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College of Engineering Manual

HYDRAULICS

BY

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PREFACE TO THE FIRST EDITION

The Engineering text book (*C. E. College Papers, No. X*) hitherto used by the junior classes of the College of Engineering having run out of print, it is in contemplation to issue new text in the shape of a series of Manuals. The present hand-book forms the first of the series. Owing to the gradual rise in the standard required from students, several of the articles in *C. E. College Papers, No. X*, such as those on *Hydraulics* and *Strength of Materials* are too incomplete and out of date to serve as a basis for the new Manuals, while others, as that on *Building Construction*, only need amplification. The present Manual accordingly contains none but new matter, which has been derived from the best and latest authorities.

To avoid the necessity of publishing a separate work for the Engineer Classes, such additional matter as is required by the senior students is incorporated in the text, but is printed in smaller type. At a first perusal, the large type only should be read. The text is freely illustrated with examples, many of which have been selected from the Madras University Examination Papers. I am indebted to Sub-Conductor W. Cames, 2nd Assistant Master, College of Engineering, for the check of the correctness of their working. The solutions of examples are all printed in small type, but those which follow large text are intended for study by junior students.

Bazin's formulæ for the co-efficients of velocity in open channels are presented in a form which can be borne in memory, and which can be readily used for the solution of examples without the aid of tables. For pipes, Darcy's co-efficients are employed, and the examples have been worked on the supposition that the pipes are old and slightly incrusted. The discharges will hence be found considerably less than those given by the various tables in ordinary use. These are for the most part calculated from a high fixed co-efficient which is really suited only to a new pipe of considerable size. Darcy's, Bazin's and Kutter's co-efficients will be found graphically represented by curves. A collection of examples for practice, selected chiefly from the final examination papers of the College of Engineering, and from the B.C.E. degree examination papers of the Madras University, is given at the end of each chapter.

The following works have been consulted in the preparation of this Manual :—

Encyclopædia Britannica, Article *Hydromechanics*.
Cunningham's Roorkee Hydraulic Experiments.
Weisbach's Mechanics of Engineering.
Fanning's Hydraulic and Water Supply Engineering.
Francis's Lowell Hydraulic Experiments.
Jackson's Canal and Culvert Tables.
Humber's Water Supply of Cities and Towns.
Jackson's Hydraulic Manual.
Spon's Dictionary of Engineering.
Higham's Hydraulic Tables.
Madras Irrigation Department Professional Circulars.
Madras C. E. College Papers, No. X.
Downing's Hydraulics.
Neville's Hydraulics.
Box's Hydraulics.
D'Aubuisson's Hydraulics.
Rankine's Civil Engineering.
Molesworth's Pocket Book.
Proceedings of the Institution of Civil Engineers, *passim*.
Professional Papers on Indian Engineering, *passim*.

Special acknowledgments are due to the author * of the division *Hydraulics* of the article *Hydromechanics* in the ninth edition of the *Encyclopædia Britannica*. The applications of the theorem of Bernoulli, the use of the principle of energy, and many other items have been adapted from this admirable work to which the student is referred for further information on the subject. Valuable particulars have also been derived from MS. notes kindly furnished by General Mullins, late Chief Engineer for Irrigation, Madras.

It is intended to issue in due course a Manual dealing with the construction of works of Irrigation, and Town Water Supply and Drainage. A chapter on Hydraulic Machinery may subsequently be added to the present hand-book, and the two Manuals will then form a text-book on Hydraulic Engineering.

MADRAS, H. D. L.
January, 1887.

* Professor Unwin.

PREFACE TO THE SECOND EDITION

In the last paragraph of the preface to the first edition it was stated that the preparation of a Manual on the construction of works of Irrigation and Town Water Supply and Drainage was in contemplation. The subsequent publication of General Mullins' *Irrigation Manual* has rendered an early fulfilment of this intention unimperative. Arrangements were made however to incorporate in the *Hydraulic Manual* a chapter on the principles of Hydraulic Machines; but owing to the latter part of the first edition having sold out with unexpected rapidity, it has become necessary to issue the second edition without waiting for new matter.

The second edition has been carefully revised, some new examples have been inserted, and the answers to the exercises on the different chapters have been supplied. I am indebted to Sergeant W. H. Goddard, Instructor in Surveying and Drawing at the College of Engineering, for the check of some of the answers.

MADRAS, H. D. L.
January, 1894.

PREFACE TO THE THIRD EDITION

The Third Edition has been supplied with Appendices containing tables for facilitating calculations regarding channels. Bazin's co-efficients of velocity are given in these tables for earthen channels, and Kutter's co-efficients for all classes of channels. The former have been specially calculated, and the latter adapted from Trautwine's Civil Engineers' Pocket Book.

The answers to the examples have been revised.

I am indebted to Sergeant B. O. Reynolds, Instructor in Engineering at the College of Engineering, for the calculations required for the revision of the present edition.

MADRAS, H. D. L.
April, 1897.

PREFACE TO THE FOURTH EDITION

The Miscellaneous Examples on pp. 93 to 98 have been supplemented by questions recently set for the College and University Examinations.

MADRAS, H. D. L.
September, 1900.

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CHAPTER I.—HYDROSTATICS

UNITS AND SYMBOLS EMPLOYED

Units.—Throughout this work 1 pound, 1 foot and 1 second are taken as the units of weight, length, and time respectively, except where otherwise expressly stated.

Symbols.—The following is a list of the principal symbols employed. In the more important formulæ these symbols are printed in thick type, as below; elsewhere, the capitals are printed in ordinary type, and the small letters in italics.

A = area of a cross section in square feet.
c = co-efficient of discharge.
d = depth of water in feet, or diameter of pipe in feet, or rainfall in inches.
g = acceleration of gravity, taken as 32 ft. per second.
H = maximum head of water in feet.
h = head of water in feet.
$\mathbf{h_a}$ = head in feet required to produce velocity of approach.
L, l = length of a notch, weir, pipe, &c., in feet.
M = area of catchment basin in square miles.
μ = co-efficient of fluid friction.
n = ratio of base to height of slopes.
p = pressure at a point in lb. per square foot.
Π = atmospheric pressure in lb. per square foot.
Q = volume of discharge in cubic feet per second.
r = hydraulic mean depth in feet.
S = area of water surface in square feet.
s = sine of slope.
t = time in seconds.
v = velocity in feet per second.
w = weight in lb. of a cubic foot of water = 62½ lb.
x = afflux in feet.
z = height of water surface in feet above datum.

HYDRAULICS

CHAPTER I

HYDROSTATICS

CONTENTS

1. **Hydraulics** is that branch of *Hydromechanics* which deals in a practical manner with the flow of fluids through orifices, and in pipes and channels. The other branches of Hydromechanics, *viz.*, *Hydrostatics* and *Hydrodynamics* treat, the first of the equilibrium of fluids at rest, the second with the mathematical theory of their motion. Fluids are either liquid or gaseous, the chief difference between these varieties being that the former are practically incompressible, while the latter are compressible to an indefinite extent. In this manual we shall not deal, except incidentally, with gaseous fluids, and the only liquid which will be considered is water. Before commencing Hydraulics it will be desirable to gain some elementary notions of the laws of Hydrostatics.

2. **Hydrostatics—Water.**—Water is an almost incompressible liquid, weighing very nearly 1,000 oz. or $62\frac{1}{2}$ lb. per cubic foot. A gallon of water weighs 10 lb. Water becomes solid in the state of ice at 32°F, and gaseous in the form of steam at 212°F. These temperatures are termed the freezing and boiling points respectively of water.*

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* The boiling temperature is 212° at the level of the sea under ordinary atmospheric pressure. If we ascend a mountain to a height of H feet, the air pressure diminishes, and the number of degrees B to be deducted from 212° for the actual boiling point is given by the relation $H = 520B + B^2$.

Plate I. **3. Hydrostatic Laws.**—The chief laws of Hydrostatics are the following:—

(i.) *The pressure of water on any plane surface is equal to the weight of a column of water whose base is the area of the surface and whose height is the depth of the centre of gravity of the surface below the surface level of the water.*

(ii.) *The direction of the pressure on a surface is perpendicular to that surface.*

(iii.) *The resultant pressure of water on a body immersed, or partly immersed in it acts vertically upwards, and is equal to the weight of the water displaced. If the body floats, it follows that the weight of the water displaced is equal to the weight of the body.*

4. Pressure at a point.—The pressure *at* a point is the pressure per unit of area. If the unit is 1 s. foot, and the depth of the point h feet, the pressure p_h at this point is the pressure on an area of 1 s. ft at a depth h; *i.e.*, by law (i), $p_h = (1 \text{ s. ft.} \times h)$ c. ft. $\times 62\frac{1}{2}$ lb., or, if we denote the weight of a cubic foot of water by w,

$$p_h = wh \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

The pressures at any two points at the same level in a liquid are evidently equal.

5. Pressure on a surface.—The above result combined with law (ii) furnishes a mode of graphically representing the pressure on a plane surface. First take a vertical surface, such as a sluice shutter or lock wall, and consider an indefinitely small horizontal length of the surface, in fact a line as AB (fig. 1). Set off BB_1 equal and perpendicular to AB. Then $BB_1 = h = \frac{p_h}{w}$, where p_h is the pressure at B. Join AB_1. Take any point Q at a depth x, and draw QQ_1 perpendicular to AB. By similar triangles $QQ_1 = x = \frac{p_x}{w}$. Hence the pressures at every point of AB are represented in magnitude and direction by the horizontal ordinates of the triangle ABB_1. If P be the whole pressure on AB, *i.e.* $\Sigma\,(p_x)$, we have $\frac{P}{w}$ = area of triangular lamina $ABB_1 = \frac{AB.\ BB_1}{2} = \frac{h^2}{2}$; $\therefore P = \frac{wh^2}{2}$. The resultant of all the pressures must pass through the centre of gravity of the triangle, and it therefore cuts AB at a point O such that $AO = \frac{2}{3}h$.

If the surface occupies the position QB, the whole pressure is $w \times$ (area of trapezoid QBB_1Q_1). and the centre of pressure is the point in which the horizontal through the centre of gravity of the trapezoid cuts QB.

If the surface, and consequently the line AB is sloping (fig. 2), set off BB_1 ($= h$) perpendicular to AB. Plate I.

$$P = w \times (\text{area } ABB_1) = w. \frac{AB.h}{2}; \text{ and } AC = \tfrac{2}{3} AB.$$

Returning to the vertical surface, let it have a definite length l (fig. 3). The triangle ABB_1 develops into a triangular wedge of length l, and

$$P = w \times (\text{volume of wedge}) = wl \times (\text{area of triangle}) = wl \frac{h^2}{2}.$$

The resultant pressure P acts horizontally through the centre of gravity of the wedge, and the centre of pressure C is at a depth of $\frac{2}{3} h$.

The value of the whole or resultant pressure can readily be obtained from law (i). Thus, in the last example, area of surface $= hl$; depth of its centre of gravity $= \frac{h}{2}$; $\therefore P = whl.\frac{h}{2}$. The advantage of the graphic method is that it indicates both the mode of distribution of the pressure, and the position of the centre of pressure.

The distribution of pressure on triangular, quadrilateral and circular surfaces immersed vertically is shewn in figs. 4, 5, 6, and 7.

Ex. 1.—A rectangular sluice shutter is 8½ ft. in height, and 4 ft. in width. Find the whole pressure when the water on one side of it stands (*a*) 6 ft., (*b*) 8 ft. on the sill, there being no water pressure on the other side.

The surface being a vertical one, $P = wl \frac{h^2}{2}$. Hence

$$(a)\ P = \frac{125}{2} \times 4 \times \frac{36}{2} = 4{,}500 \text{ lb.}$$

$$(b)\ P = \frac{125}{2} \times 4 \times \frac{64}{2} = 8{,}000 \text{ lb.}$$

Ex. 2.—A pair of lock gates sustain 12 ft. of water on the inside and 3 ft. on the outside. Each gate is 5 ft. long. The lower hinge of each gate is at the level of the sill, the upper is 12 ft. above the sill. Find the horizontal pressure which each upper hinge has to bear. See fig. 8.

Consider 1 ft. in length of the gate, and let R lb. be pressure on upper hinge. The resultant water pressures P_1, P_2 on the inside and outside of the gates respectively are balanced by the reactions of the hinges which, neglecting the weight of the gates, are horizontal.

$$P_1 = \frac{125}{2} \times \frac{(12)^2}{2} \text{ lb.}$$

$$P_2 = \frac{125}{2} \times \frac{(3)^2}{2} \text{ lb.}$$

Taking moments about the lower hinge,

$$R \times 12 = P_1 \times \frac{12}{3} - P_2 \times \frac{3}{3} = \frac{125}{4}(576 - 9) = 17{,}719. \quad \therefore R = 1{,}476 \cdot 6 \text{ lb.}$$

Since the length of each gate is 5 ft., the actual pressure on the upper hinge $= 1{,}476 \cdot 6 \times 5 = 7{,}383$ lb.

Plate II. From the first law it appears that the pressure on the horizontal base of a vessel depends only on the area of the base and the depth of water. Thus if a cylinder and cone having equal bases and equal altitudes be filled with water, the pressures on the bases are equal. But the pressure on the base of the cylinder is the weight of the contained water; so that the pressure on the base of the cone is three times the weight of the water it actually holds. The physical explanation of this fact is that the base of the cone bears, in addition to the actual weight of the contained water, the vertically resolved portion of the reaction of the curved surface to the fluid pressure thereon.

6. **Equal transmission of pressure.**—If water be confined in a closed vessel, and an external pressure be applied to any portion of the liquid, that pressure will be transmitted equally in all directions through the liquid. This principle is utilized in hydraulic presses and other machines. A large and a small cylinder containing moveable pistons are filled with water, and connected by a pipe. If the small piston be pressed down with a force of p lb. per square inch, this degree of pressure will be transmitted to each square inch of the large cylinder and piston, the latter of which carries the weight to be raised. Let A, a be the areas of the large and small piston; P the force applied to the small piston; W the weight on the large piston. $W = pA$; $P = pa$; $\therefore W = P.\frac{A}{a}$.

Ex. 3.—In a hydraulic press, the diameters of the large and small cylinders are 15 inches and 1 inch respectively. Find what weight will be balanced by a force of 10 lb. on the small piston.

$$W = P.\frac{A}{a} = 10 \times \frac{(15)^2}{(1)^2} = 2250 \text{ lb.}$$

7. **Atmospheric Pressure.**—This is due to the weight of an air column extending to the surface of the atmosphere. It is a fluid pressure, and at any point acts equally in all directions. Take a tube about 33 inches long of the form shown in fig. 9, closed at A and open at B. Fill it with mercury, a fluid whose weight is about $13\frac{1}{2}$ times that of an equal volume of water, and place the tube vertically. The mercury will descend a short distance, leaving a vacuum at A. The pressures at B and B_1, points at the same level, must be equal, or motion would take place. The pressure at B is the atmospheric pressure Π lb. per sq. ft.; that at B_1 is the pressure due to the column of mercury AB_1 which is about 30 inches high.

Hence $\Pi = \left(1 \text{ s. ft.} \times \frac{30}{12} \text{ ft.}\right) \times (13\frac{1}{2}\, w) = 2{,}110$ lb. or nearly 15 lb. per sq. in.

This apparatus is called a *Barometer*, and measures the atmospheric pressure. If we ascend a mountain, the height of the column of air above us diminishes, and the mercury falls, thus furnishing a means of estimating the height ascended. A rough formula is Plate II

$H = 60,000 (\log R - \log r)$ ....(2)

where H is the height in feet, R, r the readings in inches at the lower and upper stations respectively. If accuracy is required, a correction on account of temperature must be applied.

Ex. 4.—Simultaneous barometric readings at Salem and the Shevaroys were 29·1 and 25·2 inches. Find roughly the difference of level.

$H = 60,000 (\log 29{\cdot}1 - \log 25{\cdot}2) = 60,000 (1{\cdot}4639 - 1{\cdot}4014) = 3,750$ ft.

Since mercury is $13\frac{1}{2}$ times as heavy as water, the height of the water column which can be supported by atmospheric pressure is $13\frac{1}{2} \times \frac{30}{12}$ ft. or 34 ft. nearly.

The atmospheric pressure generally acts on all portions of free water surface, and can therefore in most practical cases be left out of calculation. Thus, suppose a vessel of water to have a small orifice h feet below the surface. The atmospheric pressure Π is transmitted to all points of the liquid. The pressure in the vessel at the orifice is therefore $\Pi + wh$, and the pressure outside the orifice is Π; so that the resultant pressure producing flow is wh, *i.e.*, the pressure due to the height h—or *head* as it is often called—of water.

8. Siphon.—Let a bent tube ABC (fig. 10) be filled with water, and have its ends closed. Let one leg AB be placed in a vessel of water, and let the ends A, C be then opened. Water will flow out through C until the level of the water surface in the vessel falls to C or A, whichever is higher.

The liquid in the tube being continuous, the pressures at any two points in it at the same level are equal. Thus the pressures at D and D_1 must each be Π. But this is the pressure at C; so that the column of water CD_1 is unsupported, and must fall out. The rest of the water in the tube must follow it; for, if there were a break of continuity, a vacuum would be formed, which is obviously impossible unless the point B rise 34 ft. above the water surface at D. The pressure in the portion DBD_1 of the tube is less than Π, so that if a hole were made in this portion the air would rush in, the water would fall in both legs, and the siphon could not act.

9. Specific gravity.—The specific gravity of a substance is the ratio of the weight of any volume of the substance to the weight of an equal volume of water. Thus the specific gravity of mercury

Plate II. is 13·6, that of water being 1. Knowing the specific gravity of a substance, the weight of any given volume of it can at once be determined.

Ex. 5.—Find the weight of a 4-inch cube of cast-iron, its specific gravity being 7·25.

$$\text{Volume} = \left(\frac{1}{3}\right)^3 \text{c. ft.} \quad \text{Weight} = \frac{1}{27} \times 7\tfrac{1}{4} \times 62\tfrac{1}{2} \text{ lb.} = 16{\cdot}78 \text{ lb.}$$

10. Flotation.—It is obvious from law (iii) that a body will float or sink in water according as its specific gravity is less or greater than unity.

Ex. 6.—A canal boat, 33 ft. long, is constructed of $\frac{3}{16}$ inch sheet-iron. Allowing for contractions at bow and stern, she may be considered, for purposes of calculation, as 30 ft. in length, and of rectangular cross section, 6 ft. wide and 3 ft. deep throughout. Adding 50 per cent of weight for frames, rivets, etc., find the load in tons which the boat will carry so that she may float with 9 inches of her side above water. Specific gravity of wrought-iron 7·75. See fig. 11.

Let W be load in tons.

Area of sides and ends = 72 × 3 = 216 s. ft.

" bottom = 30 × 6 = 180 "

$$\therefore \text{vol. of iron} = 396 \times \frac{3}{16 \times 12} \text{ c. ft} = \frac{99}{16} \text{ c. ft.}$$

$$\text{Weight of boat} = \frac{99}{16} \times 7\tfrac{3}{4} \times 62\tfrac{1}{2} \times \frac{150}{100} = 4{,}496 \text{ lb.}$$

$$\text{Weight of water displaced} = 30 \times 6 \times 2\tfrac{1}{4} \times 62\tfrac{1}{2} = 25{,}312 \text{ lb.}$$

$$\therefore W = 20{,}816 \text{ lb.} = 9{\cdot}3 \text{ tons nearly.}$$

Since a substance immersed in water loses, in effect, the weight of the fluid displaced, the relative value of building materials for submerged works, which rely on their weight for stability, is ascertained by subtracting 62½ lb. from their weight per c. ft. Thus, approximately,

	Weight in air. lb.	Weight in water. lb.
Brickwork	112	50
Rubble Masonry	125	63
Concrete	125	63
Granite or Limestone	170	108

11. Hydrodynamic Laws.—When a stream of water is in motion, the following laws hold good:—

i. *If the motion is rectilinear and uniform, and if the effect of eddies produced by the roughness of the boundaries of the stream be neglected, the pressure at any point is the same as if the fluid were at rest.*

ii. *If the fluid particles take the same accelerations which they would have if independent, the pressure is uniform.* Plate II

Thus, in a jet falling freely in the air, the pressure throughout any cross section is uniform, and equal to the atmospheric pressure.

EXAMPLES ON CHAPTER I

1. The upper edge of a sluice gate is $10\frac{1}{2}$ feet below the surface, and the dimensions of the gate are 3 ft. vertical and 18 inches horizontal. Calculate the pressure upon it. (Univ. 1871.) *Ans.* 3,375 lb.

2. What is the total pressure of the water on a pair of lock gates, each gate being 10 ft. in width, and the water on the upper side standing to the height of 6 ft. above the bottom of the gate, the water on the lower side being below the bottom of the gate. (Univ. 1865.) *Ans.* 22,500 lb.

3. State what property in water is utilized in the hydraulic press, and give the proportions of a press capable of lifting a ton weight for every 10 lb. of pressure applied to it. (Univ. 1865.) *Ans.* Pistons of diameters in the ratio of 15 to 1.

4. A cubical vessel whose capacity is 19·683 c. ft. is filled with water; a vertical tube, also filled with water, whose internal diameter is x inches, and length 8 ft. is inserted in the top. Find the respective pressures against the bottom and one of the sides of the vessel. (Univ. 1872.) *Ans.* (1) 4,875 lb. (2) 4,260 lb.

5. A barge, supposed to be of rectangular cross section, is 60 ft. long, 10 ft. broad, and 4 ft. deep, outside measure. The thickness of the sides and bottom averages 0·1 ft. and the weight of the material of which they are composed averages 100 lb. per c. ft. Find how many ton load would sink the barge 3 ft. (Univ. 1869.) *Ans.* 45 ton.

6. A vertical shutter revolving about a horizontal axis sustains a pressure of 10 ft. of water. At what depth should the axis be placed in order that the pressures on the portions of the shutter above and below the axis may be equal? *Ans.* 7·07 ft.

7. A reservoir wall 16 ft in height has in section the form of a right angled triangle, the base being 12 ft. The depth of water is 14 ft. Compare the pressures per running foot of wall according as the sloping face or the vertical face is turned towards the water *Ans.* 1·25: 1.

CHAPTER II

HYDRAULICS—PRELIMINARY. DISCHARGE FROM SMALL ORIFICES

CONTENTS

Plate II. 12. **Volume of Flow.**—A stream of water flowing in a pipe or channel may be conceived to be made up of a great number of fluid threads, flowing more or less parallel to one another. These threads do not all flow with equal velocities, partly owing to the direct frictional resistance of the boundaries, but mainly because the roughness of the boundaries sets up eddies whereby the fluid filaments cross one another, and thus have their velocities modified. It will be seen at once that the actual motion of a stream is very complex; and there is no theory which takes account of the actual motion of every thread. Although however the velocity at a given point varies from moment to moment in magnitude and direction, it appears from observation that the average velocity for a period of, say, a few minutes is constant. Suppose the average velocity of each thread in the cross section to be determined, and that v feet per second is the mean of all these velocities, the discharge Q c. ft. per sec. flowing through the area A s. ft. is (fig. 12)

$$Q = Av. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (3)$$

Ex. 7. The cross section of a stream measures 1·2 s. ft. and the mean velocity is determined to be 70 ft. per min. Find the discharge in cub. ft. per sec.

$$\text{Here } A = 1{\cdot}2\,;\ v = \frac{70}{60}\,;\ \therefore Q = 1{\cdot}2 \times \frac{70}{60} = 1{\cdot}4 \text{ c. ft. per sec.}$$

The motion conceived above, in which the cross section of the stream is divided into very small areas each of which is the section of a fluid thread, is called *stream line motion*. If each fluid thread or stream line be supposed to have an unchanging velocity, it occupies a fixed position in space, and the motion of the stream is termed *steady motion*.

13. Principle of continuity.—If any space in a stream be taken having fixed boundaries, the space will generally remain constantly filled, and the in-flow will be equal to the out flow. This is termed the *principle of continuity*. If A, A_1 are the areas of two cross sections of a stream, v, v_1 the mean velocities at those sections, the in-flow to the water space between A and A_1 is Av c. ft. per sec., and the out-flow is $A_1 v_1$ c. ft. per sec. By the principle of continuity these are equal. Plate I

$$\therefore \frac{v}{v_1} = \frac{A_1}{A} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

or the velocities are inversely as the areas. If the bed of a stream has a varying slope, the velocity will be greatest where the slope is steepest. Hence in these parts the cross section will be least.

Ex 8.—A channel with a uniform slope of bed has a velocity of 1·5 ft. per sec. at a section measuring 150 s. ft. Find the velocity at a section measuring 125 s. ft.

Here $v \times 125 = 1{\cdot}5 \times 150$; $\therefore v = 1{\cdot}8$ ft. per sec.

14. Discharge from small orifices.—Velocity of discharge.—Suppose a short pipe projecting from the side of a vessel full of water, to be turned up at the end, and closed but for a small orifice at a depth h below the surface. The water will issue vertically from this in a thin stream or jet, which will be found to rise very nearly to the level of the surface of water in the vessel. The difference is so small that it will be at once suspected to arise from friction and other resistances. Neglecting for the present this difference, the velocity of each particle at the orifice is sufficient to carry the particle through the height h, *i.e.*, the particle has the velocity which would be acquired by falling freely from the surface of the water to the orifice. By dynamics, this velocity is $v = \sqrt{2gh}$ which is termed the theoretic velocity due to the *head* h. Since $h = \frac{v^2}{2g}$ the expression $\frac{v^2}{2g}$ is called the head due to the velocity v. Also, since $p = wh$, $\frac{p}{w}$ may be termed the pressure head at the orifice.

If A be the sectional area of the jet at the orifice, the discharge is $Q = Av = A\sqrt{2gh}$. This is termed the theoretic discharge of an orifice of area A.

15. Co-efficient of velocity.—The actual velocity v_a, as already explained, differs by a small quantity from the theoretic velocity v Let $v_a = c_v v$, where c_v is called the *co-efficient of velocity*.

$$v_a = c_v\sqrt{2gh} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Plate II. The co-efficient of velocity, which is determined by experiment, is nearly constant for different heads. Its mean value is 0·97. If the head is very great however, the co-efficient may be as high as 0·99.

The co-efficient of velocity may be determined by measurement of the parabolic path of a jet. Let the direction of the jet at the orifice be horizontal, x, y the measured co-ordinates of any point of the path of the jet (fig. 13).

$$x = v_a t; \quad y = \frac{gt^2}{2} = \frac{g}{2}\left(\frac{x}{v_a}\right)^2; \quad \therefore v^2_a = \frac{gx^2}{2y}$$

$$\text{But } v_a = c_v\sqrt{2gh}; \quad \therefore c^2_v 2gh = \frac{gx^2}{2y}; \quad \therefore c_v = \frac{x}{2\sqrt{yh}}.$$

The difference between the theoretic and actual velocity may also be estimated in terms of the head. Let h be the total head, h_1 the height to which the jet rises (fig. 14). Then h_1 is the head utilized in producing velocity, h_2 is the head utilized in overcoming the resistances of viscosity and friction. The latter, *i.e.*, $h-h_1$ is termed *the loss of head.*

Theoretic velocity $v = \sqrt{2gh}$. Actual velocity $v_a = c_v v = c_v\sqrt{2gh}$.

But $v_a = \sqrt{2gh_1}$ $\therefore c_v^2 h = h_1$

$h_2 = h - h_1 = h(1-c_v^2)$. If $c_v = \cdot 97$, $h_2 = \cdot 06h$

i.e., about 6 per cent of the total head is expended in overcoming resistance, leaving 94 per cent to produce velocity.

16. Co-efficient of contraction.—If the orifice be in a thin plate, or if the edges of the orifice be bevelled off sharp, the cross section of the jet is, at a short distance from the orifice, less than the area of the orifice. The cause is that the fluid threads, which arrive from all directions, have for the most part their directions changed at the orifice. The inertia of the threads prevents the change from taking place suddenly, and the paths of the threads are accordingly curved, as shewn in fig. 15. The greatest contraction occurs at a distance from the orifice of about half its diameter. If A be the area of the orifice, and c_cA that of the jet, c_c is called the *co-efficient of contraction.* For a well-placed, sharp-edged orifice in a plane surface, it is nearly constant for different heads and different forms of orifice. Its value is 0·64 as obtained by direct measurement.

If the thickness of the plate be greater than the diameter of the orifice, the capillary attraction of the sides of the orifice causes the condition shewn in fig. 16, and the co-efficient rises in value.

17. Co-efficient of discharge.—The expression $Q = Av$ assumes that the fluid threads have a mean velocity v in a direction at right angles to the cross section. This, in a jet, is the case only at the section of greatest contraction. If A be the area of the orifice, c_cA is the area of the jet, and $c_v\sqrt{2gh}$ is its velocity. Hence $Q = (c_cA)(c_v\sqrt{2gh})$ or **Plate II.**

$$Q = cA\sqrt{2gh} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where c is the *co-efficient of discharge* $= c_c\, c_v$.

For an orifice in a thin plate $c_c = \cdot 64$; $c_v = \cdot 97$; $\therefore\ c = \cdot 62$

Hence $Q = 5A\sqrt{h}$ approximately.

The co-efficient of discharge may be determined directly by allowing the flow to issue into a gauge basin. The discharge per second, $cA\sqrt{2gh}$, is observed, and compared with the theoretic discharge $A\sqrt{2gh}$, thus giving c.

The co-efficient of discharge is sometimes expressed in terms of the head. The discharge $Q = cA\sqrt{2gh}$ may be conceived to be made up of an area A and a velocity $c\sqrt{2gh}$. Let h_1 be the head due to this velocity. Then $h_1 = \dfrac{c^2\, 2gh}{2g} = c^2\, h$.

For a thin plate $c = \cdot 62$; $\therefore\ h_1 = (\cdot 62)^2\ h = \cdot 385\ h$.

Thus, $38\frac{1}{2}$ per cent of the total head is employed in producing velocity, and $61\frac{1}{2}$ per cent is lost by contraction and resistance.

Ex. 9.—Find the head of water which will ensure a discharge of 8 c. ft. per sec. through an orifice 6 inches square in a thin plate. (Univ. 1886).

$Q = cA\ \sqrt{2gh}$, where $Q = 8$, $c = \cdot 62$, $A = \frac{1}{4}$.

$\therefore\ 8 = \cdot 62 \times \frac{1}{4} \times 8\sqrt{h}$; $\therefore\ \sqrt{h} = 6\cdot 45$; $\therefore\ h = 41\cdot 6$ feet.

18. **Bellmouths.**—If the orifice be of the form of the contracted vein (fig. 17), the whole of the contraction occurs within the orifice; and if the area of the orifice be measured at its smaller end, $c_c = 1$. Hence for this form of orifice the co-efficient of discharge $c = 1 \times c_v = \cdot 97$. The entrances to pipes from reservoirs are often bell-mouthed to avoid the contraction, and consequent loss of head which would otherwise occur.

19. **Suppressed Contraction.**—Since contraction is caused by the convergence of fluid threads, any circumstance which tends to diminish that convergence, such as the application of an internal rim to the border of the orifice, or the near approach of the orifice to the bottom or sides of the vessel, will tend to increase the co-efficient of discharge. The expression $c = \cdot 62\left(1 + 0\cdot 14\ \dfrac{n}{m}\right)$ will meet this case, $\dfrac{n}{m}$ being the fraction of the perimeter of the orifice over which the contraction is suppressed. The co-efficient is also modified by the application to the orifice of shoots or channels of discharge.

Plate II **20. Mouthpieces or Adjutages.**—If a cylindrical tube, of length not less than $1\frac{1}{2}$ times the diameter of the orifice, be applied externally to the orifice, the jet, after contraction, again fills the tube; and the co-efficient of discharge attains the value 0·82.

If the cylindric mouthpiece be applied internally instead of externally, the co efficient becomes 0·52.

If the sides of the mouthpiece converge conically towards the outside, the co-efficient attains a high value. With a length of mouthpiece of $2\frac{1}{2}$ times the smallest diameter, and with an angle of convergence of 5°, the co-efficient of discharge is 0·92.

If the mouthpiece have the form of the contracted vein, and if its sides then diverge conically, the tube runs full. It can be shewn * that the maximum discharge through such a mouthpiece is theoretically the discharge through its smallest section into a vacuum, *i.e.* $Q = A\sqrt{2g\,(h + 34)}$. In practice however the discharge is less than this, owing to the disengagement of particles of air held in suspension in the water, and the consequent check to the continuity of flow. With an adjutage of the above form, whose length is nine times the smallest diameter, and having an angle of divergence of 5°, the actual discharge is nearly 1·5 times the theoretic discharge through the smallest area of the tube, and therefore $\frac{1·5}{·62}$ or 2·4 times the discharge through an equal area in a thin plate.

21. Short Pipes.—As the cylindric adjutage is gradually increased in length so as to become a short pipe, the frictional resistance increases, and the co-efficient diminishes as follows:—

Length in diameters.	2	5	10	25	50	100	150	200	250	300
Co-efficient ...	·82	·79	·77	·71	·64	·55	·49	·44	·41	38

Ex. 10. Find the discharge per second through a pipe whose length is 4 ft. and diameter 12 inches, the head or depth from the surface of water to centre of pipe being 12 ft. (Univ. 1886).

This is the case of a cylindric adjutage or short pipe whose length is four diameters. The co-efficient may therefore be taken as 0·80. $Q = cA\sqrt{2gh}$, where $h = 12$; $A = \frac{\pi d^2}{4} = ·785$ s. ft. $\therefore Q = ·8 \times ·785 \times 8 \times 2\sqrt{3} = 17·4$ c. ft. per sec.

* Art. 37.

22. Values of co-efficient of discharge.—The values of the co-efficients are here collected:—

Internal cylindric mouthpiece	...	0·52
Orifice in a thin plate	...	0·62
External cylindric mouthpiece	...	0·82
Conical convergent (5°) mouthpiece	...	0·92
Mouthpiece of form of contracted vein	...	0·97
Conical divergent (5°) mouthpiece	...	1·50

EXAMPLES ON CHAPTER II

1. Explain what is known in Hydraulics as the *principle of continuity*. In a stream in steady motion, the mean velocity at a certain section is 2 ft. per second, and the area of the section is 500 sq. ft. Find the volume of flow. At a second station, one mile distant from the first, the cross section diminishes to 300 sq. ft. Find the velocity. (Coll. 1883). *Ans.* (1) 1,000 c. ft. per sec., (2) 3·33 ft. per sec.

2. State generally the laws of efflux from simple orifices, explaining the meaning of the terms *co-efficient of velocity*, *co-efficient of contraction* and *co-efficient of discharge*, and their relation to each other.

A sluice opening is 3 ft. wide and 1 ft. high, the depth from the surface of the water to the lower edge of the orifice is 7 ft., and the discharge takes place freely into air. Find the discharge in c. ft. per second, considering the sluice as an orifice in a thin plate. (Coll. 1883). *Ans.* 38 c ft.

3. Water is kept at a constant depth of 3 ft. in a large wrought-iron cistern from which a discharge of 14 46 gals. per minute takes place through a hole 1 inch in diameter in one of the sides. How high is the hole above the bottom of the cistern? (Coll. 1886). *Ans.* 1 ft.

4. Find the diameter of an orifice in a thin plate sufficient to discharge 80,000 c. ft. per day with a head of 50 ft. (Coll. 1885). *Ans.* 2·2 in.

5. The discharge from an orifice 1 inch square with a head of 9 ft. of water is 7 c. ft. per minute. Find the co-efficient. (Coll. 1885). *Ans.* ·7.

6. An orifice 1 ft. square, whose centre is 36 ft. below water surface, is found to discharge 30 c. ft. per second. What is its co-efficient of contraction? What will the discharge be when the head is reduced to 25 ft. and 16 ft.? (Univ. 1883). *Ans.* (1) ·64, (2) 25 c. ft. s. (3) 20 c. ft. s.

7. What head would be necessary to give a discharge of $3\frac{1}{2}$ c. ft. per minute through an orifice 1 inch in diameter in a thin plate? How much would the discharge be increased if an adjutage of the form giving the maximum discharge were attached, the head remaining as at first? (Univ. 1884). *Ans.* (1) 4 ft. (2) 4·5 c. ft. per min.

8. Deduce the formula for the discharge of sluices $Q = 5 A \sqrt{h}$, and state how the formula must be varied if the contraction be suppressed along part of the perimeter of the orifice. (Univ. 1876).

9. What form of adjutage gives the maximum discharge? State the proportions of its various parts, and the ratio in which the discharge thus obtained exceeds the theoretic discharge. (Univ. 1875).

10. What would be the discharge per minute under a constant head of 6 ft. from

(i) a square orifice of 1·039 sq. inches area in a thin plate?

(ii) a cylindric adjutage 1 inch in diameter, and 3 inches long? (Univ. 1876). *Ans.* (1) 5·3 c. ft. (2) 5·3 c. ft.

CHAPTER III

DISCHARGE FROM LARGE ORIFICES AND NOTCHES

CONTENTS

23. Large orifices.—Hitherto we have dealt only with small orifices, *i.e.*, those in which the head is nearly the same for all the issuing threads. Suppose the orifice to be in a vertical plane, and to have but small height. Then if h be the head measured to the centre of the orifice, the velocities of all the threads are nearly equal to $c_v\sqrt{2gh}$, so that, (art. 17), $Q = cA\sqrt{2gh}$. In large orifices, on the other hand, the velocity cannot be taken the same for all the threads of the jet, owing to the heads differing widely for threads at the top and bottom of the orifice respectively. It will be seen later however that the mean velocity of all the threads differs but slightly from the velocity of the thread at the centre of the orifice; so that the expression for the discharge from a small orifice may be used without serious error. Plate III.

24. Bernoulli's theorem.—Let a stream be in steady motion, and let BC, (fig. 18) be an elementary stream line; z, z_1 the heights of B and C above the datum line OO; a, p, v the area of the cross section, the pressure, and the velocity at B; a_1, p_1, v_1 the corresponding quantities at C. In any short time t let the mass of fluid BC reach B_1C_1. Then $BB_1 = vt$; $CC_1 = v_1t$; and the quantity of equal in-flow and out-flow is $Qt = avt = a_1v_1t$. Since the thread is surrounded by other threads moving with nearly the same velocity, the viscous resistance may be neglected. The work done by the external forces must be equal (the fluid being incompressible), to the kinetic energy developed. The normal pressures on the whole surface of the thread, excepting the ends, are normal to the direction of motion, and do no work. The only external forces to be considered are therefore gravity and the pressures at the ends.

The energy due to gravity is that developed by the transfer of the volume Qt from the height z to the height z_1, *i.e.*, $wQt\,(z - z_1)$.

Plate III. The pressure energy is the work done by the pressure at B moving through the space BB_1, less that of the pressure at C exerted through CC_1, *i.e.*, $pavt - p_1a_1v_1t$, or $Qt(p - p_1)$

The change of kinetic energy is $\frac{w.Qt}{2g}(v_1^2 - v^2)$.

$$\text{Hence } w.Qt\ (z - z_1) + Qt\ (p - p_1) = \frac{w.Qt}{2g}(v_1^2 - v^2).$$

$$\therefore z - z_1 + \frac{p - p_1}{w} = \frac{v_1^2 - v^2}{2g} \quad \ldots \quad (7)$$

$\therefore \frac{v^2}{2g} + \frac{p}{w} + z = \frac{v_1^2}{2g} + \frac{p_1}{w} + z_1$; or, since B and C are any two points in the stream line,

$$\frac{v^2}{2g} + \frac{p}{w} + z = \text{constant} \quad \ldots \quad (8)$$

Now $\frac{v^2}{2g}$ is the head due to the velocity, $\frac{p}{w}$ is the head due the pressure, and z is the height above datum; so that z is the work which would be done by 1 lb. of water falling to the datum line, and $\frac{p}{w}$ and $\frac{v^2}{2g}$ are the quantities of work which would be done by 1 lb. of water under the pressure p and with the velocity v. The sum of the three is therefore the total energy per lb. of water estimated with reference to the datum line. Hence the total energy per lb. of water is uniformly distributed along the stream line.

If h be the depth of any point measured from a datum line O_1O_1, eq. (8) becomes $\frac{v^2}{2g} + \frac{p}{w} - h =$ constant.

25. Hydraulic gradient.—Let two vertical tubes be placed so as to meet the stream line at B and C (fig. 19). The water will rise in them to heights due to the pressures at B and C, *i.e.*, to heights $\frac{p}{w}$ and $\frac{p_1}{w}$. The difference of level of the free surfaces in the tubes is, from the diagram, $h = z + \frac{p}{w} - \left(z_1 + \frac{p_1}{w}\right)$. Substituting in (7), we get $h = \frac{v_1^2 - v^2}{2g}$, *i.e.*, the fall of surface level between two sections is the difference of the heights due to the velocities at those sections. The line D ... is termed the *hydraulic gradient;* but this expression is also used in cases where friction is taken into account.

26. Velocity of threads issuing in a jet.- We are now in a position to prove that the velocity of threads issuing in a jet is, neglecting the viscosity of the fluid, equal to that acquired by a particle falling freely from the surface of the fluid to the orifice;—a result which has hitherto (art. 11), depended on experiment.

The jet is composed of elementary threads which start into motion at some point in the interior of the vessel. One such thread is shewn in fig. 20. Let h_1 be the head at the point B, where the velocity is insensibly small; h the head and v the velocity at the orifice.

At B, the head is h_1, the pressure is $\Pi + wh_1$, and the velocity is zero.

At C, " h, " " Π, " v.

Hence, by Bernoulli's theorem, $\frac{v^2}{2g} + \frac{\Pi}{w} - h = 0 + \frac{\Pi + wh_1}{w} - h_1$.

$$\therefore \frac{v^2}{2g} = h; \ \therefore v = \sqrt{2gh} = 8\sqrt{h} \text{ nearly}$$

If the orifice is small in dimensions compared with h, the threads will all have nearly the same velocity; and if h be measured to the centre of the orifice, the expression $v = \sqrt{2gh}$ gives very nearly the mean velocity of the jet. Plate III

27. **Rectangular notch.**—Consider a rectangular notch of length l, and of depth $BC = h$, in the vertical side of a vessel filled with water (fig. 21). A fluid thread at depth x has a theoretic velocity $\sqrt{2gx}$. Lay off PQ horizontally equal to $\sqrt{2gx}$ to represent the velocity at P. The outer extremities of all such lines as PQ can be shewn * to lie on the parabola BQD, where $CD = \sqrt{2gh}$. Hence the figure BDC is a graphic representation of the velocities of all the threads issuing in a vertical line BC. The mean velocity of all the threads $= \frac{\Sigma(PQ)}{BC} = \frac{\text{area BDC}}{h} = \frac{\frac{2}{3} h \sqrt{2gh}}{h} = \frac{2}{3}\sqrt{2gh}$, *i.e.*, the mean velocity is $\frac{2}{3}$ of the bottom velocity. The theoretic discharge is $Av = hl \times \frac{2}{3}\sqrt{2gh}$ The actual discharge is

$$Q = \frac{2}{3} clh\sqrt{2gh} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (9)$$

In this expression, the co-efficient c is not constant, but varies for different values of l and h. A mean value of c for a thin plate is ·62. As this is the co-efficient for orifices with complete contraction, it may be thought that a higher value of c should be expected for a notch. As a matter of fact, however, the water surface falls towards the notch as shewn in fig. 24, and the head is for convenience measured from the bottom of the notch to the surface of still water. Practical examples of rectangular notches are gauge boards, tank surplus weirs, and river anicuts.

The discharge can be found at once, by the aid of the Integral Calculus, as follows:—

Consider a horizontal strip of the jet, of thickness dx, and situated at a depth x

Velocity of strip is $\sqrt{2gx}$, and the area of its cross section is $l.dx$.

Hence discharge of strip is $cl\sqrt{2gx}.\,dx$.

Whole discharge $Q = cl\sqrt{2g}\int_0^h x^{\frac{1}{2}}\,dx = \frac{2}{3}cl\sqrt{2g}.\,h^{\frac{3}{2}}$.

28. The cause of the variation of c may be explained thus:—Let l, h be the length and depth of the jet, L, H those of the notch (H being measured to the surface

* Let $PQ = y = \sqrt{2gx}$. Then $y^2 = 2gx$, which is the equation to a parabola whose axis is BC and vertex at B.

Plate III of still water some little way back from the notch, since the water surface falls towards the notch). Taking the co-efficient of velocity as unity, we have :—

For the jet, $Q = \frac{2}{3} l.h \sqrt{2gh}$.

For the notch, $Q = \frac{2}{3} c\, L.H\sqrt{2gH}$. $\therefore c = \frac{lh^{\frac{3}{2}}}{L.H^{\frac{3}{2}}}$.

But this co-efficient differs from the ordinary co-efficient of contraction which is $\frac{\text{area of jet}}{\text{area of orifice}} = \frac{l.h}{L.H}$. The latter co-efficient is approximately constant, so that *c* varies with the dimensions of the notch.

The jet discharged through the notch has a smaller section than L.H owing to (*a*) the fall of water surface, (*b*) the bottom contraction, (*c*) the end contractions. The diminution of the jet due to (*a*) and (*b*) is proportional to L, while that due to (*c*) varies with H. Mr. Francis found from experiments at Lowell with rectangular notches in a thin plate, the length of the notch being not less than three times the head, that the length of the jet was (L – 0·2H) for two end contractions. Taking cH as the depth of the jet on the weir, he obtained.

$$Q = \frac{2}{3} c\,(L - 0{\cdot}2\,H)\,H\sqrt{2gH} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (10)$$

The co-efficient *c* in eq. (10) is more nearly constant for different heads and lengths than it is in the ordinary formula, eq. (9). Its mean value is ·62.

29. Rectangular orifice.—Let l be the length of the orifice, h_1, h_2 the heads to the bottom and top respectively, (fig. 22). The velocities at these points are $\sqrt{2gh_1}$ represented by CD, and $\sqrt{2gh_2}$ by EF.

Hence the mean theoretic velocity is $\frac{\text{area EFDC}}{\text{EC}} =$

$\frac{\frac{2}{3}h_1\sqrt{2gh_1} - \frac{2}{3}h_2\sqrt{2gh_2}}{h_1 - h_2}$. The theoretic discharge is

$$Av = l\,(h_1 - h_2)\frac{\frac{2}{3}\sqrt{2g}\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right)}{h_1 - h_2}. \quad \therefore \text{Actual discharge}$$

$$Q = \frac{2}{3}\, c l\sqrt{2g}\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

If h_2 be put equal to zero, we get the discharge from a rectangular notch.

As in the case of the notch, and for a similar reason, the co-efficient *c* is not constant, but varies for different heads and areas of orifice. The values for sharp-edged orifices range from ·600 to ·633, the highest values being obtained with small heads.* The mean value of the co-efficient is ·62. Practical examples of rectangular orifices are sluice openings in anicuts, tank bunds, locks, etc. Plate III

With the aid of the Calculus the discharge can be obtained directly as in the case of the notch :—

$$Q = cl\sqrt{2g}\int_{h_2}^{h_1} x^{\frac{1}{2}}dx = \frac{2}{3}cl\sqrt{2g}\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right).$$

Unless the head over the upper sill is less than the depth of the orifice, it will be sufficiently correct in practice to use the expression for the discharge from a small orifice, *viz.*, $Q = cA\sqrt{2gh}$, the head h being measured to the centre of the orifice. The greatest error that can occur is when $h_2 = 0$, *i.e.*, when the orifice becomes a notch. The head to the bottom of the notch is $2h$. Hence :—

$$\text{Correct discharge } Q_1 = \frac{2}{3}cl\sqrt{2g}\ (2h)^{\frac{3}{2}} = cl.2h\sqrt{2g\tfrac{8}{9}h} = cA\sqrt{2g\tfrac{8}{9}h}$$

$$\text{Approximate discharge } Q_2 = cA\sqrt{2gh}.$$

So that, assuming the co-efficients equal in the two expressions, the ratio $\frac{Q_1}{Q_2} = \sqrt{\frac{8}{9}} = \cdot 94$; and the maximum error introduced by adopting the approximate formula is 6 per cent.

Ex. 11. The discharge from sluices is generally calculated on the assumption that the velocity at the mean depth is the mean velocity. What difference in cubic feet per minute would this make in the calculated discharge from a sluice 4 ft. long and 2 ft. deep, with a head of 12 ft. on the sill, the co-efficient being taken as $\frac{5}{8}$? (Univ 1881.)

$$\text{True discharge } Q_1 = \frac{2}{3}cl\sqrt{2g}\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right) = \frac{2}{3}\times\frac{5}{8}\times 4\times 8\left(12^{\frac{3}{2}} - 10^{\frac{3}{2}}\right)$$
$$= 132{\cdot}619.$$

Velocity at mean depth is $\sqrt{2g \times 11}$.

$$\text{Approximate discharge } Q_2 = cA\sqrt{2g \times 11} = \frac{5}{8}\times 8\times 8\sqrt{11} = 132{\cdot}665.$$

Difference is 0·046 c. ft. per sec., or $2\frac{3}{4}$ c ft per minute.

* The following table will furnish an idea of the mode in which the co-efficient varies.

Head to centre of orifice.	Ratio of height to width (width being 1 foot).				
Feet.	4	2	1	½	¼
0·5	..	..	..	·615	·631
1·0	..	..	·601	·617	·632
2·0	..	·613	·604	·617	·630
3·0	·627	·617	·605	·615	·627
5·0	·621	·612	·604	·611	·620
10·0	·604	·602	·601	·601	·603
30·0	·605	·603	·601	·603	·604
50·0	·609	·606	·602	·605	·607

Plate III. **30. Circular orifice.**—The circle may be divided into a number of vertical strips which are approximately rectangles. The mean velocity through each rectangle is very nearly the velocity at a point situated at half the depth of the rectangle, *i.e.*, on a horizontal diameter of the circle. Hence, if h be the depth of the centre of the circle, the mean velocity $= \sqrt{2gh}$ approximately, and therefore

$$Q = cA\sqrt{2gh} \qquad \ldots \qquad (12)$$

The greatest error that can occur by using this formula, *viz.*, when the circumference of the circle touches the surface, is 4 per cent.

In a similar manner may be treated the case of any orifice whose form is symmetrical above and below a horizontal axis.

Ex. 12. Deduce the formula $Q = 3{\cdot}9\, d^2 \sqrt{h}$, where
Q = discharge in cubic feet per second
d = diameter of orifice in feet
h = head in feet
for the flow of water through a circular orifice in a thin plate. (Univ. 1880.)

$$Q = cA\sqrt{2gh} = {\cdot}62 \times \frac{\pi d^2}{4} \times 8\sqrt{h} = {\cdot}62 \times \frac{22}{7} \times 2d^2\sqrt{h} = 3{\cdot}9 d^2\sqrt{h}.$$

31. Triangular notch.—In this form of notch, if l be the top width and h the depth to the vertex, (fig. 23), the ratio $\frac{l}{h}$ remains constant for different heads, and the co-efficient c is consequently subject to but little variation. This form of notch is therefore a good one for the measurement of the discharge of small streams. The approximate formula $Q = cA\sqrt{2gH}$, where H is the head to the centre of gravity of the water section, gives a result about 8 per cent too high.

Ex. 13. If discharge takes place through a triangular notch in a waste board placed on a dam, the two sides of the notch being equally inclined, and meeting at a right angle, find the co-efficient when $Q = \frac{1}{3}\sqrt{h^5}$, where h is the height in inches of still water above bottom of notch, and Q is the discharge in cubic feet per minute. (Univ. 1881.)

The discharge in c. ft. per sec. is $cA\sqrt{2gH}$, where H is head in feet to centre of gravity of water section $l = 2h$.

$$\text{Now H ft.} = \frac{1}{12}\cdot\frac{h}{3}\text{ ft.} = \frac{h}{36}\text{ ft.}$$

$$A \text{ s. ft.} = \frac{1}{2}\left(\frac{2h}{12} \times \frac{h}{12}\right) = \frac{h^2}{144}\text{ s. ft.}$$

$$\therefore Q \text{ per minute} = 60\,c\,\frac{h^2}{144} \times 8\sqrt{\frac{h}{36}} = \frac{5}{3}\,c\,\frac{\sqrt{h^5}}{3}.$$

$$\text{By question, } Q = \frac{1}{3}\sqrt{h^5}. \quad \therefore \frac{5}{3}c = 1;\ \therefore c = 0{\cdot}60.$$

32. The true discharge of a triangular notch can be found as follows:—Consider a horizontal layer of fluid filaments at a depth x below the surface, (fig. 23). Let the length of the layer be y and its depth dx. $\frac{y}{l} = \frac{h-x}{h}$. Plate III

Velocity of layer $= \sqrt{2gx}$. Area of its cross section $= y.\,dx$.

$\therefore$ Discharge of layer $= cy\,dx\sqrt{2gx} = c\,\frac{l}{h}\sqrt{2g}\left(h\sqrt{x} - x\sqrt{x}\right)dx$.

$\therefore$ Whole discharge of notch $= c\sqrt{2g}\,\frac{l}{h}\int_0^h \left(h\sqrt{x} - x\sqrt{x}\right)dx =$

$$c\sqrt{2g}\,\frac{l}{h}\left(\frac{2}{3}h^{\frac{5}{2}} - \frac{2}{5}h^{\frac{5}{2}}\right)$$

$$i.e.,\quad Q = \frac{4}{15}cl\sqrt{2g}\;h^{\frac{3}{2}} \quad \ldots \quad (13)$$

The approximate formula gives $Q = c\,\frac{lh}{2}\sqrt{2g\frac{h}{3}} = \frac{1}{2\sqrt{3}}\,cl\sqrt{2g}\;h^{\frac{3}{2}}$

$$\therefore \frac{\text{Approximate discharge}}{\text{True discharge}} = \frac{1}{2\sqrt{3}} \div \frac{4}{15} = \frac{289}{267} = 1{\cdot}08.$$

33. Velocity of approach.—If the water passing through a notch or orifice has a velocity of approach, as in the case of streams or rivers flowing over weirs or anicuts, this velocity (which tends to increase the discharge) can be taken account of by supposing the head which would be required to produce it to be added to the actual head. Take the case of a rectangular notch, and let v_a be the velocity of approach. The head h_a required to produce this velocity is $\frac{v_a^2}{2g}$. Hence the rectangular notch of depth h becomes practically a rectangular orifice, (fig. 24), the heads to bottom and top of which are $(h+h_a)$ and h_a. Hence

$$Q = \frac{2}{3}\,cl\sqrt{2g}\left\{(h+h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\} \quad \ldots \ldots (14)$$

The construction of a weir across a river causes an increase of water section immediately above the weir; consequently the velocity of approach is less than the natural velocity of the stream.

Let A be the natural section, v the velocity.

A_a the section just above weir, v_a the velocity.

Then, by eq. (4) $v_a A_a = vA$. $\quad \therefore v_a = v\,\frac{A}{A_a}$.

Ex. 14. What will be the discharge per minute through a rectangular notch, in a thin plate 6 ft. wide, head 8 inches, velocity of approach 2 miles per hour? (Univ. 1877.)

$v_a = \frac{2 \times 5280}{60 \times 60} = 2{\cdot}93$ ft. per sec.; $h_a = \frac{(2{\cdot}93)^2}{64} = 0{\cdot}13$ ft.

$Q = \frac{2}{3}\,cl\sqrt{2g}\left\{(h+h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\}$, where $c = {\cdot}62 = \frac{5}{8}$, $l = 6$, $h = {\cdot}67$, $h_a = {\cdot}13$.

$\therefore Q = \frac{2}{3} \times \frac{5}{8} \times 6 \times 8\left\{({\cdot}80)^{\frac{3}{2}} - ({\cdot}13)^{\frac{3}{2}}\right\} = 20 \times 0{\cdot}667 = \frac{40}{3}$.

$\therefore$ Discharge per minute $= \frac{40 \times 60}{3} = 800$ c. ft.

Plate III. **34. Submerged orifice.**—Let h_1, h_2, (fig. 25), be the heads on either side of the orifice, tending to produce discharge in opposite directions. The effective head is $(h_1 - h_2)$, the difference of level between the water surfaces above and below the opening. Calling this head h and the area of the sluice A,

$$Q = cA \sqrt{2gh} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15).$$

The proof of this is as follows :—

Let BC (fig. 26) be an elementary fluid thread, the velocity at B being insensibly small. Then, with the notation shewn on the diagram we have:—

At B, the head is h_1, the pressure $\Pi + wh_1$, the velocity zero.

At C, „ h_2, „ $\Pi + wh_2$, „ v.

$$\therefore \frac{v^2}{2g} + \frac{\Pi + wh_2}{w} - h_2 = 0 + \frac{\Pi + wh_1}{w} = h_1.$$

$$\therefore \frac{v^2}{2g} = h_1 - h_2 = h$$

35. Partially submerged orifice.— Let h be the difference of water levels above and below the orifice, (fig. 27), h_1, h_2, the heads to bottom and top of the orifice respectively. The discharge may be divided into two portions, *viz.*, Q_1 taking place through a simple rectangular orifice of depth $(h - h_2)$, and Q_2 taking place through a submerged orifice of depth $(h_1 - h)$

$$Q_1 = \frac{2}{3} cl \sqrt{2g} \left(h^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right)$$

$$Q_2 = cl (h_1 - h) \sqrt{2gh}.$$

Assuming the co-efficient c to have the same value in each case,

$$Q = cl \sqrt{2g} \left\{ \frac{2}{3} \left(h^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right) + (h_1 - h)\, h^{\frac{1}{2}} \right\} \ldots\ldots\ldots (16).$$

36. Submerged notch.—Let h_1 be the head to the bottom of notch, (fig. 28); h the difference of level between the water surfaces.

$$Q_1 = \frac{2}{3} clh \sqrt{2gh}.$$

$$Q_2 = cl (h_1 - h) \sqrt{2gh}.$$

$$\therefore Q = cl \sqrt{2gh} \left(h_1 - h + \frac{2}{3} h\right) = cl \sqrt{2gh} \left(h_1 - \frac{h}{3}\right) \ldots\ldots (17),$$

taking the co-efficient the same for the two portions of the orifice.

37. Adjutages.—The theorem of Bernoulli enables us to explain the cause of the high discharge given by divergent adjutages. Suppose a gradual enlargement takes place in a horizontal pipe through which water is flowing steadily; the velocity gradually diminishes. But $\frac{v}{2g} + \frac{p}{w} + z$ is constant, by eq. (8), and z is constant; hence the pressure increases as the velocity diminishes. If a gradual contraction occurs in the pipe, the velocity increases, and the pressure diminishes. We say *gradual* enlargement or contraction, so that there may be no sudden change in velocity, and consequent loss of energy by shock. Plate IV.

Take a mouthpiece of the form of the contracted vein, the sides of which diverge conoidally, and terminate cylindrically, so that the threads of the jet may emerge parallel, (fig. 29).

Let A, v, p be the area, velocity and pressure at the smallest section C; A_1, v_1, p_1 the corresponding quantities at the exterior D. Consider a filament B C D.

At B, the velocity is zero, the pressure is $\Pi + wh_o$, the head is h_o.
At C, „ v, „ p, „ h.
At D, „ v_1, „ p_1, „ h.

$$\text{Hence } 0 + \frac{\Pi + wh_o}{w} - h_o = \frac{v^2}{2g} + \frac{p}{w} - h = \frac{v_1^2}{2g} + \frac{p_1}{w} - h.$$

$$\therefore \frac{\Pi}{w} + h = \frac{v^2}{2g} + \frac{p}{w} = \frac{v_1^2}{2g} + \frac{p_1}{w} \quad \text{.. .. .. ..} \quad (18).$$

If the jet discharges into air, $p_1 = \Pi$; $\therefore h = \frac{v_1^2}{2g}$ or, allowing for friction, $v_1 = c_v \sqrt{2gh}$. $\therefore Q = \cdot 97 A \sqrt{2gh}$.

Hence the discharge does not depend on the section at C, which may be as small as we please, provided the velocity at C does not become so great as to cause the pressure to fall below zero. Should this occur, the water pressure becomes a tension, continuity of flow would be impossible, and the mouthpiece could not run full at D. Putting $p = 0$, we have from eq. (18), $\frac{\Pi}{w} + h = \frac{v^2}{2g} + 0$.

But $\frac{\Pi}{w}$ is equivalent to a head of water of 34 ft. $\therefore v = \sqrt{2g(h + 34)}$; and

$$Q = A \sqrt{2g(h + 34)} \quad . \quad \text{..} \quad (19).$$

Thus the maximum discharge is the theoretic discharge into a vacuum through an orifice of section A under a head h.

In practice the maximum discharge can never be as great as this, owing to the disengagement of particles of air suspended in the water, which would check continuity of flow before the pressure at C fell to zero.

In cylindrical and convergent adjutages, there is a sudden contraction at the inner orifice, and a subsequent expansion to fill the tube. A loss of energy consequently takes place, which reduces the co-efficient of velocity, although the tube may run full at its external orifice, and the co-efficient of contraction be equal to unity.

Ex. 15. The external area of a divergent conoidal adjutage is 4 s. in. Find the theoretical area of the smallest cross section which will allow the adjutage to run full (1) when the head of water is 2 ft. (2) when it is 15 ft. See fig. 29.

(1) At D, the velocity is $\sqrt{2g \times 2}$. The area is 4 s. in.
At C, „ $\sqrt{2g(2 + 34)}$ „ A „

$$\therefore A\sqrt{2g \times 36} = 4\sqrt{2g \times 2}; \ \therefore A = 0{\cdot}91 \text{ s. in.}$$

(2) $$A\sqrt{2g \times 49} = 4\sqrt{2g \times 15}; \ \therefore A = 2{\cdot}21 \text{ s. in.}$$

Plate IV **38. Internal cylindric tube.**—In this case the co-efficient can be theoretically determined. Let the tube project internally to such a distance as will just allow the jet to spring clear of the external orifice. Then the velocity at the points B, C, (fig. 30), will be practically zero, and the pressures at those points will be the hydrostatic pressures due to the depths at B and C. Let the areas of the orifice and jet be A, A_1 respectively, and let the mass of fluid occupying the space between OO and D, occupy after a small interval t the space between O_1O_1 and E.

The hydrostatic pressures on the sides of the vessel balance one another everywhere except opposite the orifice. The atmospheric pressure acts on a section of the jet whose area is that of the orifice, as well as over the free water surface. Hence the unbalanced horizontal pressure is whA. Its impulse in the time t, *i.e.*, $whAt$, must be equal to the change in horizontal momentum in the moving mass. Since the motion is steady, there is no such change between O_1O_1 and D, and there is no horizontal momentum between OO and O_1O_1. Hence the whole change is in the momentum between D and E.

Volume of space $= A_1vt$. Mass of liquid $= \frac{w.A_1vt}{g}$. Momentum $= \frac{wA_1v^2t}{g}$,

$\therefore whAt = \frac{wA_1v^2t}{g}$. $\therefore \frac{A_1}{A} = \frac{gh}{v^2} = \frac{gh}{2gh} = \frac{1}{2}$.

But $\frac{A_1}{A} = c_c$. Therefore, neglecting friction, $c_c = 0·5$.

The best experiments give $c_c = 0·52$.

EXAMPLES ON CHAPTER III.

1. Find the discharge in cubic feet per minute from an orifice 2′ 6″ long and 7″ deep, its upper edge being 3″ below the water surface. (Coll. 1885.) *Ans.* 318 c. ft.

2. If a rectangular notch of width l be cut in the side of a cistern in which water is kept at a constant height h above the sill, shew that the theoretical discharge (neglecting contraction) is $\frac{2}{3} lh \sqrt{2gh}$, the mean velocity $\frac{2}{3}\sqrt{2gh}$, and the mean head $\frac{4}{9} h$. (Univ. 1883.)

3. If it were found that 381 c. ft. of water passed in 15 seconds through a rectangular notch 10 ft. wide with a head of 10 inches, what would be the value of the co-efficient? (Univ. 1880.) *Ans.* ·63.

4. Find the value of the co-efficient c in the formula $h = c\sqrt[3]{\left(\frac{Q}{L}\right)^2}$, in which Q is the actual discharge through a rectangular notch in a thin plate in c. ft. per sec., L the length of the notch in feet, and h the height in inches of still water above the notch crest (Univ. 1875.) *Ans.* 5·4.

5. Assuming the mean velocity of water discharging from a rectangular slit in the vertical side of a reservoir is two-thirds of the bottom velocity, obtain the expression for the discharge over a drowned notch when the water approaches with a surface velocity v_a. (Univ. 1884.)

6. Explain the terms, *head of approach, velocity of approach* and state how the expression for discharge through a rectangular notch is modified by the velocity of approach. (Coll. 1884.)

7. (*a*) A rectangular orifice 4 ft. wide and 3 ft. high, in a thin plate has the water surfaces on either side at 3 ft. 9 in. and 4 ft. 3 in. respectively above the lower edge of the opening. Estimate the discharge. *Ans.* 42·4 c. ft. s.

(*b*) If the water has a velocity of approach of 5 ft. per second, by how much will the discharge be increased? *Ans.* 14·2 c. ft. s.

(*c*) Estimate the discharge when the water levels are at 2 ft. 9 in. and 2 ft. 6 in. respectively, there being no velocity of approach. *Ans.* 26·6 c. ft. s.

8. A notch is of the form of a right-angled triangle. Estimate the discharge when the width of the notch at the water surface measures 15 inches. *Ans.* 0·87 c. ft. s.

9. Prove the theorem of Bernoulli. Thence shew that the effective head producing discharge through a waterway in a railway embankment which is run across a tank is the difference of level between the water surfaces on the two sides of the embankment. (Univ. 1891.)

10. A sheet-iron cistern, with vertical sides $\frac{3}{8}$ in. thick, has on one side a right-angled triangular orifice, apex upwards, whose horizontal base is 1 ft. wide, and 4 ft. 4 in. below water surface; and on two other sides circular openings 6 in. in diameter, whose centres are 9 ft. below water level; to the outside of one of these a pipe of 1 ft. length and 6 in. internal diameter is fixed, and to the inside of the other is fastened a similar pipe. What will be the discharge from each orifice, and the quantity of water necessary to keep the water level in the cistern constant? (Univ. 1893.) *Ans.* (1) 2·53 c. ft. s., (2) 3·87 c. ft. s., (3) 2·45 c. ft. s., (4) 8·85 c. ft. s.

CHAPTER IV

PRACTICAL CASES OF DISCHARGE FROM ORIFICES AND NOTCHES

CONTENTS

39. The preceding articles enable us to deal with all the practical cases usually occurring. The only difficulty lies in the choice of a suitable co-efficient. In the case of the discharge through a thin plate, the co-efficient is certainly known with some precision; but, except in the gauging of small streams, discharges take place in practice through masonry works whose varying details of construction render it impossible to assign co-efficients which will always suit each type of work.

40. Tank surplus weirs.—A tank *surplus weir, waste weir, escape, or kalingula* consists of a masonry wall, occupying a portion of the length of the bund, but at a considerably lower level. The elevation of the wall is horizontal at top, and is bounded at the ends by vertical wing walls which support the earthwork of the bund. The top or *crest* of the weir is from $1\frac{1}{2}$ to 3 ft. broad, and generally slopes slightly upwards from the tank. The level of the crest of the weir is termed *full tank level* (F.T.L.). The weir is made of such a length as to discharge the greatest inflow to the tank, with a certain assigned depth on the crest of the weir. This depth or *head* on the weir is generally from 2 to 4 ft, and the surface level at this head is called *maximum water level* (M.W.L.). The top of the bund is not less than 3 ft. above M.W.L. The inflow to the tank is determined by its *catchment basin* or drainage area, and by the maximum rainfall which observation shews can come on that area in a given time, say 24 hrs. A certain proportion of this rainfall, depending on the nature of the soil and slope of the ground, will flow into the tank; and, assuming the tank to be full when the rainfall commences, this is the maximum discharge which the weir should pass with the given head. Owing to the partial nature of heaviest rainfall, the discharge to be provided for is not directly proportional to the area of the catchment basin. Ryves' empiric formula $Q = cM^{\frac{2}{3}}$, where M is the area of the basin in square miles, and c is a local co-efficient ranging from 350 to 650, is generally used in Southern India. Dickens' formula $Q = c_1M^{\frac{3}{4}}$ is also occasionally employed. Plate XV

The discharge of a tank weir is that through a rectangular notch, viz., $Q = \frac{2}{3}clh\sqrt{2gh}$. As the water surface for some little distance above the weir falls towards the weir, the head should be measured to the surface of still water. For this purpose a vertical scale is placed against the wing wall, some feet in front of the crest. The value of the co-efficient c has not been well determined. It varies with the head, with the length and thickness of the weir crest, and with the depth of water in front of the weir. For a thin edge c varies from about ·66 to ·59 with changes of length and head. The experiments of Castel and Blackwell on added channels * and broad-crested weirs give mean co-efficients ·53 and ·51 respectively. These experiments were however conducted on a small scale; and it appears probable that the co-efficients are higher for the large weirs of tanks and rivers. Professor Unwin † proposed ·577, or $\frac{1}{\sqrt{3}}$, on theoretical grounds, and this value will be adopted in the examples which follow. It agrees well with the results of the Lowell hydraulic experiments, see note to art. 41. The formula thus becomes

$$Q = \frac{2}{3} \times \frac{1}{\sqrt{3}} \times 8\, lh^{\frac{3}{2}} = 3{\cdot}1\, lh^{\frac{3}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (20).$$

Ex. 16. What length of kalingula should a tank have for each square mile of a small catchment basin in order to carry off a rainfall of 1 inch an hour, of which 60 per cent reaches the tank; it being assumed that the tank is full, that the supply from each square mile is uniform, and that the height of still water above the crest of the kalingula is 4 feet? (Univ. 1880.)

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* Castel's "added channels" were short channels or shoots, having the same section as the notch, which were placed outside the notch.

† *Encyclopædia Britannica*, 9th edition, Article *Hydromechanics*.

Plate IV. Rainfall per sq. mile per hour = $5280 \times 5280 \times \frac{1}{12}$ c. ft.

$\therefore$ **Discharge per sec. per. sq. mile of catchment basin** = $\left(\frac{60}{100}\right)\frac{(5280)^2}{12 \times (60)^2}$
= **387·2 c. ft.**

$Q = 3{\cdot}1\, lh^{\frac{3}{2}}$, where $Q = 387{\cdot}2$; $h = 4$; $\therefore l = 15{\cdot}7$ ft.

Thus, if the drainage area were 10 s. miles, the length of the weir would be 157 ft.

41. Weirs with broad sloping crests.—Suppose the weir, (fig. 31), rounded at the crest to suppress contraction. Let h be the total head on the crest, measured from centre of crest to level of still water. Let BC be any filament extending from still water to centre of crest; h_1, h_2 the heads at B and C; x the depth of water on centre of weir; z the depth of water over C.

At B, the head is h_1; the pressure wh_1; the velocity zero.

At C, „ h_2; „ wz; „ v.

$\therefore \frac{v^2}{2g} + \frac{wz}{w} - h_2 = 0 + \frac{wh_1}{w} - h_1.$

$\therefore \frac{v^2}{2g} = h_2 - z = h - x$; $\therefore v = \sqrt{2g(h-x)}$. Hence, if l be the length of the weir, $Q = lx\sqrt{2g(h-x)}$.

If $x = 0$, $Q = 0$; and if $x = h$, $Q = 0$; so that Q is a maximum for some value of x between 0 and h.

$Q = l\sqrt{2g}\left\{x(h-x)^{\frac{1}{2}}\right\}$; $\frac{dQ}{dx} = l\sqrt{2g}\left\{-\frac{\frac{1}{2}x}{\sqrt{h-x}} + \sqrt{h-x}\right\} = 0.$

$\therefore -x + 2(h-x) = 0$; $\therefore x = \frac{2}{3}h.$

$$\text{Hence } Q = \frac{2}{3}lh\sqrt{2g\frac{h}{3}} = 0{\cdot}385\, lh\sqrt{2gh} \quad\quad (21).$$

Experiment shows* that the actual discharge is approximately equal to this maximum. The ordinary weir formula is $Q = \frac{2}{3}clh\sqrt{2gh}$. Hence the value of c for broad-crested weirs will be about $\frac{3}{2} \times {\cdot}385 = {\cdot}577 = \frac{1}{\sqrt{3}}$.

42. Drowned tank weirs.—If the crest is low, and the surplus channel confined, the tail water may sometimes rise above the level of the weir crest. This is the case of a submerged notch, the discharge from which is given by eq. (17) provided the co-efficient be the same for the orifice and notch portions of the opening. It appears however from observations on large weirs † that the co-efficient for the orifice portion is considerably higher than that for the notch portion. In the absence of precise data, the co-efficients may be taken as ·8 and ·577 respectively.

* The Lowell experiments give $c = {\cdot}563$ for weirs with crests 3 ft. broad, and contractions suppressed, heads 6 to 18 inches.

† Pro. Inst C. E., Vol. lxxxv, 1885-86, Rhind on co-efficients of discharge.

Ex. 17. A weir has a head of 4 ft. of water on its crest, and the tail water rises 3 ft. above the crest. Find the discharge per second for each length of 15·7 feet. Plate IV

Let d be depth of tail water on crest;

h difference of water levels above and below weir.

$$Q_1 = \frac{2}{3} c_1 l h \sqrt{2gh}. \quad Q_2 = c_2 l d \sqrt{2gh}. \quad c_1 = \cdot577; \quad c_2 = \cdot8$$

$$\therefore Q = 15\cdot7 \times 8 \left\{ \frac{2}{3} \times \cdot577 + \cdot8 \times 3 \right\} = 350 \text{ c. ft. per sec.}$$

Comparing this with the previous example, it will be seen that the free overfall discharges 87 c. ft. per sec. more than the drowned weir for each length of 15·7 feet.

43. Gauging weirs.—When it is desired to accurately measure the discharge of a stream, (as for water supply projects, &c.), a dam made of piles B and planks C, (figs. 32), is formed across the stream, and puddled on the inside to prevent leakage. A notch, generally rectangular, of suitable size to carry the discharge, is made in the weir, and is protected by a metal plate D, one-tenth inch thick, so as to secure accuracy of form and permanent sharpness of edge. The air must have free access behind the sheet of falling water. Fig. 32 (*a*) is a half elevation of, and fig. 32 (*b*) a cross section through the weir. Fig. 32 (*c*) is an enlarged section to show plank and plate. It will be seen that this is a case of discharge through a rectangular notch with a velocity of approach. If however care be taken that the section of the jet does not exceed one-fifth of the water section above the weir, the velocity of approach may be neglected. The head is measured by a scale placed on a pile E, the zero of the scale being accurately level with the crest of the notch. The pile should be driven some distance back from the weir, say 5 feet for small weirs to 25 feet for large ones, so as to ensure the head being measured to the surface of still water.

A more accurate mode of measurement is by the *hook gauge*. A sharp-pointed metal hook at the foot of a vertical bar, is capable of being moved up and down by a slow motion screw, the whole apparatus being attached to a pile. The bar carries an index which is at the same height above the point of the hook that the zero of the scale is above the level of the notch crest. When a reading is to be taken, the hook is depressed below the surface of the water, and then slowly raised. The instant of its reaching the surface is distinctly marked by the reflexion from a film of water formed on the point of the hook. The scale is then read. In ordinary light, differences of level of a hundredth of an inch can be detected.

If the head is variable, the scale should be read at intervals of 12 hours, and the discharge of any interval be calculated by the

Plate IV. mean of the heads at the beginning and end of that interval. The discharge can be estimated by eq. (9),

$$Q = \frac{2}{3} clh \sqrt{2gh}, \text{ where } c = \cdot 62 \text{ for ordinary heads.}$$

Ex. 18. A rectangular notch is 1·5 ft. wide, and the head to still water is 0·64 ft. Find the discharge per second.

$$Q = \frac{2}{3} \times \cdot 62 \times 1\cdot 5 \times \cdot 64 \times 8 \times \cdot 8 = 2\cdot 54 \text{ c. ft. per sec.}$$

For greater accuracy, Francis formula, eq. (10), $Q = \frac{2}{3}c(L - 0\cdot 2H) H \sqrt{2gH}$ may be used. This would give for the above example

$$Q = \frac{2}{3} \times \cdot 62 (1\cdot 5 - 0\cdot 2 \times \cdot 64) \cdot 64 \times 8 \times \cdot 8 = 2\cdot 82 \text{ c. ft. per sec.}$$

44. Anicuts.—An anicut is a masonry dam built across a river so as to raise the water surface to a sufficient height in the dry season to admit of the water being carried by gravitation to places which it otherwise could not reach. The dam serves also to regulate the often capricious shiftings of the river in its course, and thus to conduct the water to the point of offtake. The anicut terminates at each end in wing walls to support the flood banks of the river. The supply of water required is taken off by a channel, on one or both banks, just above the anicut, the opening from the river being provided with a masonry *head sluice* to regulate the admission of the water. If the full supply level in the channel is to be maintained when little water comes down the river, the anicut crest should be at this level, or indeed slightly above it, since a small additional head is needed to force the supply through the head sluices, even when the shutters are fully open. All surplus water passes over the anicut. In ordinary seasons the surplus may discharge with a free overfall; but in floods the tail water rises above the crest of the anicut, and the water on the upper side becomes heaped up until there is a sufficient head to drive the river discharge through the contracted section. In both cases, the velocity of approach must be taken into account.

45. Anicut with clear overfall.—This is the case of free discharge from a rectangular notch, with velocity of approach, eq. (14). Taking $c = \cdot 577$, the co-efficient applicable to broad-crested weirs, we have

$$Q = 3\cdot 1l. \left\{ (h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}} \right\} \text{..................(22)}$$

where h_a, (fig. 33), is the head due to the velocity of approach.* The chief use of this formula is to check estimates of the discharge

* See note to art. 92.

of the river for measured values of h. The velocity of approach is less than the measured mean velocity of the river, owing to the increase of water section just above the anicut; but it can be determined from eq. (4), $v_a A_a = vA$. Plate IV.

Should h be required for a given height of anicut, and a known discharge, the increased water section above the anicut is unknown and we must proceed by approximation. First assume an approximate value for the velocity of approach, and solve for h. From the increased water section thus obtained, deduce a nearer value of the velocity of approach, and re-solve for h. In practice however, the silting up of the river bed above the anicut tends to prevent this increase of water section, and for most practical purposes the velocity of approach may be taken as equal to the mean velocity just below the anicut.

Ex. 19. A river 200 ft. wide and flowing 5 ft. deep with a mean velocity of 4 ft. per second, passes over an anicut whose height is 8 ft. above bed, with a clear overfall. Estimate the depth on the crest.

$Q = (200 \times 5)$ s. ft. $\times 4 = 4{,}000$ c. ft. per sec.

The increased section above the anicut is unknown. Assume it to be 8×200, or 1,600 s. ft. approximately.

The velocity of approach is approximately $\dfrac{5 \times 200}{8 \times 200} \times 4 = 2{\cdot}5$ ft. per sec.

$h_a = \dfrac{(2{\cdot}5)^2}{64} = {\cdot}1$; $\therefore (h_a)^{\frac{3}{2}} = {\cdot}03$. Inserting these values in eq. (22),

$$4000 = 3{\cdot}1 \times 200 \times \left\{ (h + {\cdot}1)^{\frac{3}{2}} - {\cdot}03 \right\}$$

$\therefore \log (h + {\cdot}1) = \dfrac{2}{3} \log 6{\cdot}525 = \log 3{\cdot}49$; $\therefore h = 3{\cdot}39$.

This result is sufficiently correct for practical purposes. If we assume that there has been no silting, the value of h may be corrected as follows:—

The velocity of approach is $\dfrac{5}{8 + 3{\cdot}39} \times 4 = 1{\cdot}8$.

$\therefore h_a = {\cdot}05$, and $(h_a)^{\frac{3}{2}} = {\cdot}01$.

$\therefore \log (h + {\cdot}05) = \dfrac{2}{3} \log 6{\cdot}505 = \log 3{\cdot}48$; $\therefore h = 3{\cdot}43$.

If there were no velocity of approach, the requisite head would be found to be 3·48 ft.

46. Drowned anicut.—This is the case of a submerged rectangular notch with velocity of approach. Let d, (fig. 34), be the depth of the tail water on the crest, h the actual head, h_a the head due to the velocity of approach, Q_1, Q_2 the discharges through h and d respectively.

$$\text{Then } Q_1 = \frac{2}{3} c_1 l \sqrt{2g} \left\{ (h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}} \right\}$$

$$Q_2 = c_2\, l d \sqrt{2g\,(h + h_a)}.$$

Plate IV. The values of the co-efficients are not well known; but there is no doubt that owing to the slope of the water surface in the notch portion, and to the comparative absence of contraction in the sluice portion, c_2 is considerably higher than c_1. For the reasons mentioned in art. 42 the co-efficients accepted for drowned tank weirs, viz., $c_1 = \cdot577$, $c_2 = \cdot8$ will be here adopted, and we have then

$$Q = l\left[3{\cdot}1\left\{(h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\} + 6{\cdot}4d\,(h + h_a)^{\frac{1}{2}}\right] \cdot (23)$$

If the anicut were absent, i.e., h indefinitely small compared with d, the co-efficient would obviously be 1. If there were a clear overfall, i.e., d indefinitely small compared with h, the co-efficient would be about ·577. Hence we are justified in concluding that the average co-efficient for the whole discharge $Q_1 + Q_2$ may vary between these limits, increasing as the ratio $h : d$ diminishes. This conclusion indicates the imperfections of the formula; nevertheless, it is believed that eq. (23) will give good results in ordinary cases.

The chief use of equation (23) is to determine h for a known maximum flood discharge. The maximum flood discharge is obtained by observation of the rainfall on the catchment basin, checked by measurements of the velocity and cross section of the river. The value of h deduced from the formula is added to the given maximum depth at which the river flows, and this enables us to fix the height of the wing walls, head sluices, and flood banks, so that there may be no risk of their being overtopped by the flood. In this case, the increased water section above the anicut is unknown, and h_a must be obtained by approximation.

Other questions can of course be solved by equation (23), such as (a) the amount of flood discharge for observed values of d and h; (b) the height to which an anicut must be built in order to raise the water by a given quantity when the river is flowing at a given depth. In this latter case we solve for d. The difference between d and the depth of the river gives the height of the anicut.

Ex. 20. The maximum flood discharge of a river is estimated at 5,000,000 c. yds. per hour, the mean velocity being 500 ft. per minute. An anicut is to be built across the river, 450 ft. long, with its crest $4\frac{1}{2}$ ft. above bed. What must be the height of the wing walls and head sluice, so that a maximum flood may not rise within three feet of their tops?

$$Q = l\left[3{\cdot}1\left\{(h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\} + 6{\cdot}4d\,(h + h_a)^{\frac{1}{2}}\right]$$

Here $Q = \dfrac{5000000 \times 27}{60 \times 60} = 37500$ c. ft. per sec.; $v = \dfrac{500}{60}$.

$A = \dfrac{Q}{v} = 4500$ s. ft. Mean depth $= \dfrac{A}{l} = 10$ ft.

$d = 10 - 4{\cdot}5 = 5{\cdot}5$ ft. Taking the velocity of approach equal to the mean velocity, $h_a = \dfrac{(8{\cdot}33)^2}{64} = 1{\cdot}08$; $\therefore (h_a)^{\frac{3}{2}} = 1{\cdot}12$.

Let $\sqrt{h + 1{\cdot}08} = x$; $37500 = 450\,[3{\cdot}1\,(x^3 - 1{\cdot}12) + 6{\cdot}4 \times 5{\cdot}5\,x]$. **Plate VI.**

$\therefore x^3 + 11{\cdot}3x = 28$. Try $x = 2$; $8 + 22{\cdot}6 = 30{\cdot}6$.

" $x = 1{\cdot}9$; $6{\cdot}86 + 21{\cdot}47 = 28{\cdot}3$.

$\therefore x = 1{\cdot}9$; $\therefore h + 1{\cdot}08 = (1{\cdot}9)^2 = 3{\cdot}61$; $\therefore h = 2{\cdot}53$.

$\therefore$ Top of wing walls must be $3 + 2{\cdot}53 + 5{\cdot}5$ or 11 ft. above anicut crest.

Ex. 21.—An anicut 1,500 ft. long is to be built across a river. The full supply depth in the channel, above the floor of the head sluice, is 7·0 ft., and the area of the sluice openings is such that a head of 6 inches is requisite to furnish the given supply. The normal water section of the river is 7,500 s. ft., and the estimated normal discharge 30,000 c. ft. per sec. What should be the height of the crest of the anicut above level of sluice floor, the latter being placed at the bed of the river?

Mean depth of river $= \dfrac{A}{l} = \dfrac{7500}{1500} = 5$ ft.

The water surface has to be raised through $(7{\cdot}0 - 5{\cdot}0) + 0{\cdot}5$, or 2·5 ft.

$$Q = l\left[\,3{\cdot}1\left\{(h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\} + 6{\cdot}4\,d\,(h + h_a)^{\frac{1}{2}}\right].$$

Here $Q = 30000$; $l = 1500$; $h = 2{\cdot}5$ ft. Taking the velocity of approach equal to the mean velocity, *i.e.*, equal to $\dfrac{Q}{A} = 4$, we have $h_a = \dfrac{(4)^2}{64} = {\cdot}25$.

$$30000 = 1500\left[\,3{\cdot}1\left\{(2{\cdot}75)^{\frac{3}{2}} - ({\cdot}25)^{\frac{3}{2}}\right\} + 6{\cdot}4\,d\,(2{\cdot}75)^{\frac{1}{2}}\right].$$

$\therefore 20 = 3{\cdot}1\,(4{\cdot}56 - {\cdot}13) + 6{\cdot}4\,d \times 1{\cdot}66$. $\therefore d = {\cdot}6$.

Hence height of crest above sluice floor is $5{\cdot}0 - 0{\cdot}6 = 4{\cdot}4$ ft.

If d had come out negative, it would indicate that the anicut crest must be above tail water. Equation (22) for a clear overfall should in this case be solved for h. Then $7{\cdot}5 - h$ will give height of anicut above sluice floor.

Ex. 22.—A jungle stream has a depth of 3 ft. and mean velocity of 12 ft. per second. What must be the height of an anicut to raise the water 6 ft., supposing that the bed silts up, above the anicut, so as to give a depth of 6 ft.?

It is evident from the figures given that the crest level will be above the tail water, *i.e.*, that the anicut has a clear overfall.

$$Q = \frac{2}{3}\,c\,l\sqrt{2g}\left\{(h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\}.$$

Here $Q = Av = 3l \times 12$; $v_a = \dfrac{3}{6} \times 12 = 6$ ft. per sec. $\therefore h_a = {\cdot}56$

$$3l \times 12 = \frac{2}{3\sqrt{3}}\,l \times 8\left\{(h + {\cdot}56)^{\frac{3}{2}} - ({\cdot}56)^{\frac{3}{2}}\right\}.$$

$\therefore 11{\cdot}69 = (h + {\cdot}56)^{\frac{3}{2}} - {\cdot}42$; $\therefore (h + {\cdot}56) = (12{\cdot}11)^{\frac{2}{3}} = 5{\cdot}27$.

$\therefore h = 4{\cdot}71$ ft. Height of anicut $= 6{\cdot}0 + 3{\cdot}0 - 4{\cdot}7 = 4{\cdot}3$ ft.

47. Sluices.—Sluices are constructed in many forms. The *head sluices* regulating the admission of water to channels, and the *under sluices* of anicuts, which serve to scour away the silt in front of the channel entrance, generally consist of rectangular openings, each from 3 to 6 feet in breadth, closed by vertical shutters sliding in grooves. The sluice openings or *vents* as they

Plate IV. are termed, are separated from one another by piers which are generally provided with cutwaters. The sluice floor is usually on the level of the bed of the river or channel; and the bottom and lateral contractions being thus suppressed to a great extent, the co-efficient may ordinarily be taken as ·8. The openings of river bridges, and the waterways in railway and other embankments constructed across tanks or in tracts liable to floods, may also be regarded as sluices, with the same or even a higher co-efficient.* In all these cases the discharge takes place into water, and the head is the difference of level between the water surfaces above and below the sluice.

Tank escape sluices are rectangular openings in tank weirs, to assist in the removal of flood water, and they are closed by vertical sliding shutters Being unprovided with piers and cutwaters, the co-efficient may be taken as ·62. The discharge from these sluices usually takes place freely into air.

Lock sluices will be referred to later, art. 58. They generally consist of culverts in the side walls of the lock chamber having a cross section considerably greater than that of their vents, so as to reduce velocity. The vents are closed by sliding shutters. The lower sluices sometimes consist of openings in the gates, closed in a similar manner. In either case the co-efficient may be taken as ·62.

Tank Irrigation sluices are culverts made through the bund, measuring about 2′ – 6″ × 2′ – 0″, and are rectangular in section with an arched top. The culvert communicates with the tank

(1) by a vent at the inner end which is closed by a shutter,

(2) by vertical openings in the masonry of the sluice closed by coned plugs fitting circular holes cut in horizontal stones which are built at different levels.

The area of the vent at the inner end is small compared with the section of the culvert in order that the velocity in the latter may not be too high Each plug is attached to a staff or *spear*, and can be lifted through fixed intervals, so as to partially or completely open its vent. The plug holes vary from 4 to 12 inches in diameter, and should be coned 1 in 4. When the tank is full, one or more of the uppermost plugs are lifted; as the water sinks, those lower down may be withdrawn; and at last the shutter can,

* In sluice and bridge openings provided with curved cutwaters and wing walls, a co-efficient of ·9 may be used, *vide* Professional Papers on Indian Engineering, 2nd series, vol. IX, Appold's experiments.

if necessary, be raised. The co-efficient for the shutter opening may be taken as 0·62; that for the plug holes 0·7. Plate V.

Ex. 23.—The following are levels in connexion with the head of a channel, the full supply of which is 600 c. ft. per sec.

Floor of sluice	43·26
Full supply level of channel (F. S. L.)	51·26
Anicut crest	51·56

Determine the number of vents each 6 ft. high and 4 ft. wide which will be required for the head sluice.

Let n be this number.

$Q = cA\sqrt{2gh}$. Here $Q = 600$; $c = \cdot 8$; $A = n \times 6 \times 4$; $h = 0\cdot 3$.

$\therefore 600 = \cdot 8 \times 24n \times 8\sqrt{0\cdot 3}$. Whence $n = 7$.

Ex. 24.—When there is 10 ft. of water flowing over the anicut in the preceding example, find to what height the shutters should be raised off the sluice sill.

Let x be this height. $A = 7 \times x \times 4$; $h = 10\cdot 3$.

$\therefore 600 = \cdot 8 \times 28x \times 8\sqrt{10\cdot 3}$; $\therefore x = 1\cdot 04$ ft.

Ex. 25.—A tank irrigation sluice is provided with tiers of plug holes, three holes in each tier. When the head on a tier is reduced to 4 ft., the water surface is sufficiently low to permit of the plugs of the next tier being withdrawn. What should be the diameter of the holes to irrigate 400 acres of rice at the rate of 1 c. ft. per sec. for every 30 acres.*

$Q = cA\sqrt{2gh}$ Here $Q = \frac{400}{30} = 13\cdot 3$; $c = 0\cdot 7$; $h = 4$.

$\therefore 13\cdot 3 = 0\cdot 7\, A \times 8 \times 2$; whence $A = 1\cdot 2$. If d be diameter of holes in feet, $A = 3\,\frac{\pi d^2}{4} = 1\cdot 2$; $\therefore d^2 = \cdot 51$; $\therefore d = \cdot 72$. Hence 9-inch holes will be required.

Ex. 26.—A surplus sluice in a tank weir consists of 8 vents each 4 ft. wide. When there is 9 ft. of water on the sluice sill, find the discharge per second if the shutters are lifted 5 ft., the discharge taking place into air.

$Q = \frac{2}{3}cl\sqrt{2g}\,(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}})$. Here $l = 8 \times 4$; $h_1 = 9$; $h_2 = 4$.

$\therefore Q = \frac{2}{3} \times \frac{5}{8} \times 32 \times 8\,(27 - 8) = 2{,}027$ c. ft. per sec.

48. Discharge of bridge openings.—If the shutters of a sluice are drawn up clear of the water surface, there will remain such a difference between the surface levels above and below the sluice as will suffice to give the actual velocity of discharge, whatever this may be. This is the case of waterways in railway or other embankments constructed across tanks, or in tracts liable to standing floods. The case of ordinary river bridge openings is

* Note.—For large areas of 5,000 acres and upwards the usual allowance is 1 c. ft. per second for every 60 acres.

Plate V. similar, except that it is complicated by a velocity of approach. This will now be considered.

Let v_a be the velocity of approach (fig. 35), v_1 the velocity at the cross section A_1 of greatest contraction, x the actual heading up or *afflux*. The total head producing the velocity v_1 is $(x + h_a)$;

$\therefore v = \sqrt{2g\,(x + h_a)}$.

$$Q = cA_1\sqrt{2g\,(x + h_a)} \;\ldots\quad\ldots\quad\ldots\quad\ldots\;(24)$$

To obtain h_a the head due to the velocity of approach, it must be remembered that this velocity is less than the normal velocity of the river, owing to the increase of water section above the bridge. If v, l, d be the normal velocity, breadth, and depth of the river, we have v_a, l, $(d + x)$ the corresponding quantities above the bridge, so that $v_a = \dfrac{d}{d + x}\,v$. Hence if x, v, and A_1 be observed, Q can be determined. It will be noted that A_1 is practically the actual area of the water section, there being no surface or bottom contraction, and, if cutwaters are present, but little lateral contraction. The co-efficient has therefore a high value, and may be taken equal to ·9.*

If x is the unknown quantity, we must proceed by approximation since h_a involves x. This case will be dealt with under "afflux," art. 49.

If there is no velocity of approach, we have

$$Q = cA_1\sqrt{2gx} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

Ex. 27.—The country on each side of a railway embankment, which crosses the drainage, is flooded. A waterway of 25 ft. discharges with depths of 6 and 4 ft. on the upper and lower sides respectively. Estimate the discharge.

$$Q = \cdot 9 \times (25 \times 4) \times 8\sqrt{2} = 1018 \text{ c. ft. per sec.}$$

Some authorities consider a bridge opening equivalent to a submerged weir, and they deal separately with the discharges through the sluice and notch portions of the area. Thus the formula $Q = cl\sqrt{2gx}\,(d + \frac{2}{3}x)$ is in use in the Madras Irrigation Department for waterways in embankments crossing tanks. As, however, observation shews that the lowest water level is ordinarily† reached under the bridge, it is clear that the whole discharge must take place through the smallest section A_1. The velocity through that section cannot be greater than that due to the actual head x, or, if there be a velocity of approach, to the head $x + h_a$. Hence the mode of calculation adopted in the text seems preferable to that now described, though very similar numerical results might be obtained by the suitable selection of co-efficients.

* Under favourable circumstances ·95 may be used.

† Except when the afflux is very great.

49. Afflux.—When a stream meets with an obstruction which contracts the normal cross section, the water becomes heaped up on the upper side of the obstruction, until the head or *afflux* is sufficient to drive the whole discharge through the contracted section. The obstruction may consist of a wall of limited height, extending across the river, as in the case of an anicut; or a series of lofty detached walls, with openings at intervals of the full depth of the stream, as in the case of bridge piers. The afflux can be determined by solving eq. (22) or (23) for h, or (24) or (25) for x. In the case of bridge openings however, we may proceed directly as follows:— Plate V

Let l, d be the mean width and depth below the bridge; l_1 the lineal waterway of the bridge; x the afflux (fig. 35).

Below the bridge the velocity is v, and the water section ld.

Above " " v_a " " $l(d+x)$.

Under the arches " v_1 " " cl_1d.

—where c is the co-efficient of contraction, which is usually taken as ·95.

The maximum velocity under the arches is due to the head $x + h_a$, *i.e.*,

$$v_1 = \sqrt{2g(x+h_a)}, \quad \therefore x = \frac{v_1^2 - v_a^2}{2g}$$

But $v_1 = \frac{l}{cl_1}v$, and $v_a = \frac{d}{d+x}v$.

$$\therefore x = \frac{v^2}{2g}\left\{\frac{l^2}{c^2l_1^2} - \frac{d^2}{(d+x)^2}\right\} \quad \text{.......................(26)}$$

a cubic equation for x which may be solved by trial or approximation. To approximate, assume the velocity of approach equal to the velocity below bridge, *i.e.*, neglect x on the right. We have then, substituting the value of c,

$$x = \frac{v^2}{2g}\left\{\frac{1{\cdot}1\, l^2}{l_1^2} - 1\right\} \quad \text{.......................(27)}$$

a result which is sufficiently accurate for most practical purposes. If a closer approximation is required, insert the value of x so found in (26) and recalculate. If l_1 is the unknown quantity, we get, by transposing (26).

$$l_1 = \frac{vl(d+x)}{c\sqrt{2gx\,(d+x)^2 + v^2d^2}} \quad \text{.......................(28)}$$

If there is no velocity of approach, $x = \frac{v_1^2}{2g} = \frac{v^2}{2g}\left(\frac{1{\cdot}1\ l^2}{l_1^2}\right)$

Plate V. **Ex. 28.**—**A bridge of seven arches of 20 ft. span is constructed across a river which has in flood a mean width of 200 ft., mean depth of 6 ft. and mean velocity of 5 ft. per second. Find the afflux.**

$$x = \frac{v^2}{2g}\left(\frac{1{\cdot}1\, l^2}{l_1^2} - 1\right) = \frac{25}{64}\left\{1{\cdot}1 \times \left(\frac{200}{140}\right)^2 - 1\right\} = 0{\cdot}48 \text{ ft.}$$

For a second approximation, we have from (26)

$$x = \frac{25}{64}\left\{1{\cdot}1 \times \left(\frac{200}{140}\right)^2 - \left(\frac{6}{6{\cdot}48}\right)^2\right\} = 0{\cdot}51 \text{ ft.}$$

50. Backwater.—If the water were stationary against an obstruction, the surface BC (fig. 36), would be horizontal, and the length of the *backwater*, or distance from the obstruction to which the effect of the afflux x can be perceived, would be x cosec i, where i is the surface fall. But if this were the case in a flowing stream, there would be no surface fall between C and B to develop velocity and overcome frictional resistance (see chap. vii). The velocity, and with it the resistance are, it is true, both reduced by the increase of section above the obstruction; still, some head is required, and the result is that the actual water surface DFB becomes curved, the curve touching the normal water surface at D, and the horizontal at B. Thus at any section EFG, the fall EG would be required to overcome the normal resistance, and produce the ordinary velocity in the length DG of the unobstructed stream. As it is, however, only the head EF is required. If the curve DFB were a circular arc, the length of the backwater would be $2x$ cosec i; but observation shews that $1{\cdot}5\,x$ cosec i furnishes a result sufficiently accurate for practical purposes, provided the river bed has a fairly uniform breadth and slope.

Ex. 29.—**A stream of uniform breadth has a normal depth of $2\frac{1}{2}$ ft., and a fall of 2 ft. per mile. Find the length of backwater caused by a weir which raises the surface by $3\frac{1}{2}$ ft.**

$$\text{Length required} = 1{\cdot}5\, x \operatorname{cosec} i = \frac{3}{2} \times \frac{7}{2} \times \frac{5280}{2} \text{ ft.} = 2{\cdot}6 \text{ miles.}$$

51. Separating weirs.—In cases of town supply, it may be desirable to divert the often discoloured flood water from the supply channel. When a moderate quantity of water only is discharging, it drops over the lip C (fig. 37) into a culvert D communicating with the supply channel. In floods, however, the increased velocity due to the greater depth causes the water to leap across the opening. Assuming that the velocity of all the filaments is their mean velocity, or $\frac{2}{3}\sqrt{2gh}$, which is sufficiently accurate for practical purposes, we have $x = \frac{2}{3}\sqrt{2gh}.\,t$; $y = \frac{gt^2}{2}$; $\therefore\ y = \frac{9}{16}\frac{x^2}{h}$. This gives y for any assigned values of x and h.

52. Modules.—In irrigation districts in India, the ryot is charged for water according to the area cultivated. In Europe however the water is sold by volume, and the *Module* is the apparatus employed to measure the quantity issued. The chief difficulty is to secure a constant supply with a variable head. Plate V.

Italian Module.—The water is admitted by a sluice, from the main channel into a masonry basin, and flows thence through a rectangular notch into the distributing channel. The sluice is regulated by hand, and an approximately constant head on the notch in the chamber can thus be secured. These modules, not being self acting, are imperfect.

Spanish Module.—The area of the orifice is made to vary as the head changes by suspending a conoidal plug, attached to a float, in a circular opening. The circular orifice is made in the horizontal floor of a masonry chamber B (fig. 38), constructed in the canal bank. A brass plug C is connected with a hollow brass float D, and works vertically in guides. The water falls into a masonry well E below the chamber, and is thence conducted through the bank to the distributing channel. If r be the assigned radius of the opening, and x the radius of the plug at any point, the area of the water space is $\pi (r^2 - x^2)$, so that $Q = c\pi (r^2 - x^2) \sqrt{2gh}$, whence successive values of x can be found for different values of h, and the plug can be designed. The chief objection to this form of module is the great fall required. This has been avoided in the Jamaica water works by placing the plug horizontally, and connecting it with the float by link work.*

EXAMPLES ON CHAPTER IV

1. A tank has a drainage area of 20 square miles. Estimate the maximum flood from the expression $Q = 450\ M^{\frac{3}{4}}$, and find the length of escape weir to discharge this maximum supply with a depth of $2\frac{1}{2}$ feet on the crest. (Coll. 1884.) *Ans.* (1) 4,256 c. ft. s., (2) 347 ft.

2. A tank has a drainage area of 45 square miles, and a weir 250 ft. in length with a clear overfall. Required the height of bund above escape crest in order to give a margin of 6 ft. above maximum flood level. $Q = 500\ M^{\frac{2}{3}}$. (Coll. 1885.) *Ans.* 10 ft.

3. A weir with a free overfall 100 ft. long discharges with a depth of 3 ft. To what extent will this depth be increased if the weir be shortened to 50 ft.? What length should it be if drowned by the tail water rising one foot above the crest, so that the total depth flowing over may remain unaltered? (Univ. 1882.) *Ans.* (1) 1·75 ft., (2) 91 ft.

* Pro. Inst. C. E., Vol. lxv, 1880-81.

4. The upper and lower water surfaces are 6 ft. and 2 ft. respectively above the crest of a submerged weir 100 ft. long; and the mean velocity of approach is 4 ft. per second. Calculate the discharge; and the depth required to pass the same quantity of water over an anicut of the same length with a free overfall without velocity of approach. (Univ. 1883.) *Ans.* (1) 5,312 c. ft. s., (2) 6·6 ft.

5. Find the height of water passing over a kalingula crest 400 feet long with a velocity of approach of 9 ft. per second, the discharge being 14,000 c. ft. per sec. (Coll. 1885.) *Ans.* 4·2 ft.

6. Describe fully the way in which the discharge of an irrigation channel about 9 ft. wide and 2 ft. deep should be measured when it is desirable to obtain a very accurate result. (Univ. 1875.)

7. A river 100 ft. wide and 10 ft. deep has a mean velocity of 4 ft. per second. Find the height of an anicut to raise the water 3 ft. *Ans.* 8 ft.

8. A river 300 ft. wide with vertical banks, and 10 ft. deep, has a mean velocity of 8 ft. per second. At what height above an anicut with a clear overfall would the whole be discharged? *Ans.* 8 ft.

9. The area of a certain locality draining into the Cheyar is 1,000 s. miles. An anicut 800 ft. long is constructed on this river; and the crest is 3 ft. below the flood surface level of the water as it was before the dam was built. The mean velocity of the current is 8 ft. per second. How much will the level of the surface of the water in the river be raised above crest of anicut? $Q = 560\ M^{\frac{3}{4}}$. (Coll. 1884.) *Ans.* 6·8 ft.

10. The maximum flood discharge of a river is 60,000 c. ft. per sec., which passes over a weir built under a bridge of 15 arches of 32 ft. span. The crest is 9 ft. above bed. Velocity of approach 8 ft. per sec. The gauge on the apron below the weir reads 15 ft. To what height will the flood rise on the crest? $c_1 = \cdot 50, c_2 = \cdot 75$. (Univ. 1892.) *Ans.* 11 ft.

11. A tank has two sluices irrigating 1,800 and 1,250 acres respectively. Find the width of each opening to give the required discharge with a depth of 4 ft. on the sills, the height of each being 1 foot, and the quantity of water needed being 1 c. ft. per second for 50 acres. (Coll. 1880.) *Ans.* (1) 3·8 ft., (2) 2·7 ft.

12. A tank has three sluices irrigating 500, 800, and 1,200 acres, their sills being respectively 18, 20 and 25 ft. below F.T.L. The

width of each opening being 1 ft., required the heights of the openings in order that they may discharge the necessary supply of 1 c. ft. per second for 60 acres when the water is 12 ft. below F. T. L. (Coll. 1882.) *Ans.* (1) 8·5 in., (2) 11·8 in., (3) 13·6 in.

13. Wishing to ascertain the rate of discharge from a tank sluice, I found the orifice a circular one 4 inches in diameter, and 20 ft. below the surface of the water. At what rate was the water escaping? (Univ. 1870) *Ans.* 2 c. ft. s.

14. Find the dimensions of a submerged rectangular sluice to discharge with a head of 9 inches a supply of water for irrigating 2,000 acres of land at the rate of 2 c. yards per hour per acre. (Univ. 1874.) *Ans.* 2 ft. by 3½ ft.

15. How many square feet of waterway must a head sluice have to supply 50,000 c. yds. per hour with a head of 9 inches? (Univ. 1878.) *Ans.* 68 s. ft.

16. A road is to be carried across a tank, and provision made to allow of the maximum discharge being passed through the road bank. The following are the data :—

Estimated discharge from drainage basin 2100 c. ft. per sec.

Top of bund	50·50	above M.S.L.
M. W. L.	47·00	
F. T. L.	45·00	
Ground level of floor of opening	40·00	

The water level above bridge opening is not to be more than 6 inches above M. W. L. in tank. Find length of waterway required? (Coll. 1884.) *Ans.* 59 ft.

17. A kalingula 200 yards long has 3 ft. of water passing over it with a velocity of approach of 8 ft. per second. There are 100 vents in it, each 3 ft. wide and 4 ft. high, with their tops coincident with the kalingula crest. Find the discharge when the sluices are all open, the discharge taking place into air. *Ans.* 27,642 c. ft. s.

18. Deduce a formula which gives the additional height to which the level of the water in a river will be raised when the piers of a new bridge curtail the sectional area between the banks. (Univ. 1881.)

19. Find the rise in surface level of a river 200 ft. broad, and having velocity of 5 miles per hour, which would be caused by the erection of a bridge of 4 spans, having piers 6 ft. in breadth? (Coll. 1885.) *Ans.* 3·3 in.

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20. A river 200 ft. broad with practically vertical banks has to be bridged. What waterway is necessary in order that the afflux may not exceed 6 inches? In flood the river is 10 ft. deep, and the mean velocity is ascertained to be 4 ft. per sec. with this depth. (Univ. 1889.) *Ans.* 124 ft.

21. A bridge is built across a river, and causes a heading up of 9 inches. The mean velocity below the bridge is 5 ft. per second, and the depth is 8 ft. Find the velocity with which the stream goes through the arches, and the proportion between width of river and waterway of bridge. (Coll. 1884.) *Ans.* (1) 8·3 ft. s., (2) 1·66 to 1.

22. How may a constant discharge of water be obtained from an irrigation sluice if the head is subject to occasional variation? (Univ. 1871.)

23. During the flood which destroyed the Kali Nadi aqueduct in 1885, the depths of water in the river on the upstream and downstream sides of the aqueduct were 37 ft. and 24 ft. respectively, thus giving an afflux of 13 ft. The lineal waterway was 275 ft., and the velocity of approach 3 ft. per sec. Estimate the flood discharge of the river. Co-efficient ·9. (Univ. 1890.) *Ans.* 172,260 c. ft. s.

24. The following are levels connected with a tank weir $62\frac{1}{2}$ ft. long:—

Top of bund	29·57
M. W. L.	28·32
F. T. L.	24·32
Bed of surplus channel	20·00

It is proposed to lengthen the weir so as to increase the margin between M. W. L. and top of bund to 3 ft. What length of weir must be added? (Univ. 1890.) *Ans.* $85\frac{1}{2}$ ft.

25. A tank has a catchment basin of 10 sq. miles. What length of weir will it require to carry off with a head of 2 ft. a rainfall of 4 inches in 24 hours, 50 per cent of which reaches the tank? (Univ. 1889.) *Ans.* 61 ft.

CHAPTER V

DISCHARGE UNDER VARIABLE HEAD

CONTENTS

53. We have hitherto supposed the head under which discharge takes place to be constant. If, however, a vessel of water does not receive a supply to compensate for its discharge through an orifice, the surface level falls, and the head diminishes gradually to zero. The vessel may be prismatic or otherwise, and the orifice may discharge to waste, or into another vessel. We shall confine ourselves mainly to prismatic vessels, and, for the present, to orifices which discharge to waste. Plate V.

54. **Free discharge from prismatic vessels.**—It has been explained (arts. 14, 26), that the theoretical velocity of efflux under any head or height is that which would be acquired by a particle falling freely through that height, or which must be given to a particle thrown upwards so as to just enable it to reach that height. Hence, as the water surface descends to the orifice, or ascends from the orifice the velocity of efflux varies with the head in precisely the same manner as does the velocity of the particle.

Let the velocity vary from 0 to v in the time t seconds. After 1, 2, 3,......t seconds the velocity will be g, $2g$, $3g$,......gt feet. Let BC (fig. 39), represent t seconds, CD represent gt ($= v$) feet. The velocity at any instant is evidently given by the corresponding ordinate of the triangle BCD. Hence the mean of all the velocities

Plate V. throughout the time t is $\frac{gt}{2} = \frac{v}{2} = \frac{\sqrt{2gh}}{2}$, or half the maximum velocity. Hence the discharge, when the head varies from h to 0 or from 0 to h is halt the discharge which would take place in the same time under a constant head h.

Since the mean velocity is $\frac{1}{2}\sqrt{2gh} = \sqrt{2g\frac{h}{4}}$, it appears that the mean head is $\frac{h}{4}$.

55. Time of emptying or filling.—Let S be the area of the water surface in the vessel, H its maximum depth above the orifice, t the time required for change of head from H to 0 or from 0 to H.

The mean discharge of the orifice per second is $cA\frac{1}{2}\sqrt{2gH}$.

∴ The whole discharge from the orifice is $cA.\frac{1}{2}\sqrt{2gH} \times t$.

But the whole discharge from the vessel is SH.

$$\therefore t = \frac{2SH}{cA\sqrt{2gH}} \quad \dots \quad \dots \quad \dots \quad \dots (29),$$

or the time is double that which would be required to discharge the same volume under a constant head H.

If the head diminishes from H to h or increases from h to H we have:—

Time from H to 0, or 0 to H is $\frac{2SH}{cA\sqrt{2gH}}$.

Time from h to 0, or 0 to h is $\frac{2Sh}{cA\sqrt{2gh}}$.

But the last mentioned interval is unexpended;

∴ the actual time is $\frac{2SH}{cA\sqrt{2gH}} - \frac{2Sh}{cA\sqrt{2gh}}$; or

$$t = \frac{2S}{cA\sqrt{2g}}(\sqrt{H} - \sqrt{h}) \quad \dots \quad \dots \quad \dots (30)$$

Ex. 30.—A cylindric vessel having a diameter of 5·747 inches has an orifice 0·2 inches in diameter, and the fluid surface is observed to sink from 16 inches to 12 inches in depth in 53 seconds. Find the co-efficient of discharge, taking $g = 32{\cdot}1948$.

$$c = \frac{2S}{t \times A\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right) = \frac{2 \times (5{\cdot}747)^2}{53 \times (0{\cdot}2)^2 \times 8{\cdot}024}\left(\sqrt{\frac{4}{3}} - 1\right)$$

$$= \frac{33{\cdot}028}{53 \times {\cdot}04 \times 4{\cdot}012}(1{\cdot}155 - 1) = 0{\cdot}60.$$

56. The subject of Arts. 54 and 55 can be dealt with as follows by the aid of the Calculus.

Let dx be the surface fall in the vessel in the time dt. The change of volume in the vessel is Sdx; and the discharge from the orifice is $cA\sqrt{2gx}.\,dt$. These are equal; $\therefore \frac{dt}{dx} = \frac{S}{cA\sqrt{2gx}}$ Plate V

$$\therefore t = \frac{S}{cA\sqrt{2g}}\int_h^H x^{-\frac{1}{2}}\,dx = \frac{2S}{cA\sqrt{2g}}(\sqrt{H} - \sqrt{h}).$$

57. Discharge during a given time.—Let t be the given time during which the head changes from H to h, and let either H or h be known. The volume discharged is S(H—h).

From eq. (30), $\sqrt{H} - \sqrt{h} = \frac{cA\sqrt{2g} \times t}{2S}$; whence either H or h can be determined if the other is known.

Ex. 31.—A square prismatic basin whose side is 3 ft. has an orifice ·09 ft. in diameter, 6 ft. below the surface. Find the discharge in $4\frac{1}{2}$ minutes, taking $c = \frac{5}{8}$.

$\sqrt{H} - \sqrt{h} = \frac{cA\sqrt{2g} \times t}{2S}$; Here $t = 270''$; H = 6 ft.; A = 0·0636 s. ft.; S = 9 s. ft.

$$\therefore \sqrt{6} - \sqrt{h} = \frac{\frac{5}{8} \times \cdot 00636 \times 8 \times 270}{2 \times 9} = \cdot 477.$$

$\therefore \sqrt{h} = 2{\cdot}449 - {\cdot}477 = 1{\cdot}972$; $\therefore h = 3{\cdot}89$.

Discharge required is S (H − h) = 9 (6 − 3·89) = 19 c. ft.

58. Canal locks.—A lock (see skeleton diagram, figs. 40), is a rectangular chamber of masonry constructed at the junction of two canal reaches B, C, which are at different levels, its object being the transfer of boats from one level to the other. The difference of level of the water surfaces in the two reaches is termed the *lift* of the lock. The lock chamber is closed at each end by a pair of stout gates D, E, and neither pair can be opened unless the water surface on each side of the pair is at the same level. When full, the lock can be emptied by means of sluices F placed in the lower gates below the water surface in the lower reach, or by culverts in the side walls. When empty, it can be filled by culverts which take off from the upper reach at G, and open into the sides of the lock at H either above or below the water level in the lower reach. The vents are closed by sliding shutters. The emptying or filling of the lock does not sensibly affect the levels in the canal reaches.

Suppose a boat is to be transferred from the lower to the upper reach. If the lock chamber is full, it must first be emptied by opening the sluices F in the lower gates. These sluices are then closed, the gates E are opened, the boat passes into the chamber and the gates are shut. The upper sluices are then opened, and the chamber is gradually filled. When it is full, the upper gates D are opened, and the boat passes into the upper reach.

Plate V. **59.** In designing locks, it is necessary to estimate the time which will be occupied in filling and emptying.

Let S be the area of the water surface in the lock; H the lift; h the depth from surface of upper reach to centre of discharging sluice of upper sluice. A_1, A_2 the areas of the upper and lower sluice openings.

(1) *To empty the lock.*—The sluice being submerged, the head varies from H to 0. Time required is, from eq. (29),

$$t = \frac{2SH}{cA_2 \sqrt{2gH}} \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

(2) *To fill the lock.*—From the level of the lower reach to centre of sluice opening the head is constant, *viz.* h.

The time to centre of sluice is therefore, eq. (6)

$$t_1 = \frac{S(H-h)}{cA_1 \sqrt{2gh}}.$$

From the centre of sluice opening to level of upper reach, the head varies from h to 0. The time for this portion is therefore, eq. (29), $t_2 = \frac{2Sh}{cA_1 \sqrt{2gh}}$. Hence the total time is

$$t = t_1 + t_2 = \frac{S(H+h)}{cA_1 \sqrt{2gh}} \quad \ldots \quad \ldots \quad (32)$$

60. Three standard sizes of lock chamber are used by the Madras Irrigation Department, *viz.* 150′ × 20′, 105′ × 15′ and 70′ × 10′. The sluices, whether constructed as culverts or gate valves, are closed by sliding shutters in the ordinary way; and the discharge may be considered as that through a thin plate, $c = \cdot 62$ *. The cross section of the side culverts is greater than that of their vents, so that the velocity may be reduced to safe limits.

Ex. 32. A lock 80 ft. by 15 ft., with a lift of 9 ft., is filled by two sluices, each 4 ft. wide and 2 ft. deep, whose centres are 6 ft. below upper reach water level; and emptied by two sluices, each 2 ft. square, whose centres are 4 ft. below lower reach water surface. How long will it take to pass a boat which arrives at the upper gate when the lock is empty, supposing 5 minutes are required to open and close the gates, and to haul the boat through? (Univ. 1882.)

Here S = 80 × 15 = 1200 s. ft.; H = 9; h = 6; A_1 = 16 s. ft.; A_2 = 8 s. ft.

Time of filling $= \frac{S(H+h)}{cA_1 \sqrt{2gh}} = \frac{1200 \times 15}{\frac{5}{8} \times 16 \times 8\sqrt{6}} = 92$ secs.

Time of emptying $= \frac{2SH}{cA_2 \sqrt{2gH}} = \frac{2 \times 1200 \times 9}{\frac{5}{8} \times 8 \times 8 \times 3} = 180$ secs.

Total time is 1*m*. 32*s*. + 5*m*. 0*s*. + 3*m*. 0*s*. = 9*m*. 32*s*.

* Experiments made by D'Aubuisson tended to shew that the discharge from two contiguous equal sluices is less than double the discharge from one. The co-efficient is accordingly sometimes assumed as low as ·55.

Ex. 33. A lock 150 ft. long and 16 ft. wide is filled by means of two sluices, each 3 ft. wide and 2 ft. deep, in the upper gate. The levels of the water in the upper and lower reaches, and the centres of the sluices are 12 ft., 5 ft. and 7 ft. respectively above the floor of the lock. How many cub. ft. of water will enter the lock in the third minute after the sluices are opened? (Univ. 1884.) **Plate V.**

Here $S = 150 \times 16$; $H = 7'$; $h = 5'$; $A = 12$ s. ft.

Let h_1, h_2 be heads at beginning and end of 3rd minute. Time to fill up to centre of sluices under constant head of 5 ft. is $\frac{S\,(H - h)}{cA\sqrt{2gh}} = \frac{(150 \times 16)(7 - 5)}{\frac{5}{8} \times 12 \times 8\sqrt{5}}$ = 35·8 secs.

∴ Interval from time at centre of sluice to commencement of 3rd min. is 120 − 35·8 = 84·2 secs.

Hence 84·2 secs. $= \frac{2S}{cA\sqrt{2g}}\left(\sqrt{h} - \sqrt{h_1}\right) = \frac{2 \times 150 \times 16}{\frac{5}{8} \times 12 \times 8}\left(\sqrt{5} - \sqrt{h_1}\right)$;

whence $\sqrt{h_1} = 1{\cdot}184$; ∴ $h_1 = 1{\cdot}392$.

Again, 60 secs. $= \frac{2S}{cA\sqrt{2g}}\left(\sqrt{h_1} - \sqrt{h_2}\right)$, whence $h_2 = {\cdot}188$.

Discharge required $= S\,(h_1 - h_2) = 150 \times 16 \times 1{\cdot}204 = 2890$ c. ft.

61. Discharge through a rectangular notch.—If the discharge takes place from a prismatic vessel through a rectangular notch, let t be interval during which the head diminishes from H to h, x be the head at any instant, dx the surface fall during time dt.

Change of volume in vessel is S. dx. Discharge from notch is $\frac{2}{3}cl\sqrt{2g}\,x^{\frac{3}{2}}\,dt$.

These are equal. ∴ $\frac{dt}{dx} = \frac{S}{\frac{2}{3}cl\sqrt{2g}\,x^{\frac{3}{2}}}$

$$\therefore t = \frac{3}{2}\frac{S}{cl\sqrt{2g}}\int_h^H x^{-\frac{3}{2}}\,dx = \frac{3S}{2cl\sqrt{2g}}(-2)\left\{\frac{1}{\sqrt{H}} - \frac{1}{\sqrt{h}}\right\};\ \text{or}$$

$$t = \frac{3S}{cl\sqrt{2g}}\left(\frac{1}{\sqrt{h}} - \frac{1}{\sqrt{H}}\right) \quad \cdots \quad (33)$$

Ex. 34.—A tank, the waterspread of which is one-fourth of a square mile, is provided with a kalingula 60 ft. long, which discharges with a maximum depth of 3 ft. on its crest. Supposing no water to enter the tank, find the time in which the surface will be lowered by 1 foot. (Univ. 1878.)

Here $S = \frac{5280 \times 5280}{4}$; $l = 60$; $H = 3$; $h = 2$; $c = \frac{1}{\sqrt{3}}$.

$t = \frac{3S}{cl\sqrt{2g}}\left(\frac{1}{\sqrt{h}} - \frac{1}{\sqrt{H}}\right) = \frac{3 \times 5280 \times 1320}{\sqrt{3} \times 60 \times 8}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)$ secs.

= 54·4 minutes.

62. Discharge from non-prismatic vessels.—If the discharging vessel is not prismatic, the ratio of the time of emptying under

Plate VI. variable head to the time under constant head is no longer 2. Thus, the ratio for wedge-shaped vessels is $1\frac{1}{3}$, that for pyramidal vessels $1\frac{1}{5}$, the base of the wedge or pyramid being the water surface.

To illustrate the mode of calculation adopted, the case of a paraboloid of revolution may be taken (fig. 41). Take the vertex as the origin.

Let l, r be co-ordinates at surface level.

l_1, r_1 " orifice "

x, y " any "

Let the surface descend through dx in time dt. Volume discharged in time dt is $\pi y^2 . dx$. But, since the head is $x - l_1$, the volume of discharge through the orifice of area A is $cA\sqrt{2g(x - l_1)}\, dt$. Now $y^2 = \frac{r^2}{l}x$.

$$\therefore \frac{dt}{dx} = \frac{\pi r^2}{cAl\sqrt{2g}} . \frac{x}{\sqrt{x - l_1}}; \therefore t = \frac{\pi r^2}{cAl\sqrt{2g}}\int_{l_1}^{l}\frac{x}{\sqrt{x - l_1}}dx.$$

$$\text{Let } \sqrt{x - l_1} = z;\ \frac{dx}{dz} = 2z. \therefore \int\frac{x}{\sqrt{x - l_1}}\,dx = \int\frac{z^2 + l_1}{z} . 2z . dz.$$

$$= 2\left(\frac{z^3}{3} + l_1 z\right) = \frac{2}{3}\left(x - l_1\right)^{\frac{3}{2}} + 2l_1\left(x - l_1\right)^{\frac{1}{2}}.$$

$$\text{Hence } t = \frac{\pi r^2}{cAl\sqrt{2g}}\left\{\frac{2}{3}\left(l - l_1\right)^{\frac{3}{2}} + 2l_1\left(l - l_1\right)^{\frac{1}{2}}\right\}.$$

If the orifice is at the vertex, $l_1 = 0$;

$$\therefore t = \frac{2}{3}\frac{\pi r^2\sqrt{l}}{cA\sqrt{2g}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (34).$$

Now, the volume of the paraboloid being $\frac{1}{2}\pi r^2 l$, the time of discharge under a constant head l would be $\frac{\frac{1}{2}\pi r^2 l}{cA\sqrt{2gl}}$. Comparing this with (34), we see that the ratio of the times is $\frac{2}{3} \div \frac{1}{2} = 1\frac{1}{3}$, or the same as that for wedge-shaped vessels.

63. Discharge from irregular basins.—The slopes of the basin should be contoured, when the reservoir is empty, for each foot or two feet of depth, and the area of the waterspread at each contour estimated. The discharge from any layer lying between consecutive contours may then be considered as approximately that from a prismatic vessel whose area is the mean of the two waterspreads bounding the layer.

Let s_0, s_1 ......s_4 (fig. 42), be the areas of the water spreads at successive contours of a tank;

h_0, h_1, ... h_4 the heads over the discharging orifice;

t_1, t_2,......t_4 the times of discharge from successive layers.

$$t_1 = 2\tfrac{1}{2}\frac{(S_0 + S_1)}{cA\sqrt{2g}}(\sqrt{h_0} - \sqrt{h_1});$$

$$t_2 = \frac{S_1 + S_2}{cA\sqrt{2g}}(\sqrt{h_1} - \sqrt{h_2})\text{; and so on.}$$

$$\therefore t = t_1 + t_2 + \ldots\ldots t_4 \quad \frac{1}{cA\sqrt{2g}}\Big\{S_0(\sqrt{h_0} - \sqrt{h_1}) + S_1(\sqrt{h_0} - \sqrt{h_2})$$

$$+ S_2(\sqrt{h_1} - \sqrt{h_3}) + S_3(\sqrt{h_2} - \sqrt{h_4})$$

$$+ S_4(\sqrt{h_3} - \sqrt{h_4})\Big\} \ldots \quad \ldots (35).$$

Ex. 35.—Find in what time a reservoir of the form given below would lose 6 ft. of depth by discharging through a culvert in the dam, the area of the vent being 1 s. ft., and the co-efficient ·5;—

A_0 = 600,000 s. ft.		h_0 = 20 0 ft.	
A_1 = 495,000	,,	h_1 = 18·5	,,
A_2 = 410,000	,,	h_2 = 17·0	,,
A_3 = 325,000	,,	h_3 = 15·5	,,
A_4 = 265,000	,,	h_4 = 14 0	,,

Here
$S_0(\sqrt{h_0} - \sqrt{h_1})$ = 600000 (4 472 – 4·301) = 102600
$S_1(\sqrt{h_0} - \sqrt{h_2})$ = 495000 (4·472 – 4·123) = 172755
$S_2(\sqrt{h_1} - \sqrt{h_3})$ = 410000 (4·301 – 3·936) = 149650
$S_3(\sqrt{h_2} - \sqrt{h_4})$ = 325000 (4·123 – 3·742) = 123825
$S_4(\sqrt{h_3} - \sqrt{h_4})$ = 265000 (3·936 – 3·742) = 51410

600240

$$t = \frac{600240}{\cdot 5 \times 1 \times 8} = 150060 \text{ secs.} = 41{\cdot}7 \text{ hours.}$$

64. Notch discharge from irregular basins.—The time of discharge can be estimated by dividing the basin into thin horizontal layers, and successively applying equation (33). With the notation of art. 63, the student can readily obtain the following result:—

$$t = \frac{3}{2cl\sqrt{2g}}\Big\{S_0\Big(\frac{1}{\sqrt{h_1}} - \frac{1}{\sqrt{h_0}}\Big) + S_1\Big(\frac{1}{\sqrt{h_2}} - \frac{1}{\sqrt{h_0}}\Big) + S_2\Big(\frac{1}{\sqrt{h_3}} - \frac{1}{\sqrt{h_1}}\Big)$$

$$+ S_3\Big(\frac{1}{\sqrt{h_4}} - \frac{1}{\sqrt{h_2}}\Big) + S_4\Big(\frac{1}{\sqrt{h_4}} - \frac{1}{\sqrt{h_3}}\Big)\Big\} \quad \ldots \quad \ldots (36).$$

65. Discharge from one prismatic vessel into another.—In this case, as the surface falls in one vessel, it rises in the other, and the effective head, or difference of level between the two surfaces, diminishes more rapidly than in the case of free discharge from one vessel.

Let S_1, S_2 be the water surfaces in the vessels B, C respectively (fig. 43). Let the heads at any instant be H, h. It is required to late VI. find the time from that instant until the water has a common surface in the two vessels. Let x be the depth from the original level in B to the common surface. The outflow from B equals the inflow to C, *i.e.*

$$S_1 x = S_2 (H - h - x); \quad \therefore x = \frac{S_2}{S_1 + S_2}(H - h).$$

Hence the whole discharge during the time t under a head diminishing gradually from $(H-h)$ to 0 is $S_1 x = \frac{S_1 S_2}{S_1 + S_2}(H - h)$.

But this discharge is half that which would have taken place in the same time had the head remained constant at $(H-h)$, *viz.*, $\frac{1}{2} \times cA\sqrt{2g(H-h)} \times t$, where A is the area of the orifice.

$$\therefore t = \frac{2S_1S_2\sqrt{H-h}}{cA\sqrt{2g}\,(S_1 + S_2)} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (37)$$

It will be seen from this expression that the time is the same whichever be the discharging vessel. Eq. (37) is useful in dealing with double locks. If the vessels are connected by a pipe, take c from Art. 21.

Ex. 36.—A rectangular wrought iron tank (fig. 44), 7 ft. deep, is divided into two parts by a thin vertical partition. The larger part, which is full of water, has a horizontal area of 213 s. ft.; the other part, which is empty, an area of 27 s. ft. If a rectangular orifice 12 inches wide, and 6 inches deep, with its bottom 2 ft. above the bottom of the tank, be opened in the partition, how many seconds will elapse before the water stands at the same level in both parts? (Univ. 1881.)

The discharge, until the water surface in the smaller vessel rises to centre of the orifice is the free discharge from a prismatic vessel. That from orifice centre to a common surface is the discharge from one prismatic vessel into another.

Let t_1, t_2 be the times of these discharges respectively.

If H be the head above orifice in the larger vessel at end of time t_1, the outflow from large vessel during t_1 is $213(4\frac{3}{4} - H)$. The inflow to small vessel is $27 \times 2\frac{1}{4}$; whence $H = 4.465$ ft.

During time t_1 the head diminishes from 4·750 to 4·465;

$$\therefore t_1 = \frac{2S_1}{cA\sqrt{2g}}\left\{\sqrt{4.750} - \sqrt{4.465}\right\} = \frac{2 \times 213}{\frac{5}{8} \times \frac{1}{2} \times 8}(2.1794 - 2.1131) = 11.30 \text{ secs.}$$

$$t_2 = \frac{2S_1S_2\sqrt{H-0}}{cA\sqrt{2g}\,(S_1+S_2)} = \frac{2 \times 213 \times 27\sqrt{4.465}}{\frac{5}{8} \times \frac{1}{2} \times 8 \times 240} = 40.51 \text{ secs.}$$

Total time $t = t_1 + t_2 = 51.81$ secs.

66. If the head diminishes from $(H - h)$ to $(y - z)$, we have (fig. 45),

Time from $(H - h)$ to 0 is $\dfrac{2S_1S_2\sqrt{H-h}}{cA\sqrt{2g}\,(S_1+S_2)}$

Time from $(y - z)$ to 0 is $\frac{2S_1 S_2 \sqrt{y-z}}{cA\sqrt{2g}\,(S_1 + S_2)}$

$\therefore$ Time from $(H-h)$ to $(y-z)$ is $\frac{2S_1S_2}{cA\sqrt{2g}\,(S_1+S_2)}\left(\sqrt{H-h} - \sqrt{y-z}\right)$.

But $S_1(H-y) = S_2(z-h)$; $\therefore z = h + \frac{S_1}{S_2}(H-y)$.

$\therefore$ Time from $(H-h)$ to $(y-z) = t =$

$$\frac{2S_1S_2}{cA\sqrt{2g}\,(S_1+S_2)}\left\{\sqrt{H-h} - \sqrt{y-h-\frac{S_1}{S_2}(H-y)}\right\}.$$

$$\therefore t = \frac{2S_1\sqrt{S_2}}{cA\sqrt{2g}\,(S_1+S_2)}\left\{\sqrt{S_2(H-h)} - \sqrt{(S_1+S_2)\,y - S_1H - S_2h}\right\} \quad ..(88).$$

67. These results may be obtained directly as follows :—

In the time dt, the fall of surface level in the larger vessel is dy;

$\therefore$ Change of volume is $S_1.dy$.

Discharge through orifice in time dt is $cA\sqrt{2g\,(y-z)}.\,dt$.

$$\therefore \frac{dt}{dy} = \frac{S_1}{cA\sqrt{2g\,(y-z)}}. \quad \text{But } z = h + \frac{S_1}{S_2}(H-y).$$

$$\therefore \frac{dt}{dy} = \frac{S_1\sqrt{S_2}}{cA\sqrt{2g}\,(S_1+S_2)} \times \frac{1}{\sqrt{\left\{y - \frac{S_1H + S_2h}{S_1+S_2}\right\}}} = p\,(y-q)^{-\frac{1}{2}} \text{ suppose.}$$

$$\therefore t = p\int_y^H (y-q)^{-\frac{1}{2}}\,dy = 2p\left\{\sqrt{H-q} - \sqrt{y-q}\right\}.$$

$$\therefore t = \frac{2S_1\sqrt{S_2}}{cA\sqrt{2g}\,(S_1+S_2)}\left\{\sqrt{S_2(H-h)} - \sqrt{(S_1+S_2)\,y - S_1H - S_2h}\right\}.$$

If the time is required to the instant when the two surfaces are on the same level, we have $y = z = h + \frac{S_1}{S_2}(H-y)$.

$$\therefore y = \frac{S_1H + S_2h}{S_1+S_2}; \text{ and } t = \frac{2S_1S_2\sqrt{H-h}}{cA\sqrt{2g}\,(S_1+S_2)}.$$

EXAMPLES ON CHAPTER V

1. Find the area of a submerged sluice to empty a lock chamber 120 ft. long, 20 ft. broad, and with 10 ft. lift, in 5 minutes. (Coll. 1880.) *Ans.* 10 s. ft.

2. Required the time of filling a lock 85 ft. long, 15 ft. wide, and 10 ft. lift, by two submerged sluices, each 2 ft. square. (Coll. 1882.) *Ans.* 3 m. 21 s.

3. A lock 189′ × 20′, lift 12′ is filled by two culverts on each side with simple openings 3′ × 3′, and also by two valves 2′ × 2′ in each of the upper gates. The floors of the culverts are 6′ below upper water surface, and the sills of the gate openings are 6″ higher than those floors. If the gate valves are opened one minute before the culverts, how long will it take to fill the lock? (Univ. 1892.) *Ans.* 2 m. 35 s.

4. A canal lock is filled by two sluices, each two feet square, the sills of which are 1 ft. above the floor of the lock, and 12 ft. below the surface of the water in the upper reach. If the water rises from a depth of 6 ft. to the level of full lock in 4 minutes, what is the area of the lock? (Coll. 1886.) *Ans.* 1810 s. ft.

5. A canal lock is 81 ft. long, $7\frac{3}{4}$ ft. wide, and the lift is 7 ft. The water enters the lock chamber through a culvert 2 ft. in diameter, the top of the orifice of which is just on a level with the water in the lower reach of the canal. Find the time of filling the lock, taking the co-efficient of discharge as unity. (Coll. 1883.) *Ans.* 2 m. 12 s.

6. In what time can a lock 200 ft. long and 20 ft. wide be filled by two sluices, each 3 ft. square, in the gate, the water in the lock, the water in the upper reach, and the bottoms of the sluices being respectively 4 ft., 12 ft., and 6 inches above the floor of the chamber? (Univ. 1875.) *Ans.* 4 m. 12 s.

7. (*a*) A canal lock 200 ft. long and 30 ft. wide is being emptied by two sluices, each 3 ft. wide and 2 ft. high, the lower sides of which are 2 ft from the bottom of lock. In what time will the depth of water be reduced from 9 ft. to 8 ft.? The depth of water in the lower reach is 4 ft. *Ans.* 48 s.

(*b*) If H be the head at the beginning of the time, and h the head at the end, shew that the depth would be equally reduced in the same time if the discharge took place under a constant head $K = \left(\frac{\sqrt{H}+\sqrt{h}}{2}\right)^2$ (Univ. 1876.)

8. A lock 150 ft. long and 16 ft. wide is emptied by two sluices in the lower gate each 2 ft. deep, with their centres 3 ft. above the floor of the lock. The levels of the water in the upper and lower reaches are 12 ft. and 5 ft. respectively above the floor of the lock. What must be the width of the sluices so that the depth of the water in the chamber may be reduced from 12 ft. to 6 ft. in $2\frac{1}{2}$ minutes? (Univ. 1884.) *Ans.* 2·6 ft.

9. A cylindrical vessel 5·74 inches in diameter has an orifice ·2 inches in diameter at a depth of 16 inches below the water surface. It is found that the water sinks 4 inches in 51 seconds. What is the co-efficient of discharge? *Ans.* ·606.

10. The water in a reservoir 600 sq. ft. in horizontal section sinks 4 ft. in 1 hour. The head at starting was 25 ft. Find the side of the square discharging orifice whose co-efficient is ·62. *Ans.* 2 in.

11. A cylindrical cistern, whose horizontal section is 6 ft. in diameter, communicates with a second cistern 3 ft. in diameter by a submerged pipe 1 inch in diameter. When the pipe is opened the water in the smaller cistern is 4 ft. above that in the larger one. In what time will the surfaces become level? $c = ·75$. *Ans.* 11 m. 31 s.

12. One cistern discharges water into another through a submerged pipe having a section of 4 s. inches. The discharging cistern is 6 ft. square, and the receiving cistern 2 ft. square. If the initial difference of level be 9 ft., how long will it take for the water surfaces to reach the same level? $c = ·7$. *Ans.* 2 m. 19 s.

13. Two docks with vertical walls have superficial areas of 10 acres and 6 acres, and communicate with each other by gates containing two sluices, each 4 ft. square, with their sills at bed level. When the water in the larger dock has a depth of 29 ft., and in the smaller a depth of 4 ft., the shutters are opened. After what interval will the water attain the same level in both docks, and what will then be the depth? (Univ. 1891.) *Ans.* (1) 2 hrs. 50 m., (2) 19·635 ft.

14. A lock 150 ft. by 20 ft., with a lift of 10 ft. is filled by two sluices, each 4 ft. deep by $2\frac{1}{2}$ ft. wide, whose centres are 6 ft. below the water level of the upper reach; and is emptied by two submerged sluices of the same size. Find the time required to fill and empty the lock. (Univ. 1889.) *Ans.* (1) 3 m. 16 s., (2) 3 m. 10 s.

CHAPTER VI

FLOW OF WATER IN PIPES

CONTENTS

68. Fluid friction.—When a stream of water enters a pipe or channel which has a fixed inclination or *slope*, it is observed that whatever that slope may be, the velocity very soon becomes uniform, shewing that the force due to gravity is exactly balanced by the resistance to motion offered by the boundaries of the stream, and that the amount of resistance depends on the velocity. The nature of the resistance, which it is convenient to call frictional, will be understood from the fact that the roughness of the boundaries sets up eddies in the stream, whereby the fluid filaments cross one another, and their velocities in the channel line are checked. The velocity of filaments near the boundaries is less than the velocity of those near the centre of the water section. The mean velocity of all the filaments is however uniform, and the fluid may be supposed to flow in plane layers lying parallel to the cross section of the stream.

The laws of friction between a liquid and a solid surface are the following:—

I. *The frictional resistance varies with the nature of the solid surface, but is independent of the pressure.*

II. *For large surfaces, it is proportional to the areas of the surfaces.*

III. *The frictional resistance varies, for ordinary velocities, nearly as the square of the velocity. At very low velocities, not exceeding* 1 *inch per second, it varies nearly as the velocity.* Plate VI

Let a be the area of the surface of contact, k the resistance in lb. at a velocity of 1 ft per sec., R the resistance at a velocity of v ft. per sec. Then, at ordinary velocities, by the above laws, $R = k.a.v^2$. If $\mu = \frac{2gk}{w}$, we have

$$R = \mu wa\frac{v^2}{2g} \quad \ldots \quad (39)$$

where μ is called the *co-efficient of friction*.—Its values (which do not differ widely from those of k), are determined by experiment. Thus, for well painted iron, $\mu = \cdot 0019$; for a varnished surface, $\mu = \cdot 0026$.

69. Velocity in pipes.—Let a be the inclination of the pipe to the horizon; h the vertical fall in feet in length l; A the area of the water section, B its wetted border; and let the pressure be uniform throughout the length of the pipe. The frictional resistance varies as the surface, and as the square of the velocity, i.e. $R = kBlv^2$, where k is some constant. The work done by the quantity of water Al falling through the height h is $wAlh$. The resistance is overcome through the length l. The work done on the resistance is $R.l = kBl^2v^2$. These must be equal, $\therefore \frac{kv^2}{w} = \frac{A}{B}.\frac{h}{l}$, $\therefore \frac{2gk}{w}.\frac{v^2}{2g} = \frac{A}{B}.\frac{h}{l}$, or if we write μ for $\frac{2gk}{w}$, we have $\mu\frac{v^2}{2g} = \frac{A}{B}.\frac{h}{l}$, where μ is the co-efficient of friction, which is to be determined by experiment. The ratio $\frac{A}{B}$ is termed the *hydraulic mean depth* (H.M.D.) or *hydraulic mean radius* (H.M.R.), because, if the wetted border were rolled out flat, and the stream spread over it, $\frac{A}{B}$ is the depth which the water would have throughout. The hydraulic mean radius is usually symbolized as r. The ratio $\frac{h}{l}$ is the sine of the slope, and is denoted by s. Hence

$$\mu\frac{v^2}{2g} = rs \quad \ldots \quad (40)$$

70. Virtual slope.—Transposing (40) we have $h = \mu.\frac{l}{r}.\frac{v^2}{2g}$ as the head required to overcome resistance in the pipe. A further head h_1 is required to produce the velocity v, and to overcome contraction at the entrance to the pipe. Let CD (fig. 46), be a pipe from a reservoir, discharging into air. The total head over the pipe exit is EG. Let EF represent h_1; then FG is the head h required to overcome resistance. Join FD. Since the resistance head h is, by eq. (40), proportional to l, the ordinate KLM of the triangle FDG

Plate VI. represents the head required at any point L of the pipe to overcome resistance in the portion LD. Hence, if a vertical pipe were introduced at L, the water would rise in it to the level K; and the pressure in the pipe at that point would be w.KL. The line FD is called the *virtual slope* or *hydraulic gradient* of the pipe. If the pipe were laid along FD, it would give the same velocity and discharge, but the water would flow without pressure throughout. Similarly the pipe may occupy any line whatever, straight or curved, between D and the reservoir, provided that line lies wholly below the virtual slope FD. If however the pipe line NOD rises above the virtual slope, the pipe will be in the condition of a siphon (Art. 8), and should run full provided that OP does not exceed 34 ft. In practice however, air becomes disengaged from the water, and tends to collect at O; for the pressure along the virtual slope being the atmospheric pressure, the pressure at O must be less than this. The pipe, therefore, will not run full. The mode of dealing with this case will be explained later (Art. 76).

Since KM is the head required to overcome resistance in the length LD, KQ must be the head required to overcome resistance in LC. Now RQ is the head required to produce velocity. Hence, if at any point L of a pipe the total loss of head RK in the length CL be set off from ER, a point K on the virtual slope is determined; and any further interval between K and the pipe represents the pressure head at L. If the end D were closed by a valve, the water would rise in the vertical pipe to R, and the whole head above L would be utilized in producing pressure at L.

If the pipe has to deliver under pressure, as is generally the case in town supply, set off the head DS corresponding to that pressure, and the virtual slope will be FS. For an effective fire service in towns, DS should be from 50 to 75 ft.

In long pipes, the head EF bears so small a proportion to the total head that it may be neglected; and we have then only to solve the equations $\mu\frac{v^2}{2g} = rs$; $Q = Av$, to obtain the velocity and discharge. The case of short pipes will be treated later (Art. 74). It must be borne in mind that the slope s in the above expression is the virtual slope, which is not necessarily the inclination of the pipe.

71. Velocity and virtual slope.—The result (40) may be obtained as follows, taking the pressure into account. Consider a length of pipe $C_1C_2 = l$ (fig. 47), and suppose that in the time t, the volume C_1C_2 occupies the position D_1D_2. Let A be the area of the cross section of the pipe, Q the discharge per second.

Let p_1, p_2 be the pressures at C_1, C_2; z_1, z_2 the heights of these points above datum. The water enters and leaves the portion of pipe considered with the same velocity v, so that there is no change of kinetic energy. Hence the energy due to gravity *plus* the energy due to pressure = the energy expended in overcoming resistance. The transfer of the volume C_1C_2 to the position D_1D_2 is equivalent to the transfer of C_1D_1 to C_2D_2, *i.e.*, a weight $wA\,(C_1D_1) = w.Qt$ falls through a height $z_1 - z_2$. Plate VI.

$\therefore$ Energy due to gravity $= w.Qt\,(z_1 - z_2)$.

Energy due to pressure $= p_1A\,(C_1D_1) - p_2A\,(C_2D_2) = (p_1 - p_2)\,Qt$.

Resistance of surface $B.l$ is, by eq. (39), $= \mu w B l \dfrac{v^2}{2g}$.

Energy of resistance $= \mu w B l \dfrac{v^2}{2g}\,(C_1D_1) = \mu w B l \dfrac{v^2}{2g} vt$.

Hence $w.\,Qt\,(z_1 - z_2) + (p_1 - p_2)\,Qt = \mu w B l \dfrac{v^2}{2g} \cdot \dfrac{Q}{A} t$.

$$\therefore z_1 - z_2 + \frac{p_1}{w} - \frac{p_2}{w} = \mu \frac{B}{A} l \frac{v^2}{2g}.$$

But $\left(\dfrac{p_1}{w} + z_1\right) - \left(\dfrac{p_2}{w} + z_2\right)$ is the surface fall h.

$$\therefore \mu \frac{v^2}{2g} = \frac{A}{B} \cdot \frac{h}{l} = rs.$$

72. Co-efficient of friction.—The co-efficient of friction μ is not constant for a particular nature of surface, but varies with the velocity. Hence many writers, as D'Aubuisson, Eytelwein, and Prony, have proposed the form $\mu = a + \dfrac{b}{v}$. The experiments of Darcy at Paris shew that, as regards pipes which have been some time in use, the nature of the original surface does not much affect the value of μ, which depends mainly on the velocity. Now the velocity varies as $\sqrt{rs}$, and Darcy found that, for practical purposes, the co-efficient may be expressed in terms of the H.M R., or of the diameter of the pipe, thus ;—$\mu = a\left(1 + \dfrac{\beta}{d}\right)$, where d is the diameter of the pipe in feet, $\beta = \cdot 084 = \dfrac{1}{12}$ nearly, $a = \cdot 005$ for new iron pipes, or $\cdot 01$ for pipes which have been some time in use.

Hence, for new pipes, $\qquad \mu = \cdot 005 \left(1 + \dfrac{1}{12d}\right) \quad \ldots \textbf{(41)}$

for old pipes, $\qquad \mu = \cdot 01 \left(1 + \dfrac{1}{12d}\right) \quad \ldots \textbf{(42)}$

These values are applicable only to ordinary velocities exceeding 4 inches per second, *vide* Art. 68.

73. Velocity and discharge.—From eq. (40), we have

$$v = \sqrt{\frac{2g}{\mu}}\,\sqrt{rs} = c\,\sqrt{rs}.$$

Plate VII. If d be the diameter of the pipe in feet, the H.M.R. is

$$\frac{\pi d^2}{4} \div \pi d = \frac{d}{4},$$

$$\therefore v = \frac{c}{2}\sqrt{ds}\text{; and } Q = \frac{\pi d^2}{4}.v$$

The values of c for different sizes of pipe can readily be obtained from eq. (41) or eq. (42). Thus :—

Diameter of Pipe.			Values of c.	
			New pipes.	Old pipes.
$\frac{1}{2}$-inch	$d = \frac{1}{24}$	...	65	46
1-inch	$d = \frac{1}{12}$	...	80	56
3-inch	$d = \frac{1}{4}$	...	98	70
6-inch	$d = \frac{1}{2}$	...	105	74
12-inch	$d = 1$	...	109	77
24-inch	$d = 2$	...	111	78
36-inch	$d = 3$	...	112	79

The co-efficients are exhibited graphically in Plate VII.

For rough or trial calculations regarding water mains c may be taken as 78.

We have then for old pipes (since provision must be made, when designing, for the pipe to furnish the required discharge after it has been some time in use),

$$v = 39\sqrt{d.s} \quad \dots \quad \dots \quad \dots \quad \dots \quad (43)$$

$$Q = \frac{\pi d^2}{4}.v \quad \dots \quad \dots \quad \dots \quad \dots \quad (44)$$

From (43) and (44), we have $Q = \frac{22 \times 39}{7 \times 4} d^2 \sqrt{ds}$; and

$$d = \cdot 2545 \sqrt[5]{\frac{Q^2}{s}} \quad \dots \quad \dots \quad \dots \quad \dots \quad (45)$$

From these equations, if any two of the quantities d, s, v, Q be given, the other two can be determined.

To solve for a new pipe, eq. (43) becomes $v = 55\sqrt{d.s}$, whence $d = \cdot 2220 \sqrt[5]{\frac{Q^2}{s}}$; so that the effect of doubling the co-efficient of friction μ is to increase the diameter of the pipe requisite for a given discharge by only about 13 per cent.

The following are the maximum velocities permissible in main and distribution pipes. Plate VII.

Diameter in inches ...	4	8	12	15	24	36
Velocity in ft. per sec. ...	2·5	3·0	3·5	4·0	5·5	6·5

Ex. 37.—(*a*) What will be the discharge from a pipe 4 ft. in diameter, and one mile long, having a fall of 1 in 5280, and a head of 11 ft. over the centre of its inlet orifice?

(*b*) To what extent must this head be increased in order to double the discharge?

(*c*) How many pipes 1 ft. in diameter would give the same discharge as the 4 ft. one? (Univ. 1883.)

(*a*) $v = 39\sqrt{ds} = 39\sqrt{4 \times \frac{1}{440}} = \frac{39}{\sqrt{110}}$.

$$Q = \frac{\pi d^2}{4}.v = \frac{22}{7} \times \frac{16}{4} \times \frac{39}{\sqrt{110}} = \frac{490\cdot 3}{\sqrt{110}} = 49 \text{ c. ft. per sec.}$$

When the pipe is new, the discharge will be $\frac{55}{39} \times 49 = 69$ c. ft. per sec.

(*b*) The discharge is proportional to the velocity, and the head varies as the square of the velocity. Hence to double the discharge, the head must be quadrupled.

(*c*) Q varies as $d^{\frac{5}{2}}$. Let Q_1 be the discharge of a 1 ft. pipe. Then $Q_1 = \left(\frac{1}{4}\right)^{\frac{5}{2}} Q = \frac{1}{32}$ Q, so that 32 12-inch pipes would be required.

Ex. 38.—Find the diameter of a pipe 12,100 ft. long with 9 ft. head over its lower end, required to deliver 10 gals. per head in 6 hours to a population of 400,000. (Univ. 1886.)

$$Q = \frac{400{,}000 \times 10 \times \frac{1}{6\frac{1}{4}}}{6 \times 60 \times 60} = \frac{800}{27} \text{ c. ft.}; \; s = \frac{9}{12100}.$$

$$d = \cdot 2545 \sqrt[5]{\left(\frac{800}{27}\right)^2 \times \frac{12100}{9}} = 4\cdot 17 \text{ ft. or } 50 \text{ inches.}$$

A new pipe giving the required discharge would have a diameter of

$$\frac{2220}{2545} \times 50 = 44 \text{ inches.}$$

If greater accuracy be desired, or if the pipes be small, we must use the expressions

$$v = \sqrt{\frac{2g}{\mu} rs} = \sqrt{\frac{g}{2\mu}}\sqrt{ds} \quad .. \quad .. \quad .. \quad .. \quad (46)$$

$$\mu = \cdot 01\left(1 \times \frac{1}{12d}\right), \text{ or } \mu = \cdot 005\left(1 + \frac{1}{12d}\right) \quad .. \quad (47)$$

$$Q = \frac{\pi d^2}{4}.v \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (48)$$

Plate VI If d and s, d and v, d and Q, or v and s be given, the other two quantities can be at once obtained. If however s and Q are given, which is the usual practical problem in pipe design, d must be determined by approximation. The approximate equation $d = ·2545\sqrt[5]{\frac{Q^2}{s}}$ will, on being solved for d, furnish a sufficiently near value of μ. Then $Q = \frac{\pi}{4}d^2\sqrt{\frac{g}{2\mu}.d.s}$, whence $d^5 = \frac{Q^2\mu}{\pi^2 s}$

Ex 39.—Take Ex. 37 (a).—$\mu = ·01\left(1 + \frac{1}{48}\right) = ·0102$.

$$Q = \frac{22 \times 16}{7 \times 4}\sqrt{\frac{32}{·0204} \times 4 \times \frac{1}{440}} = 47·46 \text{ c. ft. per sec.}$$

Ex. 40.—Take Ex. 38.—By the approximate formula, we get $d = 4·17$;

$$\therefore \mu = ·01\left(1 + \frac{1}{50}\right) = ·0102.$$

$$d^5 = \frac{Q^2\mu}{\pi^2 s} = \left(\frac{800}{27}\right)^2 \times ·0102 \times \left(\frac{7}{22}\right)^2 \times \frac{12100}{9}; \text{ whence } d = 4·14 \text{ ft.}$$

Ex. 41.—A 4-inch branch main (fig. 48), laid along a street, supplies water to each house by means of a ¾-inch lead service pipe. The highest point of one of these service pipes, 72 ft. long, is 34 ft. above the main. If the pressure in the main is 15½ lbs. per square inch, what delivery in gallons per minute may be expected from the top o the service pipe? How many houses would the main supply?

Head in branch main is $\frac{p}{w} = \frac{15\frac{1}{2} \times 144}{62\frac{1}{2}} = 36$ ft.

Virtual slope of service pipe is $\frac{2}{72} = \frac{1}{36}$

$$d = \frac{3}{4} \text{ in.} = \frac{1}{16} \text{ ft.} \quad \mu = ·01\left(1 \times \frac{1}{12}\right) = ·023. \; c = \sqrt{\frac{2g}{\mu}} = \frac{8}{·15} = 53.$$

$$Q = \frac{\pi d^2}{4} c \sqrt{\frac{d}{4}.s} = \frac{22}{28} \times \frac{1}{(16)^2} \times \frac{53}{2}\sqrt{\frac{1}{16} \times \frac{1}{36}}.$$

Whence discharge per minute = ·203 c. ft. = 1¼ gals. nearly. Number of houses supplied, i.e., number of service pipes ¾-inch in diameter which will give approximately the same discharge as a main 4-inches in diameter, is $(4 + \frac{3}{4})^{\frac{5}{2}} = 65$.

74. Short pipes.—In short pipes, the head required to produce velocity and to overcome the contraction at entry must be taken into account With the usual cylindric entry (Art. 20) $c = ·82$ and $c_c = 1$ $\therefore c_v = ·82$. Hence if v be the actual velocity in the pipe, and h' the head required to produce this velocity and to overcome the resistance of entry, $v = ·82\sqrt{2gh'}$, $\therefore h' = 1·5\frac{v^2}{2g}$.

The head required to overcome resistance is, from eq. (40), $h = \mu.\frac{4l}{d}\frac{v^2}{2g}$. Hence $H = h + h' = \left(1·5 + \frac{\mu.4l}{d}\right)\frac{v^2}{2g}$, and

$$v = \sqrt{\frac{2g.Hd}{1·5d + 4\mu l}} \quad \ldots \quad \ldots \quad \ldots \quad (49)$$

The actual velocity may be calculated from this expression; or we may proceed thus:—The velocity, and therefore the discharge is proportional to $\sqrt{H}$. To find the velocity v in a given pipe, assume the velocity to be u, estimate the heads required to produce u and to overcome resistance, and sum these heads. Then Plate V.

$$\frac{v}{u} = \sqrt{\left\{\frac{\text{actual head}}{\text{estimated head}}\right\}}.$$

Ex. 42.—Find the discharge of a 12-inch pipe, 15 ft. long, with a head of 4 ft.

Assuming the velocity to be 10 ft. per sec.,

$$(10)^2 = (39)^2\, ds = 1521\frac{h}{15};\ \therefore h = \frac{1500}{1521} = 0{\cdot}99$$

$$h' = (1{\cdot}5)\frac{v^2}{2g} = \frac{3}{2} \times \frac{100}{64} \qquad = 2{\cdot}34$$

$\therefore$ Total head $= 3{\cdot}33$ ft.

But the actual head is 4 ft.; $\therefore$ actual velocity $= 10\sqrt{\frac{4}{3{\cdot}33}} = 10{\cdot}95$ ft. per sec.

$$Q = \frac{\pi d^2}{4}.\, v = \frac{22}{7} \times \frac{1}{4} \times 10{\cdot}95 = 8{\cdot}6 \text{ c. ft. per sec.}$$

Observe that in this example the head required to produce the velocity is $\frac{2{\cdot}34}{0{\cdot}99}$ or 2·4 times the head required to overcome the resistance.

If all resistances were neglected, the theoretic discharge from the pipe under a head of 4 ft. would be (Art. 14), $\frac{\pi d^2}{4}\sqrt{2gh} = 12{\cdot}5$ c. ft. per sec. Considering the pipe, whose length is 15 diameters, as a simple orifice, the co-efficient of discharge is $c = \frac{8{\cdot}6}{12{\cdot}5} = 0{\cdot}7$ nearly. Compare this with Art. 21.

It will be seen from eq. (49) that if the pipe is a long one, the first term of the denominator is very small compared with the second, and may be neglected. Since the velocity in water pipes generally ranges from about 2 to 5 ft. per second, the greatest head which can ordinarily be utilized to produce velocity is $1{\cdot}5\frac{(5)^2}{2g} = 0{\cdot}6$ ft. nearly. This head is, in a long pipe, small compared with the total head.

If the entry is bell-mouthed, the co-efficient of discharge for the orifice may be as high as ·97, so that $h' = 1{\cdot}08\frac{v^2}{2g}$.

Plate VI. 75. Siphon sluice.—This consists of a bent iron tube CDEF (fig. 49), by which water can be discharged over a tank bund or canal bank. This type of sluice is in use at the Nagpur Waterworks, and it was proposed for the Periar project* to carry off the supply water during the construction of the masonry dam. A siphon tube upwards of 6 ft. in diameter was designed for this purpose. It will readily be seen from fig. 49, that the effective head is the difference of water levels in the discharging and receiving basins. Suppose a depth of 34 ft. of water to be substituted at C and F for the air pressures at those points, the siphon becomes a submerged orifice, and the effective head is the difference between the imaginary water surfaces, which is equal to the difference h between the actual water surfaces. It is obvious from this that the velocity and discharge are unaffected by variations in the height CD of the bend above the upper water surface, provided always this height is less than 34 ft. The siphon can be put in action either by exhausting the air or by filling with water. When in action, the air disengaged from the flowing water tends to collect in the bend, and an air vessel must therefore be provided in order that the full discharge may be maintained.

The use of siphon surplus weirs has been proposed for tanks and channels. Directly the water rises above the lower margin of the bend, the siphon discharges as a simple weir with a head due to the depth of water on that margin. When the water reaches the upper margin of the bend, the siphon acts as such, the head and therefore the discharging power being limited only by the length of the outer branch.

The term *siphon sluice* is sometimes applied to a culvert, having the bend downwards, and carrying water under the bed of a channel. This is not a true siphon.

Ex. 43.—Estimate the discharge of a siphon $3\frac{1}{2}$ ft. in diameter and 240 ft. in length, the difference of water levels being 12 ft.

Assume the velocity to be 10 ft. per sec. Then $(10)^2 = (39)^2 ds = 1521 \times 3\frac{1}{2} \times \frac{h}{240}$.

$$\therefore \text{ Resistance head } h = \frac{100 \times 240 \times 2}{1521 \times 7} = 4{\cdot}51 \text{ ft.}$$

$$\text{Velocity head } h' = 1{\cdot}5 \times \frac{v^2}{2g} = \frac{3}{2} \times \frac{100}{64} = 2{\cdot}34 \text{ ft.}$$

$$\therefore \text{ Total head} = 6{\cdot}85 \text{ ft.}$$

But actual total head is 12 ft. $\therefore$ Actual velocity $= 10\sqrt{\frac{12}{6{\cdot}85}} = 13{\cdot}23$ ft. per sec.

$$Q = Av = \frac{\pi}{4}(3\tfrac{1}{2})^2 \times 13{\cdot}23 = 127 \text{ c. ft. per sec.}$$

* Selections from the Records of the Government of India, P.W.D., No. CCXV, 1886.

76. Inclination of pipes.—In practice, pipes must follow the section of the ground in which they are laid, and must therefore be in segments laid at various slopes. Suppose a given discharge is required at the end of the pipe. If the diameter of the pipe is uniform throughout, the virtual slope is practically a straight line, whatever be the slopes of the pipe segments; for the resistance heads are proportional to the lengths, i.e., very nearly proportional to the horizontal projections of those lengths. If however the segments have not all the same diameter, each has its own virtual slope; for the discharge varies as d^2v, i.e., as $d^2\sqrt{ds}$, $\therefore$ s varies as $\frac{1}{d^5}$ if the discharge is constant. Hence with a pipe whose segments are of given length and diameter, it is possible to determine the virtual slope of each segment, so as to give a constant discharge, the pipe running full throughout. If each segment begins and terminates on its own virtual slope, the intermediate portion may either coincide with the slope or be below it. On the other hand, if the actual pipe line is fixed, the line may be divided into segments, and the diameter of each segment determined so that the hydraulic gradient of each may commence and terminate on the pipe line. The latter case is the ordinary practical problem.

Let CDEF (fig. 50), be a pipe line following the ground section. The virtual slope for the whole pipe, supposing it to be of uniform diameter, would be GF; but the point D rises above this slope, so that air would collect here, and the pipe would not run full. Hence the diameter of the portion CD must be calculated for the slope GD. For the remainder of the pipe, DF may be taken as the virtual slope, since the whole of the portion DEF is below this line, and the diameter be calculated for this slope. Or, the portion DEF may be divided into two or more segments as DE, EF, and the requisite diameters calculated for the slopes DE, EF respectively. When a diameter has been fixed for any slope, that for any other slope can readily be found, for

$$Q \propto d^2\sqrt{ds}, \text{ and } Q \propto d_1^2\sqrt{d_1 s_1}$$

$$\therefore \frac{s_1}{s} = \left(\frac{d}{d_1}\right)^5 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (50)$$

Ex. 44.—A pipe 2 ft. in diameter has a fall of 1 in 1000 for half a mile, after which it falls at the rate of 1 in 250 for a quarter of a mile. If the level of the water in the supply cistern stands at 5·13 ft. above the centre of the pipe at the upper end, what will be the discharge per minute? (Univ. 1881.)

Let CDE (fig. 51), be the pipe line. The heads over the points C, D, E are 5·13, 7·77 and 13·05 ft. The point D rises above the mean gradient FE. Hence the gradient FD regulates the discharge of the whole pipe.

Plate VII.

$$s = \frac{7 \cdot 77}{2640};\ Q = \frac{\pi d^2}{4} \times 39\sqrt{ds} = \frac{22}{7} \times 39\sqrt{\frac{7 \cdot 77 \times 2}{2640}} = 9 \cdot 4 \text{ c. ft. per sec.}$$

∴ Discharge per minute = 564 c. ft.

The segment DE does not run full, and might advantageously be made of smaller diameter, so that its virtual slope may be the same as its actual slope, viz., $\frac{5 \cdot 28}{1320}$

$$\frac{d}{2} = \left(\frac{7 \cdot 77}{10 \cdot 56}\right)^{\frac{1}{5}};\ \therefore d = 1 \cdot 88 \text{ ft., or say 22 inches.}$$

Ex. 45.—A pipe is laid from a service reservoir along the ground, with a fall of 42·8 ft. in the first mile, and 23·5 ft. in the second mile. The head over the centre of the inlet end of the pipe is 10 ft. What diameter should be given to the pipes in each mile, in order that the discharge may be 236 c. ft. per minute? What would be the pressure per square inch at the end of the pipe when discharging freely, and when stopped by a plug? (Univ. 1882.)

In this case, the whole of the pipe lies below the gradient FE (fig. 52), and might therefore be of uniform diameter throughout. The question however requires that the diameters shall be different in the first and second miles.

The virtual slopes will be FD and DE, i.e. $\frac{52 \cdot 8}{5280}$ and $\frac{23 \cdot 5}{5280}$.

For CD, we have $d = \cdot 2545\sqrt[5]{\frac{Q^2}{s}}$; where $Q = 3 \cdot 933$, $s = \frac{1}{100}$.

$$d = \cdot 2545\,(39 \cdot 33)^{\frac{2}{5}} = 1 \cdot 11 \text{ ft. Say a 14-inch pipe.}$$

For DE, we have $\left(\frac{d_1}{1 \cdot 11}\right)^5 = \frac{52 \cdot 8}{23 \cdot 5}$; ∴ $d_1 = 1 \cdot 30$. Say a 16-inch pipe.

If the pipe is discharging freely, its end E is on the virtual slope, and the pressure is therefore (neglecting the atmospheric pressure, *vide* Art. 7), zero. If the end E is stopped by a plug, the whole of the head 76·3 ft. produces pressure, and the pressure per square inch is $\frac{wh}{144} = \frac{62 \cdot 5 \times 76 \cdot 3}{144} = 33 \cdot 1$ lbs.

77. Minor Losses of head.—Small losses of head are caused by sharp elbows or curved bends in the pipe, and by sudden enlargements or contractions.

Elbows.—The loss of head at elbows is due to a contraction in the stream (fig. 53). If ϕ be the angle formed by the bent portion of the pipe with the prolongation of the original portion, an empiric expression for the loss of head

$$h = (\tfrac{1}{2}\sin.^2\phi)\frac{v^2}{2g}.$$

Bends.—The loss of head at bends (fig. 54), is due to a similar cause. Weisbach's empiric expression for the loss of head is $h = \left\{0 \cdot 13 - 1 \cdot 85\left(\frac{d}{2\rho}\right)^{\frac{7}{2}}\right\}\frac{v^2}{2g}$, where $\frac{d}{2\rho}$ is the ratio of the radius of the pipe to that of the bend.

Enlargements.—When a sudden enlargement occurs in a pipe, eddies are produced which dissipate energy, and cause loss of head. If v, v_1 be the velocities in the small and large portions of the pipe (fig. 55), every particle of weight w, moving with velocity v, impinges on a body of water of weight w_1, moving with velocity v_1. The fluid being incompressible and therefore inelastic, the velocity after impact is $u = \frac{wv + w_1v_1}{w + w_1}$. Plate VIII.

Energy before impact $= w\frac{v^2}{2g} + w_1\frac{v_1^2}{2g}$.

" after " $= (w + w_1)\frac{u^2}{2g}$.

$\therefore$ Loss of energy $= w\frac{v^2}{2g} + w_1\frac{v_1^2}{2g} - (w + w_1)\frac{u^2}{2g} = \frac{ww_1}{w + w_1}\frac{(v - v_1)^2}{2g}$.

Now w is indefinitely small compared with w_1, $\therefore$ Loss of energy $= w_1\frac{(v - v_1)^2}{2g}$

$\therefore$ Loss of energy per c. ft. of water $= w\frac{(v - v_1)^2}{2g}$, and loss of head $h = \frac{(v - v_1)^2}{2g}$. Hence the loss of head is the head due to the relative velocity.

Let d, d_1 be the diameters of the smaller and larger portions of the pipe.

$\frac{v}{v_1} = \left(\frac{d_1}{d}\right)^2$; $\therefore$ Loss of head $h = \frac{v_1^2}{2g}\left\{\left(\frac{d_1}{d}\right)^2 - 1\right\}^2$.

If for instance $d_1 = 2d$, we have $h = 9\frac{v_1^2}{2g}$.

Contractions.—A contraction of the stream occurs within the constricted section of the pipe as shown in fig. 56. If A be the area of the smaller portion of the pipe, the smallest stream section is c_cA, and the velocity at this section is $\frac{A}{c_cA}v = \frac{v}{c_c}$. Hence the head lost by shock is $\frac{1}{2g}\left\{\left(\frac{v}{c_c}\right) - v\right\}^2$ or $\left(\frac{1}{c_c} - 1\right)^2\frac{v^2}{2g}$. The value of the co efficient of contraction appears in this case to be about ·6, so that the loss of head $h = {\cdot}44\frac{v^2}{2g}$.

Ex. 46.—A line of pipes 4,000 ft. long is to be laid from a service reservoir. The fall in the first half of the length is 20 ft., that in the second half 15 ft., and the minimum head of water in the reservoir over the orifice is 5 ft. There are 3 horizontal elbows of 30°, 4 of 40°, and 4 of 50°. The discharge required is 300 c. ft. per minute. Find the head required to generate the velocity of entry, and to overcome the resistance of the elbows.

We must first find an approximate value for the velocity.

$s = \frac{40}{4000} = \frac{1}{100}$. Q = 5 c. ft. per sec. $d = {\cdot}2545\sqrt[5]{\frac{Q^2}{s}} = 1{\cdot}22$.

$v = 39\sqrt{\frac{1{\cdot}22}{4} \times \frac{1}{100}} = 2{\cdot}15$ ft. per sec.: or say 2 ft.

Loss of head for velocity of entry $= 1{\cdot}5\frac{v^2}{2g}$.

" " elbows $= \frac{1}{2}\frac{v^2}{2g}\left\{3\sin^2 30° + 4\sin^2 40° + 4\sin^2 50°\right\}$

$\therefore$ Total loss $= \frac{4}{64 \times 2}(3 + {\cdot}75 + 1{\cdot}64 + 2{\cdot}44) = {\cdot}25$ ft., or 3 inches.

78. Branched main supplying two or more service reservoirs.—Let the pipe CD (fig. 57), from the main reservoir C, supply the service reservoirs E,F by the branch pipes DE, DF. Let Q_1, Q_2 be the discharges required at E and F; then $Q = Q_1 + Q_2$ is the discharge of the pipe CD. The height of water in a pressure column at D must be below C in order to secure flow in CD, and above E to secure flow in DE. Within these limits, assume any convenient head DK. Then the virtual slopes of the three pipes are CK, KE, KF, and the discharges $Q_1 + Q_2$, Q_1, Q_2 are known, whence the diameters can be calculated.

Ex. 47.—The depths of the points D, E, and F (fig. 57), below water surface of main reservoir are 18 ft., 10 ft., and 25 ft. The lengths of the segments are 800 yds., 160 yds., and 250 yds. A discharge of 60 gals. per min. is required at E, and 120 gals. at F. Design the pipes.

$Q_1 = \cdot 16$ c. ft. per sec.; $Q_2 = \cdot 32$; $\therefore Q = \cdot 48$.

Assume DK = 15 ft. Then $s = \frac{5}{900}$, $s_1 = \frac{5}{480}$, $s_2 = \frac{20}{750}$.

$d = \cdot 2545 \sqrt[5]{\frac{Q^2}{s}} = \cdot 2545 \sqrt[5]{41 \cdot 5} = \cdot 536$ i.e. a $6\frac{1}{2}$-inch pipe.

$d_1 = \cdot 2545 \sqrt[5]{2 \cdot 45} = \cdot 304$ i.e. a 4-inch pipe.

$d_2 = \cdot 2545 \sqrt[5]{3 \cdot 84} = \cdot 328$ i.e. a 4-inch pipe.

79. Pipes not running full.—If a pipe does not run full, a condition which is possible only when the pipe lies on its own virtual slope, the H.M.R. is not $\frac{d}{4}$, but must be expressed in general terms as $\frac{A}{B}$, where A is the area of the water section, and B the wetted arc. The discharge varies as $A\sqrt{\frac{A}{B}}$, i.e., as $\sqrt{\frac{A^3}{B}}$. Now, it will readily be seen that as the water level descends from E towards CD (fig. 58), the arc diminishes at a more rapid rate than the area does; in fact, up to a certain limit, B diminishes faster than A^3 does. It can be shewn that the maximum discharge is given when the angle COD is about 54°.

Ex. 48.—The discharge from a full pipe 20″ in diameter is 597 c. ft. per minute; find the discharge when the depth of water is 19 inches. (Univ. 1880.)

Let r be the radius of the pipe; A, B the area and border when the pipe runs full; A_1, B_1 similar quantities when the pipe runs partly full.

$A = \pi r^2 = 3 \cdot 14 \times 100 = 314$ s. in. $B = 2\pi r = 20 \times 3 \cdot 14 = 62 \cdot 8$ in.

If $\angle$ COE $= \theta$, arc OED $= r . 2\theta$; $\therefore B_1 = 2r(\pi - \theta)$

The sector COD $= \theta r^2$, and the triangle COD $= r^2 \sin\theta\cos\theta$.

$\therefore$ Segment CED $= r^2(\theta - \sin\theta\cos\theta)$, $\therefore A_1 = r^2(\pi - \theta + \sin\theta\cos\theta)$.

In present case, OF = 9″, OD = 10″; $\therefore \cos\theta = \cdot 9 = \cos 25° 50'$.

$\therefore \theta$ (in radians) $= \frac{25\frac{5}{6}}{180} \times 3{\cdot}1416 = {\cdot}451$; $\therefore \pi - \theta = 2{\cdot}690$. Plate VIII

$B_1 = 2r(\pi - \theta) = 20 \times 2{\cdot}690 = 53{\cdot}8$.

$A_1 = r^2(\pi - \theta + \sin\theta\cos\theta) = 100\left(2{\cdot}690 + \frac{\sin 51^\circ 40'}{2}\right) = 308{\cdot}0$

Now $\frac{Q_1}{597} = \sqrt{\frac{A_1^3}{B_1}} \div \sqrt{\frac{A^3}{B}}$; $\therefore Q_1 = 597\left(\frac{308}{314}\right)^{\frac{3}{2}}\left(\frac{62{\cdot}8}{53{\cdot}8}\right)^{\frac{1}{2}} = 627$ c.ft. per min.

To find the maximum value of θ, we have $\frac{A^3}{B}$ a maximum; or if, $\pi - \theta = \theta_1$,

$y = \frac{(\theta_1 - \sin\theta_1\cos\theta_1)^3}{\theta_1}$ is a maximum.

$$\frac{dy}{d\theta_1} = \frac{\theta_1 \times 3(\theta_1 - \sin\theta_1\cos\theta_1)^2(1 - \cos^2\theta_1 + \sin^2\theta_1) - (\theta_1 - \sin\theta_1\cos\theta_1)^3}{\theta_1^2} = 0.$$

$\therefore (\theta_1 - \sin\theta_1\cos\theta_1)^2 \{6\theta_1\sin^2\theta_1 - \theta_1 + \sin\theta_1\cos\theta_1\} = 0$. If $2\theta_1 = \phi$,

$3\phi\frac{1 - \cos\phi}{2} - \frac{\phi}{2} + \frac{\sin\phi}{2} = 0$; $\therefore 2\phi - 3\phi\cos\phi + \sin\phi = 0$.

Whence, by trial, $\phi = 306^\circ$; $\therefore 2\theta = 54^\circ$.

80. **Dupuit's Equation.**—In making approximate calculations regarding a long main constructed in segments of different lengths, diameters, and slopes, it is sometimes convenient to determine the length of the equivalent uniform main of given diameter which would have the same total resistance head for a given discharge.

Let $l_1, l_2, \ldots$ be the lengths of the segments, $d_1, d_2, \ldots$ their diameters, $s_1, s_2, \ldots$ their slopes, $v_1, v_2, \ldots$ their velocities, l, d, s, v, the length, diameter, slope, and velocity of the equivalent uniform main.

Resistance head in uniform main $h = sl = \mu\frac{4l}{d}\cdot\frac{v^2}{2g}$.

" " segments $h = s_1 l_1 + s_2 l_2 + \ldots\ldots$

$$= \mu\frac{4l_1}{d_1}\cdot\frac{v_1^2}{2g} + \mu\frac{4l_2}{d_2}\cdot\frac{v_2^2}{2g} + \ldots\ldots$$

$\therefore \frac{l}{d}\cdot v^2 = \frac{l_1}{d_1}v_1^2 + \frac{l_2}{d_2}v_2^2 + \ldots\ldots$ But $v_1 = \frac{d^2}{d_1^2}v$; $v_2 = \frac{d^2}{d_2^2}$ ..

$\therefore \frac{l}{d}v^2 = \frac{l_1}{d_1}\cdot\frac{d^4}{d_1^4}v^2 + \frac{l_2}{d_2}\cdot\frac{d^4}{d_2^4}v^2 + \ldots$

$$\therefore l = l_1\left(\frac{d}{d_1}\right)^5 + l_2\left(\frac{d}{d_2}\right)^5 + \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (81)$$

81. **Jets.**—When water issues from a small orifice under pressure, it forms a jet, as in the case of an ornamental fountain, or a fire-engine. In order that a jet, supplied by a pipe, may ascend to the greatest height, the pipe adjutage should be of a form which gives a high co-efficient of velocity. A conically convergent adjutage is the form of nozzle usually applied to the conducting pipe, and there should obviously be no sudden diminution of diameter at the junction of the pipe and nozzle.

Let v be the velocity of efflux, v_1 the velocity in the conducting pipe, d the diameter of the nozzle, d_1 the diameter of the pipe, l_1 its length. Then $v_1 = \left(\frac{d}{d_1}\right)^2 v$

The head required to produce the actual velocity of discharge is $\frac{v^2}{2g}$. The head required to overcome resistance in the pipe is $\mu\left(\frac{4l_1}{d_1}\frac{v_1^2}{2g}\right)$. Neglecting any small losses of head due to bends, or to the resistances of the orifices of entry and discharge,

Total head $H = \frac{v^2}{2g}\left\{1 + \mu\frac{4l_1}{d_1}\left(\frac{d}{d_1}\right)^4\right\}$. Height of jet is

$$h = \frac{v^2}{2g} = \frac{H}{1 + \mu\frac{4l_1}{d_1}\left(\frac{d}{d_1}\right)^4} \quad \dots \quad \dots \quad (52)$$

It is obvious from this expression that, to obtain a great height of jet, d_1 should be large as compared with d. The equation (52) requires correction for the resistance of the air; and the actual height of the jet may be taken, according to Weisbach, as $h(1 - \cdot 003h^2)$.

Ex. 49.—A 2-inch pipe for a fountain is 350 ft. long. If the available head is 30 ft., find the height to which a half-inch jet from a well formed coned nozzle will ascend.

Here $H = 30$ ft., $l_1 = 350$ ft., $d_1 = \frac{1}{6}$ ft., $d = \frac{1}{24}$ ft., $\mu = \cdot 01\left(1 + \frac{1}{12d_1}\right) = \cdot 015$.

$h = \frac{30\,(4)^4}{(4)^4 + \cdot 015 \times 4 \times 350 \times 6} = 20$ ft. Actual height $= 20 - \cdot 003\,(20)^2 = 18\cdot 8$ ft.

EXAMPLES ON CHAPTER VI.

1. What is the hourly discharge in gallons of a pipe 4 ft. in diameter running full with a fall of 1 ft. per mile? (Coll. 1884.) *Ans.* 304,500.

2. A town of 200,000 inhabitants is to be supplied with water from a reservoir 1 mile distant, and it is stipulated that one-half of the daily supply of 30 gals. per head should be delivered in 8 hours. What must be the size of the pipe to furnish this supply, supposing that the head of water above the outlet of the pipe is 13½ ft.? (Coll. 1883.) *Ans.* 30 in.

3. A pipe discharges 2,250 gals. per min. with a fall of 4 ft. per mile. Find its diameter. (Coll. 1885.) *Ans.* 27 in.

4. A horizontal pipe 1,000 ft. in length and 6 inches in diameter internally, leads from a reservoir which is kept constantly full, the surface of the water being 10 ft. above the axis of the pipe,

At what rate will the water be discharged through the pipe? (Univ. 1865.) *Ans.* 0·54 c. ft. per s.

5. Calculate the diameter of one large main to convey the same quantity of water as three mains each 2·5 ft. in diameter, with a length of 2½ miles and head of 140 ft. (Coll. 1885.) *Ans.* 47 in.

6. What must be the diameter of a pipe to discharge 30 c. ft. per second with a fall of 1 in 100? With a pipe 2 ft. in diameter, what fall must be given in order that the discharge may be the same? (Univ. 1874.) *Ans.* (1) 30 in. (2) 1 in 33.

7. There are two proposals for a water scheme; one for a double line of pipes of equal diameter, and one for a single line of pipe. Supposing the larger pipe to have one-fifth greater thickness of metal than either of the two smaller pipes, compare approximately the weights of the pipes required in the two cases. *Ans.* 1·26 to 1.

8. The discharge from two pipes each having a fall of 2 ft. per mile is 28·8 c. ft. per sec. Find their diameters, the diameter of one being double that of the other. (Univ. 1876.) *Ans.* 53·0 in., 26·5 in.

9. What will be the discharge in c. ft. per minute at the end of a service main 1 ft. in diameter, and 2 miles long, having a fall of 10·2 ft. in the first mile, 23·5 ft. in the second mile, and a head of 3 ft. over the centre of its orifice of entry? By what amount must this head be increased in order to double the discharge? (Univ. 1882.) *Ans.* (1) 92 c. ft. (2) 39 6 ft.

10. A new 40 in. pipe is to be laid from the Redhills to Madras, a distance of 32,000 ft., and is to give a discharge of 10,000,000 gals. per 24 hrs. Level of water at Redhills 51·50; level of pipe at Madras 3·50. Find (1) the loss of head due to resistance in the new pipe, (2) the pressure per sq. inch at the Madras end of the pipe. (Univ. 1891.) *Ans.* (1) 13·9 ft. (2) 15 lb.

11. A pipe main 4,800 ft. long is to be laid with a fall of 1 in 192, and is required to deliver 3,250 gals. per min. at a pressure of 10 lb. per sq. inch. The head over the inlet orifice is 10 ft. What should be the diameter of the pipe? (Univ. 1890.) *Ans.* 24 in.

12. A siphon discharging over a channel bank is 60 ft. in length and 12 in. in diameter. What will be its discharge when the effective head is 6 ft.? (Univ. 1890.) *Ans.* 7·7 c. ft. s.

CHAPTER VII

FLOW OF WATER IN CHANNELS

CONTENTS

82. **Flow of water in open channels.**—The flow of water in an open channel is similar to that in a pipe laid at its own virtual slope, *i.e.*, with free upper surface. The velocity varies from point to point of the water section, being least in the vicinity of the boundaries. The mean velocity of all the filaments in a length of channel of uniform section is however uniform; and the flow may therefore be supposed to take place in plane layers parallel to successive cross sections. The remarks and investigations of Articles 68, 69 and 71 are thus applicable, and we have

$$v = \sqrt{\frac{2g}{\mu}}\sqrt{rs} = c\sqrt{rs} \quad \dots \quad \dots (53),$$

where r is the hydraulic mean depth, s the slope of the water surface, and c a co-efficient which depends on

(1) the roughness of the boundaries,
(2) the form of the water section,
(3) (to a small extent) the bed slope.

The second condition is introduced to modify the error due to the neglect of the variation of the velocity from point to point of the section. In artificial channels, to which we shall at present confine ourselves, the section and bed slope are generally uniform, so that the depth is constant, *i.e.*, the surface is parallel to the bed, In rivers however this does not hold good, and the depth varies with every change in either the width or the bed slope. Plates VIII and IX

If the bed is not parallel to the water surface, as is the case in the vicinity of obstructions, the effective fall of every filament is still that of the water surface. Let CD (fig. 59), be a filament whose extremities are at depths z_1, z_2 below the surface, and let h be the surface fall in the length CD. The actual fall of CD is $(z_2 + h) - z_1$. The pressures at C and D are wz_1, wz_2 respectively, so that the difference of pressure head is $z_1 - z_2$. The effective fall is the sum of these heads, viz. $(z_2 + h - z_1) + (z_1 - z_2) = h$.

83. **Co-efficients.**—The experiments and investigations of M. Bazin shew that the co-efficient μ (neglecting any slight variation due to the longitudinal slope), may be expressed in the form $\mu = a\left(1 + \frac{\beta}{r}\right)$, where r is the H.M.D. of the water section, and a and β are quantities depending on the nature of the boundaries. Dividing all channels into four classes, according to their roughness, the values of a and β are as follow.*

		a	β
I.	Very smooth channels :—cement, planed planks ...	·003	·1
II.	Smooth channels :—ashlar, brickwork	·004	·2
III.	Rough channels :—rubble masonry, stone pitching	·005	·8
IV.	Very rough channels :—earth	·006	4·0

Ex. 50.—A cement plastered channel has a H.M.D. of ·5 inches. Find the value of c.

$$\mu = \cdot 003\left(1 + \frac{\cdot 1}{\cdot 5}\right) = \cdot 0036 ; \quad c = \sqrt{\frac{2g}{\mu}} = \frac{8}{\cdot 06} = 133.$$

The co-efficients of all four classes are shewn graphically in Pl. IX.

The fourth class of channels is that with which we have generally to deal. The values of c for this class are here tabulated.

Co-efficients for Earthen Channels.

H.M.D.	c	H.M.D	c	H.M.D.	c
0·25	25	3·0	68	6·5	81
0·5	34	3·5	71	7·0	83
0·75	41	4·0	73	7·5	84
1·0	46	4·5	75	8·0	85
1·5	54	5·0	77	8·5	85
2·0	60	5·5	79	9·0	86
2·5	64	6·0	80	10·0	88

* NOTE.—Bazin's values for a are carried to five decimal places, and those for β to two; but as it is desired here to give figures which can be borne in mind, these values have been rounded off. The table above is calculated for the true values of a and β, which will be found given in Plate IX. For a more complete table, see Appendix I.

Plate X Bazin's co-efficients are not applicable to large rivers. For such channels Kutter's formula for the value of c in the expression $v = c\sqrt{rs}$ should be used. The formula is—

$$c = \frac{41{\cdot}6 + \frac{1{\cdot}811}{N} + \frac{{\cdot}00281}{s}}{1 + \left(41{\cdot}6 + \frac{{\cdot}00281}{s}\right)\frac{N}{\sqrt{r}}}$$

where s is the longitudinal slope, and N is a co-efficient of roughness, some of whose values are—

Fine plaster	·010	Rivers and canals in good order ...	·025
Ashlar and brickwork	·013	Do. in fair order ...	·030
Rubble masonry ...	·017	Do. in bad order ...	·035
Firm gravel	·020		

This formula is suited to streams of all sizes, from a tiny rivulet to the largest river, but cannot conveniently be used without the aid of tables. Values of the co-efficients are given in Appendix II and a selection from them is shewn graphically in Plate X.*

For artificial channels, such as are dealt with in this chapter, Bazin's co-efficients are suitable, and are employed in the examples.

84. Two classes of problems present themselves, the direct and the inverse. In the former, the dimensions of the channel are given, so that r is known, and the proper co-efficient can be determined. In the latter r, and therefore c, is unknown, and we must proceed by approximation. Before solving examples however, allusion must be made to the ordinary forms of channels.

85. Channel section.—Earthen channels are trapezoidal in section. There is a flat bottom, which may vary in width from 1 foot for small distributaries to 180 feet for the largest main canals, and there are sloping sides, the inclination of which depends chiefly on the angle of repose of the earth. The inclination is frequently made 1 to 1, or $1\frac{1}{2}$ to 1, in the first instance; but with the lapse of time there is a tendency for the side slopes to wear steeper and to approach perhaps $\frac{1}{2}$ to 1. Generally, if the side slope is n to 1, the bottom width b, and the depth d, we have the area of water section $A = (b + nd)d$, and the wetted border $B = b + 2d\sqrt{n^2 + 1}$. The depth may vary from a few inches to 10 or 12 feet.†

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* Plotted from Jackson's Canal and Culvert Tables. W. H. Allen, 1884.

† A canal at Holyoke on the Connecticut river has a depth of 22 ft., and width 140 ft.

Masonry channels, such as aqueducts, are generally rectangular, or nearly so, in section. Channels cut in rock, or executed in concrete, may be semi-circular, this being the most economical form.

86. Discharge of channels.—If the velocity and discharge of an existing channel are required, its section and inclination are measured, so that b, d, n, and s are known. The proper value of c can then be determined, and the velocity v, and discharge Q ascertained.

Ex. 51.—An earthen channel has a bottom width of 6 ft., side slopes 1 to 1, depth 3 ft., and inclination 1 foot per mile. Find the velocity and discharge.

Here $A = (6 + 3)\,3 = 27$ s. ft.; $B = 6 + 2 \times 3\sqrt{2} = 14{\cdot}5$ ft.;

$$\therefore r = \frac{27}{14{\cdot}5} = 1{\cdot}86.$$

$$\mu = {\cdot}006\left(1 + \frac{4}{1{\cdot}86}\right) = {\cdot}019\,;\; c = \sqrt{\frac{2g}{\mu}} = 57.$$ (Its true value as taken from the table, Art. 83, would be 60).

$$v = 57\sqrt{1{\cdot}86 \times \frac{1}{5280}} = 1{\cdot}07. \quad Q = Av = 28{\cdot}9 \text{ c. ft. per sec.}$$

Ex. 52.—What would be the discharge of the above channel if the sides and bottom were pitched with rough stone?

Here $\mu = {\cdot}005\left(1 + \frac{{\cdot}8}{1{\cdot}86}\right) = {\cdot}0072\,;\; \therefore c = \sqrt{\frac{2g}{\mu}} = 94.$

$$Q = \frac{94}{57} \times 28{\cdot}9 = 47{\cdot}6 \text{ c. ft. per sec.}$$

Ex. 53.—What would be the discharge of a semicircular channel plastered with cement, whose cross section measures 27 s. ft., and whose inclination is 1 ft. per mile?

Let d be the diameter; $\frac{\pi d^2}{8} = 27$; $\therefore d = 8{\cdot}3$. H.M.D. $= \frac{d}{4} = 2{\cdot}08$.

$$\mu = {\cdot}003\left(1 + \frac{{\cdot}1}{2{\cdot}08}\right) = {\cdot}003\,;\; c = \sqrt{\frac{2g}{\mu}} = 146.$$

$$v = 146\sqrt{\frac{2{\cdot}08}{5280}} = 2{\cdot}89. \quad Q = Av = 78 \text{ c. ft. per sec.}$$

87 Design of trapezoidal channels in earth.—We have three equations :—

$$v = c\sqrt{rs} \quad \ldots \quad (54)$$

$$Q = Av \quad \ldots \quad (55)$$

$$c = \sqrt{\frac{2g}{\mu}} \quad \ldots \quad (56),$$

where $A = (b + nd)d$; $r = \frac{(b + nd)d}{b + 2d\sqrt{n^2 + 1}}$; $\mu = {\cdot}006\left(1 + \frac{4}{r}\right)$; so that of the seven quantities b, d, n, s, c, v, Q any three can be determined if the rest are given. The value of c expressed in terms of

b and d is so complex however, that it cannot be conveniently substituted in the other equations except in a numerical form. We may therefore divide the cases which can occur into two classes, one in which the data permit of c being determined directly from eq. (56), the other in which they do not. The first class can be solved without difficulty. The second can be best treated as follows:—a value of c is assumed, the channel dimensions are calculated, the H.M.D. found, and the corresponding value of c determined. If it agrees with the assumed value, the solution is complete; but, if not, it serves as a guide to a second assumed value, from which the channel dimensions must be recalculated. The design of channels is facilitated by the use of Tables, as Higham's or Jackson's, * or those given in the Appendices.

Having thus isolated eq. (56), there remain only two equations to deal with. The quantity n moreover is always given, since it depends on the nature of the soil. We have therefore five quantities, b, d, s, v, Q, any two of which can be found, if the other three are given. There are thus ten cases which can occur, the first of which has been already dealt with in Art. 86. In five of these cases the breadth and depth are either given, or are immediately deducible from the data; and the value of c is consequently directly calculable. In the remaining five, c must be obtained by successive approximation.

	Given.	Required.
Class I.—c directly calculable.	b d s	Q v
	b d Q	s v
	b d v	s Q
	d Q v	s b
	b Q v	s d
Class II.—c to be assumed.	Q v s	b d
	Q s d	b v
	Q s b	d v
	v s d	Q b
	v s b	Q d

Ex. 54.—A main canal is to have a discharge of 2,500 c. ft. per sec., with a velocity of 2·5 ft. per sec., and a depth of 5 ft. The side slopes are 1 to 1. Find the bottom width and the fall.

* Hydraulic Tables for open Channels, by T. Higham. Spon, London, 187'..
Canal and Culvert Tables, by L. d'A. Jackson, W. H. Allen London, 1884.

$A = \frac{Q}{v} = 1000$ s. ft. Mean width $= \frac{A}{d} = 200$ ft.

Bottom width $b = 200 - 5 = 195$ ft. $B = 195 + 10\sqrt{2} = 209{\cdot}14$.

$r = \frac{A}{B} = \frac{1000}{209{\cdot}14} = 4{\cdot}78$; whence $\mu = {\cdot}011$, and $c = 76$.

$v = 76\sqrt{4{\cdot}78\,s}$; $\therefore \sqrt{s} = \frac{2{\cdot}5}{76 \times 2{\cdot}19} = {\cdot}015$;

$\therefore s = 0{\cdot}225$ per 1000 ft., or 1 ft. 2 in. per mile.

Ex. 55.—An irrigation channel is to discharge 500 c. ft. per sec. with a velocity of 3 ft. and fall of 1 in 2500. The side slopes are 1 to 1. Find the depth and bottom width.

$A = \frac{Q}{v} = \frac{500}{3} = 167$ s. ft. ; $v = c\sqrt{rs}$, $\therefore c\sqrt{r} = 150$.

Try $r = 3$, $\therefore c = 70$, $\therefore c\sqrt{r} = 121$.

" $r = 4$, $\therefore c = 76$, $\therefore c\sqrt{r} = 152$.

" $r = 3{\cdot}9$, $\therefore c = 76$, $\therefore c\sqrt{r} = 150$.

We have $(b + d)\,d = 167$....................(i).

$\frac{167}{b + 2d\sqrt{2}} = 3{\cdot}9$....................(ii).

From (ii), $b = 42{\cdot}8 - 2{\cdot}8d$. Substituting in (i), $42{\cdot}8d - 1{\cdot}8d^2 = 167$; whence $d = 4{\cdot}9$, or say 5 ft.

From (i), $(b + 4{\cdot}9)\,4{\cdot}9 = 167$; $\therefore b = 29$.

Hence the required dimensions are $b = 29$ ft., $d = 5$ ft.

Ex. 56.—The discharge of a channel whose depth in $3\frac{1}{2}$ ft., fall 18 inches per mile, and side slopes 1 to 1, is 180 c. ft. per sec. Find the bottom width and the velocity.

The channel having a depth of $3\frac{1}{2}$ ft., assume $r = 3$ ft., i.e., $c = 70$. $Q = 180$.
$s = \frac{1}{3520}$. $v = c\sqrt{rs} = 2{\cdot}05$.

Mean width $= \frac{180}{3\frac{1}{2} \times 2{\cdot}05} = 25$. $\therefore b = 21\frac{1}{2}$. $\therefore B = 31{\cdot}4$, $A = 87{\cdot}5$.

Hence corrected value of r is $\frac{87{\cdot}5}{31{\cdot}4} = 2{\cdot}8$; $\therefore c = 69$; $v = c\sqrt{rs} = 1{\cdot}95$; $\therefore$ Mean width $= \frac{180}{3\frac{1}{2} \times 1{\cdot}95} = 26{\cdot}4$. Whence $b = 23$ ft.

Ex. 56a.—A channel is 7 ft. wide at bottom ; the length of each sloping side is 6·8 ft. ; the width at water surface is 18 ft. ; the depth 4 ft. ; and the inclination of surface 4 inches per mile : what is the discharge per minute? (Univ. 1886.)

Here $A = 50$ s. ft. $B = 20{\cdot}6$ ft., $\therefore r = 2{\cdot}4$, $\therefore c = 66$.

$s = \frac{1}{8 \times 5280}$. $v = c\sqrt{rs} = 66\sqrt{\frac{2{\cdot}4}{3 \times 5280}} = \frac{66}{10\sqrt{66}} = {\cdot}812$.

$\therefore Q = 50 \times {\cdot}812 = 40{\cdot}6$ c. ft. per sec. Discharge per minute $= 40{\cdot}6 \times 60 = 2436$ c. ft.

88. Practical data.—Irrigation channels generally have Q, v, n, and s, assigned, leaving d and b to be determined. Sometimes however d is assigned, leaving b and s, or b and v to be determined. The discharge Q is fixed by consideration of the area to be irrigated, an allowance of 1 c. ft. per sec. for 60 acres being usual. The velocity v should be as high as is admissible, so as to reduce the cross section to be excavated. It must not however be so high as to cause erosion of bed and banks, nor so low as to allow of the growth of weeds and free deposit of silt. It lies generally between $1\frac{1}{2}$ and 3 ft. per sec. The side slope n depends on the nature of the soil, and is generally made 1 to 1, or $1\frac{1}{2}$ to 1 in the first instance. The longitudinal slope s cannot be greater than the natural fall of the country, but it can be made as much less as is desirable by suitably aligning the channel, or by introducing at intervals sudden drops or *falls*, with arrangements for annihilating the additional velocity generated. The bed slope does not generally exceed 20 ft. per mile for small channels, 5 ft. per mile for large channels, or $1\frac{1}{2}$ ft. per mile for very large channels. If the velocity is assigned, an increase of slope implies a reduction of r, that is a diminution of depth.

If the velocity is excessive, holes are scoured in the bed. Small rapids are formed above the holes, which cause the scouring to cut back, and so to gradually work its way upwards towards the head of the channel, thus producing the effect known as *retrogression of levels.**

Navigation channels generally have all dimensions, and the velocity, assigned, leaving s and Q to be determined. The dimensions are fixed by the requirements of the traffic, while the velocity must be kept as low as possible, from 1·5 to 1·75 ft. per second, for convenience of haulage.

89. Channels of minimum border.—A channel which has a maximum area for a given border, or a minimum border for a given area is termed a *channel of maximum discharge*, or a *channel of minimum border*. The determination of the form of such channels has a practical bearing on the amount of excavation required. The discharge varies as $A\sqrt{r}$, i.e., as $\sqrt{\frac{A^3}{B}}$, so that, for a given discharge, $\frac{A^3}{B}$ is constant, i.e., $A \propto \sqrt[3]{B}$. Hence, other things being the same, the excavation will be least when the border is least.

(a) *Closed channels.*—If the channel is closed, i.e., has boundaries on all sides of the water section, the circle furnishes the best form, since this is the figure which has the least border for a given area. In this case the H.M.D. $= \frac{\text{diameter}}{4} = \frac{d}{4}$, where d is the greatest depth. This form is generally adopted for pipes.

A pipe running partly full (Art. 79), is not a closed channel in the sense here used.

* See Professional Papers on Indian Engineering, 2nd series, Vol. II, 1878.

(*b*) *Open channels.*—If the channel is open, and has its greatest breadth at the water surface, the semicircle is the best form. In this case the H.M.D. $= \frac{\text{diameter}}{4} = \frac{d}{2}$, where d is the greatest depth. This form is adapted to channels cut in rock, or executed in concrete. Plate XI.

(*c*) *Trapezoidal channels.*—If the section of an open channel is polygonal, the best form is that which approaches nearest to the semicircle, viz., a circumscribing regular semi-polygon with an indefinitely large number of sides. As the number of sides is in practical cases limited to three, the best section for a trapezoidal channel is a semi-hexagon.

The H.M.D. $= d^2\sqrt{3} \div 2\sqrt{3}d = \frac{d}{2}$. This form may be adopted in masonry, but the slopes, nearly 1 to $1\frac{3}{4}$, are too steep for earthwork unless revetted.

(*d*) *Trapezoidal channels with given side slopes.*—In practice, as already pointed out in Arts. 87 and 88, the side slopes of earthen channels must always be a given quantity. If x be the bottom width, and y the depth of a channel with given side slopes n, it can readily be shewn that the best form is obtained when

$$x = 2y\ (\sqrt{n^2+1} - n).$$

We have $A = (x + ny)\,y$ .. .. (i)

$B = x + 2y\sqrt{n^2+1}$ .. .. (ii)

In a channel of maximum discharge, we may consider B constant and A a maximum; or A constant and B a minimum; or, if the discharge is supposed fixed, A and B as both minima. In any case, the differential co-efficients of A and B must be zero. Differentiating with respect to y,

From (i), $x + y\frac{dx}{dy} + 2ny = 0.$

(ii), $\frac{dx}{dy} + 2\sqrt{n^2+1} = 0.$

Whence $x - 2y\sqrt{n^2+1} + 2ny = 0$; $\therefore x = 2y\,(\sqrt{n^2+1} - n).$

$$\text{The H.M.D.} = \frac{A}{B} = \frac{(x+ny)\,y}{x + 2y\sqrt{n^2+1}} = \frac{y^2\,(2\sqrt{n^2+1} - n)}{2y\,(2\sqrt{n^2+1} - n)} = \frac{y}{2}.$$

Let EFGH (fig. 60), be the channel required. From C the middle point of EF, drop perpendiculars p, y, p on the three sides. Then $A = \frac{1}{2}\,(EH.p + HG.y + GF.p)$; $B = EH + HG + GF$.

But $\frac{A}{B} = \frac{y}{2}$; $\therefore p$ must be $= y$; and therefore a circle described with centre C and radius y will touch the three sides of the trapezoid.

Plate XI. Draw FK perpendicular to HG; then the triangle CLF is similar to FGK; $\therefore \frac{CF}{CL}=\frac{FG}{FK}$. But CL = FK = y; $\therefore$ CF = FG, i.e., the side slope is equal to half the top width, and therefore the border is equal to the sum of the top and bottom widths. Hence the following constructions, see fig. 60.

(i) *Depth and side slope given.*—Set off CD vertical, equal to the given depth, and through C and D draw horizontals EF, HG. With C as centre, and radius CD, describe a semicircle. Draw FG, EH at the given inclination to touch the semicircle. The channel EFGH is that required.

(ii) *Top width and side slope given.*—Draw EF horizontal equal to the given top width, and bisect it in C. Lay off FG, EH at the given inclination. With centre F, and radius FC, describe an arc cutting FG in G. Draw GH horizontal, and the channel is completed.

(iii) *Bottom width and side slope given.*—Draw the bottom width HG, and bisect it in D. Draw GF, HE at the given inclination. From G set off GL equal to GD, and draw LC, DC perpendicular to GF, HG respectively to meet in C. Through C draw EF horizontal to meet the side slopes in E and F.

It should be noted that since EF $= 2d\sqrt{n^2+1}$, HG = EF $-2nd$,

$$A = \frac{(EF+HG)d}{2} = (EF-nd)d = d^2(2\sqrt{n^2+1}-n).$$ Therefore

$$d = \sqrt{\frac{A}{2\sqrt{n^2+1}-n}} \quad \dots \quad \dots \quad (57)$$

so that if any two of the quantities d, A, n are given, the third can be found.

The section of maximum discharge thus obtained can in practice be employed only for small channels. With large channels the depth becomes so great that the increased price-rate for excavation neutralizes the saving effected by the reduced area.

(*e*) *Rectangular channels.*—A rectangular channel is a trapezoid with a given slope of 90°. The figure for maximum discharge will, therefore, be a half square. The H.M.D. $= \frac{2d^2}{4d} = \frac{d}{2}$. This form is employed for aqueducts of timber or masonry

90. Design of channels of minimum border.—In designing small earthen channels, we generally have the discharge, velocity, and side slopes fixed; while the cross section and longitudinal slope have to be ascertained.

From eq. (57) we have $d = \sqrt{\dfrac{A}{2\sqrt{n^2+1}-n}}$. Also $A = \dfrac{Q}{v}$.

Thus d is determined, and the top and bottom widths can thence be found. Knowing the H.M.D., i.e., $\dfrac{d}{2}$, we can find c; and can then obtain s from the equation $v = \sqrt{rs}$.

Ex. 57.—An earthen channel of best form is to discharge 60 c. ft. per second, with a velocity of 2 ft. per sec. and side slopes $1\frac{1}{2}$ to 1. Design the channel.

$A = \dfrac{60}{2} = 30$ s. ft. $d^2 = \dfrac{A}{2\sqrt{n^2+1}-n} = \dfrac{30}{\sqrt{13}-1\cdot5} = 14\cdot25$; $\therefore\ d = 3\cdot8$ ft.

Mean width $= \dfrac{A}{d} = \dfrac{30}{3\cdot8} = 7\cdot9$ ft. Bottom width $= 7\cdot9 - nd = 2\cdot2$ ft.

Top width $= 7\cdot9 + nd = 13\cdot6$ ft.

$r = \dfrac{d}{2} = 1\cdot9$; whence $\mu = \cdot0186$, and $c = 59$,

$\therefore\ s = \dfrac{v^2}{c^2r} = \dfrac{4}{(59)^2 \times 1\cdot9} = \dfrac{1}{1654}$.

If however the channel is large, the depth comes out impracticably high.

Ex. 58.—Find the dimensions of a minimum channel of best section to carry 50,000 c. yds. per hour with a surface fall of 6 inches per mile. Side slopes 1 to $1\frac{1}{2}$ (Univ. 1878.)

Here $n = \dfrac{2}{3}$; $A = d^2(2\sqrt{n^2+1}-n) = 1\cdot73d^2$; $Q = 375$ c. ft. per sec.

$v = \sqrt{\dfrac{d}{2} \times \dfrac{1}{10560}} = \dfrac{c}{142}\sqrt{d}$. $Q = Av$; $\therefore\ 375 = 1\cdot73d^2 . \dfrac{c}{142}\sqrt{d}$.

Try $c = 80$. $d = 10\cdot8$; $\therefore\ r = 5\cdot4$; whence $\mu = \cdot0104$, and $c = 78$, which is sufficiently near the assumed value.

$A = 1\cdot73 \times (10\cdot8)^2 = 202$ s. ft.; $\therefore$ Mean breadth $= \dfrac{202}{10\cdot8} = 19$ ft. nearly.

Bottom width $= 19 - nd = 19 - 7\cdot2 = 11\cdot8$ ft. Top width $= 19 + 7\cdot2 = 26\cdot2$ ft.

Ex. 59.—A canal is 3 ft. deep, 40 ft. wide at the bottom, with slopes of $1\frac{1}{2}$ to 1, and a fall of 1 in 5875. Design a suitable branch channel to carry one-sixth part of the water, and find the fall necessary to give the water the same velocity as in the canal. (Univ. 1884.)

Here $d = 3$; $b = 40$; $n = \dfrac{3}{2}$; $s = \dfrac{1}{5875}$.

$A = (b + nd)\,d = 133\cdot5$ s. ft. $B = b + 2d\sqrt{n^2+1} = 50\cdot8$.

$r = \dfrac{A}{B} = 2\cdot63$; $\mu = \cdot006\left(1 + \dfrac{4}{r}\right) = \cdot0151$; $c = \sqrt{\dfrac{2g}{\mu}} = 65$.

$Q = cA\sqrt{rs} = 183\cdot5$. $v = \dfrac{Q}{A} = 1\cdot37$ ft.

For the branch channel, $Q = \dfrac{183\cdot5}{6} = 30\cdot5$; $v = 1\cdot37$; $A = \dfrac{Q}{v} = 22\cdot3$ s. ft.

Plate XI. This being a small channel, let us ascertain the depth, supposing it a channel of minimum border. $d = \sqrt{\frac{A}{2\sqrt{n^2+1}-n}} = 3{\cdot}25$, which is greater than the depth in the main channel, and therefore inconvenient.

Making bed of branch, at the junction, at same level as bed of main channel, and allowing a head of 3 inches for the branch head sluice, we have 2·75 as a suitable depth for the branch. Mean width $= \frac{22{\cdot}3}{2{\cdot}75} = 8{\cdot}1$ ft.

Taking $n = \frac{3}{2}$ as before :—Bottom width $= 8{\cdot}1 - 4{\cdot}1 = 4{\cdot}0$ ft.

Top width $= 8{\cdot}1 + 4{\cdot}1 = 12{\cdot}2$ ft.

$B = b + 2d\sqrt{n^2+1} = 13{\cdot}9$; $\therefore r = 1{\cdot}6$; whence $\mu = {\cdot}0210$, and $c = 55$.

$\therefore s = \frac{v^2}{c^2 r} = \frac{1}{2575}$.

Channels other than earthen can readily be designed of specified form from the data given in Art. 89.

Ex. 60.—An aqueduct of best form of rectangular section, with sides and bottom of planed timber, is to carry 12 c. ft. per second with a velocity of 4 ft. per second. Design it.

Here $n = 0$; $A = \frac{Q}{v} = 3$ s. ft. ; $d^2 = \frac{A}{2\sqrt{n^2+1}-n} = 1{\cdot}5$; $\therefore d = 1{\cdot}22$.

Width $= 2d = 2{\cdot}44$. H.M.D. $= \frac{d}{2} = {\cdot}61$; $\mu\ {\cdot}003\left(1 + \frac{{\cdot}1}{{\cdot}61}\right) = {\cdot}0035$; $\therefore c = 133$.

$s = \frac{v^2}{c^2 r} = \frac{16}{17689 \times {\cdot}61} = \frac{1}{674}$. Hence the channel should measure 1′·22 × 2′·44, and have a fall of 1 in 674.

91. Channels for a variable discharge.—When a channel has to carry a variable volume, it is desirable that the velocity should be nearly constant, i.e. (neglecting the variation of c), the H.M.D. should be constant, or the border should increase at the same rate as the area. This condition cannot be conveniently secured in earthen channels, as the slopes above the minimum water level would become very flat, and slightly convex on their upper surface.* The principle is however adopted to some extent in ovoidal sewers, which are intended for the constant discharge of sewage as well as for the occasional discharge of a comparatively large volume of rain water.

Two ovoidal sections are shewn in figs. 61 and 62, which indicate their construction. In the Metropolitan ovoid (fig. 61), the radius of the invert is one-half that of the crown; in Hawksley's ovoid (fig. 62), nearly three-fifths of that of the crown. These culverts are generally executed in brickwork, and have transverse diameters up to 6 ft. Drains of this form, though covered in at the top, are technically open channels, since they are not intended to discharge under pressure.

* Professional Papers on Indian Engineering, 2nd series, Vol. VII, 1878.

Ex. 61.—A metropolitan ovoid culvert, in brickwork, cement plastered, measures 3'-2" × 4'-9". Compare the velocities and discharges when it runs with depths of one-third and two-thirds the vertical diameter. Plate.XI.

Drawing the ovoid to scale, we find by measurement,—

For one-third full, $A_1 = 2{\cdot}85$; $B_1 = 4{\cdot}38$; $\therefore r_1 = {\cdot}65$.

„ two-thirds „ $A_2 = 7{\cdot}58$; $B_2 = 7{\cdot}58$; $\therefore r_2 = 1{\cdot}00$.

$$\mu_1 = {\cdot}003\left(1 + \frac{{\cdot}1}{{\cdot}65}\right) = {\cdot}0035; \quad \therefore c_1 = 135.$$

$$\mu_2 = {\cdot}003\left(1 + \frac{{\cdot}1}{1{\cdot}0}\right) = {\cdot}0033; \quad \therefore c_2 = 140.$$

$$\text{Hence } \frac{v_1}{v_2} = \frac{c_1\sqrt{r_1}}{c_2\sqrt{r_2}} = \frac{135 \times {\cdot}81}{140 \times 1{\cdot}0} = \frac{109}{140}.$$

$$\frac{Q_1}{Q_2} = \frac{A_1 v_1}{A_2 v_2} = \frac{2{\cdot}85}{7{\cdot}58} \times \frac{109}{140} = \frac{311}{1061}.$$

So that the velocity only increases by one-fourth, while the discharge is trebled.

92. Variation of velocity in a cross section.—As might be expected from the nature of the resistances, the velocity is least in the neighbourhood of the bed and banks, and greatest in the axis of the stream near the surface. If v_s be the greatest surface velocity, v_b the bottom velocity, and v the mean velocity, experiment shews that, approximately,

$$v = {\cdot}8 v_s = 1{\cdot}3 v_b \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (58)$$

The relation between v and v_s is useful, owing to the facility with which v_s can be measured by surface floats. The relation between v and v_b enables us, in designing a channel, to assign a velocity which will not be injurious to bed and banks of known consistency of soil. The following mean velocities in feet per second should generally be not exceeded:—

Clay	0·75	Boulders	4·0
Sand	1·5	Stratified rock	6·0
Pebbles	3·0	Hard rock	10·0

On the hypothesis of simple frictional and viscous resistance, it can be shewn that the velocities at points in a vertical line CD (fig. 63), are represented by the abscissæ of a parabola whose axis lies in the surface. The true motion is however complicated by the existence of eddies, which are most numerous near the surface; and experiment shews that the curve is a parabola whose vertex is $0{\cdot}3d$ below the surface.* Bazin found the following relation between the greatest surface velocity v_s and the mean velocity v.

$v = v_s - 25\sqrt{rs}$. Now $v = c\sqrt{rs}$. Therefore

$$v = \frac{c}{c + 25} v_s \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (59)$$

* Note.—This distribution of the velocity applies only to unobstructed sections. In dealing with the discharge over a submerged weir, some authorities take the velocity of approach for the notch portion as equal to the surface velocity of the stream, and the velocity of approach for the orifice portion as equal to the mean velocity of the stream. For the reason above given, this practice has not been followed in the text of Chap. IV.

Ex. 62.—The maximum surface velocity in an earthen channel whose H.M.D. is 4 ft. is observed to be 5 ft. per sec. Find the mean velocity.

$$\mu = \cdot 0\ldots\left(1 \times \frac{4}{4}\right) = \cdot 0120\ ;\ c = \sqrt{\frac{2g}{\mu}} = 73.$$

$$v = \frac{73}{73 + 25} \times 5 = 3 \cdot 72 \text{ ft. per sec.}$$

Many writers have asserted that the water surface of the cross section of a stream is slightly convex upwards, i.e., higher at the axis than at the banks. Experiments made at Roorkee however, give no support to the statement.

93. Minor losses of head.—Nearly the whole of the fall of a channel is effective to overcome resistance. Small heads are however required to produce the velocity of entry, and to pass obstructions, such as bends. These losses must be compensated for by giving so much extra fall to the channel.

Velocity of entry.—The entrance to a channel may be open to the source of supply, or it may be closed by a head sluice. In the former case, there is for a short distance a rapid surface slope sufficient to generate the velocity which the H.M.D. and fall impose on the channel lower down; in the latter the head is the difference of level on the upper and lower sides of the sluice.

Let A be the area of channel section, v the channel velocity, h the actual head required to produce this velocity. For an open inlet $Q = c\,A\sqrt{2gh} = v.A$; whence $h = \frac{v^2}{c^2.2g} = 1 \cdot 5\frac{v^2}{2g}$, if c be taken ·8. For a closed inlet, let A_1 be the area of the sluice openings, v_1 the velocity through them. $v_1 = \frac{A}{A_1}v$. $h = \frac{v_1^2}{c^2.2g}$ $= \left(\frac{A}{cA_1}\right)^2.\frac{v^2}{2g} = 1 \cdot 5\left(\frac{A}{A_1}\right)^2.\frac{v^2}{2g}.$

In both cases the fall can be distributed, if desired, by widening the channel near the entrance, and thus reducing the velocity to be at first produced.

Bends.—Bends in artificial channels are generally curves of large radius. In the absence of conclusive experiments as to the loss of head due to them, the following modification of Humphrey and Abbot's Mississippi formula is adopted.

For a bend having an arc subtending $a°$ the loss of head $h = \frac{a}{90} \times \cdot 36\frac{v^2}{2g}$.

Ex. 63.—In the first reach of a branch channel there are 9 bends of 30°, and 3 of 45°. The vent area of the head sluice is half the area of the channel section. The velocity in the channel is to be 2 ft. per sec. Find the additional head to be provided in designing the channel.

At entry, $h = 1 \cdot 5\,(2)^2.\frac{v^2}{2g}$

At bend of 30°, $h = \frac{30}{90} \times \cdot 36\frac{v^2}{2g}$

At bend of 45°, $h = \frac{45}{90} \times \cdot 36\frac{v^2}{2g}$

$\therefore$ Total loss $= \frac{v^2}{2g}(6 + 9 \times \cdot 12 + 3 \times \cdot 18) = 0 \cdot 5$ ft. nearly.

94. Falls.—When the natural ground slope is much greater than the inclination of the channel, length is saved by the introduction of sudden drops or falls in the channel bed. These generally consist of a low weir, with steps on the down-stream side to break up the force of the falling water, or with a single vertical fall on to a water cushion. The object in both constructions is to annihilate the additional velocity due to the fall, and to deliver the water into the lower reach with the normal channel velocity. If there is no weir, it is found that the depth in the channel begins to diminish a great distance back from the weir, and the velocity consequently increases, and erodes the bed. To ascertain the height to which the weir should be built, we have from equations (11) and (53), $\frac{2}{3} cl \sqrt{2g} \left\{ \left(h+h_a\right)^{\frac{3}{2}} - h_a^{\frac{3}{2}} \right\} = Q = cA\sqrt{rs}.$ Solve for h. Then, if d is the normal depth of the channel, the weir must be built to a height $(d—h)$. Plate XI.

The type of fall in which the water drops vertically on to a water cushion (fig. 65), is one often seen in nature; and the depth of the pool at the foot of natural falls is a guide to us in fixing the depth of the water cushion. The formula adopted for canal falls is $x = 1{\cdot}5\sqrt{H}\ \sqrt[3]{d}$, where x is the depth of the cushion, d that of the channel, and H the difference of water level in the two reaches. Plate XII* illustrates an experimental fall of this description, constructed on the Bari Doab Canal. The form of the water surface on the crest, that of the jet of falling water, and that of the wave below should be noted.

In *Ogee* falls, which consist of a double curve, fig. 66, the object is to deliver the water at the foot of the fall without vertical velocity. Excessive horizontal velocity may be checked by widening the channel below the fall, or by placing brushwood spurs or other obstacles in the tail water. The double chord CD has a slope of about 6 to 1, the chord CE of the upper arc being one-third of CD.

On the Ganges canal as first constructed, the ogee falls had their crests at bed level of upper reach. The scour produced for some miles above the falls was so great that it was soon found necessary to raise the crests.†

95. Standing Waves.—It can be shewn from the differential equation of steady varied motion ‡ that if the depth at which a stream is flowing be less than $\frac{v^2}{2g}$, and if the depth be increased by any obstruction, then at the point where $d = \frac{v^2}{g}$ the water surface tends to become normal to the bed, and a *standing*

* Taken from Professional Papers on Indian Engineering, 1st series, Vol. III, 1866. A plank weir can be fitted in the recesses shown in the side walls.

† Crofton's Report on the Ganges Canal.

‡ *Vide* Encyclopædia Britannica, 9th ed., Article *Hydromechanics*.

wave is produced. This condition may occur either above or at the foot of a weir, and it may also be noticed on the down-stream side of bridges discharging in flood.

Thus, in fig. 67, at the section C, d is $< \frac{v^2}{g}$. As the water section increases towards the obstruction, v diminishes, and eventually $d = \frac{v^2}{g}$, between C and D, when a standing wave is formed. Again at E the depth is so small, and velocity so great that d may be $< \frac{v^2}{g}$. As the bed generally consists here of a rough stone apron, the velocity rapidly diminishes, and a standing wave will be found between E and F, as soon as $d = \frac{v^2}{g}$.

The height of the wave may be found as follows:—

Let the mass CD (fig. 68), occupy the position C_1 D_1 after time t. Consider, for simplicity, a rectangular section of width l, and depths d_1, d_2. The horizontal change of momentum is $\frac{w}{g}\left(A_1 v_1^2 - A_2 v_2^2\right) t = \frac{wl}{g}\left(d_1 v_1^2 - d_2 v_2^2\right) t$.

The impulse is the difference of the pressures on CC, DD acting for the time t, i.e., $w\left(\frac{d_2^2}{2} - \frac{d_1^2}{2}\right) l.t$. Hence $d_2^2 - d_1^2 = \frac{2}{g}\left(d_1 v_1^2 - d_2 v_2^2\right)$. But $v_2 = \frac{d_1}{d_2} v_1$;

$\therefore d_2^2 - d_1^2 = \frac{2}{g} v_1^2 . \frac{d_1}{d_2}(d_2 - d_1)$; $\therefore d_2 + d_1 = \frac{2 v_1^2}{g} . \frac{d_1}{d_2}$; whence

$$d_2 = \sqrt{\frac{2 v_1^2}{g} d_1 + \frac{d_1^2}{4}} - \frac{d_1}{2} \quad \text{.. .. ..} \quad (60)$$

Ex. 64.—A bridge discharging in flood has depths of 10 ft. and 6 ft. of water on its up-stream and down-stream sides respectively, and a velocity of approach of 8½ ft. per second. Determine whether a standing wave will be formed, and if so, what its height will be.

$$\text{Here } v_1 = 8{\cdot}5; \; v_2 = \frac{10}{6} \times 8{\cdot}5 = 14{\cdot}2; \; \sqrt{gh} = \sqrt{32 \times 6} = 13{\cdot}9.$$

Hence v_2 is slightly greater than $\sqrt{gh}$. It will however diminish below the bridge, and when it reaches the value 13·9, a standing wave will be formed. The height of the wave will be

$$d_2 = \sqrt{\frac{2 \times (8{\cdot}5)^2}{32} \times 10 + \frac{(10)^2}{4}} - \frac{10}{2} = 8{\cdot}4 \text{ ft.}$$

The primary condition $d < \frac{v^2}{g}$ implies that $\frac{\mu d}{2} < rs$. In broad shallow channels r approximates to d; so that $s > \frac{\mu}{2}$ is the primary condition in such channels for the formation of a standing wave. For earthen channels $\mu = {\cdot}006\left(1 + \frac{4}{r}\right)$, its least value being consequently ·006. Hence, for a standing wave to be possible s must be > 003, or the inclination must be not less than about 16 ft. per mile.*

* For experimental researches on standing waves, see *La Propagation des Ondes*, Bazin.

EXAMPLES ON CHAPTER VII.

NOTE.—The co-efficients used (Bazin's) are those tabulated.

1. Find the fall in feet per mile of a channel with a bottom width of 40 ft., and side slopes of 2 to 1, to discharge 300 c. ft. per sec. with a depth of 4 ft. What will be the discharge of the channel with a depth of 5 ft.? (Coll. 1882.) *Ans.* (1) 10 in. per mile, (2) 457 c. ft. s.

2. What are the velocity and discharge of a channel which has a depth of $3\frac{1}{2}$ ft., bottom width 35 ft., side slopes 1 to 1, and bed fall 18 in. per mile? *Ans.* (1) 2 ft. per sec., (2) 270 c. ft. s.

3. What is the hydraulic mean depth of a channel? A channel cut through hard rocky ground is to carry 100 c. ft. of water per sec. with a mean velocity of 3 ft. per sec. Supposing its section a half square, find the fall in feet per mile. (Coll. 1883.) *Ans.* 2·5 ft.

4. Required approximately the bottom width of a channel with side slopes 1 to 1, and a fall of 2 ft. per mile, to discharge 400 c. ft. per sec. with a depth of 3 ft. (Coll. 1882.) *Ans.* 60 ft.

5. What must be the bottom width of a channel 4 ft. deep, with slopes of $1\frac{1}{2}$ to 1, and a fall of 3 ft. per mile, in order that it may discharge 195 c. ft. per sec.? (Univ. 1877.) *Ans.* 13 ft.

6. What will be the discharge in c. ft. per minute of a channel having a fall of 6 inches per mile, bottom width of 30 ft., and slopes 1 to 1, when flowing 6 ft. deep? What will be the mean, surface, and bottom velocities? (Univ. 1882.) *Ans.* (1) 20,450 c. ft., (2) 1·57, 1·97, 1·21 ft. s.

7. What are the principal circumstances to be considered in fixing the cross section and inclination to be given to a new irrigation channel to carry a given quantity of water; and why is the form for maximum discharge not so desirable or economical as others for channels formed in ordinary soils? (Univ. 1882.)

8. What should be the width of a rectangular brick aqueduct 220 yds. long, to convey 56,700 c. yds. of water per hour, the depth of water being 5 ft., and the fall through the aqueduct 3 inches? *Ans.* 20 ft.

9. A channel is 80 ft. wide at bottom with side slopes 1 to 1, fall 1 ft. per mile, mean velocity 3 ft. per sec. Find the depth and discharge of the channel. *Ans.* (1) 8·15 ft., (2) 2,155 c. ft. s.

A branch channel takes off one-third of this discharge, and the bottom width of the main channel is in consequence reduced to

60 ft. What fall must be given to the main channel in order that the velocity of 3 ft. per sec. may be maintained? (Univ. 1874.) *Ans.* 1·5 ft. per mile.

10. A channel is to be designed to carry a discharge of 350 c. ft. per sec. with a velocity of $2\frac{1}{2}$ ft. per sec. and side slopes of 1 to 1. The levels of the country are such that a fall of 1 in 3,600 is considered suitable. Sketch the section of the channel. (Univ. 1890.) *Ans.* $b = 16$ ft. $d = 6\frac{1}{4}$ ft.

11. A channel is constructed from a river to irrigate 2,000 acres. The river records shew that during September, that month of the irrigation season when the river is lowest, there is an ample supply of water from freshes during 16 days. In the intervals between the freshes not more than 12 c. ft. per sec. is available for use. What should be the capacity, i.e., the discharging power of the channel, allowing 1 c. ft. s. per 50 acres?

If it be decided to assign a velocity of 3 ft. per sec. and to give a depth of 2 ft. for full supply, and side slopes 1 to 1, determine the bottom width and the requisite fall. (Univ. 1890.) *Ans.* (1) $64\frac{1}{2}$ c. ft. s., (2) $b = 8\frac{3}{4}$ ft., (3) $s = 1$ in 470.

12. Find the dimensions of a channel to carry 4,000 c. ft. per sec. with a fall of 2 ft. per mile, side slopes 1 to 1, ratio of depth to mean width 1 : 15. (Univ. 1879.) *Ans.* $d = 8{\cdot}4$ ft. $b = 118$ ft.

13. A rectangular channel of rubble masonry, having a fall of 1·5 ft. per mile, is required to discharge 700 c. ft. per second with a width of 15 times its depth. What must the latter be? (Univ. 1883.) *Ans.* $3\frac{1}{2}$ ft.

14. In symmetrical trapezoidal channels of maximum discharge, what relation does the hydraulic mean depth bear to the depth of water? What are the geometric properties of such channels? (Univ. 1883.)

15. Determine the minimum section of a channel to carry 1,000 c. ft. per sec. with a fall of 2 ft. a mile. Side slopes $1\frac{1}{2}$ to 1, (Univ. 1874.) *Ans.* $d = 11{\cdot}3$ ft. $b = 6{\cdot}8$ ft.

16. Draw a cross section of a trapezoidal channel of best form, being given depth 4 ft. inclination of sides 2 to 1. *Ans.* $b = 1{\cdot}9$ ft.

Compare the velocity of its stream with that of a channel of the same depth and inclination, having a bottom width of 3·3 ft. and slopes of 1 to 1. (Univ. 1877.) *Ans.* The velocities are equal.

17. A maximum discharging channel of best section has a depth of 8 ft., and fall 2 ft. per mile. Calculate the discharge,

and draw a section of the channel with side slopes of 1 to 1. (Univ. 1879.) *Ans.* Q = 333 c. ft. s. b = 6·6 ft.

18. The mean width of a channel of minimum border is 14 ft., and the depth 8 ft. Find the side slopes. *Ans.* $\frac{3}{4}$ to 1.

19. Draw the following section of ground to scales of 1,000 ft. to 1 inch horizontal, and 5 ft. to 1 inch vertical; and shew on it the bed of a channel with depth 2 ft., and bed fall 1 ft. per mile, so laid out that the water surface may be everywhere below the natural ground level except at the points A and B, where it should reach the ground level. (Coll. 1884.)

Distance in feet.	Depth below datum in feet.	Remarks.
0	1·0	Point A.
1000	1·2	
2000	2·6	
3000	3·4	
4000	4·9	
5000	5·2	Point B.

20. A channel 30 ft. wide at bed with side slopes 3 vertical to 4 horizontal, and with a fall of 1 in 10,000, draws a fluctuating supply from a river. Find the velocity and discharge with depths of 2, 4 and 6 ft. (Univ. 1869.) *Ans.* (1) 0·78 ft. s., 50 c. ft. s., (2) 1·26 ft. s., 178 c. ft. s., (3) 1·60 ft. s., 365 c. ft. s.

21. Find the discharge in cubic feet per minute from an ovoid sewer of brickwork, running full, whose inclination is 1 in 1,000, transverse diameter 5 ft., vertical diameter $7\frac{1}{2}$ ft., radius of the invert one-eighth of the transverse diameter, and the radius of the sides one and one-third times the transverse diameter. (Univ. 1888.) *Ans.* 6,600 c. ft.

CHAPTER VIII.

FLOW OF WATER IN RIVERS

CONTENTS

96. **Rivers.**—The principles which govern the flow of water in natural channels are the same as those which have been already laid down for artificial channels. The conditions in the former case are however more complex owing to constant variation in the channel section and consequently in the velocity, as well as to variation in the discharge at different seasons of the year. In hilly tracts, streams have considerable fall, high velocity, and great energy of motion; their courses are therefore direct, and well marked by the depressions through which they flow. In plains, these conditions are reversed; a slight obstacle suffices to change the direction of the river, the course accordingly becomes winding, and the longitudinal slope and the velocity are still further diminished. The solid matter detached from the bed and banks in the upper part of the river's course, and suspended in the water, is gradually deposited as the velocity diminishes, the level of the bed near the mouth becomes raised, periodic floods carry the silt over the surrounding country, and the coast line moves seaward. At length, during some especially heavy flood the river makes fresh channels for itself to the sea. The same process is then repeated here, and in the course of ages a *delta* of rich alluvial soil is formed, traversed by branches of the river, whose beds are generally at a higher level than the adjacent country. Thus the Kistna has in the upper part of its course a bed slope of $4\frac{1}{2}$ ft. per mile, lower down 2 ft. per mile, and in the delta about 1 ft. per mile, the fall of the country away from the river being $1\frac{1}{2}$ ft. per mile in this last part of its course.

The surface slope of any portion of a river depends on the bed slope, on any variation of breadth which may occur in that portion, and on the state of the discharge, i.e., whether the river is in flood or otherwise. For a given discharge and bed slope the depth of a river alters with the breadth; so that whenever the banks approach each other, the water heads up to produce the velocity necessary to carry the discharge through the contracted section. Hence the surface slope, on which the velocity depends, is not generally parallel to the bed slope. The Godaveri in the delta has a bed slope of 0·5 ft. per mile, the surface slope being 0·7 per mile in dry weather and 1·25 in floods.

To utilize the water of a river for cultivation, it must be conveyed from the river to the cultivable area by artificial channels. In ordinary country, where the river flows in a valley, this can only be accomplished by taking off the water at a point far above the area in question, and laying out the canal with a slope less than that of the river, so that it may command the country where the water is required. This is the usual problem in Upper India. In the great deltaic districts of Southern India, *viz.*, the Godaveri, Kistna, and Kaveri, the problem is simpler, since the canal has only to be taken off at the head of the delta, and, with its branches, to be conducted along the subsidiary watersheds in order to command the whole of the surrounding country.

In India the main channels of irrigation are made to subserve the purposes of navigation. In England canals are constructed solely for navigation.

97. Discharge of rivers.—In drawing up a project for the supply of water from a river, it is necessary to estimate the minimum, ordinary, and maximum discharges in order to fix the dimensions of the weir, head sluices, flood banks and other works. In designing a bridge, the maximum discharge only is required. There are three principal modes of estimating the discharge, which should be used as checks on one another.

(i) Measurement of the mean cross section and longitudinal slope, and application of Kutter's or Bazin's formula for the velocity.

(ii) Direct measurement of the velocity.

(iii) Measurement of the drainage area, observations of the rainfall, and estimation of the amount which reaches the river.

Methods (i) and (ii) are suited for any discharge, method (iii) is best adapted to flood discharges only. If a weir already exists higher up the river, a further check is obtained by calculation of its discharge.

98. Discharge by velocity calculation.—A straight reach of the river, with a regular transverse section, is selected, from $\frac{1}{2}$ to $\frac{3}{4}$ mile long. Four equidistant cross sections are taken, and connected by longitudinal levels. The differences between the water levels

Plate XIII. at the sections give the surface fall for the then discharge of the river. For maximum flood discharges, reliance must be placed on flood marks on the banks and on the information furnished by village testimony; and the cross section and surface fall must be measured to the points so arrived at. The H.M.D. of each cross section is then calculated, and Kutter's or Bazin's formula used to obtain the proper co-efficient. The velocities deduced from the cross sections are then compared; and, if the product Av is approximately the same for each section, the calculation may be deemed satisfactory.

The value of N to be used in Kutter's formula (Art. 83), must be chosen with reference to that which has been found applicable to rivers similarly situated. It may vary from ·020 to ·035. The following are a few local values:—

Ohio at Point Pleasant	·021
Seine at Paris	·025
Mississippi	·027
Rhine at Basle	·030

99. Cross sections.—Cross sections are taken as follows:—A wire, marked off with pendants at equal distances, is stretched across the stream at right angles to its axis, and the depth at each pendant is measured with a wooden staff, provided with a disc at the end to prevent penetration of the bed. If the river is too broad or too rapid for the convenient adoption of this plan, staves may be set up at C,D,E, (fig. 69), the angle DCE being a right angle. A boat is allowed to drop down towards the cross section. At the instant of its reaching CD a sounding is made with an ordinary "lead line," previously suspended within a short distance of the bottom, the position F of the boat being determined simultaneously by an angular measurement made from the boat with a pocket sextant, or from E with a theodolite. When a sufficient number of soundings has been taken, the cross section can be plotted, and the area and border estimated.

100. Measurement of velocity.—In the second method of obtaining the discharge, the preliminary work of taking cross sections and levelling for the longitudinal slope is the same as before, except that the sections may be closer together. If the stream is a small one, it will be sufficient to mark out two lines across it 50 ft. apart, and to take several observations of the time occupied by a float, placed in the axis of the stream, in traversing the distance between the lines. The mean of the observations gives the maximum surface velocity $\mathbf{v}$, and the mean velocity v can then be obtained by Bazin's relation (Art. 92), $v = \frac{c}{c + 25}\mathbf{v}$, where c is the co-efficient proper to the H.M.D. of the channel. The cross sections should be at least three in number one at each end of the "run," and one Plate XIII midway between.

If the stream is a large one, *velocity rods* should be used. Two wires furnished with pendants at convenient intervals are stretched across the extremities of a 50 ft. run.* A hollow rod, sufficiently long to extend from the surface nearly to the bottom, is started above any pendant at the upper section, and the time interval to the corresponding pendant at the lower section is noted with a stop watch. Unless the rod passes near the proper pendant at the lower section, the observation should be discarded. It has been shewn by experiment † that such a rod measures very approximately the mean velocity of the vertical fluid plane in which it moves. The rods are made in different lengths so as to suit variations of depth, the proper length of rod to use at any pendant being determined by the soundings previously made at the cross sections. The rods (fig. 70), are cylindrical, 1 inch in diameter, and may be made of sheet tin loaded at the base with iron and adjusted with shot, so as to float with about 2 inches out of water. The top is closed, and should be marked with a tuft of cotton wool. When the observations are complete, the discharge can be readily found as follows.

Suppose the river breadth to be divided into convenient segments CD, DE, EF, &c. (fig. 71), of lengths l_1, l_2, &c. Pendants 1, 2, 3, 4, &c., mark the middle points of the segments and the courses of the velocity rods. The mean depth d along the course of each rod is ascertained by sounding, and the velocity v of the rod is determined by observation. The discharge of any segment is $(ld)v$, and the whole discharge is

$$Q = \Sigma\,(ldv) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

The mean velocity is $Q \div A$, where $A = \Sigma\,(ld)$.

Ex. 65.—Find the discharge of a river from the data given in the first three lines of the following table:—

—	ft.	ft.	ft.	ft.	ft.	ft.	ft.
Lengths of segments	$l_1 = 16{\cdot}5$	$l_2 = 20{\cdot}0$	$l_3 = 25{\cdot}0$	$l_4 = 32{\cdot}0$	$l_5 = 30{\cdot}0$	$l_6 = 26{\cdot}8$	$l_7 = 18{\cdot}3$
Mean depths	$d_1 = 4{\cdot}8$	$d_2 = 9{\cdot}7$	$d_3 = 12{\cdot}4$	$d_4 = 15{\cdot}7$	$d_5 = 12{\cdot}0$	$d_6 = 9{\cdot}7$	$d_7 = 4{\cdot}8$
Mean velocities	$v_1 = 2{\cdot}25$	$v_2 = 3{\cdot}80$	$v_3 = 4{\cdot}62$	$v_4 = 5{\cdot}0$	$v_5 = 4{\cdot}65$	$v_6 = 3{\cdot}75$	$v_7 = 2{\cdot}00$
	c. ft.	c. ft.	c. ft.	c. ft.	c. ft.	c. ft.	c. ft.
Discharges	$Q_1 = 178{\cdot}2$	$Q_2 = 737{\cdot}2$	$Q_3 = 1432{\cdot}2$	$Q_4 = 2512{\cdot}0$	$Q_5 = 1674{\cdot}0$	$Q_6 = 974{\cdot}9$	$Q_7 = 175{\cdot}7$

∴ Whole discharge Q = 7684 c. ft. per sec.

* If a superior time-keeper is not available, the run should be 100 ft.

† Roorkee Hydraulic Experiments, Cunningham. Roorkee, 1881.

Plate XIII. Greater accuracy may be secured by placing the pendants at equal intervals throughout the breadth of the stream, and using either the parabolic formula (Simpson's), or the sextic (Weddle's), in place of the trapezoidal formula employed above. Simpson's rule requires that the number of intervals should be a multiple of 2, Weddle's that it should be a multiple of 6; and the latter gives the best results. Let the length of each interval (fig. 72), be k. Then, the velocity at the banks being taken zero,

$$\text{Simpson's rule gives } Q = \frac{k}{3}\left(0 + 4d_1v_1 + 2d_2v_2 + 4d_3v_3 + 2d_4v_4 + 4d_5v_5 + 0\right)$$

$$= \frac{2k}{3}\left\{d_2v_2 + d_4v_4 + 2(d_1v_1 + d_3v_3 + d_5v_5)\right\} \quad ..(62)$$

$$\text{Weddle's rule gives } Q = \frac{3k}{10}\left(0 + 5d_1v_1 + d_2v_2 + 5d_3v_3 + d_4v_4 + 5d_5v_5 + 0\right)$$

$$= \frac{3k}{10}\left\{v_2d_2 + v_3d_3 + v_4d_4 + 5(v_1d_1 + v_3d_3 + v_5d_5)\right\} \quad ..(63)$$

If the cross section is of approximately uniform depth between the feet of sloping banks, it may be advantageously divided into a central portion of six intervals, and side portions each of two intervals. The discharge of the central portion can be calculated by Weddle's rule, and that of the side portions by Simpson's.

101. Other velocity meters.—Many instruments have been devised to measure the velocity at a point; but they are not of much service in the ordinary operation of river gauging. Among the best known are:—

(1) *Screw current meter.*—This consists of a small screw, like the propeller of a steam ship, which is operated on by the current and records the number of its revolutions on a counter. The screw is kept with its head against the current by a large vane place in rear. The instrument is lowered on a rod to the position required, and can be put in or out of gear at will. The objection to it is that the relation between number of revolutions and current velocity has to be determined previously by drawing the instrument through still water at known velocities, and that this relation is liable to change by the moving parts becoming clogged.

(2) *Pitôt tube.*—This is a graduated glass tube bent to a right angle near one end, the shorter arm being coned so as to present a small orifice to the current. The difference of water level inside and outside the tube measures the velocity. At a depth h in the stream the total head is $\left(h + \frac{v^2}{2g}\right)$. There being no velocity in the tube, the head is $(h + h_1)$ where h_1 is the difference of water levels. Hence $h_1 = \frac{v^2}{2g}$*. This instrument was used by Darcy in his velocity experiments. The objection to it is that, as the velocity at a point varies from instant to instant, and as the tube must be closed and removed from the water to be read, there is no certainty that a mean reading has been obtained It cannot be used moreover for very low velocities.

(3) *Perrodil's Hydrodynamometer.*—This is a torsion balance. A vane placed at right angles to the current twists a vertical wire, the angle of twist being read on an arc above water. The index oscillates with the variation of the velocity, and the mean angle can readily be observed. The relation between the angle and the current velocity can be obtained by calculation.

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* Pitôt used a bell-mouthed tube as shewn in fig. 73. He found by experiment that h_1 was equal to $1{\cdot}5\,\frac{v^2}{2g}$.

102. Maximum Flood Discharge.—To obtain the *maximum flood discharge*, the cross sections are carried up to the highest flood marks, and their areas, and hydraulic mean depths estimated. Then

$$\frac{Q_1}{Q} = \frac{cA_1\sqrt{r_1}}{cA\sqrt{r}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (64)$$

where Q is the discharge calculated from the measured velocity. Q_1 is the required maximum flood discharge. It should be borne in mind however that the bed level during maximum floods is often lowered by scour.

103. Flood Discharge from Catchment Basins.—The catchment basin of a river or tank is the whole area whose rainfall tends to flow into the river or tank. The area can readily be found from a contoured map, since its boundary is marked by a watershed, the drainage on the inner side of which flows to the basin in question, while that on the outer side discharges into other basins. The size of the basins with which we have to deal varies from 115,000 square miles for the Godaveri, to a fraction of a square mile for a small tank. Part of the rainfall fails to reach the point of final discharge, owing to absorption by the soil and evaporation. The amount lost depends chiefly on the nature of the soil, the fall of the country, and the shape of the basin. The maximum rainfall in, say, 24 hours, for a given basin, must be obtained from the register kept at the nearest station where a rain-gauge is set up. The rate of discharge from the basin cannot, however, be based directly on this fall because

(1) heaviest rainfall is very partial, *i.e.*, the record at a station for a particular storm applies only to a very limited area, perhaps about 5 sq. miles, around the station. Equally heavy rainfall may occur at other points in the basin, but not at the same time.

(2) as the basin area increases, the greater is the probability that the flow from ground near the point of discharge will have ceased before the flow from remoter portions has had time to come in.

To meet the proportionate reduction thus required for large areas, various empirical formulæ have been proposed, those chiefly employed in Southern India being

$$\text{Ryves' formula} \quad Q = cM^{\frac{2}{3}} \ldots\ldots\ldots\ldots \quad (65)$$

$$\text{Dickens'} \quad ,, \quad Q = c_1M^{\frac{3}{4}} \ldots\ldots\ldots\ldots \quad (66)$$

where M is the catchment area in square miles, c, c_1 local coefficients depending on the rainfall, soil and slope of the district.

Values of the co-efficients suited to particular districts can be deduced from measured maximum flood discharges from known catchment basins. Thus if the maximum flood discharge of a stream draining 80 s. miles is measured as 9,500 c. ft. per se., it follows that $c = 510$, $c_1 = 355$.

To obtain the flood discharge of one of a group of tanks in the same drainage basin, the practice of the Madras Irrigation Department is as follows:—if M be the area feeding the tank in question, M_1 the area feeding the tanks above it, then $Q = cM^{\frac{2}{3}} - \frac{c}{5}M_1^{\frac{2}{3}}$.

General values of c in Ryves' formula are—

In flat districts near the coast .... $c = 450$
In districts extending from 20 to 50 or 100 miles from the east coast .... $c = 550$
For limited areas near hills .... $c = 700$

In any particular case however, where the maximum flood discharge is required, it is best to have recourse to the rainfall registers. Let d inches be the heaviest fall recorded in 24 hours. The volume received by the standard area of 5 sq. miles is $\frac{d}{12} \times (5280)^2 \times 5$. In order to be on the safe side, assume that the whole of this passes to the point of discharge. Then the discharge in c. ft. per sec. from the standard area is

$$Q_1 = \frac{d}{12} \times \frac{(5280)^2 \times 5}{24 \times 60 \times 60} = 135d \text{ nearly. But } Q_1 = c(5)^{\frac{2}{3}} = c_1(5)^{\frac{3}{4}}$$

$$\therefore c = \frac{135d}{5^{\frac{2}{3}}} = 46d \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (67)$$

$$\therefore c_1 = \frac{135d}{5^{\frac{3}{4}}} = 40d \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (68)$$

The heaviest rainfall recorded for shorter periods than 24 hrs. would of course give a higher rate of discharge; and in the case of small catchment basins, it might be desirable to take 12 hrs. or even 6 hrs. as the basis of calculation.

Ex. 66.—The catchment basin of a river above a given point in its course is 150 s. miles. The maximum recorded rainfall at a meteorological station in the vicinity is 11 inches in 24 hrs. Estimate the probable maximum flood discharge of the river at the given point.

$c = 46 \times 11 = 506$. $\qquad Q = 506\,(150)^{\frac{2}{3}} = 14{,}285$.

If Dickens' formula were used, the discharge obtained would be 18,840.

It will be observed that these empirical formula are very imperfect; and that trust must not be placed unreservedly on the results deduced from them. The rainfall records are not readily applied to minimum discharges because the water in rivers during the dry season is largely derived from bed springs.

Attempts have been made to take account of the form of the basin. Burge* proposed $Q = c\frac{M}{\sqrt[3]{L^2}}$, where L is the extreme length of the catchment area in miles, and c is a co-efficient which may be taken at 1,300 for Madras. Craig* divides the basin into a series of triangles each of which has one angle at the point of discharge and one side on the perimeter of the basin. Calling 2B miles the length of the side, and L miles the distance from its middle point to the point of discharge, he obtains for the area in s. ft. of the unobstructed flood section of the river at the point of discharge, $A = 184\sum\left(B \log \frac{L^2}{B}\right)$. This expression appears to give good results, although it has been incorrectly deduced from the premises.

Ex. 67.—The catchment basin above a bridge near Chickli, Berar,† can be divided into three triangles, the dimensions of which, in miles, are

L	B	
1·84	0·61	whence area of basin is 8·5 s. miles nearly.
2·60	0·68	
1·66	0·34	

$$\text{Area of flood section} = 184\left\{\cdot 61 \log \frac{8(1\cdot 84)^2}{\cdot 61} + \cdot 68 \log \frac{8(2\cdot 60)^2}{\cdot 68} + \cdot 34 \log \frac{8(1\cdot 66)^2}{\cdot 34}\right\}$$

$$= 184\left\{(\cdot 61 \times 1\cdot 657) + (\cdot 68 \times 1\cdot 900) + (\cdot 34 \times 1\cdot 812)\right\} = 587 \text{ s. ft.}$$

The actual cross section as measured by the highest flood at the bridge was 557 s. ft.

104. River bends.—Bends existing in rivers flowing in alluvial plains tend to become constantly sharper, the bank on the outer side of the curve being cut away, while silt is deposited on the inner side. This action tends to increase the length of the river's course, and thus to diminish its fall per mile, and consequently its velocity. Ultimately the bends of a river may approach each other so closely that a cut through takes place.

The erosion of the outer bank is due to the centrifugal force developed as the water passes round the curve. At any radial cross section the surface water flows from the inner towards the outer side of the bend, while the fluid near the bed takes its place by movement in the opposite direction, and thus deposits silt on the inner bank.

105. Regime of rivers.—A river is said to be in a state of *régime* or *stability* when its form changes but little from year to year. Owing to variations of discharge and the consequent erosions and siltings which occur at different seasons of the year, a condition of permanent stability is difficult of attainment; and this is especially the case in Indian rivers which generally have sandy beds, and which are subject to heavy floods. A wide field is thus opened up for river improvement, which consists in the protection of the banks, the prevention of inundations, and the removal of obstructions. This subject is dealt with in the Manual on *Irrigation Works*.

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* Pro. Inst. C. E., Vol. LXXX, 1884-85. Craig on the Discharge of Catchment Areas.

† Taken from Professional Papers on Indian Engineering, 3rd series, Vol. IV, 1886.

EXAMPLES ON CHAPTER VIII

Plate XIII. 1. Give a brief description of the peculiar conditions which make deltaic rivers, such as the Godaveri, particularly suitable for irrigation purposes. What considerations in such cases would fix the position and height of the anicut?

2. What do you understand by the term *catchment basin* of a river? How is it determined? State briefly two independent methods of ascertaining the flood discharge of a river at any given point. (Coll. 1883).

3. Give a description of the process of the formation of a delta, and draw an imaginary section parallel to the sea coast of a delta formed by two main branches of a river with several intermediate minor branches.

Shew from this that natural channels in a delta are generally more advantageous for irrigation purposes than artificial channels. (Univ. 1874).

4. How would you ascertain the water-way required for a bridge (*a*) when the stream is dry; (*b*) when the river is in flood, and over 100 yds. wide? (Coll. 1884).

5. If the hydraulic mean depth of a river is 6·62 ft. and the fall per mile 6·36 ft., what will be the mean velocity of the stream in miles per hour? (Univ. 1876). *Ans.* 5·2 miles per hour.

6. How can the velocity of a river during flood be ascertained by observations made at the time; and how can it be calculated approximately from data obtainable after the flood has subsided? (Univ. 1875).

7. A river is 270 ft. wide by 10 ft. deep, with banks which for all practical purposes are vertical, and with a fall of 2 ft. per mile. How high should an anicut be to raise the water 3 ft.? (Univ. 1870). *Ans.* 7·1 ft.

8. In a channel 20 ft. wide at bottom, side slopes $1\frac{1}{2}$ to 1, four observations were made when the water was flowing 3 ft. deep, by a surface float in midstream passing between two points 100 ft. apart. The times observed were 46, 49, 50 and 48 seconds. What was the discharge in c. ft. per sec.? *Ans.* 110 c. ft. s.

9. If deputed to the work of ascertaining how much water escaped to the sea by the Cooum river in the months of October and November, how would you set to work so as to obtain the best result? Explain fully all the practical operations you would go through, and then the calculations you would make? (Univ. 1870).

10 Determine approximately the flood discharge of a river, of the section given in fig. 74, when the mid-surface velocity is determined by observation to be 2·4 ft. per sec. *Ans.* 1,600 c. ft. s. Plate XIII.

11. Enumerate the modes you are acquainted with for determining the discharge of rivers.

The catchment of the Cooum above Madras is 265 sq. miles. Some distance up the river, at Koratur anicut, above which the catchment area is 200 sq. miles, the flood discharge, calculated from the head on the anicut in maximum flood, is 10,600 c. ft. per sec Estimate the maximum flood discharge at Madras. (Univ. 1890.) *A*. 12,790 c. ft. s.

MISCELLANEOUS EXAMPLES

1. A channel irrigates 59,580 acres and the duty of water is 60 acres per c. ft. per sec. The fall is $\frac{1}{2000}$, depth of water 5 ft., side slopes 1 to 1. What should be the bed width? Co-efficient c in Kutter's formula = 78.

In the channel there is a drop in bed level of 6 ft. Ascertain the height to which the fall should be built above bed level in the upper reach in order that water may be delivered into the lower reach with the normal channel velocity. Length of fall equals bed width, $c = \frac{5}{8}$. (Univ. 1900.)

2. A channel leads to a tank the capacity of which is 3,000 millions of c. ft.; fall of channel $1\frac{1}{2}$ ft. per mile. Depth generally permissible in channel 7 ft. The tank is to be filled in 12 days. Give a cross section of the channel. (Univ. 1899.)

3. A tank system consists of four tanks, A, B, C, D. Tank A with catchment area 5 s. miles, discharges over two weirs, one 75 ft. long into tank B, and the other 30 ft. long into tank C. Tank B has a catchment area of 4 s. miles, and tank C of 6 s. miles, and they both surplus into tank D, the catchment area of which is 8 s. miles What will be the maximum flood discharge from each tank according to Ryves' formula, using the co-efficients 450 and 90. (Coll. 1898.)

4. Calculate the flood discharge in the following case:—A bridge of 15 arches of 40 ft. span, 5 ft. rise, piers 5 ft. thick, is built in connexion with a dam with crest 9 ft. above bed. Depth of water in flood on crest 12 ft., afflux 4 ft., velocity of approach 8 ft., springing line of arches 7 ft. above crest of dam. Give reasons for the adoption of the co-efficients you use. (Univ. 1897.)

5. It is required to excavate a channel to irrigate 15,000 acres, and the duty of water is fixed at 2 c. yds. per acre per hour. The channel takes off above an anicut the crest of which is at + 25·00. Assuming that the water level in front of the head sluice at which full discharge required will be obtained is + 24·00, sill of head sluice + 18·00, water level in rear of head sluice + 23·00, calculate ($c = \frac{5}{8}$) the length of vent 4 ft. high, which is required, and determine the section of the channel assuming a fall $\frac{1}{10000}$ and side slopes 1 to 1 ($c = 60$). (Coll. 1898.)

6. The catchment basin of the Periyar lake is 350 s. miles. A maximum fall of 12 inches in 12 hours has been observed at one station in the basin. Ascertain the flood discharge, assuming that the whole of the rainfall on a standard area of 10 s. miles reaches the surplus. (Univ. 1897.)

7. A channel is to be excavated to irrigate 15,000 acres at the rate of 2 c. yds. per acre per hour with a velocity of about 2 ft. per sec. Assuming that the fall available is $\frac{1}{5000}$ in the first reach and $\frac{1}{4000}$ in the second reach, determine the bed width. Side slopes 1 to 1. N in Kutter's formula = ·025. (Coll. 1897.)

8. A sluice irrigates 5,000 acres. Determine the duty of water from the following data :—Sill + 10·00, top of vent + 13·00, water level in front + 15·00, water level in rear + 13·00, width of vent $2\frac{1}{2}$ ft., $c = \frac{5}{8}$. (Coll. 1895.)

9. The water-supply of a town is derived from a reservoir in which water stands 30 ft. above the centre of the inlet end of the pipe. The water main is 2 miles long and 18 inches diameter, and is laid at a slope of 50 ft. per mile. If the allowance for each inhabitant is 60 gals. per day, of which two-thirds must be delivered in 8 hours, what population can be supplied? Water to be delivered 50 ft. above the end of the main, c = 78. (Coll. 1895.)

10. A tank has a catchment area of 27 s. miles, and the maximum flood discharge is to be disposed of over the crest and through the vents of an escape. The vents are 2 ft. deep, and their lower sills are 2 ft. below crest level. Calculate, using the formula $Q = 450\ M^{\frac{2}{3}}$, the flood discharge of the catchment area and the length of escape required on the assumptions that (1) one-fourth of the maximum flood discharge is to be passed through the vents when water has risen to F.T.L., (2) maximum water level is 2 ft. above F.T.L. Use $\frac{5}{8}$ as co-efficient in both notch and orifice formulæ, and assume that water level in rear is always below sill of vents. (Coll. 1893.)

11. At a certain place on a main irrigation and navigation canal there is a lock, and a masonry drop. The canal irrigates 84,700 acres, and has a bottom width of 100 ft., side slopes of 1 to 1, and bed fall of 1 in 10,000. Determine (1) the depth of water required in the canal, (2) the crest level of the drop, with reference to bed level of canal, in order that depth of water may be maintained, (3) the area of vents required for the lock sluices, in order that no boat arriving when lock is empty may be detained for more than 15 minutes, of which 5 are required to open and close gates

and haul the boat through the lock, 4 minutes for filling and 6 minutes for emptying the lock. The data are—

(a) Duty of water, 70 acres per 1 c. ft. s.
(b) Velocity in main canal = $80\sqrt{rs}$.
(c) Length of drop, 75 ft.
(d) Co-efficient for drop and sluices, $\frac{5}{8}$.
(e) Dimensions of lock, 150 ft. by 20 ft.
(f) Lift of lock, 9 ft.
(g) Centres of lock sluices, 4 ft. below water level in upper and lower reaches. (Coll. 1893.)

12. A large high level cistern can be emptied by a pipe 200 ft. long and 1 inch diameter, proceeding vertically downward from the bottom of the cistern. The water on leaving the pipe falls 6 ft. further into a river. If there is 5 ft. of water in the tank, find the discharge from it in c. ft. per minute when the pipe is open; and find also the velocity of the water jet as it reaches the river. (Univ. 1891.)

13. An aqueduct of rectangular section, built in brickwork, is 20 ft. wide and has a fall of 0·3 ft. per 1,000. Estimate the mean velocity when the water runs 4 ft. deep. In Bazin's formula $a = \cdot 004$, $\beta = \cdot 2$ for brickwork. (Univ. 1891.)

14. A channel is to be designed to carry a full supply of 528 c. ft. per sec. with a depth of 4 ft. A full supply velocity of 3 ft. per sec. is considered suitable, and the soil permits of the side slopes being at 1 to 1. Sketch a dimensioned section of the channel, and calculate what the fall per mile should be. The co-efficient of velocity can be selected from the following table:—

H.M.D.	2·5	3·0	3·5	4·0	4·5
c.	67	70	73	76	78

(Univ. 1891.)

15. Sixteen thousand acres are to be irrigated for rice cultivation from a river in which freshes occur at intervals of 30 days, the freshes lasting on an average 10 days. What storage should be provided; and what should be the discharge of the supply channel, allowing 2 c. yds. per hour per acre as normal supply? (Univ. 1892.)

16. A lock chamber is 150 ft. long, and has a mean width, allowing for face batter, of $21\frac{1}{2}$ ft. The levels connected with it are—

Floor of lock	6·88
F.S.L. lower gates	12·88
Upper sill (bed of upper reach) ...	9·72
F.S.L. upper gates	15·72

The lock is filled by two tunnels, one in each side wall. Each tunnel is 3 ft. square in section at the off take, the sill of the opening being at bed level of upper reach. The tunnel then turns at right angles, so as to run parallel to the axis of the lock. There is a sudden drop to the lock floor, the roof of the tunnel remaining at its original level. The deep part of the tunnel communicates with the lock chamber by three arched openings, 3 ft. wide and $3\frac{1}{2}$ ft. high with their sills at floor level. The sluice shutter is placed above the drop, and is worked by a rack and pinion so as to give a full opening of 3 ft. by 3 ft. in a few seconds. Neglecting loss of time due to gradual opening of shutters, find the time required to fill the lock when the canal carries its full supply. Both shutters are opened simultaneously. (Univ. 1891.)

17. A sluice of three vents, 16 ft. long by 9 ft. deep, situated at the end of a drain, the water in which has a velocity of $1\frac{1}{2}$ ft. per sec., discharges 1,200 c. ft. per sec. into a river. Allowing 4 per cent for friction, what will be the head at the sluice? (Univ. 1892.)

18. A reservoir with a contour of 55 acres at R. L. 20·00 and 32 acres at R. L. 17·00 has a sluice with a vent 1 ft. square at R. L. 10·00, discharging free. Assuming the area decreases uniformly with the depth, calculate the time it will take to fall each foot down to R. L. 17·00. Co-efficient for sluice, ·62. (Coll. 1892.)

19. A town of 50,000 inhabitants is to be supplied at the rate of 15 gals. per head per day from a service reservoir with its floor at + 100, depth of water 12 ft. The town consists of two parts, and the supply is to be divided in the proportion of 1 to 4, the division taking place at a distance of $\frac{1}{4}$ mile from the reservoir. The smaller part of the town is distant $2\frac{1}{2}$ miles and the larger $1\frac{1}{2}$ miles from the reservoir. What would be suitable diameters for the main and sub-mains? What would be the pressure per sq. in. at the town, the level of the pipes in both parts being + 12? The pipes should be made capable of discharging one-half the supply in 8 hours. (Univ. 1892.)

20. The Pilandorai channel at its tenth mile crosses a stream draining an area of 5 s. miles. It is proposed to pass the drainage by an inlet and outlet, and by an inverted siphon with the same outfall channel. The siphon is to be capable of discharging 2 in. and the outlet 10 in. of a maximum rainfall of 12 in. in 24 hours. There is 3 ft. depth of water during flood on crests of inlet and outlet. The rear floor of outlet is at level of channel bed, and the gauge on it reads 3 ft. The crests are 3 ft. above bed of channel, and the velocity of approach is 4 ft. per sec. What should be the size of the siphon, and the length of the outlet? (Univ. 1892.)

21. A head sluice and channel are to be designed to supply 4,000 acres of rice cultivation at 2 yards per hour per acre. The depth of water on front floor of sluice varies from 3 ft. 9 in. to 10 ft.; the depth of water on rear floor (on the same level as front floor), from which the channel starts with a fall of $1\frac{1}{2}$ ft. per mile, is to remain constant as nearly as possible at 3 ft. Show how this may be accomplished. What size would you propose to make the vents of the head sluice? (Univ. 1892.)

22. The rainfall over the catchment basin, 20 s. miles in extent, of a tank was half an inch an hour; after a time the depth flowing over the escape, 100 ft. long, was observed to remain steady at 5 ft. with the tail water one foot above the weir crest. Calculate the discharge, and the percentage of rainfall running into the tank. (Univ. 1893.)

23. A railway embankment, having a girder bridge 40 ft. span with splayed wings in it, divides a tank in two. While the tank is filling, the water is found to be 6 ft. deep on the upper side of the bridge, and 5 ft. on the lower side. What quantity of water is passing through the bridge? State the approximate bottom velocity, and whether a masonry floor is necessary. (Univ. 1893.)

24. Find the time necessary to fill a service reservoir with vertical sides, 100 ft. square inside, supplied from a storage reservoir by a pipe 12,544 ft. long, one foot in diameter:—

Centre of pipe inlet	10	ft. below	water level.
Do. at exit in service reservoir	196	do.	do.
Bottom of service reservoir	199	do.	do.
Full water level in service reservoir	169	do.	do.

(Univ. 1893.)

25. Find the fall in feet per mile of a channel to discharge 400 c. ft. per sec. with a bottom width of 40 ft., a depth of 3 ft., and side slopes of 2 to 1.

The above channel is supplied by a head sluice having four vents, each 4 ft. wide and $2\frac{1}{2}$ ft. high, with its sill on the same level as the bed of the channel. Find the height of the anicut crest above the bed of the channel in order that no water may pass over the anicut until the channel discharge reaches 300 c. ft. per sec. (Coll. 1880.)

26. What is the maximum aggregate width which can be given to the piers of a bridge crossing a river 200 ft. wide with vertical banks, and a fall of $3\frac{1}{2}$ ft. per mile, so that the velocity of a flood 10 ft. deep shall not be greater than 6 miles an hour when passing through the bridge? To what height would the water be headed

back on the upper side of the piers, if the latter were built of this maximum width? (Univ. 1880.)

27. From a large distributing reservoir, which is fed by a rubble masonry channel 6 ft. wide and 2 ft. deep, having vertical sides and a slope of 9 ft. per mile, it is proposed to supply a town, distant half a mile, with water, to be drawn off from the side of the reservoir by a circular pipe with its centre 30 ft. below the surface of the water in the reservoir. The head of the pipe is 50 ft. above the town. What diameter should the pipe have in order that it may discharge the same quantity of water as the channel brings this being the quantity required by the inhabitants? (Univ. 1870.)

28. A river is 300 ft. wide at water surface; area of cross section 1,474 s. ft.; length of wetted border 335 ft.; fall 16 inches per mile. Assuming the banks to be vertical above the original surface of the water, to what height must an anicut be built to raise the level of the water 3 ft.? (Univ. 1877.)

29. A new channel is 20 ft. wide at bottom with slopes of 2 to 1; the surface velocity with 4 ft. of water was found to be 117 ft. per minute. What is the least span of bridge which will pass a flood 5 ft. deep, the surface velocity being limited to 200 ft. a minute? What amount of heading up will this bridge cause? (Univ. 1882.)

APPENDICES

APPENDIX I. Bazin's co-efficients for earthen channels.

" II. Kutter's co-efficients for pipes, channels and rivers.

" III. Values of $\sqrt{s}$ for use in the formula $v = c\sqrt{rs}$.

APPENDIX I

BAZIN'S CO-EFFICIENTS
APPLICABLE TO EARTHEN CHANNELS

Table of values of c in the expression $v = c\sqrt{rs}$ where

$$c = \sqrt{2g} \div \sqrt{\cdot 00592\left(1 + \frac{4\cdot 10}{r}\right)}$$

r	c	r	c	r	c	r	c
1	16	1·75	57	3·4	70	5·1	77
·2	22	1·8	57	3·5	71	5·2	78
·25	25	1·9	59	3·6	71	5·25	78
·3	27	2·0	60	3·7	72	5·3	78
·4	31	2·1	61	3·75	72	5·4	78
·5	34	2·2	61	3·8	72	5·5	79
·6	37	2·25	62	3·9	73	5·6	79
·7	40	2·3	62	4·0	73	5·7	79
·75	41	2·4	63	4·1	74	5·8	80
·8	42	2·5	64	4·2	74	5·9	80
·9	44	2·6	65	4·25	74	6·0	80
1·0	46	2·7	66	4·3	74	6·5	81
1·1	48	2·75	66	4·4	75	7·0	83
1·2	49	2·8	66	4·5	75	7·5	84
1·25	50	2·9	67	4·6	76	8·0	85
1·3	51	3·0	68	4·7	76	8·5	85
1·4	52	3·1	68	4·75	76	9·0	86
1·5	54	3·2	69	4·8	76	10·0	88
1·6	55	3·25	69	4·9	77		
1·7	56	3·3	69	5·0	77		

APPENDIX II

KUTTER'S CO-EFFICIENTS

APPLICABLE TO PIPES, CHANNELS AND RIVERS

* Table of values of **c** in the expression $\mathbf{v} = \mathbf{c}\sqrt{\mathbf{rs}}$ where

$$\mathbf{c} = \frac{41{\cdot}6 + \dfrac{1{\cdot}811}{N} + \dfrac{{\cdot}00281}{s}}{1 + \left(41{\cdot}6 + \dfrac{{\cdot}00281}{s}\right)\dfrac{N}{\sqrt{r}}}$$

where **r** is the H. M. D., **s** the longitudinal slope, N a co-efficient of roughness.

	N.
Channels of well-planed timber	·009
Do. of neat cement; glazed pipes; very smooth iron pipes	·010
Do. of plaster; smooth iron pipes	·011
Do. of unplaned timber; ordinary iron pipes ...	·012
Do. of ashlar or brickwork	·013
Do. of rubble masonry	·017
Canals in very firm gravel	·020
Canals and rivers in moderately good order, free from stones and weeds	·025
Do. having stones and weeds occasionally	·030
Do. bad order, overgrown with vegetation and strewn with stones ...	·035

Slope S = ·000025 per unit of length = 1 in 40000 = ·132 foot per mile.

H.M.D. r	Co-efficients N of roughness.												H.M.D. r
	·009	·010	·011	·012	·013	·015	·017	·020	·025	·030	·035	·040	
	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	
·1	65	57	50	44	40	33	28	23	17	14	12	10	·1
·2	87	75	67	59	58	45	38	31	24	19	16	14	·2
·4	111	97	87	78	70	59	51	42	32	26	22	19	·4
·6	127	112	100	90	81	69	60	49	38	31	26	22	·6
·8	138	122	109	99	90	77	66	55	43	35	30	25	·8
1	148	131	118	106	97	83	72	60	47	38	32	28	1
1·5	166	148	133	121	111	95	83	69	55	45	38	33	1·5
2	179	160	144	131	121	104	91	77	61	50	43	37	2
3	197	177	160	147	135	117	103	88	70	59	50	44	3
4	209	188	172	158	146	127	113	96	78	65	56	49	4
6	226	206	188	174	161	142	126	108	88	74	64	57	6
8	238	216	199	184	171	151	135	117	96	82	71	63	8
10	246	225	207	192	179	159	142	124	102	87	76	68	10
12	253	231	214	198	186	165	149	129	107	92	81	72	12
16	263	242	223	208	195	174	157	138	115	100	88	79	16
20	271	249	231	215	202	181	164	144	121	106	94	84	20
30	283	261	243	228	215	193	176	157	133	117	104	95	30
50	297	274	257	241	228	207	190	170	147	130	117	107	50

* Taken from Trautwine's Civil Engineers' Pocket Book.

Co-efficients N of roughness.

Slope S = ·00005 per unit of length = 1 in 20000 = ·264 foot per mile.

H.M.D. r	·009	·010	·011	·012	·013	·015	·017	·020	·025	·030	·035	·040	H.M.D. r
	C.	C.	C.	C.	C.	C.	C.	C.	C.	C.	C.	C.	
·1	78	67	59	52	47	39	33	26	20	16	13	11	·1
·15	91	79	69	62	56	46	38	31	23	19	16	13	·15
·2	100	87	77	68	62	51	44	35	26	21	18	15	·2
·3	114	99	88	79	71	59	50	41	31	25	21	18	·3
·4	124	109	97	88	79	66	57	46	35	28	24	20	·4
·6	139	122	109	98	90	76	65	53	41	33	28	24	·6
·8	150	133	119	107	98	83	71	59	46	37	31	27	·8
1	158	140	126	114	101	89	77	64	49	40	34	29	1
1·5	173	154	139	126	116	99	87	72	57	47	40	34	1·5
2	184	164	148	135	124	107	94	79	62	51	44	38	2
3	198	178	161	148	136	118	104	88	71	59	50	44	3
4	207	187	170	156	145	126	111	95	77	64	56	49	4
6	220	199	182	168	156	137	122	105	85	72	63	56	6
8	228	206	189	175	163	144	129	111	91	78	68	61	8
10	234	212	195	181	169	149	134	116	96	82	72	64	10
12	238	217	200	185	173	153	138	120	99	86	75	68	12
16	245	223	206	191	180	160	144	126	106	91	81	73	16
20	250	228	211	196	184	165	149	131	110	96	85	77	20
30	258	236	219	204	192	172	157	139	118	103	92	84	30
50	266	245	228	213	201	181	165	148	127	112	101	93	50

Slope S = ·0001 per unit of length = 1 in 10000 = ·528 foot per mile.

H.M.D. r	·009	·010	·011	·012	·013	·015	·017	·020	·025	·030	·035	·040	H.M.D. r
·1	90	78	68	60	54	44	37	30	22	17	14	12	·1
·2	112	98	86	76	69	57	48	39	29	23	19	16	·2
·3	125	109	97	87	78	65	56	45	34	27	22	19	·3
·4	136	119	106	95	86	72	62	50	38	31	25	22	·4
·6	149	131	118	105	96	81	70	57	44	35	30	25	·6
·8	158	140	126	114	103	88	76	63	48	39	33	28	·8
1	166	147	132	120	109	93	81	67	52	42	35	31	1
1·5	178	159	144	130	120	103	89	75	59	48	41	35	1·5
2	187	168	151	138	127	109	96	81	64	53	45	39	2
3	198	178	162	149	137	119	104	89	71	59	51	45	3
4	206	186	169	155	143	125	111	94	76	64	55	49	4
6	215	195	178	164	152	134	119	102	84	71	61	54	6
8	221	201	184	170	158	139	124	107	88	75	66	59	8
10	226	205	188	174	162	143	128	111	92	78	69	62	10
15	233	212	195	181	169	150	135	118	98	85	75	68	15
20	237	216	200	185	173	154	139	122	102	89	79	71	20
30	243	222	206	191	179	160	145	128	108	95	84	77	30
50	249	227	211	197	185	166	151	134	114	100	91	83	50

Slope S = ·0002 per unit of length = 1 in 5000 = 1·056 foot per mile.

H.M.D. r	·009	·010	·011	·012	·013	·015	·017	·020	·025	·030	·035	·040	H.M.D. r
·1	99	85	74	65	59	48	41	32	24	18	15	12	·1
·2	121	105	93	83	74	61	52	42	31	25	21	17	·2
·3	133	116	103	92	83	69	59	48	36	29	24	20	·3
·4	143	125	112	100	91	76	65	53	40	32	27	23	·4
·6	155	138	122	111	100	85	73	60	46	37	31	26	·6
·8	164	145	131	118	107	91	79	65	50	41	34	29	·8
1	170	151	136	123	113	96	83	69	54	44	37	32	1
1·5	181	162	146	133	122	105	91	77	60	49	42	36	1·5
2	188	170	154	140	129	111	97	82	64	54	45	40	2
3	200	179	163	149	137	119	105	89	72	59	51	45	3
4	205	185	168	155	143	125	111	94	76	63	55	48	4
6	213	193	176	162	150	132	117	100	82	69	60	53	6
8	218	198	181	167	155	137	122	105	87	73	64	57	8
10	222	201	185	170	158	140	125	108	89	76	67	60	10
15	228	207	190	176	164	145	131	113	95	82	72	65	15
20	231	210	194	180	168	149	134	117	98	85	76	68	20
30	235	215	198	184	172	154	139	122	103	89	80	73	30
50	240	220	20[illegible]	189	177	158	143	126	108	94	85	78	50

Slope	H.M.D. r	·009	·010	·011	·012	·013	·015	·017	·020	·025	·030	·035	·040	H.M.D. r
		Co-efficients N of roughness.												
		C.	C.	C.	C.	C.	C.	C.	C.	C.	C.	C.	C.	
Slope S = ·0004 per unit of length = 1 in 2500 = 2·112 feet per mile.	·1	104	89	78	69	62	50	43	34	25	19	16	13	·1
	·15	116	101	90	80	71	59	50	40	29	23	19	16	·15
	·2	126	110	97	87	78	65	54	44	32	25	21	18	·2
	·3	138	120	107	96	87	73	62	50	37	30	24	21	·3
	·4	148	129	115	104	94	79	68	55	42	33	27	23	·4
	·6	157	140	126	113	103	87	75	62	47	38	31	27	·6
	·8	166	148	133	121	110	93	81	67	51	42	35	30	·8
	1	172	154	138	125	115	98	85	70	55	45	37	32	1
	1·5	183	164	148	135	124	106	93	78	61	50	42	37	1·5
	2	190	170	154	141	130	112	98	83	65	54	45	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	94	76	63	55	48	4
	6	211	191	175	161	149	130	116	99	81	69	60	53	6
	10	219	199	183	168	157	138	123	107	88	75	66	59	10
	20	227	207	190	176	164	146	131	115	96	83	73	66	20
	50	235	215	198	184	173	154	139	123	104	91	82	75	50
Slope S = ·001 per unit of length = 1 in 1000 = 5·28 feet per mile.	·1	110	94	83	73	65	54	45	36	27	21	17	14	·1
	·15	121	105	92	82	74	61	51	41	31	24	20	16	·15
	·2	129	113	99	89	81	66	57	45	34	27	22	18	·2
	·3	141	124	109	98	89	74	63	51	39	30	25	21	·3
	·4	150	131	117	105	96	80	69	56	43	34	28	24	·4
	·6	161	142	127	115	104	88	76	63	48	39	32	27	·6
	·8	169	150	134	122	111	94	82	68	52	42	35	30	·8
	1	175	155	139	127	116	99	86	71	56	45	38	33	1
	1·5	184	165	149	136	124	108	93	78	62	50	43	37	1·5
	2	191	171	155	142	130	112	98	83	66	54	46	40	2
	3	199	179	163	149	138	119	105	89	71	59	51	45	3
	4	204	184	168	154	142	124	110	93	75	63	54	48	4
	6	211	190	174	160	149	130	116	99	81	68	59	52	6
	10	218	197	181	167	155	136	122	105	87	74	65	58	10
	20	225	205	188	175	163	144	129	113	94	81	72	65	20
	50	232	212	196	182	170	151	137	120	101	89	79	72	50
Slope S = ·01 per unit of length = 1 in 100 = 52·8 feet per mile.	·1	110	95	83	74	66	54	46	36	27	21	17	14	·1
	·15	122	105	93	83	75	62	52	42	31	24	20	17	·15
	·2	130	114	100	90	81	67	57	46	34	27	22	19	·2
	·3	143	125	111	100	90	76	64	52	39	31	25	22	·3
	·4	151	133	119	107	98	82	70	57	44	35	29	24	·4
	·6	162	143	129	116	106	90	77	64	49	39	33	28	·6
	·8	170	151	135	123	112	95	82	68	53	43	35	31	·8
	1	175	156	141	128	117	99	87	72	56	45	38	33	1
	1·5	185	165	149	136	125	107	94	79	62	51	43	37	1·5
	2	191	171	155	142	130	112	99	83	66	55	46	40	2
	3	199	179	162	149	138	119	105	89	71	59	51	45	3
	4	204	184	167	154	142	123	109	93	76	63	55	48	4
	6	210	190	173	160	148	129	115	99	81	68	59	52	6
	10	217	196	180	166	154	136	121	105	86	74	65	58	10
	20	225	204	187	173	161	143	128	112	93	80	71	64	20
	50	231	210	194	181	168	150	135	119	100	87	78	71	50

APPENDIX III

VALUES OF $\sqrt{s}$ FOR USE IN THE FORMULA $v=c\sqrt{rs}$

Table I. **For fall per mile.**

Fall per mile.	$\sqrt{s}$	Fall per mile.	$\sqrt{s}$	Fall per mile.	$\sqrt{s}$	Fall per mile.	$\sqrt{s}$	Fall per mile.	$\sqrt{s}$
′ ″		′ ″		′ ″		′ ″		′ ″	
0 3	·0069	2 3	·0206	4 3	·0284	6 3	·0344	8 3	·0395
0 6	·0097	2 6	·0218	4 6	·0292	6 6	·0351	8 6	·0401
0 9	·0119	2 9	·0228	4 9	·0300	6 9	·0358	8 9	·0407
1 0	·0138	3 0	·0238	5 0	·0308	7 0	·0364	9 0	·0413
1 3	·0154	3 3	·0248	5 3	·0315	7 3	·0371	9 3	·0419
1 6	·0169	3 6	·0257	5 6	·0323	7 6	·0377	9 6	·0424
1 9	·0182	3 9	·0267	5 9	·0330	7 9	·0383	9 9	·0430
2 0	·0195	4 0	·0275	6 0	·0337	8 0	·0389	10 0	·0435

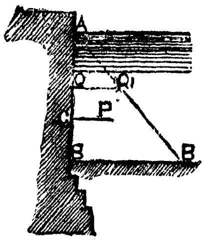
Table II. **For fall per 5,000 feet.**

Fall per 5,000 ft.	$\sqrt{s}$	Fall per 5,000 ft.	$\sqrt{s}$	Fall per 5,000 ft.	$\sqrt{s}$	Fall per 5,000 ft.	$\sqrt{s}$	Fall per 5,000 ft.	$\sqrt{s}$
′ ″		′ ″		′ ″		′ ″		′ ″	
0 3	·0071	2 3	·0212	4 3	·0292	6 3	·0354	8 3	·0406
0 6	·0100	2 6	·0224	4 6	·0300	6 6	·0361	8 6	·0412
0 9	·0122	2 9	·0235	4 9	·0308	6 9	·0367	8 9	·0418
1 0	·0141	3 0	·0245	5 0	·0316	7 0	·0374	9 0	·0424
1 3	·0158	3 3	·0255	5 3	·0324	7 3	·0381	9 3	·0430
1 6	·0173	3 6	·0265	5 6	·0332	7 6	·0387	9 6	·0436
1 9	·0187	3 9	·0274	5 9	·0339	7 9	·0394	9 9	·0442
2 0	·0200	4 0	·0283	6 0	·0346	8 0	·0400	10 0	·0447

INDEX

PLATE I.

Fig. 1.



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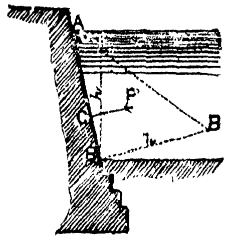

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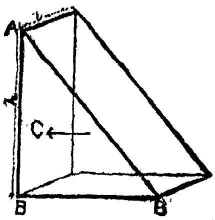

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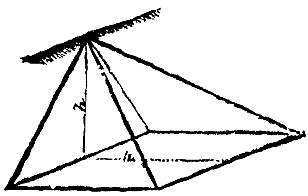

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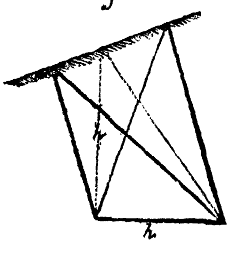

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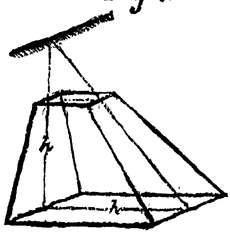

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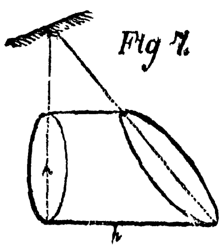

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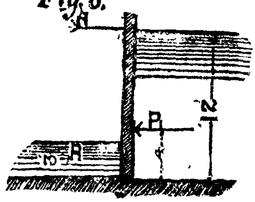
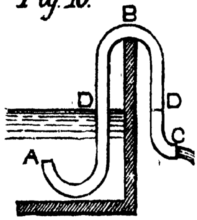


PLATE II.

Fig. 9.

Fig. 10.



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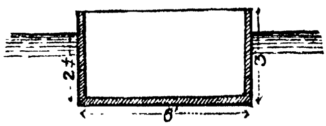

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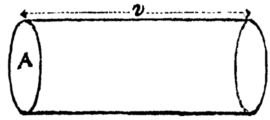

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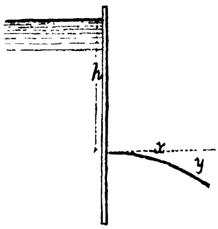

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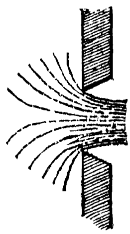

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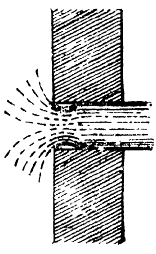

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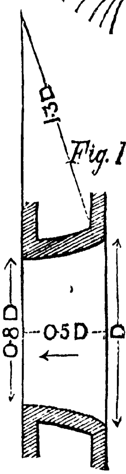

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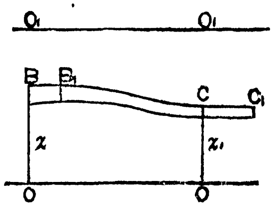

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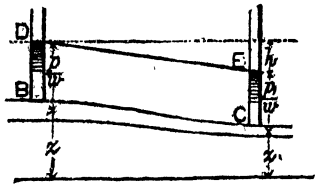

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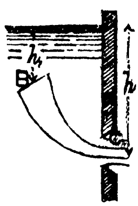

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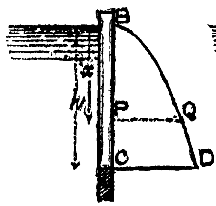

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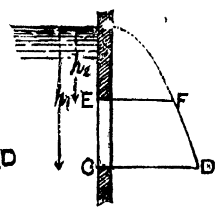

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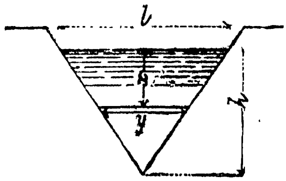

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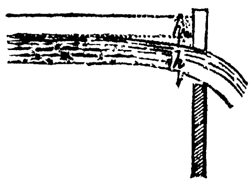

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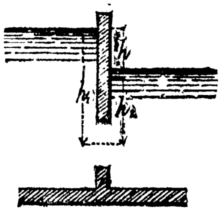

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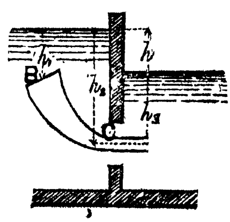

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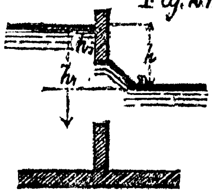

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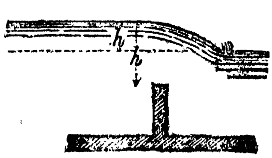
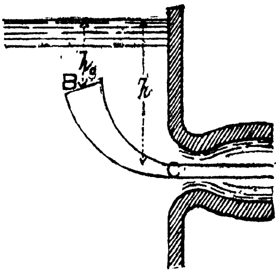


PLATE IV.

Fig. 29



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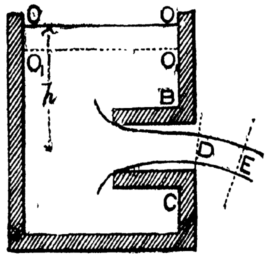

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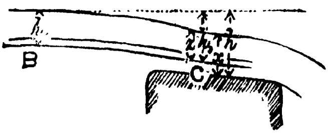


Fig. 32 (a).

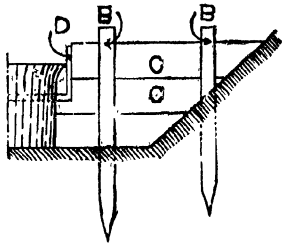


Fig. 32(b)

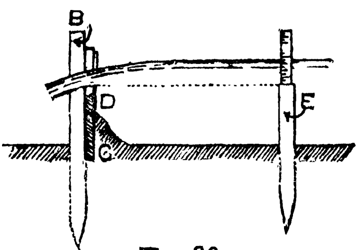
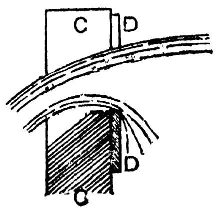


Fig. 32(c)



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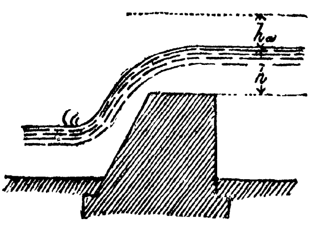

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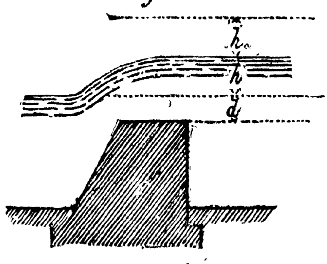

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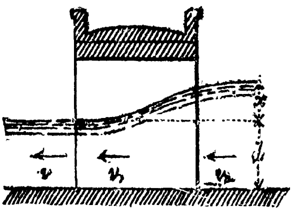

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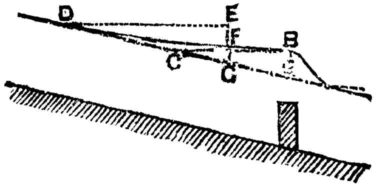

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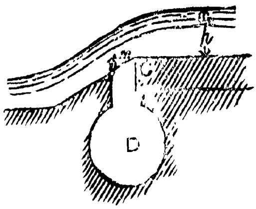

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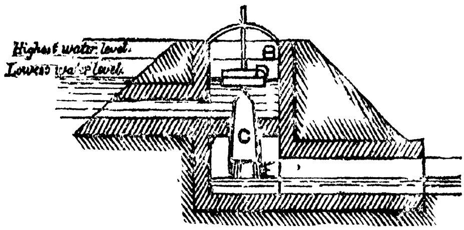

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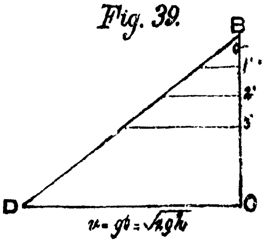
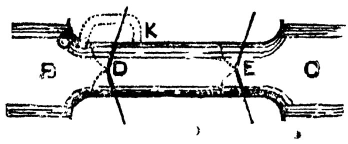


Fig. 40(a).



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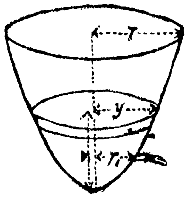

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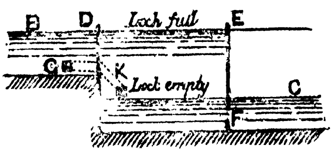
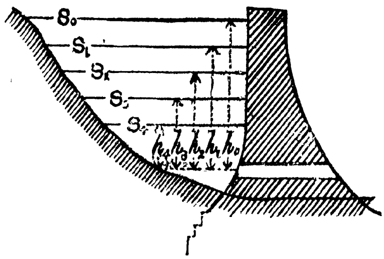


PLATE VI.

Fig. 42.



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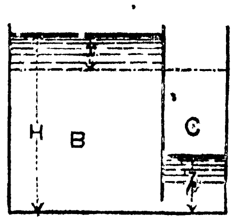

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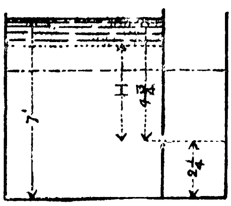

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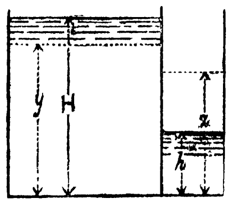

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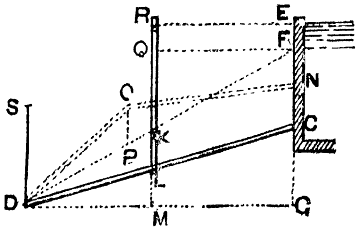

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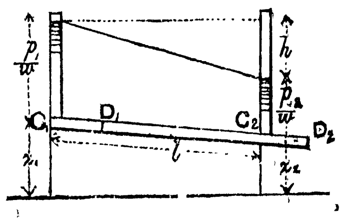

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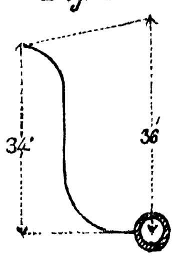

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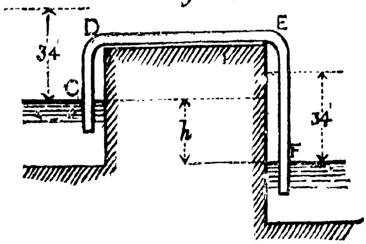


DARCY'S FORMULA FOR PIPES. PLATE VII.

GRAPHIC REPRESENTATION OF C IN THE EXPRESSION

$$v = \frac{C}{2}\sqrt{ds}$$

Two curves are drawn; the upper for new iron pipes, the lower for pipes which have been some time in use and which have become slightly incrusted. The values of C are measured vertically, the pipe diameters in inches horizontally.

Darcy's formula is $C = \sqrt{\frac{2g}{\alpha(1+\frac{\beta}{d})}}$ *where d is the diameter of the pipe in feet,*

α *and* β *are coefficients depending on the roughness of the pipe*

For smooth wrought and cast iron pipes, $\alpha = .005$ $\beta = .084$

For pipes altered by slight incrustation, $\alpha = .010$ $\beta = .084$

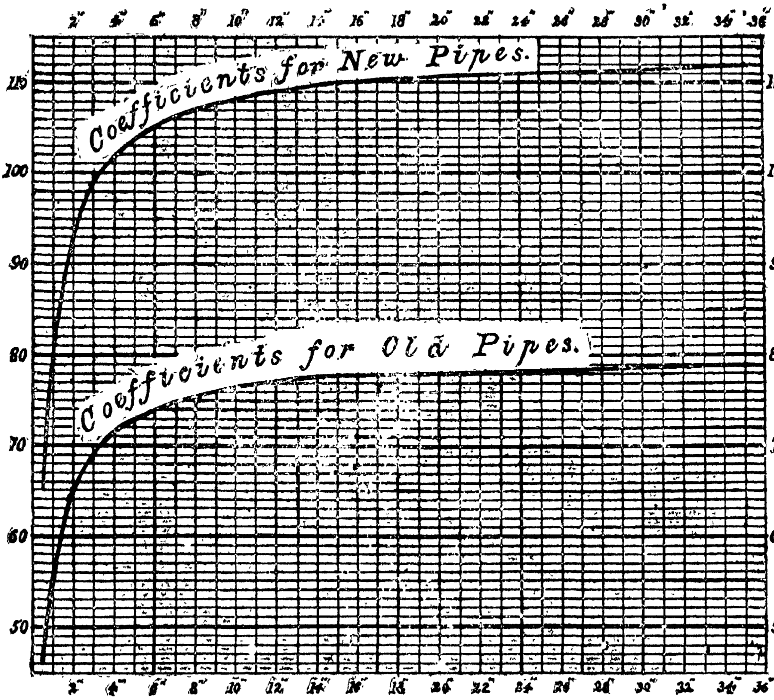


Fig. 50.

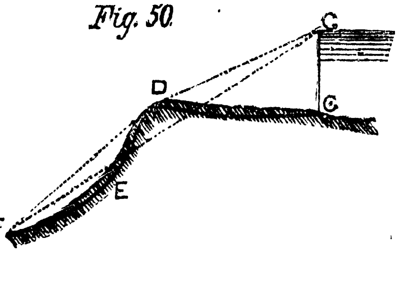


Fig. 51.

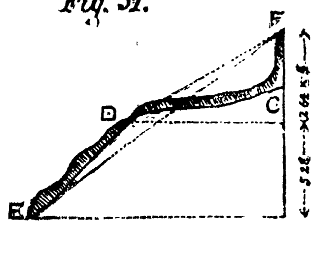
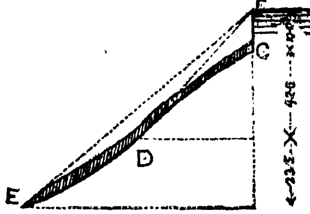


Fig. 52.



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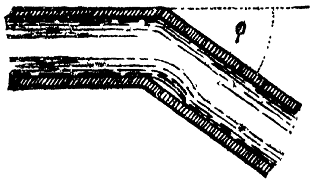

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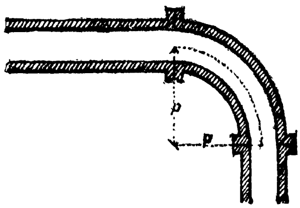

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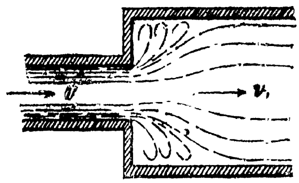

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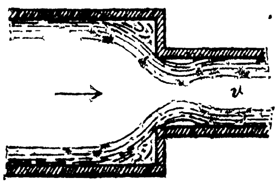


Fig. 57

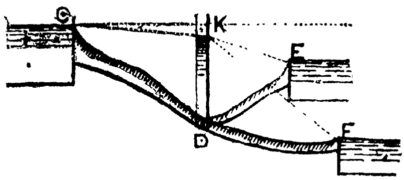


Fig. 58.

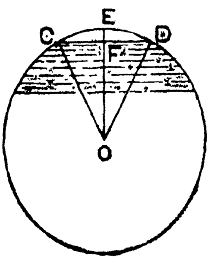
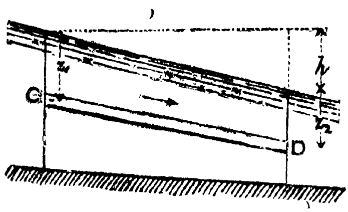


Fig. 59.



BAZIN'S FORMULA FOR CHANNELS

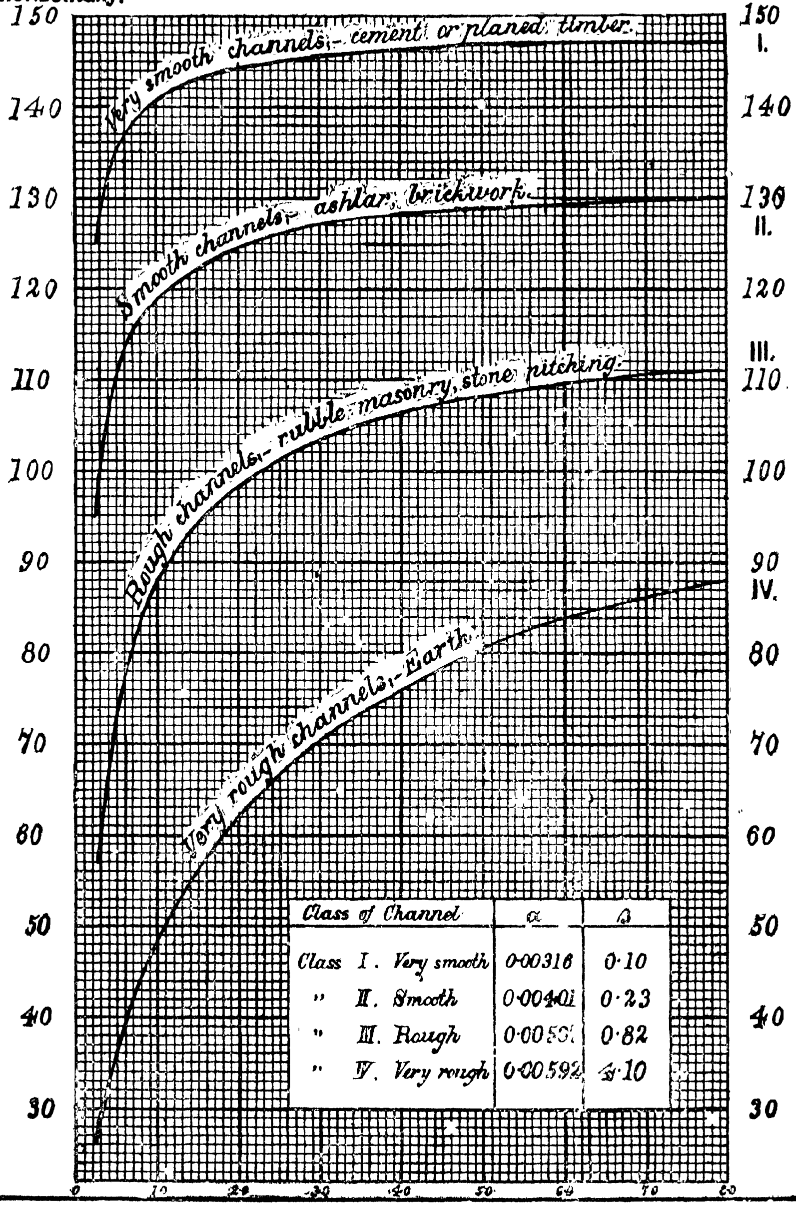
PLATE IX.

GRAPHIC REPRESENTATION OF C IN THE EXPRESSION $v = C\sqrt{rs}$

Bazin's formula is $C = \sqrt{\frac{2g}{\alpha\left(1+\frac{\beta}{r}\right)}}$. Where r is the hydraulic mean depth in feet, and β are co-efficients depending on the roughness of the Channel.

Four curves are drawn, I, II, III, IV, corresponding with the materials of which Channels, Culverts, &c. are usually constructed.

The Values of C are measured vertically; those of the hydraulic mean depth horizontally.



Class of Channel	α	β
Class I. Very smooth	0·00316	0·10
" II. Smooth	0·00401	0·23
" III. Rough	0·005[illegible]	0·82
" IV. Very rough	0·00592	4·10

KUTTER'S FORMULA FOR CHANNELS AND RIVERS

GRAPHIC REPRESENTATION OF C IN THE EXPRESSION $v = C\sqrt{rs}$

PLATE X

Kutter's formula is $C = \dfrac{41{\cdot}6 + \dfrac{1{\cdot}811}{N} + \dfrac{{\cdot}00281}{s}}{1 + \left(41{\cdot}6 + \dfrac{{\cdot}00281}{s}\right)\dfrac{N}{\sqrt{r}}}$, where

N is a coefficient varying from ·010 to ·030, and depending on the roughness of the channel; r is the hydraulic mean depth; s is the sine of the longitudinal slope.

Six curves are drawn, A, B, C, D, E, F, corresponding with the various materials of which channels, culverts, &c. are usually formed.

The values of C are measured vertically, those of the hydraulic mean depth horizontally.

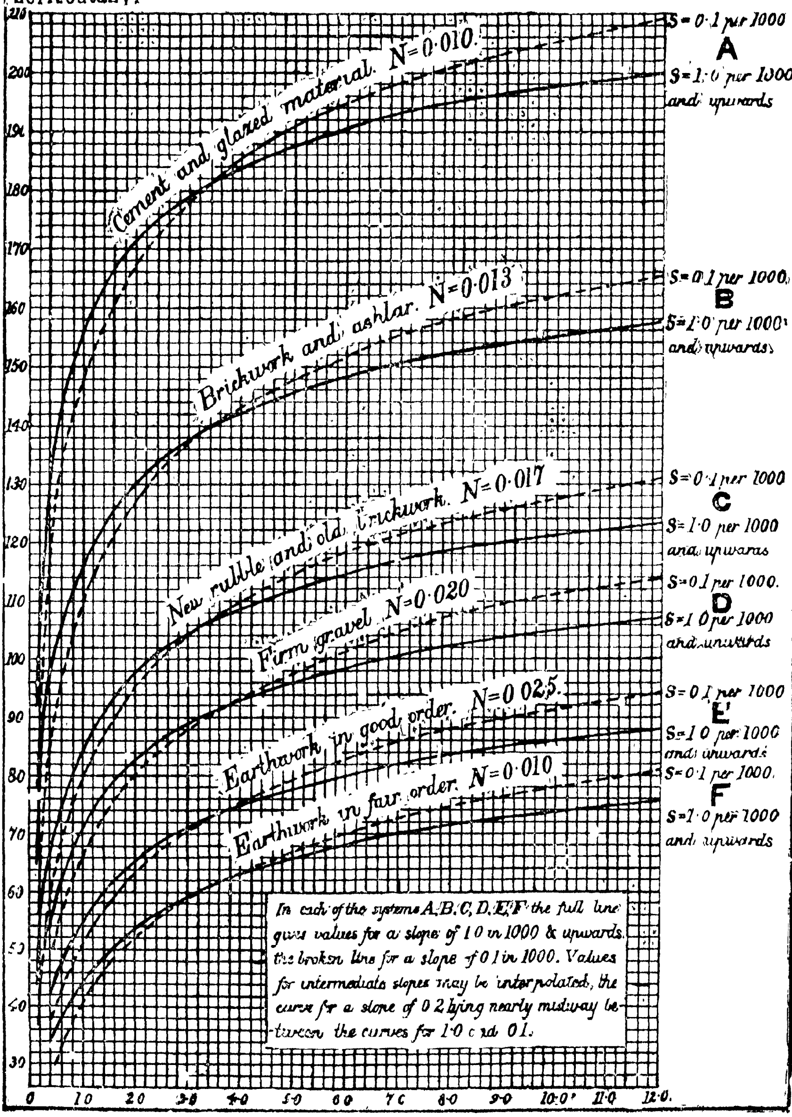


PLATE XI.

Fig. 60

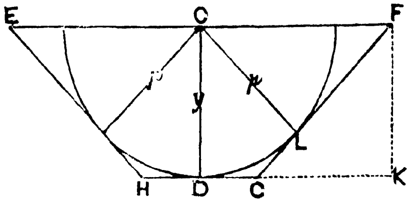


Fig. 61.

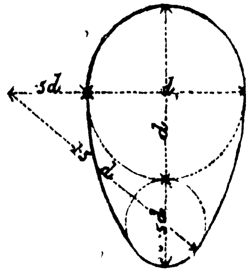


Fig. 62.

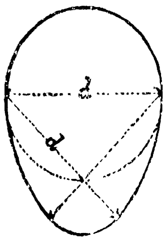


Fig. 63.

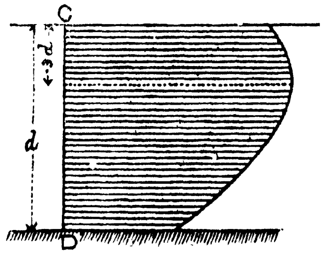


Fig. 64.

Fig. 65.

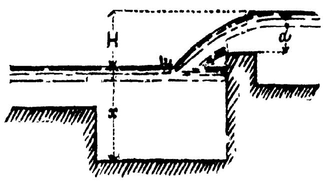


Fig. 66.

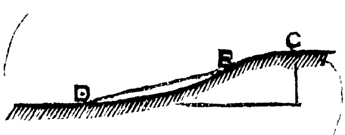


Fig. 67

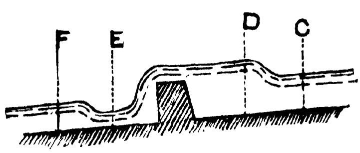


PLATE XII.

EXPERIMENTAL FALL, BARI DOAB CANAL.

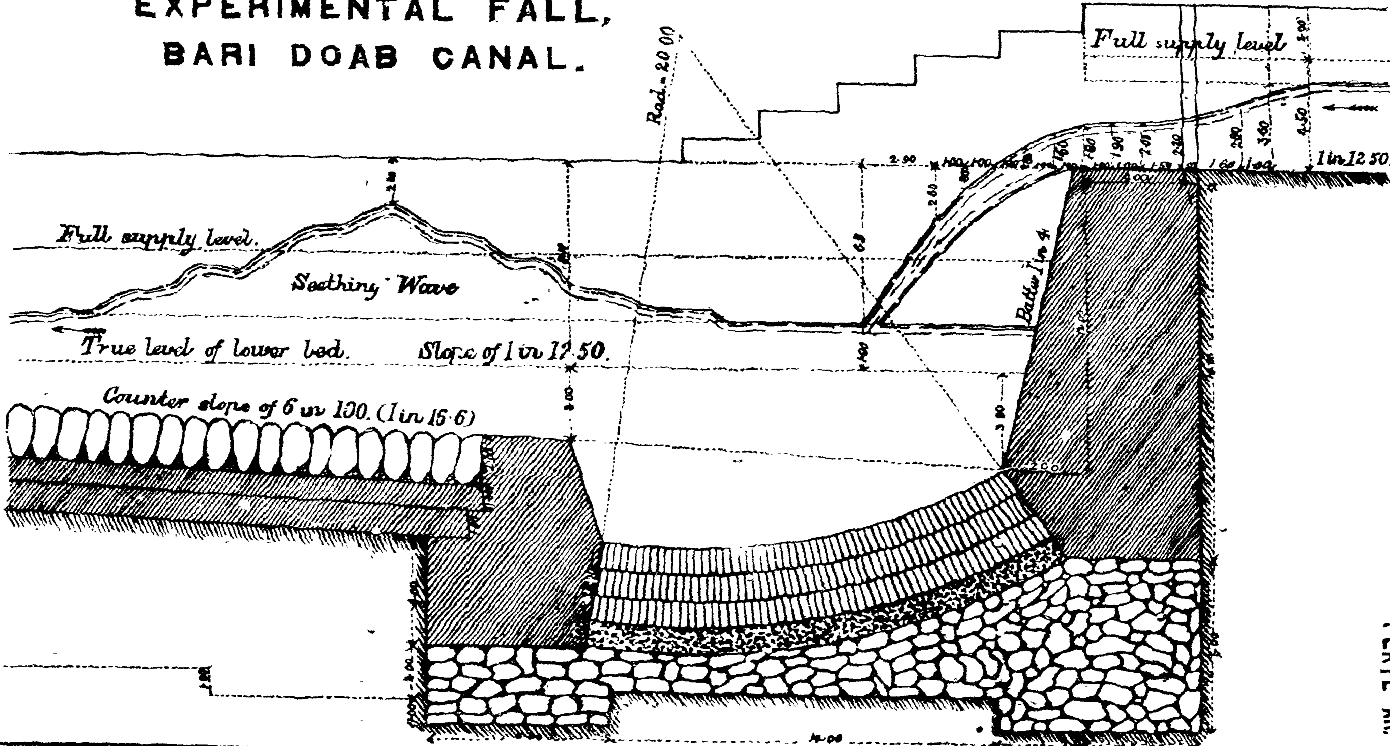


Fig. 68.

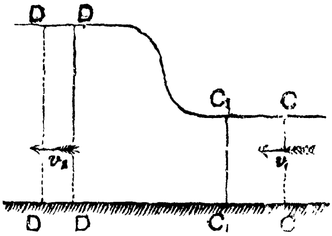


Fig. 69.

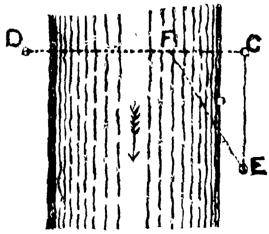


Fig. 70.

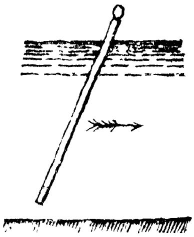


Fig. 71.

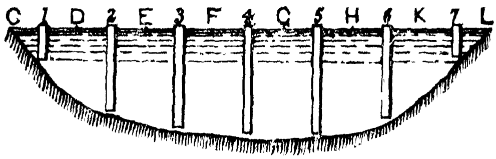


Fig. 72.

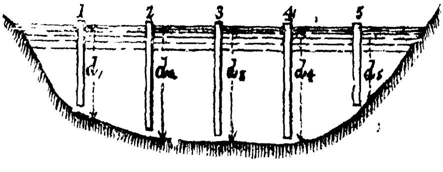


Fig. 73.

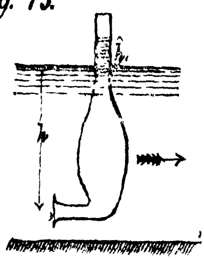


Fig. 74.