

UNIVERSAL
LIBRARY

OU_158854

UNIVERSAL
LIBRARY

OSMANIA UNIVERSITY LIBRARY

Call No. 510.0 Accession No. 7651

Author WSE
Wielstendelme, Joseph

Title Examples for Practice.

This book should be returned on or before the date last marked below.

EXAMPLES FOR PRACTICE IN THE USE OF
SEVEN-FIGURE LOGARITHMS.



EXAMPLES FOR PRACTICE

IN THE USE OF

SEVEN - FIGURE

LOGARITHMS

For the Use of Schools and Colleges

BY

JOSEPH WOLSTENHOLME, Sc.D.

PROFESSOR OF MATHEMATICS AT THE ROYAL INDIAN ENGINEERING COLLEGE, COOPER'S HILL ;
LATE FELLOW AND TUTOR OF CHRIST'S COLLEGE, CAMBRIDGE ; AND SOMETIME
FELLOW OF ST. JOHN'S COLLEGE, CAMBRIDGE

London

MACMILLAN AND CO.

AND NEW YORK

1888

PREFATORY NOTE.

CONSIDERABLE practice is required by most students before they acquire the accuracy which is absolutely necessary in the use of logarithms. According to my experience, mistakes are more frequently made by taking out a wrong logarithm than by blundering in the arithmetic. It would seem therefore that a student should always be expected to take out himself all the logarithms, whether of numbers or of Trigonometrical Functions, which are needed in the solution of any question; and that the practice, which has been so common, of supplying him with all these in addition to the data should be abandoned. It is mainly with the wish of bringing about this result that I have put together the accompanying collection of examples. As the answers are given in each case, this little book is the result of a very considerable amount of work. I have taken all pains to secure accuracy, but cannot hope to have completely succeeded, and shall be grateful to anyone who will point out to me any mistakes I may have made.

I have given what may be thought a disproportionate number of examples of the calculation of the parts of a tetrahedron when

the lengths of its edges are given. This was done, chiefly because I am convinced that no other kind of examples so soon teaches the habitual accuracy which ought to be attained, but also to some extent to give examples for the use of those who have occasion for practice in the solution of Spherical Triangles. In the calculation of each tetrahedron is involved the obtaining all the parts of four Spherical Triangles, and this has induced me, after considerable hesitation, to abstain from giving explicit examples for such solutions.

I have principally, but not exclusively, used Chambers' Mathematical Tables in my working out of these examples. Of course the seventh significant place in numbers, and the second decimal place in seconds of angles, are not absolutely to be relied upon. If this book should be used with tables of six or five-figure logarithms, the corresponding number of places should be struck off my results, the usual correction being made when necessary in the remaining final figure.

COOPER'S HILL COLLEGE, *Sept. 4th*, 1888.

EXAMPLES FOR PRACTICE IN THE USE OF SEVEN-FIGURE LOGARITHMS

*I.—Calculation of L sin, L tan, etc., when the angle is given,
and conversely.*

If it is required to find the $L \sin$, $L \cos$, $L \tan$, ..., of an angle not exactly given in the Tables, it is best to take out from the Tables the $L \sin$, $L \cos$, or $L \tan$ of that angle in the Tables which is next below the proposed angle; and then to calculate the "proportional part" for the seconds by a small sum in Practice worked at the side: this proportional part being then added (with the proper sign) to the logarithm taken out. Thus to calculate $L \tan 28^\circ 40' 49.2''$, $L \sin 27^\circ 45' 43.54''$, $L \cos 54^\circ 57' 58.2''$.

$$\begin{array}{r|l}
 L \tan 28^\circ 40' = 9.7377714 & 60'' = 3001 \\
 \text{diff. for } 49.2'' = \underline{2461} & 48'' = \frac{1}{10} = 2400 \\
 L \tan 28^\circ 40' 49.2'' = 9.7380175 & 1.2'' = \frac{1}{80} = 60 \\
 & \underline{2461}
 \end{array}
 \left| \begin{array}{l} \\ 8 \\ 60 \\ \hline \end{array} \right.$$

$$\begin{array}{r|l}
 L \sin 27^\circ 45' = 9.6680265 & 60'' = 2400 \\
 \text{diff. for } 43 \text{ } 54'' = \underline{1742} & 42'' = \frac{1}{10} = 1680 \\
 L \sin 27^\circ 45' 23 \text{ } 54'' = 9.6682007 & 1.5'' = \frac{1}{40} = 60 \\
 & .04'' = \frac{1}{2500} = 1 \\
 & \underline{1742}
 \end{array}
 \left| \begin{array}{l} \\ 1680 \\ 60 \\ 1 \\ \hline \end{array} \right.$$

$$\begin{array}{r|l}
 L \cos 54^\circ 57' = 9.7591321 & 60'' = 1800 \\
 \text{diff. for } 58.2'' = \underline{1746} & 58.2'' = \frac{1}{10} = 54 \\
 9.7589575 & \underline{1800} \\
 & 1.8'' = \frac{1}{50} = 54
 \end{array}$$

Each of these examples might also be done by multiplying in each case the number of seconds by the quotient obtained by dividing by 60 the difference for 60". Thus, in the last example, this quotient is 30 and $58.2 \times 30 = 1746$. When the difference for 60" contains only three figures, this method is generally the shorter one. When the $L \sin$, $L \cos$, $L \tan$, ... of an angle is given, and the angle is required, look out in the Tables the $L \sin$ (or $L \cos$ or $L \tan$, ...) *nearest* to the given one, whether greater or less, and write this under the given one, *omitting* those figures which are the same in both. Then take out the corresponding angle (putting + or - after it according as the number of seconds due to the difference is to be added or subtracted), and

the difference for 60"; and calculate the number of seconds by the usual proportion. Thus having given—

$$(1) \quad \begin{array}{l} L \sin x = 9.8775429 \\ 48^\circ 58' - \} \\ 1100 \end{array} \left| \begin{array}{l} 1100 \\ 601 \\ 172 \end{array} \right. \begin{array}{l} 10320(9.38'', \text{ or } x=48^\circ 57' 50.62''; \\ 42 \\ 90 \end{array}$$

$$(2) \quad \begin{array}{l} L \cos x = 9.7076837 \\ 59^\circ 20' - \} \\ 2130 \end{array} \left| \begin{array}{l} 2130 \\ 064 \\ 773 \end{array} \right. \begin{array}{l} 46380(21.78, \text{ or } x=59^\circ 19' 38.22''; \\ 378 \\ 165 \\ 159 \end{array}$$

$$(3) \quad \begin{array}{l} L \sec x = 10.1324081 \\ 42^\circ 30' + \} \\ 1158 \end{array} \left| \begin{array}{l} 1158 \\ 3091 \\ 390 \end{array} \right. \begin{array}{l} 25400(18.48, \text{ or } x=42^\circ 30' 18.48''; \\ 9820 \\ 556 \\ 928 \end{array}$$

The method of "proportional parts" must not be employed for the $L \sin$ or $L \tan$ of an angle less than $3^\circ 30'$, the methods for "small" angles being then employed; as also for the $L \cos$ or $L \sec$ of an angle greater than $86^\circ 30'$.

[Not many examples for working are given under this heading, as every example in all the subsequent sections will furnish one or more.]

EXAMPLES. I.

1. Find the $L \sin$ of the following angles :—

$$68^\circ 47' 41.5'', \quad 73^\circ 21' 23.91'', \quad 78^\circ 29' 43.85'', \quad 139^\circ 21' 10.74''.$$

$$[\text{Ans. } 9.9695517, \quad 9.9814136, \quad 9.9911858, \quad 9.8138458.]$$

2. Find the $L \cos$ of the following angles :—

$$25^\circ 45' 33.61'', \quad 22^\circ 24' 19.04'', \quad 18^\circ 49' 46.32'', \quad 66^\circ 59' 38.97''.$$

$$[\text{Ans. } 9.9545452, \quad 9.9659912, \quad 9.9761128, \quad 9.59919823.]$$

3. Find the $L \tan$ of the following angles :—

$$23^\circ 48' 41.12'', \quad 46^\circ 0' 43.79'', \quad 66^\circ 11' 18.88'', \quad 43^\circ 59' 16.21''.$$

$$[\text{Ans. } 9.6447248, \quad 10.0153473, \quad 10.3552752, \quad 9.9846527.]$$

4. Find the value of x from each of the following equations :—

$$(a) \quad L \sin x = 9.5740674, \quad L \sin x = 9.8091259, \quad L \sin x = 9.8281920, \quad L \sin x = 9.9860122;$$

$$(\beta) \quad L \tan x = 9.3600730, \quad L \tan x = 9.8353762, \quad L \tan x = 10.0562416, \quad L \tan x = 10.1321843;$$

$$(\gamma) \quad L \cos x = 9.9670225, \quad L \cos x = 9.8610172, \quad L \cos x = 9.7544211, \quad L \cos x = 9.3701332.$$

$$[\text{Ans. } \left\{ \begin{array}{l} (a) \quad 22^\circ 1' 34.47'', \quad 40^\circ 7' 2.6'', \quad 42^\circ 19' 13.01'', \quad 75^\circ 32' 9.83''; \\ (\beta) \quad 12^\circ 54' 18.65'', \quad 34^\circ 23' 30.51'', \quad 48^\circ 41' 58.59'', \quad 53^\circ 35' 16.23''; \\ (\gamma) \quad 21^\circ 51' 1.23'', \quad 43^\circ 26' 12.02'', \quad 59^\circ 22' 56.98'', \quad 76^\circ 26' 17.35''. \end{array} \right.$$

5. Find the values of the following fractions :—

$$(1) \quad \frac{\sin 58^\circ 9' 20.9'' \sin 49^\circ 26' 47.92'' \sin 65^\circ 18' 24.84''}{\sin 57^\circ 11' 25.7'' \sin 53^\circ 17' 18.06'' \sin 60^\circ 29' 59.54''}$$

$$(2) \quad \frac{\sin 57^\circ 11' 25.7'' \sin 61^\circ 20' 39.56'' \sin 68^\circ 33' 21.04''}{\sin 58^\circ 9' 20.9'' \sin 57^\circ 30' 9.46'' \sin 73^\circ 21' 46.38''}$$

$$(3) \quad \frac{\sin 61^\circ 24' 17.1'' \sin 57^\circ 30' 9.46'' \sin 60^\circ 29' 59.54''}{\sin 53^\circ 56' 29.5'' \sin 61^\circ 20' 39.56'' \sin 65^\circ 18' 24.84''}$$

$$(4) \quad \frac{\sin 53^\circ 56' 29.5'' \sin 53^\circ 17' 18.06'' \sin 73^\circ 21' 46.38''}{\sin 61^\circ 24' 17.1'' \sin 49^\circ 26' 47.92'' \sin 68^\circ 33' 21.04''}$$

[Each fraction=1.]

6. Prove the truth of the equations :—

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = 2\sqrt{\frac{\sin A \sin (s-a) \sin (s-b) \sin (s-c)}{\sin a \sin b \sin c}}, (2s = a + b + c),$$

for the following systems :—

$$(1) \begin{cases} a = 27^{\circ}28'41.02'', & A = 49^{\circ}24' 1.48'' \\ b = 33^{\circ}18'34.24'', & B = 64^{\circ}38'45.10'' \\ c = 35^{\circ}42'29.32'', & C = 73^{\circ}49'49.80'' \end{cases}; \quad (2) \begin{cases} a = 50^{\circ} 0'30.76'', & A = 49^{\circ}24' 1.48'' \\ b = 62^{\circ}59'51.52'', & B = 62^{\circ} 0'25.14'' \\ c = 81^{\circ} 9'58.50'', & C = 101^{\circ}40'57.08'' \end{cases};$$

$$(3) \begin{cases} a = 81^{\circ}17'39.16'', & A = 77^{\circ}15' 8.76'' \\ b = 66^{\circ}19'31.38'', & B = 64^{\circ}38'45.10'' \\ c = 97^{\circ} 2'17.90'', & C = 101^{\circ}40'57.08'' \end{cases}; \quad (4) \begin{cases} a = 65^{\circ}31'27.26'', & A = 77^{\circ}15' 8.76'' \\ b = 55^{\circ}29' 1.08'', & B = 62^{\circ} 0'25.14'' \\ c = 63^{\circ}39'57.86'', & C = 73^{\circ}49'49.80'' \end{cases};$$

also prove that the value of each member of the equations is (1) 1.64556, (2) .991042, (3) .9867222, (4) 1.071654.

II.—Calculation of $\sqrt{a^2 + b^2}$, when a, b are given numbers.

Taking u to denote $\sqrt{a^2 + b^2}$, then if θ be an angle determined by the equation $\tan \theta = \frac{b}{a}$, $u = a \sec \theta = b \operatorname{cosec} \theta$. The value of u may be determined from either of these equations; if both be used, the calculation is thereby checked. The work may be conveniently arranged as in the following example.

| | | | | | |
|------------------------------|--|---|--|--|---|
| $a=47993.27$ $b=38932.89$ | $\frac{\log b=4.5903167}{\log a=4.6811804}$ $L \tan \theta = 9.9091363$ $39^{\circ}3' - \}$ $2582 \quad $ 440 77 | $\frac{4620(1.79)}{2038}$ 2306 $\theta = 39^{\circ}2'58.21''$ | $\log \sec \theta = .1098046$ 31 $\frac{\log a=4.6811804}{\log u=4.7909819}$ | $1025=60''$ $1.8 = 1^{\circ}18' = 30$ $-.01 = -$ 31 | $\left. \begin{array}{l} 8 \\ 2 \end{array} \right\}$ |
| | $\log \operatorname{cosec} \theta = .2006606$ $+46$ $\frac{\log b=4.5903167}{\log u=4.7909819}$ | $\frac{1558=60''}{1.8 = 46}$ 7 $-.01 = -$ 46 | | | |

The two calculations agree in giving $\log u = 4.7909819$, which is therefore almost certainly correct. This gives $u = 61799.07$.

EXAMPLES. II.

| DATA. | ANSWERS. |
|--------------------------------------|---|
| 1. $a = 571.4284$, $b = 428.5713$. | 714.2854 ($\theta = 36^{\circ}52'11.65''$). |
| 2. $a = 93921.58$, $b = 37917.43$. | 101286.7 ($\theta = 21^{\circ}59'4.87''$). |
| 3. $a = 4285.714$, $b = 5714.285$. | 7142.855 ($\theta = 36^{\circ}52'11.65''$). |
| 4. $a = 23727.92$, $b = 7873.11$. | 25000.01 ($\theta = 18^{\circ}21'22.5''$). |
| 5. $a = 7139.247$, $b = 3143.892$. | 7800.827 ($\theta = 23^{\circ}46'1.68''$). |
| 6. $a = 52791.39$, $b = 37387.41$. | 64689.64 ($\theta = 35^{\circ}18'23.46''$). |
| 7. $a = 171428.4$, $b = 169999.8$. | 241428.3 ($\theta = 44^{\circ}45'36.92''$). |
| 8. $a = 3081.708$, $b = 212.7818$. | 3089.046 ($\theta = 3^{\circ}56'59.34''$). |
| 9. $a = 9853.08$, $b = 2127.818$. | 10080.22 ($\theta = 12^{\circ}11'10.14''$). |
| 10. $a = 13790.4$, $b = 13790.3$. | 19502.5 ($\theta = 44^{\circ}59'59.26''$). |
| 11. $a = 803.761$, $b = 803.76$. | 1136.689 ($\theta = 44^{\circ}59'59.88''$). |

EXAMPLES. II.—Continued.

| DATA. | | ANSWERS. |
|----------------------|------------------|---|
| 12. $a = 7118.12$, | $b = 606.13$, | 7143.878 ($\theta = 4^\circ 52' 1.82''$). |
| 13. $a = 34285.68$, | $b = 9999.99$, | 35714.25 ($\theta = 16^\circ 15' 36.75''$). |
| 14. $a = 7372.46$, | $b = 6989.37$, | 10158.95 ($\theta = 43^\circ 28' 19.35''$). |
| 15. $a = 1185.184$, | $b = 345.6796$, | 1234.567 ($\theta = 16^\circ 15' 36.88''$). |
| 16. $a = 273.0004$, | $b = 135.9992$, | 305. ($\theta = 26^\circ 28' 51.17''$). |
| 17. $a = 272.9$, | $b = 130.2022$, | 305.0008 ($\theta = 26^\circ 31' 24.3''$). |
| 18. $a = 5488.42$, | $b = 2273.378$, | 5940.623 ($\theta = 22^\circ 30'$). |
| 19. $a = 378.4125$, | $b = 487.4775$, | 615.37 ($\theta = 52^\circ 5' 4.83''$). |
| 20. $a = 27891.83$, | $b = 36789.17$, | 46167.06 ($\theta = 52^\circ 49' 56.27''$). |
| 21. $a = 262.087$, | $b = 151.316$, | 302.632 ($\theta = 29^\circ 59' 59.98''$). |
| 22. $a = 56471.9$, | $b = 97812.2$, | 112943.8 ($\theta = 60^\circ$). |
| 23. $a = 8055.837$, | $b = 5924.819$, | 10000 ($\theta = 36^\circ 20'$). |
| 24. $a = 7601.54$, | $b = 6497.43$, | 10000 ($\theta = 40^\circ 31' 20''$). |
| 25. $a = 7968.81$, | $b = 23695.95$, | 25000 ($\theta = 71^\circ 24' 45.02''$). |
| 26. $a = 1236.113$, | $b = 716.1294$, | 1428.572 ($\theta = 30^\circ 5' 7.5''$). |
| 27. $a = 74511.63$, | $b = 19109.62$, | 76923.095 ($\theta = 14^\circ 23' 3.6''$). |
| 28. $a = 4313.540$, | $b = 3999.424$, | 5882.351 ($\theta = 42^\circ 50' 9.6''$). |
| 29. $a = 4127.817$, | $b = 1365.548$, | 4347.825 ($\theta = 18^\circ 18' 18.17''$). |
| 30. $a = 437.7493$, | $b = 796.7571$, | 909.0909 ($\theta = 61^\circ 12' 54''$). |
| 31. $a = 6151.67$, | $b = 1104.292$, | 6250.001 ($\theta = 10^\circ 10' 36.6''$). |
| 32. $a = 4761.903$, | $b = 13468.55$, | 14285.67 ($\theta = 70^\circ 31' 42.88''$). |

III.—Solution of Equations of the form $a \cos x + b \sin x = c$.

The two values of x which satisfy the equation $a \cos x + b \sin x = c$ being denoted by α , β , $\frac{1}{a} \cos \frac{1}{2}(\alpha + \beta) = \frac{1}{b} \sin \frac{1}{2}(\alpha + \beta) = \frac{1}{c} \cos \frac{1}{2}(\alpha - \beta)$. First find $\frac{1}{2}(\alpha + \beta)$ from the equation $\tan \frac{1}{2}(\alpha + \beta) = \frac{b}{a}$, and then $\frac{1}{2}(\alpha - \beta)$ from either or both of the equations $\cos \frac{1}{2}(\alpha - \beta) = \frac{c}{a} \cos \frac{1}{2}(\alpha + \beta) = \frac{c}{b} \sin \frac{1}{2}(\alpha + \beta)$. Writing the equation so that c is positive, then if a be negative and b positive, put $x = \pi - y$; if a be positive and b negative, put $x = -y$; and if both a and b be negative, put $x = \pi + y$. We shall thus always have an equation in which the coefficients are all positive. Arrange the work according to the following pattern.

| | | |
|---|---|---|
| $a = 27839$ $b = 29273$ $c = 28565$ | $\left. \begin{array}{l} 14.4664672 \dots (10 + \log b) \\ - 4.4446536 \dots (\log a) \\ \hline 10.0218136 \\ 7380 \\ \hline 756 \times 6\phi \\ 4536 (17.93) \\ 2006 \\ 235 \\ 73 \end{array} \right\} \begin{array}{l} 9.8383441 \\ - 397 f \\ 4.4558342 \\ - 4.4446536 \\ \hline 9.8494850 \\ = L \cos \frac{1}{2}(\alpha - \beta) \\ = L \cos 45^\circ \end{array}$ | $\begin{array}{r l} (1) & \\ \hline 1329-60'' & \\ \hline 18=398 & 7 \\ -.067-1 & 3 \\ -.01=- & 2 \\ \hline & 397 \end{array}$ |
| | | $\begin{array}{r l} (2) & \\ \hline 9.8600821 & \\ 359 f & \\ 4.4558342 & 358.6 \\ - 4.4664672 & 3 \\ \hline 9.8494850 & 359 \\ = L \cos \frac{1}{2}(\alpha - \beta) & \end{array}$ |

Thus
$$\left. \begin{aligned} \frac{a+\beta}{2} &= 46^{\circ}26'17.93'', \\ \frac{a-\beta}{2} &= 45^{\circ}, \end{aligned} \right\} \text{whence } a = 91^{\circ}26'17.93'', \beta = 1^{\circ}26'17.93''.$$

If the values of a, b, c be such that $c^2 > a^2 + b^2$, the values of x will be impossible. Should such data be given, the value found for $I. \cos \frac{1}{2}(a - \beta)$ would be greater than 10, and the corresponding angle impossible.

EXAMPLES. III.

| DATA. | | | ANSWERS. | | |
|-------|-----------------|-----------------|-----------------|-------------------------------|---------------------------------|
| 1. | $a = 11175.3,$ | $b = 5600.24,$ | $c = 10548.25.$ | $a = 59^{\circ}4',$ | $\beta = 5^{\circ}50'.$ |
| 2. | $a = 58533.32,$ | $b = 49909.95,$ | $c = 62137.14.$ | $a = 70^{\circ}34'24.7'',$ | $\beta = 4^{\circ}20'.$ |
| 3. | $a = 67313.97,$ | $b = 58129.13,$ | $c = 62721.45.$ | $a = 85^{\circ}57'38.3'',$ | $\beta = 4^{\circ}20'9.6''.$ |
| 4. | $a = 39713.76,$ | $b = 29254.24,$ | $c = 18397.98.$ | $a = 104^{\circ}28'34.01'',$ | $\beta = 31^{\circ}43'24.07''.$ |
| 5. | $a = 6342.667,$ | $b = 3914.217,$ | $c = 6549.328.$ | $a = 60^{\circ}11'27.25'',$ | $\beta = 3^{\circ}10'7.07''.$ |
| 6. | $a = 549.1238,$ | $b = 508.1741,$ | $c = 618.3272.$ | $a = 77^{\circ}2'50.77'',$ | $\beta = 8^{\circ}30'59.71''.$ |
| 7. | $a = 35.67812,$ | $b = 28.73407,$ | $c = 40.13598.$ | $a = 67^{\circ}40'1.8'',$ | $\beta = 10^{\circ}1'36''.$ |
| 8. | $a = 7891.293,$ | $b = 889.312,$ | $c = 7926.027.$ | $a = 9^{\circ}58'39.75'',$ | $\beta = 2^{\circ}52'55.13''.$ |
| 9. | $a = 3712.484,$ | $b = 1139.115,$ | $c = 3795.726.$ | $a = 20^{\circ}14'59.2'',$ | $\beta = 4^{\circ}51'56.79''.$ |
| 10. | $a = 861.3982,$ | $b = 116.4593,$ | $c = 869.154.$ | $a = 8^{\circ}28'58.52'',$ | $\beta = 6^{\circ}54'58.52''.$ |
| 11. | $a = 25139.85,$ | $b = 23846.16,$ | $c = 27389.27.$ | $a = 81^{\circ}15'31.62'',$ | $\beta = 6^{\circ}23'20.84''.$ |
| 12. | $a = 1790.009,$ | $b = 2811.935,$ | $c = 2956.161.$ | $a = 85^{\circ}2'25.2'',$ | $\beta = 30''.$ |
| 13. | $a = 78913.47,$ | $b = 59134.72,$ | $c = 67314.95.$ | $a = 83^{\circ}47'49.92'',$ | $\beta = 10^{\circ}6'15.32''.$ |
| 14. | $a = 3581.72,$ | $b = 3312.93,$ | $c = 4645.07.$ | $a = 60^{\circ}34'48.14'',$ | $\beta = 24^{\circ}57'17.40''.$ |
| 15. | $a = 428.5714,$ | $b = 571.4288,$ | $c = 714.2857.$ | $a = 53^{\circ}7'48.35'',$ | $\beta = 53^{\circ}7'48.35''.$ |
| 16. | $a = 20785.2,$ | $b = 20611.99,$ | $c = 20958.41.$ | $a = 89^{\circ}2'13.18'',$ | $\beta = 0^{\circ}29'0.70''.$ |
| 17. | $a = 9999.79,$ | $b = 8654.33,$ | $c = 11235.84.$ | $a = 72^{\circ}42'19.56'',$ | $\beta = 9^{\circ}2'37.46''.$ |
| 18. | $a = 37912.37,$ | $b = 28776.52,$ | $c = 39148.21.$ | $a = 71^{\circ}51'49.02'',$ | $\beta = 2^{\circ}32'7.22''.$ |
| 19. | $a = 84513.27,$ | $b = 9837.12,$ | $c = 79125.64.$ | $a = 28^{\circ}12'10.98'',$ | $\beta = 14^{\circ}55'28.70''.$ |
| 20. | $a = 831.2798,$ | $b = 415.6399,$ | $c = 888.8931.$ | $a = 44^{\circ}25'31.66'',$ | $\beta = 8^{\circ}22'32.54''.$ |
| 21. | $a = 813908.5,$ | $b = 600305.1,$ | $c = 1000000.$ | $a = 45^{\circ}0'0.04'',$ | $\beta = 27^{\circ}49'18.4''.$ |
| 22. | $a = 7933.533,$ | $b = 6087.614,$ | $c = 9914.449.$ | $a = 45'',$ | $\beta = 30''.$ |
| 23. | $a = 7261.248,$ | $b = 6875.629,$ | $c = 9915.867.$ | $a = 50^{\circ}52'30'',$ | $\beta = 36^{\circ}0'0.0''.$ |
| 24. | $a = 35170.77,$ | $b = 35538.95,$ | $c = 48363.02.$ | $a = 60'',$ | $\beta = 30^{\circ}35'48.0''.$ |
| 25. | $a = 786.1347,$ | $b = 563.9187,$ | $c = 803.4125.$ | $a = 69^{\circ}31'40.06'',$ | $\beta = 1^{\circ}46'41.84''.$ |
| 26. | $a = 43759.16,$ | $b = 24189.58,$ | $c = 44576.71.$ | $a = 92^{\circ}0'0.00'',$ | $\beta = 34^{\circ}8'0.00''.$ |
| 27. | $a = 137904,$ | $b = 137903,$ | $c = 195025.$ | $a = 44^{\circ}59'59.24'',$ | $\beta = 44^{\circ}59'59.24''.$ |
| 28. | $a = 803.76,$ | $b = 803.761,$ | $c = 1136.689.$ | $a = 45^{\circ}0'0.12'',$ | $\beta = 45^{\circ}0'0.12''.$ |
| 29. | $a = 70,$ | $b = 24,$ | $c = 37.$ | $a = 78^{\circ}55'28.72'',$ | $\beta = 41^{\circ}4'31.28''.$ |
| 30. | $a = 70,$ | $b = 24,$ | $c = 64.08587.$ | $a = 48^{\circ}55'28.72'',$ | $\beta = 11^{\circ}4'31.28''.$ |
| 31. | $a = 39405.98,$ | $b = 17622.89,$ | $c = 21583.55.$ | $a = 21^{\circ}5'54'18.56'',$ | $\beta = 95^{\circ}54'18.6''.$ |
| 32. | $a = 97.8122,$ | $b = 56.4719,$ | $c = 112.9438.$ | $a = 30'',$ | $\beta = 30''.$ |
| 33. | $a = 27442,$ | $b = 11366.89,$ | $c = 29703.12.$ | $a = 22^{\circ}30',$ | $\beta = 22^{\circ}30'.$ |
| 34. | $a = 5488.43,$ | $b = 273.382,$ | $c = 5940.635.$ | $a = 22^{\circ}30',$ | $\beta = 22^{\circ}30'.$ |
| 35. | $a = 43155.16,$ | $b = 68971.5,$ | $c = 71859.22.$ | $a = 85^{\circ}55'55'',$ | $\beta = 30''.$ |
| 36. | $a = 395.76,$ | $b = 228.48,$ | $c = 323.12.$ | $a = 75^{\circ}0'3.18'',$ | $\beta = 15^{\circ}0'12.68''.$ |
| 37. | $a = 193.19,$ | $b = 198.79,$ | $c = 196.01.$ | $a = 90^{\circ}49'6.6'',$ | $\beta = 0^{\circ}49'6.6''.$ |
| 38. | $a = 27.839,$ | $b = 28.273,$ | $c = 28.565.$ | $a = 89^{\circ}23'44.83'',$ | $\beta = 1^{\circ}29'25.83''.$ |
| 39. | $a = 239.9,$ | $b = 3992.1,$ | $c = 3568.9.$ | $a = 113^{\circ}23'13.59'',$ | $\beta = 59^{\circ}44'5.71''.$ |
| 40. | $a = 11.599,$ | $b = 49.121,$ | $c = 35.689.$ | $a = 121^{\circ}42'50.56'',$ | $\beta = 31^{\circ}42'50.56''.$ |
| 41. | $a = 3496.7,$ | $b = 6399.7,$ | $c = 5125.7.$ | $a = 106^{\circ}41'30.68'',$ | $\beta = 16^{\circ}0'18.28''.$ |
| 42. | $a = 34967,$ | $b = 63497,$ | $c = 51257.$ | $a = 106^{\circ}9'32.31'',$ | $\beta = 16^{\circ}9'32.31''.$ |

IV.—Calculation of $\sqrt{b^2+c^2-2bc \cos A}$.

Denoting the expression by u , its value may be calculated by any one of the three following methods :—

$$(1) u_1 = (b+c) \cos \theta, \text{ where } \sin \theta = \frac{2\sqrt{bc} \cos \frac{1}{2}A}{b+c};$$

$$(2) u_2 = (b-c) \sec \theta, \text{ where } \tan \theta = \frac{2\sqrt{bc}}{b-c} \sin \frac{1}{2}A; \quad (b > c)$$

$$(3) u_3 = (b+c) \sin \frac{1}{2}A \sec \theta, \text{ where } \tan \theta = \frac{b-c}{b+c} \cot \frac{1}{2}A;$$

The calculation of one example by each of these three methods is subjoined.

| | | DATA. | |
|----------------------------|---------------|-----------------------------------|------------|
| | b=12369.12 | log b=4.0923388 | |
| | c=4123.04 | log c=3.6152175 | |
| | A=40° 55' 55" | 2)7.7075563 | |
| | b+c=16492.16 | log \sqrt{bc} =3.8537782 | |
| | b-c=8246.08 | log (b+c)=4.2172775 | |
| | A | log (b-c)=3.9162475 | |
| | 2 | | |
| | =20° 28' 2.5" | | |
| (1) | | | |
| .30103 | 471 | log cos θ =1.7667739 | 1753 |
| 3.8537782 | 2.4-18 8 | 432 | |
| 9.9716820 | .1- 7 | log (b+c)=4.2172775 | 10-392 2 |
| 19) | 2.5-19 5 | log u_1 =3.9840946 | 4.8-140 2 |
| -4 2172775 | | 05 | -.03=1 2 |
| L sin θ =9.9092146 |)1350(14.84" | 9640.3 | 14.84=4.32 |
| 54° 14' - } 371 | 440 | 9- | 41 |
| 910 } 225 | 76 | <u>u_1=9640.390</u> | |
| | 32 | | |
| (2) | | | |
| .3010300 | 3386 | log sec θ =.0678489 | 765 |
| 3.8537782 | 2 -113 | -16 | |
| 9.5430489 | .5- 28 | log (b-c)=3.9162475 | 1.2 =15 3 |
| -141) | 2.5=141 | log u_2 =3.9840948 | .02= 2 |
| -5.9162475 | | 15 5 | |
| L tan θ =9.7821955 |)3480(1.22 | <u>u_2=9640.394</u> | |
| 31° 12' - } 2013 | 629 | | |
| 2851 } 58 | 588 | | |
| (3) | | | |
| 3.9162475 | 3858 | log sec θ =.2232324 | 1693 |
| 10.4280331 | 2 -128 6 | -498 | |
| 161) | .5- 32 2 | log sin $\frac{1}{2}A$ =1.5436348 | 15 =423 3 |
| -4.2172775 | 2.5=161 | log (b+c)=4.2172775 | 2 = 56 4 |
| L tan θ =10.1270192 |)46560(17.66 | log u_3 =3.9840949 | .66= 18 6 |
| 53° 16' - } 968 | 2020 | <u>u_3=9640.396</u> | 17.66=498 |
| 2636 } 776 | 1748 | | |
| | 1664 | | |

The three values of $\log u$, found according to the three methods, will generally have slight differences in the last place. In the examples given, the answers will exhibit the three values of u , denoted by u_1, u_2, u_3 ; and the mean of these is taken for the value of u . Thus, in the one calculated, $u_1 = 9640.390, u_2 = 9640.394, u_3 = 9640.396, u = 9640.393$.

EXAMPLES. IV.

| DATA. | ANSWERS. |
|---|---|
| 1. $b=45123.76, c=34179.24, A=24^\circ 57' 18.6''$. | $u_1=20193.17, u_2=u_3=20193.20, u=20193.18$. |
| 2. $b=2357.126, c=2075.352, A=67^\circ 31' 55''$. | $u_1=2474.689, u_2=u_3=2474.691, u=2474.69$. |
| 3. $b=2357.126, c=2075.382, A=67^\circ 31' 55''$. | $u_1-u_2=u_2-u=2474.705$. |
| 4. $b=47392.16, c=25497.83, A=56^\circ 37' 55.2''$. | $u_1=398583.89, u_2=u_3=398583.92, u=398583.9$. |
| 5. $b=38914.86, c=34819.73, A=120^\circ$. | $u_1=63888.85, u_2=63888.91, u_3=63888.89,$ $u=63888.88$. |
| 6. $b=73869.13, c=9347.25, A=60^\circ$. | $u_1=69667.42, u_2=u_3=69667.40, u=69667.41$. |
| 7. $b=543.2971, c=276.8139, A=36^\circ 55' 57.6''$. | $u_1=362.4475, u_2=362.4477, u_3=362.447,$ $u=362.4474$. |
| 8. $b=47394.91, c=21382.75, A=26^\circ 39' 45''$. | $u_1=29869.01, u_2=u_3=29869.03, u=29869.02$. |
| 9. $b=7984.637, c=4138.363, A=31^\circ 37' 52.8''$. | $u_1=4961.009, u_2=u_3=4961.012, u=4961.011$. |
| 10. $b=73182.2, c=47297.4, A=39^\circ 19' 5.62''$. | $u_1=47297.42, u_2=u_3=47297.39, u=47297.4$. |
| 11. $b=670.344, c=586.551, A=120^\circ$. | $u_1-u_2=u_2-u=1080.309$. |
| 12. $b=3791.298, c=413.115, A=4^\circ 30'$. | $u_1=3379.611, u_2=3379.69, u_3=3379.612,$ $u=3379.611$. |
| 13. $b=3789.371, c=1134.199, A=23^\circ 6' 16.8''$. | $u_1=2781.982, u_2=u_3=2781.98, u=2781.981$. |
| 14. $b=890.8565, c=534.1539, A=53^\circ 7' 48.38''$. | $u_1=718.6850, u_2=718.6852(=u), u_3=718.6855$. |

Other examples for practice may be taken from VII.

V.—Triangles in which are given two angles and one side.

Let a be the given length of one side; then whichever two of the three angles are given, the third is at once found from the equation $A + B + C = 180^\circ$; then find R the radius of the circumscribed circle from the equation $2R = a/\sin A$, and the other sides b, c from the equations $b = 2R \sin B, c = 2R \sin C$. The radii of the inscribed and escribed circles r, r_1, r_2, r_3 , if required, may best be found from the equations $r = (s - a) \tan \frac{A}{2}, r_1 = s \tan \frac{A}{2}, r_2 = (s - c) \cot \frac{A}{2}, r_3 = (s - b) \cot \frac{A}{2}$, or the corresponding system involving $\tan \frac{B}{2}$ or $\tan \frac{C}{2}$, taking that one of the three half angles which is nearest to 45° . The equation $r_1 + r_2 + r_3 - r = 4R$ should then be employed to check the calculations. An example is worked out at full length below.

| DATA. | | |
|-------------------------------|-----------------------------|-----------------------------------|
| $a=27837.93$ | $\log a = 4.4446369$ | $\frac{277}{-}$ |
| $A=77^\circ 36' 57.6''$ | $-\log \sin A = -1.9897766$ | $2.4 = \frac{4}{10} = 11 \quad 1$ |
| $B=54^\circ 22' 12.48''$ | | |
| | $\log 2R = 4.4548614$ | |
| | | 01 |
| hence $C=48^\circ 0' 49.92''$ | 28501.09 | 13 |

| | | | |
|--|--|--|--|
| $\log 2R=4.4548614$ $\log \sin B=1.9099634$ <hr/> $\log b=4.3648436$ <hr/> $23165.60.$ <hr/> 113 | 905 $12''=181$ $.48=72$ | $\log 2R=4.4548614$ $\log \sin C=1.8710735$ <hr/> $\log c=4.3260295$ <hr/> $21185.05.$ <hr/> 10 | 1137 $48=909$ 6 $1.8=34$ 1 $.12=2$ 3 |
| $a=27837.93$ $b=23165.60$ $c=21185.05$ <hr/> $2s=72188.58$ <hr/> $s=36094.29$ | $s-a=8256.36$ $s-b=12928.69$ $s-c=14909.24$ <hr/> $s=36094.29$ | $\frac{A}{2}=38^{\circ}48'28.8''$ $L \tan \frac{1}{2}A=9.9052672$ <hr/> 1242 $\log \tan \frac{1}{2}A=1.9053914$ <hr/> $\log \cot \frac{1}{2}A=0.0946086$ | 2587 <hr/> $30''=1293$ 5 $-1.2''=-51$ 7 |
| $\log s-a=3.9167886$ $\log \tan \frac{1}{2}A=1.9053914$ <hr/> $\log r=3.8221800$ <hr/> $6640.182.$ <hr/> -12 | $\log s=4.5574385$ $\log \tan \frac{1}{2}A=1.9053914$ <hr/> $\log r_1=4.4628299$ <hr/> $29028.85.$ <hr/> 128 | $\log s-c=4.1734555$ $\log \cot \frac{1}{2}A=0.0946086$ <hr/> $\log r_2=4.2686041$ <hr/> $18538.05.$ <hr/> 12 | $\log s-b=4.1115545$ $\log \cot \frac{1}{2}A=0.0946086$ <hr/> $\log r_3=4.2061631$ <hr/> $16075.45.$ <hr/> 121 |

ANSWERS.

| | |
|-------------------------|-----------------|
| $C=48^{\circ}0'49.92''$ | $r=6640.18.$ |
| $b=23165.66.$ | $r_1=29028.85.$ |
| $c=21185.05.$ | $r_2=18538.05.$ |
| $R=14250.54.$ | $r_3=16075.45.$ |

CHECK.

| |
|----------------|
| $-r=-6640.18$ |
| $r_1=29028.85$ |
| $r_2=18538.05$ |
| $r_3=16075.45$ |
| $4R=57002.17$ |
| $R=14250.54$ |

The distance between the circumcentre, orthocentre, and incentre are to be found, when required, from the equations $\Delta_1^2=R^2-2Rr$, $\Delta_2^2=R^2+2\rho^2$, $\Delta_3^2=\rho^2+2r^2$, where $\rho^2=-4R^2\cos A \cos B \cos C$.

EXAMPLES. V.

| DATA. | ANSWERS. |
|---|---|
| 1. $a=7$ $R=38^{\circ}36'$ $C=129^{\circ}23'$ | $A=12^{\circ}1'$ $b=20.97616$ $c=25.98715$ $R=16.81107$ $r=2.103095$ $r_1=2.839854$ $r_2=9.448844$ $r_3=57.05867$ |
| 2. $a=86602.54$ $B=80^{\circ}42'17.3''$ $C=39^{\circ}17'42.7''$ | $A=60^{\circ}$ $b=98686.93$ $c=63331.6$ $R=50000$ $r=21770.725$ $r_1=71771.02$ $r_2=105618.62$ $r_3=44381.38$ |
| 3. $a=91035.91$ $B=37^{\circ}43'$ $C=66^{\circ}43'41.9''$ | $A=65^{\circ}33'18.1''$ $b=61175.72$ $c=91864.15$ $R=50000$ $r=20788.34$ $r_1=88339.88$ $r_2=41683.78$ $r_3=90764.7$ |

EXAMPLES. V.—Continued.

| DATA. | ANSWERS. | |
|---|--|--|
| 4. $a=96840.39$ $B=37^{\circ}43'$ $C=66^{\circ}43'28.3''$ | $A=75^{\circ}33'31.7''$ $b=61175.71$ $c=91861.56$ $R'=50000$ | $r=21779.2$ $r_1=96840.57$ $r_2=42674.64$ $r_3=82264$ |
| 5. $a=11935.26$ $B=41^{\circ}24'34.3''$ $C=24^{\circ}17'42.3''$ | $A=114^{\circ}17'43.4''$ $b=8661.392$ $c=5387.97$ $R'=6547.5$ | $r=1636.927$ $r_1=20119.62$ $r_2=4910.528$ $r_3=2796.707$ |
| 6. $a=22733.78$ $B=90^{\circ}$ $C=67^{\circ}30'$ | $A=22^{\circ}30'$ $b=59406.23$ $c=54884.19$ $R'=29703.115$ | $r=9105.87$ $r_1=13627.91$ $r_2=68512.1$ $r_3=45778.32$ |
| 7. $a=6347.287$ $A=78^{\circ}33'14.7''$ $B=39^{\circ}59'43.5''$ | $C=61^{\circ}27'1.8''$ $b=4162.347$ $c=6404.116$ $R'=3238.04$ | $r=1725.267$ $r_1=6916.216$ $r_2=2510.033$ $r_3=5251.178$ |
| 8. $a=12791.38$ $A=62^{\circ}37'15.5''$ $B=68^{\circ}25'37.24''$ | $C=48^{\circ}57'7.26''$ $b=13395.9$ $c=10863.65$ $R'=7202.49$ | $r=3487.813$ $r_1=11208.31$ $r_2=12596.28$ $r_3=8433.18$ |
| 9. $a=7674.893$ $A=97^{\circ}18'47.1''$ $C=40^{\circ}14'33.5''$ | $B=42^{\circ}26'30.1''$ $b=5222.048$ $c=4998.842$ $R'=3868.919$ | $r=1446.811$ $r_1=10169.62$ $r_2=3474.63$ $r_3=3278.239$ |
| 10. $a=44569.33$ $A=80^{\circ}32'57.54''$ $C=45^{\circ}36'2.36''$ | $B=53^{\circ}51'0.1$ $b=30483.84$ $c=32282.09$ $R'=22591.29$ | $r=10250.9$ $r_1=48014.57$ $r_2=28780.24$ $r_3=23821.26$ |
| 11. $a=8984.798$ $A=72^{\circ}48'48.6''$ $C=36^{\circ}36'12.72''$ | $B=70^{\circ}34'58.68''$ $b=8869.832$ $c=5607.804$ $R'=4702.37$ | $r=2025.334$ $r_1=8651.132$ $r_2=8303.546$ $r_3=3880.131$ |
| 12. $a=381.437$ $A=67^{\circ}10'25.8''$ $C=75^{\circ}31'14.5''$ | $B=37^{\circ}18'19.7''$ $b=250.818$ $c=400.7024$ $R'=206.9235$ | $r=89.677$ $r_1=342.9778$ $r_2=174.3436$ $r_3=400.0496$ |
| 13. $a=176.01$ $A=66^{\circ}19'31.38''$ $B=63^{\circ}39'57.86''$ | $C=58^{\circ}0'30.76''$ $b=172.24$ $c=147.24$ $R'=96.0921$ | $r=46.8754$ $r_1=161.8895$ $r_2=153.8063$ $r_3=115.5479$ |
| 14. $a=443.387$ $B=27^{\circ}28'41.02''$ $C=55^{\circ}29'1.08''$ | $A=97^{\circ}2'17.90''$ $b=206.136$ $c=368.109$ $R'=223.3767$ | $r=74.00344$ $r_1=575.4995$ $r_2=124.4027$ $r_3=267.6083$ |
| 15. $a=1266.82$ $A=81^{\circ}9'58.5''$ $C=33^{\circ}18'34.24''$ | $B=65^{\circ}31'27.26''$ $b=1166.82$ $c=704.04$ $R'=641.0137$ | $r=258.7082$ $r_1=1343.8572$ $r_2=1009.5812$ $r_3=469.3244$ |

, EXAMPLES. V.—Continued.

| | DATA. | ANSWERS. | |
|-----|--|--|---|
| 16. | $a=525.069$ $A=81^{\circ}17'39.16''$ $B=62^{\circ}59'59.52''$ | $C=35^{\circ}42'29.32''$ $b=473.283$ $c=310.032$ $R=265.5946$ | $r=110.8561$ $r_1=561.644$ $r_2=400.8712$ $r_3=210.7192$ |
| 17. | $a=122.073$ $A=64^{\circ}31'45.28''$ $B=65^{\circ}48'27.70''$ | $C=49^{\circ}39'47.02''$ $b=123.34$ $c=103.068$ $R=67.60763$ | $r=32.93385$ $r_1=109.9998$ $r_2=112.73784$ $r_3=80.62681$ |
| 18. | $a=786.33$ $B=21^{\circ}33'9.62''$ $C=57^{\circ}14'20.48''$ | $A=101^{\circ}12'29.90''$ $b=294.48$ $c=674.11$ $R=400.8097$ | $r=110.9599$ $r_1=1068.3954$ $r_2=167.0089$ $r_3=478.7944$ |
| 19. | $a=2358.99$ $A=80^{\circ}10'22.64''$ $B=67^{\circ}33'33.80''$ | $C=26^{\circ}10'3.50''$ $b=2155.23$ $c=1040.34$ $R=1118.131$ | $r=408.0798$ $r_1=2614.54$ $r_2=1068.843$ $r_3=652.2222$ |
| 20. | $a=70.48$ $A=28^{\circ}46'16.82''$ $B=67^{\circ}1'50.08''$ | $C=84^{\circ}11'53.10''$ $b=134.822$ $c=145.682$ $R=73.21606$ | $r=26.9545$ $r_1=45.0119$ $r_2=116.223$ $r_3=158.5638$ |
| 21. | $a=4322.56$ $A=72^{\circ}19'11.54''$ $C=51^{\circ}21'10.96''$ | $B=56^{\circ}19'37.50''$ $b=3775.64$ $c=3543.32$ $R=2268.424$ | $r=1094.909$ $r_1=4253.906$ $r_2=3116.297$ $r_3=2798.406$ |
| 22. | $a=1204.66$ $B=74^{\circ}14'44.72''$ $C=37^{\circ}20'44.36''$ | $A=68^{\circ}24'30.92''$ $b=1240.9$ $c=785.92$ $R=647.7839$ | $r=281.4538$ $r_1=1100.2722$ $r_2=1225.26$ $r_3=547.0578$ |
| 23. | $a=187.035$ $A=84^{\circ}46'54.70''$ $B=61^{\circ}58'11.62''$ | $C=33^{\circ}14'53.68''$ $b=165.783$ $c=102.972$ $R=93.90667$ | $r=37.29846$ $r_1=208.0306$ $r_2=136.8516$ $r_3=68.04294$ |
| 24. | $a=20077.67$ $A=82^{\circ}31'16.78''$ $C=32^{\circ}1'18.18''$ | $B=65^{\circ}27'25.04''$ $b=18420.33$ $c=10737.33$ $R=10124.96$ | $r=3982.967$ $r_1=21597.22$ $r_2=15821.55$ $r_3=7064.05$ |
| 25. | $a=5618.156$ $B=71^{\circ}46'44.56''$ $C=54^{\circ}46'38.52''$ | $A=53^{\circ}26'36.92''$ $b=6643.4$ $c=5713.6$ $R=3497.05$ | $r=1696.249$ $r_1=4524.564$ $r_2=6503.416$ $r_3=4656.465$ |
| 26. | $a=152.294$ $A=60^{\circ}58'15.38''$ $B=78^{\circ}51'16.72''$ | $C=40^{\circ}10'27.9''$ $b=170.89$ $c=112.3631$ $R=87.08746$ | $r=38.54804$ $r_1=128.2040$ $r_2=179.0552$ $r_3=79.63866$ |
| 27. | $a=7710.144$ $B=51^{\circ}22'32.94''$ $C=63^{\circ}34'12.98''$ | $A=65^{\circ}3'14.08''$ $b=6643.4$ $c=7614.7$ $R=4251.738$ | $r=2087.92$ $r_1=7004.926$ $r_2=5283.446$ $r_3=6806.502$ |

EXAMPLES. V.—Continued.

| | DATA. | ANSWERS. | |
|-----|--|--|--|
| 28. | $a=854.45$ $B=61^{\circ}45'17.70''$ $C=40^{\circ}45'14.78''$ | $A=77^{\circ}29'27.46''$ $b=771.0144$ $c=571.36$ $R=437.6132$ | $r=195.7687$ $r_1=881.4260$ $r_2=656.7996$ $r_3=407.9952$ |
| 29. | $a=1999.17$ $B=60^{\circ}46'54''$ $C=52^{\circ}6'26.74''$ | $A=67^{\circ}3'39.26''$ $b=1894.38$ $c=1714.08$ $R=1085.274$ | $r=533.2875$ $r_1=1857.858$ $r_2=1044.306$ $r_3=1372.217$ |
| 30. | $a=856.41$ $A=75^{\circ}13'54.42''$ $B=59^{\circ}17'26.38''$ | $C=45^{\circ}28'39.2''$ $b=761.47$ $c=631.40$ $R=442.8341$ | $r=206.7065$ $r_1=816.0095$ $r_2=640.079$ $r_3=471.3543$ |

VI.—Triangles in which three sides are given.

First find r the radius of the inscribed circle by the formula $r = \sqrt{\frac{s(s-a)(s-b)(s-c)}{s}}$; then the three half angles from the equations $\tan \frac{1}{2}A = \frac{r}{s-a}$, $\tan \frac{1}{2}B = \frac{r}{s-b}$, $\tan \frac{1}{2}C = \frac{r}{s-c}$, correcting the results so as to make the sum of the three half angles exactly 90° . In general this will not involve alterations beyond $\pm .02''$; should greater changes be required, the work must be suspected. The values of r_1, r_2, r_3 may then be obtained from the equations $r_1 = \tan \frac{1}{2}A, r_2 = \tan \frac{1}{2}B, r_3 = \tan \frac{1}{2}C$, and $R = \frac{1}{2}(r_1 + r_2 + r_3 - r)$. Should a further check be desired, $2R$ may be calculated from one of the three $\frac{a}{\sin A}, \frac{b}{\sin B}, \frac{c}{\sin C}$, taking the angle nearest 45° . An example is worked out below.

| DATA. | | |
|----------------|----------------|----------------------|
| $a=6634.39$ | $s-a=1426.719$ | $\log s-a=3.1543384$ |
| $b=5646.67$ | $s-b=2414.439$ | $\log s-b=3.3828162$ |
| $c=3841.158$ | $s-c=4219.951$ | $\log s-c=3.6253075$ |
| $2s=16122.218$ | $s=8061.109$ | $-\log s=3.9063948$ |
| $s=8061.109$ | | $2 \log r=6.2560673$ |
| | | $\log r=3.1280337$ |

| $L \tan \frac{1}{2}A$ | $L \tan \frac{1}{2}B$ | $L \tan \frac{1}{2}C$ |
|---|---|---|
| 13.1280337 | 13.1280337 | 13.1280337 |
| 3.1543384 | 3.3828162 | 3.6253075 |
| 9.9736953 | 9.7452175 | 9.5027262 |
| $43^{\circ}16' - \left. \begin{array}{l} 7073 \\ 2532 \end{array} \right\} 120$ | $29^{\circ}5' - \left. \begin{array}{l} 403 \\ 2975 \end{array} \right\} 228$ | $17^{\circ}39' + \left. \begin{array}{l} 6721 \\ 4371 \end{array} \right\} 541$ |
| 7200(2.84) | 13680(4.6) | 32460(7.43) |
| 2136 | 1780 | 1863 |
| 110 | | 115 |

EXAMPLES FOR PRACTICE IN THE USE

| | | | | |
|-------------------------------------|---------------------------------|----------------------|--------------------------|----------------------|
| $\frac{1}{2}A=43^{\circ}15'57.16''$ | $\log r=3.1280337$ | 1.9736953 | $\bar{1}.7452175$ | $\bar{1}.5027262$ |
| $\frac{1}{2}B=29^{\circ}4'55.40''$ | 113 | 3.9003948 | 3.9003948 | 3.9003948 |
| $\frac{1}{2}C=17^{\circ}39'7.44''$ | 224 | $\log r_1=3.8800901$ | $\log r_2=3.6516123$ | $\log r_3=3.4091210$ |
| $90^{\circ}0'0''$ | <u>1342.87</u> | 873 | 075 | 2 |
| | | 7587.348 | 4483.449 | 2565.199 |
| | | 28 | 48 | -2 |
| $-r=-1342.87$ | $\log c=3.5844621$ | 1784 | | |
| $r_1=7587.348$ | $-\log \sin C=-\bar{1}.7618208$ | 15-446 | | |
| $r_2=4483.449$ | 442j | -.12=-3 | 6 | |
| $r_3=2565.199$ | $\log 2R=3.8225971$ | 30 | | |
| $4R=13293.126$ | <u>6646.562</u> | 41 | | |
| | | | $A=86^{\circ}31'54.32''$ | $r_1=7587.348$ |
| | | | $B=58^{\circ}9'50.8''$ | $r_2=4483.449$ |
| | | | $C=35^{\circ}18'14.88''$ | $r_3=2565.199$ |
| | | | $r=1342.87$ | $R=3323.281$ |

EXAMPLES. VI.

DATA.

ANSWERS.

| | | | |
|---|--|---|--|
| 1. $a=159928.8$ $b=108792.1$ $c=183626.3$ | $A=59^{\circ}59'59.8''$ $B=36^{\circ}5'39.6''$ $C=83^{\circ}54'20.6''$ $r=28240.42$ | $r_1=130581.3$ $r_2=73094.15$ $r_3=203310.9$ $R=92334.98$ | |
| 2. $a=8175.801$ $b=9387.248$ $c=5561.611$ | $A=60^{\circ}$ $B=83^{\circ}54'20.5''$ $C=36^{\circ}5'39.5''$ $r=1955.214$ | $r_1=6675.515$ $r_2=10393.55$ $r_3=3707.335$ $R=4720.299$ | $\Delta_1=3825.637$ $=b-c$ |
| 3. $a=9943.485$ $b=8660.254$ $c=5891.161$ | $A=83^{\circ}54'20.5''$ $B=60^{\circ}$ $C=36^{\circ}5'39.5''$ $r=2071.068$ | $r_1=11000.415$ $r_2=7071.068$ $r_3=3990.584$ $R=5000$ | $\Delta_1=2071.067$ $\Delta_2=4052.325$ $\Delta_3=2071.067$ |
| 4. $a=28466.08$ $b=24792.47$ $c=16865.15$ | $A=83^{\circ}54'20.4''$ $B=60^{\circ}$ $C=36^{\circ}5'39.6''$ $r=5929.031$ | $r_1=31517.59$ $r_2=20242.97$ $r_3=11424.21$ $R=14313.93$ | $\Delta_1=5929.008$ $\Delta_2=11600.93$ $\Delta_3=5929.019$ |
| 5. $a=2356.186$ $b=2365.324$ $c=1383.01$ | $A=72^{\circ}36'20.57''$ $B=73^{\circ}19'40.76''$ $C=34^{\circ}3'52.67''$ $r=511.3707$ | $r_1=2242.342$ $r_2=2272.171$ $r_3=935.0524$ $R=1234.549$ | |
| 6. $a=38714.69$ $b=33539.37$ $c=28471.26$ | $A=76^{\circ}46'57.14''$ $B=57^{\circ}29'52.58''$ $C=45^{\circ}43'10.28''$ $r=9229.178$ | $r_1=39904.455$ $r_2=20628.73$ $r_3=21232.35$ $R=19884.09$ | |
| 7. $a=468.2137$ $b=331.2970$ $c=258.9733$ | $A=104^{\circ}18'19.36''$ $B=43^{\circ}17'8.40''$ $C=32^{\circ}24'32.24''$ $r=78.54318$ | $r_1=681.13255$ $r_2=209.9995$ $r_3=153.8038$ $R=241.5982$ | |
| 8. $a=2846.608$ $b=2479.247$ $c=1687.515$ | $A=83^{\circ}53'9.77''$ $B=59^{\circ}59'46.97''$ $C=36^{\circ}7'3.26''$ $r=593.149$ | $r_1=3151.134$ $r_2=2024.438$ $r_3=1143.37$ $R=1431.446$ | $\Delta_1=592.3827$ $\Delta_2=592.4747$ $\Delta_3=1159.2203$ |

EXAMPLES. VI.—Continued.

DATA.

ANSWERS.

| | | | | |
|-----|--|--|---|--|
| 9. | $a=47674.27$ $b=46639.05$ $c=33611.96$ | $A=70^{\circ}46'47.08''$ $B=67^{\circ}28'53.44''$ $C=41^{\circ}44'19.48''$ $r=11571.21$ | $r_1=45438.86$ $r_2=42723.55$ $r_3=24385.79$ $R=25244.25$ | $\Delta_1=7284.168$ |
| 10. | $a=466.3904$ $b=456.2141$ $c=270.8213$ | $A=75^{\circ}0'0''$ $B=70^{\circ}52'57.7''$ $C=34^{\circ}7'2.3''$ $r=100$ | $r_1=457.8737$ $r_2=424.7102$ $r_3=183.1016$ $R=241.4214$ | $\Delta_1=100$ $\Delta_2=159.8853$ $\Delta_3=60.32828$ |
| 11. | $a=19674.04$ $b=15361.02$ $c=9837.02$ | $A=100^{\circ}21'30.58''$ $B=50^{\circ}10'45.12''$ $C=29^{\circ}27'44.3''$ $r=3312.615$ | $r_1=26908.75$ $r_2=10504.845$ $r_3=5899.026$ $R=10000$ | |
| 12. | $a=9682.458$ $b=9990.51$ $c=2919.436$ | $A=75^{\circ}31'20.68''$ $B=87^{\circ}30'11.67''$ $C=16^{\circ}58'27.65''$ $r=1250$ | $r_1=8749.988$ $r_2=10814.376$ $r_3=1685.044$ $R=5000$ | |
| 13. | $a=47674.27$ $b=46639.05$ $c=33611.96$ | $A=70^{\circ}46'47.08''$ $B=67^{\circ}28'53.44''$ $C=41^{\circ}44'19.48''$ $r=11571.21$ | $r_1=45438.76$ $r_2=42723.55$ $r_3=24385.78$ $R=25244.25$ | |
| 14. | $a=375.3259$ $b=312.7715$ $c=271.0686$ | $A=79^{\circ}39'36.48''$ $B=55^{\circ}3'53.24''$ $C=45^{\circ}16'30.28''$ $r=86.95647$ | $r_1=400$ $r_2=250$ $r_3=200$ $R=190.7608$ | |
| 15. | $a=14538.87$ $b=34645.37$ $c=30139.84$ | $A=24^{\circ}42'1.5''$ $B=95^{\circ}16'15.9''$ $C=60^{\circ}1'42.6''$ $r=5500.802$ | $r_1=8684.138$ $r_2=43489.62$ $r_3=22312.04$ $R=17396.25$ | |
| 16. | $a=7406.25$ $b=31560.45$ $c=30137.34$ | $A=13^{\circ}32'3.74''$ $B=94^{\circ}13'58.62''$ $C=72^{\circ}13'57.64''$ $r=3221.169$ | $r_1=4100.008$ $r_2=37203.835$ $r_3=25210.84$ $R=15823.38$ | |
| 17. | $a=10927.86$ $b=8517.08$ $c=3642.62$ | $A=122^{\circ}47'45.58''$ $B=40^{\circ}55'54.4''$ $C=16^{\circ}16'20.02''$ $r=1129.583$ | $r_1=21171.03$ $r_2=4308.21$ $r_3=1650.345$ $R=6500$ | |
| 18. | $a=5761.367$ $b=5017.848$ $c=3413.403$ | $A=83^{\circ}54'20.57''$ $B=60^{\circ}0'0''$ $C=36^{\circ}5'39.43''$ $r=1200$ | $r_1=6378.981$ $r_2=4097.055$ $r_3=2312.188$ $R=2897.056$ | $\Delta_1=\Delta_2=r$ $\Delta_3=2347.964$ $=a-c$ |
| 19. | $a=94371.5$ $b=124351.6$ $c=106381.7$ | $A=47^{\circ}30'$ $B=76^{\circ}17'14.52''$ $C=56^{\circ}12'45.48''$ $r=30000.31$ | $r_1=71524.76$ $r_2=127657.63$ $r_3=86817.93$ $R=64000$ | $\Delta_1=15998.76$ |
| 20. | $a=4157.855$ $b=4812.793$ $c=3608.528$ | $A=57^{\circ}2'28''$ $B=76^{\circ}13'21.6''$ $C=46^{\circ}44'10.4''$ $r=1158.427$ | $r_1=3417.89$ $r_2=4933.676$ $r_3=2717.593$ $R=2477.683$ | |

EXAMPLES. VI.--Continued.

| | DATA. | ANSWERS. | | | |
|-----|--|---|---|--|--|
| 21. | $a=9401.45$ $b=7360.12$ $c=5318.79$ | The sides in $A, P.$, and the greatest angle exceeds the least by 60° . $8r=3A$. | $A=94^\circ 20' 27.92''$ $B=51^\circ 19' 4.16''$ $C=34^\circ 20' 27.92''$ $r=1767.844$ | $r_1=11910.02$ $r_2=5303.53$ $r_3=3411.289$ $A=4714.249$ | $\Delta_1=2357.125$ |
| 22. | $a=2692538$ $b=1860498$ $c=1028458$ | | $A=135^\circ 31' 20.04''$ $B=28^\circ 57' 18.08''$ $C=15^\circ 31' 20.98''$ $r=240189.2$ | $r_1=6125317$ $r_2=720507.9$ $r_3=380361.9$ $A=1921514$ | $\Delta_1=1664081$ |
| 23. | $a=964.701$ $b=955.078$ $c=945.455$ | The sides in $A, P.$, $A-C=2''$. | $A=61^\circ 0' 18.14''$ $B=59^\circ 59' 23.70''$ $C=59^\circ 0' 18.16''$ $r=275.6513$ | $r_1=843.9606$ $r_2=820.9536$ $r_3=810.6189$ $A=551.4705$ | $\Delta_1=9.622469$ |
| 24. | $a=2089.88$ $b=2413.68$ $c=1206.96$ | | $A=59^\circ 58' 46.83''$ $B=90^\circ 1' 1.35''$ $C=30^\circ 0' 11.82''$ $r=441.7114$ | $r_1=1647.81$ $r_2=820.9536$ $r_3=765.1524$ $A=1206.84$ | $\Delta_1=624.7486$ $\Delta_2=1208.062$ $\Delta_3=625.855$ |
| 25. | $a=4794.217$ $b=4329.393$ $c=3655.232$ | | $A=73^\circ 15' 10.41''$ $B=59^\circ 51' 12.13''$ $C=46^\circ 53' 37.46''$ $r=1185.846$ | $r_1=4749.779$ $r_2=3078.041$ $r_3=2771.157$ $A=2503.283$ | |
| 26. | $a=544.36$ $b=426.39$ $c=425.09$ | | $A=79^\circ 28' 50.42''$ $B=50^\circ 21' 53.56''$ $C=50^\circ 9' 16.02''$ $r=127.6707$ | $r_1=580.2548$ $r_2=328.155$ $r_3=326.5914$ $A=276.8326$ | |
| 27. | $a=51509.95$ $b=72027.91$ $c=63926$ | | $A=44^\circ 1' 38.62''$ $B=76^\circ 22' 16.08''$ $C=59^\circ 36' 5.3''$ $r=17070.53$ | $r_1=37896.22$ $r_2=73721.52$ $r_3=53682.39$ $A=37057.2$ | |
| 28. | $a=10301.99$ $b=10825.782$ $c=9205.4$ | | $A=61^\circ 16' 11.50''$ $B=67^\circ 8' 35.18''$ $C=51^\circ 35' 13.32''$ $r=2880.918$ | $r_1=8981.956$ $r_2=10065.804$ $r_3=7329.696$ $A=5874.135$ | |
| 29. | $a=5844.305$ $b=7202.791$ $c=4602.7$ | | $A=54^\circ 4' 4.86''$ $B=86^\circ 18' 43.14''$ $C=39^\circ 37' 12''$ $r=1520.916$ | $r_1=4503.11$ $r_2=8274.385$ $r_3=3178.898$ $A=3608.869$ | |
| 30. | $a=1168.861$ $b=1082.5782$ $c=1278.52$ | | $A=58^\circ 38' 46.14''$ $B=52^\circ 16' 21.68''$ $C=60^\circ 4' 52.18''$ $r=334.8419$ | $r_1=991.3952$ $r_2=806.0432$ $r_3=1214.8782$ $A=684.3687$ | |
| 31. | $a=1262.92$ $b=1332.60$ $c=1142.72$ | | $A=60^\circ 46' 54.01''$ $B=67^\circ 3' 39.26''$ $C=52^\circ 9' 26.73''$ $r=355.525$ | $r_1=1096.2042$ $r_2=1238.5722$ $r_3=914.8112$ $A=723.5157$ | |
| 32. | $a=18943.8$ $b=25692.3$ $c=22844.1$ | | $A=45^\circ 28' 39.20''$ $B=75^\circ 13' 54.42''$ $C=59^\circ 17' 26.38''$ $r=6201.195$ | $r_1=14140.63$ $r_2=25998.28$ $r_3=19202.37$ $A=13285.02$ | |

EXAMPLES. VI.--Continued.

| DATA. | | ANSWERS. | | |
|-------|--|---|--|--|
| 33. | $a=70137$ $b=66630$ $c=76147$ | $A=58^{\circ}23'20.49''$ $B=54^{\circ}09.25''$ $C=71^{\circ}36'30.26''$ $r=20293.99$ | $r_1=59483.42$ $r_2=54245.50$ $r_3=71278.06$ $K=41178.20$ | |
| 34. | $a=3506.8$ $b=4282.05$ $c=2856.8$ | $A=54^{\circ}31'24.61''$ $B=83^{\circ}55'0.40''$ $C=41^{\circ}33'34.99''$ $r=935.773$ | $r_1=2742.83$ $r_2=4785.72$ $r_3=2019.82$ $K=2153.15$ | |
| 35. | $a=786.934$ $b=604.81$ $c=431.5957$ | $A=97^{\circ}23'42.2''$ $B=49^{\circ}39'21.88''$ $C=32^{\circ}56'55.92''$ $r=141.9714$ | $r_1=1037.641$ $r_2=421.7921$ $r_3=269.6064$ $K=396.767$ | |
| 36. | $a=786.934$ $b=748.887$ $c=431.5957$ | $A=78^{\circ}35'37.44''$ $B=68^{\circ}53'4.20''$ $C=32^{\circ}31'18.36''$ $r=161.04$ | $r_1=805.0664$ $r_2=674.625$ $r_3=286.9277$ $K=401.3948$ | |
| 37. | $a=43783$ $b=43397$ $c=43012$ | $A=60^{\circ}53'9.3''$ $B=59^{\circ}59'29.07''$ $C=50^{\circ}7'21.63''$ $r=12525.76$ | $r_1=38257.23$ $r_2=37576.69$ $r_3=36921.59$ $K=25057.44$ | $\Delta_1=385.1494$ $\Delta_2=771.168$ $\Delta_3=399.6372$ |
| 38. | $a=96.264$ $b=114.735$ $c=106.848$ | $A=51^{\circ}20'53.50''$ $B=68^{\circ}33'38.16''$ $C=60^{\circ}5'28.34''$ $r=30.12111$ | $r_1=76.39623$ $r_2=108.3302$ $r_3=91.92327$ $K=61.63215$ | $\Delta_1=9.255574$ |
| 39. | $a=224.626$ $b=218.323$ $c=199.92$ | $A=64^{\circ}47'27.34''$ $B=61^{\circ}34'17.14''$ $C=53^{\circ}38'15.52''$ $r=61.428976$ | $r_1=203.9497$ $r_2=191.5018$ $r_3=162.4983$ $K=124.1302$ | |
| 40. | $a=14724$ $b=27601$ $c=17224$ | $A=50^{\circ}0'30.76''$ $B=66^{\circ}10'31.38''$ $C=63^{\circ}39'57.86''$ $r=4687.54$ | $r_1=11554.79$ $r_2=16188.95$ $r_3=15380.63$ $K=9609.21$ | |
| 41. | $a=63341$ $b=52587$ $c=29448$ | $A=97^{\circ}2'17.00''$ $B=55^{\circ}29'1.08''$ $C=27^{\circ}28'41.02''$ $r=10571.92$ | $r_1=82214.21$ $r_2=469.3244$ $r_3=17771.81$ $K=31910.96$ | |
| 42. | $a=1266.82$ $b=704.04$ $c=1166.82$ | $A=81^{\circ}9'58.50''$ $B=33^{\circ}18'34.24''$ $C=65^{\circ}31'27.26''$ $r=258.7082$ | $r_1=1343.8572$ $r_2=469.3244$ $r_3=1009.5812$ $K=641.0137$ | |
| 43. | $a=1750.23$ $b=1577.61$ $c=1033.44$ | $A=81^{\circ}17'39.16''$ $B=62^{\circ}59'51.52''$ $C=35^{\circ}42'29.32''$ $r=399.5202$ | $r_1=1872.1467$ $r_2=1336.2372$ $r_3=702.3975$ $K=885.3153$ | |
| 44. | $a=13467.87$ $b=8287.92$ $c=7251.93$ | $A=120^{\circ}$ $B=32^{\circ}12'15.22''$ $C=27^{\circ}47'44.78''$ $r=1794.387$ | $r_1=25121.42$ $r_2=4186.903$ $r_3=3588.774$ $K=7775.68$ | $\Delta_2=15539.85$ $=b+c$ |

VII.—Triangles in which two sides and the included angle are given.

Let b, c be the two given lengths ($b > c$), A the included angle, then we find $\frac{1}{2}(B - C)$ by the equation $\tan \frac{1}{2}(B - C) = \frac{b - c}{b + c} \cot \frac{1}{2}A$; and $\frac{1}{2}(B + C) = 90^\circ - \frac{A}{2}$; $B = \frac{1}{2}(B + C) + \frac{1}{2}(B - C)$, $C = \frac{1}{2}(B + C) - \frac{1}{2}(B - C)$. The value of $2R$ is then calculated from each of the expressions $b/\sin B$, $c/\sin C$, which should agree ($p.p.$). The formulae $a = 2R \sin A$, $r = (s - a) \tan \frac{A}{2}$, $r_1 = s \tan \frac{A}{2}$, $r_2 = (s - c) \cot \frac{1}{2}A$, $r_3 = (s - b) \cot \frac{1}{2}A$ enable us to find a, r, r_1, r_2, r_3 ; and the equation $4R = r_1 + r_2 + r_3 - r$ supplies another check.

| DATA. | | EXAMPLE WORKED OUT. | | | |
|---|--|---------------------------------|-------------|-----------------------------|---------------------|
| $b = 33417$ | $b - c = 9990$ | $I. \cot \frac{1}{2}A = 10$ | 5228579 | 4592 | |
| $c = 23427$ | $b + c = 56844$ | | $880 f$ | $10'' = 765$ | 3 |
| $A = 33^\circ 23' 37''$ | $A = 16^\circ 41' 48.5''$ | $\log(b - c) = 3$ | 9995655 | $1.5 = 114$ | 8 |
| | | $-\log(b + c) = 4$ | 7546816 | | |
| | | $I \tan \frac{1}{2}(B - C) = 9$ | 7678268 | $4560(1.58$ | |
| | | $30'' 22' -$ | 344 | 1664 | |
| | | $2896 f$ | 76 | 216 | |
| $\frac{1}{2}(B - C) = 30^\circ 21' 58.42''$ | | | | $\log b = 4$ | 5239675 |
| $\frac{1}{2}(B + C) = 73^\circ 18' 11.5''$ | | | | $-\log \sin B = -1.9874955$ | 257 |
| $B = 103^\circ 40' 9.92''$ | $180^\circ - B = 76^\circ 19' 50.08''$ | | | | $308 = 5 + 10 + 10$ |
| $C = 42^\circ 56' 13.08''$ | | | | | 250 |
| | | | | $\log 2R = 4$ | 5364463 |
| | | | | | 48 |
| | | | | 34391.12 | 15 |
| | | | | | 1916 |
| $\log c = 4.3697167$ | 1358 | | | $\log 2R = 4.5364463$ | 1916 |
| $-\log \sin C = 1.8332408$ | $12 - 271$ | 6 | | $\log \sin A = 1.7405505$ | 1181 |
| | $1.2 = 27$ | 2 | | | $36 = 1149$ |
| | $\log 2R = 4.5364463$ | $-1.2 = -2$ | 7 | $\log a = 4.2771149$ | 047 |
| | | | | 18928.44 | 102 |
| $\log \cot \frac{1}{2}A = 0.5229459$ | $a = 18928.44$ | $s - a = 18957.78$ | | $\log(s - a) = 4.2777877$ | |
| $\log \tan \frac{1}{2}A = 1.4770541$ | $b = 33417$ | $s - b = 4409.22$ | | $\log(r/b) = 3.6502317$ | |
| | $c = 23427$ | $s - c = 14459.22$ | | $\log(s - c) = 4.1601449$ | |
| | 75772.44 | $s = 37886.22$ | | $\log r = 4.5784813$ | |
| | $s = 37886.22$ | | | | |
| $\log r$ | $\log r_1$ | $\log r_2$ | $\log r_3$ | | |
| 1.4770541 | 1.4770541 | $.5229459$ | $.5229459$ | $-r = 5686.457$ | |
| 4.2777877 | 4.5784813 | 4.1601449 | 3.6502317 | $r_1 = 11364.11$ | |
| 3.7548418 | 4.0555354 | 4.6830908 | 4.1731776 | $r_2 = 48204.86$ | |
| 374 | 12 | 831 | 571 | $r_3 = 14899.70$ | |
| 5686.457 | 11364.11 | 48204.86 | 14899.70 | $4R = 68782.22$ | |
| 44 | 42 | 77 | 250 | | |

ANSWERS.

| | | | |
|----------------------------|----------------|------------------|------------------|
| $B = 103^\circ 40' 9.92''$ | $a = 18928.44$ | $r = 5686.457$ | $r_2 = 48204.86$ |
| $C = 42^\circ 56' 13.08''$ | $R = 17195.56$ | $r_1 = 11364.11$ | $r_3 = 14899.70$ |

EXAMPLES. VII.

DATA.

ANSWERS.

| | | | | |
|-----|---|--|---|--|
| 1. | $b=23571.26$ $c=20753.82$ $A=67^{\circ}31'55''$ | $B=61^{\circ}39'53.64''$ $C=50^{\circ}48'11.36''$ $a=24747.05$ $A=13389.91$ | $r=6544.85$ $r_1=23090.21$ $r_2=20614.16$ $r_3=16400.11$ | |
| 2. | $b=8438.715$ $c=4219.357$ $A=59^{\circ}37'56.4''$ | $B=90^{\circ}22'6.07''$ $C=29^{\circ}59'57.53''$ $a=7281.067$ $R=4219.444$ | $r=1540.727$ $r_1=5713.363$ $r_2=10033.872$ $r_3=2671.274$ | |
| 3. | $b=378.9217$ $c=269.1351$ $A=88^{\circ}59'2.4''$ | $B=55^{\circ}17'30.72''$ $C=35^{\circ}43'26.88''$ $a=460.8674$ $R=230.4699$ | $r=91.94954$ $r_1=544.71025$ $r_2=200.4319$ $r_3=178.6812$ | |
| 4. | $b=13347.61$ $c=11258.73$ $A=47^{\circ}31'57''$ | $B=77^{\circ}8'44''$ $C=55^{\circ}19'19''$ $a=10099.1$ $R=6845.362$ | $r=3194.125$ $r_1=7641.255$ $r_2=13839$ $r_3=9095.308$ | |
| 5. | $b=99144.49$ $c=60876.14$ $A=45''$ | $B=97^{\circ}30'$ $C=37^{\circ}30'$ $a=70710.68$ $R=50000$ | $r=18496.69$ $r_1=47786.01$ $r_2=131549.34$ $r_3=39161.36$ | |
| 6. | $b=5.6794$ $c=5.6318$ $A=44^{\circ}59'6.26''$ | $B=68^{\circ}5'23.27''$ $C=66^{\circ}55'30.47''$ $a=4.327479$ $R=3.060782$ | $r=1.445844$ $r_1=3.237680$ $r_2=5.283127$ $r_3=5.168167$ | |
| 7. | $b=87.92154$ $c=29.30718$ $A=70^{\circ}31'43.62''$ | $B=90'$ $C=19^{\circ}28'16.38''$ $a=82.89322$ $R=43.96077$ | $r=12.13143$ $r_1=70.75379$ $r_2=100.06097$ $r_3=17.16775$ | |
| 8. | $b=99905.1$ $c=29194.36$ $A=75^{\circ}31'20.7''$ | $B=87^{\circ}30'11.65''$ $C=16^{\circ}58'27.65''$ $a=96824.54$ $R=50000$ | $r=12500$ $r_1=87499.9$ $r_2=108143.67$ $r_3=16856.4$ | |
| 9. | $b=1243.516$ $c=1003.817$ $A=47^{\circ}30'$ | $B=76^{\circ}17'14.56''$ $C=56^{\circ}12'45.44''$ $a=943.715$ $R=640$ | $r=300.0031$ $r_1=715.2478$ $r_2=1276.5762$ $r_3=868.1792$ | $\Delta_1=159.9976$ $\Delta_2=343.1666$ $\Delta_3=185.617$ |
| 10. | $b=57777.78$ $c=47222.22$ $A=129^{\circ}18'27.52''$ | $B=28^{\circ}4'21.01''$ $C=22^{\circ}37'11.47''$ $a=95000$ $R=61388.89$ | $r=10555.56$ $r_1=211111.1$ $r_2=25000$ $r_3=20000$ | |
| 11. | $b=5237.828$ $c=6673.472$ $A=87^{\circ}50'57.8''$ | $B=38^{\circ}56'32.8''$ $C=53^{\circ}12'29.4''$ $a=8327.466$ $R=4166.667$ | $r=1725.888$ $r_1=9746.503$ $r_2=3577.745$ $r_3=5068.313$ | $\Delta_1=1725.894$ |
| 12. | $b=14045.86$ $c=8109.38$ $A=30^{\circ}$ | $B=120^{\circ}$ $C=30^{\circ}$ $a=8109.38$ $R=8109.38$ | $r=1881.79$ $r_1=4054.69$ $r_2=26209.93$ $r_3=4054.69$ | |

EXAMPLES. VII.—Continued.

| DATA. | | ANSWERS. | | |
|-------|---|---|--|---|
| 13. | $b=1404.59$ $c=810.94$ $A=30^{\circ}0'0.07''$ | $B=119^{\circ}59'59.965''$ $C=29^{\circ}59'59.965''$ $a=810.941$ $R=810.9405$ | $r=188.1794$ $r_1=405.4705$ $r_2=2621.0007$ $r_3=405.4702$ | |
| 14. | $b=4568.39$ $c=864.29$ $A=110^{\circ}34'11.96''$ | $B=60^{\circ}$ $C=9^{\circ}25'48.04''$ $a=4938.8$ $R=2637.561$ | $r=356.427$ $r_1=7484.971$ $r_2=2993.989$ $r_3=427.712$ | $\Delta_1=2253.115$ $\Delta_2=4074.51$ $\Delta_3=2253.115$ |
| 15. | $b=9237.97$ $c=6598.55$ $A=81^{\circ}47'12.46''$ | $B=60^{\circ}$ $C=38^{\circ}12'47.54''$ $a=10557.68$ $R=5333.545$ | $r=2285.805$ $r_1=11429.025$ $r_2=7014.35$ $r_3=4571.61$ | $\Delta_3=2015.891$ $\Delta_2=3959.131$ $\Delta_1=2015.891$ |
| 16. | $b=47898.32$ $c=39817.65$ $A=65^{\circ}26'23.6''$ | $B=65^{\circ}26'23.6''$ $C=49^{\circ}7'12.8''$ $a=47898.32$ $R=26331.48$ | $r=12791.04$ $r_1=43564.76$ $r_2=43564.76$ $r_3=30987.44$ | |
| 17. | $b=48127.93$ $c=36085.28$ $A=57^{\circ}2'28''$ | $B=76^{\circ}13'21.62''$ $C=46^{\circ}44'10.38''$ $a=41578.55$ $R=24776.83$ | $r=11584.27$ $r_1=34178.9$ $r_2=49336.76$ $r_3=27175.93$ | |
| 18. | $b=4157.855$ $c=3608.528$ $A=76^{\circ}13'21.6''$ | $B=57^{\circ}2'28''$ $C=46^{\circ}44'10.4''$ $a=4812.793$ $R=2477.683$ | $r=1158.427$ $r_1=4933.676$ $r_2=3417.89$ $r_3=2717.593$ | |
| 19. | $b=23571.26$ $c=20753.52$ $A=67^{\circ}31'55''$ | $B=61^{\circ}39'55.86''$ $C=50^{\circ}48'9.14''$ $a=24746.85$ $R=13389.84$ | $r=6544.721$ $r_1=23090.03$ $r_2=20614.22$ $r_3=16399.73$ | |
| 20. | $b=30901.7$ $c=58778.53$ $A=18^{\circ}$ | $B=18^{\circ}$ $C=144^{\circ}$ $a=30901.7$ $R=50000$ | $r=4654.803$ $r_1=9549.152$ $r_2=9549.152$ $r_3=185556.5$ | |
| 21. | $b=12369.12$ $c=4123.04$ $A=40^{\circ}55'55''$ | The triangle in which the greatest side is 3 times the least, and the greatest angle 3 times the mean. <i>q.p.</i> $B=122^{\circ}47'44.84''$ $C=16^{\circ}16'20.16''$ $a=9640.399$ $R=7357.262$ | $r=1278.566$ $r_1=4876.439$ $r_2=23963.16$ $r_3=1868.021$ | $\Delta_1=5942.711$ $\Delta_2=14974.37$ $\Delta_3=9397.907$ |
| 22. | $b=13571$ $c=8965$ $A=79^{\circ}55'58.5''$ | $B=63^{\circ}44'18.57''$ $C=36^{\circ}19'42.93''$ $a=14899.98$ $R=7566.486$ | $r=3199.883$ $r_1=15687.6$ $r_2=11636.99$ $r_3=6141.241$ | $\Delta_1=2971.191$ $\Delta_2=6021.614$ $\Delta_3=3159.603$ |
| 23. | $b=18791.69$ $c=11275.05$ $A=68^{\circ}35'57.6''$ | $B=75^{\circ}49'39.51''$ $C=35^{\circ}34'22.89''$ $a=18045.27$ $R=9690.8$ | $r=4100.197$ $r_1=16409.691$ $r_2=18736.413$ $r_3=7717.292$ | $\Delta_1=3800.416$ |
| 24. | $b=133.7127$ $c=206.3273$ $A=58^{\circ}21'37.4''$ | $B=39^{\circ}53'33.16''$ $C=81^{\circ}44'49.44''$ $a=177.4979$ $R=104.2432$ | $r=45.384$ $r_1=144.5037$ $r_2=93.9095$ $r_3=223.9436$ | |

EXAMPLES. VII.—Continued.

| DATA. | | ANSWERS. | | |
|-------|--|--|---|---|
| 25. | $b=8233.59$ $c=6618.853$ $A=83^{\circ}8'1.8''$ | $B=55^{\circ}25'20.65''$ $C=41^{\circ}26'37.55''$ $a=9928.281$ $R=5000.006$ | $r=2183.392$ $r_1=10987.9$ $r_2=6508.179$ $r_3=4087.339$ | |
| 26. | $b=23870.56$ $c=10775.56$ $A=41^{\circ}24'34.6''$ | $b+c=2a, B+C=90^{\circ}$ $R=4r, r_1=3r$ $B=114^{\circ}17'42.7''$ $C=24^{\circ}17'42.7''$ $a=17323.06$ $R=13095$ | $r=3273.749$ $r_1=9821.247$ $r_2=40239.337$ $r_3=5593.201$ | |
| 27. | $b=5361.796$ $c=5957.551$ $A=23^{\circ}33'23.3''$ | $B=64^{\circ}3'19.87''$ $C=92^{\circ}23'10.83''$ $a=2383.022$ $R=2981.364$ | $r=931.6764$ $r_1=1428.571$ $r_2=4285.715$ $r_3=7142.852$ | |
| 28. | $b=138.74$ $c=116.82$ $A=34^{\circ}33'37''$ | $B=88^{\circ}8'4.07''$ $C=57^{\circ}18'18.93''$ $a=78.7452$ $R=69.4068$ | $r=27.50223$ $r_1=51.99869$ $r_2=161.7969$ $r_3=91.33386$ | |
| 29. | $b=1687.48$ $c=2531.22$ $A=104^{\circ}28'39.04''$ | $B=28^{\circ}57'18.08''$ $C=46^{\circ}34'2.88''$ $a=3374.96$ $R=1742.822$ | $r=544.6318$ $r_1=4901.688$ $r_2=980.3372$ $r_3=1633.896$ | $\Delta_1=1067.256$ $\Delta_2=2586.858$ $\Delta_3=1555.78$ |
| 30. | $b=31428.54$ $c=35714.25$ $A=69^{\circ}47'25.64''$ | $74r=20r_1=30r_2=24r_3$ $B=49^{\circ}52'33.45''$ $C=60.20^{\circ}0.91''$ $a=38571.41$ $R=20550.91$ | $r=9964.067$ $r_1=36867.08$ $r_2=24578.06$ $r_3=30722.57$ | |
| 31. | $b=273.8163$ $c=262.1835$ $A=8^{\circ}13'57.6''$ | $B=102^{\circ}39'55.14''$ $C=69^{\circ}6'7.26''$ $a=40.18648$ $R=140.323$ | $r=17.8412$ $r_1=20.73332$ $r_2=360.02$ $r_3=198.3801$ | |
| 32. | $b=24.1368$ $c=12.0696$ $A=59^{\circ}58'48''$ | $B=90^{\circ}1'0.17''$ $C=30^{\circ}0'11.83''$ $a=20.89888$ $R=12.068405$ | $r=4.417119$ $r_1=16.47823$ $r_2=28.56097$ $r_3=7.651542$ | $\Delta_1=6.247486$ $\Delta_2=12.08062$ $\Delta_3=6.25855$ |
| 33. | $b=856.41$ $c=761.47$ $A=45^{\circ}28'39.2''$ | $B=75^{\circ}13'54.53''$ $C=59^{\circ}17'26.27''$ $a=631.4601$ $R=442.834$ | $r=206.7065$ $r_1=471.3542$ $r_2=866.6096$ $r_3=640.079$ | |
| 34. | $b=786.934$ $c=748.887$ $A=32^{\circ}31'18.36''$ | $B=78^{\circ}35'37.44''$ $C=68^{\circ}53'4.20''$ $a=431.5957$ $R=401.3948$ | $r=161.04$ $r_1=286.9277$ $r_2=805.0664$ $r_3=674.6250$ | |
| 35. | $b=3734.67$ $c=3593.19$ $A=52^{\circ}31'49.2''$ | $B=65^{\circ}58'31.37''$ $C=61^{\circ}29'39.43''$ $a=3245.25$ $R=2044.444$ | $r=1007.332$ $r_1=2608.78$ $r_2=3431.515$ $r_3=3144.813$ | $\Delta_1=246.7459$ $\Delta_2=489.2463$ $\Delta_3=243.3927$ |
| 36. | $b=487.139$ $c=479.363$ $A=48^{\circ}39'39.6''$ | $B=66^{\circ}41'20''$ $C=64^{\circ}36'0.4''$ $a=398.2614$ $R=265.2195$ | $r=128.4674$ $r_1=308.5446$ $r_2=448.9992$ $r_3=431.8017$ | $\Delta_1=46.87498$ $\Delta_2=85.8304$ $\Delta_3=38.99333$ |

EXAMPLES. VII.—Continued.

| DATA. | | ANSWERS. | | |
|-------|--|---|--|---|
| 37. | $b=433.97$ $c=430.12$ $A=60^{\circ}53'9.3''$ | $B=59^{\circ}59'29.06''$ $C=59^{\circ}7'21.64''$ $a=437.83$ $R=250.5744$ | $r=125.2576$ $r_1=382.5723$ $r_2=375.7669$ $r_3=309.2159$ | $\Delta_1=3.851494$ $\Delta_2=3.996372$ $\Delta_3=7.71168$ |
| 38. | $b=1057.81$ $c=1220.55$ $A=27^{\circ}47'44.76''$ | $B=60^{\circ}0'0''$ $C=92^{\circ}12'15.24''$ $a=569.59$ $R=610.7271$ | $r=211.4054$ $r_1=352.3423$ $r_2=822.1324$ $r_3=1479.839$ | $\Delta_1=338.7705$ $\Delta_2=650.9600$ $=c-a$ $\Delta_3=338.7705$ |

VIII.—Triangles in which two sides and the angle opposite the greater are given.

Let a, b be the two given lengths ($a > b$), and A the given angle. First calculate R by the equation $2R = a/\sin A$; B must be an acute angle and is found from $\sin B = b/2R$; C from $A + B + C = 180^{\circ}$; c from $c = 2R \sin C$. Calculate r, r_1, r_2, r_3 exactly as in V.

EXAMPLE WORKED OUT.

| DATA. | | log $a=3.6823972$ | | 310 | log $b=3.5573301$ | | |
|-------------------------|-------------------------|----------------------------|------------|-------------------|-------------------------------------|-------------------|------------|
| $a=4812.793$ | $b=3608.528$ | $-\log \sin A = 1.9873103$ | 102 | 18-93 | $-\log 2R = 3.6950757$ | | |
| $A=76^{\circ}13'21.6''$ | | $\log 2R = 3.6950757$ | 00 | 3.6=18 6 | $L \sin B = 0.8622544$ | 12360(10.4" | 480 |
| | | <u>4955.366</u> | 57 | 101 6 | $46^{\circ}44' +$ | | |
| | | | | | 1188 } | | 206 |
| $A=76^{\circ}13'21.6''$ | $B=46^{\circ}44'10.4''$ | $\log 2R = 3.6950757$ | 382 | 819 | $A = 38^{\circ}6'40.8''$ | | |
| $C=57^{\circ}2'28.0''$ | | $\log \sin C = 1.9237554$ | 36 | 24=327 6 | $\log \tan \frac{A}{2} = 1.8943715$ | 2602 | |
| | | $\log c = 3.6188693$ | 36 | 4- 54 6 | | | 36=1561 2 |
| | | <u>4157.855</u> | 57 | | $= 1.8945484$ | | 4.8= 208 2 |
| | | | | | $\log \cot \frac{A}{2} = 0.1054516$ | | |
| $a=4812.793$ | $b=3608.528$ | $c=4157.855$ | | $s-a=1476.795$ | $s-b=2681.06$ | $s-c=2131.733$ | |
| $2s=12579.176$ | | $s=6289.588$ | | $\log -3.1693202$ | $\log -3.4283065$ | $\log -3.3287329$ | |
| $s=6289.588$ | | | | $\log -3.7986222$ | | | |
| $\log r$ | $\log r_1$ | $\log r_2$ | $\log r_3$ | | | | |
| 3.1693202 | 3.7986222 | 3.3287329 | 3.4283065 | | | $-r = -1158.427$ | |
| 1.8945484 | 1.8945484 | .1054516 | .1054516 | | | $r_1 = 4933.676$ | |
| 3.0638686 | 3.6931706 | 3.4341845 | 3.5337581 | | | $r_2 = 2717.593$ | |
| 585 | 639 | 696 | 466 | | | $r_3 = 3417.89$ | |
| 1158.427 | 4933.676 | 2717.593 | 3417.89 | | | $4R = 9910.732$ | |
| 101 | 67 | 149 | 115 | | | | |

ANSWERS.

| | | | |
|-------------------------|----------------------|----------------|----------------|
| $R=2477.683$ | $C=57^{\circ}2'28''$ | $r=1158.427$ | $r_2=2717.593$ |
| $B=46^{\circ}44'10.4''$ | $c=4157.855$ | $r_1=4933.676$ | $r_3=3417.89$ |

EXAMPLES VIII.

| DATA. | | ANSWERS. | | |
|-------|--|--|---|---------------------|
| 1. | $a=266.0007$ $b=138.7214$ $A=62^{\circ}27'27.4''$ | $R=150$ $B=27^{\circ}32'32.6''$ $C=90^{\circ}$ $c=300$ | $r=52.36105$ $r_1=213.63965$ $r_2=86.36035$ $r_3=352.36105$ | $\Delta_1=82.41167$ |
| 2. | $a=2620.87$ $b=1513.16$ $A=120^{\circ}$ | $R=1513.16$ $B=30^{\circ}$ $C=30^{\circ}$ $c=1513.16$ | $r=351.13$ $r_1=4890.611$ $r_2=756.5801$ $r_3=756.5801$ | $\Delta_1=1107.84$ |
| 3. | $a=82893.16$ $b=29307.18$ $A=70^{\circ}31'43.55''$ | $R=43960.735$ $B=19^{\circ}28'16.45''$ $C=90^{\circ}$ $c=87921.47$ | $r=12139.435$ $r_1=70753.725$ $r_2=17167.745$ $r_3=100060.905$ | $\Delta_1=29414.78$ |
| 4. | $a=2731.843$ $b=1600.277$ $A=72^{\circ}58'7.5''$ | $R=14285.715$ $B=34^{\circ}3'45''$ $C=72^{\circ}58'7.5''$ $c=2731.843$ | $r=591.7336$ $r_1=2612.039$ $r_2=1081.942$ $r_3=2612.039$ | $\Delta_1=591.7373$ |
| 5. | $a=27442.1$ $b=11366.89$ $A=67^{\circ}30'$ | $R=14851.56$ $B=22^{\circ}30'$ $C=90^{\circ}$ $c=29703.12$ | $r=4552.935$ $r_1=22889.165$ $r_2=6813.955$ $r_3=34256.055$ | $\Delta_1=11629.12$ |
| 6. | $a=968.2454$ $b=291.9436$ $A=75^{\circ}31'20.68''$ | $R=500$ $B=16^{\circ}58'27.68''$ $C=87^{\circ}30'11.64''$ $c=999.05114$ | $r=125.0001$ $r_1=874.9987$ $r_2=168.564$ $r_3=1081.4372$ | $\Delta_1=353.5534$ |
| 7. | $a=24150.16$ $b=22202.21$ $A=57^{\circ}41'57.5''$ | $R=14285.72$ $B=50^{\circ}59'37.48''$ $C=71^{\circ}18'25.01''$ $c=27064.26$ | $r=6918.089$ $r_1=20222.015$ $r_2=17506.506$ $r_3=26332.425$ | $\Delta_1=2534.167$ |
| 8. | $a=8571.427$ $b=7142.856$ $A=73^{\circ}44'23.3''$ | $R=1464.286$ $B=53^{\circ}7'18.35''$ $C=53^{\circ}7'48.35''$ $c=7142.856$ | $r=2412.857$ $r_1=8571.427$ $r_2=5714.284$ $r_3=5714.284$ | $\Delta_1=892.8586$ |
| 9. | $a=673.1282$ $b=336.5642$ $A=92^{\circ}27'53.4''$ | $R=336.8757$ $B=29^{\circ}53'36.39''$ $C=57^{\circ}38'30.21''$ $c=569.13$ | $r=121.3962$ $r_1=824.1232$ $r_2=211.003$ $r_3=433.7731$ | |
| 10. | $a=47124.88$ $b=35623.91$ $A=65^{\circ}37'48.6''$ | $R=25867.18$ $B=43^{\circ}31'7.76''$ $C=70^{\circ}51'3.64''$ $c=48871.84$ | $r=12048.9$ $r_1=42436.38$ $r_2=26268.18$ $r_3=46813.06$ | |
| 11. | $a=9401.45$ $b=7360.12$ $A=94^{\circ}20'27.9''$ | $R=4714.249$ $B=51^{\circ}19'4.12''$ $C=34^{\circ}20'27.98''$ $c=5318.792$ | $r=1767.845$ $r_1=11910.022$ $r_2=5303.53$ $r_3=3411.291$ | $\Delta_1=2357.121$ |
| 12. | $a=7791.368$ $b=5387.123$ $A=69^{\circ}31'48.12''$ | $R=4158.254$ $B=40^{\circ}12'23.22''$ $C=70^{\circ}15'48.66''$ $c=7827.959$ | $r=1882.335$ $r_1=7290.425$ $r_2=3854.223$ $r_3=7370.705$ | |

EXAMPLES VIII.—Continued.

| DATA. | | ANSWERS. | |
|-------|--|---|--|
| 13. | $a=45217.25$ $b=22608.62$ $A=45^{\circ}40'24.6''$ | $R=31604.12$ $B=19^{\circ}40'52.88''$ $C=114^{\circ}38'42.52''$ $c=57450.47$ | $r=8087.68$ $r_1=25676.23$ $r_2=11174.34$ $r_3=97653.6$ |
| 14. | $a=967.8191$ $b=559.4229$ $A=75^{\circ}25'30''$ | $R=500$ $B=34^{\circ}0'58.6''$ $C=70^{\circ}33'31.4''$ $c=942.983$ | $r=206.6802$ $r_1=955.0335$ $r_2=377.801$ $r_3=873.846$ |
| 15. | $a=8045.638$ $b=4257.129$ $A=82^{\circ}37'42.62''$ | $R=4056.343$ $B=31^{\circ}39'5.01''$ $C=65^{\circ}43'12.37''$ $c=7395.099$ | $r=1585.028$ $r_1=8656.84$ $r_2=2791.74$ $r_3=6361.823$ |
| 16. | $a=2428.571$ $b=2142.857$ $A=58^{\circ}59'50.2''$ | $R=1416.667$ $B=50^{\circ}42'25.9''$ $C=70^{\circ}17'43.9''$ $c=2667.425$ | $r=670.2642$ $r_1=2097.227$ $r_2=1691.417$ $r_3=2548.286$ |
| 17. | $a=576.1367$ $b=501.7848$ $A=83^{\circ}54'20.57''$ | $R=289.7056$ $B=60^{\circ}0'0''$ $C=36^{\circ}5'39.43''$ $c=341.3403$ | $r=120$ $r_1=637.898$ $r_2=409.7056$ $r_3=231.2187$ |
| 18. | $a=241.368$ $b=120.696$ $A=90^{\circ}1'1.35''$ | $R=120.684$ $B=30^{\circ}0'11.88''$ $C=59^{\circ}58'46.77''$ $c=208.988$ | $r=44.17114$ $r_1=285.6109$ $r_2=76.51524$ $r_3=164.781$ |
| 19. | $a=4562.141$ $b=2708.213$ $A=70^{\circ}52'57.7''$ | $K^2 - 2Kr = r^2, q.p.$ $\Delta_1 = r$ $R=2414.214$ $B=34^{\circ}7'2.3''$ $C=75^{\circ}0'0''$ $c=4663.904$ | $r=1000$ $r_1=4247.10$ $r_2=1831.016$ $r_3=4578.739$ |
| 20. | $a=2157.87$ $b=2132.19$ $A=64^{\circ}46'37.2''$ | $R=11.92647$ $B=63^{\circ}21'58.4''$ $C=51^{\circ}51'24.4''$ $c=1875.96$ | $r=586.8513$ $r_1=1955.67$ $r_2=1902.85$ $r_3=1498.918$ |
| 21. | $a=4891.32$ $b=4377.24$ $A=63^{\circ}56'43.96''$ | $R=2722.309$ $B=53^{\circ}30'34.76''$ $C=62^{\circ}32'41.28''$ $c=4831.4$ | $r=1347.455$ $r_1=4400.659$ $r_2=3554.227$ $r_3=4281.804$ |
| 22. | $a=10978.38$ $b=9662.8$ $A=68^{\circ}3'31.5''$ | $R=5917.834$ $B=54^{\circ}43'38.54''$ $C=57^{\circ}12'49.96''$ $c=9950.22$ | $r=2915.29$ $r_1=10328.493$ $r_2=7916.244$ $r_3=8341.887$ |
| 23. | $a=719.29$ $b=698.76$ $A=57^{\circ}41'11.7''$ | $R=425.5465$ $B=55^{\circ}11'11.44''$ $C=67^{\circ}7'36.86''$ $c=784.17$ | $r=210.2832$ $r_1=606.4242$ $r_2=575.4812$ $r_3=730.5638$ |
| 24. | $a=8625.87$ $b=7818.03$ $A=67^{\circ}37'52.48''$ | $R=4663.873$ $B=56^{\circ}56'42.14''$ $C=55^{\circ}25'25.38''$ $c=7680.2$ | $r=2301.68$ $r_1=8079.603$ $r_2=6541.669$ $r_3=6335.902$ |

 $\Delta_2=234.7964$ $\Delta_1=1000.0007$
 $\Delta_2=603.2828$
 $\Delta_3=1598.853$

EXAMPLES VIII.—Continued.

| DATA. | | ANSWERS. | | |
|-------|--|---|---|--|
| 25. | $a=11878.92$ $b=10630.44$ $A=63^{\circ}27'15.2''$ | $R=6639.403$ $B=53^{\circ}10'59.98''$ $C=63^{\circ}21'44.82''$ $c=11869.4$ | $r=3283.28$ $r_1=10627.62$ $r_2=8604.674$ $r_3=10608.6$ | |
| 26. | $a=8129.16$ $b=9246.77$ $B=63^{\circ}57'31.52''$ | $R=5145.795$ $A=52^{\circ}10'28.76''$ $C=63^{\circ}51'59.72''$ $c=9239.49$ | $r=2535.525$ $r_1=6515.731$ $r_2=8308.923$ $r_3=8294.053$ | |
| 27. | $a=8704.287$ $b=7895.319$ $A=67^{\circ}34'13.06''$ | $R=4708.338$ $B=56^{\circ}58'32.84''$ $C=55^{\circ}27'14.1''$ $c=7756.236$ | $r=2324.09$ $r_1=8147.836$ $r_2=6608.74$ $r_3=6400.864$ | |
| 28. | $a=2777.331$ $b=2028.932$ $A=81^{\circ}2'39.88''$ | $R=1405.803$ $B=46^{\circ}11'20.42''$ $C=52^{\circ}45'59.7''$ $c=2238.537$ | $r=636.8488$ $r_1=3010.775$ $r_2=1502.031$ $r_3=1747.254$ | |
| 29. | $a=6004.551$ $b=4940.314$ $A=52^{\circ}16'48.72''$ | $R=3795.486$ $B=40^{\circ}36'10.9''$ $C=87^{\circ}7'0.38''$ $c=7581.363$ | $r=1599.1825$ $r_1=4545.994$ $r_2=3426.808$ $r_3=8808.323$ | |
| 30. | $a=72027.91$ $b=63926$ $A=76^{\circ}22'16.08''$ | $R=37057.4$ $B=59^{\circ}36'5.30''$ $C=44^{\circ}1'38.62''$ $c=51599.95$ | $r=17070.53$ $r_1=73721.52$ $r_2=53682.39$ $r_3=37896.22$ | |
| 31. | $a=10301.99$ $b=9205.4$ $A=61^{\circ}16'11.5''$ | $R=5874.135$ $B=51^{\circ}35'13.32''$ $C=67^{\circ}8'35.18''$ $c=10825.782$ | $r=2880.918$ $r_1=8081.956$ $r_2=7329.696$ $r_3=10065.804$ | |
| 32. | $a=72027.91$ $b=58443.05$ $A=86^{\circ}18'43.14''$ | $R=36088.7$ $B=54^{\circ}4'4.86''$ $C=39^{\circ}37'12''$ $c=46027$ | $r=15209.16$ $r_1=82743.85$ $r_2=45031.10$ $r_3=31788.98$ | |
| 33. | $a=12785.2$ $b=11688.61$ $A=69^{\circ}4'52.18''$ | $R=6843.687$ $B=58^{\circ}38'46.20''$ $C=52^{\circ}16'21.62''$ $c=10825.782$ | $r=3348.419$ $r_1=12148.782$ $r_2=9913.952$ $r_3=8660.432$ | |

IX.—Triangles in which two sides and the angle opposite the lesser arc given. Ambiguous Case.

Let the two given lengths be a, b ($a > b$), and B the given angle. First calculate R by the equation $2R = b/\sin B$; then $\sin A$ may be found from $\sin A = a/2R$, but either the acute or the obtuse angle having this sine may be taken, and there are two triangles having the assigned data in all cases except when $A = 90^{\circ}$. Thus there will be two values of A , supplementary to each other. We shall denote the acute angle by A_1 , the obtuse by A_2 . The equation $A + B + C = 180^{\circ}$ supplies two corresponding values of C , which we denote by C_1, C_2 . The work is otherwise exactly the same as in VIII. The equation $b^2 = a^2 + c^2 - 2ac \cos B$ shows us that, if c_1, c_2 be the two values of c , $2a + c_1 = c_2 \cos B$, $c_1 c_2 = a^2 - b^2 \equiv (a - b)(a + b)$, and either of these may be used

to check the work. Since the angle B is common to both triangles, this angle should be used in calculating r, r_1, r_2, r_3 for the two triangles.

EXAMPLE WORKED OUT.

| | | | |
|----------------|----------------------------|------------------------|------------------------------|
| DATA. | $\log b = 4.1673297$ | $\log a = 4.2273377$ | $2760(20.44''$ |
| $a = 16878.65$ | $-\log \sin B = 1.9375306$ | $-\log 2R = 4.2297991$ | $A_1 = 83^\circ 54' 20.44''$ |
| $b = 14700.42$ | $\log 2R = 4.2297991$ | $L \sin A = 9.9975386$ | $C_1 = 36^\circ 5' 39.56''$ |
| $B = 60^\circ$ | 842 | 40 | $A_2 = 96^\circ 5' 39.56''$ |
| | 16974.58 | 135 | $C_2 = 23^\circ 54' 20.44''$ |
| | 149 | 8 | |
| | | 104 | |
| | | 2 | |
| | | 01 | |
| | | 29 | |
| | | 01 | |
| | | 29 | |
| | | 01 | |
| | | 29 | |
| | | 01 | |
| | | 29 | |

10000.005

6878.645

$$\log(a-b) = 1.381038$$

$$\log(a+b) = 4.4993994$$

$$\log(a^2 - b^2) = 7.8375032$$

$$\log r_1 r_3 = 7.8375032$$

$$\log \tan \frac{A}{2} = 1.7614394$$

$$\log \cot \frac{1}{2} B = 2.2385606$$

| | | | | |
|-------------------|----------------------|--------------------|------------|------------------|
| $a = 16878.65$ | $s_1 - a = 3910.89$ | $\log = 3.5922756$ | | |
| $b = 14700.42$ | $s_1 - b = 6089.12$ | $\log = 3.7845545$ | | |
| $c = 10000.01$ | $s_1 - c = 10789.53$ | $\log = 4.0330024$ | | |
| | $s_1 = 20789.54$ | $\log = 4.3178449$ | | |
| $2s_1 = 41579.08$ | | | | |
| $s_1 = 20789.54$ | | | | |
| I. | $\log r$ | $\log r_1$ | $\log r_2$ | $\log r_3$ |
| | 3.7845545 | 4.0330024 | 4.3178449 | 3.5922756 |
| | 1.7614394 | .2385606 | 1.7614394 | .2385606 |
| | 3.5459939 | 4.2715630 | 4.0792843 | 3.8308362 |
| | 871 | 28 | 536 | 24 |
| | 3515.555 | 18088.01 | 12002.85 | 6773.86 |
| | 68 | 2 | 307 | 38 |
| | | | | $r = 3515.555$ |
| | | | | $r_1 = 18688.01$ |
| | | | | $r_2 = 12002.85$ |
| | | | | $r_3 = 6773.86$ |
| | | | | $4R = 33949.165$ |

| | | |
|-------------------|----------------------|--------------------|
| $a = 16878.65$ | $s_2 - a = 2350.21$ | $\log = 3.3711067$ |
| $b = 14700.42$ | $s_2 - b = 4528.44$ | $\log = 3.6559486$ |
| $c = 6878.65$ | $s_2 - c = 12350.21$ | $\log = 4.0916743$ |
| | $s_2 = 19228.86$ | $\log = 4.2839535$ |
| $2s_2 = 38457.72$ | | |
| $s_2 = 19228.86$ | | |

| | | | | | |
|-----|-----------|------------|------------|------------|------------------|
| II. | $\log r$ | $\log r_1$ | $\log r_2$ | $\log r_3$ | |
| | 3.6559486 | 4.0916743 | 4.2839535 | 3.3711067 | $r = 2614.497$ |
| | 1.7614394 | .2385606 | 1.7614394 | .2385606 | $r_1 = 21391.19$ |
| | 3.4173880 | 4.3302349 | 4.0453929 | 3.6096673 | $r_2 = 11101.79$ |
| | 720 | 11 | 621 | 584 | $r_3 = 4070.683$ |
| | 2614.497 | 21391.19 | 11101.79 | 4070.683 | $4R = 33949.166$ |
| | 160 | 38 | 308 | 89 | |

ANSWERS (I).

| | |
|----------------------------|------------------|
| $R = 8487.29$ | $r = 3515.555$ |
| $A = 83^\circ 54' 20.44''$ | $r_1 = 18688.01$ |
| $C = 36^\circ 5' 39.56''$ | $r_2 = 12002.85$ |
| $c = 10000.005$ | $r_3 = 6773.86$ |

ANSWERS (II).

| | |
|----------------------------|------------------|
| $R = 8487.29$ | $r = 2614.497$ |
| $A = 96^\circ 5' 39.56''$ | $r_1 = 21391.19$ |
| $C = 23^\circ 54' 20.44''$ | $r_2 = 11101.79$ |
| $c = 6878.645$ | $r_3 = 4070.683$ |

EXAMPLES IX.

DATA.

ANSWERS (1).

ANSWERS (2).

| | DATA. | ANSWERS (1). | ANSWERS (2). | |
|----|---|--|---|---|
| 1. | $a=3830.683$ $b=3396.549$ $B=62^{\circ} 27' 27.4''$ | $R=1015.342$ $A=90^{\circ}$ $C=27^{\circ} 32' 32.6''$ $c=1771.326$ | $r=668.596$ $r_1=4499.279$ $r_2=2727.953$ $r_3=1102.73$ | Same as (1) |
| 2. | $a=270.5115$ $b=116.5868$ $B=25^{\circ} 31' 43.6''$ | $R=135.2558$ $A=90^{\circ}$ $C=64^{\circ} 28' 16.4''$ $c=244.1011$ | $r=45.0852$ $r_1=315.5967$ $r_2=71.4936$ $r_3=199.0159$ | Same as (1) |
| 3. | $a=3650.401$ $b=2107.56$ $B=30^{\circ}$ | $R=2107.56$ $A=60^{\circ}$ $C=90^{\circ}$ $c=4215.12$ | $r=771.4205$ $r_1=2878.9805$ $r_2=1336.1395$ $r_3=4986.5405$ | $r=489.0611$ $r_1=6811.7405$ $r_2=1053.78$ $r_3=1053.78$ |
| 4. | $a=24150.16$ $b=22202.21$ $B=50^{\circ} 59' 37.5''$ | $R=14285.71$ $A=57^{\circ} 41' 57.5''$ $C=71^{\circ} 18' 25''$ $c=27064.26$ | $r=6918.088$ $r_1=20222.02$ $r_2=17506.51$ $r_3=26332.42$ | $r=1260.025$ $r_1=45099.07$ $r_2=11848.4$ $r_3=1455.472$ |
| 5. | $a=10316.36$ $b=6685.65$ $B=39^{\circ} 53' 33''$ | $R=5212.165$ $A=81^{\circ} 44' 47.09''$ $C=58^{\circ} 21' 39.91''$ $c=8874.97$ | $r=2269.206$ $r_1=11197.104$ $r_2=4695.632$ $r_3=7225.329$ | $r=1920.904$ $r_1=13841.724$ $r_2=4347.183$ $r_3=4580.709$ |
| 6. | $a=10316.36$ $b=6683.635$ $B=39^{\circ} 53' 33''$ | $R=5210.6665$ $A=81^{\circ} 55' 58.3''$ $C=58^{\circ} 14' 31.17''$ $c=8860.933$ | $r=2267.026$ $r_1=11213.669$ $r_2=4692.83$ $r_3=7203.212$ | $r=1923.817$ $r_1=13819.64$ $r_2=4349.864$ $r_3=4597.275$ |
| 7. | $a=346.4537$ $b=301.3984$ $B=60^{\circ} 153.4''$ | $R=173.95725$ $A=81^{\circ} 44' 52.3''$ $C=35^{\circ} 13' 14.3''$ $c=200.6515$ | $r=70.97449$ $r_1=387.0413$ $r_2=245.09745$ $r_3=134.6647$ | $r=55.00991$ $r_1=434.8742$ $r_2=229.1328$ $r_3=86.8318$ |
| 8. | $a=3156.045$ $b=3013.734$ $B=72^{\circ} 13' 57.6''$ | $R=1582.339$ $A=85^{\circ} 46' 13''$ $C=22^{\circ} 01' 1''$ $c=1185.525$ | $r=484.4273$ $r_1=3415.513$ $r_2=2683.390$ $r_3=714.8735$ | $r=322.116$ $r_1=3720.385$ $r_2=521.085$ $r_3=410.002$ |

EXAMPLES IX.—Continued.

| DATA | | ANSWERS (1). | | | ANSWERS (2). | | |
|------|--|--|---|---|--|--|--|
| 9. | $a=7631.229$ $b=4973.381$ $B=39^{\circ}53'46.45''$ | $A=3876.978$ $A=79^{\circ}47'32.86''$ $C=66^{\circ}18'40.69''$ $c=6736.086$ | $r=1704.741$ $r_1=8684.589$ $r_2=3509.805$ $r_3=5618.257$ | $R=3876.978$ $A=100^{\circ}12'27.14''$ $C=39^{\circ}53'46.41''$ $c=4973.38$ | $r=1384.858$ $r_1=10512.927$ $r_2=3189.923$ $r_3=3189.920$ | | |
| 10. | $a=4731.231$ $b=2718.108$ $B=32^{\circ}3'57.6''$ | $R=2560.001$ $A=67^{\circ}31'40.31''$ $C=86^{\circ}24'22.09''$ $c=5948.411$ | $r=1014.619$ $r_1=4177.595$ $r_2=1795.749$ $r_3=5281.324$ | $R=2560.011$ $A=112^{\circ}28'19.69''$ $C=35^{\circ}27'42.71''$ $c=2970.436$ | $r=716.047$ $r_1=7793.115$ $r_2=1497.173$ $r_3=1665.803$ | | |
| 11. | $a=481.2793$ $b=360.8528$ $B=46^{\circ}44'10.4''$ | $R=247.7683$ $A=76^{\circ}13'21.66''$ $C=57^{\circ}27'94''$ $c=415.7854$ | $r=1115.8427$ $r_1=493.3678$ $r_2=271.7591$ $r_3=341.7890$ | $R=247.7683$ $A=103^{\circ}46'38.34''$ $C=29^{\circ}29'11.26''$ $c=243.9119$ | $r=78.7113$ $r_1=692.2594$ $r_2=234.6279$ $r_3=142.8972$ | | |
| 12. | $a=59496.24$ $b=22733.78$ $B=22^{\circ}30'$ | $R=29703.12$ $A=90^{\circ}$ $C=67^{\circ}30'$ $c=54884.2$ | $r=9105.87$ $r_1=68512.11$ $r_2=13627.91$ $r_3=45778.33$ | Same as (1) | Same as (1) | | |
| 13. | $a=964701$ $b=955978$ $B=59^{\circ}59'23.7''$ | $R=551470.5$ $A=61^{\circ}0'18.17''$ $C=59^{\circ}0'18.13''$ $c=915454.9$ | $r=275651.2$ $r_1=843960.8$ $r_2=80953.5$ $r_3=810618.8$ | $R=551470.5$ $A=118^{\circ}59'41.83''$ $C=1^{\circ}0'54.47''$ $c=19539.84$ | $r=8416.88$ $r_1=1645989.8$ $r_2=559719.5$ $r_3=8589.98$ | | |
| 14. | $a=337.573$ $b=294.0684$ $B=60^{\circ}$ | $R=169.74584$ $A=83^{\circ}54'20.44''$ $C=36^{\circ}5'39.56''$ $c=200$ | $r=70.31104$ $r_1=373.76052$ $r_2=240.05694$ $r_3=135.47704$ | $R=169.74584$ $A=96^{\circ}5'39.56''$ $C=23^{\circ}54'20.44''$ $c=137.57292$ | $r=52.28994$ $r_1=427.82382$ $r_2=222.03568$ $r_3=81.41366$ | | |
| 15. | $a=6142.887$ $b=6127.56$ $B=59^{\circ}45'12''$ | $R=3546.597$ $A=60^{\circ}$ $C=60^{\circ}14'48''$ $c=6158.099$ | $r=1773.267$ $r_1=5319.862$ $r_2=5293.450$ $r_3=5346.341$ | $R=3546.597$ $A=120^{\circ}$ $C=0^{\circ}14'48''$ $c=30.539$ | $r=13.17464$ $r_1=10652.965$ $r_2=3533.367$ $r_3=13.23971$ | | |
| 16. | $a=775.2704$ $b=674.2277$ $B=60^{\circ}$ | $R=389.2655$ $A=84^{\circ}45'15''$ $C=35^{\circ}14'45''$ $c=449.2793$ | $r=158.8643$ $r_1=866.2148$ $r_2=548.13$ $r_3=301.5821$ | $R=389.2655$ $A=95^{\circ}14'45''$ $C=24^{\circ}45'15''$ $c=325.9911$ | $r=123.2741$ $r_1=972.2954$ $r_2=512.5396$ $r_3=194.8110$ | | |

EXAMPLES IX.—Continued.

| DATA. | ANSWERS (1). | | | ANSWERS (2). | | |
|---|--|--|---|--|-----|-----|
| | λ | r | c | λ | r | c |
| 17. $a=701.37$ $b=666.3$ $B=54^{\circ} 9.25''$ | $\lambda=411.7826$ $A=58^{\circ} 23' 20.5''$ $C=67^{\circ} 36' 30.25''$ $c=761.47$ | $r=202.9399$ $r_1=594.8341$ $r_2=542.4556$ $r_3=712.7807$ | $\lambda=411.7826$ $A=121^{\circ} 36' 39.5''$ $C=43^{\circ} 11.25''$ $c=629.89$ | $r=24.9832$ $r_1=1280.219$ $r_2=364.4987$ $r_3=27.3955$ | | |
| 18. $a=393.75$ $b=391.54$ $B=64^{\circ} 46' 15.36''$ | $\lambda=216.4135$ $A=65^{\circ} 27' 58.58''$ $C=49^{\circ} 45' 46.06''$ $c=330.41$ | $r=1205.4844$ $r_1=358.589$ $r_2=353.8239$ $r_3=238.7252$ | $\lambda=216.4135$ $A=114^{\circ} 32' 1.42''$ $C=61^{\circ} 43.22''$ $c=325.84$ | $r=2.36661$ $r_1=614.9157$ $r_2=250.7062$ $r_3=2.39848$ | | |
| 19. $a=5265.68$ $b=4822.3$ $B=65^{\circ} 28' 2.86''$ | $\lambda=2650.415$ $A=83^{\circ} 23' 53.3''$ $C=31^{\circ} 8' 3.84''$ $c=2740.78$ | $r=1023.421$ $r_1=5714.817$ $r_2=4123.293$ $r_3=1786.97$ | $\lambda=2650.415$ $A=96^{\circ} 36' 6.7''$ $C=17^{\circ} 55' 50.44''$ $c=1631.947$ | $r=667.0308$ $r_1=577.292$ $r_2=376.902$ $r_3=924.4924$ | | |
| 20. $a=688.9$ $b=627.77$ $B=64^{\circ} 16' 8.94''$ | $\lambda=348.4346$ $A=81^{\circ} 19' 36.02''$ $C=34^{\circ} 24' 15.04''$ $c=393.75$ | $r=142.8642$ $r_1=734.6451$ $r_2=537.1917$ $r_3=264.7658$ | $\lambda=348.4346$ $A=98^{\circ} 40' 23.98''$ $C=17^{\circ} 3' 27.08''$ $c=204.414$ | $r=83.40002$ $r_1=885.3568$ $r_2=477.7268$ $r_3=114.0542$ | | |
| 21. $a=313.885$ $b=165.205$ $B=35^{\circ} 44' 5.06''$ | $\lambda=190.2383$ $A=55^{\circ} 35' 10.03''$ $C=98^{\circ} 40' 44.32''$ $c=376.12$ | $r=59.94535$ $r_1=225.3852$ $r_2=97.67835$ $r_3=497.8303$ | $\lambda=190.2383$ $A=124^{\circ} 24' 49.38''$ $C=29^{\circ} 51' 5.56''$ $c=189.384$ | $r=38.61218$ $r_1=34.1209$ $r_2=76.35015$ $r_3=89.09454$ | | |
| 22. $a=10978.38$ $b=9662.8$ $B=54^{\circ} 43' 38.54''$ | $\lambda=5917.834$ $A=68^{\circ} 3' 31.5''$ $C=57^{\circ} 12' 49.96''$ $c=9950.22$ | $r=2915.29$ $r_1=10328.493$ $r_2=7916.244$ $r_3=8341.887$ | $\lambda=5917.834$ $A=111^{\circ} 56' 28.5''$ $C=13^{\circ} 19' 52.96''$ $c=2729.1$ | $r=1046.656$ $r_1=17304.79$ $r_2=6047.607$ $r_3=1365.556$ | | |
| 23. $a=1908.237$ $b=633.911$ $B=39^{\circ} 20' 21.35''$ | $\lambda=500$ $A=86^{\circ} 35' 50.4''$ $C=54^{\circ} 3' 48.25''$ $c=809.0609$ | $r=209.8239$ $r_1=1150.473$ $r_2=436.4476$ $r_3=622.9357$ | $\lambda=500$ $A=93^{\circ} 24' 9.6''$ $C=47^{\circ} 15' 29.05''$ $c=734.4185$ | $r=106.375$ $r_1=1255.73$ $r_2=422.9687$ $r_3=517.6794$ | | |
| 24. $a=4457.15$ $b=4164.894$ $B=54^{\circ} 18.24''$ | $\lambda=2573.337$ $A=60^{\circ}$ $C=65^{\circ} 38' 41.76''$ $c=4700.928$ | $r=1272.674$ $r_1=3846.01$ $r_2=3395.787$ $r_3=4324.224$ | $\lambda=2573.337$ $A=120^{\circ}$ $C=5^{\circ} 58' 41.76''$ $c=536.0318$ | $r=211.116$ $r_1=7931.125$ $r_2=2334.23$ $r_3=239.1052$ | | |

X.—Tetrahedra, the lengths of the edges being given.

In a tetrahedron $OABC$ let the lengths of OA , OB , OC be denoted by a , b , c ; the lengths of the respectively opposite edges BC , CA , AB by x , y , z ; the dihedral angles opposite to a , b , c , x , y , z respectively by A , B , C , X , Y , Z . Also, let any angle of a triangular face which is opposite to OA or BC be denoted by α , one opposite to OB or CA by β , one opposite to OC or AB by γ ; all the angles at O having the suffix 1, those at A the suffix 2, those at B the suffix 3, and those at C the suffix 4. Then, in each of the four triangular faces we have the lengths of the three sides given, and may calculate the angles as in VI. In the face ABC opposite O the lengths are x , y , z , the angles α_2 , β_2 , γ_1 ; in the face opposite A the lengths are x , b , c , the angles α_1 , β_3 , γ_3 ; in the face opposite B the lengths are a , y , c , the angles α_4 , β_1 , γ_2 ; lastly, in the face opposite C the lengths are a , b , z , the angles α_3 , β_3 , γ_1 . The sums of the lengths are denoted by $2s_1$, $2s_2$, $2s_3$, $2s_4$. The sum of the plane angles at O is similarly $2\sigma_1$, and the sums of the angles at A , B , C are $2\sigma_2$, $2\sigma_3$, $2\sigma_4$. The volume V may be found from each of the equations

$$\begin{aligned} 3V &= abc\sqrt{\sin\sigma_1\sin(\sigma_1-\alpha_1)}\sqrt{\sin(\sigma_1-\beta_1)}\sqrt{\sin(\sigma_1-\gamma_1)} \\ &= ayz\sqrt{\sin\sigma_2\sin(\sigma_2-\alpha_2)}\sqrt{\sin(\sigma_2-\beta_2)}\sqrt{\sin(\sigma_2-\gamma_2)} \\ &= abz\sqrt{\sin\sigma_3\sin(\sigma_3-\alpha_3)}\sqrt{\sin(\sigma_3-\beta_3)}\sqrt{\sin(\sigma_3-\gamma_3)} \\ &= ayx\sqrt{\sin\sigma_4\sin(\sigma_4-\alpha_4)}\sqrt{\sin(\sigma_4-\beta_4)}\sqrt{\sin(\sigma_4-\gamma_4)}. \end{aligned}$$

The most convenient manner of arranging the work will be obvious on inspecting the one calculated. The dihedral angles at O are X , Y , Z , and these are to be determined by the equations

$$\begin{aligned} \tan \frac{1}{2}X \sin(\sigma_1 - \alpha_1) &= \tan \frac{1}{2}Y \sin(\sigma_1 - \beta_1) = \tan \frac{1}{2}Z \sin(\sigma_1 - \gamma_1) \\ &= \sqrt{\frac{\sin(\sigma_1 - \alpha_1) \sin(\sigma_1 - \beta_1) \sin(\sigma_1 - \gamma_1)}{\sin\sigma_1}}, \end{aligned}$$

no fresh logarithms needing to be taken out. This part of the work is almost exactly the same as that of calculating the tangents of the half angles of a triangle when the sides are given. The dihedral angles at A are X , B , C ; at B are A , Y , C ; and at C are A , B , Z ; and are to be found in the same way from the angles with suffixes 2, 3, 4 respectively. Thus the volume is calculated from four different expressions, and each dihedral angle from two; and the agreement of the results is sufficient to guarantee the correctness of the calculations. From each tetrahedron we get the six parts of each of four plane triangles, and the six parts of each of four spherical triangles; and these may be used to supply any deficiency in any of the preceding sets of examples; or for practice in solving spherical triangles. To make these plane and spherical triangles complete, I have given always the values of all the plane angles of each face of every tetrahedron, or rather the halves of these angles. The order of magnitude of $A+X$, $B+Y$, $C+Z$ being always the same as that of $a+x$, $b+y$, $c+z$; that of $A-X$, $B-Y$, $C-Z$, the same as that of $a-x$, $b-y$, $c-z$; that of $A-X$, $Y-B$, $Z-C$, the same as that of $a-x$, $y-b$, $z-c$, etc.; the results should be tested accordingly. When

two terms of one system of lengths are equal, the two corresponding terms of the corresponding system of angles will be also equal. In many cases, the data will be found to satisfy one, or two, of such equations, and it will be seen that the calculated angles satisfy the corresponding equations, *quam prox.* Many other properties of tetrahedra are illustrated, attention being directed to such cases by a very brief entry in column of remarks. The tetrahedron in which a, b, c are the lengths of three conterminous edges, and x, y, z the lengths of the edges respectively opposite

to a, b, c , is here denoted by $\left\{ \begin{array}{l} a \ x \\ b \ y \\ c \ z \end{array} \right\}$. The same tetrahedron will thus be also

denoted by $\left\{ \begin{array}{l} a \ x \\ y \ b \\ z \ c \end{array} \right\}$, $\left\{ \begin{array}{l} x \ a \\ b \ y \\ z \ c \end{array} \right\}$, or $\left\{ \begin{array}{l} x \ a \\ y \ b \\ c \ z \end{array} \right\}$. When six lengths are given, there will

be thirty different tetrahedra having these for edges, but some of these may be impossible, even when any set of three of the lengths are sides of a real triangle. In dealing with such a system of tetrahedra, it seems most convenient to arrange them in five groups of six, the longest edge having the same edge opposite to it in all the tetrahedra of any one group.

The sums of the dihedral angles at O, A, B, C respectively are denoted by $2S_1, 2S_2, 2S_3, 2S_4$; and the order of magnitude of S_1, S_2, S_3, S_4 corresponds to that of s_1, s_2, s_3, s_4 . When $a - x = b - y = c - z$, $s_2 = s_3 = s_4$, $S_2 = S_3 = S_4$; and, if $\sigma_2 > \sigma_3 > \sigma_4$, it will be observed that

$$\sin \sigma_2 \sin (\sigma_2 - \alpha_2) \sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2) < \sin \sigma_1 \sin (\sigma_1 - \alpha_1) \sin (\sigma_1 - \beta_1) \sin (\sigma_1 - \gamma_1) \\ < \sin \sigma_4 \sin (\sigma_4 - \alpha_4) \sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4);$$

also

$$\sin (\sigma_2 - \alpha_2) \sin (\sigma_2 - \beta_2) \sin (\sigma_2 - \gamma_2) / \sin \sigma_2 < \sin (\sigma_3 - \alpha_3) \sin (\sigma_3 - \beta_3) \sin (\sigma_3 - \gamma_3) / \sin \sigma_3 \\ < \sin (\sigma_4 - \alpha_4) \sin (\sigma_4 - \beta_4) \sin (\sigma_4 - \gamma_4) / \sin \sigma_4.$$

Many cases of maximum volume under certain conditions will be found.

From the equations of which $\frac{\partial V}{\partial a} = \frac{1}{8} a x \cot A$ is the type, we see that when a, x vary subject to the condition that $a + x$ is constant, the volume (V) will be a maximum when $A = X$; when subject to the condition that $a - x$ is constant, V will be a maximum when $A + X = 180^\circ$; and similarly with the sum or difference of any pair of opposite edges. When $a, x, b + y, c + z$ are given, V will be a maximum when $B = Y, C = Z$, which involves $b = y, c = z$. When $a, x, b + y, c - z$ are given, V will be a maximum when $B = Y, C + Z = 180^\circ$. As the lengths of the edges are varied so as to approach this maximum value, it appears that the variations of b and y do not alter $C + Z$; nor those of c and z affect $B - Y$. When $a, x, b - y, c - z$ are given, it should seem that V will be a maximum when $B + Y = C + Z = 180^\circ$, but this maximum will be when b, y, c, z are infinite, and consequently V .

In any tetrahedron in which $\alpha_1 + \beta_1 + \gamma_1 = \alpha_2 + \beta_2 + \gamma_2 = \pi$, it will be found that

$$\gamma_1 + \alpha_1 - \beta_1 = \gamma_4 + \alpha_4 - \beta_4; \quad \alpha_1 + \beta_1 - \gamma_1 = \alpha_4 + \beta_4 - \gamma_4; \\ \gamma_2 + \alpha_2 - \beta_2 = \gamma_3 + \alpha_3 - \beta_3; \quad \alpha_2 + \beta_2 - \gamma_2 = \alpha_3 + \beta_3 - \gamma_3;$$

$$\begin{cases} a=3.5683, & x=4.9792 \\ b=3.6767, & y=5.0876 \\ c=4.2235, & z=5.6344 \end{cases}$$

Calculation of the Angles of the Faces.

| | | | | | | | | | |
|--------------|----------------|--------------------------|-------------------------|----------------|-------------------------|-----------------|------------------------|----------------|-------------------------------------|
| $x=4.9792$ | $s_1-x=2.8714$ | $\log(s_1-x)=.4580937$ | 10.1750904 | $57960(18.81)$ | 10.1750904 | $90360(29.95)$ | 10.1750904 | $30540(11.21)$ | $\frac{1}{2}a=27^{\circ}31'41.18''$ |
| $y=5.0876$ | $s_1-y=2.7630$ | $\log(s_1-y)=.4413869$ | $.4580937$ | 27140 | $.4413869$ | 30020 | $.3456090$ | 3300 | $\frac{1}{2}b=28^{\circ}26'30.04''$ |
| $z=5.6344$ | $s_1-z=2.2162$ | $\log(s_1-z)=.3456090$ | 2484 | 18 | 9.7337095 | 152 | 9.8291814 | 31 | $\frac{1}{2}c=34^{\circ}1'48.78''$ |
| 15.7012 | $s_1=7.8506$ | $-\log s_1=-.8919028$ | $27^{\circ}32' - 70933$ | 8601 | $25^{\circ}27' - 3017$ | 1506 | $34^{\circ}2' - 5353$ | 2724 | $90^{\circ} 0' 0''$ |
| $s_1=7.8506$ | | $2) .3501808$ | 3082 | 966 | 3017 | 1506 | 2724 | 509 | |
| $x=4.9792$ | $s_2-x=1.4605$ | $\log(s_2-x)=.1645016$ | 10.0713130 | $4440(1.72)$ | 10.0713130 | $1422(4.36)$ | 10.0713130 | $17760(5.83)$ | $\frac{1}{2}a=38^{\circ}53'58.26''$ |
| $b=3.6767$ | $s_2-b=2.7630$ | $\log(s_2-b)=.4413869$ | $.1645016$ | 1855 | $.4413869$ | 22 | $.3456090$ | 2525 | $\frac{1}{2}b=23^{\circ}55.93''$ |
| $c=4.2235$ | $s_2-c=2.2162$ | $\log(s_2-c)=.3456090$ | 9.9068114 | 45 | 9.6299321 | | 9.7257040 | 87 | $\frac{1}{2}c=28^{\circ}0'5.81''$ |
| 12.8794 | $s_2=6.4397$ | $-\log s_2=-.8688656$ | $35^{\circ}54' - 2585$ | 88 | $25^{\circ}6' - 3500$ | 558 | $28^{\circ}0' + 6744$ | | $90^{\circ} 0' 0''$ |
| $s_2=6.4397$ | | $2) .1426259$ | 2585 | 74 | 3500 | 237 | 3047 | 296 | |
| $a=3.5683$ | $s_3-a=2.8714$ | $\log(s_3-a)=.4580937$ | 10.0629230 | $74160(20.34)$ | 10.0629230 | $57780(22.59)$ | 10.0629230 | $52410(17.02)$ | $\frac{1}{2}a=21^{\circ}55'39.64''$ |
| $y=5.0876$ | $s_3-y=1.3521$ | $\log(s_3-y)=.1310088$ | 4580937 | 1220 | $.1310088$ | 6620 | $.3456090$ | 2163 | $\frac{1}{2}b=40^{\circ}31'37.40''$ |
| $c=4.2235$ | $s_3-c=2.2162$ | $\log(s_3-c)=.3456090$ | 9.6048293 | 126 | 9.9319142 | 1504 | 9.7173140 | 63 | $\frac{1}{2}c=27^{\circ}32'42.96''$ |
| 12.8794 | $s_3=6.4397$ | $-\log s_3=-.8688656$ | $21^{\circ}56' - 3647$ | 9329 | $40^{\circ}32' - 20105$ | 225 | $27^{\circ}33' - 3081$ | | $50^{\circ} 0' 0''$ |
| $s_3=6.4397$ | | $.1258159$ | 3647 | 1236 | 2558 | 963 | 3081 | 874 | |
| $a=3.5683$ | $s_4-a=2.8714$ | $\log(s_4-a)=.4580937$ | 9.9982834 | $33560(8.17)$ | 9.9982834 | $106200(26.82)$ | 9.9982834 | $48120(18.62)$ | $\frac{1}{2}a=19^{\circ}7'51.82''$ |
| $b=3.6767$ | $s_4-b=2.7630$ | $\log(s_4-b)=.4413869$ | $.4580937$ | 704 | $.4413869$ | 2702 | 1.9059577 | 22280 | $\frac{1}{2}b=19^{\circ}49'26.81''$ |
| $c=5.6344$ | $s_4-c=2.8053$ | $\log(s_4-c)=.1.9059577$ | 9.5401867 | | 9.5569025 | 366 | 10.0923237 | 1608 | $\frac{1}{2}c=31^{\circ}2'41.37''$ |
| 12.8794 | $s_4=6.4397$ | $-\log s_4=-.8688656$ | $19^{\circ}8' - 4082$ | 2453 | $19^{\circ}49' + 7255$ | 99 | $51^{\circ}31' + 4059$ | 58 | $90^{\circ} 0' 0''$ |
| $s_4=6.4397$ | | 1.9965667 | 4082 | 556 | 3959 | 1770 | 2584 | 802 | |

Calculation of the Volume.

| | | | |
|--|--|--|--|
| $\alpha_1 = 77^\circ 47' 56.52''$ $\beta_1 = 8^\circ 3' 14.80''$ $\gamma_1 = 102^\circ 5' 22.74''$ | $\alpha_2 = 55^\circ 3' 22.36''$ $\beta_2 = 39^\circ 38' 53.62''$ $\gamma_2 = 55^\circ 5' 25.92''$ | $\alpha_3 = 38^\circ 15' 43.61''$ $\beta_3 = 56^\circ 53' 0.08''$ $\gamma_3 = 56^\circ 0' 11.62''$ | $\alpha_4 = 43^\circ 51' 19.28''$ $\beta_4 = 46^\circ 11' 51.86''$ $\gamma_4 = 68^\circ 3' 37.56''$ |
| $1079 : 12 = 215$ $5 = 89$ $.03 = 5$ | $341 : 6 = 56 + \frac{1}{6}$ $305 : 7 = 43 + \frac{1}{7}$ -17 $+8$ | $325 : 5 \frac{1}{2} = 117$ $138 : 4 = 34$ $9 : 2 = 4$ $2 : 3 = 0$ | $244 : 4 \frac{1}{2} = 97$ $4 : 1 = 4$ $1 : 6 = 0$ |
| $260^\circ 56' 34.06''$ $\sigma_1 = 130^\circ 28' 17.03''$ | $149^\circ 47' 41.90''$ $\sigma_2 = 74^\circ 53' 50.95''$ | $151^\circ 8' 55.34''$ $\sigma_3 = 75^\circ 34' 27.67''$ | $158^\circ 6' 48.76''$ $\sigma_4 = 79^\circ 3' 24.35''$ |
| $1082 : 2 = 36$ $.2 = 3$ $.03 = 5$ | $1788 : 54 = 1609$ $3 = 89$ $.3 = 8$ $.03 = 9$ | $20 : 12 = 16$ $7 : 5 = 14$ $.09 = 5$ | $1956 : 30 = 978$ $2 : 4 = 78$ $.09 = 2$ $9 = 9$ |
| $86^\circ - \sigma_1 = 49^\circ 31' 42.97''$ | $3508 : 20 = 1769$ $5 = 292$ $.03 = 1$ $8 = 8$ | $3553 : 15 = 888$ $1 = 59$ $.03 = 3$ | $6504 : 48 = 5203$ $-1.2 = 130$ $-1.01 = -1$ $1 = 1$ |
| 1.6696069 $\log \sigma_1 = .5324614$ $\log \beta_1 = .5654582$ $\log \sigma_2 = .6256725$ $-\log \gamma_1 = .4771213$ $\log V_1 = .9360777$ $V_1 = 8.631332$ | 1.4033770 $\log \sigma_2 = .5524014$ $\log \beta_2 = .7065130$ $\log \sigma_3 = .7508477$ $-\log \gamma_2 = .4771213$ $\log V_2 = .9360778$ $V_2 = 8.631332$ | 1.3697336 $\log \sigma_3 = .6971596$ $\log \beta_3 = .5654582$ $\log \sigma_4 = .7508477$ $-\log \gamma_3 = .4771213$ $\log V_3 = .9360778$ $V_3 = 8.631331$ | 1.3638539 $\log \sigma_4 = .6971596$ $\log \beta_4 = .7065130$ $\log \sigma_5 = .6256725$ $-\log \gamma_4 = .4771213$ $\log V_4 = .9360777$ $V_4 = 8.631332$ |
| 1.6696069 $\log \sigma_1 = .5324614$ $\log \beta_1 = .5654582$ $\log \sigma_2 = .6256725$ $-\log \gamma_1 = .4771213$ $\log V_1 = .9360777$ $V_1 = 8.631332$ | 1.4033770 $\log \sigma_2 = .5524014$ $\log \beta_2 = .7065130$ $\log \sigma_3 = .7508477$ $-\log \gamma_2 = .4771213$ $\log V_2 = .9360778$ $V_2 = 8.631332$ | 1.3697336 $\log \sigma_3 = .6971596$ $\log \beta_3 = .5654582$ $\log \sigma_4 = .7508477$ $-\log \gamma_3 = .4771213$ $\log V_3 = .9360778$ $V_3 = 8.631331$ | 1.3638539 $\log \sigma_4 = .6971596$ $\log \beta_4 = .7065130$ $\log \sigma_5 = .6256725$ $-\log \gamma_4 = .4771213$ $\log V_4 = .9360777$ $V_4 = 8.631332$ |

Calculation of the Dihedral Angles.

| | | | | | | | | | |
|---|---------------|------------|------------|------------|---|------------|------------|------------|------------|
| $-L \sin \sigma_1 = -9.8812306$ | 8 | 19.7883763 | 19.7883763 | 19.7883763 | 5 | 19.4186421 | 19.4186421 | 19.4186421 | 19.4186421 |
| $L \sin(\sigma_1 - \alpha_1) = 9.9004660$ | 2 | 9.9004660 | 9.8805092 | 9.6770079 | 3 | 9.5307318 | 9.5307318 | 9.7612771 | 9.5300101 |
| $L \sin(\sigma_1 - \beta_1) = 9.8805092$ | 2 | | | | 4 | 9.7612771 | | | |
| $L \sin(\sigma_1 - \gamma_1) = 9.6770079$ | 4 | | | | | 9.5300101 | | | |
| | 2) 19.5767525 | | | | | | | | |
| | | 9.8879103 | 9.9078671 | 10.1113684 | 6 | 9.8879103 | 9.8879103 | 9.6573650 | 9.8886320 |
| $-L \sin \sigma_2 = -9.9860869$ | 9 | 19.4136466 | 19.4136466 | 19.4136466 | | 19.3918239 | 19.3918239 | 19.3918239 | 19.3918239 |
| $L \sin(\sigma_2 - \alpha_2) = 9.7825859$ | 6 | 9.7825860 | 9.5037796 | 9.5250146 | 4 | 9.7607634 | 9.7607634 | 9.7344588 | 9.2804556 |
| $L \sin(\sigma_2 - \beta_2) = 9.5037795$ | 9 | | | | 1 | 9.7344588 | | | |
| $L \sin(\sigma_2 - \gamma_2) = 9.5250146$ | 5 | | | | | 9.2804556 | | | |
| | 2) 18.8272932 | | | | | | | | |
| | | 9.6310666 | 9.9078670 | 9.8886320 | 5 | 9.6310665 | 9.6310665 | 9.6573651 | 10.1113683 |
| $L \tan \frac{1}{2} A = 9.6310666$ | 33.240(9.32 | | | | | | | | |
| | 0.32 | 1803 | | | | | | | |
| $23' 9'' +$ | 554 | | | | | | | | |
| $3493 \}$ | | | | | | | | | |
| $L \tan \frac{1}{2} X = 9.8879103$ | 320.40(12.62 | | | | | | | | |
| | 68.30 | | | | | | | | |
| $37^\circ 41' +$ | 8554 | | | | | | | | |
| $2611 \}$ | | | | | | | | | |
| $L \tan \frac{1}{2} B = 9.6573651$ | 900(.27 | | | | | | | | |
| | 229 | | | | | | | | |
| $37^\circ 44' -$ | 36 | | | | | | | | |
| $2611 \}$ | | | | | | | | | |
| $L \tan \frac{1}{2} C = 9.8886320$ | 3960(1.52 | | | | | | | | |
| | 86 | | | | | | | | |
| 43 | | | | | | | | | |
| $L \tan \frac{1}{2} Y = 9.9078671$ | 8760(3.39 | | | | | | | | |
| | 1008 | | | | | | | | |
| $52^\circ 16' +$ | 233 | | | | | | | | |
| $2611 \}$ | | | | | | | | | |
| $L \tan \frac{1}{2} Z = 10.1113684$ | 4200(1.61 | | | | | | | | |
| | 14 | | | | | | | | |
| 1589 | | | | | | | | | |
| 70 | | | | | | | | | |

ANSWERS.

| | | | |
|---|--|--|------------------------------------|
| $\frac{1}{2} A = 23^\circ 9' 9.50''$ | $\frac{1}{2} B = 24^\circ 26' 0.25''$ | $\frac{1}{2} C = 37^\circ 13' 58.48''$ | $\sigma_1 = 130^\circ 28' 17.03''$ |
| $\frac{1}{2} X = 37^\circ 41' 12.63''$ | $\frac{1}{2} Y = 38^\circ 58' 3.38''$ | $\frac{1}{2} Z = 52^\circ 16' 1.61''$ | $\sigma_2 = 74^\circ 53' 50.95''$ |
| $\frac{1}{2}(A + X) = 60^\circ 50' 22.13''$ | $\frac{1}{2}(B + Y) = 63^\circ 24' 3.63''$ | $\frac{1}{2}(Z + C) = 90^\circ 0' 0.09''$ | $\sigma_3 = 75^\circ 34' 27.67''$ |
| $\frac{1}{2}(X - A) = 14^\circ 32' 3.13''$ | $\frac{1}{2}(Y - B) = 14^\circ 32' 3.13''$ | $\frac{1}{2}(Z - C) = 14^\circ 32' 3.13''$ | $\sigma_4 = 79^\circ 3' 24.35''$ |
| $a + x = 8.5475$ | $b + y = 8.7643$ | $z + z = 9.8579$ | $360^\circ 0' 0''$ |
| $x - a = 1.4109$ | $y - b = 1.4109$ | $z - c = 1.4109$ | |

EXAMPLES. X.

| DATA. | $\log a$ | $\log b$ | $\log c$ | $\log d$ | $\log e$ | $\log f$ | $\log g$ | $\log h$ | $\log i$ | $\log j$ | $\log k$ | $\log l$ | $\log m$ | $\log n$ | $\log o$ | $\log p$ | $\log q$ | $\log r$ | $\log s$ | $\log t$ | $\log u$ | $\log v$ | $\log w$ | $\log x$ | $\log y$ | $\log z$ | VOLUME. | REMARKS. | | | |
|--|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|---------|----------|--|----------|--|
| 1. $\begin{cases} a=5.4669, \\ b=3.6185, \\ c=5.6061, \end{cases}$ | 43.3759.08 | 28.16.2.20 | 20.5114.07 | 20.5340.59 | 24.2610.46 | 43.5110.86 | 47.33.7.24 | 30.4933.47 | 27.2934.59 | 47.2029.40 | 28.44716 | 47.3434.13 | 21.3312.17 | 23.9.0.95 | 30.5424.33 | 27.28.3.86 | 47.28.0.18 | | | | | | | | | | | | | | $S_1 = \sigma_1 = 138^\circ 45' 40.44''$ |
| 2. $\begin{cases} a=7.4779, \\ b=8.1235, \\ c=8.4367, \end{cases}$ | 19.1531.04 | 35.5348.79 | 32.4458.22 | 30.4618.83 | 35.2448.19 | 36.1344.26 | 17.1832.99 | 40.3913.97 | 28.2749.45 | 33.4328.56 | 41.2524.17 | 16.3547.81 | 41.5514.18 | 37.0595.0 | 25.3821.76 | 43.958.40 | 28.2834.01 | | | | | | | | | | | | | 25.39085 | |
| 3. $\begin{cases} a=8.4778, \\ b=8.1235, \\ c=9.2436, \end{cases}$ | 18.121.39 | 35.5348.79 | 32.4458.22 | 26.5116.49 | 35.3345.21 | 35.2119.15 | 15.1140.71 | 40.3913.97 | 28.2749.45 | 30.2256.90 | 39.357.60 | 16.3547.81 | 47.27.2.86 | 41.3541.65 | 25.3821.76 | 43.958.40 | 28.2834.01 | | | | | | | | | | | | | 25.18782 | $S_1 = \sigma_1 = 124^\circ 0' 5.55''$ |
| 4. $\begin{cases} a=3.3333, \\ b=3.1111, \\ c=4.668152, \end{cases}$ | 14.1114 | 29.5059.97 | 41.1847.86 | 23.1048.39 | 20.2049.08 | 38.5650.18 | 19.5442.40 | 28.1850.73 | 23.1048.39 | 20.2049.08 | 27.1245.74 | 19.5034.68 | 47.2430.55 | 39.3610.95 | 25.3023.75 | 30.2239.20 | 38.5541.93 | | | | | | | | | | | | | 4.370334 | $S_2 = \sigma_2 = 117^\circ 2' 15.12''$ |
| 5. $\begin{cases} a=5.2669, \\ b=7.8712, \\ c=7.8836, \end{cases}$ | 28.1847.07 | 45.3916.88 | 20.54.4.10 | 20.5132.04 | 24.2740.25 | 43.52.5.60 | 21.36.9.84 | 47.3047.98 | 23.1225.39 | 30.48.5.79 | 47.2421.59 | 21.35.7.92 | 47.3218.12 | 30.53.7.14 | 23.847.73 | 47.2555.78 | 27.2827.88 | | | | | | | | | | | | | 25.49455 | $S_2 = \sigma_2 = 138^\circ 42' 22.97''$ |
| 6. $\begin{cases} a=2.6335, \\ b=2.7093, \\ c=2.7031, \end{cases}$ | 44.3031.86 | 29.37.2.01 | 21.3717.11 | 21.3956.95 | 26.044.50 | 44.3225.39 | 45.5043.40 | 22.2425.69 | 30.9.2.48 | 22.4644.25 | 27.3257.90 | 45.5817.20 | 22.2319.59 | 22.4243.89 | 30.1355.51 | 27.3122.62 | 45.5721.25 | | | | | | | | | | | | | 3.210399 | $S_1 = \sigma_1 = 136^\circ 25' 32.52''$ |
| 7. $\begin{cases} a=2.6155, \\ b=2.6093, \\ c=2.6031, \end{cases}$ | 45.2413.12 | 30.3953.85 | 22.5626.50 | 22.5925.01 | 28.1819.42 | 45.2540.03 | 44.9341.68 | 22.3214.79 | 29.3734.91 | 22.1954.39 | 27.21.8.69 | 44.1118.05 | 22.51.0.31 | 22.1532.49 | 29.4231.29 | 27.1926.23 | 44.1034.47 | | | | | | | | | | | | | 2.958072 | $S_1 = \sigma_1 = 133^\circ 45' 6.48''$ |
| 8. $\begin{cases} a=4.4877, \\ b=5.4877, \\ c=6.6877, \end{cases}$ | 32.3913.33 | 44.5627.89 | 26.1318.39 | 21.424.79 | 30.566.67 | 44.5528.11 | 23.52.5.53 | 37.5342.32 | 24.349.2 | 24.18.5.36 | 37.5341.77 | 23.5259.29 | 45.329.68 | 33.241.31 | 20.5942.19 | 43.430.01 | 29.5454.77 | | | | | | | | | | | | | 15.95682 | $S_2 = \sigma_2 = 127^\circ 53' 39.89''$ |
| 9. $\begin{cases} a=4.6739, \\ b=4.5877, \\ c=4.4435, \end{cases}$ | 47.41.6.93 | 30.4739.37 | 23.1423.60 | 22.4228.08 | 28.2250.84 | 47.3943.28 | 45.5819.34 | 22.3022.25 | 30.143.41 | 21.2928.01 | 20.3544.25 | 44.1514.06 | 22.1832.58 | 20.4925.06 | 29.1037.22 | 25.114.87 | 44.2033.60 | | | | | | | | | | | | | 15.69689 | $S_1 = \sigma_1 = 137^\circ 55' 20.33''$ |
| 10. $\begin{cases} a=8.7737, \\ b=7.8837, \\ c=7.7745, \end{cases}$ | 30.1359.05 | 26.5234.55 | 31.5448.09 | 31.5051.73 | 36.5445.66 | 34.2248.48 | 31.5117.90 | 26.5226.72 | 31.5456.80 | 30.1334.25 | 34.2233.72 | 31.1245.19 | 26.1750.37 | 29.3226.10 | 31.1228.65 | 33.1922.88 | 35.5149.59 | | | | | | | | | | | | | 66.38926 | $a + x = b + y, y = a - c$ |

EXAMPLES. X.—Continued.

| DATA. | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | VOLUME. | REMARKS. |
|---|---|----------|---|
| 11. $\begin{cases} a=5.0837, & x=9.9998 \\ b=5.5789, & y=9.5046 \\ c=0.0741, & z=9.0094 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 13.33528 | $a+x=b+y=z$ |
| 12. $\begin{cases} a=9.9667, & x=9.8554 \\ b=7.5449, & y=4.3336 \\ c=7.4337, & z=7.3224 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 40.50799 | $a-x=b-y=z$ |
| 13. $\begin{cases} a=4.5567, & x=5.5298 \\ b=4.1235, & y=5.9630 \\ c=3.0903, & z=6.14996 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 11.54943 | $a+x=b+y$ $S_1=S_2=136.011.94$ |
| 14. $\begin{cases} a=2.5155, & x=3.8118 \\ b=3.42184, & y=2.5093 \\ c=3.7416, & z=2.5033 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 2.581718 | $S_1=S_2=140.3646$ |
| 15. $\begin{cases} a=7.8530, & x=9.8528 \\ b=6.4705, & y=9.1375 \\ c=5.6899, & z=8.6897 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 52.14756 | $A+x=180$, vol. a max. for variations of a, x , so that $\Delta x - \Delta x=0$ |
| 16. $\begin{cases} a=4.3719, & x=3.2608 \\ b=5.3726, & y=4.2615 \\ c=5.6464, & z=4.5353 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 9.274307 | $C+Z=180$, vol. a max. for variations of c, z , such that $\Delta x - \Delta x=0$. |
| 17. $\begin{cases} a=2.1995, & x=3.4633 \\ b=2.3207, & y=3.5845 \\ c=2.6116, & z=3.8754 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 1.949617 | |
| 18. $\begin{cases} a=4.8613, & x=4.7940 \\ b=4.9233, & y=4.7320 \\ c=5.5121, & z=4.1432 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 12.85487 | $A+x=B+y+z$ |
| 19. $\begin{cases} a=4.3723, & x=3.2612 \\ b=5.3723, & y=4.2612 \\ c=5.6463, & z=4.5352 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 9.275219 | $A-x=B-y+z$ |
| 20. $\begin{cases} a=4.8624, & x=4.7929 \\ b=4.9233, & y=4.7320 \\ c=5.5121, & z=4.1432 \end{cases}$ | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | 12.85487 | (18) differs from this by $\Delta x = \text{corr.}$, $\Delta x = \text{corr.}$, and since $A = \Delta x$, x is a max., and $\Delta x = 0$. |

EXAMPLES, X.—Continued.

| | DATA. | $\log a$ | $\log b$ | $\log c$ | $\log d$ | $\log e$ | $\log f$ | $\log g$ | $\log h$ | $\log i$ | $\log j$ | $\log k$ | $\log l$ | $\log m$ | $\log n$ | $\log o$ | $\log p$ | $\log q$ | $\log r$ | $\log s$ | $\log t$ | $\log u$ | $\log v$ | $\log w$ | $\log x$ | $\log y$ | $\log z$ | VOLUME. | REMARKS. |
|-----|--|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------|---|
| 21. | $\begin{cases} a=2.3309, \\ b=5.2250, \\ c=4.1705 \end{cases}$ | $\log a=0.3671$ | $\log b=0.7212$ | $\log c=0.6722$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 5.53466 | $A + X = B + Y = C + Z$ |
| 22. | $\begin{cases} a=7.1186, \\ b=4.2250, \\ c=4.1705 \end{cases}$ | $\log a=0.8529$ | $\log b=0.6253$ | $\log c=0.6199$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 7.857833 | $A + X = B + Y = C + Z$ |
| 23. | $\begin{cases} a=7.1186, \\ b=4.2250, \\ c=4.1705 \end{cases}$ | $\log a=0.8529$ | $\log b=0.6253$ | $\log c=0.6199$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 7.857836 | $A + X = B + Y = 2R = C + Z.$ ΔY (in 22) very small |
| 24. | $\begin{cases} a=7.1187, \\ b=4.2237, \\ c=4.1697 \end{cases}$ | $\log a=0.8529$ | $\log b=0.6253$ | $\log c=0.6199$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 7.857187 | $A + X = B + Y = C + Z$ |
| 25. | $\begin{cases} a=8.8524, \\ b=7.6523, \\ c=8.6119 \end{cases}$ | $\log a=0.9463$ | $\log b=0.8829$ | $\log c=0.9342$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 69.37919 | $C + Z = 180^\circ$ very nearly |
| 26. | $\begin{cases} a=8.8529, \\ b=7.6523, \\ c=8.6140 \end{cases}$ | $\log a=0.9463$ | $\log b=0.8829$ | $\log c=0.9342$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 69.37917 | $A = X, B = Y$ |
| 27. | $\begin{cases} a=8.8529, \\ b=7.6514, \\ c=8.6119 \end{cases}$ | $\log a=0.9463$ | $\log b=0.8829$ | $\log c=0.9342$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 69.37920 | |
| 28. | $\begin{cases} a=8.8529, \\ b=7.6523, \\ c=8.6119 \end{cases}$ | $\log a=0.9463$ | $\log b=0.8829$ | $\log c=0.9342$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 69.37921 | $A = X, B = Y, C = Z = 180^\circ, Y$ a max. subject to $a + x = 17.7058, Y + y = 15.3046, z - c = 1.6228$ |
| 29. | $\begin{cases} a=7.1182, \\ b=4.2394, \\ c=4.1705 \end{cases}$ | $\log a=0.8529$ | $\log b=0.6253$ | $\log c=0.6199$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 7.858762 | $A + X = B + Y = C + Z$ $\log r = 121.594372$ |
| 30. | $\begin{cases} a=3.5703, \\ b=3.6707, \\ c=4.2233 \end{cases}$ | $\log a=0.5511$ | $\log b=0.5634$ | $\log c=0.6244$ | $\log d=1.2422$ | $\log e=1.2422$ | $\log f=1.2422$ | $\log g=1.2422$ | $\log h=1.2422$ | $\log i=1.2422$ | $\log j=1.2422$ | $\log k=1.2422$ | $\log l=1.2422$ | $\log m=1.2422$ | $\log n=1.2422$ | $\log o=1.2422$ | $\log p=1.2422$ | $\log q=1.2422$ | $\log r=1.2422$ | $\log s=1.2422$ | $\log t=1.2422$ | $\log u=1.2422$ | $\log v=1.2422$ | $\log w=1.2422$ | $\log x=1.2422$ | $\log y=1.2422$ | $\log z=1.2422$ | 8.638628 | $X' = Y' - R = Z - C$ $: C - Y'$ |

EXAMPLES. X.—Continued.

| | DATA. | $\angle a$, $\angle b$, $\angle c$, $\angle d$ | $\angle a_1$, $\angle b_1$, $\angle c_1$, $\angle d_1$ | $\angle a_2$, $\angle b_2$, $\angle c_2$, $\angle d_2$ | $\angle a_3$, $\angle b_3$, $\angle c_3$, $\angle d_3$ | $\angle A$, $\angle B$, $\angle C$ | $\frac{1}{2}A$, $\frac{1}{2}B$, $\frac{1}{2}C$ | VOLUME. | REMARKS. |
|-----|---|--|--|--|--|--|--|----------|---|
| 31. | $\begin{cases} a=7.1182, \\ b=4.1092, \\ c=4.1697, \end{cases}$ | $\begin{cases} 16^\circ 21' 0.83'' \\ 24 \quad 7' 9.93'' \\ 23 \quad 38' 46.75'' \end{cases}$ | $\begin{cases} 12^\circ 40' 35.66'' \\ 17 \quad 26' 29.93'' \\ 17 \quad 48' 37.61'' \end{cases}$ | $\begin{cases} 48^\circ 54' 43.32'' \\ 39 \quad 23' 18.62'' \\ 37 \quad 36' 10.72'' \end{cases}$ | $\begin{cases} 48^\circ 54' 43.32'' \\ 39 \quad 23' 18.62'' \\ 37 \quad 36' 10.72'' \end{cases}$ | $\begin{cases} 50^\circ 40' 1.17'' \\ 34^\circ 56' 52.18'' \\ 36^\circ 30' 8.93'' \end{cases}$ | $\begin{cases} 22^\circ 20' 16.69'' \\ 38^\circ 3' 56.68'' \\ 36^\circ 30' 8.93'' \end{cases}$ | 7.833902 | $A+X=B+Y=C+Z$ $C=Z$ I^a max. for vari- ations of c , z , such that $\Delta C+\Delta Z=0$ |
| 32. | $\begin{cases} a=7.1182, \\ b=4.1092, \\ c=4.1717, \end{cases}$ | $\begin{cases} 16^\circ 20' 44.95'' \\ 24 \quad 7' 15.22'' \\ 23 \quad 37' 56.69'' \end{cases}$ | $\begin{cases} 12^\circ 40' 43.43'' \\ 17 \quad 26' 24.55'' \\ 17 \quad 49' 16.47'' \end{cases}$ | $\begin{cases} 48^\circ 55' 39.36'' \\ 39 \quad 24' 30.07'' \\ 37 \quad 33' 50.95'' \end{cases}$ | $\begin{cases} 48^\circ 55' 39.36'' \\ 39 \quad 24' 30.07'' \\ 37 \quad 33' 50.95'' \end{cases}$ | $\begin{cases} 50^\circ 40' 11.50'' \\ 34^\circ 55' 26.06'' \\ 36^\circ 31' 48.45'' \end{cases}$ | $\begin{cases} 22^\circ 20' 9.30'' \\ 38^\circ 3' 54.74'' \\ 36^\circ 28' 32.35'' \end{cases}$ | 7.833893 | As compared with (31) |
| 33. | $\begin{cases} a=7.1182, \\ b=4.1092, \\ c=4.1697, \end{cases}$ | $\begin{cases} 16^\circ 20' 55.64'' \\ 24 \quad 7' 5.87'' \\ 23 \quad 38' 42.59'' \end{cases}$ | $\begin{cases} 12^\circ 40' 31.69'' \\ 17 \quad 26' 27.26'' \\ 17 \quad 48' 35.05'' \end{cases}$ | $\begin{cases} 48^\circ 54' 30.15'' \\ 39 \quad 23' 15.81'' \\ 37 \quad 32' 20.86'' \end{cases}$ | $\begin{cases} 48^\circ 54' 30.15'' \\ 39 \quad 23' 15.81'' \\ 37 \quad 32' 20.86'' \end{cases}$ | $\begin{cases} 50^\circ 40' 7.06'' \\ 34^\circ 56' 53.02'' \\ 36^\circ 30' 14.48'' \end{cases}$ | $\begin{cases} 22^\circ 20' 12.45'' \\ 38^\circ 3' 26.57'' \\ 36^\circ 30' 8.81'' \end{cases}$ | 7.833233 | As compared with (31) $X=C=0.02$, $\Delta X=-.002$, $\Delta Y=-.000069$ |
| 34. | $\begin{cases} a=7.1181, \\ b=4.1093, \\ c=4.1698, \end{cases}$ | $\begin{cases} 16^\circ 21' 1.98'' \\ 24 \quad 7' 9.75'' \\ 23 \quad 38' 46.63'' \end{cases}$ | $\begin{cases} 12^\circ 40' 38.53'' \\ 17 \quad 26' 32.99'' \\ 17 \quad 48' 40.67'' \end{cases}$ | $\begin{cases} 48^\circ 54' 40.38'' \\ 39 \quad 23' 12.06'' \\ 37 \quad 32' 17.38'' \end{cases}$ | $\begin{cases} 48^\circ 54' 40.38'' \\ 39 \quad 23' 12.06'' \\ 37 \quad 32' 17.38'' \end{cases}$ | $\begin{cases} 50^\circ 39' 58.69'' \\ 34^\circ 56' 52.29'' \\ 36^\circ 30' 8.71'' \end{cases}$ | $\begin{cases} 22^\circ 20' 18.42'' \\ 38^\circ 3' 24.82'' \\ 36^\circ 30' 8.40'' \end{cases}$ | 7.834277 | As compared with (31) $\Delta X=0.001$, $\Delta Y,$ $\Delta Z=0.001$, $\Delta X,$ $\Delta Y=0.001$, $\Delta Z,$ $\Delta X=0.001$, $\Delta Y,$ $\Delta Z=0.001$, $\Delta X,$ $\Delta Y=0.001$, $\Delta Z,$ $\Delta X=0.001$, $\Delta Y,$ $\Delta Z=0.001$ |
| 35. | $\begin{cases} a=2.3314, \\ b=5.3403, \\ c=5.2798, \end{cases}$ | $\begin{cases} 4^\circ 5' 6.62'' \\ 24 \quad 4' 24.10'' \\ 24 \quad 8' 42.83'' \end{cases}$ | $\begin{cases} 59^\circ 17' 30.80'' \\ 53^\circ 31' 9.61'' \\ 53^\circ 25' 37.71'' \end{cases}$ | $\begin{cases} 12^\circ 20' 7.56'' \\ 15^\circ 13' 46.96'' \\ 23^\circ 46' 36.13'' \end{cases}$ | $\begin{cases} 12^\circ 20' 7.56'' \\ 15^\circ 13' 46.96'' \\ 23^\circ 46' 36.13'' \end{cases}$ | $\begin{cases} 14^\circ 31' 55.25'' \\ 60^\circ 27' 22.45'' \\ 60^\circ 24' 33.25'' \end{cases}$ | $\begin{cases} 64^\circ 0' 6.71'' \\ 18^\circ 4' 39.51'' \\ 18^\circ 7' 28.71'' \end{cases}$ | 4.800386 | An ill-conditioned tetra- hedron. $A+X=B+Y$ $=C+Z=78^\circ 32' 1.98''$ |
| 36. | $\begin{cases} a=3.5679, \\ b=3.8767, \\ c=3.7893, \end{cases}$ | $\begin{cases} 4^\circ 49' 5.28'' \\ 43^\circ 43' 22.38'' \\ 55^\circ 12' 1.78'' \end{cases}$ | $\begin{cases} 28^\circ 56' 15.44'' \\ 22^\circ 29' 1.67'' \\ 24^\circ 22.84'' \end{cases}$ | $\begin{cases} 21^\circ 38' 56.55'' \\ 29^\circ 57' 50.95'' \\ 24^\circ 34' 25.72'' \end{cases}$ | $\begin{cases} 21^\circ 38' 56.55'' \\ 29^\circ 57' 50.95'' \\ 24^\circ 34' 25.72'' \end{cases}$ | $\begin{cases} 26^\circ 9' 52.94'' \\ 28^\circ 0' 27.61'' \\ 30^\circ 24' 22.18'' \end{cases}$ | $\begin{cases} 41^\circ 46' 28.39'' \\ 43^\circ 37' 3.06'' \\ 46^\circ 0' 57.63'' \end{cases}$ | 8.220333 | $X-A=Y-B=Z-C$ |
| 37. | $\begin{cases} a=3.5683, \\ b=3.6767, \\ c=4.2220, \end{cases}$ | $\begin{cases} 38^\circ 54' 33.39'' \\ 40^\circ 35' 15.50'' \\ 51^\circ 1' 33.44'' \end{cases}$ | $\begin{cases} 27^\circ 31' 59.08'' \\ 19^\circ 50' 1.62'' \\ 27^\circ 31' 59.08'' \end{cases}$ | $\begin{cases} 10^\circ 8' 24.94'' \\ 28^\circ 26' 19.20'' \\ 27^\circ 59' 22.78'' \end{cases}$ | $\begin{cases} 10^\circ 8' 24.94'' \\ 28^\circ 26' 19.20'' \\ 27^\circ 59' 22.78'' \end{cases}$ | $\begin{cases} 23^\circ 9' 55.42'' \\ 33^\circ 6' 3.83'' \\ 37^\circ 42' 14.16'' \end{cases}$ | $\begin{cases} 37^\circ 42' 14.16'' \\ 38^\circ 59' 11.04'' \\ 52^\circ 14' 32.82'' \end{cases}$ | 8.631324 | $X-A=Y-B=Z-C$; $C=X$ |
| 38. | $\begin{cases} a=4.9793, \\ b=7.2877, \\ c=7.8345, \end{cases}$ | $\begin{cases} 13^\circ 29' 34.11'' \\ 24^\circ 16' 45.21'' \\ 23^\circ 29' 58.05'' \end{cases}$ | $\begin{cases} 18^\circ 21' 43.65'' \\ 44^\circ 57' 5.85'' \\ 46^\circ 0' 16.09'' \end{cases}$ | $\begin{cases} 21^\circ 35' 56.10'' \\ 33^\circ 58' 7.44'' \\ 42^\circ 32' 18.45'' \end{cases}$ | $\begin{cases} 21^\circ 35' 56.10'' \\ 33^\circ 58' 7.44'' \\ 42^\circ 32' 18.45'' \end{cases}$ | $\begin{cases} 21^\circ 35' 53.86'' \\ 40^\circ 18' 31.71'' \\ 46^\circ 44' 43.42'' \end{cases}$ | $\begin{cases} 18^\circ 20' 8.16'' \\ 37^\circ 8' 5.70'' \\ 39^\circ 23' 51.68'' \end{cases}$ | 15.51394 | $A-X=B-Y$; $B+Y=C+Z$ |
| 39. | $\begin{cases} a=4.9793, \\ b=5.0877, \\ c=5.6345, \end{cases}$ | $\begin{cases} 10^\circ 13' 31.16'' \\ 19^\circ 58' 6.84'' \\ 18^\circ 0' 11.31'' \end{cases}$ | $\begin{cases} 31^\circ 18' 50.85'' \\ 36^\circ 53' 30.14'' \\ 39^\circ 50' 12.48'' \end{cases}$ | $\begin{cases} 35^\circ 0' 18.55'' \\ 33^\circ 6' 15.51'' \\ 39^\circ 32' 49.87'' \end{cases}$ | $\begin{cases} 35^\circ 0' 18.55'' \\ 33^\circ 6' 15.51'' \\ 39^\circ 32' 49.87'' \end{cases}$ | $\begin{cases} 36^\circ 24' 19.54'' \\ 43^\circ 29' 20.72'' \\ 45^\circ 17' 25.22'' \end{cases}$ | $\begin{cases} 32^\circ 8' 55.00'' \\ 34^\circ 13' 56.18'' \\ 39^\circ 23' 51.68'' \end{cases}$ | 8.12844 | $A-X=B-Y$; $B+Y=C+Z$ |
| 40. | $\begin{cases} a=2.4897, \\ b=9.1439, \\ c=9.4175, \end{cases}$ | $\begin{cases} 5^\circ 27' 4.27'' \\ 29^\circ 50' 3.36'' \\ 29^\circ 43' 23.37'' \end{cases}$ | $\begin{cases} 6^\circ 5' 47.64'' \\ 52^\circ 39' 53.54'' \\ 52^\circ 47' 20.72'' \end{cases}$ | $\begin{cases} 7^\circ 36' 43.09'' \\ 46^\circ 20' 13.26'' \\ 46^\circ 39' 53.08'' \end{cases}$ | $\begin{cases} 7^\circ 36' 43.09'' \\ 46^\circ 20' 13.26'' \\ 46^\circ 39' 53.08'' \end{cases}$ | $\begin{cases} 6^\circ 10' 29.56'' \\ 45^\circ 15' 52.66'' \\ 46^\circ 35' 37.45'' \end{cases}$ | $\begin{cases} 7^\circ 36' 55.66'' \\ 45^\circ 39' 26.56'' \\ 42^\circ 48' 41.77'' \end{cases}$ | 5.816192 | $A-X=B-Y$, $B+Y$; $C+Z$. A most ill-con- ditioned tetrahedron. |

EXAMPLES. X.—Continued.

| DATA. | $\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | $\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | $\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | VOLUME. | REMARKS. |
|---|---|---|---|---|--|---|-----------|---|
| 41. $\begin{cases} a=4.3723, \\ b=5.3723, \\ c=5.6335, \end{cases}$ | $\begin{cases} 17\ 11' 3.80'' \\ 24\ 12\ 31.63'' \\ 27\ 3\ 55.49'' \end{cases}$ | $\begin{cases} 21\ 43' 52.58'' \\ 37\ 8\ 43.27'' \\ 40\ 45\ 33.90'' \end{cases}$ | $\begin{cases} 25\ 47\ 21.33'' \\ 32\ 0\ 3.40'' \\ 38\ 36\ 7.28'' \end{cases}$ | $\begin{cases} 25\ 3\ 54.47'' \\ 34\ 12\ 48.02'' \\ 30\ 16\ 3.96'' \end{cases}$ | $\begin{cases} 26\ 37\ 39.64'' \\ 38\ 4\ 24.15'' \\ 47\ 4\ 24.65'' \end{cases}$ | $\begin{cases} 21\ 57\ 34.94'' \\ 33\ 24\ 39.45'' \\ 42\ 24\ 39.95'' \end{cases}$ | 9.274728 | $A - X = B - Y = C - Z$ |
| 42. $\begin{cases} a=3.3003, \\ b=3.4803, \\ c=3.7209, \end{cases}$ | $\begin{cases} 46\ 9\ 5.70'' \\ 49\ 35\ 51.48'' \\ 55\ 54\ 51.48'' \end{cases}$ | $\begin{cases} 28\ 10\ 58.45'' \\ 17\ 33\ 26.70'' \\ 21\ 35\ 15.76'' \end{cases}$ | $\begin{cases} 16\ 31\ 41.82'' \\ 29\ 44\ 20.25'' \\ 22\ 50\ 35.97'' \end{cases}$ | $\begin{cases} 18\ 36\ 52.76'' \\ 21\ 40\ 31.47'' \\ 33\ 4\ 41.30'' \end{cases}$ | $\begin{cases} 19\ 10\ 42.11'' \\ 50\ 44\ 11.68'' \\ 27\ 14\ 44\ 72'' \end{cases}$ | $\begin{cases} 48\ 14\ 22.32'' \\ 50\ 44\ 11.68'' \\ 56\ 18\ 24.93'' \end{cases}$ | 6.474979 | $A - X = B - Y = C - Z$ |
| 43. $\begin{cases} a=4.8801, \\ b=4.9233, \\ c=5.5117, \end{cases}$ | $\begin{cases} 29\ 3\ 19.06'' \\ 26\ 32\ 47.33'' \\ 25\ 0\ 7.41'' \end{cases}$ | $\begin{cases} 32\ 21\ 0.42'' \\ 32\ 46\ 10.77'' \\ 34\ 58\ 43.77'' \end{cases}$ | $\begin{cases} 32\ 13\ 41.82'' \\ 31\ 48\ 44.90'' \\ 34\ 37\ 20.63'' \end{cases}$ | $\begin{cases} 28\ 8\ 28.03'' \\ 28\ 19\ 20.31'' \\ 25\ 50\ 14.68'' \end{cases}$ | $\begin{cases} 35\ 27\ 47.66'' \\ 35\ 49\ 1.77'' \\ 31\ 19\ 23.43'' \end{cases}$ | $\begin{cases} 35\ 14\ 59.77'' \\ 34\ 53\ 45.66'' \\ 31\ 31\ 24.00'' \end{cases}$ | 12.855546 | $A + X = B + Y = C + Z$ |
| 44. $\begin{cases} a=9.41135, \\ b=8.76269, \\ c=8.07525, \end{cases}$ | $\begin{cases} 16\ 32\ 11.88'' \\ 24\ 53\ 46.20'' \\ 19\ 48\ 21.18'' \end{cases}$ | $\begin{cases} 20\ 5\ 15.62'' \\ 32\ 16\ 1.72'' \\ 27\ 5\ 38.78'' \end{cases}$ | $\begin{cases} 37\ 55\ 37.10'' \\ 42\ 6\ 33.62'' \\ 32\ 49\ 1.09'' \end{cases}$ | $\begin{cases} 37\ 13\ 35.02'' \\ 40\ 38\ 47.03'' \\ 27\ 48\ 20.79'' \end{cases}$ | $\begin{cases} 38\ 32\ 5.39'' \\ 45\ 0\ 0.00'' \\ 33\ 8\ 42.47'' \end{cases}$ | $\begin{cases} 22\ 48\ 11.85'' \\ 45\ 0\ 0.00'' \\ 28\ 18\ 0.18'' \end{cases}$ | 38.61827 | $B = 1=90^\circ; I'$ a max. for variations in b, y |
| 45. $\begin{cases} a=6.4321, \\ b=5.5433, \\ c=5.3391, \end{cases}$ | $\begin{cases} 16\ 52\ 49.23'' \\ 19\ 27\ 37.17'' \\ 20\ 24\ 45.17'' \end{cases}$ | $\begin{cases} 22\ 20\ 31.21'' \\ 29\ 9\ 15.60'' \\ 27\ 54\ 28.51'' \end{cases}$ | $\begin{cases} 40\ 25\ 59.23'' \\ 32\ 7\ 8.27'' \\ 34\ 47\ 26.13'' \end{cases}$ | $\begin{cases} 45\ 37\ 53.78'' \\ 38\ 19\ 44.04'' \\ 35\ 32\ 20.52'' \end{cases}$ | $\begin{cases} 44\ 15\ 31.89'' \\ 38\ 45\ 6.93'' \\ 35\ 49\ 36.00'' \end{cases}$ | $\begin{cases} 26\ 53\ 40.11'' \\ 32\ 54\ 5.07'' \\ 35\ 49\ 36.00'' \end{cases}$ | 10 51411 | $C = \angle, A + X = B + Y = C + Z. I'$ a max. for variations of c, z , such that $\angle C + \angle z = 90^\circ$ |
| 46. $\begin{cases} a=7.809322, \\ b=6.09328, \\ c=8.810394, \end{cases}$ | $\begin{cases} 31\ 32\ 54.01'' \\ 29\ 4\ 5.27'' \\ 29\ 23\ 0.72'' \end{cases}$ | $\begin{cases} 32\ 10\ 56.83'' \\ 24\ 7\ 6.85'' \\ 33\ 33\ 26.32'' \end{cases}$ | $\begin{cases} 36\ 29\ 52.44'' \\ 20\ 57\ 2.49'' \\ 37\ 30\ 3.51'' \end{cases}$ | $\begin{cases} 27\ 22\ 28.40'' \\ 20\ 57\ 2.49'' \\ 23\ 26\ 23.53'' \end{cases}$ | $\begin{cases} 39\ 0\ 46.96'' \\ 26\ 33\ 54.19'' \\ 40\ 34\ 30.21'' \end{cases}$ | $\begin{cases} 37\ 52\ 29.94'' \\ 33\ 41\ 24.24'' \\ 34\ 9\ 57.27'' \end{cases}$ | 49.16382 | $\sigma_1 = \sigma_2 = \pi; \text{ whence } \sigma_1 - \beta_1 = \sigma_2 - \beta_1$ $\sigma_1 - \gamma_1 = \sigma_2 - \gamma_1$ $\sigma_2 - \beta_1 = \sigma_3 - \beta_1$ and $\sigma_2 - \gamma_1 = \sigma_4 - \gamma_1$ |
| 47. $\begin{cases} a=7.069273, \\ b=7.375329, \\ c=7.315225, \end{cases}$ | $\begin{cases} 29\ 4\ 40.45'' \\ 26\ 38\ 39.03'' \\ 34\ 10\ 40.52'' \end{cases}$ | $\begin{cases} 28\ 35\ 42.85'' \\ 28\ 45\ 4.73'' \\ 32\ 39\ 12.42'' \end{cases}$ | $\begin{cases} 26\ 58\ 14.75'' \\ 24\ 43\ 23.96'' \\ 30\ 14\ 59.77'' \end{cases}$ | $\begin{cases} 39\ 42\ 8.55'' \\ 30\ 40\ 19.78'' \\ 36\ 40\ 53.19'' \end{cases}$ | $\begin{cases} 33\ 1\ 28.12'' \\ 32\ 59\ 14.62'' \\ 39\ 51\ 15.35'' \end{cases}$ | $\begin{cases} 32\ 45\ 55.20'' \\ 29\ 35\ 54.76'' \\ 42\ 53\ 11.45'' \end{cases}$ | 43.1705 | |
| 48. $\begin{cases} a=3.671925, \\ b=3.440879, \\ c=3.917612, \end{cases}$ | $\begin{cases} 31\ 43\ 2.88'' \\ 35\ 46\ 57.13'' \\ 22\ 30\ 0.00'' \end{cases}$ | $\begin{cases} 30\ 5\ 30.94'' \\ 31\ 30\ 28.70'' \\ 28\ 23\ 40.36'' \end{cases}$ | $\begin{cases} 35\ 59\ 31.30'' \\ 41\ 8\ 29.03'' \\ 32\ 8\ 0.30'' \end{cases}$ | $\begin{cases} 23\ 49\ 22.52'' \\ 26\ 8\ 56.52'' \\ 18\ 45\ 40.06'' \end{cases}$ | $\begin{cases} 36\ 50\ 7.50'' \\ 37\ 45\ 40.55'' \\ 32\ 41\ 16.20'' \end{cases}$ | $\begin{cases} 35\ 15\ 31.78'' \\ 45\ 0\ 0.00'' \\ 24\ 5\ 41.36'' \end{cases}$ | 5 21751 | |
| 49. $\begin{cases} a=3.405534, \\ b=3.299274, \\ c=3.914063, \end{cases}$ | $\begin{cases} 29\ 34\ 6.34'' \\ 35\ 23\ 25.83'' \\ 25\ 2\ 27.83'' \end{cases}$ | $\begin{cases} 28\ 23\ 40.36'' \\ 31\ 30\ 28.69'' \\ 30\ 5\ 50.95'' \end{cases}$ | $\begin{cases} 33\ 27\ 3.48'' \\ 25\ 50\ 38.79'' \\ 34\ 29\ 14.87'' \end{cases}$ | $\begin{cases} 24\ 30\ 43.22'' \\ 26\ 39\ 3.91'' \\ 20\ 39\ 9.91'' \end{cases}$ | $\begin{cases} 34\ 8\ 46.35'' \\ 37\ 45\ 40\ 45\ 0\ 0.00'' \\ 35\ 15\ 51.81'' \end{cases}$ | $\begin{cases} 32\ 41\ 16.20'' \\ 27\ 9\ 26.42'' \\ 27\ 9\ 26.42'' \end{cases}$ | 4 825503 | |
| 50. $\begin{cases} a=6.064344, \\ b=4.950051, \\ c=5.3391, \end{cases}$ | $\begin{cases} 28\ 54\ 0.57'' \\ 23\ 47\ 32.88'' \\ 22\ 1\ 56.84'' \end{cases}$ | $\begin{cases} 33\ 56\ 45.16'' \\ 26\ 57\ 57.45'' \\ 28\ 59\ 58.80'' \end{cases}$ | $\begin{cases} 41\ 0\ 5.71'' \\ 29\ 53\ 0.18'' \\ 32\ 30\ 34.75'' \end{cases}$ | $\begin{cases} 37\ 12\ 28.32'' \\ 28\ 35\ 20.68'' \\ 26\ 10\ 14.66'' \end{cases}$ | $\begin{cases} 47\ 41\ 37.89'' \\ 30\ 8\ 55.09'' \\ 32\ 50\ 33.88'' \end{cases}$ | $\begin{cases} 42\ 18\ 22.11'' \\ 30\ 8\ 55.90'' \\ 27\ 27\ 18.10'' \end{cases}$ | 13.65186 | $A + X = 180^\circ, B = 1; I'$ a max. if $\sigma_1 - x = \sigma_2 - z = 90^\circ$ $b + y = c + z = 9.9981$ |

EXAMPLES. X.—Continued.

| DATA. | $\frac{a}{b}$, $\frac{a^2}{b^2}$, $\frac{a^3}{b^3}$, $\frac{a^4}{b^4}$, $\frac{a^5}{b^5}$, $\frac{a^6}{b^6}$, $\frac{a^7}{b^7}$, $\frac{a^8}{b^8}$, $\frac{a^9}{b^9}$, $\frac{a^{10}}{b^{10}}$, $\frac{a^{11}}{b^{11}}$, $\frac{a^{12}}{b^{12}}$ | VOLUME. | REMARKS. |
|--|--|-----------|--|
| 61. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{array} \right.$ | 10° 3'50.01", 22 37' 3.57", 37 ² 23' 41.37", 47 10' 17.32", 45 19' 31.89", 25 ⁵ 15' 13.49", 20 25' 47.66", 32 30' 52.49", 33 10' 59.37", 50 6' 38.54", 49 38' 31.91", 35 50' 26.60", 19 59' 26.14", 22 23' 55.02", 23 49' 31.45", 34 11' 57.00", 24 59' 26.10", 34 24' 16.94" | 13.09054 | This group of ten tetrahedra, (61) to (70), have all the same six lengths for their edges; and if these, in order of descending magnitude, be denoted by l, m, n, p, q, r , the following relations will hold $3l^2 + 2r^2 = 3m^2 + 2q^2$ $= r^2 + m^2 + n^2 + p^2$ $+ q^2 + r^2$. |
| 62. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{array} \right.$ | 16° 3'50.01", 22 37' 3.57", 37 ² 23' 41.37", 46 14' 41.03", 44 40' 28.11", 25 9' 19.61", 20 49' 36.58", 32 43' 41.30", 34 11' 57.66", 50 6' 38.54", 49 49' 12.08", 37 24' 6.41", 19 40' 57.75", 22 25' 39.39", 23 49' 31.45", 33 10' 59.37", 24 55' 43.10", 33 18' 4.32" | 13.69054 | |
| 63. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=5.1220, \end{array} \right.$ | 22° 28' 7.17", 34 11' 57.06", 40 40' 27.22", 47 10' 17.32", 49 49' 2.08", 40 16' 5.13", 20 25' 47.66", 32 43' 41.30", 34 11' 57.66", 50 6' 38.54", 49 49' 12.08", 37 24' 6.41", 19 40' 57.75", 22 25' 55.02", 24 16' 30.86", 22 37' 5.57", 24 11' 57.00", 23 17' 38.76" | 13.69054 | |
| 64. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{array} \right.$ | 15° 40' 18.85", 21 48' 38.61", 36 35' 30.81", 49 9' 4.63", 47 5' 35.70", 23 59' 54.11", 20 14' 28.81", 32 7' 34.10", 31 20' 5.14", 52 51' 37.01", 52 0' 43.10", 31 15' 7.3", 21 16' 55.00", 20 36' 26.56", 21 28' 4.14", 36 44' 30.25", 22 36' 20.02", 37 32 32.38" | 12.977715 | |
| 65. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{array} \right.$ | 15° 40' 18.85", 21 48' 38.61", 37 35' 30.97", 46 44' 44.03", 42 54' 24.36", 23 25' 13.22", 22 25' 39.30", 32 43' 41.30", 36 44' 30.25", 52 51' 37.01", 52 30' 33.76", 40 45' 31.82", 19 40' 57.75", 20 49' 36.58", 21 28' 4.14", 31 36' 51.14", 22 19' 25.25", 31 24' 7.35" | 12.977715 | Denoting these tetrahedra by the suffixes I to 10, it will be seen that $l^2 = I^2$ $-l_1 + l_2 = B_1 + B_2$ $B_1 = 180^{\circ}$ $I^2 = I_1^2 - I_2^2$ $-l_4 + l_5 = B_4 + B_5$ $B_4 = 180^{\circ}$ |
| 66. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{array} \right.$ | 21° 16' 36.86", 36 44' 30.25", 40 40' 27.22", 49 9' 4.63", 52 56' 33.76", 44 36' 48.96", 20 14' 28.81", 34 14' 11.92", 31 26' 51.14", 43 15' 15.97", 37 59' 16.84", 30 0' 1.56", 15 5 20.86", 20 36' 26.56", 22 28' 7.17", 21 48' 38.61", 21 42' 12.81", 21 3 20.27" | 12.97772 | |
| 67. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{array} \right.$ | 15° 34' 40.76", 21 40' 53.76", 37 23' 41.37", 47 10' 17.32", 43 13' 41.35", 23 10' 4.21", 22 23' 55.02", 32 36' 52.40", 30 17' 13.87", 53 33' 31.47", 53 28' 7.04", 40 21' 56.65", 19 59' 26.14", 20 25' 47.66", 20 53 38.77", 32 1 22.37", 21 47' 7.55", 32 4 47.03" | 12.83348 | |
| 68. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{array} \right.$ | 15° 3' 49.76", 21 40' 53.76", 39 35 30.81", 49 9' 4.63", 46 46' 15.65", 23 38 49.86", 20 36' 26.56", 32 7' 34.19", 32 1 22.37", 53 33' 31.47", 52 42' 44.16", 34 49 9.49", 21 16 55.00", 20 14 28.81", 20 53 38.77", 30 17 43.87", 22 0 22 90", 37 7 31.06" | 12.83348 | |
| 69. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{array} \right.$ | 23° 49' 31.44", 32 18' 35.14", 37 23' 41.37", 58 53' 26.61", 58 13' 5.36", 41 10' 16.67", 17 44' 30.19", 32 36 52.49", 28 30 37.55", 50 6' 38.54", 52 46 28.06", 25 33 28.52", 19 59 26.14", 13 22' 3.20", 16 3 59.02", 29 10 47' 31", 14 47' 5.83", 29 45' 4.60" | 11.22593 | |
| 70. $\left\{ \begin{array}{l} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{array} \right.$ | 21° 28' 4.14", 20 10 47.31", 36 35 30.81", 58 53' 26.61", 56 51' 51.45", 35 8 53.00", 17 44' 30.19", 32 7 34.19", 28 30 37.55", 52 51' 37.01", 47 53 13.72", 26 40 23.45", 21 16 55.00", 13 22' 3.20", 15 0 48 85", 32 18 35 14", 14 54 59.08", 34 36 10.81" | 11.22593 | |

EXAMPLES. X.—Continued.

| DATA. | a_1, b_1, c_1, γ_1 | a_2, b_2, c_2, γ_2 | a_3, b_3, c_3, γ_3 | a_4, b_4, c_4, γ_4 | a_5, b_5, c_5, γ_5 | a_6, b_6, c_6, γ_6 | a_7, b_7, c_7, γ_7 | a_8, b_8, c_8, γ_8 | VOLUME. | REMARKS. |
|--|--|--|--|--|--|--|---------------------------|---------------------------|-----------|---|
| 71. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=5, 1220, \end{cases}$ | $22^\circ 4' 19.49''$ $20^\circ 49' 36.58''$ $15^\circ 5' 20.86''$ | $33^\circ 10' 59.37''$ $34^\circ 14' 11.92''$ $22^\circ 25' 39.39''$ | $40^\circ 40' 27.22''$ $41^\circ 17' 57.06''$ $21^\circ 17' 59.87''$ | $46^\circ 44' 44.03''$ $47^\circ 40' 61.11''$ $23^\circ 37' 3.57''$ | $48^\circ 49' 59.22''$ $49^\circ 19' 16.21''$ $24^\circ 22' 49.11''$ | $38^\circ 48' 19.55''$ $34^\circ 22' 45.17''$ $22^\circ 24' 48.43''$ | | | 13 77206 | |
| 72. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=5, 1220, \end{cases}$ | $22^\circ 4' 19.49''$ $14^\circ 18' 6.32''$ $19^\circ 59' 26.14''$ | $33^\circ 10' 59.37''$ $32^\circ 36' 52.49''$ $20^\circ 51' 11.06''$ | $37^\circ 23' 14.37''$ $22^\circ 37' 3.57''$ $24^\circ 17' 59.87''$ | $54^\circ 50' 42.62''$ $43^\circ 37' 40.61''$ $34^\circ 11' 57.00''$ | $56^\circ 13' 19.51''$ $59^\circ 20' 13.50''$ $22^\circ 37' 49.36''$ | $40^\circ 48' 28.45''$ $21^\circ 25' 25.05''$ $32^\circ 56' 27.75''$ | | | 12 84365 | |
| 73. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=4, 8265, \end{cases}$ | $22^\circ 4' 5.66''$ $22^\circ 25' 39.37''$ $15^\circ 5' 20.86''$ | $31^\circ 26' 51.14''$ $31^\circ 14' 11.02''$ $20^\circ 49' 36.58''$ | $40^\circ 40' 27.22''$ $35^\circ 41' 30.25''$ $22^\circ 31' 9.65''$ | $46^\circ 44' 44.03''$ $45^\circ 22' 11.09''$ $21^\circ 48' 38.61''$ | $47^\circ 7' 58.93''$ $43^\circ 42' 58.08''$ $22^\circ 40' 12.81''$ | $36^\circ 27' 59.00''$ $37^\circ 37' 46.06''$ $21^\circ 47' 8.56''$ | | | 13-47945 | |
| 74. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=5, 1220, \end{cases}$ | $22^\circ 4' 5.66''$ $13^\circ 34' 58.03''$ $21^\circ 16' 55.00''$ | $31^\circ 26' 51.14''$ $32^\circ 7' 34.19''$ $18^\circ 21' 32.60''$ | $36^\circ 35' 30.81''$ $31^\circ 48' 38.61''$ $22^\circ 31' 9.65''$ | $58^\circ 3' 29.31''$ $45^\circ 22' 44.69''$ $30^\circ 44' 30.35''$ | $58^\circ 32' 26.51''$ $41^\circ 15' 54.61''$ $20^\circ 34' 45.94''$ | $39^\circ 14' 51.79''$ $19^\circ 57' 32.53''$ $35^\circ 51' 56.23''$ | | | 12 03492 | |
| 75. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=3, 6557, \end{cases}$ | $20^\circ 53' 38.77''$ $20^\circ 51' 11.06''$ $19^\circ 59' 26.14''$ | $28^\circ 30' 37.55''$ $32^\circ 36' 52.49''$ $14^\circ 18' 6.32''$ | $37^\circ 23' 14.37''$ $32^\circ 18' 35.14''$ $15^\circ 32' 49.76''$ | $54^\circ 50' 42.62''$ $49^\circ 13' 2.84''$ $29^\circ 10' 47.31''$ | $51^\circ 28' 55.49''$ $49^\circ 13' 2.84''$ $15^\circ 43' 5.96''$ | $33^\circ 1' 24.98''$ $32^\circ 55' 14.21''$ $30^\circ 53' 33.48''$ | | | 11, 86436 | (71) to (84) are also tetrahedra with the same six lengths for their edges. The six tetrahedra in which the two longest edges are opposite each other are all impossible. |
| 76. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=5, 1220, \end{cases}$ | $20^\circ 53' 38.77''$ $18^\circ 21' 32.66''$ $21^\circ 16' 55.00''$ | $28^\circ 30' 37.55''$ $32^\circ 36' 52.49''$ $13^\circ 34' 58.03''$ | $36^\circ 35' 30.81''$ $29^\circ 10' 47.31''$ $15^\circ 32' 49.76''$ | $58^\circ 3' 29.31''$ $53^\circ 33' 31.47''$ $32^\circ 18' 35.14''$ | $55^\circ 39' 49.09''$ $48^\circ 44' 29.8''$ $15^\circ 5' 6.60''$ | $33^\circ 12' 51.86''$ $27^\circ 58' 17.71''$ $34^\circ 48' 7.14''$ | | | 11, 34094 | |
| 77. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=5, 1220, \end{cases}$ | $22^\circ 28' 7.17''$ $14^\circ 18' 6.32''$ $19^\circ 40' 57.73''$ | $34^\circ 11' 57.06''$ $32^\circ 43' 11.30''$ $20^\circ 51' 11.06''$ | $37^\circ 35' 20.97''$ $22^\circ 37' 3.57''$ $24^\circ 16' 36.86''$ | $54^\circ 50' 42.62''$ $43^\circ 15' 15.97''$ $33^\circ 10' 59.37''$ | $56^\circ 38' 17.11''$ $38^\circ 26' 2.57''$ $22^\circ 42' 35.79''$ | $42^\circ 14' 54.68''$ $21^\circ 12' 50.88''$ $31^\circ 40' 45.14''$ | | | 12 7562 | |
| 78. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=4, 7557, \end{cases}$ | $22^\circ 31' 9.65''$ $22^\circ 23' 55.02''$ $15^\circ 5' 20.86''$ | $32^\circ 1' 22.31''$ $34^\circ 14' 11.02''$ $20^\circ 25' 47.66''$ | $40^\circ 40' 27.22''$ $36^\circ 17' 43.87''$ $22^\circ 6' 5.66''$ | $47^\circ 10' 17.32''$ $45^\circ 22' 44.69''$ $21^\circ 40' 53.76''$ | $47^\circ 45' 45.65''$ $43^\circ 14' 56.18''$ $22^\circ 17' 30.93''$ | $37^\circ 22' 19.32''$ $36^\circ 56' 27.33''$ $21^\circ 38' 0.51''$ | | | 13, 39033 | |
| 79. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=4, 7557, \end{cases}$ | $22^\circ 31' 9.65''$ $13^\circ 21' 3.20''$ $21^\circ 16' 55.00''$ | $32^\circ 1' 22.31''$ $32^\circ 7' 34.19''$ $17^\circ 44' 30.19''$ | $36^\circ 35' 30.81''$ $21^\circ 40' 53.76''$ $22^\circ 6' 5.66''$ | $58^\circ 33' 26.61''$ $48^\circ 22' 44.69''$ $36^\circ 17' 43.87''$ | $50^\circ 39' 8.08''$ $40^\circ 43' 2.21''$ $10^\circ 47' 31.02''$ | $40^\circ 22' 54.14''$ $19^\circ 26' 3.19''$ $35^\circ 19' 41.99''$ | | | 11, 7694 | |
| 80. $\begin{cases} a=7, 2485, \\ b=6, 8205, \\ c=3, 6857, \end{cases}$ | $21^\circ 28' 4.14''$ $20^\circ 51' 11.06''$ $19^\circ 40' 57.73''$ | $29^\circ 10' 47.31''$ $32^\circ 43' 11.30''$ $14^\circ 18' 6.32''$ | $37^\circ 35' 20.97''$ $32^\circ 18' 35.14''$ $15^\circ 40' 48.85''$ | $54^\circ 50' 42.62''$ $50^\circ 51' 37.01''$ $28^\circ 30' 37.55''$ | $51^\circ 47' 4.39''$ $48^\circ 10' 54.29''$ $15^\circ 46' 7.56''$ | $34^\circ 14' 46.68''$ $32^\circ 38' 55.92''$ $30^\circ 0' 14.77''$ | | | 11, 9202 | |

EXAMPLES. X.—Continued.

| DATA. | \log_1 | \log_2 | \log_3 | \log_4 | \log_5 | \log_6 | \log_7 | \log_8 | \log_9 | \log_{10} | \log_{11} | \log_{12} | \log_{13} | \log_{14} | \log_{15} | \log_{16} | \log_{17} | \log_{18} | \log_{19} | \log_{20} | VOLUME. | REMARKS. | |
|---|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|----------|---|--|
| 81. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.8265, \end{cases}$ | $24^{\circ}16'36.86''$ | $36^{\circ}44'30.25''$ | $37^{\circ}35'20.97''$ | $58^{\circ}3'29.31''$ | $60^{\circ}53'22.78''$ | $47^{\circ}34'3.05''$ | $35^{\circ}27'11.69''$ | $18^{\circ}40'55.74''$ | $19^{\circ}11'54.74''$ | $28^{\circ}43'0.00''$ | | | | | | | | | | | 11.47017 | | |
| 82. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.7557, \end{cases}$ | $24^{\circ}17'59.87''$ | $36^{\circ}17'43.87''$ | $37^{\circ}23'41.37''$ | $58^{\circ}53'26.61''$ | $61^{\circ}28'32.90''$ | $47^{\circ}17'4.47''$ | $43^{\circ}37'40.64''$ | $18^{\circ}21'26.04''$ | $18^{\circ}39'53.35''$ | $29^{\circ}19'2.54''$ | | | | | | | | | | | 11.29084 | See previous page. | |
| 83. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=4.7757, \end{cases}$ | $24^{\circ}17'59.87''$ | $36^{\circ}17'43.87''$ | $37^{\circ}23'41.37''$ | $58^{\circ}53'26.61''$ | $61^{\circ}28'32.90''$ | $47^{\circ}17'4.47''$ | $43^{\circ}37'40.64''$ | $18^{\circ}21'26.04''$ | $18^{\circ}39'53.35''$ | $29^{\circ}19'2.54''$ | | | | | | | | | | | | | |
| 84. $\begin{cases} a=7.2485, \\ b=6.8205, \\ c=3.6857, \end{cases}$ | $23^{\circ}49'31.44''$ | $32^{\circ}18'35.14''$ | $37^{\circ}35'20.97''$ | $58^{\circ}3'29.31''$ | $60^{\circ}53'22.78''$ | $47^{\circ}34'3.05''$ | $35^{\circ}27'11.69''$ | $18^{\circ}40'55.74''$ | $19^{\circ}11'54.74''$ | $28^{\circ}43'0.00''$ | | | | | | | | | | | 11.97171 | | |
| 85. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.8265, \end{cases}$ | $43^{\circ}15'15.97''$ | $20^{\circ}14'28.81''$ | $20^{\circ}14'28.81''$ | $20^{\circ}51'11.06''$ | $20^{\circ}33'38.77''$ | $24^{\circ}16'36.86''$ | $21^{\circ}57'56.74''$ | $25^{\circ}53'32.64''$ | $25^{\circ}53'32.64''$ | $79^{\circ}21'27.65''$ | | | | | | | | | | | Imp. | | |
| 86. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.8265, \end{cases}$ | $43^{\circ}15'15.97''$ | $20^{\circ}14'28.81''$ | $20^{\circ}14'28.81''$ | $20^{\circ}51'11.06''$ | $20^{\circ}33'38.77''$ | $24^{\circ}16'36.86''$ | $21^{\circ}57'56.74''$ | $25^{\circ}53'32.64''$ | $25^{\circ}53'32.64''$ | $79^{\circ}21'27.65''$ | | | | | | | | | | | Imp. | | |
| 87. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.8265, \end{cases}$ | $43^{\circ}15'15.97''$ | $20^{\circ}14'28.81''$ | $20^{\circ}14'28.81''$ | $20^{\circ}51'11.06''$ | $20^{\circ}33'38.77''$ | $24^{\circ}16'36.86''$ | $21^{\circ}57'56.74''$ | $25^{\circ}53'32.64''$ | $25^{\circ}53'32.64''$ | $79^{\circ}21'27.65''$ | | | | | | | | | | | 5.223158 | These six, (86) to (91), are those, out of the thirty which have the same six lengths for their edges, in which the two longest edges are opposite. They are all "ill-conditioned," and their volume is much less than in any similar group of six. | |
| 88. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.7557, \end{cases}$ | $43^{\circ}15'15.97''$ | $20^{\circ}14'28.81''$ | $20^{\circ}14'28.81''$ | $20^{\circ}51'11.06''$ | $20^{\circ}33'38.77''$ | $24^{\circ}16'36.86''$ | $21^{\circ}57'56.74''$ | $25^{\circ}53'32.64''$ | $25^{\circ}53'32.64''$ | $79^{\circ}21'27.65''$ | | | | | | | | | | | 5.81898 | | |
| 89. $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=4.7557, \end{cases}$ | $43^{\circ}15'15.97''$ | $20^{\circ}14'28.81''$ | $20^{\circ}14'28.81''$ | $20^{\circ}51'11.06''$ | $20^{\circ}33'38.77''$ | $24^{\circ}16'36.86''$ | $21^{\circ}57'56.74''$ | $25^{\circ}53'32.64''$ | $25^{\circ}53'32.64''$ | $79^{\circ}21'27.65''$ | | | | | | | | | | | 5.167364 | | |
| 90. $\begin{cases} a=6.8205, \\ b=5.1220, \\ c=5.3856, \end{cases}$ | $40^{\circ}27'9.94''$ | $45^{\circ}22'44.69''$ | $47^{\circ}10'17.32''$ | $45^{\circ}3'58.03''$ | $45^{\circ}30'51.12''$ | $78^{\circ}51'15.86''$ | $8^{\circ}15'59.98''$ | $8^{\circ}40'30.20''$ | $7^{\circ}40'25.27''$ | $7^{\circ}44'12.92''$ | | | | | | | | | | | | 5.825749 | |
| | $20^{\circ}52'26.27''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | $22^{\circ}23'55.02''$ | | | | | | | | | | | | 5.247945 | |

EXAMPLES. X.—Continued.

| | DATA. | $\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | VOLUME. | REMARKS. |
|------|--|--|--|--|--|--|--|
| 91. | $\begin{cases} a=7.2485, \\ b=5.1220, \\ c=5.3856, \end{cases}$ $\begin{cases} A=6.8205, \\ B=4.7557, \\ C=4.8205 \end{cases}$ | $\begin{cases} 40^{\circ}27' 9.94'' \\ 20^{\circ}29' 50.39'' \\ 20^{\circ}49' 36.58'' \end{cases}$ $\begin{cases} 45^{\circ}22' 44.69'' \\ 22^{\circ}56' 5.06'' \\ 23^{\circ}59' 18.49'' \end{cases}$ | $\begin{cases} 46^{\circ}44' 44.03'' \\ 22^{\circ}6' 5.06'' \\ 23^{\circ}56' 58.48'' \end{cases}$ $\begin{cases} 45^{\circ}30' 51.12'' \\ 23^{\circ}35' 51.58'' \\ 22^{\circ}31' 9.65'' \end{cases}$ | $\begin{cases} 79^{\circ}50' 21' 87'' \\ 7^{\circ}1' 9.65'' \\ 7^{\circ}52' 12.49'' \end{cases}$ $\begin{cases} 79^{\circ}50' 21' 87'' \\ 7^{\circ}1' 9.65'' \\ 7^{\circ}52' 12.49'' \end{cases}$ | $\begin{cases} 79^{\circ}17' 30.88'' \\ 7^{\circ}1' 9.65'' \\ 7^{\circ}52' 12.49'' \end{cases}$ $\begin{cases} 79^{\circ}17' 30.88'' \\ 7^{\circ}1' 9.65'' \\ 7^{\circ}52' 12.49'' \end{cases}$ | $\begin{cases} 5.305218 \\ 26.76496 \\ 27.04064 \end{cases}$ $\begin{cases} 5.305218 \\ 26.76496 \\ 27.04064 \end{cases}$ | See previous page. |
| 92. | $\begin{cases} a=7.3871, \\ b=5.9835, \\ c=6.1635, \end{cases}$ $\begin{cases} A=6.9870, \\ B=5.9634, \\ C=5.7634 \end{cases}$ | $\begin{cases} 35^{\circ}10' 9.55'' \\ 25^{\circ}37' 31.80'' \\ 24^{\circ}52' 47.74'' \end{cases}$ $\begin{cases} 36^{\circ}33' 34.19'' \\ 26^{\circ}5' 8.05'' \\ 20^{\circ}51' 23.75'' \end{cases}$ | $\begin{cases} 39^{\circ}2' 4.21'' \\ 27^{\circ}22' 44.36'' \\ 28^{\circ}5' 8.06'' \end{cases}$ $\begin{cases} 39^{\circ}2' 4.21'' \\ 27^{\circ}22' 44.36'' \\ 28^{\circ}5' 8.06'' \end{cases}$ | $\begin{cases} 49^{\circ}51' 29.66'' \\ 27^{\circ}32' 38.29'' \\ 28^{\circ}24' 19.82'' \end{cases}$ $\begin{cases} 49^{\circ}51' 29.66'' \\ 27^{\circ}32' 38.29'' \\ 28^{\circ}24' 19.82'' \end{cases}$ | $\begin{cases} 48^{\circ}16' 23.66'' \\ 27^{\circ}40' 55.35'' \\ 26^{\circ}49' 13.82'' \end{cases}$ $\begin{cases} 48^{\circ}16' 23.66'' \\ 27^{\circ}40' 55.35'' \\ 26^{\circ}49' 13.82'' \end{cases}$ | $\begin{cases} 26.76496 \\ 27.04064 \\ 27.117785 \end{cases}$ $\begin{cases} 26.76496 \\ 27.04064 \\ 27.117785 \end{cases}$ | |
| 93. | $\begin{cases} a=5.1471, \\ b=5.9669, \\ c=6.1635, \end{cases}$ $\begin{cases} A=6.7470, \\ B=5.9600, \\ C=5.7634 \end{cases}$ | $\begin{cases} 33^{\circ}16' 37.23'' \\ 20^{\circ}17' 5.52'' \\ 25^{\circ}35' 23.03'' \end{cases}$ $\begin{cases} 35^{\circ}7' 27.48'' \\ 27^{\circ}36' 10.53'' \\ 28^{\circ}45' 9.39'' \end{cases}$ | $\begin{cases} 37^{\circ}31' 35.85'' \\ 27^{\circ}14' 44.44'' \\ 28^{\circ}45' 9.39'' \end{cases}$ $\begin{cases} 37^{\circ}31' 35.85'' \\ 27^{\circ}14' 44.44'' \\ 28^{\circ}45' 9.39'' \end{cases}$ | $\begin{cases} 46^{\circ}37' 29.15'' \\ 29^{\circ}29' 35.41'' \\ 30^{\circ}22' 39.48'' \end{cases}$ $\begin{cases} 46^{\circ}37' 29.15'' \\ 29^{\circ}29' 35.41'' \\ 30^{\circ}22' 39.48'' \end{cases}$ | $\begin{cases} 44^{\circ}58' 17.27'' \\ 30^{\circ}36' 31.69'' \\ 28^{\circ}43' 27.60'' \end{cases}$ $\begin{cases} 44^{\circ}58' 17.27'' \\ 30^{\circ}36' 31.69'' \\ 28^{\circ}43' 27.60'' \end{cases}$ | $\begin{cases} 27.04064 \\ 27.117785 \\ 27.117785 \end{cases}$ $\begin{cases} 27.04064 \\ 27.117785 \\ 27.117785 \end{cases}$ | (92) ex- ply the process of finding the tetra- hedron of maximum volume when $a = x,$ $b = y,$ $c,$ and z are given. |
| 94. | $\begin{cases} a=7.0871, \\ b=5.9769, \\ c=6.1635, \end{cases}$ $\begin{cases} A=6.6870, \\ B=5.9500, \\ C=5.7634 \end{cases}$ | $\begin{cases} 33^{\circ}24' 43.34'' \\ 26^{\circ}24' 9.16'' \\ 25^{\circ}45' 7.03'' \end{cases}$ $\begin{cases} 34^{\circ}48' 5.32'' \\ 27^{\circ}7' 43.21'' \\ 27^{\circ}48' 9.75'' \end{cases}$ | $\begin{cases} 37^{\circ}7' 10.76'' \\ 28^{\circ}15' 20.69'' \\ 27^{\circ}40' 8.82'' \end{cases}$ $\begin{cases} 37^{\circ}7' 10.76'' \\ 28^{\circ}15' 20.69'' \\ 27^{\circ}40' 8.82'' \end{cases}$ | $\begin{cases} 45^{\circ}52' 4.32'' \\ 29^{\circ}58' 33.44'' \\ 30^{\circ}49' 22.42'' \end{cases}$ $\begin{cases} 45^{\circ}52' 4.32'' \\ 29^{\circ}58' 33.44'' \\ 30^{\circ}49' 22.42'' \end{cases}$ | $\begin{cases} 44^{\circ}12' 24.45'' \\ 30^{\circ}0' 31.53'' \\ 29^{\circ}9' 42.55'' \end{cases}$ $\begin{cases} 44^{\circ}12' 24.45'' \\ 30^{\circ}0' 31.53'' \\ 29^{\circ}9' 42.55'' \end{cases}$ | $\begin{cases} 27.117785 \\ 27.117785 \\ 27.117785 \end{cases}$ $\begin{cases} 27.117785 \\ 27.117785 \\ 27.117785 \end{cases}$ | |
| 95. | $\begin{cases} a=7.0841, \\ b=5.9807, \\ c=6.1635, \end{cases}$ $\begin{cases} A=6.6840, \\ B=5.9462, \\ C=5.7634 \end{cases}$ | $\begin{cases} 33^{\circ}23' 1.17'' \\ 26^{\circ}23' 32.04'' \\ 25^{\circ}45' 19.88'' \end{cases}$ $\begin{cases} 34^{\circ}47' 49.00'' \\ 27^{\circ}9' 27.60'' \\ 27^{\circ}49' 4.82'' \end{cases}$ | $\begin{cases} 37^{\circ}5' 12.32'' \\ 28^{\circ}14' 41.88'' \\ 28^{\circ}37' 44.80'' \end{cases}$ $\begin{cases} 37^{\circ}5' 12.32'' \\ 28^{\circ}14' 41.88'' \\ 28^{\circ}37' 44.80'' \end{cases}$ | $\begin{cases} 45^{\circ}49' 45.62'' \\ 30^{\circ}0' 48.71'' \\ 30^{\circ}50' 36.71'' \end{cases}$ $\begin{cases} 45^{\circ}49' 45.62'' \\ 30^{\circ}0' 48.71'' \\ 30^{\circ}50' 36.71'' \end{cases}$ | $\begin{cases} 44^{\circ}10' 45.62'' \\ 30^{\circ}0' 52.12'' \\ 29^{\circ}11' 4.12'' \end{cases}$ $\begin{cases} 44^{\circ}10' 45.62'' \\ 30^{\circ}0' 52.12'' \\ 29^{\circ}11' 4.12'' \end{cases}$ | $\begin{cases} 27.117785 \\ 27.117785 \\ 27.117785 \end{cases}$ $\begin{cases} 27.117785 \\ 27.117785 \\ 27.117785 \end{cases}$ | |
| 96. | $\begin{cases} a=7.0841, \\ b=5.9807, \\ c=6.1635, \end{cases}$ $\begin{cases} A=6.6840, \\ B=5.9462, \\ C=5.7634 \end{cases}$ | $\begin{cases} 33^{\circ}23' 0.26'' \\ 26^{\circ}23' 29.95'' \\ 25^{\circ}45' 19.18'' \end{cases}$ $\begin{cases} 34^{\circ}47' 50.76'' \\ 27^{\circ}9' 29.52'' \\ 27^{\circ}49' 5.29'' \end{cases}$ | $\begin{cases} 37^{\circ}5' 11.30'' \\ 28^{\circ}14' 39.36'' \\ 28^{\circ}37' 43.80'' \end{cases}$ $\begin{cases} 37^{\circ}5' 11.30'' \\ 28^{\circ}14' 39.36'' \\ 28^{\circ}37' 43.80'' \end{cases}$ | $\begin{cases} 45^{\circ}49' 46.14'' \\ 30^{\circ}0' 50.02'' \\ 30^{\circ}50' 36.16'' \end{cases}$ $\begin{cases} 45^{\circ}49' 46.14'' \\ 30^{\circ}0' 50.02'' \\ 30^{\circ}50' 36.16'' \end{cases}$ | $\begin{cases} 44^{\circ}10' 45.62'' \\ 30^{\circ}0' 52.12'' \\ 29^{\circ}11' 4.12'' \end{cases}$ $\begin{cases} 44^{\circ}10' 45.62'' \\ 30^{\circ}0' 52.12'' \\ 29^{\circ}11' 4.12'' \end{cases}$ | $\begin{cases} 27.117785 \\ 27.117785 \\ 27.117785 \end{cases}$ $\begin{cases} 27.117785 \\ 27.117785 \\ 27.117785 \end{cases}$ | |
| 97. | $\begin{cases} a=4.886136, \\ b=3.967768, \\ c=4.3557, \end{cases}$ $\begin{cases} A=3.775036, \\ B=3.572332, \\ C=3.2146 \end{cases}$ | $\begin{cases} 26^{\circ}58' 18.90'' \\ 22^{\circ}33' 53.45'' \\ 20^{\circ}28' 30.21'' \end{cases}$ $\begin{cases} 33^{\circ}40' 27.35'' \\ 26^{\circ}59' 59.41'' \\ 29^{\circ}33' 10.32'' \end{cases}$ | $\begin{cases} 42^{\circ}31' 24.38'' \\ 30^{\circ}35' 32.63'' \\ 33^{\circ}33' 10.88'' \end{cases}$ $\begin{cases} 42^{\circ}31' 24.38'' \\ 30^{\circ}35' 32.63'' \\ 33^{\circ}33' 10.88'' \end{cases}$ | $\begin{cases} 48^{\circ}26' 47.83'' \\ 30^{\circ}14' 47.59'' \\ 33^{\circ}41' 35.39'' \end{cases}$ $\begin{cases} 48^{\circ}26' 47.83'' \\ 30^{\circ}14' 47.59'' \\ 33^{\circ}41' 35.39'' \end{cases}$ | $\begin{cases} 41^{\circ}33' 12.17'' \\ 23^{\circ}29' 14.39'' \\ 26^{\circ}47' 59.73'' \end{cases}$ $\begin{cases} 41^{\circ}33' 12.17'' \\ 23^{\circ}29' 14.39'' \\ 26^{\circ}47' 59.73'' \end{cases}$ | $\begin{cases} 6.445679 \\ 5.912593 \\ 166.9554 \end{cases}$ $\begin{cases} 6.445679 \\ 5.912593 \\ 166.9554 \end{cases}$ | $A - \lambda = C - Z,$ $B + \lambda = A + 26,$ $C - \lambda = 6,$ a similar case of maximum volume. |
| 98. | $\begin{cases} a=3.147, \\ b=4.0357, \\ c=4.3557, \end{cases}$ $\begin{cases} A=4.2036, \\ B=3.5146, \\ C=3.2146 \end{cases}$ | $\begin{cases} 30^{\circ}9' 27.67'' \\ 20^{\circ}35' 27.40'' \\ 18^{\circ}33' 42.30'' \end{cases}$ $\begin{cases} 38^{\circ}35' 14.65'' \\ 24^{\circ}32' 54.36'' \\ 27^{\circ}4' 4.68'' \end{cases}$ | $\begin{cases} 46^{\circ}53' 23.39'' \\ 31^{\circ}41' 24.20'' \\ 31^{\circ}41' 24.20'' \end{cases}$ $\begin{cases} 46^{\circ}53' 23.39'' \\ 31^{\circ}41' 24.20'' \\ 31^{\circ}41' 24.20'' \end{cases}$ | $\begin{cases} 58^{\circ}15' 33.72'' \\ 24^{\circ}11' 41.63'' \\ 26^{\circ}38' 44.13'' \end{cases}$ $\begin{cases} 58^{\circ}15' 33.72'' \\ 24^{\circ}11' 41.63'' \\ 26^{\circ}38' 44.13'' \end{cases}$ | $\begin{cases} 52^{\circ}39' 1.46'' \\ 23^{\circ}29' 14.39'' \\ 21^{\circ}2' 11.89'' \end{cases}$ $\begin{cases} 52^{\circ}39' 1.46'' \\ 23^{\circ}29' 14.39'' \\ 21^{\circ}2' 11.89'' \end{cases}$ | $\begin{cases} 5.912593 \\ 166.9554 \\ 18.889605 \end{cases}$ $\begin{cases} 5.912593 \\ 166.9554 \\ 18.889605 \end{cases}$ | $A - \lambda = C - Z;$ $B + \lambda = C + Z;$ |
| 99. | $\begin{cases} a=11.99045, \\ b=12.0002, \\ c=15.01087, \end{cases}$ $\begin{cases} A=10.96995, \\ B=9.7858, \\ C=10.70947 \end{cases}$ | $\begin{cases} 23^{\circ}7' 58.6'' \\ 20^{\circ}17' 26.61'' \\ 26^{\circ}30' 47.00'' \end{cases}$ $\begin{cases} 32^{\circ}15' 47.00'' \\ 31^{\circ}46' 0.51'' \\ 43^{\circ}16' 16.71'' \end{cases}$ | $\begin{cases} 31^{\circ}43' 12.49'' \\ 20^{\circ}49' 11.31'' \\ 46^{\circ}41' 11.19'' \end{cases}$ $\begin{cases} 31^{\circ}43' 12.49'' \\ 20^{\circ}49' 11.31'' \\ 46^{\circ}41' 11.19'' \end{cases}$ | $\begin{cases} 31^{\circ}35' 16.82'' \\ 31^{\circ}35' 16.82'' \\ 31^{\circ}35' 16.82'' \end{cases}$ $\begin{cases} 31^{\circ}35' 16.82'' \\ 31^{\circ}35' 16.82'' \\ 31^{\circ}35' 16.82'' \end{cases}$ | $\begin{cases} 166.9554 \\ 18.889605 \\ 18.889605 \end{cases}$ $\begin{cases} 166.9554 \\ 18.889605 \\ 18.889605 \end{cases}$ | | Tetrahedron of max. vol. when $a = x, b, y,$ and $c = z$ are given. |
| 100. | $\begin{cases} a=5.966997, \\ b=6.028293, \\ c=7.5063, \end{cases}$ $\begin{cases} A=5.4836, \\ B=4.8314, \\ C=4.3556 \end{cases}$ | $\begin{cases} 23^{\circ}6' 38.24'' \\ 20^{\circ}17' 45.64'' \\ 21^{\circ}17' 22.23'' \end{cases}$ $\begin{cases} 36^{\circ}14' 24.17'' \\ 34^{\circ}43' 53.23'' \\ 43^{\circ}26' 9.60'' \end{cases}$ | $\begin{cases} 33^{\circ}58' 44.54'' \\ 29^{\circ}8' 23.94'' \\ 46^{\circ}37' 17.00'' \end{cases}$ $\begin{cases} 33^{\circ}58' 44.54'' \\ 29^{\circ}8' 23.94'' \\ 46^{\circ}37' 17.00'' \end{cases}$ | $\begin{cases} 34^{\circ}43' 13.30'' \\ 31^{\circ}43' 13.30'' \\ 46^{\circ}38' 17.92'' \end{cases}$ $\begin{cases} 34^{\circ}43' 13.30'' \\ 31^{\circ}43' 13.30'' \\ 46^{\circ}38' 17.92'' \end{cases}$ | $\begin{cases} 36^{\circ}13' 46.82'' \\ 29^{\circ}36' 59.18'' \\ 31^{\circ}39' 31.09'' \end{cases}$ $\begin{cases} 36^{\circ}13' 46.82'' \\ 29^{\circ}36' 59.18'' \\ 31^{\circ}39' 31.09'' \end{cases}$ | $\begin{cases} 18.889605 \\ 18.889605 \\ 18.889605 \end{cases}$ $\begin{cases} 18.889605 \\ 18.889605 \\ 18.889605 \end{cases}$ | $A = B$ |

EXAMPLES. X.—Continued.

| | DATA. | $\beta_1, \beta_2, \beta_3$ | $\beta_4, \beta_5, \beta_6$ | $\beta_7, \beta_8, \beta_9$ | $\beta_{10}, \beta_{11}, \beta_{12}$ | $\beta_{13}, \beta_{14}, \beta_{15}$ | $\beta_{16}, \beta_{17}, \beta_{18}$ | $\beta_{19}, \beta_{20}, \beta_{21}$ | $\beta_{22}, \beta_{23}, \beta_{24}$ | $\beta_{25}, \beta_{26}, \beta_{27}$ | $\beta_{28}, \beta_{29}, \beta_{30}$ | $\beta_{31}, \beta_{32}, \beta_{33}$ | $\beta_{34}, \beta_{35}, \beta_{36}$ | $\beta_{37}, \beta_{38}, \beta_{39}$ | $\beta_{40}, \beta_{41}, \beta_{42}$ | $\beta_{43}, \beta_{44}, \beta_{45}$ | $\beta_{46}, \beta_{47}, \beta_{48}$ | $\beta_{49}, \beta_{50}, \beta_{51}$ | $\beta_{52}, \beta_{53}, \beta_{54}$ | $\beta_{55}, \beta_{56}, \beta_{57}$ | $\beta_{58}, \beta_{59}, \beta_{60}$ | $\beta_{61}, \beta_{62}, \beta_{63}$ | $\beta_{64}, \beta_{65}, \beta_{66}$ | $\beta_{67}, \beta_{68}, \beta_{69}$ | $\beta_{70}, \beta_{71}, \beta_{72}$ | $\beta_{73}, \beta_{74}, \beta_{75}$ | $\beta_{76}, \beta_{77}, \beta_{78}$ | $\beta_{79}, \beta_{80}, \beta_{81}$ | $\beta_{82}, \beta_{83}, \beta_{84}$ | $\beta_{85}, \beta_{86}, \beta_{87}$ | $\beta_{88}, \beta_{89}, \beta_{90}$ | $\beta_{91}, \beta_{92}, \beta_{93}$ | $\beta_{94}, \beta_{95}, \beta_{96}$ | $\beta_{97}, \beta_{98}, \beta_{99}$ | $\beta_{100}, \beta_{101}, \beta_{102}$ | VOLUME. | REMARKS. | | | | | |
|------|--|--|--|--|--|--|--|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|---|---------|----------|--|----------|----------|----------|--|
| 111. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893. \end{cases}$ $\begin{cases} x=4.0188, \\ y=4.1941, \\ z=3.9794. \end{cases}$ | 22°39'55.74", 23°51'55.51", 20°35'48.29" | 29°24'59.22", 34°22'51.05", 24°20'44.08" | 35°1'20.66", 31°37'31.51", 24°41'24.64" | 42°1'20.41", 42°38'39.62", 28°57'29.27" | 42°57'30.25", 44°4'51.88", 29°5'23.85" | 33°17'4.01", 35°11'0.06", 29°5'23.85" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10.14892 | |
| 112. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893. \end{cases}$ $\begin{cases} x=4.0188, \\ y=3.9794, \\ z=4.1941. \end{cases}$ | 22°39'55.74", 22°11'35.74", 21°45'47.46" | 29°24'59.22", 33°48'59.21", 24°28'3.70" | 34°25'13.33", 28°57'29.27", 24°41'24.64" | 43°20'20.56", 43°11'50.97", 31°37'31.51" | 44°19'28.80", 33°38'1.83", 27°13'45.53" | 33°43'2.81", 33°38'1.83", 31°42'19.75" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10.17192 | | |
| 113. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941. \end{cases}$ $\begin{cases} x=4.0188, \\ y=3.2893, \\ z=3.9794. \end{cases}$ | 22°42'40.72", 20°35'48.29", 22°17'0.28" | 29°02'5.82", 34°22'51.05", 23°27'55.51" | 35°1'20.66", 32°25'47.57", 24°0'35.31" | 43°1'20.41", 43°16'43.97", 28°33'40.61" | 42°33'55.17", 36°49'31.66", 25°59'58.65" | 32°45'3.72", 36°49'31.66", 28°38'34.07" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10.06527 | | | |
| 114. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.1941. \end{cases}$ $\begin{cases} x=4.0188, \\ y=3.9794, \\ z=4.2893. \end{cases}$ | 22°42'40.72", 22°13'23.52", 22°17'0.28" | 29°02'5.82", 33°33'50.81", 23°46'53.71" | 34°9'8.91", 28°31'46.61", 24°0'35.31" | 43°59'42.77", 43°16'43.97", 32°25'47.57" | 44°41'49.10", 43°34'11.58", 26°33'54.34" | 33°25'12.21", 32°20'2.08", 32°27'50.67" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10.10116 | | | |
| 115. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$ $\begin{cases} x=4.1941, \\ y=4.2893, \\ z=4.0188. \end{cases}$ | 22°45'46.37", 23°46'53.71", 22°17'0.28" | 28°16'10.80", 33°33'50.81", 22°13'23.52" | 34°9'8.91", 30°16'2.76", 22°28'41.68" | 43°59'42.77", 45°14'34.29", 31°27'46.44" | 43°59'41.32", 34°43'55.66", 24°43'50.08" | 32°26'24.22", 34°43'55.66", 31°27'46.44" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 9.94747 | | |
| 116. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=3.9794. \end{cases}$ $\begin{cases} x=4.0188, \\ y=4.2893, \\ z=4.1941. \end{cases}$ | 22°45'46.37", 24°28'3.70", 21°45'47.46" | 28°16'10.80", 35°48'59.21", 22°11'35.74" | 34°25'13.33", 31°27'46.44", 22°28'41.68" | 43°20'20.56", 44°45'31.95", 30°16'2.76" | 43°13'42.23", 45°39'43.55", 30°15'20.49" | 32°12'15.21", 36°10'42.75", 30°15'20.49" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 9.93456 | | |
| 117. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893. \end{cases}$ $\begin{cases} x=4.1941, \\ y=4.0188, \\ z=3.9794. \end{cases}$ | 23°50'32.39", 22°27'24.92", 20°35'48.29" | 31°37'31.51", 34°22'51.05", 24°27'0.33" | 35°1'20.66", 29°24'59.22", 24°34'0.17" | 43°5'34.73", 41°35'27.44", 28°57'29.27" | 44°55'55.77", 42°10'10.90", 26°40'57.36" | 36°13'5.68", 32°46'26.66", 29°3'34.06" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10.13696 | | |
| 118. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.2893. \end{cases}$ $\begin{cases} x=3.9794, \\ y=4.0188, \\ z=4.1941. \end{cases}$ | 23°50'32.39", 21°45'47.46", 20°48'36.44" | 31°37'31.51", 34°16'39.91", 24°28'3.70" | 34°34'43.65", 31°37'29.27", 24°34'0.17" | 43°20'20.56", 41°35'27.44", 29°24'59.22" | 45°7'4.23", 42°1'8.22", 26°50'14.24" | 36°17'50.69", 32°15'16.27", 29°29'45.45" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10.13696 | | |
| 119. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.0188. \end{cases}$ $\begin{cases} x=4.1941, \\ y=3.2893, \\ z=3.9794. \end{cases}$ | 24°0'35.31", 24°27'0.33", 20°35'48.29" | 30°24'38.60", 34°22'51.05", 22°27'24.92" | 35°1'20.66", 31°37'17.14", 22°42'40.72" | 43°5'34.75", 43°16'43.97", 27°58'4.26" | 43°51'37.22", 44°12'36.86", 24°36'32.72" | 34°43'30.92", 35°49'48.26", 28°1'42.23" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 9.956056 | In (107), (119), (120) $I_1 = I_2 = 1$ $A = B_1$ $A_2 + J = 180^\circ$ $R_1 + R_2 = 180^\circ$ |
| 120. | $\begin{cases} a=5.6794, \\ b=5.6318, \\ c=4.0188. \end{cases}$ $\begin{cases} x=4.1941, \\ y=3.9794, \\ z=4.2893. \end{cases}$ | 24°0'35.31", 22°14'22.20", 22°17'0.28" | 30°24'38.60", 33°35'50.81", 22°31'9.76" | 34°0'8.91", 27°58'4.26", 22°42'40.72" | 45°14'28.04", 41°16'43.97", 31°37'17.14" | 46°8'22.78", 42°57'0.90", 24°59'52.83" | 35°28'14.10", 31°29'26.40", 31°34'42.73" | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 9.956056 | |

EXAMPLES. A.—Continued.

| | DATA. | $\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ | $\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ | $\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ | $\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | $\frac{1}{2}V, \frac{1}{2}Y, \frac{1}{2}Z$ | VOLUME. | REMARKS. |
|------|--|--|--|--|--|---|--|----------|--|
| 131. | $\begin{cases} a=5.6794, & r=5.6318 \\ b=4.0188, & r=4.2893 \\ c=4.9794, & r=4.1941 \end{cases}$ | $\begin{cases} 44^\circ 45' 31.95'' \\ 23^\circ 26' 3.70'' \\ 23^\circ 46' 13.46'' \end{cases}$ | $\begin{cases} 41^\circ 35' 27.44'' \\ 22^\circ 19' 33.12'' \\ 22^\circ 50' 32.39'' \end{cases}$ | $\begin{cases} 43^\circ 44' 13.42'' \\ 24^\circ 34' 0.17'' \\ 23^\circ 50' 32.39'' \end{cases}$ | $\begin{cases} 43^\circ 20' 20.56'' \\ 23^\circ 45' 46.37'' \\ 23^\circ 50' 32.39'' \end{cases}$ | $\begin{cases} 70^\circ 48' 58.17'' \\ 13^\circ 10' 7.71'' \\ 13^\circ 12.00'' \end{cases}$ | $\begin{cases} 70^\circ 34' 55.59'' \\ 14^\circ 044.58'' \\ 13^\circ 41' 8.23'' \end{cases}$ | 5.247219 | |
| 132. | $\begin{cases} a=5.6794, & r=5.6318 \\ b=4.0188, & r=4.1941 \\ c=3.9794, & r=4.2893 \end{cases}$ | $\begin{cases} 44^\circ 45' 31.95'' \\ 23^\circ 46' 13.46'' \\ 24^\circ 27' 0.33'' \end{cases}$ | $\begin{cases} 41^\circ 35' 27.44'' \\ 22^\circ 27' 24.93'' \\ 22^\circ 13' 23.52'' \end{cases}$ | $\begin{cases} 43^\circ 54' 34.75'' \\ 23^\circ 40' 32.39'' \\ 22^\circ 28' 41.08'' \end{cases}$ | $\begin{cases} 43^\circ 59' 42.77'' \\ 23^\circ 45' 46.37'' \\ 21^\circ 34' 0.17'' \end{cases}$ | $\begin{cases} 70^\circ 50' 12.18'' \\ 13^\circ 8' 27.08'' \\ 13^\circ 1' 29.00'' \end{cases}$ | $\begin{cases} 70^\circ 41' 35.48'' \\ 13^\circ 40' 48.36'' \\ 13^\circ 59' 35.62'' \end{cases}$ | 5.242459 | The tetrahedra from (129) to (134) are corrected for small angles by taking the average. |
| 133. | $\begin{cases} a=5.6794, & r=5.6318 \\ b=4.0188, & r=4.2893 \\ c=4.1941, & r=4.2893 \end{cases}$ | $\begin{cases} 43^\circ 16' 43.97'' \\ 22^\circ 13' 53.52'' \\ 24^\circ 27' 0.33'' \end{cases}$ | $\begin{cases} 42^\circ 53' 10.41'' \\ 22^\circ 27' 24.92'' \\ 23^\circ 46' 53.71'' \end{cases}$ | $\begin{cases} 43^\circ 54' 34.75'' \\ 22^\circ 24' 9.2'' \\ 24^\circ 0' 35.31'' \end{cases}$ | $\begin{cases} 43^\circ 59' 42.77'' \\ 22^\circ 42' 4.72'' \\ 24^\circ 42' 44.07'' \end{cases}$ | $\begin{cases} 70^\circ 35' 5.64'' \\ 13^\circ 16' 50.86'' \\ 13^\circ 53' 27.93'' \end{cases}$ | $\begin{cases} 70^\circ 25' 44.83'' \\ 13^\circ 8' 31.16'' \\ 14^\circ 13' 47.61'' \end{cases}$ | 5.314359 | |
| 134. | $\begin{cases} a=5.6794, & r=5.6318 \\ b=4.0188, & r=4.2893 \\ c=3.9794, & r=4.2893 \end{cases}$ | $\begin{cases} 43^\circ 16' 43.97'' \\ 22^\circ 14' 22.20'' \\ 24^\circ 20' 44.08'' \end{cases}$ | $\begin{cases} 42^\circ 53' 10.41'' \\ 22^\circ 31' 9.76'' \\ 23^\circ 37' 55.51'' \end{cases}$ | $\begin{cases} 45^\circ 14' 28.04'' \\ 24^\circ 42' 44.07'' \\ 24^\circ 0' 35.31'' \end{cases}$ | $\begin{cases} 42^\circ 1' 20.41'' \\ 22^\circ 42' 4.72'' \\ 22^\circ 24' 5.52'' \end{cases}$ | $\begin{cases} 70^\circ 52' 51.89'' \\ 13^\circ 9' 21.25'' \\ 13^\circ 41' 49.68'' \end{cases}$ | $\begin{cases} 70^\circ 40' 7.64'' \\ 14^\circ 1' 10.07'' \\ 13^\circ 0' 15.65'' \end{cases}$ | 5.245816 | |
| 135. | $\begin{cases} a=3.4567, & r=4.5678 \\ b=3.004518, & r=4.115618 \\ c=2.8993, & r=4.0104 \end{cases}$ | $\begin{cases} 50^\circ 46' 48.45'' \\ 40^\circ 5' 35.35'' \\ 38^\circ 11' 17.29'' \end{cases}$ | $\begin{cases} 34^\circ 11' 43.79'' \\ 23^\circ 21' 40.55'' \\ 21^\circ 58' 47.78'' \end{cases}$ | $\begin{cases} 38^\circ 25' 53.16'' \\ 28^\circ 26' 53.16'' \\ 19^\circ 14' 30.42'' \end{cases}$ | $\begin{cases} 27^\circ 35' 36.87'' \\ 20^\circ 4' 41.13'' \\ 27^\circ 21' 23.05'' \end{cases}$ | $\begin{cases} 52^\circ 10' 5.38'' \\ 24^\circ 40' 38.14'' \\ 23^\circ 10' 1.34'' \end{cases}$ | $\begin{cases} 38^\circ 25' 21.76'' \\ 38^\circ 25' 21.76'' \\ 36^\circ 54' 44.96'' \end{cases}$ | 4.656314 | $V = A^2 Y - B^2 Z - C^2$ $a_3 = \frac{1}{2} b_3; A = Y$ |
| 136. | $\begin{cases} a=3.468882, & r=2.879982 \\ b=3.243232, & r=3.105632 \\ c=3.932198, & r=3.363298 \end{cases}$ | $\begin{cases} 27^\circ 56' 38.41'' \\ 24^\circ 28' 23.73'' \\ 30^\circ 0' 55.38'' \end{cases}$ | $\begin{cases} 26^\circ 20' 40.57'' \\ 28^\circ 19' 37.53'' \\ 36^\circ 50' 5.40'' \end{cases}$ | $\begin{cases} 31^\circ 39' 27.09'' \\ 29^\circ 31' 42.08'' \\ 40^\circ 445.11'' \end{cases}$ | $\begin{cases} 28^\circ 41' 30.81'' \\ 26^\circ 58' 36.18'' \\ 34^\circ 7' 37.35'' \end{cases}$ | $\begin{cases} 32^\circ 20' 52.34'' \\ 30^\circ 6' 10.24'' \\ 47^\circ 14' 42.10'' \end{cases}$ | $\begin{cases} 27^\circ 51' 28.14'' \\ 30^\circ 6' 10.24'' \\ 42^\circ 45' 17.90'' \end{cases}$ | 4 | $V = A^2 Y - B^2 Z - C^2$ $a_3 = \frac{1}{2} b_3; A = Y$ $A + Y = B + Z; A - Y = C - Z; B = Y; C + Z = 160; Y$ a max. where $a + x = b + y = 6.348664$ $a - x = c - z = 0.3889$ |
| 137. | $\begin{cases} a=4.9916, & r=4.8206 \\ b=4.8548, & r=5.0258 \\ c=3.5152, & r=3.4476 \end{cases}$ | $\begin{cases} 34^\circ 5' 19.96'' \\ 34^\circ 59' 23.90'' \\ 20^\circ 28' 31.49'' \end{cases}$ | $\begin{cases} 33^\circ 10' 13.54'' \\ 33^\circ 41' 39.78'' \\ 20^\circ 32' 31.41'' \end{cases}$ | $\begin{cases} 35^\circ 49' 28.75'' \\ 36^\circ 22' 1.66'' \\ 21^\circ 18' 10.40'' \end{cases}$ | $\begin{cases} 34^\circ 28' 4.69'' \\ 34^\circ 56' 20.61'' \\ 20^\circ 27' 44.80'' \end{cases}$ | $\begin{cases} 40^\circ 42' 46.83'' \\ 30^\circ 42' 16.83'' \\ 41^\circ 5' 6.28'' \end{cases}$ | $\begin{cases} 42^\circ 5' 36.28'' \\ 22^\circ 25' 29.34'' \\ 21^\circ 58' 44.36'' \end{cases}$ | 8.593858 | |
| 138. | $\begin{cases} a=4.9916, & r=4.8206 \\ b=4.8548, & r=5.0258 \\ c=3.5152, & r=3.4476 \end{cases}$ | $\begin{cases} 31^\circ 5' 19.96'' \\ 21^\circ 49' 53.13'' \\ 30^\circ 40' 56.98'' \end{cases}$ | $\begin{cases} 33^\circ 10' 13.54'' \\ 28^\circ 59' 17.05'' \\ 22^\circ 22' 20.79'' \end{cases}$ | $\begin{cases} 30^\circ 45' 97.89'' \\ 34^\circ 30' 29.63'' \\ 21^\circ 18' 10.40'' \end{cases}$ | $\begin{cases} 45^\circ 47' 46.08'' \\ 34^\circ 30' 29.63'' \\ 36^\circ 22' 1.66'' \end{cases}$ | $\begin{cases} 40^\circ 17' 13.17'' \\ 33^\circ 49' 18.60'' \\ 25^\circ 4' 56.90'' \end{cases}$ | $\begin{cases} 43^\circ 38' 47.64'' \\ 23^\circ 59' 48.63'' \\ 35^\circ 25' 19.94'' \end{cases}$ | 8.593857 | |
| 139. | $\begin{cases} a=4.9916, & r=4.8206 \\ b=4.8548, & r=5.0258 \\ c=3.5152, & r=3.4476 \end{cases}$ | $\begin{cases} 22^\circ 35' 43.5'' \\ 31^\circ 59' 23.90'' \\ 29^\circ 18' 11.27'' \end{cases}$ | $\begin{cases} 20^\circ 27' 44.80'' \\ 29^\circ 38' 21.89'' \\ 20^\circ 52' 31.41'' \end{cases}$ | $\begin{cases} 31^\circ 3' 26.84'' \\ 22^\circ 11' 6.66'' \\ 33^\circ 10' 13.54'' \end{cases}$ | $\begin{cases} 34^\circ 28' 4.69'' \\ 41^\circ 12' 13.03'' \\ 33^\circ 10' 13.54'' \end{cases}$ | $\begin{cases} 33^\circ 49' 18.60'' \\ 46^\circ 9' 39.66'' \\ 24^\circ 36' 51.15'' \end{cases}$ | $\begin{cases} 21^\circ 30' 30.29'' \\ 46^\circ 9' 39.66'' \\ 32^\circ 35' 55.73'' \end{cases}$ | 8.593857 | See next page. |
| 140. | $\begin{cases} a=4.9916, & r=4.8206 \\ b=4.8548, & r=5.0258 \\ c=3.4476, & r=3.3132 \end{cases}$ | $\begin{cases} 34^\circ 17' 51.19'' \\ 35^\circ 12' 7.16'' \\ 20^\circ 54' 6.54'' \end{cases}$ | $\begin{cases} 32^\circ 59' 41.69'' \\ 33^\circ 30' 56.45'' \\ 20^\circ 7' 46.86'' \end{cases}$ | $\begin{cases} 35^\circ 57' 27.01'' \\ 36^\circ 7' 16.98'' \\ 20^\circ 53' 25.74'' \end{cases}$ | $\begin{cases} 34^\circ 40' 5.08'' \\ 34^\circ 49' 43.07'' \\ 20^\circ 53' 1.33'' \end{cases}$ | $\begin{cases} 40^\circ 17' 27.16'' \\ 41^\circ 6' 24.24'' \\ 22^\circ 2' 2.18'' \end{cases}$ | $\begin{cases} 39^\circ 43' 34.02'' \\ 42^\circ 4' 16.48'' \\ 22^\circ 22' 8.55'' \end{cases}$ | 8.596395 | |

EXAMPLES. X.—Continued.

| DATA. | $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ | $\beta_7, \beta_8, \beta_9, \beta_{10}, \beta_{11}, \beta_{12}$ | $\beta_{13}, \beta_{14}, \beta_{15}, \beta_{16}, \beta_{17}, \beta_{18}, \beta_{19}, \beta_{20}$ | $\beta_{21}, \beta_{22}, \beta_{23}, \beta_{24}, \beta_{25}, \beta_{26}$ | $\beta_{27}, \beta_{28}, \beta_{29}, \beta_{30}, \beta_{31}, \beta_{32}$ | $\beta_{33}, \beta_{34}, \beta_{35}, \beta_{36}, \beta_{37}, \beta_{38}, \beta_{39}, \beta_{40}$ | $\beta_{41}, \beta_{42}, \beta_{43}, \beta_{44}, \beta_{45}, \beta_{46}$ | VOLUME. | REMARKS. |
|--|--|--|--|--|--|--|--|----------|---|
| 151. $\left\{ \begin{array}{l} a=4.8370 \\ b=8.5602, \\ c=8.0746, \end{array} \right.$ | $16^{\circ}40'25.87",$ $25^{\circ}8'46.85",$ $19^{\circ}57'47.85",$ | $19^{\circ}54'24.82",$ $31^{\circ}18'25.89",$ $27^{\circ}48'15.68",$ | $38^{\circ}43'46.26",$ $42^{\circ}35'53.04",$ $33^{\circ}49'51.15",$ | $37^{\circ}2'57.47",$ $39^{\circ}20'42.98",$ $27^{\circ}29'42.14",$ | $39^{\circ}12'3.07",$ $43^{\circ}35'2.42",$ $34^{\circ}44.147",$ | $23^{\circ}2'11.94",$ $44^{\circ}57'41.02",$ $28^{\circ}16'8.41",$ | | 38.5523 | |
| 152. $\left\{ \begin{array}{l} a=4.8370 \\ b=8.5602, \\ c=8.0746, \end{array} \right.$ | $16^{\circ}40'25.87",$ $20^{\circ}10'30.92",$ $24^{\circ}37'29.66",$ | $19^{\circ}54'24.82",$ $29^{\circ}44'25.39",$ $29^{\circ}10'18",$ | $35^{\circ}38'4.95",$ $37^{\circ}29'42.14",$ $33^{\circ}49'51.15",$ | $40^{\circ}42'18.90",$ $39^{\circ}20'42.98",$ $45^{\circ}35'53.04",$ | $41^{\circ}5'26.07",$ $39^{\circ}37'49.63",$ $37^{\circ}41'16.24",$ | $27^{\circ}27'13.26",$ $29^{\circ}28'49.69",$ $43^{\circ}23'24.62",$ | | 38.90001 | Set of thirty tetrahedra having all the same six lengths for their edges. (151) to (156) is Group I, in which 9.4106, 4.8370 are opposite. (157) to (162) is Group II, in which 9.4106, 6.1874 are opposite. (163) to (168) is Group III, in which 9.4106, 7.5278 are opposite. (169) to (174) is Group IV, in which 9.4106, 8.0746 are opposite; and lastly, Group V, in which the two longest edges 9.4106, 8.5602 are opposite. The average volume is considerably less in this group; indeed, 5 out of the 6 least are in this group. |
| 153. $\left\{ \begin{array}{l} a=4.8370 \\ b=8.5602, \\ c=7.5278, \end{array} \right.$ | $17^{\circ}7'4.08",$ $27^{\circ}47'15.68",$ $19^{\circ}57'47.85",$ | $18^{\circ}21'48.60",$ $31^{\circ}18'25.89",$ $25^{\circ}8'40.85",$ | $35^{\circ}43'46.26",$ $40^{\circ}41'6.12",$ $30^{\circ}33'22.03",$ | $37^{\circ}2'57.47",$ $44^{\circ}19'33.89",$ $24^{\circ}5'2.8",$ | $38^{\circ}59'33.15",$ $43^{\circ}14'57.84",$ $29^{\circ}42'21.74",$ | $21^{\circ}0'2.88",$ $50^{\circ}31'14.11",$ $24^{\circ}52'30.62",$ | | 35.81829 | |
| 154. $\left\{ \begin{array}{l} a=4.8370 \\ b=8.5602, \\ c=8.0746, \end{array} \right.$ | $17^{\circ}7'4.08",$ $20^{\circ}29'44.11",$ $26^{\circ}34'18.74",$ | $18^{\circ}21'48.60",$ $29^{\circ}0'41.80",$ $26^{\circ}28'18.40",$ | $34^{\circ}24'59.46",$ $24^{\circ}57'5.8",$ $30^{\circ}33'22.03",$ | $43^{\circ}1'57.49",$ $42^{\circ}19'33.89",$ $46^{\circ}41'6.12",$ | $41^{\circ}43'50.80",$ $38^{\circ}15'37.49",$ $24^{\circ}32'17.15",$ | $22^{\circ}16'40.71",$ $27^{\circ}26'35.89",$ $46^{\circ}52'52.44",$ | | 37.21727 | |
| 155. $\left\{ \begin{array}{l} a=4.8370 \\ b=8.5602, \\ c=8.0746, \end{array} \right.$ | $16^{\circ}49'58.84",$ $26^{\circ}28'18.40",$ $26^{\circ}34'18.74",$ | $17^{\circ}57'6.85",$ $29^{\circ}0'41.80",$ $20^{\circ}29'44.11",$ | $34^{\circ}24'59.46",$ $34^{\circ}57'14.05",$ $22^{\circ}34'53.39",$ | $43^{\circ}1'57.49",$ $50^{\circ}35'7.77",$ $39^{\circ}6'39.10",$ | $41^{\circ}43'50.80",$ $51^{\circ}8'26.42",$ $24^{\circ}32'17.15",$ | $21^{\circ}14'48.00",$ $38^{\circ}15'37.49",$ $38^{\circ}34'57.26",$ | | 35.83271 | |
| 156. $\left\{ \begin{array}{l} a=4.8370 \\ b=8.5602, \\ c=7.5278, \end{array} \right.$ | $16^{\circ}49'58.84",$ $29^{\circ}1'10.18",$ $24^{\circ}37'29.66",$ | $17^{\circ}57'6.85",$ $29^{\circ}44'25.39",$ $20^{\circ}16'30.92",$ | $35^{\circ}38'4.95",$ $39^{\circ}6'39.10",$ $22^{\circ}34'53.39",$ | $40^{\circ}42'18.90",$ $50^{\circ}35'7.77",$ $33^{\circ}36'14.05",$ | $37^{\circ}2'57.47",$ $40^{\circ}42'40.59",$ $23^{\circ}11'29.52",$ | $20^{\circ}23'8.24",$ $45^{\circ}57'4.25",$ $31^{\circ}35'24.39",$ | | 34.86761 | |
| 157. $\left\{ \begin{array}{l} a=6.1874 \\ b=8.5602, \\ c=8.0746, \end{array} \right.$ | $21^{\circ}46'30.73",$ $25^{\circ}8'16.85",$ $15^{\circ}22'57.43",$ | $27^{\circ}29'42.14",$ $32^{\circ}25'44.16",$ $27^{\circ}48'15.68",$ | $42^{\circ}11'18.41",$ $42^{\circ}35'53.04",$ $32^{\circ}1'21.46",$ | $37^{\circ}2'57.47",$ $36^{\circ}12'7.81",$ $19^{\circ}54'24.82",$ | $43^{\circ}2'14.82",$ $40^{\circ}42'40.59",$ $32^{\circ}10'9.35",$ | $31^{\circ}43'55.28",$ $43^{\circ}4'28.89",$ $20^{\circ}48'57.02",$ | | 38.16771 | |
| 158. $\left\{ \begin{array}{l} a=6.1874 \\ b=8.5602, \\ c=7.5278, \end{array} \right.$ | $21^{\circ}46'30.73",$ $15^{\circ}24'54.53",$ $24^{\circ}37'29.66",$ | $27^{\circ}29'42.14",$ $29^{\circ}44'25.39",$ $29^{\circ}32'53.45",$ | $35^{\circ}38'4.95",$ $35^{\circ}44'58.47",$ $32^{\circ}1'21.46",$ | $44^{\circ}59'12.02",$ $36^{\circ}12'7.81",$ $42^{\circ}35'53.04",$ | $41^{\circ}43'50.80",$ $36^{\circ}10'16.71",$ $35^{\circ}49'12.37",$ | $32^{\circ}28'40.22",$ $21^{\circ}15'38.60",$ $42^{\circ}29'2.81",$ | | 38.24435 | |
| 159. $\left\{ \begin{array}{l} a=6.1874 \\ b=8.5602, \\ c=4.8370, \end{array} \right.$ | $22^{\circ}34'53.39",$ $29^{\circ}32'53.45",$ $24^{\circ}37'29.66",$ | $23^{\circ}16'55.53",$ $29^{\circ}44'25.39",$ $15^{\circ}27'54.53",$ | $35^{\circ}38'4.95",$ $35^{\circ}44'58.47",$ $32^{\circ}1'21.46",$ | $44^{\circ}59'12.02",$ $50^{\circ}35'7.77",$ $31^{\circ}1'51.03",$ | $41^{\circ}43'50.80",$ $51^{\circ}20'27.93",$ $17^{\circ}47'57.02",$ | $27^{\circ}39'39.12",$ $42^{\circ}9'3.20",$ $30^{\circ}44'1.05",$ | | 34.71445 | |
| 160. $\left\{ \begin{array}{l} a=6.1874 \\ b=8.5602, \\ c=4.8370, \end{array} \right.$ | $22^{\circ}34'53.39",$ $26^{\circ}18'8.88",$ $26^{\circ}34'18.74",$ | $23^{\circ}16'55.53",$ $29^{\circ}0'41.80",$ $15^{\circ}20'53.15",$ | $34^{\circ}24'59.46",$ $31^{\circ}1'51.03",$ $16^{\circ}49'58.84",$ | $48^{\circ}20'58.04",$ $50^{\circ}35'7.77",$ $35^{\circ}41'13.44",$ | $46^{\circ}41'19.92",$ $49^{\circ}47'7.25",$ $18^{\circ}12'0.79",$ | $28^{\circ}47'20.53",$ $35^{\circ}31'2.79",$ $36^{\circ}7'58.78",$ | | 34.84775 | |

EXAMPLES. X.—Continued.

| DATA | $\log a$ | $\log b$ | $\log c$ | $\log d$ | $\log e$ | $\log f$ | $\log g$ | $\log h$ | $\log i$ | $\log j$ | $\log k$ | $\log l$ | $\log m$ | $\log n$ | $\log o$ | $\log p$ | $\log q$ | $\log r$ | $\log s$ | $\log t$ | $\log u$ | $\log v$ | $\log w$ | $\log x$ | $\log y$ | $\log z$ | VOLUME. | REMARKS. |
|---|----------|----------|----------|----------|----------|-----------|-----------|-----------|-----------|----------|----------|---|---|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|---------|----------|
| 181. $\begin{cases} a=9.4826, & x=7.5998 \\ b=8.3612, & y=8.0736 \\ c=6.1874, & z=4.8370 \end{cases}$ | 29 44 | 24 28 | 33 22 | 42 39 | 19 01 | 41° | 6' 12.48" | 48° | 3' 55.23" | 41° 57' | 5.90" | 35.69917 | In these three tetra hedra $a-x=1.88-8$ $b+y=16.6348$ $c=618.74, z=4.8370$ and (181) is the one of maximum volume under these condi- tions, having $A+\lambda$ $=186°, B-1=6°.$ | | | | | | | | | | | | | | | |
| 182. $\begin{cases} a=0.482093, & x=7.599293 \\ b=8.501929, & y=8.072871 \\ c=6.1874, & z=4.8370 \end{cases}$ | 28 45 | 30 32 | 32 59 | 38 43 | 17 00 | 38° | 0' 5.73" | 38° 25' | 18.84" | 38° 25' | 39.51" | 35.69918 | | | | | | | | | | | | | | | | |
| 183. $\begin{cases} a=9.482061, & x=7.599261 \\ b=8.501973, & y=8.072827 \\ c=6.1874, & z=4.8370 \end{cases}$ | 29 43 | 43 78 | 33 22 | 44 06 | 41° | 6' 13.45" | 48° | 3' 24.77" | 41° 56' | 38.85" | 35.69918 | | | | | | | | | | | | | | | | | |
| 184. $\begin{cases} a=5.239019, & x=5.239999 \\ b=3.806876, & y=3.900688 \\ c=3.506886, & z=3.442099 \end{cases}$ | 32 50 | 54 94 | 32 50 | 53 27 | 32 | 32 | 45.13" | 36° | 34 | 38.90" | 11.25615 | | | | | | | | | | | | | | | | | |
| 185. $\begin{cases} a=4.600696, & x=4.705 \\ b=3.763106, & y=4.681051 \\ c=3.591406, & z=4.616923 \end{cases}$ | 29 20 | 28 02 | 30 23 | 49 33 | 32 | 50 | 23.27 | 26° | 53 | 54.08" | 10.78158 | | | | | | | | | | | | | | | | | |
| 186. $\begin{cases} a=13.2480, & x=6.0361 \\ b=11.5857, & y=11.4048 \\ c=9.7425, & z=9.5616 \end{cases}$ | 15 45 | 11.24 | 16° | 2' 3.76" | 38° 34' | 4.24" | 36° 51' | 50.22" | 18° 55' | 26.67" | 90.22281 | A group of six tetra- hedra, all having one common pair of op- posite edges; and the other four edges having the same lengths in each, but differently disposed. The volumes are equal to two and two; and in any pair of equal volume, the sum of two corre- sponding dihedral angles is supplement- ary. Several rela- tions of the form $A+X=B+\lambda$ hold. | | | | | | | | | | | | | | | | |
| 187. $\begin{cases} a=13.2480, & x=9.5616 \\ b=11.5857, & y=11.4048 \\ c=9.7425, & z=6.0361 \end{cases}$ | 28 31 | 52.05 | 30 27 | 59.69 | 45 | 29 | 13.42 | 36° | 52 | 42.84" | 90.22281 | | | | | | | | | | | | | | | | | |
| 188. $\begin{cases} a=13.2480, & x=6.0361 \\ b=11.5857, & y=11.4048 \\ c=9.5616, & z=9.7425 \end{cases}$ | 15 43 | 58.45 | 16° | 2' 7.71" | 38° | 7' 40.21" | 38° | 54 | 50.80" | 48° 54' | 24.01" | 90.14519 | | | | | | | | | | | | | | | | |
| 189. $\begin{cases} a=13.2480, & x=9.7425 \\ b=11.5857, & y=11.4048 \\ c=9.5616, & z=6.0361 \end{cases}$ | 26 55 | 6.71 | 29 20 | 3.57 | 45 | 56 | 24.78 | 38° | 54 | 50.80" | 90.14519 | | | | | | | | | | | | | | | | | |
| 190. $\begin{cases} a=13.2480, & x=9.5616 \\ b=11.5857, & y=11.4048 \\ c=6.0361, & z=9.7425 \end{cases}$ | 27 45 | 2.00 | 29 20 | 3.57 | 45 | 56 | 24.78 | 38° | 54 | 50.80" | 89.43556 | | | | | | | | | | | | | | | | | |

EXAMPLES, X.—Continued.

| | DATA. | $\frac{1}{2}a$, $\frac{1}{2}b$, $\frac{1}{2}c$, $\frac{1}{2}d$, $\frac{1}{2}e$, $\frac{1}{2}f$, $\frac{1}{2}g$, $\frac{1}{2}h$, $\frac{1}{2}i$, $\frac{1}{2}j$, $\frac{1}{2}k$, $\frac{1}{2}l$, $\frac{1}{2}m$, $\frac{1}{2}n$, $\frac{1}{2}o$, $\frac{1}{2}p$, $\frac{1}{2}q$, $\frac{1}{2}r$, $\frac{1}{2}s$, $\frac{1}{2}t$, $\frac{1}{2}u$, $\frac{1}{2}v$, $\frac{1}{2}w$, $\frac{1}{2}x$, $\frac{1}{2}y$, $\frac{1}{2}z$ | VOLUME. | REMARKS. | |
|------|---|---|--|----------|--|
| 201. | $\left\{ \begin{array}{l} a=7, 39443, \\ b=6, 52081, \\ c=7, 39442, \end{array} \right.$ $\left\{ \begin{array}{l} x=6, 5208 \\ y=5, 51718 \\ z=4, 77357 \end{array} \right.$ | $27\ 43\ 47\ 22''$, $38\ 29\ 3\ 70''$, $40\ 5\ 2\ 96''$, $33\ 46\ 32\ 74''$, $22\ 20\ 54\ 85''$, $30\ 9\ 50\ 54''$, $28\ 46\ 3\ 07''$, $27\ 43\ 47\ 45''$, $19\ 45\ 0\ 50''$, $33\ 40\ 33\ 41''$, $34\ 32\ 25\ 33''$, $22\ 44\ 52\ 63''$ | $15\ 45\ 35\ 70''$, $43\ 41\ 54\ 57''$, $31\ 29\ 47\ 43''$, $29\ 26\ 6\ 21''$, $35\ 41\ 29\ 42''$, $25\ 14\ 24\ 22''$ | 26.65187 | Another system of six tetrahedra, in all of which $a = \sqrt{3}b$, $x = 4/3q$; the other four lengths of edges being $\sqrt{3} + 2q$, $\sqrt{3} - 2q$, $4\sqrt{3} + 2q$, $4\sqrt{3} - 2q$. |
| 202. | $\left\{ \begin{array}{l} a=7, 39443, \\ b=6, 52081, \\ c=5, 64718, \end{array} \right.$ $\left\{ \begin{array}{l} x=6, 5208 \\ y=4, 77357 \\ z=7, 39442 \end{array} \right.$ | $32\ 10\ 13\ 83''$, $30\ 9\ 50\ 49''$, $31\ 55\ 6\ 61''$, $15\ 0\ 0\ 00''$, $20\ 6\ 14\ 20\ 26''$, $26\ 9\ 47\ 06''$, $19\ 45\ 0\ 60''$, $32\ 10\ 14\ 15''$, $31\ 55\ 6\ 33''$, $24\ 53\ 45\ 80''$, $25\ 39\ 32\ 02''$, $40\ 5\ 2\ 91''$ | $47\ 23\ 15\ 74''$, $40\ 13\ 4\ 22''$, $31\ 57\ 54\ 41''$, $22\ 27\ 52\ 63''$, $30\ 2\ 22\ 52''$, $39\ 33\ 24\ 30''$ | 25.92999 | $\frac{1}{2}p = \sqrt{3} \cdot \frac{1}{2}q$, so that $\frac{1}{2}p + \frac{1}{2}q = 4\sqrt{3}$, $\frac{1}{2}p - \frac{1}{2}q = 2\sqrt{3}$. |
| 203. | $\left\{ \begin{array}{l} a=7, 39443, \\ b=6, 52081, \\ c=5, 64718, \end{array} \right.$ $\left\{ \begin{array}{l} x=6, 5208 \\ y=4, 77357 \\ z=7, 39442 \end{array} \right.$ | $32\ 10\ 13\ 83''$, $30\ 9\ 50\ 49''$, $40\ 5\ 2\ 06''$, $33\ 46\ 32\ 74''$, $33\ 46\ 32\ 41''$, $30\ 9\ 50\ 54''$, $40\ 5\ 2\ 91''$, $32\ 10\ 14\ 15''$, $19\ 45\ 0\ 50''$, $22\ 26\ 54\ 85''$, $25\ 39\ 32\ 02''$, $19\ 45\ 0\ 60''$ | $42\ 30\ 44\ 26''$, $38\ 11\ 36\ 44''$, $38\ 11\ 36\ 53''$, $42\ 36\ 44\ 17''$, $40\ 6\ 11\ 55''$, $21\ 39\ 3\ 89''$ | 25.92999 | $\frac{1}{2}p + \frac{1}{2}q = 4\sqrt{3}$, $\frac{1}{2}p - \frac{1}{2}q = 2\sqrt{3}$. |
| 204. | $\left\{ \begin{array}{l} a=3, 46771, \\ b=3, 43555, \\ c=4, 0137, \end{array} \right.$ $\left\{ \begin{array}{l} x=2, 8788 \\ y=3, 1110 \\ z=3, 4248 \end{array} \right.$ | $22\ 30\ 58\ 52''$, $25\ 59\ 54\ 52''$, $31\ 20\ 16\ 87''$, $28\ 16\ 41\ 44''$, $24\ 14\ 4\ 04''$, $27\ 50\ 41\ 20''$, $20\ 11\ 22\ 30''$, $26\ 28\ 0\ 00''$, $30\ 40\ 1\ 93''$, $37\ 29\ 14\ 52''$, $40\ 55\ 0\ 58''$, $34\ 48\ 43\ 18''$ | $31\ 25\ 42\ 06''$, $26\ 59\ 41\ 23''$, $29\ 20\ 37''$, $29\ 15\ 54\ 92''$, $48\ 44\ 42\ 06''$, $44\ 18\ 41\ 23''$ | 3.987851 | In all the four tetrahedra, (204) to (207), $\frac{1}{2}p + \frac{1}{2}q = 4\sqrt{3}$, $\frac{1}{2}p - \frac{1}{2}q = 2\sqrt{3}$. |
| 205. | $\left\{ \begin{array}{l} a=3, 46771, \\ b=3, 42421, \\ c=3, 9489, \end{array} \right.$ $\left\{ \begin{array}{l} x=2, 8788 \\ y=3, 1044 \\ z=3, 3600 \end{array} \right.$ | $22\ 57\ 9\ 14''$, $26\ 21\ 15\ 27''$, $31\ 20\ 11\ 03''$, $28\ 42\ 16\ 88''$, $24\ 28\ 50\ 39''$, $28\ 20\ 9\ 78''$, $29\ 32\ 28\ 59''$, $26\ 59\ 25\ 54''$, $29\ 59\ 39\ 19''$, $36\ 48\ 57\ 18''$, $40\ 3\ 25\ 02''$, $34\ 6\ 16\ 14''$ | $32\ 22\ 34\ 58''$, $27\ 52\ 36\ 49''$, $30\ 7\ 46\ 31''$, $30\ 7\ 46\ 31''$, $47\ 12\ 4\ 69''$, $42\ 42\ 26\ 60''$ | 3 995449 | $\frac{1}{2}p + \frac{1}{2}q = 4\sqrt{3}$, $\frac{1}{2}p - \frac{1}{2}q = 2\sqrt{3}$. |
| 206. | $\left\{ \begin{array}{l} a=3, 46771, \\ b=3, 24074, \\ c=3, 95078, \end{array} \right.$ $\left\{ \begin{array}{l} x=2, 8788 \\ y=3, 104426 \\ z=3, 36188 \end{array} \right.$ | $22\ 56\ 33\ 59''$, $26\ 20\ 40\ 36''$, $31\ 39\ 33\ 53''$, $28\ 41\ 31\ 85''$, $24\ 28\ 20\ 38''$, $28\ 19\ 39\ 91''$, $29\ 31\ 45\ 60''$, $20\ 58\ 37\ 30''$, $30\ 0\ 46\ 56''$, $36\ 50\ 5\ 77''$, $40\ 4\ 49\ 11''$, $34\ 7\ 34\ 04''$ | $32\ 20\ 58\ 08''$, $27\ 51\ 27\ 92''$, $30\ 6\ 13\ 00''$, $30\ 6\ 13\ 00''$, $47\ 14\ 40\ 77''$, $42\ 45\ 10\ 61''$ | 3.995455 | volume under these conditions. Observe that in passing from (206) to (207) $B - Y$ is unaltered by the changes in c , z . |
| 207. | $\left\{ \begin{array}{l} a=3, 46771, \\ b=3, 242074, \\ c=3, 95083, \end{array} \right.$ $\left\{ \begin{array}{l} x=2, 8788 \\ y=3, 104126 \\ z=3, 36193 \end{array} \right.$ | $22\ 56\ 32\ 66''$, $26\ 20\ 39\ 46''$, $31\ 39\ 32\ 53''$, $28\ 41\ 32\ 75''$, $24\ 28\ 20\ 38''$, $28\ 19\ 39\ 15''$, $29\ 31\ 44\ 44''$, $26\ 58\ 36\ 05''$, $30\ 0\ 48\ 32''$, $36\ 50\ 7\ 72''$, $40\ 4\ 51\ 29''$, $34\ 7\ 36\ 10''$ | $32\ 20\ 55\ 54''$, $27\ 51\ 25\ 56''$, $30\ 6\ 10\ 55''$, $30\ 6\ 10\ 55''$, $47\ 14\ 44\ 99''$, $42\ 45\ 15\ 01''$ | 3.995456 | $A - X = B - Y$, $C + Z = B + 1$, $B - X = A - C - Z$, $S_1 = S_2 = 120$, $S_2 = 164$ |
| 208. | $\left\{ \begin{array}{l} a=9, 807817, \\ b=8, 52081, \\ c=8, 431859, \end{array} \right.$ $\left\{ \begin{array}{l} x=5, 55317 \\ y=4, 7268 \\ z=5, 571241 \end{array} \right.$ | $18\ 45\ 16\ 12''$, $32\ 21\ 47\ 08''$, $40\ 33\ 56\ 83''$, $47\ 52\ 59\ 00''$, $14\ 19\ 35\ 39''$, $32\ 21\ 47\ 08''$, $25\ 7\ 55\ 31''$, $39\ 45\ 29\ 02''$, $17\ 4\ 16\ 09''$, $27\ 47\ 25\ 68''$, $31\ 29\ 13\ 96''$, $32\ 30\ 17\ 61''$ | $50\ 44\ 18\ 49''$, $37\ 47\ 45\ 05''$, $37\ 47\ 45\ 05''$, $24\ 51\ 11\ 61''$, $31\ 2\ 13\ 66''$, $31\ 36\ 43\ 00''$ | 31.10119 | $A + X = B + 1$, $A - X = C - Z$, $\frac{1}{2}p = \sqrt{3} \cdot \frac{1}{2}q$, so that $\frac{1}{2}p + \frac{1}{2}q = 4\sqrt{3}$, $\frac{1}{2}p - \frac{1}{2}q = 2\sqrt{3}$. |
| 209. | $\left\{ \begin{array}{l} a=5, 8311, \\ b=5, 29313, \\ c=6, 3758, \end{array} \right.$ $\left\{ \begin{array}{l} x=2, 9148 \\ y=3, 4838 \\ z=3, 3805 \end{array} \right.$ | $13\ 37\ 59\ 26''$, $24\ 59\ 0\ 56''$, $40\ 12\ 54\ 87''$, $32\ 30\ 51\ 93''$, $16\ 23\ 50\ 93''$, $31\ 43\ 53\ 01''$, $32\ 30\ 7\ 57''$, $42\ 57\ 07''$, $18\ 3\ 12\ 12''$, $41\ 5\ 17\ 14''$, $48\ 39\ 3\ 07''$, $32\ 30\ 17\ 61''$ | $37\ 55\ 8\ 72''$, $37\ 55\ 8\ 72''$, $37\ 55\ 8\ 72''$, $37\ 55\ 8\ 72''$, $51\ 22\ 41\ 57''$, $51\ 22\ 41\ 57''$ | 795052 | $A + X = B + 1$, $A - X = C - Z$, $\frac{1}{2}p = \sqrt{3} \cdot \frac{1}{2}q$, so that $\frac{1}{2}p + \frac{1}{2}q = 4\sqrt{3}$, $\frac{1}{2}p - \frac{1}{2}q = 2\sqrt{3}$. |
| 210. | $\left\{ \begin{array}{l} a=7, 2841, \\ b=6, 71393, \\ c=7, 860354, \end{array} \right.$ $\left\{ \begin{array}{l} x=2, 9448 \\ y=3, 49497 \\ z=3, 521254 \end{array} \right.$ | $10\ 40\ 36\ 21''$, $24\ 48\ 56\ 15''$, $42\ 9\ 50\ 67''$, $33\ 47\ 45\ 43''$, $13\ 9\ 58\ 42''$, $33\ 27\ 32\ 14''$, $32\ 21\ 36\ 40''$, $28\ 34\ 25\ 40''$, $14\ 22\ 37\ 19''$, $43\ 2\ 16\ 15''$, $50\ 38\ 58\ 39''$, $32\ 49\ 27\ 45''$ | $39\ 3\ 40\ 64''$, $23\ 42\ 54\ 06''$, $31\ 23\ 17\ 35''$, $31\ 23\ 17\ 35''$, $52\ 40\ 23\ 29''$, $37\ 19\ 36\ 71''$ | 10.09838 | $A + X = B + 1$, $C + Z = B + 1$, $S_1 = S_2 = 120$, $S_2 = 164$. |

EXAMPLES. X.—Continued.

| DATA. | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | $\beta_1, \beta_2, \beta_3$ | VOLUME. | REMARKS. |
|--|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|----------|---|
| 211. $\begin{cases} a=7.71042, \\ b=8.5445, \\ c=7.6147, \end{cases}$ $\begin{cases} x=5.618158, \\ y=6.6434, \\ z=5.7136 \end{cases}$ | $20^\circ 5' 13.98''$ | $26^\circ 43' 18.50''$ | $30^\circ 52' 38.84''$ | $32^\circ 31' 37.04''$ | $33^\circ 36' 14.42''$ | $33^\circ 36' 14.42''$ | $33^\circ 36' 14.42''$ | $33^\circ 36' 14.42''$ | $33^\circ 36' 14.42''$ | $33^\circ 36' 14.42''$ | 35.00578 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 212. $\begin{cases} a=9.31534, \\ b=8.9763, \\ c=8.01066, \end{cases}$ $\begin{cases} x=5.06584, \\ y=4.7268, \\ z=5.69244 \end{cases}$ | $17^\circ 3' 4.15''$ | $28^\circ 38' 34.36''$ | $30^\circ 34' 18.51''$ | $45^\circ 57' 16''$ | $46^\circ 14' 22.54''$ | $46^\circ 14' 22.54''$ | $46^\circ 14' 22.54''$ | $46^\circ 14' 22.54''$ | $46^\circ 14' 22.54''$ | $46^\circ 14' 22.54''$ | 30 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 213. $\begin{cases} a=9.8933, \\ b=8.9763, \\ c=8.0107, \end{cases}$ $\begin{cases} x=5.6138, \\ y=4.7268, \\ z=5.6924 \end{cases}$ | $19^\circ 8' 23.15''$ | $32^\circ 24' 40.59''$ | $40^\circ 45' 5.24''$ | $49^\circ 18' 53.88''$ | $49^\circ 18' 53.88''$ | $49^\circ 18' 53.88''$ | $49^\circ 18' 53.88''$ | $49^\circ 18' 53.88''$ | $49^\circ 18' 53.88''$ | $49^\circ 18' 53.88''$ | 31.06101 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 214. $\begin{cases} a=9.8671, \\ b=8.9763, \\ c=8.1807, \end{cases}$ $\begin{cases} x=5.6176, \\ y=4.7268, \\ z=5.5224 \end{cases}$ | $18^\circ 56' 0.90''$ | $32^\circ 58' 16.63''$ | $41^\circ 1' 31.00''$ | $48^\circ 0' 12.39''$ | $51^\circ 14' 52.86''$ | $51^\circ 14' 52.86''$ | $51^\circ 14' 52.86''$ | $51^\circ 14' 52.86''$ | $51^\circ 14' 52.86''$ | $51^\circ 14' 52.86''$ | 31.12113 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 215. $\begin{cases} a=9.86936, \\ b=8.9763, \\ c=8.18092, \end{cases}$ $\begin{cases} x=5.61986, \\ y=4.7268, \\ z=5.52218 \end{cases}$ | $18^\circ 56' 29.39''$ | $32^\circ 59' 15.00''$ | $41^\circ 2' 19.83''$ | $48^\circ 1' 7.06''$ | $51^\circ 16' 21.93''$ | $51^\circ 16' 21.93''$ | $51^\circ 16' 21.93''$ | $51^\circ 16' 21.93''$ | $51^\circ 16' 21.93''$ | $51^\circ 16' 21.93''$ | 31.12115 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 216. $\begin{cases} a=9.869433, \\ b=8.9763, \\ c=8.180921, \end{cases}$ $\begin{cases} x=5.619933, \\ y=4.7268, \\ z=5.522179 \end{cases}$ | $18^\circ 56' 30.38''$ | $32^\circ 59' 16.77''$ | $41^\circ 2' 21.34''$ | $48^\circ 1' 8.99''$ | $51^\circ 16' 24.88''$ | $51^\circ 16' 24.88''$ | $51^\circ 16' 24.88''$ | $51^\circ 16' 24.88''$ | $51^\circ 16' 24.88''$ | $51^\circ 16' 24.88''$ | 31.12115 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 217. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.4025, \end{cases}$ $\begin{cases} x=8.8901, \\ y=9.0797, \\ z=6.3040 \end{cases}$ | $34^\circ 20' 5.56''$ | $33^\circ 55' 43.65''$ | $35^\circ 16' 16.08''$ | $34^\circ 31' 35.95''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | 52.31824 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 218. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.4025, \end{cases}$ $\begin{cases} x=8.8901, \\ y=9.0797, \\ z=6.3040 \end{cases}$ | $34^\circ 20' 5.56''$ | $33^\circ 55' 43.65''$ | $35^\circ 16' 16.08''$ | $34^\circ 31' 35.95''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | $40^\circ 47' 59.34''$ | 52.31824 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 219. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.4025, \end{cases}$ $\begin{cases} x=8.8901, \\ y=9.0797, \\ z=6.3040 \end{cases}$ | $22^\circ 28' 34.72''$ | $20^\circ 31' 42.24''$ | $30^\circ 31' 46.02''$ | $34^\circ 31' 35.95''$ | $33^\circ 49' 28.48''$ | $33^\circ 49' 28.48''$ | $33^\circ 49' 28.48''$ | $33^\circ 49' 28.48''$ | $33^\circ 49' 28.48''$ | $33^\circ 49' 28.48''$ | 52.31824 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |
| 220. $\begin{cases} a=9.0481, \\ b=8.9217, \\ c=6.3040, \end{cases}$ $\begin{cases} x=8.8901, \\ y=9.0797, \\ z=6.4025 \end{cases}$ | $34^\circ 30' 7.54''$ | $33^\circ 46' 30.08''$ | $35^\circ 5' 38.49''$ | $34^\circ 31' 35.95''$ | $40^\circ 47' 27.35''$ | $40^\circ 47' 27.35''$ | $40^\circ 47' 27.35''$ | $40^\circ 47' 27.35''$ | $40^\circ 47' 27.35''$ | $40^\circ 47' 27.35''$ | 52.31891 | $\begin{cases} A+B+C=Z, \\ B+C=Z, \\ \sigma_2=97' 158.77'' \end{cases}$ |

See next page.

$$\begin{aligned} V &= V_1 = V_2 = V_3 \\ A_1 + A_2 &= 180^\circ \\ B_1 + B_2 &= 180^\circ \\ B_3 - A_3 &= 0 \end{aligned}$$

EXAMPLES. X.—Continued.

| DATA. | \log_2 | \log_3 | \log_4 | \log_5 | \log_6 | \log_7 | \log_8 | \log_9 | \log_{10} | \log_{11} | \log_{12} | \log_{13} | \log_{14} | \log_{15} | \log_{16} | \log_{17} | \log_{18} | \log_{19} | \log_{20} | VOLUME. | REMARKS. |
|--|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|----------|--|
| 221. $\begin{cases} x=8.8901 \\ y=6.4205 \\ z=8.9217, \\ =6.3040, \\ =9.0797 \end{cases}$ | $34^{\circ}30'7.54''$ | $33^{\circ}46'30.08''$ | $30^{\circ}10'14.03''$ | $45^{\circ}24'11.02''$ | $49^{\circ}12'32.65''$ | $44^{\circ}25'26.33''$ | $34^{\circ}40'14.55''$ | $33^{\circ}58'48.20''$ | $24^{\circ}37'51.40''$ | $24^{\circ}27'9.81''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | 52.31891 | $V_1=V_2=V_3$ $A_1+B_1=180^{\circ}$ $B_2+A_2=180^{\circ}$ $B_3-A_3=0$ |
| 222. $\begin{cases} x=6.4025 \\ y=9.0797 \\ z=8.8901 \end{cases}$ | $22^{\circ}55'36.30''$ | $20^{\circ}51'51.46''$ | $30^{\circ}31'46.02''$ | $34^{\circ}41'25.81''$ | $33^{\circ}58'48.29''$ | $24^{\circ}54'9.05''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | $34^{\circ}31'16.72''$ | 52.31891 | $V_1=V_2=V_3$ $A_1+B_1=180^{\circ}$ $B_2+A_2=180^{\circ}$ $B_3-A_3=0$ |
| 223. $\begin{cases} x=8.8901 \\ y=6.4025 \\ z=6.3040 \end{cases}$ | $29^{\circ}35'55.61''$ | $44^{\circ}23'49.76''$ | $35^{\circ}16'16.98''$ | $34^{\circ}31'35.95''$ | $45^{\circ}0'32.60''$ | $48^{\circ}32'28.71''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | 52.62805 | $V_1=V_2$ $A_1+A_2=180^{\circ}$ |
| 224. $\begin{cases} x=8.8901 \\ y=6.3040 \\ z=6.4025 \end{cases}$ | $29^{\circ}35'55.61''$ | $44^{\circ}23'49.76''$ | $35^{\circ}16'16.98''$ | $34^{\circ}31'35.95''$ | $45^{\circ}0'32.60''$ | $48^{\circ}32'28.71''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | $29^{\circ}35'55.61''$ | 52.62805 | $V_1=V_2$ $A_1+A_2=180^{\circ}$ |
| 225. $\begin{cases} x=6.4205 \\ y=9.0797 \\ z=6.3040 \end{cases}$ | $21^{\circ}3'58.89''$ | $22^{\circ}24'51.21''$ | $35^{\circ}16'16.98''$ | $30^{\circ}13'38.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $30^{\circ}13'38.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | 52.01483 | $V_1=V_2$ $B_1+B_2=180^{\circ}$ |
| 226. $\begin{cases} x=9.0481 \\ y=9.0797 \\ z=6.4205 \end{cases}$ | $20^{\circ}43'37.91''$ | $21^{\circ}58'45.79''$ | $35^{\circ}5'38.49''$ | $30^{\circ}13'38.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $20^{\circ}43'37.91''$ | $21^{\circ}58'45.79''$ | $35^{\circ}5'38.49''$ | $30^{\circ}13'38.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $20^{\circ}43'37.91''$ | $21^{\circ}58'45.79''$ | $35^{\circ}5'38.49''$ | $30^{\circ}13'38.96''$ | $34^{\circ}16'1.55''$ | $24^{\circ}39'22.96''$ | $20^{\circ}43'37.91''$ | 52.01483 | $V_1=V_2$ $B_1+B_2=180^{\circ}$ |
| 227. $\begin{cases} x=9.952135, \\ y=4.284763, \\ z=5.047310, \\ =8.365096, \\ =5.41454 \end{cases}$ | $16^{\circ}47'59.09''$ | $32^{\circ}53'44.95''$ | $41^{\circ}57'41.22''$ | $49^{\circ}17'31.84''$ | $51^{\circ}56'52.78''$ | $38^{\circ}3'7.22''$ | $16^{\circ}47'59.09''$ | $32^{\circ}53'44.95''$ | $41^{\circ}57'41.22''$ | $49^{\circ}17'31.84''$ | $51^{\circ}56'52.78''$ | $38^{\circ}3'7.22''$ | $16^{\circ}47'59.09''$ | $32^{\circ}53'44.95''$ | $41^{\circ}57'41.22''$ | $49^{\circ}17'31.84''$ | $51^{\circ}56'52.78''$ | $38^{\circ}3'7.22''$ | $16^{\circ}47'59.09''$ | 26.39783 | $A-X=B-1, A+X=180^{\circ}$ $B+Y=C+Z, C=Z, B=$ $X: 5$ equations, sufficient to determine shape. |
| 228. $\begin{cases} x=4.1451 \\ y=3.4069 \\ z=2.39966, \\ =5.41404 \end{cases}$ | $44^{\circ}40'58.35''$ | $24^{\circ}18'24.67''$ | $20^{\circ}56'40.82''$ | $28^{\circ}10'57.79''$ | $28^{\circ}25'5.02''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | $44^{\circ}40'58.35''$ | 4.925029 | $A+X=B+Y=C+Z$ $S_1=S_2$ |
| 229. $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=2.5582, \\ =5.41454 \end{cases}$ | $43^{\circ}25'51.50''$ | $24^{\circ}55'53.50''$ | $21^{\circ}58'10.24''$ | $28^{\circ}10'57.79''$ | $28^{\circ}25'5.02''$ | $44^{\circ}40'58.35''$ | $43^{\circ}25'51.50''$ | $24^{\circ}55'53.50''$ | $21^{\circ}58'10.24''$ | $28^{\circ}10'57.79''$ | $28^{\circ}25'5.02''$ | $44^{\circ}40'58.35''$ | $43^{\circ}25'51.50''$ | $24^{\circ}55'53.50''$ | $21^{\circ}58'10.24''$ | $28^{\circ}10'57.79''$ | $28^{\circ}25'5.02''$ | $44^{\circ}40'58.35''$ | $43^{\circ}25'51.50''$ | 5.28296 | $S_1-S_2=12^{\circ}54'88''$ |
| 230. $\begin{cases} x=3.6683, \\ y=4.4069, \\ z=5.5575, \\ =5.2592 \end{cases}$ | $43^{\circ}24'55.28''$ | $24^{\circ}55'51.64''$ | $21^{\circ}58'10.24''$ | $28^{\circ}10'57.79''$ | $28^{\circ}25'5.02''$ | $44^{\circ}40'58.35''$ | $43^{\circ}24'55.28''$ | $24^{\circ}55'51.64''$ | $21^{\circ}58'10.24''$ | $28^{\circ}10'57.79''$ | $28^{\circ}25'5.02''$ | $44^{\circ}40'58.35''$ | $43^{\circ}24'55.28''$ | $24^{\circ}55'51.64''$ | $21^{\circ}58'10.24''$ | $28^{\circ}10'57.79''$ | $28^{\circ}25'5.02''$ | $44^{\circ}40'58.35''$ | $43^{\circ}24'55.28''$ | 5.287298 | $S_1-S_2=12^{\circ}53'9''$ |

EXAMPLES. X.—Continued.

| | DATA. | $\frac{1}{2}a_1, \frac{1}{2}b_1, \frac{1}{2}c_1$ | $\frac{1}{2}a_2, \frac{1}{2}b_2, \frac{1}{2}c_2$ | $\frac{1}{2}a_3, \frac{1}{2}b_3, \frac{1}{2}c_3$ | $\frac{1}{2}a_4, \frac{1}{2}b_4, \frac{1}{2}c_4$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | VOLUME. | REMARKS. |
|------|--|---|--|--|--|--|----------|--|
| 231. | $\begin{cases} a=3.6683, \\ b=3.4069, \\ c=2.5535, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.2662 \end{cases}$ | $43^{\circ}26'47.72''$ $41^{\circ}8'53.76''$ $47^{\circ}59'33.55''$ | $24^{\circ}54'55.35''$ $20^{\circ}3'2.79''$ $17^{\circ}41'48.35''$ | $21^{\circ}57'23.69''$ $27^{\circ}34'28.59''$ $37^{\circ}55'21.53''$ | $28^{\circ}9'17.56''$ $28^{\circ}39'9.86''$ $24^{\circ}35'13.44''$ | $29^{\circ}17'2.61''$ $43^{\circ}20'32.46''$ $43^{\circ}58'45.21''$ | 5.278617 | $A+B+C=125^{\circ}57'0''$ $\sigma_1 - \sigma_2 = 125^{\circ}57'0''$ |
| 232. | $\begin{cases} a=3.6683, \\ b=3.4069, \\ c=3.5485, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.2632 \end{cases}$ | $43^{\circ}29'8.44''$ $44^{\circ}11'8.50''$ $48^{\circ}3'11.86''$ | $24^{\circ}53'44.91''$ $20^{\circ}1'20.91''$ $17^{\circ}39'27.76''$ | $21^{\circ}55'27.23''$ $27^{\circ}8'20.37''$ $37^{\circ}57'54.69''$ | $28^{\circ}9'23.14''$ $28^{\circ}37'47.10''$ $24^{\circ}32'42.89''$ | $43^{\circ}23'00.90''$ $44^{\circ}1'3.59''$ $48^{\circ}6'7.80''$ | 5.267735 | $\sigma_1 - \sigma_2 = 125^{\circ}56.54''$ being the max. value when a, b, x, y , and $c+z$ have the values here assigned them. |
| 233. | $\begin{cases} a=3.6683, \\ b=3.4069, \\ c=3.5481, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.2656 \end{cases}$ | $43^{\circ}29'19.73''$ $44^{\circ}11'19.30''$ $48^{\circ}3'29.36''$ | $24^{\circ}53'39.30''$ $20^{\circ}1'12.74''$ $17^{\circ}39'16.50''$ | $21^{\circ}55'17.90''$ $27^{\circ}8'13.74''$ $37^{\circ}58'6.04''$ | $28^{\circ}9'24.20''$ $28^{\circ}34'40.23''$ $24^{\circ}32'30.81''$ | $43^{\circ}23'32.83''$ $44^{\circ}1'14.68''$ $48^{\circ}6'24.33''$ | 5.266861 | $\sigma_1 - \sigma_2 = 125^{\circ}54.30.32''$ |
| 234. | $\begin{cases} a=3.6683, \\ b=3.4069, \\ c=2.0007, \end{cases}$ $\begin{cases} x=4.1454 \\ y=4.4068 \\ z=5.8130 \end{cases}$ | $48^{\circ}15'58.69''$ $48^{\circ}30'17.22''$ $55^{\circ}13'6.02''$ | $22^{\circ}39'30.65''$ $16^{\circ}39'21.14''$ $13^{\circ}22'11.05''$ | $18^{\circ}7'32.84''$ $27^{\circ}24'58.38''$ $41^{\circ}19'23.34''$ | $27^{\circ}47'31.73''$ $27^{\circ}21'37.97''$ $42^{\circ}47'30.97''$ | $50^{\circ}1'39.56''$ $50^{\circ}23'2.84''$ $55^{\circ}52'9.85''$ | 3.810566 | $\sigma_1 - \sigma_2 = 3^{\circ}54'30.32''$ |
| 235. | $\begin{cases} a=3.3457, \\ b=2.94202, \\ c=3.4569, \end{cases}$ $\begin{cases} x=4.4568 \\ y=4.85958 \\ z=4.3456 \end{cases}$ | $43^{\circ}36'48.39''$ $45^{\circ}35'3.04''$ $43^{\circ}35'16.57''$ | $28^{\circ}47'43.41''$ $21^{\circ}16'52.73''$ $22^{\circ}40'0.20''$ | $35^{\circ}7'50.70''$ $33^{\circ}30'9.13''$ $25^{\circ}24'28.98''$ | $21^{\circ}41'56.80''$ $20^{\circ}38'42.63''$ $27^{\circ}42'7.46''$ | $28^{\circ}18'54.85''$ $45^{\circ}38'14.18''$ $28^{\circ}39'59.43''$ | 5.660726 | $\sigma_1 = \sigma_2$ |
| 236. | $\begin{cases} a=3.72374, \\ b=5.19556, \\ c=4.87204, \end{cases}$ $\begin{cases} x=3.672916 \\ y=4.87204 \\ z=3.44460 \end{cases}$ | $22^{\circ}6'11.54''$ $39^{\circ}0'26.71''$ $20^{\circ}44'0.26''$ | $24^{\circ}23'47.02''$ $46^{\circ}24'36.89''$ $29^{\circ}20'57.66''$ | $22^{\circ}51'22.87''$ $45^{\circ}8'57.53''$ $37^{\circ}42.41''$ | $23^{\circ}38'35.69''$ $40^{\circ}35'0.05''$ $22^{\circ}26'15.45''$ | $18^{\circ}21'7.27''$ $62^{\circ}16'7.63''$ $17^{\circ}17'6.82''$ | 5.310628 | |
| 237. | $\begin{cases} a=3.72374, \\ b=5.19556, \\ c=4.87204, \end{cases}$ $\begin{cases} x=3.55213 \\ y=4.33034 \\ z=3.67424 \end{cases}$ | $20^{\circ}34'54.15''$ $29^{\circ}20'57.66''$ $22^{\circ}30'2.36''$ | $25^{\circ}57'2.89''$ $44^{\circ}36'36.29''$ $37^{\circ}0'26.71''$ | $22^{\circ}53'21.35''$ $36^{\circ}48'16.75''$ $32^{\circ}15'48.71''$ | $23^{\circ}38'55.69''$ $37^{\circ}9'17.14''$ $27^{\circ}44'40.36''$ | $24^{\circ}9'13.62''$ $50^{\circ}37'13.20''$ $35^{\circ}4'58.08''$ | 7.30914 | |
| 238. | $\begin{cases} a=7.3782, \\ b=9.2160, \\ c=8.1664, \end{cases}$ $\begin{cases} x=6.5108 \\ y=5.4688 \\ z=5.7472 \end{cases}$ | $21^{\circ}50'40.74''$ $20^{\circ}24'3.68''$ $19^{\circ}16'47.88''$ | $35^{\circ}38'56.37''$ $44^{\circ}8'38.64''$ $38^{\circ}40'52.69''$ | $26^{\circ}34'33.48''$ $26^{\circ}11'4.04''$ $29^{\circ}47'40.98''$ | $30^{\circ}55'3.53''$ $38^{\circ}21'38.28''$ $28^{\circ}9'59.59''$ | $32^{\circ}40'30.24''$ $46^{\circ}14'55.80''$ $38^{\circ}37'57.94''$ | 35.68824 | See next page. |
| 239. | $\begin{cases} a=7.3782, \\ b=8.0000, \\ c=4.4722, \end{cases}$ $\begin{cases} x=7.73156 \\ y=6.9282 \\ z=6.0000 \end{cases}$ | $35^{\circ}6'22.85''$ $32^{\circ}15'35.19''$ $22^{\circ}38'1.11''$ | $36^{\circ}31'49.43''$ $35^{\circ}38'55.72''$ $17^{\circ}49'15.76''$ | $31^{\circ}43'3.17''$ $29^{\circ}30'9.76''$ $16^{\circ}29'16.00''$ | $39^{\circ}55'9.11''$ $38^{\circ}24'21.15''$ $23^{\circ}58'0.81''$ | $44^{\circ}3'56.80''$ $40^{\circ}40'23.11''$ $48^{\circ}43'47.09''$ | 28.84288 | |
| 240. | $\begin{cases} a=4.2893, \\ b=5.6794, \\ c=5.6318, \end{cases}$ $\begin{cases} x=3.931876 \\ y=3.9794 \\ z=4.1911 \end{cases}$ | $20^{\circ}20'23.19''$ $22^{\circ}24'5.52''$ $23^{\circ}37'55.51''$ | $28^{\circ}43'4.96''$ $42^{\circ}1'20.41''$ $42^{\circ}53'10.41''$ | $24^{\circ}20'44.08''$ $29^{\circ}16'6.80''$ $34^{\circ}30'17.68''$ | $24^{\circ}42'44.07''$ $35^{\circ}9'10.13''$ $32^{\circ}0'48.24''$ | $26^{\circ}46'28.99''$ $42^{\circ}48'50.12''$ $44^{\circ}23'20.75''$ | 9.999143 | |

EXAMPLES. X.—Continued.

| DATA. | $\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | $\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | $\frac{1}{2}a, \frac{1}{2}b, \frac{1}{2}c$ | $\frac{1}{2}A, \frac{1}{2}B, \frac{1}{2}C$ | VOLUME. | REMARKS. |
|---|--|--|--|--|--|--|---|----------|
| 241. $\begin{cases} a=4.8625, & x=4.826543 \\ b=4.9233, & y=4.7320 \\ c=5.5121, & z=4.1432 \end{cases}$ | $27^{\circ}22'25.81''$ $26^{\circ}54'57.65''$ $25^{\circ}2'46.92''$ | $32^{\circ}44'54.89''$ $32^{\circ}51'27.70''$ $35^{\circ}32'7.03''$ | $32^{\circ}5'43.38''$ $31^{\circ}34'12.75''$ $34^{\circ}25'21.59''$ | $28^{\circ}1'35.32''$ $28^{\circ}12'12.00''$ $29^{\circ}40'52.36''$ | $35^{\circ}38'55.79''$ $34^{\circ}43'33.39''$ $31^{\circ}23'13.57''$ | 12.90015 | In all the tetrahedra from (236) to (250) the following equations are all true:— | |
| 242. $\begin{cases} a=7.3782, & x=5.5408 \\ b=9.2160, & y=5.4658 \\ c=8.1664, & z=5.7472 \end{cases}$ | $21^{\circ}50'40.74''$ $20^{\circ}24'3.68''$ $19^{\circ}16'47.88''$ | $35^{\circ}58'56.37''$ $44^{\circ}38.04''$ $38^{\circ}40'52.69''$ | $26^{\circ}34'33.48''$ $26^{\circ}11'4.04''$ $29^{\circ}47'40.98''$ | $30^{\circ}35'3.63''$ $32^{\circ}14'55.80''$ $28^{\circ}9'59.59''$ | $35^{\circ}36'23.16''$ $35^{\circ}47'33.97''$ $29^{\circ}20'33.45''$ | 35.68824 | $a_1 + a_2 = a_3 + a_4$ $\beta_1 + \beta_2 = \beta_3 + \beta_4$ $\gamma_1 + \gamma_2 = \gamma_3 + \gamma_4$ $\sigma_1 + \sigma_2 = \sigma_3 + \sigma_4$ (=180°) | |
| 243. $\begin{cases} a=4.441884, & x=4.617116 \\ b=5.7663, & y=4.6568 \\ c=3.9749, & z=5.0844 \end{cases}$ | $26^{\circ}23'20.10''$ $33^{\circ}27'31.31''$ $29^{\circ}0'39.09''$ | $28^{\circ}11'31.15''$ $37^{\circ}4'42.36''$ $25^{\circ}52'15.99''$ | $23^{\circ}54'38.55''$ $28^{\circ}33'47.37''$ $31^{\circ}28'13.00''$ | $30^{\circ}40'12.70''$ $41^{\circ}58'26.30''$ $33^{\circ}14'41.48''$ | $29^{\circ}41'52.37''$ $31^{\circ}57'11.76''$ $33^{\circ}14'8.20''$ | 11.39663 | $\sigma_1 - \beta_1 = \sigma_2 - \beta_2$ $\sigma_1 - \gamma_1 = \sigma_2 - \gamma_2$ $\sigma_1 - \beta_1 = \sigma_2 - \beta_2$ $\sigma_2 - \gamma_2 = \sigma_3 - \gamma_3$ also $\lambda_1 + \Delta_1 = \lambda_2 + \Delta_2$ where $\lambda_1, \lambda_2, \Delta_1, \Delta_2$ | |
| 244. $\begin{cases} a=6.838052, & x=6.480348 \\ b=8.6241, & y=6.7030 \\ c=7.6147, & z=5.7130 \end{cases}$ | $23^{\circ}20'20.89''$ $27^{\circ}28'17.30''$ $20^{\circ}44'40.24''$ | $31^{\circ}12'35.02''$ $43^{\circ}1'26.09''$ $34^{\circ}12'39.86''$ | $26^{\circ}13'53.07''$ $33^{\circ}5'29.35''$ $29^{\circ}18'24.47''$ | $28^{\circ}19'2.84''$ $37^{\circ}21'14.61''$ $25^{\circ}38'55.03''$ | $30^{\circ}55'33.29''$ $41^{\circ}23'50.34''$ $26^{\circ}42'15.68''$ | 35.7079 | | |
| 245. $\begin{cases} a=5.685056, & x=5.310241 \\ b=5.4128, & y=5.0205 \\ c=4.6027, & z=6.3926 \end{cases}$ | $31^{\circ}43'0.02''$ $44^{\circ}0'35.51''$ $35^{\circ}8'48.09''$ | $22^{\circ}44'59.65''$ $26^{\circ}25'43.71''$ $19^{\circ}50'53.02''$ | $28^{\circ}25'28.20''$ $37^{\circ}40'7.73''$ $25^{\circ}24'48.49''$ | $26^{\circ}2'31.47''$ $33^{\circ}52'11.49''$ $29^{\circ}34'52.63''$ | $31^{\circ}10'19.56''$ $49^{\circ}5'16.24''$ $34^{\circ}23'57.09''$ | 19.67475 | | |
| 246. $\begin{cases} a=3.92024, & x=2.87306 \\ b=7.3046, & y=5.5095 \\ c=2.2027, & z=4.5926 \end{cases}$ | $36^{\circ}8'2.67''$ $53^{\circ}29'36.14''$ $43^{\circ}17'50.16''$ | $17^{\circ}12'13.01''$ $17^{\circ}28'57.96''$ $12^{\circ}23'19.80''$ | $29^{\circ}13'11.88''$ $40^{\circ}32'33.72''$ $33^{\circ}25'59.75''$ | $24^{\circ}7'3.80''$ $36^{\circ}26'0.59''$ $33^{\circ}15'13.27''$ | $27^{\circ}18'31.39''$ $54^{\circ}31'19.21''$ $40^{\circ}18'45.83''$ | 3.410436 | denote the areas of the triangular faces. Every one of this system of equations is true for any tetrahedron in which any one is true. | |
| 247. $\begin{cases} a=8.02597, & x=8.84993 \\ b=7.3046, & y=8.5104 \\ c=7.0238, & z=8.8512 \end{cases}$ | $32^{\circ}2'20.92''$ $34^{\circ}15'8.01''$ $35^{\circ}2'2.91''$ | $26^{\circ}51'34.10''$ $25^{\circ}43'53.32''$ $25^{\circ}5'0.71''$ | $29^{\circ}14'3.77''$ $30^{\circ}27'37.56''$ $27^{\circ}26'15.28''$ | $33^{\circ}39'51.28''$ $29^{\circ}31'23.77''$ $32^{\circ}13'43.81''$ | $35^{\circ}38'54.04''$ $37^{\circ}18'0.76''$ $38^{\circ}27'56.54''$ | 57.3202 | | |
| 248. $\begin{cases} a=6.816076, & x=6.854894 \\ b=7.123412, & y=6.54312 \\ c=6.451523, & z=7.21334 \end{cases}$ | $30^{\circ}12'33.34''$ $29^{\circ}31'22.74''$ $31^{\circ}9'35.26''$ | $20^{\circ>45'48.97''$ $30^{\circ}28'46.63''$ $28^{\circ>51'53.00''$ | $27^{\circ>40'35.46''$ $32^{\circ>6'42.34''$ $27^{\circ>27'52.75''$ | $31^{\circ>36'44.20''$ $33^{\circ>19'33.91''$ $33^{\circ>33'55.57''$ | $35^{\circ>12'4.06''$ $36^{\circ>26'2.73''$ $33^{\circ>40'31.14''$ | 37.32572 | In the eight from (243) to (250) the equations $A+X=C+Z$ $B-Y=C-Z$ or $A+X=B+Y=C+Z$ | |
| 249. $\begin{cases} a=7.66209, & x=7.64821 \\ b=8.4325, & y=8.0294 \\ c=7.8567, & z=7.45116 \end{cases}$ | $27^{\circ>56'10.01''$ $31^{\circ>9'1.66''$ $27^{\circ>27'43.23''$ | $29^{\circ>31'56.21''$ $33^{\circ>1'0.34''$ $30^{\circ>5'1.8.55''$ | $28^{\circ>58'16.43''$ $32^{\circ>36'19.66''$ $29^{\circ>7'30.33''$ | $28^{\circ>40'49.79''$ $33^{\circ>36'10.66''$ $28^{\circ>21'21.45''$ | $32^{\circ>59'14.06''$ $46^{\circ>18'38.33''$ $33^{\circ>42'0.76''$ | 56.03472 | al-o hold. | |
| 250. $\begin{cases} a=7.520224, & x=7.451176 \\ b=8.4374, & y=7.5484 \\ c=7.9302, & z=7.0412 \end{cases}$ | $25^{\circ>1'35.21''$ $29^{\circ>12'34.81''$ $25^{\circ>59'22.71''$ | $30^{\circ>39'11.00''$ $35^{\circ>22'1.78''$ $31^{\circ>45'13.59''$ | $28^{\circ>38'35.51''$ $31^{\circ>21'6.77''$ $27^{\circ>59'41.33''$ | $29^{\circ>2'11.66''$ $33^{\circ>13'29.82''$ $27^{\circ>59'41.33''$ | $35^{\circ>23'45.66''$ $40^{\circ>45'35.54''$ $34^{\circ>50'10.28''$ | 51.728 | | |

GLASGOW :
Printed at the University Press, by
ROBERT MACLEHOSE, WEST NILE STREET.

