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ELEMENTS OF ASTRONOMY

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ASTRONOMY

WITH

Numerous Examples and Examination Papers

BY

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OF TRINITY COLLEGE, DUBLIN

SEVENTH EDITION

LONGMANS, GREEN, AND CO.

39, PATERNOSTER ROW, LONDON

NEW YORK AND BOMBAY

1925

PRINTED AT THE

DUBLIN UNIVERSITY PRESS

BY PONSONBY & GIBBS.

PREFACE.

THE present volume is intended to meet the wants of those Students whose knowledge of Mathematics is limited to an acquaintance with the Elements of Euclid, Algebra, and Plane Trigonometry. In a few cases easy formulæ in Dynamics are introduced, but the articles containing these may, if necessary, be omitted without a breach in the continuity of the work.

Many of the examples have been selected from papers set to third and fourth year Students of Trinity College, Dublin; while a considerable number have been chosen with a view to assist those reading for Degrees in the London and Royal Universities.

The book forms, to some extent, a connecting link between the many popular works on Astronomy and more advanced treatises on the subject. The author, therefore, hopes that it may be found useful, not only by those for whom it has been specially written, but also by many others among the general public.

The author is much indebted to Mr. Piers Ward, M.A., LL.B., for his kind assistance in reading the proof-sheets.

13, Trinity College, Dublin,
July 19th, 1894.

CONTENTS.

CHAPTER I.

PROPERTIES OF THE SPHERE. DEFINITIONS.

CHAPTER II.

THE EARTH.

CHAPTER III.

THE OBSERVATORY.

CHAPTER IV.

ATMOSPHERIC REFRACTION.

CHAPTER V.

THE SUN.

CHAPTER VI.

THE MOTIONS OF THE PLANETS. THE SOLAR SYSTEM.

CHAPTER VII.

PARALLAX.

CHAPTER VIII.

DETERMINATION OF THE FIRST POINT OF ARIES. PRECESSION, NUTATION, AND ABERRATION.

CHAPTER IX.

THE MOON.

CHAPTER X.

ECLIPSES.

CHAPTER XI.

TIME.

CHAPTER XII.

APPLICATION TO NAVIGATION.

CHAPTER XIII.

THE FIXED STARS. SPECTRUM ANALYSIS.

CHAPTER XIV.

MASSES OF THE HEAVENLY BODIES.

CHAPTER I.

PROPERTIES OF THE SPHERE. DEFINITIONS.

1. **Definition.**—A SPHERE is a solid bounded by one surface, such that all right lines drawn to that surface from a certain point within it are equal. That point is called the centre.

A sphere might also be defined as being generated by causing a circle to revolve round one of its diameters. Thus if a circular hoop or ring be taken, and if, having fixed two diametrically opposite points in its circumference, we make it revolve round the diameter joining those points, we see that the circumference will trace out the surface of a sphere.

2. Every plane meeting a sphere cuts its surface in a circle.

For, let DEF represent a plane section of a sphere.

From O, the centre of the sphere, let fall OO' perpendicular to the plane DEF; take any point P in the circumference DEF. Now, since OO' is perpendicular to the plane DEF, it must be perpendicular to $O'P$ which lies in that plane; therefore (EUCLID, I. 47),

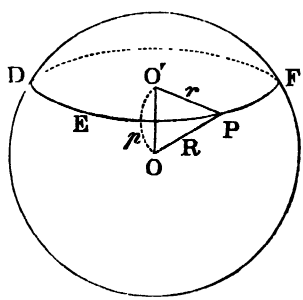

FIG. 1.

$$R^2 = p^2 + r^2;$$

$$\therefore \quad r^2 = R^2 - p^2.$$

But R is of constant length, being the radius of the sphere, and p is constant; therefore r is of constant length, and therefore as P changes its position along the curve DEF, its distance from O' is constant. Therefore DEF must be a circle.

The reader can illustrate this experimentally by taking an apple as nearly spherical as possible, and, with a knife, cutting a section through. On examining the interior of the apple thus brought to view, he will find that the shape of the section is circular.

3. **Definition.**—A *great circle* on the surface of a sphere is that whose plane passes through the centre of the sphere. Thus the circle $AmnB$ (fig. 2) is a great circle.

A *small circle* is such that its plane does not pass through the centre of the sphere. Thus (fig. 1) DEF is a small circle.

It is evident that all great circles on a sphere are equal in magnitude, but small circles are not, as they vary from being almost great circles till they dwindle down to mere points.

Definition.—If at the centre of a circle on a sphere a perpendicular be erected to its plane and produced out both ways, the two points in which it cuts the sphere are called the *poles* of that circle.

Thus, if at the centre O we erect PQ perpendicular to the plane of the great circle AB, the two points P, Q are the poles of the great circle AB.

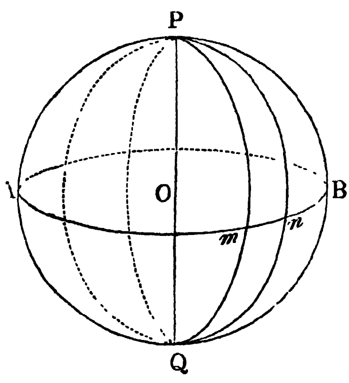

FIG 2.

Definition.—Great circles which pass through the poles of another great circle are called *secondaries* to that great circle. Thus, if through P and Q (fig. 2) great circles PmQ, PnQ be drawn, these circles are secondaries to the great circle AB.

N.B.—A great circle and its secondaries cut at right angles. Also the arcs of secondaries Pm and Pn drawn from P to AB are equal, and each is equal to a quadrant $= 90°$.

Apparent Diurnal Motion of the Heavens. Celestial Sphere.

4. To an observer situated in the middle of a level plane the appearance which the heavens present to him is that of a vast hollow dome of hemispherical shape, whose base appears to rest on the plane on which he stands, meeting it in a circle. This circle which bounds his view of the heavens is called the *sensible* or *apparent horizon*, the plane in which he stands being the *plane* of the *sensible horizon*.

If now the time of observation be a cloudless night, there will be seen, apparently spangled over the concave surface of this hollow dome, a great number of shining bodies or specks of light called stars.

By far the greater number of these bodies always maintain nearly the same situation with respect to each other; that is, if the angle which any two subtend at the eye of an observer be measured, it is found never to undergo any alteration in magnitude, except such changes as are of so minute a character as only to be observable after considerable intervals of time. These bodies are therefore called *fixed stars*.

To even the most careless observer it will at once appear that all these fixed stars appear to move across the sky in a common direction without altering their positions relative to one another. Some rise in the eastern horizon, ascend in the sky, and, after describing arcs which seem circular, they sink below the western horizon, only to reappear again in the east on the next night at the same point as before, the whole revolution of each being completed in about $23^h\ 56^m\ 4^s$.

Many others make smaller circuits in the sky such that they never reach the horizon, and can therefore be seen throughout their whole course. The paths of these stars also seem circular, the time taken by each to complete a revolution being about $23^h\ 56^m\ 4^s$, the same as for those which rise and set. Moreover, their apparent diurnal paths all seem to be round one common point as pole. That point is called the *celestial pole*. Those stars which circle so closely to the pole that they do not rise and set are called *circumpolar stars*.

In order to more accurately observe this apparent diurnal motion of the heavenly bodies, it is necessary to have a telescope suitably mounted, called an *equatorial*, such as is to be found in every observatory. This telescope, of which a full description will be given later on, can be directed to any part of the heavens, so that any star desired may be brought into the field of view. Moreover, by means of a clockwork arrangement, the field of view can be made to move uniformly round the celestial pole in the same direction as the stars appear to move, completing its revolution in $23^h\ 56^m\ 4^s$. It is now found that whatever star is chosen, once the revolving clockwork apparatus is set going, it is possible to keep it in the field of view throughout its whole course above the horizon. If the reader bear in mind that every point in the field of view of the telescope moves uniformly in a small circle round the celestial pole, and that the time of completing a revolution is $23^h\ 56^m\ 4^s$, he will at once arrive at the following conclusions:—

(1) The stars appear to move in small circles round the celestial pole.

(2) This apparent motion is uniform.

(3) The time occupied in completing one revolution is the same for all, viz. $23^h\ 56^m\ 4^s$.

5. **Celestial Sphere.**—If two observers be situated at diametrically opposite points of the earth, say in England and somewhere about New Zealand, each will see an apparently concave hemispherical surface of the heavens having a celestial pole. If, therefore, the observer could see the whole

heavens at a glance, the appearance would be that of a complete sphere, on the concave surface of which all the heavenly bodies would *seem* to be situated, and revolving round two diametrically opposite points, the north and south celestial poles. This apparently spherical surface of the heavens is called the *celestial sphere.*

It is usual, from a mathematical point of view, to regard the celestial sphere as a sphere of infinitely great radius compared with any distance on the earth, so that when we say that the earth occupies a position in the centre of this imaginary sphere, we mean that we may regard it as a mathematical point, *i.e.* of no dimensions.

The Sun, Moon, and Planets.

6. **The Sun.**—Let us now change the time of observation to the day-time, and see if the sun shares in the diurnal revolution common to the fixed stars. At first sight we might come to the conclusion that his apparent diurnal motion is exactly the same; he rises in the east, describes an arc in the sky, sets in the west, and reappears again in the east next morning. But there is a difference; the time taken by him to make a complete revolution is not $23^h\ 56^m\ 4^s$, but 24 hours, or about 4 minutes longer than for the fixed stars. This can be verified roughly by waiting any day until one edge of the sun is in a line with a vertical wall, or, better still, with two vertical strings hung with weights at the ends of them. The interval which elapses until he gets into the same position next day is then noted, and is found to be about 24 hours; whereas, as we have seen above, the same experiment applied to a fixed star would give $23^h\ 56^m\ 4^s$.

The apparent position of the sun among the fixed stars cannot therefore be the same from day to day, but must be slowly shifting from *west to east*, so that he, as it were, hangs behind the fixed stars in his diurnal revolution from east to west, taking a slightly longer time to complete the circuit.

If we could observe the fixed stars during the day-time with our naked eyes as can be done through an astronomical telescope, we could actually see the sun's slow change of position from west to east, and also that he returns to the same position among the fixed stars at the end of a period of time which is called a year. But we can demonstrate it without the aid of a telescope, thus :—

If we note each evening some group of stars which sets in the west some time after sunset, it will be seen, if the observations be continued for some weeks, that the interval that elapses between sunset and the disappearance of the stars becomes shorter and shorter until eventually the stars set before the sun, and therefore cease to be visible in the early part of the night. If, however, we get up before dawn and look towards the eastern part of the sky we will see that these stars have risen before the sun.

But if we continue our observations for 365 days, we shall find at the end of that time the sun in the same position relatively to the fixed stars as before, and the group in question will again be visible in the early part of the night. We thus say that the sun has two apparent motions :—

(1) A daily revolution from east to west in common with all the heavenly bodies.

(2) A slow yearly revolution from west to east among the fixed stars.

7. **The Moon.**—Besides its apparent diurnal motion, the moon also appears to move from west to east among the fixed stars, but much more quickly than the sun, as it seems to complete a revolution relative to the sun and earth in a period of time which is called a month.

8. **The Planets.**—Besides the fixed stars, sun and moon, there are five other bodies visible to the naked eye whose apparent motions among the fixed stars are, as it were, so whimsical, that it is difficult to reduce them to any general laws. On this account they are called *Planets* or wandering stars. Sometimes they appear to move among the fixed stars in the same direction as the sun and moon, when their motion is said to be direct: sometimes in the opposite direction, when their motion is retrograde, and occasionally for a short time they appear stationary among the fixed stars.

A fixed star to the naked eye burns with a twinkling light, a planet shines with a steady light. Also when a fixed star is examined through even the most powerful telescope it does not seem increased in size, the only difference being that its brilliancy is intensified. On the other hand, the disc of a planet appears enlarged when seen through a telescope.

Four of the planets, Venus, Mars, Jupiter, and Saturn, are as bright or brighter than the most brilliant fixed star. If the planet appear shining in the south it is Mars, Jupiter, or Saturn. Venus is an evening or a morning star; in the former case it is seen in the west after sunset, in the latter case in the east before sunrise.

The only other heavenly bodies visible at any time to the ordinary observer are comets and shooting stars or meteors, which complete the list of those bodies of which the following chapters give a more detailed description.

9. **The Ptolemaic System.**—The different celestial phenomena mentioned above—the diurnal revolution of all heavenly bodies, the yearly path of the sun, the monthly motion of the moon, and the apparently irregular courses taken by the planets were accounted for by Ptolemy, who lived in the second century after Christ, on apparently so

satisfactory a basis that it was not until the sixteenth century that the true explanation was accepted.

The whole celestial sphere was supposed to revolve round an axis passing through the north and south celestial poles, and called the celestial axis, the earth being in the centre. The sun, besides this daily revolution *with* the celestial sphere, was supposed to have a motion of its own in the opposite direction *on* the sphere describing a circle round the earth as centre once every year, the moon similarly completing a circuit once every month. The retrograde and stationary stages in a planet's motion they accounted for in a rather ingenious way by supposing the planets to describe circles round the sun, which in its turn described a circle round the earth.

The Copernican System.—The true explanation, however, first given by Copernicus, and now so well known, shows that the diurnal revolution of the heavens is only apparent, and that it is really the earth which rotates in the opposite direction, from west to east, round an axis which, if produced out, would pass through the north and south celestial poles, thus causing the plane of the observer's horizon by its motion to uncover and bring into view new stars in the east, when these stars are said to rise, and to cover up and hide from view other stars in the west which are then said to set.

Copernicus also referred the apparent yearly motion of the sun round the earth to a motion of the earth round the sun, and showed that the earth and planets form one system revolving round the sun, from which they derive their light and heat.

This explanation was at first received by astronomers with the greatest suspicion, and it was only subsequent discoveries which placed it beyond any doubt; we should therefore not take it for granted, but examine carefully the successive steps which led up to these conclusions. Although the diurnal revolution of the earth on its axis was not generally believed in until about three centuries ago, it is, however, not by any means a new idea. Cicero mentions that it was the opinion of Hicetas of Syracuse, who lived 400 years before Christ. Copernicus says that it was this statement of Cicero's which first led him to consider the earth's motion.

10. For the purpose of reference, the relative positions of the different fixed stars and the apparent path of the sun among them are mapped out on the surface of a globe, such that the arc joining the positions of any two stars on the surface of this miniature globe subtends the same angle at its centre as the two stars in question subtend at the eye of the observer. Such a celestial globe, the observer being supposed at its centre, would then serve as a representation of the appearance of the heavens.

The reader should bear in mind that such a celestial globe only represents the angular distances of the heavenly bodies from one another, and not their distances from the earth: for the fixed stars are immensely further distant than the sun or planets, while being on the surface of a globe they are represented as being at the same distance from the observer.

Definitions.

(1) The great circle in which the plane of the horizon cuts the celestial sphere is called the *celestial horizon.*

N.B.—At sea, it is easy to observe the position of a heavenly body with respect to the horizon; but on land, on account of the inequalities of the earth's surface, the horizon cannot be seen; but we can determine it by taking a plane perpendicular to the direction in which a plumb line hangs, and note the position of the body with reference to that plane. The surface of a small portion of liquid at rest, such as mercury, is also used by astronomers to determine the plane of the horizon.

(2) If we imagine the direction of a plumb line produced upwards, that point in which it would cut the celestial sphere is called the *zenith*.

(3) If we imagine the direction of the plumb line produced downwards so as to cut the celestial sphere in the diametrically opposite point, that point is called the *nadir*.

N.B.—It is evident that the zenith and nadir are the two poles of the celestial horizon.

(4) *Celestial meridian.*—That great circle in the heavens drawn through the zenith and celestial pole is called the *meridian*.

(5) Great circles drawn perpendicular to the horizon, *i.e.* secondaries to the horizon, are called *verticals*.

That vertical drawn due east and west at right angles to the meridian is called the *prime vertical*.

(6) The four points in which the meridian and prime vertical cut the horizon are called the four *cardinal points*—the north, south, east, and west points.

(7) The *celestial equator* is that great circle in the heavens whose plane is at right angles to the direction of the celestial pole.

The north and south celestial poles are evidently the poles of the equator.

The small circles described round the celestial pole by the stars in their apparent diurnal motion are all parallel to the celestial equator.

It is evident since any two great circles bisect each other (having a common diameter) that one-half of the celestial equator is above the horizon and half below, so that if any star or other heavenly body be situated in the equator it will, in its diurnal revolution, remain equal times above and below the horizon, rising at the east point, and setting at the west point.

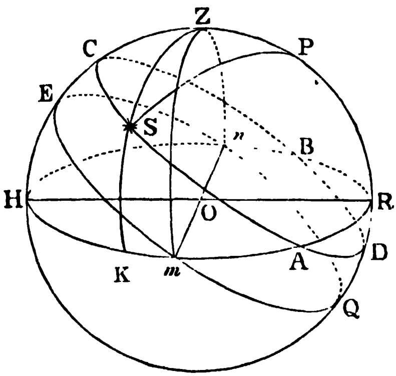

FIG. 3.—DIAGRAM OF CELESTIAL SPHERE, THE OBSERVER BEING AT ABOUT THE LATITUDE OF DUBLIN.

ASCB is the diurnal parallel of a star *S*, *A* being the point where the star rises, *B* where it sets, and *C* the point where it crosses the meridian.

O . .	Observer.	*R* . .	North Point.
HmRn . .	Horizon.	*H* . .	South Point.
Z . .	Zenith.	*m* . .	East Point.
P . .	Celestial Pole.	*n* . .	West Point.
HZPR . .	Meridian.	*S* . .	A Star.
EmQn . .	Celestial Equator.	*ZSK* . .	Vertical through *S*.
mZn . .	Prime Vertical.	∠*SPZ* . .	Hour angle of *S*.

(8) **Definition.**—The *ecliptic* is the apparent path of the sun among the fixed stars in the course of a year.

When this apparent annual path of the sun is traced out on the celestial sphere it is found that it can be represented by a great circle. Its name arises from the fact that if the moon in her monthly revolution happen to cross the plane of the ecliptic when it is full or new moon, there will be an *eclipse*, in the former case of the moon, and in the latter of the sun.

Obliquity of Ecliptic to Equator. Equinoxes.

11. The angle at which the planes of the ecliptic and equator cut is about 23° 28′ which is called the obliquity of the ecliptic to the equator. These two great circles must

intersect in two points, therefore on two days each year the sun is in the act of crossing the equator, and on those days his diurnal path almost coincides with the equator, rising due east and setting due west (fig. 3); one-half of his diurnal path is therefore above horizon and half below, and day and night are of equal duration all over the world. From this latter circumstance these two periods are called the *Equinoxes*, the two points of intersection of the ecliptic with the equator being called the *equinoctial points*. One of these points is called the *first point of Aries* (♈), the other the *first point of Libra* (♎), because when these points were first named by the ancient astronomers they were in the constellations of Aries and Libra respectively. The sun is at the first point of Aries on the 21st March when crossing from the south to the north side of the equator; this date is called the vernal or spring equinox, and is at Libra on the 23rd September, in his passage from the north to the south side of the equator, this date being the autumnal Equinox.

The Signs of the Zodiac.

12. Ancient astronomers found by observation that the moon and planets were never at any time at a very great angular distance from the ecliptic; they therefore conceived an imaginary belt in the heavens extending for about 8° on either side of the ecliptic. Inside this space the moon and planets, and of course the sun were always to be found. They called this belt the *zodiac* from their imagining certain forms of animals situated within it, which they named the *signs* of the zodiac. There are twelve signs of the zodiac: these together with the symbols which represent them are as follows:—

Aries.	Taurus.	Gemini.	Cancer.	Leo.	Virgo.
♈	♉	♊	♋	♌	♍
Libra.	**Scorpio.**	**Sagittarius.**	**Capricornus.**	**Aquarius.**	**Pisces.**
♎	♏	♐	♑	♒	♓

Altitude, Azimuth.

13. The *altitude* of a heavenly body is its distance from the horizon measured on the arc perpendicular to the horizon drawn through the body (*i. e.* on the vertical drawn through the body).

The *azimuth* of a body is the arc intercepted on the horizon between the foot of the vertical drawn through the body and the meridian.

Thus (fig. 3) SK = altitude, and HK = azimuth of the star S.

Of course it is immaterial whether we call RK or HK the azimuth of the star S, provided we mention whether we are measuring it from the north or south point. In northern latitudes the azimuth is generally measured east and west from the south point, and in southern latitudes from the north point. Thus if the arc $HK = 30°$, the azimuth of $S = 30° E$.

The arc SZ is called the *zenith distance* of the body, and is evidently the complement of the altitude.

The position of a body on the celestial sphere with respect to the observer's horizon and meridian can be described by knowing its altitude and azimuth, but as the horizon of the observer is, owing to the earth's rotation, changing every instant, and, moreover, both the horizon and meridian are different for different places on the earth, therefore the altitude and azimuth of a heavenly body only describe its position at some particular instant and observed from a certain definite place on the earth.

Declination and Right Ascension.

14. Instead of describing the position of a body with reference to the horizon we may refer it to the equator. The measurements by which its position is then indicated are

independent of the position of the observer on the earth, and do not change appreciably each instant, but, as we shall subsequently see, only after comparatively long periods of time.

The *declination* of a heavenly body is its distance from the equator measured on an arc perpendicular to the equator drawn through the body.

The *right ascension* is the arc of the equator intercepted between the first point of Aries and the perpendicular to the equator drawn through the body.

The right ascension is reckoned from ♈ eastward from 0° to 360°.

Thus (fig. 4) let EQ represent the equator, AB the ecliptic; draw a common secondary $AQBP$ to both passing through P the pole of the equator (celestial pole) and P' the pole of the ecliptic. Then if S be the position of a heavenly body we have:

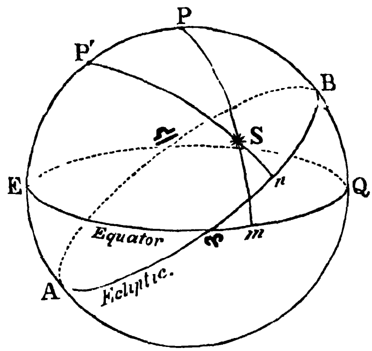

FIG. 4.

Sm = declination of body measured along PM.

♈m = right ascension of body measured along the equator.

The arc SP is called the polar distance of the body and is evidently the complement of the declination.

Celestial Latitude and Longitude.

The position of a body may be indicated also with reference to the ecliptic.

The *latitude* of a heavenly body is its distance from the ecliptic measured on a perpendicular arc to the ecliptic.

The *longitude* is the arc of the ecliptic intercepted between the first point of Aries and a perpendicular arc to the ecliptic drawn through the body.

Thus (fig. 4) Sn = latitude of S and ♈n = longitude.

The terms *celestial* latitude and longitude are applied to these measurements to distinguish them from *terrestrial* latitude and longitude with which they are not in any way connected.

The longitudes of heavenly bodies are, like their right ascensions, measured from ♈ eastward from 0° to 360°.

Both the declinations and latitudes of heavenly bodies vary from 0° to 90° on either side of the equator and ecliptic respectively. They are counted north or south according to whichever celestial pole happens to be on the same side of the great circle from which they are measured.

Declination Circles. Hour Angle.

Secondaries to the equator are called *declination circles* because it is on these circles that the declinations of heavenly bodies are measured.

The angle which the declination circle through a star makes with the meridian is called the *hour angle* of the star, because when this angle is known we are able to calculate the time that must elapse before the star crosses the meridian or the time which has elapsed since it last crossed it, from the fact that the star completes a revolution of 360° round the celestial pole in $23^h\ 56^m\ 4^s$.

Thus (fig. 3) the angle SPZ = hour angle of star S.

Declination circles are on this account also called *hour circles*.

Changes in the Sun's Declination during the year as he describes the Ecliptic.

15. At the spring equinox the declination of the sun is zero, he being at ♈ (fig. 4). Each day, however, on account of his slow annual motion his declination increases until, some time about 21st June, he reaches his greatest declination, viz. the arc BQ (fig. 4). But BQ is the arc intercepted by the ecliptic and equator on their common secondary, and must therefore measure the angle between those great circles, which is 23° 28′.

Therefore the arc BQ = greatest declination of sun at midsummer = 23° 28′ north. This period is called the *summer solstice* (*sol, stare*), because the sun before descending to ♎ seems for some time to stand still.

After midsummer the sun's declination gradually decreases until at ♎ (about 23rd September) it is again zero. Between 23rd September and the 21st of the following March the declination of the sun is south, reaching at mid-winter (21st December) a value AE which = 23° 28′ south. This period is called the winter solstice; therefore we have:—

The sun's declination at the vernal equinox			= 0
,,	,,	summer solstice	= 23° 28′ N.
,,	,,	autumnal equinox	= 0
,,	,,	winter solstice	= 23° 28′ S.

During this period which is called a year the sun's right ascension and longitude increase from 0° at vernal equinox to 360° immediately before the following vernal equinox, each being 90° on 21st June, 180° on 23rd September, and 270° on 21st December.

It is hardly necessary to mention that the sun being in the ecliptic has his latitude always zero.

N.B.—When we speak of the sun's declination, &c., we mean that of the centre of the sun's disc.

Tropics of Cancer and Capricorn.

If on the celestial sphere we draw two small circles parallel to the equator, and distant from it 23° 28′ north and south, these small circles will nearly coincide with the sun's apparent diurnal path on 21st June and 21st December. They are called the *tropics* because the sun seems to be on the point of *turning* at these periods. The northern circle is the *Tropic of Cancer*, the southern the *Tropic of Capricorn*.

The *Equinoctial Colure* is the secondary to the equator passing through the equinoctial points.

The *Solstitial Colure* is the secondary to the equator passing through the solstices, and hence is also a secondary to the ecliptic.

The altitude of a Star is greatest when on the Meridian.

Let S represent the star in the meridian, and S' its position at any other time. Join ZS' and PS'. Now since any two

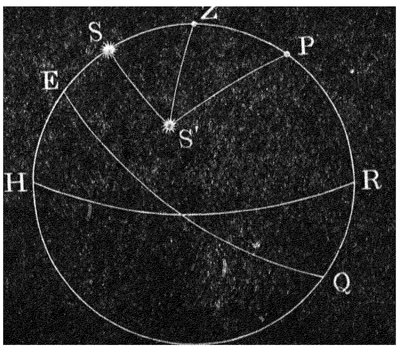

Fig 5.

sides of a spherical triangle are together greater than the third, therefore we have

$$ZS' + ZP > PS';$$

but $PS' = PS$, since a star always maintains the same distance from the pole;

$$\therefore\ ZS' + ZP > PS.$$

Take away the common part ZP, therefore we get ZS'

greater than ZS, that is, the zenith distance is least when on the meridian, and hence the meridian altitude is greatest.

In the same way it can be shown that the depression of a body below the horizon is greatest when on the meridian.

EXERCISES.

1. What are the altitude and hour angle of the zenith? *Ans.* 90° · 0.

2. What are the declination and latitude of the celestial pole? *Ans.* 90 · 66° 32′ (90 − 23° 28′)

3. How far is the pole of the ecliptic from the celestial pole, or, in other words, what is the magnitude of the arc PP in fig. 4? *Ans.* 23° 28′

4. What are the declination, right ascension, latitude, and longitude of ♎? *Ans.* 0 : 180° : 0 · 180°.

5. What point in the heavens has its declination, right ascension, latitude, and longitude each equal to zero? *Ans.* First point of Aries (♈).

6. If a certain star cross the meridian at 11 o'clock P.M. to-night, at what o'clock will it cross the meridian—(1) to-morrow night; (2) 15 days hence, assuming the sun's change of right ascension throughout the year to be uniform? *See* Arts. (5) and (6)) *Ans.* (1) About 10.56 P.M.
(2) About 10 P.M.

7. At what hour will the same star cross the meridian a year hence? *Ans.* 11 P.M. again.

8. A star is in the meridian 10° above the pole at midnight to-night, where will it be at midnight—(1) six months hence; (2) a year hence, supposing the sun's apparent motion in the ecliptic to be uniform?

9. What is the sun's right ascension on 21st March, 21st June, 23rd September, 21st December. *Ans.* 0 : 90° : 180° : 270°.

10. Calculate what would be the declination and right ascension of the sun on 21st April if the changes in these quantities were uniform throughout the year *Ans.* 7° 49′ 20″ N. : 30°.

11. Making the same assumption as in the last question—(1) Find at what time the sun's right ascension should be 120°; (2) at what time should his declination be 15° 38′ 40″ N. *Ans.* (1) 21st July.
(2) 21st May or 21st July.

N.B.—The reader can, by reference to a celestial globe or the Nautical Almanac, see that the results obtained in Examples (10) and (11) are not the correct values of the right ascension and declination of the sun on the dates mentioned, which shows that the changes in these quantities throughout the year are not at all uniform.

12. What is the time of sunrise and sunset at any place during the equinoxes? *Ans.* About 6 A.M. and 6 P.M.

13. What is the hour angle of the sun at sunrise on 21st March? *Ans.* 90°.

CHAPTER II.

THE EARTH.

16. THAT the earth's shape is approximately spherical has been known from the earliest times. It will not here be necessary to do more than mention the different reasons which lead us to this conclusion. They are:—

(1) The hull of a ship disappears first, which shows that the ship is sailing on a convex surface.

(2) The outline of the earth's shadow, as seen on the surface of the moon during an eclipse, *always* seems an arc of a circle, and no body but a sphere can project a circular shadow in *all* positions.

(3) The most conclusive proof, however, depends on the fact, which is found by observation, that equal distances gone over by the observer due north or south produce almost equal variations in the meridian altitude of any chosen star (or of the celestial pole). This could not happen except on the supposition that the earth is nearly spherical.

Celestial Pole Constant in Direction.

17. The celestial pole being supposed to be situated at an indefinitely great distance away compared with any distance on the earth, therefore, as the observer changes his position on the earth's surface, the lines drawn from those positions in the direction of the celestial pole are practically parallel.

Earth's Axis. Terrestrial Equator. Terrestrial Latitude and Longitude.

That diameter of the earth which is parallel to the constant direction of the celestial pole is called the *earth's axis.*

The earth's axis cuts the surface of the earth in two points called the north and south poles of the earth.

That great circle drawn round the earth whose plane is perpendicular to the earth's axis is called the *terrestrial equator.*

Great circles drawn through the poles of the earth are called *terrestrial meridians.*

Therefore, every place on the earth's surface may be supposed to have its meridian.

The meridian of Greenwich is called the *first meridian.*

The *latitude* of a place is its distance north or south of the equator measured on the meridian through the place.

The *longitude* of a place is its distance east or west of the first meridian, and is measured by the number of degrees in the arc intercepted on the equator between the meridian of the place and the first meridian.

All places situated on the same parallel to the equator have evidently the same latitude, and situated on the same meridian have the same longitude. Latitude is measured north and south from 0° to 90°, and longitude east and west from 0° to 180°.

Corresponding to the Tropics of Cancer and Capricorn on the celestial sphere, we imagine two small circles on the earth parallel to the equator, one north the other south, and distant from it about 23° 28′: these small circles are also called the Tropics of Cancer and Capricorn. The two small circles drawn round the north and south poles of the earth at a distance of 23° 28′ are called the *arctic* and *antarctic circles* respectively.

The portion of the earth's surface enclosed between the two tropics is called the *torrid zone*, between the tropics and the arctic and antarctic circles the *temperate zones*, and between the arctic and antarctic circles and the poles the *frigid zones*.

18. *The altitude of the celestial pole at any place is equal to the latitude of the place.*

For let O be the position of the observer; EOQ the meridian of the place, cutting the equator in E and Q. If OP represent the direction of the celestial pole as seen from O, then the line CP drawn from C, the centre of the earth, in the direction of the celestial pole, will be parallel to OP (the pole being so far distant). The horizon of the observer will be represented by a tangent plane OH drawn to the earth at O.

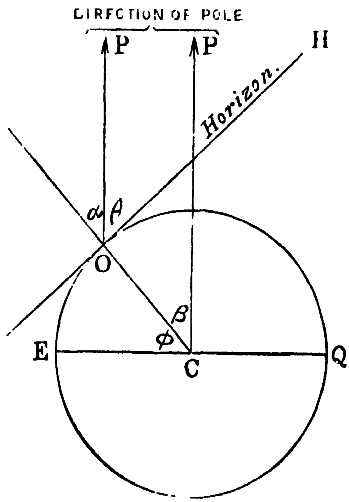

FIG. 6.

Then we have to prove that the angle θ which is the altitude of the pole = the arc EO or the angle ϕ, which is the latitude of the place. Since OP is parallel to CP, the angle α = the angle β; but θ is the complement of α, and ϕ is the complement of β; therefore $\theta = \phi$, or altitude of pole = latitude of place. From this it follows that the change in the altitude of the pole must equal the change in the latitude of the observer as he proceeds north or south.

Length of a degree of Latitude. Magnitude of Earth. Shape of Earth.

19. The measurement of the length of a degree of latitude on the earth is an operation of much practical difficulty. A position is chosen on the earth, and the altitude of the pole observed. Another station is chosen due north or south of the former position at such a distance from it that the altitude of the pole is increased or diminished by 1°, as the case may be. The length of the arc of the meridian between the two stations is then measured, and is found to have a mean value of about $69\frac{1}{10}$ miles, which must be the length of a degree. The length of a degree has thus been calculated at about twenty different places on the earth, and the results have not been found to differ to any very great extent, which is confirmatory evidence of the earth's approximate spherical shape.

It has, however, been found that the length of a degree near the poles is somewhat greater than near the equator, which shows that the curvature of the earth is not so great at the poles as at the equator, or, in other words, that the earth is slightly flattened at the poles. In fact the figure of the earth is what is called an *oblate spheroid*, differing but little from a sphere. The lengths of a degree of latitude at different parts of the earth have been found to be as fol ows :—

At the equator, . .	68·704 miles.
At latitude 20°, . .	68·786 ,,
,, ,, 40°, . .	68·993 ,,
,, ,, 60°, . .	69·230 ,,
,, ,, 80°, . .	69·386 ,,

The length of a degree being about $69\frac{1}{10}$ miles, an approximate value for the circumference and diameter of the earth can thus be found.*

$$1^\circ = 69\tfrac{1}{10} \text{ miles};$$

$$\therefore 360^\circ = \text{somewhat under } 25{,}000 \text{ miles} = \text{circumference of earth.}$$

$$\text{Diameter of earth} = \frac{25000}{3 \cdot 14159} = \text{somewhat under } 8000 \text{ miles.}$$

The polar diameter of the earth is found to be about 26 miles shorter than the equatorial.

Appearances of the Celestial Sphere due to Observer's Change of Place on Earth.

20. If the observer, starting from any place north of the equator, move due north along the meridian of the place, the celestial pole will appear to rise in the sky as his latitude increases (Art. 18). If he reach the north pole the celestial pole will appear right overhead in the zenith; the celestial equator will therefore coincide with his horizon (see fig. 7). The apparent diurnal paths of the stars will appear as small circles parallel to the horizon; therefore, *all* the stars visible will be circumpolar. Those stars, on the other hand, whose positions in the heavens are south of the celestial equator will never rise into view. Therefore, an observer at the north pole will never see more than half the heavens, no part of which, however, ever sinks below his horizon. Such a celestial sphere is called a *parallel sphere*. The sun for half the year (21st March to

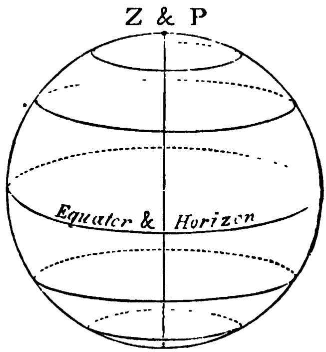

FIG. 7.—*Parallel Sphere, observer being at either pole of Earth.*

* The above method was that employed by Eratosthenes (230 B.C.) in determining the magnitude of the earth, except that he measured the meridian altitude of the sun instead of the altitude of the pole.

23rd September) being north of the equator, will during this period appear above the observer's horizon. For these six months he will appear to make a circuit every 24 hours in the heavens, which would be parallel to the horizon but for his continual change of declination. His greatest altitude is reached on the 21st June, and is then 23° 28′. For the remaining six months the sun keeps below the horizon, reaching a distance below it of 23° 28′ on 21st December. Therefore, at the north pole the day and night are each six months long. However, of the six months' night at the north pole a considerable portion is twilight.

Observer at Equator.

Let the observer now proceed southwards, and he will find that the celestial pole gradually falls in the sky until at the equator it will appear on the horizon coinciding with the north point, the south celestial pole coinciding with the south point. The celestial equator will therefore pass through the zenith and nadir, and, cutting the horizon at right angles, will coincide with the prime vertical (fig. 8).

The apparent diurnal paths of the stars being parallel to the celestial equator will be bisected at right angles by the horizon; the stars will therefore be an equal time above and below the horizon. There are therefore no stars circumpolar, but every star in the heavens appears for nearly twelve hours above the horizon.

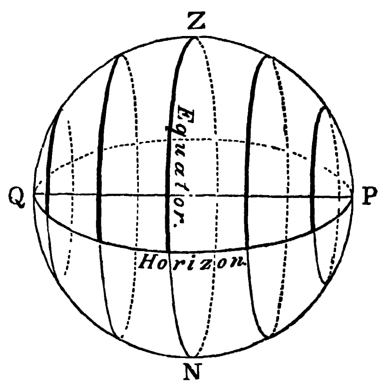

FIG. 8.—*Right Sphere, observer being at Equator.*

It is evident, also, that day and night are equal throughout the year at the equator.

Since the horizon bisects the diurnal paths of the stars at *right angles*, this sphere is called a *right sphere.*

Similarly, in the southern hemisphere the south celestial pole will increase its altitude as the south latitude increases

Observer at about Latitude of Dublin.

As the latitude of Dublin is about 53° 20′N., the celestial pole will have an altitude $PR = 53° 20'$ (fig. 3). The apparent diurnal paths of the stars cut the horizon obliquely. Some stars are circumpolar, and some rise and set, while other stars whose declinations are south become visible for a small portion of their daily circuit. The sun's apparent diurnal path during the summer months may be represented by the circle ACB north of the equator, of which there is more than half above the horizon; therefore during the *summer* we have *a long period of daylight and a short night.* Also in the *winter*, when the sun's declination is south, we have a *short period of daylight and a long night.*

The sphere in this position is called *an oblique sphere.*

Diurnal Rotation of the Earth.

21. That the apparent diurnal motion of the heavens from east to west is really due to the earth rotating round an axis from west to east appears from the following considerations:—

(1) From simplicity.

(2) From analogy.

(3) From centrifugal force.

(4) The experiment of letting a body fall from the top of a high tower.

(5) Foucault's pendulum experiment.

Under the first three of these headings are comprised the arguments which show that it is extremely probable that the earth does rotate; but (4) and (5) are experimental *proofs* of its rotation.

From Simplicity.—At the time of Copernicus, the only argument in favour of the earth's rotation was, that this was a much simpler, and therefore a much more probable, explanation than that all the stars and other heavenly bodies should be connected in such a complicated manner as to perform each its revolution round the celestial pole in the same time.

From Analogy.—However, the subsequent invention of telescopes (1609) supplied an additional argument. By the aid of the telescope we can see that many of the planets, as well as the sun and moon, are spherical bodies rotating about axes, from which we conclude that it is probable the earth also rotates.

From Centrifugal Force.—If the sun and planets, not to speak of the fixed stars, really described circles of such large radius in such a short period as one day, it would need an enormous attracting force acting towards the centres of those circles to keep them from flying off in a tangent.

For we know from mechanics that if m be the mass of a body moving in a circle of radius r with a periodic time $= T$, the necessary force acting towards the centre to keep it in its circular path would be given by the formula:—

$$F = m \cdot \frac{4\pi^2 r}{T^2}.$$

But here r would be very large and T very small, therefore F would be enormously great. But there are no bodies we know whose attractions could be as great as this; therefore the idea of the diurnal motion of the sun and planets, as well as the stars, is very improbable.

Experimental Proof from Falling Bodies.

22. Newton first suggested that if the earth rotate from west to east, a body, on being let fall from a considerable height above the earth's surface, should fall to the *east* of the vertical line.

For, let P be the place from which the body is let drop, suppose the top of a tower (fig. 9); PAC the vertical through P passing through C, the centre of the earth. Then, if PQ represent the arc described by the top of the tower while the body is falling, AB will represent the arc described by the base of the tower which, being less than PQ, shows that the base moves with a less velocity than the top. But while the body is falling in the air it preserves the same velocity *towards the east* which it had at starting in common with the top of the tower, which, being slightly greater than that of the base of the tower, will cause the body to deviate slightly to the east. Thus, if we cut off $AB' = PQ$, B will represent the position of the base of the tower, and B' of the body, when it reaches the ground.

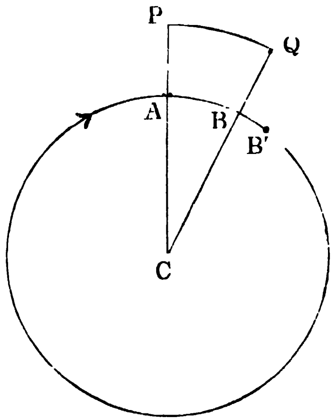

FIG. 9.

If now we let a body fall from a high tower, and we find by actual measurement that during its fall it has deviated to the east of the vertical, the only cause we can give for this deviation is that the earth rotates from west to east.

It is, however, very difficult to perform the experiment so as to give a decided result, the height of the tower being so small compared with the radius of the earth that the deviation would be very slight. It has been tried at Boulogne and Hamburg, and the deviation was found to be one-third of an inch in a fall of 250 feet.

Pendulum Experiments.

23. The experimental proof of the earth's rotation which is most striking is that first performed by Foucault in Paris in 1851, and very many times since by different observers.

Before entering upon the details of the experiment, we will first suppose that the earth *does* rotate from west to east, and see what effect this rotation would have on a pendulum swinging at the north pole. We know, from mechanics, that if a pendulum vibrate under the action of gravity alone, the plane of oscillation will remain *fixed in space*, for there is no force to make it deviate from that plane.

Therefore, if it were possible to have a pendulum vibrating at the north pole, the observer and the plane in which he stands would be carried by the rotation of the earth round the fixed plane of the pendulum through 360° in $23^h\ 56^m\ 4^s$. The observer, however, would be altogether unconscious of his own motion and that of the plane in which he stands, and it would appear to him as if the plane of the pendulum turned round in the opposite direction, making a complete circuit in $23^h\ 56^m\ 4^s$ (fig. 10).

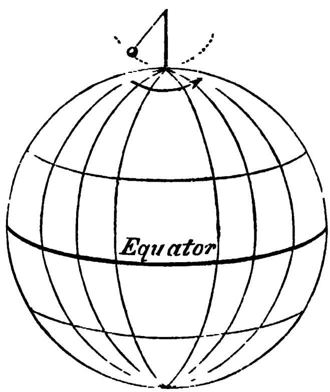

FIG. 10.

On the contrary, if a pendulum be set in motion at the equator, the plane of vibration, together with the observer and the surrounding surface of the earth will be carried bodily round in one common motion; therefore there will be no disturbance in the relative positions of the plane of the pendulum and the landmarks about it (fig. 11).

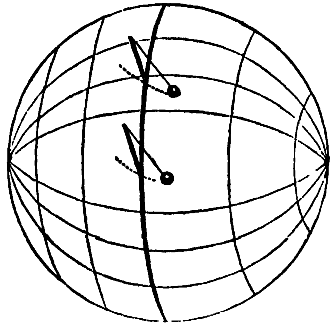

Fig. 11.

At a place intermediate between the equator and pole, the parts of the earth in the immediate neighbourhood of the pendulum which are nearest the equator will have a greater velocity towards the east than the parts nearest the pole; therefore the plane in which the observer stands will really revolve beneath the pendulum, or, in other words, the plane of vibration of the pendulum will *seem* to revolve in the opposite direction with respect to the observer and surrounding landmarks. The time of the apparent revolution of the pendulum will get greater the nearer we approach the equator, until on the equator itself, as we have seen above, the plane of vibration does not seem to change at all.

It is easy to prove, supposing that the earth does rotate, that at a place whose north or south latitude is λ, the time of apparent revolution of the pendulum would be T cosec λ, where T = time of revolution of the earth on its axis.

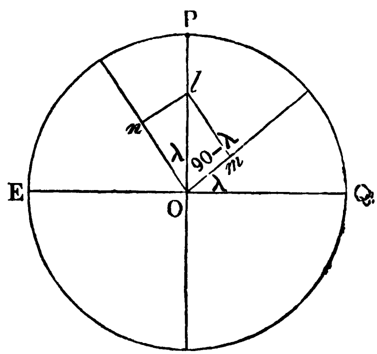

Fig 12.

For, let Om be the direction of the observer, P the north or south pole of the earth, EQ the equator, and λ = latitude of observer.

Now, the earth revolves round OP through $360°$ in T units of time; therefore it revolves through $\frac{360°}{T}$ in 1 unit of time. This angular velocity of $\frac{360°}{T}$ per unit of time can be resolved into two components in direction at right angles to one another; for we know, from dynamics, that rotations round axes can be resolved in exactly the same way as forces. Therefore, if we cut off Ol to represent an angular velocity of $\frac{360}{T}$ round OP, we find that this rotation is equivalent to a rotation represented by Om round the radius drawn to observer, and a rotation of On round a line at right angles to that radius; but

$$Om = Ol \cos (90 - \lambda) = Ol \sin \lambda ;$$

therefore to an observer at A the plane of a vibrating pendulum will appear to revolve through $\frac{360°}{T} \sin \lambda$ in 1 unit of time, and the time of making a complete revolution would therefore be

$$= \frac{360}{\frac{360}{T} \sin \lambda} = \frac{T}{\sin \lambda} = T \operatorname{cosec} \lambda = (23^h\, 56^m\, 4^s) \operatorname{cosec} \lambda.$$

Foucault's Experiment.

24. Foucault took a heavy iron ball and let it hang from the roof of the Pantheon by means of a wire about 200 feet long. A circular ridge of sand was placed in such a position that at every swing of the pendulum a pin attached to the lower part of the ball just scraped a mark in the sand. The ball was then drawn aside by means of a cord, and when at rest the cord was burnt off so that the pendulum should swing in as true a plane as possible.

It was then observed that the marks made in the sand at each swing did not coincide, but that the plane of the pendulum seemed to be slowly turning round with a watch-hand rotation. What actually happened, however, was that the whole Pantheon, together with the observer and the circular ridge of sand, slowly rotated in the opposite direction.

The wire was taken of this great length (200 feet) in order that the pendulum might move very slowly, thus meeting with very small resistance from the air, which enables it to keep up its motion for a long time. The reason a long wire ensures a long time of vibration is that the time of vibration is proportional to the square root of the length of the pendulum

$$T = \pi \sqrt{\frac{l}{g}};$$

therefore the longer the pendulum is made the greater is the time of one vibration.

If it were possible to keep the pendulum vibrating long enough at Paris to enable its plane to appear to make a complete revolution the time taken would be about 32 hours.

We can account for this phenomenon on no other supposition than that the earth revolves round an axis, and as the plane of the pendulum does not seem to change at the equator we know that the axis of revolution of the earth must be perpendicular to the equator.

There are various other phenomena which can be explained on the hypothesis of the rotation of the earth, such as trade winds and certain constant currents in the ocean; also the revolution of cyclones which in the southern hemisphere move with a watch-hand rotation, and in the northern hemisphere in the opposite direction.

EXAMPLES.

1. Find the lowest latitude at which it is possible to have a midnight sun.
Ans. 66° 32′ north or south.

2. What circles on the earth correspond to the latitudes 66° 32′ north or south? *Ans.* The arctic or antarctic circles.

3. What is the highest latitude north or south at which it is possible to see the sun in the zenith at noon? *Ans.* 23° 28′.

4. What is the latitude of a place at which the celestial equator and horizon coincide? *Ans.* 90°, at the poles.

5. What is the latitude of a place at which the ecliptic coincides with the horizon? *Ans.* 66° 32′.

6. Why is the sun never seen in the zenith at Dublin?

7. If a pendulum be made to vibrate at a place whose latitude is 30°, in what period of time will the plane of vibration appear to make a complete revolution?

Here
$$T = (23^{h}\ 56^{m}) \operatorname{cosec} 30^\circ$$
$$= (23^{h}\ 56^{m})\ 2$$
$$= 47^{h}\ 52^{m}.$$

8. At what part of the earth would a body have no deviation towards the east when let drop from a height? *Ans.* At the poles.

9. If a person travelling eastward go round the world, he will at the end of his journey appear to have gained a day. On the other hand, if he travel westward, he will appear to lose a day. Explain this.

10. How far should a man travel northwards from the equator in order that the altitude of the pole might become 10°? Assume the radius of the earth to be 4000 miles (J. S., T. C. D.).

Here
$$1^\circ = \frac{2 \times 3{\cdot}14159 \times 4000}{360};$$
$$\therefore\ 10^\circ = \frac{2 \times 3{\cdot}14159 \times 4000 \times 10}{360} = 698{\cdot}13 \text{ miles}.$$

CHAPTER III.

THE OBSERVATORY.

Astronomical Clock.

25. We have seen in Chapter I. that the apparent uniform revolution of the stars round the celestial pole is completed in about 4 minutes less time than the apparent diurnal revolution of the sun. This latter interval of time (more accurately its mean value throughout the year) is what is taken as the ordinary day, and is called a *mean solar day*. It is divided into 24 mean solar hours.

On the other hand, the interval of time taken by the fixed stars to complete a revolution round the pole is called a *sidereal day*. The sidereal day is divided into 24 sidereal hours, which are reckoned from 1 to 24, therefore we have—

$$24 \text{ sidereal hours} = 23^h\, 56^m \text{ mean solar time.}$$

The *astronomical clock* is regulated so as to mark sidereal time, and as the sidereal day commences when the first point of Aries is on the meridian, the clock should then be set to mark $0^h\, 0^m\, 0^s$. It will then indicate the sidereal hours up to 24 when the next transit occurs.

Definition.—The *sidereal time* at any instant is the interval that has elapsed since the preceding transit of the first point of Aries expressed in sidereal hours, minutes, &c.

As the right ascensions of heavenly bodies are measured eastwards along the equator from the first point of Aries, it follows that those stars which have a small right ascension will cross the meridian before those stars whose right ascensions are greater. In fact, 360° of right ascension correspond to 24 sidereal hours or 15° to 1 sidereal hour. Right ascensions may therefore be expressed in degrees or in time, the former being reduced to the latter by dividing by 15. Hence we might define the right ascension of a body as *the sidereal time of its passage across the meridian.*

The hour angle of the first point of Aries (Art. 14) at any instant reduced to time (by dividing by 15) is evidently the sidereal time at that instant.

The Transit Instrument

26. The object of this instrument is to determine the exact instant at which a body crosses the meridian. It consists of a telescope rigidly fixed to a horizontal axis. At the extremities of this horizontal axis are two cylindrical pivots, of the same diameter, which move in sockets fixed on two piers of solid masonry. In order to diminish the pressure of the pivots on the sockets, and consequently the wear caused by friction, a great part of the weight of the telescope is balanced by two weights which are hung at the extremities of a pair of levers, the other extremities being attached to the cylindrical pivots (see fig. 13).

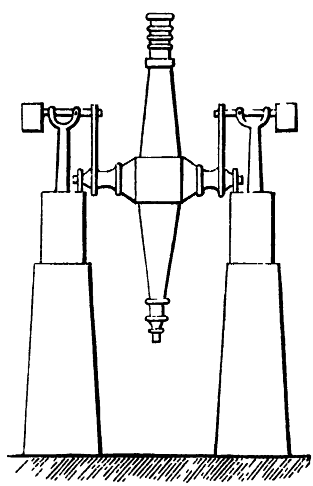

FIG. 13.

In the plane of the principal focus of the object-glass is placed a framework of five, or seven, or a greater number of vertical wires or spider lines (see fig. 14) placed at equal intervals apart. These are intersected at right angles by two horizontal lines, to which the path of the image of a star or other body across the field of view will be almost parallel and in a position midway between them.*

* In some portions of this Chapter, to avoid complexity, mention is only made of one horizontal line supposed to be situated midway between the two mentioned above.

Since the principal focus of the object-glass is in the same plane as that in which the lines are stretched, the image of a star under observation and the spider lines can be seen at the same time, and for purposes of adjustment the framework of lines admits of various movements by means of screws. When the instrument is used at night it is necessary to illuminate the spider lines. This is done by means of a lamp placed opposite one of the cylindrical pivots, the light from which by means of mirrors is reflected down the tube on to the lines.

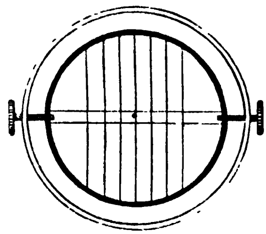

FIG. 14.

It is the object of the observer that the telescope be so adjusted that the middle vertical spider line may coincide as nearly as possible with the meridian. The time at which the star crosses the meridian can therefore be estimated by observing the instant, as indicated by the astronomical clock, at which it crosses this line. But as there is always a small error in noting the time of transit over one line, the seven are used in order that the observer may note the time at which the image crosses each of them, when the mean of these being taken will, in all probability, give a more accurate result than one observation could afford, as the observer may in some cases be too precipitate, in others too tardy, the positive and negative errors thus to some extent neutralizing one another.

Line of Collimation.—When the image of an object is formed at the principal focus of the object-glass of a telescope, then the direction in which it is viewed is the same as its true direction as seen by the naked eye; this line along which the object is viewed is called the *line of collimation*, or line of sight. For practical purposes the line of collimation of a telescope may be defined as the line joining the optical centre of the object-glass with that point of the central vertical spider line midway between the two horizontal lines.

Collimation, Level, and Deviation Errors, with the corresponding Adjustments.

27. Every transit instrument, to be perfectly adjusted, must satisfy the three following conditions:—

(1) The line of collimation should be perpendicular to the axis of rotation of the telescope.

(2) The axis of rotation should be horizontal.

(3) This horizontal axis should point due east and west, that is, the line of collimation should be due north and south.

Corresponding to the above conditions we have therefore in every instrument three errors—(1) collimation error; (2) level error; (3) deviation error; and to correct these we have three corresponding adjustments.

*Collimation Error.**

The error of collimation may be defined as the amount by which the angle between the line of collimation and the axis of revolution of the telescope falls short of a right angle.

Let XY (see fig. 15) represent the axis of revolution of the telescope, and AB the line of collimation supposed not at right angles to XY.

Let the telescope be pointed at an object or mark on the earth placed at some distance away, and let P be a point of the object which coincides with the middle vertical spider line. The telescope is then reversed in its bearings, the right hand pivot being

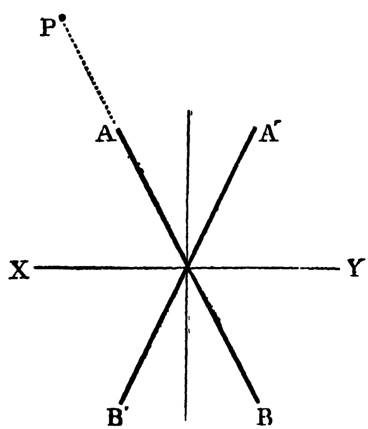

FIG. 15.

* *Besides the above method of correcting the collimation error two other methods, which are more frequently used in observatories, are given in Arts.* 36, 36A.

placed in the left socket, and *vice versâ*. If the point P still coincide with the central spider line, there is no collimation error. If not, the line of collimation, on the telescope being reversed, occupies the position $A'B'$, making an equal angle with the perpendicular on the other side, the error of collimation being half the angle between AB and $A'B'$.

To correct for this error the spider lines must be moved by means of screws until the central line coincides with the same point before and after reversing the telescope. When this adjustment has been made we know that the line of collimation sweeps out a great circle in the heavens. The object of the next two adjustments will be to insure that this great circle shall coincide with the meridian.

Level Error.

28. This error is due to the axis of revolution not being horizontal. To correct for it we make use of a spirit-level which is long enough to reach from one extremity of the axis to the other. The level is first hung on to the axis by means of hooks, and the position of the bubble is noted by means of a scale attached. The level is then reversed and the reading of the scale again noted. If the bubble occupies the same position as before there is no level error. But if not, one end of the axis must be raised or lowered by means of a screw until the reading of the bubble is the mean of the two former readings.

After correcting for this error we know that the line of collimation not only describes a great circle, but that this circle passes through the zenith, or in other words, is a vertical circle.

Deviation Error or Error of Azimuth.

29. This error is due to the line of collimation not pointing due north and south. The middle vertical spider line will therefore not coincide with the meridian, but with a vertical

such as ZX (see fig. 16). The error is detected by observing the interval between the upper and lower transits of a circumpolar star (preferably the pole star), and again the interval between its lower and upper transits. These two intervals should be equal, as the meridian bisects the circle described by the star round the pole. But if not equal, the line of collimation cannot coincide with the meridian, the star appearing to transit at m and n (fig. 16).

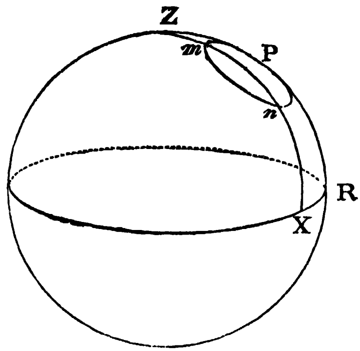

FIG. 16.

To correct for this error one extremity of the axis must be moved horizontally by means of a screw, until the two intervals above mentioned are identical.

Observing a Transit. Eye and Ear Method.

30. As the apparent motion of a star across the field of view is greatly magnified by the telescope, the star may appear at one side of a wire or spider line at the termination of one second, and at the other side before the end of the following second. An expert observer, however, can estimate to a small fraction of a second the instant of crossing the wire. When the star appears in the field of view he writes down the hour and minute from the clock, and then, without again looking up from his observation, keeps counting the seconds by the beats of the clock. The relative distances of the image of the star to the right and left of the wire at the end of two consecutive seconds enables him to determine the exact time of its passage across the wire. A similar observation is made for each of the seven wires.

This method is called the "eye and ear method."

The Chronograph.

A more accurate method of observing a transit is now coming into general use, by means of the *chronograph.* The clock is so arranged that at every beat an electric circuit is broken, which causes a dot to be made on a sheet of paper wrapped round a uniformly-revolving cylinder. The cylinder, besides revolving, let us say round a vertical axis, has a slow motion, either up or down in the direction of its length, so that the dots corresponding to seconds of time are arranged at equal intervals in a spiral or corkscrew curve on the cylinder.

The observer at the instant of transit across a wire, presses a button, which causes a dot to be made on the cylinder in addition to those caused by the clock-beats. The position of this dot relatively to the two dots caused by the clock immediately before and after, enables him, by direct measurement, to determine the time of transit to a very small fraction of a second.

Meridian Circle.

31. The meridian circle, or, as it sometimes called, the

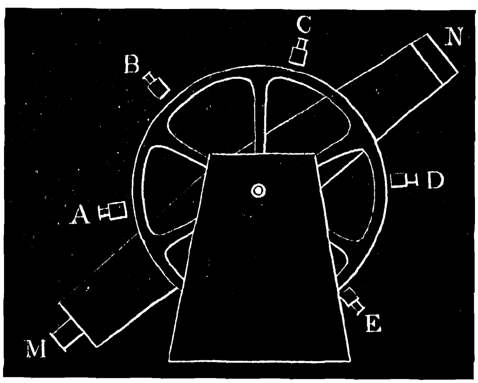

FIG. 17.

transit circle, consists of a transit instrument *MN* (fig. 17) such as we have already described, with the addition of a pair of graduated circles placed one on either side of the telescope. These circles are fixed with their planes at right angles to the horizontal axis, and revolve with the telescope. The rim of each circle is graduated by means of fine lines usually into intervals of 5′. As mechanical subdivision cannot go much further than this, the intermediate minutes and seconds are determined by means of a microscope. Usually, however, six microscopes placed at equal intervals are used, each of them being read off and the mean of them all taken. These are represented in fig. 17 by the letters *A*, *B*, *C*, &c. In addition to these there is a microscope *of low magnifying power* called the *Pointer*, which is used to read off the degrees and graduations corresponding to the intervals of 5′. These microscopes are all fixed, and therefore, as the circles revolve, the graduations pass across the field of view of each of them. The pointer microscope should read zero when the line of collimation points to the zenith.

Reading Microscopes.

In the focal plane of the object-glass of each of the six microscopes is fixed a small metal scale *mn*, cut into fine notches and called a comb. This scale and the image of the graduations on the circle are both seen together in the field of view as represented in fig. 18. There are 5 notches to each interval *ab* of the graduated circle, therefore each notch corresponds to 1′. A small aperture *p* in the scale is placed to mark the central notch, so that when the pointer is opposite a graduation *p* shall coincide with one also. A spider line *xy*,

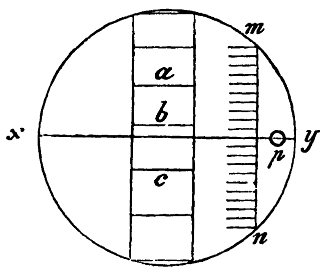

FIG. 18.

stretched across the field of view, can be moved parallel to itself by means of a micrometer screw, the head of which is divided into 60 equal parts. Five turns of the screw-head serve to bring the spider line through an interval *ab* of the graduated circle; therefore we have

5 turns screw-head = 5′ of graduated circle;
∴ 1 turn ,, ,, = 1′ ,, ,,
∴ 1 division ,, ,, = 1″ ,, ,,

Now, suppose we have to take the reading of the circle in any position. The pointer microscope gives the reading roughly in intervals of 5′. The odd minutes are given by the number of notches in the comb from the central notch *p* to the preceding graduation, say at *c*. The number of divisions through which the screw-head must be turned in order to bring the spider line into coincidence with *c* from the preceding notch will give the number of seconds.

Error of Eccentricity avoided by taking the mean of two microscope readings by diametrically opposite points.

32. The error of eccentricity is caused by the graduated circle revolving round some point not its centre. This is

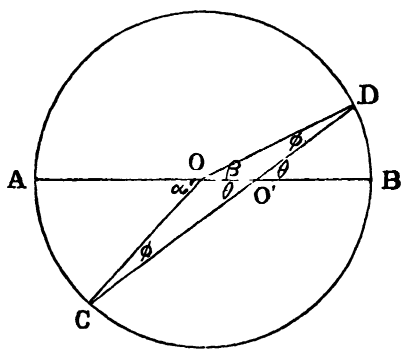

FIG. 19.

entirely eliminated by taking the mean of the readings at diametrically opposite points. For let O be the centre of the circle (fig. 19), O' the point round which the circle revolves. Now let the circle revolve round O' through an angle θ, so that the line AB occupies the position CD; then the angles α and β subtended at the centre by the arcs AC and BD will correspond to the two readings at opposite sides. We have to prove that the mean of α and β will equal θ.

By Euclid (I. 32) we have

$$\beta + \phi = \theta;$$

$$\therefore \quad \beta = \theta - \phi,$$

also

$$\alpha = \theta + \phi;$$

$$\therefore \quad \alpha + \beta = 2\theta; \qquad \therefore \quad \theta = \tfrac{1}{2}(\alpha + \beta).$$

To find the Zenith Point on the Meridian Circle.

33. We have already stated that the pointer should read zero when the line of collimation points to the zenith. But the mean reading of the six microscopes is not generally zero at the same time. We have, therefore, in every transit circle to find the *zenith point* or the reading of the circles corresponding to the zenith.

In order to find this point the telescope is directed vertically downwards to a basin of mercury placed beneath it. It is then moved until the fixed horizontal wire and its image reflected by the mercury are perfectly coincident. The line of collimation then points directly to the nadir. The reading of the pointer and microscopes, therefore, gives the nadir point, 180° from which is the zenith point.

Another method of finding the zenith point is by observing the pole star (which is chosen because on account of its slow motion it remains a long time in the field of view), and taking the reading of the circle when the star appears on the horizontal wire. The telescope is then depressed until the image of the star reflected from the surface of a basin of mercury coincides with the horizontal wire, and the reading again taken. As the telescope in the two instances must have been inclined at equal angles above and below the horizon, the mean of the two readings gives the horizontal point, 90° from which is the zenith point.

It is also evident that half the difference of these two readings, corresponding to the direction of a star and of its image in a basin of mercury, is the meridian altitude of the star.

The Polar Point on the Meridian Circle.

33A. To find the *polar-point*, *i.e.* the reading of the meridian circle when the telescope is pointed to the pole.

A circumpolar star is observed in its upper and lower transits, and in each case the reading of the meridian circle is taken.

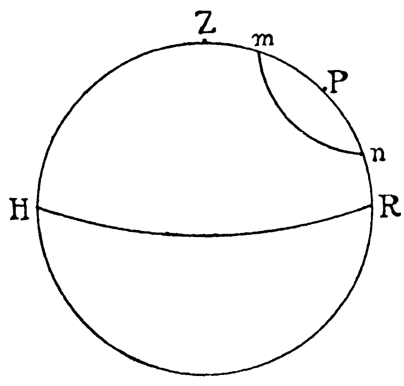

FIG. 19A.

Thus, if m and n represent the positions of the star at its upper and lower culminations, we have

$$Pm = Pn = \text{polar distance of star} = \Delta.$$

$$\text{Let polar point} = x.$$

$$\text{Then } x + \Delta = \text{reading of circle when star is at } n,$$

$$\text{and } x - \Delta = \text{reading of circle when star is at } m.$$

Therefore the polar point x = half the sum of the two readings.

Since PR = altitude of pole = latitude of place; $\therefore ZP$ = the complement of the latitude = colatitude.

The latitude of the observatory or place can now be found, for *the difference between the zenith point and the polar point is the colatitude ZP which, subtracted from 90°, gives the latitude.*

Meridian Zenith Distance. Meridian Altitude. Declination.

34. In order to measure the zenith distance of a star when in the meridian, the telescope is pointed to the star and the reading of the circle taken. The difference between this reading and the zenith reading gives the meridian zenith distance of the body, which must be corrected for refraction and other errors.

The meridian altitude is obtained by subtracting the

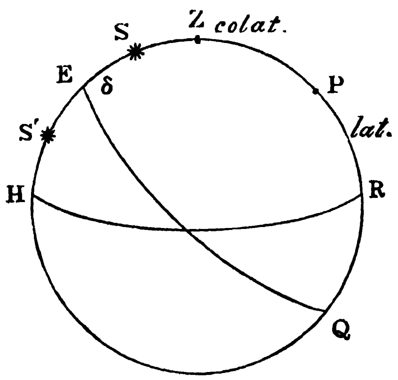

FIG. 20.

observed zenith distance from 90°. Having obtained the meridian altitude of the star we can now, the latitude of the place being known, determine its declination. For let S be the position of a star in the meridian, then :—

SH = meridian altitude = a;

SE = declination = δ.

Also EH = colatitude ZP (both having a common complement ZE). Now we have

$$EH + SE = SH,$$

$$\text{or} \quad \text{colat} + \delta = a.$$

Similarly, if the star be at S', we have

$$\text{colat} - \delta = a.$$

Therefore colatitude ± declination = meridian altitude, the plus or minus sign being taken in the northern hemisphere, according as the star's declination is north or south.* From this equation, knowing the meridian altitude of the star and the latitude of the place, its declination is determined.

35. **Standard Stars.**—In the Nautical Almanac, which is published every year, is a list of stars whose right ascensions and declinations are recorded for each day. Their declinations are determined by the method we have just indicated, and a method of finding their right ascensions will be given later on (see Flamsteed's Method, Chapter VIII.). These stars are called *standard stars.* The declination of any other star can be found by comparing the reading of the transit circle when the telescope is directed to the star with the corresponding reading for a standard star. The difference of the two readings being the difference of their declinations, the declination of the body in question can therefore be found.

Regulation of the Clock.

As the first point of Aries is an imaginary point in the sky, such that we cannot *observe* its passage across the meridian, we are not therefore able to tell by direct observation when to set the clock at $0^h\ 0^m\ 0^s$. But the time of transit of

---

* In case the star transits between Z and P the equation becomes

$$\text{colat} + \delta = 180 - a.$$

a standard star is known from its right ascension, and the clock can therefore be set at correct sidereal time when one of these stars is observed in the meridian.

The *rate* of the clock, *i.e.* the amount it gains or loses each day, can be determined by noting the interval between the transits of a fixed star on two successive nights. The interval should be 24 sidereal hours, from which the daily gain or loss of the clock is found. A good clock should be such that its rate of gain or loss is uniform.

To find the Right Ascension of a Body.

The clock being set correctly and its *rate* being known, then the sidereal time at which a body crosses the meridian is its right ascension, which can be reduced to degrees, minutes, and seconds by multiplying by 15.

Collimating Telescopes.

36. In order to correct the error of collimation, we have seen that the axis of the telescope has to be reversed in its sockets, and the direction of a distant mark observed before and after reversal. This method has, however, now been superseded by the use of two small telescopes, called *collimating telescopes*, fixed one to the north, and the other to the south side of the transit telescope. Each of these is furnished with cross wires, so that, on looking through one into the other, which can be managed by means of an opening in the tube of the large telescope, the images of the cross wires (illuminated) appear coincident. If now the cross wires of the transit instrument itself be so adjusted as to coincide with those of the north collimator, and it be found on rotating the telescope round that they are also coincident with those on the south, the line of collimation must be perpendicular to the horizontal axis. By this method the troublesome operation of reversing the axis of the telescope is avoided.

36A. There is yet another method of determining the collimation error by pointing the telescope vertically downwards towards a basin of mercury. If the axis be perfectly horizontal and there be no collimation error, the spider lines should coincide with their image formed by reflection from the surface of the mercury ; for the rays of light diverging from the spider lines (which are illuminated), after passing through the object glass, fall in parallel lines on the surface of the mercury, from which they are again reflected in parallel lines, which are converged back again to its focus by the object glass. If, therefore, the level error having been previously corrected, the real system of spider lines do not coincide with the reflected system, the difference, which may be measured by a micrometer, is twice the error of collimation.

The Equatorial.

37. Most of the large telescopes in observatories are mounted equatorially. This arrangement consists in an axis *AB* (fig. 21) which points to the celestial pole, called

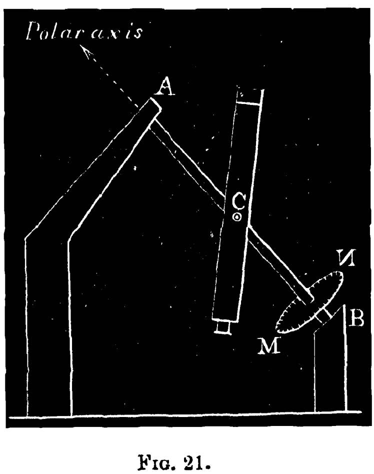

FIG. 21.

the *polar axis*. This polar axis turns in fixed bearings *A* and *B* attached to two fixed piers. The telescope can be turned round an axis *C*, so as to be set at any angle to the polar axis. A clockwork apparatus is generally attached to the larger instruments, by means of which the polar axis is made to revolve uniformly in its bearings in the same direction as the diurnal motion of the heavens, the revolution being completed in $23^h\ 56^m$, so that once the telescope is pointed at a star, and the clockwork apparatus set going, it is possible to keep that star in the field of view for a prolonged period.

By combining the two motions which it is possible to give the equatorial, it can be pointed at any star in the heavens, which need not, as in the case of the transit instrument, be in the meridian. It is therefore for observation of bodies not in the meridian that this instrument is used.

A graduated circle *mn*, whose plane is at right angles to the polar axis, serves to set the telescope at any required right ascension. It is called the *hour circle*. The axis *C*, round which the telescope turns, also carries a graduated circle, which is not drawn on the figure. It is called the *declination circle*, as by means of it the telescope can be set at any required declination. Both circles are read off by pointer microscopes.

The equatorial, on account of its high magnifying power, enables us to observe the nature of the moon and planets and other heavenly bodies. It is also used in stellar photography and in the spectroscopic analysis of the stars.

Micrometers.

38. Every equatorial is furnished with a micrometer for measuring small angular distances such as the angle subtended at the observer by two neighbouring stars in the field of view of the telescope. The kind most commonly used is the parallel wire or spider line micrometer. It consists of a

rectangular framework (fig. 21A) with a graduated screw-

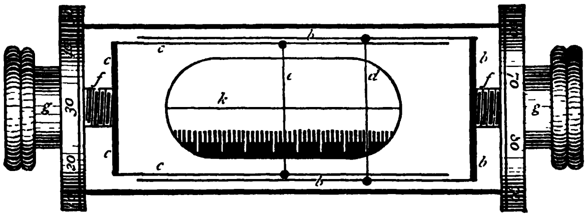

FIG. 21A.

head *gg* at each end; *bbb*, *ccc* are two metal forks which slide within one another on which are fixed two parallel spider lines *d* and *e*; two fine screws *f*, *f* having milled heads *g*, *g* connected with graduated circles, are attached one to each fork, so that by turning these milled heads each fork can be drawn out or pushed in according to the direction in which the head is turned, and thus the spider lines can be brought as wide apart or placed as close together as we please. There is also a fixed transverse spider line *k* at right angles to *d* and *e*. The circumference of each of the circles in connexion with the screw-heads is divided into 100 equal parts. There is also a fine scale cut into notches, every fifth notch being cut deeper than the others as is seen in the above diagram; the distance between two consecutive teeth being equal to the interval between the threads of each of the screws, therefore a complete revolution of one of the screw-heads just moves the corresponding spider line through a distance equal to the common interval between the teeth.

In order to measure the angle subtended by two neighbouring stars at the observer, the micrometer is placed in the focal plane of the telescope and rotated until the fixed transverse spider line passes through the images of the two stars. The two parallel lines are then shifted by means of the screws until each coincides with an image of a star. The distance between the two wires can now be found by noting, by means of the teeth cut in the scale, how many turns must be given to the screw-head (or screw-heads) to make the parallel lines coincide. The fractional parts of a turn can be read off on the graduated circle attached to each screw-head; and the angular value of each turn being known, we are able to calculate the angle subtended by the stars.

The micrometer also serves to measure the angular diameters of the sun, moon, or planets, one of the parallel lines being placed so as to touch one limb, and the other the diametrically opposite limb of the circular disc presented by the body, and the distance between them is measured as before.

38A. *To find the angular value of each turn of the micrometer screw*, a circumpolar star is chosen, preferably the pole star, for, on account of its very small distance from the pole, its motion is very slow, and can therefore be most accurately observed. The micrometer is then adjusted so that the diurnal motion of the star is along or parallel to the fixed spider line. The two movable lines are then separated by a certain number of turns of the screw, and the time taken by the image of the star to pass from one line to the other is noted, from which, knowing that the star describes 360° of a small circle in 24 sidereal hours, the angular value of the distance between the wires is easily found, and hence the angle, expressed in seconds of a small circle, corresponding to one turn is known: but the relative magnitude of the small circle described by the pole star to a great circle can be found since the declination of the star is known; hence the number of seconds of the arc of a great circle corresponding to each turn is obtained.

The Alt-Azimuth Instrument.

The alt-azimuth may be described as an equatorial, of which the axis points to the zenith instead of the celestial pole. It admits of a double motion in altitude and azimuth, just as the equatorial does in right ascension and declination. Like the equatorial, it is used in ex-meridian observations.

Given the zenith distances of a circumpolar star at its upper and lower transits to calculate the latitude of the place and the star's declination.

Let the zenith distances* Zn and Zm (fig. 19A) be represented by z and z', also

$$\text{Polar distance } Pm = Pn = \Delta, \text{ and } ZP = \text{colat};$$

$$\therefore\ z = \text{colat} + \Delta$$

$$z' = \text{colat} - \Delta;$$

$$\therefore\ z + z' = 2 \text{ colat},$$

and

$$z - z' = 2\Delta;$$

$$\therefore\ \text{colat} = \frac{z + z'}{2}; \text{ hence lat.} = 90 - \frac{z + z'}{2},$$

and polar distance $\Delta = \dfrac{z - z'}{2}$; hence decln. $\delta = 90 - \dfrac{z - z'}{2}$.

N.B.—If the star in one of its transits *souths*, i.e. if it cross the meridian south of the zenith, its zenith distance at this transit is to be considered negative.

EXAMPLES.

1. Supposing the earth to rotate with the same angular velocity as at present, but in the opposite direction, what would be the length of a mean solar day and the number of mean solar days in the year? *Ans.* $23^h\ 52^m$; $367\frac{1}{4}$.

2. How many sidereal days are there in the year? *Ans.* $366\frac{1}{4}$.

3. What is the meridian altitude of the sun at Dublin on the 21st June, the latitude of Dublin being 53° 20′? *Ans.* 60° 8′.

Here $\text{colat} \pm \delta = \alpha$ (Art. 34):

but $\delta = 23^\circ\ 28'$ N. and $\text{colat} = 90^\circ - 53^\circ\ 20' = 36^\circ\ 40'$;

$$\therefore\ 36^\circ\ 40' + 23^\circ\ 28' = \alpha;$$

$$\therefore\ \alpha = 60^\circ\ 8'.$$

4. What is the meridian altitude of the sun at Dublin—(1) during the winter solstice; (2) at the equinoxes? *Ans.* (1) 13° 12′.
(2) 36° 40′.

N.B.—At winter solstice $\delta = 23^\circ\ 28'$ S. (minus).

* *In all cases these zenith distances when measured by the meridian circle are to be corrected for refraction and other errors.*

5. The zenith distances of a circumpolar star as it crosses the meridian above and below the pole are found, after correcting for refraction, &c., to be 13° 7′ 16″ and 47° 18′ 26″. Calculate from this the latitude of the place and the declination of the star. (See Art. 38A). *Ans.* 59° 47′ 9″; 72° 54′ 25″.

6. The latitude of John o' Groat's house is 58° 59′ N. Find the sun's meridian altitudes at that place on midsummer and midwinter days, respectively. *Ans.* 54° 29′; 7° 33′.

7 Find the latitude of a place where the greatest elevation of the sun above the horizon at midsummer is 76° 42′. *Ans.* 36° 46′.

8. The declination of Canopus is 52° 38′ S.; if we travel southwards, where shall we first find it attain a meridian altitude of 10°? *Ans.* 27° 22′ North latitude.

9. Find the declination of a star whose corrected meridian zenith distance, as observed at Dublin (lat. 53° 20′), is 72° 18′ 40″. *Ans.* 18° 58′ 40″ S.

10. What is the sun's midnight depression below the horizon at Dublin during midsummer and midwinter, respectively. *Ans.* 13° 12′; 60° 8′.

11. The zenith distances of a star at lower and upper culminations are found, after correcting for refraction, &c., to be 76° 1′ and 2° 52′ S. respectively.

Find the latitude of the place, and the declination of the star.

N.B. —Apply formulæ in Art. 38A, but 2° 52′ being south is given a minus sign. *Ans.* 53° 24′; 50° 32′.

12. The declination of Vega (α Lyræ) is 38° 41′ N.; does it cross the meridian of Dublin (lat. 53° 20′) north or south of the zenith? *Ans.* Upper transit, 14° 39′ S. of zenith. Lower transit, 87° 59′ N. of zenith.

CHAPTER IV.

ATMOSPHERIC REFRACTION.

39. We have seen (Chapter III.) how the altitude of a star can be found by observation. This observed altitude, however, is liable to some error, owing to the rays from the star being bent in passing through the atmosphere before they reach the eye of the observer, thus leading him to think that the star is in a different direction than is really the case. This apparent displacement is due to the refracting power of the atmosphere.

We know from optics that when a ray of light passes from a rarer to a denser medium, it is refracted or bent towards the perpendicular to the common surface of the two media. Thus AOB would represent the path of such a refracted ray, the angle i being the angle of incidence, r the angle of refraction, while $i - r$ is the amount of the refraction.

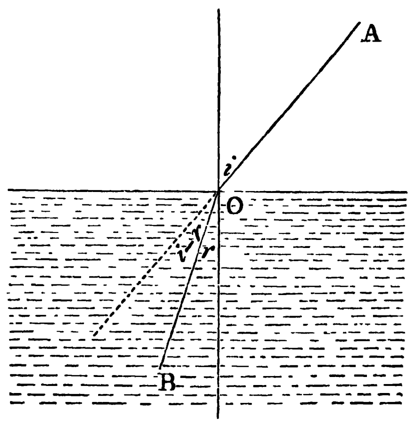

FIG. 22.

It is also a law of optics that the angles of incidence and refraction are such that their sines are in a constant ratio; therefore,

$$\frac{\sin i}{\sin r} = \text{a constant} = \mu.$$

Now the atmosphere is a gaseous fluid, subject to the action of gravity. Its density in its upper layers is very small; but as we approach the earth its density increases as the weight of the superincumbent air on any given area increases. Therefore, when a ray of light from a star *S* strikes the atmosphere, we may suppose it in its passage to the earth to pass through an indefinite number of media each denser than the preceding, like a number of concentric spherical shells.

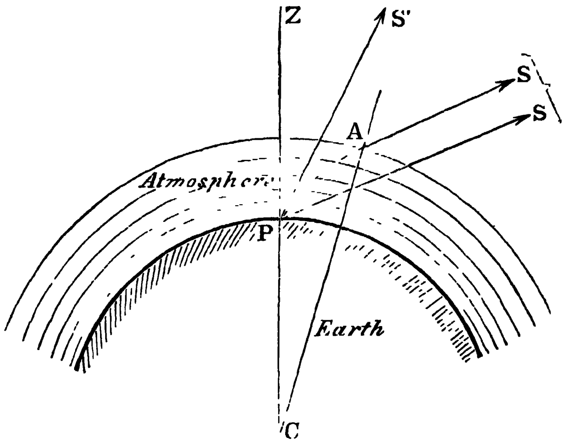

FIG. 23.

The path *AP* of the ray through the atmosphere thus being continually bent will be curved. To an observer at *P* the star will appear in the direction *PS'*, a tangent to the curve at the point *P*; whereas its real direction, if there were no atmosphere to refract the ray, would be *PS*, a parallel drawn through *P* to *AS*; for, being so far distant, the lines drawn from *A* and *P* to the star will be practically parallel.

The angle *SPS'* between the apparent direction of the star and the direction in which it would appear if there were no atmosphere is called the *refraction*.

The effect of refraction, therefore, on the position of a heavenly body, is to raise it in the sky, so as to increase its altitude and diminish its zenith distance. But as this apparent displacement takes place in a vertical plane, the azimuth of the body is not affected.

Therefore, in all observations of the altitudes of heavenly bodies each apparent altitude must be diminished by the amount of the refraction, in order to get the true altitude. This correction is called the *correction for refraction.*

The amount of refraction is greatest when the angle of incidence is greatest, *i.e.* when the body is on the horizon. The refraction is then called the *horizontal refraction.*

The refraction is zero at the zenith, as the rays from a body situated right overhead strike the different layers of the atmosphere at right angles, and, therefore, do not get bent. The horizontal refraction is about 35′. Therefore, a body on the horizon will appear a little more than half a degree above the horizon. As the angular diameter of the sun is about 32′, or a little more than half a degree, we are able to form some sort of idea as to how much a body on the horizon is displaced by refraction, by remembering that it is through an arc nearly equal to the breadth of the sun's disc. From this we conclude that when it appears to us that the sun is about to set he has in reality just set, and we would not see him at all were there no atmosphere to refract his rays.

The amount of refraction is influenced by the changes in the pressure and temperature of the atmosphere. A rise in the barometer is accompanied by an increase in the amount of refraction, provided the altitude of the body remain the same. On the contrary, an increase of temperature produces a diminution of refraction under the same circumstances. In an observatory it is necessary, in estimating the error due to refraction, to take into account not only the zenith distance of the body, but also the pressure and temperature of the atmosphere as indicated by the barometer and thermometer respectively. We have seen that the horizontal refraction is about 35′: therefore, how rapidly it decreases as the zenith distance decreases is seen from the fact that the refraction at an altitude of 45° has a mean value of only 58″·2.

Law of Refraction.

40. As the height of the atmosphere is so very small compared with the radius of the earth, we may assume that the lines drawn from A and P (fig. 23) to the centre of the earth are parallel, or, in other words, that the surface of the earth is a horizontal plane, with an indefinite number of horizontal layers of atmosphere of gradually decreasing density resting on it. We can now very easily deduce a law according to which the refraction varies; for the ray will get bent through the same amount if, instead of passing through a number of layers of varying density, we suppose it to pass through a homogeneous atmosphere of the same density throughout as the layer in contact with the earth, when we can imagine it to get bent once for all at its entrance into the atmosphere, and then proceed in a straight line to the observer.

The refraction of a heavenly body, the temperature and pressure being constant, varies as the tangent of the apparent zenith distance.

Let SAP represent the path of a ray from a star to an observer at P (fig. 24). The apparent direction of a star will then be PS', the angle z being the apparent, and $z + x$ the real zenith distance, while angle SAS' = amount of refraction $= x$.

Now $$\frac{\text{sine (angle of incidence)}}{\text{sine (angle of refraction)}} = \text{a constant} = \mu.$$

or $$\frac{\sin (z + x)}{\sin z} = \mu, \text{ that is, } \sin (z + x) = \mu \sin z;$$

$$\sin z \cos x + \cos z \sin x = \mu \sin z.$$

But x is a very small angle, and we know from trigonometry that the cosine of a very small angle is almost = 1, and as the perpendicular and arc almost coincide, its sine = its circular measure;

$$\therefore \quad \sin x = x \text{ (expressed in circular measure)},$$

and $$\cos x = 1;$$

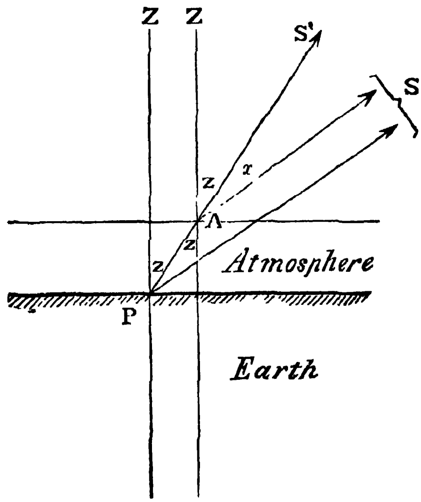

FIG. 24.

therefore the above equation becomes

$$\sin z + x \cos z = \mu \sin z;$$

$$\therefore \quad x \cos z = \mu \sin z - \sin z = (\mu - 1) \sin z;$$

$$\therefore \quad x = (\mu - 1) \frac{\sin z}{\cos z},$$

or $$x = (\mu - 1) \tan z.$$

Let $$\mu - 1 = K;$$

$$\therefore \quad x = K \tan z; \quad \therefore x \text{ varies as } \tan z.$$

This law has been found to be approximately true for zenith distances up to 75°. Nearer the horizon the law does not hold, as the constitution of the different layers of the atmosphere will affect it. It is evident at once that the law could not hold at the horizon where the zenith distance = 90°, for tan 90° = infinity.

The amount of the refraction at any observed zenith distance less than 75° can be found by substituting for tan Z its value, provided the value of the constant K be known, for finding which we give the following methods.

41. *To find the constant coefficient of refraction when the latitude of the place is known.*

This is done by observing with the meridian circle the zenith distances of a circumpolar star as it crosses the meridian above and below the pole.

Let m and n represent the true positions of the star at its two culminations, then to the observer the star will appear at m' and n' as raised by refraction. Let the observed zenith distances Zm' and Zn' be represented by z and z':

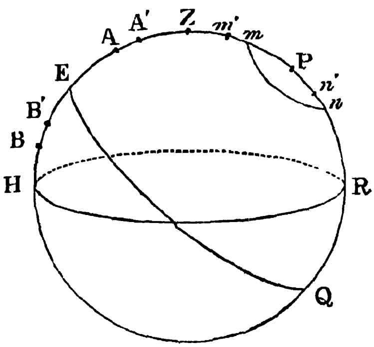

FIG. 25.

$$\therefore \quad Zm = Zm' + \text{the refraction} = z + K \tan z,$$

$$Zn = Zn' + \text{the refraction} = z' + K \tan z';$$

adding, we get $\quad Zm + Zn = z + z' + K(\tan z + \tan z').$

But $\quad Zm + Zn = 2 \text{ colat} \quad$ (Art. 38A)

(for PR = latitude of place) = 180° − 2 lat:

$$\therefore \quad 180° - 2 \text{ lat} = z + z' + K(\tan z + \tan z');$$

$$\therefore \quad K = \frac{180° - 2 \text{ lat} - z - z'}{\tan z + \tan z'}.$$

But the latitude of the place is known, and z and z' are the observed apparent zenith distances; therefore K is determined.

42. **Bradley's Method.**—The coefficient of refraction can be found when the latitude of the place is not known by the method of Dr. Bradley, who, besides observing the zenith distances of a circumpolar star at its two culminations, measured the zenith distances of the sun when in the meridian at the summer and winter solstices, when the sun's declination is 23° 28′ north and 23° 28′ south respectively. Let these observed zenith distances be denoted by s and s'. If now A and B (fig. 25) represent the real positions of the sun, the apparent positions when observed will appear raised to A' and B';

$$\therefore \quad ZA = ZA' + \text{refraction} = s + K \tan s,$$

$$ZB = ZB' + \text{refraction} = s' + K \tan s';$$

adding, we get

$$ZA + ZB = s + s' + K(\tan s + \tan s');$$

but $ZA + ZB = 2ZE$ as before, and arc $ZE = PR$ (having a common complement ZP) = lat;

$$\therefore \quad 2\text{ lat} = s + s' + K(\tan s + \tan s').$$

We have also from observations on a circumpolar star, as in the last method,

$$180° - 2\text{ lat} = z + z' + K(\tan z + \tan z').$$

By adding these two equations, we eliminate the latitude thus:—

$$180° = z + z' + s + s' + K\{\tan z + \tan z' + \tan s + \tan s'\};$$

$$\therefore \quad K = \frac{180° - z - z' - s - s'}{\tan z + \tan z' + \tan s + \tan s'}.$$

But z, z', s, and s' are observed; therefore K is determined. The fact that the latitude need not be known is an advantage in Bradley's method, but it takes six months to complete the observation.

By these methods the constant of refraction has been estimated at about 58″·2; $\therefore$ $r = 58''{\cdot}2 \tan z$.

42A. The coefficient of refraction may also be found and the latitude of the place determined at the same time, by observing the apparent zenith distances of two circumpolar stars at their transits above and below the pole.

Thus we have for one star:—

$$180^\circ - 2 \text{ lat} = z + z' + K(\tan z + \tan z');$$

the second circumpolar star will similarly give

$$180^\circ - 2 \text{ lat} = z_1 + z_1' + K(\tan z_1 + \tan z_1');$$

$$\therefore \quad z + z' + K(\tan z + \tan z') = z_1 + z_1' + K(\tan z_1 + \tan z_1');$$

$$\therefore \quad K(\tan z + \tan z' - \tan z_1 - \tan z_1') = z_1 + z_1' - z - z',$$

$$\therefore \quad K = \frac{z_1 + z_1' - z - z'}{\tan z + \tan z' - \tan z_1 - \tan z_1'};$$

the value of K being thus found, the latitude may be determined by substitution in one of the above equations.

43. A curious effect of refraction is the oval shapes which the sun and moon appear to have when near the horizon. The reason of this phenomenon is, that the lower limb being nearer the horizon than the upper limb will be raised to a greater extent by refraction. The vertical diameter AB

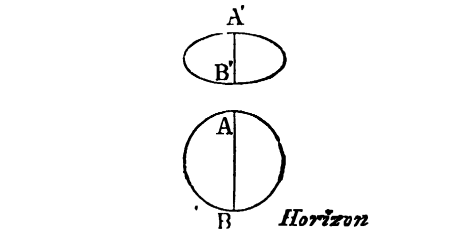

FIG. 26.

will therefore appear shortened as $A'B'$, while the horizontal diameter remains the same. This apparent dimmution in the vertical diameter of the sun and moon when near the horizon amounts to about one-sixth part of the whole, or about 5'.

EXAMPLES.

1. The apparent zenith distance of a star is 30°: calculate the true zenith, assuming the coefficient of refraction to be 58″·2.

Here the refraction = 58″·2 tan 30°.

$$= 58''\cdot 2 \times \frac{1}{\sqrt{3}} = 33''\cdot 6;$$

∴ True zenith distance = 30° 0′ 33″·6.

2. The apparent altitude of a star is 30°; calculate the true altitude, the coefficient of refraction being 58″·2. *Ans.* 29° 58′ 19″·2.

3. An altitude of a star is observed, and found to be the angle whose sine is $\frac{5}{13}$; calculate the true position of the star, assuming the amount of refraction at an altitude of 45° to be 58″·2 (J. S., T.C.D.).

Here the refraction = $K \tan Z$,

$$\text{but} \quad K = 58''\cdot 2 \text{ for } \tan 45° = 1, \text{ and } \tan Z = \cot(\text{alt}) = \frac{12}{5};$$

$$\therefore \quad \text{refraction} = \frac{12}{5} \times 58''\cdot 2 = 2'\ 19''\cdot 7.$$

Therefore the true altitude is 2′ 19″·7 less than the observed altitude

4. The meridian altitudes of a circumpolar star are 20° and 30°, and the corresponding corrections for refraction are 1′ 40″ and 1′ 9″; find the latitude of the place (Degree, T.C.D.). *Ans.* 24° 58′ 35″·5.

5. If α, α' be the true and apparent altitudes of a body affected by refraction, prove the equation $\alpha = \alpha' - 58''\cdot 2 \cot \alpha'$.

6. Find the latitude of a place at which the observed meridian zenith distances of a circumpolar star were 47° 28′ and 22° 18′, given that the tangents of these angles are 1·09 and ·41 respectively, and taking the coefficient of refraction to be 58″·2.

Here (Art. 41) $2 \text{ colat} = Z + Z' + K(\tan Z + \tan Z')$;

or
$$\begin{aligned} 2 \text{ colat} &= 47°\ 28' + 22°\ 18' + 58''\cdot 2\ (1\cdot 09 + \cdot 41) \\ &= 69°\ 46' + 58''\cdot 2 \times 1\cdot 5 \\ &= 69°\ 47'\ 27''\cdot 3; \\ \therefore \text{ colat} &= 34°\ 53'\ 43''\cdot 6; \\ \therefore \text{ lat} &= 55°\ 6'\ 16''\cdot 4. \end{aligned}$$

CHAPTER V.

THE SUN.

44. To an inhabitant of the earth the sun is by far the most important of all the heavenly bodies. His rays supply light and heat not only to the earth, but to the other planets, and his attraction controls their motions, causing them to describe their respective orbits. It is therefore hardly to be wondered at, that from the most ancient times a body of such splendour, whose influence on earthly affairs was so supreme, should have been an object of great awe and veneration.

The sun is an intensely hot and luminous body, distant from the earth by about 92,700,000 miles. The angle which the diameter of his disc subtends at the earth, when measured by a micrometer, is found to have a mean value of about 32′. From this the sun's diameter in miles can be obtained, for

$$\frac{32' \times 60}{206265''} = \frac{d}{92,700,000}.$$

From which we get d the diameter of the sun to be about 860,000 miles, or about 110 times the earth's diameter.

As the volumes of two spheres are to one another as the cubes of their diameters, this would give

$$\text{vol. of sun} = (110)^3 \times \text{vol. of earth}$$

$$= 1,331,000 \times \text{vol. of earth.}$$

So that if 1,331,000 spheres like the earth were massed together into one sphere, the resulting volume would about equal that of the sun.

The sun's density, however, owing to his physical state, is only about one-fourth that of the earth, from which we conclude that his mass is about 333,000 times the mass of the earth.

The Sun's Apparent Diurnal and Annual Motions.

45. We have seen in Chapter I., that besides a comparatively rapid diurnal motion from east to west which the sun has in common with all the other heavenly bodies, he seems to have a slow motion from west to east among the fixed stars at the rate of about 1° daily, so as to make a complete revolution each year. A mean daily change in right ascension of 1° is equivalent to 4 minutes of time, for 15° corresponds to one hour. The mean solar day is thus 4 minutes longer than the sidereal day.

We have seen, in Chapter II., that the apparent diurnal motion of the heavenly bodies is really due to a revolution of the earth on its axis, and it will presently be shown that the sun's apparent annual motion in the ecliptic is due to a motion of the earth in an orbit round the sun.

On account of the sun's change of position among the fixed stars, the appearance which the heavens present to us each night at a certain fixed hour goes through a regular cycle of changes in the course of the year. For instance, stars and constellations which are visible at, say 11 o'clock at night during winter—such as Sirius, Aldebaran, the Pleiades, and the constellation of Orion, will be below the horizon at the same hour in summer. The reason of this is evident if it is borne in mind that 11 P.M. means 11 *hours after the sun has been in the meridian*; and therefore, when observed at the same hour each night, each star will have shifted with reference to both meridian and horizon.

To Trace the Annual Path of the Sun on the Celestial Sphere.

46. On account of the sun's brightness it is impossible, even in an observatory, to see those stars which are at all close to his disc, and therefore the sun's position with reference to them cannot be directly measured. How, then, can the ecliptic be traced out? To the ancient astronomers, who were without instruments of great accuracy, this was indeed a difficult problem. Hipparchus (160 B.C.) noted the sun's position relative to the moon during the daytime, and then, during the night, he determined the moon's position among the fixed stars, from which he deduced the position occupied by the sun. But in a modern observatory, by means of the transit circle and astronomical clock, we can find the right ascension and declination of the sun's centre, from which we are able to note on the celestial globe his position among the fixed stars. By repeating these observations at noon each day, his annual path can be traced out.

When the ecliptic is thus mapped out on the celestial sphere it is found to be a *great* circle, that, is, its plane passes through the earth, which is situated at its centre. But let us not for a moment suppose that the sun's apparent yearly motion could be explained by supposing it to describe a circle round the earth as centre merely because the projection of that path on the imaginary celestial sphere is a circle. For if the sun were to move in a circle round the earth as centre, the angle subtended by the diameter of its disc should be always the same, that is, supposing that the sun itself does not undergo any change in volume. However, we find that this angle is not constant, but goes through a regular cycle of changes throughout the year, being greatest on the 31st December, when it is 32′ 36″, and least on 1st July, when it has a value 31′ 32″.

From this it is seen that the sun is nearest the earth on

31st December, and furthest away on 1st July, but that the difference is not very great. From this we may conclude that if the sun moves round the earth, his path must be nearly, but not quite, circular.

Apparent Annual Motion of the Sun due to a Motion of the Earth.

47. As the apparent annual motion of the sun in the ecliptic, together with the changes in the seasons, could be explained on the supposition that the earth describes an annual orbit about the sun, we have therefore one or other of two alternatives to choose—

Either the sun revolves round the earth in an orbit nearly circular; or

The earth revolves round the sun in an orbit nearly circular.

That the second explanation is the only admissible one appears from the following considerations:—

(1) It is known (Chapter VI.) that the planets, which are opaque bodies, receiving light and heat from the sun like the earth, revolve round the sun in orbits nearly circular. That some of these are much larger and some smaller than the earth; some at greater and some at less distances from the sun; also the earth's periodic time ($365\frac{1}{4}$ days), and its mean distance from the sun (92,000,000 miles) satisfy Kepler's 3rd Law (Chapter VI.), viz. that the squares of the periodic times of the planets vary as the cubes of their mean distances from the sun. We therefore argue from analogy that the earth, like the other planets, revolves round the sun.

(2) We know from dynamical principles that the sun, earth, and planets, on account of their mutual attractions, must either come together or revolve round the common centre of gravity of the whole system. But the sun's mass

being much greater than that of all the planets put together, the common centre of gravity of all is a point within the sun not far removed from his centre; and round this point the earth and planets must revolve.

(3) The aberration of the fixed stars (Chapter VIII.) cannot be explained on any other hypothesis except on the supposition that the earth moves round the sun.

Parallelism of the Earth's Axis.

48. We find that although the earth revolves round the sun, the position of the celestial pole among the fixed stars remains very nearly constant throughout the year: we therefore conclude that the axis of the earth is constant in direction, *i.e.* remains parallel to itself, while the earth moves round the sun.

Since the plane of the ecliptic, or, in other words, the plane of the earth's orbit, is inclined to the equator at an angle of 23° 28′, therefore the earth's axis, which is perpendicular to the equator, must be inclined to the plane of its orbit at an angle of 66° 32′, the complement of 23° 28′.

The Seasons.

49. *The changes of the seasons are due to this constant obliquity of the earth's axis to the plane of its orbit* (66° 32′).

Let fig. 27 represent the orbit of the earth round the sun. *NS* represents the axis of the earth, of which there are four parallel positions taken corresponding to the summer and winter solstices, and the autumnal and vernal equinoxes. *EQ* represents the equator, *ab* and *cd* the arctic and antarctic circles, *O* the centre of the earth, and *H* the sun.

Position (1) *Winter Solstice* (*left side of figure*).

This represents the position of the earth when the northern portion of its axis is turned from the sun, *i.e.* when the ∠ *NOH* is greatest, which happens at about 21st December, when the sun is vertical to the Tropic of Capricorn *mn*. Since the $\angle\, bOH = 90°$, the $\angle\, NOH$ therefore $= 90° + 23°\ 28'$ $= 113°\ 28'$. To an observer at the north pole *N* this period coincides with the middle of the long night lasting for six months, for it is evident, on looking at the figure, that the diurnal revolution of the earth on its axis could not bring any part not distant from *N* more than 23° 28′ into sunlight.

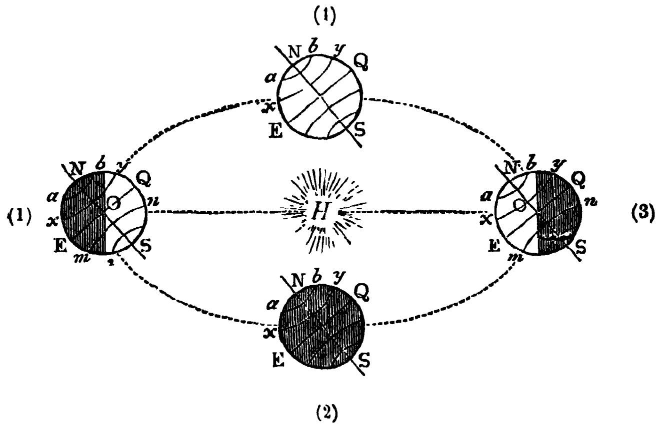

FIG. 27.

If we draw a small circle round *N* at a distance from it of 23° 28′, just reaching the line of demarcation of light and darkness, this circle coincides with the arctic circle. The reverse is the case round the south pole, where this period corresponds to the middle of the long period of daylight. Similarly, at this period the sun will not set even at 12 o'clock, P.M., to any observer within the antarctic circle.

Position (3) *Summer Solstice* (*right side of figure*).

Here the conditions are reversed: the north pole of the earth is turned towards the sun, such that the $\angle\, NOH$ has its least value, viz. $90° - 23°\ 28' = 66°\ 32'$. The sun in this case is vertical to the Tropic of Cancer *xy*. This period corresponds to the middle of the six months' daylight at the north pole and six months' night at the south pole.

Positions (2) and (4).

These two positions represent the earth at two intermediate periods when the plane of the equator passes through the sun, which therefore occupies a position on the celestial equator at one or other of the equinoctial points. Here the $\angle NOH = 90°$; therefore the line of demarcation of light and darkness will pass through the north and south poles of the earth, and day and night are of equal duration all over the world. These two periods are therefore called the two equinoxes, position (2) corresponding to the vernal, and (4) to the autumnal equinox.

Amount of Heat received daily from the Sun.

50. The average amount of heat received from the sun each day in summer is greater than in winter. There are two reasons for this :—(1) The sun remains a longer time above the horizon each day in summer than in winter; and (2) he attains a greater meridian altitude in summer than in winter. But why should we get more heat from the sun when he has a great meridian altitude than when he is low

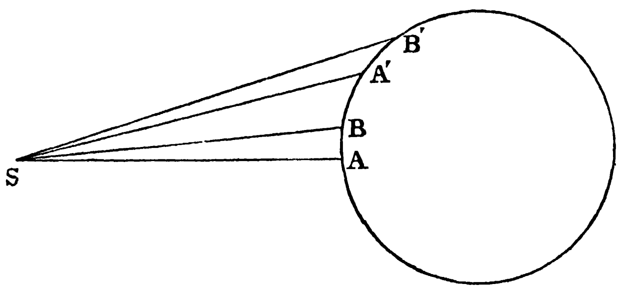

FIG. 28.

down near the horizon? Could the explanation be that he is then nearer to us?—No, for he is at practically the same distance from us at noonday when his rays are warm as at sunset, when he seems to give out very little heat; and moreover he is, as we have seen, nearer to us at midwinter than at midsummer. The explanation, however, depends on the fact that, when the sun has a great altitude in the sky, his rays strike the earth *directly*; on the other hand, when low down near the horizon, they strike *obliquely*. Why the efficiency of his rays in warming the earth should be greater in the former case than in the latter appears at once from fig. 28. Let S represent a point on the sun, SAB and $SA'B'$ two cones having equal vertical angles at S, the former striking the earth directly, the latter obliquely, so that to an observer on the earth situated inside the area AB the sun will appear high up in the heavens, and viewed from a point within $A'B'$ he will appear quite close to the horizon. We may now assume, since the cones have equal vertical angles that equal quantities of heat radiate from S along the cones SAB and $SA'B'$, and that therefore the areas AB and $A'B'$ receive the same amount of heat, but the area $A'B'$ being an oblique section of the cone is greater than AB, which is a direct section; therefore, as the same amount is distributed over both, the quantity of heat per unit of area must be less inside $A'B'$ than AB.

This explanation accounts both for the fact that the average amount of heat derived from the sun each day in summer is greater than in winter, and also that, other conditions being the same, the sun should feel hotter at noon on any day than at any other hour. This difference is still further increased owing to more heat being absorbed by the atmosphere when the sun is near the horizon, for the rays of the sun have a greater thickness of atmosphere to pass through when coming almost horizontally than vertically.

From this we should expect that in northern latitudes June should be the hottest month of the year, and December the coldest. But we generally find that the mean temperature is higher in August than in June, and lower in February than in December. The reason of this is, that during June the earth has not had sufficient time to regain the heat lost during the winter; but for some months after June, the earth gains more heat during the day than it loses at night; there is, therefore, a continuous increase in the mean temperature until the amounts of heat gained and lost during the twenty-four hours become equal. Again, for some time after the winter solstice, the amount of heat lost during the night exceeds that gained during the day; therefore, during this period the earth is losing heat, the lowest mean temperature being in general registered when the gain and loss during the twenty-four hours become exactly equal. This is the explanation of the old saying: "as the day lengthens the cold strengthens."

For the same reason, mid-day is not generally the warmest hour of the day, as there is in general a continuous gain in heat for some time into the afternoon: nor is the coldest period of the night generally reached for some hours after midnight.

The mean temperature at any place is, however, greatly influenced by other conditions, such as prevailing winds, insular or continental position, proximity to the gulf-stream, height above sea-level, &c.

Rotation of Sun. Sun Spots.

51. When the disc of the sun is observed through a telescope, dark spots are very often seen on its surface. These appear first at the eastern edge, move slowly across the bright face of the sun, and after disappearing behind the western edge, reappear again on the same side as before. Moreover, the times of appearance and disappearance are equal, each being about 13½ days. From these observations we are led to one or other of two conclusions—

(1) Either they are due to bodies revolving round the sun, so that they, coming between the sun and observer, appear as dark spots projected on the sun's surface; or

(2) They are due to actual appearances on the surface of the sun itself, the sun rotating round an axis.

That the first conclusion is in the highest degree improbable appears at once. For let *FCD* (fig. 29) represent the supposed orbit of such a body round the sun.

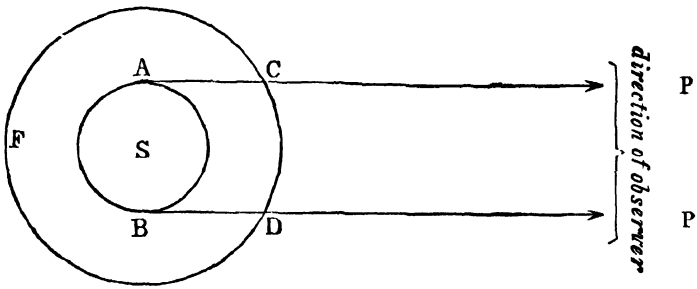

FIG. 29.

AP and *BP* are tangents drawn to the sun from the observer on the earth; these are almost parallel, as the observer is so far distant compared with the diameter of the sun. Now it is evident that the time during which a body moving in the orbit *FCD* would appear on the sun's surface would be while passing through the arc *CD*, the time of disappearance corresponding to the arc *CFD*; therefore, assuming that the velocity of the body is uniform, the time of appearance would be much less than that of disappearance. But observation shows that these two periods are almost equal. From this we conclude that the sun rotates round an axis. The period of rotation is, however, somewhat less than the apparent period of revolution of the spots, as allowance must be made for the motion of the earth in its orbit. The period for the spots is about 27 days, while the sun rotates once in $25\frac{1}{2}$ days.

These spots are darker at the centre than round the margins. The dark central portion is called the *umbra*, surrounding which is the *penumbra*, of a somewhat lighter hue, apparently composed of radiating filaments. Apart altogether from the motion due to the sun's rotation they are observed to undergo changes in their size and shape, and after some weeks or months to disappear altogether. "The inference from these various facts is irresistible." (I here quote Sir Robert Ball.) "It tells us that the visible surface of the sun is not a solid mass—is not even a liquid mass—but that the sun, as far as we can see it, consists of matter in the gaseous or vaporous condition."

"It often happens that a large spot divides into two or more smaller spots, and these parts have been sometimes seen to fly apart with a velocity, in some cases, of not less than 1000 miles an hour."

In the case of some of the largest spots the umbra has been found to subtend an angle of 1′ 30″ at the eye of the observer, which would give a diameter of about 40,000 miles—about five times greater than that of the earth.

The Sun a Sphere.

52. We have seen that the sun rotates round an axis; we also know that the shape of the disc which he turns to the observer is always circular, for all the diameters, when measured in different directions with a micrometer, are found to be equal. The sun must therefore be a sphere, as no body rotating in the same way as the sun does could always present a circular margin unless it were spherical.

Twilight.

53. After sunset a considerable time elapses before complete darkness sets in. We call this interval *twilight*. There is a corresponding interval before sunrise, which we call *dawn*.

Twilight is caused by the diffused reflection of the sun's rays from the upper layers of the atmosphere. After the sun sets, when his rays, on account of the curvature of the earth, can no longer reach us, he still continues to illumine the atmosphere or particles suspended therein, which reflect the light down to us.

Twilight is considered at an end when minute stars of the sixth magnitude can be seen in the zenith. Of course, atmospheric conditions will alter considerably the interval of time after sunset which must elapse before this takes place; but generally stars of the sixth magnitude appear when the perpendicular distance of the sun below the horizon exceeds 18°. Therefore, *twilight lasts until the perpendicular distance of the sun below the horizon exceeds* 18°.

Twilight is shortest at the equator. The student will see this at once by referring to the diagram of the celestial sphere for an observer at the equator (Art. 20). The sun's diurnal path here cuts the horizon at right angles, and therefore he takes a very short time to get 18° below the horizon. On the other hand, for an observer in the British Isles, the sun in setting cuts the horizon at an acute angle (which gets more acute the further north we go), and therefore he has to skim below the horizon a much greater distance, and for a much longer time, before his *perpendicular* distance below the horizon reaches 18°.

54. **Twilight at the North and South Poles.**—At the north pole we have seen (Chap. II.) that the sun remains for about six months below the horizon, from 23rd of September until 21st of the following March. He is never, however, during that period, at a very great perpendicular distance below the horizon, the greatest depth being 23° 28′, which he reaches on 21st December. However, of this six months of so-called night a great portion is twilight, for it will not be altogether dark as long as the sun is within 18° of the horizon. That the period during which twilight lasts will be as great a portion of the six months as 18° is of 23° 28′ we can by no means say, for the sun's change in declination is not uniform. Assuming, however, that this is the case, we would have about four out of the six months during which twilight lasts, viz. two months after the 23rd

September and two months before the 21st of the following March.

Of course, the above will also apply to the south pole during the period when the sun is below the horizon, viz. from 21st March till the 23rd September.

To find the duration of Twilight at the Equator during the Equinoxes.

55. During the Equinoxes the sun's diurnal path almost coincides with the celestial equator, which, for an observer at the earth's equator, cuts the horizon at right angles, passing through the zenith and nadir. Let S represent the sun at sunset (fig. 30). Let S' represent the sun at end of twilight.

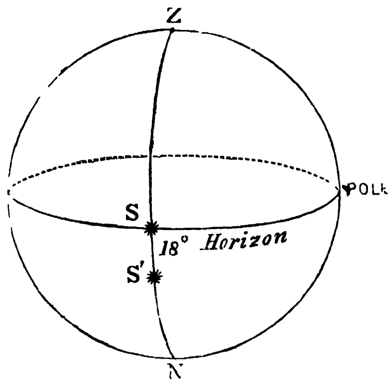

FIG. 30.

We have therefore to find the interval of time corresponding to SS' or 18° of his daily course.

But 360° correspond to 24 hours ;

$$\therefore \quad \text{As } 360^\circ : 18^\circ :: 24^h : x,$$

$$\therefore \quad x = \frac{18 \times 24}{360} = 1\tfrac{1}{5} \text{ hours} = 1^h\, 12^m.$$

56. To calculate the duration of twilight at any place we have to solve two spherical triangles, the three sides being given.

For, let S represent the sun at sunset, let S' represent the sun at end of twilight. Join S and S' to zenith and pole by four arcs of great circles.

Now, the sides of the $\triangle\ ZPS'$ are known; for $ZP = 90°$ − lat = colatitude; $ZS' = 90° + 18° = 108°$, since S' is 18° below horizon, and $PS' = PX - S'X = 90°$ − declination of sun. But the declination of the sun is known for each day in the year from the Nautical Almanac; therefore PS' is known; therefore by solving we are able to calculate the $\angle\ ZPS'$, which is the hour angle of the sun at end of twilight.

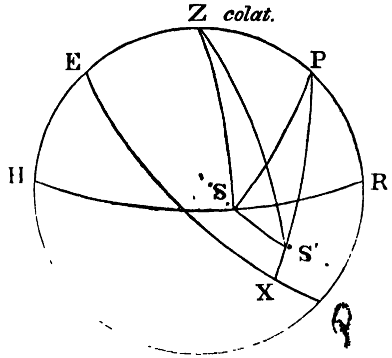

FIG. 31.

Similarly, the sides of the $\triangle\ ZPS$ are known, and therefore we can solve for the $\angle\ ZPS$, which is the hour angle of the sun when setting. Subtracting these two angles, we get the $\angle SPS'$, which measures the duration of twilight. Converting this into time at the rate of 360° to 24 hours, or 15° to 1 hour, gives the duration.

It is obvious that the duration of twilight depends upon *the latitude of the place and the declination of the sun*, for these quantities alone are involved in the solution of the above spherical triangles. That is, it depends on the part of the earth at which the observer is situated, and, even in the same place, it varies according to the season of the year.

57. It is evident that twilight cannot last all night at or near the equator, the sun's diurnal path cutting the horizon at nearly a right angle. The question then arises as to what are the conditions which must hold, in order that twilight may last all night:—

Twilight will last all night at any place, provided *the* *latitude of the place plus the declination of the sun is not less than 72°.*

For let S represent the sun at midnight when in meridian below the horizon, then

PR = alt. of pole = lat. of place = l, and

SQ = decl. of sun = δ.

Now $PQ = 90°$;

that is,

$$l + SR + \delta = 90°.$$

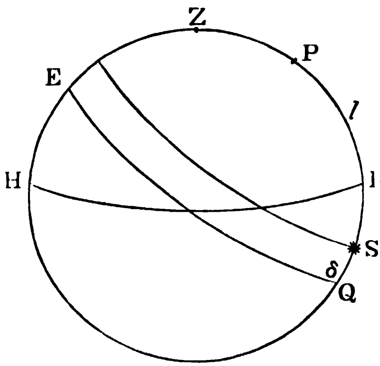

FIG. 32.

But if twilight *just* lasts all night, $SR = 18°$ at midnight;

$$\therefore\quad l + 18° + \delta = 90°,$$

$$\therefore\quad l + \delta = 72°,$$

$$\therefore\quad \text{if } l + \delta \text{ is} = \text{or} > 72°,$$

twilight lasts all night.

This rule holds when the latitude of the place and the declination of the sun are both north or both south. When the latitude is north and the declination of the sun south, or *vice versa*, then the condition becomes

$$l - \delta \text{ not} < 72°.$$

EXAMPLES.

1. What effect would be produced upon the seasons if the earth's axis were in the plane of the ecliptic or were perpendicular to it?

2. If the declination of the sun be 10°, find the lowest latitude at which twilight lasts all night.

$$\text{Here}\quad l + \delta = 72°,$$

$$\text{or}\quad l + 10° = 72°;$$

$$\therefore\quad l = 62°.$$

3. Find the latitude of the place for which twilight just lasts all night when the sun's declination is 16° N. (Degree Exam., T.C.D.). *Ans.* 56° N.

4. How does the duration of twilight at a given place alter with the season of the year? (S. S., T.C.D.). *Ans.* See Art. 56.

5. Determine the limits of the latitudes of places at which twilight lasts all night long, when the sun's declination is 10° 15′ N.

Ans. At lat. 61° 45′ and places further north.

6. Find the declination of the sun when twilight begins to last all night at Dublin (lat. 53° 20′). *Ans.* 18° 40′ N.

7. Find the lowest latitude at which it is possible for twilight to last all night. *Ans.* 48° 32′.

8. Upon what does the duration of twilight depend?

Ans. The latitude of the place and the declination of the sun.

9. Can twilight last all night at Paris (lat. 48° 50′)? (See question 7.)

Ans. Yes, but only for several nights before and after the summer solstice.

10. Show how, by solving a spherical triangle, the time of sunset or sunrise can be calculated for any place at a given date.

Ans. See Art. 56.

CHAPTER VI.

THE MOTIONS OF THE PLANETS. THE SOLAR SYSTEM.

58. We mentioned in a previous chapter that those planets which are visible to the naked eye and with which the ancients, who were not possessed of telescopes, were acquainted, are—Mercury, Venus, Mars, Jupiter, and Saturn.

If the ordinary observer wish to find out whether a bright object in the sky be a planet or a fixed star he has only to note its position with reference to the neighbouring fixed stars; for instance, it may happen to be in a line with two stars, or form an equilateral triangle with them. If, after several weeks, the body seems to have altered its position with reference to these fixed stars, it is probably one of the above planets.

Since the invention of telescopes several other large planets, with some hundreds of very small ones, have been discovered. The names of the planets at present known are, in their order, from the sun outwards—

Inferior Planets	Mercury	Interior Planets.
	Venus	
	Earth	
Superior Planets	Mars	
	The Asteroids	
	Jupiter	Exterior Planets.
	Saturn	
	Uranus	
	Neptune	

Those planets whose orbits lie between the earth and the sun are called *inferior planets*. Those whose orbits lie outside that of the earth are called *superior planets*. Thus Mercury and Venus are inferior planets, whereas Mars, Jupiter, &c., are superior planets.

There are also *interior* and *exterior* planets—those whose orbits lie between the Asteroids and the sun being interior, and those outside exterior.

The orbits of the Planets cut the Plane of the Ecliptic at very small angles.

59. It is to be observed that, throughout the whole of a planet's path round the sun, it never appears more than a few degrees above or below the ecliptic. The conclusion from this is obvious—that the orbits of the planets round the sun are nearly in the plane of the ecliptic, *i.e.* the plane of the earth's orbit; in fact they cut the plane of the ecliptic at very small angles.

Definition.—A planet is said to be in *inferior conjunction* when it comes between the earth and the sun, and in *superior conjunction* when the sun is between the earth and the planet. Thus if *E* represents the earth, *V* will be the position of a planet in inferior conjunction, and *V′* in superior conjunction (fig. 33).

A planet is said to be in *opposition* when the earth comes between the sun and the planet. Thus *M* represents the position of a planet in opposition.

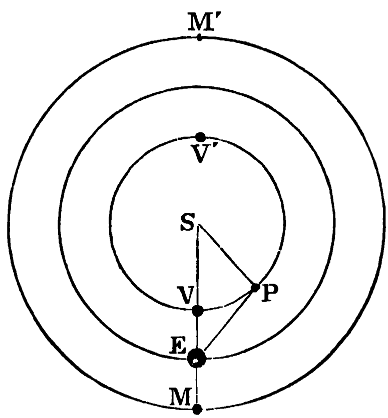

FIG. 33.

It is evident that only an inferior planet can be in inferior conjunction, and only a superior planet in opposition.

By the *nodes* of a planet are meant the two points where its orbit cuts the plane of the ecliptic, *i.e.* the plane of the earth's orbit. That point through which the planet passes in going from the southern to the northern side of the ecliptic is called the *ascending node*, the other the *descending node*.

N.B.—It is evident that if the orbits of the planets were in the plane of the earth's orbit instead of cutting it at small angles, it would be possible to see the transit of an inferior planet across the sun's disc each time inferior conjunction occurs. But this phenomenon is of very rare occurrence ; for although the inferior planets are very often in inferior conjunction, they, not being at the same time in the plane of the earth's orbit, will be situated above or below the sun ; and when these planets are in the plane of the earth's orbit, which occurs when they are passing through their nodes, they do not happen to be in inferior conjunction. Therefore, the planet must be in conjunction, and at one of its nodes at the same time, in order that a transit may occur.

Definition.—By the *elongation* of a planet from the sun is meant the angle subtended at the earth by the sun and planet. Thus (fig. 33), the elongation of the planet P from the sun is the angle SEP.

Corollary.—It is evident that the elongation of an inferior planet must always be an acute angle. It has a maximum value at a point near P, where EP becomes a tangent to the orbit of the planet. On the other hand, the elongation of a superior planet may have any value from 0° to 180°, reaching the latter value when in opposition.

The reader should bear this in mind, for it accounts for the fact that the superior planets may be seen at all angular distances from the sun, occupying, when in opposition, a diametrically opposite point on the celestial sphere, so as to cross the meridian at midnight. The inferior planets, Mercury and Venus, on the other hand, on account of their small angular distances from the sun, can only be seen either in the west after sunset or in the east before sunrise, according to their position relative to the earth and sun.

So small is even the maximum elongation of Mercury from the sun that it is only possible to see it with the naked eye on very rare occasions, and for a very short time indeed after sunset or before sunrise.

Phases of the Planets.

60. As the planets are bodies like the earth, not self-luminous, but deriving their light from the sun, they can have only half their surface lit up at once, the other half being dark, just as when we hold a globe up before a lamp, the half which is next the light is bright, while the half which is turned away from the lamp is in the shade. Now as it is plain that, by getting into different positions with respect to the globe and the lamp, we can see as much or as little as we choose of the half which is lighted up, so it is with respect to the planets which show more or less of their illuminated surface to us, as they vary their positions with respect to the earth and sun. These changes in the amount which we see of the illuminated surface of a planet are called its *phases*. It is needless to say that these phases cannot be distinguished with the naked eye.

61. *The greatest breadth of the portion of the illuminated surface of a planet which is turned towards the earth is proportional to the exterior angle subtended at the planet by the earth and sun.*

For let *PS* and *PE* represent the directions of the sun and earth, as seen from *P*, the centre of the planet.

Then CD, drawn perpendicular to PS, separates light from darkness. Erect AB perpendicular to PE, then the angle APD will measure the arc AD, which is the greatest breadth of the visible illuminated surface, as seen from the earth. The angle SPE is the angle subtended by the sun and earth at P, the angle SPF being the external angle We have to prove the $\angle APD =$ the $\angle SPF$.

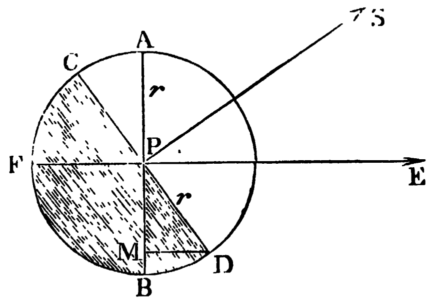

FIG. 34.

The $\angle SPD = \angle APF$, both being right.

To each add the $\angle SPA$.

$\therefore$ the $\angle APD =$ the $\angle SPF$, $\therefore$ &c.

The apparent breadth of the illuminated surface of a planet varies as the versed sine of the exterior angle subtended at the planet by the earth and sun.

62. We have already seen that the greatest breadth AD is measured by the exterior angle SPF. But the *apparent* breadth of AD will be measured by AM, the projection of AD on a line perpendicular to PE, the direction of the observer. For the earth being so far away compared with the breadth of the planet, we may take all lines drawn from the observer to the surface of the planet as being parallel, and each perpendicular to AB.

$\therefore$ apparent breadth varies as AM.

but $AM = r + PM = r + r\cos BPD$

$= r\,(1 + \cos BPD)$

$= r\,(1 - \cos APD) = r$ versin APD

$= r$ versin SPF.

Phases of Inferior Planets.

63. Let *ACDF* represent the orbit of an inferior planet, *E* being the earth, and *S* the sun. We will now suppose the earth to be at rest, and that the planet revolves round the sun with an angular velocity equal to the excess of its real angular velocity over that of the earth · for we will see later

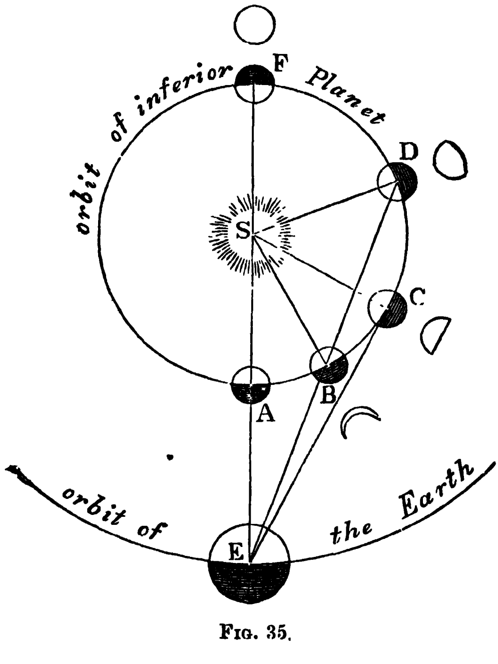

Fig. 35.

on, that of two planets the nearer to the sun has the greater angular velocity. This, of course, will represent exactly the apparent motion of the planet, as seen from the earth.

When the planet is in inferior conjunction at *A*, none of its illuminated surface is seen.

At *B* a small crescent is visible, its greatest breadth being measured by the exterior angle *SBD*, which is an acute angle.

When the planet reaches its greatest elongation from the sun at *C*, where *EC* is a tangent to its orbit, the exterior angle subtended at the planet is a right angle, and the planet appears as a semicircle, like the moon at first or third quarter. It is then said to be *dichotomized.*

At *D*, the exterior angle, being $180° - SDE$, is an obtuse angle, and the disc appears nearly, but not quite, full. It is then said to be *gibbous.*

The *full* phase is reached at *F*, and the above changes are then repeated in inverse order until inferior conjunction is again reached.

Superior Planets' Phases.

64. It is evident that a superior planet must always appear either full or gibbous; for as its orbit is outside that of the earth, the observer is always on the same side of the planet as the sun is situated, and therefore must have all, or very nearly all, its illuminated surface turned towards him.

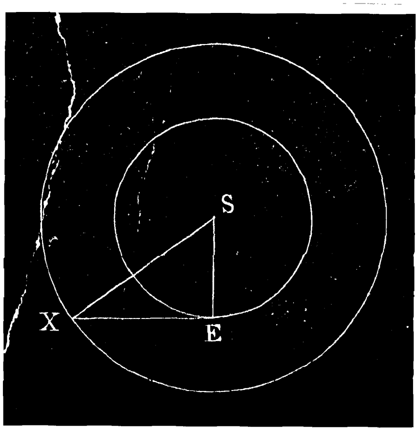

FIG. 36.

This also appears from the fact that the exterior angle subtended at the planet is always obtuse, and therefore the illuminated disc which the observer sees is greater than a semicircle.

It is easy to see that a superior planet will present the smallest portion of illuminated surface to the earth when the sun and planet subtend a right angle at the earth; that is to say, *a superior planet is most gibbous in quadrature.*

For let X be the planet (fig. 36). Now it is evident that the planet will appear most gibbous when the exterior angle at X is least, that is, when the angle SXE is greatest, which is the case when XE is a tangent to the orbit of the earth (supposed circular). For supposing X to be fixed, and that the motion is due to the earth alone, then the elongation of the earth from the sun as seen from X, viz. the angle SXE, will be greatest when XE becomes a tangent.

Brightness of Planets.

65. The brightness of a planet depends on the amount of illuminated surface turned towards the earth and also on its distance. For, assuming the illuminated surface to remain the same, it will appear brighter the nearer it approaches the earth, the intensity of the brightness being inversely as the square of the distance. Thus, at twice any given distance it would only appear one-quarter as bright; at three times the distance one-ninth as bright, &c.

The inferior planets are not brightest near superior conjunction, for though they are then nearly full they are at their greatest distances from the earth: Venus, for instance, when at superior conjunction, being about six times as far away as when in inferior conjunction; its disc in the former case subtending an angle of only 11″, and in the latter of 66″. It has been calculated that Venus is brightest when at an elongation of about 40° from the sun, near inferior conjunction, for although on being viewed through a telescope it then only appears as a thin crescent, still, owing to its proximity to the earth, that crescent appears to have a much larger area than even the full illuminated circle which it presents at superior conjunction.

A superior planet is evidently brightest when in opposition, for not only does it then appear full, but it is at the same time at its least distance from the earth.

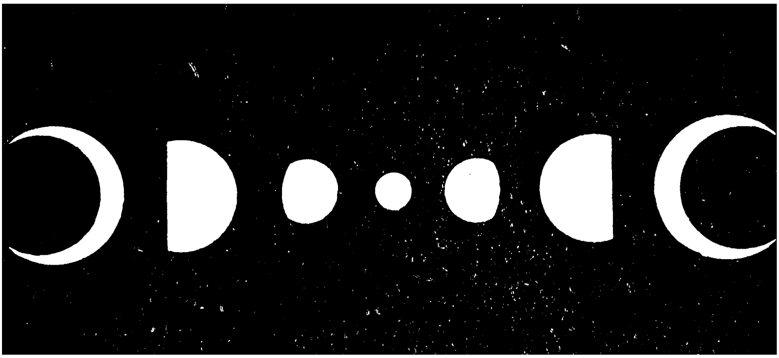

FIG. 37.

Apparent size of Venus at its extreme and mean distances from the earth.

To find the ratio of the distances of an Inferior Planet and the Earth from the Sun at any time.

66. Let E, V, and S represent the earth, Venus, and the sun, the orbits being supposed circular and in the same plane.

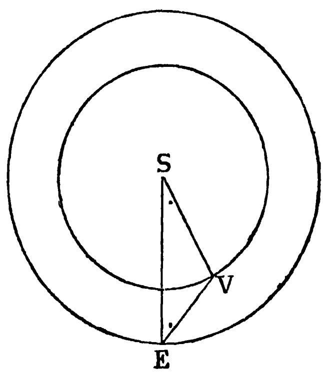

FIG. 38.

The $\angle$ SEV is got by observation, and the $\angle$ ESV by calculation, for it is the angle gained by Venus on the earth since the last inferior conjunction, which can be calculated as follows:—

Let T = interval between two inferior conjunctions, in days, and x = number of days since last inferior conjunction;

$\therefore$ Venus gains 360° on earth in T days;

$\therefore$ " " $\frac{360°}{T}$ " " 1 day;

$\therefore$ " " $\frac{360°}{T} \times x$ " " x days;

$\therefore$ the $\angle ESV$ is known and all the angles of the triangle SEV are known; but—

$$\frac{SE}{SV} = \frac{\sin SVE}{\sin SEV};$$

$\therefore$ the ratio of SE to SV is found.

Similarly the ratio of the distances of a superior planet and the earth from the sun can be found, the proof being the same, the earth in this case gaining on the planet.

Definition.—The *periodic time* of a planet or, as it is often called, its *sidereal period*, is the time taken by the planet to make one revolution round the sun.

The *synodic period* is the interval that elapses between two conjunctions of the same kind (both inferior or superior) or in the case of a superior planet, between two oppositions.

To find the Periodic Time of a Planet when the Synodic Period is known.

Inferior Planet.

67. Let P = periodic time of planet expressed in days,

E = " " earth " "

T = synodic period;

$\therefore \frac{360°}{P} = \angle$ moved through by planet in 1 day,

and $\frac{360°}{E}$ = " " earth "

$\therefore \frac{360°}{P} - \frac{360°}{E}$ = angle gained by planet in 1 day, for the inferior planet goes at the greater rate;

but $\frac{360^\circ}{T}$ = angle gained by planet in 1 day also;

$$\therefore\ \frac{360^\circ}{P} - \frac{360^\circ}{E} = \frac{360^\circ}{T};$$

$$\therefore\ \frac{1}{P} - \frac{1}{E} = \frac{1}{T};$$

but $E = 365{\cdot}25$ days, and T being known, P is determined.

EXAMPLE.

The interval between two inferior conjunctions of Mercury is 116 days: find its periodic time.

$$\frac{1}{P} - \frac{1}{365{\cdot}25} = \frac{1}{116},$$

$$\therefore\ \frac{1}{P} = \frac{481{\cdot}25}{365{\cdot}25 \times 116}; \quad \therefore\ P = 88 \text{ days nearly.}$$

Similarly for a superior planet we have the formula

$$\frac{1}{E} - \frac{1}{P} = \frac{1}{T},$$

the earth going at the greater rate. Thus by noting the interval between two inferior conjunctions (or oppositions) of a planet we can calculate its periodic time, supposing its orbit and that of the earth circular.

Kepler's Three Laws.

68. Kepler, the Danish astronomer, who lived at the commencement of the seventeenth century, first enunciated the following laws :—

I. Each planet moves in an elliptic orbit with the sun in one of the foci.

II. The straight line drawn from the sun to a planet (the planet's "radius vector") sweeps out equal areas in equal times.

III. The squares of the periodic times of the planets are to one another as the cubes of their mean distances from the sun.

Definition.—An ellipse is a plane figure bounded by one line called the circumference, such that the sum of the distances of any point on that circumference from two fixed points within it is constant. Those two fixed points are called the *foci*.

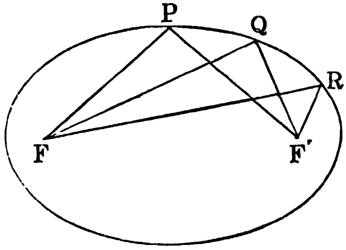

FIG. 39.

Thus (see fig. 39) if F and F' be the two foci we have

$$EP + F'P = FQ + F'Q = \text{a constant.}$$

The following is therefore a mechanical method of describing an ellipse :—

Let two pins be taken and fixed into a plane board or table, say at F and F' (fig. 39), and round these let a loose endless string be thrown. If now a pencil be taken and, keeping the string tightly stretched, let it be carried round, occupying successively the points P, Q, R, &c., the curve traced out will be an ellipse, the two pins F and F' being situated at the foci.

Kepler's Second Law asserts that the line drawn from the sun to a planet sweeps out equal areas in equal times, that is, if the times of describing the distances AB and PQ are equal, then the area SAB = area SPQ. From this we can conclude that the nearer a planet approaches the sun the greater must be its velocity, for if we regard the arcs AB and PQ as being described in a small unit of time, they, being small compared with the planet's distance from the sun, may be taken as straight lines. Now if the distance from S to AB be greater than from S to PQ, the

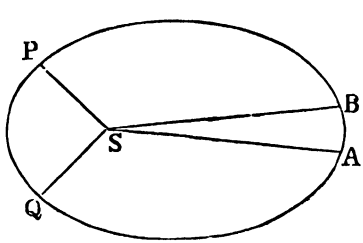

FIG. 40.

base PQ must in its turn be greater than the base AB in order that the two triangles may be equal, and therefore the velocity at PQ is greater than at AB.

Corollary.—As the earth is nearest the sun at midwinter (Art. 46), we now see that its velocity at midwinter is greater than at any other part of its orbit.

Verification of Kepler's Laws.

69. As regards the earth it can be seen by measurement of the sun's apparent diameter with the parallel wire or other micrometer that the orbit is not a true circle, and that therefore the earth's distance from the sun is not constant, being greatest when the angle subtended by the sun's diameter is least. We can now therefore construct the curve which represents the earth's orbit, for if lines be drawn from a point S (fig. 40), the lengths of these lines being inversely proportional to the different angles which the sun's diameter subtends at the earth, measurements of which can be made daily, the extremities of these lines will be found to trace out an ellipse with S in one of the foci.

Kepler determined the orbit of Mars before that of the earth had been ascertained. He determined the position of Mars relative to the earth and sun by a method somewhat similar to that in Art. 66, and by this means he arrived at the conclusion that the orbit of Mars was elliptic. The fact that he considered the orbit of the earth as circular in these calculations did not give rise to very serious error, as the eccentricity of the earth's orbit is very small and much less than that of the orbit of Mars.

70. Newton showed that Kepler's Third Law was a direct consequence of the law of universal gravitation which may be enunciated as follows :—

Every particle in the universe attracts every other particle with a force directly proportional to the mass of each, and inversely proportional to the square of their distance apart.

To deduce Kepler's Third Law from the Law of Gravitation.

Let M = mass of the sun.

Let r and r' be the distances of two planets from the sun whose periodic times are T and T' respectively. Now by the law of gravitation the attractions of the sun at distances r and r' from its centre are in the proportion—

$$\frac{M}{r^2} : \frac{M}{r'^2}.$$

But the centrifugal acceleration of a body moving in a circle of radius r is given by the equation—

$$f = \frac{4\pi^2 r}{T^2};$$

therefore assuming the orbits of the planets circular we have

$$\frac{M}{r^2} : \frac{M}{r'^2} :: \frac{4\pi^2 r}{T^2} : \frac{4\pi^2 r'}{T'^2}.$$

Multiplying the extremes and means we get eventually—

$$\frac{r'}{r^2 T'^2} = \frac{r}{r'^2 T^2};$$

$$\therefore \quad r'^3 T^2 = r^3 T'^2;$$

$$\therefore \quad T^2 : T'^2 :: r^3 : r'^3,$$

which is Kepler's Third Law.

Bode's Law.

71. There is a remarkable relation between the distances of the different planets from the sun which bears the name of the astronomer Bode. Write down the following numbers in which each after the first is doubled:—

0 1 2 4 8 16 32 64 128

Now multiply by 3 and add 4 to each and we get

4 7 10 16 28 52 100 196 388

corresponding to Mercury, Venus, Earth, Mars, Asteroids, Jupiter, Saturn, Uranus, and Neptune.

These numbers are approximately proportional to the distances of the different planets from the sun, that of the earth being 10. There is, however, a serious discrepancy in the case of Neptune, which is represented by the number 388, whereas to represent its actual distance it should 300·369.

This law received a remarkable confirmation from the discovery of the Asteroids, which consist of a number of small planets whose orbits lie between the orbits of Mars and Jupiter. They are over 300 in number. Before their discovery there was no planet known whose distance from the sun corresponded to the number 28. However this number is found to approximately represent the mean distance of the different asteroids from the sun.

Bode's Law can be expressed by means of the general formula $D = 4 + 3 \times 2^{n-1}$, where D represents the distance of a planet from the sun, and n the number of the planet beginning with Venus. By giving to n the values 1, 2, 3, &c., the numbers corresponding to the distances of the different planets from the sun commencing with Venus, are found to be the same as those mentioned above.

					True Distance.
Mercury,	.	4	=	4	3·871.
Venus,	.	$4 + 3 \times 2^0$	=	7	7·233.
Earth,	.	$4 + 3 \times 2^1$	=	10	10·000.
Mars,	.	$4 + 3 \times 2^2$	=	16	15·237.
Asteroids,	.	$4 + 3 \times 2^3$	=	28	22 to 31.
Jupiter,	.	$4 + 3 \times 2^4$	=	52	52·028.
Saturn,	.	$4 + 3 \times 2^5$	=	100	95·388.
Uranus,	.	$4 + 3 \times 2^6$	=	196	191·826.
Neptune,	.	$4 + 3 \times 2^7$	=	388	300·369.

Direct and Retrograde Motion. Stationary Points.

72. A planet's apparent motion is said to be *direct* when it seems to move in the same direction as the sun in the ecliptic, and *retrograde* when it appears to move in a contrary direction. In other words, its motion is direct when its longitude is increasing, and retrograde when diminishing.

As the earth E moves in its orbit in the direction indicated by the arrow (fig. 41) the sun *appears* to move as if the line ES were rotating round E from right to left in a direction contrary to the hands of a watch. Therefore we will, for greater clearness, regard a contra-watch-hand rotation round E as *direct*, and a watch-hand rotation as *retrograde*, E being supposed fixed.

An **inferior planet** moves with greater velocity than the earth (Art. 47); therefore when the planet is at inferior conjunction the extremity V of the line VE moving with greater velocity than E, the line will appear to revolve round

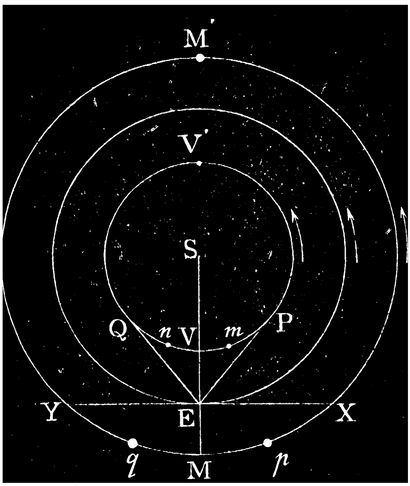

FIG. 41.

E like the hands of a watch, and therefore *the apparent motion of a planet at inferior conjunction is retrograde.* At the points of greatest elongation P and Q the planet's own velocity will produce no change in its direction, as it is along the lines EP and EQ respectively, but the earth's motion will make the lines EP and EQ appear to revolve in a direction contrary to that of the hands of a watch. Hence the planet's apparent motion at P and Q is *direct*. At any point on the arc $PV'Q$ the motion will appear direct, for both the planet's own velocity and that of the earth will combine to make the line joining them revolve round E in a direction contrary to that of the hands of a watch.

Again, as the planet's motion appears retrograde at V, and direct at P and Q, it must pass through two points m and n, at which the retrograde motion is on the point of changing into direct or *vice versa*, and at which the planet does not seem to move. These two positions are called the *stationary points*.

A **superior planet** on the other hand moves with a velocity which is less than that of the earth; therefore when the planet is in opposition at the point M (fig. 41) the line EM will appear to rotate round E like the hands of a watch. Hence *the motion of a superior planet in opposition is retrograde.*

Again, when the planet is in quadrature at X and Y the velocity of the earth will have no effect, as it is in the direction of the line joining the observer to the planet. The planet's own motion, however, will cause EX or EY to revolve round E in a contra-watch-hand direction. *Hence the apparent motion of a superior planet in quadrature is direct.*

Also at any position along the arc $XM'Y$ both the planet's velocity and that of the earth combine to cause the line joining them to appear to revolve with a contra-watch-hand rotation, *i.e.* the planet's motion will be direct.

As the planet appears retrograde at M and direct at X and Y, there will be two points p and q at which the retrograde motion is on the point of changing into the direct, and *vice versa*, these points being the two stationary points of the planet.

Rotations of the Planets round their Axes.

73. We have already seen that the earth and sun rotate. It has also been shown by observing the markings and spots

on the surfaces of the planets that most of them, and probably all, rotate in the same manner. Mars rotates once in $24^h\ 37^m$, so that a day in Mars is almost of the same length as a day on the earth. Jupiter takes $9^h\ 55^m$, and Saturn $10^h\ 29^m$.

It is much more difficult to find the period of rotation of an inferior than of a superior planet; for the latter when in opposition can be seen all night, whereas an inferior planet only appears as an evening or morning star, and observations can only be made at intervals of 24 hours. Now supposing that markings are observed on the surface of Venus after sunset, it is found that they occupy nearly the same position on the following night. From this we might be led to one or other of two conclusions—(1) either Venus makes a revolution on its axis in about 24 hours, or (2) it takes a very long period to complete a revolution so that the angle turned through in 24 hours would be very small. In either case it is evident the markings would not be much changed during 24 hours. Until quite recently it was believed that the first conclusion was true. From observations by Schräter the period for Venus was believed to be $23^h\ 21^m$, and for Mercury $24^h\ 5^m$. Professor Schiaparelli, however, has recently contended that Mercury and Venus take the same time to rotate on their axes as they do to revolve round the sun, the period for the former being 88 and the latter 224 days, and that therefore they turn always nearly the same face towards the sun, just as the moon does to the earth, large portions of each being in perpetual sunlight, and other portions always in darkness.

As these planets are only to be seen close to the horizon after sunset or before sunrise the changes in the temperature and density of the lower strata of the atmosphere render it very difficult to observe the markings on their surface with sufficient accuracy to determine the exact truth; but more recent observations would seem to show that Schiaparelli's conclusion is, at all events, false in the case of Venus.

To prove that the Velocities of Two Planets round the Sun are inversely as the square roots of their distances from the Sun.

74. For by Kepler's Third Law we have

$$T^2 : T'^2 :: r^3 : r'^3;$$

but circumference of orbit = velocity × time.

$$\therefore \quad 2\pi r = vT;$$

$$\therefore \quad T = \frac{2\pi r}{v};$$

therefore we have $\left(\frac{2\pi r}{v}\right)^2 : \left(\frac{2\pi r'}{v'}\right)^2 :: r^3 : r'^3;$

$$\therefore \quad \frac{r^2 r'^3}{v^2} = \frac{r'^2 r^3}{v'^2};$$

$$\therefore \quad \frac{r'}{v^2} = \frac{r}{v'^2};$$

$$\therefore \quad v^2 : v'^2 :: r' : r;$$

$$\therefore \quad v : v' :: \sqrt{r'} : \sqrt{r}.$$

Corollary.—Hence of two planets the nearer to the sun has the greater velocity.

75. We shall conclude this chapter with a brief review of the different bodies which constitute the solar system.

Mercury ☿.

This is the nearest planet to the sun. Its diameter is about 3000 miles, being much smaller than that of the earth (⊕), whose diameter is 8000 miles. The orbit of Mercury is much more eccentric than those of the other principal planets, that is, it does not so nearly approach a circular shape. At one time the planet approaches to within 28,000,000 miles of the sun, and again in the opposite point of its orbit recedes to a distance of 43,000,000 miles. It is also distinguished by the great inclination of its orbit to the ecliptic, namely, about 7°. Its periodic time about the sun is about 88 days.

Venus ♀.

Venus has a diameter almost equal to that of the earth. Its orbit also like that of the earth differs but little from a circle. The inclination of its orbit to the ecliptic is 3° 23′. We have seen how Mercury and Venus being inferior planets are only to be seen within certain angular distances on the east or west side of the sun, and are therefore morning or evening stars; and also that their discs, when seen through a telescope, show phases like those of the moon. In both these planets it has been observed that the line of separation of the light from the dark portions is not continuous but notched, and also that the horns of the crescents they present are sometimes cut off abruptly. This is caused by mountains on their surfaces, which have been calculated to rise in both planets to heights considerably greater than those on the earth. The periodic time of Venus is 224·7 days.

Transits of Venus and Mercury.

76. We have already seen that the transit of Venus or Mercury can only occur when the planet is in inferior conjunction at or near one of its nodes.

If a transit of one of these planets occur at any time another transit *at the same node* will not occur until the earth and the planet shall have each made an exact number of revolutions.

Now 8 revolutions of the earth expressed in days are almost equal to an exact number of revolutions of Venus, viz. 13, there being only a difference of one day, for

$$8 \times 365{\cdot}242 = 2922 \text{ days nearly,}$$

and

$$13 \times 224{\cdot}7 = 2921{\cdot}1 \text{ days.}$$

Hence, if a transit of Venus occur at any time there *may* be another at the same node 8 years afterwards if one has not already occurred 8 years before. There will not, however, be a transit 16 years afterwards, as, on account of the above difference of one day, the distance from the node when in conjunction will be too great. In fact, a transit at the same node cannot in this case occur for another 235 years, which is the next number of years which corresponds to an exact number of revolutions of Venus, for

$$235 \times 365{\cdot}242 = 85835 \text{ days nearly,}$$

and

$$382 \times 224{\cdot}7 = 85835 \quad ,, \quad ,,$$

The first transit of Venus ever observed was that seen by Horrox, in 1639, which occurred at the ascending node. A transit at this node did not again occur for 235 years, viz. in 1874, and again in 1882. Transits at the descending node have been observed in the years 1761 and 1769, the next occurring in the year 2004.

Transits of Mercury occur more frequently than those of Venus, for its periodic time is such that it more frequently happens that an exact number of revolutions of the planet correspond to an exact number of years. Thus transits of Mercury at the same node may happen at intervals of 7, 13, 33, or 46 years.

At present the earth in its orbital motion is opposite a node of Venus on the 5th of June, and again on the 7th of December. Hence, for a very long period of time, transits of Venus will occur in December and June. For a similar reason transits of Mercury will occur in May and November.

Transits of Venus are of great practical interest, as their observation furnishes the most accurate methods of determining the sun's parallax and distance (Chapter VII). Transits of Mercury cannot be used in the same manner, as its distance from the earth approaches too nearly that of the sun to give reliable results. Besides it moves too rapidly across the sun's disc to give time for accurate observations;

and also as its orbit does not so nearly approach a circular shape as that of Venus, the ratio of its distance from the sun to that of the earth cannot be so easily calculated.

Mars ♂.

77. The nearest of the superior planets is Mars. Its distance from the sun varies from 127,000,000 to 153,000,000 miles, and therefore its orbit is much more eccentric than that of the earth. If the orbits of Mars and the earth were both circular the planet would be closest to us at opposition, its distance being then only the difference of the radii of the orbits. But the distance of Mars from the sun, as we have seen above, is very variable, and that of the earth from the sun changes from 90,500,000 miles at midwinter to 93,500,000 miles at midsummer. We can, therefore, see how some oppositions are much more favourable for observation than others. For, suppose during opposition that Mars were at its least distance from the sun, and the earth at its greatest distance, the planet would only be distant from us by 34,000,000 miles; and astronomers would then have the opportunity of viewing it under most favourable conditions.

If we imagine the two points where Mars is nearest to and at its greatest distance from the sun to be joined to one another, it is found that the earth in its orbital motion passes close to this line on August 26, and again on February 22. On August 26 the earth passes that point of its orbit which is in a line between the sun and the position which Mars would occupy when closest to the sun (*perihelion*); and on February 22 it crosses the line between the sun and the point in which Mars would be situated when at its greatest distance from the sun (*aphelion*). Therefore if we regard the orbit of the earth as being circular (for it is much more nearly so than that of Mars), the nearer the date of an opposition approaches to August 26, the more favourable are the conditions under which the planet can be observed, and the closer that date is to February 22 the more unfavourable is such an opposition for accurate observation.

The periodic time of Mars is about 687 days, and the inclination of its orbit to the ecliptic is about 2°. Two small satellites of Mars were discovered by Mr. Hall, of Washington, during the opposition which occurred on eptember 5, 1877, when the planet was very close to the earth, the date of the discovery being only ten days after the best date possible. They have been named Deimos and Phobos,* the former, which is the outer, completing a revolution round Mars in about $30^h\ 18^m$, and the latter in $7^h\ 39^m$. As Mars completes a revolution on its axis in $24^h\ 37^m$, we have in Phobos an example of a satellite revolving round the primary planet much more quickly than the latter rotates on its axis, a case which is without a parallel in the solar system.

At the beginning of this century a number of very small planets were discovered with orbits lying between Mars and Jupiter. They are called the *Asteroids*. There are at present considerably over 300, the smaller ones being only a few miles in diameter. The four largest are Vesta, Juno, Ceres, and Pallas. Their orbits are generally very eccentric, some of them being also inclined at considerable angles to the ecliptic.

Jupiter ♃.

78. This is the largest of all the planets, its diameter being 11 times that of the earth. Its orbit is nearly circular like that of the earth, and is inclined to the ecliptic at an angle of about $1\frac{1}{3}$°. When observed through a telescope, a number of bright belts or bands are seen encircling it parallel to its equator, which are probably belts of clouds or vapours

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* In Homer Deimos and Phobos are represented as the attendants of Mars. The passage in the *Iliad* which first suggested the names of the satellites has been thus construed by Professor Tyrrell :—

> " Mars spake and called Dismay and Rout
> To yoke his steeds, and he did on his harness sheen."
>
> *Il.* 15, 119, 120.

in its atmosphere. It has five satellites or moons, which may be seen with a good field glass. The periodic times of these five moons, and their mean distances from Jupiter, satisfy Kepler's third law, which we have seen is true for the orbits of the planets round the sun. This is true of the satellites of all the planets. They are frequently *eclipsed* when they enter the shadow cast by Jupiter on the side opposite the sun. An eclipse must not, however, be confounded with an *occultation* which happens when a satellite is in a line with Jupiter and the earth so as to be hidden from the observer's view. Again, a very curious phenomenon is observed when a satellite comes between Jupiter and the sun. The shadow cast by the satellite will then be observed as a dark spot moving across the face of Jupiter, which is indeed a wonderful sight, illustrating, as it does, the appearance which the earth would present if viewed from Mercury or Venus during a total eclipse of the sun by our satellite the moon. A transit of the satellite may also occur when the satellite is in a direct line between Jupiter and the earth.

Saturn ♄.

79. The orbit of Saturn is also very nearly circular. It is inclined to the ecliptic at an angle of about $2\frac{1}{2}°$. At certain periods Saturn when viewed through a telescope presents a most wonderful appearance. It is surrounded by a series of circular rings which do not touch the surface of the planet; indeed through the interval between the rings and the body of the planet fixed stars are sometimes seen. The plane of these rings is inclined at a constant angle of about 28° to the plane of Saturn's orbit, and, therefore, they being seen obliquely by us, will not appear circular but oval. They are supposed to be formed of immense numbers of small satellites. They become invisible—(1) when the plane of the rings, when produced, passes through the earth, for being very thin when the edge is turned towards us, it is not possible to see

them except through the most powerful telescopes; (2) when their plane passes through the sun, for they, deriving their light from the sun, have only their edge illuminated; (3) when their plane passes between the earth and the sun, for their dark surface being towards us it is not possible to see them.

Saturn has, besides, eight satellites, all situated external to the rings. The seven nearest move in orbits whose planes almost coincide with the plane of the rings, but that of the eighth is inclined to this plane at an angle of about 10°.

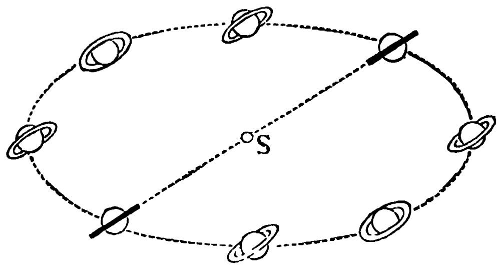

FIG. 42.—Phases of Saturn's Rings.

Uranus ♅.

Uranus was discovered in 1781 by Herschel. Its orbit is nearly circular, and inclined at a very small angle to the ecliptic. Four satellites have been discovered which revolve in orbits nearly perpendicular to the plane of the orbit of Uranus.

Neptune ♆.

The discovery of Neptune is one of the most brilliant in the history of Astronomy. It was found that the positions which it was calculated Uranus should occupy, after making allowance for all known disturbing forces, did not coincide with the observed positions. It was therefore thought that there must be some unknown planet whose attraction produced these disturbances. After the most laborious calculations the position which this unknown body should occupy was determined at almost the same time by Leverrier in France, and Adams in England, in the year 1846. One satellite of Neptune has been discovered.

Comets.

80. The solar system includes a number of other bodies which differ widely from the planets, both in their physical state and in the nature of the orbits described by them round the sun. These bodies are called *comets*. Comets differ very much as regards their shape, and even the shape and size of the same comet may change considerably at different parts of its orbit; but we generally find at one end a brilliant nucleus surrounded by nebulous matter stretching out into an elongated tail. The tails of some comets which have appeared have been of enormous dimensions; that of 1811 had a tail 23° in length, another in 1843 had a length of 40°, while that of 1618 extended across the sky through an arc of 104°.

Comets generally appear suddenly in the sky, remaining visible for some weeks, or months, during which time they approach the sun with great velocity; they then recede from it, and finally disappear from view.

The mass and density of comets are extremely small; it is even possible to see faint stars shining through them almost as if no material body were interposed between.

By far the greater number of comets describe orbits of such great eccentricity that we may regard them as parabolas described round the sun as focus, the other focus being practically at an infinite distance. But there are a few comets whose orbits, although much more elongated than those of the planets, are sufficiently small to be contained within the solar system. The motions of these can be calculated and the dates of their return predicted from knowing the magnitudes of the ellipses which they describe. These are called *periodic comets*.

The orbits of comets, besides being much more eccentric, also differ from those of the planets in that they may be inclined at any angle to the plane of the ecliptic; moreover all the planets go round the sun in the same direction as the earth moves, whereas the motion of some comets is direct and of others retrograde.

Periodic Comets.

81. **Halley's Comet.**—Of the periodic comets, that known as Halley's is, perhaps, the most remarkable. It was observed that the comets which appeared in the years 1531, 1607, and 1682 were almost identical as regards the position of the nodes, the perihelion distance from the sun, the inclination of the orbit to the plane of the ecliptic, and certain other measurements, from which Halley concluded that they were really one and the same comet, having a periodic time about the sun of 75 years, and he therefore predicted its return in 1758. Clairaut, having calculated that, owing to perturbations caused by the attractions of Jupiter and Saturn, it would be retarded 518 days and 100 days respectively, predicted that it would be closest to the sun, or, in other words, at perihelion, at about the middle of April, 1759. No allowance was made for disturbances caused by the attractions of Uranus and Neptune, as these planets were not then discovered. It actually appeared at the end of 1758, and reached the perihelion at the middle of March, 1759.

Halley's Comet has since appeared in 1835, and it may be again expected in 1910. One of the previous visits of Halley's Comet was on a very memorable occasion in the year 1066, the date of the Norman conquest; the picture of this comet is depicted in the Bayeux tapestry.

Encke's Comet.—This periodic comet is also known to describe an elliptic path round the sun. At perihelion it is closer to the sun than Mercury; and at aphelion, at its greatest distance, it is not altogether as far from the sun as Jupiter; so that its orbit is well within the limits of the solar system. Its periodic time is about $3\frac{1}{3}$ years. The motion of this comet has been most carefully observed, and the perturbations in its movements due to the attractions of the earth and the other planets have been calculated. But it is found that after making allowance for all these disturbing forces, there is still a diminution in its periodic time of

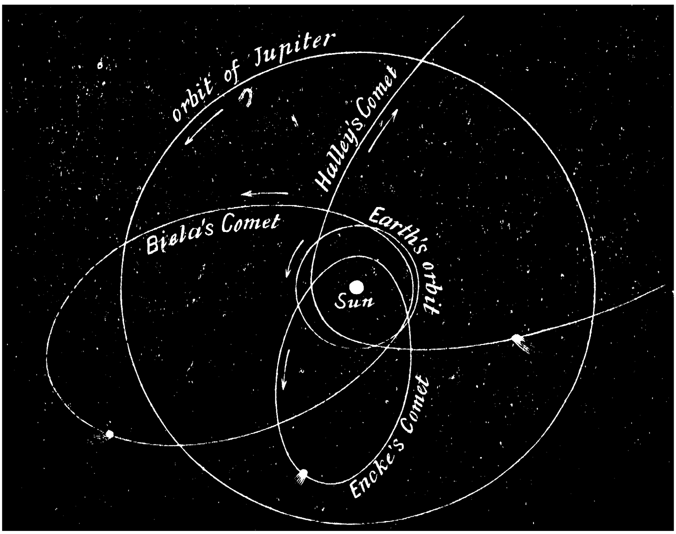

FIG. 43.—Orbits of some Periodic Comets.

$2\frac{1}{2}$ hours in each successive revolution. Encke accounted for this by supposing that there exists for a considerable distance round the sun a medium which, although of extreme tenuity, is still capable of offering sufficient resistance to the passage of a body of such small density as a comet as to appreciably diminish its periodic time.

Non-periodic comets are much more numerous than periodic. To this class belonged the great comet of 1843, Donati's Comet, which appeared in 1858, and the comet of 1881.

Meteors or Shooting Stars.

82. In addition to the different members of the solar system which we have already enumerated there are an innumerable number of minute bodies which are called *meteors.* When these bodies, which move with great velocity, impinge on the earth's atmosphere in a direction opposite to that in which the earth is moving, the relative velocity is so large that the heat developed by the resistance of the air is sufficient to consume them, and they appear as a streak of light in the sky. Others, whose relative velocity is not so great, on rare occasions, fall to the earth unconsumed. These are called *meteorites* or *meteoric stones.* The heights of meteors have been found to vary from 16 to 160 miles.

Although it is possible to see many stray meteors on almost any night, there are three periods in the year at which they occur in very considerable numbers, viz. August 9–11, November 12–14, and November 27–29.

Radiant Point —During these August and November meteoric showers the apparent paths on the celestial sphere of most of the meteors seem to spring from one common point called a *radiant point.* This is merely an effect of perspective. For, as we will shortly see, all the meteors which compose the swarm through which the earth happens to be passing are, for the short period they are under observation, approximately moving in parallel straight lines. If now we imagine planes drawn through these lines and the observer they will cut the celestial sphere in a number of great circles all having a common diameter, viz. the line drawn through the observer parallel to the common direction of the motion of the meteors. This line when produced will cut the celestial sphere in the two common points of intersection of all the circles, one of which is the radiant point. The radiant point for the August meteors is in the constellation of Perseus, and those for the two showers which take place in November are in the constellations of Leo and Andromeda respectively. Hence we have the three showers—

The Perseids,	Aug. 9–11,	radiant point in Perseus.
The Leonids,	Nov. 12–14,	,, in Leo.
The Andromedes,	Nov. 27–29,	,, in Andromeda.

Connexion between Comets and Meteors.

83. The meteors which we see dashing into the atmosphere at about the 14th November are believed to be portions of a train of an innumerable number of minute bodies whose orbits are almost identical with that of Temple's Comet. The orbit of this comet actually cuts that of the earth, the earth arriving each year at the point of intersection on the 14th November. There are portions of this elliptic belt where these minute bodies are crowded together into groups; and on certain occasions, separated by long intervals of time, the earth passes through one of these groups or shoals, as, for instance, on November 13, 1866, when the appearance of the heavens, lit up by myriads of meteors, was of the most wonderful description.

As the periodic time of this shoal is 33 years, it was fully expected that an equally brilliant display would be observed in November, 1899. But although a few individual members of the band were observed from several places on the earth, the result, for reasons which we can at present only conjecture, was very disappointing.

The showers known as the Andromedes and Perseids can be similarly accounted for, the former being due to the fact that the orbit of Biela's Comet cuts that of the earth at a point corresponding to November 27.

84. **Line of Apsides.**—Those points where the earth, or a planet, in its orbit is nearest to and most remote from the sun (perihelion and aphelion) are called *apsides*. The earth

is nearest the sun at the apse A (fig. 44) on the 31st December, and arrives at the opposite apse B on the 1st July, the line AB being called *the line of Apsides.* This line evidently coincides with the major axis of the ellipse.

N.B.—The sun's apparent diameter is greatest when the earth is at the apse A, and least when at B. Also as observed from two positions E and E' such that the $\angle ASE = \angle ASE'$ the sun will have the same apparent diameter, for from the symmetry of the ellipse it is evident that the distances SE and SE' are equal.

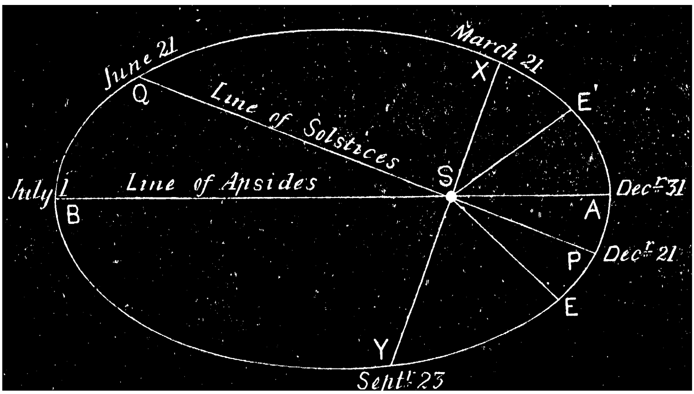

FIG. 44

To find the Direction of the Apse Line.—It might at first suggest itself that the direction of the line of Apsides could easily be found by noting the position of the sun in the ecliptic when its apparent diameter is least or greatest. It is, however, very difficult to tell when this occurs, as the apparent diameter remains very nearly constant for some time before and after the earth's passage through the apse. The following is the method employed:—

When the earth is at E some considerable time before the apse is reached the sun's apparent diameter is measured and its position in the ecliptic noted by calculating its longitude. The longitude is again noted when its angular diameter measures the same as before, the earth being at E'. The mean of these two longitudes will give the point on the ecliptic occupied by the sun during the earth's passage through the apse, the line joining this point to the earth giving the direction of the line of apsides.

Slow Motion of the Apse Line.—By observing the position of the apse line for a number of years it is found that it has a slow direct motion in the plane of the ecliptic at the rate of 11·25″ each year.

85. **Lengths of the Seasons.**—If P and Q represent the positions of the earth at the two solstices, a perpendicular

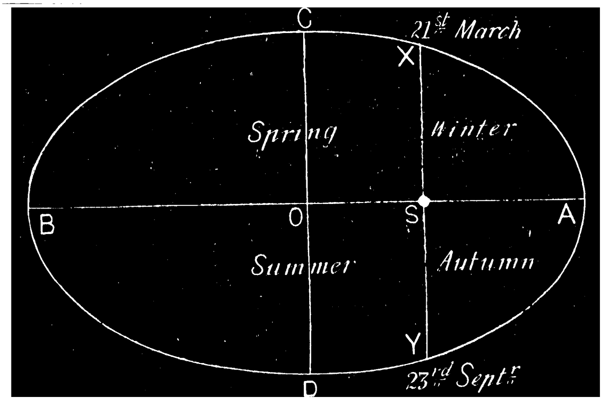

FIG. 15.

XY erected at S to PQ (fig. 44) will give the positions X and Y of the earth at the vernal and autumnal equinoxes respectively. The orbit of the earth is thus divided into the four arcs XQ, QY, YP, and PX, corresponding to the four seasons, Spring, Summer, Autumn, and Winter, respectively.

The four seasons are unequal in length, spring and summer lasting from 21st March till the 23rd September, being about 8 days longer than autumn and winter, which last from the 23rd September till the 21st of the following March. This inequality can very easily be explained from Kepler's Second Law, thus:—

Let AB represent the line of solstices (fig. 45), which for simplicity is supposed to coincide with the apse line, XY being the line of equinoxes, and CD the axis minor of the ellipse, *i.e.* perpendicular to AB erected at O the centre of the ellipse Now since CD bisects the area of the ellipse we have—

$$\text{area } CBD = \text{area } CAD;$$

$$\therefore \quad \text{area } CBD \text{ is} > \text{area } XAY;$$

$$\therefore \quad \textit{à fortiori } \text{area } XBY \text{ is} > \text{area } XAY.$$

But since equal areas are described in equal times, it follows that the combined length of spring and summer is greater than that of autumn and winter.

The lengths of the four seasons are as follows:—

Spring.	Summer.	Autumn.	Winter.
$92^d\ 20\frac{3}{4}^h$.	$93^d\ 14\frac{1}{2}^h$.	$89^d\ 18\frac{1}{2}^h$.	$89^d\ 0\frac{1}{2}^h$.

Eccentricity of the Earth's Orbit.

86. **Definition**.—The ratio of the distance of the centre of the ellipse from the focus to the semiaxis major is called the eccentricity. Thus—

$$\text{Eccentricity } E = \frac{OS}{OA} \text{ (fig. 45).}$$

We can express the eccentricity of the earth's orbit in terms of the greatest and least apparent diameters of the sun. For, since

$$OS = \tfrac{1}{2}(SB - SA),$$

$$OA = \tfrac{1}{2}(SB + SA);$$

$$\therefore \ E = \frac{SB - SA}{SB + SA}.$$

But SA and SB are inversely proportional to the apparent diameters of the sun at A and B respectively. Therefore, if d and d' represent these diameters, we have

$$E = \frac{\frac{1}{d'} - \frac{1}{d}}{\frac{1}{d'} + \frac{1}{d}} = \frac{d - d'}{d + d'}.$$

Therefore *the eccentricity of the earth's orbit is equal to the* *difference between the greatest and least apparent diameters of the sun divided by their sum.*

Example.—The greatest apparent diameter of the sun being 32′ 36″ and the least 31′ 32″, calculate from this the eccentricity of the earth's orbit.

Here $$E = \frac{d - d'}{d + d'} = \frac{1956 - 1892}{1956 + 1892} = \frac{1}{60} \text{ nearly.}$$

GENERAL EXAMPLES.

1. A planet is found to have an elongation from the sun of 150°. Is it an inferior or superior planet? *Ans.* Superior (Corollary, Art. 59).

2. A planet is found to be in quadrature. Is it inferior or superior? *Ans.* Superior.

3. Two planets are observed through a telescope. One appears as a thin crescent, the other appears dichotomized. State whether they are inferior or superior planets. *Ans.* Both inferior (Art. 64).

4. If the exterior angle at a planet formed by lines drawn from the earth and sun be 120°, find what part of the hemisphere which is turned towards the earth is illuminated. *Ans.* $\frac{2}{3}$rd (Art. 61).

5. In question 4 find the ratio of the *apparent* breadth of the visible illuminated portion at its widest part to the apparent diameter of planet's complete disc (Art. 62).

Here

$$\frac{\text{apparent breadth}}{\text{diameter of planet}} = \frac{r \text{ versin } 120°}{2r} = \frac{r(1 - \cos 120°)}{2r} = \frac{r(1 + \frac{1}{2})}{2r} = \frac{3r}{4r} = \frac{3}{4}.$$

6. Find what should be the radius of a planet's orbit in order that its greatest elongation from the sun, as seen from the earth, should be 30°, assuming the distance of the earth from the sun as 92,000,000 miles.

Ans. 46,000,000 miles.

7. Calculate approximately in miles per second the velocity of the earth in its orbit (J. S., T. C. D.). *Ans.* 18·3.

8. If there be 378 days between two successive oppositions of Saturn; find the length of Saturn's year. (Degree, T. C. D.).

Here $$\frac{1}{365 \cdot 25} - \frac{1}{P} = \frac{1}{378} \text{ (Art. 67);}$$

and solving, we get $$P = 10828 \cdot 6 \text{ days,}$$
$$= 29 \cdot 6 \text{ years.}$$

9. The periodic time of Mercury being 88 days; find the interval between two successive inferior conjunctions of this planet.

Here $$\frac{1}{88} - \frac{1}{365 \cdot 25} = \frac{1}{T};$$

and solving, we get $T = 115 \cdot 9$ days.

10. Assuming the mean distance of Venus to be ·72, that of the earth being unity, apply Kepler's Laws to find the periodic time of Venus.

Here, by Kepler's Third Law, we have

$$T^2 : T'^2 :: r^3 : r'^3,$$

or $$T^2 : (365 \cdot 25)^2 :: (\cdot 72)^3 : (1)^3;$$

$$\therefore \quad T = \sqrt{(365 \cdot 25)^2 \times (\cdot 72)^3} = 223 \text{ days nearly.}$$

11. Assuming the distances of the different planets from the sun as given by Bode's Law, calculate from this the periodic time—(1) of Mercury, (2) of Saturn. *Ans.* (1) 90·11 days,
(2) 11550·3 days

12 Supposing a planet were to revolve round the sun at a distance of half a million miles, find what should be its periodic time.

Ans. $3\frac{1}{3}$ hours nearly.

13. Why do comets move with much greater velocity when at perihelion than at other parts of their eccentric orbits? (Kepler's Second Law).

14. The two satellites of Mars have periodic times, which are about 30 hours and $7\frac{1}{2}$ hours respectively; find the ratio of their mean distances from Mars.

Since Kepler's Laws apply to the motions of the satellites, we have:—

$$T^2 : T'^2 :: r^3 : r'^3;$$

that is, $$(30)^2 : (7\tfrac{1}{2})^2 :: r^3 : r'^3,$$

or $$4^2 : 1^2 :: r^3 : r'^3;$$

$$\therefore \quad \sqrt[3]{16} : 1 :: r : r'.$$

Hence their mean distances from Mars are in the ratio of

$$\sqrt[3]{16} \text{ to } 1 \quad \text{or} \quad 2\sqrt[3]{2} \text{ to } 1.$$

15. The velocity of Mercury in its orbit is 30 miles per second; hence calculate the velocity of Saturn.

Here, $$v : v' :: \sqrt{r'} : \sqrt{r},$$

or, $$30 : v' :: \sqrt{100} : \sqrt{4} \quad \text{(Bode's Law)},$$

or, $$30 : v' :: 10 : 2;$$

$$\therefore \quad 10\,v' = 60; \quad \therefore \; v' = 6 \text{ miles per second.}$$

16. Why was September 5, 1877, when the satellites of Mars were discovered, a date particularly favourable for observing that planet? (Art. 77.)

TABLE OF NAMES, PERIODS, ETC., OF THE PLANETS.

NAME.	Symbol.	Mean distance from Sun.	Periodic Time	Synodic Period.	Diameter. Earth's being 10	Velocity in miles per second.	Angle of inclination of orbit.	Arcs which they retrograde.
Mercury, . .	☿	3·8	88 days.	115·9 days.	3·8	30	7° 0′	12°
Venus, . .	♀	7·2	224 ,,	584 ,,	9·6	21½	3° 23′	16°
Earth, . .	⊕	10	365¼ ,,	—	10	18½	—	—
Mars, . .	♂	15·2	686 ,,	780 ,,	5·3	15	1° 51′	18°
Asteroids, . .		26·5	3–8 years.	—	—	—	—	—
Jupiter, . .	♃	52	12 ,,	399 ,,	110·4	8	1° 19′	9°
Saturn, . .	♄	95·3	29½ ,,	378 ,,	91	6	2° 30′	6°
Uranus, . .	♅	191·8	83 ,,	369·7 ,,	41·6	4⅓	0° 46′	4°
Neptune, . .	♆	300·5	165 ,,	367·5 ,,	46	3½	1° 47′	3°

CHAPTER VII.

PARALLAX.

87. **Definition.**—By the *diurnal parallax* of a heavenly body is meant the angle subtended at the body by that radius of the earth which is drawn to the observer.

Thus, if C be the centre of the earth, O the observer, the parallax of the body M is the angle subtended by CO at M, viz. the $\angle p$.

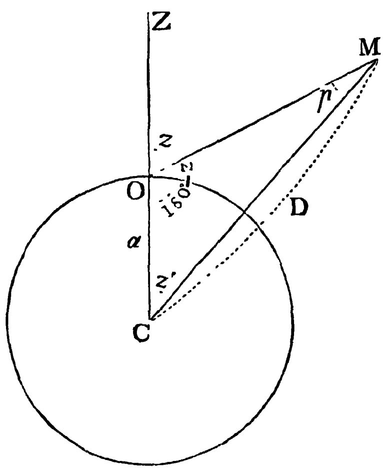

FIG. 46.

The fixed stars are so very far away that we may regard the lines joining one of them to the observer and to the centre of the earth as being so nearly parallel that the parallax is practically zero. To illustrate how small this angle becomes, let the reader take a marble one inch in diameter, and try to imagine what angle its radius could subtend at a point, say 1000 miles away. The most delicate instrument we possess would be unable to measure it; and yet this angle is more than one hundred times as great as the angle which the radius of the earth could subtend at even the nearest fixed star.

The planets, however, as well as the sun and moon, are comparatively so near us that this difference in the direction of the lines drawn from a point on the surface and from the centre of the earth to the planet is large enough to be measured.

Also the directions in which these bodies are observed from any two positions on the earth's surface are not exactly the same. All observers therefore, wherever situated, reduce their observations to what they would be if situated at the centre of the earth. This reduction is what is called the *correction for parallax.* The declinations, right ascensions, &c., of bodies which we see noted in the Nautical Almanac are those which they would have if seen from the centre of the earth.

Definition.—The *horizontal parallax* is the parallax of a body when on the horizon. Thus, if the body *M* be on

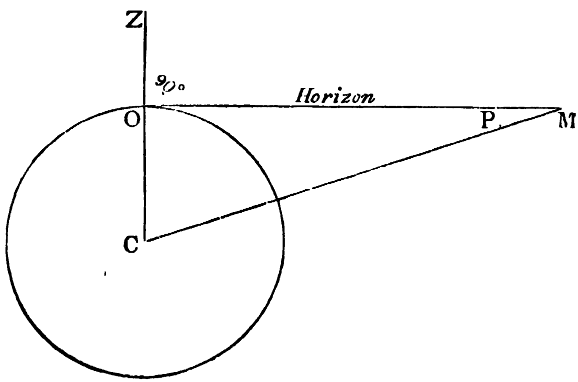

FIG. 47.

the horizon of the observer *O*, the ∠ *P* is the horizontal parallax.

88. **The effect of Parallax on a heavenly body is to depress it in the heavens.**

For, if *O* be the position of the observer (fig. 46), then *OZ*, the production of the radius drawn to *O*, is the direction of the zenith, and the ∠ *z* is the zenith distance of the body

M as seen from O. Also the $\angle z'$ would be its zenith distance if the observer were at the centre of the earth;

but $$\angle z = \angle z' + \angle p;$$
that is,

apparent zenith distance = true zenith distance + parallax;

therefore, as seen from O, the body appears lower down in the heavens than if seen from C.

To find the parallax of a body for a given zenith distance.

a = radius of earth (fig. 46),
$D = CM$ = distance of body.

Since, from Trigonometry, we know that the sides of the $\triangle\ COM$ are as the sines of the opposite angles;

$$\therefore \quad \frac{\sin p}{\sin(180 - z)} = \frac{a}{D},$$

$$\text{or} \quad \frac{\sin p}{\sin z} = \frac{a}{D};$$

$$\therefore \quad \sin p = \frac{a}{D} \sin z;$$

but p being in all cases a very small angle, therefore $\sin p = p$ (expressed in circular measure);

$$\therefore \quad p = \frac{a}{D} \sin z.$$

When the body is on the horizon $z = 90°$, and p becomes the horizontal parallax P;

$$\therefore \quad P = \frac{a}{D} \sin 90° = \frac{a}{D};$$

therefore substituting, we have

$$p = P \sin z,$$

or, parallax = horizontal parallax × sine of apparent zenith distance.

Hence **the parallax of a heavenly body varies as the sine of its apparent zenith distance.**

As $\sin z$ is a maximum when $z = 90°$, we see that the parallax is a maximum when the body is on the horizon.

EXAMPLES.

1. Supposing the sun's observed altitude to be 60° and the parallax 4″·4, find his true altitude.

Here, since parallax depresses a body,

$$\text{true altitude} = \text{observed altitude} + \text{parallax}:$$

therefore $$\text{true altitude} = 60° + 4''\cdot 4 = 60°\ 0'\ 4''\cdot 4.$$

2. Given the moon's horizontal parallax as being 57′ 6″, find its true altitude corresponding to an observed altitude of 60°.

Here $$p = P\sin z \quad \text{and} \quad z = 90° - 60° = 30°;$$

$$\therefore\ p = (57'\ 6'')\sin 30° = (57'\ 6'')\tfrac{1}{2} = 28'\ 33'';$$

therefore $$\text{true altitude} = 60°\ 28'\ 33''.$$

3. The sun's horizontal parallax being 8″·8, find the true zenith distance corresponding to an observed zenith distance of 60°.

Here $$p = P\sin z,$$

$$\text{or} \quad p = 8''\cdot 8 \sin 60° = 8''\cdot 8 \times \frac{\sqrt{3}}{2} = 7''\cdot 6;$$

therefore $$\text{true zenith distance} = 60° - 7''\cdot 6 = 59°\ 59'\ 52''\cdot 4.$$

Given the horizontal parallax of a body, to find its distance, and vice-versa.

We have just seen that $P = \frac{a}{D}$, but P is expressed in circular measure. Hence, if expressed in seconds, we have :—

$$\frac{P''}{206265''} = \frac{a}{D}.$$

EXAMPLES.

1. Given that the moon's horizontal parallax is 57′ 6″; find its distance from the earth, the earth's radius being 4000 miles.

Ans. About 240,000 miles.

2. The sun's horizontal parallax being 8″·8, find its distance from the earth. *Ans.* About 93,700,000 miles.

3. The moon's distance being 60 times the earth's radius, find the moon's horizontal parallax.

Here $$\frac{P''}{206265''} = \frac{a}{D} = \frac{1}{60};$$

$$\therefore \quad P = 57' \, 17''.$$

89. The displacement of a heavenly body due to parallax like that from refraction is in the direction of the vertical drawn through the body. Hence the azimuth of a body is not affected by either parallax or refraction. We have seen, (Art. 39) that refraction does not depend on the distance of the body from us, for the rays only get bent on their entrance into the atmosphere; the parallax, however, becomes less the greater the distance of the body, the moon's horizontal parallax being about 57′, while that of the sun, which is much further away, is only about 8″, and the fixed stars are so remote that their parallax is zero. All bodies except the moon are much more elevated by refraction than depressed by parallax. For instance, horizontal refraction amounts to about 34′, whereas the sun when on the horizon, as we have seen above, is only depressed by parallax through about 8″. For the moon, however, parallax is much greater than refraction; hence the combined effect of both in this case produces a depression.

To find the Angle which two distant places on the Earth's Surface, nearly in the same Meridian, subtend at the Moon or a Planet.

90. Let A and B be two distant places on the earth, as nearly as possible in the same meridian (Greenwich and the Cape of Good Hope are favourably situated for the purpose);

M represents the moon or planet when in the meridian of *A* and *B*. Let a fixed star be observed from *A* and *B*, *in nearly the same part of the heavens as M*, so that their right ascensions and declinations differ very slightly. The lines joining *A* and *B* to the star are nearly parallel, the star being so distant.

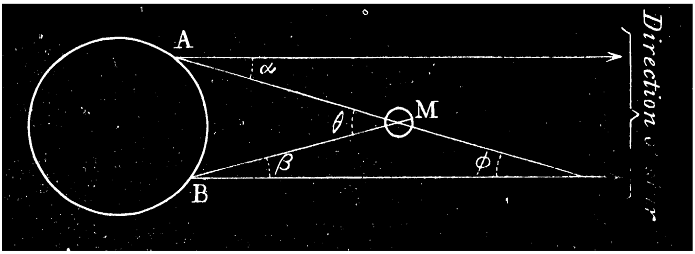

FIG. 48.

The angles α and β, the angular distances of the star from *M*, are measured at *A* and *B* respectively by means of micrometers; but

$$\angle\theta = \angle\beta + \angle\phi,$$

and $\angle\phi = \angle\alpha$ by parallel lines;

$$\therefore \quad \angle\theta = \angle\alpha + \angle\beta,$$

and α and β being known, θ is determined.

If the two places *A* and *B* are not in the same meridian, then the two observers, not making their measurements at the same time, a correction must be made for the small distance moved by the moon or planet (owing to the orbital motion) in the interval between its passages over the two meridians.

To find the Horizontal Parallax of the Moon or a Planet.

91. Two positions *A* and *B* (fig. 49) are chosen as before on the same meridian of the earth in the northern and southern hemispheres respectively. The meridian zenith

distances of the moon or planet M are then measured simultaneously by means of the meridian circle. Let these be z and z', the lines OZ and OZ' being the directions of the

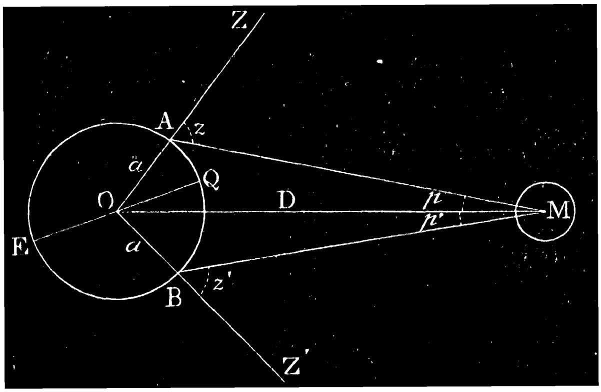

FIG. 19.

zenith at A and B respectively; also let P be the horizontal parallax. Now (Art. 88) we have—

$$p = \frac{a}{D} \sin z = P \sin z,$$

and
$$p' = \frac{a}{D} \sin z' = P \sin z';$$

$$\therefore \quad p + p' = P (\sin z + \sin z');$$

$$\therefore \quad P = \frac{p + p'}{\sin z + \sin z'};$$

but $p + p'$ is known, being angle subtended at M by A and B (Art. 90), and z and z' being observed, the horizontal parallax P can be found. This method is free from any serious errors due to refraction, for in Art. 90 the moon and fixed star are nearly in the same position in the sky, and therefore almost equally affected by refraction, and therefore the value of $p + p'$ is got with great accuracy

The above result for the horizontal parallax can be put in another form. For, draw the equator EQ, and let l and l' be the latitudes of A and B respectively;

$$\therefore \quad \angle ZOQ = l, \text{ and } \angle Z'OQ = l',$$

but (Euclid, I. 32),

$$\angle z + \angle z' = \angle ZOM + \angle Z'OM + \angle p + \angle p',$$

or

$$z + z' = l + l' + p + p':$$

$$\therefore \quad p + p' = z + z' - l - l';$$

$\therefore$ by substitution $\quad P = \dfrac{z + z' - l - l'}{\sin z + \sin z'}.$

It is not possible to determine the sun's parallax in this manner, for, owing to the intensity of his rays the neighbouring stars cannot be observed. It can, however, be calculated indirectly. For, let the parallax of Mars when in opposition be observed by the above method, from which the distance of that planet from the earth can be found (Art. 88), this distance is the difference of the distances (r and r') of Mars and the earth from the sun or $r - r'$. But the ratio of r to r' is known from Kepler's Third Law; hence we can solve for r' the distance of the earth from the sun, and the sun's parallax is determined by Art. 88.

But the most accurate methods of obtaining the sun's parallax, and hence his distance from the earth, are from observations of the transit of Venus across his disc (Art. 76), as follows:—

Delisle's Method of finding the Sun's Parallax.

92. Two stations A and B (fig. 50) are chosen, both near the earth's equator, but separated as far apart as possible, the circle AB being the equator of the earth. Let us now suppose, in order to simplify the explanation, that the sun and the orbit of Venus VV' are in the plane of the equator AB. Draw tangents AS and BS to the sun.

The observer at A notes the instant at which internal contact takes place, which occurs when Venus is at V, touching the line AS internally; a similar observation is made at B, internal contact occurring when Venus is at V'.

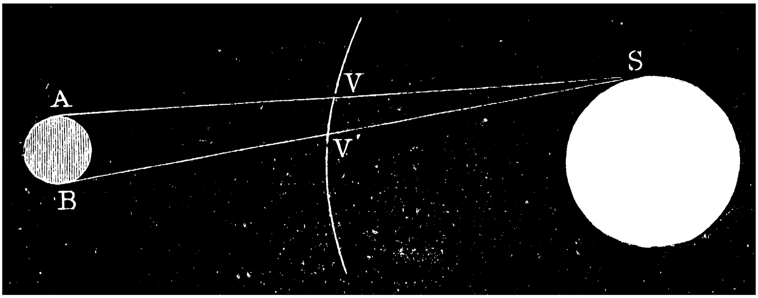

FIG. 50.

The time of each observation is reduced to Greenwich time, which corrects for the difference in longitude of A and B. The difference of the two results will give the interval of time during which Venus *appears* to move round the sun through the angle VSV'. (The reader must remember that we are supposing the earth to be at rest, and that Venus has an angular velocity round the sun equal to the excess of its real angular velocity over that of the earth.) But the rate at which the angle VSV' is described is known, being 360° in each synodic period: we can therefore calculate the angle VSV' or ASB.* Thus, knowing the angle which two distant places on the earth's surface subtend at the sun, we can calculate the sun's horizontal parallax as in Art. 91, and hence his distance from the earth.

In actual practice a great many difficulties have to be met, the principal being the inclination of the orbit of Venus to the ecliptic.

* The point S is here taken as practically coincident with the sun's centre.

In Delisle's method the longitudes of the places have to be very accurately known. In the following method, proposed by Halley in 1716, it is not necessary to know the longitudes of the places, as only the *duration* of the transit is observed at each place, the fact that the clocks indicate different hours being of no consequence.

Halley's Method or the Method of Durations.

93. In this method the *duration* of the transit is observed from two places *A* and *B* on the earth, separated as far apart as possible, one in a high northern and the other in a high southern latitude so that there may be as large a difference as possible in the observed length of time during which the transit lasts, as seen from the two places

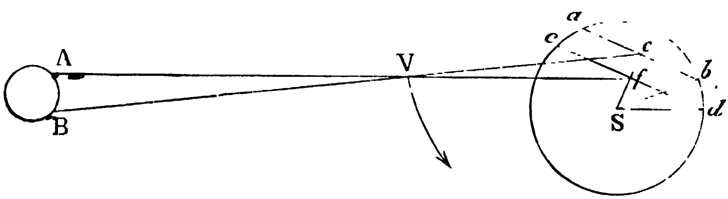

FIG. 51.

Let *V* represent Venus, the plane containing *A*, *B*, and *V* being the plane of the paper, while the reader must bear in mind that the plane of the circle, of which *S* is the centre, and which represents the sun's disc, is at right angles to this plane. To an observer at *A*, Venus, in her apparent motion in the direction indicated by the arrow, will appear to cross the sun's face in the direction *cd*, and to the observer at *B* in the direction *ab*, the time occupied being noted in each case. But the rate at which Venus appears to cross the sun's face can be calculated (Ex. 4, p. 125), being at the rate of 4″ in each minute of time, we can therefore, by a statement in simple proportion, calculate the number of seconds in *ab* and *cd* respectively, very much more accurately than if measured by a micrometer. Therefore the halves of these chords, viz. *eb* and *fd* are known in seconds. But the sun's semidiameter *sb* or *sd* is also known in seconds; therefore we can find the number of seconds in *se* and *sf*, for we have approximately, by Euclid (I. 47),

$$se^2 = sb^2 - be^2;$$

also $$sf^2 = sd^2 - df^2.$$

Knowing *se* and *sf*, we find, by subtraction, the number of seconds in *ef*.

Again, the *number of miles* in *ef* can be found from knowing the number of miles between the two stations *A* and *B*, for, regarding the two triangles *AVB* and *eVf* as similar, we have

$$ef : AB :: Vf : AV.$$

But the ratio of *Vf* to *AV* can be found (Art. 66), being 723 to 277; therefore *ef* in miles is known. Lastly, knowing *ef* in miles and the angle in seconds subtended by it at the earth, the distance of the sun can be found thus:—

$$\frac{ef''}{206265} = \frac{ef \text{ in miles}}{\text{sun's distance}},$$

and consequently his parallax is determined.

To find the Radius of the Moon in Miles.

94. Having shown how to determine the parallax of the

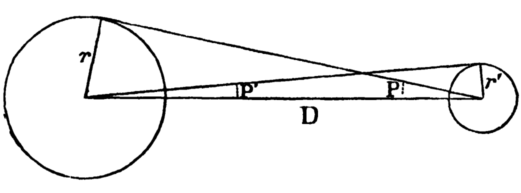

FIG. 52.

moon, sun, or a planet, we can calculate the radii of these bodies in miles by a comparison with the radius of the earth.

Let r = radius of earth.

r' = radius of moon, or other body.

P = moon's horizontal parallax = earth's angular semidiameter as seen from the moon.

P' = moon's angular semidiameter.

Now $$\frac{r}{d} = P \text{ (in circular measure),}$$

$$\frac{r'}{d} = P' \text{ (in circular measure);}$$

$$\therefore \quad r : r' :: P : P',$$

or (**radius of earth**) : (**radius of moon**) : : (**moon's parallax**) : (**moon's semidiameter**).

Examples.

1. Taking the moon's horizontal parallax as 57′, and its angular diameter as 32′, find its radius in miles, assuming the earth's radius to be 4000 miles.

Here moon's semidiameter = 16′;

$$\therefore \quad 4000 : r' :: 57' : 16'; \quad \therefore r' = \frac{4000 \times 16}{57} = 1123 \text{ miles}$$

2. The sun's horizontal parallax being 8″·8, and his angular diameter 32′, find his diameter in miles. *Ans.* 872,727 miles.

3. The synodic period of Venus being 584 days, find the angle gained in each minute of time on the earth round the sun as centre.

Ans. 1″·54 per minute.

4. Find the angular velocity with which Venus crosses the sun's disc, assuming the distances of Venus and the earth from the sun are as 7 to 10, as given by Bode's Law.

Since (fig. 50) $SV : VA :: 7 : 3$. But SV has a *relative* angular velocity round the sun of 1″·54 per minute (see Example 3); therefore, the relative angular velocity of AV round A is greater than this in the ratio of 7 : 3, which gives an approximate result of 3″·6 per minute, the true rate being about 4″ per minute.

Annual Parallax.

95. We have already seen that no displacement of the observer due to a change of position on the earth's surface could apparently affect the direction of a fixed star. However, as the earth in its annual motion describes an orbit of about 92 million miles radius round the sun, the different positions in space from which an observer views the fixed stars from time to time throughout the year must be separated from one another by very great distances indeed. For instance, any two diametrically opposite points in the orbit of the earth are separated by an interval of about 184 million miles, the earth proceeding from one of these points to the other in about six months. We should therefore expect that, when viewed from two points separated by such a distance as this, the fixed stars should not occupy exactly the same position with respect to one another, those which are nearer the earth being more displaced than those further away. This is to a certain extent quite true; but to such vast depths are the fixed stars sunk in space, that only in the case of some of those nearest to us can any appreciable displacement be detected; or, in other words, a base line of 184 million miles is much too small a distance to take in an attempt to measure the distances of by far the greater number of these bodies.

Owing to the small displacements in the apparent directions of some of the fixed stars, due to the earth's changes of position throughout the year, we refer their directions on the celestial sphere to what they would have if viewed from the centre of the sun, which is fixed. This direction, as seen from the centre of the sun, being called the star's *heliocentric* direction, the correction, which must be applied to reduce the apparent or geocentric to the heliocentric direction being called the *correction for annual parallax.*

Definition.—The annual parallax of a star is the angle subtended at the star by the line joining the earth and sun. Thus, if E represent the earth, H the sun, and S a star, the annual parallax of S is the angle subtended at S by EH or the angle p

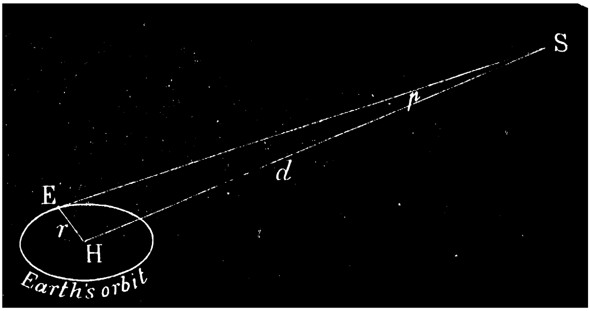

FIG. 93

96. The law according to which the annual parallax of a star should vary can be deduced by a method similar to that applied to the diurnal or geocentric parallax of the moon or planets. For

$$\frac{\sin p}{\sin E} = \frac{r}{d};$$

$$\therefore \quad \sin p = \frac{r}{d} \sin E;$$

but p being small, $\sin p = p$ (in circular measure),

$$\therefore \quad p = \frac{r}{d} \sin E;$$

therefore, the annual parallax varies as the sine of the angular distance of the sun from the star.

It is evident that the parallax of a star is a maximum when $E = 90°$, which happens twice a year for each star. Let P represent this maximum value, and we have—

$$P = \frac{r}{d} \sin 90° = \frac{r}{d}; \text{ also } p = P \sin E.$$

N.B.—Generally speaking, by a star's parallax is meant this maximum value of the parallax, unless otherwise stated.

As P is expressed in circular measure in the above formula; therefore, when expressed in seconds, we have—

$$\frac{P''}{206265} = \frac{r}{d}.$$

Thus, when the parallax of a star is known, we can deduce its distance from the solar system as r the distance of the earth from the sun is known (Art. 92). In the following questions, r may be taken as 92 million miles.

EXAMPLES.

The parallax of *a Centauri* being 0″·8, find its distance from the solar system.

Here

$$\frac{\cdot 8''}{206265} = \frac{92000000}{d}.$$

$$\therefore\ d = \frac{92000000 \times 206265}{8}\ \text{miles}.$$

2. Supposing the parallax of a star to be 0″·2, find how long a ray of light would take to travel to the earth, being given the velocity of light as 190,000 miles per second. *Ans.* 16 years nearly.

The effect of annual parallax on a star is to cause it to appear to move in a small ellipse throughout the year.

97. For as each displacement of the earth in its orbit produces a corresponding small displacement in the apparent position of the star in the sky, the star will therefore seem to describe a small yearly orbit round its heliocentric position (which is fixed) parallel to the plane of the earth's orbit. If now we *assume the earth's orbit to be circular*, we will consider the effect of parallax on a star, according as it is situated—(1) near the pole of the ecliptic, (2) on the ecliptic, (3) at any point of the sky.

(1) If a star be situated in the pole of the ecliptic, the plane of the small arc which we may suppose it to describe being at right angles to our line of sight will, when projected on the celestial sphere, still appear circular.

(2) If situated on the ecliptic, it will appear to move back and forward along the ecliptic in a straight line, this being explained by the fact that a circle seen edgeways appears as a line.

(3) If situated in any other part of the sky, the apparent path throughout the year will appear as a diminutive ellipse, as a circle seen obliquely will appear elliptic.

To determine the Annual Parallax of a Star—Bessel's Method.

98. Bessel's method, otherwise called the *differential* method, consists in choosing a very *faint star* very close to the star whose parallax is sought. Being very faint, it is

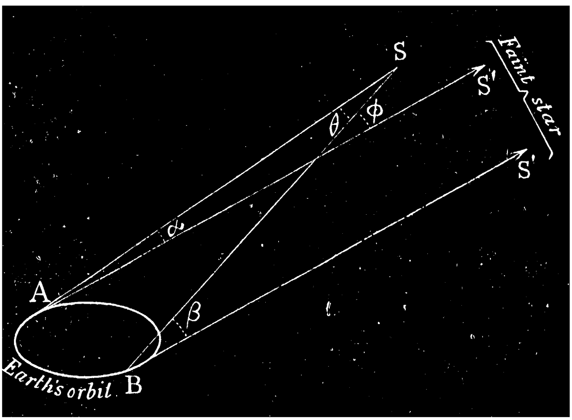

FIG. 54.

presumably much further away than the star in question; and we may, therefore, assume that its own parallax is so very small that any changes which take place throughout the year in the angular distance of the two stars from one another must be due almost entirely to the parallax of the near one. The actual measurement of these changes enables us to determine the annual parallax.

N.B.—The faint star is chosen *very close* to the star, whose parallax we want, in order that both may be equally affected by errors due to *refraction*, *aberration*, &c., so that we have not to correct for these errors. The following will illustrate the method employed:—

Let A and B be two diametrically opposite points in the earth's orbit. AS and BS (fig. 51) are the directions of the star S, as viewed from A and B; and AS' and BS' the directions of the faint star, these lines being taken as parallel, owing to the much greater distance of S'. A, B, S, and S' are supposed in the same plane.

At A the observer measures, by means of the micrometer or heliometer, the angle α between S and S'; and again at B, six months afterwards, he measures the angle β. But by Euclid (I. 32), we have—

$$\angle \phi = \angle \alpha + \angle \theta,$$

but $$\angle \phi = \angle \beta \text{ by parallel lines;}$$

$$\therefore \quad \angle \beta = \angle \alpha + \angle \theta;$$

$$\therefore \quad \angle \theta = \angle \beta - \angle \alpha;$$

but α and β are known, therefore θ is determined; and θ being the angle subtended by the diameter of the earth's orbit, is twice the annual parallax which can therefore be found.

N.B.—The reader can compare this method with that employed to find the angle subtended at the moon by two distant places on the earth's surface (Art. 90) by means of which the moon's *diurnal* or *geocentric* parallax was determined.

Strictly speaking, the lines AS' and BS' have some inclination to each other; therefore, the error to which Bessel's method is open is, that it determines not the parallax of the near star, but the difference in parallax of the two stars; hence the parallax of a star thus determined is always too small, but never too great, and therefore the distance of a star from us is found to be greater than is actually the case.

Bessel, by this method, first measured the parallax, and hence the distance of 61 Cygni in the year 1838, and in the following year that of α Centauri was found.

Instead of measurements with the micrometer, photography is now being very successfully employed in noting the changes in the angular distance of the two selected stars from one another.

Absolute Method.

99. This method consists in measuring the star's right ascension and declination when in the meridian at different times throughout the year, and after making, with as much accuracy as possible, all the corrections for precession, nutation, &c., the different results are compared together when any small differences which they may show give sufficient data to calculate the annual parallax of the star.

EXAMPLES.

1. (*a*) Where must a star be situated so as to have no displacement due to parallax, (*b*) where must it be situated so that the effect of parallax may be greatest

Ans. (*a*) In a line with earth and sun.
(*b*) At an angular distance of 90° from sun.

2. If the parallax of 61 Cygni be 0''·5, find the parallax of a star which is ten times as far away from our solar system. *Ans* 0''·05.

3. The parallax of α Centauri being 0''·75, compare its distance with that of 61 Cygni, whose parallax is 0''·5.

Ans. (dist. α Centauri) : (dist. 61 Cygni) : : 2 : 3.

To find the Annual Parallax of Jupiter.

100. As the distance of Jupiter at opposition is more than four times as great as that of the sun, its diurnal parallax is therefore very small, so that it is not possible to observe

it with the same degree of accuracy as in the case of Mars (Art. 91). Its annual parallax, however, may be found thus:—

Let S, E, and J represent the sun, earth, and Jupiter, respectively when Jupiter is in quadrature, *i.e.* when the angle SEJ is a right angle. Again, let S, E', J' be their positions when Jupiter is again in quadrature, the earth having moved through EE' and Jupiter through JJ'.

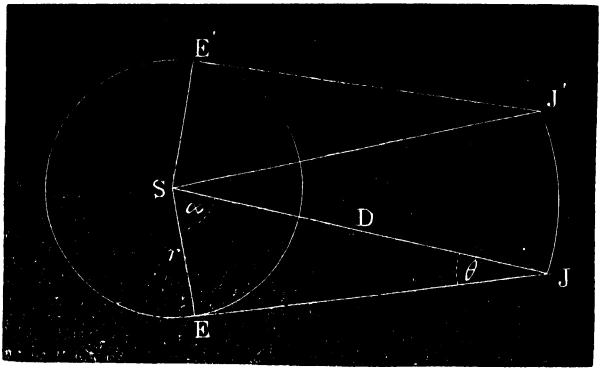

FIG. 55.

The number of days between the two observations being known, we therefore know the $\angle ESE'$ described by the earth in that time. Similarly, we know the $\angle JSJ'$. Therefore, the $\angle \alpha$, which is half the difference of these two angles (the two triangles SEJ and $SE'J'$ being equal in every respect), is known.

But the angle α is the complement of the $\angle \theta$. Therefore θ is found; that is, the angle subtended at Jupiter by the radius of the earth's orbit is known.

$$\text{Again,} \quad \sin\theta = \frac{r}{D}; \quad \therefore D = \frac{r}{\sin\theta},$$

which determines the distance of Jupiter.

This method also applies also to any planet outside the orbit of Jupiter.

It can be easily shown that if T represent the synodic period of Jupiter, and Q the interval between its eastern and western quadratures, the annual parallax

$$= 90°\left(1 - \frac{2Q}{T}\right),$$

for $\frac{360°Q}{T} = \angle$ gained by earth on Jupiter in Q days;

$$\therefore \text{(fig. 55) } 2a = \frac{360°Q}{T}; \quad \therefore a = \frac{180°Q}{T};$$

$$\therefore \text{ annual parallax } \theta = 90° - \frac{180°Q}{T} = 90°\left(1 - \frac{2Q}{T}\right).$$

Example.

The interval between eastern and western quadratures of Jupiter is 175 days, and between two oppositions 400 days, find the annual parallax of this planet.

Ans. 11° 15'.

CHAPTER VIII.

DETERMINATION OF THE FIRST POINT OF ARIES. PRECESSION, NUTATION, AND ABERRATION.

101. The first point of Aries being the zero point from which the right ascensions of all heavenly bodies are measured, it is therefore necessary to know its position with reference to the fixed stars with very great accuracy. Once having fixed this point, and the astronomical clock being set at zero when it crosses the meridian, then the time at which any other star crosses the meridian will, on being reduced to degrees (by allowing 15° for each hour), give the right ascension of that star.

It is evident that if we could find independently the right ascension of any one star, the position of the first point of Aries is immediately determined, and, consequently, the right ascensions of all other stars. The following method was first used by Flamsteed, the star selected being α Aquilæ.

Flamsteed's method of finding the Right Ascension of a Star.

Let σ be the star whose right ascension is sought (fig. 56): we have, therefore, to find X ♈ (X being the foot of the declination circle drawn through σ).

The declination of the sun SM is measured (Art. 34) at noon on some day shortly after the vernal equinox, this being done by measuring his meridian zenith distance. At the same time the interval between his transit across the meridian and that of the star σ is noted, this interval being the difference of their right ascensions MX, which we will denote by a.

Again, it can be ascertained at what time the sun shall have an equal declination shortly before the Autumnal Equinox by observing his meridian zenith distance at noon on successive days previous to September 23rd. But here we will have to do a little calculation; for it is very improbable that the sun will have an equal declination exactly at noon on any one of these days; but we can observe his declination at noon on two successive days, at which, in one case, it is greater, and, in the other case, less than SM; and

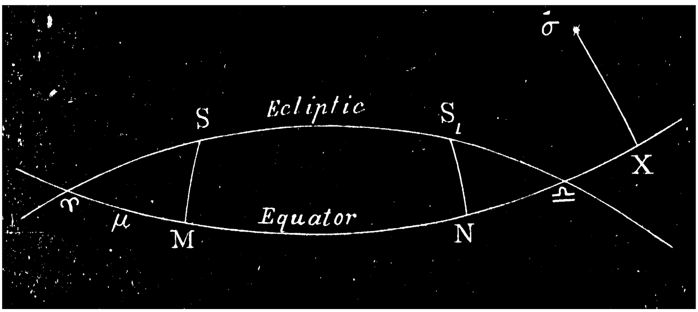

FIG. 56.

then, if we assume that for short periods his changes in right ascension and declination are proportional to one another, we can, by a simple statement in proportion, calculate the exact time at which his declination S_1N shall be equal to SM. In this case also the difference between his right ascension and that of the star σ is noted. This difference is NX, which we will call β. It is evident that $M♈ = N♎$.

Let x = the required right ascension ♈ X of star,

μ = right ascension of sun at $S = M♈$;

$\therefore$ $180° - \mu$ = right ascension of sun at $S_1 = N♈$,

but $X♈ - M♈ = MX$,

or $x - \mu = a$;

also $x - (180° - \mu) = \beta$, or $x - 180° + \mu = \beta$.

Thus we have two simultaneous equations, the unknown quantities being x and μ. Adding, we get

$$2x - 180^\circ = \alpha + \beta,$$

$$\therefore \quad x = \frac{180^\circ + \alpha + \beta}{2};$$

but α and β being known, x is therefore determined.

The above formula is open to some error, as during the interval between the two observations there is a slight increase in the right ascension of the star, owing to precession. It can, however, be corrected as follows:—

Let p = the increase in the R.A. of a star during the interval, and our two equations become

$$x - \mu = \alpha,$$

and $$x + p - 180^\circ + \mu = \beta;$$

$$\therefore \quad x = \frac{180^\circ + \alpha + \beta - p}{2}.$$

The uncorrected value of x gives, not the star's right ascension during the first observation near the vernal equinox, but its mean value between the two observations or the value it would have at about the 21st June.

The advantages of Flamsteed's method are that it is not necessary to know exactly the sun's declination; it is quite sufficient to observe when the declinations at the two observations are equal, so that any uncertainty in the latitude of the place due to instrumental errors, which will affect both observations equally, is of no consequence. Also, as the sun has nearly the same zenith distance during each observation, he will be equally affected by refraction and parallax, and hence these errors are avoided.

Determination of the Obliquity of the Ecliptic to the Equator.

102. This angle is measured by observing the meridian zenith distances of the sun at the summer and winter solstices. Let these be z and z', and let the latitude of the place be l. Now if S be the position of the sun at one of the solstices, its declination SM is equal to the inclination ω of the ecliptic to the equator (fig 57), for the angle between two great circles is measured by the arc they intercept on a circle perpendicular to both.

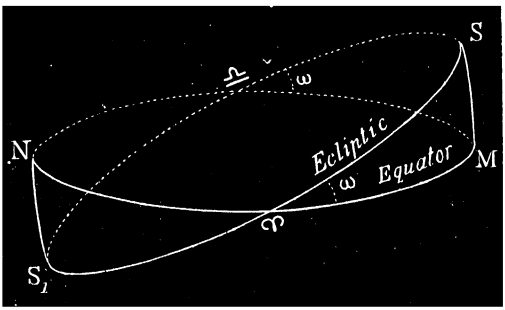

FIG. 57.

But latitude = zenith distance ± declination* (Art. 34);

$$\therefore \quad l = z + \omega \text{ for summer solstice,}$$

also
$$l = z' - \omega \text{ for winter solstice.}$$

Subtracting, we get
$$\omega = \frac{z' - z}{2};$$

and, therefore, ω is found.

In the above observations, it is not probable that the sun will be exactly at the solstitial point when in the meridian, but allowance can be made for his change of declination during the interval.

* This equation is obviously identical with that given in Art. 34, viz. colat $\pm\, \delta = a$.

Precession of the Equinoxes.

103. Repeated observations of the right ascensions and declinations of the stars extending over a long period of time show us that the first point of Aries is not a fixed point in the sky, but has a very slow movement among the fixed stars along the ecliptic in a direction opposite to that of the yearly motion of the sun. This backward motion of Aries to meet the sun, in which the first point of Libra also takes part, causes the equinoxes, as it were, to *precede* their due time each year. Hence this slow movement is called the *precession of the equinoxes.*

The rate of *precession* is 50″·24 in one year, or about 1° in 72 years. The time taken by Aries to complete one revolution of the heavens would, therefore, be about 26,000 years; for

$$\frac{360^\circ \times 60 \times 60}{50''\cdot 24} = 26{,}000 \text{ years nearly.}$$

Owing to precession, the longitude of each fixed star increases at the rate of 50″·24 each year. The right ascensions and declinations of the stars are also found to be slowly changing, but their latitudes remain almost constant. From this latter consideration we are led to the conclusion that the ecliptic is very nearly fixed in the heavens, but that the equator must be slowly shifting on the ecliptic, thus causing their intersections ♈ and ♎ to move in the manner described above.

This movement of the equator on the ecliptic is accompanied by a corresponding gradual displacement of the celestial pole, which describes a circular path round the pole of the ecliptic at a distance from it of 23° 28′, one revolution being completed in 26,000 years. Hence, in some thousands of years the star which is now our pole star will be at a considerable distance from the celestial pole. The bright star α Lyræ will, in about 10,000 years, be distant about 5° from the point round which the heavens will then seem to revolve, and will, like our present pole star, appear almost stationary in the sky.

Physical Cause of Precession.

104. The precession of the equinoxes is almost entirely caused by the attraction of the moon and sun on the protuberant portions of the earth at the equator. If the earth's shape were perfectly spherical, the attractions of the sun and moon could each be represented by a single force passing through its centre, and would, therefore, not disturb the axis of rotation of the earth nor the plane of the equator. However, the shape of the earth is spheroidal, not unlike that of a sphere with an additional layer or belt of matter round the equatorial regions. In the adjoining figure, let *S* represent the sun, and *PP'* the axis of rotation of the earth (fig. 58).

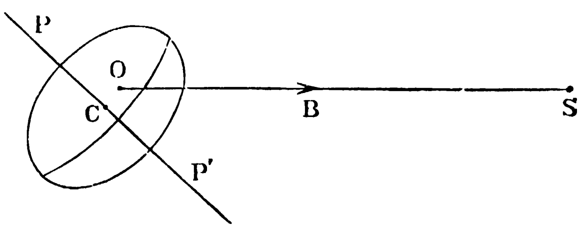

Fig. 58

Now, the attraction of the sun on the protuberant portions of the earth being greater on the nearer than on the more remote side, the resultant attraction will, therefore, be represented by a single force, *OB* acting at a point *O* above the centre of gravity *C*, the effect of which would be to cause a disturbance of the axis of rotation of the earth. On first thoughts we might imagine that this would result in a change in the plane of the equator, so as to make it eventually coincide with the ecliptic, and set the earth's axis at right angles to the plane of the ecliptic. And certainly this would be the case were it not that the earth is at the same time rotating rapidly round its axis, the resultant effect of these two rotations being that the axis of the earth is indeed disturbed, but in such a manner as not to alter its angle of inclination to the plane of the ecliptic. In fact, the axis of the earth has, as it were, a slow "wobbling" motion, so that the point in the heavens to which it is directed, viz. the celestial pole, describes the circle round the pole of the ecliptic, which we have previously mentioned

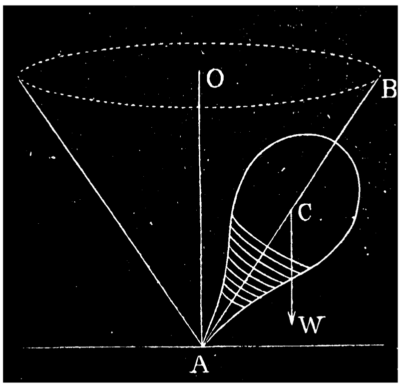

FIG. 59.

This motion of the earth's axis can be very well illustrated by the "wobbling" of the axis of rotation of a spinning-top. The weight of the top acting vertically downwards tends to pull the axis of rotation AB away from the vertical; but if the top be spinning sufficiently rapidly, it will not fall to the ground, but, as we all know, the axis of rotation describes a cone round the vertical AO, keeping at a constant angle to the ground in precisely the same manner as in the case of the earth the celestial pole, which is the extremity of its axis, revolves round the pole of the ecliptic.

The disturbing effect of the moon's attraction is more than twice as great as in the case of the sun, the ratio being as 7 : 3, the reason of this being on account of the greater proximity of the moon to the earth.

In either case, the disturbance is greatest when the attracting body reaches its greatest north or south declination, and is zero when the body is on the celestial equator.

The precession caused by the sun and moon is sometimes called the *lunar-solar precession.* It amounts to 50″·35 annually. This has, however, to be diminished by a very small amount called the *planetary precession*, which, acting in the opposite direction, is found to be 0″·11 each year, and leaves an annual *general* precession of 50″·24. The planetary precession is caused by the action of the planets, which tends to disturb the earth's orbit, and therefore the plane of the ecliptic, producing at the same time a diminution of the obliquity of the ecliptic to the equator of about half a second each year. However, this change of obliquity will never exceed a certain fixed limit, viz. about 1½° on either side of the mean value.

At present the vernal equinoctial point, though still retaining the name "First point of Aries," is not in the constellation of Aries, but, owing to precession, has shifted about 30° into the neighbouring constellation Pisces. Also the autumnal equinoctial point is not now in the constellation of Libra but in Virgo.

Nutation.

105. So far we have dealt with precession as if the celestial pole moved uniformly in a circle round the pole of the ecliptic, and this would certainly be the case if the disturbance due to the attractions of the sun and moon were

constant; but in consequence of a want of uniformity in this disturbance the celestial pole really describes a wavy path (see fig. 60). This *nodding*, as it were, of the celestial pole to and from the pole of the ecliptic is called *nutation*. The result is, that the precession is sometimes more and at other times less than its mean value, and there also results a small periodic increase and diminution of the obliquity of the ecliptic to the equator, according as the celestial pole *P* approaches or recedes from the pole of the ecliptic.

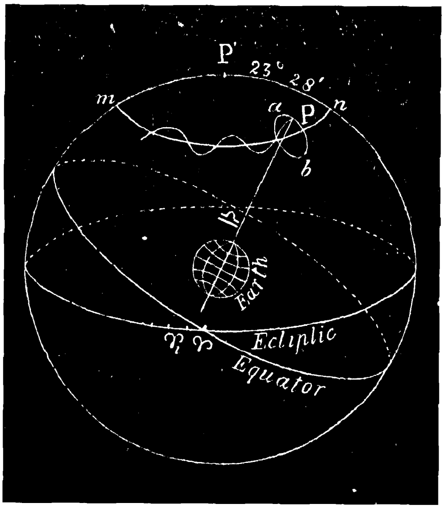

FIG. 60.

Nutation is almost altogether caused by the variable action of the moon depending on the position of the moon's nodes (the points where its path cuts the ecliptic) which make a complete revolution of the heavens in 18⅔ years.

The wavy motion of the celestial pole may be graphically represented in the following manner:—

Round the mean position of the celestial pole as centre (fig. 60) describe a small ellipse *ab*, with a major axis $ab = 18''\cdot 5$ directed towards the pole of the ecliptic, and a minor axis $13''\cdot 7$ along the circle *mn*; then if we imagine the mean pole which is the centre of the ellipse, to move along the arc *mn*, the true pole *P* will move in the ellipse round it as centre, completing a revolution in 18⅔ years.

Bradley first discovered nutation by observing that, after correcting for aberration, &c., the apparent displacements in the fixed stars with reference to the equator and ecliptic could not be accounted for on the supposition of a uniform precession.

The Velocity of Light.

106. That the propagation of light is not instantaneous was discovered by Roemer, in 1675 from observing the eclipses of Jupiter's satellites. The times at which these eclipses should occur were predicted from a great number of previous observations, and would therefore correspond to the mean distance of the planet from the earth. It was found, however, that when Jupiter was in opposition, or, in other words, nearest to the earth, the eclipses appeared to occur about eight minutes before the calculated time. On the other hand, when Jupiter was in superior conjunction or furthest from the earth the observed time was about eight minutes later than that predicted; from which it appeared that this difference of about sixteen minutes, or, more accurately, sixteen minutes and thirty-six seconds, was the time taken by a ray of light to move through the diameter of the earth's orbit. Taking this distance as 185,000,000 miles we get that the velocity of light is about 186,000 miles per second. As the velocity of the earth in its orbit is 18½ miles per second, we see that the velocity of light is about 10,000 times greater than that of the earth.

We see from the above that light takes about $8^m\ 18^s$ to pass from the sun to the earth. This interval is sometimes called the *equation of light*. The velocity of light has since been measured directly by M. Fizeau, and afterwards by M. Foucault.

Aberration.

107. The *Aberration of Light* is the apparent displacement in the directions of the heavenly bodies due to a combination of the velocity of the earth with that of light. The velocity of the earth, although small compared with that of light, is still large enough to produce a sensible deflection in the direction of the rays of light coming to us, so that the direction in which we have to point a telescope in order to observe a star is not the same as if the earth were at rest.

We may illustrate the effect of aberration in the following manner:—A man standing still in a shower of rain when the drops are falling vertically will, in order to shield himself, hold an umbrella right over his head. But if he proceed to walk or run he will find that the drops seem to strike him in the face, so that he has to hold the umbrella before him. Also the more he increases the rate at which he is moving the greater will be the deflection in the direction of the rain drops. This deflection we might call the *aberration* of the rain.

Effect of Aberration on a Star.

108. Let O (fig. 61) be the position of the earth; draw OA a tangent to the earth's orbit at O, and cut off OA to represent v, the velocity of the earth. Then let OS be the direction of a star and, produce SO to B, so that OB may represent V, the velocity of light. Now in order to be able to consider the question as if the earth were at rest, let us apply a velocity equal and opposite to v to both the earth and to light. This leaves the relative motion unaltered. The point O is thus reduced to a state of rest, while the light may be supposed to have two velocities OC and OB which give a resultant velocity OD. The star will therefore appear in the direction OS' the production of OD and the angle SOS' or a, which measures the amount of the displacement, is called the aberration of the star.

Definitions.

(1). The angle a between the real and apparent directions of the star is called the *aberration.*

(2). The angle between the real direction of a star and the direction of the earth's motion is called the *earth's way.* Thus the $\angle\ SOA$ or the $\angle\ E$ is the earth's way.

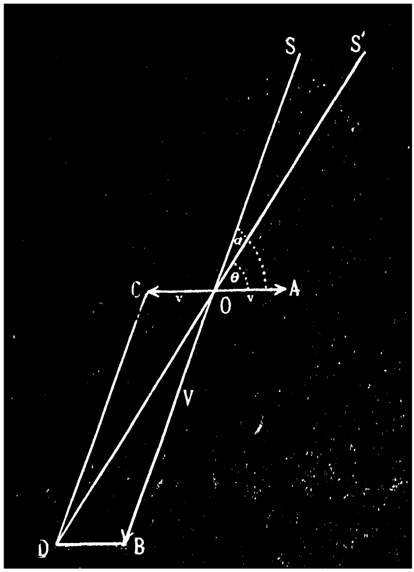

FIG. 61.

From the above it is seen that *the effect of aberration is to displace a star in the direction of the earth's motion.* As the direction of the earth's motion, being a tangent to its orbit, must be at right angles to the direction of the sun; therefore, at any instant the earth seems to be moving to a point on the ecliptic 90° behind the sun; and the displacement of each star in the heavens, owing to aberration, takes place along the great circle joining its position on the celestial sphere to this point.

Since the sun's apparent motion in the ecliptic is from west to east, and as the longitudes of all heavenly bodies are measured from Aries in the same direction, hence the point on the ecliptic 90° behind the sun is that point whose longitute is less than that of the sun by 90°. Thus if the sun's longitude be 120°, all stars will aberrate towards that point of the ecliptic whose longitude is 30°.

Aberration Varies as the sine of the Earth's Way.

109. We have, in the triangle OCD,

$$\frac{\sin CDO}{\sin COD} = \frac{OC}{CD} = \frac{v}{V} = K,$$

or
$$\frac{\sin \alpha}{\sin \beta} = K,$$

$$\therefore \quad \sin \alpha = K \sin \beta;$$

but α being small, $\sin \alpha = \alpha$ (in circular measure), and β may be taken equal to the angle E or the earth's way, as they differ by a very small amount;

$$\therefore \quad \alpha = \text{aberration} = K \sin E.$$

K is called the *coefficient of aberration*, and, when expressed in circular measure, may be defined as the ratio of the velocity of the earth to that of light.

If the aberration be expressed in seconds, we have

$$\frac{a''}{206265} = \frac{v}{V} \sin E$$

$$= \frac{1}{10000} \sin E; \text{ (Art. 106)}$$

$$\therefore \quad a'' = 20''{\cdot}6 \sin E \text{ nearly.}$$

Therefore, the coefficient of aberration expressed in seconds is about $20''{\cdot}6$, a more accurate value being $20''{\cdot}49$. It is evident that the aberration is a maximum when the earth's way $= 90°$;

$$\therefore \quad \text{maximum aberration} = 20''{\cdot}49 \sin 90° = 20''{\cdot}49.$$

EXAMPLE.

A star in the ecliptic has a longitude of 75°, obtain the change in the position of the star owing to aberration, when the longitude of the sun is 135°, assuming the constant of aberration to be $20''{\cdot}49$.

Here the angular distance of star from sun $= 135° - 75° = 60°$, $\therefore$ the earth's way $= 30°$ since the direction of earth's motion is at right angles to direction of sun;

$$\therefore \quad a = K \sin E = 20''{\cdot}49 \sin 30°$$
$$= 10''{\cdot}245.$$

110. The effect of aberration is to cause each star to appear to describe a small ellipse round its true position in

the course of a year. This can be shown in somewhat the same way as in the case of annual parallax. For we may regard the earth's orbit as being approximately a circle, and that its velocity throughout the year is uniform. We may therefore suppose each star to move in a circle, parallel to the earth's orbit, round its true position as centre. But when this imaginary circle is projected obliquely on the surface of the celestial sphere it becomes an ellipse of which the semi-axis major is parallel to the ecliptic, and equal to 20″·49 (the maximum aberration), the semi-axis minor being 20″·49 sin l, when l is the latitude of the star. Therefore, summing up, we have—

(1) *Each star aberrates towards a point on the ecliptic* 90° *behind the sun.*

(2) *The displacement varies as the sine of the earth's way.*

(3) *A star situated at the pole of the ecliptic (that is, with latitude* = 90°) *will, in the course of a year, appear to revolve round its true position in a circle whose angular radius is* 20″·49.

(4) *A star situated on the ecliptic (that is, with zero latitude) will appear to oscillate through an arc on the ecliptic of* 20″·49 *on either side of its true position, the total annual displacement being* 40″·9.

(5) *In general, a star whose latitude is l will, throughout the year, appear to describe a small ellipse round its true position as centre, the semi-axis major being* 20″·49, *and parallel to the ecliptic and the semi-axis minor* 20″·49 *sin l.*

The student will easily see that these results differ considerably from those obtained in the case of annual parallax, although they have some points of similarity. For the annual parallax of a star depends on its distance from us, whereas the constant of aberration is the same for all stars, irrespective of their distances. Also in the particular case, when a star is either in the same part of the celestial sphere as the sun or on the diametrically opposite point, the annual parallax is zero, but the aberration is a maximum. The displacement due to parallax takes place towards the sun, and that due to aberration towards a point on the ecliptic 90° behind the sun.

111. The aberration of a planet differs somewhat from that of a star, being due to two causes—(1) That due to the velocity of the earth, and (2) to the velocity of the planet. If the planet's motion were equal to that of the earth, and in the same direction, there would be no aberration. In general, it is easy to calculate the aberration due to these two causes separately.

As the velocity of the moon about the earth is very small compared with the velocity of light, we may regard the aberration due to this velocity as zero. Neither is there any aberration on account of the earth's orbital motion round the sun, for this motion is shared in by the moon. We may, therefore, regard the moon as having practically no aberration.

Discovery of Aberration.—Bradley was first led to the discovery of aberration while attempting to find the annual parallax of γ Draconis. Observing that the latitude of this star was subject to small annual variations for which he could not account by attributing them to any known cause, he was eventually led to adopt the above explanation.

112. **Diurnal Aberration.**—Owing to the earth's rotation on its axis, a point on the equator turns through 25,000 miles in $23^h\ 56^m$. This is at the rate of $\frac{3}{10}$th of a mile per second, or $\frac{1}{60}$th of the velocity of the earth in its orbit. Any other point on the earth not on the equator will have, of course, a less velocity than this.

The aberration due to this motion is called *diurnal aberration.* It is, however, as we can easily see by comparing the above velocity of rotation with that of light, so small as to be almost inappreciable.

CHAPTER IX.

THE MOON.

113. Next to the sun the moon is to us the most important of all the heavenly bodies. Besides its diurnal motion from east to west, which is imparted to it in common with all the other heavenly bodies in consequence of the rotation of the earth on its axis, it has, like the sun, a motion among the fixed stars in the opposite direction, making a complete revolution of the heavens in about $27^d\ 7^h\ 43^m$. As the sun appears to make a complete revolution of the ecliptic in one year, we see that the moon's motion among the fixed stars is about thirteen times faster than that of the sun. So rapid is this motion, that its change of position with respect to bright stars in its neighbourhood can be easily seen, even after as short an interval as two or three hours.

The moon's path, on being mapped out on a celestial globe, is found to be represented by a great circle, cutting the ecliptic at an angle of $5° 9'$, from which it follows that, like the planets, it is always to be found near the ecliptic, its north or south latitude never exceeding $5° 9'$.

The moon's motion among the fixed stars is due to an orbital motion round the earth. In fact, the moon is the earth's satellite. We must not, however, suppose that its orbit round the earth is a circle, because the projection of this orbit on the celestial sphere, on being traced out, is represented by a great circle. No, for just as in the case of the sun, we find that the moon's distance from the earth is not constant. We are led to this conclusion by the fact that its angular diameter, on being measured at different times by means of a micrometer, is found to undergo periodic changes, which shows that its distance from the earth must be changing also, being least when the apparent diameter is greatest. Its greatest angular diameter is 33½′; least 29½′, and the mean 31½′, or a little more than half a degree. These changes in the apparent angular diameter lead us to the conclusion—(1) that the moon's orbit round the earth is approximately elliptic with the centre of the earth situated in one of the foci, (2) the radius vector joining the centres of the earth and moon sweeps out equal areas in equal times.

From this we might infer that the moon's motion among the fixed stars is not uniform In fact, it varies from a maximum of 33′ 40″ per hour to a minimum of 27′, its mean hourly velocity being 32′ 56″. So that we may say that the moon in its motion among the fixed stars moves through an arc equal to its own diameter in one hour.

The mean distance of the moon from the earth is 238,000 miles, or about 60 times the earth's radius. As this distance is much less than the radius of the sun, which is 110 times the radius of the earth (Art. 44), we see that if the sun were placed with its centre at the centre of the earth its mass would extend considerably beyond the moon, a consideration which will perhaps enable the mind to form some idea of the magnitude of the body which forms the centre of our system.

The Moon's Phases

114. One of the most interesting phenomena to be seen in the heavens is the series of changes which the visible portion of the moon's illuminated surface presents during its orbital motion about the earth. These appearances are called its phases. They prove that the moon is an opaque spherical body deriving its light from the sun. As only one hemisphere of the moon can be illuminated at once, viz. that half which is turned towards the sun, an observer will therefore see a variable amount of this bright surface depending on the relative positions of the sun, moon, and earth.

Let *ACMD* (fig. 62) represent the orbit of the moon, *E* the earth, and *S* the direction of the sun. In the eight positions of the moon, which we have here depicted, the line *mn*, which is perpendicular to the direction of the sun,

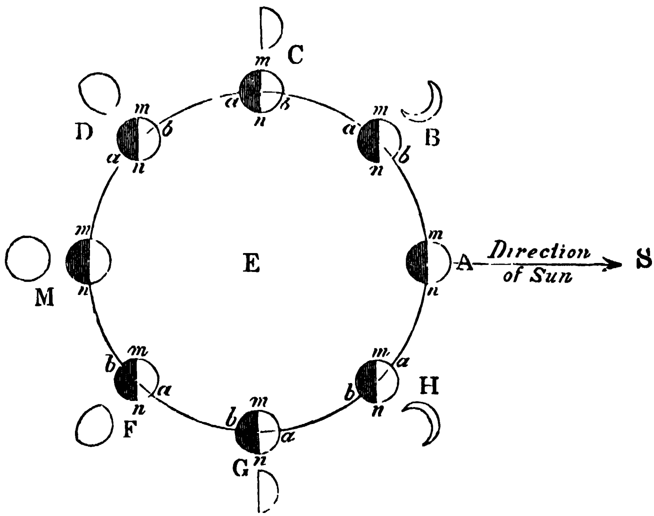

FIG 62.

separates the illuminated half of the moon from the unilluminated half, and all the positions of *mn* are drawn as if parallel to one another, the sun being so far distant. The line *ab* may be taken as separating the half of the moon which is turned towards the observer from that which is turned away from him.

When the moon is in conjunction at *A*, its dark hemisphere is turned towards the earth, and no portion is visible to the observer. It is then said to be *new moon.*

Some four or five days afterwards, when the moon is at *B*, the observer will see a small portion of the illuminated surface which will appear as a thin crescent in the sky, seen in the west after sunset.

When the moon is at *C*, 90° from the sun; that is, in quadrature, it will appear in the sky as a bright semicircle. This is said to be *first quarter*, and the moon is then said to be *dichotomized.*

At *D* it is *gibbous*, and when in opposition at *M*, which occurs at about 15 days after conjunction, the whole of the illuminated hemisphere is turned towards the observer. The moon will then present a complete circular disc in the sky. This is said to be *full moon.*

After full moon, these phases are repeated in reverse order, the moon being again in quadrature at *G*, which is called *third quarter*, and finally, conjunction is once more reached at *A*.

When in conjunction and opposition, the moon is said to be in *syzygy*. Its elongation from the sun is then 0° and 180°, respectively. When in quadrature at first quarter its elongation is 90°, and at third quarter 270°.

Definitions.

(1). The time taken by the moon to make a complete revolution with reference to the fixed stars is called its *periodic time* or *sidereal period*. This period is $27^d\ 7^h\ 43^m$.

(2). The interval between two successive conjunctions or oppositions, or, in other words, the time taken to make a complete revolution with reference to the sun is called the *synodic* period or a *lunation*. This period is $29\frac{1}{2}$ days, or, more accurately, 29·5305887 days.

It is obvious that if the sun had no apparent motion in the ecliptic, the synodic and sidereal periods would be exactly the same, so that the full moons would follow one another at intervals of $27^d\ 7^h$, instead of $29\frac{1}{2}$ days. But while the moon is making a complete revolution round the earth, which it does in $27^d\ 7^h$, the sun moves through an arc of about 27° on the ecliptic in the same direction (roughly at the rate of 1° daily), so that it takes the moon an additional two days to arrive at the same position relative to the sun and earth as when it started. In the above diagram illustrating the phases of the moon we have, for the sake of simplicity of explanation, supposed the sun and earth fixed, and that the moon moves with its relative velocity with respect to the sun, completing the revolution in $29\frac{1}{2}$ days.

To determine the Moon's Synodic Period.

115. We know that when an eclipse of the moon takes place, the moon must be in opposition. Therefore, if we observe the exact interval of time that elapses between the middle of two eclipses, and divide by the number of lunations between them we get the length of a single lunation or synodic period.

The mean length of a lunation can be calculated very accurately from the records of ancient eclipses. The earliest observations of eclipses of which there is an accurate account are those taken at Babylon in the years 720 and 719 B.C. The number of lunations between one of these eclipses and an eclipse at the present day being known, we are able to calculate the mean value of a lunation over a very long period of time.

To find the Moon's Sidereal Period.

116. Knowing the moon's synodic period, we are able to calculate its sidereal period, or periodic time, in the same way as in the case of a planet (Art. 67). In fig. 63 E represents the earth, the inner circle being the orbit of the moon, and the outer circle the apparent orbit of the sun about the earth. A and B are the positions of the

moon and sun at conjunction, and A' and B' their positions one day after conjunction.

Let

S = Period of sun's motion about earth = $365\frac{1}{4}$ days.

P = Moon's periodic time or sidereal period.

L = Interval between two conjunctions = $29\frac{1}{2}$ days.

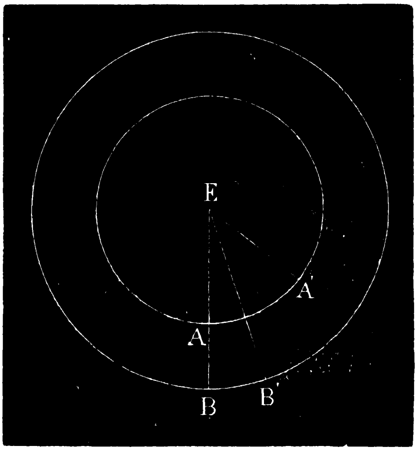

FIG. 63

$$\therefore \quad \frac{360}{P} = \angle \text{ described by moon in 1 day} = \angle AEA',$$

$$\frac{360}{S} = \angle \text{ described by sun in 1 day} = \angle BEB';$$

$$\therefore \quad \frac{360}{P} - \frac{360}{S} = \angle \text{ gained by moon in 1 day} = \angle B'EA',$$

but $$\frac{360}{L} = \angle \text{ gained by moon in 1 day, also};$$

$$\therefore \quad \frac{360}{P} - \frac{360}{S} = \frac{360}{L};$$

$$\therefore \quad \frac{1}{P} - \frac{1}{S} = \frac{1}{L},$$

or $$\frac{1}{P} - \frac{1}{365 \cdot 25} = \frac{1}{29 \cdot 5};$$

therefore, solving for P, we find the periodic time to be $27^d\ 7^h$ nearly.

A more accurate value for the periodic time is $27^d\ 7^h\ 43^m\ 11^s$, while that of a lunation is 29·5305887 days.

Metonic Cycle.

117. Meton first discovered, B.C. 433, that 19 years expressed in days is an almost exact multiple of a lunation, for $365{\cdot}25 \times 19 = 6939{\cdot}75$ and $29{\cdot}5305887 \times 235 = 6939{\cdot}688$. So that in every 19 years there are almost exactly 235 lunations. Therefore, at the end of every 19 years the sun and moon, returning to the same positions with respect to the fixed stars, all the phases of the moon will occur again on the same days of the month as for the previous 19 years, the only difference being that they will occur about one hour sooner. This is called the *Metonic Cycle*. The discovery of the Metonic cycle was of considerable importance, as it afforded a ready method of predicting the dates of the full moons, etc., without the trouble of calculation. It has been much used in order to find the date on which Easter should fall in a given year, because this festival occurs on the Sunday following the first full moon after the 21st March. For this reason, the nineteen numbers, from 1 to 19, are called the *golden numbers*. The golden number, or the number in the Metonic Cycle, for any year, is the remainder got after dividing the year increased by unity by 19. Thus the golden number for 1901 is the remainder when 1902 is divided by 19, *i. e.* 2. Where zero is the remainder, then 19 is the golden number.

Apparent area of illuminated Surface of Moon.

118. It can be shown in exactly the same way as in the case of a planet (Art. 62) that the apparent area of the bright portion turned towards the earth is proportional to

the versed sine of the exterior angle subtended at the moon by the earth and sun. Thus, if (fig. 64) *M* represent the moon, *E* the earth, and *S* the sun, the external angle at the moon is the angle a; therefore apparent area varies as versin a; but (Euclid, I. 32), $a = \beta + \theta$. Therefore the angle a is very nearly equal to β, for the moon, being so near the earth, θ is always a very small angle, being never more than 10′. Therefore the apparent area varies approximately as versin β where β is the angle of elongation of the moon from the sun. Of course this approximation is not true in the case of a planet, for its distance from the earth being so very much greater than that of the moon, we could by no means neglect the angle θ.

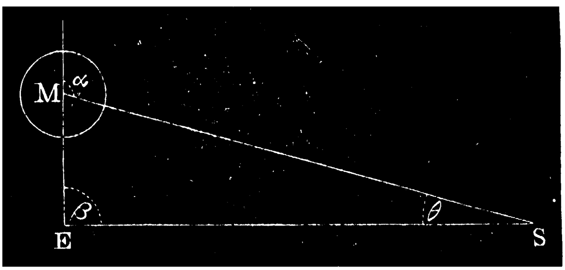

FIG. 64.

119. **Earth-shine.**—It is evident that if the earth were seen from the moon it would appear to pass through the same phases as the moon does when observed from the earth, but in inverse order. During new moon the earth, as seen from the moon, would appear full. When the moon appears as a crescent the earth would appear gibbous, and *vice versâ*. This accounts for the phenomenon which doubtless everyone has observed, that when the moon appears as a thin crescent in the sky the remainder of its surface can be seen shining with a dull grey light, caused by the *earth-shine* on the moon, which is reflected back again to the earth.

To find the Sun's distance by observing when the Moon is Dichotomized.

120. In Chapter VII. we described the different methods by which the sun's distance can be calculated. There is another method however, which, although not susceptible of the same degree of accuracy as those employed in modern times, is of great historical interest, as it was used by Aristarchus at Alexandria about 280 B.C., being the first attempt at determining the sun's distance.

The angle of elongation β (fig. 64) of the moon from the sun is observed when the moon is dichotomized, or, in other words, when the $\angle SME = 90°$.

Now, $$\cos\beta = \frac{EM}{ES},$$

and β being known, the ratio of the moon's distance to the sun's is known, from which, knowing that of the moon, the distance of the sun is determined.

It is not possible to obtain accurate results by this method, as, owing to inequalities in the moon's surface, the line which separates the bright from the dark portion is, when seen through a telescope, very uneven, so that the observer is unable to tell the exact instant when the moon is dichotomized. Aristarchus deduced by this method that the sun was 19 times more distant than the moon, instead of 400 times, which modern observations give us.

The Moon rotates round an Axis.

121. It is a remarkable circumstance in connexion with the moon that it always turns nearly the same face to the observer. The mountains and other markings which are to be seen on its surface are always to be found nearly in the same position with respect to the circumference of the moon's disc, and also

relative to the plane of its orbit. From this circumstance we are led to conclude that—

(1) The moon revolves round an axis which is nearly perpendicular to the plane of its orbit.

(2) The period of its rotation round its axis must be equal to the time of completing a revolution round the earth, viz $27^d\ 7^h$. On first thoughts it might appear to the reader as if the fact that the moon keeps the same face turned towards the earth proves that it has no rotation. The following illustration will serve to show how erroneous is such a conclusion:—Let the reader place a lamp or other body in the middle of a room, and let him proceed to walk round it in a circle so as to keep his face turned towards it all the time. Now, let us suppose that at first his face is turned towards the north, and he will find, while he is completing a circuit, that he faces in turn towards all the points of the compass. He will be looking towards the south when he has moved through a semicircle, and will again face the north when he arrives at the point from which he started. In other words, in order to keep his face turned constantly towards the lamp he will have to rotate his body through 360° for every circuit he makes. So it is in the case of the moon's revolution round the earth.

Moon's Librations—Libration in Latitude.

122. The axis of rotation of the moon is not quite perpendicular to the plane of its orbit, being inclined to it at an angle of $83\frac{1}{2}°$, or about $6\frac{1}{2}°$ to the perpendicular. Therefore while the moon revolves about the earth its north and south poles are alternately turned slightly towards or from the observer. At one part of its orbit we see about $6\frac{1}{2}°$ beyond the north pole, and at another time about $6\frac{1}{2}°$ beyond the south pole. This phenomenon is called *libration in latitude.*

Libration in Longitude.

We have seen that the period of the moon's rotation on its axis is equal to the time taken to go round the earth. But its motion round the earth is not uniform, as, owing to the elliptic form of its orbit, its distance from the earth is not constant. On the other hand its rotation on its axis is perfectly uniform. The consequence is, that although the two periods of making a complete revolution are the same for each, still at one time we are able to see a little more of the eastern side, and at another time a little more of the western side. This is called *libration in longitude.* Its maximum amount is 7° 45′.

Diurnal Libration.

There is also a *diurnal libration* which is really due to parallax. For, from the time the moon rises until it sets, the observer, on account of the rotation of the earth, has changed his point of observation, and therefore he does not in each case see exactly the same face. When the moon is rising in the east he sees a little more of its western side, and when setting in the west, a little more of its eastern side, than when it is high up in the sky crossing the meridian.

The total effect of these librations is such that we are at different times able to see a total of about 59 per cent. of the moon's surface instead of about 50 per cent.

Path of the Moon round the Sun.

123. We have seen that the moon's orbit relative to the earth is an ellipse, but, as it also follows the earth in its motion round the sun, we see that the path of our satellite round the sun is due to a combination of two motions, a monthly motion about the earth, and a yearly motion about the sun. If we neglect the small angle at which its orbit cuts the plane of the ecliptic, and assume them both in the same plane, the moon's path may be represented by the

dotted line in the adjoining figure, going alternately inside and outside the orbit of the earth AEE', and crossing it about 25 times in the course of the year. M represents the moon, and E the earth at new moon, while M' and E' would be their positions at the next full moon after an interval of about a fortnight. It is to be observed that this path of the moon is always concave to the sun.

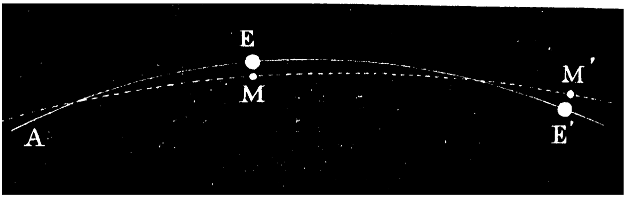

FIG. 65.

More Moonlight in Winter than in Summer

The moon, when full, being in opposition, must be at almost the diametrically opposite point of the celestial sphere to that in which the sun is situated. Hence at full moon at midsummer the sun's declination being north the moon must have an equal southern declination, and therefore remains but a short time above the horizon (Art. 20). Again, at full moon during midwinter the conditions are reversed, the sun's declination is south, and the moon's north; hence we have the moon a long time above the horizon. This happens just when the days are shortest and we are most in need of light.

Moon's Retardation.

124. The moon moves from west to east *with reference to the sun* through 360° every $29\frac{1}{2}$ days, or about $12\frac{1}{2}°$ daily. Therefore its time of rising will be later and later each night by an interval whose mean value is about 50 minutes.* This *retardation*, as it is called, of moonrise is not by any means uniform throughout the year; it may be as great as 1 hour 16 minutes or as small as 17 minutes.

* Since 15° correspond to 1 hour, $12\frac{1}{2}°$ are equivalent to 50 minutes.

Harvest Moon.

125. At the full moon nearest the autumnal equinox it is found that the retardation in the time at which the moon rises is less than at any other full moon throughout the year Therefore for several nights in succession the moon will rise very shortly after sunset, rising on the night of full moon, as it always does, at sunset. As this happens when the farmers are getting in their crops, thus enabling them to prolong their work into the night, it is called the *Harvest Moon.*

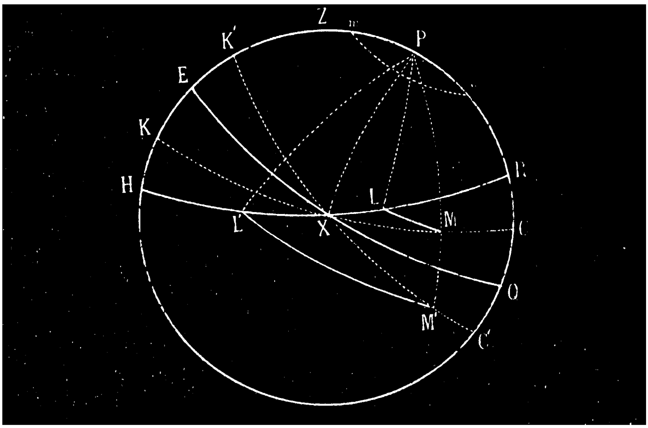

FIG. 65A.

In order to explain this phenomenon more clearly we shall suppose that the moon's path is along the ecliptic instead of being inclined to it at a small angle, and that it moves uniformly in the ecliptic at the rate of 13½° daily (360° in $27^d\ 7^h$), and we may remark that, owing to the apparent diurnal revolution of the heavenly bodies the angle at which the ecliptic cuts the horizon is continually changing, its greatest and least values being colat + 23° 28′ and colat − 23° 28′ respectively. The reason of this is that the pole of the ecliptic, which, in its diurnal motion, describes a small circle *mn* (fig. 65A) of angular radius 23° 28′ round the celestial pole, is closest to the zenith at m when $Zm = ZP - Pm$ = colat $- 23° 28'$, and at its greatest zenith distance at n when Zn = colat $+ 23° 28'$. Hence the angle between the ecliptic and horizon (being equal to the angular distance between their poles) must vary between the same limits, being least when ♈ is rising at the east point X when the ecliptic KC passes between the horizon and equator, the order being *horizon*, *ecliptic*, *equator*; and greatest when ♎ is at X when the ecliptic takes the position $K'C'$, the order being *horizon*, *equator*, *ecliptic*.

During the full moon nearest the autumnal equinox the sun is in Libra, and the moon, being in opposition, is in Aries, crossing from the south to the north side of the equator; the moon will therefore rise at X, when the ecliptic KC is at its smallest inclination (colat $- 23° 28'$) to the horizon. But after an interval of $23^h 56^m$ when, owing to the diurnal revolution of the celestial sphere, the point X returns to the same position as on the previous night, the moon will have moved about 13°. Hence, if XM be cut off on the ecliptic equal to the moon's daily rate, and through M an arc of a small circle ML be drawn parallel to the equator, the moon, on the night following full moon, will rise at L, the amount of retardation being measured by the arc of the small circle LM or by the angle LPM. This retardation, expressed in time, is found to be in our latitudes only $18\frac{1}{2}$ minutes, or $14\frac{1}{2}$ minutes with reference to the sun (allowing for the sun's daily retardation of 4 minutes). This phenomenon of course occurs each time the moon is in Aries or, in other words, every month; but it is only during the harvest that the moon is in Aries and full at the same time.

Similarly we might show that the reverse occurs when the sun is in Aries and the moon in Libra, the daily retardation being then a maximum. For when the moon is rising, the ecliptic takes the position $K'C'$, cutting the horizon at the greatest angle possible (colat $+ 23° 28'$): if XM' be now cut off equal to the moon's daily rate of motion, and a parallel $M'L'$ be drawn to the equator, the moon, on the following night, will rise at L', the retardation being measured by the arc of the small circle $M'L'$ or by the angle $M'PL'$ (the arc PM when produced passing through M'). This, expressed in time, corresponds to a retardation of about $1^h\ 10^m$ with reference to the fixed stars or $1^h\ 6^m$ with reference to the sun. The retardation is thus a maximum each month when the moon is in Libra, and therefore is a maximum during the full moon nearest the vernal equinox. It will, however, be a minimum for observers in the southern hemisphere, and to them the full moon at this period is a harvest moon.

At the arctic circle there is even an acceleration in the time at which the moon rises on successive nights when the moon is in Aries. For, when Aries is rising, the ecliptic actually coincides with the horizon (Ex. 5, page 32), and the distance ML (fig. 65A) vanishes, and therefore the interval between two successive risings is only $23^h\ 56^m$. So that, as measured by solar time, at the arctic circle, the moon after passing through Aries actually rises four minutes earlier than she did on the previous night.

During the October full moon the same phenomena occur, but in a less marked degree. This moon is called the Hunter's Moon.

Revolution of the Moon's Nodes.

126. The moon's nodes are not fixed points, but have a retrograde motion along the ecliptic at the rate of about 19° each year, completing a revolution in about $18\frac{2}{3}$ years. This backward motion is similar to that of the equinoctial points ♈ and ♎, but is very much more rapid, as the period for the precession of the equinoxes is about 26,000 years (Art. 103). The moon's motion is therefore very complicated, moving, as it appears to do, in a circle which is inclined to the ecliptic at an angle of 5°, at the rate of rather more than half a

degree each hour, while the plane of this circle is carried backwards on the ecliptic at the rate of 19° each year, or about 8″ an hour.

127. **Synodic Revolution of the Moon's Nodes.**—We have just seen that the sidereal period of the revolution of the moon's nodes is $18\frac{2}{3}$ years. The synodic period of revolution, *i.e.* the time taken to return from any position to the same position again with respect to the sun and earth, can now be calculated in the same way as that of a planet, for—

$$\frac{360^\circ}{365{\cdot}25} = \text{arc traversed by sun in 1 day,}$$

and
$$\frac{360^\circ}{18\frac{2}{3} \times 365{\cdot}25} = \text{arc traversed by node in 1 day;}$$

$$\therefore \quad \frac{360^\circ}{365{\cdot}25} + \frac{360^\circ}{18\frac{2}{3} \times 365{\cdot}25} = \frac{360^\circ}{T} \quad \text{(see Art. 67),}$$

where T represents the synodic period. The plus sign is taken on the left-hand side of the equation, as the relative velocity of the sun and node is the sum of their angular velocities, the motion of the node being retrograde.

On solving the above equation, T is found to be 346·62 days, which is the synodic period.

The line of apsides of the moon's orbit is not fixed, but, like that of the earth's orbit, it has a slow progressive motion, making a complete revolution of the moon's orbit in about nine years, the period in the case of the earth being about 108,000 years.

To find the Height of a Lunar Mountain.

128. It has been known, from the time of Galileo, that the surface of the moon is covered with mountains. Some of these mountains have been calculated to rise to heights of four or five miles above the surface of the surrounding plains, which shows that, considering the smaller size of the moon, its mountains are comparatively much

more lofty than those of the earth. When the sun shines obliquely on the mountains, they cast long shadows over the surrounding plains on the side remote from the sun in exactly the same way as we are familiar with on the earth. Also, wherever there is a high mountain, its topmost peak catches the first rays of the rising and the last of the setting sun, when all the surrounding parts are still in complete darkness. Small points of light are for this reason sometimes seen on the dark portion of the moon's disc, separated by a measurable distance from the line of separation of light and darkness.

129. **First Method.**—The method employed by Messrs. Beer and Mädler in 1837 for finding the height of a lunar mountain consists in measuring, by means of a micrometer, the length of the shadow cast by the mountain when illuminated by the sun's rays. By comparing this angular measurement with the angle subtended by the moon's diameter, the length of the shadow in miles can be found (for the moon's diameter in miles is known); from which, knowing the inclination of the sun's rays, the height of the mountain can be determined, just as the height of a tower on the earth can be found by knowing the length of its shadow and the altitude of the sun. In applying this method, allowance must be made for the effect of foreshortening, as the shadow being generally viewed obliquely, the micrometer measures merely the projection of its actual length on a plane perpendicular to the line of vision.

130. **Second Method.**—This method consists in measuring, by means of a micrometer, the angular distance between the bright summit of the mountain-top appearing on the dark portion of the moon's disc and the line of separation of light and darkness. This measurement is made in a direction perpendicular to the line joining the extremities of the horns, and therefore parallel to the plane of the moon, earth, and sun.

Let AB represent the line of separation of light and darkness on the moon, and P the top of a mountain when just illuminated by the ray PBS, the line PS being perpendicular to AB, and touching the moon's surface at B. Let E be the earth, and ES' the direction of the sun, as

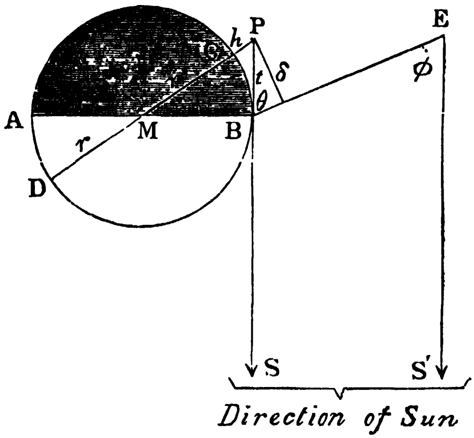

FIG. 66.

seen from E, which may be taken as parallel to PS, on account of the sun's great distance. The radius of the moon is denoted by r, and the height PC of the mountain by h; then by Euclid (III. 36),

$$PD \times PC = PB^2,$$

or $$(2r + h)\, h = t^2 \text{ (see fig. 66)};$$

that is, $$2rh + h^2 = t^2.$$

But h being very small compared with r, its square may be neglected;

$$\therefore \quad 2rh = t^2 \text{ or } h = \frac{t^2}{2r}.$$

Now the distance t is not measured directly; what is actually measured being the projection of t on a plane perpendicular to the line of sight, viz. the perpendicular δ let fall from P on BE (fig. 66).

$$\text{But} \quad \frac{\delta}{t} = \sin\theta = \sin\phi \text{ (by parallel lines)};$$

$$\therefore \quad t = \frac{\delta}{\sin\phi}.$$

But the angle ϕ is known, being the angle of elongation of the moon from the sun as seen from the earth; therefore, t is known. Substituting this value of t, we have—

$$\text{Height of mountain } h = \frac{\delta^2}{2r\sin^2\phi}.$$

131. **Lunar Craters.**—Perhaps the most striking objects to be seen in lunar landscapes are what are to all appearances enormous craters of what were once volcanoes, but which are now probably quite extinct. The typical lunar crater consists of an immense circular plain surrounded by a high wall or rampart. In the centre of the plain there generally rises a mountain, or sometimes more than one. Among the most characteristic of these craters are Tycho, having a diameter of fifty-four miles, and Archimedes, whose diameter is sixty miles. Another immense crater is Schickard, with a diameter of over 130 miles, and a surrounding wall, which, in some parts, attains a height of 10,000 feet. It has been pointed out that an observer situated in the centre of this walled space would think himself in the midst of a boundless desert, for, on account of the curvature of the moon's surface, the summit of the lofty surrounding wall would be altogether beneath his horizon.

132. **Lunar Atmosphere.**—All observers of the moon have come to the conclusion that it either possesses no atmosphere at all, or, if any such gaseous covering exist, that it is of very extreme tenuity indeed. No change is observed in the intensity of the light from a fixed star as it approaches

the dark edge of the moon, such as there would be were there any appreciable thickness of atmosphere for its rays to penetrate. Also, when the moon passes between the observer and a fixed star, the observed time during which the occultation of the star lasts is found not to be less than the calculated time, as would be the case if the moon had an atmosphere of any considerable density; for the star would still be visible for some time after being actually covered by the moon, owing to its rays being refracted in their passage through the lunar atmosphere, if such existed, just as, owing to refraction by the earth's atmosphere, the sun remains visible to us for some time after he has sunk below our horizon.

In addition to having no atmosphere, astronomers have not been able to detect water in any form on the moon's surface, which renders the existence of life, such as is known to us, altogether impossible.

CHAPTER X.

ECLIPSES.

133. Eclipses are of two kinds, *lunar* and *solar*.

Lunar Eclipses.—A lunar eclipse is caused by the passage of the moon through the shadow of the earth. This can only happen when the earth is between the sun and moon, or, in other words, when the moon is in opposition.

If the plane of the moon's orbit coincided with the plane of the ecliptic instead of being inclined to it at an angle of about 5°, we should have an eclipse of the moon at every opposition. However, on account of the above angle of inclination of its orbit, it generally happens that the moon, when in opposition, is either so far above or below the plane of the ecliptic that it fails to pass through the shadow of the earth. So we see that, in order that an eclipse may take place, the moon must be very nearly in the ecliptic, that is, at or near one of its nodes. Therefore, the conditions for a lunar eclipse are :—

(1) The moon must be in opposition, *i.e.* full.

(2) It must be at, or near, one of its nodes.

There are two kinds of lunar eclipse, *total* and *partial*. It is total when the whole surface of the moon passes through the shadow, and partial when only part of its surface is involved.

134. Let S and E (fig. 67) represent the centres of the sun and earth, respectively. Draw a pair of direct common tangents AB and CD to the sun and earth, meeting SE

produced in *V*, and a transverse pair *AD* and *BC* meeting *SE* in X. If these lines be now supposed to revolve round *SE* as axis they will generate cones, and there is thus a conical shadow *BVD*, having *V* as vertex, into which no direct ray from the sun can enter. This conical space is called the *umbra*.

The spaces represented by *VBL* and *VDN* form what is called the *penumbra*, from which only part of the light of the sun is excluded. It is to be remembered that the passage of the moon through the penumbra does not give rise to any eclipse, but only to a diminution of brightness.

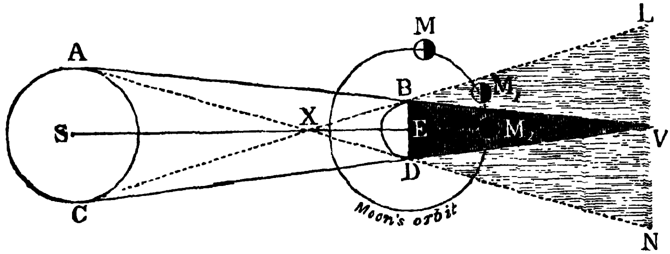

FIG. 67.

Thus the moon when at M_1 (fig. 67) receives light from portions of the sun next *A*, but rays from parts near *C* will not reach the moon, owing to the interposition of the earth; consequently, the brightness of the moon is somewhat diminished, the diminution being greater the nearer the moon approaches the edge of the umbra. An eclipse, properly so-called, however, does not commence until a portion of the moon's surface shall have entered the umbra.

Phenomena due to Refraction.

135. As everyone who has seen a total eclipse is aware, the moon appears of a dull-red or brownish colour. It must, therefore, receive light from some source. That it is not due to earth-shine (Art. 119) is certain, for the moon

being in opposition, the dark hemisphere of the earth is turned towards it. The phenomenon is caused by the refracting power of the earth's atmosphere, owing to which those rays from the sun, which nearly touch the earth, are bent round, and thus reach the moon's surface.

Another curious phenomenon, due to refraction, is seen when an eclipse occurs at sunset or sunrise; for both the sun and moon being elevated by refraction, it is possible to see the moon eclipsed when the sun still appears shining in the heavens, a phenomenon which was observed in 1666, 1668, and 1750.

N.B.—Throughout the remainder of this Chapter we shall occasionally denote the sun by the symbol ☉, and the moon by ☾.

To find the Diameter of the Section of the Earth's Shadow where the Moon crosses it.

136. The angular diameter of the cross-section of the cone of shadow is represented by the arc MN. Let the semiangle MEV subtended by MN at the centre of the earth be α (fig. 68).

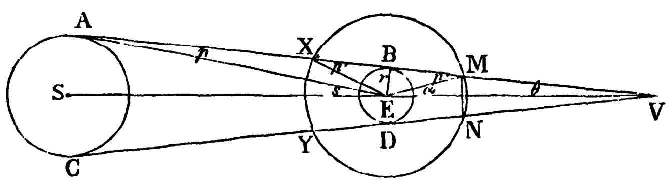

FIG. 68.

Let p = ☉'s hor. parallax = $\angle EAX$ (fig. 68).

p' = ☾'s hor. parallax = $\angle EMB$ or $\angle EXB$.

s = angle subtended by ☉'s semidiameter at $E = \angle SEA$.

$\theta = \angle EVB$, the semiangle of cone of shadow.

Now by Euclid (I. 32) we have:—

$$a + \theta = p'; \quad \therefore \; a = p' - \theta.$$

For the same reason $\theta = s - p$;

$$\therefore \; a = p' - (s - p) = p' + p - s.$$

But p, p', and s are known; therefore $2a$, the angle subtended by MN at E, is determined.

If the moon's horizontal parallax be taken as 57′, the sun's as 8″, and the sun's semidiameter as 16′, the breadth of the shadow $2a$, or $2\,(p' + p - s)$ is found to be about 82′.* As the moon's angular diameter has a mean value of about half a degree, or 30′, we see that the breadth of the section of the shadow at the distance of the moon is nearly three diameters of the moon; and since the moon moves through an arc equal to its own diameter in about an hour (Art. 113), we see that when the moon passes through the axis of the shadow, that is, when the eclipse is *central*, it may remain totally eclipsed for about two hours.

137. In the above we see that the semiangle θ of the cone

$$= s - p = (\odot\text{'s semidiam.}) - (\odot\text{'s hor. parallax}).$$

In the same way as the breadth of the section of the cone at MN has been found, it can be shown that the semidiameter of the section at XY, where the moon crosses it when in conjunction, is equal to $p' - p + s$. For, by Euclid (I. 32),

$$\angle XES = p' + \theta = p' - p + s.$$

To find the Length of the Earth's Shadow.

138. The distance EV (fig. 68) from the centre of the earth to the vertex of the cone is called the length or height of the earth's shadow. Its magnitude can now be found,

---

* Or to be more accurate, the breadth of the shadow varies from a maximum of 89′ 14″ to a minimum value of 75′ 38″, the maximum value being reached when the moon is nearest the earth (perigee) and the earth at the same time farthest from the sun (aphelion), the minimum value being attained when these conditions are reversed.

knowing the earth's radius and the semiangle θ of the cone. For, since the angle θ is so small (being equal to $s - p$), we may assume that r, the radius of the earth, coincides with an arc of the circle whose centre is V and radius VE (fig. 68). Therefore it follows at once from circular measure that

$$\frac{\theta''}{206265''} = \frac{r}{EV};$$

$$\therefore\ EV = \frac{206265''\,r}{\theta''} = \frac{206265''\,r}{s'' - p''}.$$

Taking r, the radius of the earth, roughly as 4000 miles, s the semidiameter of the sun as $16'$ or $960''$, and p, the sun's parallax as $8''$, we have

$$EV = \frac{206265 \times 4000}{960 - 8} \text{ miles}$$

$$= \text{about } 860{,}000 \text{ miles,}$$

or 215 times the earth's radius.

Since the moon's distance from the earth is only about sixty times the earth's radius, we see that the moon's orbit extends for a much less distance from the earth than the length of the cone of shadow, and therefore a lunar eclipse must happen if the moon is at one of its nodes and full at the same time.

139. **Solar Eclipses.**—An eclipse of the sun is caused by the interposition of the moon between the sun and the observer. As in the case of a lunar eclipse the moon must be nearly in the plane of the ecliptic. The two conditions for a solar eclipse are therefore :—

(1) The moon must be in conjunction, *i.e.* it must be new moon.

(2) It must be at, or near, one of its nodes.

In a lunar eclipse, the moon, on entering the umbra, loses its light, and consequently the eclipse is visible from any part of the hemisphere of the earth which is turned towards the moon. On the other hand, in the case of a solar eclipse, the light of the sun is merely hidden from the observer; and the moon being much smaller than the earth, this shows that a solar eclipse can only be visible over a very limited area at the same time.

There may be an eclipse of the sun visible from some portion of the earth if any part of the moon come within the arc *XY* (fig. 68); and there may be a lunar eclipse if it enter *MN*. Since the arc *XY* is greater than *MN* we should expect that more solar eclipses should occur than lunar if we count the eclipses observed over the whole earth; and this in fact is the case; but, as we have just seen, an eclipse of the sun is only visible over a very limited area of the earth, and therefore it happens that there are more lunar than solar eclipses seen from any particular place.

140. There are three kinds of solar eclipses—(1) total; (2) annular; and (3) partial. When the eclipse is total the whole of the sun's disc is hidden from view, whereas in the case of an annular eclipse only the central portion is darkened, with a bright ring surrounding it.

In order to arrive at a clear idea as to how the moon, coming between the sun and the observer, can sometimes hide the whole of the sun's surface from view, and at other times only the central portion, it should be borne in mind that the moon's apparent angular diameter is not constant, for, on account of its elliptic orbit its distance from the earth is variable (Art. 113); the apparent diameter of the sun also varies, the mean values of both being very nearly equal. The moon's angular diameter varies from 33′ 22″ when nearest the earth (perigee) to 28′ 48″ when at its greatest distance (apogee), and in the case of the sun the variation is from 32′ 36″ to 31′ 32″. When the moon's apparent diameter is greater than that of the sun, which occurs when it is closest to the earth, it will hide the whole of the sun's surface from the view of an observer situated on the line of centres of the two bodies, causing thus a total eclipse. When, however, the moon's apparent diameter is less than that of the sun, as it is when at its greatest distance from the earth, it will, under the same conditions, hide only the central portion of the sun, giving rise to an annular eclipse.

A very simple experiment will render the above explanation perfectly clear. If the reader take a coin, and, closing one eye, hold it in such a position before the other eye as to just completely hide the sun's surface from view, the position of the coin now is similar to that occupied by the moon when totally eclipsing the sun. If, however, the coin be removed to a greater distance from the eye, keeping its centre still in a direct line with that of the sun, it will be found, owing to the diminution in its apparent diameter, due to the increase of distance, that it only hides the central portion of the sun from view, thus illustrating how the moon, when farthest from the earth, causes an annular eclipse.

When a partial eclipse takes place, only a portion of the sun's disc at one side becomes darkened, owing to the centres of the two bodies not being in a direct line with the observer. It is evident that all total and annular eclipses must begin and end as partial eclipses.

To find the Length of the Cone of Shadow cast by the Moon.

141. Let S denote the centre of the sun, M of the moon, and R and r the radii of the sun and moon respectively (fig. 69). The apex of the shadow cast by the moon will be at O where the common tangents CH and DF meet. It is required to find the distance MO.

Since the triangles OSC and OMH are similar we have, by Euclid (VI. 4),

$$\frac{OS}{OM} = \frac{R}{r}, \text{ that is, } \frac{OM + SM}{OM} = \frac{R}{r};$$

therefore solving for OM and denoting SM by d we have

$$OM = \frac{rd}{R - r}.$$

But r, the radius of the moon, is about 1076 miles, while d, the distance between the centres of the sun and moon varies from 11,717 to 11,713 times the earth's diameter. Substituting these values we find that OM varies from 28·94 to 28·93 times the earth's diameter.

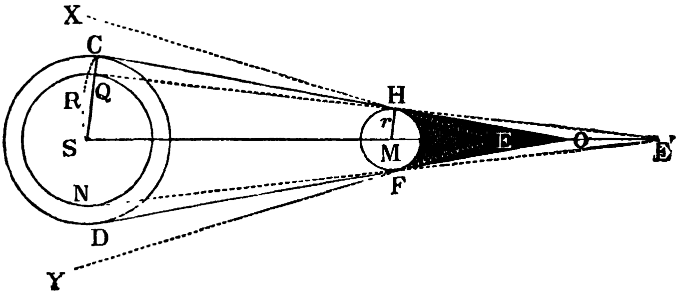

FIG. 69.

Since the distance from the moon's centre to the surface of the earth varies from 28 to 31 diameters of the earth, it follows that the observer may sometimes be situated at E (fig. 69) inside the cone of shadow, and at other times at E', beyond the point O, where the cone tapers to a vertex. In the former case a total eclipse takes place, the moon subtending a greater angle than the sun, and in the latter case an annular eclipse occurs, the portion of the sun's surface hidden from view being represented by the inner circle QN (fig. 69), marked off by tangents drawn from E' to the surface of the moon, and produced out to meet the sun.

To calculate the Conditions for a Lunar or Solar Eclipse.

142. **Lunar Eclipse.**—Let O (fig. 70) represent the centre of the section of the earth's shadow at the distance of the moon, and M the centre of the moon when touching the shadow externally. MN represents the apparent path of the moon, NO the ecliptic, and N the position of the node.

Now it is evident that an eclipse of no portion of the moon's surface can take place unless the distance between the centres of the moon and shadow becomes less than MO.

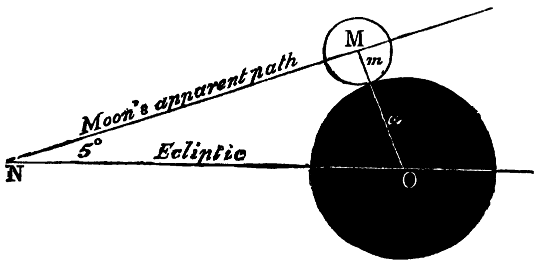

FIG. 70.

But MO = (semidiam. of shadow) + (semidiam. of moon)
$= a + m$.

But $a = p' + p - s$ (Art. 136);

$\therefore \quad MO = p' + p - s + m,$

where p = ☉'s hor. parallax = 8″,
p' = ☾'s hor. parallax = 57′,
s = ☉'s semidiam. = 16′ (mean value),
m = ☾'s semidiam. = 15′ (mean value).

Therefore, we have

$$MO = 57' + 8'' - 16' + 15' = 56' \text{ (roughly)}.$$

Similarly, for a *total* lunar eclipse the moon will, in the limiting position, touch the shadow *internally*, and we shall have—

$$MO = \text{(radius of shadow)} - \text{(semidiam. of moon)}$$
$$= a - m$$
$$= p' + p - s - m = 26' \text{ (roughly)}.$$

Therefore it is impossible for a lunar eclipse to occur if the distance between the centres of the moon and shadow exceeds 56′, and for a total eclipse the distance cannot exceed 26′.

143. **Solar Eclipse.**—We have seen (Art. 137) that the angular radius of the section of the cone where the moon

crosses it in conjunction at *XY* (fig. 68) is $p' - p + s$: therefore, it is evident that the limiting distance of the moon from the centre of the section for a partial eclipse of the sun is $p' - p + s + m$ or about 88′, the limiting distance for a total eclipse being $p' - p + s - m$ or 58′.

As the moon's orbit is inclined at such a small angle to the ecliptic (5°), the distance *MO* must be nearly perpendicular to the ecliptic, and therefore is almost equal to the latitude of the moon; but, as the latitude of the moon varies from 0° to 5°, we see that an eclipse can only take place very near a node.

144. **Definition.**—The greatest distance (measured along the ecliptic) of the moon from the node, when in opposition, at which an eclipse can happen is called the *Lunar Ecliptic Limit*. Thus, in fig. 71, the apparent path of the moon is represented by *MN*, the moon being taken just touching the shadow when nearly in opposition; then the distance *NO* will represent the distance, measured along the ecliptic, of the moon from the node when in opposition (*i.e.* the projection on the ecliptic of the moon's distance from the node when in opposition), and is therefore the ecliptic limit.

To find the Lunar Ecliptic Limit.

145. In order to calculate the ecliptic limit *NO*, in the

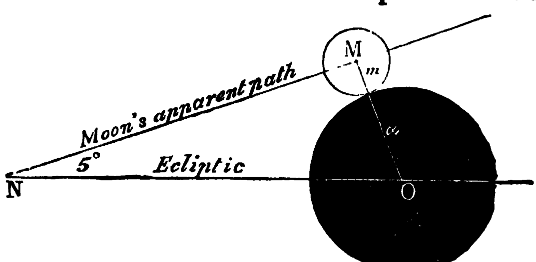

FIG. 71.

spherical triangle *MON*, the arc *MO* is known, being the sum of the semidiameters of the shadow and the moon (by

Art. 142, $MO = p' + p - s + m$); the angle N, the inclination of the moon's orbit to the ecliptic is also known, being about 5°, and the angle M is a right angle (since OM is the shortest distance from O to MN); therefore, the arc NO can be calculated.

Major and Minor Limits.—The lunar ecliptic limit is not a constant quantity, as the parallax and semidiameter of both the sun and moon are variable. Moreover, the inclination of the moon's orbit varies from 5° 20′ to 4° 57′. All these causes combine to produce considerable variations in the limit. When the moon is nearest the earth and the earth farthest from the sun, and at the same time the angle of inclination of the moon's orbit least, the circumstances are then *most favourable* for an eclipse, which may, therefore, take place at a greater distance from the node than at any other time. Under these circumstances the magnitude of ON is found to be 12° 5′, and is called the *Major Ecliptic Limit.*

On the other hand, when the moon is farthest from the earth, the earth nearest the sun, and the angle at N (fig. 71) greatest, the circumstances are *most unfavourable* for an eclipse, and the moon must be much closer to the node than in the former case, in order that an eclipse should occur. Under these conditions ON is found to be 9° 30′, and is called the *Minor Ecliptic Limit.*

When the distance of the moon from the node at opposition is within the **major limit** the eclipse **may** take place, but within the **minor limit** it **must** take place.

146. **Solar Ecliptic Limits.**—There is also an ecliptic limit for the sun, viz. the greatest distance (measured along the ecliptic) of the moon from the node, when in conjunction, consistent with a solar eclipse. The maximum and minimum values are also called the major and minor limits; the

former being 18° 31′ within which a solar eclipse **may** take place, and the latter 15° 21′ within which it **must** occur.

147. In (Art. 126) it was seen that the moon's nodes have a retrograde motion on the ecliptic, making a complete revolution in $18\frac{2}{3}$ years. From this it was proved (Art. 127), that the synodic period of revolution of the line of nodes is 346·62 days, or, in other words, we might say that the sun separates from the line of nodes through 360° in 346·62 days. Therefore, in one synodic lunar month of $29\frac{1}{2}$ days, the sun separates from a node through an angle

$$\frac{360° \times 29{\cdot}5}{346{\cdot}62} = 30° \, 38' = 30°\tfrac{2}{3} \text{ nearly.}$$

As the comparison of this result with the solar and lunar ecliptic limits enables us to calculate the frequency of eclipses, it is of importance that the student should remember the approximate values of the following quantities:—

		Major.	Minor.
Lunar ecliptic limits,	.	12°	$9°\frac{1}{2}$.
Solar ecliptic limits,	.	$18°\frac{1}{2}$	$15°\frac{1}{2}$.
Relative motion of sun	=	$30°\frac{2}{3}$ in each lunation.	
Period of sun's revolution	=	346 days.	
Period from node to node	=	173 days.	
Six lunations = $6 \times 29\frac{1}{2}$	=	177 days.	

N.B.—It is evident that during either a lunar or solar eclipse the distances of the sun and moon from the nearest node are nearly equal.

To determine the Frequency of Eclipses.

148. **Least Possible Number.**—Let N and n represent the moon's nodes (fig. 72). Cut off, on the ecliptic EC, distances NL, NL', nl, nl' each equal to the lunar ecliptic

limit; and similarly *NS*, *NS'*, *ns*, *ns'* each equal to the solar ecliptic limit. Now as the sun moves with reference to the nodes through 30°$\frac{2}{3}$ in one synodic month, it follows that he will take more than a month in moving through the arc *SS'* or *ss'*; for the least value of these arcs, being double the sun's minor limit, is 31°. Hence at least one new moon, and therefore one solar eclipse, must occur within each of these arcs.

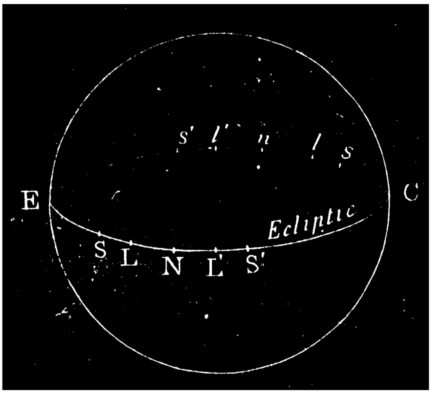

FIG. 72.

On the other hand, the least value of *LL'* or *ll'*, being twice the moon's minor limit, is only 19°, and the sun traverses each of these arcs in much less than a month (about 18 days); therefore, it is possible that there may be no full moon near either node, and therefore no lunar eclipse during the year. Hence *the least possible number of eclipses in a year is two, both of the sun.*

149. **Greatest Possible Number.**—The sun takes 173 days to pass from *N* to *n* (fig. 72), or 4 days less than six lunations (177 days); therefore, when the moon happens to be full two days before the sun arrives at *N*, there will also be a full moon 2 days after his passage through *n*, thus rendering a lunar eclipse very close to each node a certainty.

But if a lunar eclipse occur 2 days before or after the sun's passage through a node, there may also be two solar eclipses near that node, viz. at the preceding and following new moon; for, in half a lunation, or $14\frac{3}{4}$ days, the sun moves through $15^{\circ}\frac{1}{3}$; and even if to this we add the arc gone through in 2 days, the result is still well within the sun's major ecliptic limit. Therefore, there may be one lunar and two solar eclipses at each node in a period of 346 days. But if the eclipses within SS' (fig. 72) occur in January, there will be ample time before the year is completed for the sun to arrive a second time within SS'. There will now be another solar eclipse near S, followed by a lunar eclipse 6 *days after the sun's passage through* N. There will not now, however, be a solar eclipse near S', as the following new moon will take place outside the sun's major limit. In all, we have counted eight eclipses in $12\frac{1}{2}$ lunations or 368 days, viz. five of the sun and three of the moon. But all these eight eclipses cannot happen in a year (365 days); therefore one, either a solar or lunar, will have to be omitted. Hence *the greatest possible number of eclipses in a year is seven, five of the sun and two of the moon, or four of the sun and three of the moon.*

In every 18 years there are generally 41 eclipses of the sun to 29 of the moon.

Chaldean Saros.

150. The synodic period of the moon's nodes being 346·62 days, and a lunation being 29·53 days, we therefore have—

$$19 \text{ synodic revolutions of node} = 19 \times 346{\cdot}62 \text{ days} = 6585 \text{ days};$$

also $\quad 223 \text{ lunations} = 223 \times 29{\cdot}53 = 6585 \text{ days.}$

Therefore, we see that after every period of 6585 days, which are equivalent to 18 years 11 days or 18 years

10 days, according as there are four or five leap years in the interval, the sun and moon will return to nearly the same positions relative to the nodes (each having made an exact number of revolutions), and therefore the eclipses will repeat themselves in the following cycle in the same order as in the previous one. This period is called the Chaldean Saros, as, by means of it, the Chaldeans were enabled to foretell the occurrence of eclipses.

CHAPTER XI.

TIME.

Mean and Apparent Time. Equation of Time.

151. We explained in Chapter III. the difference between *sidereal* and *solar* time. The sidereal day is of constant length as the rotation of the earth on its axis is uniform. The length of the apparent solar day, however, is variable, as the sun's rate of change of right ascension is not uniform throughout the year. On account of this inequality a clock cannot be regulated to point to 12 o'clock just when the sun is in the meridian. Accordingly our clocks, instead of keeping *apparent solar time* keep *mean solar time* as indicated by the motion of an imaginary body called the *mean sun*, which is supposed to move uniformly in the equator at the same mean rate as that of the true sun in the ecliptic.

Definition.—A *mean solar day* is the interval between two successive transits of the mean sun across the meridian.

As the mean sun changes its right ascension at a uniform rate we see that the length of a mean solar day is constant. The hour-angle of the mean sun at any instant measures the mean time at that instant, while that of the apparent or real sun gives the apparent time, or time as indicated by a sun-dial.

152. **Definition.**—The *equation of time* is the difference between the mean and the apparent time. It is counted positive when the mean exceeds the apparent time, and

negative when the latter exceeds the former. Therefore we have—

(Mean Time) - (Apparent Time) = (Equation of Time),
or (Clock Time) - (Dial Time) = (Equation of Time).

As the true sun moves in the ecliptic and the mean sun in the equator, the motion of the former being variable, and of the latter, uniform, we see that the equation of time is due to two causes:—

(1) *The variable motion of the true sun in the ecliptic owing to the eccentricity of the earth's orbit.*

(2) *The obliquity of the ecliptic to the equator.*

We shall now consider each of these causes separately, and by combining their effects we shall be able to see how the equation of time varies throughout the year, when it reaches a maximum, and at what periods it vanishes.

Equation of Time due to Unequal Motion of Sun.

153. On December 31st, at perihelion, or, in other words, when the earth is nearest the sun, its velocity is greatest (Art. 68), and therefore the rate at which the sun moves from west to east along the ecliptic will, at this period, be greater than the mean rate. As the earth turns on its axis from west to east this would cause the apparent or true solar days to exceed in length the mean solar days, and, if a sun-dial and a clock be started together at perihelion, the *apparent time will gradually get behind mean time*, so that the sun-dial will lose compared with the clock. This will continue for about three months until the rate of the sun in the ecliptic becomes equal to its mean rate. Hence that component of the equation of time, due to the unequal motion of the sun, reaches at the end of March its greatest positive value, viz. about 7 minutes. The sun-dial will now begin to gain on the clock what it lost in the preceding three months, until aphelion (July 1st) is reached, when it will coincide with

the clock, and the equation of time, in so far as it is due to the unequal motion of the sun, vanishes.

Similarly we can see that from aphelion to perihelion the equation of time, due to this cause, is negative, having a maximum value of − 7 minutes at the end of September.

Equation of Time due to the Obliquity of the Ecliptic.

154. Even if the sun's motion along the ecliptic were uniform, his rate of change in right ascension would still be variable on account of the obliquity of the ecliptic to the equator. Let us now suppose that the true sun S and the mean sun S_1 (fig. 73) start together at the vernal equinox ♈, the former moving in the ecliptic and the latter in the equator; they will again be together at the autumnal equinox ♎, and also their right ascensions will coincide at the two solstices. Hence that portion of the equation of time due solely to the obliquity of the ecliptic becomes zero four times each year, at the equinoxes and solstices.

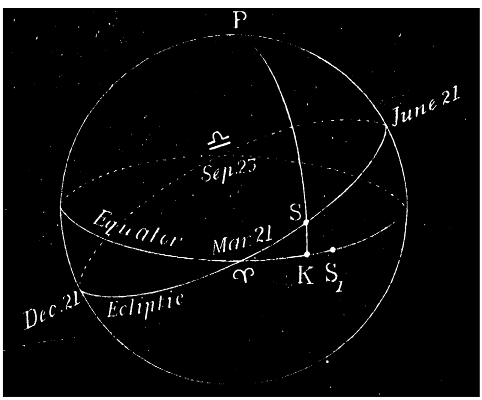

FIG. 73.

Again, when the true sun is at S (fig. 73) his right ascension is ♈ K (PSK being the arc passing through the celestial pole and the sun's centre. But the position of the mean sun S_1 (as affected by the obliquity of the ecliptic alone) would be obtained by cutting off $\Upsilon S_1 = \Upsilon S$, S_1 falling to the east of K, since ΥS, being the hypotenuse of the right-angled spherical triangle ΥSK, is greater than ΥK. Therefore the true sun, being to the west of the mean sun, will each day cross the meridian first, *the sun-dial being faster than the clock.* Hence this portion of the equation of time is *negative*, its greatest value being about − 10 minutes. Similarly we may see that from solstice to equinox *the dial will be slower than the clock*, and the component of the equation of time will be *positive*, having a maximum value of + 10 minutes.

Combination of the Two Components.

155. Let X = that portion of equation of time due to the unequal motion of the sun.

Y = that due to obliquity of the ecliptic.

If we summarize the above results we have—

(1) X vanishes twice each year, on December 31st and July 1st, and varies from a maximum of + 7 minutes at end of March to − 7 minutes at end of September. (See dotted curve, fig. 74.)

(2) Y vanishes four times each year, at the equinoxes and solstices. From equinox to solstice Y is negative, and from solstice to equinox, positive, varying from a maximum of + 10 minutes to − 10 minutes at intermediate points. (See continuous curve, fig. 74.)

(3) The sum or difference of X and Y, according as they are of the same or opposite sign, gives the equation of time at any instant. (See curve, fig. 75.)

The Equation of Time vanishes four times each Year.

156. We have seen that the equation of time is equal to the algebraic sum of X and Y; the maxima values of Y being

$$+10m, \quad -10m, \quad +10m, \quad -10m,$$

which occur in the months

February, May, August, November.

Now as X has never a greater numerical value than ± 7 minutes it follows that, in the four months mentioned above, the equation of time $(X + Y)$ must have the same sign as Y, whether X is positive or negative. Hence it follows that the equation of time has at least four changes of sign throughout the year, viz.:

$$+, \quad -, \quad +, \quad -,$$

and therefore, on changing from positive to negative or *vice versa*, must at least on four occasions pass through a zero value.*

The dates on which the equation of time vanishes are about April 16, June 15, September 1, and December 25. The greatest positive value is $14^m\ 28^s$ on February 11, and the greatest negative value $16^m\ 21^s$ on November 3. (See the curve in fig. 75.)

157. We can now represent graphically how the equation of time varies throughout the year, when it reaches a maximum, and at what periods it vanishes.

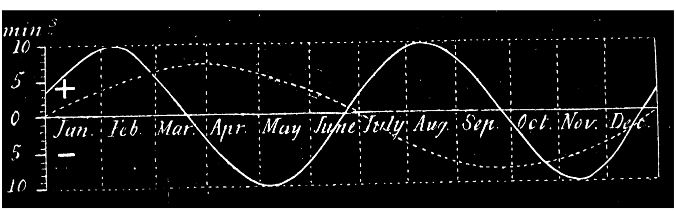

FIG. 74.

These curves represent the variations in the two components of the equation of time.

* That it does not vanish oftener than four times each year appears from an examination of the curve in fig. 75.

In fig. 74 the dotted curve represents the component due to the unequal motion of the sun, and the continuous line that due to the obliquity of the ecliptic. In fig. 75 is a single curve representing the combined effect of the other two, the equation of time corresponding to any point on the curve being represented by the perpendicular distance of the point from the zero line. Thus the equation of time corresponding to *p* is represented by *pm*. All portions of the curve below the zero line represent negative values. The periods at which the equation of time vanishes are represented by the points where the curve cuts the zero line.

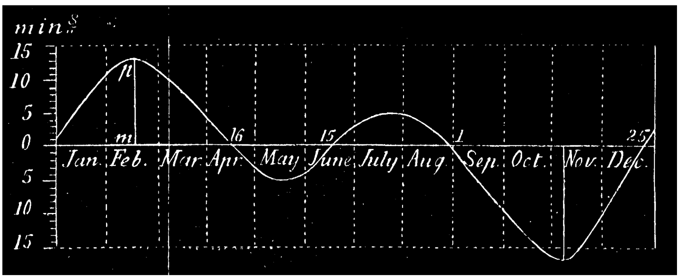

FIG. 75.

The curve represents the variations in the equation of time found by combining the two components whose curves are given in the preceding figure.

Morning and Afternoon unequal in Length.

158. The interval of time from sunrise until the sun is in the meridian (apparent noon) is equal to the interval from apparent noon to sunset, neglecting the small change in the sun's declination throughout the day. But mean and apparent noon do not in general coincide; therefore, morning and afternoon, as measured by our clocks, are not of equal length, the former being less (algebraically) than half the interval between sunrise and sunset by the equation of time, the latter being greater (algebraically) by the same amount.

Hence the lengths of the morning and afternoon always differ by twice the equation of time, so that

(length afternoon) − (length morning) = 2 (equation of time).

Immediately after the winter solstice, the afternoons begin to lengthen, while the mornings still continue to get shorter. The explanation of this is simple.

For the sun being at the winter solstice he, as it were, stands still for some days, during which time we may regard his declination as constant. The interval between apparent noon and sunset, therefore, remains constant. But as the equation of time is at this period increasing (see curve, fig. 75), mean noon precedes apparent noon by a greater amount each day. Hence the mean time of sunset increases and the afternoons get longer.

Similarly, it may be shown that while the apparent time of sunrise remains the same, the mean time of sunrise increases each day, and, consequently, the mornings are shortened. It is, however, to be borne in mind that very soon the increasing declination of the sun causes the mornings to lengthen as well as the afternoons.

EXAMPLES.

1. Given mean time = $5^h\ 12^m\ 20^s$ p.m., and the equation of time = $+\ 5^m\ 25^s$; find apparent time.

Ans. $5^h\ 6^m\ 55^s$ p.m.

2. Given apparent time = $10^h\ 4^m\ 15^s$ a.m., on November 3, when the equation of time has its greatest negative value, viz. $16^m\ 21^s$; find mean time.

Ans. $9^h\ 47^m\ 54^s$ a.m.

3. Find the mean time of apparent noon in questions 1 and 2.

Ans. 1. $5^m\ 25^s$ p.m.
2. $11^h\ 43^m\ 39^s$ a.m.

4. On Nov. 3, the sun-dial is $16^m\ 21^s$ faster than the clock. Given that the sun rose at $6^h\ 57^m$ a.m.; find the time of sunset.

Ans. $4^h\ 30^m\ 18^s$ p.m.

5. Given that the sun rose on a certain date $6^h\ 54^m$ a.m., and set at $4^h\ 33^m$ p.m.; find the equation of time.

Ans. $-\ 16^m\ 30^s$.

6. In question 5, by how much does the length of the morning exceed the length of the afternoon?

Ans. 33^m.

Local Time.

159. As the earth rotates uniformly on its axis from west to east, it is evident that the further east a place is situated the sooner will the sun cross the meridian of that place, and, therefore, the later will be the local time.

When it is noon at any place it will be 1 o'clock p.m. 15° to the eastward, and 11 o'clock a.m. 15° to the westward of that place. For:—

360° correspond to 24 hours;

∴ 15° „ to 1 hour.

Given the longitudes of two places (A and B), and the time at one of them (A), to find the time at the other (B).

Rule.—*Divide the algebraic difference of the longitudes by 15, which gives the difference of the local times. Add this result to, or subtract from, the given time at A, according as B is east or west of A, and the result will be the time at B.*

Example.—Find the time at New York (long. 74° 10′ W.) when it is 3 o'clock, P.M. at Dublin (long. 6° 20′ W.).

Here the difference of longitudes = 67° 50′.

On dividing by 15, we get:—

Difference in times = $4^h\ 31^m\ 20^s$.

New York being west of Dublin, the time is earlier. Therefore, subtract $4^h\ 31^m\ 20^s$ from 3 p.m., that is, from 15 hours.

	H.	M.	S.
	15	0	0
	4	31	20
New York time =	10	28	40 a.m.

Should one longitude be east and the other west, their algebraic difference will be got by adding the longitudes.

N.B.—In the same way, when given the sidereal time at one of two places whose longitudes are known, the sidereal time at the other place can be found; for the earth turns on its axis through 360° relative to the fixed stars in 24 sidereal hours, and therefore 15° relative to the fixed stars correspond to 1 sidereal hour.

To reduce a given Interval of Mean Time to Sidereal Time, and vice versa.

160. There are $365\frac{1}{4}$ mean solar days in the year, and $366\frac{1}{4}$ sidereal days, the sun making one less diurnal revolution than the fixed stars, on account of his annual motion in the ecliptic;

$$\therefore \quad 365\tfrac{1}{4} \text{ mean solar days} = 366\tfrac{1}{4} \text{ sidereal days.}$$

Therefore, if m be any interval of mean time, and s the corresponding interval of sidereal time, we have—

$$365\tfrac{1}{4} : 366\tfrac{1}{4} :: m : s.$$

From which, if m be given, s can be found, and *vice versa*.

EXAMPLES.

1. Express in sidereal time an interval of $16^h\ 15^m\ 23^s$ mean time. *Ans.* $16^h\ 18^m\ 3^s$.

2. Express in mean time an interval of $12^h\ 16^m\ 26^s$ sidereal time. *Ans.* $12^h\ 14^m\ 16^s$.

N.B.—We cannot convert, by means of the above formula, the actual mean time at any instant in sidereal time, or *vice versa*. This is done as follows:—

Given the Sidereal Time at any instant at Greenwich, to find the Mean Time at that instant.

161. Let AQ (fig. 76) represent the meridian, ΥQ the celestial equator, and m the mean sun.

Then $Q\Upsilon$ = sidereal time (expressed in arc),

Qm = mean time (expressed in arc),

$m\Upsilon$ = right ascension mean sun.

But $Qm = Q\Upsilon - m\Upsilon$.

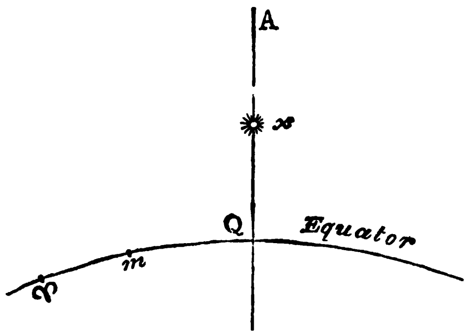

FIG. 76.

Now, let

M = mean time at any instant expressed in mean hours,

S = the corresponding sidereal time expressed in sidereal hours,

R = the right ascension of the mean sun at the instant expressed in sidereal hours,

and

R_0 = the right ascension of the mean sun at Greenwich mean noon expressed in sidereal hours.

Now

arc mQ = 15 M degrees, because the mean sun passes over 15° in one mean hour;

arc ΥQ = 15 S degrees, because Aries passes over 15° in one sidereal hour;

and

arc Υm = 15 R degrees, because 15° of right ascension equals one sidereal hour.

$$\therefore \quad 15M = 15S - 15R,$$

or

$$M = S - R;$$

or Mean time (expressed in mean hours) = Sidereal time (expressed in sidereal hours) − Right Ascension of mean sun at the instant (expressed in sidereal hours).

Again, the interval between the sidereal time at the instant and the sidereal time at the preceding mean noon $= S - R_0$ sidereal hours.

But in 1 sidereal hour ♈ passes over 15°;

$$\therefore \text{ in 1 sidereal hour } m \text{ passes over } 15° \times \frac{365\frac{1}{4}}{366\frac{1}{4}};$$

$$\therefore \text{ in 1 sidereal hour ♈ gains } 15°\left(1 - \frac{365\frac{1}{4}}{366\frac{1}{4}}\right) \text{ on } m;$$

$$\therefore \text{ in } S-R_0 \text{ sidereal hours ♈ gains } 15°(S-R_0)\left(1 - \frac{365\frac{1}{4}}{366\frac{1}{4}}\right) \text{ on } m;$$

$$\therefore \quad 15°R = 15°R_0 + 15°(S - R_0)\left(1 - \frac{365\frac{1}{4}}{366\frac{1}{4}}\right);$$

or

$$R = R_0 + (S - R_0)\left(1 - \frac{365\frac{1}{4}}{366\frac{1}{4}}\right);$$

but

$$M = S - R$$

$$\therefore \quad M = S - R_0 - (S - R_0)\left(1 - \frac{365\frac{1}{4}}{366\frac{1}{4}}\right)$$

or

$$M = (S - R_0)\frac{365\frac{1}{4}}{366\frac{1}{4}}.$$

Therefore to reduce sidereal time to mean time at any instant at Greenwich, we have the following rule :—

Rule.—*From the given sidereal time subtract the right ascension of the mean sun at noon, and reduce the result to mean time.*

The Nautical Almanac gives the right ascension of the mean sun at noon for each day at Greenwich. The right ascension of the mean sun at noon is evidently the same as the sidereal time of mean noon.

EXAMPLE.—Given sidereal time = $5^h\ 32^m\ 37^s$ and the right ascension of the mean sun at mean noon = $7^h\ 37^m\ 32^s$, find mean time.

		H.	M.	S.	H.
Here sidereal time	=	5	32	37	(+ 24)
R. A. mean sun at noon	=	7	37	32	
∴ Mean time	=	21	55	5	

(Expressed in sidereal hours, &c.)

Reducing this to mean hours, &c., we get the mean time = $21^h\ 51^m\ 30^s$ or $9^h\ 51^m\ 30^s$ a.m.

Given the mean time at any instant at Greenwich, to find the sidereal time at that instant.

Proceeding as before we obtain

$$15\,S \text{ degrees} = 15\,M \text{ degrees} + 15\,R \text{ degrees},$$

$$\therefore\quad S = M + R,$$

or sidereal time (expressed in sidereal hours) = mean time (expressed in mean hours) + Right Ascension of mean sun at the instant (expressed in sidereal hours).

Again, in 1 mean hour m passes over 15° ;

$$\therefore\quad \text{in 1 mean hour } \Upsilon \text{ passes over } 15° \times \frac{366\frac{1}{4}}{365\frac{1}{4}},$$

$$\therefore\quad \text{in 1 mean hour } \Upsilon \text{ gains } 15°\left(\frac{366\frac{1}{4}}{365\frac{1}{4}} - 1\right) \text{ on } m,$$

$$\therefore\quad \text{in } M \text{ mean hours } \Upsilon \text{ gains } 15°\,M\left(\frac{366\frac{1}{4}}{365\frac{1}{4}} - 1\right) \text{ on } m,$$

$$\therefore\quad 15°R = 15°R_0 + 15°M\left(\frac{366\frac{1}{4}}{365\frac{1}{4}} - 1\right),$$

or

$$R = R_0 + M\left(\frac{366\frac{1}{4}}{365\frac{1}{4}} - 1\right),$$

but

$$S = M + R,$$

$$\therefore\quad S = M + R_0 + M\left(\frac{366\frac{1}{4}}{365\frac{1}{4}} - 1\right).$$

This may be expressed as follows:—

Rule I.—*To R_0, the right ascension of the mean sun at noon in sidereal hours, add* $M\left(\frac{366\frac{1}{4}}{365\frac{1}{4}} - 1\right)$ *i.e., the change expressed in sidereal hours, in this right ascension during M mean solar hours, which gives the right ascension of the mean sun at the instant expressed in sidereal units. If we add this corrected right ascension (expressed in sidereal units) to the mean time (expressed in mean units), we obtain the sidereal time at the instant (expressed in sidereal units).*

This rule can, however, be simplified as follows:—
The formula

$$S = M + R_0 + M\left(\frac{366\frac{1}{4}}{365\frac{1}{4}} - 1\right)$$

reduces to $$S = M \cdot \frac{366\frac{1}{4}}{365\frac{1}{4}} + R_0.$$

Hence the above rule becomes:—

Rule II.—*Reduce the mean time to sidereal units, and add the result to the right ascension of the mean sun at noon (expressed in sidereal units); the result is the required sidereal time (expressed in sidereal units).*

EXAMPLE.—Given mean time = $3^h\ 20^m\ 50^s$ p.m., when the right ascension of the mean sun at mean noon is $16^h\ 32^m\ 9^s$; find sidereal time.

First method, according to Rule I.—

Sidereal time = mean time + R.A. mean sun.

		H.	M.	S.
Here R. A. mean sun at noon	=	16	32	9
Change in $3^h\ 20^m\ 50^s$	=	0	0	33
		16	32	42
Mean time	=	3	20	50
∴ Sidereal time	=	19	53	32

Second method, according to Rule II.—

		H.	M.	S.
Mean time	=	3	20	50
Mean time reduced to sidereal units	=	3	21	23
R. A. mean sun at noon	=	16	32	9
∴ Sidereal time	=	19	53	32

To convert the sidereal time at a given meridian, not that of Greenwich, into the corresponding mean solar time, and vice versa.

162. As the Nautical Almanac only gives the right ascension of the mean sun for mean noon *at Greenwich*, therefore, for any other meridian, we will have to proceed as follows:—

Rule.—*Reduce the given time, whether it be sidereal or mean solar, to the corresponding Greenwich time* (Art. 159). *Then, knowing the R. A. of mean sun at noon, the Greenwich sidereal time can be changed as above into mean solar time, or* vice versa, *and the result reduced back again to the meridian of the given place.*

EXAMPLE.—Assuming that on a certain day at Greenwich the right ascension of the mean sun was 10 hours at 12 o'clock, find for a place whose longitude is 60° west, the time by an ordinary clock on that same day, when the time by an astronomical clock at the place was 14 hours. (Degree, T.C.D., Hilary, 1893.)

Here, local sidereal time = 14 sidereal hours;

∴ Greenwich ,, $= 14 + \frac{60}{15} = 18$ sidereal hours.

But mean time = sidereal time − R. A. mean sun

$= 18 - 10 = 8$ sidereal hours.

But 8 sidereal hours = $7^h\ 58^m\ 41^s$ mean time (Art. 160);

∴ Greenwich mean time = $7^h\ 58^m\ 41^s$ p.m.

But difference between Greenwich and local mean times = 4 mean solar hours.

∴ Local mean time = $3^h\ 58^m\ 41^s$ p.m.

To find at what time a Star will cross the Meridian.

163. Let x (fig. 77) represent a star in the meridian AQ. Then, m being the mean sun, we have—

$$♈Q = \text{R. A. of star},$$
$$♈m = \text{R. A. mean sun},$$
$$Qm = \text{mean time of star's transit (in arc)}.$$

But $$Qm = ♈Q - ♈m.$$

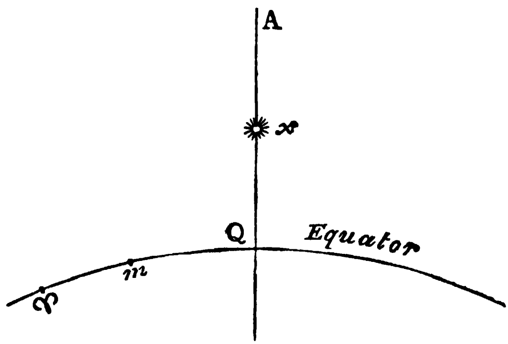

FIG. 77.

Therefore, we have the following equation, all the quantities being supposed *expressed in arc*:—

(Mean time of transit) = (R. A. star) – (R. A. mean sun).

EXAMPLE.—Find at what time α Aquilæ will cross the meridian of Greenwich, being given the right ascension of the star = $19^h\ 43^m\ 51^s$, and the right ascension of the mean sun at Greenwich mean noon = $0^h\ 6^m\ 40^s$.

	H.	M.	S.	
R. A. star =	19	43	51	
R. A. mean sun =	0	6	40	
∴ Mean time of transit =	19	37	11	(in sidereal units).

Reducing $19^h\ 37^m\ 11^s$ to mean time (Art. 160), we get—

	H.	M.	S.	
Mean time of transit =	19	33	58	after mean noon
=	7	33	58	a.m.

N.B.—When the transit is across some other meridian than that of Greenwich, a correction must be made for the change in the position of the mean sun corresponding to the difference of longitude of the two places. This correction is made in the same way as in Art. 162.

Equinoctial Time.

164. In addition to the apparent, mean solar, and sidereal times, another kind of time is sometimes used which is independent of the position of the observer on the earth.

The *Equinoctial Time* at any instant is the interval, measured in mean solar days, hours, &c., since the preceding vernal equinox.

The Calendar.

165. The ordinary civil year contains an exact number of days, viz. 365. But the time taken by the sun to complete a revolution in the ecliptic is about 365¼ days. The exact interval between two successive vernal equinoxes is $365^d\ 5^h\ 48^m\ 45{\cdot}5^s$. This period is called a *Tropical Year*. A *Sidereal Year*, or the time taken by the sun to return to the same position relative to the fixed stars, is slightly longer than the tropical year on account of the precession of the equinoxes.

Thus, by taking the civil year as 365 days, there is an error compared with the tropical year of $5^h\ 48^m\ 45{\cdot}5^s$, which in four years amounts to $23^h\ 15^m\ 2^s$, or very nearly a day. If this error were not corrected, the result would be that the dates of the equinoxes and solstices would be later by one day every four years.

166. The first exact attempt at approximating the length of the civil to that of the tropical year was made in the time of Julius Cæsar. It was then agreed that an additional day should be given to every fourth year, which was to contain 366 days. Such a year is called a *bissextile* or *leap year*.

Those years are chosen as leap years, which are divisible by 4 without remainder, such as 1888, 1892, &c.

The Calendar constructed in this manner is called the *Julian Calendar.*

According to the Julian Calendar a correction is made of one day every four years. But one day, or 24 hours, is in excess of $23^h\ 15^m\ 2^s$ by about 45 minutes. Thus the correction by means of leap year leads to a new but very much smaller error of about 45 minutes in four years, or an average of rather more than 11 minutes each year. This error, in 400 years, would amount to nearly three days.

Hence we have the *Gregorian correction* to the Julian Calendar adopted by Pope Gregory XIII. in 1582, according to which each year which is a multiple of 100, such as 1700, 1800, 1900, which, by the Julian Calendar, are leap years, should be ordinary years, with the exception of those years in which the number of the century is divisible by 4 without remainder, such as 2000, 2400, which should remain leap years. This arrangement evidently makes the required correction of three days in 400 years.

Even with the Gregorian correction there is still a very small error, which, however, would amount to not more than a day in 20,000 years. The Gregorian correction was not adopted in England until the year 1752, when the accumulated error, as compared with the corrected calendar, amounted to eleven days. Eleven days of that year were therefore skipped, the 2nd of September being called the 13th.

In Russia, where the Julian Calendar is still adhered to, the dates are thirteen days behind those of the rest of Europe.

The Sun-dial.

167. In a sun-dial the apparent time is indicated by means of the shadow cast by a rod of metal on a horizontal

plane. The rod is called a *gnomon* or *stile*. The gnomon points to the celestial pole, and, therefore, makes an angle with the horizontal dial-plate equal to the latitude of the place.

The principle on which the sun-dial is constructed can be easily understood by supposing the observer situated at *O* (fig. 78), the centre of a sphere, *ON* being the direction of the celestial pole. If 12 equidistant great circles be drawn through *N* and *S*, the sun *H* will, in his diurnal motion, be in the plane of one of these circles at the termination of each hour,

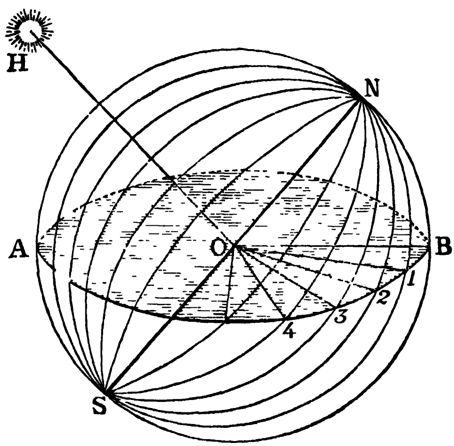

FIG. 78.

and the shadow cast by a gnomon fixed in the direction *ON* will coincide each hour with one of the numbers 1, 2, 3, &c., which mark the intersection of the hour-circles with the horizontal circle *AB*. A horizontal sun-dial is, therefore, graduated into intervals proportional to the spaces between these intersections. Hence these graduations on a dial-plate which mark the hours are not generally at equal intervals apart, as they would be if the plane of *AB* were at right angles to *NS*. The 24 hour-angles, however, subtended at the celestial pole *N* are all equal.

Some sun-dials are constructed with the dial-plate vertical instead of horizontal. In this case the graduations correspond to the intersections of a vertical circle, whose centre is O, with the hour-circles passing through N and S.

EXAMPLES.

1. The times of the sun's rising and setting on November 1st are $6^h\ 56^m$ and $4^h\ 32^m$, respectively; find approximately the equation of time.

Ans. -16^m.

2. Convert $22^h\ 26^m\ 1^s$ sidereal time into mean solar time for the meridian of Greenwich, being given the R. A. of mean sun at mean noon as $24^h\ 4^m\ 17^s$.

Ans. $2^h\ 21^m\ 21^s$.

3. In question 2, convert $2^h\ 26^m\ 12^s$ mean solar into sidereal time for the same meridian.

Ans. $20^h\ 30^m\ 53^s$.

4. At New York, in longitude 74° 10′ W., an observation is made on August 25th, 1893, at $6^h\ 3^m\ 4^s$ mean solar time; find the corresponding sidereal time, being given from the "Nautical Almanac" that the sidereal time of mean noon (R. A. mean sun at noon) at Greenwich on above date is $10^h\ 15^m\ 54^s$.

Ans. $16^h\ 20^m\ 46^s\cdot3$.

5. Mars revolves on its axis in $24^h\ 37^m$, and round the sun in 686 days; find by how much the mean solar day on Mars exceeds that of the sidereal day.

Ans. $2^m\ 9^s$.

6. Find the R. A. of the true sun at true noon on November 25th, 1893, being given the following from the "Nautical Almanac" for 1893:—

Equation of time at mean noon Nov. 25th = $-12^m\ 45^s$.

Sidereal time of mean noon on Sept. 2nd = $10^h\ 47^m\ 26^s$.

Here the R. A. of mean sun at mean noon on September 2nd = $10^h\ 47^m\ 26^s$.
But increase in the R. A. from September 2nd to November 25th

$$= 24 \times \frac{84}{365\frac{1}{4}} = 5^h\ 31^m\ 10^s.$$

$\therefore$ R. A. of mean sun at mean noon on November 25th = $16^h\ 18^m\ 36^s$.

But change in R. A. in $12^m\ 45^s = 2^s$;

$\therefore$ R. A. of mean sun at true noon = $16^h\ 18^m\ 34^s$.

But true sun's R. A. − mean sun's R. A. = equation of time

$\therefore$ true sun's R. A. − $16^h\ 18^m\ 34^s = -12^m\ 24^s$;

$\therefore$ true sun's R. A. at true noon = $16^h\ 6^m\ 10^s$.

7. (1) The maximum values of the equation of time due to the obliquity of the ecliptic being ± 10 minutes, and due to the eccentricity of the earth's orbit ± 7 minutes, show that the equation of time vanishes four times a year.

(2) How many times a year would it vanish were the magnitudes of the maxima reversed?

Ans. (1) see Art. (156).
(2) twice.

8. Find the mean solar time at Madras, longitude 80° 14′ 19″ E., corresponding to apparent time 8 p.m. there on September 6th, 1893, being given the following from the "Nautical Almanac" for 1893:—

At Greenwich, mean noon.

Equation of time on September 6th = $-1^m\ 52^s{\cdot}22$.

" " " 7th = $-2^m\ 12^s{\cdot}42$.

Ans. $7^h\ 58^m\ 5^s.54$.

9. The longitude of Dublin being 6° 40′ W., and that of Paris 2° 20′ E.; find the time at Paris when it is 11.30 a.m. at Dublin.

Ans. $12^h\ 6^m$.

10. The longitude of Pulkowa being 30° 19′ 40″ E., and that of New York 74° 1′ W.; find the time at New York when i is 3.30 p.m. at Pulkowa.

Ans. $8^h\ 32^m\ 37^s$ a.m.

CHAPTER XII.

APPLICATION TO NAVIGATION.

168. By observing the heavenly bodies, we are enabled to determine the latitude and longitude of a place on the earth or of a ship at sea. But the instruments used in a fixed observatory would be altogether useless at sea, owing to the motion of the ship. For the same reason, the artificial horizon we avail ourselves of on land, viz. the surface of a vessel of mercury, would not at sea remain a horizontal plane. What is required in such observations as taking the altitude of a heavenly body, or finding the angular distance between two bodies, is an instrument by means of which the measurement can be made by *observing both objects at once*, and not by two successive adjustments to each object, of which the instability of the ship would not allow. Such measurements, unaffected by the motion of the ship, can be made with Hadley's Sextant.

Hadley's Sextant.

169. This instrument consists of a fixed framework, formed by a graduated arc *AB* (fig. 79), and two fixed arms *AM* and *BM*, *M* being situated at the centre of the circle formed by *AB*. Another arm *VM* turns round the centre *M*, its other extremity *V* travelling along the graduated arc *AB*. This movable arm carries a small mirror at *M*, called the *index glass*, which moves with the arm, its plane being perpendicular to the plane of the instrument. On one of the fixed arms *BM* is a fixed mirror *N* called the *horizon glass*, whose plane is parallel to the other fixed arm *AM*, so

that when the movable arm VM coincides with AM the two mirrors are parallel, that is, their angle of inclination is zero. The point A of the graduated arc is, for this reason, the zero point of the scale. Half of the horizon glass is silvered, so as to act as a mirror, the other half being plain glass, and therefore transparent. A small telescope T is fixed to the arm AM, and directed to the horizon glass.

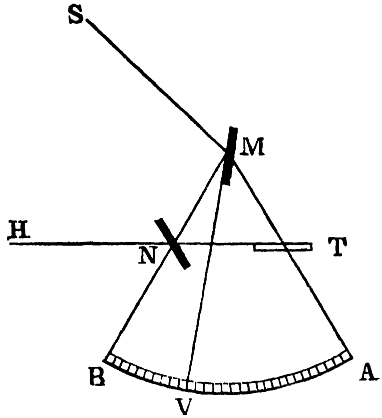

FIG. 79

170. Let us now see how, by means of this instrument, the angular distance between two objects such as H and S (fig. 79) can be found. The sextant is held so that its plane passes through both objects, and in such a position that, in looking through the telescope, one of the objects H can be seen through the unsilvered half of the mirror N. The mirror M is then rotated by means of the arm VM until the image of S coincides with that of H. In this case the rays from S are doubly reflected, first from the index glass and then from the horizon glass, along the lines $SMNT$. Having got the images of S and H coincident, the arm VM is clamped, and the arc AV is read off by means of a vernier at V. This reading, on being doubled, gives the angular distance between the objects. Usually, however, the half degrees in the arc are numbered as whole ones, so that the reading of the vernier at once gives the required distance.

The arc AV (fig. 79) evidently measures the angle at which the mirrors are inclined to one another, for it measures the angle between AM and the mirror at M, AM being parallel to the mirror N.

171. The principle on which Hadley's Sextant is constructed is that the angular distance between two distant bodies, such as S and H, is double the angle between the planes of the mirrors when the image of S, after double reflection, is made to coincide with that of H. To prove this: the ray SM (fig. 80) is reflected from the mirror M in the direction MN, the incident and reflected rays making

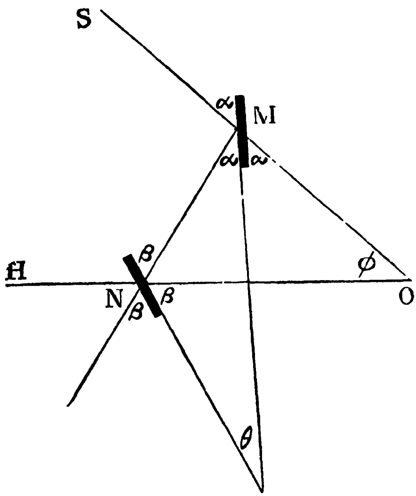

FIG. 80.

equal angles with M; let each of these angles be a. Again, the ray MN is reflected from the horizon glass along the line OH (since the two images are coincident). Let the angles made with the mirror N be each called β. Now we have (Euclid, I. 32) —

$$\text{The external angle } 2\beta = 2a + \phi \text{ (fig. 80)};$$

$$\therefore \quad \phi = 2\beta - 2a = 2(\beta - a).$$

For the same reason—

$$\beta = a + \theta,$$

$$\therefore \quad \theta = \beta - a;$$

$$\therefore \quad \phi = 2\theta.$$

But ϕ is the angular distance of S from H as observed from O, and θ is the angle between the planes of the mirrors.

172. The principal use of Hadley's Sextant is for measuring the altitude of the sun. The observer, holding the instrument upright, looks through the unsilvered portion of the horizon glass at that part of the horizon which is vertically beneath the sun; he then rotates the movable arm, and with it the index glass, until the lower edge of the sun's image just touches the horizon. Then the reading of the vernier, after correction for refraction, "dip" of the horizon, and other errors, gives the altitude of the lower limb of the sun, to which the sun's semi-diameter will have to be added, in order to determine the altitude of the sun's centre.

The instrument is called a *sextant*, because the arc AB (fig. 79) generally contains 60°. Angular distances up to 120° can therefore be measured.

The Chronometer.

173. Every ship carries one or more very accurately constructed watches, called *chronometers*, which, before leaving port, are set to Greenwich time. As it is of the greatest importance that Greenwich time should be accurately known during the voyage, it is necessary that the chronometer should go at a rate as nearly uniform as possible. It is not necessary that it should keep correct time, but only that the amount of gain or loss should be the same from day to day. So that this "rate" being known, the correct Greenwich time can be calculated by allowing for the total error accumulated since it was correctly set.

174. Chronometers differ from ordinary watches in two particulars—(1) the peculiar construction of the balance wheel, so as not to be affected by changes of temperature; (2) the "detached escapement." If the balance wheel were

to consist of an entire circle, composed of one metal alone, then an increase of temperature would cause it to expand, and the time of oscillation would be increased, causing the watch to go more slowly. However, the circumference of the wheel is generally composed of three unconnected arcs: the external portion of each arc is brass, and the internal part, steel. When the temperature rises, the brass expands more than the steel, causing the extremities of the arcs to curve inwards towards the centre, thus compensating for the expansion of the spokes, which cause the arcs to be pushed outwards. Each arc is also, for purposes of adjustment, weighted with little screws. The "detached escapement" is an arrangement by which the action of the main spring, which keeps up the motion, is suspended for the greater part of the oscillation, so that the isochronism of the balance wheel is hardly at all affected by external impressions.

Latitude at Sea. Meridian Observations.

175. First Method. The latitude at sea can be found by taking the meridian altitude of the sun with the sextant. The observations commence some time before apparent noon; and the altitude is repeatedly taken until it ceases to increase, and thus the maximum or meridian altitude is found. Also the sun's declination is given for each day at Greenwich noon in the "Nautical Almanac," together with its rate of variation per hour. Therefore, after correcting for this change in declination, due to the interval between Greenwich and local noon (Greenwich time being given by the ship's chronometers), we get the sun's declination at that instant. The latitude is now given by the formula:—

$$\text{(colat.)} \pm \text{(declination)} = \text{(meridian alt.)}. \quad \text{(Art. 34.)}$$

In our latitudes the plus or minus sign is taken according as the declination of the sun is north or south.

Similarly, the latitude may be found by observing the meridian altitude of a star (or other body) whose declination is known, the same formula colat. $+ \delta = a$, being used. If the star cross the meridian between the zenith and the pole the formula becomes colat. $+ \delta = 180 - a$ (Art. 34); and should the altitude be observed when the star crosses the meridian between P and R (fig. 20), it changes to $-$ colat. $+ \delta = a$, the sign of δ being in all cases altered when it represents a south declination.

It would be better, however, in numerical examples, for the student, in each case, instead of relying on the formula, to draw a diagram and directly deduce the result after placing the star in its proper position corresponding to the given measurements.

EXAMPLE.—The sun's meridian altitude on December 4th, 1893, is observed to be 16° 8′. The chronometer indicates $6^h\ 5^m\ 12^s$ Greenwich time. The "Nautical Almanac" gives the sun's declination at the preceding noon at Greenwich as 22° 19′ 25″ south, and his hourly change in declination as 19″·6. Find the latitude of the ship.

Here we have:—

Sun's declination at Greenwich noon = 22° 19′ 25″ S.
Hourly change (increasing) = 19″·6;
∴ Change in $6^h\ 5^m\ 12^s$ = 1′ 59″·3;
∴ Declination at local noon = 22° 21′ 24″·3.
B colat. − declination = meridian altitude;
∴ colat. − 22° 21′ 24″·3 = 16° 8′;
∴ colat. = 38° 29′ 24″·3;
∴ latitude = 51° 30′ 35″·7.

Ex-meridian Observations.

176. Second Method. *By simultaneously observing the altitudes of two known stars.*

Let S, S' be the two stars (fig. 81) when their altitudes are measured. Join Z and P with S and S' by arcs of great circles.

In order to find the latitude it is necessary to solve three spherical triangles. In the triangle SPS' the polar distances PS, PS' of the stars are known, since they are the complements of their declinations which are given in the "Nautical Almanac." Also the angle SPS' is known, since it measures the difference between the known right ascensions of the stars; hence the side SS' and the angle PSS' can be calculated.

Again, in the triangle ZSS' the zenith distances ZS, ZS' are known, being the complements of the observed altitudes, also the base SS' is known, therefore the angle ZSS' can be found, and hence the angle ZSP.

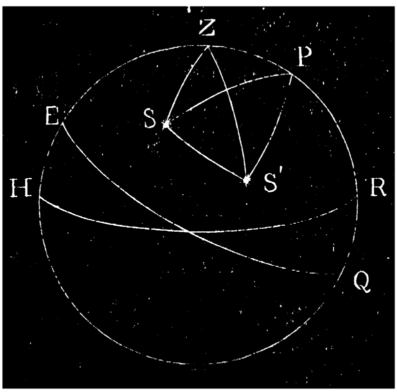

FIG. 81.

Lastly, in the triangle ZSP the two sides ZS, PS are known, and the included angle ZSP: therefore the colatitude ZP can be found.

177. Third Method. The latitude may also be found by taking two altitudes of the sun, and noting the interval of time beween the two observations. This is practically the same method as the last, for the interval of time between the two observations, reduced to degrees at the rate of 15° to each hour, gives the value of the angle SPS' when the solution of the different spherical triangles enables us to find the latitude as before.

The latitude can also be found by taking a single altitude of the sun, provided the local time be known.

For if *S* (fig. 81) be the position of the sun, we have in the triangle *ZPS* the two sides, *ZS* and *SP* are known. Also the angle *ZPS* is known, being the hour angle of the sun, which measures the local apparent time. Therefore, the colatitude *ZP* can be calculated.

To find Mean Local Time.

178. First Method. *By equal altitudes.* The mean local time, or, as it is called at sea, the *ship mean time*, can be calculated as follows :—Observe, with the sextant, the altitude of the sun some time before it crosses the meridian. Again, after its transit, note the instant at which it attains the same altitude as before. The mean of the times of the two observations (given by a chronometer) will give the time of transit, that is, apparent noon ; from which, knowing the equation of time from the "Nautical Almanac," the mean time can be found.

179. Second Method. *By observing the altitude of a known star, or of the sun, moon, or a planet (when the body is in, or near, the prime vertical).*

In this case the latitude of the place is supposed to be known.

Let *S* be a star (fig. 81) in, or near, the prime vertical whose declination is known from the "Nautical Almanac," and whose altitude is measured. Then in the triangle *ZSP* the three sides are known, *ZS* being the complement of the observed altitude, *PS* the complement of the declination of the star, and *ZP* being the colatitude; hence the hour angle *ZPS* of the star can be found. When this is reduced to time (by dividing by 15) and the result added to, or subtracted from, the known right ascension of the star according as the star is west or east of the meridian, we obtain the sidereal time of the observation, which can be reduced to mean solar time by the method explained in Chapter XI.

In all cases when the sun, moon, or a planet is chosen instead of a star, the altitude of the lower or upper limb is measured, and to this is added the semidiameter of the body (obtained from the "Nautical Almanac") which gives the altitude of the centre.

The reason the body is chosen in, or near, the prime vertical is that, in that position, the altitude of the body is most rapidly changing, and therefore a small error in the observed altitude will produce the least possible error in the calculated time.

This latter method is one very frequently used at sea.

Longitude at Sea

180. The problem of finding the longitude is reduced to finding, as accurately as possible, the Greenwich time corresponding to the ship mean time. For (Art. 159)—

Longitude (in time) = Greenwich mean time ~ ship mean time.

There are two methods by which the longitude may thus be determined :—

(1) By the chronometer, and (2) by lunar distances.

We have already seen how Greenwich time is given by the ship's chronometers, two or three being kept, in order to check one another. The ship mean time is generally found by observing a star in the prime vertical or by the method of equal altitudes. The difference between the two times multiplied by 15 gives the longitude in degrees.

EXAMPLE.—On April 6th, when the sun's altitude was first observed, the ship's chronometer indicated $10^h\ 6^m\ 4^s$; and again, when the altitude was the same as in the first observation, the indication was $4^h\ 3^m\ 12^s$. Also, it was known that the chronometer gained 5 seconds daily, it being 6 days since the ship left port (the chronometer then indicating correct Greenwich time). The equation of time on April 6th was $2^m\ 20^s$. Find the ship's longitude.

Here, if we take half the sum of the two chronometer readings, 12 hours being added to the second one, we get $13^h\ 4^m\ 38^s$, or, subtracting 12 hours, $1^h\ 4^m\ 38^s$. From this we subtract 30 seconds, the error of the chronometer, and we have—

	H.	M.	S.
Greenwich time of local apparent noon =	1	4	8
Equation of time =	0	2	20
∴ Greenwich time of local mean noon =	1	1	48

Multiplying by 15, we get longitude = 15° 27′ W.

Longitude by Lunar Distances.

181. If the chronometers on board a ship should, from any cause go astray, so as not to be available for indicating correct Greenwich time, then the moon, by its change of position with reference to the fixed stars in its motion round the earth, serves as a fairly reliable time-keeper. In fact, we may regard the whole heavens as an immense clock-face, the stars as dial figures, and the moon as a moving clock-hand.

In the "Nautical Almanac" is given a series of tables which predict, for each day, the distances of the moon's centre from certain bright stars or planets in its neighbourhood for every three hours of Greenwich mean time. The observer, whose object is to ascertain Greenwich mean time, therefore measures with the sextant the distance of one of the given stars from the edge of the moon's limb; to this must be added or subtracted the moon's semi-diameter, in order to find the distance of the star from the moon's centre. The tables in the "Nautical Almanac" will then, on being referred to, give roughly Greenwich mean time, corresponding to this distance, with an error of perhaps one or two hours. But during the above intervals of three hours we may assume that the angular distance of the moon from the star changes uniformly, and therefore, by a statement in proportion, we can calculate the exact Greenwich mean time, from which, knowing the local time, the longitude may be found as before.

182. **Clearing the Distance.**—In the above method, by lunar distances, a correction has to be made for refraction. Also the lunar tables are calculated to predict the place of the moon as seen from the centre of the earth, and therefore we must also allow for parallax. These corrections, which are somewhat involved, constitute what is called "clearing the distance."

183. The moon's motion among the fixed stars, although very much faster than that of the sun or planets, is not sufficiently rapid to determine by the above method the longitude with very great accuracy. On account of this slowness of movement, a small error in the observed distance will produce a comparatively large error in the calculated Greenwich time, and therefore in the longitude. If the moon were to revolve about the earth in two or three days, it would be possible to find the longitude as easily as the latitude.

EXAMPLE.—On January 2nd, 1893, the angular distance of Regulus from the moon's centre was observed to be 44° 15′, the local time being $6^h\ 30^m$ p.m.; at Greenwich at 3 p.m. and 6 p.m., the distances, as given in the "Nautical Almanac," were 45° 13′ 19″ and 43° 24′ 48″, respectively. Determine the longitude of the place.

Here we have—

Angular distance at 3 p.m. = 45° 13′ 19″

" " at time of observation = 44° 15′ 0″

Change in interval = 0° 58′ 19″ = 3499″

But subtracting the angular distance at 6 p.m. from that at 3 p.m., we find that the change in 3 hours is 1° 48′ 31″ = 6511″;

$$\therefore \text{ The time in which it decreases } 3499'' = 3^h \times \frac{3499}{6511}$$

$$= 1^h\ 36^m\ 44^s;$$

$$\therefore \text{ Greenwich time} = 3^h + 1^h\ 36^m\ 44^s = 4^h\ 36^m\ 44^s.$$

$$\text{But local time} = 6^h\ 30^m;$$

$$\therefore \text{ Longitude (in time)} = 1^h\ 53^m\ 16^s;$$

or, multiplying by 15, = 28° 19′ E.

Another method of determining Greenwich time, and therefore the longitude, is by observing the occultation of a star by the moon. This is merely a modification of the method by lunar distances.

184. Attempts have been made to find the longitude by means of the eclipses of Jupiter's satellites, the Greenwich time at which the eclipses commence being foretold in the "Nautical Almanac." However, it is not possible to properly observe the eclipses by means of a telescope on board ship; and even on land it is difficult to tell exactly when the eclipse begins or ends. Also certain other celestial signals such as the beginning or end of an eclipse or the bursting of meteors have been used for the same purpose.

Since the invention of the electric telegraph, the longitude of any station on land can be determined very readily by signalling the local time at any instant from some station with which it is in telegraphic communication, and whose longitude is known, the difference of the longitudes being found by multiplying the difference of the times by 15.

Examples.

1. On March 7th, the sun was observed to have equal altitudes when the chronometer indicated $10^h\ 51^m$ a.m. and $8^h\ 38^m$ p.m. Greenwich time; hence calculate the longitude, the equation of time at Greenwich noon on March 7th and March 8th being $11'\ 12''$ and $10'\ 57''$, respectively.

Ans. $53^\circ\ 42'$ W.

2. Given the sun's computed hour angle to be 75° E. when the chronometer indicated $21^h\ 9^m\ 30^s$. Find the longitude, the equation of time being $-2^m\ 10^s$.

Ans. $32^\circ\ 55'$ W.

CHAPTER XIII.

THE FIXED STARS. SPECTRUM ANALYSIS.

185. The stars are suns situated at such immense distances from the earth that they appear, even when viewed through the most powerful telescopes, as mere points of light. Each of these distant suns is in all probability the centre of a system similar to our own, a focus from which light and heat are distributed to bodies of the same nature as our planets, the motions within each system being doubtless regulated according to the same general laws which, as all our observations show, hold throughout the universe.

We have seen in Chapter VII. that in attempting to measure the annual parallax and distances of these stars our efforts to thus make a survey of the heavens are in the greater number of instances doomed to failure, the reason being that the greatest distance available to us as a basis of observation, viz. the diameter of the earth's orbit (185,000,000 miles) actually dwindles down to our conception of a geometrical point compared with the vastness to which it has to be applied.

In the present chapter the classification of the stars is dealt with, together with a short account of the principal discoveries which have been made in modern times, chiefly by means of spectroscopic analysis, into their nature and physical condition.

186. **Star Magnitudes.**—The stars are classified into different "magnitudes," according to their degrees of brightness. There are about twenty of the most brilliant stars

which are said to be of the first magnitude. Of these there are about twelve visible to observers in our latitudes. These, together with the constellations in which they are situated, are as follows:—Sirius (Canis Major); Aldebaran (Taurus); Capella (Auriga); Vega (Lyra); Betelgeux and Rigel (Orion); Procyon (Canis Minor); Spica (Virgo); Regulus (Leo); Arcturus (Boötes); Antares (Scorpio); Altair (Aquila).

187. There are also some stars of the first magnitude whose south declinations are so great that they are only visible to observers in the southern hemisphere, as Canopus (*α* Argûs); *α* and *β* Centauri; *α* Crucis; Achernar (*α* Eridani); and *η* Argûs. Those of the second magnitude are less brilliant; there are about 50 of these visible to us. An example of this class is the Pole Star. It is not possible with the naked eye to distinguish those beyond the 6th magnitude, but further divisions of telescopic stars down to the 7th, 8th, 9th, and still lower magnitudes have been made. These divisions are in a great measure arbitrary, for stars belonging to the same class differ considerably in brightness, colour, and, as we shall presently see, in their physical condition.

188. **Number of the Stars.**—The number of the stars on the whole celestial sphere which it is possible to distinguish with the naked eye is about 6000; and from any one place, even on a favourable night, not many more than 2000 are visible at the same time, numbers of stars near the horizon being obscured by the greater thickness of atmosphere through which their rays have to penetrate. This number is not nearly so great as a person who has not made an exact estimate of them would expect, for the idea conveyed to the mind by the multitudes of shining points spangled over the heavens is that they are innumerable. With the aid of telescopes the number which it is possible to see amounts to many millions.

189. **The Milky Way.**—On any dark, clear night there will be seen stretching across the sky, almost in a great circle, a faintly luminous belt, which is called the *milky way*. Its luminosity varies greatly in different parts. The telescope shows that this phenomenon is produced by the light from countless multitudes of stars which cannot be individually distinguished with the unaided eye.

190. **Star Clusters.**—In certain parts of the heavens the stars seem so densely packed together and in so marked a manner as to lead to the supposition that they are in some way connected. Such a group is called a *star cluster*. The group called the Pleiades is seen with the naked eye to consist of about six stars, but with the aid of a small telescope the number is increased to over fifty. We have another illustration in a luminous spot in Perseus which the telescope reveals as consisting of great numbers of stars grouped together, the appearance forming a most beautiful spectacle.

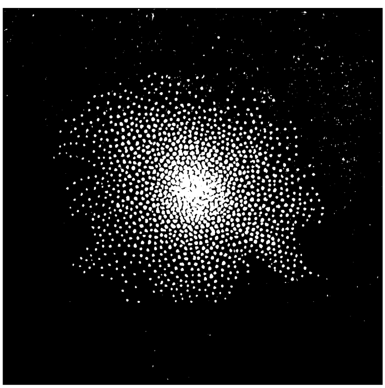

FIG. 82

Star Cluster in Hercules (SIR J. HERSCHEL).

191. **Nebulæ.**—Besides star clusters there are seen with the aid of a telescope, in different parts of the heavens, many

objects which appear as small luminous spots, and many of which cannot be considered as star clusters, as they do not seem to be resolvable into separate stars. These are called *nebulæ*. Perhaps the most remarkable illustration of these objects is the great nebula in Orion, which appears shining as a bluish mass, portions of which, under high magnifying power, are seen to contain numerous stars.

Another remarkable example is the annular nebula in the constellation of Lyra: "It consists of a luminous ring; but the central vacuity is not quite dark, but filled in with faint nebula like a gauze stretched over a hoop" (Sir Robert Ball).

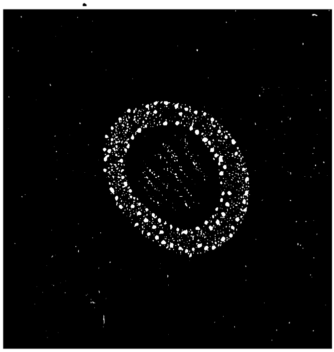

FIG 83.

Annular Nebula in Lyra (LORD ROSSE).

192. **Proper Motions of Stars.**—When the right ascension and declination of a star is observed over a long series of years it is found, after allowing for changes in these quantities due to precession, nutation, and parallax, that its position in most cases is slowly changing with reference to other stars in its neighbourhood. Each star is thus said to have a *proper motion* of its own of a character not common to the other stars. The name "fixed," therefore, applied to the stars, is not strictly accurate, and may only be used to distinguish them from the rapidly moving planets.

The proper motions of stars are due partly to a motion of the sun with the whole solar system, which is believed to be moving through space towards a point in the heavens near λ Hercules, thus causing those stars not in the line of direction of motion to appear displaced in the opposite direction. However, making allowance for displacements due to this cause there is no doubt that the stars are in actual motion themselves.

193. **Double Stars.**—With the aid of powerful telescopes it is seen that many stars which otherwise appear single are in reality double, consisting of two distinct stars. Sometimes the two components are nearly of the same magnitude, but in many double stars they are unequal, and when this is the case they are often, for some reason not as yet explained, of different colour, the smaller having a tint higher in the spectrum than the larger: for instance, if the larger be reddish, the smaller will be blue or green. About 10,000 of these double stars have been discovered. Many stars which at first appear double are only apparently so, owing to their being nearly in the same line of vision; they thus appear, when projected on the surface of the celestial sphere, very close together, when in reality they are far apart. Among the most remarkable examples of double stars are Castor, α Herculis, the Pole Star, and Sirius. Sometimes a star is seen to consist of three or four separate components; as in the case of ϵ Lyræ, in which there are four stars, three white and one red.

194. **Binary Stars.**—In many double stars the two components are seen to be in motion, each describing an ellipse round their common centre of gravity as focus, which in fact is a consequence of the law of universal gravitation (Art. 70). In the case of the double star Castor this movement is so slow that many centuries will elapse before each

will have made a complete revolution. Stars connected in this manner are called *binary* stars.

195. **Orbits of Binaries.** — The apparent angular magnitude of the regular orbit of binaries round one another can be determined by means of the micrometer; but the dimensions of the orbit in miles cannot be found unless the distance of the star, or, which amounts to the same thing, its annual parallax be known. In the case of α Centauri the parallax is about ·75″ and the semi-axis of its apparent orbit has been estimated at about 17·5″. Hence we have, by taking the ratio of the circular measures of these angles—

$$\frac{17{\cdot}5''}{{\cdot}75''} = \frac{\text{semi-axis of orbit in miles}}{92{,}000{,}000}.$$

Knowing the dimensions of the orbit of a binary star and its periodic time, it is possible to calculate the sum of the masses of the two components. For an account of the method used, see Art. 214.

196. **Variable Stars.**—There are some stars whose brightness is not constant, to which the name *variable stars* is given. Of these the most remarkable are those which are known to change their lustre periodically. It will be sufficient here to mention some of the more remarkable examples of these, each being representative of a distinct class.

197. **The "Mira Type."**—The star ο Ceti or Mira (the wonderful) goes through a regular cycle of changes in a period of 331 days, during which time it varies from the second to the sixth magnitude; it then becomes invisible for about five months, after which it gradually returns to its original brightness.

198. **The "Algol Type."**—The star Algol in the constellation of Perseus is a type of another remarkable class of periodical stars. It remains of the second magnitude for

about 2^d 13^h; its brightness then gradually diminishes till it appears of the fourth magnitude, the time taken being about $3\frac{1}{2}$ hours. For twenty minutes it remains of the fourth magnitude, when it gradually recovers its brightness until, at the end of another $3\frac{1}{2}$ hours, it appears again of the second magnitude. The whole period of this cycle of changes is 2^d 20^h 48^m 55^s. As regards stars of this type, the most probable explanation is that the loss of light is due to the interposition of some dark body which, revolving in a fixed period round the star, causes, at regular intervals, a partial eclipse to take place.

The Spectroscope.

199. We know from Optics that when a ray of ordinary white sunlight passes through a prism it emerges split up into component rays of different hues, viz. violet, indigo, blue, green, yellow, orange, and red; so that when thrown by a suitable arrangement on a screen there appears a distinct band of colours like a rainbow, having violet at one end and

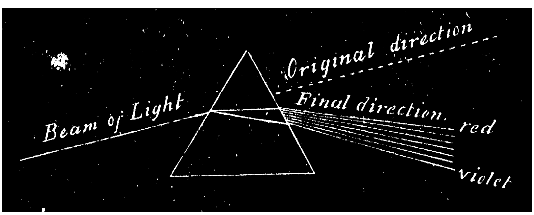

FIG. 84.

deep red at the other (fig. 84). This band constitutes what is called the spectrum of the light. In a similar manner the light from any other source may be examined. Usually the rays are admitted through a narrow slit, and, before reaching the prism are passed through a collimating lens, whose focus coincides with the slit, from which they therefore emerge as parallel rays, and, after passing through the prism, the spectrum is viewed through a telescope. Such an arrangement is called a spectroscope.

200. Thus by the aid of a simple glass prism we have the means of analysing the light from any source by decomposing it into its separate parts, and arranging these parts before our view. An examination into any deficiencies or other peculiarities in the spectrum of a beam of light thus divided enables us to learn a great deal concerning the composition and physical state of the luminous body from which the light is derived. This is the principle of the method of research known as *spectrum analysis*, which in recent years has enabled us to add so much to our knowledge of the universe.

201. **Solar Spectrum.**—Early in the present century it was discovered by Fraunhofer that the spectrum of the sun is not a continuous succession of colours, but is interrupted by thousands of fine dark lines. In certain parts of the spectrum but few of these dark lines occur, but in other portions they are so crowded together that it is difficult to distinguish them individually. It is also found that the arrangement of these lines is a characteristic and permanent feature of sunlight, so characteristic that their number and the positions which they occupy in the spectrum enable us to distinguish sunlight from that due to any other source.

From this it is evident that only those rays reach us from the sun which are of certain definite degrees of refrangibility, while, from some cause unknown at the time of Fraunhofer's discovery, other rays, corresponding to the dark lines, are absent. This phenomenon was at last explained by an examination of different kinds of artificial light.

202. When rays of light from different sources are observed by means of the spectroscope it is found that

their spectra may be divided into the two following classes:—

(1) The spectra of luminous solids and liquids are continuous, containing light of all degrees of refrangibility, and therefore show no dark lines.

(2) The spectra of the flames of burning gases, not containing solid particles in suspension, are discontinuous, consisting merely of a certain number of finite bright lines interrupted by dark bands.

203. **Reversal of Bright Lines.**—If a burning gas or vapour emitting, as we now see, only rays of certain degrees of refrangibility, be interposed between the observer and the light from a source giving a continuous spectrum, the gas *will absorb rays of the same kind as those which it emits*, and it depends on the relative brightness of the two sources of light as to whether this particular class of rays shall, in the combined spectrum, be darker or brighter than the rest. The following experiment will serve to illustrate this important principle:—

Let the spectrum of the burning vapour of sodium (got by colouring the flame of a spirit lamp with common salt, of which sodium is a constituent) be observed, and it is found to merely consist of two *bright* yellow lines. However, if very bright limelight be placed behind the sodium flame, it is found that the continuous spectrum which limelight would afford, if viewed alone (being that of an incandescent solid), is crossed by two *dark lines* corresponding to the two *bright lines* of sodium. On removing the limelight these lines flash out as bright as before. The bright lines produced by sodium are thus reversed, *i.e.* changed to dark ones; not that they are actually darker than when viewed alone, but that they *appear dark by contrast* with the brilliance of the rest of the spectrum; for, those particular rays from the limelight which are characteristic of sodium, and which would otherwise illuminate the spaces occupied by these lines, so as to make them appear as bright as the rest of the spectrum, are cut off by the interposition of the sodium flame, while all other rays pass freely through.

204. By means of the above principle, the occurrence of dark lines in the solar spectrum can now be explained by supposing the sun to be surrounded by an external vaporous layer which absorbs rays of its own peculiar kind from the light coming to us from an inner stratum of the sun called the *photosphere*, which would otherwise give a continuous spectrum. Kirchoff showed that these lines correspond to those of hydrogen, iron, zinc, nickel, copper, and other metals which, existing in a state of vapour in the solar atmosphere, cause the reversal of their characteristic rays.

205. **Surface of the Sun. Solar Prominences.**—During recent total eclipses the spectroscope has been applied to investigate the nature of the outer surface of the sun. Outside the photosphere is the chromosphere, so called on account of its bright red colour, due probably to intensely heated hydrogen, of which it is mainly composed. Beyond this lies the corona, a ring of light surrounding the sun seen during a total eclipse, whose spectrum gives but faint indications of hydrogen, and whose chief characteristic is a conspicuous green line.

During a solar eclipse, when the sun's disc is hidden by the moon, there are seen, apparently, on the edge of the moon's disc, some remarkable objects now known as *solar prominences*. At first, when their nature was unknown, they were simply called *prominences*, as it was uncertain whether they were appearances on the outer surface of the moon or sun; but it was definitely proved that they were solar during the eclipse of 1860 by means of photographs which showed that the moon's disc moved over them just as it did over other portions of the sun's surface. They appear as flame-like objects, scarlet in colour, and they vary in a wonderful manner as regards their form and magnitude. Some of them attain a height of over 80,000 miles. With what rapidity they are capable of changing will be seen from an example chronicled by Professor Young, and observed at Princeton, New Jersey, in which one of these prominences, which at first was about 40,000 miles high, suddenly shot up until it attained an elevation of 350,000 miles, when it gradually broke up, and eventually faded completely away, the whole series of changes being completed in an interval of two hours. These prominences are now known to be protuberant portions of red incandescent gas, principally hydrogen from the chromosphere. In order to observe these prominences it is not now necessary to wait until a total eclipse of the sun occurs, as by a proper arrangement of the spectroscope the changes in their structure can be studied almost as well as during an eclipse, with the additional advantage of being able to observe them for a much longer period of time.

206. The spectrum of a planet would be identical with that of the sun were it not that it is somewhat altered owing to the solar light having to pass twice through the thickness of the planet's atmosphere, which causes absorption of some of the rays. In the case of some of the planets there are characteristic lines indicating the presence of vapour of water under conditions similar to that which is present in the atmosphere of the earth. The spectrum of the moon is identical with that of the sun, which is confirmatory evidence that it possesses no atmosphere.

207. **Star Spectra.**—The stars may be divided into different classes according to their spectra :—

(1) Those whose spectra are distinguished by comparatively few lines, the most prominent corresponding to

hydrogen at a very high temperature. In this class are included all the white or bluish stars such as Sirius and Vega.

(2) In the second class are such stars as Aldebaran and Arcturus, whose spectra are similar to that of the sun, *i.e.* are intersected by large numbers of fine lines indicating the presence of many metals in addition to hydrogen.

(3) In the third class are those whose spectra are interrupted by dark broad bands. Most of these stars are red, and a large number are variable.

Many of the metals present in the sun have been observed in the stars. Thus in Aldebaran there is evidence of sodium, iron, bismuth, antimony, magnesium, calcium, mercury, and tellurium.

208. **Spectra of Nebulæ.**—It was first observed by Huggins that some of the nebulæ afford spectra which are not continuous bands of colours, crossed by dark lines, like those of stars, but merely consist of a few bright lines. Four bright lines are usually easily observed, two of which are certainly due to hydrogen, but the nature of the others remains as yet unknown. These nebulæ cannot therefore consist of aggregations of separate stars, but of glowing gaseous material, of which hydrogen is a chief constituent. The great nebula in Orion is typical of this class. Many other nebulæ, however, of which that in Andromeda is a type, afford spectra of a totally different kind; instead of a few bright lines we find a continuous spectrum, which would point to the conclusion that the light may be due to an immense cluster of minute stars. Nebulæ of this type are generally white, while the gaseous nebulæ are of a bluish tint.

209. One of the most remarkable applications of the spectroscope is to measure the velocity of a star along the line of sight either towards or from the earth. If rays of

light proceed from a body which is approaching the earth their wave-lengths will be diminished, for the number of vibrations reaching the earth in each second will be greater than if the body were at rest. On the other hand, when receding from the earth there will be a corresponding increase in the wave-lengths. But the refrangibility of each ray depends on its wave-length, a diminution in which causes the ray to fall nearer the violet end of the spectrum. Hence, by comparing the positions of the lines in the spectrum of a star corresponding to hydrogen or some other well-known substance with the positions which these lines occupy in independent observations, it can be determined whether the wave-lengths are diminished or increased, and consequently whether the star is moving towards or from the earth, and with what velocity. This method has been applied to calculate the velocities of a considerable number of the stars, and the results obtained in some cases indicate a speed along the line of sight of from 20 to 30 miles per second.

"The theory of this method is beautifully verified by observations on the sun. As the eastern edge of the sun is approaching and the western is receding, there is a corresponding difference in the spectra of the two edges, and the observed amount gives a velocity of rotation practically coincident with that otherwise known."—(Sir Robert Ball.)

Note on the Celestial Globe.

On the surface of a celestial globe are marked the apparent positions of the stars and the different circles of the celestial sphere. The globe revolves within a framework consisting of a brass meridian (graduated), which remains in a vertical plane, and a broad horizontal wooden ring which represents the horizon. On this wooden horizon are marked the months and days of the year, the equation of time for each day, and the daily longitude of the sun; there are also the twelve signs of the zodiac in their order, and, finally, a circle divided into degrees for the purpose of measuring azimuths.

The bearings on which the globe revolves are attached to two diametrically opposite points of the brass meridian corresponding to the north and south celestial poles.

At the northern pole of the globe is seen a small brass circle called the *hour index*, on which are marked the hours of the day as on the face of a clock. This circle can be turned round with the fingers, so that any hour desired may coincide with the meridian, but, once set, there is sufficient friction to cause it to turn with the globe when the latter is rotated.

As regards the different circles drawn on the globe it is to be observed that the equator and ecliptic are graduated into degrees and fractions of a degree.

To set the Celestial Globe so as to show the Appearance of the Heavens at a given place and at any given apparent time.

(1) Elevate the pole to an angle equal to the latitude of the place by means of the graduations on the meridian.

(2) Find the position of the sun in the ecliptic on the day in question (the spot can be marked with a small piece of gum-paper stuck on the globe); rotate the globe until this point coincides with the meridian, and set the hour index at XII. This position corresponds to apparent noon.

(3) Finally, turn the globe until the required hour is brought to coincide with the meridian, which gives the required position.

If the globe be set so that the plane of the meridian is due north and south, the actual direction as well as the relative position of any star in the heavens will be indicated.

To determine the Apparent Time at which a Heavenly Body rises or sets at a given place.

(1) Make the elevation of the pole equal to the latitude of the place.

(2) Rotate the globe until the position occupied by the sun in the ecliptic coincides with the meridian, setting the hour index at XII.

(3) Again rotate the globe until the heavenly body in question is brought to the eastern or western horizon, according as we wish to find the time of rising or setting, and the hour index will show the required time.

Similarly the time at which a heavenly body culminates may be found.

Exercises.

(1) Find by means of a globe, the apparent times of sunrise and sunset on the 15th April.

(2) Find how long twilight lasts on the same date.

(3) Find the length of the day at Dublin (lat. 53° 20′) on 25th November.

(4) At what hour does Sirius cross the meridian of Dublin on (1) the 10th August, (2) the 14th December?

(5) At a place whose latitude is 47° the bright star Arcturus rises on a certain date at 8 p.m. What is the date?

(6) In what part of the heavens should we expect to see the bright star α Lyræ at 8 p.m. on the 15th October, at Dublin?

To draw a Meridian Line on the Earth.

On a fixed horizontal plane, describe with a compass several concentric circles. At the common centre of the circles, erect a small piece of straight wire at right angles to the horizontal plane. In the forenoon, let the point where the shadow of the top of the wire just touches any one of the circles be marked, and also the point where the shadow just reaches the same circle again in the afternoon. If a radius be now drawn bisecting the arc between these points, this radius will coincide with the meridian. Several circles are drawn lest a cloud over the sun should interfere with the observation.

To show on the Celestial Sphere the positions of the horizon, ecliptic, equator, and the sun at 9 a.m. on the 21st June at Dublin (lat. 53° 20′).

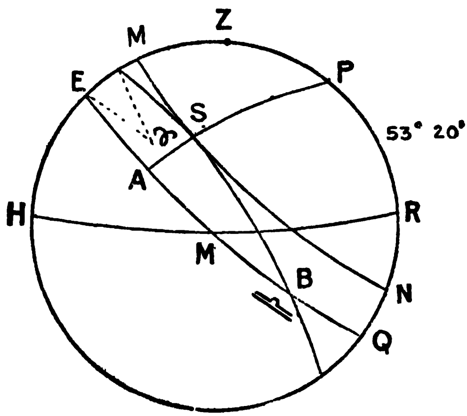

FIG. 85.

Here make *PR* (fig. 85) the altitude of the pole above the horizon HR = 53° 20′. Cut off EM = 23° 28′, and MN, drawn parallel to the equator EQ, represents the sun's apparent diurnal path on the 21st June. Then, since the hour angle of the sun at 9 a.m. is 45° east, therefore bisect EM in A, and join AP, and the angle ZPA is an hour angle of 45° east, and the position of the sun at this hour is S where PA intersects MN. Then, since the right ascension of the sun on 21st June is 90° measured eastwards from ♈, the sun is therefore 90° west of ♎, hence cut off AB equal to 90°, and ♎ is at B, and the ecliptic is SB cutting the equator in ♎ and also in ♈ on the opposite or western side of sphere.

To show on the Celestial sphere the angles which measure sidereal time, mean solar time, the right ascension of the mean sun, apparent solar time, and the equation of time.

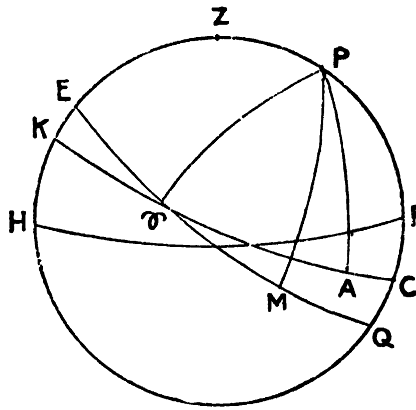

FIG. 86.

Here KC represents the ecliptic cutting the equator EQ in ♈; M and A are the mean and apparent suns respectively, the former in the equator, the latter in the ecliptic. Then the angle ZP♈ measures the sidereal time, ZPM the mean solar time, and ♈PM is the right ascension of the mean sun; the apparent solar time is measured by the angle ZPA, and the angle MPA measures the equation of time.

EXERCISES.

1. Draw diagrams of the celestial sphere as seen from Dublin (1) at sunrise, (2) at noon, (3) at 3 p.m. on the 21st March.

2. Draw diagrams of the celestial sphere as seen from a place whose latitude is 80° N. at 10 a.m., and also at noon on the 21st May.

3. If y be the length of a tropical year, s the length of a sidereal day, and m the length of a mean day, prove

$$\frac{1}{s} - \frac{1}{m} = \frac{1}{y}.$$

EXAMINATION PAPERS

AND

MISCELLANEOUS QUESTIONS.

EXAMINATION PAPERS.

The following questions from 1 *to* 100 *have been set to third and fourth year Students in the University of Dublin, each set of ten questions constituting a paper. The remainder of the questions are taken from the Degree Examination Papers set at the London and Royal Universities:—*

I.

1. State what you know of "double stars," "new stars," "periodical stars," and "nebulæ."

2. State clearly the reason why the discs of the sun and moon appear oval when near the horizon.

3. "The north celestial pole, therefore, will be, about 13,000 years hence, nearly 49° from the polar star." Give a clear explanation of this statement.

4. What is "annual parallax"?

(*a*) If p be the number of seconds in the annual parallax of a fixed star, show that the time taken by light to reach us from this star is, approximately, $\frac{16}{5p}$ years.

5. Give an account of the different explanations of the planetary motions called, respectively, the Copernican system and Ptolemaic system; and show how the former may be verified in the case of an inferior planet.

6. Assuming Kepler's laws, prove that the velocities of any two planets are connected with their distances from the sun by the relation

$$v : v' :: \sqrt{r'} : \sqrt{r}.$$

7. Show how to find the moon's sidereal period. How is the exact synodic period determined?

8. What is the cause of a lunar eclipse? Why does the phenomenon not occur at every full moon? How are the lunar ecliptic limits found?

9. Describe the Meridian Circle, and show how it may be used to find (*a*) the declination, and (*b*) the right ascension, of a star.

10. Find the time at Vienna (16° 20′ E.), and at Chicago (87° 40′ W.), when it is 10 o'clock, a.m., at Dublin (6° 15′ W.).

II.

11. Show how a degree of a meridian is measured; and assuming the length of a degree to be 69½ miles, find the earth's diameter in miles.

12. Show by a figure the effect on the position of a star of the refraction of light by the atmosphere; and prove that the amount of refraction varies approximately as the tangent of the star's zenith distance.

13. What is the cause of twilight?

(*a*) Does twilight ever last all night at Paris (lat. 48° 50′)? Give a reason for your answer.

14. A circumpolar star crosses the meridian at altitudes 10° 11′ 17″ and 72° 15′ 31″; find the latitude of the place, and the star's polar distance.

15. The interval between eastern and western quadratures of Jupiter is 175 days, and between two oppositions 400 days, approximately; find the annual parallax of this planet.

16. Show how the height of a lunar mountain may be obtained by measuring the distance from the boundary of light and darkness of a bright spot observed in the unilluminated part of the moon's disc, and prove the approximate formula—

$$\text{height in miles} = 537m^2 \operatorname{cosec}^2 e,$$

where m is the ratio of the observed distance to the moon's radius, and e the moon's elongation.

17. Prove the formula for finding the periodic time of a superior planet by means of the earth's periodic time and the observed interval between two successive oppositions.

18. Prove that more than half the disc of a superior planet is always seen, and that the planet is most gibbous in quadrature.

19. Give a general explanation of solar and lunar eclipses.

20. Define the "equation of time," and state from what causes it arises. What is its greatest value? How many times in the year does it vanish, and at what dates?

III.

21. The apparent zenith distances of γ Draconis at lower and upper culminations were $75^\circ\ 3'\ 13''\cdot 2$ and $1^\circ\ 53'\ 18''\cdot 6$ south; the amounts of refraction in the two observations being $3'\ 41''\cdot 9$ and $1''\cdot 9$, respectively. Find the declination of the star, and the latitude of the place.

22. Explain how the meridian altitude of a star can be observed correctly to the fraction of a second.

23. Give a direct explanation of the effect of aberration on the positions of stars, and indicate on the celestial sphere the point towards which they are displaced.

24. Calculate the number of seconds in the coefficient of aberration.

25. Explain the nature of the phenomena—(*a*) *precession of the equinoxes;* (*b*) *nutation.* By what observations may their existence be detected? What are their periods and amounts?

26. Determine an expression for the angle which the breadth of the shadow, cast by the earth at the distance of the moon, subtends at the earth, in terms of the semidiameter of the sun and the horizontal parallaxes of the sun and moon. When is this a maximum, and when a minimum?

27. What is meant by the term "radiant point of a meteoric shower"? By what arguments is the connexion between comets and meteors established?

28. Assuming that the planetary orbits are circles with the sun in their common centre, find an expression for the annual parallax of Jupiter in terms of the synodic period, and the interval between two consecutive quadratures.

29. Explain the Lunar Method of finding longitude at sea, and point out its disadvantages.

30. Examine the statement:—"If the moon had moved round the earth in about three days, the longitude would have been as easily found as the latitude."

IV.

31. Describe a transit instrument, and state the nature of the errors for which allowance has to be made.

32. How would you determine the angle subtended by the earth's disc at the moon?

33. Prove the following rule for determining from the true position of a star σ its apparent position σ' due to aberration:—Let S be the sun, and E a point on the ecliptic 90° behind S, then σ' lies on the great circle σE and $\sigma\sigma' = 20''\cdot 5 \sin \sigma E$.

34. Prove Bradley's formula for the coefficient of refraction, and state accurately what observations have to be made in applying it.

35. Define the terms "lunation" and "periodic time" of the moon; and find the periodic time, being given that a lunation is 29·5305887 days.

36. Verify the following statement:—"At the end of 19 years the sun and moon return to the same relative position with regard to the fixed stars, and the full moons fall again on the same days of the month, and only one hour sooner."

37. Show by a figure the relative positions of the *Plough*, *Pole star*, *Arcturus*, *Spica*, *Capella*.

38. Prove that when a transit of Venus is about to take place, Venus and the sun approach each other at the rate of 4″ per minute.

39. Describe Foucault's pendulum experiment for the latitude of Dublin, and give its explanation.

40. How would you propose to draw to scale the orbit of the earth as a result of observation?

V.

41. Describe a transit instrument, and define accurately its line of collimation.

42. Explain how an angle can be measured to a fraction of a second.

43. State some facts which have been accurately observed which can only be explained on the hypothesis of the earth's rotation on its axis.

44. Give a direct explanation of the aberration of light, and calculate the constant.

45. Assuming that the amount of refraction varies as the tangent of the zenith distance, indicate how the coefficient is determined.

46. In the "Nautical Almanac" the declination of the sun is given for times separated by intervals of three hours. Show how it can

be determined for any instant whatever, and hence deduce the latitude of the place from one observation of the sun.

47. Represent on a diagram the relative positions of the equator, ecliptic, and horizon at sunset on the evenings of the vernal and autumnal equinoxes; and for the latitude of Dublin in each case calculate the angle between the latter pair of circles.

48. Give the meanings of the terms *nutation* and *precession of the equinoxes*. How was their existence detected?

49. Prove the following statement, connecting the mean time at a given meridian with the corresponding sidereal time:—

Sidereal time = mean time + mean sun's right ascension.

50. What observations are necessary in order to determine the periodic time of the moon? Give the formula which is required in the subsequent calculation.

VI.

51. Assuming that the earth is spherical, explain how her diameter may be measured.

52. Explain, by means of a diagram, how the change in the sun's declination produces the succession of the seasons.

53. Determine the limits of the latitude of places at which twilight lasts all night long, when the sun's declination is $+10^\circ\ 15'$.

54. Assuming the horizontal parallax of the moon to be $\frac{1}{60}$, and her apparent diameter to be 1963″, find the moon's diameter in miles.

55. The interval between the inferior conjunctions of Mercury is 115·8 days; find the periodic time of Mercury.

56. Write a short account of the rising and setting of the moon at different seasons of the year.

57. Explain how the meridian of a place may be found.

58. Give an account of the lunar method of finding the longitude at sea, and the objections to it.

59. How are the retrograde and stationary appearances of the planet Venus explained?

60. What is Bode's law of the distances of the planets?

VII.

61. How would you make use of a celestial globe to find out what stars would be visible in a given place at a known date, and at a given hour of the night?

62. Assuming that all the other corrections have been made, how would you make sure that the great circle in which the line of collimation of a transit instrument moves coincides with the meridian of the place?

63. An altitude of a star is observed and found to be the angle whose sine is $\frac{6}{13}$; calculate the true position of the star, assuming the amount of refraction at an altitude of 45° to be 58″·2.

64. At mean noon on a given date, the sidereal time was 14 hours; what will be the sidereal time 50 days after, at mean noon, in the same place? You are given that the length of a tropical year is 365¼ days.

65. Explain the different causes that enable us to see somewhat more than exactly half the surface of the moon.

66. A star in the ecliptic has a longitude or 75°, obtain the change in the position of the star owing to aberration, when the longitude of the sun is 135°, assuming the constant of aberration to be 20″·45.

67. Explain a method of finding the ratio of the distances of Venus and the earth from the sun.

68. Describe accurately how the latitude can be found at sea.

69. In connexion with the satellites of Jupiter, we can observe the following: *transits of their shadows* over his disc, *eclipses*, *occultations* and *transits* of the *satellites*; explain these phenomena by means of a diagram.

70. Assuming that the orbits of the planets are circles described with uniform velocity in the same plane, prove the formula for the periodic time of a planet when the time between two conjunctions is known.

VIII.

71. Given a celestial globe, describe how you could use it to find out at what time, approximately, Regulus will culminate on January 23, 1893, in Dublin.

72. Define the terms—right ascension, declination, celestial longitude, azimuth.

73. The apparent diameter of the sun when least is 31′ 32″, and when greatest 32′ 36″·4; hence calculate the eccentricity of the orbit of the earth round the sun.

74. Give a general description of the movements of the moon round the earth.

75. Draw a figure to show the apparent relative sizes of Venus, and the portions of her disc which appear bright, just before inferior conjunction, when at her greatest brilliance, when gibbous, and when at superior conjunction, respectively.

76. Jupiter's outer satellite is at a distance of 1,170,000 miles from Jupiter, and takes 16 days 16¾ hours to complete one revolution round him; given that the innermost satellite is at a distance of 262,000 miles, find the time it takes to revolve round Jupiter.

77. Prove that the apparent motion of Mars is retrograde when we are closest to him, and direct when we are farthest from him.

78. Owing to the aberration of light, a star can at most be displaced from its true position through an angle of 20″ 3; hence calculate the velocity of light assuming that the velocity of the earth is 19 miles per second.

79. Assuming that, on a certain day at Greenwich, the right ascension of the mean sun was 10 hours at 12 o'clock, find, for a place whose longitude is 60° west, the time by an ordinary clock on that same day, when the time by an astronomical clock at the place was 14 hours. You may assume that a sidereal day contains $23^h\ 56^m\ 4^s$ mean time.

80. In what respects are the motions of the planets strikingly similar, and strikingly different from the motions of such comets as Encke's, Biela's, and Halley's?

IX.

81. Explain by what measurements and calculations the length of the earth's diameter is obtained.

82. State and prove the law of atmospheric refraction.

The horizontal refraction is about 35′. How is this proved?

83. State the arguments for the annual motion of the earth round the sun.

84. State what you know of solar spots, and the inferences drawn from observations made on them.

85. If an observer could reach the North Pole, and remain there from the autumnal equinox to the vernal equinox, what appearance of (1) sunlight, and (2) moonlight would he observe?

86. Calculate the moon's sidereal period, assuming the synodic period to be 29·53 days. How is the latter period determined accurately?

87. The retardation of rising on successive nights of the new moon nearest the vernal equinox is less than that of the new moon at any other time of the year. Explain this phenomenon.

88. State the cause of a solar eclipse, and explain under what circumstances it is (1) total, (2) partial, or (3) annular.

89. Assuming the velocity of the planet Mercury to be 30 miles per second, determine the velocity of Saturn by an application of Bode's law.

90. Explain accurately the methods of finding longitude at sea (1) by chronometers, and (2) by observations on the moon.

X.

91. What are the observations by which the distance from the earth to the sun is ascertained in terms of the standard yard? (The earth may be supposed a perfect sphere.)

92. What further observations determine the distances of certain of the fixed stars?

93. The latitude of John o' Groat's house is 58° 59′ N. Find the sun's meridian altitudes at that place on midsummer and midwinter days, respectively.

94. Why is it that some of the planets are seen at all angular distances from the sun, others only when they are within a certain angular distance from the sun?

95. How is an astronomical clock regulated?

96. How do you account for the fact that some of the great meteoric displays are periodic?

97. Explain how the fact that the moon always turns the same face towards the earth enables its time of revolution on its own axis to be calculated.

98. Why is summer longer than winter? Is it the case for the southern hemisphere?

99. How does the duration of twilight at a given place alter with the season of the year?

100. How is it shown that the elevation of a star, due to refraction, varies as the tangent of the zenith distance?

XI.

MISCELLANEOUS QUESTIONS.

101. State the law of Kepler relating to the periods of the planets, and deduce it from the law of gravitation for the case of circular orbits.

Calculate the periodic time of a meteorite describing a circular orbit round the sun close to its surface. (B.A. Lond.)

102. How long does the sun take to rise at a point on the Equator on March 21st? Is this interval greater or less than at other times and places? How much would it be altered for the case of an observer on the deck of a ship sailing due east at 10 miles an hour? [The diameter of the sun may be taken as half a degree.] (B.SC. Lond.)

103. How is the declination of a heavenly body obtained by observation?

The declination of the moon's centre is observed at the same instant at two observatories in different latitudes; show that the results will differ by an angle of the same order of magnitude as a degree, and state precisely the meaning of the moon's declination as tabulated in the "Nautical Almanac." (B.SC. Lond.)

104. The constellation of the Southern Cross is in right ascension 12°, and North Polar distance 152°; find the most northerly latitude at which it is ever visible, and the time of year at which it may be seen at such a place. (B.SC. Lond.)

105. Give an exact definition of longitude on the earth's surface, and state the principle by means of which longitudes are ascertained.

Find the ratio of the lengths of a degree of longitude at the equator and in latitude λ, supposing the earth to be a sphere. (B.A. Lond.)

106. Explain the cause of the seasons. What would be their character if the earth's axis were nearly at right angles to the ecliptic. (B.A. Lond.)

107. How is a star's parallax determined? What is the distance of a star which shows a parallax of $0''\cdot5$, correct to the number of figures that would be reliable in practice? (B.A. Lond.)

108. Given that one lunation is 29·5306 days, and the period of the synodic revolution of the moon's node is 346·66 days; prove, with full explanations, that eclipses of the sun and moon will repeat themselves after an interval of about 18 years 10 days. (B.SC. Lond.)

109. Explain the influence of the aberration of light on astronomical observations. Should observations of the stars be corrected for aberration due to the motion of the solar system through space? (B SC. Lond.)

110. The eccentricity of the earth's orbit is $\frac{1}{60}$; find what percentage of the coefficient of aberration is in consequence variable throughout the year. (B.SC. Lond.)

111. Why is it that the time of sunrise, as observed by an ordinary clock, becomes later for some days after the shortest day? (B.A., R.U.I.)

112. From what observations has the precession of the equinoxes been determined? What is its effect on the right ascension, declination, latitude, and longitude of a star. (B.A., R U.I.)

113. How would you find the sun's declination when it rises at the N.E. point of the horizon? (B.A , R.U.I.)

114. Knowing the periodic time of Venus and the earth, how, from observing the interval between the greatest eastern and western elongations of the planet, can its distance from the sun be compared with the radius of the earth's orbit? (B.A., R U.I.)

115. Given the maximum elongation of an inferior planet, calculate its periodic time. What is the synodic period of a planet? (B.A., R.U.I.)

116. Show that the moon, moving under the attraction of the earth and the sun, is always subject to a component force tending to the sun, and therefore that its orbit round the sun is concave to that body. [The radius of the moon's orbit may be taken as $\frac{1}{400}$th of the radius of the earth's orbit, and the lengths of the month and year may be assumed.] (B.SC. Lond.)

117. Richer discovered, in the 17th century, that a pendulum-clock, regulated at Paris, lost 2 minutes 28 seconds daily when taken to Cayenne; deduce the conclusion that the value of gravity is smaller at Cayenne than at Paris by ·11 of a foot-second unit approximately. (B.SC. Lond.)

118. How are the masses of the planets determined from observations of the periodic times of their satellites? (B.SC. Lond.)

119. Explain how, by observation of the transits of a circumpolar star across the meridian, to determine—(1) the declination of the star; (2) the latitude of the observatory.

An error of one second in the latitude corresponds to an error of how many feet in position on the earth's surface? (B.SC. Lond.)

120. How is the rate of error of the astronomical clock in an observatory exactly determined? (B.A. Lond.)

121. What is the *equation of time*? How does its magnitude vary throughout the year?

On November 1st the sun rises at $6^h\ 56^m$ and sets at $4^h\ 31^m$ mean time; find the equation of time for that day. (B.SC. Lond.)

122. How might the length of a degree of longitude be directly measured? How does it vary in different latitudes?

Truro is marked on a map as being 20 min. 32 sec. slow by Greenwich time, and Norwich as 5 min. 8 sec. fast; what is the difference of their longitudes in degrees? (B.SC. Lond.)

123. Explain how the moon's distance has been determined. (B.SC. Lond.)

124. Describe and explain the effect of the aberration of light on the apparent position of the stars. (B.SC. Lond.)

125. Show that a swarm of meteors passing through the earth's atmosphere in parallel lines appears to radiate from a point; and show how, by taking account of the direction of this point and the motion of the earth and the velocity of the meteors, to determine the direction of their actual motion through space. (B.SC. Lond.)

126. If the apparent meridian altitudes of a circumpolar star are 45° and 60°, find the latitude of the place and the declination of the star, the coefficient of refraction being $58''{\cdot}2$.

127. What conditions would have to be taken into account in calculating when Venus is brightest? Give her position approximately, and describe her appearance when she is brightest.

128. When the astronomical time at Dunsink (longitude $25^m\ 22^s$ W.) was $5^h\ 10^m\ 16^s$ on September 1st, 1893, what was the mean time? You are given that the right ascension of the mean sun at mean noon at Greenwich on September 1st, 1893, was $10^h\ 43^m\ 29^s$, and that the earth rotates on its axis in $23^h\ 56^m\ 4^s$.

129. If a star is situated on the ecliptic, show that its parallax is nothing when its aberration is a maximum, and *vice versa* when its aberration is nothing, its parallax is a maximum.

130. The right ascension of Sirius is $6^h\ 38^m$, and the south declination is 16° 30′. In what month would you expect it to be due south about 6 o'clock in the evening? Would you expect it to be visible in June at any time in the British Isles? State generally in what portions of the world you would expect it to be visible in that month.

131. How would you calculate the coefficient of refraction by observations on one circumpolar star whose declination is known?

132. Calculate the average retardation of the moon in rising, and show by a figure, for the latitude of Dublin, when the actual retardation is greatest and least.

133. Explain how it is that Venus appears both as an evening and a morning star. Jupiter and Venus are evening stars, and stationary; find which way they will begin to move. (R.U.I.)

134. Does the time given by a sun-dial agree with that given by an ordinary clock? Explain your answer, and examine when that part of the equation of time arising from the eccentricity of the earth's orbit is positive. (R.U.I.)

135. Prove that the displacement of a heavenly body by refraction is approximately proportional to the tangent of the zenith distance for moderate zenith distances; and show how the index of refraction of air at the earth's surface might be determined by observing the two meridian altitudes of a circumpolar star whose declination is known? (R.U.I.)

136. In an observatory, how would you proceed to determine the right ascension of a fixed star, the index error of the sidereal clock and the right ascensions of all other stars being supposed known. (B.A., R.U.I.)

137. To what is the equation of time due? Trace approximately its value throughout the year; and state whether it remains constant for the same day in successive years. (B.A., R.U.I.)

138. The declination of a given star is known. Show how, from observations of it, you would deduce your latitude. (B.A., R.U.I.)

139. Describe the phenomenon known as aberration. Show that from it and Foucault's determination of the velocity of light, the earth's distance from the sun may be deduced. (B.A., R.U.I.)

140. Assuming that the planets move round the sun in circles, use Kepler's third law in deducing the law of inverse squares as regards gravitation. (B.A., R.U.I.)

141. Determine the lowest latitude at which twilight lasts all night. Why is it that twilight does not last so long in the tropical regions as elsewhere? When is its duration shortest? (R.U.I.)

142. Explain how the latitude of a place can be determined by observing a known star when crossing the meridian. At a place in latitude 50° N., what are the limits of position of those stars which remain always above the horizon? (R.U.I.)

143. Represent, on one diagram, the celestial equator, horizon, ecliptic; the latitude, longitude, declination, right ascension, hour angle, and azimuth of a star.

144. Name the principal stars of the first magnitude visible to inhabitants of Dublin, and point out how you would recognise them.

145. Suppose you were asked to determine the errors in a transit circle, describe, in proper order, how you would proceed. Explain also what is meant by the polar and zenith points.

146. How are the coefficient of refraction at and the latitude of a place determined by the same set of observations?

147. Explain how you would determine the altitude of the sun by means of an artificial horizon.

148. About what time on the 21st March is the first point of Aries on the horizon? Within what limits does the true time lie?

149. Describe a sundi' '

150. It sometimes happens that three eclipses occur within one month. Explain.

151. State fully the arguments from which the connexion between meteoric showers and comets has been inferred.

152. How are the local time and longitude at sea determined?

153. How is the angle which the earth subtends at the moon found?

154. Describe the position and motion of the moon's orbit in space, and explain why eclipses recur.

155. The apparent zenith distances of a star at upper and lower culminations were 75° 3′ 13″, and 1° 53′ 19″ south; the amounts of refraction were 3′ 42″, and 2″, respectively. Find the latitude of the place. Draw a figure to represent the position of this star at its upper and lower transits, and insert the numbers.

156. What is the theory of meteoric showers?

157. Indicate how you would verify the following statement:—"At the end of 19 years the sun and moon return to the same position with regard to the fixed stars, and the full moon's fall again on the same days of the month, and only an hour sooner."

1 year = 365·25 days. 1 lunation = 29·53 days.

158. How is the radius of the earth determined in miles?

159. You are in an observatory, and are in possession of a "Nautical Almanac"; explain how you would test the accuracy of your watch.

160. How would you draw a meridian line at any place on the earth?

INDEX.

N.B.—The Numbers refer to the pages.

THE END.