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STAR-CLOUDS IN SAGITTARIUS.
(Barnard, the Yerkes Observatory.)

ASTRONOMY

BY

FOREST RAY MOULTON, PH.D., SC.D.

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PREFACE

No prefatory remarks can so clearly indicate the scope of this book as its Table of Contents, to which the reader is referred. Such a table is a skeleton of the subject matter presented, but it does not describe the flesh which this bare framework carries. Nor can any preliminary statement give more than mere hints of the spirit in which the book has been written.

It will be obvious, however, from the Table of Contents that an attempt has been made to unfold the subject of astronomy by proceeding step by step from the familiar earth and the evening constellations out to galaxies and super-galaxies of stars. It is hoped that the reader will often experience the thrills felt by the original discoverers of the wonders that are described. Yet these wonders are not merely awe-inspiring, for astronomy, the oldest of the natural sciences, has played an essential rôle in most of the progress of science and philosophy from ancient attempts to explain simple celestial phenomena to recent tests of the theory of relativity. It would be unfair to the reader and untrue to the subject not to indicate these connections, even at the cost of an occasional brief digression. In fact, since science consists largely of relationships, a writer, as a scientist, is under obligations to point them out.

Many of the conclusions in astronomy are among the most certain in the whole domain of human knowledge. In the discussions of those parts of the subject in which this is true, pains have been taken to make clear to the reader the thoroughness with which the theories have been established and the exhaustive tests to which they have been put. Often the reader will be called on to pursue a train of reasoning through many sequential steps before the goal of a discussion is reached. Those who have a taste for coherent thinking will derive keen satisfaction in proceeding systematically from fundamental principles and observational data to some important final conclusion. On the other hand, some parts of astronomy are on the farthest speculative frontiers of science. It has been the desire in presenting these aspects of the subject to permit the reader to feel the expectancy

and the elation of the pioneer, and at the same time to warn him of the extreme caution with which he should proceed.

In the long and honorable history of astronomy many great names appear. It has been the plan to refer explicitly to all those whose contributions to the subject have been of far-reaching importance, such as Newton and Herschel and Laplace. On the contrary, it has not been the intention to cite every contributor to the extensive astronomical literature of recent decades, for to do so would make the discussion at some places approach in character a catalogue of names. Yet to make the exposition vivid it has often seemed desirable to attach the name of a living astronomer to an observation or a speculation, even though it does not promise to be of permanent importance in the development of the science.

The author is indebted to an unusual degree to scientific friends for advice and criticisms. Among those to whom he is particularly grateful are Professor Robert H. Baker, Professor Walter Bartky, Dr. Philip Fox, Professor Oliver J. Lee, Professor William D. MacMillan, Mr. William W. Morgan, Professor Otto Struve, and Professor C. C. Wylie. He is greatly indebted to Dr. Max Wolf and to the directors and the astronomers of a number of observatories for illustrative materials, acknowledgments of which are made in connection with the reproductions. He is indebted to Dr. Oliver C. Farrington, of the Field Museum of Natural History, for a number of photographs of meteorites. He is indebted to Dr. F. E. Brasch for a portrait of Kepler and to Dr. T. J. J. See for portraits of Newton and Laplace. Finally, he is indebted to the publishers for their hearty coöperation in the printing of the book.

F. R. MOULTON.

CHICAGO, ILLINOIS,
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ASTRONOMY

CHAPTER I

PRELIMINARY CONSIDERATIONS

1. Astronomy as a Science. Within two or three generations science has revolutionized the world. Although its immediate purpose may often have been the construction of giant machines and the control of the forces of nature, its ultimate goal has been to provide men leisure for exercising their highest powers—their imagination, their reason, their aesthetic emotions, their spiritual aspirations—the faculties that distinguish them from the lower animals. By its marvelous applications to the production of the necessaries of life, science has shortened the hours of labor and furnished abundant opportunities for enjoying all that has been best in the past. At the same time, its far excursions into the nature of the universe have broken through the narrow intellectual horizons of earlier days and have led the human spirit into a new paradise. It is quite likely that a hundred thousand years from now historians will remember our period only because it was the one in which science first became a dominant factor in the progress of mankind.

It is to the glory of astronomy that in it men started on the pathway to science. Before the dawn of recorded history, such simple orderly phenomena as the seasons and the phases of the moon caught their attention, aroused their wonder, and, by regularly recurring, first suggested to them that the physical universe is orderly rather than subject to the whims of capricious gods. In the valleys of the Nile and the Euphrates rivers, where highly developed civilizations flourished continuously for many centuries preceding the Christian era, the simpler celestial phenomena had been observed so long and carefully that, at the time of Aristotle (384–322 B.C.), the lengths of the year and the month were known with an uncertainty of only a minute or two, the inclination of

the earth's equator to the plane of its orbit had been determined with much accuracy, the causes of eclipses of the sun and of the moon had been discovered and considerable progress had been made toward the development of satisfactory theories for explaining the apparent motions of the sun, the moon, and the planets among the stars. The theory was current among Greek philosophers of this early period that the earth is spherical and rotates on its axis, and that it revolves around the sun.

When the Greek civilization declined, science ceased to develop, for the philosophic and scientific genius of the Greeks was not taken over by their conquerors. The Romans held up the torch of science for a few centuries, even if they did not light new torches, but when Rome fell, science fell, and a long night came on. Eventually, however, there was an awakening. In the eighth and ninth centuries, the Arabs vigorously revived astronomy and erected well-appointed observatories for observing and recording celestial phenomena. The Crusades enormously stimulated the intellectual life of Europe with the result that science and literature and art gradually began to flourish in every quarter. Copernicus (1473–1543), in Poland, developed his theory that the planets revolve about the sun; Galileo (1564–1642), in Italy, ably supported the Copernican theory and took the first steps in the science of mechanics; and Newton (1643–1727), in England, completed the foundations of mechanics, discovered the law of gravitation, and perfected the theories of the motions of the planets and of the moon.

It is impossible within reasonable limits to give an adequate conception of the precision of many branches of astronomy, particularly of those parts that pertain to the dimensions of the solar system and the motions of its members. Every one knows that astronomers assign distances from the earth to the sun and to the moon, but most persons think that these distances are estimates that may or may not be correct. As a matter of fact, the interplanetary distances are really measured, though a bit indirectly, and the results arrived at have a percentage of accuracy comparable to that of the measurements of distances along the surface of the earth. It is in the motions of the heavenly bodies, however, that the most astonishing proofs of the precision of astronomical science are encountered. For example, the moon revolves about the earth in an orbit containing a thousand measurable irregularities, due

to the attraction of the sun, which are found by observations to be in harmony with theory. An astronomer could set a telescope now so that his successors, at some precise instant in a hundred years, could look through it and see the moon or any planet or star he might choose to have then in its field of vision.

2. The Scientific Attitude of Mind. In their earlier years, students must accept a great deal simply on the authority of their books and of their teachers, and often they continue to rely upon the opinions of others long after they should have put aside this childish attitude of mind. Certainly, in the domain of science there is no place for intellectual subserviency. The whole spirit of science is not only to learn the facts, but to describe means by which others may verify them. Evidently the readers of this book can verify only a few of the conclusions that are contained in it, because to attain them cost an enormous amount of labor on the part of hundreds of men, and to repeat the observations on which they are based would be impossible within the lifetime of an individual. Yet every reader is entitled to such a statement of the conclusions he is expected to accept that he himself may judge of their validity. In fact, he owes it to himself to demand the bases for conclusions and not to adopt them unless they appeal to his reason. It will be assumed in this book that the student who reads it has arrived at such a state of intellectual independence and uprightness that he will not accept its statements unless satisfactory reasons are given why he should accept them.

Many of the conclusions of astronomy are supported by such an abundance of reliable observational or deductive evidence that they may be confidently accepted as true; there are others which are only tentative hypotheses, awaiting complete verification or refutation; and there are still others, at present on the borders of the known, which are only hopeful speculations. A mind that is accustomed to demanding evidence for all the conclusions it is asked to accept will assign each doctrine it encounters to one or another of these classes, and it will always retain clear distinctions between what is secure, what is only a working hypothesis, and what is doubtful speculation.

3. Laws of Nature. In astronomy one hears much of the laws of motion and of the law of gravitation. The searching tests to which these laws have been put in thousands of instances, and the

marvelously successful way in which they have always met these tests, make these principles respecting the motions of the celestial bodies more worthy of our confidence than almost anything else within the whole range of our beliefs. If any so-called "law of nature" may justly be said to govern natural phenomena, then certainly this statement may be made of the laws of motion and of the law of gravitation. Let us, however, examine into what the so-called laws of nature really are.

Since the word *law* is commonly used in referring to a statute, it carries with it something of the connotation of authority and compulsion. This is the reason that laws of nature are so frequently said to "govern" certain classes of phenomena. It will be found, however, that the laws of science do not govern natural phenomena, nor are they in any way similar to the laws that have been enacted and placed on our statute books. It is, in fact, unfortunate that they are called "laws."

In attempting to make clear the real character of the laws of nature, it will be advantageous to use a rather extended illustration. Suppose a man who knew nothing whatever about railways or railway trains should find himself living beside a railway track between stations. After observing for a time the passing of trains, he would likely notice that some particular train appeared at about the same time each day. After having learned and recorded the times of its passage, he could predict its appearance, and his friends would be astonished that he could accurately foretell the occurrence of an event so extraordinary and one over which he had no control. Let it be assumed that the observer found that the appearances of other trains were similarly regular, and that he measured the speeds at which they passed.

After the observer had recorded and systematized the assumed data, he would have a complete time-table, giving accurately the times and the speeds at which all trains passed him in each direction. Therefore he could calculate the positions of the trains at any time before or after they passed him. His time-table would be his *Law of Nature* for the trains. If he were very unsophisticated, he would be likely to say that it "governed" the running of the trains. It is clear to us, however, that although the time-table describes accurately the running of the trains at and near the points of observation, yet it neither "governs" their motions nor has anything whatever to do with running them. Moreover, the

results it gives for places far distant from the point of observation may be completely erroneous.

Astronomers are fixed on this relatively tiny earth. For more than three thousand years they have turned their eyes up to the sky at night and have found regularities in the motions of the celestial bodies compared to which those in the running of the trains are trivial. From these long-continued observations they have found the ways in which the sun, the moon, and the planets move. They have learned how, in fact, to describe the motions of these bodies very simply, if somewhat indirectly, by means of the laws of motion and the law of gravitation. Although these so-called laws are on a much higher level of certainty than the time-table of the observer of the trains, they are nevertheless, like it, purely descriptive and nothing more.

Like all figures of speech, the foregoing illustration imperfectly represents the facts. On the one hand, it tends to the conclusion that the so-called laws of nature are less certain and important than most of them actually are. On the other hand, it tends to the conclusion that the observations of scientists bear more simply and directly upon their theories than is usually the case. To adopt another figure of speech, a scientific theory in a certain real sense is a model that represents experience in a limited domain, just as a photograph in black and white may represent satisfactorily a landscape as seen from a single point by a person who is color blind. But to a person who can distinguish colors, the picture is not a perfect representation of the landscape unless it is made in colors. If the observer can change his point of view, a picture in colors can no longer completely represent the landscape; it is necessary to use a three-dimensional relief model in colors.

So it is with scientific theories, or models of the observations of scientists; new information often makes it necessary to change the model. At one stage a model may correctly represent all that is known, and to this extent it is true, but it may not be the whole truth. New discoveries may require an enlargement of the means of representing the observed data, just as the supposed mobility of the observer of the landscape made it necessary to change from a plane picture in colors to a three-dimensional relief model. It is in precisely this sense that scientific theories fail from time to time and are succeeded by new theories. The

necessity for changes in scientific theories is one of the proofs that progress is being made.

While the foregoing remarks are a warning against misconceptions respecting the real nature of scientific laws and against confidence in the finality of scientific theories, much of the remainder of the book will be devoted to showing how abundant and how convincing is the evidence for many of the conclusions held by astronomers. Those who for the first time are becoming acquainted with this field will be amazed that the feeble and short-lived inhabitants of this little world have been so astonishingly successful in exploring and understanding the vast universe of which the earth is a relatively insignificant part.

4. The Scope of Astronomy. A general preliminary outline of the scope of astronomy will assist the reader in understanding the bearings of the more detailed discussions which follow, just as a preliminary outline map of an unknown country aids one in integrating the details of its separate parts into a consistent and unified whole.

(*a*) *The earth.* It may occasion some surprise that the earth is included in the scope of astronomy, for astronomy is popularly supposed to be concerned only with the stars. It is a fact, though, that such of our information respecting the earth as its diameter, its oblateness, its average density and its rigidity, to say nothing of our theories respecting its origin and its final destruction, depends essentially upon astronomical observations. Likewise, the motions of the earth are determined from astronomical observations. Moreover, the earth is one of a family of somewhat similar planets.

There is another important reason why astronomers have a scientific interest in the earth, and this is that the radius of the earth is the base line which they use in measuring the distances to the moon, the sun, and the other planets. The distance from the earth to the sun becomes, in turn, the base line which they use in determining the distances to the stars; and the distances to the stars are the units that enter into the determination of the dimensions of our galaxy and of the distances to other similar galaxies of stars.

(*b*) *The moon.* Of all the heavenly bodies, the moon is the one nearest to the earth; yet its distance compared with the familiar

terrestrial distances is enormous. In round numbers, it is on the average nearly 400,000 kilometers (240,000 miles) from the center of the earth. The diameter of the moon is nearly 3500 kilometers (2200 miles), its density is 3.4 times that of water, and its mass is about 1/80 of that of the earth.

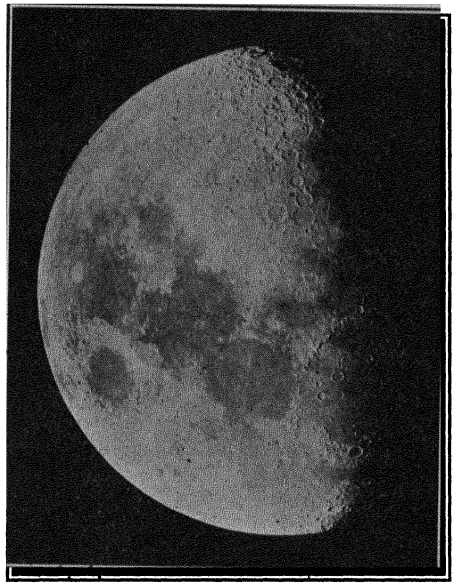


FIG. 1.—Moon as seen through a small telescope about 1.5 days after first quarter. (The Yerkes Observatory.)

Telescopes show that the moon is a barren little world, without water or an atmosphere and hence wholly unlike the earth around which it revolves. It is probable that all members of the solar system whose diameters are not much greater than that of the moon are similar to it in general characteristics.

(*c*) *The planets.* The larger bodies such as the earth, which revolve around the sun in independent orbits, are called planets. Of the nine planets at present known, three are smaller than the earth, four are larger, and the diameter of the ninth is yet uncertain. They are all so remote from the earth, even at the times of nearest approach, that to the unaided eye they appear like stars, though Venus and Jupiter sometimes seem to be very large stars. They are not stars, however, and as will be seen later, they are in no way related to the stars.

Although the distance from the earth to the moon seems to be very great, it is very small in comparison with the distance from the sun to the planets. For example, the distance from the sun to the earth is about 400 times the distance from the earth to the moon, or about 150,000,000 kilometers (93,000,000 miles); in fact, the distance is so great that the earth and moon viewed from the sun would appear to be almost touching each other. The orbits of two of the nine planets are inside that of the earth, while the orbits

of six of them are more remote than the earth from the sun. The distance from the sun to its nearest planet, Mercury, is only about one-third the distance from the sun to the earth, while the distance from the sun to the planet Pluto averages nearly forty times the distance from the sun to the earth. The diameter of the smallest planet, Mercury, is about three-eighths that of the earth, while the diameter of the largest planet, Jupiter, is nearly eleven times greater than that of the earth.

Perhaps the easiest way to get an approximately correct mental picture of the geometrical properties of the planetary system is to construct a model of it. Let the sun be located at some point on a plane surface and represent the earth's orbit by a circle about the sun as a center, the circle having a radius of one meter (40 inches). The radius of the orbit of Mercury on this scale will be 40 centimeters (16 inches), that of the orbit of Neptune will be 30 meters (98 feet), and that of the orbit of Pluto will be 40 meters (131 feet). The orbits of all of the remaining planets will be distributed somewhat regularly between the orbits of Mercury and Neptune. On the scale chosen, the diameter of the earth will be less than one-eleventh of a millimeter (one three-hundredth of an inch), and the diameter of the largest planet, Jupiter, will be only one millimeter (one twenty-fifth of an inch). Now the picture takes definite form: nine planets revolving around a central sun in orbits that are enormously far from the sun and from one another in comparison with their diameters. Although the larger planets all have satellites, the solar system is almost entirely vacant space; a room with relatively so little in it would be said to be quite empty.

(*d*) *The sun.* We regard the planets as important largely because the earth on which we live is a planet. In comparison with the sun, the planets are insignificant bodies; in fact, the mass of the sun is 700 times greater than the masses of all the planets together. The sun, nearly 1,400,000 kilometers (866,000 miles) in diameter, compares with the planets in size and mass much as a continent compares with vessels sailing off its coast.

It is not in size alone that the sun is unlike the planets; its temperature and its physical condition differ to an even greater extent. The materials of which it is composed, even at its surface, are at a temperature of approximately 6000 degrees Centigrade (10,000 degrees Fahrenheit), which is far higher than is necessary to melt

and vaporize any known substance, while in its deep interior the temperatures mount to millions of degrees. As might be expected, the activities in the sun correspond to its size and temperature. In the storms which often rage on the sun, vast masses of its substance, a hundred times the earth in volume, surge and whirl along at very great velocities; occasionally enormous eruptive prominences of glowing gases are shot up from its surface (Fig. 2) with velocities exceeding 100 kilometers per second, and they are sometimes visible until they recede as far as from the earth to the moon.

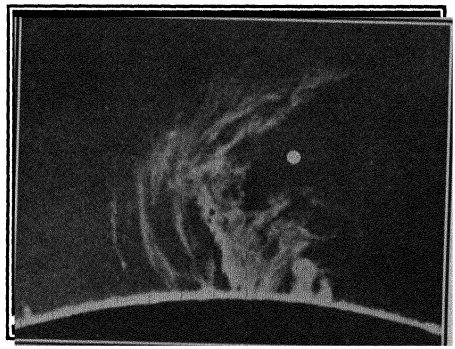


FIG. 2.—Eruptions from the sun. White circle represents the earth on the same scale. (Mount Wilson Observatory.)

As a general preliminary mental picture, the sun may be thought of as a huge body nearly 1,500,000 kilometers (1,000,000 miles) in diameter and a million times the earth in volume; a body enormously hot and self-luminous; a body torn by violent storms and eruptions; a body around which revolve, nearly in one plane, remote and widely separated tiny attendants, called planets.

(*e*) *Our galaxy of stars.* In ancient times the stars were thought to be jewels on a crystalline sphere; at a later date they were supposed by bold minds to be objects comparable to the moon, but never did imagination approach the facts that have been established by measurements. The stars are suns. About half of them are much like our sun in size and temperature and constitution; some are smaller than our sun and others are larger, in extreme cases millions of times larger in volume, but not many times greater in mass. Most of the stars which can be seen with the unaided eye are the larger and more brilliant suns, and they constitute a great cosmic unit called the galaxy.

It is natural to inquire why great suns, millions of times larger than the earth and brilliantly self-luminous, are so faint that they can be seen only in the darkness of the night. The answer to the question is that they are extremely far away (it might be better to say we are extremely far from them), and that distance alone dims

their light. At their distance our great sun would appear to be an inconspicuous star.

The distances to the stars are so great that it is difficult to describe them in comprehensible terms. Even the nearest star (Alpha Centauri) is more than 270,000 times as far from the solar system as the earth is from the sun, or 40,000,000,000,000 kilometers (25,000,000,000,000 miles). On the suggested scale of the model of the solar system, in which the distance from the sun to the earth was taken as a meter, the distance to the nearest star would be 270,000 meters (170 miles), and the diameter of the star would be approximately one centimeter (two-fifths of an inch). Since most of the stars are much more remote than Alpha Centauri, some being thousands of times as far away, it is convenient to use a larger unit of distance than the kilometer. The unit most frequently used in popular discussions is the distance light travels in a year, a distance for brevity called the *light-year*. Since the velocity of light is about 300,000 kilometers per second, the light-year is about 9,500,000,000,000 kilometers (5,900,000,000,000 miles). In terms of this enormous unit (which, at best, no one can really comprehend), the distance to Alpha Centauri is more than four light-years. The brilliant white star Sirius, the brightest one in the sky and the nearest one visible from the United States and Europe, is twice as far away. The distance to the orange-colored Arcturus is 40 light-years, while Canopus and red Antares are much more remote. Although the brighter stars on the whole are much nearer than those which are visible only with telescopic aid, when a person turns his eyes to the sky at night the light he sees from a majority of the stars within his range has been on its way since before the United States became an independent nation.



FIG. 3.—Stars in the Milky Way. (Photograph by Max Wolf Heidelberg, Germany.)

Only about 5000 stars are visible to the unaided eye in all the

heavens, including not only the half that is above the horizon but the other half as well. The telescope, however, reveals riches never dreamed of by the ancients. An instrument only five centimeters (2 inches) in diameter brings more than half a million stars into view, while the giant telescopes of the great observatories enable

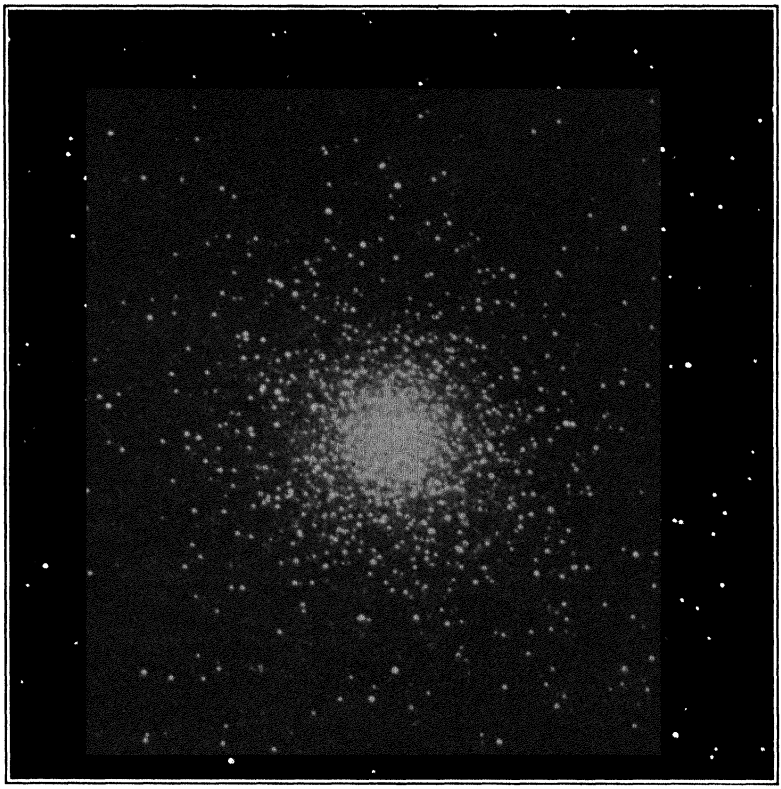


FIG. 4.—Great globular cluster M3 in Canes Venatici, distant 40,000 light-years and containing many thousands of stars. (The Lick Observatory.)

astronomers to photograph hundreds of millions of stars. There are probably more than 10,000,000,000 stars (suns) in our galaxy, though the total number is as yet uncertain. These stars occupy a disk-like, or watch-shaped, region whose thickness through its shortest diameter is perhaps 3000 light-years, and whose long diameter from edge to edge is approximately 30,000 light-years. The sun is deep in the interior of, but not at the center of, this enormous aggregation of stars.

Outside of our galaxy and at distances ranging from 20,000 to 200,000 light-years are globular clusters of thousands of stars (Fig. 4). These objects are so remote that, in spite of the number and the brilliancy of the stars which they contain, all of them except two or three are completely beyond the range of the unaided eye. Even the Great Hercules Cluster, which is one of the nearest and one of the largest, without the use of a telescope appears to be a faint and slightly hazy star.

Beyond the globular clusters there are countless galaxies in all essential respects similar to our own. They are generally spread out in thin disks and many of them contain hundreds and probably thousands of millions of suns. And far off at distances of more than a hundred million light-years, these galaxies in hundreds have been found grouped into super-galaxies.

Though the foregoing paragraphs briefly outline the scope of astronomy, they necessarily omit the greater part of the things which are of interest and importance. For example, little reference has been made to the mutual relationships among the myriads of bodies within range of the astronomer's interest, and not a word has been said respecting the origin, the evolution, and the possible destruction of satellites and planets and stars and galaxies. As much as this outline lacks of giving an adequate conception of the subject matter of astronomy, it lacks still more of giving even a glimpse of the enormous toil the science has cost, the ingenuity of the methods that have been used in developing it, the perfection of many of its theories, and the inspiration that has come from the achievements of astronomers. In no other subject have men won a better right to be considered superior beings, capable of understanding the universe about them.

I. QUESTIONS

1. What is the approximate time of the writing of Euclid's *Geometry*?
2. In what year did Gutenberg invent printing with the use of movable type?
3. In what year did Watt invent the steam engine?
4. In what year was the atomic law in chemistry formulated by Dalton?
5. In what year did Fulton put his first steamboat in operation?
6. In what year did Stephenson make his first successful locomotive?
7. In what year was the first railroad in the United States put in operation?
8. In what year did Morse invent the electric telegraph?
9. In what year did Darwin publish his *Origin of Species*?
10. In what year did Bell invent the telephone?
11. In what year did Edison invent the electric light?

12. In what year was the first steam turbine developed by Parsons?
13. In what year did the Wright Brothers make their first successful airplane flights?
14. In what year was the transmission of sound by radio waves first accomplished?
15. Represent the time from Euclid to the present by a line extending across a sheet of paper, and lay off on this line the dates contained in the answers to the foregoing questions.
16. What is the fundamental basis on which physical science rests? Does the same principle apply in the domain of mental phenomena?
17. Discuss the real meaning of the "laws of nature."
18. Make a drawing to the same relative scale of the orbits of the earth and the moon.
19. Draw on the same relative scale circles to represent the sun, the earth, and the moon.

GENERAL REFERENCES

Serial Publications

Science (weekly), New York, N. Y.
Science Progress (quarterly), London, England.
Scientific Monthly, New York, N. Y.
Scientific American (weekly), New York, N. Y.
Nature (weekly), London, England.
Scientia (monthly), Milan, Italy.
American Journal of Science (monthly), New Haven, Conn.
Popular Astronomy (monthly), Northfield, Minn.
The Observatory (monthly), London, England.
Publications of the Astronomical Society of the Pacific (monthly), Berkeley, Calif.

Recent Books

Science Remaking the World (1922). Otis W. Caldwell, Edwin E. Slosson, and others. Garden City Publishing Co., Inc., New York.
Keeping Up with Science (1925). Edwin E. Slosson. Harcourt, Brace and Company, New York.
The Nature of the World and of Man (1925). Sixteen authors. University of Chicago Press, Chicago.
Pioneers of Science (1926). Sir Oliver Lodge. Macmillan & Company, Ltd., London.
The New Universe (1926). Baker Brownell. D. Van Norstrand & Co., New York.
The Romance of Reality (1927). Beverly L. Clarke. The Macmillan Company, New York.
A Short History of Science (1927). W. T. Sedgwick and W. H. Tyler. The Macmillan Company, New York.
Modern Science (1929). J. Arthur Thomson. G. P. Putnam's Sons, New York.
The Mysterious Universe (1930). Sir James Jeans. Cambridge University Press, Cambridge, England.

CHAPTER II

THE CONSTELLATIONS

5. Origin of the Constellations. It is doubtful whether there is in the whole range of human experience any more awe-inspiring spectacle than that presented by the sky on a clear and moonless night. Under the vault of the sparkling heavens one is raised, if ever, to an actual realization of the fact that the earth beneath his feet is a relatively tiny mass in comparison with the infinite cosmos spread out above. Every one who aspires to partake to some extent of the fine things in art and science that have always been the delight of the greatest minds owes it to himself to become familiar with the glories of the evening sky.

A glance at the stars shows that they are not all equally bright, and a moment's attention reveals among them such natural groupings as the quadrilateral in Ursa Major, the diamond in Delphinus, and the cross in Cygnus. The ancients noticed these celestial figures before the dawn of history and wove about them the most fantastic romances. The names of these groups, or *constellations*, have come down to the present day by way of Ptolemy (about 140 A.D.), who enumerated, located, and described 48 of them.

The constellations of Ptolemy were somewhat irregular in outline and did not cover the entire sky. After several additions and revisions, they were finally simplified by the International Astronomical Union in 1922 (see page 32).

6. Names of the Stars. The ancients gave proper names to all the more conspicuous stars and identified many others by describing their positions in the imaginary objects in which they were located. For example, there were Sirius, Vega, Antares, Altair, etc., with proper names, and The Star at the End of the Tail of the Little Bear (Polaris), The Star in the Eye of the Bull (Aldebaran), etc., designated by their positions.

The proper names of 30 or 40 of the brightest stars have been retained and are commonly used, particularly in popular work, but the method of the ancients is much too cumbersome for scien-

tific use. In 1603, Bayer introduced the system of giving the stars in each constellation, usually in the order of their apparent brightness, the names of the letters of the Greek alphabet in their natural order. The Greek letter is followed by the name of the constellation in the genitive (possessive) case. For example, the brightest star in the sky is Sirius, in Canis Major. Its name according to the system of Bayer is Alpha Canis Majoris (Alpha of the Greater Dog). The second brightest star in the constellation Perseus is the famous variable star Algol; its name in this system is Beta Persei. After the Greek letters are exhausted, the Roman letters are generally used and then the integers, though there are many exceptions to the rule.

In about 1700, Flamsteed published an extensive catalogue of stars in which he numbered all those in each northern constellation according to their right ascension (Art. 12) regardless of their brightness. In modern catalogues the stars are simply numbered in the order of their right ascension without reference to their brightnesses or to the constellations to which they belong.

7. Star Catalogues. As will be seen later in many connections, the positions of the stars are of fundamental importance. Consequently, a star catalogue gives not only the brightness but also the right ascension and the declination (Art. 12) of each of the stars it contains. The date must be specified for which the position of a star is given, because the reference lines from which its coordinates are measured slowly change; and furthermore most of the stars have slight motions with respect to one another.

The sudden blazing out and the gradual disappearance of a star where none had previously been observed stimulated Hipparchus (about 160–105 B.C.) to catalogue 1080 of the brightest of these celestial objects to determine whether they may not all be transitory. This earliest known star catalogue was revised by Ptolemy (second century A.D.) about 250 years later. Tycho Brahe (1546–1601), whose original interest in astronomy was also aroused by a temporary star, made a catalogue of 1005 stars in 1580, the last star catalogue before the invention of the telescope. With the invention of the telescope the number of stars which can be catalogued was increased from the few thousands visible to the unaided eye to the hundreds of thousands which are visible even through a 5-centimeter (2-inch) instrument. The telescope is

important not only because it enormously increases the number of stars that may be catalogued, but also because its magnifying power enables the astronomer to determine their positions with corresponding greater precision. Without the great accuracy secured by the use of telescopes, astronomers would be quite unable to determine the distance to any star.

Up to 1890, star catalogues were based on observations of individual stars through telescopes having graduated circles attached for the purpose of determining the position of the object observed, but the great catalogues since that date have been made by photography. A single photographic plate may contain the images of a few stars whose positions are known, and also the images of thousands of others whose positions can be determined at leisure by measuring their distances and directions from those of the known stars. By these methods the positions of more than a million stars have been determined. Though catalogues will soon contain several millions of stars, there will be hundreds of millions of others which must long remain unlisted, except as one now and then becomes of interest for some special reason.

8. Stellar Magnitudes. For scientific purposes it is not sufficient to state simply that one star is brighter than another; a precise definition of the brightness of a star must be given. The word *magnitude* in the present connection refers only to the apparent brightness of a star and not to the amount of light it radiates, though there is a relation between the two quantities.

Long ago, astronomers arbitrarily divided the stars which are visible to the unaided eye into six groups, or magnitudes. They put the 20 brightest stars into the first-magnitude group, and the faintest that can be seen without optical aid into the sixth-magnitude group. They distributed the stars between these extremes in the four intervening magnitudes in such a way that the ratio of the brightness of a star in one group to that of a star in the next fainter group equals the ratio of the brightness of the latter star to that of a star in the next following fainter group. That is, the brightness of a star of one magnitude is found from that of the next fainter magnitude by multiplying by a constant factor rather than by adding a constant amount. It was found by measurements that the first-magnitude stars are 100 times brighter than the sixth-magnitude stars.

The foregoing definitions and the ratio of the brightness of a first-magnitude star to that of one of the sixth provide the basis for finding the ratio of the brightness of a star to that of a star in the next fainter magnitude. Let r represent this ratio. Then the fifth-magnitude stars are r times brighter than those of the sixth magnitude. Those of the fourth magnitude are r times brighter than those of the fifth and r^2 times brighter than those of the sixth. On continuing in this way, step by step, it is found that the brightness of a first-magnitude star is r^5 times that of a sixth-magnitude star. Since the measurements show that this ratio is 100, it follows that $r^5 = 100$, from which it is found that $r = 2.512\ldots$. That is, the brightness of a star of one magnitude is 2.512 times that of the next fainter magnitude.

Stars that are beyond the range of visibility with the unaided eye have magnitudes 7, 8, . . . in the order of decreasing brightness, with the ratio 2.512 . . . from each group to the next. The faintest stars that can be seen directly through the largest telescopes of the present day are of about the eighteenth magnitude, but very long-exposure photographs with the greatest reflectors reach down to the twenty-first or possibly to the twenty-second magnitude.

If a star were 2.512 times brighter than the first magnitude, its magnitude would be zero. Still brighter stars would have negative magnitudes. For example, the magnitude of Venus at its brightest is -4.0, while that of the sun is -26.7.

Since only exceptionally is a star of an integral magnitude, it is necessary to provide for fractional magnitudes. Ordinarily magnitudes are divided into tenths, a tenth of a magnitude being such a ratio that when multiplied by itself 10 times the product equals 2.512 . . . . It is found that a tenth of a magnitude corresponds to a ratio of brightness of 1.097 . . . . Evidently tenths of magnitudes can be divided into hundredths by the same method.

TABLE I. RATIOS OF BRIGHTNESS FOR DIFFERENT MAGNITUDES

0.1 mag. equals ratio	1.096...	1 mag. equals ratio	2.512...
0.2 " " "	1.202...	2 mags. equal "	6.310...
0.3 " " "	1.318...	3 " " "	15.850...
0.4 " " "	1.445...	4 " " "	39.810...
0.5 " " "	1.585...	5 " " "	100
0.6 " " "	1.738...	10 " " "	10,000
0.7 " " "	1.905...	15 " " "	1,000,000
0.8 " " "	2.089...	20 " " "	100,000,000
0.9 " " "	2.291...	25 " " "	10,000,000,000

Since ordinary photographic plates are more sensitive to blue light than to yellow and red light, while the human eye is more sensitive to yellow light than to blue light, there is some difficulty in keeping photographic and visual determinations of stellar magnitudes in agreement. The most serious difficulties in this respect naturally occur in connection with the faint stars, and, as will be seen later, it is with these stars that uncertainties in the magnitudes are most important.

9. The Celestial Sphere. Before the actual study of the constellations can be profitably taken up, it is necessary to consider the celestial sphere on which all the stars, whatever their distances, appear to be located. The direction of a star from the observer, which is often of great importance, can be conveniently described by giving its apparent position on the celestial sphere, for there is only one line from the place of observation to each of its points.

Let the spherical surface represented in Fig. 5 be the celestial sphere. The point O is the location of the observer and the circle $NESW$ is his horizon. The principal points of the compass are N, E, S, and W, which are assumed to be known, but which will be defined explicitly in Art. 10. In order to make further progress it is necessary to make a number of precise definitions, to comprehend exactly what they mean, and to retain them all clearly in the memory.

The *zenith* is the point on the celestial sphere at which the plumb line at the position of the observer, when extended upward, intersects the celestial sphere. Since the direction of the plumb line for every point on the surface of the earth is perfectly definite, the corresponding zenith is a perfectly definite point. The *nadir* is the point on the celestial sphere that is 180° from the zenith.

The *horizon* is the great circle on the celestial sphere that is 90° from the zenith, and its plane passes through the position of the observer. It should be kept in mind that the horizon is on the celestial sphere and not on the surface of the earth.

Suppose a star is located at the point A, Fig. 5. Draw an arc of a great circle from the zenith Z through A and continue it until it intersects the horizon at the point B. This circle is called a *vertical circle*, for it intersects the horizon perpendicularly. The arc BA, or the angle BOA, is the *altitude* of the point A.

If an object is above the horizon, its altitude is positive; if it

is below the horizon, its altitude is negative. The circle through the point A paralled to the horizon is called a *parallel of altitude*, because every point on it has the same altitude. The altitude of an object evidently does not completely define its position, just as the terrestrial latitude of a place does not completely determine its position on the surface of the earth.

If the angular distance NEB, measured eastward along the horizon from the north point N to the point B, is also given, the position of the point A is completely defined. The angular distance NEB, or the angle NOB, is called the *azimuth* of the point A.

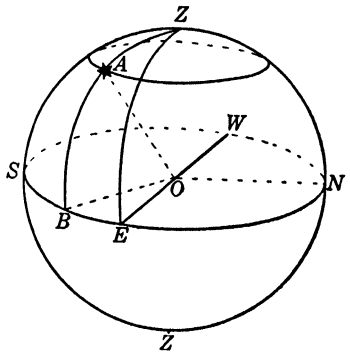


FIG. 5.—The horizon system.

In general, the azimuth of a celestial object is the angular distance measured from the north point (the south point is also sometimes used) eastward along the horizon to the point at which the vertical circle through it intersects the horizon. The azimuth of a star may be anything from 0° to 360°. If a star is directly east, its azimuth is 90°; if it is directly south, its azimuth is 180°; and if it is directly west, its azimuth is 270°.

It is evident after a little consideration that altitude and azimuth are both convenient and precise. Instead of describing vaguely the location of a star by saying, "It is up there in the northwest," one may just as easily define its position exactly by saying, for example, "Its altitude is 20° and its azimuth is 315°." A few minutes' practice with altitude and azimuth makes their use perfectly clear and simple.

10. Diurnal Motions of the Stars. If the stars and the other celestial bodies were fixed on the apparent sky, their altitudes and azimuths would remain constant and the problem of locating the constellations would be simplicity itself. But the stars are not fixed. They have motions westward across the sky, just as the sun and moon have diurnal motions, and for the same reason; namely, the eastward rotation of the earth. If a star is in the east in the evening, it will be in the south at midnight and in

the west in the morning. The next evening it will reappear in the east again.

If the sky is observed attentively, it will be found that there is one point, P in Fig. 6, at which the stars do not have diurnal motions. Stars near this point move around it in small circles with the point P at their center. For example, in Fig. 6 a star moves from N through L to M and back to N, completing a circuit in a day. Those stars which rise exactly at the east point E move along a great circle 90° from P through Q, then on to W, and finally through Q' and back to E. Whether the diurnal motions of stars are in small circles or in a great circle, they all complete their circuits in exactly the same period, as is necessarily the case since these apparent motions are due to the rotation of the earth. After these preliminaries, some precise and necessary definitions can be given.

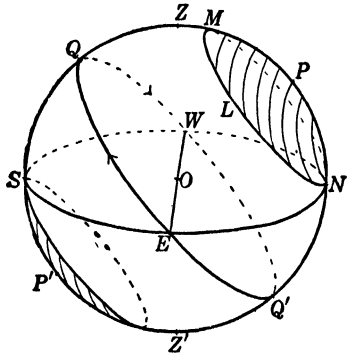


FIG. 6.—The equator system.

The point P (Fig. 6), the center of the diurnal circles of the stars, is one of the two *poles* of the celestial sphere; the other pole is P', 180° from P. Unless otherwise specified P will represent the north celestial pole; that is, the one which is near the stars of the Big Dipper.

The great circle $EQWQ'E$, 90° from the poles P and P', is the *celestial equator.*

The great circle through the zenith Z and the pole P is the *celestial meridian.* The meridian is uniquely defined except when the zenith Z is at either the celestial pole P or the celestial pole P', in which case it is not defined.

The point N at which the vertical circle from the zenith through the north celestial pole P first intersects the horizon is the *north point* of the horizon. This point is on the celestial sphere and *not* on the earth. The point E on the horizon 90° from the north point in the direction of the diurnal motions of the stars is the *east point* of the horizon. The south point S and the west point W are 180° and 270° respectively from the north point N, measured in the same direction.

The stars whose diurnal circles are within the shaded area

around the north celestial pole P are above the horizon of an observer whose zenith is Z the whole 24 hours, though they can not be seen except at night. The stars in the equal area around the south celestial pole P' are never above the horizon of an observer whose zenith is Z.

II. QUESTIONS

NOTE.—It is strongly recommended that Stetson's *A Manual of Laboratory Astronomy* be used in connection with this set of questions and the remaining ones in this chapter.

1. How many square degrees are there on the celestial sphere?
2. Suppose a photographic plate records the stars on one square degree; how many plates (square ones) will be required to cover the whole sky, each area overlapping the adjacent area by 10 per cent of its width?
3. How many square degrees are there on the average to each of the 5000 stars visible to the unaided eye?
4. What is the magnitude of a star whose brightness is 100 times that of a first-magnitude star?
5. What is the brightness of the sun (magnitude -26.7) compared to that of a first-magnitude star?
6. What is the definition of a great circle of a sphere?
7. Estimate the altitude of the sun at 10 o'clock this morning; at 10 o'clock tonight.
8. Point toward an object whose altitude is $-30°$ and azimuth 225°.
9. If the moon is visible at 8 P.M. today, estimate its altitude and azimuth.

11. Relation between an Observer's Latitude and the Altitude of the Pole. Since the celestial pole P is determined by the rotation of the earth, its position is independent of the observer. On the other hand, the zenith Z depends upon the position of the observer, for it is defined by the direction of his plumb line. It follows that the relationships among the points Z, P, and N, of Fig. 6, vary with the position of the observer.

In order to determine the important relationships among these quantities, let Fig. 7 represent the earth, the points p and p' being its poles and c its center. Suppose the observer is at the point O; then the line ON is in the plane of the observer's horizon. The equator of the earth is the great circle qq', which is 90° from the poles p and p'.

It follows from the definitions in Art. 9 that the line OZ_1 is directed toward the observer's zenith Z (the oblateness of the earth being neglected for the present). It follows from the definitions of Art. 10 that the line cp is directed toward the north

celestial pole P. Therefore the line ON is directed toward the north point N of the horizon. Let the line OP_1 be drawn parallel to the line cp; since the surface of the celestial sphere is infinitely distant from the observer, this line is also directed toward the north celestial pole P. Therefore the angle NOP_1, or a, Fig. 7, is the *altitude* of the pole.

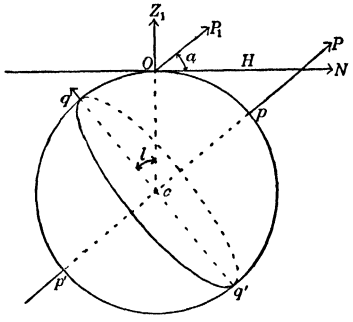


FIG. 7.—Relationship between latitude of observer and altitude of pole.

It follows from Fig. 7 that the angle qcO is the latitude l of the observer at O. Since the sides of the angle a are respectively perpendicular to the sides of the angle l, the required relation is found to be $a = l$. In words, *the altitude of the celestial pole P equals the latitude of the observer.*

Now it is easy to see how an observer might determine his latitude at night. It would be sufficient for him to observe the stars near the pole long enough to determine the center P around which they revolve. The altitude of this point equals his latitude. In practice, it is necessary to make allowance for the refraction of the earth's atmosphere (Art. 89) on the apparent altitudes of the stars.

Now return to Fig. 6. Since the celestial equator is 90° from the celestial pole, it follows that the altitude of the point Q at which the celestial equator intersects the celestial meridian is $90° - l$.

It follows from this discussion that a diagram corresponding to Fig. 6 can be constructed for an observer at any latitude on the surface of the earth. (The diagram does not depend upon the observer's longitude.) If the observer is on the earth's equator, the point Z coincides with the point Q and the point N coincides with the point P. On the other hand, if the observer is at the earth's north pole, the point Z coincides with the point P.

12. Declination and Right Ascension. It has been noted that the stars have apparent diurnal motions westward; actually the zenith Z and the horizon $NESW$ rotate eastward as the earth turns on its axis, for the zenith is defined by the position of the observer on the surface of the earth. It follows that the altitudes and the azimuths of the celestial bodies continually change.

On the contrary, the celestial pole P, Fig. 8, is fixed among the stars. It is not fixed in an absolute sense, for, as will be seen later, the direction of the earth's axis changes very slowly and, moreover, the stars have very minute motions with respect to one another; but it is fixed in the sense that, day after day and year after year, its position with respect to the stars is not appreciably altered. This fixity of the celestial pole with respect to the stars suggests defining a system of reference lines based on the celestial pole and the celestial equator. In such a reference system rotating with the apparent diurnal motions of the stars, the coördinates of the heavenly bodies are constants.

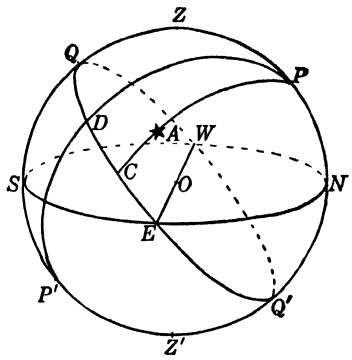


FIG. 8.—Declination and right ascension.

Positions of points on the surface of the earth are described by their latitudes and longitudes; positions of points on the celestial sphere are described similarly. The angular distance of a star north or south of the celestial equator, measured along a great circle passing through the celestial poles, is the *declination* of the star. If the star is north of the equator, its declination is positive; if it is south of the equator, its declination is negative and is preceded by the minus sign. Thus, in Fig. 8, the declination of the star at the point A is the angular distance CA, or about $+25°$.

Declination on the celestial sphere evidently corresponds to latitude on the earth. In order completely to describe the position of a star, a second coördinate corresponding to terrestrial longitude must be defined. The meridians of the earth pass through its poles and intersect the terrestrial equator at right angles; the corresponding circles on the celestial sphere, which are called *hour circles*, pass through the poles P and P', Fig. 8, and intersect the celestial equator at right angles. The half great circle PDP' is an hour circle. Longitude on the surface of the earth is measured from an arbitrary meridian, usually the one through the observatory at Greenwich, England. A corresponding reference hour circle on the celestial sphere will now be defined.

In the winter the sun is far south in declination and in the summer it is far north. Hence it crosses the celestial equator, and it

also has an apparent eastward motion among the stars. The point at which the center of the sun, in its motion northward, crosses the celestial equator is called the *vernal equinox.* The corresponding point at which the sun, in its motion southward, crosses the celestial equator is called the *autumnal equinox.*

The hour circle that intersects the celestial equator at the vernal equinox is taken as the reference hour circle, and is called the *equinoctial colure.* The second of the two coördinates of a star in this system is measured eastward from the equinoctial colure. Suppose the half circle PDP', Fig. 8, represents the equinoctial colure. Then the angular distance DC is the second coördinate, called the *right ascension,* of the star at the point A. That is, the coördinates of the point A in the equator system are the declination CA and the right ascension DC. They correspond respectively to altitude and azimuth in the horizon system and to latitude and longitude on the earth.

13. Determining the Right Ascension of the Sun on Any Date. The ultimate purpose of this discussion is to prepare for observing the stars, but it is necessary first to determine the position of the sun as an essential step in the problem.

It is found by observation that the sun is at the vernal equinox each year on March 21, or within one day of this date, the variation being due to the fact that the number of days in a year is not exactly 365. Since right ascension is measured from the vernal equinox, the right ascension of the sun is zero on March 21. The sun appears to move eastward among the stars, completing a circuit of the sky in a year of 365 days. Since the angular distance around the celestial equator is 360°, the right ascension of the sun on any date is approximately equal to the number of days from March 21 to the date. Instead of counting right ascension in degrees, astronomers find it convenient to count it in hours, an hour equaling 15°. Since an hour of right ascension equals 15°, there are 24 hours of right ascension in the circuit of the celestial sphere. The sun passes through 24 hours of right ascension in 12 months, or at the average rate of 2 hours per month. Therefore, the right ascension of the sun on any date (with sufficient approximation for present purposes) is the number of months from March 21 to the date multiplied by two.

In accordance with the foregoing rule, the right ascension of

the sun on June 21 is 6 hours; on July 4, about 7 hours; on September 21, about 12 hours; and on December 21, about 18 hours.

14. Determining the Date on Which a Star of Given Right Ascension Crosses the Meridian at a Given Time of Day. The problem has been set of determining the time a star will be on the celestial meridian, because, in general, stars are most favorably situated for observing when they are on the meridian. Similar principles may be used to find the date a star is at any angular distance from the meridian.

Since 8 o'clock in the evening is generally a convenient time for observing the stars, let this be taken as the given time of day of the problem. It has been assumed that a star of given right ascension is on the meridian; hence the right ascension of the meridian is the same as that of the stars and is known. At eight o'clock in the evening the sun has apparently moved 8 hours west of the meridian. Since right ascension is measured eastward, the right ascension of the sun is 8 hours less than that of the meridian, and is therefore known. It follows from Art. 13 that the date at which the sun has a given right ascension is found by dividing the number of hours of its right ascension by two and adding to March 21 the number of months equal to this quotient.

As an illustrative example, consider Sirius, the right ascension of which is given in star catalogues as 6 hours 41 minutes, or about 6.7 hours. When Sirius is on the meridian, the right ascension of the meridian is 6.7 hours. If the time of day is 8 P.M., the right ascension of the sun is 6.7 hours minus 8 hours, or 1.3 hours negative. Since right ascension is counted from 0 hours up to 24 hours, which is the same as zero hours, 1.3 hours negative is the same as 22.7 hours. Another way to treat the question, which is perhaps simpler, is to think of the right ascension of Sirius as being 6.7 hours+24 hours=30.7 hours, just as an angle of 5° may be considered as being 365°. If the right ascension of the meridian is 30.7 hours and the sun is 8 hours west of the meridian, then the right ascension of the sun is 30.7 hours−8 hours=22.7 hours, agreeing with the result found by the former method.

In accordance with the results given in Art. 13, the date at which the sun has the right ascension 22.7 hours is 22.7/2=11.35 months after March 21, or about March 1. That is, the bright star Sirius is on the meridian at 8 P.M. on or about March 1 each year. Since

also has an apparent eastward motion among the stars. The point at which the center of the sun, in its motion northward, crosses the celestial equator is called the *vernal equinox.* The corresponding point at which the sun, in its motion southward, crosses the celestial equator is called the *autumnal equinox.*

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the right ascension of the sun increases very slowly—only about 1° per day—Sirius is near the meridian at 8 P.M. for a week or two before and after March 1. Or, to express the facts in another way, Sirius crosses the meridian at about 8 o'clock in the evening in the latter part of February and in the early part of March each year. Before March 1 it crosses the meridian after 8 P.M., and after March 1 it crosses the meridian before 8 P.M.

15. The First-Magnitude Stars. A person can not claim reasonable familiarity with the sky until he readily knows by sight all of the first-magnitude stars that can be seen from his latitude. Since there are only 20 first-magnitude stars, they are relatively rare objects that may serve as guideposts in a study of the constellations. They are the stars, moreover, about which fanciful legend and folklore have woven the most delightful stories.

TABLE II. THE FIRST-MAGNITUDE STARS

STAR NAME	CONSTELLATION	MAGNITUDE	RIGHT ASCENSION	DECLINATION	ON MERIDIAN AT 8 P.M.	RADIAL VELOCITY
Sirius	Canis Major	−1 6	6^h7	−16°6	Mar. 1	− 9.0
Canopus	Carina	−0.9	6.4	−52.6	Feb. 24	+20.0
Alpha Centauri	Centaurus	0.1	14.6	−60.5	June 29	−22 2
Vega	Lyra	0.1	18.6	+38.7	Aug. 30	−13.7
Capella	Auriga	0.2	5.2	+45.9	Feb. 5	+31.8
Arcturus	Boötes	0.2	14.2	+19.6	June 24	− 5.0
Rigel	Orion	0.3	5.2	− 8.3	Feb. 5	+22.0
Procyon	Canis Minor	0.5	7.6	+ 5.4	Mar. 14	− 4.0
Achernar	Eridanus	0.6	1.6	−57.7	Dec. 16	+16.1
Beta Centauri	Centaurus	0.9	14 0	−60.0	June 21	+ 4.0
Betelgeuse	Orion	0.9	5.9	+ 7.4	Feb. 15	+21.0
Altair	Aquila	0.9	19.8	+ 8.6	Sept. 19	−33.1
Alpha Crucis	Crux	1.1	12.4	−62.6	May 29	+ 6.9
Aldebaran	Taurus	1 1	4.5	+16.4	Jan. 26	+55.1
Pollux	Gemini	1.2	7.6	+28.2	Mar. 15	+ 3.9
Spica	Virgo	1.2	13.4	−10.8	June 12	+ 2.0
Antares	Scorpius	1.2	16.4	−26.3	July 27	− 3.0
Fomalhaut	Piscis Australis	1.3	22.9	−30.0	Nov. 8	+ 6.8
Deneb Cygni	Cygnus	1.3	20.6	+45.0	Oct. 4	− 4.0
Regulus	Leo	1.3	10.1	+12.3	Apr. 23	+ 8.1

It is found on examining Table II that although at least one star crosses the meridian at 8 P.M. in each month in the year, the favorable opportunities for observing them in the evening hours are not distributed uniformly. Of the 20 stars in the list, 14 are on the meridian at 8 P.M. in the first 6 months, and they include most

of those which are brightest. Moreover, some of the most magnificent constellations are visible in the late winter and the spring months. The last column in Tables II gives the components of velocity from (+) or toward (−) the solar system in kilometers per second.

16. Altitudes at Which Stars Cross the Meridian. It was seen in Art. 11 that the altitude of the point at which the celestial equator intersects the celestial meridian is $90°-l$, where l is the latitude of the observer. Consequently, if a star has the declination 0° (is on the celestial equator), it crosses the meridian at the altitude $90°-l$. If a star has the declination δ (Greek letter delta), it crosses the meridian at the altitude $90°-l+\delta$, where δ may be positive or negative.

As an illustration, suppose the observer is at latitude 40° north, and consider the bright star Procyon, of Table II. The celestial equator intersects the celestial meridian of this observer at the altitude $90°-l=50°$. Since Procyon has the declination +5°.4, it crosses the meridian at the altitude $50°+5°.4=55°.4$. On the other hand, Sirius, with the declination −16°.6, crosses the meridian at the altitude $50°-16°.6=33°.4$.

Now consider the star Capella for an observer at the same latitude 40° north. According to the rule, the altitude of Capella, which has the declination +45°.9, when it crosses the meridian is $50°+45°.9=99°.9$. Since the altitude of every star lies between −90° and +90°, this result requires an explanation. By referring to Fig. 8, it is seen that Capella is 9°.9 beyond the zenith and that its altitude, measured up from the north point N, is $90°-9°.9=80°.1$. That is, while Procyon and Sirius and other stars having declinations less than +40° cross the meridian of the observer (at latitude 40° north) to the south of the zenith, Capella and other stars having declinations greater than +40° cross the meridian of the observer to the north of his zenith. The general rule is that if the declination of a star is less than the latitude of an observer, it crosses his meridian south of his zenith; if the declination of a star is greater than the latitude of an observer, it crosses his meridian north of his zenith.

If a star crosses the meridian of an observer south of his zenith, it may actually cross below the point S on the horizon, Fig. 8. In such a case the star's diurnal circle is in the shaded area around

P', Fig. 6, and the star is never visible to the observer. Since the altitude of a star whose declination is δ when it crosses the meridian of an observer whose latitude l is $90-l+\delta$, the value of δ which makes this altitude zero is $\delta = -(90°-l)$. As is evident, the declination is negative.

Suppose an observer's latitude is 40° north. Then, on referring to Table II, it is found that 5 first-magnitude stars are never visible from latitude 40° or more north. On the other hand, there are no first-magnitude stars that are always invisible from latitude 40° south of the earth's equator.

17. Altitudes and Azimuths of Stars not on the Meridian. It is obviously sometimes desirable to observe a star when it is not on the meridian. For example, a star in the western sky at sunset would descend below the western horizon somewhat later without having crossed the meridian. Since on the following day it would rise later than the sun, it could not be observed in the morning; but it could be seen in the early evening provided the observer knew where to look for it.

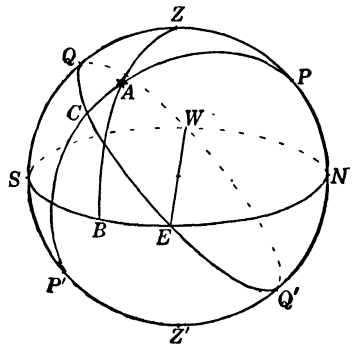


FIG. 9.—Determination of altitude and azimuth.

The problem of determining the apparent position of a star is one that can generally be solved with sufficient approximation for unaided-eye observations by the construction of a good diagram. For example, suppose the latitude of an observer is 40° north. The first step is to construct a diagram representing the meridian, the horizon, the equator, and the associated points and circles. Suppose the date is August 21; then the right ascension of the sun is 10 hours. The right ascension of the meridian is 8 hours greater, or 18 hours. Suppose the right ascension of the star is 20 hours; it is 2 hours east of the meridian on the hour circle PCP', Fig. 9. Suppose its declination is +10°; it is at the point A. Then draw a vertical circle from the zenith Z through A and perpendicular to the horizon at B. The altitude BA of the star is seen to be about 50°; its azimuth NB is seen to be about 130°. These results are so close to the truth that an observer would be

able to locate the star. On referring to Table II, it is seen that the problem applies to the star Altair.

A more convenient and accurate method than that of depending upon the construction of a diagram is the use of a celestial globe, Fig. 10. The globe is first set so that the altitude of the pole equals the latitude of the observer. Then it is rotated about its polar axis until the hour circle that has the same right ascension as that of the meridian is brought into coincidence with the meridian. With the globe in this position, the relations of the celestial sphere to the horizon at the particular time in question are evident.

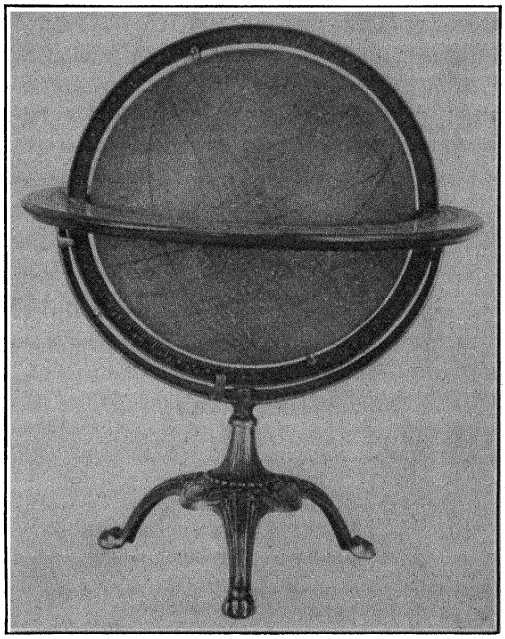


FIG. 10.—Celestial globe. (A. J. Nystrom.)

The angular distance between the meridian and the hour circle through a star at a given time is called the *hour angle* H of the star. If the star is east of the meridian, its hour angle is east; if it is west of the meridian, its hour angle is west. The hour angle of the point A, Fig. 9, is two hours east. For a given latitude of an observer and hour angle and declination of a star, the altitude and azimuth of the star are completely defined. In all cases these coördinates can be computed with any desired degree of accuracy by the use of spherical trigonometry, but results derived from a carefully-drawn figure are generally sufficient for unaided-eye observations.

Conversely, if the altitude and the azimuth of a star are known at a given date and time of day, its right ascension and declination can be estimated by the aid of a diagram or a celestial globe.

III. QUESTIONS

1. What is the relationship between the earth's equator and the celestial equator?
2. Draw a diagram corresponding to Fig. 6 for an observer at latitude 30° south; at latitude 70° north.
3. From what points on the surface of the earth can all the stars be seen in the course of a year?
4. What is the right ascension of the sun on July 21? October 21? December 21?
5. What is the right ascension of the meridian at 8 o'clock P.M. on June 21? On December 21?
6. Determine the brightnesses of the stars in Table II in comparison with the brightness of a first-magnitude star. What is the ratio of the brightness of Sirius to that of Regulus?
7. Which of the stars in Table II are always above the horizon of an observer in latitude 40° north? Which of them are always below his horizon?
8. Which of the stars in Table II are always below the horizon of an observer in latitude 40° south?
9. What would be the dates in the next to the last column of Table II if the observations were to be made at midnight?
10. Determine the altitude at which each of the stars in Table II will cross the meridian of an observer at latitude 40° north.
11. Suppose a ship's navigator finds that Sirius crosses his meridian at an altitude of 30°; what is the ship's latitude?
12. What are the altitude and azimuth of Capella for an observer in latitude 50° north when the star is 12 hours from his meridian?
13. Suppose an observer is in latitude 40° north. Make a diagram corresponding to Fig. 9, showing the position of Vega on June 21 at 9 P.M. Estimate its altitude and azimuth.
14. Suppose an observer in latitude 40° north sees a bright star whose altitude is 60° and azimuth 140° at 8 P.M. on May 21. What is the star?

18. Numbers of Stars of the First Six Magnitudes. Only those stars which are of the sixth magnitude or brighter are visible to the unaided eye. When an observer, far from city lights and haze, looks at the sky on a clear, moonless night, he seems to see the vault covered with stars. But appearances are very deceiving, for there are only about 5000 stars of the first six magnitudes in the whole celestial sphere, only half of which is above the horizon at any one time. The observer can not, in fact, see even 2500 stars at one time, for the faint ones near the horizon are dimmed beyond the point of visibility by the absorption of light by the earth's atmosphere.

The following table gives the number of stars in each of the first six magnitudes.

TABLE III. NUMBERS OF STARS OF FIRST SIX MAGNITUDES

First magnitude	20	Fourth magnitude	425
Second magnitude	65	Fifth magnitude	1100
Third magnitude	190	Sixth magnitude	3200

It is seen from Table III that, down to the sixth magnitude, the stars in each magnitude are about three times as numerous as the stars in the next brighter magnitude. If the ratio 3 held to the ninth magnitude, there would be 86,400 stars in the ninth magnitude alone and about 130,000 in the first nine magnitudes. The ratio 3 does continue to about the ninth magnitude, and there are approximately 117,000 stars in the first nine magnitudes. Since stars of the ninth magnitude are visible through a 5-centimeter (2-inch) telescope, it follows that moderate optical aid enormously increases the number of objects that can be observed. Some of the large star catalogues have, indeed, been made with the aid of telescopes only a few centimeters in diameter.

Beyond the ninth magnitude the ratio of the number of stars of one magnitude to the number of those of the next brighter magnitude steadily declines. If it did not decrease, there would be about 24,000,000,000 stars of the twentieth magnitude alone, whereas the number of stars in the first 20 magnitudes together probably does not exceed 1,000,000,000.

19. The Constellations and Their Positions. There are 90 recognized constellations, whose names and approximate locations are given in Table IV. The numbers at the top of the table show the degrees of declination between which they entirely or largely lie; the Roman numbers on the left of the table give the corresponding limits for their right ascensions. The names of those constellations that lie along the apparent annual path of the sun among the stars, the so-called *ecliptic*, are printed in italics. These constellations are called "the zodiac," for most of them were named after animals. After each constellation is printed the number of the stars in it that are easily visible without optical aid.

The altitudes and the azimuths of the constellations obviously can be determined by the method that has been explained for finding the coördinates of individual stars.

TABLE IV. THE CONSTELLATIONS

R.A \ Dec.	+90° TO +50°	+50° TO +25°	+25° TO 0°	0° TO −25°	−25° TO −50°	−50° TO −90°
I–II	Cassiopeia, 10	Andromeda, 16 Triangulum, 3	*Pisces*, 9 *Aries*, 5	Cetus, 17	Phoenix, 8 Sculptor, 3	Hydrus, 5
III–IV	Camelopardalis, 3	Perseus, 21	*Taurus*, 26	Eridanus, 32	Fornax, 3	Horologium, 2 Reticulum, 2
V–VI		Auriga, 11	Orion, 28 *Gemini*, 16	Lepus, 10	Caelum, 2 Columba, 7	Dorado, 3 Pictor, 3 Mensa, 2
VII–VIII		Lynx, 4	Canis Minor, 2 *Cancer*, 5	Canis Major, 15 Monoceros, 4	Puppis, 15	Volans, 3
IX–X		Leo Minor, 3	*Leo*, 18	Hydra, 15 Sextans, 1	Pyxis, 3 Antlia, 2	Vela, 11
XI–XII	Ursa Major, 20		Coma Berenices, 2	Corvus, 6 Crater, 4	Centaurus, 25	Carina, 14 Chamaeleon, 4
XIII–XIV		Canes Venatici, 3 Bootes, 15	Serpens Caput, 8	*Virgo*, 15	Lupus, 20	Crux, 6 Musca, 6
XV–XVI	Ursa Minor, 8	Corona Borealis, 8 Hercules, 27	Serpens Cauda, 5	*Libra*, 7	Norma, 2	Circinus, 2
XVII–XVIII	Draco, 16	Lyra, 10	Aquila, 10 Sagitta, 4	*Scorpius*, 23 Ophiuchus, 19	Ara, 8	Triangulum Australe, 5 Apus, 3
XIX–XX		Cygnus, 23	Vulpecula, 2 Delphinus, 5	*Sagittarius*, 18 Scutum, 2	Corona Australis, 2	Telescopium, 2 Pavo, 7 Octans, 2
XXI–XXII	Cepheus, 12	Lacerta, 3	Equuleus, 1	*Capricornus*, 10	Piscis Austrinus, 5 Microscopium, 1	Indus, 2
XXIII–XXIV			Pegasus, 17	*Aquarius*, 17	Grus, 7	Tucana, 7

On the pages following the Table of Constellations there are maps which show the outlines of the constellations and the more conspicuous stars, the various magnitudes being distinguished from one another by the characters that represent the stars.

Map I represents the region within 50° of the north celestial pole. To use this map, determine first the right ascension of the meridian on the date and at the time of day in question. Find the corresponding right ascension mark on the margin of the map. Turn the book until this part of the map is directly over the center; then hold up the book so that the line from the eye through the center of the map lies in the direction of the north celestial pole. When this is done, the map shows how the constellations are apparently spread out around the north celestial pole. For example, suppose the date is May 21 and the time of day is 8 P.M. The right ascension of the sun is 4 hours and the right ascension of the meridian is 12 hours. The mark XII on Map I is at the bottom of the page. In accordance with the instructions, the page is to be turned around until XII is directly above the center and the book is to be held up toward the northern sky. The map shows that the Big Dipper on this date and at this time of day is directly in the north about 30° above the celestial pole.

Maps II, III, IV, and V represent the regions within 50° of declination, north and south, of the celestial equator. The equator is the heavy horizontal line through the centers of the maps. Since Map I extends out 50° from the north celestial pole, its last 10° are also included on the equatorial maps. This removes uncertainties respecting the relative positions of stars having north declinations from 40° to 50°. On Maps II, . . . ,V the right ascensions are marked at the tops and bottoms and the declinations along the right and left margins. It will be observed that the hours of right ascension increase from the right margins of the maps toward the left; therefore west is on the right and east is on the left, contrary to the conventions in making geography maps. This arrangement is used in order that, when the observer faces the south and holds the map up to the sky, the map shall agree with the sky in the relationships of the stars.

To use the equatorial maps, first determine the right ascension of the meridian. Next find the corresponding hour circle on one of the maps. Then face the south and hold the book up with the determined hour circle in a line from the eye to the meridian, and

with the central horizontal line of the map in a line from the eye to the celestial equator. When this is done, the map and the sky agree, except for distortions due to representing a portion of the spherical surface of the celestial sphere upon the plane paper of the map.

Each of the maps is 7 hours of right ascension from its right margin to its left. Consequently, the hours of right ascension that occur at a margin of a map occur also on the opposite margin of another map. This overlapping avoids uncertainties respecting the neighbors of any star that is near the margin of a map.

20. Finding the Pole Star. In order to use the star maps or any other systematic method for observing the constellations, it is necessary to know the location of the celestial meridian. An observer generally knows his north-and-south line, but if he does not, he can easily determine it from the position of the pole star.

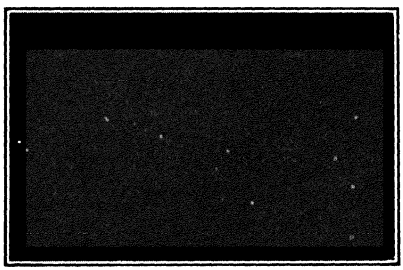


FIG. 11.—The Big Dipper.

One of the best-known and most conspicuous groups of stars in the northern heavens is the Big Dipper. Since it is in the shaded area around *P* in Fig. 6, it is always above the horizon for an observer whose latitude is 40° or more north and can be seen on any clear night. This group of 7 rather conspicuous stars has the unmistakable form of a dipper (Fig. 11) and will never be confused with any other group of stars. The stars are named Alpha (α), Beta (β), Gamma (γ), Delta (δ), Epsilon (ϵ), Zeta (ζ), and Eta (η), beginning with the star at the top of the bowl of the dipper opposite the handle, and proceeding in order to the extreme end of the handle.

To find the pole star, called Polaris, draw a line from Beta through Alpha, which are called *The Pointers*, and continue it about five times the distance between Beta and Alpha. The terminal point will be near Polaris, which is near the north celestial pole (Map I). The vertical circle through the zenith and Polaris is the meridian with sufficient approximation for unaided-eye observations.

Polaris is not only useful in defining the meridian and furnishing

a starting point for studying the northern constellations, but it is itself an interesting object. In the first place, it is probably more than 400 light-years distance from the solar system, or about 25,000,000 times the distance from the sun to the earth. At its distance a star having the intrinsic luminosity of the sun would be far beyond the range of visibility without optical aid. A good telescope shows that Polaris is accompanied by a faint companion of the ninth magnitude, which is so close to the larger star that it could not be seen as a separate object with the unaided eye even if it were brighter than the sixth magnitude. The brighter component is a spectroscopic binary (Art. 326) and is slightly variable.

21. Estimating Angular Distances. Until a person has had a little experience with angular distances, the statement that a star is 10°, for example, in a given direction from a known star is not of much assistance. Since distances on the celestial sphere are always given in degrees, it is important to have from the outset some convenient known units at hand for use in comparison.

The altitude of the pole star is equal to the latitude of the observer, which, in general, is known. Consequently, the altitude of Polaris furnishes a unit for estimating angular distances approximately equal to the latitude of the observer.

It will be found on examining Map I that the angular distance from Alpha Ursae Majoris, the star of the Big Dipper nearest the pole, to Polaris is about 30°. Since these stars are always above the horizon in mid-latitudes north, they furnish a very convenient unit for distances from 20° to 40°. For a shorter unit, the distance of 5°.3 between the pointers is equally convenient. With these units after a little practice one can estimate distances down to a degree or two with considerable approximation. If the angular distance between two stars is less than about 3′.5, they can not be seen with the unaided eye as separate objects.

22. Ursa Major (The Greater Bear). No other group of stars is better adapted to starting an acquaintance with the constellations than the Big Dipper (Fig. 11), the most conspicuous stars in Ursa Major. The outline of the Bear extends north, south, and west of the bowl of the Dipper for more than 10°. In using north, south, etc., in referring to the sky, it must be remembered that these directions are measured and respect to hour circles and declination circles, and that they do not have any necessary

connection with the points of the compass. Thus, the direction from Beta Ursae Majoris to Alpha Ursae Majoris (see Map I) is northward, but the direction of one of these stars from the other with respect to the horizon depends upon their direction from the pole when the observation is made.

The Big Dipper is involved in the folklore of many peoples. According to Greek mythology, Zeus changed the nymph Callisto into a bear in order to protect her from the jealousy of his wife Hera. While wandering in a forest, the transformed Callisto met her son Arcas who, not recognizing her, was about to slay her when Zeus intervened and saved her by placing them both among the stars, where they became the Greater Bear and the Smaller Bear (Ursa Minor). Hera was still not satisfied and prevailed on Oceanus and Thetis to cause them to pursue forever their courses around the pole without descending beneath the ocean waves for rest. In this way ancient peoples explained continuous circumpolar motions of these stars which never set.

The Pawnee Indians, a vigorous and intelligent tribe that formerly dwelt on the Central Plains, attached a much more reasonable story to the Big Dipper. According to their star-lore, the four stars of the bowl of the Big Dipper constitute a stretcher on which a sick man is being carried; the first star in the handle is the medicine man, who follows the stretcher; the next star at the bend in the handle is the medicine man's wife, who follows still farther behind; and the very faint star near the last is the medicine man's wife's dog, which trots at her heels.

Near the star Zeta, called Mizar by the Arabs, there is a faint companion. They named the faint star Alcor, a word which is said to mean "The Test," for they regarded it as a difficult object to see. It is, however, easily discernible under favorable conditions by any one with fairly good eyes. Alcor is of the fourth magnitude, and its angular distance from Mizar is 11′.5, a distance so great, astronomically speaking, that they are not regarded as constituting a double star. This star should be examined by every person interested in astronomy as a test of his sight, and particularly as an example of a distance of approximately 10′ of arc.

When observed through a telescope, Mizar and Alcor present a great surprise. In the first place, unless the telescope is of low magnifying power, these stars are apparently so far apart that they can not both be seen at the same time. And it is at once

noticed that Mizar is a double, the two components being nearly equal in brightness and apparently close together, for they are distant from each other only 14″.6. Although this is only one-fifteenth of the smallest angular distance within limits of visibility with the unaided eye, these stars can easily be seen as a double with a 7.5-centimeter (3-inch) telescope, and they are, as a matter of fact, very much farther apart than are the components of most double stars. The pair of stars of which Mizar is composed was first observed as two objects by G. B. Riccioli in 1650, the earliest telescopic discovery of a double star.

The fanciful tales woven about the stars by primitive peoples are merely interesting, but the facts established by science respecting what they actually are fill the comprehending mind with deepest awe. These little second-magnitude stars of the Big Dipper are not petulant gods and goddesses, fixed on the sky as a consequence of petty quarrels. They are great suns, much greater than our own, which appear dim only because of our enormous distance from them. In fact, if the earth were as near Mizar as it is to our sun, the surface of our world would be largely liquefied in the blazing rays of this star, for its rate of radiation is about 50 times that of the sun. According to H. Ludendorff, the earth is 4,500,000 times as far from this star as from the sun; even the distance from Alcor to Mizar is at least 15,000 times as far as from the sun to the earth. We see the system as it was 75 years ago, for 75 years are required for light to come from it to the earth.

The foregoing is not the whole story, for in 1899 E. C. Pickering found by spectroscopic methods, which will be explained in Art. 326, that the brighter of the components of Mizar is itself a double star, though the two members are too close together to be seen as separate objects even through the most powerful telescopes. Their combined mass is more than twice that of the sun, and they revolve around the center of gravity of the two at a distance of about 22,000,000 kilometers (14,000,000 miles) in a period of 20.5 days. In 1907, H. Ludendorff found by similar methods that the other component of Mizar is also a double star, though one of its two components is dark. In 1908, E. B. Frost and O. J. Lee proved that Alcor also is a double star. It follows that Mizar is in fact a quadruple sun, three of which are greater than our sun in mass and far surpass it in luminosity. And these stars, together with Alcor and its companion and nearly all the other principal

stars of the Big Dipper, constitute a majestic system of suns that probably had a common origin and which now pursue parallel paths through the galaxy of stars at a speed of nearly 100 kilometers per second. Yet they are so far from the earth that they now form sensibly the same outline as that which they formed in those far-away days when the Greeks wove about them their fanciful stories.

23. Cassiopeia (The Woman in a Chair). The Big Dipper is 30° from the celestial pole and in right ascension 12 hours; the constellation Cassiopeia is also 30° from the pole, but on the opposite side of it and in right ascension 0 hours. Consequently, when the Big Dipper is above the celestial pole, Cassiopeia is below the pole near the northern horizon, and vice versa; when the Big Dipper is to the right of the pole, as the observer faces north, Cassiopeia is to the left of the pole, and vice versa.

Cassiopeia can be easily recognized because the principal stars of which it is composed, which range from the second magnitude to the fourth, resemble the outline of a letter *W*. The name of the constellation owes its origin to the fact that when it is tilted in a certain way in its diurnal circle around the celestial pole, the wide-spreading *W* has some resemblance to a reclining chair. The brightest star in the constellation is Alpha Cassiopeiae at the bottom of the second part of the *W*. About 2° from Alpha and on the third stroke of the *W* is Eta Cassiopeiae, a fine double star whose components can be separated with a 3-inch telescope. These stars form a physical system, revolving around their common center of gravity in a period of about 500 years.

In 1572 a new star suddenly blazed forth in Cassiopeia and soon became the brightest star in the sky. A temporary star stimulated Hipparchus to make the first catalogue of stars (Art. 7); the new star of 1572 aroused the interest of Tycho Brahe. Though his observations were made before the invention of the telescope, they formed the basis for Kepler's work, which, in turn, was the foundation for Newton's brilliant discoveries (Art. 175).

24. Lyra (The Lyre, or Harp). There is apparently nothing in this group of stars which suggests its name, but it is a very interesting, though small, constellation. Since its right ascension is about 18.7 hours, it is on the meridian in the evening hours of the summer months. For observers in middle northern latitudes,

it crosses the meridian near the zenith, for its declination is about +40°. Its altitude and azimuth may be determined, however, at any time by use of the methods of Art. 17.

After the approximate position of Lyra has been found, it can easily be identified by its brilliant, bluish-white, first-magnitude star Vega, for there is no other star of equal brilliancy within 50° of it. Moreover, it forms with two fourth-magnitude stars, Epsilon and Zeta Lyrae, an equilateral triangle whose sides are about 2° in length. Since there is no other similar configuration in all the sky, it is very easy to be certain of the identity of the group.

In about 12,000 years Vega will be the pole star, as it was the pole star of our predecessors 14,000 years ago, though they probably were quite ignorant of the fact. The celestial pole, which is defined by the position of the axis of the earth, is slowly moving toward Vega as a consequence of the attractions of the moon and the sun for the equatorial bulge of the earth (Art. 97). In fact, the celestial pole describes a circle, shown in Map I, whose radius is about 23°.5, in a period of nearly 26,000 years. The period of the pole's circuit among the stars is trivial in comparison with the long ages of geological time, but a half-cycle reaches back beyond the beginning of recorded human history. One can not refrain from wondering what perils civilization will encounter in the next half-cycle nor dismiss the hope that science will lead mankind to undreamed-of levels of achievement and happiness by the time Vega is again the pole star.

Vega is more than 1,600,000 times as far from the solar system as the earth is from the sun. A star like the sun at its distance would be a faint object of magnitude 4.4. The solar system is moving nearly in the direction of Vega at the rate of about 400,000,000 kilometers (250,000,000 miles) per year. One would be quite likely to conclude that the brightness of Vega, before it again becomes the pole star in 12,000 years, will rival that of the sun. As a matter of fact, the distance of Vega is so enormously great that even this high velocity of approach will increase its apparent brightness in 12,000 years by less than one twenty-fifth of one per cent. The stars are incomprehensibly remote.

One of the stars which forms with Vega the equilateral triangle is Epsilon Lyrae, a little fourth-magnitude star, appearing at a casual glance much like other faint stars. Keen eyes on a clear night will note, however, that Epsilon Lyrae is not an ordinary

star, but twin suns whose components apparently almost touch. Only eyes having exceptional acuity of vision can detect the fact that the star is a double, for the angular distance between the pair is only 207″; hence this object offers a splendid test for excellence of sight. It is an interesting fact that neither the Greeks nor the Arabs nor any other astronomers before the invention of the telescope made any mention of having noticed that Epsilon Lyrae is a double star, and a century ago certain astronomers cited their ability to see the two stars separately without optical aid as proof of exceptional keenness of their eyes. Yet a majority of students, wholly inexperienced in observing the stars, readily discover that Epsilon Lyrae is composed of two components and determine the apparent direction they are from each other. Perhaps the severe burdens imposed on the eyes in reading the printed page, a use wholly unlike any other to which they have been put in all the ages of their evolution, actually improve the sight instead of impair it, as is often supposed.

When Epsilon Lyrae is viewed through a telescope of 10 centimeters (4 inches) or more in diameter, the observer gets a surprise, as in the case of many other stars. The two components are at opposite sides of the field of view and each of them is seen to be a double. That is, this little star is in reality four suns, each of them probably at least equal to our sun in size and luminosity. The closer pairs are twin suns, and it is probable that all four form a physical system.

There are two other remarkable objects in Lyra, one of which is the variable Beta Lyrae, a third-magnitude star about 5°.5 south and 3° east of Vega. The brightness of this star varies cyclically through nearly a magnitude, or by a ratio of more than two to one, in a period of 12 days and 22 hours. The explanation of this variability is that the star is composed of two large components, nearly in contact, which revolve around their center of gravity in a plane passing through the earth, and which periodically eclipse each other. The other remarkable object in Lyra is the Ring Nebula (Art. 347), which is so faint that its details are beyond all except large telescopes, though it is the brightest of the few nebulae of its kind in the entire sky.

25. Hercules (The Kneeling Hero). West and southwest of Lyra is the large constellation Hercules. Although it is marked

by no stars brighter than the third magnitude, it can be recognized from a trapezoid of 5 stars about 20° west of Vega. The 6°-base of this trapezoid is on the north, tipped slightly to the east, and at the northeast corner are two stars of the third and fourth magnitudes. The star that marks the southeast corner of the trapezoid is of the fourth magnitude, while the remainder of the 5 stars are of the third magnitude.

On the west side of the trapezoid, about one-third of the distance from the north end, is a fuzzy object near the limits of visibility without optical aid which, in the telescope, is one of the most magnificent sights the eyes of men have ever beheld, the Great Globular Cluster in Hercules (Fig. 208). The Hercules Cluster is composed of probably at least 1,000,000 stars, of which many thousands are much greater than our sun, but individually the brightest of them are very much too faint to be seen with the unaided eye. This group is so remote, or rather we are so far from it, that at least 30,000 years are required for the light of its stars to come to us. We see the cluster as it was before the dawn of recorded history, but not exactly as it was at any one time, for the group itself is so vast that 250 years more are required for light to come from its most remote part than from its nearest side. The imagination falters before the undoubted facts.

26. Corona Borealis (The Northern Crown). Just west of the great constellation Hercules lies little Corona Borealis whose brightest star is of the second magnitude. This group consists mainly of a semicircle, or crown, of stars which is turned toward the northeast with an opening about 8° across.

The name Corona was, of course, suggested by the outline formed by the stars of the constellation. The Pawnee Indians, however, had a name for this group which is equally appropriate. They called it the Camp Circle, and it is easy to imagine that warriors, sitting silently around their campfires on the plains at night, often looked up toward the zenith of their summer sky and saw a camp circle of the heavenly spirits, which are the stars.

27. Boötes (The Hunter). Directly west of Corona Borealis is the large constellation Boötes, in right ascension about 14 hours and extending from near the equator to within 35° of the north celestial pole.

The most conspicuous object in Boötes is the orange-colored

first-magnitude Arcturus, one of the most brilliant stars in the northern sky. Although this star is about 40 light-years distant from the solar system and is only two-thirds as hot as the sun, yet it is very bright because of its great diameter, which is nearly 300 times that of the sun. In fact, it radiates about 100 times as much energy as the sun gives forth.

In mythology Boötes is represented as leading his hunting dogs, Canes Venatici, across the sky. The constellation Canes Venatici lies northwest of Boötes, but it contains no conspicuous nor remarkable stars.

28. Scorpius (The Scorpion). In its annual circuit of the sky, the sun passes through twelve constellations, on the average one each month. The ninth and most brilliant of these zodiacal constellations is Scorpius, which lies largely between 16 and 18 hours of right ascension and between $-20°$ and $-40°$ declination. This group of stars is consequently visible in the southern sky in the evening hours during the summer months.

Scorpius can easily be recognized from its fiery-red first-magnitude star Antares, and also because it is 60° directly south of the constellation Hercules. A natural inference from the color of Antares is that its temperature is not so high as that of the sun and of the yellow and white stars in general, and the inference is correct; but it would be erroneous to suppose that this star does not radiate an enormous quantity of light, for actually it pours out radiant energy more than 3000 times as fast as the sun. Since the temperature of Antares is much below that of the sun, its surface must be enormously greater. Antares is so remote that more than 300 years are required for light to come from it to the solar system, and at that enormous distance a star a thousand times larger than the sun would appear to be a point of light, even through the largest telescopes of the present time. Yet its diameter has been measured by a remarkable instrument invented by A. A. Michelson and has been found to be fully twice that of the orbit of the earth. The volume of this enormous star, the largest one now known, is 90,000,000 times greater than that of our sun, but its density is low and its surface temperature is only half that of the yellow stars.

Scorpius lies in one of the richest and most varied parts of the Milky Way, and its numerous shining stars are seen projected

on a background of tremendous galactic clouds. About 5° northwest of Antares is a very compact and splendid globular cluster of stars known as Messier 80.

According to the Greek legend, Scorpius is the monster which killed Orion and frightened the horses of the sun so that Phaëton was thrown from his chariot when he attempted to drive them. The word Antares (anti Ares) means opposed to, or rivaling, Mars, the red planet associated with the god of war.

29. Leo (The Lion). Leo, the seventh constellation of the zodiac, lies about 60° west of Arcturus and is easily recognized by the fact that 7 of its principal stars form the outline of a sickle with the blade extending to the north and opening to the southwest. Since the right ascension of this constellation is from about 9 to 12 hours, it is visible in the evening sky during the spring months.

The first-magnitude star Regulus, the faintest of the 20 first-magnitude stars in Table II, page 26, is at the end of the handle of the sickle at its southern extremity. According to classic legends, Regulus is at the heart of the Nemean lion which was killed by Hercules as the first of his twelve great labors.

The Leonid meteors (Art. 254) derive their name from the fact that they seem to radiate from a point in the sickle of Leo. These meteors occur in considerable numbers each year about November 14 to 16, and every 33 years they appear in unusual abundance. The next great "shower" is expected in 1932 or 1933. Unfortunately, in November Leo rises at one o'clock in the morning, and the Leonids are, therefore, visible only from midnight till dawn.

30. Andromeda (The Chained Woman). Just south of Cassiopeia, in right ascension 0 to 2 hours, lies the constellation Andromeda. Although it covers a large region in the sky, it contains no really conspicuous stars. It is notable, however, because in it there is a very remarkable object, the so-called Great Nebula in Andromeda (Art. 379).

On clear and moonless nights, about 15° directly south of Alpha Cassiopeiae, a hazy patch of light can be detected which might easily pass for a fuzzy star or a comet. In the telescope, however, it is seen to be a mass of nebulous mist more than 2° long and nearly 1° wide. Until 1920, it was generally supposed to be the gaseous stuff out of which stars condense in the course of

enormous time, but now it is known to be a galaxy of stars comparable to our own. Those who take keen delight in the magnificent things encountered in nature will recall when they look at this fourth-magnitude spot of light in Andromeda that it is what we have been accustomed to call a universe; and that it consists of hundreds of millions of suns, occupying a space something like 30,000 light-years in diameter, and that its apparently feeble light is due to the fact that it is beyond our galaxy and approximately 870,000 light-years away.

31. Perseus (The Champion). The region that presents perhaps the finest view in all the sky through field glasses lies about 20° east of Andromeda in Perseus, especially in the neighborhood of its brightest star Alpha Persei. About 9° south and a little west of Alpha Persei is the second-magnitude star Beta Persei, whose more common name is Algol (the Demon). Algol owes this name to the fact that it varies in brightness every 2.867 days, declining for a few hours from magnitude 2.2 to magnitude 3.4. The star is so conspicuous, the range in its brightness is so great (about three-fold), and its period is so short that any one can verify its variability.

The variability of Algol is due to the fact that it consists of two stars, one of which is dark, revolving in a plane passing through the solar system. While the dark star is passing between the observer and the brighter star, the apparent magnitude of the latter is reduced by about 66 per cent.

Algol and the neighboring fainter stars, according to mythological tales, constitute the Medusa's head which Perseus, carrying in his hand, used in the rescue of Andromeda. He is said to have stirred up, in his haste, the dust of heaven which now appears as the Milky Way. This explanation of the Milky Way is even more fanciful than that of the Indians, who have a story that in the long-ago a horse chased a buffalo across the sky, both kicking up a cloud of dust. For part of the way they ran in parallel paths, where the Milky Way is divided into two parts in Sagittarius, Scutum, Serpens, and on into Cygnus, the horse kicking up the coarser dust represented by the brighter stars of one part, and the buffalo the finer dust of the other.

32. Auriga (The Charioteer). The next constellation east of Perseus is Auriga, in right ascension about 4.5 to 7 hours and in

declination 30° to 50°. It follows that this constellation is on the meridian in the evening hours from January to March.

Auriga contains the first-magnitude star Capella, the fifth brightest star in the sky. It was known in antiquity as *The She-goat,* and the three fainter stars near it were called *The Kids.* Capella is receding from the solar system at the rate of about 30 km. (20 miles) per second; but, since it is nearly 50 light-years away, its increasing distance will not perceptibly affect its apparent brightness in a thousand years. It follows from its distance and magnitude that Capella radiates nearly 200 times as much light as the sun.

33. Taurus (The Bull). Taurus lies about 30° southwest of Capella between the third and sixth hours of right ascension and in declination 15° to 30°. Besides its brilliant first-magnitude red star Aldebaran, it contains two remarkable groups of fainter stars, the Pleiades and the Hyades. Aldebaran is a large star whose radius is about 38 times that of the sun and which radiates about 90 times as much light as the sun.

About 10° northwest of Aldebaran, in right ascension 3.5 hours and declination + 24°, are the Pleiades, a little group of 6 stars of the fourth magnitude and one of the fifth that form the outline of a little dipper (Fig. 12). The ancients appear to have had difficulty in seeing the faintest of these stars, for there are many references in their mythology to the lost Pleiad. There is no difficulty, however, in seeing it without optical aid; indeed, persons with exceptionally keen sight can detect 3 or 4 other stars within or close to the group. A good telescope shows at least 100 additional stars which evidently belong to the same cluster, and even field glasses show an interesting region of sparkling suns.

The stars of the Pleiades cluster are distant something like 220 light-years, the diameter of the space occupied by the group is approximately 20 light-years (1,200,000 times the distance from the sun to the earth), and the individual stars range in intrinsic luminosity from 10 to 100 times that of the sun.

No group of stars in all the sky is mentioned more frequently in classic legends than the Pleiades, and in the folklore of primitive peoples as well. According to Greek mythology, the seven daughters of Atlas and Pleione; namely, Alcyone, Celaeno, Electra, Maia, Merope, Asterope, and Taygeta, were transformed into the

seven stars that bear their respective names. Merope, the lost Pleiad, concealed herself out of shame for having loved a mortal. The Pleiades were *The Many Little Ones* of the Ancient Babylonians, *The Hen and Chickens* of the early peoples of many parts of Europe, and *The Little Eyes* of the savage tribes of the South Sea Islands. The Pawnee Indians have a story of a great chief, Good Eagle, whose seven brothers were slain during a raid into an enemy's territory. When the survivors of the battle were being pursued toward their own country, Good Eagle looked up into the eastern early-morning sky and saw the faces of his brothers, the Pleiades. From their advantageous position they looked down and directed his faltering flight. And so, for the Pawnees the Pleiades are the spirits of the brothers of a great chief.

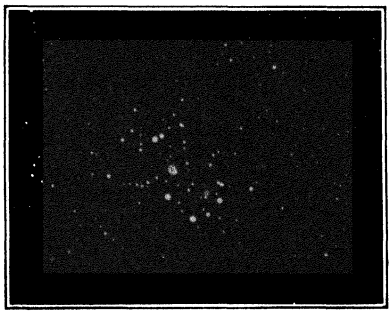


FIG. 12.—The Pleiades.

The Pleiades cross the meridian at midnight about the middle of November, and several ancient peoples began their year at that time. It is said that on the exact date, usually November 17 in the present system of counting time, no petition was ever presented in vain to the kings of ancient Persia. The religious ceremonies of the Aztecs were timed with reference to these stars, and certain of the wild Australian tribes held dances in their honor.

Aldebaran is in the midst of a group of small stars called the Hyades. The usual mythological story is that they were nymphs, daughters of Atlas and nurses of Dionysus, placed in the sky by Zeus. These stars were often referred to as *The Rainy Hyades*, for their rising with the sun coincided with the usual beginning of the rainy season in eastern Mediterranean lands. The Hyades constitute the first great group of related stars whose motions, distances, and luminosities were accurately determined. From extensive observations and an exhaustive discussion of the motions of these stars, Lewis Boss found that the principal stars of the group (exclusive of Aldebaran, which is less than half the distance of the remainder of the stars and is not physically related to them) constitute a great, open globular cluster whose diameter is about

33 light-years and whose distance from the solar system is about 130 light-years (Art. 363).

34. Orion (The Warrior). Southeast of Taurus and lying directly across the equator between the fifth and sixth hours of right ascension is the constellation Orion. This region, which is on the meridian in the evening hours during the late winter months, is the finest part of the whole sky for observing without a telescope. The legends respecting Orion are many and, in their details, conflicting. In all of them, however, Orion was a giant and a mighty hunter who, with a club in his right hand and a slain lion's skin in his left, stands facing the bull (Taurus) in the sky.

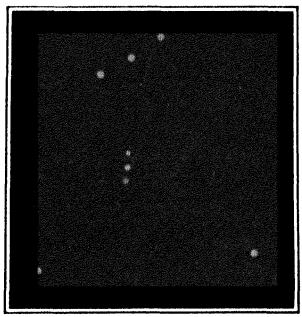


FIG. 13.—Belt of Orion.

There are two first-magnitude stars in Orion, the white star Rigel and the ruddy Betelgeuse. Rigel is in the lower part of the constellation, about 8° south of the equator. This star is very remote for one so bright, probably at a distance of 500 light-years, and its luminosity is more than 10,000 times that of the sun. About 20° northeast of Rigel is the red star Betelgeuse, which is less than half as remote as Rigel and only one-tenth as luminous, but its volume is 27,000,000 times greater than that of the sun.

About midway between Rigel and Betelgeuse and almost on the celestial equator is a row of three second-magnitude stars, lying in a southeast-northwest line (Fig. 13). This is the Belt of Orion, below which is a line of three fainter stars, lying almost parallel to an hour circle, and constituting the Sword of Orion. The central one of these three stars appears a little fuzzy as though it were seen through a mist. Viewed with a large telescope it is one of the most magnificent objects in the heavens, the Great Orion Nebula (Fig. 192). Unlike the so-called Great Nebula in Andromeda, this is an actual nebula composed of faintly luminous masses of gas of enormous volume. According to J. C. Kapteyn, the Orion Nebula is at a distance of more than 600 light-years; and, since it covers more than a square degree in the sky, its diameter is approximately 10 light-years, or 700,000 times the distance from the sun to the earth.

35. Canis Major (The Greater Dog). Lying southeast of Orion is Canis Major marked by brilliant bluish-white Sirius, the brightest star in the entire sky. Sirius is almost in line with the stars in the Belt of Orion and about 20° distant from them. This great star is comparatively near to us, as astronomical distances go, being only 83×10^{12} kilometers (52×10^{12} miles), or 8.3 light-years from the solar system.

Not only is Sirius the brightest star in the sky, but it has been historically one of the most interesting. First, in 1718, Edmund Halley found that it has a slow motion with respect to neighboring stars; then, a little more than a century later, F. W. Bessel found that its motion is not exactly in a straight line with uniform speed. He interpreted this slightly wavy motion of Sirius as meaning that this star and an unseen companion are in slow revolution around their center of gravity which describes a straight-line path at uniform speed among the stars. He concluded that the period of revolution of the pair is about 50 years. The faint companion was actually discovered by Alvan G. Clark in 1862, and a long series of observations since that time has shown that the mean distance between the two stars is about 3,000,000,000 kilometers (1,800,000,000 miles) and that their period of revolution is 48.8 years. Their combined mass is about 3.4 times that of the sun, and the mass of the faint companion is about two-fifths that of the principal star (Art. 316).

36. Canis Minor (The Smaller Dog). The principal object of interest in Canis Minor, a constellation lying directly east of Orion, is the first-magnitude star Procyon. Like Sirius, it is a telescopic double star, its fainter companion having been discovered in 1896. The period of revolution of Procyon and its companion is 39 years, the combined mass of the pair is about 1.5 times that of the sun, and the luminosity of the brighter star is about 6 times that of the sun.

37. The Milky Way. In the preceding descriptions of the constellations there have been several references to the Milky Way, a somewhat irregular, hazy band of light extending entirely around the sky. Although the Milky Way does not follow the celestial equator, nor an hour circle, nor any other ordinary circle of reference, yet it lies along a great circle, but one which cuts the celestial equator and the hour circles obliquely. On the average,

the Milky Way is about 20° wide, but its borders are irregular and for a long distance it is divided into two parts. Moreover, in places it is almost cut across by dark areas, and in others it is composed of great clouds of faint stars.

Since the Milky Way is a great circle, it has poles. Its north celestial pole is in right ascension about 12 hours and 40 minutes and in declination about + 28°, and the south celestial pole is, of course, directly opposite. It attains its greatest northern declination (45° to 65°) in the constellations Cassiopeia and Cepheus, where it is about 20° wide and of average brightness. It extends from this region southeastward across Perseus (Map I), with very irregular outlines, and it narrows down to a width of about 5° where it crosses the borders of Taurus. It then bulges wider in Monoceros and continues across the northeast corner of Canis Major. It crosses the celestial equator at right ascension 0 hours in Monoceros at an angle of 62°. Farther south, in Argo, it spreads irregularly to a width of about 30°. In this part of it there are dark lanes, one stretching entirely across it at right ascension about 9 hours and declination about − 45°. In the Southern Cross there is an area which is so dark in comparison with the surrounding regions that it has been called the "coal sack."

After reaching its southernmost extension in the constellation Crux, the Milky Way circles northeastward somewhat irregularly through Centaurus, part of Musca, Circinus, and Norma; and then it continues northward through Ara, Lupus, and Scorpius. In Scorpius and in Sagittarius, just to the east, there are some of the most remarkable star clouds in the heavens (Fig. 14). E. E. Barnard's photographs of these regions show amazing banks of countless thousands of suns, traversed by intervening darker lanes and often enveloped in a soft nebulous haze. It is in this direction that Harlow Shapley locates the center of the Milky Way system of stars.

Northeast of Scorpius through Ophiuchus, Serpens, Aquila, Sagitta, and Cygnus, the Milky Way is divided longitudinally by a rift of varying width and form. This bifurcation, extending for more than 50°, is one of the most remarkable features of the galaxy, and proves that this immense mass of stars has not arrived at a final steady state of motion in which the individual members forever pursue their journeys without changing the shape or the properties of the system as a whole. In Cygnus is the Northern

Cross, the top of which is marked by the first-magnitude star Deneb. Although this region is involved in the far-reaching star masses of the Milky Way, just south of Deneb is a spot so black

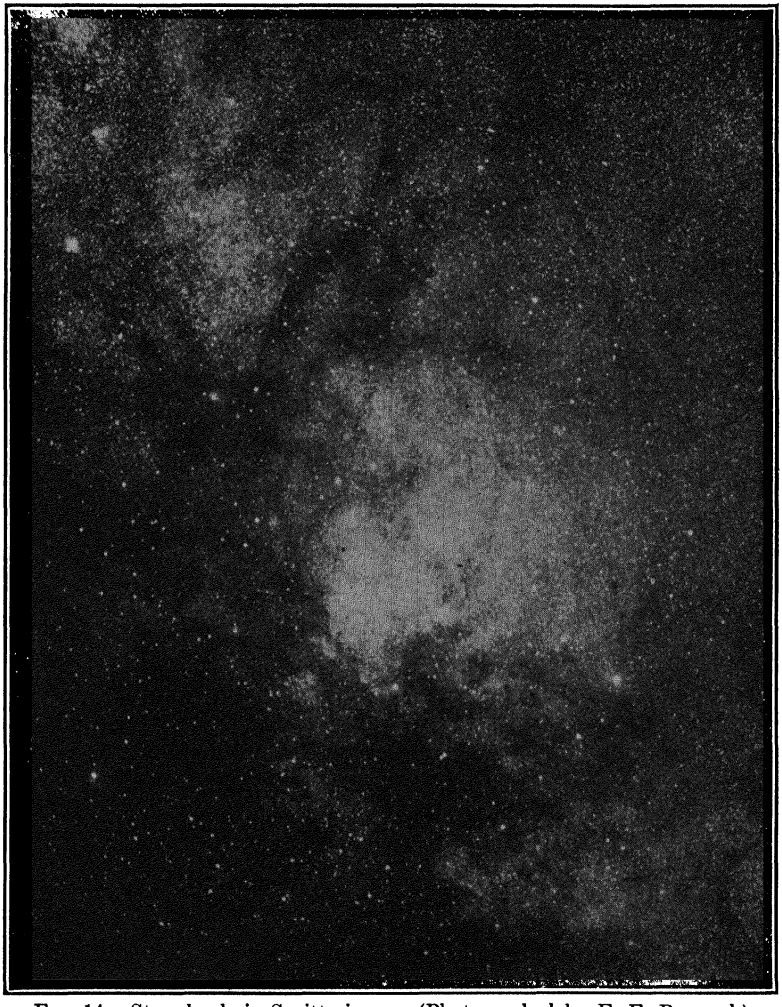


FIG. 14.—Star-clouds in Sagittarius. (Photographed by E. E. Barnard.)

that it appears as though one could here look out through a hole in the galaxy of stars to an infinite void beyond. The dark spot, however, is not a hole through the Milky Way, but is due to absorbing matter which cuts off the light of the stars which lie

beyond. In the photograph reproduced in the Frontispiece there is a very remarkable black spot in one of the richest star-clouds in the Milky Way. It, too, is undoubtedly due to dark matter lying between us and those distant parts of space.

When observed by the unaided eye, the Milky Way presents endless interesting details and is a ceaseless source of wonder. But with a telescope it loses its distinguishing characteristics, for the field of view is so limited that no idea of the star masses as a whole can be obtained. To look at it through a telescope is much like looking at a landscape through a microscope. Telescopes are necessary, however, for making the innumerable measurements from which the facts are established that show us what the Milky Way really is. Without their use we should still regard its shining clouds as star dust, or we might explain it by the childish theories of the ancients.

Up to the present point in this book, astronomy has mostly been taken in the easy and somewhat careless way that observers usually look at the sky. Its scope is so broad that no other procedure has been possible, just as a person in looking first at the Milky Way can not get an idea of its general characteristics by the painfully slow process of making measurements with telescopes. There comes a time, however, at which further progress in a subject depends upon abandoning the casual methods of the idly curious and adopting the exact and critical procedure of the expert. The point has been reached in this work at which unsupported statements must give way to the presentation of the observational evidence upon which conclusions are based and the reasoning by which they are reached.

IV. QUESTIONS

1. Make a list from Table IV of the constellations that cross the meridian for each of the hours from 8 P.M. till midnight on the twenty-first of this month. Determine the altitude at which each of them crosses the meridian.
2. Determine from Map I what constellation is directly above the north celestial pole on the seventh of this month.
3. Observe the Big Dipper and estimate the angular distance from the star Alpha to the star Eta Ursae Majoris. Estimate the angular distance from Eta Ursae Majoris to Polaris.
4. There are several stars in the bowl of the Big Dipper just fainter than the sixth magnitude. Can you see any of them without optical aid?
5. Do you have any trouble in seeing Alcor distinctly on a clear and moonless night? If you do, you should promptly consult an occulist.

6. If Lyra is visible before midnight at the time of year you are reading this chapter, be sure to observe it. Can you see that Epsilon Lyrae is a double star?
7. Can you find the Great Hercules Cluster? If so, examine it with field glasses.
8. Find the star Antares if it is above the horizon at convenient hours at the present time of year.
9. If Corona Borealis is well above the horizon in the evening hours at this time of the year, find the Crown, or Camp Circle.
10. If possible, observe the Great Nebula in Andromeda, and as you look at it, reflect on what it really is.
11. If the Pleiades are visible before midnight at this time of the year, observe them. Do you have any difficulty in seeing the seventh star? Can you see more than 7 stars? Examine the group with field glasses.
12. Observe Orion, if it is visible at this time of year. Identify Rigel, Betelgeuse, the Sword and the Belt. Examine the Great Nebula with field glasses.

REFERENCE BOOKS

Astronomy with an Opera-Glass (1923). Garrett P. Serviss. D. Appleton & Company, New York.

A Beginner's Star-Book (1927). Kelvin McKready. G. P. Putnam's Sons, New York.

A Guide to the Constellations (1928). Samuel G. Barton and Wm. H. Barton. McGraw-Hill Book Company, New York.

The Constellations and Their History (1928). Charles Whyte. Charles Griffin & Company, Ltd., London.

Star Lore of All Ages (1911). William Tyler Olcott. G. P. Putnam's Sons, New York.

The Pageant of the Stars (1929). William J. Luyten. Doubleday, Doran & Company, New York.

A Field Book of the Stars (1930). W. T. Olcott. G. P. Putnam's Sons, New York.

CHAPTER III

TELESCOPES

38. Importance of Telescopes. In astronomical work telescopes serve two quite distinct purposes: (*a*) they gather light from the object at which they are pointed and concentrate it for the use of the observer, and (*b*) they magnify the apparent size of the object.

(*a*) With the unaided eye only about 5000 stars are visible; through a telescope 10 centimeters (4 inches) in diameter more than 1,000,000 of them can be seen; and with the great 100-inch reflector of the Mount Wilson Observatory 1,000,000,000 of them can be photographed. But the importance is more than one of mere numbers, for without telescopes we should have no knowledge of the more distant stars or of the extent and the form of our galaxy. Moreover, telescope lenses and mirrors concentrate the light analyzed by spectroscopes, which are marvelous instruments upon which all our knowledge of the constitution of celestial bodies depends.

(*b*) When one thinks of the advantages of the telescopic magnification of astronomical objects, he is quite likely to have in mind the increased scale on which the surfaces of the moon and of the planets are presented. These are, indeed, important advantages, for without them our knowledge of the moon, the planets, and the sun would be much less than it is. But the magnification attainable by the use of telescopes has other advantages that are even more important. It enables us to measure small angles and to determine the distances of objects that are far away. Without telescopes, we should be unable to measure the distances to the members of the solar family, except in the roughest way; and we should be wholly incapable of sounding the depths of space beyond our own little system.

Probably no other animal has more nearly perfect eyes than those of human beings, for normal eyes approach theoretical perfection. Their degree of excellence has been acquired by the slow process of evolution through probably millions of generations of ancestors, the improvements in a thousand years being quite

inappreciable. But by the use of our reasoning faculties during a few generations we have been able to design instruments which, in effect, have improved our sight ten thousand fold. Otherwise stated, our largest telescopes are equivalent to eyes the pupils of which at the present maximum are 250 centimeters (100 inches) in diameter and the lengths of which range up to 40 meters (120 feet). Now we shall learn how our brains have outrun the processes of evolution.

39. Invention of the Telescope. The principles of the telescope appear first to have been at least partly understood and put to practical purposes, about 1608, by Jan Lippershey, a Dutch spectacle maker. He was interested in spectacles rather than in astronomy, and he did not make an instrument that would be useful in observing the heavenly bodies. An account of the work of Lippershey reached Galileo (1564–1642) in 1609. The alert mind of the great Italian at once recognized the astronomical possibilities of the marvelous invention, and he promptly made a telescope (not on the plans now used, but more like our opera glasses) and turned it on the moon, the sun, and other celestial objects with epoch-making results.

There are serious defects in the type of telescope used by Galileo, depending upon the fact that lenses bend different colored rays of light by different amounts. To avoid these defects, opticians turned to the use of mirrors for collecting light; and in the days of Newton (1642–1727) it was thought that all great telescopes of the future would be of the reflecting type. But John Dollond (1706–1761), an English optician, invented a method that largely overcomes these defects by making lenses of two pieces of different kinds of glass having compensating properties.

Since the time of Dollond both lenses and mirrors have been used for telescopes. Generally speaking, telescopes for visual purposes use lenses for concentrating light, while those for photographic purposes use mirrors. From the practical point of view it is possible to make and use larger mirrors than lenses, and therefore the largest telescopes are reflectors.

40. The Human Eye. A brief description of the elements of the optics of the eye will assist us in understanding the principles which underlie telescopes.

The eye consists essentially of a nearly spherical body into

which there is only a small opening (the pupil) P through which light may enter, a lens L, a transparent medium M, and a sensitive layer (the retina) R on which the light falls after passing through L and M.

Suppose a point-source of light is at A, Fig. 15. There is one ray from it which goes through the lens L without change of direction and falls on the retina at the point a. Other rays of light also go through the pupil and they are bent by the lens L and brought also to the point a on the retina. That is, corresponding to the point-source A there is a point-image a on the retina, and the brain sees a point of light.

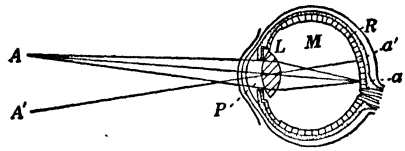


FIG. 15.—The human eye.

If the point A is too near the eye, the rays will not be brought quite to a point on R, but will be spread out over a small circle and the brain will see a small circle instead of a point. If the point A is very far away, the rays that enter the pupil P will be very nearly parallel, and in the normal eye they will be brought to a focus on the retina R. The eye has some power of adjusting its length and the convexity of its lens L to compensate for different distances of the object A. An eye is considered normal when its lens P brings parallel entering-rays strictly to a focus on the retina R.

Now suppose there were another source of light A', below A in Fig. 15. One ray from it will go centrally through the lens P and fall at a point a', above a, on the retina R. All other rays from A' passing through the pupil P will be brought by the lens L to a focus at the same point A', and the brain will see a second point of light. It is evident from the diagram that the images a and a' are inverted in order as compared with the sources A and A'. The apparent distance between A and A' is proportional to the actual distance between the images a and a'. Since the central rays from A and A' go through the lens L without change of direction, it is evident that the distance between the images a and a' is proportional to the angle between the lines from A and A' to the center of the lens L. If this angle is very small, the images a and a' are so close that the brain does not see two distant objects. The distance which must separate a and a' in order that the brain shall see two points of light depends in part upon the fineness of the

rods and cones of which the retina R is composed, and in part upon the fact that, owing to the appreciable wave-lengths of light, the images a and a' are not strictly points, but very small circles of light.

To sum up: If parallel rays enter the eye, they converge to a point on the retina and the brain sees a point of light. If two sets of parallel rays enter the eye, the brain sees two points of light, their apparent distance apart being proportional to the angle between the two sets of parallel rays. Obviously a corresponding statement is true if any number of different sets of parallel rays enter the eye.

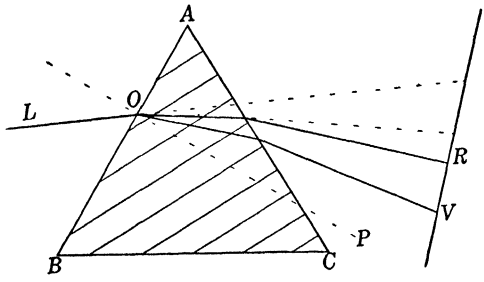


FIG. 16.—Refraction of light.

41. Refraction of Light. The Spectrum. As was stated in Art. 40, when parallel rays of light enter a normal eye they converge to a point on the retina. This convergence, upon which definiteness of vision depends, is due to the *refraction* of light by the lens L, Fig. 15.

In explaining refraction, it is simplest to start with a prism. Let ABC, Fig. 16, represent a cross section of a triangular prism of glass or other transparent medium that is denser than the air. Let L represent a beam of parallel rays of light meeting the surface of the prism at the point O. It is found by experiment that when the light enters the denser medium it is bent, or refracted, *toward* the perpendicular OP to the surface of the prism. The more obliquely the beam enters the prism the more it is refracted. Moreover, the different colors of which the beam of light is composed are refracted different amounts, the red the least and the violet the most. When the light emerges from the prism back into the rarer medium, it is refracted *from* the perpendicular to the surface, and the different colors are again refracted different amounts, the red the least and the violet the most. This spreading of light into its different colors by refraction is called *dispersion.*

If a screen S is placed across the path of the emerging beam, the visible colors are spread out in a streak extending from the

violet V to the red R. This band of light is usually considered as being divided into the seven colors, *violet, indigo, blue, green, yellow, orange,* and *red,* though there are in reality infinitely many colors, for the colors change gradually from one into another.

There are rays that are refracted less than the red rays, the so-called *infra-red* rays; and there are rays that are refracted more than the violet rays, the so-called *ultra-violet* rays. That is, after the beam of light has passed through a prism, it is spread out into a band of colors, or rays, some of which are visible and some of which are invisible. This complete band of rays constitutes what is called the *spectrum* of the beam. In general, not all colors are present in a spectrum in equal abundance; in fact, in many cases, depending upon the source of light, some colors are completely absent from a spectrum.

In a general way, the denser the prism the more it refracts, but the amount of the refraction and the extent of the dispersion depend also upon the composition of the prism. Among the very great number of kinds of glass available for optical purposes, some have the property of refracting light only moderately while others refract it to a much greater degree. Likewise, some kinds of glass produce considerable dispersion for a moderate amount of refraction, while other kinds produce much less dispersion for the same amount of refraction. Each kind of glass and other transparent medium has its own peculiar characteristics of refraction and of dispersion.

I. Refracting Telescopes

42. The Simple Telescope. If all refinements are omitted from consideration for the moment, a telescope consists of two suitable lenses, L and l, arranged as in Fig. 17, and tubes for holding them in position. The lens L, called the *objective,* is a double convex lens whose surfaces have slight curvature; the lens l, called the *eye-piece,* is a smaller double convex lens whose surfaces have strong curvature.

Let A, B, and C be three rays entering the lens L parallel to its axis. They will be brought to a *focus* F whose position depends upon the refractive power of L and the curvature of its surfaces. Let the lens l be placed at such a position in the axis of L that if parallel rays from the right should pass through it toward the left, they would be brought to a focus at the same point F. Since the

path of a ray of light is reversible, when L and l are placed in this relative position, the rays of light A, B, and C, after passing the focus F and entering the lens l, emerge in the parallel lines a, b, and c respectively.

Now suppose an eye just to the right of the eyepiece l is directed toward the left. Since the rays of light enter it in parallel lines, they are brought to a focus on the retina (Art. 40) and the brain sees a point of light just as it would if the parallel rays A, B, and C entered the eye.

Suppose a photograph plate is placed at the focus F with the emulsion toward the objective L. A point-image will be formed upon it.

43. How a Telescope Increases the Apparent Brightness of a Star. The apparent brightness of a star or other point-source of light is proportional to the number of rays that enter the pupil of the observer's eye. Suppose the pupil is just large enough to receive all the rays that emerge from the eyepiece l, Fig. 17. Since the cylinder of rays $A-B$ has been condensed to the much smaller cylinder $a-b$, it is evident that more rays enter the eye with the aid of the telescope than would enter without it. That is, the telescope increases the apparent brightness of the object.

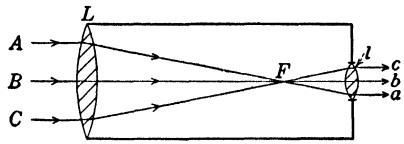


FIG. 17.—Simple refracting telescope.

It should be noted that the more convex the surface of the eyepiece lens the closer will it be to F and the smaller will be the cross section of the emerging rays. Consequently, an eyepiece can always be chosen such that the entire cylinder of emerging rays will enter the pupil of the eye.

It follows from the foregoing discussion that the apparent brightness of a star or other point-source of light is proportional to the cross section of the cylinder of rays that passes through the objective. That is, if D represents the diameter of the objective, *the brightness of the star is proportional to* D^2. This result is independent of the length of the telescope, which depends only upon the convexity of the surfaces of the objective. It is independent also of the eyepiece l, provided it is convex enough to bring the emergent cylinder of rays down to the diameter of the pupil of the

eye. If a photographic plate is placed at the focus F, the number of rays that fall upon it is also proportional to D^2. That is, both visually and photographically the brightness of a star is proportional to the square of the aperture of a telescope. There are, however, appreciable losses of light by reflection at the surfaces of the lenses and by absorption in the lenses.

44. Relation of Apertures to Visibility of Stars of Various Magnitudes. The diameter of the pupil of the human eye in darkness or feeble light is about 0.6 centimeter (0.25 inch), though it varies somewhat from one individual to another. Consequently, if the aperture of a telescope is D, the brightness of a point-source of light when observed through the telescope as compared with its brightness when seen with the unaided eye is the square of D divided by the square of 0.6, or $D^2/.36 = 2.777\ldots D^2$. In order that this shall be one magnitude brighter, D must satisfy the equation $2.778\ldots D^2 = 2.512\ldots$ from which it is found that $D = 0.904\ldots$ centimeter (.356 inch). That is, a telescope whose objective is 0.904 centimeter in diameter will just increase the brightness of a seventh-magnitude star to visibility. Some allowance should be made, of course, for losses of light by reflection and absorption.

Let D_0 and D_1 be the apertures with which stars of succeeding magnitudes are just visible. Then it follows that $D_1^2 / D_0^2 = 2.512\ldots$, whence $D_1/D_0 = 1.585$. Since a star of the seventh magnitude is just visible with an aperture $D = 0.904$, it follows that a star of magnitude m will be visible with a telescope of aperture D, where

$$D = 0.904\ (1.585)^{m-7} \text{ centimeters}$$
$$= 0.356\ (1.585)^{m-7} \text{ inches.}$$

It follows from the foregoing equations that if m is 17 the value of D is 90.4 centimeters (35.6 inches), and that if m is 19 the value of D is 227 centimeters (89.4 inches). Consequently, after making some allowance for loss of light, it is clear that stars of the seventeenth magnitude should be just visible with the 40-inch telescope of the Yerkes Observatory, and that those of the nineteenth magnitude should be just visible with the 100-inch telescope of the Mount Wilson Observatory.

45. Relation of the Brightness of the Image of an Area to the Diameter and Focal Length of an Objective. The preceding discussions apply to point-sources of light, such as the stars, all of

which are so remote that they appear like points of light. The results are quite different in the case of objects with appreciable apparent areas, such as the moon or a nebula.

In Fig. 18, let AB represent the moon, which is, of course, very far away to the left of the telescope objective L. A certain ray of light from A will pass through the center C of the lens L without having its direction changed, and will fall on the focal plane F, which is perpendicular to the axis of the telescope, at the point a.

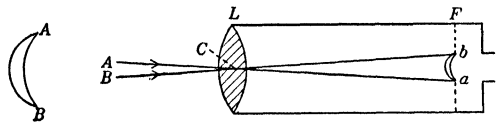


FIG. 18.—Telescopic image.

All other rays of light from A that pass through the lens L will also intersect at the point a, just as all the rays from A in Fig. 15 come to a focus on the retina at a. The distance from the objective L to the focal plane F depends upon the glass of which the lens is made and upon the curvatures of its surfaces, but not upon its diameter. That is, if the outer margin of L were covered, the rays from A that pass through its uncovered central portion would still be brought to a focus at the same point a.

Now consider another point B on the moon. There is a certain ray B from it which will pass through the center C of the objective L without having its direction changed, and this ray will fall on the focal plane at the point b. Likewise, every other ray from the point B on the moon that passes through L will intersect the focal plane at b. Similarly, all the rays from any other point on the moon that pass through L will be brought to a point in the focal plane F. That is, an optical image of the moon is formed at the focal plane F, just as an image is formed by the objective of a camera on the sensitive plate.

Now consider the image in the focal plane F, Fig. 18. It is evident that it is inverted with respect to the object at which the telescope is directed. Since the rays A and B are the ones that pass through the center of the objective without having their direction changed, the angle aCb is equal to the angle ACB, which is the apparent angular diameter of the moon as seen from the position of the observer. The diameter of the image is proportional to the angle aCb, which does not depend upon the diameter of the objective, and to the focal length CF of the telescope,

which depends upon the curvatures of the surfaces of its objective. The area of the image is proportional to the square of its diameter, and, consequently, to the square of the focal length of the telescope.

The amount of light that passes through the objective L is proportional to the square of its diameter; and, for a given amount of light, the brightness of an image is inversely proportional to its area. Therefore, *when the object under consideration is an area, the brightness of its image is proportional to the square of the diameter of the objective and inversely proportional to the square of its focal length.*

It follows that if it is desired to photograph a faint object, it is advantageous to use a large objective having a short focus, a fact which every photographer knows. But if an objective with short focus is used, the size of the image is small, for its diameter is proportional to the focal length of the objective.

Suppose a photographic plate is placed a little to the left of the focal plane F, Fig. 18. The rays from the point A, for example, strike the plate in a small circle just before they reach their focus, and similarly for rays from every other point on the moon. The result is that the images of the neighboring points overlap and impair the definition. There is a similar lack of definition if the photographic plate is placed a little to the right of the focal plane.

46. How a Telescope Magnifies. It was found in Art. 40 that the apparent diameter of an object is proportional to the distance apart on the retina of the images of its extreme points. It was found, further, that the position of the image of a point on the retina is defined by the direction of the parallel rays from the point when they enter the lens of the eye.

Figure 19 shows, on a large scale, the course of the rays from the points A and B of the moon (Fig. 18), when they are near the focal plane F. The figure ab is the image in the focal plane F of the moon AB. Three rays from A and B are shown converging at a and b respectively, the central one in each set being the one that passes through the center C of the objective without being refracted. When the eyepiece l is at its focal distance from the focal plane F, all the rays that diverge from any point in the image, say a, and pass through l emerge from it in parallel lines. Similarly, the rays diverging from b also emerge from l in parallel

lines. Consequently, if an eye is placed just to the right of l and is directed toward l, all the parallel rays a converge to a point on the retina and the brain sees A as a point. Similarly, B is seen as a point. Since the angle between the rays A and the rays B is greater, after they emerge from l, than the angle between A and B before they enter the objective, the object is magnified. The angle between the emerging rays is inversely proportional to the focal length of the eyepiece.

Since the diameter of the image in the plane F is proportional to the focal length of L, it follows that: *The magnification of a telescope is equal to the focal length of its objective divided by the focal length of its eyepiece.*

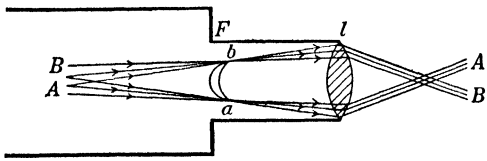


FIG. 19.—How a telescope magnifies.

A telescope is ordinarily supplied with several eyepieces, which are relatively inexpensive accessories, and it consequently has a corresponding magnification for each of them. If the image in the focal plane were perfect, it might be highly magnified; but it is imperfect and magnification beyond what is usually a moderate amount is disadvantageous.

There are two reasons, besides diffraction (Art. 49), why an eyepiece of high power is generally not desirable. One is that the diameter of a very convex lens in an eyepiece must evidently be small, and consequently the light from only a small part of the image can enter it. For example, the diameter of the image of the moon in the focal plane of the great Yerkes telescope is about 1.7 centimeters (6.7 inches) and the diameter of the lens l of an eyepiece with which it is observed often is less than 2.5 centimeters (1 inch). Consequently, it is evident that only a very small part of the moon's surface can be seen with the great telescope at one time, particularly if the eyepiece has a short focus. But the whole image can be easily photographed with a sufficiently large sensitive plate.

A second reason that it is generally not advantageous to use an eyepiece of short focus and high power is that the more the image is magnified the fainter it apparently becomes, for with large magnification the light is distributed over an apparently large area. This consideration is not important in the case of such a

bright object as the sun or even the moon, but it is important in observing fainter objects such as nebulae. On combining all the factors on which the apparent brightness of an area observed with a telescope depends, we have the rule: *The apparent brightness of an area viewed through a telescope is proportional to the product of the square of the diameter of the objective and the square of the focal length of the eyepiece divided by the square of the focal length of the objective.*

47. Spherical Aberration. There are two principal sources of imperfection in the images that are formed at the focal plane of an objective, the first of which is spherical aberration.

In Fig. 20, aa represents two parallel rays passing through the objective L near its axis. Let it be supposed that they converge to a point in the focal plane F_a. Let bb represent two parallel rays which pass through the objective near its opposite margins. If the surfaces of L are portions of spheres, as they usually are for practical reasons connected with their grinding and polishing, these two rays will converge to a point in the focal plane F_b, which is nearer the objective than F_a. Rays passing through the objective in other zones converge at corresponding other focal distances. There is no single focal plane for all the light, and the result is that the image is blurred. This is *spherical aberration.*

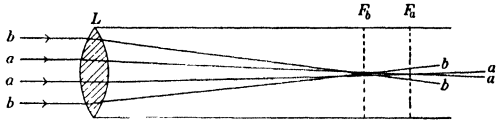


FIG. 20.—Spherical observation.

The question naturally arises whether this defect can not be avoided by using surfaces that are not spherical, and the answer is in the affirmative, at least theoretically. It is clear that the curvature of the margin of the objective must be less than that in its central portion. It is very difficult, however, to produce such surfaces with the requisite accuracy.

48. Chromatic Aberration. In Art. 41 it was explained that when light passes through a prism it is spread out into a spectrum of colors. In a precisely similar manner, light is separated into its colors when it passes through a lens, giving rise to what is known as *chromatic* (color) *aberration.*

In Fig. 21, let A and B be two entering beams of light that are parallel to the axis of the objective L. Suppose the red rays of

these beams are refracted to a point on the focal plane F_r. Since the violet rays are refracted more than the red, they will be brought to a point in the focal plane F_v, which is between the objective L and the focal plane F_r. Every other color will have a corresponding focal plane.

Consider first the problem of obtaining a photograph of the object, which may be supposed to be a star since the beams A and B are parallel. If the plate is placed in the plane F_v, its image will be sharp so far as it depends upon violet light alone, but the images in all the other colors will be small circles. Consequently, the photograph will be blurred. The result would be similar if the plate were placed in any other plane.

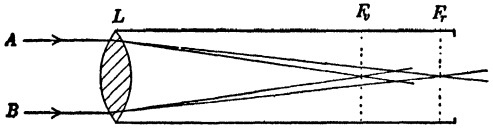


FIG. 21.—Chromatic aberration.

If the object is to be viewed through an eyepiece, the eyepiece must be placed its focal length to the right of the image. If it is placed so that the red image, for example, is in focus, the others will not be, and the star appears blurred. The results would be satisfactory only if the plate or the eye were sensitive to light of one color alone.

If the object is an area instead of a point-source of light, the results are similar, because each point of the area is, in effect, a point-source of light.

49. Diffraction. Whatever light may actually be, it has the properties of a wave motion at right angles to its direction of travel, somewhat analogous to waves on water. When two exactly similar beams of light are superimposed, they combine as waves add, doubling at the points where they are in phase, and canceling each other where they are out of phase a half wave-length.

It follows from the theory of physical optics that the image of a point-source of light, even neglecting spherical and chromatic aberrations, is not a point, but a small circle of light surrounded by a series of little rings of light which decrease rapidly in brightness with increasing distance from the center of the circle. The result is that the images formed by telescopic objectives are not perfectly sharp, and therefore that they can not be magnified to an unlimited extent. If, for example, the central circles of the images of the two components of a double star overlap, no amount of

magnification by an eyepiece will separate them and show them as two stars.

The property of optical images now under consideration is due to what is called the *diffraction* of light. Diffraction is a phenomenon that has its origin at the borders of lenses, and it is present in all optical instruments, including eyes. It sets limits to the resolving power (ability to show as distinct two points) of all optical instruments. As a consequence of diffraction, the unaided human eye could not see separately two objects subtending an angle at the eye less than about 120″, however fine the rods and cones on the retina might be. As a matter of fact, the angular distance between two stars must be about 200″ in order that they may be seen separately.

The size of the diffraction circles of a point-source of light depends upon the wave-length of light and the diameter of the objective. The diameter of the central diffraction circle of a point-source of light formed with a given objective is inversely proportional to the wave-length of light. Consequently, if our eyes were sensitive to light of shorter wave-lengths, we should be able to use microscopes and telescopes of higher powers.

For light of a given wave-length, the diameter of the central diffraction circle of a point-source of light is inversely as the diameter of the objective lens and independent of its focal length. Consequently, the larger the objective the smaller and the brighter the images it produces and the more they may be magnified. The limiting factor, however, is usually the unsteadiness of the image due to the effects of the earth's atmosphere.

50. Compound Achromatic Objectives. Although diffraction causes defects in the images formed by objectives that can not be avoided, spherical and chromatic aberrations can be largely overcome by using suitable compound lenses, as John Dollond (1706–1761) first discovered in 1758. A common form of compound objective is illustrated in Fig. 22.

The outside numbers of the objective is a double convex lens of crown glass, though usually the two surfaces do not have the same curvature. If this lens were used alone the red light would be brought to a focus at a point R between the objective and the focal plane F, and the blue light would be brought to a focus at some nearer point V. Moreover, the marginal rays would be

brought to a focus nearer the objective than those that pass through the center.

The inside member of the compound objective is made of flint glass, which refracts more strongly than crown glass; and, moreover, for a given amount of refraction, it produces more dispersion than crown glass. The first surface of the flint lens is strongly concave, and its second surface is slightly convex, plane, or slightly concave.

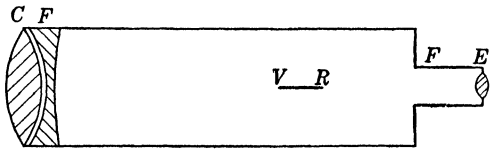


FIG. 22.—Compound objective.

The flint lens is negative and acts opposite to a double convex lens in refraction, spherical aberration, and chromatic aberration. When the flint lens is properly related to the crown lens, it partially straightens out the red rays, which the crown lens alone would bring to a point at *R*, and brings them to a focus in the focal plane *F*. It has a larger effect on the violet rays and, when the lenses are suitably matched, it brings the violet rays also to a focus in the plane *F*. Likewise, its effects on the marginal rays are greater than on those which pass through its center, and it tends to correct the spherical aberration. It can be seen in a general way from this description that the four surfaces of the two lenses can be so determined that the telescope will have a prescribed focal length and the spherical and chromatic aberrations be largely removed. In order to obtain a more nearly perfect correction of the errors, camera objectives are usually composed of from four to six lenses.

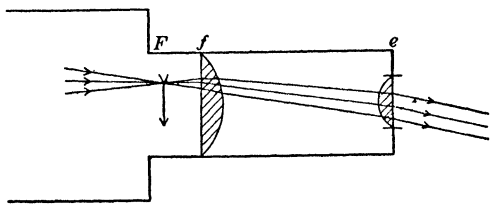


FIG. 23.—Positive eyepiece.

51. Eyepieces. Heretofore eyepieces have been spoken of as though they were single lenses of double convex glass. Such an eyepiece would be satisfactory only for a small object near the center of the field. In order to secure good definition over a large field compound eyepieces are used.

The so-called *positive* eyepiece, which was invented by Ramsden, consists of two plano-convex lenses, arranged as illustrated in Fig. 23. The focal plane of the objective is to the left of both lenses of the eyepiece at F where the rays cross. The lens f is called the *field glass*, and e the *eye glass*.

The so-called *negative* eyepiece, which was invented by C. Huyghens, consists of two plano-convex lenses, arranged as illustrated in Fig. 24.

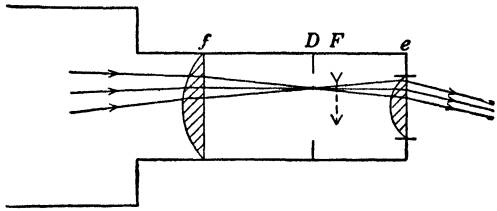


FIG. 24.—Negative eyepiece.

In this eyepiece the focal plane is between the field glass and the eye glass at f, but the rays of light from the objective do not come to a focus in that plane because they are intercepted by the lens f.

52. The Reticle. In order to measure very short angular distances, such as the separation of the two components of a double star, astronomers use a *reticle*, which, in its simplest form, consists of three spider lines arranged in the focal plane of the eyepiece, as shown in Fig. 25. The reticle is so mounted that it can be rotated around its axis, which coincides with the axis of the eyepiece.

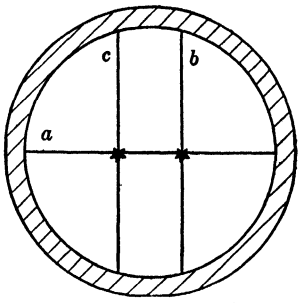


FIG. 25.—Simple reticle.

Suppose it is desired to measure the directions of the two components of a double star from each other and their angular distance apart. After the telescope has been moved so that the star has been brought into the center of the field of view, the reticle is rotated until the line a, Fig. 25, passes through the centers of the images of the two stars in the focal plane F. If the mounting of the reticle is graduated into degrees and fractions of degrees, the reading on the reticle opposite previously determined marks defines the direction of the stars from each other, or their *position angle*, as astronomers call it. The telescope and eyepiece are then moved until one of the other lines, say b, passes through the image of one of the stars. The reticle mounting is so constructed that by

turning a screw with a graduated head the other spider line may be moved until it passes through the image of the other star. It is possible to determine from the amount the screw has been rotated the angular separation of the two stars.

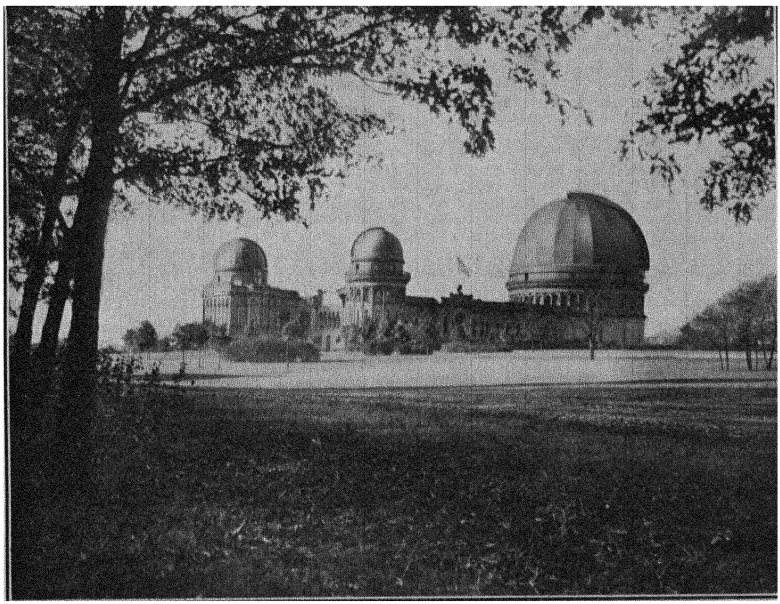


FIG. 26.—The Yerkes Observatory, Williams Bay, Wisconsin. The great 40-inch refractor is in the dome at the right.

V. QUESTIONS

1. Is the scale of the optical image in the eye of a bird greater or smaller than that in the human eye? Can birds have more acute vision with respect to fine details than human beings have?
2. Suppose a person is nearsighted because his eyes are not of normal length. Are they too long or too short?
3. Suppose a person looks at an object with both eyes. The image of one part of the object will be on the left, or nose, side of his right eye. Where will it be in his left eye?
4. When a person opens his eyes under the water, is he nearsighted or farsighted in the medium?
5. A telescope of the type described in Art. 42 can be constructed in a pasteboard tube with a spectacle lens for the objective and a small double convex magnifying glass for an eyepiece. Make one.
6. Suppose one should attempt to look at a bright star through the objective end of a telescope. Could it be seen?

7. Suppose a telescope, with its eyepiece removed, is pointed at a star and that a person places his eye at the focal plane F. When he looks toward the objective what does he see?
8. Suppose two telescopes have the same aperture of 10 cm. (4 inches) and that the focal length of one is 160 cm. (64 inches) and that the focal length of the other is 80 cm. (32 inches). If they are pointed at the moon, what is the relative size of the moon's image in the two? What is the relative brightness of the moon's image? Suppose an eyepiece of 2.5 cm. (1 inch) is used with the first and an eyepiece of 1.25 cm. (0.5 inch) with the second. What is the relative magnification in the two cases? What is the relative apparent brightness of the images as viewed through the eyepieces?
9. Does the eye necessarily have spherical aberration? Chromatic aberration? Diffraction?

II. Reflecting Telescopes

53. Reflecting Telescopes. In refracting telescopes, lenses are used to converge the light from a point-source of light to a focus; in reflecting telescopes, concave mirrors are used for precisely the same purpose.

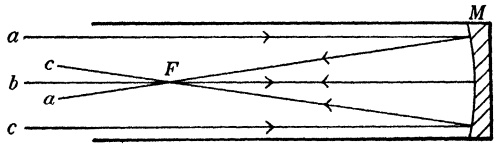


FIG. 27.—Concave mirror.

In Fig. 27, M represents a concave mirror on which fall three parallel beams of light, a, b, and c. After reflection from the surface of the mirror, these beams converge to the focus F. The mirror performs the same function as the objective L, Fig. 17. But evidently there is some difficulty in using an eyepiece without intercepting the rays of light before they reach the mirror M.

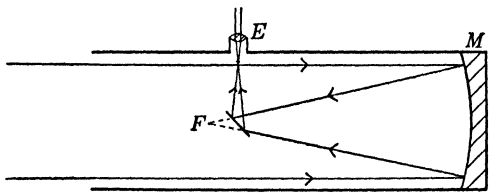


FIG. 28.—Newtonian reflector.

There are three principal types of reflectors in which the observer does not intercept the incident light. The first, due to Newton, is illustrated in Fig. 28. In this mounting, a small oblique mirror is inserted between the mirror M and the focus F at such a point that the eyepiece E will be just outside the cylinder of incident light. The mirror intercepts, of course, a little of the incident

light, and the observer looks at right angles to the actual direction of the object.

The second type, invented by Cassegrain in 1672, is illustrated in Fig. 29. In this arrangement, a small, slightly convex mirror is placed in the path of the converging rays just before they reach a focus. The convex mirror decreases their convergence and they do not come to a focus until they reach a point near the eyepiece *E*. In this mounting the observer looks in the direction of the object, as when using a refracting telescope.

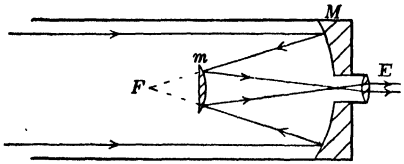


FIG. 29.—Cassegrainian reflector.

The third type, due to William Herschel, is illustrated in Fig. 30. In this mounting, the mirror is inclined a little to the incident rays, and the focus is thrown to a side of the telescope. The tilting of the mirror distorts the image to some extent, but Herschel constructed many telescopes of this type and used them with extraordinary success.

54. Comparison of Refracting with Reflecting Telescopes. Reflecting telescopes depend upon the fact that when light strikes a reflecting surface at any angle it is reflected from the surface at the same angle, whatever may be the color of the light. It follows at once that reflecting telescopes do not have chromatic aberration, and this is of great advantage in instruments used for photographic purposes.

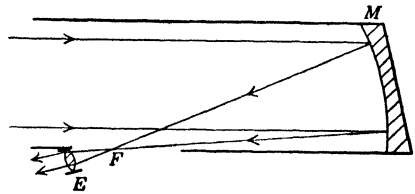


FIG. 30.—Herschellian reflector.

Reflecting telescopes are subject, however, to spherical aberration, and it is more difficult to correct it in them than in refractors. Moreover, an error in the surface of the mirror *M* has about three times as much effect on the image as an equal error in an objective *L*. This disadvantage is partly offset by the fact that in reflectors there is only one surface to produce, the collecting mirror *M*, as compared with four surfaces in the case of a compound objective.

Reflectors and refractors are both subject to diffraction, and to the same extent for instruments of the same aperture, except for the additional diffraction effects of the interposed plane mirror

and its supports in the Newtonian type, and the interposed mirror m and its supports in the Cassegrainian type.

It is not necessary that the materials for mirrors should be transparent, as they must be for objectives, nor is it so vital that they should be homogeneous. But on the other hand, the silvered surfaces of the mirrors gradually become tarnished and must be renewed from time to time, while the surface of lenses last for many years with only an occasional cleaning.

Lenses must be supported at their margins and very large ones are distorted appreciably by their weight as their positions are changed. On the other hand, mirrors may be supported over all their rear surfaces, and there are not so severe limits to their sizes and weights. Disks of optical glass have not yet been produced large enough for the construction of lenses equal in aperture to several of the largest mirrors that are actually in use.

III. The Mounting of Telescopes

55. The Meridian Circle. Of all types of telescope mountings, that of the meridian circle is simplest in principle. In this mounting, the telescope turns in the plane of the meridian about a horizontal axis placed in the east-west direction. A small transit instrument mounted in this way is shown in Fig. 31. It follows from its mounting that stars can be observed with meridian circles only when they are precisely on the meridian.

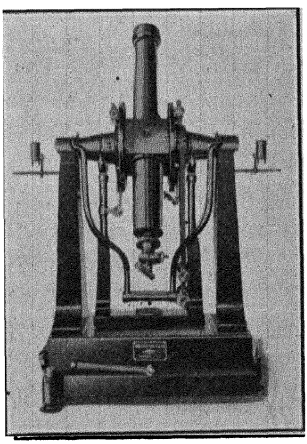


Fig. 31.—Transit instrument. (William Gaertner & Company.)

The meridian circle is the mounting which is made with the greatest precision. In the first place, it is very rigidly constructed so that it will not suffer appreciable flexure in whatever direction it may be pointed. In the next place, it is so mounted that its horizontal axis may be leveled with the utmost precision. Likewise, the telescope tube is mounted as nearly as possible at right angles to its horizontal axis, for, if it were not, when the telescope is rotated it would not follow the meridian. The pivots on which

the telescope rotates must be as nearly cylindrical as possible. Generally, counter weights carry most of the weight of the telescope, the pivots simply fixing its position. In addition, accurately and finely graduated circles are attached to the horizontal east-west axis, so that the amount the telescope has been rotated from a vertical position may be read off. The mounting is so constructed that the horizontal axis may be brought into the east-west line whenever it departs from this line. Finally, the focal plane of the objective must be provided with a reticle, which is not of the simple type described in Art. 52, but one of the most accurate construction, usually having two horizontal spider lines and from 5 to 13 (always an odd number) symmetrically-spaced vertical spider lines.

In spite of the care with which meridian circles are mounted, they are usually of moderate size. Generally speaking, their range in aperture is from about 6 centimeters (2.4 inches) to 24 centimeters (9.6 inches).

56. Determination of the Latitude of an Observer by Use of a Meridian Circle. Meridian circles are constructed with precision and mounted with care in order that they may be used for a number of important and fundamental purposes, one of which is that of determining the latitude of the observer.

Since the latitude of an observer equals the altitude of the celestial pole as seen from his position (Art. 11), the problem of determining latitude is simply that of measuring the altitude of the pole. After the meridian circle has been set up and adjusted, altitude readings of circumpolar stars are taken both when they cross the meridian under the pole and also, twelve hours later, when they cross it above the pole. The average of the two altitudes, after they have been corrected for the refraction of light by the earth's atmosphere, is the altitude of the celestial pole. The precision attainable with meridian circles is so great that latitude can be determined by their use to $0''.5$, which corresponds to about 16 meters (50 feet) measured along the surface of the earth.

In the method just described, the observer determines his latitude without using the declinations of the stars. As a matter of fact, the declinations of all the brighter stars are now known with high precision and the latitude of an observer can be determined from meridian altitudes of any known star. Suppose

the astronomer finds that the altitude of a star is $+70°$ from the south horizon when it is on the meridian. Suppose the declination of the star is $+20°$. Therefore, the altitude of the celestial equator at the point it intersects the meridian is $70°-20°=50°$, and the latitude of the observer is $90°-50°=40°$. To avoid the effects of refraction, it is advantageous to choose a star that crosses the meridian near the zenith.

57. Determination of the Declinations of Stars. Much of astronomy depends directly or indirectly upon a precise knowledge of the apparent positions of the stars; that is, upon their declinations and right ascensions. When an observer knows his latitude, he can determine the declinations of stars from observations made with the meridian circle.

If a star is observed north of the zenith and above the celestial pole, its altitude minus that of the pole (which equals the latitude of the observer) is the polar distance of the star. The declination, by definition, is $90°$ minus the polar distance. The observed altitude of the star must be corrected, of course, for the effects of refraction by the earth's atmosphere.

If the observed star is south of the zenith, its altitude minus the altitude of the equator (which is $90°-l$) is the declination of the star. If the difference is positive, the star has north declination; if it is negative, the star has south declination. In this case also the apparent altitude of the star must be corrected for the effects of refraction by the atmosphere of the earth. The refraction is zero at the zenith and increases to the horizon, where it is about half a degree. If the star is within $15°$ of the horizon, the correction for refraction, which varies with the temperature and humidity of the atmosphere, as well as with the barometer, is subject to considerable errors.

58. Determination of the Right Ascensions of Stars. Since the stars describe diurnal circles as a consequence of the rotation of the earth, each of them crosses the meridian at intervals of the period of the earth's rotation with respect to the stars, which is called the *sidereal day*.

Suppose the observer has a very accurate clock which runs at such a rate that it registers 24 hours while a star makes its diurnal circuit of the sky. Suppose, further, that the clock registers zero when the vernal equinox is crossing the meridian. Then, since right

ascension is measured eastward from the vernal equinox while the diurnal motions of the stars are westward, the time at which any star crosses the meridian equals the right ascension of the star. It is this relationship between time and angular distance east of the vernal equinox which has caused astronomers to count right ascension in hours instead of degrees.

The problem of determining the instant at which a star crosses the meridian is one involved in rather formidable difficulties. Usually the observer has a recording instrument, a *chronograph*, on which he can register the time of events by pressing an electrical key. Suppose a star is soon to cross the meridian. If the telescope is set at the right altitude, the astronomer will observe the star entering the field. He will adjust the telescope in altitude so that the star will pass midway between the two horizontal lines of the meridian circle reticle (Fig. 32). Since the telescope magnifies the motion of the star, as well as the apparent distances between the stars, the star will move steadily across the field. With the electrical switch in hand, he will push the key when the star is crossing each of the vertical lines of the reticle and the record of the time will be registered by the chronograph. Since the lines are symmetrically arranged with respect to the central one, the mean of the times they are crossed is the time the star transits the meridian, which equals the right ascension of the star. The time of crossing is not affected by the refraction of the earth's atmosphere, for refraction is in the vertical direction.

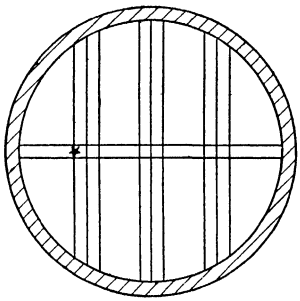


FIG. 32.—Meridian circle reticle.

While the foregoing is an outline of the general principles involved in the use of the meridian circle, there are many refinements that are necessary in order to secure results meeting modern standards of accuracy. Without the light-gathering power of objectives, faint stars could not be observed; and without the magnification produced by objectives and eyepieces jointly, the desired precision could not be obtained.

59. Determination of the Longitude of an Observer. Suppose, for simplicity, that an observer's clock registers the same time as

is registered by a clock in the observatory in Greenwich, or one on any other meridian chosen as fundamental. The times registered by the two clocks can be compared by telegraph or radio signals.

Suppose a certain star is observed to cross the meridian of Greenwich at the instant t and that the same star crosses the meridian of the observer at $t+x$, or x hours later. Since the earth rotated through x hours of right ascension between the transits, the observer's longitude is x hours west from the meridian of Greenwich.

In practice, the two clocks would not and need not register exactly the same time. Nor would they in general run at exactly the same rate, for allowance can be made for these variations from ideal conditions. Naturally, time would be determined from the transits of many stars.

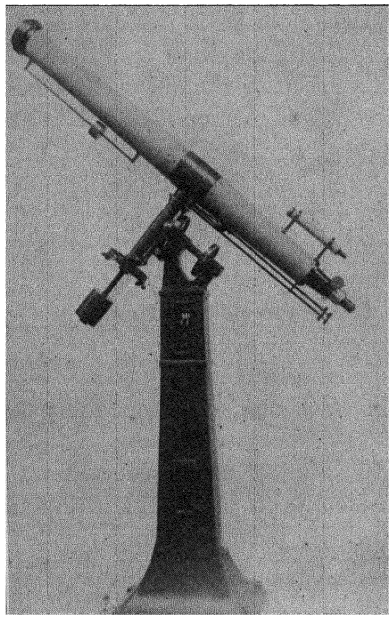


FIG. 33.—Small equatorial telescope. (William Gaertner & Company.)

60. The Equatorial Telescope. All telescopes designed for general observational use and for photography are mounted equatorially. This mounting permits the telescope to be turned to any part of the sky, and, except in very small instruments for amateurs, it is driven by a clockwork which will cause it to follow indefinitely a star that is once in the field of view; at least, it would follow the star if refraction did not vary as the altitude of the star changes as a consequence of its diurnal motion. If the telescope is to be used for such measurements as those of double stars or for securing long-exposure photographs, a clock-driven equatorial mounting is absolutely necessary.

The essentials of the equatorial mounting are shown in Fig. 33. First, there is a very rigid pedestal mounted on a pier reaching down to a stable foundation. The pedestal carries an axis called

the *polar axis*, which must be accurately parallel to the earth's axis. It is called the polar axis, because the line of its prolongation passes through the celestial poles. At right angles to the polar axis is the *declination axis*, which is so mounted that it may be rotated around the polar axis while remaining perpendicular to it. A graduated circle attached to the polar axis enables the observer to determine through what angle the declination axis has been rotated.

It is now possible to see how the equatorial mounting functions. If the telescope is rotated around the declination axis until it is parallel to the polar axis, it is in line with the celestial pole. If its relation to the declination axis is kept fixed while the declination axis is rotated about the polar axis, the telescope will remain pointed at the celestial pole, for its direction is not changed by the motion. Then if the telescope is rotated about the declination axis 90° so that it is perpendicular to the polar axis, it will be pointing toward the celestial equator. If the declination axis is rotated about the polar axis while it is in this position, the line of the telescope will trace out the celestial equator. If the telescope is set at any intermediate declination and the declination axis is rotated around the polar axis, the line of the telescope will trace out a corresponding declination circle.

61. Finding an Object with an Equatorial Telescope. The first problem is to turn the telescope until the object is in the field of view. By means of the circle on the declination axis the telescope is first set to the declination of the object and is clamped so that it can not rotate about the declination axis. The hour angle of the object is then determined (Art. 17), and by means of a circle on the polar axis the telescope is set to the hour angle of the object at the time.

The declination axis is then clamped to the polar axis so that it can not move except as the polar axis rotates. There is a clockwork which rotates the polar axis westward with the angular speed of the diurnal motions of the stars.

If the settings have all been correctly made, the object will be in the field of view, but in practice it will often not be visible, for the field of view of a large telescope is very restricted. But a small telescope, called a *finder*, is mounted on a large telescope so as to be strictly parallel to it. The object will likely be visible through

the small telescope, for its magnification is less and its field of view is correspondingly greater than that of the larger instrument. The telescope is then moved by slow-motion screws until the object is seen at the center of the finder, when it will be in the field of view of the large telescope.

When once a star is in the field of view of an equatorially mounted telescope, it will be kept there by the clock which rotates the declination axis about the polar axis. In the case of a considerable interval, however, the changing refraction with change of altitude of the star might remove it from the field. Indeed, the clock will not rotate the polar axis at precisely the required speed. The result is that when long-exposure photographs are taken, the observer must sit at the eyepiece end of the telescope, constantly watching and making slight adjustments of the position of the plate holder in both declination and right ascension.

Since the sun apparently moves eastward one degree per day among the stars, the rate of rotation of the polar axis must be changed to match its angular velocity if it is to be kept constantly in the field. The moon moves eastward about 13° per day among the stars, and consequently a considerable change in the rate of rotation of the polar axis is required in order to keep it steadily in the field of view. Moreover, the apparent motion of the moon among the stars is not perfectly uniform.

62. Determination of the Positions of Stars by Use of the Equatorial Telescope. The equatorial telescope is not adapted to determining directly the precise positions of objects that may be observed through it; its mounting does not compare in accuracy of construction or fineness of adjustment with that of the meridian circle. It is adapted, however, to determining the positions of objects by measuring their distances and directions from stars whose positions are accurately known. These measurements are made by a micrometer, which is an accurately made and mounted reticle (Art. 52).

When a comet is discovered, its apparent position is determined by the method that has just been described. The same is true of small planets, temporary stars, and other objects whose positions change or which are not permanently bright enough to be observed with meridian circles. In order to have comparison stars available for any new object that may appear, it is obviously

important to have accurate star catalogues containing many stars in every part of the sky.

If a photograph of a comet, for example, is taken, its declination and right ascension at the time can be determined by measuring the direction and distance of its image from the images of stars whose coördinates are known.

Although equatorially mounted telescopes are not adapted to determining accurately by direct means the apparent position of a celestial object, they give very precise results by the differential method just described. They are, in fact, now extensively used for determining the parallaxes of stars (Art. 359), a problem that requires an order of accuracy that is hardly surpassed in the whole domain of science. In determining the parallaxes of most stars, it is necessary to measure angular distances to within 0″.05, which corresponds to one part in 4,000,000.

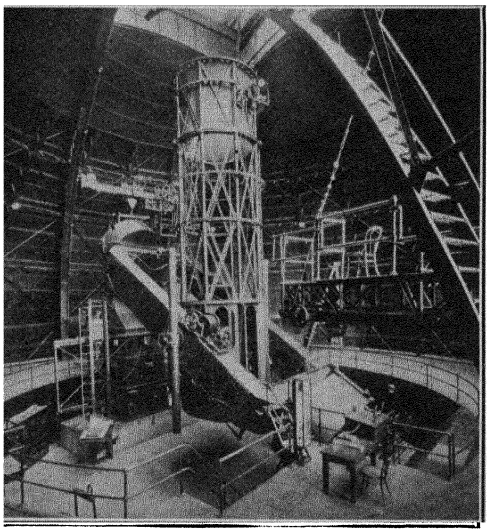


FIG. 34.—The 100-inch reflector of the Mount Wilson Observatory, California.

63. Great Telescopes. Although telescopes were invented a little more than three centuries ago, it is only within the past 75 years that large ones have been made. About the middle of the last century Lord Rosse erected a 183-centimeter (72-inch) reflector at Birr Castle, Ireland. The great light-gathering power of this instrument made it particularly valuable for observing faint objects, such as nebulae. A large number of valuable observations were made with it, but not so many as might have been expected, for the mounting was unsatisfactory and the mirror did not approach the standard of excellence attained in recent reflectors.

Shortly after the erection of Lord Rosse's great reflector, as-

4. Suppose an astronomer observes a star whose declination is + 70° and finds that its altitude is 20°; what is his latitude?
5. Suppose an astronomer's latitude is 38° north and that he observes a star crossing his meridian on the north side of the zenith at an altitude of 15°; what is the declination of the star?
6. An observer wishes to observe the transit across the meridian of a star whose right ascension is 18 hours; at what time of the night will the transit occur on the night of April 21?
7. At what points on the surface of the earth would the polar axis of an equatorial telescope be vertical? Horizontal?
8. When an observer looks through a telescope at a star on the meridian south of the zenith, will the star apparently cross the field from right to left or from left to right? If the star is north of the zenith and above the pole? If the star is north of the zenith and below the pole?

REFERENCE BOOKS

The Telescope (1922.) Louis Bell. McGraw-Hill Book Company, New York.

Amateur Telescope Making (1926). The Scientific American, New York.

Outline of Applied Optics (1912). P. G. Nutting. P. Blakiston's Sons & Co., Philadelphia.

A Compendium of Spherical Astronomy (1906). Simon Newcomb. The Macmillan Company, New York.

CHAPTER IV

THE EARTH

I. Shape and Size of the Earth

65. The Earth as an Astronomical Body. It would not be possible to develop astronomy to any considerable extent without knowing many things about the earth. For example, if the diameter of the earth were not known, it would be impossible to measure the distance to the moon and the planets, for the earth is used as a base line in these determinations. Or, if the mass of the earth were not known, it would be impossible to determine the mass of any celestial body.

Conversely, astronomical observations are necessary in order to determine the shape of the earth or to measure its diameter with much precision. Likewise, all that is known about the motions of the earth depends upon observations by astronomers. And when it comes to the origin, the evolution, and the ultimate fate of the earth, our globe is considered as simply one of a number of planets revolving about a sun which, in turn, is only one of millions of similar suns. Consequently, it is evident that it is necessary to consider the earth as an astronomical body.

The interrelation of the earth sciences and astronomy is typical of what is true in every domain of science. For convenience, we divide up the field of scientific thought into many subdivisions, but the different parts of the whole are tied into larger units by relationships which are found to be the more numerous the more knowledge increases. For example, on superficial examination there would seem to be no real connection between the kind of light radiated by the sun and the safety of airships; but, as a matter of fact, the discovery by astronomers of helium in the sun led to its much later discovery on the earth, and eventually to its use for inflating dirigibles. It is the essential unity of the physical universe that makes science possible.

66. Sphericity of the Earth. There are a number of phenomena, such as the disappearance of ships over the horizon, which prove

that the surface of the earth is curved, but they are only rough indications that the earth is spherical. We are not interested here in such inconclusive evidence; we desire demonstrations which remove all doubt that our conclusions are correct.

In order to simplify matters so that we may devote our attention entirely to methods of proof, we shall assume for the present that the earth is spherical. One of the simplest properties of a sphere is that the curvature of its surface is the same at all points. Otherwise expressed, a sphere is a solid such that the distances between perpendiculars to its surface are proportional to the angles between the perpendiculars, and a body of no other shape has this property.

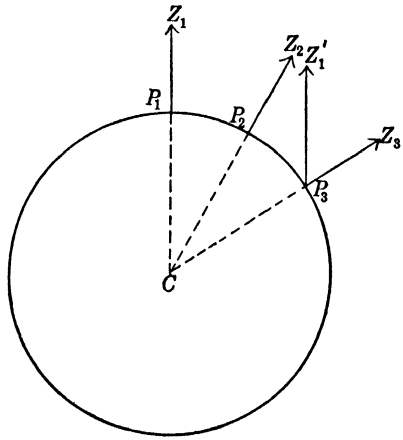


FIG. 35.—Properties of a spherical surface.

Let Fig. 35 represent a plane section through the center C of a sphere. The plane intersects the surface of the sphere in a circle P_1, P_2, P_3, at which perpendiculars are erected. By a theorem in geometry, the arcs between the points P_1, P_2, and P_3 are proportional to the corresponding angles subtended at the center C. And conversely, if the arcs are proportional to the subtended angles, whatever their length, the curve is a circle.

An obvious difficulty in the case of the earth is that the lines to the center C can not be drawn; it is at this point that astronomical observations come to the rescue. The perpendicular P_1Z_1 is directed toward the zenith of the point P_1, which is a definite known point on the celestial sphere. The zenith of the point P_3 is Z_3, which is also a definite known point on the celestial sphere. The zenith point Z_1 is visible from P_3, and hence the direction of the line P_3Z_1' can be determined by astronomical observations. Since the earth is of inappreciable dimensions in comparison with the celestial sphere, the lines P_1Z_1 and P_3Z_1' are parallel. Therefore, the angle $Z_3P_3Z_1'$ equals the angle P_3CP_1; that is, astronomical observations make it unnecessary to measure directly the angles at the center C.

Observations and measurements, essentially of the kind that have just been described, show that (except for qualifications to be made in Art. 68) the arcs between all points on the surface of the earth are proportional to the angles between the perpendiculars at the points. It follows that the surface of the earth is spherical.

67. Measurement of the Earth's Circumference. There are two kinds of measurements necessary to prove the sphericity of the earth by the method of Art. 66: (*a*) the measurement of arcs on the surface of the earth, and (*b*) the determination of the angles between the perpendiculars (or plumb lines) to the surface. The former is the work of the geodesist, and the latter is that of the astronomer.

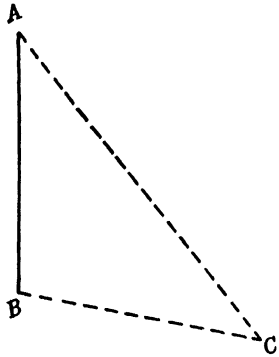


FIG. 36.—Triangulation.

(*a*) To measure accurately a long arc on the surface of the earth, the geodesist first measures a base line AB, Fig. 36, a few kilometers in length, at some favorable place. After he has measured the base line with all possible precision, he sets up a very accurate surveyor's telescope at A and directs it at a distant point C, which may be a tower erected for this purpose. After reading the direction of C on the graduated circle of the instrument, he turns the telescope until it is directed toward B and makes the corresponding reading of the graduated circle. The difference in the two readings is the angle BAC. He measures the angle at B in a similar way. Then, in the triangle ABC, the side AB and the adjacent angles are known, and the distance BC can be computed by plane trigonometry. A second point D is selected which is visible from both B and C and the process is repeated, giving the distance CD; and it is continued through as many steps as may be desired. Not only are the distances computed, but the direction of each point from its predecessor is determined. At the final step, the distance and the direction of the final point from the initial point are found by properly combining the distances and the directions determined in the separate steps.

(*b*) The astronomical part of the work consists simply in determining the angular distances between the zeniths of the various points. This part of the problem presents no difficulties whatever.

68. Proof of the Oblateness of the Earth by Arcs of Latitude. On carrying out the measurements of arcs and angles described in Art. 67, it is found that the curvature of the earth is not the same at all points nor in all directions at one point. For example, the length of a degree of longitude at the earth's equator is 111.323 kilometers (69.173 miles), while the length of a degree of latitude at the equator is 110.57 kilometers (68.71 miles). On the other hand, the length of a degree of latitude at the earth's pole is 111.70 kilometers (69.41 miles). Consequently, the earth is not strictly spherical.

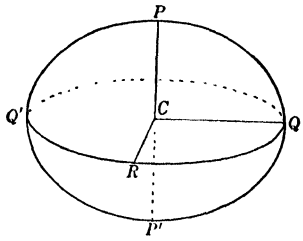


FIG. 37.—An oblate spheroid.

It follows from the relationships between arcs of latitude and the corresponding angles that the earth is an oblate spheroid, illustrated in Fig. 37. An *oblate spheroid* is a flattened figure, somewhat like an orange in shape, but it is not simply any flattened figure, for it has perfectly definite geometric properties. Simply expressed, these properties are that its equatorial section is a *circle,* while all meridian sections are *ellipses* whose polar diameters are shorter than their equatorial diameters. Thus, in Fig. 37 the equatorial section $QRQ'Q$ is a circle, and the meridian section $QPQ'P'Q$ is an ellipse whose polar diameter PP' is less than its equatorial diameter QQ'.

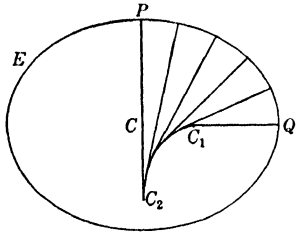


FIG. 38.—Radii of curvature of an ellipse.

If perpendiculars are drawn to the meridian ellipse E from Q to P, Fig. 38, they terminate in a curve from C_1 to C_2 such that the radii from E to this curve are the radii of the curvatures of the ellipse at the corresponding points. The length of an arc on an ellipse is approximately equal to the length of the corresponding arc on a circle having a radius equal to the radius of curvature of the ellipse at the point. Therefore the lengths of arcs of latitude of a given angular length increase in a perfectly definite way from the equator Q to the pole P. Theory and observations are completely in harmony, except for local irregularities due to mountains and lesser variations in level and to abnormalities in density near the surface of the earth.

69. Size and Shape of the Earth. The equatorial radius of the earth CQ, Fig. 37, is the major semi-axis of the meridian ellipse, and it is usually denoted by a. The polar axis CP is the minor semi-axis, and it is usually denoted by b. The ellipticity (oblateness) of the meridian ellipse is $e=(a-b)/a$. It follows that the larger the ellipticity of a spheroid the more flattened it is, while the ellipticity of a sphere is zero.

The most exhaustive discussion of the earth's dimensions is due to J. F. Hayford, whose results, as adopted in 1926 by the International Geodetic and Geophysical Union, are:

Equatorial radius a	6378.388 kilometers = 3963 34 miles
Polar radius b	6356 812 kilometers = 3949.99 miles
Difference	21 476 kilometers = 13 374 miles
Mean radius	6371 22 kilometers = 3958 82 miles
Surface area	500×10^6 square kilometers = 197×10^6 square miles
Volume	$10{,}833\times10^8$ cubic kilometers = 2599×10^8 cubic miles
Ellipticity	1/297

The mean radius is, by definition, the radius of a sphere having the same volume as the earth, and it equals the actual radius of the earth at latitude 35° 15′ 52″. It follows from the accuracy with which individual measurements are made and from the agreement among the results of measurements on all continents that the uncertainty in the radius of the earth can not exceed 50 meters. That is, the uncertainty is only one part in 100,000.

French geodesists first definitely proved the oblateness of the earth in the earlier half of the eighteenth century by the measurement of an arc of 3° in Peru, another of 9° in middle Europe, and a third of 1° in Lapland. These measurements of the French forever settled the question of the oblateness of the earth, which seems to have been somewhat in doubt up to that time.

70. Newton's Proof of the Earth's Oblateness. Newton inferred the oblateness of the earth from dynamical considerations long before it was directly proved by measurements of arcs. He had discovered and formulated the laws of motion and the law of gravitation. He had shown that if the earth were not rotating, it would be spherical. He accepted the rotation of the earth and discussed the effect of its rotation upon its shape. In order to represent the matter concretely, he imagined a tube, filled with water, extending from the surface at the equator to the center and

thence to the surface at the pole. The centrifugal acceleration of the water in the branch of the tube reaching from the center to the equator would partially counteract the attraction toward the center on each unit mass, the effect depending upon the distance of the mass from the center. On the other hand, there would be no centrifugal acceleration of the water in the branch of the tube extending to the pole. Since the pressure in the two tubes must balance in order to be in equilibrium, the equatorial tube must be the longer.

Newton took into account all the varying factors of this problem, including the fact that the attraction of the earth on a mass decreases from its surface to its center, where it vanishes, and before 1686 had arrived at a result in close agreement with that obtained more than half a century later by measurements of arcs. Newton's reasoning was quite conclusive provided the laws of motion and the law of gravitation are correct and the earth rotates on its axis.

71. Gravity Proof of the Earth's Oblateness. If the earth were spherical, gravity would be constant all over its surface. If it is rotating and is oblate, gravity will vary with latitude.

Gravity at the equator Q, Fig. 37, is less than it would be on a sphere of equal volume and mass both because the point on the spheroid is farther from the center of mass than it would be on a sphere and also because the earth's attraction is partly counterbalanced by the centrifugal acceleration due to its rotation. On the other hand, gravity is greater at the pole P than it would be on a sphere of equal volume and mass because it is nearer the center of mass than it would be on the surface of a sphere.

The variation in gravity from the earth's equator to its pole is easily measured, for a mass that weighs one kilogram (2.2 pounds) by spring balances at the equator weighs 1.0053 kilograms at the pole. The measurements, however, are not made by spring balances, but the value of gravity is calculated from the time of swing of a pendulum of a given length. The time of a complete oscillation of a simple pendulum is given in terms of gravity and its length by the equation

$$t = 2\pi \sqrt{\frac{l}{g}},$$

where $\pi = 3.1416\ldots$, l is the length of the pendulum, and g is

gravity. It follows from this equation that if l is measured and t is observed, g can be computed. The advantage in the method consists largely in the fact that the time of a single oscillation can be determined with great accuracy by observing the number of oscillations between successive transits of a star across the meridian, for the errors in determining the times of transit are divided among all the oscillations in the interval. If the pendulum oscillates in seconds, the error will be divided by 86,400, for there are 86,400 seconds in a day.

72. Geocentric, Astronomical, and Geographic Latitudes. The fact that the earth is somewhat oblate gives rise to different kinds of latitude.

Suppose an observer is at the point O, Fig. 39, which is a meridian section of the earth with the oblateness greatly exaggerated. His geocentric latitude is the angle QCO. If his plumb line OZ is continued downward, it will intersect the equatorial radius at the point A. The angular distance of the zenith from the celestial equator, and for this reason the angle QAO, is the astronomical latitude of the observer at O. This is the latitude that is determined by observations for obtaining the latitude of a point on the surface of the earth. It is apparent from Fig. 39 that the astronomical latitude of an observer is greater than his geocentric latitude except when he is on the equator or at a pole, where the two kinds of latitude are equal. The greatest difference in the two latitudes is about 11′.

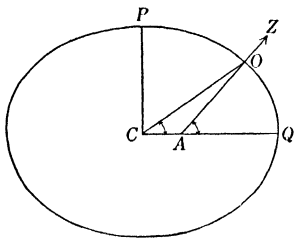


FIG. 39.—Geocentric and astronomical latitudes.

The direction of the plumb line is often affected by mountains or other irregularities in the topography of the earth's surface, or even by abnormalities in the density of the rocks below the surface. These effects are very small, however, for the attraction exerted by a mass even as large as a mountain rarely produces a deflection of the vertical of more than a few seconds of arc. After the astronomical latitude is corrected for these local irregularities it is called the geographical latitude. That is, the geographical latitude is the latitude associated with the ideal mean surface of the earth.

VII. QUESTIONS

1. If possible, observe an eclipse of the moon and note the shape of the earth's shadow as it sweeps across the moon.
2. Eratothenes (275–194 B.C.) noticed that the altitude of the pole star was less when observed from Egypt than when observed from Greece. He interpreted this as meaning that the earth is spherical; was his reasoning conclusive?
3. Is it not possible to determine the curvature of the earth's surface, at least approximately, without using astronomical observations?
4. What is the equatorial circumference of the earth?
5. Draw an ellipse with axes in correct ratio to represent a meridian section of the earth, and draw about the same center and to the same scale a circle to represent the equatorial circumference of the earth. How great is the uncertainty in the dimensions of the earth on the scale you have adopted?
6. Does the weight of an object as determined by balance scales vary with its latitude?
7. Suppose an interval of one day can be determined within 0.5 of a second; how accurately can the time of one oscillation of a second's pendulum be determined? Then how accurately can the gravity g be determined if the length of the pendulum l is so accurately known that no appreciable errors enter through it?
8. Can the latitude of an observer be determined by measuring the rate at which a pendulum clock runs at his position?
9. What is the maximum distance between two places on the surface of the earth such that the geocentric latitude of one equals the astronomical latitude of the other?

II. Mass and Density of the Earth

73. Methods of Determining Mass. There are only two essentially distinct methods of determining the mass of a body: (*a*) by its attraction and (*b*) by its inertia.

(*a*) When a body is weighed, the attraction the earth has for it is compared by means of a convenient arrangement of lever arms, constituting the scales, with the attraction the earth has for an adopted unit, such as a kilogram or a pound. Since a body attracts the earth as much as the earth attracts it, weighing a body consists simply in determining its mass by measuring its attraction for the earth.

(*b*) The mass of a body can also be determined from its inertia, which is the property by virtue of which it tends to remain at rest, if it is at rest, or to continue moving in a straight line with uniform speed, if it is not at rest. The larger a body of any kind of

material, the greater is its inertia; that is, the greater the force necessary to produce in it a given change in motion. Evidently we might determine, though somewhat inconveniently, the mass of a small body by its inertia. Experiments show that the two methods give the same results.

74. Determination of the Earth's Mass by the Torsion Balance. The problem of determining the earth's mass is, theoretically, simply that of comparing its attraction for a given body with that of some known unit mass. The practical difficulties of the problem are all due to the fact that the gravitational force exerted by a mass small enough to be manipulated in a laboratory is extremely slight. For example, the attraction of an iron ball weighing 1000 kilograms (2200 pounds) for a unit mass at its surface is less than one twenty-millionth of that of the earth for the same mass.

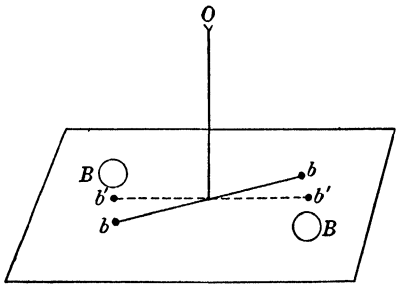


FIG. 40.—The torsion balance.

Among the methods that have been employed for determining the mass of the earth, that depending upon the use of the torsion balance has been most satisfactory. In its essentials, the torsion balance consists of a light horizontal bar suspended by a quartz fiber or a tungsten filament from a fixed point O, Fig. 40. At the ends of the bar there are two small balls b and b. The apparatus is placed in an air-tight, constant-temperature vessel from which the air has been exhausted so that there will be no air currents to disturb the balls b and b or the quartz fiber. After all oscillations of the balls have ceased, two large masses B and B are brought up near b and b and on opposite sides of these balls. These large balls, by their attractions, will gradually pull the small balls to the positions b' and b'. The forces are extremely small, however, and the displacements of b and b are so minute that they can be observed and measured only by very refined methods.

The amount of force exerted by the large balls on the small balls can be determined from the amount of their displacement and the stiffness of the fiber. Let f represent this force and F the attrac-

tive force exerted by the earth. The quantity F is simply the weight of b after allowance has been made for centrifugal acceleration. By the law of gravitation (Art. 173), the attraction f is proportional to the mass of B divided by the square of the distance from the center of B to the center of b. Let r represent this distance, which can be measured. Similarly, the attraction of the earth upon b is proportional to the mass of the earth E divided by the square of its radius R, which is known. Therefore, it follows that the ratio of f to F is

$$\frac{f}{F} = \frac{B/r^2}{E/R^2} .$$

All the quantities in this equation have been determined except the mass of the earth E. On solving the equation for E, it is found that

$$E = \left(\frac{R}{r}\right)^2 \frac{F}{f} B,$$

which expresses E, the mass of the earth, in terms of that of the ball B.

The torsion-balance method of determining the mass of the earth was first employed by Lord Cavendish in 1798. The problem is extremely difficult because the forces to be measured are very small, and hence extraordinary care must be taken in order to get results of any value.

75. Determination of the Earth's Mass by the Mountain Method. The torsion balance measures very small forces; the mountain method makes use of large masses and large forces. The torsion balance is under laboratory control; the mountain method can be used only where the natural conditions are favorable.

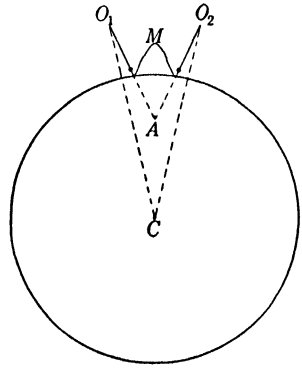


FIG. 41.—The mountain method.

Suppose plumb lines are suspended at O_1 and O_2, Fig. 41, on opposite sides of the mountain M. If it were not for horizontal components of attraction of the mountain, the lines of these plumb lines would intersect at the center

of the earth C (neglecting its oblateness). But actually the plumb lines extended downward intersect at the point A.

If the mountain could be removed at will, the problem of determining its effects on the plumb lines could be solved by observing their directions when it is present, and then after it has been removed. Since the mountain can not be removed, the problem is solved somewhat indirectly. Suppose for simplicity that the plumb lines are on the same meridian. Then the difference in their latitudes, which can be determined by astronomical observations (Art. 56), equals the angle O_1AO_2. Since the radius of the earth is known, the angle O_1CO_2 can be calculated from the distance between the two plumb lines, a distance which can be determined by triangulating around the mountain. After these measurements have been made, both the angle at A and also that at C are known. Suppose the mountain is symmetrical and let a represent the angle of deflection of each of the plumb lines. Then it easily follows by elementary geometry that

$$2a = \textit{angle } O_1AO_2 - \textit{angle } O_1CO_2.$$

Even if the conditions are not so simple as those assumed, still the corresponding deflection can easily be obtained.

The dimensions of the mountain can be determined by accurate surveys, and then its mass can be computed from the densities of its rocks. Since the angular deflections it produces have been determined, its mass relative to that of the earth can be computed by the method which was used in the corresponding problem in Art. 74.

There are several factors which impair the accuracy of the results obtained by the mountain method. The most obvious are that mountains are irregular in outline and of somewhat variable density. Then, too, such non-spherical bodies do not attract as though their masses were concentrated at their centers. Finally, in the neighborhoods of mountains there are usually unknown abnormal densities beneath the surface which affect the direction of the plumb line. As a consequence of these uncertain factors, the mountain method is much less accurate than that of the torsion balance.

In 1774, Nevil Maskelyne devised and first used the mountain method to determine the mass of the earth. He measured the deflection (about 12″) produced by Schehallien, a narrow east-west

ridge in Scotland, and obtained a result for the mass of the earth which is now known to be about 20 per cent too small.

76. Mass and Density of the Earth. It has been found by P. R. Heyl (1930) from experiments of the type described in Art. 74 that the mass of the earth is 5.97×10^{27} grams, the uncertainty in the result being now less than one-tenth of one per cent.

It follows from the volume of the earth (Art. 69) and its total mass that its average density is about 5.51 times that of water, a figure which is much more easily comprehended than the number of grams in the earth. The density of quartz (sand) is 2.75, limestone 3.2, cast iron 7.1, steel 7.8, lead 11.3, mercury 13.6, gold 19.3, and plantinum 21.5. It follows that no considerable part of the earth is composed of such heavy metals as lead, mercury, gold, and platinum. There is nothing in its density, however, to indicate that a large fraction of its mass may not be iron, though the conclusion that it is iron should not be reached from considerations of density alone.

Near its surface the density of the earth is usually between 2 and 4, with an average in the neighborhood of 2.75. Since this is only about half the average for the whole earth, the density of the deep interior must be considerably above 5.5, as would be expected both because there would be a tendency for lighter materials to work their way toward the surface, even if the earth were never liquid, and also because the materials far below the surface are subject to enormous compressive forces.

77. Distribution of the Earth's Mass. Nearly 150 years ago Laplace calculated the distribution of mass in the earth's interior from theoretical considerations, and the results he reached are probably not far from the truth. If D represents the distance below the surface and R the radius of the earth, the results of Laplace are as follows: At $D=0$, $\frac{1}{4}R$, $\frac{1}{2}R$, $\frac{3}{4}R$, and R the corresponding densities (water $=1$) are 2.72, 5.62, 8.30, 10.19, and 10.87. According to these results the density of the earth at all distances from its center is below the densities of the heavier metals.

It is clear that scientists will never be able to penetrate into the earth even one per cent of its radius; yet they are not compelled to rely wholly on theory for conclusions respecting the distribution of its mass. The rotating oblate earth is somewhat like a spinning top or gyroscope, and the attractions of the moon and the sun for

its equatorial bulge, when these bodies are not in its equatorial plane, have a tendency to tilt its axis, and they produce what is known as precession (Art. 97). The rate of precession depends upon the distribution of mass in the earth, and since the rate of precession has been accurately determined from observations, it is possible to determine whether any particular distribution of mass will produce the observed results. On examination, it is found that the Laplacian law of density is in harmony with the observed rate of precession. The inconclusiveness of the result lies in the fact that somewhat different distributions of density in the interior of the earth would also not be inconsistent with the actual rate of precession.

78. Temperature and Pressure in the Earth's Interior. That the temperature of the interior of the earth is high is proved by the existence of hot springs, geysers, and volcanoes in many parts of the world. The rate of increase of temperature with depth in mines and wells varies through a wide range, depending upon the character of the rocks penetrated. Since there are no data for rocks under the oceans and since deep mine shafts and wells are rare on the land surface, the average rate of increase of temperature with depth below the earth's surface is not well determined, but it is probably about 1° Centigrade for 50 meters (1° F. for 75 feet).

If the temperature increased at the uniform rate of 1° Centigrade for each 50 meters below the earth's surface, its value at the moderate depth of 100 kilometers (62 miles) would be 2000° Centigrade (4500° F.), and its value half way to the center of the earth would be about 130,000° Centigrade (292,500° F.), a temperature which is far beyond that required to melt and vaporize any known substance. The temperature, however, does not increase at a uniform rate with depth, for the earth loses heat by radiation only at its surface, and it follows that the temperature gradient is higher for the first few hundred or few thousand meters than it is at greater depths. In complete harmony with theory, a decline in the rate of increase of temperature is found in very deep mines and wells.

Pressures at various distances below the surface of the earth can be computed with a considerable degree of approximation. Since the density of the surface rocks is about 2.75 times that of water, the pressure for a considerable distance below the surface increases at the rate of 275,000 grams per square centimeter for each

kilometer of depth (6300 pounds per square inch for each mile of depth). At greater depths, however, the rate of increase of pressure declines, for pressure is entirely due to the overlying materials, and the weight of a given amount of matter decreases and becomes zero at the center of the earth. If the Laplacian law of density is substantially correct, the pressure at the center of the earth is about 32×10^8 kilograms per square centimeter (22,500 tons per square inch). It is probable that this result does not differ more than 5 per cent from the actual pressure at the earth's center.

The pressures in the deep interior of the earth are so far beyond the range covered by laboratory experience that we can only conjecture the properties of matter subject to them. They are enormously higher than the stresses necessary to cause any known substance to flow almost like a fluid. On the other hand, experiments have shown that pressures of the order of ten thousand kilograms or more per square centimeter greatly increase the stiffness of substances. Only by a study of the earth as a whole when it transmits earthquake waves and responds to tidal forces can we hope to determine the general properties of its interior.

79. Origin of the Earth's Heat. Until recently it was almost universally assumed that the present heat of the interior of the earth is only that which remains in a body which was originally enormously hot and gaseous, then liquid, and finally covered with a solid crust. This explanation should not be accepted without question, for there are other possibilities and probabilities to be considered.

As will be seen in Chap. XIV, there are reasons for concluding that the earth may have grown up in large part by the somewhat gradual accretion of solid particles and masses of various sizes, just as it is now growing by the infalling of several millions of meteors per day. The average density of these original masses was probably equal to that of the surface rocks, as is that of the meteors which now fall upon the earth. In contracting from this density to that of the present earth, enough heat would be generated to raise the temperature of its whole mass several thousand degrees. In short, this source of energy would adequately explain all the positively known heat of the earth.

There is an additional source of energy in the heavy radioactive elements such as uranium, thorium, and radium, which undergo

spontaneous transformations into lighter elements with the liberation of heat. Although these radioactive elements in the earth's surface rocks are rare, yet they release such enormous amounts of energy per gram that their total effect may be very important. In fact, if uranium, thorium, and radium are scattered throughout the whole earth, even in the minute percentages that they occur in the surface rocks, it is difficult to escape the conclusion that the temperature of the earth is rising. If it is now rising, apparently it should have been rising throughout the whole past geological history of the earth, for as these elements disintegrate, their effects continually decrease. To escape this conclusion, Harold Jeffreys assumes that these radioactive elements are largely limited to a zone within 19 or 20 kilometers of the surface. Yet the possibility should not be neglected that the earth may not be getting hotter, because in its deep interior there are transformations taking place which are using up energy instead of liberating it.

In mountainous regions the surface rocks are usually much folded and show clear evidences of horizontal pressures. These phenomena have been interpreted as being due to a shrinkage of the earth as a consequence of loss of heat. There are other places, however, in which there are sinks and depressions which are apparently due to a stretching of the surface. Sometimes these relatively infrequent stretched places are not far removed from mountain wrinkles. Since the surface rocks are not strong enough to transmit thrusts a quarter or a third of the way around the earth, as would be necessary to explain the distribution of mountain ranges, if they were produced simply by contraction, it follows that they are not conclusive evidence that the earth has shrunk. Clearly the whole problem of the earth's heat is one that is now far from a final solution.

80. Rigidity of the Earth Proved by the Character of Earthquake Waves. The theory that the earth consists of a relatively thin crust, perhaps 100 or 200 kilometers in thickness, floating on a liquid interior has been commonly held down to recent times. Yet for many years evidence of various kinds has been accumulating that, on the average, the earth is more rigid than steel and that it contains no large liquid masses in its interior.

Perhaps the simplest proof of the rigidity of the earth is the

fact that earthquake waves are transmitted along its surface and through its interior as they are transmitted in solid bodies. In the first place, they move at a speed ranging from 3.3 to 5.6 kilometers per second; and, in the second place, part of them are transverse to their direction of propagation. It follows that the earth responds to earthquake waves like a rather rigid body, though the degree of its rigidity is not accurately determined.

81. Rigidity and Elasticity of the Earth Proved by the Tide Experiment. In the laboratory, the rigidity and the elasticity properties of a body are determined by applying to it a deforming force and observing what happens. If a body strongly resists deformation, it has a high *rigidity*, or stiffness, which is more exactly expressed by suitably defined moduli. If a somewhat rigid body is deformed by a force and does not retake its original shape after the force is removed, it is *viscous;* if it springs back to its original shape and size, it is *elastic.* Elasticity is also more exactly expressed by suitably defined moduli. Speaking in general terms, iron is rigid and water is not rigid; thick pitch has stiffness and is viscous; and finely tempered steel has stiffness and is elastic.

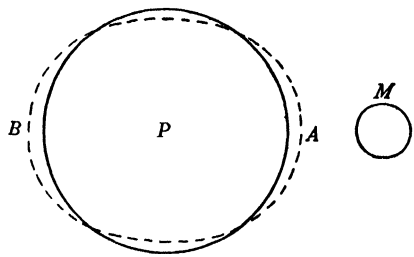


FIG. 42.—Tides generated by the moon.

Obviously, forces can not be applied to the earth to deform it as they can to a body in the laboratory. But the earth is subject to changing exterior forces, due to the moon and the sun, whose effects can be observed and from which the rigidity and the elasticity properties of the earth can be inferred. For simplicity, consider the effects of the moon alone. In Fig. 42, let P represent the pole of the earth, and M the moon (not to the same scale). The moon raises the tide A on the side toward it and an almost equal tide B (Art. 186) on the side opposite. The rotating earth carries the observer past the tidal bulge A and through the region of low tide, then past the bulge B, and finally back to the starting point. The circuit with respect to the moon is completed in $24^h\ 51^m$, during which the observer has two high tides that move over the surface of the earth from east to west.

Ocean tides are modified by so many factors that it is impossible to derive from them accurate quantitative results respecting the rigidity of the earth. Hence a number of scientists have measured the deformations of the earth from equivalent phenomena. The most accurate results have been obtained by A. A. Michelson and H. G. Gale who measured during a period of nearly two years the varying tides in horizontal pipes partly filled with water. By using pipes only 160 meters in length they controlled the conditions surrounding the experiment, but the tides they measured were at their maxima only 0.005 centimeter (0.002 inch).

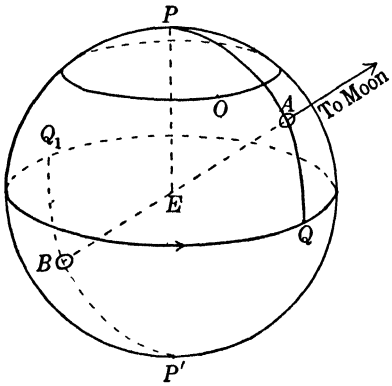


FIG. 43.—High tides at A and B.

In Fig. 43, let E represent the center of the earth and EA the line to the moon. Then there are high tides on the earth at A and B. When the moon is north of the plane of the earth's equator, as illustrated in the diagram, the tide A is in north latitude and the tide B is in equal south latitude. The tides gradually decrease from A and from B to low tide on a great circle around the earth midway between these two points.

If the earth were perfectly rigid, the position of the pipes would remain fixed and consequently tides would rise in them and with respect to them in response to the forces exerted by the moon. On the contrary, if the earth were a perfect fluid, the pipes themselves would rise and fall as much as the water which they contain, and the observer would find no movement of the water with respect to the pipes. The actual conditions are between these extremes; the earth yields somewhat and the tides are less than their theoretical maximum.

It is clear that the problem consists of three parts: (*a*) The determination by mathematical theory of the values the tides would have if the earth were absolutely rigid; (*b*) the measurements of the actual tides in the pipes; and (*c*) the deduction of the rigidity and elasticity properties of the earth from the differences between the theoretical maximum tides and the observed actual

tides. Unfortunately, the problem is difficult both theoretically and observationally. When the observer is at *O*, Fig. 43, the east end of the east-west pipe is ascending the slope of the tidal bulge *A* and there is high tide in it. Similarly, there is high tide in the south end of the north-south pipe. When *O* arrives on the meridian of *A* there is no tide in the east-west pipe, but the high tide in the south end of the other pipe has its maximum value. Although the tidal bulge at *B* is nearly as great as that at *A*, it produces smaller tides in the pipes because it is south of the equator far from the parallel of latitude of *O*.

Even the foregoing complexities are only a fraction of those encountered in the actual problem. The moon is revolving around the earth at a somewhat variable rate; its distance from the earth varies considerably; and it moves from north of the celestial equator to an equal distance south and back again each month. Moreover, the sun produces an almost equally complex set of tides which sometimes add to and sometimes partially counteract those of the moon. The tides vary from day to day and from month to month throughout the year, never being twice alike. Yet during the two years covered by the experiments there was perfect agreement between the theory and the observations except for the reduction in the observed tides due to the yielding of the earth.

The measurements of Michelson and Gale showed that the tides in the pipes were 70 per cent of the values they would have had if the earth were perfectly rigid. A mathematical discussion of the yielding of an elastic body subject to tidal forces shows that this result implies that the earth has a stiffness, or rigidity, somewhat greater than that of a sphere of steel of the same size. Therefore our globe does not have a crust floating on a fluid interior, for such a crust could not give the earth its high rigidity, even if it were several hundred kilometers in thickness.

Another important conclusion followed from the tide experiments. When the measurements were compared with the results of theory, it was found that the theoretical and observed tides coincided in time. This means that the earth is *elastic* and not viscous, as had theretofore been believed to be the case; for, if the earth were viscous, the tides in it would lag behind the water tides in the pipes, which respond within a few seconds to the forces that produce them. The scientific importance of this result is about as great as is the conclusion that the earth is rigid. For

example, it is evident that this property of the earth will have a bearing on theories of the formation of continents. And Sir George Darwin developed an elaborate and beautiful theory of the effects of the body tides of the earth upon the rotation of the earth and the orbit of the moon, the basic assumption of which was that the earth is viscous instead of elastic, as these tide experiments now prove it to be.

In summary, the principal conclusions are: (*a*) the earth's average density is about four-fifths the density of steel; (*b*) the earth's stiffness is perhaps a little greater than that of steel; and (*c*) the earth is somewhat more elastic than steel. Though these far-reaching conclusions pertain to the earth alone, they have been established by principles which depend upon observations of the stars.

82. Isostasy. As far back as 1859, Archdeacon Pratt inferred from determinations of gravity in various localities that the density of the earth beneath mountains is less than it is elsewhere. These results led him to the hypothesis that the excess in elevation in mountain regions balances the deficiencies in density beneath them, and that the depressions of the ocean basins are compensated by abnormal densities beneath their floors. Major Dutton introduced the word *isostasy* to denote a condition in which the total amount of material along a radius of the earth is independent of surface irregularities.

The theory of isostasy has been put to the most exhaustive tests in the United States, first by J. F. Hayford and later by William Bowie. Hayford and Bowie concluded from an enormous amount of observational evidence of both the intensity and the direction of gravity in a large part of the United States that isostatic compensation is complete at a depth of about 100 kilometers (62 miles). This conclusion implies, of course, a surface layer of this thickness having quite different properties from the main body of the earth, and earthquake phenomena lend some support to the theory. Harold Jeffreys thinks there is an exterior layer of lighter materials separated from the interior of heavier materials by a rather abrupt discontinuity; but instead of placing the division zone at a depth of 100 kilometers, he concludes that it is only 20 or 30 kilometers below the surface. Gravity observations are not sensitive to deep abnormalities, and earthquakes

are distorted in passing deeply through the earth. Moreover, the interpretations of both classes of phenomena cost an enormous amount of labor and are involved in numerous uncertainties. Consequently, it is advisable at present to have reservations respecting the precise depth of isostatic compensation, if, indeed, there is any such depth that is everywhere the same.

83. Age of the Earth. When geologists first inferred from geological phenomena that the earth is millions of years old, their conclusions were regarded as utterly fantastic and unreasonable. Their estimates of its age, based on the thickness of sedimentary rocks and on the amount of salt that has been dissolved and carried into the sea, ranged from 40,000,000 to 120,000,000 years. But even the geologists were much too conservative. As has always been the case in our exploration of the universe, the facts have far transcended our anticipations. Instead of counting the age of the earth in tens of millions of years, it has been found necessary to accept the conclusion that it has been approximately of its present size and mass for at least a thousand million years.

One naturally inquires how the great age of the earth has been established. The methods that are the simplest by far and apparently the most trustworthy are based upon the properties of the radioactive elements uranium, thorium, and radium and their derivatives, lead and helium. When uranium and thorium disintegrate, as they do very slowly, their final products are helium and lead. Their rates of disintegration are completely independent of such differences of temperature, pressure, and chemical combination as can be controlled in the laboratory. Lead derived from uranium has an atomic weight of 206 and that derived from thorium has an atomic weight of 208, while ordinary lead has an atomic weight of 207. Hence, if lead having an atomic weight of 206 is found in uranium ore, it is very probable that it originated from the disintegration of uranium.

Now suppose an ore of uranium or thorium is found undisturbed in a geologically old rock, and consider the problem of determining the age of the rock. From the proportion of uranium lead present and the known rate of its formation, it is evidently possible to compute the age of the crystals in which it is contained. Rocks of the same geological period have been found by this method to be of the same age, a fact which inspires confidence in its reliability.

Rocks of the carboniferous period are 300,000,000 years old; those of the upper Pre-Cambrian, 560,000,000 years; and those of the ower Pre-Cambrian, from 1,210,000,000 years to 1,340,000,000 years. That is, the Pre-Cambrian periods of the Archaean Era, in which only very low forms of life existed on the earth, extended over a longer interval than all the time since their close. Back of the Archaean Era stretches a history which there is now no very satisfactory means of measuring, but H. N. Russell concludes from the present percentages of uranium, thorium, and lead in the surface rocks, so far as they are known, that the age of the earth does not exceed 8,000,000,000 years.

There can be no doubt that the earth has been of approximately its present mass for 1,000,000,000 years, and its age may be several times this enormous interval. But the unsolved problems respecting its history are many. We do not know, for example, how the uranium and thorium originated, nor whether they formed larger and larger fractions of the earth's mass as we go farther and farther back in time.

VIII. QUESTIONS

1. Suppose only iron in the interior of the earth attracts iron on its surface; only lead in the interior of the earth attracts lead on its surface; and similarly for other elements. Would the gravity and the inertia of bodies be proportional?
2. What is the weight of a cubic meter of quartz? A cubic kilometer? A cubic mile?
3. What is the melting temperature of iron? If the temperature increased uniformly with depth below the surface of the earth at the rate of 1° C. for 50 meters, at what depth would the temperature be reached at which iron melts under surface pressures?
4. What is the pressure per square centimeter (per square inch) at the bottom of one of the deep places in the ocean, say at a depth of 7000 meters?
5. What is the speed of a compressional (sound) wave in air? In water?
6. Draw a circle to represent the earth, showing to the same scale a crust with a thickness of 200 kilometers (120 miles). If the part of the earth interior to this crust is liquid, will the crust be likely to insure any considerable rigidity for the earth as a whole?
7. If there is isostatic compensation in 100 kilometers, what is the average deficiency in density beneath a mountain 500 meters in height?

III. The Atmosphere of the Earth

84. Mass of the Earth's Atmosphere. The pressure of the atmosphere on a surface is simply the weight of the air above it. Consequently, to determine the total mass of the atmosphere, it is

sufficient to measure the pressure on a given area and to multiply this pressure by the number of such areas in the whole surface of the earth.

At sea level, the average pressure of the earth's atmosphere is about 1030 grams per square centimeter (14.7 pounds per square inch). Hence it follows from the total area of the earth's surface, given in Art. 69, that the mass of the earth's atmosphere is about 5.25×10^{21} grams (5.81×10^{15} tons). It is found by comparing these results with those given in Art. 76 that the mass of the atmosphere is a little less than one-millionth that of the entire earth; more exactly, it is $.88 \times 10^{-6}$ that of the whole earth. At sea level the density of the air is about 1/800 that of water.

85. Composition of the Earth's Atmosphere. Near the surface of the earth about 78 per cent of the atmosphere, by volume, is oxygen, about 21 per cent is nitrogen, about 0.94 per cent is argon, and the remaining substances occur in much smaller quantities. Nitrogen, oxygen, and argon are chemical elements and not compounds of two or more elements, as are most of the materials of the solid part of the earth. It is not surprising that nitrogen and argon are found in the earth's atmosphere, for these elements are chemically inert; that is, they do not readily form chemical compounds with other elements, and the compounds in which they do occur are often unstable and easily broken up. Oxygen, on the other hand, is a very active chemical element, uniting with many others in compounds which are generally very stable. For this reason, it seems that in general the atmospheres of planets should not contain free oxygen.

One of the compounds of oxygen is water, of which 16 parts are oxygen to 2 parts of hydrogen. If the water were spread uniformly over the surface of the earth, its depth would be approximately 4000 meters. The pressure at the bottom of this uniform sea would be 400,000 grams per square centimeter, eight-ninths of it due to oxygen. But the weight of the oxygen in a column of the earth's atmosphere having a cross section of a square centimeter is only 920 grams, or less than 1/400 the weight of the oxygen in the water.

If there had been originally one per cent or even one-tenth of one per cent more free hydrogen on the surface of the earth, apparently the free oxygen which is now in the air would have united

with it, in the formation of water, and left the atmosphere entirely without this element on which all higher forms of life depend. Similar conclusions are reached by considering silicon with which oxygen has united in enormous quantities in the formation of the silicates (sand rocks).

86. Climatic Effects of Water Vapor and Carbon Dioxide. In addition to the elements nitrogen and oxygen, there are in the earth's atmosphere two important compounds, water vapor (H_2O) and carbon dioxide (CO_2). The former occurs in variable quantities averaging less than one per cent; the latter constitutes only about 3.3/10,000 of the atmosphere by volume and about 1/2000 of it by weight.

Both water vapor and carbon dioxide play important rôles in determining the temperature of the earth's surface, for they are relatively transparent to the incoming light rays from the sun and relatively opaque to the outgoing heat rays from the earth. The result is that the temperature of the surface of the earth is probably somewhat higher on the average than it would be if the atmosphere did not contain these compounds. In fact, there is a theory that glacial epochs have occurred when the earth's atmosphere from time to time has been relatively poor in carbon dioxide. Water vapor and carbon dioxide also reduce the daily variations in temperature of the surface of the earth by absorbing heat during the day and radiating it at night.

Water is not only very abundant on the surface of the earth but it has exceptional thermal properties. It absorbs more heat than any other substance (except hydrogen gas) for a given rise in temperature, and it of course gives up an equal amount on cooling. Moreover, it has very high latent heats of melting and evaporation. Simply to melt ice without changing its temperature requires enough heat to raise the temperature of an equal mass of iron through 400° Centigrade, and more than six times this amount of heat is necessary to evaporate the water. In technical terms, the latent heat of melting ice is 80 calories per gram, and the latent heat of evaporation is 540 calories per gram. It is evident, therefore, that in its various forms on the earth, water absorbs great quantities of heat and has important retarding effects when temperatures are rising; and that, since it gives up equal quantities of heat on cooling, it has equal retarding effects when temperatures

are falling. It is a reservoir of enormous capacity for heat which greatly reduces variations in temperature levels on the surface of the earth. Possibly the temperatures of such planets as Mars are not regulated similarly.

A molecule of carbon dioxide consists of an atom of carbon (atomic weight 12) joined in chemical union with two atoms of oxygen (atomic weight 16). It follows that 12/44 of carbon dioxide is carbon. Since about one part in two thousand of the earth's atmosphere is carbon dioxide, it follows that there are 2.62×10^{18} grams of this compound in the air, of which 7×10^{17} grams are carbon. Living plants absorb carbon dioxide from the air, and under the influence of sunlight break it up into carbon and oxygen, retaining the carbon and exhaling the oxygen. Most of the carbon in plants is returned to the air through their decaying, but a fraction of it has been preserved in coal. The total amount of carbon in the known coal deposits is much greater than that in the air; and if the coal were all burned, apparently the atmospheric carbon dioxide would be more than doubled with possible important climatic effects. There are, however, greater quantities of it locked up in limestone rocks and dissolved in the oceans. Consequently one should be cautious in drawing conclusions respecting the variations of this important compound in the earth's atmosphere.

87. Variations in Density of the Earth's Atmosphere with Altitude. Since the atmosphere is a mixture of compressible gases, its density is greatest at the surface of the earth where the pressure of the materials above is greatest. It follows from the elasticity of these gases and the constant of gravity that the density of the atmosphere at an altitude of about 5600 meters (3½ miles) is half that at the earth's surface. Moreover, the law of change of its density is such that half of its entire mass is within the same zone.

The density of the air is diminished by the factor one-half in the next 5600 meters, and so on as long as the laws of gases are satisfied. At an altitude of 11,200 meters the atmospheric density is 0.25 of that at the surface, and at an altitude of 22,400 meters (14 miles) it is again diminished by the factor one-half to .0625. The greatest heights reached by artillery projectiles are about 40,000 meters, at which the density of the atmosphere is only .004 of its density at sea level.

The composition of the earth's atmosphere varies somewhat with

altitude above the surface of the earth. In the first place, the properties of water vapor are such that very little of it reaches levels beyond 5000 or 6000 meters (3 or 4 miles). Then, too, there is a tendency for the heavier gases to accumulate at the lower levels. The order in which they (with the exception of water vapor) have a tendency to be arranged can be seen from the molecular weights of the constituents of the air: carbon dioxide, 44; argon, 40; oxygen, 32; nitrogen, 28; water vapor, 18; and hydrogen, 2. It is generally believed that at very great heights the earth's atmosphere may be largely hydrogen.

88. Determination of Height of Earth's Atmosphere from Observed Phenomena. The height to which the earth's atmosphere extends in appreciable quantities can be determined directly from several classes of observed phenomena, as well as from theory.

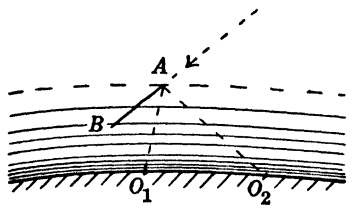


FIG. 44.—Height at which a meteor becomes visible.

(*a*) *Meteors, or shooting stars.* There are great numbers of minute masses moving in the interplanetary spaces that are completely invisible except when they plunge, as so-called shooting stars, into the earth's atmosphere and glow for a second or two with the heat produced by the resistance to their motion. The height at which the atmosphere is dense enough to make a meteor incandescent can be determined by observing from two points on the surface of the earth the directions at which it first becomes visible.

Let O_1 and O_2, Fig. 44, represent two points on the surface of the earth from which meteors are observed. Let the dotted line represent the path of a meteor before it enters the atmosphere. Suppose the meteor encounters enough resistance at the point A to cause it to become incandescent and that the observer at O_1 notes the apparent position of this point and measures the angle AO_1O_2. Similarly, suppose the observer at O_2 measures the angle AO_2O_1. Since the distance between O_1 and O_2 will be known, all parts of the triangle can be determined and the height of the point A above the surface of the earth can be computed. In a similar way, the height of the point B at which the meteor is consumed and disappears can be computed.

Observations from two stations, separated by a few kilometers, show that meteors usually first become visible at a height of from 100 to 150 kilometers (60 to 90 miles) above the surface of the earth. This is, therefore, the height to which the atmosphere extends in sufficient quantities to retard meteors enough to make them incandescent. The theoretical density at 100 kilometers is about 4/1,000,000 of that at sea level, and this value is probably substantially correct. It is found by similar observations that meteors usually disappear when they have descended to within 50 to 75 kilometers (30 to 45 miles) of the earth's surface.

(*b*) *Aurorae* are electrical phenomena of the very rare upper atmosphere that are probably due to ionized particles which, streaming out from the sun, are drawn in upon the earth. The height above the earth's surface to which an auroral streamer extends can be determined by the same method as that used in computing the altitude at which a meteor becomes visible.

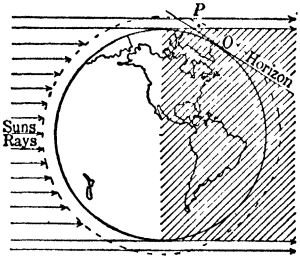


FIG. 45.—Height of the earth's atmosphere from the duration of twilight.

It has been found that the higher southern ends of the northern auroral streamers are visible at altitudes ranging from 150 to 1000 kilometers. This means simply that auroral phenomena occur in portions of the atmosphere which are far too rare to have sensible effects upon the velocities of meteors.

(*c*) *Duration of twilight.* For a considerable time before the sun rises in the morning the dawn heralds its coming, and for a considerable time after it has set in the evening the lingering twilight is a reminder of its recent departure. The dawn and the twilight are due to portions of the illuminated atmosphere of the earth which are visible although the sun itself is below the observer's horizon.

In Fig. 45, which represents the depth of the earth's atmosphere on a greatly exaggerated scale, an observer at the point *O* can see the portion of the illuminated atmosphere near the point *P*. Although it is impossible to decide precisely when twilight ceases, it has been found that under favorable weather conditions the day's afterglow does not entirely fade from the sky until the sun is about 18° below the horizon. The region *P* is

midway between the observer at O and the sunset line. Since the angular distance from O to the sunset line is known, the altitude of P can be computed, and it is found to be about 80 kilometers (50 miles). Therefore, the atmosphere at the heights of meteors or auroral displays does not reflect enough sunlight to be visible at the surface of the earth.

89. Refraction. Although the earth's atmosphere is of low density in comparison with glass, yet it refracts the light we receive from celestial objects by amounts which increase from zero at the zenith to about 35′ at the horizon, the amount depending to some extent upon its temperature, humidity, and barometric pressure. Since the density of the atmosphere increases from its outer limits to the earth's surface, refraction continually increases as light penetrates to lower and lower levels.

When light enters a denser medium, it is refracted toward the perpendicular to the surface (Art. 41); hence as light passes through the earth's atmosphere it is bent downward. Since we see objects in the direction from which light comes when it enters our eyes, refraction increases the apparent altitudes of celestial objects. In fact, it apparently raises the sun and the moon just above the horizon when they would otherwise be just below it. Their lower margins are raised somewhat more than their upper margins, with the result that they appear to be somewhat flattened when they are rising or setting.

90. The Kinetic Theory of Gases. One of the most interesting and far-reaching physical theories developed during the nineteenth century is that which explains the properties of gases. In sharp contrast to solids and liquids, gases are elastic fluids which expand and dissipate unless held inclosed by impervious vessels; and, when inclosed, their various parts ceaselessly mix and mingle without ever twice arriving at precisely the same distribution of constituent units.

The physical universe is composed of about 90 relatively fundamental units, called *atoms*. The atom is the unit of chemical action, and each of the millions of the different substances known on the earth is composed of a few or many of these 90 elements joined in chemical units called *molecules*. When atoms of one kind are not combined in chemical union with atoms of other kinds, they are

usually joined in pairs with atoms of their own kind. Thus, free oxygen consists of molecules of two atoms each.

Although molecules usually consist of several atoms, they are very small. A million of them together would be far beyond the range of the most powerful microscope that can be made. As Lord Kelvin found by a number of ingenious experiments, the distances between the centers of adjacent molecules of water do not exceed 1/200,000,000 of a centimeter (1/500,000,000 of an inch). Probably the smallest microörganism known contains more molecules than there are human beings on the whole earth.

Gases exert pressures on the walls of their retaining vessels by the countless impacts of their molecules. At sea level a cubic liter of air weighs only 1.25 grams, yet it exerts on the six walls of its retaining vessel (total area 600 square centimeters) a pressure of 618,000 grams. It has been found that for such a small mass to maintain so great a pressure the molecules of air under moderate temperatures at sea level must be moving at an average speed of about 500 meters (1600 feet) per second. Under these conditions a molecule encounters other molecules 5,000,000,000 times per second, and the average straight-line path between collisions is only 1/100,000 of a centimeter. This does not mean, however, that the molecules of a gas are almost continually in contact; more than 99 per cent of the time they are describing undisturbed straight-line paths. All these numerical results are rough average values, for the molecules in a gas move with various speeds. Most of them move with a velocity approximately equal to the square root of the average of the squares of the speeds of those in the mixture, and the numbers having different velocities decrease rapidly with departures from this value.

The temperature of a gas is proportional to the mean of the squares of the velocities of its molecules, from which it follows that the pressure a gas exerts on the walls of its retaining vessel increases as its temperature rises. In a mixture of gases of different molecular weights, the molecules of the lighter gases move with higher speeds than those of the heavier, the squares of the velocities being inversely proportional to the molecular weights of the gases. In a mixture of hydrogen and oxygen, for example, the mean of the squares of the velocities of the hydrogen molecules is 16 times the mean of the squares of the velocities of the oxygen molecules.

If the kinetic theory of gases explained only pressures, there might be some reservations respecting its acceptance. But it explains diffusion and the variations from the approximate laws of Boyle and Charles and a host of phenomena connected with viscosity, boiling points, liquefaction, etc. Those best qualified to judge regard it as an adequate explanation of phenomena under conditions even remotely resembling those in the air at the surface of the earth. When it comes, however, to the questions of the behavior of gases on the tenuous borders of atmospheres and in the deep interiors of stars, there are much greater uncertainties.

91. Escape of Atmospheres. If a mass is projected upward from the surface of the earth, the height to which it will rise depends upon the speed with which it is started. If the initial speed is great, it will rise very high; at a certain speed it will leave the earth and never return. This speed is called the *velocity of escape.*

In the case of the earth, if the resistance of the atmosphere is neglected, the velocity of escape is about 11.2 kilometers (6.95 miles) per second. In the case of the moon the velocity of escape is only 2.38 kilometers (1.48 miles) per second, while in the case of the great sun it is 611 kilometers (380 miles) per second.

Although the average speed of the molecules of the earth's atmosphere is far below the velocity of escape, a very small percentage of them at any instant will be moving at speeds above that velocity. About half of these will be moving downward toward the earth and, consequently, can not directly escape. Most of the remainder will encounter other molecules before they have gone a centimeter. A very minute percentage of them may, however, pass out through all the atmosphere above without collision and escape away into space. Evidently, the chances of escape for molecules in the high upper air are not so unfavorable as for those near the earth's surface, but those at high altitudes, where the temperatures are low, move at lower average velocities. In spite of the unfavorable conditions for the escape of the molecules from the earth, it seems probable that the earth is losing some of its atmosphere. But when a molecule escapes from the earth, it is still subject to the sun's control and continues to circulate around it. Since the earth revolves around the sun in a region almost certainly containing innumerable wandering molecules, it is likely that it long ago arrived at a condition of equilibrium in which it gains as many as

it loses. The geological and the palaeontological evidences indicate that throughout geological history there has been no progressive change in the extent or the constitution of the atmosphere.

If the theory of the escape of atmospheres is correct, it should be in harmony with the phenomena presented by other celestial bodies, and it is. The moon has a low velocity of escape and no atmosphere. Furthermore, the satellites of the other planets and all known bodies having low velocities of escape are quite devoid of atmospheres. On the other hand, all known bodies having velocities of escape higher than that of the earth have atmospheres, and in general the atmospheres are more extensive the higher the velocity of escape.

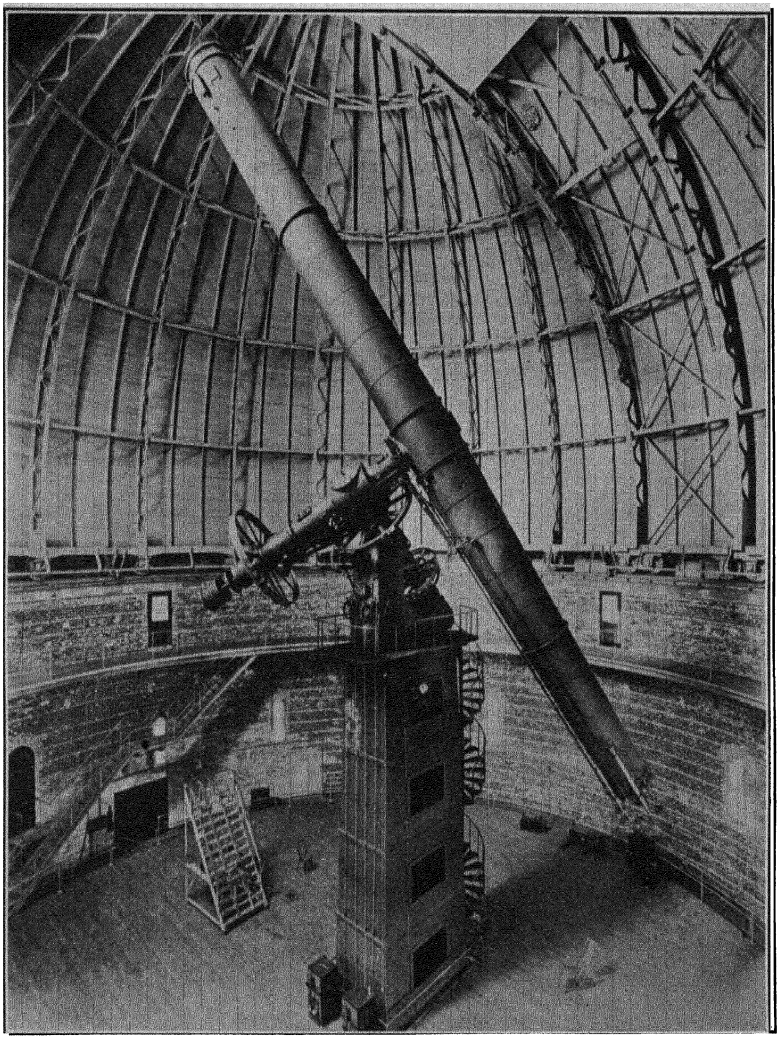
IX. QUESTIONS

1. What is the total atmospheric pressure on a square kilometer? Square mile?
2. Estimate the total atmospheric pressure on the exterior of a man.
3. At the present time about 1,000,000,000 tons of coal are burned annually on the earth, and the principal product of the combustion is carbon dioxide (CO_2). On the hypothesis that all the carbon dioxide is retained in the atmosphere, how many years will be required for the amount now in it to be doubled?
4. On the assumption that the average annual rainfall on the earth is two-thirds of a meter, how many years will be required for a mass of water equal to that of the oceans to go through the cycle of evaporation and precipitation again?
5. On the hypothesis that all the oxygen now in the earth's atmosphere has been derived by plants from carbon dioxide that was formerly a part of the atmosphere, and that half of the weight of the plants produced was carbon, how many tons of plants were involved in the transformation? If spread uniformly over the surface of the earth, how deep a layer of unit density would they make? In the process, what was the net loss to the atmosphere in tons? In percentage? Is there enough unoxidized coal and other known materials of vegetable origin now in the earth to meet the requirements of the theory?
6. What is the mean velocity of hydrogen molecules under surface conditions of temperature and pressure?
7. What is the theoretical density of the air at a height of 150 kilometers?
8. What is the theoretical density of the air at the altitude of the highest observed auroral streamers?
9. Determine the duration of twilight by observations.
10. What bearing, if any, has the theory of the escape of atmospheres on the theory that the earth's atmosphere at great heights is hydrogen?

REFERENCE BOOKS

The Earth (1929). Harold Jeffreys. The Cambridge University Press, Cambridge, England.

Physics of the Air (1929). W. J. Humphreys. McGraw-Hill Book Company, New York.



The 40-inch telescope of the Yerkes Observatory, the largest refractor in the world.

CHAPTER V

THE MOTIONS OF THE EARTH

I. The Rotation of the Earth

92. Two Explanations of the Diurnal Motions of the Celestial Bodies. The sun, the moon, and those stars which are near the celestial equator apparently rise from the eastern horizon, move westward across the celestial sphere, and set below the western horizon. The diurnal motions of equatorial stars are illustrated by Fig. 46, which is a reproduction of a photograph of Orion taken with a telescope that was fixed in position while the stars drifted westward across the field of view.

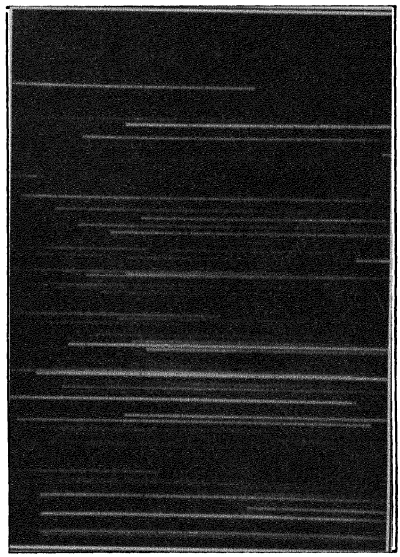


Fig. 46.—Star trails in Orion.

Stars near the celestial pole describe circles around it as a center, as is illustrated in Fig. 47. The conspicuous streak a little below and to the left of the center of the circles is the trail of the pole star, which is at present a little more than 1° from the celestial pole (Art. 24). Most of the streaks in these photographs were produced by stars far too faint to be seen with the unaided eye.

Not only do the stars apparently revolve westward around the sky, but they all make their circuits in exactly the same period whether they describe large circles near the celestial equator or small circles near the celestial poles. The sun, however, makes its apparent circuit in a period that is slightly variable (Art. 123) and nearly 4 minutes longer than that of the stars. The period of

daily motion of the moon is also somewhat variable and about 51 minutes longer than that of the stars.

There are two distinct explanations of the apparent diurnal motions of the celestial bodies which are in harmony with all the facts that have been so far enumerated in this article. The first and most natural one is that the earth is stationary and that the stars, the sun, and the moon revolve around it from east to west. In accepting this explanation, it is of course necessary to make the extraordinary assumptions that all the stars revolve in precisely the same period irrespective of their positions on the celestial sphere, their differences in brightness, and their differences in distance from the earth, and that the sun and the moon move independently.

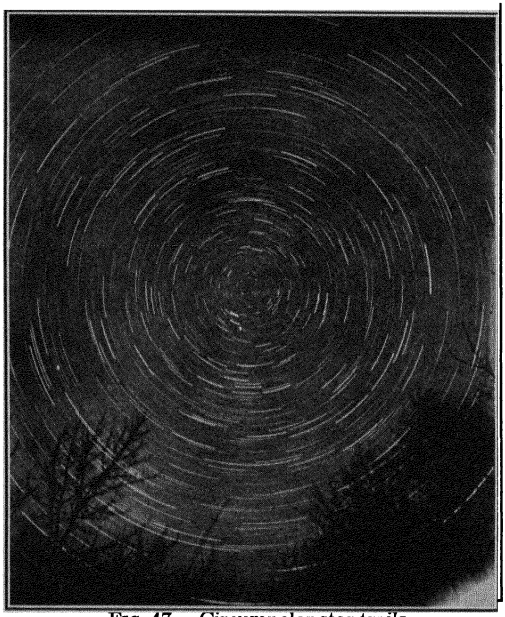


FIG. 47.—Circumpolar star trails.

The second explanation of the diurnal motions of the stars is that they remain fixed while the earth rotates from west to east. If this explanation is correct, the diurnal periods of the stars should be independent of their positions on the celestial sphere and of their distances from the earth, as they are. If we had not been taught from childhood that this apparently solid and fixed earth actually rests on nothing and rotates rapidly, we probably should reject this explanation of the diurnal motions of the celestial bodies as promptly as did nearly all our predecessors until about three centuries ago.

Since each of the two explanations of the diurnal motions of the sun, the moon, and the stars is in complete harmony with all the phenomena so far enumerated, it is impossible to determine which of them is correct without making use of essentially new

observational facts. If we were considering either of the two theories alone, we should quite likely be convinced that it is correct, for it would explain all the phenomena. It is evident from this illustration that it is not safe to accept a theory as established just because it may be in harmony with a single kind of observational evidence. Generally it is highly advisable to keep in mind more than one possible explanation of observed phenomena until additional evidence of a different kind enables us to decide among them.

The earliest philosopher who is known to have believed that the earth rotates is Philolaus, a Pythagorean, who lived in the fifth century B.C. Although his ideas had very little influence in antiquity, they were quoted by Copernicus (1473–1543) in his great work on the motions of the planets and the moon. Aristotle (384–322 B.C.), the greatest Greek philosopher and one of the profoundest thinkers of all time, recognized the fact that the diurnal motions of the stars can be explained equally well by the theory that the earth rotates. Aristarchus of Samos (310–250 B.C.) made the clearest statements of any philosopher of antiquity respecting both the rotation of the earth on its axis and also its revolution around the sun. But Hipparchus (180–110 B.C.), an astronomer whose numerous and important discoveries were not equaled by any other philosopher or scientist of ancient times, believed the earth is stationary, and he was followed in this opinion by Ptolemy (100–170 A.D.) and every other astronomer of note down to Copernicus (1473–1543). Since the time of Copernicus the theory that the earth rotates has been accepted by scientists, though the actual proofs of it date from Newton's *Principia* which was published in 1687.

93. The Laws of Motion. All the actual proofs of the rotation of the earth depend upon the laws of motion, which are, therefore, necessary in order to place the discussion on a sound foundation. Their value is not limited to this subject, however, for they are involved in the theory of the motions of every body in the heavens and on the earth as well. The importance of these so-called laws (Art. 3) in interpreting astronomical and physical phenomena can not easily be overestimated.

The laws of motion, essentially as Newton gave them in the *Principia*, are as follows:

LAW I. *Every body continues in its state of rest or of motion in a straight line at constant speed, unless it is compelled to change that state by an exterior force acting upon it.*

LAW II. *The rate of change of motion of a body is directly proportional to the exterior force acting upon it and inversely proportional to its mass, and the change of motion takes place in the line in which the exterior force acts.*

LAW III. *To every action there is an equal and opposite reaction; or, when one body exerts a force upon another body, the latter exerts an equal and opposite force upon the former.*

The first law is the law of inertia. It states the important fact that if a body is at rest it will never begin to move unless it is acted upon by some exterior force; and that if it is in motion, it will forever move in a straight line with uniform speed unless it is acted upon by some exterior force. In ancient times it was supposed that it is natural for bodies to fall toward the earth; that is, in those days it was a law of motion that bodies fall. Moreover, until the time of Galileo (1564–1642), who was an immediate predecessor of Newton, it was believed that bodies in motion gradually stop unless they are acted upon by some exterior force to keep them going.

It would be erroneous to suppose that the laws of motion are wholly free from difficulties. In the first law, for example, one may inquire how we know when a body is at rest. A large rock seems to be at rest, but it is moving at high speed with the earth. The earth revolves around the sun which seems to be at rest, but the sun is moving swiftly with respect to the stars. The center of gravity of our galaxy might be assumed to be at rest, but actually it moves with enormous speed with respect to other galaxies.

An alternative formulation of the second law of motion is: The rate of change of the product of the mass of a body and its velocity is proportional to the exterior force acting upon the body and is in the direction in which the force acts. When the exterior force is zero, the second law reduces to the first. If two or more forces act simultaneously, each produces its effects independently of the others, and the combined effect is given by the parallelogram law. For example, if a force is acting on a body at the point P, Fig. 48, in the direction PA and in magnitude is proportional to the segment PA; and if another force acts in the direction PB and is proportional to the segment PB, then the

two forces are equivalent to the one represented by the diagonal *PC* of the parallelogram. This is the law of composition of velocities; and since forces produce velocities, forces are compounded in the same way.

The first two laws of motion pertain to the motion of a single body; the third expresses the way in which two bodies change the motion of each other. When one body exerts a force upon another in any way whatever or by any means whatever, the latter exerts an equal contrary force upon the former. If a man, for example, takes a child by the hand and pulls, the child pulls equally in the opposite direction. The pull may be enough to move the child and not move the man, for the child is more easily moved than the man, but they pull equally.

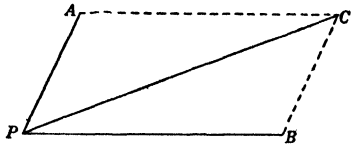


FIG. 48.—The parallelogram of forces.

Although the laws of motion can not be formulated in terms of completely known elements and can not be absolutely tested, they express very important truths and are of the highest value. As will be seen, they lead to interpretations of the motions of the heavenly bodies that for precision and completeness have no parallel in any other domain of science.

94. Rotation of the Earth Proved from Its Oblateness. The proof of the earth's oblateness by measurements of arcs of latitude (Art. 68) is purely geometrical and makes no use of its rotation. Therefore, to use the fact that the earth is oblate in demonstrating its rotation is not reasoning in a circle.

It follows from the laws of motion and the law of gravitation (Art. 173) that the earth would be spherical if it were not rotating, and that it will be oblate if it is rotating. The first law of motion, in particular, leads to the conclusion that there is a tendency for matter to fly out away from the axis of rotation, and the result is that a rotating body becomes oblate unless it is absolutely rigid. It is possible to determine mathematically the oblateness the earth should have for any given rate of rotation, and the measurements of arcs show that the actual flattening of the earth agrees precisely with the theoretical. We have, therefore, a sound proof based on the laws of motion that the earth rotates.

Since the shape of the earth at present is that which it would

have if it were fluid, the conclusion has been sometimes drawn that its rate of rotation has not changed appreciably since it was fluid (if, indeed, it has ever been fluid). At first thought, the present high rigidity of the earth would seem to justify this conclusion. Nevertheless, the argument is fallacious, for the larger a body is the greater is the rigidity necessary to maintain its shape against forces that depend upon its mass. In spite of the high rigidity of the earth, if its rate of rotation should change as much as 10 per cent it would take the corresponding form of equilibrium almost as freely as it would if it were wholly liquid.

95. Rotation of the Earth Proved by the Eastward Deviation of Falling Bodies. Scientists are not satisfied with a theory unless it is supported by several lines of evidence. Hence other proofs that the earth rotates will be outlined.

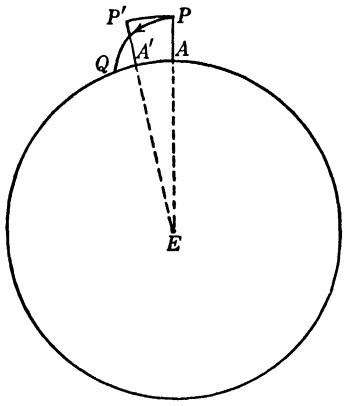


FIG. 49.—Eastward deviation of falling body.

Let P, Fig. 49, represent a point above the surface of the earth from which a ball may be dropped, and suppose that if the ball is dropped it will strike the earth after an interval of time t. Let PEP' represent the angle through which the earth rotates (assuming for the moment that it rotates) in the interval of time t. If the ball is not dropped, it will move forward with the point of suspension and arrive at the point P' after the interval t. The ball has a motion in the direction PP' and, in accordance with the first law of motion, it tends to continue moving in the same direction with constant speed even though the attraction of the earth at right angles to this direction causes it to fall to the surface of the earth. The ball strikes the surface at a point Q east of the point A' directly under P', for the distance PP' is greater than AA'.

The foregoing explanation of the eastward deviation of falling bodies is not quite complete. The attractive force which acts on the falling body always acts in a line passing through the axis of the earth and, of course, nearly toward the center of the earth. It can be proved mathematically from the laws of motion that under

these circumstances the moment of momentum (Art. 100) of the body with respect to the earth's axis remains constant, and that the eastward deviation of falling bodies is somewhat greater than would be inferred from the argument just advanced. Yet in latitude 40° the eastward deviation for a fall of 100 meters (328 feet) is only about 1.6 centimeters (.62 inch).

96. Other Phenomena Depending Upon the Rotation of the Earth. Other observed phenomena which also prove that the earth rotates include:

(*a*) *Deviations of air currents.* Suppose a mass of air is driven along the surface of the earth by differences in barometric pressure from the point *A*, Fig. 50, northward toward the point *B*. As the air arrives at the latitude of the point *B*, its distance from the axis of the earth is considerably less than was its original distance; and since its moment of momentum remains constant (neglecting any forces acting upon it in an east-west direction), it moves eastward faster than the surface at *B*. That is, the mass of air moves northeastward with respect to the surface of the earth.

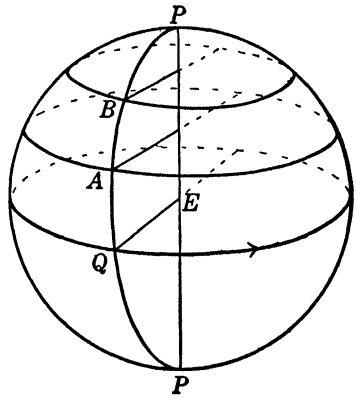


FIG. 50.—Deviations of air currents.

If a mass of air should move from the point *B* southward toward the point *A*, it would lose eastward velocity and, consequently, with respect to the surface of the earth, it would be deflected westward. That is, in the northern hemisphere there is a tendency for air currents to deviate to the right of the direction of motion; and, for similar reasons, in the southern hemisphere, there is a tendency for air currents to deviate toward the left.

The general circulation of the earth's atmosphere in trade winds and anti-trade winds conforms to the foregoing requirements of the theory that the earth rotates. The same is true of the great ocean currents.

There is another atmospheric phenomenon which is also due to the rotation of the earth. When the density of the lower strata of the atmosphere is abnormally low because of high temperature

and large water vapor content, a condition of instability exists. This condition is broken up sooner or later by an uprush of the heated layers from some limited area, accompanied by air flowing from all sides toward the region of the ascending currents. Each of the inflowing streams tends to deviate to the right (in the northern hemisphere) of the center of the uprush. The result is that a vortex is developed in which the motion is counterclockwise in the northern hemisphere (Fig. 51) and clockwise in the southern hemisphere.

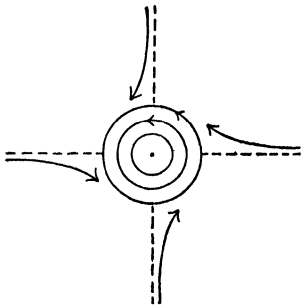


FIG. 51.—Counterclockwise cyclones in northern hemisphere.

(*b*) *Action of the gyro-compass.* Every student of physics has observed the behavior of a gyroscope when its axis is rotated. If an attempt is made to rotate the axis of a gyroscope about a second axis at right angles to its axis of rotation, the wheel resists the applied force and the gyroscope rotates about a third axis at right angles to the other two. All of this is completely in harmony with the laws of motion and there is no mystery about it whatever, contrary to statements sometimes made.

Suppose now that the axis of the wheel of a gyroscope is horizontal and in an east-west line. The rotation of the earth will cause this axis to rotate around a horizontal north-south line, and this will result in a rotation of the axis of the gyroscope about a vertical line. If, however, the axis of the gyroscope is initially in a north-south line and on the earth's equator, the rotation of the earth will only displace the axis of the gyroscope parallel to itself and no rotation of its plane will result. In other latitudes the problem is not quite so simple, but the result is similar if only the axis of the gyroscope is constrained to remain in the horizontal plane.

Obviously a gyroscope of the type described can be used for a compass. As compared with the magnetic compass, it is wholly independent of the changing of the magnetic pole and of local magnetic disturbances produced by masses of iron. The behavior of the gyro-compass and the success in its use are consequences of the earth's rotation; or, conversely, its motion proves that the earth rotates. Other adaptations of the gyroscope, depending upon the rotation of the earth and the position of its axis, can be made to give the latitude of the observer.

(*c*) *The Foucault pendulum.* In 1851, the French physicist, J. B. Foucault gave a very simple and convincing proof of the rotation of the earth by suspending a heavy ball from a long wire so that it could swing freely in any plane. If the point of suspension were directly over one of the poles of the earth, evidently the pendulum would continue to swing in one plane while the earth would rotate under it. To an observer on the surface the earth would appear stationary and the plane of the pendulum would appear to rotate from east to west. At the earth's equator there would be no apparent rotation of the pendulum. In intermediate latitudes the plane of the pendulum would apparently rotate in a period varying from a day at the poles to infinity at the equator. In fact, from the period of rotation of the plane of the pendulum the latitude of the observer could be determined.

In intermediate latitudes where the period of rotation of the pendulum is about 40 hours, the change in its plane can be detected in a few minutes, particularly if its position is observed through a surveyor's telescope.

(*d*) *The Compton circular tube.* In 1915, A. H. Compton devised a simple laboratory method of demonstrating the rotation of the earth. The apparatus consists of a circular tube filled with water. Suppose such a tube is placed in a vertical east-west plane and is then rotated 180° about a horizontal east-west line. After the rotation of the tube the water in the lower part will have a tendency to flow eastward, because it has been brought down from a place of higher eastward speed to one of lower eastward speed. On the contrary, the water in the upper part of the tube will have a tendency to flow westward for the opposite reason. The result will be a very slow circulation of the water in the tube, westward at the top and eastward at the bottom, which can be observed if the tube is transparent or has a transparent section.

97. Precession of the Equinoxes. In about 120 B.C., Hipparchus discovered that the points at which the sun crosses the celestial equator are not fixed but continually regress in right ascension. This *precession of the equinoxes*, as it is called, is not due to changes in the apparent path of the sun among the stars, but to shifts in the position of the celestial equator. There are, of course, corresponding changes in the positions of the celestial poles, which are 90° from the celestial equator. It was remarked in Art. 24 that in

about 12,000 years the north celestial pole will have moved near to the star Vega.

The cause of the precession of the equinoxes remained a mystery for nearly 2000 years until Newton explained it in his *Principia*, in 1687. As will be seen, the explanation depends upon such a great variety of phenomena and scientific principles that the wonder is not that it remained so long unknown, but that it was discovered at all.

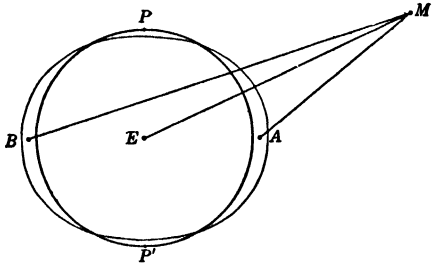


FIG. 52.—Attraction of moon for the equatorial bulge of the earth produces the precession of the equinoxes.

Although the complete theory of precession of the equinoxes is very difficult, its general nature can be indicated in non-technical terms. Let E, Fig. 52, represent the earth, with its equatorial bulge greatly exaggerated, and let M represent the moon. The moon has a greater attraction per unit mass for the bulge A than it has for the spherical part of the earth E. Consequently, the attraction of the moon tends to bring the bulge A up into the line EM. Similarly, the moon has a greater attraction, per unit mass, for E than it has for the bulge B. Therefore, the attraction of the moon tends to bring E up into the line BM; or, to bring B down into the line EM extended backward.

Since the earth is rotating, it is a sort of gyroscope. When the moon tends to change the plane of the earth's equator, it sets up a precession of this plane without changing the angle between it and the line EM. In order to assist in visualizing the facts, let the plane of a table represent the plane of the moon's orbit and hold a sheet of paper inclined to this plane by about 23°.5. Suppose the moon revolves counterclockwise in its plane. Then rotate the line of the intersection of the paper with the plane of the table in the opposite direction, keeping the angle between the two planes constant. The motion corresponds to the effects produced by the moon upon the plane of the earth's equator.

The foregoing description falls far short of giving an idea of the great complexity of precession. In the first place, the moon moves from one side to the other of the plane of the earth's equator each month, being once at the point M, Fig. 52, and about two weeks

later in the line *ME* extended in the opposite direction from the earth. The precessional effect is greatest when the moon is farthest north or south of the plane of the earth's equator, and it vanishes twice each month when the moon crosses the equator. It is a difficult problem to take accurate account of these varying effects, particularly as the distance of the moon from the earth continually changes. In the second place, the sun also causes precession. It, too, is sometimes north of the plane of the earth's equator and sometimes south of it, but the cycle of its changes is a year. Finally, the sun and the moon do not remain in the same plane. The genius of Newton is evident from the fact that he not only discovered the laws of motion and the law of gravitation upon which the explanation of the precession of equinoxes depends, but he threaded his way through its intricate mazes with unerring steps and arrived at a result agreeing with the observations.

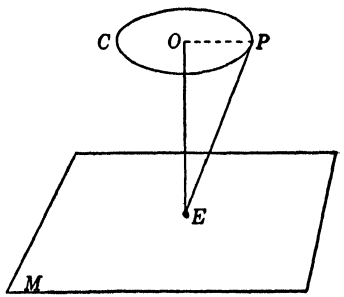


FIG. 53.—Precessional cone described by the axis of the earth.

The mass of the whole earth is very great in comparison with that of its equatorial bulge, the turning forces due to the moon and the sun are relatively small, and the rate of rotation of the earth is rapid. The result is that the precession of the equinoxes is small, amounting to only 50″.2 per year and to a complete rotation in about 25,800 years.

98. Nutation. In spite of the fact that the mass of the moon is small compared to that of the sun, our satellite is so near the earth that its precessional effects are about 2.4 times those of the sun. Consequently, the principal precessional motion of the earth's equator is with respect to the orbit of the moon. More simply stated, the axis of the earth tends to describe a cone about a line perpendicular to the plane of the orbit of the moon with an angular distance from it averaging about 23°.5.

In Fig. 53, *M* represents the plane of the orbit of the moon, *E* the position of the earth, *EP* the axis of the earth prolonged, *EO* a line perpendicular to the plane of the orbit of the moon, and *C* the precessional cone described by the axis of the earth.

A complexity enters as a consequence of the fact that the plane

of the orbit of the moon is not fixed. Let S, Fig. 54, represent the plane of the orbit of the earth and M that of the moon, the angle between the two planes being 5° 9′. The attraction of the sun for the moon causes the line of intersection AB of these two planes to rotate opposite to the direction of revolution of the moon, making a complete rotation in about 18.6 years. Consequently, the line EO, Fig. 53, describes a cone whose half-angle is 5° 9′. The result is that the curve C is not a circle but a slightly wavy line, each wave having an amplitude of 9″.2 each side of the curve C and a period of 18.6 years. These small oscillations of the axis of the earth are called *nutation* (nodding).

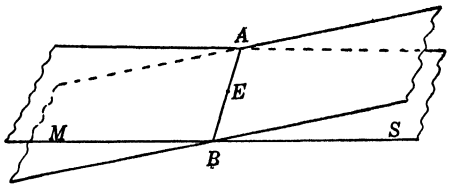


FIG. 54.—Inclination of moon's orbit to the ecliptic gives rise to nutation.

Nutation was discovered by James Bradley (1693–1762) from observations made during the period from 1727 to 1746. The cause of nutation was first worked out by d'Alembert (1717–1783), a famous French mathematician. No words can give an adequate conception of the intricacy or the beauty of the mathematical theory of nutation.

99. Variation of Latitude. In the phenomena of precession and nutation the axis of the earth describes cones in space, but it remains fixed in the earth. In the phenomena under consideration at present the axis of the earth moves in the earth itself, and consequently latitudes of points on its surface vary.

It is often supposed that the axis of rotation must remain fixed in the earth. This, however, is not the case, for the geometric axis of an oblate body may oscillate about its axis of rotation in a complicated way, depending on the distribution of its mass, its rate of rotation, and the way it is started. The period of such an oscillation for the earth was first determined by Euler (1707–1783), a great Swiss mathematician who laid the foundations of the theory of rotating bodies. Under the assumption that the earth behaves like a *perfectly rigid body*, he found that the period of its oscillation is 305 days.

Observations failed to reveal any variations in latitude until 1888, when F. Küstner and S. C. Chandler discovered indications

of such phenomena which were so minute as to be on the border line of determination. They found a period of variation of 428 days instead of 305 days, and consequently at first their conclusions were thought to be erroneous. Their results have been verified, however, by observations of great accuracy made in various longitudes around the earth. Fig. 55 shows the wanderings of the pole in the earth from 1906 to 1913. This complicated curve is a compound of a circle 8 meters in diameter, described in 433 days, and an elongated ellipse 9 meters in length, described in a period of a year. At its maximum the pole departs from its mean position by only 18 meters.

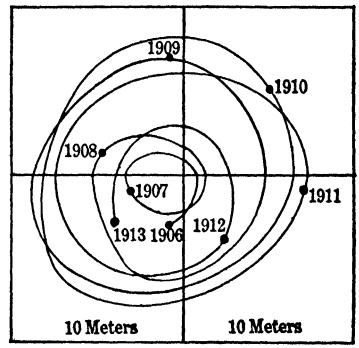


FIG. 55.—Wandering of the earth's pole from 1906 to 1913.

Simon Newcomb and others proved that the observed period of 433 days instead of the expected 305 days is due to a lack of perfect rigidity of the earth. They found, however, that in order for theory to agree with observations the rigidity must be as great as that of steel, agreeing substantially with results subsequently obtained from earthquake phenomena and tidal observations. The annual period is undoubtedly due to some annual cause, such as variations in air currents or the deposition of large masses of snow and ice in high latitudes. That is, the annual period is what is called a *forced* period, while the 433-day period is the *free* period for the earth. The mathematical theory of the subject shows that the forced oscillations can generate indirectly an oscillation with the free period and of the same amplitude. But the oscillations with the free period will be damped out if there is friction, unless they are constantly renewed. Consequently, we are forced to the conclusion that some annual phenomenon, at present not certainly known, produces the forced oscillations which, in turn, continually generate the oscillations in the free period. Although the variation of latitude is small, the shifting of enormous masses would be necessary to displace the earth's axis by the observed amount.

It should not be inferred from the present small variations in latitude that there has ever been any large and prolonged

displacement of the earth's axis with resulting radical changes in climate in certain regions of the earth. The actual variations are oscillatory with short periods and produce no such results. It is improbable that the position of the earth's pole has changed to an important extent during all geological history.

100. On the Uniformity of the Earth's Rotation. There are two possible causes of variation in the rate of rotation of the earth: (*a*) changes in its size or in the distribution of its mass, and (*b*) the action on it of exterior forces.

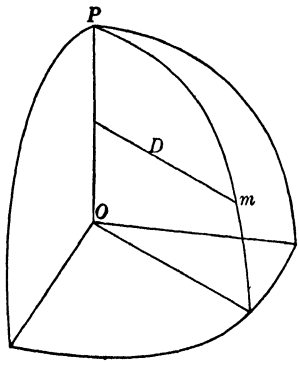


FIG. 56.—Moment of momentum.

(*a*) It can be shown from the laws of motion that if a body is not acted upon by exterior forces, the total quantity of its rotation remains constant. The quantity of rotation of a body, or its *moment of momentum*, as it is technically called, is the sum of the rotations of all its parts, where the quantity of rotation of a single part is the product of its mass m, its distance D from the axis of rotation, and the speed v with which it moves at right angles to the line from the axis. Thus, in Fig. 56, the moment of momentum of the mass m is mDv. The formula for this dynamical quantity for a body of radius R is $M = CmR^2\omega$, where ω is the rate of angular rotation and C is a number depending upon the shape of the body and the distribution of its density.

Since M is constant when a rotating body is subject to no external forces, it follows from the formula that if R decreases, ω increases. Therefore, if the earth loses heat and shrinks, it rotates faster and becomes more oblate, unless these effects are counterbalanced by the action of exterior forces. On the other hand, if the earth should expand as a consequence of the heat generated by radioactive elements (Art. 78) or for any other reason, its rate of rotation would decrease and it would become less oblate.

(*b*) The earth is subject to various exterior forces, most of which do not affect its rotation. For example, the sun's attraction does not change the rotation of the earth except through the tides which it raises, because the line of its action is through the earth's center.

But the earth in its revolution around the sun passes through a region in which there are vast numbers of meteors, several millions of which it encounters daily (Art. 245). These meteors act as a resisting medium and retard the rotation of the earth, but the earth is so large that they can not alter the length of the day by a second in a million years.

The moon and the sun raise tides in the body of the earth and also in the water and the atmosphere on its exterior. The interior of the earth is elastic (Art. 81), at least for disturbances of such short periods as those of the tides, and consequently the tides in the body of the earth do not have sensible effects upon its rotation. But there is friction in the ocean tides, particularly where they beat upon the shores or surge and eddy in shallow seas. When there is friction in the tides with resulting generation of heat, which is eventually radiated away into space, the system of the tidally-distorted and the tide-generating body loses energy. In the case of the earth-moon system, the loss of energy comes mostly at the expense of the rotation of the earth and to a lesser extent at the expense of the revolution of the moon. Estimates of the effects of the ocean tides on the rotation of the earth indicate that the length of the day is probably now decreasing a second in approximately 150,000 years. At least this is the case if other factors that might alter the earth's rate of rotation do not have appreciable effects. The tides are doing work at the rate of approximately 14×10^8 ergs per second (1,877,000,000 horsepower).

It is not easy to determine observationally whether the rate of rotation of the earth is undergoing slight variations, for it is impossible to construct a clock accurate enough to detect a change in the length of the day of one second in 150,000 years, or even in 120 years. But the rotation of the earth may be compared with other celestial motions such as the revolutions of the planets and the moon. Since the measurement of time depends upon the rotation of the earth (Art. 114), it follows that if the earth's rate of rotation varies, the apparent motions of all the observed bodies will all vary in the opposite sense; that is, if the earth gradually rotates more slowly, the other bodies will appear to move more rapidly and all will gain at the same rate.

Of all the celestial motions, that best adapted to showing the existence of a retardation in the rotation of the earth is the revolution of the moon. Eclipses of the sun occur only when the moon

is precisely between the earth and the sun, and since records of eclipses are available for a period of more than 2000 years, the effects of changes in the length of the day have greatly accumulated in the interval. Although there are some uncertainties in the data, apparently the angular velocity of the moon about the earth is increasing at the rate of 9 seconds of arc per century. If this apparent acceleration is due to a retardation of the rotation of the earth, it corresponds to a decrease in the length of the day of one second in 100,000 years.

E. W. Brown explains certain minute discrepancies between his exhaustive theory of the motion of the moon and observations of its actual motion by the hypothesis that from time to time the rate of rotation of the earth undergoes very slight and somewhat irregular variations as a consequence of changes in its moment of momentum due to shifts in its mass, possibly a contraction or an expansion. A local displacement of matter would be inadequate to produce the observed effects. Even if the whole earth should expand or contract, a variation in its radius of a meter would be necessary, but there are no known forces which could cause such displacements. From these theoretical and observational considerations it may be safely concluded that at present the length of the day is not changing more than one second in 100,000 years. Although the effect of such a change is unimportant in a few million years, it would be important in the long ages of geological time. One might infer from this line of reasoning that the earth was rotating much more rapidly and consequently was much more oblate in the earliest geological periods than it is now, but there is no geological evidence to support this conclusion.

X. QUESTIONS

1. Verify the diurnal motions of the stars.
2. Is there any proof of the rotation of the earth that does not depend upon the laws of motion?
3. Will the deviations of air currents for a given change in latitude be greater near the earth's equator or near its poles?
4. Will an uprush of air deviate toward the east or toward the west?
5. If there is an uprush of air in one region, there must be a corresponding flowing out over the region and a descent at some distance around it. What will be the character of the vortical motion, if any, of the descending currents of air.
6. Make the experiment with a Foucault pendulum.

7. Which of the phenomena enumerated in Art. 95 are not observable at the earth's poles? At the equator?
8. Would precession be the same if the earth rotated in the opposite direction?
9. Which of the proofs of the rotation of the earth also determines the direction of its rotation?
10. If movements of ocean currents and air currents are for a time not fully compensated by the motions of other currents, do they affect the rotation of the earth?
11. How many cycles of nutation are there in a precessional period?
12. Would the precession be the same if the moon revolved about the earth in the opposite direction?
13. Would the nutation be the same if the moon revolved about the earth in the opposite direction?
14. Are those international boundaries that are defined by latitude and longitude circles fixed on the surface of the earth?
15. If the earth should shrink to a considerable extent, what would be the effect upon its rate of rotation? Its oblateness? The distribution of its land and water?
16. Are there any exterior forces acting on the earth that tend to increase its rate of rotation?
17. At the rate of the present probable maximum change in the rate of rotation of the earth, how much would the length of the day have changed during geological time?

II. The Revolution of the Earth

101. Apparent Motion of the Sun with Respect to the Stars. As was explained in Art. 13, the sun has an apparent eastward motion among the stars, completing a circuit of the celestial sphere in a year. The problem now is to determine what actual motion causes this apparent motion. If we consider nothing except the apparent motion of the sun, there are two explanations that agree equally well with the observed phenomena, as was also the case with the diurnal motions of the stars.

A natural explanation of the apparent annual motion of the sun is that it revolves around the earth. In Fig. 57, let E represent the position of the earth, assumed fixed, and C the path of the sun around the earth. When the sun is at the point S_1, it appears to be among the stars at S_1'; or, since stars can not be seen in its vicinity, it appears to be 180° from the stars at S_1''. When the sun has advanced to the point S_2, it is seen in the direction S_2' and opposite to the stars at S_2''. It is clear that it would apparently revolve around the sky, in complete harmony with the observations.

In the second theory, illustrated in Fig. 58, the sun is assumed

to be fixed at S and the earth to revolve around it. When the earth is at the point E_1, the sun is apparently among the stars at S_1'. When the earth has advanced to the point E_2, the sun is apparently among the stars at S_2'. If the stars are very distant in comparison

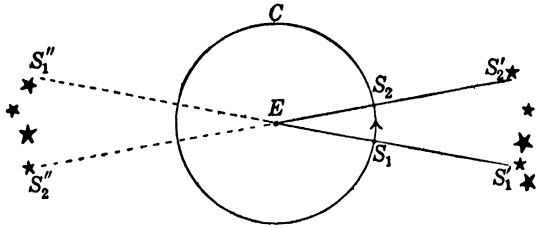


FIG. 57.—The hypothesis that the sun revolves around the earth explains the apparent eastward motion of the sun with respect to the stars.

with the sun so that the revolution of the earth does not change their apparent brightness, this theory explains the apparent motion of the sun as well as the other.

The ancients almost universally adopted the former theory to explain the motion of the sun with respect to the stars because the earth is evidently very great and apparently stationary, while the sun is apparently small and moving. The phenomena so far considered simply prove that there is a relative motion of the earth and sun, without indicating which of the two bodies moves. To determine which of the two theories is correct it is necessary to make use of something quite independent of the earth and sun.

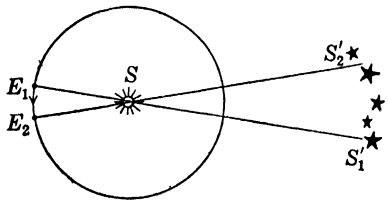


FIG. 58.—Heliocentric theory of apparent motion of the sun.

102. Revolution of the Earth Proved from the Laws of Motion (1686). Although Copernicus published a book on the motions of the sun and the planets in 1543, in which he showed that all the changing relations of these bodies can be most simply explained by the theory that the sun is the center of the system, the first actual proof of the fact was given by Newton, in his *Principia*, in 1687. Newton proved that it follows from the laws of motion that two mutually interacting bodies revolve around their center of gravity. Since it was known that the volume of the sun is enormously greater than that of the earth, it followed that, unless the

sun is extraordinarily tenuous, the center of gravity of the earth and sun is near the center of the sun, and that both the sun and the earth revolve about this point, the sun in a small orbit and the earth in a large orbit.

The conclusion as stated is correct except for slight modifications due to the presence of the other planets. The center of gravity of the two bodies is only about 450 kilometers (280 miles) from the center of the sun, and for most purposes the sun may be regarded as fixed and the earth as revolving around it.

103. Revolution of the Earth Proved by the Aberration of Light (1728). In 1728, James Bradley made a discovery that not only proves the revolution of the earth but also gives the size of its orbit. During the preceding three years, he had found that the stars apparently make small oscillations about their mean positions in the period of a year. He had expected a parallactic effect (Art. 104) which would depend upon the distance of a star, but the amplitudes of the observed displacements were the same for all stars. Consequently he was compelled to seek another explanation, which he found in *aberration*, a phenomenon depending only upon the velocity of light and the velocity of the earth in its orbit and not upon the distance of the star that is observed.

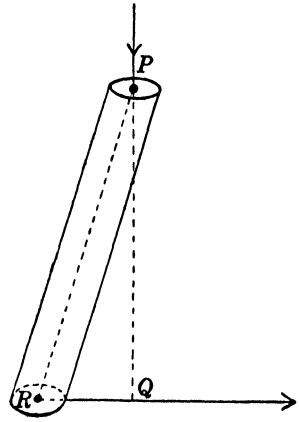


FIG. 59.—Explanation of aberration of light.

In order to illustrate the principle of aberration, suppose rain is falling vertically and that a person is standing still in it. If he holds a tube vertically, rain drops will fall directly through it without touching its sides. Suppose, however, that he is walking forward. Then the rain will appear to him to descend somewhat obliquely from the direction in which he is moving. Moreover, it will be necessary for him to tilt the top of the tube forward in the direction in which he is moving in order that the drops of rain shall fall through it without touching its sides. It is obvious that the faster he goes the more he will have to tilt the tube, and the greater the velocity of the falling drops the less he will have to tilt the tube.

In Fig. 59, the drop of rain enters the tube at the point P and emerges at the point Q. The tube must be tilted enough so that, with the velocity at which it is moving, its base R will arrive at the point Q at the same time that the rain drop arrives at Q.

If a represents the angle RPQ, v the velocity of fall of the rain drop, and V the horizontal velocity of the tube, then it follows from the relations among these quantities that (with a expressed in seconds) $a = 206{,}265\,V/v$, for there are 206,265 seconds in an arc of a circle equal to its radius.

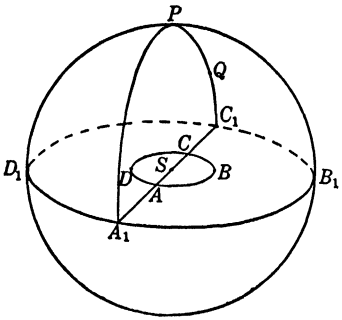


FIG. 60.—Ecliptic A_1, B_1, C_1, D_1, and orbit of earth A, B, C, D.

In aberration, light from the stars takes the place of the falling rain and the motion of the earth that of the observer on the surface of the earth. The angle a is the aberration, which is observed, V is the velocity of the earth in its orbit, and v is the velocity of light, which has been measured. Consequently, the formula gives V from which the perimeter of the earth's orbit and the distance from the sun to the earth can be determined. The angle of aberration is 20″.47 and the velocity of light V is about 300,000 kilometers (186,000 miles) per second, from which it follows that the velocity of the earth v is about 29.76 kilometers (18.5 miles) per second.

There is one important difference between the apparent displacement of the direction of falling rain and the aberration of light. The observer moves at right angles to the falling rain, but the earth does not move during its whole revolution at right angles to the light received from all stars. In Fig. 60, let A, B, C, D represent the orbit of the earth and A_1, B_1, C_1, D_1, the intersection of its plane with the celestial sphere. The light received from a star at the pole P of the circle A_1, B_1, C_1, D_1 moves perpendicularly to the direction of motion of the earth, wherever it may be in its orbit. Since the star is apparently displaced in the direction in which the earth moves, the aberrational orbit described by the star has the characteristics of the curve (P), Fig. 61, in which the star appears to be at the points a, b, c, and d when the earth is at the points A, B, C, and D, respectively, of Fig. 60. The true position of the star is near the center of the curve.

Now consider a star at the point Q, Fig. 60, midway between the ecliptic and its pole. When the earth is at the point A it moves at right angles to the direction of the light received from the star, and, consequently, the aberration is the full 20″.47. But when the earth is at the point B, it does not move at right angles to the direction of the light received from the star, and the aberration is reduced. For a star at Q the aberrational curve is an ellipse (Art. 106) of the type illustrated by (Q), Fig. 61. If the

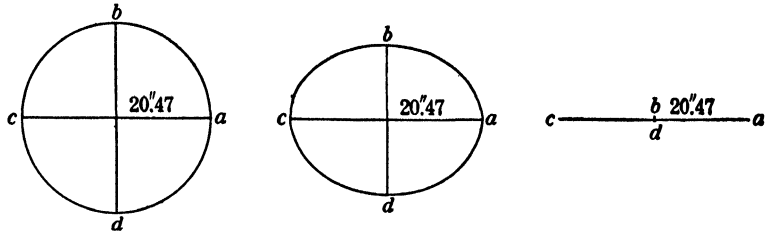


FIG. 61.—Aberrational orbits of stars at P, Q, and C_1, Fig. 60.

star were midway between the points P and B_1, Fig. 60, the aberrational ellipse would be elongated in the direction bd. If the star were at the point C_1, its aberrational curve would reduce to the straight line illustrated in (C_1), Fig. 61.

There is, similarly, a small aberration due to the earth's rotation, which, for a point on the earth's equator, amounts at its maximum to 0″.31.

104. Revolution of the Earth Proved by the Parallax of Stars (1840). It is obvious that if the earth revolves around the sun, the stars will be seen in slightly different directions at different times of the year. Since the ancients could not observe such displacements, they rejected the theory that the apparent annual motion of the sun is due to the revolution of the earth. Their failure was due to the fact that the distances of even the nearest stars are enormous in comparison with the diameter of the orbit of the earth. The difference in direction of the nearest star (Alpha Centauri) when observed from opposite sides of the earth's orbit is about as small as the difference in direction of an object at a distance of 9 kilometers (5.6 miles) when viewed first with one eye and then with the other. Yet the parallaxes of many stars have now been measured, and each of them constitutes a proof that the earth revolves around the sun.

It is necessary for later use to define precisely what is meant by the parallax of a star. Let S, Fig. 62, represent the position of the sun and A that of a star. When the earth is at the point E_1 the star is observed in the direction E_1A. A few months later, when the earth has arrived at the point E_2, the star is observed in the direction E_2A. If the line E_2B is parallel to the line E_1A, the apparent displacement of the star as a consequence of the earth's motion during the interval between the observations is the angle AE_2B, which equals the angle E_1AE_2.

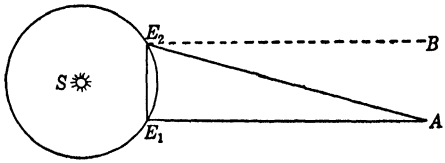


FIG. 62.—Parallax of a star.

If the distance E_1E_2 equals the mean distance from the sun to the earth and if the line E_1E_2 is perpendicular to the line from S to A, then the angle E_1AE_2 is the parallax of the star. In other words, the parallax of a star is the angle subtended at the distance of the star from the sun by a line equal to the mean distance from the sun to the earth. Of course, in measuring the parallax of a star, the observations are made whenever it is convenient to make them, and the final results are reduced to standard conditions.

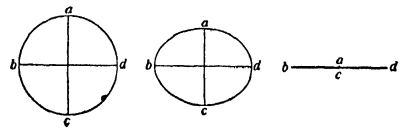


FIG. 63.—Parallactic displacement of stars.

If a star is located at the pole of the ecliptic, Fig. 60, its parallactic curve is similar in shape to the orbit of the earth with the phase relationship indicated in (P), Fig. 63. Unlike the aberrational displacement, the radius of the circle depends upon the distance of the star, but in all cases it is very small. If a star were located at the point Q or C_1, Fig. 60, its parallactic curve and phase relationship would be as illustrated in (Q) or (C_1), respectively, Fig. 63. It will be noted on comparing the curves in Fig. 61 with those in Fig. 63 that the aberrational and parallactic displacements differ by 90° in phase.

105. Revolution of the Earth Proved by the Spectroscope (1890). There is a final direct proof of the revolution of the earth around the sun which is based upon the fact that the spectroscope (Art. 276) is an instrument with which an observer can determine the rate

at which he relatively approaches or recedes from a gaseous source of light.

Suppose that when the earth is at the point A, Fig. 60, a star is in the direction B_1, and that it is approaching or receding from the sun with the velocity v, the direction of motion being indicated by the sign of v. At A the earth is approaching the star with the velocity V. Hence the relative velocity of the star and the earth, as measured with the spectroscope, is $v+V$. When the earth has revolved to the point C_1, the measured relative motion of the star and the earth is $v-V$. Since $v+V$ and $v-V$ are known, they both can be computed and then the dimensions of the orbit of the earth can be determined.

In practice, matters are not quite so simple as the foregoing discussion would indicate. In general, stars are not in the plane of the earth's orbit and the relative velocity of the star and the earth is consequently reduced; in fact, a star at the pole P of the ecliptic, Fig. 60, can not be used to determine the orbital velocity of the earth. Then, too, in general a star will not be observed at times separated by an interval of six months. But all these variations from ideal conditions introduce no fundamental difficulties, and allowance can be made for them.

The spectroscopic method of proving the revolution of the earth is independent of the distance of the stars, provided they are near enough to give the necessary amount of light. There are thousands of stars which are bright enough for the purpose and the *radial velocities*, as they are called, of thousands of them have been measured.

106. Shape of the Orbit of the Earth. It has been tacitly assumed in the preceding paragraphs that the orbit of the earth is a circle with the sun at its center. If this assumption were true, the apparent diameter of the sun would be constant. Observations show, however, that it varies from a maximum of $32'\ 36''$ early in January to a minimum of $31'\ 30''$ about the first of July. Since the apparent diameter of the sun varies inversely as its distance, it follows that the earth is nearest the sun at the beginning of January and farthest from it in July.

If the angles through which the sun apparently moves day by day with respect to the stars are laid down on drawing paper, and if distances are marked off on these lines which are inversely

proportional to the apparent diameter of the sun, the points obtained define the shape of the orbit of the earth with respect to the sun.

Observations essentially of the kinds that have been described establish the fact that the orbit of the earth is an ellipse with the sun at one of its foci. An *ellipse* is a plane curve such that the sum of the distances from two fixed points in its interior, called its *foci*, to every point on its circumference is the same. In Fig. 64, E represents an ellipse and F and F' represent its two foci. The sum of the distances FP and $F'P$ is the same whatever the point P may be on the perimeter of the ellipse. If the points F and F' are near together in comparison to the sum of the distances from F and F' to P, the ellipse is nearly circular; if F and F' are far apart, the ellipse is elongated. The orbit of the earth is nearly circular.

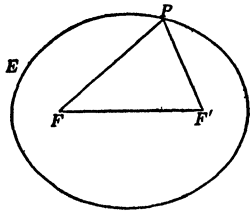


FIG. 64.—An ellipse.

107. Motion of the Earth in Its Orbit. Observations of the kinds described in the preceding article enable the astronomer to determine not only the shape of the orbit of the earth but also the way the earth moves in its orbit. Instead of revolving at uniform angular speed around the sun or at uniform linear speed along its path, the earth moves in such a way that the radius from the sun to it describes equal areas in equal intervals of time. When the motion of a body has this property with respect to a point, it is said to satisfy the *law of areas* with respect to the point.

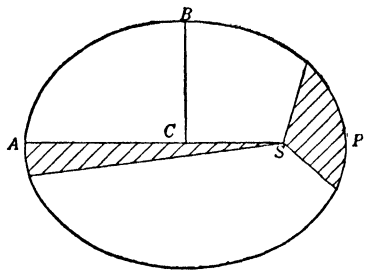


FIG. 65.—The law of areas.

In Fig. 65, the elliptical orbit of the earth is shown with its elongation greatly exaggerated. The point P nearest the sun is called the *perihelion point*, and the most distant point A is called the *aphelion point*. The radius from the sun describes the equal shaded areas in equal intervals of time. It follows that the linear velocity of the earth is greatest at the perihelion point P and least

at the aphelion point A, the ratio of the velocities being inversely as the ratio of the distances.

The degree of elongation of an ellipse is defined by its eccentricity e, which is equal to $CS \div CP$, Fig. 65. When the point C coincides with the point S, the ellipse reduces to a circle and its eccentricity is zero. As the point S approaches the point P the eccentricity approaches unity. In the case of the orbit of the earth $e = 0.01677$, from which it follows that the earth's perihelion distance is about 3 per cent less than its aphelion distance.

108. Inclination of the Plane of the Equator to the Plane of the Orbit of the Earth. The plane of the orbit of the earth is called the *plane of the ecliptic*, and the great circle in which this plane intersects the celestial sphere is called *the ecliptic*. The plane of the ecliptic is of fundamental importance as a plane of reference because it remains fixed, except for very slight and slow variations produced by the attractions of the other planets. On the other hand, the plane of the equator of the earth is subject to a precession (Art. 97) that completes a cycle in the relatively short period of about 26,000 years.

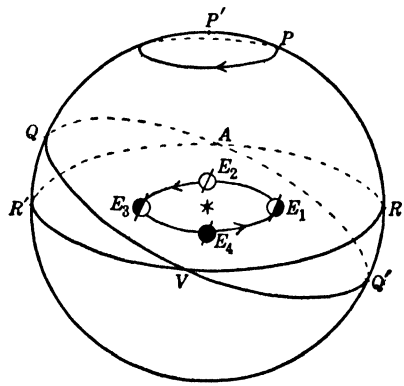


FIG. 66.—Ecliptic, celestial equator, and their poles.

In Fig. 66, the ecliptic is the circle $VRAR'$ and the celestial equator is $VQ'AQ$. The point P is the celestial pole (toward which the axis of the earth points) and P' is the pole of the ecliptic. The angle between the plane of the ecliptic and the plane of the equator is about 23°.5, and the pole P describes a circle (except for nutations) around the pole P' in the retrograde direction. The corresponding motions of the vernal equinox V and of the autumnal equinox A in the retrograde direction are known as the *precession of the equinoxes.*

109. Relation of the Ecliptic to the Celestial Equator. In spite of the fact that the ecliptic is almost exactly fixed while the equator continually changes its position with the precession of the equinoxes, it is advantageous to use the equator as a circle of

reference for most astronomical phenomena, for the celestial equator is the great circle of the diurnal motions of the celestial objects.

Since the angle between the celestial equator and the ecliptic is about 23°.5, the relation of the ecliptic to the equator may be represented as in Fig. 67. The sun apparently moves along the ecliptic eastward (from right to left when one faces south). The point V at which the sun crosses the equator from the south side to the north side is called the *vernal equinox*, for the sun is at V in the spring time (in the northern hemisphere); and when the sun

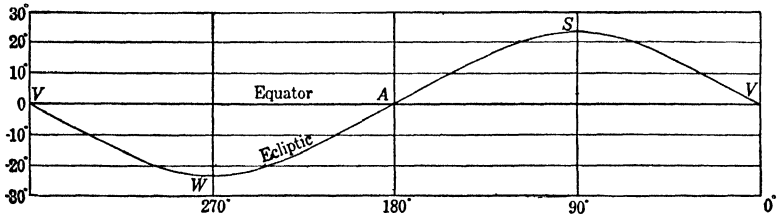


FIG. 67.—Relationship between equator and ecliptic.

is on the celestial equator the days and nights are of equal length. The sun is at the vernal equinox on March 21 (or within about a day of this date, the variation being mostly due to the leap year). The point S is called the *summer solstice*, for when the sun is at this point it ceases to move northward and for a time seems to stand still in declination. The sun is at the summer solstice on or about June 21. The point A is called the *autumnal equinox*, for the sun is at A in the autumn season (in the northern hemisphere) and the days and nights are again equal. The sun is at the autumnal equinox on or about September 23. The point W is called the *winter solstice*, for the sun stands still in declination on or about December 21.

When the earth is at the points E_1, E_2, E_3, and E_4, in Fig. 66, the sun is at the points W, V, S, and A, respectively, in Fig. 67. The annual motion of the sun is represented as being counterclockwise, as it appears to be to observers in the northern hemisphere of the earth.

110. Diurnal Circles of the Sun. The diurnal circles of the sun depend upon the sun's declination, which varies from about $+23°.5$ to $-23°5$, and upon the latitude of the observer. Fig. 68

represents the horizon coördinate system and three diurnal circles of the sun for an observer at 40° north latitude. The small circle $E_1Q_1W_1Q_1'$ is 23°.5 north of the celestial equator, and the small circle $E_2Q_2W_2Q_2'$ is 23°.5 south of the celestial equator. When the sun is at the point V or A, Fig. 67, its diurnal circle is the celestial equator. The sun, of course, is exactly on the celestial equator only for an instant, but its diurnal circle on a day on which it is at an equinox differs only a little from the celestial equator.

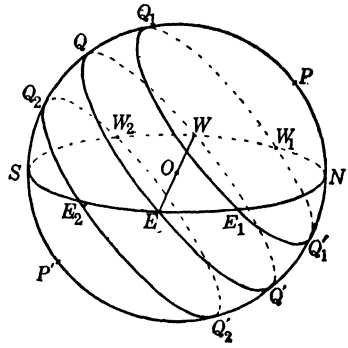


FIG. 68.—Diurnal circles of sun.

When the sun is at the point S, Fig. 67, its diurnal path is the small circle $E_1Q_1W_1Q_1'$, Fig. 68; and when it is at the point W, Fig. 67, its diurnal path is the small circle $E_2Q_2W_2Q_2'$, Fig. 68. When the sun is at other points in Fig. 67, its diurnal paths are at corresponding places between the extreme curves of Fig. 68. It is understood, of course, that the diurnal curves of the sun are not strictly circles but spirals leading slowly from one declination to another.

When the sun is at an equinox, it rises in the east at E, Fig. 68, and sets in the west at W, whatever may be the latitude of the observer. Since the horizon and the celestial equator are great circles, they bisect each other and consequently when the sun is on the celestial equator the days and nights are equal to each other for all points on the earth. When the sun is at the summer solstice S, Fig. 67, it rises in the northeast at E_1, Fig. 68, crosses the meridian at Q_1 (altitude $50°+23°.5=73°.5$), sets in the northwest at W_1, and crosses the anti-meridian at midnight at Q_1 (altitude $-50°+23°.5=-26°.5$). The sun is above the horizon more than half the 24 hours, the length of time being greater the greater the latitude of the observer up to 66°.5 north, at which the sun is on the northern horizon at midnight. When the sun is at the winter solstice, it rises in the southeast, sets in the southwest, and is above the horizon less than half the 24 hours, the length of time being lesser the greater the latitude of the observer up to 66°.5 north, at which latitude the sun is on the southern horizon at midday.

At a point on the equator of the earth, the sun is above the horizon (neglecting the effects of the refraction of its light by the earth's atmosphere) precisely half of each day in the year. At the poles of the earth, the sun is above the horizon half of the year continuously, and below the horizon the other half. It can be shown that at every point on the earth the sun is above the horizon half of each year; if at one time of the year the days are less than 12 hours in length, at another time of the year they are enough longer to make up for the deficiency. Although the total number of hours the sun is above the horizon during a year is independent of the latitude, the total amount of radiant energy received during a year per unit area varies with the latitude.

111. Seasons in Various Latitudes. There are two related subjects for consideration in this article: (*a*) The differences in mean temperature at different latitudes on the earth, and (*b*) the seasonal variations in temperature at a given latitude. The principal factors affecting temperatures are the amount and the distribution of radiant energy received from the sun; all other factors will be neglected in this discussion.

TABLE V. SOLAR ENERGY RECEIVED PER UNIT AREA PER DAY

DATE	$l=0°$	$l=22°5$	$l=45°0$	$l=67\ 5$	$l=90°$
Jan. 21	9.38	6.70	3 26	0.13	0.00
Feb. 21	9.80	7.89	4.86	1.32	0.00
Mar. 21	10.00	9.24	7.07	3.83	0.00
Apr. 21	9.80	10.28	9.29	7.11	6.26
May 21	9.38	10.85	10.93	10.15	10.85
June 21	9 17	11 01	11.54	11.57	12.53
July 21	9.38	10.85	10.93	10 15	10.85
Aug. 21	9.80	10 28	9.29	7.11	6.26
Sept. 21	10.00	9.24	7.07	3.83	0.00
Oct. 21	9.80	7.89	4.86	1.32	0.00
Nov. 21	9.38	6.70	3 26	0.13	0.00
Dec. 21	9.17	6.21	2 68	0.00	0 00
Totals	115.06	107.14	85.04	56.65	46.75

(*a*) In high latitudes the sun's rays strike the earth's surface obliquely, even at noon, and are spread out thinly, their obliquity depending upon the declination of the sun. Within the tropics they at times fall perpendicularly upon the earth's surface. Table V gives the relative amounts of solar energy received at various

northern latitudes per unit area per day on the twenty-first of each month throughout the year and the totals for all of these days. In the southern hemisphere the numbers are the same with the dates all displaced by six months.

It follows from the totals in Table V that a unit area at the earth's equator receives in a year about 2.5 times as much energy from the sun as is received by an equal area at the earth's poles. This difference is the principal reason for the difference in mean temperatures in the two latitudes.

(*b*) It also follows from Table V that the range in the seasonal variations in the energy received increases progressively from the equator to the poles. These variations are largely due to the wide differences in the number of hours the sun is above the horizon on different dates, though its altitude at noon is a factor. Table VI gives the hours per day on various dates that the sun is above the horizon in five different northern latitudes.

TABLE VI. HOURS SUN IS ABOVE HORIZON PER DAY

DATE	$l=0°$	$l=22°5$	$l=45°0$	$l=67°5$	$l=90°$
Jan. 21	12 hrs.	10.8 hrs.	9.1 hrs.	3.6 hrs.	0.0 hrs.
Feb. 21	12 "	11.4 "	10 4 "	8.1 "	0.0 "
Mar. 21	12 "	12.0 "	12.0 "	12.0 "	on horizon
Apr. 21	12 "	12.6 "	13.6 "	15.9 "	24.0 hrs.
May 21	12 "	13.2 "	14.9 "	20.4 "	24.0 "
June 21	12 "	13.4 "	15.4 "	24.0 "	24.0 "
July 21	12 "	13.2 "	14.9 "	20.4 "	24.0 "
Aug. 21	12 "	12.6 "	13.6 "	15.9 "	24.0 "
Sept. 21	12 "	12.0 "	12.0 "	12.0 "	on horizon
Oct. 21	12 "	11.4 "	10.4 "	8.1 "	0.0 hrs.
Nov. 21	12 "	10.8 "	9.1 "	3.6 "	0.0 "
Dec. 21	12 "	10.6 "	8.6 "	0.0 "	0.0 "

112. Effects of the Eccentricity of the Orbit of the Earth upon the Seasons. In the discussions in Art. 111 it was tacitly assumed that the distance of the earth from the sun remains constant, whereas it varies by about 3.4 per cent as a consequence of the eccentricity of the orbit of the earth. Since the amount of energy received by the earth from the sun is inversely proportional to the square of their distance apart, the variation in energy received due to the eccentricity of the orbit of the earth is about 6.8 per cent, or 3.4 per cent each side of the mean.

The earth is at the perihelion point of its orbit about January 1

and at the aphelion point on July 4 (the dates may vary by a day because of the leap year). Consequently, the minimum energy received in December for points in the northern hemisphere is not so low by about 3.4 per cent as would be inferred from Table V. Similarly, the maximum energy received in June lacks about 3.4 per cent of being as great as indicated in the table. That is, the annual variation in energy received at points in the northern hemisphere is decreased by the eccentricity of the earth's orbit by about 6.8 per cent. The result is that at latitude about 15° north the energy received is almost constant throughout the year. As a consequence of the precession of the equinoxes, in about 12,000 years the situation with respect to the seasons in the northern and southern hemispheres will be reversed.

If the dates in the first column of Table V are shifted forward by six months, then the resulting table gives the relative amounts of solar energy received during a day on the various dates at corresponding southern latitudes. It follows that, in the southern hemisphere, the eccentricity of the orbit of the earth decreases the energy received when it is a minimum by about 3.4 per cent, and increases it when it is a maximum by the same percentage.

There is an interesting consequence of the fact that the motion of the earth about the sun satisfies the law of areas. It can be shown from this property and the fact that the energy received varies inversely as the square of the distance from the sun to the earth that the total amounts received during corresponding seasons at points in equal northern and southern latitudes are equal. For example, the amount of solar energy received at latitude 40° north while the sun is moving from the vernal equinox to the autumnal equinox is equal to the amount received at latitude 40° south while the sun is moving from the autumnal equinox to the vernal equinox. The deficiency in the amount received per day in the northern hemisphere during its summer, as compared with the amount received per day in the southern hemisphere during its summer, is exactly counterbalanced by the greater number of days of the summer in the northern hemisphere. The interval from the time the sun is at the vernal equinox (on or about March 21) to the time it is at the autumnal equinox (on or about September 23) is 186.25 days, while the interval from the autumnal equinox to the vernal equinox is only 179 days. The same result is true for any other corresponding seasons or parts of seasons.

113. Lag of the Seasons. From the standpoint of the heat received by the earth from the sun, March 21 and September 23 are corresponding seasons, June 21 is the middle of the summer (in the northern hemisphere), and December 21 is the middle of the winter. From the standpoint of climate, however, March 21 and September 23 are not corresponding seasons, and midsummer and midwinter are later than June 21 and December 21 respectively, for the seasons have a lag.

The lag of the seasons is due to the fact that the earth does not radiate energy immediately after it is received. For example, during the spring months more and more heat is received daily, but not so much is radiated as is received, and consequently the temperature rises. As the temperature rises the rate of radiation increases, and eventually, some time after the summer solstice, it equals and then exceeds the rate at which energy is received, after which the temperature declines. The lag is greater the denser the atmosphere. In high altitudes where the air is thin and dry, the lag of the seasons is much less than it is at sea level and in humid atmospheres.

Variations in mean daily temperatures are also caused by changes in the major atmospheric currents, such as the trade winds. Since these great atmospheric movements lag considerably behind the causes that produce them, the seasonal changes also lag. In some cases, however, the effects are the opposite. For example, as the autumn approaches in southern California, the normal winds change in direction from the northwest to the southwest, and for a time the mean daily temperature rises.

XI. QUESTIONS

1. Find the time that some easily recognizable star or group of stars passes the meridian or any other fixed line, and verify that it crosses about half an hour earlier after an interval of a week.
2. All the proofs that the earth rotates depend upon the laws of motion; which of the proofs that the earth revolves depends upon the laws of motion?
3. Which of the proofs that the earth revolves furnishes the size of the earth's orbit?
4. If the constant of aberration can be determined to ″.05, what is the percentage of uncertainty in the mean distance from the sun to the earth as determined from it? In determining the distance from the sun to the earth by this method, is it necessary to take into account the fact that the sun also revolves about the center of gravity of the sun and the earth?

5. Draw an ellipse having the eccentricity of the orbit of the earth.
6. Draw a figure corresponding to Fig. 68 for an observer in latitudes 0°, 23°5 north, 66°5 north, 90° north, 23°5 south, 66°5 south, and 90° south.
7. If the lag of the seasons were less, would the variations of the seasons in high latitudes be greater or less?
8. If the inclination of the equator to the ecliptic were greater, would the seasonal variations be greater or less?
9. Discuss the seasonal variations in the northern hemisphere and in the southern if the earth is at its perihelion when the sun is at an equinox.
10. Discuss the days and the seasons in latitudes 0°, 45°, and 90° if the axis of the earth were in the plane of its orbit.

REFERENCE BOOKS

Physics of the Air (1929). W. J. Humphreys. McGraw-Hill Book Company, New York, N. Y.

The Earth, Its Origin, History and Physical Constitution (1929). Harold Jeffreys. Cambridge University Press, Cambridge, Eng.

CHAPTER VI

TIME

114. Measurement of Time. Although it seems to be impossible to define time in terms that are simpler or better understood than the word itself, yet it is worth while to examine the conditions under which we perceive it. When we recall past events (intellectual experiences), we find that they not only appear distinct and separate, but that they are arranged in our consciousness in a definite order like the numbers on a measuring rod; they seem to occupy an interval, and we call it *time*.

There is nothing absolute in our perception of time, for our estimate of the length of an interval depends upon the number and the character of the events we pass in retrospection. For example, when we attempt to measure mentally all the hours we have been asleep in a year, we find that they shrink into insignificance because they furnish no memories to give them length. But when we look back upon the many and varied experiences of a brief vacation among the mountains or at the seashore, we find that weeks seem to be longer than ordinary months. In fact, the more we examine our mental processes, the more we become convinced that, psychologically speaking, the length of an interval of time depends upon the number, variety, and intensity of the intellectual experiences we have had during it. An easy corollary is that our lives, as intelligent beings, are not truly measured by clocks, but rather by what takes place in our minds.

It is obviously necessary for practical purposes to have a means of measuring time that does not depend upon the varying mental processes of individuals, but which, on the average, substantially agrees with their perceptions of time. Such a means of measuring time is given by the First Law of Motion (Art. 93); this law is, in fact, partly a definition of equality of intervals of time. For present purposes it is convenient to reword it as follows: Two intervals of time are equal (by physical definition) if a moving body subject to no exterior forces passes over equal distances in them.

Time is not measured by direct applications of the first law of motion, for there is no body available whose motion is not influenced by exterior forces; but time is measured by an indirect consequence of the laws of motion. This consequence is that a body subject to no exterior forces and to no changes in the distribution of its mass rotates at a uniform angular rate. As was seen in Art. 100, the earth satisfies these conditions so nearly that its rotation offers an ideal means of measuring time.

The angle through which the earth rotates can be determined conveniently and accurately by observing the passage of the meridian of an observer across the stars. Hence it follows that for practical purposes two intervals of time are equal if, during them, the earth rotates through equal angles. Yet the first law of motion provides the fundamental definition of the equality of two intervals of time; if it were not so, there would be no point to the question of the uniformity of the rotation of the earth (Art. 100). When the rotation of the earth is tested practically, the method is simply that of comparing it with some other observable consequence of the first law of motion, such as the revolution of the moon.

115. Relativity of Time. In order to illustrate more completely the fact that, psychologically, the length of an interval of time is relative to our intellectual experiences, let us suppose that in some far-away place there is a clock which keeps perfect "uniform" time; but that in our neighborhood, by some miracle, everything takes place at one hundred times its normal speed. The years are one-hundredth of their normal length, and so are the days; we move one hundred times as fast; we think one hundred times as fast; in short, everything within our range goes one hundred times as fast as normal. The question is whether we should know that our lives are passing at one hundred times their normal rate.

Before answering this question two other points must be briefly considered. The first is that one may inquire how we know that the distant clock keeps normal time and that time is "running away" with us. It may be supposed that there are intelligent beings like ourselves using the clock and that they are sure that it behaves normally. Moreover, it may be assumed that, although we can not see them, they see us and note that everything with us is going at an abnormally rapid rate. We may even assume that at

a later time velocities become normal and that our distant friends then tell us what happened to us for a time.

The second point is that if time alone were changed, certain derived quantities depending in part on time, such as acceleration, would also be changed. In order that these derived quantities shall not betray the change in our time, certain other quantities also must be supposed to be modified. To analyze the necessary modifications would involve us in difficulties which can not be treated here, and hence we will simply assume that all the requisite auxiliary changes are possible and that they have accompanied the change in time. Under these circumstances, we should not know that there had at any time been anything abnormal. In fact, a thousand years could be compressed into the length of a day or a day drawn out into a geological age without our perceiving the fact.

116. Other Examples of Relativity. It has been seen that, from the psychological point of view, time is relative to our intellectual experiences; and that, from the physical point of view, it is relative to some chosen standard of reference, such as the angle through which the earth rotates. This sort of relativity appears in practically every domain we examine. For example, there is nothing absolute in size. When we consider the earth from the point of view of the non-scientific man, it appears to be enormous beyond all comprehension, but from the point of view of an astronomer it is an inconsequential particle. When we turn in the other direction and consider a microörganism, perhaps on the limits of visibility with the most powerful microscopes, we are likely to despise it because it is small (compared to ourselves). In fact, biologists have often found difficulty in conceiving of microscopical forms of life as having complicated structures. Yet the smallest living organism known is composed of countless billions of molecules and probably of thousands of distinct chemical compounds, all arranged in intricate patterns that no work of art can approach. If we were sufficiently small, these microscopic forms of life would appear larger to us than the largest animals or trees do now.

Newton's dynamics brought into view still another phase of relativity which has often been noted. It follows from the laws of motion and the law of gravitation that the interactions of the sun and the planets and their motions relative to one another would

be exactly the same as they are now if the center of gravity of these bodies were moving in any straight line with uniform speed with respect to the present system. This means that we are wholly unable to determine from its internal dynamics whether the system is at rest or moving with respect to some more fundamental basis of reference.

Naturally, astronomers undertook the problem of determining the motion of the solar system with respect to the stars, and they found that the sun is moving at a speed of about 20 kilometers (12 miles) per second toward the constellation Hercules. More exactly, this is the motion of the sun with respect to a few thousand nearer stars used in the measurements. Even if all the stars of the galaxy were used, then the question of the motion of the galaxy as a whole would arise. It has been found in most cases that the relative speed of our galaxy with respect to exterior galaxies is several hundreds of kilometers per second. Clearly, there is no final goal in this direction unless the number of stars in the universe is finite (which is doubtful) and we take the center of gravity of the whole system as a fixed point. Even then there would remain difficulties respecting what should be taken as fixed directions.

117. The Michelson-Morley Experiment. In 1887, A. A. Michelson and E. W. Morley performed an experiment which had a surprising outcome and which has led on and on to conclusions of the greatest significance; in fact, this experiment has been a turning point in physical theories comparable to that produced by the dynamics of Newton. The purpose of the experiment was to determine the motion of the earth, not with respect to the stars or any other physical bodies, but with respect to an assumed all-pervading and fundamental substratum known as the ether. As will be seen (Art. 270), light has the properties of wave-motion and it was believed that there is a medium, the so-called ether, through which it is transmitted.

As has been stated, Michelson and Morley attempted to measure the motion of the earth with respect to the ether. They devised ingenious apparatus, but found only a small fraction of the expected effects. The failure was not due to faulty experimental technique, for the experiment has been repeated several times with similar or equivalent means and in all cases the measured motions have been so small that it has been assumed that they were entirely

due to accidental errors. Dayton C. Miller, however, has obtained measurable displacements, but they do not agree in phase or amount with those expected. Aside from the anomalous results obtained by Miller, the experiments clearly indicate that the time required for light to go from one body to another depends only upon their distance apart and their motion relative to each other, and that it is wholly independent of their motion with respect to the medium through which light is transmitted, if, indeed, there is such a medium.

It is important to realize fully the unexpected and remarkable character of the results of the Michelson-Morley experiment. To assist in getting an understanding of the nature of the outcome of the experiment, consider two aviators flying one behind the other at the same level and at the same speed. If the one in the lead should shout back at the second, the latter would obviously hear the call after a shorter interval than he would if they were both at rest. If the one in the rear should shout, the interval before the first would hear him would be longer than if they were at rest. If they were flying with the speed of sound, the waves of the shout of the second would never overtake the first. In the case of sound the results are found to be precisely what would be expected. But if the aviators communicated with each other by means of light, their motion would have no effect whatever on the signals. Light appears to travel as though it were material particles although it has the properties of a wave-motion.

118. The FitzGerald Contraction Explanation. The first suggested explanation of the negative results of the Michelson-Morley experiment was FitzGerald's hypothesis that all material bodies are shortened in the direction in which they are moving by amounts depending upon the ratios of their velocities to that of light. Upon examination this hypothesis is found to be not so bizarre as it at first appears, for it follows from the electro-magnetic equations of Clerk Maxwell and reasonable assumptions respecting the electrical constitution of matter that such a contraction is to be expected and that it will be precisely of the required amount.

At first it might be thought that it would be possible to detect a shrinking of matter in the direction of its motion. An examination shows, however, that all attempts to do so must fail, for a relativity is involved analogous to the relativity of time.

119. Einstein's Special Theory of Relativity. In 1905, Albert Einstein published an epoch-making paper on relativity, which it will not be possible to describe within available limits of space, much less to make really comprehensible. The purpose of the theory was to harmonize dynamics and other physical phenomena, including the negative results of the Michelson-Morley experiment. In developing a theory in which all phenomena will be precisely the same for all systems of reference axes which move with respect to one another in straight lines with uniform speeds, Einstein found it necessary to consider space and time together.

So far as our perceptions of time are concerned, there is nothing strange that they should in some way be tangled up with our perceptions of space, for, as has been seen, they depend upon our mental experiences which are accompanied by space displacements of matter. But since the three dimensions of space and the one dimension of time are, in Einstein's theory, inseparably connected, it is necessary to treat phenomena in terms of four parameters, or coördinates. There are plenty of things in the world which depend upon four or more factors, and they do not distress us. For example, the temperature of a geographical spot depends upon its latitude, longitude, altitude, and the time of year. The writers on relativity, however, speak of space and time as being four-dimensional, and this terminology has aroused antagonism in many minds, quite without real justification. The four dimensions are four dimensions only in a manner of speaking, and Einstein's time is quite different from his space. There are, however, consequences of the four-fold basis of experience which are quite startling to our naïve ways of looking at things. Another interesting conclusion from Einstein's equations is that the mass of a body increases with its velocity, becoming infinite as the velocity equals that of light.

120. Einstein's General Theory of Relativity. In 1915, Einstein modified his theory so as to include the case in which the coördinates of reference move non-uniformly with respect to one another; that is, the case in which bodies are subject to accelerations (gravitational forces). This theory introduced curvatures of space in the neighborhoods of what we have been accustomed to call masses. The term "curvature of space" has caused even more distress than the four dimensions. Again the usual reaction has been largely

without a valid basis. No one has any objection to the curvature of a spherical surface. One of its properties is that the sum of the interior angles of a spherical triangle whose sides are arcs of great circles is greater than two right angles. We use three dimensions and explain the excess over two right angles in terms of the radius of the sphere and the lengths of the sides of the triangle. Suppose, however, that all our experiences were in two dimensions, so that we could not even conceive of three dimensions. On a plane surface we should think that space was not warped; but on a spherical surface we should conclude it was warped.

The situation is somewhat similar in the curved space of Einstein's general relativity. If we think of this space (in the mathematical sense) as being included in a space of ten dimensions, we can completely explain away the property of curvature. But the difficulty of thinking of ten dimensions is as great as that of thinking of curved four dimensions; the difficulties in both cases are largely fictitious, because we have carried over terms with familiar connotations to domains in which they do not exactly apply.

There is an important feature of Einstein's definition of the properties of space-time which merits notice. Einstein defined the properties of space in a small neighborhood of a point (by differentials, in the terminology of calculus) and not in a large region. This sort of definition puts certain limitations on the properties of space in the large, but it does not completely define the space. For example, the latitude lines and longitude lines on all maps intersect at right angles, yet in the large there are several kinds of maps satisfying this condition. The point that is being made is simply that since Einstein defines the properties of space in an infinitesimal neighborhood, he does not uniquely define space in the large. This is the explanation of the fact that various solutions of his defining equations can be derived and, indeed, have been derived, at least in certain simple cases. The danger in such a situation is that one who obtains a particular solution of the equations will erroneously assume that it is the only solution.

121. Is the Theory of Relativity Correct? Those who propound this question usually think that either the classical geometry of Euclid and the dynamics of Newton are correct or that the space-time-gravitation system of Einstein is correct. As a matter of fact, there are many other possibilities, some of which may at any time

be brought forward, just as the general theory of relativity superseded the special theory after it had been widely accepted for about ten years.

So far as the observational evidence goes, there are several phenomena near the border line of observability, such as that disclosed by the Michelson-Morley experiment, which are not consistent with the classical theory. On the other hand, there is none at present that is definitely inconsistent with Einstein's theory. It must be admitted that most of the observational evidence which has often been taken to substantiate Einstein's theory is so near the limits of observational determination that, from the practical point of view, the results are not very conclusive (Arts. 171, 184, 316). But assuming that the observational evidence is completely satisfactory, what of the theory of relativity?

It has been noted that Einstein's theory is explicit only in the infinitesimal and that there are many solutions of his equations in the large. As an illustration of possible variety, Einstein has one solution of his equations leading to the conclusion that space is finite, while W. de Sitter has quite another one in which it is infinite. There is nothing in the theory of relativity that compels a person to accept either of these solutions of its fundamental equations.

We may inquire whether even the fundamental equations of relativity must be adopted. They have not been forced on us by any logical necessity; they are merely the simplest and the most aesthetically satisfying equations so far devised for harmonizing the phenomena of the physical world. But simplicity and beauty, which are largely subjective in character, have often been found to be unsafe guides in science. For example, it was believed on these grounds for many centuries that the orbits of the planets are circles, yet the belief was quite incorrect. At any time some observational evidence may be discovered that is contradictory to our present theory of relativity. If such contradictions should be found, we shall know that the theory needs modification; if they should not be found, its correctness will remain open to question. This theory is no more absolute and final than other theories have been, and it is probable that it is only a very important step in the evolution of science (Art. 3).

The theory of relativity is considered here because its origin was connected with astronomical phenomena and because its

tests and interesting consequences are largely astronomical. In pointing out its lack of finality, there is no intention of questioning its importance. It has been of enormous service in removing the naïve habits of thought that have been common in classical physical theory. Whatever its future may be, it marks a turning point in the mingled stream of science and philosophy.

122. Sidereal Time. Sidereal time is the time that is measured by the rotation of the earth with respect to the stars. A sidereal day is the interval of time between the passage of a meridian, in its apparent eastward motion, across the vernal equinox and its next passage across the vernal equinox. Since the earth rotates at a sensibly uniform rate (Art. 100), all sidereal days are of the same length.

The sidereal day is arbitrarily divided into 24 sidereal hours, which are numbered from 1 to 24; the sidereal hour is divided into 60 sidereal seconds. The sidereal time of a place on the earth is zero when its meridian crosses the vernal equinox. It follows that all places on the earth having the same longitude have the same sidereal time.

123. True Solar Time. True solar time is the time which is defined by the rotation of the earth with respect to the sun. A solar day is the interval of time between the passage of a meridian across the center of the sun and its next passage across the center of the sun. Since the sun in its apparent annual motion moves eastward with respect to the stars, a solar day is longer than the sidereal day. The sun makes a circuit of 360° in 365.25 days, and it consequently moves eastward with respect to the stars about 1° per day. The earth rotates through 15° in a sidereal hour, from which it follows that, on the average, a solar day is nearly 4 sidereal minutes longer than the sidereal day.

124. Variations in the Lengths of Solar Days. If the apparent motion of the sun eastward among the stars were uniform, each solar day would be longer than the sidereal day by the same amount; and since all sidereal days are of equal length, all solar days would also be of equal length. But the apparent motion of the sun is somewhat variable for two reasons, and consequently the solar days are variable in length as compared with the sidereal day as a standard.

It is evident from Fig. 58 that the sun apparently moves eastward at the same angular rate that the earth revolves about the sun. Hence it follows from Fig. 65 that the sun apparently moves eastward most rapidly when the earth is at the perihelion point P. Since the earth is at P about January 2, the solar days, so far as they depend upon this factor, are longest about January 2 and shortest about July 4.

There is another reason why solar days vary in length. In order to simplify the problem, suppose for the moment that the earth revolves uniformly in a circular orbit. The difference in length between a solar and the sidereal day is proportional to the angular distance the sun moves eastward among the stars as measured along the celestial equator—northward or southward motion evidently has no effect on the length of the day. It follows from Fig. 67 that when the sun is at the vernal equinox V only a part of its motion is eastward, the remaining being northward. Similarly, when the sun is at the autumnal equinox A, part of its motion is southward and its eastward component of motion is less than the average. The sun is at the points V and A on March 21 and September 23 respectively, and these are the dates on which the solar days are shortest, at least so far as they depend upon the inclination of the ecliptic to the celestial equator.

On June 21 and December 21 the sun is about 23°.5 from the celestial equator, where the hour circles are not so far apart as they are at the celestial equator. Consequently, at these times of the year the sun moves eastward with respect to the hour circles more rapidly than it does on the average, and the solar days are then longest.

The effects of the eccentricity of the orbit of the earth and the inclination of the ecliptic to the celestial equator upon the length of the solar day are small; the longest day in the year (December 22) is only about 51 seconds longer than the shortest day of the year (September 17). The differences between the average length of the solar day and the lengths of the solar days are given in Table VII on the first of each month in terms of the sidereal second as the unit. The numbers in the line E are the effects due to the eccentricity of the orbit of the earth; those in the line I are the effects due to the inclination of the ecliptic to the celestial equator; and those in the last line are the algebraic sum of the other two. The plus sign precedes the numbers when the true

solar days are longer than the mean solar day; and the negative sign precedes when they are shorter.

TABLE VII. DIFFERENCES IN SECONDS BETWEEN APPARENT SOLAR DAYS AND THE MEAN SOLAR DAY

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
E	+ 8	+8	+ 4	0	−4	− 7	− 7	−6	− 4	0	+3	+ 7
I	+21	+1	−17	−18	−4	+12	+17	+3	−14	−18	−4	+11
$E+I$	+29	+9	−13	−18	−8	+ 5	+10	−3	−18	−18	−1	+18

125. Mean Solar Time. The length of the mean solar day is the average of the lengths of all the solar days in a year. If we imagine a fictitious sun moving eastward uniformly at the average rate of the actual sun, then mean solar time is defined by the rotation of the earth with respect to this fictitious sun. All mean solar days are of equal length with respect to the sidereal day as a standard, and mean solar time is the time in ordinary use.

The mean solar day is divided into 24 mean solar hours; the mean solar hour, into 60 mean solar minutes; and the mean solar minute, into 60 mean solar seconds. In the year of ordinary use (tropical) there are 365.2422 mean solar days and 366.2422 sidereal days. Hence, the mean solar day equals 24.06571 sidereal hours, or $24^h\ 3^m\ 26^s.555$ of sidereal time; and the sidereal day equals 23.93447 mean solar hours, or $23^h\ 56^m\ 4^s.091$ of mean solar time.

126. The Equation of Time. Although true solar days do not differ much from the mean solar day in length, these differences accumulate day after day so long as they have the same sign, and the sum of these accumulations at the maximum amounts to more than 16 minutes. The mean solar time minus the true solar time is called the equation of time, which may be positive or negative according as it is later by the fictitious mean sun or the actual sun. With this agreement respecting the meaning of the sign before the equation of time, Fig. 69 gives the variation in the equation of time throughout the year.

When the equation of time is negative, it is later by the sun than by a correct mean solar clock. For example, when it is noon by a mean solar clock in November, the sun is about 15 minutes west of the meridian.

An interesting phenomenon occurs late in December and during the first days of January which is often puzzling to those who do

not know its explanation. On December 24 the equation of time is zero; that is, when it is noon by a mean solar clock it is also noon by the sun. It follows from Table VII that at this time of the year true solar days are nearly 30 seconds longer than mean solar days. Consequently, when it is noon by the clock on December 25 it will be about 30 seconds before noon by the sun. That is, the sun has shifted about 30 seconds with respect to the clock. Since the lengths of the days are not increasing much at this time of the

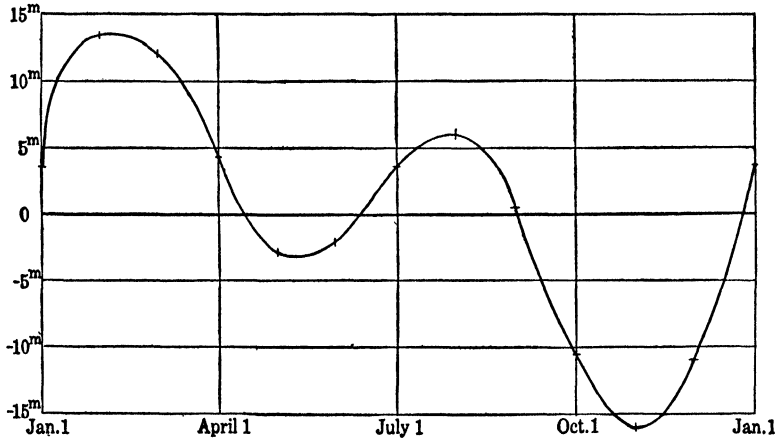


FIG. 69.—The equation of time—mean solar time minus true solar time.

year, the interval from sunrise till mean solar noon will decrease nearly 30 seconds, while the interval from noon by the clock till sunset will increase by a slightly larger amount. Hence at this time of the year we have the interesting phenomenon of the mornings decreasing in length while the afternoons and the whole periods from sunrise to sunset are increasing.

127. Standard Time. It is obvious that only places having the same longitude on the earth have the same mean solar time. In order to facilitate the conducting of business, such as running trains over east-and-west railways, it is important that the inhabitants of considerable strips of territory should use the same time. The acute problems presented by far-stretching areas and extensive travel stimulated the adoption of *standard time* in 1884.

Standard time zones, each 15° (an hour) wide, were marked off, starting with the meridian of Greenwich, England. All points in each zone were assigned the same standard time. In the United

States there are four zones. The meridian of Eastern Standard Time is 75° (5 hours) west of Greenwich and passes through Philadelphia. The meridian which defines Central Standard Time is six hours west of the meridian of Greenwich and passes through St. Louis, Mo. Mountain Time is the mean solar time of the seventh hour west of Greenwich, and Pacific Time is the eighth. The meridian of Mountain Time passes near Denver, Colorado, and that of Pacific Time about 100 miles east of San Francisco, California. In actual practice the time zones are not exactly 15° wide, but time changes from one zone to another at points at which a jump of an hour is least inconvenient.

Most of the world has now adopted standard time. Greenwich time is generally used throughout western Europe, the time of the meridian one hour east of Greenwich in Central Europe, and the time of the meridian two hours east of Greenwich in Eastern Europe, Egypt, and South Africa. Most of South America uses the time of the meridians from 3 to 5 hours west of Greenwich.

128. Determination of Time and Distribution of Time. It is obvious that the determination of mean solar time is an important practical problem. The first step in its solution is that of determining sidereal time by observing the transits of stars of known right ascensions across the meridian (Art. 14). It follows from the definition of right ascension that the sidereal time at which a star crosses the meridian is equal to the right ascension of the star. Such observations enable the observer to determine the errors of his sidereal clock, which will rarely be exactly correct. By using the transits of several stars, the error of his sidereal clock can be determined within a very small fraction of a second.

The second step is that of determining the mean solar time when the date, the sidereal time, and the longitude of the observer are all known. This is done simply by computing from the given and the observed data the time which has elapsed since the sun was at the vernal equinox, and then calculating the distance the mean sun has traveled eastward in the interval. The sidereal time and the difference between sidereal time and mean solar time being known, mean solar time is determined. It is not necessary for an astronomer to make these calculations from fundamental principles, for tables are given in the *Nautical Almanac* that aid him to go easily from sidereal time to mean solar time.

The observations of the transits of stars upon which the determination of time depends are now made almost exclusively at such national observatories as the Naval Observatory, at Washington, the Royal Observatory, at Greenwich, England, l'Observatoire de Paris, at Paris, France, etc. The calculations that give mean solar time are also made at the same observatories. Formerly, time was distributed entirely by telegraph, but it is now largely sent out much more conveniently and accurately by wireless signals. The accuracy of this method is so great that time signals are now sent out with an error of less than one-fifth of a second, and with an error of the average of several signals of only one-thirtieth of a second.

It may occasion some surprise at first thought that time is not determined by transits of the sun across the meridian. If the sun were used, only one transit per day could be observed, the error in the time of its transit because of its apparent diameter would be much greater than that for a star, and to derive mean solar time from true solar time would require as extensive computations as to derive it from sidereal time. Hence it is evident that the advantages are all with the method using transits of the stars.

129. Line Where the Day Jumps. If a person should start at any place on the earth and travel continually westward, the sun would cross his meridian a little later each day than it would if he remained at rest. After he had completed a journey around the earth and had returned to his starting point, he would find that he had lost a day (by the sun, but not, of course, in elapsed time) as compared with those who had remained at the starting point. If he had made the circuit eastward, he would have gained a day. Evidently he would find it almost necessary for practical reasons to add or subtract a day, according as he had journeyed westward or eastward.

The problem of adjusting dates became most acute where those sailing eastward from Europe and America met those sailing westward, particularly in the islands of the Pacific Ocean. The travelers in each group, having gone only half way around the earth, found themselves under no necessity of omitting or duplicating a day; yet when the groups met they differed by one in their reckoning of the day of the week and the date, those having gone eastward having a date one day later than those having gone westward.

Now, by international agreement, the change-of-date line is near the 180th meridian from Greenwich, which fortunately does not pass through any large masses of land. The line varies, however, considerably from the 180th meridian to meet the wishes of those who settled the habitable places after having gone half way around the earth in one direction or the other.

130. The Sidereal Year. One natural and fundamental unit of time is the day; another is the year. The month depends roughly upon the motion of the moon, while the week is an arbitrary division of time.

The sidereal year is the interval of time required for the sun apparently to move from any position with respect to the stars, as seen from the earth, around to the same position again. Perhaps it would be better to define the sidereal year as the period required for the earth to make a complete revolution around the sun, directions of the earth from the sun being determined with respect to the stars as reference points.

The length of the sidereal year in mean solar time is 365.25636 days, or $365^d\ 6^h\ 9^m\ 9^s.50$. If it were not for the attractions of the other planets for the earth, the sidereal years would all be of the same length. The attractions of the planets, however, cause the sidereal years to oscillate slightly in length with no permanent change in either direction. Just at present the length of the sidereal year is increasing slightly.

131. The Anomalistic Year. The anomalistic year is the interval of time required for the earth to revolve from the perihelion point of its orbit around to the perihelion point again. If the perihelion point were fixed, as it would be if it were not for the disturbance of the motion of the earth by the other planets (and neglecting possible relativity effects), the anomalistic year would equal the sidereal year in length. The attractions of the other planets, however, cause the perihelion point of the orbit of the earth to move slowly in the direction of the revolution of the earth, completing a circuit of 360° in about 111,000 years. Therefore the perihelion point of the orbit of the earth advances about 12″ per year, and the anomalistic year is about 5 minutes longer than the sidereal year. In mean solar time, the length of the anomalistic year is 365.25964 days, or $365^d\ 6^h\ 13^m\ 52^s.9$.

One of the inferences from the general theory of relativity is that the perihelion point of an elliptical planetary orbit advances in the direction of revolution of the planet. The rate of advance of the perihelion point depends upon the mean distance of the planet from the sun and the eccentricity of its orbit. Only in the case of the planet Mercury would the effect be appreciable and for this planet it is only 42″.9 per century (Art. 184).

132. The Tropical Year. Since the seasons depend upon the position of the sun with respect to the celestial equator, it is obvious that it is desirable to define a year for ordinary use as the interval of time required for the sun to move (apparently) from an equinox around to the same equinox again. An equivalent definition is the period required for the sun to move from a tropic around to the same tropic again. For this reason this year is called the *tropical* year.

It has been seen (Art. 97) that there is a precession of the equinoxes at the rate of about 50″ per year. Therefore, the tropical year is shorter than the sidereal year by the time required for the sun to move eastward 50″ in its annual motion among the stars. In mean solar time, the length of the tropical year is now 365.24220 days, or $365^{d}\ 5^{h}\ 48^{m}\ 46^{s}.08$. At present the tropical year is decreasing in length at the rate of about one-half a second in 100 years, but in the course of time its length will increase again.

133. Ancient Calendars and the Week. Most primitive peoples and even the cultured ancient Egyptians and Greeks based their calendars on the phases of the moon. Since the cycle of the moon's phases is completed about 12.4 times in a year, a calendar based upon the month can not be kept in phase with one depending upon the revolution of the earth about the sun except by interpolating two complete months every five years. The year, however, is much more fundamental in human activities than the month, and consequently calendars since the days of Julius Caesar have been based on the tropical year and have no relationship to the phases of the moon.

The week is usually regarded as being one-quarter of a month, though the length of the month is a little more than 29.5 days. If the ancients had set themselves the problem of breaking up the month into simple fractions, it would have been simpler for them to divide it into three parts of ten days each. The week, however,

may have had an origin quite independent of the phases of the moon. Certainly, the days of the week appear to be named after the seven independently moving celestial bodies known in antiquity. Sunday is the sun's day; Monday, the moon's day; Tuesday, Mars' day; Wednesday, Mercury's day; Thursday, Jupiter's day; Friday, Venus' day; and Saturday, Saturn's day. In some cases the connections are evident; in others, they are not so obvious because the English names of the days of the week have descended from Northern European mythology rather than from that of Greece and Rome. For example, in Scandinavian or Teutonic mythology Mars is Tiw, Mercury is Woden, Jupiter is Thor, and Venus is Freya. In the romance languages the connections with the characters of Roman mythology are clear; for example, Tuesday is Mardi (Mars' day) in French.

134. The Julian Calendar. In the year 46 B.C. the confused Roman calendar was reformed by Julius Caesar under the advice of the Alexandrian astronomer Sosigenes. This calendar, which was wholly independent of the moon, divided the tropical year into 12 months of nearly equal lengths. For three years in succession, the Julian year consisted of 365 days, and then, since there are 365.2422 days in a tropical year, the fourth year was made to consist of 366 days so that the year should not gradually get out of phase with the seasons.

Caesar transferred the beginning of the year from March, where it had hitherto been, to January 1, where it has since remained. He took the fifth (counting the year from March) month Quintilis and named it July after himself. His successor, Augustus, similarly took the sixth month Sextilis and named it after himself.

135. The Gregorian Calendar. It follows from the length of the tropical year (365.24220 days) that the average year of the Julian calendar is too long by .0078 days. Therefore, in 100 years the calendar gets behind over three-quarters of a day with respect to the seasons; in 400 years the error is more than 3 days. As the centuries rolled by the error in the Julian calendar gradually became important.

After much discussion and in the face of a great deal of opposition, the Julian calendar was changed, in 1582, by Pope Gregory XIII with the advice of the astronomer Clavius. By that year the vernal equinox had receded from the twenty-first of March

to the eleventh. Pope Gregory decreed that 10 days should be dropped from October of that year, the fourth of the month being followed immediately by the fifteenth. At the same time, the leap year rule was amended so that those years whose date numbers are divisible by 100 (which are leap years in the Julian calendar) are not leap years unless thay are also divisible by 400.

The Gregorian calendar was at once adopted by all Roman Catholic countries, but not by Greek Catholic nor Protestant peoples. In fact, the change to the new calendar was not made in England until Parliament passed an act in 1751 that the year 1752 should begin on January 1 (instead of on March 25, as theretofore in England), and that in the following September the second of the month should be immediately followed by the fourteenth instead of the third. The change was violently opposed and led to rioting and bloodshed in many cities, the mobs crying, "Give us back our fortnight." Russia did not adopt the Gregorian calendar until 1918, nor Roumania till 1919.

In 400 years there are 97 leap years in the Gregorian calendar. The time to be accounted for by the extra day in the leap years is $400 \times .2422 = 96.88$ days. Therefore, the years of the Gregorian calendar are too long and lead to an error of .12 of a day in 400 years, or nearly a day in 3,200 years. Evidently the Gregorian calendar will require further correction by changing the leap-year rule for years ending in 1000, 10,000, etc.

136. The Julian Day. In certain astronomical work, such as discussions of the observations of variable stars, where the intervals of time rather than dates are of primary interest, it is convenient to express time in days without reference to years. Such a system avoids all considerations of the leap year. For this purpose J. A. Scaliger, in 1582, defined the Julian Era as January 1, 4713 B.C., and the Julian Day of any date as the number of days which have elapsed since the Julian Era to that time. For example, January 1, 1930, was Julian Day number 2,425,978, and by international agreement the Julian Day begins at noon, although other days begin at midnight.

137. Reform of the Calendar. Although the Gregorian calendar provides a leap-year rule which preserves the relationships between the different parts of the year and the seasons, it has grave defects. For example, the months are not all of the same length.

Moreover, since the number of days in a year is not divisible by 7, the months in successive years begin on different days of the week. These variations from month to month and from year to year are inconvenient and costly. For example, a manufacturer must make adjustments in comparing his production during one month with that of another in the same year or with that of the corresponding month in the preceding year.

There are two inherent difficulties that must be met in order to produce a calendar which will be convenient and precisely the same from year to year. The first is that the year contains about 52 weeks, from which it follows that there must be 13 months of 4 weeks each if they are all to be of equal length. The second is that the ordinary year contains one more day (the leap year two days) than 52 weeks, from which it follows that one day (or two days) must be omitted from the counting of weeks if all years are to begin on the same day of the week.

Although a year of 12 months, because of its easy divisibility into halves, thirds, and quarters, is somewhat more desirable than one of 13 months, the important advantages of having all months of the same length make the calendar of 13 months decidedly the more preferable. It has been proposed to dispose of the extra day beyond 52 weeks of the ordinary year by having a New Year's Day which belongs to no week nor month. On leap years a corresponding special holiday would be interpolated in the middle of the year. Business and commercial organizations in increasing numbers are using a year of 13 months for their accounting, and there is a steadily growing sentiment favorable to a thorough reform of the present inconvenient calendar.

XII. QUESTIONS

1. Do your impressions of the lengths of past intervals of time depend upon your experiences during them? Why does "time drag" when you are waiting for something to occur? Does such a time later stand out in your memory as long?
2. It is seen in Art. 115 that time is relative to some standard. Do all persons have the same standard? Otherwise stated, is a year the same length, at least approximately, from the psychological point of view for everybody?
3. Why do we use the first law of motion as the ultimate basis for determining equal intervals of time?
4. In view of the fact that size is determinable only with respect to some standard, is there any more reason for assuming that an organism of a

single cell is homogeneous and without structure than it would be to make the same assumption for a star a million kilometers in diameter, but so far away that its structure, if it has any, is beyond the reach of observation?

5. Can you give an illustration in which some circumstance, condition, or event depends upon more than four factors, or parameters?
6. Is the fact that a physical theory is expressible in simple and attractive mathematical formulas presumptive evidence of its correctness?
7. Is there any advantage in dividing the day into 24 hours, the hour into 60 minutes, and the minute into 60 seconds? Suppose the circumference were divided into 400 units instead of 360°; what would be a convenient method of subdividing the sidereal day?
8. If the earth rotated in the opposite direction in the same sidereal period as at present, what would be the length of the mean solar day in sidereal hours?
9. What is the relation between standard time and mean solar time for a place 5° of longitude west of a standard time meridian?
10. Develop a rule for the leap year suitable for the next 100,000 years. This interval probably seems no greater to us than 2000 years seemed to Caesar and his contemporaries.
11. If the first day of January is on Monday and the year has 365 days, determine the number of working days in each month, allowing a half holiday on each Saturday and making allowance for all national holidays.
12. Set up a calendar of 52 weeks, beginning with January 1 on Monday and closing the year with a day not belonging to the last week. Compare this calendar with the present calendar for this year. On what days of the week do the various national holidays fall?

REFERENCE BOOKS

The A B C of Relativity (1925). Bertrand Russell. Harper and Brothers, New York.

The Nature of the Physical World (1929). A. S. Eddington. The Macmillan Company, New York.

CHAPTER VII

THE MOON

138. Apparent Motion of the Moon among the Stars. The moon not only has a westward diurnal motion similar to the diurnal motion of the sun, but it also has an eastward revolution around the sky with respect to the stars. The apparent path of the moon among the stars is a great circle which is inclined to the ecliptic at an angle of 5° 9′.

In its eastward motion with respect to the stars, the moon moves at the average rate of nearly 13°.2 per day, or approximately its diameter per hour. Since the sun's eastward motion with respect to the stars is only 1° per day (Art. 123), the moon moves eastward 12°.2 per day with respect to the sun. Consequently, on the average, it crosses the meridian (and rises and sets) about 51 minutes later each day than on the preceding day.

Since the orbit of the moon and the ecliptic are both great circles, they bisect each other. The point at which the moon, in its eastward motion, crosses the ecliptic (Art. 108) from the south to north is called the *ascending node* of its orbit; and the other intersection point is called the *descending node.* These nodes are not exactly fixed in position, for the attraction of the sun for the moon causes them continually to shift westward along the ecliptic, with a period of revolution of about 18.6 years.

It is a remarkable fact that the apparent motion of the moon, including the regression of the nodes of its orbit, was so well known by the Chaldeans at the dawn of history that they were able to predict eclipses of the sun. They had no knowledge, however, of the distance or the diameter of the moon and only a little of its real motion with respect to the earth.

139. Sidereal and Synodic Periods of the Moon. The sidereal period of the moon is the time required for it to move from any apparent longitude among the stars, as seen from the earth, around the celestial sphere back to the same longitude again. Since the sun's attraction modifies considerably the moon's motion, as does

also the equatorial bulge of the earth to a lesser extent, the sidereal month, as defined, varies somewhat from month to month. The average of these sidereal months over a long interval of time has been found to be 27.321661 mean solar days, or $27^d\ 7^h\ 43^m\ 11^s.5$, and is taken as *the* sidereal month.

The period required for the moon to move from any apparent position with respect to the sun (for example, in a line with it) around to the same relative position again is a *synodical* month. The average of the synodical months over a long interval of time has been found to be 29.530588 mean solar days, or $29^d\ 12^h\ 44^m\ 2^s.8$. This period, which is what is meant when the word *month* is popularly used, is known to within one-tenth of a second. The most accurate method of determining its length is that of dividing the intervals between very ancient eclipses and those of recent times by the numbers of revolutions of the moon in the intervals. Since the error in the recorded time of an ancient eclipse is divided by the number of months in the interval, the result is that the uncertainty in the period of a single revolution is very small, even though the time of the ancient eclipse may be uncertain by as much as an hour.

There is a simple relation connecting the year E, the sidereal period of the moon S, and its synodical period M, all of which will be supposed to be expressed in days. The fraction of a revolution that the sun moves eastward with respect to the stars in one day is $1/E$, and the fraction that the moon moves eastward with respect to the stars is $1/S$. Therefore the fraction of a revolution that the moon gains on the sun in one day is $1/S - 1/E$. But this is $1/M$, whence the equation

$$\frac{1}{S} - \frac{1}{E} = \frac{1}{M}.$$

When any two of the quantities in this equation are known, the third can be computed. For example, if E and M have been determined from observations, S can be found from the equation. All three quantities can, of course, be found directly from observations.

140. Phases of the Moon. Since the moon is dark except where it is illuminated by the sun (and to a much lesser degree by the earth), its appearance depends upon what fraction of its lighted side is toward the earth; and this depends, in turn, upon its position relative to the earth and the sun. Fig. 70 illustrates eight positions

of the moon relative to the earth and the sun's rays. Since the sun is very far away compared to the distance from the earth to the moon, the solar rays in the small region of the orbit of the moon are nearly parallel.

In Fig. 70 the illuminated sides of the earth and the moon are at the right. When the moon is at the relative position M_1, its dark side is toward the earth, a fact which is indicated by the small black circle outside the orbit of the moon. In each of the eight positions, the small circle on the moon's orbit shows the illuminated and the unilluminated portions of the moon as they actually are; the small circle outside the moon's orbit shows the corresponding phase of the moon as seen from the earth.

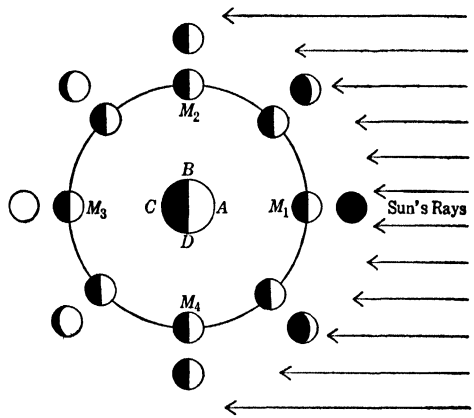


FIG. 70.—Phases of the moon.

When the moon is at M_1, it is said to be in *conjunction* with the sun and its phase is *new*. It is on the meridian A (but invisible) at noon. When the moon is at M_2, it is in *quadrature* with the sun and its phase is *first quarter*. It is evident from the diagram that the moon at this phase is on the meridian B at sunset. When the moon is at M_3, it is in *opposition* to the sun and its phase is *full*. It is then rising in the east for an observer on the sunset meridian B, it is on the midnight meridian C, and it is setting for an observer on the sunrise meridian D. When the moon is at M_4, it is in *quadrature* again and its phase is *third quarter*.

The position of the moon with respect to the earth and the sun can be expressed in terms of its phases as follows: When the right ascension of the moon is equal to that of the sun, the moon is new; when its right ascension is 6 hours (90°) greater than that of the sun, it is at first quarter; when its right ascension is 12 hours (180°) greater than that of the sun, it is full; and when its right ascension is 18 hours (270°) greater (or 6 hours less) than that of the sun, it is at third quarter.

When the moon is new as seen from the earth, the earth is evidently full as seen from the moon, for its illuminated side is toward the moon. As will be seen in Art. 144, the area of the earth is about 13.4 times that of the moon; and, moreover, the reflecting power of the earth is approximately six times that of the moon. Therefore, full earthlight on the moon is about 80 times full moonlight on the earth.



FIG. 71.—The earthlit moon.

Fig. 71 is a reproduction of a photograph taken shortly after the moon was new; the crescent on the right is that part of the earth-side of the moon which is illuminated by the sun, and the remainder is more feebly illuminated by the earth. In order to get a photograph of the part illuminated by the earth, it was necessary to overexpose the part that was illuminated directly by the sun.

141. Diurnal Circles of the Moon. Since the orbit of the moon is inclined only 5° 9′ to the ecliptic, its diurnal circles are nearly the same as those of the sun (Art. 110). The moon, however, runs through its whole cycle of diurnal circles in a month (27.3 days) instead of in a year.

Suppose for the moment that the orbit of the moon coincides with the ecliptic. When the moon is at the point V, Fig. 67, its diurnal circle is the celestial equator. About one-quarter of a month later the moon is the point S and its diurnal circle is then 23°.5 north of the celestial equator. After three-fourths of a month has passed, the moon is at the point W and its diurnal circle is 23°.5 south of the celestial equator.

The diurnal circles of the moon are of most interest when its phase is full or nearly full, for then it reflects upon the earth the greatest amount of light. It follows from Fig. 70 that, when the moon is full, the sun and the moon are 180° apart as seen from the earth. Consequently, when the sun is near the point S, Fig. 67, in the latter part of June or the fore part of July, the full moon is

near the point W, 23°.5 south of the celestial equator. At this time of the year, when places in the northern hemisphere receive the most light per day from the sun, they receive the least light per day from the full moon. On the other hand, when the sun is near the point W, late in December or early in January, the full moon is near the point S; and under these conditions places in the northern hemisphere receive the least light from the sun and the most light from the full moon. Evidently the conditions are similar in the southern hemisphere, but with the seasons displaced by six months. In both hemispheres of the earth at times of least abundant sunlight there is most abundant light from the full moon. If an observer were at a pole of the earth during a winter, the moon, while between the first quarter and the third, would light his landscape for the half of the six months that the sun would be below the horizon.

There is still another point respecting the diurnal circles of the moon which deserves a little attention. The preceding discussion has neglected the inclination of 5° 9′ of the orbit of the moon to the ecliptic, and consequently the conclusions stated require slight modifications. Suppose the ascending node of the orbit of the moon is at the vernal equinox V, Fig. 67. Then the orbit of the moon is north of the ecliptic from V to A, and south of the ecliptic from A to V. If the sun is at W, the full moon is 5° 9′ north of the point S, and the diurnal circle of the moon is then about 28°.6 north of the celestial equator. On the other hand, when the sun is at S the full moon is 5° 9′ south of the point W. Under these circumstances the moon's orbit goes from 28°.6 north declination to 28°.6 south declination, the whole range in declination covered each month being 57°.2.

Although the ascending node of the orbit of the moon is sometimes at the vernal equinox V, it does not remain there but moves opposite to the direction of revolution of the moon, completing a circuit in 18.6 years. Consequently, every 18.6 years the descending node of the orbit of the moon is at the point V, and then the orbit of the moon is south of the ecliptic between V and A and north of the ecliptic between A and V. Under these circumstances the moon passes each month from 18°.3 north declination to 18°.3 south declination, or through a total range of only 36°.6. The ascending node of the orbit of the moon may, of course, be at any other place on the ecliptic, as well as at one of the equinoxes.

142. Shape of the Orbit of the Moon with Respect to the Earth. Since the apparent diameter of an object is inversely proportional to its distance, the shape of the orbit of the moon can be determined from the variations in its angular diameter with variations in its direction from the earth. It is necessary, of course, to make allowance for the fact that our measurements of the moon's angular diameter are made from the surface, rather than from the center, of the earth.

It is found that the orbit of the moon with respect to the earth, like that of the earth with respect to the sun, is an ellipse, with the earth at one of its foci. The point on the orbit of the moon nearest the earth is called the *perigee*, and the most remote point is called the *apogee*. The eccentricity of the orbit of the moon is .05490, or more than three times that of the orbit of the earth. As a consequence of the eccentricity of its orbit, the apparent diameter of the moon, as viewed from the center of the earth (if it could be seen), varies from 29′.38 to 32′.97, with a mean of 31′.08.

It has been noted that the attraction of the sun causes a regression of the nodes of the orbit of the moon. As a consequence of the same attraction, the eccentricity of the orbit of the moon undergoes similar complicated variations about a mean value, and the perigee of the orbit of the moon has a motion which is the compound of a steady advance and numerous oscillations.

143. Distance from the Earth to the Moon. In Fig. 72, let O_1 and O_2 represent the positions of observers and M the position of the moon (or that of a point on the moon). For simplicity, suppose the points O_1, O_2, and M are on the same meridian. The zenith points of O_1 and of O_2 are in the direction OZ_1 and OZ_2 respectively. The observer at O_1 measures the angular distance Z_1O_1M that the moon is south of his zenith; the observer at O_2 measures the angular distance Z_2O_2M that the moon is north of his zenith. Since the positions of O_1 and O_2 are known, the angle O_1EO_2 is known, and hence the angles MO_1O_2 and MO_2O_1 can be found. Since the size of the earth and the positions of O_1 and of O_2 are known, the distance O_1O_2 can be computed. Now, in the triangle MO_1O_2 one side and the adjacent angles are known, and hence all the remaining parts of the triangle can be found. Then from the triangle MO_1E the distance EM can be computed, which is the required distance.

Evidently, conditions in practice are not so simple as those which have been assumed, but the fundamental principles are the same, and any one moderately skilled in mathematics can solve the problem in any case. From the observational point of view the problem is not very difficult, for the quantities to be determined are easily measured. The equatorial radius of the earth, as seen from the mean distance from the earth to the moon, subtends an angle of 57′ 2″.7 (E. W. Brown). This is the angle (corresponding to that at M, Fig. 71) on which the accuracy of the determination of the distance from the earth to the moon depends.

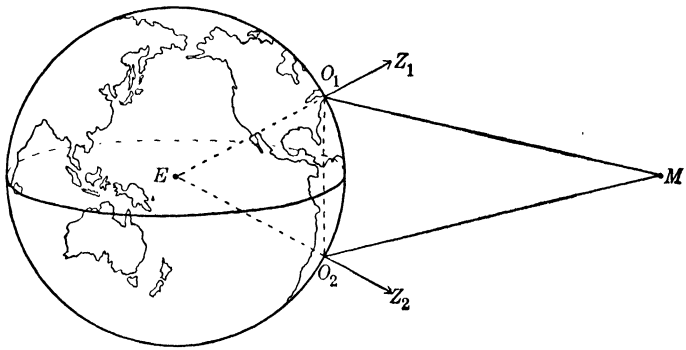


FIG. 72.—Measuring the distance to the moon.

Without optical aid, this angle could be determined with an uncertainty of about 7 per cent; with good telescopes, it can be measured with an uncertainty of less than one-tenth of one per cent. When expressed in percentages, the accuracy of our knowledge of the distance from the earth to the moon is nearly as great as that of our knowledge of the diameter of the earth, which is used in measuring the distance to the moon.

The mean distance from the center of the earth to the center of the moon is 384,403 kilometers (238,857 miles), or 60.2665 times the equatorial radius of the earth. As a consequence of the eccentricity of the orbit of the moon and the effects of the attraction of the sun, the distance of the moon varies from about 363,000 kilometers (225,000 miles) to 406,000 kilometers (252,000 miles). On dividing the perimeter of the orbit of the moon, which is approximately 2,400,000 kilometers (1,500,000 miles), by the number of seconds in a sidereal month, it is found that the mean velocity of the moon in its orbit is about 1020 meters (3350 feet) per second,

or equal to the muzzle velocity of projectiles fired from very high-power guns.

In its motion about the earth, the moon describes so large a curve that it deviates from a straight line only about one-eighth of a centimeter in 1000 meters (one-twentieth of an inch in 3280 feet). It might appear that a very feeble attraction of the earth controls the motion of the moon; but the mass of the moon is so great that it is found by computation that the gravitational pull between the earth and the moon would break a steel cable 400 kilometers (250 miles) in diameter.

144. Dimensions of the Moon. As soon as the distance of the moon from the earth is known, its linear diameter can be computed from its apparent mean angular diameter of 31′ 5.″16. Its linear diameter is found to be 3476 kilometers (2160 miles).

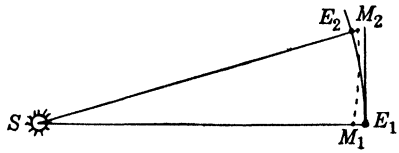


FIG. 73. Orbit of moon with respect to sun.

Since the areas of two similar bodies are proportional to the squares of their corresponding linear dimensions, it follows that the area of the earth is about 13.4 times that of the moon; and since similar volumes are proportional to the cubes of their corresponding linear dimensions, it follows that the volume of the earth is about 49.3 times that of the moon. The earth as seen from the moon would appear to be an approximate circle nearly 2° in diameter, with an apparent area 13.4 times that of the moon as seen from the earth.

145. Orbit of the Moon with Respect to the Sun. It is quite natural for one to suppose that the orbit of the moon with respect to the sun is a wavy line which crosses the orbit of the earth twice each month. The orbit of the moon is very great relative to the earth, but it is very small in comparison with the orbit of the earth, and it is at every point *concave* toward the sun.

In order to show that the orbit of the moon is never convex toward the sun, consider Fig. 73 in which E_1E_2 is the arc described by the earth while the moon goes half way around the earth from M_1 to M_2. Evidently the arc M_1M_2 is concave toward the sun. The orbital velocity of the earth with respect to the sun is 30 times greater than that of the moon with respect to the earth,

and the attraction of the sun for the moon is always much greater than that of the earth. The sun does not rob the earth of the moon because both of these bodies are free to move, and its attraction for each of them, per unit mass, is on the average the same. Its great attraction prevents them from flying away into space, while their smaller mutual attraction holds them in their relatively slow motion around their center of gravity.

146. Mass of the Moon. Although the moon is relatively near the earth, it is much more difficult to determine its mass than it is to find the masses of distant planets having satellites.

Evidently the ratio of the mass of the moon to that of the earth could be determined if the distance of the earth from the center of gravity of the earth and the moon could be found. Since it is this center of gravity which describes an elliptical orbit about the sun, it is possible to determine the distance of the earth from this point by observations of the direction of the sun from the earth. Thus, when the earth is midway between the points E_1 and E_2, Fig. 73, it is ahead of the center of gravity of the earth and the moon, and the sun has a corresponding displacement as seen from the earth. On comparing the observed position of the sun with that which it would have if there were no moon, it is possible to compute the displacement of the earth due to the moon, and then the mass of the moon.

Another method of determining the mass of the moon is from its precessional and nutational effects upon the earth. The constants of precession and nutation can be computed for various possible masses of the moon, because the mass of the earth, the oblateness of the earth, and the distance of the moon are all accurately known. Hence, from the observed values of the precession and nutation the mass of the moon can be found.

The mass of the moon, as determined by Simon Newcomb and others, is 1/81.56 of that of the earth, or 7.3×10^{25} grams (8×10^{19} tons), with an uncertainty of about one part in 2000. No other planet has a satellite which is relatively so large, though several satellites are more massive than the moon.

147. Rotation of the Moon. Observations show that the moon always presents almost exactly the same face toward the earth; therefore it rotates in the same direction it revolves and the period of its rotation equals that of its revolution. This agreement in

periods is not simply approximate; there has been no departure from equality between them in the thousands of revolutions during which the moon has been observed by astronomers.

It is extremely improbable that the equality of the periods of rotation and revolution of the moon is accidental, and the fact that some of the satellites of other planets have corresponding equalities adds to the high probability that this remarkable relationship is due to some physical cause. It is not easy, however, to discover the cause of the phenomenon. One suggested explanation is that the earth raised tides in a once viscous moon and now holds the elongated body in its present relationship by tidal friction (Art. 188). But the sun also has tidal effects upon the rotation of the moon which should make the moon's period of rotation slightly longer than its period of revolution. It is probable that there are causes as yet unknown which operate to keep the same face of the moon toward the earth.

148. Librations of the Moon. Although the period of rotation of the moon is equal to its period of revolution, our satellite does not always present *exactly* the same side toward the earth. In the first place, the axis of rotation of the moon is inclined 6° 41′ to the perpendicular to the plane of its orbit, just as the axis of the earth is inclined 23° 27′ to the plane of the ecliptic. A result of this inclination is that at one time in the month we are able to see 6° 41′ beyond one of its poles, while half a month later we can see an equal distance beyond the other pole. This is the *libration* of the moon in latitude.

In the second place, the moon rotates on its axis at a uniform rate but revolves in its orbit in such a way that the law of areas (Art. 107) is satisfied. The constant rate of rotation combined with the variable rate of revolution produces a libration in longitude as follows: Suppose the moon, starting at its perigee, revolves 90° in its orbit. Since during this part of its motion its distance from the earth is less than its mean distance, the 90° will be described, in accordance with the law of areas, in less than one-fourth of its period. Consequently, it will have rotated less than 90°, and a zone on its side toward the perigee of its orbit will be brought temporarily into view. As the moon approaches the corresponding position on the other side of its orbit, the libration makes visible an equal zone, each being about 7° 45′ wide.

Finally, the moon is not observed from the center of the earth but from various places on its surface. When the moon is rising, an observer can see about 1° farther over its eastern margin than he can when it is on his meridian; and when it is setting he can see 1° farther over its western side. On taking all the librations of the moon into account, we find that 41 per cent of its surface is always visible from the earth, 41 per cent of it is never visible, and 18 per cent of it is sometimes visible and sometimes invisible.

149. Density and Surface Gravity of the Moon. Since the volume and the mass of the moon are both known, its density can be computed. It is found that its average density is about 3.4 times that of water, or about 60 per cent of that of the earth. The average density of the moon is about 30 per cent more than that of the surface rocks of the earth.

In accordance with the law of gravitation (Art. 173), the surface gravity of a spherical body is proportional to its mass divided by the square of its radius. Therefore, if g_E and g_M represent the surface gravity of the earth and the moon respectively, and if R_E and R_M represent their respective radii, it follows that

$$\frac{g_M}{g_E} \quad \frac{M/R_M^2}{E/R_E^2} = \frac{M}{E}\left(\frac{R_E}{R_M}\right)^2 = \frac{1}{6}.$$

That is, by spring balances a body would weigh about one-sixth as much on the surface of the moon as on the surface of the earth.

If a body were thrown up from the surface of the moon with a given velocity, it would rise six times as high as it would if thrown up from the surface of the earth with the same velocity; a jumper on the moon could jump six times as high and six times as far as he could on the earth, assuming that the lunar conditions would not impair his physical activity.

XIII. QUESTIONS

1. Verify the westward diurnal motion of the moon.
2. Verify the eastward motion of the moon with respect to the stars, and estimate its rate of motion by determining its daily retardation in crossing the meridian.
3. Verify the variation of the moon in declination by determining its altitude when it crosses the meridian on successive nights.
4. If the time of an eclipse which took place 2400 years ago is uncertain by one hour, how great an uncertainty does it introduce into the length of a month based on the interval from it to a recent eclipse?

5. Suppose the moon is at first quarter. At what time of the day does the moon rise? Set?
6. Suppose an observer is on the moon. (*a*) Does the sun rise and set? If so, in which direction does it move and what is its period from meridian to meridian? (*b*) Do the stars rise and set? If so, in which direction do they move and what is their period from meridian to meridian? (*c*) Does the earth rise and set? If so, in which direction does it move and what is its period from meridian to meridian?
7. Suppose the date is December 21, the moon is at the full phase, and the ascending node of the moon's orbit is at the vernal equinox. Draw the diurnal circle of the moon (Fig. 68) for an observer in latitude 40° north. Draw the corresponding diurnal circle of the moon when the ascending node of its orbit is at the autumnal equinox.
8. Draw a quadrant of the orbit of the earth and put on the diagram the orbit of the moon to the same scale.
9. Draw the earth and the orbit of the moon (assuming that it is circular) to the same scale.
10. Draw circles representing the earth and the moon to the same scale.

150. The Moon Has No Atmosphere. Several distinct kinds of observational evidence establish the fact that the moon does not have an appreciable atmosphere. In the first place, the illuminated part of its surface is always clearly visible (if the atmosphere of the earth above the observer is transparent), with no obscuration from lunar clouds or haze from a lunar air, and its margin is as clearly seen as its center. In contrast, the surface of the planet Venus is always completely or almost completely hidden by clouds, while that of Mars is often partially dimmed by an atmospheric haze.

A more convincing proof that the moon has no atmosphere is the fact that when it passes between the earth and the sun, its margin is black and sharp and not surrounded by a luminous atmospheric ring. On the other hand, Venus is encircled by a conspicuous luminous ring when it is nearly between the earth and the sun (Art. 209).

That the moon has no atmosphere is also shown by the fact that when it passes between the earth and a star, it cuts off the light of the star suddenly and not somewhat gradually. But when a planet having an atmosphere occults a star, the light of the star is gradually reduced before it completely disappears.

Whatever may have been the origin of the moon, it is reasonably certain that an abundance of gases suitable for an atmosphere have been released on its surface. If there have been volcanoes on the

moon, they have probably poured forth enormous quantities of gaseous materials. Further, meteors strike the moon as they do the earth and give up their occluded gases, which we know from terrestrial experience often amount to several times their own volumes. Yet these sources have not given the moon an appreciable atmosphere in the many hundreds of millions of years that it and the earth have been revolving together around the sun.

It is probable that the moon lost its atmosphere a molecule at a time, for its velocity of escape (Art. 91) is only about 2.4 kilometers (1.5 miles) per second. This velocity is considerably above the mean velocities of molecules of an atmosphere under such conditions as prevail on the surface of the moon, but a very small percentage of the velocities would be beyond this limit. The theory is not sufficiently complete to place the conclusions beyond all question. Yet, as was stated in Art. 91, all known bodies having surface gravities not much exceeding that of the moon are similarly without atmospheres, while those with surface gravities exceeding that of the earth all have extensive atmospheres. Apparently the velocity of escape critical for the retention of an atmosphere lies between that of the moon and that of the earth.

151. Light Received by the Earth from the Moon. All the light received from the moon is evidently reflected sunlight, for the moon is totally dark where it is not illuminated either directly by the sun or more dimly by sunlight reflected from the earth. Moreover, when light from the moon is analyzed by the spectroscope it is found to have the qualities of sunlight.

When the full moon is high above the horizon on a clear winter's night, it appears to be so brilliant that it is difficult to realize how feeble its light is in comparison with that of the sun. It follows from its small size and great distance from the earth that even if it were a perfect mirror, at the full phase it would reflect to the earth only about 1/100,000 as much light as is received directly from the sun. It is not easy to measure precisely the ratio of moonlight to sunlight, and the results obtained by different astronomers have been far from accordant. But to the human eye, which is most sensitive to yellow light, the brightness of the full moon is approximately 1/500,000 that of the sun; and to the ordinary photographic plate, which is most sensitive to blue light, the brightness of the somewhat yellowish full moon is only about

1/650,000 that of the sun. We ordinarily much overestimate the light from the moon because our eyes automatically make a large adjustment for variations in illumination.

Light is not reflected equally in every direction from the surface of the moon, but largely in the direction of the perpendicular to its surface. The result is that when the moon is full it is much brighter, relatively to the amount of illuminated surface visible, than it is at other phases. For example, at half moon the brightness of moonlight is only one-ninth that at full moon although the visible illuminated area in the former is half as great as in the latter. The surface brightness of the moon falls off steadily from the center of its disk to its margin, and moonlight declines from the full phase to the new, even after allowing for the decreasing illuminated area.

On taking into consideration the varying amount of light received by the earth from the moon throughout the synodic month, it is found that, on the average, this light does not exceed 1/2,500,000 that received from the sun. In other terms, the earth receives more light from the sun in 13 seconds than it does from the moon in an entire year.

152. Temperature of the Moon. It is improbable that the surface of the moon receives any appreciable amount of heat from its interior. Consequently its surface temperature depends upon the amount of radiant energy it receives from the sun, the percentage it reflects, and its rate of radiation at a given temperature.

On the average the moon receives about as much radiant energy per unit area as the earth, for on the average the two bodies are the same distance from the sun. It follows from its apparent brightness that the moon reflects about 9.3 per cent of the light that falls upon it, and absorbs the remaining 90.7 per cent. If the rate of radiation of its surface materials at a given temperature were known, it would be possible to determine how high their temperature would rise before radiation would equal the radiant energy absorbed. The radiating properties of these materials are unknown, yet the temperature of the surface of the moon can be found with considerable approximation.

Two methods have been used for determining the temperature of the moon. The first depends on measuring the radiant energy received from the moon when it is totally eclipsed. Suppose the

moon is near the point M_3, Fig. 70, just entering the shadow of the earth. The center of the side toward the earth is directly under the sun, and the temperature at that point is near its maximum value. As soon as the direct sunlight is cut off, the only heat received by the earth from the moon is that which is radiated from its heated surface. S. P. Langley developed an instrument by means of which this heat can be measured, and from results obtained by its use he and others have estimated the temperature of the surface of the moon.

The second method, which was initiated and perfected by W. W. Coblentz, depends upon the fact that the reflected radiant energy is largely light, while the energy radiated is heat energy of much longer wave-lengths. In the path of the rays received from the moon there is interposed an absorbing material which cuts off the shorter reflected rays and transmits the longer radiated rays, at least over a considerable range of wave-lengths. With this instrument the heat received can be determined not only when the moon is eclipsed but also at any other time when it is visible. In fact, the amount of heat received from different parts of the surface of the moon can be measured.

Although the amount of energy the moon radiates can be readily measured, it is not easy to determine its temperature from the results, for there is no means of finding how much its rough and broken surface falls short of being a normal radiator. According to the recent results (1930) of E. Pettit and S. B. Nicholson, at lunar noon the maximum temperature of the surface of the moon is probably about 120° C. (250° F.); and just before sunrise it probably reaches a minimum as low as −100° C. (−150° F.).

It is now possible to form at least a rough idea of the climatic conditions on the moon. During the long lunar night the surface of the moon loses most of its heat, for it has no blanketing atmosphere to retard radiation, and its temperature at sunrise is far below the temperature ever reached on the surface of the earth. Since the length of the lunar day is about 29.5 times that of the terrestrial day, a lunar sunrise is a very slowly developing phenomenon, an hour being required for the sun to rise through its diameter. When the sun is finally above the lunar horizon, the moon is as light as at noon, for there is no atmosphere to reduce the intensity of slanting rays and there are never any clouds to cut them off entirely. The temperature of the surface mounts

rapidly and passes the freezing point within two or three hours after the sun rises. As the temperature becomes higher, the rate of radiation increases until it equals the rate at which energy is received from the sun; but before equality is reached the temperature exceeds the boiling point of water at all places where the sun passes near the zenith. During the long afternoon the temperature steadily declines and falls below the freezing point before the sun sets; and then, during the long lunar night, it descends far toward the absolute zero.

The remarkable differences between surface conditions of the earth and the moon are due primarily to the fact that the moon has no atmosphere, and secondly to the fact that its day is 29.5 times that of the earth. How different the sky would appear as seen from the moon! The sun and the stars would be three times as bright as they are when seen from the bottom of the atmosphere of the earth; and the earth, a circle nearly 2° in diameter, would run through half its phases as it made its leisurely journey from one lunar horizon to the other. Moreover, since the moon has no atmosphere, its sky would be as black by day as at night, and the whole vault above would always be sprinkled with about 40,000 stars visible to the unaided eye. Moreover, there would be no atmospheric waves causing the stars to twinkle, and consequently the disks of the sun and the planets could be enormously magnified by optical instruments.

153. General Surface Conditions on the Moon. On the whole, the surface of the moon is extremely rough, the side toward the earth being broken by several mountain chains, dotted by numerous isolated peaks, and largely covered by more than 30,000 craters. There are, in addition, several large relatively smooth and level areas, which were called *maria* (seas) by Galileo and other early observers.

The dark markings which produce the "man in the moon" or the "woman in the moon" to the unaided eye are due to the grayish maria; the contrasting light areas are the regions covered by the craters. For example, the large, dark, nearly circular area shown near the bottom of the photograph reproduced in Fig. 74 is Mare Imbrium; the smaller dark oval to the left of it and slightly higher is Mare Serenitatis. The bright area at the upper margin of Mare Imbrium consists of the great crater Copernicus and the

immediately surrounding rough region; the conspicuous light area near the upper central part of the photograph is made up of the great craters Clavius and Tycho and thousands of smaller ones, ranging down to and probably beyond the limits of visibility with the largest telescopes.

As to the condition of the moon beyond the range of telescopes, there are, of course, some uncertainties. But in view of its larger



FIG. 74.—Moon at 14.5 days. (The Yerkes Observatory.)

rugged features and the fact that neither water nor an atmosphere has disintegrated its rocks, it is clear that the surface of the moon contains no true soil. It is probable, however, as C. P. Olivier has pointed out, that the daily impacts of millions of small meteors throughout countless ages have produced a fine powder which covers the entire lunar surface. Moreover, such impacts continually wear down the lofty mountains and the rugged walls of the

great craters. In confirmation of this inference, large telescopes reveal some mountains and craters standing with sheer walls as though they had recently been formed, and others apparently worn and reduced to mere stumps of their original structures.

154. Lunar Mountains. There are ten ranges, or lines, of mountains on the portion of the moon which is visible from the earth. With the exception of the Apennines (Figs. 75 and 76),

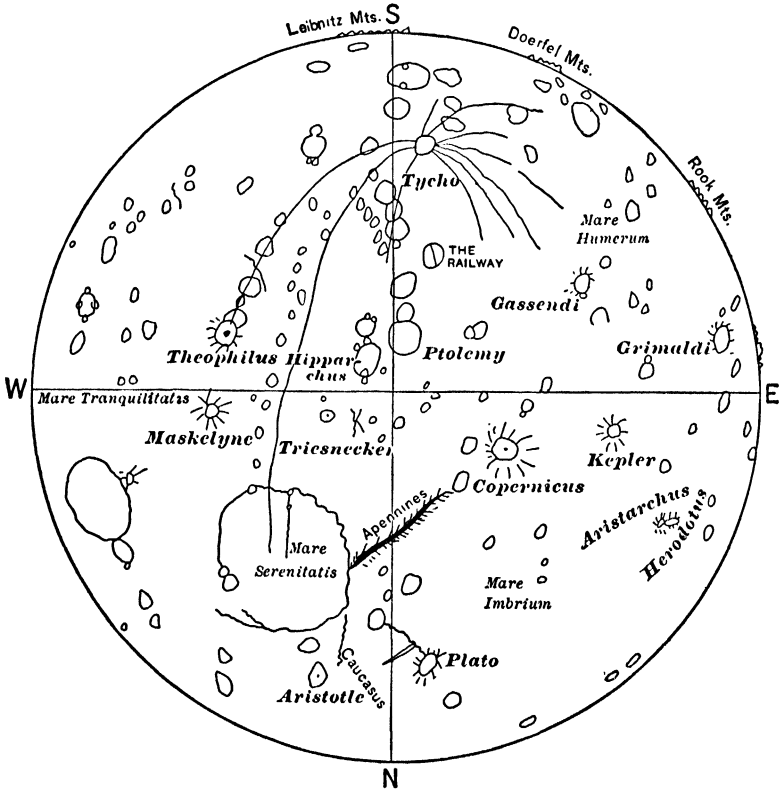


FIG. 75.—Outline map of moon as seen through an inverting telescope.

which are about 400 miles in length, they are relatively short as compared with many on the earth. Nor are the mountains of the moon great series of approximately parallel ridges, as is the case with several terrestrial mountain systems, suggesting that they may be wrinkles on a slowly shrinking body. The lunar mountain chains are remarkably narrow and sharp, like the walls

of the craters, and the loftiest peaks extend up above the surface of the moon 6000 or 8000 meters (20,000 or 26,000 feet).

There are also on the moon many isolated mountains that stand quite alone upon smooth plains. These solitary peaks, which

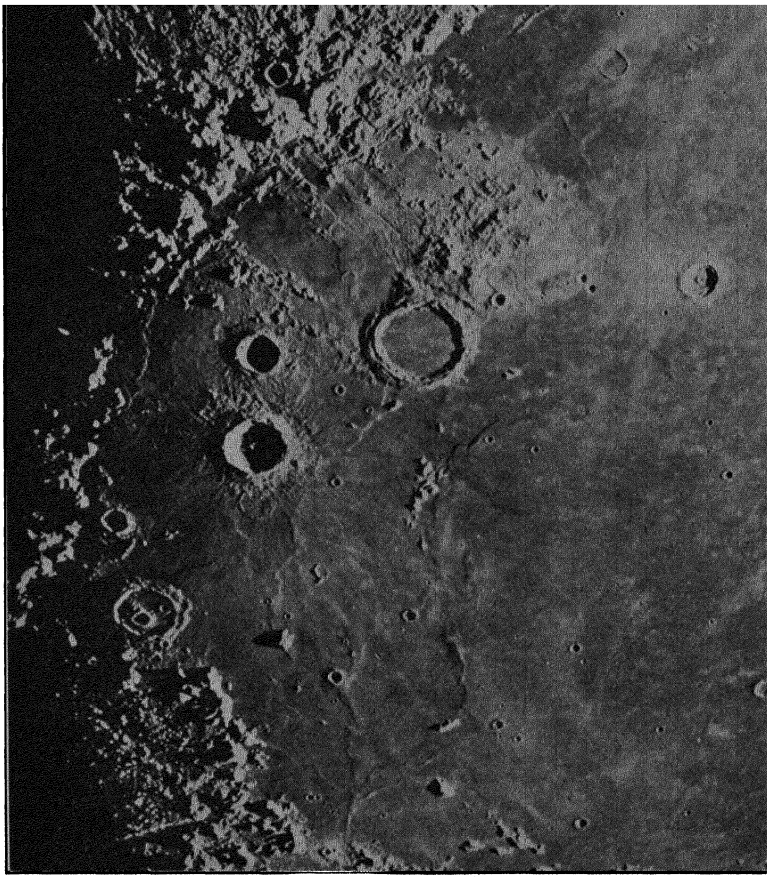


FIG. 76.—Crater Copernicus just above center, Mare Imbrium below, and the Apennines to the left. (Mount Wilson Observatory.)

have no counterpart on the earth, often rise precipitously from their bases to altitudes of 5000 to 6000 meters (16,000 to 20,000 feet). The heights of the lunar mountains are determined from the lengths of their shadows and the known angular distance of the sun above the moon's horizon when the observations are

made, or from their distance from the sunrise line when their peaks are first touched by the sun's rays.

In Fig. 76 there are visible several isolated mountains, as well as others that are involved in the rugged wall which borders Mare Imbrium, and their black shadows are seen extending toward the left. The larger lunar craters often contain one or more conical mountains in their interiors. Such mountains can be seen in nearly all the photographs of the moon, and particularly in that of the Crater Theophilus and its surrounding region (Fig. 77).

Since the moon is much smaller than the earth, it at first seems remarkable that the lunar mountains are relatively so lofty and often so spirelike. Perhaps they are due to causes which are entirely different from those which have produced terrestrial mountains. It seems particularly likely that the isolated lunar peaks have a peculiar origin not paralleled on the earth. However that may be, given forces would push matter up six times as high on the moon as on the earth, for the surface gravity of the moon is one-sixth that of the earth. Moreover, for the same reason mountain masses could be piled six times as high on the moon without crushing the underlying materials at their bases.

155. Lunar Craters. The most remarkable and conspicuous features of the lunar topography are the so-called craters, of which more than 30,000 have been mapped. They are generally circular in shape and range in diameter from about 200 kilometers (120 miles) down to the limits reached with the largest telescopes, and it is quite probable that there are hundreds of thousands of others too small to be observed from the earth. Small lunar craters are similar in appearance to terrestrial volcanic craters, but the larger ones are great walled plains of such vast extent that their rims, though they are from 3000 to 6000 meters (10,000 to 20,000 feet) in height, would be invisible from their centers because of the curvature of the surface of the moon.

It is perhaps unfortunate that the term "crater" is applied to these lunar features, for the word at once suggests that they are unquestionably similar to terrestrial craters, but they differ from our volcanic craters in a number of important respects. In the first place, the greater craters on the moon are a hundred times larger in area than the largest craters on the earth. In the second place, they are not pits on the tops or in the sides of mountain

cones, but they are great depressions whose floors are usually far below the level of the surrounding regions; and, moreover, they

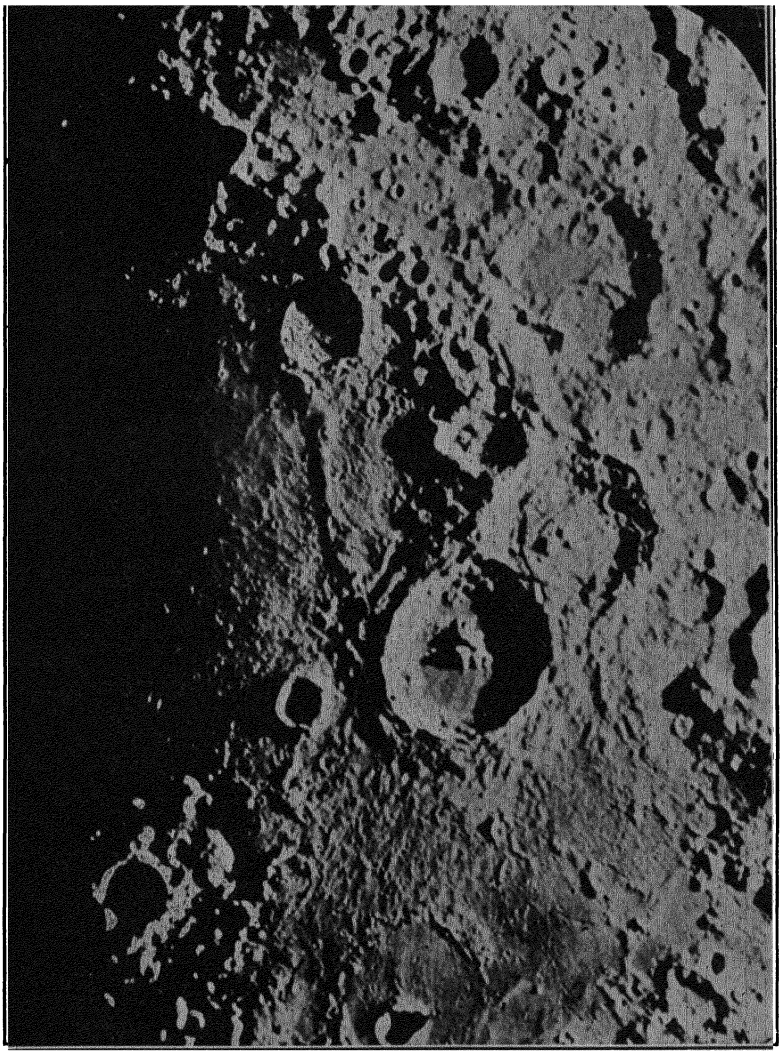


FIG. 77.—Crater Theophilus. (The Yerkes Observatory.)

often have lofty mountains in their centers. In the third place, they overlap one another in a very irregular way, little craters often appearing in the interiors of large ones, and sometimes on

their very rims (Fig. 78). Finally, there are no clear indications of lava flows or erupted cinders, and often the rims and surrounding scattered materials would not fill the crater depressions. On the

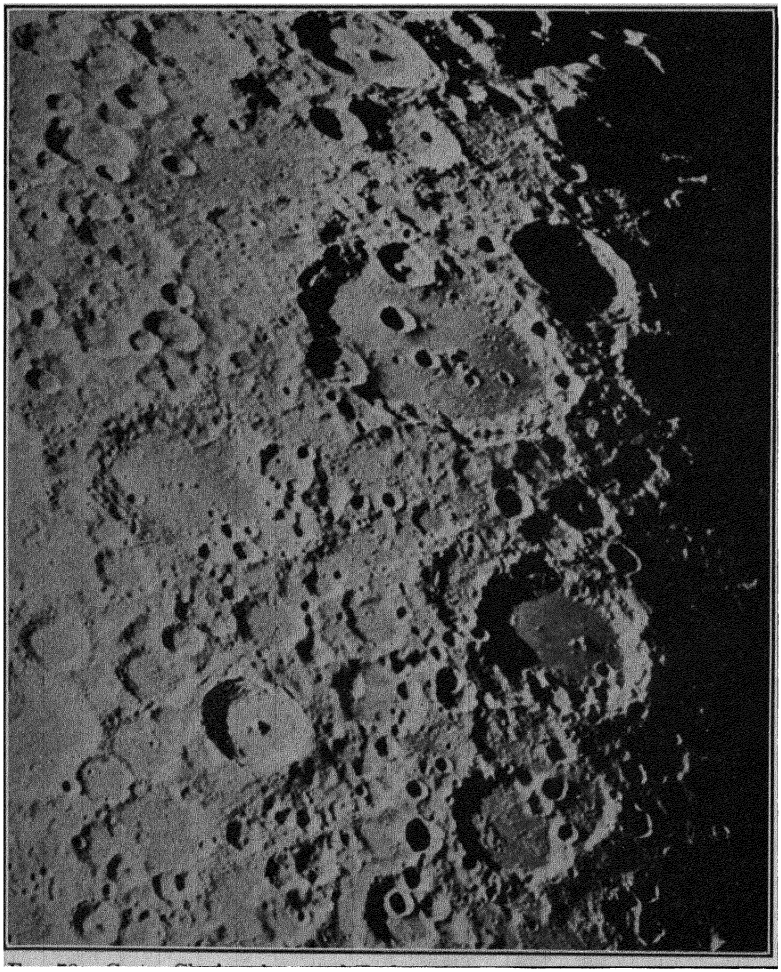


FIG. 78.—Crater Clavius above and Tycho below. (The Yerkes Observatory.)

other hand, the lunar craters are similar to volcanic craters in that they are circular in form and have steep interior walls.

It has not been found easy to formulate a wholly satisfactory theory of the origin of the craters of the moon. It has usually been assumed that they have been produced by volcanic activities, but

their dimensions, as well as their other characteristics, show that they are at best only remotely analogous to terrestrial volcanic craters. If the earth had craters on a corresponding scale, they would be gigantic circular depressions as large as some of the eastern states or the smaller European countries. The steep and crumbled walls of the lunar craters indicate that they were produced by extraordinarily violent forces (Fig. 77); yet abundant ejected material in most cases is certainly lacking (Fig. 78).

An alternative theory is that the lunar craters have been produced by the impacts of meteorites ranging in size from small

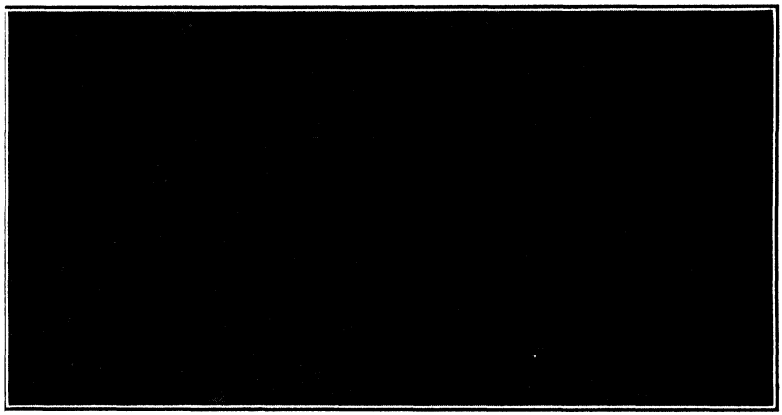


FIG. 79.—Meteor crater near Winslow, Arizona.

masses up to objects perhaps many kilometers in diameter and weighing millions of tons; or that, perhaps, the larger craters have been produced by dense swarms of meteorites having aggregate masses of millions of tons. Certainly, many millions of small meteors plunge daily into the atmosphere of the earth (Art. 245) and are consumed, while now and then larger masses reach the earth's surface and are recovered.

In Arizona there is a great circular depression over 1200 meters (4000 feet) in diameter and about 200 meters (600 feet) deep. D. M. Barringer has conclusively shown that this enormous pit was produced by a huge meteorite or swarm of meteorites, probably within a few thousand years, and many hundreds of its parts have been found in the vicinity. Nearly 400,000,000 tons of limestone and sandstone rock were shattered and pulverized by the impact of probably several hundred thousand tons of matter plunging upon

the earth from interplanetary space with a speed more than 20 times that of a projectile from a high-power gun. The photograph of this crater, Fig. 79, resembles the photographs of the smaller lunar craters and supports the hypothesis that they, too, are of meteoric origin. If this theory is correct, the maria have been produced by relatively recent impacts of large open swarms, which have obliterated all earlier surface markings where they fell and left smooth plains surrounded by high walls.

An objection often advanced against the foregoing theory is that most of the lunar craters are sensibly circular in form, although many meteorites would strike obliquely and apparently should produce elliptical scars. But the enormous kinetic energy suddenly released when a large meteor strikes the moon produces an effect equivalent to a violent explosion, and the result is a circular crater whatever the direction of impact (Art. 250). The Arizona crater is almost circular, although the evidence indicates that the meteor fell at an angle of approximately 45°.

Another important objection to the theory is that the 30,000 craters visible on the moon should be distributed nearly uniformly, according to the laws of probability, if they have been produced over a long interval of time by impacts on a rotating body. Unless the maria have been formed in relatively recent times by open swarms of meteors, as was suggested in an earlier paragraph, this objection to the theory is very grave. The objection that similar large craters are not now found upon the earth is less weighty, for such surface features, as vast as they are, would be completely destroyed by the action of air and water in periods that are brief in comparison with the geological ages. But on the moon they remain, whatever their origin, as illustrations of the magnitudes of the forces which act on these bodies in their early history.

156. Rays and Rills. From several of the large lunar craters, particularly from Tycho and Copernicus (Fig. 74), long light-colored streaks, called *rays*, radiate like spokes from the axle of a wheel. These remarkable lunar features, which are not interfered with by valley or mountain, often extend from the craters to a distance of several hundred kilometers, and even up to 2000 kilometers (1300 miles) in the case of some of those radiating from Tycho. In width they range up to 20 kilometers (13 miles). It is practically certain that these markings are neither depressions

nor elevations, for they are never darkened by shadows from their walls nor do they ever cast shadows. They are simply materials which are lighter colored than the surrounding regions; and since they are visible only because of a contrast in color, they are seen better at full moon than at any other lunar phase.

It is evident that the rays date from the time their associated craters were formed, and it seems probable that they depend in some way upon what were once cracks in the rocky structure of the moon, though it is not clear just what they are now. In any case, they bear evidence of the violence of the forces which produced the craters.

In contrast to the rays are the *rills*, which are narrow, crooked valleys, or cracks, a kilometer or two wide and of unknown depth. Maps have been made of more than a thousand of these remarkable chasms, ranging from a few kilometers up to several hundred kilometers in length. They have no apparent relationship to lunar mountains or craters; they seem to be simply cracks formed in a cooling and shrinking surface.

That the surface of the moon has been modified by so-called faulting and slipping of rocks is certain, for there are several instances in which there are perpendicular cliffs on fairly level plains. After the fracture of the rocks along a line, one side has settled hundreds of meters with respect to the other, and these perpendicular cliffs cast black shadows when the sun is in one direction, and their faces are brilliantly illuminated when it is in the other.

157. Is the Moon's Surface Now Undergoing Changes? There have been no observed changes in the larger features of the lunar topography, although, from time to time, minor alterations have been suspected. The two or three centuries during which the moon has been observed through telescopes are such a brief instant in comparison with the immense periods of the cosmic processes that no observable changes on its surface should be expected as yet. It is probable that the 30,000 craters which have been mapped have been formed during a period stretching back over 300,000,000 years; and it may well be that the forces which produced the craters are much less active now than they were in the remote past.

The moon, however, is subject to influences which have appreciable effects upon the finer structures of its surface. As has been remarked, millions of meteors rain daily upon its mountains,

plains, and craters, which are wholly unprotected by water or an atmosphere, and gradually pulverize their rocks. Moreover, the great changes in temperature from night to day have a somewhat similar effect, but the seasonal changes are relatively unimportant. Finally, the tides of the earth and the sun continually distort and knead the body of the moon, through and through, and probably promote readjustments of its mass. Although, as compared to the earth, the moon is a dead and changeless body, yet it had its birth when the earth was born; it now undergoes the slow changes of advancing age; and it will meet its final destruction with that of the earth (Art. 305).

158. Effects of the Moon Upon the Earth. Obviously the moon, particularly when it is full, reflects a large amount of light upon the earth. But in comparison with sunlight it is of little importance (Art. 151). Consequently the belief that the moon has appreciable direct effects upon the farmer's crops can have no foundation in fact. The passing of a fleecy cloud or the haze of an autumn day cuts off more heat from the sun than the earth receives from the moon in an entire season. Moreover, accurate observations of the weather extending over long periods of time show no certain relation between climatic conditions and the phases of the moon.

There are, however, several observable effects which the moon has upon the earth, the most important of them being the tides. There are also others, such as the precession of the equinoxes (Art. 97), the nutation, small variations in the magnetic forces, and small departures of the earth from elliptical motion. With the exception of the tides and the precession of the equinoxes, these effects are so small that they can be observed only by refined scientific methods.

One of the most majestic phenomena observable upon the earth is the flow and ebb of the ocean tides. Twice in a period of 24 hours and 51 minutes (the interval required for the moon to move westward from a meridian around to the meridian again) the waters of the open seas rush in upon the continental shores and recede again. The variations in water level as a consequence of the tides range from a meter or two to 20 or 25 meters. These tidal surges do not advance and recede again with no effects except their transitory action upon the shore; part of the energy of their rushing waters is changed into heat and is eventually radiated

away into space. It can be shown that this radiated and lost energy is in part at the expense of the rotation of the earth and in part at the expense of the revolution of the moon. The result is that the rate of rotation of the earth is very slowly decreased (the day becomes longer) and the length of the month is increased. These effects can not be observed in a year or a century; they accumulate and become important only with the lapse of the longer periods of geologic time. Though we have not observed them directly and have only obtained indications of their existence from the examination of ancient eclipses (Art. 100), yet we can follow them with our mathematical analysis across hundreds of millions of years.

XIV. QUESTIONS

1. Suppose the moon is passing between an observer and a star. If the lunar atmosphere were 10 miles deep, how long after it began to cut off the light of the star before the solid part of the moon would eclipse the star?
2. For an observer on the moon would the sky be blue in the daytime? Would stars be visible in the daytime?
3. The formula for the area of a sphere is $4\pi r^2$. Since a radian equals 57°3 ($360° \div 2\pi$), the formula will give the area in square degrees if 57°3 is used for r. If the whole sky as seen from the earth were covered with full moons, how would the illumination compare with that of sunlight?
4. What effect would meteoric dust upon the surface of the moon have upon its reflecting power? Upon the depth of heating during the long lunar day?
5. Are shadows darker on the moon or on the earth? If there is a difference, what is the reason for it?
6. Examine the photographs of the moon reproduced in this chapter and determine the number of lunar craters shown which depart sensibly from circularity.
7. How many instances are there of smaller craters directly on the rims of larger ones?
8. Are there examples of apparently recently formed craters interior to apparently older craters?
9. Are there instances of apparently old craters whose rims are now recognizable only with difficulty?
10. What objections are there to the theory that the lunar craters are of volcanic origin?
11. What objections are there to the theory that the lunar craters have been produced by meteorites or by swarms of meteorites?
12. How do you explain the mountains which are often found in the centers of lunar craters?
13. Are there instances in the photographs in which the rims of craters have apparently been broken by obliquely descending meteorites? Could many such instances be reasonably expected?
14. Are the rays more easily explained under the theory that the lunar craters are volcanic in origin or that they have been produced by meteorites?

CHAPTER VIII

ECLIPSES

159. Conditions Necessary for an Eclipse. Since the earth and the moon are both non-luminous, they cast conical shadows which project directly away from the sun. Thus, in Fig. 80, S represents the sun, E the earth, and CPD the earth's shadow. Four positions of the moon are shown at a, b, c, and d, together with their shadow cones. The complete shadow CPD, in which there is no sunlight except a little that may be refracted into it through the marginal zone CD of the earth's atmosphere, is the *umbra;* the regions

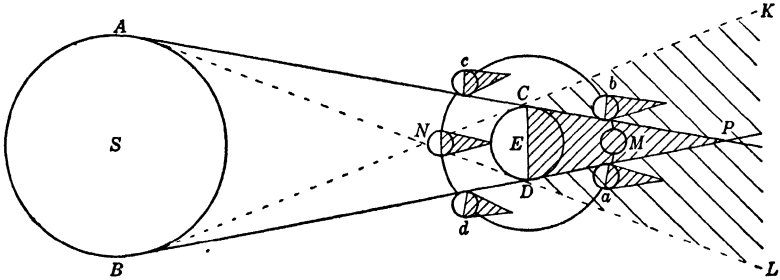


FIG. 80.—Conditions for lunar and solar eclipses.

KCP and LDP, from which light from only a part of the sun is cut off, are parts of the *penumbra.* The illumination in the penumbra decreases gradually from the full light of its outer borders to complete darkness where it merges into the umbra. The shadow of the moon consists similarly of an umbra and a penumbra.

When the moon is entirely in the earth's umbra, as at M, it is totally eclipsed; when it is partly outside the umbra, it is partially eclipsed. The eclipsed moon is visible from an entire hemisphere of the earth.

A total eclipse of the sun occurs if the umbra of the moon falls upon a portion of the earth's surface, as at N, Fig. 80. A partial eclipse of the sun occurs if the penumbra of the moon falls upon a portion of the earth's surface. It is evident from the figure that the sun is totally eclipsed at any time only in a relatively small region

along which the narrow cone of the moon's shadow strikes, and that surrounding this region there is a larger zone in which it is partially eclipsed. During the time of a solar eclipse the moon advances in its orbit and the earth rotates on its axis; hence the region in which any particular eclipse occurs is a streak across the earth, a few hundred kilometers wide at the most.

The umbra of the moon may come to a point before it reaches the surface of the earth. Then in the inverted cone beyond the point of the umbra (to the right in Fig. 80), the sun is visible entirely around the disk of the moon and there is an *annular* (ring) eclipse; elsewhere in the penumbra the eclipse is partial, but not annular.

160. Limits for Lunar Eclipses. If the orbit of the moon were in the plane of the ecliptic, as represented in Fig. 80, there would be a lunar eclipse every synodic month; but the inclination of the orbit of the moon to the ecliptic is 5° 9′, as illustrated in Fig. 81, in which E represents the earth, A and B the ascending and the descending node respectively of the moon's orbit, S_1, ..., S_4 four positions of the earth's umbra at the distance of the moon, and M_1, ..., M_4 the moon in the corresponding four longitudes. The points are so chosen that the moon just touches the umbra of the earth. It follows from the dimensions of the sun, the earth, and the moon, and their distances apart that a lunar eclipse can occur only if the moon passes the earth's umbra while it is on either the arc S_1S_2 or S_3S_4, each of which is about 21° in length.

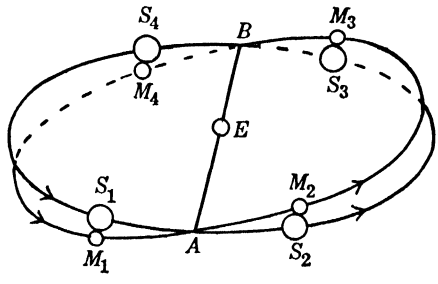


FIG. 81.—Lunar and solar eclipse limits.

Since on the average the earth moves about 29°.1 along its orbit in a synodic month, it follows that if the moon is full two or three days before the earth's umbra arrives at S_1, then it will not be full again until after the earth's umbra passes S_2. In this case there will be no lunar eclipse in the half year during which the earth passes the node A.

In five synodic months after the earth's umbra is at S_2 it will

move forward 145°.5, and in the same interval the node B of the moon's orbit will regress about 8°. Hence the moon will pass the earth's umbra near S_3 at $10°.5+145°.5=156°$ beyond A and $180°-(8°+156°)=16°$ before it arrives at B. There will, therefore, be no eclipse of the moon; nor will there be an eclipse 29.5 days later when the moon is full near S_4. On the other hand, if there is a lunar eclipse near A, there will be one six synodic months later a little beyond B.

It follows from the foregoing discussion that during a year there may be no lunar eclipse, one lunar eclipse, or two lunar eclipses. In fact, if there is a lunar eclipse on or near January 1, there may be another 6 months later, and still a third before the close of the calendar year.

161. Limits for Solar Eclipses. The conditions for solar eclipses are similar to those for lunar eclipses. Since the arc cd, Fig. 80, is greater than the arc ab, it follows that the limits for solar eclipses are wider than those for lunar eclipses. For eclipses of the sun the eclipse limits S_1S_2 and S_3S_4, Fig. 81, are on the average about 33° instead of 21°, as in the case of lunar eclipses. When Fig. 81 is used for discussing solar eclipses, $S_1, \ldots, S_4$ represent the sun as seen from the earth instead of the earth's shadow. Since the solar eclipse limits are 33° while the sun moves only 29°.1 in a synodic month, it follows that there is at least a partial eclipse in each synodic month during which the sun passes a node of the moon's orbit. Since the sun passes both nodes of the moon's orbit each year, there are at least two solar eclipses, total or partial, during each year.

If there is a solar eclipse when the sun is near S_1, Fig. 81, there will be another a synodic month (29.5 days) later near S_2. After five synodic months the sun will be eclipsed near S_3, at least partially. After a synodic month there will be still another solar eclipse near S_4. Thus there may be four solar eclipses in a year; obviously the conditions may be such that there are only three. And, if a solar eclipse occurs on or near January 1, there may be five eclipses of the sun in a calendar year.

162. Characteristics of Succeeding Lunar Eclipses. Suppose there is a central eclipse of the moon when the earth's shadow is at the node A, Fig. 81. In six synodic months (177.2 days) the earth's shadow will have advanced 174°.6 and the node B will

have regressed 9°.4. Hence there will be a lunar eclipse 4° beyond the node B, and the moon will pass south of the center of the earth's shadow. After six synodic months there will be a lunar eclipse 8° beyond the node A, but in this one the moon will pass north of the center of the earth's shadow. At the next conjunction there will in general not be an eclipse of the moon. It is to be understood that the foregoing average results may be considerably modified by the effects of the eccentricities of the orbits of the earth and the moon.

163. Characteristics of Succeeding Solar Eclipses. Suppose there is a solar eclipse when the sun is at the node A, Fig. 81. If the node A happens to be at an equinox of the earth's orbit, the point A is directly above the earth's equator and the center of the eclipse path is on the earth's equator. If the point A is at the summer solstice, it is directly above latitude 23°.5 north of the earth's equator, and the eclipse is central in latitude 23°.5 north. Since the point A is on the ecliptic, it may be over any point on the earth's surface from 23°.5 north to 23°.5 south, and eclipses near A are central in corresponding latitudes.

If there is a solar eclipse when the sun is at the node A and if it is at an equinox of the earth's orbit, then the eclipse is central on the earth's equator. After six synodic months there will be an eclipse 4° beyond the node B, and this eclipse will be central south of the earth's equator. After six additional synodic months there will be an eclipse 8° beyond the node A, and this eclipse will be central north of the earth's equator. The eclipses progress toward one pole and the other, alternately, until there are two eclipses in each synodic month during which the sun passes a node of the moon's orbit. If, however, A is not at an equinox of the earth's orbit, then the succeeding eclipses diverge alternately north and south from some other latitude.

If there is a solar eclipse when the sun is near the eclipse limit S_1, Fig. 80, it is visible only in far southern latitudes on the earth. The eclipse following after a synodic month near S_2 is visible only in far northern latitudes. Eclipses of these types are likely to be annular or only partial; if they are total they are of short duration. If there is a solar eclipse when the sun is near a node, such as A, of the moon's orbit, it is likely to be total and of relatively long (3 to 7 minutes) duration.

164. Relationships Between the Characteristics of Lunar and of Solar Eclipses. Since lunar eclipses occur when the moon is full and solar eclipses when it is new, it follows that if a lunar and a solar eclipse occur in the same synodic month they are half a synodic month apart.

Suppose there is a solar eclipse when the sun is at a node A, Fig. 81, of the moon's orbit. There can not be a lunar eclipse during the synodic month, for the lunar eclipse limits are only 10°.5 from A. After six synodic months there will be a solar eclipse 4° beyond the node B. In this case the conditions half a synodic month earlier are on the borders of those under which a lunar eclipse can occur. If the various modifying factors are such that a lunar eclipse occurs near S_3, the moon passes north of the center of the earth's shadow. Six synodic months later there will be a lunar eclipse in which the moon will pass through the southern part of the earth's shadow.

If there are two solar eclipses in a synodic month, for example, when the sun is near S_1 and S_2, Fig. 81, there will necessarily be an eclipse of the moon when the earth's shadow is near the node A.

It follows from the foregoing discussions that the principal cases respecting the number of eclipses in a year are:

(*a*) There may be two solar eclipses in a year, separated in time by six synodic months, and no lunar eclipse.
(*b*) There may be two solar eclipses in a year, and one or two lunar eclipses.
(*c*) There may be three solar eclipses in a year, and two lunar eclipses.
(*d*) There may be four solar eclipses in a year, and two lunar eclipses.
(*e*) There may be three lunar eclipses (one near January 1) in a year, and four solar eclipses.
(*f*) There may be five solar eclipses (one near January 1) in a year, and two lunar eclipses.

165. Relative Frequency of Lunar and Solar Eclipses. Since the eclipse limits for solar eclipses are 33° while those for lunar eclipses are only 21°, it follows that there are about three eclipses of the sun, on the average, to two of the moon. This does not mean, however, that from any given position on the earth solar eclipses are more frequently visible than are lunar eclipses, for the opposite

is the case. The reason for this apparent contradiction is that every lunar eclipse is visible from at least half the earth, while solar eclipses are visible only from the narrow path described by the shadow of the moon across the earth (Fig. 85).

The apparent diameter of the moon, as seen from the earth, is at most only a little greater than that of the sun, while it is actually less when the earth is at its perihelion and the moon is at its apogee. It follows that the umbra of the moon comes to a point near the earth. The average width of the region in which a solar eclipse is total is approximately 100 kilometers, while its length ranges up to 10,000 or 12,000 kilometers, or an area less than 1/400 that of the earth. Hence at a given place on the earth a total solar eclipse is visible on the average of only once in nearly 400 years, but partial eclipses occur much more frequently.

Since solar eclipses are rather startling phenomena and occur at any spot only very infrequently, the circumstances of their occurrence have been recorded from very early times; and these records have in many cases been useful in fixing what would otherwise be uncertain historic dates. For example, Herodotus tells us that during a great battle between the Lydians and the Medes, the day suddenly turned to night. The combatants, thinking the sudden darkness was a sign that they were offending the immortal gods, ceased fighting and made a treaty of peace. In spite of the fact that the descriptions of the phenomena are vague, mathematicians confidently carry their calculations back across the centuries and establish the fact that the date of the eclipse and the battle was May 28, 584 B.C.

166. The Saros. In very ancient times (ancient as history goes), the Chaldeans discovered from their records of eclipses extending over many centuries that nearly similar solar eclipses occur in cycles of 223 synodic months, or 18 years and 11.32 or 10.32 days, according as there are four or five leap years in the interval. This interval is the *saros*. Since the period is 8 hours more than 18 years and 11 days, the eclipse at the end of the interval will occur 120° farther west on the earth than that at its beginning. The next eclipse will be central 240° west of the first one, and the fourth eclipse will be in the same longitude as the first of the cycle.

The Chaldeans discovered the saros from their observations of eclipses; it can now easily be derived from the characteristics of

the motion of the moon and the sun. During the period of the saros there are usually approximately 71 solar eclipses, of which about 45 are either total or annular.

167. Phenomena of Lunar Eclipses. Lunar eclipses obviously occur at full moon. When the moon, approaching the point in its orbit opposite the sun, first enters the penumbra of the earth's shadow, no change in its appearance is noticeable. But as it nears the umbra its eastern part becomes slightly dimmed; and when it enters the umbra it is almost totally darkened. Although the borders of the umbra are not sharply defined because the sun is not a point-source of light, still it is evident at the time of a lunar eclipse that the curve of the earth's shadow is at least approximately an arc of a circle. This fact was given by Aristotle as a proof that the earth is spherical.

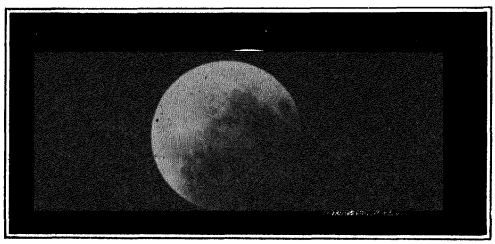


FIG. 82.—Moon at total eclipse. (E. E. Barnard.) The length of the faint star trail at the left shows that in obtaining the photograph the telescope followed the dim moon for about 12 minutes.

About an hour elapses after the moon first enters the umbra before it becomes totally eclipsed. If it passes through the center of the umbra, it is totally eclipsed for about an hour and 40 minutes, though the interval varies considerably as a consequence of the fact that the distances of the sun and the moon from the earth are different at different times. If the moon does not pass through the center of the umbra, the duration of totality is shorter. About an hour is required for the moon to emerge completely from the umbra after its eastern limb first passes into the penumbra. Consequently the interval from first contact of the moon with the umbra until the last contact may be as much as 3 hours and 40 minutes, though it is usually considerably less.

At total eclipse the moon is not entirely dark, for the earth's atmosphere acts somewhat as a lens and refracts to it enough sunlight to make it visible. Since only the red rays pass through the marginal zone of the earth's atmosphere, the eclipsed moon is copper colored.

168. Scientific Importance of Lunar Eclipses. (*a*) As was stated in Art. 152, the measurement of the heat radiated by the moon when it is totally eclipsed furnishes one of the best means for determining the temperature of the lunar surface.

(*b*) One of the most highly developed theories in the whole domain of science is that of the motion of the moon. Literally hundreds of terms are required to express the infinitely varying effects of the earth and the sun upon its position and its components of velocity. The best method of determining the constants that occur in the mathematical expressions for the moon's coördinates is to fix its position from time to time by the fact that it eclipses (occults) known stars. The brightness of the moon is so great that these occultations can be satisfactorily observed only at the times of total lunar eclipses.

(*c*) There is no dynamical reason why the moon should not now have a satellite revolving about it at a distance of a few thousand miles from its surface. If, however, the moon was once near the surface of the earth, as Sir George Darwin inferred from his researches in tidal theory, then it could have had no satellite in its early history and would not have one now. E. E. Barnard photographed the region around the moon at the time of a lunar eclipse for the purpose of discovering a lunar satellite, if one exists. Since he found no evidence of such an object, it is almost certain that the moon does not have a satellite with a diameter exceeding a kilometer.

169. Phenomena of Solar Eclipses. Solar eclipses occur precisely at new moon. The moon itself at that phase is invisible from the earth, and the first sign of the approaching eclipse is that the western edge of the sun is gradually obscured. Until the moon has eclipsed a considerable sector of the sun's disk, the fact that there is an eclipse developing would not be noticed unless one chanced to look at the sun. But when the instant of totality approaches, daylight rapidly fades, the temperature steadily falls, objects assume unusual colors, animals become restless, and birds seek their roosts. When only a thin crescent of the sun remains visible, faint rippling "shadow bands" are observed swiftly sweeping across white surfaces, such as a snow-covered field. Finally, there is the rush of the moon's shadow along the surface of the earth at the rate of more than 2000 kilometers an hour.

From a mountain top or other height it often appears like a gigantic and irresistible cloud which is swallowing up everything in its path. Just as the sun disappears, its eastern edge is for an instant broken up into "Bailey's Beads" by the irregularities of the moon's limb. Then stars flash out all over the sky, and around the apparent edge of the moon are seen the chromosphere and rose-colored prominences of luminous gas thrown up from the surface

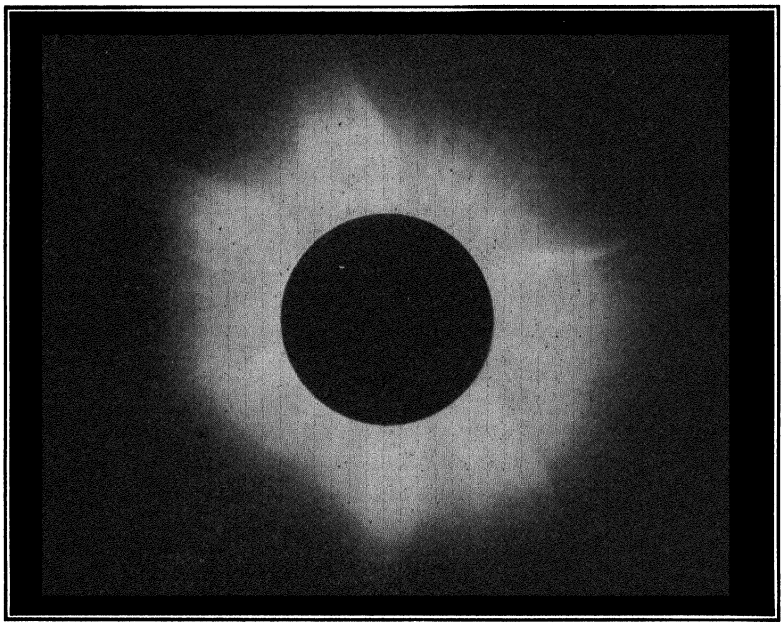


FIG. 83.—The sun's corona at total eclipse. (John A. Miller.)

of the sun to heights sometimes of 200,000 or 300,000 kilometers. And surpassing all is the corona, a crown of pearly light extending out from the sun half its diameter or more.

After about 7 minutes and 31 seconds at the most, "Bailey's Beads" appear on the western limb of the sun, the crescent sun becomes visible as the chromosphere and corona disappear, and all the phenomena of the approaching eclipse take place in the reverse order. The interval from first contact of the partial eclipse to the last is at the most about two hours.

170. Scientific Importance of Total Solar Eclipses. Total eclipses of the sun are very rarely visible at any great observatory; conse-

quently, if astronomers are to observe and photograph them, expeditions must go to the tracks of totality, even though they be half way around the world. The following are the principal reasons that observatories send out eclipse parties equipped with costly apparatus to investigate the sun during the brief intervals it is totally or partially in eclipse:

(*a*) The relative positions of the earth, the sun, and the moon can be determined more accurately from the position of the borders of the track of a total eclipse on the earth and from the times of contact, both external and internal, of the limb of the moon with that of the sun than by any other means. Such observations put the severest tests upon the accuracy of the mathematical theories of the motion of the moon. Although such highly perfected theories as that of E. W. Brown form a never-failing basis for predicting the places and times of eclipses, yet there are very slight discrepancies between the expected and the observed phenomena which may be due to shiftings of material in the earth (Art. 100). It seems that we must turn our eyes to the sky in order to find out what goes on far beneath our feet.

(*b*) If there were a planet nearer the sun than Mercury, it could be seen only with difficulty, if at all, except at the time of a total eclipse of the sun. When the moon cuts off the sun's rays from our atmosphere, even faint stars can be observed. At the times of total eclipses, photographs are taken of the region around the sun for the purpose, in part, of detecting an intra-Mercurian planet, if one exists. Photography has failed to show the existence of such an object, though it was once thought probable that one had been seen. Visual observations are hurried and made during a strange phenomenon; photography secures an impersonal and accurate record which may be examined at leisure. We conclude, therefore, that there is no planet interior to the orbit of Mercury whose diameter is as much as 100 kilometers.

(*c*) Just before the sun is totally eclipsed, only a thin crescent of its exterior layers remains visible. Astronomers avail themselves of these fleeting seconds to study the radiations of the outer layers of the sun and of its atmosphere (Art. 279).

(*d*) No feature of the sun is more remarkable or of greater interest to astronomers than the corona, which can not be detected by any means except during a total solar eclipse (Art. 292). If the moon were a little smaller than it is or a little farther away, it would

never totally cover the sun and we should be in complete ignorance of the existence of the solar corona.

171. Tests of the Predicted Curvature of Light by the Sun. One of the inferences drawn from the general theory of relativity is that rays of light are appreciably deflected when they pass close to such a massive body as the sun. The method of testing whether light is bent on passing the sun is illustrated in Fig. 84. The stars in the region in which the sun will be when an eclipse occurs are photographed either long before or long after the eclipse. Let O represent

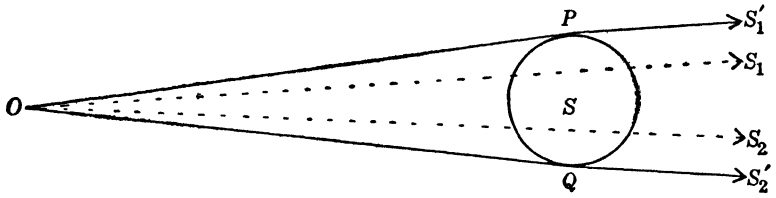


FIG. 84.—Relativity deflection of light by the sun.

the position of the observer and OS_1 and OS_2 the directions of two stars when the sun is in another part of the sky. The angle S_1OS_2 is measured, and it will be assumed that it is a little greater than the apparent diameter of the sun. When the sun has passed between S_1 and S_2 and is in eclipse, a photograph of the stars is taken again. If the sun deflects the light of stars, then certain rays PS_1' and QS_2', parallel to OS_1 and OS_2 respectively, will be bent somewhat and converge to the observer at O. The angle POQ is evidently greater than the angle S_1OS_2, and their difference is twice the angle of deflection.

Unfortunately the theoretical maximum deflection of light by the sun is only 1.″75, an angle very difficult to determine with certainty from photographs taken under the exceptional conditions prevailing at the time of an eclipse. In 1919 English astronomers photographed an eclipse which was total in Brazil, and in 1922 the Lick Observatory astronomers secured photographs of another from western Australia. After months spent in measuring the relative positions of the numerous star images on the plates and in discussing the data secured, both groups of astronomers reached the conclusion that the measurements indicate deflections of light near the sun in substantial harmony with the implications of the theory of relativity. There were, however, anomalies in the case of a

considerable percentage of the stars, some being apparently deflected too much, others at right angles to lines from the center of the sun, and some in the direction exactly opposite to that predicted from theory. Apparently the observational data are not yet fully satisfactory, and the investigation has been and is being continued as favorable eclipses occur.

172. Recent and Future Total Eclipse Paths. As a consequence of the inclination of the orbit of the moon to the plane of the earth's equator and of the rotation of the earth during an eclipse, the paths of totality streak across the earth in a great variety of curves.

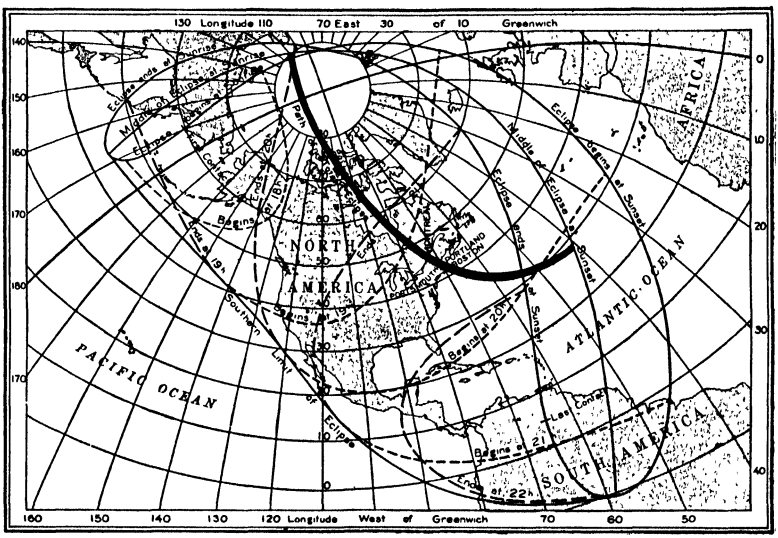


FIG. 85.—Path of total eclipse of sun, August 31, 1932.

A comprehensive view of the character of eclipse paths can be obtained from Oppolzer's great work on eclipses, in which he gave the elements of all eclipses, both lunar and solar, occurring between 1207 B.C. and 2162 A.D. His discussion includes 8000 solar eclipses and 5200 lunar eclipses, and his maps give the approximate paths of totality of the solar eclipses.

In Fig. 86, which is based on Oppolzer's work, the paths of total eclipses are given for the interval from 1918 to 2000. It is evident from this diagram that total eclipses are visible from only a very small part of the earth's surface in a period of 80 years.

When it is seen how irregular the eclipse paths are, it appears very remarkable that the ancients, nearly 3000 years ago, should have discovered such periods as the saros. Yet if the diagram is carefully examined evidences of the existence of the saros can be found. For example, on June 30, 1973, there will be a total eclipse of the sun, starting in South America, becoming central in northern Africa, and ending in the Indian Ocean. Eighteen years and 11 days later, on July 11, 1991, an almost exactly similar eclipse

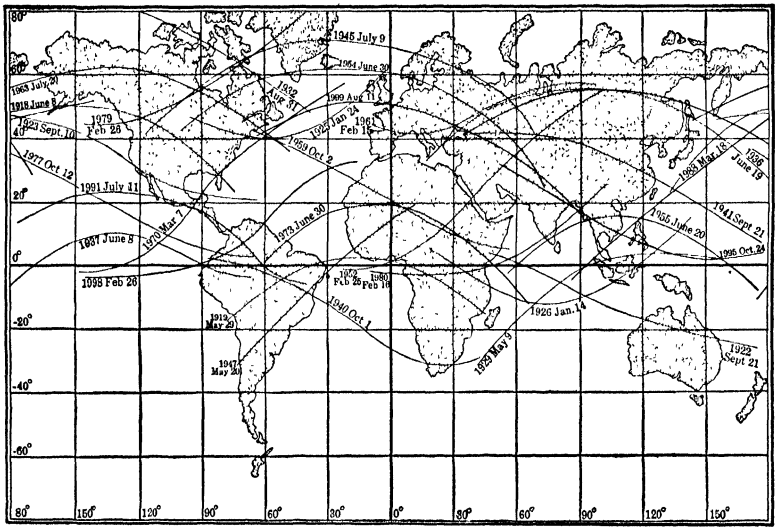


FIG. 86.—Paths of principal total solar eclipses from 1918 to 2000.

will occur about 180° in longitude west of that of 1973. Other similar pairs of eclipses differing in time by 18 years and 11 days and in longitude by 180° are those of 1919 and 1937, and also those of 1936 and 1954. It must not be expected that the saros can be followed for long intervals on this diagram, for eclipses shift with respect to the nodes of the moon's orbit, Fig. 81, until they are missed, and occasionally they are annular or partial instead of total.

XV. QUESTIONS

1. If the inclination of the moon's orbit to the ecliptic were only 3° instead of 5° 9′, what would be the lunar eclipse limits? What would be the maximum number of lunar eclipses in a year?
2. If the nodes of the moon's orbit were stationary, what would be the maximum number of lunar eclipses in a year?

3. What is the maximum inclination of the orbit of the moon in order that there should be at least a partial solar eclipse every synodic month?
4. Draw a diagram showing the part of the sun which is visible in an annular eclipse when the observer is in the axis of the moon's shadow.
5. Under what circumstances is the sun eclipsed as seen from the moon? What is the maximum duration of total eclipse? Is there ever an annular eclipse of the sun as seen from the moon?
6. Are there any positions on the moon from which an eclipse of the sun is never visible?
7. What would be the effect of the earth's atmosphere on an eclipse of the sun as seen from the moon? If its depth is 160 kilometers (100 miles), what is its angular depth as seen from the moon? Would a prominence 40,000 kilometers (25,000 miles) be likely to be visible through it? Would the corona be visible?
8. Which would give eclipses more advantageous for scientific purposes, a moon twice as large at its present distance, or one of the same diameter at half the distance?

REFERENCE BOOKS

Eclipses of the Sun (1923). S. A. Mitchell. Columbia University Press. New York, N. Y.

The Graphic Construction of Eclipses and Occultations (1924). William F. Rigge. Loyola University Press, Chicago, Ill.

Canon der Finsternisse (1887). Th. Ritter von Oppolzer. Karl Gerold's Sohn, Wien, Austria. This book contains the elements of all the eclipses, 8000 solar and 5200 lunar, occurring between 1207 B.C. and 2162 A.D., together with maps showing the approximate paths of the total and annular solar eclipses in the interval. Book is now out of print.

CHAPTER IX

THE LAW OF GRAVITATION

173. The Law of Gravitation. In simplicity, in generality, and in exactness, the law of gravitation equals or surpasses every other physical principle so far discovered; nor does any other law of nature mark so definitely the beginning of a new epoch of science. The law of gravitation is:

Every particle of matter in the universe attracts every other particle with a force which is proportional to the product of their masses, and which varies inversely as the square of the distance between the centers. As a formula, the law is:

$$f = \frac{k^2 mM}{r^2},$$

in which f is the force, m and M the masses of the two bodies, r the distance between their centers, and k^2 a constant depending on the units of time, mass, and distance used.

Strictly speaking, the law of gravitation holds exactly only for particles whose diameters are inappreciable in comparison with the distance between them. In general, bodies do not attract other bodies as if their masses were all condensed at their centers of gravity, though spheres which are homogeneous in concentric layers do attract exterior masses in precisely this way.

A law of nature is important in proportion to the number of phenomena it describes and coördinates, and in proportion to its usefulness in making predictions and in serving as a basis for drawing inferences in hitherto unknown domains. Let us measure the law of gravitation by these standards.

In generality, the law of gravitation attains the ultimate limit, for it describes the force relation between every two particles in the universe. Since the motions of bodies depend upon the forces to which they are subject, the law of gravitation is involved in the theory of the motion of every mass of matter, whether it be the distant stars, mechanical devices on the earth, or our own brains. It not only gives a basis for treating the motions of individual bodies, but it implies that the shape of every body is modified by

stresses which depend upon the positions of all other bodies. It follows that the shape of the earth is changed (of course very slightly) by a change in the position of a star a hundred thousand light-years away.

In exploring the unknown, the law of gravitation has been perhaps the most important scientific principle ever discovered. Moreover, it was the first great scientific generalization to illuminate whole domains that had hitherto been shrouded in mystery. Isaac Newton himself, the discoverer of the law, made giant strides in using it to demonstrate the orderliness of the universe. He completely analyzed the causes of the oblateness of the earth, the tides in the sea, the precession and the nutation, the peculiarities of the motion of the moon, the elliptical orbits of the planets, and the elongated orbits of the comets.

The successors of Newton not only explained additional celestial phenomena as they were discovered, but time after time their theories outran observations and they were able to foretell what practical astronomers would find with their telescopes. After the death of Newton in 1727, a galaxy of great French mathematicians appeared who devised many new mathematical methods and wrote a chapter in science the brilliancy of which has not been surpassed. So long as men cherish intellectual achievements, the names of Clairaut and d'Alembert and Euler (a Swiss) and Laplace and Lagrange will be held in highest esteem. These great scientists not only explained what had been observed and also guided observers to new discoveries, but they looked back beyond historical times and even geological times to the possible origin of the planets (Art. 301), and they raised the question of the stability of the present order and looked forward to an indefinite future (Art. 183). Their successes were not merely the accomplishments of cold logical machines. These men were fired by the spirit of pioneers; they embarked on journeys filled with stirring adventure. Though their writings are strewn with mathematical symbols and long formulas, their work had the elegance that is characteristic of the French. Lagrange, particularly in such memoirs as that on the *Problem of Three Bodies*, wrote scientific symphonies as profoundly impressive from the aesthetic point of view as are the works of the greatest composers. In fact, in the foremost scientists art and science are always combined.

The work of the French mathematicians who succeeded Newton

has been developed in various directions by scientists in many countries. As the decades have passed, the usefulness of the law of gravitation as a key for unlocking the gates to unknown regions has continually increased. It has given rise, even in recent times, to new mathematical methods. For example, the French mathematician Poincaré (1854–1912), in his epoch-making researches on the problem of three bodies, developed and used mathematical processes occupying a logical level that in this problem had never been approached. That the greatest mathematician of recent times, if not of all time, should have made such a brilliant use of Newton's discovery nearly two hundred years after Newton's death is convincing evidence of the supreme position of the law of gravitation among the great scientific principles.

174. Kepler's Laws of Planetary Motion. Since a law of nature is essentially a description of the way natural phenomena succeed one another (Art. 3), it follows that it is always based directly or indirectly upon observations. The law of Newton was the culmination of work beginning in prehistoric times and extending over a period of more than 3000 years. It was, in fact, an inference from the laws of motion (Art. 93) and from Kepler's three laws of planetary motion.

The ancient Greeks undertook to explain the apparent motions of the sun, the moon, and the planets by the theory that all these bodies revolve around the earth in circular orbits. It would be an error to assume that these attempts were crude or that they were not attended with a high degree of success. In fact, it would be difficult without optical aid to detect differences between their theories and the actual phenomena. Moreover, they formulated the theory that the sun is the central body around which all the remaining bodies revolve, a theory which was revived by Copernicus (1473–1543) and supported so convincingly that it was never again abandoned. These theories were not the vague constructions of eccentric minds; they were worked out in great numerical detail and were compared with thousands of observations.

Up to the time of Kepler (1571–1630), astronomers had universally assumed that the orbits of the celestial bodies are circles. The argument in support of the assumption ran as follows: The heavens are perfect; the circle is the only perfect curve; therefore the orbits of the heavenly bodies are circles. Kepler, too,

for a long time made the same assumption, but fortunately he had available for testing his theories the unsurpassed observations of his immediate predecessor Tycho Brahe (1546–1610). At one stage in his laborious calculations, Kepler secured almost perfect agreement between his theories and the observations of Tycho Brahe. In fact, the greatest discrepancy amounted to only about half the angular distance between Mizar and Alcor (Art. 22), a quantity near the limits of Tycho Brahe's observations without optical aid. A man with less persistence and intellectual integrity or with less confidence in the accuracy of the observations of another would have been satisfied with his results and would have assumed that the few remaining small discrepancies are of no importance. Fortunately for Newton and the future of science, Kepler had high ideals for science. He attacked the problem of the planetary motions again and again over a period of many years until he completely triumphed.

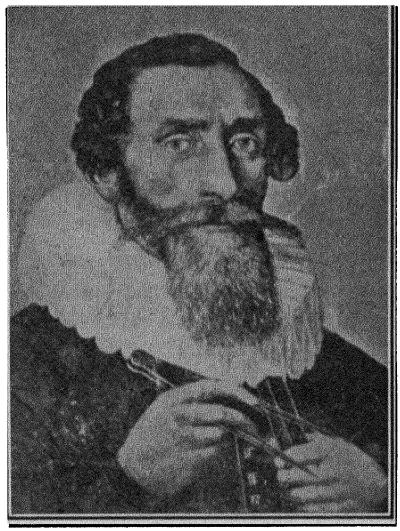


FIG. 87.—Johannes Kepler (1571–1630).

Kepler started on the way to final success when he abandoned the assumption that the orbits of the planets are necessarily circles. He naturally first examined ellipses which had been studied as mathematical curiosities two thousand years earlier by the Greeks, who probably had never a thought that these beautiful curves might some time play a conspicuous rôle in interpreting the apparent motions of the evening and the morning stars. If they had not investigated the properties of ellipses, it is improbable that Kepler would have discovered his laws of planetary motion or that Newton would have discovered the law of gravitation.

After twenty years of arduous work, Kepler condensed the essential properties of the motions of the planets, as established by observations extending over 2000 years, into the following three simple laws:

LAW I. *Every planet moves in its orbit in such a way that the line between it and the center of the sun sweeps over equal areas in equal intervals of time, whatever their length.*

LAW II. *The orbit of each of the planets is an ellipse with the sun at one of its foci.*

LAW III. *The squares of the periods of revolution of any two planets are in the same ratio as the cubes of their mean distances from the sun.*

The first and the second of the foregoing laws, which have already been encountered in discussing the motions of the earth (Arts. 106, 107) and of the moon (Art. 142), are often interchanged. The mean distance of a planet from the sun is half the major axis of its elliptical orbit.

175. Newton's Discovery of the Law of Gravitation. Occasionally a great genius appears whose intellectual achievements raise the human race to new levels. Aristotle was such a genius, and so were Galileo of Italy and Newton of England.

Isaac Newton (1643–1727) was born the year Galileo died. In 1665 he took his degree at Cambridge, having shown in the last two years of his university course extraordinary talent in mathematics and natural science. After taking his degree, he retired to his home as a consequence of a plague which was sweeping over England, and he at once entered upon a period of unparalleled intellectual activity. Within two years he had discovered the binomial theorem of algebra, he had laid the foundations of what he called Fluxions (now the calculus), he had developed his theory of colors, he had formulated the laws of motion (Art. 93), and he had conceived the idea that the moon is held in its orbit by forces of the same nature as those which cause bodies to fall near the surface of the earth.

Newton based his deduction of the law of gravitation upon his general laws of motion and upon Kepler's laws of the planetary motions. Since the planets do not move in straight lines, he concluded from his first law of motion that they are acted on by exterior forces. He then proved from Kepler's first law that each planet is subject to a force which always passes through the sun. Before the time of Newton it had generally been assumed that the planets must be subject to continually impelling forces to keep them going, and it was not suspected that the only forces acting on them are directed toward the sun.

Newton inferred from the first of Kepler's laws the direction of the force to which each planet is subject; he used the second of Kepler's laws to determine how each force varies. With a skill which has always won the admiration of mathematicians, he proved by the use of this law that the force to which each planet is subject varies inversely as the square of its distance from the sun.

A single step remained to complete the law of gravitation, at least so far as the solar system is concerned, and that was to prove that the attractions of its members depend only upon their masses and not upon their compositions, their temperatures, their densities, or any other of their properties. Newton showed that it follows from the third of Kepler's laws that gravitation depends only upon mass.

The genius of Newton was quickly recognized in England, and while he was yet a young man he was given a professorship at Cambridge University, where he lived a quiet life and made his greatest discoveries. Fortunately, Edmund Halley became acquainted with the diffident Newton, promptly recognized the brilliancy of his genius, and prevailed upon him to prepare his investigations for publication. Under Halley's encouragement, Newton produced an epoch-making work, *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*), ordinarily called simply *The Principia*, the manuscript for which was written in 1685 and 1686, and which appeared from the press in July, 1687.

It is impossible to give in many pages, much less in a single paragraph, any adequate understanding of the great discoveries contained in *The Principia*, or of the extraordinary talents required to make them. This single volume revolutionized mathematics and the whole realm of physics, as then known. The best that can be done here is to present the opinions of competent judges. The brilliant German philosopher and mathematician Leibnitz (1646–1716), who was an independent inventor of the calculus and who regarded himself as a rival of Newton, said, "Taking mathematics from the beginning of the world to the time when Newton lived, what he had done was much the better half." The great French mathematician Lagrange (1736–1813), a master of celestial mechanics who viewed the work of Newton as an expert judge and with the perspective of a later day, wrote, "Newton was the greatest genius that ever existed and the most fortunate, for we

can not find more than once a system of the world to establish." The English writer on the history of science, Whewell (1794–1866) wrote in his *History of the Inductive Sciences*, "It, [the law of gravitation] is indisputably and incomparably the greatest scientific discovery ever made, whether we look at the advance it involved, the extent of the truth disclosed, or the fundamental and satisfactory nature of this truth."



FIG. 88.—Isaac Newton (1643–1727).

Let us compare with these brilliant and merited eulogies Newton's own estimate of his achievements. In a moment of introspection he wrote, "I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, while the great ocean of truth lay all undiscovered before me." Those familiar with the work of Newton do not find in this sentence a syllable of pose or of false humility; it is the sincere and unaffected expression of a great mind which realized keenly the enormous magnitude of the unknown as compared with the little known.

In Westminster Abbey, in London, Newton lies buried among the noblest and the greatest English dead. There is a sentence in the Latin inscription on the tablet above his tomb which expresses in perfect dignity and effectiveness both the glory of his achievements and the perpetual indebtedness of his successors to him. In free translation it is, "Mortals, congratulate yourselves that so great a man has lived for the honor of the human race."

More than 200 years have passed since the death of Newton and all the rulers and warriors of his day are forgotten, but his name is familiar to every student of physical science in the whole world.

176. The Problem of Two Bodies. The problem of two bodies is that of determining the relative motion of two spherical masses which are subject to no exterior forces except their mutual attractions. Newton proved that such pairs of bodies move so that the radius from one of them to the other satisfies Kepler's first law (the law of areas), and so that the orbit of one of them with respect to the other is a conic section. As the name implies, conic sections are the curves obtained when planes intersect a circular cone at various angles.

The conic sections are illustrated in Fig. 89. When the intersecting plane is perpendicular to the axis of the cone, the conic is a circle C. When the plane is slightly oblique to the axis of the cone, the conic is an ellipse E. When the plane is parallel to one side of the cone, the conic is a parabola P. When the plane intersects both branches of the double cone, the conic is the hyperbola H. Circles and ellipses are curves of finite length, but the parabola extends to infinity in one direction while the hyperbola consists of two symmetrical branches which extend to infinity in opposite directions. A definition of the ellipse in terms of its properties was given in Art. 106

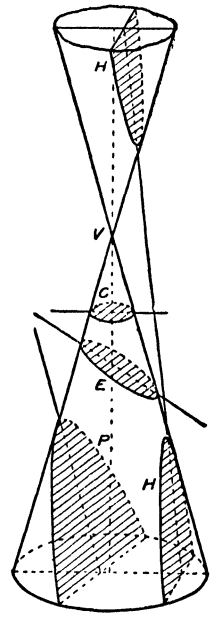


FIG. 89.—The conic sections.

The problem of two bodies consists not only in establishing the fact that the orbits are conics, but also in developing means of computing the positions and the components of velocity of the bodies in their orbits, whether they are circles, ellipses, parabolas, or hyperbolas. In the case of circular orbits the problem is simple, for the bodies move at uniform speed. In all the other cases there are formidable difficulties, but they were all overcome by Newton. The methods of Newton, however, are not so powerful nor so simple as those devised by later mathematicians.

The intricacies of the problem of two bodies will be omitted, but a few formulas will be given which are simple and which will often be of use.

Let m and M represent the masses of two bodies and r their distance apart. If v represents the velocity of one with respect to

the other when they move in a circle, and V their relative velocity when they move in a parabola, then

$$(1) \qquad v^2=\frac{k^2(m+M)}{r}, \quad V^2=\frac{2k^2(m+M)}{r},$$

where k^2 is a constant depending upon the adopted units of time, mass, and distance. If the mean solar day is taken as the unit of time, the sun as the unit of mass, and the distance from the earth to the sun as the unit of distance, then $k^2=1/3380$.

The period P of two bodies describing a circular or an elliptical orbit of major semi-axis a is given by the formula

$$(2) \qquad P=\frac{2\pi a^{3/2}}{k\sqrt{m+M}},$$

which has the important property of being independent of the eccentricity of the orbit.

177. Determination of the Masses of Planets and of Double Stars. Equation (2) can be used to determine the sum of the masses of a planet and its satellite when their mean distance r and their period P have been determined by observation, though a slight modification of the formula must be made to allow for the effect of the sun upon the motion and the period of a satellite. The masses of all planets having satellites have been accurately determined by this method, but the masses of Mercury and Venus, which do not have satellites, are not very accurately known.

By essentially the same method the sum of the masses of double stars can be found, at least in those cases in which their distances apart are known. The apparent angular distance between the components of a double star can, of course, be measured, but it is not possible to translate this angular distance into kilometers or into other linear measure unless the distance of the pair from the solar system has been determined.

178. The Problem of Three Bodies. Since two bodies which are subject only to their mutual attractions necessarily move in one or another of the various classes of conics, it is not difficult to describe the general properties of their motions. The simplicity is due to the fact that the bodies travel along fixed and relatively simple curves in accordance with the first and the second of Kepler's laws of planetary motion.

In the problem of three bodies, however, the complexities are of entirely different order. Except in special cases, the orbits are neither closed curves nor do the bodies pursue similar paths in any two circuits about their center of gravity. Nor are there any simple laws corresponding to those of Kepler that describe how the individual bodies move in their orbits. There are, however, four laws which pertain to all three bodies combined, the simplest of them being that the sum of all the kinetic and potential energies of the system remains constant, however the individual masses move. The others state that the moment of momentum of the entire system about each of three perpendicular axes through its center of gravity is constant.

Although in general the problem of three bodies is of enormous complexity, there are certain special solutions in which the motions are relatively simple. In the first of these, discovered by Lagrange in 1772, the three masses describe conic sections, just as in the case of two bodies; and the first two of Kepler's laws are satisfied by the motion of each body separately. The Lagrangian solutions are of two types: in one, the three bodies are always in a straight line; in the other, they form an equilateral triangle, whatever their relative masses.

The great difficulty of the problem of three bodies is illustrated by the fact that, after the work of Lagrange, no other special solutions of the problem were discovered until the works of G. W. Hill and H. Poincaré more than 100 years later. The researches of Poincaré in this subject were incomparably superior to those of all his predecessors. In fact, he created a new domain of science far beyond the horizon of his contemporaries. And just as happened in the case of Newton, several generations may pass before the power of his methods and the great value of his work are generally understood by astronomers.

It is sometimes erroneously stated that the problem of three bodies can not be solved. As a matter of fact, it can be solved in any case for intervals of time that are not too great. The true statement is that there is no known method of determining in general the properties of motion over a long interval of time; but any particular case can be numerically solved over any short interval. For example, in the general case it is not possible to determine whether in the course of enormous intervals of time one of the bodies will not recede to very great distances from its initial

position, or whether two or more of the bodies will not collide (Art. 183). It is evident that although in a certain sense the problem of three bodies can be solved for a restricted interval of time, it lacks much of being solved in a general sense.

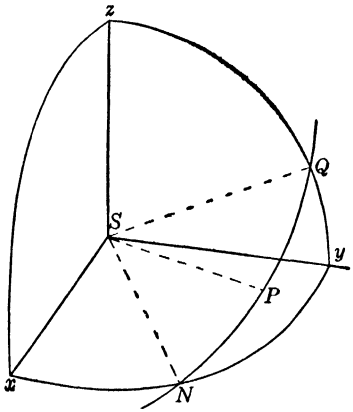


FIG. 90.—Elements of orbit.

179. The Elements of a Planetary Orbit. The motion of a planet evidently depends upon the major axis and the eccentricity of its orbit. There are also four additional quantities (or *elements*) which must be given in order that an astronomer shall be able to compute the position of a planet at any time.

In Fig. 90, let S represent the position of the sun, the arc xNy a quadrant of the ecliptic with the vernal equinox at x, the arc NPQ a portion of the projection of the orbit of the planet on the celestial sphere, the point N the ascending node (the point where the planet, as seen from the sun, crosses the ecliptic from the south side to the north side), and P the projection of the perihelion point of the planetary orbit upon the celestial sphere.

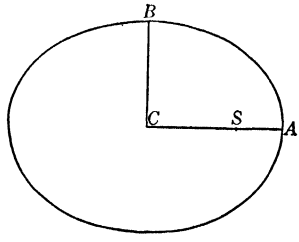


FIG. 91.—The ellipse.

The position of the plane of the orbit of the planet is defined by two elements: (1) the longitude of the ascending node, namely the angle xSN (or arc xN) and (2) the inclination of the orbit to the ecliptic; namely, the angle yNP. The orientation of the orbit in its plane is defined by one element: (3) the longitude of the perihelion from the ascending node; namely, the angle NSP (or arc NP). The size and shape of the orbit are defined in two elements: (4) the major semi-axis, and (5) the eccentricity. In Fig. 91, the major axis is CA and the eccentricity is CS/CA, where S is at the focus. The last and sixth element defines the time at which the planet is in a known part of its orbit, as at the perihelion point P, Fig. 90. When these six

elements are known, the position of a planet can be computed for any desired interval of time.

Unless the orbit of a comet is a parabola, its elements are defined precisely as those of the orbit of a planet. In the case of parabolic orbits the eccentricity is unity and the perihelion distance is given in place of the major semi-axis.

180. Determination of the Elements of an Orbit from Three Observations of Apparent Position. When an astronomer measures the apparent position of a celestial body, he determines nothing but its direction from his point of observation at the time. All he may later deduce respecting its distance and its motion is based entirely upon measurements of direction from the surface of the revolving and rotating earth.

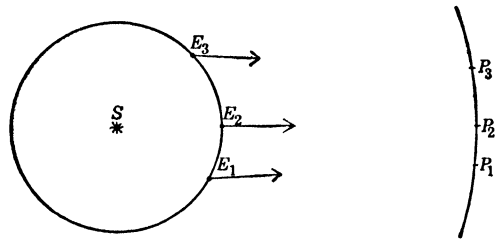


FIG. 92.—Observations of directions of body.

Figure 92 illustrates the nature of the data furnished by observations of the apparent positions of celestial objects. At a certain time the earth is at E_1, and the astronomer observes, for example, a comet in the direction of the point P_1, which is on the celestial sphere. The distance of a newly discovered comet is quite unknown. Similarly, when the earth is at the places E_2, E_3, etc., the comet is observed in the directions of the points P_2, P_3, etc., on the celestial sphere. In general, the points P_1, P_2, etc., do not lie on the ecliptic, but along some kind of twisted curve.

Since an orbit is defined by six elements (Art. 179), six observational data must be available before it can be determined. Each observation of the apparent position of a celestial body gives two coördinates, such as right ascension and declination. Therefore, three observations of the apparent position of a body give six data which are in general sufficient for determining the six elements of its orbit.

The problem of determining the elements of an orbit of a body from three observations of its apparent position was first solved by Newton in *The Principia*. Superior methods were later devised both by Laplace and by Gauss, and practically all the methods since

their day are based more or less directly upon the work of one or the other of these great mathematicians.

181. Perturbations; Variations of the Elements of an Orbit. In the solar system the sun is the dominating body, in mass more than a thousand times greater than the largest planet. Moreover, the planets are very far from one another. The result is that each planet revolves around the sun in a nearly elliptical orbit and almost as though no other planet existed.

The planets do, however, influence the motions of one another, and their mutual effects in the course of time are of considerable importance. The slight deviations from strictly elliptical orbits are called *perturbations*. Apparently the perturbations were formerly regarded as anomalies and possibly defects in the planetary motions, but they are as much in harmony with the law of gravitation as elliptical motion is in the problem of two bodies. In fact, if perturbations were not found by observation to be in agreement with theory, the law of gravitation would have to be abandoned.

If the perturbations of the planetary orbits were not small, Kepler would not have discovered his three laws of planetary motion (which are not absolutely exact), for there would not be such laws, and Newton would not have had the necessary basis (Art. 175) for discovering the law of gravitation. It is likely, too, that we should not yet have this marvelous key for unlocking the doors to celestial dynamics. The orbits of celestial bodies are not necessarily conic sections, for we know many cases of triple suns in which the motions are far from elliptical.

Some very ingenious methods have been devised for computing and describing perturbations. The one which is most used and which gives the best insight into their real nature consists in regarding the planetary orbits as ellipses, but as ellipses whose elements continually change. If we regard the planetary orbits as elastic hoops on which the planets slide, and then think of the hoops as continually changing sizes, shapes, and positions, we shall have a physical picture which roughly resembles the analysis. With this conception in mind, we can legitimately speak of the regression of the node, or the advance of the perigee, or the decrease of the eccentricity of the elliptical orbit of a planet.

182. The Discovery of Neptune. One of the most dramatic events in the history of science—the discovery of the planet

Neptune—illustrates the principles which have so far been discussed, though somewhat abstractly, in this chapter.

In 1781, William Herschel discovered the planet Uranus while carrying out his project of examining every object in the heavens within reach of his telescope. The elements of the orbit of the new planet were promptly determined and its motion was carefully followed by observers. In a few years it became necessary to take into account the perturbations of its motion by the other planets. Instead of there being perfect agreement between theory and observations, as had been confidently expected, by 1820 there were unmistakable discrepancies between them. By 1830 they were more serious; by 1840 they had become intolerable. This does not mean that the mathematical theory assigned the planet to one part of the sky and observation found it in a far distant part. On the contrary, in 1841, sixty years after the discovery of the planet, the difference between theory and observations which astronomers regarded as intolerable amounted to only two-thirds the apparent distance between the components of Epsilon Lyrae (Art. 24), an angular distance too small to be within the range of the unaided eye.

At first thought it seems absurd that astronomers should have been distressed by the fact that a distant planet, itself invisible without optical aid, had departed by so minute a distance from its predicted position in the course of sixty years. If the affairs of the world had been carried on up to 1781 without any one even knowing that the planet exists, it surely seems that the slight unexpected peculiarities in its motion should not be regarded as disquieting. But this discrepancy called into question the laws of motion, the law of gravitation, and the soundness of mathematical reasoning. It struck at our most cherished hopes by throwing doubt on our ability to discover and to apply the laws of nature.

By 1820 it was suggested that the unexplained peculiarities in the motion of Uranus might be due to the attraction of a more remote and wholly unknown planet. The problem was to find the unknown planet. The mathematical difficulties involved in the problem were enormous; in fact, Sir George Airy, Astronomer Royal of England, expressed the opinion later than 1840 that the problem was wholly unsolvable. But it was attacked and solved independently by two young men who were fired with the courage and the enthusiasm of youth. How often youth has accomplished

the apparently impossible! One of these young men was J. C. Adams, of Cambridge, England; the other was U. J. Leverrier, of Paris, France. Adams finished his work first, but the work of Leverrier led to the discovery. He communicated his results to a young German astronomer, J. G. Galle, who received the letter on February 23, 1846, and impatiently waited for night and the stars. When the twilight finally faded, he turned his telescope to the sky and found the unknown body within half a degree of the position assigned to it by Leverrier, which agreed substantially with the results obtained by Adams.

Neptune is more than four thousand millions of kilometers from the earth and wholly beyond the reach of all our senses except that of sight, and even our eyes can not detect it without optical aid. Although its distance is so great that more than four hours are required for its light to come to us, yet it is bound to the remainder of the system by the invisible bonds of gravitation. The reasoning of Adams and Leverrier followed these tenuous threads out to the planet beyond the known borders of the solar system, and their work in doing so will always stand as a monument to the perfection of the theories of the motions of the heavenly bodies.

183. The Question of Stability of the Solar System. Is the solar system stable? That is, are the planets so distributed that the gravitation of the sun and their mutual attractions will indefinitely preserve the present general characteristics of the system? Or, on the contrary, will one or more of the planets approach and finally fall into the sun, or gradually recede and at last be lost from the solar family; or will two or more of them eventually collide?

It has sometimes been stated that there is such a wonderful balance of forces among the members of the solar family that if one planet were removed the system would collapse and be destroyed. This statement is wholly erroneous, for if all the planets except the earth were blotted out no one but astronomers and attentive observers of the sky would ever notice the loss. The question of the stability of the solar system is not so simple as such conceptions imply. It is, in fact, so difficult that the answer to it has not yet been completely established.

Lagrange and Laplace were the first to attempt to determine whether or not the solar system in its present form is permanent.

Although they did not succeed in answering the question with complete logical rigor, yet they proved that the major axes of the planetary orbits (at least so far as the so-called terms of the first order are concerned) undergo only periodic variations of relatively short periods. That is, the major axis of the orbit of the earth, for example, will sometimes increase and sometimes decrease, but it will never be much greater or much smaller than it is at present. Consequently the planets will neither fall into the sun nor recede from it indefinitely. They proved also that the eccentricities of the planetary orbits and their inclinations undergo only periodic variations. On the contrary, the longitudes of the nodes and the longitudes of the perihelia progressively advance or regress, but these changes have no important effects upon the general properties of the system.

So far as the work of Lagrange and Laplace goes, it indicates that the solar system is stable. In the first enthusiasm following their conclusions these great theorems were called the *Magna Charta* of the continued existence of the planets. A more penetrating insight into their logic reveals, however, that their results pertain only to the first groups of terms of infinite series of such groups of mathematical expressions, and that the properties of the higher groups are different. Moreover, the expressions are known to be true representations of the facts only for restricted periods of time, and consequently conclusions can not be drawn from them for great intervals. Finally, these discussions neglect all such factors as a possible resisting medium, the pressure of light, the radiation away of mass, the oblateness of the bodies, the tides, the disintegration of matter, and relativity. Though all of these factors have inappreciable effects in the short intervals covered by our observations, yet they may lead to the distant destruction of the present order unless before that time arrives our planets are wrecked by a passing star. Consequently in this problem, as in the case of most others, we are left without an absolute and final answer to our question.

184. Motion of the Perihelion of the Orbit of Mercury. The theories of the planetary motions include not only the effects of the attraction of the sun but also the effects of the planets on one another. These mutual perturbations are very numerous and difficult to determine; for example, G. W. Hill spent many years

developing the theory of the interactions of Jupiter and Saturn. In spite of the complexity of the planetary motions, theory and observation are in harmony with only one or two exceptions, and in these cases the differences are very slight.

When Leverrier computed the effects of the attractions of the planets for one another and compared the results with observations, he found but a single disagreement that was large enough to raise a serious question. Observations seemed to show that the perihelion of the orbit of Mercury advances about 40 seconds of arc in 100 years more than predicted by theory; at least, the assumption was an easy way of removing the disagreement. It really seems absurd that so slight a discrepancy should cause even a comment, for 500 years are required for it to become large enough to be visible to the unaided eye. The theoretical results are secure, for analysis does not have to wait on the passage of time. The observational results are much less certain, for they depend largely on the infrequent transits of Mercury across the disk of the sun. Although Simon Newcomb confirmed the conclusion of Leverrier, the problem should be reëxamined in the light of more recent observational data.

Many attempts have been made to explain the probable anomalous motion of the perihelion of the orbit of Mercury by ascribing it to diffused matter around the sun or to an oblate sun, but without quantitative success. But when Einstein developed his theory of relativity, he pointed out that it implies a slight modification of the law of gravitation which produces forward motions of the perihelia of the orbits of the planets. In the orbit of Mercury alone is the effect appreciable, and there it is only 40″ per century.

185. Significance of Tides in Evolution. In Art. 81 a discussion was given of the method of determining the rigidity of the earth from tide experiments, and in Art. 100 it was briefly remarked that the tides raised in the earth by the moon and the sun have an effect upon the rate of rotation of the earth.

As will be shown in succeeding paragraphs, the planets and their satellites are slowly dissipating their energies through tidal friction, and these losses of energy are necessarily accompanied by changes in their orbits and in their rates of rotation. If we look forward far enough into the future, say some billions of years, we find that the planetary system may be greatly modified by the

effects of the friction of tides if no important influence, such as a close visit from another star, intervenes. And if we look back to the remote past, we find that the tides are a factor that can not safely be neglected in treating the origin of the planets and their satellites. In general, the major forces to which the heavenly bodies are subject produce cyclical results, but some of the minor forces, working always in one direction, in the long run have supremely important effects.

186. The Tide-Raising Acceleration. In order to simplify the problem, consider first the tidal acceleration of the moon upon a unit mass in line with the centers of the earth and the moon. In Fig. 93, let E represent the center of the earth, M the moon, P (or P') a unit mass upon the surface of the earth in the line EM, r the radius of the earth, and R the distance from E to M. It follows from the law of gravitation that the acceleration of M on E is k^2M/R^2, and that the acceleration of M on P is $k^2M/(R-r)^2$. The difference in these two accelerations is the tide-raising acceleration at P (which will be denoted by A), or the tendency for the moon to raise P from the surface of the earth. Hence

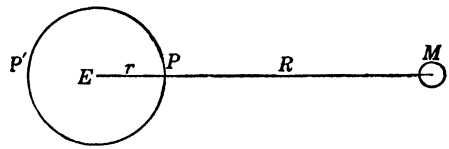


FIG. 93.—Attractions of moon on E and P.

$$A=\frac{k^2M}{(R-r)^2}-\frac{k^2M}{R^2}=\frac{k^2M}{R^2}\left[1+\frac{2r}{R}+3\left(\frac{r}{R}\right)^2\right]-\frac{M}{R^2},$$

and since r is small compared to R, it follows that a close approximation to the value of A is given by the equation

$$A=\frac{2k^2Mr}{R^3}\left[1+\frac{3r}{2R}\right]. \tag{3}$$

For the point P' the corresponding equation is

$$A'=-\frac{2k^2Mr}{R^3}\left[1-\frac{3r}{2R}\right]. \tag{4}$$

The positive sign in equation (3) indicates that the tide-raising acceleration at P is toward the right; the negative sign in equation (4) indicates that the tide-raising acceleration at P' is toward the left. Since r/R is small (1/60 in the case of the earth-moon system), the two tide-raising accelerations are nearly equal. The tide-raising

acceleration at P is, however, about one-twentieth greater in the case of the earth-moon system than that at P', as follows from the second terms in the right members of the equations. Furthermore, the tide-raising accelerations are proportional to the product of the mass of the tide-producing body and the radius of the tidally disturbed body, and inversely proportional to the cube of the distance between the centers of the bodies.

Since the acceleration of gravity at the surface of the earth is defined approximately by the equation $g=k^2E/r^2$, it follows from equation (3) that

$$\frac{A}{g}=\frac{2M}{E}\left(\frac{r}{R}\right)^3$$

approximately. Since M/E is about 1/80 and r/R averages about 1/60, it follows that in round numbers $A/g=1/8{,}640{,}000$. Consequently, the tide-raising acceleration of the moon on the earth is very small in comparison with gravity. This is the reason that the change in shape of the tidally distorted surface of the earth is very slight and that the tides are perceptible only in large bodies of water.

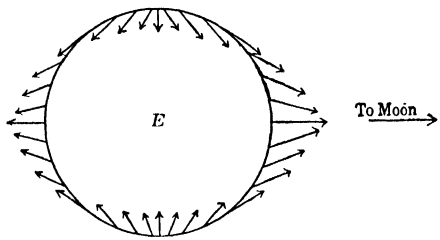


FIG. 94.—Tide-raising acceleration.

Obviously it is not sufficient to consider only unit masses P and P' in the line of the centers of E and M, for all parts of E are subject to the tide-raising accelerations. Although there is no mathematical difficulty in deriving a formula for the general case, it will be sufficient for present purposes to indicate the character of the results by Fig. 94. The arrows represent roughly the direction and the relative magnitudes of the tide-raising accelerations. They are largest on the side of E toward M and next largest on the side of E opposite to M. On the circle 90° from the line joining E and M the acceleration is toward the center of E and in magnitude about one-half that in the line EM. Obviously E and M may represent any two bodies as well as the earth and the moon.

187. Lag of the Tides. If the body E were not rotating and the body M were not revolving, the tides on E would be distributed in harmony with the arrows of Fig. 94. With one known exception in

the solar system (the inner satellite of Mars), the tidally distorted body rotates at a greater angular velocity than the tide-raising body revolves. Since the tides do not respond instantly to the forces by which they are produced, the tidal protuberances are carried forward as indicated in Fig. 95.

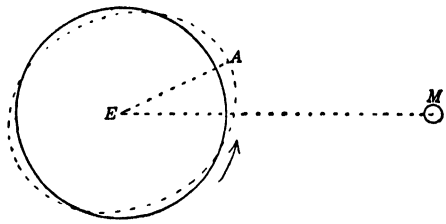


FIG. 95.—Lag of the tides.

The angle AEM by which A is carried forward from the line EM depends upon the rate of angular rotation of E, the rate of angular revolution of M, and the viscosity of E. If E were perfectly elastic, A and B would be in the line EM (Art. 81). If E is not perfectly elastic, the angle AEM is greater the greater the differences in the rate of angular rotation of E and the angular revolution of M.

Now consider a point P on the surface of the body E, carried around by the rotation of E. It is seen from Fig. 95 that if the point P passes the meridian of M at a certain instant, it will pass high tide A at a later instant; that is, the tide apparently lags with respect to the tide-raising body. The oceanic tides on the earth do not lag by any fixed amount, for they are modified by the varying depths of the oceans; and they are deflected and reflected by the continents until they present a complexity that is almost beyond analysis.

188. Tidal Evolution. Lord Kelvin seems first to have investigated the possible effects of tidal friction upon the mutual relations of the earth and moon. The problem was fully developed, however, by Sir George Darwin, whose theory of tidal evolution approaches in interest the theory of the origin of species developed by his illustrious father, Charles Darwin. It reaches back to the birth of the earth and the moon, and it looks forward to a time when their mutual relations will be far different from what they are at present.

In the tidal evolution of mutually interacting bodies, the moment of momentum (Art. 100) of the system remains constant; but its total energy decreases, for the tides generate heat which is radiated away into space. In the case of such a simple system as that of the earth and the moon, the conditions of these two

principles can be expressed in terms of the day and the month alone as variables. It follows from these expressions that both the day and the month, under present relations between the earth and the moon, are increasing in length, the month at a greater rate than the day. But as the moon recedes from the earth, this condition will eventually be reversed and the day will gain on the month in length. These conclusions are certain so far as tidal friction alone is concerned.

Let us, with Darwin, follow the history of the earth-moon system backward in time. The farther we go the shorter the day and the month and the nearer the moon to the earth. Finally, we reach a limit at which both the day and the month were about 4.8 of our present hours in length and the moon was about 14,000 kilometers (9,000 miles) distant from the center of the earth. Darwin conjectured that at this time the moon separated from the earth partly as a consequence of the rapid rotation of the combined mass and partly because of the tidal effects of the sun.

Now let us look to the future, during which the day and the month continually increase in length. At present the month is increasing faster than the day, but eventually the condition will be reversed and the system will approach a state in which the day and the month will both be equal in length to about 47 of our present days. Hence at this time the earth and the moon will each keep the same face toward the other, and their tidal evolution will cease because there will then be no lag in the tides.

In the foregoing discussion there has been no reference to the intervals of time required for the great changes to take place. Darwin assumed that the deep interior of the earth is viscous and that the bodily tides are and have been much more important in tidal evolution than those of the oceans and the atmosphere. Under these assumptions he reached the conclusion that the past history of the system as a minimum could not have been less than an interval of approximately 60,000,000 years, a period in agreement with the ideas then current respecting the age of the earth.

The earth, however, is not viscous, as has been proved from the tide experiment (Art. 81) and from other evidence. Hence tidal evolution has been much slower than Darwin thought probable. G. I. Taylor calculated the rate of dissipation of energy by the tides in the Irish Sea, and Harold Jeffreys extended the same methods to most of the shallow ocean basins on the earth. On the

basis of these calculations, it appears that the day has probably increased only one second in length during the last 120,000 years, and that the interval since the moon may have separated from the earth is of the order of 4,000,000,000 years. Even this enormous interval of time is only a moment in comparison with that required for the day and the month to approach 47 of our present days, for the tidal forces (Art. 186) vary inversely as the cube of the distance between the earth and the moon, and the dissipation of energy varies inversely as the sixth power of the distance.

189. Is the Theory of Tidal Evolution True? There are two principal methods of testing the correctness of a scientific theory. The first is to determine whether it forms a complete and logically self-consistent body of doctrine; the second is to find whether it is in harmony with observed phenomena.

The theory of tidal evolution is by no means so simple as the foregoing brief outline of it seems to imply; Darwin's mathematical work on it fills hundreds of printed pages. In developing his theory, it was necessary for him to assume physical properties for the earth and to take into account both the eccentricity and the inclination of the orbit of the moon. Moreover, he had to consider at the same time the effects of the tides raised by the sun. His assumption of a viscous earth was erroneous, but the other data of the problem were secure. Although he pursued these intricate investigations with great skill and perseverance, he did not succeed in assuming properties for the earth which would remove all inconsistencies, but those which remained did not seem to him to be very serious. At the time of Darwin's death (1912) the conclusion that the age of the earth is several billions of years had not been generally accepted, yet in respect to the origin of the moon he was cautious. In his last reference to the theory of the fission of the moon from the earth, he wrote ". . . it is only a wild speculation, incapable of verification." With the elasticity of the earth established and the immense intervals of its duration to be considered, there is reason now for still greater caution; and there is reason for caution particularly because we have omitted from consideration all such possibilities as a resisting medium, variable magnetic forces, and increases or decreases in the masses of the earth and the moon.

There is, finally, the evidence presented by the remaining planets

and their satellites. Darwin examined thoroughly all other planet-satellite systems and proved that tidal friction can not have been important in any of them, and that none of the 25 other known satellites can have separated from its primary by fission.

XVI. QUESTIONS

1. Do you know any other law as general as the law of gravitation? How many years have elapsed since Newton published the law in *The Principia*? What other laws of physical science have endured without appreciable change for so long a period?
2. What other criteria, besides those given in Art. 173, are useful in measuring the value of a natural law?
3. What other laws besides that of gravitation have quickly reduced large numbers of phenomena to orderliness?
4. What is the binomial theorem of algebra?
5. Enumerate the steps in Newton's deduction of the law of gravitation.
6. Show that Kepler's third law follows from equation (2) for two planets when m is very small in comparison with M.
7. Show how equation (2) can be used to determine the sum of the masses of Mars and one of its satellites in terms of the sum of the masses of the earth and the moon. For the distance and the period of a satellite of Mars see Art. 215.
8. How many observations of an unknown planet or comet are in general necessary in order that its orbit may be determined?
9. What fraction of the orbit of Uranus was described between the time the planet was discovered by Herschel and the discovery of Neptune?
10. Determine in the case of the earth-moon system the ratio of the tide-raising acceleration at P to that at P', Fig. 93.
11. Determine by equation (3) the ratio of the tide-raising acceleration of the moon to that of the sun.

REFERENCE BOOKS

Newton's *Principia*. There are many editions.

The Tides (third edition, 1911). Sir George Darwin. John Murray, London, England.

The Earth (1929), Chapter III. Harold Jeffreys. The Cambridge University Press, Cambridge, England.

CHAPTER X

THE SOLAR SYSTEM

190. The Solar Family. In this chapter we shall consider the solar family as a group of mutually related bodies; in the next two chapters we shall examine its members as individuals.

In ancient times the earth was supposed to constitute the greater part of the physical universe, and it occupies the same relative position in the careless thoughts of the world today. Even the sun appears to be a small object which each evening sinks below the horizon of the great earth, and which may be completely obscured by a cloud. The stars are ordinarily regarded as objects so tiny and faint that they pale with the rising sun and are dimmed by the moon. In the present discussion, however, the whole order of our ordinary thought will be reversed. The earth will play no major rôle, for we shall stand afar and regard it as only one member of the solar family, and a minor one at that. We shall see it held in its course by the gravitation of the mighty sun and pulled somewhat from its elliptic path by the attractions of the other planets.

From every point of view the sun is the dominant member of the solar family. In mass, it is 700 times greater than all its attendants combined, and it holds them in their orbits by its gravitative power. Moreover, it illuminates and warms all the members of its family, and if it were not for its abundant rays the entire system would be cold and invisible.

Among the secondary members of the solar family the nine planets easily occupy first place. Their masses are relatively large, their nearly circular orbits are comparatively simple and lie nearly in the same plane, and six of the nine have one or more satellite attendants. Aside from the earth, five of the planets, when favorably located, are conspicuous objects to the unaided eye and have been known from prehistoric antiquity; three, Uranus, Neptune, and Pluto were discovered in 1781, 1846, and 1930, respectively. There are also more than 1000 known small

planetoids which revolve around the sun in orbits similar to those of the planets.

Not all the attendants of the sun are so regular in their habits as the planets and planetoids. There are also the comets, bizarre and little-understood objects, which in many instances come in from beyond the orbits of the most remote planets known and pass around the sun and then recede, to return again, if at all, only after an interval of many thousands of years. Their elongated paths lie in all planes and loop through the orbits of the planets and of one another without apparent order or system. Meteors present as many puzzling problems as the comets, and at least a part of them are disintegrated comets. Comets and meteors will be discussed in Chapter XII.

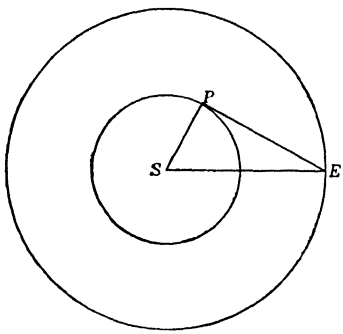


FIG. 96.—Relative distance of inferior planet from sun.

191. Determining the Relative Dimensions of the Planetary Orbits. It is an interesting fact that it is much easier to determine the dimensions of the orbit of any planet compared to that of the earth than it is to determine any of these distances in kilometers or miles. Even in the days of the ancient Greek astronomers, the comparative distances of the planets from the sun were known with considerable approximation, but the actual distances were quite unknown.

Consider first a planet whose orbit is interior to that of the earth, a so-called *inferior planet.* In Fig. 96, let S represent the sun, E the earth, and P the planet. An observer on E measures from night to night the angle SEP in order to find the greatest value it attains. If the orbits of the earth and the planet were circles, this angle would have its greatest value when the angle SPE is a right angle. Then all the angles of the triangle SPE would be known, and the distance SP could be determined by trigonometry in terms of the distance SE.

Since the orbits of the planets are not exactly circular, the method of the foregoing paragraph would be not quite sufficient for the solution of the problem. It is clear, however, in a general way that many observations furnish a basis for determining the

relative distance of a planet whose orbit is interior to that of the earth, even though both orbits are elliptical.

In the case of a planet whose orbit is exterior to that of the earth, a so-called *superior planet,* the problem of determining its distance from the sun in terms of the earth's distance from the sun is a little more complicated. The first step is to find the period of revolution of the planet.

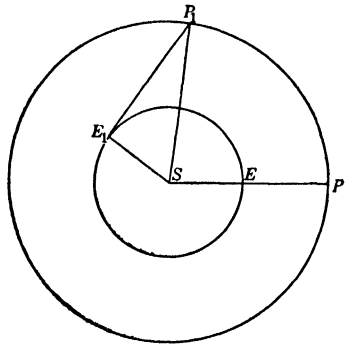


FIG. 97.—Relative distance of a superior planet from sun.

When the sun, the earth, and the planet are in a straight line, as at S, E, and P, respectively, in Fig. 97, the planet is said to be in *opposition,* obviously because it is opposite the sun as seen from the earth. After a certain interval, called the *synodical period,* the planet will be in opposition again. Obviously the synodical period can be easily determined from observations, and after it has been obtained, the sidereal period can be computed as follows. Let E represent the earth's sidereal period, P that of the planet, and S its synodical period. Then it follows by a discussion analogous to that given in Art. 139 of the synodical and sidereal periods of the moon that

$$\frac{1}{P}=\frac{1}{E}-\frac{1}{S},$$

from which the period P can be computed when the year E and the synodical period S are known.

Now suppose the planet is observed until the angle SE_1P_1, Fig. 97, is 90°. The angles ESE_1 and PSP_1 are both known because the sidereal periods of the earth and the planet are both known. Therefore the angle P_1SE_1 is known, and in the triangle SE_1P_1 the distance SP_1 can be computed in terms of the earth's distance SE_1. Naturally the eccentricity of the orbit of the earth and of that of the planet add some complexities to the problem, but the difficulties are not insurmountable.

192. Distance from the Sun to the Earth. From our geocentric point of view we naturally speak of the distance from the earth to the sun, but in considering the solar system it is much more

appropriate to speak of the sun as the center from which to measure distances. It follows from Art. 191 that after the distance from the sun to the earth has been determined, the distances of all the other members of the solar system from the sun and from one another can easily be found. It is obvious after a little thought that the distance between any other member of the solar system and the earth would serve just as well as a basis for determining all the remaining distances.

Various methods for finding the dimensions of the solar system have been employed. The simplest theoretically is the direct method similar to that used in finding the distance from the earth to the moon (Art. 143). Although this method is fully satisfactory in case of the moon, it does not furnish very accurate results in the case of the sun, partly because of the great distance from the sun to the earth with a resulting small angle to be measured, partly because the sun has no fixed point on which to focus in making measurements, and partly because the heat of the sun disturbs the measuring instruments. The same method can be applied, however, in determining the distance from the earth to Mars and to the planetoids. The planetoid Eros (Art. 204) is the most convenient to use, because at times it is nearer the earth than any other known celestial body except the moon, and also because it is so small that it has no atmosphere and presents no appreciable disk.

There is another completely different method which depends upon a class of perturbations of the motion of the moon by the sun. As Laplace long ago pointed out, the magnitudes of certain terms in the expressions for the effect of the sun upon the motion of the moon around the earth depend upon the ratio of the moon's distance from the earth to that of the sun. Since the distance from the earth to the moon is known with a high degree of approximation, the determination of the magnitude of these terms by observations furnishes a basis for computing the distance from the sun to the earth. A similar method, giving excellent results, depends upon the perturbations of the motion of Eros by the earth, which determine the ratio of the mass of the earth to that of the sun. Then from the radius of the earth, the value of gravity at its surface, and the length of the year the distance of the earth from the sun can be computed.

A third completely independent method of obtaining the dimen-

sions of the planetary orbits depends upon the use of the spectroscope for measuring the relative velocity of the earth and a star in the line of sight (Art. 105), and the best determinations by this method of the distance from the sun to the earth rank high in order of precision.

Instead of using the distance from the sun to the earth, astronomers more frequently employ the sun's parallax, which is the angle subtended by the earth's equatorial radius at the mean distance of the earth from the sun. The methods of the first of the three classes discussed above (the direct geometrical) give a mean of about 8″.806 for the solar parallax, with a probable error of about ±0″.004. The gravitational methods give an average of about 8″.802, with a probable error of about ±0″.002. The result obtained with the spectroscope is 8″.802, with a probable error of ±0″.004. It is evident that the solar parallax is known with certainty to only three figures, or to about one part in a thousand, and that the distance from the sun to the earth is known only with a corresponding approximation. It follows that there are also proportional uncertainties in all other distances in the solar system, including the diameters of the planets (other than the earth) and the distances of their satellites. Of course, in many instances there are greater uncertainties due to other causes. Since the distance from the sun to the earth is directly or indirectly the base line used in exploring the sidereal system, the distances to the stars are subject to uncertainties of at least one part in a thousand. In most cases the stellar distances are for other reasons subject to much greater probable errors.

That the solar parallax has been obtained with only a moderate degree of accuracy is due to the fact that 8″.8 is a small angle. It is approximately the slight difference in direction of an object at a distance of 1500 meters (4900 feet) when seen first with one eye and then with the other.

Although the solar parallax appears to be a little greater than 8″.8, this value has been used by international agreement in the Nautical Almanacs since 1900. With Hayford's value of the equatorial radius of the earth, it corresponds to a mean distance from the sun to the earth of 149,504,000 kilometers (92,897,000 miles). This distance is so great in comparison with the distances which come within the range of our ordinary experience that it is entirely beyond our comprehension, and only by illustrations are

we able to approach an understanding of it. For example, we now hear much of the destruction of distance by our rapid means of travel. Suppose an airplane could make a circuit of the earth at the equator in 10 days, and that a child at birth began to make such circuits and continued making them all his life. Unless he lived well beyond 100 years he would not travel a distance so great as that from the sun to the earth.

193. Distance from the Sun to the Planets. In the following table there are given the mean distances of the planets from the sun, calculated under the assumption that the mean solar parallax is 8″.8.

TABLE VIII. DISTANCES OF THE PLANETS FROM THE SUN

PLANET	ASTRONOMICAL UNITS	KILOMETERS	MILES
Mercury	0.3871	57,870,000	35,960,000
Venus	0.7233	108,140,000	67,200,000
Earth	1.0000	149,504,000	92,897,000
Mars	1.5237	227,800,000	141,540,000
Jupiter	5.2028	777,820,000	483,310,000
Saturn	9.5388	1,426,100,000	886,120,000
Uranus	19.1910	2,869,100,000	1,782,700,000
Neptune	30.0707	4,495,600,000	2,793,400,000
Pluto	39.5967	5,920,000,000	3,680,000,000

Even though we have aided ourselves by illustrations in obtaining a comprehension of the great distance from the sun to the earth, it is difficult to translate this table into a correct mental picture. It is not easy to realize how widely scattered the planets are—the distance of Pluto is more than 100 times that of Mercury. It is not easy to realize how rapidly the distance between adjacent planets increases as we go out from the sun. For example, the interval which separates the orbits of Uranus and Neptune is greater than the distance from the sun to Saturn. We find to our surprise that we can not make a diagram of all the planetary orbits to the same scale on an ordinary page and have the orbit of Mercury large enough to be discernible. We can, however, represent the orbits of the first four planets to the same scale, as is done in Fig. 98 with the eccentricities neglected.

In Fig. 99, the diagram is continued to Neptune on a reduced scale, the orbit of the innermost planet shown being that of Mars. If we imagine Fig. 98 diminished in size until its outermost circle

coincides with the innermost circle of Fig. 99, we shall have a diagram of eight planetary orbits to scale, that of Pluto being omitted because it is so large.

194. On the Surface Temperatures of the Planets. Since the planets are at widely different distances from the sun, they receive from it widely different amounts of radiant energy (heat and light). As a consequence their surface temperatures differ greatly.

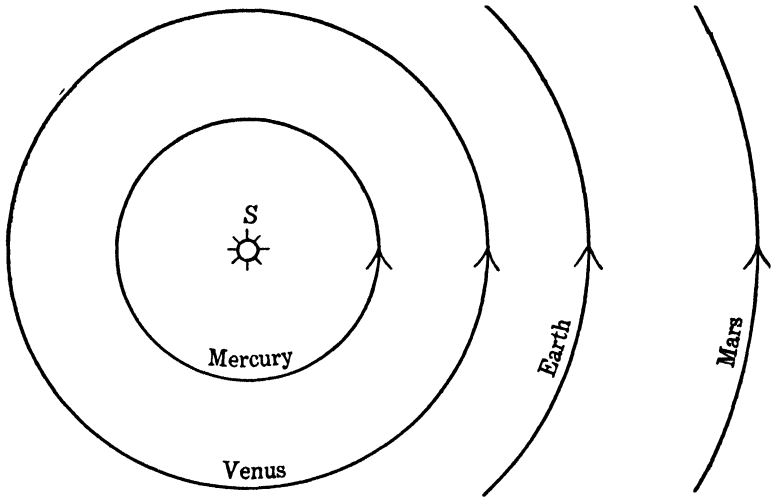


FIG. 98.—Orbits of four inner planets to same scale.

The surface temperature of a planet depends, however, not only upon the radiant energy it receives, but also upon the extent and the composition of its atmosphere and the character of its surface materials. In this article all differences among the planets will be ignored except those depending on their distances from the sun. Consequently the results obtained will be only very roughly indicative of their surface temperatures.

During the period of a year the earth receives a certain amount of radiant energy from the sun, and on the average it radiates the same amount in a year; for if it did not, the temperature of the earth's surface would year after year either increase or decrease, whereas it remains sensibly constant. If the earth were at a different distance from the sun, the amounts of energy it would receive and radiate would also be equal. Since the amount it would receive

at any distance can be computed, the amount it would radiate at that distance is known. The problem is to determine what temperature would give rise to an equal rate of radiation.

The amount of solar energy received per unit area by a body varies inversely as the square of its distance from the sun. When expressed as a formula, the relationship is $e=A/r^2$, where e represents the radiant energy received, r the distance of the body from the sun, and A a constant depending upon the units of time, distance, and energy used.

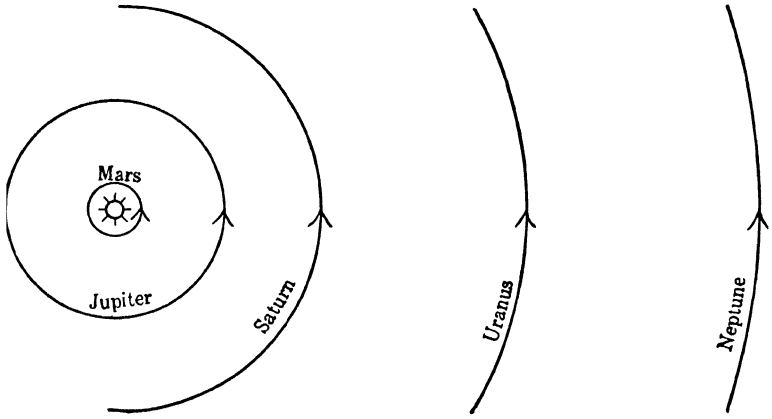


FIG. 99.—Orbits of major planets to same scale.

Now consider radiation. According to Stefan's law, a body radiates energy in proportion to the fourth power of its absolute temperature; or, as a formula, $e=BT^4$, where T is the absolute temperature of the radiating surface and B is a constant depending upon the units employed. Consequently it follows from the two formulas that $BT^4=A/r^2$; or, if we let $A/B=C^4$, that $T=C/\sqrt{r}$.

There is a corresponding formula for the earth which may be written $T_E=C/\sqrt{r_E}$. It follows from the two equations that

$$T=\sqrt{\frac{r_E}{r}}\,T_E,$$

an equation which contains no unknown constants.

Absolute temperatures are measured from absolute zero, which is 273° Centigrade (491° Fahrenheit) below the freezing point of pure water. The mean temperature of the earth is about 15°.5 C. (60° F.), with zero at the freezing point of water (32° F. below the freezing point). Hence the absolute mean temperature of the

earth T_E is about 288°.5 C. (519°.3 F.). With this value of T_E the value of T can be computed by the formula for any distance of a planet from the sun. The results for the distances of the various planets are given in the following table:

TABLE IX. THEORETICAL SURFACE TEMPERATURES OF PLANETS

PLANET	T (ABS. C.)	T (CENTIGRADE)	T (FAHRENHEIT)
Mercury	463°.7	90°.7	195°.3
Venus	339.2	66.2	151.2
Earth	288.5	15.5	60.0
Mars	233.7	− 39.3	− 38.7
Jupiter	126.5	−146.5	−231.7
Saturn	93.4	−179.6	−291.3
Uranus	65.9	−207.1	−340.8
Neptune	52.6	−220.4	−364.7
Pluto	45.9	−227.1	−376.8

It should be kept in mind that the foregoing table gives the temperature that the earth would have at the distances of the various planets only under the hypothesis that the radiation characteristics of its surface would remain just as they are at present. This hypothesis probably differs considerably from what would be true if the earth were much nearer the sun or much farther from it. For example, if the earth were at the distance of Mercury, it is likely that enormous clouds would continually shield its surface from the rays of the sun. If, on the other hand, the earth were at the distance of Jupiter, it is probable that a perpetual mantle of snow and ice would reflect back into space a large fraction of the solar rays falling upon its surface. Consequently the table gives only a measure of the great importance of taking into account the distance of a planet from the sun when such questions as the suitability of its surface as an abode for life are considered.

Even the rate of rotation of a planet has an effect upon its average temperature. Consider the extreme case in which a planet always presents the same face to the sun, as the moon does to the earth. Then the constant A of the foregoing equations is doubled because the heat received by the planet is on one-half its surface. Therefore the constant C is multiplied by the fourth root of 2, or by 1.19. Hence if the earth always presented the same face to the sun, the temperature of that face would be $1.19 \times 288°.5 = 343°.3$ absolute Centigrade, or 70°.3 C. (158°.6 F.). The temperature

of the face which would receive no radiant energy whatever from the sun would approach absolute zero. The actual conditions would, of course, be quite different from these theoretical results because of the enormous amount of energy that would be carried from the illuminated to the dark side by atmospheric currents.

195. Calculating Convenient Dates for Observing Planets. Mercury and Venus are most conveniently situated for observation when they are evening stars and as far as possible from the brilliant rays of the sun. In Fig. 100, S represents the sun, and V, E,

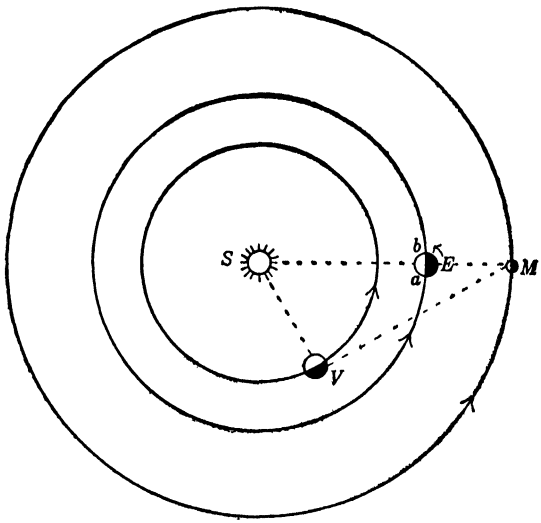


FIG. 100.—Planet V in greatest eastern elongation and M in opposition.

and M Venus, the earth, and Mars respectively. Since the earth rotates in the same direction as that in which it revolves around the sun, a is its evening side and b its morning side.

When Venus is at the point corresponding to V in the diagram, it is at its greatest eastern angular distance from the sun (about 46°), and is situated most conveniently for observation in the early evening hours. The greatest angular distance of Mercury from the sun is only 28° (as a consequence of the eccentricity of Mercury's orbit it ranges from 18° to 28° and averages about 23°), and hence it is never visible long after sunset. When Mercury and Venus are in greatest elongation, they are at the half phase as seen from the earth.

An exterior planet, such as M in Fig. 100, is nearest the earth and most favorably situated for observation when it is exactly in opposition to the sun. The planet is then full and on the meridian of the observer at midnight.

Suppose Venus is at greatest eastern elongation, as indicated by V in Fig. 100. After a synodical period it will be in greatest eastern elongation again. Consequently if the date of such an elongation of Venus is known, all future dates when it will be in the same relative position again can be obtained by adding multiples of its synodical period. Similarly, the dates of opposition of a planet farther from the sun than the earth can be obtained by adding multiples of its synodical period to the date of a known opposition.

The following table gives the sidereal and the synodical periods of each of the planets, and it also gives dates on which Mercury and Venus were at greatest eastern elongation and the other planets were in opposition.

TABLE X. DATES OF GREATEST EASTERN ELONGATIONS OR OF OPPOSITION OF THE PLANETS

PLANET	SIDEREAL PERIOD (YEARS)	SYNODICAL PERIOD (YEARS)	DATE OF EASTERN ELONGATION OR OPPOSITION
Mercury	0.24085	0.31726	Aug. 8, 1930
Venus	0.61521	1.59872	Aug. 13, 1930
Earth	1.00004	.....	.....
Mars	1.88089	2.13539	Jan. 28, 1931
Jupiter	11.86223	1.09211	Jan. 6, 1931
Saturn	29.45772	1.03518	July 13, 1930
Uranus	84.01529	1.01209	Oct. 12, 1930
Neptune	164.78829	1.00614	Feb. 24, 1930
Pluto	249.16610	1.00403	Jan. 7, 1931

Suppose it is desired to find on what date Jupiter will be in opposition in 1935. If four synodical periods of the planet are added to the date January 6, 1931, May 20, 1935, is obtained. For most purposes the exact date of opposition (or of greatest elongation) of a planet is not important, for a planet is in approximately the same position relative to the sun and the earth for several weeks (except in the case of Mercury).

196. Eccentricities and Inclinations of the Orbits of the Planets. Next to the major axes, the most important elements of the planetary orbits are the eccentricities and the inclinations to some plane

of reference such as the ecliptic, which is usually taken as the fundamental plane. The eccentricities define the shapes of the orbits of the planets. If the eccentricities were very great, the stability of the system (Art. 183) might be imperiled. The inclinations of the orbits of the planets to the plane of the ecliptic provide a measure of the closeness of approach of the planetary system to a single plane.

The following table gives the eccentricities of the planetary orbits and the inclinations of their planes to the ecliptic on January 0.0, 1930, Greenwich mean solar time. As was stated in Art. 181, these elements undergo periodic changes.

TABLE XI. ECCENTRICITIES AND INCLINATIONS OF PLANETARY ORBITS

PLANET	ECCENTRICITY	INCLINATION TO ECLIPTIC		
Mercury	0 20562	7°	0′	12″
Venus	0.00681	3	23	38
Earth	0 01674	0	0	0
Mars	0.09334	1	51	0
Jupiter	0 04839	1	18	25
Saturn	0.05579	2	29	28
Uranus	0.04713	0	46	22
Neptune	0 00855	1	46	35
Pluto	0.25374	17	8	57

It follows from this table that the eccentricities of the orbits of all the planets except those of Mercury and Pluto are very small. Since the eccentricity of the orbit of Mercury is about 0.2, this planet is 50 per cent farther from the sun at aphelion than at perihelion. Therefore it has a very pronounced season as a consequence of the variations in its distance from the sun; when it is at perihelion it receives more than twice as much heat as it receives when it is at aphelion.

It also follows from the table that the orbits of all the planets except those of Mercury and Pluto are nearly in the same plane. As Laplace and Lagrange showed, this relationship of the planes of the planetary motions tends to their stability. The suggestion has many times been made that the planets were distributed in this particular way in order to preserve the system. It is more nearly in harmony with present thought to regard these characteristics of the planetary orbits as dynamical fossils, so to speak, by the help of which we may hope to trace the planetary system back to its origin.

197. Dimensions, Masses, and Periods of Rotation of the Planets. Since the distances of the planets from the sun are known, their distances from the earth at any time can be computed. After their distances have been calculated and their apparent diameters have been measured, their linear diameters can be determined. There is no serious difficulty except in the case of Pluto which is so small and so far away that it presents no sensible disk; its diameter is estimated from the amount of light it reflects.

The mass of a planet which has a satellite can be computed by the formula of Art. 177 from the distance of the satellite from the center of its primary and the period of its revolution about its primary. The masses of Mercury, Venus, and Pluto, which have no known satellites, must be determined by other means, such as from their effects upon the motions of passing comets or upon the orbits of each other and the orbit of the earth. The masses of all the planets except Mercury, Venus, and Pluto are known with an error not exceeding one per cent; the masses of Mercury and Venus are subject to considerably greater uncertainties, and the mass of Pluto is quite unknown.

After the diameters and the masses of the planets have been found, the determination of their average densities is clearly a simple matter of computation. It is an interesting fact that the earth is the densest planet, and that the larger planets are much less dense than the smaller ones.

Mars, Jupiter, and Saturn, as seen through a good telescope, under favorable conditions present such conspicuous markings that it has been possible to determine their periods of rotation with a high degree of accuracy. Uranus and Neptune, however, are always so remote from the earth that their surface markings, which at times can be detected, can not be followed. Their periods of rotation, however, are inferred, at least roughly, from their degrees of oblateness, which obviously depend upon their rates of rotation. The problem of determining the periods of rotation of Mercury and Venus is even more difficult, for the orbits of these planets are interior to that of the earth, and their dark sides are toward the earth when they are nearest. Moreover, Venus appears to be covered with such a dense canopy of clouds that its surface is rarely, if ever, visible from the earth. All these data for Pluto are quite unknown, though it is probably not larger than the earth. The data for the other planets, as adopted

by the American Nautical Almanac, are condensed in the following table:

TABLE XII. DIMENSIONS, MASSES, DENSITIES, AND PERIODS OF ROTATION

BODY	MEAN DIAMETER (KILOMETERS)	MEAN DIAMETER (MILES)	MASS (EARTH = 1)	DENSITY (WATER=1)	SURFACE GRAVITY (g = 1)	PERIOD OF ROTATION
Sun	1,391,107	864,392	333,430	1.41	27.9	24.65 days
Moon	3,476	2,160	0.0123	3.33	0.16	27.32 "
Mercury	4,842	3,009	0.04 (?)	3.80 (?)	0.27 (?)	88 " (?)
Venus	12,192	7,575	0.81 (?)	5.09 (?)	0.88 (?)	?
Earth	12,742	7,918	1.000	5.52	1.00	24.00 hrs.
Mars	6,784	4,216	0.108	3.95	0.38	24.60 "
Jupiter	139,576	86,728	316.94	1.33	2.64	9.9 "
Saturn	116,565	72,430	94.92	0.73	1.19	10.2 "
Uranus	49,693	30,878	14.58	1.36	0.96	10.7 " (?)
Neptune	52,999	32,932	16.93	1.30	0.98	15.0 " (?)

It appears from this table that, omitting Pluto, the planets in respect to size fall into two quite distinct groups, the inner four and the outer four. The members of the inner group are a few thousand kilometers in diameter; the diameters of the members of the outer group are about 10 times as great. It follows that the surfaces and the volumes of the members of the outer group are 100 and 1000 times greater respectively than those of the inner group, as is illustrated in Fig. 101. The differences in dimensions are so great that it is quite likely that if we lived on one of the larger planets of the outer group, we should express our lack of respect for the four members of the inner group by calling them something inferior to planets.

In respect to mass, the eight planets fall into the same two groups, though the differences in masses are not so great as they are in volume, for the smaller planets of the inner group are about four times as dense as the larger planets of the outer group. It has often been thought that this difference in density is due to the fact that the earth-like planets are farther advanced in their evolutions than are the planets of the outer group. It may mean, however, that they are composed of essentially heavier materials as a consequence of the manner of their origin. Since the pressures in the interiors of the planets are greater the greater their masses, it would be expected, if there were no observational evidence to the contrary, that the larger outer planets would be the denser.

Surface gravities of bodies depend upon both their masses and their dimensions, and it is seen from the table that although the large planets have relatively low densities, their surface gravities are comparable to that of the earth.

The periods of rotation of the planets present a surprise, for the large planets rotate more rapidly than the smaller ones, and the giant Jupiter has the shortest day of all. The periods of rota-

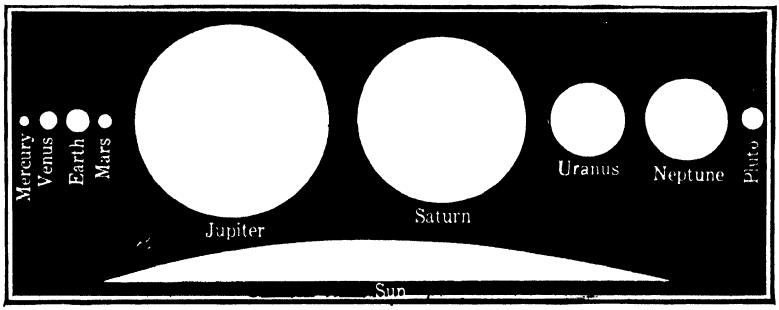


FIG. 101.—Relative dimensions of the planets and the sun.

tion of the planets, as well as the characteristics of their orbits, are probable dynamical inheritances from their origins and early histories.

198. The Discovery of Pluto. Since the discovery of Neptune in 1846, many astronomers have considered the possibility of the existence of a planet more remote from the sun and have discussed methods of finding it if it does exist. They knew that there might be such a planet, for the gravitative control of the sun extends a thousand times as far as the most distant planet; indeed, some of the comets recede to those great distances.

The discovery of Neptune from its effects upon the motion of Uranus suggested the same method for proving the existence of still another world. But at those remote distances the planets are far apart and their gravitational effects are small. A number of astronomers attempted to determine the distance of a trans-Neptunian planet by its possible effects upon comets which recede to great distances, and conclusions were reached that there are probably several such bodies. Among all who have been interested in the subject, Percival Lowell made the most thorough study of the pertinent data and deduced precise results respecting

the probable position of an unknown planet. Moreover, he founded and endowed an observatory for the purpose, among other things, of searching for the planet whose existence and approximate position he believed he had demonstrated.

On January 21, 1930, the Lowell astronomers secured a plate on which C. W. Tombaugh found the image of a new planet which they had sought during 25 years. It was nearly in the direction predicted by Lowell, but at a considerably smaller distance. Moreover, it was only of the fifteenth magnitude, from which it follows that even if its reflective power is as low as that of the moon (9 per cent) its diameter is much less than that of the earth. Consequently, unless it is improbably dense its mass is so small that it can not have produced the effects on which Lowell based his prediction. It seems likely that Lowell's prediction that the planet would be near the place at which it was found was good fortune; but that the Lowell astronomers made the discovery was the result of a well-conceived and thoroughly executed plan of search. Since a distant planet is distinguishable from the millions of faint stars among which it appears only by its very slow motion, the enormous difficulty of discovering such a body is apparent.

In respect to both eccentricity and inclination, the orbit of Pluto is quite different from the orbits of the other planets. At its nearest approach to the sun it is closer than Neptune, but the inclination of its orbit is high and its path does not cross that of Neptune. Perhaps it is the most remote member of the solar family, but more probably it is related to the planetoids which we shall consider in the next article.

199. The Planetoids (or Asteroids). On the very first day of the nineteenth century, January 1, 1801, G. Piazzi, at Palermo, noticed a faint object, apparently a star, where none had been seen a few days earlier. His observations on succeeding nights showed him that it was moving with respect to the stars, and, therefore, that it is a member of the solar system. Piazzi named the new body Ceres.

Shortly after the discovery of Ceres, Piazzi became ill, and by the time he had recovered, the sun had moved eastward among the stars so near to the new body that it was no longer visible. After nearly a year it should have been favorably situated for

observing again, but it had moved forward among the numerous small stars from which it was not readily distinguishable, and its recovery by direct observational search promised to be a very laborious task. Fortunately, C. F. Gauss, a brilliant young German mathematician, during the interval had perfected a method for computing the orbit of a comet or of a planet from three observations of its apparent position. Gauss applied his method to computing the orbit of Ceres and to determining its apparent position on dates when it became again observable, and the planetoid was easily rediscovered.

Three similar objects, Pallas, Juno, and Vesta, were discovered by 1807, but not another was found till 1845. Since 1847 at least one planetoid has been discovered each year and now (1931) more than 1150 of them are known.

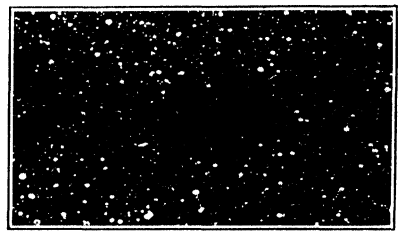


FIG. 102.—Two planetoid trails.

In the early days of searching for planetoids, they were discovered by the excessively laborious method of comparing the sky through a telescope with stellar maps to determine whether a strange object had moved in among the stars. After photography was introduced into this field by Max Wolf, in 1891, the images of hundreds and even thousands of objects were obtained in a few hours with wide-angle cameras mounted equatorially and driven accurately by clockwork. Moving objects, such as the planetoids, could then be found by the simpler process of comparing plates taken at different dates. In fact, the images of planetoids obtained during exposure extending over a few hours are short streaks (Fig. 102), instead of points such as the star images, and hence are readily identified. This is, indeed, the method by which most of the planetoids have been discovered.

A variation in the foregoing method consists in making the telescope follow the expected and usual motion of the planetoids instead of that of the stars. The star images are then short streaks and the planetoid images, if they are found, are small dots or possibly very short streaks if their motion is exceptional. By this method the images of planetoids can be secured which are so faint that if they were permitted to drift on the plates they would not produce sensible streaks.

200. Orbits of the Planetoids. Gauss found that Ceres moves around the sun in an elliptical orbit between the orbit of Mars and that of Jupiter, and the orbits of nearly all other known planetoids lie between the orbits of these planets. One, however, crosses within the orbit of Mars, several are at about the distance of Jupiter, and one is known which recedes to the distance of Saturn. The orbits of the planetoids, however, are not concentric nor are they regularly spaced. They are, in fact, so interwoven that if a model of them were made of wire and one were removed, nearly all the remainder would be taken with it.

Some of the orbits of the planetoids are nearly circular, while others are much elongated, the highest eccentricity being at present about 0.65. The orbits of a number of them are more eccentric than the less eccentric cometary orbits, and the average eccentricity is about 0.15, which is much beyond the eccentricities of the orbits of planets.

The inclinations of the orbits of the planetoids to the plane of the ecliptic are as diverse as are their eccentricities. Some orbits are nearly coincident with the ecliptic while many are highly inclined, the inclinations ranging up to 30° and even higher in one or two cases. The average inclination of the orbits of the known planetoids is about 9°, or higher than that of the orbit of Mercury. Usually high eccentricities and high inclinations to the plane of the ecliptic go together.

201. Families of Planetoids. Although the distances of the planetoids from the sun range from the orbit of Mars to that of Jupiter, there are zones in which these strange objects are very infrequent or entirely lacking. As Daniel Kirkwood pointed out long ago when only 88 planetoids were known, these gaps occur at the distances at which the period of revolution of a planetoid would be 1/2, 1/3, 2/3, . . . of Jupiter's period. The numerous discoveries by photography have emphasized these remarkable vacant spaces.

If a planetoid were moving in an orbit with a period bearing such a simple relationship to that of Jupiter as one-half or one-third, its motion would be exceptionally affected by the attraction of Jupiter. It is probable, though it has not been rigorously proved, that the disturbing action of Jupiter has cleared the planetoids from the zones in which these relationships would

exist. In this connection it is worthy of note that there are vacancies in Saturn's rings where there would be similar commensurabilities with its larger satellites.

There are also families of planetoids in a stronger sense. Since elliptical orbits are closed, it follows that if a planet should break up into a number of separate masses, the fragments would all return to the original point of rupture, though after different periods depending upon the forces which caused the catastrophe. Consequently it is natural to seek for families of planetoids whose members all pass near a given point, possibly a point from which they once separated.

It is evident, however, that the perturbations by Jupiter would destroy any such simple relationship as that which has just been mentioned. But as K. Hirayama has shown in notable investigations of the subject, certain more complicated relationships among the orbits would persist, however much the orbits might be changed by the attraction of Jupiter. On examining the orbits of the planetoids, he finds that there are at least five of these families of the type his theory predicts. It is likely that the members of each group had a common origin near one another, though it is improbable that they are fragments of a larger mass which was broken up by some violent force.

202. The Jupiter (Trojan) Planetoids. There is another family of planetoids whose members revolve around the sun at the distance of Jupiter and, consequently, in the same period as that of Jupiter. Moreover, they move so that they form with Jupiter and the sun the vertices of nearly equilateral triangles. Five of these bodies oscillate about the vertex in advance of Jupiter, and four about the corresponding point 120° behind Jupiter. As E. W. Brown has shown, these bodies illustrate (Fig. 103) approximately one of the solutions of the problem of three bodies discovered by Lagrange (Art. 178).

The Trojan planetoids are very faint objects, ranging from the twelfth magnitude to the fourteenth, and they are visible only through powerful telescopes. Their apparent faintness is due in part to their great distance from the sun and from the earth and partly to the fact that they are probably not more than 100 to 150 kilometers in diameter. It is likely that there are many smaller members of this group which are as yet undiscovered.

203. The Planetoids Themselves. E. E. Barnard measured the diameters of the brightest four planetoids with the 36-inch refractor of the Lick Observatory and repeated the measurements with the great Yerkes telescope. He obtained for the diameters: Ceres, 780 kilometers (485 miles); Pallas, 489 kilometers (304 miles); Vesta, 391 kilometers (243 miles); Juno, 190 kilometers (118 miles). It is probable that only a few of the remainder have diameters exceeding 160 kilometers (100 miles), and the smallest of those known can hardly be greater than 10 to 15 kilometers in diameter.

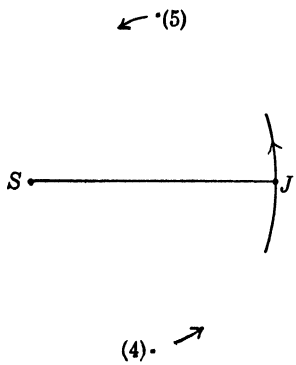


FIG. 103.—The Trojan planetoids which form equilateral triangles with Jupiter and the sun.

On taking into account the different diameters, distances, and magnitudes of these four planetoids, it is found that they differ greatly in reflecting power per unit area. The reflecting power of Vesta is twice that of Juno, three times that of Pallas, and more than four and one-half times that of Ceres, which is about equal to that of the moon. These bodies are comparable in diameter and in distance from the sun, and they probably had a similar origin; yet their reflecting powers and consequently the characteristics of their surfaces differ enormously.

Among the planets and satellites it is found that the bodies with extensive atmospheres, such as Venus and the large planets, have high reflecting power, while such smaller bodies as the moon and Mercury without appreciable atmospheres have low reflecting power. It can not reasonably be supposed that Vesta can retain an atmosphere; yet its reflecting power per unit area, when it is at the full phase, is almost equal to that of Venus and Jupiter. Ceres reflects about the same percentage of light as does the moon, and Pallas about the same percentage as Mars.

In a number of cases it has been found that the brightness of a planetoid varies periodically in a period of a few hours, its variation being undoubtedly due to its rotation. When the earth moves forward in its orbit from E_1 to E_2 and the planetoid from P_1 to P_2 (Fig. 104), their direction from each other is changed by the angle E_1PE_2. If the planetoid rotates in the forward (or retrograde)

direction, its phases will occur earlier (or later) by a corresponding amount. Since they are observed to occur earlier under the conditions illustrated in the diagram, these planetoids rotate in the same direction as that in which they revolve.

There is no method available for determining the densities or the masses of the planetoids. They are obviously solid bodies, for a gaseous body of their dimensions would not have enough self-gravitation to hold together. We shall probably not be much in error if we assume that they have the density of the exterior of the earth, or 2.70 with water as the standard. Under this assumption, the mass of Ceres is about 1/10,000 that of the earth, and the mass of all the planetoids known probably does not exceed 1/40 that of the moon. Of course, there may be, and probably are, tens of thousands of other planetoids which are too small to be detected by any means now available, but it is very unlikely that their entire mass is as great as 1/20 that of the earth, for if it were its perturbing effects on the motion of Mars would be observable.

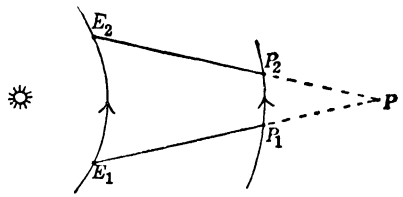


FIG. 104.—Change of phases of planetoids.

204. The Planetoid Eros. Since most of the planetoids are very small, they are not distinguishable from one another except by their positions, which can be computed only at the cost of very great labor because of the perturbations of their motions by Jupiter. Just as astronomers were considering the advisability of giving up the task of following all the smaller planetoids, a very important discovery was made by G. Witt, at Berlin.

In August, 1898, Witt photographed a small body which at times comes nearer the earth than any other celestial object except the moon. The sidereal period of this planetoid, which he named Eros, is 643.23 mean solar days and its synodic period is 845 days, which is longer than that of any other known planetoid. Its mean distance from the sun is 1.458 times that of the earth and its orbit has the high eccentricity 0.223. At its nearest possible approach to the earth, it comes within 22,300,000 kilometers (13,800,000 miles) of our planet; and at its aphelion, it is 38,600,000 kilometers (24,000,000 miles) beyond the orbit of Mars.

There is no danger, however, of its colliding with Mars, for the inclination of its orbit to the ecliptic is 10° 49′.

The discovery of Eros aroused great interest both because of the exceptional nature of its orbit and also because, at times when it approaches near to the earth, it affords an unexcelled opportunity for determining the dimensions of the solar system (Art. 192). Only exceptionally, however, is Eros in opposition at the point in its orbit which is nearest the orbit of the earth. In the opposition of 1901 its minimum distance from the earth was about 48,000,000 kilometers, and the oppositions since that date have all been at points at which its orbit is farther from that of the earth. In the early months of 1931 Eros was again very near the earth, its minimum distance on January 30 having been about 26,100,000 kilometers (16,200,000 miles).

Eros is remarkable not only because of the position of its orbit, but also for the fact that both in 1901 and also in 1931 it varied in brightness both extensively and rapidly, the range being about three to one. In an interval of $5^h\ 15^m$ it went through two maxima and two minima. The surprise produced by the variability of Eros was followed by an even greater surprise when, after about three months, its changes in brightness greatly declined and soon ceased entirely. The variability of Eros is explained by the theory that different parts of its surface differ greatly in reflecting power, and that it rotates in a period of $5^h\ 15^m$ with the plane of its equator highly inclined to the orbit of the earth. At present these suggestions are only tentative, and a satisfactory and complete explanation of its variability awaits the discussion of observations made under favorable conditions.

205. The Zodiacal Light and the Gegenschein. On clear, moonless nights just as twilight fades, a soft, hazy wedge of light can be seen stretching up along the ecliptic from the western horizon, or from the eastern horizon just before dawn appears. At its base on the horizon, this so-called *zodiacal light* is 20° or 30° wide, and it gradually tapers to a width of a few degrees at a distance of 60° to 75° from the sun. Sometimes it can be followed as a faint, narrow band extending along the ecliptic from one horizon to the other.

Under favorable conditions the zodiacal light is not difficult to observe, but it is most conspicuous when the ecliptic makes a

large angle with the horizon, for it is then reduced least in brightness by the absorption of the dense lower strata of the air. In the spring of the year the sun is near the vernal equinox where the ecliptic, measured eastward, crosses the equator from the south side to the north side. Consequently in northern latitudes the zodiacal light is most easily seen in the evening during the spring months, and in the morning during the autumn months.

Around the point on the ecliptic 180° from the sun there is a very faint elongated oval patch of soft haze, usually a few degrees wide and 10° or 15° long. This *gegenschein* (or counterglow) is so faint that it is never visible in moonlight or when it falls in the Milky Way.

The zodiacal light has been ascribed to reflections of sunlight from innumerable small particles, circulating around the sun near the plane of the ecliptic. In harmony with this theory, the gegenschein is explained as being a condensation in these revolving particles at a point in the direction opposite to the sun and distant from the earth 1,530,000 kilometers (930,000 miles) where, as a consequence of the combined attractions of the sun and the earth, wandering particles tend to congregate in a sort of dynamic whirlpool. But V. M. Slipher has found that the spectrum of the zodiacal light is both reflected sunlight and also in part bright lines and bands, among them being the bright auroral line in the yellow, which are due to unknown tenuous gases. In fact, the whole sky is somewhat luminous with a very faint glow which resembles the zodiacal light and which, according to P. J. van Rhijn, accounts for nearly 60 per cent of the light of the sky on a moonless night. Moreover, H. T. Stetson and others have observed unmistakable fluctuations in extent and brightness of the zodiacal light in the course of a few minutes. These phenomena indicate that the zodiacal light and the gegenschein may be due, at least in part, to the effects of electrified particles streaming out from the sun through a tenuous medium involving the entire planetary system.

XVII. QUESTIONS

1. What is the ratio of the volume of the earth to that of a sphere whose radius equals the distance from the sun to the earth?
2. Show how, in the case of an inferior planet, the angle *ESP* (Fig. 96) can be determined at any time from the periods of the earth and the planet and the time the planet is between the earth and the sun (assuming the orbits are circles).

3. At how many meters from the eye will a disk one centimeter in diameter subtend the same angle as the earth as seen from the sun?
4. How many independent methods are there for determining the distance from the sun to the earth?
5. How long would be required for a train running at the rate of 100 kilometers per hour to describe a distance as great as the perimeter of the earth's orbit around the sun?
6. Make a diagram of the orbits of all the planets on a single sheet of paper.
7. From Tables VIII and IX make a diagram in which horizontal distances represent distances of the planets from the sun and vertical distances their temperatures.
8. Determine the next dates at which Mercury and Venus will have greatest eastern elongation; at which the superior planets will be in opposition. Look for Mercury, Venus, Mars, Jupiter, and Saturn. If possible, observe them through a telescope.
9. Compute the orbital velocities of the planets.
10. Compute the equatorial rotational velocities of the sun, the moon, and the planets.
11. What would be the apparent brightness of a planet in comparison with Jupiter if it were similar to Jupiter but 20 times as far from the sun?
12. If the orbits of 1000 planetoids were uniformly distributed between the orbits of Mars and Jupiter, what would be the distance between the adjacent ones?
13. Show from the periods of revolution of the Earth and Eros that favorable oppositions of Eros occur once in 30 years.
14. Observe the zodiacal light.

CHAPTER XI

THE PLANETS

I. Mercury and Venus

206. Mercury and Venus as Seen With the Unaided Eye. Occasionally in the western sky in the evening there is visible a brilliant silver-white "star," far surpassing all other stars in splendor (−4.3 mag.). Sometimes this beautiful object, the planet Venus, is seen directly in the west; at others, it is south of west; and at still others, in the far northwest. For many weeks it adorns the evening sky; then it gradually pales and quickly follows the sun below the western horizon. Finally, it is seen no more as the evening star.

More frequently an attentive observer will see not far from the western horizon, shortly after the sun has set, a less brilliant object, yet one which surpasses in brightness all stars except Sirius. This is the planet Mercury which for a few days comes out far enough from the sun to be easily visible; then it, too, declines in brightness and disappears as it approaches the sun.

Within a few weeks after either Mercury or Venus disappears from the western evening sky, a similar object appears in the eastern sky shortly before the dawn. In fact, it is the same object, reappearing after it has passed by the sun. In mythology, Venus as evening star and Venus as morning star were supposed to be different bodies. The Greek scientists, however, and also some of the American Indian tribes, knew that the evening and the morning stars are actually the same object.

When Venus is evening star it becomes visible in the western sky about sunset and shares in the westward diurnal motions of the celestial bodies until it sinks below the horizon. On the contrary, when it is morning star it rises in the east some time before dawn begins and ascends the eastern sky ahead of the sun. Often it can be easily followed with the unaided eye until after the sun has risen; in fact, it is so brilliant when near its maximum that it

can be seen even at midday provided one focuses his eyes on a distant point and looks precisely in its direction.

207. Phases of Mercury and Venus. Since the orbits of Mercury and Venus are interior to that of the earth, these planets in the course of their synodical periods run through all phases from that in which their dark sides are toward the earth to that in which their illuminated sides are fully visible. The reasons for the phases of an inferior planet are illustrated in Fig. 105, in which E represents the earth, and P_1, . . . , P_4 four positions of the planet.

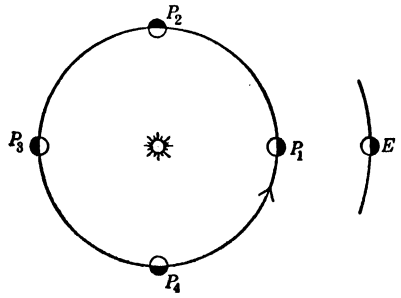


FIG. 105.—Four phases of an inferior planet.

Since the moon revolves around the earth as a center, its apparent dimensions are about the same at all phases. The inferior planets, however, revolve around the sun and hence their apparent diameters vary with their phases. The apparent dimensions of Venus change through a much greater range than those of Mercury because its orbit is not far interior to that of the earth. Thus, when Venus is at P_1, its distance from the earth averages 41,363,000 kilometers (25,702,000 miles), while its distance at P_3 averages 257,644,000 kilometers (160,100,000 miles). On the other hand, the corresponding average distances for Mercury are 91,630,000 kilometers (56,937,000 miles), and 207,376,000 kilometers (128,860,000 miles). In other words, the greatest distance of Venus is nearly 6.3 times its least distance, while the greatest distance of Mercury is only a little more than 2.2 times its least distance. These limits vary considerably, especially in

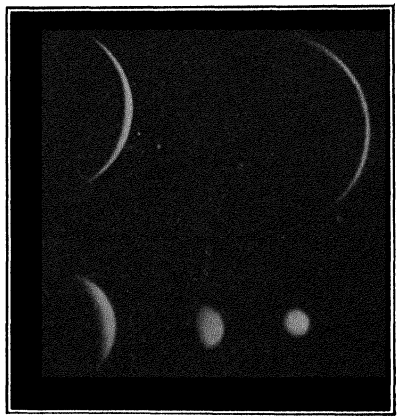


FIG. 106.—Relative diameters of Venus at five phases. (From a photograph by E. C. Slipher, Lowell Observatory.)

the case of Mercury, because of the eccentricities of the orbits of these planets.

208. Transits of the Sun by Mercury and Venus. If the orbits of Mercury and Venus were exactly in the plane of the orbit of the earth, each of these planets would apparently pass in front of the sun once each synodical revolution. But the orbit of Mercury is inclined 7° to the plane of the ecliptic and the inclination of the orbit of Venus is 3° 24′. These angles are so great in comparison with the apparent radius of the sun (about 16′) that there are transits of these planets only when they are near the nodes of their orbits.

The sun is near one of the nodes of the orbit of Mercury in May and near the other in November. Consequently transits of Mercury occur only in these months. Since the synodical period of this planet (0.31726 years) does not have a simple relationship to the year, its transits do not occur frequently. When we examine the numbers, we find that a transit of Mercury is followed by another at the same node after an interval of 7, 13, or 46 years, according to circumstances, for these intervals are approximately 22, 41, and 145 synodical periods respectively. The next transit of Mercury will occur on May 10, 1937.

Transits of Venus, which occur in June and December, are even more infrequent than those of Mercury. They appear in cycles whose intervals, starting with a June transit, are 8, 105.5, 8, and 121.5 years. The most recent two transits of Venus occurred on December 8, 1874, and December 6, 1882, and the next will occur on June 8, 2004, and June 5, 2012.

Transits of the inferior planets are of scientific interest, for they aid in determining very precise positions of these bodies at known times, and consequently provide material for checking and correcting the elements of their orbits. The difficult determinations of the motion of the perihelion of the orbit of Mercury by Leverrier and by Newcomb were largely based on discussions of transits of this planet. The transits of Venus in 1874 and 1882 were used to measure the solar parallax, though other methods now give more nearly accurate results (Art. 192).

209. Atmospheres of Mercury and Venus. In respect to their atmospheres, Mercury and Venus are wholly unlike, the former having no observable atmosphere, while the latter has a gaseous

envelope which in extent probably approaches that of the earth. The surface gravity of Mercury is probably too low to hold an atmosphere; the surface gravity of Venus is 0.85 that of the earth.

When Mercury transits the disk of the sun, it is seen as a round black spot with perfectly sharp margins, like those of the moon in eclipse; but Venus, on the other hand, is surrounded by a ring of light as it approaches transit and just after it passes from in line with the sun. This ring of light is undoubtedly due to refraction of the sun's rays by the atmosphere of the planet, just as the atmosphere of the earth refracts some light to the moon at the time of a total lunar eclipse (Art. 167).

The conclusion that Mercury has little or no atmosphere is also confirmed by the fact that its reflecting power (albedo) is only about 0.07, or a little less than that of the moon. On the contrary, the albedo of Venus is 0.59, or probably somewhat higher than that of the earth, and its atmosphere is so completely filled with clouds that only rarely if ever do we get glimpses of its surface. But an observer on Venus (if he could get above its clouds and had a telescope) would see our planet only half covered with brilliant reflecting clouds, and the remainder, except where there happened to be snow, would appear to be about one-third as bright. In the zone of the equatorial rains there would be found a brilliant cloud belt whose position would vary with the seasons; on each side of this belt there would be observed a relatively cloudless zone in the neighborhood of the tropics; and in the higher latitudes there would be seen spotted and ever-changing areas up to the polar regions of perpetual snows.

As seen from Venus, the brilliant earth would always be accompanied by a tiny, yellowish object one-ninetieth as bright, the moon. The apparent distance between the earth and the moon would depend upon their distance from Venus and upon the position of the moon in its orbit. At its maximum the angular distance between our planet and its satellite as seen from Venus would be about 1°.

210. Surface Markings and Rotation of Mercury. Since Mercury, as seen from the earth, is never more than 28° from the sun, its surface is never observed under favorable conditions. As a consequence of this fact, very little is yet known of its surface markings. Aside from some doubtful early observations, the first

observer to announce discernible features on the surface of the planet was G. V. Schiaparelli. In 1880, the Italian astronomer reported certain dark permanent markings from which he inferred that the planet rotates on its axis in its period of revolution in such a way that, except for librations, one of its sides is permanently toward the sun. E. E. Barnard also at times with the great Lick and Yerkes telescopes detected features resembling those seen on the moon without optical aid. Percival Lowell, observing with the 24-inch refractor at Flagstaff, Arizona, drew more detailed and definite maps of the surface of Mercury than either Schiaparelli or Barnard, and he, too, believed it probable that this planet always presents the same face to the sun. The evidence, however, is as yet inconclusive.

It is natural to think of observing the planets only when the sun is below the horizon. Astronomers have found, however, that it is preferable to observe Mercury during full daylight when its altitude is considerable, for the illumination of the sky is a much less serious obstacle than are the absorption and the irregularities of refraction which are encountered when the planet is near the horizon.

211. Surface and Temperature Conditions of Mercury. Since Mercury is not surrounded by an appreciable amount of atmosphere, it is probable that its surface is very rough and comparable in a general way to that of the moon. It apparently has not been subject to the weathering and smoothing action of air and water. Yet in the long run the small meteors which probably fall upon the planet in countless millions may have gradually beaten its surface smooth, though an occasional very large one would make it perhaps as rugged as the face of the moon.

As was stated in Art. 194, if the surface, atmosphere, and rotation of Mercury were in every respect similar to those of the earth, its mean surface temperature would be 190°.7 C. (375°.3 F.). But since Mercury has no atmosphere, it loses by reflection much less of the incident energy from the sun; on the other hand, it has no gaseous blanket to retain the heat which is absorbed. The latter factor is probably more important than the former, and it is likely that the surface temperature of Mercury is somewhat lower than it would be if it had an atmosphere similar to that of the earth.

If Mercury rotates in such a way that one of its sides is always toward the sun, that side must be very hot and the opposite one extremely cold. From theoretical considerations we infer that the average surface temperature of the hot side would be about 280° C. (540° F.), while the temperature of the dark side would be near absolute zero. The measurements of Edison Pettit and S. B. Nicholson of the amount of heat radiated by the illuminated side of the planet confirm the conclusion that its maximum temperature is 382° C. (656° F.), or quite sufficient to melt such metals as lead.

The temperature of the illuminated side of Mercury is, of course, highest where the sun is at the zenith, and it diminishes steadily to the circle at which the sun is on the horizon. Since the planet has librations of $23\overset{\circ}{.}7$ each side of its mean position, as a consequence of the eccentricity of its orbit, its position of highest temperature moves 47° in longitude during half its period of revolution, or in 44 terrestrial days. The line on which the sun is on the horizon of Mercury varies by an equal angular distance, and there are only 133° in longitude on which the sun never shines. All this, of course, applies only if it is true that the planet rotates exactly in its period of revolution.

212. Surface Markings and Rotation of Venus. It appears to be impossible to determine the period of rotation of Venus from observations of its permanent markings, for an almost perpetual canopy of clouds veils the surface of the planet from our view. Barnard, however, at times saw bright spots which he thought might be the lofty summits of snow-clad mountains, reaching above the level of the denser clouds, but he never found it possible to follow such spots long enough to throw any light upon the period of rotation of the planet.

Venus is so near the earth when it transits the sun that its disk subtends an angle of more than a minute, and through a good telescope it appears like a large object. If the planet were rotating in a period approximately equal to that of the earth, its polar diameter would be about 1/150 less than its equatorial diameter, and the difference could be readily measured. Since no such difference has been found in spite of attempts to detect it, it is probable that the period of rotation of Venus is at least a considerable number of terrestrial days in length.

A rapid rotation of Venus could also be detected by determining the motion of its margins toward or from the earth by means of the spectroscope. The fact that no rotational effects upon the spectral lines could be found either at the Lowell Observatory or at the Mount Wilson Observatory confirms the conclusion that the period of rotation of the planet is long in comparison with that of the earth.

On the other hand, F. E. Ross finds on his photographs of Venus taken in ultra-violet light a banded structure in the clouds of the

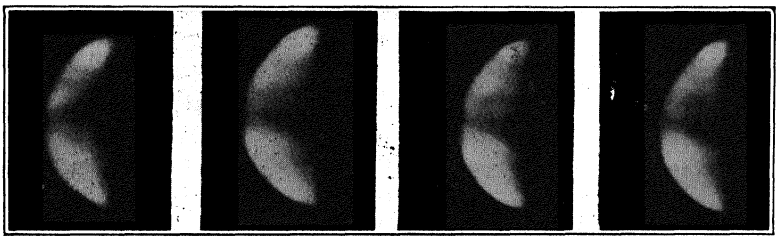


FIG. 107.—Photographs of Venus in ultra-violet light with the 60-inch reflector of the Mount Wilson Observatory, June 29 and 30, 1927. (F. E. Ross.)

planet's atmosphere, dimly suggestive of those in the atmosphere of Jupiter (Fig. 118). Such bands of course imply rotation, and Ross suggests a period of about 30 terrestrial days.

213. The Atmosphere of Venus. That Venus is covered by an atmosphere of considerable extent is certain. As has been mentioned, it is surrounded by a luminous ring when it is about to begin a transit of the sun. When it is farther from the sun the horns of its crescent extend appreciably beyond the line of its diameter. H. N. Russell has shown from the extent of these protrusions of the horns at various phases of Venus that the thickness of the atmosphere which produces the phenomenon is about 1300 meters (4000 feet). Probably the phenomenon is due only to that part of the atmosphere of Venus which lies above the level of the clouds.

Although it is certain that an atmosphere surrounds Venus, it is difficult to determine how it compares in extent with that of the earth. Since the surface gravity of the planet is less than that of the earth, and since molecular velocities in its atmosphere are greater than they are in our air because of its higher temperatures, it is probable that its atmosphere is less extensive than the earth's.

There is no observational evidence which throws a serious doubt on this conclusion.

With respect to the constitution of the atmosphere of Venus very little is known. There is, of course, no reason why it should be composed of precisely the same gases in the same proportions as those in the atmosphere of the earth. But it could not be composed of substances that would be liquid or solid under the conditions prevailing on the planet, nor could it contain a substance having a strong chemical affinity for other atmospheric constituents or for the surface materials of the planet.

In the sun's atmosphere there are vaporous iron and other elements whose existence is established by their absorption lines in the solar spectrum (Art. 278). There is no difficulty in inferring that these elements are in the sun, for they are not in the earth's atmosphere. But the atmosphere of Venus can not be composed of much else than the elements and compounds which are in the earth's atmosphere, and consequently its absorption lines are blended with, and not readily distinguishable from, those due to the atmosphere of the earth. There is, however, one favorable circumstance which partially separates the effects at times. Sometimes the earth and Venus are relatively approaching toward or receding from each other as a consequence of their orbital motions. At such times the absorption lines of the atmosphere of Venus are relatively displaced with respect to those of the earth's atmosphere by the so-called Doppler effect (Art. 277), and there is a chance of distinguishing one source from the other. An exhaustive investigation of the absorption lines of Venus has failed to show the presence of either water vapor or oxygen in the part of its atmosphere which is traversed by the light that is reflected to the earth. Perhaps this result should not be surprising, for the light which has been analyzed probably did not penetrate at all into the cloud-filled lower levels.

214. On the Temperature of Venus. Our ignorance of the period of rotation of Venus and the uncertainties respecting the extent and the constitution of its atmosphere combine to make it impossible for us to reach any very definite theoretical conclusion about its surface temperature. If these factors were all similar to those on the earth, its mean temperature would average approximately 66° C. (151° F.). At the equator, it would probably often

be 20° to 30° C. higher, or near the boiling point of water under the conditions assumed; at the poles, it would rarely, if ever, descend to the freezing point.

If, however, the day of Venus were very long in comparison with that of the earth, and, particularly, if the planet rotated in its period of revolution, then its temperature conditions would be much more extreme. The highest temperatures would probably be above the boiling point of water and the lowest far below the freezing point.

Fortunately, there is some observational evidence respecting the temperature of Venus. The measurements by W. W. Coblentz and C. O. Lampland of the radiant energy received from the planet indicate a temperature for its radiating layer (on the illuminated side) of 50° C. (122° F.). That is, the radiometric observations do not contradict the results tentatively obtained by theory. On the night side, Edison Pettit and S. B. Nicholson obtained measurements corresponding to about 30° C. below the freezing point of water.

Obviously it is not possible at present to draw any positive conclusions respecting the atmosphere and the surface temperature of Venus, though we are justified in thinking of it tentatively as a planet that in a general way is similar to the earth, but considerably warmer. The question whether there is life on Venus at once arises, for to us life seems to be supremely important. Unfortunately, so many of the factors on which life depends are as yet unknown in the case of this planet that we can not have even the partial satisfaction of fixing our minds on probabilities.

II. The Planet Mars

215. Mars and Its Satellites. As compared with the earth, the diameter of Mars is 0.53, its mass is 0.11, its mean density is 0.72, and its surface gravity is 0.38. That is, in dimensions and mass Mars is far inferior to the earth, but is much superior to the moon.

In view of the small mass of Mars, it is perhaps surprising to find that it has two satellites. These satellites are not large bodies, comparable to our moon, but tiny objects, the greater probably not exceeding 15 kilometers (10 miles) in diameter and the smaller half as great. They were discovered by Asaph Hall, at Washington, in 1877. The satellites of Mars are difficult to observe not only

because they are small but also because they are close to the planet. The inner one, Phobos, revolves in a period of $7^h\ 39^m$ in an orbit whose mean radius is 9380 kilometers (5828 miles); the outer satellite, Deimos, revolves in a period of $30^h\ 18^m$ in an orbit whose mean radius is 23,460 kilometers (14,577 miles). The distance of Phobos from the surface of Mars is only 5995 kilometers (3725 miles), and that of Deimos is 20,075 kilometers (12,475 miles). The relative dimensions of Mars and the orbits of its satellites are illustrated in Fig. 108.

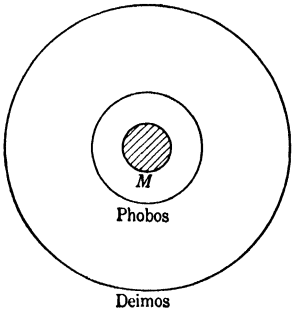


FIG. 108.—Mars and its two satellite orbits.

The satellites of Mars revolve in the plane of the planet's equator and in the direction in which the planet rotates. The inner one (Phobos) revolves eastward at a greater angular speed than the planet rotates, for its period of revolution is less than one-third the day of Mars. Hence, this satellite, as observed from the surface of Mars, rises from the western horizon, passes eastward along the celestial equator of the planet, and sets in the east. Its synodic period is 11 hours. Deimos, on the other hand, rises in the east and sets in the west, for its period of revolution is longer than that of the rotation of Mars, but the two periods are so nearly equal that the synodic circuit of the satellite is 132 hours, or more than four of its months. During the 66 hours which elapse between its rising and its setting it runs through all its phases twice.

216. Rotation of Mars. In 1666, Robert Hooke, in England, and G. D. Cassini, in Paris, detected dark markings on the ruddy disk of Mars which are so definite and so permanent that they can be identified by astronomers of the present time. Some of them can be seen in Fig. 109, which is from a photograph of the planet. It follows that the surface of Mars is solid and that it is not hidden from our view by clouds or a dense atmosphere.

Observations for a few hours of the markings of Mars are sufficient to show that its period of rotation is about 24 terrestrial hours; and observations over a month determine its period with an error not exceeding a minute or two. Mars has now been observed by astronomers for so long a time that its day has been

found to be $24^h\ 37^m\ 22^s.6$ in mean solar time with an uncertainty not exceeding one-tenth of a second.

The inclination of the plane of the equator of Mars to the plane of its orbit is approximately equal to the inclination of the plane of the earth's equator to the ecliptic. Moreover, the relation of the axis of rotation to the major axis of the orbit is nearly the same in both planets, for Mars arrives at its perihelion point shortly before midsummer in its southern hemisphere. Consequently the summer seasons of the northern hemisphere of Mars, like those of the earth (Art. 111), are longer than the winter seasons; and in the southern hemisphere the conditions are reversed. Since the eccentricity of the orbit of Mars is nearly six times that of the orbit of the earth, the difference in the lengths of the Martian seasons is much greater than the difference in the lengths of the terrestrial seasons.

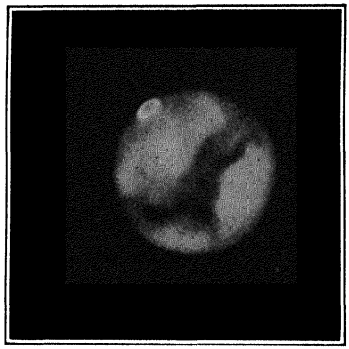


FIG. 109.—Mars as photographed with the 40-inch telescope of the Yerkes Observatory.

The orbits of the earth and Mars are nearest each other at the perihelion point of the relatively eccentric orbit of Mars. Hence when these planets are in nearest approach, the southern hemisphere of Mars is tilted about 23°.5 toward the sun and the earth. For this reason its southern hemisphere is somewhat better known than its northern hemisphere, a fact which perhaps is fortunate, for its southern hemisphere appears to have the more interesting markings and the more extensive polar caps.

217. The Atmosphere of Mars. Mars, as seen from the earth, never transits the sun, and consequently the observations which prove that it has an atmosphere are different from those which are available in the case of Mercury and Venus.

Mars reflects about 15 per cent of the sunlight which falls upon it, or about twice the fraction reflected by the moon and Mercury, which indicates that it probably has an atmosphere. Other phenomena establish the conclusion beyond a doubt. The most striking of these is the appearance and the disappearance of its

white polar caps, which cover its polar regions during their winter seasons. A polar cap sometimes comes on in the course of two or three days, covering an area extending down from the pole 20° to 35°. It remains undiminished in whiteness during the long winter of the hemisphere of the planet; as spring advances, it gradually contracts in size, often breaking up somewhat irregularly on its margins (Fig. 110); and before the summer passes, it sometimes completely disappears. There can be no reasonable doubt that these white mantles are precipitated from an atmosphere and that they evaporate into an atmosphere again.

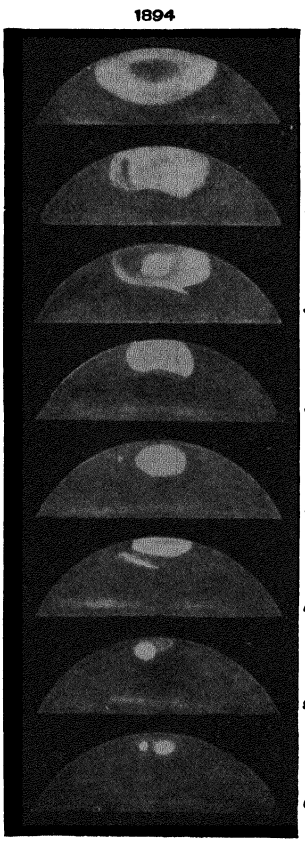


FIG. 110.—Disappearing polar caps of Mars. (Barnard.)

Although Mars does not appear to have clouds similar to those on Venus and the earth, yet faint patches, possibly of dust and mist, have many times been observed on its surface and have been photographed by E. C. Slipher. These faint patches are clearly in the atmosphere of the planet, for occasionally they project above its limb. They are particularly common along the borders of the polar caps. Moreover, the surface markings of Mars seem partially obscured by an atmosphere as they are carried by the rotation of the planet toward its limb. Finally, photographs by W. H. Wright with plates sensitive only to blue light showed no markings on the planet except the polar caps, for the atmosphere absorbed the blue rays reflected from the solid surface; on the other hand, plates which were sensitive to red light, to which the atmosphere is transparent, showed the surface markings distinctly.

There is an important difference in the density gradient of an atmosphere on the earth and that on a planet having the low surface gravity of Mars (38 per cent of that of the earth). For the same temperature, the rate of decrease in density of its atmosphere with increase of altitude above its surface would be only 38 per cent of that for the earth's atmosphere. Consequently, for a given atmospheric mass, its atmosphere will extend much higher than our own.

Many attempts have been made to determine the constitution of the atmosphere of Mars by means of the spectroscope, but the results on the whole have been quite contradictory. Recent investigations with powerful instruments and under favorable atmospheric conditions, both at the Lowell Observatory and at the Mount Wilson Observatory, indicate that the atmosphere of Mars contains 5 per cent as much water vapor and 15 per cent as much oxygen as are normally in the atmosphere of the earth. Carbon dioxide and nitrogen, however, do not give absorption spectra by which their presence in planetary atmosphere can be determined.

218. The Larger Surface Markings on Mars. In addition to the general brick-red areas, of which the surface of the planet is largely composed, there are fairly conspicuous permanent darker regions lying mostly along its equator and irregularly south of it. The early observers of Mars supposed that these grayish-green areas are oceans, but it has been found with the great telescopes of modern times that they change somewhat with the seasons and occasionally contain a great amount of finer details. Consequently they can not be great seas, but it is within the realm of possibility that, as Lowell suggested, they are marshy regions in which the waters collect when the polar caps melt.

On the whole, the surface of Mars appears to be much smoother than that of the earth. Certainly, there are no great mountain ranges like those on the earth nor isolated peaks like those on the moon. Yet the surface of the planet is not a level plain. The dark areas indicate appreciable differences in surface conditions, and when the southern polar cap recedes there are left behind isolated white patches, which also slowly disappear. Since season after season these peculiar areas have the same position and shape, it can hardly be doubted that they are due to differences in surface

level. Perhaps they are on elevated plateaus from which the snows melt more slowly than in lower altitudes.

Sometimes whitish areas in lower latitudes are observed on the surface of Mars shortly after the sun has risen on the part of the planet in which they are located. They appear to be regions from which perhaps night frosts disappear slowly after the sun rises.

219. The Canals of Mars. In 1877, the Italian astronomer G. V. Schiaparelli announced the discovery of strange and hitherto unseen markings on the surface of Mars. Instead of being large, irregular areas like those which had been observed by earlier astronomers, these markings were described as being long, narrow, straight, dark streaks (Schiaparelli called them "canali"), each extending along an arc of a great circle of the planet, but crossing one another without apparent relationship or design. Since Schiaparelli's observations were made with a telescope of only moderate dimensions, astronomers thought he was mistaken respecting what he had seen. But at the next opposition in 1879, and particularly at the one in 1881, he not only confirmed his earlier observations but he found certain instances in which the "canali" were double; that is, he found cases in which two "canali," separated from each other by a few hundred kilometers, continued parallel for 4000 or 5000 kilometers. The canals themselves were described as being line-like in their fineness, but the distance of the planet from the earth is always so great that it is not likely that streaks presenting so little contrast in color would be visible unless they are several kilometers in diameter.

After the observations of Schiaparelli, the canals of Mars were not seen for more than 10 years until a number of other observers in several countries, using larger instruments and in some cases enjoying better atmospheric conditions, verified in a general way the observations of the keen-eyed Italian and added a large amount of additional detail. Of all who have observed the planet Mars, Percival Lowell spent by far the largest amount of effort in unraveling its mysteries. He was thoroughly convinced of the objective reality of the fine geometrical network of canals which he observed. During the years he worked on the subject, he and his associates located more than 400 canals extending across the ruddy regions and the darker areas as well. About 50 of the canals are double, the two components being separated from each other

by 150 to 300 kilometers. Fig. 111 is a photograph of a globe on which Lowell drew all the canals he had observed. When a number of them intersect in a point, there is generally a dark spot, or "oasis," at the junction point. Lowell located about 200 oases upon his maps.

In spite of the observations of Schiaparelli, Lowell, and other astronomers, there are good grounds for hesitating to accept the

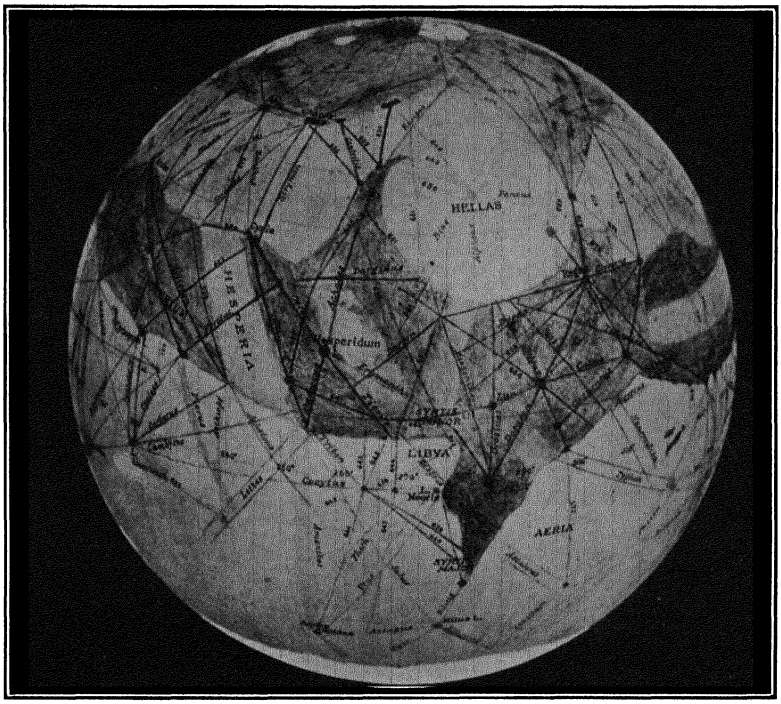


FIG. 111.—Lowell's map of Mars on a globe.

precise geometrical network of canals as objective realities. In the first place, many of the foremost observers, working with the greatest instruments and under the most favorable conditions, do not describe the numerous markings they have observed on Mars as at all resembling the line-like features depicted by those who drew the canals. In fact, some of them regard their observations as completely disposing of the theory that there are any such things as canals on Mars.

There is no subject in observational astronomy respecting which

there has been greater diversity of opinion than that of the finer markings on Mars. By themselves the observations of a number of experienced observers seem conclusive; but they are squarely contradicted by the observations of other equally distinguished astronomers. The fact is that the Martian details which are seen by some as canals are so near the limits of visibility, even under the most favorable conditions, that the *interpretations* of the stimuli they produce in the optical nerves play a dominant rôle. It may be that the mind in some way integrates rather irregular streaks and dots into lines, just as we often pick out geometrical patterns among haphazard groups of objects. Astronomers on the whole are not inclined to believe that Lowell's drawings are really descriptive of the Martian surface, although they accept the conclusion that it is rich in details which are near the limits attainable under our atmospheric conditions. Moreover, astronomers generally believe that a strict geometrical pattern of the planet's surface features is inherently improbable, whether the markings are due to inanimate forces or to the direct intervention of intelligent creatures; and they would be willing to accept such extraordinary networks as real only on the basis of the most direct and unquestioned evidence.

At once the query arises why astronomers do not photograph Mars and settle the question of the existence of its canals. The answer is that the planet has been photographed many times with the greatest telescopes, but the image of the planet is so small and the irregularities of refraction of light during the exposures are so numerous that all the finer details are lost. For studying the planetary markings, the human eye, which avails itself of the fleeting instants of steady seeing, far surpasses the photographic plate, which records a composite of all that falls upon it during the interval of exposure. Yet the photographs of E. C. Slipher and other astronomers show many markings on Mars, and so far as they go they agree with the drawings of Lowell.

220. The Temperature of Mars. It was shown in Art. 194 that if the earth were at the distance of Mars from the sun, its average temperature would be (disregarding resulting changes in its surface as a consequence of precipitation of snow) approximately $-39°$ C. ($-38°$ F.). Since the atmosphere of Mars is less than that of the earth, it is reasonable to conclude that the mean surface tempera-

ture of the planet does not exceed −39° C., but apparently it is somewhat higher.

It can not be doubted that the polar caps of Mars shrink as spring advances and disappear during the summer as a consequence of the sun's effects upon the planet. But the disappearance of the polar cap does not necessarily mean that its temperature rises above the freezing point and that snow actually melts, for in a dry atmosphere, whatever its temperature may be, snow evaporates (technically *sublimes*) a molecule at a time and vanishes without melting to the liquid state. Hence the polar caps of Mars may be due to a sort of hoar frost which is precipitated from its relatively dry atmosphere only when the temperature falls below a certain minimum, and which evaporates again when the temperature rises above this limit.

Fortunately, there is now important additional evidence respecting the surface temperature of Mars. The measurements of W. W. Coblentz and C. O. Lampland, at the Lowell Observatory, and of S. B. Nicholson and Edison Pettit, at the Mount Wilson Observatory, apparently show conclusively that, in spite of the difficulties in translating their observations of energy received into surface temperatures, the degree of frigidity of Mars is less than has generally been assumed probable. In fact, they conclude that the equatorial temperatures of Mars at noon are considerably above the freezing point of water, rising possibly to 30° C. (86° F.). Even though there are considerable uncertainties respecting the actual figures, marked variations of temperature with position on the planet have been established. For example, the temperature of the sunrise limb is found to be about 40° C. (72° F.) lower than that of the center of the illuminated disk. On the other hand, the temperature of the sunset limb is only 10° C. (18° F.) lower than that of the point directly under the sun. The dark areas are warmer than the light areas in the same latitudes; the temperatures increase with advancing seasons; the poles in winter are much colder than those of the earth; and the temperature, even on the equator, descends every night considerably below the freezing point of water.

221. Hypotheses Respecting the Canals of Mars. W. H. Pickering suggested the hypothesis that the canals of Mars are strips of vegetation growing along the banks of water courses, just

as green fields border the river Nile after its annual flood waters recede. It is probable that, if the earth could be observed from the planet Venus, the valley of the Nile at a certain time of the year would be seen as a slender, greenish streak extending across the yellow African desert. The streak would be, not the river itself, but the vegetation watered by it.

Percival Lowell greatly elaborated the theory that the canals of Mars are due to vegetation, and he defended his position vigorously and ably. With his telescope he followed the changes of these remarkable streaks through the Martian year from the winter season, when they are absent, to winter again. He states that as spring on a hemisphere of the planet advances, the canals first become visible along the borders of the dark (marshy?) areas; that they progressively develop even down across the equator and to regions having the opposite latitude; and that they disappear as winter approaches. He explains their progressive development by the theory that vegetation can grow on the desert surface of Mars only as water is carried slowly (4 kilometers per hour) along through canals from sources in the marshy regions which are annually filled by melting snows. He interprets the oases at the intersections of canals as simply large irrigated areas.

Lowell infers from the straightness, the uniformity, and in some cases, the parallelism of the canals that their central water courses (the real canals) were produced and are controlled by intelligent beings. He pictures the planet as being in the last stages before it will have lost its water vapor, molecule by molecule, into space. According to his views, the inhabitants of Mars maintain a precarious existence in their desert home by conserving and utilizing to the best possible advantage the remnants of what may once have been extensive seas. Death awaits them all, according to Lowell, in the relatively near future, and he interprets their melancholy prospects as a prophecy of similar ones which our successors on the earth will eventually face.

Few astronomers agree with Lowell's theories and interpretations of the canals on Mars. As has been stated, most scientists who have considered the subject do not think it probable that the geometric network of the so-called canals corresponds closely with anything actually on the surface of the planet. In fact, experiments show that human eyes in some way integrate faint, irregular marks, just beyond the limits of distinct vision,

into regular geometric patterns resembling those observed on the planet Mars.

Astronomers, moreover, do not regard the straightness and the uniform width of the canals of Mars through several thousand kilometers, regardless of irregularities in the surface of the planet and variations in its climate and soil, as evidence of intelligent design and construction. Nor do they find the exact parallelism of two canals, separated by hundreds of kilometers, easily explainable on rational grounds. Moreover, they are unable to account for the energy which would be necessary to force water along canals extending more than one-fourth the distance around the planet. They know of no observational nor reliable theoretical evidence that the atmosphere of either the earth or Mars is being progressively lost without a corresponding gain from the interplanetary spaces or from the interiors of the planets themselves. They do not regard it as probable that the earth will inevitably tend toward the state of apparent senility in which Mars is found today. Yet, in spite of the reservations respecting Lowell's explanation of the canals of Mars which cold reason demands, our emotions eagerly respond and our pulses quicken at the thought that we possibly have been looking at magnificent engineering works of intelligent beings on another world.

222. On the Conditions for the Existence of Life. That which we call life involves so much that is unknown that it can not be satisfactorily defined. It appears in myriads of plant and animal forms ranging from microscopical organisms to towering trees and great beasts. It flourishes in oceans and in deserts, in torrid and frigid climates, and it lives as individuals a few hours or perhaps a hundred years. It may be without nerves and the organs of sense or it may be endowed with the brain of man. It transmits its numberless essential qualities from one generation to another, at least in its higher forms, through tiny bits of protoplasm in ova and in sperm.

In comparison with living organisms, the inanimate world is very simple. For example, the molecules of marble contain five atoms, while those of such a relatively simple organic substance as sugar contain more than forty. Even the simplest forms of living organisms are composed of hundreds, if not thousands, of highly complicated molecules and chains of molecules which are

arranged in such relationships that they have the property of maintaining their identity and reproducing themselves so long as the stream of living matter to which they belong is not broken. Evidently life can persist and reproduce itself only under those physical conditions in which its complex molecules and chains of molecules are not destroyed.

Low forms of life can not grow and reproduce themselves in temperatures which are continually much below the freezing point of water. Since the higher forms are dependent for their food upon the lower forms, the higher animals could not live permanently in subfreezing temperatures. On the other hand, the relatively unstable organic molecules are broken up by moderately high temperatures. The difference in behavior between a bit of stone and a bit of sugar on a hot stove illustrates the point. Hence those complex structures which are life or on which life depends can not flourish in an environment which is permanently either much below the freezing point or above the boiling point of water. There are, of course, other necessary conditions such as the availability of suitable elements and sources of energy.

On the basis of the foregoing considerations and the determinations of the surface temperatures of Mars, it does not appear to be impossible that the planet is the abode of various kinds of life. This does not mean, of course, that in its exterior characteristics Martian life resembles the living organisms on the earth. In order that life shall be able to perpetuate itself, it must be at least approximately adapted to its environment. Although the conditions presented on the surface of the earth and in the oceans are extraordinarily diverse, there are certainly far different ones on the surface of Mars in respect to moisture, weight, atmospheric pressure, and daily range of temperature. And life on Mars, if there is life on the planet, is necessarily adapted to all these variations from terrestrial conditions and to others about which we do not know.

If there is life on Mars, it is very improbable that its highest forms are at the same stage of evolution as the highest terrestrial forms. Nor is it at all likely that the highest manifestations of intelligence on other worlds are similar to those on the earth. Even among the animals on our planet there is an extraordinary diversity of types of mental activity. It is difficult for us to understand the mind of an African pygmy; and that which moves

migratory birds at the proper seasons to make flights of thousands of kilometers, often over long stretches of water, is to us a complete mystery. And between the apparently rational life of such insects as ants and bees and the mental processes of men there is a bridgeless gulf.

There is slight probability, therefore, that we could find a level of mutual understanding with life on another world, even if there were no difficulties of communication as a consequence of distance and difference of physical conditions. Yet men will continue to think of intelligent beings on other planets simply as other and perhaps better men, as indeed they have nearly always thought of their gods. It is not intended to imply that such thoughts are futile because they are necessarily mixed with error; they always tend to raise us above our little selves.

III. Jupiter

223. The Planet Jupiter. As was stated in Art. 197, Jupiter is the giant among the planets, but relatively insignificant in comparison with the sun. In volume, it is equal to a thousand earths; and in mass, to 317 such bodies as our planet. But the volume and the mass of the sun are in turn a thousand times that of Jupiter.

Jupiter revolves around the sun at a distance 5.2 times that of the earth; its period is 11.862 years. If its surface and its atmosphere were similar to those of the earth, it would be perpetually frozen, for it receives from the sun on the average only about one-tenth as much radiant energy as is received at the poles of the earth. According to the measurements of E. Schönberg, Jupiter directly reflects 44 per cent of the sunlight which falls upon it.

The average density of Jupiter is less than one-fourth that of the earth, or 1.34 times that of water; yet its total mass is so great that gravity on its surface is 2.64 times gravity at the surface of the earth. Surface gravity on Jupiter is 15 per cent greater at its poles than at its equator, for the planet rotates rapidly and is very oblate. On the earth, gravity at the poles is only about one-half of one per cent greater than at the equator.

224. Jupiter's Satellite System. When Galileo began to use his little telescope in 1610, the first celestial objects he discovered were the largest four moons of Jupiter. These satellites are barely within the limits of visibility without optical aid, but they could

be readily seen with the unaided eye if they were not lost in the more brilliant rays of the planet.

No other satellite of Jupiter was discovered until 1892, when E. E. Barnard, then at the Lick Observatory, detected a fifth one very close to the planet. It is so close to Jupiter and so faint that it can be seen only through the greatest telescopes and under favorable atmospheric conditions.

In December, 1904, and in January, 1905, C. D. Perrine photographed two very small additional moons of Jupiter, satellites VI and VII, which are much more remote from the planet than the five previously known. Their orbits are approximately of the same size, but their mutual inclination is 28° and the two satellites never pass within less than about 3,000,000 kilometers of each other. Not only are the planes of the two orbits highly inclined to each other, but the plane of each of them is inclined to the plane of Jupiter's equator by about 28°; on the contrary, the orbits of the first five satellites lie almost in the plane of the planet's equator. It is equally remarkable that the orbits of the fifth and sixth satellites have considerable eccentricities (about 0.16 and 0.21 respectively), while the orbits of the first five satellites are almost exactly circular.

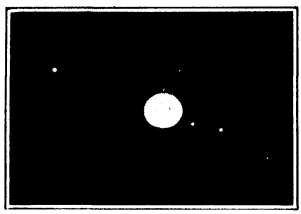


FIG. 112.—Jupiter and its largest four satellites. (The Yerkes Observatory.)

In 1908, P. Melotte, at Greenwich, England, discovered photographically a very faint and more remote eighth satellite of Jupiter. The first seven revolve around the planet in the direction in which it revolves and rotates, but the eighth revolves in the opposite direction. Moreover, a ninth satellite, discovered by S. B. Nicholson at the Lick Observatory in 1914, also revolves in the retrograde direction. The planes of the orbits of these more remote satellites are still more highly inclined to the plane of the planet's equator. According to the theories respecting the origin of the planets that were held by many astronomers until a number of years after the discovery of these remarkable satellites, retrograde satellites would be quite impossible.

The diameters of the four large Galilean satellites were measured by Barnard and they have since been determined, perhaps more accurately, by the time it takes them to emerge from Jupiter's

shadow. The diameters of the remaining satellites are so small that they can not be directly measured, but it is evident from the faintness of these bodies that they are very small. If their respective reflecting powers were known, their diameters could be determined from their apparent brightnesses.

The four Galilean satellites in the order of their distance from Jupiter are numbered I, II, III, and IV, and they have been given the respective names Io, Europa, Ganymede, and Callisto. The remaining five satellites are numbered V to IX in the order of their discovery. Table XIII gives the distances, the periods of revolution, and the measured or estimated diameters of the nine satellites:

TABLE XIII. JUPITER'S SATELLITE SYSTEM

SATELLITE	DISTANCE FROM CENTER OF JUPITER		PERIOD OF REVOLUTION			DIAMETER	
	KILOMETERS	MILES	DAYS	HRS.	MINS.	KILOMETERS	MILES
V	181,200	112,600	0	11	57.4	160?	100?
I	421,300	261,800	1	18	27.6	3730	2320
II	670,500	416,600	3	13	13.7	3150	1960
III	1,069,300	664,200	7	3	42.6	5150	3200
IV	1,881,000	1,168,700	16	16	32.2	5180	3220
VI	11,450,000	7,114,000	250	16		130?	80?
VII	11,730,000	7,292,000	260	1		40?	25?
VIII	23,500,000	14,600,000	738	21		25?	16?
IX	24,100,000	15,000,000	745	0		25?	16?

Satellites I and II are approximately equal to the moon in size, while III and IV slightly exceed Mercury in diameter. It is possible to make rough determinations of their masses and densities from their mutual perturbations. The first three Galilean satellites are a little less dense than the moon; the fourth is only 0.6 as dense as water. It is not likely that any of them has an atmosphere.

225. Rotation of Jupiter's Satellites. In spite of the great distance of Jupiter from the earth, markings on the surfaces of its four Galilean satellites can be seen with large telescopes under very favorable atmospheric conditions.

In 1890, Barnard noticed that Satellite I appeared to be elongated parallel to the equator of Jupiter when transiting its darker portions and elongated, or double, in the opposite direction when passing over its lighter colored parts. He interpreted this as meaning that the poles of the satellite are dark colored and that

its equatorial belt is lighter colored. Barnard's drawing, reproduced in Fig. 113, shows in the upper part the appearance of the satellite when transiting a light colored region on Jupiter, and below, its appearance when transiting a darker area. The observed phenomena are represented at the left and the explanation of them at the right. If the drawing is held a meter or two from the eye, the right and left representations of the satellite appear the same.

Although the markings on Satellite I are more easily seen than those on the other satellites, they are far from conspicuous. It seems probable, however, from the observations of faint markings that each of the four satellites always presents the same face to Jupiter. Photometric measurements of the satellites at various phases prove that their surfaces are spotted, and, moreover, that these bodies rotate and revolve at the same angular velocities.

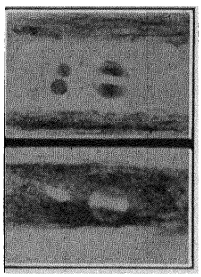


FIG. 113.—Barnard's drawing of Satellite I.

If the diameters of the satellites given in Table XIII are substantially correct, then it follows from the apparent brightnesses of the largest four satellites that the reflecting power of Satellite I is about equal to that of Jupiter; that of Satellite II is greater by about 20 per cent; that of Satellite III is less by about 20 per cent; and that of Satellite IV is about one-third that of Satellite I. These differences in reflecting power are remarkable, for these satellites are approximately equal in size, they exist under similar conditions, and they undoubtedly originated under similar circumstances. Perhaps the facts should not be surprising, for the planetoids exhibit even greater differences in reflecting power (Art. 203).

226. Discovery of the Finite Velocity of Light. In 1675, the Danish astronomer O. Römer made the important discovery from observations of eclipses of Jupiter's largest four satellites that light traverses the interplanetary spaces with a finite, though very great, velocity.

When the relations of the earth to the sun and Jupiter are as indicated in Fig. 114, the satellite S is just passing into Jupiter's shadow. The interval from the time the satellite disappears in the shadow until it disappears again is its sidereal period. After the period of the satellite has been determined, the times of future

eclipses can be computed. Suppose that when the earth is at E_1, Fig. 115, and Jupiter is at J_1, eclipses of a satellite are observed and that from the interval between them the times of eclipses are computed for the future. Then, six months later when the earth has arrived at E_2, the observed eclipses will be later than the predicted ones by the time required for light to travel approximately across the earth's orbit. It will of course be necessary to make rather simple corrections for the fact that when the earth is at E_1, the satellite disappears (is eclipsed) by passing into the shadow of Jupiter; but that when the earth is at E_2, the satellite disappears (is occulted) behind Jupiter, as seen from the earth, before it enters the shadow.

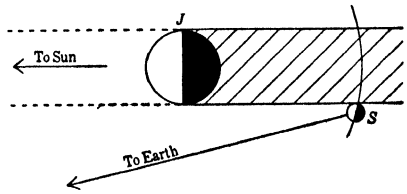


FIG. 114.—Eclipse of a satellite of Jupiter.

Römer inferred from his observations that about 600 seconds are required for light to travel a distance equal to that from the sun to the earth. Later observations showed that the actual time-interval for the mean distance from the sun to the earth is 498.58 seconds. Since the velocity of light can now be determined with a high order of precision by physical experiments, it can be used in connection with the variations in the times of the eclipses of Jupiter's satellites, as the distance from Jupiter to the earth varies, to determine the dimensions of the solar system (Art. 192).

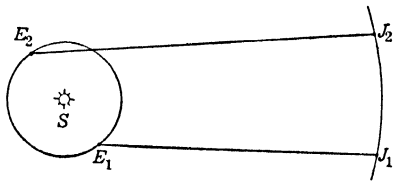


FIG. 115.—Measurement of velocity of light from eclipses of Jupiter's satellites.

227. Rotation of Jupiter. In view of the fact that Jupiter is the largest planet, it might be expected that its period of rotation would be long. It has been found, however, from rather conspicuous semi-permanent markings on its surface that it rotates faster than any other planet. This giant planet completes a rotation in a little less than 10 hours, with an equatorial speed of nearly 50,000 kilometers (30,000 miles) per hour.

Spots near the planet's equator complete a sidereal circuit in about $9^h\ 50^m$; those in higher latitudes require about $9^h\ 57^m$.

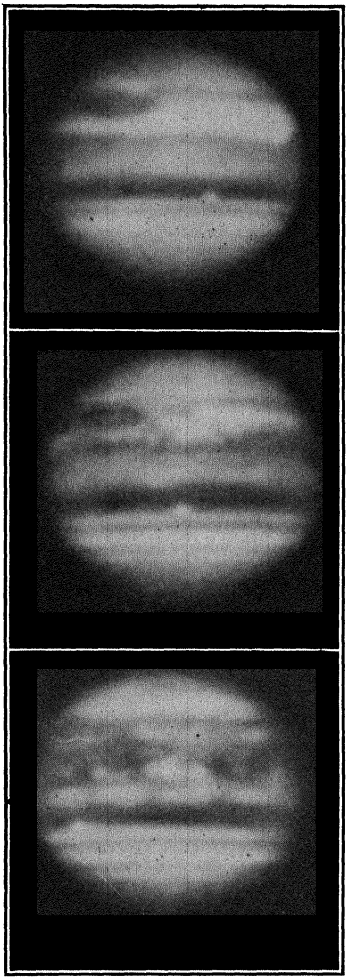


FIG. 116.—Photographs of Jupiter by E. C. Slipher. (The Lowell Observatory.)

Hence between the equatorial zone and high latitudes there is a difference of about 1/85 of a period; that is, in 85 rotations, or about 35 terrestrial days, a spot at the equator gains a complete rotation on those in higher latitudes. Neighboring zones in some cases differ in rotational velocity as much as 200 kilometers per hour. Moreover, Barnard found that the rates of rotation in corresponding northern and southern latitudes are quite different in several zones. In fact, the zones vary from year to year both in position and in the speed of rotation.

The upper photograph, Fig. 116, shows a dark oval (Art. 230) in the upper (southern) left quadrant and a small bright spot on the lower border of the dark belt below the equator of the planet. In the second photograph the difference in longitude of the oval and the bright spot has decreased, and in the third photograph the oval and the spot are in the same longitude near the left margin. The photographs were taken on September 14, October 6, and November 30, 1928, and they illustrate the rapid relative drift in longitude of the different markings on the surface of Jupiter.

228. Oblateness of Jupiter. Although Jupiter at its nearest is about 620,000,000 kilometers (380,000,000 miles) from the earth, it is so large that it subtends an angle of nearly 50″. Therefore when it is observed through a telescope giving a moderate mag-

nification of even 50 diameters, it subtends a larger angle than the moon does to the unaided eye; through a large telescope magnifying 500 diameters, it appears larger than the bowl of the Big Dipper, in Ursa Major.

When Jupiter is observed through a good telescope of more than 10 centimeters (6 inches) aperture, it appears to be considerably oblate. Measurements show that its polar diameter is about one-fifteenth less than its equatorial diameter, a large difference obviously due to its low density and high rate of rotation. It is an interesting fact, however, that if the constitution of Jupiter were similar to that of the earth its oblateness would be still greater than it actually is. It can be shown from theoretical considerations respecting the forms of equilibrium of rotating bodies that this fact implies that the mass of Jupiter is strongly condensed near its center. Therefore, although its mean density is only one-fourth that of the earth, its central density may be greater than that of the earth, while its exterior layers to a depth of thousands of kilometers may have a density comparable to that of the earth's atmosphere.

There is a difficulty, however, in assuming that the visible surface of the planet is comparable in density to our atmosphere, and that the density remains low to great depths; for the surface gravity of Jupiter is 2.6 times that of the earth, and therefore the increase in density with depth of a compressible atmosphere around the planet would be more rapid than on the earth. Since the density of the earth's atmosphere doubles in a descent of about 6 kilometers, it is evident that the more rapid change in density of Jupiter's atmosphere in a depth of 6000 or even 1000 kilometers would be enormous. It is not probable that the visible surface of Jupiter is excessively rare, for it contains cloud-like forms which often preserve their dimensions and shapes for days and months; in fact, some of them maintain their identity for years. Evidently much remains to be learned respecting the physical condition and the distribution of mass of Jupiter.

229. Major Surface Markings of Jupiter. Along the equator of Jupiter there is a somewhat irregular light-colored belt whose width usually is between 12,000 and 18,000 kilometers. On each side of this equatorial belt there is an approximately equal belt

of reddish-brown color, and these belts are paralleled by several less conspicuous ones in higher latitudes.

It would be an error to suppose that the belts of Jupiter are constant in width or exactly fixed in position, for they vary greatly in both respects. Since these changes in the surface markings of Jupiter have no relation to the period of revolution of the planet, they are not produced by such exterior influences as the radiant energy received by it from the sun. There are not, in fact, marked seasons on Jupiter because the plane of its equator is inclined to the plane of its orbit by only 3° 7′ and the eccentricity of its orbit is only 0.048. The belts unquestionably have their origin in the deep-seated properties of the planet's constitution and rotation, properties probably inherited from its origin and early history.

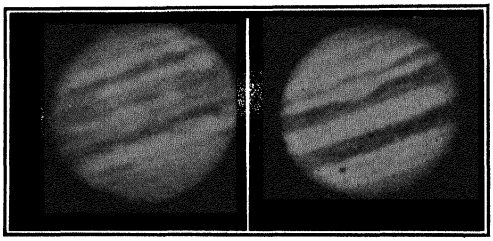


FIG. 117.—Photographs of Jupiter on Feb. 12, 1921, and on May 29, 1922, showing marked changes in belts. (Photographed with the 100-inch reflector, Mount Wilson Observatory.)

230. The Great Red Spot. Among the markings so far observed upon Jupiter, the most remarkable is the great red spot which was first noticed in 1878 on the southern border of the principal southern dark belt. When first discovered, it was an oval, pinkish area, extending a distance of nearly 50,000 kilometers in longitude and about one-third as much in latitude. In a year the spot had changed to a decided red color and was the most conspicuous marking visible on the surface of the planet; then it gradually lost its color and became somewhat less elongated. Most of the time for the past 30 years it has been scarcely visible as a curved indentation in the border of the southern belt. The red oval did not actually first appear in 1878, for W. F. Denning identified the associated hollow in the southern belt on drawings made in 1831 and the spot itself in drawings made in 1859. It simply became very conspicuous for a few years beginning in 1878.

It might be supposed that the great red spot of Jupiter is an incipient continent, rooted in solid materials in the depths of the planet, if it were not for the fact that its period of rotation has

changed considerably since 1878. In the first eight years after its discovery its period increased by 7 seconds, an amount sufficient to correspond to a relative drift backward in longitude of about 80 kilometers in a single rotation, or 70,000 kilometers in a terrestrial year. It lost about half a rotation with respect to the neighboring belt by 1885, but since that time its period has on the average been equal to that of the surrounding material, though

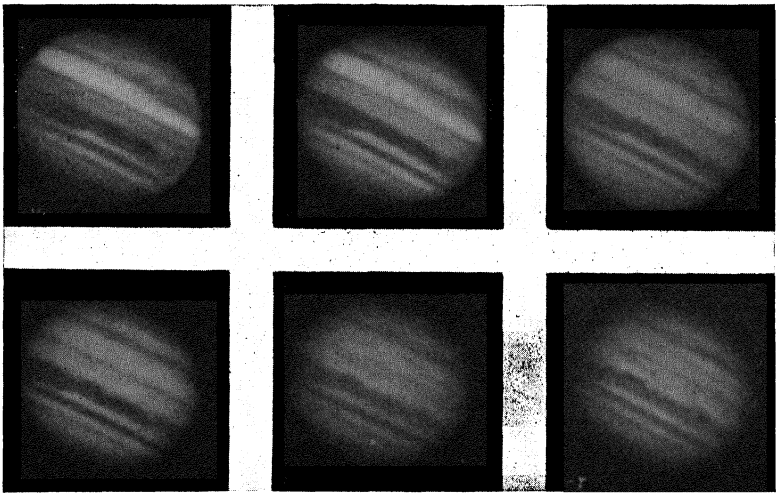


FIG. 118.—Photographs of Jupiter on September 19, 1927, in light of various colors ranging from ultra-violet, in the upper left-hand corner, to infra-red, in the lower right-hand corner. Since the blue rays are absorbed by an atmosphere, the photographs show the belts at successively lower levels in the atmosphere of the planet. (W. H. Wright, the Lick Observatory.)

it has at times drifted one way and then the other 20,000 or 30,000 kilometers in longitude and nearly one-third as much in latitude.

In the belt just south of the great red spot there has been observed since 1901 a large dark area, some 70,000 kilometers in length, which is known as the south tropical disturbance. Although this disturbed area is nearly in the same latitude as that of the great red spot, its period of rotation is about 20 seconds shorter. Consequently it gains a rotation on the red spot in a little more than two years. When it passes the red spot it drifts by with a relative speed of about 24 kilometers per hour. At these passages the red spot is drawn several thousand kilometers toward the belt in which the "disturbance" rotates, only to return to its original

position after the encounter; and as the "disturbance" passes the red spot, it moves with somewhat more than its normal speed of rotation.

It is natural to inquire what the cloudy belts of Jupiter, the great red spot, and the south tropical disturbances are, but no certain answer is at hand. The changes they undergo preclude the theory that they are of solid materials, even if the low density of the planet were not opposed to such a conclusion. Their semi-permanence makes it difficult to regard them simply as clouds. The spectroscope shows strong absorption bands in the sunlight reflected from Jupiter, but it is not possible to identify with certainty what the absorbing medium is. The strongest band, which is in the red, coincides in position with the strongest band of water vapor, but V. M. Slipher, its discoverer, thinks it probably is due to some other compound, because the other absorption bands of water are not found in the spectrum. At present our knowledge respecting both the constitution and the physical condition of Jupiter is very meager.

IV. Saturn

231. The Planet Saturn. To the unaided eye Saturn appears to be a yellowish star of about the first magnitude, but through a good telescope it is one of the most interesting objects in the sky. Its wonderful ring system presents a spectacle not approached by any other planet. Although Saturn is somewhat smaller than Jupiter and considerably farther from the earth, its apparent diameter varies from 14″ to 20″, and through a telescope giving a magnification of 100 diameters or more it presents a conspicuous, very oblate disk.

In volume, Saturn is 734 times greater than the earth; and in mass, it is 94.9 times greater. Its mean density of 0.715 is only about one-eighth that of the earth and is less than that of any other planet, yet the mass of Saturn is so great that its surface gravity averages 1.17 times greater than gravity on the earth's surface. Gravity at the poles of this oblate planet is about 30 per cent greater than gravity at its equator.

It follows from the rate of rotation of Saturn and its oblateness that its mass must be largely condensed toward its center, for it has almost exactly the shape that a tenuous atmosphere would

assume over a dense nucleus having the planet's mass and rate of rotation.

The plane of Saturn's equator is inclined to the plane of its orbit by 28° 6′. Consequently if its surface conditions depended primarily upon the energy received from the sun, as is true in the case of the earth, the planet would have marked seasons. But in the sense that the term is used when referring to the earth, this gaseous body does not have seasons. Its equator is its brightest region and is yellowish; its poles are much darker and of a slightly greenish hue. Its principal markings are belts parallel to its equator, somewhat similar to those of Jupiter but much less definitely defined and less variable. Saturn, according to E. Schönberg, reflects 42 per cent of the sunlight which falls upon it.

232. Saturn's Satellite System. Saturn, like Jupiter, has nine known satellites, the largest of which was discovered by C. Huyghens in 1655. Four additional ones were discovered by D. Cassini between 1671 and 1684; two by William Herschel in 1789; one by W. C. Bond, and independently by W. Lassell two nights later, in 1848; and a ninth by W. H. Pickering, in 1899. There may be, of course, other satellites which are quite unknown, for these bodies are comparatively feebly illuminated by the sun because of the great distance of Saturn; and, moreover, they are faint because they are always very far from the earth. Saturn is always so far from the earth that it is not possible to measure the diameters of its satellites, but it seems probable that even the smallest satellite is as great as the largest planetoid for otherwise it would be so faint as to have escaped detection.

The names of the satellites of Saturn in the order in which they appear in the table are Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Hyperion, Japetus, and Phoebe. The planes of the orbits of the first seven are nearly coincident with the plane of the planet's equator, that of the eighth is inclined by 13° 51′, and the ninth revolves in the retrograde direction with an inclination of 31° 6′. Here again, as in the case of Jupiter, we find a remote satellite revolving in the direction opposite to that of rotation of the planet which controls its motion.

Table XIV gives the distances of the satellites of Saturn from the center of the planet, their periods of revolution, and estimates of their diameters.

TABLE XIV. SATURN'S SATELLITE SYSTEM

SATELLITE	DISTANCE FROM CENTER OF SATURN		PERIOD OF REVOLUTION			DIAMETER	
	KILOMETERS	MILES	DAYS	HRS.	MINS.	KILOMETERS	MILES
I	185,700	115,300	0	22	37	650?	400?
II	237,900	147,800	1	8	53	800?	520?
III	294,500	183,000	1	21	18	1300?	800?
IV	377,200	234,400	2	17	41	1200?	750?
V	526,700	327,300	4	12	25	1750?	1100?
VI	1,220,000	758,800	15	22	41	4200?	2600?
VII	1,480,000	919,700	21	6	38	500?	310?
VIII	3,558,000	2,210,000	79	7	56	1800?	1100?
IX	12,930,000	8,034,000	550	10	34	250?	160?

The sun would rob a planet of a satellite if the satellite were too remote from its primary. Although we can not determine at just what distance such a loss of a satellite would necessarily occur, we know from mathematical considerations that the motion of a retrograde satellite is more securely stable at a given distance than that of a satellite which revolves around its planet in the same direction that the planet revolves around the sun. This fact probably explains why the motions of the very remote satellites are retrograde, but it does not explain why all the nearer satellites revolve in the same direction as the planet rotates.

Several of the satellites of Saturn vary in reflecting power as they revolve around the planet, always having the same brightness when in the same parts of their orbits. The eighth satellite, Japetus, at its western elongation is about five times as bright as at its eastern elongation, and the remaining satellites vary to lesser degrees. These variations in brightness with positions in their orbits imply that the satellites of Saturn have different reflecting powers on various parts of their surfaces, and that at least most of them revolve in such a way as always to present the same face toward their primary. In this remarkable respect they are like the moon and the largest four satellites of Jupiter. There is evidently some factor whose long-continued action has brought about this precise agreement between the respective periods of revolution and of rotation, for it can not reasonably be assumed that so many exact equalities would be consequences of the conditions at the time of the origin of these bodies.

The masses of the satellites of Saturn have been determined with some approximation from their mutual perturbations. By this

method it was found by G. W. Hill that the mass of Titan, the largest satellite, is 1.86 times that of the moon; its density is about 3.5 times that of water. The remaining satellites are much smaller in mass; their masses are even smaller than would be inferred from their apparent brightnesses. For example, Mimas is 1/33 as bright as Titan, but its mass is only 1/4000 as great. If this determination of its mass is correct, the mean density of the satellite is less than half that of water!

233. The Rings of Saturn. A great system of encircling rings distinguishes Saturn from all other planets. In fact, the rings of Saturn are the most remarkable and the most beautiful features of the planetary system. Like thin sheets of silver, they lie in concentric circles in the plane of the planet's equator, the nearest being 9,000 kilometers from its surface and the farthest extending out to a distance of 138,000 kilometers from its center. In spite of the vast lateral extent of the rings they are very thin, probably not exceeding 50 kilometers in thickness.

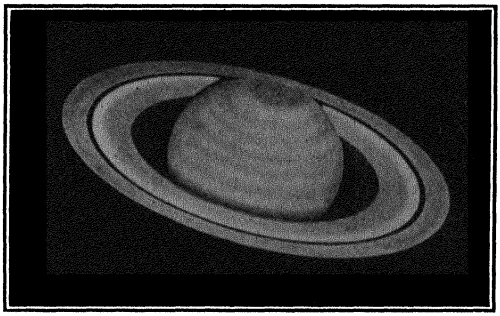


FIG. 119.—Saturn with rings tilted at greatest angle. (Drawn by E. E. Barnard.)

One of the first discoveries made by Galileo with his telescope in 1610 was the rings of Saturn. It is true that the great Italian did not perceive the structure of what he saw, for through his simple instrument the rings seemed to be merely wings, or appendages, on the sides of the planet. Their true nature was established, however, by C. Huyghens in 1655, and twenty years later D. Cassini found that a dark rift (since known as Cassini's division) separates the rings into two distinct parts, as is illustrated in Fig. 119. Percival Lowell and other more recent observers have caught glimpses of still narrower rifts in both of the bright rings. In 1850, W. C. Bond and W. R. Dawes independently discovered an inner faint ring which is known as the "crape" ring. Although the rings of Saturn are considerably beyond the range of the unaided

eye, under favorable conditions they are easily visible through a telescope 10 centimeters (4 inches) in diameter and give the observer an impression of reality which no picture can approach.

According to the measurements of Barnard, the distance from the surface of the planet to the inner edge of the crape ring is 7,500 kilometers; the width of the principal bright ring, which joins the crape ring, is 29,000 kilometers; the width of Cassini's division is 3,600 kilometers; the width of the outermost ring is 17,800 kilometers; and the extreme diameter of the whole ring system is 276,800 kilometers (172,600 miles). As seen from the medium latitudes on Saturn the rings would stretch from horizon to horizon, a more striking spectacle than a thousand comets seen from the surface of the earth. But, alas, the physical conditions are such that there can be no life on Saturn to be inspired by the sight.

234. Phases of the Rings of Saturn. Since the plane of Saturn's equator is inclined to the plane of its orbit by 26° 45′, the rings are presented to the earth at a great variety of angles. As a consequence of the angle of 2° 29′ between the planes of the orbits of the earth and Saturn the maximum inclination is about 28°.

A superior planet is most favorably located for observation when it is in opposition, for it is then nearest the earth and its illuminated side is directly toward the earth. When Saturn is in opposition in a certain part of its orbit, its rings are tilted to the line of sight to the earth by 28°; when it is in opposition 180° from this point, they are tilted equally in the opposite direction. At certain intermediate points the plane of the rings passes through the earth, and for a few hours while the earth is precisely in their plane they are entirely invisible. The rings are faintly visible when their plane passes almost exactly, but not quite, through the earth; and, as Barnard noted, those places are brightest which normally are faintest. He suggested that the bright regions are due to the sunlight which is shining through the gaps and is being reflected back from the denser parts of the rings.

When Galileo first saw the rings of Saturn in 1610, they presented an appearance similar to that illustrated in Fig. 119; when he examined the planet some years later, he was astonished to find that the appendages to the planet had disappeared. (Their plane at that time passed through the earth.) It is fortunate that

the rings of Saturn are seen at various angles, for, if they were always edgewise to us, we should have no suspicion of their magnificent extent; and if they were always highly inclined, we should not know how thin they are.

235. Constitution of the Rings of Saturn. The two bright rings of Saturn appear to be as continuous and as substantial as the planet itself, and they were originally supposed to be sheets of matter comparable in substance and in density to the mass about which they revolve. The fact that they cast strong shadows on the planet supported this view. But appearances of solidity are often misleading: clouds floating over a distant mountain look as solid as its ice-capped summit, but they are only fleecy banks of mist tossed by the wind. And the apparently solid rings of Saturn are tenuous masses, probably no denser than a cloud. They are not clouds of water particles, but swarms of bodies which are individually small and probably dust-like in character.

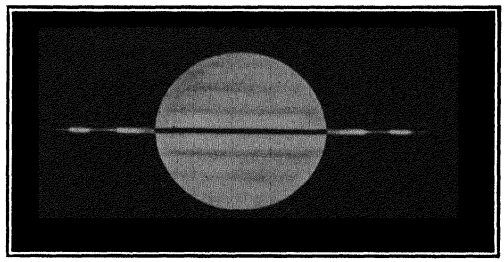


FIG. 120.—Rings of Saturn almost edgewise to the earth, December 12, 1907. (Drawn by E. E. Barnard.)

In 1715, J. Cassini suggested that the rings of Saturn may be composed of meteors. At the close of the eighteenth century Laplace virtually established the correctness of this suggestion by proving that a symmetrical, solid ring, whirling about Saturn at any speed, would be dynamically unstable under the enormous attraction of the planet and the centrifugal forces due to the motion of the revolving mass. James Clerk Maxwell showed in 1857 that the rings could not revolve as rigid masses whether they were solid or fluid, but that they must be made up of discrete particles, each revolving independently of all the others.

A direct demonstration of the meteoric character of Saturn's rings was made by J. E. Keeler in 1895. He made use of the fact that the motion of a source of light in the line of sight can be measured by means of the spectroscope (Art. 277). He found that the line-of-sight component of velocity of each of the rings is

precisely that which meteors revolving around Saturn under its gravitation would have. The particles of the inner edge of the crape ring revolve around the planet in about 5 hours; those of the outer bright ring, in about 13.7 hours.

An upper limit to the possible total mass of the ring system can be determined by a consideration of its feeble gravitational effects upon the motions of the satellites. H. Struve has shown that its entire mass can not exceed 1/27,000 that of the planet, or about 1/280 that of the earth, and it is probably very much less. That the crape ring in particular is a tenuous swarm of particles is proved by the fact that the brighter satellites are visible through it.

236. Gaps in the Rings of Saturn. Cassini's division and the lesser gaps in Saturn's rings which have been glimpsed naturally call for an explanation. If a small mass were revolving about Saturn within the zone covered by Cassini's division, its period would be about one-half that of Mimas, one-third that of Enceladus, and one-fourth that of Tethys. Under these circumstances the disturbing effects of these satellites on the motion of the small mass would be very great and would probably remove it permanently from the zone. It will be recalled that there are similar vacancies in the planetoids at zones in which their periods of revolution about the sun would be expressible to that of Jupiter in simple fractions such as 2/3.

The other divisions in the rings of Saturn which have been observed are also at distances at which the periods of revolution of particles would be simply commensurable with the periods of its larger satellites. These are evidently celestial illustrations of resonance phenomena analogous to those encountered in many familiar physical problems. The gaps in the ring system separate the space around Saturn into zones which have distinct dynamical characteristics, but at present our understanding of the general properties of these dynamic zones is very imperfect.

237. On the Origin and the Permanency of Saturn's Rings. It is probable that the rings of Saturn originated much as the satellites of the various planets originated, except that in this case the matter spread out into a thin sheet instead of condensing into one or more spherical masses. It is not easy to account for the gathering of matter into satellites, but an explanation of the reason the rings of Saturn did not condense is at hand. As E. Roche proved

in 1850, a fluid satellite within 2.44 radii of a planet of the same density would be broken up by the tidal forces due to the planet. Hence a satellite could not condense from fluid masses within this limit of Roche, nor is it probable that solid particles could unite to form a large mass.

Since the outer limit of Saturn's ring system is only 2.30 times the radius of the planet from its center, all the rings are within Roche's limit. On the other hand, Mimas is distant 3.11 times the radius of Saturn. In fact, every satellite in the solar system is well outside the Roche's limit of its planet.

It is clear that the particles of which the rings of Saturn are composed may from time to time collide with one another. When a collision of two masses occurs, especially if it is at a considerable relative speed, heat is generated and is radiated away. This energy that is lost is at the expense of the orbital energy of one or both of the revolving masses, and the orbit of at least one of them is reduced in size. Consequently there must be a gradual shrinkage of the rings, and it may well be that the crape ring is composed of masses whose orbits have been reduced by collisions. On the other hand, the largest ring extends far out toward Roche's limit and is brighter and apparently denser than any other. In view of the fact that Saturn has probably existed for more than 1,000,000,000 years, it follows that this diminution of the orbits of the separate particles of its rings has gone on very slowly; and we infer from this fact, too, that collisions have been infrequent and that the rings are very tenuous.

238. Rotation of Saturn. In a general way Saturn's markings, from which its rotation is determined, resemble those of Jupiter. The planet has a bright equatorial belt and broad, darker belts in higher latitudes, but they are less enduring and much less conspicuous than those of Jupiter. They rarely present sufficiently definite points for use in measuring the period of rotation of the planet. Nothing resembling the great red spot on Jupiter has been observed, but occasionally bright spots have been noted and Lowell saw fine gray wisps obliquely crossing the brighter zones.

In spite of the observational difficulties, a number of observers from the time of William Herschel have determined the period of rotation of Saturn. The results have ranged from a minimum of about $10^h\ 14^m$ to a maximum of $10^h\ 38^m$, depending largely upon

the latitude of the markings upon which the determination depended. Saturn, like Jupiter, rotates most rapidly in its equatorial zones, and in some cases neighboring regions have velocities relative to each other of as much as several hundred kilometers per hour.

It is evident from the large differential motions on the surface of Saturn that the planet is mostly in a fluid, and probably a gaseous, state. This conclusion follows also from its low density of 0.715 of that of water. Finally, the spectrum of Saturn has strong absorption bands in the orange and the red similar to those in the spectrum of Jupiter, but more pronounced. Since these absorption bands are absent in the spectrum of the ring, they are undoubtedly due to the planet's atmosphere.

In 1915, R. W. Wood photographed Saturn in light from various parts of the spectrum and obtained extraordinary results, and several astronomers have continued the investigations. When all except the red light is filtered out, the surface markings are almost completely lacking. With yellow light the equator is light and the polar regions are dark. With violet light the equatorial belt is dark and the polar regions are black, but the limb of the planet is not so much reduced in brightness in comparison with the center of the disk as with red and yellow light. The rings are brightest relative to the planet when photographed with violet light and faintest when photographed with red light. The light received from Saturn is reflected sunlight modified by the absorption of certain of its rays by its gaseous envelope.

V. Uranus

239. The Discovery of Uranus. All the planets whose orbits are interior to that of Uranus have been known from prehistoric antiquity. Uranus was discovered in 1781 by William Herschel while he was sweeping the heavens with a reflecting telescope of his own construction. Herschel's keen eye was attracted to the object by the fact that it did not have quite the point-like sharpness of a star. Observations extending over a few days showed him that it was moving with respect to the stars. At first Herschel supposed it was a comet, apparently not suspecting that it might be a hitherto unknown planet; but the character of its motion presently proved that it was a planet revolving around the sun beyond the orbit of Saturn.

On following back the positions of Uranus by the mathematical theory of its motion, it was found that the planet had been observed and recorded on many occasions as a star. In fact, P. Lemonnier had observed it on twelve nights and barely missed recognizing the fact that it is a member of the solar system. These earlier observations were very useful in precisely fixing its position over a long interval and in laying a foundation for the discovery of Neptune (Art. 182).

The mean distance of Uranus from the sun is about twice that of Saturn and 19.19 times that of the earth. The sun seen from the distance of Uranus by the unaided human eye would appear as a point of light, but one of such dazzling brilliancy that in illuminating power it would surpass 3000 full moons as seen from the earth. The plane of the orbit of Uranus is more nearly coincident with that of the earth than that of any other planet, the angle between the two planes being only 46′. The eccentricity of the orbit is 0.047, which is a little less than the eccentricities of the orbits of Jupiter and Saturn, but nearly three times that of the orbit of the earth.

240. The Planet Uranus. It is difficult to determine the diameter of Uranus with much precision, for the planet is always very remote from the earth and presents a very small disk. The measurements of different observers are not very accordant, but they show that the diameter of the planet is approximately 51,000 kilometers (31,700 miles). In spite of the uncertainty respecting its diameter, several observers have not only found that the planet is oblate, but have agreed that its equatorial diameter is one-twelfth greater than its polar. This great flattening implies a rapid rotation with a period comparable to that of Saturn. In 1912, Percival Lowell and V. M. Slipher determined the period of rotation of Uranus by spectroscopic means and found that it is $10^h\ 45^m$. Substantially the same result was obtained by J. H. Moore and D. H. Menzel in an exhaustive reinvestigation (1931) by the same method. The rotation of the planet is retrograde and the plane of its equator probably coincides with the plane of the orbits of its satellites.

Although few surface markings have even been glimpsed on Uranus and its diameter is somewhat uncertain, its mass has been accurately determined both from its effects upon the motion of

Saturn and also from the periods of its satellites. Its mass is 14.7 times that of the earth with an uncertainty not exceeding one per cent, and its density is less than one-fourth that of the earth, or 1.27 times that of water.

The reflecting power of Uranus is 0.45, about the same as that of Jupiter and Saturn. Its spectrum contains strong absorption bands in the red and the orange, similar to those in the spectra of Jupiter and Saturn. The planet is apparently much like Jupiter and Saturn, but resembling Jupiter the more nearly, for its density is 1.27 on the water standard compared to 1.34 for Jupiter. Uranus must be largely gaseous, probably with great belts of clouds parallel to its equator.

241. The Satellites of Uranus. Four known satellites revolve about Uranus in a plane which is inclined about 98° to the plane of the planet's orbit. This means that if the plane of the orbits of the satellites is thought of as being rotated out of coincidence with the plane of the planet's orbit, the rotation is continued nearly 8° beyond perpendicularity.

TABLE XV. SATELLITE SYSTEM OF URANUS

SATELLITE	DISTANCE FROM CENTER OF URANUS		PERIOD OF REVOLUTION			DIAMETER	
	KILOMETERS	MILES	DAYS	HOURS	MINS.	KILOMETERS	MILES
I	191,700	119,100	2	12	29	900?	560?
II	267,000	165,900	4	3	28	700?	430?
III	438,000	272,200	8	16	56	1700?	1000?
IV	586,000	364,000	13	11	7	1500?	900?

Of the four satellites of Uranus, the outer two were discovered by William Herschel in 1787, and the inner two by W. Lassell in 1851. The names of the satellites in the order of their distance from Uranus are Ariel, Umbriel, Titania, and Oberon. The planet is so remote from the earth that the diameters of its satellites can be inferred only roughly from their apparent brightnesses, and their masses are subject to even greater uncertainties.

VI. NEPTUNE

242. The Planet Neptune. In Art. 182 the romantic discovery of Neptune was described. Adams and Leverrier supposed the unknown planet would be found at approximately twice the

distance of Uranus from the sun, just as Uranus is at twice the distance of Saturn, but its distance is only about 50 per cent greater than that of Uranus. The plane of its orbit is inclined only 1° 46′ to the plane of the ecliptic; and the eccentricity of its orbit is only 0.00855, which is about half that of the orbit of the earth and considerably less than that of the orbit of any other planet except Venus.

Neptune is so remote that it is not visible without optical aid. Naturally its diameter can not be determined with a high degree of precision. From a discussion of all the observations made up to 1912, G. Abetti concluded that it is about 50,000 kilometers (31,000 miles), or perhaps a little less than that of Uranus.

Since Neptune has a satellite whose distance and period of revolution are known, it is easy to determine its mass; and its mass can also be determined with considerable precision from its effects upon the motion of Uranus. It is found that the mass of Neptune is 17.16 times that of the earth, corresponding to a density 0.29 times that of the earth, or 1.6 on the water standard.

It is not possible to determine the period of rotation of Neptune by direct observations, for the planet is so remote from the earth that markings on its surface can not be detected and its oblateness can not be measured. Nevertheless astronomers have found its period of rotation, at least approximately, by a very interesting indirect method. The planet has one satellite whose orbit can be determined and followed as it changes. Now suppose the plane of the orbit of the satellite were not coincident with the plane of the planet's equator. Then the satellite would produce a precession in the planet, just as the moon produces a precession in the plane of the earth's equator (Art. 97). This precession is slow because the mass of the planet is great, but there is a reciprocal effect that produces a precession in the plane of the orbit of the satellite, an effect which has been observed. It implies that the plane of the orbit of the satellite is inclined to that of the planet's equator by about 20°, for the pole of the plane of the orbit describes a circle having a radius of 20°. At the same time it is shown that the plane of the planet's equator is inclined to the plane of its orbit by about 29°.

A step remains before the rate of rotation of Neptune is determined. The rate of precession of the plane of the satellite's orbit depends upon the oblateness of Neptune, and the oblateness

of Neptune depends upon its rate of rotation. Unfortunately the unknown distribution of density in the planet is a factor in the result. Under what seem to be reasonable assumptions, particularly in view of the corresponding data for Jupiter and Saturn, it is found that the period of rotation of Neptune lies between 15 and 20 hours. This conclusion has been confirmed by a spectroscopic examination of the planet by J. H. Moore and D. H. Menzel, who obtained a period in the neighborhood of 16 hours (1928). The dynamical method does not determine the direction of rotation of the planet, but the spectroscopic method indicates that it rotates in the same direction as that in which it revolves, and hence contrary to the direction of revolution of its satellite.

243. The Satellite of Neptune. About a month after the discovery of Neptune, Lassell found a satellite revolving around the planet at a distance of 354,000 kilometers (220,000 miles). The plane of the orbit of the satellite is inclined to the plane of the orbit of the planet by about 140°, and the satellite revolves in the retrograde direction in a period of $5^d\ 21^h$.

The diameter of Neptune's satellite can be inferred only from its apparent brightness. If its reflecting power equals the mean of the widely varying reflecting powers of the satellites of Jupiter and Saturn, its diameter is approximately 4000 kilometers (2500 miles) and its mass is comparable to that of the moon. It may be that Neptune has other satellites comparable in size to the smaller satellites of Jupiter and Saturn, but they are beyond the reach of present instruments.

XVIII. QUESTIONS

1. How great would be the theoretical variation in the temperature of Mercury as a consequence of the eccentricity of its orbit?
2. What would be the theoretical temperature of the day side of Venus if it always presented the same face toward the sun?
3. If the phase of Phobos is full when it rises, what is its phase when it next sets for an observer on the equator of Mars?
4. In view of the rare atmosphere of Mars, would the daily variation in its temperature be greater or less than that on the earth?
5. What do you regard as the strongest evidence in favor of the theory that the canals of Mars have been produced by intelligent beings? The strongest evidence opposed to the theory?
6. If there are animals on Mars, what are the probabilities of the relationship of lung capacity to size as compared with those on the earth? Strength of skeleton and muscles? Protective covering? Means of travel?

7. What are the probabilities about sources of energy for the use of intelligent beings on Mars?
8. How do you account for the absence of mountains on Mars?
9. Draw Jupiter and the orbits of its satellites to the same scale.
10. What are the ratios of the masses of the largest four satellites of Jupiter to the mass of Jupiter? To the mass of Mercury?
11. What objections are there to the suggestion that the great red spot on Jupiter is the beginning of a continent?
12. Draw circles representing Saturn, its rings, and the orbits of its satellites to the same scale.
13. How frequently are the rings of Saturn presented edgewise to the earth? In what year following 1610, in which Galileo discovered Saturn's rings, were the rings first presented edgewise to the earth?
14. What differences would there probably be in Saturn's ring system if the planet had no satellites?

REFERENCE BOOKS

Mars as the Abode of Life (1908). Percival Lowell. The Macmillan Company, New York.

Astronomy I (1926). Henry Norris Russell, Raymond Smith Dugan, John Quincy Stewart, Ginn & Company, Boston.

Astronomy (1930). Robert H. Baker. D. Van Nostrand Company, Inc., New York.

CHAPTER XII

METEORS AND COMETS

I. Meteors

244. Celestial Visitors. Many a story has been written of some miraculous journey to the moon or to Mars. These romances have found eager readers in every generation, for there are few who have not wished they might some time fly away from this earth and visit those distant worlds. There is no hope, however, that such a wish will ever be realized. The difficulty of escaping the earth's gravity is insuperable; the problem of directing a journey through the celestial spaces and that of descending gently to rest on the surface of another gravitating body are equally formidable. Only those who are unfamiliar with the physical factors involved believe that such adventures will ever pass beyond the realms of fancy.

Although we are not able to go to other worlds, visitors from the interplanetary spaces come to the earth. We call them *meteors*. On clear evenings we see them flash for an instant as they plunge into our atmosphere; sometimes they are so large that they reach the surface of the earth, and then they can be studied at our leisure. These celestial visitors are not fragments of the moon or of the planets, but it is probable that they are similar to the materials out of which these bodies and our earth have grown. Moreover, they are in many cases, if not in all, related to the mysterious comets. Therefore, instead of being merely minor bodies which must be mentioned only for the sake of completeness, these objects are important for an understanding of the origin and the evolution of the planetary system.

Small meteors are commonly called *shooting stars*, though they have no relationship to the stars except in appearance from the surface of the earth. Larger ones are known as *fire balls* or *bolides*. Sometimes fire balls are more brilliant than Venus and maintain their visible flight for several seconds; occasionally they surpass the moon. Still larger meteors which survive passage through the earth's atmosphere and reach its surface are called *meteorites*.

Meteorites have been known from remote antiquity. In the tenth chapter of the Book of Joshua there is a brief account of "great stones from heaven" which fell upon the enemies of the children of Israel and killed them. In ancient Chinese annals there is an unmistakable reference to meteorites which fell in 687 B.C., and the Chinese record fifteen other meteoric falls before our era. The Greek and the Roman writers gave accounts of four meteorites. In ancient times these celestial visitors were often objects of superstitious reverence. For example, the Phrygian meteorite which was transferred to Rome in 204 B.C. had long been worshiped at Pessinus as "the mother of the gods." It is believed that the image of Diana, at Ephesus, the sacred shield of Numa, at Rome, and the image of Venus, at Cyprus, were meteorites. And there is a meteoric stone, sacred to Mohammedans, imbedded in the northeast corner of the Kaaba, at Mecca. In America, a meteorite was found in an Aztec temple, and several Indian tribes had meteorites among their sacred relics.

On November 7, 1492, there fell near Ensisheim, in Alsace, the earliest meteorite still preserved whose place and date of fall are known. From that time onward meteorites have been occasionally recovered after having been seen in flight through the earth's atmosphere. On July 24, 1790, at least 300 persons made written statements of having seen a fall of meteoric stones in southwestern France, and fragments were recovered and sent to scientists for examination. Yet members of the French Academy of Sciences denied the possibility of such events and cited the accounts of them as examples of the extremes to which popular ignorance and mass credulity will go. Another meteoric fall, however, in 1803 near L'Aigle, in France, stimulated an investigation of the subject by E. Biot under the auspices of the French Academy. The inquiry of this eminent scientist established at last the indisputable and amazing fact that masses of iron and stone do rather frequently come in from the interplanetary spaces and fall on the surface of the earth.

245. Masses and Numbers of Meteorites. At the present time there are about 1000 meteorites in scientific museums—a number large enough to constitute a fair sample for determining their characteristics. They range in weight from less than a gram to about 33 metric tons (36.5 tons of 2000 pounds) in the case of the great

meteorite brought by Peary from Greenland to the United States. A meteorite was discovered at Bacubirito, Mexico, which weighs 25 metric tons; another, at Chupederos, Mexico, which weighs 24 tons; and another, near Willamette, Oregon, which weighs about 15 tons. In all, there are about 15 known meteorites each of which weighs a ton or more. A meteorite whose mass W. J. Luyten estimates to be from 50 to 70 tons lies where it fell in Africa; and it is probable that the meteoric masses which produced the great crater in Arizona (Fig. 79) originally weighed in the aggregate several hundred thousands of tons. On June 30, 1908, a swarm of meteorites fell in north central Siberia, producing earthquakes and an air blast which were observable to a distance of 900 kilometers. L. A. Kulik, who visited the region in 1927, found many craters in the swampy grounds and noted that the forests were destroyed over several square kilometers.

In the nineteenth century, 316 meteorites were seen during at least a part of their descent through the atmosphere to the surface of the earth, and at the present time on the average about four meteorites are observed to fall annually. The number of those which are large enough to survive passage through the earth's atmosphere is certainly much greater, probably several hundred per year, for most of them escape notice because three-quarters of the surface of the globe is covered with the oceans and a large part of the remainder is not inhabited by civilized men. At this rate on the average a meteorite falls on every square kilometer of the surface of the earth once in about 5,000,000 years. It is therefore not surprising that they have never been identified in sedimentary rocks.

O. C. Farrington found from an examination of the dates of fall of 350 meteorites that 75 per cent more struck the earth in April, May, and June than in October, November, and December, the three months half a year later. There was a secondary maximum in August after a low minimum in July. Probably the data are too few to give very significant results. He also found that 192 meteorites have been observed to fall in the daytime and only 81 at night, with the maximum between three and four o'clock in the afternoon. It may be that this distribution of observed meteoric falls is due to the greater chances of seeing them in the daytime than at night, but there is no obvious explanation of the fact that only 68 fell before noon and 124 after noon.

246. Kinds of Meteorites. When classified according to their composition, meteorites are found to be of three principal kinds: (1) the *all-metal* meteorites (siderites), (2) the *iron-stone* meteorites (siderolites), and (3) the *stony* meteorites (aerolites). The three classes are not completely distinct, for the second class grades into each of the others, and, moreover, each meteorite has characteristics peculiar to itself.

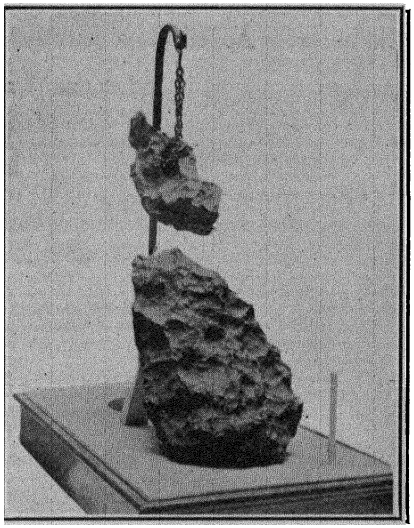


FIG. 121.—Iron meteorites from Canyon Diablo, Arizona. Weights 120 and 460 kilograms. (From the Field Museum of Natural History, Chicago.)

The all-metal meteorites are composed of about 90 per cent of metallic iron in alloy with variable percentages of nickel, ranging from 5 to 25, and accompanied by smaller quantities of cobalt, copper, phosphorus, sulphur, etc. These elements are joined in a number of peculiar minerals which are not found in the rocks native to the earth. In fact, most of the metallic meteorites now in our museums have been recognized as being of extra-terrestrial origin by their appearance and by the characteristics of their minerals.

The great meteorites found in Greenland, Mexico, Oregon, Brazil, Australia, and many from other parts of the world belong to the all-metal class. Although meteorites of this class are observed to fall much less frequently than those that are stony, they constitute the greater part of those in our museums, for they are hard and relatively permanent. On the other hand, stony meteorites disintegrate rapidly and lose their identity. Figure 121 is a reproduction of a photograph of one of the numerous iron meteorites found near Meteor Crater in Arizona; Fig. 122 is a reproduction of a photograph of a polished surface, etched with weak acid, showing an arrangement of iron crystals not found in minerals of terrestrial origin.

In the iron-stone meteorites the alloys of metallic iron and nickel

are usually arranged in a sort of network, or sponge, whose interstices are filled with silicate minerals peculiar to meteorites. Occasionally the metals are isolated in small masses a centimeter in diameter which show, when polished and etched, the crystalline structure characteristic of the metal meteorites. In certain types the silicates appear to occur in such forms as to suggest their

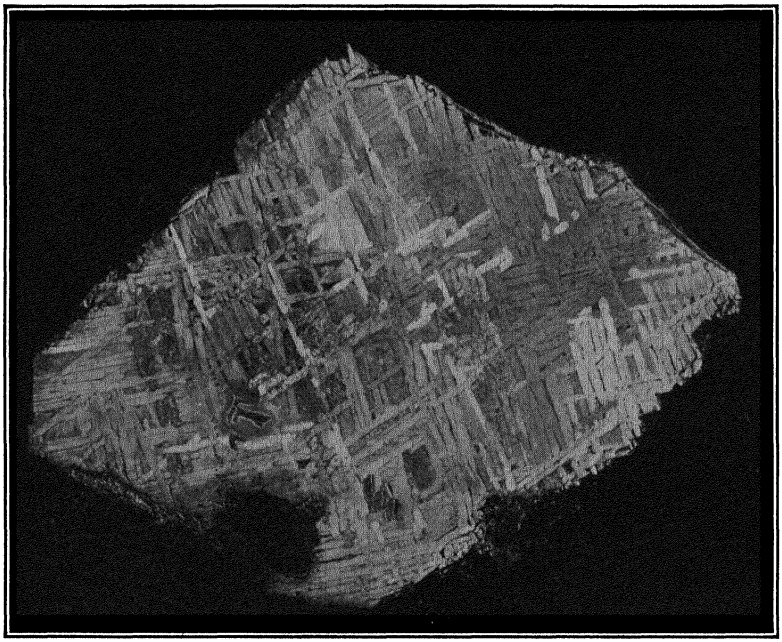


FIG. 122.—Polished and etched surface of iron meteorite. (From the Field Museum of Natural History, Chicago.)

crystallization out of a liquid mass, though the conditions under which this could have happened are not easy to formulate.

The chemical constitution of the iron-stone meteorites varies widely from one to another. Usually they contain from 25 to 50 per cent of iron, one-fifteenth as much nickel, a large percentage of silicon and magnesium oxides, and lesser quantities of sodium, calcium, aluminum, copper, manganese, and other familiar terrestrial elements.

The stony meteorites contain less iron and nickel than the first two classes and more silicon and magnesium oxides. The iron occurs largely in the metallic form in alloy with nickel and is

usually scattered in small masses, a few millimeters in diameter, throughout the stony mass. An analysis of 63 stony meteorites gave an average of more than 62 per cent of magnesium and silicon oxides; there were, in addition, more than 20 other familiar terrestrial elements in appreciable amounts. In hardness, meteorites range from those which approach stone in solidity to those which can be crushed between the fingers; in color, they range from light to nearly black.

247. Characteristic Structure of Stony Meteorites. Most stony meteorites are composed largely of small spherical masses, called *chondrules* (from the Greek word for a grain), which are imbedded in matrices of various materials. The chondrules of stony meteorites are made up of various minerals intermingled irregularly with one another. In some cases large chondrules are composed of smaller ones (Fig. 123) arranged in fan-like assemblages, indicating a growth under unsymmetrical conditions.

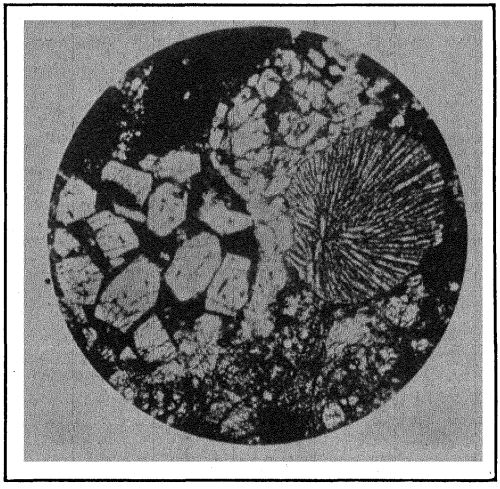


FIG. 123.—Fan-shaped chondrule magnified 180 diameters. (From the Field Museum of Natural History, Chicago.)

The chondritic structure of most stony meteorites is not more remarkable than the fact that under the microscope the matrices in which they are imbedded are found to be loosely cemented masses of sharply angular splinters, mostly silicates. Even the chondrules give unmistakable evidence of having been subject to fracturing forces. Their structure is not that of matter crystallizing freely in a molten mass, but is instead suggestive of a partial recrystallization of heterogeneous fragments of various kinds of crystals and glasses.

248. Occluded Gases in Meteorites. It is a significant fact that when meteorites are heated in a vacuum, large quantities of gas

are usually expelled from their interiors. This is true of the iron, as well as of the stony, meteorites. Usually the expelled gases at atmospheric pressure are in volume several times greater than the meteorite itself, though the amounts vary from one-half to 60 times the volume of the meteorite. The gases in various proportions are carbon dioxide, carbon monoxide, hydrogen, nitrogen, and methane, several of which readily unite with oxygen.

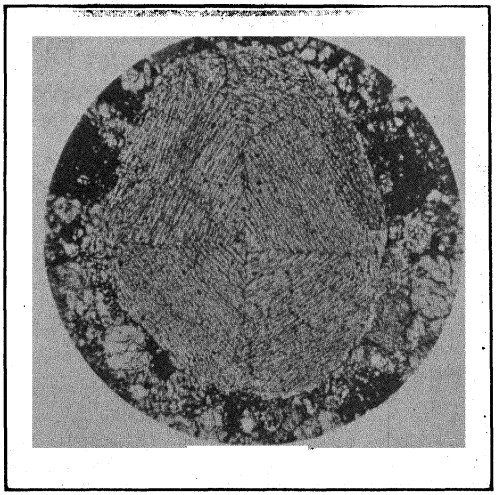


FIG. 124.—Nearly symmetrical chondrule imbedded in a glassy matrix, magnified 60 diameters. (From the Field Museum of Natural History, Chicago.)

The gas helium is generally found in small quantities in meteorites. As is well known, one of the sources of helium is the disintegration of radium, but the stony meteorites are less than one-fourth as radioactive as terrestrial granites, and the all-metal meteorites are almost wholly free from radioactive elements. Whether this fact indicates something important respecting their origin or that they are so excessively old that uranium and radium have almost entirely disintegrated, remains to be determined in the future.

249. Passage of Meteorites Through the Earth's Atmosphere. It can be proved mathematically that the lowest possible velocity at which a meteorite can strike the earth is 11.2 kilometers (6.95 miles) per second. Only under very exceptional conditions can the velocity of impact with the atmosphere be anywhere near this minimum, and it can not be greater than 79 kilometers (49 miles) per second unless the meteorite came into the planetary system from the distance of the stars. In view of the very exceptional conditions under which the velocity of impact would even approximate 11.2 kilometers per second, it would be unsafe to

assume that any particular meteorite has entered our atmosphere with a relative speed below 20 kilometers per second.

When a meteorite enters the atmosphere it has a kinetic energy $1/2\ mv^2$, where m is its mass and v is its velocity; in coming to rest, its kinetic energy is changed into some other form. If a body is moving at the rate of 20 kilometers per second, its kinetic energy is equivalent to enough heat to raise its temperature more than 50,000° C. Since this is more than enough to volatilize the mass, we inquire how a meteorite can survive its flight through the atmosphere.

A meteorite encounters an atmospheric resistance which is proportional to the square of its velocity and to the density of the air. At a speed of 20 kilometers per second at sea level, this resistance is about 16,000,000 grams per square centimeter, or more than enough to crush steel. But at high altitudes the atmospheric density is low and the pressure is moderate. As the meteorite descends, it does work in compressing the air in its path and it loses a corresponding amount of energy. A little of this energy fuses the surface of the meteorite; the remainder is dissipated in space. By the time the meteorite approaches the surface of the earth, its velocity is so much reduced that the pressures it encounters are not enough to rupture it, in many cases even when it strikes the soil or rock. In fact, the velocities of those whose falls have been observed have been so low during the last stages of their flight that they have not been even incandescent.

Meteors owe their brilliancy on entering the atmosphere to the work they do in compressing the air. This work equals the product of the resistance to which the meteor is subject and the distance through which it moves. It is therefore proportional to the cube of the velocity. The work done by a meteor moving through an atmosphere of sea-level density at a speed of 20 kilometers per second is at the rate of 5,660,000 kilowatts per second per square centimeter of front surface. Since this enormous amount of energy given up in a second would initially be delivered to less than 2600 grams of air, it is evident that the temperature of the gases in front of the meteor would be raised to many thousands of degrees; and that, consequently, a large percentage of the energy would be dissipated as ultra-violet light. At greater altitudes, where the atmosphere is rare, less energy would be dissipated; but since it would initially be delivered to a correspondingly

lesser amount of air, the temperature would be comparable. Even at 122 kilometers above the earth's surface, each square centimeter on the forward side of a meteor moving at the speed of 20 kilometers per second encounters a resistance that dissipates energy at the rate of 50 kilowatts. Consequently, the luminosity of meteors at high altitudes and the reductions of their speeds as they approach the surface of the earth are satisfactorily explained.

250. Fate of Meteors of Various Masses. The smaller a meteor, the greater is its surface relative to its volume and mass. Consequently small meteors suffer greater retardation than large ones and are more rapidly consumed by the heat which they generate in the air on their forward sides. The portions of them which are combustible are burned and either become a part of the atmosphere or settle to the surface of the earth as iron oxide or other compounds. The incombustible parts are melted or vaporized and also become a part of the atmosphere (helium, for example) or gradually descend to the surface of the earth.

Since the heat generated by a meteor is proportional to the cube of its velocity with respect to the air, the higher the speed of a meteor the greater the altitude at which it will become visible and the greater the probability that it will be entirely consumed before reaching the surface of the earth. Consequently meteors that meet the earth are less likely to survive passage through the atmosphere than are those which overtake the earth or are overtaken by the earth. Moreover, the higher the speed of a meteor, the more likely it is to explode as a consequence of excessive pressures.

For a given speed there is a limiting mass and rigidity above which a meteor will reach the surface of the earth without being consumed or broken up. It is not known what this limiting mass is, but meteorites have been recovered which weigh only a fraction of a gram. It is probable that in most cases they are fragments produced by the rupture of a larger meteor in the air.

Not enough attention has been given to what may happen to meteorites when they strike the earth's surface. If they are so small as barely to survive passage through the atmosphere, they will, of course, strike at low velocities. If the material at the surface is soft and they weigh a few kilograms, they will become imbedded in it; if it is hard rock, they may be shattered. Generally

meteorites weighing 100 kilograms or somewhat more and falling upon soil penetrate several meters. But a meteorite weighing thousands of tons would not be greatly retarded by the air and would strike the surface at a high velocity. At a speed of 20 kilometers per second the resistance of surface soil or rock, due to its inertia alone, would amount to 32×10^9 grams per square centimeter; at 14 kilometers per second the resistance would be about half as great. Either of these pressures would be sufficiently great to cause the material of the meteorite to flow as though it were a gas. The energy given up in a tenth of a second would be sufficient to vaporize both the meteorite and the material it encountered—there would be in effect a violent explosion that would produce a circular crater, regardless of the direction of impact, which alone would remain as evidence of the event.

251. The Number of Meteors. If a person watches the sky for an hour and sees only a few meteors, he is likely to conclude that the number striking the earth daily is not very large. The conclusion, however, is erroneous; for although an observer can see the stars in a hemisphere, he can see meteors only if they are within 40 or 50 kilometers from the point at which he is located. The reason for the difference is illustrated in Fig. 125. An observer at *O* can see only those meteors whose trails are within the cone *AOB*, which evidently includes only a small part of the earth's atmosphere.

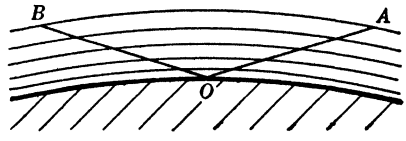


FIG. 125.—Meteors are visible from *O* only in the cone *AOB*.

On taking into account the number of meteors which can be seen from a single point and the small fraction of the atmosphere within visible range of an observer, it is found that more than 20,000,000 meteors strike the earth daily. Besides these which are brilliant enough to be observed without optical aid, there are fainter ones visible only through a telescope.

Meteors do not strike the earth with uniform frequency throughout the year but are about 1.5 times as numerous in the second half of the year as they are in the first. Moreover, on the average they are about twice as numerous from midnight to dawn as they are from the beginning of evening darkness to midnight. This

difference is due to the fact that after midnight an observer is on the forward side of the earth in its motion around the sun, and he observes both those meteors which the earth overtakes and also those which it meets. In the hours before midnight he observes only those which overtake the earth.

252. Heights and Velocities of Meteors. If the apparent point at which a meteor first becomes visible were observed from two stations (Fig. 44), its height and position above the surface of the earth could be computed. Its point of disappearance could be similarly determined. There is much difficulty, however, in making the necessary observations with any considerable degree of precision, for meteors appear at wholly unexpected times and at unexpected places in the sky, and they rarely last more than a second or two.

An alternative and theoretically much superior method is to photograph from two stations the apparent paths of meteors among the stars. The practical difficulty with this method is that meteors are so fleeting that the trails of only very bright ones are recorded, and very bright ones are rare.

In spite of the inherent observational difficulties, a number of observers have determined at least roughly the heights at which many meteors first become visible. The average height for 106 meteors in a list published by W. F. Denning was 118.5 kilometers (73.6 miles). The results for a more extensive list by G. von Niessl are about the same. He found the interesting fact that the very brilliant meteors become visible at greater heights than the fainter ones.

The altitudes at which meteors disappear are various. Evidently their masses and constitutions are factors affecting the result, and their velocities with respect to the atmosphere also have an influence. Von Niessl found from 390 meteors that the lower the velocity of the body, the deeper it penetrates into the atmosphere before it is consumed and disappears. Meteors moving with the highest velocities disappeared at altitudes approximating 100 kilometers (62 miles); those with the lowest velocities, at altitudes ranging from 20 to 40 kilometers. Even detonating fire balls did not come within 20 kilometers of the surface of the earth.

If the end points of the path of a meteor and the interval of time during which it was visible are known, its velocity can of

course be computed. Unfortunately, it is very difficult to determine the duration of such a suddenly appearing and fleeting object as a meteor; estimates for the same object by different experienced observers are often quite discordant, and the percentages of error in meteoric velocity determinations in general are very large.

It is a remarkable fact that nearly all investigators agree in assigning to meteors high velocities with respect to the air. The average velocity for the 106 meteors discussed by Denning was 43.3 kilometers (26.9 miles) per second. Von Niessl obtained velocities ranging from about 20 kilometers per second up to about 80 kilometers per second. He found, too, in the case of certain fire balls with long paths that the velocities decreased as the meteors approached the earth.

253. Masses of Meteors. It is probable that most of the kinetic energy with which a meteor enters the earth's atmosphere is transformed into light and radiated away. Since it is fairly easy to estimate the apparent brightness of a meteor, the total light radiated by one of these objects can be computed if its distance and duration have been determined. Since this total light is equivalent to the kinetic energy E of the meteor, and since $E = 1/2\, mv^2$, where v is the determined velocity, the mass m of the meteor can be computed.

A number of scientists have determined the masses of meteors by the method that has just been described, and they agree in the result that these bodies are very small. F. A. Lindemann and G. M. B. Dobson conclude that it would take 150 first-magnitude meteors to weigh a gram—ten thousand could be held in one's hand. Yet each of them gives up energy during its brief flight at the rate energy is radiated by 100,000 ordinary incandescent electric lights!

254. Meteoric Showers. On the latter half of the night of November 12, 1833, observers at many places in the eastern part of the United States saw thousands of meteors. One observer counted 650 of them in a small part of the sky in 15 minutes; other competent witnesses estimated that they appeared at the rate of 10,000 an hour. Some of the meteors were simply faint streaks similar to those which are visible on any clear moonless night; others were as brilliant as Jupiter or Venus, and one was reported to have appeared nearly as large as the moon. The flashes at times

were so brilliant that many persons were said to have been awakened from sleep, and some of the meteors left luminous trains that persisted for several minutes.

The great meteoric shower of 1833 terrified the ignorant and superstitious who took the spectacle to be a portent that the world was about to come to an end. Like the stars or the aurora or the rainbow, it was for the scientists a magnificent and thrilling phenomenon which demanded a rational explanation. A number of observers noted the important fact that the meteors all seemed to radiate from the same point on the celestial sphere; or, rather, that if their trails were prolonged backward they all passed through a point, called the *radiant*. D. Olmsted and others quickly perceived that this remarkable property of their trails was a consequence of the fact that all of them were moving in parallel lines through the earth's atmosphere.

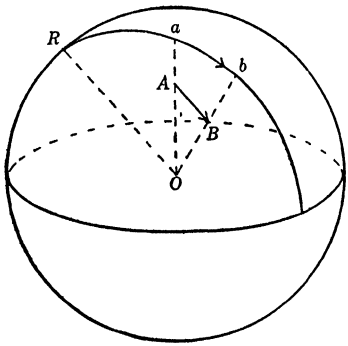


FIG. 126.—Explanation of the radiant.

In Fig. 126 the reason for this interesting effect of perspective is illustrated. The observer is at the point O, and R is the radiant point on the celestial sphere in which the prolongations backward of the apparent meteor trails intersect. The meteors move parallel to the line RO. Consider, for example, a meteor which is visible while it describes the path AB, parallel to RO. The projection of the line AB on the celestial sphere is the arc ab, which is the apparent path of the meteor. Since RO and AB are parallel, the arc ab extended backward passes through the point R. Similarly, the apparent path of every other meteor which moves parallel to RO is also an arc of a circle which passes through R. A meteor moving directly toward O from R appears as a fixed point, and the nearer a meteor is to R the shorter is its apparent path.

The radiant R for the meteoric shower which occurred in 1833 is in the constellation Leo, and hence these meteors are known as the Leonids. Since 1833 many other meteoric radiants have been established by Heis, Schiaparelli, Denning, Olivier, and other observers. The groups whose meteors are most numerous are the Perseids (radiant in Perseus), which appear in greatest numbers

about August 11; the Lyrids (radiant in Lyra), which are central around April 20; the Andromedes, which are spread over several nights from November 20 to November 27; the Aquarids, visible in early May; the Orionids, which occur on October 19; and the Geminids, which are seen about December 11. There are in addition more than 500 other radiants which have been defined by relatively small numbers of meteors, rather than by the hundreds and thousands appearing in the great showers.

It should not be assumed that the Leonids appear in equal numbers every year. Relatively small numbers were seen on

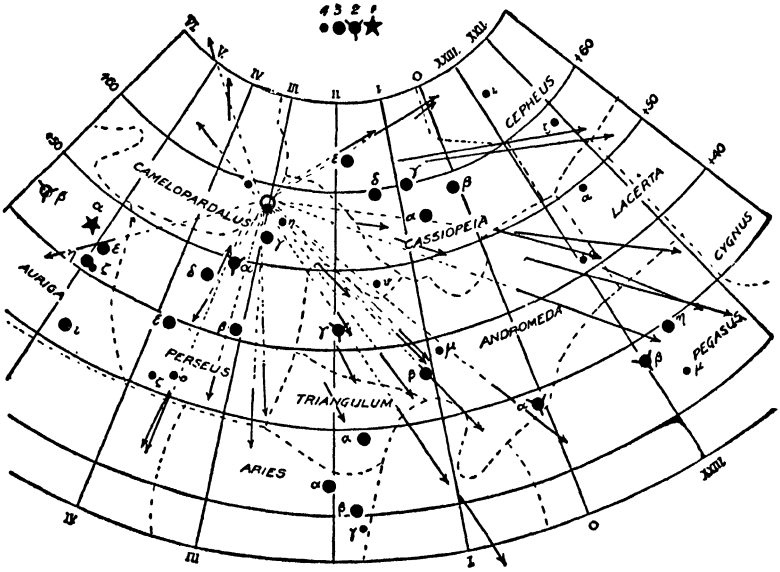


FIG. 127.—Meteor trails defining a radiant. (Charles P. Olivier.)

November 12 in the years following 1833, but it was recalled that the great naturalist Alexander von Humboldt had recorded a numerous meteoric shower which he had observed on November 11–12, 1799, at Cumana, in South America, where he was exploring at the time. He stated that the meteors were observed in thousands, moving from north to south (as we now know, away from the radiant in Leo in the northern sky), and that there was no part of the sky twice as large as the moon which was not crossed each instant by a meteor or fire ball. On the basis of these observations and from a theoretical study of the probable paths of the

Leonids in the solar system, H. A. Newton predicted a repetition of the shower in 1866. The prediction was fulfilled, though the shower was not so remarkable as those of 1799 and 1833. It was then expected that the next Leonid shower would occur in 1899 or 1900. Although many meteors were observed on November 14 for several years at the beginning of the century, their numbers fell very far short of those of the earlier showers. A considerable number of Leonids were observed in 1930, and it is hoped that they will appear in greater abundance in 1932 or 1933.

255. Paths of Meteors in the Solar System. It is theoretically possible to determine the path of a meteor with respect to the sun if its direction of motion and its velocity are known when it enters the earth's atmosphere. In the case of a meteoric shower the common direction of motion of the meteors is known, for they come from the direction of their radiant; but their velocities can not be determined with much accuracy. The paths of the meteors of several showers have been well determined, however, and it has been found that the members of these groups are associated with comets or with the remains of comets which have disappeared.

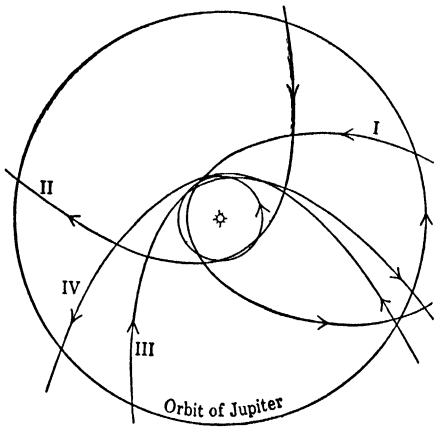


FIG. 128.—Orbit I, the Andromedes (Biela's Comet); II, the Perseids (Tuttle's Comet, 1862 III); III, the Leonids (Temple's Comet, 1866 I); IV, the Lyrids (Comet, 1861 I).

In 1866, Schiaparelli published an important book on meteors in which he reached the conclusion that the Perseid (August) meteors move in orbits essentially identical with that of Tuttle's comet of 1862. In the following year, the work of Leverrier showed that the Leonids move in the orbit of Temple's comet of 1866, which had just been computed by Oppolzer. The Andromedes (late November meteors) are associated with the orbit of Biela's comet, a small comet which broke up in 1846 and disappeared in 1852. Denning, Olivier, and others have brought forward strong

evidence that the Aquarid meteors, of early May, have some relationship to Halley's comet (Art. 265), though this comet itself does not pass within about 6,500,000 kilometers (4,000,000 miles) of the earth. Probably in many other cases, and possibly in all, meteor radiants are produced by the disintegrated remains of comets which have been invisible during historic times.

In Fig. 128, the orbits of the Leonids, Perseids, and Andromedes in the planetary system are shown. It will be observed that at their nearest approach to the sun the Leonids are at the distance of the earth's orbit and that they move in the direction opposite to that of the earth. The earth arrives at the point of contact of the two orbits on or about November 14, which is consequently the only date when these meteors can produce a shower. Since the earth and the meteors meet, these showers occur in the morning hours, as is illustrated in Fig. 129, in which *M* represents the path of a meteor before it strikes the earth on the side near the sunrise point *A*. And it follows, too, that the relative velocity of the earth and these meteors is the sum of their separate velocities, increased by the gravitational effects of the earth upon the meteors, or nearly 80 kilometers (49 miles) per second. As might be expected, these meteors move swiftly across the sky; and they are bluish-white in color, probably as a consequence of the excessively high temperatures to which they are raised by their great speed in the atmosphere. On the other hand, the earth and the Andromedes move in the same direction around the sun, the meteors overtaking the earth on its evening side with a relatively low velocity of about 18 kilometers (11 miles) per second. These meteors are much less brilliant than the Leonids and are yellow or reddish in color.

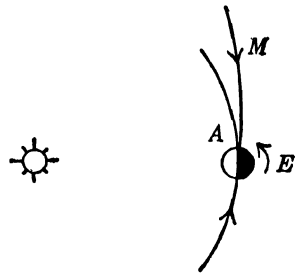


FIG. 129.—Meteor *M* meets earth on morning side.

Aside from the meteors which move in recognized showers or are associated with established radiants, there are numerous others which together constitute the greater part of those which flash in the sky. Such determinations of their velocities as have been made usually indicate that they come into the solar system from the distances of the stars, but the conclusions as yet are doubtful.

II. Comets

256. General Appearance and Characteristics of Comets. From time to time objects are visible in the sky which are wholly unlike the sun, the moon, the planets, or the stars. They often appear unexpectedly, and usually, after being visible during an interval ranging from a few days to a few months, they recede from the sun and completely disappear. Instead of being apparently stable, opaque masses like the sun and the planets, these transitory visitors consist of (*a*) large glowing *heads* in the midst of which there are usually (*b*) small *nuclei*, a few hundred kilometers in diameter, and normally (*c*) *tails* streaming out from the heads to distances of millions of kilometers. As a consequence of their nebulous heads and streaming tails (*comae*) these objects are called *comets*.

Until after the time of Newton (1687) comets were usually regarded with superstitious terror and were thought to be malevolent visitors to the upper air, portending evil and actually exerting baneful influences upon human affairs. It was easy for those believing in signs and omens to point to an earthquake or a war or a pestilence or some other disaster which occurred near the time of visitation of every great comet, for the woes of mankind have been many. But, in precise harmony with the law of gravitation, comets come in toward the sun on elongated orbits, pass around it, and then recede again to the regions from which they came. Aside from the fears they arouse in the minds of the ignorant and the entrancing problems they present to scientists, they have less influence on human affairs than a cloud which floats from a mountain top.

Comets vary in brightness from the few which are so brilliant that, at their maxima, they can be seen in full daylight without optical aid to those which are so faint that they are on the limits of visibility (and probably beyond) through large telescopes. Although the heads of comets range from a few thousand to more than a million kilometers in diameter, they are so transparent that stars shine through them with undiminished luster. The nuclei, though not always present, are relatively sharp points, but so tenuous that they are invisible when they transit the disk of the sun. The tails of the great comets reach out many degrees from the heads from which they emanate, sometimes a third of the distance across the sky; in actual length they extend at the maximum to distances greater than that from the sun to the earth.

About 450 comets were seen and recorded before Galileo first used a telescope to observe the heavens in 1609; since that date about 500 have been observed, most of which were visible only with telescopic aid. During the past few decades from 3 to 11 comets have been visible each year, about one-third of which have been returns of those already known from earlier visits to the neighborhood of the sun. Very brilliant comets approach the sun at irregular intervals, usually from 3 to 5 appearing during a century. The comets of each year are lettered a, b, c, . . . in the order of their discovery (as 1930 c), usually prefixed by the name of the discoverer, and are numbered with the Roman numerals I, II, III . . . in the order in which they pass their perihelia (as 1930 II). A particularly noteworthy comet, such as Donati's or Halley's, is given simply either the name of its discoverer or that of some one who made important investigations of it or of its orbit.

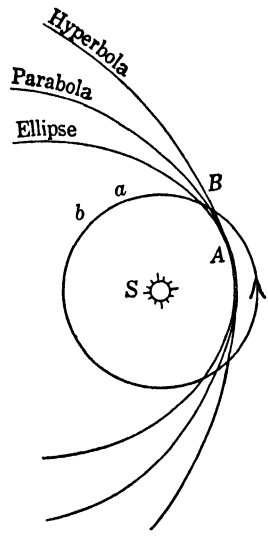


FIG. 130.—Parabola, ellipse, and hyperbola.

257. Orbits of Comets. Although Kepler proved that the planets revolve around the sun in elliptical orbits, he assumed that the comets are malevolent visitors which pass through the solar system in straight lines. Unsupported conjectures gave way to science in 1687, when Newton published a method for computing a parabolic orbit of a comet moving about the sun in harmony with the law of gravitation; superior and more general methods were later devised by Laplace and Gauss, who laid the foundations for all that has followed in this field.

Of the orbits of about 400 comets which have been computed, nearly 300 have been found to be sensibly parabolic. Perhaps it would be better to state that the small arcs near the sun in which the comets have been observed do not depart by measurable amounts from parabolas. The orbits remain slightly uncertain because comets are not objects of star-like sharpness whose positions can be exactly determined at any time; and, furthermore, they are observed while they describe only very small arcs of

their orbits. For example, a comet might be observed describing the arc AB, Fig. 130, while the earth moves over the arc ab of its orbit. It is obvious that the parabola through AB does not differ greatly from the elongated ellipse or the hyperbola with which it nearly coincides. This diagram is fairly representative of the short arcs during which the comets are usually visible, for only exceptionally can they be followed, even with telescopes, much beyond the orbit of Mars.

If the orbits of comets are strictly parabolic, it follows that these strange objects come from the distances of the stars, and moreover that at great distances their velocities with respect to the sun are vanishingly small. If, however, comets come from the distances of the stars and if their original velocities with respect to the sun are not approximately zero, then their orbits are hyperbolic. Therefore if there are any parabolic orbits we should expect to find many more which are hyperbolic, for it is not reasonable that out among thousands of moving suns comets should be at rest only with respect to our own sun.

The computations of E. Strömgren and G. Fayet show that all known comets, as they entered the planetary system, moved in either sensibly parabolic or distinctly elliptic orbits, though in a few cases, as they neared the sun, their orbits became slightly hyperbolic as a consequence of the attractions of the planets. We therefore draw the very important conclusion that, though many comets come near the center of our system from distances perhaps thousands of times that of Neptune or of Pluto from the sun, yet they are permanent members of the solar family rather than wanderers among the stars. Those whose paths are sensibly parabolic in the relatively short arcs during which they are observed make circuits of their elongated orbits in periods which are expressible in terms of tens of thousands and perhaps even hundreds of thousands of years.

Some comets have perihelia which are only a few hundred thousands of kilometers from the surface of the sun, and certain of them have actually passed through the sun's corona (Art. 260). About 25 known comets have perihelia nearer the sun than the orbit of Mercury; nearly three-fourths of those observed approached nearer to the sun than the orbit of the earth; very few have been found to be permanently more distant from the sun than Mars; and no comet has been observed whose perihelion

point was more remote than Jupiter. It is probable, however, that comets with great perihelion distances have not been observed simply because of their great distance from the earth and their faintness when they are far from the sun.

After the conclusion has been reached that comets are permanent members of the solar system, it is natural to expect their orbits to have the simplicity and similarities of the planetary orbits. Such expectations, however, are not realized. Their planes have all possible inclinations to the plane of the ecliptic, and their perihelia are in every direction from the sun. This apparently random distribution of their orbits was the principal reason that astronomers long assumed that these bodies are stray wisps of matter temporarily under the control of the gravitation of the sun in its journey in the interstellar spaces. Now that comets are believed to be permanently attached to the sun, astronomers have before them the difficult problem of discovering the origin and the evolution of a system which includes not only a thousand relatively orderly planets and planetoids, but also an approximately equal number of bizarre objects whose motions exhibit nearly every possibility compatible with the law of gravitation.

258. Periodic Comets. About 110 of the 400 comets whose orbits have been computed are known certainly to describe elongated ellipses about the sun in periods ranging from 3.3 years, in the case of Encke's comet, to several thousands of years. Most of those whose periods are short have been observed at more than one approach to the neighborhood of the sun, and their orbits are consequently well known. Unfortunately, they are all so faint that they can be seen only with telescopic aid. Those whose periods are hundreds or thousands of years have been observed over only one short arc and hence the elements of their orbits are less certainly determined.

About half of the periodic comets complete the circuits of their orbits in less than a century, and nearly 40 in periods ranging from 5 to 7.5 years. All these short-period comets revolve around the sun in the same direction as that in which the planets move, and the inclinations of the planes of their orbits to the plane of the ecliptic are small, averaging only 14°. On the contrary, the orbits of comets having longer periods show no such strict relationship to the orbits of the planets; their planes are inclined at

various angles and some of these comets, like Halley's, revolve in the retrograde direction.

259. The Capture of Comets. It is a significant fact that the aphelion points and one of the nodes of the orbits of the short-period comets are at approximately the distance of the orbit of Jupiter from the sun. It follows that at long intervals, depending upon their periods, Jupiter and each of these comets will be relatively near each other, and then the massive planet will greatly modify the motion of the comet.

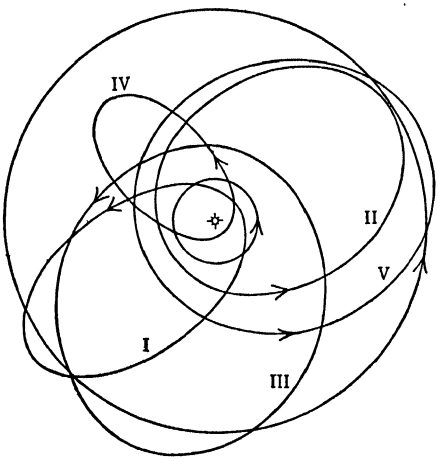


FIG. 131.—Members of Jupiter's family of comets. Inner circle represents the orbit of the earth; the outer circle the orbit of Jupiter. The ellipse I represents the orbit of Brorsen's comet; II, that of DeVico's comet; III, that of Faye's comet; IV, that of Encke's comet; and V, that of Brooks' comet.

Laplace first discussed the effects of the attraction of a planet upon the motion of a comet. He found that if a comet approaching the sun on an elongated orbit, extending perhaps far beyond the known limits of the planetary system, passes closely in front of a planet, its speed will be retarded so that it will subsequently move in a shortened elliptical orbit, at least until its motion is disturbed again. The comet is then said to be *captured* by the planet. On the other hand, the circumstances of close approach of a planet and a comet might be such that the latter would be expelled from the solar system on a hyperbolic orbit and never return. It is probable that the comets of Jupiter's family have been captured in several stages. At one near approach of a comet the aphelion point of its orbit might be reduced from some enormous distance to that of the remote planets; at some subsequent near approach the capture might be completed. H. A. Newton showed that a comet must pass within about 320,000 kilometers (200,000 miles) of Jupiter in order that its orbit may be reduced from a parabola to one of Jupiter's family in a single near approach.

There are several comets whose motions are definitely known to have been greatly altered by near approaches to Jupiter. For example, the period of Brooks' comet, 1889 V, was reduced by Jupiter, in 1886, from 29 years to 7 years. At the time of this close approach the comet passed inside the orbit of the Fifth Satellite and within about 90,000 kilometers (56,000 miles) of the surface of the planet. Lexell's comet of 1770 furnishes an example in which the disturbance was in the opposite direction, for in 1779 its orbit was greatly enlarged by a close approach to Jupiter, and it has never been seen again.

It is to be understood that a comet, after its capture by a planet, continues to revolve around the sun in an orbit of reduced dimensions. The question arises, however, whether a planet might not capture a comet in the more complete sense of reducing it to a satellite. Contrary to what might be conjectured, such is not the case. For such a capture to take place it would be necessary for the comet to encounter a resisting medium that would reduce its kinetic energy to half that which it would otherwise have had. It is not probable that there is any such medium in the solar system; and, moreover, such a resistance would completely disintegrate a comet.

260. Groups of Comets. In a certain sense, those comets whose orbits have aphelion points near the orbit of Jupiter constitute a family, but they have become members of Jupiter's family by force. There are, however, groups of comets whose members are similar in all characteristics and which travel in almost identical paths, one far behind another. The great comets of 1668, 1843, 1880, and 1882 belong to a group of the type now under consideration, and the smaller comet of 1887 should probably be added to the list. These comets were not simply different appearances of the same one, for they revolved in very elongated ellipses with a period of 600 or 800 years. The perihelion points of their orbits were within the sun's corona about 200,000 kilometers from its surface, and consequently they passed more than half way round the sun in a few hours at a speed of over 500 kilometers per second. They came in from the direction of Sirius and receded and became invisible in the same direction again. This does not mean that they came from the vicinity of Sirius, for as they recede from the sun their speed is decreased, and even in 1,000,000 years they would

not cover an appreciable fraction of the enormous distance to this relatively near star.

The comets of this group were objects of dazzling splendor, though as a consequence of their near approach to the sun most of them were not long situated favorably for observation. Their tails extended out from their heads in directions opposite to that of the sun to distances of 150,000,000 kilometers, or as far as from the sun to the earth. It is probable that these brilliant objects were fragments of a greater comet which was partially broken up at some ancient visit to the interior of the solar system.

261. Disintegration of Comets. Since comets are tenuous bodies held together by the weak gravitational attractions of their small masses, they are easily disrupted by disturbing forces. They are subject, in the first place, to solar tides which vary inversely as the cubes of their distances from the sun. Moreover, they are subject to tides that are raised by the planets. Finally, they traverse regions through which are moving countless millions of meteors that are perhaps the remains of other comets that long ago disappeared. For all these reasons, as well as because of the disrupting forces generated in them by the heat of the sun, comets have a tendency to disintegrate when they are in the interior of the planetary system.

There are several conspicuous cases in which astronomers actually saw the partial disintegration of comets. Perhaps the most noteworthy of such examples is that of Biela's comet, a member of Jupiter's family with a period of about 6.6 years. When it was observed in 1772, 1806, 1826, and 1832, it presented no exceptional phenomena. On its return in 1846 it was normal in appearance until December 20, when it became considerably elongated. By January 1, it had separated into two distinct bodies which traveled in parallel paths about 250,000 kilometers apart, each having its own coma and a short tail. Apparently the two parts had an influence on each other, for they alternately became brighter and fainter, and at times they were connected by a faint arc of light. At their return in 1852, the two components were 2,400,000 kilometers from each other, and they have never been seen again though at several returns the positions where they should have appeared have been searched with the utmost care. The Andromedes meteors (Art. 255), however, move substantially in the path

of the lost comet and furnished exceptionally fine showers of thousands of meteors in 1872 and in 1885, years when the comet should have been near the earth.

Other comets have histories somewhat similar to that of Biela's. Brorsen's comet of 1846 disappeared after having been observed at four approaches to the neighborhood of the sun, and 16 members of Jupiter's family have been seen but once. Four companion comets accompanied Brooks' comet of 1889, the separation probably having taken place when it was near Jupiter in 1886, and one of the fragments for a time was brighter than the principal mass. The nucleus of Taylor's comet of 1915 divided into two parts while it was favorably situated for observations. In all these cases, except that of Brooks' comet, the disintegrations took place while the comets were remote from the sun and the planets, and it is improbable that the disruptions were due to tidal distortions. In support of the conclusion that the disturbances of these comets were not tidal is the fact that in several instances unseen matter or forces of unknown origin have abruptly cut off and brushed aside portions of the tails of comets.

The great comet of 1882, however, was partially disintegrated by the sun. This comet, one of the most splendid of modern times, was observed both before and also after its perihelion passage, and at its maximum it was plainly visible in full daylight. After its perihelion passage close to the surface of the sun, its nucleus was found to have divided into five parts, separated by short distances along its path; and Barnard and other observers saw as many as six or eight comet-like masses, near to and apparently broken from the large body and traveling parallel to its orbit. In five months the comet and its attendants had passed beyond the range of the unaided eye; in a year it was invisible through the largest telescopes. When it returns again in approximately 1000 years, it will probably consist of several parts whose perihelion passages may differ in time by as much as 100 years.

262. Masses and Densities of Comets. The spectroscope shows that when comets are as far from the sun as the orbit of Mars they shine almost entirely by reflected sunlight. Hence if we knew the reflecting power of the particles of which they are composed, we could compute their total reflecting areas, for the amount of light they reflect can be measured. From the total reflecting areas we

could compute the volumes and then the masses, if we knew the densities of the particles.

Although not all the data necessary for determining precisely the masses of comets by the foregoing method are at hand, still its application gives results which are almost certainly of the right order of magnitude for the larger comets. Under the assumption that the particles of which Halley's comet was composed were a few millimeters in diameter and had the same reflecting power as that of the moon, it is found that if their density was equal to that of stony meteorites the total mass of this great comet was only 1/50,000,000 of that of the earth. Though its mass, according to this determination, is small compared to that of the earth, yet it is 10^{12} tons. This result may easily be a hundred times too large or too small. The masses of such great comets as that of 1811 are undoubtedly many times larger; on the contrary, the masses of such small comets as those of Jupiter's family are certainly many times smaller.

A more nearly direct method of determining limits to the masses of comets is that of investigating their gravitational effects upon the motions of the planets and their satellites. In 1770, Lexell's comet passed within about 2,400,000 kilometers of the earth, and though its orbit was greatly changed by the earth's attraction, it produced no appreciable effect in return. If its mass had been even 1/13,000 of that of the earth, the length of our year would have been altered by about a second.

It was remarked in Art. 259 that in 1886 the period of Brooks' comet was reduced by the attraction of Jupiter from 29 to 7 years. The great effect upon the motion of the comet is indicated by the fact that the aphelion of its orbit after the encounter was at approximately the distance from the sun of its perihelion before the event. For several months the comet was within the limits of Jupiter's satellite system; yet, in spite of the effects of Jupiter upon the motion of the comet, neither the motion of Jupiter nor that of any of its satellites was altered by an observable amount. This method of investigating the masses of comets indicates that they can not exceed 1/1,000,000 the mass of the earth; it gives no indication of how much smaller they may be.

It follows from the small masses and great volumes of comets that their average densities are very low, although they may be composed largely of solid particles. If the mass of a typical comet

100,000 kilometers in diameter were 1/1,000,000 of that of the earth, its average density would be only 1/640,000 that of air at sea level. Even this low density is probably much too high, for the relatively concentrated nuclei of the Great Comet of 1882 and of Halley's comet in 1910 were quite invisible when they passed between the earth and the sun.

George van Biesbroeck established directly the extremely low density of comets when he found that the light received from stars was not refracted by an appreciable amount after it had passed through 100,000 kilometers of the head of the Pons-Winnecke comet.

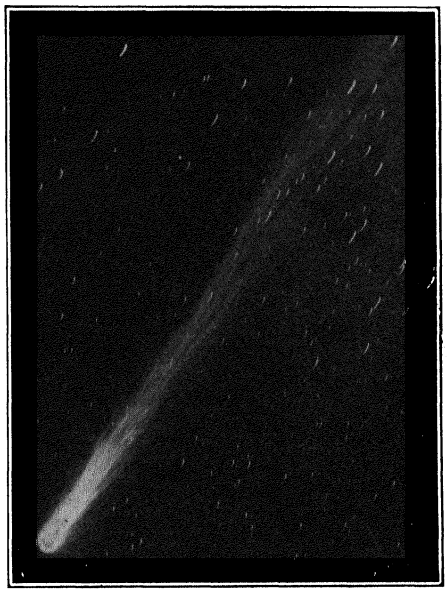


FIG. 132.—Brooks' comet, Oct. 23, 1911. (E. E. Barnard, Yerkes Observatory.)

263. Tails of Comets. Nothing distinguishes comets as a class more completely from the other members of the solar system than their tails. Comets do not have appreciable tails when their distances from the sun are as great as that of Jupiter; but as they approach the sun to within the orbit of Mars or of that of the earth, at least the larger comets normally acquire these remarkable appendages.

Comets apparently expel from their nuclei the materials from which their tails are formed. It seems probable that gases are evolved as a direct or indirect consequence of the sun's effects upon loose swarms of particles which hold on their surfaces and in their interiors vast quantities of materials that are volatile at moderate temperatures. After their ejection, these materials are repelled by the sun as a consequence of the pressure its light exerts upon the individual particles; theory shows that this repulsion may be, in the case of molecules of gas absorbing the radiation falling upon them, many times greater than the sun's attraction.

Consequently, with respect to the nucleus of the comet regarded for the moment as stationary, the ejected particles move in paths of the type illustrated in Fig. 133.

The comet, however, is moving forward in its orbit at high speed with the result that the particles are emitted from a continually advancing nucleus. It follows that a comet's tail does not extend precisely away from the sun, as it would if the comet were not moving in its orbit; but it is curved backward with respect to the direction of the motion of the nucleus, as illustrated in Fig. 134. The diagram shows the position of the tail before the perihelion passage of the comet, and the curvature in the opposite direction when the comet is receding after having passed around the sun.

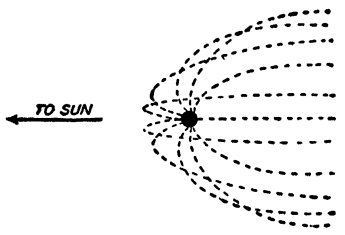


FIG. 133.—The repulsion theory of the origin of comets' tails.

Unfortunately, conditions are not so clear and simple as those which have been outlined. In the first place, no explanation is at hand for the expulsion of gases from a comet's nucleus at the occasionally observed rate of several kilometers per second, for there is neither any known force which would expel them with such a speed nor would they diffuse into a vacuum at such a rate. Nor is there any explanation of the fact that many comets have ejected materials very irregularly. For example, Morehouse's comet from time to time emitted great volumes of tail-stuff which was followed for several days by photography as it receded from the comet and the sun, until it finally became too tenuous and faint to record its image on the sensitive plate. It is from the motions of such knots of material that the repulsive forces of the sun are computed. Equally mystifying is the fact that in several instances the directions of the tails of comets have departed radically from what would be expected according to this theory of their origin. For example, Barnard photographed Brooks' comet 1893 on November 2 and again on November 3 of that year. In the interval, the line joining the comet with the sun changed in direction only about 1°, but the direction of the comet's tail changed by 16°! There are likewise sudden and great variations in the luminosity of comets' tails which no theory explains. In fact, it is not easy to account for the luminosity of a comet's

tail, for the particles of which it is composed not only reflect sunlight but continue to radiate for many days while they recede perhaps 100,000,000 kilometers from the nucleus. Apparently the tail-stuff absorbs the solar energy which falls upon it, being subjected to a pressure in the process, and then radiates it again; but it is not clear why the tenuous exterior atmospheres of Venus and the earth are not similarly affected. Evidently much remains to be learned about the tails of comets.

264. Spectra of Comets. When the light from the head of a comet is analyzed with the spectroscope, it is found to be in part reflected sunlight, for its spectrum is continuous except where it is crossed by the characteristic Fraunhofer absorption lines of the solar spectrum (Art. 277). Even a small part of the light received from the tails of comets, in the few cases in which they were bright enough to make the examination possible, is reflected sunlight, though the spectrum of the head of a comet is usually quite different from that of its tail.

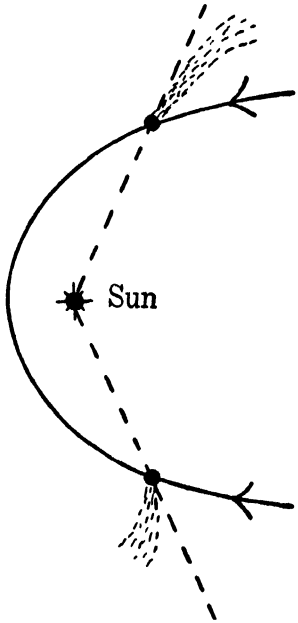


FIG. 134.—Curvature of comets' tails.

Comets shine, however, partly as a consequence of their own radiation of light. When they are at a considerable distance from the sun their spectra contain a number of bright bands which have been identified as due to radiation from nitrogen, carbon monoxide, cyanogen, and from hydro-carbon compounds. As they approach the sun, the yellow light emitted by vaporous sodium becomes conspicuous, while the light radiated from the carbon compounds relatively diminishes. When the Great Comet of 1882 was near its perihelion not far from the surface of the sun, its spectrum contained the light radiated by heavier metals among which iron seems to have been included. Unfortunately no great comet in recent decades has approached close to the sun under conditions favorable for prolonged spectroscopic examination.

From the spectroscopic evidence, rather meager though it is, we

appear to be entitled to draw the conclusion that comets are composed of familiar materials, such, for example, as those which are contained in the meteorites which fall upon the earth. Their spectra have given direct evidence of the presence of hydrogen, carbon, nitrogen, oxygen, and sodium.

265. Celebrated Comets. A number of comets have exhibited such unusual phenomena or have been so intimately associated with the development of theories respecting these bodies that they have become celebrated in astronomical literature. A brief description of some of the more important of them will illustrate the general statements contained in the preceding paragraphs.

Halley's Comet. The great comet of 1682 was named after Edmund Halley because he investigated its orbit by the methods

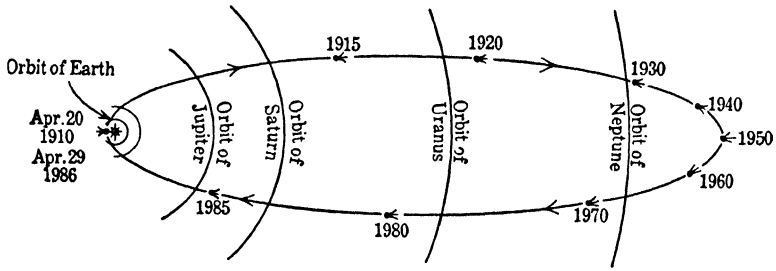


FIG. 135.—Orbit of Halley's comet.

developed by Newton, and he established the fact that it revolves about the sun in an elongated ellipse in a period of about 75 years. On examining the earlier records of the appearances of comets, he found that similar objects had been observed in 1607 and 1531, and in nearly all the expected years back to 11 B.C. and possibly to 240 B.C. Among the appearances of the comet which have been associated with important historical events are those of 451, 1066, and 1456.

Halley confidently predicted that the comet would return in 1758. The contemporaries of Halley were skeptical, but before the 75 years had passed the triumphs of the law of gravitation had been so numerous that the return of the comet was eagerly awaited by astronomers. The comet came as expected, though a little late because of the slight uncertainties in its orbit and the effects of the planets upon its motion. It passed the sun again in 1835; and if the next revolution had been made in the same period,

it would have passed its perihelion in July, 1912. The perturbations of the planets, however, reduced its period by more than two years; and in spite of the fact that it was invisible in the remote parts of the solar system for more than 70 years, P. H. Cowell and A. C. D. Crommelin, of Greenwich, predicted the time of its return (April 19, 1910) within eleven days.

When Halley's comet was at its perihelion and had its longest tail, the earth was more than 180,000,000 kilometers distant. The two bodies, however, were approaching each other and the tail rapidly increased in apparent size until it extended up from the eastern horizon half the distance across the sky. On May 18 the comet passed between the earth and the sun, but its nucleus was entirely invisible as it crossed the sun's disk. On the same date the tail of the comet swept past the earth without producing the slightest observable effect. In a few months the comet disappeared as it pursued its path out toward its aphelion point beyond the orbit of Neptune. Day by day the backward pull of the sun decreases its speed, and shortly before the middle of the century it will round the most remote part of its orbit, and it will then begin its return journey to the vicinity of the sun. Although it has been wholly invisible since 1911 and will remain invisible for many years more, astronomers follow it with their analysis and they will predict the date it will pass its perihelion again, probably within a day.

The Great Comet of 1811. This remarkable comet, which was visible from March 26, 1811, until August 17, 1812, was carefully observed by William Herschel. He found that its variations in brightness implied it was shining partly by its own light, for as it approached the sun its brilliancy increased more rapidly than it would if it were shining entirely by reflected light. At one time, its tail was more than 160,000,000 kilometers in length and 24,000,000 kilometers in diameter. The phenomena presented by this comet led Olbers to the repulsion theory of the origin of comets' tails.

Encke's Comet (1819). Encke's comet, which at its brightest was an inconspicuous telescopic object, was the first member of Jupiter's family to be discovered (1786), and it has a shorter period (3.3 years) than any other known comet. It is unique and noteworthy because its period was reduced about 2.5 hours each revolution until 1868; since that time the reduction in period has

been at a much lower rate. It is supposed that the comet encountered some sort of resisting material in the part of its orbit nearest the sun, but there is no direct evidence respecting its nature or location.

The changes in volume of Encke's comet were extraordinary. On October 28, 1828, when it was about 220,000,000 kilometers from the sun, its diameter was 500,000 kilometers; on December 24, less than two months later, its distance was 80,000,000 kilometers and its diameter was 22,000 kilometers. In 57 days its volume decreased to less than 1/500 of its earlier value. The comet did not lose nearly all its substance as it approached the sun; it simply contracted in volume, as many other comets have done to a lesser extent. Even this was not the extreme contraction, for at its perihelion passage on December 17, 1839, its diameter was less than 5000 kilometers and its volume was less than 1/1,000,000 of its value on October 28, 1828.



FIG. 136.—Encke's comet, Oct. 29, 1914. (Barnard.) The long star trails are due to the rapid motion of the comet. Note that this comet had no appreciable tail.

Donati's Comet (1858). Donati's comet, one of the greatest in the nineteenth century, was visible with the unaided eye for 112 days and through a telescope of moderate power for more than 9 months. Its tail, which was more than 90,000,000 kilometers long when it was near the sun, at one time stretched more than 30° among the stars. Like Halley's comet, Donati's comet revolves in the retrograde direction about the sun, but with a period of more than 2000 years. The aphelion point of its orbit is about 10 times the distance of Neptune from the sun, and most of its existence is spent in the frigid regions beyond the orbit of this planet.

Morehouse's Comet (1908). On September 1, 1908, D. W. Morehouse found the image of a comet on a long-exposure photograph he had taken of the stars, one of the very few discoveries of a

comet by photography although tens of thousands of photographs of the stars have been taken.

Although Morehouse's comet at its brightest barely reached the limits of visibility without optical aid and did not approach much within the orbit of the earth, its varied and extraordinary activities made it one of the most remarkable and interesting comets ever observed. Fortunately, it was favorably located for examination and it was extensively photographed by many astronomers.



FIG. 137.—Morehouse's comet (1908 III), Oct. 15, 1908. (Barnard.)

These records show that tail-stuff was often evolved from its nucleus and head with startling rapidity. For example, during the fore part of the night of September 30 it presented the normal cometary appearance; before morning cyclonic activities distorted the tail which was joined to a star-like nucleus by a curved and tapering neck; on the following night the tail was entirely detached from the head. Fifteen days later another outburst of material occurred (Fig. 137) which photographs on successive nights showed to be swiftly receding from the sun. N. T. Bobrovnikoff has found by a recent investigation of the photographs taken of this comet during the four days following the great explosion of October 14 that the tail-stuff receded on two different hyperbolas

which were convex to the sun. The repulsive forces for one corresponded to the pressure which would be exerted on carbon monoxide gas when it absorbs solar radiation; the origin of the other repulsive forces has not been identified.

Not only was Morehouse's comet noteworthy for the extraordinary activities exhibited by its tail, but its brightness varied in a very remarkable manner. Without apparent cause it from time to time flashed up in brightness for a day or two until it was almost visible to the unaided eye, and then quickly sank back into relative obscurity. Although a number of brighter comets have been observed in recent years, none has exhibited more varied and interesting phenomena or placed before scientists a greater number of puzzling unanswered questions.

266. On the Origin of Meteors and Comets. Nearly every astronomical investigation closes with a question respecting the origin, the evolution, and the final end of the objects under consideration. And so it is with meteors and comets. What were the circumstances respecting their birth? What changes have they undergone? What is their goal?

As was explained in Art. 254, a number of meteoric showers are known to be associated with comets or with the remains of comets, and the scores of other radiants may be the remains of unknown comets which have disintegrated within a few thousand years. As for the more numerous sporadic meteors which are not known to be associated with any radiant, it may be that they are the thoroughly scattered fragments of comets whose light went out during the millions of years of the geologic ages; or it may be that meteors and comets are different phases of something more fundamental than either.

If the assumption is true that meteors and comets are intimately related, then it follows from the elliptical nature of the orbits of comets that meteors, too, have originated in the solar system. This does not imply, of course, that they have always been within the distances of the orbits of the known planets; it means simply that though they may have made excursions out to thousands of times the distance of Neptune, they have originated and remained under the sun's dominion. Since comets disintegrate rather rapidly when they are in the neighborhood of the sun and among the orbits of the planets, it follows that they must continually be

forming, for otherwise they all would have been disintegrated ages ago. Moreover, it can not reasonably be assumed that those comets which almost graze the sun's surface and pass through its corona have made more than a few circuits of their orbits, for in the vicinity of their perihelion points they are subject to violent disruptive forces.

A theory that appears to explain the foregoing phenomena has been formulated by T. C. Chamberlin. He concluded that meteors and comets have probably developed from matter which is frequently ejected from the sun with high speed. The ejected matter leaves the sun as vaporous iron, magnesium, oxygen, etc., and arrives at the distance of the earth within a few weeks, its heavier materials having condensed into chondrules. If its aphelion point is only 32,000 times the distance from the sun to the earth, it does not arrive at its far turning point until after the lapse of about 2,000,000 years. For more than 1,000,000 years its velocity with respect to the sun is less than a kilometer per second, and during this long interval its chondrules are drawn together by their attraction into a compact swarm. The attractions of the stars through these long ages give them slight cross-components so that they do not fall back upon the sun. When they pass near the sun, they collide with one another and are fractured, their volatile materials are expelled, their glassy substances are melted, and their more refractory materials are cemented into meteors, in some cases of large dimensions. At one end of the process meteors and comets are being formed, and at the other end they are being dissipated.

It is clear that the tentative hypothesis which has been outlined in a general way meets the principal requirements of a theory of meteors and comets. It provides a continual supply, it gives the observed diversity of orbit planes and orientations, it explains the abundance of meteors, and it accounts for the characteristics of meteorites.

Many other hypotheses respecting the origin of comets have been formulated, ranging from the suggestion of R. A. Proctor many years ago that they were shot out by the planets to the recently proposed theory of C. P. Olivier that they were born billions of years ago with the planets. They still exist in considerable numbers because, according to this theory, their original perihelion distances were great, and only very gradually have they

been brought into the interior of the solar system by the perturbations of their orbits by the planets.

267. Effects of Meteors and Comets upon the Planetary System. Whatever may be the true explanation of the origin of meteors and comets, it is evident that meteors are continually adding to the masses of the planets. These additions, however, are relatively so small at present that they are not of much importance, even in the course of geological ages. It is not likely that the total mass of the meteors striking the earth daily averages more than a few tons; yet at the rate of 100 tons per day more than 400,000,000 years would be required for meteoric material to accumulate to an average depth of only one centimeter over the surface of the earth.

Meteors act as a resisting medium and consequently they reduce the radii of the orbits of the planets; as a secondary effect they decrease the eccentricities of the planetary orbits. If meteors come from beyond the exterior limits of the solar system, they decrease the periods of the planets by adding to the mass of the system; if they originate from the sun, there is no such effect. But in any case the effects are so small as to be of no appreciable consequence, even in hundreds of millions of years. In the early history of the system, however, the planets may have grown rapidly from the accession of meteor-like masses (Art. 303), and their present nearly circular orbits may be an evidence of this growth.

It is obvious that if meteors occupied the interstellar spaces with such frequency that 20,000,000 of them strike the earth per day, they would to some extent reduce the light of the enormously distant stars. Most meteors, however, are very small, perhaps something like a millimeter in diameter, and they are very far apart. In the region of space traversed by the earth there is now only one meteor, on the average, to 16,700,000 cubic kilometers (4,000,000 cubic miles). Computation shows that at this frequency they would not seriously reduce the light of the stars in our galaxy, though many of the stars are so distant that thousands of years are required for light to come from them to us. Moreover, the mass of all the meteors in a sphere with the sun as a center and extending out half way to the nearest stars would not exceed that of the sun. Consequently their effects upon the general dynamics of the galaxy would not be of much importance.

XIX. QUESTIONS

1. Make a count of meteors for an hour on a clear, moonless night. If possible, observe at the time of one of the principal showers and attempt to locate a radiant.
2. If possible, visit some museum in which there are meteorites and examine their structure.
3. At a velocity of 20 kilometers per second and at angle of fall of 45°, how long would a meteor be visible in descending from an altitude of 100 kilometers to within 30 kilometers of the earth's surface?
4. What would be the atmospheric pressure per square centimeter on a meteor moving at a velocity of 20 kilometers per second at an altitude of 100 kilometers?
5. Why could not a meteorite weighing thousands of tons survive and be recovered if it should strike the earth?
6. Why should we expect to find hyperbolic orbits of comets if there are comets whose orbits are parabolas?
7. Compute the period of a comet whose perihelion distance is near the sun and whose aphelion distance is 4000 astronomical units (Art. 176).
8. At the time the tail of Morehouse's comet was most active this body was more than an astronomical unit from the sun. Could the formation of its tail have been a purely temperature effect of the sun's rays?
9. At about what time will Halley's comet again pass within the orbit of the earth?
10. What objection is there to the theory that meteors and comets come in from the interstellar spaces?

REFERENCE BOOKS

Meteorites (1915). Oliver C. Farrington. The Field Museum of Natural History, Chicago, Ill.

Meteors (1925). Charles P. Olivier. Williams & Wilkins Company, Baltimore, Md.

The Story of Comets (1909). G. F. Chambers.

The Two Solar Families (1928). T. C. Chamberlin, The University of Chicago Press, Chicago, Ill.

Comets (1930). Charles P. Olivier. Williams & Wilkins Company, Baltimore, Md.

CHAPTER XIII

THE SUN

268. Dimensions and Mass of the Sun. In comparison with the planets, the sun is a body of enormous size and mass. In fact, its diameter is nearly twice that of the moon's orbit, as is illustrated in Fig. 138, in which the small spot at the center represents the earth, and the circle the orbit of the moon. Its mass is greater than would be that of a solid sphere of iron large enough to fill the orbit of our satellite.

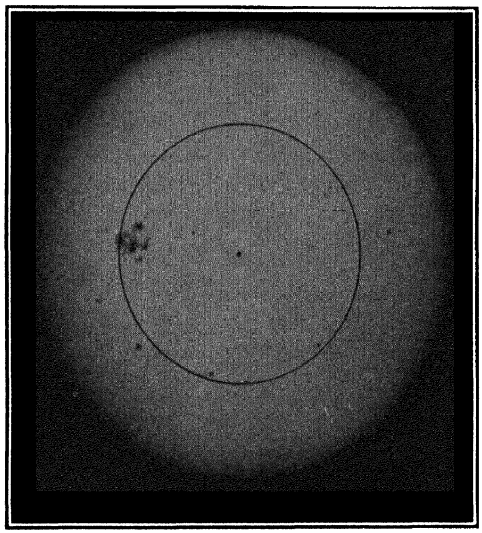


FIG. 138.—Orbit of moon drawn on a photograph of the sun.

As was stated in Table XII, the diameter of the sun is 1,391,107 kilometers, or nearly 110 times that of the earth. It follows that the area of the sun's surface is 12,000 times that of the earth, and that its volume is more than 1,300,000 times that of our relatively little planet. The sun, however, is only about one-fourth as dense as the earth (1.41 on the water standard); consequently its mass is only 332,000 times that of the earth. For scientific purposes, however, it is more convenient to have its mass expressed in metric units, in which it is 1.982×10^{33} grams.

The mass of the sun is of course determined by comparing its attraction with that of some other body of known mass, such as the earth. The attraction of the sun is just sufficient to hold the earth in its orbit, and the attraction of the earth similarly holds the moon. On taking into account the radii of these orbits and

the speeds at which the earth and the moon move, the relative attractions of the sun and the earth can be determined, and therefore their relative masses. Then, since the mass of the earth is known, the mass of the sun can be computed.

Although the density of the sun is only 1.41 times that of water, its mass is so enormous that its surface gravity is 27.9 times that of the earth. Therefore a person weighing 70 kilograms (154 pounds) on the surface of the earth would weigh nearly a ton on the surface of the sun. Such illustrations, of course, are not of scientific interest. But the fact that matter ejected by the sun is held back by a gravity about 28 times that of the earth is of scientific consequence when we attempt to explain the phenomena associated with the eruptive prominences (Art. 290).

269. Rotation of the Sun. If the sun were a solid body with conspicuous surface markings, the problem of determining its rotation would be simple; but it has no permanent surface features. It has been found, however, that its equatorial zone completes a circuit eastward in approximately 25 terrestrial days, and that the period increases with latitude to about 34 days at the poles. The axis of rotation of the sun is inclined about 7° to the perpendicular to the plane of the ecliptic, its north end being directed to a point whose right ascension is $18^h\ 44^m$ and whose declination is $+64°$. It will be found on referring to Map I that this point is almost exactly midway between Vega and Polaris.

In the middle of the nineteenth century R. C. Carrington and G. Spoerer made systematic investigations of the sun's rotation from observations of the apparent motions of sun spots across its disk. They determined the inclination of its axis and found that its period of rotation increases from the equator to latitude $\pm 45°$, beyond which the spots are not seen.

A number of other methods of determining the solar rotation have given substantially the same results. One consists in measuring the motions of exceptionally bright areas, called *faculae* (Art. 284), which are found particularly in the neighborhood of the sun spots. In another method the spectroheliograph (Art. 291) is used for photographing bright clouds of calcium vapor, called *flocculi*. G. E. Hale, Philip Fox and other astronomers have determined the rotation of the sun from both bright and relatively dark hydrogen flocculi, as well as from bright calcium

flocculi, and have reached results in harmony with those obtained by the other methods, although the flocculi generally lie in regions tens of thousands of kilometers above the level in which the spots occur. Finally, the rate of rotation of the sun has been determined spectroscopically from the Doppler-Fizeau effect (Art. 277) by W. S. Adams from the solar equator to latitudes 75° north and south.

The phenomenon of a celestial body rotating most rapidly at its equator again presents itself for explanation. It has generally been supposed that the present peculiarities of rotation of Jupiter, Saturn, and the sun are inheritances from conditions produced when these bodies were formed (Art. 304). Fluid friction would, of course, tend to reduce the differences in rates of rotation of different parts; but it can be shown that in the case of a mass as large as the sun the effects would not be important in the lifetime of the planets.

270. Some Properties of Radiant Energy. Light of every color travels in a vacuum, such as the interplanetary spaces, with a fixed speed of 299,796 kilometers (186,285 miles) per second. It is not propagated like material particles, but rather like waves whose oscillations are at right angles to their direction of motion, as is illustrated in Fig. 139. In this diagram the oscillations are in the vertical direction and the line of propagation is to the right. It is only by the hypothesis that light is composed of trains of waves of this sort that the phenomena of interference and spectrum analysis have been explained. Yet there are some phenomena, such as the Compton effect, which point toward the corpuscular theory of light. Moreover, light is radiated in finite amounts, called *quanta*, rather than continuously. For most purposes, however, we may think of light as consisting of waves transverse to the direction of its motion, and perhaps being transmitted through an ether.

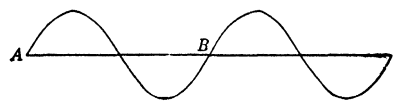


FIG. 139.—Transverse wave.

The distance between successive waves of a ray of light, as AB in Fig. 139 is its *wave-length*, which varies with its color; the number of waves which pass a fixed point per second is the *frequency*. Of the light to which the human eye is sensitive, red has the greatest wave-length (8×10^{-5} centimeter) and violet the

least (4×10^{-5} centimeter), or about 1/32,000 and 1/64,000 of an inch respectively. But beyond these narrow limits in both directions there are similar waves which differ essentially from light only in their wave-lengths. Those longer than the red rays are called the *infra-red;* those shorter than the violet, the *ultra-violet.* Together they constitute the class of electromagnetic radiations which Clerk Maxwell predicted from theoretical considerations (1860) and H. R. Hertz verified experimentally (1888) in some of the most brilliant chapters of science. At present scientists have covered experimentally the interval, in the direction of long waves, from the red to Hertzian radio waves several kilometers in length; and in the direction of short waves, from the violet to X-rays 1/10,000 as long, and recently to the so-called cosmic rays (Art. 307) less than 1/2,000,000 as long. In the terminology of sound, our eyes are sensitive to only one octave of light, but our science has revealed and uses more than 40 other octaves of similar radiations, the existence of which our predecessors never suspected.

Astronomers and physicists usually let c represent the velocity of radiant energy in a vacuum, λ its wave-length, and ν its frequency. It follows that $\nu=c/\lambda$. Since c in centimeters is nearly 3×10^{10} and λ for yellow light is 6×10^{-5}, the frequency for yellow light is 5×10^{14} per second. Therefore there are more transverse oscillations per second in a ray of yellow light than there are seconds in 15,000,000 years.

Wave-lengths of light are generally expressed in terms of a unit known as the *angstrom* (defined by A. J. Ångström in 1853), which is 10^{-8} centimeter, or 0.3937×10^{-8} inch. In this unit, the rays to which human eyes are sensitive range in wave-lengths from about 3900 angstroms, in the violet, to nearly 8000 angstroms, in the red. The *quanta,* in multiples of which radiant energy of various wave-lengths is radiated, are $h\nu$, in which h is M. Planck's constant, a fundamental quantity independent of the frequency ν. The quantum for yellow light is so minute that more than 1.25×10^{19} of them would be required to raise the temperature of a gram of water one degree centigrade; yet physicists have devised several methods of measuring both h and ν with an error not exceeding one per cent.

In general the transverse vibrations of a ray of light or other radiant energy are in all directions perpendicular to the line of its

propagation. If the transverse vibrations, however, are all in one plane, as is illustrated in Fig. 139, the light is *plane polarized.* Light is partially polarized when it is reflected from a surface, such as that of the moon; and it is more completely polarized when it is passed through certain crystals or solutions. In fact, the compositions of such solutions as those of sugar are conveniently determined by their polarizing effects upon light. There is also a *circular polarization* in which the plane of transverse motion rotates uniformly as the ray advances.

271. Amount of Radiant Energy Received by the Earth from the Sun. A discussion of solar energy must evidently rest primarily upon the determination of the amount and character of the radiant energy received by the earth from the sun. Unfortunately, the ozone in the earth's upper atmosphere absorbs and cuts off all radiations whose wave-lengths are less than about 2900 angstroms, and it partially absorbs radiations of several other wave-lengths. It is possible, however, to allow for these absorptions and to measure with an uncertainty not exceeding about one per cent the total radiant energy received from the sun at the outer limits of the earth's atmosphere.

According to the investigations of C. G. Abbot and his associates of the Smithsonian Institution, a square centimeter exposed pendicularly to solar radiation at the mean distance of the earth from the sun receives 1.938 calories of energy per minute; that is, enough energy to raise the temperature of water (at 15° C.) 1.938 degrees Centigrade per minute. This result becomes more easily comprehensible when it is translated into familiar units. By a suitable transformation, we find that a square meter placed perpendicularly to the solar rays receives radiation from the sun at the astonishing rate of 1.35 kilowatts per square meter (1.51 horse power per square yard). Since the area of the spherical earth ($4\pi r^2$) is 4 times the area of the cylinder of solar rays it intercepts (πr^2), the energy received by the earth from the sun averages $1.35/4 = 0.34$ kilowatts per square meter (0.38 horse power per square yard), or enough to maintain in continuous operation six 60-watt lamps. Although about 45 per cent of this energy is reflected from or absorbed by the earth's atmosphere, yet the amount received at the surface of the earth averages about 187,000 kilowatts per square kilometer (646,000 horse power per

square mile). If we could make efficient use of solar energy, we should need no other source of power.

Although the geological record proves that for hundreds of millions of years the earth has received radiant energy from the sun at approximately the present rate, the measurements of Abbot, Fowle, and Aldrich seem to indicate that the solar radiation sometimes varies several per cent in the course of a few days. In order to eliminate the effects of local atmospheric changes, they made measurements of the solar energy received simultaneously at several widely separated stations. There are difficulties inherent in measuring minute variations in such great amounts of energy as are received from the sun. In order to avoid them, Joel Stebbins and T. S. Jacobsen measured the reflected sunlight received from Uranus, and concluded that, in the range of wave-lengths which they used, there is no short-period variability of the solar radiation of as much as one per cent.

It follows from Stefan's law (Art. 274) that if the solar radiation should change by a certain fraction and then remain constant, the surface temperature of the earth would change by approximately one-fourth of the same fraction. Since the absolute mean temperature of the earth's surface is about 288° C. (519° F.), it follows that if the rate of the sun's radiation should decrease 10 per cent, the mean surface temperature of the earth would fall only 7° C. (13° F.) Evidently such small and transitory variations in the rate of solar radiation as may have been observed can not have important effects on terrestrial temperatures.

272. Sources of Terrestrial Energies. It is an interesting fact that although we do not use solar energy directly to drive our machines, most of the useful work of the world is done by energy that originally came from the sun. For example, the energy in the winds that even yet drive our commerce over the seas and spin the wheels of millions of windmills is available because winds are developed in restoring equilibrium in the elastic atmosphere after the sun's rays have heated it unequally in different places. There is, in fact, much more than enough energy in the winds to meet all the needs of mankind. Similarly, water is available as a source of energy only as it plunges over precipices after it has been raised up by solar radiation and transported far by winds.

All the work animals do, as well as the heat they generate,

depends upon the animal or plant food they eat; and hence their energies can be traced back to the vegetable world in whose countless tiny cells of chlorophyl solar energy is stored. Since coal and petroleum are of vegetable origin, their locked-up energies also came from the sun in some far-off geological ages before man evolved on the earth.

There are, however, a few manifestations of energy on the earth that are not of solar origin, except as the earth itself may be a fragment of the sun. The most obvious of these are the heat energies which come from the interior of the earth, such as are released by volcanoes, geysers, and hot springs. More important, however, at the present time are those in the ocean tides which surge and beat on the shores. These tides are raised primarily by the moon and to a lesser extent by the sun (Art. 186); but when we trace back to its ultimate source the energy they expend, we find that it is largely derived from the rotation of the earth. At present these sources of energy are of very little practical value.

273. Total Energy Radiated by the Sun. Since the intensity of energy radiating from a point or from a uniformly luminous sphere is inversely as the square of the distance from the point or from the center of the sphere, it is a simple matter to compute the rate of solar radiation from the measured amount received by the earth. On letting R represent the mean distance from the center of the sun to the earth and r the radius of the sun, the formula defining the rate of solar radiation E becomes

$$E=\left(\frac{R}{r}\right)^2 1.938=\left(\frac{14{,}945\times10^4}{695{,}554}\right)^2 1.938=89{,}470$$

calories per minute per square centimeter of its surface, or for the entire sun, 9.07×10^{25} calories per second. This is equivalent in round numbers to a rate of 63,000 kilowatts per square meter (70,000 horse power per square yard).

Energy is radiated fron the sun at a rate which is quite beyond our comprehension. To assist the imagination, suppose the sun were entirely covered with a layer of ice 1000 meters deep; it would be melted by the energy the sun radiates in less than 1.5 hours. In fact, the sun radiates enough energy to melt a sphere of ice as large as the earth in 16.6 minutes. Although the earth intercepts only 5×10^{-10} of the energy radiated by the sun, yet it receives solar radiation at the rate of 17×10^{13} kilowatts, or at the

rate of nearly 100,000 kilowatts for every human being on the earth. All the planets together intercept only about 5×10^{-9} of the energy which the sun radiates; the remainder streams off into space to the distances of the stars, just as energy from the stars comes to us. Even the stars of our galaxy, though there are hundreds of millions of them, intercept only a small fraction of the energy radiated by the sun. What becomes of the remainder we do not know, unless it keeps on simply as radiant energy, though we feel that in some way it must be integrated again.

274. Temperature of the Sun. There are three distinct methods of determining the temperatures of the radiating layers of the sun.

(*a*) *Stefan's law.* J. Stefan's law expresses the relationship between the temperature and the rate of radiation of a perfect radiating surface. When the radiated energy E is expressed in calories per minute per square centimeter and the temperature T is expressed in degrees centigrade measured from absolute zero, the formula of this law is

$$E=\frac{8.20}{10^{11}}T^4.$$

With the value of E given in Art. 273, the value of T is found from this equation to be 5750° C. (9450° F.) absolute. Since the solar radiations are emitted from many layers rather than from a single surface, this result is simply a measure of the order of temperature of the exterior of the sun. Even at its surface the temperature is high enough to volatilize any known substance.

(*b*) *Wien's law.* This law, which was developed by W. Wien, is a relationship between the temperature of a perfect radiating surface and the wave-length of its maximum radiation. A piece of iron, for example, at ordinary temperatures radiates only infra-red rays; at a higher temperature it becomes dull red; at a still higher temperature, bright red; and at the melting point, bluish-white. When the wave-length of maximum radiation λ_m is expressed in angstroms, the formula for Wien's law is

$$\lambda_m=\frac{289\times10^5}{T}.$$

It is found by observation that for the sun $\lambda_m=4700$ angstroms; therefore this formula gives $T=6150°$ C. (11,070° F.) absolute as the temperature of the surface of the sun under the hypothesis that it is a perfect radiator.

In Fig. 140 the relationship between T in degrees centigrade absolute and λ, the wave-length of maximum radiation in angstroms, is shown graphically. Since the longest wave-lengths (in the red) to which our eyes are sensitive are about 8000 angstroms, it follows that a body having a temperature much below 4000° would have its maximum radiation in the infra-red; at $T = 2000°$ its radiation would be nearly all invisible.

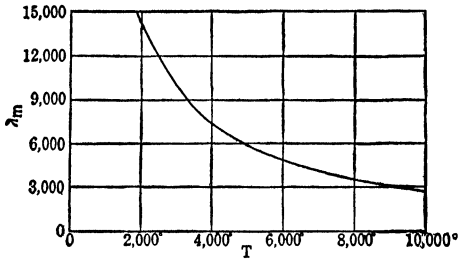


FIG. 140.—Wien's law.

(*c*) *Planck's law.* There is a fundamental theoretical equation, derived by M. Planck, that gives the amount of energy of all wave-lengths radiated by a perfect radiator for any absolute temperature, and the results computed by the formula agree excellently with laboratory measurements. Fig. 141 shows in full curves the theoretical distribution of radiated energy for T equal to 2000°, 4000°, 5000°, and 6000°, and in a dotted curve the measured radiation of the sun.

It is evident from the diagram that the effective radiating temperature of the sun is about 6000° C. absolute (10,800° F.). The departures of the measured radiation from its theoretical value, as shown in Fig. 141, are probably a consequence of the fact that the sun radiates from various levels and that the higher levels absorb energy from the radiating lower levels in varying amounts, depending upon its wave-length. Moreover, radiations of very short wave-lengths are absorbed by the upper atmosphere of the earth.

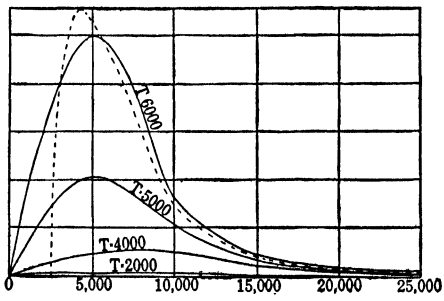


FIG. 141.—Planck's law of distribution of radiant energy at various temperatures.

When we consider the difficulties which are met in determining the temperature of the exterior layers of the sun, we are likely to conclude that the problem of finding the temperature of its hidden

deep interior, hundreds of thousands of kilometers below its visible parts, is wholly insoluble. Science has many ways, however, of penetrating apparently inaccessible regions. Many years ago (1870) J. H. Lane, of Washington, determined the enormous gravitational pressures to which the gases in the interior of the sun are subject; then he computed the temperatures and the densities they must have in order that their expansive forces should balance these pressures.

The work of Lane was greatly extended by A. Ritter, though he too neglected some factors depending on more recent discoveries which are now known to be important. A. S. Eddington, E. A. Milne, and other investigators have attempted to take these factors into account. One factor neglected by Lane is the outward pressure due to the enormous outpouring of radiant energy which occurs at temperatures of a million degrees or more; another is that the outer rings of negative electrons are believed to be stripped from their atoms and to assist, like a perfect gas, in withstanding the pressures of the overlying masses. The pressure at the center of the sun is approximately 1.33×10^9 times atmospheric pressure at sea level, or about 1,400,000 tons per square centimeter. According to Eddington, the temperature at the sun's center must be of the order of 10,000,000° C. in order to balance this enormous pressure; Milne derives a result many times higher; and J. H. Jeans concludes that the deep interiors of stars have the properties of liquids, rather than of gases, for otherwise they would be unstable. These divergent results illustrate the uncertainties that still remain respecting the conditions in the interiors of the sun and other stars.

Since according to Stefan's law the rate of radiation of a body is proportional to the fourth power of its absolute temperature, the radiation per unit mass in the sun's interior is prodigious. By Wien's law, its maximum is in very short wave-lengths, the so-called X-rays, for which matter is very opaque. Hence the radiations emitted deep in the sun are quickly absorbed, to be emitted again in all directions at various wave-lengths depending upon the temperature of the radiating mass. Thus after a succession of emissions and absorptions, in number so great as to be beyond our comprehension and occupying perhaps millions of years, radiant energy originating near the center of the sun finally arrives at its surface and then flies away to the planets or beyond them to

the distances of the stars and even beyond our galaxy at a speed of 300,000 kilometers per second.

275. Source of the Sun's Energy. Few subjects illustrate more strikingly the difficulty of changing established opinions than the history of the ideas of scientists respecting the source of the sun's energy and the related question of the sun's duration as a great radiating star.

A century ago when our predecessors were thinking of the lifetime of the universe in terms of thousands of years, it would not have seemed unreasonable to assume that the sun is simply a burning mass of carbon and oxygen, though now we find that such combustion would provide the energy it radiates for only 1500 years. But in the middle of the nineteenth century, in connection with the formulation of the principle of the *Conservation of Energy* by Robert Mayer and J. P. Joule, the whole problem of energy came prominently into the consciousness of physicists. In 1854, H. von Helmholtz developed the theory that the heat and the radiant energy of the sun are due to its contraction and he worked out in quantitative detail the consequences of such a contraction. The underlying principle of the theory is simply that as a body contracts it loses potential energy and gains an exactly equivalent amount of kinetic (heat) energy.

For many years after the work of Helmholtz, scientists accepted the contraction theory as being fundamentally true and complete. They found it highly satisfactory because it was simple and numerically exact. It set a limit of about 25,000,000 years to the whole past duration of the earth, an interval within which all geological and biological evolutions on its surface must have taken place. It likewise predicted a future, not more than 10,000,000 years away, when the sun will have become so dense that it can no longer contract. It was believed, in harmony with the theory, that the sun's light will then fail and that the planets, cold and lifeless, will forever revolve about a dark and invisible central body as it pursues its endless wanderings among the stars.

A geologist first challenged the adequacy of the contraction theory. In 1899, T. C. Chamberlin showed conclusively that it falls far short of providing sufficient time for the geological processes, and he boldly maintained, therefore, that the principal sources of stellar energies had not been discovered. With prophetic

insight, he predicted that they would be found in hitherto unknown kinetic and potential energies within the atoms themselves. His predictions were soon supported by revolutionary physical discoveries, for within the succeeding decade the foundations were laid for the whole theory of atomic structure and the enormous subatomic energies. When eventually (after 1920) astronomers generally abandoned the contraction theory and accepted a time scale of an entirely different order, they changed also their entire conception of the evolution of the stars.

Although it can not easily be doubted that the energy radiated by the stars is of subatomic origin, it is not entirely clear just how this energy becomes available nor how much there is of it. There is no difficulty in showing that its source is not the energy released by the radioactive elements uranium, radium, etc., when they break up, for no mixture of them would provide the observed rate of radiation even for a few thousand years. Just now scientists generally accept a formula published by Einstein in 1905 in which energy and mass are regarded as fundamentally equivalent and related by the equation $E=mc^2$, c being the velocity of light. The energy equivalent of mass, according to Einstein's formula, is enormous; a gram of matter, whatever be its chemical constitution, becomes in the transformation 2.15×10^{13} calories, or enough heat to melt 14×14^{9} grams (14,000 tons) of iron. If this theory is correct, the sun is losing 42×10^{11} grams (4,200,000 tons) of mass every second as a consequence of its radiation. There is, however, no occasion to fear that it will soon radiate itself away, for at this rate its mass of 1.978×10^{33} grams would decrease by one per cent only after the lapse of about 150,000,000,000 years.

Previous to 1920 many astronomers seemed to take a melancholy satisfaction in looking forward, in accordance with the contraction theory, to a time a few million years in the future after which the sun will be forever cold and the earth lifeless. Now the Einstein formula extends the period of solar activity a hundred thousand fold; yet the sun, according to it, will eventually come to an equally inglorious end, a large part of its mass being scattered as radiant energy beyond our galaxy of stars, never to be assembled again. It is likely, however, that scientists are once more too eagerly seizing on a formula that for a time appears to give us the ultimate truth for which we strive, but which we never attain. That the future of the sun in approximately its present state will extend to

thousands of millions of years there can be little doubt. But the theory that it will perish in a finite time never to revive again, either as a unit or as parts of other suns, is based on too little knowledge for so broad a conclusion and is inharmonious with the existence of galaxies of stars.

276. Production of the Spectrum. In the popular mind, the telescope is the supreme astronomical instrument; to astronomers, the spectroscope is equally important. Each is essential in exploring the universe, and together they are a thousand times as valuable as either would be alone.

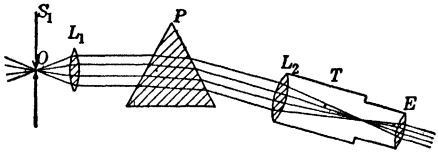


FIG. 142.—The prism spectroscope.

Refracting telescopes are subject to the defect of chromatic aberration because glass refracts light of different wave-lengths by different amounts; the simplest type of spectroscopes owe their possibility to precisely this property of light. A spectrum might be formed by the method illustrated in Fig. 16, but it would be much too faint for use. In the simplest form of the spectroscope, the spectrum is produced and observed as is illustrated in Fig. 142.

Suppose light of only one wave-length is brought to a focus on the slit O in the screen S_1. The lens L_1 is placed at such a distance from the screen S_1 that the rays diverging from the slit O emerge from L_1 in parallel lines and fall on the prism P. They are refracted both on entering P and also on emerging from it. Since the light is assumed to be of one wave-length, all rays emerge from P in parallel lines. The lens L_2 is the objective of the telescope T through which the rays are condensed and made parallel as they emerge from the eyepiece E. An eye to the right of E sees the light as a line parallel to the slit O; it sees, in fact, the image of the slit O. By means of the lens L_1, called a *collimator*, and the viewing telescope T a much larger number of rays enter the eye than would be the case if the arrangement illustrated in Fig. 16 were used; that is, the spectrum is brighter.

Now suppose rays of two wave-lengths pass through the slit O at the same time. They remain mixed until they enter the prism P after which all those having the shorter wave-length are refracted more than those having the longer wave-length. On emerging

from P, they diverge still more, but all the rays of each wavelength are parallel to one another. Consequently the eye sees two images of the slit O in different directions and in different colors. The situation is similar for a mixture of any number of colors.

In practical astronomical applications, a telescope of large aperture brings the light of the celestial object under examination to a focus on the slit O in the screen S_1. The collimator L_1 is adapted in diameter and focal length to the size of the prism P. If the object is bright, such as the sun or the moon, several prisms in series may be used to secure large dispersion; if it is a faint star, it is not advantageous to use more than one. If the spectrum is observed, a telescope T is used; if it is photographed, the photographic plate is exposed in the focal plane of L_2. On this plane the images of the slit O are formed in the different colors.

A variation which has advantages in securing small-dispersion spectra of many stars consists in putting the prism outside the telescope objective. With this arrangement, the various wavelengths are separated before they pass through the telescope.

Another method of producing spectra is by the use of so-called *gratings*, which are reflecting surfaces of glass or metal on which are ruled from 10,000 to 20,000 parallel equidistant lines. The rays of radiant energy which have been rendered parallel by a collimator, when reflected from the reflecting surfaces between the lines on such a grating, interfere at certain places and add at others, giving a spectrum. The details of the reasons a spectrum will be formed in this way are explained in works on physical optics. If the grating is a plane surface, a telescope is used for viewing the spectrum; but a concave grating forms its own image of the slit.

The prism spectroscope is generally used in astronomy for producing the spectra of faint stars, comets, and nebulae. The grating spectroscope has important advantages in producing spectra of bright sources of light, such as the sun or an electric arc. A third type of spectroscope, the *echelon*, which was invented by A. A. Michelson, consists of a series of equally thick pieces of plane glass piled up like a stairway. Although the echelon has important advantages in producing high-dispersion spectra, it has not been used much in astronomy.

277. Principles of Spectrum Analysis. More than 250 years ago (1666) Newton passed sunlight through a prism and broke it up

into its constituent colors; and he also recombined them again into white light by passing them through a prism reversed in position. For more than a century after the time of Newton little progress toward the spectrum analysis of the present day was made, largely because the light was passed through a small circular opening, before it reached the prism, rather than through a narrow slit whose edges are separated by perhaps one-twentieth of a millimeter. The images in different wave-lengths of the circular opening overlap and impair definition; the images of a narrow slit are distinct.

In 1802, W. H. Wollaston introduced the slit in examining the sun's light. J. Fraunhofer vigorously used the improved instrument and found that the sun's spectrum is crossed by dark lines, the positions of 324 of which he determined in 1817. Since that date they have been called *Fraunhofer lines* in his honor. All that remained was the formulation of the principles of spectrum analysis in order to explain the meaning of the Fraunhofer dark lines and to place in the hands of astronomers a new means of investigation of extraordinary and wholly unexpected importance. These principles were first approached by A. J. Ångström in 1853; they were completed in their present form by R. Bunsen and especially by G. R. Kirchhoff in 1859–62. William Huggins, A. Secchi, and H. C. Vogel were leading pioneers in developing solar and stellar spectroscopy, which received an enormous impetus in 1895 when H. A. Rowland published his great table giving the positions of about 14,000 solar spectral lines.

There are four laws of spectrum analysis which follow from Kirchhoff's principles and which contain in compact form their principal consequences.

Law I (Continuous spectrum). *A radiating solid, or a liquid, or a gas under high pressure gives a continuous spectrum, the position of whose maximum intensity depends upon the temperature of the source.*

The converse of this law is also true; that is, if a spectrum is continuous, the source of radiant energy is a solid, a liquid, or a gas under high pressure, but which of the three is not determined. Moreover, the position of maximum intensity of radiant energy in a spectrum enables us to determine the temperature of the source by Wien's law. In case the radiating material is a solid, a liquid, or a gas under high pressure, the spectrum is indepen-

dent of its chemical constitution; consequently the chemical constitution of the source can not be determined from a continuous spectrum.

It follows from Art. 276 that when a source emits radiant energy of all wave-lengths, its spectrum is continuous. Hence in solids, liquids, and gases under high pressure the molecules are under such restraints and interferences from one another that they do not emit radiations of particular frequencies, but of all frequencies, though in amounts depending upon their temperatures.

LAW II (Bright-line spectrum). *A radiating gas under low pressure gives a spectrum which consists of bright lines whose positions and intensities depend upon the chemical constitution of the gas.*

The converse of the second law of spectrum analysis is also true; that is, if a spectrum consists of bright lines, its source is a gas under low pressure, and the chemical constitution of the gas can be determined from the positions of the spectral lines. For example, Fig. 143 is a reproduction of part of a photograph of the spectrum of gaseous titanium. The spectra of some gases contain only a few lines, as in the case of sodium; the spectra of others contain hundreds of lines, as in the case of iron. No difficulty results from a mixture of gases, for no two substances have been found to have even a single spectral line in common, though the differences in wave-lengths are in some cases small. Even when a spectral line of one substance is nearly coincident with a line of another there is no difficulty, for the two substances have other lines which are not in common.

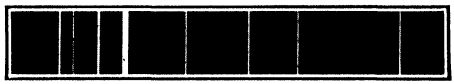


FIG. 143.—Part of titanium bright-line spectrum.

Although the spectral lines of incandescent gases depend primarily upon the chemical constitution of the gases, there are other factors which have appreciable effects. For example, at moderate temperatures the spectrum of a gas does not contain all the lines which appear at several successively higher temperatures. The highest temperatures are produced in the laboratory by the electric arc and the electric spark, both of which add the effects of powerful electrical disturbances to those produced by temperatures alone. High pressures, or perhaps densities, of radiating gases cause their spectral lines to be broadened and usually, though not always, displaced toward the red by various amounts. In 1896, P. Zeeman found that when a radiating gas is in a powerful

magnetic field, most of its spectral lines are broken up into a number of closely neighboring lines, the light of each of which is polarized. Finally, in 1913, J. Stark found analogous effects on the character of light from a source in a strong electric field. For a time these various effects were regarded as unfortunate disturbing factors; now, as we shall see, they often furnish very important information respecting the source of light.

It is evident from the second law of spectrum analysis that the nature of a radiating gas under low pressure, however distant it may be, can be determined from the positions of its spectral lines. Moreover, considerable information can be obtained respecting

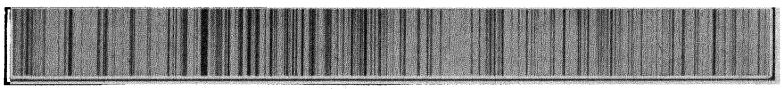


FIG. 144.—An absorption (dark-line) spectrum.

its temperature and whether it is in a magnetic or an electric field. It is from this law that the chemical compositions of comets, certain stars, and nebulae are determined.

Upon reflection, the reason that incandescent gases give bright-line spectra becomes evident. In a gas the molecules are far apart relative to their diameters and are quite independent of one another most of the time in spite of their very frequent collisions (Art. 90). Consequently they vibrate, or at least emit vibrations, in certain characteristic frequencies, just as any oscillating body, such as a tuning fork, vibrates in a period peculiar to itself. Light of a given frequency gives rise to a bright line in the spectrum; light of various frequencies produces the various spectral lines, all of which are characteristic of the radiating source.

LAW III (Dark-line absorption spectrum). *When radiant energy of all wave-lengths passes through elementary gases under low pressure before reaching the spectroscope, the spectrum is continuous except for dark lines where there would be bright lines if the intervening gases were the radiating source.*

It is evident that if it were not for the gases through which the radiant energy passes, the spectrum in this case would be continuous. Hence the interposed gases absorb precisely the wave-lengths they would radiate if they were the source of energy. It follows that in this case the temperature of the source can be determined by Stefan's law, and that the composition of the absorbing gases

can be determined from the positions of the various dark lines they produce.

If the intervening gas is a compound instead of an element, the spectrum is usually made up of absorption bands rather than isolated lines, as is illustrated in Fig. 145.

There are in physics many analogues to the absorption spectrum. For example, a musical instrument will absorb tones of the same pitch as those which it can produce, and no others. The radio

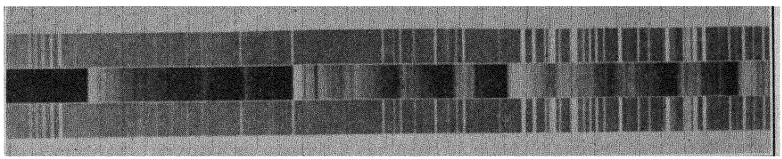


FIG. 145.—Band spectrum of R Leonis from λ 4395 to λ 5000 in center and comparison spectrum above and below. (Yerkes Observatory.)

responds only when the receiving set is tuned to the sending instrument. The underlying principle is simply that of resonance which was considered in other connections in Art. 201.

LAW IV (Doppler-Fizeau law). *If a gaseous source of radiant energy (in the case of a bright-line spectrum) or an absorbing gas (in the case of a dark-line spectrum) and a spectroscope are relatively approaching toward (or receding from) each other, the spectral lines are displaced toward the ultra-violet (infra-red) end of the spectrum, the amount of the displacement being proportional to the relative velocity of approach (or recession).*

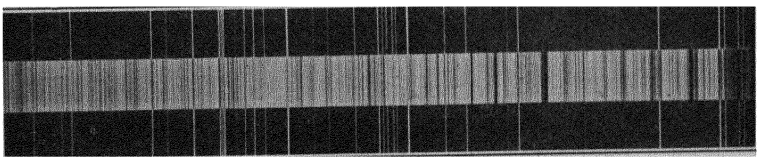


FIG. 146.—Absorption spectrum in center due to many elements; comparison spectrum of titanium above and below.

When a gaseous source of radiant energy moves toward the receiver, its waves evidently follow one another more closely than they would if it were stationary. That is, they are effectively shortened and the corresponding spectral lines are displaced toward the ultra-violet. If the source is stationary and the spectroscope moves toward it, the result is nearly (but not exactly) the same. If

v represents the relative velocity of the source (or absorbing gas) and the spectroscope and $\Delta\lambda$ the displacement of a spectral line of normal wave-length λ, the formula for $\Delta\lambda$ is $\Delta\lambda = v\lambda/c$.

Since c equals 300,000 kilometers per second and v for the orbital motion of the earth is about 30 kilometers per second, the displacement of the spectral lines of a star due to the revolution of the earth can not exceed about 1/10,000 of their wave-lengths. Evidently refined means must be employed to determine the speed of revolution of the earth by spectroscopic means (Art. 105).

In order to determine conveniently and accurately the Doppler-Fizeau displacement of spectral lines a spectrum of a stationary source in the observatory is photographed immediately above or below that of the star, as is illustrated in Fig. 146. The lines of the spectrum of the stationary source furnish a known standard of reference from which the positions of the spectral lines of the moving source can be measured under a microscope by suitable micrometer means. By this method the relative velocity of the earth with respect to a star whose spectrum has sharp lines can be determined with an error not exceeding about one-half of a kilometer per second.

278. The Solar Spectrum. If sunlight is passed through such a simple instrument as that illustrated in Fig. 142, the resulting spectrum is found to consist of a background of color which is continuous except where it is crossed by a number of dark lines. If the spectrum is formed by a grating of high resolving power, thousands of dark lines are present.

It follows from the first and third laws of spectrum analysis that the part of the sun which emits most of the radiated solar energy is a solid, a liquid, or a gas under high pressure. In view of the enormous temperature of the sun, there can be no doubt that even the exterior layers are gaseous.

By far the most interesting features of the solar spectrum are the dark lines. It follows from the third law of spectrum analysis that absorbing gases, which are called the *reversing layer*, lie between the principal radiating layers of the sun and the surface of the earth. It is obvious at once that the absorption lines of oxygen, nitrogen, and water vapor may be due to the atmosphere of the earth, but there are thousands of other absorption lines which can only be due to elements in the atmosphere

of the sun, for they correspond to such heavy and refractory elements as iron, copper, aluminum, etc.

At first thought, it seems remarkable that iron hot enough to be in a vaporous state in the atmosphere of the sun should yet be cool enough to absorb energy from the radiating layers below. The question, however, is simply one of relative rates of absorption and radiation outward: the iron vapors, for example, receive and absorb radiant energy at an enormous rate, but instead of radiating all of it directly outward, they radiate it in every direction. The result is that the outward flow of energy of the wave-lengths which they absorb is decreased and *relatively* dark lines are produced.

There is no difficulty in proving from the absorption lines of the solar spectrum that there is iron in the atmosphere of the sun, but the conclusion is not reached so simply for oxygen and nitrogen, for these elements are abundant in the atmosphere of the earth. There are two methods, however, of distinguishing between the absorption by oxygen in the earth's atmosphere and its absorption in the solar atmosphere. The first depends upon the fact that the density of absorption lines increases as the amount of absorbing gas increases. When the sun is observed at different altitudes its light passes through different amounts of the earth's atmosphere; and similarly, light from various parts of the solar disk passes through various amounts of the sun's atmosphere. From the absorptions of these various thicknesses of terrestrial and solar atmospheres it is possible to determine whether or not the oxygen lines are due entirely to these elements in the atmosphere of the earth. The second of the methods depends upon the fact that if there is oxygen in the atmosphere of the sun, as well as in that of the earth, its lines of solar origin from the eastern limb of the sun will be displaced slightly toward the ultra-violet end of the spectrum, and those from the western limb toward the infra-red. By both of these methods it has been established that oxygen is present in the atmosphere of the sun.

Nitrogen is known to exist in the sun because the lines of the compound cyanogen (CN) are found in the solar spectrum. Moreover, the lines of atomic oxygen, as distinguished from the molecular oxygen in the earth's atmosphere, occur in the solar spectrum.

279. The Flash Spectrum. At the total solar eclipse in 1870, C. A. Young made an interesting observation that verifies the

third law of spectrum analysis. He placed the slit *S* of his spectroscope parallel to the advancing limb of the moon, as is illustrated in Fig. 147. Until the disk of the sun was nearly eclipsed by the moon, the usual dark-line spectrum was observed. But just as the opaque solar disk was completely covered, the only sunlight received was from the atmosphere above the sun's surface, and then a bright-line spectrum flashed out precisely where the dark-line spectrum had been an instant before. In a few seconds the bright-line spectrum disappeared, for the reversing layer is very thin, but it reappeared on the western limb for an equally brief interval just before the sun was uncovered again.

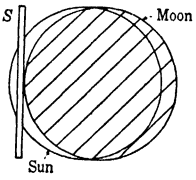


FIG. 147.—Position of slit when flash spectrum is photographed.

The flash spectrum is now photographed at every solar eclipse if the conditions are favorable. The purpose is not to verify the third law of spectrum analysis, for it long ago was confidently accepted as being true, but to determine the depth of the reversing layer and the distribution of the most abundant gases in it. It has been found from the interval during which the flash spectrum is visible and the speed with which the moon apparently travels across the sun's disk that the depth of the reversing layer is between 600 and 1000 kilometers, the lack of precision being due in part to the fact that it is difficult to determine just when the flash spectrum vanishes. Those elements which produce the principal lines in the flash spectrum appear to be distributed in uniform proportions throughout the reversing layer; but calcium and hydrogen are present up to a height of 12,000 kilometers.

280. The Theory of Ionization. Recent discoveries in atomic structure and the properties of radiant energy have laid a new foundation for the interpretation of solar and stellar spectra. Instead of being structureless units, atoms have been found to be highly complicated organizations of positively and negatively charged entities, called *electrons*. Although the mass of the positive electron (or *proton*) is 1847 times that of the negative electron, it carries an equal electric charge and its diameter appears to be much the smaller.

In 1911, E. Rutherford organized a large amount of observational

evidence in support of the theory that an atom consists of a compact nucleus of positive and negative electrons, the number of positive electrons being the greater, with enough negative electrons normally revolving around the nucleus to render the entire unit electrically neutral. N. Bohr soon explained the fact that atoms radiate only in quanta by the theory that the negative electrons can revolve around their nuclei only at certain distances, corresponding to "energy levels" differing by multiples of quantities depending upon associated frequencies. When a negative electron passes from an orbit of higher energy level to one of lower level, its atom radiates an amount of energy equal to the difference in potential; when an atom absorbs energy from an outside source, at least one of its electrons passes from an orbit of lower energy level to one of higher level.

Electrons may not only pass from one energy level in an atom to another, but they may escape from their atom entirely. After an atom has lost one or more electrons it is said to be *ionized.* A neutral atom radiates energy of certain wave-lengths; with each successive stage of its ionization the wave-lengths abruptly change. Therefore spectrum analysis enables the astronomer to determine not only what elements are in the atmosphere of the sun or of a star but also to what extent they are ionized. This information, in turn, leads to important results respecting the physical state of the radiating material, for M. N. Saha proved in 1920 that the fraction of a gas which is ionized to various stages depends jointly upon its temperature and the pressure of all the free electrons in which its atoms move.

Under given conditions of temperature and pressure the fraction of the atoms which are ionized to various degrees differs from one element to another. It is clear, therefore, without going into details that this fact furnishes a basis for determining from its spectrum both the temperature and the density of a gaseous mixture of several elements and the fraction each element is of the whole. The observational data are the presence and the density, or the absence, of the spectral lines of each element, both in its neutral and in its various ionized states; the theory consists of formulae for translating the observations into quantitative conclusions. On the basis of the theory of E. A. Milne, Miss Cecilia H. Payne has determined from a study of the spectra of several classes of stars the relative abundance of the 18 elements which occur in their

atmospheres most frequently. By a method resting on the same fundamental principles, H. N. Russell has made an investigation of the chemical composition of the sun.

281. Chemical Composition of the Sun. Of the 92 elements which theoretically may exist, ranging from hydrogen to uranium inclusive, 90 are now known on the earth, but information respecting the spectra of 10 of them is entirely lacking or too meager to be of use in establishing their presence in the sun. Of the remaining 80 elements (including hydrogen), Russell finds that 50 certainly exist in the sun. In the case of 9 elements the identification of solar lines with those obtained in the laboratory is doubtful, and 21 elements do not appear to be present in the atmosphere of the sun, at least in quantities sufficient for the detection of their spectra. With the exception of boron and nitrogen, the elements discovered in the sun exist in its atmosphere in the atomic state; these elements occur in compounds.

Although hydrogen is by far the most abundant element in the atmospheres of stars, for technical reasons it is difficult to determine what its percentage is. Russell estimates that 46 per cent of the sun's atmosphere by weight is hydrogen alone; Miss Payne arrived at more than 90 per cent of the same element for the stars in general. These differences are largely due to the present uncertainties in the methods, though the sun is quite distinct from certain other types of stars. Russell's tentative conclusions respecting the composition of the sun's atmosphere by volume are: hydrogen, 0.91; helium, 0.03; oxygen, 0.03; metals, 0.02; and free electrons, 0.01. By weight, the corresponding proportions are: hydrogen, 0.46; helium, 0.06; oxygen, 0.24; and metals, 0.24.

Among the metals that constitute 24 per cent of the sun's atmosphere, there are 15 which are present in considerable abundance. Russell's tentative determination of their relative abundance is as follows, their atomic weights being inclosed in parentheses: magnesium (24.3), 0.359; iron (55.8), 0.239; silicon (28.1), 0.143; sodium (23.0), 0.095; potassium (39.1), 0.060; calcium (40.1), 0.048; aluminum (27.0), 0.014; nickel (58.7), 0.014; manganese (54.9), 0.010; chromium (52.0), 0.006; cobalt (59.0), 0.006; titanium (47.9), 0.002; copper (63.6), 0.002; vanadium (51.0), 0.001; and zinc (65.4), 0.001.

Although a number of heavier elements, such as silver and even

lead with an atomic weight of 207, have been identified in the atmosphere of the sun, their combined mass does not exceed one-hundredth of one per cent of that of the metals listed. Of course, it is not impossible that the heavier elements occur in greater relative amounts in the depths of the sun beyond the range of spectroscopic examination.

As compared with the exterior part of the earth and with meteorites, the sun apparently contains an enormous excess of hydrogen and helium. In respect to metal constituents, the composition of the sun is similar to that of meteorites but different from that of the earth, for the latter has a greater percentage of aluminum, silicon, and titanium and a deficiency of magnesium. These differences may be limited, however, to the exterior zones of the earth, for rearrangements of its materials have been going on throughout the geological ages.

282. General Structure of the Exterior of the Sun. There are four layers, or zones, of the sun's exterior that are accessible to direct investigation; the conditions of the deeper interior can be inferred only from theoretical considerations.

(*a*) *The photosphere* (*light-sphere*). When the sun is observed either with or without optical aid, it presents an opaque surface which appears to be almost as sharply defined as that of the moon. Since this is the surface from which most of the sun's energy is radiated, it is called the photosphere, or light-sphere. It is not a uniform smooth surface, for it has a mottled structure and it is often disturbed by violent storms.

(*b*) *The reversing layer.* Immediately above the photosphere lies the layer of relatively cooler and rarer gases that produce the absorption spectrum from which the sun's chemical constitution is inferred. The lower limits of the reversing layer are the well-defined photosphere; at an altitude of several hundred kilometers it gradually merges into sensibly different higher layers. Since eruptions of vaporous gas from the photosphere often break through the reversing layer and doubtless mingle with it on cooling and descending again, it is probable that this mantle of absorbing gas has essentially the same chemical constitution as that of the photosphere itself.

(*c*) *The chromosphere* (*color-sphere*). Lying over the reversing layer and extending up to altitudes of 10,000 or 15,000 kilometers,

there is an extensive envelope of tenuous gases which derives its name from the fact that it radiates scarlet light. These gases are largely hydrogen, helium, and calcium.

(*d*) *The corona* (*crown*). At the times of total solar eclipses the sun is seen to be surrounded by a halo of pearly light which extends

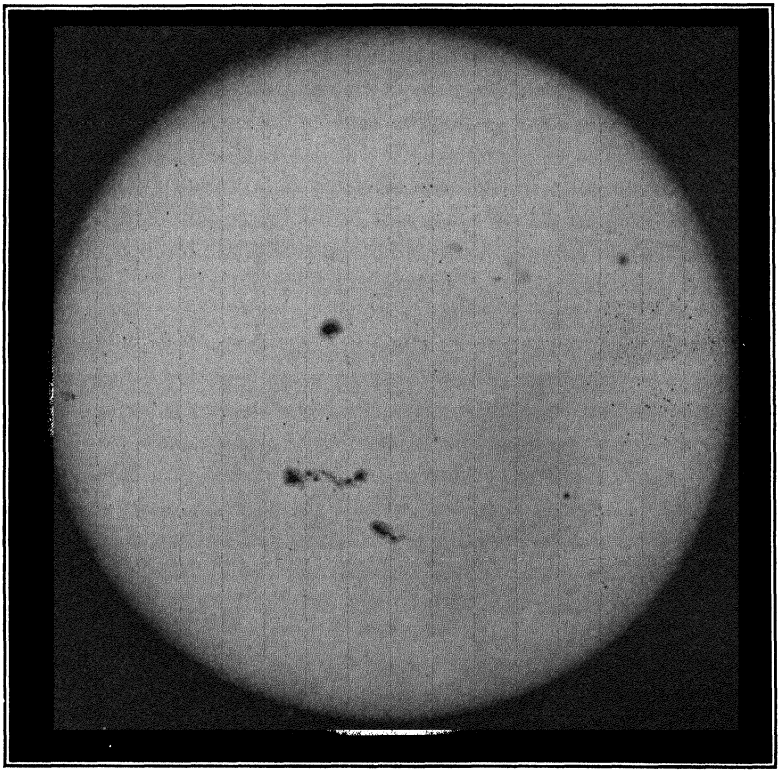


FIG. 148.—The photosphere of the sun and several sun spots. (C. D. Perrine, Lick Observatory.)

out somewhat irregularly to distances of 1,000,000 kilometers or more, particularly in the plane of the sun's equator, before it gradually fades out into the blackness of the sky. The corona is visible only during the brief intervals of total solar eclipses.

283. Characteristics of the Photosphere. Although the photosphere of the sun appears to present a sharply defined outline, it undoubtedly lacks much of being a smooth surface similar to that of the sea. In interpreting the appearance of the sun, as shown in

Fig. 148, it should be remembered that at its great distance a gradual transition of density and opacity through a depth of 50 or 100 kilometers would appear to be an abrupt change from one layer to another.

That the surface of the photosphere is decidedly uneven is indicated by its mottled appearance. When it is observed under good conditions with a telescope of considerable power, it is found strewn with innumerable brilliantly luminous areas, usually a few hundred kilometers in diameter, which have been likened in appearance to rice grains floating in a darker medium (Fig. 149). Under the best observing conditions, the individual bright areas are partially resolvable into finer structure. In general they are roughly circular in form, but in the vicinity of sun spots they are often elongated radially with respect to these disturbances.



Fig. 149.—Photosphere highly magnified. (Yerkes Observatory.)

Until recently there has been no satisfactory explanation of the rapid transition from the relatively transparent chromosphere and reversing layer to the perfectly opaque photosphere. The intensely luminous photosphere can not be composed of clouds of liquid drops of refractory materials, for its temperature is far too high. Moreover, its density is undoubtedly much below that of the earth's atmosphere at sea level. What, then, is the explanation of its opacity? J. Q. Stewart suggested that the radiant energy from the lower levels may be absorbed by the free electrons of the ionized gases. In a masterly investigation of the subject, E. A. Milne concluded that the photosphere would be effectively opaque as a consequence of photo-electric ionization of its atoms at a pressure .0001 of that of the earth's atmosphere at sea level. It is very probable that the photosphere is composed of negative electrons and highly ionized gases which absorb the short wavelength radiations from the depths of the sun and radiate the continuous spectrum which is observed. The bright clouds which

mottle the apparent surface of the sun are probably produced in some way by vast convection currents which in extreme cases develop spots.

284. Sun Spots. One of the first discoveries made by Galileo with his little telescope in 1609 was that of dark spots on the surface of the sun. The announcement of this discovery brought severe censure on Galileo, for our central luminary was assumed to be perfect and spots on it were interpreted as blemishes. It is

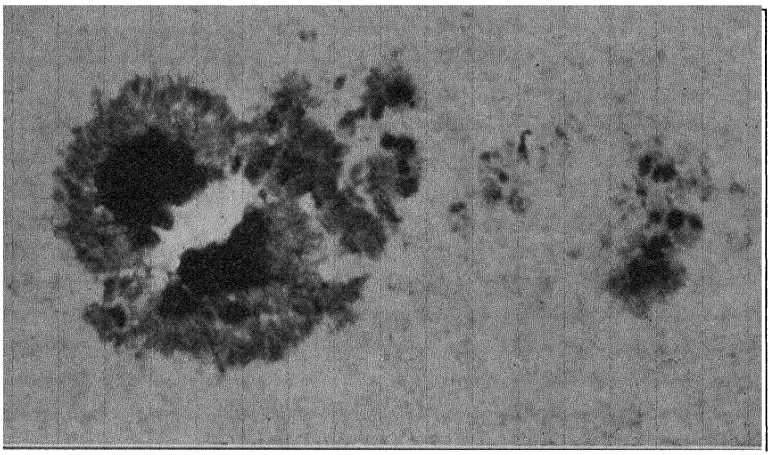


FIG. 150.—Great sun spot July 17, 1905, photographed by Fox with the 40-inch telescope of the Yerkes Observatory.

remarkable that sun spots should not have been noticed before the invention of the telescope, except by the Chinese, for occasionally they are of such enormous extent that they are easily visible with the unaided eye if the intensity of the sun's light is reduced by partially opaque glass or by the strongly absorbing air near the horizon when the sun is near rising or setting.

A typical sun spot consists of a nearly circular, *relatively* black (but actually very bright) center, called the *umbra*, and a surrounding lighter zone, called the *penumbra*. The normal penumbra is composed of filaments arranged radially around the umbra, and they look as though they are columns of intensely luminous photospheric material which have tipped over so that their sides are visible. There is a fairly definite line of demarcation between the umbra and the penumbra, and an equally definite one between

the umbra and the surrounding photosphere, as is illustrated in Fig. 150. In spite of the fact that the umbra of a sun spot is very dark in comparison with the photosphere, it is extremely brilliant by ordinary standards; in fact, an electric light projected on it appears to be black.

In the neighborhood of sun spots, the brightness of the photosphere is usually above the average; and, moreover, in their vicinity there are nearly always intensely brilliant and somewhat elevated large areas, called *faculae*. When a spot is near a limb of the sun its associated faculae are particularly conspicuous, for then the brightness of the lower-lying surrounding photosphere is reduced by the absorption of the solar atmosphere, above which these bright areas lie.

In diameter, the umbrae of sun spots range from the limits of observability with a telescope to 100,000 kilometers; the penumbrae and outlying spots are two or three times as great. Therefore the areas of spot disturbances are often a hundred times that of the earth. In about two out of three cases a spot disturbance consists of two principal umbrae, one east of the other, as well as a retinue of smaller spots. According to F. W. Dyson and E. W. Maunder, the umbra which is in advance is usually the more regular in shape, and in the early life of the disturbance is likely to move forward with respect to the remainder of the group. Recent studies by O. A. Akesson indicate that the drifts of sun spots in direction and in amount are somewhat more irregular than has often been supposed to be the case. He finds that their average daily motion in longitude with respect to the surrounding photosphere is about 9000 kilometers, and in latitude, 5000 kilometers. That is, they move with a speed of several kilometers per minute.

285. Development and Duration of Sun Spots. According to the observations of A. Secchi made many years ago, the first evidence of the approaching development of a sun spot is the formation of conspicuous faculae interspersed with small, dark points. In the course of a few hours or days the dark points expand and coalesce into one or more umbrae whose accompanying penumbrae are formed from elongations of the granules of the neighboring photosphere. Then the disturbance ordinarily becomes elongated in solar longitude with two principal umbrae.

Sun spots vary greatly in duration, ranging from a few hours up to several months, and even up to 18 months in the case of the extraordinary disturbance of 1840–1841. The numerous small spots ordinarily last only a day or two; the large spots persist longer, but fewer than 10 per cent of the thousands which have been observed have endured long enough to reappear after having been carried to the invisible side of the sun by its rotation. Since the period of rotation of the sun is about 25 days, only a small fraction of sun spots remain visible for as much as 20 days, though in the past three centuries more than 100 spots have been followed for 60 days or longer.

Normally, sun spots develop rapidly and decline slowly. When the disturbance which has caused a group of spots has about run its course, the penumbral filaments gradually invade the umbra and the surrounding photosphere eventually covers all. There remain, however, brilliant and changing faculae strewn about the region, evidence that there are still titanic forces acting beneath the surface on which masses of world dimensions were lately tossed about in wild confusion. In some cases new sun spots develop where earlier ones have disappeared.

286. Distribution and Periodicity of Sun Spots. Instead of being distributed with approximate uniformity over the surface of the sun, spots rarely appear outside of two zones between latitudes 6° and 40° on each side of the solar equator. Moreover, sun spots are not distributed uniformly in time, but occur in approximate cycles, averaging about 11.13 years in length. For two or three years spots appear in exceptional numbers, after which they gradually decline in frequency to a minimum for three or four years, and then they increase in numbers again. Although the average length of the cycle is a little more than 11 years, the intervals between maxima range from about 7 years to 16 years. The numbers of spots at maxima and minima are subject to equal irregularities that do not appear to have any relationship with the variations in the lengths of the cycles.

After systematically recording his observations of sun spots for nearly 20 years, S. H. Schwabe, in 1843, discovered their approximate periodicity. The observations of nearly 100 years since his discovery have naturally added to our knowledge of the cycles in which they occur in largest numbers. The investigations of

C. Wolf, A. Schuster, D. Alter, and many others have shown the probable existence of a number of lesser cycles superimposed on the principal cycle of about 11.13 years. There is apparently a cycle of approximately 5.6 years whose amplitude is about one-fifth that of the principal cycle; and there is apparently another cycle of approximately 82 years whose amplitude is about two-

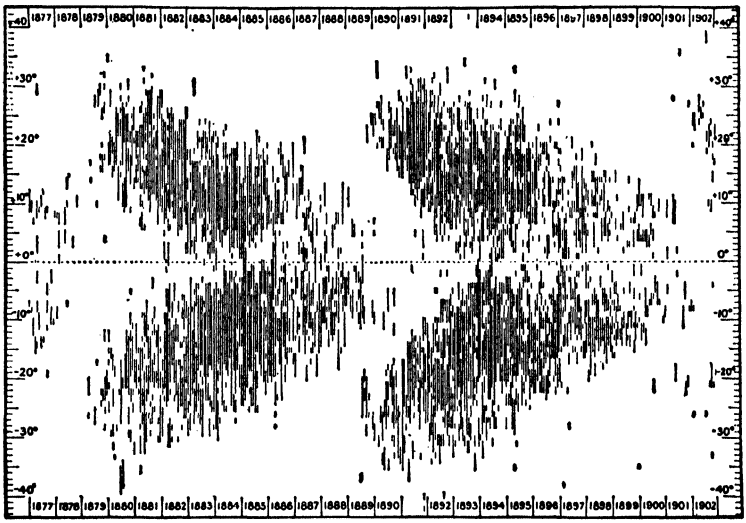


FIG. 151.—Distribution and magnitudes of sun spots. (Maunder.)

thirds that of the 11-year cycle. This long-period term, compounding with the 11-year cycle, accounts for the fact that during various intervals of 30 or 40 years in the last three centuries spots have been relatively few, with intervening intervals in which they have been relatively numerous. There are also indications of possible other periods, but the problem is so complex that observations must be continued over many more years before we shall be sure that sun spots appear in a number of definite cycles and shall know the periods and the amplitudes of these cycles.

There is a remarkable relationship between the phase of the sun-spot cycle and the distribution of spots in solar latitude which was first noted by G. Spoerer. When spots appear in maximum number they are almost entirely limited to latitudes $10°-20°$ on each side of the sun's equator. As the numbers of spots decline from a maximum, they occur year after year in successively lower

latitudes. After about 6 years on the average, a few spots are still found in latitudes $\pm 6°$; at the same time the first stages of a new cycle appear in latitudes about $\pm 35°$. Presently, the old cycle disappears entirely and the new cycle descends in latitude with increasing numbers of spots.

The distribution of spots for the interval from 1876 to 1902 is shown by Maunder's diagram in Fig. 151. This remarkable diagram represents the characteristics of phenomena which will undoubtedly be found sometime to be of deep significance, but at present we can only wonder what it may mean except for the fact that spots, instead of being simply isolated phenomena, are expressions of general dynamical properties of the sun.

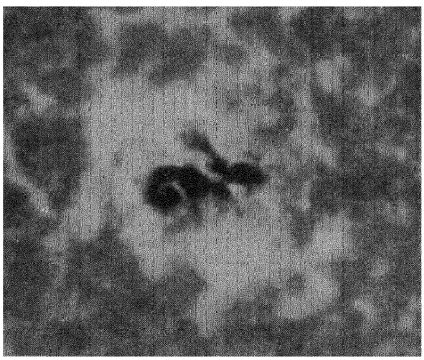


FIG. 152.—Spiral spot. (Yerkes Observatory.)

During the past 50 years sun-spot activity south of the sun's equator has been about 30 per cent greater than that north of the equator, and the less perfect records of earlier years indicate that the same conditions have generally prevailed since the days of Galileo. It will be recalled that the southern hemispheres of Jupiter and Saturn have also been more largely disturbed than their northern.

287. Motions in Sun Spots. The general characteristics of sun spots—their huge dimensions, their cyclic appearance, their distribution in zones, their drifts with respect to photosphere—imply that these remarkable phenomena are produced by, or at least are accompanied by, motion on an enormous scale.

In some cases the fact that there are great spiral whirls in sun spots is directly evident (Fig. 152). In most cases, however, the superficial appearances do not indicate such gyrations; yet the fact that sun spots are huge vortices has been established by a fine example of the combination of splendid observational technique and physical theory. The history of the subject began in 1896 when P. Zeeman found by laboratory experiments that if

light moves in and *parallel* to a powerful magnetic field, each of its spectral lines is normally divided into two components which are circularly polarized in opposite directions; and that if light moves in and at *right angles* to a powerful magnetic field, each of its spectral lines is normally divided into three components, the outer two of which are plane polarized at right angles to the magnetic field, and the central one of which is plane polarized parallel to the magnetic field. The effects of a magnetic field are the same on an absorption as on an emission spectrum.

In 1906, G. E. Hale found that when a spot is near the center of the sun's disk, its spectral lines are doubled and circularly polarized in opposite directions. He interpreted this as meaning that the spot is in a powerful magnetic field acting in the line of sight; that is, in a line perpendicular to the surface of the sun. He also found that when a spot is near the sun's limb, the spectral lines are tripled and plane polarized, the outer two at right angles to the central one. He interpreted this as meaning that the spot in this case is in a magnetic field acting in a plane perpendicular to the line of sight; or, since the spot is near the sun's limb, also in a line perpendicular to the surface of the sun.

Hale next took up the problem of explaining the magnetic fields involving sun spots and having their lines of force roughly perpendicular to the surface of the sun. He recalled that H. S. Rowland had proved in 1876 that static electric charges in motion produce perpendicular magnetic fields analogous to those produced by electric currents. Hence he drew the conclusion that the magnetic fields associated with sun spots are produced by the circular motion of electric charges in planes parallel to the surface of the sun. It is probable that these electric charges are ionized atoms, a part of huge cyclonic motions which always involve sun spots.

The work of Hale and his associates at the Mount Wilson Observatory did not stop with proving that sun spots involve vast vortical motions. They found the important fact that the polarities of neighboring spots are nearly always opposite, implying that their gyratory motions are in opposite directions (Fig. 153). This fact suggests that neighboring sun spots are usually connected by a huge vortex cylinder, the spots themselves being merely the surface ends out of which the rotating material rises. It is clear that this explanation implies opposite rotations at the ends of the tubes, just as a rotating hose turns in the right-hand direction for

a person holding one of its ends and in the left-hand direction for a person holding its other end. Moreover, Hale and his associates found that if the leading members of pairs of sun spots in the northern hemisphere of the sun have one polarity, then the leading members of pairs in the southern hemisphere have the opposite polarity. Such facts suggest the conclusion that there are under-surface connections between the sun's northern and southern

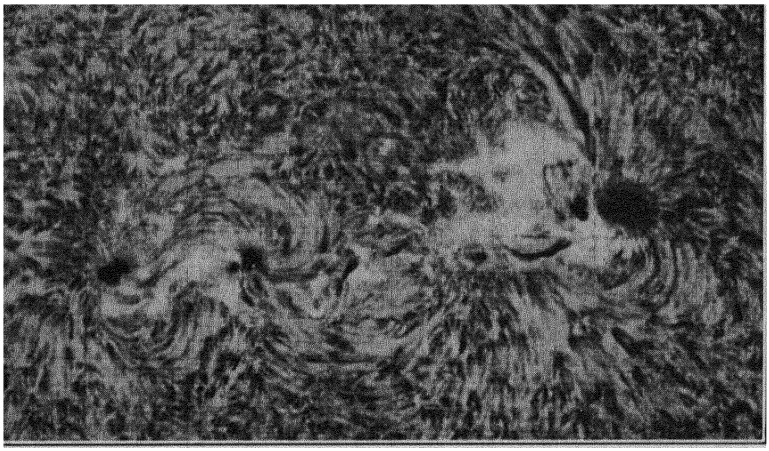


FIG. 153.—Polarities of sun spots. (Mount Wilson Observatory.)

hemispheres, perhaps due to a rotation of the sun in cylindrical layers about its axis. And when one sun-spot cycle disappears and a new one begins in higher latitudes, the polarities in the northern and southern hemispheres are interchanged. Therefore the complete cycle of polarities is two sun-spot cycles, or about 22.2 years. It is not unlikely that this is the fundamental solar cycle.

Beginning in 1909, J. Evershed made an exhaustive investigation of radial velocities, by the Doppler-Fizeau principle, of material in and around sun spots. He found that gases appear to be flowing out of sun spots and spreading outward from them in all directions. The velocities are usually of the order of one kilometer per second. Consequently it appears to be a safe inference from the various lines of evidence that spots are the exterior funnels, as it were, through which gases rise out of rotating cylinders and spread out on the sun's surface. As the gases ascend, they cool with expansion. Direct proof of the cooling is the fact that the spectral lines from

the umbrae of sun spots are related to the corresponding lines from the photosphere as those of low-temperature electric furnaces are related to those of very high-temperature arcs. Moreover, there are absorption bands and flutings in the spot spectra due to compounds which can not exist under the high temperatures of the photosphere. It is probable that the umbrae of sun spots are 2000° C. cooler than the normal photosphere, or 4000° absolute, but the total radiation of a spot and the surrounding disturbed region is probably fully up to the average of the surface of the sun as a consequence of the excessive brilliancy of neighboring faculae and prominences.

288. The Question of the Cause of Sun Spots. Many suggestions have been made respecting the cause of sun spots, but none has been worked out satisfactorily in quantitative detail or has been generally accepted. The fact that the principal sun-spot period is nearly equal to that of the revolution of Jupiter about the sun has often given rise to the vague hypothesis that these solar disturbances are in some way due to the great planet. At once the question arises as to how such prodigious and violent storms can be produced by what is after all a relatively insignificant world which is so far away that as seen from the sun it would appear to be only a point in the sky.

Since the moon and the sun raise important tides on the earth, it is natural to suspect that sun spots may be due to tides raised in the sun by the planets. This explanation would account for the approximate equality of the sun-spot period and that of Jupiter's revolution, though it should be insisted that the length of the sun-spot cycle is *not* equal to the year of Jupiter, but nearly 9 months shorter. When this theory is examined quantitatively, however, it completely fails, for all the planets combined do not raise a tide in the sun more than a meter in height; and it is quite incredible that such insignificant disturbing forces should produce results expressible in terms of thousands and tens of thousands of kilometers. A related theory is that the planets tend to shift the slight equatorial bulge of the sun which is probably produced by its rotation.

Many astronomers think it likely that sun spots are not due to exterior forces, but that they are superficial manifestations of the fundamental dynamics of the sun. If this view is correct, sun

spots are probably minor, though spectacular, phenomena resulting from the relatively rapid rotation of the sun's equatorial zone. Such an explanation appears to offer prospects of finally coördinating the characteristics of sun spots themselves, the shifting of the sun-spot zones, and the differential rotation of the sun's equatorial zone. At present, however, the question of the cause of sun spots has no certain answer.

289. The Chromosphere. At first thought it seems reasonable that the sun should have a vast and tenuous atmosphere like the deep photospheric envelope which surrounds it. A difficulty arises, however, when we take into account the fact that the surface gravity of the sun is nearly 28 times that of the earth, for the rate at which the density of a compressible gas varies is directly proportional to the gravity to which it is subject and inversely proportional to its absolute temperature. Since the absolute temperature of the chromosphere (about 5000°) is less than 20 times that of the earth's atmosphere (273°±) the density gradient of the chromosphere according to the unmodified theory of gases should be greater than that of the earth's atmosphere; that is, it should double with a descent of about 5 kilometers, quadruple at 10 kilometers, etc. Hence its density where it is in contact with the reversing layer should be millions of times greater than at 1000 kilometers above. But the density of the chromosphere does not vary greatly with depth.

In explaining the slowly varying density of the chromosphere with altitude, astronomers have made an important contribution to the physics of radiating stars. Obviously some outward force approximately balances the inward force of the sun's gravity. Such an outward force has been found in the pressure of the radiant energy which the sun emits. Now the pressure that radiant energy exerts upon the atoms of a gas is directly proportional to the energy the gas absorbs, and the amount of energy a gas absorbs depends upon whether or not its absorption lines are in a part of the spectrum in which radiation is abundant. The wavelengths of absorption lines depend upon the nature of the gas, its temperature, and the degree of its ionization. It is obvious from the various factors involved that the theory of the constitution of the chromosphere is one of great complexity. Yet Milne's investigations appear to coördinate all the varied phenomena presented

in the spectrum of the chromosphere and explain why iron is found only in the lower levels, sodium to a height of about 1500 kilometers, neutral calcium to a height of 5000 kilometers, ionized calcium to a height of 12,000 kilometers and even to the tops of the prominences, and helium to the upper limits of the chromosphere, as is shown in the flash spectrum (Art. 279).

It is an interesting fact that helium was first discovered (by William Jenner, in 1868) in the chromosphere of the sun (whence its name from *helios* = sun) from its bright yellow line in the solar spectrum near the D lines of sodium at the time of the eclipses. More than 20 years later, Hillebrand (1889) and Ramsay (1895), finding the spectral line of helium in the mineral clevite, isolated the element. Next to hydrogen, helium is the lightest element, having only two protons in its nucleus. It enters into no known chemical combinations with other elements, it has the lowest known refractive index for a given density, it is an excellent conductor of electricity, it is not soluble in water, it is liquefied with the greatest difficulty, and it is one of the products of the disintegration of uranium, radium, and other radioactive elements. Formerly helium was a rare element; now it is obtained in large quantities from some of the natural gas wells in the southwestern part of the United States, and its principal use is in the inflation of dirigible balloons.

290. Solar Prominences. When the sun is totally eclipsed, scarlet projections, called *prominences*, are seen here and there scattered along its margin and extending up from its surface to heights of tens of thousands of kilometers. Occasionally 15 or 20 of them are simultaneously visible. The prominences owe their brilliancy and red color largely to the intense spectral lines of the light from hydrogen gas. This fact suggested to J. Janssen and N. Lockyer, in 1868, a method of observing prominences at any time instead of only during total solar eclipses.

Prominences can not be seen through telescopes, except during eclipses, because the brilliant illumination of the sky (the earth's atmosphere) in the immediate neighborhood of the sun makes them quite invisible, just as stars are not visible in daylight. If, however, the light from the sun's limb is passed through a spectroscope, that which comes from the earth's atmosphere, consisting of all colors, is spread out into a spectrum and is correspondingly

enfeebled; on the contrary, that which comes from the prominences, consisting of a few pure colors, is not diminished so much in intensity. Consequently, when the dispersion is considerable, the atmospheric illumination is spread out and reduced until the prominences stand out conspicuously in contrast. The method introduced by Janssen and Lockyer has been extended and used



FIG. 154.—Solar prominence, height over 150,000 kilometers. (John A. Miller.)

for more than 80 years in investigating these remarkable solar protuberances.

Prominences are of two quite different kinds, the eruptive and the quiescent. The former occur only in the sun-spot zones and particularly in the neighborhood of active spots; the latter are found irregularly over the surface of the sun, even in high latitudes.

Eruptive prominences. As a rule, eruptive prominences originate from faculae in the vicinity of the more active sun spots. With the suddenness of an explosion, vast masses of flaming hydrogen, helium, and calcium, interspersed with gaseous iron, magnesium, sodium, barium, titanium, and other metals, are projected up from the photosphere to altitudes ranging from a few thousand to a million kilometers. Occasionally they rise in the form of huge arches whose ends reach points on the surface of the sun separated by hundreds of thousands of kilometers. In some cases the changes in position and form are so rapid that they are conspicuous even in a few minutes, as is illustrated in Fig. 155. Since at the distance of the sun from the earth 40,000 kilometers subtends an angle of less than one minute of arc, it follows from these obvious displacements that the velocities are sometimes

enormous. Direct measurements of movements in prominences projecting from the limb of the sun, as well as measurements of the Doppler-Fizeau displacements of their spectral lines, prove that these flaming gases move with velocities often as great as 100 kilometers per second and sometimes as high as 250 or 300 kilometers per second.

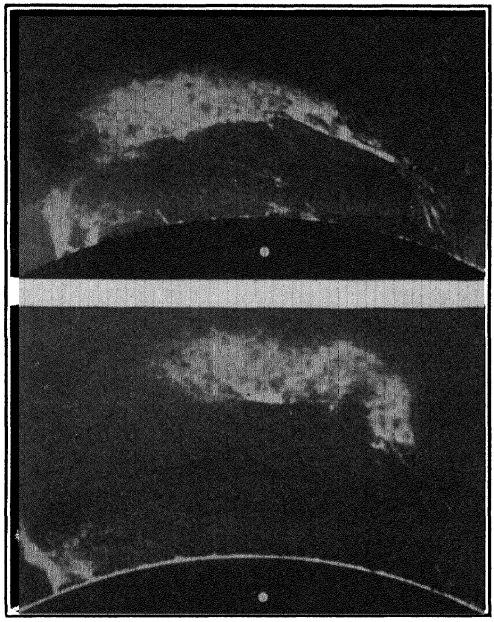


FIG. 155.—Great Eruptive Prominence of May 29, 1919. The small white circles near the lower margin of the photographs represent the earth. In the upper photograph the prominence extended up to a height of 210,000 kilometers; when the lower one was taken, $2^h\ 36^m$ later, it had attained an altitude of 320,000 kilometers. (The Yerkes Observatory.)

Astronomy presents so many large numbers that we are likely to become insensible to their real meaning. Let us inquire for a moment what the significance is of a velocity of even 100 kilometers per second. If we compare the eruptions in prominences with projectiles shot from guns, we find that the velocities of the former are 100 times greater than those of the latter. It is, however, the kinetic energy ($^1/_2mv^2$) rather than the velocity that is really significant, for it measures the work that has been done on a mass and the height to which it will ascend against gravity. In bodies having velocities in the ratio of 100 to 1, their kinetic energies per unit mass are in the ratio of $100^2 = 10{,}000$ to 1. That is, the work done per unit mass in giving the prominences their amazing velocities is 10,000 greater than that done by powder in propelling a projectile from the confined chamber of a high-pressure gun. It is altogether improbable that the eruptive prominences are due to explosions in the ordinary sense of the word, for none of the necessary conditions

for such an action appears to be present. It is much more likely that for some reason at the time of an eruption these gases absorb radiant energy to an unusual degree and are consequently driven out from the surface of the sun; the fact that the velocities sometimes increase as the matter ascends supports the hypothesis. There remains, however, much work to be done before even this theory can be regarded as very probable.

A mass moving at the rate of 100 kilometers per second has enough kinetic energy to change it from a solid to a gaseous state and even to break up its molecules into atoms. In the case of the eruptive prominences the material is not in the solid state nor is its kinetic energy changed to some other form by collision. Doubtless in many cases the forces which drive the materials up from the surface of the sun gradually diminish and the sun's gravity resumes its control. It is not unlikely that in others the ejected materials reach the exterior portions of the sun's atmosphere with a speed sufficient to cause them to recede to the distances of the planets and possibly to those of the stars. For such an escape from the sun a speed of about 600 kilometers per second is necessary, a speed which is not greatly beyond the velocities occasionally observed. If such high-speed ejections and escapes do occur, the matter involved may become the raw materials out of which meteors and comets develop (Art. 266).

Quiescent prominences. Not all solar prominences are accompanied by high velocities and evidences of violent forces. Those which are called quiescent spread out lazily far above the sun's surface, usually with a stem here and there reaching down to the photosphere below. Somewhat like terrestrial clouds, only tens of thousands of kilometers in height and diameter, they apparently float for days, notwithstanding the sun's gravity, with little change in form or position. It is probable, however, that they are not actually floating, but that they are held aloft by radiation pressure. The spectroscope shows that they are composed of hydrogen, helium, and calcium.

Quiescent prominences are not associated with sun spots or limited to the sun-spot zones, for they are found all the way from the sun's equator to its poles. They are, however, connected with the general dynamics of the sun, as is proved by the fact that their number varies somewhat with the number of sun spots; and they are most numerous in zones at latitudes 25° and about 40° on each

side of the solar equation, the latitude of the latter zone gradually increasing as the sun-spot cycle advances.

291. The Spectroheliograph. In 1890, G. E. Hale and H. Deslandres independently invented a remarkable instrument, the spectroheliograph, which has been of enormous value in investigating the surface of the sun. By its use an astronomer can photograph the sun with light of a single wave-length, and hence with light from a single element. Such a photograph of the surface of the sun shows the distribution of the element whose light was used. In fact, as will be explained, it also provides a means of learning something of the distribution of gases in altitude as well as over the sun's surface.

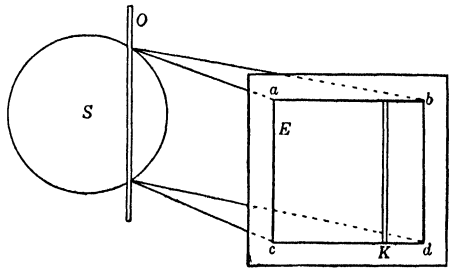


FIG. 156.—Principle of the spectroheliograph.

In order to illustrate the use of the spectroheliograph concretely, consider the K spectral line of the element calcium which is produced in the reversing layer by an abundance of calcium gas. But high above the reversing layer there is often brilliantly luminous calcium at much lower density than that in the reversing layer. The radiation from this higher calcium produces a bright spectral line in the center of the dark absorption line, a reversed line as it is called. The spectroheliograph enables the astronomer to photograph the sun with the light from the central part of the reversed line, and hence with light from the elevated calcium gas from which it was radiated. Even if a line is not reversed it may be used, for it is not actually dark but only less brilliant than the photosphere.

The essential elements of the spectroheliograph are as follows. Let S in Fig. 156 represent the image of the sun formed in the plane of the slit O of the spectroscope. On referring to Fig. 142, it will be seen that the lens L_1 picks up the rays of light diverging from O and passes them in parallel lines into the first of one or more prisms of the spectroscope. A second lens then forms a separate image in each color of the slit on the plane E.

In Fig. 156, the image of O is represented by the spectral band

abcd on the plane *E*. Suppose the image of *O* in the K-light falls at *K*, and that a screen is interposed in the plane *E* with a slit at *K*. If a photographic plate is placed directly back of the plane *E*, it will receive only the K-light which passes through the slit *K*. Suppose now that the slit *O* passes steadily across the image of the sun's disk, and that at the same time the photographic plate is moved at a corresponding rate at right angles to the slit

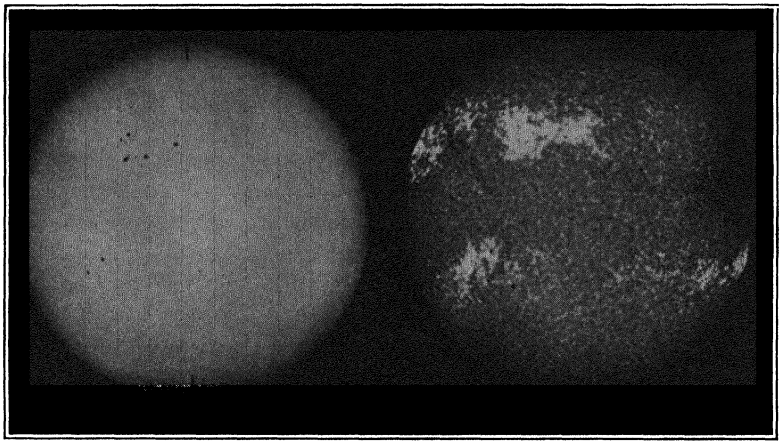


FIG. 157.—Direct photograph on left; spectroheliogram in calcium (K) light on right. (Mount Wilson Observatory, January 11, 1926.)

K. It is readily seen that as the slit *O* traverses successively different parts of the image of the sun's disk, the K-line image of the slit falls on successively different parts of the plate, and that a photograph of the sun is obtained with the K-line alone.

The photograph reproduced in Fig. 157 is an illustration of the results that are obtained by the methods which have just been described. The sun's surface is covered with brilliant calcium clouds, called *flocculi* by Hale.

As has been stated, the central parts of the reversed K-lines are believed to be due to calcium at very high levels. Calcium which lies lower where the pressure and the density are greater has broader lines. Consequently a photograph showing the distribution of these lower-lying gases can be obtained by setting the slit *K*, Fig. 156, slightly to one side of the exact center of the line. The application of the spectroheliograph and the interpretation of the rich results it furnishes are yet in their infancy.

Obviously, the spectroheliograph can be used with any solar lines whatever, provided they are bright enough to give a photograph in a reasonable length of exposure. The results obtained with light from hydrogen and from iron have been especially successful and interesting. Hydrogen often appears as dark absorbing clouds of enormous extent, as shown in Fig. 158.

292. The Corona. Visual observations of the sun's corona, and photograph of it as well, are limited to the brief intervals of total

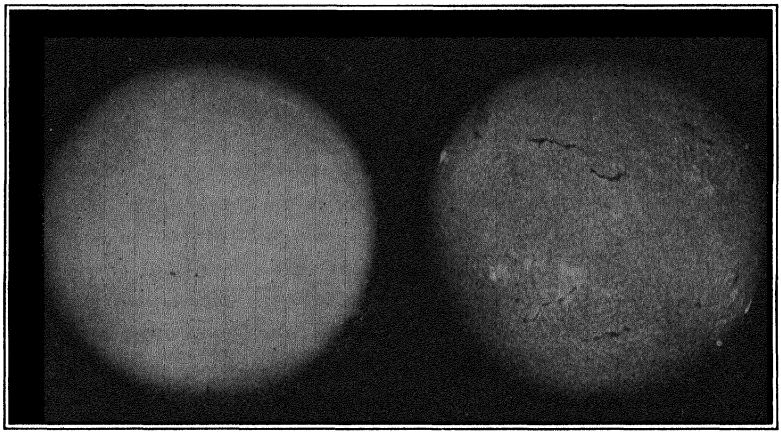


FIG. 158.—Direct photograph on left; spectroheliogram in hydrogen (Hα) light on right. (Mount Wilson Observatory, June 16, 1926.)

solar eclipses, which together have amounted to less than an hour since photography was first used in investigating the sun. The reason that the corona can not be seen at any time is that it is fainter than the illumination of the sky through which it must be observed. If it were shining with strong monochromatic light, it could be observed, as are the prominences, by spreading out with a spectroscope the white light of the earth's atmosphere and reducing its intensity relative to that of the corona.

The corona is not arranged in concentric layers around the sun, like an atmosphere, but is composed of long streamers extending out radially to distances of a million kilometers or more, particularly in the plane of the sun's equator (Fig. 159). These streamers are often curved and sometimes doubly curved; in the neighborhood of the sun's poles they spread out as though they were following lines of magnetic forces.

In general, well-defined coronal streamers extend out from regions of active eruptive prominences, though apparent exceptions have been noted. They are at least associated with sun spots or the sun-spot cycle, for when spots are at their maximum activity coronal streamers originate in all latitudes from the sun's equator to well beyond the limits of the sun-spot zones; but as the sun-

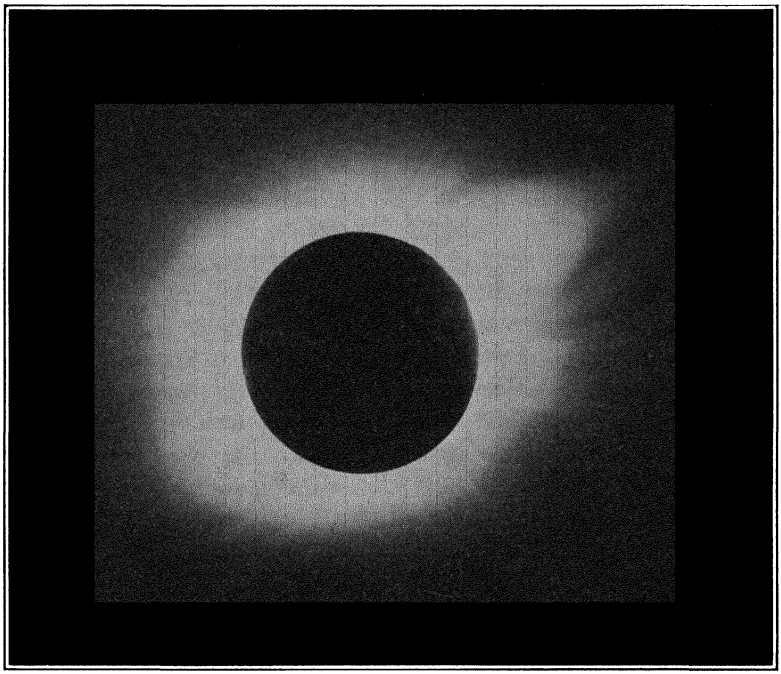


FIG. 159.—The sun's corona, Sept. 10, 1923. (John A. Miller.)

spot activity declines, the long coronal streamers follow equatorward and terminate at the sun-spot minimum largely in the plane of the sun's equator.

Near the sun the corona is pale yellow in color, contrasting sharply with the scarlet prominences; farther out from the sun, it is pearly white. Measurements made during recent eclipses show that the corona radiates about 1/1,000,000 as much light as the sun; the total light it reflects and radiates to the earth is approximately one-half that reflected to the earth by the moon. Consequently the sky is far from dark during a total eclipse.

It is found by means of the spectroscope that the corona emits

three kinds of light. The first is known to be reflected sunlight, for its spectrum shows faintly the Fraunhofer absorption lines; and it is polarized, indicating that it is reflected. The second is white light, producing a continuous unpolarized spectrum. The third is a bright-line spectrum of about 30 lines which have not been observed in physical laboratories. Formerly it was often supposed that these lines are due to an unknown element; now it is thought more likely that they are unfamiliar lines of known elements that appear only under conditions of tenuity and volume which can not be reproduced in a laboratory.

From all this evidence—the extent and shape of the corona, its relation to sun spots, the character of the light it radiates—it appears probable that this interesting appendage of the sun is made up of gases whose ionized molecules are driven out radially by the pressure of radiant energy, and possibly by other forces, but measurements indicate comparatively low radial velocities. Apparently there are magnetic or electric forces that curve the particles in the streamers from the straight lines which they would otherwise take. Whether these particles are ultimately held by the gravitation of the sun or stream out into the interplanetary and interstellar regions, we do not now know. In view of the probable nature of the earth's aurorae and their relationship with solar storms, as will be mentioned in the next article, it seems not at all improbable that at least a part of the coronal materials completely escape from the sun, and that what we observe is the paths along which they depart.

293. The Eleven-Year Cycle. Sun spots not only vary in numbers in cycles of about 11.13 years, but several other phenomena are cyclical in the same period. They even follow the sun-spot curves, having high maxima or low minima, or unusually long or short intervals between corresponding phases, in almost exact harmony with the spots themselves. There can be no doubt that all these phenomena are intimately related.

As would be expected, the faculae undergo cyclic variations in unison with sun spots; for, although they are not limited to the sun-spot zones, they are forerunners of the development of spots, they surround spots during the existence of these storms, and they generally remain for a time as conspicuous evidence of the places at which spots have disappeared.

Eruptive prominences similarly are associated with sun spots, and they undergo corresponding cyclic variations both in position of greatest numbers and also in extent and violence. The quiescent prominences have a less conspicuous relationship with spots, as was remarked in Art. 290.

Since the sun's corona also undergoes changes in form and extent in the sun-spot cycle, all major solar phenomena vary in unison. It is probable that the underlying cause is, or finds its principal expression in, the relatively rapid rotation of the sun's equatorial zone. And since the polarities of spots complete a cycle in 22.26 years, this may be the fundamental period.

The supremacy of the sun in the solar system is illustrated by the fact that the throbs of its mighty phenomena find echoes in the earth and probably in the other planets as well. When the sun-spot forces rage, the components of the earth's magnetism are agitated; when the sun's rotation carries storm areas across the solar disk, the earth is often visited by brilliant auroral displays. From an exhaustive examination of the observational data, Maunder has concluded that the solar disturbances which produce magnetic storms on the earth are limited in area and propagate their effects radially from the sun. Only when the rotating sun causes these cones of radiation, perhaps of ionized atoms, to sweep across the earth are the terrestrial disturbances produced. As a consequence of the relationship among solar phenomena, these sources of disturbance are usually associated with sun spots.

Many investigations have been undertaken for the purpose of discovering whether other terrestrial phenomena, such as the weather and the quality of radio reception, vary in the sun-spot cycle. A number of investigators have found some evidence of such relationships, with mean temperatures slightly below normal when sun spots are most numerous and radio reception best when sun spots are least numerous. Although it is not improbable that many other such relationships exist to a slight extent, longer series of observations will be required to establish them.

In view of the fact that solar phenomena are completely different in character and scale from those with which we are familiar on our little planet, astronomers have obtained an astonishing amount of information respecting the sun. The problems that they have investigated relate to enormous masses, excessive temperatures, stupendous forces, the fundamentals of the structure of atoms,

and the nature of radiant energy. Though the steps we have taken in the solution of these problems are in many instances only preliminary, they have led us into a marvelous world of which our predecessors did not dream.

XX. QUESTIONS

1. How does the volume of the sun compare with that of all the planets?
2. What is the linear velocity of rotation of the sun at its equator? Centrifugal force is proportional to v^2/R, where R is the radius of rotation. How does the centrifugal force at the equator of the sun compare with that at the equator of the earth?
3. How many light waves of yellow light in an angstrom?
4. Eighty calories are required to melt a gram of ice. If the sun's energy were received uniformly over the earth and were all used to melt ice, how thick a layer would it melt per year?
5. Compute by means of Stefan's law (Art. 274) the average theoretical temperature of the earth from the amount of energy received from the sun and radiated again by the earth.
6. What would be the wave-length of maximum radiation by the earth, as determined by Wien's law? What relationship has it to visible light?
7. How much energy per gram does the sun radiate in a year?
8. If the reversing layer is 1000 kilometers thick, how long does the flash spectrum last?
9. Show that if the sun rotates in cylindrical layers with connection between its northern and southern hemispheres, the polarities in the two hemispheres would be opposite.
10. What is the interpretation of each of the three kinds of light emitted by the solar corona?

REFERENCE BOOKS

Eclipses of the Sun (1922). S. A. Mitchell, Columbia University Press, New York, N. Y.

The Sun (1929). C. G. Abbot. D. Appleton & Company, New York, N. Y.

CHAPTER XIV

EVOLUTION OF THE SOLAR SYSTEM

294. Evolution. Before we leave the solar system and turn our attention to the great problems presented by millions of other suns in the galaxy, let us consider the question of the origin and the evolution of the planets. Let us from the summit of our present scientific knowledge look far back and attempt to reconstruct the circumstances of the birth of these bodies; let us follow in outline the long course of their development; and let us look forward to their probable ultimate destruction. In this undertaking we shall not only unify our information respecting the members of the solar family, but we shall throw on the canvas a picture of such heroic proportions that it could be painted only with the materials which modern science places at our command.

The great events attending the birth of the planets, as well as the lesser ones of their subsequent development, are steps in an evolution which we shall attempt to trace from well-established scientific data and principles. As we use the word *evolution*, we recall how often it has aroused opposition, particularly in the minds of those who have unhesitatingly accepted the doctrine of an abrupt creation of everything out of nothing in the darkness of the completely unknown. When we examine the matter, however, we find that this opposition to evolution is due to misconceptions respecting the meaning of the term.

That the physical and the biological worlds are orderly in space, rather than haphazard and chaotic, no one will deny. Nor will anyone deny that everything is in a state of constant change. The heavenly bodies ceaselessly pursue their courses; mountains are raised up and during the geologic ages are disintegrated by air and water; plants and animals grow, die, and decay; and human institutions—governments, social order, philosophies, and religions—are established, flourish for a time, and eventually are succeeded by others. That all these changes take place continuously and in an orderly manner, rather than discontinuously and chaotically, is the foundation of the general doctrine of evolution.

That is, the essence of the doctrine of evolution is that the universe is orderly in *time* as well as in *space*.

Sometimes an evolution is from the relatively simple to the complex, as when a tree grows from the soil, the air, and water. Sometimes it is from the relatively complex to the simple, as when a tree dies and disintegrates into the elements of which it is composed. Often evolutions in both directions are going on in the same object. Sometimes an evolution is relatively slow, as when rivers wash a continent into the sea. Sometimes it is relatively rapid, as when an eruptive volcano hurls millions of tons of ashes above the clouds. But whether the changes are from the simple to the complex or from the complex to the simple, whether they are slow or rapid, they are orderly in time. Orderliness in time is the essential element in evolution.

295. Nature of Our Observational Data. When a biologist develops a theory of evolution of a species of plants or animals, he bases his conclusions in part upon sequences of fossil forms which he finds preserved in the rocks. He also bases them in part upon comparisons of the organisms in which he is interested with those of numerous other species, and he often checks his hypotheses by experiments.

In astronomy there are no fossil remains of planets, nor are there any other planetary systems within reach of our instruments which may be used for comparison; and obviously no experiments can be made on planetary origins or development. Hence at first thought the problem of discovering the mode of evolution of the planets seems quite insoluble. The laws of astronomy, however, are very simple in comparison with those of the biological world. For example, Newton's three simple laws describe very precisely the fundamental characteristics of the motions of material bodies; but the biologist can describe only in vague terms what takes place when a plant or an animal grows, or reproduces its kind, or dies. It is the relative simplicity of the changes in the inorganic world that gives us courage to attempt to look back across hundreds of millions of years to the birth of the planets.

We have been able to determine from our observations the mass of the solar system and the distribution of its mass among its various members. These data are particularly important, because, as we shall see, neither the total mass nor its distribution can have

greatly changed during the existence of the planets. We know, too, that the geometrical arrangement of the planetary orbits has remained substantially fixed. There are, in addition, certain dynamical properties of the system that are fully as important for our purposes as the distribution of its mass and the geometrical characteristics of the planetary orbits. For example, the sum of the kinetic and the potential energies of rotation and of revolution of the planets remains constant. Likewise, the total moment of momentum (Art. 100) of the system is invariable. It is evident that these changeless properties are of vast importance in our problem, because a theory of the origin of the planets to be satisfactory must start with an original distribution of mass and motions which in respect to these invariants agrees exactly with the present system. For this reason we shall carefully examine these changeless properties, and we shall find that they compensate for the astronomer's lack of fossil remains and of other planetary systems for comparison.

296. Distribution of the Mass of the Solar System. For the purposes of our present inquiry, it is important to know whether from the time of their origin the planets have been as extremely small relative to the sun as they are at present. The sun may be decreasing in mass as a consequence of its radiation (Art. 275), but the planets radiate so little energy that they are not undergoing corresponding losses. Hence, so far as this factor is concerned, the disparity between the mass of the sun and that of the planets is decreasing very slowly with time rather than increasing.

In their journey through the interstellar regions, the members of the solar system are quite possibly picking up scattered materials. Since the smaller a body is the more matter it will encounter in comparison with its own mass, the planets have always grown in this way relatively faster than the sun. Consequently this factor also tends to reduce the inequalities of mass in the solar systems. But before we conclude that from the time of their origin the planets have revolved in their present far-flung orbits, we must raise the question whether there are forces in operation which have gradually brought about their present distribution.

If the sun is radiating away its mass, its attraction for the planets is gradually decreasing and their orbits are gradually

increasing in size. But the fraction of the sun's mass radiated away during the lifetime of the planets, even if they have existed a hundred times as long as seems probable from geological evidence, is so small that the effects on the planetary orbits have not been important. Moreover, the sweeping up of meteoric matter by the sun and the planets increases their mutual attractions and decreases the distances of the planets from the sun. Nor can the mutual interactions of the planets have increased the distances of all of them from the sun and from one another, for such a change would require a decrease of their total energy, whereas it follows from the laws of motion and the law of gravitation that the total orbital energy of the planets remains constant.

A final question to be considered is whether attractions of the stars may not have enlarged the orbits of the planets. If the sun should pass near a star, the orbits of some of the planets might possibly be increased in size, but the orbits of most of them would certainly be made highly eccentric. Moreover, such a close approach would in general cause the planets to revolve in widely different planes. Hence we conclude from the properties of the planetary orbits that no other star has passed near the sun since the planets were formed.

It follows from the foregoing considerations that throughout all the long history of our planetary system the masses of the planets have been insignificant in comparison to that of the sun, and that these relatively minute bodies have been separated by enormous distances from the sun and from one another. We therefore seek a theory of the origin of the planets which implies both their relatively small masses and their widely separated orbits.

297. Significant Geometric Properties of the Planetary Orbits. It is often supposed that the small eccentricities of the planetary orbits are in some way a direct consequence of the law of gravitation. Such is not the case, however, for there is no dynamical reason why the planets should not move in orbits as elongated as those of some of the planetoids or even as those of the comets. Indeed, in the case of certain double stars we find two suns revolving around their center of gravity in such eccentric orbits that at their greatest distance they are more than 20 times as far from each other as they are at their least distance. Consequently

the approximate circularity of the planetary orbits is a property inherited from the mode of origin of the planets.

There remains, of course, the question whether initially eccentric planetary orbits might not have been reduced, in the course of hundreds of millions of years, to their present shapes by the mutual attractions of the planets. Although we are perhaps not justified in stating unequivocally that such a transformation of the orbits of the planets can not have taken place, yet there are very good mathematical reasons for believing that no considerable reductions in eccentricities have been produced. Moreover, many of the planetoids revolve in orbits of considerable eccentricity in the midst of the planetary system. We are therefore forced to reject any theory of the origin of the planets which does not provide for nearly circular orbits from their beginning, or at least since they had acquired substantially their present masses.

Another equally significant property of the planetary orbits is the small mutual inclinations of their planes. There is no reason of a dynamical character why the planets might not revolve around the sun in planes with any inclinations whatever to each other, or even in opposite directions. If the planes of the planetary orbits had been initially highly inclined to one another, there would not now be the close approach to coincidence among them which is found. The considerable inclinations of the planes of the orbits of a number of the planetoids to the planes of the orbits of the planets is concrete evidence that the mutual attractions of the planets would not gradually reduce the planes of their orbits to approximate coincidence. A theory of the origin of the planets will be satisfactory only if it meets the severe conditions of providing planetary orbits having small eccentricities and nearly coincident planes.

298. The Satellite Systems. Six of the nine planets are attended by one or more satellites which revolve around their respective primaries in or near the planes of their equators. The satellite systems of Jupiter and Saturn are in a general way miniatures of the planetary system itself. There is, however, the important difference that the outermost satellite in each of these systems revolves in the direction opposite to that of the rotation of its planet and of the revolution of the remaining satellites.

In the early theories of the origin of the planets, the fact that

the sun rotates in a period shorter than that of revolution of the innermost planet Mercury was thought to be important. In fact, the theory of Laplace (Art. 301) could not have been held if the sun's period of rotation had been longer than Mercury's period of revolution. The corresponding relationship, however, does not always exist in the satellite systems, for the period of revolution of Phobos is less than one-third of the period of rotation of Mars. Similarly, the periods of revolution of the particles which make up the inner portion of Saturn's crape ring are only about one-half the period of rotation of the planet.

In order that a theory of the origin of the planetary system shall be satisfactory, it must not only provide for the planets but also for their satellite systems, including the possibility both of retrograde revolutions and also of periods of revolution shorter than the periods of rotation of their respective primaries.

299. Distribution of the Moment of Momentum of the Solar System. One of the fundamental dynamical properties of an isolated system of bodies is that its total moment of momentum (Art. 100) about any axis remains unchanged, however much the bodies may interact upon one another and however much the configuration of the system may be altered. The fact that the planetary orbits are nearly circular, as well as the fact that their planes are nearly coincident, proves that the solar system has not been subject to important exterior forces, such as the attraction of a closely-approaching star, since the planets were formed. Consequently the moment of momentum of the system at the time of the birth of the planets was equal to that which it has at present.

In a system of widely scattered bodies, such as that of the sun and the planets, the distribution of moment of momentum is of deep significance, for there are limitations on the extent to which it can be transferred from one body to another. For example, the tides that the planets raise in the sun reduce the moment of momentum of the sun's rotation; and the sun, by reciprocal action, adds equal amounts to the moments of momentum of the revolutions of the planets, but the bodies are so far from one another and the tidal forces are so feeble that the effects would not be important in ten thousand million years. Nor can the mutual interactions of the planets have transferred a large amount of moment of momentum from one planet to another, for it follows

from the investigations of Laplace and Lagrange (Art. 183) that the major axes of the planetary orbits undergo only periodic changes, and that their ecccentricities and inclinations vary only within restricted limits. Therefore we conclude that the distribution of the moment of momentum of the solar system has been essentially constant throughout the whole existence of the planets as separate bodies.

In the following table, it has been assumed that the mass of the sun is distributed according to a law of increase of density toward the center which was developed by Laplace. The result derived from it for the sun is certainly approximately correct, though perhaps a little too large. The moments of momentum of the planets are due almost entirely to their revolution about the sun and only to a very slight extent to their rotations. For example, that part of the moment of momentum of the earth which is due to its revolution is more than 3,000,000 times greater than that due to its rotation. Consequently the uncertainties in the distribution of densities of the planets can have no appreciable effects upon the results given in the table.

TABLE XVI. DISTRIBUTION OF MOMENT OF MOMENTUM

BODY	MASS	MOMENT OF MOMENTUM	MOMENT OF MOMENTUM PER UNIT MASS
Sun	0.9986591	0 02742	1
Mercury	0 0000001 (?)	0 00002	6081
Venus	0.0000024 (?)	0 00057	8485
Earth	0.0000031	0 00083	9976
Mars	0 0000003	0 00011	12,260
Jupiter	0.0009535	0.59877	22,760
Saturn	0.0002855	0 24192	30,810
Uranus	0 0000441	0.05285	43,710
Neptune	0.0000516	0 07700	54,700
Pluto	0 0000003 (?)	0 00051	60,700
Total	1.0000000	1.00000	

It appears from the foregoing table that although the sun contains nearly 99.9 per cent of the entire mass of the system, its moment of momentum is less than 3 per cent of that of the planets. Perhaps it is better for later purposes to emphasize the large moment of momentum of the relatively small planets. For example, the mass of Jupiter is less than one-thousandth that of the sun, yet its moment of momentum is more than 20 times greater.

In the case of a rotating body the moment of momentum expresses the quantity of its rotation. Similarly, the moment of momentum of a revolving body expresses the magnitude of its orbital motion. Therefore if the planets are fragments of the sun, as has been generally believed for more than 100 years, then their moments of momentum per unit mass should be comparable to that of the exterior of the sun from which they came, unless they were torn from the sun by exterior agencies.

300. Hypotheses of Wright and Kant Respecting the Origin of the Planets. Naturally, it was not possible for scientists to make any progress in theories of the origin of the planets until after the publication of the laws of motion and the law of gravitation in 1687. Even the astronomers did not attack the problem for more than 100 years, for their abilities were taxed to the utmost in explaining the motions of the planets and the satellites under these laws. But during the interval Thomas Wright, of England, and Immanuel Kant, of Germany, neither of whom was an astronomer, made bold attempts to reconstruct the past history of the solar system and of the stars.

The speculations of Wright, published in 1750, were daring and suggestive. They had the defects, however, to be expected in the work of one who was not a master of mathematical analysis. In 1751, the work of Wright fell into the hands of Kant and fired the imagination of this universal genius, but it had no apparent effect on any one else. Kant at once turned his penetrating mind to the question of the origin and evolution of the solar system, leaving for the future the consideration of the stars which make up the galaxy. Nearly every page of the book that he published on the subject in 1755 gave proof of the brilliant intellect which in his maturer years made him the foremost philosopher of his time.

Kant started with a vast aggregation of simple elements, without any general mass motion and subject only to gravitational and chemical forces and the repulsions of molecules in the gaseous state. He reasoned that chemical affinities and mutual attractions would set up motions among the molecules, that the large molecules would draw to themselves smaller molecules, and that these aggregates would grow and eventually form planets.

Since the system of Kant was assumed to be initially at rest, it had no moment of momentum and it could never acquire any,

for the moment of momentum of an isolated system does not change. Consequently the cloud of stationary particles of Kant could not develop by interior forces into a huge central sun and a far-spreading family of planets, all revolving about it nearly in the same plane and in the same direction, nor could it provide the satellite systems. Naturally, Kant violated the laws of dynamics, for his training was not primarily in the physical sciences. Moreover, many of his arguments were of an *à priori*, rather than of a scientific, nature, particularly in those sections in which he commented on the habitability of other worlds and on the future life. We may well be charitable, however, in respect to these defects, for Kant wrote nearly two centuries ago and his work antedated Darwin's *Origin of Species* by more than 100 years.

Kant regarded his *Natural History of the Universe* as being so radical that he published it anonymously, and in an introduction to it he went to much trouble to explain that his theory of the gradual aggregation of the planets from the materials of an initial chaos was in no way subversive of religion; in fact, he insisted that it was quite the opposite.

301. The Nebular Hypothesis of Laplace. For more than a century after the publication of Newton's *Principia* (1687), mathematicians were absorbed in the difficult problems of proving that the intricate motions of the planets and their satellites can all be explained by the laws of motion and the law of gravitation. Foremost among the triumphant scientific explorers of this period was Pierre Simon Laplace (1749–1827). Toward the close of the eighteenth century, Laplace published his *Mécanique Céleste*, a monumental series of volumes containing detailed expositions of his profound mathematical investigations. After the completion of this great work he wrote a popular book on astronomy, in an appendix of which he briefly sketched some speculations on the origin of the planets and their satellites. Not only did these hypotheses occupy an inconspicuous place in a minor work of Laplace, but they were suggested "with that distrust which everything ought to inspire that is not a result of observation or of calculation."

What a commentary on the fickleness of fame the work of Laplace presents! Within a century after his death all his profound discoveries in celestial mechanics had become at the best

merely a vanishing tradition among laymen and astronomers alike, except for the very few who find delight in those recondite fields. Yet his hesitatingly expressed speculations of a pensive hour had not only gradually come to dominate astronomical opinion respecting the origin of the planets and the evolution of the stars, but had given geologists authority for their belief in an originally liquid earth, and echoes of the theory were being reflected in every level of scientific, philosophic, and theological thought. Perhaps no theory in the history of science has had a more general acceptance or a wider influence than the Nebular Hypothesis of Laplace had at the beginning of the present century.

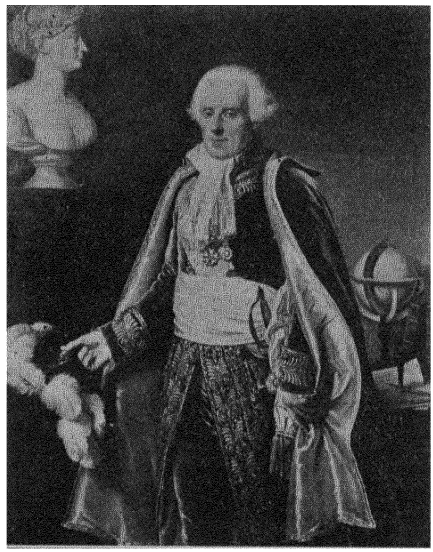


FIG. 160.—Pierre Simon Laplace.

In their speculations on cosmogony, philosophers have generally started with a chaos, though it is not evident why this is a more probable beginning state than one of organization. Laplace began with a sun having an atmosphere extending out beyond the orbit of the most remote planet and in equilibrium under the gravitation of the central mass and its own gaseous expansive forces. He assumed that the entire mass was in slow rotation and that its temperature was enormously high. He concluded that as heat was lost by radiation the mass would shrink in volume, and that as it decreased in volume it would rotate more rapidly so as to keep constant the moment of momentum, a conclusion that was supported by the work of Helmholtz on the contraction of the sun (Art. 275). He assumed without detailed examination that a stage would be reached in which the centrifugal acceleration at the equator of the rotating mass would balance the attraction toward the center; and that at this place a ring of gaseous matter would

be left behind by the continually shrinking central mass. He assumed that the loss of a ring of matter would temporarily restore equilibrium, but that after further contraction a balance between centrifugal and centripetal accelerations would again occur with the abandonment of a second ring, and so on to the present size of the sun. He assumed that a ring would not be perfectly symmetrical, but that, separating at some point, it would unite at some other, because of the mutual attraction of its parts, into a great gaseous ball. Such a globe of gas would eventually shrink into a planet, perhaps leaving off one or more rings during its contraction to condense into satellites.

It is clear that the Laplacian theory explains such fundamental properties of the solar system as (*a*) the approximate coincidence of the planes of the orbits of the planets, (*b*) the revolution of all the planets in the same direction, (*c*) the approximate circularity of the orbits of the planets, and (*d*) the agreement in the direction of the rotation of the sun with that of the revolution of the planets. It provides for initially gaseous, and then liquid, planets, the outermost being the oldest. It was an easy inference from the theory that the surface of the earth is a relatively thin crust covering the primeval fires which have not yet died out in the interior of our planet. Almost within the limits of a paragraph the Laplacian hypothesis outlines great events reaching from the birth of worlds to their final death when the heat of the sun fails. Many scientists have felt that a theory which is so simple and which so stirs the imagination should be true.

Our feelings, however, are unsafe guides in seeking the truth, for they reflect only the limited past experience of the race as epitomized in ourselves rather than the broad universe which we seek to encompass. The Laplacian theory has failed to satisfy both observational evidence and theoretical considerations, though for two generations astronomers clung to it with such modifications as they were forced to accept. In 1900, T. C. Chamberlin and the author exhaustively examined the nebular hypothesis with the hope that it might be shown to be satisfactory, but arrived at the positive conclusion that it would have to be abandoned. In 1918, Jeans arrived at the same conclusion on the same grounds; by 1925 practically all astronomers had finally given up completely the ring theory of the origin of the planets.

It will be sufficient to point out the more obvious of the respects

in which the Laplacian hypothesis fails. (*a*) It does not provide for such orbits as that of Eros, which approaches near the earth and reaches beyond the orbit of Mars. Nor does it provide for such interlacing orbits as those of the planetoids. (*b*) It does not provide for both direct and retrograde satellites, for the direction of rotation of a planet could not change during its period of shrinking. (*c*) It does not provide for satellites revolving in shorter periods than the periods of rotation of their respective planets, examples of which are found in the case of the inner satellite of Mars and in the particles of the inner ring of Saturn, for the period of rotation of a planet progressively becomes shorter as the planet contracts. (*d*) It does not provide a dynamical means by which a ring containing less than one-tenth of one per cent of the mass of the system (the hypothetical ring antecedent of Jupiter) can take with it more than 95 per cent of the moment of momentum of the matter interior to the orbit of Saturn (Table XVI).

There are other fundamental objections to the hypothesis of Laplace. For example, if there were once a balance between the centrifugal and the centripetal accelerations at the equator of the contracting solar nebula, a continuous thin disk of particles would be left behind instead of a succession of isolated rings. Moreover, a gaseous ring would dissipate instead of contract into a planet, for the feeble mutual attractions of its parts would be much less than the forces of gaseous expansion.

If Laplace were alive today, he undoubtedly would recognize the inadequacy of the speculations which he published with distrust because they were not based on observations or calculations. If he had examined his hypothesis more carefully even in his own day, he probably would have discovered its dynamical defects. It is fortunate, however, for the progress of science that he did not, for his simple and concrete theory helped bring about the revolutions in geological, biological, and astronomical thought that took place in the nineteenth century.

302. A Change in Point of View. In the interval from 1900 to 1925, the point of view of astronomers underwent a radical change, as far-reaching in its consequences as was that which followed the demonstrations of Copernicus and Galileo that the sun, rather than the earth, is the center of the solar system. In the previous interval, extending from the publication of Helmholtz's theory of

the sun's heat (1854), until the close of the century, it was universally believed that the nebular hypothesis gives a substantially correct picture of the origin of the planets, and that the amount of heat generated by the contraction of the sun sets a limit of a few million years to their duration. It was believed that all other stars contracted similarly from nebulae in the course of a few millions or tens of millions of years, and that after they once pass their maxima of brilliancy, depending upon their masses, their temperatures and rates of radiation forever decrease. So great are the distances which separate the stars and so brief are the periods of their evolution (according to this view) that normally a star contracts from a nebula to a dead sun without being appreciably influenced by another star. The small, dense groups of stars, as well as the great star clouds of the Milky Way, have no significance in this theory except to suggest that their members all condensed from the same great nebula. By some miracle, they all started at about the same time, and in a few tens of millions of years they all will have run their courses.

Such conceptions as the foregoing are wholly erroneous, as erroneous as were those of the geologists, who, a century or two ago, saw in the strata of the rocks the records of only a few thousands of years. As T. C. Chamberlin showed in 1899, the contraction theory of the sun's energy is entirely inadequate, even for the demands on time made by the earth itself. And the magnitudes of star-clusters and star-clouds and the galaxy, compared to which the whole solar system shrinks to a particle of dust, imply correspondingly greater periods of time for stellar evolution. There was, however, at the beginning of this century no known store of energy adequate to maintain the radiation of stars for millions of millions of years, but the discovery of subatomic energies during its first decade provided such a source. Finally, Einstein's formula for the energy equivalent of mass completed the psychological background that was apparently necessary for so radical a change of point of view as was required.

If the enlarged time-scale of celestial evolution were all there is to the new ideas, it would be important, but it is only a part. The stars are no longer considered to be permanent independent units; they are thought of as constituting great families of mutually dependent members, and, as we shall see, the planets probably are children of two parents instead of one.

303. Outline of the Planetesimal Hypothesis. Immediately after the rejection of the nebular hypothesis in 1900, T. C. Chamberlin and the author undertook to develop a theory of the origin of the planets that would be in harmony with the dynamical principles and the observational data which have been set forth earlier in this chapter. It was not found possible to derive our system of planets, satellites, and planetoids from an isolated nebula or from a star uninfluenced by a second star. In introducing the



FIG. 161.—Solar eruption, Aug. 19, 1928. (Mount Wilson Observatory.)

intervention of a second body as a factor in the birth of the planets, the ideas of Kant and of Laplace and their followers were completely abandoned.

According to the planetesimal hypothesis, a few billions of years ago a star passed by our sun at hyperbolic speed. At its nearest approach it was probably within the limits of the present planetary system. As a consequence of the great mass of the visiting star and the fluid state of the sun, two great tides were raised on the latter, one toward the visiting star and the other in the opposite direction. Moreover, it is assumed that the sun was then subject to violent eruptive forces such as those which now eject matter up from its surface to hundreds of thousands of kilometers (Fig. 161). These eruptions were most numerous and violent in the directions of the tidal bulges A and B, as represented in Fig. 162; moreover, they were somewhat irregular, as they are at present.

If the visiting star S' had been stationary, the material ejected

from A or B would have receded a certain distance from S and then would have fallen back upon the sun; or, if the speed of ejection had been sufficiently great, it would have escaped from S and would never have returned to it. But since S' moved rapidly forward in its orbit, it soon exerted a cross-pull on the material ejected from A; and an analysis of the forces involved shows that it exerted a similar cross-pull on the material ejected from B and in the same direction. Even if the ejections were in other directions, the final results would be much the same. Consequently after S' had passed from the neighborhood of the sun, the materials ejected from both A and B were left moving around S in the orbits of the type illustrated in Fig. 163. The full lines represent the paths described by the ejected masses; the dotted lines represent the elliptical paths in which they were left revolving.

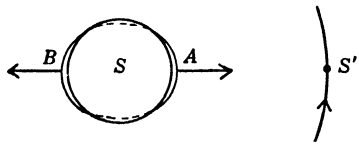
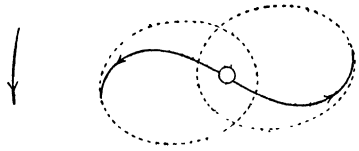


FIG. 162.—Tidal bulges A and B.

In Fig. 163 the orbits of only two ejected masses are shown. Ejections of masses of various dimensions and at various speeds took place, however, during the whole time the passing star was in the neighborhood of the sun. Moreover, the visiting star may have been subject to similar activities. The materials ejected at low velocities fell back upon the sun; those ejected at higher velocities were left revolving in elliptical orbits, as illustrated in Fig. 163; those ejected at very high velocities, if there were such, either passed to the opposite star or were lost to both. Obviously the planes of the orbits of all the ejected masses must have been near the plane of the orbit of the passing star, which is the present plane of the planetary orbits.


FIG. 163.—Paths of masses ejected from the sun.

After the recession of the visiting star, the ejected materials were left revolving around the sun in orbits of various sizes, eccentricities, and orientations. Some of the masses were large and probably in a gaseous or a liquid condition; the great majority, however, were small and hence they speedily cooled to the solid state. Since they revolved around the sun like little planets, they are called *planetesimals*. The larger nuclei grew into planets by sweeping up the planetesimals whose orbits they crossed. Some of

the meteors the earth now encounters may be remnants of the original planetesimals that have survived the hazards of hundreds of millions of years; the craters on the moon may be scars produced by large planetesimals during the youth of our satellite. The planetoids occupy a zone in which no predominant nucleus revolved, and they are possibly representative of conditions in an early state of the system.

304. Confronting the Planetesimal Hypothesis with the Observational Data. In our galaxy there are several thousand millions of stars, and they are moving among one another somewhat irregularly rather than in simple closed orbits like those of planets. It follows from the distances which separate the stars and their average relative speeds that a given star, such as our sun, will pass within planetary distances of another star on the average only once in an interval of the order of 10^{15} years. Although the near approach of two stars is a rare event, yet such partial encounters are certain to occur. The planetesimal hypothesis therefore rests upon an assumption that is quite in harmony with the facts. A similar close approach of our sun to another star in the future would destroy the planetary system, but the infrequency of such events implies that this danger is very small.

(*a*) *Distribution of mass.* The fact that the total mass of the planets is only approximately one-tenth of one per cent of that of the sun is quite in harmony with the theory, for the sun would not eject a large fraction of its substance. The cause of ejection remains somewhat vague, as does that of the present eruptions from the sun. The wide distribution of the planetary masses is also a direct consequence of the theory. There remain, of course, numerous unanswered questions respecting the reasons for the particular distribution of mass among the various planets.

(*b*) *Geometric properties of the planetary orbits.* According to the hypothesis, the eccentricities of the original orbits of the planetesimals must have been considerable, possibly comparable to those of the planetoids today. As the nuclei swept up the smaller planetesimals, the eccentricities of the orbits were steadily decreased; by the time the planets were full grown, their orbits had become nearly circular. It appears to be a significant fact that the larger planets which presumably have grown the more have orbits with the smaller eccentricities. Although it is true that in respect

to the eccentricities of the planetary orbits there is a general agreement between theory and observations, the hypothesis as yet lacks much in quantitative details.

When the passing star had receded to a great distance, the planetesimals were left revolving in planes not greatly inclined to the plane of its orbit—the planes of the orbits of the planetoids perhaps illustrate the situation. As the larger nuclei grew into planets by sweeping up the smaller planetesimals, the planes of their orbits were gradually reduced toward coincidence with the average plane of the ejected material, which was the plane of the orbit of the passing star. It is worthy of note that the planes of the orbits of the larger planets, which presumably have grown the more, are nearly coincident.

(*c*) *Satellites.* It is believed that the satellites have grown up around secondary nuclei which were either associated with their respective primaries from the time they left the sun or, perhaps in certain cases, became entangled with outlying parts of the planetary nuclei and were captured. Although this hypothesis permits retrograde satellites, as well as those which revolve in the forward direction, it at present lacks much of explaining families of several satellites revolving around planets in nearly circular orbits and in the same plane.

(*d*) *Distribution of moment of momentum.* The large moments of momentum of the planets in spite of their relatively small masses, and the small moment of momentum of the great sun are the rock on which all theories break which rely on the rotation of the parent body to separate the planets. This distribution, however, is an immediate consequence of the origin suggested by the planetesimal hypothesis. It is, in fact, one of the principal considerations which originally led to the development of the planetesimal hypothesis.

(*e*) *Rotation of the sun.* It will be recalled that the plane of the sun's equator is inclined to the average plane of the planetary orbits by about 8°. According to the Laplacian theory, the two planes should be coincident; according to the planetesimal hypothesis, the present rotation of the sun is a consequence of its rotation before the visit of the star S' and of the effects of the ejected material which fell back upon its surface, as is illustrated in Fig. 164. Since all this material had some cross-motion in the direction of motion of S', it tended to produce in the sun a rotation in the

direction in which the planets revolve. The materials which fell upon the sun in the plane of the planetary orbits had the greatest effects and hence produced a relatively rapid rotation of the sun's equatorial zone, remains of which still persist. The rapid rotations of the equatorial zones of Jupiter and Saturn are explained in a similar way.

(*f*) *Rotation of the planets.* No previous theory has given a satisfactory explanation of the fact that at least the earth, Mars, Jupiter, and Saturn rotate in the direction in which they revolve around the sun. Some investigators have concluded that if the Laplacian theory were correct, then the planets would necessarily rotate in the retrograde direction. But it can be shown that, according to the planetesimal hypothesis, if a nucleus were revolving around the sun and sweeping up planetesimals describing a great variety of orbits, then the net result of all the collisions would be to give the growing planet a forward rotation. Whether these effects would be sufficient to account for the present rotations of the planets is not known. It is to be noted, however, that the larger planets which have presumably grown the more by sweeping up planetesimals rotate more rapidly than the smaller ones.

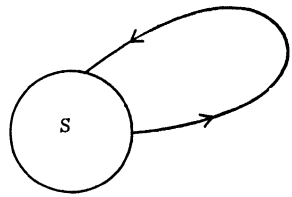


FIG. 164.—Explanation of the rotation of sun under the planetesimal hypothesis.

305. Brief Outline of the History of the Earth. If the planetesimal hypothesis provides a substantially correct description of the origin of the planets, the earth grew up around a nucleus which left the sun at the time of the passage of the visiting star. In its early stages this nucleus may have been a mass of gas or perhaps a dense swarm of liquid and solid masses which condensed from the ejected materials; for in view of the probable violence of the activities that attended the birth of the planets and scattered the materials of which they are composed out to hundreds of millions of kilometers from the sun, it is not unlikely that some of the planetary nuclei were initially aggregates of discrete masses. Such a dense swarm would gradually concentrate into a continuous mass, for the collisions of its members with one another would waste their kinetic energy and precipitate them toward its center.

There appears to be no way now of determining the mass of the

original nucleus of the earth. Since a thousand little planetoids are known and only four large planets, it is probable that it was small compared to the earth at present. This conjecture is supported by the fact that the growing earth swept up enough planetesimals to reduce its orbit almost to a circle. In the beginning, the nucleus must have been as barren as the moon is now, for its gravity could not have held an atmosphere on its surface. Eventually, however, its mass became sufficient to retain the beginnings of an atmosphere and of a hydrosphere, and then the geological processes which have transformed its surface became active. It is difficult to determine the length of time before this condition was reached, for 20,000,000 years would hardly have been sufficient under the most favorable conditions for the growing nucleus to sweep up half the planetesimals between the orbits of Venus and Mars. At some later state, probably more than one thousand million years ago, life appeared on the earth.

Biologists so far have sought in vain for a clue to the origin of life. It may have first emerged from the inorganic world on the shores of some primitive sea where pulsing tides and lapping waves produce the rhythmic changes characteristic of living organisms. At least, there the leaching rocks gave rise to an enormous variety of chemical solutions and furnished conditions most favorable for the origin of complex organic molecules. There are those who prefer to assume the "creation" of living creatures, though the difference between the two conceptions is largely one of words, for the growth of every organism and the reproduction of its kind are as miraculous as a creation.

Whatever may have been the mode of origin of life on the earth, the most ancient sedimentary rocks of the Archeozoic Era contain indications of only the simplest kinds of organisms. A hundred millions or more of years later, in the Cambrian period, trilobites lived in abundance; in the Ordovician period the types of low forms greatly increased and the vertebrates began to appear; in the Silurian, the vertebrates became firmly established; in the Devonian, they were still further developed; and eventually mammals and man evolved on the surface of the earth. During all these long periods of geological time the earth, heterogeneous as a consequence of its origin, was continually shifting toward a condition of thermal and gravitational equilibrium. Sometimes its surface was scarred by volcanic activities and wrinkled by

rising mountains; at others, for long intervals, it was relatively quiescent while the slow processes of erosion washed its continents largely into the seas. Perhaps, as Wegener and others have recently suggested, its continents were always floating and drifting on the denser materials of its deep interior.

Having determined the outlines of a great past, we may hopefully attempt to penetrate the future for long periods of time. The evidence that the earth a million years in the future will be physically much as it is at present is of the same kind and as convincing as that which makes us confident that a new morning will follow tonight and that summer will again drive away winter's snows. In fact, hundreds of millions of years are promised both by past history and by the forces which determine the state of the earth. What glorious hopes for the future of the human race flood us when we contemplate the enormous changes that have taken place in life on the earth from the lowest forms to the highest during comparable periods of time in the past! Yet it seems certain that ultimately terrestrial life and the earth itself will come to an end. Even if nothing else makes some day their last, the planets will all be wrecked when the sun again passes near another star. Then out of their scattered fragments new planets will be formed, illustrating on a cosmic scale the passage of the individual but the survival of the type.

306. Hypotheses Related to the Planetesimal. Two theories of the origin of the planets have been propounded which have essential elements in common with the planetesimal hypothesis. One, the *Encounter Theory*, was formulated by A. W. Bickerton, of New Zealand, in 1878, and thus antedates the planetesimal hypothesis by more than 20 years. The other, the *Tidal Theory*, was advanced by J. H. Jeans in 1918, and further developed by Harold Jeffreys in 1924, and hence follows the planetesimal hypothesis by about 15 years.

The encounter theory. Since Bickerton was a chemist and published his work in a journal chiefly devoted to biology, his theory was quite unknown in Europe and America until it was named and ably expounded by A. C. Gifford, in 1926. Bickerton appears to have had his attention directed actively to astronomy by the sudden appearance of a so-called temporary star (Art. 344), and his hypothesis respecting the origin of the planets was an

outgrowth of his explanation of these remarkable outbursts of stellar activity. He attributed the origin of the planets to the grazing collision of a passing star, instead of appealing to eruptions from the sun or to the effects of tides.

In 1749, Buffon published the hypothesis that the planets are fragments of the sun, torn off by an oblique impact of a comet. Most of the objections of this theory which led Laplace to reject it apply also to the hypothesis of Bickerton. In the first place, even a grazing collision of a star with the sun is much less probable than simply a near approach—250,000 times less probable than an approach only to the orbit of Jupiter. Then, too, the fragments in the beginning would all have had their perihelia at the surface of the sun, and it is difficult to account quantitatively for the present approximate circularity of their orbits with perihelia thousands of millions of kilometers away. The difficulty in accounting, under the theory, for the present relatively large moment of momentum of such small masses as those of the planets is equally great, for the present moment of momentum of Jupiter, for example, is nearly 25 times greater than it would be if the perihelion of its orbit were at the surface of the sun. In spite of these apparently serious difficulties, the theory of Bickerton should not be rejected without a thorough quantitative examination of its implications.

The tidal theory. In his original statement of the tidal theory, in 1918, Jeans assumed that about 300,000,000 years ago the sun, then in an early stage of its evolution, extended out to the distances of the present most remote planets. While it was in this enormously expanded condition, a more massive star, according to the theory, passed near its surface. At its nearest approach the tidal forces of the passing star neutralized the attraction of the sun on the outer part of its atmosphere in a line with the visiting body. It was assumed that when this condition was reached a large volume of solar atmosphere for a time followed the passing star, and that later this gaseous material broke up into a number of separate masses which condensed into the planets.

When Jeffreys discussed the tidal theory in 1924, he also assumed that the planets must have been formed when the sun was in a giant stage. In 1928, however, Jeans in his *Astronomy and Cosmogony* abandoned the theory that the heat of the sun is primarily due to its contraction and that the sun a few hundred

millions of years ago was a giant star; and in the 1929 edition of his book on the earth, Jeffreys also adopted the assumption that the sun was in its present state when the planets were born.

In both the planetesimal and the tidal theory the near approach of a visiting star is an essential factor. The dynamics of the origin of the planets from the sun, however, is quite different in the two hypotheses, for Jeans assumed that the separation of matter from the sun was entirely due to tides raised by the passing star. He assumed, moreover, that the cross-motion of the separated material which now appears in the moment of momentum of the planets was also a tidal effect. In tides, however, it is the wave-form which advances and not the matter itself, as Jeffreys also has recently (1929) concluded. That is, the imparting of cross-motion was not a tidal effect but a direct perturbative effect, as was explained in the original development of the planetesimal theory.

According to the tidal theory as published, the first time the tenuous, gaseous planets passed their perihelia, the sun took the satellites from them by a tidal action similar to that exerted on the sun by the passing star. It is not clear that this theory can account for the inner satellite of Mars, or the retrograde satellites of Jupiter and Saturn, or the absence of satellites of Mercury and Venus. Moreover, Jeffreys has pointed out more serious dynamical difficulties (1929) in the tidal theory of the origin of the satellites that he states will require its complete revision; in fact, he has now (1931) abandoned the tidal theory of the origin of satellites and expressed himself as favorable to the encounter theory.

Although from the astronomical point of view there are few, if any, remaining essential differences between the planetesimal and the tidal hypothesis, from the geological point of view their differences are important. According to the planetesimal hypothesis, the earth grew from a nucleus of some unknown size to its present mass by acquiring planetesimals, just as it now sweeps up 20,000,000 meteors per day. If a considerable part of its mass has been gathered together in this way, as is quite possible if not probable, it grew up a solid body, at least in all its exterior layers. On the contrary, according to the tidal theory the earth was originally gaseous and cooled to a liquid condition in a few revolutions. Its subsequent development under this theory is therefore identical with that under the Laplacian theory. Many geologists hold

the inherited view that the earth was originally fluid, but they do not satisfactorily explain how it became solid.

Two hundred and fifty years have not yet passed since Newton gave us the laws of motion and the law of gravitation, the only means we have for working back a thousand million years to the origin of the planets and of looking forward perhaps a greater time to their final destruction. We who have heard the cosmic clock tick but once are attempting to determine when and how it started and when and why it will stop. That many uncertainties remain is not so remarkable as that there is so general an agreement on fundamentals.

307. Long-time Effects of Variations in the Masses of the Sun and the Planets. In an earlier chapter we considered the theory that the sun's radiation is at the expense of its mass; in this chapter, as well as in that on meteors and comets, we have discussed the growth of planets by their sweeping up of planetesimals. In none of them, however, have we pursued the consequences of these variations in mass over extremely long intervals of time, for heretofore we have been interested in periods comparable to the age of the planets.

W. D. MacMillan has shown, however, that in sufficiently great time-intervals the changes to which we are referring may be of the first order of importance. For example, if the sun should continually radiate away its mass for 10^{12} years without being affected by some compensating factor, it would gradually lose its gravitative control over the planets and they would spiral out to great distances. On the other hand, if the sun, in its journeys in the sidereal system, should pass through a region rich in scattered nebular or meteoric material and double its mass, the planets would spiral inward. The wings of the imagination falter at the prospect of the unaccustomed flights in time demanded by these problems, but the limitations are our own and not those of the subject we are discussing. We have been somewhat like a traveler who sees only dust in the dust of the desert, while the geologist sees in it the disintegrated remains of mountains. In our problem we must now shift from the point of view of the traveler to that of the geologist. The intervals of time we must consider have as great a ratio to the present age of the earth as that has to the brief moment of human history.

There is abundant evidence of scattered interstellar materials. In the first place, there are enormous diffuse light and dark nebulae (Arts. 348–351). Then, too, there are very tenuous calcium and sodium clouds in the entire galactic system which produce characteristic absorption lines in the spectra of very distant stars of certain types (Art. 321). Finally, the meteors may be in part interstellar rather than limited to the planetary regions. Even if they originate in the solar system and in other similar systems, it is highly probable that some of them have passed near to the planets in such special ways that they have been expelled and driven to stellar distances.

The origin of nebulae is quite unknown, though MacMillan has argued very persuasively that they may be the product of the integration into matter again of the energy radiated by the stars. At present we do not know how such an integration can take place; but that is not a fatal objection to the suggestion, for if our whole experience were limited to the hours of an afternoon, we should see night approaching and we should not know how day could come again. R. A. Millikan has advocated the theory that the so-called "cosmic rays" which come to the earth from every direction in space are signals of the birth of atoms in the interstellar regions. Whether these speculations respecting the origin of matter are true or not, there are certainly materials of various kinds in the galactic regions traversed by the sun and other stars.

Let us start with the solar system as it is at present. The sun is radiating its mass away, according to the widely accepted theory, and it is acquiring matter in its journey among the stars. MacMillan has shown that it will draw in upon itself the materials which are in a cylinder around its path of about 22,000,000 kilometers (14,000,000 miles) radius, and that the mass gained will equal the mass radiated if the density of the interstellar medium is 10^{-19}. On the basis of present data it seems likely that the average density of the interstellar regions is much below this figure, and that the sun is now radiating away its mass more rapidly than it is acquiring mass. The planets, however, are similarly acquiring mass, but do not radiate away important amounts. Consequently the planets are probably very slowly spiraling outward from the sun and growing in mass. If this process should continue a sufficiently long period of time, Jupiter would approach the sun in

size and the system would become a double star attended by several enormous planets.

Suppose, however, that the sun will some time pass through a region whose density is greater than 10^{-19}. Then its mass will increase, its temperature will rise, and its rate of radiation will increase until there is an equality between loss and gain in mass. The planets will also increase in mass and slowly spiral inward toward the sun. Since the sun will cease to grow when its rate of radiating mass equals its rate of acquiring mass, the density of the interstellar regions sets superior limits on the masses and the luminosities of the stars. According to the hypothesis of MacMillan, there is an approximate equilibrium between mass in the stellar state and mass diffused in the interstellar regions, whether it is nebular, meteoric, or in the form of radiant energy.

As has been seen, all stars which have planets evolve toward binary or multiple systems of stars. Observations show that about one star in four is a binary, but the question arises why all stars, including our sun, have not arrived at the multiple stage. For the answer we return to the birth of the planets at the time of a passing star. Such a catastrophic event would destroy planets and might wreck a multiple star, though the difficult problem of determining the numerous things that such a dynamic encounter might produce has never been solved. It seems probable, however, that these near approaches, once in approximately 10^{15} years for a single star, maintain a certain average degree of simplicity in the stellar system. It should be stated that there are quite different theories of the origin of binary stars (Art. 335).

In harmony with the foregoing hypotheses, we may think of our planets as having been born but yesterday, as it were, when our sun passed near another star. With its family of relatively insignificant members, the sun pursues its trackless path through the star-clouds of which the galaxy is composed. Sometimes, as at present, it is in the deep interior of the galactic system; at other times, it is far toward its borders, held back from escaping by the attractions of hundreds of millions of suns. Unlike the planets, it describes no simple curve, but wanders irregularly through the galaxy in response to its inertia and the forces to which it is subject. For tens of thousands of millions of years it radiates its mass away and declines toward extinction; for equal periods it grows and becomes perhaps a giant sun and then declines again. The

planets are destroyed and renewed by successive near approaches; or they may grow to the stature of companion stars, only to be eventually merged into other suns or torn from their larger parent and sent away on solitary paths.

Is such a system eternal, always changing in its individual parts but on the average always the same? How satisfying it would be if it were! There are, however, enormous spaces between the galaxies into which the stars pour their radiant energy and from which they get no appreciable mass in return. That is, the galaxies appear to be dissipating their masses. To avoid this conclusion, MacMillan assumed that the average density of matter throughout space is of the same order as that in our galaxy, and that radiant energy is reintegrated into matter. There are serious dynamical difficulties with this hypothesis and our knowledge respecting the enormous vacant regions between the galaxies is opposed to it (Arts. 379–382). Hence in spite of our efforts we have not yet formulated a completely satisfactory theory of a cyclical and eternal system.

XXI. QUESTIONS

1. Is a flash of lightning an example of evolution, in the sense defined in Art. 294? Do you have some other definition which you prefer?
2. What factors tend to reduce the ratios of the mass of the sun to the masses of the planets? To increase these ratios?
3. What are the reasons for believing that the present geometrical properties of the planetary orbits have remained essentially unchanged since the origin of the planets?
4. Would the near approach of a star be likely to change the moment of momentum of the solar system?
5. In what respect does the inner satellite of Mars contradict the Laplacian theory? In what, the retrograde satellites of Jupiter and Saturn?
6. How is the forward rotation of several of the planets explained under the Laplacian theory? Under the planetesimal?
7. How is the inclination of the sun's axis to the planes of the orbits of the planets explained under the Laplacian theory? Under the planetesimal?
8. In what respects are the encounter theory, the planetesimal theory, and the tidal theory similar? In what respects do they differ?
9. What objections are there to the encounter theory that do not apply to the planetesimal?
10. Does the tidal theory apply as readily to a small relatively dense star, such as the sun at present, as to the large tenuous sun originally assumed by Jeans?

CHAPTER XV

STARS AND NEBULAE

308. Stellar Astronomy. During the eighteenth century, astronomers devoted their attention largely to the solar system. Observers carefully examined the moon, the planets, and the sun; mathematicians proved that the motions of these bodies are in harmony with the laws of motion and the law of gravitation; and until the beginning of the nineteenth century only a few astronomers regarded the stars as being much more than convenient reference points on the celestial sphere.

During the nineteenth century, astronomical interest rapidly extended beyond the relatively narrow limits of the solar system to the enormously distant stars. Astronomical literature was more and more devoted to double stars, variable stars, stellar motions, and stellar distances. In this century, too, spectroscopy had its birth, for in 1824 Fraunhofer first observed absorption lines in stellar spectra and in 1859 Bunsen and Kirchhoff laid down the fundamental principles of spectrum analysis (Art. 277). Moreover, in 1864 William Huggins and Angelo Secchi began detailed examinations and classifications of stellar spectra, and in the last decade of the century astronomers took up the absorbing problems which depend upon the radial velocities of the stars.

Now stellar astronomy has become the center of astronomical thought. With a rapidity unsurpassed in science we have gone out from our little island, the solar system, and have taken long steps in exploring the galaxy beyond. We have, in fact, recently overleaped its borders, though they are thousands of light-years away, and have begun the investigation of exterior galaxies and systems of galaxies. The problems of sidereal astronomy are numerous and so closely interwoven that every conclusion depends directly or indirectly upon many others. It will tend, however, to clarity in the exposition to treat in this chapter the stars and the nebulae simply as individual objects, and to reserve for the next chapter the determination of their distances and their organization into cosmic systems.

I. Stellar Spectra

309. Classification of Stellar Spectra. Between 1860 and 1870, William Huggins examined in detail the spectra of a few stars and recognized in them a number of the absorption lines which Bunsen and Kirchhoff had shown by laboratory experiments are due to such familiar elements as sodium, magnesium, calcium, and iron. Secchi examined in less detail the spectra of about 4000 stars and arranged them in four classes. H. C. Vogel began extensive investigations of stellar spectra at Potsdam in the following decade,

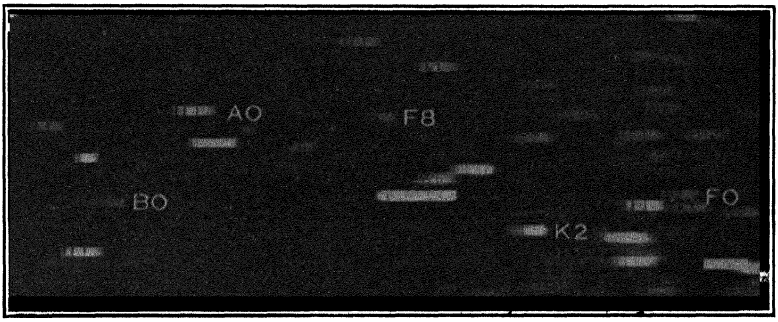


Fig. 165.—Objective prism spectogram. (Harvard College Observatory.)

and in 1885, E. C. Pickering inaugurated at Harvard an enormous program of recording stellar spectra by photography that required nearly forty years for its completion. In the nine volumes of results obtained at Harvard and at its stations in the southern hemisphere, there are recorded the positions, the magnitudes, and the spectral classifications of 225,300 stars. These volumes were published in 1924 as a memorial to Henry Draper who first studied stellar spectra in America.

The spectra in the Draper catalogue were not photographed by bringing the images of the stars, one after another, on the slit of the spectroscope, but by placing a large prism outside of its objective. By this method all the light from a star is spread out into a spectrum and the spectra of all the stars in the field are obtained simultaneously. The result is that the method is perhaps a thousand times more rapid than that of using a slit spectrograph at the eye end of the telescope, but it does not provide a comparison spectrum of a known substance, as may be secured with the slit spectroscope (Fig. 142).

Miss Annie J. Cannon is primarily responsible for examining and classifying the spectra of more than 200,000 stars, and the system of classification she developed is in universal use among astronomers. In the Harvard classification, the stars are arranged in an orderly sequence depending primarily on the characteristics of the absorption lines in the violet and blue regions of their spectra, as illustrated in Fig. 166. The spectral classes arranged in the order of simplicity of their absorption lines are designated by the letters O, B, A, F, G, K, M, R, N, S. Each of these classes is divided into subclasses by adding numerals to the letters. The Class F5, for example, is midway between the Classes F0 and G0.

As will be seen in the following articles, the Draper classification of stellar spectra is undoubtedly related to fundamental properties of the various classes of stars, but it falls short of making use of all the characteristics of stellar spectra. In fact, W. S. Adams, A. Kohlschütter and other astronomers of the Mount Wilson Observatory use a number of additional properties of spectra in classifying the stars.

310. The Spectral Sequence. In the spectra of the Harvard classification, the stars from Class O to those of Class M form a sequence that is fairly continuous with respect to the characteristics used as basic; those of Classes R, N, and S appear to be branches of Class K of different kinds. Formerly many astronomers believed that the ages of the stars are in the same order as their spectra, the O stars being newly formed and the M stars being close to extinction. It is now realized that such ideas are naïve and that the possibility of arranging the stars in a continuous linear sequence must be explained on other grounds. It seems probable from the work of M. N. Saha, as extended by R. H. Fowler, E. A. Milne, and H. N. Russell, that the spectra of the stars simply reflect the degree of ionization (Art. 280) of the elements in their atmospheres, the degree of ionization depending upon the temperatures and the pressures of the radiating gases. The temperatures and pressures of the stellar atmospheres in turn depend upon the masses and the densities of the stars. There is thus a physical explanation of a classification of stellar spectra which was purely descriptive when it was first developed.

Class O stars. It follows from the lines in the spectra of these stars that their atmospheres contain hydrogen, ionized helium,

and doubly and triply-ionized oxygen and nitrogen. These facts imply that these stars are very hot (30,000° C.) or that their atmospheres are extremely tenuous. But the distribution of their radiated energy points to a lower temperature, on the basis of Planck's law (Art. 274), than has heretofore seemed probable. Besides the

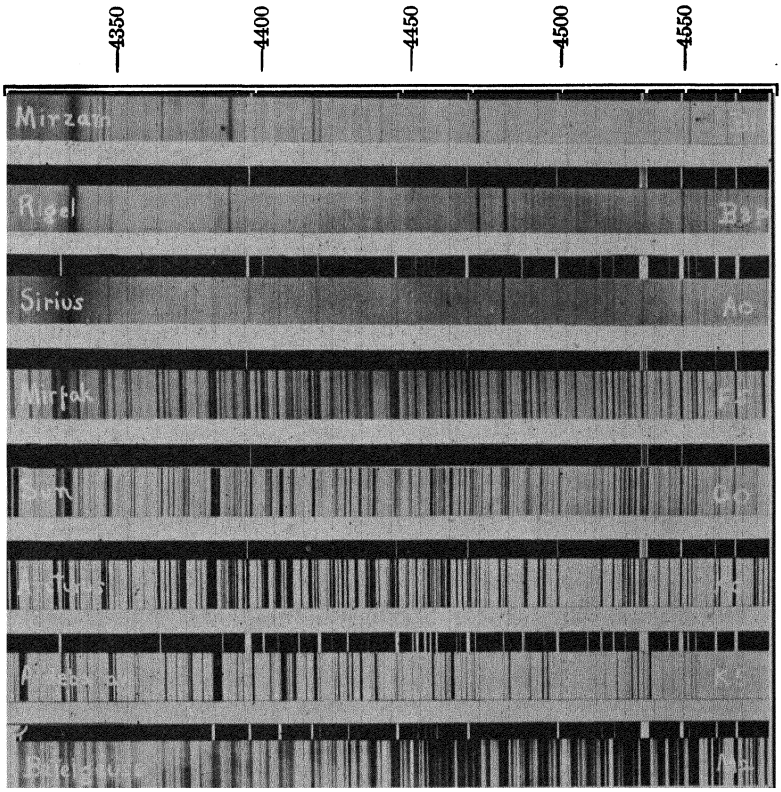


FIG. 166.—Typical spectra (in the blue from 4325 to 4575 angstroms) of spectral classes B1, B8, A0, F5, G0, K0, K5, and Ma. (The Yerkes Observatory.)

absorption lines, the spectra of many of these stars contain bright emission lines whose origin is not fully understood.

Class B stars. In the atmospheres of these stars there is hydrogen, neutral helium, and singly-ionized oxygen and nitrogen. The surface temperatures of these stars are probably very high (20,000° C.) and their atmospheres are tenuous. A number of the stars in Orion belong to this class.

Class A stars. The lines of helium are not present in the spectra of these stars, but the hydrogen lines are very strong. The characteristics of their spectra show that the atmospheres of these stars contain *ionized* calcium, magnesium, iron, and other metals. These stars have surface temperatures of about 10,000° C. and their atmospheres are of low density. The typical star of this spectral class is Sirius.

Class F stars. In the spectra of these stars the hydrogen lines are less conspicuous than in those of the Class A stars. The atmospheres of these stars contain ionized calcium and evidence of *neutral* atoms of the metals begins to appear. Canopus and Procyon are examples of stars of this class, and their surface temperatures are of the order of 8000° C.

Class G stars. In the atmospheres of these stars, of which the sun and Capella are examples, there are *neutral* atoms of many metals, such as magnesium, iron, titanium, and nickel. The temperatures of the atmospheres of these stars are 5500° C. to 6000° C.

Class K stars. In the spectra of stars of Class K the lines characteristic of lower temperatures are abundant. Absorption bands due to the compound titanium oxide are first present, and the temperatures are of the order of 4000°, or about that of the umbrae of sun spots. Arcturus and Aldebaran are conspicuous stars in this spectral class.

Class M stars. The temperatures of these stars are of the order of 3000°, and their spectra contain strong bands of titanium oxide and the absorption lines of neutral atoms of many metals. These stars, of which Betelgeuse and Antares are examples, owe their red color to the strong absorption of the blue end of their spectra.

Classes N, R, S. The spectra of the stars of these classes show strong low-temperature lines of the metals. Those of the first two classes contain, in addition, absorption bands of carbon and carbon compounds, and the spectra of stars of Class S contain lines of zirconium oxide, and in some cases titanium oxide as well. The stars of these classes seem to branch from the main sequence at K2, but their relationship to the stars of Class K is not clear. Only a very few of the stars of Class N are bright enough to be visible to the unaided eye, and photographically they are much fainter because they are deep red in color. The brightest star of Class R is of magnitude 7.04, and the brightest of Class S is of magnitude 6.65, both of which are invisible without optical aid.

311. Distribution of Stars Among the Spectral Classes. Of the stars brighter than magnitude 4.25, fully half belong to the spectral Classes B and A, while nearly all the remainder are distributed among the Classes F, G, K, and M. As fainter magnitudes are considered, however, the percentage of stars of Class B progressively declines, that of Class K increases, and those of the other classes remain essentially constant. The distribution of spectra for the 225,000 stars catalogued by the Harvard astronomers is as given in the following table:

Class	B	A	F	G	K	M
Percentage	2	29	9	21	33	6

Of the more than 6000 stars in the catalogue brighter than magnitude 6.25, only 20 were of Class O, 8 of Class N, and none of Classes R and S.

There is a temptation to draw the conclusion from the decline in the numbers of stars of Class B with successively fainter magnitudes that these stars are more numerous in our part of the galaxy than in equal volumes at remote distances. It is probable, however, that the conclusion would be erroneous. The stars of Classes B and A are intrinsically very luminous and for given magnitudes are more remote than those of the spectral Classes F, G, and K. They suffer, therefore, rapid diminution of light from the obscuring matter which, as we shall see, pervades our galaxy, while the stars intrinsically less luminous are so much nearer that they are not greatly affected. Hence the table probably overstates somewhat the percentages of stars of Classes B and A and correspondingly understates the percentages of Classes F, G, and K.

It is an interesting and significant fact that stars of all spectral classes except N, R, and S are found in the same star-clusters and star-clouds. Hence the spectral classes can not be simply a measure of the ages of the stars, nor can those of each class be due to some common origin. It is possible that they are phases which a star may go through more than once (Art. 307), and that their relative numbers are proportional to the fraction of the time in which stars are, on the average, in their respective conditions. At any rate, these percentages are significant data for any theory of stellar evolution.

312. Visual, Photographic, Radiometric, and Bolometric Magnitudes. *Visual* and *photographic* magnitudes of stars are their

magnitudes as determined by visual observations and photography respectively. Since the human eye is most sensitive to light in the yellow-green part of the spectrum while the ordinary photographic plate is most sensitive to light in the blue-violet end of the spectrum, it follows that the two kinds of magnitudes do not in general agree. The zero of the photographic magnitude scale is so chosen that the visual and the photographic magnitudes are equal for stars of magnitude 5.5–6.5 and Class A0.

The photographic magnitude of a star minus its visual magnitude is called its *color index*. For stars bluer than those of Class A0 the color index is evidently negative, because the fainter the star the larger the numerical value of its magnitude. Negative color indices usually do not numerically exceed -0.3; positive color indices for stars of Class M may be as much as $+2.0$ magnitudes, or even more. It is evident that the color and the spectral class of a star are determined with considerable approximation simply by its color index.

The *radiometric magnitude* of a star is its magnitude as determined by the total amount of its radiant energy of all wave-lengths which reaches the surface of the earth. The zero of the scale is so chosen that the radiometric magnitude agrees with the visual magnitude for a star of spectral Class A0. The total radiation received from a star is measured by its heating effects on a thermocouple of extraordinary sensitivity, the essential parts of which weigh but little more than a drop of ink the size of a period.

The *bolometric magnitude* of a star is its magnitude as determined by the total amount of its radiant energy of all wave-lengths which reaches the upper surface of the earth's atmosphere, the zero of the scale being so chosen that this magnitude coincides with the visual magnitude for a star of spectral Class G0. Hence the radiometric magnitude differs from the bolometric magnitude because the earth's atmosphere reflects and absorbs part of the radiant energy which comes from the star, and also because the two magnitudes agree with the visual for stars of different spectral classes.

Since the amount of absorption depends upon the wave-length of the radiant energy, the relationship between the radiometric and the bolometric magnitude of a star depends upon its spectral class. For example, the radiometric and bolometric magnitudes of the bluish-white Sirius (Class A) are -1.3 and -1.4 respectively, while the corresponding magnitudes of the red star Antares (Class

M) are -1.3 and -1.0. If heed were not given to the fact that different zero points are used in defining the two magnitudes, the conclusion might be drawn that more radiant energy is received from Antares at the surface of the earth than at the outer borders of its atmosphere.

A number of types of energy-measuring devices have been used to determine the energy received from a celestial body, either as a whole or in its separate parts after it has been resolved into a spectrum. One of these instruments, developed by Joel Stebbins, depends upon the fact that the electrical resistance of selenium varies with the intensity of its illumination. A very sensitive type which was introduced by P. Guthnick and developed by Stebbins uses the photo-electric cell.

In considering the properties of a star, the characteristics of the energy which it radiates, rather than those of the fraction which penetrates through our atmosphere, are important. Hence the bolometric magnitude is one of the data of primary interest.

313. The Absolute Magnitude of a Star. In investigating stars as physical bodies, we are interested in their actual luminosities rather than in their magnitudes as observed from the earth. In order to compare conveniently the actual luminosities of the stars, their magnitudes are determined for a standard distance which astronomers have agreed shall be that at which the parallax of a star (Art. 104) is $0''.1$, or the distance light travels in 32.58 years. The absolute magnitude may be visual, photographic, radiometric, or bolometric, according to the observed magnitude from which the reduction to standard distance is made.

It follows from the definition of observed and absolute magnitudes that if m represents the observed magnitude of a star, M its absolute magnitude, and p its parallax expressed in seconds of arc, then M is given by the equation

$$M = m + 5 + 5 \log p.$$

The visual magnitude of the sun is -26.72; at a distance of 2,062,648 times greater, at which its parallax would be $0''.1$, its visual magnitude would be $+4.85$; that is, the sun's absolute magnitude is $+4.85$. The absolute magnitude of Sirius is $+1.27$, from which it follows that the luminosity of Sirius is 27 times that of the sun. Rigel has an absolute magnitude of -5.5, and its luminosity is therefore about 14,000 times that of the sun. Many

stars, on the contrary, are less luminous than our sun, an interesting example being a faint and relatively near star, discovered by E. E. Barnard, which the sun exceeds in luminosity nearly 2500 times. Only a few stars, such as S Doradi, are known which certainly exceed Rigel in luminosity, but there are stars less luminous than Barnard's; indeed, some stars may be entirely dark, as is suggested by eclipsing variables (Art. 327). The known range in luminosity extends over 21 magnitudes, or a ratio of more than 250,000,000 to 1.

Early in this century, J. C. Kapteyn took up the problem of determining the numbers of stars of various absolute magnitudes, and other astronomers have greatly extended his investigations. In a space surrounding the sun large enough to contain 1,695,000 stars, the number of stars of various absolute magnitudes, according to F. H. Seares, is as given in Table XVII.

TABLE XVII. NUMBER OF STARS OF VARIOUS ABSOLUTE MAGNITUDES

ABSOLUTE MAGNITUDE	NUMBER OF STARS	ABSOLUTE LUMINOSITY	LUMINOSITY (SUN = 1)
−5.0	1	250	6920
−2.5	90	25	692
0.0	3,300	2.5	69.2
+2.5	42,000	0.25	6.92
+5.0	200,000	0.025	0.692
+7.5	350,000	0.0025	0.069
+10.0	500,000	0.00025	0.0069
+12.5	600,000	0.000025	0.0007

It follows that only one star in about two million is a super-giant, six or seven thousand times as luminous as the sun, while 600,000 stars have a luminosity less than 1/1000 that of the sun. Apparently our sun stands well up among the less numerous brilliant stars, yet far from the maximum of brilliancy.

314. Relationship Between Absolute Magnitudes and Spectral Classes of Stars. It is evident that the two principal factors upon which the absolute magnitude of a star depends are the temperature, and therefore the luminosity, of its atmosphere and the area of its radiating surface. Since the spectrum of a star depends primarily upon the temperature of its atmosphere, there would necessarily be a simple relationship between the luminosities and the spectra of the stars if their radiating surfaces were all equal to

one another. Although the stars differ very greatly in diameter, there is nevertheless a fairly well-defined relationship between their absolute magnitudes and their spectra, a fact brought out clearly by H. N. Russell in 1913 (Fig. 167).

Since stars of absolute magnitude −5 are of enormous luminosity while those of absolute magnitude +15 are of low luminosity, the diagram means that with few exceptions the blue and bluish-white stars are of the highest luminosity, while the redder stars are progressively of lower luminosity. Since the absolute magnitude of the sun is +4.85 and its spectral class is G0, its representative point lies in the main band near the center of the diagram. There are, however, a few points far below the principal band, and a considerable number above it.

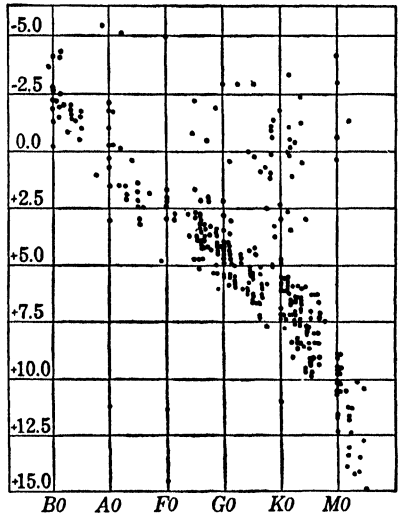


FIG. 167.—Relationship between spectral class and absolute magnitude. (Russell.)

315. Dwarf Stars and Giant Stars. Those stars whose representative points are near the bottom of Fig. 167 are of low absolute magnitude. It is seen that there is a star of spectral Class A0 and that there are two stars of spectral Class F0 which are of as low luminosity as those of spectral Class M0. These are dwarf stars. On the other hand, there are many stars of spectral Classes G0, F0, and M0 which are of as high luminosity as those of Class B0. These are giant stars. The terms dwarf and giant are defined solely by the luminosities of the stars and not by their diameters or their volumes.

E. Hertzsprung, in 1905, first pointed out the fact that the red (Class M) stars consist of two types, the feebly luminous and the highly luminous, and he designated them as dwarfs and giants; Russell extended the investigations to greater numbers of stars and to stars of all spectral classes. According to Fig. 167, the red stars are grouped on the basis of luminosity into two widely

separated classes; the divisions are less marked in the stars of other spectral types. It should be remembered, however, that the determination of the absolute magnitude of a star depends upon a knowledge of its distance, as well as upon its apparent magnitude, and that the distances of all except a few stars are subject to considerable uncertainties. It is evident that better and more numerous data may somewhat alter the characteristics of the diagram, though it is probable that it represents the normal relationship between spectral class and absolute magnitude.

It is natural to inquire how red stars, for example, can include both dwarfs and giants. The differences in luminosity can not be due to a difference in surface temperatures, for their red color implies a relatively low temperature for both dwarfs and giants. The true explanation is undoubtedly that the giants have enormously greater diameters and radiating surfaces than the dwarfs; in fact, measurements of the diameters of a number of giant stars (Art. 317) unmistakably confirm this conclusion.

The giant stars are not only large, but they are very tenuous. Consequently there is more ionization of the atoms in their atmospheres for a given temperature than there is in the atmospheres of smaller dwarfs with denser atmospheres. This difference in the degree of ionization has distinct effects upon the characteristics of the spectral lines which have been observed and found to be quite in harmony with theory. In fact, the spectra of so many stars have been examined that it is now possible to determine from the characteristics of the spectral lines of a star whether it is a dwarf or a giant. That is, the characteristics of the spectrum of a star indicate its approximate absolute magnitude. Since its apparent magnitude can be determined by direct measurements, its distance can then be computed.

316. Super-Giant Stars and White Dwarfs. In the course of the Harvard survey, Miss Antonia C. Maury noted an occasional star having a spectrum with certain exceptionally sharp absorption lines and with its other lines unusually dark. The Harvard astronomers distinguished these stars from the remainder by prefixing the letter c to the usual designation, as cF0. These c-stars are the super-giants of approximate absolute magnitude -5. Examples of these super-giant c-stars are Rigel (spectral Class cB8), Deneb (spectral Class cA2), Canopus (spectral Class cF0),

and Antares (spectral Class cM0). It is thus found that several of the most conspicuous and best-known stars are super-giants. It should be noted, however, that as a consequence of their enormous luminosity they are visible from great distances and hence appear to be relatively more numerous than they actually are.

Even more remarkable than the super-giants are the white dwarfs whose representative points are near the lower left-hand corner of Fig. 167. Their white color implies that they are very hot and very brilliant per unit area, and their high positive absolute magnitudes imply that they are of very low total luminosity, not more than 1/200,000,000 of that of the super-giants. Therefore they must be very small stars. These stars are the white dwarfs, the opposite extreme from the super-giants. Unfortunately, the data on which the conclusions rest are very insecure and at present (1931) there are only three known stars of the type in question, though several others are suspected.

At first the foregoing conclusions seem simple enough, but upon considering them a difficulty arises. A very hot star has a strong tendency to expand as a consequence of the kinetic energy of its gases. But a small star (unless it is extraordinarily dense) has a low surface gravity and therefore small resistance to expansion. Hence we are driven to the conclusion that these stars are very dense, for otherwise they would expand because of their high temperatures. But how dense must they be? Computation seems to indicate that in order to fit the observations they must average nearly 400 times as dense as platinum! How can a star be 400 times as dense as platinum? The explanation, according to A. S. Eddington, is that the white dwarfs are composed of atoms that have lost all of their exterior electrons. The effective diameters of the atoms, which are supposed to be determined by the orbits of their revolving electrons, are consequently greatly reduced, and the atoms simply pack more closely together than is possible for those that are neutral; yet according to the theory they behave like perfect gases. A cubic centimeter of these gases weighs 7.7 kilograms!

These ideas are startling, but once the theory that the earth, resting on nothing, rotates on its axis was equally startling; it seemed wholly absurd. We admit that the theory of the density of white dwarfs is possibly correct, but we wish to check it by independent means. Fortunately, the faint companion of the star

Sirius inspires hope of a check. According to the recent (1930) determinations of the magnitude (mag. 7.1) of the faint star by A. Vyssotsky, Sirius is more than 2500 times as luminous as its companion, but its mass is only 2.5 times as great, a fact that is determined from the motions of the two stars about their center of gravity. Moreover, the spectrum of the faint companion appears to prove that its temperature and surface brilliancy must be high and comparable to those of the principal star. If this conclusion is sound, the star must be very small and of great mass; in fact, it must be only twice as great as the earth in diameter, but about 4000 times as dense.

The climax, however, to the story of the companion of Sirius is yet to come. According to the general theory of relativity, a large mass pulls back, as it were, on the energy it radiates and increases its wave-length. If the mass were great enough, radiant energy, according to the theory, could not be emitted at all. In the case of an ordinary star, the backward pull on radiation does not produce observable effects; but Eddington showed that in the case of the dense companion of Sirius, with the magnitude of the companion as then determined, there should be a displacement of the spectral lines toward the red of about 20 kilometers per second. W. S. Adams, at the Mount Wilson Observatory, in 1924, secured what he regards as distinct evidence of the predicted displacement of 20 kilometers per second, and the results have been verified by J. H. Moore at the Lick Observatory.

When the spectroscopic measurements of the companion of Sirius confirmed the predicted relativity effect of 20 kilometers per second, many astronomers regarded the conclusions respecting the density of this star as quite trustworthy. But now that more careful determinations of the magnitude of the star have increased its total luminosity by the factor 3.3, the predicted displacement becomes only 10 kilometers per second, or half that which has been measured. Moreover, the companion itself may be a close double star; and if it is, the conclusions may be quite erroneous, for a pair in general would radiate much less light than the same mass united in a single star. That the companion of Sirius is a close double is more than a mere suspicion, for irregularities in its motion indicate the probable effects of a companion. Moreover, Philip Fox (1920) and R. T. A. Innes and his associates (1926, 1928) observed the star as a double, though the observations should be confirmed.

Hence a very good story about the great density of this star may be spoiled, but we may be confident that if it is, the truth will eventually be found to be even more interesting.

317. Direct Measurement of Stellar Diameters. If the sun were at the distance of the nearest star, it would subtend an angle far too small to be observed even with the largest telescope; nor does it seem probable that a telescope will ever be constructed with which the diameter of any star (except the sun) can be measured. This does not mean, however, that scientists are completely baffled, for in studying the principles which underlie the resolving power of optical instruments, A. A. Michelson found that for measuring small diameters (or the distance between two very close stars) a certain form of *interferometer* is greatly superior to a telescope. Instead of forming an image of the object, as the telescope does, the stellar interferometer passes its light over two paths and then superposes it at the eyepiece. When the two paths differ a certain amount in length, depending upon the diameter of the star and the distribution of brightness over its disk, the two beams combine in such a way as to give a series of light and dark interference fringes. It is therefore possible to calculate the angular diameter of a star from the distance between the mirrors of the interferometer when its interference fringes appear.

A great interferometer was mounted on the 100-inch reflector of the Mount Wilson Observatory, and F. G. Pease has measured with its aid the diameters of seven stars. The first attempt was made on Betelgeuse, which is a super-giant, and its angular diameter was found to be 0.″047, almost in perfect agreement with the predictions from theory. Betelgeuse is so remote that its parallax can not be measured with high precision, but according to the best results for its distance available, the measured angular diameter of 0.″047 corresponds to a linear diameter of 418,000,000 kilometers (260,000,000 miles). Betelgeuse therefore would almost fill the orbit of Mars, and its volume is more than 27,000,000 times that of the sun. The largest known star is not Betelgeuse, however, but Antares, which exceeds the sun in volume more than 90,000,000 fold. The other stars whose diameters have been measured are α Herculis, Mira Ceti, β Pegasi, Aldebaran, and Arcturus; and their respective volumes are 64,000,000, 27,000,000, 64,000,000, 55,000, and 20,000 times that of the sun.

It has not been possible to make the same sort of verification of the small diameters of the dwarfs, for they are all much below the limits attainable by our instruments. But the diameters of certain eclipsing double stars have been determined from their periods, their distances apart, and the durations of the obscuration of one of the stars of a pair by the other (Art. 327). The diameters obtained by this method are quite in harmony with those which would be inferred from their absolute magnitudes and their spectra.

318. Relationship Between Spectral Classes and Masses of Stars. At present the masses of comparatively few stars are known. But from the data at hand it appears that, generally speaking, the masses of the Class O stars are the greatest and those of the Class M stars are the least, with a steady decrease through Classes B, A, F, and G of the main sequence of spectral classes. The masses of stars of Classes N, R, and S are quite unknown.

Of all the stars whose masses have so far been determined, that of 27 Canis Majoris is the greatest, at least if the displacement of its spectral lines is due to orbital motion. According to the measurements of O. Struve, its combined mass appears to be 1000 times that of the sun.

The next greatest stellar masses are the components of a Class O double star which was investigated by J. S. Plaskett, the masses being 75 and 63 times that of the sun. In fact, they may even be greater, for these are the values if their orbit is edgewise to the earth, a condition which gives the least possible values (Art. 331). In terms of the sun's mass, the masses of the Class B stars probably range from 3 to 20; those of Class A, from 1.5 to 3; those of Class F, from 0.5 to 1.5; those of Class G, from 0.4 to 1.2; those of Class K, from 0.3 to 0.8; and those of Class M, from 0.2 to 0.6.

The giant stars of all spectral classes from F to M may be much greater than the figures which have been given for the stars of the regular spectral sequence. It does not seem improbable that eventually there will be a close correlation between the masses of the stars and the finer characteristics of their spectra than can be expressed in terms of the present classification.

In 1924, A. S. Eddington found some evidence of a correlation between the masses and the absolute magnitudes of 45 stars, and derived a theoretical basis for the relationship. The data are too few, however, to justify positive conclusions; and, moreover, E. A.

Milne has rejected the theoretical discussion. It does not seem probable that there is a relationship between masses and luminosities which is independent of densities.

319. Relationship Between Spectral Classes and Rotations of Stars. Since the stars are so remote that up to the present it has been possible to measure the diameters of only 7 of them, it may at first occasion surprise that astronomers have been able to find out anything about their rotations. Our information respecting stellar rotations has not been obtained by observing them turn on their axes, as in the case of the sun, but by an interesting application of the principles of spectrum analysis.

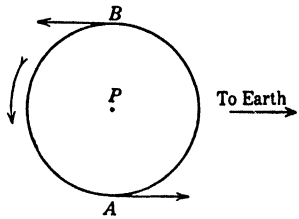


FIG. 168.—Rotation of star.

For simplicity, suppose the axis of rotation of a star is at right angles to the line from the star to the earth, with its pole at P, Fig. 168. The limb A is approaching the earth and the limb B is receding. If the star were an eclipsing binary (Art. 327), the dark member of the pair would cover first one limb and then the other, and at these times the rotational velocity could be determined (Art. 328). In general, however, light from the entire disk of the star passes through the spectroscope, that from A being displaced toward the violet, that from B toward the red, and that from the center not at all. The result is that the spectral lines are widened; the motion of the star as a whole displaces its spectral lines but does not affect their widths.

The problem of determining the rotations of stars by spectroscopic methods is complicated by the fact that spectral lines normally have appreciable widths due to such factors as the radial components of the velocities of the atoms in the source of light or of absorption. Moreover, allowance must be made for the relative areas and brightnesses of the zones at different distances from the axis of rotation of a star. O. Struve and G. Shajn have developed the theory of the effects of all these factors, and C. T. Elvey has found that the distribution of density in the spectral lines agrees with the inferences from theory. Consequently it is possible to determine in many cases the rotations of the stars by this application of spectrum analysis.

Although the investigations of the rotations of the stars are recent developments, the limb velocities of more than 60 stars have been determined (1931). It has been found that stars of classes B and A in many instances are in rapid rotation; that stars of Class F rotate more slowly; and that most giant stars of classes G, K, and M show in their spectral lines no effects of rotation.

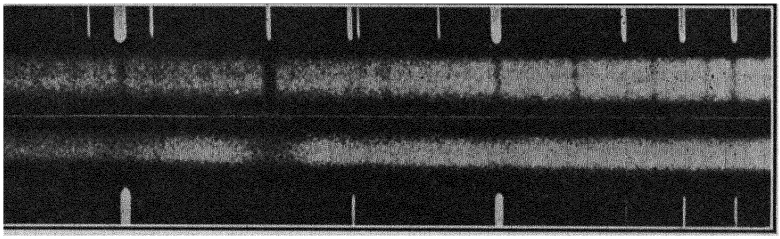


FIG. 169.—Above, spectrum of Vega (slow rotation); below, spectrum of Altair (rapid rotation). The difference in the absorption lines under the fourth bright comparison line (at 4481 angstroms) is conspicuous.

In the case of Spica (α Virginis) the velocity-component of the limb is of the order of 200 kilometers per second, indicating that the star must be very oblate, and the rotational speed of the Class O8 star Altair (α Aquilae) is equally high.

There is an upper limit to the speed at which a star of given mass and size can rotate without breaking up into two or more bodies, and it appears probable that Spica is near this limit. If Spica grew to be a massive star by sweeping up scattered material (Art. 307), we should expect to find it rotating very slowly, for such matter would act as a resisting medium. On the other hand, if Spica has been formed by the union of two stars whose growing masses drew them together, we should expect to find a rapid rate of rotation. Evidently, an important new field is being explored in the determinations of the velocities of rotation of the stars.

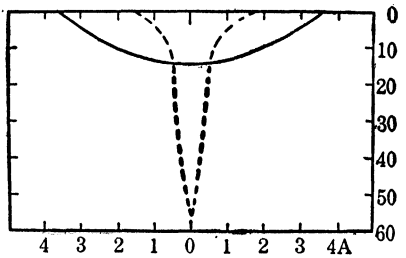


FIG. 170.—Dotted curve and full curve are contours of absorption lines of Vega and Altair at 4481 angstroms.

320. Relationship Between Spectral Classes and Velocities of Stars. There are two distinct methods of determining the

velocities of the stars with respect to the solar system: (*a*) by measuring their proper motions at right angles to the line of sight, and (*b*) by measuring their velocities in the line of sight with the spectroscope. The former measurements evidently do not give the linear velocities, in kilometers per second for example, unless the distances of the stars are known.

Lewis Boss showed from an exhaustive examination of the observations of 6188 stars that the average velocities transverse to the line of sight are least for the stars of Class B and that they increase to Class G and remain essentially the same for stars of Classes K and M. The average velocities in kilometers per second of the stars of the various spectral classes, as determined by Boss are: Class B, 6.0; Class A, 10; Class F, 16; Class G, 18.5; Class K, 15; and Class M, 17. That is, the stars of Class B have velocities only about one-third as great as those of the stars of Classes F, G, K, and M.

W. W. Campbell found a similar relationship between spectral classes and velocities for the 1193 stars whose radial velocities were measured at the Lick Observatory and at its branch in South America. His results are as follows, the velocities being expressed in kilometers per second: Class B, 6.5; Class A, 11; Class F, 14; Class G, 15; Class K, 16.8; and Class M, 17. Although the stars used by Boss and by Campbell were on the whole quite different, their results are in remarkable agreement. J. Halm suggested that the large stars move more slowly than the small ones for the same reason that heavy molecules in a mixture of gases move more slowly than lighter ones. In the case of the stars the conclusion is doubtful, partly because the stars of various spectral classes are at different distances and partake of group motions.

321. Spectra of Interstellar Elements. In 1904, J. Hartmann found that the K line of calcium in the spectrum of the star α Orionis does not vary in position as do the other lines of this revolving binary (Art. 324). The same phenomenon has since been observed not only for the K line but also for the H line of calcium and the D of sodium in the spectra of many other stars. In fact, J. S. Plaskett has shown that these stationary lines are found in the spectra of practically all Class O and Class B binary stars; and there is clear evidence that they are present in the spectra of single stars as well, for these lines are narrow and sharp even though the

other lines in a star's spectrum be wide and poorly defined. That is, these stationary lines differ in character from those which are due to the atmosphere of the star, and consequently they are recognizable even though the lines of the star's spectrum are fixed in position.

At once the question arises respecting the origin of these stationary absorption lines. Hartmann suggested the hypothesis that they may be due to interstellar calcium clouds, and A. S. Eddington developed the theory that they may be produced by extremely tenuous ionized calcium and sodium gases scattered throughout the sidereal system. It appears that the feeble energy of starlight is sufficient to ionize these gases at their very low density, which is probably of the order of 10^{-26} on the water standard. O. Struve investigated the observational evidence and found that these lines are stronger the more distant the star whose light is analyzed, indicating that the amount of absorption is proportional to the distance traversed by light. This result implies that this tenuous material probably pervades all of interstellar space. It is evident that the use of the observational data may now be reversed, and that the distances of the stars may be inferred from the densities of their interstellar calcium and sodium lines.

It is an interesting and important fact that although there are tenuous calcium and sodium, and probably other elements, throughout our stellar system, it does not appear that they extend to the regions between the galaxies (Art. 379).

II. Double Stars

322. Visual Double Stars. In the history of double-star astronomy no other star has played a more conspicuous rôle than Mizar (ζ Ursae Majoris). In the first place, it and its close neighbor Alcor constitute an unaided-eye double star which has been known at least since the time astronomy flourished among the Arabs a thousand years ago. Moreover, Mizar itself was found by G. B. Riccioli in 1650 to be a double star, the first one discovered with telescopic aid. Finally, one of its two components is the first known spectroscopic double star, having been discovered by E. C. Pickering in 1889.

A definition of what constitutes a double star is of course essential. R. G. Aitken has proposed limits of apparent separation within which stars are to be classified as doubles, and outside of

which they are not to be considered as doubles unless they are otherwise of special interest. For two stars which together are brighter than the second magnitude he sets a limit of angular distance apart of 250″, or about the smallest angle observable with the unaided eye. With this definition Mizar and Alcor do not constitute a double star, but the two components of α Centauri do constitute a double star for their angular separation is about 17″. Since fainter stars are on the average more remote from the earth than the brighter, Aitken sets for them progressively narrower limits of separation in order that they should be classified as doubles. At the fourth magnitude his limit is 100″; at the sixth, 25″; at the ninth, 10″; at the eleventh, 4″; and for all fainter stars, 1″. This is, of course, only a general rule that is not to be strictly followed in case a star has a large proper motion (implying that it is relatively near) or possesses other exceptional characteristics.

Even with the narrow limits of separation set by Aitken, double stars are found to be very numerous. One star in nine of those bright enough to be visible to the unaided eye is a visual double, and one in eighteen of those brighter than the ninth magnitude. This does not mean that a smaller percentage of the fainter stars are doubles; the faint stars are on the average more remote than the brighter ones and hence the components of doubles must be at greater linear distances apart in order that they may be distinguished as two objects. That is, the remote stars can be seen as doubles only if they are very far apart.

323. Double-Star Catalogues. Although Mizar was discovered to be a double star in 1650 and several other stars were found to be multiple in the succeeding hundred years, no one made a systematic search for these objects until William Herschel began his epoch-making sweeps of the sky in 1779. By 1784 he had discovered about 700 pairs, many of which, however, are outside the limits set by Aitken. In 1827, F. G. W. Struve, in Russia, published a catalogue of 3110 double-stars, and since that time much attention has been given to double-star astronomy in many countries. In 1906, S. W. Burnham, the discoverer of many double stars, published a great catalogue of 13,655 of these objects, nearly all of which are north of declination −31°; similar catalogues by R. T. A. Innes cover the remainder of the southern sky. R. G. Aitken has more recently published (1931) a new catalogue containing complete

information for 17,190 pairs, all except a few of which come within the limits he prescribed for the inclusion of such objects in lists of double stars.

A catalogue of double stars is not simply a list of the names or the catalogue designations of such objects. In addition, it contains as a minimum their right ascensions and declinations, the visual magnitudes of their components, the angular distances of their components apart, and their directions from each other. Modern catalogues contain also a history of the discovery of each pair, references to the measurements of it, information respecting the relative motion of the two objects, the types of their spectra, the relative orbits which may have been computed, and other known significant data.

The smaller the angular separation between two stars, the larger the diameter of the telescope required to show them as two distinct objects, the minimum measurable angle varying inversely as the aperture. Telescopes six centimeters in diameter under the best atmospheric conditions resolve into distinct components a double star whose members are separated by about 2″. The great Lick and Yerkes telescopes reach down to angles of 0″.15 or a little less, and Aitken with the Lick telescope has found the overlapping star images giving an elongated pattern when they are separated by only 0″.10. The stellar interferometer (Art. 317) measures distances one-tenth as great, or of the order of 0″.01. In fact, the distance between the two components of one of the spectroscopic doubles of Mizar has been measured by this method, although the angle is only about 0″.012.

Of the double stars in Aitken's catalogue, about 83 per cent are separated by less than 2″, 62 per cent by less than 1″, and 29 per cent by less than 0″.5. As the use of the stellar interferometer is extended, the percentages of stars having very small angular separations will increase.

324. Binary Stars. When the two components of a double star are not only apparently in nearly the same direction from the earth but are actually close together and revolve about their center of gravity, they are called a *binary star*. The final proof that a double star is a binary system is their revolution about their center of gravity, though if they have equal parallel proper motions the probability of their forming a physical system is very great. In

fact, if two stars are apparently within 10″ of each other it is likely that they constitute a binary system, for if 1,000,000 stars were distributed uniformly over the celestial sphere the angular distance between adjacent stars would be about 400″. If 100,000 stars were scattered at random over the sky, the chances are that in not more than six cases would two stars be less than 10″ from each other.

When Herschel began his search for double stars, he did it under the assumption (contrary to probabilities) that such pairs are not related but are simply in almost the same direction from the earth.

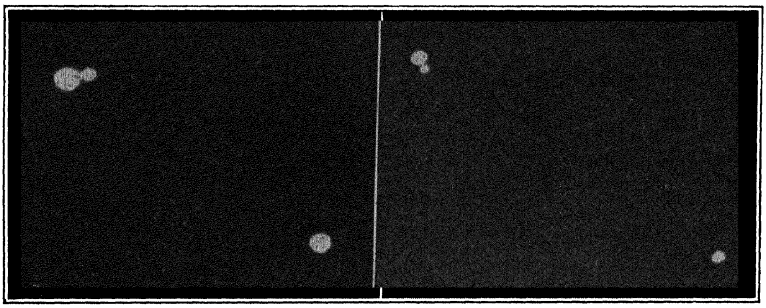


FIG. 171.—Photographs of wide binary star Krueger 60 and distant star, showing orbital motion of the binary between 1908 and 1920. (Barnard.)

His plan was to measure the angular distance apart of the two components of each pair for the purpose of determining the relative parallax of the nearer one (Art. 359). By 1803, he had found to his astonishment that in certain cases the two members of a pair revolve about their center of gravity and constitute a double sun (Fig. 171).

In a few cases the periods of visual binary stars are five or six years, but most of them are much longer. In fact, in many cases the relative motions of the members of binary pairs are so slow that no conclusions respecting their orbits can be drawn from observations extending over so short an interval as 100 years. The periods of binary stars are generally long not because their masses are small, but because their linear distances apart are very great compared to the distances from the sun to the earth. In spite of these great linear separations, the stars are so enormously remote from the earth that the angular separations of more than 17,000 double systems come within Aitken's narrow limits; and, as we shall

presently learn, large numbers of other pairs are so close that their double character has been established only by means of the spectroscope.

325. Apparent and Actual Orbits of Binary Stars. It is obvious that only very exceptionally is the plane of the orbit of a binary star perpendicular to the line of sight. Consequently the *observed* path described by one star with respect to another is not its *actual* orbit, but the projection of its actual orbit on a perpendicular surface, a small portion of the celestial sphere.

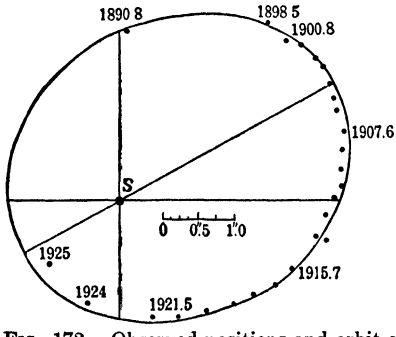


FIG. 172.—Observed positions and orbit of Krueger 60. (Aitken.)

It follows from the law of gravitation that the relative orbit of the components of a binary star is an ellipse. Since the projection of an ellipse inclined at any angle to the line of sight remains an ellipse, it follows that the apparent orbits of binary stars are ellipses, with the straight line as a limiting case for an orbit edgewise to the earth. In the projected orbit, however, the principal star is not in the focus of the orbit described by its companion, though it is in the focus of the actual orbit.

In Figure 172 the observed positions of the companion of Krueger 60 are plotted with respect to the principal star of the binary as an origin. The scale of this diagram is very great, as is evident on comparing it with the photograph (Fig. 171), and the errors of the observations are very small. In determining the apparent orbit, use is made of the fact that, aside from errors of observation and possible disturbing effects of a third star, the companion describes an apparent ellipse with respect to the principal star in such a way as to fulfill the law of areas (Art. 175).

After the apparent orbit of one component of a double star with respect to the other has been established by observations extending over at least a large part of one revolution, the problem remains of determining the actual orbit. In the actual orbit one star is in a focus of the ellipse described by the other. The problem is to find such an ellipse that, when it is rotated about a line to be

determined through an angle also to be determined, the result will be the observed ellipse with the focus of the actual ellipse projected into the principal star in the apparent ellipse. The line of rotation is called the *line of nodes*, and the angle of rotation is called the *inclination* of the orbit.

As would be expected, the lines of nodes of the orbits of binary stars are distributed at random and their inclinations range from zero in the case of γ Virginis to 90° in the case of 42 Comae Berenices. The eccentricities of the orbits of binary stars range from nearly zero to 0.95 in the case of γ Virginis, the average eccentricity, according to T. J. J. See, of those whose orbits are known with considerable accuracy (about 100 in number) being approximately 0.50. Therefore in most binary pairs the distances between their members vary greatly in a revolution. For example, the distance between the components of γ Virginis at their apastron is nearly 40 times greater than it is at their periastron. Therefore their mutual tides, which vary inversely as the cube of their distance apart, are 64,000 times greater at their periastron than at their apastron.

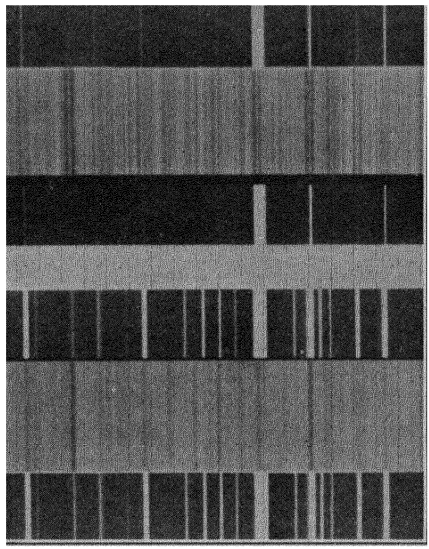


FIG. 173.—Spectrum of Mizar, lines double above and single below. (Yerkes Observatory.)

326. Spectroscopic Binary Stars. In 1889, E. C. Pickering found that the spectral lines of one of the components of the star Mizar are periodically doubled, as shown in Fig. 173. The explanation of the phenomenon is that the star is composed of two components revolving in a plane passing near the solar system, and that at certain times one of the stars is moving toward the earth while the other is receding (Fig. 174). The spectral lines of the approaching star are displaced toward the violet end of the spectrum;

those of the receding star, toward the red end of the spectrum, the result being double lines twice each complete revolution of the pair. Hundreds of similar stars have since been discovered at the many observatories which have pursued this line of research.

That the explanation of the doubling of the lines is correct is proved by the fact that in the case of a number of stars the planes of the orbits pass so nearly through the earth that each of the stars partially or totally eclipses the other once in each revolution. When the stars are in a line with the earth, the spectral lines are not doubled and the eclipse occurs; when the stars are at right angles to the line of the earth, the spectral lines are doubled and neither star eclipses the other. The time relationships between the eclipses and the doubling of the spectral lines are in exact harmony with the theory that these stars are binaries.

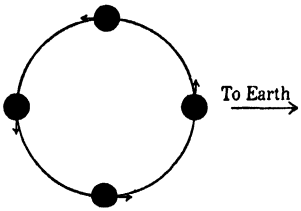


FIG. 174.—Spectroscopic binary star.

There are other spectroscopic binaries besides those whose lines are periodically doubled. If one component is less than one-third as bright as the other, its spectrum is not ordinarily observable; but the spectral lines of the brighter component are displaced alternately toward the red end and the blue end of the spectrum as a consequence of its revolution around the center of gravity of the pair. In some of these cases, as in that of the variable star Algol, the relatively dark star eclipses its brighter companion in the period of the displacement of the spectral lines and in the phase relationship required by the theory that the star is binary. In the *Third Catalogue of Spectroscopic Binaries* of the Lick Observatory (1924) there are listed 1054 stars, and in only one pair in six were the spectral lines of both of the companions photographed.

Spectroscopic binary stars have relatively short periods, the smallest period known, that of W Ursae Majoris, being about one-third of a day. The periods of revolution are so short in the case of some of the giant stars of Class B that the surfaces of the two components must be nearly or quite in contact.

327. Eclipsing Binary Stars. Historically, eclipsing binary stars were observed and interpreted long before the discovery of spectro-

scopic binary stars. In 1670, G. Montanari noticed that the apparent brightness of the second-magnitude star Algol (β Persei) at times is greatly reduced. A little more than a century later (1782), J. Goodricke found that the reductions in brightness of this star recur at regular intervals of $2^d\ 20^h\ 49^m$, and he suggested the hypothesis that the phenomenon may be due to partial eclipses of Algol by a relatively dark companion revolving around it. The explanation was nearly forgotten until 1889 when H. C. Vogel, at Potsdam, found that the spectral lines of Algol are alternately displaced toward the red and the violet in the period of its variations in brightness and in phase relationships agreeing with the eclipse theory. Since the observations of Vogel, astronomers have found that the positions of the spectral lines of every eclipsing variable bright enough for spectroscopic examination are periodically displaced toward the red and the blue in the period of light variations. In 1927, according to the data collected by D. B. McLaughlin, the spectra of 48 of about 200 eclipsing binary stars had been measured for radial velocities. In 28 pairs, the spectra of both stars had been measured; in 17 pairs, the spectrum of only one component was observable; and in 3 pairs, the spectral lines were broadened, presumably as a consequence of the revolutions of the stars.

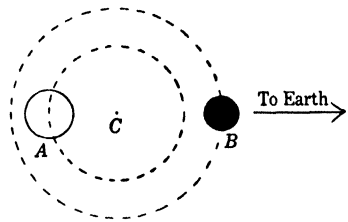


FIG. 175.—Eclipsing binary star.

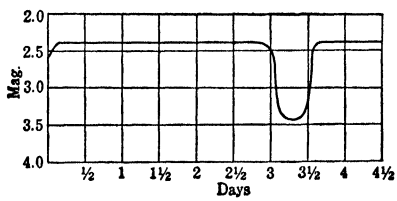


FIG. 176.—Light curve of eclipsing binary star.

The converse evidence is equally convincing, for Joel Stebbins and C. M. Huffer have found with the sensitive photoelectric photometer that nearly all spectroscopic binaries having short periods are variables, at least to some slight extent. Moreover, the phases of the radial velocities and the variations in brightness are in harmony with the theory that these stars are pairs of suns.

There are many varieties of eclipsing binary stars. If a pair consists of a bright star A, Fig. 175, and a dark star B revolving

about their center of gravity C in a plane passing through the earth, then when the eclipse is central the conditions will be as represented in Fig. 175. The light curve for such a binary star, which is of the Algol type, is illustrated in Fig. 176. The depth of the minimum depends upon the relative diameters of the two stars; the period from minimum to minimum, upon the masses and the distance of the stars apart; the duration of the eclipse, upon the ratio of the diameters of the stars to the perimeter of their relative orbit; and the shape of the light curve during the eclipse, upon the oblateness of the bright star and the rate of diminution of its brightness from its center to its limb.

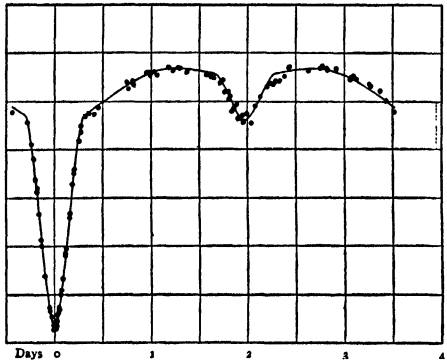
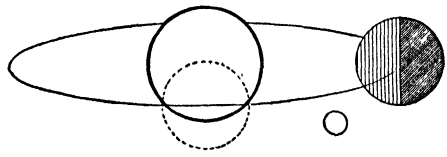


FIG. 177.—Light curve of λ Tauri. (Joel Stebbins.)

If the dark star is as large as the bright one or larger, a central eclipse is total. If two stars are equal in diameter and brightness and pass each other in central eclipse as seen from the earth, their light at minimum is half that at maximum, or about 0.75 of a magnitude. In this case the period of revolution of the binary is the interval from a minimum to the second succeeding minimum. Variations from this simple case are those in which the stars are of unequal brightness, or have unequal diameters, or only partially eclipse each other. If an eclipse is not central, there are variations in the light curve which are easy to determine. The actual problem, of course, is the converse, for the light curve is obtained from the observations and the nature of the eclipse is to be inferred from them. Nearly all imaginable cases have been found by observation. In the light curves of some stars the illumination of the darker component by the brighter, as well as the tides raised by each star in the other, apparently has appreciable effects.


FIG. 178.—System of λ Tauri, small circle below represents the sun. (Stebbins.)

In all the foregoing discussion no mention has been made of the fact that the orbits of eclipsing binary stars are eccentric and in some cases considerably elongated. It is evident that their light curves are affected by the eccentricities of their relative orbits. In all cases, however, the light curves are found from observations and the physical conditions are to be inferred. It is evident that the problem of interpreting the light curves of eclipsing binaries is one of considerable difficulty because of the great variety of possible factors which may be involved.

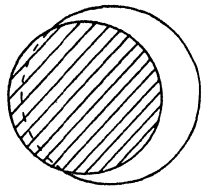


FIG. 179.—Eclipsing binary.

328. Rotations of Eclipsing Binary Stars. Presumably the axes of rotation of the members of a binary star are approximately perpendicular to the plane of their relative orbits, just as the axes of the planets are nearly perpendicular to their planes of revolution. If this presumptive relationship exists in binary systems, eclipsing binaries offer the most favorable possible conditions for the determination of axial rotations (Art. 319), for the planes of the equators of their members pass near the earth.

Eclipsing binaries in which one of the two stars is dark provide a precious test of the theory of determining the rates of rotation of binary stars from the distributions of density in their spectral lines. When the dark component of a binary pair has eclipsed all except one margin of the bright star, as is illustrated in Fig. 179, light is received only from the approaching or the receding part of the star, depending upon the direction of its rotation. Consequently if the theory of the broadening of its spectral lines by the rotation of a star is correct, the lines under these circumstances should be broadened in one direction but not in the other.

When the eclipsing star is slightly smaller than the brighter and the eclipse is central, light is received from both margins of the bright star and the spectral lines should be double. As the eclipse begins to wane, light is received from the other margin of the bright star, and the spectral lines should be broadened in the opposite direction. All these implications of the theory have been verified in the spectra of eclipsing binary stars by F. Schlesinger and other astronomers, and therefore there can be little doubt that the spectroscopic method of determining the rotation of stars,

single and double alike, is both sound in theory and reliable in practice. The method fails, of course, for those stars whose axes of rotation are approximately in line with the solar system.

329. Distribution of Binary Stars Among the Visual, Spectroscopic, and Eclipsing Classes. Although some visual double stars are spectroscopic binaries and some spectroscopic binary stars are eclipsing binaries, the three classes in general are not identical. In the first place, only those binary stars whose orbital planes pass through, or at least near, the earth can be either spectroscopic or eclipsing binaries. Then, too, the farther apart are the components of a pair, the better the chance of detecting the two stars as a double with the telescope; but the greater the distance between two stars, the longer their period of revolution, the lower their orbital velocity, and the more nearly must their orbital plane pass through the earth in order that there may be an eclipse. Moreover, if an eclipse of one star by another occurs only once in an interval of many years, the chance of its being observed is very small. Only a small fraction of the many thousands of known double stars have yet moved enough with respect to each other to prove that they are binaries; and of the visual doubles which are known to be binaries, only a small fraction are spectroscopic or eclipsing binaries.

If the plane of the orbit of a binary star is perpendicular to the line from the star to the earth, it can not be a spectroscopic binary because it has no component of velocity toward or from the earth. It is not necessary, however, that the plane of the orbit should pass exactly through the earth in order that a Doppler displacement of the spectral lines shall be observable, for, if the components of the binary move with high velocity, an inclination of the plane of their orbit of 20° or 30° will give an observable radial component, especially if the spectral lines of the stars are sharp. If the components of a binary are far apart and move with velocities comparable to that of the earth in its orbit about the sun, a higher inclination of the planes of its orbit is necessary in order that the Doppler effect on their spectral lines shall be great enough to be measured. It follows that most spectroscopic binary stars are not eclipsing binaries. On the contrary, all eclipsing binaries are necessarily spectroscopic binaries, though in most cases eclipsing variables are so faint that photographs of their spectra have not been secured.

330. Masses of Visual Binary Stars. As has been emphasized in several connections, the masses of the celestial bodies are determinable only from their gravitative effects upon other bodies. Consequently, we are able to determine directly the masses of only those stars that are binaries. If the masses of a binary system are represented by m_1 and m_2, the semi-axis of the ellipse one describes with respect to the other by a, and their period of revolution by P, then the relationship among these quantities is (Art. 177)

$$m_1+m_2=\frac{a^3}{P^2},$$

in which P is expressed in years when a is expressed in terms of the semi-axis of the earth's orbit and m_1 and m_2 in terms of the mass of the sun.

In the case of a visual binary star, the observations give the period P and the angular distance between the components as seen from the earth. In case the distance of a binary pair from the solar system is known, the distance of its two components from each other can be computed, for the distance from the observer and the apparent angular separation of a pair determine their linear separation. Then the foregoing formula can be used for the determination of the sum of the masses of the two stars.

As promising as this method at first appears to be, it has not yet led to many reliable results, for the periods of the visual binaries are so long that not over 100 are known with considerable accuracy, and the stars are all so remote that the distances of only a few of them have been directly measured. According to the statistical studies (1929) of visual binary stars by J. H. Pitman, there are only 33 pairs whose parallaxes have been determined with probable errors not exceeding 15 per cent of the quantities measured, and there are only 20 additional pairs whose parallaxes have been determined with probable errors lying between 15 per cent and 30 per cent of the quantities measured. It follows from the formula that the percentage error in the sum of the masses is about three times as great as the percentage error in the mean distance a of the stars apart; and the percentage error in a is approximately equal to the percentage error in the parallax of the pair, for a varies inversely as the parallax. Consequently it is not surprising that the masses of only a few purely visual binaries have so far been determined.

Although the foregoing formula gives only the sum of the masses of a binary pair, it will be ultimately possible in most cases to determine the individual masses of the two stars. The first problem is to find from the proper motions of the stars the center of gravity C, Fig. 180, on the line joining the stars m_1 and m_2 which describes a straight line L with uniform speed. The masses of the stars are inversely as their distances from their center of gravity C; or $m_1 = rm_2$, where r is the inverse ratio of the distances of m_1 and m_2 from C. Then when the sum m_1+m_2, is known, m_1 and m_2 are separately determined, for they must satisfy two independent linear equations.

In only about 20 cases have the individual masses of visual binary stars been so far determined, and they range from 0.2 to 4 times the mass of the sun. The stars of smallest known mass are the components of Krueger 60, whose masses are 0.18 and 0.27 in terms of the sun's mass as unity. As would be expected, these are low-temperature stars. They are of spectral Class M3, and their absolute magnitudes are 13.4 and 11.8 respectively. The luminosity of the sun (abs. mag.+4.85) is therefore more than 2600 that of the fainter of these stars and more than 600 times that of the brighter. It is highly probable, particularly in view of the dark stars found in eclipsing binaries, that there are many fainter binary stars having still smaller masses, but they are as yet unknown.

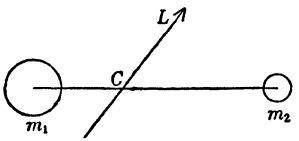


FIG. 180.—Motion of center of gravity of binary star.

Sirius is one of the most massive of the visual binaries (there are much more massive spectroscopic binaries). In terms of the sun, its mass is 2.50 while that of its faint companion is 0.97. These stars are of spectral class A0 and probably F0 respectively and of absolute magnitudes 1.3 and 7.1. It follows that the luminosity of Sirius is 26 times that of the sun, and that the luminosity of the sun is 100 times that of the fainter companion of the pair. Only in the case of α Aurigae among visual binaries are the masses greatly in excess of that of Sirius, and in this pair they are 4.2 and 3.2 times that of the sun. These stars are giants of the spectral classes G0 and F5 respectively, and their absolute magnitudes are -0.2 and 0.1 respectively, corresponding to luminosities 105 and 86 times that of the sun.

331. Masses of Spectroscopic Binary Stars. It is certainly surprising that we are able to determine the masses of binary stars which are so remote in most cases that they appear as single objects when observed through the largest telescopes in the world. As we shall see, it is not even necessary to know the distances of these stars, as it was in the case of visual binaries, in order to apply the method by which their masses are found.

(*a*) For simplicity, consider first a spectroscopic binary having the spectral lines of both stars visible, and suppose it is also an eclipsing binary. It follows that the plane of the orbits of the revolving stars passes through the earth. The maximum separation of the spectral lines at the time they are double determines the relative velocity of the stars in their orbit. Since the period P of revolution is known, the speed of one star with respect to the other enables us to compute the perimeter and the major axis of the orbit. Then by the formula of Art. 330 we determine the sum of the masses of the stars. The fact that the orbits are more or less eccentric introduces, of course, some complexities which we are at present ignoring.

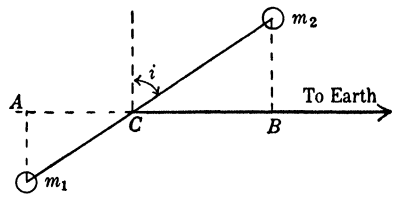


FIG. 181.—Component of radial velocity.

The radial velocity of the center of gravity of a binary system is determined by the displacement of the spectral lines when the stars are in a line with the earth and their spectral lines are single. The displacements of the lines of the separate stars with respect to the position of these combined lines determine their respective velocities relative to their center of gravity, and hence their relative distances from their center of gravity and therefore their masses. That is, it is possible in these cases to find the mass of each star of a spectroscopic binary entirely by spectroscopic means.

(*b*) Consider next the case in which the spectral lines of both members of a binary are visible, but in which the stars are neither a visual nor an eclipsing pair. In this case the inclination of the orbits of the stars to the line of sight remains unknown, and the observed velocity is only the radial component of the actual velocity. If i represents the angle between the plane of the orbits of the stars and the plane perpendicular to the line from the star to the earth, as illustrated in Fig. 181, the observed components of radial

velocity are the same as the actual components would be if the plane of the orbits passed through the earth and the distance between the two stars was AB. Consequently the derived major semi-axis a_0 in this case will be $a_0 = a \sin i$, where a is the true major semi-axis, i the unknown inclination, and the formula for the sum of the masses becomes

$$m_1 + m_2 = \frac{a_0^3}{P^2 \sin^3 i}.$$

This formula gives the smallest value for $m_1 + m_2$ when $i = 90°$. Hence in the case of an individual binary we have only the lower limit to the sum of the masses, but in statistical investigations involving many stars, the order of the magnitude of $m_1 + m_2$ can be determined because the average value of $1/\sin^3 i$ can be computed for many orbits oriented at random.

(*c*) If a double star is a visual binary whose orbit has been determined, the value of i is known. Since a_0 is supposed to have been determined by the displacement of the spectral lines, the formula in this case gives the value of the sum of the masses within the limits of accuracy of the various kinds of observations.

(*d*) In most spectroscopic binary stars the spectrum of only the brighter component is visible and measurable. Consequently the measured maximum radial velocity v_0 is related to the relative velocity v of the pair in the plane of their orbit by the equation

$$v_0 = \frac{m_2 \sin i}{m_1 + m_2} v,$$

and a_0 and a are related by a similar equation. Hence in this case the formula for the sum of the masses of the two components is

$$m_1 + m_2 = \frac{a_0^3 \left(1 + \frac{m_1}{m_2}\right)^3}{P^2 \sin^3 i},$$

in which i and m_1/m_2 are both unknown. Since by hypothesis, m_2 is fainter than m_1, the ratio m_1/m_2 will usually be greater than unity. Hence the sum $m_1 + m_2$ will be greater than $8a_0^3/P^2$.

In general, spectroscopic binary stars are pairs in which the components are near each other and have large masses, for these are the conditions under which large and the most easily observable variations in radial velocities occur. Among these stars, masses are seldom found smaller than that of the sun. Aside from the somewhat doubtful case of 27 Canis Majoris (Art. 318), the most

massive pair so far discovered is known as BD 6°1309, a sixth-magnitude star in Monoceros, whose lines are periodically double. The masses of these stars are at least 75 and 63 times the mass of the sun, for these results are obtained by taking $i = 90°$ in the formula under (*b*) in the foregoing discussion.

332. Densities of Eclipsing Binary Stars. Only in the case of stars which are both spectroscopic binaries and eclipsing variables is it possible to determine directly stellar densities. Suppose the period of revolution and the major axis of a binary star have been determined. Since the speeds of the stars in their orbits are therefore known, the durations of the minima determine the diameters of the stars. In the simple case, illustrated in Fig. 182, in which the eclipse is central and the star disks are uniformly bright, the light of the pair begins to decrease when the star m_2 has arrived at the point 1 and it reaches its minimum when it moves its diameter and arrives at the point 2. Since the speed of m_2 with respect to m_1 is known, the diameter of m_2 is determined. The minimum continues until m_2 moves to the point 3, a distance equal to the diameter of m_1 less the diameter of m_2, and the diameter of m_1 is therefore known. When m_1 eclipses m_2 a similar determination of the diameters of the stars can be made. The densities of the stars can be easily computed from their masses and their diameters.

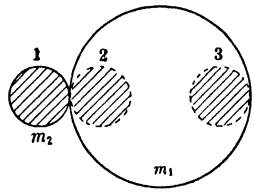


FIG. 182.—Eclipsing binary star.

In practice the problem is considerably more complicated than is indicated in the foregoing discussion. In general, an eclipse is not central and the astronomer has only the light curve of the pair to guide him in inferring its character. Moreover, the orbits of binary stars are not circular and they may be very eccentric. Finally, the apparent disks of stars are not uniformly luminous, and the stars may be oblate. In spite of these complexities, astronomers coördinate the observational data respecting light curves and radial velocities and derive results worthy of our acceptance.

When D. B. McLaughlin made a catalogue of eclipsing binary stars in 1927, there were 41 pairs whose masses had been determined. In 38 of these cases the dimensions and the densities of the

stars had also been found. In the greatest pair, a star of spectral Class 08.5, known as HD 1337, the masses, according to J. A. Pearce, are 36.3 and 33.8 times greater than that of the sun, and the luminosities are 21,000 and 7,000 times greater than that of the sun. The period of revolution of this pair is only 3.52 days, and the densities of the stars, in comparison with the density of the sun, are 0.003 for the primary and 0.009 for the secondary. The smallest masses are those of Castor C, a star of spectral Class M1, each of whose components is only 0.6 of the sun in mass. The average mass of all these stars is about 8 times that of the sun.

It is an interesting fact that the densities of the stars whose masses and diameters are known are nearly all much less than the density of the sun, though the density of W Ursae Majoris is 2.8 times that of the sun. The result may be due in part to the fact that the greater the volume of a star, the more likely it is to be an eclipsing binary and to provide the data necessary for the determination of its density.

333. Relationships of Spectra to Other Properties of Binary Stars. Since more properties can be determined of binary stars than of those which are single, these stars provide the best opportunities for correlating stellar spectra with mass and other physical characteristics. In the case of a number of visual doubles whose spectra could be separately determined, F. C. Leonard found that if the two components of a pair are approximately equal in magnitude, their spectra, with few exceptions, are similar. If the two stars of a pair differ in magnitude, they differ in spectra and the differences in spectra increase with differences in brightness. As a rule the fainter star of a pair is redder than its brighter companion, and hence of a so-called later spectral class. In a considerable number of stars, however, the relationship is reversed, with the fainter star the bluer. These latter stars belong to the giant class, while those in which the fainter star is the redder belong to the main stellar sequence (Art. 310). Moreover, double-star observers are generally of the opinion that the fainter of the two components of binary pairs are the bluer.

(*a*) Among the spectroscopic binary stars, there is a significant relationship between spectral class and period. In the case of stars of Classes O and B, 71 per cent of the known binaries have periods of less than 10 days. For Class A stars, the percentage is 64; for

Classes F and G stars, it is 52; and for Classes K and M stars, it is 16. Moreover, the percentage of stars of Classes O, B, and A which are binaries is much greater than that of the stars toward the other end of spectral sequence.

The short periods of the spectroscopic binary stars of Classes O and B are in part a consequence of their relatively large combined masses, which are from 8 to 150 times that of the sun. The stars at the other end of the spectral sequence are comparable to the sun in mass.

(*b*) There is also an interesting relationship between the masses (and the spectral types) of binary stars and the eccentricities of their orbits. The stars with largest masses describe the least eccentric orbits. In terms of periods and eccentricities, it is found that the average eccentricity of the orbits of the stars whose periods are less than 6 days is only 0.06; for periods between 6 and 20 days, it is 0.20; and for spectroscopic binaries with longer periods, it is 0.32. The increase of eccentricity with period continues among the visual binaries. For stars whose average period is of the order of 100 years, the average orbital eccentricity is about 0.5, and it appears to increase with the period beyond this limit, though the orbits of binaries with long periods are as yet poorly determined.

(*c*) If eclipsing binary stars are sufficiently bright, the spectral classes to which they belong can be determined directly with the telescope; if they are faint, their spectral classes in some cases can be inferred from their visual and photographic magnitudes at their minima. Suppose, for simplicity, that the two stars of a pair are of equal diameter but of different spectral types, and that they centrally eclipse each other. When m_1 eclipses m_2, only light from m_1 is received at the earth. The difference in the visual and the photographic magnitudes at this minimum determines the color of m_1. Similarly, when m_2 eclipses m_1, the color of m_2 can be determined. There is the check on the results that the combined radiation must have the quality of the light received from the pair when neither star is in eclipse.

Investigations of the spectra of eclipsing binary stars confirm the theory that the spectrum of a star depends upon its mass and density, the temperature being determined by these data, but the subject is yet in its early stages. It may confidently be expected that the study of the spectra of binary stars, and particularly of

eclipsing binaries, will eventually lead to such a correlation between the spectra and the other characteristics of stars that the spectrum alone of a star will contain implicitly its complete description. Then our detailed exploration of space will be limited only by the power of our spectroscopes combined with telescopes (Art. 276).

334. Multiple Stars. Several cases have been noted in which more than two stars are associated. For example, Mizar is a telescopic double, and each of its components is a spectroscopic binary. It is probable that all four stars are physically related, though the unaided-eye components are so far from one another that no relative motions of one pair with respect to the other pair have yet been observed.

There is another type of triple star, consisting of a bright pair and a relatively distant faint companion. One of the most interesting systems of this kind is α Centauri which has a distant faint companion discovered by R. T. A. Innes in 1915. The two bright components of α Centauri are similar to the sun in mass and brightness; the red faint companion is not more than one-millionth as luminous. According to H. L. Alden, its distance from the brighter pair is 14,000 times as far as from the sun to the earth, and its period of revolution about them is of the order of 1,000,000 years. As it happens, this faint companion is nearer (4.16 light-years) the solar system than the brighter pair; in fact, it is the nearest known star, and hence has been named *Proxima*. The star Castor is a similar system, consisting of an easy telescopic double having a faint attendant at a distance of 73″. In this case, however, each of the three stars is a spectroscopic binary. Usually triple-star systems consist of a binary and a relatively remote companion, though there are a few cases in which the distances of the three stars from one another do not differ greatly.

Besides the double, triple, and quadruple stars, there are groups comprising larger numbers. For example, in the Hyades group there are 40 or 50 stars and in the Pleiades 100 or more. In these particular clusters the stars are much farther from one another than they are in the doubles and triples heretofore considered, but there appears to be no clear line of demarcation between the two classes of systems of multiple stars. Moreover, there are compact clusters of tens of thousands of suns such as the Great Cluster in Hercules and the one known as ω Centauri.

335. Origin and Evolution of Multiple-Star Systems. In this article we are interested primarily in considering whether binary and multiple stars originate from single stars or the opposite, and not in the origin and the evolution of stars as individual bodies. Of the two hypotheses that have been formulated respecting the genesis of binary stars, the earlier and the more completely developed is that a binary, like a simple living organism, is born by the fission of a parent mass into two parts. The other is that a single star acquires a mate of an independent origin.

(*a*) In the fission theory the division of a star into two parts is ascribed to the effects of increasing rotation as it radiates energy and contracts. If it were not losing mass and were contracting, the principle of the constancy of its moment of momentum would require it to rotate at a continually increasing rate. Its oblateness would increase for a time, then it would take a somewhat elongated form, and finally it would become unstable and break up. At this point in the theory it is assumed that the fragments would reassemble into two stable globular masses at some distance from each other. Each of these parts might continue to contract until another condition of instability should set in, followed by its fission. In this way multiple stars are explained under the hypothesis.

Most of the facts established by the observations do not obviously contradict this qualitative outline of the fission theory. For example, the surfaces of the components of many pairs of spectroscopic binaries must be nearly or quite in contact, for their short periods and moderate velocities imply large masses moving in relatively small orbits. Moreover, there are eclipsing binaries, of which β Lyrae is the typical example, in which the apparent magnitudes of the stars vary continuously as though the two members of the pairs were revolving in contact, and the characteristics of the displacements of their spectral lines are in harmony with the interpretation. Hence we have a tentative explanation of the binaries of large masses and short periods, and the pairs having short periods are supposed to be transformed into binaries having long periods by the effects of tidal friction.

Although the fission theory at first appears satisfactory, further examination shows that it is subject to serious difficulties. In the first place, when a star contracts its increasing density tends to make it *less* oblate, while its more rapid rotation tends to make it *more* oblate. The latter effect predominates, but by so slight a

margin that the sun, for example, would not become so oblate as Saturn is now until it had shrunk to a diameter of 150 kilometers. In the second place, it is a pure assumption that a star having arrived by contraction at an *unstable* state will break up into two *stable* and approximately *equal* masses. In fact, as far as theory goes, it suggests that if there were only two parts after fission they would be very unequal. In the third place, the mutual tides of an approximately equal pair would be wholly incapable of increasing their period from a few hours or days to the many years found in the visual binaries. In the fourth place, if a star did divide into two masses, each of its parts could not divide again until its density had increased a thousand-fold, with a similar increase in density before each subsequent fission.

J. H. Jeans has examined the fission hypothesis and has recognized the difficulties that have been enumerated. To avoid them he appeals to the decrease in the masses of the stars by radiation, a process that results in increases in their periods. He ascribes the greater eccentricities of binary orbits having longer periods to the disturbing effects of the galaxy of stars, at least for binaries whose periods of revolution are less than 55 days. He concludes that those having periods longer than 55 days were formed from condensations in an ancient parent nebula. He even suggests that our galaxy of stars owes its form and structure to the fact that it has descended from a giant nebula by successive divisions. Yet there still remain many gaps in the fission theory of the origin of binary stars that must be filled by quantitative investigations before it can be accepted as anything more than a hypothesis to be considered.

(*b*) Many vague suggestions have been made that one star may capture another when they chance to pass near one another, but most of them violate essential dynamical conditions. For example, if two stars approach each other from a distance, they will return to the same distance unless they at least partially collide; and if they collide, at each succeeding approach they will collide again. Hence it is evident that enormous orbits of the visual binaries can not have originated in this manner. To overcome the objections of this hypothesis, it has been suggested that the original close approach of a pair occurred when at least one of the stars was attended by remote planets or nebular masses with which the other star collided. It follows from the high relative velocities of the stars that

such a collision would not produce a binary at all unless the material encountered were in mass a considerable fraction of that of the visiting star.

As was explained in Art. 307, MacMillan has made an important modification of the capture theory. He assumes that only planets are born at the time of a passing star, and that at least one of these planets grows by the accretion of scattered material into a rival of the principal star. The mass of the planet would gain on that of the star both because a small body would grow relatively faster than a large one and also because only a hot star would lose mass appreciably by radiation. According to this theory, the remote visual binaries moving in elongated orbits are of recent origin; the closer spectroscopic and eclipsing binaries are mature pairs that for long periods of time have traveled in regions rich in the materials on which stars grow; and the giant binaries whose surfaces are almost in touch represent the last stage before their union into a single mass. After such a union, the rate of radiation would increase and exceed that of the acquisition of mass. On passing through a region of abnormally low density, the mass of the star would decrease and, if long continued, it would attain the dwarf stage. At any time in its career it might acquire new planets as a consequence of the effects of a passing star and start again on its way toward becoming a binary pair.

It is obvious that this hypothesis, too, lacks much of having the theoretical and observational verifications that are necessary to raise the tentative explanation to the level of an established theory.

III. Variable Stars

336. Principal Classes of Variable Stars. In spite of the variety of the light curves of eclipsing variables, which range from those having such short, sharp minima as that of Algol to those continually varying in brightness like β Lyrae, there are several classes of stars whose variability is clearly due to other causes. The principal other classes are:

(*a*) *The Cepheids*, which we shall later see have been of very great importance in determining the dimensions of our galaxy and the distances to other galaxies.

(*b*) *Cluster variables*, which are at least related to the Cepheids, and which have enabled astronomers to measure the distances and

the diameters of the great aggregations of stars in which they are found.

(*c*) *Variables of long period,* whose phenomena, though yet unexplained, promise to throw light on the evolution of stars.

(*d*) *Irregular variables,* whose anomalous behavior restrains us from prematurely concluding that our understanding of the stars is essentially complete.

(*e*) *Temporary stars, or novae,* which flash out brilliantly in the sky for a few days and then rapidly fade away, accompanied by extraordinary and puzzling phenomena.

There are now (1931) known between 5000 and 6000 stars which vary in brightness, and the number is being increased very rapidly by new discoveries. In many cases the range of variation in magnitude is small and the properties of the light curves have not been determined. Among these variables, about 300 are eclipsing binaries, nearly 200 are Cepheids, about 900 are cluster variables, approximately 1100 are variables of long period, and the remainder are either irregular variables or of unknown light-curve characteristics.

337. Characteristics of Cepheid Variables. The Cepheid variables owe their name to the fact that δ Cephei is the first star of the class whose light curve was thoroughly investigated. Although these stars do not vary in brightness as the eclipsing stars vary, their luminosity is nevertheless periodic, with periods ranging from approximately a day to about 40 days in one or two extreme cases. Moreover, their spectral lines are periodically displaced in the periods of their light variability. Therefore it is not surprising that for a long time astronomers erroneously assumed that they are unusual examples of eclipsing and spectroscopic binary stars.

Instead of descending to a minimum in brightness and ascending nearly symmetrically to a maximum, as eclipsing variable stars do (Fig. 176), typical Cepheid variables descend gradually and often with some irregularities from sharp maxima to relatively flat minima, and then increase rapidly and more smoothly to their maxima. The descending branches of their light curves usually cover about three-quarters of their periods, as illustrated in Fig. 183. In some Cepheid variables, however, such as ζ Geminorum, the light curves are nearly symmetrical with respect to their maxima. At their maxima, most of these stars are not more than twice

as bright visually as they are at their minima and the range is often much less, though in rare cases it may be somewhat greater. The photographic variability in magnitude is considerably greater than the visual, a fact that implies that these stars become redder as they diminish in brightness.

Although the spectral lines of Cepheid variables are displaced in the periods of their light variations, the phase relationships are not in accord with the eclipse theory. At the time of maximum brightness of a Cepheid variable its spectral lines have their largest displacement toward the violet, indicating that they have then a maximum velocity of approach; and at the time of minimum brightness the lines are shifted toward the red, indicating recession. According to the eclipse theory, there should be no displacements of spectral lines at the minimum brightness, for the motion of the two stars would then be perpendicular to the line of sight. Moreover, the radial velocities of the Cepheids are so low that the orbits of the bodies would be too small to provide room for stars of their probable dimensions. Finally, their spectral-line displacements, interpreted as Doppler effects of radial velocities, do not vary in harmony with elliptic motion.

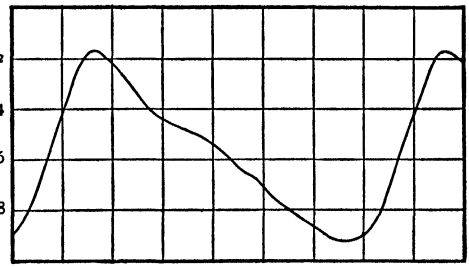


FIG. 183.—Light curve of typical Cepheid.

There is a general correlation between the periods of Cepheid variables and their spectral classes. The spectra of stars having periods not exceeding a day are usually of Class A; those of stars having periods between 1 and 20 days are of Classes F and G; and those of stars having periods longer than 20 days are of spectral Class about G5. It is a significant fact that the spectral types of Cepheid variables change periodically with their brightness, their spectra at their maxima being characteristic of higher temperatures than those at their minima. Moreover, the spectra of the Cepheids have the characteristics of stars which have been found to be super-giants of very high luminosity (c-stars), and it is probable that at their maxima they are at least 500 times as intrinsically luminous as the sun.

Since Cepheid variables are very luminous stars, they are visible at great distances, yet the number of them known is not very large. Probably not more than one star in a million in our galaxy of stars is a Cepheid variable. They are concentrated in the galactic zone, they have small proper motions, and their radial velocities (variable parts eliminated) are low, all of which confirms the conclusion that these stars are super-giant suns widely dispersed throughout the galaxy.

338. Characteristics of Cluster Variables. About 1890, a few variable stars were discovered on photographs of two or three of the largest globular clusters, but our knowledge of these objects really began in 1895 with the work of S. I. Bailey at the South American branch of the Harvard Observatory. In the great ω Centauri cluster he found 125 variables out of the 3000 stars which he examined. There are now 10 clusters in each of which more than 25 typical cluster variables are known, and there are more than 30 other clusters in each of which at least one has been discovered. The characteristics of these objects are well established from the 900 known examples of them. The considerable number of variables in at least 10 clusters and their scarcity elsewhere proves that they really belong to the clusters instead of appearing in them only by projection.

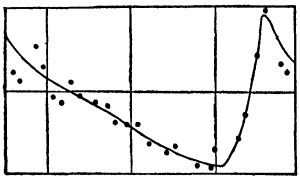


FIG. 184.—Light curve of cluster variable.

The general characteristics of the light curves of cluster variables are similar to those of the Cepheids. The most important difference between the two classes is that the periods of the cluster variables are nearly all less than a day; in fact, they are usually in the neighborhood of 12 hours. In the case of these stars the increases from minima to maxima, as illustrated in Fig. 184, are generally somewhat more rapid than in typical Cepheid variables though there are two subclasses in which this is not the case. The variations in brightness are of the order of a magnitude or two. In a given cluster the median magnitudes of the variables are almost exactly equal to one another, even though there be considerable differences in periods or in the characteristics of their light curves. It is a fact whose significance is not yet understood that most cluster variables, at median brightness, are from 1.5

to 2 magnitudes fainter than the brightest stars in their respective groups. Hence at their maxima they approach in luminosity the most brilliant super-giant stars.

339. The Period-Luminosity Relationship for Cepheid Variables. In 1912, a very important relationship between the periods and the luminosities of Cepheid variables was discovered by Miss Henrietta S. Leavitt from investigations of the periods and the magnitudes of more than 100 of these stars in the Small Magellanic Cloud. Since this whole cloud is extremely remote from the solar system (95,000 light-years), the percentage differences in the distances of its various parts are very small; consequently the actual luminosities of its stars are sensibly proportional to their observed brightnesses.

The period-luminosity law is represented in Fig. 185, in which the ordinates are the visual absolute magnitudes and the abscissae are the logarithms of the periods in days. The figure includes not only the representative points for variable stars in the Small Magellanic Cloud, but also for those in the great ω Centauri and Hercules clusters, and for those in three other clusters as well. The fact that all of these points fall so nearly on a smooth curve confirms the conclusion that there is a close relationship between normal Cepheid variables and cluster variables. It should be noted that since the ordinates are magnitudes, the diagram includes stars having a range of luminosity of 100 to 1.

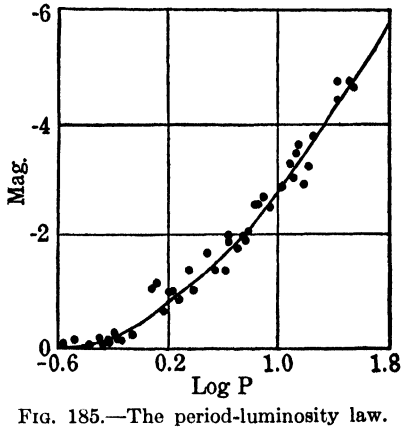


FIG. 185.—The period-luminosity law.

It is easily seen that the period-luminosity relationship is of the highest importance in finding the distance of a cluster, for example, in which a Cepheid variable may be known, for this relationship determines the absolute magnitude of a star after its period has been found from observations of its varying brightness; and its absolute and apparent magnitudes together enable us to compute its distance by the formula in Art. 313.

340. Hypotheses Respecting Cepheid Variables. Since the characteristics of Cepheid variables prove that they are not eclipsing binaries, the question arises respecting the causes of the peculiar variations in rates of radiation of these giant stars. The changes in their color and their spectra imply changes in their temperatures. Hence we are led to look to their interiors for the cause of their variability.

In 1879, A. Ritter advanced the hypothesis that Cepheid stars owe their variability to alternate expansions and contractions under the combined effects of gravity and gaseous expansion, a suggestion which is known as the *pulsation theory.* If a star is expanding rapidly, its inertia will carry it somewhat beyond the equilibrium stage, after which gravity will contract it again. The contraction will likewise go in the opposite direction beyond the condition of equilibrium, after which it will expand once more. The oscillations are roughly analogous to those of a weight suspended by an elastic string.

When a star expands, its potential energy increases and its heat energy decreases by an equivalent amount so that the sum of the two remains constant. When a star contracts, its potential energy decreases and its heat energy increases, as in Helmholtz's contraction theory of the heat of the sun (Art. 275). It is found that variations in radius of a few per cent will account for the range of variations in radiation of Cepheid variables. When a star is contracting and changing from minimum brightness to maximum, its surface should have the greatest velocity of recession from the earth, but we have seen that the spectral lines are displaced most toward the red when a Cepheid is near its minimum brightness. It has been suggested that the general displacement of phase of a quarter of a period is due to a lag in the generation and the radiation of heat.

A rapid rise from minimum to maximum and a slow decline back to a minimum are not easily explained by the simple pulsation theory, nor are the secondary oscillations in the falling branch of the light curve. Perhaps the difficulties of the theory in this simple form are due to the fact that a rotating oblate star could not undergo simple pulsations of the type postulated, for as the equatorial materials rose they would lag in rotation behind the remainder in harmony with the requirements of the constancy of moment of momentum of the mass. Hence it is probable that in all cases there would also be oscillations from an oblate form to

the prolate, the period of which would be shorter than that of expansion and contraction. Such superimposed oscillations might account for the phenomena which are not in harmony with the theory of simple periodic dilatations.

341. Long-Period Variable Stars. Nearly 1100 stars are known which belong to the class of long-period variables, and several hundred other stars are probably of the same type. These numerous variable stars are characterized only in part by the lengths of their light cycles, which range from a few days to several hundred days. Among their other distinguishing characteristics is the fact that they undergo great changes in brightness, the range from minima to maxima often exceeding a hundred-fold and in some cases a thousand-fold. Moreover, their cycles of light changes are only roughly periodic; their maxima and their minima are far from constant (Fig. 186); and they all are red stars, generally belonging to the spectral Class M, though a few are of Classes N, R, and S.

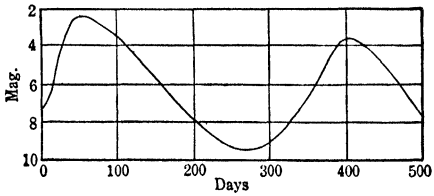


FIG. 186.—Light curve of long-period variable.

The spectra of typical variables of this class show the absorption bands of titanium oxide and the low-temperature lines of various metals, particularly when the stars are near their minima. When they are at their maxima, the bright lines of hydrogen are likely to be present also. The combination of absorption bands and bright hydrogen lines has led the Harvard astronomers to the discovery of many long-period variable stars from their spectra alone.

It has been found that the proper motions of long-period variables are very small, a fact which implies that these stars are far from the solar system. Since many of them are fairly bright, their luminosity is high, probably at their maxima 100 times that of the sun. They are giant stars of low mean densities and low surface temperatures.

Astronomers have sought in vain for an explanation of the changes in the brightness of long-period variables. Their perplexity has been increased by the fact that some of these stars partake of the characteristics of Cepheid variables, and by the fact that in some other cases their approximate periods of variability suddenly

change by large amounts. For example, G. P. Gerasimovič found from an examination of Harvard plates that UU Herculis, which has a range of variation of more than a magnitude, during the past 30 years has changed its period three times from approximately 45 days to 72 days and back; and he found that Z Canĉri has had the three periods of 80, 114, and 166 days. The cause of these remarkable changes in luminosity is believed by most astronomers to be within the stars themselves, and the displacements of their spectral lines in the periods of their variability suggest pulsations perhaps similar to those postulated for explaining the Cepheid variables. Their spectral lines, however, are displaced toward the red when they are near their maxima, while the corresponding displacements for the Cepheid variables occur when they are near their minima. At every turn these stars lead to contradictions.

342. The Long-Period Variable Omicron Ceti. Omicron Ceti, the first known variable star, was discovered by Fabricius in 1596; and although it has been followed through more than 300 of its cycles, its future behavior can not be confidently predicted. William Herschel saw it in 1779 when it was nearly as bright as the first-magnitude star Aldebaran; four years later it was not visible through his telescope. Its visual magnitude at maximum ranges from about 1.5 to 5.6, while its magnitude at minimum usually lies between 8.0 and 10.0. Its period averages about 330 days, though it varies from about 300 days to 360 days.

It is reasonable to assume from what is known of stars of the class to which *o* Ceti belongs that its absolute visual magnitude is 0. Under this assumption, its observed visual magnitude enables us to compute its distance, which is found to be 165 light-years. Then, using its angular diameter, which has been measured by F. G. Pease with the stellar interferometer, we can determine its linear dimensions. It is found from these calculations that the diameter of this remarkable star is of the order of 490,000,000 kilometers, from which it follows that its volume is 44,000,000 times that of the sun.

The theory that the variability of *o* Ceti is due to internal oscillations is supported by J. H. Joy's measurement of the changes in its radial velocity; he concluded that its diameter varies by 50,000,000 kilometers from the mean. In the case of Betelgeuse (an irregular variable), the measurements of F. G. Pease with the

stellar interferometer indicated variations in diameter of as much as 20 per cent. If this star is undergoing simple gravitational oscillations, it follows from its period that its mass is about 4 times that of the sun and that its density is of the order of 1/10,000 of that of our atmosphere at sea level. It may be that these inconceivably huge masses of gas are undergoing tremendous and somewhat irregular oscillations in size and shape because they are now in a state of transition from one condition of equilibrium to another.

In 1923, R. G. Aitken found that Mira is accompanied by a faint companion at a distance of 0″.9 from the principal star. The existence of this faint attendant of the tenth magnitude, which apparently is a white dwarf of spectral Class B8, had been anticipated by Joy a number of years before its visual discovery because of the presence of bright hydrogen and helium lines (due to the faint companion) which became visible when the principal star was near its minima and which were displaced with respect to the Class M spectrum of Mira. The existence of an intensely hot and probably dense, white dwarf in close proximity to the tenuous and relatively cool Mira only adds to the perplexing problems which have been raised by this star for more than 300 years since its discovery.

343. Irregular Variable Stars. In all the classes of variable stars which we have so far considered, the changes in brightness are periodic or nearly periodic. There are, however, many other stars whose magnitudes vary so irregularly that they have no semblance of periodicity. Nearly all of these stars are red and they include Betelguese as their most conspicuous example. Like o Ceti, they are probably of enormous dimensions and very low density.

Some irregular variable stars, such as RV Tauri, resemble the Cepheids and point to a relationship among all non-eclipsing variables. Others, such as U Geminorum, are normally faint, but at irregular intervals measured in months become several-fold brighter than normal for a few days. The light variations and the spectra of these stars are suggestive of the Novae (Art. 344). A few other stars, such as R Coronae Borealis, remain at constant brightness for months or even years and then rapidly decline several magnitudes in luminosity. After irregular intervals of minor irregularities, often extending over months, these stars increase to their normal brightness. Although the light changes of many

irregular variables at present appear as capricious as the weather, it is very probable that they are associated with fundamental properties of stellar constitution which will some day lead to important discoveries.

344. Temporary Stars. In 1572, a previously unknown star suddenly blazed out in Cassiopeia and for some time was as bright as Venus; in 1604, a similar and nearly as bright star appeared in Ophiuchus. The former turned the attention of Tycho Brahe to astronomy; the latter stimulated the interest of Kepler. Since these and similar stars appeared where none had been previously known, they are called *novae*, or new stars.

It should not be assumed that all temporary stars are as brilliant as those observed by Tycho Brahe and Kepler, for only a few rise to such splendor. Until the use of photography in astronomy only the brighter temporary stars were discovered, about 20 having been observed without optical aid. Now many fainter ones are also recorded, nearly all of which are in or near the galactic zone. S. I. Bailey concluded from studies of Harvard plates that from 10 to 20 temporary stars having maxima brighter than the ninth magnitude appear in the Milky Way each year.

Temporary stars are not only often discovered by photography, but in a number of cases complete histories of their increases in brightness have been found recorded on plates taken for other purposes before their discovery. The first of these stars was Nova Persei, discovered by T. D. Anderson on February 22, 1901, when it was already a first-magnitude star. That its rise to great brilliancy was meteoric in swiftness is proved by the fact that on plates taken by E. C. Pickering and Stanley Williams three nights earlier it did not exceed magnitude 13 in brightness. Therefore in less than three days its luminosity increased 100,000-fold. In 24 hours, however, it lost one-third of its brightness and within a year it faded to the twelfth magnitude.

Stars with somewhat similar histories are Nova Geminorum (1912), which increased in brightness from magnitude 15 to 3.7; Nova Aquilae (1918), from magnitude 11 to -1.1 within two days; Nova Cygni (1920), from below magnitude 15 to 1.8; and Nova Pictoris (1925), from magnitude 12.7 to 1.2. In each of these cases except the last there was an increase in brightness within a day or two of approximately 12 magnitudes, or about 60,000-fold, and

in each case the maximum was succeeded by a very rapid decrease in luminosity for a few days, followed by a slower decline that carried the star in a few months or years back approximately to the magnitude from which it rose.

The light curves of most of these stars are similar to that of Nova Persei (Fig. 187). There have been, however, a few abnormal temporary stars whose characteristics differ widely from those which have been described. Two or three of these stars have remained near their maxima for a number of months, and one star (T Pyxidis) increased from magnitude 14 to above magnitude 8 three different times (in 1890, 1902, and 1920). Such stars suggest the possibility that the novae do not rise only once from obscurity and then fade away forever, but that at long intervals tremendous energies again cause them to blaze out so brilliantly as to be visible throughout the galaxy. Several of the stars that have been novae were observed before their outbursts of radiation, and at these earlier times they showed no unusual characteristics of brightness or of spectra.

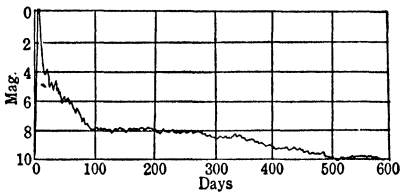


FIG. 187.—Light curve of Nova Persei.

345. Spectra and Envelopes of Temporary Stars. The spectral changes of novae are as remarkable as their variations in luminosity. When these stars are approaching their maxima their spectra are like those of the super-giants, consisting of absorption lines. At least this was the case with Nova Aquilae, the only star adequately observed before it reached its maximum. As they attain their maxima, broad emission bands appear in their spectra and rapidly increase in brightness, and on the violet borders of these bands are familiar dark absorption lines whose displacements correspond to enormous velocities of approach. In the case of Nova Aquilae, indicated velocities of as much as 1700 kilometers per second were attained, but the bright emission bands were not similarly displaced. Moreover, the dark lines in some cases are doubled, and occasionally isolated absorption lines are in their normal positions. Up to this stage the spectra of novae appear to be similar to those of Class A stars.

As temporary stars pass their maxima, their spectra approach

those of the Class B stars in type. The dark lines are increasingly displaced toward the violet for two or three days, and in the case of Nova Aquilae attained a maximum indicated velocity of 2300 kilometers per second. As the novae decrease in luminosity, their bright lines broaden and gradually spread over and efface the dark absorption lines.

Within a few weeks after a temporary star begins to decline in brightness, its spectrum changes to the nebular type (Art. 348). Finally, the bright nebular lines disappear and the nova becomes a star of spectral Class O, though with bright lines of the subclass known as Wolf-Rayet stars. During this sequence of spectral variations, the star changes in color from white to red, then to a greenish nebular hue, and then back to white in the final stage. It is not improbable that all of the relatively few Wolf-Rayet stars were formerly novae.

Seven months after Nova Persei passed its maximum in 1901, a faint nebulous ring around it was found on photographs taken by Max Wolf. Later photographs by G. W. Ritchey and C. D. Perrine showed that this ring was expanding at the rate of several seconds of arc per week. In the meantime it had been found that the star is enormously remote, so remote, in fact, that the velocity corresponding to the expanding ring is far beyond all known velocities except that of light. J. C. Kapteyn promptly suggested that the apparently expanding ring was in reality a shell of light in a previously dark nebula. During the brief interval while the star was near its maximum brightness it sent out a wave of light which week after week illuminated more and more distant parts of the otherwise invisible nebula (Fig. 188). At the enormous distance of the star from the earth, the velocity of light across the line of sight is apparently so low that its advancing front would not move an angular distance equal to the diameter of the moon in three years. On the somewhat reasonable assumption that the expanding ring was actually due to the velocity of light, we can compute the distance of the star and then its absolute magnitude. By this process we find that it is distant 350 light-years and that its absolute magnitude at maximum was -5, corresponding to a luminosity nearly 10,000 times greater than that of the sun.

Nova Aquilae, in 1918, was also surrounded by an expanding luminous shell which moved outward at the rate of $2''$ per year. On combining this result with a radial velocity of 1700 kilometers

per second, as determined from the displacements of the spectral absorption lines of the expanding shell, it is found that this star is distant 1200 light-years and that its luminosity is 300,000 times that of the sun. Observations of Edwin Hubble and J. C. Duncan indicated that the shell was still expanding in 1930 with a sensibly constant velocity. In the case of this star it has been assumed that there is an actually expanding shell of gas, instead of a shell of

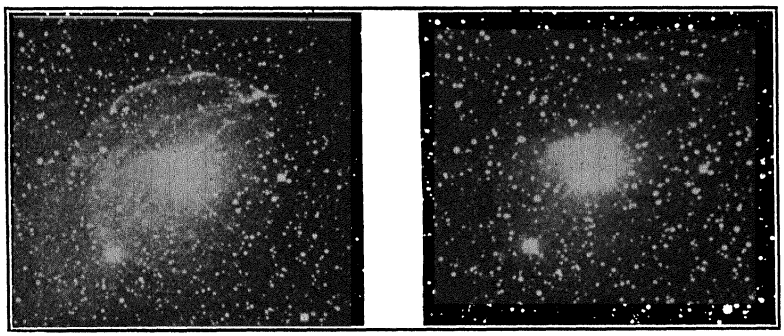


FIG. 188.—Nebulous ring around Nova Persei on Sept. 20 and Nov. 13, 1901. (G. W. Ritchey, Yerkes Observatory.)

light, as was assumed in the case of Nova Persei. It is probable, however, that the rings or shells of light of the two stars were fundamentally of the same nature.

Many suggestions have been made for explaining temporary stars. One hypothesis, that of Bickerton (Art. 306), is that these sudden outbursts are produced by collisions of stars, their kinetic energies being turned in part into heat. Perhaps the most serious objection to this hypothesis is that if it were the true explanation, novae would appear only once in our galaxy in hundreds of thousands of years, instead of 10 per year, for the distances between the stars are enormous in comparison with their diameters. To avoid this difficulty, it has been suggested that temporary stars are due to collisions of bodies of planetary dimensions with suns, but the theory has not been sufficiently developed to determine whether such a cause would be adequate to produce the observed phenomena.

H. Seeliger suggested another variation of the collision theory. He assumed that a nova is produced when a star enters a swarm of meteoric dust or a nebula. Its surface is suddenly heated to an enormous temperature on penetrating the medium, and when it

passes out of such a swarm this superficial heat is quickly radiated away. It is very improbable that such a cause would give rise to many temporary stars so similar in their sudden rise in brightness, range in magnitude between minimum and maximum, short duration of maxima, change of spectra, and final state.

Another type of hypothesis is that the temporary stars are due to the sudden releasing of internal energies. It has been suggested that before their outbursts they may be white dwarfs which suddenly expand, accompanied by cataclysmic convulsions, to stars of more usual types.

Finally, a nova may be the outward manifestation of the cataclysm which accompanies the breaking up of a star when its rotation is so rapid as to cause it to be unstable. Instead of dividing into two masses with unchanged total luminosity, it may blaze out briefly during the process of fission and then quickly cool as it gains potential energy at the expense of kinetic as a consequence of the separation.

All these suggestions appear to be based on the assumption that a given star is a nova but once. Since at least 10 temporary stars appear per year in our galaxy, several thousand millions of suns, according to this theory, have been novae during the relatively brief existence of the earth as a geological body; and therefore novae must be occurring now in abnormal members or all stars are novae within a millionth of the lifetime of a star. The temporary star T Pyxidis, which has already had three observed outbursts of radiation, probably points the way to a better hypothesis. This hypothesis is that novae are an extreme class of variables, probably irregular and with long intervals between sudden outbursts to their maxima.

IV. Nebulae

346. General Characteristics of Nebulae. From the time of Newton, searchers of the sky occasionally found small, faint patches of light which they called *nebulae*. The early observers had little doubt that these objects are clouds of gas, possibly the materials which ultimately contract into stars. But occasionally a nebula was resolved into numerous points of light, and then the opinion grew that all nebulae are swarms of stars, though some of them may be so remote that their constituent units are not individually discernible. Although no one resolved more so-called

nebulae into stars than did William Herschel, he was convinced by the soft, smooth glow of some of these objects that in these cases they are shining masses of continuous fluids.

In 1864, William Huggins brought conclusive support to Herschel's views when he found that the spectra of some of the nebulae consist of bright lines characteristic of radiating gases. He found, on the other hand, that the spectra of the resolvable nebulae are similar to those of the stars. This is the earliest of a long series of instances in which the spectroscope has played an essential rôle in unraveling the mysteries of the stellar system.

Since the time of Huggins, nebulae of various types have been discovered and investigated. At one extreme, astronomers have found small, sharply defined nebulae surrounding stars and having some resemblances to those which spread out from Nova Persei and Nova Aquilae. They have also found many examples of the shining gases which gave Huggins the bright-line spectra. E. E. Barnard and other astronomers have observed and photographed far-reaching wisps of nebulous materials, and they have found that even whole regions of the stars, hundreds of light-years in diameter, are involved in a faint nebulous glow. Barnard has shown, too, that there are numerous dark clouds, perhaps of dust, which in some areas blot out the stars. In fact, the *Coal Sack* near the Southern Cross, the black spot south of Deneb in Cygnus, and even the long rifts in the Milky Way are produced by nonluminous, absorbing materials lying between us and those distant parts of space.

Nebulae are widely spread throughout our galaxy and they are an essential part of its structure. Undoubtedly they play a rôle in its evolution and they may be, perhaps, one stage in some unknown great cycle of changes. They are also found in exterior galaxies. In the Magellanic Clouds they appear much as in our own galaxy; in the more distant galaxies (Fig. 197) only the dark nebulae are evident, but their eclipsing effects are often of great importance. In fact, by studying these distant galaxies we are able to learn more about the extent and the distribution of absorbing materials in galactic systems than we should learn if we limited our investigations to the galaxy in whose interior we are immersed.

347. Planetary Nebulae. The name of this class of nebulae is quite misleading, for these objects have no relationship to planets;

they simply appear in the telescope to have faint disks. The largest planetary nebulae are a few minutes of arc in diameter, but most of the hundred known do not exceed 1′ and some of them are recognized to be planetaries only by their spectra. The apparently larger planetaries are distributed generally over the sky; the apparently smaller ones are confined to the Milky Way and are most numerous in the Sagittarius region where it extends farthest. The larger ones evidently are in our local star-cloud, and the smaller ones are those which are more distant in the other star-clouds which make up our galaxy.



Fig. 189.—Ring nebula in Lyra. (Lick Observatory.)

Planetary nebulae are circular or oval faint masses, usually with well-defined margins and exhibiting considerable detail of internal structure. The typical planetary has a faint star at its center and one or more rings or shells of matter around it. In extreme cases, such as that of the ring nebula in Lyra (Fig. 189), the nebulous material constitutes a nearly perfect ring. Usually these nebulae are more like the planetary N.G.C. 7662, in Andromeda (Fig. 190).

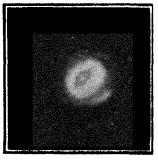


Fig. 190.—Planetary nebula N.G.C. 7662. (Lick Observatory.)

That planetary nebulae are in rotation is implied by their shape, and the inference has been fully established by V. M. Slipher. In 23 cases W. W. Campbell and J. H. Moore found unmistakable evidences of rotation. When they placed the slit of their spectroscope along the greater axes of oval planetaries, they found that the spectral lines were given an inclination due to the Doppler effect, and the amounts of the inclinations determined the velocities. But when they placed the slit along the shorter axes, the effects disappeared; and in the case of the circular planetaries whose planes are presumably perpendicular to the line of sight, no such effects were observed for any position of the slit. The linear velocities of rotation ranged from 1.4 to 18 kilometers per second.

If we could find the distances of the planetary nebulae, we could determine their actual dimensions from their apparent diameters, and their periods of rotation from velocities indicated by the inclinations of their spectral lines. Unfortunately, it is difficult to determine their distances and no very reliable results have so far been secured. But such measurements as have been made



FIG. 191.—Nebulosities surrounding the Pleiades. (Yerkes Observatory.)

indicate that their diameters are thousands of times greater than the distance from the sun to the earth, that their periods of rotation are of the order of 10,000 years, and hence that their masses must be of the order of 10 to 100 times that of the sun in order to balance the centrifugal accelerations of their outlying parts.

There is, however, one difficulty with the distances adopted, for the observed apparent magnitudes of their central stars imply that their absolute photographic magnitudes are about +5. Now these stars are of spectral Class O whose absolute photographic magnitudes in other cases are −4. That is, from their spectral

class we should expect their luminosity to be 4000 times greater than that which we infer from their observed magnitudes and our imperfect knowledge of their distances. It has been suggested that there are great reductions in apparent brightness of the central stars as a consequence of absorptions of light by the surrounding nebulous shells. But we note that such energy as is absorbed is necessarily radiated again, perhaps at other wavelengths, and there is no evidence that their gaseous envelopes radiate thousands of times as much energy as apparently are radiated by their central stars. More information is necessary before positive conclusions about these stars can safely be drawn.

Whether or not the central stars of planetary nebulae are fainter than we should expect them to be on the basis of their spectral class, the luminosity of their gaseous rings or shells presents a puzzling problem. It is not easy to see how the large amount of energy which a planetary nebula radiates could be generated within itself. Nor could the energy come from outside of the nebula except from the central star, and this source seems doubtful because apparently the nebulae radiate more energy than is emitted by their associated stars.

348. Bright Diffuse Nebulae. All planetary nebulae appear to be much alike in shape and size and general characteristics; the bright diffuse nebulae, on the contrary, are extraordinarily varied. Some are definitely associated with stars or groups of stars, as in the case of the nebulous clouds which surround the Pleiades (Fig. 191), or in that of the vaster masses in Orion (Fig. 192); others are irregular veil-like wisps which, apparently quite independent of the stars, extend inconceivable distances through the galactic region (Fig. 193); and still others, as diffuse and vague as an autumn haze, stretch like a faint mist over areas including whole constellations. A soft glow spreads over the Pleiades and most of the remainder of Taurus, as well as the similar gossamer veils in Orion, Sagittarius, and many other parts of the Milky Way.

For many years astronomers were at a loss to account for the luminosity of the diffuse nebulae, for evidently such tenuous masses can not have high temperatures. It would be going too far to state that now the source of the energy they radiate is fully understood, but progress has been made. In 1912, V. M. Slipher suggested that the nebulae surrounding the Pleiades shine by

light reflected from the stars in the group. Edwin Hubble has shown that the nebulae in the Pleiades and in Orion are not isolated examples of such masses in association with stars, but that the brighter nebulae are almost invariably near brilliant stars. He found, moreover, that the variations in brightness of nebulae with differences in distance from their associated stars are in harmony with the theory that the nebulae derive their energy



FIG. 192.—The Great Orion Nebula. (Mount Wilson Observatory.)

from the stars. Part of the light received from nebulae is reflected starlight and part is energy which has been absorbed and transformed to other wave-lengths before it is radiated. The theory that the diffuse nebulae receive their energy from neighboring stars is strongly supported by the fact that the spectra of nebulae

and associated stars have characteristics in common throughout a spectral range from Class O to Class K8.

It should not be assumed that nebulae must be near to stars in any except the galactic sense in order to be made luminous. A star having the moderate brilliancy of the sun renders a nebula at a distance of 14,000 astronomical units capable of being photographed with the Mount Wilson reflector in 160 minutes. A supergiant of absolute magnitude −5 is similarly effective to a distance



FIG. 193.—Veil-like nebulae in the constellation Cygnus. (Photograph by W. A. Cogshall at the Lowell Observatory.)

of 33 light-years, or four times the distance from the earth to Sirius.

Until recently, the brightest lines in the bright-line spectra of nebulae have been ascribed to an unknown element "nebulium," for they do not coincide with any known lines of the elements. Many of the spectral lines of nebulae are due, however, to such familiar elements as hydrogen and helium, and all the nebular lines have the sharpness characteristic of the spectra of gases under low pressure. These considerations suggested to H. N. Russell that the unidentified green nebular lines, which give to the nebulae their color, and other and fainter unknown lines as well, may be

radiant energy emitted by familiar elements under the conditions of extreme tenuity prevailing in the nebulae and which can not be duplicated in our laboratories. This conjecture was verified by the theoretical investigations of I. S. Bowen, who found that the unexplained lines are due to radiations from singly and doubly ionized atoms of oxygen and nitrogen which have absorbed energy while in the ionized state. A long unexplained mystery seems to have been cleared up without the introduction of a new element.

349. Dimensions and Distribution of Bright Diffuse Nebulae. If nebulae were not related to the stars it would be difficult, if not impossible, to determine their distances or dimensions. But since they usually are associated with the stars, their distances are known when the distances to their neighboring stars have been found. For example, the nebulae about the Pleiades and in Orion are at the distances of these respective groups of stars. In many cases, of course, the distances to particular stars are uncertain or quite unknown.

Many investigations have been made of the parallax of the Pleiades and they have led to the conclusion that these stars are about 220 light-years from the solar system. The brighter part of the nebula which surrounds them has an apparent diameter of 8° or 10°, and consequently a linear diameter of 30 light-years. If such a nebula surrounded the solar system as a center, it would extend far out beyond α Centauri, Sirius, Vega, Capella, Arcturus, Procyon, Altair, Regulus, Castor, and many other stars which are visible to the unaided eye. Even if its mass were as great as 10,000,000 suns such as ours, its average density would be less than 1/1,000,000,000 of the best approach to a vacuum we can produce in our laboratories. Its density and total mass are, however, quite unknown, though it is not improbable that its mass is enormous compared to that of a star.

Orion is distant nearly 600 light-years and its nebula is 10 light-years in diameter. Even this stupendous volume of gas is small compared to the *Looped Nebula* in the Large Magellanic Cloud, whose diameter, according to Shapley, is 130 light-years. Since similar volumes are proportional to the cubes of their diameters, this nebula is about 2200 times greater than that in Orion.

According to Hubble, the diffuse bright nebulae are distributed in two main belts. One is in the Milky Way, and the other is



FIG. 194.—Large-scale photograph of the right-hand nebula in Fig. 193. The nebula is very faint and its structure is brought out only by long-exposure photographs with large reflectors. (Yerkes Observatory.)

inclined about 20° to the plane of the galaxy. The latter nebulae are interpreted as belonging to a local star-cloud of which the sun is a member, though their inclination is about 8° higher than would be expected from the distribution of its stars. The former nebulae belong to the many star-clouds which constitute the far-reaching parts of our galaxy. As was stated in the beginning (Art. 346), the nebulae are essential parts of our galaxy and of other galaxies, but, significantly, they are not found in the intergalactic spaces.

350. Velocities of Nebulae. It is possible to determine the radial velocities of nebulae with considerable accuracy because their spectral lines are sharp; the principal difficulty is that their light is feeble. As might be expected, their radial velocities are comparable to those of the stars. Some nebulae are approaching and others are receding, usually with velocities not exceeding 20 kilometers per second. The nebula in the Large Magellanic Cloud, however, is receding at the rate of 276 kilometers per second, doubtless at the same rate as that of the cloud of stars itself.

The internal motions of the nebulae are as interesting as their motions as a whole, for the relative velocities of their parts will ultimately throw light upon the question of their masses, if we may assume that they have enough gravitation to prevent their being dissipated. In view of the well-defined outlines of some of them, such as the far-reaching wisps in Cygnus, this assumption does not appear unreasonable. Relative internal motions were first found in the Orion nebula by Bourget, Fabry, and Buisson. E. B. Frost and others have shown that the nebula on the whole is receding from the solar system at the speed of about 17.5 kilometers per second, and that there are relative internal velocities ranging from 4 to 10 kilometers per second. If the relative velocities were simply due to a rotation of the nebula as a whole, centrifugal acceleration would cause it to dissipate unless its mass is more than 30,000 times that of the sun.

351. Dark Nebulae. For a long time astronomers assumed that the dark spots in the Milky Way are cylindrical openings through those vast clouds of stars, extending out into the emptiness of infinite space. In view of the thickness of these star clouds, the hypothesis that many of these vacant cylinders should be directed toward the present position of the solar system is of course absurd. Barnard gave conclusive evidence, however, that the dark areas

are due to clouds of obscure matter which cut off the light of all except a few of the nearer stars. In many instances these dark nebulae gradually merge into diffuse light nebulae when they



FIG. 195.—Dark nebula in Orion. (Exposure 3 hours with the 100-inch reflector, Mount Wilson Observatory.)

extend to the vicinity of bright stars. Even the nebula surrounding the Pleiades apparently eclipses most of the stars, for the faint stars in the background of this group are less numerous than they

are in neighboring regions. In some cases dark nebulae partially eclipse bright nebulae, and there is a coal-black mass near ζ Orionis (Fig. 195) which is passing in front of, or possibly into, the soft nebulous haze which covers that part of the sky.

In Barnard's *Star Atlas of the Milky Way*, there are catalogued and described 349 dark nebulae. Some of them are conspicuous to the unaided eye; others are too small to be detected without telescopic aid. Some of the small ones are sharply defined and

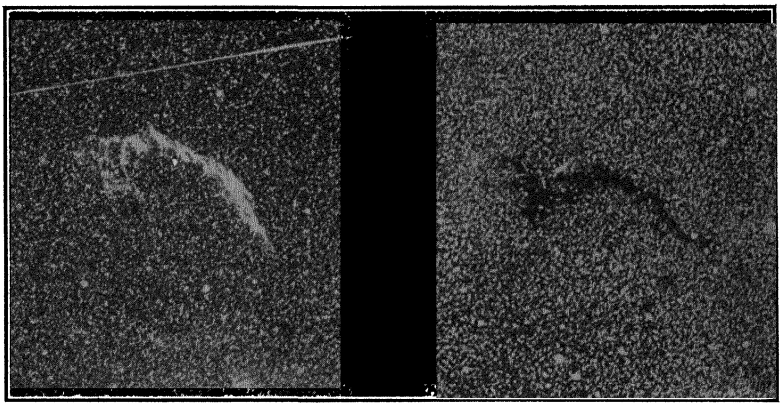


FIG. 196.—**Bright nebula in Cygnus** (Fig. 193) and similarly shaped dark nebula. There is a meteor trail on the photograph at the left. (Barnard.)

very black. In some cases they have the shapes and dimensions of bright nebulae, as illustrated in Fig. 196.

For nearly 120° in longitude, from Cygnus in the northern sky to Centaurus in the southern, the central part of the Milky Way is obscured by dark nebulous clouds of appalling dimensions. These dark nebulae not only cut off the light of the individual stars, but they completely eclipse the great galactic clusters which are most numerous in that general part of space. Some of the nearest of these eclipsing clouds are associated with stars which are known to be distant many hundreds of light-years. It is probable that they are distributed throughout all the star-clouds of our galaxy. In fact, when exterior galaxies are presented edgewise to us, we find that they are apparently cut entirely through by obscure matter around their peripheries, as is illustrated in Fig. 197.

352. Variable Nebulae. If the bright nebulae owe their luminosity to energy radiated by stars, they should be variable when they

envelope variable stars. As yet no nebula is known to change in brightness in the period of a neighboring variable star, but there are nevertheless several nebulae which undergo changes both in luminosity and in apparent form.

There is an irregular variable star in the constellation Monoceros, known as N.G.C. 2261, which has a small comet-like nebula, spreading out from it as a head (Fig. 198). The luminosity of the star varies irregularly at long intervals; the attending nebula varies frequently and by large amounts. Moreover, it varies simultaneously in apparent shape, often undergoing marked changes within a day or two. It is probable that the changes in shape are not due to actual displacements of matter, but to variations in the luminosity of relatively fixed nebulous masses. This conclusion is confirmed by the fact that the same apparent structure recurs time after time.



FIG. 197.—Exterior galaxy (N.G.C., 891) edgewise to earth. (Mount Wilson Observatory.)

One is tempted to conclude that the changes in apparent form are due to waves of radiant energy sweeping over the mass, possibly emanating from the variable star; but unless the star and the nebula are much nearer to us than seems probable, the speed of varying luminosity is far beyond that of light. Moreover, there is no correlation between the variability of the star and that of the nebula.

Another somewhat similar variable nebula is N.G.C. 6729, in Corona Austrina. The spectra of the associated stars in the two objects are, however, quite different, that of the former having the characteristics of a spent nova and that of the latter being Class G with unusual bright lines. The spectra of the two nebulae resemble those of their respective stars, but they have also the characteristic bright nebular lines. The nebula N.G.C. 6729, in Corona Austrina, is in a part of the sky which is veiled by heavy

absorbing clouds. Possibly if this screen should pass away we could learn more about this remarkable mass which, according to an estimate by Hubble, is distant more than 300 light-years.

353. Evolution of Stars. In view of the uncertainties that remain respecting the source of stellar energies, the internal constitution of stars, the origin of binaries, the cause of variability of all except members of eclipsing pairs, the real nature of novae, the connection of planetary nebulae with ordinary stars, and the rôle played by the diffuse nebulae, it may seem presumptuous to formulate hypotheses respecting so transcendental a subject as the evolution of the stars—and it is. But speculations nearly always outrun certainties and often they anticipate and lead the way to substantial conclusions. At the least, they help break down our inherited prejudices and stimulate our minds to contemplate new and greater possibilities.

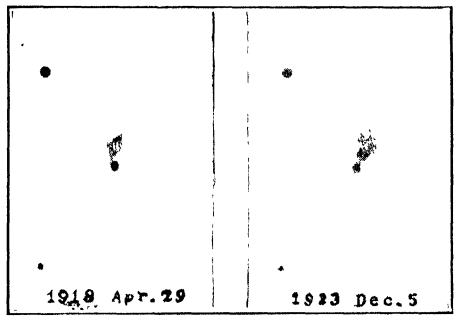


FIG. 198.—Variable nebula in Monoceros. (Mount Wilson Observatory.)

Since the days of the Greek philosophers the idea has been current that the present order of the physical universe has developed out of a primeval chaos. The Kantian and the Laplacian hypotheses both started with extended nebulae and derived out of them the sun and its planetary attendants. After the development of the Helmholtzian contraction theory of the sun's heat, astronomers for more than 60 years looked forward to a time when the solar system, its energy having been radiated away, would become forever dark and cold. The same order of ideas was quite naturally carried over into theories of the evolution of the stars. For two or three decades, astronomers supposed that the stars proceed steadily from their present high temperatures to dark bodies, until Norman Lockyer, making use of the consequences of the contraction theory, suggested the hypothesis that contracting nebulae grow hotter until they become so dense that their cores become incompressible; and that in the process the stars rise from feeble

luminosity to the greatest brilliancy, and then gradually reverse their thermal history back through yellow and red stars to dead suns.

This theory of Lockyer was ably and extensively developed by H. N. Russell. When E. Hertzsprung established the existence of giant and dwarf stars and Russell discovered (1913) the relationship between the absolute magnitudes of stars and their spectral classes (Art. 314), the story was for a time regarded as essentially complete. According to this theory, stars of large mass pass from the nebular state to the red giants, then through the spectral sequence to Class B stars, at which stage they attain their greatest brilliancy. From the B stars they reverse the order of spectral sequence and become red dwarfs of Class M just before they cease to be luminous.

When it finally became recognized that the durations of the stars are not measured in tens of millions of years but in millions of millions of years, and probably in intervals of a higher order, the time-scale of events at least was radically changed, though the general order of ideas was preserved. The researches of R. Emden, and since 1915, those of A. S. Eddington, J. H. Jeans, and E. A. Milne on the internal constitution of stars have raised serious questions respecting the circumstances under which energies are released in their interiors and respecting the conditions for their equilibrium. These investigators have sought an explanation, for example, of the apparent fact that luminous stellar bodies in mass have a limited range from one-tenth that of the sun to about 75 times that of the sun.

These problems of stellar constitution are so difficult and their solutions rest upon so many assumptions which are not directly verifiable that no agreement whatever among the various investigators has been reached; yet the general pattern of the earlier hypothesis has been preserved by all. Jeans, in particular, has formulated a grandiose theory that starts with all our galaxy in the form of a great rotating nebula which, through successive instabilities and fissions, has evolved into our present system of hundreds of millions of suns. Its magnificent scale in space and time and mass and energy stirs the imagination; but this system, too, is wasting its energy, and possibly its mass, in the immensity of space, and it tends always toward a uniform distribution of heat and a consequent condition of stagnation and death.

Many astronomers have searched for some restoring factor which would reassemble the dissipated energies of an apparently dying universe, but no suggestion except that of W. D. MacMillan (Art. 307) makes any approach to describing concrete processes



FIG. 199.—The Great Orion Nebula, distant 600 light-years and 10 light-years in diameter. (G. W. Ritchey, Yerkes Observatory.)

by which the stars may restore their wasted masses. Even this suggestion does not provide a means for reintegrating the large fraction of their radiant energy that they pour out beyond the borders of our galaxy into the void of the intergalactic spaces (Art. 374). The innumerable members of our galaxy assure us, however, that though the stars may be evaporating, so to speak, like the dew, their energies will in some way be integrated again. But, in truth, we actually know very little about the evolution of the stars.

XXII. QUESTIONS

1. Enumerate the evidences that the atmospheres of stars of Class M are lower in temperature than those of Class A.
2. Why are stars of Classes B and A apparently more numerous relative to stars of Classes G and K than they actually are?
3. In what sense, if any, is the absolute magnitude of a star absolute?
4. Why do stars of negative absolute magnitude appear to be more numerous than they actually are?
5. Enumerate the steps by which the great density of the companion of Sirius was inferred? Is relativity involved in the inference?
6. How do eclipsing variables provide a test of the spectroscopic method of determining the rotations of stars?
7. How is interstellar calcium distinguished from calcium in the atmospheres of stars?
8. What reason is there for believing that most double stars are binaries?
9. In what way do eclipsing variable stars confirm the conclusion that the so-called spectroscopic binary stars are actually binaries?
10. Why is it practically more difficult to determine the masses of visual binaries than of spectroscopic binaries?
11. From what observational data can the density of a star be determined?
12. Enumerate the objections to each of the two hypotheses respecting the origin of binary stars.
13. What reasons are there for thinking that novae are in reality variables which blaze out repeatedly?
14. What is the probable source of light radiated by the nebulae?

REFERENCE BOOKS

Modern Astrophysics (1924). Herbert Dingle. The Macmillan Company, London, England.

Astronomical Physics (1925). F. J. M. Stratton. Methuen & Co. Limited, London, England.

The Internal Constitution of the Stars (1926). A. S. Eddington. Cambridge University Press, Cambridge, England.

Astronomy and Cosmogony (1928). J. H. Jeans. Cambridge University Press, Cambridge, England.

An Introduction to Variable Stars (1915). Caroline E. Furness. Houghton, Mifflin Company, New York, N. Y.

Astronomy (1927). H. N. Russell, R. S. Dugan, J. Q. Stewart. Ginn & Company, Boston, Mass.

Astronomy (1930). R. H. Baker. D. Van Nostrand Company, Inc., New York, N. Y.

New General Catalogue of Double Stars within 120° of the North Pole (1931). R. G. Aitken. The Carnegie Institution of Washington, Washington, D. C.

CHAPTER XVI

THE SIDEREAL STRUCTURE

354. Introductory Outline. In the preceding chapter we considered the stars as individuals—their brightnesses, their spectra, their temperatures, their masses, their diameters, their densities, their types of variability. Only in the case of binary stars were their relationships to one another of importance. Now we are taking up the problem of finding the number of stars in our galaxy, the nature of their organization in a system, the characteristics of other galaxies, and all ascertainable properties of the sidereal structure.

About 250 years ago the diameter and the shape of the earth were first measured with considerable precision. With the radius of the earth as a base line, astronomers measured the distances to the planets; and with the radius of the orbit of the earth as a base line, they have been measuring the distances to the stars of our galaxy, most of which are millions of times as remote as the sun. Within the last decade, their surveys have passed the limits of the Milky Way and have reached across starless voids to other galaxies and clusters of galaxies, to distances at which a system of a billion suns shrinks to the apparent dimensions of a particle of dust.

It is evident from the foregoing outline that we start with our earth and by successive steps go out to remote parts of space. The first step, that of determining the distances of the members of the solar system, is simple in theory and relatively simple in practice (Art. 192). But the stars are so enormously far away that there are serious practical difficulties in measuring their distances, though we use the radius of the orbit of the earth as a base line. In fact, the distances to only a few hundred stars have been directly determined, but astronomers have made this precious information the basis for calibrating a number of other ingenious methods which fail to be applicable only when stars are so remote that not sufficient light for analysis is received from them. It follows that the direct measurements of the parallaxes of the nearest stars are worth the enormous labor they cost, for they not only

determine the characteristics of our part of space but they aid us indirectly to sound the far depths of the sidereal system.

355. Deductions from Star Counts. A surprising amount of information can be obtained from so simple data as the number of stars of different magnitudes. It is easy to count the brighter stars on the whole celestial sphere, for there are not very many of them. The fainter stars, however, are so numerous that detailed counts of them are made only in a limited number of sample areas distributed uniformly over the sky. Counts are not made visually, but on long-exposure photographs, the magnitudes (photographic) of the stars being determined from their photographic images.

Let us first consider the matter theoretically under the assumption that all stars are equally luminous and that they are distributed uniformly throughout transparent infinite space. Suppose the stars at the distances R_{n-1}, R_n, and R_{n+1} from the observer are of magnitudes $n-1$, n, and $n+1$ respectively, n representing an arbitrary magnitude. Stars of magnitude $n-1$ are 2.512... times brighter than those of magnitude n, and stars of magnitude n are 2.512...times brighter than those of magnitude $n+1$. Since the brightness of a star varies inversely as the square of its distance from the observer, it follows that $R_{n-1}^2 = 2.512\ R_n^2$ and $R_n^2 = 2.512\ R_{n+1}^2$. Therefore $R_{n-1} = 1.585\ R_n$ and $R_n = 1.585\ R_{n+1}$.

Stars whose magnitudes are $n-1$, n, and $n+1$ will be on the surfaces of the spheres whose radii are R_{n-1}, R_n, and R_{n+1} respectively (Fig. 200). Let N_n represent the number of stars between magnitudes $n-1$ and n, and N_{n+1} the number between n and $n+1$. These numbers are proportional to the volumes of the spherical shells between corresponding radii. Let the volumes of the spheres of radii R_{n-1}, R_n, and R_{n+1} be represented by V_{n-1}, V_n, and V_{n+1} respectively. Then it follows that

$$\frac{N_{n+1}}{N_n} = \frac{V_{n+1} - V_n}{V_n - V_{n-1}} = \frac{R_{n+1}^3 - R_n^3}{R_n^3 - R_{n-1}^3} = 3.98\ldots,$$

as is easily found by using the formula for the volume of a sphere and the relations among R_{n+1}, R_n, and R_{n-1}.

We have derived the important result that, under the assumptions of *equal* luminosity and *uniform* distribution, the number of stars between any two adjacent magnitudes whatever is nearly 4 times greater than the number of stars between the next two

magnitudes fainter. That is, if our assumptions are true, we shall find 4 times as many stars between magnitudes 14 and 15 as between magnitudes 13 and 14.

Let us find next the ratio of the total amount of light received from stars of two adjacent one-magnitude intervals. Since the light received from a star varies inversely as the square of its distance, and since the mean distances of the magnitude intervals are in the ratio 1.585 to 1, it follows (letting L_{n+1} and L_n represent the total light received) that

$$\frac{L_{n+1}}{L_n} = \frac{N_{n+1}}{N_n} \frac{1}{1.585^2} = 1.585.$$

That is, under our assumptions, we should receive 1.585 times as much light from the stars in each one-magnitude interval as from those of the next brighter one-magnitude interval. We should receive 10,000 times as much light from stars of magnitude 21 as from those of the first magnitude.

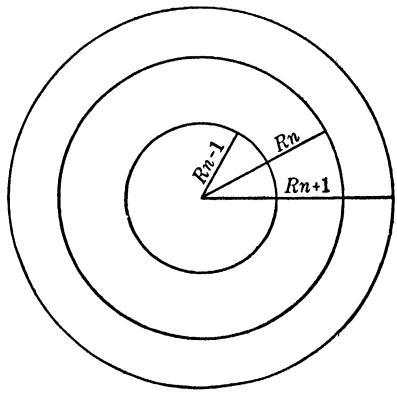


FIG. 200.—Volumes occupied by stars of successive magnitudes.

Since under the adopted assumptions there would be an infinite number of magnitudes, the whole sky would be as bright as a star, the sun, for example, but not infinitely bright because eclipses of the more remote stars by the nearer ones would eventually become an important factor. From this luminous sky the earth would receive nearly 200,000 times as much radiant energy as it receives from the sun.

Since the conclusions reached are not even remotely in harmony with experience, at least one of the hypotheses on which they are based must be false. It can be shown that the assumption that the stars are all of the same luminosity does not affect the results for the fainter magnitudes. However, as we already know, space is not perfectly transparent, for there are dark nebulae in the Milky Way; and the stars are not uniformly distributed, but are clustered in gigantic galaxies. Then we inquire whether the galaxies may not

be scattered throughout infinite space. The answer is that they can not be distributed *uniformly*, for then they would apparently cover the celestial sphere as the stars would under the former hypothesis. There are, however, other possibilities of an infinite physical universe to which we shall return (Art. 382).

Before closing this phase of the discussion, let us consider specifically the results that have been furnished by observations. In 1784, William Herschel began extensive star-counts of sample areas, and similar star-counts were continued in the southern heavens by John Herschel. Eventually, photography completely superseded visual observations in this work, for a single photographic plate may contain a permanent record of both the positions and the magnitudes of thousands of stars. Although all stars down to about the eleventh magnitude are known and catalogued, those fainter than the fifteenth or the sixteenth magnitude are so numerous that it is not practicable to count them in the entire sky. Consequently J. C. Kapteyn, the father of modern research in the galactic structure, proposed that detailed investigations of the stars be limited to those in 206 sample areas distributed uniformly over the celestial sphere. In conformity with the plan of Kapteyn, these selected areas have been studied at many observatories. For example, as many of them were photographed as were within reach of the 100-inch reflector of the Mount Wilson Observatory. Although the total surface of the selected areas is less than one-thousandth of that of the celestial sphere, there are recorded on the photographs of these regions more than 70,000 stars of various magnitudes down to the twenty-first, the limit attainable by the 100-inch reflector with the exposure-durations used. On multiplying the numbers of stars of various magnitudes recorded on these plates by the appropriate factor, approximate results are obtained for the entire celestial sphere. The results derived by F. H. Seares and P. van Rhijn from the star-counts of plates of the selected areas are given in Table XVIII.

In this table, the ratio of the number of stars in any one-magnitude interval to the number in the corresponding next brighter interval is not only less than the theoretical ratio 3.98, but it steadily decreases as the fainter magnitudes are reached. In the first 21 photographic magnitudes there are, according to the star-counts, 890,000 stars, and the total light received from them is equal to that which would be received from about 825

first-magnitude stars. If the numbers increased with fainter magnitudes according to the theoretical ratio 3.98, a total of 3,500,000,000,000 stars would have been found in the same magnitudes and the total light received would be equal to that from 100,000 first-magnitude stars. The difference between this theoretical number and the number found by observation is due partly to absorbing material in the Milky Way, but much more to the fact that our galaxy is composed of a limited number of stars occupying a limited space.

TABLE XVIII. NUMBERS OF STARS OF VARIOUS MAGNITUDES

MAGNITUDE LIMITS	NUMBER OF STARS	RATIO OF INCREASE	MAGNITUDE LIMITS	NUMBER OF STARS	RATIO OF INCREASE
4– 5	670		13–14	3,780,000	2 33
5– 6	1,910	2.85	14–15	8,500,000	2 25
6– 7	5,260	2.75	15–16	18,000,000	2 12
7– 8	14,600	2.77	16–17	37,000,000	2 06
8– 9	39,200	2.68	17–18	73,000,000	1.97
9–10	104,000	2.65	18–19	132,000,000	1 81
10–11	265,000	2 55	19–20	230,000,000	1.74
11–12	669,000	2.52	20–21	385,000,000	1.67
12–13	1,620,000	2 42	1–21	890,000,000	

Until recently astronomers believed that the number of stars in our galaxy does not exceed a few hundred millions, but now much larger limits are thought probable, the highest estimate being of the order of 100,000,000,000 suns. Although the uncertainties in the number are great, it is becoming evident that even the hundreds of millions of stars now individually within range of our greatest telescopes are only a small fraction of those which make up the galactic system.

356. Apparent Distribution of the Stars. Since the Milky Way is the dominant feature of the stellar system, it is desirable to base on it a system of galactic coördinates for use in describing the positions of the stars. The *galactic poles* are the two points on the celestial sphere most distant from the median line of the Milky Way, though the outlines of the Milky Way are so irregular that this line is not very well defined. To make the system of coördinates precise, astronomers have agreed to take for the north galactic pole the point in Coma Berenices whose right ascension is $12^h\,40^m$ and declination $+28°$. The south galactic pole is at the opposite point in the sky in the constellation Sculptor. The *galactic*

equator is the great circle midway between the galactic poles. It follows from the foregoing definitions that the galactic equator intersects the celestial equator from south to north, on going eastward, at right ascension $18^h\,40^m$. The *galactic latitude* of a star is its angular distance north or south of the galactic equator, and the *galactic longitude* of a star is its angular distance eastward along the galactic equator from its intersection with the celestial equator.

The star-counts give not only the numbers and the magnitudes of the stars in the selected areas but also their coördinates. In Table XIX the entries are the average number of stars per square degree of all magnitudes within the limits set in the first column, according to the Mount Wilson star-counts, and the galactic concentration in the last column is the number of times that the stars between given magnitude limits at the galactic equator exceed those in an equal area at the galactic poles.

TABLE XIX. DISTRIBUTION OF STARS IN GALACTIC LATITUDE

MAGNITUDE LIMITS	GALACTIC LAT. = 0°	GALACTIC LAT. = ±30°	GALACTIC LAT. = ±60°	GALACTIC LAT. = ± 90°	GALACTIC CONCENTRATION
0– 5	0.045	0.021	0 015	0.013	3 4
0– 7	0.36	0 17	0.12	0.10	3.6
0– 9	2.81	1.31	0.87	0 72	3.9
0–11	20.8	9.1	5.5	4.3	4.8
0–13	146 0	54 0	29.0	21.0	6.8
0–15	910 0	272.0	123.0	87.0	10.4
0–17	4,780 0	1,090 0	428.0	288.0	16.6
0–19	20,800 0	3,440 0	1,190.0	770.0	27.0
0–21	73,600.0	8,690.0	2,650.0	1,670.0	44.0

Three obvious conclusions follow from Table XIX: (*a*) Stars of all magnitudes down to the twenty-first are condensed near the plane of the Milky Way; (*b*) the relative condensation is greater the fainter the stars; and (*c*) the decrease in numbers of stars is progressive from the galactic equator to the galactic poles.

(*a*) Since the brighter stars partake of the galactic concentration, some of them must be so distant that their distribution is influenced by the general structure of the galaxy. Some of them are therefore of great intrinsic luminosity, a fact that is established also by direct means.

(*b*) Since the galactic concentration is greater the fainter the stars, it follows that the stars are not only more numerous in the

plane of the Milky Way than at its poles but they extend to much greater distances in this plane. That is, the galaxy can not be approximately spherical in shape with a plane of exceptional star density extending through it; it must be actually an extended disk whose equatorial diameter is much greater than its polar diameter.

(*c*) Since the decrease in stellar density is progressive from the galactic equator to the galactic poles, it follows that the sun is deep in the interior of the galactic system, though not necessarily near its center. In fact, there are variations from uniformity of star density with galactic longitude which indicate that the sun is far from the galactic center. In galactic longitude 320° and latitude 0°, in the constellation Sagittarius, the stars of the fainter magnitudes are about five times as numerous as they are in the opposite part of the sky, a fact which implies that the center of the galactic star-clouds which make up the Milky Way is in the direction of Sagittarius as seen from the present position of the solar system.

357. The Local Star-Cloud. It has been known since the time of John Herschel, or for more than 100 years, that the brighter stars have a tendency to cluster near a plane that is inclined by about 12° to the plane of the Milky Way and that intersects the galactic equator in galactic longitudes 70° and 250°. Since the brighter stars are on the whole less distant than the fainter ones, this fact implies the existence of a local cloud of stars analogous to the many star-clouds of which the Milky Way is composed. It should not be assumed that all members of this local group are conspicuous objects, for it is probable that it contains millions of fainter stars which are not readily distinguishable from the hundreds of millions of those which make up the Milky Way.

According to F. H. Seares, the local cloud is a flattened aggregation of stars whose greatest and least diameters are of the order of 20,000 and 7,000 light-years respectively, though its dimensions are yet very much in doubt (Art. 372). As nearly as can be determined at present, the solar system is distant 300 light-years from its center, which, from our present position, is in the direction of the constellation Carina at galactic longitude 236°. Moreover, the solar system is north of the center of the local cloud, but sensibly in the plane of the remote parts of the Milky Way. At least this conclusion is a satisfactory explanation of the fact that down to the ninth magnitude there are 50 per cent more stars south of the

galactic equator than north of it, while the numerous still fainter stars are symmetrically distributed with respect to the galactic plane. Figure 201, in which G represents the galaxy, C the cloud, and the dot the present position of the solar system, illustrates this conception of the relation of the local clouds to the Milky Way.

358. Relative Distances of Various Classes of Objects. It is not altogether unfortunate that we are deep in the interior of an enormous local star-cloud, for the inclination of its equatorial plane to the plane of the Milky Way provides us with an additional means of determining the relative luminosities of various classes of objects. Consider, for example, stars of some class which are not luminous enough to be visible if they are beyond the limits of our local cloud. They will evidently appear most numerous in the equator of our oblate local system. The actual situation is, of course, reversed. We observe that the stars of a certain class are concentrated in the equator of our local cloud, rather than in the plane of the galaxy, and we infer that they are relatively near. If, on the contrary, the objects of any kind are apparently limited to the Milky Way, with no appreciable concentration in the plane of the equator of our local system, we conclude that they are in remote parts of the galaxy and that they are therefore of high luminosity.



FIG. 201.—The galaxy G and local star-cloud C according to the conception of Seares.

It was pointed out in Art. 314 that the spectra of Class O stars imply that these objects are of very high temperatures and therefore presumably of great intrinsic brilliancy. This conclusion is confirmed by the fact that these stars are found only on or near the galactic equator, their concentration in the plane of the galaxy being due to the fact that they are luminous enough to be visible in the far star-clouds of the Milky Way. They are also found in the exterior galaxies known as the Magellanic Clouds.

Stars of spectral Class B also cluster in the plane of the Milky Way, but those brighter than the sixth magnitude show strongly the effects of our local cloud. The inference is that they are objects of very high luminosity, though not equal in brilliancy to stars of Class O. The stars of Class A also show a galactic concentration, particularly those of the fainter magnitudes, but not so much as

those of Class B. Stars of spectral Classes F and G show very little galactic concentration, from which it follows that they are not sufficiently luminous to be within the range of our instruments if they are beyond the limits of our local cloud. In fact, they are not visible even from the far parts of our local group, for they show no appreciable concentration in the plane of its equator. The stars of Classes K and M, however, do show some tendency to follow the Milky Way, while stars of Class N are almost limited to the plane of the galaxy.

Other classes of celestial objects that are limited almost exclusively to the galactic zone are the Cepheid variables, the temporary stars, the planetary nebulae, and the dark clouds which veil parts of the Milky Way. On the contrary, the spiral nebulae (Art. 379) are not found at all in the Milky Way. They are independent galaxies, far beyond the borders of the Milky Way whose dark obscuring clouds occult all the spirals which are in its plane.

From counts of stars of various magnitudes and from their apparent distribution we have deduced important conclusions respecting the galactic system. Moreover, we have been able to determine the relative luminosities of the principal classes of stars. These investigations do not give us, however, the actual dimensions of the sidereal structure. In order to determine its scale we must measure the distances of enough stars of various classes to provide us with indirect means for spanning its abysmal depths.

359. Direct Determination of Stellar Parallaxes. By definition, the parallax of a star is the angle subtended at the star by the major semi-axis of the orbit of the earth about the sun (Art. 104). An equivalent definition is that the parallax of a star is the angular difference in direction of the star when it is observed from two points separated by the mean distance from the sun to the earth and such that the line joining them is perpendicular to the line from the sun to the star.

Since 1903, stellar parallaxes have been directly determined almost entirely by the photographic method, the geometric basis for which is illustrated in Fig. 202. In this figure E' and E'' represent two positions of the earth which, for convenience, will be assumed to be separated by the mean distance from the sun to the earth. It will be assumed further that the line joining E' and E''

is perpendicular to the line from the sun to the star S_1 whose parallax is to be measured. Let S represent a portion of the celestial sphere. The star S_1 as seen from E' appears to be on the celestial sphere at S_1'. The star S_1 as seen from E'' appears to be on the celestial sphere at S_1''. The distance between S_1' and S_1'' is of course greatly exaggerated by representing S_1 as relatively near the sun. In general there are many other stars in nearly the same direction from the sun as S_1, but only one additional star S_2 is represented in the diagram.

According to the first form of the definition of parallax, the parallax of S_1 is the angle $E'S_1E''$, which we shall represent by p_1. The parallax p_2 of S_2 is the angle $E'S_2E''$. The angle $S_1E'S_2$ is equal to the distance expressed in seconds of arc, between the photographic images S_1' and S_2'. The angle $S_1E''S_2$ is similarly equal to the arc between S_1'' and S_2''. Let the arcs S_1' to S_2' and S_1'' to S_2'', which are measured on the photograph, be represented by d' and d''. Then it follows from the simple geometry of the figure that

$$p_1 - p_2 = d' - d'',$$

though it may take a more exaggerated figure to show the relationship clearly. If the star S_2 is so far from the sun that its parallax p_2 is inappreciable, the equation gives the desired parallax p_1 as the difference of the measured arcs d' and d''. Corresponding measurements will of course be made for S_1 paired with all the other stars in its neighborhood. If they are faint stars, the chances are that most of them have negligible parallaxes.

Although the photographic determination of stellar parallaxes is simple in theory, there are several practical difficulties. In the first place, the plates for the parallax of a given star must be taken at times separated by several months, and during these intervals climatic and possibly other conditions may considerably change. In the second place, the stars often have appreciable proper motions which must be eliminated by extending the measurements over several years. In the next place, the photographs are usually secured when the star is at various distances above the horizon, and therefore there are various corrections for atmospheric refraction and dispersion to be applied. And if the star whose parallax is to be determined is much brighter than the comparison stars, its light must be dimmed by some means so that its photographic image will be comparable with those of the remainder. Further-

more, a telescope objective does not give a picture on precisely the same scale over the entire plate, and corrections must be made for these minute distortions. Nor will the stellar images be circular except at the center of the plate, a fact which adds to the difficulty of measuring the distances between them. Occasionally a comparison star is nearer than the star in which interest is centered and the difference $d'-d''$ is found to be negative.

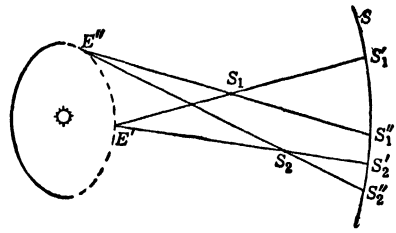


FIG. 202.—Relative parallax.

It is not expected that any but specialists will remember all the practical difficulties there are in determining the parallaxes of the stars. Some of the principal difficulties have been enumerated in order to make clear how serious is the problem of determining the dimensions of the stellar system and how excessively costly it is in labor. According to the method developed by F. Schlesinger, who laid the practical foundations for this important branch of astronomy, a dozen or more photographs of a region are taken, extending over a period of two or three years. The plates are measured under a specially devised measuring machine, and about half a dozen comparison stars are generally used for the determination of the parallax of a star.

360. Limits Attainable by the Direct Method of Measuring Parallaxes. Since all our knowledge of stellar distances depends directly or indirectly upon trigonometric parallaxes of the stars, it is important to determine as many of them as possible. The limits of accuracy attainable naturally depend upon the instruments used; the longer the focal length of the telescope, other things being equal, the greater the scale of the photographs and the more accurate the results.

When astronomers determine a parallax by measurements, they get a result subject to a *probable error*. For example, they might find $p = 0''.216 \pm 0''.011$, which means that, according to the measurements, the most probable value of the parallax is $0''.216$; and, further, that the probability of its being within $0''.011$ of $0''.216$ equals the probability of its differing from $0''.216$ by more than $0''.011$. In other words, in this case the chances that the parallax

lies between 0″.205 and 0″.227 equal those that it lies outside these limits. Under the most favorable conditions, the probable error of a trigonometric parallax determination usually lies between ±0″.005 and ±0″.015, whatever the distance of the star. Consequently, if the parallax of a star is 0″.10, a single determination of it is usually not in error more than 10 or 15 per cent. If the parallax of a star is 0″.05, the mean of the determinations of it at several observatories is probably not in error more than 20 or 30 per cent. But the parallaxes of stars which are less than 0″.02 are not very trustworthy. For example, the trigonometric parallax of Algol, as measured at various observatories, has ranged from 0″.011±0″.010 to 0″.020±0″.014. It is evident that the parallax of Algol is small and quite uncertain.

Let us translate the foregoing results into distances. It is convenient for theoretical work to have a unit of distance corresponding to a parallax of 1″.0. Astronomers have named this unit the *parsec* (*par*-allax *sec*-ond), which is 206,265 times the distance from the sun to the earth, or 3.258 light-years. The general formula connecting the distance D (in parsecs) of a star and its parallax p (in seconds of arc) is

$$D = 1/p \text{ parsecs} = 3.258/p \text{ light-years.}$$

It follows from the formula that a parallax of 0″.1 corresponds to a distance of 10 parsecs, or 32.58 light-years. If the probable error of the parallax is 0″.01, the chances are one-half that the distance to the star is between 29.6 and 36.2 light-years. But if the parallax of a star is only 0″.05 with a probable error 0″.01, the most probable value of its distance is 65.2 light-years, with the chances even that it lies between 55.1 and 80.1 light-years. The uncertainty in the distance increases rapidly as the parallax decreases (distance increases). For a parallax of 0″.02 and a probable error 0″.01, the probability is one-half that the distance of the star is between 109 and 326 light-years. In this case our knowledge of the distance of the star is evidently very uncertain.

Up to the present time (1931) the parallaxes of approximately 2500 stars have been measured, many of them at several observatories. About one-third of the measured parallaxes are so small, however, that they merely indicate that the stars are very remote, but even this information is important for it enables us to determine lower limits to the luminosities of these stars. When we consider the hundreds of millions of stars in our galaxy, the number of

those whose distances can be directly measured seems pitifully small, as indeed it is. The precious results that have been secured are used, however, in many ingenious ways and to the fullest possible extent; and, moreover, they have been supplemented by information obtained by a variety of indirect methods, some of which are of great power and generality.

361. Stellar Density Near the Solar System. There is no reason for believing that the frequency of stars in the neighborhood of the solar system is exceptional. It is true that there are great clusters in our galaxy in which the distances between neighboring stars are less than normal, and there are other regions in which stars are relatively infrequent; but it seems reasonable to assume that in terms of very large volumes they are distributed nearly uniformly throughout the space which they occupy. If there is any large-scale departure from such a distribution, it is that the densities of the great star-clouds are greatest at their centers and least near their exteriors. The hypothesis of uniform distribution is not an unsupported assumption, for the stars have undoubtedly circulated among one another and have been subject to their mutual attractions during millions of millions of years, and a consequence of their mutual interactions is a tendency toward the homogeneity which we are assuming.

Stellar density in the neighborhood of the solar system can be determined from the trigonometric parallaxes of the nearer stars. Then, under our assumption, the volume occupied by the stars can be calculated from their total number, and even their distribution can be worked out from their numbers in various directions. It is not possible to carry out at present this ambitious program with any considerable degree of approximation because it is probable that a considerable fraction of the stars of our galaxy are beyond the reach of our instruments, and dark nebulae obscure many which would otherwise be visible.

If we determine stellar density in a relatively small volume, we run the risk of not getting a fair sample of the average; if we use too large a space, we get beyond the distances at which trigonometric parallaxes are reliable. Table XX is a list of the stars whose parallaxes are not less than 0″.25. It is noticed that a number of them are very faint; there may be others within the limits of the distance set whose parallaxes have not yet been measured.

TABLE XX. STARS WHOSE PARALLAXES ARE NOT LESS THAN 0.″25

STAR	PARALLAX	DISTANCE LIGHT-YEARS	MAGNITUDE	LUMINOSITY (SUN = 1)	SPECTRAL CLASS	VELOCITY (KM. PER SEC.)
α Centauri	0.″758	4.3	0.3	1.15	G0	31.8
Barnard's	0.555	6.1	9.7	0.0004	M5	135.
Wolf 359	0.404	8.1	13.	0.000003	M4	106.
Lalande 21185	0.392	8.3	7.6	0.002	M6	104.
Sirius	0.371	8.8	−1.6	27.0	A0	33.6
B.D. −12°4523	0.350	9.3	10.	0.0007	M5	10. +
Innes'	0.340	9 6	12.	0.00012	M	38. +
Kapteyn's	0.317	10.3	9.2	0.0018	M0	131. +
τ Ceti	0.315	10.3	3.6	0.32	K0	33.0
Procyon	0.312	10.4	0.5	5.65	F5	19.3
ε Eridani	0.310	10.5	3.8	0.27	K0	21.8
61 Cygni	0.300	10.9	5.6	0.055	K5	104.
Lacaille 9352	0.292	11.1	7.4	0.011	M0	113.
Σ 2398	0 287	11.4	8.7	0.0035	M4	38.4
Groombridge 34	0.282	11.5	8.1	0.005	M2	48.6
ε Indi	0.281	11 6	4.7	0.145	K5	88.2
Krueger 60	0.257	12.7	9.6	0.0019	M3	29.2
Van Maanen's	0.255	12 8	12.3	0.00016	F0	56. +
Cordoba 29191	0.253	12.9	6.6	0.031	M0	68.5

Of the stars in Table XX, Sirius, Procyon, 61 Cygni, Σ 2398, and Krueger 60 are doubles, and α Centauri is a triple. It follows that, including our sun, there are at least 27 stars in a sphere about the solar system whose radius is 4 parsecs. In view of the fact that several of these stars are very faint, it is probable that there are still others within the sphere which are as yet undiscovered. Let us assume that there are 30 stars within 4 parsecs of the sun. If they were uniformly distributed in this space, the distance between adjacent stars would be 2.58 parsecs, or 8.4 light-years. If the ratio of shortest to longest diameter of our local star-cloud (Art. 357) is 7 to 20, as seems probable from the apparent distribution of the stars of which it is composed, and if there are 40,000,000 stars uniformly scattered throughout this star-cloud, then its shortest and longest diameters are found to be 630 and 2180 light-years respectively. The volume of this cloud is only 1/780 of the volume assigned to this vast aggregation of stars by F. H. Seares.

Similarly, if the diameters of the Milky Way are in the ratio of 1 to 10 and if there are in this galaxy 30,000,000,000 stars with the average stellar frequency of our immediate vicinity, then its least and greatest diameters are 2800 and 28,000 light-years respec-

tively. Although these numerical results are very uncertain and probably somewhat too small, they make it evident that the hundreds of millions of stars with which the Milky Way is strewn imply that our galaxy is a cosmic unit whose diameters are thousands of light-years.

362. Other Conclusions from Table XX. Before we pursue still further the subject of the distances of the stars and of the properties of the sidereal structure, we will note a few other conclusions which follow immediately from Table XX.

(*a*) Perhaps the most obvious fact exhibited by the table is that the stars cover an enormous range in luminosity. Although there does not happen to be even one of the relatively few giant Class O or Class B stars within 4 parsecs of the solar system, yet the range from brilliant Sirius to the feebly luminous star Wolf 359 is 9,000,000 to unity. If the super-giants were included, the range would be probably a hundred times greater. Moreover, it is highly probable that there are many stars of such low luminosity that they have not been observed at all.

(*b*) There is a relationship between the luminosities of the stars and their spectral classes, though the table does not include enough objects fully to establish the law. When more extensive data from more remote stars are included in the discussion, it is found that the Class O stars are super-giants and that there is a steady decrease in luminosity through the spectral Classes B, A, F, G, and K to M. At least this is the rule, though there are giants in every spectral class (Art. 315) and three white (Class A) dwarfs are known (Art. 316).

The fact that great care must be taken in statistical investigations in order not to derive erroneous conclusions is simply illustrated in the problem of determining the relative numbers of stars of the various spectral classes. Suppose, for example, that the number of stars down to the ninth magnitude has been determined for each spectral class separately. It obviously would be quite erroneous to conclude that these numbers represent the relative proportions of stars of the various classes in the galaxy as a whole, for the very luminous Class O and Class B stars are included from a much larger volume than that occupied by the relatively dull Class K and Class M stars contained in the statistical data. If, for example, the Class B stars average 10,000 times more

luminous than the Class K stars, then the volume the former occupy down to a given magnitude is 1,000,000 times greater than that occupied by the latter. Hence if no correction were made for the greater luminosity of the Class B stars, it would be inferred that they are a million times more numerous relative to the Class K stars than they actually are.

The necessity for care in interpreting the data of the foregoing simple problem is due to the fact that there is a correlation between the characteristics involved; namely, the magnitudes and the spectral classes of the stars counted. In every stellar statistical investigation there is necessarily a basis for choice of material, and often this basis of choice has an effect that must be analyzed and taken into account in order that erroneous conclusions may not be drawn.

(*c*) Stellar velocities with respect to the sun can be resolved into cross components and radial components. The former can be determined from proper motions and the distances of the stars; the latter, by means of the spectroscope if the stars are not too faint. In the last column of Table XX the velocities of those stars are given for which both the cross components and radial components are known; in the case of those stars for which the radial velocities have not been determined, the cross components are given, followed by the plus sign.

It follows from the table that in general the less the luminosities of the stars the higher their velocities with respect to the sun. This conclusion is supported by more extensive data including stars to much greater distances. It follows also from the correlation between luminosities and spectral classes that the velocities of the stars increase from a minimum for stars of Class B through the Classes A, F, G, and K to M.

(*d*) Four stars in Table XX are visual binaries with known periods; consequently their masses are also known (Art. 330). Table XXI contains these double stars and several other pairs whose parallaxes, distances apart, and periods have been determined with considerable approximation. It follows from this table that the blue and the yellow stars of spectral Classes A, F, and G have larger masses than the orange and the red stars of spectral Classes K and M. At least this is the rule for stars of the main spectral sequence (Art. 310), but it may not be true of the giant stars.

TABLE XXI. VISUAL BINARIES WITH KNOWN PARALLAXES AND PERIODS

STAR	PARALLAX	MAGNITUDES	SPECTRAL CLASSES	PERIOD (YEARS)	MASSES (SUN = 1)
α Centauri	0″.76	0.3; 1.7	G0; K5	78.8	1.10; 0.94
Sirius	0.37	−1.6; 8.4	A0; F0	50.0	2.44; 0.96
Procyon	0.31	0.5; 13.0	F5; ?	39.0	1.1 ; 0.4
Krueger 60	0.26	9.7; 11.3	M3; ?	44.3	0.27; 0.18
70 Ophiuchi	0.19	4.3; 6.0	K0; K6	87.7	0.9 ; 0 8
β 416	0.17	6.1; 8.1	K2; ?	42.2	0.4 ; 0.3
ξ Boötis	0.17	4.8; 6.7	G6; K4	153.	0.53; 0.47
ξ Urs. Maj.	0 15	4 4; 4.9	F9; G2	59.8	0.7 ; 0.7
ζ Herculis	0.11	3.0; 6.5	G0; ?	34.5	1.1 ; 0.5
μ Herculis BC	0.11	10.0; 11.0	M2; ?	43.2	0.5 ; 0.4

363. Distances of Moving Groups of Stars. In order that the parallax of an individual star may be directly determinable with any considerable precision, its distance must not exceed 100 to 150 light-years. There is a method, however, that is applicable to groups of stars moving as a unit which is not subject to these severe limitations as to distance, and which, moreover, may give more accurate results than the direct method except for stars which are within 15 to 20 light-years of the sun.

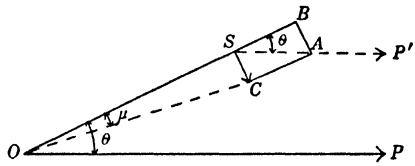


FIG. 203.—Resolution of motion of a star into radial velocity and cross component.

If the members of a group of stars move in parallel lines, their paths on the celestial sphere, as determined from their proper motions, apparently diverge from one point and converge toward another 180° distant. The effect is one of perspective, just as it is in the case of the apparent divergence of meteors from a radiant (Art. 254). The line from the earth to the convergent point is parallel to the line of motion of the group of stars. It will now be shown that this direction of motion and the radial velocity of the group, which can be measured provided only that the stars are not too faint, enable us to determine the distance of the cluster.

Let *O*, Fig. 203, represent the position of the observer, and *OP* the line directed toward the convergent point of a moving group of stars. Let *S* represent one of the stars of the group, and *SP′*, which is parallel to *OP*, the direction of its motion. The speed of the star is as yet unknown, but let *SA* represent the distance the

star moves in a unit of time. The motion represented by SA can be resolved into the radial velocity SB and the cross component SC. Now suppose the radial velocity SB is measured by means of the spectroscope. Then, since the angle θ is known, the speed SA can be computed; and, likewise, the cross component SC can be computed from SA and the angle ASC. In fact, it easily follows that $SC = SB \tan \theta$.

A final step remains in determining the distance OS of the star from the solar system. Since the proper motions of the stars are used in determining the convergent point of the group, the proper motion μ of the star S is known. Now since the distance SC has been computed and the angle μ has been measured, the distance OS of the star can be computed.

In applying the method under discussion, only two quantities have to be determined from observations: the angle θ and the radial velocity SB. If the cluster is spread over several degrees and if the proper motions of its members are considerable, the convergent point is well determined and the value of θ can be found to within perhaps one degree. The greater the interval over which the observations extend, the longer are the proper-motion arcs described and the more accurately is the convergent point located. It is necessary, of course, to measure also the radial velocity of the star. The value obtained can be checked by comparing it with the radial velocities of the other members of the group, for they all are necessarily very nearly equal. It follows from the formula $SC = SB \tan \theta$ that if the radial velocity is determined within 5 per cent and the angle θ within 2 or 3 degrees, then the distance of the star can be found with an error not exceeding 5 per cent. The parallaxes of only the few stars which are within 5 parsecs of the solar system can be individually determined with so great accuracy.

Since stars are very numerous in every part of the sky, one may wonder how astronomers know they are dealing with the members of a related group. In the first place, there are obvious clusters in the sky, such as the Pleiades, the Hyades, and Coma Berenices, which presumably are physically related. If the stars of a cluster are of approximately the same magnitude and especially if they have similar spectra, the presumption is increased. The real test, however, is the agreement in the direction and the speed of their proper motions, for there is only about one chance

in a thousand that a star not belonging to a group would accidentally have a proper motion agreeing in direction within 3 degrees and in amount within 10 per cent with that of the system as a whole. Even if a foreign star had a proper motion agreeing with that of the members of the group, it would almost certainly be excluded by a different radial velocity.

364. The Taurus, Ursa Major, Scorpius-Centaurus, and Other Moving Groups of Stars. (*a*) *The Taurus group.* About 20 years ago Lewis Boss found that 39 stars in Taurus, including most of the brighter stars of the Hyades (except Aldebaran) and a number of others besides, are apparently moving toward a point a few degrees east of Betelgeuse, in Orion. About 80 stars are now known to belong to this system; and since they spread over more than 15° in both right ascension and declination, they accurately define the point toward which their apparent motions converge. Many of these stars, moreover, are so bright (magnitudes 4 to 7) that there is no difficulty in determining their radial velocities.

At present the center of the Taurus group is at a distance of 130 light-years from the solar system. Consequently the parallaxes of its stars, which are in the neighborhood of 0″.025, can not be very accurately determined by the direct trigonometrical method. Nevertheless, as a consequence of their belonging to a moving cluster, their distances are known with a precision not exceeded even by the stars within 5 parsecs of the sun. About 800,000 years ago the Taurus cluster, then at its nearest approach, was 65 light-years from the solar system; 65,000,000 years in the future it will have receded until from the earth it will appear to be a compact group of stars of the ninth magnitude and fainter, and only one-third of a degree in diameter.

It follows from the distance and the apparent distribution of the stars of the Taurus cluster that most of them are within a nearly spherical space about 35 light-years in diameter, though a few pursue parallel paths at much greater distances from the center of the group. Even in the deep interior of this tremendous cluster, the average distance between adjacent stars is as great as the distance from the sun to Sirius. Its brightest members are comparable to Sirius in luminosity, and as seen from one another they would be comparable to Sirius in magnitude. Although these stars attract one another, their distances apart are so great that their

gravitational effects are exceedingly feeble and their mutual velocities are very low; otherwise they would not move in parallel lines at equal speeds. It is quite probable that several foreign stars, having come from different directions in space, are now passing through this great family of suns. Although they will be guests, as it were, of this group for something like a million years, their paths will not be appreciably changed and they will eventually

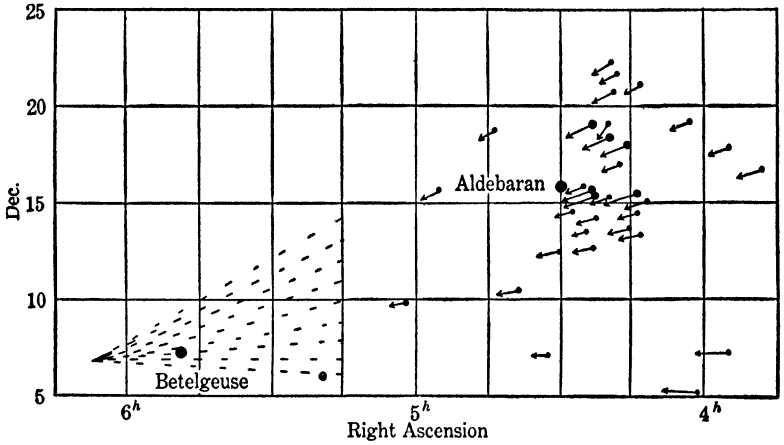


FIG. 204.—Motions of stars in the Taurus group. (Lewis Boss.)

leave the neighborhood of these stars and pursue their courses to remote parts of the galactic regions. In fact, there is no physical reason why two such clusters might not pass through one another without having any important mutual effects.

As we reflect on the significance of the foregoing statements, we are likely to assume, at least for the moment, that such clouds as that in Taurus are simply accidental communities of stars of no more import than the groupings into constellations which caught the fancy of the ancients. Such a conclusion, however, would be quite erroneous. They undoubtedly constitute a family in the strictest sense of the word, for their common dynamical properties imply a common origin, perhaps from a nebula a million million years ago. But as they wander through the wide dominion of our galaxy they are subject to continual minute disturbing forces from other stars which eventually will break all their family ties.

(*b*) *The Ursa Major group.* Our solar system is at present in the interior of a moving group of stars which includes all those of the

Big Dipper (except the first and the last), Sirius, α Coronae Borealis, and β Aurigae, as well as a number of fainter stars. The sun is not a member of this system of stars, for it does not partake of their motions nor is it of the same spectral class, but it happens now to be passing among them.

Since the sun is in the interior of the Ursa Major group, the distances to its various members naturally differ greatly. Sirius, the nearest of these stars, is distant only 8.8 light-years, while those in the Big Dipper are 70 to 80 light-years away. It follows that this cluster is much more open than that in Taurus. Perhaps it is older and is only the scattered remains of what was once a more numerous and compact family of suns.

(*c*) *The Scorpius-Centaurus group.* In apparent area and in number of members, this moving group of stars is one of the greatest of the type known. It includes many of the brightest southern stars, such as the Southern Cross, α Centauri, Antares, and Spica. One would be likely to infer from its wide apparent extent and the brightness of its stars that it is relatively near, but as a matter of fact many of its members are brilliant suns of spectral Class B and it is actually very remote. The stars of the Southern Cross are distant more than 200 light-years, while the members of the bright group in Scorpius are nearly twice as far away.

It is fortunate that Class B stars are found in these moving groups so that their distances and luminosities can be determined, for there are no examples of these stars within the range of accurate trigonometrical parallaxes. These results help us to calibrate the means by which, as we shall see, the more distant parts of space are explored.

(*d*) *Other groups.* There are several other moving groups for which the fundamental data, on which the computations of distances depend, have not yet been accurately determined. In some cases the stars cover so small an area that their convergent points are not well defined; in others, longer intervals of time must be used in order to determine accurately their proper motions. Among these other systems are the Perseus group, at a distance of more than 300 light-years; the Orion group of Class B stars, some 500 light-years away; the Pleiades, at a distance of 220 light-years; and the Coma Berenices and Praesepe clusters, for which convergent points have not yet been well determined.

In the problem of directly determining trigonometrical parallaxes one year is as good as another, for the base line on which the measurements depend is the radius of the orbit of the earth. But in the case of moving groups of stars time works with the astronomer, for as the years go by their proper-motion arcs steadily increase in length until ultimately they accurately define the points in the sky toward which they apparently converge.

365. Determination of Mean Stellar Parallaxes from Proper Motions and Radial Velocities. In the problem of determining the distances to the stars, astronomers are not limited to the relatively few whose parallaxes can be directly measured and to those which belong to well-defined moving groups. There are several indirect statistical methods of finding the average parallaxes of classes of stars. Since in determining the characteristics of the sidereal structure, individual stars sink into insignificance and only the common properties of millions of them are of interest, these statistical methods that we are about to describe give fully satisfactory results.

In order to make clear the essential principles of the statistical methods, we shall consider first an ideal problem which is somewhat simpler than the actual. Suppose that at the time the investigation is undertaken there are many stars between two concentric spherical surfaces about the sun as a fixed center, as is illustrated in Fig. 205. Let it be assumed, further, that the stars are moving at random both in direction and in speed. Under the hypotheses that have been adopted, the algebraic sums of both the radial velocities and also the proper motions parallel to every direction are zero.

Each star has a motion with respect to the spherical surfaces, called its *peculiar motion*, which is the same as its motion with respect to the average of the entire system under consideration. The velocity of each star can be resolved into two components, one of which is toward or from the surface and the other of which is perpendicular to the line to the surface. The former is the *radial velocity* of the star; the latter is the *tangential velocity* which, together with the distance of the star, determines its proper motion. In the case of an individual star it is, of course, not possible to determine what part of its entire motion is radial and what is tangential; but it can be shown by a mathematical investigation that the aver-

age of the radial velocities, without regard to sign, of all the stars considered is $\frac{1}{2}V$, where V represents the average speed of all the stars. If the radial velocities of all the stars are measured by means of the spectroscope and their average is taken, we have $\frac{1}{2}V$, and hence the average speed V with which the stars of the class move.

Now consider the tangential motions. It can be proved by a mathematical discussion that the average tangential velocity of all the stars under consideration is 0.7854 V, which is known after V has been determined by measurements of radial velocities. The average of the proper motions of the stars can be determined, at least if the observations extend over a long interval of time. Now we have the average tangential motion expressed linearly, namely 0.7854 V, and the same expressed in angle, namely, the average proper motion. Hence we can compute the average parallax of the stars of the class considered; and since the stars do not differ greatly in distance, we have the approximate distance of the individual stars.

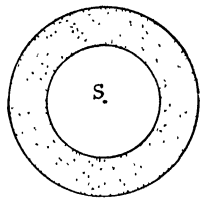


FIG. 205.—Homogeneous group of stars.

In spite of the great generality of the statistical method, it has some practical limitations. It is obviously important to use both the radial and the tangential components of precisely the same class of stars. Since the displacements due to proper motions increase with time, proper motions can ultimately be found however small they may be; but the determination of radial velocities is limited by the faintness of the stars, for the spectroscope spreads out and attenuates their light to such an extent that excessively long exposures are required for stars below the eighth or the ninth magnitude. Therefore it is very costly at present to determine the radial velocities of any considerable class of faint stars, although their proper motions present no insuperable difficulties. Another complication in the statistical method is due to the fact that there are large groups of stars having equal, parallel motions, and even great aggregations having similar motions. In such groups the relationship between the average radial and tangential components of motion is quite unknown. Finally, the sun is not stationary at the center of any extensive class of stars, but has a peculiar motion of its own.

366. Motion of the Sun with Respect to the Stars. Before going further into statistical discussions pertaining to stellar distances, it is necessary to consider the motion of the sun with respect to the stars. There are two distinct methods of investigating the solar motion: (*a*) from proper motions and (*b*) from radial velocities.

(*a*) *Proper motions.* More than a century ago (1783), William Herschel found on comparing early observations with those of his own day that the brighter stars in the direction of the constellation Hercules are apparently moving radially outward from a center, while those in the opposite part of the sky are slowly converging toward a similar center. He correctly interpreted these very slight apparent motions as being due to an actual motion of the sun toward Hercules. It is clear that this explanation is fully in harmony with the observed phenomena.

Herschel's discussion enabled him to determine the direction of the motion of the sun with respect to the brighter stars, but it did not provide a basis for calculating its speed. Of course, if the distances of the stars in the direction of Hercules were known, the speed of the sun could be found from the rate at which they appear to separate. In Herschel's day not a single stellar parallax had been measured, and even now not enough of them have been determined to make a satisfactory basis for measuring in this way the speed of the sun with respect to even the nearer stars.

In the early part of this century, Lewis Boss inferred from an analysis of the proper motions of 6188 stars distributed widely over the celestial sphere that the solar apex is at right ascension $270^{\circ}.5$ and declination $+34^{\circ}.3$, with a probable error of about $\pm 1^{\circ}.5$ in each coördinate. Essentially the same results were obtained when he made computations separately for stars of various galactic latitudes, but H. C. Wilson's discussion of revised proper motions of 2748 of the same stars reduced the declination of the solar apex to $+27^{\circ}$. Moreover, a determination by F. W. Dyson and W. G. Thackeray of the solar apex from the proper motions of 3707 stars from the Groombridge catalogue gave a declination ranging from $+16^{\circ}$, for the brighter stars, to $+43^{\circ}$ for those between magnitudes 8 and 9. Finally, Boss and also Dyson and Thackeray found that the declination of the solar motion is about 12° less when it is determined from the proper motions of stars of spectral Classes B and A than it is when determined from those of spectral Classes F, G, and K. All this is an indirect result,

of course, of the fact that the stars of the spectral Classes B and A are of high luminosity and mostly at great distances, while the stars of spectral Classes F, G, and K are fainter and for equal magnitudes much nearer the sun. Obviously the sun might move at a different speed and in a different direction with respect to groups of stars at different distances, for the near stars might have motions as a whole with respect to the remote stars.

(*b*) *Radial velocities.* The second method of determining the motion of the sun gives not only the direction in which it moves but also its speed relative to the stars. This method is simply that of measuring the radial velocities of the stars. On the average the radial velocities are negative (indicating relative approach) for the stars in the direction in which the sun is moving, positive for those in the opposite direction, and zero for those midway between.

For more than 30 years radial velocities of the stars have been accumulated at many of the great observatories of the world. The latest (1928) results obtained by W. W. Campbell and J. H. Moore from an analysis of the radial velocities of more than 2000 stars scattered widely over the sky place the solar apex at right ascension $270^{\circ}.6$ and declination $+29^{\circ}.2$, and give a velocity of the sun with respect to the stars of 19.65 kilometers per second.

The position of the solar apex as determined from radial velocities agrees excellently with that based on proper motions. Evidently the motion of the sun with respect to the local star-cloud is approximately in the direction defined by right ascension 270° and declination +30°, and its speed with respect to these stars is about 20 kilometers per second.

367. Determination of Mean Parallaxes of Stars from Their Parallactic Motions. When a person observes exterior objects from a swiftly moving train, he sees them apparently passing to the rear, the more slowly the farther they are away. It is obvious that if he knew the speed of the train and the apparent angular rates at which different objects drift backward, he could compute their perpendicular distances from the railway tracks. Similarly, the stars have backward components of proper motion due to the motion of the sun. These backward components of proper motion are called the *parallactic motions* because their magnitudes depend upon the parallaxes of the stars. The observed radial velocities and the proper motions of the stars are evidently compounds of the

peculiar velocities of the stars and of their backward components due to the solar motion. The statistical problem is to disentangle these two effects in the averages for large numbers of stars.

It is evident that the motion of the solar system has no effect upon the radial velocities of the stars which are 90° from the solar apex. Moreover, if the proper motion of a star is resolved into two components, one parallel to and the other perpendicular to the direction of the sun's motion, the second will not be affected by the motion of the sun. It can be proved that the latter of these two components averages 0.637 of the total average proper motion. Hence the method of Art. 365 is applicable for stars 90° from the apex of the sun's way, provided only that the component of proper motion perpendicular to the direction of the sun's motion is used. The objection to this special use of the method is, of course, that it is not based on all the stars in the class under consideration.

In the general problem, three things are to be determined: (*a*) The coördinates of the solar apex, (*b*) the peculiar velocity of the sun, and (*c*) the mean parallax of the stars.

(*a*) The solar apex is defined by the conditions that both the sum of the components of radial velocities and also of proper motions parallel to its direction from the sun shall be a maximum, and that corresponding components perpendicular to this direction shall be zero.

(*b*) The peculiar velocity of the sun is equal to the average of the components of the radial velocities of the stars parallel to the solar motion, for if the sun were stationary this sum would be zero. The sun's velocity can not be determined from proper motions alone, for they are only angles.

(*c*) The mean parallax of the stars of the class under consideration is determined from their mean parallactic motions and the peculiar motion of the sun. The former is obtained from the components of the proper motions of the stars parallel to the solar motion; the latter is determined in (*b*). The principles involved in these steps are easily grasped; to work out their details would lead us too far into trigonometric formulas.

Distances corresponding to mean parallaxes can be computed from the formula (Art. 360) $D = 3.258/p$ light-years, but these numbers will not be mean distances because the reciprocal of the mean of a set of numbers is in general not the mean of their reciprocals.

368. Mean Parallaxes of Stars of Various Magnitudes. J. C. Kapteyn first used statistical methods of the kind we have been discussing for determining mean parallaxes of various classes of stars. His basis for selecting classes of stars was primarily their visual magnitudes, though he made separate discussions for stars of the spectral Classes B, A and F, G, and K. With somewhat improved observational data, F. H. Seares has derived the somewhat different results which are given in Table XXII for all stars from magnitudes 1 to 12. For convenience, the mean parallaxes are translated into corresponding distances expressed in light-years.

TABLE XXII. MEAN PARALLAXES OF STARS

MAGNITUDE	MEAN PARALLAX	LIGHT-YEARS	MAGNITUDE	MEAN PARALLAX	LIGHT-YEARS
1	0.″060	54	7	0.″0090	362
2	0.044	74	8	0.0065	501
3	0.032	100	9	0.0047	693
4	0.023	142	10	0.0034	960
5	0.017	192	11	0.0025	1300
6	0.012	271	12	0.0018	1800

Although this table probably gives rough approximations to the mean parallaxes of stars of various magnitudes, its results should not be accepted as definitive, for the observational data are too few to give entirely trustworthy conclusions. Moreover, the important condition that the radial velocities and proper motions shall pertain to the same stars in each class is not satisfied; in fact, the radial velocities of the stars of the fainter magnitudes are wholly unknown. And we know, too, that the stars of the various magnitudes are unhomogeneous as to luminosity and as to distance. For example, the first-magnitude stars consist only of relatively brilliant suns, some of which are very distant, and they contain no stars at all of feeble luminosity, such as Barnard's star (Table XX). On the other hand, the fainter magnitudes include some of the nearest stars of feeble luminosity, as well as giants and super-giants a thousand times as far away.

In a certain rough sense the table gives the volume occupied by stars of each magnitude, but these volumes are by no means proportional to the numbers of stars in the respective magnitude groups. As was pointed out in Art. 365, the ideal condition for an application of the statistical method is that in which the stars of a

class under consideration shall all be at approximately the same distance from the sun, and that both radial velocities and proper motions be known for each of the stars. Since the stars of each normal spectral class are approximately equal in luminosity, those of a given normal spectral class and magnitude are at approximately equal distances from the solar system, and therefore constitute a fairly homogeneous group, ideal for the application of the statistical method. In fact, a number of astronomers have already made preliminary investigations on this basis. Numerous abnormalities of motion have been found which for the time are troublesome, but which eventually will probably be of very great service in determining the organization of the Milky Way.

369. Star-Streaming. In 1904, J. C. Kapteyn announced the important fact that, instead of moving at random, a large fraction of the stars investigated belong to two streams whose members have the same general direction of motion. When the motion of the sun is eliminated, the two classes of stars necessarily move in opposite directions, for the origin of reference is then at rest with respect to the average of the two systems of stars. The apex of motion of Stream I, containing about 60 per cent of the stars, is at right ascension 94° and declination +12°, in Orion; the apex of Stream II, containing the remaining 40 per cent of the stars, is opposite at right ascension 274° and declination −12°, in Scutum. It is a significant fact that the lines of motion of these two streams are almost exactly in the central plane of the Milky Way.

Stars in all parts of the sky, of all spectral classes, and of all magnitudes, so far as the proper motions have been determined, partake of these *preferential motions*, as they are called. The two streams do not occupy separate parts of space, but appear to be intermingled as though two great star-clouds were passing through each other. Their relative velocity is about 40 kilometers per second, or about 8 astronomical units per year. Although the original discovery was made from a study of proper motions of the stars, the same results have been reached from their radial velocities.

It would be erroneous to conclude that the star-streams of Kapteyn are similar to the moving groups whose members move in parallel lines with equal velocities, for the members of neither star-stream move in exactly the same direction or at one definite

speed, but they simply exhibit a general tendency toward similar motions. It is not probable that all the stars in our galaxy belong to these streams, for their existence and properties have been established from a discussion of the motions of only a few thousands out of thousands of millions of stars which make up the Milky Way. It is quite possible that the phenomena are properties of our local cluster or of our position in the Milky Way, and that there are hundreds of similar star-streams in the galactic system.

We have spoken of the two star-streams as though they were actually two great swarms of stars which at present are passing through each other. Such a theory fits the observed facts and is, indeed, the explanation of the phenomena suggested by Kapteyn. This is not, however, the only tenable hypothesis. K. Schwarzschild proposed the theory that at least our local star-cloud is a much flattened ellipsoid in an approximately steady state of motion. To maintain this form it would be necessary for the stars to move with greatest velocities in and parallel to the plane of its equator. The result would obviously be star-streaming of the kind observed. As yet it is impossible to determine which of these two theories corresponds the more nearly with reality, and a third quite different one has been proposed (Art. 371).

370. Variations from Simple Star-Streaming. It is true that on the whole the stars have preferential motions in two opposite directions, but this simple description of stellar movements is only a first approximation to the galactic organization. The speed of the sun relative to the stars varies from one class to another, which is the same as saying that different classes of stars have relative motions with respect to one another. For example, among the stars brighter than magnitude 8, those whose velocities with respect to one another and with respect to the sun do not exceed 60 kilometers per second have heterogeneous motions in various directions. Such motions are characteristic of the heterogeneity of a permanent system. Most of those, however, whose peculiar velocities exceed 100 kilometers per second have been found by Benjamin Boss to be moving toward a point near the Milky Way about 40° from the apex of one of the star-streams, indicating their association with one another and their independence of the local group.

On arranging the stars in various groups depending upon their spectra, or their peculiar motions, or their characteristics as

variables, or other characteristics, G. Strömberg found remarkable variations in their motions. With respect to the stars of low velocities relative to the sun, the sun moves with a speed of about 15 kilometers per second in the direction of right ascension 265°, declination +22°, or about 12° from the apex of the sun's way defined in Art. 366. With respect to the stars of spectral Class O, the sun has a higher speed; with respect to variables of long period, a still higher speed; and, finally, with respect to the globular clusters, at the end of the list of objects considered, it has the enormous speed of 286 kilometers per second. In all the higher speeds, the direction of the sun's relative motion is approximately in right ascension 300° and declination +62°.

These variations of solar motion from one class of stars to another emphasize the importance of using the same objects for both radial velocities and proper motions in determining statistical parallaxes by the method described in Art. 367. This is a practical consideration, but the phenomena undoubtedly have also a theoretical significance. It is probable that the stars with low relative velocities constitute a permanent swarm whose members are held together by their mutual attractions. Eventually, when we shall know better their numbers and the space they occupy, as well as their relative velocities, the hypothesis can be verified or disproved by calculation; for there is a relationship between attractions and motions which must be satisfied in order that the members of a system shall not fall toward its center or scatter in space. Those groups with respect to which the sun has a higher velocity constitute similar independent swarms. The fact that in most cases the velocities of their members with respect to one another are far below their velocities as a whole with respect to the local star-cloud supports the hypothesis. These great aggregations of stars, moving at various speeds, are an evidence of the enormous extent of the Milky Way and the effectiveness of its gravitative control.

371. Rotation and Dimensions of the Galactic System. When star-streaming was first discovered it amazed astronomers, but the ease with which its phenomena can be described soon lulled them into a feeling that the stellar system is after all fairly simple and pretty well understood. After a time, however, such discordant motions were found as those that have just been described. Though for a brief period they may have been disquieting, they

are leading to the discovery of important properties of the galactic system.

In 1926, B. Lindblad outlined the consequences of the theory, often vaguely entertained before, that our galaxy is a thin sheet of star-clouds, like those of the exterior galaxies, which are revolving around their common center of gravity. In the following year, J. H. Oort developed the mathematical formulas which express the quantitative relationships and undertook to compare their implications with the observed phenomena. In spite of the fact that his observational data were limited, his success was marked. Finally, in 1930, J. S. Plaskett brought into the discussion the rich results of the radial velocities of the luminous Class O and Class B stars which had been accumulated at the Dominion Observatory during 20 years. The reality of a general galactic rotation is assured also by the fact that it is indicated by Doppler displacements of the spectral lines of interstellar calcium.

A disk of stars with the large central star-clouds attracts its remote parts somewhat as our central sun attracts its planets. Stars nearest its center are subject to the greatest attraction and move most rapidly, as planets nearest the sun move fastest. If from our earth we could observe, instead of Venus and Mercury, great clouds of planets between us and the sun, but not beyond the sun, and also similar ones in the regions of Mars and Jupiter, we should have conditions geometrically analogous to those in the stellar system. Whether we looked toward the sun or away from the sun, some of the planets would appear to be moving faster than the others; or, if we had no way of determining directions except from the planets themselves (as is the case when we consider the stars), we should take their average as defining fixed directions, and then some of them would appear to be moving in one direction and the remainder in the opposite direction.

We do not refer the motions of the planets to any absolute frame, but to the stars, and with reference to this exterior frame we find that the planets all revolve in the same direction with various speeds. Similarly, we may refer the motions of the star-clouds to the globular clusters, which are remote masses of stars outside our galaxy. On considering the analogy with the apparent motions of the planets, it is clear without going into mathematical details that observations of the relative motions of stars within our system and of the sun with respect to the exterior globular

clusters will enable us eventually to determine (*a*) the direction of the center of our galaxy, (*b*) our distance from its center, (*c*) the period of revolution of our local cloud around its center, and (*d*) the total mass of the central star-clouds corresponding to the period of revolution of our local cloud around this central mass.

At present only rough approximations have been made to some of these quantities. The indications are that the galactic longitude of the gravitational center of the system is 330°, agreeing with Harlow Shapley's determination of the direction of the geometric center of the galaxy (Art. 373). On the basis of present data and this method of discussion, our distance from this center appears to be of the order of 20,000 or 30,000 light-years. The local cloud is moving around the center with a speed of revolution of 250 to 300 kilometers per second, and if the calculation of its distance from the galactic center is correct, it performs a revolution in 150 to 250 million years. As for the total mass of the central star-clouds, it follows from the other data that it is probably between 10,000 and 60,000 millions of times that of the sun. Such is one of the present pictures of the circle of hazy light which we call the Milky Way, though the numerical details are subject to very large uncertainties.

372. Measuring the Galactic System by Means of Galactic Clusters. If an explorer in an unknown region observes snow-covered peaks reaching up through the clouds, he can infer from his knowledge of mountains their approximate heights and hence their distances, and finally the area they occupy. Similarly, the astronomer can most conveniently and most accurately explore the depths of the galactic system by centering his attention on its most distinctive parts. In our galaxy the best-defined constituents are the clusters, minor examples of which are the Pleiades, the Hyades, and Praesepe. There are 334 of these aggregations of stars in R. J. Trumpler's catalogue of such objects, which are relatively open clusters, not to be confused with the great globular clusters which are outside of our galaxy and possibly far beyond its borders. Figure 206 is a reproduction of a photograph of a moderately condensed galactic cluster containing more than 100 stars.

It is of course important to know that the clusters whose distances we determine are in our galactic system and not aggregations of stars far beyond its borders. We are confident that they

belong to the galactic family because they are very strongly condensed in the plane of the Milky Way and are found in all galactic longitudes, though with a tendency to be most numerous in the direction of Sagittarius and Scorpius except where dark nebulae absorb their light. They are, indeed, more strongly condensed in the plane of the Milky Way than are the stars, a fact which indicates that they are visible at very great distances. It is believed that the galactic clusters now known extend throughout our entire stellar system, for increasing telescopic power does not bring additional ones within our range. On the contrary, increases in light-gathering power or longer exposures add greatly to the numbers of faint stars, most of which are probably of moderate or low luminosity. Therefore we feel confident that these clusters are actually within our galaxy, and we think it highly probable that they are scattered throughout essentially its entire volume.

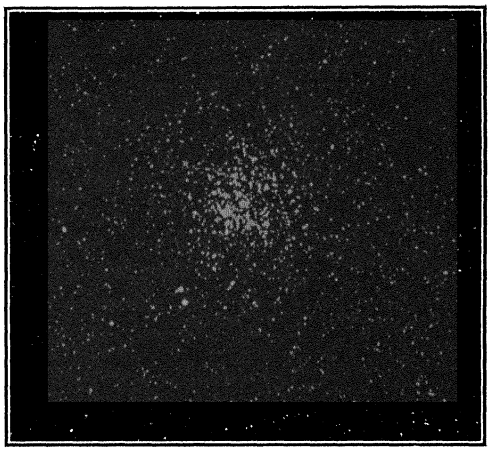


FIG. 206.—Galactic cluster N.G.C. 6705. (Lick Observatory.)

Now we come to the important problem of determining the distances to the galactic clusters. With the exception of a very few, they are so exceedingly remote (distances measured in thousands of light-years) that there is no hope of measuring the parallaxes of any of their stars by direct means. Nor are the statistical methods explained in Arts. 365–367 generally available. Instead, Trumpler determined their distances from the magnitudes and spectral types of their brighter stars. This method is worthy of our confidence because we know from the investigations of the nearer stars that there is a fairly well-defined relationship between the spectral classes and the absolute luminosities of the stars (Art. 314). That is, when we know the spectral class of a star, we know its luminosity; and then, from its apparent magnitude, we can

compute its distance. Though the result might be considerably in error for a single star, the method is reliable when applied to many of them. Moreover, the more distant the clusters, according to this method, the less their diameters are found to be, as should be the case if they are of the same average dimensions in all parts of the galaxy.

Since there are dark nebulae and also the absorbing materials which produce the stationary calcium lines (Art. 321), the stars are fainter than they would be if space were transparent. The calcium absorption, however, is not a misfortune, for it is selective and its amount may be made a basis for calculating the distances of the stars. Similarly, the effects of the scattering of light by interstellar dust on the color indices of the stars of known spectral classes provide a means of determining their relative distances. By using all these methods and cross checks for 100 clusters, Trumpler derived what he believes is a basis for computing with substantial accuracy the distances of the 334 clusters in his catalogue, and consequently the dimensions and the form of our galaxy.

According to Trumpler, our galactic system is a disk of at least 334 clusters and thousands of millions of stars, interspersed with vast luminous and dark nebulae, whose thickness is about 3300 light-years and whose equatorial diameter is about 33,000 light-years. Our sun is now 1150 light-years from its center and 30 light-years north of its central plane.

Outside of our galaxy, but within a distance of the order of 200,000 light-years, there are the Magellanic Clouds and 93 globular clusters, each consisting of many thousands of stars. Together, all these vast aggregations of stars constitute a sort of super-galaxy, perhaps roughly similar to others which are distant millions of light-years (Art. 381). In Fig. 207 our galaxy, the Magellanic Clouds, and the distances to the globular clusters, according to this conception, on the same scale are projected on the plane of the paper.

373. Measuring the Galactic System by Means of Globular Clusters. In 1917, Harlow Shapley published the first of a series of papers which have revolutionized the ideas of astronomers respecting globular clusters and have had a profound influence upon all interpretations of the nature of the galactic structure.

None of the usual methods of measuring celestial distances is

applicable in the case of globular clusters, for these masses of stars are all extremely remote. In fact, the only means available of determining their parallaxes is that depending upon the period-luminosity law of Cepheid variables (Art. 339). It will be recalled that this relationship rests entirely upon an observational basis, having been originally discovered from the periods and luminosities of these stars in the Magellanic Clouds. The evidence is strong that the method determines the relative distances of groups of

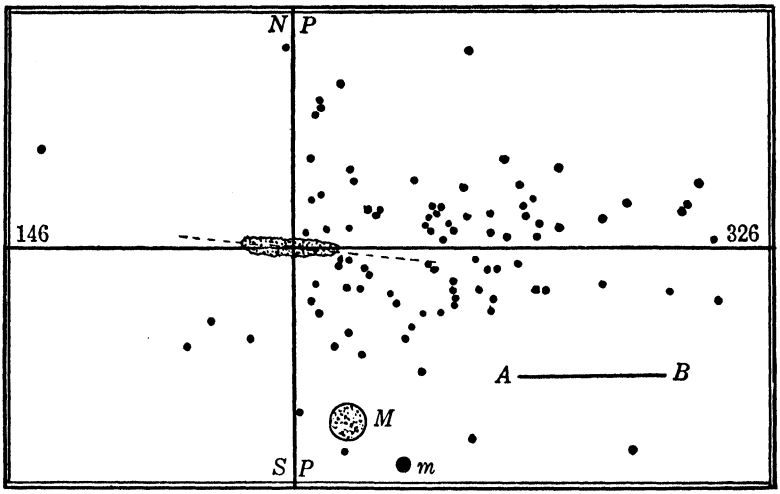


FIG. 207.—At center, our galaxy; below, the Greater Magellanic Cloud (*M*) and the Smaller Magellanic Cloud (*m*); the dots, the globular clusters, not to scale, projected on a plane. *AB* represents 50,000 light-years. (Trumpler.)

stars with which they are associated, but there remains considerable uncertainty respecting the absolute distances because all Cepheids are so remote that it is difficult to calibrate the process by direct parallaxes in even a few particular cases. There are, however, various statistical methods of checking the results, all of which naturally have been employed.

In only 19 of the 93 globular clusters have Cepheid variables been found, but in these clusters 730 Cepheids are known, 524 of which have been so thoroughly investigated that they are useful in determining distances. Fortunately, the 19 clusters containing Cepheids are a good sample of the whole 93. They fix the luminosities of the brightest stars in clusters of different average spectral classes, they furnish diameters of these great masses of stars, and

they give a measure of the total light they radiate. Under the assumption that the remaining clusters are similar in these properties to those of corresponding types among the 19, their distances can be computed from these data. We are, of course, pyramiding our hypotheses to a considerable extent, but the results obtained satisfactorily meet so many indirect tests that it is believed they are not greatly in error.

When Shapley translated his observational data into distances, he found that the globular clusters are enormously remote, even in comparison with the great moving groups in Taurus and Scorpius. The nearest of them, such as ω Centauri, are 22,000 light-years away; the most distant are about ten times as remote. It follows that we see these gigantic groups of suns, not as they are now but as they were in the days of our Neanderthal predecessors. It is not to be understood, however, that they have changed appreciably in the interval except in some slight relative motions of their members, for a million years is only an instant in the existence of these enormous systems of stars.

Now we come to the question of the relationship of the globular clusters to our galaxy. The first fact of great significance is that they are not found at all in the plane of the galaxy, while the galactic clusters on which Trumpler based his conclusions are largely limited to the zone of the Milky Way. It follows that the globular clusters are beyond the obscuring dark nebulae of our stellar system and are eclipsed by it, for they are symmetrically distributed with respect to the galactic equator with marked frequency in galactic latitudes from 5° to 15°. There is, moreover, a remarkable condensation around galactic longitude 327° (right ascension 262°, declination $-29°$). In fact, all except 6 globular clusters are within 90° of this point, while the galactic clusters are distributed nearly uniformly in galactic longitude. This fact, too, indicates that they are not in the interior of our galaxy. Shapley assumes that they are on its immediate exterior and constitute a sort of framework which marks out its tremendous dimensions. The Milky Way largely fills up the space inside the globular clusters whose positions are projected on the plane of the paper in Fig. 207.

According to this view, the galactic star-clouds, the millions of isolated stars, and the globular clusters constitute a super-galaxy whose least and greatest diameters are about 20,000 and 200,000

light-years respectively, and from whose center the solar system is distant about 52,000 light-years. The difference in the conceptions of Trumpler and Shapley consists largely in the fact that in one the Milky Way system is regarded as being only the largest and the most oblate member of a group of nearly 100 isolated aggregations of stars, while in the other it is regarded as embracing not only all of our galactic star-clouds but the globular clusters as well. The galactic system according to Trumpler is comparable to such exterior galaxies as that known as the Great Nebula in Andromeda (Art. 379), while according to Shapley it is more like the combined group of exterior galaxies in Coma-Virgo (Art. 381).

374. Transparency of Interstellar and Intergalactic Space. Since the parallaxes of remote star-clouds and of the globular clusters are computed from the periods and the apparent brightnesses of the Cepheid variables which they contain, it follows that it is important to know whether space is transparent; for the magnitudes of stars are reduced if their light is absorbed, and their calculated distances are correspondingly too large. We may well have some misgivings on the subject because dark nebulae cause an apparent rift for many degrees in the Milky Way and effectively eclipse remote objects in one or two per cent of the entire celestial sphere.

If there is fine dust widely distributed throughout the celestial spaces, it will not only absorb light from distant stars, but it will scatter their blue rays much more than their red. Hence the more remote the stars the redder they will appear to be, for only their red rays will penetrate the cosmic fog, just as when the sun is setting only its red rays reach the observer. But observations indicate that the most distant globular clusters are not appreciably redder than those which are nearer, and therefore it follows that the intervening regions are sensibly free of fine dust. It is true that coarser materials would not have such differential effects upon light, but by their collisions they would in time produce the finer materials that do not seem to be present.

Another method of determining whether there are interstellar absorbing materials in appreciable amounts depends upon the fact that, if space is perfectly transparent, the total light received from the globular clusters at all distances will be proportional on the average to their apparent areas; while if interstellar space is not

perfectly transparent, the light received from the more distant clusters will be less in proportion to their apparent areas than that received from those which are nearer. When this method is applied, it also shows that there is no appreciable absorption of light which comes from clusters even at a distance of 200,000 light-years. In fact, the same method points to the conclusion that space is almost perfectly transparent out to exterior galaxies which are 50,000,000 light-years from our system.

At first these results appear to contradict the fact that there are dark nebulae and calcium clouds in our galaxy, but they do not really do so, for these obscuring masses are purely local, so to speak. Though the dark nebulae may extend out thousands of light-years from the present position of the solar system, they are in our galaxy and perhaps largely around its periphery. At least, many exterior galaxies which are presented edgewise to us show the eclipsing effects of such obscure masses (Fig. 212). These globular clusters and those exterior galaxies which are in the plane of the Milky Way, if there are such, are totally eclipsed; those which are far from its plane are not affected appreciably.

In Art. 307 we were considering the possible dissipation of the masses of the stars by radiation, and of the integration of these energies into matter again and of its absorption into stars. When we limit our attention to our galaxy, the hypothesis appears promising; but, when we find the vast intergalactic regions are sensibly transparent, we encounter a serious difficulty, for the stars pour their radiant energy out into these limitless unoccupied spaces between the galaxies, as well as into their own galaxies. Even if radiant energy in our galaxy should be to some extent reintegrated into matter which is swept up by the stars, so far as we can see at present on the whole it becomes progressively more nearly uniformly distributed between the galaxies. While this is the trend so far as we can see, we are conscious of the fact that our knowledge is wholly inadequate for the solution of such a far-reaching problem as that of the origin and the disintegration of matter.

375. On the Dynamics of the Galaxy. As the first step in considering the dynamics of the galaxy, let us compute the number of stars it would contain under the principal conclusions which have been reached respecting its dimensions. According to Trumpler,

the polar and equatorial diameters of the Milky Way system are respectively of the order of 3300 and 33,000 light-years. Since there are approximately 30 stars within 13 light-years of the sun, if the average stellar density through the galaxy is the same, the Milky Way system contains 48,000,000,000 stars. Under the assumption of the same average stellar density, the enormously larger galaxy (20,000 by 200,000 light-years) of Shapley would contain at least 10,000,000,000,000 stars. The smaller of these two numbers at present seems decidedly the more probable, but the direct observational evidence as yet is quite inadequate for a positive conclusion.

Now let us see what light a consideration of stellar velocities can throw on the problem of the dimensions and the structure of the galaxy. There is no reasonable doubt that the stars move in conformity to the laws of motion and the law of gravitation. As a practical matter, however, the stars are so numerous that we could not follow them individually even if we had the data necessary for the solution. We must content ourselves with a consideration of the motion of our galactic system as a whole and with statistical studies of the general properties of the internal motions of its stars.

When we consider the rotation of our galaxy, we can check our deductions only by referring to such exterior objects as the globular clusters. The period of the galactic rotation is so long that no appreciable drifts of exterior objects can be expected within 1000 years; consequently our observational data are limited to radial velocities. The radial velocities of 18 globular clusters, most of which were measured by V. M. Slipher, range from 350 kilometers per second of approach to about an equal speed of recession. Though the data are meager, they point to a speed of rotation of our galaxy at the distance of the sun from its center of perhaps 200 or 300 kilometers per second.

If the galaxy were *spherical* and the stars were *uniformly* distributed throughout its volume, they would all describe ellipses about its center in the same period, and this period would depend only upon the average density of matter in the system. For a density such as that which we find within 13 light-years of the sun, this period of revolution would be about 130,000,000 years. If this were the organization of the galaxy and the solar system were 1150 light-years from its center, as Trumpler concludes, our neighboring stars would be moving around the center at a speed

of about 17 kilometers per second, which is probably too low. On the other hand, if the solar system were 52,000 light-years from its center, as Shapley infers it to be, the speed of stars in our neighborhood would be about 770 kilometers per second, which is probably too high.

Our results, however, are based on somewhat erroneous hypotheses and possibly do not give even the order of magnitude of the velocities which might be expected. The galaxy is very much flattened and the star density appears to be greatest at its center. If it consists of 48,000,000,000 suns with one-quarter of them in great central star-clouds, then a star circulating at a distance of 1150 light-years would complete a revolution in about 5,400,000 years with a speed of the order of 390 kilometers per second, which does not seem to be wholly unreasonable. For the same number of stars in the central nucleus and the distance from its center of 52,000 light-years derived by Shapley, the speed would be reduced to 58.1 kilometers per second, which is probably too low. But if one-fourth of the 10^{13} stars of the Shapley galaxy, under our assumption of average density, were interior to the distance of the sun from its center, the speed of revolution at the distance of 52,000 light-years would be 5640 kilometers per second, which is certainly much too high.

Up to the present we have considered only the average velocities of the stars, and we have not referred to the interesting problems raised by the high velocities of some of them. It is evident that if a star moves with very high speed, it will escape from the gravitative control of the galaxy, just as a molecule of our atmosphere may escape from the earth. In fact, the escape of a star is a simpler matter than the escape of a molecule, for the former in its passage outward into the intergalactic spaces is quite unlikely to pass very near another star, while the latter is quite likely to undergo many collisions. It is found both from proper motions and also from radial velocities that a few of the stars in our part of the galaxy are moving relative to the sun with a speed of 200 to 300 kilometers per second. Since these stars move in various directions, it follows that some of them are moving with respect to the frame of reference defined by the globular clusters with the speed of the sun, whatever it may be, plus 200 or 300 kilometers per second, or probably at the rate of 500 kilometers per second. Since velocities high enough to carry stars beyond the gravitative control of the galactic system

would occur extremely infrequently, it follows that the mass and the dimensions of our galaxy are such that the speed of escape in the neighborhood of the sun probably equals at least 500 kilometers per second. The foregoing assumption of a total of 48,000,000,000 suns in our galaxy with one-fourth of them in the central star-clouds would give a velocity of escape of this order at a distance of 1150 light-years from its center.

In the foregoing discussions we have spoken of the Milky Way as though it were fairly homogeneous in structure; as a matter of fact, it is made up of many vast star-clouds in a somewhat irregular assemblage. There are in addition many smaller groups of associated stars, some of which are moving at velocities far above the average. These irregularities in distribution and in velocities are expressions of the dynamical immaturity of the galaxy, for the interactions of its stars always tend toward homogeneity in concentric spheroidal layers and toward velocities distributed according to the probability law around a mean value. The galaxy is youthful, of course, only in an astronomical sense, and it approaches the dull uniformity of maturity at a leisurely rate compared to which the geological processes are very rapid.

If the masses of the stars are being radiated away without being restored in some way, the galaxy may never arrive at dynamical maturity. Let us, however, neglect this possibility and follow our system of stars. As a consequence of near approaches, occasionally a star will acquire a velocity sufficient to take it permanently out of our system into the intergalactic regions. Escaping stars will in general leave our rotating Milky Way from its equatorial plane, and they will in general carry away more than their proportion of its angular momentum. The result will be to decrease its rate of rotation and its oblateness. Hence as our galaxy tends toward uniformity of structure, it also tends toward sphericity, the limit of its dynamical evolution being a globular cluster. Even a globular cluster is not permanent, for occasionally one of its stars will acquire high speed as a consequence of close approaches to other stars and will then fly away into space. If there are no neglected factors that will change the evolution which we are considering, these spherical systems will lose their stars, one by one, as a drop of dew loses its molecules. A star escaping from a galaxy will wander in the intergalactic regions, perhaps to join later a foreign galaxy millions of light-years from its earlier home

376. On the Origin of the Milky Way System. Since our galaxy is dynamically young, we are tempted to try to look back to its birth. The farther we go back in time, the more largely we find it consisting of sharply differentiated groups and swarms of stars, each of its subsystems pursuing its individual way. Perhaps the Milky Way originated from the rushing together under their mutual attractions of one or more great globular clusters, not necessarily center to center but more likely in such a way that the stars of their far-reaching exteriors mingled in great swarms. This suggestion accounts at least roughly for the oblateness of the Milky Way and for its numerous great star-clouds. That such partial collisions of great clusters occur seems certain; but unless these streaming masses of stars lose an enormous amount of kinetic energy while they are mingled, their gravitative embrace is broken and they separate to the distance from whence they came, as comets recede from the sun. Yet for long aeons after their separation their oblate forms are clear evidence of the dynamic adventures that they have undergone.

Another quite different hypothesis has been suggested by J. H. Jeans. He thinks it not wholly improbable that the parent of our galaxy was a vast, gaseous, spiral nebula in rapid rotation. With loss of heat and consequent shrinking, this nebular mass rotated more rapidly and finally became unstable. It first broke up into huge gaseous masses, perhaps thousands of light-years in diameter, and these separated similarly into smaller masses and finally into stars. The first separation gave rise to the great divisions of the Milky Way now recognized as star-clouds, and the later ones to the galactic clusters and the moving groups. If we assume the existence of the nebula, we have an explanation of the oblateness of the galactic system, but we have no origin for the nebula because the process is never reversed. Moreover, according to this hypothesis all our varied stars and nebulous masses are of precisely the same age, a conclusion very difficult to accept.

377. Globular Star-Clusters. Until the distances of some of the globular clusters were determined a few years ago, little was known of these aggregations of stars except that the larger of them contain many thousands of members and that they appear to be sensibly spherical in form. Now we can determine their diameters, the absolute magnitudes of their visible stars, the average distances

of their stars from one another, and the average relative velocities of their component parts. The distances of the clusters range from 20,000 to 200,000 light-years; the clusters themselves, though they subtend small angles and with few exceptions are invisible to the unaided eye, in some cases rival, and perhaps exceed, the greatest star-clouds in our galaxy.

Although the clusters we are considering are called "globular" and indeed appear to be globular upon casual examination, most of them are actually somewhat oblate. They do not exhibit the extreme flattening of our galaxy, for their polar axes are usually at least 80 or 90 per cent of their equatorial. Some clusters are relatively open, while others are very compact; some contain a few hundred observable stars, and others many thousands. Those whose integrated spectra have been determined are with one or two exceptions of spectral Class F, G, or K; their color indices indicate the same variations in average spectral class.

There are two great globular clusters which, though not conspicuous, are visible on cloudless nights to the unaided eye. One of them, ω Centauri, is a somewhat open cluster far in the southern sky; the other, the Great Cluster in Hercules, is a more compact mass of stars visible from northern latitudes. Their distances from us are respectively 22,000 and 33,000 light-years. Since the clusters are apparently imbedded in countless stars of our own galaxy, it is not possible to determine precisely where they terminate. Moreover, the greater the light-gathering power of the telescope with which they are photographed and the longer the exposure, the greater the fields they cover and the more numerous their stars. The cluster in Centaurus subtends an angle approximately equal to that subtended by the moon; that in Hercules is apparently half or two-thirds as great. Consequently it follows from their distances that their linear diameters are of the order of 200 light-years, a diameter which is only about 5 times the average of the 93 clusters. Edmund Halley, who discovered both of these clusters, the former in 1677 and the latter in 1714, after reflecting on the significance of these objects, wrote:

> ". . . . yet since they [the clusters] are among the fixed stars . . . they can not fail to occupy spaces immensely great, and perhaps not less than our whole solar system."

Although the Great Cluster in Hercules is somewhat above the average in size and in number of stars, let us consider it as

representative of these amazing cosmic units. At least 40,000 star images can be counted on the original of such photographs as that reproduced in Fig. 208. It follows from the distance of the cluster and the magnitudes of its brightest stars that they are several thousand times as luminous as our own sun. In fact, all the thousands recorded individually on the photograph are giant stars. Since normal stars in our galaxy outnumber the giants thousands to one, it is probable that the Hercules cluster contains, in addition to the great stars which are visible individually, millions of other stars comparable in luminosity to those near the sun. In order to have an explicit number on which to fix our attention, let us assume that the cluster contains 10,000,000 stars whose individual masses on the average are equal to that of the sun. Under these assumptions, which may be considerably too high or too low, the average distance between adjacent stars in the cluster is nearly a light-year, or 60,000 times the distance from the sun to the earth. Yet 200 years ago Halley thought this cluster might be as large as the entire solar system, and as recently as 25 years ago many astronomers regarded it as an interesting object in which for some reason matter had concentrated into many small suns rather than into one or a few large ones. Rarely, if ever, have our conjectures even remotely approached the realities.

Quite unlike our Milky Way system, the globular clusters are dynamically mature. With few exceptions, they have arrived at a nearly symmetrical distribution of stars which will be changed very slowly, if at all, by their mutual attractions. In such great aggregations, the stars do not move in simple elliptical orbits, such as those of the planets, but in more varied and complex curves. Ordinarily a star, subject to the general attraction of the whole cluster, moves from its outer regions into its deep interior and out again at some other place. Occasionally, however, it passes relatively near another star, and then the character of its path is entirely changed; but statistically the cluster as a whole remains essentially unchanged.

It is not difficult to determine limits to the periods of motion of stars from the interior to the exterior of a globular cluster, and consequently limits to the relative velocities with which the stars move. The mass of a globular cluster is neither uniformly distributed nor concentrated entirely at its center, but is scattered in its stars between these extremes. In the former limiting case, the

stars would all move in concentric elliptical orbits and their common period of revolution in a cluster composed of 10,000,000 suns and occupying a sphere 200 light-years in diameter would be 5,000,000 years. In the latter limiting case, the orbits would be ellipses with a focus in common at the center of the cluster and the periods would depend on the major axes of the ellipses. For stars



FIG. 208.—The Great Globular Cluster in Hercules. Photographed by G. W. Ritchey with the 60-inch reflector of the Mount Wilson Observatory on June 6, 7, and 8, 1910. The total exposure was 11 hours. On an enlargement of the original plate the images of 40,520 stars were counted.

moving from the outer zones, the periods in this case would range from 1,700,000 years to 5,000,000 years. In an actual cluster the stars move from its outer regions into its deep interior and out again in intervals of time of the order of a few millions of years. It is inferred from these dynamical considerations that the actual velocities of the stars in the clusters are something like 20 or 30 kilometers per second.

Ordinarily, a star makes its wide circuits through a cluster almost as though all the remaining stars were uniformly distributed throughout their respective spherical zones. Occasionally, however, it passes near another star and then its orbit is abruptly changed, the character of the change depending upon the circumstances of the close approach. It is these close approaches, which are dynamically similar to the encounters of molecules in a gas, that are effective in reducing initial irregularities to an ultimate uniformity in both mass and motions. It is impossible to determine how long the globular clusters have been in their present steady state, and it is difficult to calculate at what rate they have approached their present condition from possible earlier ones of considerable heterogeneity. We can, however, determine something of the frequency of these near approaches on which the dynamical evolution primarily depends.

In order to treat the problem numerically we must first define what constitutes a *close* approach of two stars. We shall agree that, for the purpose of this discussion, two stars have been in close approach if each has changed the direction of motion of the other by at least 10°. We find that in a system of stars having relative velocities of 30 kilometers per second and masses equal to that of the sun, two stars will have a close approach, according to this definition, only if their paths are such that, neglecting the effects of their mutual attractions, they would pass within about 10 astronomical units of each other. Consequently we have reduced the problem to that of computing the probability of a star having one close approach in a single circuit through the cluster, a journey requiring several millions of years. We find that, in spite of the great numbers of stars in the clusters, a particular star is in close approach with another star only once, on the average, in many thousands of circuits. Hence it follows that even in great clusters made up of millions of suns, close approaches of two stars are of infrequent occurrence. There is abundant room for planets in these clusters and abundant time between close approaches for their evolution.

We are interested just now, however, in the time necessary for the clusters to evolve dynamically from a state of possible considerable heterogeneity to that of the symmetry which exists at present. It is probable that many close approaches for each star are necessary for such a dynamical transformation, and hence

the ages of these globular clusters since their last encounters with other great masses of stars must be counted in millions of millions of years. Neither the stratified rocks of the earth nor the radioactive minerals which it contains use terms of the same order of magnitude as do the globular clusters in describing the duration of suns. Yesterday, as it were, men were discussing the creation of the universe a few thousands of years ago; today we see vast systems of suns in whose very constitution there is written the fact that they have been isolated from other systems for millions of millions of years. Yet they will not go on forever as they are. Each of them sooner or later, in its swift flight through the intergalactic regions, will approach some similar system and, under the urge of their mutual attractions, will rush to mingle with it and to give birth perhaps to a distant Milky Way somewhat similar to our own.

378. The Magellanic Clouds. Far in the southern heavens there are two great isolated and exterior clouds of stars, mostly of the eleventh magnitude and fainter, which help to bridge the gap between the Milky Way and distant galaxies. They are near enough to be resolvable into their individual clusters, nebulae, and brighter stars; they are remote enough to be observed and studied in their entirety and as vast stellar organizations. Investigations of them are throwing light on the nature of both our own galaxy and more distant galaxies.

It was from the Cepheid variables in the Small Magellanic Cloud that Miss Leavitt discovered the period-luminosity law. Having determined its zero-point as nearly as possible from the data for similar variables in our own galaxy, Shapley finds that the Large Magellanic Cloud is distant 86,000 light-years and has a diameter of 10,800 light-years. The Small Cloud is found similarly to be at a distance of 95,000 light-years and to be 6,000 light-years in diameter. Results of the same order of magnitude have been found from considerations of 10 objects in these clouds which appear to be typical globular clusters, though their nature is as yet somewhat in doubt. It follows that though the Magellanic Clouds are great aggregations of stars, they are inferior to our own galaxy in dimensions.

There are probably at least 500,000 stars brighter than the eighteenth magnitude in the Magellanic Clouds. Hence there is abundant material within the range of large telescopes available

for statistical comparison. For example, it is possible to determine the relative numbers of various classes of the more luminous stars. Our knowledge of the absolute magnitudes of the super-giant Class O stars is based on their occurrence in these star-clouds, for in our own galaxy they are not associated with stars whose distances are known. It has been found that some of these stars are 500,000 times as luminous as our sun. Moreover, since all the stars in each of the Magellanic Clouds are at approximately the same distance from the solar system, the question of the equality of the luminosities of the stars within the various spectral classes can be answered by finding whether the members of each of them are all of the same apparent magnitude. In fact, the relative luminosities of various classes of stars can be directly determined as well as the variations in the magnitudes of the novae. As yet these investigations of far-reaching importance are in their early stages.

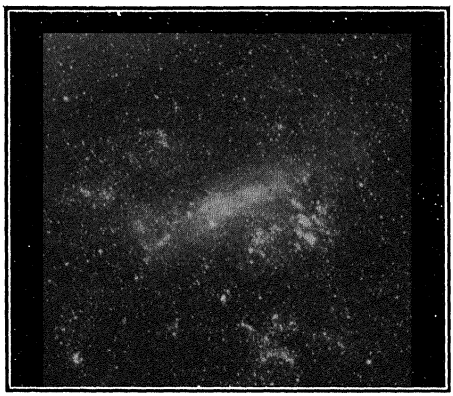


FIG. 209.—Large Magellanic Cloud. (Lick Observatory, Southern Branch.)

In discussing the star-clouds of our own galaxy, there is always a difficulty in distinguishing between faint stars which are near at hand and the more distant ones which are under consideration. In the Magellanic Clouds such difficulties are greatly reduced, for the structures of these great systems are spread out before us like a map. We observe the degree of their central condensation, the distribution of their secondary star-clouds, the relative numbers of their stars of various luminosities, and the arrangement and magnitudes of their nebulae. We find in those galaxies, as in our own, diffuse and irregular nebulae and a meager sprinkling of planetaries. Although the Magellanic Clouds are much smaller than the system of the Milky Way, they contain many nebulous masses of enormous extent and high total luminosities. In fact, the luminosities of the brighter of these nebulae, particularly such ones as 30 Doradi in the Large Cloud, seriously challenge the

theory that the energy given off by these tenuous clouds is derived from the stars (Art. 348), for this one nebula radiates 12,000,000 times as much energy as is radiated by our sun. Moreover, we often get better estimates of the dimensions of nebulae in the Magellanic Clouds than we do of those in our own galaxy. The great loop 30 Doradi, 100 light-years in diameter, is incomparably greater in



FIG. 210.—The Great Nebula in Andromeda, a great galaxy of stars comparable to our own; distant 900,000 light-years. (The Yerkes Observatory.)

volume and in light radiated than, for example, such vast objects as the Great Nebula in Orion (Art. 349).

Although the Magellanic Clouds have many characteristics in common with our galaxy, it would be unsafe to assume that they are counterparts of the Milky Way, but on a smaller scale. They do not exhibit any suggestion of the spiral structure which many investigators believe they have discerned in our galaxy and which

certainly is the most marked characteristic of the exterior galaxies known as spiral nebulae. Nor do they resemble the globular clusters. They are not specialized to either extreme, a fortunate circumstance for astronomers, since now they furnish a basis for wide comparisons without great extrapolations. These clouds are receding from us, the Large Cloud at the rate of 276 kilometers per second and the Small Cloud at the rate of 170 kilometers per second, part of the apparent motion probably being due to the rotation of our galaxy. As great as these velocities are, the distances of these clouds of stars will not increase by so much as one per cent in 100,000 years.

Dynamically, the Magellanic Clouds appear to have progressed a considerable distance toward maturity. They do not have the marked irregularities of many of the more distant galaxies, nor the beautiful symmetries of the globular clusters. Until they pass near other great masses of stars, they will become more and more nearly symmetrical, perhaps eventually becoming somewhat oblate globular clusters. We infer this conclusion from well-established principles and support it by citing the existence of other systems farther advanced in their dynamical evolution; but we shall not be able to verify it by direct observations, because our little world will probably not endure long enough, though it continue to exist a thousand million years, for appreciable changes to take place in these great aggregations of stars.

379. Exterior Galaxies. There is in Andromeda a hazy spot of light of about the fourth magnitude, which, through a telescope, is seen to be not a star but a nebula consisting of a relatively bright nucleus surrounded by a less luminous cloud. When photographed, it is found to be an elongated oval nearly 3° in length and one-fourth as broad; and if the scale is large, it is resolved into a complex aggregation of bright clouds with intervening dark rifts, presenting clear evidence of a spiral structure. This is the Great Nebula in Andromeda (Fig. 210), which recent investigations have shown is an exterior galaxy comparable to our Milky Way system in dimensions and in number of stars.

The Great Nebula in Andromeda is an example of hundreds of thousands of somewhat similar objects which photography has revealed. Although a number were discovered by visual telescopic observations, our extensive knowledge of them really began about

1900 with the work of J. E. Keeler, who photographed thousands of these objects. They ranged in brightness from the relatively conspicuous ones which had been visually observed down to minute and faint ones barely distinguishable on the photographic plate from faint star disks. He estimated that there are 120,000 of these objects within reach of the Crossley reflector which he used.

Subsequent photographs by H. D. Curtis and other astronomers with greater telescopes have increased the number of known extra-galactic nebulae to hundreds of thousands. Over 95 per cent of them have symmetries about their nuclei, indicating rotations, and are called *regular nebulae* by Edwin Hubble in his classification of these objects. The planes of some of these nebulae are perpendicular to the line of sight, as in Fig. 211; some are tilted, as in the case of the Great Nebula in Andromeda; and some are edgewise, as illustrated in Fig. 213. They have various degrees of oblateness, ranging from nearly spherical masses to very thin disks. Those which are nearly spherical or are elliptical in shape decrease smoothly in brightness from their centers to their margins, with no suggestion of resolution into subgroups or clusters of stars. They may possibly be gaseous, but it is much more probable that they are dynamically mature aggregations of stars from which all important local irregularities have disappeared, particularly as their spectra prove that their light is like that radiated by the stars.



FIG. 211.—Exterior spiral galaxy M51, in Canes Venatici. (Lick Observatory.)

The very oblate regular nebulae are generally well-defined spirals, often with two arms coiling out from opposite sides of their nuclei, though sometimes there are dark bars across their centers. The degree of central condensation varies from those in

which most of the matter appears to be in the nuclei to those in which it is nearly all in the spiral arms. Dynamically, the spiral nebulae are immature, perhaps in many cases even more immature than our Milky Way system. Their shapes and their structures indicate high rates of rotation, an inference which is supported by V. M. Slipher's measurements of the radial velocities of some of them, for when the slit of the spectroscope is placed along their major axes, their spectral lines are found to be inclined (Art. 235). In the Andromeda nebula the rotational velocity at less than one-tenth the distance from the center to the end of the major axis is 75 kilometers per second, and even much higher velocities have been recorded in several others. Exposures up to 80 hours are required to get photographs of the spectra of even the central parts of the brighter of these nebulae, but the important results they furnish can be obtained in no other way. To prove the rotations of these objects by direct observations of the relative positions of their parts would probably require tens of thousands of years. The fact that they do turn on their axes supports the conclusion that our own galaxy owes its shape to its rotation: by journeying abroad we understand better what we have at home.



FIG. 212.—Exterior galaxy (HV 24) in Coma Berenices, which is edgewise to the solar system. (Lick Observatory.)

The very much flattened exterior galaxies which are presented edgewise to us are nearly always found to be crossed lengthwise through their centers by dark streaks, as is illustrated in Fig. 212. When they are somewhat tilted to the line of sight, the dark belts are displaced to one side of their centers, as is illustrated in Fig. 213. It follows from a comparison of a series of these barred nebulae that their dark lanes are due to absorbing materials surrounding

their equatorial peripheries. These materials are probably of the same nature as the dark nebulae which produce the rift and the vacant lanes in our Milky Way.

Besides the elliptical and the spiral nebulae that we have been describing, there are about three per cent as many others which are classed as irregular. They are neither approximately spherical in form nor coiled with the symmetries of spirals. The Magellanic Clouds are the nearest and the most conspicuous objects of this class. If their irregularities are due to encounters of two aggregations of stars, apparently the meeting was nearly central or perhaps one of the swarms was small in comparison with the other. There are evidently many possibilities, but at present we can only conjecture what influences actually gave rise to the structural characteristics of these myriads of suns.

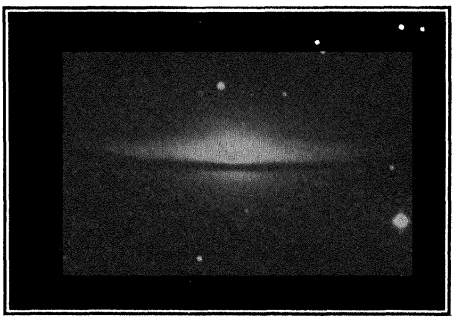


FIG. 213.—Exterior galaxy N.G.C. 4594. (Lick Observatory.)

380. Distances to the Exterior Galaxies. We have so far based our conclusion that the nebulae we have been considering are exterior to our galaxy primarily on the facts that none of them is found in the Milky Way and that their light is like that of the stars. Our conclusion, however, is not left with this vague support, for Hubble has discovered a number of Cepheid variables in the Great Nebula in Andromeda and in a few others of the more conspicuous and nearer exterior nebulae, and he has determined their light curves. Then he has determined from the period-luminosity law and the apparent magnitudes of these stars the distances of the galaxies in which they are located. That the results derived are of the right order of magnitude is confirmed by the brightness of their super-giant stars and of the considerable numbers of novae which have been discovered in them.

The nearest exterior galaxies are the Magellanic Clouds (Art. 378); then follows N.G.C. 6822, an irregular nebula beyond Sagittarius at a distance of 700,000 light-years. At such an enormous

distance only giant stars are individually visible or capable of being photographed with our most powerful telescopes. This is a small galaxy whose greatest diameter is only 4000 light-years.

After Messier 33, which is in the direction of Triangulum at a distance of 850,000 light-years, comes the Great Nebula in Andromeda at a distance of 900,000 light-years. The diameter of this great galaxy is about 42,000 light-years, not differing greatly from that determined for the Milky Way by Trumpler. It follows from the amount of light we receive from this galaxy that it radiates 2,000,000,000 times as much energy as is radiated by the sun. If the gravitative pull of its interior nucleus of stars just balances the centrifugal forces due to its measured rate of rotation, the mass of its central core is about 300,000,000 times that of the sun. There can be little doubt that the Andromeda nebula, which our predecessors saw only as a faint and fuzzy star, is a galaxy in all respects generally similar to our own. It seems probable, however, that these two galaxies are considerably above the average in size and in number of stars.

No other exterior galaxies than the five so far mentioned are known to be within 1,000,000 light-years of the Milky Way. The methods by which these results have been secured are directly applicable only within the distances at which the Cepheids can be observed as individual stars and followed in their variations in magnitude. Since the brightest stars, the super-giants, are of the same absolute magnitude in our galaxy and in all the other galaxies whose distances have been determined by the aid of Cepheid variables, we may reasonably assume that they are the same in all galaxies. Hence, under this assumption, we can go somewhat farther and compute the distances of all those galaxies which are not so remote that their super-giant stars are individually beyond the reach of our most powerful telescopes. By this method it will be possible to penetrate to distances of about 10,000,000 light-years from our system with the 100-inch reflector of the Mount Wilson Observatory.

Even 10,000,000 light-years is not the limit to which the distances of exterior galaxies can be approximately measured, for we can determine from those which are within this distance the average diameters and the average total luminosities of these huge aggregations of stars. Then, when more remote ones are photographed to which the previous methods do not apply, we can make at least

approximate estimates of their distances from their apparent diameters and magnitudes. In this way we can sound the depths of space with present instruments to a distance of the order of 500,000,000 light-years.

381. Super-Galaxies. Fortunately, there are enough exterior galaxies within the reach of our telescopes to enable us to determine something of the characteristics of their distribution in space. Instead of being scattered with approximate uniformity, they exhibit the gregarious tendencies of the stars. Although the investigations of this subject are all very recent, Harlow Shapley and Miss Adelaide Ames have found more than 40 clusters of galaxies, or *super-galaxies*, as we shall call them. Some of these super-galaxies consist of hundreds of galaxies bright enough to be recorded on photographic plates. It is not improbable that these clusters which are photographed are the greatest galaxies in their groups, and that such smaller members as the Magellanic Clouds are beyond the reach of the instruments used.

One of the greatest super-galaxies now known is located in Centaurus; or, more correctly, in the direction of Centaurus as now seen from the earth, but thousands of times as far away. The plates of this region which Shapley used in his investigation cover 30 square degrees and contain, outside of three clusters of galaxies, an average of 18 galaxies per square degree. The principal super-galaxy in the region is an oval 2°.8 by 0°.8 and contains 315 galaxies, ranging in magnitude from 16.0 to 18.1. Such a great instrument as the 100-inch Mount Wilson reflector might reveal many more. The distance of this super-galaxy is of the order of 150,000,000 light-years, and its longest diameter is over 7,000,000 light-years. The diameter of the largest galaxy it contains is about 45,000 light-years, and the average diameter of all its observed galaxies is 10,000 light-years.

No other part of the sky appears to be so rich in super-galaxies as the Coma-Virgo region. In a limited area in the direction of these constellations there are four principal clouds of galaxies whose constituent galaxies are estimated by Shapley and Miss Ames to range in diameter from 5,000 to 25,000 light-years. If these dimensions are somewhat underestimated, as is quite possible, the following distances and dimensions of the super-galaxies themselves are too small in the same proportion:

Super-Galaxy	Distance	Diameter
A	10,500,000 light-years	2,000,000 light-years
B	39,000,000 light-years	2,000,000 light-years
C	139,000,000 light-years	2,100,000 light-years
D	169,000,000 light-years	2,400,000 light-years

It follows that though these super-galaxies happen to appear close together because they are nearly in the same direction from our galaxy, they are actually many millions of light-years from one another. Even the galaxies within each of these super-systems are separated, on the average, from their nearest neighbors by more than a million light-years.

It is only recently that the existence of super-galaxies has been established and as yet we have not become accustomed to their immensity. It is probable, however, that our Milky Way is a member of such a super-system, consisting of our galaxy, the Magellanic Clouds, the globular star clusters, the irregular nebula N.G.C. 6822, the spiral nebula in Triangulum, the Great Nebula in Andromeda, and possibly a few others. The units of our discussion are not satellites, nor planets, nor stars, but galaxies of hundreds of millions of stars. These immense aggregations of suns wander in stupendous paths through their super-system as the stars make wide excursions in their own galaxies. The periods of their circuits are probably of the order of 10,000,000,000 years and their relative velocities about 1000 kilometers per second.

It is too early to derive very definite numerical results, but a relationship exists among the masses, relative velocities, and distances apart of the members of our super-galaxy in order that these vast systems shall be semi-permanent. Eventually this relationship will serve as a check on our determination of these quantities, as it does now in the corresponding problem in our own galaxy of stars (Art. 375).

It has already been remarked that the Magellanic Clouds are receding with respect to our solar system at respective speeds of 170 and 276 kilometers per second. The Great Nebula in Andromeda is approaching at the rate of 300 kilometers per second. Even this high velocity has been greatly exceeded by many others, as has been verified by V. M. Slipher, W. H. Wright, F. G. Pease, and M. L. Humason. In a super-galaxy described by W. Baade, Humason found a galaxy that is apparently receding at the rate of 11,500 kilometers per second, and Hubble has recently (1931) found a displacement of spectral lines corresponding to a velocity

of 19,000 kilometers per second. Not only are these radial velocities very great, but with few exceptions the exterior galaxies appear to be receding, and, according to Hubble, the more rapidly the farther they are away. W. de Sitter concludes that, after allowing for the effects of the rotation of our galaxy, they are still receding. It is improbable that a large majority of exterior galaxies in every direction are actually moving away from our system. The apparent recession may be due to a gradual increase in the wave-lengths of light in its passage through the enormous intergalactic spaces; or it may be an effect of relativity, as has been suggested. But any explanation of an apparent large rate of recession is embarrassed by the fact that in some cases there are large velocities of approach.

382. Beyond the Super-Galaxies. Only yesterday, as it were, the horizon of even the partially known was limited to the surface of the earth; today it lies beyond a hundred million light-years and includes galaxies of galaxies of stars. Last evening the pyramids were regarded as old; this morning antiquity has retreated across millions of millions of years to the far times when the globular clusters were dynamically young. With the suddenness of the coming of day in the tropics, science has begun to illuminate the world and for the moment dazzles our unaccustomed eyes. Yet our achievements in science, and in the arts of living as well, do not place us among the gods; for though we have rapidly pushed back the frontiers of space and of time, the physical and biological universe without and the mental and spiritual universe within lie before us almost entirely unknown. The hope of the progress of our race rests upon a never-satisfied curiosity that ceaselessly impels us to attempt to explore beyond every limit we actually attain. Just now it makes us inquire what is beyond the super-galaxies of stars.

It is obvious that any answer we may give to our question is now, and perhaps will always remain, beyond the range of direct observational verification or disproof. Yet to refrain from attempting an answer would be contrary to our nature, for we universally reach out for even Omnipotence and a First Cause. We shall not, however, draw dogmatic conclusions where our information is least, but in humility and by analogy we shall construct a model universe which we hope will have some resemblance to reality. Those who feel a vague terror of infinity in space or time, or find

comfort and satisfaction in the finality of arbitrary authority will hesitate to join in this speculation.

There are now within the range of our knowledge several orders of physical units which we partially understand. The least are the positive electrons which, in volume, bear a smaller ratio to our own bodies than our bodies bear to the super-galaxies of stars. A few or many positive and negative electrons in proper organizations, but far apart in comparison with their diameters, constitute systems known as atoms. Somewhat similarly, a few or many atoms in proper organizations, but far apart in comparison with their diameters, constitute molecules. Another step takes us to planets, and the next to the stars. We have been considering in this chapter the evidence that the stars are not unrelated suns floating idly in the immensity of space, but that they are the units of which galaxies are organically composed. And finally, we have found that the galaxies, at least in many instances, are themselves relatively small components of super-galaxies. Cosmic units of seven ranks we know—electrons, atoms, molecules, worlds, suns, galaxies, super-galaxies—each an extremely minute and relatively isolated constituent of the next one above.

Now we return to the question with which we started: What is beyond the super-galaxies? As we have seen, our super-galaxy is made up of the Milky Way, the globular clusters, the Magellanic Clouds, and many others of the nearer galaxies. We shall call this a super-galaxy of the first order. Other known super-galaxies of the first order are the one in Centaurus and those in the Coma-Virgo region of the sky. Now we infer from analogy extending back through seven classes of physical bodies that the super-galaxies of the first order are the component units of super-galaxies of the second order. Our own super-galaxy and those in Centaurus and Coma-Virgo, together with an unknown number of others which we have not yet discovered, constitute our super-galaxy of the second order. We do not know its dimensions, but we conjecture by analogy that its diameter is 1,000,000,000 light-years, and possibly very much more.

Beyond the borders of our super-galaxy of the second order there is not, according to the hypothesis we are outlining, limitless unoccupied space. At distances that are enormous in comparison with its diameter there are other super-galaxies of the second order, in a general way similar to our own. If they could be analyzed

downward, they would be found to consist sequentially of super-galaxies of the first order, then galaxies, and stars, and worlds, and molecules, and atoms, and electrons. These super-galaxies of the second order are not unrelated cosmic masses lying in the vast void of space, but they constitute similarly a super-galaxy of the third order whose diameter may be a million times that of a super-galaxy of the second order. Now the process of ascent is clear. Each cosmic group, however great, is a minute unit in that of the next higher order, and the sequence continues without limit. If we should pursue it sufficiently far, we should arrive at super-galaxies of the thousandth, and the millionth, and the billionth order, and then we should have not really begun. Our universe is *infinite* (the first time the word has been used in all this discussion) in space and, as a little examination shows, in the number of its stars, and of its galaxies, and of its super-galaxies of every order.

At this place some minds will draw back in terror at the magnitude of the universe they are asked to consider, though they may not hesitate to adopt positive conclusions respecting Eternity and an Infinite Being. Their present fears are probably due to the fact that this relatively concrete picture now puts more than accustomed content into such inclusive words as *infinity*. And there are those who will reject and possibly resent the hypothesis because they despairingly realize that they can not fully grasp the far-reaching implications of its terms. If it were legitimate to accept or reject conclusions of the greatest importance on the basis of our vague fears and the weakness of our quickly-wearying minds, we should positively decide that the earth is not spherical and whirling unsupported around the sun, because such theories even now are wholly beyond the bounds of conceivability for a considerable fraction of the human race. We ask that only one step at a time be considered and taken, and that it be analogous to all that have gone before. A refusal at any stage to take such an additional step can be justified only by the assumption that we have arrived at a cosmic unit beyond whose borders there are no others in all the boundless realms of space, or by the assumption that if there are others they do not constitute similar orderly systems. During the past few centuries such negative assumptions as these have been made, one after another, with respect to the borders of the unknown, ranging from terrestrial continents to super-galaxies of stars; and these assumptions that ultimate limits have been reached

have successively been found to be only reflections of our timid and childish minds.

It is not feasible to go into details, but let us enumerate some of the simpler properties of the universe that we have defined. We note first that it has no boundaries beyond which it does not continue in a general way unchanged; nor has it any center having special properties not possessed by infinitely many other similar points. The stars in each of its galaxies describe great curves in enormous periods of time, and they occasionally pass near one another and give rise to planets. The galaxies in each of its super-galaxies describe similar curves on a vaster scale in space and in time, and passing near one another spin out the flattened disks of the spiral nebulae. Similarly, the cosmic units of every order constitute circulating and mutually interacting subsystems of those of the next higher order. Under plausible assumptions respecting the masses of the cosmic groups of various orders, the mutual velocities are higher the higher the order to which they belong. That is, the mutual velocities of the stars are less than those of the galaxies, and so on to super-galaxies of higher orders. In such a universe there is enough energy to last an infinite time, though it be continually scattered in space; that is, the universe is infinite in time as well as in space. Moreover, the apparent distribution of stars under the hypothesis would be precisely that which we actually find, and the sky is not a blaze of light as it would be if the stars were scattered *uniformly* throughout infinite space (Art. 355).

It would be decidedly misleading to leave the impression that this particular model of the universe is free from far-reaching unanswered questions or that it necessarily corresponds closely with what actually exists. It is not so important to have exactly attained reality in our wide excursion as it is to have persuaded ourselves that within the realm of possibility there is a universe which is infinite in space and in time and which is completely in harmony with all our experience. If our predecessors had had such a conception of the cosmos, they would not have looked on the mountains as having endured only a few thousands of years, nor would they have believed that all the stars will soon become cold.

So far as we can see, the near approaches of stars and the origin of planets are relatively rare events. It has been thought probable that only one star in a million of those in our galaxy is accompanied

by planets. Even if only one star in a galaxy has planets, there are yet infinitely many places in our infinite universe suitable for the abode of life; in fact, there are infinitely many such places at every stage of their evolution. With sublime egotism, our predecessors of a few generations ago assumed that the earth is the principal object of a special creation and that the human race is its ultimate goal. Though astronomical discoveries have toppled the insignificant earth from the preëminent position it was assigned, man is yet widely regarded as a creation unique and supreme. There is, of course, no basis except our amazing egotism for this assumption; it is more likely that there are infinitely many other worlds on which there is life, and that intelligence on some of them has advanced immeasurably beyond our own. This conception takes from us nothing of value, but inspires us with high aims. We rejoice in the possession of memory, imagination, reasoning powers, and a longing to know the truth, each as inexplicable as a galaxy of stars. Now we find ourselves a part of a Universal Order of which we did not dream and whose alphabet we are just beginning to learn. Instead of shrinking it to our measure, we contemplate its infinite orderliness and set no limits to the goal our race may hope to attain.

XXIII. QUESTIONS

1. Discuss the problem of Art. 355 for a mixture of stars of two different luminosities. In what respects, if any, do the conclusions differ?
2. Prove from the star-counts that our galaxy is limited in space.
3. Prove from the star-counts that our galaxy is an oblate disk of stars.
4. Prove from the star-counts that the stars differ greatly in luminosity.
5. What evidence is there for the existence of a local star-cloud?
6. Translate the parallax of Algol, $p = 0''.020 \pm 0''.014$, into light-years.
7. Find the relationship between the masses and the luminosities for the stars that are common to Tables XX and XXI.
8. How many of the methods of determining the distances to the stars are independent of spectroscopic measurements? How many depend in part upon spectroscopic measurements? How many depend only upon spectroscopic measurements?
9. Is the motion of the sun an advantage or a disadvantage in determining the parallaxes of the stars by statistical methods?
10. How are the distances to the galactic and globular clusters determined?

NOTE.—For a discussion of the dynamics of an infinite universe, see the last Chapter of *Differential Equations* (1930). F. R. Moulton, The Macmillan Company, New York.

TABLE OF CONSTANTS

Kilometer	0.62137 mile
Light-year	9.461×10^{12} kilometers (5.88×10^{12} miles)
Parsec	3.258 light-years (3.082×10^{13} kilometers)
Angstrom	10^{-8} centimeter (0.3937×10^{-8} inch)
Equatorial radius of earth	6378.388 kilometers (3963.34 miles)
Polar radius of earth	6356.812 kilometers (3949.99 miles)
Mean radius of earth	6371.200 kilometers (3958.82 miles)
Distance earth to moon	384,403 kilometers (238,857 miles)
Distance sun to earth	149,504,201 kilometers (92,897,416 miles)
Gram	0.002205 pound
Mass of earth	5.97×10^{27} grams (1.32×10^{25} pounds)
Mass of moon	7.3×10^{25} grams (1.6×10^{23} pounds)
Mass of sun	1.98×10^{33} grams (4.36×10^{30} pounds)
Sidereal day	$23^h\ 56^m\ 4^s.091$ mean solar time
Tropical (ordinary) year	$365^d\ 5^h\ 48^m\ 46^s.0$ mean solar time
Anomalistic year	$365^d\ 6^h\ 13^m\ 53^s.01$ mean solar time
Sidereal year	$365^d\ 16^h\ 9^m\ 9^s.5$ mean solar time
Synodical (ordinary) month	$29^d\ 12^h\ 44^m\ 2^s.8$ mean solar time
Sidereal month	$27^d\ 7^h\ 43^m\ 11^s.5$ mean solar time
Inclination of equator to ecliptic	$23° 27' 8''.26 - 0''.4684$ (t − 1900)
General precession	$50''.2564 + 0''.000222$ (t − 1900)
Constant of aberration	$20''.47$
Constant of nutation	$9''.21$
Solar parallax	$8''.80$
Lunar parallax	$57' 2''.7$
Velocity of light	299,796 kilometers (186,284 miles) per second
Constant of gravitation	6.670×10^{-8} dyne
Gravity (meters per second)	$9.8060 - 0.0260 \cos 2\phi$ (ϕ = latitude)
Planck's constant h	6.56×10^{-27} erg second
Mass of electron	9.05×10^{-28} gram
Mass of proton	1.663×10^{-24} gram

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