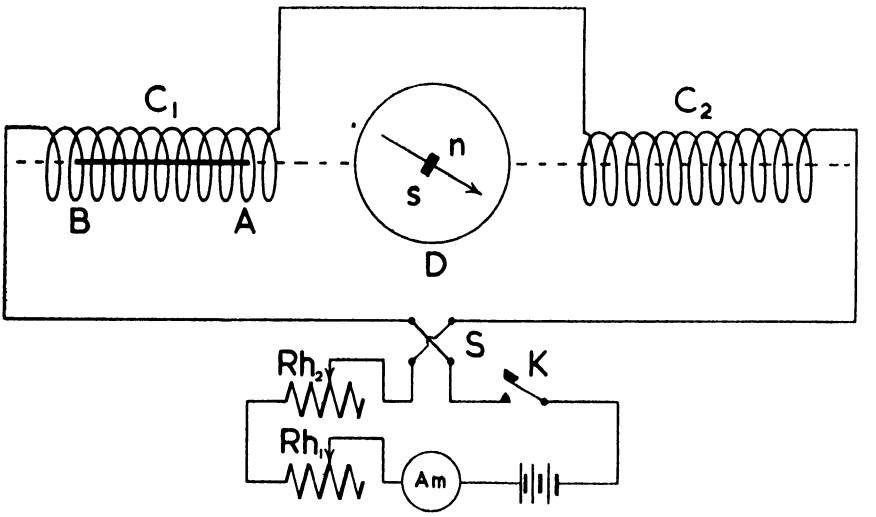


Fig. 10.4

After demagnetization, AB is fixed along the axis of C_1 and the rheostats adjusted so that a small current ($I = 0{\cdot}2$ amperes) flows. The deflection θ of the magnetometer needle is noted.

If the number of turns per metre of C_1 is n, the magnetizing field strength is:

$$H = nI \text{ amp.m.}^{-1} \text{ [(19) Chapter 5]} \quad . \quad . \quad (10)$$

The specimen behaves like a dipole and the field strength causing the deflection θ is:

$$F = \frac{2Md}{4\pi\mu_o(d^2 - l^2)^2} \text{ amp.m.}^{-1}$$

where d m. is the distance from the centre of AB to the centre of the magnetometer, $2l$ m. is the lengt of the specimen, and M is the induced magnetic moment. For the magnetometer:

$$F = H_H \,.\, \tan\theta$$

Hence:
$$M = \frac{4\pi\mu_o H_H \,.\, (d^2 - l^2)^2 \,.\, \tan\theta}{2d} \quad . \quad . \quad (11)$$

By definition:
$$J = \frac{M}{V} = \frac{M}{2al}$$

where a is the cross-sectional area in m.^{2}. Hence:

$$J = \frac{4\pi\mu_o H_H (d^2 - l^2)^2 \tan\theta}{4ald} = c \tan\theta \quad . \quad . \quad (12)$$

where c is a constant.

The experiment is continued by adjusting Rh_1 to increase I in 0·2 amp. steps and noting the corresponding magnetometer deflection, until θ is constant. The curve of tan θ (J) against $I(H)$ is plotted to give OA of Fig. 10.5.

The experiment is continued by reducing the current strength by similar increments and noting the values of I and θ. This is repeated until $I = 0$ (approx.) and then this small current is reversed using S. The process is repeated until θ is once more constant. The full curve, representing the change in J with H, is given by $ABCD$ of Fig. 10.5.

The negative current strength is now reduced to zero by similar increments, reversed and then increased positively until θ is once more a positive maximum. The complete series of changes is known as a *Cycle of Magnetization*. The complete curve $ABCDEFA$ is called the "$J - H$" curve or hysteresis loop of the specimen.

For each positive value of H, J is greater when H is decreasing than when increasing. The same thing happens when H is in the reverse direction. The specimen thus shows a "reluctance" to being demagnetized after being magnetized, and this is known as *Magnetic Hysteresis*.

When J is a maximum it is called the *Saturation Intensity of Magnetization*, (J_s in Fig. 10.5); J_s varies with the material, (cf. table p. 319).

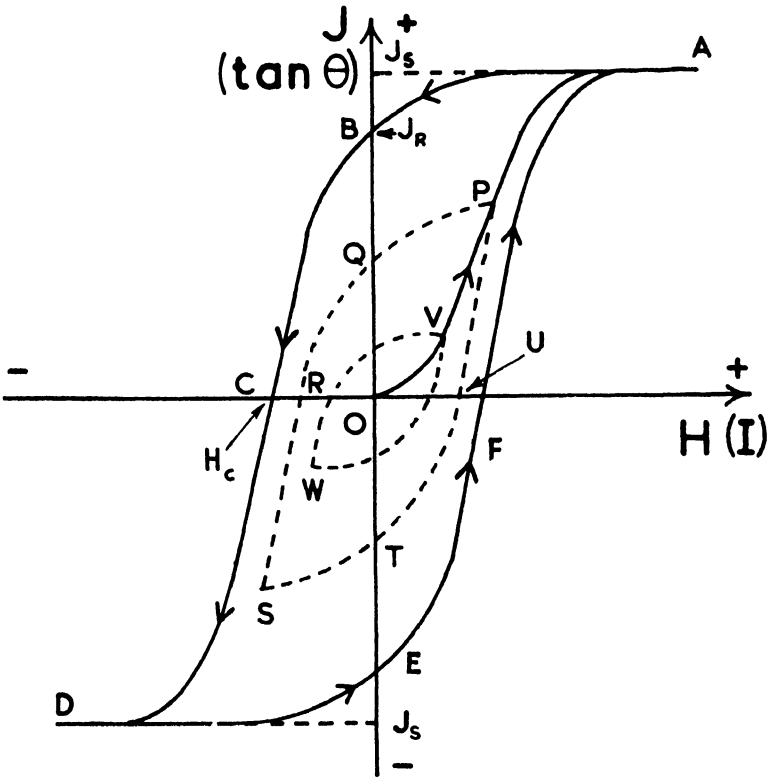


Fig. 10.5

10.10. Hysteresis and Demagnetization

If the specimen is not magnetized to saturation, but to some lower value P (Fig. 10.5), before it is taken round a cycle, the hysteresis curve is a smaller loop: $PQRSTUP$. If the cycle ends at a slightly lower value than the initial value of H, and the process is repeated with an ever decreasing starting value, the continuous variation of J is represented by $PQUSTUVW$ . . . etc., the area of the cycle ultimately becoming zero.

This is effected when the specimen is placed in a demagnetizing coil through which A.C. of ever-decreasing strength flows.

10.11. Residual Magnetism and Coercive Force

When H is reduced to zero, after magnetization to saturation, J is given by OB (Fig. 10.5). This is called the *Residual Magnetization* or *Remanent Magnetization* (J_r). If the specimen is magnetised to a value less than J_s, the residual magnetization left when the field is removed is less than J_r, and given by the intercept on the J-axis of one of the smaller hysteresis loops of Fig. 10.5. Thus, a specimen cannot be demagnetised simply by reducing a magnetizing field to zero.

The magnetizing field strength required to reduce the intensity of magnetization to zero after being completely magnetized is given by the intercept OC. This is called the *Coercive Force* (H_c). A specimen is not demagnetized by making $H = -H_c$, because J does not remain at zero when the field or the specimen is removed but resorts to the value corresponding to $H = 0$ on the hysteresis cycle of which the point: $J = 0$, $H = H_c$ is a member, i.e., the specimen has *residual magnetism.*

10.12. Permanent Magnet Materials

Ferromagnetics having large values of J_r and H_c are used for the manufacture of permanent magnets. A high coercive force is essential since the free poles of the magnet tend to demagnetize it. If the demagnetizing field is greater than the coercive force the magnetization is reduced to a very low value. Apart from this, any small adverse field, such as the earth's field, tends to demagnetize the magnet.

Pure iron is a " soft " magnetic material, i.e. it has a high residual magnetization but a very low coercive force. It cannot therefore be used for permanent magnets. Apart from this, minute quantities of impurities, such as carbon, oxygen, nitrogen, etc., affect its magnetic properties considerably. Special steels containing chromium and tungsten, or cobalt, have been developed for permanent magnets. They are " hard " ferromagnetics with a high residual magnetization and high coercive force (about 6000 amp.m.$^{-1}$). Special alloys like alnico (*Al, Ni* and *Co*), are also used; they have coercive forces as high as 50,000 amp.m.$^{-1}$ (cf. table p. 319).

Even with hard ferromagnetics the demagnetising effect of free poles

tends to destroy the permanence of the magnetization. This effect is reduced when the magnet is not in use, by employing "*keepers*", consisting of short pieces of soft iron. They are placed across the pole faces of two similar magnets lying parallel and having their N and S poles adjacent to each other. A complete "magnetic circuit" is thus formed and no free poles are exposed, (cf. p. 311).

10.13. Electromagnets etc.

An *electromagnet* in its simplest form consists of a coil of wire wrapped round a bar or rod of soft-ferromagnetic material such as soft-iron. When D.C. passes, the bar is magnetized like a bar magnet. The chief requirement is a large intensity of magnetization for a given current strength. A low coercive force is desirable so that the magnetic field may be destroyed when the current is switched off. Iron containing up to 4 per cent of silicon (*silicon steel*) is used; it has a very low coercive force (cf. table p. 319). This is also used for *electromagnetic relays* and *valves*. In the relay, an iron piece (armature) is attracted to an electromagnet when energized and its movement breaks or makes an external circuit. The relay acts as a remote controlled switch. In electromagnetic valves the movement of the iron piece opens or closes a valve.

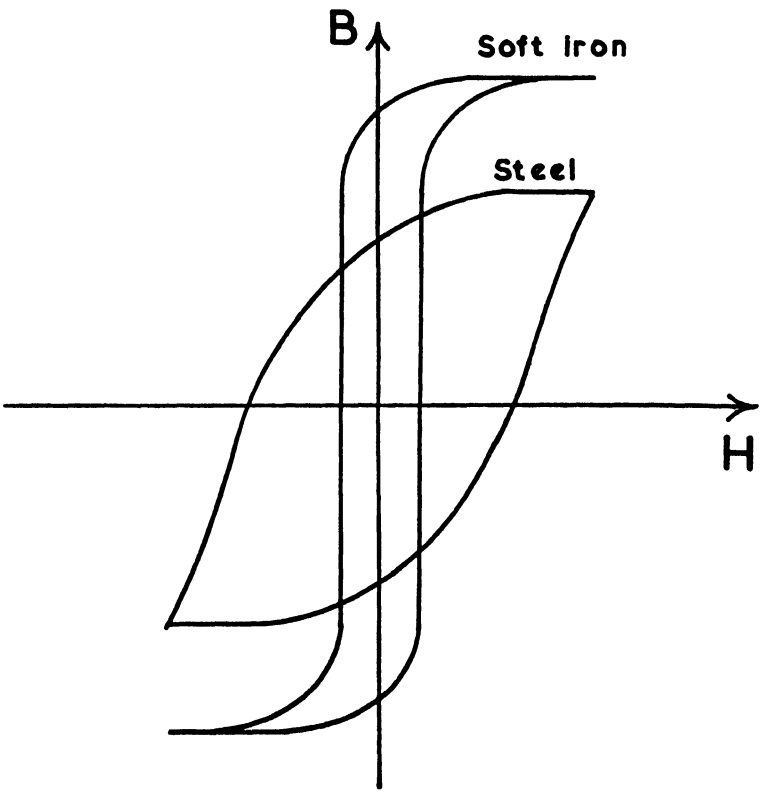


Fig. 10.6

10.14. B — H Curves

It is possible to transform $J - H$ curves into $B - H$ curves by using the relation: $B = \mu_o H + J$, but they are best determined by experiment using a ballistic galvanometer. The specimen is used in the form of a ring which is magnetized by means of current in a coil wrapped round it. Using a ring specimen no free poles are produced and magnetization is uniform, lines of induction forming concentric circles inside the material. The flux through a secondary coil wrapped round the ring and connected to a ballistic galvanometer is determined by reversing the primary current. Since $\Phi \propto \theta_1$, $B \propto \theta_1$, for the given H.

The $B - H$ curve is similar to the $J - H$ curve, except that B does not remain constant after saturation. When $H > H_s$, J remains constant (J_s), but B increases since: $B = \mu_o H + J_s$, (this is the equation of a straight line with gradient μ_o). Two typical $B - H$ curves, for a soft iron and a hard steel, are plotted in Fig. 10.6 (p. 305).

10.15. Maximum Induction and Permeability

The variation of B with H from zero magnetization to saturation cannot be determined by ballistic galvanometer method but is obtained by transforming OA of Fig. 10.5. This curve is called the *magnetizing curve* of a specimen. A typical curve is given in Fig. 10.7. From

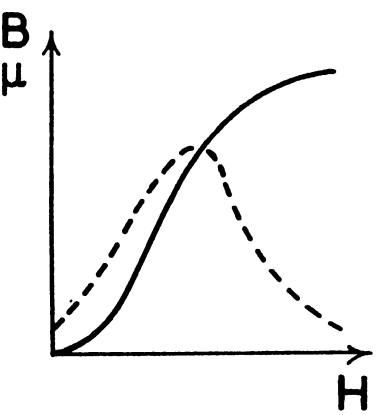


Fig. 10.7

$B = \mu\mu_o H$, the variation of μ with H may be drawn, as illustrated (dotted line); μ has a maximum value (μ_{max}). The induction corresponding to $\mu_{max.}$ is $B_{max.}$ This is not the largest induction possible since B increases with H always. Both $\mu_{max.}$ and $B_{max.}$ are important characteristics in the manufacture of permanent magnets.

10.16. Remanence and Coercivity

The value of B remaining when H is reduced to zero, after magnetization to saturation, is the *Remanence* or *Remanent Induction* (B_r). It

corresponds to residual magnetization. The strength of the magnetizing field which operates when the value of B is reduced to zero on the same hysteresis cycle is called the *Coercivity*. This is not quite the same thing as the Coercive Force, since, when $J = 0$, $B = H_c$, i.e. B is finite when the specimen is demagnetized, and likewise J is finite when B is zero. However, Coercivity is often called Coercive Force and represented by H_c.

10.17. Energy Dissipation in a Hysteresis Cycle

Hysteresis is explained by assuming that rotation of the elementary magnets is opposed by friction. During magnetization these forces oppose alignment and energy is lost in doing work against them. The same happens on demagnetization; work is again done against friction. The energy lost is derived from the magnetizing field and, ultimately, from the source of e.m.f. supplying the current producing the field. The energy lost per hysteresis cycle is of great importance in applications involving magnetization.

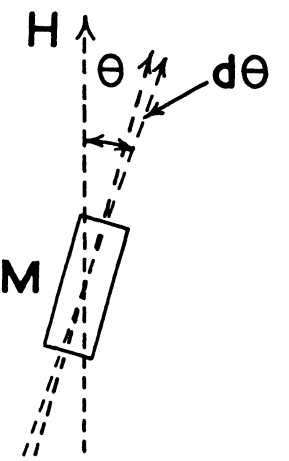


Fig. 10.8

Modern theory attributes the magnetization of a ferromagnetic material to the alignment of "domains" which may contain up to about 10^{15} molecules, but each domain is equivalent to a dipole with a magnetic moment, say M. Suppose a specimen is contained in a uniform magnetizing field H amp.m.$^{-1}$. A typical domain is aligned with its vector M inclined at an angle θ to the direction of H (Fig. 10.8). The domain is in equilibrium, the couple tending to align it in the direction of H being:

$$G = M.H. \sin \theta \text{ newton.m.} \quad . \quad . \quad . \quad (13)$$

The forces opposing alignment, caused by temperature and hysteresis, give rise to an equal and opposite couple. On increasing H by a small amount dH, the vector M is rotated through the small angle $d\theta$. The work done is:

$$dw = -MH \,.\, \sin \theta \,.\, d\theta \text{ joules} \quad . \quad . \quad . \quad (14)$$

The negative sign is inserted since $d\theta$ represents a decrease in θ.

From (14): $$dw = H \,.\, d\,(M \cos \theta) = H \,.\, dM_1 \text{ joules} \quad . \quad (15)$$

where $M \cos \theta = dM_1$ is the component of M in the direction of H and $d\,(M \cos \theta)$ is the change in this component.

The work done per unit volume is given by summing expressions like dw for all the domains in unit volume, viz:

$$\Sigma dw = H \,.\, \Sigma dM_{\mathrm{I}} \ (H = \text{const.}) \quad . \quad . \quad (16)$$

ΣdM_{I} is the vector sum of the changes in M_{I} of all the domains per unit volume and is, therefore, the change in the intensity of magnetization, i.e. dJ.

Hence: $$\Sigma dw = H \,.\, dJ \quad . \quad . \quad . \quad . \quad (17)$$

The work done per unit volume on increasing J from J_1 to J_2 is hence:

$$W = \int \Sigma dw = \int_{J_1}^{J_2} H \,.\, dJ \text{ joule.m.}^{-3} \quad . \quad . \quad (18)$$

When J is increasing W represents work done by the field on the specimen, and vice versa. Because of hysteresis W is positive for a complete cycle and represents a loss of energy, which appears as heat.

Referring to Fig. 10.9, $H' \,.\, dJ$ is equal to the area of the strip of width dJ and mean ordinate H'; $\int_{J_1}^{J_2} H' \,.\, dJ$ is equal to the area beneath the

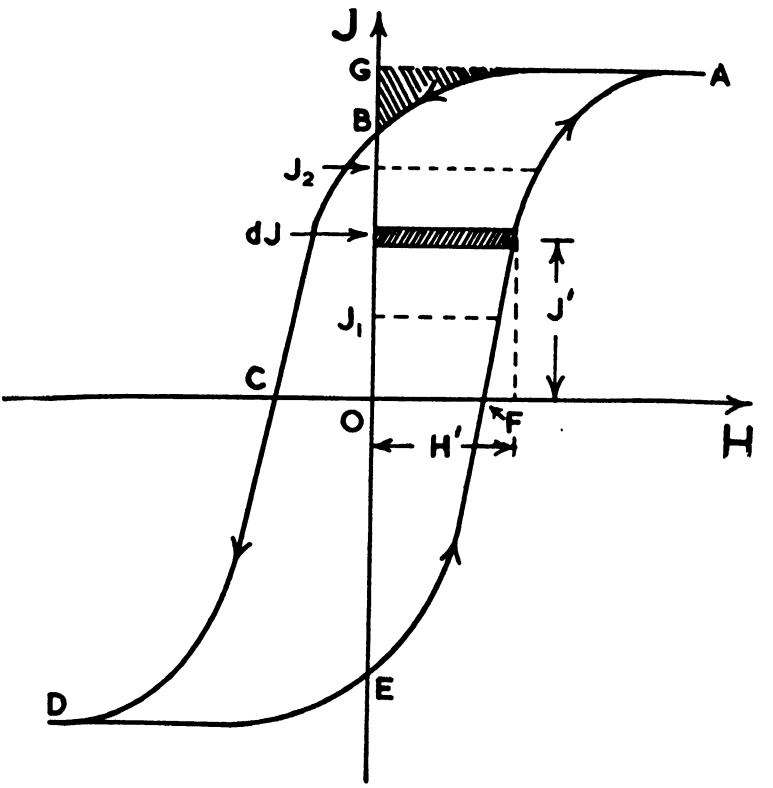


Fig. 10.9

increasing $J - H$ curve, between the limits J_1 and J_2. The energy loss during a cycle of magnetization is estimated by considering the quadrants of the $J - H$ curve separately. The work done per unit volume in taking the specimen from point F to A is positive, since J is increasing, and is equal to the area $OFAG$. The work done in taking the specimen from A to B is negative, (J decreasing), and is given by the hatched area BAG. The net work done in this quadrant is positive and given by area $OFAB$.

Treating the other quadrants similarly, *the work done per unit volume in taking the specimen round the cycle is given by the area enclosed by the hysteresis curve.* This is equal to the energy lost by the source, which appears as heat.

Alternative Formula

The work done during a cycle of magnetization is expressed in terms of B and H by substituting for J. From (4):

$$J = B - \mu_o H$$

so that:

$$dJ = dB - \mu_o dH \quad . \quad . \quad . \quad . \quad (19)$$

For a complete cycle:

$$W = \oint H \,.\, dJ = \oint H \,.\, dB - \oint \mu_o H \,.\, dH \quad . \quad (20)$$

For a complete cycle $\oint H \,.\, dB$ is given by the area enclosed by the cycle, whilst $\oint H \,.\, dH$ is zero, since the limits of H are the same. Hence:

$$W = \oint H \,.\, dB \text{ joule.m.}^{-3} \quad . \quad . \quad (21)$$

i.e. the area enclosed by the $B - H$ loop.

Ferromagnetic material is constantly being magnetized and demagnetized in transformers, dynamos, alternators, etc., and the hysteresis loss must be reduced because (i) power is lost and (ii) the heat produced necessitates cooling. For such applications soft ferromagnetic materials, having narrow hysteresis loops and low coercivity, are used, e.g. silicon irons, transformer iron, and alloys of special composition (p. 351).

10.18. Effect of Temperature

When the temperature of a ferromagnetic substance is raised it loses its characteristic properties gradually, until at a temperature known as the *Curie Point* it ceases to act like a ferromagnetic, i.e. its permeability is reduced to unity (approximately), it shows no ability to retain any induced magnetism, and hysteresis disappears. It behaves as a paramagnetic substance.

The Curie Point for iron is 770°C., for nickel 358°C., and for cobalt 1,121°C. A ferromagnetic specimen may be demagnetized by heating it to a temperature above the Curie Point, e.g. the magnetism of steel

magnets is removed by heating to red heat. For complete demagnetization care must be taken to avoid the remagnetizing effects of any external field (e.g. earth's field) when the specimen cools.

Example

Explain the terms permeability *and* hysteresis.

Draw hysteresis curves for material suitable for use (a) *in a transformer,* (b) *as a permanent magnet. Explain how the curve shows that the material is suitable for the purpose indicated.*

A long iron rod, area of cross-section 0·03 *cm.*^{2}, *is placed in a vertical solenoid* 20 *cm. due magnetic W. of a magnetometer needle. When a current is passed through the solenoid the deflection of the needle is* 30°. *Assuming that the induced north pole is in the same horizontal plane as the needle, that the magnetizing field is* 3200 *amp.m.*$^{-1}$ *and the horizontal component of the earth's magnetic field is* $50/\pi$ *amp.m.*$^{-1}$, *calculate the value of the permeability of the iron. Ignore the effect of the south pole of the rod, and assume that the magnetic effect due to the solenoid is compensated.* (Northern H.S.C.).

The first two parts of the question are discussed in the text.

In the problem the effect of the current in the solenoid is to magnetise the specimen so that free poles appear at its ends. Since the magnetizing field is compensated the magnetometer deflection is caused by the specimen alone. Since the magnet is long the upper induced S pole may be ignored. If the strength of the induced N pole is m wb., the field strength at the needle is:

$$H = \frac{m}{4\pi\mu_o r^2} \text{ amp.m.}^{-1}$$

For the deflection of the magnetometer:

$$H = H_H \tan\theta$$

so that:

$$m = 4\pi\mu_o r^2 H_H \tan\theta \text{ wb.}$$

The magnetization of the rod is uniform and $J = \sigma =$ pole strength per unit area of end-face. If the cross-section is A m^2, $J = m/A$ wb.m.$^{-2}$

From:

$$B = \mu_o H + J$$

$$\mu_o\mu H = \mu_o H + J$$

i.e.

$$\mu = 1 + \frac{m}{\mu_o AH} = 1 + \frac{4\pi r^2 H_H \tan\theta}{AH}$$

Using $r = 0{\cdot}20$ m., $H_H = 50/\pi$ amp.m.$^{-1}$, $\theta = 30°$ and $\tan\theta = 1/\sqrt{3}$, $A = 3 \times 10^{-6}$m.^{2}, $H = 3{,}200$ amp.m.$^{-1}$:

$$\mu = 1 + \frac{4 \times 0{\cdot}04 \times 50}{3 \times 10^{-6} \times 3200} \times \frac{1}{1{\cdot}732}$$

$$= 1 + 481$$

i.e.

$$\underline{\mu = 482 \text{ (approx.)}}.$$

Example

Define magnetic field strength, intensity of magnetization *and* susceptibility.

Draw a diagram to show, in the case of iron, how the intensity of magnetization varies with the magnetizing field, H, (a) *when* H *is small and* (b) *when* H *is large.*

A uniform rod of iron 12 *sq. mm. in cross section and* 4 *cm. long is placed horizontally*

and carries 5 amps., the field strength inside the coil is given approximately by using the formula for the long solenoid:

$$H = nI = 10^3 . 5 = 5{,}000 \text{ amp.m.}^{-1}$$

When the same current is passed through the coil on the cut ring, the field strength in the air-gap is obtained from (33):

$$H_a = \frac{B'}{\mu_o} = \frac{NI\mu}{(2\pi r + \mu l)} = \frac{100 . 5 . 500}{(2\pi\ 0{\cdot}1 + 500 \times 10^{-3})} \text{ amp.m.}^{-1}$$

$$= \frac{250000}{1{\cdot}13} = 221000 \text{ amp.m.}^{-1}$$

Thus, a much higher magnetic field strength is produced, even though this is only available in a very narrow air-gap.

This principle is used for the production of high field strengths. The apparatus is a magnetic circuit containing an air-gap and called an electromagnet because of the induced poles at the air-gap.

A simple electro-magnet capable of producing magnetic fields of 10^5 amp.m.$^{-1}$ is illustrated in Fig. 10.11. The magnetic circuit consists

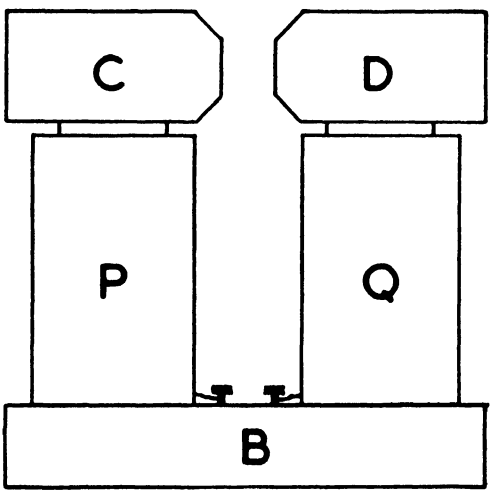


Fig. 10.11

of a soft-iron bed B on which are fixed two similar upright pieces of soft-iron, P and Q. The air-gap is formed between the pole pieces C and D, which are specially shaped to create maximum field strength. On P and Q are wound coils of heavy gauge wire capable of carrying several amperes. The coils are connected in series to a D.C. supply.

10.23. Magnetic Screening

If a hollow cylinder of a ferromagnetic substance, possessing a high permeability, is placed in a uniform magnetic field, the cylinder becomes

magnetized so that lines of induction are formed inside the material as shown in Fig. 10.12. The hollow space inside the cylinder is relatively free from lines of induction. This effect is caused by the alignment of domains created by the field in one section of the cylinder affecting the whole cylinder, so that lines of induction travel inside rather than outside the specimen. This is similar to the magnetization of a magnetic circuit.

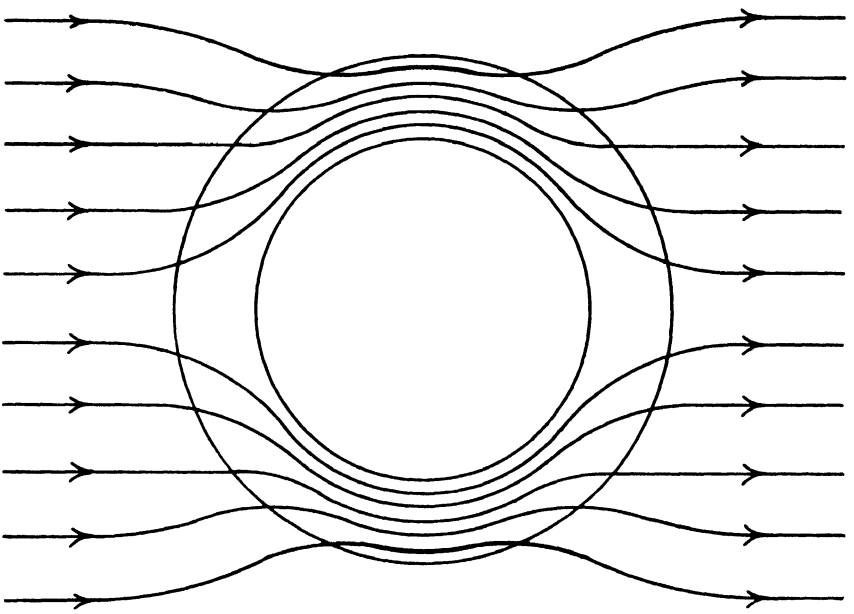


Fig. 10.12

A similar sort of thing happens when the cylinder is placed in *any* magnetic field, so that the cylinder shields the internal space from the effects of magnetic fields. The shielding effect is greater with a hollow sphere. This phenomenon of *magnetic shielding* is used to protect sensitive galvanometers from small changing magnetic fields. Special alloys such as *mumetal* have been developed for this purpose, being distinguished by high permeability and low coercive force. Mumetal screens are used in electronic circuits to protect components from varying magnetic fields, a common example of its use being around the cathode ray tube of a television set.

10.24. Magnetic Properties of Common Ferromagnetic Materials

Below are tabulated the composition and important properties of representative hard and soft magnetic materials. The value of permeability quoted is the maximum permeability ($\mu_{max.}$) defined in the text. The saturation magnetization (J_s) and Curie Point (T) are given where known.

TABLE

Magnetically Hard Materials

Material	*Composition* (approx.)	B_r (wb.m.$^{-2}$)	H_c (amp.m.$^{-1}$)	J_s (wb.m.$^{-2}$)	T (deg. C.)
Carbon Steel	0·9 — 1% C., rest Fe.	0·95	4 × 10^3	1·98	770
Tungsten Steel	0·7% C, 6%W., 0·3% Cr. rest Fe.	1·05	5·2 ,,	1·65	760
Chromium Steel	0·9% C., 3·5% Cr. rest Fe.	0·98	5·6 ,,	1·7	745
2% Cobalt Steel	1% C., 0·5% W., 3·8% Cr., 2% Co., rest Fe	0·98	6·6 ,,	1·85	775
40% ,, ,,	0·7% C, 5% W., 5% Cr., 40% Co., rest Fe.	1 0	19·0 ,,	—	—
Alnico	10% Al, 17% Ni, 12% Co, 6% Cu, rest Fe.	0·8	40·0 ,,	1·25	800
Alcomax IV	8% Al, 13·5% Ni, 24% Co, 3% Cu, rest Fe.	1·2	62·0 ,,	1·3	860
Ticonal	8% Al, 14% Ni, 24% Co, 3% Cu, 3% Ti, rest Fe.	1 25	45·0 ,,	1·4	—
Hycomax	9%Al, 21% Ni, 20% Co, 1·6% Cu, rest Fe.	0·95	66·0 ,,	1 31	865

Magnetically Soft Materials

Material	*Composition* (approx.)	μ *max.*	H_c (amp m^{-1})	W (joules m.$^{-3}$)	J_s (wb.m.$^{-2}$)
Electrolytic Iron	Impurities of C, S, P Mn, Si, & Cu, (total <0 1%)	20,000	210—1,600	—	—
Silicon Iron	3% Si, 0·005% C, rest Fe	63,000	5 6	22	—
Transformer Steel	0 03% C, 4% Si, rest Fe.	8,000	—	163	1 95
Permendur	50% Co, 50% Fe.	5,000	80—160	300	2 45
Permalloy C	77·4% Ni, 3·7 Mo, 5% Cu, rest Fe	75,000	2·4	4·5	0·8
Mumetal	75% Ni, 5% Cu, 2—4% Cr, rest Fe	100,000	2 1	3 85	0·6

10.25. Detection of Paramagnetics and Diamagnetics

The difference between ferro-, para- and dia-magnetic substances is shown by suspending small rod-like specimens between the poles of an electromagnet. A ferromagnetic specimen is attacted towards one of the poles, whilst a paramagnetic sets itself with its longer axis parallel to the field.

The ferromagnetic is strongly magnetized and the force of attraction between the induced pole nearer to a magnet pole-face is greater than that between the opposite induced pole and the other pole-face, so that the specimen is attracted to a pole-face. This is the behaviour of iron filings on a large scale.

The magnetism induced in the paramagnetic (e.g. platinum, etc.), is in the same direction as the ferromagnetic, but because of its weakness there is no net attraction. The specimen has a weak magnetic moment and the field causes the specimen to set with its axis parallel to it.

A diamagnetic specimen (e.g. bismuth, aluminium, etc.) is magnetised in the opposite direction to the ferro- or para-magnetic specimens. If it is initially aligned with its longer axis parallel to the field the specimen becomes unstable with its induced N pole adjacent to the N pole of the magnet. Any slight orientation relative to the field causes a couple to act on the specimen causing rotation. As the specimen rotates the induced magnetism changes, causing a reduction in the value of the couple. A stable state of equilibrium is reached when the specimen is

at right angles to the field. The magnetization is then at right angles to the axis and there is no appreciable restoring force when the specimen is orientated relative to this position. For any other initial orientation, forces are called into play which cause rotation of the specimen to the stable position at right angles to the field.

10.26. Susceptibility of Para- and Dia-magnetic Substances

Since the permeability of para- and dia-magnetic substances is approximately unity it is more convenient to use magnetic susceptibility. By definition, $\kappa = J/H$, and since J is the magnetization per unit volume, κ is called a *volume* susceptibility. Owing to the very small value of κ special experimental methods have to be used for its measurement.

Gouy's Method for Determining κ

In this experiment the specimen, in the form of a thin rod of about 1 mm. diameter, is suspended from one arm of a sensitive balance and counterpoised. The lower end of the specimen is placed between the poles of an electromagnet which gives a well defined and extremely uniform field (Fig. 10.13). When the magnet is energised, the specimen

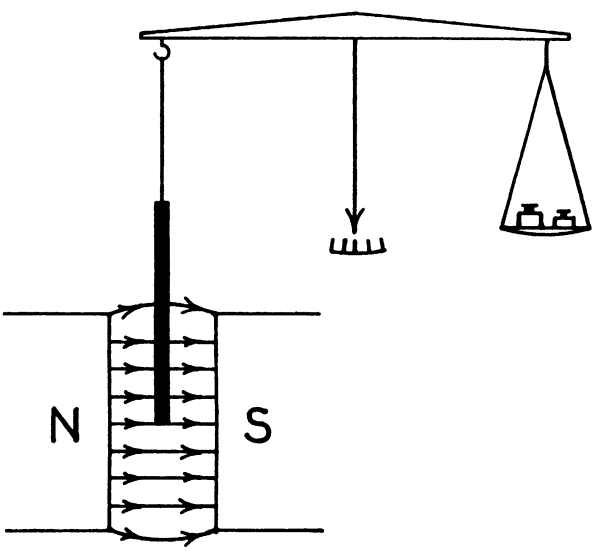


Fig. 10.13

experiences a force which pulls it further into the field if a paramgnetic, and pushes it out of the field if a diamagnetic. Weights are added to, or removed from, the balance to restore the original equilibrium position. The change in weight gives the magnitude of the force F which acts vertically on the specimen. It can be shown that:

$$F = \tfrac{1}{2}\kappa A H^2 \quad . \quad . \quad . \quad . \quad (35)$$

where A is the cross-section of the specimen and H is the strength of the uniform field. Thus, the volume susceptibility is found.

EXERCISES

1. (*a*) Give an explanation of the observation that a steel needle is attracted to a magnet when placed end-on near one of the poles.
   (*b*) State how a uniform rod orients itself when it is magnetized and freely suspended by a torsionless thread attached to the middle of the rod. Why does it take up this position.
   (*c*) Describe (i) the behaviour of rods of soft iron, of hard steel and of bismuth when freely suspended between the poles of a strong electromagnet, (ii) the magnetic condition of each rod when it is withdrawn from the field. (Northern G.C.E. Adv. L.).

2. What is meant by the magnetic field (H) and the magnetic induction (B) inside a bar of iron? How is magnetic permeability (μ) defined?
   Describe the magnetometer method of finding the relation between induction and magnetizing field for soft iron.
   Sketch a B-H curve for soft iron and derive from it the form of a J-H curve using the same scale of H. (London Inter. B.Sc.).

3. Give a description of the magnetic properties of iron and show how far they may be accounted for by a simple physical theory.
   Point out the chief differences in the magnetic behaviour of soft iron and hard steel and give a brief account of the experiments you would perform to verify your statements. (London G.C.E. Adv. L.).

4. Describe in detail the magnetometer method of finding the relation between magnetizing field and intensity of magnetization for a long thin soft iron rod. Explain the method of finding the values of the two quantities and give a graph to indicate the results you would expect. (London G.C.E. Adv. L.).

5. Explain fully how the magnetic properties of a ferro-magnetic material can be investigated by a magnetometer method.
   Write a brief account of the properties and uses of modern magnetic materials. (London Inter. B.Sc.).

6. Distinguish between *intensity of magnetization* and *intensity of a magnetic field*.
   Show by means of a graph how the intensity of magnetization of soft iron varies with the intensity of the magnetising field (*a*) as the field is increased from initial value zero, (*b*) for a cycle of magnetization.
   How would the results for cobalt steel differ from those for soft iron?
   Discuss the importance of the shape of the graph in selecting materials for (i) permanent magnets, (ii) transformer cores. (Assoc. Exam. Bd. G.C.E. Adv. L.).

7. Define *intensity of magnetization, permeability, susceptibility*. Describe how you would obtain the J/H graph for a specimen of ferromagnetic

material taken round a complete cycle of magnetization, and explain as fully as you can the information that such a graph would furnish. (Oxford Local G.C.E. Schol.).

8. Write an essay on the magnetic properties of iron and steel. (Oxford Local G.C.E. Schol.).

9. An iron bar is magnetized by passing a direct current through a coil wound on the bar. Draw a diagram showing how the magnetization of the iron varies (*a*) as the current is gradually increased from zero to a high value, and (*b*) as the current is reduced again to zero.

Discuss, with the help of this diagram, the properties of a material which would be suitable for the construction of a permanent magnet. (Cambridge Open Schol.).

10. Describe an experiment for determining the hysteresis curve of a ferromagnetic substance.

Draw on the same diagram (*a*) a hysteresis curve of a material suitable for a permanent magnet, and (*b*) a hysteresis curve of a material suitable for a transformer core. Point out the features of each of these curves which show that the material for which it is drawn is suitable for the purpose indicated. (Northern G.C.E. Adv. L.).

11. Define *susceptibility*, *permeability* and state the relation between them.

Describe briefly how you would test a small rod to find if it is diamagnetic, paramagnetic or ferromagnetic. Draw intensity of magnetization graphs for each type of material and comment on their special features. (Northern G.C.E. Adv. L.).

12. Define *intensity of magnetization.*

Give an account of a method by which a hysteresis curve could be obtained for a specimen of iron in the form of a long thin rod.

Two identical unmagnetized iron rods, each of mass 70 gm , and area of cross section 0·3 cm.^{2}, are suspended by vertical strings 100 cm. long so that they hang in contact side by side within and parallel to the axis of a horizontal solenoid. When a current is passed through the solenoid the rods repel one another to a distance of 4 cm. Assuming that they are uniformly magnetized, calculate a value for the intensity of magnetization. (Northern H.S.C).

13. Explain the term *permeability* and draw a graph showing the general relation between permeability and magnetic field strength for soft iron.

A soft iron ring has a mean circumference of 250 cm. and a uniform circular cross section of 1·0 cm. diameter. Under the conditions of the problem we may assume the permeability to remain constant at 2,500. A flux of 5×10^{-4} webers is required in the iron. How many turns of wire carrying a current of 2 amperes must be wound around the iron ring? What will the flux become if an air gap of mean length 2 mm. is cut across the iron ring? (London G.C.E. Schol.).

14. Describe how you would investigate the hysteresis of a specimen of iron, and give the theory of your experiment.

A current of i amps. traverses a long solenoid having n turns per metre. Show that the magnetic field at a point well inside the solenoid is ni amp.m.$^{-1}$

A primary coil of 600 turns is wound uniformly on an iron ring of mean radius 8·0 cm. and cross-sectional area 4 sq.cm. A secondary coil of 5 turns is wound on the top of the primary, but insulated from it, and is connected with a ballistic galvanometer, the total resistance of the secondary circuit being 400 ohms. What quantity of electricity will be discharged through the galvanometer when a current of 3 amps. is reversed in the primary coil, assuming that the permeability of the iron is 500? (Oxford & Cambridge Jt. Bd. G.C.E. Schol.).

15. What is meant by (*a*) magnetic field strength, (*b*) intensity of magnetization, (*c*) magnetic induction?

Describe in detail a method of determining experimentally the hysteresis curve for a ferromagnetic substance.

A transformer coil has a soft iron core of volume 30,000 cm.^{3} Around it there is a water coil to cool it. When conditions are steady, it is found that the rate of flow of the water is 10 litres per min. and the inlet and outlet temperatures of the water 15·7°C. and 18·1°C. respectively. Determine the value for soft iron of $\int H.dI$ taken once round the hysteresis loop, H being the magnetic field strength and I being the magnetization per unit volume of the iron core. (The transformer coil operates on the 50 cycle mains supply). (Cambridge Local G.C.E. Schol.).

16. The pole pieces of an electromagnet consist of two similar cylindrical pieces of soft iron of diameter 8 cm. which are aligned so that their ends are exactly opposite with a uniform air gap 0·2 cm. wide between them. Assuming that the two pole pieces are uniformly magnetized and that the force of attraction between them is 300 kilogram weight, calculate (*a*) the intensity of magnetization of the iron, (*b*) the intensity of the magnetic field between the poles and the magnetic flux in the circuit, (*c*) the energy stored in the magnetic field between the poles. Indicate how any formulae you use are obtained.

Suggest a method by which the intensity of field between the poles might be measured.

Upon what factors does the magnetic flux of an electromagnet depend? (Northern H.S.C. Schol.).

17. Three long coils are wound, one on top of another, round a long iron rod. One coil is used to provide an adjustable magnetic field, another carries an alternating current of fixed magnitude, and the third is connected to an A.C. voltmeter. If the adjustable field is small enough, the voltmeter registers a large voltage, but if a sufficiently large steady field is applied the voltage falls to a very much smaller value. Explain this behaviour.

What would be the behaviour of the voltmeter if, with the adjustable field at zero, the magnitude of the alternating current were steadily increased from zero? (Cambridge Open Schol.).

CHAPTER 11

ALTERNATING CURRENT, GENERATORS & MOTORS

11.1. Alternating Current

The electric current passing through a device connected to the "mains" is generally an *alternating current* (A.C.). Its direction is reversed a definite number of times per second called the (*mains*) *supply frequency* (50 cycles per second in Gt. Britain). The supply of A.C. has advantages over direct current (D.C.) as discussed later. The laws governing the relation between current strength and p.d. are somewhat different to those for D.C.

11.2. Production of A.C.—Rotating Coil

The production of A.C. is based on the principles of electromagnetic induction. The simplest method consists of rotating a coil in a uniform magnetic field.

Consider a flat coil of N turns, having a cross-section A m.^{2}, mounted so that its axis is at right angles to a uniform magnetic field of strength H amp.m.$^{-1}$ (Fig. 11.1). Let the coil rotate with constant angular

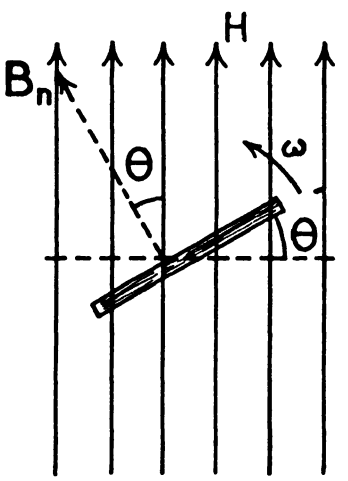


Fig. 11.1

velocity ω radians per sec. Let the angular displacement of the coil after t sec. be θ, $(t = 0, \theta = 0)$. The component of magnetic induction normal to the plane of the coil is: $B_n = \mu_o\mu\ H \cos\theta$ wb.m.$^{-2}$. The number of flux turns through the coil is hence:

$$N\Phi = B_nAN = \mu_o\mu\, HAN\,.\cos\theta \text{ weber-turns} \quad . \qquad (1)$$

by definition: $\omega = (d\theta/dt)$ and $\theta = \omega t$, so that:

$$E = N \cdot \frac{d\Phi}{dt} = \mu_0\mu\, HAN\,\omega \cdot \sin \omega t = \mu_0\mu\, HAN\,\omega \cdot \sin \theta \quad . \qquad (2)$$

The factor: $\mu_0\mu_0\, HAN\,\omega$ is constant and may be replaced by E_0, so that:

$$\underline{E = E_0 \cdot \sin \omega t, \text{ or: } E = E_0 \cdot \sin \theta} \quad . \quad . \quad (3)$$

E is plotted as a function of t in Fig. 11.2; E is a sinusoidal function of the time. The *amplitude* or *peak value* of the e.m.f. is the maximum value of E, i.e. E_0. This occurs when the plane of the coil is *parallel to the field* ($\theta = \pi/2$). The *period* is the time between any two maxima on the same side of the axis and given by: $T = (2\pi/\omega)$, since: $E_0 \cdot \sin \omega\left(t + \frac{2\pi}{\omega}\right) = E_0 \cdot \sin \omega t$, (Fig. 11.2). The frequency of the e.m.f. is: $f = 1/T = \omega/2\pi$, i.e. $\omega = 2\pi f$. Thus, f is the same as the number of revolutions of the coil per second; ω is called the *angular frequency* of the supply.

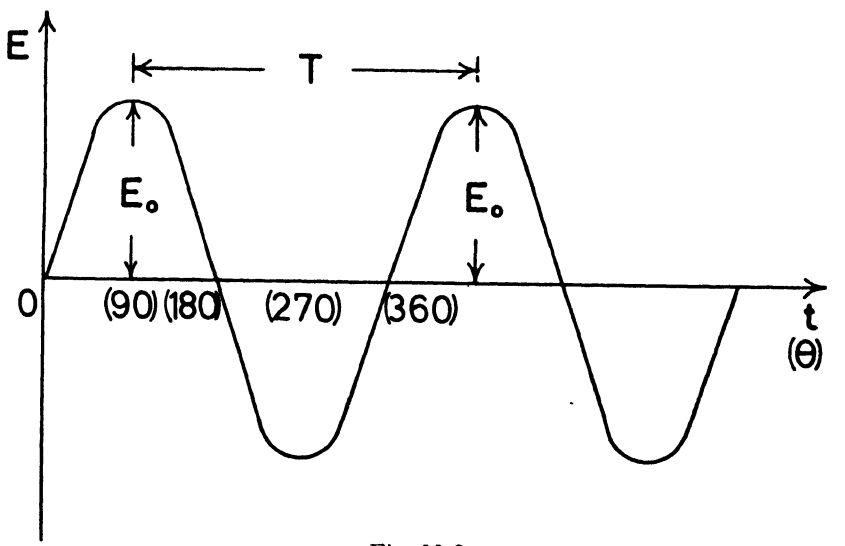


Fig. 11.2

The peak value of the e.m.f. is dependent on the constants of the coil and the values of μ, H and ω. The limits imposed on these variables by virtue of constructional difficulties or expense does not allow large e.m.f.'s to be produced by this method.

Example

The coil of an earth inductor having 100 *turns of radius* 15 *cm. is mounted so as to rotate about a vertical axis at a locality where the earth's magnetic field strength is* 40 *amp.m.*$^{-1}$ *and the angle of dip is* 70°. *If the coil rotates at* 200 *revs. per minute, what is the maximum value of the e.m.f. produced across its ends?*

Since the coil is assumed to be in air: $\mu = 1$; $\mu_o = 4\pi \times 10^{-7}$ M.K.S. units.

The magnetic flux through the coil is due solely to the horizontal component of the earth's field so that:

$$B_n = \mu_o \mu H_H = 4\pi \times 10^{-7} \times 40 \times \cos 70^\circ \quad \text{wb.m.}^{-2}$$

The angular velocity of the coil: $\omega = 2\pi \times$ no.revs. per sec. $= 2\pi \times \frac{200}{60}$ rad.sec.$^{-1}$

i.e. $$\omega = \frac{20}{3} \cdot \pi \quad \text{radians.sec.}^{-1}$$

The cross-section of the coil is: $A = \pi r^2 = \pi(0{\cdot}15)^2 = 0{\cdot}0225\pi$ m^2

The maximum value of the e.m.f.: $E_o = \mu\mu_o\omega H_H A N$ volts

i.e. $$E_o = 4\pi \times 10^{-7} \times 40 \cos 70^\circ \times 0{\cdot}0225 \times \frac{20}{3}\pi \times 100 \text{ volts}$$

hence: $E_o = 4.97 \times 10^{-4}$ volts.

This peak e.m.f. is of little use for practical purposes.

11.3. A.C. Generators

The design of A.C. generators is based on relative motion between a conductor and a magnetic field, but the number of lines of induction cut by any conductor is increased enormously by filling most of the space through which the lines pass with iron, leaving a gap for the relative motion. The conductors which cut the lines of induction are the turns of coils wound on a laminated iron *armature*, the coils being called the armature windings. The magnetising field producing the lines of induction is created by D.C. in coils wound on another laminated iron core (field windings). To obtain relative motion either the armature or field windings are rotated by external power derived from a fast moving stream of water (hydro-electricity), or from steam obtained by burning fuel. The rotating and stationary members are called the *rotor* and *stator*, respectively.

11.4. Synchronous Generator

This generator has the armature as stator and field windings as rotor. The rotor R has an even number of iron " poles " N_1, S_1, N_2, S_2, etc., symmetrically fixed round the circumference (Fig. 11.3(*a*)). The field coils wound on N_1, S_1, etc., form north and south pole faces when current flows. The coils are connected in series to " slip-rings " mounted on the rotor shaft; D.C. is supplied to carbon " brushes " which make contact with the slip rings.

The armature A is a laminated hollow iron cylinder. Cut in the surface of A are groups of slots parallel to the axis. The slots contain insulated wires which are connected at their ends according to a definite plan, shown diagrammatically in Fig. 11.3(*b*). Wire 1 is connected to 4 of an adjacent group of conductors, whilst 4 of this group is connected

to 1 of the next group, etc. A similar group of armature windings exists for each pole of the rotor.

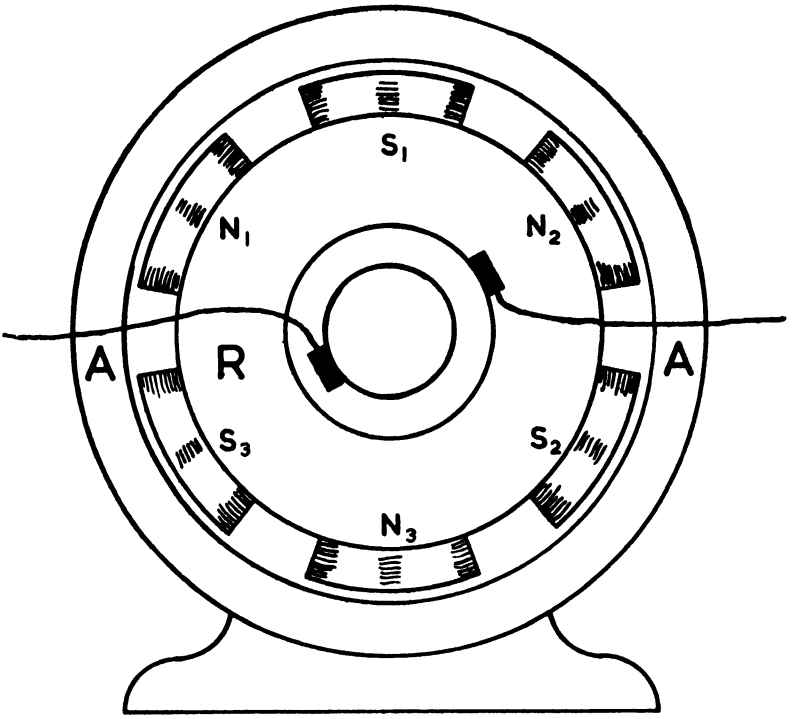


Fig. 11.3(*a*)

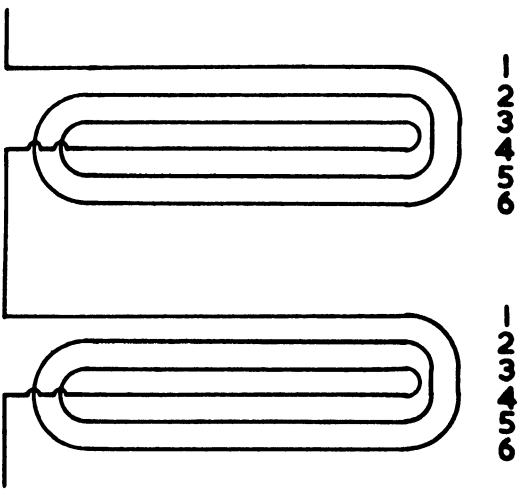


Fig. 11.3(*b*)

When the rotor is at rest the lines of induction form closed paths linking A and R (Fig. 11.3(*c*)). When R rotates each conductor of A cuts these lines and induced e.m.f.'s are produced across their ends. With this arrangement half a sine wave variation in e.m.f. is produced

across each *group* of wires whilst one pole of the rotor passes the group. The half-sine waves are in phase so that the resultant e.m.f. is m times the e.m.f. across each group, where m is the number of poles. If the rotor makes n revolutions per second, the output frequency is: $f = (mn/2)$ c/s.

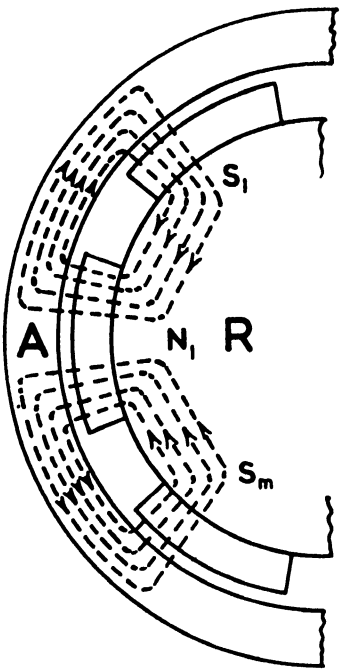


Fig. 11.3(*c*)

11.5. Counter Torque

When current flows a force is exerted on each armature conductor since it lies in the field of the rotor. The fixed armature cannot move, but reaction forces are exerted on the rotor creating a couple which opposes the motion—*counter torque*. When the generator is running at constant speed the fraction of power dissipated in overcoming counter torque is converted into electrical energy to give the output power. This output is, however, reduced by energy losses.

11.6. Energy Losses

The chief sources of energy loss are: (i) heating of armature windings (*copper losses*); (ii) hysteresis in the armature—(*hysteresis losses*); and (iii) eddy currents in the armature—(*eddy current losses*). Copper losses are minimised by using thick copper wire of low resistance. Hysteresis losses are reduced by using iron having a very narrow B—H curve (Chapter 10). Eddy currents are created by the changing magnetic flux in the iron and these are reduced in strength by using a laminated armature.

11.7. D.C. Generators or Dynamos

(i) *Rotating Coil with Commutator*

Direct current for power purposes is produced most cheaply by a *dynamo*. The simplest form consists of a coil such as $JKLM$ rotating in a magnetic field provided by the pole pieces N and S, as in Fig. 11.4. The ends of the coil are connected to the separate halves of a copper ring C fixed to the rotating shaft. The copper ring is called a *split-ring commutator*. The output terminals T_1 and T_2 are connected to the carbon brushes B_1 and B_2 which make contact with C.

As $JKLM$ rotates the magnetic flux through it changes and an e.m.f. is induced across its ends. The direction of the induced e.m.f. is reversed every half-revolution. When the plane of the coil is vertical the flux is a maximum so that the e.m.f. is zero. When $JKLM$ is horizontal the *rate of change* of flux is a maximum and the e.m.f. is a maximum. To maintain unidirectional output the separate halves of C change brushes when $JKLM$ is vertical. For the rotation shown in Fig. 11.4, B_1 is always negative and B_2 positive.

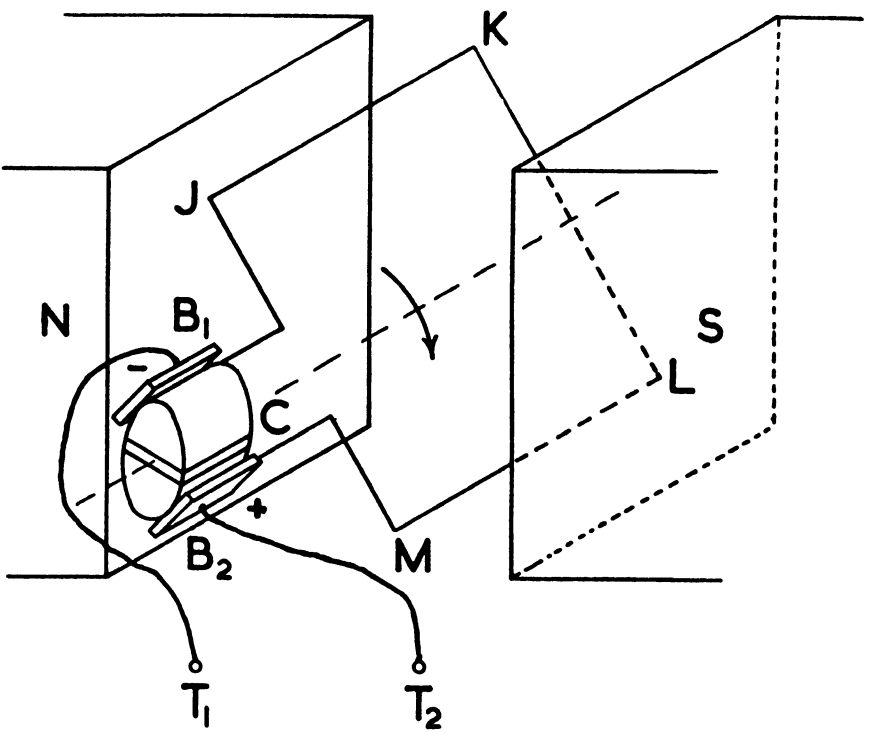


Fig. 11.4

The induced e.m.f. varies with the orientation of the coil (p. 324) and the output thus consists of half-sine waves on the same side of the t-axis (Fig. 11.5). The dotted curve is produced when slip-rings are used. The complete sine-wave is said to be *rectified*.

This method is demonstrated by rotating an earth inductor provided with a split-ring commutator. If the speed of rotation is small the variation of current is followed by connecting the terminals to a galvanometer.

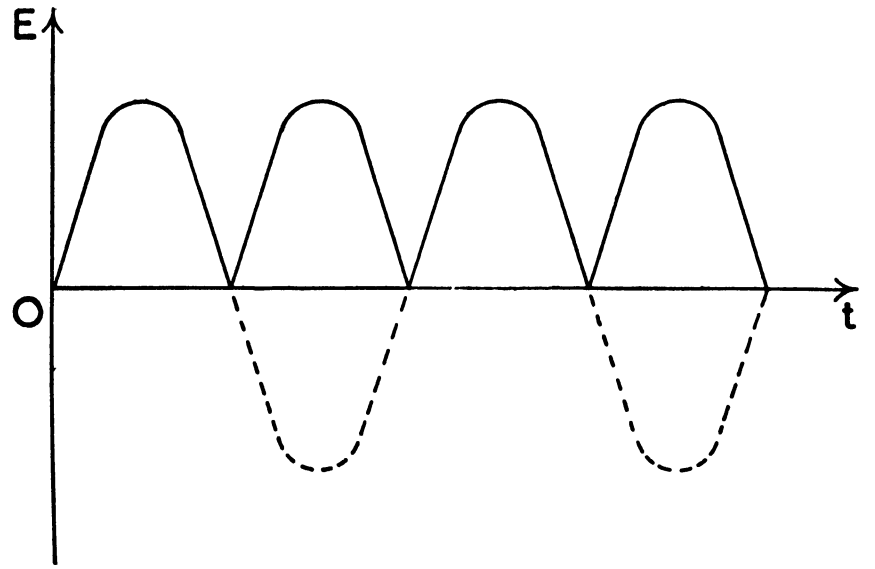


Fig. 11.5

(ii) *Drum Armature Dynamo*

A practical dynamo is based on similar principles but the magnetic field is produced by D.C. flowing in " field " coils wound on pole pieces fixed on the inner surface of a hollow iron stator (S), (Fig. 11.6). The rotor consists of conductors embedded in slots on the surface of a laminated iron cylinder A called a *drum armature.* The ends of the conductors of A are connected to the segments of a commutator mounted on the shaft which is driven by external power. The number of commutator segments is half the number of armature slots. The commutator connections are complex but are such that a continuous current path is always obtained throughout the armature. The e.m.f. developed is produced across the carbon brushes B_1 and B_2 which rest on the commutator.

As the armature rotates in the field of the stator its conductors cut lines of induction and e.m.f.'s are produced across their ends. The commutator connections ensure a steady resultant e.m.f. at any time.

11.8. Field Connections

The D.C. for the dynamo field windings is supplied by the dynamo itself in three different ways. In Fig. 11.7(a), the field coils are in series with the armature windings (*series-wound dynamo*). When the rotor begins to revolve the residual magnetism in the stator causes a small

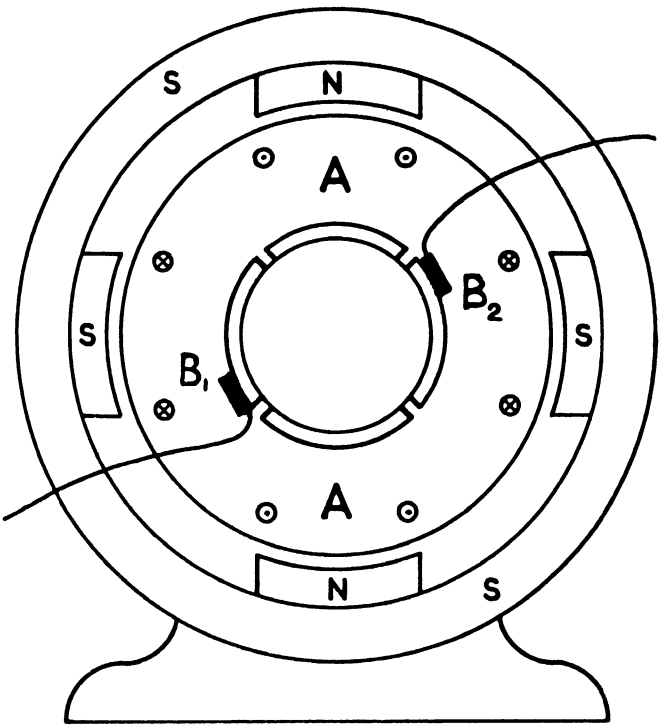
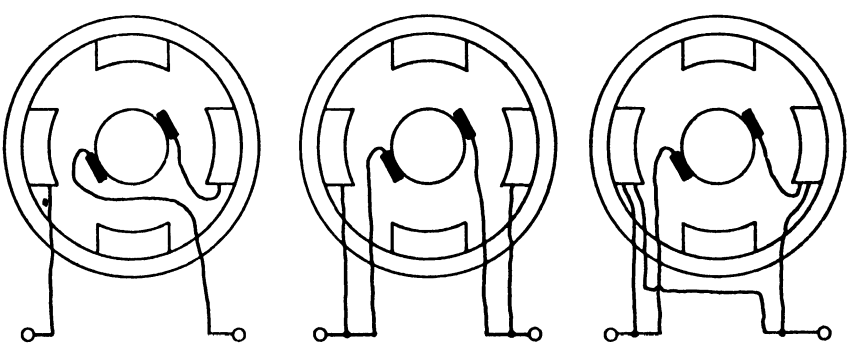


Fig. 11.6

induced e.m.f. in the armature windings and hence a small current in the field coils. Magnetisation of the stator then increases, the induced e.m.f. increases and maximum e.m.f. is soon produced.

In Fig. 11.7(*b*) the field coils are in parallel with the armature windings (*shunt-wound dynamo*). When the armature begins to rotate a small e.m.f. is produced as before, and, since this appears across the field coils, current flows which increases the magnetisation and so on. This is effective even on open circuit, whereas the series-wound dynamo cannot function unless there is a load across the terminals.

In Fig. 11.7(*c*) two separate field windings are used; one connected in



(*a*) (*b*) (*c*)

Fig. 11.7

series and the other in parallel with the armature (*compound-wound dynamo*). This gives characteristics which are a combination of those of the other two types. For example, it gives very little variation in output voltage with output current, whereas the shunt-wound dynamo voltage decreases with current output and the series wound dynamo voltage increases with the current output.

The dynamo has the same defects as the A.C. generator. A counter torque is developed and losses are caused by heating, hysteresis and eddy currents in the armature. Similar precautions minimise these losses. The efficiency of both generators is, by definition:

$$e = \frac{\text{Useful electrical power developed}}{\text{Mechanical power used.}}$$

The power output is: EI watts, where E is the terminal voltage and I is the current strength (amps.) in the external circuit. The power used is the sum of the power output and all the power losses caused by heating, hysteresis and eddy currents. This follows from the principle of conservation of energy, since all the mechanical energy used is converted into electrical energy or dissipated in the form of heat associated with the sources of loss. If the total sum of the power losses is W watts:

$$e = \frac{EI}{EI + W} \times 100\%$$

11.9. Generation of Unidirectional e.m.f.—Faraday's Disc

By rotating a copper disc in a uniform magnetic field Faraday demonstrated the production of D.C. Faraday's disc is represented diagrammatically in Fig. 11.8; A is a thin copper disc mounted on an

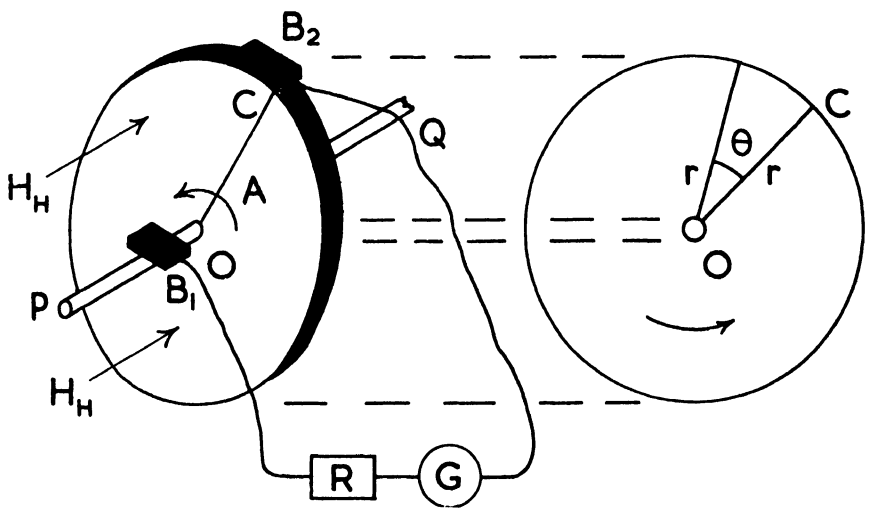


Fig. 11.8

axle PQ; B_1 and B_2 are thick brass strips (" brushes ") resting on the edge of the disc and axle respectively; they are connected to the external circuit consisting of a resistance R and galvanometer G. The disc is placed so that it is at right angles to the earth's horizontal field (H_H) and rotated with a constant angular velocity ω radians.sec.$^{-1}$ A steady e.m.f. is produced between B_1 and B_2 which gives rise to a constant deflection on G.

The induced e.m.f. is caused by the change in magnetic flux through the disc. This is determined by considering any radius such as OC: OC is equivalent to a straight wire which cuts the lines of induction of the earth's field, so that an induced e.m.f. is produced across it. Let OC rotate through the angle θ in the time t secs. The area swept out by this line is: $A = \frac{1}{2}r^2\theta$ m.^{2}, where $r = OC$ in metres. The lines of induction are normal to the disc and the number cut by OC in this time is: $\Phi = \frac{1}{2}r^2\theta H_H\mu_o$ webers. Hence, the rate of change of flux is:

$$\frac{d\Phi}{dt} = \tfrac{1}{2}r^2\mu_o H_H \frac{d\theta}{dt} = \tfrac{1}{2}r^2\mu\omega_o H_H \text{ volts}$$

i.e. the induced e.m.f. is $\underline{E = \frac{1}{2}r^2\mu_o\omega H_H \text{ volts}}$ . . . . (4)

Each radius cuts lines of induction at the same rate, so that a steady p.d. exists between the axle and the edge of the disc. The magnitude of the e.m.f. is very small.

Example

A Faraday disc has a radius of 10 *cm. and rotates at* 100 *r.p.m. in a locality where the earth's horizontal field strength is* 16 *amp.m.*$^{-1}$ *What is the value of the steady e.m.f. produced?*

The angular velocity of the disc is: $\omega = 2\pi \times \frac{100}{60} = \frac{200\pi}{60} = \frac{10\pi}{3}$ radians.sec.$^{-1}$

Since $r = 0{\cdot}1$ m., $H_H = 16$ amp.m.$^{-1}$; $\mu_o = 4\pi \times 10^{-7}$ M.K.S.:

$$E = \tfrac{1}{2} \times 4\pi \times 10^{-7} \times 0{\cdot}01 \times 16 \times \frac{10\pi}{3} \text{ volts}$$

$$= \frac{3{\cdot}2}{3} \cdot \pi^2 \times 10^{-7}$$

i.e. $\underline{E = 1{\cdot}06 \times 10^{-6} \text{ volts.}}$

Example

In the diagram the shaded circle represents a cross-section of a horizontal copper wire which is free to move between two magnetic poles.

(*a*) *Describe and explain in terms of magnetic lines of force what will happen to the wire when a circuit consisting of the wire, a rheostat, and a battery is completed.*

(*b*) *What happens when the wire is moved up or down, if it is connected in series with a galvanometer?*

Describe one *appliance the working of which depends upon the latter phenomenon.* (Cambridge Local G.C.E. Adv. L.).

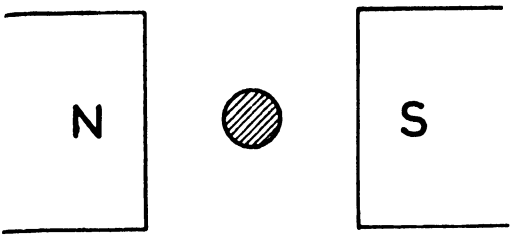


Fig. 11.9(*a*)

(*a*) When the circuit is completed a current flows which creates a magnetic field consisting of concentric circular lines of force surrounding the wire. The original field and that of the wire are superimposed in Fig. 11.9. A force is exerted on the wire given by $\mu_o HI$ newtons per metre, where H is the strength of the original field between N and S in amp.m.$^{-1}$ and I the current strength in amps., (I flows into the paper). The direction of the force is downwards. The force is upwards when the current is reversed.

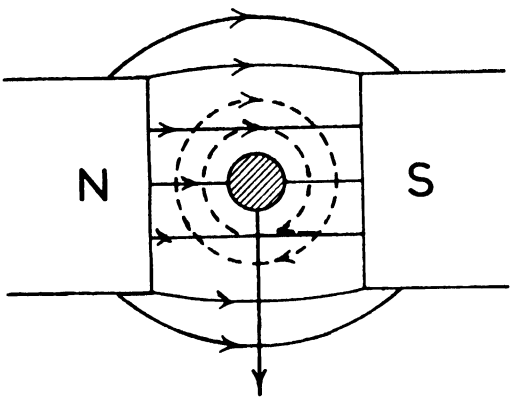


Fig. 11.9(*b*)

(*b*) When the wire is moved upwards it cuts lines of induction and an e m.f. is produced across its ends. The direction of this e.m.f. is such that the end nearer to the paper is negative. Current flows, creating a magnetic field which tends to prevent the upward motion, (Lenz's law). The galvanometer thus shews a deflection. At the end of the upward movement the rate of cutting lines of induction is reduced to zero, so that the deflection of the galvanometer falls to zero. An induced e.m.f. in the opposite direction is produced when the wire moves downwards so that the galvanometer deflection increases in the opposite sense. As the motion continues up and down, the galvanometer deflection is alternately positive and negative about its zero position. Thus, an alternating current flows in the circuit of wire and galvanometer.

The most important application of this phenomenon is the alternating current generator described above.

Example

A circular coil of wire, fitted with a commutator, is rotated at a uniform speed about a diameter at right angles to a uniform magnetic field. Describe in a general way how the e.m.f. varies during one revolution.

If the above coil has 100 *turns of radius* 5 *cm. and the field has an intensity of*

25×10^3 *amp.m.*$^{-1}$, *calculate the average e.m.f. in volts when the coil makes* 1800 *r.p.m.* (Cambridge Local G.C.E. Adv. L.).

The rotating coil is discussed above. Each half-revolution of the coil produces half a complete sine wave of e.m.f. represented by $E = E_0 \sin \omega t$; E_0 is the maximum e.m.f. generated and given by (2) above, viz. $E_0 = \mu_0 AHN\omega$.

The average value of the e.m.f. during each half-period is found by using the formula cf. p. 342:

$$\bar{E} = \frac{\int_0^{T/2} E \, . \, dt}{T/2}$$

where T is the time for one complete revolution. Since, $T = 2\pi/\omega$:

$$\bar{E} = \frac{\omega}{\pi} \int_0^{\pi/\omega} E \, . \, dt = \frac{\omega}{\pi} E_0 \int_0^{\pi/\omega} \sin \omega t \, . \, dt$$

integrating: $$\bar{E} = \frac{\omega E_0}{\pi} \left[\frac{-\cos \omega t}{w} \right]_0^{\pi/\omega} = \frac{\omega E_0}{\pi} \frac{(1+1)}{\omega} = \frac{2E_0}{\pi}$$

Substituting for E_0:

$$\bar{E} = 2\mu_0 r^2 \omega HN \quad (A = \pi r^2)$$

Since: $r = 0{\cdot}05$ m., $H = 25 \times 10^3$ amp.m.$^{-1}$, $N = 100$, $\omega = 2\pi f = 2\pi \, . \, \frac{1800}{60}$ $= 60\pi$ radians sec.$^{-1}$, and $\mu_0 = 4\pi \times 10^{-7}$ M.K.S.:

$$\bar{E} = 2 \times 4\pi \times 10^{-7} \times 25 \times 10^{-4} \times 60\pi \times 25 \times 10^3 \times 100 \text{ volts}$$

i.e. $\bar{E} = 2{\cdot}96$ volts

Since the same e.m.f. variation is repeated each half-revolution, $\bar{E}$ is constant.

11.10. The External Circuit—(i) Effect of Resistance on A.C.

If a " pure " resistance is connected in series with an alternating e.m.f. the current strength at any time is given by Ohm's law:

$$I = \frac{E}{R} = \frac{E_0}{R} \, . \, \sin \omega t = I_0 \, . \, \sin \omega t. \quad . \quad . \quad (5)$$

The current follows a similar variation with time to the applied e.m.f., the amplitude of current strength being: $I_0 = E_0/R$.

Apart from resistance, inductance and capacity are important components in A.C. circuits. They limit the current strength available from a given supply e.m.f., and also introduce a *phase difference* between the current strength and applied e.m.f.

11.11. (ii) Circuit with Inductance and Resistance: L—R Circuit

Suppose an inductance L henries is connected in series with a resistance R ohms and an e.m.f.: $E = E_0 \, . \, \sin \omega t$ volts, (Fig. 11.10). The current strength (I amps.), alternates with the supply frequency so that a self-induced e.m.f. is produced continuously across L. This e.m.f. opposes an increasing current and supports a decaying current

(Lenz's law). Thus, the current strength does not reach its maximum value at the same time as the applied e.m.f. and a phase difference exists between the two.

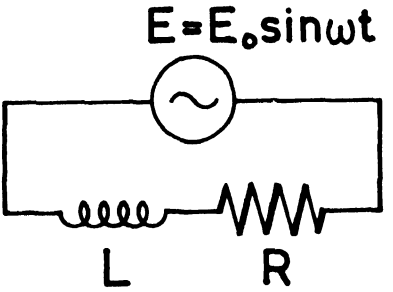


Fig. 11.10

The value of I at any time is found by an analysis of the circuit to be given by:

$$I = I_0 \,.\, \sin(\omega t - \phi) \text{ amps.} \qquad (6)$$

where: $$I_0 = \frac{E_0}{\sqrt{R^2 + L^2\omega^2}}; \text{ and: } \tan\phi = \frac{L\omega}{R} \,. \qquad (7)$$

Both I and E are plotted against t in Fig. 11.11; the sine wave representing I is displaced by ϕ/ω secs. relative to that representing E. The e.m.f. "*leads*" the current by the phase difference ϕ.

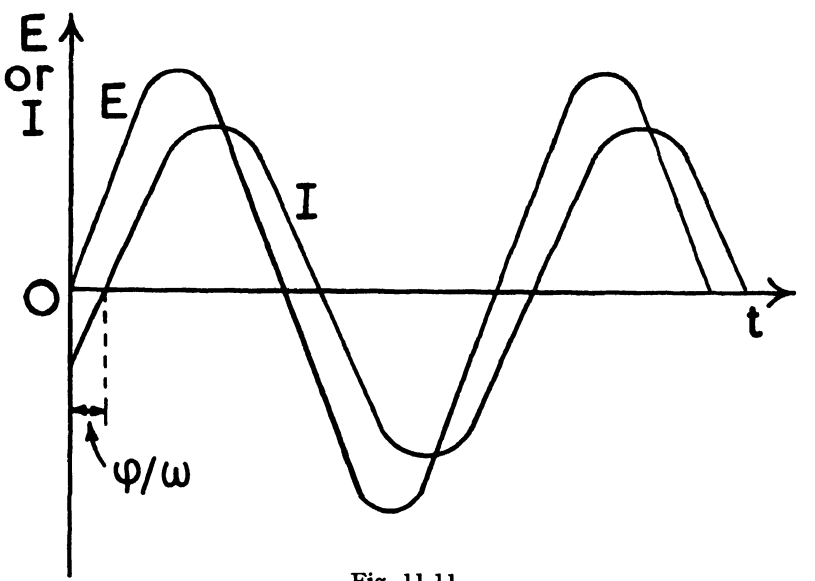


Fig. 11.11

**Reactance and Impedance*

The ratio of e.m.f. to current strength at any time is:

$$\frac{E}{I} = \frac{\sqrt{R^2 + L^2\omega^2} \,.\, \sin\omega t}{\sin(\omega t - \phi)} \quad . \quad . \quad . \qquad (8)$$

It follows that E/I is a function of the time. In D.C. circuits E/I is constant (Ohm's law). The *instantaneous* values of E and I do not, therefore, obey Ohm's law.

The ratio of the *amplitudes* of e.m.f. and current strength is:

$$\frac{E_o}{I_o} = \sqrt{R^2 + L^2\omega^2} \quad . \quad . \quad . \quad . \quad (9)$$

$$\therefore \qquad I_o = \frac{E_o}{\sqrt{R^2 + L^2\omega^2}} \text{ amps.} \quad . \quad . \quad . \quad (10)$$

Equation (10) is analagous to Ohm's law except that resistance is replaced by: $\sqrt{R^2 + L^2\omega^2}$. This quantity limits the maximum current strength obtainable and is called the *total impedance* of the circuit (Z); Z^2 is made up of two terms: R^2 and $L^2\omega^2$; the square root of the latter is called the *reactance* (X). The value of X determines the phase difference since:

$$\tan\phi = (L\omega)/R = X/R.$$

From (10):

$$I_o = \frac{E_o}{Z} = \frac{E_o}{\sqrt{R^2 + L^2\omega^2}} \; ; \; \tan\phi = \frac{X}{R} \; ; \; Z^2 = R^2 + X^2 \quad (11)$$

**Component p.d.'s of L—R Circuit*

The p.d. across R at any time is given by:

$$V_R = IR = I_o R \,.\, \sin(\omega t - \phi) = \frac{E_o}{Z} R \,.\, \sin(\omega t - \phi) \qquad (12)$$

i.e. $V_R = V_{Ro} \sin(\omega t - \phi)$, where $V_{Ro} = \dfrac{E_o R}{Z} = E_o \sin\phi$ (13)

The induced e.m.f. across L is given by:

$$E_L = -L \,.\, \frac{dI}{dt} = -L\omega I_o \,.\, \cos(\omega t - \phi)$$

This *e.m.f.* corresponds to a *p.d.* (V_L) in the same direction as V_R, hence the *p.d.* across L is:

$$V_L = -E_L = +LI_o\omega \,.\, \cos(\omega t - \phi) = V_{Lo} \,.\, \sin\left(\omega t - \phi + \frac{\pi}{2}\right) \quad (14)$$

Thus, V_L and I differ in phase by $\pi/2$, and:

$$V_{Lo} = L\omega I_o = \frac{L\omega E_o}{Z} = \frac{X}{Z} E_o = E_o \,.\, \cos\phi \quad . \quad (15)$$

From (13) and (15): $V_{Ro}^2 + V_{Lo}^2 = E_o^2$ . . . . . (16)

Example

A lamp having a hot resistance of 10 *ohms is not allowed to pass more than* 5 *amps. What is the value of the inductance which must be used in series with the lamp if the A.C. mains supply e.m.f. has a maximum value of* 320 *volts and its frequency is* 50 *c/s?*

The circuit is that of Fig. 11.10, with $E_o = 320$ volts and $R = 10$ ohms.

Hence: $V_{Ro} = I_o R = 5 \times 10 = 50$ volts.

Since: $E_o^2 = V_{Ro}^2 + V_{Lo}^2$; $V_{Lo}^2 = (320)^2 - (50)^2 = 99{,}900$

i.e. $V_{Lo} = 316$ volts.

The value of L required is given by using (15): $V_{Lo} = L\omega I_o$

and since $I_o = 5$ amps., and: $\omega = 2\pi \times 50$ rads.sec.$^{-1}$:

$$L = \frac{316}{500\pi} = 0{\cdot}201 \text{ henries}$$

The inductance acts as a *voltage-dropping* device. No power is absorbed by an inductance so that this is more economical than a resistance. In practice the small resistance associated with the inductance wire absorbs power.

*11.12. (ii) Capacity and Resistance in A.C. Circuits: C—R Circuit

In A.C. circuits condensers act as impedances. Suppose a capacity C farads is connected in series with a resistance R ohms to an A.C. supply e.m.f.: $E = E_o \cdot \sin \omega t$, as in Fig. 11.12. Current cannot flow through the dielectric of the condenser, but as the applied e.m.f. increases positive charge flows into one plate and leaves the other—charging the condenser. The flow of charge round the rest of the circuit constitutes a current, which creates a p.d. across R and prevents the condenser charging-up instantaneously to the value of the *e.m.f.* at that time. If the current strength at time t is I amps., the p.d. across R is: $V_R = IR$ volts. Applying Kirchhoff's law, the p.d. across C is: $V_C = E - IR$; V_C corresponds to a charge Q (say) coulombs on either condenser plate. Since $I = (dQ/dt)$:

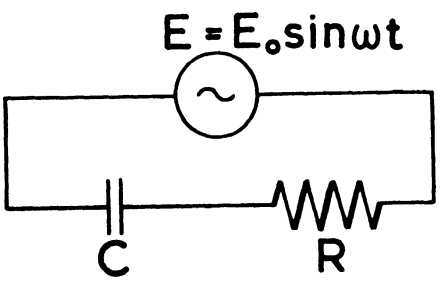


Fig. 11.12

$$\frac{Q}{C} = E - R \cdot \frac{dQ}{dt}$$

i.e. $$R \cdot \frac{dQ}{dt} + \frac{Q}{C} = E_o \cdot \sin \omega t \quad . \quad . \quad . \quad (17)$$

The solution for Q is:

$$Q = Q_o \cdot \sin (\omega t + \alpha) \quad . \quad . \quad . \quad . \quad (18)$$

where: $$Q_o = \frac{-E_o}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}; \text{ and } \tan \alpha = -\omega CR \quad . \quad . \quad (19)$$

From (18): $$I = \frac{dQ}{dt} = Q_o \,.\, \cos(\omega t + \alpha) \quad . \quad . \quad . \quad . \quad (20)$$

i.e. $$I = I_o \,.\, \sin(\omega t - \phi)$$

where:

$$I_o = \frac{E_o}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \text{ and } \phi = \left(\frac{\pi}{2} - \alpha\right) \quad \left[\tan \phi = \cot \alpha = \frac{-1}{\omega CR}\right] \quad (21)$$

Since CR is always positive, ϕ is always negative and I *leads* the applied e.m.f. by the phase angle ϕ.

By analogy with the $L-R$ circuit, $\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$ is the total impedance of the circuit Z; $\frac{1}{\omega C}$ is the reactance X. It follows that:

$$I_o = \frac{E_o}{Z}; \tan \phi = \frac{X}{R}; Z^2 = R^2 + X^2; \text{ as for the } L-R \text{ circuit.}$$

Component p.d.'s

For the resistance:

$$V_R = IR = E_o \frac{R}{Z} \,.\, \sin(\omega t - \phi) = V_{Ro} \,.\, \sin(\omega t - \phi)$$

where: $$V_{Ro} = E_o \frac{R}{Z} = E_o \,.\, \cos \phi \quad . \quad . \quad . \quad (22)$$

For the condenser:

$$V_C = \frac{Q}{C} = \frac{-E_o}{CZ} \,.\, \sin(\omega t + \alpha) = -E_o \frac{X}{Z} \,.\, \sin\left(\omega t + \frac{\pi}{2} - \phi\right)$$

$$= V_{Co} \,.\, \sin\left(\omega t - \phi - \frac{\pi}{2}\right)$$

where: $$V_{Co} = E_o \frac{X}{Z} = E_o \,.\, \sin \phi \quad . \quad . \quad . \quad (23)$$

hence: $$E_o^2 = V_{Co}^2 + V_{Ro}^2 \quad . \quad . \quad . \quad . \quad (24)$$

11.13. L—C—R Circuit

In the general A.C. circuit containing inductance, capacitance and resistance in series, the current strength is limited by all three components. It can be shown that the current strength at any time t, when the applied e.m.f. is $E = E_0 \sin \omega t$, is sinusoidal and given by:

$$I = I_0 \sin (\omega t - \phi) \quad . \quad . \quad . \quad . \quad (25)$$

where:

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}} \quad . \quad . \quad . \quad (26)$$

and ϕ, the phase difference (in radians) between I and E, is given by:

$$\tan \phi = \frac{L\omega - \frac{1}{\omega C}}{R} \quad . \quad . \quad . \quad . \quad (27)$$

The strength of the maximum current (I_0) can be written in the form:

$$I_0 = \frac{E_0}{\sqrt{R^2 + X^2}} = \frac{E_0}{Z} \quad . \quad . \quad . \quad (28)$$

where $X = L\omega - \frac{1}{\omega C}$, is the *total reactance* and Z is the *total impedance* of the circuit. The reactance is composed of two terms depending on the values of inductance and capacitance.

By working out the component p.d.'s as for the $L-R$ and $C-R$ circuits, it is found that the amplitude of the p.d. across the total reactance, given by: $V_{X0} = V_{L0} - V_{C0}$, is always $\pi/2$ out of phase with I, and furthermore:

$$V_{X0}^2 + V_{R0}^2 = (V_{L0} - V_{C0})^2 + V_{R0}^2 = E_0^2 \quad . \quad (29)$$

i.e. the sum of the squares of the amplitudes of the reactive and resistive p.d.'s gives the square of the applied e.m.f.

Example

A $10\mu F$ condenser is connected in series with a coil having an inductance of 4 henries and a resistance of 100 ohms and the mains supply which has a maximum e.m.f. of 320 volts and a frequency of 50 cycles.sec.$^{-1}$. What is the maximum value of the current which flows?

Since the frequency $f = 50$ cycles.sec.$^{-1}$, the angular frequency $\omega = 100\pi$ radians.sec.$^{-1}$

The reactance of the circuit is given by:

$$X = L\omega - \frac{1}{\omega C} = 400\pi - \frac{1}{1000\pi \cdot 10^{-6}} = 1257 - 318{\cdot}3 \text{ ohms}$$

i.e. $X = 938{\cdot}7$ ohms

From (28) above:

$$I_0 = \frac{E_0}{\sqrt{R^2 + X^2}} = \frac{320}{\sqrt{10^4 + (938 \cdot 7)^2}} = \frac{320}{939 \cdot 2} \text{ amps.}$$

i.e. $I_0 = 0 \cdot 341$ amps.

Hence, the strength of the maximum current is 0·341 amps.

11.14. Measurement of Alternating Current Quantities — Current Strength

If A.C. is passed through a moving-coil ammeter the deflection is zero unless the frequency of the alternations is very low, because the natural frequency of suspension is far less than the A.C. frequency. The A.C. gives rise to " forced " oscillations of the coil of negligible amplitude.

If a suitable moving-coil instrument were designed for A.C., the pointer of the instrument would move rapidly on either side of the zero so that the eye could not be used to take an instantaneous reading. In any case, the maximum current strength (I_0) only, is of interest.

In seeking a solution to the problem of A.C. measurement, advantage is taken of the fact that some current effects are independent of the direction, i.e. their magnitude is proportional to the square of the current strength. Notable among these effects is the heating effect given by I^2R joules per second (p. 105). Similarly, the magnetic force between two coils passing the same current is proportional to I^2. The value of I^2 varies periodically with time but the steady rise in temperature caused by the current is independent of this, owing to the relatively long time taken to heat a wire compared with the period of the A.C. Likewise, the magnetic effect between two coils varies with time, but, owing to inertia, the coils cannot follow this variation and a mean effect is observed.

11.15. Root Mean Square Current Strength

Both the steady temperature of a heated wire and the steady deflection between two coils are proportional to the average value of the square of the current strength ($\overline{I^2}$), which is called the *Mean Square Current.* This is related to the maximum current strength I_0 by:

$$\overline{I^2} = \frac{I_0^2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

A.C. instruments could be calibrated to read Mean Square Current, but instead they are calibrated to read the square root of this quantity, viz. $\sqrt{\overline{I^2}}$; this is called the *Root Mean Square* (*R.M.S.*) *Current Strength* and represented by the symbol I_{rms}. This is more convenient to use since it is proportional to I_0, $\left(I_{rms} = \frac{I_0}{\sqrt{2}}\right)$.

11.16. Derivation of I_{rms} in terms of I_o

The variation of the square of an alternating current strength with time is shown in Fig. 11.13. The curve is a plot of $I_o^2 . \sin^2\omega t$. It is seen that positive maxima occur every half-cycle. The time average of I^2 cannot be determined by integrating between two arbitrary time

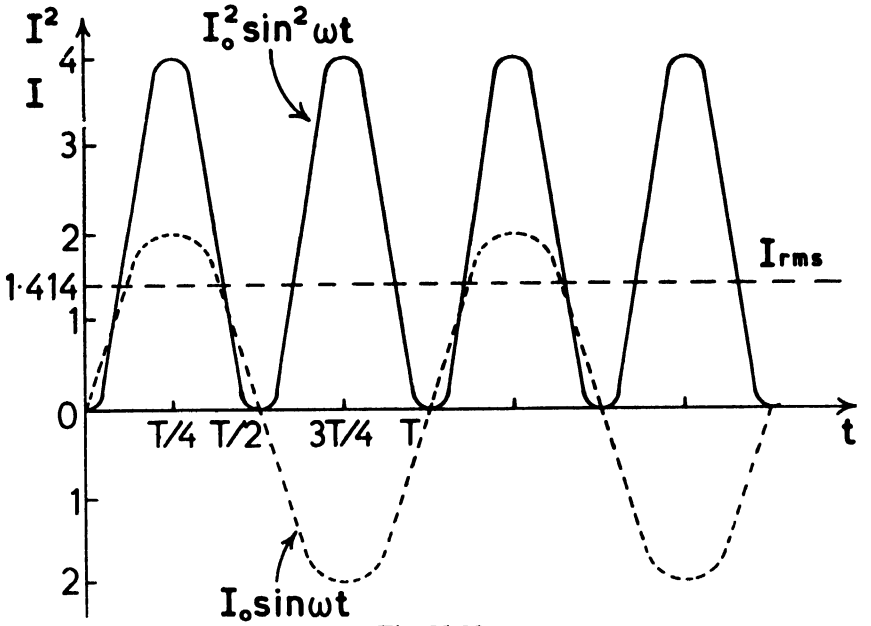


Fig. 11.13

limits and dividing by the time because this quantity varies with the time interval chosen. If $I_o^2 . \sin^2\omega t$ is integrated over an interval of one period (T) a constant value is obtained which represents the area beneath the curve between these limits. This quantity divided by T gives $\overline{I^2}$. Hence:

$$\overline{I^2} = \int_0^T \frac{I_o^2 . \sin^2\omega t}{T} . dt = \frac{\omega I_o^2}{2\pi} . \int_0^{2\pi/\omega} \sin^2\omega t . dt \quad . \quad (31)$$

To perform the integration, $\sin^2\omega t$ is put in the form: $\frac{1}{2} - \frac{1}{2} . \cos . 2\omega t$, so that:

$$\overline{I^2} = \frac{\omega I_o^2}{2\pi} . \int_0^{2\pi/\omega} \tfrac{1}{2} . dt - \frac{\omega I_o^2}{2\pi} . \int_0^{2\pi/\omega} \tfrac{1}{2} . \cos . 2\omega t . dt \quad . \quad (32)$$

$$= \frac{\omega I_o^2}{2\pi} . \left[\frac{t}{2} - \frac{\sin . 2\omega t}{2} \right]_0^{\frac{2\pi}{\omega}}$$

Since $\sin 2\omega t = 0$ when $t = 0$ and $t = 2\pi/\omega$, it follows that:

$$\overline{I^2} = \frac{\omega I_o^2}{2\pi} \cdot \frac{2\pi}{2\omega} = \frac{I_o^2}{2} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and:

$$I_{rms} = \sqrt{\overline{I^2}} = \frac{I_o}{\sqrt{2}} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

If a meter is calibrated to read I_{rms}, I_o can be obtained by using (34). In the derivation of (33) the phase angle was assumed to be zero for simplicity but the formula is readily obtained by using: $I = I_o \sin(\omega t - \phi)$. Furthermore, two special limits of time were chosen, viz. O and T, but again (33) is obtained by taking general limits t and $t + T$. The value of I_{rms} is plotted as a straight line parallel to the t-axis in Fig. 11.13.

11.17. A.C. Ammeters

The common A.C. ammeters are: (i) the hot-wire ammeter, (ii) the moving-iron ammeter, (iii) the thermo-ammeter, and (iv) the dynamometer type of ammeter. The thermo-ammeter is based on the heating effect of a wire, using a thermocouple in conjunction with a galvanometer as described on p. 212. The dynamometer type utilizes the interaction between the magnetic fields of a fixed and pivoted coil as described on p. 147.

Hot-Wire Ammeter

This instrument consists of a fine strip of platinum stretched between two points A and B (Fig. 11.14). The A.C. enters at terminal T_1, and after passing through AB, leaves the instrument via terminal T_2.

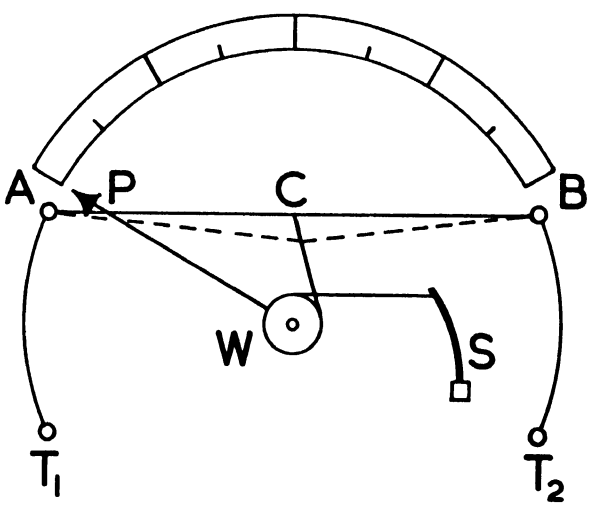


Fig. 11.14

Connected to the mid-point C of AB is a fine thread, which passes round a small pulley wheel W before being joined to the free end of a short leaf-spring S. Attached to the face of W is a light aluminium pointer P, which moves over a scale calibrated to read in amperes, the deflection of P giving the r.m.s. current strength.

When current flows the temperature of AB rises and the wire expands. The expansion is taken up by S pulling the thread, so that AB assumes the shape shown by the dotted lines. As S takes up the expansion W rotates and P gives a deflection dependent on the final temperature of the wire. The meter reading thus depends on $\overline{I^2R}$ and since the resistance of AB is more or less constant the meter reading $\propto \overline{I^2}$. When current ceases to flow the wire cools and contracts, AB moves back to its original shape and the pointer resumes its zero reading.

The main drawback of the instrument is the thermal delay between the application of the current and the final steady deflection. This necessitates using a fine strip of wire of small thermal capacity. Draughts must be excluded to prevent a spurious cooling.

Moving-Iron Ammeter (i)

In this type the A.C. passes round a solenoid S, wound on a non-conducting former, (Fig. 11.15). The current creates a uniform magnetic field inside S, the direction being reversed every half-cycle. Inside S are two rods of soft iron, AB and CD. The latter is fixed whilst the former is attached to a metal framework so that it can rotate about the axis EF. The rotation of AB is opposed by a couple caused by gravitation acting on the small mass M attached to BF. A form of spring control is equally effective.

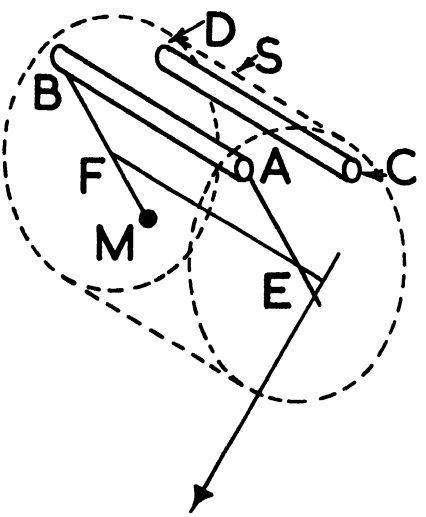


Fig. 11.15

When current flows, AB and CD become magnetized with similar poles at adjacent ends. The sign of the poles changes every half cycle, but they are always the same at similar ends, so that AB and CD always repel each other and AB rotates. The mean repelling force between the rods depends on the average value of the magnetization in the rods and since this depends on the intensity of the magnetizing field, which in turn is proportional to the current strength, the average repelling force $\propto \overline{I^2}$. The movement of the pointer attached at E thus depends on $\overline{I^2}$ and the scale is calibrated to give the r.m.s. values.

Moving-Iron Ammeter (ii)

This form uses one piece of iron only (Fig. 11.16). Current enters the instrument and passes through a short coil C having a thin rectangular cross-section. A flat piece of soft iron F, is pivoted at O, so that it can move into or out of C. The movement of F is controlled by the fine spring, one end of which is attached to F.

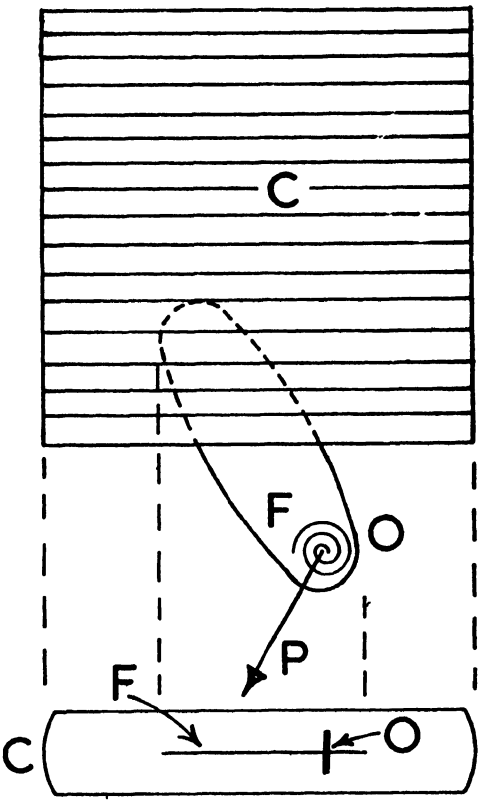


Fig. 11.16

When current flows, F is magnetized by induction and attracted into C. The magnetization is always such that F is attracted into C. The pointer P is attached to the spindle and moves over the calibrated scale, the deflection being proportional to $\overline{I^2}$. The scale is calibrated in r.m.s. values.

The moving-iron meter has the advantage of being quick reading, providing oscillations of the moving iron part are damped. One method of damping is to attach an arm to the spindle so that when the latter rotates a small piston is pushed in or pulled out of an enclosing cylinder. The movement of the piston is opposed by a relatively strong force due to air viscosity and this damps any oscillations.

Calibration: A.C. meters are calibrated by using known direct currents,

since an alternating current produces the same deflection as the D.C. having the same strength as the r.m.s. value of the A.C. Maximum A.C. current strengths (I_o) are obtained by multiplying scale readings by $\sqrt{2}$. ($I_o = \sqrt{2}\, I_{rms}$).

11.18. A.C. Voltmeters

An alternating voltage is measured in a similar way to a direct voltage, by using an A.C. ammeter in series with a high resistance across the two points. Providing the series resistance is sufficiently high, the voltage being measured is not appreciably affected by the small current going through the meter. Assuming the impedance of the voltmeter is caused by resistance only, the current strength in the meter is given by:

$$I_V = \frac{V}{R_A + R_V} \text{ amps.}$$

where V is the voltage being measured, R_A and R_V are the resistances in ohms of the ammeter and series resistance, respectively. Since $V \propto I_V$, the meter current may be used as a measure of the voltage, as in D.C. methods. Since I_V is a r.m.s. current, V is a r.m.s. voltage — V_{rms}; r.m.s. voltages are defined in an exactly similar manner to r.m.s. current strengths and are related to maximum alternating voltages by a similar formula: $V_{rms} = \frac{V_o}{\sqrt{2}}$

Since: $$I_o = \frac{E_o}{Z},\ I_{rms} = \frac{E_{rms}}{Z} \quad . \quad . \quad . \quad (35)$$

N.B. Alternating current strengths and p.d.'s are usually quoted as r.m.s. values and when this is not stated explicitly it is implied, e.g. the A.C. mains voltage is 230 volts, signifying that $V_o = \sqrt{2} \times 230 = 325$ volts.

11.19. Power in A.C. Circuits

By an argument similar to that for D.C. circuits, it can be shown that the rate of working in an A.C. circuit, whose supply e.m.f. has the instantaneous value E volts, is given by: IE joules per second (watts), where I amps. is the instantaneous value of the current strength. This is called the *power* (P). If $E = E_o \sin \omega t$, and $I = I_o \sin (\omega t - \phi)$:

$$P = I_o E_o \sin \omega t \sin (\omega t - \phi) \quad . \quad . \quad (36)$$

P varies in time in a regular manner, as seen in Fig. 11.17, where P is plotted for a value of $\phi = 30°$ (i.e. $\pi/6$ radians), and $I_o < E_o$. The variation of P is periodic, and an average is obtained by integrating (36) over a complete period and dividing by the period $\left(T = \frac{2\pi}{\omega}\right)$. This

is called the *mean power* of the circuit, and is given by:

$$P = \int_o^T \frac{P}{T} \cdot dt = \frac{\omega}{2\pi} \cdot \int_o^{\frac{2\pi}{\omega}} I_o E_o \cdot \sin \omega t \cdot \sin (\omega t - \phi) \cdot dt$$

$$= \frac{\omega}{2\pi} I_o E_o \int_o^{\frac{2\pi}{\omega}} \{ \sin^2 \omega t \cdot \cos \phi - \sin \omega t \cdot \cos \omega t \cdot \sin \phi \} \cdot dt$$

$$= \frac{\omega}{2\pi} I_o E_o \left\{ \int_o^{\frac{2\pi}{\omega}} \sin^2 \omega t \cdot \cos \phi \cdot dt - \int_o^{2\pi/\omega} \frac{\sin 2\omega t}{2} \cdot \sin \phi \cdot dt \right\}$$

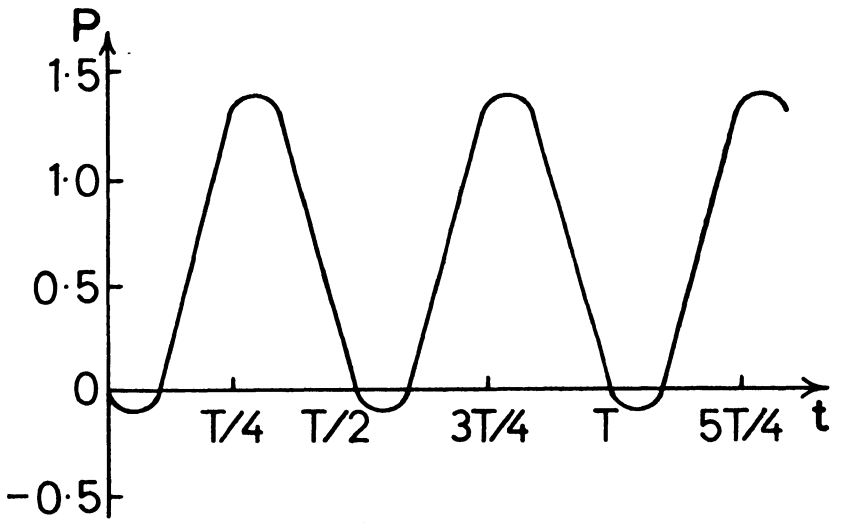


Fig. 11.17

Integrating $\sin 2\omega t$ gives: $-\cos 2\omega t$, which taken between the limits $t = 0$ and $t = 2\pi/\omega$, gives zero. The integration of $\sin^2 \omega t$ between the limits $t = 0$ and $t = 2\pi/\omega$, gives the value: π/ω, (p. 342), so that:

$$P = \frac{\omega}{2\pi} \cdot \frac{\pi}{\omega} \cdot I_o E_o \cdot \cos \phi = \tfrac{1}{2} \cdot I_o E_o \cdot \cos \phi \quad . \quad (37)$$

Since $I_o = \sqrt{2} \cdot I_{rms}$ and $E_o = \sqrt{2} \cdot E_{rms}$, the mean power can be expressed in the form:

$$P = I_{rms} E_{rms} \cdot \cos \phi \quad . \quad . \quad . \quad . \quad (38)$$

The term $\cos \phi$ is called the *Power Factor* of the circuit, whilst the product $(I_{rms} E_{rms})$ is called the *Virtual* or *Apparent Power* of the circuit. The relation (38) then reads:

True Power = Apparent Power × Power Factor

From A.C. theory, $\cos \phi = R/Z$ (p. 340), so that (38) can be written:

$$P = I_{rms} E_{rms} \frac{R}{Z} = I^2_{rms} R \quad \left(I_{rms} = \frac{E_{rms}}{Z} \right) \quad . \quad (39)$$

This formula involves resistance only, so that the mean power in the reactive part of the circuit must be zero. This is in accordance with the ideas already developed, in which inductances and condensers were conceived as storehouses of energy without ever absorbing energy.

From (38) the power in any *part* of an A.C. circuit is given by: $\overline{P} = I_{rms}V_{rms}\cos\phi$ where ϕ is the phase difference between current strength and *p.d.* For reactive components $\phi = \frac{\pi}{2}$ always, so that $\cos\phi = 0$ and $\overline{P} = 0$.

11.20. Power Factor Correction

Electrical energy is supplied from a power station for driving large machinery, etc. Such circuits have reactance as well as resistance and generally give rise to a phase angle which represents a lag of current behind the supply e.m.f. The power factor for the circuit is thus less than unity. The mean power depends on resistance only, and since the power station e.m.f. is constant, the only variables in (39) are I_{rms} and $\cos\phi$. If, therefore, $\cos\phi$ is low, I_{rms} must be relatively high. A high current strength requires large cross-section supply lines which is uneconomical. To reduce I_{rms} and the costs of power supply, $\cos\phi$ is made approximately unity by connecting large capacity condensers across the machinery. The condensers form a reactance in parallel with the device, thus reducing ϕ and increasing $\cos\phi$. This process of *power factor correction* is often resorted to by manufacturers using large electrical machinery.

11.21 Transmission of Electric Power

Most power stations in this country generate A.C. at a high potential of some tens of thousands of volts (rms). The power is transmitted to various parts of the country by transmission lines which, in the simplest form, consist of two parallel conductors for current flow to and from the station. To avoid an excessive loss of p.d. and power caused by line resistance, the generator output p.d. is first *transformed* to a much higher value. A *step-up transformer* is used, which in the ideal case converts electrical power at low p.d. into the same power at higher p.d. without power-loss. As a consequence the maximum available current strength is reduced.

To appreciate the economy in transmitting power at high p.d.'s, suppose the maximum mean power available from a given generator is P_0 watts. If the generator e.m.f. is E_0 volts, the maximum current strength available is: $I_0 = P_0/E_0$ amps. If this power is transmitted over two lines of total resistance R ohms and negligible inductance and capacity, to a load which absorbs all the power available, the loss in *p.d.* along the lines is I_0R volts. The loss of power in the lines is $I^2{}_0R$ watts. If these losses are not to be excessive, R must be made as small as possible. Assuming copper is used for the transmission lines and the

proviso is made that the potential drop along the lines must not exceed a given small percentage of the output p.d. (say, 5 per cent), the cross-sectional area A of the cables is worked out; $R = \rho L/A$ and hence: $\triangle V = I_o . \rho L/A$, where L is the sum of the lengths of the lines and ρ is the specific resistance of copper. Expressed as a percentage of the generator e.m.f. this becomes:

$$\frac{\triangle V}{E_o} \times 100 = \frac{I_o \rho L}{E_o A} \times 100 = \frac{P_o \rho L}{E_o^2 A} \times 100\%$$

If P_o, L and ρ are constant, the percentage voltage drop along the lines is inversely proportional to: (generator e.m.f.)^2. Likewise, under the same conditions, the power loss along the lines may be expressed in the form:

$$\triangle P = I_o^2 R = \frac{P_o^2 . \rho L}{E_o^2 A}$$

If the maximum power loss is fixed, the cross-sectional area of the transmission lines is inversely proportional to: (generator e.m.f.)^2. To avoid expense in using very large diameter cables it is obviously advantageous to *generate* A.C. at very high e.m.f.'s. This involves difficult insulation problems in the generators, and instead, the output from the generator is transformed into a conveniently high voltage before it is transmitted across country.

A typical power alternator produces 1,000 K.watts at an output p.d. of 6,600 volts, which, without transformation, necessitates using cables sufficiently thick to carry a maximum of 1,000/6·6 amps. In practice, the generator e.m.f. is transformed to 132,000 volts (i.e. 20 × 6,600) before transmission; the maximum current is reduced to 1/20th of the previous value, and requires a relatively small cross-section. The cables used for transmitting the power over long distances are suspended, by means of large porcelain or glass insulators, from large steel pylons which are a common feature of the countryside. The pylons usually carry more than the two lines because A.C. is usually generated by *three-phase alternators* which require at least three lines for transmission.

The main transmission lines from the power station form part of the *National Grid*, which covers the whole country. This consists of a comprehensive network of lines which allows power from generating stations to be fed into a common system and abstracted where needed. This is most economical and allows power distribution to be controlled by a central authority. From the grid, branch lines conduct the power to large cities at lower potentials, such as 33,000 or 66,000 volts, the potential transformation being carried out at a transformer station outside the city. The supply is then transformed again to 6,600 volts at a main sub-station, and may be supplied to large consumers such as factories at this voltage, the consumer transforming down again to

the common mains voltage of 230 volts for lighting, heating, etc. For ordinary domestic consumers smaller transformers contained in kiosks, are used to reduce the voltage to the mains value of 230. Although the main grid and branch line supplies are carried by overhead cables, the 6,600 volt supplies and mains supplies in cities and towns have to be carried underground, which necessitates the use of expensive lead covered insulated cables. In rural areas supplies are carried more cheaply by overhead cables in a manner similar to telegraph wires.

11.22. Transformers

Transformers are based on the principle of electromagnetic induction. The ideal transformer consists of two coils having perfect coupling such that the magnetic flux created by a current in one passes through the other. This ideal is closely approached by winding the two coils on opposite limbs of an iron yoke of high magnetic permeability, (Fig. 11.18).

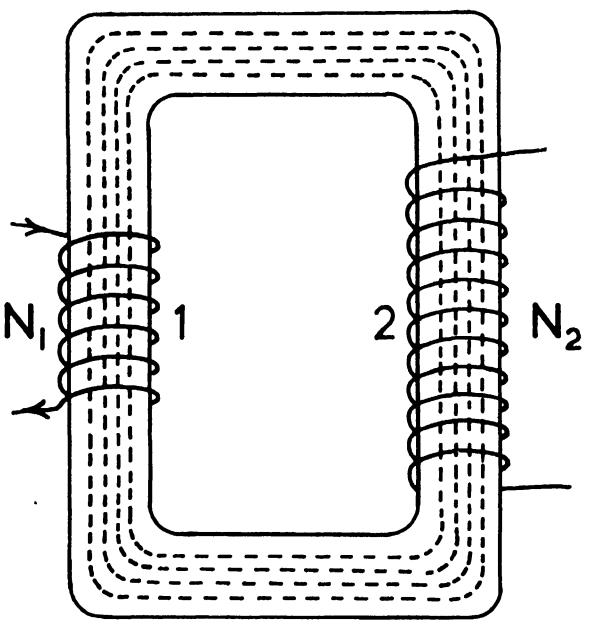


Fig. 11.18

When a current flows in coil 1 (the primary), lines of induction form closed paths in the iron circuit, as shown. Assuming all the lines pass through coil 2, (the secondary), the theory becomes very simple.

Let the primary and secondary turns be N_1 and N_2, respectively, and let the cross-section of each coil be A m.^{2} If the magnetic induction inside the iron is B weber.m.$^{-2}$, the magnetic flux is: $\Phi = BA$ wb. The number of *flux-turns* of the primary is: $N_1\Phi = N_1BA$, whilst the flux-linkages with the secondary is: $N_2\Phi = N_2BA$. When the primary

current changes in time an induced e.m.f. is produced across the primary given by the usual relation:

$$E_1 = -\frac{d(N_1\Phi)}{dt} = N_1 A \cdot \frac{dB}{dt} \quad . \quad . \quad . \quad (40)$$

Similarly, an induced e.m.f. is produced across the secondary given by:

$$E_2 = -\frac{d(N_2\Phi)}{dt} = N_2 A \cdot \frac{dB}{dt} \quad . \quad . \quad . \quad (41)$$

Hence, the ratio of secondary to primary e.m.f. is given by:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

A transformer can be used with A.C. only. If the internal resistance of the source of e.m.f. and the primary coil resistance are negligible, E_1 is equal and opposite to the source e.m.f. (Kirchhoff). Providing that the secondary coil is not connected to a load, i.e. it is on open circuit, E_2 represents the *output p.d.* of the transformer. The ratio of the terminal *p.d.'s* is then given by $\frac{N_1}{N_2}$, which is called the *transformation ratio.*

When current is taken by the secondary and power is dissipated in the secondary circuit the theory of the transformer becomes rather complicated, because of the effects of self-inductance in the secondary circuit and mutual inductance in the primary circuit. However, with perfect coupling between the coils the power dissipated in the secondary circuit is the same as that transferred from the primary circuit. Hence, using rms. (or peak) values: $E_1 I_1 = E_2 I_2$, so that:

$$\frac{I_2}{I_1} = \frac{E_1}{E_2} = \frac{N_1}{N_2} \quad . \quad . \quad . \quad . \quad (43)$$

This relation holds to a very good approximation in practice since transformer efficiencies, defined by the ratio: (output power)/(input power), are usually greater than 95 per cent.

11.23. Defects of the Transformer

Flux leakage occurs but this is small and does not cause much loss of power. The total power loss is attributed to (i) the *iron loss* and (ii) the *copper loss.* As in generators, A.C. causes periodic magnetisation of the iron and an energy loss depending on the area of the hysteresis cycle. For transformer cores as small an area as possible is required for the iron hysteresis loop. Furthermore, in order that the varying flux shall follow the current variations faithfully, the hysteresis loop should have almost straight sides. Special soft iron alloys, such as permalloy and silicon steel are used (cf. p. 309).

Eddy currents in the iron contribute to the iron loss. They are minimised by using a laminated core made of thin iron sheets (stampings), which are interleaved and insulated from each other by thin varnish or oxide coatings. The copper loss is reduced by using sufficiently thick wire.

11.24. Transformer Ratings

Transformers are rated according to the power they can accept or deliver without becoming too hot. This rating is expressed as a product of the maximum r.m.s. output voltage and amperage. Large power transformers rated in kilovoltamps (kVA), are usually enclosed in oil which acts as an insulator and coolant.

Smaller transformers, which are used to convert the mains voltage into lower or higher supply voltages, have much lower power ratings and the heat developed is effectively dissipated by air convection. *Step-down* transformers of this type are used extensively for supplying $12V$ galvanometer lamps in physics laboratories. Such a transformer, supplied from a mains socket ($230V$ r.m.s.) and fused at 5 amps. (r.m.s.) delivers: $230 \times 5 = 1{,}150$ watts. If the secondary of the transformer delivers $12V$ rms. and each lamp takes 24 watts (say), about 20 lamps can be supplied at once, assuming a certain amount of power loss in the transformer.

11.25. Electric Motors

If the flat coil depicted in Fig. 11.19, is supplied with current rather than being used to deliver current, each horizontal limb JK and LM, is submitted to forces causing the coil to rotate as in a moving-coil galvanometer. The directions of the forces are given by Fleming's Left-Hand Rule. In the diagram, the coil rotates in an anti-clockwise direction.

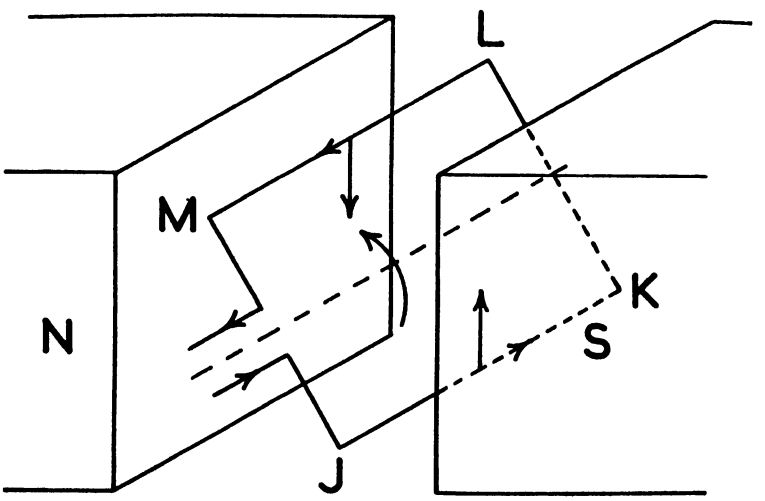


Fig. 11.19

When the plane of the coil is vertical, no torque acts since the forces acting on JK and LM are vertical. Providing the coil has sufficient inertia and friction forces are small, however, the angular momentum gained in its earlier rotation carries it past this zero torque position. Beyond the vertical position the torque acts in the opposite direction, so that in order to maintain continuous rotation the current direction must be reversed as the coil passes the vertical position. This is achieved by using a split ring commutator, as in the dynamo.

The principles involved in the motor are the converse of those used for the D.C. dynamo and the latter may be used as an electric motor by supplying the appropriate current to the output terminals. With A.C. supplies the situation is more difficult, as explained later.

11.26. D.C. Motors

The design of a D.C. motor, which is the most common type of small motor found in laboratories, is similar to that of the D.C. dynamo. A drum-type armature, having windings similar to those of Fig. 11.6, is caused to rotate by the torque exerted on the various conductors carrying the direct current from a segmented commutator. The direction of rotation is that given by the counter-torque developed in the dynamo.

Back-e.m.f.: An e.m.f. is created in the motor armature windings as these rotate in the magnetic field of the stator. The e.m.f. always opposes the applied p.d. and is called a *back-e.m.f.* For a motor whose armature supply is separate from the stator supply and whose back-e.m.f. is E' volts (for a particular speed), the current (I amps.) used by the armature, is given by applying Khirchoff's law:

$$IR_a = E - E' \quad . \quad . \quad . \quad . \quad (44)$$

where R_a is the armature resistance, E the applied p.d. and the e.m.f. internal resistance is neglected.

Multiplying (44) by I: $\quad I^2R_a = EI - E'I \quad . \quad . \quad . \quad . \quad (45)$

It follows that of the power supplied (EI), $E'I$ is returned and the only power dissipated is I^2R_a—this appears as heat in the armature windings.

For a given loading with a constant field current, the speed depends on I which is itself governed by the speed of the armature because it is dependent on the back-e.m.f. E'. A steady speed is obtained when part of the mechanical energy acquired by the rotor is utilised in overcoming all opposing forces due to load, friction, etc., and the rest is converted into the electrical energy of the back-e.m.f.

If the load is suddenly reduced whilst running at speed, the torque derived from the magnetic field becomes temporarily too large and the rotor is accelerated. This causes an increase in the back-e.m.f., and

according to (44), a reduction in I. The reduction in armature current causes the mechanical torque to be decreased and the speed of the motor is adjusted to its previous value. The converse occurs when the load is suddenly increased. The D.C. motor which is provided with a constant magnetic flux is thus a self-regulating device.

Starter Resistance: The resistance of armature windings is very small, (less than one ohm), and extremely high currents flow on starting with the full supply e.m.f. When the motor is running at speed, the back-e.m.f. reduces the effective p.d. and limits the current flow. To prevent excess current flowing when the motor is switched on, a special starting mechanism is used which introduces a high resistance in the armature circuit. As the motor gains speed the resistance is gradually cut out manually.

11.27. Shunt, Series and Compound Excitation

The field windings of a motor are usually provided with current from the same power supply as the armature and they are connected in series, in parallel, or part in series and part in parallel, with the supply. Thus, there are *series-*, *shunt-* and *compound*-excited motors, analogous to the generators. The shunt-excited windings have a fairly constant current, since their resistance is much higher than that of the armature coils. The magnetic flux is virtually constant and the motor behaves as above, the variation of motor speed with load being very small. Speed is controlled by using a rheostat in series with the field windings.

In the series-excited motor the same current flows through field and armature windings; when the motor starts-up a high torque is obtained because there is no back-e.m.f. When the motor is not loaded very little torque is required to drive it and excessively dangerous speeds may be developed before the back-e.m.f. is sufficient to exert a controlling effect. Thus, it is always used in the loaded state. The motor is very useful for the large variable loads encountered in electric traction, lifts, cranes, etc., because of the initial high torque.

The compound motor combines the characteristics of both series and shunt excited motors and is designed to give variations of speed or torque with armature currents to cope with varying loads.

11.28. A.C. Motors—Synchronous Motor

If an A.C. supply is provided, by means of slip-rings, to the coil $JKLM$ of Fig. 11.19, continued rotation is possible if the supply frequency is the same as the speed of rotation. The current then changes its direction every half-revolution and the driving torque acts in the same direction all the time. This is the basis of the A.C. *synchronous motor.* It can only operate at one speed and cannot start from rest without some device which brings the rotor speed up to synchronous speed. Synchronous motors correspond to synchronous generators.

11.29. The Electric Clock

A mains electric clock is a simple kind of synchronous motor, represented schematically in Fig. 11.20. The rotor is a small permanent magnet N–S. The stator is represented by two soft iron cored solenoids AB and CD, connected in series to the mains and wound to give induced magnetism in the same direction. The end-faces B and C attract and repel the ends of the magnet NS, giving rise to a driving couple. If NS rotates with the frequency of the mains supply, this motion is maintained. The clock is not self-starting and one common method of starting it is to give the rotor an impulse by mechanical means.

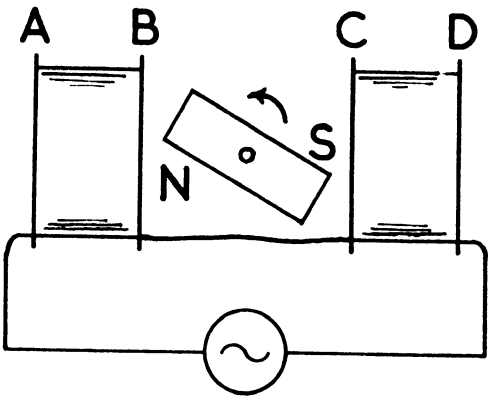


Fig. 11.20

In the clock the motor drives a gear train which eventually causes hands to rotate over a clock face. Keeping good time depends only on the mains frequency. If this increases the rotor speed increases and the clock gains and vice versa. Electric clocks use very little power (less than 0·5 watts).

11.30. Induction Motors

The principle of this motor is illustrated in Fig. 11.21. A permanent magnet NS is attached to an arm A which revolves about the same axis as the spindle PQ on which is mounted a thin copper disc C. The plane of C lies in the air-gap of the magnet. If C is initially stationary and A revolves, the magnetic field associated with NS moves round the edge of C and any radius of the disc cuts lines of force. Induced currents flow in C and the magnetic field caused by these opposes the relative motion between magnet and disc (Lenz's Law). In order to oppose the motion the disc rotates in the same direction as A. If A rotates at constant speed the disc eventually rotates at a lower speed. This demonstration really illustrates the effect of eddy currents in the copper causing rotation. This apparatus does not constitute a motor, but is a clutch, since motion is transmitted from one member to another.

The essential part of the induction motor is the rotating magnetic field, and this is achieved by using *three-phase* A.C. supply. Such supply is produced at large power stations using "*three-phase*" *alternators.*

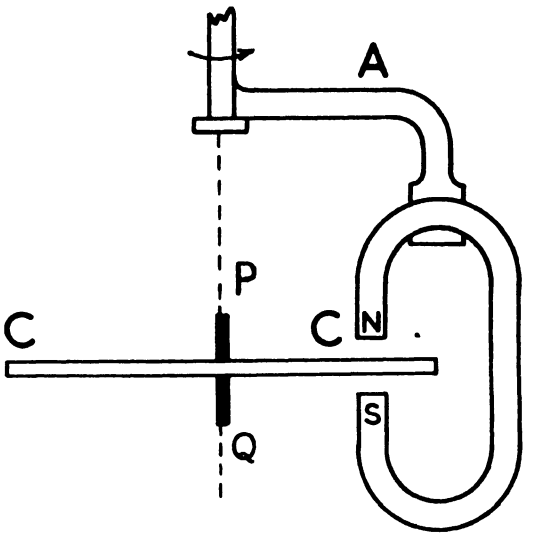


Fig. 11.21

These generators produce alternating e.m.f.'s at three separate terminals, whose values at any time relative to a fourth common terminal, are represented by: $E_1 = E_0 . \sin \omega t$. $E_2 = E_0 \sin (\omega t + 120°)$ and $E_3 = E_0 . \sin (\omega t + 240°)$. Power is transmitted as three-phase A.C. by means of wires connected to the three terminals and one to the common terminal (*neutral wire*). A single phase supply, which is most common for domestic purposes, is obtained from one of the three wires and the neutral wire.

A typical motor is shown in Fig. 11.22. Diametrically opposite pole-faces of the stator are supplied with e.m.f. of the same phase, but alternate coils have p.d.'s 120° out of phase with each other. Thus diametrically opposite poles have opposite polarity whilst the pole-faces on either side of any pole may be the same or different, depending on the time. At any given time, the resultant magnetic field is symmetrical about an axis joining two diammetrically opposite poles. As time advances this axis appears to rotate. The magnetic field is said to rotate because the pattern of the field moves with a regular angular velocity about the axis of the rotor.

A "squirrel-cage" rotor is used consisting of copper rods embedded in insulated slots on the surface of an iron armature. The ends of the copper rods are joined to copper rings. Induced currents flow in the copper rods and the end rings, caused by the relative motion between magnetic field and rotor. The magnetic interaction between the fields

of these induced currents and that of the stator causes the rotation of the rotor.

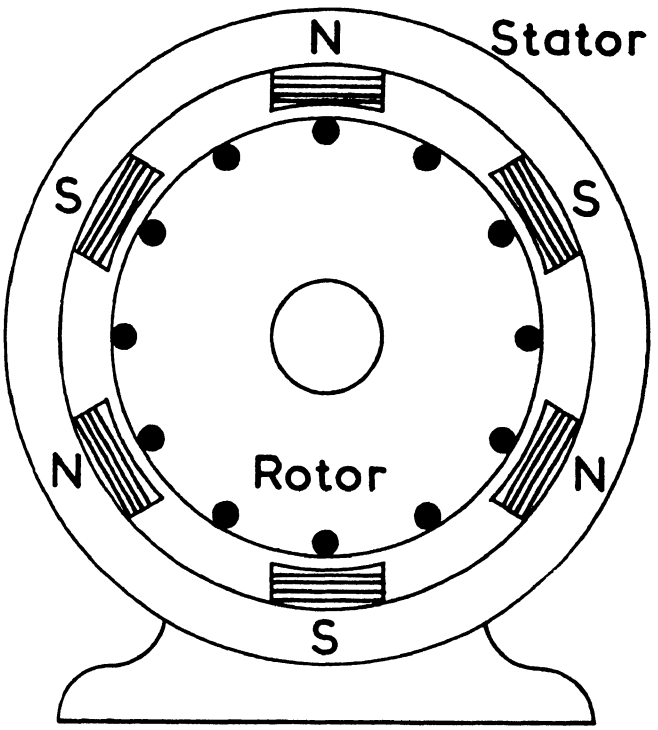


Fig. 11.22

EXERCISES

1. State the laws of electromagnetic induction and show how you would demonstrate them using a circular coil mounted to rotate about an axis at right angles to a uniform magnetic field, and connected to a galvanometer. If such a coil rotating in a uniform field of strength H, produces an e.m.f. $E = \mu_0 NA\omega H \sin\omega t$, where A is the *effective* area of the coil and ω is the angular velocity of rotation of the coil, find the maximum value of the e.m.f. produced, *in volts*, when the coil rotates at 2 revolutions per second in a field of 3×10^6 amp.m.$^{-1}$ There are 500 turns in the coil each of radius 3 cm. (London G.C.E. Adv. L.).

2. State the laws of electromagnetic induction. Hence derive an expression for the time variation of the electromotive force induced in a single turn of wire rotating about an axis in its plane, the axis being perpendicular to a uniform magnetic field. Explain the action of a simple alternating current generator.

   What modification of this generator is required to produce a direct current? Indicate by a sketch how the e.m.f. across the output terminals of a single coil would vary with time in the case of (*a*) an

A.C. and (*b*) a D.C. generator. How may a more uniform output e.m.f. be obtained in the latter case?

A rectangular coil of wire having 100 turns, of dimensions 30 cm. × 40 cm., is rotated at a constant speed of 600 r.p.m. in a magnetic field of 10^5 amp.m.$^{-1}$ the axis of rotation being in the plane of the coil and perpendicular to the field. Calculate the maximum induced e.m.f. (London G.C.E. Adv. L.).

3. State the laws of electromagnetic induction. A closed circular coil is mounted so that it can be rotated about a diametral axis. By considering the flux changes which occur when the coil is rotated steadily in the earth's magnetic field with the axis of rotation vertical show that an alternating current is produced.

Describe how an earth inductor may be used to determine the angle of dip in the laboratory. (Northern H.S.C.).

4. State the laws of electromagnetic induction.

A circular coil of N turns rotates with constant angular velocity about a diameter which is perpendicular to a uniform magnetic field of strength H amp.m.$^{-1}$ Show that an alternating electromotive force will be induced in the coil, and indicate on a diagram the position of the coil in which this e.m.f. is (*a*) a maximum, (*b*) zero. What factors determine (i) the magnitude of the maximum e.m.f., (ii) the frequency of alternation?

Describe a form of ammeter suitable for the measurement of an alternating current of mains frequency, illustrating your answer by diagrams. (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

5. State the laws of electromagnetic induction, and show how they are applied in a simple form of A.C. generator.

Explain what is meant by the *root mean square* value of an alternating current. Describe briefly the construction and mode of action of one type of A.C. ammeter. (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

6. Describe some simple device for the production of an alternating e.m.f., and give the theory of its action.

Explain without going into detailed quantitative calculation, the action of (*a*) a condenser, and (*b*) a coil wound on a soft iron core, when it is included in a circuit to which an alternating e.m.f. is applied. (Oxford Local G.C.E. Adv. L.).

7. Derive an expression for the electrical capacity of a condenser consisting of two concentric conducting spheres separated by an insulating medium of dielectric constant k, the inner sphere of radius b being insulated, and the outer sphere of radius a earthed. If $a = 50$ cm., $b = 40$ cm., and $k = 1$, how much electrostatic energy will be stored in the condenser when it is charged to a potential difference of 30,000 volts?

A 230 volt electric lamp and a 10μF condenser, joined in series, are connected to 230 volt mains. Explain, in general terms why the lamp glows steadily when the supply is alternating, but not when it is direct current. (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

8. What is meant when a coil of wire is said to have a large *self-inductance*? Would you expect the self-inductance to increase or decrease if the cross-sectional area or the number of turns in the coil were increased? Describe the effects that occur when a D.C. battery is (*a*) connected to, (*b*) disconnected from, a circuit of large self-inductance.

How can an alternating voltage be generated? If such a voltage be applied to a circuit consisting of a length of resistance wire wound in solenoidal form on a soft-iron ring, why is the current which flows less than if the resistance wire were stretched out straight? (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

9. How would you show that a change in the number of lines of magnetic force, however produced, threading through a circuit produces an induced e.m.f.?

A magnet is suspended by a thin wire so that its axis is horizontal and its centre is above the centre of a circular copper disc, mounted horizontally. Explain what happens to the magnet when the disc is rotated.

What would be the effect of replacing the disc by one of identical dimensions but made of a substance of high resistivity. (London G.C.E. Adv. L.).

10. Explain what is meant by *self-induction*.

Describe one experiment in each case to illustrate the effect of a coil having self-inductance in (*a*) a direct current circuit, (*b*) an alternating current circuit.

Describe the essential features of an instrument for measuring alternating current. (Northern H.S.C.).

11. A coil is rotated with constant angular velocity about an axis set at right angles to a uniform magnetic field. Describe and explain how the magnitude and direction of the e.m.f. induced in the coil varies with its position.

An e.m.f. $E = 200 \sin 100\pi t$ volts, where t represents time in sec, is applied to a coil of resistance 10 ohms and self-inductance 50 mH.

What is the maximum value of the current through the coil? State also the maximum value of the applied e.m.f. and its frequency.

What would be the effect on the value of the self-inductance of the coil of putting within it a bar of (*a*) iron, (*b*) copper? (Assoc. Exam. Bd. G.C.E. Adv. L.).

12. Describe and explain the action of a simple form of direct current dynamo.

In practical forms of dynamo the armature is in some cases *series wound* and in others *shunt wound*. Explain the reason for this. (Durham G.C.E. Adv. L.).

13. State the laws of electromagnetic induction and explain the essential features in the construction of a dynamo for producing direct current.

A metal bar of length 1 metre is dropped with its length horizontal and pointing E-W. Just before it hits the ground the induced potential difference between its ends is 5×10^{-4} volts. If the horizontal

component of the earth's field is 16 amp.m.$^{-1}$, from what height must the bar have dropped? (g = 9·81 m.sec.$^{-2}$). (Oxford Open Schol. Subsid. Physics).

14. Explain what is meant by *the R.M.S. value of an alternating current.*

Describe an ammeter which gives R.M.S. values when used in an A.C. circuit, explain why it gives these values and name *two* electrical characteristics desirable in such an instrument.

Unless a wire resistance is carefully wound it may have both inductance and capacitance. Discuss the effect of each of these (separately) on the reading of an ammeter placed in series with the resistance in a circuit to which an alternating voltage of fixed frequency is applied. (Northern G.C.E. Schol.).

15. Show that the heating effect of a sinusoidal alternating current of peak value I_o passing through a resistance is the same as that of a steady current $I_o/\sqrt{2}$.

A resistance of 100 ohms, a 40 microfarad condenser, and an inductance of 20 millihenries are connected in series across the 230-volt 50-cycle mains. Find the current flowing in this circuit. (Oxford Local G.C.E. Schol.).

16. What factors determine the magnitude and direction of the mechanical force acting upon a wire carrying an electric current when it is placed in a magnetic field at right angles to the lines of force? Show on a diagram the relation between the directions of the current, field, and force. Describe *two* experiments to illustrate this effect.

Give a short account of the construction and mode of action of a simple form of D.C. series motor, and explain the purpose of the starting resistance. (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

17. Explain the following phenomena:

(*a*) the development of an e.m.f. in Faraday's disc;

(*b*) the rotation of a horizontal magnet suspended centrally above a horizontal metal disc when this disc is rotated about a vertical axis through the centre of the magnet, a sheet of glass being placed between them;

(*c*) the development of heat in a sheet of metal placed with its plane perpendicular to the field of an electromagnet through which alternating current is passed. (Northern G.C.E. Schol.).

18. An alternating e.m.f., $E = E_o \sin pt$ is applied to a circuit consisting of a coil of wire of resistance R and self-inductance L.

What form will the expression for the current in the coil take? What will be (*a*) the maximum value, (*b*) the average value, and (*c*) the root mean square value of the current?

Explain the terms *impedance* and *reactance.* (London G.C.E. Schol.).

19. A rectangular coil of wire of resistance R ohms is mounted on an axis which is at right angles to a uniform magnetic field H amp.m.$^{-1}$, the plane of the coil being parallel to the magnetic lines of force. A

potential difference E volts is applied between the ends of the coil. Find an expression for the couple on the coil. Explain what must be done in order to convert the system into a simple electric motor which will rotate continuously. What will determine the current in the coil at any time? (London G.C.E. Adv. L.).

20. Describe with the aid of diagrams (*a*) a transformer and (*b*) an induction coil. Explain the action of each.

Draw diagrams to show, in a general way, how the voltage output from each of these appliances varies with the time. (Northern G.C.E. Adv. L.).

21. State the laws of electromagnetic induction.

Describe the structure and explain the action of an electrical transformer. Give an account of the energy losses in a transformer and the means by which they are reduced to a minimur . (London G.C.E. Adv. L.).

22. Describe and explain the mode of action of (*a*) an ammeter, (*b*) a voltmeter, each suitable for use with alternating current circuits.

Compare and contrast the effect of a coil, possessing resistance and self-inductance, when used in (i) a direct current, (ii) an alternating current circuit. (Northern H.S.C. Schol.).

23. Write a short essay on the fundamental physical principles employed in the production and distribution of electric power. (Northern H.S.C. Schol.).

24. Write an essay on the generation, transmission, and rectification of an alternating current. (Oxford Local G.C.E. Schol.).

25. What experiments would you carry out to verify the laws of electromagnetic induction?

Explain, by reference to the laws of electromagnetic induction why,

(*a*) the current in the primary winding of a transformer changes when the resistance of the secondary circuit is changed,

(*b*) the current taken by an electric motor increases when the mechanical load is increased. (Oxford Open Schol.).

26. Explain the action of the transformer, and explain how and why the core is laminated. Show that a generator will supply the same power to a resistance R ohms connected directly across it as it will to a resistance n^2R ohms connected across the secondary of an ideal transformer whose primary is connected to the generator, n being the secondary/primary turns ratio of the transformer.

Discuss briefly the factors affecting the transmission of electric power from power-stations to consumer by A.C. and D.C. methods. (Oxford Open Schol.).

27. Explain what is meant by the *coefficient of mutual induction* between two coils, and define the practical unit in which it is measured.

A circular coil of n turns of mean radius r m. is placed inside a

solenoid of length l m. and N turns, the axes of the coil and solenoid being parallel. Calculate the coefficient of mutual induction between the coil and the solenoid. You may assume that the length of the solenoid is large compared with its diameter and hence that the magnetic field inside it due to a current i amp. is equal to Ni/l amp.m.$^{-1}$

An alternating current transformer has a primary of N_1 turns connected to the A.C. supply, and a secondary of N_2 turns. Discuss in general terms the relation between (*a*) the primary and secondary voltages when the secondary circuit is open, (*b*) the primary and secondary currents when the secondary circuit is closed. Why does the closing of the *secondary* circuit cause an increase in the *primary* current? (Oxford & Cambridge Jt. Bd. G.C.E. Schol.).

28. L_1 and L_2 are the primary and secondary coils of a transformer. A resistance R and a condenser C are connected in series across L_2 to form a closed circuit. An alternating potential difference (with a frequency of, say, 50 cycles per second) is applied across L_1. Describe an instrument suitable for measuring the current in the secondary circuit, and indicate the type of instrument (or instruments) required to measure the potentials across each of the components L_2, R and C.

Indicate, in a qualitative manner, the effect on the secondary current when (*a*) the value of R is increased, (*b*) the frequency of the applied potential difference is increased continuously to a high value, (*c*) C is short-circuited. (Northern G.C.E. Schol.).

CHAPTER 12

CATHODE RAYS, ELECTRONS AND ELECTRON PHYSICS

12.1. Introduction—Conduction in Gases

It has been assumed that atoms contain electrons. The discovery of the electron arose from experiments carried out to discover the laws of conduction of electricity through gases. Gases can conduct only when they contain *ions*, which are positively and negatively charged atoms of the gas. The process of conduction is analogous to that in electrolysis: positive ions travel to points at lower potential and negative ions to points at higher potential. The movement of ions in opposite directions constitutes an electric current. On reaching oppositely charged conductors, ions give up their charges. In electrolysis, the neutralized ions are deposited on the electrodes and chemical effects occur, but gas ions become ordinary gas atoms once more and return to the gas.

12.2. Natural Ionization

The presence of a small number of ions in a gas at ordinary pressures must be assumed to explain the slow discharge of insulated charged conductors. Cosmic rays, consisting of high speed particles from outer space, are the major cause of this "natural" ionization. Other ionizing agents are also present at the earth's surface. These include: (i) radiations from radio-active substances, which are present in surface minerals—in particular, potassium; (ii) ultra-violet light; and (iii) flames produced by combustion.

Positive and negative gas ions are able to neutralize each other's charge when they collide and produce two neutral atoms (*recombination*). Ions are also neutralized when they give up their charge at conducting surfaces. As a consequence ions are continually being produced and destroyed by natural means. This leads to a fairly constant number of ions being available per unit volume of the atmosphere (one or two per c.c.).

12.3. Spark Discharge

The conduction of electricity through air at ordinary pressures was investigated in a controlled manner by scientists as soon as high voltages became available from batteries of cells. Faraday and others studied the phenomenon and found that when a sufficiently high p.d. was applied between electrodes a spark passed between them. The *sparking potential* for air at N.T.P. depends on the shape and distance between the electrodes. For spherical electrodes of 2 cm. diameter

with an air gap of 2 mm., the sparking potential is 8·5 kV., and for similar electrodes 2 cm. apart the potential is 52·1 kV. The variation of sparking potential with distance may thus be used for measuring high p.d.'s.

The production of a spark " discharge " on a large scale is observed as lightning. During thunderstorms the charges created on clouds produce very high potentials relative to earth and lightning follows as a succession of enormous spark discharges. Small spark discharges are often produced in friction processes which generate static electricity, e.g. combing the hair, brushing a nylon shirt, etc.

12.4. The Discharge Tube

For investigations at different pressures discharge tubes are used consisting of two electrodes sealed in a glass tube containing gas at a given pressure. With a sufficiently high p.d. between the electrodes, gas at any pressure eventually " breaks down " and conducts electricity freely, i.e. discharge occurs. The current strength is dependent only on the gas pressure and the resistance in the external circuit. The breakdown potential depends on the gas pressure, the distance between the electrodes and the gas itself.

At pressures of a few cm. of mercury the *glow discharge* occurs. It may be demonstrated with a discharge tube consisting of two aluminium electrodes, C (cathode) and A (anode), welded to wire supports passing through tight fitting rubber bungs in a glass tube which may be evacuated by means of a filter pump (Fig. 12.1). The electrodes are connected to the terminals of an induction coil capable of giving several kV. A microammeter μA is included in the external circuit. The fairly unidirectional p.d. produced by the induction coil is measured with an electrostatic voltmeter V.

With a distance of some 20 cm. between A and C and the induction coil operating no current flows at atmospheric pressure. With the pump working and the tap T adjusted to give a slow evacuation of the tube, a few wraithlike pink streamers eventually cross the tube from C to A and a small current is detected. The streamers fuse as the gas pressure is reduced further and are suddenly replaced by a mass of brilliantly glowing gas; a current of milliamps. flows whilst the glow discharge continues.

A close inspection of the tube at this stage reveals two thin dark spaces near the cathode, separated by a sheath of purplish coloured gas. The dark space nearer C is known as the *Crooke's* or *Cathode Dark Space*, whilst the one further away is called the *Faraday Dark Space*. The glowing gas between the dark spaces is known as the *Negative Glow*. The intensely glowing gas between the Faraday Dark Space and the anode is called the *Positive Column*. In Fig. 12.2, the positive column is shown as a series of separate glows (*striae*). They are produced when a steadier source of p.d. than an induction coil is used.

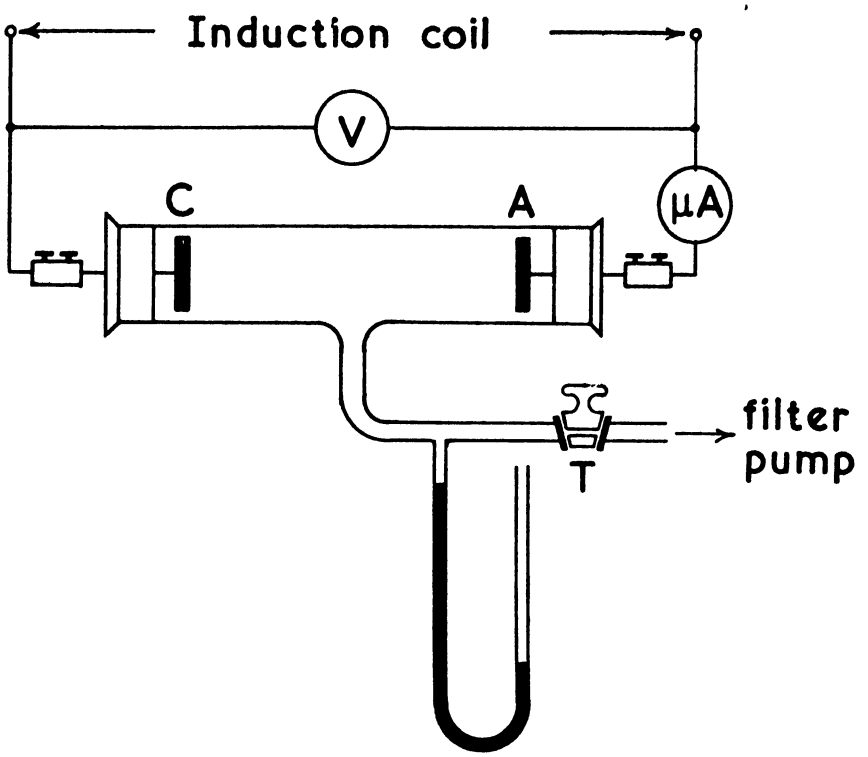


Fig. 12.1

On reducing the gas pressure further the positive column contracts, the Faraday Dark Space moves towards A and the Crooke's Dark Space expands. At a pressure of 10^{-2} mm. Hg. or less, the Faraday Dark Space and Negative Glow disappear and the Crooke's Dark Space almost fills the tube. A faint bluish-green glow is seen on the glass walls near the anode and the current strength is much reduced. Below this pressure the discharge ceases, unless a higher p.d. is made available.

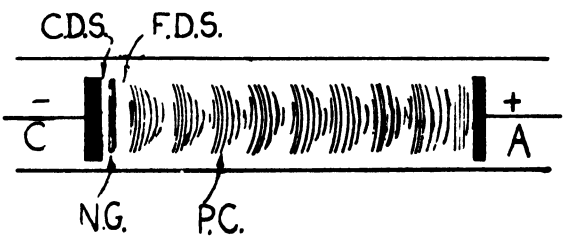


Fig. 12.2

12.5. Variation of Current with p.d.

A more refined experiment with the discharge tube consists of measuring the current strength as the p.d. between A and C is increased from zero and the gas pressure is maintained constant. With p.d.'s

of a few volts, an extremely small ($<10^{-15}$ amps.) but steadily increasing current is detected—with sufficiently sensitive meters (e.g. quadrant electrometer). At a p.d. of the order of tens of volts this current becomes constant. With increasing p.d. no variation in current strength is detected until the kilovolt region is reached. The current then begins to rise as the p.d. increases and eventually begins to rise very rapidly, culminating in a sudden breakdown of the gas (Fig. 12.3). This type of curve is characteristic of all gases. The experiment is made more easy by artificially supplying an initial concentration of ions in the gas before applying the p.d. This is achieved by passing a beam of X-rays through the gas. The steady current at low p.d.'s is then much higher and allows less sensitive meters to be used.

**Characteristic Discharge Curve—Discussion*

(i) *Region* 1: The initial current observed in region 1 of Fig. 12.3, is caused by the collection of the ions already present in the gas (natural ionization). The applied p.d. produces a fairly uniform electric field and the ions are directed to oppositely charged electrodes. As in

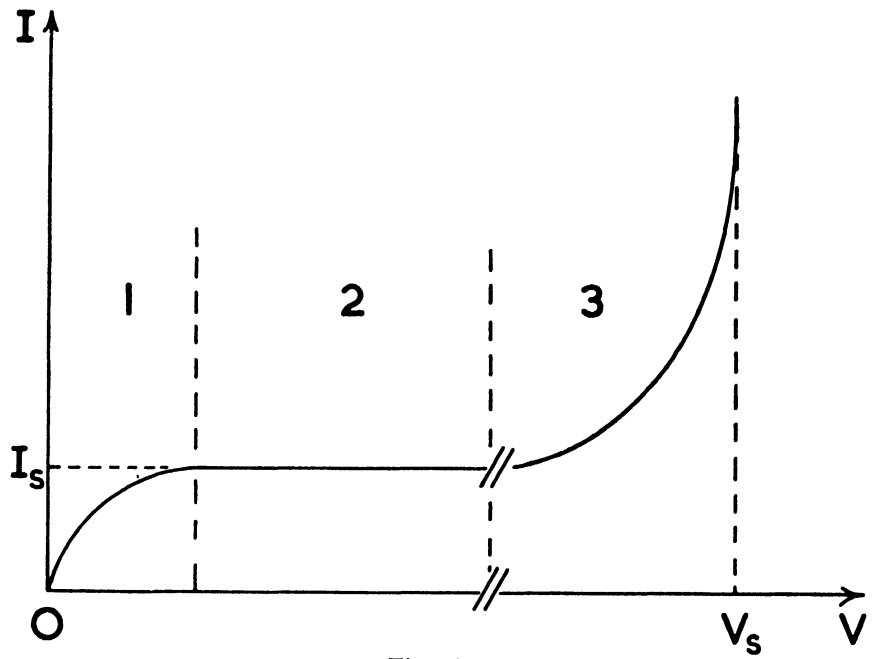


Fig. 12.3

electrolysis, the "drift" velocity of the ions is constant and proportional to the field strength E. If the p.d. between the electrodes is V volts and the separation is d m., $E = V/d$ volt.m.$^{-1}$ The drift velocity for each sign of ion is then given by:

$$v - k_{\pm} E = k_{\pm} \frac{V}{d} \text{ m.sec.}^{-1} \quad . \quad . \quad . \quad (1)$$

The constants k_+ and k_- are the *mobilities* of the ions. As in electrolysis, the *mobility of a gas ion is the drift velocity acquired in an electric field of unit strength.* (Mobilities are usually measured in *cm.*sec.$^{-1}$ per volt.*cm.*$^{-1}$).

For the low p.d.'s of region 1, recombination is effective during the time the ions are in the electric field and the number of ions per unit volume drifting towards the electrodes will be the constant number resulting from the production by natural ionization and removal by recombination. If the numbers of each sign per m.^{3} are n_+ and n_-, and the charge carried by each is " e " coulombs, the current strength, given by the quantity of charge collected by each electrode per second, is:

$$I = A(n_+ev + n_-ev) = \frac{AeV}{d}(n_+k_+ + n_-k_-) \text{ amps.} \qquad (2)$$

where A is the cross-section of the electrodes in m.^{2} It follows that: $I \propto V$, as in electrolysis, i.e. Ohm's Law is valid. The linear variation of current strength with applied p.d. typifies region 1.

(ii) *Region* 2: In this part of the curve the ion drift velocities are sufficiently high to prevent recombination to any appreciable extent and all the ions produced by natural ionization are collected per second. Thus, the current strength is equal to the total charge on all the ions of one sign created per second in the volume of gas contained in the tube. Assuming A to be the same as the cross-section of the tube, the volume of gas is Ad m.^{3} and:

$$I = An_0ed \text{ amps.} \quad . \quad . \quad . \quad . \qquad (3)$$

where n_0 is the number of ions of each sign *created* per m.^{3} per second. Thus the current strength is constant and called the *saturation ionization current* (I_s).

The saturation ionization current created by an ionizing radiation in a gas between suitable electrodes, is used as a measure of the ionizing ability of that radiation.

(iii) *Region* 3: The initial steady increase of current strength in this region was explained by Townsend, who suggested that the negative ions acquired sufficiently high velocities to ionize other gas atoms in their passage towards the anode. Townsend supposed that each negative ion (now known to be electrons) was able to ionize α atoms in each metre of its path, where α is a constant depending on the gas, the field strength E and the gas pressure p. On this assumption Townsend worked out a formula expressing the current strength in terms of the distance between the electrodes:

$$I = I_0e^{\alpha d} \qquad (4)$$

where I_0 is a constant. This formula was verified by experiment for the first part of this region but failed at higher p.d.'s. The faster

increase of I with V in the latter part of the region was explained by Townsend, who supposed that at sufficiently high field strengths the positive ions acquired sufficient energy to ionize gas atoms in their path towards the cathode. Using a second coefficient β, describing the average number of new ions created per m. of path by the positive ions, Townsend deduced a new formula:

$$I = I_0 \cdot \frac{e^{\alpha d}}{1 - \beta(e^{\alpha d} - 1)} \quad . \quad . \quad . \quad (5)$$

This formula was found to agree very well with experimental results.

From (5) it follows that $I = \infty$, when $\beta(e^{\alpha d} - 1) = 1$. This is interpreted as indicating the conditions under which a gas breaks down to produce a spark or glow discharge. For a given gas pressure and constant electrode separation, α and β are dependent on the field strength only (i.e. applied p.d.), so that the condition allows an estimation of breakdown potential. The values of breakdown potentials found experimentally agree well with theory.

12.6. Applications of Electric Discharge

(i) *Display Lighting*: " Neon signs ", etc., are long discharge tubes containing gas at low pressure. Small nickel electrodes at the ends of the tubes are connected to the secondary of a mains transformer giving a p.d. of several kV. The applied p.d. creates a glow discharge which is reversed with mains frequency. Under the conditions used, the positive column almost fills the tube and the discharge reversal is of no consequence, a brilliant glow being seen continually.

(ii) *Fluorescent Lamps*: These are discharge tubes containing mercury at low pressure and having a thin coating of a fluorescent salt, such as calcium tungstate, zinc orthosilicate, etc., on the inner wall of the tube. With mercury much of the radiation from the positive column is in the ultra-violet region of the spectrum and this creates a brilliant fluorescence on striking the tube coating. The process is one of absorption of ultra-violet and the re-emission of visible radiation by the atoms of the fluorescent salt. Ordinary mercury discharge tubes are used as sources of ultra-violet radiation.

12.7. Cathode Rays

The most striking feature of the discharge tube at low pressures—such that the Crooke's Dark Space almost fills the tube—is the greenish fluorescence of the glass wall beyond the anode. The shape of the fluorescent patch suggested to the early workers that it was caused by radiations emanating from the cathode. These radiations were called *cathode rays*. By modifying the discharge tube, as described below, the following properties can be ascribed to cathode rays:

(i) They cause fluorescence of certain salts (zinc sulphide, barium platinocyanide, calcium tungstate, etc.).

(ii) They travel in straight lines from a region near the cathode.
(iii) They are deflected by electric and magnetic fields.
(iv) They cause heating of thin metal foils.
(v) They possess a negative charge.

Demonstration Experiments

(1) *Fluorescence and Rectilinear Propagation*: A discharge tube having a flat end-face near the anode A is used (Fig. 12.4). This face has an internal coating of zinc sulphide, forming a screen S. The anode is in the form of a Maltese cross and the tube contains air at about 10^{-2} mm.Hg. A sufficiently high p.d. is applied between cathode and anode from a small induction coil and a sharp shadow of the Maltese cross is produced on a brightly fluorescing background at S. The sharpness of the shadow indicates straight line propagation of the rays from the cathode.

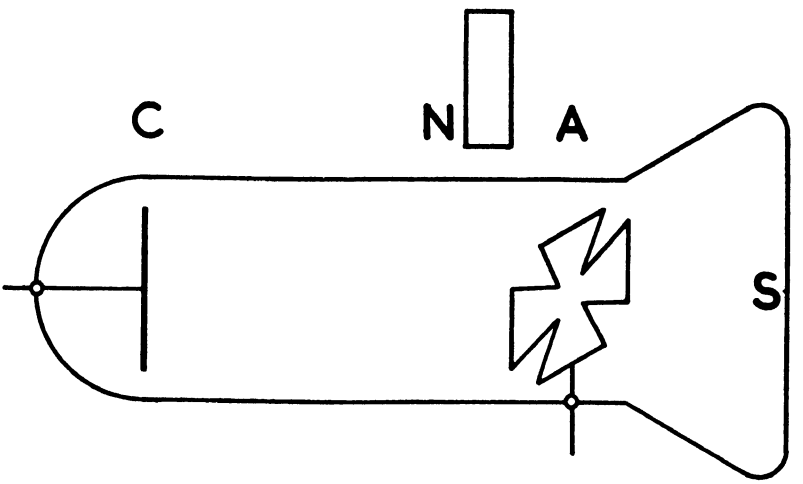


Fig. 12.4

(2) *Magnetic Deflection*: Using the same tube, one end of a bar magnet is brought near the centre of the tube. The shadow of A on S is displaced. If the magnet is held horizontally with its axis at right angles to the tube, the shadow moves upwards when the S pole of the magnet is nearer the tube and vice versa.

(3) *Heating Effect*: A discharge tube is used containing an anode of thin platinum foil and a cathode consisting of a concave disc of aluminium. After applying sufficient p.d. to initiate the discharge, a spot on the anode reaches red heat. In this experiment the rays are " focused " by the shaping of the cathode.

(4) *Negative Charge*: A discharge tube in the form of a large bulb with a small cathode (C) and pierced anode (A) is used (Fig. 12.5). The cathode rays pass through A in the form of a fine beam, missing the small metal cylinder F which is sealed into the tube off-axis, as shown.

Inside F is an insulated electrode P, which is connected externally to a positively charged electroscope. The cylinder "screens" P so that the electroscope deflection is unaffected by stray potentials. By using a magnet outside the tube, the beam of rays from A is deflected towards the mouth of F and the electroscope is discharged. This is caused by their negative charge neutralising the positive charge on P.

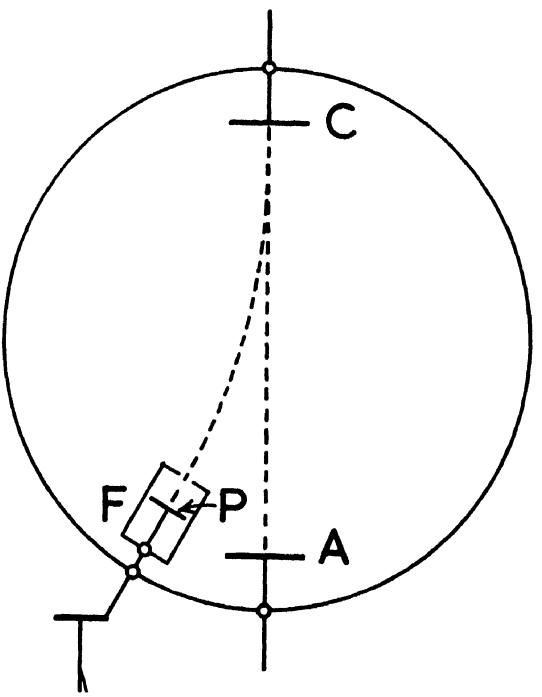


Fig. 12.5

12.8. The Nature of Cathode Rays

Before J. J. Thomson investigated cathode rays in 1897, two points of view existed regarding their nature: (i) they were small particles possessing a negative charge; (ii) they were a form of radiation like light. The correctness of the former viewpoint was proved by J. J. Thomson in 1897, by the determination of the ratio of the charge to mass (e/m) possessed by each "ray".

12.9. The Determination of e/m

The apparatus used by J. J. Thomson in one experiment is sketched in Fig. 12.6. The discharge tube T_1 contains gas at a pressure less than 10^{-2} mm.Hg. It is connected to the larger tube T_2, by a narrow neck containing the anode A. The latter consists of a cylindrical metal block containing a fine hole drilled along its axis; T_2 is evacuated to as low a pressure as possible. The flat end-face of T_2 is coated internally with zinc sulphide to form the fluorescent screen S. Near to A,

inside T_2, are two horizontal parallel plates P_1 and P_2. That part of the tube occupied by the plates is placed between the pole faces of an electro-magnet (indicated by the smaller dotted circle).

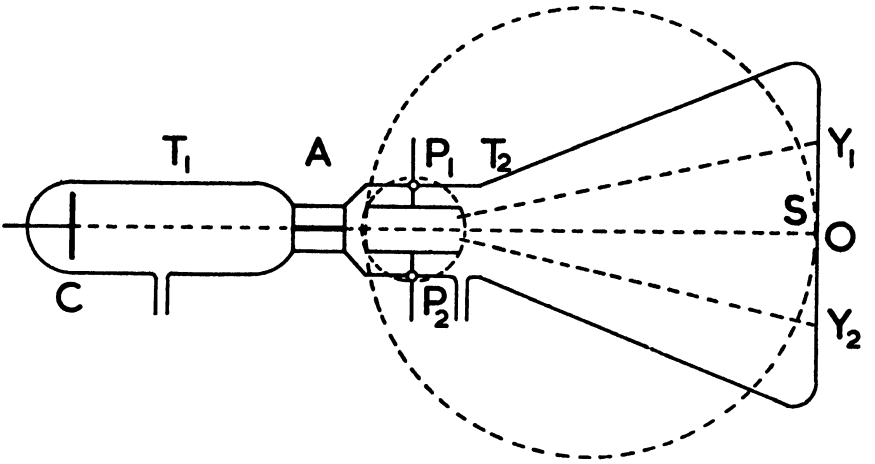


Fig. 12.6

Cathode rays are produced by a high p.d. between C and A. A narrow beam emerges from A and strikes S, producing a small fluorescent spot at O. When a p.d. of some hundreds of volts (V) is placed between P_1 and P_2, so that P_1 is positive, the spot is deflected upwards to Y_1. If this p.d. is reversed, the spot is deflected downwards to Y_2, ($OY_1 = OY_2$). When P_1 is positive the electromagnet is switched on, producing a uniform magnetic field (H amp.m.$^{-1}$) at right angles to the electric field between the plates (E volt.m.$^{-1}$). If H is directed inwards the spot at Y_1 is displaced downwards. By altering H the spot is brought back to O. Thus, under the influence of H and E at right angles, the beam is undeflected. This allows the velocity of the rays to be determined:

Theory

(i) *Electric Effect*

Assume each cathode ray is a particle of mass m kgm. possessing a charge "e" coulombs; the force acting on each particle in the electric field is:

$$F_e = Ee \text{ newtons} \quad . \quad . \quad . \quad . \quad (6)$$

Assuming the electric field is uniform, $E = V/d$, where d is the plate separation.

Whence: $$F_e = \frac{Ve}{d} \text{ newtons} \quad . \quad . \quad . \quad . \quad (7)$$

This force, at right angles to the initial direction of the rays, causes the deflection upwards.

(ii) *Magnetic Effect*

The moving charges constitute an electric current whose strength I is found by supposing that at any instant of time there are N particles per metre of the beam. Assuming each particle has a linear velocity of v m.sec.$^{-1}$:

$$I = Nev \text{ amps.} \quad . \quad . \quad . \quad . \quad (8)$$

(each particle travels v metres per second, i.e. Nv particles pass a point in the path of the beam per second).

From (27) of Chapter 5, the force acting on a small element (ds) of a current is given by: $f \,.\, ds = BI \,.\, ds$ newtons, hence, the force acting at right angles to an element ds of the beam is $BI \,.\, ds$ newtons. Since there are $N \,.\, ds$ particles in the element ds, the force acting on *each* particle is:

$$F_H = \frac{BI \,.\, ds}{N \,.\, ds} = \frac{BI}{N} \text{ newtons}$$

Substituting for I: $F_H = \dfrac{BNev}{N} = Bev$ newtons . . . (9)

The direction of this force is given by the Fleming Right-Hand Rule (p. 130), remembering that the beam of negatively charged particles constitutes a conventional current going in the opposite direction. The initial force on each particle is thus vertically downwards.

Using the above apparatus, J. J. Thomson adjusted the electro-magnet current, i.e. H, until the fluorescent spot returned to its undeflected position, whence:

$$F_e = F_H$$

i.e.

$$\frac{eV}{d} = Bev$$

hence:

$$v = \frac{V}{dB} = \frac{V}{d\mu_o H} \text{ m.sec.}^{-1} \quad . \quad . \quad . \quad (10)$$

To complete the determination of e/m the electric field is removed and the electro-magnet dispensed with. A pair of Helmholtz coils of large radius is used to give a large uniform magnetic field H' covering the whole of T_2 in Fig. 12.6, at right angles to the paper (inwards). Each particle entering T_2 is submitted to a force F'_H, which always acts at right angles to its motion. The trajectory of the particles is thus the arc of a circle, the centripetal force being F'_H. From dynamics, if the radius of the trajectory is r m., the centripetal force $= mv^2/r$, hence:

$$F'_H = \mu_o H'ev = \frac{mv^2}{r} \text{ newtons} \quad . \quad . \quad . \quad (11)$$

whence: $$\frac{e}{m} = \frac{v}{\mu_0 H' r} \text{ coulomb . kgm.}^{-1} \quad . \quad . \quad . \quad (12)$$

Using (10): $$\frac{e}{m} = \frac{V}{\mu_0^2 H H' dr} \text{ coulomb . kgm.}^{-1} \quad . \quad (13)$$

The radius r is found by using the geometry of the circle. In Fig. 12.7 the dotted line represents the circular path that the particles would

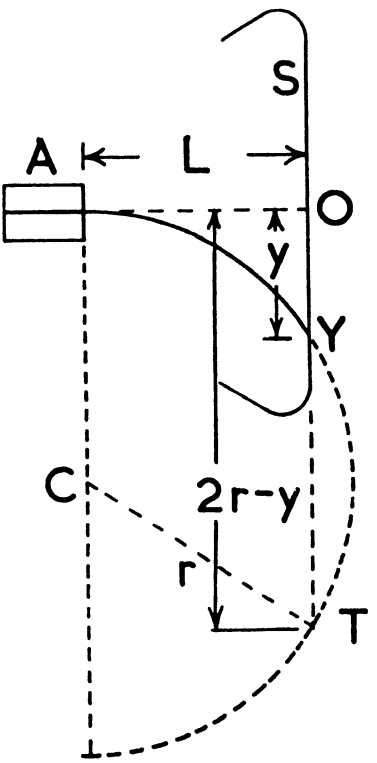


Fig. 12.7

have followed if they had not been intercepted by S. Since AO is a tangent to the circle:

$$OY \,.\, OT = AO^2$$

Further, $OT = 2r - y$, and $AO = L$, hence:

$$(2r - y)y = L^2$$

In the experiment $y \ll r$, and y^2 may be neglected by comparison with $2r$, whence:

$$r = \frac{L^2}{2y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and: $$\frac{e}{m} = \frac{2yV}{\mu_0^2 H H' d L^2} \text{ coulomb.kgm.}^{-1} \quad . \quad (15)$$

The value obtained by J. J. Thomson was $6{\cdot}2 \times 10^{10}$ coulomb.kgm.$^{-1}$ The modern accepted value is $1{\cdot}759 \times 10^{11}$ coulomb . kgm.$^{-1}$, but the early result was sufficient evidence for J. J. Thomson to assert that cathode rays were negatively charged *particles*.

12.10. Cathode Rays and Electrons—Elementary Charge

Experiments on electrolysis suggest that the charge carried by an electrolytic ion is a common charge "e" multiplied by the valency (p. 229). Assuming this idea the ratio of charge to mass for the proton (e/m_H) could be found by experiment. The value of e/m_H was found to be some thousand or so times less than e/m for cathode rays. If it be assumed that the charge carried by the proton is the same as that of cathode rays, the mass of a cathode ray must be a thousand times less than the mass of the proton (using modern values $m/m_H = \frac{1}{1836}$).

This inference was supported by the fact that cathode rays pass through thin metal foils. If their mass was similar to that of atoms this would not be possible. Apart from this, if the charge on cathode rays was not the same as that carried by monovalent ions in solution a general theory of ionization could not be developed.

J. J. Thomson determined e/m for photo-electrons emitted by metals irradiated by ultra-violet light (p. 379), and showed that it was the same as for cathode rays. Later it was discovered that the negatively charged particles emitted by hot metal wires in vacuo had the same value of e/m (p. 377). As a consequence it was assumed that particles like cathode rays were constituents of all atoms and they were generally called *electrons*.

12.11. Ionization and Discharge Processes

Electrolytic and gaseous ions are formed when one or more electrons are removed from, or acquired by, atoms (or molecules). In gas discharges ions are formed as the result of collision processes. In the process of ionization a positive ion is always formed but the formation of a negative ion is not guaranteed. To a certain extent negative ion formation is dependent on the gas atoms, since some atoms have a far greater ability to acquire electrons than others, i.e. a greater electron affinity.

Cathode rays are electrons resulting from ionization near the cathode of a discharge tube. Owing to the high field strength near the cathode the rays travel in straight lines towards the anode, suffering few collisions with neutral atoms. Some of the electrons formed near the cathode are deflected after collisions and strike the glass walls to produce the characteristic fluorescence. Some of these electrons produce additional ions after collision. The additional electrons have smaller velocities than the undeflected cathode rays, with the result that the electron stream arriving at the anode has a velocity distribution. It is for this reason that J. J. Thomson's early results for e/m were not accurate.

In modern determinations of e/m an electron beam having more uniform velocities is obtained by using a thermionic emitter.

Example

How are cathode rays produced and what are their properties? An electron starts from rest and moves freely in an electric field of intensity 2,400 *volts per metre. Determine* (a) *the force on the electron,* (b) *its acceleration, and* (c) *the kinetic energy acquired and the velocity attained if the electron moves through a potential difference of* 90 *volts. (Charge of an electron* $= 1{\cdot}6 \times 10^{-19}$ *coulombs, mass of an electron* $= 9{\cdot}13 \times 10^{-31}$ *kgm.).* (London Inter. B.Sc.).

The production and properties of cathode rays are discussed in the text. (*a*) The force acting on an electron in a field E volt.m.$^{-1}$ is:

$$F = Ee \text{ newtons}$$

Since $E = 2{,}400$ volt.m.$^{-1}$ and $e = 1{\cdot}6 \times 10^{-19}$ coulombs:

$$F = 2400 \times 1{\cdot}6 \times 10^{-19} = \underline{3{\cdot}84 \times 10^{-16} \text{ newtons}}$$

(*b*) The acceleration of the electron (f) is given using Newton's Law:

$$F = mf$$

hence:
$$f = \frac{F}{m} = \frac{3{\cdot}8 \times 10^{-16}}{9{\cdot}12 \times 10^{-31}} = \underline{4{\cdot}21 \times 10^{14} \text{ m.sec.}^{-2}}$$

(*c*) The potential energy lost by the electron = kinetic energy gained.

Potential energy $= eV = 1{\cdot}6 \times 10^{-19} \times 90 = 1{\cdot}44 \times 10^{-17}$ joules

$$\therefore \text{ kinetic energy } = \tfrac{1}{2}mv^2 = eV = 1{\cdot}44 \times 10^{-17} \text{ joules}$$

hence:
$$v = \sqrt{\frac{2eV}{m}} = \frac{2{\cdot}88 \times 10^{-17}}{9{\cdot}12 \times 10^{-31}} = \underline{5{\cdot}62 \times 10^{6} \text{ m.sec.}^{-1}}$$

Example

Describe a simple form of cathode-ray oscilloscope tube, and explain the general principles on which it works.

A beam of electrons while traversing a uniform magnetic field of 400 *amp.m.*$^{-1}$ *at right angles to its path is caused to describe an arc of a circle of radius* 10 *cm. Calculate the velocity of the electrons, and the accelerating voltage required to impart this velocity. (The ratio e/m of the charge on an electron to its mass is* $1{\cdot}76 \times 10^{11}$ *coulombs per kilogram).* (Oxford Local G.C.E. Adv. L.).

The cathode ray oscilloscope is discussed later in this chapter.

The radius of the electron trajectory is given by:

$$Bev = \mu_o Hev = \frac{mv^2}{r}$$

hence:
$$v = \frac{\mu_o Her}{m} \text{ m.sec.}^{-1}$$

Using $e/m = 1{\cdot}76 \times 10^{11}$ coulomb.kgm.$^{-1}$; $H = 400$ amp.m.$^{-1}$; $r = 0{\cdot}1$ m.; $\mu_o = 4 \times 10^{-7}$ M.K.S.

$$v = 4 \times 10^{-7} \times 400 \times 1{\cdot}76 \times 10^{11} \times 0{\cdot}1 \text{ m.sec.}^{-1}$$

i.e.
$$v = 8{\cdot}85 \times 10^{6} \text{ m.sec.}^{-1}$$

As in the previous example: $\tfrac{1}{2}mv^2 = eV$

Hence the accelerating voltage required to give the same velocity is:

$$V = \frac{\tfrac{1}{2}mv^2}{e} \text{ volts}$$

whence: $V = \frac{1}{2} \times \frac{(8{\cdot}85 \times 10^6)^2}{1{\cdot}76 \times 10^{11}}$ volts

i.e. $V = 222$ volts

Example

Describe one *experiment by which the ratio of charge to mass (e/m) for the electron has been determined.*

A narrow horizontal beam of electrons passes symmetrically between two metal plates mounted one on each side of the beam. The velocity of the electrons is 3·00 × 10^9 *cm.sec.*$^{-1}$, *the plates are* 3·00 *cm. long and* 1·00 *cm. apart. It is found that when a battery of* 568 *volts is connected to the plates the electron beam just strikes the end of one of them. Calculate the value of e/m.* (Northern G.C.E. Adv. L.).

The determination of e/m is discussed above.

The electron beam is deflected between the plates so that it follows the trajectory *SY* (Fig. 12.8). Since the plate separation is small, assume the electric field is

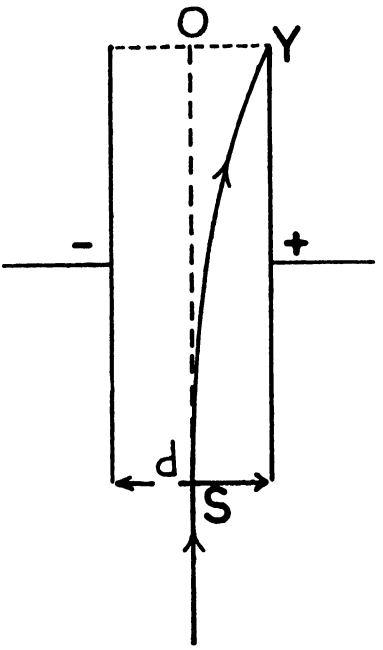


Fig. 12.8

always at right angles to the trajectory. The force acting on each electron is given by:

$$F = Ee = \frac{eV}{d} \text{ newtons}$$

The acceleration at right angles to the beam is:

$$f = \frac{F}{m} = \frac{eV}{md} \text{ m.sec.}^{-2}$$

From dynamics: $OY = ut + \frac{1}{2}ft.^2$

The initial velocity at right angles to the beam is zero ($u = O$), whilst the time taken to travel to Y is approximately the same as the time taken for the electrons to travel from S to O when E is absent, i.e. $t = (SO)/v$ secs., where v is the initial electron velocity (given). Hence:

$$OY = \tfrac{1}{2}ft.^2 = \tfrac{1}{2} \times \frac{eV}{md} \times \frac{(OS)^2}{v^2} \text{ m.}$$

and: $$\frac{e}{m} = 2 \times \frac{OY}{V} \times \frac{dv^2}{(OS)^2} \text{ coulomb.kgm.}^{-1}$$

Using: $V = 568$ volts; $OY = 5 \times 10^{-3}$m.; $d = 10^{-2}$m.; $OS = 3 \times 10^{-2}$m.; $v = 3 \times 10^{7}$m.sec.$^{-1}$

$$\frac{e}{m} = \frac{2 \times 5 \times 10^{-3} \times 10^{-2} \times 10^{14} \times 9}{568 \times 9 \times 10^{-4}} = 1{\cdot}76 \times 10^{11} \text{ coulomb.kgm.}^{-1}$$

The same result is obtained by assuming E is always at right angles to the motion, causing a circular trajectory. Then:

$$Ee = \frac{mv^2}{r} \text{ and } (OS)^2 = OY(2r - OY) = 2r \times OY \text{ (cf. p. 373)}$$

and: $$\frac{e}{m} = \frac{2v^2 \times OY}{E(OS)^2} = \frac{2dv^2 \times OY}{V(OS)^2} \text{ coulomb.kgm}^{-1}$$

The approximations are justified with the small deflections involved.

12.12. Thermionic Emission

A stream of negative electricity emitted by an incandescent carbon filament in vacuum was observed by Edison as early as 1883. In 1901 Richardson investigated the general phenomenon of the emission of electrons by heated metals in vacuo and developed a fundamental theory of this so-called *Thermionic Effect.*

Richardson's apparatus consisted of a thin wire C (cathode), stretched between two supports sealed into the evacuated glass tube (Fig. 12.9).

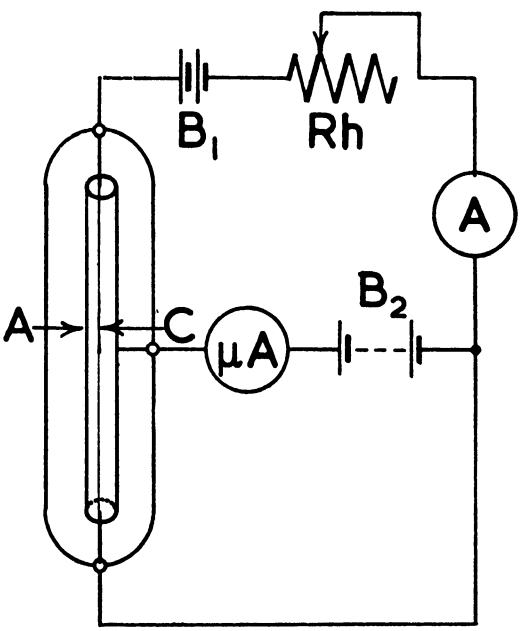


Fig. 12.9

The wire is heated electrically by battery B_1 to a temperature estimated by using C as a resistance thermometer. The metal cylinder A (anode)

is fixed so that C lies along its axis; A is maintained at a positive potential relative to C, by the battery B_2. The two electrode tube is known as a *diode*. The microammeter μA measures the electron emission current from C to A.

For a sufficiently high positive potential on A, Richardson found that the electron emission current was constant at a given temperature. This saturation current (I_s) was measured as a function of the absolute temperature (T) and found to obey the law:

$$I_s = Ae^{-B/T} \text{ (A and B constants)} \quad . \quad . \quad (16)$$

12.13. Theory of Thermionic Emission

Richardson supposed that some of the electrons in a metal behave as though they were free to wander through the interstices of the metal structure (cf. Introduction). Since electrons are not emitted from metal surfaces at low temperatures, Richardson also supposed that the " free " electrons were normally maintained inside the metal by repulsive surface forces.

On this theory an electron escapes from a metal surface when it has sufficient kinetic energy to overcome the surface forces. At low temperatures a negligible number of electrons have sufficient energy to escape. When the temperature is increased the free electrons gain kinetic energy by collision with the more energetic metal atoms and some are able to escape. The process is somewhat similar to the evaporation of molecules from a liquid surface. If no accelerating electric field is provided at the surface the electrons that escape form a " vapour " of negative charge which repels other escaping electrons and prevents further emission. Thus, a p.d. is required between emitting cathode and anode to maintain a continuous emission. On reaching the anode the electrons enter a metal again and current flows in the " anode circuit ".

Richardson's original theory gave a formula for the saturation current as a function of the absolute temperature: $I_s = A'T^{\frac{1}{2}}e^{-B/T}$, where A' is a constant. This differs from (16) by the factor $T^{\frac{1}{2}}$. More recent theory based on similar ideas gives the following formula:

$$I_s = AT^2e^{-B/T} \quad . \quad . \quad . \quad . \quad (17)$$

Since the factor T^2 changes by a comparatively small amount relative to the corresponding changes in $e^{-B/T}$, the theoretical formula (17) is equivalent to (16), i.e. the theory agrees with experiment.

12.14. Work Function of a Metal

The constant B in the above formulae is a characteristic of a metal. Theory shows that B is related to the minimum kinetic energy required

by an electron to escape from the surface ($\frac{1}{2}mv_o^2$), by the formula:

$$B = \frac{\frac{1}{2}mv_o^2}{k} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where k is a universal constant (Boltzmann's Constant), which is equal to the perfect gas constant R divided by Avogadro's Number ($k = R/N$). The minimum escape energy of the electron is usually expressed in the form: $\frac{1}{2}mv_o^2 = e\phi$, where e is the charge on the electron in coulombs, and ϕ is the p.d. in volts through which an electron must pass, starting from rest, in order to acquire the same amount of kinetic energy*. Hence:

$$B = \frac{e\phi}{k} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The constant ϕ is known as the *Work Function* of the metal; it is a measure of the work which the electron must do in overcoming the repulsive surface forces on escaping. For most metals ϕ lies between 1·81 volts for caesium and 6·27 volts for platinum.

12.15. The Photoelectric Effect

This effect consists of the emission of electrons from substances irradiated by light, and may be demonstrated by using a small insulated zinc plate connected to an electroscope and irradiated by ultra-violet light from a mercury discharge tube. If the electroscope is charged negatively the leaf falls when the lamp is switched on. If charged positively the electroscope leaf remains divergent. The photo-electrons can only be released continually when the plate is negatively charged, otherwise their loss creates a positive charge on the metal which attracts emitting electrons back.

*12.16. Experimental Investigation

The apparatus used for an investigation of this effect is sketched in Fig. 12.10. The semi-cylindrical electrode C (photo-cathode), is made of the metal whose photo-electric effect is to be investigated. The rectangular wire electrode A (anode) is maintained at a positive potential relative to C to ensure the collection of photo-electrons. Both A and C are mounted inside an evacuated glass tube which is provided with a small quartz window W. The quartz lenses L_1 and L_2, the quartz prism P and light source S, form a spectrometer transmitting ultra-violet as well as visible light. An exit slit attached to the spectrometer allows light of a given frequency to be selected from the spectrum

* The energy acquired by an electron in falling through a p.d. of 1 volt defines a unit of energy known as the *electron-volt*; this unit is used extensively in atomic and nuclear physics. (Abbrev. = e.v.).

and transmitted through W on to C. The photo-current of electrons released from C, for this frequency, is measured with a very sensitive galvanometer G.

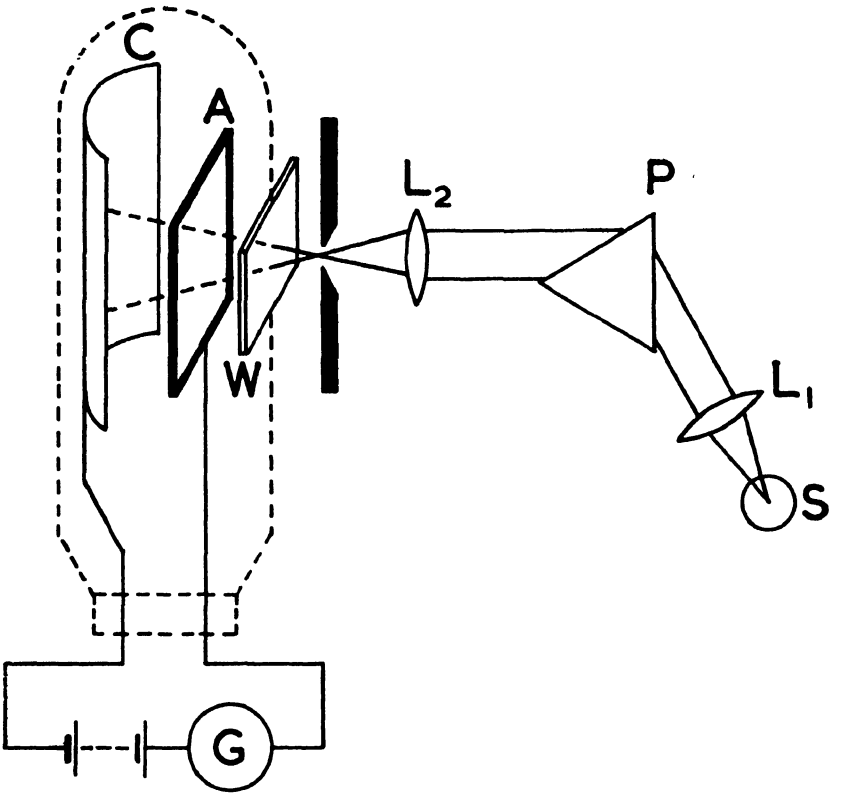


Fig. 12.10

Using a p.d. of several volts between A and C, sufficient to collect the *saturation* electron current, the latter is measured over a wide range of light frequencies and intensities. The experimental results show that: (i) the current is proportional to the light intensity for a constant frequency; (ii) there is a minimum frequency (maximum wavelength), below which it is impossible to obtain a photo-current from a given metal; (iii) the photo-current strength depends on the metal used.

The second experimental result is investigated in more detail by determining the emission energies of the electrons. This is achieved by making the anode progressively negative relative to the cathode and noting that p.d. (V volts) which just prevents a current being collected. The kinetic energy possessed by the electrons on emission ($\frac{1}{2}mv^2$) is then all used up in overcoming the adverse p.d. and they just fail to reach the anode, i.e. $\frac{1}{2}mv^2 = eV$. Thus, the " retarding " potential V is a measure of the electron emission energies. It is found that the emission energies are independent of the light intensity. Furthermore, if V is

plotted as a function of the light frequency v a straight line graph is obtained, for a given metal, which can be represented by:

$$V = av + b \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where a and b are constants.

12.17. Theory of Effect

The photoelectric effect is explained by supposing that free electrons just inside a metal surface can absorb energy from the light shining on that surface. Result (ii) and the independence of emission energies of the light intensity cannot be explained by the wave theory of light since an electron should, on this theory, sooner or later be able to absorb sufficient energy from any light wave to be emitted.

In 1905, Einstein applied the concepts of *quantum theory* to the photoelectric effect. The quantum theory was introduced by Planck in 1902 to explain the characteristic emission of radiant heat by "black" bodies. Planck assumed that atoms could only absorb or emit radiant energy in separate small quantities called *quanta*. As a consequence, a beam of radiant heat is considered as a stream of quanta of energy travelling with the velocity of light. Each *quantum* has an equivalent energy given by the formula:

$$E = hv \text{ joules} \quad . \quad . \quad . \quad . \quad (21)$$

where h is a universal constant called *Planck's Constant*, ($h = 6{\cdot}624 \times 10^{-34}$ joule.sec.), and v is the frequency of the radiation.

Since light is an electromagnetic radiation like radiant heat, it behaves as a stream of "packets of energy"; light quanta are known as *photons*. The energy of each photon is given by (21). This conception differs from the older classical ideas, which regard light as energy propagated in a continuous stream.

Einstein assumed that a photon is absorbed completely by an electron near the surface of a metal so that it gains energy hv joules. The energy thus acquired is partially used up in overcoming surface forces on escape. Since the minimum escape energy is $e\phi$ joules, the electron retains the excess energy as kinetic energy, so that its emission velocity is given by:

$$\tfrac{1}{2}mv^2 = hv - e\phi \quad . \quad . \quad . \quad . \quad (22)$$

Using the equivalent retarding potential for $\frac{1}{2}mv^2$:

$$eV = hv - e\phi \quad . \quad . \quad . \quad . \quad (23)$$

i.e. $$V = av + b$$

where $a = h/e$ and $b = -\phi$. Thus, (23) is exactly the same as (20), i.e. the theory agrees with experiment. Furthermore, from (22), if

$h\nu < e\phi$, no electrons can escape, since negative kinetic energies are impossible. Thus, there exists a minimum frequency for escape given by: $h\nu_0 = e\phi$. Since h can be determined from the gradient of the V *vs* . ν graph ($a = h/e$), and ϕ can be found from the intercept of this graph ($b = \phi$), ν_0 may be calculated. Alternatively, ϕ can be determined knowing ν_0, etc. The values of ϕ and h thus determined agree excellently with other methods of determination.

12.18. Photoelectric Cell

This consists of a photo-cathode and anode in an evacuated tube, similar to that of Fig. 12.10. The cathode is chosen so that emission is possible with the light frequencies used, e.g. if Ni is used having a work function of 4 volts (approx.), the minimum frequency that can be used (ν_0) is given by: $h\nu_0 = 4e$, i.e. $\nu_0 = 10^{17}$; this frequency corresponds to a wavelength of 3100 Å, which is in the ultra-violet region of the spectrum. Thus, nickel cathodes cannot be used for visible light and a metal with a much smaller work function like caesium ($\phi = 1{\cdot}81$ volts) must be used. In commercial cells, alkali metals such as caesium are deposited on the inner wall of the tube by evaporation in vacuo. A wide variety of composite photo-cathodes are now manufactured covering the infra-red, visible and ultra-violet regions of the spectrum. They are used for the accurate comparison of light intensities, since the photo-current is proportional to the light intensity of a given wavelength.

12.19. Photo-voltaic Cells

The main disadvantages of evacuated photo-cells are their fragility and extremely small photo-current. More sensitive and robust photo-cells are developed which depend on the *photo-voltaic effect*. In this phenomenon, electrons are released within the material of an irradiated substance and collect at its metal base interface to create an e.m.f. within the substance. Such cells are sources of e.m.f. They give rise to an appreciable current in a low resistance circuit connected across the irradiated substance. They are more correctly referred to as photo-voltaic cells.

*12.20. Determination of Electronic Charge—Millikan's Oil-Drop Experiment

The absolute measurement of the charge on the electron was made by Millikan in 1911. His apparatus is represented diagrammatically in Fig. 12.11. The circular metal plates P_1 and P_2 are maintained at a fixed distance apart by means of an insulating ring. In the upper plate P_1 are drilled several small holes which are surmounted by a cylindrical chimney C. Oil-drops produced by the atomizer A fall through the holes in P_1 and are observed in the microscope M by the light they scatter as they fall in the light beam from the lamp S. The drops are seen as points of light against a relatively dark background. An electric

field may be maintained between P_1 and P_2 by means of batteries and the switch *Sw*.

Some of the drops that pass into the chamber carry a negative charge produced by friction with the air. In the absence of a field the drops fall with various velocities. When a sufficiently high p.d. is applied some negatively charged drops are forced to travel upwards towards P_1. One of these is selected for continuous observation by removing and applying the p.d. at intervals, to allow most of the slower or faster drops to be collected by P_2.

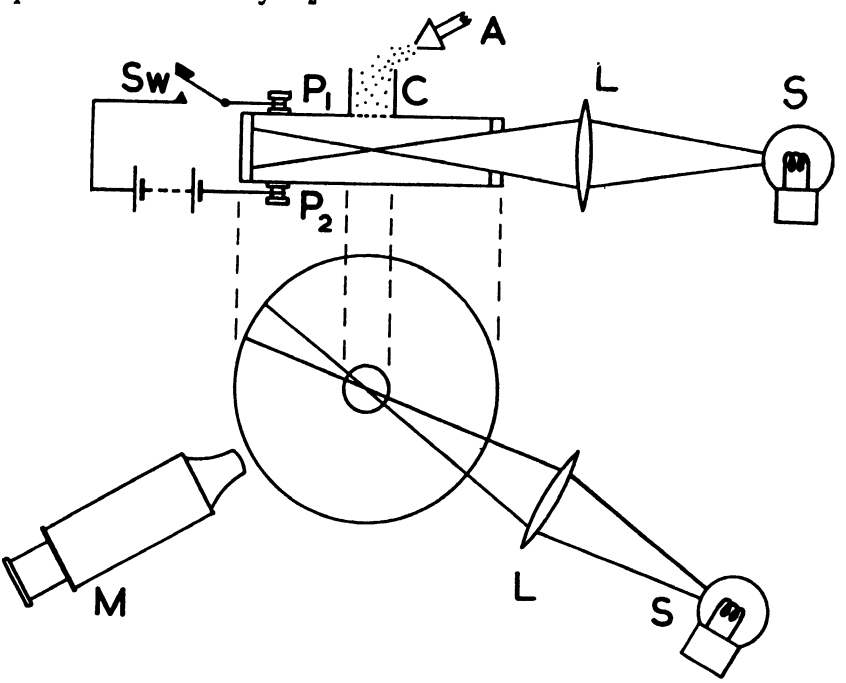


Fig. 12.11

The charge carried by the selected drop is generally greater than e; let it be e'. To determine e' the time is taken for the drop to fall under gravity between two divisions on a scale in the eye-piece of M. Knowing the magnification of M, the velocity is found, say v_1 m.sec.$^{-1}$ A sufficiently high p.d. is then applied to make the drop travel upwards towards P_1, and the upward velocity is determined, say v_2 m.sec.$^{-1}$ The field is removed, and the downward velocity redetermined. These measurements are repeated and average values of v_1 and v_2 found. Apart from these experimental observations the following are measured or taken from tables:

V = p.d. between P_1 and P_2 in volts.
d = distance between P_1 and P_2 in m.
ρ = density of oil used in kgm.m.$^{-3}$
σ = density of the air in the chamber in kgm.m.$^{-3}$
η = viscosity of air inside chamber in kgm.m.$^{-1}$sec.$^{-1}$

The electric field strength between the plates is given by: $E = V/d$ volt.m.$^{-1}$

Assuming the selected drop is spherical of radius " a " m., its weight is $w = mg = (4/3)\pi a^3 \rho g$ newtons. The force acting on the drop when it falls under gravity is its weight minus the weight of air displaced (Archimedes' Principle). Thus:

$$F_1 = \frac{4}{3}\pi a^3 g\ (\rho - \sigma) \text{ newtons} \quad . \quad . \quad . \quad (24)$$

After an initial acceleration the drop reaches a steady " terminal " velocity due to the equalization of the viscous retarding force and F_1. The terminal velocity, which is the measured velocity v_1, is related to the viscous retarding force by Stokes' formula; thus:

$$F_1 = \frac{4}{3}\pi a^3 g\ (\rho - \sigma) = 6\pi\eta a v_1 \quad . \quad . \quad . \quad (25)$$

whence: $a = \sqrt{\frac{9}{2} \cdot \frac{\eta}{(\rho - \sigma)g}} \cdot v_1^{\frac{1}{2}} = k_1 v_1^{\frac{1}{2}}$ (k_1 = constant).

Hence: $$F_1 = 6\pi\eta k_1 v_1^{\frac{1}{2}} v_1 = 6\pi\eta k_1 v_1^{3/2} \text{ newtons} \quad . \quad . \quad (26)$$

When the drop is forced upwards the forces acting are: (i) the apparent weight downwards (F_1), (ii) the opposing viscous force downwards ($6\pi\eta a v_2$) and (iii) the electric force acting upwards ($Ee' = Ve'/d$). Hence:

$$Ee' = F_1 + 6\pi\eta a v_2 = 6\pi\eta k_1 v_1^{\frac{3}{2}} + 6\pi\eta k_1 v_1^{\frac{1}{2}} v_2$$

i.e. $$\frac{Ve'}{d} = 6\pi\eta k_1 v_1^{\frac{1}{2}}(v_1 + v_2)$$

hence:

$$e' = \frac{d}{V} \cdot 6\pi\eta k_1 v_1^{\frac{1}{2}}(v_1 + v_2) = k_2(v_1 + v_2) \ (k_2 = \text{constant}) \quad (27)$$

Millikan found that the values of e' for different drops were always multiples of a common value. This result was independent of the manner in which the charged drops were produced, (e.g. in some cases X-rays were used to ionize the air near C, so that the drops could acquire charges from the ions). This was evidence for the so-called atomicity of electric charge, i.e. charge existing only in multiples of an elementary charge. The fundamental charge is assumed to be that of the electron. By taking the L.C.M. of the values of e' obtained from many observations, Millikan determined the charge on an electron, viz. $e = 1{\cdot}59 \times 10^{-19}$ coulombs. More recent determinations have shown that this value is slightly in error because of the inaccurate value of η used. The modern value for e is: $e = 1{\cdot}601 \times 10^{-19}$ coulombs.

ELEMENTARY ELECTRONICS

12.21. The Diode

The diode is a vacuum-filled tube containing a heated metal filament (cathode) and a collecting electrode (anode), cf. p. 378. The electron current between cathode and anode depends upon the temperature of the filament and the p.d. between the electrodes. If the filament temperature is constant the variation of current strength (I) with p.d. (V) gives the diode characteristic of Fig. 12.12. For low p.d.'s the

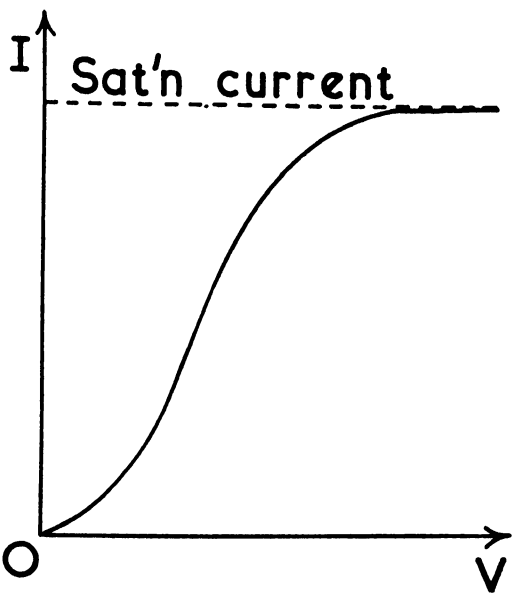


Fig. 12.12

current is less than the saturation current and varies with p.d. according to:

$$I = cV^{3/2} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

where c is a constant depending on the shape and distance apart of the electrodes. This relation is known as the " Three-Halves Law " or the Langmuir-Child's Law.

The saturation current is not quite constant in practice and I increases slowly with increasing p.d.'s. One important reason for this is the reduction in work function of the cathode as the electric field strength increases at its surface.

12.22. Diode Rectifier

(i) *Half-Wave Rectification*

The diode is able to rectify an alternating current since current can flow in one direction only through the valve, (conventionally from anode

to cathode]. A simple diode rectifying circuit is given in Fig. 12.13*a*. The diode is represented by the filament cathode C and anode A. The filament is heated electrically by a separate battery or transformer supply. The A.C. supply to be rectified is placed in series with A and C; R_L represents the load on the supply. With the circuit connected as shown, the D.C. voltmeter V across R records a steady reading.

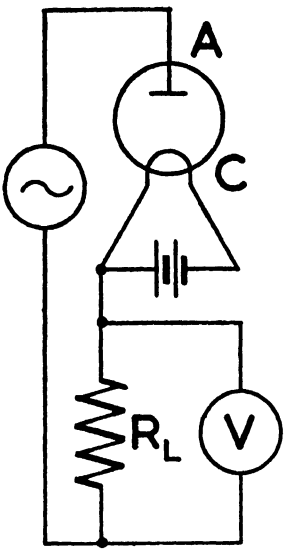


Fig. 12.13(*a*)

The current in R_L consists of a series of separate pulses which flow during alternate half-cycles of the mains supply. If the diode characteristic was perfectly linear, the pulses would consist of half-sine waves: suppose the A.C. e.m.f. is given as: $E = E_0 \sin \omega t$, and the equation of the diode characteristic is $I = kV$, where k is a constant; the p.d. across R_L at a time t, during a half-cycle when A is positive relative to C, is given by: $V = IR_L$. The p.d. across the diode at this time is hence: $V' = E - IR_L = E_0 \sin \omega t - IR_L$, whence:

$$I = kV' = kE_0 \sin \omega t - kIR_L$$

i.e.
$$I = \frac{kE_0 \sin \omega t}{1 + R_L} \quad . \quad . \quad . \quad . \quad (29)$$

i.e. the current is sinusoidal during the half-cycle. Since the diode characteristic only approximates to a straight line the current pulses are distorted half-sine waves (Fig. 12.13*b*). This method is known as *Half-wave Rectification.*

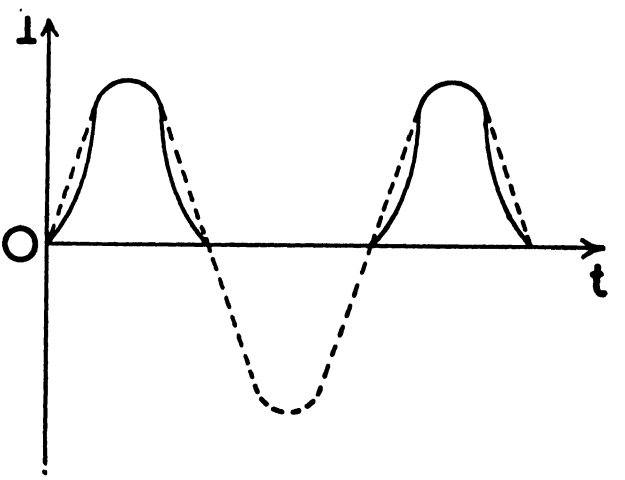


Fig. 12.13(*b*)

12.23. (ii) **Full-Wave Rectification and Steady D.C.**

In full-wave rectification a second diode is used which only operates during the half-cycles when the first diode is not passing current. This necessitates the use of a transformer whose secondary winding is tapped at the centre and connected to the load resistor as in Fig. 12.14*a*.*

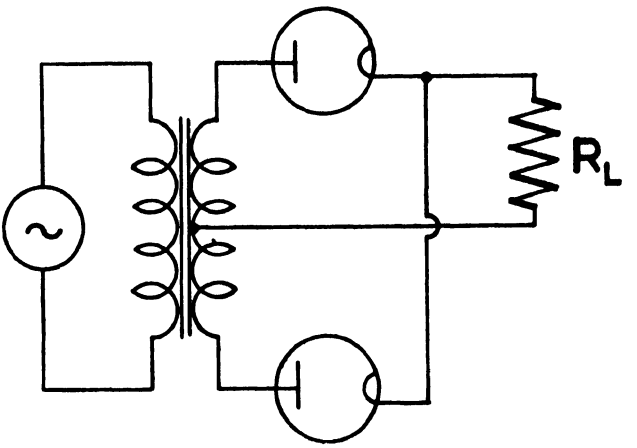


Fig. 12.14(*a*)

With this arrangement each valve conducts every alternate half-cycle. The valve current flows in the same direction in each case and current in R_L consists of pulses having mains frequency (Fig. 12.14*b*).

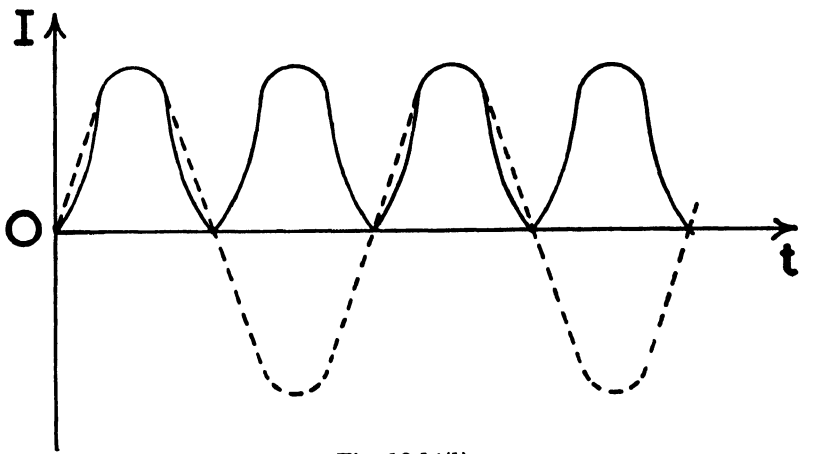


Fig. 12.14(*b*)

For steady output p.d.'s a "filter" or "smoothing" circuit is added. The simplest type of filter consists of a condenser C (several microfarads), connected in parallel with R_L (Fig. 12.15*a*). When current flows through R_L, C is charged to the maximum p.d. across R_L. Since the p.d. across the diodes is comparatively small the condenser is charged

* It is usual to omit the valve filament supplies in circuit diagrams.

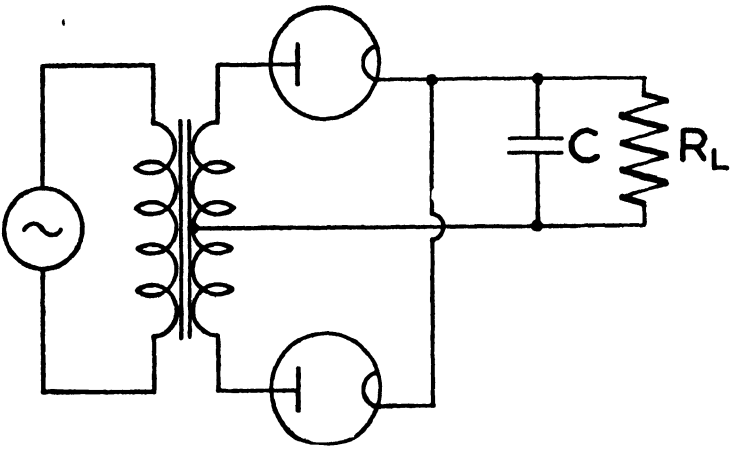


Fig. 12.15(*a*)

to a p.d. which is almost the same as the maximum e.m.f. across each half of the secondary of the transformer ($E_o/2$). When the secondary e.m.f. falls below its maximum value the diode current decreases, but the p.d. across R_L is maintained by the condenser which begins to discharge through the resistor as soon as the applied p.d. becomes less than maximum. Providing the time constant of the circuit ($T = CR_L$) is sufficiently large, the p.d. across C does not fall excessively and the voltage variation across R_L is as shown in Fig. 12.15*b*. The small variations in output p.d. (ripple voltage) are not a serious handicap in the small power applications, such as high tension supplies for X-ray tubes, cathode ray tubes, etc.

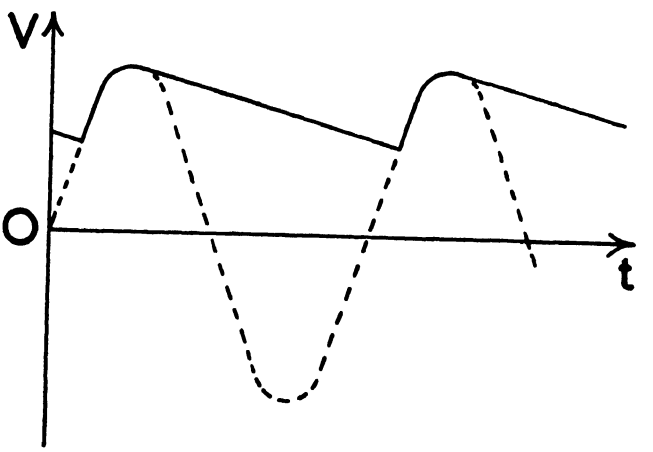


Fig. 12.15(*b*)

Example

Describe a simple form of alternating-current generator, and explain the physical principles underlying its action.

Describe also one method for the rectification of alternating current.

An alternating voltage of peak value 20 volts is applied to a full-wave rectifier, and

the ouput of the rectifier passed through a circuit of total resistance 11 *ohms, which includes a copper voltameter. It is found that* 1·37 *gm. of copper is liberated in one hour. Assuming that there is no internal voltage drop in the rectifier, find the ratio of the half-cycle average to the peak value of the voltage applied to the rectifier.*

(*The electrochemical equivalent of copper is* 0·00033 *gm. per coulomb*). (Oxford Local G.C.E. Adv. L.).

The diode or the split ring commutator are used for rectification. The A.C. generator is discussed earlier.

From Faraday's laws of electrolysis the amount of copper deposited is:

$$M = zQ$$

where z is the e.c.e. of copper and Q is the total charge. The latter is given by the average current strength multiplied by the time, i.e. $Q = \bar{I}t$. Hence:

$$M = z\bar{I}t$$

If R is the total resistance in the circuit the average half-cycle voltage is:

$$\bar{V} = \bar{I}R = \frac{MR}{zt} \text{ volts}$$

Since $M = 1{\cdot}37$ gm.; $z = 0{\cdot}00033$ gm.coulomb^{-1}; $R = 11$ ohms; $t = 3{,}600$ seconds:

$$\bar{V} = \frac{1{\cdot}37 \times 11}{0{\cdot}00033 \times 3600} = 12{\cdot}7 \text{ volts}$$

Hence, *half-cycle average voltage*: *peak voltage*: :12·7:20, i.e. 0·635

12.24 The Triode

A triode is an evacuated tube containing a thermionic filament C (cathode), an anode (A) and a third electrode, situated between cathode and anode, called the grid (G). The grid is represented as a dotted line in symbolic form (Fig. 12.16*a*). In a typical triode the cathode C is mounted vertically and the grid G, consisting of a number of turns of fine wire welded to two rigid supports, surrounds C (Fig. 12.16*b*). The anode A is a concentric cylinder surrounding G and C. All three electrodes are mounted on metal pins sealed into the tube.

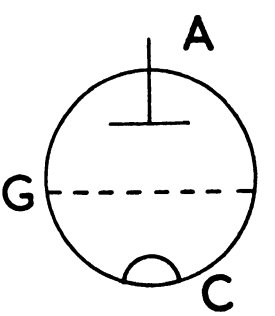


Fig. 12.16(*a*)

12.25. Triode Characteristics

With a steady cathode temperature and A at a high positive potential relative to C (100–200 volts), a steady electron current flows between C and A, whose strength depends on the potential of the grid relative to C (V_g). The latter is generally negative and the adverse electric field created by G reduces the electron current which would otherwise flow. When V_g is sufficiently negative (several volts), the electron current is reduced to zero and this value of V_g is known as the *cut-off voltage*;

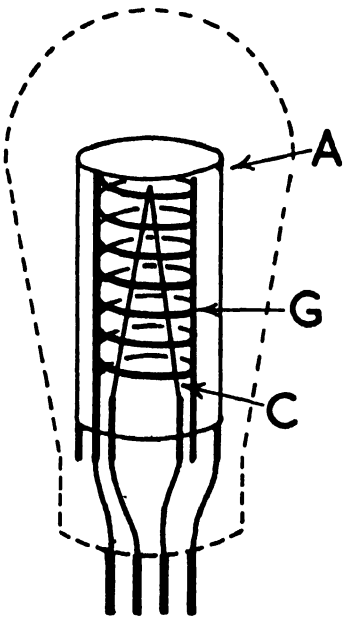


Fig. 12.16(*b*)

this depends on V_a. The variation of anode current (I_a) with V_g, when V_a remains constant, gives a characteristic which is important to the applications of the triode (Fig. 12.18).

12.26. Experimental Determination of Triode Characteristic

The apparatus required is shown in Fig. 12.17. The filament heater voltage is provided by accumulators; the grid voltage is variable and

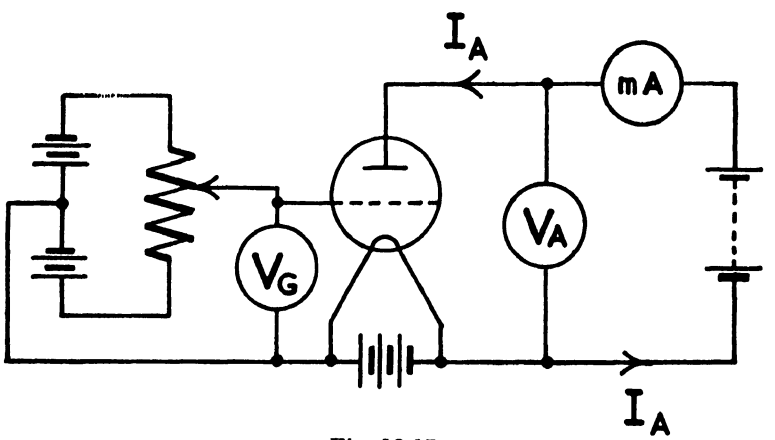


Fig. 12.17

supplied by dry batteries, whilst the anode voltage is supplied by high tension batteries. Voltages and current strengths are measured with the meters indicated.

The grid voltage V_g is first made fully negative (— 12 volts say), and V_a is maintained constant (150 volts say); no current flows through mA, i.e. $I_a = 0$. Readings of I_a are then taken as V_g is increased by one volt increments to a maximum value of 6 volts, say. This is repeated for other values of V_a, and the family of curves plotted as in Fig. 12.18. These triode characteristics are all similar in shape but have different cut-off values A_1, A_2, etc. The linear part of each curve is parallel (B_1C_1, B_2C_2, etc.).

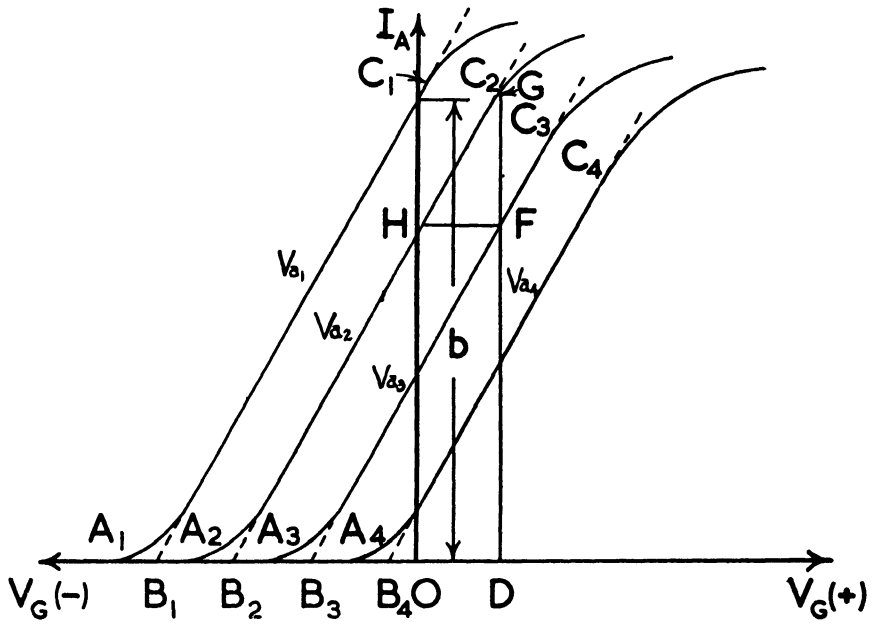


Fig. 12.18

The equation relating I_a and V_g in the linear region of a characteristic is of the form:

$$I_a = aV_g + b. \quad . \quad . \quad . \quad . \quad (30)$$

where " a " is the gradient of the line and b the intercept on the I_a axis. Since " a " is constant the increase in b for each curve is proportional to the increase in anode voltage. Hence, b is related to V_a by the equation:

$$b = cV_a + d$$

where c and d are constants. Hence, for any characteristic:

$$I_a = aV_g + cV_a + d \quad . \quad . \quad . \quad . \quad (31)$$

12.27. The Triode as an Amplifier

The triode is used for the amplification of small alternating voltages, using the circuit of Fig. 12.19. The anode voltage is maintained by batteries and the grid voltage kept at a small negative potential by

means of a small dry battery. The resistance R (anode load) is carefully chosen as indicated later. The A.C. to be amplified (input) is connected between grid and cathode and the amplified A.C. (output) is produced across R.

The input causes periodic variations in V_g and hence I_a. The variations in I_a produce corresponding variations in the p.d. across R. The ratio of the output amplitude across R to the input amplitude between grid and cathode is called the gain of the valve.

**Theory*: When V_g changes by a small amount δV_g, corresponding changes in I_a and V_a occur, viz. δI_a and δV_a. From (31) it follows that:

$$\delta I_a = a\delta V_g + c\delta V_a \quad . \quad . \quad . \quad . \quad (32)$$

If V_a is maintained constant whilst V_g varies:

$$\delta I_a = a\delta V_g \quad (\delta V_a = 0)$$

i.e.
$$a = \left(\frac{\delta I_a}{\delta V_g}\right)_{\delta Va=0} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

This quantity is called the *mutual conductance* of the valve and represented by g_m; the units are ohm^{-1} or mho. It is determined from the characteristic curves, e.g. in Fig. 12.18, $g_m = GF/HF$.

If V_g is maintained constant whilst V_a varies:

$$\delta I_a = c\delta V_a \quad (\delta V_g = 0)$$

hence:
$$c = \left(\frac{\delta I_a}{\delta V_a}\right)_{\delta Vg=0} \quad . \quad . \quad . \quad . \quad (34)$$

The quantity $1/c = \rho$ is called the *anode impedance* or internal resistance of the valve; the unit is ohms. In Fig. 12.18, $c = (GF)/(V_{a3} - V_{a2})$. It follows that (32) can be written in the form:

$$\delta I_a = g_m\, \delta V_g + \frac{1}{\rho}\, \delta V_a \quad . \quad . \quad . \quad (35)$$

If, therefore, I_a is maintained constant and V_g is varied:

$$0 = g_m\delta V_g + \frac{1}{\rho}\delta V_a$$

i.e.
$$\left(\frac{\delta V_a}{\delta V_g}\right)_{\delta Ia=0} = -\, g_m\rho = \mu \quad . \quad . \quad (36)$$

The quantity μ is known as the *amplification factor* of the valve. In Fig. 12.18, $\mu = (V_{a3} - V_{a2})/HF$.

When the triode is used as an amplifier δV_g represents the amplitude of the A.C. input, whilst δI_a and δV_a are the amplitudes of the corresponding A.C. changes produced in I_a and V_a. If E is the e.m.f. of the battery supplying the anode voltage: $V_a = E - I_a R$, where I_a is the total current strength in R. It follows that:

$$\delta V_a = -R \,.\, \delta I_a = \text{amplitude of output p.d. across } R \qquad (37)$$

The gain of the valve is defined by:

$$P = \frac{\delta V_a}{\delta V_g}$$

Since: $$\delta V_a = -R\delta I_a = -Rg_m\delta V_g - \frac{R}{\rho}\delta V_a \quad \text{[from (35)]}$$

$$P = \frac{-g_m R}{1 + \frac{R}{\rho}} = \frac{-\rho g_m R}{\rho + R} = \frac{\mu R}{R + \rho} \quad . \quad . \quad (38)$$

If $R \gg \rho$, $P = \mu =$ maximum gain possible. It is not practicable to make $R \gg \rho$, otherwise I_a would be reduced excessively, so that in practice R is made four or five times larger than ρ.

The total p.d. across R consists of the amplified A.C. p.d. superimposed on the steady D.C. p.d. The alternating output p.d. is isolated and used, either for further amplification or for a particular purpose,

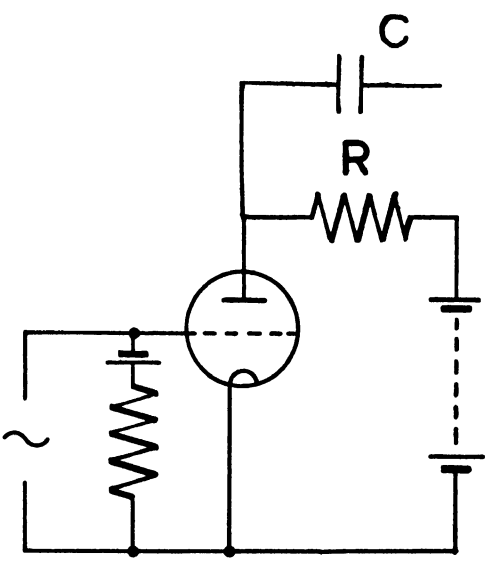


Fig. 12.19

by connecting a *coupling* condenser C, as in Fig. 12.19; C acts as a low impedance to A.C. but effectively blocks any D.C. For additional amplification the isolated A.C. voltage is passed to a second triode,

as shown in Fig. 12.20. The final amplified voltage is given by P_1P_2v, where P_1 and P_2 are the gains of the two stages of the amplifier and v is the initial voltage.

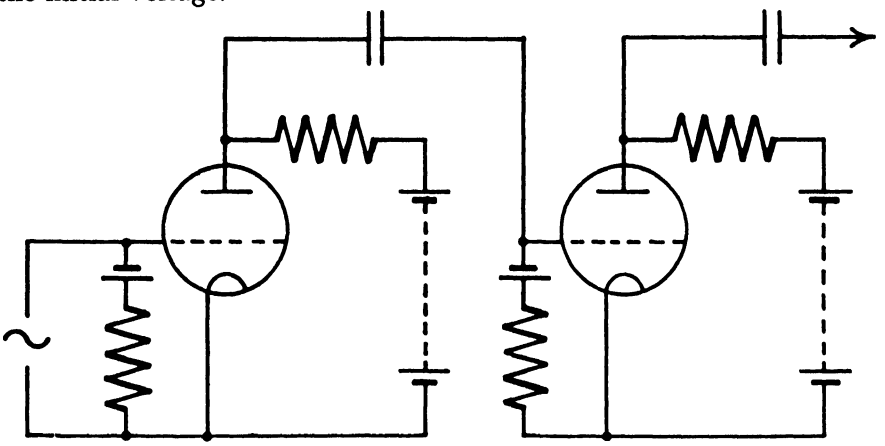


Fig. 12.20

Example

A triode with an amplification factor of 10 *and an internal resistance of* 10,000 *ohms is connected in series with an anode resistance of* 50,000 *ohms and a small voltage v is applied between grid and cathode. What change in voltage takes place across the anode resistance?* (Cambridge Open Schol.).

In this example the anode impedance is called the internal resistance and the load resistance (R above) is called the anode resistance.

Since $\mu = 10$, $\rho = 10{,}000$ ohms, the gain, given by (38) is:

$$P = \frac{\mu R}{R + \rho} = \frac{10 \times 50{,}000}{50{,}000 + 10{,}000} = 8{\cdot}33$$

Hence, the change in voltage across the anode resistance is: $8{\cdot}33v$.

12.28. The Triode Oscillator Circuit

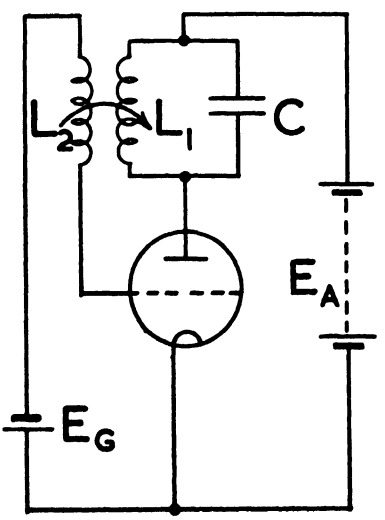


Fig. 12.21

The triode is used for the continuous production of electromagnetic oscillations. A simple oscillator circuit is given in Fig. 12.21. The steady anode potential is supplied by the battery E_A, whilst the steady grid potential is supplied by the battery E_G, so that if the inductances L_1 and L_2 and the condenser C were not present, a steady electron current would flow in the anode circuit. The steady operating conditions correspond to a point on the linear part of the triode characteristic.

The sudden introduction of a p.d. across L_1 or C produces an oscillatory current in the L—C circuit, which would normally decay rapidly, owing

to the presence of resistance (p. 291). The function of the valve is to maintain the oscillations. This is achieved by a transfer of energy during each oscillation by the mutual inductance formed by L_1 and L_2. The energy transferred to the L—C circuit is just sufficient to replace that dissipated in the resistance of this circuit. As a result, the anode potential varies sinusoidally with a frequency which is almost the same as the natural frequency of the L—C circuit, viz. $f = 1/(2\pi\sqrt{L_1 C})$.

12.29. Wireless Waves

Energy is radiated from an oscillatory circuit as electromagnetic waves (p. 291). This energy is derived from the power supply of the circuit, which, in the triode oscillator circuit, is the high tension battery. It can be shown that the rate at which electromagnetic energy is radiated increases with the frequency of the oscillations. The invention of the triode oscillator meant that waves of short wavelength, and hence high frequency (wireless waves), could be generated continuously.

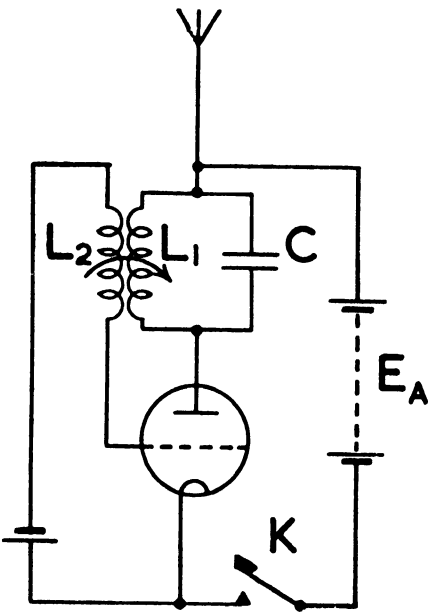


Fig. 12.22

The circuit of Fig. 12.21 is not an efficient transmitter of wireless waves because of the geometrical nature of the components, e.g. the magnetic field of the inductance coil is mainly contained within the region of the coil itself, so that variations in field strength at distant points are very small. For maximum efficiency of propagation an *aerial*, consisting of a long wire is connected to the oscillatory circuit (Fig. 12.22). The variation of p.d. across L_1 or C causes the p.d. along the aerial to vary with the same frequency. Thus, an electron current flows in the aerial, creating an associated electric and magnetic field in space. The fields associated with the aerial are much stronger at distant points than those associated with L_1 or C, and this results in more energy being propagated as an electromagnetic radiation, i.e. a wireless wave.

12.30. The Cathode Ray Tube

A typical cathode ray tube is sketched in Fig. 12.23. The electrically heated filament F is a thermionic source of electrons. This is contained

in an evacuated tube having a flat end-face coated internally with a fluorescent material to form a screen S. Immediately in front of F is an electrode having a small hole in it to allow a limited electron beam to pass through. This is called the grid (G). The potential of G relative to F is negative; this may be varied to control the strength of the electron beam passing through. The two cylindrical electrodes A_1 and A_2, (first and second anode), are maintained at relatively high positive potentials relative to F. Their function is to provide an electrostatic field which focuses the electron beam on to S. The filament and its associated electrodes form an " electron gun " for producing a focused beam of electrons at S. The beam gives rise to a small spot of light on S and by varying the potential of G the intensity of this may be altered.

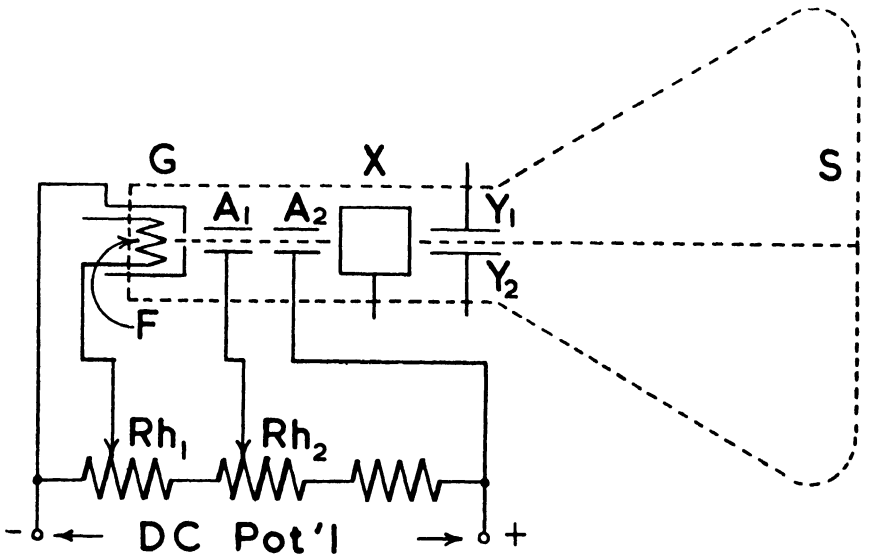


Fig. 12.23

The vertical parallel plane electrodes X_1 and X_2 are known as the X-deflection plates. When a p.d. is placed across them the electron beam is deflected horizontally and the spot on S moves accordingly. Likewise, the horizontal parallel electrodes Y_1 and Y_2 are known as the Y-deflection plates. When the electron gun is maintained with constant potentials the deflections of the spot on S in the horizontal and vertical directions are directly proportional to the p.d.'s applied across X_1X_2 and Y_1Y_2, respectively.

12.31. The Cathode Ray Oscilloscope (C.R.O.)

This is a device which uses a cathode ray tube (C.R.T.), to display the variation with time of a periodic voltage, i.e. a " waveform ". For this purpose the C.R.T. is provided with a *time-base*, i.e. a circuit which causes the electron beam to be deflected at a uniform velocity horizont-

ally across the screen and then returned very quickly to the origin, the process being repeated with a given frequency. The time-base applies a periodic *saw-tooth* voltage V, between the X-plates (Fig. 12.24). Since the deflection on S is proportional to V, the spot moves across S uniformly as the voltage increases during a period such as OB. At the end of each period, when the p.d. is suddenly reduced to zero, the spot is rapidly returned to its origin.

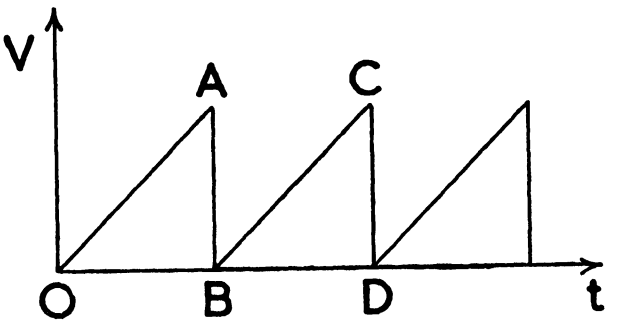


Fig. 12.24

An approximate saw-tooth waveform is obtained by charging a condenser through a high resistance and then discharging it through a low resistance, periodically. The p.d. across the condenser increases exponentially ($V=E(1-e^{-t/RC})$—p. 248), but the initial part of the curve is approximately linear. To effect the periodic charge and discharge of the condenser, a neon lamp (N) is used as in Fig. 12.25. The battery B charges the condenser C via the high resistance R. When the voltage across C reaches a particular value, known as the striking potential of the lamp (V_S), the gas in the lamp breaks down and a glow discharge is created between its two electrodes. This causes C to discharge, since N is equivalent to a low resistance when it is conducting. The discharge proceeds until the maintenance p.d. of the lamp (V_M) is reached when the lamp is extinguished and it becomes an infinite resistance. The condenser begins to charge via R once more and the discharge is repeated when V_S is reached again.

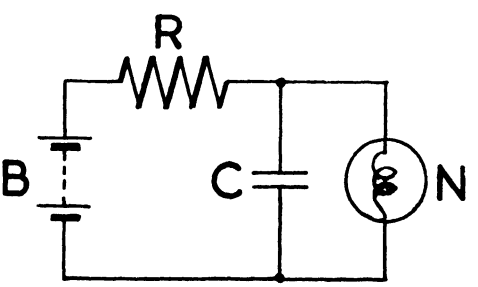


Fig. 12.25

The period of the approximate saw-tooth variation of p.d. across C is governed by the values of R and C. Assuming the time of the lamp

discharge to be negligible, the period T is the time taken for the p.d. across C, i.e. V_C, to increase from V_M to V_S. This is obtained by solving the simultaneous equations:

$$V_S = E(1 - e^{-t_s/RC}) \text{ and } V_M = E(1 - e^{-t_m/RC}) \text{ (Fig. 12.26)}$$

whence:

$$T = t_m - t_s = RC\left\{\log_e \frac{E}{E - V_M} - \log_e \frac{E}{E - V_S}\right\} \text{ (E = battery e.m.f.)} \quad (39)$$

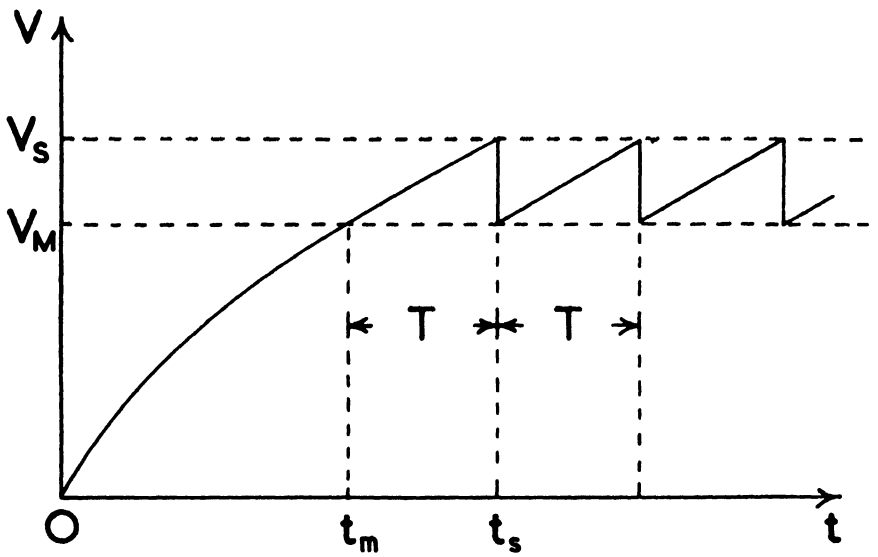


Fig. 12.26

12.32. Examination of Waveforms

Oscilloscopes are provided with time-bases of variable frequency, from a few cycles to many kilocycles per second. If the time-base frequency across the X-plates is adjusted so that it is the same as that of a sinusoidal p.d. placed across the Y-plates, the waveform of the latter is displayed on S. As the electron beam moves horizontally under the influence of the time-base p.d. it is deflected vertically by the applied p.d. across the Y-plates. The vertical deflection is proportional to the alternating applied p.d. so that a sinusoidal pattern is traced on the screen by the electron spot. The pattern appears to be continuous because of persistence of vision and afterglow of the fluorescent material. For a stationary pattern the frequency of the applied p.d. must be the same or an integral number times the time-base frequency.

Example

Explain what is meant by electromagnetic induction and describe two *experiments from which the quantitative law of induction can be deduced.*

A search coil of 10^3 *cm.*^{2} *area turns is placed in a field of* 10^6 *amp.m.*$^{-1}$ *and rotated* 50 *times per second about an axis perpendicular to the field; its ends are connected to the Y-plates of a cathode ray oscillograph of sensitivity* 0·1 *cm. per volt, whose X-plates are connected to a circuit which sweeps the electron beam at uniform speed from left to right across the screen* 50 *times per second. Describe exactly what you would expect to see on the screen. If the oscillograph were connected to the secondary of a mutual inductance of* 10 *millihenry instead of to the search coil, what current would have to be passed in the primary to produce exactly the same effect?* (Cambridge Open Schol.).

Electromagnetic induction is discussed in Chapter 9.

The induced alternating e.m.f. generated by the rotating coil causes the electron beam to be deflected up and down 50 times per second so that, as indicated above, one complete sine wave will be seen on the oscillograph screen. This will appear to be stationary.

The induced e.m.f. is given by:

$$E = E_0 \sin \omega t, \text{ where } E_0 = \mu_0 \omega HAN \text{ (p. 325).}$$

Since $f = 50$, $\omega = 100\pi$ radians.sec.$^{-1}$; $H = 10^6$ amp.m.$^{-1}$; the area-turns $= AN = 10^3$ cm.^{2}. turns $= 0{\cdot}1$ m.^{2}. turns; $\mu_0 = 4\pi \times 10^{-7}$ M.K.S.:

$$E_0 = 4\pi \times 10^{-7} \times 100\pi \times 10^6 \times 0{\cdot}1 = 4\pi^2 = 39{\cdot}5 \text{ volts.}$$

The Y-sensitivity of the C.R.O. is 0·1 cm. per volt, meaning that the spot on the screen is deflected by 0·1 cm. per applied volt across the Y-plates. Hence the maximum deflection of the beam (at the peak of the sine curve) is:

$$y = 0{\cdot}1 \times 39{\cdot}5 = 3{\cdot}95 \text{ cm.}$$

To produce the same effect with the inductance, an A.C. current of similar frequency is passed through the primary, having a strength such that the induced e.m.f. across the secondary has the same amplitude as E_0. The induced e.m.f. across the secondary is given by:

$$E = M \cdot \frac{dI}{dt}$$

Assuming the primary current to be given by: $I = I_0 \sin \omega t$:

$$E = \omega M I_0 \cos \omega t \text{ volts}$$

whose amplitude is: $E_0 = \omega M I_0$ volts

whence: $I_0 = \dfrac{E_0}{\omega M}$ amps.

Since $\omega = 100\pi$ radians.sec.$^{-1}$; $M = 0{\cdot}01$ henries:

$$I_0 = \frac{4\pi^2}{100\pi \times 0{\cdot}01} = 4\pi = 12{\cdot}6 \text{ amps.}$$

The r.m.s. value of the current strength is: $I_{rms} = \dfrac{12{\cdot}6}{\sqrt{2}} = 8{\cdot}89$ amps.

12.33. Radar

The word *radar* is an abbreviation of the term Radio Direction Finding and Range. The term is used to describe the method of determining the bearings of ships and aircraft by means of electromagnetic

waves of very high frequency (V.H.F.). Such waves are reflected by the ships, aircraft, etc., and received back at the radar transmitting station, the time taken between emission and reception giving a measure of the distance of the object from the transmitter. Instead of using a continuous source of waves, trains of V.H.F. waves are used, each train (pulse) consisting of many complete oscillations. The pulses are transmitted with a definite frequency called the pulse repetition frequency, the value of which is much less than that of the electromagnetic wave.

Electromagnetic waves are reflected by all solid objects to a varying degree. The reflected pulse is amplified on return to the transmitting station and eventually displayed on an oscilloscope. The individual waves in a pulse cannot be distinguished on the C.R.O. because of their high frequency but the envelope of the wave train appears as a rectangular pulse. Providing the time-base of the C.R.O. is sufficiently fast, the initial outgoing and reflected pulses are received during a single period and both are displayed on the screen. They appear to be stationary if the reflecting object is stationary since they are being received many times per second and persistence of vision gives rise to a continuous picture.

Thus, the appearance of a reflected pulse on an oscilloscope screen is sufficient to detect an approaching aircraft, the amplitude of the pulse increasing with the nearness of approach. The distance on the screen between the two images is interpreted as a distance between transmitter and reflecting object, since the time taken for a complete horizontal deflection of the electron beam across the screen is known (time-base period), and during this time the pulse of radiation has travelled twice the distance between transmitter and reflector. The oscilloscope is an essential feature of radar.

For radar transmission it is thus necessary to devise an oscillator which can produce pulses of radiation, sufficiently intense to travel long distances and be detected after reflection. In the early days of the recent war the radar wavelength was a few metres and the oscillators incorporated a special triode operating at very high anode voltages and giving a large power output. The triode was allowed to pass current for a few microseconds only (i.e. the time of the pulse), a trigger circuit being used to initiate and stop the conduction through the valve. By this means the wave train was given a rectangular envelope.

With the triode oscillators it was found that enemy aircraft and surfaced submarines could be detected by their reflected pulses at distances of 20 to 30 miles from a transmitter. However, the short wave receivers used by the enemy detected the transmitted pulses and used them as a warning. To remove this possibility transmitters using shorter wavelengths (centimetric waves) were devised. It is impossible to use a triode for the generation of centimetric waves because the period

of the oscillations is appreciable compared with the time taken for the electrons to travel from cathode to anode. Instead, special types of valve such as the *magnetron* were developed.

Special means have to be used for directing centimetric waves from the oscillator to the transmitter aerial, and for this purpose " wave-guides " are employed. They consist of hollow metal tubes. Electro-magnetic energy is directed through the tubes in a manner somewhat analogous to the transmission of sound through hollow pipes.

*Television

The essential requirements for television are: (i) a camera which views the scene to be televised and converts the light from each element of the scene into electric signals, (ii) a radio transmitter which can propagate the signals over long distances, and (iii) a receiver for reconverting the electric signals into light signals.

One type of camera is called the *Image Iconoscope.* Light from the scene being televised is focused on to a transparent photo-cathode (A) (Fig. 12.27), consisting of a thin film of caesium and antimony on a thin glass disc. The photoelectrons emitted from each point on A are accelerated to the thin transparent plate B on which is deposited a film of caesium. For this purpose a suitable p.d. exists between A and B. The electrons are magnetically focused on B by the field created in the coil CC by a steady current.

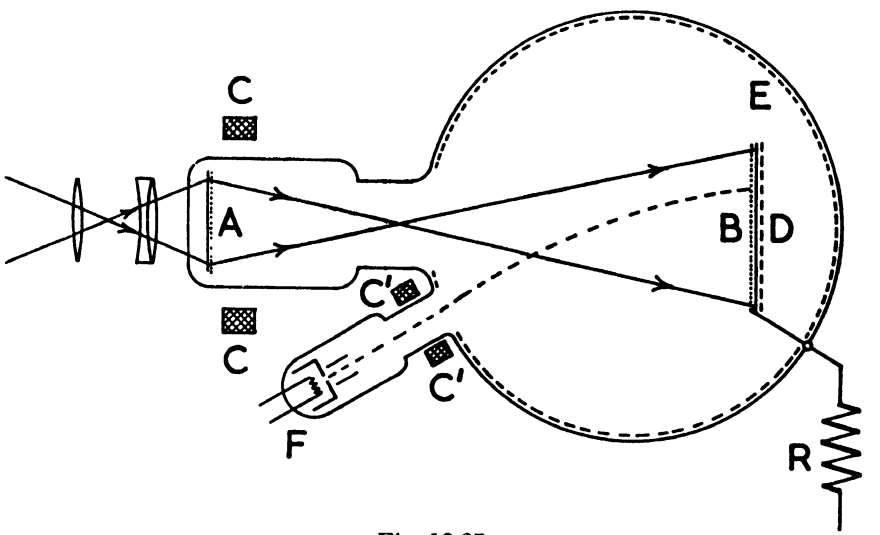


Fig. 12.27

The film on B consists of a large number of regularly spaced spots of caesium insulated from each other. The number of electrons striking each caesium spot is proportional to the light received by the corresponding point of A. Each electron causes the emission of " secondary " electrons from B by a process known as secondary electron emission,

which involves the interaction of the impacting electron with the metal surface. The secondary electrons are collected at the earthed metal film E coated on the inside of the tube. The number of secondary electrons ejected per incident electron is greater than unity, with the result that each caesium spot is left with a net positive charge proportional to the light intensity striking the corresponding element of A.

Each spot of B forms the charged plate of a very small condenser whose opposite plate is the metal screen D. The condensers are discharged in turn by a fine beam of electrons passing over B. The change of voltage across each condenser causes a momentary small current to flow through the resistance R. The electron beam which effects the discharge of the condensers is produced by the heated filament F in the side-tube of Fig. 12.27. The beam is caused to " scan " B by a periodic current passing through the coil $C'C'$. The scanning process involves a horizontal sweep across B followed by a rapid return to a new starting point just below the previous line. The electron beam thus follows a zigzag course until all the film has been covered, when it returns to its initial starting point. A complete scan (frame) takes place 25 times per second and is effected by a suitably designed time base. The number of separate small electric currents that pass through R per second is hence given by 25 times the number of caesium spots on B.

The small currents in R create voltage changes of a pulse-like nature. They are used to " modulate " a powerful oscillator delivering an electromagnetic wave of short wavelength. Thus, the original light signals are converted into variations of a " carrier " wave amplitude. The transmitter aerial is placed on the top of a very high mast to avoid absorption and reflection at intervening hillsides.

In a television receiver the variations in carrier wave amplitude are passed to the grid of a cathode ray tube so that the beam of focused electrons fluctuates in intensity. In order to reproduce the original picture the electron beam of the C.R.T. must go through exactly the same procedure as the electron beam of the camera. As the latter scans a line of B, the receiver electron beam must pass across the screen of the C.R.T. at the same corresponding height and with the same velocity. The variations in fluorescence emitted from the screen correspond to the light changes in the original camera image. The continuous scanning at the camera scanning frequency builds up a picture which appears to be continuous and stationary because of persistence of vision and screen afterglow.

EXERCISES

1. Write a brief account of the phenomena associated with the conduction of electricity in gases indicating particularly those experiments and results which have led to advances in our knowledge of the constitution of matter. (Northern H.S.C. Schol.).

2. Describe the variation in the appearance of the electric discharge, between electrodes sealed into a tube, as the pressure is reduced from atmospheric to the point where cathode rays traverse the whole length of the tube. Illustrate the description with at least one diagram.

   Outline the essential steps in the discovery of the electron. On what grounds is it believed that the charge on the electron is the same as that on a monovalent ion in electrolysis? (Northern H.S.C. Schol.).

3. Give a brief summary of the experimental methods of producing ionization in gases.

   Describe one method of ionizing a gas at a constant rate.

   If this gas is contained between two parallel metal plates how would you find the relation between the potential difference applied to the plates and the current which passes between them? (London G.C.E. Schol.).

4. Give an account of the relation between potential and current in the case of (*a*) electrolyte, (*b*) gases under constant ionization. (London G.C.E. Schol.).

5. How may cathode rays be produced? Describe experiments to demonstrate the chief properties of cathode rays. (London G.C.E. Adv. L.).

6. Discuss the experimental evidence for the view that cathode rays are negatively charged particles moving at high speed. Describe how particles of this kind can also be obtained from metals by the action (*a*) of heat, (*b*) of light. Describe briefly a practical application of either (*a*) or (*b*). (London Inter. B.Sc.).

7. What are electrons? Briefly describe *one* method by which a beam of electrons may be produced.

   Describe and explain experiments to demonstrate:

   (*a*) the effect of passing a beam of electrons through (i) a magnetic field, (ii) an electric field;

   (*b*) the effect of focusing the rays on a metal target.

   (Cambridge Local G.C.E. Adv. L.).

8. Describe and explain a method of measuring e/m for an electron. (London G.C.E. Schol.).

9. What properties of cathode rays are utilized in finding the ratio of their charge to their mass? Describe a method of finding this ratio.

   The ratio e/m for cathode rays is found to be $1{\cdot}76 \times 10^{11}$ coulombs per kilogram; the electrochemical equivalent of hydrogen is $1{\cdot}05 \times 10^{-8}$ kgm. per coulomb. Compare the mass of the electron with the mass of the hydrogen atom (expressing the result as a fraction). (Northern H.S.C.).

10. Describe an experiment to determine the charge of an electron.

    A narrow beam of electrons, accelerated by a potential difference of 400 volts, passes between two parallel metal plates 4.0 cm. long and 0·60 cm. apart in an evacuated tube, and then strikes a fluorescent

screen 15 cm. from the centre of the plates (as in a cathode ray oscillograph). Calculate the deflection of the spot of light on the screen when a difference of potential of 15 volts is established between the plates assuming that the electric field between them is uniform. (Northern H.S.C. Schol.).

11. Give an account of the appearance of a discharge tube across which a fixed high potential difference is maintained, as the pressure of the gas inside it is steadily reduced.

A beam of electrons, accelerated from the cathode of a discharge tube under a p.d. of 500 volts, passes through the perforated anode and strikes a fluorescent screen beyond it. After leaving the anode and before reaching the screen it passes between two parallel plates 2 cm. apart, across which a p.d. can be applied so as to give an electric field at right angles to the path of the electrons. A p.d. of 1,200 volts is applied between these plates, and the beam is deflected down. The spot on the screen is restored to its original position by applying a magnetic field of 3,580 amp.m.$^{-1}$ in the appropriate direction in the space between the deflecting plates, across which the voltage is still maintained. Calculate the value of e/m for the electron, in coulombs per kilogram, from this information. (Oxford Local G.C.E. Schol.).

12. Explain with the aid of diagrams the structure and mode of action of *three* of the following:

(*a*) A telephone receiver; (*b*) a wattmeter; (*c*) an induction coil; (*d*) a fluorescent lamp and (*e*) a thermionic valve. (London G.C.E. Adv. L.).

13. Answer *two* of the following:

(*a*) Explain the use of a laminated soft iron core in a transformer.

(*b*) Explain the unidirectional conductivity of a diode and give *one* application.

(*c*) Describe a photo-cell and give *one* application. (Northern H.S.C.).

14. Describe the thermionic diode and explain its action. Draw a diagram to show the way in which the anode current varies with the anode potential for such a diode. Give an explanation of the shape of the curve.

Describe *one* practical application of the diode, showing how this application depends on the shape of the characteristic curve. (Northern G.C.E. Adv. L.).

15. Describe the high vacuum diode, and explain how it can be used to produce a steady voltage from an alternating supply.

Find the velocity with which an electron strikes the anode of a diode when the potential difference across the diode is 100 volts. (Specific charge of the electron = $1{\cdot}767 \times 10^{11}$ coulombs per kgm.) (Oxford Open Schol.).

16. How would you verify Ohm's law for a piece of eureka wire (of negligible resistance coefficient) and show that the law is not obeyed by a thermionic diode. The current through a diode depends upon

the voltage according to the law $i = fv^2$ for positive values of v, and $i = 0$ for negative values of v (f being a constant). Sketch the form of the current wave when an A.C. potential is applied to the diode, and calculate the mean D.C. current when 2 volts R.M.S. is applied. (Oxford Open Schol.).

17. A condenser is connected to a source of alternating voltage. Explain why the current flowing in the leads is a maximum when the value of the applied voltage is zero.

What is the effect of introducing a resistor in series with the condenser? In a circuit of this type it is found that when the r.m.s. voltage of the alternating source is 250 volts, the r.m.s. voltage across the condenser and resistor are 150 volts and 200 volts respectively; explain this observation.

A suitable instrument for making such voltage measurements consists of a diode rectifier in series with a large condenser. A high-resistance D.C. voltmeter connected across the condenser will be found to give a reading which is a measure of the potential across the diode and condenser. Explain how this circuit works. Does the voltmeter read r.m.s. or peak volts? (Oxford & Cambridge Jt. Bd. G.C.E. Schol.).

18. Describe the changes in the distribution of energy in a series circuit containing a self-inductance, a capacitance and a spark gap when an oscillatory current is set up in it.

Describe and explain how such an oscillatory current may be produced in a circuit using (*a*) a source of alternating current, (*b*) a triode valve. (Northern H.S.C. Schol.).

19. Describe the action of the grid in controlling the anode current in a triode operating with constant anode potential, when (*a*) the potential of the grid is made progressively negative with respect to the cathode and (*b*) when a small alternating voltage is superimposed on a fixed negative grid potential.

Draw a circuit diagram showing the use of a triode as an oscillator. Point out the function of each component indicated on the diagram. (Northern G.C.E. Adv. L.).

20. Draw a labelled diagram of a triode and explain its mode of action.

Describe how you would obtain the static characteristics of a triode.

Give a short account of *one* use to which it may be put. (Northern H.S.C.).

21. Describe the construction of a simple triode valve. Why is the grid normally given a small negative potential (bias)? Draw curves to show roughly how the anode current depends on the potentials of the anode and the grid.

Draw a circuit showing how the voltage developed by a gramophone pick-up can be amplified and applied to a second valve, and discuss how the circuit operates. (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

22. Describe the construction and explain the principle of a cathode ray

oscillograph. How could such an instrument be used to investigate the voltage wave-form of the alternating supply?

A particle of mass m, carrying a charge e, moves with velocity v in a direction perpendicular to a uniform magnetic field of strength 1,000 amp.m.$^{-1}$, and is found to describe an arc of a circle of radius 34·5 cm. A uniform electrostatic field is then superimposed on the magnetic field so as to exert a force on the particle opposite to that due to the magnetic field. When the electrostatic field is adjusted to a strength of 960 volts per cm., it is found that the path of the particle is a straight line. Calculate the values of v and e/m for the particle. (Oxford & Cambridge Jt. Bd. G.C.E. Schol.).

23. Summarize the experimental evidence which led to the concept of the electron. Describe a cathode ray oscillograph and explain its use in one application. (Durham G.C.E. Adv. L.).

24. What properties of cathode rays are used in the cathode ray oscillograph? Draw a labelled diagram to show the construction of this instrument and describe *one* application. (Northern H.S.C.).

25. A closed flat coil is rotated with uniform angular velocity about a diametral axis which is perpendicular to a uniform magnetic field. Show that an alternating e.m.f. is produced in the coil. Indicate by a diagram the relative magnitude and direction of the e.m.f. for various positions of the coil during one revolution.

Describe how these deductions can be confirmed by the use of a cathode ray oscillograph. (Northern H.S.C.).

26. Derive an expression for the magnitude of the alternating e.m.f. which is produced in a closed flat circular coil when it is rotated uniformly about a diametral axis which is perpendicular to a uniform magnetic field.

Discuss the effect of applying an alternating potential difference to an inductive circuit.

By means of circuit diagrams and brief comments indicate how you would demonstrate with the aid of a cathode ray oscillograph (*a*) the variation with time of the e.m.f. produced in the flat circular coil, and (*b*) the effect produced in the inductive circuit. (A description of the oscillograph is *not* required). (Northern G.C.E. Schol.).

27. Between two points on the surface of the human body differences in potential occur, at times synchronous with the heart beats, with a magnitude of about 50 millivolts and lasting a few milliseconds.

Outline a method you would use, employing a thermionic valve or valves, cathode-ray tube and camera, to determine the magnitude and duration of the potential changes and to record them photographically. Give a description of the mode of action of the apparatus. (Cambridge Open Schol.).

28. Write an essay on "Some practical applications of electron beams".

Diagrams should be clear and the essay should give some details of two, rather than a catalogue of many, applications. (London G.C.E. Schol.).

CHAPTER 13

X-RAYS, RADIOACTIVITY AND ATOMIC STRUCTURE

13.1. X-Rays

Röntgen discovered X-Rays in 1895, before the discovery of the electron: Röntgen operated a discharge tube at high p.d. and low gas pressure and found that fluorescence was produced in zinc sulphide screens outside the tube. The fluorescence appeared when the tube was covered with black paper. Using screens in different positions it became apparent that the radiation causing the fluorescence emanated from the anode. The unknown radiation was called X-Rays.

In the original discovery it was found that X-rays penetrate matter. This property is dependent on the density of the material penetrated and the type of X-radiation produced. This led to the use of X-rays for investigating bone fractures: the rays are transmitted more easily through the less dense flesh and a fluorescent screen receiving the transmitted beam of X-rays shows a shadow of the bone structure.

Röntgen also discovered that X-Rays blacken a photographic plate to an extent depending on the intensity. By placing a leg, say, between an X-Ray tube and a photographic plate, less radiation passes through the bony structure than the flesh, causing less dense blackening on the plate. This is the common practice of " taking an X-Ray ".

13.2. Production of X-Rays

(i) *Gas-filled Tube*

The older method uses a large glass discharge tube with bulb, filled with air at a pressure of about 10^{-2}mm.Hg. (Fig. 13.1). The cathode (C) consists of a concave piece of aluminium, whilst the anode (A) is a fairly large cylinder of copper, whose end face nearer to C is inclined at an angle to the axis of the tube. On this face is soldered or welded a thin sheet of metal such as tungsten or molybdenum. This metal facing is known as the target since it is bombarded by cathode rays. The bombardment causes excess heat which is conducted away rapidly by the copper base.

A p.d. of tens of kilovolts from an induction coil is applied between A and C to produce cathode rays which strike the target, causing X-Ray emission in a divergent beam. The concave shaped cathode focuses the cathode rays on to the target by means of the non-uniform electric field produced in its vicinity.

The gas-filled tube has serious disadvantages: after operating the discharge for some time the gas pressure falls owing to the absorption

of gas ions on the walls of the tube. This necessitates a higher discharge voltage, otherwise the discharge current and X-ray intensity fall. Using a higher p.d. alters the quality (i.e. penetrating power) of the X-Rays. At higher p.d.'s the rays are " harder " i.e. more penetrating. At lower p.d.'s the rays are " soft ", i.e. more easily absorbed.

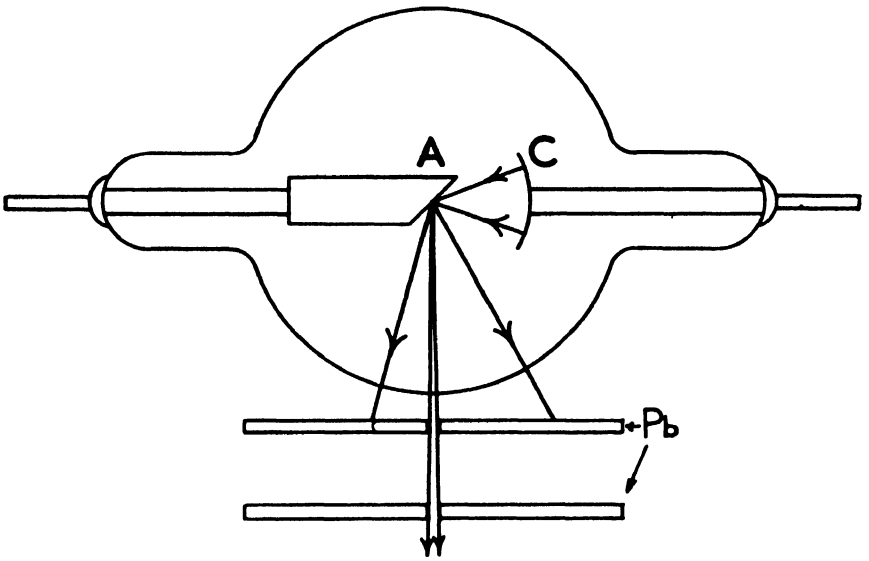


Fig. 13.1

With the gas-filled tube it is difficult to control the X-Ray intensity owing to the voltage-pressure requirements for the discharge. Beam intensities are varied by using aluminium absorbing screens but this results in "*filtering*", i.e. the preferential absorption of the softer radiation.

(ii) *Coolidge Tube*

X-Rays are produced, nowadays, with a *Coolidge Tube*, which is vacuum-filled. A beam of electrons is produced by a heated tungsten cathode (C) housed in a metal holder. The cathode is heated by low voltage A.C. from a transformer (T_1). The target assembly (A) consists of a block of metal with a facing of target metal, and is often supplied with internal water cooling. The whole block is sealed to the glass tube containing the cathode (Fig. 13.2).

The accelerating p.d. is supplied by a high voltage transformer (T_2) (Fig. 13.2). This is rectified by the tube itself which functions as a diode, the unidirectional current flow being given by the milliameter *mA*. The target assembly is earthed. This obviates any difficulties with cooling water at high potentials but necessitates the filament transformer being carefully insulated. Hospital tubes are Coolidge tubes which are not normally water-cooled because they are used

intermittently. The whole tube is immersed in oil to allow convection cooling.

In the Coolidge tube the electron beam intensity, and hence the X-ray intensity, is controlled by varying the filament heater current. The quality of the X-Ray output is also controlled by altering the p.d. across the tube. In general, hard X-rays are preponderant at high voltages.

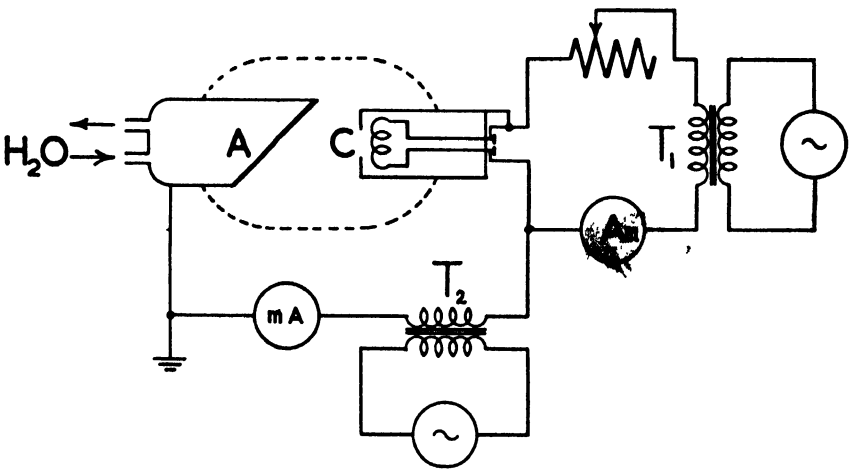


Fig. 13.2

Example

Draw a labelled diagram of an X-ray tube. Explain its mode of operation, and indicate the electrical arrangements that are needed in order to work it from the normal mains supply.

If the potential difference applied across the tube is 5,000 *volts and the current through it is* 2 *milliamperes, calculate the number of electrons striking the target per second, and the speed at which they strike it. (Take the electronic charge to be* $1{\cdot}6 \times 10^{-19}$ *coulombs, and assume that the mass of an electron remains constant at* 9×10^{-31} *kgm.).* (Oxford Local G.C.E. Adv. L.).

The first part of the question is covered in the text.

The current consists of electrons all having the same charge e, so that if n electrons strike the anode per second the current strength is given by:

$$I = ne$$

Since: $e = 1{\cdot}6 \times 10^{-19}$ coulombs and $I = 2 \times 10^{-3}$ amps.:

$$n = \frac{I}{e} = \frac{2 \times 10^{-3}}{1{\cdot}6 \times 10^{-19}} = 1{\cdot}25 \times 10^{16}$$

The kinetic energy of the electrons striking the anode is equal to the potential energy lost in travelling from the cathode, hence:

$$\tfrac{1}{2}mv^2 = eV$$

i.e. $$v = \sqrt{\frac{2eV}{m}}$$

Since: $V = 5{,}000$ volts, and $m = 9 \times 10^{-31}$kgm.:

$$v = \sqrt{\frac{2 \times 1{\cdot}6 \times 10^{-19} \times 5000}{9 \times 10^{-31}}} \text{ m.sec.}^{-1}$$

i.e. $v = 4{\cdot}21 \times 10^{7} \text{ m.sec.}^{-1}$

13.3. Properties of X-Rays

Three important properties already mentioned, are: (i) ability to cause fluorescence in certain salts, (ii) ability to penetrate matter and (iii) the blackening of photographic plates. The fourth important property is the ability to ionize gases. The latter is easily demonstrated by charging a gold leaf electroscope and placing it in front of an X-Ray tube. When the set is switched on the gold leaf falls rapidly, indicating the collection of charges from gas ions produced in the air.

13.4. The Nature of X-Rays

X-Rays are unaffected by electric and magnetic fields, whence it was suspected that they were a form of electromagnetic radiation like light, rather than a stream of particles like electrons. If X-radiation is a similar phenomenon to light, it should show the same properties such as rectilinear propagation, reflection, refraction, interference, polarization, etc. The sharpness of shadows on fluorescent screens indicates a rectilinear propagation. Partial polarization was produced by Barkla in 1906, using the property of reflection from various substances. Early diffraction and interference experiments suggested that X-Ray wavelengths were far less than those of light.

Von Laue's Experiment

In 1911, von Laue suggested that if crystals were made up of atoms arranged in a symmetrical network, the atoms should act like the apertures of diffraction grating, each atom becoming a secondary source of radiation. Interference is thus possible in particular directions if the phase differences between the rays from successive atoms are correct.

Friedrich and Knipping carried out an experiment to verify von Laue's theory in 1912, using the simple arrangement of Fig. 13.3. A fine beam of X-Rays falls on a thin crystal of zinc blende (ZnS). The photographic plate P receives the diffracted beams. On developing the plate, black spots are found in a symmetrical pattern, representing the positions where the diffracted beams struck the plate. Measurements of the relative positions of the spots confirmed von Laue's ideas.

13.5. Bragg's Law

In 1912, Bragg shewed how X-Ray diffraction could be regarded as a simple two-dimensional phenomenon. To understand Bragg's conception of X-Ray diffraction, suppose the dots in Fig. 13.4 represent an array of atoms in the cross-section of a crystal. If a parallel beam of X-Rays is directed on to the face AB at an angle θ, the scattered X-Rays

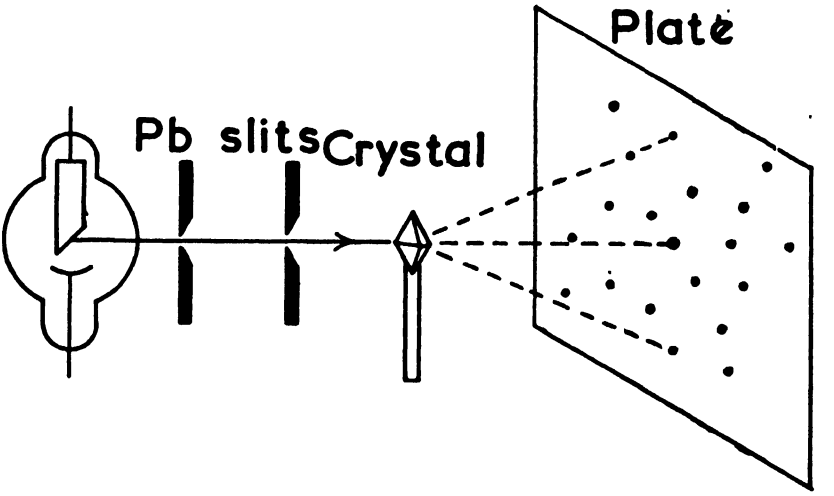


Fig. 13.3]

from the atoms in successive parallel planes give rise to a beam of maximum intensity at the same angle θ because they are in phase. The diffracted beam appears to have been *reflected* from the successive *planes*.

The phase condition for " reflection " is that the path difference between two rays scattered from successive planes is an integral number of wavelengths ($n\lambda$). If the distance between successive planes is d cm. (this is the *lattice contant*) and the wavelength of the X-Rays is λ cm., the required condition is:

$$n\lambda = 2d \,.\, \sin\theta \quad — \quad \text{Bragg's Law} \qquad . \quad . \quad (1)$$

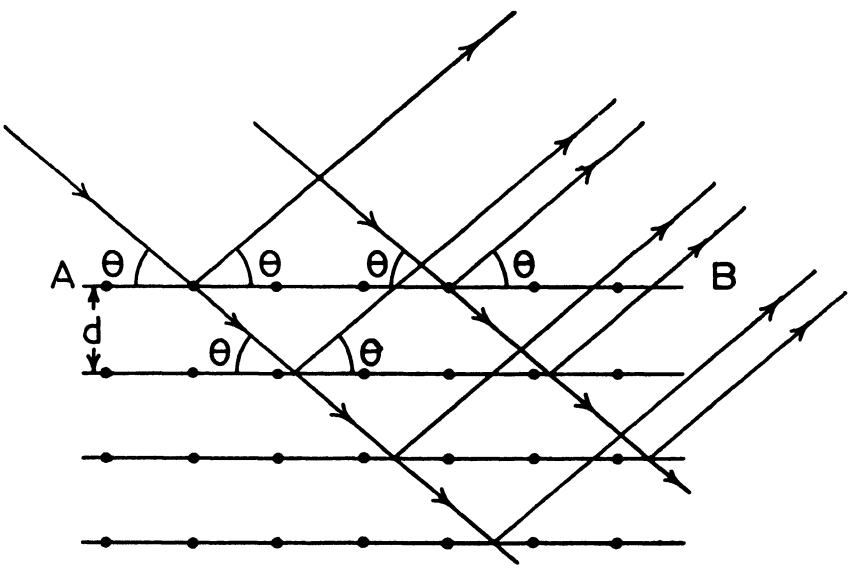


Fig. 13.4

13.6. Bragg's Determination of X-Ray Wavelengths

Bragg's major contribution to crystallography was the simultaneous determination of d and λ using crystals of sylvine (KCl) and rock-salt ($NaCl$). He assumed that these crystals had a cubic network of atoms (more correctly ions), alkali or chlorine ions being fixed at the corners of the cubes, an alkali atom always being adjacent to a chlorine atom, e.g. a part of a crystal of $NaCl$, is represented in Fig. 13.5. X-rays are "reflected" from "planes" of atoms such as 1, 2, or 3, by using different orientations of the crystal. The diffracted beams obey Bragg's Law, each series of planes having a different value of d. The separate values of d are related by geometry, so that the values of θ for each series are related to each other. These relations were confirmed by Bragg, thus proving that KCl and $NaCl$ had the cubic structure suggested.

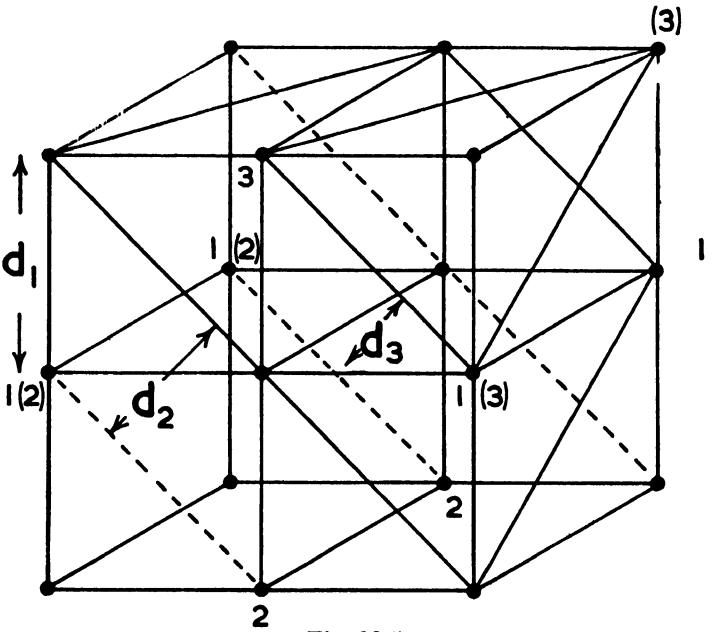


Fig. 13.5

Since each atom of a cubic crystal is a member of eight neighbouring cubes of atoms the mass that can be associated with one cube is one eighth of the aggregate mass of the atoms in the cube. For $NaCl$, the masses of the Na and Cl atoms are given by multiplying the mass of the hydrogen atom (m_H grams) by the atomic weights of the atoms, say Na and Cl, respectively. Hence, the mass of a small cube is:

$$M = \tfrac{1}{8}(4Na + 4Cl)m_H = \tfrac{1}{2}(Na + Cl)m_H \text{ (grams)} . \qquad (2)$$

The volume of the cube is d^3 cubic cm. so that if the density of the crystal is ρ gm.cm.$^{-3}$, the mass of the cube is also given by: $M = d^3\ \rho$ gm.

Hence: $$d^3 = \frac{1}{2\rho}(Na + Cl)m_H = \frac{1}{2\rho N}(Na + Cl) \text{ cm.} \quad . \quad (3)$$

where N is Avogadro's Number, $(m_H = \frac{1}{N} \text{ gm.})$.

By means of (3), Bragg found the value of d for horizontal planes like 1 in Fig. 13.5, and then determined λ by measuring θ and using (1).

The *X-Ray spectrometer* was used for wavelength determination. This is sketched in Fig. 13.6. A small crystal C is counted on a rotating

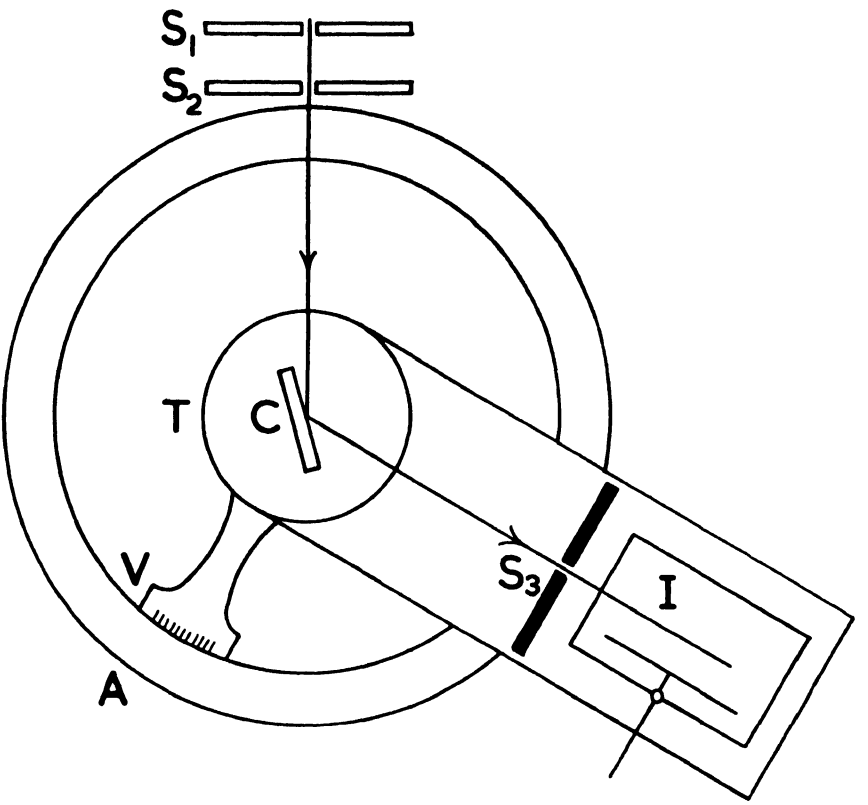


Fig. 13.6

table T. Attached to the table is a vernier arm. V, which moves over the circular scale A. A narrow beam of X-Rays is produced by the lead slits S_1 and S_2. The diffracted beams pass through the lead slit S_3, into the *ionization chamber* I. The chamber is mounted on an arm which rotates about the same axis as T, the motion being so arranged that if T rotates through the angle θ, I rotates through 2θ. Hence, for the correct *glancing* angle θ, the ionization current is a measure of the diffracted beam intensity.

Bragg used X-Ray tube targets of nickel, copper, rhodium, palladium, osmium, and iridium. From his results Bragg deduced that each target

gave a continuous series of wavelengths (*continuous spectrum*), and groups of wave-lengths of much greater intensity (*line spectrum—characteristic X-Rays*). (cf. p. 424). The groups of smallest wavelength became known as the *K-lines* and the groups of longer wavelengths were called the *L*, *M*, etc., " lines ".

13.7. Moseley's Law

In 1913 Moseley investigated the wavelengths of the characteristic *K* and *L* rays of a large number of target elements. His results showed that the square root of the *K* or *L*, X-Ray *frequencies* (ν_K, ν_L) plotted against the number of each element in the periodic table (Z) gives a straight line graph. Moseley called Z the *Atomic Number*. Hence:

$$\sqrt{\nu_K} = a(Z - b) \quad — \textit{ Moseley's Law } . \quad . \quad (4)$$

Moseley had to assume that the gaps in his curve corresponding to $Z = 43$, 61, 72 and 75, belonged to undiscovered elements. They have since been discovered and their *K* and *L*, X-radiations shown to have the surmised frequencies.

13.8. X-Ray Intensities and Dose

Because of their ability to ionize atoms, X-Rays are a source of danger to the body. The biological effects of ionizing radiations is not fully understood, but it is known that they can cause the destruction of cells. This property not only causes serious lesions to form on healthy parts of the body, but is paradoxically used to destroy cancerous growths on diseased parts of the body. The effects are dependent on the intensity of the radiation and the time of exposure, i.e. *dose* = intensity × time.

The absolute unit of dose, i.e. quantity of radiation received, is the *Röntgen*:

> *The Röntgen* (R) *is that quantity of radiation, which, on passing through* 1 *cm.*^{3} *of air at N.T.P., causes ionization such that the total charge on the ions, independent of sign, is* 1 *e.s.u. of charge* (*i.e.* 3×10^{-19} *coulombs*).

This unit is related to the ionizing ability of the rays, which is the most important property from the point of view of biological effects.

13.9. Radioactivity

Bequerel discovered in 1897 that uranium salts caused the blackening of nearby photographic plates in the dark and concluded that the blackening was caused by radiations emitted by the salt. Other uranium compounds gave the same effect. Bequerel also showed that charged electroscopes were discharged by the radiations and concluded that they ionized gases. The phenomenon was called *radioactivity* and the uranium compounds were said to be *radioactive*.

In 1898, Mme. Curie studied radioactivity quantitatively, using the ionizing ability of the radiations as a measure of the " activity " of a specimen. She shewed that the activity depended on the amount of uranium contained in a specimen. She discovered that thorium compounds gave rise to similar effects. Mme. Curie also discovered that particular ores possessing a low percentage of uranium often gave a relatively large activity and deduced that a more radioactive element than uranium was present in the ores. With her husband Pierre Curie, Mme. Curie isolated the elements *polonium* and *radium* which were much more radioactive than uranium or thorium.

13.10. α-, β- and γ-Rays

Rutherford showed by means of absorption experiments that the radiations emitted by natural radioactive substances consisted of at least two types which he called α- and β-radiations. The α-rays were easily absorbed so that β-rays were isolated by " filtering " with absorbers. Both α- and β-rays were deflected by a magnetic field. Using lead absorbers some ionization was always detected even when the α- and β-rays were completely absorbed, and a third type of radiation was assumed to be present. This was called γ-radiation; it is undeflected by a magnetic field. The three types of radiation are thus distinguished by the effects of a magnetic field. In Fig. 13.7, the field (H) is at right angles to the paper (outwards).

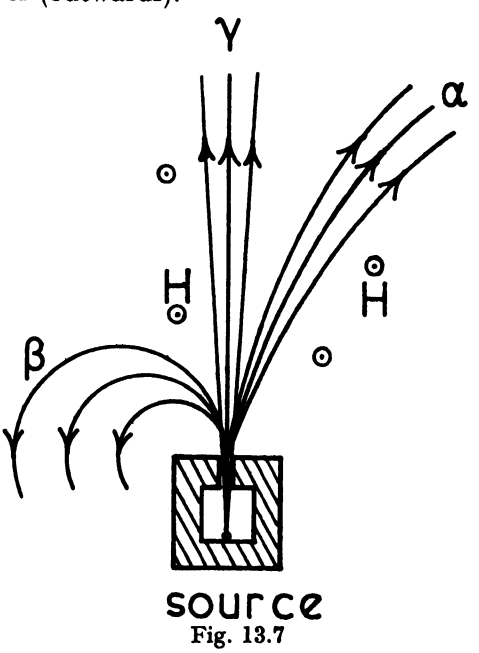


Fig. 13.7

13.11. The nature of α-, β- and γ-rays

(i) *β-rays*: The deflection of β-rays by a magnetic field is always in a direction expected of a beam of negatively charged particles. The value

of e/m for the particles was shown to be the same as for cathode rays by Bequerel. The velocity of the rays was estimated to approach the velocity of light. Thus, β-rays are high speed electrons.

(ii) *α-rays*: These are more difficult to deflect in a magnetic field than β-rays (Fig. 13.7) and because they are deflected in a direction opposite to β-rays, Rutherford suggested that α-rays consisted of positively charged particles of much greater mass than electrons. Rutherford found the ratio of charge to mass of α-rays (e/m_α) to be about half the value of the ratio for hydrogen ions in solution. Believing in the atomicity of electricity (p. 231), Rutherford assumed that α-rays possessed one or two electronic charges so that their mass was proportionately greater than that of the electron. Since helium is always found to be occluded in radioactive materials but never hydrogen, he suggested that α-rays were helium atoms carrying a charge $+ 2e$; this gives the correct result for the e/m_α value.

Rutherford confirmed this interpretation by allowing the α-rays to pass through an extremely thin walled glass tube into an evacuated space containing two electrodes. The transmitted rays gave up their charge inside the exhausted tube and became neutral atoms; these were shown to be helium atoms by effecting a discharge between the electrodes using a high p.d. The characteristic spectrum of helium was observed.

(iii) *γ-rays*: γ-radiation is an electromagnetic radiation with similar properties but more penetrating than X-Rays. This conception was confirmed by measurements of γ-ray wavelengths by Rutherford and Andrade in 1914 using a modified X-Ray spectrometer; γ-ray wavelengths are much smaller than X-Ray wavelengths, the longest being about 10^{-11}cm.

13.12. Properties of α-, β- and γ-Rays—Summary

α-Rays

1. Cause fluorescence of luminescent materials like zinc sulphide.
2. Blacken photographic plates.
3. Ionize gases readily (producing about 10^5 ion-pairs in air).
4. Easily absorbed by matter: they penetrate between 10^{-2} and 10^{-3}cm. of aluminium or a few cm. of air at N.T.P. Their penetration in air is unique and the rays cannot be detected beyond a distance called the *Range*; this depends on the initial energy of the rays which is characteristic of a given source.
5. Deflected by magnetic and electric fields.
6. Have high emission velocities—about 10^7 m. sec.$^{-1}$
7. Possess a charge: $+ 2e$ and mass: $4m_H$; they are hence helium nuclei.

β-Rays

1. Have a much smaller fluorescent effect than α-rays.

2. Blacken photographic plates.

3. Ionize gases—less efficiently than α-rays—about 100 ion-pairs per cm. of path in air.

4. Penetrate matter more easily than α-rays but are absorbed completely by one or two mm. of aluminium or a few metres of air; they have no definite range owing to the successive deflections caused by collisions with atoms.

5. Easily deflected by magnetic fields; also deflected by electric fields.

6. Emission velocities approach the velocity of light; are fast electrons.

γ-Rays

1. Cause fluorescence but this is not easily observed by eye.

2. Blacken photographic plates like X-Rays.

3. Cause relatively small ionization in gases; this is not effected directly by the γ-Rays but by *secondary* electrons released from gas atoms and at metal surfaces; ionization is thus a secondary process (cp. X-Rays).

4. Penetrate matter readily; " hard " γ-rays pass through cm. of lead.

5. Unaffected by magnetic or electric fields.

6. Are an electromagnetic radiation of very short wavelength; like light and X-Rays they are considered as " photons " whose energy is given by: $E = h\upsilon$; these energies are characteristic and generally several mev.*

13.13. Early Detection of Radiations

1. *Ionization Methods*

The common property of α-, β- and γ-rays is the ability to ionize or cause the ionization of gases. The ionization current produced by the rays is used for quantitative detection.

An ionization chamber consists of two insulated electrodes mounted in an earthed box. An electric field sufficiently intense to collect all the ions formed by radiations passing through the chamber (i.e. saturation ionization current), is created by a p.d. between the electrodes. The ionization current is measured by an electroscope or electrometer (p. 466).

Small portable electroscopes possessing a metallised quartz fibre are used as *dose-meters*. The change in deflection of the charged fibre gives a measure of dose in röntgens (also used for X-Rays).

2. *Scintillations*

Crooke shewed that an α-ray causes a small flash of light on hitting a *ZnS* screen; these " scintillations " are visible when viewed with a low-powered microscope in total darkness. The modern detector is the *scintillation counter*; this detects the flashes of light produced by α-,

*mev. = 10^6 e.v. (electron.volts) cf. p. 379; γ-ray energies are generally expressed in this unit.

β- or γ-rays in certain crystals. The light flashes strike a special photo-cathode to produce photo-electrons. These are collected to give small pulses of current which can be amplified and counted.

3. *Wilson Cloud Chamber*

The cloud chamber was devised by C. T. R. Wilson in 1896. In its simplest form it consists of a cylindrical vessel A containing a piston P (Fig. 13.8). Between the glass face B and P the air is saturated with water vapour and illuminated by a beam of light from a source S. When the piston is suddenly withdrawn an adiabatic expansion occurs. The sudden cooling causes the water vapour to become supersaturated and a cloud is created in A. In dust free air a cloud forms only when gas ions are present. Radioactive rays cause ionization along their paths in the chamber and after an expansion small water drops condense on the ions formed. With suitable illumination the water drops become visible against a dark background, showing the characteristic paths of the different rays, i.e. *tracks*.

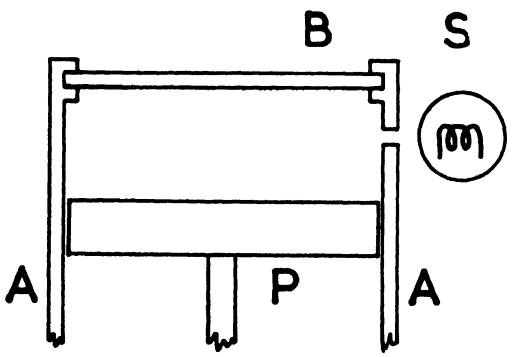


Fig. 13.8

13.14. Structure of Atoms

The radioactivity of a pure element decays in time but is independent of temperature and unaffected by chemical combination. This suggests that radioactivity is not a property of that part of the atom responsible for such processes described as atomic, e.g. ionization, magnetism, emission of light spectra, etc. Hence, the atom, apart from possessing electrons, which are relatively easy to remove, must have a special type of structure to explain the radioactive properties.

More light was shed on the nature of atomic structure when Rutherford's colleagues Geiger and Marsden shewed (1919) that α-rays could be deflected, (i.e. scattered), through large angles on passing through very thin metal foils.

Rutherford explained the results by supposing that the α-rays are deflected on collision with a concentration of positive charge at the centre of the atom, its volume being much smaller than the atom itself.

Thus arose the nuclear conception of the atom; the central positive charge is associated with most of the mass of the atom and called the *nucleus*; this is surrounded by *extra-nuclear* electrons to maintain electric neutrality.

Rutherford worked out a thoery of α-particle scattering assuming the charge on the nucleus to be $+ Ze$, where e is the electronic charge. The theory agreed with the experimental results and Z turned out to be the same as the atomic number defined by Moseley (p. 414). An estimate of the radius of nuceli was 10^{-12} cm. approx.

The emission of α- and β-rays is regarded as a result of the spontaneous disintegration of radioactive nuclei caused by instability. The disintegrations are difficult to influence by physical or chemical means since the surrounding electrons act as a protecting shield and, in general, nuclei take no part in ordinary physical and chemical processes.

13.15. Displacement Law for Radioactive Changes

Since an α-ray has a charge $+ 2e$ and mass $4m_H$ the element resulting from an α-emission has an atomic number $Z - 2$ and atomic weight $A - 4$, and is, therefore, displaced two places to the left of the parent element in the periodic table. The emission of a β-particle is equivalent to an increase in nuclear charge by one unit and hence an increase in atomic number by unity. The new element formed has the same atomic weight but is displaced one place to the right of the parent element in the periodic table. This is the essence of the *Displacement Law* deduced by Soddy and Fajans in 1913.

13.16. Radioactive Series

On the basis of the displacement law it is possible to draw up tables of the radioactive transformations which naturally occurring radioactive elements undergo, (cf. Table p. 506). The tables are referred to as the uranium, thorium and actinium series, these elements being the initial members of the series. The final element of each series is lead, which is found to accompany uranium minerals in small quantities.

Some members of the radioactive series, such as *RaA* and *RaC'*, *ThA* and *ThC'*, *AcA* and *AcC'*, have the same atomic number but different atomic weights. Such atoms, having the same atomic number but different atomic weights, are called *Isotopes* (p. 428).

13.17. Decay of Radioactivity

Experiment shows that the time taken for the activity of an element to be reduced by a definite fraction is always the same. This is characteristic of an exponential change represented by:

$$A = A_0 \cdot e^{-\lambda t} \quad . \quad . \quad . \quad . \quad (5)$$

where A and A_0 represent the activities of a source when the time is t and zero respectively, and λ is a constant. Rutherford and Soddy

explained this result in their mathematical theory of radioactivity (1902). They supposed that if N active atoms exist at time t and the number which disintegrate in the succeeding small time interval dt is dN, the rate of disintegration is proportional to the number of atoms existing at that time, i.e. $(dN/dt) \propto N$. Hence:

$$\frac{dN}{dt} = -\lambda N \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the negative sign is inserted because (dN/dt) is a decrease; the constant λ is called the *disintegration* constant. Integrating (6):

$$\int \frac{dN}{N} = -\int \lambda dt$$

i.e. $$\log_e N = -\lambda t + \text{const.}$$

If: $N = N_o$ when $t = 0$, the constant of integration is : $\log_e N_o$, so that:

$$\log_e N - \log_e N_o = -\lambda t$$

i.e. $$\underline{N = N_o \cdot e^{-\lambda t}} \quad . \quad . \quad . \quad . \quad (7)$$

From (7): $$\frac{dN}{dt} = -\lambda N_o e^{-\lambda t} \quad . \quad . \quad . \quad . \quad (8)$$

when $t = 0$: $$\left(\frac{dN}{dt}\right)_o = -\lambda N_o \quad . \quad . \quad . \quad . \quad (9)$$

Hence (8) may be written:

$$\frac{dN}{dt} = \left(\frac{dN}{dt}\right)_o \cdot e^{-\lambda t} \quad . \quad . \quad . \quad . \quad (10)$$

i.e. $$\underline{A = A_o \cdot e^{-\lambda t}} \quad . \quad . \quad . \quad . \quad (11)$$

where $A = (dN/dt)$ and $A_o = (dN/dt)_o$, (A = Activity which is proportional to disintegration rate).

13.18. The Disintegration Constant and Half-Life

The Rutherford-Soddy theory helped to unravel the complex series of changes that the uranium, thorium, etc. atoms underwent in their series of transformations.

From (10), taking logs:

$$\log_e\left(\frac{dN}{dt}\right) = -\lambda t + \log_e\left(\frac{dN}{dt}\right)_o \quad . \quad . \quad . \quad (12)$$

so that by plotting the log of an ionization current ($I \propto dN/dt$) against time, a straight line graph is obtained whose gradient is $-\lambda$. Since

λ is characteristic of a given element the determination of λ identifies the presence of that element.

A more convenient quantity than λ is the *half-life* ($t_{\frac{1}{2}}$), i.e. *the time taken for a given number of radioactive atoms to be reduced by a half.* From (7) putting: $N = N/2$, $t = t_{\frac{1}{2}}$; gives:

$$\frac{N}{2} = N_o \cdot e^{-\lambda t_{\frac{1}{2}}}$$

i.e. $$e^{\lambda t_{\frac{1}{2}}} = 2$$

Taking logs: $$t_{\frac{1}{2}} = \frac{\log_e 2}{\lambda} = \frac{0{\cdot}693}{\lambda} \quad . \quad . \quad . \quad . \quad (13)$$

Since λ can be determined experimentally, the half-life is found using (13).

13.19. Radioactive Equilibrium

A radioactive element with a very long half-life (e.g. uranium: $t_{\frac{1}{2}} = 4{\cdot}5 \times 10^9$ yrs.), decays so slowly that it is virtually constant. It follows that the daughter element is formed at a constant rate. If this "daughter" element has a much smaller half-life than the parent element its rate of decay is much faster. Hence starting with a pure parent element the daughter element accumulates until the rate at which it decays is the same as the rate at which it is formed. This state is called *radioactive equilibrium.*

13.20. Source Strength or Intensity

Radium has a very long half-life (1590 yrs.) and because of radioactive equilibrium the amount of daughter element in equilibrium with it is constant. The daughter element is a rare gas *radon* (*Rn*). The volume of radon in equilibrium with a gram of pure radium is measureable and was formerly used as a standard unit of activity for radium sources, called the *Curie.*

The Curie has been given a new interpretation in the light of counting techniques now used for measuring radioactive source strengths. It is known that one Curie gives rise to $3{\cdot}7 \times 10^{10}$ disintegrations per second; using counting techniques the strength of a source is determined in Curies by dividing its total disintegration rate by $3{\cdot}7 \times 10^{10}$. The millicurie and micro-curie are equivalent to $3{\cdot}7 \times 10^7$ and $3{\cdot}7 \times 10^4$ disintegrations per second.

*13.21. Bohr Theory of the Hydrogen Atom—Atomic Spectra

The Rutherford conception of the atom was used by Bohr to explain the production of *atomic spectra,* i.e. the characteristic light spectrum produced by atoms of a substance.

The hydrogen atom is the simplest to discuss since it has a nucleus consisting of a proton surrounded by one electron. Let the mass of the proton be: m_H kgm., that of the electron: m kgm., the charge on the proton: $+e$ coulombs and the charge on the electron: $-e$ coulombs. Bohr supposed that the electron travels in a circular orbit with the hydrogen nucleus at its centre (Fig. 13.9). The angular velocity of the electron and the radius of the orbit are assumed constant, the centripetal force being provided by the Coulomb force of attraction between nucleus and electron.

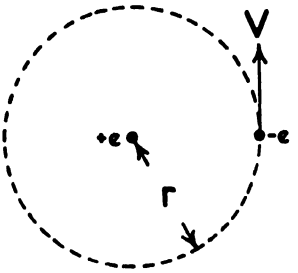


Fig. 13.9

If the tangential velocity of the electron is v m.sec.$^{-1}$ and the radius of the orbit is r m. the centripetal force is mv^2/r newtons. The attractive force between nucleus and electron is: $e^2/(4\pi\epsilon_0 r^2)$ newtons.

Hence:
$$\frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$mv^2 = \frac{e^2}{4\pi\epsilon_0 r} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The energy of the electron is partly kinetic and partly potential. The former is $\frac{1}{2}mv^2$ joules. Since the potential at all points distant r m. from the nucleus is $e/(4\pi\epsilon_0 r)$, the potential energy possessed by the electron is $-e^2/(4\pi\epsilon_0 r)$ joules. Hence:

$$\text{total energy: } E = \tfrac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r} \text{ joules} \quad . \quad . \quad (15)$$

From (14):
$$\tfrac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 r}$$

Hence in (15):
$$E = \frac{-e^2}{8\pi\epsilon_0 r} \text{ joules} \quad . \quad . \quad . \quad (16)$$

From (16), if the electron acquires or loses energy by some means, the radius of the orbit must change.

To explain the production of atomic spectra Bohr made two sweeping postulates, viz. (i) only particular values of r are possible, defining "*stationary*" *states* of the atom; (ii) the angular momentum possessed by the electron in its stationary states has values which are an integral number of units, each unit having the value $h/2\pi$, where h is Planck's Constant.

Applying (ii), the angular momentum of the electron, is given by:

$$mvr = \frac{nh}{2\pi} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where n is an integer known as the *quantum number* of the atom.

By squaring (17) and dividing (14) by this:

$$\frac{1}{mr} = \frac{e^2\pi}{\epsilon_0 n^2 h^2}$$

i.e.

$$\frac{1}{r} = \frac{\pi m e^2}{\epsilon_0 n^2 h^2}$$

Substituting for $\frac{1}{r}$ in (16):

$$E_n = \frac{-me^4}{8\epsilon_0^2 n^2 h^2} \text{ joules} \quad . \quad . \quad . \quad . \quad (18)$$

For any other stationary state the energy is given by:

$$E_p = \frac{-me^4}{8\epsilon_0^2 p^2 h^2} \text{ joules} \quad . \quad . \quad . \quad . \quad (19)$$

where p is the quantum number corresponding to that state.

The electron is normally in the orbit having the smallest radius and minimum angular momentum (*ground state*); this corresponds to: $n = 1$. If the electron gains energy it must satisfy (19); the atom is said to be *excited*. According to Bohr, the excited state lasts for a small fraction of a second and then reverts to the ground state. In this *transition* to the ground state the excess energy of the electron is emitted in the form of an electromagnetic radiation (light photon), whose frequency is given by:

$$\triangle E = E_p - E_1 = \frac{me^4}{8\epsilon_0^2 h^2} \cdot \left(\frac{1}{1} - \frac{1}{p^2}\right) = h\nu \quad . \quad (20)$$

Since $p = 2, 3, \ldots \infty$, it is possible to produce a whole series of spectral frequencies. The state $n = 1$ is not the only one to which excited atoms return after excitation and, in general, a spectral " line " is produced by excited atoms reverting to a state represented by $n = 2, 3, \ldots (p > n)$. Thus separate series of " lines ", known as the Lyman, Balmer, Paschen and Brackett spectra are produced. The wavelengths of lines of the Balmer spectrum of hydrogen when measured optically agree almost exactly with values obtained from the formula using known constants. The Bohr theory is thus admirably confirmed.

13.22. The Rutherford-Bohr Atom

In heavier atoms it is supposed that the extra-nuclear electrons exist in orbits of restricted energies. Each electron belongs to a " shell ", covering a range of energy. The shells are denoted by the letters K, L, M, N, O and P, the K shell having the smallest radius and hence the lowest energy (i.e. maximum negative potential energy). The electrons in any shell are limited in number; the maximum number in the K shell

is 2, in the L shell 8, in the M shell 18, in the O and P shells 32. For any atom the total number of electrons is Z.

On this basis the helium atom ($Z = 2$) has two electrons in the K shell only; lithium ($Z = 3$) has 2 electrons in the K shell and 1 electron in the L shell, and so on (Fig. 13.10).

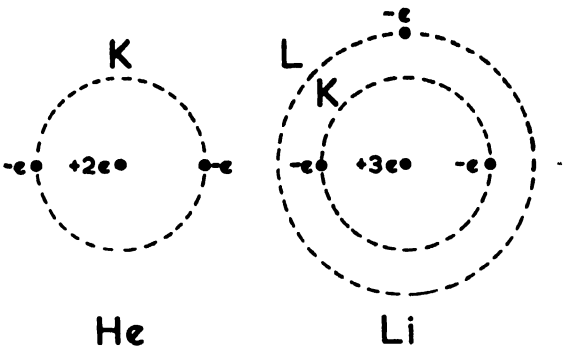


Fig. 13.10

*13.23. Atomic Spectra—X-Rays

Characteristic X-Rays are photons emitted when electrons in the innermost shells of an atom undergo transitions from one *shell* to the other. The negative energies of these tightly bound electrons are much larger than those of the optical electrons. An essential condition for the generation of an X-Ray photon is the prior removal of an electron from one of the innermost shells. This is achieved by bombardment with high velocity electrons—the part played by the cathode rays in the gas-filled X-Ray tube or by thermionic electrons in the Coolidge tube. An electron from an outer shell falls into the vacant space of the inner shell, and in the transition from the outer to the inner shell the excess energy of the electron is emitted as an electromagnetic radiation, whose frequency is:

$$E_K - E_L = h\nu_K; \quad E_L - E_M = h\nu_L; \text{ etc.} \quad . \quad (21)$$

where the subscripts indicate the particular shell concerned. Transitions from the L shell to a vacancy in the K shell give the K lines of the X-Ray spectrum (ν_K), whilst transitions from M to L shells give L lines (ν_L). Because of the larger values of $\triangle E$, X-Ray frequencies are much higher (wavelengths smaller) than those of visible light. By assuming this simple picture of the atom it is relatively easy to deduce Moseley's Law relating X-Ray frequency and atomic number for the K and L lines.

EXERCISES

1. Write an account of the production and properties of cathode rays. How are X-rays produced from cathode rays and how do they differ from them? (Northern G.C.E. Adv. L.).

2. Distinguish between cathode rays and X-rays as regards their origin, nature and properties.

   Compare the modern type of " thermionic " X-ray tube with the original " gas " tube. (Durham G.C.E. Adv. L.).

3. Contrast the gas-filled and Coolidge types of X-ray tube both as regards design and working.

   Give an account of the observable properties in which X-rays (*a*) resemble, (*b*) differ from, cathode rays. (London G.C.E. Adv. L.).

4. Describe the apparatus used in one method of producing X-rays.

   Give a *concise* account of the nature of X-rays and their more important properties. (London G.C.E. Adv. L.).

5. Give an account of the properties of X-rays.

   Draw a labelled diagram of a modern form of X-ray tube and show by means of a circuit diagram how the tube is used in conjunction with its associated electrical equipment. (Assoc. Exam. Board G.C.E. Adv. L.).

6. You are provided with a glass tube containing an electrode at each end, an exhaust pump and a source of high potential. Under what conditions (*a*) does the gas within the tube become a relatively good conductor, (*b*) is a beam of electrons (cathode rays) produced within the tube?

   What modifications must be made in the tube in order that a strong beam of X-rays may be produced.

   What would happen in each case if the potential applied was increased?

   What experiment would you perform to show the effect of a magnetic field on the conducting particles in (*a*) and (*b*) and on a beam of X-rays? State the result you would expect. (London G.C.E. Adv. L.).

7. Give an account, illustrated by diagrams, of the part played by electrons in *two* of the following: (*a*) the production of X-rays, (*b*) the photo-cell, (*c*) the cathode ray oscillograph. (Northern H.S.C. Schol.).

8. Write a concise essay on the electron, dealing with its discovery, properties and the part it plays in atomic structure. (Northern H.S.C. Schol.).

9. Draw (*a*) a large annotated diagram of some form of X-ray tube, (*b*) a diagram of the circuit required to work it.

   An X-ray tube passes 5 milliamps. at a potential difference of 100,000 volts. Calculate the maximum speed of the cathode rays produced, and the rate of production of heat at the target (anticathode) if only 0·1 per cent of the energy is converted into X-radiation.

   The ratio of the charge to the mass of an electron is $1{\cdot}77 \times 10^{11}$ coulomb per kgm.; 4·18 joules are equivalent to one calorie. (Northern H.S.C.).

10. What is the linear velocity of an electron describing a circular orbit of radius 10^{-8} cm. about a proton? Assume the proton to remain

stationary. (Electronic charge = $1{\cdot}6 \times 10^{-19}$ coulombs; mass of electron = $9{\cdot}1 \times 10^{-31}$ kgm. (Cambridge Open Schol.).

11. Describe experiments (one in each instance) with a discharge tube containing air at low pressure to determine (*a*) the ratio of the charge to the mass of an electron, (*b*) the ratio of the mass of a nitrogen atom to the mass of an oxygen atom.

State, in each case, the quantities that have to be measured to enable the ratios to be calculated. The calculations themselves are *not* required. (Northern G.C.E. Adv. L.).

12. Explain how the velocity and the ratio of charge to mass of atomic or sub-atomic particles can be measured. Describe what you consider to be a practicable arrangement for measuring these quantities for an electron, estimating, as far as you can, the magnitudes involved (e.g. field strengths).

A proton has a velocity of 4×10^7 m.sec.$^{-1}$, a charge (e) of $1{\cdot}6 \times 10^{-19}$ coulombs and a mass of 2×10^{-27} kgm. It moves in a circular orbit in a uniform magnetic field of strength $10^7/(4\pi)$ amp.m.$^{-1}$ What is the radius of the orbit.

For a proton of this velocity how could the radius be substantially reduced? (Cambridge Open Schol.).

13. What are *isotopes*, and how was their existence first discovered? Discuss, with the aid of kinetic theory, how the physical properties of a gaseous substance should vary from one isotope to another. (Cambridge Open Schol.).

14. (*a*) *Compare* and *contrast* the properties of α-, β- and γ-rays (unrelated lists of the properties of each are not sufficient).

(*b*) Outline the evidence for identifying the β-particle as an electron.

(*c*) Give a reasoned statement of the changes in atomic weight and atomic number of radioactive nuclei when (i) α-particles, (ii) β-particles, (iii) γ-rays are emitted. (Northern G.C.E. Schol.).

15. Distinguish between α-, β- and γ-rays and point out the chief characteristics of each.

What experimental evidence is there (*a*) that an atom contains a comparatively massive nucleus, (*b*) that the nucleus is surrounded by a number of electrons? (Northern G.C.E. Schol.).

16. Give an account of the nature, properties and methods of detection of the " rays " emitted by radioactive substances. (Northern G.C.E. Adv. L.).

17. What do you understand by the term *radioactive half-life*?

The half-life of the cobalt radioisotope (Co^{60}) is 5·3 years. What strength will a millicurie source of this isotope have after a period of one year?

18. What is the experimental evidence that (*a*) atoms contain electrons, and (*b*) atoms contain a small positive heavy nucleus? (Oxford Open Schol.).

CHAPTER 14

NUCLEAR PHYSICS

14.1. Introduction

Prior to the end of the first world war very little was known about atomic nuclei. From α-ray scattering experiments Rutherford showed that nuclei have radii less than 10^{-12} cm. It was thought that a nucleus was made up of smaller particles because β- and α-rays are emitted by radioactive nuclei. The energies of emission of β's and α's give evidence of large forces existing between nuclear constituents.

14.2. Artificial Disintegration of a Nucleus

In order to investigate the constitution of nuclei, Rutherford bombarded nitrogen gas with the α-rays of *RaC'* which have extremely high velocities—about 2×10^9 cm.sec.$^{-1}$ Rutherford used relatively simple apparatus: a small radium *C'* source *P*, is placed on a disc on the holder *B*, inside the metal box *A* (Fig. 14.1). Any radiation or particle with sufficient energy can escape from the box through a very thin aluminium window *W*. Adjacent to *W* is a zinc sulphide screen *S*, scintillations being detected by the microscope *M*.

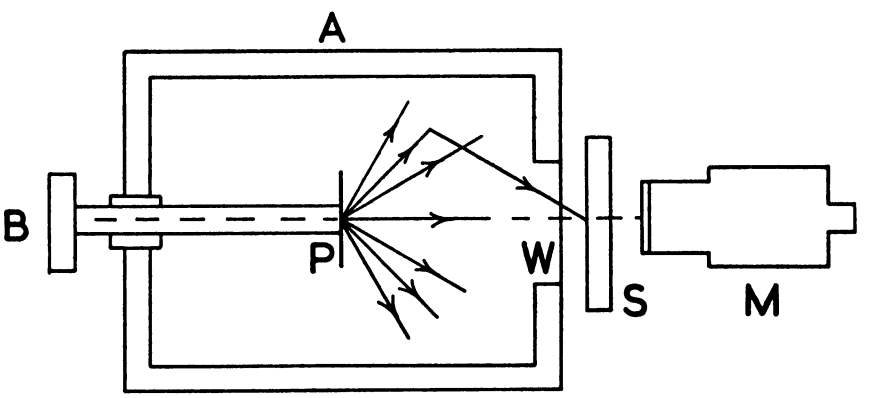


Fig. 14.1

When *A* was filled with nitrogen, scintillations were observed even when *P* was at a distance from *W* far greater than the range of the α-rays. Thus, the scintillations were not caused by the α-rays from *P* but by particles emitted by the nitrogen atoms. Investigation using magnetic deflection suggested that the particles were protons. This was later confirmed by Blackett (1926) who used an α-source inside a cloud chamber containing nitrogen; photographs shewed that an

α-track occasionally stopped short in the gas and divided into two tracks of dissimilar character; a short thick track—identified as that of the nitrogen nucleus and a longer one—that of a proton.

Other light elements emit protons when bombarded with α-particles and it was deduced that nuclei must contain protons.

14.3. Masses of Isotopes

Aston investigated the accurate measurement of atomic masses using a *mass spectrograph*, following earlier work initiated by Sir J. J. Thomson in 1911.

Aston's mass spectrograph uses a gas discharge tube (CA) as a source of positive ions and a system of slits (S_1, S_2) to define a parallel beam which passes into an evacuated enclosure (Fig. 14.2). The beam is deflected separately by a uniform electrostatic field E and a uniform magnetic field H, both at right angles to the beam and to each other. The theory of the instrument shows that the beam of ions is split into separate beams containing ions of the same mass after leaving the magnetic field. The separate beams come to a line focus on a suitable placed photographic plate P. After long exposures, developed plates show lines corresponding to the ions of different masses.

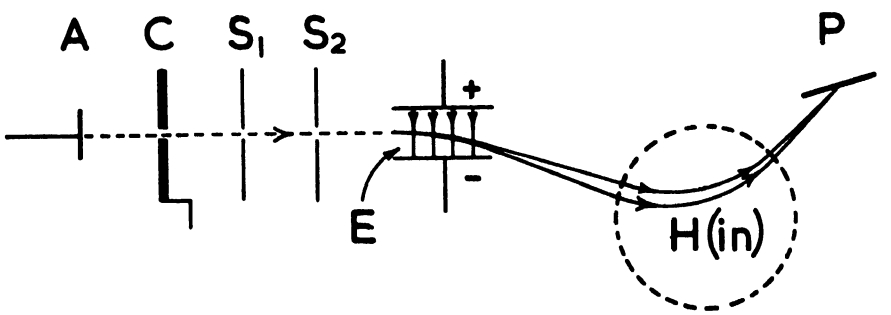


Fig. 14.2

With neon in the discharge tube, two lines corresponding to masses $20m_H$ and $22m_H$, where m_H is the mass of the hydrogen atom, were obtained. The conclusion was that ordinary neon gas contains two different types of atoms whose chemical properties are identical, (otherwise their presence would be revealed in chemical experiments). The different types of neon atom are called *Isotopes* (p. 419).

Aston found that many other elements consisted of isotopes whose masses relative to hydrogen were approximately whole numbers. The whole number to which the isotopic mass approximates is called the *Mass Number* of the isotope.

From the integral nature of atomic weights known in 1816 Prout had suggested that all atoms were made up of hydrogen atoms. The hypothesis was discounted when non-integral atomic weights like chlorine (at. wt.: 35·46) were discovered. The approximate integral

nature of relative isotopic masses revived this hypothesis.* The non-integral nature of some atomic weights is explained by isotopic mixture.

14.4. Construction of Nucleus

Early theory supposed that a nucleus consisted of A protons (A = mass number), and $(A - Z)$ electrons (Z = atomic number). The positive charge on a nucleus is then $+ Ze$, as required, and the mass of the atom (including Z outer electrons), is Am_H, in accordance with Aston's result.

Such a nucleus easily accounts for β-rays—as electrons ejected by an unstable nucleus. However, it was felt that the two oppositely charged particles could not retain their separate identities in such close contact. As early as 1920, Rutherford suggested that a proton and an electron were combined inside the nucleus as a neutral particle which he called the *Neutron*. These particles were discovered by Chadwick in 1932.

14.5. The Discovery of the Neutron

In 1930, radiation of an unknown kind was found to be emitted when beryllium was bombarded by an intense beam of α-particles. This radiation was capable of traversing large thicknesses of lead. It appeared to be uncharged, since it was unaffected by a magnetic field. It also possessed the remarkable property of knocking protons out of hydrogenous materials such as water, paraffin, etc.

In 1932, Chadwick shewed that all the observed effects were explained if were it assumed that the radiation consisted of high speed particles having a mass similar to that of the proton and possessing no charge. From the results of his experiments Chadwick determined the mass of the particle to be similar to that of the proton and concluded that the particles were neutrons.

The neutron is incapable of ionizing gas atoms directly because it has no charge and is therefore unable to push or pull electrons out of atoms using electric forces. In collisions with gas atoms, however, kinetic energy is transferred to the atoms and the latter can ionize by collision using electric forces. Thus, the original neutron beams were detected by ionization.

When a neutron collides with a hydrogen atom the collision is similar to that between billiard balls of similar mass, and theory shews that the neutron gives up almost all its energy in head-on collision. Thus, the hydrogen atom, which exists as a proton in molecules, is ejected with high energy.

14.6. The Function of the Neutron in the Nucleus

The neutron is considered to be a constituent of all nuclei. Accordingly, the earlier structure of the nucleus is revised. The nucleus is now considered to contain Z protons (charge $= + Ze$) and $(A - Z)$ neutrons. Because the masses of proton and neutron are similar the

mass of the nucleus $= Am_H$, as required. Thus, A is the total number of protons and neutrons (collectively called *nucleons*) in the nucleus. The various isotopes of an element have atoms containing identical numbers of protons (Z) but differing numbers of neutrons ($A - Z$), so that their mass numbers are different. Their chemical nature is the same since it is governed solely by Z.

14.7. Symbolic Representation of Nuclei

A nucleus is represented by its mass number A and atomic number Z; A and Z are used as superscript and subscript, respectively, of the chemical symbol representing the element: e.g.: ${}^{22}Ne_{11}$ represents the heavier neon isotope and ${}^{20}Ne_{11}$ the lighter isotope. Similarly, α-particles, (helium nuclei) are represented by: ${}^{4}He_{2}$. The fundamental particles are variously represented, e.g. ${}^{1}H_{1}$ or p for proton, ${}^{1}n_{0}$ or n for neutron, and β_{-1} or ${}^{0}e_{-1}$ for electron (relative mass is negligible, hence $A = 0$, charge $= -e$, $\therefore Z = -1$).

14.8 Nuclear Reactions

The nuclear disintegrations of light elements by α-ray bombardment are particular examples of *nuclear reactions*. Nuclear reactions are represented by equations similar to those of chemical reactions. The bombarded nucleus and bombarding particle are represented by symbols as above. Whereas the atoms that appear on the L.H.S. of a chemical equation must also appear on the R.H.S., the same nuclear symbols do not necessarily appear on the R.H.S. of a nuclear equation.

In nuclear reaction equations the following laws are always obeyed:

(i) the sum of the mass numbers on the L.H.S. must be the same as the sum of those on the R.H.S.

(ii) the sum of the atomic numbers on the L.H.S. must be the same as the sum on the R.H.S.

The first law signifies that the number of nucleons taking part in the reaction is unchanged. The second law expresses the conservation of electric charge.

Following these rules, equations can be devised for any nuclear reactions, e.g. the reacting particles in the first artificial disintegration discovered by Rutherford are a nitrogen nucleus: ${}^{14}N_{7}$ and an α-particle: ${}^{4}He_{2}$. A proton is emitted during the reaction so that the other product of the reaction has a total mass number: $(14 + 4 - 1) = 17$, and an atomic number: $(7 + 2 - 1) = 8$. These values identify the product nucleus as an oxygen isotope: ${}^{17}O_{8}$. Hence:

$$ {}^{14}N_{7} + {}^{4}He_{2} \longrightarrow {}^{1}H_{1} + {}^{17}O_{8} \quad . \quad . \quad . \quad (1) $$

The production of neutrons by the bombardment of beryllium and boron with α-rays is represented by:

$$ {}^{9}Be_{4} + {}^{4}He_{2} \longrightarrow {}^{12}C_{6} + {}^{1}n_{0}; \; {}^{11}B_{5} + {}^{4}He_{2} \longrightarrow {}^{14}N_{7} + {}^{1}n_{0} \quad (2) $$

14.9. Cockcroft and Walton Experiment

The scope of nuclear bombardments was enlarged by using high speed charged particles. In their celebrated experiment Cockcroft and Walton used protons which were produced in a hydrogen gas discharge tube and extracted through a slit in the cathode. The protons were accelerated down a long evacuated column by means of a p.d. as high as 700,000 volts maintained between the cathode and a target, containing atoms to be bombarded, at the bottom of the tube. In travelling down the tube the protons convert all their potential energy into kinetic energy.

In the Cockcroft and Walton experiment the target contained lithium atoms. The protons bombarded the lithium atoms to produce a nuclear reaction. The products of the reaction were α-particles, which were detected by scintillations produced on nearby ZnS screens. Two α-particles were produced per reaction, according to:

$$^7Li_3 + {}^1H_1 \longrightarrow 2{}^4He_2 \quad . \quad . \quad . \quad (3)$$

The kinetic energies of the α-particles were estimated from range measurements and their sum found to be much greater than the initial kinetic energy of the bombarding proton.

14.10. Einstein's Law: The Equivalence of Mass and Energy

To explain their results, Cockcroft and Walton used the Einstein Special Theory of Relativity (1905). The main deduction from Einstein's theory is that the mass of any particle increases as its velocity increases, according to the equation:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where m_o is the mass of the particle at rest, v is the velocity of the particle and c the velocity of light (in vacuo). From this result it is possible to show that whenever a particle acquires an energy E joules, a change in mass ($\triangle m$ kgm.) occurs, given by:

$$E = \triangle mc^2 \ (c \text{ in m.sec.}^{-1}) \quad . \quad . \quad . \quad (5)$$

This equation is known as *Einstein's Mass-Energy Equation.* It implies that mass and energy are equivalent and under suitable circumstances each can be converted into the other.

Equations (4) and (5) are perfectly general, but for masses other than those of atomic particles the relative changes experienced are negligible.

The later results obtained by Aston had shown that the masses of atoms were not exactly integral compared to the mass of the hydrogen atom. Furthermore, if the exact masses of the reacting and product particles are inserted in a nuclear reaction equation their sum is not the

same on each side of the equation. In other words, mass is not conserved in a nuclear reaction. It is quite clear from the Cockcroft and Walton results that energy is not conserved either. By assuming the equivalence of mass and energy it is found that *mass and energy together are conserved.*

Following Cockcroft and Walton's interpretation, the exact masses are inserted beneath the appropriate particle in the nuclear reaction:

$$^{7}Li_{3} + {}^{1}H_{1} \longrightarrow 2 \times {}^{4}He_{2} \quad . \quad . \quad . \quad (6)$$

$$7{\cdot}0182 \quad 1{\cdot}0081 \quad 2 \times 4{\cdot}0039$$

These masses are referred to an arbitrary mass scale on which the oxygen atom* has 16·00 *atomic mass units* (a.m.u.). They include the masses of the extra-nuclear electrons, but this is of no consequence since the total number of extra-nuclear electrons is unchanged. The total mass on the L.H.S. of (6) is 8·0263 a.m.u., that on the R.H.S. is 8·0078 a.m.u. The mass difference ($\triangle m = 0{\cdot}0185$ a.m.u.) is converted into energy during the nuclear reaction. One atomic mass unit = $1{\cdot}66035 \times 10^{-27}$ kgm., and the energy equivalent given by: $E = \triangle mc^2$, is $14{\cdot}94 \times 10^{-11}$ joules. Since 1 *ev* is equal to $1{\cdot}6 \times 10^{-19}$ joules†, *one atomic mass unit is equivalent to* 931 *mev.* Hence, $\triangle m \equiv 17{\cdot}2$ *mev.*, which is nearly the sum of the α-particle kinetic energies observed by Cockcroft and Walton.

The Einstein mass-energy relation is now accepted as a fundamental law in nuclear physics.

14.11. Artificial Radioactivity

The next most important step in the progress of nuclear physics occured in 1934, when Mme. Joliot-Curie and her husband M. Joliot found that aluminium foil appeared to be radioactive after being bombarded by α-particles. The radioactivity thus "induced" in the aluminium was shown to consist of the emission of particles exactly similar to β-rays but possessing a positive charge. Such particles are called *positrons* or β_{+}-rays.‡

The positron activity discovered by the Joliot's, decayed with a half-life of several minutes. By performing chemical separation experiments they showed that the activity belonged to a phosphorus isotope, produced according to:

$$^{27}Al_{13} + {}^{4}He_{2} \longrightarrow {}^{30}P_{15} + {}^{1}n_{o} \quad . \quad . \quad (7)$$

* Since oxygen contains isotopes with mass numbers 17 and 18 also, the definition of a.m.u. should refer to the major isotope of oxygen as 16·00 a.m.u.

† 1 e.v. (electron-volt) = energy acquired by "e" coulombs falling through 1 volt = $1{\cdot}6 \times 10^{-19} \times 1$ joules.

‡ First detected by Anderson in 1933 in cosmic radiation.

The phosphorus isotope $^{30}P_{15}$ is not found in nature and decays to form a stable silicon isotope:

$$^{30}P_{15} \longrightarrow {}^{30}Si_{14} + {}^{0}\beta_{+1} \quad . \quad . \quad . \quad (8)$$

where ${}^{0}\beta_{+1}$ represents a positron.

Thus, the radioactivity is not induced in the aluminium but is caused by the disintegration of a product of a nuclear reaction. The phenomenon is generally known as *Artificial Radioactivity*, because it does not occur naturally.

Many nuclear reactions are now known in which radioactive isotopes are produced. All artificial radioisotopes obey the ordinary laws of natural radioactivity and their half-lives are characteristic of the isotopes.

14.12. Production of High Speed Particles

The field of nuclear reactions was greatly enlarged by the invention and operation of machines designed to accelerate protons and other charged particles to very high speeds. These machines are roughly divided into two groups: (i) *D.C. Accelerators* and (ii) *Resonant Accelerators*.

14.13. D.C. Accelerators—Van de Graaf Generator

These machines accelerate charged particles by allowing them to travel from a high potential terminal to earth in a vacuum tube, as in the Cockcroft-Walton experiment. An important example is the Van de Graaf generator This was invented in 1931 by Van de Graaf in America. It is based on electrostatic principles. The endless belt *A* (Fig. 14.3), is made of insulating material such as thin rubber sheet. It is driven at high speed past the metal points *B*, which are connected to a D.C. source of p.d. of several kilovolts. The charged points cause electric charge to be " sprayed " on to the belt. The charge is carried to the inside of the hollow metal terminal *D*, where it is taken off by another set of metal points *C*. The latter are connected to *D*, which is supported on an insulating column, so that its potential increases relative to the earth.

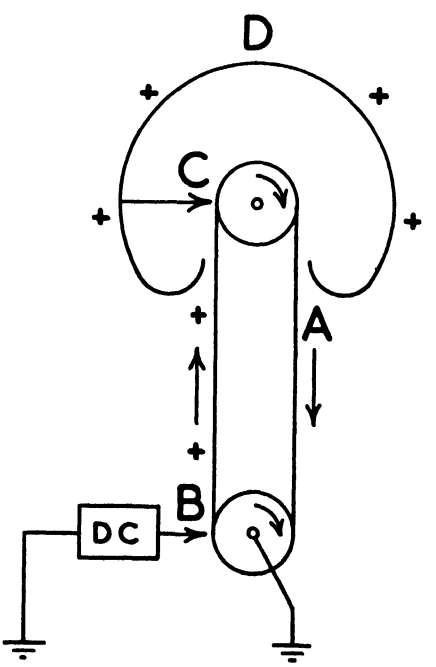


Fig. 14.3

The accelerated particles are usually positive ions produced by means of a gas discharge in a tube located inside D. Since D is almost a complete hollow sphere all points inside it are at the potential of the sphere (p. 24). If this is a high positive potential, the positive ions which emerge through a slit in the cathode of the discharge tube, are accelerated down an evacuated glass column to an earthed target. The target is coated with the substance to be bombarded.

The action of the generator depends on gas discharge at B and C. The air surrounding B is partially ionized because of the intense electric field that exists near the points (p. 27). Negative ions flow to B whilst positive ions are driven to the belt and carried away. At C the charges on the belt have a higher potential than C because they have the potential of the belt in addition to that which they acquire on entering D. Hence the converse of the charging process at B takes place at C. In effect, therefore, the charge on the belt is transferred to D.

The ultimate potential of D is limited by discharge which occurs when the surface density of charge is such that the electric field strength at the surface exceeds the breakdown strength (p. 27). The machine is run continuously and the potential of D remains constant, the charge lost by breakdown being equal to that gained from the moving belt. For dry air at normal atmospheric pressure the maximum p.d. may be several million volts. This is increased by inserting the whole machine in a pressure vessel containing gas, such as freon or sulphur dioxide, at high pressure; this reduces the leakage from D.

14.14. Resonator Accelerators—Cyclotron

In this class, charged particles pass to and fro in an alternating electric field with a similar frequency, gaining energy each time they do so. A typical example is the cyclotron which was first operated by Lawrence and Livingston in 1931.

The diagram of Fig. 14.4, shews the main features in plan and elevation. It is used to accelerate positive ions which are produced by ionizing a fine stream of gas entering at the centre O between the " dees " D_1 and D_2. The ionization is effected by an electron current from a heated filament. The dees are hollow semi-circular metal boxes whose cross-section has the shape of a capital D. They are supported inside a large evacuated chamber A, and are connected externally to a high frequency oscillator having an amplitude of several thousand volts. The chamber A lies between the pole faces of an electro-magnet producing a uniform magnetic field of about 10^6 amp.m.$^{-1}$

A positive ion is directed to D_1 or D_2, by the electric field existing across the gap G. If D_1 is negative the ion enters D_1. The magnetic field acting normally to the plane of motion of the ion causes it to be bent in a circular path whose radius r m. is given by:

$$Bev = \frac{mv^2}{r} \text{ (cf. p. 372)} \quad . \quad . \quad . \quad . \quad (9)$$

(B is the magnetic induction in wb.m.$^{-2}$, e is the charge on the ion in coulombs, m is the mass in kgm., and v the linear velocity in m.sec.$^{-1}$).

If the p.d. between D_1 and D_2 is reversed by the time the ion re-enters G, it is accelerated again across the gap. The ion enters D_2 with an increased linear velocity, so that it follows a semi-circular path of larger radius. If the ion leaves D_2 when the field is reversed again, it is accelerated once more on crossing G and follows a semi-circular path in D_1 of increased radius. This process continues until the ion reaches the periphery of the dees, providing the resonance condition holds, i.e. the frequency with which the ion crosses G is the same as the applied p.d. frequency.

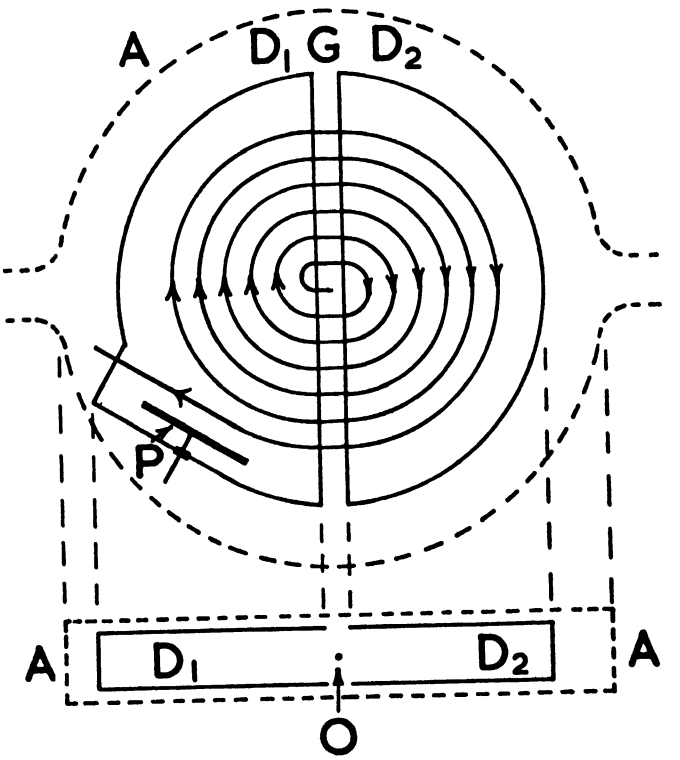


Fig. 14.4

From (9) the angular velocity of the ion is:

$$\omega = \frac{v}{r} = \frac{Be}{m} \text{ radians.sec.}^{-1} \quad . \quad . \quad . \quad (10)$$

The time taken for an ion to travel in the dees round any semi-circle is:

$$t = \frac{\pi}{\omega} = \frac{\pi m}{Be} \text{ secs.} \quad . \quad . \quad . \quad . \quad (11)$$

For resonance to hold t must be half the period of the oscillator. If the oscillator frequency is ν c/s., $t = \frac{1}{2\nu}$ secs. For resonance, therefore:

$$\frac{1}{2\nu} = \frac{\pi m}{Be}$$

i.e.
$$\nu = \frac{Be}{2\pi m} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Before reaching the edge of the dees the ions are deflected out of the chamber by applying a fairly high D.C. potential to the deflector plate P. The ions leave via a thin metal foil window W.

14.15. Modern Detection of Nuclear Particles—Geiger Counter

Ionization of a gas (directly or indirectly) is the common property of nuclear radiations most widely used in modern detectors, the most common of which is the Geiger-Müller tube. This consists of a fine metal wire A stretched along the axis of a glass tube containing gas at low pressure (few cm. Hg). The wire is called the anode since it is maintained at a high positive potential (several hundred volts) relative to a cylindrical metal cathode C. The tube together with a basic counting circuit is shown in Fig. 14.5.

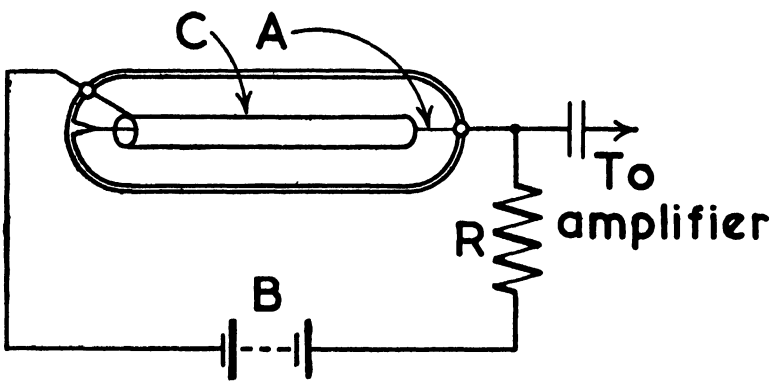


Fig. 14.5

When an ionizing radiation passes through the gas inside the tube positive ions and electrons are formed. The electrons are accelerated towards A. Within the close vicinity of A the electric field strength increases rapidly ($E \propto 1/r$). The electrons rapidly gain energy from this field and ionize gas atoms by collision, producing more positive ions and electrons. The additional electrons also gain energy from the field and ionize. The ionization rapidly becomes cumulative and in a time interval, less than 10^{-6} seconds, an extremely large number of electrons is formed very close to A and along the whole of its length. This rapid

production of a large number of electrons by each single primary electron is called an " avalanche ". The electrons are collected at A and give rise to a small current in R.

The transient current in the external circuit gives rise to a rapid change in voltage across R. This voltage pulse is amplified and eventually made to operate a mechanical counter or cause a loudspeaker to emit a characteristic click. By such means the ionizing particles (or photons) can be counted.

14.16. Nuclear Energy

Mass Defect and Binding Energy

The mass of the nucleus is not exactly integral compared with the hydrogen atom, but, apart from this, it is not exactly equal to the sum of the separate nucleon masses; it is always less. In other words:

$$(A - Z)m_n + Zm_H > M \quad . \quad . \quad . \quad (13)$$

where, m_n, m_H and M, are the masses of a neutron, proton and nucleus. From this it is inferred that if a nucleus were created from its separate nucleons by some process, there would be a loss in mass given by:

$$\triangle M = (A - Z)m_n + Zm_H - M \quad . \quad . \quad . \quad (14)$$

the difference in mass, $\triangle M$, is called the *mass defect* of the nucleus.

Theory supposes that the mass defect is converted into energy when a nucleus is created. The energy equivalent, given by: $\triangle E = \triangle Mc^2$, is called the *binding energy* of the nucleus. The same amount of energy must be provided before a nucleus can be broken up into all its separate nucleons, so that it is a measure of the cohesion of the nucleus.

The binding energies of nuclei increases with mass number, it being about 28 mev. for helium and about 1800 mev. for uranium. The curve relating binding energy (BE) and mass number (A) is roughly a straight line (Fig. 14.6), indicating that the binding energy *per nucleon* is fairly constant and equal to about 8·0 mev. Since all nuclei can be imagined being built up by adding more nucleons to a lighter nucleus, this means that on the average each nucleon loses mass equivalent to 8·0 mev. energy. There is a most important deviation from this simple scheme: for the heavier nuclei the binding energy per nucleon falls somewhat as the number of nucleons is increased, e.g. for uranium the binding energy per nucleon is about 7·5 mev.

The rough-constancy of binding energy per nucleon suggests that the nucleons are held together by *short-range binding forces*. Similar forces are responsible for the cohesion of molecules of a liquid. The latter forces are effective over a range comparable to the size of molecules ($\sim 10^{-8}$cm.), whilst in the nucleus the binding forces are effective over the range comparable to the size of the nucleus ($\sim 10^{-12}$cm.).

The cohesive forces of a liquid are said to be unsaturated at the surface because the surface molecules are not completely surrounded by other molecules and, hence, have a net force acting normally to the surface inwards giving rise to surface tension. The surface molecules are thus loosely bound and require less energy to overcome the restraining forces and evaporate. A similar state of affairs exists in a nucleus which is compared to a very small liquid drop. As the size of the nucleus increases the surface nucleons become more loosely bound and hence the binding energy per added nucleon is reduced.

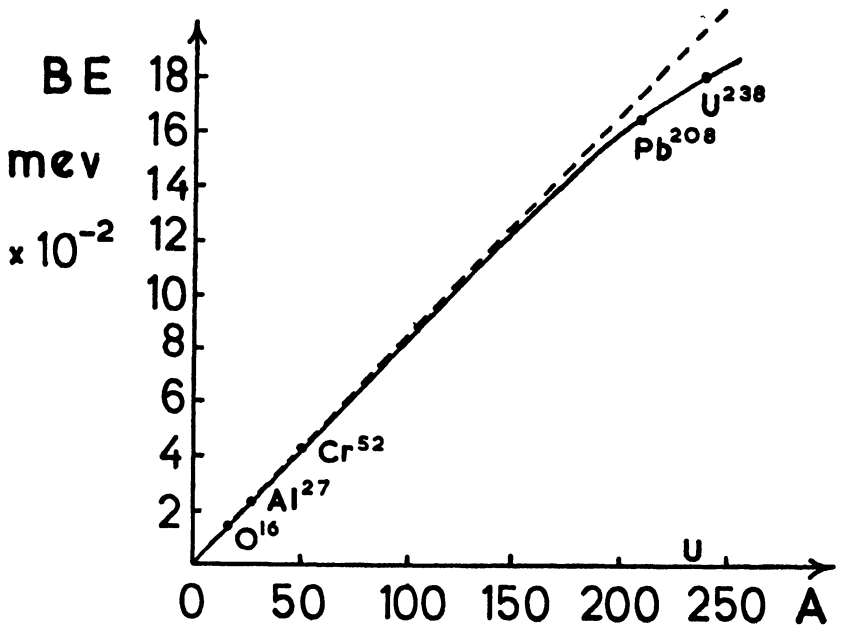


Fig. 14.6

Example

The masses of a proton and neutron are 1·0076 *and* 1·0089 *atomic mass units, respectively. If the mass of a helium nucleus is* 4·0029 *atomic mass units what is the binding energy of the helium nucleus?* (1 a.m.u. = 931 mev.).

The helium nucleus ($^4He_2$) contains two protons and two neutrons, so that if a helium nucleus were formed by combining these nucleons together the loss in mass would be:

$$\Delta m = 2m_H + 2m_n - m_{He} = 2 \times 1{\cdot}0076 + 2 \times 1{\cdot}0089 - 4{\cdot}0029$$

i.e. $\Delta m = 0{\cdot}0301$ a.m.u.

Hence: $E = 931{\cdot}\Delta m = 27{\cdot}96$ m.e.v.

i.e. the binding energy of the *He* nucleus is 27·96 m.e.v.

14.17. Nuclear Forces and Nuclear Disintegration

The origin of the nuclear binding forces is unknown. Because of the forces the nucleons are not fixed and are pictured as being in constant

motion, converting potential energy into kinetic energy and vice versa. In their motions the nucleons collide with each other, gaining or losing energy during the collision. Radioactive nuclei are unstable due to the particular number of neutrons and protons they possess and a greater degree of stability of the nucleons is produced by ejecting particles.

The emission of α-, β_- or β_+-rays involves a change which results in the formation of a new nucleus. The new nucleus may find that it has more energy than it should possess for its given complement of nucleons. The excess energy is dissipated as a γ-photon whose frequency is given by: $\triangle E = h\nu$, where $\triangle E$ is the excess energy of the " excited " nucleus. Thus, γ-emission generally accompanies α- and β-emissions.

14.18. Theory of Nuclear Reactions

In a nuclear reaction a bombarding particle enters a nucleus and loses mass which is converted into binding energy. This energy and any excess kinetic energy it may have is shared among the other nucleons by collision. As a result, one of the nucleons may gain sufficient energy to overcome surface forces and escape.

The penetration of a nucleus by a bombarding particle results in the formation of an unstable compound nucleus called an *intermediate nucleus*. A γ-photon, a proton, a neutron or an α-particle may be emitted as a result of energy sharing in the intermediate nucleus. It is thus possible for several different types of reaction to occur for a nucleus bombarded by a particle. This is found to be true in practice, but each type of reaction is governed by energy considerations, since a minimum energy is required for the emission of any particle.

14.19. Neutron Reactions: Capture

Neutrons are able to penetrate nuclei easily because they have no charge. They give rise to various reactions but the most common is the retention of the neutron by a nucleus and the emission of a γ-photon. This results in a nucleus of the same atomic number but mass number increased by unity, i.e. the product nucleus is an isotope of the bombarded nucleus. If the isotope occurs in nature it is stable, eg.:

$$^1H_1 + {}^1n_0 \longrightarrow {}^2H_1 + \gamma \quad . \quad . \quad . \quad . \quad (15)$$

In this reaction hydrogen is converted into the heavy isotope *deuterium*.

If the isotope formed by such a *capture collision* is not found in nature, it is artificially radioactive, emitting a β_- or a β_+ ray, e.g.:

$$^{115}In_{49} + {}^1n_0 \longrightarrow {}^{116}In_{49} + \gamma \quad . \quad . \quad . \quad (16)$$

$$^{116}In_{49} \longrightarrow {}^{116}Sn_{50} + {}^0\beta_{-1}$$

Most *radio-isotopes* are formed by this process in an atomic pile.

Capture reactions are most easily effected by neutrons which have

been slowed down by repeated collisions with light elements, i.e. *slow neutrons*. Neutrons that are emitted in reactions are called *fast neutrons*, their energies being in the mev. region. When neutrons are passed through light material they are eventually slowed down to such an extent that they have similar energies to the molecules of the material. Such neutrons are called *thermal neutrons*; their energies are about 0·028 ev. at ordinary temperatures.

Particle Ejection Reactions

Slow neutrons are sometimes able to effect the emission of an α-particle from a bombarded nucleus. This process takes place very readily with an isotope of boron, viz.: $^{10}B_5$; the isotope is present in ordinary boron to the extent of 19 per cent. The reaction is represented by:

$$^{10}B_5 + {}^1n_o \longrightarrow {}^7Li_3 + {}^4He_2 \quad . \quad . \quad . \quad (17)$$

This reaction is utilized for the efficient detection of neutrons. An ionization chamber containing boron trifluoride gas (BF_3) is used: α-particles are emitted according to (17) and the ionization caused by the α-rays is a measure of the neutron activity.

14.20. Nuclear Fission

Fermi and his associates carried out experiments in Italy in 1934, in which uranium was bombarded with slow neutrons. They found several β-emitting nuclei were formed with different half-lives. Fermi supposed that the β_- rays came from successive transformations of the intermediate $^{239}U_{92}$ nucleus, according to the scheme:

$$^{239}U_{92} \longrightarrow {}^{239}X_{93} + {}^0\beta_{-1}; \quad {}^{239}X_{93} \longrightarrow {}^{239}Y_{94} + {}^0\beta_{-1}, \text{ etc.} \quad (18)$$

The unknown elements X, Y, etc., were called *transuranics*.

Different observers could not agree on the number of transuranics formed by the neutron bombardment of uranium. The problem of what really happened in such experiments was solved by Hahn and Strassmann in 1938. They proved, by chemical methods, that radioactive atoms of barium and lanthanum, having masses of 137 and 138, were being formed. It was suggested by Frisch and Meitner in 1939 that the uranium nucleus was split up into roughly equal parts in the bombardment and the process was called *nuclear fission*. Almost immediately the idea was confirmed by noting the intense ionization produced by the very energetic products of the reaction called *fission products*. Further experiment shewed that the uranium nucleus splits up into two parts having a mass ratio 4:3 approx.

14.21. Nuclear Energy and Fission

Nuclear fission can be likened to the splitting up of a liquid drop into two smaller drops. The process is caused by the neutrons hitting the

drop and setting up an oscillatory motion which ends in the division of the drop. The process is governed by energy considerations.

The average binding energy per nucleon of the heaviest nuclei is appreciably less than that of atoms of medium atomic weight ($A = 30 - 150$). Hence, if the uranium nucleons form two smaller nuclei of medium atomic weight, they have to lose more mass as binding energy of the new nuclei. The difference in binding energy per nucleon multiplied by the number of nucleons involved, viz. 238, gives the total energy released per fission. This is about 200 mev., which is enormous compared to the few mev. generally released in a nuclear reaction.

In practice about 160 mev. is retained as kinetic energy of the fission products. Some is liberated as γ-photons and some as kinetic energy of neutrons released on fission. The kinetic energy of the fission products is about 10^{-11} joules per fission. If every atom of a kilogram of uranium is fissioned it means the release of: $2{\cdot}53 \times 10^{24} \times 160$ *mev.* $\equiv 2{\cdot}53 \times 10^{13}$ joules, i.e. about 6×10^{12} calories. This is equivalent to the energy derived from the detonation of 20,000 tons of T.N.T. or the burning of 7,000 tons of coal.

14.22. Nuclear Chain Reactions

The successful applications of nuclear energy to atomic weapon and power production rely on the free neutrons which are released at each fission of uranium. On the average about 2·5 neutrons are released per fission. These are fast neutrons with energies of several mev. They can initiate fissions with neighbouring nuclei so that more free neutrons are produced and so on. The process can thus lead to a *chain reaction.*

A chain reaction is one which is initiated at some point in a mixture of reactants and spreads without further assistance from external sources. The burning of a match is a familiar example. This reaction is initiated at some point in the match head by the heat of friction. It proceeds throughout the whole of the combustible mixture, ultimately causing the burning of the match stem. In the atomic bomb a mass of uranium is rapidly fissioned producing an enormous quantity of heat in a very short time. In nuclear power reactors the fission of uranium, etc., is maintained in a steady controlled chain reaction; the heat evolved is used to generate steam and drive normal electric generators.

EXERCISES

1. Describe some form of electrostatic generator for the production of high voltages, and explain how it works.

   Water-drops 6 mm. in diameter fall from a jet maintained at a potential of 600 volts into a hollow spherical vessel 5 cm. in diameter, which is well insulated. Estimate the highest voltage reached by the vessel, and the electrical energy gained in the process. What is the source of this energy? (Oxford Local G.C.E. Adv. L.).

2. Describe and explain the action of a machine used to produce a high electrostatic potential.

Two flat parallel plates are at a potential difference of 3,000 volts and are situated in a vacuum. If an electron of mass 9×10^{-28} gm. and charge $1{\cdot}6 \times 10^{-19}$ coulombs leaves the plate at lower potential from rest, calculate the velocity with which it will strike the other plate. (Durham G.C.E. Adv. L.).

3. Describe the principles of any electrostatic generator (such as a Wimshurst or Van der Graaf generator) capable of reaching a high electrical potential. Explain carefully how mechanical power is converted to electrostatic energy, and describe an experiment to verify this principle in the laboratory. A sphere 2 m. in diameter is charged to a potential of 30,000 volts negative. Calculate the gain in mass. (e/m for electrons is $1{\cdot}76 \times 10^{11}$ coulombs.kgm.$^{-1}$.) (Oxford Open Schol.).

4. Write an essay on *one* of the following:
   (*a*) the constitution of the sun.
   (*b*) the investigation of the upper atmosphere.
   (*c*) Methods of *either* producing *or* detecting high energy particles.
   (Oxford Open Schol.).

5. Explain the term *binding energy of a nucleus*. Indicate how the binding energy per nucleon varies with the atomic mass number and give your reasons for believing that nuclear fission is possible only with heavy elements.

6. Write an essay on nuclear fission and the production of nuclear power.

7. Write short notes on *three* of the following:
   (*a*) α-rays.
   (*b*) artificial radioactivity.
   (*c*) cathode rays.
   (*d*) Planck's constant h.
   (*e*) the Wilson cloud chamber.
   (Oxford Open Schol.).

8. Write short notes on *three* of the following:
   (*a*) isotopes.
   (*b*) neutrons.
   (*c*) photons.
   (*d*) the Geiger counter.
   (*e*) the cyclotron.
   (Oxford Open Schol.).

CHAPTER 15

ADDITIONAL MAGNETISM AND ELECTROSTATICS

Magnetic Measurements

15.1. The Deflection Magnetometer

The deflection magnetometer consists of a small pivoted magnet (*ns*) enclosed within a glass covered draughtproof brass case (*C*), (Fig. 15.1). Attached at right angles to *ns* is a light aluminium pointer (*P*) which moves over a circular scale graduated in degrees. Attached to *C* are two long wooden arms *A* and *B*.

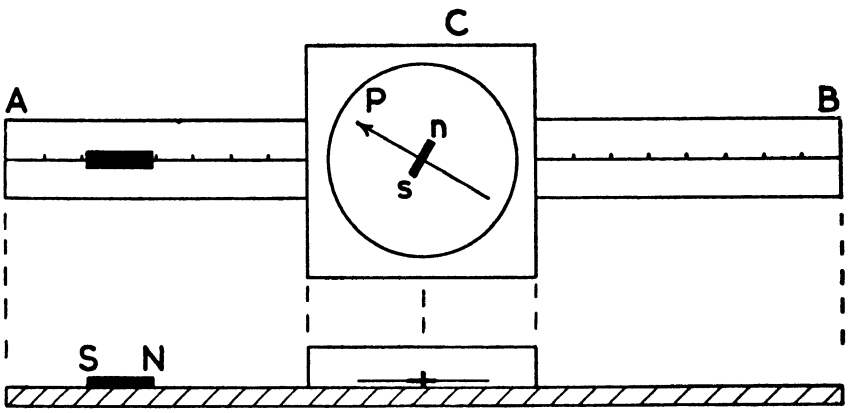


Fig. 15.1

15.2. Determination of Field Strength of Permanent Magnet

The deflection magnetometer is set with its arms pointing east and west. The magnet *ns* is placed on one arm with its axis parallel to the scale (Fig. 15.1). The field of the magnet (*F*) is sensibly uniform at *ns* and at right angles to the earth's field (H_H). Hence the deflection of *ns* is given by:

$$F = H_H \tan \theta \quad . \quad . \quad . \quad . \quad (1)$$

For points on the perpendicular bisector of the magnet the arms of the magnetometer are arranged to point N–S and the magnet is placed on one arm with its axis pointing E–W. The deflection is again given by (1).

Sources of Error and Corrections

(i) The axis of the magnet may not coincide with its geometric axis. This source of error is avoided by repeating the reading with the magnet turned over. (ii) The arms *A* and *B* may not be quite perpendicular

to H_H. The error is avoided by repeating the readings with the magnet at the corresponding position on the opposite arm. (iii) The angular scale may not be orientated properly, i.e. the line joining the zeros may not be parallel to H_H. The error is avoided by reading both ends of the pointer always. Thus, 4 readings are taken with the magnet on each arm of the magnetometer and the mean of these 8 readings is used.

Limitations of Magnetometer

Because of the rapid variation of $\tan\theta$ with θ a small inaccuracy in measuring θ outside the range 30° to 70°, leads to large errors in F. Using $H_H = 15$ amp.m.$^{-1}$ (approx.), this limits F to the range: 15 tan 30° to 15 tan 70°, i.e. 6 to 41 amp.m.$^{-1}$ (approx.). Since H_H is very dependent on the presence of iron in the form of pipes, girders, etc., the magnetometer is restricted to comparative measurements.

15.3. Reflection Magnetometer

The range and sensitivity of the deflection magnetometer is increased by using a very small magnet mounted on a small piece of mica sheet and suspended inside a draughtproof case. Small deflections of the magnet are measured accurately by reflecting a light beam from a small concave mirror attached to the fibre. By this means differences in field strengths as small as 0·1 amp. m.$^{-1}$ can be measured.

15.4. Comparison of Magnetic Moments

Each magnet is placed in turn on the magnetometer as in **15.2.** In this case $F = \dfrac{2Mr}{4\pi\mu_o(r^2 - l^2)^2}$ and the deflection of the magnetometer needle is given by:

$$\frac{2Mr}{4\pi\mu_o(r^2 - l^2)^2} = H_H \tan\theta \quad . \quad . \quad . \quad (2)$$

where r is the distance from the centre of the magnet to the centre of the magnetometer. It follows from (2) that the ratio of the moments is given by:

$$\frac{M_1}{M_2} = \frac{r_2 \cdot (r_1^2 - l_1^2) \cdot \tan\theta_1}{r_1 \cdot (r_2^2 - l_2^2) \cdot \tan\theta_2} \quad . \quad . \quad . \quad (3)$$

Inaccuracy results from the uncertainty in determining l_1 and l_2 (see example below). If the bar magnets are " short ", i.e. $r \gg l$, (3) may be simplified:

$$\frac{M_1}{M_2} = \frac{r_1^3 \cdot \tan\theta_1}{r_2^3 \cdot \tan\theta_2} \quad . \quad . \quad . \quad . \quad (4)$$

Example

Describe, how, using a tangent galvanometer and an ammeter, you would find the magnetic moment of a bar magnet.

If a deflection magnetometer and a bar magnet, moment M, *magnetic length* 2l, *is used in the end-on (Gauss A) position, the deflection is* θ_1 *when the centre of the bar*

is placed at a distance d_1 *from the centre of the needle and* θ_2 *when the distance is* d_2 ($d_2 > d_1$). *Show that, neglecting terms involving the fourth and higher powers of* l/d,

$$\frac{M}{H} = \frac{2\pi\mu_o\,(d_2^5 \tan\theta_2 - d_1^5 \tan\theta_1)}{d_2^2 - d_1^2}$$

What form would this equation take if used in the experiment referred to in the first part of the question? (Northern H.S.C. Schol.).

The galvanometer is set with the plane of its coil parallel to the magnetic meridian and the magnet is placed on the axis of the coil with its centre, d m. say, from the centre of the coil O (Fig. 15.2). The galvanometer deflection θ is given by:

$$H_A = \frac{2Md}{4\pi\mu_o\,(d^2 - l^2)^2} = H_H \tan\theta \quad . \quad . \quad . \quad (5)$$

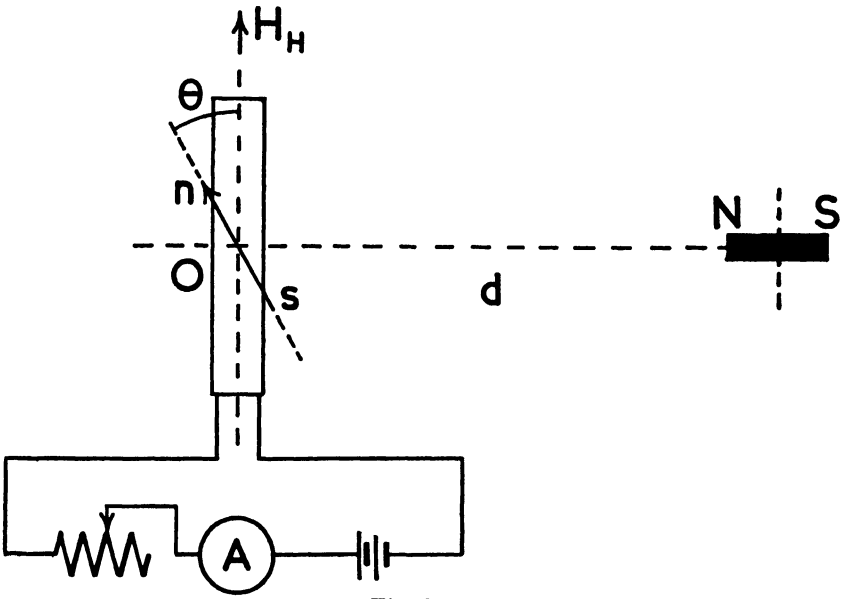


Fig. 15.2

The magnet is now removed and the galvanometer connected to the circuit shown. The current is adjusted to give the same galvanometer deflection θ. If this current strength is I amps:

$$\frac{NI}{2r} = H_H \tan\theta \quad \text{(cf. p. 121)}. \quad . \quad . \quad . \quad . \quad (6)$$

Hence:
$$\frac{NI}{2r} = \frac{2Md}{4\pi\mu_o\,(d^2 - l^2)^2}$$

i.e.
$$M = \frac{\pi\mu_o NI}{r}\,\frac{(d^2 - l^2)^2}{d} \text{ (wb.m.)} \quad . \quad . \quad . \quad . \quad (7)$$

Providing the equivalent length of the magnet ($2l$) is known, M is found from this expression. The measurement of $2l$ is avoided below.

In the problem, the magnetometer deflection for the first Gauss A position is given by:

$$\frac{2Md_1}{4\pi\mu_o\,(d_1^2 - l^2)^2} = H \,.\, \tan\theta_1 \quad . \quad . \quad . \quad . \quad (8)$$

For the second position:

$$\frac{2Md_2}{4\pi\mu_o (d_2^2 - l^2)^2} = H \,.\, \tan\theta_2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From (8): $$\frac{2M}{4\pi\mu_o d_1^3}\left(1 - \frac{l^2}{d_1^2}\right)^{-2} = \mathrm{H} \tan\theta_1$$

i.e. $$\frac{2M}{4\pi\mu_o d_1^3}\left(1 + \frac{2l^2}{d_1^2} + \frac{3l^4}{d_1^4} + \ldots\right) = H \tan\theta_1$$

making use of the approximation given:

$$\frac{2M}{4\pi\mu_o d_1^3}\left(1 + \frac{2l^2}{d_1^2}\right) = H \tan\theta_1$$

i.e. $$\frac{2M}{4\pi\mu_o d_1^3} + \frac{4Ml^2}{4\pi\mu_o d_1^5} = H \tan\theta_1$$

hence: $$4Ml^2 = 4\pi\mu_o d_1^5 H \tan\theta_1 - 2Md_1^2$$

From (9): $$4Ml^2 = 4\pi\mu_o d_2^5 H \tan\theta_2 - 2Md_2^2$$

Hence: $$2\pi\mu_o H (d_2^5 \tan\theta_2 - d_1^5 \tan\theta_1) = M (d_2^2 - d_1^2)$$

i.e. $$\frac{M}{H} = 2\pi\mu_o \frac{(d_2^5 \tan\theta_2 - d_1^5 \tan\theta_1)}{d_2^2 - d_1^2} \quad Q.E.D. \quad . \quad . \quad . \quad (10)$$

If, in the experiment, the magnet is used in two positions, distance d_1 and d_2m. from O, and the current strengths required to give the same corresponding deflections θ_1 and θ_2, are I_1 and I_2 amps., respectively, then $H \tan\theta_1 = NI_1/2r$ and $H \tan\theta_2 = NI_2/2r$, and (10) becomes:

$$M = \frac{\pi\mu_o N(d_2^5 I_2 - d_1^5 I_1)}{r(d_2^2 - d_1^2)}$$

This allows M to be determined without a knowledge of $2l$.

15.5. The use of the Vibration Magnetometer

1. *The Field Strength on the axis of a Bar Magnet*

The specimen magnet is arranged with its axis pointing N–S. If F is the field strength of the magnet alone at some point on its axis, the resultant field strength is: $H = F \pm H_H$. The time for 10 or more complete oscillations is found when the vibration magnetometer magnet is displaced through a small angle and released. Hence, the period T and the frequency $n = 1/T$, are found. The magnet is then removed and the period T_o (frequency n_o), is determined for vibrations in the earth's field alone.

Supposing the specimen N pole points N:

$$\frac{1}{n} = T = 2\pi\sqrt{\frac{I}{M(F + H_H)}} \text{ and } \frac{1}{n_o} = 2\pi\sqrt{\frac{I}{MH_H}}$$

whence:
$$\frac{n^2}{n_0^2} = \frac{F + H_H}{H_H} = \frac{F}{H_H} + 1$$

$$\therefore \quad \frac{F}{H_H} = \frac{n^2}{n_0^2} - 1 = \frac{n^2 - n_0^2}{n_0^2} \quad . \quad . \quad . \quad (11)$$

The ratio of field strengths at two points on the axis is:

$$\frac{F_1}{F_2} = \frac{n_1^2 - n_0^2}{n_2^2 - n_0^2} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

2. *Comparison of Magnetic Moments*

The frequencies (n_1 and n_2) of the magnetometer vibrations are determined separately at points on the axes of two magnets, each arranged with its N pole pointing north. From (12):

$$\frac{F_1}{F_2} = \frac{2M_1 r_1}{4\pi\mu_0(r_1^2 - l_1^2)^2} \cdot \frac{4\pi\mu_0(r_2^2 - l_2^2)^2}{2M_2 r_2} = \frac{n_1^2 - n_0^2}{n_2^2 - n_0^2}$$

i.e.
$$\frac{M_1}{M_2} = \frac{n_1^2 - n_0^2}{n_2^2 - n_0^2} \cdot \frac{(r_1^2 - l_1^2)^2}{(r_2^2 - l_2^2)^2} \cdot \frac{r_2}{r_1} \quad . \quad . \quad . \quad (13)$$

If the magnets are " short " so that $r_1 \gg l_1$ and $r_2 \gg l_2$, (13) becomes:

$$\frac{M_1}{M_2} = \frac{n_1^2 - n_0^2}{n_2^2 - n_0^2} \cdot \frac{r_1^3}{r_2^3} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For very strong magnets the earth's field may be neglected and (14) then becomes:

$$\frac{M_1}{M_2} = \frac{n_1^2 r_1^3}{n_2^2 r_2^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

15.6. Neutral Points

Because of the superimposed horizontal field of the earth the field of a magnet depends on the orientation of the magnet in the earth's field. Depending on the orientation there are usually two points at which the earth's field is equal and opposite to the field of the magnet alone. These are known as *neutral points*. The magnetic field varies slowly from zero in the vicinity of a neutral point and this allows the rough location of a neutral point with a small compass needle. The needle shows no tendency to point in a particular direction near the neutral point. The position of a neutral point becomes obvious when lines of force are plotted. The most accurate method of finding a neutral point uses *directional loci*.

15.7. Directional Loci Method of Finding Neutral Points

This method is based on the fact that the locus of points having a given direction of magnetic intensity must pass through the neutral

point. The neutral point lies at the intersection of two ci. The method is best illustrated by an example, viz. the neutral points in the field of a magnet pointing E–W.

The magnet is placed with its axis on a large sheet of squared paper the lines being parallel and perpendicular to the magnetic meridian. Lines (directional loci) are drawn on the paper joining points at which the compass needle points in one of two directions; for convenience the directions chosen are parallel and perpendicular to the meridian (Fig. 15.3). The intersections of the loci give the neutral points P and Q.

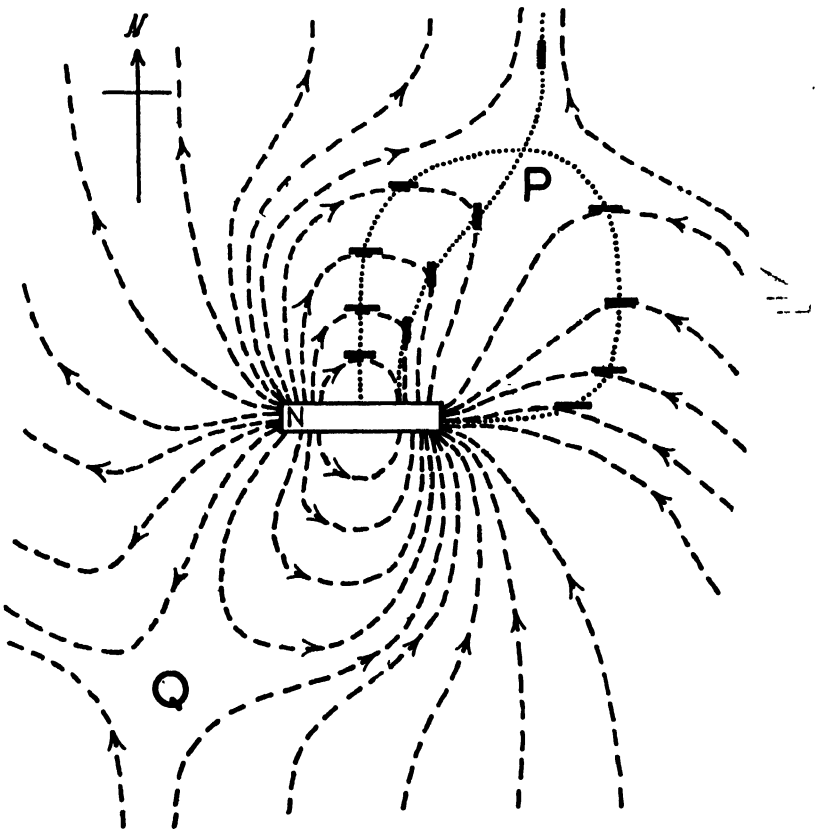


Fig. 15.3

Example

Describe and explain how, if you were provided with a large sheet of paper, a pl. compass, and a suitable bar magnet, you would determine the magnetic moment o bar magnet, given the value of the horizontal component of the earth's magnetic fi.d and the absolute permeability of free space.

A small powerful bar magnet lies in the magnetic meridian with its N pole pointi.a north. An oscillation magnetometer placed 20 cm. to the north of the magnet mi.. 100 oscillations in 100 seconds. When the magnet is removed the time of 100 oscil tions is found to be 200 seconds. Calculate the magnetic moment of the bar mag.. given that the horizontal component of the earth's magnetic field is 16 amp.m.- (Cambridge Local G.C.E. Adv. L.).

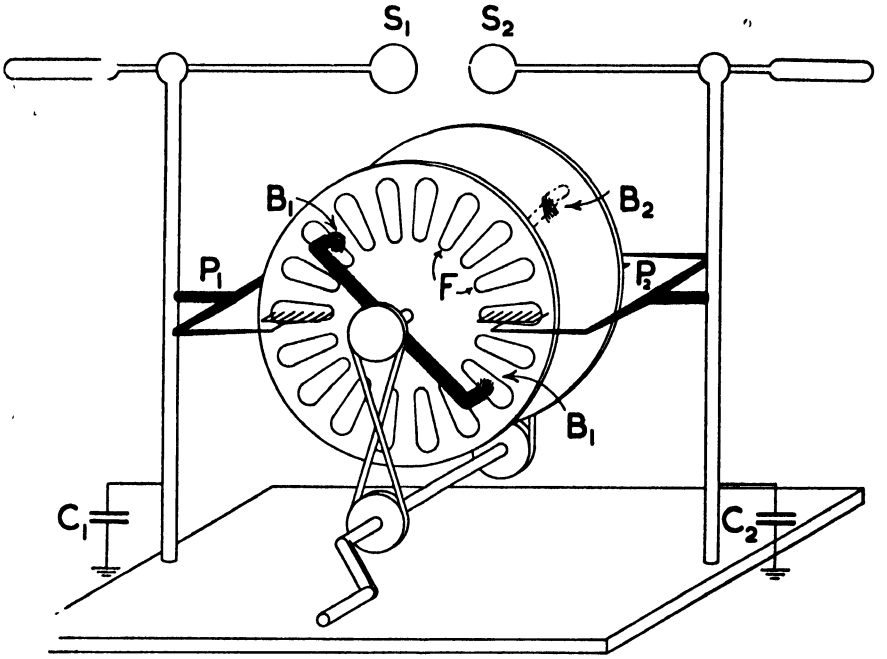


Fig. 15.12

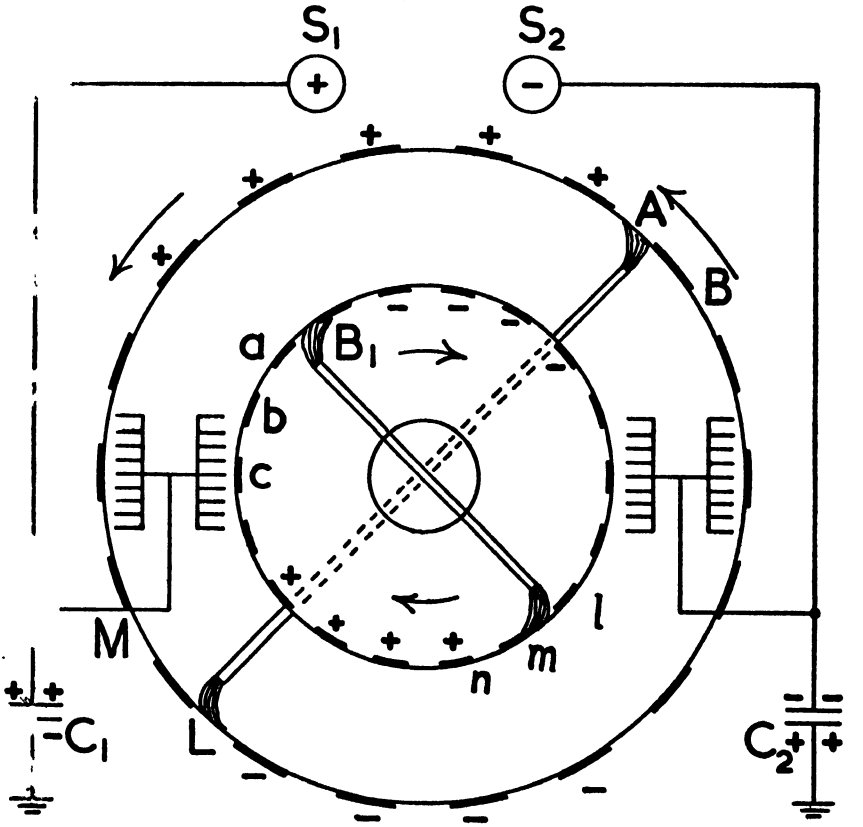


Fig. 15.13

discs. Suppose the region near B_1 is at a higher potential than the rest of the apparatus. As foil "a" passes this region it is in momentary contact with "l" and positive charge flows from "a" to "l" until both are at the same potential, each foil forming one half of a composite conductor.

As the disc rotates the two foils "a" and "l" are isolated again and "a" is left with a negative charge and "l" with a positive charge. The same happens to successive pairs of foils "b", "c", . . . and "m", "n",. . . . When "a" reaches a point opposite A of the second disc, A is in momentary contact with L, which is then opposite "l". A and L are in the fields of "a" and "l" and induction occurs, resulting in charge (positive) leaving L and flowing to A in order to equalise the p.d. across the connecting wire. Thus, L becomes negatively charged and A positively charged by induction. The charges remain as the disc rotates. Successive foils such as B and M pass under "b" and "m" and acquire charges in a similar manner. The charged foils subsequently pass beneath the collecting points where the charge is removed, the R.H.S. acquiring negative charge and the L.H.S. positive charge. Thus, a p.d. is created between S_1 and S_2, as indicated by their charges.

The Wimshurst machine relies on the fact that some point near the apparatus is at a higher (or lower) potential than all other points. This is generally true and the machine operates without any external influence. The efficiency of the machine depends on the loss of charge by leakage over the insulating surfaces. A maximum p.d. is produced when the rate of collection of charge by the points is equal to the rate of loss of charge.

Since the quantity of charge produced by a Wimshurst machine is extremely small it cannot supply appreciable power. The maximum current which might be provided without lowering the p.d. appreciably, is probably less than one microampere.

Professor Felici of Grenoble has recently produced electrostatic generators, based on similar principles, to provide currents up to a few milliamps at about 250 kilovolts. The distinctive features of these machines are (i) the use of compressed gas as the dielectric surrounding the machine and (ii) the high speed of rotation of the conducting elements. The use of compressed gas raises the breakdown potential so that loss of charge by gas ionization is much reduced. The machines can be used for motor-car ignition, supplying high voltages for cathode ray tubes, etc.

15.17. Electrostatic Instruments—Electroscopes and Electrometers

The deflection of the gold leaf electroscope is proportional to the voltage applied to the electroscope cap (approximately). Unfortunately, the robust type described earlier (Chap. 1) requires a few hundred volts to give an appreciable deflection. To measure lower p.d.'s the electroscope is modified or an electrometer used instead.

(i) *The Condensing Electroscope*

This is a modified gold-leaf electroscope in which a varnished metal plate (P), which is attached to an insulating handle, is placed in contact with the cap C of the electroscope (Fig. 15.14). The small potential to be measured is momentarily connected to C and P is earthed as shown. The gold-leaf is not deflected appreciably. P is now removed and the leaf shews a deflection depending on the p.d. initially connected between P and C.

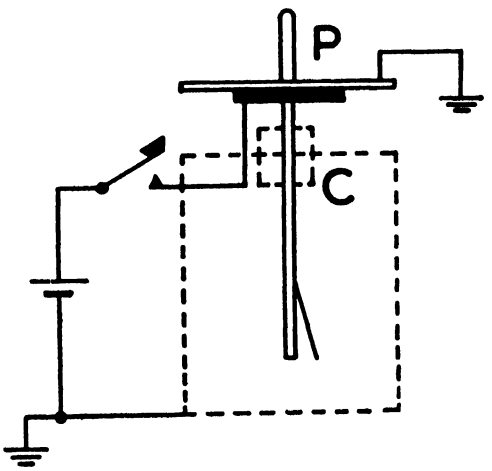


Fig. 15.14

Theory

Let the capacity of the electroscope without P be C_0 farads. When P is placed on C the two plates form an additional condenser of relatively large capacity (C'), the thin coating of varnish acting as dielectric. The two capacities are charged in parallel, the common " plate " being the electroscope cap. Let the p.d. being measured be V_1 volts. Since the total capacity is $(C_0 + C')$ farads, the charge acquired is: $Q = (C_0 + C')V_1$ coulombs. When the connection to the p.d. is broken and P is raised, C' is reduced to C'', say, (distance between " plates " increased). Since Q is constant the p.d. across each capacity is increased to V_2 volts, say, where $V_2 = Q/(C_0 + C'')$. Hence:

$$Q = (C_0 + C')V_1 = (C_0 + C'')V_2$$

$$V_2 = \frac{(C_0 + C')V_1}{(C_0 + C'')} \quad . \quad . \quad . \quad (32)$$

Since $C' \gg C_0$ and $C'' \ll C_0$, (32) may be simplified to become:

$$V_2 = \frac{C'V_1}{C_0} \quad . \quad . \quad . \quad . \quad (33)$$

Thus, the original p.d. is effectively multiplied by C'/C_0, which may be several hundreds.

The electroscope deflection may be calibrated in (applied) volts by using numbers of cells of known e.m.f. in series.

(ii) *Electroscope for Current Measurement*

A sensitive electroscope can be used for measuring small ionization currents, (created by radioactive substances, etc.). If the initial charge on the electroscope is Q coulombs, its capacity C farads and its initial potential V volts, the collection of ions of opposite sign gives an ionization current of strength:

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} \text{ amps.}$$

Assuming the capacity of the instrument does not change appreciably as the leaf falls:

$$I = C \cdot \frac{dV}{dt}$$

If the divergence of the leaf θ is proportional to the voltage, dV/dt is proportional to $d\theta/dt$, hence:

$$I \propto \frac{d\theta}{dt} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The C. T. R. Wilson electroscope was specially designed for this purpose. The gold leaf L is attached to a rod A, which is insulated from the earthed box B, (Fig. 15.15). Near to the free end of L is a metal plate P charged to a suitable potential. The leaf is attracted towards P and takes up a position of equilibrium. If B, which is inclined at an angle (α) to the horizontal, is raised or lowered slightly, a position of instability of the leaf is approached when any further change in α causes a sudden deflection. In this position the electroscope is most sensitive. When measuring ionization currents, the rate of change of the leaf position with time is observed with a low-powered microscope. By this means currents less than 10^{-15} amps. can be measured.

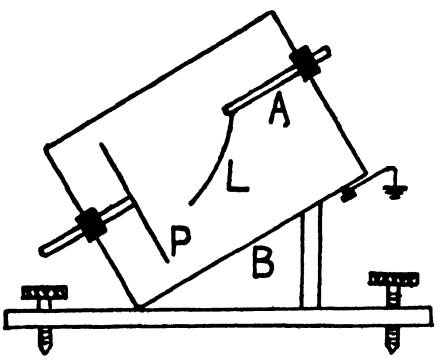


Fig. 15.15

(iii) *The Quadrant Electrometer*

The Dolazalek type quadrant electrometer is sketched in Fig. 15.16. It consists of four equal sections of a hollow circular box (quadrants), each mounted on a small pillar of amber. Opposite quadrants A and A' or B and B', are connected together. Suspended symmetrically

in the hollow space inside the quadrants is a light aluminium vane (C), which has the shape of a paddle. The suspending fibre, made of phosphor-bronze or metallised quartz, is attached at the upper end to a torsion head (T). The latter is able to rotate relative to the earthed case (E) which rests on the metal base (D). Levelling screws are

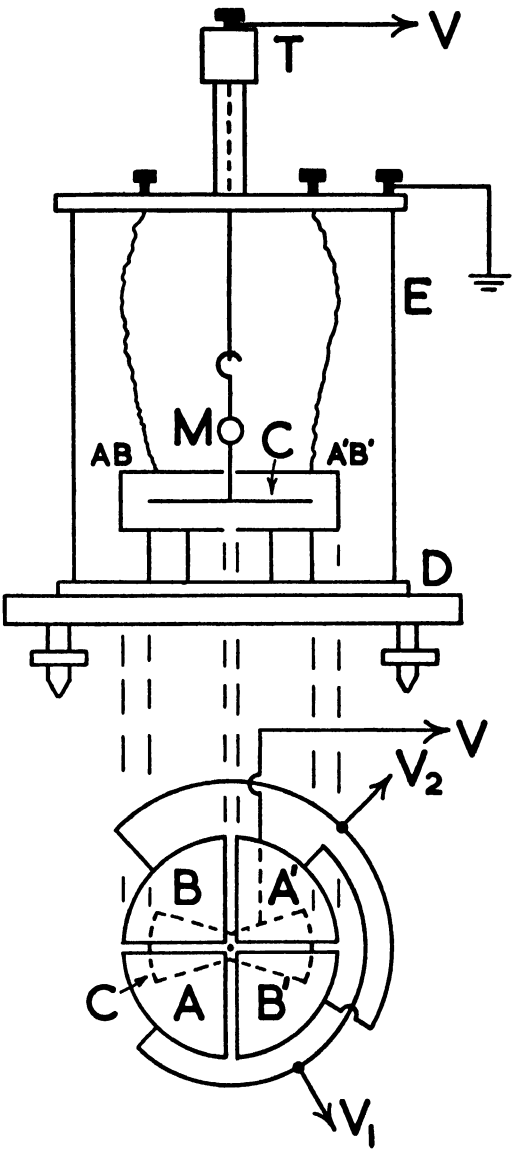


Fig. 15.16

provided. Small deflections of C are measured by using a light beam reflected from the mirror (M) attached to the suspension.

When the sets of quadrants are maintained at fixed potentials (V_1 and V_2), and the vane C at a positive potential (V), the charge on C is

attracted to the set of quadrants at the lower potential. A steady deflection (θ) results when the couple caused by the attraction is equal to the twist in the fibre. The rather complex theory of the instrument shews that:

$$\theta = p\{(V - V_2)^2 - (V - V_1)^2\} \quad (V > V_1 > V_2) \qquad (35)$$

where p is a constant depending on the capacity of the instrument and the torsional constant of the suspending fibre.

Heterostatic use of Electrometer

A small p.d. (<1 volt) is measured by connecting it between the pairs of quadrants and arranging for the vane potential to be a relatively high value (100 volts, say), i.e. $V \gg (V_1 - V_2)$. Writing (35) in the form:

$$\theta = p\{2V - (V_1 + V_2)\}(V_1 - V_2)$$

and neglecting $(V_1 + V_2)$ by comparison with $2V$, leads to:

$$\theta = p \,.\, (V_1 - V_2) \,.\, 2V \quad . \quad . \quad . \quad (36)$$

i.e. the deflection is proportional to the p.d. measured. The *heterostatic* method of use is most sensitive ($\theta \to 100$ cm. per volt).

Idiostatic use of Electrometer

If the vane is connected to one pair of quadrants, say the AA' quadrants, $V = V_1$ and (35) becomes:

$$\theta = p \,.\, (V_1 - V_2)^2 \quad . \quad . \quad . \quad . \quad (37)$$

i.e. the deflection is proportional to the square of the p.d. between the quadrants. The *idiostatic* method of use is often used for measuring alternating p.d.'s, since the deflection is independent of sign. The electrometer is far less sensitive than when used heterostatically ($\theta \to 1$ cm. per volt).

For accurate use it cannot be assumed that the scale of the quadrant electrometer is linear. Calibration is achieved by applying a series of known potentials between the pairs of quadrants and plotting a graph of deflection against p.d.

15.18. Measurement of Ionization Currents

(i) *Rate of Deflection Method*

Assuming the capacity of the electrometer is constant and charge is collected at a constant rate by one pair of quadrants when the instrument is used heterostatically:

$$I = \frac{dQ}{dt} = C \,.\, \frac{dV}{dt} = kC \,.\, \frac{d\theta}{dt} \quad . \quad . \quad . \quad (38)$$

where k is a constant of proportionality. An ionization current, is thus measured by connecting one pair of quadrants to one plate of the ionization chamber, (Fig. 15.17) and noting the time taken for θ to change from θ_1 to θ_2, say. This gives a measure of $d\theta/dt$ and hence I.

For the absolute determination of current strength, C and K must be known; k is obtained from the calibration curve of the electrometer, and C is determined by experiment (p. 265).

(ii) *Constant Deflection Method*

Method (i) is not suitable for continuous measurement because the deflection utimately goes off the scale and, furthermore, C and k depend on the position of θ on the scale. In the second method, the current (I) flows through a high resistance (R), one end of which is connected to one pair of quadrants and the other earthed (Fig. 15.18). The p.d. across

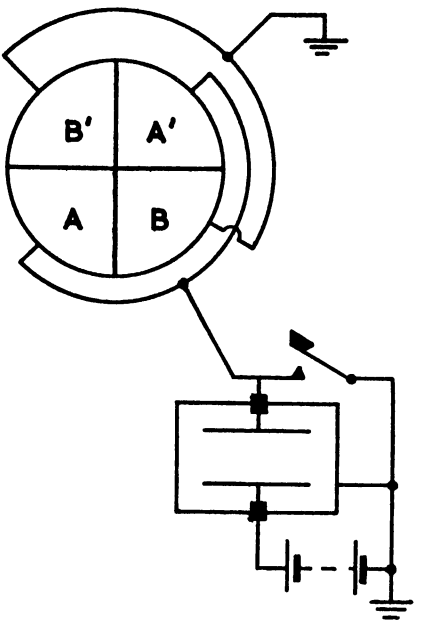


Fig. 15.17

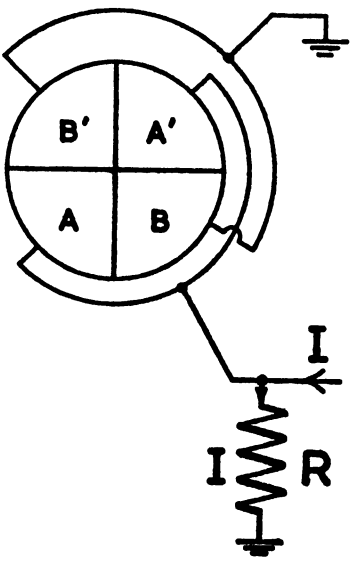


Fig. 15.18

R is: $V = IR$, and this gives a steady deflection θ. Since the electrometer is used heterostatically:

$$V = IR = k\theta$$

whence:
$$I = \frac{k\theta}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

where k is a constant of proportionality.

For currents of the order of 10^{-12} amps., R must be of the order of 10^{10} ohms, assuming the smallest measureable p.d. is 10^{-2} volts. The

exact value of such high resistors is difficult to determine. Both methods are thus best used for comparison rather than absolute determination.

15.19. Electrostatic Experiments

The following experiments utilize an electrostatic instrument for the measurement of p.d. Although a quadrant electrometer is specified, any other type of electrostatic voltmeter, such as a calibrated electroscope, may be used.

1. *Comparison of Capacities—Method of Sharing Charge*

The two condensers A and B have one plate earthed (Fig. 15.19). A is first charged to the e.m.f. of the cell E, using switch S_2. This

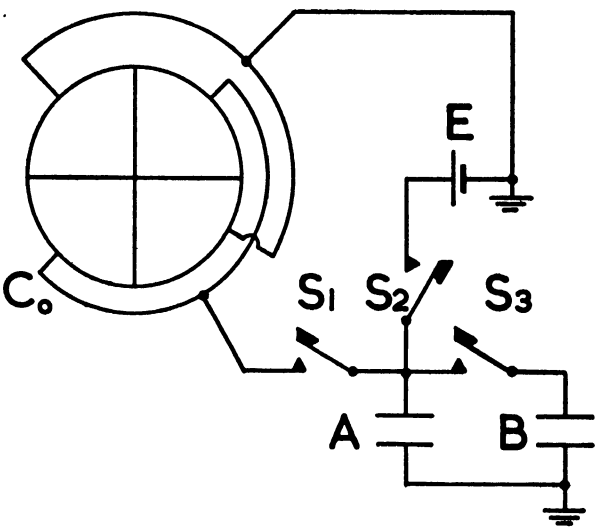


Fig. 15.19

connection is broken and A connected to the electrometer (used heterostatically) by using S_1. The steady deflection θ_1 is proportional to the p.d. across A, say V_1; ($V_1 < E$, because A has shared its charge with the electrometer capacity C_0).

If the original charge given to A is Q:

$$Q = (C_A + C_0)V_1 = (C_A + C_0)k\theta_1 \quad . \quad . \quad (40)$$

S_3 is now closed to connect A and B in parallel with the electrometer. The deflection is reduced to θ_2, since Q is now distributed among C_A, C_B and C_0, and:

$$Q = (C_A + C_B + C_0)V_2 = (C_A + C_B + C_0)k\theta_2 \quad . \quad (41)$$

From (40) and (41):

$$(C_A + C_0)k\theta_1 = (C_A + C_B + C_0)k\theta_2$$

i.e. $$(C_A + C_o)(\theta_1 - \theta_2) = C_B\theta_2$$

hence: $$\frac{C_A + C_o}{C_B} = \frac{\theta_2}{\theta_1 - \theta_2} \quad . \quad . \quad . \quad . \quad (42)$$

The capacity of the electrometer is usually smaller than the capacitances being compared and may be neglected in (42), whence:

$$\frac{C_A}{C_B} = \frac{\theta_2}{\theta_1 - \theta_2} \quad . \quad . \quad . \quad . \quad (43)$$

2. *Determination of Electrometer Capacity*

The switch S_2 is first connected so that the standard condenser (C_S) is charged to the e.m.f. of the cell (E), (Fig. 15.20). Switch S_2 is then

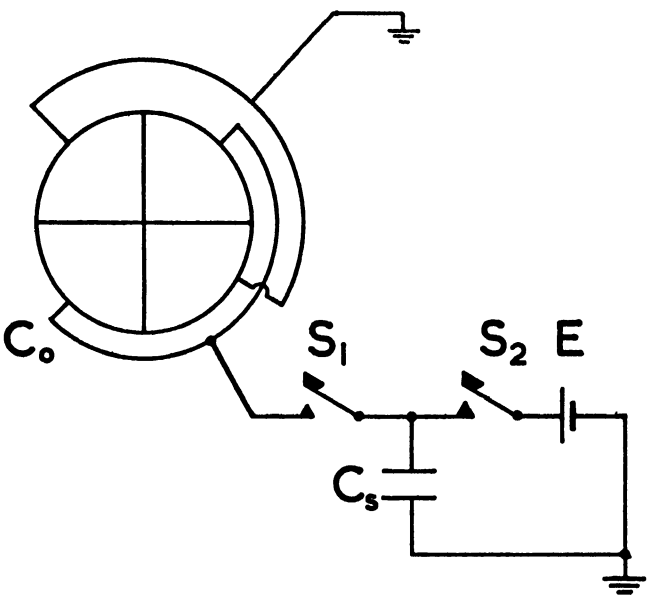


Fig. 15.20

disconnected and S_1 closed so that C_S and C_o share charge. The deflection (θ_1) is given by:

$$Q = (C_S + C_o)k\theta_1 \quad . \quad . \quad . \quad . \quad (44)$$

Switch S_1 is now opened and the electrometer quadrants are earthed. The charge lost by C_o is: $Q' = C_o k\theta_1$. After this, S_1 reconnects C_o to C_S and the electrometer shares the charge remaining on C_S, viz. $Q - Q'$. The new deflection is θ_2, where:

$$Q - Q' = (C_S + C_o)k\theta_2$$

from (44), and substituting for Q':

$$C_S k\theta_1 = (C_S + C_o)k\theta_1$$

i.e. $$C_o = C_S \frac{(\theta_1 - \theta_2)}{\theta_2} \quad . \quad . \quad . \quad . \quad (45)$$

C_S must be similar in value to that of the electrometer, otherwise the difference in deflections $(\theta_1 - \theta_2)$ will not be appreciable.

Example

An electroscope with a calibrated scale is joined to a charged condenser and the reading indicates 150 *volts. A second condenser is now joined to the first so that the charge is shared and the potential falls to* 70 *volts. On joining a third condenser the potential falls to* 30 *volts. Find the ratio of the capacity of the third condenser to that of the second, assuming perfect insulation.* (Inter B.Sc. London).

Let the capacities of the electroscope and the condensers be C_o, C_1, C_2 and C_3 farads respectively. When the charge possessed initially by the first condenser, say Q coulombs, is shared with the electroscope, the p.d. across each becomes V_1 volts, say, where:

$$Q = (C_o + C_1)V_1 = (C_o + C_1)150 \quad . \quad . \quad . \quad (46)$$

On sharing the charge with the second and third condensers:

$$Q = (C_o + C_1 + C_2)70 \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

and: $$Q = (C_o + C_1 + C_2 + C_3)30 \quad . \quad . \quad . \quad . \quad . \quad (48)$$

From (46) and (47): $$(C_o + C_1)150 = (C_o + C_1 + C_2)70$$

i.e. $$8(C_o + C_1) = 7C_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

From (46) and (48): $$12(C_o + C_1) = 3(C_2 + C_3) \quad . \quad . \quad . \quad . \quad (50)$$

Multiplying (49) by 3 and (50) by 2:

$$24(C_o + C_1) = 21C_2$$

and: $$24(C_o + C_1) = 6(C_2 + C_3)$$

whence: $$15C_2 = 6C_3$$

i.e. $$\frac{C_2}{C_3} = \frac{15}{6} = 2{\cdot}5$$

*3. *Comparison of Large and Small Capacitance*

When the charge on a large condenser is shared with a much smaller one the change in p.d. is too small to detect, and this simple procedure cannot be used for the comparison of the two capacitances. However, if the smaller capacitance is earthed after each sharing with the larger condenser, a large number of times, the change in p.d. becomes appreciable. This forms the basis of determining the capacity of an electrostatic instrument, such as an electroscope or electrometer, when a small standard condenser is not available.

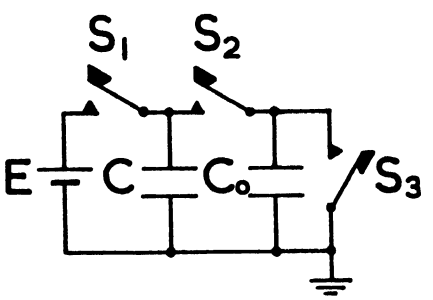


Fig. 15.21

The circuit of Fig. 15.21, is used, the capacitance C_o being much less than the standard C. The standard

condenser is first charged using S_1 and then isolated. Its charge is then shared with C_o. The charge remaining on C is:

$$Q_1 = CV_1$$

C_o is now earthed, using S_3, and then re-connected across C. The new p.d. across each condenser is V_2 where:

$$Q_1 = CV_1 = (C + C_o)V_2$$

hence:
$$V_2 = \frac{CV_1}{C + C_o}$$

The charge remaining on C is:

$$Q_2 = CV_2 = \frac{C^2}{C + C_o} \cdot V_1$$

C_o is again earthed and reconnected across C. The new potential V_3 is given by:

$$Q_2 = CV_2 = (C + C_o)V_3$$

hence:
$$V_3 = \frac{C^2}{(C + C_o)^2} \cdot V_1$$

The process of earthing of C_o and sharing with C is repeated a definite number of times, say n, when the final p.d. across C is given from above by:

$$V_n = \left(\frac{C}{C + C_o} \cdot \right)^{n-1} V_1$$

Whence:
$$\frac{C + C_o}{C} = \left(\frac{V_1}{V_n}\right)^{\frac{1}{n-1}}$$

and:
$$\frac{C_o}{C} = \left(\frac{V_1}{V_n}\right)^{\frac{1}{n-1}} - 1 \quad . \quad . \quad . \quad . \quad . \quad (51)$$

When C_o is the capacitance of an electrometer or electroscope, for which $V \propto \theta$, (51) may be written:

$$\frac{C_o}{C} = \left(\frac{\theta_1}{\theta_n}\right)^{\frac{1}{n-1}} - 1 \quad . \quad . \quad . \quad . \quad . \quad (52)$$

15.20. Dielectric Constant (Relative Permittivity) Determination

1. *Faraday's Method*

For liquids Faraday used a spherical condenser, the hollow space between the two spheres being half-filled with liquid. The condenser

is thus equivalent to two condensers in parallel, one having half the capacity of an air-filled spherical condenser and the other, half the capacity of a spherical condenser filled with the given liquid. If C is the capacity of a complete air-filled condenser and the liquid dielectric constant is ϵ, the capacity of the composite condenser is: $C(1 + \epsilon)/2$.

An air-filled spherical condenser of the same dimensions is first charged and connected to an electroscope of negligible capacity. If the p.d. across the condenser is V and the deflection θ_1:

$$V_1 = p\theta_1 \ (p = \text{const.})$$

The charge on the condenser is thus:

$$Q = CV_1 = Cp\theta_1$$

The inner sphere of this condenser is now connected to the inner sphere of the half-filled condenser. Charge is shared and the new deflection is θ_2 where:

$$Q = Cp\theta_1 = \left\{C + \frac{C}{2}(1 + \epsilon)\right\}p\theta_2$$

whence:
$$\epsilon = \frac{2\theta_1 - 3\theta_2}{\theta_2} \quad . \quad . \quad . \quad . \quad (53)$$

2. *Boltzmann's Method*

Dielectric constants may be determined by finding the ratio of the capacities of a condenser containing dielectric and vacuum. The experiment is thus the comparison of two capacities. The experiment is not easy because it may involve undesirable static effects when inserting or removing the dielectric. This is avoided in Boltzmann's Method. In this experiment a dielectric slab is inserted in a parallel plate air condenser without filling the space completely.

Let the area of each condenser plate be A m.^{2}, the separation t m., the thickness of the dielectric slab d m., and the dielectric constant ϵ. Let the slab be placed with its upper surface h m. from A (Fig. 15.22). When the condenser is charged to a p.d. (V volts), the lines of induction are normal to both plates (assuming $A \gg t$). If the density of charge on each plate is σ coulomb.m.$^{-2}$, $D = \sigma$ coulombs, at all points.

In the air spaces between the plates and dielectric, $\epsilon = 1$, and the electric intensity $E_a = D/(\epsilon_o) = \sigma/\epsilon_o$ volt.m.$^{-1}$ In the space occupied by the slab the electric intensity is $E_d = D/(\epsilon_o\epsilon) = \sigma/(\epsilon_o\epsilon)$ volt.m.$^{-1}$. Hence, the p.d. between the plates is given by:

$$V = E_a h + E_d d + E_a(t - h - d) \text{ volts}$$

i.e.
$$V = \frac{\sigma}{\epsilon_o} \cdot \left\{t - d + \frac{d}{\epsilon}\right\} = \frac{\sigma}{\epsilon_o}\left\{t - d\left(1 - \frac{1}{\epsilon}\right)\right\} \text{ volts}$$

The charge on each condenser plate is $A\sigma$ coulombs, hence the capacity is:

$$C = \frac{Q}{V} = \frac{A}{\epsilon_0\left\{t - d\left(1 - \frac{1}{\epsilon}\right)\right\}} \text{ farads}$$

It follows that the same capacity is obtained if the dielectric is removed and the plate separation reduced by $d(1 - 1/\epsilon)$ metres.

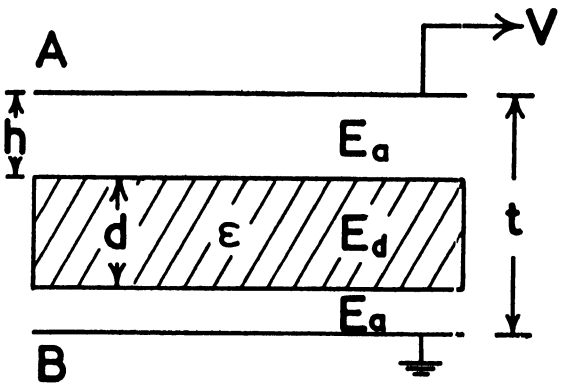


Fig. 15.22

In the experiment the condenser containing the slab of dielectric is charged, connected to a sensitive electrometer and the deflection noted. The slab of dielectric is then removed. The electrometer deflection is increased because the capacity is decreased but the charge is unaltered. The lower plate of the condenser is now moved a measured distance x m. towards the upper plate until the electrometer deflection is restored to its initial value. Then:

$$\epsilon = \frac{d}{d - x}$$

*15.21. The Determination of ϵ_0

The experiment consists of charging a parallel plate air condenser with a known voltage V and discharging it through a sensitive galvanometer, the charging and discharging process being repeated a definite number of times per second, say p. Providing that p is sufficiently large compared with the natural frequency of the galvanometer, the charge which flows through the galvanometer causes a steady deflection which is a measure of current strength. The charge acquired by the condenser each time it is charged is:

$$Q = CV = \frac{A\epsilon_0 V}{d} \text{ coulombs}$$

The amount of charge passing through the galvanometer per second is thus:

$$I = pQ = \frac{A\epsilon_0 pV}{d} \text{ amps.}$$

If the steady deflection of the galvanometer is θ divisions and the current sensitivity of the instrument is λ amps. per scale division:

$$I = \lambda\theta = \frac{A\epsilon_0 pV}{d}$$

whence:

$$\epsilon_0 = \frac{d\lambda\theta}{ApV} \text{ farad.m.}^{-1} \quad . \quad . \quad . \quad . \quad (54)$$

A suitable circuit is shown in Fig. 15.23; C is a parallel plate air condenser consisting of two large glass plates (30 cm. square), coated with tin-foil and separated from each other with 1 mm. thin perspex spacers; S is a vibrating reed type of switch, magnetically operated by A.C., so that it makes contact with points 1 and 2 alternately with mains

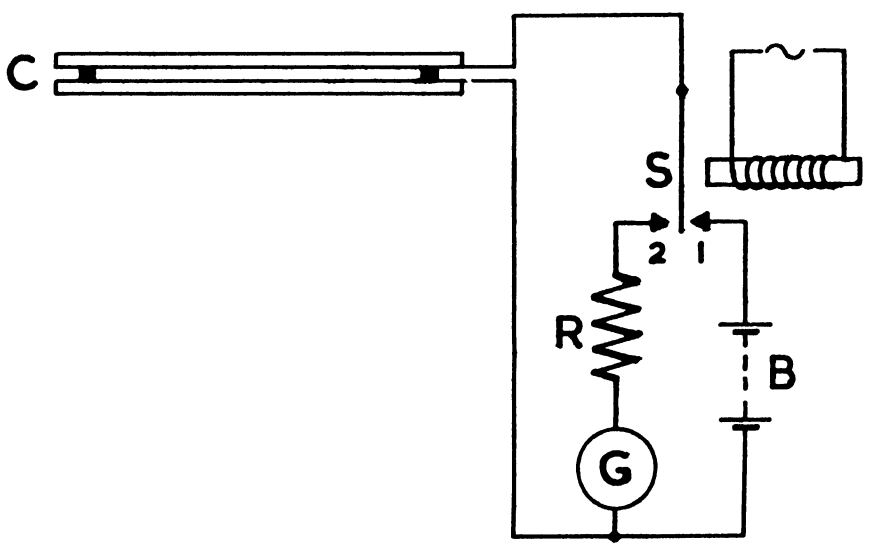


Fig. 15.23

frequency. When S is connected to 1, the condenser is charged by the H.T. battery B; when 2 is connected, it is discharged through the galvanometer G; R is a high resistance (about 10^5 ohms) which prevents an accidental "short" across G causing the coil to be burnt out. The time constant of the circuit containing C, R and G, is very small because C is very small (about 10^{-9} farads), so that C is fully discharged each time S is connected to 2.

Divergence of the lines of force near the edges of C makes the actual value of the capacitance somewhat greater than that given by (54), the

discrepancy amounting to about 1 per cent. The error in the measurement of d is reduced using a micrometer screw gauge to measure the thickness of the perspex spacers.

The permittivity (ϵ_0) is determined absolutely providing that λ and V are known. Since $\mu_0 = 4\pi \times 10^{-7}$ M.K.S. units, by definition, the Maxwell relation: $\epsilon_0\mu_0 = 1/c^2$, may thus be verified.

Example

How would you use a condensing electroscope and the usual accessories to find the polarity of the copper and zinc elements of a Daniell cell?

The two pairs of quadrants of a quadrant electrometer are connected to two insulated terminals and the instrument is used as an electrostatic voltmeter. A source of potential difference is connected to the terminals and then removed, leaving the instrument charged and showing a steady deflection θ. The plates of an uncharged parallel plate condenser are joined to the terminals and the deflection is thereby reduced to $\theta/5$. When the plates of the condenser are separated by a further 5 mm. the deflection increases to $\theta/2$. How far apart were the plates of the condenser in the first case? (London G.C.E. Adv. L.).

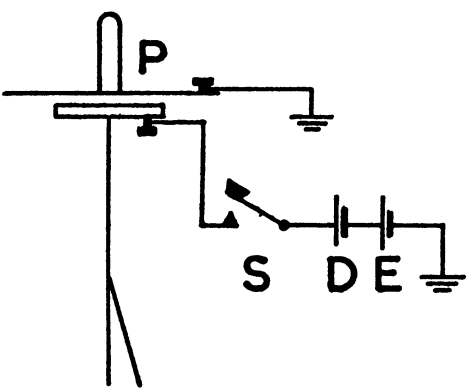


Fig. 15.24

(i) In order to determine the polarity of a Daniell cell use the circuit of Fig. 15.24. The switch S is closed; the electroscope potential (V) is given by the net e.m.f. of the Daniell cell D and the accumulator E, depending on whether the cells are in series or in opposition. After charging, S is opened and P is momentarily earthed prior to its removal. When P is removed the electroscope deflection is a measure of V. The accumulator is now reversed and the experiment repeated. The size of the second deflection relative to the first indicates whether the cell and accumulator were connected in series or opposition. Since the polarity of the accumulator is known the polarity of the cell is determined.

(ii) Let the capacity of the electrometer be C_e farads and the initial capacity of the condenser C farads. For the electrometer alone:

$$V_e = p\theta_e \ (p = \text{const.})$$

The charge on the electrometer is hence:

$$Q_e = C_eV_e = pC_e\theta_e \text{ coulombs}$$

When the condenser is connected to the electrometer the total capacity is: $C_1 = C + C_0$ farads, but the charge is unchanged so that the new p.d. is:

$$V_1 = \frac{Q}{C_1} = \frac{pC_0\theta_0}{C_1} = \frac{p\theta_0}{5} \text{ volts.}$$

hence: $$\frac{C_0}{C_0 + C} = \frac{1}{5}$$

i.e. $$C = 4C_0$$

Let the area of the plates be A m.^{2} and the separation d m.

Then: $$C = \frac{A\epsilon_0}{d} \text{ farads}$$

When the plate separation is increased by 5×10^{-3}m. the capacity becomes:

$$C' = \frac{A\epsilon_0}{d + 5 \times 10^{-3}} \text{ farads}$$

hence: $$\frac{C}{C'} = \frac{d + 5 \times 10^{-3}}{d}$$

i.e. $$C' = \frac{Cd}{d + 5 \times 10^{-3}} = \frac{4C_0 d}{d + 5 \times 10^{-3}} \quad . \quad . \quad (55)$$

The p.d. across the modified condenser and electrometer is given by:

$$V_2 = \frac{Q_0}{C_0 + C'} = \frac{pC_0\theta_0}{C_0 + C'} = \frac{p\theta_0}{2}$$

i.e. $$C_0 = C'$$

Hence from (55):

$$C_0 = \frac{4C_0 d}{d + 5 \times 10^{-3}}$$

i.e. $$d + 5 \times 10^{-3} = 4d$$

$$\therefore d = 1{\cdot}67 \times 10^{-3}\text{m.} = 1{\cdot}67 \text{ mm.}$$

Example

Define the terms potential and capacity of a conductor as used in electrostatics. Describe how you would use an electroscope or electrometer to compare the capacities of two condensers, each of small capacity.

The plates of an oil filled condenser are charged to a potential difference of 200 volts. When an air condenser of the same dimensions is connected in parallel with it, the potential falls to 150 volts. What is the dielectric constant of the oil? (Northern H.S.C.).

(i) The potential at a point is the work done in bringing up a unit charge from infinity to that point (p. 13).

The capacity of a conductor is the quantity of charge required to raise the potential of the conductor, relative to an arbitrary zero, by unity (p 40).

(ii) For the comparison of two small capacitances the electrometer or calibrated gold leaf electroscope is used, as outlined on p. 468.

(iii) Let the capacity of the oil-filled condenser be C farads and the dielectric constant ϵ. The capacity of the air-filled condenser is thus C/ϵ farads.

The initial charge on the oil-filled condenser is:

$$Q = CV = 200 \, . \, C \text{ . coulombs.} \quad (V = 200 \text{ volts}).$$

When the air-filled condenser is connected in parallel the total capacity is:

$$C' = C + \frac{C}{\epsilon} = \frac{C(\epsilon + 1)}{\epsilon} \text{ farads.}$$

Since the p.d. is 150 volts:

$$Q = C'V' = 150 \cdot \frac{C(\epsilon + 1)}{\epsilon} \text{ coulombs.}$$

The total charge before and after the connection is unaltered, hence:

$$150 \cdot C \frac{(\epsilon + 1)}{\epsilon} = 200 \cdot C$$

i.e. $$150\epsilon + 150 = 200\epsilon$$

hence: $$\epsilon = 3$$

*15.22. Theory of Dielectrics

Insulating materials are distinguished from conductors by having no free electrons. Hence, when a source of e.m.f. is connected across a dielectric no current flows. Since all insulators are imperfect and contain a small number of free electrons a very small current flows when a p.d. is applied. Here, however, it is assumed that a dielectric is a perfect insulator.

Suppose two large plane parallel plates (A and B) in vacuo, are connected to a battery. The plates become charged, A positively and B negatively, and a uniform electric field is created between the plates whose intensity is $E = V/d$ volt.m.$^{-1}$. This is achieved by electrons leaving A and flowing to B and constituting a transient current in the connecting wires, the energy dissipated being supplied by the battery.

During the creation of the electric field a magnetic field is also created. The magnetic field is exactly the same as would be created by the charging current if it flowed between the plates as well as in the connecting wire. To all intents, therefore, the electron charging current may be considered to flow from B to A across the vacuum. The apparent current in the vacuum is called the *displacement current.* The charge acquired by each plate per unit area is the *displacement* of the dielectric, represented by the vector D_0. The displacement current density (i.e. current strength per unit area) is given by:

$$i = (dD_0)/(dt) \quad (cp \cdot I = dQ/dt)$$

There is a displacement D_0 at all points between the plates, which represents the apparent charge displaced per unit area. In Chapter 1 the electric induction was shown to be: $D_0 = \epsilon_0 E = \sigma_0$, so that induction and displacement are the same. Thus, induction (D) is given physical significance as an apparent charge displacement.

Suppose a slab of dielectric of constant ϵ is interposed between the plates. The capacity of the system is reduced by the factor ϵ, but since

the p.d. is maintained by the battery, the charge on the plates must be increased by the factor ϵ. Hence, the charge per unit area $= D = \epsilon\sigma_0$. In order to effect this change another transient current flows in the connecting wire, energy again being supplied by the battery. In accordance with the ideas of displacement a further quantity of charge is displaced per unit area between the plates, the time rate of this representing an apparent current, which is called the *polarization current.* The additional charge displaced per unit area at any point between the plates is represented by a vector P, called the *polarization.*

The total displacement at any point D, is now given by: $D = D_0 + P$. Since $D_0 = \epsilon_0 E$, it follows that:

$$D = \epsilon_0 E + P \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Furthermore, $D = \epsilon\sigma_0 = \epsilon D_0 = \epsilon_0 \epsilon E$, (the field intensity is unchanged) hence:

$$P = \epsilon_0(\epsilon - 1)E \quad . \quad . \quad . \quad . \quad (57)$$

Polarization is explained by atomic theory: a dielectric consists of molecules, the atomic nuclei of which are virtually fixed relative to each other. The electrons of the atoms are acted upon by the electric field between the plates. This causes a movement of the electrons which is opposed by the attractive force between nuclei and electrons. The resultant effect is to separate the positive and negative charges in each molecule (displacement), so that they behave like electric dipoles. The polarized dielectric thus consists of layers of dipoles, (Fig. 15.25).

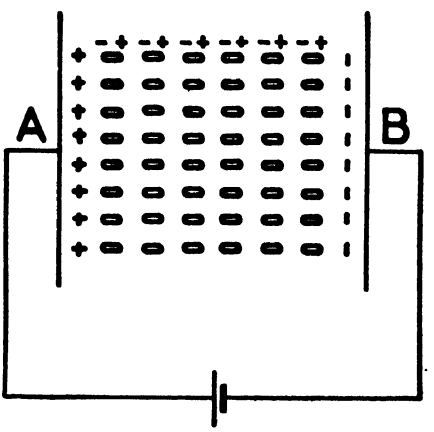


Fig. 15.25

The dielectric surface charges give rise to a field within the dielectric opposite in direction to the original field. Since the original field strength is unchanged, additional charge must be supplied to the plates by the battery to counteract this effect.

Insulated Plates

Suppose A and B are charged initially in vacuo to a potential V and then insulated. The displacement D_0 = charge per unit area of each plate $= \sigma_0$, say. If the dielectric is inserted between the plates, polarization occurs and a polarization current is produced. The original displacement D_0 is unaltered since σ_0 is unchanged (no charge flows externally). The surface charges on the dielectric caused by the polarization produce a field within the dielectric which is opposite to

the original field (E_0), so that the latter is reduced to E_1, say. This reduces the potential between the plates as may be observed with an electroscope or electrometer.

If the plates were kept *in vacuo* with a reduced charge density (σ') to give the same reduced intensity E, then $\sigma' = \sigma_0 - P$, i.e. $\sigma' = D_0 - P$.

Since: $E = \sigma'/\epsilon_0$:

$$\sigma' = E\epsilon_0 = D_0 - P$$

i.e.

$$D_0 = E\epsilon_0 + P$$

Thus (56) applies in both cases.

Example

Two condensers, each consisting of a pair of plates, area A *sq.m. separated by* d *m. are connected in series. One condenser is air spaced, the other is filled with a sheet of material of dielectric constant* ϵ. *Find an expression for the capacity of the series combination.*

The condensers, so combined are connected to a battery of potential difference V *volts, by means of copper wires. The sheet of dielectric filling the second condenser is withdrawn at a uniform rate, the process of withdrawal occupying a time* t_0 *seconds. What current flows through the copper wires during the process? What would be the current if a* very large *resistance,* R *ohms, were included in series with one of the connecting wires?* (Cambridge Open Schol.).

For the air-spaced condenser A, say, the capacity is: $C_0 = A\epsilon_0/d$ farads, ($\epsilon = 1$); for the dielectric filled condenser, B, the capacity is: $C' = A\epsilon_0\epsilon/d$ farads. The equivalent capacity C is given by:

$$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C'} = \frac{d}{A\epsilon_0} + \frac{d}{A\epsilon_0\epsilon}$$

hence:

$$C = \frac{A\epsilon_0\epsilon}{d(1+\epsilon)} \text{ farads}$$

When the condensers are connected to the battery the circuit is as shown in Fig. 15.26. Let the length of each condenser plate be h metres and suppose the

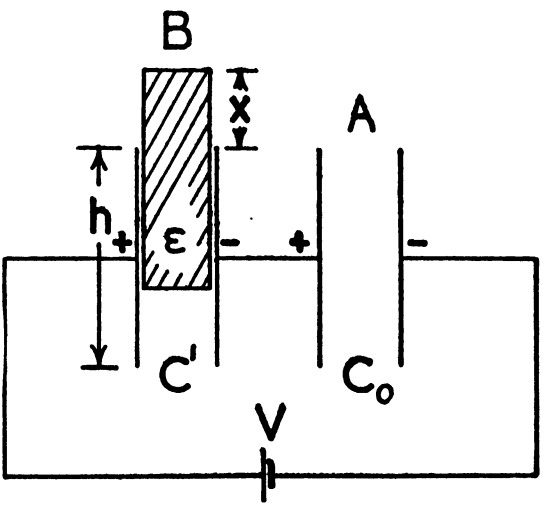


Fig. 15.26

dielectric has moved a distance x metres at the time t seconds. Since the complete removal takes t_o seconds:

$$\frac{h}{t_o} = \frac{x}{t} \text{ and: } x = \frac{ht}{t_o} \text{ metres}$$

At time t, B consists of two capacitances in parallel, one air-filled (C_1) and the other dielectric filled (C_2). If the breadth of the condenser plates is b metres, the areas of the plates of C_1 and C_2 are bx and $b(h - x)$ sq.m., respectively. Since $bh = A$, the areas are Ax/h and $A(h - x)/h$ sq.m., respectively, and hence:

$$C_1 = \frac{Ax\epsilon_o}{hd} \text{ and } C_2 = A\frac{(h - x)}{hd}\epsilon\epsilon_o \text{ farads.}$$

Hence the capacity of B is now:

$$C' = C_1 + C_2 = \frac{A\epsilon_o}{hd}\{x + h\epsilon - x\epsilon\} = \frac{A\epsilon_o}{hd}\{h\epsilon - x(\epsilon - 1)\} \text{ farads}$$

The total capacity of A and B is given by:

$$\frac{1}{C} = \frac{1}{C_o} + \frac{1}{C'} = \frac{d}{A\epsilon_o} + \frac{hd}{A\epsilon_o\{h\epsilon - x(\epsilon - 1)\}} = \frac{hd(\epsilon + 1) - dx(\epsilon - 1)}{A\epsilon_o\{h\epsilon - x(\epsilon - 1)\}}$$

i.e.
$$C = \frac{A\epsilon_o\{h\epsilon - x(\epsilon - 1)\}}{hd(\epsilon + 1) - dx(\epsilon - 1)} \quad . \quad . \quad . \quad . \quad (58)$$

The removal of the dielectric from B causes a change in its polarization, and as a consequence a flow of charge in the circuit. The time rate of charge on the combination gives the strength of the current which flows in the connecting wires, i.e.:

$$I = \frac{dQ}{dt} = V \cdot \frac{dC}{dt} \text{ amps.}$$

From (58):

$$I = V \cdot \frac{d}{dt}\left[\frac{A\epsilon_o\{h\epsilon - x(\epsilon - 1)\}}{dh(\epsilon + 1) - dx(\epsilon - 1)}\right] = \frac{A\epsilon_o V}{d} \cdot \frac{d}{dt}\left[\frac{h\epsilon}{h(\epsilon + 1) - x(\epsilon - 1)} - \frac{x(\epsilon - 1)}{h(\epsilon + 1) - x(\epsilon - 1)}\right]$$

Using $x = \frac{ht}{t_o}$:

$$I = \frac{A\epsilon_o V}{d} \cdot \frac{d}{dt}\left[\frac{\epsilon t_o}{t_o(\epsilon + 1) - t(\epsilon - 1)} - \frac{t(\epsilon - 1)}{t_o(\epsilon + 1) - t(\epsilon - 1)}\right]$$

$$= \frac{A\epsilon_o V}{d}\left[\frac{\epsilon t_o(\epsilon - 1)}{\{t_o(\epsilon + 1) - t(\epsilon - 1)\}^2} - \frac{\epsilon - 1}{t_o(\epsilon + 1) - t(\epsilon - 1)} + \frac{t(\epsilon - 1)^2}{\{t_o(\epsilon + 1) - t(\epsilon - 1)\}^2}\right]$$

Re-arranging: $$I = \frac{A\epsilon_o V}{d}(\epsilon - 1) \cdot \frac{[2t(\epsilon - 1) - t_o]}{\{t_o(\epsilon + 1) - t(\epsilon - 1)\}^2} \text{ amps.}$$

When a very large resistance is included, the current which flows as a result of the dielectric removal creates a p.d. across the resistance. This reduces the current strength to negligible proportions and the dielectric is removed before any appreciable change in charge distribution occurs. The problem now becomes one of decay of charge on a condenser with time. An ever decreasing small discharge current flows until the p.d. across B reaches the value required for vacuum between the plates.

EXERCISES

1. Obtain from first principles an expression for the magnetic field strength at a point on the axis of a bar magnet.

   Describe how you would use a deflexion magnetometer to compare the magnetic moments of two bar magnets, not necessarily short ones.

   A tangent galvanometer is carrying a steady current, and it is found that, when it is set with the plane of the coil at an angle of 20° to the magnetic meridian, the needle makes an angle of exactly 90° with the meridian. What will be the deflexion when the plane of the coil is set up in the magnetic meridian? (Oxford Local G.C.E. Adv. L.).

2. Describe and explain how you would use a short bar magnet and a deflection magnetometer to verify the inverse square law of magnetic force.

   A short bar magnet, placed 30 cm. due magnetic E. of a magnetometer needle and pointing towards it, produces a deflection of 45°. The same magnet suspended by a torsionless fibre in a horizontal plane in the place formerly occupied by the magnetometer is found to have a period of oscillation of 2·0 sec. If the moment of inertia of the magnet about the axis of oscillation is 4×10^{-6} kgm.m.^{2}, find the horizontal component of the earth's magnetic field. (Northern H.S.C.).

3. Define *strength of a magnetic field, neutral point.*

   Show approximately on a diagram the positions of the neutral points when a bar magnet is placed with its axis horizontal in a magnetic E-W direction, and describe briefly the " directional loci " method for locating a neutral point.

   A neutral point occurs 16·5 cm. from a short bar magnet, of magnetic moment 10^{-6} weber-metre, placed with its magnetic axis horizontal in the magnetic meridian and with its N-pole pointing north. What is the value of the horizontal component of the magnetic field of the earth? (London Inter. B.Sc.).

4. Indicate briefly how the magnetic, chemical, and heating effects of current can be made the basis of current measurement.

   A vibration magnetometer makes 10 vibrations in 30 sec. when placed at a point 10 cm. due E. of a long vertical wire carrying a current and 10 vibrations in 35 sec. when placed the same distance due W. of the wire. The magnetic field at these points due to the current exceeds the horizontal component of the earth's intensity. Given that the magnetic field at a point due to the conductor is inversely proportional to the distance of the point from it determine the position of a neutral point in the magnetic field. (Durham G.C.E. Adv. L.).

5. State with reference to a diagram, the quantities which are necessary to define the earth's magnetic field in direction *and* magnitude at any point on its surface.

Give a brief description of two different methods of determining the horizontal component of the earth's field. (London G.C.E. Adv. L.).

6. Define three quantities which specify completely the magnetic field due to the earth at any point on its surface. Illustrate your answer by a diagram.

Describe how any two of the quantities may be determined. (London G.C.E. Adv. L.).

7. How would you determine the value of the horizontal component of the earth's magnetic field?

A long straight wire is mounted vertically, and carries a steady current of 2·5 amps. A vibration magnetometer is placed 10 cm. away from the wire in such a position that the vertical plane containing the wire and the magnetometer suspension is at right angles to the magnetic meridian. The magnetometer needle makes 50·0 vibrations per minute when the current is in one direction, and 35·3 per minute when it is reversed. What is the value of the earth's horizontal component? (Oxford Local G.C.E. Adv. L.).

8. Describe how a dip circle is set up for use. Why is it necessary to read both ends of the needle when finding the angle of dip?

The actual angle of dip at a place where $H = 45/\pi$ amp.m.$^{-1}$ is 68°. In an experiment with a dip circle the needle is found to come to rest making an angle of dip of 66°. The dip needle weighs 9·5 gm. and measures 10·6 cm. from pole to pole. The strength of each pole is $8{\cdot}4\pi \times 10^{-6}$ weber and the needle is pivoted at the mid-point of a line joining its poles, but the centre of gravity of the needle, which is in this line, is a short distance, x, from the point of suspension. If this is the only error in the instrument, find the value of x. (London G.C.E. Adv. L.).

9. A thin circular disc of cobalt steel is strongly magnetized in the direction of one of its diameters, symmetrically about the centre.

How would you find, as accurately as possible, the direction and length of the equivalent bar magnet? (London G.C.E. Schol.).

10. Give a brief account of the earth's magnetism and outline methods of investigating it.

The vertical plane in which the needle of a dip circle is free to swing is inclined at 15° to the meridian plane. The observed angle of dip is 69° 20′. What is the true angle of dip? (Durham G.C.E. Adv. L.).

11. What is meant by (*a*) *declination*, (*b*) *inclination or dip*?

Give a general account of how these vary with time at any place (e.g. London) on the earth's surface.

A freely suspended magnet oscillates in a horizontal plane at two places, A and B. The time per oscillation at A is 2·8 sec. The horizontal component of the earth's magnetic field at A is 12·57

amp.m.$^{-1}$ and the angle of dip is 70°12′. What will be the time for an oscillation at B where the *total* magnetic intensity of the earth's field is 37·15 amp.m.$^{-1}$ and the angle of dip is 66°0′. What is the magnitude of the total intensity of the earth's magnetic field at A? (London G.C.E. Adv. L.).

12. Explain what is meant by the moment of a magnet. How is it that it is possible to find an accurate value of the moment of a magnet although the distribution of the poles can only be found approximately? Show from first principles that two small bar magnets far apart compared with their lengths attract each other approximately according to an inverse fourth power law. (Oxford Open Schol.).

13. You are supplied with two short magnets, an empty cotton-reel, and a light wooden lath. One magnet is placed on the lath near one end and the lath balanced on the reel so as to form a " see-saw " which moves in a vertical plane. Explain how you would use this apparatus to investigate how the force between the magnets depends on the distance apart of their centres, the axes of the magnets being maintained parallel to each other. You may, in addition to the material mentioned, use any other which is likely to be found in a school physics laboratory. (Durham G.C.E. Schol.).

14. Write an essay on *The principles underlying the methods of measuring magnetic field strength.* (Oxford Open Schol.).

15. Write an essay on *one* of the following subjects:
   (*a*) The earth's magnetism.
   (*b*) Colour.
   (*c*) Soap bubbles.
   (*d*) Interplanetary travel.
   (Oxford Open Schol. Subsid.).

16. Describe and explain the action of an electrostatic machine for producing continuously a high potential difference.
   Describe experiments which demonstrate the identity in nature of the electricity produced by the machine and that produced by other means (e.g. a cell). (London G.C.E. Adv. L.).

17. Define *unit charge* and *unit potential difference* in the M.K.S. system of units.
   Describe *either* a condensing electroscope *or* a quadrant electrometer and show how the instrument you describe may be used to compare the e.m.f.'s of two cells (each of the order of 12 volts). (London G.C.E. Adv. L.).

18 Describe the quadrant electrometer and explain, with circuit diagrams, how you would use the instrument (*a*) to compare the e.m.f.'s of two cells and (*b*) to measure a current. (London G.C.E. Adv. L.).

19. Describe the construction of the quadrant electrometer, and explain

how it may be used to compare the capacities of two condensers.

Two condensers of capacities 3 and 5 microfarads respectively are connected in series across a 200-volt dry battery. The condensers are separately taken out of the circuit without discharging them, and are then joined to one another in parallel. Calculate the potential difference between the plates, and the final total energy of the system. (Oxford Local G.C.E. Adv. L.).

20. Describe the method you would use to measure the dielectric constant of glass.

Discuss the difficulties of the experiment and possible sources of error. (London G.C.E. Schol.).

21. Derive an expression for the energy which is stored in a condenser of capacitance C farads when it has a charge Q coulombs.

Show that there is always a loss of energy when a charged condenser shares its charge with an uncharged condenser. Account for this loss.

Describe an experimental method for the comparison of the capacitances of two condensers of the order of magnitude of $1\mu F$. (Northern G.C.E. Adv. L.).

22. Define *capacitacne* of a condenser.

Describe how you would compare the capacitances of two condensers using an electrometer.

A uniform slab of glass of thickness 0·25 cm. is inserted between the charged insulated plates of a parallel plate air condenser. Will it be necessary to increase or decrease the distance between the plates, and by how much, in order to maintain the same difference of potential between them? You are to assume that the area of the slab is at least equal to that of each of the plates and that the dielectric constant of the glass is 5·0. (London Inter. B.Sc.).

23. Explain what is meant by the *electrical capacity* of a condenser, and define the practical unit in which it is measured. Show how to calculate the capacity of a parallel plate air condenser.

Two circular metal plates, each of radius 10 cm., are placed 0·1 cm. apart in air so as to form a parallel plate condenser. One plate is earthed and the other connected to an electrometer, which is then charged to a potential of 80 volts. When the distance between the plates is increased to 0·5 cm., the electrometer is found to read 300 volts. Explain this observation, and calculate the capacity to earth of the electrometer. (Oxford & Cambridge Jt. Bd. G.C.E. Adv. L.).

24. Define *electrical capacitance* (*capacity*). Describe experiments to demonstrate the factors which determine its value for a parallel plate condenser.

The plates of a parallel plate air condenser consisting of two circular plates, each of 10 cm. radius placed 2 mm. apart, are connected to the terminals of an electrostatic voltmeter. The system is charged to give

a reading of 100 on the voltmeter scale. The space between the plates is then filled with oil of dielectric constant 4·7 and the voltmeter reading falls to 25. Calculate the capacitance of the voltmeter. You may assume that the voltage recorded by the voltmeter is proportional to the scale reading. (Northern G.C.E. Adv. L.).

25. Deduce expressions for the combined capacity of two condensers (*a*) connected in series, (*b*) connected in parallel.

Describe how *one* of these expressions may be verified by experiment.

A fixed condenser of capacity 10^{-4} microfarads is connected in series with a variable condenser the capacity of which may be varied from zero to 10^{-4} microfarads in steps of 10^{-6} microfarads. These two in series are connected in parallel with a third condenser of capacity 5×10^{-4} microfarads. Calculate (i) the maximum capacity of the whole combination and (ii) the smallest change in this capacity which can be produced by the arrangement. (Give the second result to two significant figures). (Northern H.S.C.).

26. Describe with any theory you consider necessary, two of the following:
(*a*) a vibration magnetometer,
(*b*) an attracted disc electrometer,
(*c*) a moving coil galvanometer,
(*d*) an induction coil.
(Oxford Open Schol. Subsid.).

27. State and prove Gauss's law of total normal induction over a closed surface. Apply this to deduce the field at the surface of a conductor with surface density of charge σ.

A flat conducting surface 3 metres square flush with the earth's surface but insulated from it is connected to the earth through a ballistic galvanometer. Initially the vertical field at the earth's surface is 200 volts metre^{-1}, but as a result of a lightning discharge, it changes rapidly to 100 volts metre^{-1}. What charge in coulombs flows through the galvanometer? (Oxford Open Schol.).

28. Show that the electric intensity at a point outside an isolated charged conducting sphere is the same as if all the charge were concentrated at the centre.

The plate of an electrophorus acquires a charge "q". When the plate is brought in contact with an uncharged insulated conductor, a fraction "f" of this charge is transferred to it. Show that the maximum charge that can be acquired by this conductor after repeated contacts with the charged electrophorus plate is: $fq/(1-f)$. (Oxford Open Schol.).

29. Describe and explain an experiment to demonstrate the change in the capacity of a parallel plate condenser produced by inserting a sheet of dielectric between the plates.

Derive the expression for the energy stored in a condenser of capacity C when a potential difference V is applied across its plates.

A parallel plate condenser in air has a capacity of $1/3 \times 10^{-9}$ farads.

Calculate the change in energy of the condenser when the space between the plates is filled with an insulator of dielectric constant 5, (*a*) if the potential difference between the plates is kept constant at 600 volts, (*b*) if the charge on the plates is kept constant at 2×10^{-7} coulombs. State the unit in which the results are expressed. (Northern G.C.E. Adv. L.).

Bearing in mind that the dimensions can be manipulated like algebraic quantities, it follows that:

$$[k]^{-\frac{1}{2}}[p]^{-\frac{1}{2}} = [L][T]^{-1} \quad . \quad . \quad . \quad (21)$$

The dimensions on the R.H.S. of (21) are those of velocity. The above dimensional analysis shews, therefore, that the only measurements that need to be made in order to determine the quantity $(kp)^{-\frac{1}{2}}$, are one of length and one of time. If the C.G.S. system of units was used for these measurements the quantity $(kp)^{-\frac{1}{2}}$ would be measured in cm.sec.$^{-1}$, whilst if the M.K.S. system of units was used $(kp)^{-\frac{1}{2}}$ would be measured in m.sec.$^{-1}$

It is impossible to use any other defining relation involving k and p, which allows the dimensions of each quantity to be determined separately in terms of the fundamental dimensions. Since the dimensions of all electric and magnetic quantities involve those of k and p, it follows that it is impossible to express the dimensions of any electric or magnetic quantity in terms of the fundamental dimensions alone. However, by choosing either $[k]$ or $[p]$ to be an additional fundamental dimension this difficulty is resolved. It is preferred, nowadays, to choose $[p]$ as the fourth fundamental dimension, so that: $[k] = \frac{[T]^2}{[p][L]^2}$. It follows that the dimensions of charge and pole strength can be written as:

$$[q] = [F]^{\frac{1}{2}}[k]^{\frac{1}{2}}[L] = \frac{[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}}{[T]} \cdot \frac{[T]}{[p]^{\frac{1}{2}}[L]} \cdot [L]$$

$$\text{i.e.} \quad [q] = [M]^{\frac{1}{2}}[L]^{\frac{1}{2}}[p]^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and:

$$[m] = [F]^{\frac{1}{2}}[p]^{\frac{1}{2}}[L] = \frac{[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}}{[T]} \cdot [p]^{\frac{1}{2}}[L]$$

$$\text{i.e.} \quad [m] = [M]^{\frac{1}{2}}[L]^{3/2}[T]^{-1}[p]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Maxwell's Theory of Light

In his theory of light Maxwell supposed that the transmission of light through any transparent medium was effected, *at any given time*, by the sinusoidal variation in electric field strength (E) and magnetic induction (B) at all points along the direction of propagation, and by the sinusoidal variation with time of E and B *at any given point* in the medium. The equations used in the theory express both E and B as functions of the time at any given point in the medium, and a solution to the equations gives the velocity of propagation of the energy associated with the variations in E and B. This velocity is known as the velocity of light in that medium.

If all the quantities required in Maxwell's theory are measured in the C.G.S. electrostatic system of units, so that the fundamental force laws take the form:

$$F = \frac{q_1 q_2}{\epsilon r^2} \; (k = 1) \text{ and } F = \frac{m_1 m_2}{\mu_0' \mu r^2} \; (p = \mu_0')$$

the velocity of propagation of light in the medium is given by:

$$v = \frac{1}{\sqrt{\mu_0' \mu \epsilon}} \text{ (cm.sec.}^{-1}) \quad . \quad . \quad . \quad . \quad (24)$$

If the C.G.S. electromagnetic system of units is used the force laws take the form:

$$F = \frac{q_1 q_2}{\epsilon_o' \epsilon r^2} \ (k = \epsilon_o') \text{ and } F = \frac{m_1 m_2}{\mu r^2} \ (p = 1)$$

and the velocity of light becomes:

$$v = \frac{1}{\sqrt{\epsilon_o' \epsilon \mu}} \text{ cm.sec.}^{-1} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

For a given medium, ϵ and μ are constant, independent of the system of units used, so that from (25) and (24) it follows that $\epsilon_o' = \mu_o'$, i.e. the conversion factors required in the inverse square laws to enable either system of units to be used are the same. In vacuo, $\epsilon = \mu = 1$, and the velocity of light has the well-known value: $v = c = 3 \times 10^{10}$ cm.sec.$^{-1}$ (approx.), so that:

$$\epsilon_o' = \mu_o' = \frac{1}{c^2} = \frac{1}{9 \times 10^{20}}$$

It would appear from these results that the dimensions of $1/\sqrt{\epsilon_o'}$ and $1/\sqrt{\mu_o'}$ are those of velocity, but this is incorrect owing to the fact that the dimensions of the constant of proportionality appearing in each force law are suppressed when their values are assumed to be unity. From (24) and (25), however, it follows that:

$$v^2 = \frac{1}{\epsilon_o' \mu_o' \epsilon \mu}$$

so that dimensionally:

$$\frac{[L]^2}{[T]^2} = \frac{1}{[k]\,[p]} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

This result is in agreement with the dimensional analysis noted earlier. Maxwell's theory is thus supported by this evidence, but apart from this, it has been shown by experiment that electromagnetic waves of the type envisaged by Maxwell can be generated artificially and their velocity has been found to be the same as that of light.

When all the quantities in Maxwell's equations are measured on the M.K.S. system of units the velocity of propagation of light is given by:

$$v = \frac{1}{\sqrt{\epsilon_0 \epsilon \mu_0 \mu}} \text{ m.sec.}^{-1} \quad . \quad . \quad . \quad . \quad (27)$$

The dimensional form of this equation is the same as (26) and no suppression of dimensions occurs using M.K.S. units. Since $\epsilon = \mu = 1$ in vacuo, and $v = c = 3 \times 10^8$ m.sec.$^{-1}$, it follows that:

$$\epsilon_0 \mu_0 = \tfrac{1}{9} \times 10^{-16} \text{ m.}^{-2}\text{sec.}^2 \quad . \quad . \quad . \quad . \quad (28)$$

Since Maxwell's theory has been verified by experiment the information derived about the values of ϵ_o', μ_o', ϵ_0 and μ_0 using the results of this theory may be utilized to determine the conversion factors relating the sizes of the

coulomb (unit of charge), 4
Counter torque, 328
Coupling coefficient, 277
Crystal structure, 412
Curie, 421
  Point, 309, 319
Current, (xiii), 17, 94
  strength, 98, 112
Cycle of Magnetization, 303
Cyclotron, 435
Cylindrical condenser, 45

D.C. Generator, 329
Damping, 137, 263, 291
Daniell cell, 237, 240
Decay of current (C-R), 285
  of current (L-R), 274
Declination, 450
Decomposition voltage, 231
Deflection magnetometer, 78, 443
Demagnetising field, 81, 300
Demagnetization, 304
Depolarization, 237
Deuterium, 439
Diamagnetic, 297, 319
Dielectric, 477 et seq.
  Constant, 3, 471
  breakdown, 27
Dimensional analysis, 490 et seq.
Diode, 378, 385
Dip, 77, 449, 453
Dipole, 69, 85, 298
Directional Loci, 447
Discharge (electric), 27, 364
  tube, 364
  tube applications, 368
Disintegration constant, 420
Displacement, 7, 477
  current, 291, 477
  law, 419
Dissociation, 227
Domains, 298, 307
Drum armature, 330
Dry battery, 239

e.c.e. (electro-chemical equivalent) 221, 222, 229
E.M.F. (electro-motive force) 94, 96, 103, 171
  back, 224, 353
  induced, 252 et seq.
e.m.u., 67, 113, 488
e.s.u., 3, 113, 487
e/m, 373
e/m_H, 232
Earth (as capacitor), 43
  's magnetic field, 77, 449
  inductor, 267
Eddy currents, 270
Einstein, 381, 431
Electric charge, xi, 1, 487
  clock, 355
  double layer, 96, 240
  field strength, 6, 14, 27
  induction, 7
  potential 12 et seq.
Electrode, 219
  potential, 239
  processes, 230
  standard, 240
Electrolysis, 219 et seq.
Electrolyte, 96, 219
Electrolytic condenser, 48
Electromagnet, 305, 317
Electromagnetic induction, 252, et seq., 324
  units (e.m.u.) 67, 113, 488
  wave, 290, 395, 494
Electrometer (quadrant), 464
Electron, xii, 94, 374
  gun, 396
  shell, 423
Electrophorus, 460
Electroplating, 247
Electroscope, 18, 462
Electrostatic focusing, 396
  induction, 20
  units (e.s.u.), 3, 113, 487
  voltmeter, 36
Energy, 12, 57
  density, 58
Equilibrium, radioactive, 421
Equipotential, 17
Equivalence of mass & energy, 431
Equivalent weight, 221
Excited state, 423
Extra-nuclear electrons, 419

Faraday (the), 231
  dark space, 364
  's disc, 332
  's Ice-pail experiment, 22
  's determination of dielectric constant, 471
  's law of electromagnetic induction, 255
  's laws of electrolysis, 220
Ferromagnetic, 86, 297 et seq.
Féry radiation pyrometer, 211
Field windings, 326, 330, 354
Filtering, 408, 415
Fission products, 440
Fleming rule, 129
Fluroescent lamp, 368
Flux, electric, 10
  magnetic, 88, 311
  magnetic, density, 88
  magnetic, leakage, 314
  -turns, 255
Force on surface of charged conductor, 32
  on current, 128
  line of, 6, 69

tube of, 57
Frequency (A.C.), 324
natural, 290, 395
Fuel cell, 247

γ-rays, 407 et seq.
Galvanometer, 99, 120 et seq., 159, 260
Gauss positions, 70
's theorem, 10 et seq., 24, 30, 89
Geiger-Muller counting tube, 436
Glow discharge, 364
Gouy's method (susceptibility), 320
Gram-atom, 221
Ground state, 423
Growth of current (C-R), 282
of current (L-R), 272
Guard-ring, 30, 47

Half-life, 420
Half-wave rectification, 385
Heating effect of current, 180 et seq., 343
Helmholtz galvanometer, 124
Henry, 271, 277
Hot-wire ammeter, 343
Hysteresis, 301, 307
energy losses, 307, 351

Image iconoscope, 401
Impedance, 377
Induced magnetism, 86, 297 et seq.
Inductance, 271 et seq., 335
Induction coil, 279
motor, 355
Insulator, xiii, 1
Intensity of magnetization, 298 et seq.
Intermediate metals (law), 198
nucleus, 439
temperatures (law), 199
International ampere, 236
ohm, 162
Inverse square law, 4, 66, 72 (verification by expt.), 487
Ion, 226, 363
Ionic Mobility, 228, 367
Ionic theory, 319, 226 et seq.
Ionization, 27, 363, 374, 410, 466
chamber, 413, 417, 467
Ionizing agent, 363
Iron ring, 314
Isotopes, 419, 428

J (Mechanical equivalent), 188
J-H curve, 301 et seq.

K-Lines, 414, 424
Keepers, 305
Kirchhoff's laws, 163
Kohlrausch, 225

L-Lines, 414, 424
L-C circuit, 289
L-R circuit, 272, 335
L-C-R circuit, 291, 340
Lamp filaments, 192, 195 (example)
Langmuir-Childs law, 385
Laplace rule, 115 et seq.
Law of intermediate metals, 198
of intermediate temperatures, 199, 203
Leakage resistance (condenser), 289
Leclanché cell, 238
Lenz's law, 255
Lightning, 27, 364
Local action, 237
Logarithmic decrement, 264

M.K.S. units, 3, 67, 114, 488
M.M.F. (Magneto-motive force), 312
Magnet, 66, 76 et seq., 304
Magnetic circuit, 311
dipole, 69, 85, 115, 298
domains, 298, 307
elements, 451
equator, 450
field, 68 et seq., 109 et seq., 316, 443 et seq.
field strength, 68 et seq., 116, 443
hysteresis, 303, 307
induction, 85, 88 et seq., 266, 299
line of force, 69
meridian, 77, 450, 453
moment, 69, 299, 452
pole, 66, 115, 298
pole strength, 66
potential, 91, 112
reluctance, 313
screening, 317
shell, 111, 112
storm, 451
susceptibility, 301, 320
Magnetism, induced, 86, 297
origin of, 115
residual, 304
terrestrial, 77, 449
Magnetization, 85, 297
cycle of, 303
intensity of, 298
residual, 304
saturation, 303, 319
Magnetometer, deflection, 78, 109, 443, 452
reflection, 444
vibration, 82, 109, 446
Magnetomotive force, 312
Mass defect, 437
-energy equivalence, 431
number, 428, 429

spectrograph, 428
Maxwell, 57, 291, 475, 493
Mobility, 228, 367
Moseley's law, 414
Motor (electric), 353 et seq.
Moving-coil galvanometer, 134, 159, 260
Mutual conductance, 392
inductance, 276 et seq., 395

National grid, 349
Natural frequency, 290, 395
Neon signs, 368
Neptunium series, 508
Neumann's law, 255
Neutral point, 298, 447, 448
Neutron, xii, 429
reactions, 439
Newton (unit of force), 3
Nickel-iron cell, 244
Nuclear reactions, 430, 439
reactors, 441
Nucleons, 430
Nucleus, 427 et seq.

Ohms law, 99, 226, 229, 335, 337, 367
Oscillator (triode), 394
Oscillatory circuit, 290

Paper condenser, 48
Paramagnetic, 297, 298, 301, 309, 320
Peak (A.C.), 325
Peltier coefficient, 197 et seq.
effect, 196, 197, 200, 204
Periodic Table, 419, 509
Permanent magnet materials, 304, 319
Permeability, 67, 87, 113, 306, 489
of free space, 67, 489
Permittivity, 3, 227, 471
of free space, 4, 473, 488
Phase difference, 335
Photo-electric cell, 382
-electric effect, 379
-electrons, 374
-voltaic effect, 382
Photon, 381, 417, 423, 439
Planck's constant, 381
Platinum resistance thermometer, 185
Plutonium, 508
Polarization (electric), 223, 237, 478
(magnetic) 86
Positive column, 364, 368
Positron, xii, 432
Post-office box, 154 et seq., 180
Potentiometer, 170 et seq.
Power, 104, 145, 346, 348
-factor, 347
-factor correction, 348
Proton, xii, 232, 422, 429, 431
Pulse, 400
Pyrometer, radiation, 211
thermo-electric, 210

Quadrant electrometer, 99, 464 et seq.
Quality, of X-Rays, 408
Quantum theory, 381
number, 422

R.M.S. (root mean square) values, 341
Radar, 399
Radioactive decay, 419
equilibrium, 421
series, 419, 506
Radioactivity, 414 et seq.
Radiomicrometer (Boys), 212
Ratio arms, 155
Rationalised M.K.S. units, 4, 67, 114, 488
Rayleigh potentiometer, 174
Reactance, 337
Recombination, (ionic), 227, 363
Rectification, 329, 385
Reduction factor, 121
Reflection magnetometer, 444
Relative permeability, 67, 489
permittivity, 3, 471, 487
Reluctance, 313
Remanence, 306
Residual magnetism, 304
Resistance, 98, 152 et seq.
of galvanometer, 138, 159
internal, 103, 160, 172
temperature coefficient of, 181
thermometry, 185
Resistivity, 100, 156
Rest-mass, 431
Retarding potential, 380
Rhumkorff coil, 279
Richardson, 377
Röntgen, 407, 414
Rotating coil, 324, 329
Rotor, 326, 330, 355
Rutherford, 415, 416, 418, 419, 420, 421, 423, 427
Rutherford-Bohr atom, 423 et seq.

Saturation current, 367, 378, 385
magnetic, 298, 303
Sawtooth waveform, 397
Schuster's method (H), 453
Scintillations, 417
Secondary cells, 236, 242 et seq.
circuit, 253, 278
Secular variation, 451
Seebeck effect, 196, 200
Self-inductance, 271 et seq.
Semi-conductor, 187
Sensitivity of galvanometer, 136, 138
of C.R.T., 399 (example)
Series-wound dynamo, 330

-wound motor, 354
Sharing of charge, 18, 286, 468
Shell, electron, 423
magnetic, 111, 112
Shielding, magnetic, 318
Shunt, 140
-wound dynamo, 331
-wound motor, 354
Silver voltameter, 235
Simple cell, 236
Slip-rings, 326
Soddy, 419
Soft magnetic material, 304, 319
Spark discharge, 363, 368
Sparking potential, 363
Spherical condenser, 43, 44
Split-ring commutator, 329
Standard cell, 248
condenser, 47
inductance, 277
Starter resistance, 354
Stationary states, 422
Stator, 326, 355
Stefan's law, 193
Striae, 364
Sulphation, 244
Susceptibility, 301, 320
Synchronous generator, 326
motor, 354

Television, 401
Thermionic emission, 377, 378
Thermistors, 187
Thermo-ammeter, 212
-E.M.F., 198, 200
-electric couple, 196, 210
-electric effect, 196 et seq.
-electric diagram, 207
-electric lines, 208
-electric power, 207
-electric pyrometer, 210
Thermopile, 211
Thomson, Sir J. J., xi, xii, 370, 372, 374
Thomson effect, 196, 205
coefficient, 205
Three-halves law, 385
Three-phase supply, 349, 356
Time-base, 396
constant, 274, 283, 285
Torsion balance, 1, 66
Townsend, 367
Transformer, 348, 350 et seq.
Transient current, 94, 282
Transition (electron), 423
Transuranic, 440
Triode, 389 et seq.

Units, 3, 67, 113, 487 et seq.
Uranium, 414, 437, 440, 441

Valency, 221, 229
Van der Graaff generator, 433
Vibration magnetometer, 82, 109, 446, 452
Virtual power, 347
Volt, 13,
Voltage divider, 176
Voltaic pile, 236
Voltameter, 220, 222, 235
Voltmeter, 13, 36, 141, 346
Von Laue, 410

Water voltameter, 220
Watt, 105
Wattmeter, 146,
Waveform, 396, 398
Weber, 68
Weston cell, 248
Wheatstone bridge, 152 et seq.
Wilson cloud chamber, 418
electroscope, 464
Wimshurst machine, 460
Wireless waves, 395
Work function, 379

X-Deflection plates, 396
X-Rays, 407 et seq.
X-Ray dose, 414
intensity, 414
spectrometer, 413
spectrum, 414

Y-Deflection plates, 396