SKELETON PROBLEMS IN GEOMETRY

By

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PREFACE

This book, intended as a companion to *Skeleton Theorems*, is designed to introduce to the pupil the more usual devices employed in the solution of geometrical problems.

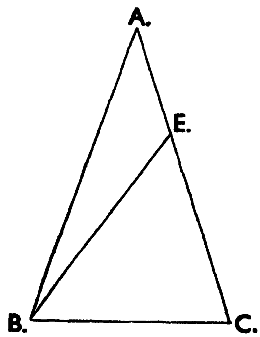
In many cases the skeleton is followed by problems using the same device which the pupil should solve for himself.

I am indebted to the various examining bodies for permission to use and quote questions taken from recent papers. These are indicated in the text as follows :—

Cambridge Local Examinations Syndicate [C].
University of London School Certificate [L].
Oxford Delegacy for Local Examinations [O].

W.E.W.

1. ABC is a triangle in which AB = AC. E is a point in AC such that CB = CE. Find the value of ∠ABE in terms of ∠BAC.



Since AB = AC.

∠ABC = = (180° −) = 90° −

Since CB = CE.

$= \frac{1}{2}($ $- \angle ACB) = \frac{1}{2}\left(90° + \frac{A}{2}\right)$

=

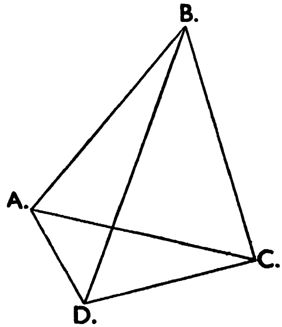
∴ ∠ABE = −

= .

2. ABC is a triangle in which the angle ACB is obtuse. The internal bisector of the angle BAC meets BC at D, and the external bisector of this angle meets BC produced at E. If AD = AE, prove (i.) that the angle ADE is 45°, and (ii.) that the angle BCA exceeds the angle ABC by 90°. [O]

3. In a certain triangle ABC, AB = AC and D is a point in AB such that AD = DC = BC. Prove that angle ABC = 2 angle BAC and find angle BAC in degrees. [L]

4. ABCD is a quadrilateral in which the side AB is equal to the diagonal AC ; prove that the side CD is less than the diagonal BD. [C]



Join BD.

Since AB = AC

$\angle$ $= \angle$

$\therefore$ $\angle$DBC = –

is less than

= $\angle$ACB + .

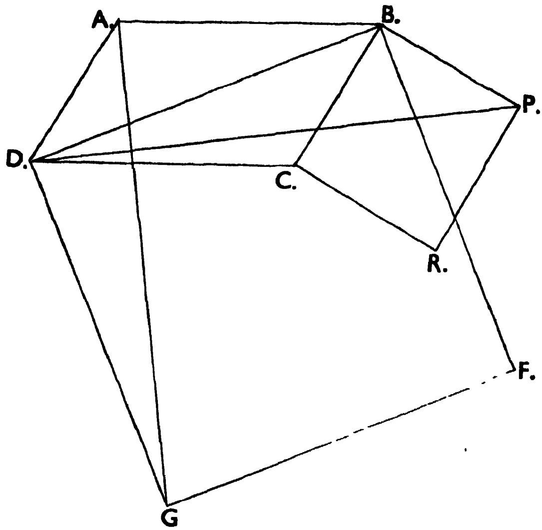
$\therefore$ DC < BD

because lesser side is opposite smaller angle.

5. ABC is a triangle in which AB is greater than AC and the angle ACB is acute. The perpendicular drawn to BC at C meets BA produced at D. Prove that AC is greater than AD. [O]

6. The internal bisector of the angle A of a triangle ABC, of which AB is greater than AC, meets BC at D. Prove that the angle ADB is obtuse. [L]

7. ABCD is a parallelogram with squares BPRC and BFGD drawn on the side BC and diagonal BD respectively both being on the same side of BD. Prove AG = DP.



BP =	Sides of a square
= AD	. .. .
∠PBD = 90° +	
= + ∠BDA	.. .. = ... Alternate
=	

In the △s ADG and DBP

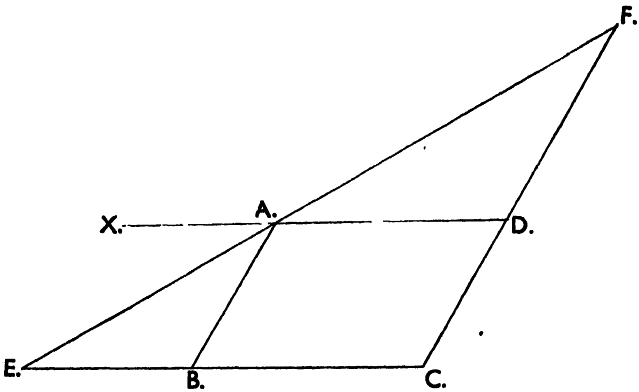
1. . =	Proved
2. . = .	. . . .
3. .. . = . . .	Sides of a square
∴ △s are congruent	. .

∴ AG = DP

8. Equilateral triangles AEC and BFC are drawn on the sides AC, BC of a triangle ABC. Prove AF = BE.

9. AC is a diagonal a square ABCD. P is any point within the triangle ACD and APRS is a square such that R and D are on on the same side of AC. Prove triangles APB, ASD are congruent. [O]

10. ABCD is a parallelogram. DA is produced to X and the bisector of the angle BAX is drawn to meet CB produced at E and CD produced at F. Prove CE = CF = AB + AD. [O]



$\angle$CFE = ...	Corresponding
=	Given
= $\angle$AEC	

$\therefore$ CE = CF

Also EB = ... since .... = ...

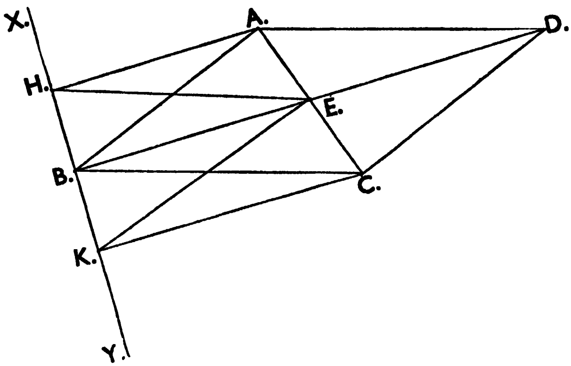
$\therefore$ CE = ... + ...

= AB + AD since = ... | Opp. sides of parallelogram

11. D is the mid-point of the side AB of a triangle ABC. DE is drawn parallel to BC to meet the bisector of the angle C in E. Prove that the points A, E, C lie on a circle.

12. PQRS is a parallelogram ; the internal bisectors of the angles SPQ, PSR meet QR, produced if necessary, at X and Y respectively. Prove that the triangle PQX is isosceles and that QY = RX. [L]

13. The diagonals of a parallelogram ABCD intersect at E ; through B the line XBY is drawn at right angles to DB and H, K are the feet of the perpendiculars drawn to XBY from A and C respectively. Prove that HE = KE. [L]



Since HA, BE and KC are all perpendicular to XY they are

AE = ...

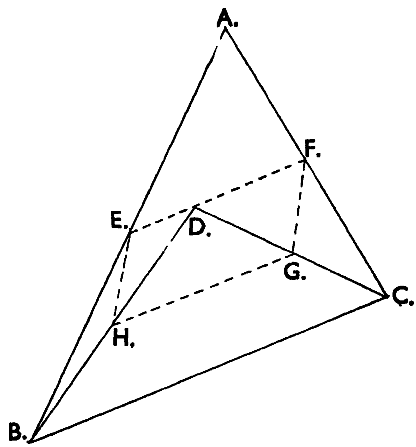
∴ HB = ...

Hence BE is the . . . . of HK

∴ HE = KE.

14. Two parallel chords AR and CS are drawn in a circle centre O. Through B, the mid-point of AC, a chord is drawn parallel to AR and CS cutting RS in M. OM produced cuts CS or CS produced in N. Prove triangle RNS is isosceles.

15. ABC is a triangle and D any point inside it. E, F, G, H are the mid-points of AB, AC, DC, DB respectively. Prove that EG bisects FH.



Since E is the mid-point of AB and F is the mid-point of AC

.. is to and . .. .. . of BC.

Similarly

Since .. ..

. . . .. is .. to and ... .. . of BC.

∴ is a parallogram.

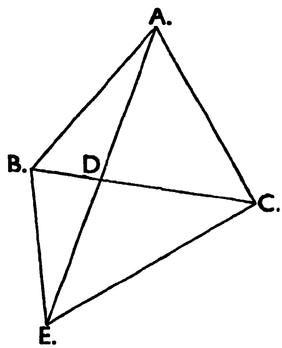
∴ EG bisects FH because . . . .

. ...

16. P, Q are the middle points of the sides AB, BC respectively of a convex quadrilateral ABCD, and O is the middle point of the diagonal BD. If PO produced meets CD at M, and QO produced meets AD at N, prove MN passes through the middle point of OD. [C]

17. ABC is a triangle ; any line through B meets the perpendicular from C to AB at H ; P, Q, R are the mid-points of BH, BC and AC respectively. Prove that PQR is a right angle. [L]

18. Any point D is taken in the base BC of a triangle ABC and AD is produced to E making DE = AD. Prove that the triangle BCE is equal in area to the triangle ABC. [L]



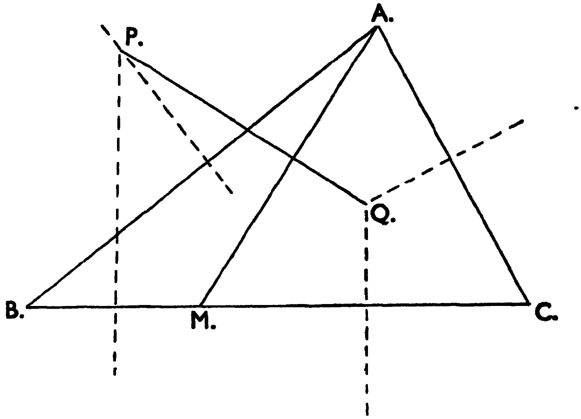
Area △ ABD = Area △ .	On equal bases and between same parallels
Area △ = Area △	....

Adding Area △ ABC = Area △ BEC.

19. ABCD is a quadrilateral whose opposite sides AB, DC are parallel. Any line through the point of intersection of the diagonals AC, BD intersects the parallels to AC through D and B in X and Y respectively. Prove (i.) that areas of XAY and DAB are equal, (ii.) that AY and XC are parallel. [O]

20. ABCD is a parallelogram. A line through A cuts CB produced at P, and CD produced at R ; the line BR cuts AD at Q. Prove that the triangles PQR, BCD are equal in area. [C]

12. M is any point on the base BC of a triangle ABC. The perpendicular bisectors of AB, BM intersect in P and the perpendicular bisectors of AC, CM intersect in Q. Prove PQ is perpendicular to AM. [O]



Join PA, PM, QA, QM.

Since P is on the perpendicular bisector of AB, it

.. ..

Since P is on the perpendicular bisector of BM, it is

. . ..

∴ P is equidistant from A, B and M

∴ PA = PM

Similarly

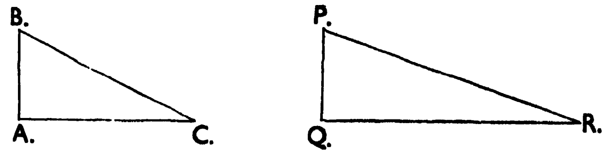
QA =

Hence both P and Q are equidistant from A and M

∴ PQ is the . .. of AM.

22. M is any point on the base BC of a triangle ABC. The bisectors of the angles BAM, ABM meet in L; the bisectors of the angles CAM, ACM meet in N. Prove ∠LMN = 90°. [O]

23. ABC is a right-angled triangle with angle A = 90°. A right-angled triangle is constructed in which the lengths of the sides containing the right angle are the sum and difference of the lengths of AB and AC. Prove that the hypotenuse is equal to the diagonal of the square on BC. [O]



I. II.

PQ = AC − AB QR = AC + AB

$PR^2 = PQ^2 + QR^2 = (\quad)^2 + (\quad)^2$

= Expanding

= 2(+)

∴ PR =

$BC^2 =$ +

BC = (+)

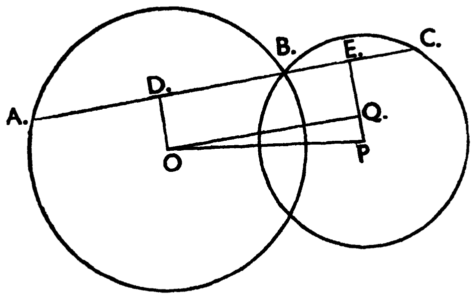
Now diagonal of a square is equal to times the side

∴ Diagonal of square on BC = (+).

24. In a rectangle ABCD, AC = 2BC. F is a point in BC such that BF = $\frac{1}{4}$BC. Prove AF = 7BF.

25. ABCD is a rectangle, in which AB = 24 in., BC = 9 in. ; the point E is taken on CB produced so that BE = 7 in., and F is the middle point of CD. Prove angle AFE is a right angle. [C]

26. Prove that the straight line drawn through one of the points of intersection of two circles and terminated by those circles is greatest when it is parallel to the straight line joining their centres. [O]



Drop OD, PE perpendicular to AC.

Draw OQ parallel to AC.

DB = ... AB | ....

BE = BC | . . . .

Adding DE = ...

But DE = OQ | ... .

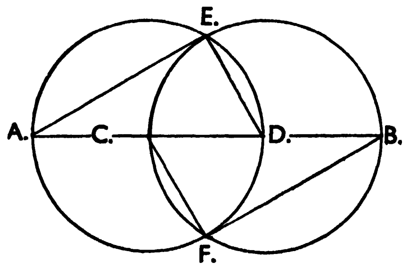
∠OQP = 90°

∴ OP is always ... . than OQ.

∴ DE is largest when Q and P . .

or AC is parallel to OP.

27. A, C, D, B are 4 points on a straight line such that AC = CD = DB. The circle with centre C and radius CA cuts the circle with centre D and radius DB in E and F. Prove AEBF is a parallelogram. [L]



Join CF and DE

CF = ...	Radii of equal circles

∴ In the triangles AED and CFB

1. AD = ...	....
2. . . = ∠CFB	....
3. ED = ...	....
∴ △s are congruent	....

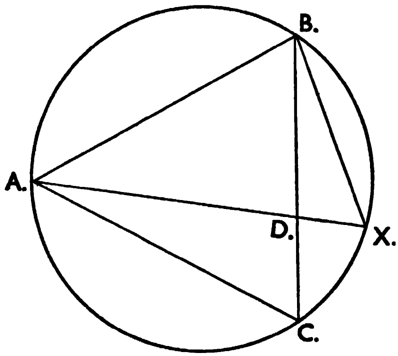
∴ AE = FB and ∠EAD = ...

but these are alternate

∴ AE is parallel to FB

∴ .... is a ....

28. The vertices of a triangle ABC in which AB = AC lie on a circle. X is any point on the arc BC. Of AX cuts BC in D. Prove ∠ADB = ∠ABX.

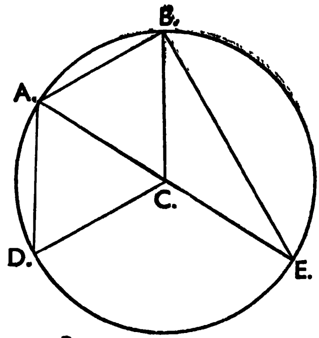


∠ADB = ∠ACD +	Exterior angle of triangle is equal to sum of interior opposite angles
∠ACD =	
.. = ∠DBX	. .
But adding	
. ... + = ∠ABX	
∴ ∠ADB = ∠ABX	

29. ABC is a triangle inscribed in a circle. The bisector of the angle BAC meets BC at D and the circle at E. Prove that the triangles EAC and ECD are equiangular. [O]

30. ABC is a triangle in which AB = AC. P is any point on the circumcircle of ABC, P and B being on opposite sides of AC. CP is produced to Q so that CQ = BP. Prove AP = AQ. [O]

31. ABCD is a parallelogram such that the circle with centre C passes through A, B, D. The diagonal AC is produced to cut the circle at E. Prove angle DAB is four times the angle AEB.



B

Since ABCD is a parallelogram

AB = . and AD =

Since C is the centre of the circle

AC = and

∴ triangles ABC and ADC are . . . .

∴ = 120°.

But ∠BEC = ½ ..... | ... .

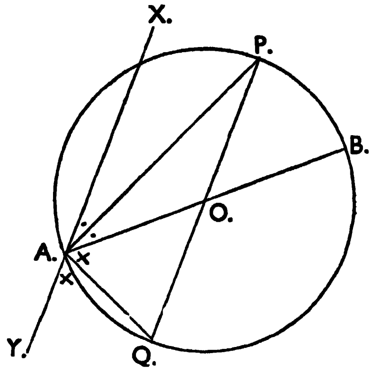
=

∴ ∠DAB = 4∠AEB .

32. ABC is a triangle insribed in a circle, and the centre O of the circle lies inside the triangle ; BO, CO produced meet CA, AB in E, F respectively ; shew that the sum of the angles BFC, BEC is equal to three times the angle A. [C]

33. ABC are three points on a circle whose centre is O such that the angle ABC is obtuse ; AD is a diameter and BD meets AC at X. Prove that $\angle BXA + \angle OAB = \angle ABC$. [L]

34. AB is a diameter of a circle ; XAY is any line through A, and bisectors of the angles BAX, BAY cut the circle at P and Q respectively. Prove that PQ is a diameter of the circle and that it is parallel to XY. [L]



The lines PA and QA are the internal and external bisectors of $\angle XAB$.

$\therefore \angle PAQ =$ ....

$\therefore$ PQ is a diameter | . .. .

Diameters AB and PQ will intersect at the . . of the circle.

$\therefore$ AO = | ... .

$\therefore \angle OQA =$ | . . . .

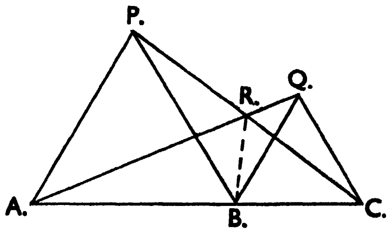
= | . ..

But $\angle OQA$ and . ... are .

$\therefore$ PQ is parallel to XY.

Note.—This device is the reverse of the one used in problems 10, 11, 12.

35. ABC are three points in order on a straight line. P and Q are on the same side of the line ABC such that PAB, QBC are equilateral triangles. Prove that the triangles PBC and QBA are congruent and if AQ and PC intersect in R, angle ARB = angle BRC = 60°. [O]



In $\triangle$s ABQ and PBC

1. AB =	.. ..
2 . = BC	. ...
3. .. . .. = . .	Each is $\angle$PBQ + ...
∴ $\triangle$s are congruent	... .

Since the triangles are congruent

$\angle$RQB = ...

∴ R, Q, C, B are concyclic

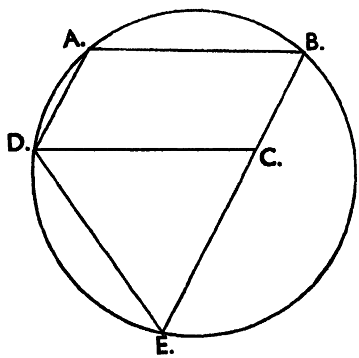
∴ $\angle$BRC = ... = 60° | .. ..

Similarly A, P, R, B are concyclic

∴ .. .. = . . = 60° | . . ..

36. AB is a diameter of a circle whose centre is O and P any point inside the circle. PN is drawn perpendicular to AB, and AP, BP are joined and produced to cut the circle again at C and D respectively. Prove that PN bisects the angle DNC, and that D, O, N, C lie on the circle. [L]

37. In the figure ABCD is a parallelogram, prove that DE = DC. [C]

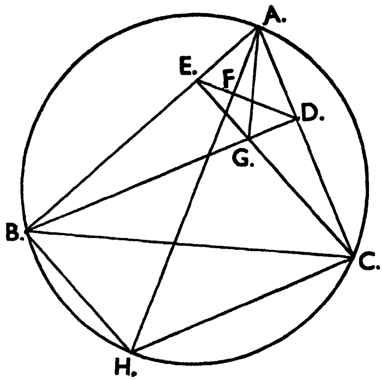


∠DAB = ...	....
= 180° − ∠DCE	....
But ∠DAB = ...	Opposite angles of cyclic quadrilateral
∴ .... = ...	
∴ DE = DC	

38. Two circles cut at Q and Y, and through Q and Y are drawn straight lines PQR, XYZ, which cut one circle at P and X and the other circle at R and Z. Prove that PX is parallel to RZ. [C]

39. If in Question 38 ZR and PX are produced to cut QY produced in G and H respectively, prove that triangles PQG and YZH are similar. [L]

40. The triangle ABC is inscribed in a circle, and BD, CE are the perpendiculars from B and C to the opposite sides meeting CA and AB respectively in D and E. The perpendicular from A to DE meets it at F. Prove that angles EAF and ACB are complementary, and hence deduce that AF lies along a diameter of the circle. [L]



Quadrilateral AEGD is cyclic since. ……

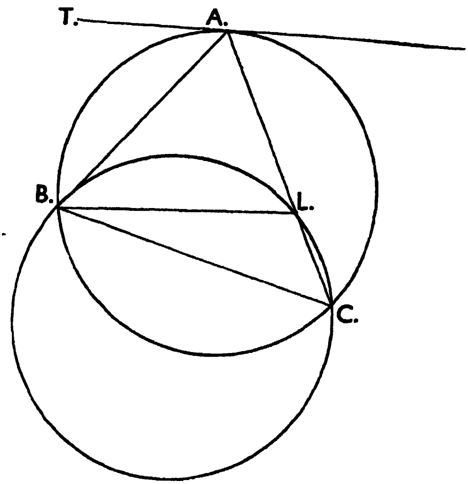
Quadrilateral BEDC is cyclic since ……

(a) $\angle$EAF	= 90° – ……	……
	=	AEG = 90°
	= $\angle$DBC	……
	= 90° – ……	……
(b) $\angle$BHA	= $\angle$ACB	……
	= 90° – ……	Proved
∴	= 90°	Remaining angle of △ ABH

∴ AFH is a diameter.

41. Line 1 in this proof is unnecessary for this particular problem, but it enables you to prove $\angle$DAG = $\angle$DBC and $\angle$EAG = $\angle$ECB. Do this.

42. AB, AC are two chords of a circle ABC. BL, drawn parallel to the tangent at A, cuts AC at L. Prove that the circle through BLC touches AB at B. [L]



Join BC

$\angle TAB =$	Alternate
$\angle TAB =$.	Angle in the alternate segment

$\therefore$ = .

But these for circle BLC are

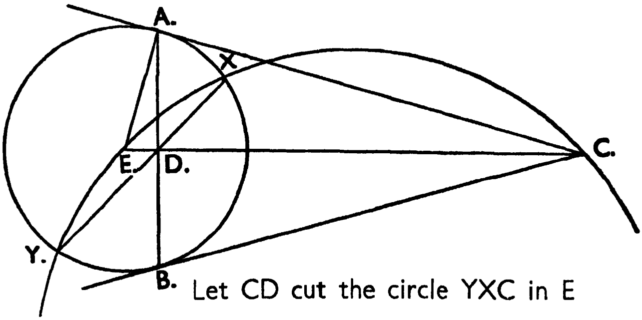
... .

$\therefore$ BA is a tangent to circle BLC.

43. Two circles APB, ABQ which intersect at A and B, are such that AP, AQ, the tangents at A to the circles ABQ, ABP respectively, are at right angles. Prove PBQ is a straight line. [C]

44. ABC is a triangle inscribed in a circle. The angle BAC is trisected and the trisector nearest to AB cuts the circle at D. The circle with centre D and radius DC cuts CB produced at E. Prove that BE = BD, and that DE is a tangent to the circle ABC. [L]

45. The tangents to a circle at the points A and B meet at the point C, and a diameter of the circle through C meets the chord AB at D. Any chord if the circle drawn through D meets the circle at X and Y. Prove that the circle which can be drawn to pass through C, X and Y passes through the centre of the first circle. [O]



Let CD cut the circle YXC in E

If E is centre of circle AXB

$\angle EAC =$ . . . . | . .. .

and EC^2 = | . ...

This must be proved.

Since CD is a diameter, AD = DB and ADE = 90°

$AE^2 = AD^2 + ED^2$ | .. . .

$= AD.DB + ED^2$ since AD = DB

= . . . . $+ ED^2$ since AXBY are concyclic

$= ED.DC + ED^2$ . . . .

= ED (. + .)

= ED. .

Also AC^2 = ... | Pythagoras' Theorem

= ED.DC + ...

= ... ... (... ...... + ...)

= ....

∴ $AE^2 + AC^2$ = ... + ...

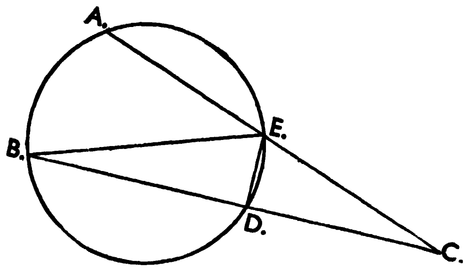
= EC (.. + ...)

= . .

∴ $\angle EAC = 90°$

∴ E is centre of circle AXB.

46. BE are the opposite ends of a diameter of a circle. AD are points on the circle on opposite sides of BE, and EA, DB when produced meet at C. Prove that the square on DE = the difference of the rectangles BD.DC and AE.EC. [L]



CA.CE = ...	Rectangles contained by segments of intersecting chords

(CE + AE).CE = (............ ...... + ...)... ..

CE^2 + AE.CE = + ...

CE^2 – . = ... – AE.CE

But ∠BDE = ...

∴ ∠EDC =

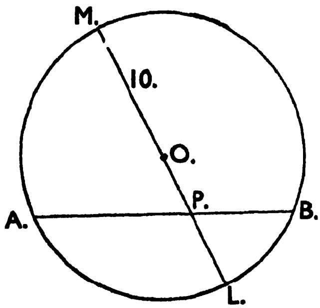
∴ CE^2 – = DE^2 .

∴ DE^2 = ...

47. E is any point on the base CB produced of an isosceles triangle ABC. Prove $AE^2 - AB^2$ = EB.EC.

48. Two circles which do not intersect are cut by a third circle in H and K and M and N respectively. HK and MN intersect in O. Prove tangents from O to the non-intersecting circles are equal. [O].

49. P is a point inside a circle of radius ten inches, and AB is a chord passing through P. The area of the rectangle contained by AP, PB is 25 square inches. Calculate in inches (to two places of decimals) the distance of P from the centre of the circle. [C]



Let O be centre of circle

Draw ML any diameter through P

AP.PB.= .... | Rectangles contained by segments of intersecting chords

$\therefore$ 25 = (10 + OP) (. . ..)

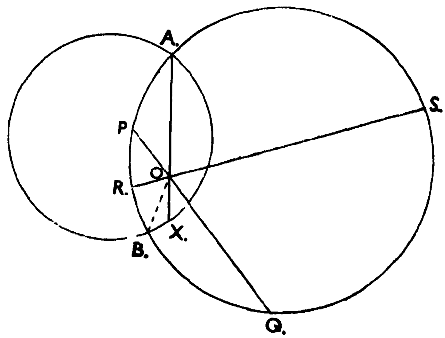
$\therefore$ 25 = ... – ... .

$\therefore$ OP^2 = . .. square inches

OP = ... inches.

50. P is a point 3 cm. from the centre of a circle radius 5 cm. AB is a chord which passes through P. Calculate the area of the rectangle AP.PB. [L]

51. Two circles intersect at points A and B. O is a point inside both circles such that the rectangles contained by the segments of chords of either circle drawn through O are equal. Prove that the line joining O to A must pass through B. [O]



Rectangles PO.OQ and OR.OS are both equal to ....
where X is the point where AO produced cuts the circles

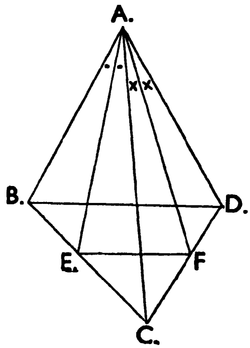
AO is fixed

∴ OX is ... .

∴ X and B must coincide.

52. A and B are two fixed points in the plane of a circle. A straight line drawn at random through B meets the circle at the points P and Q. A circle is drawn through A, P and Q, and the straight line joining A and B meets this circle at C. Show that the point C is the same for all positions of the straight line BPQ. [O]

53. ABCD is a quadrilateral having AB = AD. The angles BAC and CAD are bisected by lines meeting BC and CD at E and F. Prove EF is parallel to BD. [C]



Since AE bisects $\angle$BAC

$$\frac{BE}{EC} = \text{——}$$

Since AF bisects $\angle$CAD

$$\text{——} = \text{——}$$

But AB = AD ... .

$\therefore$ —— = ——

$\therefore$ EF divides two sides of triangle BCD proportionally

$\therefore$ . . ..

54. ABCD is a parallelogram. The bisector of the angle BAD meets BD at E, and the bisector of the angle ABC meets AC at F. Prove that $\frac{AF}{FC} = \frac{BE}{ED}$, and hence prove that EF is parallel to AB. [O]

55. The internal bisector of $\angle$A of the triangle ABC cuts BC at D; from D a line is drawn parallel to BA cutting AC at E; if AB = 3 cm., AC = 7 cm. and BC = 6 cm., calculate the length of CD and of EC. [L]

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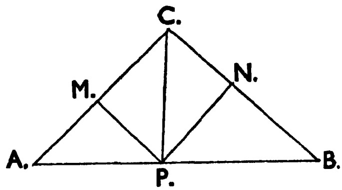
56. The triangle ABC has AB = 8 cm., BC = 5 cm., AC = 6 cm. Calculate the distance from A of a point P in AB such that the triangle ACP may be similar to the triangle ABC. Find also the ratio of the perpendicular from P to the sides AC and BC. [L]



(a) Since triangles ABC and APC are similar, corresponding sides are proportional and

$$\frac{AC}{AB} = \frac{\quad}{\quad}$$

$$\therefore \frac{6}{8} = \frac{\quad}{\quad}$$

$\therefore$ AP = ... cms. PB = ... cms.

(b) Areas of triangles of equal height are proportional to their

..

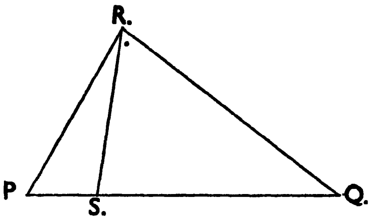
$$\therefore \frac{\text{Area } \triangle \text{ ACP}}{\text{Area } \triangle \text{ CPB}} = \frac{\quad}{\quad}$$

$$\therefore \frac{\frac{1}{2} \ldots \ldots .PM}{\frac{1}{2} \ldots\ldots .PN} = \frac{\quad}{\quad}$$

$$\therefore \frac{PM}{PN} = \frac{\quad}{\quad} \times$$

$$= \frac{\quad}{\quad}$$

57. In a triangle PQR, in which PRQ is the greatest angle, RS is drawn cutting PQ internally at S and making the angle QRS equal to the angle RPQ. By considering the areas of two triangles, prove that $\frac{PQ}{QS} = \frac{PR^2}{RS^2}$ [L]



In triangles PRQ and SRQ

1. $\angle RPQ$ = ...	.. . .
2. .. = ...	....
3. ... = . . .	. . ..

∴ Δs are equiangular

∴ they are similar

∴ $\frac{\text{Area } \Delta \text{ PRQ}}{\text{}} = \frac{}{RS^2}$ . . ..

But since Δs PRQ and RSQ have same altitude

$\frac{\text{Area PRQ}}{\text{}} = \frac{}{QS}$

∴ $\frac{}{QS} = \frac{}{RS^2}$

58. A straight line parallel to the side BC of a triangle ABC meets AB and AC at D and E respectively and divides the area of the triangle into two equal parts. Prove that the ratio of AD to AB is that of a side of a square to a diagonal. [O]

59. You are given the following facts about a pentagon ABCDE :—

(i.) The vertices of the pentagon lie on a circle.

(ii.) AB = 2″, BC = 3″, the angle ADB is 30°.

(iii.) The area of triangle ABC is equal to the area of the triangle ABE.

(iv.) BD bisects the angle CBE.

From (ii.) you have sufficient data to enable you to draw the circle. Draw it, and then proceed to the construction of the pentagon. [O]

Draw a rough figure, first making O the centre of the circle.

From (ii.) $\angle AOB =$

$\therefore$ $\triangle$ AOB is

$\therefore$ Radius of circle = inches.

Draw the circle and put in chord AB = 2″.

How will you fix the point C ?

From (iii.) CE is to AB.

Why ?

How will you fix the point E ?

Use (iv.) to complete the construction.

60. Draw a triangle ABC, in which BC = CA = 2·8 in., AB = 2·5 in. Construct a circle to touch AB at A and to pass through C. [C]

Draw a rough figure.

Since BA is a tangent, the centre of the required circle will lie

on a line through A and   to AB. The

centre will also be   . . .   from A and C. What is the locus of all points satisfying this condition ?

....

Complete the construction.

61. State a method of constructing a circle that shall touch the hypotenuse of a right-angled triangle, have its centre in a side of the triangle, and pass through the vertex opposite the hypotenuse. [O]

62. AB are two points 3·4 inches apart and BC is a line such that ABC = 55°. By geometrical constructions and measurement find the distances from A of (i.) a point P in BC which is equidistant from A and B ; and (ii.) a point Q in BC which is 1·2 inches nearer B than A. [L]

63. ABCD is a square, side 2 inches. E is a point in BC 1.8 inches from B. Find a point F in CD produced such that the area of triangle FBE is equal to the area of the square. [O]

Produce AB to G so that AB = BG.

Put a circle round AEG.

Produce CB backwards to cut the circle at H.

Then GB.BA = | Intersecting chords of a circle.

$\therefore$ BA^2 = ...

For a triangle to be equal in area to a rectangle on the same base, the height of the triangle must be .... the height of the rectangle.

Complete the construction and proof.

i.e. cut off from CD produced CF =

Join BF, EF.

Then area $\triangle$ BEF = $\frac{1}{2}$ × ....

= . . .

= BA^2. Proved.

64. Draw a square ABCD with AB = $2\frac{1}{2}$ inches. Without any further use of the graduations on the ruler construct a rectangle AEFD whose area is two-thirds of the area of the square. [L]

65. A parallelogram, area 8 sq. in., has diagonals 4″ and 5″ long. Construct it.

Draw a right-angled isosceles triangle in which AO = OP = 2″.

Through P draw PQ parallel to OA.

Centre O, radius OD = $2\frac{1}{2}$″, describe an arc cutting PQ in D.

Join AD, OD.

Produce AO to C making OC = AO. Join CD.

Area △ AOP = sq. in.

∴ Area △ AOD = sq. in. | ....

∴ Area △ ADC = sq. in.

To complete the parallelogram produce .... to B making

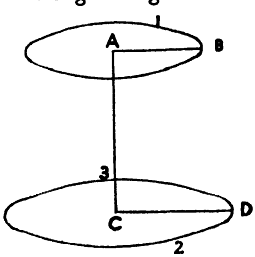
.... = Join .... and ....

The figure ABCD is a parallelogram because ....

....

66. Two circles of radii 1 inch and 2 inches lie in parallel planes which are at right angles to the line joining their centres. The distance between their centres is 3 inches. Find by drawing the radius of the sphere which contains the two circles. [O]

Rough Diagram.



Centre of sphere will lie

(1) on ....

(2) On .... of BD.

This should enable you to complete the construction.

67. Two circles which are sections of a sphere have their planes at right angles. Their radii are 3 inches and 5 inches and that of the sphere is 7 inches. Find the distance between the centres of the circles. [O]

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