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Applied Physics Series

ELECTRICITY

AND ITS PRACTICAL APPLICATIONS

ELECTRICITY

AND ITS PRACTICAL APPLICATIONS

BY

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PREFACE

This book is designed to cover the subjects in a first-year course of Electrical Engineering. The elements of Electricity and Magnetism are included to meet the requirements of evening-class students who may not have already taken a course in Physics; and for those who have done so a convenient source of reference to the theoretical parts of Magnetism and Electricity may be found serviceable.

Special thanks are due to Mr. F. H. Downie, B.Sc., and Mr. Duncan J. MacKellar, B.Sc. They have given most valuable assistance with the illustrations, and have otherwise helped in the production of the book.

In an appendix will be found useful tables of constants, as well as specimen sets of questions and exercises. For laboratory work the former will be found of considerable help, and the working of numerical examples similar to those given will, it is hoped, enable the student to obtain a thorough grasp of the subject.

MAGNUS MACLEAN.

August, 1923.

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Symbols

A, *a*.	Cross-sectional area.
B.	Flux density.
b.	Breadth.
C.	Capacity of a condenser.
D.	Diameter; length.
E, *e*.	Electromotive force.
F.	Force; field intensity (electrostatics).
f.	Force; frequency in cycles per second.
G.	Galvanometer constant.
H.	Magnetizing force; field intensity.
I, *i*.	Current; intensity of magnetization.
J.	Joule's dynamical equivalent.
K.W.	Kilowatts.
k.	Radius of gyration; specific inductive capacity.
L.	Inductance in henrys.
l.	Length.
M.	Magnetic moment.
m.	Pole strength; mass.
$\aleph$ or Φ.	Flux; flux per pole crossing the air-gap.
n.	Revolutions per second; number.
p.	Number of pairs of poles.
Q, *q*.	Quantity of electricity.
R, *r*.	Resistance; length; radius.
S.	Surface area; number of turns in a coil.
s.	Length; resistance of shunt.
T.	Period or number of turns in a coil.
t.	Thickness; time in seconds.
V.	Potential difference; volume; vertical component of earth's magnetic field.
v.	Linear velocity.
W.	Watts; weight.
W_{cu}.	Copper I^2R loss in watts.
W_h.	Hysteresis loss in watts.
W_i.	Total iron loss; watts input.
W_e.	Eddy current loss in watts.
W_o.	Watts output.
X.	Ampere-turns.
y_{av}.	Mean winding pitch.
y_b.	Back pitch of winding.

y_f. Front pitch of winding.
Z. Number of surface conductors.
z_1. Number of winding elements.

α. Resistance temperature coefficient; angle.
β. Pole arc/pole pitch; eddy current coefficient.
δ. Angle of dip.
η. Hysteretic coefficient; efficiency.
θ. Rise of temperature; angle.
κ. Susceptibility.
λ. Leakage coefficient.
μ. Permeability.
ρ. Specific resistance.
σ. Surface density.
τ. Pole pitch; torque.
ϕ. Angle.
ω. Angular velocity $= 2\pi n$ radians per second.

ELECTRICITY AND ITS PRACTICAL APPLICATIONS

CHAPTER I

INTRODUCTORY PHENOMENA

Fundamental Phenomena.—If a glass rod be rubbed with a silk handkerchief, both rod and rubber being very dry, the rod will be found to have acquired the property of attracting light bodies. If the part rubbed be held near to small scraps of paper, pieces of cut straw, sawdust, &c., these objects will move to the rod; sometimes they remain in contact with it, sometimes they are alternately attracted and repelled. If the rod be brought near the face, the result is a sensation similar to that produced by the contact of a cobweb. If the knuckle be held near the rod, a peculiar crackling noise is heard, and a bright *spark* passes between the rod and knuckle. Thus friction has caused the rod to acquire peculiar properties. It is said to be *electrified*, and the name of *electricity* is given to the agent to which the various phenomena just described are attributed. Glass is not the only substance which can be electrified by friction; the same property is possessed also by resin, sulphur, precious stones, amber, &c. The Greek name of this last substance (ἤλεκτρον) is the root from which the word *electricity* is derived. Ebonite rubbed with flannel gives especially powerful effects.

At first sight it appears that this property of becoming electrified by friction is not common to all bodies; for if a

bar of metal be held in the hand and rubbed with wool, it does not acquire the properties of an electrified body. But we should be wrong in concluding that metals cannot be electrified by friction; for if the bar be fitted on to a glass rod, and, while held by this handle, be struck with flannel or catskin, it may be very sensibly electrified. There is therefore no basis for the distinction formerly made between electrics and non-electrics, that is, between substances capable and incapable of being electrified by friction; for all bodies, as far as at present known, are capable of being thus excited. There is, however, an important difference of another kind between them, which was first pointed out by Stephen Grey in 1729.

Conductors and Non-conductors.—In certain bodies, such as glass and resin, electricity does not spread itself beyond the parts of the surface where it has been developed; while in other bodies, such as metals, the electricity developed at any point immediately spreads itself over the whole body. Thus, in the last-mentioned experiment, the signs of electricity are immediately manifested at the end of the metal bar which is farthest from the glass rod, if the end next the rod be submitted to friction. Bodies of the former kind, such as glass, resin, &c., are said to be *non-conductors*. Metals are said to be good *conductors*. A non-conductor is often called an *insulator*, and a conductor supported by a non-conductor is said to be *insulated*. The appropriateness of these expressions is evident. No substance is perfectly non-conducting, but the difference in conducting power between what are called non-conductors and good conductors is enormous. The following are lists of conductors and non-conductors, arranged, at least approximately, in order of their conducting powers. In the list of conductors the best conductors are put first; in the list of non-conductors the worst conductors (or best insulators) are put first.

CONDUCTORS

All metals.	Metallic ores.	Living vegetables.
Well-burned charcoal.	Animal fluids.	Flax.
Plumbago.	Sea water.	Hemp.
Concentrated acids.	Spring water.	Living animals.
Dilute acids.	Rain water.	Flame.
Saline solutions.	Snow.	Moist earth and stones

NON-CONDUCTORS

Shellac.	Gems.	Leather.
Amber.	Ebonite.	Baked wood.
Resins.	Caoutchouc.	Porcelain.
Sulphur.	Gutta percha.	Marble.
Wax.	Silk.	Camphor
Jet.	Wool	Chalk.
Glass.	Feathers.	Lime.
Mica.	Dry paper.	Oils.
Diamond.	Parchment.	Metallic oxides.

The human body is a good conductor of electricity. If a person standing on a stool with glass legs be struck with a catskin, he becomes electrified in a very perceptible degree, and sparks may be drawn from any part of his body.

When an insulated and electrified conductor is allowed to touch another conductor insulated but not electrified, it is observed that, after the contact, both bodies possess electrical properties, electricity having been communicated to the second body at the expense of the first. If the second body be much the larger of the two, the electricity of the first is greatly diminished, and may become quite insensible. This explains the disappearance of electricity when a body is put in connection with the earth, which, together with most of the objects on its surface, may be regarded as constituting one enormous conductor. On account of its practically inexhaustible capacity for furnishing or absorbing electricity, the earth is often called *the common reservoir*.

It will now be easily understood why it is not possible to electrify a metal rod by rubbing it while it is held in the hand; since the electricity, as fast as it is generated, passes off through the body into the earth.

Air, when thoroughly dry, is an excellent insulator; and electrified conductors exposed to it, and otherwise insulated, retain their charge with very little diminution for a considerable time. Dampness in the air is, however, a great obstacle to insulation, mainly, or (as it would appear from Lord Kelvin's experiments) entirely, by reason of the moisture which condenses on the insulating supports. Electrical experiments are accordingly very difficult to perform in damp weather. The difficulty is sometimes met by employing a stove to heat the

air in the neighbourhood of the supports, and thus diminish its relative humidity. Dry frosty days are the best for electrical experiments, and next perhaps to these is the season of dry cutting winds in spring.

Duality of Electricity.—The elementary phenomena which we have mentioned in the beginning of this chapter may be more accurately studied by means of the electric pendulum, which consists of a pith ball suspended by a silk fibre from an insulated support. When an electrified glass rod is brought near the insulated ball, the latter is attracted; but as soon as it touches the rod, the attraction is changed to repulsion, which lasts as long as the ball retains the electricity which it has acquired by the contact. A similar experiment can be shown by employing, instead of the glass rod, any other body which has been electrified by friction, for example, a piece of resin which has been rubbed with flannel.

If, while the pendulum exhibits repulsion for the glass, the electrified resin is brought near, it is attracted by the latter; and conversely, when it is repelled by the resin, it is attracted by the glass.[1] These phenomena clearly show that the electricity developed on the resin is not of the same kind as that developed on the glass. They exhibit opposite forces towards any third electrified body, each attracting what the other repels. They have accordingly received names which indicate opposition. The electricity which glass acquires when rubbed with silk is called *positive*; and that which resin acquires by friction with flannel, *negative*. The former was also called *vitreous*, and the latter *resinous*. On repeating the experiment with other substances, it is found that all electrified bodies behave either like the glass or like the resin.

Without making any assumption as to what electricity is, we may speak of an electrified body as being *charged with electricity*, and we may compare quantities of electricity by means of the attractions and repulsions exerted. Bodies oppositely electrified must then be spoken of as charged with *electricities of opposite kind*, or *of opposite sign*; and experiment shows that, whenever electricity of the one kind is developed, whether by friction or by any other means, electricity of the opposite sign

[1] The experiment can be more effectively exhibited by employing as the attracting and repelling bodies the knobs of two small Leyden jars oppositely charged.

is always developed in exactly equal quantity. If a conductor receives two charges of electricity of equal quantity but opposite sign, it is found to exhibit no traces of electricity whatever.

Electricities of like sign repel, and those of unlike sign attract one another.

The magnitude of the force exerted upon each other by two electrified bodies is not altered in amount by reversing the sign of the electricity of one or both of them, provided that the quantities of electricity, and their distribution over the two bodies, remain unchanged. If the sign of one only be changed, the mutual force is simply reversed, and if the signs of both be changed, the force is not changed at all.

The simultaneous development of both kinds of electricity is illustrated by the following experiment: Two persons stand on stools with glass legs, and one of them strikes the other with a catskin. Both of them are now found to be electrified, the striker positively, and the person struck negatively, and from both of them sparks may be drawn by presenting the knuckle.

The kind of electricity which a body obtains by friction with another body depends on the nature of their surfaces. If we take two disks, one of glass and the other of metal, and, holding them by insulating handles, rub them briskly together, we shall find that the metal becomes negatively and the glass positively electrified; but if the metal be covered with a catskin, and the experiment repeated, it will be the glass which will this time be negatively electrified. In the subjoined list the substances are arranged in such order that, generally speaking, each of them becomes positively electrified by friction with those which follow it, and negatively with those which precede it.

Fur of cat.	Feathers.	Silk.
Polished glass.	Wood.	Shellac.
Woollen stuffs.	Paper.	Rough glass.

Hypotheses regarding the Nature of Electricity.—Two theories regarding the nature of electricity must be described on account of the historical interest attaching to them.

The two-fluid theory, originally propounded by Dufaye, and reduced to a more exact form by Symmer, maintains that the opposite kinds of electricity are two fluids. Positive electricity is called the *vitreous fluid*, and negative electricity the *resinous*

fluid. Fluids of like name repel, and those of unlike name attract each other. The union of equal quantities of the two fluids constitutes a neutral fluid, which is supposed to exist in very large quantity in all unelectrified bodies. When a body is electrified, it gains an additional quantity of the one fluid, and loses an equal quantity of the other, so that the total amount of electric fluid in a body is never changed; and (as a consequence of this last condition) when a current of either fluid traverses a body in any direction, an equal current of the other fluid traverses it in the opposite direction.

This theory is in complete agreement with all electrical phenomena so far as at present known; but as it is conceivable that the two electricities, instead of being two kinds of matter, may be two kinds of motion, or, in some other way, may be opposite states of one and the same substance, it is more philosophical to avoid the assumption involved in speaking of *two electric fluids*, and to speak rather of *two opposite electricities*. They may be distinguished indifferently by the names *vitreous* and *resinous*, or *positive* and *negative*.

The *one-fluid theory*, as originally propounded by Franklin, maintained the existence of only one electric fluid, which unelectrified bodies possess in a certain normal amount. A positively electrified body has more, and a negatively electrified body less than its normal amount. The particles of this fluid repel one another, and attract the particles of other kinds of matter, at all distances. The one-fluid theory in this form is, like the two-fluid theory, consistent with the explanation of all known phenomena. But it is to be remarked that there is no sufficient reason, except established usage, for deciding which of the two opposite electricities should be regarded as corresponding to an excess of the electric fluid.

Franklin was the author of the terms positive and negative to denote the two opposite kinds of electrification; but the names can legitimately be retained without accepting the one-fluid theory, understanding that opposite signs imply forces in opposite directions, and that the connection between the *positive* sign and the forces exhibited by *vitreous* electricity is merely conventional.

In speaking of electric currents, the language of the one-fluid theory is almost invariably employed. Thus, if A is a

conductor charged positively, and B is a conductor charged negatively; when the two are put in connection by a wire, we say that the direction of the current is from A to B; whereas the language of the two-fluid theory would be, that vitreous or positive electricity flows from A to B, and resinous or negative from B to A.

CHAPTER II

ELECTROSTATIC INDUCTION

Induction.—In the preceding chapter we have spoken of movements of material bodies caused by electrical attractions and repulsions. We have now to treat of the movement of electricity itself in obedience to the attractions or repulsions exerted upon it by other electricity This kind of action is called *induction.*

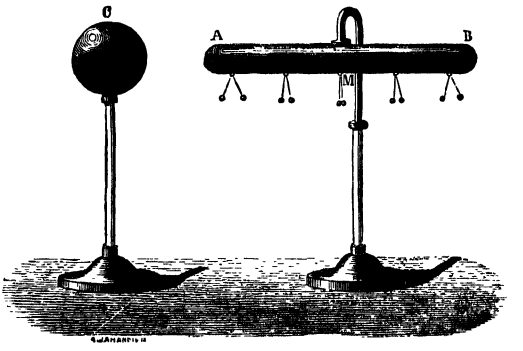


Fig. 1.—Electrification by Influence

It may be illustrated by means of the arrangement shown in fig. 1. The apparatus consists of a sphere C which is electrified positively, suppose, and of a conducting insulated cylinder AB placed near it. From this latter are suspended at equal distances a few pairs of pith balls. When the cylinder is brought near the sphere, the balls are observed to diverge. The divergence of the different pairs is not the same, but goes on decreasing from the pair nearest the sphere until a point M is reached, where there is no divergence. Beyond this the divergence goes on increasing. The neutral point M does not exactly bisect the length of the cylinder, but is nearer the end A than the end B, and the former end is found to be more strongly electrified than the latter.

It is easy to show that the two ends of the cylinder are

charged with opposite kinds of electricity; the end A being negatively, and the end B positively electrified. We have only to bring an electrified stick of resin near the pith balls at A, when these will be found to be repelled; if, on the contrary, it be held near those at B, they will be attracted.

The explanation is, that the positive electricity with which C is charged attracts the negative electricity of AB to the end A, and repels the positive to the end B. This action is more powerful at A than at B, on account of the greater proximity of the influencing body, and for the same reason the effect falls off more rapidly in the portion AM than in MB.

If the cylinder be brought closer to the sphere, the divergence of the balls increases; if it be removed farther from it, the divergence diminishes. Finally, all signs of electricity disappear if the sphere be taken away, or connected with the earth.

If, while the cylinder is under the influence of the electricity of C, the end B is connected with the earth, the pith balls at this end immediately collapse, while the divergence of those at A increases. The explanation is that the electricity which was repelled to the end B escapes to the earth, and thus affords an opportunity for a fresh exercise of induction on the part of the sphere, which increases the accumulation of negative electricity at A. We may also remark that the whole of the cylinder is now negatively electrified, the neutral line being pushed back to the earth. If the earth connection be now broken, and the sphere C be then removed, the cylinder will remain negatively electrified, and will be in the same condition as if it had been touched by a negatively electrified body. This mode of giving a charge to a conductor is called *charging by induction*, and the charge thus given is always opposite to that of the *inducing body* C.

If a series of such conductors as AB be placed in line, without contact, and the positively electrified body C be placed opposite to one end of the series, all the conductors will be affected in the same manner as the single conductor in the last experiment. They will all be charged with negative electricity at the end next C, and with positive electricity at the remote end, the effect, however, becoming feebler as we advance in the series. In this experiment each of the conductors acts inductively upon those next it; for example, if there be two conductors

AB, A′B′, as in fig. 2, the development of electricity at A′ and B′ is mainly due to the action of the positive electricity in MB. If the conductor AB be removed, the pith balls at A′ and B′ will diminish their divergence.

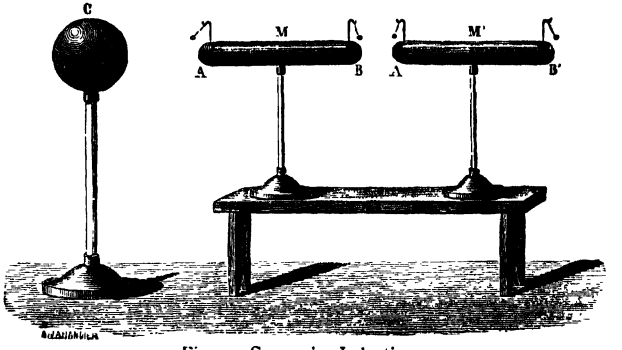


Fig. 2.—Successive Induction

Electrical Attraction and Repulsion.—The attraction which is observed when an electrified is brought near to an unelectrified body is dependent upon induction. Suppose, for instance, that a body C, which is positively electrified, is brought near to an insulated and uncharged pith ball. Negative electricity is induced on the near side of the pith ball, and an equal quantity of positive on the farther side. The former, being nearer to the body C, is more strongly attracted than the other is repelled. The ball is therefore upon the whole attracted.

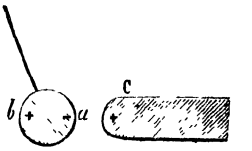


Fig. 3.—Electrical Attraction

If the pith ball, instead of being insulated, is suspended by a conducting thread from a support connected with the earth, it will be more strongly attracted than before, as it is now entirely charged with negative electricity.

In the case of any insulated conductor, the algebraic sum of the electricities induced upon it by the presence of a neighbouring electrified body must be zero. If the pith ball be insulated, and have an independent charge of either kind of electricity, the total force exerted on the pith ball is the resultant[1] of the two following:—

[1] We here suppose C to be a non-conductor, so that the distribution of its electricity is not affected by the presence of the pith ball. If C be a conductor, the effect of induction upon it will be to favour attraction, so that an attractive force must be added to the two forces specified in the text.

1. The force which the ball would experience if it had no independent charge. This force, as we have just seen, is always attractive.

2. The force due to the independent charge when distributed over the ball as it would be if C were removed. This second force is attractive or repulsive, according as the independent charge is of unlike or like sign to that of C. In the latter case, repulsion will generally be observed at distances exceeding a certain limit, and attraction at nearer distances, the reason being that the force due to the induced distribution increases more rapidly than the second force as the distance is diminished.

It is important to remember this in testing, by the electric pendulum, or by any other electroscope, the kind of electricity with which a body is charged. In bringing the body towards the electroscope the first movement produced is that which is to be observed, and repulsion is in general a more reliable test of kind of electricity than attraction.

Electroscopes.—An electroscope is an apparatus for detecting the presence of electricity, and determining its sign. The insulated electric pendulum is an electroscope. If the pith ball, when itself uncharged, is attracted by a body brought near it, we know that the body is electrified. To determine the kind of electricity, the body is allowed to touch the pith ball, which is then repelled. At this moment an excited glass rod is brought near. If it repels the ball, this latter, as well as the body which touched it, must be electrified positively. If the glass rod attracts it, or, still more decisively, if excited resin or sealing-wax repels it, the ball and the body which touched it are electrified negatively. The loss of electricity from the pith ball is often so rapid as to render this test of sign somewhat uncertain.

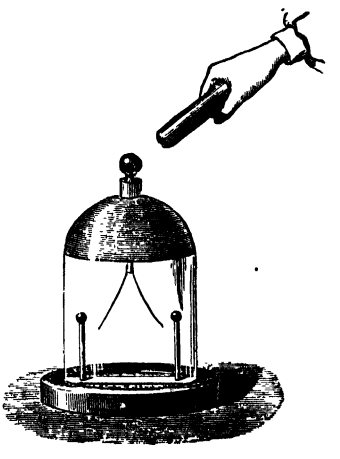


Fig. 4.—Gold-leaf Electroscope

The *gold-leaf electroscope* (fig. 4) is constructed as follows:

Two small gold leaves are attached to the lower end of a metallic rod, which passes through an opening in the top of a bell glass, and terminates in a ball. The metallic rod is sometimes, for the sake of better insulation, enclosed in a glass tube secured by sealing wax or some other non-conducting cement, and, for the same purpose, the upper part of the bell glass is often varnished with shellac, which is less apt than glass to acquire a deposit of moisture from the air. The bell glass is attached below to a metallic base, which excludes the external air.

To test whether a body is electrified, it is brought near the ball at the top of the electroscope. The like electricity is repelled into the leaves, and makes them diverge, while the unlike is attracted into the ball. The sign of the body's charge may be determined in the following manner: While the leaves are divergent under the influence of the body, the operator touches the ball with his finger. This causes the leaves to collapse, and gives to the insulated conductor composed of leaves, rod, and ball, a charge opposite to that of the influencing body. The finger must be removed while the influencing body remains in position, as the amount of the induced charge depends upon the position of the influencing body at the instant of breaking connection. On withdrawing the influencing body, the charge of unlike electricity is now no longer attracted to the ball, but spreads over the whole of the conductor, and causes the leaves to diverge. If, while this divergence continues, an excited glass rod, when gradually brought towards the ball, diminishes the divergence, we know that the body in question was electrified positively. If it increases the divergence, the body was electrified negatively.

Great caution must be used in bringing electrified bodies near the gold-leaf electroscope, as the leaves are very apt to be ruptured by quick movements. If they diverge so widely as to touch the sides of the bell glass, it is often difficult to detach them from the glass without tearing. To prevent this contact, two metallic columns are interposed, communicating with the ground. If the leaves diverge too widely, they touch these columns and lose their electricity.

CHAPTER III

QUANTITATIVE LAWS

Coulomb's Torsion Balance.—Coulomb and Cavendish were the first to make electricity a quantitative science, and the former employed in his researches an instrument which is often called after his name. It depends on the principle that the torsion of a wire is simply proportional to the twisting couple or *torque*. We shall first describe it, and then point out some of its applications.

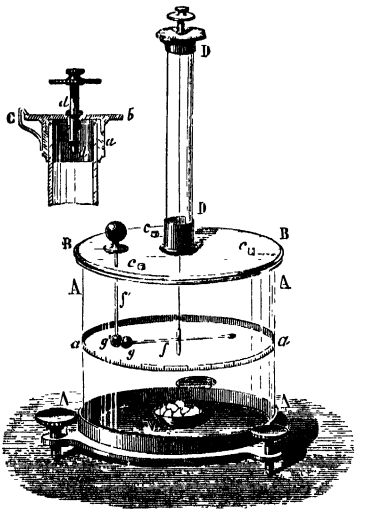


Fig. 5.—Coulomb's Torsion Balance

It consists of a cylindrical glass case AA (fig. 5), from the upper end BB of which rises another glass cylinder DD of much smaller diameter. This small cylinder is fitted at the top with a brass cap *a*, carrying an index C. Outside of this, and capable of turning round it, is another cap *b*, the top of which is divided into 360 equal parts. In the centre of the cap *b* is an opening through which passes a small metal cylinder *d*, capable of turning in the opening with moderate friction, and having a holder for gripping a wire. When the cap *b* is turned, the cylinder *d* turns with it; but the latter can also be turned separately, so as not to change the reading. These parts compose the *torsion head*. A very fine wire, held as above described, supports a small piece of metal, through which passes a light needle of shellac *f*, carrying at one end a small gilt ball *g*. A circular scale runs round the outside of the large cylinder in the plane of the needle. Finally, opposite the zero of this scale there is a fixed ball *g'* of some conducting material, supported by a rod *f'* of shellac, which passes through a hole in the cover of the cylindrical case.

Laws of Electric Repulsion.—To illustrate the mode of

employing this apparatus for electrical measurements, we shall explain the course followed by Coulomb in investigating the law according to which electrical repulsions and attractions vary with the distance. The index is set to the zero of the scale. The inner cylinder *d* is then turned, until the movable ball just touches the fixed ball without any torsion of the wire. The fixed ball is then taken out, placed in communication with an electrified body, and replaced in the apparatus. The electricity with which it is charged is communicated in part to the movable ball, and causes the repulsion of this latter through a number of degrees indicated by the scale which surrounds the case. In this position the force of repulsion is in equilibrium with the force of torsion tending to bring back the ball to its original position. The graduated cap *b* is then turned so as to oppose the repulsion. The movable ball is thus brought nearer to the fixed ball, and at the same time the amount of torsion in the wire is increased. By repeating this process, we obtain a number of different positions in which repulsion is balanced by torsion.[1] But we know, from the laws of elasticity, that the torque exerted by the wire is proportional to the angle of torsion. Hence we have only to compare the total amounts of torsion with the distances of the two balls. By such comparisons Coulomb found that the force of electrical repulsion varies *inversely as the square of the distance.*

In measuring attractions, it was necessary to employ special precautions to prevent the movable ball from coming into contact with the fixed ball and sharing its charge.

Influence of Amount of Charge.—We may assume as evident, that when an electrified ball is placed in contact with a precisely equal and similar ball, the charge will be divided equally between them, so that the first will retain only half the charge which it had before contact. Suppose that an observation on repulsion has just been made with the torsion balance, and that we touch the fixed ball with another exactly equal insulated ball, which we then remove. It will be found that the amount of torsion requisite for keeping the movable ball in

[1] The repulsive force on the movable ball is equivalent to an equal and parallel force acting at the centre of the needle (the point of attachment of the wire), and a couple whose arm is the perpendicular from this centre on the line joining the balls. This couple must be equal to the torque of the wire. The other component produces a small deviation of the suspending wire from the vertical.

its observed position is just half what it was before. We conclude that if the charge of either body be altered, the attractive or repulsive force between the bodies at given distance will be altered in the same ratio.

Coulomb's method of experiment is not capable of giving anything more than rough indications of the exact law. If by "the distance between the two balls" we understand the distance between their nearest points, very large discrepancies will be found. If we understand it to mean the distance between their centres, the agreement will be closer, but will still be far from exact when the balls are nearly in contact. The observed attraction or repulsion of two electrified bodies is the resultant of an infinite number of forces acting along lines drawn from different points of the one to different points of the other. The law of inverse squares applies directly to these several components, and not to the resultant which they yield. The latter usually requires for its computation an elaborate mathematical process. The strongest evidence for the rigorous exactness of the law of inverse squares is indirect.

Definition of Unit Quantity.—When two bodies, very small in all their dimensions as compared with the distance D between them, have charges Q and Q′ of electricity, their mutual force is represented by the expression

$$\frac{QQ'}{D^2}.$$

If this expression has the positive sign the force is repulsive, if negative, attractive. If the three magnitudes, Q, Q′, D, are each unity, the mutual force is unity. In the c.g.s. system D is expressed in centimetres, and the mutual force in *dynes*. Accordingly, the c.g.s. unit of electricity in electrostatics is *that quantity which, if collected on a particle, repels an equal quantity collected on another particle at the distance of one centimetre, with a force of one dyne* (*medium air*).

Electricity resides on the Surface.—Electricity (subject to the exceptions mentioned below) resides exclusively on the external surface of a conductor. This is shown by the following experiments:—

A hollow insulated sphere, with an orifice in the top, is charged with electricity (fig. 6). A *proof plane*, consisting of

a small disk of gilt paper insulated by a thin handle of shellac, is then applied to the interior surface of the sphere, and, when tested by an electroscope, is found to exhibit no trace of electricity. But if the disk be applied to the external surface of the sphere, it will be found to be electrified, and capable of attracting light bodies. Faraday varied this experiment by substituting a cylinder of wire-gauze for the sphere. This cylinder rested on an insulated disk of metal. The disk was charged with electricity, and it was found that no trace of the electricity could be detected by applying the proof plane to the interior surface of the cylinder.

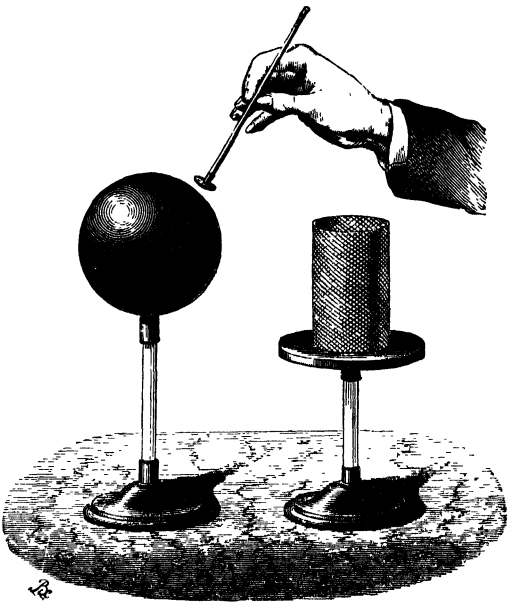


Fig. 6.—Proof Plane and Hollow Sphere

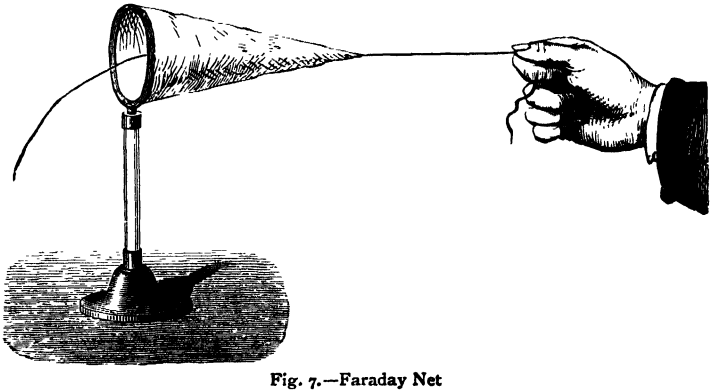


Fig. 7.—Faraday Net

The following experiment is also due to Faraday. A metal ring is fixed upon an insulating stand (fig. 7). To this ring is attached a cone-shaped bag of fine linen, which is a conductor

of electricity. A silk thread, attached to the apex of the cone, and extending both ways, enables the operator to turn the bag inside out as often as required, without discharging it. When the bag is electrified, the application of the proof plane always shows that there is electricity on the outer, but not on the inner surface. When the bag is turned inside out, the electricity therefore passes from one surface of the linen to the other.

Limitations of the Rule.—There are two exceptions to the rule that electricity is confined to the external surface of a conductor.

1. It does not hold for electric currents. We shall see hereafter, in connection with current electricity, that the resistance which a wire of given length opposes to the passage of electricity through it depends not upon its circumference but upon its cross-sectional area. A hollow wire will not conduct electricity so well as a solid wire of the same external diameter.

2. Electricity may be induced on the inner surface of a hollow conductor by the presence of an electrified body insulated from the conductor itself. If an insulated body charged with electricity be introduced into the interior of a hollow conductor, so as to be completely surrounded by it, but still insulated from it, it induces upon the inner surface a quantity equal to its own charge, but of opposite sign. If the conductor is insulated, an equal quantity, but of the same sign as the charge of the enclosed body, is repelled to the outside, and this is true whether the conductor has an independent charge of its own or not. In this case, then, we have electricity residing on both the external and the internal surfaces of a hollow conductor, but it still resides only on the surfaces.

If a conducting body connected with the earth be introduced into the interior of a hollow charged conductor, so as to be partially surrounded by it, the body thus introduced will acquire an opposite charge by induction, and, by the reciprocal action of this charge, electricity will be induced on the inner at the expense of the outer surface of the hollow conductor, just as in the preceding case.

Ice-pail Experiment.—The effect of introducing a charged body within a hollow conductor is well illustrated by the following experiments of Faraday. Let A (fig. 8) represent an insulated pewter ice pail, connected by a wire with a delicate

gold-leaf electroscope E, and let C be a round brass ball insulated by a dry thread of silk, 3 or 4 feet in length, so as to remove the influence of the hand holding it from the ice pail below. Let A be perfectly discharged, and let C, after being charged at a distance, be introduced into A as in the figure. If C be positive, E also will diverge positively; if C be taken away, E will collapse perfectly, the apparatus being in good order. As C enters the vessel A, the divergence of E

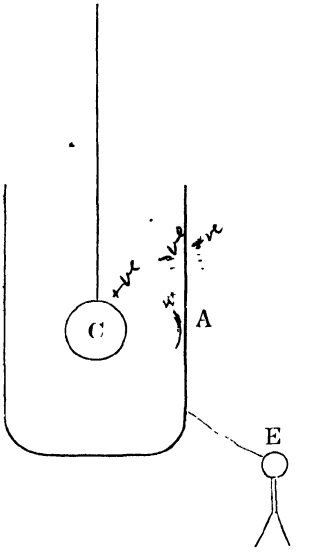


Fig. 8.—Ice-pail Experiment

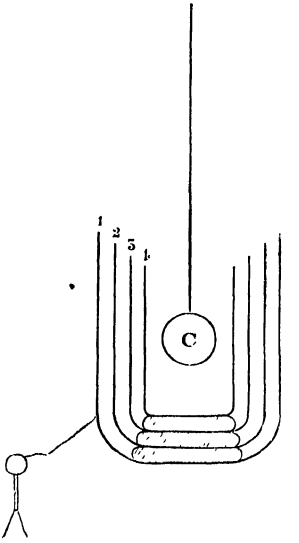


Fig. 9.—Experiment with Four Ice Pails

will increase until C is well below the edge of the vessel, and will remain quite steady and unchanged for any greater depression. If C be made to touch the bottom of A, all its charge is communicated to A, and C, upon being withdrawn and examined, is found perfectly discharged. Now Faraday found that at the moment of contact of C with the bottom of A not the slightest change took place in the divergence of the gold leaves. Hence the charge previously developed by induction on the outside of A must have been precisely equal to that acquired by the contact, that is, must have been equal to the charge of C.

He then employed four ice pails (fig. 9), arranged one within the other, the smallest innermost, insulated from each other by

plates of shellac at the bottom, the outermost pail being connected with the electroscope. When the charged carrier ball C was introduced within the innermost pail, and lowered until it touched the bottom, the electrometer gave precisely the same indications as when the outermost pail was employed alone. When the innermost was lifted out by a silk thread, after being touched by C, the gold leaves collapsed perfectly. When it was introduced again, they opened out to the same extent as before. When 4 and 3 were connected by a wire let down between them by a silk thread, the leaves remained unchanged, and so they still remained when 3 and 2 were connected, and finally when all four pails were connected.

No Force within a Conductor.—When a hollow conductor is electrified, however strongly, no effect is produced upon pith balls, gold leaves, or any other electroscopic apparatus in the interior, whether connected with the hollow conductor or insulated from it, provided, in the latter case, that they have no communication with bodies external to the hollow conductor. Faraday constructed a cubical box, measuring 12 feet each way, covered externally with copper wire and tinfoil, and insulated from the earth. He charged this box very strongly by outside communication with a powerful electrical machine; but a gold-leaf electrometer within showed no effect. He says: "I went into the cube and lived in it, using lighted candles, electrometers, and all other tests of electrical states. I could not find the least influence upon them, or indication of anything particular given by them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every part of its outer surface."

The fact that electricity resides only on the external surface of a conductor, combined with the fact that there is no electrical force in the space enclosed by this surface, affords a rigorous proof of the law of inverse squares. For if the conductor be a sphere removed from the influence of external bodies, its charge must be distributed uniformly over its surface. Now it admits of proof, and is well known to mathematicians, that a uniform spherical shell exerts no attraction at any point of the interior space, if the law of attraction be that of inverse squares, and that the internal attraction does not vanish for any other law.

Surface Density and Distribution.—When the proof plane

is applied to different parts of the surface of a conductor, the quantities of electricity which it carries off are not usually equal. But the electricity carried off by the proof plane is simply the electricity which resided on the part of the surface covered by it, for the proof plane during the time of its contact is virtually part of the surface of the conductor. We must therefore conclude that equal areas on different parts of the surface of a conductor have not equal amounts of electricity upon them. It is also found that if the charge of the conductor be varied, the electricity resident upon any specified portion of the surface is changed in the same ratio. The ratio of the quantities of electricity on two specified portions of the surface is in fact independent of the charge, and depends only on the form of the conductor. This is expressed by saying that *distribution* is independent of charge, and that the distribution of electricity on the surface of a conductor depends on its form.

By the *average electric density* on the whole or any specified portion of the surface of a conductor is meant the quantity of electricity upon it, divided by its area. By the *electric density at a specified point* on the surface of a conductor is meant the average electric density on an exceedingly small area surrounding it, in other words, the *quantity of electricity per unit area* at the point. The name is appropriate, from the analogy of ordinary material density, which is mass per unit volume, and is not intended to imply any hypothesis as to the nature of electricity. The name was introduced by Coulomb, who first investigated the subject in question.

The following are some of Coulomb's results. The dotted line in each of the figures is intended to represent, by its distance from the outline of the conductor, the electric density at each point of the latter. In all cases it is to be understood that the conductor is so far removed from external bodies as not to be influenced by them:—

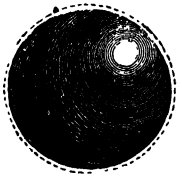


Fig. 10.—Distribution on Sphere

1. *Sphere* (fig. 10). The electric density is the same for all points on the surface of a spherical conductor.

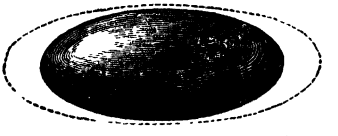


Fig. 11.—Distribution on Ellipsoid

2. *Ellipsoid* (fig. 11). The density is greatest at the ends of

the longest, and least at the ends of the shortest axis; and the densities at these points are simply proportional to the axes themselves.

3. *Flat Disk* (fig. 12). The density is almost inappreciable over the whole of both faces, except close to the edges, where it increases almost *per saltum*.

Fig. 12.—Distribution on Disk

Fig. 13.—Distribution on Cylinder with Rounded Ends

4. *Cylinder with Hemispherical Ends* (fig. 13). The density is a minimum, and nearly uniform, at parts remote from the ends, and attains a maximum at the ends. The ratio of the density at the ends to that at the sides increases as the radius of the cylinder diminishes, the length of the cylinder remaining the same.

Power of Points.—The distribution of electricity on a conductor of any form may be roughly described by saying that the density is greatest on those parts of the surface which project most, or which have the sharpest convexity, and that in depressions or concavities it is small or altogether insensible. Theory shows that at a perfectly sharp edge, such, for example, as is formed by two planes meeting at any angle however obtuse, but *not rounded off*, the density must be infinite, and *a fortiori* it must be infinite at a perfectly sharp point, for example at the apex of a cone, however obtuse, *if not rounded off*. Practically, the points and edges of bodies are always rounded off; the microscope shows them merely as places of very sharp convexity (that is, of very small radius of curvature), and hence the electric density at these places is really finite; but it is exceedingly great in comparison with the density at other parts, and this is especially true of very acute points, such as the point of a fine needle. The consequence is, that if a pointed conductor is insulated and charged, the concentration of a large amount of repulsive force within an exceedingly small area produces very rapid escape of electricity at the points. Conductors intended to retain a charge of electricity must have no points or edges, and must be very smooth. If of considerable length in proportion to their breadth, they are usually made to terminate in large knobs.

Dissipation of Charge.—When an insulated conductor is charged and left to itself, its charge is gradually dissipated, and at length completely disappears. This loss takes place partly through the supports, and partly through the air.

As regards the supports, the loss occurs chiefly at their surface, especially if (as is usually the case) they are not perfectly dry. It is diminished by diminishing their perimeter, and by increasing their length; for example, a long fibre of glass or raw silk is an excellent insulator.

As regards the air, we must distinguish between conduction and convection. Very hot air and highly rarefied air probably act as conductors; but air in the ordinary condition acts chiefly by contact and convection. Successive layers of air become electrified by contact with the conductor, and are then repelled, carrying off the electricity which they have acquired. It is by an action of this kind that electricity escapes into the air from points, as is proved by the wind which passes off from them. Particles of dust present in the air, in like manner, act as carriers, being attracted to the conductor, charged by contact with it, and then repelled. They also frequently adhere to the conductor, and thus constitute pointed projections through which electricity is discharged into the air.

Coulomb deduced from his observations on dissipation of charge a law precisely analogous to Newton's law of cooling, namely, that when all other circumstances remain the same, *the rate of loss is simply proportional to the charge*, so that the charges at equal intervals of time form a decreasing geometric series. Subsequent experience has confirmed this law as approximately true for moderate charges of the same sign. Negative charges are, however, dissipated more rapidly than positive.

Preliminary Definitions.—It will be convenient at this stage to define a few mathematical terms which we shall have to employ.

A *scalar* is a magnitude which has no direction. Mass is an example.

A *vector* is a directed magnitude which follows the parallelogram law of composition. A vector requires three numbers for its specification. They may specify its three rectangular components; or one may specify its magnitude, and the other two its direction in space.

The *line integral of a vector* F *along a path* s is the algebraic sum of all the terms $F.ds.\cos\theta$ for every element ds of the path, θ denoting the angle between the direction of F and the direction in which ds is traced. A familiar example is the work done upon a particle moving along the path s by a force F acting on the particle.

The *surface integral of a vector* F *over a given surface* S is the algebraic sum of all the terms $F.dS.\cos\theta$ for every element dS of the surface, θ denoting the angle between the direction of F and the normal drawn towards a specified side of the surface. As an example, if F denote the velocity of a liquid, $F dS\cos\theta$ is the volume of liquid that passes through the small area dS per unit time. This is called the *flux* through dS, and the surface integral measures the whole flux through the surface, fluxes opposite ways through the surface being regarded as having opposite signs.

Potential in General. — In many instances (for example, when F denotes the force of gravity on a particle) the line integral vanishes for every closed path, or, what amounts to the same thing, the line integral is the same along all paths from the same starting-point to the same finishing-point. In such cases the vector is said to *have a potential*, and the line integral is the difference of the potentials at the two ends of the path. The potential is a scalar.

One important instance is the general case of a force directed to or from a fixed centre, and having a magnitude depending only on distance from that centre. Let the force at distance r (regarded as positive when repulsive) be written $f(r)$. Then the work which it does in the elementary path ds is $f(r)ds.\cos\theta$, where θ is the angle between ds and the force, so that $\cos\theta$ is dr/ds. Hence the work done is $f(r)dr$. Suppose this to be expressible as $d.\phi(r)$, where $\phi(r)$ denotes a function of r. Then the work in any finite path is $\phi(r_2) - \phi(r_1)$, where r_1 and r_2 are the values of r for the beginning and end of the path. The work is therefore the same along all paths from a given point A to a given point B. A particular case is:

$$f(r) = \frac{m}{r^2}; \qquad \phi(r) = -\frac{m}{r}.$$

Electric Field. — The influence which an electrified body

exerts in the region of air around it can be specified by stating (in direction and magnitude) the force of attraction or repulsion which the body would exert upon a small charged body placed in various parts of this region or field (which is accordingly called an *electric field*). For simplicity the charge of the small body must be supposed small enough to produce no sensible disturbance in the distribution of the electricity to which the field is due; then, if q be the small charge (of positive electricity), and Fq the force of attraction or repulsion which it experiences at a given point of the field, F is called the *intensity of the field* at the point in question. It is more frequently called the *electric force* at the point, but it is not *force* in the strict sense of the word. Fq is a force, strictly so called, and the "dimensions" of F can be calculated by dividing the dimensions of force by the dimensions of quantity of electricity. F is clearly a vector.

Difference of Potential—P.D.—When the small body with charge q is moved in the electric field, work (either positive or negative) is done upon it by the electric forces which pervade the field. As q is supposed too small to disturb the field, the field remains unchanged by the motion of the small body; and if the body travels along any path ACBDA which ends at the starting-point A, the whole work done upon it is zero, since the charged bodies to which the field is due are in the same condition as at first, and the field has thus neither lost nor gained energy. Since the work in BDA just cancels the work in ACB, it follows that the work in ADB is equal to the work in ACB; that is to say, *the work from* A *to* B *is the same for all paths which begin at* A *and end at* B. This work, when q is taken as unity, is called the *difference of potentials* between A and B. As regards sign, the work done by the electric forces when a unit of positive electricity moves from a point A to a point B is the potential of A *minus* the potential of B. "Difference of potentials" has been abbreviated by some recent writers into "potential difference", and is now commonly denoted by the initial letters P.D.

Force = Slope of Potential.—If V denote the potential at a point, and $V - \delta V$ the potential at a neighbouring point, δV is the work which electrical attractions and repulsions do upon a unit of positive electricity in its passage from the first point

to the second; and since work is equal to force multiplied by distance, the average force along the joining line can be computed by dividing δV by the distance, which we will call δs. Hence, the limit of $\frac{\delta V}{\delta s}$, as the two points are taken nearer together, is the component force in the direction δs; that is,

$$-\frac{dV}{ds} = F \cos \theta$$

or the rate of diminution of potential in any direction is equal to the component force in that direction. In the direction of the resultant force we have

$$-\frac{dV}{ds} = F.$$

The P.D. between two points is the line integral of the field intensity F along any path joining them. As regards sign, the line integral of F from the first of two points to the second is the potential of the first *minus* that of the second.

Lines of Force and Equipotential Surfaces.—A line traced by travelling always in the direction of F is called a *line of force* (strictly a line of electric force). At every point of a line of force the direction of F is tangential.

An *equipotential surface* is a surface over the whole of which there is the same value of potential. In travelling along such a surface, dV/ds is zero, hence F cos θ is zero, and F is normal to the surface. *Equipotential surfaces* accordingly *cut lines of force at right angles.*

When we are dealing with gravitational instead of electrical forces, equipotential surfaces are called *level surfaces*, and lines of force are called *verticals.*

Force between Equipotential Surfaces.—Let V_1 and V_2 be the potentials of two equipotential surfaces; then the work from any point of the first to any point of the second is $V_1 - V_2$. This is accordingly the work done in the portion of any line of force intercepted between them, and the average value of F for any one of these portions must be inversely as the length of the portion. Hence the force at different points of the narrow region lying between two consecutive equipotential surfaces varies inversely as the breadth of the region. The force is greatest where the surfaces are closest.

Potential of a Conductor. — When electric potential is uniform throughout a given space, there is no electric force in that space; and conversely, if there is complete absence of electric force from a given space the potential throughout that space must be uniform. These conditions prevail through the whole substance of a conductor in electrical equilibrium. If there are hollows within the conductor, the potential is the same in them as in the conductor itself; unless there are charged bodies within them insulated from the conductor. When a conductor is not in electric equilibrium, currents set in, tending to restore equilibrium, and the direction of such currents is always from places of higher to places of lower potential.

It is usual to assume as the zero of potential the potential of the earth. But this assumption is not consistent with itself, since the existence of earth currents proves that there are differences of potential between different parts of the earth. The absolute zero of potential is the potential of places infinitely distant from all electricity.

Energy of a Charged Conductor. — When positive electricity is allowed to run down from a conductor of higher to one of lower potential, there is a loss of potential energy, just as there is a loss of potential energy in the running down of a heavy body from a higher to a lower level; and, on the other hand, to make positive electricity pass from a conductor of lower to one of higher potential, work must be expended from some external source, just as work must be expended to raise a heavy body. In the case of the heavy body, the work expended in the latter case, or the energy which runs down in the former, is equal to the weight multiplied by the difference of levels; and in the analogous electrical operation the work or the energy is the product of the quantity of electricity which passes from one conductor to the other by the difference of potentials of the two conductors, provided that these potentials remain sensibly constant during the operation.

When a conductor is charged in the ordinary way its charge is drawn from the earth, the potential of which remains constant. If we suppose the charge to be given in a succession of small equal parts, the potential of the conductor, which is initially zero, is increased by a succession of equal steps till

it attains its final value; hence it is only the last part that is raised through the full difference of potential, and the mean rise of potential for all the parts is the half of this. Hence the work done in charging a conductor, or the energy which runs down in discharging it to the earth, is half the product of its potential and its charge. In like manner, the work done in raising bricks to build a uniform wall is the total weight multiplied by half the height of the wall.

Potential defined as $\Sigma\frac{Q}{R}$.

Let a quantity Q of electricity be collected at a point O, and let A, B be any two points very near together, the surrounding medium being air. The forces at A and B due to Q will be $\frac{Q}{OA^2}$ and $\frac{Q}{OB^2}$, and these will be nearly equal to each other or to $\frac{Q}{OA \cdot OB}$. When a unit of electricity is carried from A to B its motion can be resolved into two components, one of them in the direction of the force and equal to OB − OA, and the other perpendicular to the direction of the force. Hence the work done will be $\frac{Q(OB - OA)}{OA \cdot OB}$, that is, $\frac{Q}{OA} - \frac{Q}{OB}$; or if R denote the distance of any point from O, the *work done in a* small *movement is equal to the change in the value of* $\frac{Q}{R}$. Since any movement can be resolved into a succession of small movements, we may omit the word *small*, and the proposition will still be true. As R increases to infinity, $\frac{Q}{R}$ will diminish to zero. Hence, $\frac{Q}{R}$ denotes the work from distance R to infinite distance.

As regards sign, $\frac{Q}{R}$ is the work done by electrical force when a unit of positive electricity is carried from distance R to infinite distance.

Next, suppose several quantities Q_1, Q_2, &c., to be collected at different points, O_1, O_2, &c. Let P be any other point, and let $O_1P = R_1$, $O_2P = R_2$, &c. Then in the passage of a unit of electricity from P to infinite distance, the electrical work is, by the preceding section,

$$\frac{Q_1}{R_1} + \frac{Q_2}{R_2} + \quad \&c.$$

which we will denote by $\Sigma\frac{Q}{R}$, the symbol Σ being read "the sum of such terms as".

$\Sigma\frac{Q}{R}$ is therefore the general expression for the potential at a point due to any quantity of electricity distributed in any manner; in other words, the potential is equal to the sum of the quotients obtained by dividing each element of electricity by its distance from the point. The distances are essentially positive. If the electricity is not all of one sign, some of the quotients $\frac{Q}{R}$ will be positive and others negative, and their algebraical sum is to be taken.

Application to Sphere.—In the case of a charged conducting sphere, all the elements Q are equally distant from the centre of the sphere, and the sum of the quotients $\frac{Q}{R}$ when we are computing the potential at the centre, will be $\frac{Q}{R}$, Q denoting the charge, and R the radius of the sphere. But the potential is the same at all points in a conductor. $\frac{Q}{R}$ is therefore the potential of a sphere of radius R, with charge Q, when uninfluenced by any other electricity than its own.

Capacity.—The electrical capacity of a conductor is *the quantity of electricity required to charge it to unit potential*, when it is not influenced by any other electricity besides its own charge and the electricity which this induces in neighbouring conductors. Or since, in these circumstances, potential varies directly as charge, capacity may be defined as *the quotient of charge by potential.* Let C denote capacity, V potential, and Q charge, then we have

$$C = \frac{Q}{V}; \quad V = \frac{Q}{C}; \quad Q = VC.$$

But we have seen that, for a sphere of radius R, at a distance from other conductors or charged bodies, $V = \frac{Q}{R}$. Hence $C = R$; that is, the *capacity of a sphere is numerically equal to its radius.*

Connection between Potential and Induced Distribution.—In the circumstances represented in fig. 1, if we suppose the influencing body C to be positively charged, the potential due

to this charge will be algebraically greater at the near end A of the influenced conductor than at the remote end B. The induced electricity on AB must be so distributed as to balance this difference, in fact the potential due to this induced electricity is negative at A and positive at B. All cases of induced electricity upon conductors fall under the rule that *the potential at all parts of a conductor must be the same, and hence, wherever the potential due to the influencing electricity is algebraically greatest, the potential due to the electricity on the influenced conductor must be algebraically least.*

As there can be no force in the interior of a conductor, the force at any point in the interior, due to the influencing electricity, must be equal and opposite to the force due to the electricity on the surface of the conductor. This holds whether the conductor be solid or hollow. A hollow conductor thus completely screens from external electrical forces all bodies placed in its interior.

Electrical Images.—If a very large plane sheet of conducting material be connected with the earth, and an electrified body be placed in front of it near its middle, the plate will completely screen all bodies behind it from the force due to the electrified body. The induced electricity on the plate therefore exerts, at all points behind the plate, a force equal and opposite to that of the electrified body, or, what is the same thing, a force identical with that which the electrified body would exert if its electricity were reversed in sign. But electricity distributed over a plane surface must act symmetrically towards both sides. Hence the force which the induced electricity exerts in front is identical with that which would be exerted by a body precisely similar to the given electrical body, symmetrically placed behind the plane, and charged with the opposite electricity. The total force at any point in front of the plane is the resultant of the force due to the given electrified body and the force due to this imaginary image. The name and the idea of *electrical images*, of which this is one of the simplest examples, are due to Lord Kelvin.

CHAPTER IV

ELECTROSTATIC MACHINES

Electrical Machines.—The first electrical machine was invented by Otto Guericke. It consisted of a ball of sulphur which was turned upon its axis by one person while another held his hands upon the ball, thus causing the friction necessary for the production of electricity. The result was that the globe was negatively electrified, and the positive electricity escaped into the earth through the hands of the operator. This machine, however, was capable of producing only very feeble effects, and the sparks obtained from it were visible only in the dark. An English philosopher, Hawksbee, substituted a globe of glass for the globe of sulphur; the electricity thus obtained was positive, and the sparks obtained by the new machine were of considerable brightness.

Ramsden's Machine.—The kind of friction machine most commonly employed is the plate machine, invented by Ramsden about 1768. The most usual form of this machine is shown in fig. 14. It has a circular plate of glass, which turns on an axis supported by two wooden uprights. On each side of the plate, at the upper and lower parts of the uprights, are two cushions, which act as rubbers when the plate is turned. In front of the plate are two metallic conductors, supported on glass legs, and terminating in branches which are bent round the plate at the middle of its height, and are studded with points projecting towards it. The plate becomes charged with positive electricity by friction against the cushions, and gives off its electricity through the points to the two conductors, or, what amounts to the same thing, the conductors give off negative electricity through the points to the positively electrified plate. In order to avoid loss of electricity from that portion of the plate which is passing from the cushions to the points, sector-shaped pieces of oiled silk are placed so as to cover it on both sides. The cushions become negatively electrified by the friction, and the machine will not continue working unless this negative electricity is allowed to escape. The cushions are accordingly connected with the earth by means of metal plates let into their supports.

Before using the machine the glass legs which support the conductors should be wiped with a warm dry cloth. The plate

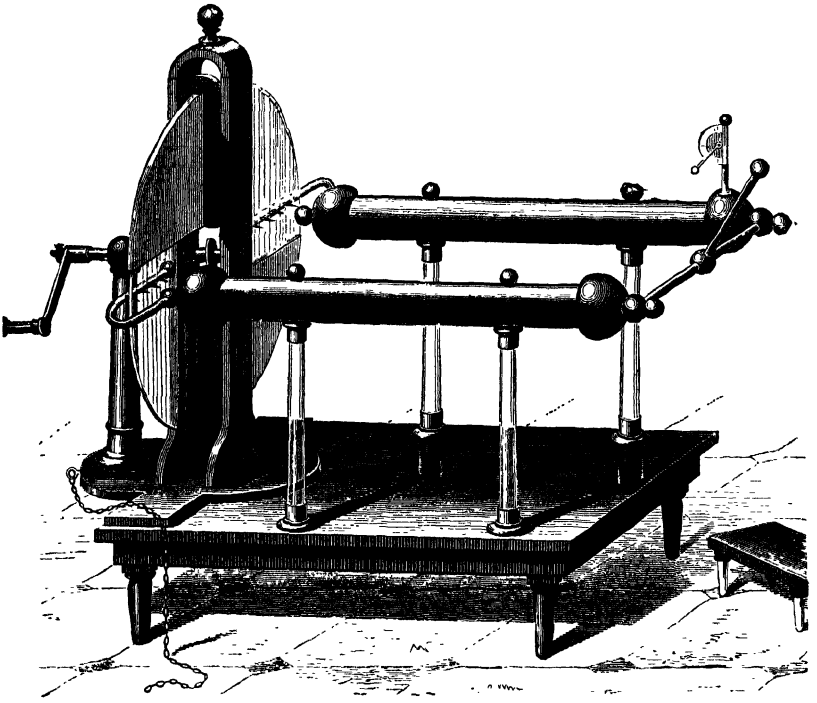


Fig. 14.—Ramsden's Electrical Machine

must also be cleaned from any dust or portions of amalgam which may adhere to it, and lastly dried with a hot cloth or paper. When these precautions are taken, the machine, if standing near a fire, will always work; but the charging of Leyden jars, and especially of batteries, may be rendered impossible by bad weather.

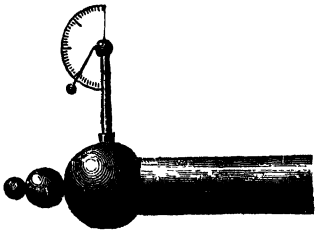


Fig. 15.—Quadrant Electroscope

The variations of charge are indicated by the *quadrant electroscope* (fig. 15), which is attached to one of the conductors. It consists of an upright conducting stem supporting a quadrant, or more commonly a semicircle of ivory, at whose centre a light needle of ivory is jointed, carrying a

pith ball at its end. When there is no charge in the conductor this pendulum hangs vertically, and as the charge increases it is repelled farther and farther from the stem. In damp weather it will be observed to return to the vertical position almost immediately on ceasing to turn the machine, while in very favourable circumstances it gives a sensible indication of charge after two or three minutes.

Electrophorus.—When electricity is required in comparatively small quantities it is readily supplied by the simple apparatus called the *electrophorus*. This consists (fig. 16) of a disk of shellac, or some other material easily excited by friction, and of a polished metal disk B with an insulating handle CD. The shellac disk is electrified by striking or rubbing it with catskin or flannel, and the metal plate is then laid upon it. In these circumstances the upper plate does not receive a direct charge from the lower, but if touched with the finger (to connect it with the earth) receives an opposite charge by induction. On lifting it away by its insulating handle it is found to be charged, and will give a spark. It may then be replaced on the lower plate (touching it at the same time with the finger), and the process repeated an indefinite number of times without any fresh excitation, if the weather is favourable. The shellac plate has usually a base of metal with a pin reaching to the surface of the shellac, which is in connection with the earth while the electrophorus is being worked. The cover thus receives its positive charge without touching it with the finger.

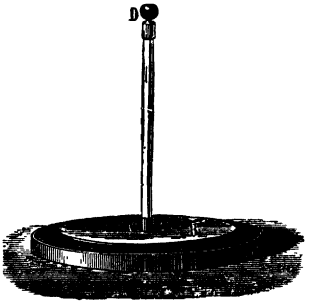


Fig. 16.—Electrophorus

Wimshurst's Machine.—The best of all the electrostatic machines of the present day is that of Mr. Wimshurst, fig. 17. It has two glass plates revolving with equal speeds in opposite directions, each plate carrying on its outer face a number of equidistant patches of tinfoil. The number should not be less than sixteen, and must be the same for both plates. Sometimes each patch is furnished with a projecting stud to make contact with the brushes, but this is of doubtful benefit. The curved bar of metal seen in front of the machine carries brass brushes

at its two ends which make simultaneous contact with a pair of diametrically opposite patches. This bar is adjustable, and is usually set to a slope of about 45°. At the back of the machine, just visible in the figure, is a similar bar sloping the opposite

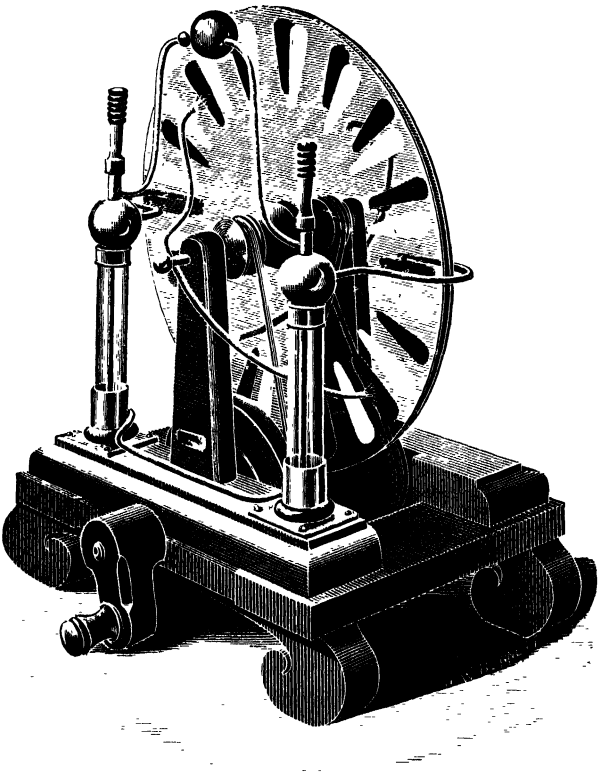


Fig. 17.—Wimshurst's Electrical Machine

way, so that the two bars are nearly at right angles to one another.

The electricity furnished by the machine is collected by brass combs embracing the plates at the ends of a horizontal diameter and connected with the two conductors, one positive and the other negative. In the figure each of the two conductors consists of a fixed sphere of brass, in which turns, by means of an insulating handle, a curved arm of brass terminating in a ball. The two balls are of unequal sizes, and can be interchanged so as to make either of them positive at

pleasure. Their distance apart is regulated by rotating the insulating handles.

As shown in the figure, the conductors are connected with the inner coatings of two Leyden jars, whose outer coatings are connected with each other. To disconnect the conductors from the jars, the vertical rods seen in the centres of the jars are lifted out.

The working of the machine will be understood from the accompanying diagram, fig. 18, which indicates the directions of rotation of the two plates and the signs of the charges on their tinfoil patches when the right-hand conductor is the positive one. At the moment when the brushes at A and B make contact with a pair of tinfoil patches on the front plate these two patches, together with the connecting rod AB, form one conductor, which is influenced by the negative electricity on the portion of the back plate opposite to A, and the positive on the portion opposite to B. Thus the patch at A acquires a positive and that at B a negative charge, and these charges are carried on till they are partly delivered up to the combs. Similar reasoning applies to the patches on the back plate. In passing C they become negatively and in passing D positively charged by the inductive action of the charges on the front plate. Thus the right-hand combs are supplied with positive electricity by both plates and the left-hand combs with negative.

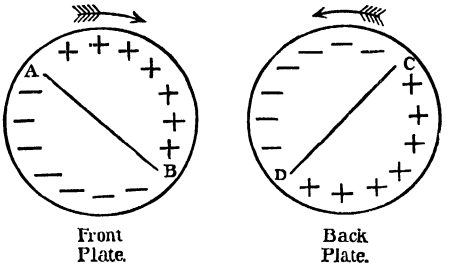


Fig. 18.—Working of Wimshurst Machine

CHAPTER V

EXPERIMENTS ON ELECTRIC DISCHARGE

Spark, Brush, and Glow.—Luminous discharges of electricity are usually classed as of three kinds: the spark, the brush, and the glow. The spark, as furnished by ordinary

electrical apparatus, may be of any length, from a small fraction of an inch to several inches. When short it is usually straight, and accompanied by an instantaneous sound which may be described as a crack. The crack becomes louder in proportion as the length of the spark is increased, and still more as the capacity of the discharging conductor is increased, the discharge of a battery of Leyden jars even across a small distance being of deafening loudness. When the spark is long and loud it is usually crooked, resembling on a small scale a flash of fork lightning. Such sparks are easily obtained from an induction machine when its Leyden jars are in connection with the terminals. When the jars are out of connection there is no loud crack, but the discharges succeed each other so rapidly as to produce a sizzling sound, and a continuous stream of fire (often consisting of several nearly parallel threads) plays across the interval. As the knobs are drawn farther apart, a dark interval is left near the negative knob. Lines of fire extend from the positive knob to a certain distance, where they terminate in mid-air; the negative knob exhibits a luminous glow, and the intervening region is dark. When the room is darkened, the direct discharge is seen to be surrounded by a nebulous luminosity of purple tint, shaped like an egg, with its ends at the two knobs, and an interruption or defect of brightness at the dark space above-mentioned.

When a conductor is giving off electricity from points or sharp edges into the air, it exhibits in the dark an appearance of lines of fire issuing from the points and spreading out into broader streams of fainter luminosity. This is called the *brush* discharge. It is best seen when the electricity which is escaping from the conductor is positive.

The glow discharge as seen in the dark renders the surface of the conductor luminous, but the luminosity does not extend into the surrounding air.

When an induction machine is working in the dark, the discharges which take place between the combs and the revolving glass plates appear very different at the two combs. The positive conductor is receiving positive or giving off negative electricity through its comb. The points of its comb are accordingly covered with a glow, and some luminosity is seen on the surface of the plate. The negative conductor is receiv-

ing negative or giving off positive electricity through its comb. Lines of fire proceed from its points and spread out into a large sheet of light on the surface of the plate.

Very interesting variations in the appearance of the discharge between the terminal knobs can be obtained by making the knobs very unequal in size and interchangeable, as in the Wimshurst machine. It is probable that the passage of a spark is always preceded by a state of stress in the intervening air, the stress being most intense along the path of least resistance, which at last gives way, the spark passing along the line of rupture.

Duration of the Spark.—We can form no judgment of the duration of the electric spark from what we see with the unaided eye; for impressions made upon the retina remain uneffaced for something like $\frac{1}{10}$ of a second, and the duration of the spark is incomparably less than this. The discharges of a Wimshurst or Voss machine with Leyden jars cause the revolving plates with their tinfoil sectors to be seen as if stationary. As this appearance can be seen even in full daylight, it shows that the spark, though very brief, is an extremely powerful illuminant while it lasts.

Wheatstone, in a classical experiment, measured the duration of the spark of a Leyden jar by means of a revolving mirror, and found it to be $\frac{1}{24000}$ of a second.

Oscillatory Discharge.—Feddersen, by taking instantaneous photographs of the spark of a Leyden jar as seen in a revolving mirror, has shown that it consists not of one discharge only, but of a number of successive discharges, a result which was predicted by Lord Kelvin from theory as early as 1853. The successive discharges are in opposite directions, and are analogous to the successive swings of a pendulum, first to one side and then to the other, in a medium whose resistance produces a rapid decrease of amplitude.

Physiological Effects of the Spark.—When the finger is held between the knobs of an induction machine, with the jars disconnected, the sparks produce a sharp pricking sensation. In general a pricking sensation more or less strong is felt at the place where a spark enters or leaves the body.

When the spark is taken from a Leyden jar, a shock is felt; especially if one hand is kept in contact with the outer coating

of the jar while the other hand is brought to the knob. The sensation experienced is altogether unique, and is accompanied by spasmodic contraction of the muscles.

At the distance of a few feet from a machine in powerful action a tickling sensation is felt on the exposed parts of the body, due to the movement of the hairs in obedience to electrical force. These phenomena are exhibited in a still more marked manner when a person stands on a stool with glass legs, and keeps his hand upon the conductor. He thus becomes highly charged with electricity. His hair stands on end, and is luminous if seen in the dark. If a conductor connected with the earth is presented to him, a spark passes, and his hair falls again.

Electricity has frequently been resorted to for medical purposes. The electrical machine was first employed, and afterwards the Leyden jar, but both have now been abandoned in favour of magneto-electric machines and other apparatus for obtaining induced currents, which will be described later.

Heating and Mechanical Effects.—The heating power of the electric spark may be conveniently shown by taking the discharge of an induction machine (with or without jars attached) across gas escaping from a burner. The gas is instantly ignited.

If a person standing on a stool with glass legs keeps one hand in permanent contact with one conductor of the machine, while the other conductor is connected to the earth, he can light gas by giving a spark from his other hand to the burner. The luminosity of the spark is doubtless due, either wholly or in part, to the very high temperature which is produced in the particles of air traversed by the discharge.

The spark of an induction machine with jars attached will puncture card, the hole being larger or smaller according to the strength of the spark. Thin glass can be punctured by the discharge of a battery of jars.

Wind from Points.—If a metallic rod terminating in a point be attached to the conductor of the electrical machine, electricity escapes in large quantity from the point, which, accordingly, when viewed in the dark, is seen to be crowned with a tuft of light. A layer of air in front of the point is electrified by contact, and then repelled, to make way for other portions of air,

which are in their turn repelled. A continuous current of air is thus kept up, which is quite perceptible to the hand, and produces a very visible effect on the flame of a taper.

The *electric whirl* (fig. 19) consists of a set of metallic arms, radiating horizontally from a common centre about which they can turn freely, and bent, all in the same direction, at the ends, which are pointed. When the central support is mounted on the conductor of the machine, the arms revolve in a direction opposite to that in which their ends point. This effect is due to the mutual repulsion between the pointed ends and the electrified air which flows off from them.

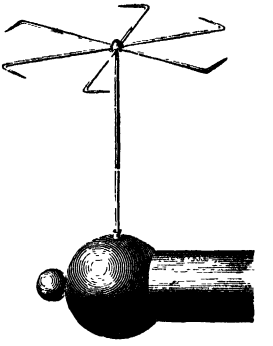


Fig. 19.—Electric Whirl

It is instructive to remark that if, by a special arrangement, the rotating part be enclosed in a well-insulating glass case, the rotation soon ceases, because in these circumstances the enclosed air quickly attains a state of permanent electrification.

CHAPTER VI

ELECTRICAL CONDENSERS

Condensers.—The process called *condensation of electricity* consists in increasing the capacity of a conductor by bringing near it another conductor connected with the earth, or with some other source from which electricity can be supplied. The two conductors are usually thin plates or sheets of metal, with a larger plate of non-conducting material between them.

Let A and B (fig. 20) be two conducting plates, of which A, called the *collecting plate*, is connected with the conductor of the machine, and B, called the *condensing plate*, with the earth; and let C be the non-conducting plate (or dielectric) which separates them. Then if the machine is turned, and a charge thereby given, the surface of B which faces towards A is covered with negative electricity drawn from the earth and held

by the attraction of the positive electricity of A; and, conversely, the surface of A which faces towards B is covered with positive electricity, held there by the attraction of the negative of B, in addition to the charge which would reside upon it if the conductor were at the existing potential, and B and C were absent. In fact, the electrical density on the face of A, as well as the whole charge of A, would, in this latter case, be almost inappreciable, in comparison with those which exist in the actual circumstances. By condensation of electricity, then, we are to understand *increase*—usually enormous increase—*of electrical density on a given surface, attained without increase of potential.*

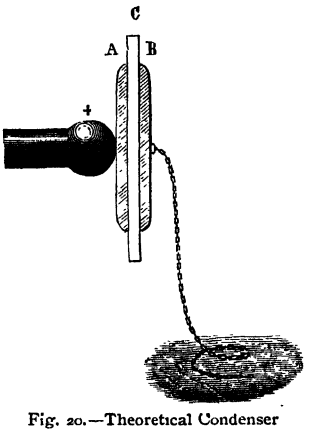


Fig. 20.—Theoretical Condenser

If two conducting plates, in other respects alike, but one with and the other without, a condensing plate, be connected by a wire, and the whole system be electrified, the two plates will have the same potential, but nearly the whole of the charge will reside upon the face of that which is accompanied by a condensing plate.

The lines of force between the two plates A and B are everywhere sensibly straight and perpendicular to the plates, with the exception of a very small space round the edge, which may be neglected. The tubes of force are cylinders, and the intensity of force is constant at all parts of their length. Also, since the potential of the plate B is zero, if we take V to denote the potential of the plate A, which is the same as the potential of the conductor, and t to denote the thickness of the intervening plate C, the rate at which potential varies along a line of force is $\frac{V}{t}$, which is the expression for the force at any point between the plates A, B. The whole space between the plates may be regarded as one cylindrical tube of force of cross-section S equal to the area of either plate, the two ends of the tube being the inner faces of the plates. The quantities of electricity $\pm Q$ residing on these faces are therefore equal, but of opposite sign; and as the force changes from nothing to $\frac{V}{t}$ in passing from one side to the other of the

electricity which resides on either of these surfaces, we have

$$\frac{V}{t} \cdot S = 4\pi Q.^{1}$$

Hence the capacity of the plate A, being, by definition, equal to $\frac{Q}{V}$, is equal to

$$\frac{S}{4\pi t}.$$

We should, however, explain that, if the intervening plate C is a solid or liquid, we are to understand by t not the simple thickness, but the thickness reduced to an equivalent of air, in a sense which will be explained farther on. This reduced thickness is, in the case of glass, about half the actual thickness.

If s denote an element of area of A, and q the charge residing upon it, it is evident, from considering the tube of force which has s for one of its ends, that

$$\frac{V}{t} \cdot s = 4\pi q;$$

and the electric density $\frac{q}{s}$ on the element is equal to $\frac{V}{4\pi t}$, which is constant over the whole face of the plate.

To give a rough idea of the increase of capacity obtained by the employment of a condensing plate, let us compare the capacity of a circular disk of 10 in. diameter, accompanied by a condensing plate at a distance of $\frac{1}{20}$ in., with the capacity of a globe of the same diameter as the disk. The capacity of the globe is equal to its radius, and may therefore be denoted by 5. The capacity of the disk is $\frac{25\pi}{4\pi \times \frac{1}{20}} = 125$, or 25 times the capacity of the globe. It is, in fact, the same as the capacity of a globe 250 in. (or 20 ft. 10 in.) in diameter.

The form of condenser chiefly employed for electrostatic experiments is the *Leyden jar* (figs. 21, 22). It consists of a glass

[1] If we conceive a narrow tube bounded on all sides by lines of force and call it a tube of force, we can lay down the following remarkable rules for the comparison of the forces which exist at different points in its length: (1) In any portion of a tube of force not containing electricity the intensity of force varies inversely as the cross section of the tube, or the product of intensity of force by section of tube is constant. This is obviously analogous to the law which applies to the comparison of the velocities of a liquid in different parts of a tube through which it flows, since the product of area by velocity is the volume of liquid which flows past any section in unit time. (2) When a tube of force cuts through electricity this product changes, from one side of this electricity to the other, by the amount of $4\pi Q$, where Q denotes the quantity of the electricity enclosed by the tube.

jar coated both inside and out with tinfoil for about four-fifths of its height. The mouth is often closed by a cork, through which passes a metallic rod, terminating above in a knob, and connected below with the inner coating, either by a chain hanging from it, or by pieces of metallic foil with which the jar is filled. The interior of the jar must be thoroughly dry before it is closed, and the cork and neck are usually covered

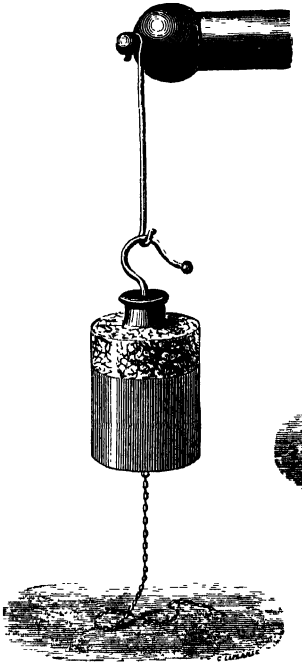


Fig. 21.—Leyden Jar

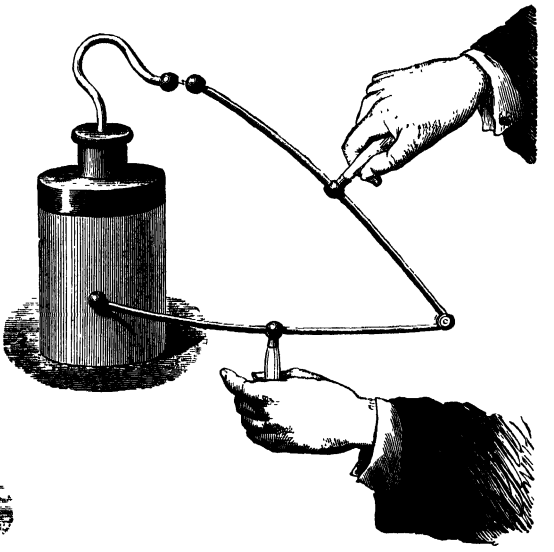


Fig. 22.—Discharge of Leyden Jar

with sealing wax and shellac varnish, which are less hygroscopic than glass. The two coatings of tinfoil perform the parts of a collecting plate and a condensing plate. The outer coating is usually connected with the earth, and the knob and inner coating with the conductor (or one of the two conductors) of an electrical machine. The inner coating will then acquire a charge of one sign, and the outer coating an equal (or practically equal) charge of the opposite sign.

Discharge of Leyden Jar.—To exhibit the discharge of a Leyden jar, it is usual to employ a discharger consisting of a glass handle or pair of handles attached to two brass rods

hinged together and terminating in knobs (fig. 22). The outer coating is first touched with one knob, and while this contact is maintained the other knob is brought into contact with the knob of the jar. Before this last contact is completed, a bright spark leaps across the interval with a loud report.

Discharge by Alternate Contacts.—Instead of discharging a Leyden jar at once by connecting its two coatings, we may gradually discharge it by alternate contacts. To do this we must set it on an insulating stand (or otherwise insulate both coatings from the earth), and then touch the two coatings alternately. At every contact a small spark will be drawn. The coating last touched has always rather less electricity upon it than the other, but the difference is an exceedingly small fraction of the whole charge, and, after a great number of sparks have been drawn by these alternate contacts, we may still obtain a powerful discharge by connecting the two coatings.

Residual Charge.—A jar which has been kept charged for some time cannot at once be completely discharged. After connecting the coatings, then disconnecting them, and then waiting for a time, another discharge can be obtained, very much smaller than the first, but often sufficient to produce a visible and audible spark. Other and smaller discharges can be subsequently obtained in the same way. It is as if the charge soaked into the glass and gradually oozed out again. Elastic solids exhibit a mechanical analogy; for instance, a wire subjected to long-continued torsion, and then set free, does not instantly lose all its torsion, but continues for some time a slow process of recovery. The charging of a Leyden jar puts the glass between the coatings into a strained condition, which takes some time to subside.

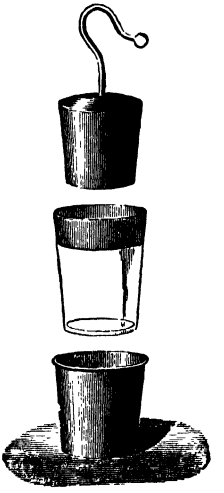


Fig. 23.—Jar with Movable Coatings

Franklin's experiment of the *jar with movable coatings* (fig. 23) is interesting as showing that the glass of a jar can retain its charge after the coatings are removed. The jar is charged in the ordinary way and placed on an insulating stand. The

inner coating is then lifted out by means of a stick of sealing wax or other insulator, and touched with the hand to discharge it of any electricity which it may retain. (The glass may also be lifted out and the outer coating discharged, but this is scarcely necessary.) On reinserting the inner coating by means of the insulator, the jar will be found to give nearly as strong a spark as it would have given originally.

Influence of the Dielectric.—Faraday discovered that the capacity of a condenser is changed by changing the material between the plates (called the dielectric), being always greater with a solid or liquid dielectric than with air; and he gave the name *specific inductive capacity* to the multiplier which must be applied to the capacity of an air condenser, to obtain the capacity when the air is replaced by the substance in question. His measurements of its value were made with the pattern of condenser shown in fig. 24. It is a kind of Leyden jar, containing a metallic sphere A, attached to the rod M, and forming with it the inner conductor. The outer conductor consists of the hollow sphere B divided into two hemispheres which can be detached from each other. The interval between the outer and inner conductor can be filled, either with a cake of solid non-conducting material, or with gas, which can be introduced by means of the cock R. The method of observation and reduction will be best understood from an example.

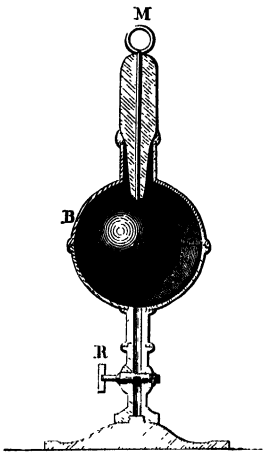


Fig. 24.—Apparatus for Specific Inductive Capacity

The interval being occupied by air, the apparatus was charged, and a carrier ball (a brass ball with insulating handle), having been made to touch the summit of the knob M, was introduced into a Coulomb's torsion balance, and found to be charged with a quantity of electricity represented by 250° of torsion. With a second uncharged apparatus precisely similar to the first, it was found that, on contact of the two knobs, the charge divided itself equally, and the carrier ball, if applied to either knob, took a charge represented very nearly by 125°.

The conditions were then changed in the following way.

The first jar still containing air, the interval between the two conductors in the second was filled with shellac. It was then found that the air jar, being charged to 290°, was reduced, by contact of its knob with that of the shellac jar, to 114°, thus losing 176°. If no allowance be made for dissipation the capacities of the air jar and shellac jar would therefore be as 114 : 176, or as 1 : 1.54, and the specific inductive capacity of shellac would be 1.54.

With the improved methods now available much more accurate measurements have been made, and they give larger values than those obtained by Faraday. The best determinations are made by very rapid charges and discharges, which have the advantage of minimizing the residual charge.

Polarization of the Dielectric.—As the interposed non-conductor, or dielectric, modifies the mutual action of the two electricities which it separates, and does not play the mere passive part which was attributed to it before Faraday's experiments, it is natural to conclude that the dielectric must itself experience a peculiar modification. According to Faraday, this modification consists in a polarization of its particles, which act inductively upon each other along the lines of force, and have each a positive and a negative side, the positive side of each facing the negative side of the next. This polarization is capable of being sustained for a great length of time in good non-conductors; but in good conductors it instantly leads to discharge between successive particles, and the opposite electricities appear only at the two surfaces.

Condensers for Voltaic Electricity.—The capacity of a condenser varies inversely as the distance between the collecting and the condensing plate. But if this distance is very small, the resistance of the interposed dielectric may be insufficient to prevent discharge, and it will not be practicable to establish a great difference of potential between the two coatings. We may practically distinguish two sorts of condensers, one having a dielectric thick enough to resist the tensions of powerful electrostatic machines, the other having a very thin dielectric and very large capacity. Condensers of the latter kind are much used in connection with voltaic batteries. They are constructed by arranging a number of sheets of tinfoil (sometimes several hundreds) in a pile, according to the plan sketched in fig. 25,

with either mica or paraffined paper between them. The first, third, fifth, &c., sheets of tinfoil are connected at one end, and are to be regarded as forming one coating; the second, fourth, sixth, &c., are connected at the other end, and constitute the other coating. The condenser may be charged by connecting each coating with one pole of a voltaic battery.

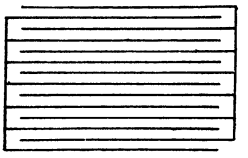


Fig. 25.—Condenser of Tinfoil and Paraffined Paper

Volta's Condensing Electroscope. — This instrument, which has rendered very important services to the science of electricity, differs from the simple gold-leaf electroscope in having at its top two metal plates, of which the lower one is connected with the gold leaves, and is covered on its upper face with insulating varnish, while the upper is varnished on its lower face, and furnished with a glass handle. These two plates constitute the condenser. In using the instrument one of the two plates (it matters not which) is charged by means of the body to be tested, while the other is connected with the earth. They thus receive opposite and sensibly equal charges. The upper plate is then lifted off, and the higher it is raised the wider do the gold leaves diverge. The separation of the plates diminishes the capacity and strengthens the potential of both, one becoming more strongly positive and the other more strongly negative. This involves increase of potential energy, which is represented by the amount of work done against electrical attraction in separating the plates. No increase in quantity of electricity is produced by the separation; hence the instrument is chiefly serviceable in detecting the presence of electricity which is available in large quantity but at low potential.

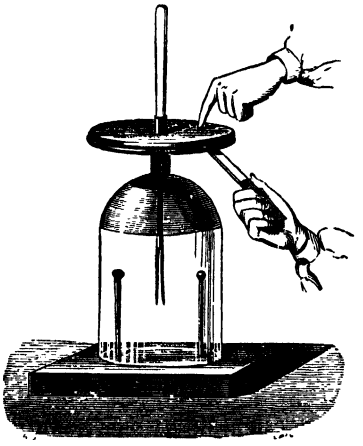


Fig. 26.—Condensing Electroscope

Leyden Battery.—The Leyden battery consists of a number of Leyden jars placed in compartments of a box lined with tin-

foil, which serves to establish good connection between their outer coatings, while their inner coatings are connected by brass rods. In charging by an influence machine the outer coatings are connected with one terminal and the inner coatings with the other. The capacity of a battery is the sum of the capacities of the jars which compose it, and its discharges are correspondingly powerful.

Cascade Arrangement.—Instead of connecting all the inner coatings together and all the outer coatings together, as in the battery arrangement, we may connect the outer coating of the first jar to the inner of the second, the outer of the second to the inner of the third, and so on; the inner coating of the first being connected with one terminal of the machine, and the outer coating of the last with the other terminal or with the earth. All the jars except one must be on separate insulating supports. When the machine is worked, a charge (which we will suppose to be positive) is given to the inner coating of the first, and a sensibly equal negative charge is induced upon its outer coating, this negative charge being drawn from the inner coating of the second, which accordingly is left with an equal positive charge. The second jar acts in the same way upon the third, and so on. Each inner coating thus acquires a charge $+Q$ (all equal), and each outer coating a charge $-Q$. If we further suppose all the jars to have the same capacity C (defined as *charge* of either coating *divided by difference of potentials* of the two coatings), and if we denote the potentials of the inner coatings by V_1, V_2, . . . V_n, we shall have (if the outer coating of the last jar is earthed)

$$\frac{Q}{C} = V_1 - V_2 = V_2 - V_3 = , \&c., = V_n - 0.$$

By adding the n equal differences, we obtain $V_1 - 0$, which is accordingly equal to n times Q/C. Hence each of the n differences is equal to V_1/n, and $Q/V_1 = C/n$.

Jars connected in this way are said to be *in cascade*, the name being suggested by the successive falls of potential from jar to jar.

CHAPTER VII

ATMOSPHERIC ELECTRICITY

Resemblance of Lightning to the Electric Spark.—The resemblance of the effects of lightning to those of the electric spark struck the minds of many of the early electricians. Lightning, in fact, ruptures and scatters non-conducting substances, inflaming those which are combustible; heats, reddens, melts, and volatilizes metals; and gives shocks, more or less severe, and frequently fatal, to men and animals; all of these being precisely the effects of the electric spark with merely a difference of intensity. We may add that lightning leaves behind it a characteristic odour precisely similar to that which is observed near an electrical machine when it is working, and which we now know to be due to the presence of ozone. Moreover, the form of the spark, its brilliancy, and the detonation which attends it all remind one forcibly of lightning.

To Franklin, however, belongs the credit of putting the identity of the two phenomena beyond all question, and proving experimentally that the clouds in a thunderstorm are charged with electricity. This he did by sending up a kite, armed with an iron point with which the hempen string of the kite was connected. To the lower end of the string a key was fastened, and to this again was attached a silk ribbon intended to insulate the kite and string from the hand of the person holding it. Having sent up the kite on the approach of a storm, he waited in vain for some time even after a heavy cloud had passed directly over the kite. At length the fibres of the string began to bristle, and he was able to draw a strong spark by presenting his knuckle to the key. A shower now fell, and by wetting the string improved its conducting power, the silk ribbon being still kept dry by standing under a shed. Sparks in rapid succession were drawn from the key, a Leyden jar was charged by it, and a shock given.

Shortly before this occurrence, Dalibard, acting upon a published suggestion of Franklin, had erected a pointed iron rod on the top of a house near Paris. The rod was insulated from the earth, and could be connected with various electrical appa-

ratus. A thunderstorm having occurred, a great number of sparks, some of them of great power, were drawn from the lower end of the rod.

These experiments were repeated in various places, and Richmann, of St. Petersburg, while conducting an investigation with an apparatus somewhat resembling that of Dalibard, received a spark which killed him on the spot.

Duration of Lightning.—It appears that thunder clouds must be regarded as charged masses of considerable conducting power. The discharges which produce lightning and thunder occur sometimes between two clouds, and sometimes between a cloud and the earth. The duration of the illumination produced by lightning is certainly less than the ten-thousandth of a second. This has been established by observing a rapidly rotating disk (fig. 27) divided into sectors alternately black and white. If viewed by daylight the disk appears of a uniform grey; and if lightning, occurring in the dark, renders the separate sectors visible, the duration of its light must be less than the time of revolving through the breadth of one sector. The experiment has been tried with a disk divided into sixty sectors, and making 180 revolutions per second, so that the time of turning through the space occupied by one sector is $\frac{1}{60}$ of $\frac{1}{180}$ of a second, that is, $\frac{1}{10800}$. When the disk, turning with this velocity, is rendered visible by lightning, the observer sees black and white sectors with grey ones between them. For the black and white sectors to be seen sharply defined, without intermediate grey, it would be necessary that the illumination should be absolutely instantaneous.

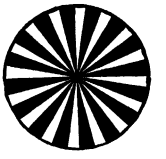


Fig. 27.—Duration of Flash

Thunder.—Thunder frequently consists of a number of reports heard in succession. This can be explained by the crookedness of the line of discharge. The sound from the nearest portions will be heard earliest; and if any considerable portion of the path of discharge is at a uniform distance from the observer, the simultaneous arrival of the disturbances propagated from all this portion will produce a specially loud burst of sound. Persons near whom a flash of lightning passes, frequently experience a severe shock by induction.

Lightning Conductors.—Experience having shown that

electricity travels in preference through the best conductors, it is easy to understand that, if a building be fitted with metallic rods terminating in the earth, lightning will travel through these instead of striking the building. But further, if these rods terminate above in a point, they may exercise a preventive influence by enabling the earth and clouds to exchange their opposite electricities in a gradual way, just as the conductor of a machine is prevented from giving powerful sparks by presenting to it a sharp point connected with the earth. The following conditions should always be complied with:—

1. The connection with the ground should be continuous.

2. The conductor must be everywhere of so large a section that it will not be melted by lightning passing through it.

3. The earth contact must be good. The conductor may be connected at its base with the iron pipes which supply the neighbourhood with water; or it may terminate in the water of a well or pond. Failing these, it should be provided with branches traversing the soil in different directions and surrounded by coke, which is a good conductor.

4. At no part of its course above-ground should it come near to the metal pipes which supply the house with water or gas, nor to any large masses of metal in the house. All large masses of metal on the outside of the house, such as lead roofing, should be well connected with the conductor.

5. It should terminate above in a sharp point or points, which should project above the highest parts of the building.

The conditions which determine the path of a sudden and violent discharge like lightning are not quite the same as those which apply to a steady current. There is a tendency to leap across a short air gap rather than take a circuitous course through a conductor. The discharge also tends to travel along the skin of a conductor instead of pervading its whole substance. Hence flat bands have an advantage over solid rods.

Ordinary Electricity of the Atmosphere.—The presence of electricity in the upper regions of the air is not confined to thunder clouds, but can be detected at all times. In fine weather this electricity is almost invariably positive, but in showery or stormy weather negative electricity is as frequently met with as positive; and it is in such weather that the indica-

tions of electricity, whether positive or negative, are usually the strongest.

Methods of obtaining Indications.—A method of collecting electricity from the air was long ago devised by Volta, who employed for this purpose a burning match attached to the top of a rod connected with the gold leaves or straws of his electroscope. If there is positive electricity overhead, its influence causes negative electricity to collect at the upper end of the rod, whence it passes off by convection in the products of combustion of the match, leaving the whole conducting system positively electrified. In like manner, if the electricity overhead be negative, the system will be left negatively electrified.

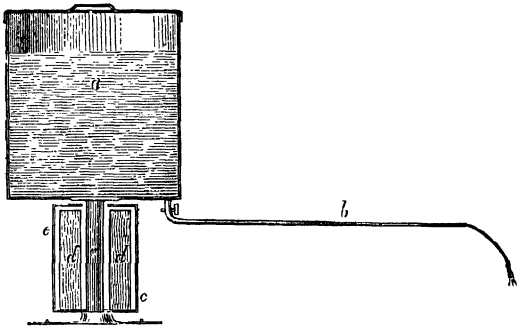


Fig. 28.—Water-dropping Collector

Another method was introduced by Lord Kelvin. It consists in allowing a fine stream of water to flow from an insulated metallic vessel, through a pipe, which projects through an open window or other aperture in the wall of a house, so that the nozzle from which the water flows is in the open air. The apparatus for this purpose, called the water-dropping collector, is represented in fig. 28. *a* is a copper can, containing water, which can be discharged through the brass pipe *b* by turning a tap. The mode of insulation is worthy of notice. The can is supported on a glass stem *c*, which is surrounded without contact by a ring or rings of pumice *d d*, moistened with sulphuric acid. These are protected by an outer case of brass *e e*, having a hole in its top rather larger than the glass stem, the brass being separated from the moist pumice by an inner case of gutta percha. The acid needs renewal about once in two months.

In severe frost, burning matches can be used instead of water, and give nearly identical indications. Whether water or match be used, the principle of action[1] is that as long as any difference of potential exists between the insulated conductor and the point of the air where the issuing stream (whether of water or smoke) ceases to be one continuous conductor, and begins to be a non-conductor or a succession of detached drops, so long will each drop or portion that detaches itself carry off either positive or negative electricity, and thus diminish the difference of potential. The time required to reduce the system to the potential which exists at the point above specified is practically about half a minute with the water jet, and from half a minute to a minute or more, according to the strength of the wind, with a match.

Interpretation of Indications.—We have seen that the collecting apparatus, whether armed with water jet or burning match, is merely an arrangement for reducing an insulated conductor to the potential which exists at a particular point in the air. An electrometer will then show us the difference between this potential and that of any other given conductor, for example, the earth. The earth offers so little resistance to the passage of electricity that any temporary difference of potential which may exist between different parts of its surface must be very slight in comparison with the differences of potential which exist between different points in the non-conducting atmosphere above it. As there is no possible method of determining absolute potential, since all electric phenomena would remain unchanged by an equal addition to the potentials of all points, it is convenient to assume, as the zero of potential, that of the

[1] The following quotation from an article by Lord Kelvin puts the matter very clearly: "If, now, we conceive an elevated conductor, first belonging to the earth, to become insulated, and to be made to throw off, and to continue throwing off, portions from an exposed position of its surface, this part of its surface will quickly be reduced to a state of no electrification, and the whole conductor will be brought to such a potential as will allow it to remain in electrical equilibrium in the air, with that portion of its surface neutral. In other words, the potential throughout the insulated conductor is brought to be the same as that of the particular equipotential surface in the air, which passes through the point of it from which matter breaks away. A flame, or the heated gas passing from a burning match, does precisely this: the flame itself, or the highly heated gas close to the match, being a conductor which is constantly extending out, and gradually becoming a non-conductor. The drops into which the jet issuing from the insulated conductor, on the plan introduced by the writer, produce the same effects, with more pointed decision, and with more of dynamical energy to remove the rejected matter, with the electricity which it carries, from the neighbourhood of the fixed conductor."—*Papers on Electrostatics and Magnetism*, pp. 207, 208.

most constant body to which we have access, namely, the earth; and under the name earth we include trees, buildings, animals, and all other conductors in electrical communication with the soil.

Now we find that, as we proceed farther from the earth's surface, whether upwards from a level part of it, or horizontally from a vertical part of it, such as an outer wall of a house, the potential of points in the air becomes more and more different from that of the earth, the difference being, in a broad sense, simply proportional to the distance. Hence we can infer that there is electricity residing on the surface of the earth; the density of this electricity, at any moment, in the locality of observation, being measured by the difference of potential which we find to exist between the earth and a given point in the air near it. Observations of so-called atmospheric electricity made in the manner we have described are in fact simply determinations of the quantity of electricity residing on the earth's surface at the place of observation. The results of observations so made are, however, amply sufficient to show that electricity residing in the atmosphere is really the main cause of the variations observed. A charged cloud or body of air induces electricity of the opposite kind to its own on the parts of the earth's surface over which it passes; and the variations which we find to occur in the electrical density at the parts of the surface where we observe are so rapid and considerable that no other cause but this seems at all adequate to account for them. We may therefore safely assume that the difference of potential which we find, in increasing our distance from the earth, is mainly due to electricity induced on the surface of the earth by opposite electricity in the air overhead.

Results of Observation.—In fine weather the potential observed (taking that of the earth as zero) is positive, and more and more positive as we go higher; the electric force is therefore directed from above downwards, indicating a positive charge somewhere in the air above, and a negative charge on the earth's surface below. In wet weather the potential is often negative, and a change of sign not infrequently accompanies a sudden downpour of rain.

MAGNETISM

CHAPTER VIII

LAWS AND EXPERIMENTAL DETAILS

Magnets, Natural and Artificial. — Natural magnets, or *lodestones*, are exceedingly rare, although a closely allied ore of iron, capable of being strongly acted on by magnetic forces, and hence called *magnetic iron ore*, is found in large quantity in Sweden and elsewhere. Artificial magnets are usually pieces of steel which have been permanently endowed with magnetism by methods which we shall hereafter describe. Magnets are chiefly characterized by the property of attracting iron, and by the tendency to assume a particular orientation when freely suspended.

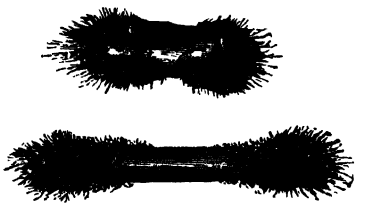


Fig. 29.—Magnets dipped in Filings

Force Greatest at the Ends. —The property of attracting iron is very unequally manifested at different points of the surface of a magnet. If, for example, an ordinary bar magnet be plunged in iron filings, these cling in large quantities to the terminal portions, and leave the middle bare, as in the lower diagram of fig. 29. If the magnet is very thick in proportion to its length, we may have filings adhering to all parts of it, but the quantity diminishes rapidly towards the middle. The name *poles* is used in a somewhat loose sense to denote the two terminal portions of a magnet, or to denote two points, not very accurately defined, situated in these portions. The middle portion to which the filings refused to adhere is called *neutral*.

Lines formed by Filings.—If a sheet of card is laid horizontally upon a magnet, and wrought-iron filings are sifted over it, these will, with the assistance of a few taps given to the card, arrange themselves in a system of curved lines, as shown in fig. 30. These lines give very important indications both of the direction and intensity of the force produced by the magnet at different points of the space around it. They cluster very closely about the two poles *p*, *p*, and thus indicate the places where the force is most intense.

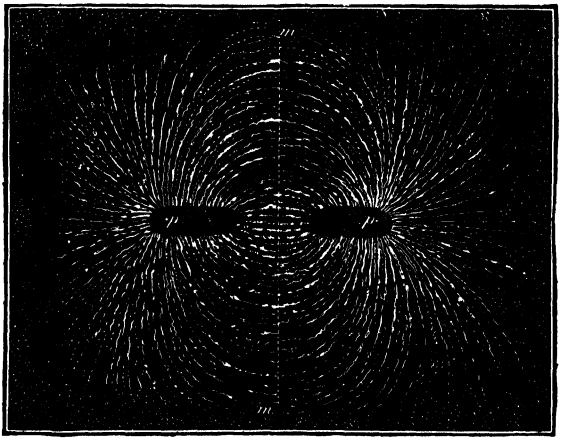


Fig. 30.—Magnetic Curves

Curve of Intensities.—Some idea may be obtained of the relative intensities of magnetic force at different points in the length of a magnet by measuring the force required to detach an iron ball at various points. Better determinations can be obtained by counting the number of vibrations made by a small magnetized needle when suspended opposite different parts of the bar, the bar being in a vertical position, and the vibrations of the needle being horizontal. The intensity of the force is nearly as the square of the number of vibrations; on the same principle that the force of gravity at different places is proportional to the square of the number of vibrations of a pendulum. Both these methods of determination were employed by Coulomb, who was the first to make magnetism an accurate science; and the results which he obtained are represented by the curve of intensities AMB (fig. 31). M is

the middle of the bar, O one end of it, and the ordinates of the curve (that is, the distance of its points from the line OX) represent the intensities of force at the different points in its length. Perfect symmetry was found between the intensities over the two halves of the length. In the figure the curve for one-half is inverted, in order to indicate an opposition of properties. The curves of intensities for two magnets of different sizes but of the same form are usually similar.

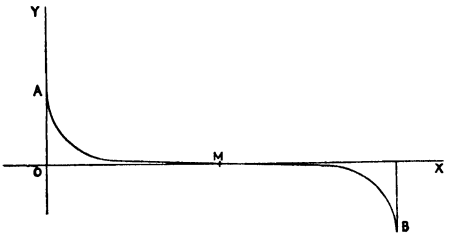


Fig. 31.—Curve of Intensities

Magnetic Needle.—Any magnet freely suspended near its centre is usually called a *magnetic needle*, or more properly a *magnetized needle*. One of its most usual forms is that of a very elongated rhombus of thin steel, having very near its centre a concavity or *cup* by means of which it can be balanced on a point. When it is thus balanced horizontally it does not, like a piece of ordinary matter, remain in equilibrium in all azimuths,[1] but assumes one particular direction, to which it always comes back after displacement. In this position of stable equilibrium one of its ends points to magnetic north, and the other to magnetic south, which differ in general by several degrees from geographical (or true) north and south. This is the principle on which compasses are constructed.

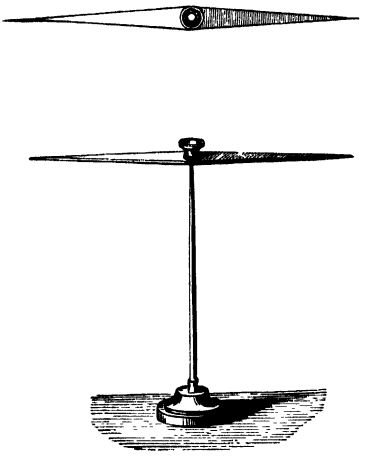


Fig. 32.—Magnetized Needle

[1] All lines in the same vertical plane are said to have the same *azimuth*. Azimuthal angles are angles between vertical planes, or between horizontal lines. The azimuth of a line, when stated numerically, is the angle which the vertical plane containing it makes with a vertical plane of reference, and this latter is usually the plane of the meridian. The plane of the *meridian* means a vertical plane passing through the place of observation, and through or parallel to the earth's axis. A horizontal line in this plane is a meridian line. The *magnetic*

Declination. — The difference between magnetic and true north, or the angle between the magnetic meridian and the geographical meridian, is called *magnetic declination*.[1] It is very different at different places, and at a given place undergoes a gradual change from year to year, besides smaller changes, backwards and forwards, which are continually taking place. At Greenwich its value is about 15° W., that is, magnetic north is west of true north by this amount.

Inclination or Dip. — If, before magnetizing a needle, we mount it on an axis passing through its centre of gravity, and support the ends of the axis, as in fig. 33, by a torsionless fibre, the needle will exhibit no preference for any particular position. If it be then magnetized, it will no longer be indifferent, but will place itself in a particular vertical plane (the magnetic meridian) and at a definite inclination to the horizon. Supposing the experiment to be accurately performed (which is difficult in practice), this inclination to the horizon is called the *dip* or the *magnetic inclination*. In the northern hemisphere generally, the end which points north points down, and in the southern hemisphere the opposite rule holds. The dip at Greenwich is about 67°.

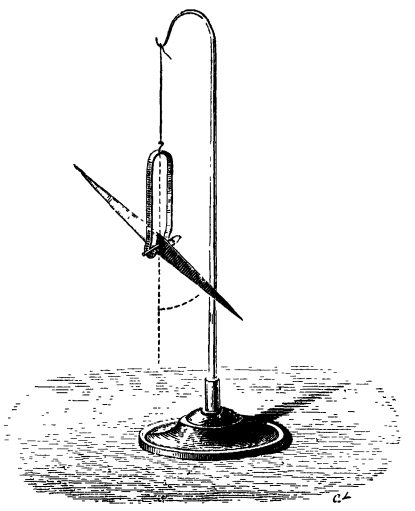


Fig. 33.—Dip

If a magnetized needle is to be balanced in a horizontal position (as in mariners' compasses), the point of support must be between the centre of gravity and the end which tends to dip. Needles thus balanced are called *declination needles*.

Mutual Action of Poles. — On presenting one end of a magnet to one end of a needle thus balanced, we obtain either

meridian is the vertical plane in which a magnetized needle, when freely suspended, tends to place itself.

[1] The nautical name for magnetic declination is *variation*; but it is inconvenient to denote the element itself by the same name as the variations of the element.

repulsion or attraction, according as the pole which is presented is similar or dissimilar to that to which it is presented. *Poles of contrary name attract each other; poles of the same name repel each other.*

This property furnishes the means of distinguishing a body which is merely magnetic (that is, capable of temporary magnetization) from a permanent magnet. The former, a piece of soft iron for example, is attracted by either pole of a magnet; while a body which has received permanent magnetization has, in ordinary cases, two poles, of which one is attracted where the other is repelled. Magnetic attractions and repulsions are exerted without modification through any body which may be interposed, provided it be not magnetic.

Names of Poles.—The phenomena of declination and inclination, above described, evidently require us to regard the earth in a broad sense as a magnet, having one pole in the northern and the other in the southern hemisphere. Now, since poles which attract one another are dissimilar, it follows that the magnetic pole of the earth which is situated in the northern hemisphere is *dissimilar* to that end of a magnetized needle which points to the north. Hence there is some doubt as to which end of a needle or magnet is to be called the *north* pole. We shall avoid this ambiguity by calling that end of it which seeks the north the *north-seeking* end or pole, and the other the *south-seeking* end or pole. Lord Kelvin called the north-seeking pole the *true south* pole, and the other the *true north* pole, because the former is similar to the south, and the latter to the north pole of the earth. Popular usage calls the north-seeking end the *north*, and the other the *south* pole, a nomenclature which tends to introduce confusion whenever we have to reason respecting the earth regarded as a magnet. Faraday, to avoid the ambiguity which has attached itself to the names north and south pole, calls the north-seeking end the *marked*, and the other the *unmarked* pole. Airy, for a similar reason, employed, in his *Treatise on Magnetism*, the distinctive names *red* and *blue* to denote respectively the north-seeking and south-seeking ends, these names, as well as those employed by Faraday, being purely conventional, and founded on the custom of marking the north-seeking end of a magnet with a transverse notch or a spot of red paint.

Magnetic Induction.—When a piece of iron is in contact with a magnet, or even when a magnet is simply brought near it, it becomes itself for the time a magnet, with two poles and a neutral portion between them. If we scatter filings over the iron they will adhere to its ends. If we take away the influencing magnet, the filings will fall off, and the iron will retain either no traces at all or only very faint ones of its magnetization. If we apply similar treatment to a piece of steel, we obtain a result similar in some respects, but with very important differences in degree. The steel, while under the influence of the magnet, exhibits much weaker effects than the iron; it is much more difficult to magnetize than iron, and does not admit of being so powerfully magnetized; but, on the other hand, it retains its magnetization after the influencing magnet has been withdrawn. This difference is expressed by saying that steel has great and iron little *coercive force.* The hardest steel is at one extreme, and the softest iron at the other.

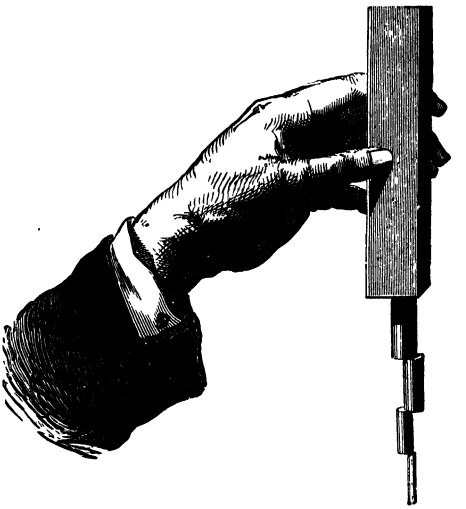


Fig. 34.—Magnetic Chain

In magnetization by influence, which is also called *magnetic induction*, it will be found on examination that the pole which is next the inducing pole is of contrary name to it; and it is on account of the mutual attraction of dissimilar poles that the iron is attracted by the magnet. The iron can, in its turn, support a second piece of iron; this again can support a third, and so on through many steps. A magnetic chain can thus be formed, each of the component pieces having two poles. An action of this kind takes place in the clusters of filings which attach themselves to one end of a magnetized bar, these clusters being composed of numerous chains of filings.

In comparing the phenomena of magnetic induction with those of electrical induction we find both points of resemblance

and points of difference. In the case of electricity, if the influencing and influenced body are allowed to come in contact the former loses some of its own charge to the latter. In the case of magnetism there is no such loss, a magnet after touching soft iron is found to be as strongly magnetized as it was before.

Effect of Rupture on a Magnet.—If a magnet is broken into any number of pieces, every piece will be a complete magnet with poles of its own. In the case of an ordinary bar magnet or needle the similar poles of the pieces will all be turned the same way, as in fig. 35, which represents a magnet AB broken into four pieces. The ends *a*, *a*, *a*, *a* are of one name, and the ends *b*, *b*, *b*, *b* of the opposite name.

Fig. 35.—Broken Magnet

Imaginary Magnetic Fluids: Magnetic Force.—All mutual forces between magnets can be reduced to attractions and repulsions between different portions of two imaginary fluids, called *magnetisms*,[1] one of which may be called *positive* and the other *negative*. Neither magnetism can exist apart from the other; every magnet possesses equal quantities of both; quantity being measured by force of attraction or repulsion at given distance, just as in the case of electricity; like portions repelling, and unlike portions attracting each other inversely as the square of the distance. Equal quantities of the two magnetisms, when coexisting at the same place, produce no resultant effect, and may be regarded as destroying each other.

The direction of magnetic force at a point can either be defined as the direction in which a pole of a magnet would be urged if brought to the point, or as the direction in which a small magnetized needle, if brought to the point and balanced at its centre of gravity, would place its line of poles; and lines of magnetic force are lines to which this direction is everywhere tangential. Any region pervaded by magnetic force is called a *magnetic field*.

[1] Poisson, following Coulomb, spoke of *two magnetic fluids*, and laid down a theory of their action. Lord Kelvin, avoiding the hypothetical parts of Poisson's theory, spoke of *imaginary magnetic matter* of two dissimilar kinds.

The force which is specified by magnetic "lines of force" is the force which *one pole* of a permanent magnet would experience; and it is the same in intensity, but opposite in direction, for dissimilar poles. The two poles of a small magnet (temporary or permanent) in the position of stable equilibrium as regards rotation, are pulled in nearly opposite directions; and the force which tends to produce movement of translation is the resultant of these two nearly opposite pulls.

Specification of Magnetization.—A piece of steel is said to be *uniformly magnetized* if equal and similar portions, cut in parallel directions from all parts of it, are precisely alike in their magnetic properties.

If a piece of magnetized steel be suspended at its centre of gravity, so as to be free to turn all ways about it, the effect of the earth's magnetism upon it consists in a tendency for a particular line through this centre of gravity to take a determinate direction, which is the direction of terrestrial magnetic force. When the line is placed in any other position, the couple tending to bring it back is proportional to the sine of the angle between the two positions, and is the same for all directions of deviation. The line which possesses this property is the *magnetic axis* of the body, and the name is sometimes given to all lines parallel to it. If the piece of steel is uniformly magnetized, this axis is the direction of magnetization; or the *direction of magnetization is the common direction of all those lines which tend to place themselves along lines of force* in a field where the lines of force are parallel.

Ideal Simple Magnet: Thin Bar, uniformly and longitudinally Magnetized.—The mutual actions of magnets admit of very accurate expression when the magnets are very thin in comparison with their length, uniform in section, and uniformly magnetized in the direction of their length. Such bars, which may be called *simple magnets*, behave as if their forces resided solely in their ends, which may therefore in the strictest sense be called their poles. The two poles of any one such bar are equal in strength; that is to say, one of them attracts a pole of another simple magnet with the same force with which the other repels it at the same distance. They have no free magnetism except at the two ends, and the quantities at the two ends are equal but of opposite sign. The same number which

denotes the quantity of free magnetism at either pole denotes the *strength of the pole*, or, as it is often called, the *strength of the magnet.* Its definition is best expressed by saying that the force between a pole of one simple magnet and a pole of another is the product of their strengths divided by the square of the distance between them.

The force which a pole of a simple magnet experiences in a magnetic field is the *product of the strength of the pole and the intensity of the field.* This rule applies to the force which a pole experiences from the earth's magnetism, the intensity of the field being in this case the intensity of terrestrial magnetic force; and, from the uniformity of the field, the forces on the two poles are in this case equal, constituting a couple, whose arm is the line joining the poles multiplied by the sine of the angle which this line makes with the lines of force.

The product of the line joining the two poles by the strength of either pole is called the *moment of the magnet*, and it is evident, from what has just been said, that the continued product of the *moment of the magnet, the intensity of terrestrial magnetic force, and the sine of the angle between the length of the magnet and the lines of force*, is equal to the moment of the couple which the earth's magnetism exerts upon the magnet.

Magnetic Units.—In order to enable us to study *quantitatively* the laws of magnetic attraction and repulsion, it

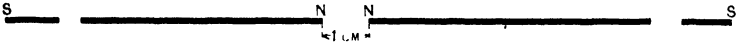


Fig. 36

becomes necessary to define a *unit magnetic pole* (also called unit quantity of magnetism). If we attempt to isolate a single magnetic pole by breaking a magnet, we find that this merely results in the production of two complete magnets, each with a north and south pole. In order to enable us to study the effects due to a *single* pole, we have to use an extremely long magnet. If, then, we confine our attention to points in the immediate neighbourhood of one of the poles, the effect due to the other may, on account of its distance, be neglected.

Imagine, then, two very long magnets, similar in every respect, placed in line, and with their like poles facing each other, as in fig. 36, and let suitable arrangements be provided

for measuring accurately the force exerted between them when the poles are 1 cm. apart.[1] Suppose, further, that by suitable means we are able to vary simultaneously and equally the degree of magnetization of the two magnets, and let this be so adjusted that the repulsion between them amounts exactly to 1 dyne. We then agree to say that each pole is a *unit magnetic pole*. Hence the following definition:—

A unit magnetic pole is one which, when placed at a distance of 1 cm. from an equal pole, acts on it with a force of 1 dyne, the intervening medium being supposed to be air.

If we suppose that a unit pole is placed at a distance of 1 cm. from another pole, and that the force exerted between them amounts to m dynes, we should say that the strength of the second pole is m. From this definition it follows that the force exerted between two poles varies as the product of their pole strengths.

As a result of experiment, we find that the force exerted between two poles of given pole strength varies *inversely as the square of the distance between them.*[2]

Consider a straight bar magnet of pole strength m, and let the distance between the poles be l. Then the product ml is termed the *magnetic moment* of the magnet.

If v be the volume of the magnet, then $\frac{ml}{v}$ is the *intensity of magnetization* (or magnetization simply) of the magnet (= magnetic moment per unit volume).

Let a = area of cross section of the magnet. Then since $v = la$, we have $\frac{ml}{v} = \frac{ml}{la} = \frac{m}{a}$, i.e. the intensity of magnetization may also be defined as the *pole strength per unit of area* of cross section of the magnet.

If we imagine a unit north pole (i.e. one end of a very long magnet of unit pole strength) placed at any point of space, then, on account of the earth's magnetic action, or that due to magnets in the neighbourhood, or to conductors conveying

[1] The medium surrounding the magnets is supposed to be air, and their lengths are assumed to be very great in comparison with 1 cm., so that the action due to the distant poles S, S may be left out of consideration.

[2] The law of the inverse square of the distance is sometimes called the *law of nature*, on account of its very general applicability. It holds not only for magnetic, but also for gravitational and electric attractions and repulsions, and in other cases. The researches which led up to the experimental verification of this law are fairly numerous, the most important being those of Coulomb and Gauss, especially the latter.

electric currents, the unit pole will experience a force of definite magnitude and direction, which is defined to be the *magnetic intensity*[1] (or *magnetic force*) at that point.

Any region throughout which magnetic force is exerted is termed a *magnetic field*, and the magnetic force or intensity is sometimes called *field intensity*.

Any line drawn so that the direction of the tangent to it at every point gives the direction of the magnetic force at that point is called a *line of magnetic force*. It is obvious from this definition that a line of magnetic force is the curve which would be traced out by a magnetic pole if free to move in the field.

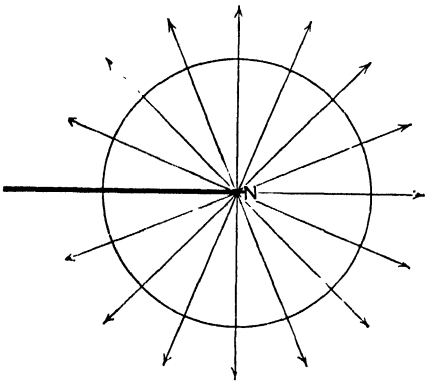


Fig. 37.—Unit Pole Lines

An infinite number of magnetic lines might be drawn in any given region. For purposes of measurement, however, it is convenient to adopt a convention according to which the number of lines is regarded as limited. Let a surface be drawn through any given point so as to be everywhere at right angles to the direction of the lines. Then—simply as a convenient way of expressing certain facts—we agree to say that the number of lines passing through a square centimetre of the surface in the immediate neighbourhood of the point is numerically equal to the magnetic force (or intensity) at that point.[2] Thus, magnetic force, field intensity, and number of lines per square centimetre of a surface drawn at right angles to the lines, are all equivalent expressions.[3]

Consider now the lines of force due to a unit north magnetic pole. Since another unit pole if placed in its neighbourhood would experience a force which is along the line joining the two, it follows that the lines of force due to the unit pole are

[1] *Magnetic intensity* (more generally called *magnetic force*) must be carefully distinguished from *intensity of magnetization*.

[2] Instead of speaking of magnetic *lines*, some writers prefer the expression *tubes* of magnetic force. A tube of force may be defined as a tubular surface so drawn that the lines of force lie *along* it at every point. [3] The medium being supposed to be an air space.

everywhere radial, as shown in fig. 37. In order to ascertain their *number*, according to the convention explained above, imagine a spherical surface described of 1 cm. radius and having the unit pole for its centre. It is evident that every line which proceeds from the unit pole cuts the spherical surface, so that the total number of lines to which the unit pole gives rise is equal to that crossing the spherical surface. Now (by the definition of unit pole and unit field intensity) the field intensity at every point of the spherical surface is unity; and since this surface is everywhere at right angles to the lines, it follows that *one* line passes through every square centimetre of the surface. But the total surface has an area of 4π sq. cm.[1] Hence *a unit pole gives rise to* 4π *lines*.

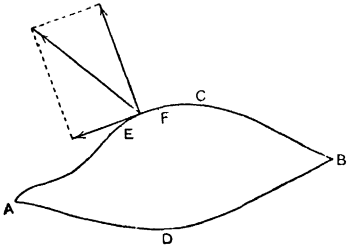


Fig. 38.—Difference of Magnetic Potential

Our next magnetic unit, that of *difference of magnetic potential*, is one of extreme importance. Consider any two points in space, A and B, fig. 38, and let a unit north magnetic pole be transferred along any path, such as ACB, from A to B, against the action of any magnetic forces which may happen to exist in that region. In moving the unit pole from A to B against the action of the magnetic forces, a definite amount of work will be done. This is *independent of the path* joining the two points, and (expressed in ergs per unit pole) is defined to be *the difference of magnetic potential* between A and B. If the work done is positive, i.e. requiring expenditure of energy, B is said to be at a higher magnetic potential than A; but if the work is negative, i.e. if the magnetic forces themselves tend to produce the transference from A to B, then A is said to be at a higher potential than B. Otherwise, a unit north pole tends to move from places of high to places of low potential.

In order to determine the total amount of work done we may imagine the path ACB divided up into a large number of small parts or elements, such as EF. By resolving the magnetic force into two components, one along, and the other at right angles to EF, and taking the product of the length of

[1] The area of a sphere is $4\pi r^2$, r being the radius (= 1 in the case considered).

EF and the component of magnetic force along it, we get the work done in moving over the distance EF. The sum of all such products will give the magnetic potential difference (or P.D.) between A and B. It is obvious from this that *the mean value of the magnetic force along any curve is equal to the difference of magnetic potential between its ends, divided by the length of the curve.*

Let us next suppose that instead of transferring our unit pole from A to B, we take it round a closed curve such as ACBDA, thus returning to the starting-point. Since—so long as we exclude the case of magnetic fields due to electric currents—the work done in passing from A to B along ACB is equal and opposite in sign to that done in passing from B to A along BDA, it follows that the total work done in describing the closed path vanishes. *The work done in carrying a unit pole round a closed curve is defined to be the magnetomotive force round that curve*, and, so long as we exclude the case of fields due to conductors conveying electric currents, the magnetomotive force vanishes round every closed curve.

The unit of magnetomotive force (or M.M.F.) is obviously identical with that of magnetic P.D., and is represented by 1 erg per unit pole.

Horizontal, Vertical, and Total Intensities of the Earth's Field.—If m denote the pole strength of a magnet, and I the intensity of terrestrial magnetic force, each pole of the magnet experiences a force mI, which acts parallel to the dipping needle; in other words, in the direction of the lines of force due to terrestrial magnetism. Let δ denote the dip, that is the inclination of the lines of force to the horizon, then the force mI can be resolved into mI cos δ horizontal, and mI sin δ vertical. Hence the horizontal and vertical intensities H and V are connected with the total intensity and dip I and δ by the two equations

$$H = I \cos \delta, \quad V = I \sin \delta$$

which are equivalent to the following two:—

$$\frac{V}{H} = \tan \delta, \qquad V^2 + H^2 = I^2.$$

Measurement of Declination. — Magnetic declination has been observed with several different forms of apparatus.

At sea the most common method of determining it has consisted in observing the magnetic bearing of the rising or setting sun, and comparing this with its true bearing as calculated by a well-known astronomical method.

At fixed observatories more accurate methods of observation are employed. Fig. 39 shows the arrangement adopted at Greenwich. A bar magnet B carries at one end a cross of fine threads C, and at the other a lens D, the distance between them being equal to the focal length of the lens, thus forming a kind of inverted telescope, whose line of collimation is the line joining the cross to the optical centre of the lens. The bar is

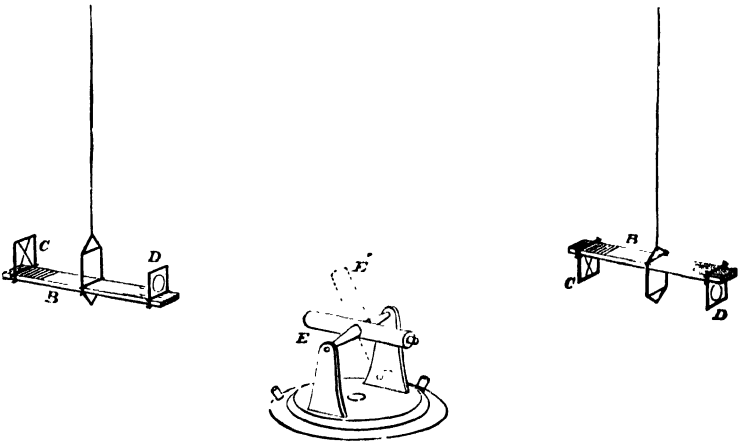


Fig. 39.—Declination Magnet

suspended by means of a stirrup from a torsionless thread, and sets its magnetic axis in the magnetic meridian. The telescope E, with theodolite mounting, is stationed opposite the end which carries the lens, and is so adjusted at each observation that its line of collimation is parallel to that of the inverted telescope carried by the magnet, an adjustment which is identified by seeing the cross C coincident with a similar cross fixed in the interior of the telescope E. When the observation has been made with the magnet in one position, it must be repeated with the magnet turned upside down as shown in the figure. Error of parallelism between the magnetic axis of the bar and the line of collimation of the inverted telescope which it carries will affect these two observations to the same extent in opposite

directions, and will therefore disappear from their mean. The readings are taken on the horizontal circle at the base of the instrument, and astronomical observations must be made once for all to determine what reading corresponds to the geographical meridian.

Another very accurate method consists in rigidly attaching to the bar, instead of the lens and cross, a small vertical mirror. This can either be viewed through a telescope, so as to show the reflection of a horizontal scale of equal parts, which will appear to travel across the field of view of the telescope as the magnet turns, or it can be employed to throw the image of a spot of light either upon a screen viewed by the observer, or still better upon photographic paper drawn by clockwork, which leaves a permanent record of continuous changes. Both these methods of employing mirrors for the observation of small movements of rotation are now extensively employed in many applications.

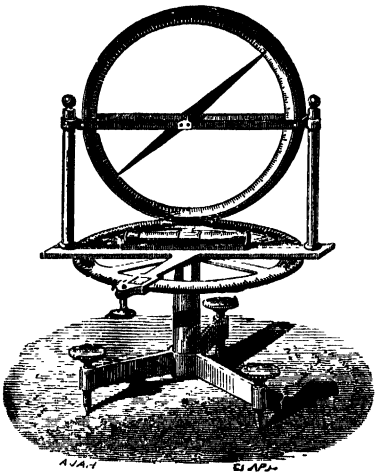


Fig. 40.—Dip Circle

Measurement of Dip.—An early form of dip circle or inclination compass is represented in fig. 40. It consists essentially of a magnetized needle, very accurately and delicately mounted on a horizontal axis through its centre of gravity, in the centre of a vertical circle on which the positions of the two ends of the needle can be read off. This circle can be turned with the needle into any azimuth, the amount of rotation being indicated by a horizontal circle. It is obvious that, if the vertical circle is placed in the plane of the magnetic meridian, the needle, being free to move in this plane, will directly indicate the dip. On the other hand, if the vertical circle is placed in a plane perpendicular to the magnetic meridian, the horizontal component of terrestrial magnetism is prevented from moving the needle, which, accordingly, obeys the vertical component only, and takes a vertical position. In intermediate

positions of the vertical circle the needle will assume positions intermediate between the vertical and the true angle of dip. In fact, if θ be the angle which the plane of the vertical circle makes with the magnetic meridian, the component H sin θ of terrestrial magnetism, being perpendicular to this plane, merely tends to produce pressure against the supports, and the horizontal component influencing the position of the needle is only H cos θ, which lies in the plane of the circle. As none of the vertical force is destroyed, the tangent of the apparent dip will be $\frac{V}{H \cos \theta} = \frac{\tan \delta}{\cos \theta}$. The most accurate method of setting the vertical circle in the magnetic meridian consists in first adjusting it so that the needle takes a vertical position, and then turning it through 90°.

The instrument having thus been set, and a reading taken at each end of the needle, it should be turned in azimuth through 180°, and another pair of readings taken. By employing the mean of these two pairs of readings several sources of error are eliminated, including non-coincidence of the axis of magnetization with the line joining the ends of the needle. One important source of error — deviation of the centre of gravity from the axis of suspension in a direction parallel to the length of the needle—is, however, not thus corrected. It can only be eliminated by remagnetizing the needle in the reverse direction so as to interchange its poles. The mean of the results obtained before and after the reversal of its magnetization will be the true dip.

Measurement of H and M.—To determine experimentally the value of H at a given time and place two observations must be made. One of them consists in timing the horizontal vibrations of a hard-steel magnet, and the other consists in observing the deflection which this magnet produces in a magnetic needle.

In the deflection observation the deflecting magnet is either placed "end on" or "broadside on".

In the former case it is magnetic east or west of the centre of the needle, and points directly towards it. Let r be the distance from centre to centre, and $2l$ the distance between the poles of the magnet, the strength of each pole being m. Then the force at the centre of the needle due to the magnet is

$\frac{m}{(r-l)^2} - \frac{m}{(r+l)^2} = \frac{4mrl}{(r^2-l^2)^2} = \frac{4ml}{r^3}$ nearly, since l is small compared with r.

But $2ml$ is the moment of the magnet; denoting this by M, the force is $\frac{2M}{r^3}$, and its direction is perpendicular to the earth's force, H. If we regard the field as uniform, the tangent of the deflection will be the ratio of these two forces, say

$$\tan\theta = \frac{2M}{Hr^3}.$$

By combining this equation with $T^2 = 4\pi^2\frac{wk^2}{MH}$, T denoting the time of a complete vibration, w the weight and k the radius of gyration of the magnet, we find

$$H^2 = \frac{8\pi^2}{T^2}\frac{wk^2}{r^3\tan\theta}, \text{ and } M^2 = \frac{2\pi^2}{T^2}wk^2r^3\tan\theta.$$

In the "broadside on" position the line of centres is magnetic north and south, and the deflecting magnet points magnetic east and west. With the same notation as above the force at the centre of the needle due to either pole is approximately $\frac{m}{r^2}$, and its east-and-west component is approximately $\frac{l}{r}$ of this, or is $\frac{ml}{r^3}$. This is in the same direction for both poles of the magnet, hence the total east-and-west force is $\frac{2ml}{r^3}$ or $\frac{M}{r^3}$. This is the whole disturbing force, for the north and south components destroy one another. Accordingly we have now

$$\tan\theta = \frac{M}{Hr^3}, \text{ and } H^2 = \frac{4\pi^2}{T^2}\frac{wk^2}{r^3\tan\theta}.$$

To eliminate errors arising from want of symmetry in the magnetization, the position of the disturbing magnet should be reversed in three ways, viz. by turning it upside down, by turning it end for end, and by shifting it to the opposite side of the needle, keeping the distance r unchanged. This makes eight positions in all, and the mean of the eight deflections obtained is to be adopted. These remarks apply alike to the "end on" and the "broadside on" positions.

The above formulæ may be regarded as the first approximations; for closer approximations we must refer to fuller treatises.

As regards the moment of inertia, wk^2, if the magnet is a

rectangular bar, $k^2 = \frac{1}{3} l^2 + \frac{1}{4} b^2$ where b is half the breadth of the magnet. The distances r, l, b, should be expressed in centimetres, and the mass w in grammes, the time T being in seconds. The value of H will thus be obtained in c.g.s. measure. Its average value for the British Isles is about 0.17.

The observations must be made at a distance from all other magnets and from all large masses of iron.

Force in a Crevasse; H and B.—When a piece of iron is in a magnetic field the magnetic force at the centre of a small cavity in the iron will depend partly on the shape and direction of the cavity. It will be a maximum for a narrow cavity with plane parallel sides perpendicular to the direction of magnetization of the iron, and will be a minimum for a similar cavity with its sides parallel to the direction of magnetization. Such cavities are briefly designated a *transverse crevasse* and a *longitudinal crevasse.*

The surface density on the sides of a transverse crevasse is I, and the repulsion of one surface layer and attraction of the other are both in the same direction as the magnetizing force to which I is due. Each of them amounts to 2πI. Hence, if we denote the magnetizing force by H, and the force in the transverse crevasse by B, we have

$$B = H + 4\pi I,$$

whereas in a longitudinal crevasse the force is simply H. The modern name for B is *induction*, or, more fully, "induction at a point", or "intensity of induction" or "flux density". We shall call H the *magnetizing force.* It is often called the *magnetic force.* In non-magnetic matter, such as air or copper, B and H are the same, and I is zero.

When we speak of the values of B and H at a point in any medium, we mean the forces which would exist in crevasses excavated at that point. H may be more simply defined as the *force due to the actual free magnetism.*

The ratio of I to H is called magnetic *susceptibility*, and is denoted by the Greek letter κ. The ratio of B to H is called magnetic *permeability*, and is denoted by the Greek letter μ. The above equation for the value of B in terms of H and I may accordingly be written

$$\mu = 1 + 4\pi\kappa.$$

The Earth as a Magnet.—The intensity and direction of terrestrial magnetic force at different places may be *roughly* represented by supposing that there is a magnet $\pi\pi'$ (fig. 41) at the earth's centre, having a length very small in comparison with the earth's radius, and making an angle of about 20° with the earth's axis of rotation. The points A and B obtained by producing this magnet longitudinally to meet the surface would be the magnetic poles, and at any other place the magnetic meridian would be the vertical plane containing the magnetic axis AB. At places situated on the great circle whose plane contains both the axis of rotation and the magnetic axis, the magnetic meridian would coincide with the geographical meridian, and the declination would be zero. At any other place M, the two meridians would cut each other at an angle which would be the angle of declination. At all places on the great circle $\epsilon\epsilon'$ whose plane is perpendicular to the magnetic axis, a needle suspended at its centre of gravity would place itself parallel to this axis, and consequently the dip would be zero. This circle would be the magnetic equator. It would cut the geographical equator at an angle of 20°. Proceeding from the magnetic equator towards the north magnetic pole B, the needle would dip more and more, until at B it became vertical. A declination needle at B would remain indifferently in all positions. Similar phenomena would be observed at the other magnetic pole A. The end of the needle which would dip at B, and which at other parts of the earth would point to magnetic north, is that which is similar to the southern pole π' of the terrestrial magnet $\pi\pi'$, and the pole which is similar to π would dip at A.

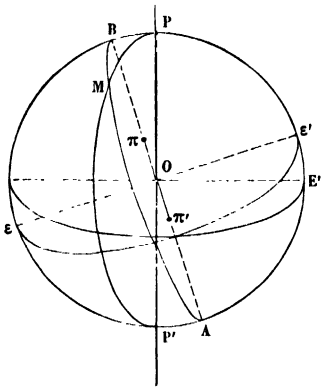


Fig. 41.—Biot's Hypothesis

Secular Changes.—Declination and dip vary greatly not only from place to place but also from time to time. For example, at London, taking a few representative results for the declination and dip respectively:

in 1580	the declination was	10° E.	and	the dip	72°
,, 1660	,,	zero	,,		74°
,, 1800	,,	24° W.	,,		71°
,, 1900	,,	16° W.	,,		67°

It would seem that the magnetic poles are rotating from east to west round the geographical poles, the period of rotation being about five hundred years.

Periodic and Irregular Fluctuations.—Besides the gradual changes which occur in terrestrial magnetism, both as regards direction and intensity of force, in the course of long periods of time, there are smaller fluctuations continually traceable. To a certain extent these are regular, depending on time of day and time of year—in other words, there is a *diurnal variation* and an *annual variation*. But there is also a large amount of irregular fluctation, which occasionally becomes so great as to constitute what is called a *magnetic storm*. Magnetic storms "are not connected with thunderstorms or any other known disturbance of the atmosphere; but they are invariably connected with exhibitions of aurora borealis and with spontaneous galvanic currents in the ordinary telegraph wires; and this connection is found to be so certain, that, upon remarking the display of one of the three classes of phenomena, we can at once assert that the other two are observable (the aurora borealis sometimes not visible here, but certainly visible in a more northern latitude)".[1] They are sensibly the same at stations many miles apart, for example at Greenwich and Kew, and they affect the direction and amount of horizontal much more than of vertical force.

Methods of Magnetization.—A bar of steel can be magnetized by stroking it from one end to the other with one pole of a magnet. A usual method is to stroke it simultaneously with unlike poles of two equal bar magnets, beginning at the middle and finishing at the ends. In every case the end at which the stroke finishes is dissimilar to the stroking pole.

Another method is to place the bar in a magnetic field with its length parallel to the lines of the field, and tap it with a hammer. A bar of iron or steel can be magnetized to a sensible degree in the earth's field by holding it parallel to the dipping needle and tapping it.

[1] Airy, on *Magnetism*, p. 204.

Pieces of iron and steel are often accidentally magnetized, temporarily or permanently, by the influence of the earth's magnetism, especially when the direction of their length nearly coincides with that of the dipping needle. If fireirons which have usually stood in a nearly vertical position be tested by their influence on a needle they will generally be found to have acquired some permanent magnetism, the north-seeking end being that which has been pointing downwards.

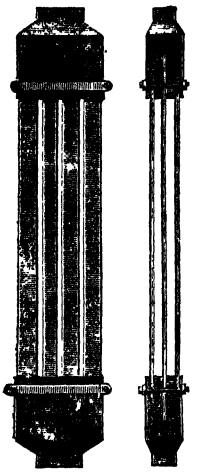


Fig. 42.—Compound Magnet

Thin bars can be more thoroughly magnetized than thick ones; and hence it has been found advantageous to construct composite magnets, consisting of a number of thin bars laid side by side with their similar poles all pointing the same way. Fig. 42 represents such a composite magnet composed of twelve elementary bars arranged 4 × 3. Their ends are inserted in masses of soft iron, the extremities of which constitute the poles of the system.

Electromagnets. — The strongest magnets are made by causing an electric current to circulate several times round a piece of iron. The insulated wire through which the current is to be sent may be wound like thread upon a reel, and all the convolutions will contribute to magnetize the bar the same way. If the bar has the horseshoe form, as in fig. 43, the windings on the two limbs must be such that if the bar were straightened out the current would circulate in the same direction round both portions. The connection between the direction of the current and the direction of magnetization may be expressed by the rule that if a corkscrew be supposed to turn in the direction in which the current circulates, the north-seeking pole of the bar is at the end towards which the corkscrew advances. Thus, in the two cases 1 and 2, shown in fig. 44, the direction of the current being shown by the arrows, the north-

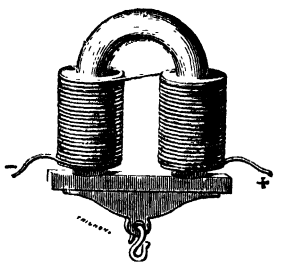


Fig. 43.—Horseshoe Electromagnet

seeking poles will be at the ends marked *a*, and the south-seeking poles at the ends marked *b*. In case 3, where the direction of winding is reversed at the two intermediate points *ab*, there will be "consequent poles" in the bar at these points of reversal. There will be poles of one sign at the two points, *aa*, and of the opposite sign at the two points *bb*.

Besides the enormous power that can be given them, electromagnets have the advantage of being readily made or unmade instantaneously, by completing or interrupting the circuit to which the coiled wire belongs.

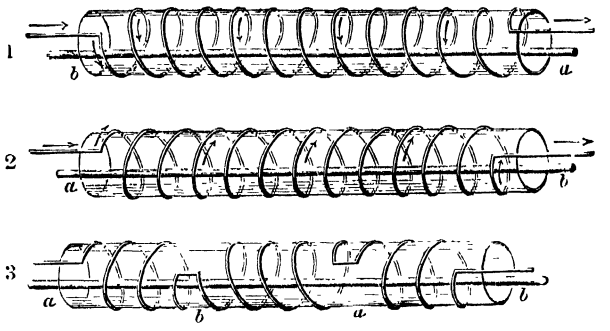


Fig. 44

1, Right-handed Helix. 2, Left-handed Helix. 3, Arrangement for Consequent Poles.

Molecular Changes accompanying Magnetization. — Joule performed experiments from which he inferred that when a bar of iron is magnetized longitudinally it acquires a slight increase of length, compensated by transverse contraction, so that its volume is unchanged.

Shelford Bidwell has shown that the increase of length observed by Joule occurs only in the early stage of magnetization, and that with further magnetization there is a decrease of length far exceeding the previous increase. In subsequent experiments he found that, even in the first stage of magnetization, *annealed* iron contracted both in length and breadth.

If the magnetization of a bar is effected suddenly by completing an electric circuit, an ear close to the bar hears a clink and another clink is heard when the current is stopped.

The moment of a hard steel magnet increases with falling and diminishes with rising temperature, returning to its original value when the original temperature is regained. At

temperatures above 790° C. or thereabouts (corresponding to a dull red heat) iron and steel cease to be magnetic, and steel magnets are permanently demagnetized.

An iron or steel wire slowly raised to a red heat by sending an electric current through it undergoes a momentary check in its heating and a momentary contraction of length in passing this temperature (about 790° C.). When heated higher and then left to cool, it exhibits on arriving at this temperature a sudden increase of luminosity (indicating spontaneous elevation of temperature) accompanied by momentary elongation.

These phenomena seem to indicate that an important molecular rearrangement occurs in iron on reaching this temperature either from above or below. It is called the *critical temperature* for iron.

Action of Magnetism on all Bodies. — It has long been known that iron and steel are not the only substances which can be acted on by magnetism. Nickel and cobalt especially were known to be attracted by a magnet, though very much more feebly than iron, while bismuth and antimony were repelled. Faraday, by means of a powerful electromagnet, showed that all or nearly all substances in nature, whether solid, liquid, or gaseous, were susceptible of magnetic influence, and that they could all be arranged in one or the other of two classes, characterized by opposite qualities. This opposition of quality is manifested in two ways.

1. As regards attraction and repulsion, iron and other *paramagnetic* bodies are attracted by either pole of a magnet, or more generally they tend to move from places of weaker to places of stronger force. On the other hand, bismuth and other *diamagnetic* bodies are repelled by either pole of a magnet, and in general tend to move from places of stronger to places of weaker force.

2. As regards orientation, a paramagnetic body, when suspended between the poles of a magnet, tends to set *axially*; that is to say, tends to place its length along the line joining the poles; whereas a diamagnetic body tends to set *equatorially*, that is, to place its length at right angles to the line joining the poles.

CURRENT ELECTRICITY

CHAPTER IX

PRIMARY BATTERIES

Voltaic Electricity.—If two plates, one of zinc and the other of copper (fig. 45), are immersed in water acidulated by the addition of sulphuric acid, and are not allowed to touch each other within the acid, but are connected outside it, either by direct contact or by a metallic wire M, as in the figure, a continuous current of electricity flows round the circuit thus formed, the direction of the positive current being from copper to zinc in the portion external to the liquid, and from zinc to copper through the liquid. Chemical action at the same time takes place, the zinc being gradually dissolved by the acid, and hydrogen being given out at the copper plate.

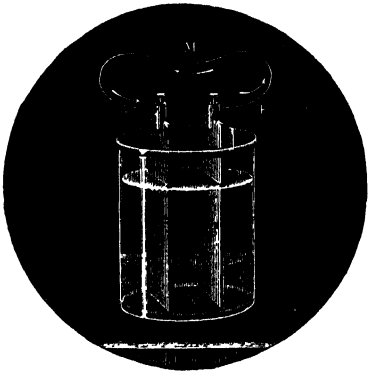


Fig. 45.—Voltaic Element

Every electric current may be regarded as a flow of positive electricity in one direction, and of negative electricity in the opposite direction. The direction in which the positive electricity flows is always spoken of as the *direction of the current.*

Galvanic Battery.—By connecting the plates of successive elements in the manner represented in fig. 46, we obtain a battery. The copper of the first cell on the left hand is connected with the zinc of the second; the copper of the second with the zinc of the third; and so on to the end of the series.

If two wires of the same metal be connected, one with the first zinc and the other with the last copper, the difference of potential between these wires is independent of the particular metal of which they are composed, and is called the *electromotive force* of the battery.

The zinc of the first and the copper of the last cell (or wires proceeding from them) are called the *electrodes* or *poles* of the battery, the zinc being the negative and the copper the positive electrode. The current flows through the connecting wire from the positive to the negative electrode, and is forced through the battery from the negative to the positive.

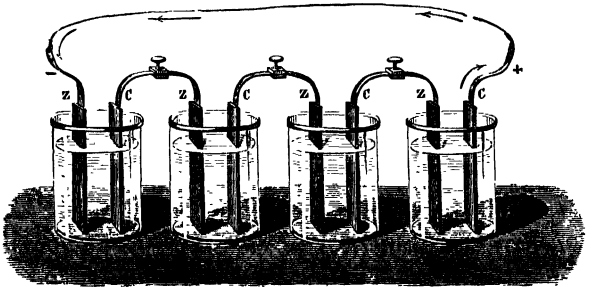


Fig. 46.—Battery of Four Elements

Polarization and Local Action.—One great objection to the simple voltaic cell is the waste of zinc taking place during the time that the cell is not in use, since ordinary commercial zinc is readily attacked by sulphuric acid. In order to remedy this defect, the practice of amalgamating the zinc, or coating it with a thin layer of mercury, was introduced in 1828 by Kemp. The coating of amalgam is found to afford a very effective protection to the zinc. The readiness with which ordinary zinc is attacked by the acid is due to impurities in the zinc; these impurities when in contact with the acid form little voltaic cells with the zinc, and since the circuit of such cells is closed, through the contact of the impurity with the zinc, a local current flows from the impurity to the zinc through the point of contact, and from the zinc back to the impurity through the acid, causing the zinc to go into solution. For this reason the waste of zinc which goes on when there is no current being taken out of the cell is generally referred to as "local action".

There is, however, another very serious defect in the simple voltaic cell under consideration. As soon as the cell is allowed to send a current, the sulphuric acid is split up by it into its constituent ions—H_2 and SO_4—the former appearing at the copper plate, the latter at the zinc plate, which it attacks, forming $ZnSO_4$. The hydrogen which appears at the copper plate partly escapes, but some of it adheres to the plate and gives rise to an effect known as "polarization", which consists in a lowering of the E.M.F. of the cell. A plate of copper coated with hydrogen bubbles does not, in fact, behave like an ordinary copper plate. A simple experiment is sufficient to show the difference. For if into a vessel filled with dilute sulphuric acid we introduce a plate of zinc and two newly cleaned plates of copper, then on connecting a galvanometer between the two copper plates, no appreciable current will, in general, be found to flow, showing that there is no difference between them. But if one of the copper plates be connected for a time to the zinc plate through an external resistance, and a deposition of small bubbles of hydrogen be thereby produced, then on introducing a galvanometer between the two copper plates a current will be found to flow round the external circuit from the unused plate to the one coated with hydrogen. The hydrogen, then, introduces an opposing E.M.F. into the circuit of the cell, and as a consequence there is a rapid fall in the current. In order to overcome this "polarization" of the cell some means must be adopted for preventing the deposition of the hydrogen on the cathode plate. The various forms of primary cell differ from each other mainly in the means adopted for counteracting polarization. We shall here describe only the more important types of primary cell.

The Daniell Cell and its Modifications.—In this form of cell, which is shown in fig. 47, polarization is prevented by the use of two electrolytes—copper sulphate and sulphuric acid—separated from each other by a porous pot (of unglazed earthenware). The cylindrical plate of zinc and dilute sulphuric acid[1] are contained in the porous pot, while the external containing vessel is filled with a saturated solution of copper sulphate, the copper plate surrounding the porous pot.

When a current passes through the cell, SO_4 appears at the

[1] This gradually becomes contaminated with $ZnSO_4$ during use.

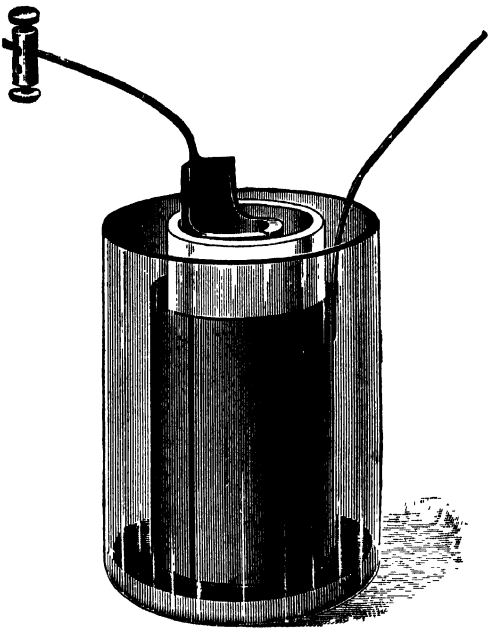


Fig. 47.—Daniell Cell

zinc plate, and causes it to go into solution. The H_2 which is split off from the last molecule of H_2SO_4 before the $CuSO_4$ is reached attacks this latter, reforming H_2SO_4, and the action is transmitted through a chain of $CuSO_4$ molecules, until the last molecule is reached, whose Cu is deposited in the form of metallic copper on the copper plate. The final result, then, is the solution of Zn, and the deposition of copper on the copper plate. At the same time the outer liquid gets poorer in $CuSO_4$, and in order to supply the deficiency it is usual to have a supply of $CuSO_4$ crystals in the outer vessel, so as to maintain the solution saturated.

The substance, then, which prevents polarization in the Daniell cell, or, as it is technically termed, the *depolarizer*, is a liquid—copper sulphate solution.

Numerous modifications of this cell have been devised. One of the most important of these is the *Minotto* cell, shown in fig. 48. It consists of a cylindrical glass jar, with a disk of copper (to which is riveted a connecting wire) at

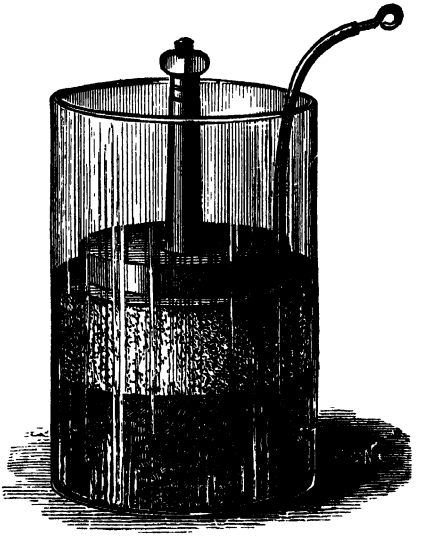


Fig. 48.—Minotto Cell

the bottom of the jar, a layer of crystals of $CuSO_4$ above the disk, then a layer of blotting paper or cloth, followed by a thick layer of sawdust or clean river sand, another thickness of blotting paper, and finally a disk of zinc with a column supporting the terminal. The cell is, to start with, filled with water slightly above the level of the disk of zinc. This cell is largely used in India for telegraph circuits.

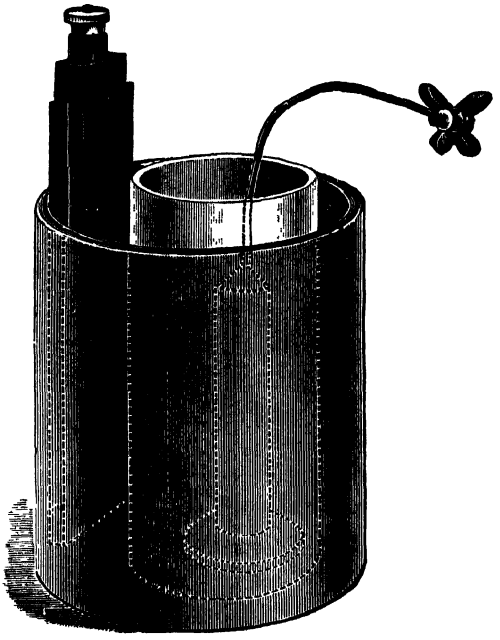


Fig. 49.—Bichromate Cell

The Bichromate Cell.—This is shown in fig. 49, which represents the form known as the "porous-pot" variety of this cell. The zinc and sulphuric acid are contained in a porous pot, and outside this is a block of carbon in a solution of chromic acid. This latter is frequently prepared by introducing into the outer vessel a solution of potassium bichromate—$K_2Cr_2O_7$—and adding to it sulphuric acid, when the following reaction takes place:—

$$K_2Cr_2O_7 + H_2SO_4 = K_2SO_4 + 2\,CrO_3 + H_2O.$$

The chromic acid forms the depolarizer, the hydrogen, which would otherwise appear at the carbon plate, being attacked by the CrO_3, according to the equation

$$2\,CrO_3 + 3\,H_2 + 3\,H_2SO_4 = Cr_2(SO_4)_3 + 6\,H_2O.$$

The zinc in the type of cell illustrated in fig. 49 is made with an expanded base which rests on a layer of mercury at the bottom of the porous pot. This mercury, by creeping up the sides of the zinc, keeps it well amalgamated.

Another form of this cell is also made in which no porous

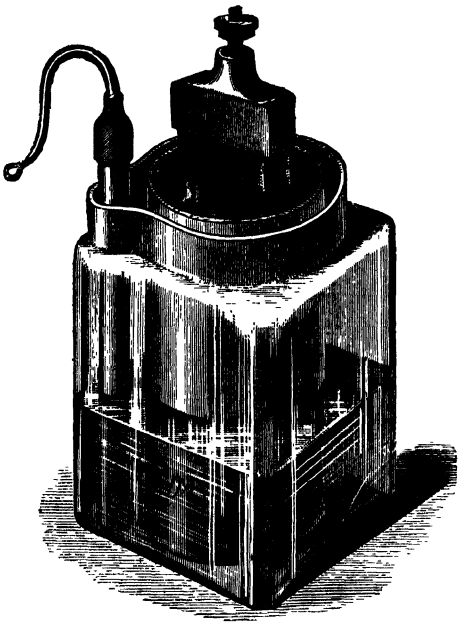


Fig. 50.—Leclanché Cell

of carbon. Further, the liquid used is in each case a solution of sal-ammoniac, and the depolarizer is manganese dioxide (MnO_2). The only difference between the two types consists in the method of applying the MnO_2, which is an example of a *solid* depolarizer. In the porous-pot form the space inside the porous pot around the carbon block is tightly packed with a mixture of granulated MnO_2 and carbon, and the top of pot is used. In this form arrangements must be provided for lifting the zinc out of the liquid when the cell is not in use, in order to prevent the formation of an insoluble chromium salt on the surface of the zinc.

The Leclanché Cell and its Modifications.—One form of Leclanché cell, known as the porous-pot variety, is shown in fig. 50; while fig. 51 shows the "agglomerate block" type.

In each case the zinc is in the form of a cylindrical rod, and the other plate consists of a block

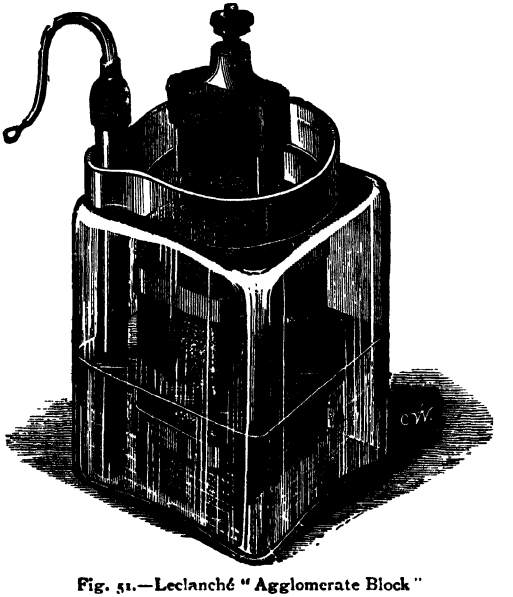


Fig. 51.—Leclanché "Agglomerate Block"

the porous pot is sealed, a small opening being left for the escape gases. In the agglomerate pattern the carbon plate is held between two blocks, formed under great pressure, of a mixture of MnO_2 and carbon.

When the cell is in use the zinc dissolves, forming $ZnCl_2$, and the NH_4—which is the cation of an NH_4Cl molecule—is acted on by the MnO_2, according to the equation

$$2\,NH_4 + 2\,MnO_2 = Mn_2O_3 + H_2O + 2\,NH_3.$$

Various other forms of Leclanché cell have been devised, and the so-called "dry cells" are simple modifications of it.

The Leclanché cell (fig. 50), if used for intermittent work, is the least troublesome of all primary cells, and will work for months without requiring any attention. It is to a very large extent used for bell and telephone work and for testing purposes.

Standard Cells—The Clark Cell.—The construction of the cell which has been chosen by the Board of Trade as the practical standard of E.M.F., and which is known as the Clark cell, will be understood by reference to fig. 52, which shows (full size) the type recommended by the Board of Trade. The containing vessel consists of a glass tube, and contact is made with the mercury by means of a platinum wire sealed into a glass tube, as shown. Above the mercury is a paste consisting of mercurous sulphate and a saturated solution of zinc sulphate. The insoluble mercurous sulphate, which acts as the depolarizer, settles down on the top of the mercury, leaving a layer of clear solution of zinc sulphate above it. In order to maintain the solution saturated at all temperatures a few crystals of the salt are added, and these may be generally seen resting on the top of the mercurous sulphate.

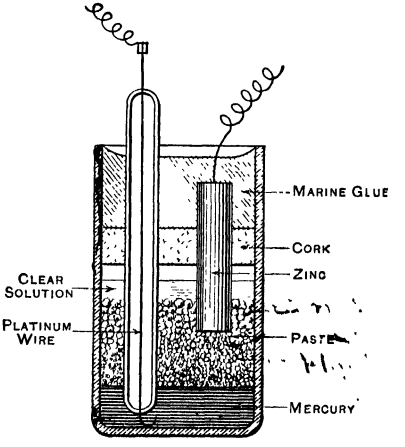


Fig. 52.—Clark Cell

The cell constructed as described possesses several disadvan-

tages, and numerous modifications of it have been suggested and used. In order to render it more portable, Dr. Muirhead dispenses with the large mass of mercury at the bottom of the cell, and for it substitutes a flattened spiral of platinum wire, which has been amalgamated by heating to redness and plunging into mercury; the small amount of mercury retained by the spiral is quite sufficient, and this arrangement renders the cell much more convenient to handle.

The E.M.F. of a Clark cell at 15° C. is 1.434 volts, and the E.M.F. at any other temperature $t°$ C. is given by

$$1.434\{1 - .00077(t - 15)\}.$$

Cadmium or Weston Cell.—This cell is usually made up in the H form, as shown in fig. 53. The negative element consists of a cadmium amalgam—1 part of cadmium to 6 parts of mercury. The positive element is formed of pure mercury over which is placed a paste of mercurous sulphate, and cadmium sulphate. The remainder of the cell is filled up with a saturated solution of cadmium sulphate, crystals of cadmium sulphate being placed on the top of the negative element to ensure that the solution is always saturated. Connection is made to positive and negative elements by platinum wires sealed through the glass of the side tubes at their lower ends. The tubes are closed by a layer of paraffin wax, a cork, and then sealed by sealing wax.

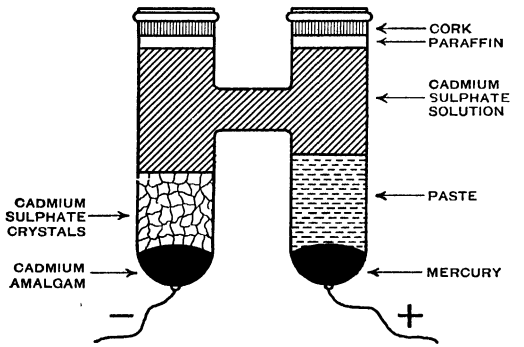


Fig. 53.—Cadmium Cell

The E.M.F. of this cell is 1.0195 volts at 15° C., and only changes .00005 volt per degree Centigrade change of temperature. This gives the cell for use as a standard of E.M.F. a great advantage over the Clark cell, which has a larger variation of E.M.F. with temperature.

Grouping of Cells in Battery.—Suppose that we have a number n of precisely similar cells, each having electromotive

force E and resistance r, and that we connect them in a series, as in fig. 54, with a conductor of resistance R joining their poles. The whole electromotive force in the circuit will then be nE, and the whole resistance will be $nr + R$; hence the strength of current will be

$$I = \frac{nE}{nr + R}.$$

This formula shows that, if the external resistance R is much greater than the resistance in the battery nr, any change in the

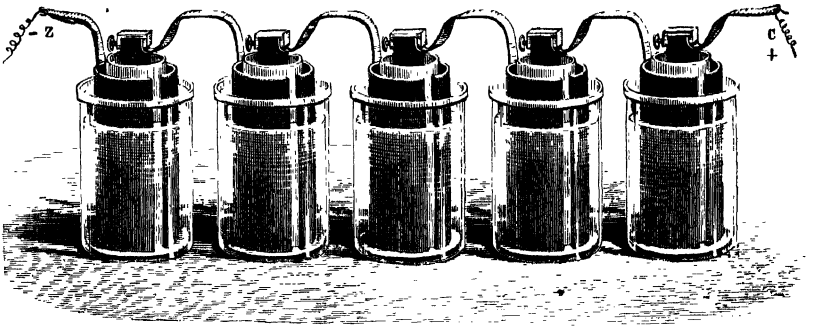


Fig. 54.—Bunsen's Battery

number of cells will produce a nearly proportional change in the current; but that when the external resistance is much less than that of one cell, as is the case when the poles are connected by a short thick wire, a change in the number of cells affects numerator and denominator almost alike, and produces no sensible change in the current. It is impossible, by connecting any number of similar cells *in a series*, to obtain a current exceeding $\frac{E}{r}$, which is precisely the current which one of the cells would give alone if its plates were well connected by a short thick wire.

It is possible, however, by a different arrangement of the cells, to obtain a current about n times as strong as this, namely, by connecting all the zinc plates to one end of a conductor, and all the carbons or coppers to the other end, as in fig. 55. In the arrangement of three cells here figured the current which passes through the spiral connecting wire is the sum of the currents which the three cells would give separately. The arrangement is equivalent to a single cell with plates three

times as large superficially, and at the same distance apart. The electromotive force with n cells so arranged is simply E. but the resistance is only $\frac{r}{n} + R$, so that the current is

$$I = \frac{E}{r/n + R} = \frac{nE}{r + nR}.$$

This system of arrangement may be called *arranging the cells as one element*, or arranging them *in parallel.*

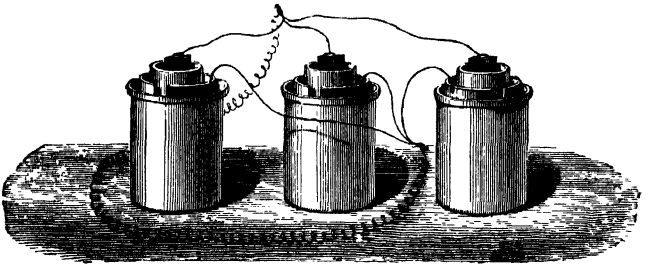


Fig. 55.—Cells with Similar Plates connected

If in fig. 55 we substitute for each of the three cells a *series* consisting of four cells, the electromotive force in circuit will be 4E, and the resistance in circuit will be $\frac{4r}{3} + R$, for each series has a resistance of $4r$, and three parallel series connected at the ends are equivalent to a single series, of the same electromotive force as one of the component series, and of one-third the resistance. The current will therefore be

$$I = \frac{4E}{4r/3 + R} = \frac{12E}{4r + 3R} = \frac{E}{r/3 + R/4}.$$

The question often arises: What is the best manner of grouping a given number of cells in order to give the strongest possible current through a given external conductor? The answer is: They should be so grouped that the internal and external resistance should be as nearly as possible equal; for example, if we have 12 cells as above, and the resistance R in the given conductor is $\frac{4}{3}$ of the resistance of one of these cells, the arrangement just described is the best.[1]

[1] Put x for the number of series, and y for the number of cells in a series. Then the current will be $\frac{e}{r/x + R/y}$, and will vary inversely as $\frac{r}{x} + \frac{R}{y}$. Now the product of $\frac{r}{x}$ and $\frac{R}{y}$ is given, being the quotient of rR by the whole number of cells; and when the product of two

CHAPTER X

ELECTROCHEMISTRY

Electrolysis.—When a current is passed through a compound liquid, decomposition is frequently observed, two of the component substances being separated, one at the place where the current enters and the other at the place where it leaves the liquid. This decomposition is called *electrolysis*, and the substance decomposed or *electrolysed* is called the *electrolyte*. The action only occurs in the case of liquids, and these must be conductors.

The process may be illustrated by the decomposition of water as represented in fig. 56. The apparatus consists of a

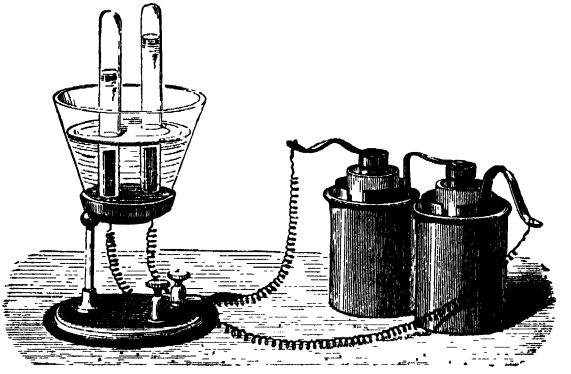


Fig. 56.—Voltameter

vessel containing water to which a little sulphuric acid has been added, and in which two strips of platinum are immersed, connected respectively with the two poles of a battery. When the connections are completed bubbles make their appearance at the surfaces of the two strips and rapidly rise to the surface. If two tubes filled with the liquid are inverted over the two strips the gases will bubble up through the liquid into the upper part of these tubes, and the level of the liquid will gradually fall as shown in the figure. It will be found that the volume of the hydrogen is about double that of the oxygen.

variables is given, their sum is least when they are equal, and increases as they are made more and more unequal. As x and y must be integers, exact equality cannot generally be obtained.

The two strips of platinum are called the poles or *electrodes* of the decomposing cell; the one in connection with the positive pole of the battery is called the *anode* (literally the *way up*) and the one in connection with the negative pole the *cathode* or *kathode* (*way down*). The direction of the current through the liquid is from the anode to the cathode. The gas which is given off at the anode (in the present case oxygen) is called the *anion* (*that which goes up*), and that which is given off at the cathode the *kathion* or *cation* (*that which goes down*). The anion is often called the *electronegative* element, because it moves as if attracted by the positive and repelled by the negative pole. For a similar reason the cation is called the *electropositive* element.

In many cases the separation effected by the direct action of the current is followed by secondary actions due to chemical affinities.

Thus, in the decomposition of acidulated water above described, the first effect, according to modern theory, is a breaking up of the sulphuric acid (SO_3, H_2O) into hydrogen and sulphion (2 H and SO_4), the latter being a substance which has never been obtained by itself. The hydrogen travels to the negative pole and there escapes. The sulphion goes to the positive pole, but instead of escaping enters into combination with the hydrogen of the liquid, forming again the primitive compound (SO_3, H_2O) and leaving the oxygen of the liquid to escape.

Transport of Elements.—It is a remarkable fact that the separated elements never make their appearance except at the electrodes. Nothing is seen of them, nor is any action exhibited, at intermediate points. The appearance is as if the gases could vanish from one extremity and appear at the other without passing through the intermediate space. A very possible explanation of this phenomenon seems to be what is known as Grotthus' hypothesis, that all the particles of the water in the course of the current undergo continual decomposition and recomposition. Thus if fig. 57 represent a line of particles traversed by the current from left to right, there will

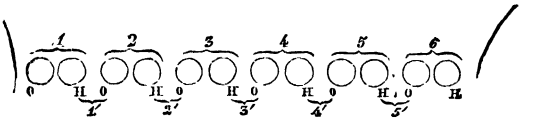


Fig. 57.—Grotthus' Hypothesis

be a continual stream of hydrogen particles along this line from left to right, and a stream of oxygen[1] particles from right to left. The hydrogen of molecule 1 will combine with the oxygen of molecule 2 to form a new molecule 1′; the hydrogen of molecule 2 will combine with the oxygen of molecule 3 to form a new molecule 2′, and so on. The oxygen of molecule 1 is given off at the left-hand extremity, which we suppose to be the point of contact with one of the strips of platinum, and the hydrogen of molecule 6 at the other strip. The molecules 1′, 2′, 3′ . . . are then in their turn decomposed to form a new set. In actual cases the number of molecules, instead of being only six as represented in the figure, is of course many millions.

Discovery of Potassium.—When a compound formed by the union of a metal with a non-metallic substance is submitted to electrolysis, the metal always comes to the negative pole. It was in this way that several of the metals were first obtained from their oxides by Sir Humphry Davy. Potassium, for example, was obtained by placing a piece of potash on a platinum disk connected with the negative pole of a battery of 250 cells, and then applying a platinum wire connected with the positive pole to its upper surface. The potash, which had been allowed to absorb a little moisture from the atmosphere, in order to give it sufficient conducting power, soon began to fuse at the points of contact of the electrodes. A violent effervescence occurred at the upper or positive electrode; while at the lower surface small globules appeared resembling quicksilver, some of which instantly burst into flame, while others merely became tarnished and afterwards coated over with a white film.

Electrolysis of Salts.—When a salt of any of the less-inflammable metals is submitted to electrolysis, a continual deposition of the metal is observed on the negative electrode; while, at the positive electrode, oxygen is disengaged, and acid set free. These effects occur, for example, if platinum electrodes are plunged in a solution of sulphate of copper. If copper is employed as the positive electrode, the oxygen will combine with it instead of being given off, and the oxide thus formed will be dissolved by the acid.

[1] Or sulphion particles according to modern theories. The explanation as it stands in the text represents the views held by Faraday and his contemporaries.

Voltameters.—The quickness with which a given electrolyte is decomposed is simply proportional to the strength of the current; and any apparatus designed for measuring the strength of a current on this principle is called a *voltameter*. The apparatus in fig. 56 is one of its forms. The commonest form consists of a cell containing a solution of sulphate of copper, with two copper plates for electrodes. As the current passes, the anode is gradually dissolved away, and an equal quantity of copper is deposited on the cathode. The quantity of electricity that has passed through the cell can accordingly be determined by weighing the plates. Copper, however, has the drawback that it is to some extent acted on by the solution when no current is passing. When the greatest possible accuracy is required for small currents, two plates of silver immersed in a solution of nitrate of silver are preferred.

Laws of Electrolysis and Electrochemical Equivalents.—It is an experimental result that the amount of an ion liberated from a compound by a constant current flowing through the electrolyte is proportional to the time during which the current is allowed to flow. It is further found that if different currents be passed through a given electrolyte during equal intervals of time, the amounts of either ion liberated are *proportional to the currents*. The total amount of chemical decomposition is therefore proportional to the product of the current into the time. It is convenient to give to this product a special name—it is called the *quantity of electricity*. Thus, we say that when a current of i absolute units has been allowed to flow for t seconds, it absolute units of electric quantity have been passed round the circuit. Making use of this definition of electric quantity, we may express the two facts stated above as follows:—

The amount of an ion liberated is proportional to the quantity of electricity which has passed through the electrolyte. This result was first established by Faraday, and is known as Faraday's First Law of Electrolysis.

The practical unit of quantity is the *ampere-second* or *coulomb*. One coulomb = 10^{-1} C.G.S. unit of quantity.

The amount of an ion liberated by the passage of 1 coulomb is called the *electrochemical equivalent* of that ion.

Faraday's Second Law of Electrolysis states that *the electrochemical equivalent of an ion is proportional to its chemical*

equivalent. The chemical equivalent is equal to the atomic weight—which is definite for each element—divided by the valency. The valency expresses the number of atoms of hydrogen which are replaced by one atom of the substance. Thus, if we once for all carefully determine the electrochemical equivalent of *one* ion, the electrochemical equivalents of all the other ions may be easily calculated from the known atomic weights of the elements and the valencies of the ions.

If the electrochemical equivalent of an ion is known, then we have a means of measuring a current by determining the amount of that ion decomposed during a given time, and dividing this by the product: time × electrochemical equivalent.

The most suitable electrolytes for this purpose are silver nitrate ($AgNO_3$) and copper sulphate ($CuSO_4$). The former may be used for the measurement of small, and the latter of large currents.

The determination of the electrochemical equivalent of an ion is one of the most important measurements in physics, furnishing us as it does with a cheap and comparatively simple and easy mode of measuring currents in absolute measure. Hence this determination has formed the subject of numerous researches, one of the most important of which is that carried out by Lord Rayleigh and Mrs. Sidgwick, who determined the electrochemical equivalent of silver. The value arrived at by them was about 0.001118 gr. per coulomb (or .01118 gr. per absolute unit of quantity). The corresponding value for copper is about .000328 gr. per coulomb.

Definition of Practical Unit of Current.—The following is the Board of Trade definition of the ampere:—

The ampere, which has the value $\frac{1}{10}$ in terms of the centimetre, the gramme, and the second of time, is represented by the unvarying electric current which, when passed through a solution of nitrate of silver in water, . . . deposits silver at the rate of .001118 of a gramme per second.

Chemical Relations of Electromotive Force.—The energy of the current produced by a battery is equal to the potential energy of chemical affinity which runs down in its production; and this latter is measured by the heat of combination due to the chemical action which goes on in the battery. If there are decomposing cells in circuit, they contribute negative heat of

combination, and there can be no current unless the total heat of combination for the whole circuit be positive.

The energy of the current for a given time is equivalent to the total heat of combination due to the action which takes place in this time; and if this energy be divided by the quantity of electricity conveyed by the current in the time, the quotient is called the *electromotive force* of the circuit.

The quantity of electricity conveyed bears a definite relation of equivalence to the zinc dissolved in any one cell; and hence the electromotive force of a cell is proportional to the heat of combination due to the action which takes place during the consumption of a given quantity of zinc.

Ionization.—Numerous facts in connection with the influence of dissolved salts upon the physical properties of the solvent—facts relating to osmosis, lowering of freezing-point, lowering of vapour pressure, and diminution of surface tension—seem to indicate that in very dilute solutions each ion produces its own definite effect independently of the rest (just as in a mechanical mixture of gases the pressure is the sum of the separate pressures). It is accordingly believed that the ions which are held together by chemical affinity in a molecule of the undissolved salt are set free in solution, and move about independently. This process of dissociation is called *ionization*.

Electrons.—The ions in a solution move about, carrying definite electric charges, which are proportional to their valencies and are the same for different ions of the same valency. The charge of an ion of valency 1 is called an *electron*, and is believed to be the smallest quantity of electricity that occurs in nature. Ions carrying positive electrons tend to move from higher to lower potential, and are called *cations*. Ions carrying negative electrons tend to move from lower to higher potential, and are called *anions*.

The passage of electricity through an electrolyte is believed to be entirely a process of convection, effected by this transfer of electrons, which are carried by the oppositely moving ions, and given up by them to the oppositely charged electrodes. The quantity of positive electricity thus carried in one direction and given up to the cathode, is equal to the quantity of negative carried in the opposite direction and given up to the anode, each of these quantities being equal to the current multiplied by the time.

Secondary Batteries. — In some forms of battery electromotive force is produced by the deposition of gas surface of the plates, and this action is called *polariza* A similar effect occurs in the voltameter of fig. 56; and the oxygen and hydrogen, as they are given off, are collected in separate tubes, each platinum plate being in contact both with one of the gases and with the liquid, the voltameter, after being disconnected from the source of the original current, can be used as a battery cell giving a current in the opposite direction. This is the principle of *Grove's Gas Battery*, in which the tubes may be filled with oxygen and hydrogen either by electrolysis or in any other manner, and any number of the cells may be connected in series. During the flow of the current which this battery gives, the two gases of each cell gradually unite through the medium of the acidulated water between them.

Any cell which is first subjected to electrolysis by a current from an external source, and then gives of itself a current in the opposite direction, is called a *secondary cell*, and a combination of such cells is a *secondary battery*. As they store up energy, to be used when and where it is wanted, they have received the name of *storage batteries* or *accumulators*.

Electrometallurgy. — Electrolysis is largely used in connection with the reduction of metals from their ores, especially in the case of copper. Electrolytically deposited copper is remarkable for its purity, and is almost universally employed when very high conductivity is required.

Electrolysis is also used still more extensively for the two kindred purposes of electroplating and electrotyping. In the former the electrolytic deposit is intended as a permanent covering, and should adhere perfectly so as to form one mass with the body which it covers. In the latter the adhesion is temporary, and must not be too close, the object being merely to obtain an exact copy of the original form.

Electrogilding and Electroplating. — The deposition of a coating of gold or silver on the surface of a less-precious metal is merely an example of the electrolysis of a salt. The metal in solution is always deposited on the negative electrode; hence we have merely to make the negative electrode consist of the article which we wish to coat. The only points to be decided

y relate to the means of making the deposit solid and adherent.

ne solutions are always alkaline, and usually consist of cyanide or chloride of the metal, dissolved in an alkaline cyanide.

To prepare the *gold bath*, 50 gr. of fine gold are dissolved in aqua regia; and the solution is evaporated till it has the consistence of syrup. Water is then added, together with 50 gr. of cyanide of potassium, and the mixture is boiled. The quantities named give about 50 litres of solution.

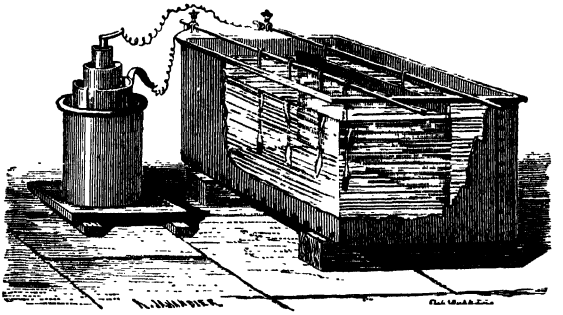


Fig. 58 —Apparatus for Electrogilding

The negative electrode consists of the article to be gilded. The positive electrode is a plate of fine gold, which constitutes a soluble electrode, and serves to keep the solution at a constant strength. In order that the gilding may be well done, the bath must be maintained during the operation at a temperature of from 60° to 70° C.

Fig. 58 represents a form of apparatus which is very frequently employed. The poles of the battery are connected with two metallic rods resting on the top of the cistern which contains the bath. The articles to be gilded are hung from the negative rod. From the positive rod is hung a plate of gold, whose size should be proportional to the total surface of the articles which form the negative electrode.

The *silver bath* is a solution containing 2 parts of cyanide of silver, 10 of cyanide of potassium, and 250 of water. The operation of plating is the same as that of gilding, except that the apparatus is usually on a larger scale, and that the temperature may be lower.

In both cases the surfaces to be coated must be thoroughly cleansed from grease. For this purpose they are subjected to the processes of pickling and dipping, which need not be described in detail.

Other bodies, as well as metals, can be coated if their surfaces are first covered with some conducting material. Baskets, fruits, leaves, &c., have thus been gilded or silvered.

Similar processes are employed for depositing other metals, of which copper is the most frequent example.

Electrotype. — Electrotyping consists in obtaining copper casts or facsimiles of medals, engraved plates, &c., by means of electrolytic deposition. The first successful attempts in this direction were made about 1839 by Jacobi at St. Petersburg and Spencer in England. The art is now very extensively practised.

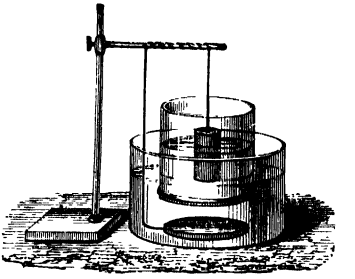


Fig. 59.—Bath and Battery in one

If a facsimile of a medal is required, a cast is first taken of it, either in fusible alloy, plaster of Paris, or gutta percha softened by heating to 100° C., this last material being the most frequently employed. The fusible alloy is a conductor; the other materials are not, and their surfaces are therefore rendered conducting by rubbing them over with plumbago. The mould thus prepared is made to serve as the negative electrode in a bath of sulphate of copper, a copper plate being used as the positive electrode. When the current passes, copper is deposited on the surface of the mould, forming a thin sheet, which, when detached, is a facsimile of one side of the original medal. A similar process can be applied to the other side, and thus a complete copy can be obtained.

In operations of this kind the bath itself is often made to serve as the battery. Fig. 59 represents such an arrangement.

In the interior of a vessel containing a saturated solution of sulphate of copper a second vessel is supported, consisting either of porous earthenware or of a glass cylinder closed below by a membrane. In this second vessel is placed acidulated water, with a cylinder of zinc suspended in it. The mould is placed in the outer vessel under the bottom of the porous

cylinder, and is connected with the zinc by a stout wire which completes the circuit. The arrangement is evidently equivalent to a Daniell's cell. The current passes through the liquids from the zinc to the mould, electrolysing the solution of sulphate of copper; and as the metal travels with the current it is deposited on the surface of the mould. The strength of the solution is kept up by suspending in it crystals of sulphate of copper contained in a vessel pierced with holes.

Applications of Electrotype.—One of the commonest applications of electrotype is the production of copies of wood engravings. The original blocks, as they leave the hand of the engraver, could not yield a large number of impressions without being materially injured by wear. When many impressions are required they are not taken directly from the wood, but from an electrotype taken in copper from a gutta-percha mould. The process of deposition is continued only for twenty-four hours, and the plate of copper thus obtained is very thin. It is strengthened by filling up its back with melted type metal. Such plates will afford about 80,000 impressions, and it is from them that nearly all the illustrations in popular works are printed. Postage stamps, which must be exactly alike in order to prevent counterfeits, are also printed from electrotypes; and, on account of the great number of impressions required, the electrotypes themselves need frequent renewal; but the operations necessary for this purpose do not sensibly injure the original.

Copperplate engravings and even daguerreotypes can be very accurately reproduced in copper. No preparation of the surface is necessary, as the thin film of oxide which is present is quite sufficient to prevent the deposit from adhering too closely.

CHAPTER XI

ELECTROMAGNETISM

Œrsted's Experiment.—Instruments for measuring the strength of a current by means of its tendency to deflect a magnetized needle are called *galvanometers*.

The discovery by the Danish philosopher Œrsted, in 1819, that a magnetized needle could be deflected by a current was the first indication of a definite relation between magnetism and electricity.

Œrsted's experiment can be repeated by means of the apparatus represented in fig. 60. Two insulated metallic wires are placed in the magnetic meridian, one of them above and the other below a magnetized needle. If a current be sent through one of these wires the needle will be deflected; and if the current be strong the deflection will amount nearly to a right angle. The direction of the deflection will be reversed if the current be passed through the lower instead of the upper wire. It will also be reversed by reversing the direction of the current. In the figure the current is supposed to be passing above the needle from south to north. In this case the north end of the needle moves to the west and the south end to the east. On making the current pass in various directions, either horizontally, vertically, or obliquely, near one pole of the needle, it will be found that deviation is always produced except when the plane containing the pole and current is perpendicular to the length of the needle.

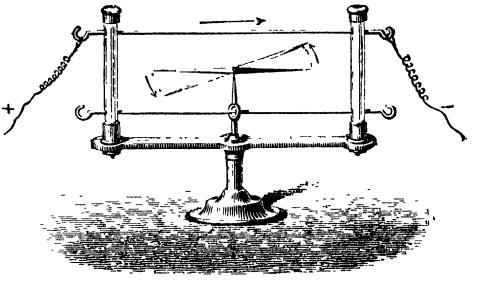


Fig. 60.—Œrsted's Experiment

Ampere's Rule.—The direction in which either pole of a needle is deflected by a current, whatever their relative positions may be, is given by the following rule, which was first laid down by Ampère.

Imagine an observer to be swimming with the current and looking at the needle, then the deflection of the *north-seeking pole* will be *to his left.* The deflection of the south-seeking pole will be in the opposite direction. The two figures (61, 62) illustrate the application of this rule to the two cases just considered. A is the north-seeking pole of the needle, and B the south-seeking pole.

The lines of magnetic force due to a current in a long

straight wire are circles having the wire for their common axis. This is illustrated by fig. 63, where the current flows from the end marked + to the end marked −, and the arrows show the direction in which a north-seeking pole will be urged along

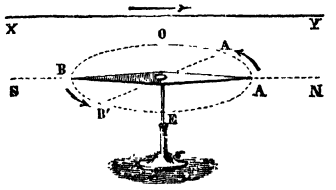


Fig. 61

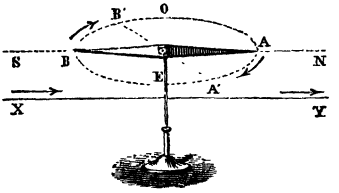


Fig. 62

Ampère's Rule

each of the circles. By universal convention the direction in which a north-seeking pole would be urged is regarded as the direction of the lines of magnetic force. *The direction of a current, and the direction in which its lines of magnetic force run*

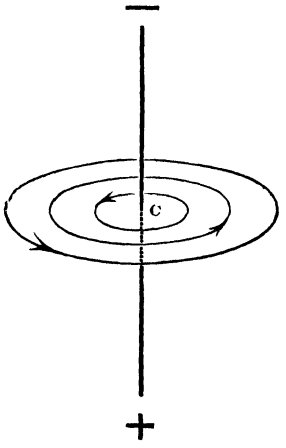


Fig. 63.—Lines of Force due to Current

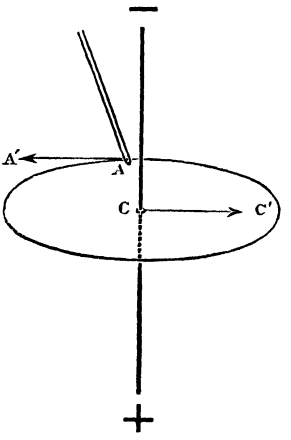


Fig. 64.—Reaction on Current

round it, are related like the advance and rotation of a right-handed screw.

Reaction of Magnet on Wire.—Let A in fig. 64 represent a north-seeking pole, the current being supposed to flow through the wire from + to − as before. The mutual action between the wire and the pole is such that, while the pole (as we have seen) is urged to the left along AA′, the wire is urged to the right along CC′.

This mechanical force on the wire may be regarded as due to the lines of magnetic force which run out from A. A line of magnetic force running in the direction AC accordingly gives the wire a tendency to move in the direction CC'.

Definition of C.G.S. Unit of Current.—If by means of a magnetic needle we determine the direction of the magnetic force produced by a conductor conveying a current, and then reverse the connections of the conductor to the source producing the current (battery or dynamo), we find that the magnetic force has also been reversed. An electric current therefore possesses a definite directive property, which is expressed by saying that the current flows in a definite direction along the conductor.

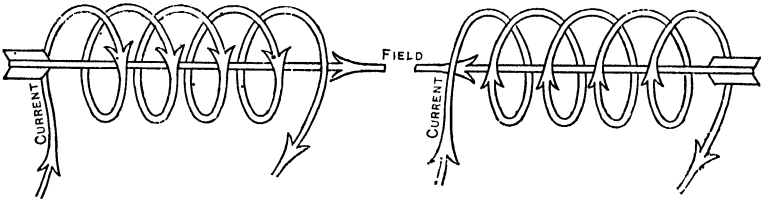


Fig. 65.—Right-handed Screw Rule

It must be clearly understood that this expression does not necessarily imply the flow or transference of any material substance along the wire. By a generally adopted convention the direction of the current is taken to be such that the relation connecting this direction with the direction of the magnetic force is identical with the relation which exists between the thrust and the twist of a right-handed screw. This relation is shown in fig. 64 for the case of a straight wire. The same "right-handed-screw rule" enables us to find the connection between the direction of the current in a coil or solenoid and the direction of the magnetic force due to it. This case is illustrated in fig. 65.

The magnitude of a current is proportional to the magnetic force which it produces. Thus, if a certain current flowing in a given coil produces a definite magnetic force at a given point, and if we increase the current until the magnetic force at the same point is doubled, we agree to say that the current in the second case is twice as great as in the first.

The C.G.S. or "absolute" unit of current is arrived at as

follows: Imagine a circular conductor of 1 cm. radius, as shown in fig. 66, to be conveying a current whose magnitude may be varied at will, and let a unit north pole be placed at the centre of the circular conductor. The force acting on this pole will, by a simple application of the right-handed-screw rule, be from right to left, as shown in the sketch. Let us now suppose that the magnitude of the current is adjusted until the force acting on the unit pole becomes equal to 2π dynes.[1] The current is then said to be a unit C.G.S. current. We thus have the following definition:—

The absolute unit of current is a current which, flowing round a circular conductor of 1 cm. radius, produces a magnetic force of 2π units at the centre of the circle.

At first sight it might appear as if a more convenient constant than 2π—say unity—were preferable. The reason for choosing 2π is, that the total length of the circular conductor is 2π cm., so that, corresponding to unit length of the conductor, we have unit magnetic force at the centre.

Dynamical Action on Conductor Conveying Current when Placed in Magnetic Field.—The action between the circular conductor and the unit pole at its centre is, by Newton's Third Law of Motion, a mutual one. The conductor tends to move the unit pole from right to left, and the unit pole tends to move the conductor from left to right. But the only effect produced by the presence of the unit pole at the centre of the conductor is a magnetic field at every point of the conductor which, as shown in fig. 67 (*a*), is everywhere radial, and therefore everywhere perpendicular to the length of the conductor. Further, since every point of the conductor is at unit distance from the unit pole, it follows that the field intensity (due to the pole) at every point of the conductor is unity. We may say that the conductor tends to move *because it is placed in a magnetic field*, and we see that the direction of motion is perpendicular to both the current and the field. The total force acting on the conductor is 2π, and since this is distributed over a length of conductor equal to 2π, the force per unit length of conductor is unity. Since this force is solely due to the presence of a unit

[1] The force is supposed to be that due solely to the current in the circular conductor; the earth's field, or any other magnetic field, being supposed to be either non-existent or suitably compensated.

magnetic field at every point of the conductor, it follows that if the conductor be opened out straight, as in fig. 67 (*b*), and if, by suitable means, a unit field intensity, which is everywhere

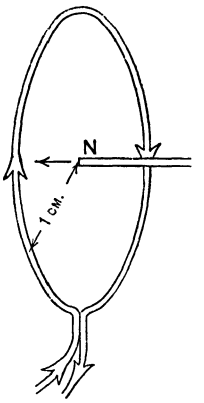


Fig. 66.—C.G.S. Unit Current

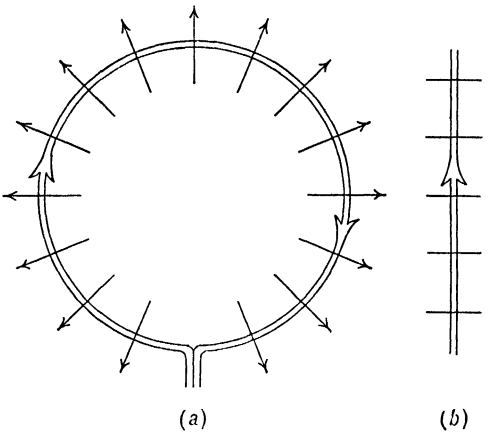


Fig. 67.—Unit Pole and Unit Current

perpendicular to the conductor, be maintained as before, the force acting on the conductor will remain unaltered. We thus have the following important result:—

A conductor of unit length and conveying a unit current when placed in a field of unit intensity which is everywhere at right angles to the conductor, experiences unit force in a direction perpendicular to both the conductor and the field.

If the current amount to I units, the length of the conductor to *l* cm., and the field intensity to H, the total force acting on the conductor is I*l*H dynes.

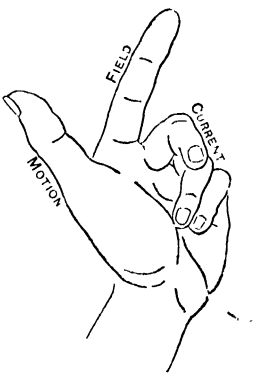


Fig. 68.—Left-hand Rule

A very convenient rule for determining the relation connecting the direction in which the conductor tends to move with the direction of the field and current has been given by Dr. Fleming, and is illustrated in fig. 68. If the thumb, the forefinger, and the middle finger of the *left* hand be held so as to form a system of three mutually perpendicular lines, the forefinger pointing along the

direction of the magnetic field, and the middle finger along that of the current, then the thumb will indicate the direction of the force acting on the conductor.

Work Done in Increasing Magnetic Flux through Circuit. —Let us suppose that a conductor of length l cm., conveying a current of I units, and placed in a field of intensity H (with the length of the conductor perpendicular to the field), is moved parallel to itself through a distance d cm. in a direction opposed to that of the force acting on it. Since this force amounts to lH dynes, and is overcome through a distance d cm., the work done in displacing the conductor is lHd ergs. Now ld represents the area of the rectangle swept out by the conductor, and since H also denotes (by the definition of a unit line) the number of lines passing through every square centimetre of that area, ldH represents the total number of lines cut by the conductor during the motion. We thus see that the work done equals the product of the current into the number of lines cut. This result holds generally, so that in every case in which a circuit conveying a current has the number of lines linked with it increased, whether by mechanical displacement, increase of area of the circuit, or by increase of current, the corresponding expenditure of energy in bringing about this increase is given by the product of the current into the increase in the number of lines.

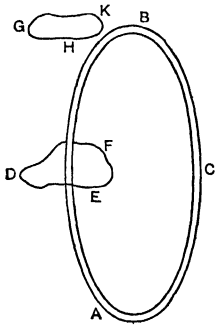


Fig. 69.—M,M.F. Theorem

Fundamental Theorem regarding M.M.F. —In many of the practical applications of electromagnetism it becomes necessary to provide magnetic fields of definite intensity, and this is generally accomplished by means of coils carrying currents. It is a matter of extreme importance to the designer to be able to predetermine, with a sufficiently high degree of approximation, the magnetic intensity in any given region which will be produced by a coil of given size and shape when supplied with a known current. Such calculations are rendered possible by the use of an extremely important theorem, which we now proceed to consider.

Imagine a simple circuit, ABC (fig. 69), conveying a current of I absolute units, and consider any closed curve, DEF, which

is so drawn as to be *linked* with the electric circuit. Let a unit magnetic pole be carried round the closed curve. By the definition of M.M.F., the work done in carrying the unit pole round DEF gives the M.M.F. round that curve. Now, as the pole is carried round the curve, every magnetic line proceeding from the pole will cut the circuit, and the total number of lines cut will be equal to that proceeding from the unit pole, i.e. to 4π. But in order to make the circuit cut 4π lines, an expenditure of energy is required of amount $4\pi I$. Hence the M.M.F. round the closed curve is equal to $4\pi I$.[1]

The result obtained, it will be observed, is quite independent of the *shape* of the curve, since the reasoning employed does not involve a consideration of the shape. The essential point is the *linkage* of the closed curve with the circuit; for if we consider a closed curve, such as GHK, which is *not* linked with the circuit, then although, in carrying a unit pole round it, the lines of force cut the circuit in certain direction during part of the journey, they cut it (or sweep across it) *in the opposite direction* during the remainder of the journey. Thus, if the first part of the journey involves an *expenditure* of energy, the second corresponds to a *gain* of energy, the total gain or loss of energy becoming zero, i.e. the M.M.F. round the closed curve vanishing.

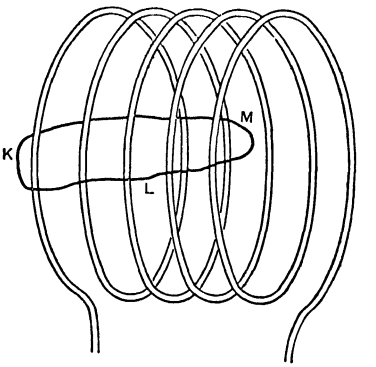


Fig. 70.—M.M.F. Theorem

If instead of a simple circuit we consider a coil of S turns conveying a current I, and determine the M.M.F. round any closed curve which is linked with *every* turn of the coil, such as the curve KLM in fig. 70, then since the work done in carrying a unit pole round a single turn of the coil is $4\pi I$, and since the coil consists of S turns, the total work done in going round KLM is $4\pi IS$.

The expression IS still denotes the total flow of current

[1] The algebraical sign of the M.M.F. is determined by the direction followed in going round the curve: a positive sign being taken if the M.M.F. corresponds to a *gain* of energy, and a negative sign if it corresponds to an *expenditure* of energy.

through the closed curve (or total current linked with it). The relation established is a perfectly general one, and may be briefly put thus:—

The M.M.F. round any closed curve is equal to 4π times the total current linked with it.

In practical calculations it is usual to estimate M.M.F. in C.G.S. units, while the current is measured in amperes. If we are considering a coil of wire, then the product of the current in amperes and the number of turns is spoken of as the *ampere turns* of the coil. Since an ampere = 10^{-1} C.G.S. units, we see that the M.M.F., in C.G.S. units, round any closed curve linked with a given coil is equal to $\frac{4\pi}{10}$, or *1.257 times the ampere turns* of the coil. Conversely, the ampere turns required to produce a given M.M.F. equal $\frac{1}{1.257}$, or, roughly, .8 time the M.M.F.

Applications of Theorem regarding M.M.F.—In order to illustrate the importance of the theorem just considered, we shall apply it to the solution of a few important problems.

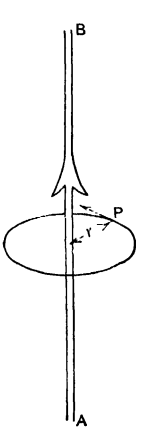


Fig. 71.—Magnetic Force near Straight Wire

The first of these consists in finding the magnetic intensity (or magnetic force) at any point outside a long straight wire conveying a current of I C.G.S. units. Let, in fig. 71, AB be the wire, and P the point at which the intensity is required. Through P draw a circle having its centre on the axis of the wire, and its plane perpendicular to it. We know as a result of experiment that the lines of magnetic force will form circles surrounding the wire, and having their centres on its axis. It is clear from symmetry that the intensity at every point of the circle passing through P is the same; let H stand for this intensity. Then the M.M.F. round the circle is $H \times 2\pi r$, where r is the distance of P from the axis of the wire. But by the theorem regarding M.M.F., the M.M.F. is also equal to $4\pi I$. Since the two expressions for the M.M.F. must give the same result, we have

$$H \times 2\pi r = 4\pi I, \text{ or } H = \frac{2I}{r}.$$

The magnetic force is thus seen to vary inversely as the distance from the wire.

The second problem is that of finding the magnetic intensity at any point inside a coil wound in the form of an anchor ring, as shown in fig. 72.

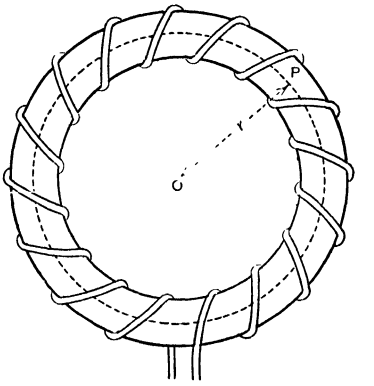


Fig. 72.—Anchor Ring Coil

Let P be the given point. Through P draw a circle whose plane is perpendicular to, and whose centre is on, the axis of the ring. Let r be the radius of this circle (= distance of P from axis of ring). As before, considerations of symmetry indicate that the intensity will have the same value at every point of the circle, the intensity being tangential to the circle. If H = intensity, then M.M.F. = $H \times 2\pi r$. But M.M.F. also = $4\pi SI$, S being number of turns in coil and I current in C.G.S. units. Hence

$$H \times 2\pi r = 4\pi SI, \text{ or } H = \frac{2SI}{r}.$$

Thus the intensity is not quite constant over the cross section of the anchor ring, but diminishes as the distance from the axis increases. If, however, the radius of the ring is large in comparison with its cross section, the intensity may be taken as approximately constant over the cross section.

The third problem which we shall consider is that of determining the intensity inside a straight coil of length great in comparison with its cross section. Such a coil, or *solenoid*, may be regarded as forming part of an anchor ring of infinite radius. Applying the result obtained above for an anchor ring, we get $H = \frac{4\pi SI}{2\pi r}$. This may be written in the form $H = 4\pi \frac{SI}{2\pi r}$. Now $\frac{S}{2\pi r}$ denotes the number of turns S_1 per unit of length of the ring, and $H = 4\pi S_1 I$. So long, therefore, as $S_1 I$ is constant, H is independent of the value of r. We may now suppose r to increase indefinitely; then any finite portion of the ring, of length great in comparison with the

cross section, may be regarded as approximately straight, forming a *solenoid.* If—which is permissible in the case of a very long solenoid—we neglect the effect of the distant portions of the anchor ring, we may suppose these to be removed without materially affecting the intensity inside the solenoid at points not near the ends. We thus get $H = 4\pi S_1 I$ approximately, for points well inside the solenoid.

If, in the last problem, the current be measured in amperes, we see that $H = 1.257$ times the ampere turns per centimetre length of the solenoid.

Difference of Electric Potential and E.M.F. — If a conductor be connected by means of a wire to one terminal of a battery or other source of current, a *momentary* current is found to flow, the duration of which is in general extremely short. The conductor which has been so connected is found to possess peculiar properties, and is said to be *electrified*, or to have received an *electric charge.* The amount of this charge, or the *quantity of electricity* which has passed into the conductor, is measured by the product of the average value of the current into the time during which the current has lasted.

Electric charges are of two kinds: positive and negative. A conductor is said to receive a positive charge if this is communicated by the flow of a current *into* the conductor; if the current flows *out of* the conductor, the corresponding charge acquired by the conductor is said to be *negative.*

Conductors which have both acquired either positive or negative charges are found to repel each other, while a positively charged conductor is found to attract a negatively charged one. In this respect charged conductors behave precisely like magnetic poles. But while electrical repulsions and attractions may be exhibited by all known substances, magnetic polarity is confined to the relatively small class of "magnetic" substances.

The law of electrical attraction and repulsion is also identical with the corresponding magnetic law—it is that of the inverse square of the distance. The analogy between the two cases holds still further. For just as it is impossible to produce a north-seeking pole without at the same time producing a south-seeking one of equal strength, so it is impossible to impart a positive charge to any conductor without at the same time imparting an equal negative charge to some other conductor:

positive and negative charges being always produced in equal amount.

The charge of a conductor is found to reside entirely on its surface. If, by means of suitable experimental arrangements, the outer layers or skin of a conductor be removed, the conductor ceases to exhibit any electrification, thus showing that its charge resided entirely in its surface layers.

The phenomena of electrical attraction and repulsion are exhibited by conductors which have been momentarily, or are permanently, connected to the terminals of a battery. In general, however, these attractions or repulsions are in this case so feeble as to require specially constructed instruments for their detection. There is, however, another way of producing electrification—by friction. In this latter case the attractions and repulsions are so powerful that they may be readily studied without the use of any very delicate instruments. The method of electrification by friction has been known for ages, long before the existence of electric currents was even suspected.

We have already had occasion to consider the mathematical abstraction to which we gave the name of unit magnetic pole. It will now be convenient to introduce a corresponding conception into the electrical case, to which we shall give the name of a *unit electric charge* or *unit quantity of electricity*. Imagine a conductor of indefinitely small dimensions (say a very small sphere), and suppose that by connecting it to a battery a momentary current is allowed to flow into the conductor. Suppose, further, that either the strength of the battery or the duration of the contact is so adjusted that the product of the average value of the current into the time during which it has been flowing is equal to unity. Then the conductor has received a unit positive charge, and since the dimensions of the conductor are assumed to be indefinitely small, the charge is concentrated into a point, forming a *point charge*.

If such a unit point charge be brought into the neighbourhood of any other charged conductors, it will experience a definite force acting in a definite direction, and this force (measured in dynes) gives the value of the *electric intensity* at the point where the unit charge is placed.[1] Any region throughout which electric intensity is exerted is spoken of as an *electric field*.

[1] Electric intensity is measured in *dynes per unit charge*.

Consider any two points, A and B, in an electric field, and let a unit charge be taken from A to B along any path. In moving the charge from A to B against the action of the electric forces, a definite amount of work will be done, and this amount (measured in ergs) gives us the *difference of electric potential*, or electric P.D., between the two points. Hence the definition:—

The difference of electric potential between two points is the work done in transferring a unit electric quantity from the one point to the other.

If the transference of unit charge from A to B involves expenditure of energy, B is said to be at a higher potential than A; and vice versa.

Let us next consider the case of a closed curve, and let unit electric quantity be carried completely round the closed curve. The work done is then defined to be the *electromotive force*, or E.M.F., round the curve.

Practical Unit of P.D. and E.M.F.—The unit of P.D. and E.M.F. employed in practice is termed the *volt*, and is a very large multiple of the C.G.S. unit.

$$1 \text{ volt} = 10^8 \text{ C.G.S. units.}$$

Capacity and Units of Capacity.—Any arrangement of two conductors placed a definite distance apart forms what is known as a *condenser*. If the two conductors forming the condenser be connected to the terminals of a battery, a momentary current will flow, each conductor acquiring a charge—positive in the one case, negative in the other. If we divide the charge on either conductor by the P.D. between them, we get the *capacity* of the condenser. The capacity of a condenser depends not only on the size and shape of the conductors and their relative positions, but also on the nature of the intervening medium.

If we measure the quantity in coulombs (or ampere-seconds), and the P.D. in volts, then the ratio $\frac{\text{coulombs}}{\text{volts}}$ gives us the capacity in terms of a unit known as the *farad*, which is taken as the practical unit of capacity. Since 1 coulomb $= 10^{-1}$ C.G.S. unit, and 1 volt $= 10^8$ C.G.S. units, it follows that 1 farad $= \frac{10^{-1}}{10^8}$ C.G.S. unit, or

$$1 \text{ farad} = 10^{-9} \text{ C.G.S. unit.}$$

The capacity represented by a farad is very large, and hence in practice it is usual to express capacities in terms of the *microfarad.*

$$1 \text{ microfarad} = 10^{-6} \text{ farad} = 10^{-15} \text{ C.G.S. unit.}$$

Activity in an Electric Circuit, and Units of Activity.—Let a current of i C.G.S. units flow in a circuit, and let A and B be any two points in the circuit, between which there is a P.D. of V units (C.G.S.). In accordance with the definition of unit electric current, we may say that i units of quantity are transferred in every second from A to B. But by the definition of P.D., the transference of *each* unit from A to B involves the expenditure of V ergs of work. Thus the total work per second, or the activity, in the part of the circuit lying between the points A and B, is Vi ergs per second. We therefore see that

$$\text{Activity or Power} = \text{P.D.} \times \text{current.}$$

If, instead of considering a portion of the circuit, we consider the entire closed circuit, then the work done on every unit in carrying it completely round the circuit gives us the E.M.F., which we shall denote by E. Hence the activity in the entire circuit is Ei.

If the P.D. be expressed in volts, and the current in amperes, then the product P.D. × current will give us the activity in *volt-amperes* or *watts*. The watt is taken as the practical unit of activity, and since one volt = 10^8 C.G.S. units, and 1 ampere = 10^{-1} C.G.S. unit, we find that 1 volt-ampere = $10^8 \times 10^{-1}$, or 10^7 C.G.S. units.

$$1 \text{ watt} = 10^7 \text{ ergs per second.}$$

One horse-power corresponds to 550 ft.-lb. per second; and since 1 ft.-lb. = 13.56×10^6 ergs, it follows that 1 horse-power = $550 \times 13.56 \times 10^6 = 746 \times 10^7$ ergs per second approximately. Hence

$$1 \text{ horse-power} = 746 \text{ watts.}$$

The watt forming only a very small fraction of a horse-power, it is usual to employ 1000 watts, or a *kilowatt*, as a unit of power.

$$1 \text{ kilowatt} = 1000 \text{ watts} = \frac{1000}{746}, \text{ or } 1.34 \text{ H.P.}$$

Units of Energy.—Electrical energy is generally reckoned in terms of the Board of Trade unit (frequently called "unit" simply), which corresponds to a *kilowatt-hour*, or 1000 watt-hours (= 1.34 H.P. hours).[1]

Fundamental Theorem regarding E.M.F.—In 1831 Faraday and Henry made the important discovery that every change in the number of magnetic lines passing through a circuit results in the production or *induction* of an E.M.F. in that circuit. This E.M.F. lasts only so long as the number of lines is varying, and disappears as soon as that number becomes constant. Imagine that into a circuit which originally contains no E.M.F. there are introduced N lines during t seconds, and that the average current i is due to an average induced E.M.F. of amount E.[2] The mechanical energy expended in introducing N lines into a circuit in which the current is i is given by Ni. But since the introduction of the N lines has taken t sec., the average power is $\frac{Ni}{t}$. This power is transformed into electrical power, and must be equivalent to this latter. Now the electrical power is given by Ei. Hence $Ei = \frac{Ni}{t}$, or

$$E = \frac{N}{t}.$$

In other words, *the E.M.F. induced in a circuit by variations in the number of magnetic lines passing through the circuit is equal to the rate at which the number of lines is changing.*

In cases where the change in the number of lines is produced by the displacement of a circuit in a magnetic field in such a manner that one of the conductors forming the circuit is made to cut the magnetic lines, while the motion—if any—of the remainder of the circuit does not result in any cutting of lines, the rate at which the number of lines is changing is equal to that at which the *active* conductor is cutting lines. We may then regard the active conductor as being the seat of an E.M.F., and say that the E.M.F. induced in the conductor is equal to the rate at which it is cutting lines.

The direction of the E.M.F. induced is always such as to tend to oppose the change which gives rise to it. If, for in-

[1] One kilowatt-hour = 1.34 × 33,000 × 60 ft.-lb. Another unit of energy which has been occasionally used is the *joule*; this is equal to a *watt-second*.

[2] C.G.S. units being used in each case.

stance, the change consists in an increase in the number of lines, the E.M.F. tends to produce a current which will decrease the number of lines. A convenient "hand rule" has been given by Dr. Fleming for the case of a conductor moving parallel to itself perpendicularly across a magnetic field. The *right* hand is used, the forefinger pointing along the field, and the thumb along the direction of motion; the middle finger (which is at right angles to the other two) then gives the direction of the Induced E.M.F.

If a conducting mass be rotated in a magnetic field, the induced E.M.F.'s give rise to currents which tend to stop the motion, and thus exert a powerful *damping effect.* This damping effect is very frequently made use of in practice.

CHAPTER XII

AMMETERS AND VOLTMETERS

Methods of Comparison of Electric Currents.—In several ways, of which the most important consists in moving copper wires across the lines of a magnetic field, it is possible to produce what is called an *electric current.* Electric currents are known to us by their effects. These are of several kinds. One is the generation of heat (as in electric lighting). Another is the decomposition of electrolytes (as in electrometallurgy, electroplating, and electrotyping). Another is the deflection of magnetized needles (as in galvanometers and needle telegraphs). Another is the excitation of electromagnets.

In order that quantitative comparison between currents may be possible, some effect must be selected to furnish the standard of comparison. The selection is made easier by the well-ascertained fact that two of the most easily measured effects are in absolute agreement so far as the most searching examination has been able to ascertain. These effects are: (1) the tendency to deflect a needle, or, to speak more precisely, the intensity of the magnetic field produced at a given point by a current in a given fixed circuit; (2) the quantity of a given electrolyte decomposed by the current in a given time. If a circuit in which

the current can be varied at pleasure is so arranged that in one part it traverses an electrolyte, and in another part it traverses the coil of a galvanometer, experiment shows that when we increase the strength of the current so much as to double the rate of decomposition, we at the same time double the intensity of the magnetic field.

Further, by giving electrostatic charges to a condenser of large capacity, and discharging the condenser through a delicate galvanometer, we obtain an effect indicative of a current of brief duration; and experiment shows that, when we double the quantity of electricity thus discharged, the effect is equivalent to doubling the current as estimated by intensity of field.

Here, then, we have three distinct modes of estimating the numerical ratios of currents; and as far as present knowledge goes they are in exact agreement. Electricians have unanimously adopted these modes of estimating currents. We accordingly assert:

I. Currents in a given circuit are proportional to the magnetic forces which they produce at a given point.

II. Currents are proportional to the quantities of a given electrolyte which they decompose in a given time.

III. Currents are proportional to the quantities of electricity which they convey in a given time.

Any one of these three statements can be adopted as the definition of current ratio, and the other two will then be statements of fact.

The units of current in practical use are based upon I; but II has been extensively employed for purposes requiring simplicity and certainty; while III lies at the foundation of the original conception expressed in the name "electric current".

IV. When the quantities of heat generated per unit time in one and the same conductor by two currents are compared, they are found to be proportional, not to the currents themselves, but to their squares.

V. In like manner, in the electrodynamometer, which contains a fixed coil and a movable coil both traversed by the same current, the mechanical force or torque required to keep the movable coil in its place during the passage of a current is proportional to the square of the current.

Currents can accordingly be compared by observing either of these effects and taking the square root.

An electric current has not only strength but direction. As regards effect I, the direction of force in a field is reversed by reversing the direction of the current. As regards effect II, the two constituents into which an electrolyte is decomposed come out at opposite ends, and would change ends if the current were reversed. As regards III, a current may indifferently be regarded as conveying positive electricity in one direction or as conveying an equal quantity of negative electricity in the opposite direction. By universal convention the *direction of a current* is the direction in which the positive electricity may be regarded as travelling. As regards IV, the rate at which heat is generated in a uniform conductor is not affected by reversing the current; and the same remark applies to the indication of the electrodynamometer in V.

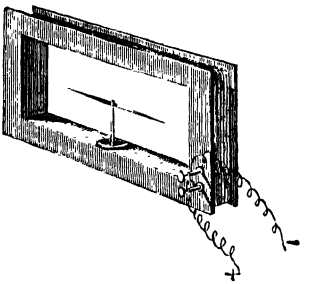


Fig. 73. Schweigger's Multiplier

Multiplier.—The idea of carrying a wire several times round a needle in a vertical plane is due to Schweigger. The form of apparatus designed by him, called *Schweigger's multiplier*, is represented in fig. 73. The name *multiplier* is derived from the fact that, if the current is not sensibly diminished by increasing the number of convolutions of wire through which it has to pass, the force exerted on the needle is n times as great with n convolutions as with only 1, since each convolution exerts its own force on the needle independently of the rest. The difference between the rectangular and the circular form is merely a matter of detail. When the coil is circular, and the needle so small that each pole is nearly in the centre, equal lengths of the current, in whatever parts of the circle they may be situated, exert equal forces upon the needle, and all alike urge the poles in directions perpendicular to the plane of the coil.

Tangent Galvanometer.—The tangent galvanometer consists of a short declination needle mounted in the centre of a coil whose plane is that of the magnetic meridian. The needle is placed in the centre, because the central part of the field is

the most uniform; and it is made short that its ends (both before and after deflection) may be within the region of approximate uniformity.

The magnetic field at any point is proportional to the current I; call it GI for the central part of the field. The multiplier G depends on the coil and not on the current, and is called the *constant of the galvanometer*. If H denote the field due to the horizontal component of terrestrial magnetism, M the magnetic moment of the needle, and α the deflection from the meridian, the couple HM *sin* α due to the earth is balanced by the couple GIM *cos* α due to the current; hence

$$H \tan \alpha = GI, \quad I = \frac{H}{G} \tan \alpha.$$

The needle usually carries a long light pointer whose ends move along a graduated circle.

When a more uniform field is desired than can be obtained with a single coil, the plan adopted (due to Helmholtz) is to place the needle midway between two equal and parallel circular coils whose distance apart is equal to the radius of either, the two coils being connected in series so that the same current flows through both in the same direction. If the coils consist of more than one convolution each, it will be necessary to wind them, not on cylinders, but on two equal frusta of a double cone, the centre of the needle being at the common apex, and the semi-vertical angle of the cone being the angle whose tangent is 2. Elaborate calculation shows that this angle gives a more uniform field than any other.

Kelvin's Mirror Galvanometer.—A more sensitive instrument is the mirror galvanometer of Lord Kelvin. Its needle, which is very short, is rigidly attached to a small light concave mirror, and suspended in the centre of a vertical coil of very small diameter by a silk fibre. A divided scale is placed in a horizontal position in front of the mirror, at the distance of about a yard, and the image of an illuminated slit, which is thrown by the mirror upon this scale, serves as the index. The arrangement of the mirror and scale is exhibited in fig. 74. M is the mirror of silvered glass, slightly concave, with a small piece of magnetized watch spring attached to its back,[1] the two

[1] Instead of a single piece, four or five pieces of smaller spring are now usually employed.

together weighing only 1/10 gr., and suspended by a few fibres of unspun silk. AA is a divided scale forming an arc of a horizontal circle about the mirror as centre. Immediately below the centre of this scale is a circular opening S with a fine wire

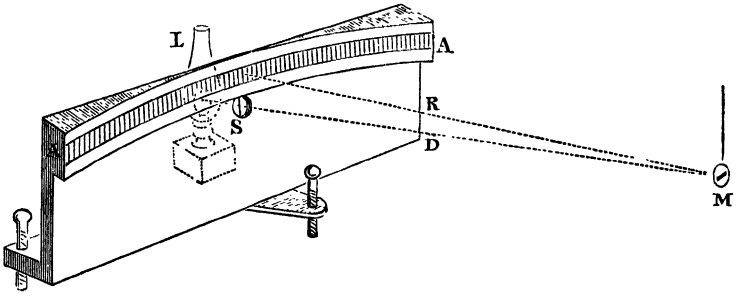


Fig. 74.—Mirror and Scale

stretched vertically at the back of it. A paraffin lamp L or electric lamp is placed directly behind this opening, so as to shine through it upon the mirror, which is at such a distance as to throw upon the screen a bright image of the flame with a sharply defined dark image of the wire in its centre. The image of the wire or of the electric lamp filament is the index in taking the readings.

In order to obviate the necessity of keeping the needle in the meridian, with the lamp east or west of it, and to admit of other positions, a controlling magnet M (fig. 75) is provided, which can be raised or lowered, and can also be turned round. When it is low down it overpowers the earth's magnetism, and compels the needle to take any position that may be required.

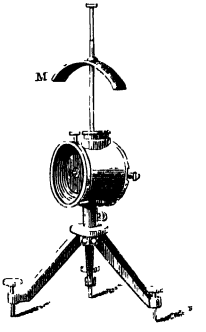


Fig. 75.—Kelvin's Mirror Galvanometer

For maximum sensitiveness the magnetic field around the needle should be made as weak as possible. For this purpose the current should be placed in or not far from the meridian, and the magnet, after being turned into such a position as directly to oppose the earth's action on the needle, should be lowered till its force is a very little less than that of the earth. The operator in making the adjustment watches the vibrations of the needle, as indicated by the movements of the image on the scale, and knows that

the force on the needle is diminishing when he sees the vibrations becoming slower.

For use at sea the galvanometer is modified by fastening the supporting fibre of silk at both ends, so as to keep it tight, with the needle and mirror attached at its centre, care being taken to make the direction of the fibre pass through the common centre of gravity of the needle and mirror, in order that the rolling of the ship may not tend to produce rotation. In this form it is called the *marine galvanometer.*

Moving-coil Galvanometers. — The fact that a conductor conveying a current is subject to a definite dynamical force when placed in a magnetic field is utilized in the construction of a type of galvanometer known as the *moving-coil galvanometer.* An instrument of this type was devised originally by Lord Kelvin in connection with submarine telegraphy; it was modified for use in the laboratory by Deprez and D'Arsonval,[1] and is nowadays very largely employed in preference to a needle instrument. The construction of one form of such instrument is shown in fig. 76. Essentially, it consists of a coil of insulated wire suspended between the poles of a powerful steel magnet, a mass of soft iron (marked "iron core" in the figure) being introduced between the pole pieces with the object of concentrating the lines and so obtaining a more intense field in the annular space which is occupied by the coil. The suspensions of the coil consist of two very fine strips of phosphor bronze, which also serve the purpose of conveying the current into and out of the coil. The deflections are read off by means of the usual lamp and scale arrangement.

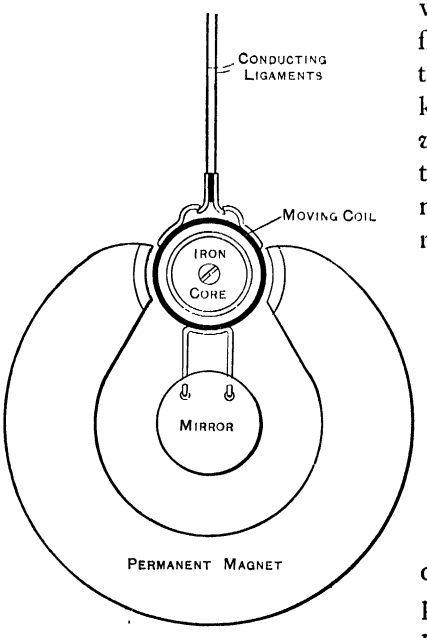


Fig. 76.—Moving-coil Galvanometer

[1] Hence such an instrument is frequently called a Deprez-D'Arsonval galvanometer.

One of the main advantages of this type of instrument lies in its being practically unaffected by external magnetic disturbances, so that it may be used in the neighbourhood of dynamos and under other conditions which would render the use of an ordinary needle galvanometer extremely troublesome, if not impossible.

Ammeters.—In order that the introduction of an ammeter may not disturb the existing conditions of a circuit, the resistance of the instrument must be negligible in comparison with that of the remainder of the circuit. This is one reason why it is desirable to make the resistance of an ammeter *as low as possible.*

There is also, however, another reason. The power absorbed by an ammeter is spent in producing heat in the instrument, and the amount of power so used is given by ri^2, where r is the resistance of the ammeter, and i the current. The power wasted is therefore reduced to its smallest practicable amount by making r as small as possible.

Hot-wire Ammeters.—A hot-wire ammeter consists essentially of a short piece of platinum-silver wire which is heated by the current, and a suitable arrangement for magnifying the expansion of the wire when hot. The principle of their action is exceedingly simple. The current to be measured flows through the wire, heats it, and causes it to expand, and the amount of expansion serves as a measure of the current flowing. When the current is a very large one a shunt is used in parallel with the wire, so that only a small fraction of the total current flows through it. It is important to remember that the fraction of the total current flowing through the hot wire is not a constant quantity, unless both the shunt and wire heat up to the same temperature and are of the same material. In most cases the shunt is arranged to become less heated than the wire, so that with the materials employed a smaller fraction of the total current flows through the wire for large currents than for small. Such an instrument is therefore only accurate for the particular shunt for which it is calibrated. A hot-wire instrument cannot safely be shunted to decrease its sensibility, nor can the shunt be removed to increase its sensibility, and the instrument used for measurement, unless a calibration is performed *throughout the whole scale.*

One of the most popular instruments of this class is that made by Messrs. Johnson & Phillips, an instrument originally devised by Messrs. Hartmann & Braun (fig. 77). In this a platinum-silver wire AA is stretched between two blocks T_1T_2, one of which is adjustable. To the centre of the platinum-silver wire is attached a much thinner one BB of phosphor-bronze, which is fixed at its other end to a terminal T; to the phosphor-bronze wire is attached a piece of cocoon fibre C, which is wrapped round a grooved metal roller R, to which the pointer is fixed, the other end terminating in an eyelet attach-

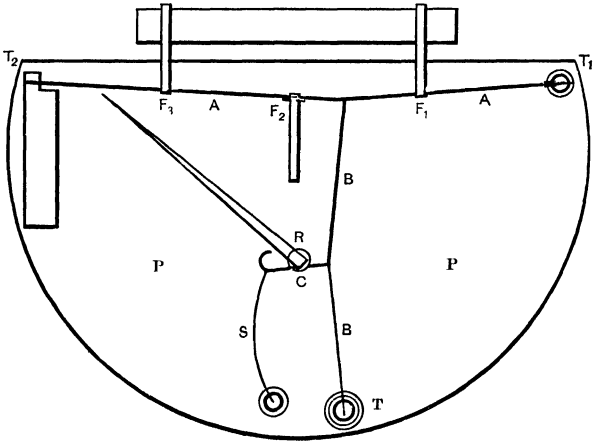


Fig. 77.—Hot-wire Ammeter

ment to a flat steel spring S. When the current flows through the wire the sag increases, thus enabling the phosphor-bronze wire to become more deflected. The increased deflection of the latter is measured by the movement of the thin cocoon fibre, which turns the pointer. The whole of the hot-wire movement is mounted on a compensation plate PP, whose coefficient of expansion is the same as that of the wire.

The wire AA is divided into either two or four equal parts, arranged electrically in parallel with one another by means of silverfoil connecting strips, the object being to divide up the current passing through the measuring wire and thus to diminish its effective resistance (fig. 78). With a two-segment wire the current enters at the centre and leaves at the ends, as shown in the figure, while with a four-segment wire the current

enters at two points a quarter of its length from either end, and leaves at the centre and ends. By this arrangement 4 or

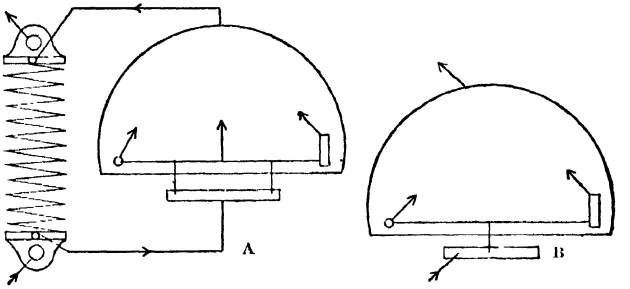


Fig. 78.—A, Diagram of Connections in an Ammeter with the Wire divided up into Four Segments. B, Connections with a Two-segment Wire

5 amperes may be passed through the wires with a drop of potential of less than $\frac{1}{4}$ volt. A thin metal plate is fixed near the wire to protect it from any disturbance due to air draughts.

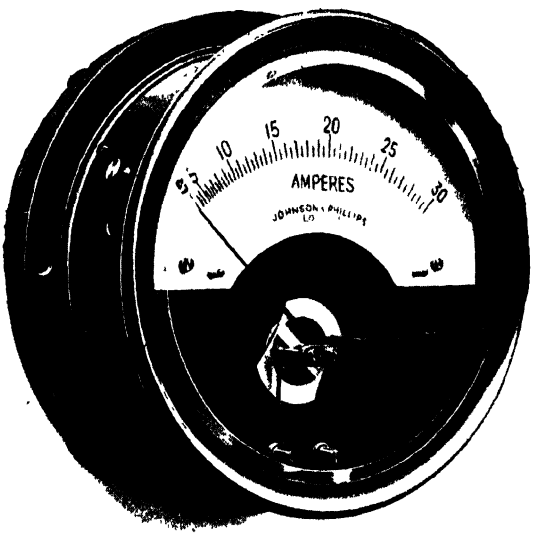


Fig. 79.—Johnson & Phillips Ammeter

The shunts used with these instruments when designed for measuring large currents are made of constantan, a substance which has a negligible temperature coefficient, so that the indications of the instruments are independent of temperature variation.

The instrument comes to its proper reading somewhat slowly, owing to the time taken by the wire to attain a steady temperature; for many purposes this is rather an advantage, and for the measurement of steady alternating currents considerable accuracy can be attained. A damping device is attached to the spindle of the needle, consisting of an aluminium disk (see fig. 79) moving between the poles of a strong magnet, which is said to assist the working of the instrument, but as the motion of the needle is slow under ordinary conditions the effect produced is not very great. When the current is suddenly switched off, however, the damping prevents the needle from swinging back too quickly, thus obviating any tendency of the pulley, to which the pointer is attached, to slip.

Electrodynamic Ammeters—Siemens Electrodynamometer.—The dynamical action which exists between two conductors conveying currents is utilized in the construction of an important class of instruments which we may call *electrodynamic* instruments. One of the oldest and best-known of these is the *Siemens electrodynamometer*, illustrated in fig. 80. It consists of a fixed and a movable coil arranged with their planes at right angles to each other, and connected in series. The movable coil is suspended by a silk thread, and the current enters and leaves it by two mercury cups into which the ends of the coil dip. This coil carries a pointer bent so that its extremity is capable of moving, through a small range limited by two stops, over a graduated horizontal circular scale. Over the same scale may be moved by hand a pointer attached to the torsion head of a fine gilt-steel spiral spring whose lower end is rigidly fixed to the top of the movable coil. When a current passes through the instrument, the pointer attached to the movable coil is thrown against the right-hand stop; but by turning the torsion head,

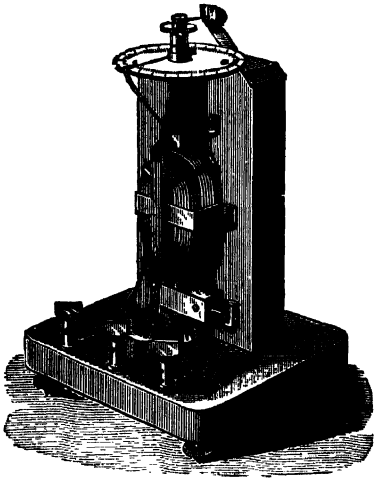


Fig. 80.—Siemens Electrodynamometer

and so twisting the spiral spring, the movable coil may be restored to its normal or zero position.

The Siemens electrodynamometer obeys a very simple law. When the movable coil is in its zero position the couple due to the torsion of the spring, which is proportional to the angle of torsion, balances the electrodynamic couple due to the current. This latter, however, varies as the *square* of the current; for it is proportional to the product of the current through the movable coil into the field in which this coil is placed; the field, however, is that produced by the fixed coil, and is proportional to the current flowing through it, which is the same as the current through the movable coil. Thus the current = constant $\times \sqrt{\text{angle of torsion}}$.

In order to prevent the earth's field from exerting a couple on the movable coil, the instrument must be so placed that the plane of this coil is perpendicular to the earth's field.

In the illustration three terminals are shown. The middle terminal, which is always used, is connected to the lower mercury cup, and so to one end of the movable coil. The other end of this coil dips into the upper cup, which is connected to the junction of two fixed coils, only one of which is in use at a time. The other ends of these coils are connected to the two extreme terminals. One of the fixed coils consists of a comparatively large number of turns of thin wire, and would be used for the measurement of small currents, while the other is wound with thick wire, has only a few turns, and is intended for the measurement of larger currents.

Kelvin's Current Balances.—Electrodynamic instruments known as *Current Balances* have been constructed by Lord Kelvin. They possess a very high degree of accuracy and are frequently used as *standard* instruments. The general appearance of such an instrument will be seen from fig. 81. As in the Siemens electrodynamometer, we here have to deal with a system of fixed and movable coils. By using two movable coils which carry currents in opposite directions, an *astatic* movable system is obtained. The two movable coils are mounted at the ends of a balance beam, and above and below each movable coil is a fixed coil. The direction of the currents is shown in fig. 82, and it will be seen that each pair of fixed coils produces a vertical field in which is placed the movable coil. The

connections are such that the left-hand coil tends to move down while the right-hand one tends to move up. By reasoning similar to that employed in the description of the Siemens electrodynamometer, it is easy to see that the deflecting couple

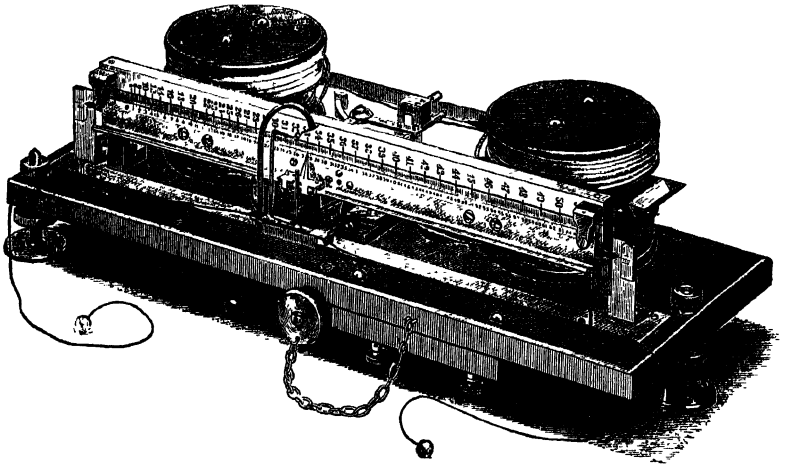


Fig. 81.—Kelvin's Deciampere Balance

acting on the movable system is proportional to the *square* of the current. This deflecting couple is balanced by sliding a weight along the finely divided or movable scale attached to the balance beam. Since the balancing couple varies as the displacement of the weight, it follows that the current is propor-

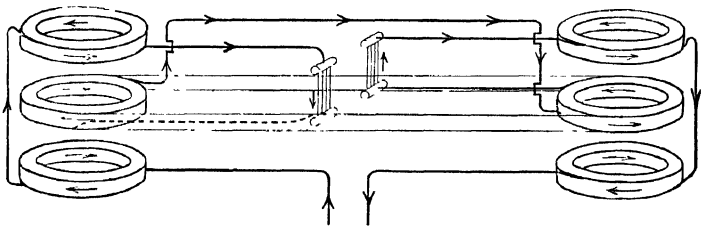


Fig. 82.—Sketch of Kelvin Balance

tional to the *square root* of the displacement. The fixed or inspectional scale seen above the finely divided scale in fig. 81 has divisions proportional to the square roots of the corresponding divisions on the movable scale, and the value of the current is obtained by multiplying the fixed-scale readings by a certain constant.

One of the chief features of these instruments is the ingenious manner in which large currents may be conveyed into and out of the movable system of coils without interfering with the flexibility of the suspensions. The method adopted is shown diagrammatically in fig. 82. The weight of the balance beam is borne by a double set of suspending "ligaments", which consist of a number of very fine copper wires laid side by side and soldered to suitably insulated contact pieces. The current enters the movable coils by one set of ligaments, and leaves by the other. Hundreds of fine copper wires are required in the case of instruments intended for the measurement of very large currents.

Kelvin's current balances are among the most accurate and reliable electrical measuring instruments constructed, and are very frequently employed for checking other instruments. They are not intended to be used as ordinary commercial measuring instruments.

The scale of a Kelvin balance is rendered very open over the greater portion of the range of the instrument by the simple device of providing three or four different sets of sliding weights which are simple multiples of each other. A different constant will then correspond to each sliding weight. In order to maintain equilibrium of the balance beam in the zero position of each weight, a set of counterpoise weights is provided. These latter are made cylindrical, and are supported in the triangular trough seen to the right of fig. 81, a cross pin which passes through the counterpoise fitting into a hole at the bottom of the trough and thus giving the weight a perfectly definite position with respect to the beam. The sliding weights fit over a carriage (clearly shown in fig. 81), which may be moved along the beam by pulling one or other of the two strings attached to a self-releasing pendant and passing outside the glass case which is placed over the instrument when in use.

The Siemens electrodynamometer and the Kelvin balance are both examples of *zero instruments*, i.e. instruments requiring an adjustment before a reading can be taken. In an instrument intended for ordinary commercial work the necessity for such an adjustment would be considered a very serious disadvantage. Hence all ordinary commercial instruments are direct-reading *deflectional* instruments; i.e. they indicate the value of the

current by the position of a pointer moving over a scale marked in amperes, no preliminary adjustment or reference to tables or multiplication by a constant being required.

An important feature of the electrodynamic class of instruments (which they share with instruments of the hot-wire type) is that they are capable of giving correct readings on alternate-current circuits without requiring recalibration.

Permanent-magnet Ammeters — Weston Ammeter. — One of the most commonly used types of ammeter is that in which

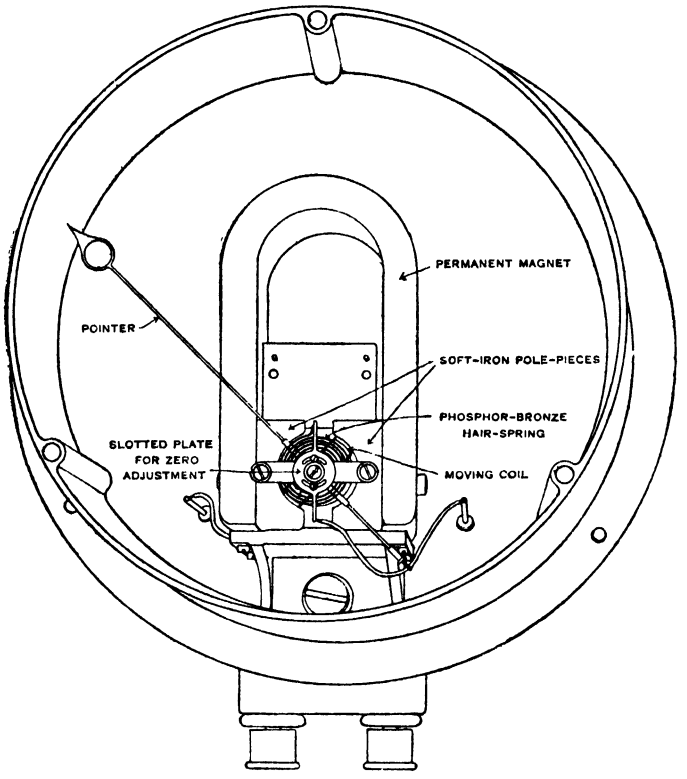


Fig. 83.—Permanent-magnet Ammeter

a coil of wire is pivoted between the poles of a permanent magnet, and by its deflection when conveying a current indicates the amount of current passing. This type of ammeter has recently come much into favour, and is being manufactured in a variety of different forms. Such an instrument is really a

moving-coil galvanometer, similar to the one illustrated in fig. 76. The deflecting couple due to the current is balanced by the controlling couple due to the coiling of two fine hair-springs, the current entering the coil by the one, and leaving by the other (fig. 83). It is evident that the cross section of the springs must always be small, otherwise a very powerful deflecting couple—due to the great rigidity or stiffness of the springs—would be required to produce the necessary deflection; and a difficulty thus arises with large currents, which would either

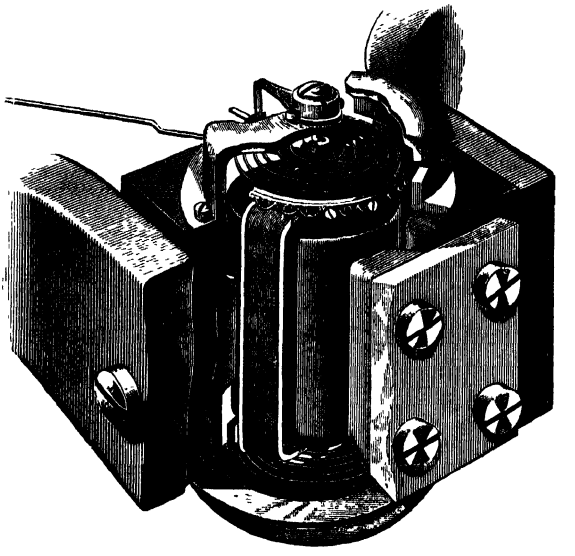


Fig. 84.—Coil of Weston Ammeter

fuse or seriously damage the fine springs if allowed to pass through them. This difficulty is overcome in a very simple and effective manner by allowing only a small fraction of the current to pass through the moving coil, the remainder flowing through a special resistance or shunt placed across the terminals of the instrument. Where very large currents, amounting to hundreds of amperes, have to be measured, it is usual to make the shunt quite separate from the instrument proper, which is then connected across the shunt terminals by means of flexible leads supplied for the purpose. For moderately large currents, however (say below 100 amperes), it is usual to enclose the shunt in the ammeter case.

One of the best-known forms of permanent-magnet ammeter is the Weston ammeter. The construction of one of these instruments is shown in figs. 83 and 84, which represent a switchboard type of ammeter with the front cover removed. To the poles of a horseshoe magnet are attached soft-iron pole pieces, hollowed out so as to form a cylindrical cavity, into which is introduced a soft-iron cylinder whose diameter is less than that of the cavity, an annular space being thereby formed. The moving coil embraces the cylindrical core, its sides being capable of moving in the annular space. The hairsprings attached to the ends of the coil are made of phosphor bronze. The outer end of the top spring is soldered to an arm projecting from a small slotted circular plate, clearly seen in the illustration. By slackening the two small screws which fix this plate the plate may be rotated through a small angle, and thus the position of the pointer adjusted to the zero of the scale when no current is passing through the instrument. The pointer is of aluminium, and the coil is wound on an aluminium frame which, by the damping effect produced during its motion across the field, renders the instrument extremely dead-beat. The springs are connected to two straight wires seen projecting through the back plate of the instrument, and on the other side of this plate the wires are soldered to the shunt terminals, as shown in fig. 85.

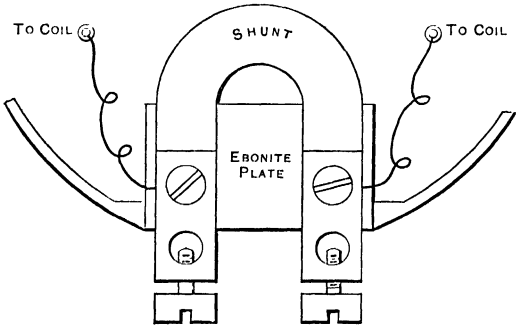


Fig. 85.—Ammeter Shunt

Permanent-magnet instruments belong to the class of *polarized* instruments, i.e. a reversal of current produces a reversal of the deflection. In a well-constructed instrument there should be no appreciable weakening of the magnet with age; a sudden weakening may take place if an excessive current is momentarily passed through the instrument, as such a current exerts a demagnetizing effect on the magnet.

If the magnetic field in which the coil moves be uniform, the

deflecting couple will vary in proportion to the current, and the scale will be one of equal divisions.

Soft-iron Ammeters.—Ammeters in which soft iron is used form a very numerous class. They are generally of relatively simple construction, and some forms are obtainable at very low prices. All soft-iron instruments suffer more or less from a defect which is known as the *hysteresis error*, and on account of which the instrument gives higher readings when descending than when ascending the scale.[1]

The principle on which all such instruments act may be very briefly stated by saying that a mass of soft iron always tends to move from weak to strong regions of a magnetic field. The movable part of the instrument consists of a piece of soft iron, and the fixed part is some device—either a simple coil or a coil provided with a core—for producing a field of varying intensity by means of the current to be measured.

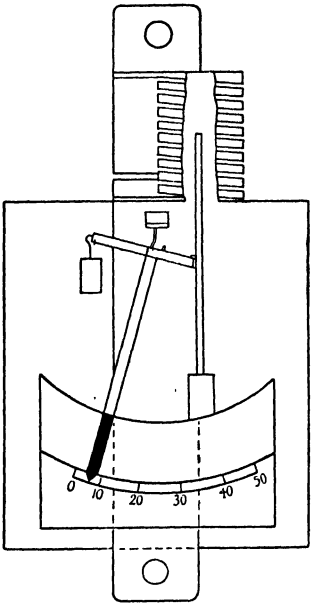


Fig. 86.—Kelvin's Ampere Gauge

As an example of this type of instrument we shall consider—

Kelvin's Ampere Gauge.—This is illustrated in fig. 86, and consists of a fixed coil of wire and a movable, very thin, soft-iron wire suspended from a knife-edge carrier. In the normal position of the movable part the top end of the wire enters the coil; and this latter exerts a sucking action on the wire when a current is sent through it. The controlling force is furnished by gravity.

Calibration of Ammeters.—An ammeter may be *calibrated*—i.e. the true values of its readings ascertained—by one of the following methods:—

1. By the use of a voltameter.
2. By comparison with a standard instrument.
3. By the potentiometer method.

[1] The reader will readily understand how the hysteresis error is caused by referring to Chapter XVI, on magnetic measurements.

Use of Copper Voltameter.—Although in the legal definition of the ampere special reference is made to the silver voltameter, and a schedule is appended which contains detailed instructions regarding the use of such a voltameter, we shall here consider only the copper voltameter, as this is more suitable for the measurement of large currents. The silver voltameter can be conveniently used for currents up to 1 ampere; its cost would be prohibitive for larger currents.

Although the electrochemical equivalent of an ion is an absolute constant of nature, it is found that the values obtained by experiment in the case of copper deposited from a solution of copper sulphate vary appreciably under different conditions of working. Such variations are not real, but only *apparent*, and are due to the solvent action of the electrolyte on the freshly deposited copper. This solvent action depends on the density of the solution and on the temperature, and in order to obtain reliable results with a copper voltameter the following precautions should be observed:—

The solution is prepared by dissolving pure recrystallized sulphate of copper in clean ordinary tap water until a density of 1.18 is reached, and then adding 1 per cent by volume of strong sulphuric acid. It should be replaced by a freshly prepared solution when the aggregate time during which it has been in use amounts to ten hours.

The area of the cathode plate or plates should be such that there are not less than 50 sq. cm. per ampere of current, and the area of the anode plates should always be in excess of that of the cathodes.

The deposition is allowed to go on for about one hour, the current being maintained constant by means of a suitable rheostat. The cathode plates are taken out of the solution immediately after the current has been switched off, and are plunged into a vessel filled with water which has been acidulated with a few drops of sulphuric acid. They are then thoroughly washed and dried, first by the application of warm white blotting paper and then by being held in front of a clear fire or over a spirit lamp, care being taken not to allow them to get appreciably hot.

The copper-voltameter method of calibrating an ammeter is especially suitable in the case of instruments obeying a known

law, such as the Siemens electrodynamometer and Kelvin balance. In the case of other instruments a single determination with the voltameter only enables us to verify the accuracy of the instrument for one particular scale reading. The voltameter method, which is the one actually employed in the standardization of Lord Kelvin's balances, is thus unsuitable for calibrating commercial instruments over their entire scale. For this purpose the next method is more convenient.

Calibration of Ammeters by Comparison with a Standard Instrument.—The instruments used as standards for this purpose are generally Lord Kelvin's current balances. The instrument to be calibrated is connected in series with the Kelvin balance, a suitable rheostat, and a number of secondary cells, and a series of simultaneous readings is obtained with both instruments. A calibration curve may then be plotted, instrument readings being measured horizontally and their correct values vertically; or an "error curve" may be plotted instead, whose abscissæ (horizontal distances) represent instrument readings, and ordinates the amount to be added to, or subtracted from, these readings in order to obtain the correct values of the current as given by the standard instrument.

Potentiometer Method of Calibrating Ammeters.—In this method the ammeter is joined in series with a known standard resistance (preferably an exact submultiple of 10), and the value of the P.D. across this resistance is measured by means of an instrument known as a "potentiometer" (described below). The P.D. divided by the value of the standard resistance gives the true value of current corresponding to the reading of the ammeter.

In the calibration of ammeters it is important to have all the connecting wires or cables arranged so that there is no direct magnetic effect produced on the instruments by the current flowing along the connecting wires. The latter should be preferably twisted so as to form a twin conductor, all large loops or coils being carefully avoided. Neglect of this precaution may give rise to serious error.

Definition of Practical Unit of P.D. and E.M.F.—The following is the legal definition of a volt:—

The volt, which has the value 10^8 in terms of the centimetre, the gramme, and the second of time, is the electrical pressure

that, if steadily applied to a conductor whose resistance is 1 ohm, will produce a current of 1 ampere, and which is represented by 0.6974 ($\frac{1000}{1434}$) of the electrical pressure at a temperature of 15° C. between the poles of the voltaic cell known as Clark's cell.

Resistance of Voltmeters and Temperature Error.—One consideration which determines the amount of resistance to be given to a voltmeter whose action depends on the passage of a *current* is that the introduction of the instrument into the circuit should not materially affect the previously existing conditions of the circuit—i.e. that the total resistance of the part of the circuit included between the two points across which the P.D. is to be measured should not be materially affected by connecting up the voltmeter. Now the change in the total resistance between the two points will be made as small as possible by making the resistance of the voltmeter *as high as possible.*

Suppose the voltmeter resistance to be denoted by R, and the P.D. across its terminals by V; then the power absorbed by the instrument is V^2/R. Hence for a given value of V the power wasted in heating the instrument will be reduced by increasing R. This furnishes another reason for giving voltmeters a high resistance.

In the case of all voltmeters except those of the electrostatic type the deflection is produced by the *current*, and since this latter depends on the P.D. and the *resistance* of the instrument, it follows that if the latter is, under all conditions, to give the same deflection with the same P.D. across its terminals, its resistance must be constant, and must not be affected by either temperature changes in the surroundings or temperature changes brought about by the heating effect of the current. In order to secure this condition the bulk of the resistance of a voltmeter usually consists of a resistance coil wound with a wire having a very small temperature coefficient (German silver, platinoid, manganin, &c.), while the *working coil* consists of copper wire, the use of which enables this coil to be made much lighter and more compact than if the high-resistance and low-temperature-coefficient material were employed throughout.

The resistance of the voltmeter circuit cannot be maintained absolutely constant at all temperatures, and any error arising from temperature changes is referred to as the *temperature or heating error.*

Hot-wire Voltmeters.—Hot-wire voltmeters may be constructed on the same principle as hot-wire ammeters. In common with electrostatic instruments hot-wire voltmeters possess the property of reading correctly on alternate-current circuits when calibrated with continuous currents.

Electrodynamic and Electromagnetic Voltmeters.—All the instruments under this head which have already been described as ammeters may be wound as voltmeters. Fine wire would be used in winding the working coil, and a large additional resistance having a negligible temperature coefficient would be connected in series with this coil. The Weston instrument, for example, illustrated in fig. 83, could be transformed into a voltmeter by removing the shunt across its terminals, and substituting for it a very high resistance connected *in series* with the working coil.

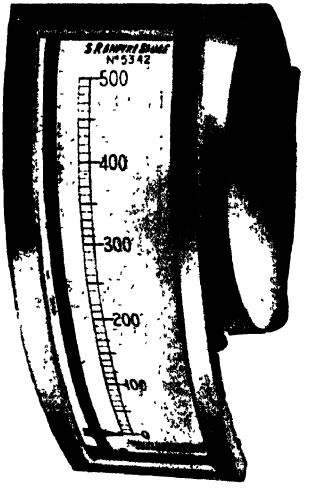


Fig. 87.—Kelvin's S.R. Edgewise Voltmeter

Kelvin's S.R. Type Voltmeter.—A form of switchboard voltmeter known as the "edgewise" pattern[1] (a form of construction first introduced by Mr. Evershed), one example of which is illustrated in fig. 87, has recently come much into favour on account of the small space which it takes up on a switchboard. The instrument shown in fig. 87 is of the S.R. (siphon recorder) or moving-coil type, and is shown in fig. 88 with the outer case removed. The magnets are circular in shape. The moving coil and phosphor-bronze hairspring are clearly seen in fig. 88, and other details of construction will be readily understood by an inspection of this figure.

Calibration of Voltmeters.—Voltmeters may be calibrated by comparison with a standard instrument, or by the potentiometer method. It is needless to remark that in comparing the readings of two voltmeters these instruments should be connected *in parallel*, and not in series as would be the case with

[1] Ammeters of a similar pattern are also largely used.

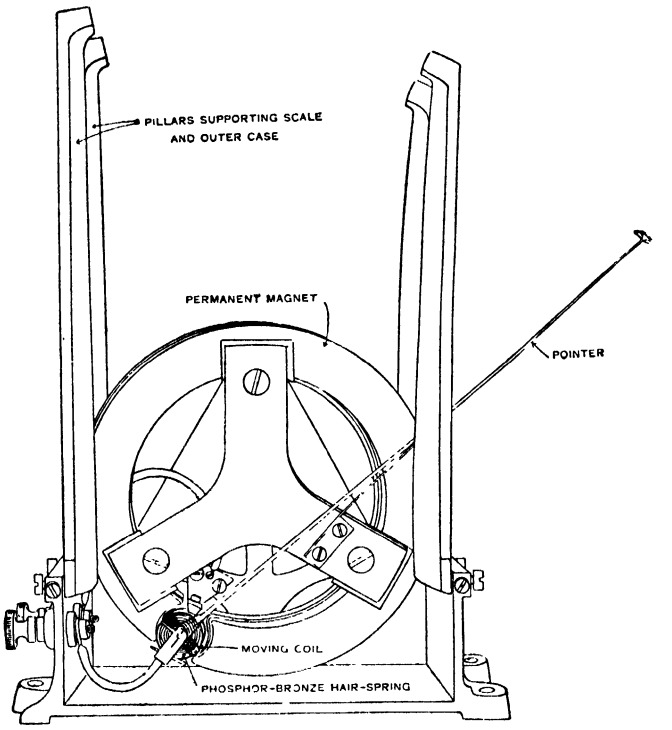


Fig. 88.—Kelvin's S.R. Edgewise Voltmeter

ammeters. A Kelvin Centi-ampere Balance connected in series with a high resistance, and one of Kelvin's electrostatic voltmeters, are frequently used as standard instruments.

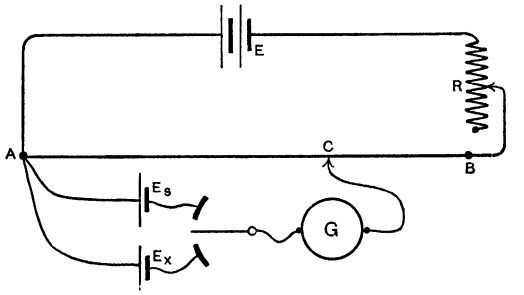


Fig. 89.—Polentiometer

The Potentiometer.—The principle underlying the potentiometer method was originally introduced by Poggendorff, and is illustrated in fig. 89. It is essentially a method for determining the ratio of two E.M.F.'s or P.D.'s. Let E be a source of constant E.M.F. (a battery of secondary cells) which maintains a constant current in the conductor AB,

and let this conductor be so arranged that contact may be established at any point C of it (by means of a slider), the value of the resistance AC being accurately known. If while the current is flowing along AB we arrange a branch circuit between the points A and C, including a galvanometer G in this circuit, then the P.D. which exists between the points A and C will also produce a current in the branch circuit from A to C, and this current will be indicated by the galvanometer. Suppose next that an E.M.F. E_s is introduced into the branch circuit, the direction of this E.M.F. being from C to A, i.e. in opposition to the P.D. existing between those two points. If we suppose that this E.M.F. is steadily increased, then the effect will be to steadily decrease the current in the branch circuit, and for a certain value of E_s—when it becomes just equal to the P.D. between A and C—the current will vanish, and the galvanometer will show no deflection. We then say that E_s *balances* the P.D. across AC. If the value of E_s be still further increased, a current will flow from C to A, the galvanometer deflection being reversed.

Instead of supposing that E_s is varied while the P.D. across AC remains constant, let us now take E_s to have a definite fixed value, and to represent the known E.M.F. of a standard cell. Balance may in this case be obtained by varying the P.D. across AC. This may be done in two ways: either by shifting the point of connection C along the wire AB while the current in AB remains constant, or else by keeping C fixed and altering the current along AB by means of an adjustable resistance R placed outside AB. Let us suppose that by either of these two methods balance has been obtained: then we know the value of the P.D. across AC, for this must equal the E.M.F. E_s of the standard cell. Again, if r_s be the resistance of AC corresponding to a state of balance, then the current along AC (or along AB) is given by E_s/r_s. Let, next, an unknown E.M.F. or P.D. of amount E_x be substituted for E_s[1] (this may be quickly done by means of the two-way switch shown in the diagram), and let balance be once more obtained *by shifting the point* C, *the current along* AB *remaining constant.* If r_x is the resistance of AC corresponding to balance in this second case, then E_x/r_x gives the current along AB, and since this has the

[1] Care being taken to connect it up the right way—in opposition to the P.D. across AC.

same value as before, we must have $E_x/r_x = E_s/r_s$, whence $E_x = \frac{r_x}{r_s} E_s$. We thus have a method of obtaining an unknown E.M.F. in terms of a known one, and the arrangement shown diagrammatically in fig. 89 for doing this is known as a *potentiometer.*

A simple but crude and not very accurate form of potentiometer is one in which AB is represented by a bare wire of high-resistivity material stretched along a graduated scale, the diameter of the wire being as uniform as possible, so that the resistance of any portion of it may be assumed to be proportional to its length, and hence the ratio r_x/r_s equal to that of the corresponding two lengths of the potentiometer wire; these lengths may be read off directly on the scale. The point C would correspond to a knife-edge contact attached to a suitable slider.

The main objection to such an arrangement is the want of uniformity in the wire, and its liability to become accidentally damaged or destroyed. For these reasons, in all accurate forms of potentiometer, the slide-wire either forms only a small fraction of the total resistance corresponding to AB, or else is entirely absent.

A good though not very convenient form of potentiometer may always be improvised by using two resistance boxes, one to represent AC, the other CB, and, balance having been obtained with the standard cell and the value of AC noted, again obtaining balance with the unknown E.M.F. or P.D., by varying the resistances AC and CB in such a manner as to keep their sum constant. This is done by plugging an equivalent resistance in CB when a certain resistance in AB has been unplugged and vice versa.

It will be noticed that when balance has been obtained, the standard cell is not required to give any *current.* If it is allowed to send large currents, its E.M.F. is lowered, and an error introduced into the measurement. In order to prevent the possibility of large currents passing through the standard cell, it is advisable, during the early stages of the measurement, to introduce a large resistance into the galvanometer and cell circuit, and then to cut this out as balance is approached.

The potentiometer itself as usually constructed is suitable for

the measurement of only relatively small E.M.F.'s—not exceeding about 1.5 volts. In combination with a standard resistance, it may be employed for the calibration of ammeters as already mentioned, the standard resistance taking the place of E_x in fig. 89 (E_x would be represented by the P.D. across the standard resistance). The complete diagram of connections is given in fig. 90.

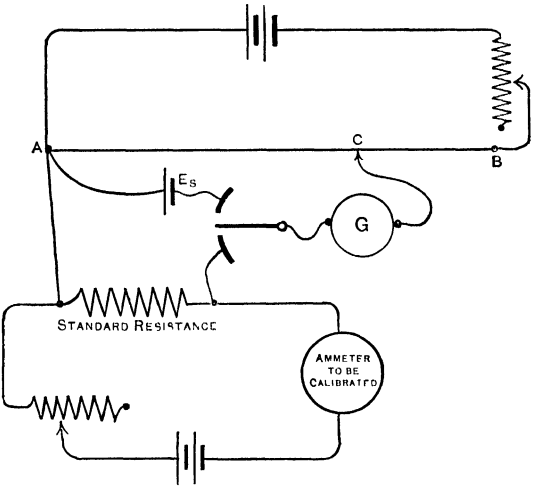


Fig. 90.—Ammeter Calibration

The range of the instrument may be enormously extended, so that it may be used for the measurement of very high P.D.'s, by the simple device of using a small, accurately *known* fraction, of the unknown high P.D., and balancing this on the potentiometer.

Subdivision of P.D. by Means of Resistance—Calibration of Voltmeters.—A known fraction of an unknown P.D. is easily obtained by introducing between the two points across which the P.D. exists a resistance of known value, and using a known fraction of the total resistance. The P.D. across this section of the resistance then corresponds to the same fraction of the total P.D. This arrangement enables

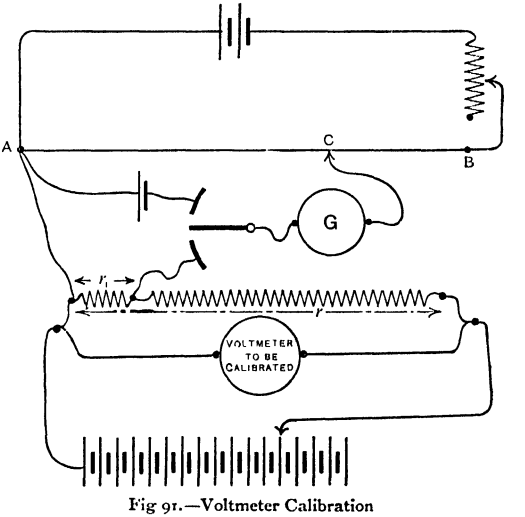


Fig 91.—Voltmeter Calibration

us to calibrate high-reading voltmeters by means of the potentiometer.

The diagram of connections is shown in fig. 91. A resistance r is connected in parallel with the voltmeter, so that the P.D. across this resistance is the same as that which exists between the voltmeter terminals. The P.D. across r_1, which is a known fraction of r, is then balanced on the potentiometer; if the value corresponding to balance is E_x volts, then the true value of the voltmeter reading is $E_x r/r_1$.

Recording Ammeters and Voltmeters.—In many cases it is desirable to have a continuous record of the current in a circuit, or of the P.D. across its terminals, and in such cases *recording* instruments are employed. The electrical and magnetic arrangements of such an instrument present no peculiarities, and are similar to those used in ordinary indicating instruments. But the simple pointer is replaced by a tracing point or pen which traces out a record on a sheet of paper moved at a constant rate by clockwork, in a direction which is at right angles to the motion of the tracing pen.

Short-scale Instruments.—Electrical energy is generally supplied at a constant P.D., and many of the voltmeters found in central stations are intended to enable the switchboard attendants or engine drivers to maintain a constant P.D. between certain points. It is evident that under these conditions only a limited portion of the scale is used, readings which are either much higher or much lower than the normal P.D. not being required. It is an obvious advantage to make the working part of the scale as open as possible, and to suppress the remainder. This may be done in a variety of ways. For instance, in the case of a moving-coil instrument the field in which the coil moves may be made of variable strength by making the air-gap of variable length, the shortest length, and hence most intense field, corresponding to the normal reading; a given change in the current through the coil at the normal reading will then produce a much larger displacement of the coil across the field than at a reading either lower or higher than the normal.

Although short-scale instruments are mainly represented by voltmeters, the construction is equally applicable to ammeters, should such be required.

Electrostatic Voltmeters.—The best instruments for measuring ordinary voltages are, however, those of the electrostatic type. They differ from all the others described in that they require little, if any, current and no power to actuate them. They depend for their action on the known principle that when two bodies are charged to different potentials they attract one another. The simplest instrument of this type is the gold-leaf gauge adapted by Lord Kelvin. It consists of a single thin sheet of gold leaf fastened along its top edge to a metal plate (fig. 92). The leaf and plate are mounted in a quadrantal box with a glass front, being insulated from the case by ebonite supports. One terminal is placed in electrical connection with the gold leaf, the other with the case, and when there is a potential difference between these the gold leaf is drawn upwards toward the top of the case, the amount of deflection being observed on a scale engraved both on the face and back of the instrument. This gauge is, of course, not accurate, and will only measure pressures of over 1500 volts. A more sensitive instrument is the multicellular voltmeter (figs. 93, 94), also devised by Lord Kelvin. In this case the moving part is a needle, made up

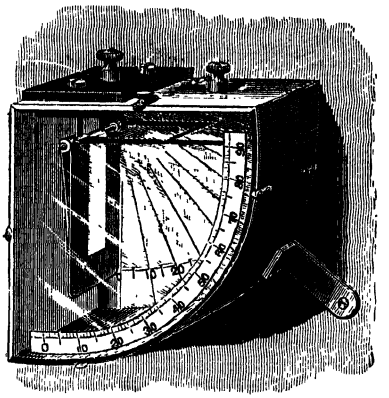


Fig. 92.—Kelvin Gold-leaf Gauge

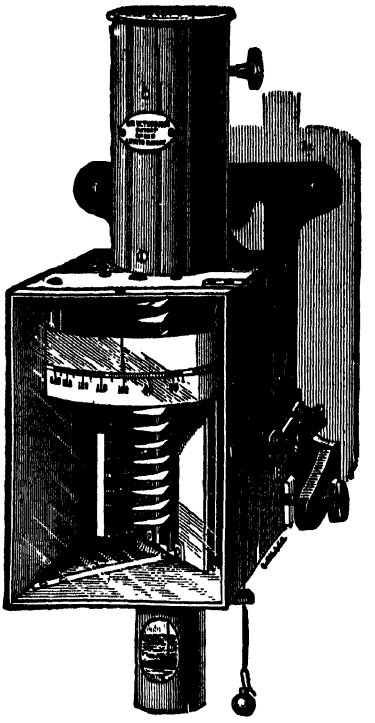


Fig. 93.—Kelvin Multicellular Voltmeter with front removed

of a number of horizontal parallel triangular plates rigidly attached to a metallic rod, and suspended by an iridio-platinum wire (a small coach spring being interposed between the vanes and the needle to prevent the latter from being damaged by vibration). The fixed part of the instrument consists of two sets of cells, which attract into them the vanes of the needle. The cells are also triangular in shape, and are made by inserting brass plates in saw-cuts in a brass back block, thus ensuring parallelism and equidistance between them.

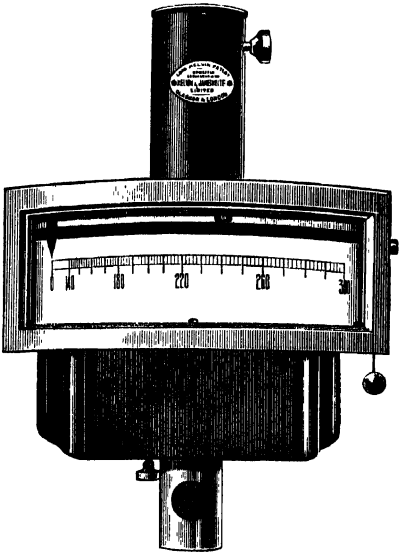


Fig. 94.—Kelvin Multicellular Voltmeter

The arrangement of the cells and needle is well shown in the figure. Attached to the bottom end of the needle is a small horizontal plate moving in a bath of heavy oil, which effectually damps the motion of the needle and prevents undue oscillation.

CHAPTER XIII

WATTMETERS AND SUPPLY METERS

Wattmeters.—The power spent in any part of a circuit is, as we have seen, measured by the product of the P.D. into the current. This power is, in the most general case, used up in two ways: first, in producing *heat* in the part of the circuit considered; if r stand for the resistance of this part, and i for the current, then the rate of heat production is in every case given by ri^2 (not unfrequently, this is the only way in which the power is employed, and if so, ri^2 becomes identical with Vi, where V is the P.D. across the part of the circuit considered); secondly, the power may be used in effecting some

transformation of electrical energy into a form other than heat —e.g. mechanical or chemical energy. It is important to bear in mind that the second kind of energy transformation is *always* accompanied by the production of a counter E.M.F. (or opposing E.M.F.), and that the corresponding rate of transformation, or power, is given by $E\,i$, where E is the counter E.M.F. Assuming the existence of such a counter E.M.F., the total power is given by $r\,i^2 + E\,i = V\,i$.

Instruments for measuring power are termed *wattmeters*. In the case of continuous-current circuits, if a voltmeter and ammeter are available, a simple multiplication of the volts and amperes gives us the power in watts. A wattmeter would, therefore, in this case be superfluous, although it might sometimes be convenient to replace the volt- and am-meter by a single instrument. In the case of alternating-current circuits, however, the *mean* power is not in general given by the product of the volts and amperes, and the wattmeter in this case performs a much more complicated mathematical operation than a simple multiplication. Hence, although wattmeters are sometimes used in connection with continuous currents, their chief use is on alternating-current circuits.

A wattmeter is essentially a combination of a volt- and ammeter. It is provided with two windings, one being a high-resistance winding, which represents the voltmeter part of the instrument; and the other a low-resistance winding, which corresponds to the ammeter part. Four terminals are frequently provided, one pair to each winding; in some cases, however, there are only three terminals (two large and one small), one of these corresponding to the junction of the two windings.

The high-resistance winding is frequently spoken of as the *pressure coil*, and the low-resistance one as the *current coil*, of the instrument. As in the case of voltmeters, the high-resistance winding consists of two parts, one being the working coil of copper wire, and the other, by far the larger portion of the resistance, a winding of high-resistivity, low-temperature-coefficient material, arranged so that it produces no appreciable magnetic effect (wound *non-inductively*, as it is termed).

Practically every form of wattmeter is an instrument of the *electrodynamic* type. The thick wire or current coil is fixed, and produces a field proportional to the current. In this field

is placed the movable pressure coil, which carries a current proportional to the P.D. Hence for any given relative positions of the two coils, the couple acting at any instant on the movable coil is proportional to the product of the P.D. and current at that instant, i.e. to the instantaneous power. If the power fluctuates rapidly, as it does in alternating-current circuits, the mean couple acting on the movable coil will be proportional to the mean power.

Fig. 95.—Weston Wattmeter

As a typical example of a good wattmeter, we shall consider the Weston Wattmeter, the general appearance of which is shown in fig. 95. The two heavy strips of copper seen projecting to the left of the figure are the ends of the current coil, while the small terminals at the top of the case are in connection with the ends of the pressure circuit. The method of arranging the connections when using the wattmeter is shown in fig. 96, where T_c, T_c' are the terminals of the current coil, and T_p, T_p' those of the pressure circuit. The coiled line L represents that part of the circuit in which power is being measured. The case of the instrument is made of mahogany,[1] with the exception of the top, which is of ebonite, and which carries the coils. Fig. 97 shows the instrument with the top unscrewed and laid upside down on the case, so as to exhibit clearly the current and pressure coils. The controlling couple is furnished by the usual phosphor-bronze hairsprings.

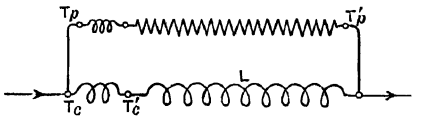


Fig. 96.—Connections of Wattmeter

Electric Supply Meters—Quantity and Energy Meters.—

[1] In the case of all wattmeters intended for use with alternating currents it is extremely important to avoid the use of metallic cases or metallic supports for the coils, as the currents induced in such masses of metal may seriously affect the readings of the instrument.

A form of electrical measuring instrument which is used in larger numbers than any other, is the instrument which is connected to the wiring of a consumer of electrical energy for the purpose of measuring the total amount of energy supplied to him. From the electrical point of view such instruments may be divided into two large classes, known as *quantity* meters (coulomb-meters, or integrating ammeters) and *energy* meters (watt-hour meters, or integrating wattmeters).

Electrical energy is supposed to be supplied to a consumer at a constant, or approximately constant, P.D. Now, if the P.D. be assumed constant, it is evident that the power will be

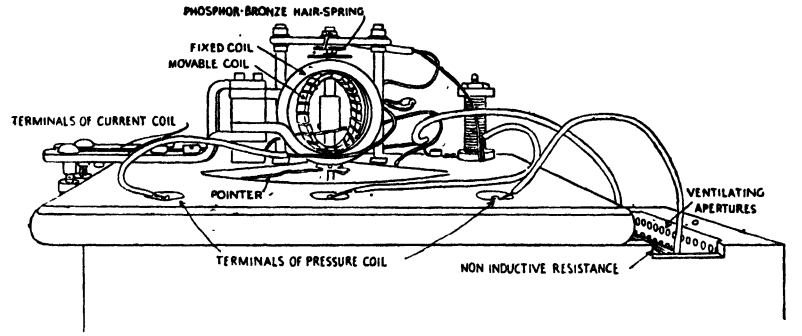


Fig. 97.—Weston Wattmeter

simply proportional to the current, and hence the energy to the quantity of electricity (in coulombs or ampere-hours) supplied. On the assumption of constancy of P.D. an instrument capable of measuring the quantity of electricity supplied may be used as an energy meter. Many forms of such *quantity* meters are in use.

A more satisfactory form of meter is, however, one capable of measuring correctly the actual energy, and not merely the quantity.[1] Such a meter is called an energy meter.

The mechanism of an electric supply meter is so arranged as to make the meter read directly in Board of Trade units (kilowatt-hours), either on a series of dials—resembling those used in a gas meter—or on some other suitably arranged scale.

The fundamental law which a thoroughly satisfactory meter

[1] If the P.D. at which a consumer is supplied falls below the specified value, a quantity meter will register an amount which is in excess of the actual energy received by the consumer.

must fulfil is that the rate of registration at any instant should be proportional to the current in the case of quantity meters, and to the power in the case of energy meters; and the great difficulty which manufacturers have always had to face is the construction, at a reasonable cost, of meters capable of obeying this law with sufficient accuracy.

Electrolytic Meters.—Numerous forms of electrolytic quantity meters have been devised. Edison used copper and, at a later stage, zinc voltameters; by weighing the cathode plates after the meter had been in use for some time, the amount of copper or zinc deposited could be determined, and from this the quantity of electricity supplied calculated. Such meters, however, are not direct-reading, and involve the troublesome operations of collecting and weighing the plates.

Two direct-reading electrolytic quantity meters are in use at the present time. One of these, the Bastian meter, consists of a glass vessel fitted with platinum electrodes which reach down to the bottom of the vessel. The vessel is filled with acidulated water, and a layer of oil is floated on the top of the water to prevent loss by evaporation. The current passing through the meter decomposes the water, and the quantity of electricity supplied is directly proportional to the amount of water decomposed, and may be read off on a suitably arranged vertical scale.

The main defect of the Bastian meter is its large counter E.M.F. (about 1.5 volts), and consequent large absorption of power; hence it is only suitable for small currents.

In the Wright electrolytic meter, illustrated in figs. 98 and 99,[1] a double iodide of mercury and potassium is used as the electrolyte, and the counter E.M.F. is so small as to be negligible. D is a reservoir filled with mercury, and connected with a flat glass vessel whose lower part is provided with an annular groove A C containing mercury. This circle of mercury forms the anode, and as the mercury is used up by going into solution, its level is maintained by the flow at intervals of additional mercury from the reservoir D, the place of this mercury being taken by the electrolyte, which forces its way past the narrow

[1] From the *Journal of the Institution of Electrical Engineers*, by permission of the Council of the Institution. In order to prevent any mercury from being shaken down the measuring tube from the annular anode by mechanical vibration, a guard cylinder of platinum gauze is fixed into the glass lip which forms the inner boundary of the annulus.

neck connecting the two vessels when the mercury level in A C falls slightly. The entire vertical glass tube seen below A C is filled with the electrolyte. The conical cathode B is made of iridium foil, and the mercury which collects on it drops into the funnel F at the top of the siphon tube E, and gradually fills this tube. When the tube has been completely filled (which corresponds to 100 units), the mercury automatically siphons out and falls to the bottom of the outer tube, as shown at G in the figures. The current which produces electrolysis is only a small fraction of the total current, the bulk of which passes through the manganin shunt R. An iron-wire resistance r_1 is connected in series with the

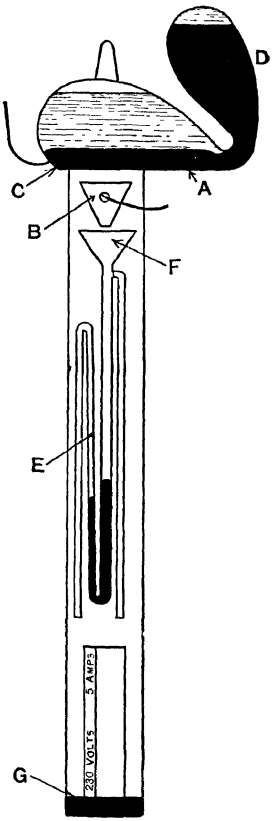


Fig. 98.—Wright Electrolytic Meter

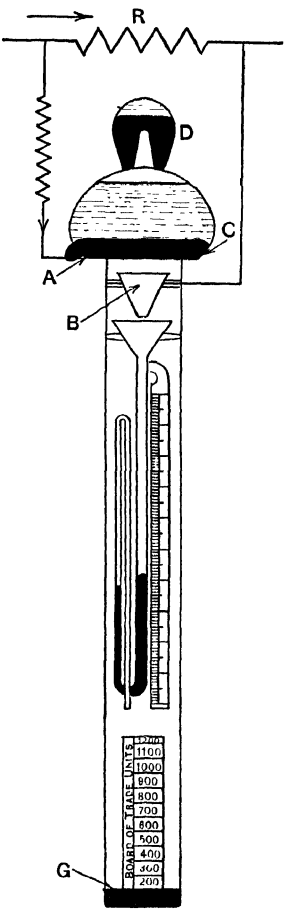


Fig. 99.—Wright Electrolytic Meter

voltameter, the value of r_1 being so adjusted as to make the total resistance of the voltameter circuit independent of temperature changes. When all the mercury contained in the reservoir D has been used up, the meter is easily reset by tipping it up and allowing the mercury to flow back into C.

Clock Meters.—These meters depend for their action on the attracting force which is exerted between two coils carrying current in the same direction. One of these coils, which is made of fine wire and has in series with it a large non-inductive resistance, is connected across the mains, so as to carry a current proportional to the pressure applied to the distributing circuit. The other coil is made of thick wire and carries the main current. The relative position of the coils is shown in fig. 100. As in the case of the wattmeter the total attraction between the coils will depend, not only on the magnitudes of the currents flowing through them, but on their phase relationship, and the average attraction between the coils is proportional to the power supplied to the circuit. The effect of this attraction between the coils will be to increase the downward pull on the bob of the pendulum, which will be equivalent to increasing the gravitational force. Thus the rate at which the pendulum will vibrate will be more rapid when the current is passing than when it is not; also, since the attraction between the coils will be *proportional* to the power supplied, the rate at which the pendulum will swing will depend on the total power transmitted.

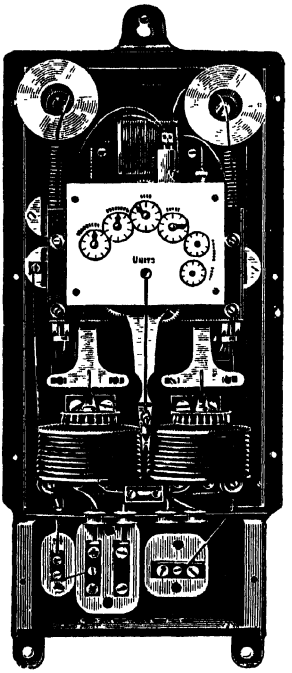


Fig. 100.—General View of Aron Meter with case removed

In the first meters of this type that were developed the clock used was first "rated" by comparison with a chronometer when no current was passing, and the total gain of the clock was then used to determine the total energy supplied to the consumer. This arrangement was, however, not at all satisfactory, and

before many years elapsed two clocks were substituted. These two clocks were adjusted so as to go at exactly the same rate, and their mechanism connected by what is known as a differential gear. The two driving wheels of this gear, shown in fig. 101, revolve in opposite directions. As long as the rate of the clocks is the same the central wheel will revolve but will not move bodily, but as soon as one wheel moves faster than the other the central wheel will move at a speed equal to the difference in rate at which the two driving wheels are revolving. The translational movement of the central wheel therefore measures the amount of gain of one clock on the other, and the support for this wheel is geared to a counting mechanism which records its motion. The train of wheels which connects the central wheel to the counters is so arranged that the instrument records directly the number of kilowatt-hours supplied through it.

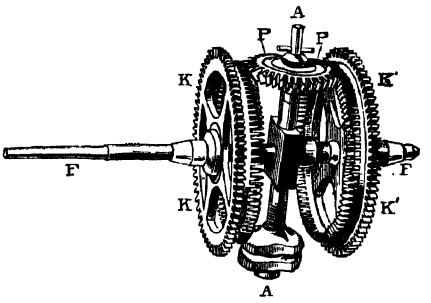


Fig. 101.—Differential Gear of Aron Meter

The instrument has, during recent years, been greatly modified from its original form, the most considerable alterations being as follows: Both pendulums are now acted on by the current, the connections being arranged so that an increase in the apparent force of gravity is produced on the one, and a repulsion or decrease in the apparent force of gravity on the other. The accuracy of the instrument is increased and its range is enlarged by this method of connection. The clocks are now electrically driven, and are wound by a simple arrangement every thirty seconds. An automatic cut-off is also provided, so that if for any reason either of the clocks stop, the record is not spoiled.

Motor Meters—Ferranti Meter.—This is a very numerous class, and includes a number of well-known types. The examples which we have selected for a detailed description are the Ferranti, the Elihu Thomson, and the Chamberlain and Hookham meters.

The general appearance of the Ferranti meter is illustrated in fig. 102*a*, which shows a meter with the cover and the

terminal door removed. The instrument consists essentially of a copper disk which rotates in a mercury bath between the poles of a double permanent magnet, the limbs being connected by cross-bars above and below the pole pieces, as shown in fig. 102*b*. The mercury chamber is closed by an insulating ring, through the right-hand side of which a lead passes and makes contact with the mercury.

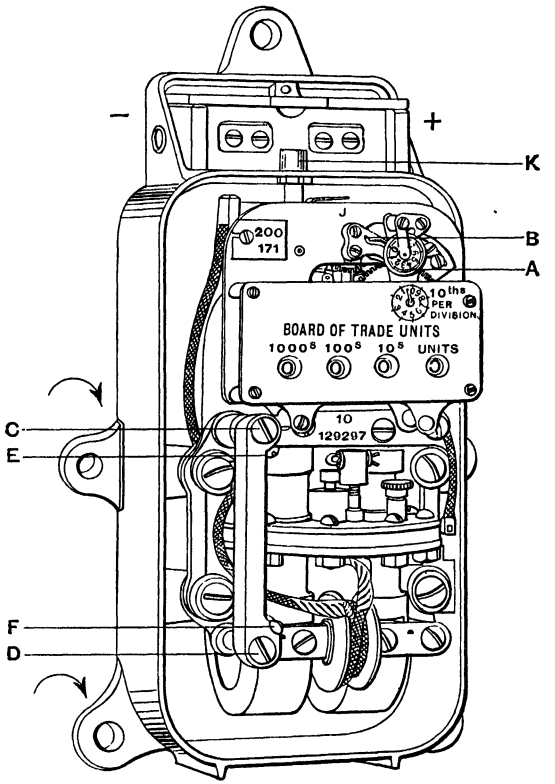


Fig. 102*a*.—Interior of Ferranti C.C. Meter

A second lead makes contact with the mercury at the bottom of the chamber, and is continued in the form of a coil round the lower cross-bar. The current passes through the copper disk from the side connection to the connection at the bottom of the chamber. This current flowing from the edge to the centre of the disk is at right angles to the magnetic flux between the right-hand pair of poles, and the disk is thus subjected to a driving torque proportional to the current. The controlling torque is supplied by the interaction of the magnetic flux between both pairs of poles with the eddy currents produced in the copper disk, and is proportional to the speed of the disk.

The speed at which the disk runs is thus proportional to the current passing through the meter. An additional retarding torque is supplied by the fluid friction of the mercury, and since this increases slightly as the speed rises, it has to be compen-

sated for. This is accomplished by the coil on the lower cross-bar, which, by increasing the magnetic flux between the right-hand pair of poles and diminishing it between the left-hand pair, increases the driving torque and at the same time diminishes the retarding torque at the higher speeds. An iron bar secured by iron screws connects the left-hand ends of the cross-bars. This provides a means of adjusting the speed of the meter. For example, if the meter is found to be reading low, the bar is tightened up by means of the screws. The flux between the left-hand pair of poles is thereby reduced, and consequently the retarding torque is diminished, allowing the meter to run faster. By slackening the screws the meter may be made to run slower.

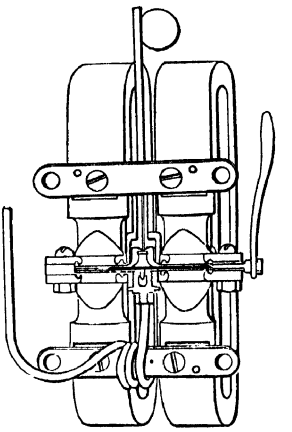


Fig. 102*b*.—Motor Element Ferranti Meter

Elihu Thomson Meter.—The best-known example used for alternating current in this country is that devised by Prof. Elihu Thomson (fig. 103). It consists essentially of a small motor in which the field coils F carry the main current, and the armature A the shunt current. There is no iron in the field-magnet system, so that the magnetic field produced by the current is always directly proportional to it. The force f acting on the armature conductors at any instant is dependent on the strength of field H, and the strength of the shunt current i. Hence $f \propto i \times H$, and H being proportional to I, the main current, f is proportional to iI.

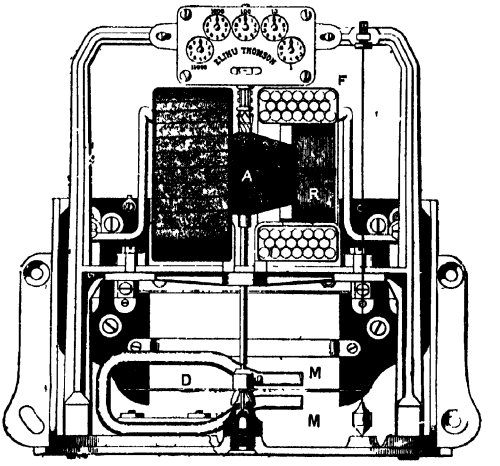


Fig. 103.—Part Section of Thomson Meter, showing principle of construction

Now $i = \frac{E}{R}$, if the inductance of the shunt circuit is negligible, R being a constant, we have torque $\propto$ IE at any instant. If there were no resisting torque the motor would increase in speed until the back E.M.F. it produced was equal to the E.M.F. applied. In order that the number of revolutions of the meter may register energy, the speed at which the meter is running must be proportional to the power supplied through it. This is effected if a retarding torque is applied which is *proportional to the speed.* When such an arrangement is adopted the speed will increase until the driving and retarding torques are equal, and consequently, the speed, being proportional to the retarding torque, will be proportional to the driving torque, that is to the power supplied.

This retarding torque is produced usually by attaching to the arbor of the armature a flat copper sheet D (fig. 103), which moves between the poles of a strong permanent magnet. Eddy currents are induced in the copper by its motion, and these eddy currents react on the magnetic field, producing a retarding torque, which is always exactly proportional to the speed at which the plate is moving. In order that the meter may record accurately, many details have to be looked to; the chief difficulty arises from the necessary friction at the bearings. To reduce this to the smallest possible amount, the bottom of the arbor, which is made of hard steel, is pivoted on a sapphire jewel, and the top supported in a cup of hard steel. The spindle is steadied at both ends by collar bearings, loosely fitting. The friction of the brushes on the commutator (which is usually of eight parts) is eliminated in the latest type by using silver strip, which is arranged to press edgewise on the commutator surface.

Chamberlain and Hookham Meter.—This form of motor meter is shown, partly in section, in fig. 104. AA is a powerful permanent magnet to whose poles are attached the soft-iron pole pieces BB, separated from each other in the middle by the insertion of a brass distance piece C. Below these pole pieces is placed a piece of soft iron DD, so that some of the magnetic lines pass downwards on one side and upwards on the other across the gap which separates DD from BB. In this gap is placed the horizontal copper disk N, which forms the "motor disk", the remainder of the space being filled up with mercury.

The remaining magnetic lines pass up two cylindrical pole pieces E on one side, across iron bridge pieces G, and then

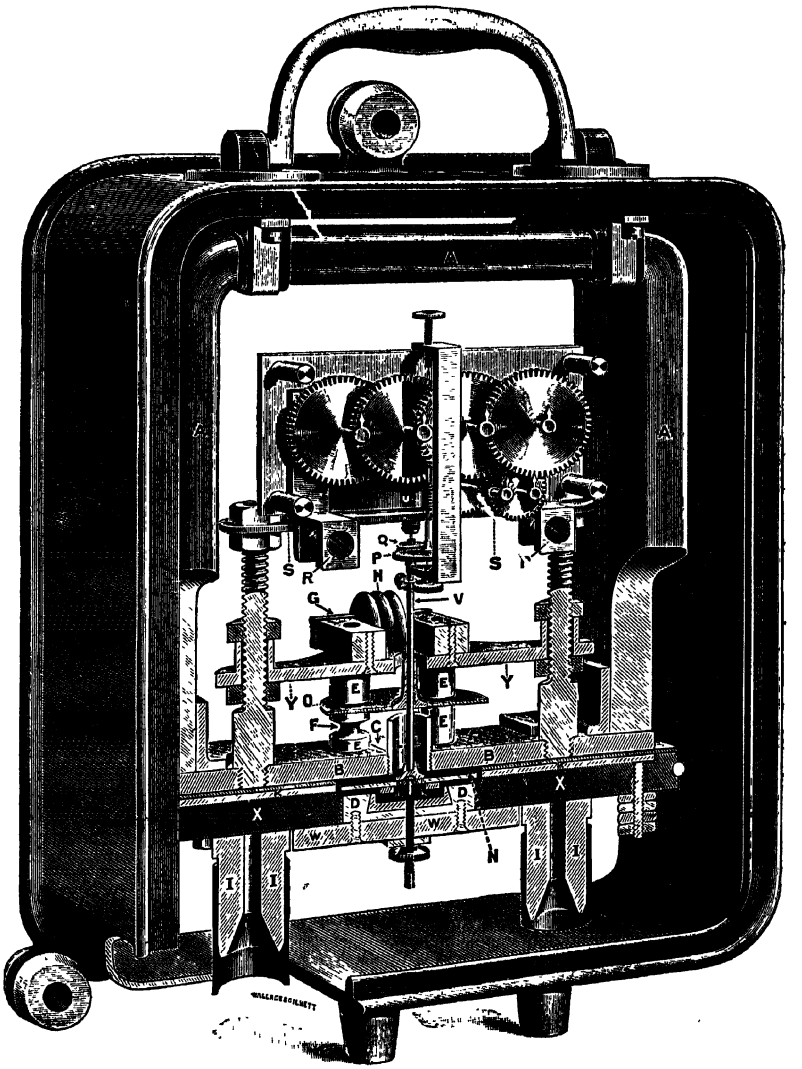


Fig. 104.—Chamberlain and Hookham Meter

down two pole pieces E on the other side. In so doing the lines have to cross the air-gaps interposed in their path halfway up each pole piece. Into these air-gaps is introduced the

"brake disk" O, which is made of a low-temperature-coefficient alloy, and which is mounted on the same vertical spindle as the motor disk. This spindle is geared with the counting mechanism.

The current flows along the left-hand copper strip K, enters the mercury, and then flows through the motor disk in a diametral direction, leaving it by the right-hand copper strip K. The magnetic field in which the disk is placed causes the latter to rotate—the speed of rotation being, even at full load, slow, and hence the friction of the mercury inconsiderable. Since the field is constant, the driving torque acting on the motor disk will be simply proportional to the current. The retarding torque is provided by the currents induced in the brake disk on account of its motion across a non-uniform magnetic field, and since the intensity of this field remains constant, the retarding torque is simply proportional to the speed. Hence the speed varies in direct proportion to the current, and the quantity is proportional to the number of revolutions.

The mercury friction cannot, however, be altogether neglected, and in order to balance it a compensating coil H is wound round the upper iron bridge piece; this coil is supplied with the main current, and weakens the field in which the brake disk is placed, thereby tending to make it run faster and thus counteracting the effect of fluid friction.

CHAPTER XIV

ELECTROMOTIVE FORCE AND RESISTANCE

Electromotive Force.—Electrostatics gave us the conception of a field of electric force. When an insulated conductor is carried into such a field, a movement of electricity takes place in the conductor, ending in a definite distribution of positive and negative electricity on different parts of its surface. A movement of electricity, whether of a transitory or a steady character, can only be brought about by electric force.

In a uniform electric field, if we consider a line drawn in the direction of the force, the electric potential along it varies from point to point, and varies by equal amounts in travelling equal

distances. The product of the force by the distance we travel is the *line integral of the force*, and is clearly a difference of potential. As regards sign, it is a *fall* of potential if we travel in the direction of the force. In dealing with electric currents, we have usually to consider not the force at a point but the line integral of the force along a wire or other conductor. This line integral is called *electromotive force*. As regards its nature and "dimensions" it is a difference of electrical potential.

As regards the energy of a current: When a quantity Q of electricity moves from a place where the potential is V_1 to a place where it is V_2, the quantity of potential energy which runs down is $Q(V_1 - V_2)$, which we may describe as the product of the quantity of electricity displaced, by the line integral of electric force along its path. Let I denote the strength of a steady current, and E the electromotive force which maintains it; then the quantity of electricity which passes in time t is It and the line integral of electric force is E; hence the energy which runs down is EIt, and the energy in unit time is EI.

Distribution of Potential in the Circuit of a Battery.—In a circuit consisting of a battery of four similar cells and a

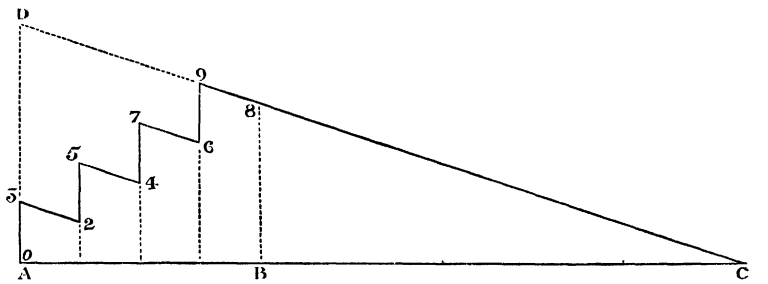


Fig. 105.—Curve of Potential for Closed Circuit

connecting wire, the distribution of potential is as shown in fig. 105. Resistance is represented by horizontal distance, potential by vertical height, and current by tangent of slope. We are to imagine the diagram bent round cylindrically till the point C coincides with the point A. The direction of the current is the direction ABC. As we travel in this direction there is a downward slope of potential in the whole circuit, with the exception of a limited number of points, where the change of level is abrupt. Possibly there is in each cell an abrupt rise

at one point and a smaller abrupt fall at another. For our purpose it is only necessary to attend to their algebraic sum for each cell. This is represented by any one of the four lines 03, 25, 47, 69. Calling it E, the current I, the resistance of each cell r, and the resistance of the wire R, the gradual fall in each cell will amount to Ir, in the wire to IR, and in the whole circuit to I $(4r + R)$. This must be equal to the total abrupt rise 4E. Hence I is equal to $\frac{4E}{4r+R}$; that is, to the whole electromotive force divided by the whole resistance. This is equal to the tangent of the angle ACD, and all the sloping lines in the figure are parallel. AB represents the total resistance of the four cells, BC the resistance of the wire, and AD the total electromotive force 4E.

The fall of potential in the wire (which is the same thing as the difference of potential between the poles of the battery) is represented by the line B8, and is IR, or $\frac{4E}{4r+R}$ R. The gradual fall in any one of the cells amounts to Ir, or $\frac{4E}{4r+R}$ r. If the external resistance BC is continually increased, the sloping lines tend to horizontality as the limit, and the difference of potential between the poles tends to the limit 4E, which is its value when the poles are not connected.

Resistance—Ohm's Law.—Let a current of I C.G.S. units flow in a conductor between whose ends there exists a P.D. of V units (C.G.S.). The ratio $\frac{V}{I}$ is then defined to be the *resistance* of the conductor to the current, and is usually denoted by R. Thus $R = \frac{V}{I}$.

Since the P.D. is V, and the current I, the power is $VI = RI^2$. This power is employed entirely in heating the conductor, since no other transformation of energy is supposed to be taking place.[1] Hence RI^2 gives us the rate at which heat is being produced in the conductor. We thus arrive at the following *alternative* definition of resistance:—

By the resistance of a conductor is meant the coefficient which, when multiplied by the square of the current, gives the rate of heat production in the conductor.[2]

[1] It is assumed that the conductor is not the seat of an E.M.F.

[2] The heat being measured in dynamical units (ergs).

There is no *a priori* reason why the quantity which we have just defined as the resistance of a conductor should not have different values for different currents; and whether such is the case or not is a matter to be decided entirely by experiment. Very careful experimental researches have shown that, so long as the *physical state* of the conductor (as regards temperature and stress) remains unaltered, *its resistance is independent of the current.* This extremely important result is known as *Ohm's Law*, and the reader must be careful to discriminate between the mere definition of resistance and Ohm's Law.

If Ohm's Law did not hold, then in dealing with the resistance of a conductor we should have to specify the current for which the resistance has been determined. But since the resistance is found to be independent of the current, the latter need not be specified.

The resistance of a conductor is thus a definite physical quality of it, which depends solely on the size and shape of the conductor and on its physical state.

The absolute unit of resistance is the resistance of a conductor in which unit current is produced by unit P.D. between its terminals.

If the P.D. be measured in volts, and the current in amperes, then the ratio $\frac{\text{volts}}{\text{amperes}}$ is taken as the resistance of the conductor in practical units or *ohms*.

$$1 \text{ ohm} = \frac{10^8}{10^{-1}} \text{ or } 10^9 \text{ C.G.S. units.}$$

Heating of Wires.—The heating of a wire by the passage of a current may conveniently be exhibited by the aid of the apparatus represented in fig. 106. Two uprights mounted on a stand are furnished, at different heights, with pairs of insulated binding-screws a, a', b, b', c, c', having wires

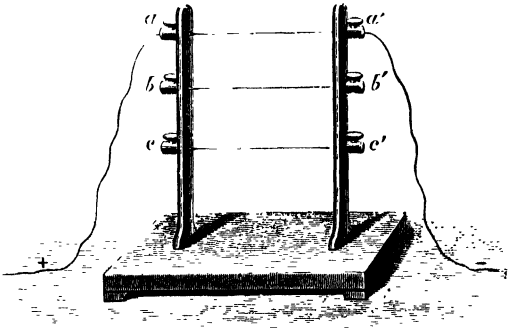


Fig. 106.—Stand for Heating Wires

stretched between them. A current can thus be sent through any one of the wires by connecting the terminals of a battery with the binding-screws at its extremities. When this is done with a battery of suitable power, the wire is first seen to droop in consequence of expansion, then to redden, and finally to melt, becoming inflamed if the metal is sufficiently combustible.

If a file is attached to one of the terminals of a battery, and the other terminal is drawn along the file, a rapid succession of sparks will be obtained; and, if the battery be sufficiently powerful, globules of incandescent metal will be scattered about with brilliant effect.

Joule's Law.—The energy of a current is equal to the product of the quantity of electricity that passes and the electromotive force that drives it. As the numerical measure of a current is the quantity of electricity which passes in unit time, it follows that the energy of a current I lasting for a time t is EIt, E denoting the electromotive force. But again, by Ohm's law, E is equal to the product of the current I and the whole resistance R. The expression for the energy therefore becomes

$$I^2Rt,$$

and this energy is all transformed into heat in the circuit, subject to a small correction for the Peltier and Thomson effects. It has accordingly been found, first by Joule, and afterwards by Lenz, Becquerel, and others, that the formula I^2Rt represents the quantity of heat generated by a current under ordinary circumstances. The experiments have usually been conducted by passing a current through a spiral of wire immersed in water or alcohol, and observing the elevation of temperature of the liquid.

This law of Joule's, like that of Ohm, may be applied to any part of a circuit, as well as to the circuit considered as a whole; that is to say, if the circuit consists of parts whose resistances are r_1, r_2, . . . , the quantities of heat generated in them are respectively I^2r_1t, I^2r_2t, . . . , and are therefore proportional to the resistances r_1, r_2 . . . of the respective parts, since I and t are necessarily the same for all.

Relation of Heat in Circuit to Chemical Action in Battery.—The energy of a current, and consequently the heat developed in the circuit, is the exact equivalent of the potential

energy of chemical affinity which runs down in the cells of the battery. This fact, first verified approximately by Joule, was more accurately confirmed by the experiments of Favre, who introduced into the muffle of his mercurial calorimeter a small voltaic cell with its poles connected by a fine wire.

His experiments also served to verify the application of Joule's law to each part of the circuit considered separately. By introducing the cell into the muffle whilst a spiral of fine wire connecting the poles was outside, and then introducing the spiral while the cell was outside, he was able to measure separately the heat generated in the cell and in the spiral, and these were found to be proportional to their resistances.

Elevation of Temperature.—If wires of different diameter or of different electrical conductivity form parts of the same circuit, so as to be traversed by the same current, the bad conductors will become more heated than the good, and the fine wires more than the coarse. All parts of the length of a uniform wire will be uniformly heated. The specific resistance of platinum is about six times greater than that of copper; hence six times as much heat will be generated in a platinum as in a copper wire by a given current, if the diameters of the two wires be the same.

The *elevation of temperature* is greater in a fine than in a coarse wire, not only because of its greater resistance, which leads to the development of a greater quantity of heat in it, but also on account of its smaller capacity for heat, and its smaller surface. When the current is passed for so short a time that the heat emitted may be neglected, the elevation of temperature varies directly as the resistance per unit length, and inversely as the capacity per unit length. The resistance varies inversely and the capacity directly as the section of the wire, and hence the elevation of temperature is inversely as the square of the section, or as the fourth power of the diameter.

On the other hand, if the current be continued till the permanent temperature is attained, capacity ceases to have any influence, and the heat emitted in unit time must be equal to the heat received. If θ denote the elevation of temperature, the heat emitted is approximately $2\pi r B\theta$ by Newton's law, r being the radius of the wire, and B a constant. The heat received is $\frac{A}{\pi r^2}$, A being another constant. By equating these two expres-

sions we find that $r^3\theta$ is equal to a constant, and hence θ varies inversely as r^3, that is, the elevation of temperature is inversely as the cube of the diameter.

To obtain the most rapid production of heat in the circuit considered as a whole, we must reduce the resistance to a minimum; for the heat produced in unit time is EI, which, by Ohm's law, is the same as $\frac{E^2}{R}$, and therefore varies inversely as R the total resistance.

Series and Parallel Arrangement of Resistances—Shunts. —Let two resistances, r_1 and r_2, be connected end to end, or

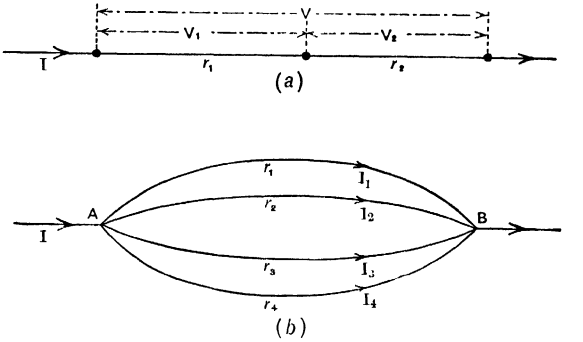


Fig. 107.—Resistances in Series and in Parallel

in series with each other, as in fig. 107 (*a*), and let a current I be sent through them. Let V, V_1, and V_2 stand respectively for the P.D.'s across the two resistances, and across the first and second taken separately. Then from the definition of P.D. it follows that $V = V_1 + V_2$; but by the definition of resistance, $V_1 = r_1 I$, and $V_2 = r_2 I$. Hence $V = (r_1 + r_2)I$, or $\frac{V}{I} = r_1 + r_2$. Again, by the definition of resistance $\frac{V}{I}$ is the total resistance of the two resistances connected in series. If this be denoted by r, then we have $r = r_1 + r_2$, i.e. the total resistance is the *sum* of the two resistances. Similar reasoning would apply to any number of resistances joined in series, so that in general

The total resistance of any number of resistances connected in series with each other is equal to the sum of the separate resistances.

From this it follows that the resistance of a conductor of uniform cross section is *proportional to the length* of the con-

ductor, and that the fall of potential over it when conveying a given current is also proportional to its length.

Consider next a number of conductors, whose resistances are r_1, r_2, r_3, r_4, placed side by side, as in fig. 107 (*b*), and connected between two common points A and B. Such an arrangement is called a *parallel* connection of conductors. If x = joint resistance of the system of conductors, V = P.D. between A and B, and I = total current, then $x = \frac{V}{I}$. But if I_1, I_2, I_3, I_4 denote the currents in the various conductors, then $I = I_1 + I_2 + I_3 + I_4$, and $I_1 = \frac{V}{r_1}$; $I_2 = \frac{V}{r_2}$: $I_3 = \frac{V}{r_3}$; $I_4 = \frac{V}{r_4}$, so that $I = V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}\right)$, whence

$$x = \frac{V}{I} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}}.$$

The reasoning may be extended to any number of conductors; hence the following general rule:—

The joint resistance of a number of conductors connected in parallel is equal to the reciprocal of the sum of the reciprocals of the separate resistances.

From this it follows that the resistance of a conductor of given length varies *inversely as the cross section* of the conductor.

In practice we very frequently have to deal with a parallel connection of two conductors. In such a case, one of them is said to form a shunt to, or to be shunted by, the other. Applying the rule given above, we easily find that in this special case the joint resistance is equal to the product of the two resistances, divided by their sum.

The reciprocal of the resistance of a conductor or system of conductors is termed the *conductance*, and in some problems it is more convenient to consider conductances than resistances.

If r_1 r_2 r_3 are three resistances in parallel, and I_1 I_2 I_3 the currents through them, we have $I_1r_1 = I_2r_2$, $= I_3r_3$, each of these products being equal to the difference of potential between the two points where the branches join. Hence the partial currents are inversely as the resistances through which they flow. When the number of branches is only two, we

have $\frac{I_1}{r_2} = \frac{I_2}{r_1} = \frac{I}{r_1 + r_2}$, I denoting the total current. Hence $I_1 = \frac{r_2}{r_1 + r_2} I$.

The name *shunt* is often applied to one of two parallel branches. The shunt has usually much less resistance than the other branch, and therefore takes the greater part of the current. A delicate galvanometer is often accompanied by a set of shunts, whose resistances are respectively $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of that of the galvanometer. Calling the resistance of the galvanometer g, and putting these fractions of it for r_2, we obtain $\frac{1}{10}$ I, $\frac{1}{100}$ I, and $\frac{1}{1000}$ I, as the portions of the whole current I that go through the galvanometer when one of these shunts is employed. Any one of the three shunts can be brought into use at pleasure by inserting a brass plug in the proper hole. The galvanometer can thus be employed for measuring currents which are too strong to be sent through it entire.

Universal Shunt-box.—Let a galvanometer of resistance g be joined up in closed circuit with a wire of resistance r. Let A be one of the junctions of g with r, and B a variable point on the wire, the resistance of the portion of r lying between A and B being x.

The resistances of the two branches which connect A and B are x and $g + r - x$, which have the constant sum $g + r$.

When a current passes between A and B, the fraction of it that goes through the galvanometer is $\frac{x}{g + r}$, which is simply proportional to x.

Ayrton and Mather have designed a shunt-box on this principle, in which, by inserting a plug in the proper hole, x can be made to have any of the four values r, $\frac{1}{10} r$, $\frac{1}{100} r$, $\frac{1}{1000} r$; so that if the whole current is sensibly independent of x, the current through the galvanometer can be changed at pleasure in the ratios of 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$. The same shunt-box can be used for different galvanometers, as no special relation is required between r and g.

Standards of Resistance.—Standards of resistance generally take the form of coils of wire of definite length and cross section. There is, however, a certain objection to the use of such standards. For the molecular structure of the material of which the wire is composed is liable to undergo slow changes, and

these bring about corresponding changes of resistance. The standard is therefore liable to change. Although in ordinary commercial work such changes need hardly be considered, yet in accurate determinations they must be guarded against. Hence it is that, in defining the practical unit of resistance, the Board of Trade has chosen to refer it to a material which possesses a perfectly definite molecular structure—pure mercury. The Board of Trade definition of the ohm reads as follows:—

The ohm has the value 10^9 *in terms of the centimetre and the second of time, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice,* 14.4521 *grammes in mass, of a constant cross-sectional area, and of a length of* 106.3 *centimetres.*

Resistivity and Temperature Coefficient.—We have already seen that, so far as its dimensions are concerned, the resistance of a conductor is directly proportional to its length, and inversely proportional to its cross section. The other two factors determining the resistance are the nature of the material forming the conductor, and the temperature.[1]

If we consider a conductor of length l and cross section a, then its resistance r may be written in the form

$$r = \rho \frac{l}{a},$$

where ρ, which depends on the material and temperature of the conductor, represents the resistance of a conductor of the same material, but of unit length and cross section, and is termed the *resistivity* of the material at the given temperature. The *resistivity* of a material may be defined as the resistance between opposite faces of a cube of it whose edge is unity.

The reciprocal of resistivity is termed *conductivity.*

The resistivity of a material in general depends on the temperature, and for moderately small changes of temperature the changes of resistivity may be taken as proportional to changes of temperature. It is usual to take 0° C. as the standard temperature, and if ρ_0 stand for the resistivity of a material at this temperature, its resistivity ρ_t at any other temperature t may be written as

$$\rho_t = \rho_0 (1 + \alpha t),$$

[1] We omit the consideration of the effects of mechanical *stress.*

where t = temperature, in degrees C., and α is a constant (roughly) known as the *temperature coefficient* of resistance.

As a general rule, α is positive for good conductors (metals), and negative for bad conductors (electrolytes and insulators). In the case of certain metallic alloys, α is so small that within a certain range of temperature the resistivity may be regarded as independent of temperature. Such alloys are particularly suitable for the construction of standard resistance coils, since it becomes unnecessary to apply any "temperature corrections".

Network of Conductors: Kirchhoff's Laws. — It is frequently required to calculate the steady currents in the various branches of a network of conductors, one or more of the branches being supposed to include a battery of given E.M.F., and the resistances of all the branches being supposed to be known. The necessary equations can be written down by employing the two following principles:—

I. *At any point of meeting of different branches, the inflow must equal the outflow*; or the algebraic sum of the currents is zero. This follows from the assumption of steady currents.

II. *In travelling round any portion that forms a closed circuit* (any mesh of the network) *the algebraic sum of the electromotive forces is equal to the algebraic sum of the products of current by resistance.*

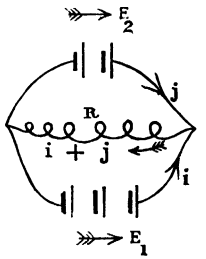


Fig. 108.—Kirchhoff's Laws

When the only electromotive forces are those of batteries in the branches, this amounts to the assertion that in travelling round a mesh the algebraic sum of the gradual falls is equal to the algebraic sum of the sudden rises; or that the algebraic sum of all the changes of potential in going round a mesh is zero. These two principles are known as Kirchhoff's Laws.

The following is an example (see fig. 108):—

A coil of resistance R connects the terminals of a battery of resistance R_1 and electromotive force E_1, and also connects the terminals of a second battery R_2, E_2, like terminals being connected to the same end of the coil. Required the currents in the system.

Let I_1 denote the current through the first battery, reckoned positive when it has the same direction as E_1; and I_2 the current through the second battery, reckoned positive when it has the same direction as E_2. Then the first law shows that the current through the coil will be $I_1 + I_2$, reckoned positive when it has the direction due to E_1 and E_2. The second law, applied to the circuit formed of the first battery and the coil, gives

$$E_1 = I_1R_1 + (I_1 + I_2)\,R,$$

and for the circuit formed of second battery and coil,

$$E_2 = I_2R_2 + (I_1 + I_2)\,R.$$

These two equations suffice to give I_1 and I_2. A third equation might be written down for the circuit formed of the two batteries; but it would be an equation derivable from the other two.

Solving the equations, we find

$$I_1 = \frac{(E_1 - E_2)\,R + E_1R_2}{RR_1 + RR_2 + R_1R_2},\quad I_2 = \frac{(E_2 - E_1)\,R + E_2R_1}{RR_1 + RR_2 + R_1R_2},$$

$$I_1 + I_2 = \frac{E_1R_2 + E_2R_1}{RR_1 + RR_2 + R_1R_2}.$$

Supposing E_1 to be greater than E_2 the expression for I_2 shows that the current through the second battery will be reversed if E_2 is less than $E_1R/(R + R_1)$.

Measurement of Resistance—Very Low Resistances.—The methods of measuring resistance are so numerous and varied, that it will be impossible here to attempt more than an outline of some of the more important of them.

If the resistance is a very low one (such as that of the armature or series coils of a dynamo), one of the most satisfactory methods is to send a known current I through the resistance, and to measure the corresponding fall of potential V. The resistance is then given by V/I.

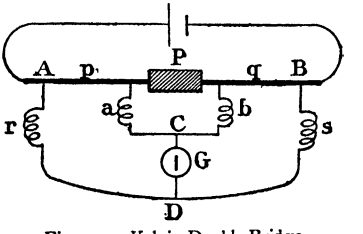


Fig. 109.—Kelvin Double Bridge

Kelvin's Method of Measuring Low Resistances.—To compare the resistances per unit length of two rods or thick wires p, q (fig. 109).

Connect one end of each to a metallic block P of very small

esistance. Connect their other ends to the terminals of a battery. Connect two points A, B, at a few diameters distance from these latter ends, through two known resistances r, s; and connect two points at a few diameters distance from the other ends through two known resistances a, b, proportional to r and s. If p and q denote the resistances of the portions of the two rods between the contacts, and are roughly known beforehand, it is necessary that approximately

$$\frac{p}{q} = \frac{a}{b} = \frac{r}{s}.$$

Connect the junction C of a, b with the junction D of r, s through a galvanometer; and if there is a deflection, shift the connections so as to alter the lengths of p and q until the deflection is annulled. Then the required ratio of p to q is equal to the known ratio r to s, as is shown by the following proof.

Imagine P divided by an equipotential section having the potential of C and D. The equal currents in p and q are made up of two branch currents, one flowing through a and b and the other through P, and these two branch currents are inversely as the resistances of the two branches. If we include in P the terminal portions of the two rods measured from the contacts with a, b, the two parts into which P is divided by the equipotential section are $Pa/(a+b)$ and $Pb/(a+b)$. Hence the resistance between A and the equipotential surface through C, P is found to be $p + aP/(a+b+P)$ and the resistance between B and the same surface $q + b/P(a+b+P)$. These may be regarded as two branches of a Wheatstone's bridge, the other two being r, s. Equating products of opposite branches, we easily deduce

$$ps - qr = (br - as)\frac{P}{a+b+P}.$$

If br is approximately equal to as and $P/(a+b)$ is small, $ps - qr$ is a small quantity of the second order. We have therefore a well-conditioned determination of the ratio of the two small quantities p, q.

This method is appropriate for comparing the resistances per unit length of two stout wires or rods. The junctions with r, a, b, s are to be far enough from the ends for the equipotential surfaces to be normal sections. The resistances r, a, b, s are to

be large enough to be unaffected by the resistances of the contacts. P should be as small as possible, and should at all events be small compared with a and b. a and b should be as nearly as possible proportional to r and s.

It is frequently used for measuring the resistance of an electric light cable wound on a drum, by comparing it with the portion of a linear resistance between two sliding contacts.

Moderately High Resistances—Wheatstone's Bridge.—For the measurement of moderately high resistances the most satisfactory method is that known as the Wheatstone's Bridge. The usual diagrammatic way of representing this arrangement is shown in fig. 110, where r_1, r_2, and r are three resistances having known values, and x is the unknown resistance. A battery and key is connected between A and D, and a galvanometer and key between B and C. In carrying out the measurement, r is varied until the galvanometer shows no deflection when both battery and galvanometer keys are closed. Balance is then said to have been obtained, and there is no P.D. between B and C. In order that this condition may be satisfied it is obvious that the fall of potential from A to B must equal that from A to C, and the fall from B to D that from C to D. These conditions are expressed by the equations

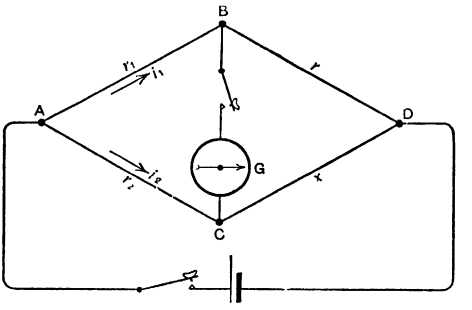


Fig. 110.—Wheatstone Bridge

$$r_1 i_1 = r_2 i_2$$
$$\text{and } r i_1 = x i_2,$$

where i_1, i_2 stand for the currents along ABD and ACD respectively. Dividing the first equation by the second, we get for the condition of balance

$$x = \frac{r_2}{r_1} r,$$

which gives us x in terms of the known resistances. In order to avoid spurious and misleading deflections of the galvano-

meter *the battery key is always pressed before the galvanometer key, and released after.*

In practice, the known resistances usually take the form of coils of platinum silver, German silver, platinoid, or manganin silk-covered wire wound on bobbins of boxwood and thickly coated with paraffin wax. These bobbins are attached to the lower surface of a horizontal slab of ebonite, which is polished on its upper surface, and the ends of the coils are connected to a set of heavy brass contact blocks arranged on the upper surface of the ebonite. Neighbouring contact blocks may be connected by means of a plug, whereby the coil between them is short-circuited. It is a matter of the highest importance to have the plugs thoroughly clean and fitting well into the conical spaces intended to receive them.

Insulation Resistances — Direct-deflection Method.—One of the most commonly employed methods for the measurement of very high resistances is that known as the direct-deflection method. A high E.M.F. is generally necessary in such cases, and the resistances dealt with are so high that the resistance of the battery and galvanometer may be neglected in comparison with them. The galvanometer and testing battery are joined in series with the unknown high resistance x, and the galvanometer reading d_x is noted. The unknown resistance is next replaced by a known high resistance r, and the new reading d_r is noted. If the galvanometer and battery resistances be negligible, then, assuming the same E.M.F. to be used in both cases, and the galvanometer readings to be proportional to the currents, we have $\frac{x}{r} = \frac{d_r}{d_x}$, or $x = \frac{d_r}{d_x} r$. In practice, if the same E.M.F. is used in obtaining the two readings, it is generally necessary to shunt the galvanometer, in which case its reading must be multiplied by the "multiplying power" of the shunt;[1] if, in addition, the E.M.F.'s are different in the two cases, say E_x and E_r, then the factor $\frac{E_x}{E_r}$ must be introduced into the formula. Thus, in the most general case we have

$$x = \frac{E_x}{E_r} \cdot \frac{d_r}{d_x} \cdot n \cdot r,$$

where n stands for the multiplying power of the shunt.

[1] The "multiplying power" of a shunt is the number by which the current passing through the galvanometer must be multiplied in order to obtain the current in the main circuit. Corresponding to shunts of $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ the multiplying powers are 10, 100, and 1000.

In carrying out any measurements of very high resistances it is extremely important to have all the instruments and connecting wires thoroughly well insulated.

Kelvin's Testing Set.—This testing set is used for measuring the insulation resistance of wiring installations and of electrical machinery. The principle on which it works is that of measurement of resistance by substitution.

Referring to fig. 111 it will be seen that when the switch GS is on E the galvanometer G is in series with the battery. When the switch GS is on L the galvanometer and the insulation resistance to be measured are in series with each other. The galvanometer resistance in this case is the known high resistance (usually 50000 ohms). The arrangement of shunts and compensating resistances is such that the resistance between GS and B_2 is always equal to the resistance of the galvonometer no matter what position SS may be in.

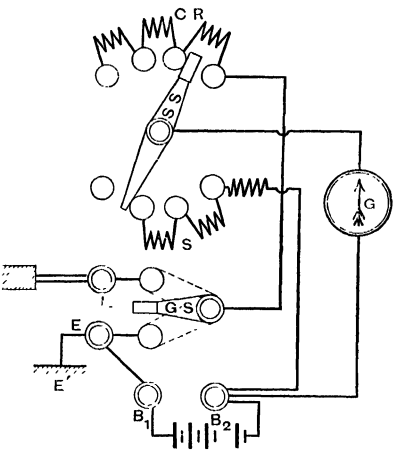


Fig. 111.—Kelvin Testing Set

If d_1 is the deflection on the galvanometer when the switch GS is on E, and n_1 is the multiplying power of the shunt in use, then the current through the galvanometer

$$= kd_1 = \frac{1}{n_1} I_1 = \frac{E}{gn_1} \quad (1).$$

If d_2 is the deflection on galvanometer when the switch GS is on L, and n_2 is the multiplying power of the shunt in use, then the current through the galvanometer

$$= kd_2 = \frac{1}{n_2} I_2 = \frac{E}{(g+x)n_2} \quad (2).$$

Dividing (1) by (2).

$$\therefore \frac{d_1}{d_2} = \frac{(g+x)n_2}{gn_1}.$$

Simplifying, we have

$$x = \frac{d_1 n_1 g}{n_2 d_2} - g,$$

where x is in the insulation resistance of the main under test.

Evershed's Ohmmeter.—The idea of an instrument capable of giving the values of resistances directly in ohms was originally suggested by Professors Ayrton and Perry. The general principle of such an instrument is illustrated in fig. 112. Imagine a magnetic needle placed in a field which is the resultant of a constant field and a variable field of constant direction. Then as the variable field (of constant direction) assumes all possible values between a zero and a certain maximum value, the position of the needle, which is always along the resultant field, will vary continuously between certain limits depending on the relative directions of the two fields. Since the direction

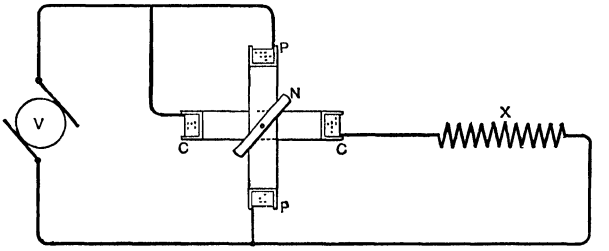


Fig. 112.—Diagram showing Principle of Ohmmeter

V, Source of Pressure; CC, Current Coil in series with External Resistance; PP, Pressure Coil directly connected to Pressure; N, Magnetized Needle; X, Resistance under test.

of the resultant remains unaltered if both fields be altered in the *same ratio*, the position of the needle will depend simply on the *ratio* of the two fields.

Suppose, next, that one of the fields is produced by a coil of constant resistance connected to a source of E.M.F., while the other is produced by a coil joined in series with a variable resistance and connected to the *same* source of E.M.F. As this variable resistance is indefinitely increased from a zero to a very large value, the position of the needle steadily changes, and from what has been said above it follows that the value of the E.M.F. supplying both coils does not affect the position of the needle, which is determined solely by the value of the variable resistance. By using a set of known resistances the instrument may be "calibrated" or made direct reading, and on then substituting any unknown resistance the value of this latter may at once be read off.

The great defect of such an instrument is the comparatively

small degree of accuracy attainable, since the scale extends from zero to infinity. Hence, although there is no reason why such an instrument should not be used for the approximate measurement of any range of resistances, practically it is only employed in cases where a very high degree of accuracy is not required, viz. for the measurement of very high or insulation resistances, such as that of the wiring in a building. An instrument devised

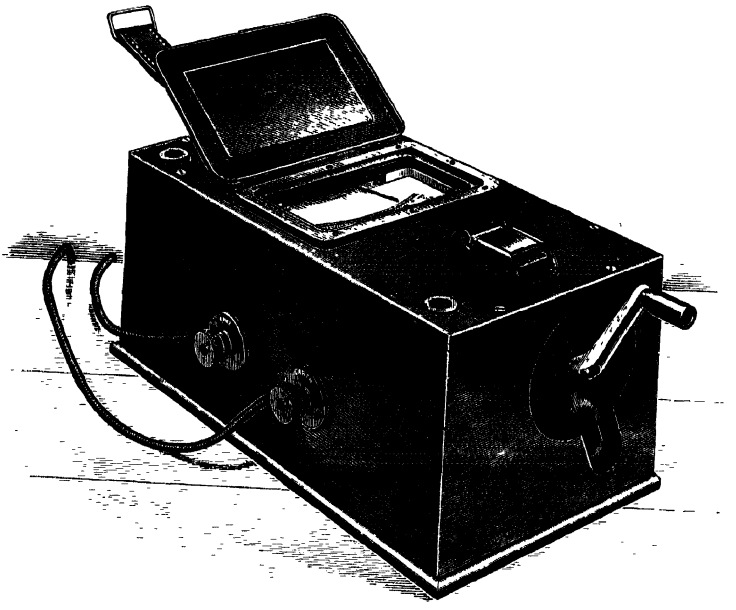


Fig. 113.—Evershed's Patent Megger

for this purpose by Mr. Evershed, and known as Evershed's "Megger", is very largely employed.

The general appearance of the instrument is shown in fig. 113. The unknown high resistance is connected across the two terminals seen to the left, and marked "line" and "earth" respectively. The handle in front is for driving the small high-voltage generator which supplies the testing voltage. The generator and measuring instrument are combined in a single case, measuring $6\frac{1}{4} \times 6\frac{1}{4} \times 12$ in. The entire "Megger" weighs about 18 lb. The scale is fitted immediately below the top of the case, and is viewed through a plate-glass window provided with a suitable cover. The driving shaft is fitted with an auto-

matic clutch, which begins to slip when a certain speed is reached.

The instrument is used as follows: It is stood on a steady base, and the driving handle is turned in a clockwise direction, the speed being gradually increased until the clutch is felt to slip. The value of the unknown high resistance is then read off on the scale.

In the original "ohmmeter" devised by Ayrton and Perry, and shown in fig. 112, two *fixed coils* and a *moving magnet* or needle were provided. The instrument was thus of the

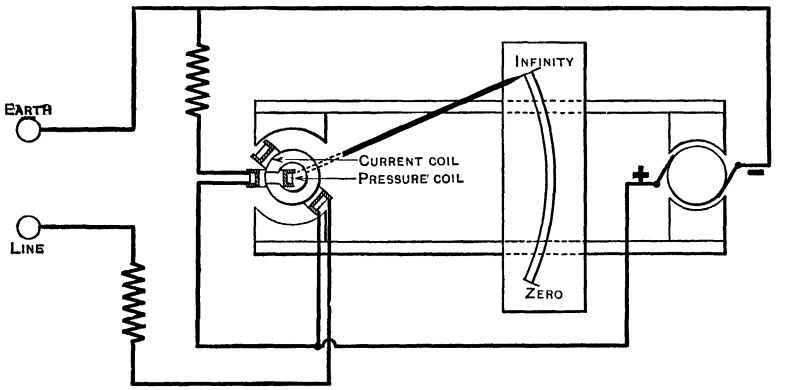


Fig. 114.—Diagram showing Principle of Megger

"needle" type. In Evershed's Megger this arrangement is inverted, there being a powerful *fixed magnet* and two *moving coils*. The "Megger" thus belongs to the moving-coil type of instrument so largely represented nowadays by voltmeters and ammeters for continuous currents. The general arrangement of the instrument is shown in fig. 114. It will be noticed that the two permanent bar magnets are fitted with pole pieces at each end. The pole pieces at the left-hand end provide the instrument field, while those at the right-hand end provide the generator field. By making the same set of magnets serve this double purpose, great economy of space is effected, and a very compact form of construction is secured. The two moving coils are rigidly attached to a common axle, which carries the pointer of the instrument. The coils make a certain angle with each other. One of them, known as the *current coil*, is connected in series with the unknown resistance and the generator, a safety

resistance being also included in the circuit. The other coil, known as the *pressure coil*, is connected in series with a very high resistance and across the generator terminals. The hollow, soft-iron cylindrical core placed between the pole pieces is slotted on one side as shown, so as to admit one-half of the pressure coil. It is evident that, since this coil will tend to move so as to make its axis coincide with that of the field in

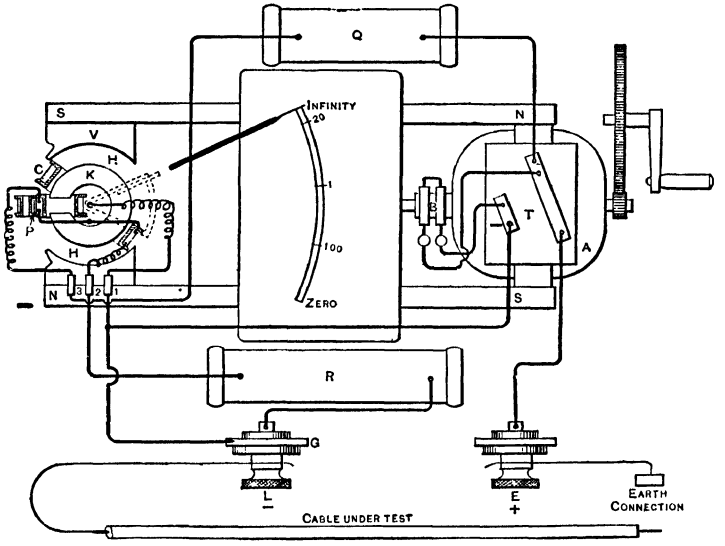


Fig. 115.—Evershed's Patent Megger. Explanatory Diagram

A, Generator Armature; B, Generator Commutator; C, Ohmmeter Current Coil; E, Earth Terminal; G, Guard Plate; HH, Annular Air-gaps; K, Iron Core; L, Line Terminal; NS, Field Magnets; P, Pressure Coils; Q, Resistance in series with Pressure Coils; R, Resistance in series with Current Coils; T, Terminals of Generator; V, Pole pieces of Ohmmeter; 1, Common Terminal of Current and Pressure Coils; 2, Terminal of Current Coil; 3, Terminal of Pressure Coil.

which it is placed, it will take up the position shown in the figure when there is no current flowing through the current coil; this will happen when the external resistance is infinite, and the pointer will then stand at the "infinity" division of the scale. If, however, the unknown resistance is not infinite, a current will pass through the current coil and this coil will be deflected in a clockwise direction, the deflection being opposed by the couple acting on the pressure coil, which moves into a field of increasing intensity as the deflection increases. The coil system is quite free to move under the action of the resultant couple due to the two coils, there being no control, as the cur-

rents are led into the coils by means of extremely fine strips, whose control is negligible.

Some further details of the instrument are shown in fig. 115. In order to prevent the instrument from being affected by external fields, the pressure coil is made astatic[1] with respect to such fields by having a small external compensating coil attached to it on one side, as shown in fig. 115. The number of turns in this compensating coil is so adjusted that the couple exerted on it by any uniform field is equal and opposite to the couple exerted by the same field on the main pressure coil.

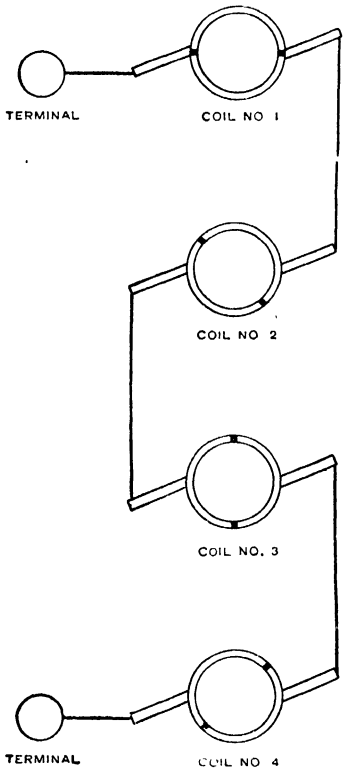


Fig. 116.—Diagram of Connections of Evershed's Portable Magneto-generator

The connections of the generator are shown in fig. 116. Depending on the useful range of the instrument, the generator is constructed to give voltages of 100, 250, 500, and 1000 at the speed when slipping begins to take place. A high-testing voltage is essential, as a defect in the insulation under test may not be detected with a low-testing voltage. The armature of the generator consists of several independent coils spaced at equal angular distances from each other, and each coil is provided with a two-part commutator. The several commutators are then connected in series, as shown in fig. 116. The E.M.F. developed by such a generator is not absolutely steady, but fluctuates between certain limits. Double-reduction toothed gearing is used for driving the generator, as shown in fig. 115. The normal speed of the driving handle is 100 revolutions per minute.

Instruments of this type are made having a useful range up to 2000 megohms.

[1] i.e. constructed so as to be unaffected by any *uniform* field.

CHAPTER XV

ELECTROMAGNETIC INDUCTION AND PASSAGE OF ELECTRICITY THROUGH GASES

Induced Currents.—Induced currents may be described as currents produced in conductors by the influence of neighbouring currents or magnets. Their discovery by Faraday in 1831 constitutes an epoch in the history of electrical science. We shall first describe some modes of producing them; and then state their general laws.

Currents induced by Commencement and Cessation of Currents.—Let two coils be wound upon the same frame B, one of them, called the secondary coil, having its ends connected

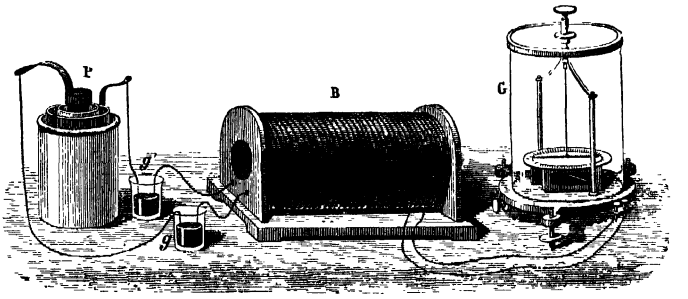


Fig. 117.—Current induced by Commencement or Cessation

with the binding-screws of the galvanometer G, while the ends of the other, which is called the primary coil, dip in two cups of mercury *gg'* connected with the two plates of the voltaic element P. As long as the current is passing steadily in the primary coil, the needle of the galvanometer remains undeflected; but if the current be stopped, by lifting a wire out of one of the mercury cups, the needle is immediately deflected, indicating the existence of a current in the same direction as that which was previously circulating in the primary coil. This effect is very transitory. The needle appears to receive a sudden impulse which immediately passes away. If the current be then re-established, there is a deviation to the other side, indicating a current in the opposite direction to that in the primary coil; and this deviation, like that which occurred before, is merely the

rect of an instantaneous impulse, the needle making a few oscillations from side to side, and then remaining steadily at zero. This experiment, which is substantially the same as that by which Faraday first made the discovery, establishes the following proposition: *When a current begins to flow, it induces an inverse current in a neighbouring conductor; when it ceases, it induces a direct current; and both the currents thus induced are merely instantaneous.*

Currents induced by Variations of Strength of Primary Current.—Employing the same apparatus, let us, while the primary current is passing, connect the two mercury cups by a wire, thus dividing the circuit, and causing a great diminution of the current in the primary coil. At the instant of making this connection the needle of the galvanometer is affected, moving in the same direction as if the primary current were stopped; and on lifting the connecting wire out of one of the cups, so as to produce a sudden increase in the current in the primary coil, the needle moves in the opposite direction. *When a current receives a sudden increase, this produces an inverse current in a neighbouring conductor; and when it is suddenly decreased, a direct current is induced.*

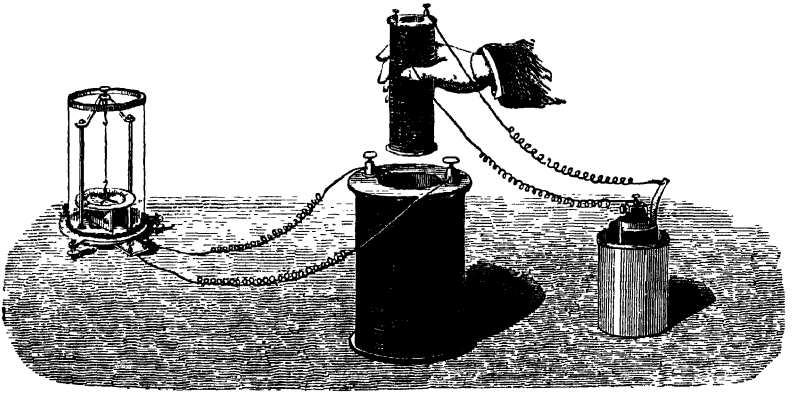


Fig. 118.—Current induced by Change of Distance

Currents induced by Variations of Distance.—Currents may also be induced by change of distance between the primary and secondary conductors. Let the secondary coil, for example, be hollow, as in fig. 118, and let the primary coil, with the

current passing in it, be suddenly introduced into its interior. The galvanometer will indicate the production of an inverse current in the secondary coil. When the needle has come to rest, let the primary coil be withdrawn, and a direct current will be indicated by the galvanometer. These currents differ from those previously mentioned in being less sudden. They last as long as the relative motion of the two coils continues. *When a conductor conveying a current approaches or is approached by a neighbouring conductor, an inverse current is induced in the latter; and when one of these conductors moves away from the other, a direct current is induced.*

Induction by Magnet.—Let a hollow coil be connected with a galvanometer, and a magnet held over it, as in fig. 119. As

Fig. 119.—Current induced by Motion of Magnet

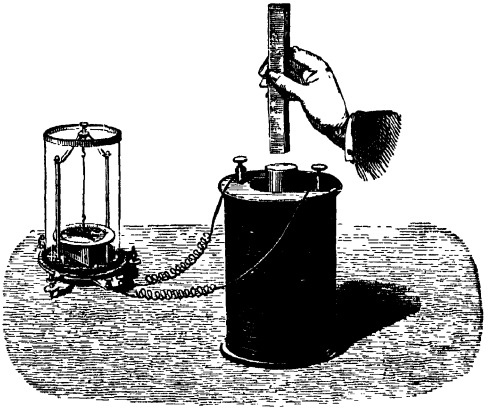


Fig. 120.—Current induced by Magnetization of Soft Iron

long as the magnet remains stationary, no current is indicated; but when one pole of the magnet is thrust into the interior of the coil, the needle is deflected by an impulse which lasts only as long as the motion of the magnet. If the magnet is allowed to remain at rest in this position, the needle, as soon as it has time to recover from its oscillations, stands at zero; but on withdrawing the magnet another current will be indicated in the opposite direction to the former.

Currents may also be induced, with even more striking effect, by moving one pole of a magnet towards or from one end of a soft-iron bar previously placed in the interior of the coil (fig. 120). These currents are due to the magnetism pro-

duced and destroyed in the soft iron. *When the intensity of magnetization of a piece of iron or steel undergoes changes, currents are induced in neighbouring conductors.* The directions of these currents can be inferred from the preceding rules by supposing a solenoid to be substituted for the magnet.

Lenz's Law.—*The currents induced by the relative movement either of two circuits or of a circuit and a magnet are always in such directions as to produce mechanical forces tending to oppose the movement.* For example, when two parallel wires, through one of which a current is passing, are made to approach, an opposite current is induced in the other; and opposite currents by their mutual repulsion resist approach. This general law as to the direction of induced currents was first distinctly enunciated by Lenz, a Russian philosopher.

Ruhmkorff's Induction Coil.—Induced currents capable of producing very striking effects are furnished by the apparatus

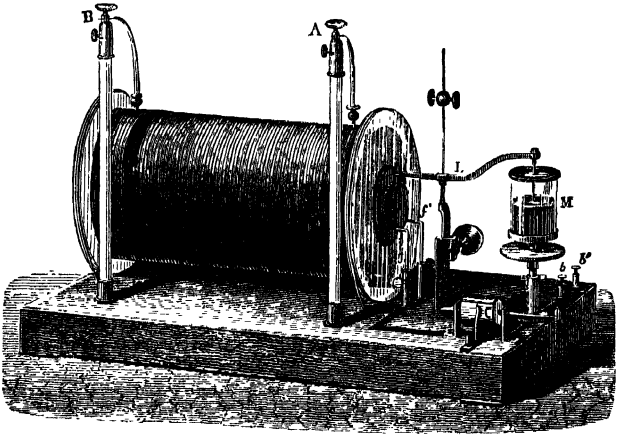


Fig. 121.—Ruhmkorff's Coil

first successfully constructed by Ruhmkorff, and hence known as Ruhmkorff's coil (fig. 121).

It contains two coils of wire, one of them forming part of the circuit of a battery, and called the primary coil; while in the other, called the secondary coil, the induced currents are generated. In the axis of the coils is a bundle of stout straight wires of soft iron, with a disk of the same material at each end, to which the wires are united. Around this core is wound the

primary coil, consisting of a copper wire about two millimetres in diameter. The ends of this wire are shown at *f* and *f'*. The secondary coil consists of much finer wire (about a quarter of a millimetre in diameter) and of much greater length. In large instruments the primary coil may have a length of 80 metres, and the secondary a length of 150 kilometres (94 miles). Special precautions must be taken to insulate the different convolutions of the secondary coil from one another, and from the primary coil. The two ends of the secondary wire are at the binding-screws A, B, which are supported on glass or ebonite pillars. It is obvious that if currents are alternately passed and stopped in the primary coil, there will be an alternate generation of currents (or at all events of electromotive forces) in opposite directions in the secondary coil. The stoppage is produced by suddenly breaking the primary circuit. The effect of the core is to increase the permeability of the field and thus increase the flux of induction.

The wooden stand which forms the base of the instrument is hollow, and contains a condenser, its two coatings being connected with the primary coil at the two sides of the break. It mitigates the spark which occurs at the break, and the two opposite electricities, after rushing into its two coatings, rebound, so as to produce a reverse current in the primary, thus doubling the inductive effect of a mere stoppage. When the primary current is supplied by an alternating dynamo (without any breaking of contact) the condenser is not required.

The wires from the battery are attached at *b* and *b'*; and by means of the commutator C, the current can be sent in either direction at pleasure through the primary coil, or can be turned off.

The successive makes and breaks are effected automatically. The completion of the circuit magnetizes the core, causing one end of it to attract a neighbouring piece of soft iron previously held back by a spring; and this movement breaks the circuit. In the arrangement shown in the figure the break occurs at the surface of the mercury M, by the lifting out of an immersed point. A thick layer of alcohol is usually poured on the surface of the mercury, and serves by its non-conducting power to make the interruptions and renewals of the current more sudden, besides hindering the oxidation of the mercury by the spark.

The core being thus demagnetized, the spring produces a reverse movement, and the circuit is again completed. The lever L, which is supported by the spring, is thus kept in perpetual vibration; and the rapidity of vibration can be regulated by an adjustable weight, shown near the top of the vertical wire.

The construction of the commutator C is shown, in elevation and plan, in figs. 122 and 123. There is a cylinder of insulating material, turning by means of metallic axle ends on metallic supports connected with the two ends of the primary coil. One axle end is permanently connected by means of the screw *g* with the brass plate C on the surface of the cylinder, and the other axle end is in like manner connected with the plate C′ diametrically opposite to C. The contact springs *ff′* are in permanent connection with the two binding-screws A A′ which receive the wires from the battery. When *f* presses against C, and *f′* against C′, as shown in the figure, A is connected with one end of the primary coil and A′ with the other; and when the commutator is turned (by its milled head) through 180°, these connections will be reversed. If it is turned through 90°, the connections will be interrupted, as the contact springs will bear against insulating portions of the cylinder The milled head is, of course, insulated from the axle ends so as to protect the operator.

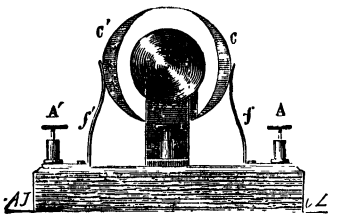


Fig. 122.—Commutator (Elevation)

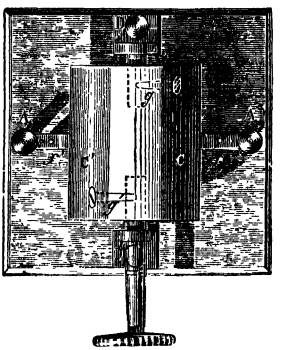


Fig. 123.—Commutator (Plan)

Discharge in Air.—When the ends of the secondary coil are connected, currents traverse it alternately in opposite directions, as the primary circuit is made and broken. These opposite currents convey equal quantities of electricity, and if they are employed for decomposing water in a voltameter, the same proportions of oxygen and hydrogen are collected at both electrodes. There is, however, an important difference. The

E.M.F. at each instant in the secondary is proportional to the rate of change of induction, and this change is much more sudden at stoppage than at recommencement. The consequence is that if the ends of the secondary are disconnected, the E.M.F. at starting is unable to produce a spark, and discharge only occurs at breaking. The discharges, accordingly, are all in one direction. This direction can be reversed by reversing the position of the commutator. The sparks are usually from 1 in. to about 18 in. long, according to the size and power of the apparatus, and exhibit effects comparable to those obtained by electrostatic machines. A Leyden battery may be charged, glass pierced, or combustible bodies inflamed.

Fig. 124.—Statham's Fuse

The great electromotive force of the induced current, which enables it to produce these striking effects, depends on the great number of convolutions of the secondary coil, and on the suddenness of the interruptions of the primary current. The average electromotive force is the product of the number of convolutions by the number of lines of induction, divided by the time occupied.

The discharges from a Ruhmkorff's coil become more violent and detonating if the two electrodes of the secondary coil are connected respectively with the two coatings of a Leyden jar, but the length of the spark is very much diminished.

Induction coils are often used for firing mines, by means of Statham's fuse, which is represented in the annexed figure (fig. 124). Two copper wires covered with gutta percha have their ends separated by a space of a few millimetres, and enclosed in a little cylinder of gutta percha containing sulphide of copper. This, again, is enclosed in a cartridge, CD, which is filled up with gunpowder. The two wires are connected with the two ends of the secondary coil, and when the instrument is set in action, sparks pass between the ends A, B, heating the sulphide of copper to redness, and exploding the powder.

Discharge in Rarefied Gases.—When the ends of the secondary coil are connected with the electrodes of the electric egg (fig. 125), which has first been exhausted as completely as possible by the air pump, a luminous sheaf, of purple colour,

is seen extending from the positive ball to within a little distance of the negative ball. The latter is surrounded by a bluish glow. The blue and purple lights are separated by a small interval of darkness. If other gases are used instead of air, the tints change, but there is always a decided difference of tint between the positive and negative extremities. By the aid of the commutator it is easy to reverse the current, and thus produce at pleasure an interchange of the appearances presented by the two terminals. When the pressure of the air in a tube is about a centimetre of mercury, the discharge is a luminous column stretching from the two aluminium electrodes, anode and cathode. As the pressure is reduced by gradual exhaustion (1) the luminous column breaks up into alternate bright and dark patches with a glow on the cathode; (2) the glow on the cathode moves from it, leaving a dark space between it and the cathode; (3) this dark space goes on extending until it practically fills the tube, and the glass becomes phosphorescent, the colour of the phosphorescence depending on whether lead is used in the manufacture of the glass or not. Certain substances, such as uranium glass, and solution of sulphate of quinine, become luminous in the presence of the electric light, and are called *fluorescent*. Such substances are often introduced into Geissler tubes for the sake of the brilliant effects which they produce.

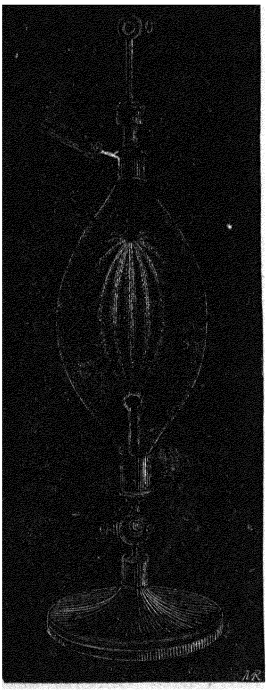


Fig. 125.—Electric Egg

If a Geissler tube is fastened to the front of a disk which can be rapidly rotated, the instantaneousness of the discharges, combined with the persistence of visual impressions, cause it to present the appearance of the rays of a star. The star appears stationary if an exact number of discharges occur in the time of one revolution. A slight increase or diminution in the speed

of rotation will then make the star appear to revolve forwards or backwards.

The discharge in a Geissler tube is capable of deflecting a magnetic needle, and can itself be deflected to one side of the tube by the action of a magnet, the direction of deflection being the same as for a wire carrying a current.

Experiments of Gassiot and De La Rue.—By means of a battery of some thousands of cells, discharge in rarefied gases can be obtained without the use of an induction coil, and with the advantage of greater steadiness. This was done by Mr. Gassiot, and on a larger scale by Dr. De La Rue. The stratifications were still observed, and appeared absolutely fixed in position to the naked eye. When examined by a revolving mirror they were found to exhibit the appearance of a rapid succession of discharges.

Dr. De La Rue's battery consisted of 11,000 small chloride of silver cells; and when all the cells were used, a steady stream of fire passed between the terminals as soon as they were brought within about half an inch of each other, in air at atmospheric pressure. Geissler tubes can also be worked by a Wimshurst machine used without its Leyden jars.

Spectra.—The spectrum of an electric discharge usually consists of the spectrum of the gas in which it occurs, superposed on the spectrum of the metallic terminals, a small portion of the latter being vaporized. The source of electricity usually employed is the Ruhmkorff coil. As the discharge is rendered more violent, whether by increasing the strength of the primary current, or by connecting the terminals of the secondary with a Leyden jar, the number of lines in the spectrum is usually increased. The spectra of gases are usually observed by means of Geissler tubes; which, for this purpose, are sometimes constructed of such a shape as to admit of directing the spectroscope along the axis of the tube, so as to obtain greater concentration of light.

Change of Resistance with Diminution of Pressure.—The resistance of a gas to electric discharge diminishes as the pressure diminishes. A coil which can only give a $\frac{1}{2}$-in. spark in air at atmospheric pressure may give a discharge between terminals more than a foot apart in highly rarefied air. But this diminution does not go on indefinitely. At a certain very

small pressure (different for different gases) the resistance is a minimum, and at the lowest pressures attainable by modern methods the resistance is many times greater than at atmospheric pressure. This fact was for a long time disputed, but the earlier experiments of Gassiot and Alvergniat have been confirmed by the later ones of De La Rue and Crookes, and the result is now conclusively established.

De La Rue employed a vacuum tube connected with a bulb containing caustic potash which had absorbed the last traces of carbonic acid gas, after the tube, originally filled with this gas, had been exhausted by a Sprengel pump. No discharge passed until heat was applied to the bulb, so as to drive off some of the gas, and thus diminish the perfection of the vacuum. The discharge ceased as the potash cooled down, and could be renewed any number of times at pleasure.

Sir William Crookes, who has given increased command of vacua by his improvements in the Sprengel pump, has succeeded in measuring the pressures at which the resistances of air and some other gases are a minimum, and has arrived at the conclusion that across an absolute vacuum discharge would be impossible. The explanation, stated roughly, seems to be that the particles of gas act as carriers of the discharge. When there are too many carriers they hamper one another, and when there are no carriers nothing is carried.

Crookes's "Radiant Matter".—Crookes, by carrying exhaustion to a higher point than had before been reached, has obtained a new set of phenomena. An influence, to which he gives the name of *radiant matter*, and which is also called the *cathode stream*[1] or *cathode rays*, emanates from the negative terminal, and proceeds only in straight lines normal to the emitting surface. Glass and precious stones exposed to this influence become strongly phosphorescent, and a delicately balanced glass wheel with blades like a paddle wheel is set in rotation by the successive impacts of the "radiant matter" upon its blades. The influence is only exerted on surfaces which are directly visible from the negative terminal, and is stopped by glass, so that a piece of glass interposed casts a shadow. By employing as the negative terminal a platinum saucer, shaped like a concave mirror, the influence can be made to converge

[1] The positive terminal is called the *anode* and the negative terminal the *cathode*.

on an object placed at its centre of curvature, and platinum foil can thus be heated to incandescence. No special effect is exhibited by the positive terminal.

Tesla's Experiments with High Frequencies.—Tesla has shown that the character of the discharge is greatly altered by raising the frequency of alternation in the primary to a high pitch. This he has done sometimes by employing a specially constructed alternating dynamo giving many thousand reversals per second, but more usually, and with more strongly marked effects, by employing the oscillatory discharge of a jar or battery of jars, the oscillations in this case ranging from hundreds of thousands to some millions per second. It is necessary so to regulate the dimensions of the primary and secondary conductors that the electric oscillations in the one correspond in natural period with those of the other, so that the effects are exalted by means of resonance, as in Hertz's experiments.

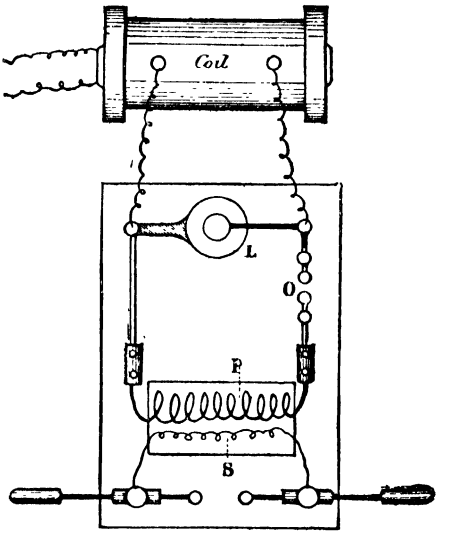


Fig. 126.—Tesla Apparatus

The apparatus used consists essentially of a large induction coil, one or two Leyden jars, and either a helix of stout copper wire or a second induction coil of comparatively few turns immersed in a bath of oil. Wires are carried from the terminals of the induction coil to the inner and outer coatings of the Leyden jar, L, and thence through a spark-discharge apparatus, O, to the wire helix or to the primary of the oil induction coil, PS (fig. 126).

Another plan is to connect the induction coil to the inner coatings of two Leyden jars, and then to a spark-discharge apparatus, and to connect the two outer coatings of the jars by the wire helix (fig. 127).

The induction coil charges the jars very rapidly, and the dis-

charge of the jars across the spark-gap appears to be almost of a continuous nature; this discharge is an oscillatory one with a periodicity of millions per second. In the helix connecting the outer coatings of the jars, or in the secondary of the oil induction coil, induced currents of very high potential and of great frequency of alternation are set up.

A discharge in the form of powerful brushes and luminous streams can thus be obtained, covering an area of several square feet, and this not only in exhausted spaces but in ordinary air.

To produce discharge in a Geissler tube by such means it is

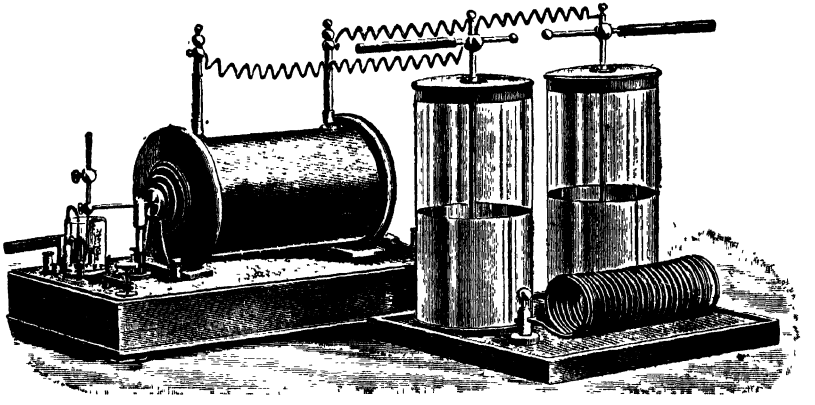


Fig. 127.—High-frequency Apparatus

sufficient to connect one terminal of the tube with a terminal of the coil, and to bring within a moderate distance of the other terminal of the tube a large conductor, such as the human body.

When the frequency is sufficiently high the effects are mainly due to the electrostatic induction of the alternating charges at the terminals of the secondary; and as this action can take place through glass, there is no necessity for wires sealed through the glass. It is only necessary to have two small pieces of metal inside the tube, one at each end, and to have two conductors of considerable size serving as terminals of the secondary coil and brought near the ends of the tube.

It is noteworthy that the currents which produce the brilliant discharges above mentioned can traverse the human body without producing injury or inconvenience. The Geissler tube can conveniently be excited by placing one hand on a terminal of

the coil and the other on a terminal of the tube. The absence of shock may be due to the fact that currents with excessively rapid alternations are merely superficial; or it may be analogous to the fact that the eye is not sensitive to the bobbing of a gas flame when the bobs are quick enough to give a musical tone.

Röntgen Rays.—By employing the arrangment shown in fig. 128, in which the cathode stream in a Crookes's tube falls upon an obliquely placed platinum reflector forming the anode, we obtain outside the tube the remarkable radiation discovered by Professor Röntgen, and called by him the X-rays (X standing as a symbol for the unknown). The vacuum tube, which

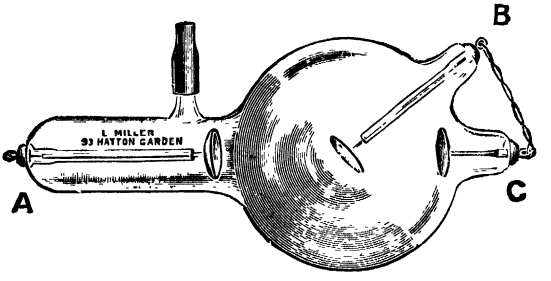


Fig. 128.—Focus Tube

should be made of soda glass, and not of lead glass, is a cylinder of about 2 in. internal diameter with a bulb of about 8 in. diameter blown near one end. Through the closed extremities of the cylinder pass two platinum wires, which are connected by aluminium rods with the internal electrodes. One of these, the cathode, consists of a small concave aluminium disk, the other, the anode, of a flat piece of platinum inclined at an angle of forty-five degrees and placed slightly beyond the focus of the concave cathode.

When such a tube has been sufficiently exhausted, say to somewhere about a 6-in. alternative spark-gap, then when the current from a suitable source is sent through it, there will be seen an even hemisphere of greenish-yellow phosphorescence on that part of the wall of the tube opposite the platinum target. If the tube be wrongly connected, so as to make the anode a cathode, and vice versa, this even hemisphere will be wanting, and there will be a patchy green phosphorescence. Such a tube is known as a focus tube, because the cathode rays

or stream of electrons are focused upon the platinum anode or target. When very powerful discharges are passed through a tube the impact of the electrons may raise the platinum to a white heat, and even perforate it. To avoid this, tubes are furnished with a water- or air-cooling apparatus round the anode; or the latter may be made of tantalum, which has a higher melting-point than platinum. Through use the vacuum and resistance of a tube tend to rise. This may be due to a diminution in the electrons either from their consumption or from their being shot bodily into or through the walls of the tube. Modern tubes are therefore provided with a regenerative arrangement. This may consist of a substance enclosed in an annex B (fig. 129) which will give off a little gas when heated by the passage of the current, or there may be a capillary tube of platinum whose open end communicates with the interior of the X-ray tube. If the platinum be brought to a red heat by warming the free end with a spirit lamp, it becomes permeable to hydrogen, which passes through it and lowers the vacuum of the X-ray tube.

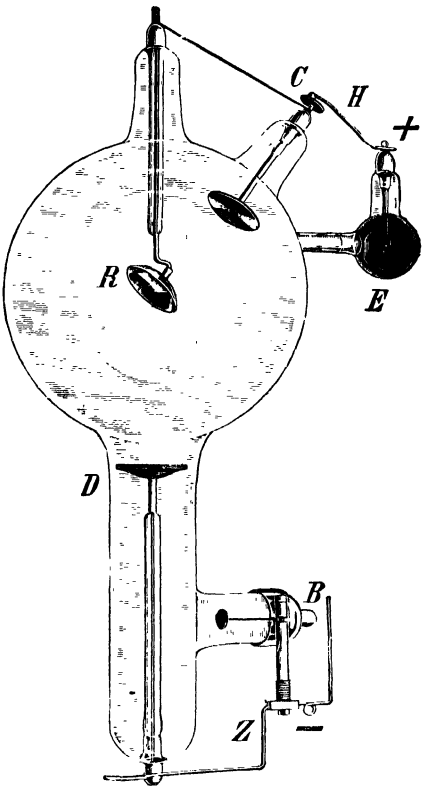


Fig. 129.—Tube with Regenerative Arrangement

Cardboard, wood, flesh, and many other substances opaque to ordinary light are transparent to these rays. There is no change of direction by refraction when the rays pass out of one medium into another, their course is always straight. They do not produce the sense of vision when they fall upon the eye, and they do not render ordinary objects visible; but they produce fluorescence in certain substances, especially the platino-

cyanides, and they affect an ordinary photographic plate, which for this purpose need not be taken out of its dark slide, as the wooden cover is transparent to them. The interior structure of many opaque objects, such as a human hand, or a block of wood with nails in it, can be rendered visible by throwing the shadow of the object either on a photographic plate, to be afterwards developed, or more simply by throwing it on a fluorescent screen forming the front of a dark box. The observer, protected from extraneous light by dark curtains, looks in at the back of the box towards the fluorescent screen, which has its active side turned towards him, its other side (which faces the object) being blackened. The screen is seen as a luminescent background with shadows upon it of different depth, according to the varying capacity of different parts of the object. From the nature of shadows the definition will improve with the nearness of the object to the screen, and with the remoteness of the source of the rays.

All substances seem to be to some extent transparent to the rays, the transparency being least when the density is greatest. Bone is opaque compared with flesh, but is transparent compared with lead. The penetrative power of the rays increases with the power of the induction coil which is used to work the tube. Coils giving an 18-in. spark are suitable for obtaining photographs of the thick parts of the human body.

The cathode stream in a Crookes's tube, like the discharge in a Geissler tube, can be deflected by a magnet, the direction of the deflection being the same as for a wire carrying a current from the anode to the cathode. The Röntgen rays are not deflected by a magnet. There are, however, other rays mingled with them which are so deflected. These are called, from their discoverer, the Lenard rays. In air at a pressure of less than about a quarter of an atmostphere the Lenard rays form a fairly definite beam, but in air at ordinary pressure they are much scattered, like light in a turbid medium. When the pressure is low enough to give a well-defined beam a magnetic field deflects them, and at the same time broadens the beam in the direction of deflection, indicating unequal deflectibility, analogous to unequal refrangibility in the rays of a spectrum. This is what would happen if the beam consisted of a stream of electrically charged particles moving with different velocities.

Those which move fastest would be least deflected, the comparison being not for equal times, but for equal lengths of path.

The nature of the Röntgen rays has been much discussed. The view which is finding most favour was suggested by Sir George Stokes. It is that they are non-periodic disturbances of the ether, produced by irregular impacts of the particles of the cathode stream against the walls of the tube. This view is supported by the fact that all attempts to obtain "interference" phenomena from X-rays have been unsuccessful.

Ultra-violet radiation falling on the negative terminal of a spark-gap promotes discharge. It enables a longer spark to be taken, or with a given distance causes the spark to pass with lower P.D., and this action is accompanied by the disintegration of the negative terminal.[1] Small particles of its substance seem to be torn off, carrying their negative charges with them. Radiation suitable for producing this effect is furnished by the spark of a Leyden jar, or by an electric arc light. The fact that the ultra-violet rays are the efficient rays was proved by Hertz; first, by interposing various absorbing media, and observing to what extent they diminish the effect. Quartz was almost perfectly transparent to the influence. Glass and water were comparatively opaque. Secondly, and more directly, by employing a quartz prism to produce a spectrum. The place of strongest action was about as far beyond the violet as the violet was from the red.

Becquerel Rays.—M. Henri Becquerel has discovered that compounds of uranium emit rays capable of passing through bodies opaque to light. These rays affect a photographic plate, discharge an electrometer brought near, and are deflected by a magnet. They are called "Becquerel rays" or "uranic rays", and bodies which emit them are called *radioactive*. The rare metal *thorium* is found to resemble uranium in this respect, and the same properties are exhibited in a much higher degree by recently discovered substances called *polonium* and *radium*. Of natural minerals the most strongly radioactive is *pitchblende*, which is largely composed of uranium.

[1] Metal knobs between which sparks are taken become rapidly roughened at the place where the spark passes, and roughness, by diminishing the suddenness of the discharges, tends to prevent them from being oscillatory. Frequent polishing of the knobs is necessary in Hertz's experiments.

CHAPTER XVI

MAGNETIC MEASUREMENTS

Magnetic Intensity and Magnetic Induction—Susceptibility and Permeability.—One of the terms which we have frequently had occasion to use is magnetic force or magnetic intensity, and we have defined this as the force which would act on a unit pole if placed at the point considered, the assumption being tacitly made that the point lies in an air space. Let us now suppose that this restriction is removed, and that we consider a point inside a mass of iron. If we attempt to determine the force acting on a unit pole by scooping out a cavity in the region immediately surrounding the point, and introducing our unit pole into this region, then we find that the force depends on the shape of the cavity, and varies from a very small to a very large value, according to the shape. Unless, therefore, we specially restrict the meaning of the term "magnetic force" or "magnetic intensity", this term ceases to have any definite meaning. It has been assigned a perfectly definite meaning by specifying the manner in which the cavity surrounding the given point is to be shaped. We agree, namely, to restrict the term to denote that particular value of the force acting on the unit pole which is obtained by forming the walls of the cavity so that they are everywhere *along the direction of magnetization* of the iron. The main effect of this arrangement is the absence of magnetic polarity on the bounding surfaces of the mass of iron which immediately surrounds the point—in other words, the effect due to the magnetization of the mass of iron immediately surrounding the point is eliminated. The magnetic force so obtained is usually denoted by **H**.

Let us now take the other extreme, and suppose the cavity so shaped that the development of magnetic polarity on its bounding surfaces is as great as possible. This will be obtained by making the walls everywhere *perpendicular to the direction of magnetization*. Let the magnetic polarity per unit of area of the wall be **I**.[1] Then the ratio **I/H**, generally denoted by κ,

[1] This is identical with intensity of magnetization, or magnetic moment per unit volume.

is defined to be the *susceptibility* of the material. We thus have

$$\kappa = \frac{\mathbf{I}}{\mathbf{H}}, \text{ or } \mathbf{I} = \kappa\mathbf{H}.$$

Let us now introduce our unit pole into this second cavity, and determine the force which acts on it. This force is defined to be the *flux density* at the point considered, and is denoted by **B**. The ratio **B**/**H**, denoted by μ, is called the *permeability* of the material. Thus

$$\mu = \frac{\mathbf{B}}{\mathbf{H}}, \text{ and } \mathbf{B} = \mu\mathbf{H}.$$

Instead of saying that the magnetic induction has the value **B** at a given point, we sometimes say that there are **B** lines of magnetic induction per unit of area of a plane drawn at right angles to the direction of magnetization at that point.

If we take any surface bounded by a closed curve, divide it up into a large number of small elements of area, multiply each little area by the value of the magnetic induction in a direction perpendicular to the area,[1] and then take the sum of all the products so formed, then this sum gives us what is termed the *total number of magnetic lines* crossing the surface, or the *total magnetic flux* through it. For this reason **B** is sometimes spoken of as the magnetic *flux density*, since it corresponds to the flux per unit of area.

We may now obtain a relation connecting **H**, **I**, and **B**. The force which acts on the unit pole in the second case (walls perpendicular to direction of magnetization), and which we denote by **B**, may be regarded as made up of two amounts: (1) the force **H**, which exists independently of magnetic polarity in the immediate neighbourhood of the point; (2) the additional amount due to the development of polarity on the walls of the cavity in the second case. In order to find this additional amount, we notice that the pole strength per unit of area being **I**, and the lines of force going straight across the cavity, the number of these lines per unit of area is $4\pi\mathbf{I}$, since each pole gives rise to 4π lines. But the number of lines of force per unit of area is the same as the force acting on the unit pole. Hence,

[1] It must be remembered that magnetic induction is a quantity possessing *direction* as well as magnitude.

this latter amounts to $4\pi\mathbf{I}$. Thus the total force acting on the unit pole **B** is the sum of **H**, the force existing quite apart from the presence of any magnetic material in the neighbourhood of the point considered, and $4\pi\mathbf{I}$, which is the additional amount contributed by the development of magnetic polarity on the walls of the cavity. We thus have

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}.$$

From this we at once deduce the relation

$$\mu = \frac{\mathbf{B}}{\mathbf{H}} = 1 + 4\pi\frac{\mathbf{I}}{\mathbf{H}},$$

$$\text{or } \mu = 1 + 4\pi\kappa.$$

Stress between Two Magnetized Surfaces.—Imagine two plane magnetized surfaces separated by a short air-gap, the intensity of magnetization being **I** and the magnetic induction **B**. Then a unit pole introduced into the gap would experience a force of **B** dynes. This force may be regarded as due partly to the north and partly to the south pole face, the amount contributed by each being one-half of the total, i.e. **B**/2. Now imagine the unit pole placed *on* one of the surfaces. Then this surface will exert no force on the pole in a direction at right angles to the surface, so that the total force on a unit pole which is *on* either surface is that due to the other surface only, viz. **B**/2. But the pole strength of either surface per square centimetre is **I**. Hence the pull per square centimetre of the surface is $\frac{1}{2}\mathbf{BI}$. If **H** be negligible in comparison with **B**, then $\mathbf{I} = \frac{\mathbf{B}}{4\pi}$, and the stress between the two magnetized surfaces, per square centimetre of area, becomes

$$\tfrac{1}{2}\mathbf{B} \times \frac{\mathbf{B}}{4\pi} = \frac{\mathbf{B}^2}{8\pi}.$$

Relation connecting B and H—Magnetization Curves.—If we take a piece of iron which is in a non-magnetic condition to start with, and apply to it a known magnetic force **H** whose value is steadily increased, measuring **B** (by any of the methods to be explained presently) at the same time; then, on plotting a curve connecting values of **B** and **H**, we find that this curve, as shown in fig. 130, may be described roughly as consisting of

three approximately straight-line portions united by curves. Such a curve is called a **B–H** or *magnetization curve*. The upper bend in the curve is spoken of as the *knee*, and for any point lying well beyond the knee, and corresponding to the region in which a relatively large increase of **H** produces only a relatively small increase in **B**, the iron is said to be *saturated*.

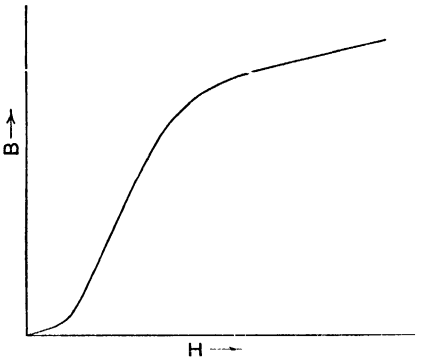


Fig. 130.—Magnetization Curve

The Magnetic Circuit—Calculation of Ampere-turns Required to Produce Given Magnetic Flux.—Every line of magnetic induction forms a closed loop or curve, and the assemblage of the paths followed by the magnetic lines is spoken of as the *magnetic circuit*. In order to produce a given magnetic flux in a circuit a definite M.M.F. must be applied to it, and by analogy to an electric circuit the ratio $\frac{\text{M.M.F.}}{\text{magnetic flux}}$ is termed the magnetic *reluctance* of the circuit.

We shall now briefly indicate how the ampere-turns necessary to produce a given flux may be approximately calculated. Let us suppose that we proceed around a closed loop which is a line of induction, and that the value of the magnetic flux through every cross section of the circuit is given. Then by dividing the flux by the cross section we obtain the value of **B** in every part of the circuit. In the most general case, in passing along any line of induction we find that **B** varies continuously from point to point. In order to simplify the problem it is usual to divide the line of induction into a number of parts, and to assume that **B** has a constant value along each part. Then if the values of **H** corresponding to those of **B** be determined by reference to the **B–H** curves, the sum of the products obtained by multiplying each value of **H** by the corresponding length of the curve gives the M.M.F. Finally, .8 times the M.M.F. gives the ampere-turns which must be provided.

A numerical example will make the matter clear. Let it be

required to find the ampere-turns necessary to produce a flux of 40000 C.G.S. lines across the space between the parallel plane faces of the pole pieces PP in fig. 131, the area of each pole face being 8 sq. cm., the pole pieces being of cast iron, and the sum of the lengths of the portions AB and CD of the dotted line of induction amounting to 10 cm. Let the length of the air-gap between the pole pieces be .5 cm. Further, let the exciting coil be wound on the cylindrical core M, made of wrought iron, 4.5 cm. long and 4 sq. cm. in cross section.

Of the lines produced by the exciting coil at the middle cross section of the core M, only a certain fraction will pass across the plane faces of the pole pieces, a large number *leaking* out into the surrounding air space, forming closed loops of induction such as the one indicated by the chain-dotted curve in the figure. Before we can solve the problem we must know what fraction of the total lines produced in M will cross the air-gap, where they are wanted. The ratio of the maximum flux through M to the useful flux across PP is termed the *leakage coefficient* of the magnetic circuit. Let us assume that in the case considered its value is 1.3. We further assume that instead of a uniformly distributed leakage, we have to deal with a leakage which takes place suddenly along the edges of the pole pieces. This is tantamount to assuming that **B** is constant along each of the portions DA, AB, BC, and CD of the magnetic circuit.

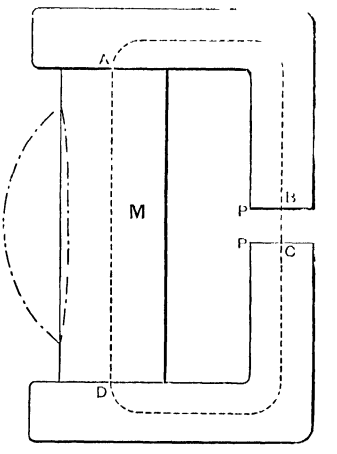


Fig. 131.—Magnetic Circuit

Now consider each of these portions separately, and find the corresponding values of **H**. In the air-gap $\mathbf{H} = \mathbf{B} = \frac{40000}{8}$ = 5000. The flux through the core M and the pole pieces is, on account of leakage, 1.3 × 40000 = 52000. Hence, **B** in the pole pieces = $\frac{52000}{8}$ = 6500. Referring to the **B**–**H** curve for cast iron, we find the corresponding value of **H** to be 45.5. Next, in the wrought-iron core we have $\mathbf{B} = \frac{52000}{4}$ = 13000.

A reference to the curve for this material gives **H** = 10. The total M.M.F. round the circuit is thus

$$\underbrace{5000 \times .5}_{\text{Air-gap.}} + \underbrace{45.5 \times 10}_{\text{Cast-iron pole pieces.}} + \underbrace{10 \times 4.5}_{\text{Wrought-iron core.}} = \underbrace{3000}_{\text{Total.}}$$

The corresponding ampere-turns are .8 × 3000 = 2400 approximately.

It will be noticed that by far the largest portion of the M.M.F. is employed in maintaining the flux across the air-gap. Such is frequently the case in practice.

Calculation of Winding to Produce Given Number of Ampere-turns.—A coil may be required to produce a given number of ampere-turns either with a given exciting current, or with a given P.D. across its terminals. In the former case, the number of turns is at once obtained by dividing the ampere-turns by the amperes. Thus, in the numerical example just considered, if the exciting current is given as 10 amperes, the turns which must be wound on the coil are 240. The *size* of the wire under these conditions does not, obviously, affect the result, and is determined solely by considerations regarding the permissible rise of temperature.

Let us next suppose that V, the P.D. at which the excitation is to be produced, is given. In order to calculate the winding, we have to make an assumption regarding the probable *depth* of winding, and from this to find the *mean* length of a turn.[1] Let the length of turn so determined be l, and let the size of wire be such that the resistance of this length is r. If we denote the current by i, and the number of turns by S, then, since the total resistance of the coil is Sr, we have $i = \frac{V}{Sr}$, or $r = \frac{V}{Si}$; i.e.

$$\text{resistance of mean turn} = \frac{\text{P.D.}}{\text{ampere-turns}}.$$

Knowing the length l of the turn, we find the resistance of the wire per unit of length, r/l; a reference to table XII then gives the *size* of wire required.

It will be noticed that in this second case (when the P.D. is given) the ampere-turns depend on the *size* of wire used, and

[1] In electromagnets of moderate size the depth of winding lies between 1 and 3 in.

are (so long as the length of the mean turn is not appreciably altered) independent of the number of turns, or the amount of wire used; this latter is determined solely by considerations regarding heating.

Experimental Determination of B–H Curve—Ballistic-ring Method.—By far the most satisfactory method of finding the relation which connects **B** and **H** is to make a sample of the material to be tested in the form of a ring, such as the one shown in fig. 132. A convenient size is one having an external diameter of about 6 in., a radial depth of $\frac{1}{4}$ in., and an axial length of 1 in., the section being rectangular (it would be an advantage to arrange the area of the cross section to correspond to an exact number of square centimetres). This ring is wound with some 100 or 200 turns of fine silk-covered copper wire, carefully insulated; this winding, which forms the secondary or exploring coil, need not be uniformly distributed, but may be concentrated over a small portion of the ring. Another, *uniformly distributed* winding, consisting of several layers, and forming the primary or magnetizing coil, is then wound round the ring.

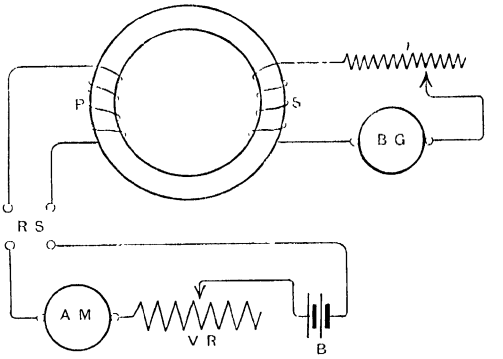


Fig. 132. - B-H Ballistic Method

The arrangement of connections is shown in fig. 132, where P and S are the primary and secondary coils,[1] RS a reversing switch, AM an ammeter for measuring the exciting current, VR a variable resistance, and B the battery (of secondary cells) supplying the current. The secondary coil is joined in series with a resistance r of convenient amount and a *ballistic* galvanometer BG—i.e. a galvanometer having a long period of vibration and subject to as little damping as possible.[2]

[1] For the sake of clearness in the diagram, P is shown concentrated over a portion of the ring; in reality, it is *uniformly* distributed.

[2] The most convenient form of ballistic galvanometer for carrying out such tests is a moving-coil galvanometer of the *undamped* type (i.e. without any metallic frame supporting the coil).

The problem is to find **H**, and the corresponding value of **B**. If i = exciting current in amperes, S_1 = number of primary turns, and l = mean circumference of ring, then the mean value of **H** over the cross section of the ring is given by

$$\mathbf{H} = \frac{1.257\, S_1 i}{l}.$$

In order to find **B** we observe the throw of the galvanometer obtained on reversing the primary current. If a = cross section of ring in square centimetres, then the change in the magnetic flux through each turn of the secondary coil on reversal amounts to $2\,\mathbf{B}a$. If we suppose that the reversal takes t sec., then the mean rate of change of flux—which is numerically equal to the mean induced E.M.F. in C.G.S. units—amounts to $2\,\mathbf{B}a/t$, and the total mean E.M.F. (in C.G.S. units) in the secondary coil is $2\,\mathbf{B}S_2a/t$, where S_2 = number of turns in secondary coil. If now we suppose that the total resistance of the secondary circuit (including secondary coil, galvanometer, and the extra resistance r) amounts to r_2, then the mean value of the induced secondary current is $\frac{2\,\mathbf{B}S_2a}{tr_2}$, and as this current lasts t sec., the total quantity q discharged through the galvanometer amounts to

$$q = \frac{2\,\mathbf{B}S_2a}{r_2}.$$

whence

$$\mathbf{B} = \frac{r_2}{2\,S_2a}q.$$

All the quantities on the right-hand side are known except q. The final step, then, is to find the connection between q and the corresponding throw of the galvanometer, i.e. to calibrate the galvanometer.

There are several ways of doing this. The most satisfactory is to use a *standard solenoid* (some 4 ft. long and 3 in. in diameter), uniformly wound and having a small exploring or secondary coil *inside* it at the middle. When all the readings have been taken with the sample ring, the standard solenoid is substituted for it, and the throws obtained on the galvanometer scale by reversing a number of gradually increasing primary currents are noted. The corresponding quantities may now, however, be easily calculated. For since **H** has the same

numerical value as **B** in this case (there being no iron core), the flux through each turn of the secondary coil is given by

$$\frac{1.257\ S_1' i_1 a_2}{l_1}.$$

where S_1' = turns in solenoid, i_1 = current in amperes, a_2 = area in square centimetres of one turn of secondary coil, l_1 = length in centimetres, of solenoid.

By reasoning similar to that employed above in connection with the ring we find for the quantity discharged through the galvanometer when a primary current of i_1 amperes is reversed the value

$$\frac{2 \times 1.257\ S_1' S_2' i_1 a_2}{l_1 r_2},$$

where S_2' = number of turns in secondary coil of solenoid. Everything in this expression being known, we can calculate the quantity.

A curve may now be plotted giving the relation connecting galvanometer throw with quantity, and this *calibration curve* may be used for determining the values of q corresponding to the various throws obtained in the experiment with the sample ring. Formula (1) may then be used for calculating **B**.[1]

Traction Methods—S. P. Thompson's Permeameter.—The ballistic-ring method just described, although requiring a considerable amount of time and experimental skill, is the most reliable method as yet devised for the magnetic testing of iron or steel. A rough *workshop* method, in which the value of **B** is calculated from the pull required to separate two magnetized surfaces originally in contact, has been devised by Prof. S. P. Thompson, and will now be briefly described.

The instrument is shown in fig. 133, and is termed by its inventor a "permeameter". The specimen is in the form of a cylindrical rod, which passes through one side of a massive rectangular block of iron, technically termed a "yoke", into a magnetizing solenoid, the lower faced end of the rod butting against the faced inner surface of the yoke. The cross sectional area of the yoke being very much larger than that of the rod, **B** inside the yoke will have a small value, and **H** in the yoke

[1] It must be particularly noted that all the quantities on the right-hand side of (1) are in C.G.S. units. So that if the resistance of the secondary circuit is given in ohms, then before substituting in the formula it must be multiplied by 10^9.

will be very small in comparison with its value inside the rod. We may therefore neglect the work done in carrying a unit pole along a line of induction inside the yoke, and assume that the M.M.F. consists entirely of the work done in conveying the unit pole inside the rod from end to end of the solenoid. On this assumption $\mathbf{H} = 1.257\, Si/l$, where S = turns in solenoid, i = current in amperes, l = distance between inner faces of yoke.

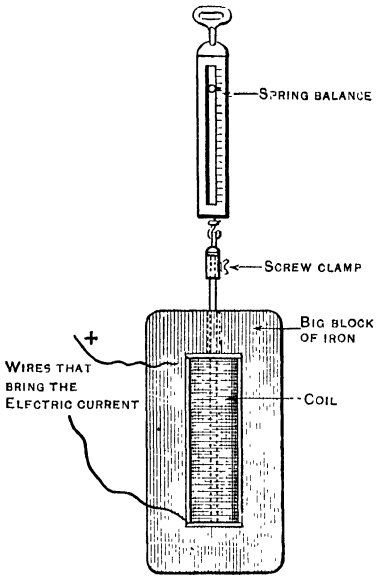


Fig. 133.—Thompson's Permeameter [1]

The pull required to detach the rod from the yoke having been found, we divide this pull by the area of contact (or area of cross section of the rod), thus obtaining the stress per unit of area. If this stress be expressed in dynes per square centimetres, then, as we have already shown, its value is $\mathbf{B}^2/8\pi$, and we have a means of calculating **B**.

Residual Magnetism—Coercive Force—Hysteresis.—When a piece of iron is subjected to the action of a magnetic force which is subsequently withdrawn, the induction does not disappear, and in general a very large fraction of it is retained by the specimen. The curve connecting **B** and **H** is, in fact, for descending values of **H** totally different from the curve for ascending values. This is shown in fig. 134, where OPB is the ascending, and BC the descending branch of the curve. The induction OC which remains in the specimen is termed *residual induction*, and its value depends, for a given specimen, on the maximum induction previously reached by it. Thus, if the maximum induction had been carried up to the point D in fig. 134, the corresponding residual induction would have been represented by OE. The property possessed by the magnetic materials of retaining a certain amount of magnetization after the withdrawal of the magnetic force is sometimes spoken of

[1] From Prof. Ewing's *Magnetic Induction in Iron and Other Metals.* By permission.

as *retentiveness*. In order to wipe out the residual induction a definite magnetic force OF must be applied in the reverse direction, and this force—which will, of course, vary with the residual induction—is termed the *coercive force* for that induction.

If in fig. 134 we consider any value of **H**—such as OG—then corresponding to this value of **H** we get a value of **B** which is represented by GH if the specimen is thoroughly demagnetized to start with, and **H** is simply increased from zero to OG. But we get the very much larger value GK if **H** is first increased to OM, and then brought down to OG; and the still larger value GL if it is increased to ON, and then reduced to OG. In fact, corresponding to any value of **H** we have an *infinite number* of values of **B**, depending on how the particular value of **H** has been arrived at. There is thus no definite connection between these two quantities, unless we specify the manner in which the changes in **H** and **B** take place. When we speak of the "**B**–**H** curve", we always mean the particular curve obtained when the specimen is thoroughly demagnetized to start with,[1] and when **H** is *steadily increased*, and never allowed first to increase, and then to decrease, or vice versa.

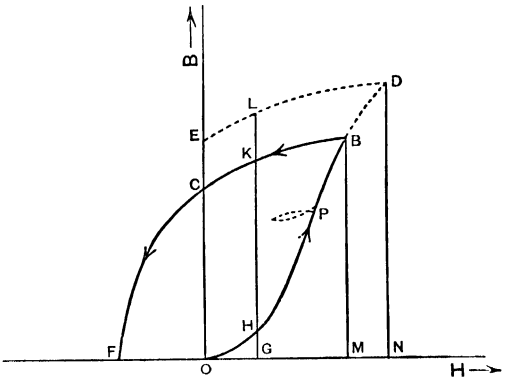


Fig. 134.—Coercive Force and Residual Magnetism

If, for example, when the point P (fig. 134) is reached we first decrease **H** and then restore it to its original value, the curve connecting **B** and **H** will trace out the loop shown by the dotted line, and when **H** has regained its original value, **B** will have increased by a small amount.

The facts which we have been considering may be summed up by saying that a magnetic material always tends to retain its magnetic state—the changes in the magnetic force being always

[1] Such demagnetization is effected by subjecting the specimen to the action of a magnetic force which undergoes rapid reversals while its amount slowly diminishes and ultimately vanishes.

ahead, as it were (not as regards time, but as regards phase), of those in the induction. The term *hysteresis*, introduced by Prof. Ewing, is used to describe this species of magnetic inertia.

Hysteresis Loops and their Experimental Determination.—If a piece of iron be subjected to a series of cyclical changes in the magnetic force, consisting of repeated applications of it in opposite directions, the magnetic force increasing continuously from a zero to a fixed maximum value, then the curve connecting **B** and **H** will take the form of a closed loop, as shown in fig. 135, which consists of an ascending and a descending branch, the two branches enclosing a certain area. Such a closed loop, representing a complete cycle of magnetic operations, is termed a *hysteresis loop*.

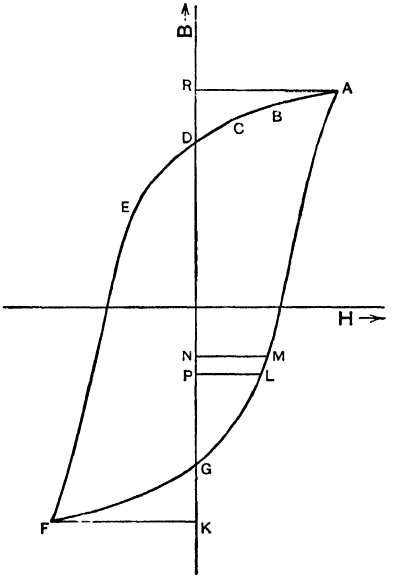


Fig. 135.—Hysteresis Loop

The experimental determination of the hysteresis loop corresponding to a given maximum value of **B** or **H** is a matter of extreme importance, and we shall now explain how this determination may be carried out.

The loop being symmetrical, it will be completely determined by finding either of its two branches. The most convenient method is always to start from the corner or peak of the loop, A in fig. 135, and to observe the throws of a ballistic galvanometer obtained on passing from A to a number of points B, C, D, &c., on the descending branch, the calculation of the changes in **B** and **H** being carried out as already explained in connection with the **B**–**H** curve.[1] In order to pass from A to B, for example, all that is necessary is to suddenly reduce the current by an amount corresponding to the value of **H** at that point; and this may be easily done by suddenly unplugging resistance

[1] The specimen is supposed to be in the form of a ring, with primary and secondary coils arranged as explained above.

in the primary circuit. By varying this resistance a series of points between A and D may be obtained. In order to reach a point (such as E) which lies lower down, to the left of the B-axis, a *reversal* of current is necessary, but the reversed current must have a value much smaller than the maximum corresponding to the points A or F. The operation of passing from A to E involves, then, a sudden increase in the resistance of the primary circuit, accompanied by a reversal.

In order to be able to effect these changes conveniently a reversing switch may be used which is connected to a variable resistance R as shown in fig. 136—an arrangement due to Prof. Ewing. RS is the reversing switch, the dotted lines showing the change of connections due to throwing over the switch. It will be noticed that when the resistance R is plugged up, throwing over the switch from right to left produces a simple reversal; but if some of the resistance is unplugged, the introduction of this into the primary circuit accompanies the act of reversal.

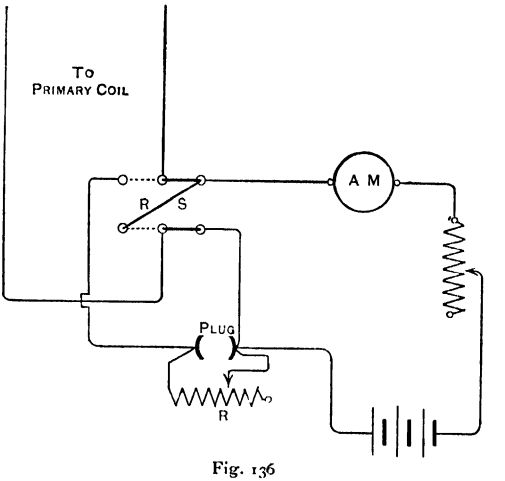


Fig. 136

Between every two readings R is plugged up, and the switch worked to and fro a number of times, so as to produce a series of simple reversals and thereby restore the iron to a steady cyclic state.

While taking readings corresponding to points lying between A and D in fig. 135 the switch is kept in the left-hand position, the reduction of current being obtained by suddenly unplugging any desired amount of resistance. For points lying beyond D, such as E, the switch would be in the right-hand position, and the point E would be reached by throwing it over to the left.

Energy Dissipated by Hysteresis — Steinmetz's Law.—

When a mass of iron is carried through a rapid succession of magnetic cycles it is found to grow hot. The development of heat is due to the dissipation of part of the energy spent in producing the magnetization, and it may be shown that the amount of energy dissipated in every cubic centimetre of the iron during a complete magnetic cycle is directly proportional to the area of the corresponding hysteresis loop.

Referring to fig. 137, let us suppose that at a certain stage the point L has been reached, and that H increases from PL to NM, the increase being supposed small, and the mean value of the current, *in C.G.S. units*, during this change being i. Let b = NP be the corresponding change in B. If a = cross section of the ring in square centimetres, then the total change of magnetic flux through each turn of the primary coil is ba, and if this takes place during t sec., the average induced E.M.F. is Sba/t, where S = number of turns in primary coil. Since the current is increasing, the induced E.M.F. opposes it, and the power spent in maintaining the current against this E.M.F. is $iSba/t$; the power being exerted during t sec., the total *energy* spent in producing the change of flux is $iSba$. Now if l = mean circumference of ring, then the value of H corresponding to i is $H = \frac{4\pi Si}{l}$, so that $Si = \frac{1}{4\pi} Hl$. Substituting this value of Si in the expression just obtained for the energy, we find that (fig. 137)

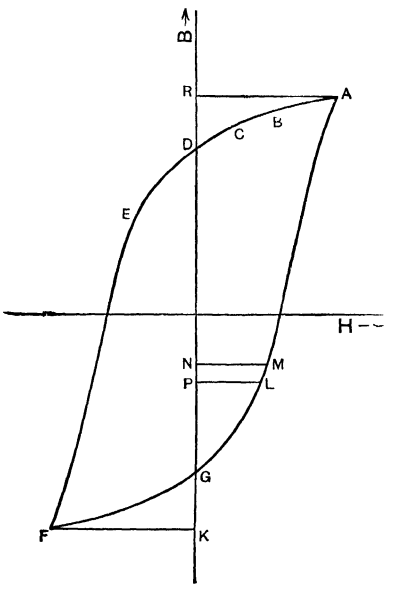


Fig. 137

$$\text{energy spent in passing from L to M} = \frac{1}{4\pi} Hlba.$$

But since la = total volume of iron in cubic centimetres, we see that in passing from L to M we communicate to every cubic

centimetre of the iron an amount of energy equal to $\frac{1}{4\pi}\,\delta\mathbf{H}$ $= \frac{1}{4\pi} \times$ area of strip PLMN.

Hence if we pass from G to A, the total energy given to the iron will be $\frac{1}{4\pi}$ times the area GAR.

Now in going back from A to D, the current is decreasing; the induced E.M.F. will help to maintain it, and the ring in giving up its magnetism will act as a generator of energy. The amount of the stored energy so returned (by each cubic centimetre of the iron) is given by $\frac{1}{4\pi}$ times the area RAD. The remainder, $\frac{1}{4\pi}$ times the area GAD, is not given up. Similarly, as the cycle is completed by passing from D through E and F to G, a further amount of stored energy, represented by $\frac{1}{4\pi}$ times the area DFG, is not returned. Hence during a complete cycle there is an amount of the stored energy, per cubic centimetre of the iron, represented by $\frac{1}{4\pi}$ *times the area of the hysteresis loop*,[1] which is not returned, and which is used up in producing heat.

We thus see that $\frac{1}{4\pi}$ times the area of a hysteresis loop gives the energy (in ergs) dissipated in every unit volume of the material subjected to the magnetic cycle. The area of the loop will, of course, depend on the maximum value of the induction. As a result of a very large number of measurements, Steinmetz found that if **B** = maximum induction, then the energy dissipated, in ergs per cubic centimetre per cycle, could be represented by the formula

$$\eta \mathbf{B}^{1.6},$$

where η is a constant for a given material, called the *hysteretic coefficient*, and has a value varying from .001 to .003 for soft iron.

The result that for a given material the hysteresis loss varies as the 1.6th power of the maximum induction is known as *Steinmetz's Law*. This law only holds within a certain range of induction, and is not correct for either very low or very high inductions.

Ewing's Hysteresis Tester.—If we attempt to rotate a well-laminated mass of iron in a magnetic field, then although, on

[1] This energy will be expressed in C.G.S. units, i.e. ergs.

account of the lamination, there will be no appreciable resistance due to induced eddy currents, yet a considerable resistance will be encountered by reason of the dissipation of energy by hysteresis. The resistance to the motion will increase in proportion to the energy dissipated per cubic centimetre per cycle.

A highly ingenious yet simple instrument for the comparative testing of the hysteresis loss in specimens of sheet iron or

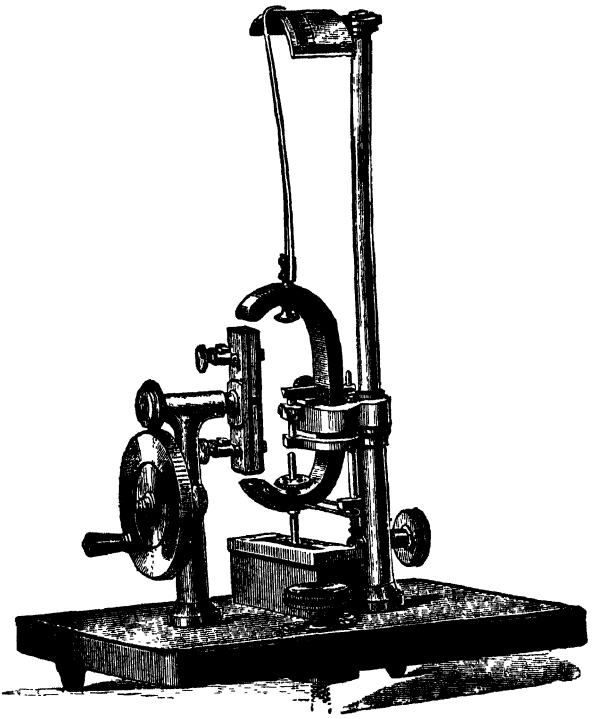


Fig. 138.—Ewing's Hysteresis Tester[1]

steel has been constructed on this principle by Prof. Ewing, and is shown in fig. 138. Between the poles of a permanent magnet supported on a horizontal knife-edge and provided with a pointer which moves over a scale is rotated the specimen to be tested, which takes the form of a few strips of the sheet metal held in a suitable clamp. The greater the hysteresis loss the larger will be the angle through which the pointer is deflected.

[1] From Prof. Ewing's *Magnetic Induction in Iron and Other Metals.* By permission.

CHAPTER XVII

ELECTRIC STORAGE BATTERIES

Introduction.—A secondary storage or accumulator cell is the name given to that particular class of apparatus which, although quite inert in itself, yet, on the passage of an electric current through it, is capable of receiving, retaining, and re-developing a certain amount of electrical energy. The energy is stored in the form of chemical energy due to chemical changes caused by the passage of the current through the cells. A distinction must be made between storage cells and condensers, since in the first the electricity is stored as chemical energy, while in the second it is stored as electrical energy. In a storage cell the ratio between the amount of energy put in and the amount returned is dependent upon the chemical nature of the materials employed and the mechanical construction of the cell.

The idea of electrical storage is not new, as it may be traced back to 1801, when Gautherot demonstrated the fact that platinum or silver wires, when used in the electrolysis of saline solutions, developed secondary currents.

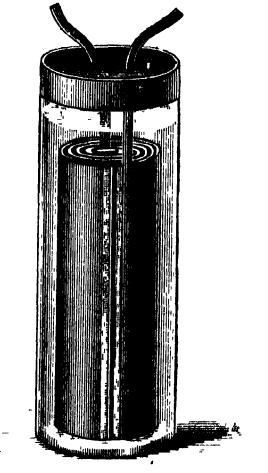


Fig. 139.– Planté Cell

In 1842 Sir William Grove constructed his classical gas battery, which derived its power from the difference of polarity of the constituents of water. The electrodes used were strips of platinum immersed in acidulated water. He found that better results were obtained by covering the metal strips with a metallic oxide. Wheatstone and Siemens, working on the same lines, found that peroxide of lead gave the best results when used in the way suggested by Grove.

In 1860 the great step forward in the production of a commercial storage cell was made by Gaston Planté, who invented the lead-peroxide cell.

Planté's original form of cell (fig. 139) consisted merely of

two sheets of lead 60 cm. long., 20 cm. wide, and 1 mm. thick, placed one upon the other. To prevent metallic contact a piece of coarse felt was placed between them. The compound plate was then rolled up into the form of a spiral, and was then immersed in a solution of 1 part, by volume, of pure sulphuric acid added to 10 parts of water. The "forming" process, as suggested by Planté, was carried out in the following way.

A current from two Grove's cells was passed through the cell till gas was given off freely from both plates. On breaking the forming circuit and connecting through a resistance a current in the reverse direction of great intensity but of small duration was obtained. On examination, the plate which had been connected to the positive pole of the primary cells was found to be covered with a thin film of lead peroxide. The plate which had been joined to the negative pole of the charging battery was scarcely altered in appearance.

At each successive charging the peroxide on the positive plate sinks deeper and deeper into the metal. The operation proceeds until there is formed a sufficient thickness of lead oxide to protect the lead from further action.

Since the publication of Planté's original invention a very large number of improvements and alterations have been made in storage cells. Most of these are of a mechanical nature, and relate more especially to (1) methods of accelerating the formation of the elements, (2) means of preventing the disintegration of the active material, (3) attempts to overcome that great drawback of all lead accumulators, the warping and buckling of the plates. Much also has been done in the way of improving the real active material.

The active materials are porous, and are therefore mechanically weak. They must therefore be supported by massive lead grids, which also serve to conduct the current to the active materials. These grids are made from cast or rolled lead, which must be very pure, since a small percentage of impurity reduces the capacity of the cell by setting up local action. Occasionally a small percentage of antimony is added to harden and strengthen the lead. The structure of the grid should be sound and homogeneous.

Lead storage cells fall naturally into two classes, viz. those in which the active material is formed from the substance of the

element itself either by direct chemical or electrochemical action, and those in which the chemical formation is accelerated by the application of some easily reducible compound of lead. Elements of the former type are usually called Planté, and those of the latter Faure or pasted plates, Messrs. Planté and Faure being the respective inventors.

Planté Plates.—In forming Planté plates by the original method of repeated chargings the active material is only formed slowly, and the process is therefore costly. It was found that the process could be quickened by first charging, then allowing the cell to rest and charging in the reverse direction. Now at each reversal of current the peroxide is reduced by the nascent hydrogen liberated at its surface during the decomposition of the electrolyte, and is reduced first to a lower oxide and finally into metallic lead in a spongy state. At each successive reversal the oxidizing and reducing action sinks more deeply into the lead, and by careful treatment any desired depth of active material may be obtained. When preparing plates by this method it is found necessary to allow a period of several days or even weeks to elapse before reversal of polarity is effected, since, if the peroxide is not allowed to assume a crystalline state before its reduction, the soft oxide may be forced away from its support by the evolution of gas on the true metallic surface of the plate. The formation of the plates is still further accelerated by boiling them in dilute nitric acid, or by electrically forming them in a solution of sodium nitrate or nitrite in sulphuric acid and water. After this treatment the plates are well washed and dried to remove all traces of nitric acid, the forming being finished by the old method. Usually all the plates are formed against dummy negatives, those which are to be used as negatives being reversed before the final charge.

When properly formed the positive plate should be of a dark chocolate brown, while the negative plate is of a deep grey colour. Since the depth of the active material is small, the ampere-hour capacity for a flat plate is small per unit weight of plate. To increase the surface exposed to the electrolyte Planté plates are usually ribbed, as shown in figs. 140, 141, showing an Epstein Planté positive and a Tudor Planté positive plate respectively.

Another method of increasing the active surface is that adopted by the Chloride Storage Company for their positive plates. The plates, which are cast in a mould, have a large number of holes in them. These are about three-quarters of an inch in diameter, and are slightly tapered outwards on both sides of the plate. Into the cast holes are fitted small rosettes of lead, which are formed of strips of pure lead gimped and rolled up into the form of small coils. When all the apertures are filled in, the plates are subjected to pressure in a hydraulic

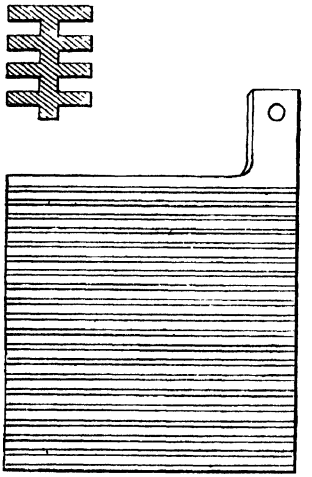


Fig. 140.—Epestein's Element

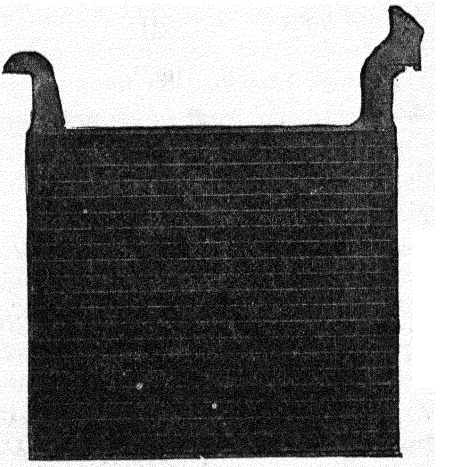


Fig. 141.—Tudor Positive Plate

press, which operation keys the rosettes in their place. The plates are then formed in the usual way.

Faure Plates.—In order to reduce the time and cost of formation by the Planté method, Faure applied a layer of minium or red oxide of lead (Pb_3O_4), made into an adherent paste with dilute sulphuric acid. When the paste was sufficiently hardened the cell was subjected to electrolytic action, when the oxide on the positive plate was oxidized to lead peroxide, and that on the negative reduced to spongy lead.

Afterwards it was found to be more advantageous to use litharge (PbO) for the paste for the negative plate. The litharge paste requires less energy for its reduction to spongy lead.

When the red lead is mixed with the dilute sulphuric acid,

a mixture of lead peroxide and lead sulphate is formed; as shown by the following equation:—

$$Pb_3O_4 + 2H_2SO_4 = PbO_2 + 2PbSO_4 + 2H_2O.$$

In the case of the litharge paste, lead sulphate alone is formed; as follows:—

$$PbO + H_2SO_4 = PbSO_4 + H_2O.$$

Faure plates when formed are similar to plates in a discharged condition, and when a battery, built with these plates, is installed, the cells require to be charged at a slow rate for about 48 hours continuously.

The paste does not adhere well to the plate, so that various methods are adopted to hold it mechanically in position.

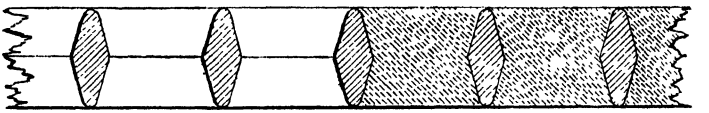


Fig. 142.—E.P.S. Grid

Fig. 142 shows one of the earliest devices used. The grid is cast in an iron or steel mould. The plates are perforated with a large number of square apertures, which are tapered inwards from both surfaces of the plates, thus forming a double dovetail, which is filled with the active material. Owing to their peculiar shape, the pellets of active material tend to key themselves firmly into the countersunk holes.

Fig. 143 shows a more recent type of plate which has a series of ledges cast upon it on both sides, and these ledges are filled with active material. Plates made in this manner are capable of giving larger discharges per pound weight of plate.

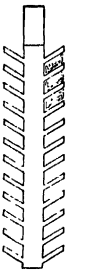


Fig. 143

Faure plates are lighter than Planté plates, and are also very much cheaper to manufacture. When used as negatives they are quite satisfactory, since in a negative plate the paste contracts on reduction to spongy lead, and in subsequent use the active material tends to contract still further. On the other hand, the paste on the positive plates tends to expand with use, and therefore to break away from

its support. For this reason positive plates for heavy work are made of Planté plates.

Construction of a Storage Cell.—To increase the capacity of a storage cell, and at the same time to reduce the internal resistance, plates are connected in parallel by burning the lugs on to a common bar. This common bar terminates in a lug by which two cells may be joined in series.

There should always be one more negative plate than positive in a cell, so that there is a negative plate outside at both ends of the cell. If this is not done it is found that the

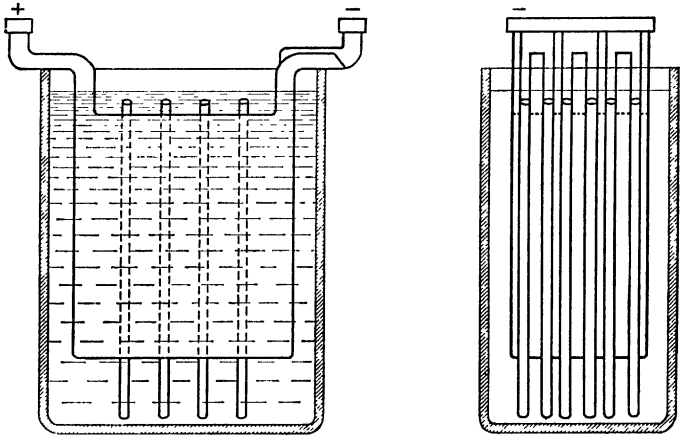


Fig 144

positive plate at the end "buckles" or warps very badly, and that the active material is often forced off its surface, thus reducing the life of the cell. The negative plate when outside does not behave in this manner, and is therefore always put at the end.

The plates are placed in a glass or lead-lined teak vessel which contains the electrolyte. As shown in fig. 144 they are supported from the top by lugs resting on the top of the containing vessel. A space of from 4 to 8 in., depending on the depth of the plates, is left at the bottom, so that any deposit formed during working may fall to the bottom, and so reduce the risk of short-circuiting. The plates are free to expand in all directions, and are separated by glass, ebonite, or sheet-wood separators; the distance between them varying from $\frac{1}{4}$ to

½ in. This allows the detached active material to fall to the bottom of the cell, and at the same time allows the plates to buckle slightly without short-circuiting.

The plates are delivered dry and uncharged. In making

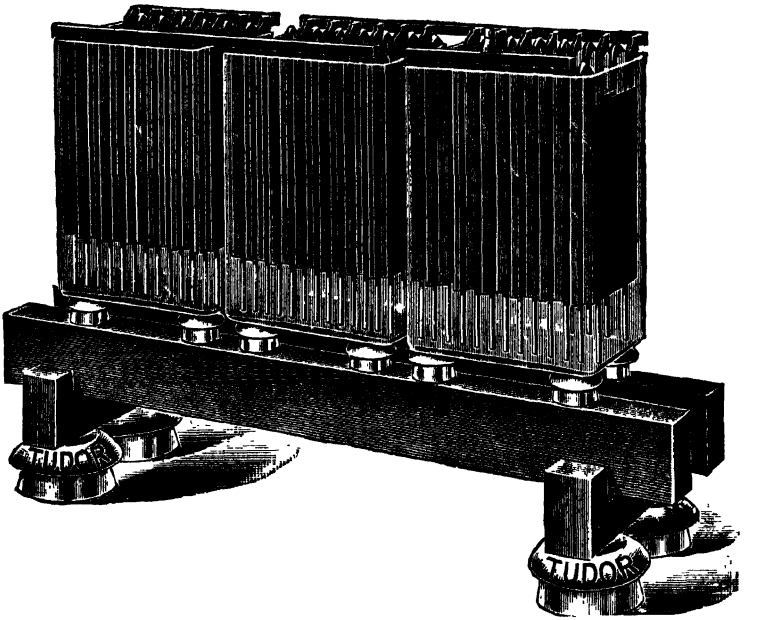


Fig. 145.—Tudor Elements in Glass Boxes

up the cells prior to the first charge the solution is poured in to a height of about ½ in. above the plates. The specific gravity of the solution, which is a mixture of distilled water and pure brimstone sulphuric acid, should be about 1200, water being 1000.

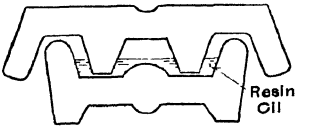


Fig. 146

In order to increase the voltage available, storage cells are usually connected in series to form a battery. The cells are mounted on stands in the following manner (fig. 145). The cells are mounted in a tray containing sawdust to absorb any acid spray which runs down the side of the cells. These trays stand on two-part glass insulators similar to that shown in fig. 146. The lower half of the insulator contains resin oil, which is non-conducting and non-evaporative. This insulator prevents

earths, caused by the acid spray, given off during charging, running down the side of the cells.

Connections between cells are made by burning the lugs together or by using lead-covered iron bolts. Brass and iron bolts should not be used for bolting the lugs together, since they are attacked by the acid spray given off at the end of charge. This action may be retarded by giving the bolts a coat of vaseline.

To reduce the loss of acid caused by bubbles of gas carrying acid with them a plate of glass is placed on the top of the cell.

Chemical Actions during Charge and Discharge.—Various explanations have been given of the chemical changes during charge and discharge, but the actual changes are still doubtful. The reactions involved are highly complicated, and vary considerably under various conditions incidental to charging and discharging, rates of charge and discharge, variations of temperature, and also on the density and composition of the electrolyte.

The main reactions, however, are as follows. We will assume that the cells are fully charged, so that the positive plate is covered with a layer of lead peroxide, while the negative plate is covered with a layer of spongy lead.

The state of the cell can be represented as follows:—

$$\underset{\text{Positive plate.}}{PbO_2} + \underset{\text{Electrolyte.}}{H_2SO_4} \ldots\ldots \underset{\text{Negative plate.}}{H_2SO_4} + Pb.$$

During the discharge the plates are partially converted into lead sulphate, $PbSO_4$, and the changes are

$$\begin{aligned} &PbO_2 + H_2SO_4 \ldots H_2SO_4 + Pb \\ = &\ PbSO_4 + H_2O \ldots H_2O + PbSO_4. \end{aligned}$$

During discharge, therefore, sulphuric acid is decomposed, forming water. When the active material of the plates has been more or less converted into lead sulphate the action ceases and no more current can be obtained. The lead sulphate is a non-conductor, and is insoluble in the sulphuric acid, so that it forms an insulating covering, and prevents the active material from being attacked.

During charge the actions are reversed, and we have

$$\underset{\text{Positive plate.}}{PbSO_4} + H_2O \ . \ . \ . \ \underset{\text{Negative plate.}}{H_2O + PbSO_4}$$
$$= \underset{\text{Positive plate.}}{PbO_2} + H_2SO_4 \ . \ . \ . \ \underset{\text{Negative plate.}}{H_2SO_4 + Pb.}$$

The lead sulphate on the positive plate is oxidized to lead peroxide, while the lead sulphate on the negative plate is reduced to spongy lead. At the same time sulphuric acid is formed.

These reactions only affect part of the plates, in fact the active portion of cell is usually much smaller than that which remains passive.

Since water is formed during discharge the specific gravity of the electrolyte falls to 1.185 or 1.195. During charge the specific gravity rises to 1.200 or 1.220, due to the sulphuric acid formed. The density of the electrolyte therefore gives some indication of the state of the cell, so that a hydrometer is a necessity.

Variation of E.M.F. during Charge and Discharge.—It has been shown by Gladstone and Tribe that the E.M.F. of a

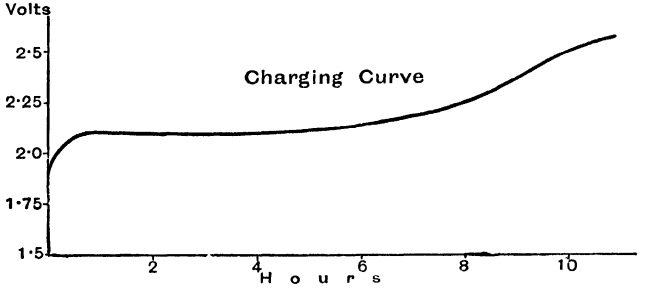


Fig 147

storage cell depends on the density of the acid. It is therefore evident that there will be a variation in the E.M.F. during charge and discharge, the E.M.F. rising during charge and falling during discharge, due to variations in the density of the electrolyte. In addition to this general variation there are changes in the E.M.F. due to local alterations in the density of the electrolyte.

In general the characteristic E.M.F. curves for a storage cell, charging and discharging at the normal rate, have the forms shown in figs. 147, 148.

At the beginning of the charge the E.M.F. rises rapidly to about 2.2 volts, and then remains approximately constant at this value till the charge is nearly finished. It then rises rapidly to 2.5 to 2.7 volts at the end of charge. This rapid increase is due to the fact that all the active material has been oxidized or reduced as the case may be, and oxygen and hydrogen are being given off. This change is easily detected, since the electrolyte in the cell appears milky as a result of the great evolution of gas. The charge should be stopped at this point, since a prolonged overcharge, especially with a small current, is injurious to the cells.

When the charging circuit is broken the E.M.F. falls rapidly to about 2.2 volts due to the diffusion of the stronger acid near

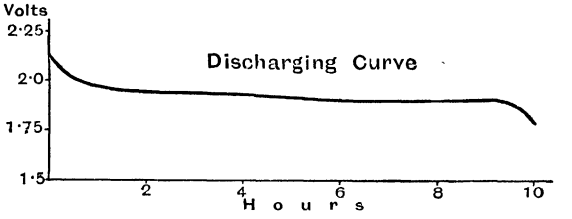


Fig 148

the plates into the remainder of the electrolyte. On discharge the E.M.F. falls rapidly to 2.0 volts. This first drop is due to the fact that some hydrogen is occluded in the pores of the negative plate, and till this hydrogen is oxidized the E.M.F. of the cell is that of a lead-peroxide-hydrogen cell. The E.M.F. remains nearly constant till very near the end of the discharge, when it begins to fall rapidly. The discharge should be stopped when the E.M.F. drops to 1.85 to 1.8 volts. Any attempt to discharge the cell below this limit causes a very rapid drop of the E.M.F. to zero, and usually results in permanent damage to the cell.

Maintenance of Storage Cells.—The life and efficiency of a storage battery depend solely on the care which is taken of it. It is therefore a matter of great importance to pay great attention to the following rules, which practical experience has shown to be necessary for the proper working of the battery.

The battery must never be discharged below 1.8 volts per cell measured when the battery is discharging. If the discharge is

carried beyond this point a hard non-conducting layer of the white sulphate of lead is formed which is very difficult to remove. The formation of this sulphate, which can only be removed slowly by electrolytic action, is known as "sulphating" of the cell. Sulphating also occurs in cells which are allowed to stand fully discharged for any length of time.

Sulphating reduces the capacity of the cell by insulating the active material. It also causes the plates to "buckle" or warp, since pellets of active material which are sulphated expand. This expansion, if the supporting grid is strong enough, may cause the pellets to crack and fall out of the plate.

Cells must not be discharged at a greater rate than the maximum specified by the makers; too rapid a discharge causes the active material to expand, and consequently the plates are buckled. It may also result in sulphating, loss of active material, and internal short-circuiting.

The charge should be continued till all the cells are gassing freely and the voltage of each cell has risen to 2.5 or 2.6, with the charging current flowing. Overcharging, especially with a small current, is injurious to the cells, and entails a waste of energy. At regular intervals, however, a short overcharge at normal charging current may be given to the cells. If it is necessary to leave a battery idle for some time it should be charged till all the cells gas freely, and if possible a short charge, till the cells gas, should be given once a fortnight.

The level of the electrolyte should be about half an inch above the plates. Owing to evaporation, and loss of acid through spray carried off by the gas, it is necessary to add water to the electrolyte from time to time to maintain the level. Distilled water must be used, since tapwater or rainwater usually contains impurities which injure the cell. Acid must not be added without the maker's consent. A record should be taken every week of the specific gravity and E.M.F. of each cell of the battery at the end of the charge, and while the charging current is still flowing. If it is found that some of the cells are lower than the rest, they should be treated with great care until they come back to the normal condition. The bad cells should be charged in the ordinary way with the rest of the battery, then during discharge these cells are cut out of circuit completely, and so by continued charging may be brought into

good condition. In large batteries the more usual plan is to give the bad cells an additional charge by means of a small motor-driven dynamo, known as a "milking booster". This obviates any breaking of connections, which is very troublesome when all the cell connections are burnt together. Before giving the cell this additional charge it should be examined for internal short-circuits. These may be caused by the plates buckling, or by particles of the active material falling out of the plates and lodging between adjacent plates and so forming a more or less conducting bridge between the plates, which discharges the cells. These short-circuits should be removed by means of a clean rod or strip of glass. In some types of cells the separators are made from a thin sheet of wood which covers the whole of the plates and so prevents internal short-circuits.

Capacity and Efficiency of Storage Cells.—The capacity of a cell is the number of ampere-hours it will give when discharged at a given rate before the P.D. drops to 1.8 volts.

In specifying capacity the normal rates of charge and discharge must be given, since a cell has a smaller ampere-hour capacity the higher the discharge rate. This is shown in fig. 149. Fig. 150 shows the discharge characteristic for different rates of discharge. From the curves it is seen that the higher the discharge rate the more rapid is the fall in E.M.F. This rapid fall is probably due to a diminution in the strength of the acid in the pores of the active material. The acid in the pores is weakened by the formation of water, and diffusion of the stronger solution cannot take place quickly enough. A cell which has been rapidly discharged to 1.8 volts will often show an increase of E.M.F. to 1.9 or 1.95 volts after it has stood idle for some time.

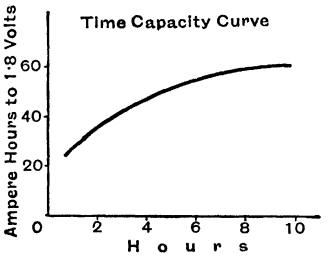


Fig. 149

Fig. 149 shows the variation in capacity with time of discharge.

The efficiency of storage cell is often defined in two ways: (*a*) the energy efficiency, which is the true efficiency, and (*b*) the ampere-hour efficiency or quantity efficiency.

The energy efficiency is the ratio of watt-hours output to

watt-hours input. It varies between 40 and 70 per cent when charged and discharged at normal rates. An average value is about 60 per cent. If a large cell be discharged very slowly, the energy efficiency may rise to 80 per cent.

The quantity efficiency is the ratio of ampere-hours output to ampere-hours input. It varies from 75 to 90 per cent.

Gassing during charge reduces both efficiencies, but if the charge is stopped before gassing occurs there is a risk of sulphating the plates. Leakage and local action in the cells also reduce the efficiency.

For central electric-light stations, or for such work as level-

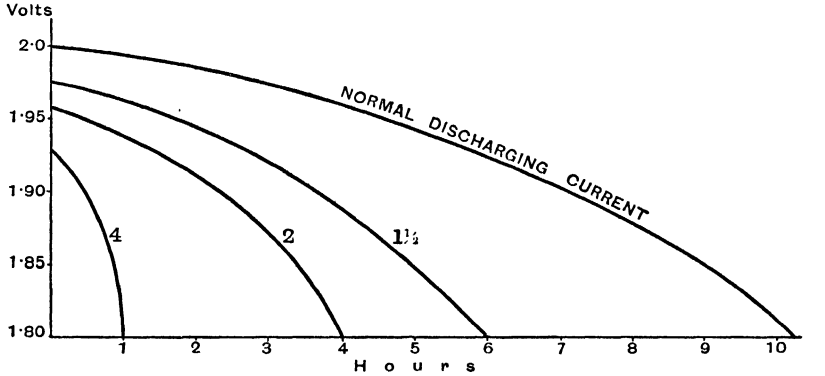


Fig. 150

ling up the loads on electric tramways, where moderately constant work is required, a battery whose weight is large, or whose bulk is great, may be safely used. For self-contained electric-light installations, weight and space occupied by the battery is of some moment; and where intermittent charging only is resorted to, absence of local action is desirable. The battery under these circumstances may have to stand idle for considerable periods, and it would not do for it to lose its charge while thus waiting. For traction purposes, where charging each day, or even several times per day, may be practised, local action is not such an objection, and the loss sustained thereby may be neglected. For some forms of vehicles, such as broughams, dogcarts, tricycles, and the like, which may be only occasionally required, and may have considerable waits between the periods of use—waits which may extend over days and even weeks—the absence of local action is essential. A

fifth class of apparatus, in which storage batteries are now playing a most important part, is that of self-contained portable electric lamps, so largely used for medical and other purposes. In this case absence of local action, lightness and compactness, together with great mechanical strength, is absolutely essential. In many cases such cells are required to hold their current for many months, and this can only be obtained where there is little or no local action. Again, as the plates used in these small batteries are of small dimensions, the form of construction may be such as is not permissible in any other class of apparatus.

Edison Cell.[1]—The electrolyte of the Edison cell is a 21-per-cent solution of potassium hydrate, having, in addition, a small amount of lithium hydrate. The active metals of the electrodes, which will oxidize and reduce in this electrolyte without dissolution or chemical deterioration, are nickel and iron. These active elements are not put in the plates as *metals*; but one, nickel, in the form of a hydrate, and the other, iron, as an oxide. The negative or iron plate has a flat-pocket construction. The positive or nickel plate consists of two rows of round rods or pencils, held in a vertical position by a steel frame. The mean voltage of full discharge is approximately 1.1 volts.

Arrangements for Charging.—Storage cells are usually charged from a dynamo. This dynamo must be shunt wound or separately excited, since in this case, should the prime mover break down or fall off in driving power, the dynamo will slow down and will finally be driven as a motor by the cells, taking only a small current. However, if a series dynamo is used, and a breakdown occurs, the battery will send a current through the machine, which tends to drive the machine as a motor in the opposite direction. In the process of slowing down to rest the battery might send a current large enough to be injurious to the cells and to the machine.

A shunt dynamo used for cell charging must be capable of having its voltage varied over a wide range. Since the E.M.F. of a cell during charge varies from 2.0 to 2.6 volts, the voltage of the dynamo must be capable of being varied through a similar range.

Very often, when a battery is installed in a lighting station,

[1] Paper by W. E. Holland in *Electrical World*.

a separately excited dynamo or "booster", connected in series between one main and one pole of the battery, is used for charging the cells. The voltage of the booster then varies from 0 to 0.6 × number of cells in series.

Usually some form of reverse-current circuit breaker is used to open the circuit if the dynamo E.M.F. falls below the cell E.M.F. A reverse-current cut-out suitable for low voltage and moderate current circuits is shown in fig. 151. This automatic switch has three coils, D, S, and M. The coils D and M have a common core of soft iron. The core I of S is also of soft iron, and is free to rotate through a small vertical angle, and is pivoted at O. This core has attached to it two balance weights and a lever with a copper fork B which dips into two cups containing mercury. One of these cups is connected to the negative terminal of the cells, while the other is connected through coil M to the negative terminal of the dynamo. When the main switch is closed, coil S, which is wound with fine wire, is connected across the cells, thus converting the core I into an electromagnet. The balance weights are so arranged that normally the fork B is just clear of the mercury in the cups. Coil D is wound with fine wire and connected between the two cups. It is so arranged that when the E.M.F. of the dynamo is less than that of the cells a current flows from the cells through D, making the core an electromagnet. The core attracts I, and so lifts the fork out of contact. By adjusting the speed and excitation of the dynamo the dynamo E.M.F. can be made greater than that of the cells, and the current in D reverses; core I is therefore repelled by coil D, and thus makes contact between the cells and the battery. The charging current flowing through the thick wire coil M holds the core I in this position. If the current in M should reverse, core I is again repelled, breaking circuit, and is held in the off position by coil D. The cut-out is so sensitive that the circuit is broken when only a small current is flowing, thus preventing sparking.

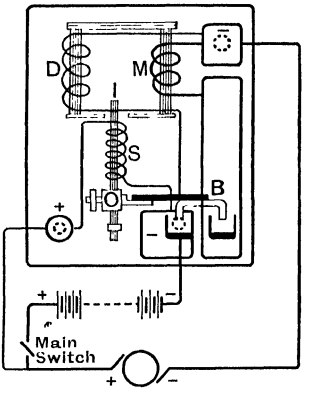


Fig. 151.—Automatic Cut-out

CHAPTER XVIII

ARC LAMPS

Historical.—It is a well-known fact that when a circuit carrying current is broken, the current does not die away immediately, but has a tendency to continue to flow by bridging across the gap thus formed, showing its existence by a flame playing between the two ends of the broken circuit. Although this fact may have been known to many of the early experimenters, the first careful study of the electric arc was made by Sir Humphry Davy in 1808, when he produced his 2000-volt electric arc by utilizing the energy of a 2000-cell galvanic battery. Sir Humphry Davy soon found, however, that the arc formed between two metallic points had far too weak illuminating properties for any practical purpose, and therefore tried other substances than metals, and was finally convinced that carbon sticks were the most suitable for this purpose.

Up to the present day no substance has been discovered superior to carbon for use in the production of an electric arc. As the carbon vapour which carries the current across the arc is a poor conductor of electricity a large difference of potential is required between the carbon points; but, on the other hand, under the action of the arc a stream of small carbon particles passes from one carbon to the other, and in consequence, for a given voltage, a longer arc can be maintained than with metal electrodes.

Carbon has a very high melting-point, and being also a bad conductor of heat the tips are kept at a very high temperature, and very little of the heat is conducted away by the carbon rods.

Since that time many scientists closely investigated the subject, and between the years 1840 and 1860 many patents on arc lamps were taken and carried out, the operation of these lamps being chiefly controlled by clock-work mechanism. It is, however, only during the last twenty-five years that arc-lamp mechanism has been brought to such perfection as to bring its application into practical use, its theory becoming better known at the same time.

Striking of the Arc.—The first thing to be considered in an

arc lamp is the striking of the arc. It will require a voltage of about 4500 to 8000 volts to bridge across a gap of 1 to 2 mm. Since the electric arc requires about 40 to 50 volts to maintain it, this excessively high voltage cannot be available for starting it, and therefore some means must be provided for producing the arc with the difference of potential available. For this purpose Sir Humphry Davy used the apparatus illustrated in fig. 152, which shows the complete circuit. B represents a voltaic battery, C_1 and C_2 the positive and negative carbons respectively, A_1 and A_2 two metallic posts connected to the two poles of the battery, and through which slide the two carbon holders. When the carbons are brought into contact the circuit is completed and a current is allowed to flow; by slightly pulling apart the carbons an arc is formed.

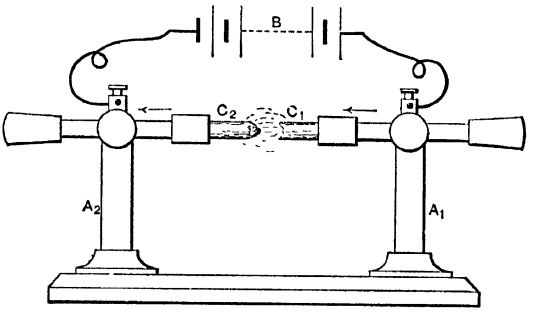


Fig. 152.—Davy's Apparatus for producing an Arc

Changes at the Electrodes.—On maintaining this arc we find that the carbons are both consumed, thus increasing the air-gap between their tips, and the flame, getting thinner, will eventually die out if we do not constantly readjust it, an operation which is called feeding the arc. After the arc has been burning for some time the positive carbon will have its end hollowed in a cup-shaped form, and is called the crater, while the negative carbon assumes a pointed shape, as shown in fig. 153.

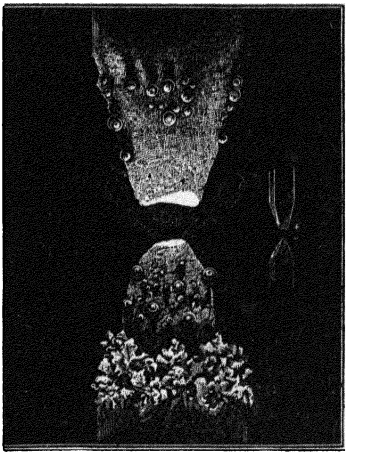


Fig. 153.—Positive and Negative Carbons

On examining an arc we notice that the ends of the carbons

become brilliantly incandescent, and that both carbons are worn away—the positive about twice as fast as the negative—in the open arc, that is, the arc formed in the air. The oxidization of the carbon forms a vapour round the carbon tips, which maintains the arc. Particles of carbon are detached from one pole and fly immediately over to the opposite pole, the stronger stream taking place from the positive to the negative pole, changing the hard carbon into graphite.

Illumination.—On turning our attention to the illuminating properties of the arc we find that all parts are not homogeneously bright, but show at least five distinct parts. The brightest part is the crater, which gives out the greatest amount of light; the point of the negative carbon comes next; thirdly, the arc and gaseous vapour round it; and finally, the red parts of the two carbons. From what we have just said it is obvious that the arc itself gives but a very small fraction of light, its function being indeed only to heat to a very high degree the carbon tips which give out the light. The cup-shape form of the positive carbon is most usefully employed when placed at the top, thus reflecting all the light downwards; but as the negative carbon partly interferes with the amount of light reflected it is usual to employ larger carbons for the positive electrodes, and small carbons, as far as the current density permits, for the negative electrodes. Owing to this, the light given out by an arc lamp is not uniform in intensity all round, but is usually a maximum at an angle of about forty-five degrees with the axis of the carbons. Fig. 154 shows the polar curves of candle-power for a magazine arc lamp. The lengths of the radii drawn from the arc in various directions to points on the curve are measures of the intensity of light in those directions. The direction of the maximum intensity will vary according to the relative sizes of the two carbons and the shape of the crater.

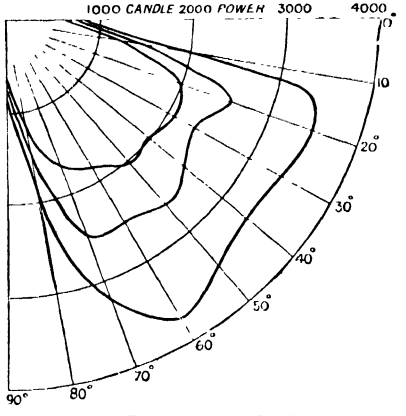


Fig. 154.—Polar Curves of Candle-power

The illuminating power of the crater varies from 150 candle-power per square millimetre in small arcs to 220 candle-power per square millimetre in very large ones, its temperature being about 3500° C.

Behaviour of the Arc.—The arc, when formed, moves round the carbon tips, and tends to maintain itself between the two nearest points, thus causing a flickering in the light. In order to reduce the shifting of the arc, cored carbons, with better conducting materials at their centre, are used, thus tending to keep the arc central. Another advantage gained by using cored carbons is that the potential difference at the carbon tips is not greatly increased as the carbons wear away. The length of the arc varies between $\frac{3}{32}$ and $\frac{1}{8}$ in., and requires about 40 volts to maintain it. This voltage must, however, be increased at the terminals of the lamps, to compensate for the drop of volts in the internal resistance of the lamps, and in the extra resistance necessary to steady the arc. An increase of voltage at the terminals of the lamp would cause a larger current to flow through it, and the arc would hiss or sputter; noises are also very often produced by improper distance between the electrodes, or by impurities in the carbons.

During the last few years experiments have been made with low voltage and long arcs, attaining as much as $\frac{5}{8}$ in. in length. The satisfactory running of these lamps greatly depends on the sensitiveness of the feeding mechanism, and the quality of the carbons, which must be very soft and made of materials of high conductivity. Other manufacturers control the arc flame by a magnet, the arc being very susceptible to magnetism.

Nature of the Arc.—It is generally believed that the arc does not produce any back electromotive force, but only offers a resistance to the flow of current, which varies considerably according to the substances employed in the core of the carbons. The apparent resistance of the arc varies, not only with the diameter of the carbons and the length of the arc, but also with the quality of the carbons and the gases in which they burn.

The arc does not follow Ohm's Law, as an increase of current increases the section of the arc, thus decreasing its resistance. This causes an instability in the arc, and it would die out if some special compensating devices were not used to

steady it. For this reason a resistance is used in series with the arc which varies the difference of potential available at the terminals of the lamp inversely as the current. In practice, from 10 to 30 per cent of E.M.F. required at the terminals is found necessary for steadying the arc, this greatly depending on the type of lamp and the method of connecting up. The voltage required to maintain an arc of length l mm. between two carbon rods with a current I amperes is given by:

$$E = 38.88 + 2.07l + \frac{11.66 + 10.54l}{I}.$$

For example, if I is 10 amperes and l 1 mm. E = 43.17.

Rating.—Arc lamps could be rated by their candle-power, viz. 600, 1200, 2000 candle-power, but, as the intensity of light is different in every direction, this method is too uncertain to be used in practice, and therefore it is usual to rate arc lamps by the current they take. The voltage required by an arc lamp varies according to the grouping employed. An open arc lamp requires about 40 volts at its terminals, but, as explained above, it is necessary to allow from 45 to 50 volts for each lamp on account of the steadying resistance in series with it. The following is a practical grouping:—

One lamp in single parallel on 60-volt circuits.
Two lamps in series on 110-volt circuits.
Four ,, ,, 200 ,, ,,
Five ,, ,, 240 ,, ,,

And 48 volts only being allowed for each lamp above that number. The power absorbed is easily obtained by multiplying the terminal voltage by the current absorbed by the lamp. For instance, a 10-ampere lamp in single parallel absorbs 50 × 10 = 500 watts, and gives approximately 600 m.h.c.p.

Size of Carbons.—The size of carbons varies according to the current absorbed; the table on p. 221 gives the approximate sizes of positive and negative carbons used for different currents.

The length of time for which a lamp will burn depends on the length and quality of the carbons employed. Arc lamps are usually adjusted to burn fourteen hours with 9-in. carbons, and eighteen hours with 12-in. carbons, or thirty-six hours if the double-carbon type is employed. After a lamp has been

Current.	Diameter of Top Carbon.	Diameter of Bottom Carbon.
5 amperes	13 millimetres cored	9 millimetres solid
6 ,,	13 ,, ,,	9 ,, ,,
8 ,,	15 ,, ,,	11 ,, ,,
10 ,,	18 ,, ,,	12 ,, ,,
12 ,,	18 ,, ,,	12 ,, ,,
15 ,,	20 ,, ,,	13 ,, ,,
18 ,,	20 ,, ,,	13 ,, ,,
20 ,,	22 ,, ,,	15 ,, ,,

burning during that period of time new carbons have to be fitted, an operation which is called the trimming of the lamp.

Enclosed Arc.—In order to save the labour required for trimming lamps, and to reduce the cost of maintenance, another arc lamp has been introduced, in which the life of the carbon is considerably increased by enclosing the arc. Oxidization takes place freely in the open arc, while the enclosed arc lamp is so arranged as to limit the access of oxygen to the arc, thus reducing the rate of combustion of the carbons. One set of carbons will only last about eighteen hours at the most in the open arc lamp, while the same set would burn from 80 to 150 hours if the arc were enclosed. The increase of life of the carbons is obtained by placing a small glass globe around the arc, which, however, does not exclude all air from the arc.

On striking the arc the oxygen in the small globe is soon absorbed, leaving an atmosphere of heated and rarefied gases. The gases produced by combustion escape at the top, and a small opening at the bottom allows a constant but limited circulation of air in the globe. The carbon tips being thus surrounded by an atmosphere containing a limited amount of oxygen, the combustion takes place very slowly. The negative carbon in an enclosed arc lamp wastes away at the rate of about $\frac{1}{50}$ in. in one hour and the positive at about double this rate. Enclosed arc lamps are generally connected direct on to 100- to 110-volt circuits, about 80 volts being required across the arc.

Alternating-current Arc.—Up to the present we have only considered the continuous-current arc. A steady arc is, however, produced by alternating currents having a frequency of from 40 to 100. It has been found in practice that lamps burning on circuits having a frequency lower than 40 periods produce an arc in which the fluctuations are perceptible to the eye,

while there is no dynamo machinery built to-day with frequencies above 100 periods. The alternating-current differs essentially from the continuous-current lamp in that the current in the circuit is constantly changing in direction, and consequently each carbon is alternatively positive and negative. Both carbons are therefore cratered in turn, and give out the same amount of light, the distribution of which is very different from that of a continuous-current arc. Fig. 155 shows the distribution of light given by an alternating-current arc. As it will be noticed, the amount of light is about equal above and below the horizontal. There is therefore a certain amount of light which, not being reflected downwards, is lost, thus reducing the illuminating power of the alternating-current as compared with the continuous-current arc absorbing the same power. In an alternating-current arc lamp about 26 per cent larger current is required for a given illuminating power than in a continuous current. Both carbons are of the same diameter and are consumed at approximately the same rate. The following table gives the sizes of carbons generally employed for different currents in alternating-current open lamps:—

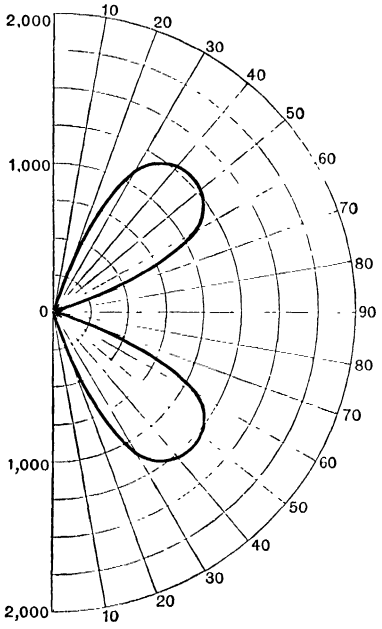


Fig. 155.—Alternating-current Arc. Distribution of Light

Current.	Diameter of Top Carbon.	Diameter of Bottom Carbon.
5 amperes	9 millimetres cored	9 millimetres solid
6 ,,	9 ,, ,,	9 ,, ,,
8 ,,	11 ,, ,,	11 ,, ,,
10 ,,	13 ,, ,,	13 ,, ,,
12 ,,	13 ,, ,,	13 ,, ,,
15 ,,	15 ,, ,,	15 ,, ,,
18 ,,	18 ,, ,,	18 ,, ,,
20 ,,	20 ,, ,,	20 ,, ,,

Another feature of the alternating arc lamp is that the steadying resistance is replaced by a choking coil, which chokes back a great portion of E.M.F. available instead of dissipating it as heat, thus reducing the waste of power.

Functions of the Arc-lamp Mechanism.—An arc-lamp mechanism has to fulfil the following conditions: On the current being switched on, the carbons must be first brought into contact with each other and then drawn apart to a certain distance. As the carbons burn away they must be gradually moved towards each other so that the length of the arc remains constant.

One of the essential functions to be performed by an arc-lamp mechanism is, therefore, to keep the distance between the carbon tips constant, and to perform this operation smoothly and uniformly without sudden movements which will cause great variations in the illumination. This operation may be performed either by causing the top carbon to move only, the focus of the arc being thus constantly moved towards the bottom; or by causing the two carbons to move together towards each other as they are consumed and keeping the arc in the same position.

Many devices have been adopted for this purpose. In the early days clockwork mechanism was exclusively employed, but had to be abandoned owing to its complication and the delicacy of the gearing. Motor arc lamps have succeeded, and at this time are still to be found on the market, principally on the Continent. Arc lamps, however, in which the action of the mechanism is controlled by electromagnets are used almost universally at the present time. An electromagnet of this kind consists of a solenoid, which, when carrying a current, attracts an iron core which passes through it. The pull thus exerted on the core depends upon the number of ampere turns on the coil. This property of the solenoid, in conjunction with the electrical resistance of the arc, has been taken advantage of to regulate the arc. Two coils are very often employed, one being wound with many turns of thin wire and connected across the terminals of the lamp, its strength depending on the voltage across the arc; and another coil wound with few turns of thick wire and being in series with the arc, its strength depending upon the current taken by the arc; this combination being

called the differential winding. The two iron cores are connected in such way as to cause the carbons to move towards each other or away from each other according to the current passing through the arc. A long arc will increase the resistance of the circuit and decrease therefore the current through it, thus reducing the strength of the series coil and causing the mechanism to act in one way, while a short arc will increase the current through the arc, strengthening the solenoid and causing the mechanism to act in the opposite way.

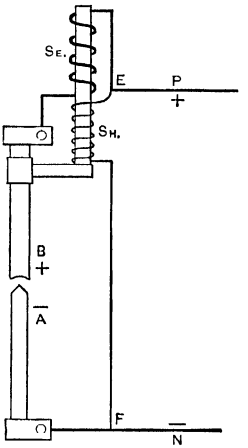


Fig. 156.—Differential Arc Lamp

The mechanism is so adjusted that when the proper length of arc is attained and the proper current is flowing through it, the system is balanced. Fig. 156 represents diagrammatically the differential arc lamp. A represents the negative carbon, which is fixed, and B represents the positive carbon, which is free to move. The latter, as shown, is fixed to an iron core or plunger, which constitutes the core of the two coils, SE. being the series coil and SH. the shunt coil connected across the line at the points E, F; the total current flowing from P through the lamp to N divides into two paths, a large amount flowing through the series coil and the arc and a small amount through the shunt coil as a by-pass round the arc.

When the current is switched on, assuming that the carbons are apart, there is no current flowing through the coil SE., the circuit being broken at A. There is, however, a complete path for the current through the shunt coil, which is immediately energized, thus pulling the core downwards. This motion brings the two carbons into contact, and on the circuit being completed a large current is caused to flow through the series coil owing to the comparatively low resistance of the path; this current will cause the series coil to get very much stronger than the shunt coil, and will result in an upward motion of the plunger, thus pulling the carbons apart, and to such a distance as to equalize the strength of the two coils and to keep the iron core floating between the two balanced attractions. As the

carbons are consumed the current through the arc decreases, the SH. solenoid becomes more powerful and causes the carbons to come close together, thus tending to re-establish the equilibrium. On these principles, modified for practical requirements, an arc lamp can be made which, after being connected to the circuit, will strike the arc and feed the carbons towards each other as they are consumed.

Differential Plungers and Coils.—Before starting the description of modern arc-lamp mechanism, it would be useful to consider the action and construction of the differential coil and plunger in its practical applications. The principal advantage of the coil and plunger is its long range of action. When a current is sent round a solenoid it will be found that on lowering an iron core into it the attraction will increase as the core or plunger descends, reaching a maximum when half the plunger is surrounded by the solenoid, and from that point onward the pull becomes less until the ends of the plunger coincide with the ends of the coil, when there is no more pull exerted. This long range of action and the even and smooth change of strength make the solenoid and plunger particularly suitable for an arc-lamp mechanism, hence its extensive application.

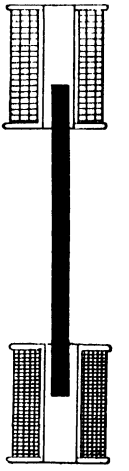


Fig. 157.—Differential Coils

One of the early forms of differential coils is shown in fig. 157. The plunger consists of a solid iron core, one end of which is drawn into one coil, which is in series with the main circuit, and the other end into another coil, which is in shunt. In this form of plunger and coil the attraction exerted on the plunger varies with the strength and direction of the currents in the two coils. The resultant pull in this arrangement is also affected by the magnetic interference between the two coils. Further, when an alternating current is used, the plunger extending through both coils would act like the core of an induction coil, and the coils would set up induced currents in each other. With alternating current it is also found necessary to use laminated cores, as the Foucault currents set up in a solid core would cause considerable heating, and the energy lost would be detrimental to the efficiency of the

mechanism. In order to give greater independence of action to the two coils the differential coil and plunger has been modified, as shown in fig. 158. Instead of one core we have here two separate ones, mechanically connected together by a rod of non-magnetic material.

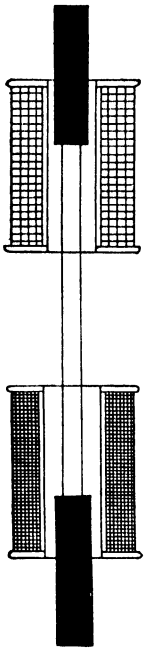


Fig. 158.—Differential System

Another way of obtaining a differential system is to wind a single solenoid with two windings, one in series and the other in shunt, using only one core, as shown in fig. 159. The action of this plunger is due to the difference between the ampere turns in the two windings.

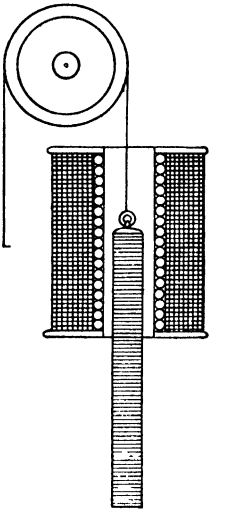


Fig. 159.—Differential Single Solenoid

Fig. 160 shows another form of differential coils with separated cores connected together through a brass lever, the two magnetizing actions being independent. This latter form is very extensively used in this country.

Arc lamps can be divided into two general classes—namely, the open and enclosed types. In the first category are included all the lamps burning in the air, or surrounded by an open globe. In the second category we can place all the different types of lamps called long-burning lamps, in which the arc is enclosed in a chamber, so as to limit the access of air, thus considerably reducing the combustion.

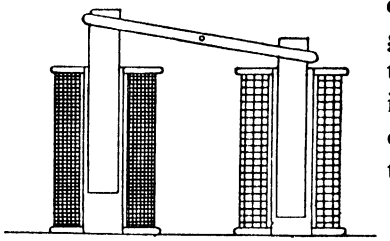


Fig. 160.—Differential Coils with Separate Cores

Arc lamps, except some special types, are suitable for either continuous- or for alternating-current circuits. The slight modifications required do not affect the mechanism generally, which remains the same, and it is unnecessary to consider them under a separate heading.

Among the open arc lamps there are various types which we can distinguish according to their mechanism.

We have in the first instance the lamps containing no mechanism whatever, the operation being simply controlled by two differentially wound solenoids acting direct on the carbons.

Lamps in which the solenoids act on a brake wheel regulating the feed.

Lamps in which the solenoids act on a clockwork mechanism which in turn regulates the feed.

Motor lamps, which are operated by motors regulating the carbons direct.

Enclosed arc lamps are very simple in construction and differ very little from each other.

The hot-wire arc lamp, in which there is no mechanism whatever, the operation of the lamp being controlled by the contraction and expansion of a steel wire; and the Bremer arc lamp, which is a departure on new lines entirely different from other lamps both in its mechanical construction and working principles.

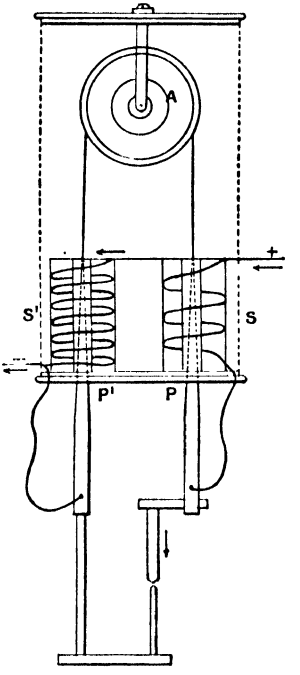


Fig. 161.—Mechanism of Pilsen Lamp

The Pilsen Lamp.—As shown diagrammatically in fig. 161, this lamp is differentially wound, one coil S being in series with the arc and the other S′ in shunt. The two iron plungers, P and P′, are cone shaped, so as to slightly compensate for the difference in the attractions exerted on them as they are displaced in the solenoids on the carbons burning away. The two cores are suspended by a flexible cord passing over the pulley A, the other ends of the plungers P and P′ being directly connected to the positive and negative carbons respectively. When the current is switched on, the carbons being in contact, a large current passes through the series coil S, which draws its core upwards, bringing the carbons apart. As the carbons are consumed the difference of potential across the shunt coil increases, while the current through the arc, and therefore through the

series coil, diminishes, and thus both coils combine to bring the carbons closer together, the shunt coil pulling its core upwards and allowing the feed to take place until balance is re-established between the two coils and the arc is again normal.

This type of lamp is very simple in construction, and is extensively manufactured on the Continent with slight modifications according to the make, but is very seldom found in use in this country.

The Johnson and Phillips's Lamp.—Fig. 162 represents a line diagram of Messrs. Johnson and Phillips's open-type, single-carbon, focusing arc lamp. Its operation is based on the differential system, the only mechanism employed for striking and feeding the arc consisting of a brake wheel and two solenoids. A represents the positive terminal, which is connected to the frame of the lamp, and the current is conducted to the brake wheel C through a flexible copper strip wound spirally round the spindle carrying the brake wheel, thus taking up any slack. Another flexible copper strip D, connected to the brake wheel and wound round a drum, carries the current to the positive-carbon holder and serves also to suspend the same. The current passes through the arc to the negative carbon and carbon holder to an insulated terminal on the base of the lamp; to this terminal is connected one end of the series coil, the other end being connected to the terminal B, which is also insulated from the frame, the circuit being thus completed. The shunt coil is connected across the terminals of the lamp.

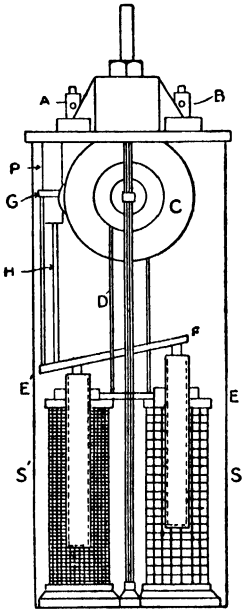


Fig. 162.—Mechanism of Johnson and Phillips's Arc Lamp

The negative carbon is suspended by a silk cord wound in the opposite direction to the positive flexible copper strip, on a drum which is fixed on the same spindle as the brake wheel. The two iron cores E, E′ of the solenoids are connected together through a brass yoke F, thus rendering their respective magnetic pulls quite independent from each other. This yoke is

pivoted at its centre. One end of the yoke is connected through a lever to the brake G, which consists of two blocks working eccentrically to the centre of the brake wheel, and which grip the rim of the wheel tightly when moved upwards, or release it when in the position shown. Another lever H connects the yoke to an air dashpot P, and prevents any sudden motion. The lamp is suspended by a porcelain insulator at the top of the lamp.

The following is the operation of the lamp: When the lamp is not in use an adjustable weight attached to the positive holder brings the carbons together. When the current is switched on, the iron core of the series coil is strongly magnetized and pulled downwards; this causes an upward motion of the other end of the yoke against the resistance of the dashpot, and the brake grips the wheel and turns it round, thus winding up the flexible copper strip of the positive carbon, unwinding the silk cord of the negative carbon, drawing the carbons apart, and striking the arc. The potential difference across the arc goes on increasing, strengthening the shunt solenoid S', which, acting in opposition to the series coil, brings the mechanism to a standstill when the arc has reached its proper length and the pull is equal on both sides.

As the carbons burn away, the potential difference across the arc increases, while the current decreases through the arc and series coil. The shunt coil becomes stronger, pulls its core downwards, and, releasing the brake slightly, allows the feed to take place and keeps the arc at its normal length. These lamps are designed to take 43 volts across their terminals, and work satisfactorily singly on 50-volt circuits or ten in series on 480-volt circuits, 5 to 7 volts per lamp being absorbed by the steadying resistance. These lamps are manufactured for stock complete with their shunt coils, the series coils being adjusted according to the current required, from 5 to 20 amperes, and the proper carbons fitted. These lamps are also adaptable to alternating-current circuits, the only alteration being that the iron cores are laminated in order to reduce the eddy current loss set up by an alternating field. These lamps require 38 to 40 volts across their terminals, a choking coil being used in series with them on constant potential circuits, which chokes back 10 to 12 volts without any watts being lost in a resistance.

Messrs. Johnson and Phillips are also makers of a double lamp made to burn 36 hours. This lamp is designed on the same principle as the single lamp, but two sets of carbons are placed side by side and there are two brake wheels, one for each set of carbons, although there is only one pair of solenoids. The two brakes are so set in relation to each other that one set grips the brake wheel a little in advance of the other and causes the carbons to come apart, while the others are still in contact, thus breaking that particular circuit and causing the other set to strike the arc. When these carbons are burned out the arc is struck immediately by the other set as the first set of carbons are apart and the circuit is open. In order to keep the first set of carbons apart and to prevent the burning of the carbon holders, the lengths of the flexible strips are adjusted so as to prevent the feed from taking place any further; both carbons coming therefore to a standstill, the arc dies out. As it has been explained, the two sets of carbons are set at work one after the other, and come into action automatically, thus reducing considerably the expense of trimming, a very important consideration when lamps are placed at a long distance from the central station.

The globe is fixed to a ring which is attached to the lamp body by two spring cams; two internal rods sliding through the bottom frame of the lamp serve to steady it. For trimming, the two spring cams are disengaged and the globe gear slides down, remaining suspended to the lamp body by the two rods mentioned above.

The Crompton Lamp.—The Crompton lamp shown in fig. 163 is of the focusing type, that is, both the carbons are fed towards each other in such a way that the arc is always in the same position. For regulating the feed two differentially wound solenoids, *a*, *a'*, are provided, one having a shunt and the other a series winding, and each enclosing a soft-iron core attached to the ends of a rocking lever *b*, the action of which is thus governed by the currents passing in the solenoids.

The carbons with their holders are suspended between the guide rods *d* by a flexible connector C insulated from the positive carbon and passing over the feed pulley *e*, which is fitted with guide rollers. A flexible cord or brake band fastened at one end of the rocking lever by a screw and insulator, and at

the other end to a flat spring fixed to the frame, controls the movements of the brake wheel *g* and feed pulley, which are combined but insulated from one another and carried on the frame. To steady its movements the rocking lever is connected to a piston and dashpot by a screw. The negative carbon is clamped in its holder by a spring *i*, and fixed to the carrier rod *j*. The positive carbon is insulated from its carrier by a mica disk, both carriers being attached to the flexible copper cord passing over the feed pulley.

The positive holder is made somewhat heavier than the negative, so that the carbons come together by gravity, and they are always in that position when the lamp is not in action.

On switching on the lamp there is an excess of current at first owing to the carbons touching one another, and the series solenoid, being the most powerful, draws down its core, causing the rocking lever to pull the brake band, the flat spring bends, and the brake wheel is turned slightly, thus drawing the carbons apart and striking the arc. As the arc becomes longer the current falls and the voltage rises, the shunt solenoid, being the most effective, draws down its core. This releases the tension of the brake band, and the brake wheel revolves owing to the greater weight of the positive holder, and brings the carbons nearer together. In this way the action of the feed pulley is always controlled by the currents in the solenoids.

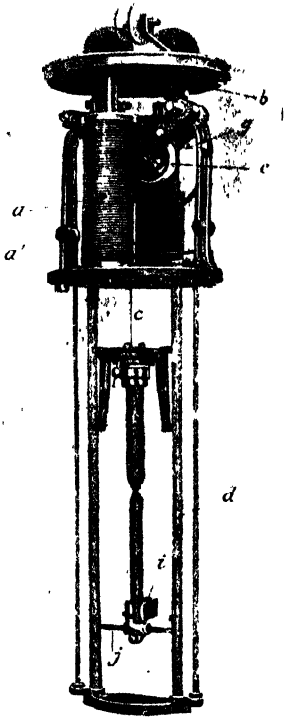


Fig. 163.—Mechanism of Crompton Arc Lamp

The Ark Lamp.—The ark is an enclosed type of lamp manufactured by Messrs. Johnson and Phillips, and designed for long burning, one pair of carbons lasting for about 100 hours.

Being enclosed, the lamp works satisfactorily direct on cir-

cuits of 80 to 100 volts, taking about 5 amperes, and has been employed extensively on 500-volt circuits arranged in groups of five in series. Its efficiency is slightly lower than that of an open-type arc lamp, but the long hours of burning, which means a saving in carbons and economy in labour, is a more important consideration, and largely compensates for the slight inefficiency of the lamp. A feature worth noticing, which is common to all enclosed types, is that the light is quite free from the yellow rays formed in the open arc.

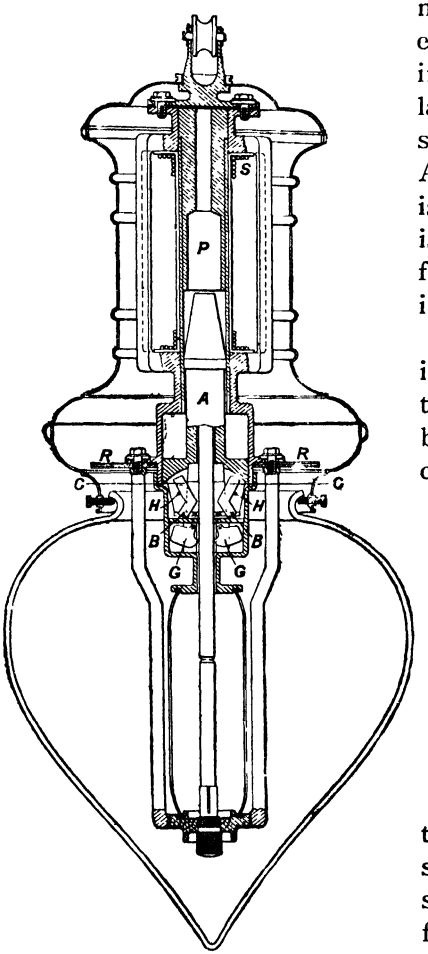


Fig. 164.—Section of Ark Lamp

A sectional view of the lamp is shown in fig. 164; the positive carbon is fed downward by gravity, the focus of the arc constantly moving towards the bottom of the lamp as the carbons wear away. The action of the feeding mechanism is controlled by a single-series coil S, wound on a brass tube, in the upper part of which is inserted a fixed pole piece P.

The armature A consists of an iron casting running through the lower part of the solenoid, its upper end being so shaped that under the influence of the pole piece it can slide freely up and down. The lower end of the armature is greater in diameter and forms a piston working in a cylindrical casting, which, while serving as a dashpot, is really the body or frame of the lamp.

The positive carbon fits loosely into the armature, and is held by four clutch cams C, which rest against it by gravity.

These cams, placed below the piston, are attached to a plate by the pins B. The current is conveyed to the positive carbon by the four copper contact pieces G contained in a casting which screws on to the dashpot. The negative carbon is fixed in a holder at the bottom of the globe enclosing the arc, and has the current conveyed from it by two arms, which are screwed to, but insulated from, the frame of the lamp. The upper part of the lamp body is therefore positive and the lower part negative, the two halves being insulated by a large mica washer, the screws clamping the two parts together being also bushed with mica. The path of the current through the lamp is as follows: Entering by the series coil it passes out to the frame, and through the contact pieces, which press lightly by gravity against the carbon, and so through the negative carbon and arms to the other terminal of the lamp.

The parts are shown here as they would appear before switching on the current. On the circuit being closed the armature is attracted upwards, and the clutch cams are forced inwards by the pins H on the armature and grip the carbon, pulling it upwards, thus striking the arc. As the carbons are consumed the current is diminished, and the armature descends until the lower corners of the cams press against the bottom of the frame, causing them to release the carbon, which slides down. As the current increases, the armature is raised slightly, causing the cams to hold the carbon and stop the feed.

The upper end of the carbon is grooved, and when the contact pieces engage in this groove they grip it, thus automatically preventing the carbon from sliding down any farther, and the lamp goes out. The steadying resistance is fixed on the lamp and is wound on an ambroin ring R, giving a large radiation surface. The lamp is also wound differentially, the only alteration in this case being that the solenoid is wound with a few layers of wire in series, and on the top, and insulated from it, is placed the shunt winding.

In the alternating-current lamp the series coil is embedded in soft-iron stampings, forming with the armature a closed magnetic circuit. That part of the armature under the influence of the coil is laminated, and the brass tube surrounding it has four longitudinal slots to reduce the eddy currents induced by the alternating field.

Only one size of carbon is necessary for the ark lamp, and the unburnt portion of the positive can be used for a new negative whenever the lamp is trimmed, thus reducing the waste of carbon.

The trimming of the lamp is effected by unscrewing the negative carbon holder and removing the inner globe. The old carbons are then taken out, a new positive is slipped into position, the remaining portion of the old one being fixed in the negative holder.

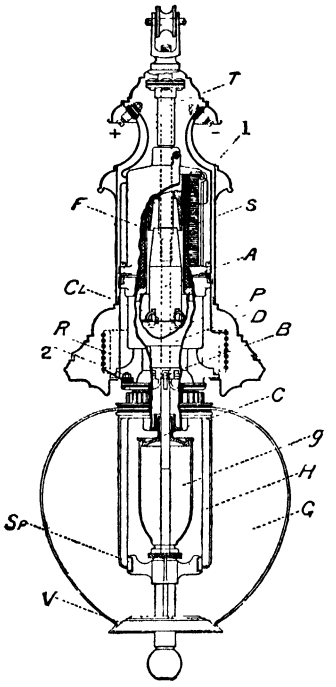


Fig. 165.—Section of Jandus Arc Lamp

The Jandus Arc Lamp.—This lamp contains only one moving part, i.e. the armature A, as shown in fig. 165. The bottom of the armature forms the piston P working in the dashpot D. The armature has a deadweight of nearly 4 lb., and the diameter of the piston is $3\frac{1}{2}$ in. These details are both unique, and the great weight of the armature, together with the large dash-pot, form a very powerful and certain controlling mechanism, not liable to derangement by dirt or dust.

The upper or positive carbon C is gripped, when the armature rises, by three radially disposed clutch rings *Cl* working on inclined rings. These rings at the same time centre and clutch the positive electrode, lifting it with a smooth motion. The customary carbon rod has been dispensed with, the carbons being carried by a loose sheath, and both the holder and carbon P feed through the clutch mechanism.

The armature is surrounded by a differentially wound solenoid S contained within a cast-iron frame F, forming a completely ironclad construction. The lamps for working in series are provided with a shunt coil of high resistance, 600 ohms. The series coil is, in addition, provided with a number of loops for adjusting the lamp, forming a positive connection that cannot get out of order.

The resistance R is enclosed in the lamp case for small currents and low voltages, but the more usual arrangement is a separate resistance.

The current passes from the positive terminal + through the series winding of the solenoid S by screw *l* to the lamp frame F. It is led to the upper carbon by gravity brush rings B, which give a smooth and frictionless contact. From the lower carbon it passes by the negative frame H to an insulated bolt 2 to the resistance R, and thence to the negative terminal − of the lamp. The shunt winding is connected directly across the carbons.

An important feature of the lamp is the double arc enclosure. The carbons burn in a small inner globe *g*, covered by a cap of special shape. The inner globe acts as a chamber of high temperature. On the formation of an arc the enclosed air within the small globe is heated and the oxygen is soon reduced to carbon monoxide. This, with the chemically inert nitrogen, is heated and expelled into the outer large globe. This outer globe G is air-tight at all points except at the bottom, where it is closed by a valve V. The surplus air escaping from the inner globe remains in the upper part of the outer globe, and a corresponding amount of cold air is displaced through the valve V at the bottom of the outer globe. The slight variations in the voltage, that occur in nearly all electrical circuits, cause changes in the temperature of the inner globe, and consequently in the density of the enclosed gases. A certain portion of the contents of the inner globe is continually being expelled or injected, and is drawn from the accumulation of warm inert gases in the upper portion of the outer globe.

The lamp is trimmed without removing the outer globe, by the complete withdrawal of the inner globe and carbons through the hand hole at the bottom of the outer globe, normally closed by the valve V. This withdrawal is effected by giving the porcelain knob attached to the valve a quarter turn, which releases the spider *Sp* from the frame H, and the whole of the inner globe and carbons can then be lowered through the outer globe.

The Hot-wire Arc Lamps.—An extremely simple and ingenious arc lamp is shown in fig. 166. This lamp contains neither coils nor clockwork, the striking of the arc and the feed-

ing being effected by the simple expansion and contraction of a flat steel wire.

The whole of the mechanism consists of a brake collar, which presses against and grips the upper carbon and which is held up by a special spring, and the flat wire mentioned above, which acts in opposition to the spring.

The current enters the lamp at the terminal A, and passes thence through the resistance coil B to the end of the flat steel wire C; through this wire it is carried to the terminal plate fixed on the frame of the lamp, and from there to the upper carbon, and through the lower carbon to the other terminal G.

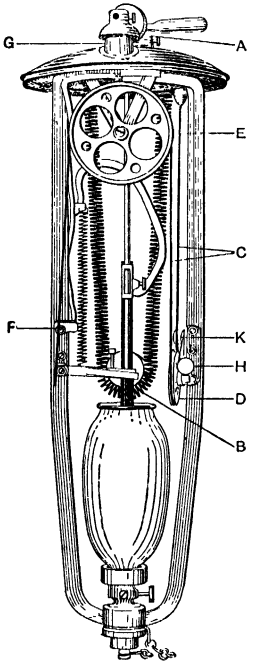


Fig. 166.—Mechanism of Hot-wire Arc Lamp

D and E are wheels over which the steel wire C passes, and H is a setscrew with a milled head, by means of which the little roller K is pressed more or less tightly against the steel wire; this is the only adjustment in the lamp.

When the current is switched on, the steel wire C expands rapidly, allowing the special spring to draw up the brake collar, which grips the carbon and draws it up, thus striking the arc. As the carbons burn away, the steel wire gradually cools, due to the decrease of current, and, contracting, slightly pulls down the brake collar against the tension of the spring, the carbon being gradually released and allowed to feed forward.

This lamp, which is a continuous-current enclosed arc lamp, is guaranteed to burn for 150 hr. on a 100–110-volt circuit.

The Flame Arc Lamp.—Fig. 167 represents an early form of a flame lamp. This is a continuous-current differentially-wound focusing clockwork lamp, the two most notable features in it being the great length of the arc, and the reflector or furnace controlling it.

The usual form of lever would not give a sufficiently long travel to strike the arc, and a special arrangement has therefore been adopted with this object. The shunt and series coils are

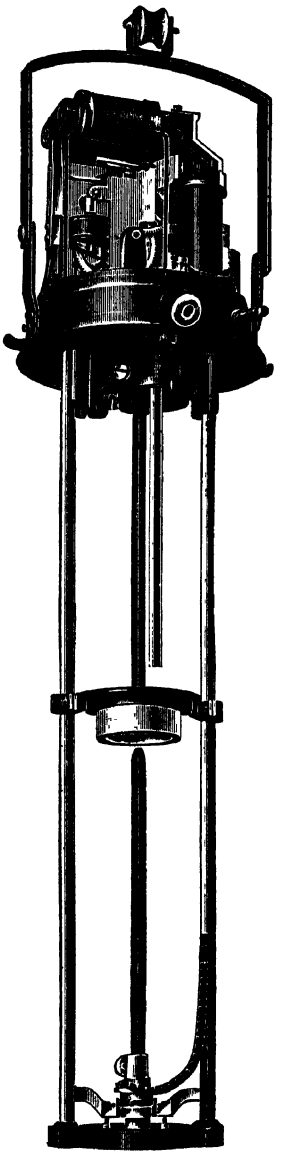


Fig. 167.—Mechanism of the Flame Arc Lamp

wound on two separate cores placed at right angles to each other as shown in fig. 168. The armature floats between the horns of these two cores, moving about the axis of the chain wheel. The two carbon holders are joined together by a chain, the weight of the upper carbon bringing the two carbons together when the lamp is not in use; on switching in the lamp the current through the series coil, which is placed horizontally at the top of the lamp, draws the armature upwards, thus separating the carbons to a distance of about $\frac{1}{2}$ or $\frac{5}{8}$ in. and striking the arc. As the lamp burns, and the resistance of

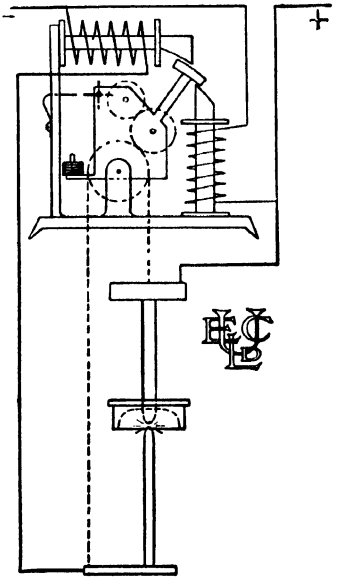


Fig. 168.—Special Form of Lever

the arc increases, the current through the shunt coil increases, while that in the series coil diminishes; the armature is thus drawn downwards, moving the frame which carries the gear

wheels until the flier, which, with the detent, is of the same type as that used in all the clockwork lamps made by this firm, is disengaged from the detent, and the carbons are fed forward. The motion of the mechanism is regulated by an adjustable weight consisting of thin washers placed on a spindle attached to the rocking frame, and to ensure further steadiness a dashpot is provided in the usual manner.

The leads are taken to the terminals of the lamp through two porcelain bushes, the positive terminal being connected to the upper carbon holder by means of a flexible wire which is wound spirally round a guide rod, and the negative terminal to the lower carbon by a lead which passes through one of the rods of the lamp frame. The lamp rods, base plate, and frame of this lamp are all insulated from the circuit by means of mica, and the whole lamp is further insulated from its suspension by a porcelain reel. The lead connecting the lower carbon to the negative terminal is insulated by means of glass beads. Fixed to the frame rods by means of a crossbar is the furnace or reflector; this consists of a small fireclay dish, placed just above the arc with its concave surface downwards, and through its centre is a hole just larger than the upper carbon. The object of this furnace is twofold: in the first place it prevents the carbon vapour from cooling, keeping the arc at a very high temperature, the small draught produced keeping the arc on the edge of the carbons, which are very small in diameter, and preventing any flickering; secondly, it acts as a reflector, throwing down the rays which would otherwise be wasted in an upward or horizontal direction. In this capacity the reflector is greatly assisted by the coating of brilliantly white powder which is deposited all over its concave surface. This powder is formed during the burning of the special carbon, which contains certain salts, the exact constitution of these being a speciality of the makers.

The cores of the carbons used in this lamp contain certain salts, which give to the light a very agreeable yellowish-white colour, quite different from the distressing bluish tint of an ordinary arc lamp.

A curious feature of this lamp is that a crater is formed in both the positive and negative carbons, and it requires an expert to distinguish between the two after they have been burning for some time.

When this lamp was first manufactured some trouble was experienced from the particles of carbon which were deposited on the whole of the mechanism, and it was found that after some time the little detent and flier became seriously corroded, with the result that the feeding of the lamp became uncertain and irregular; to overcome this difficulty an air-tight shield was placed round the top of the furnace which protected the mechanism from the deposit, and the difficulty has entirely been removed. The long life of this lamp has been obtained through the very careful way in which the strengths required for the different parts of the mechanism have been calculated; the gear wheels are all accurately machined and graduated, the chain wheel being the strongest and the flier the lightest, the strength of the intermediate wheels being proportional to the strain they have to stand.

As regards trimming, this lamp is exceedingly simple; the lower carbon holder is so arranged that it can be moved sideways in any direction, so that the accurate alignment of the carbons is very easily effected; nothing further is required for trimming than the cleaning of the rods and the insertion of new carbons in the carbon holders.

The Westinghouse-Bremer Lamp.—As a rule the numerous types of arc lamps differ from each other only so far as the details of the lamp mechanism are concerned, and consequently they all give approximately the same output of light per unit of power absorbed.

The Westinghouse-Bremer lamp is quite different from any other, both in its mechanical design and in its working principles.

It is well known that the electric arc is very susceptible to magnetism, that an arc flame between two electrodes can be directed or distorted at will by the definite movement of a magnet in its vicinity, or, similarly, by varying the strength of a magnet in the neighbourhood of the arc. In the Westinghouse-Bremer lamp the arc is controlled by an ingenious magnetic arrangement, whereby the flame is compelled to assume a position and shape impossible under ordinary free conditions. The advantages gained by the magnetic control of the arc are twofold: excellent arrangements of the lamp were rendered possible by its skilful adoption, and at the same time

the efficiency of the lamp as a light diffuser immensely increased.

The carbons or electrodes of the lamp are almost vertical and side by side. They converge towards their lower ends at an angle of about 20 degrees. The two ends of the carbons project through the base of the lamp for about $\frac{3}{4}$ in. into the globe, and between these ends the arc is maintained. Fig. 169 shows this lamp with its globe removed. The arc is forced by magnetism to stay at the carbon ends, and is also compelled to spread itself out in the form of a fan. This is obtained by two coils wound in opposite directions, the one being in series and the other in shunt with the arc magnetizing two cores built up of soft-iron wire, the pole faces being curved a little below the tips of the carbons. The arc is inside a little conical reflector, whose inner surface is of infusible material. The carbons are composed of metallic salts which give a special colouring to the light, allow of the spreading of the arc, and increase its illuminating power. These substances on being volatilized are deposited in the form of a white powder on the reflector, thus constantly renewing its surface and ensuring a perfect reflection. The carbons being both practically vertical, and feeding downwards, no care is required in fixing them; they are simply dropped loosely into tubes through the top of the lamp, and feed down by gravity as required. Short pieces of carbon can be used, and will be consumed down to the last $\frac{1}{2}$ in. or less.

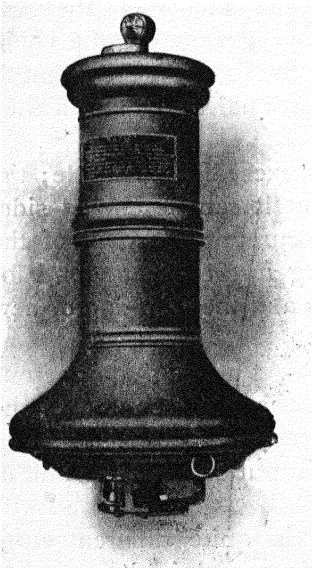


Fig. 169.—Westinghouse-Bremer Lamp

A particular feature of the Bremer lamp is the great surface of the arc flame and the small crater. While in an ordinary lamp the crater gives out about 85 per cent of the light, in the Bremer lamp the crater gives only 25 per cent of the total light. When used on alternating-current circuit both positive and negative carbons are of special composition, while on con-

tinuous-current circuits only the positive is made of special material.

The globe of the lamp is illuminated evenly over its entire surface; there are no shadows or dark areas, nor is there any point of vivid light to be seen. The lamp when burning is in effect a luminous sphere, radiating a warm diffused light all round.

The lamps are of the differential type, and may be used

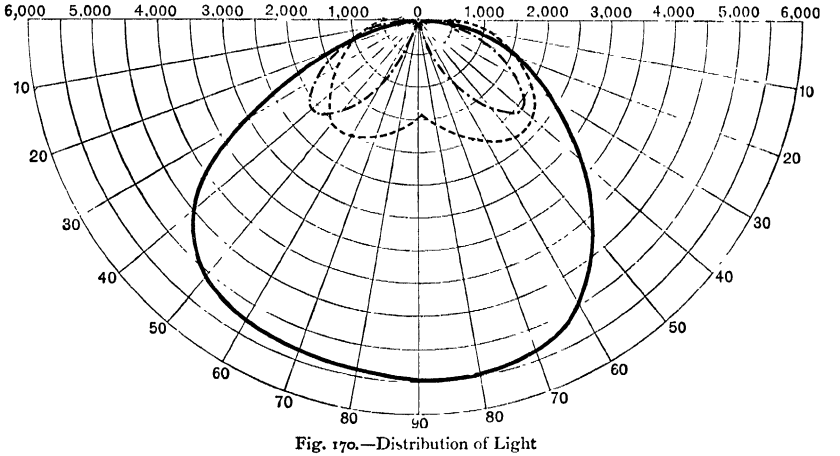


Fig. 170.—Distribution of Light

either singly or in groups of many in series. They are manufactured in several sizes varying from 5 to 12 amperes, and for 12- and 20-hour carbons.

The curves shown in fig. 170 are plotted from the results of actual comparative tests, and show the great economy obtained by the Westinghouse-Bremer lamp.

Curve thus ———— is of a 9-ampere Westinghouse-Bremer lamp without globe; circuit, 8.8 amperes 50 volts.

Curve thus —·—·— is of an ordinary 10-ampere arc lamp without globe; circuit, 500 watts.

Curve thus ---------- is of a 9-ampere Westinghouse-Bremer lamp with globe; circuit, 8.8 amperes 50 volts.

The tests on the Westinghouse-Bremer lamp gave the following typical results:—

	With Globe.	Without Globe.
Maximum candle-power	2400	5500
Mean hemispherical candle-power	1550	3300
Watts per maximum ,, ...	.180	.081
,, mean hemispherical candle-power	.289	.133

Angold Magazine Flame Lamp.—This lamp was designed to meet the demand for a lamp having long burning hours, low carbon cost, low maintenance expenses, and allowing for easy trimming. Before the flame lamps of the magazine type appeared on the market the number of burning hours obtainable in flame lamps depended on the length of the carbons used. In order to allow for steady burning of the flame lamps, the carbons must have a comparatively small diameter, and naturally their length had to be increased in order to get burning hours of sufficient duration for practical purposes. Naturally these long flame carbons had to be provided with some means to reduce the drop of voltage caused by their resistance, and for this reason they are either coated with copper or provided with a metal wire running through the whole length of the carbon. Carbons of this nature were very expensive, and the risk of breakage was excessive considering the rather unwieldy lengths of the rods, which reach for certain lamps 30 in. Even if two pairs of such carbons were combined in a lamp the maximum burning hours which could be reached were only between 30 and 40. The magazine lamp has dealt successfully with the question of carbon cost, because carbons of comparatively short length, maximum 15 in., and without copper coating or metal core, can be used. This naturally also had great influence on the risk of breakage, and as nine or ten pairs of these carbons can be filled into the magazine at one time, and thereby, according to the current, 60 to 100 burning hours obtained, the trimming expenses are greatly reduced as compared with flame lamps of the original type.

There are two types of the Angold magazine lamp at present on the market, one using carbons of a 15-in. length and giving with one trim of nine pairs 60 to 65 burning hours with 10 amperes, and 90 to 100 hours with 8 amperes. The other type uses 12-in. carbons, and gives accordingly shorter burning hours. The lamp can also be used for alternating current, but only in parallel connection with a transformer for each lamp. The lamp is provided with a self-contained equivalent resistance and differential cut-out whereby each lamp is made a complete independent unit even if burning in long series.

Arc-lamp Circuits.—Having dealt with the theory of the arc and described various types of lamps on the market, we

must now consider the practical working of arc lamps on various circuits and describe some of the accessories used in their installation.

In the old days only low voltages were used, and they are still to be found in private installations, the pressure varying from 50 to 70 volts; in these cases the lamps are run in single parallel.

Fig. 171 shows a circuit of five 10-ampere lamps running on a 50-volt circuit. Each lamp has the full voltage across its terminals, the generator supplying 50 amperes, which are equally divided among the lamps as shown. This system entails the use of heavy mains, and has therefore been abandoned for large installations.

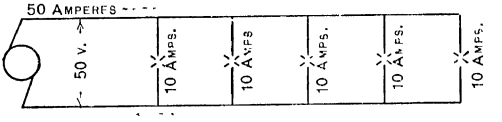


Fig. 171.—Arc Lamps in Parallel

The most economical system as regards cost of mains is the series or constant-current system.

The lamps to the number of 30 to 50 are connected in series, and are supplied with a constant current from a dynamo known as an arc lighter at a pressure of 1500 to 2500 volts according to the number of lamps, each requiring about 50 volts across its terminals. As shown in fig. 172, a current, say of 10 amperes, passes through each lamp in turn. An objection to the series system is the high voltage required, which is considerably more dangerous than a lower voltage.

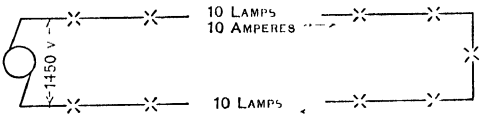


Fig. 172.—Arc Lamps in Series

A system used most extensively, however, is the series-parallel, constant-potential distribution, from 250- or 500-volt circuits. This consists of groups of five or ten lamps respectively connected in series, each group being run in parallel from the supply mains.

Alternating-current circuits, which are becoming more generally used, present great advantages for grouping the lamps, although the efficiency of the alternating lamp is lower than that of the continuous. High voltages can be used for trans-

mission, and many lamps could be run in series direct from the high-tension mains, or the voltage could be reduced by transformers in order to run few lamps in series; and again, lamps can be run in single parallel by the use of economy coils off low-tension mains.

Fig. 173 illustrates these three different methods, and clearly demonstrates the flexibility of alternating-current distribution.

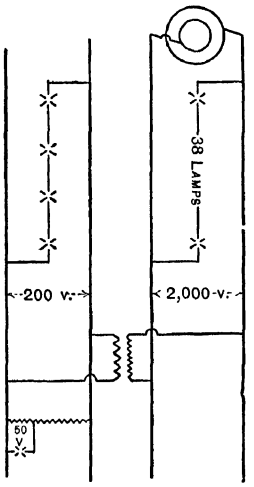


Fig. 173.—Alternating-current Circuits

Series Resistances. — A form of series resistance employed on arc-lamp circuits is illustrated in fig. 174. It consists of two iron plates bolted together, the resistance consisting of German-silver or other suitable material wound on four grooved slates running longitudinally along the frame. The ends of the resistance wires are connected to the two terminals shown at the top, and to which are brought the leads going to the lamp; this frame is provided with two lugs for fixing in the lamp-post.

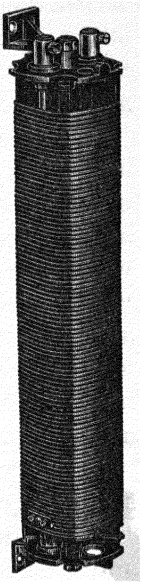


Fig. 174.—Series Resistance

Automatic Cut-outs and Substituting Resistances.—Automatic cut-outs are usually fitted to arc lamps burning in series, which automatically come into action, short-circuiting the lamps and offering a by-pass to the current should the carbons be consumed or separated too far, thus preventing the circuit from being broken and allowing the current to continue to flow through the other lamps. They usually consist of a small magnet wound with a few turns of thin wire connected across the arc, and so adjusted that a potential difference beyond the normal causes a little armature to be pulled up, thus offering a by-pass for the current. On constant potential circuits the automatic cut-out substitutes a resistance equal to that of the arc, so that the other lamps burning on the same circuit are not disturbed, the E.M.F. dropped across this resistance being equal to that absorbed by the lamp when burning. Fig. 175

illustrates an automatic cut-out suitable for fixing in the base of the lamp-post, and fig. 176 a substitution resistance and switch.

On constant-current circuits no series resistance is required, but the automatic cut-out, which in this case is usually contained in the lamp, is wound with a few additional turns of thick wire, which are connected to the circuit only when the cut-out short-circuits the lamp. The magnet is then energized by the current flowing in the circuit after the lamp has been short-circuited, the high difference of potential across the arc which brought the cut-out into action being no longer available.

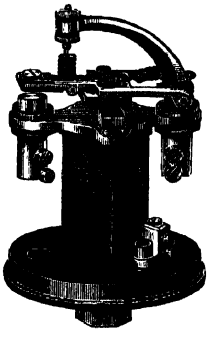


Fig. 175.—Substitution Resistance and Switch

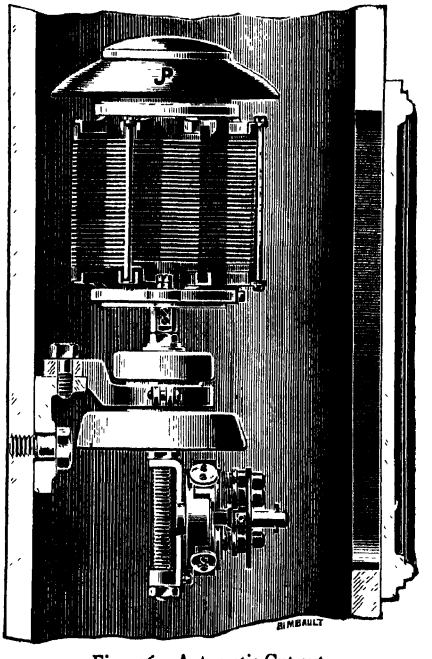


Fig. 176.—Automatic Cut-out

Choking Coils.—As already stated above, on alternating-current circuits the series resistance is replaced by a choking coil. This apparatus consists of a laminated iron core, round which are wound a few turns of wire connected to the main circuit. The magnetic circuit being opened, it results in a choking effect on the E.M.F. which is proportional to the number of ampere turns round the core, which is movable, so that it can be adjusted to choke the proper amount of volts according to the circuit.

Transformers and Economy Coils.—When high-voltage

alternating-current circuits are used it may be more suitable for certain conditions of lighting to run only one, two, or three lamps in series. In this case an alternating-current transformer is used, the primary of which is connected to the supply mains and the secondary is adjusted so as to suit the number of lamps required. In this way the line voltage is reduced to the required amount through the medium of a very efficient apparatus instead of wasting energy in a resistance.

On low-voltage circuits an economy coil is used; this is

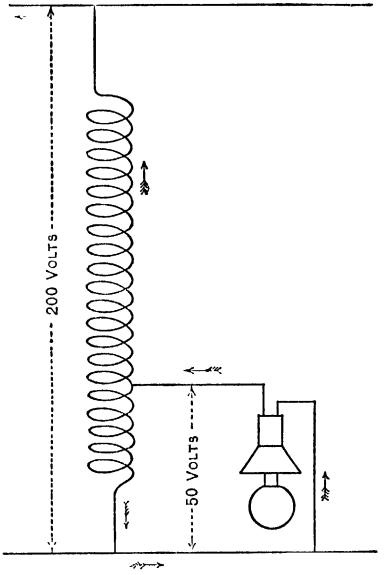


Fig. 177. -Economy Coil

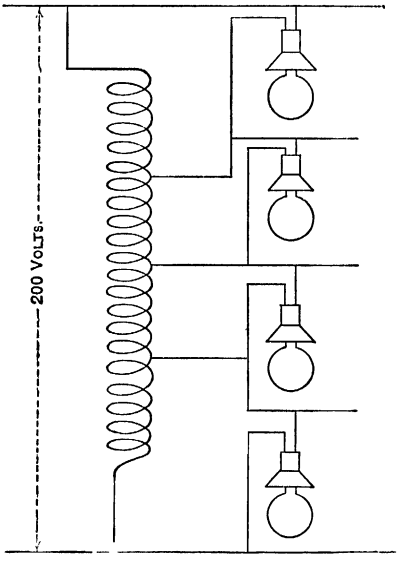


Fig. 178.—Economy Coil with Four Lamps

really a transformer with one winding only surrounding a closed magnetic circuit made up of iron laminæ. Fig. 177 illustrates diagrammatically the connections of an economy coil, and the direction of flow of the current for a 50-volt 10-ampere lamp on 200-volt circuit. It is easily seen that all the turns tend to magnetize the core, the proper voltage being obtained by tapping the winding at a suitable point. In the case illustrated assume the economy coil to be wound with 200 turns; each turn giving therefore approximately 1 volt, the required voltage would be obtained by tapping the coil on the fiftieth turn, and connecting the lamp between this point and the line. Another

advantage of the economy coil, or autotransformer as it is sometimes called, is that four 50-volt lamps, for instance, could be connected on 200-volt circuits by tapping the economy coil at three equidistant points, as shown in fig. 178; the lamps will burn quite independently of each other. When the four lamps are all burning they take their current directly off the mains, the economy coil taking only its magnetizing current.

Globes.—The design and density of the glass must be chosen according to the effect required. Rough glass globes, although very effective, are difficult to keep clean, and when dirt accumulates in the grooves it considerably reduces the light. Clear glass globes, which are the most efficient, absorbing only from 10 to 30 per cent of the light, are used in such places as railway yards, but they are not suitable for street lighting owing to the light not being properly diffused. For the latter purpose obscured glass is generally used, and may reduce the light by 40–70 per cent. The following table gives the approximate amount of light absorbed by various kinds of glass, and can be used as a guide in the selection of a globe:—

Kind of Globe.	Loss of Light.
Clear glass globe	10–30 per cent.
Ground glass globe	25–50 ,,
Opal globe	40–70 ,,
Cut-glass globe (holophane)	5–20 ,,

Street Lighting.—For street lighting the lamps are 18 to 25 ft. above the ground level. Short columns give bad distribution of light, but they are employed in cases where the roadway is spanned by a bridge. The lamps may be placed in line in the middle of the roadway, in which case the columns are about 25 ft. in height, and are placed at a distance of 60 to 100 yd. from each other. This is no doubt the most effective position for lamps, but cannot be adopted in narrow thoroughfares where space is limited. In these cases alternate lamps can be placed on opposite sides of the road at the edge of the footpath, when they are usually 20 ft. high, the distance between the columns on each side being about 100 yd. Usually 10-ampere continuous- or 15-ampere alternating-current lamps are used for street lighting, 480 watts being absorbed by a continuous-current and 720 watts by an alternating-current lamp.

Lamp Columns.—The designs of lamp columns and sup-

ports are numerous, but many points have to be considered before deciding which is the most suitable for the purpose in view. The columns should be ornamental, should be sound and solid in design, and should not be in the way of the traffic. On the other hand, they must be effective from the lighting point of view, and conveniently arranged for facilitating trimming. The columns are fixed in position by a foot piece or base, which, after being placed in the ground and levelled, is surrounded with concrete. A space is provided in the lower part of the column where the switches, resistances, transformers, or any accessories required for the lamp can be locked in out of reach. The supply cables come up from below and are connected on to the lamp leads, a door or shutter giving access to the connections. Any part carrying current ought to be well insulated from the column, and the latter should be well earthed.

Trimming.—The trimming of the lamps consists in recarboning them and cleaning the rods. The lamps have either to be fitted with tackle for lowering, or a ladder of some kind has to be provided to enable the trimmer to do his work.

The lamps can be lowered to the ground by means of a winch fixed inside or on the post. When this arangement is adopted street posts must be fitted with overhanging brackets for the lamps. To prevent the conductors from looping when the lamp is in position a special block may be used for winding up the conductors when the lamp is raised in position. It is, however, much better to employ some form of contact gear arranged in such a way that when the lamp is raised in position connection is made with the circuit, and the lamp is at the same time locked in position. An upward pull is sufficient to disengage the lamp and to allow it to be lowered.

Fig. 179 illustrates a contact gear. It consists of two parts—one fixed to the post, and the other to the top of the lamp, from which it is insulated by two porcelain insulators 1. The diagram shows the gear locked in position. To lower the lamp the wire rope which passes over the pulley R is pulled, raising the lamp and contact gear until the pawls *a* enter the upper tube *b*, which forces them inwards against the pressure of the spring *s*, and are locked in this position by the catches *c* falling on the pins *e*. The lamp can now be lowered, the pawls clearing the recess in the lower tube. Before raising the lamp the

trimmer lifts the catches, thus releasing the pawls, which fly apart as far as the pins *f* will allow. On the pawls entering the lower tube they are forced inwards slightly, and on reaching the recess they fly outwards and lock the lamp in position.

The lamp at the same time is connected to the circuit through the brushes P and N, which make contact with the two rings O connected to the mains.

In the case where lamps are fixed in position various devices are in use. Columns are fitted with short pieces of iron rod, with a round head at each end passing through holes in the columns on alternate sides. All but the heads are concealed, and they are pulled out by the trimmer as he ascends, and pushed back into the column when the trimming is completed.

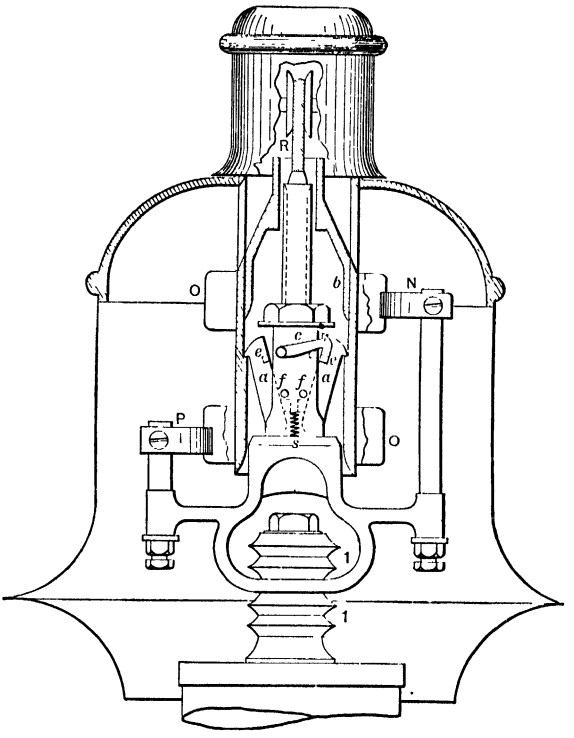


Fig. 179.—Section showing Contact Gear

The columns may be fitted with a structure like a parallel ruler concealed and embedded in each side when shut up, and which forms, when opened, a ladder with the column at its centre.

A third and simpler method consists of inserting iron pins in holes provided for them on each side of the post, thus forming successive steps which the trimmer removes on coming down. A short ladder which he carries with him enables him to reach above the ornamental base of the lamp-post.

In order to be able to use both hands he secures himself by a safety belt provided with two hooks which hook on to the side arms of the carriers.

In the above methods only one man is required, and he can trim about fifty lamps per day when spaced for ordinary lighting, while with ladders two men are required.

Ladders are either of the fire-escape type, and can be used when the lamp is supported by a carrier right above the centre of the post, or tower ladders mounted on trolleys or wheels, the trimmer standing on the platform provided at the top. The latter type is generally used for trimming lamps lighting interiors, railway stations, picture galleries, museums, &c.

Further Applications of the Arc Light.—For indoor and factory lighting the even distribution of light is obtained by the use of inverted lamps, which are designed to reflect the light on to the ceiling, which is usually whitened, and a perfectly diffused light is thrown all round. Although a great amount of light is lost, the arc is concealed from the eye, and the inconvenience caused by patches of light and shade is eliminated. These lamps have their carbons reversed, so that, the positive being underneath, the light is thrown upwards, and the arc is surrounded by a conical reflector.

In some instances a beam of light is required, as in a projector used in a naval searchlight, or for coast defence; an arc light is readily adapted for this purpose. The arc is made to burn with the crater facing a parabolic reflector, and is so arranged that the source of light is approximately in the focus of the parabola, with the result that the rays coming from the crater are reflected in a parallel beam. In such lights the arc takes from 5 to 10 kilowatts, the carbons being proportionally larger than those of the ordinary lamp. Both automatic and hand-feeding regulation is used.

A similar lamp, though of a smaller size, is used in theatres for producing light effects on the stage. These lamps are fireproof, and are mounted on a universal joint so that the light can be thrown in the required direction. No reflector is used, and the light passes through a lens, by means of which either a beam or a diffused light can be obtained by moving the arc towards or away from the lens; 9-mm. carbons are used for both positive and negative; the lamps take about 10 amperes,

and are fed by hand by means of a rack and pinion. Coloured lights are obtained by a circular screen fitted with sectors of coloured gelatine, which can be brought in front of the lamp as desired.

The high temperature of the arc has been applied in the commercial production of calcium carbide, carborundum, in the reduction of aluminium ore, and for other purposes.

In these electric furnaces very large currents are used, ranging from 1000 to 6000 amperes or more, and the size of the carbons attain as much as 12 to 18 in. square.

CHAPTER XIX

INCANDESCENT LAMPS

History.—No one person can lay claim to the invention of the glow lamp, as it has been gradually evolved by many inventors. In the history of the glow lamp the starting-point was the knowledge of the fact that a metallic wire, or other conductor, could be heated by an electric current and raised to a sufficient temperature to give out rays of light. From this starting-point invention was directed to discover a conductor which would be capable of being raised to a high temperature without fusion; and it was further necessary that such a substance should have a high specific resistance, and be capable of being shaped into such a form as to conveniently provide small units of light.

The earliest mention of an electric glow lamp was in 1836. The first to patent such a lamp was De Moleyns, in 1841. He used incandescent platinum in a vacuum. A further advance was made in 1845 by J. W. Starr, who patented the use of carbon heated to incandescence by electric current in a vacuum. The next step was by Roberts, who, in 1852, first pointed out that the carbon conductor should be made "as thin as possible, and in a vacuum as perfect as possible".

Up to this period primary batteries were the only source of current, and, this method being very costly, it was not until the dynamo of Gramme became a commercial success that any real

want for the incandescent lamp was felt. It was then that inventors directed their attention to obtaining substances capable of being subdivided into small units of light, and one of the earliest patents pointing out the essentials of economic lighting by small incandescent lamps in parallel is that of Lane-Fox in 1878, who advises a pressure of about 100 volts. The next patent is by Edison in 1878 wherein he goes over much the same ground. In 1878 Sawyer and Mann point out in their patent that the use of carbon in a vacuum is generally known, but they describe for the first time a process of "flashing", for improving carbon by heating it in a carbon-containing gas or liquid. They also explain the important process of ridding carbon of its occluded gases by first heating it in a carbon-preserving atmosphere. Their filament, however, was of low resistance, and used in nitrogen gas and not in a vacuum.

The first lamp which most nearly approached that in use to-day was patented by Lane-Fox in 1879. He there describes the use of high-resistance filaments, regulated by the Sawyer-Mann "flashing" process. This is again followed by Edison, who introduces cotton thread, wood splints, and millboard as suitable materials from which to obtain high-resistance carbon filaments. The next patents of note are those by Swan and Lane-Fox in 1880. In Swan's is mentioned the parchmentizing of the cotton thread, while that of Lane-Fox is for a complete lamp and the processes of its manufacture. Lamps made by several of these last inventors were almost simultaneously placed on the market about 1881. From this period steady progress was made in the perfecting of the various details of manufacture, more especially in the composition and process of manufacture of the carbon filaments.

General Remarks. — The incandescent lamp is simply a transformer of energy, and the vibratory motion of the ether which it produces (felt by the eye as a luminous sensation) requires a calculable expenditure of energy for its support. With oil or gas lamps the energy is derived from the combustible; in glow lamps it is furnished by the current. The transformation of this energy is, in each case, accompanied by losses, and depends upon the radiant employed. Each source of light has therefore a mechanical efficiency which indicates what fraction of the total energy expended and transformed into

a vibratory movement of the ether is capable of producing a luminous impression. Table A shows the comparative values of the "mechanical equivalent" of the candle-power obtained by the best-known illuminants, these values being converted from calories into the equivalent number of watts required to give 1 candle-power.

TABLE A

Gas burner (butterfly)	consumes	93	watts	to give	1	candle-power.
Candle	,,	86	,,	,,	,,	,,
Oil lamp	,,	57	,,	,,	,,	,,
Incandescent lamp	,,	from $2\frac{1}{4}$-$4\frac{1}{2}$	,,	,,	,,	,,
Enclosed arc lamp	,,	,, $2\frac{1}{2}$-4	,,	,,	,,	,,
Metallic filament	,,	,, 1-1.7	,,	,,	,,	,,
Arc lamp	,,	,, 0.8-$1\frac{1}{2}$	,,	,,	,,	,,

Table B gives the working temperatures of incandescent filaments, and shows that tantalum and tungsten have a greater steadiness of light on a variable voltage supply.

TABLE B

	Melting-point.	Temperature for Electric Light.	Variation of Candle-power with Voltage.
Carbon... ...	Sublimation from 1900° C.	1850° C.	Candle-power = $K(E)^{6\cdot8}$
Tantalum ...	2300° C.	2000° C.	Candle-power = $K(E)^{4\cdot3}$
Tungsten ...	2950° C.	2300° C.	Candle-power = $K(E)^{4\cdot1}$

The first three illuminants in Table A have an extremely low efficiency, owing to the enormous losses undergone in combustion. The luminous body is the particle of carbon not yet consumed, and whose incandescence is maintained by the combustion of the gas, and with these illuminants there are heavy losses from incomplete combustion and the convection of heat to the surrounding air. The only loss with the incandescent lamp is by radiation, as its incandescence is maintained by the electric current, and the filament being in a vacuum there is no loss by convection. The temperature to which the carbon particles in each of these sources can be raised determines their brightness and mechanical efficiency.

Although the glow lamp appears from the above table to be efficient in comparison with the other illuminants enumerated,

yet the "luminous or optical efficiency" of all these seven sources is very low. By optical or luminous efficiency is meant the ratio of the energy of the luminous radiations to that of the total radiations. When any body is below 400° C. its luminous efficiency is zero, as the vibrating movement of the ether is not rapid enough to be sensible to the eye as light. With a glow lamp the total energy radiated by it can be estimated by immersing it in water in a blackened metal calorimeter, and measuring the elevation of the temperature attained by the water in a given time. The lamp is then placed in a clear-glass calorimeter, in which the elevation of the temperature will be due almost solely to the action of the obscure (heat) rays. The difference between the amounts of heat given by each of these two methods is a measure of the luminous efficiency, i.e. of that portion of the total energy which is converted into luminous radiation.

By these methods Blatner has found the luminous efficiency of glow lamps to be about 5 per cent. Tyndall gives that of an oil lamp as 3 per cent, and of a gas burner as 4 per cent. The arc lamp is found to be about 15 per cent. A Geissler tube (equivalent to a mercury-vapour lamp), according to Staub, has an efficiency of 33 per cent, and thus heads the list. The efficiency of a glow lamp being only 5 per cent, the other 95 per cent of the energy is wasted in producing heat, i.e. in radiations of the ether of too long a wave length to be sensible as light, consequently as far as producing light is concerned this energy is lost. Thus, with a glow lamp giving 1 candle-power for 4 watts, about 3.8 of these watts produce heat only, and the remaining 0.2 watt produces light. This result, however, is common to all the illuminants, as, being based on the incandescence of carbon, its temperature is about the same in all of them. In this respect alone little progress has been made since the commencement of civilization, when prehistoric man used a resinous torch whose luminous efficiency was nearly equal to the arc lamp. After allowing for all losses, from the coal to the engine, dynamos, lamps, &c., we get an optical or luminous efficiency of barely 1 per cent.

There seems to be a practical limit to obtaining light by incandescence, owing to the fact that the eye becomes fatigued and is harmed by increasing the temperature of incandescence.

Also, it must be remembered that at an increased temperature the proportion of luminant energy to the total radiant energy does not increase at nearly the same rate, and consequently by raising the temperature there is a large addition to the "candle-power" obtained by photometry, although a comparatively small amount of energy is required to obtain an increased temperature. Bodies such as the alkaline earths—magnesium, thorium, zirconium, cerium, &c.—withstand higher temperatures than carbon without fusion. With regard to all known bodies it seems an incontrovertible fact that greater light can only be obtained by increasing their temperature, although the supply of energy required to maintain a given temperature varies with different bodies according to their varying emissivities (rate of heat dissipation). In this respect white and shiny bodies appear to be the best, as they have the least (heat) emission. It may, however, be possible to find a substance other than carbon which emits at the same temperature a greater quantity of light, and is of such a nature that its vibrations shall give the ether a more rapid vibrating movement. The most interesting source of illumination seems to be indicated in the experiments of Professor Langley with fireflies. He found that the energy given out by these insects was expended entirely in the visible part of the spectrum.

Phosphorescence and fluorescence are assumed to be due to a class of vibrations distinct from those which cause incandescence, and it may be possible to excite phosphorescence (called "luminescence") in an economical way directly by electrical means. Maxwell first pointed out that electrical vibration coincided with luminous vibration when its wave length was sufficiently short, and the experiments of Hertz confirmed this point. From these results it can be seen that the most efficient light would be obtained if one could vibrate the ether at a period equal to that of luminous undulations, and it may some day be possible to maintain this vibration direct from electrical vibration.

Filament Preparation.—To-day most lamp factories employ for the filament a solution of cellulose in one form or another. (1) A large number use cotton wool or wood fibre dissolved in chloride of zinc. (2) Some use cotton converted into gun-cotton and dissolved into collodion. (3) Others, again, use

cotton converted into guncotton and dissolved in acetic acid. It is equally interesting to note that nearly every factory ejects each of these solutions through graduated jets into a liquid capable of "setting" the solution of cellulose into a semi-solid gelatinous thread. In the first and more commonly used method pure zinc chloride with an acid reaction is dissolved in water to a specific gravity of about 1800. To the zinc chloride solution best bleached cotton wool or Swedish filter paper cuttings are added, and the mass when cold resembles clear thick treacle, and is ready for squirting through jets. For this operation the flasks containing the cold mass are each fitted with two tubes—one connected to a source of air pressure, which must be kept constant and be able to be regulated to any desired amount; the other, leading to the bottom of the flask, connected by distributing tubes to various arms terminated with fittings to which are attached vertical graduated jets. Through these tubes and jets the mass is forced by air pressure. The jets, carefully gauged and proportioned in diameter to the various diameters of filament required, are now arranged over tall glass cylinders which contain the setting fluid—a mixture of methylated alcohol of full strength and hydrochloric acid. The hydrolised cellulose mass is dehydrated by means of the alcohol. Coagulation having thus been caused, the mass issues from the jets in a colloidal state and possessing a certain amount of strength. It falls as a continuous thread to the bottom of the cylinders, where it winds itself in a continuous hollow spiral. The thread is then carefully removed from the cylinders and passed through baths of alcohol, acid, and water, and by means of the acid any oxychloride of zinc is removed.

The wet thread is now wound on drums, or other arrangements, where it can freely contract as it dries. When dry the resulting material should be transparent, round, and strong, somewhat resembling a clear violin string. In this dry state the filament material is quite strong, and lends itself readily to being wound on the carbon blocks or formers used for carbonizing the dried mass. Some makers place the filtered mass in a taper flask with a jet or jets at the lowest part, the air pressure being introduced through a joint at the upper part of the flask.

Carbonizing.—The various threads are wound when dry on

to carbon moulds or formers, which are so shaped as to give the desired form of filament, such as horseshoe, curls, zigzag, &c. The winding is done either by hand or by a machine which enables the moulded block to be slowly revolved. The manner of finishing off the block ready for carbonization is shown in fig. 180. The upper part of this block is an arc-light carbon of the same diameter as the filament curl desired, while the lower part is solid moulded or retort carbon. When wound, the block is bound round with thread or string, and the filament material is cut away from the lower edge, that it may freely contract when undergoing carbonization in the furnace. Some makers use carbon rods with wooden side pieces or supports; others, closed-in moulds of the desired form; while others split the rods, which are separated by a material that will contract when carbonized. But in all cases great care must be taken that full and unrestricted allowance for shrinkage of all parts is obtained in the carbonizing process, otherwise the resulting filaments will be warped, of bad shape, or of uneven diameter.

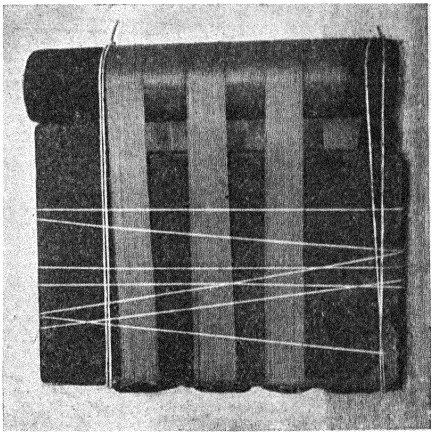


Fig. 180.—Finished Block ready for Carbonizing

The wound blocks are now placed in pots or boxes, which are filled in with plumbago or charcoal in fine powder, to keep the filaments of good form during their contraction by carbonization, and also at the same time to exclude all access of air. These pots or boxes (provided with a small opening in their covers, to allow of the escape of the gases produced by the first heating of the filament) are now placed in furnaces of various forms, and heated slowly and carefully by gradual and successive increments of temperature to 2000° C., during a period of from twenty to thirty hours. Previous to the final carbonization the pots or boxes are hermetically closed, and when all are quite cold, the blocks are removed from the pots; and, in

the case of charcoal or plumbago packing, the filaments are well brushed before and after removal from the blocks or formers. Fig. 181 shows a bundle of carbonized filament as it appears when removed from the block (fig. 180) used for obtaining the usual curl filaments. In appearance the filaments should now be smooth, even, shiny, steely-black, very hard, and flexible. If short lengths be mounted, they should be able to scratch glass, being veritably "black diamonds".

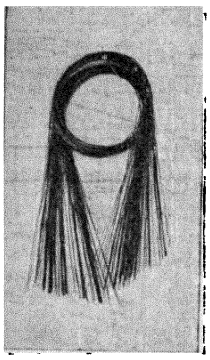


Fig. 181.—Bundle of Carbonized Filaments

Gauging and Cutting. — After the filaments have been removed from the carbon blocks they should be tested, in order to ascertain their specific resistance, as this forms a criterion by which the success of the carbonization can be judged. It is also necessary to know the specific resistance, so as to check diameters, lengths, and the amount of preparation that the filament should subsequently have. Each filament must be measured to find out its diameter, as an accurate knowledge of this is necessary if lamps are to be turned out to a given voltage, candle-power, and efficiency. This gauging process is generally carried out by some form of micrometer gauge, capable of being adjusted for wear. The filaments are usually assorted into half-thousandths of a millimetre.

At a given efficiency in watts per candle-power, and for filaments prepared in the same way, the final surface for a given candle-power is the same irrespective of the voltage. To obtain these conditions each diameter of filament requires a distinct length. The surface necessary is a constant k multiplied by the required candle-power, the constant being procured by the photometry of lamps with carefully measured filaments. The surface of a filament is equal to πld (where $\pi = 3.14$, l = length, d = diameter). Therefore, the length for a known diameter is given by the equation $l = \frac{c.p. \times k}{\pi d}$.

This is only applicable, as previously stated, to lamps of the same efficiency and same method of preparation. Photometry of test lamps at different efficiencies, or with filaments of different preparation, will give quite a different value to the

constant k. With unflashed filaments, which are to be run at the same efficiency, the above formula holds good; but in the case of flashed filaments, the amount and kind of deposit alters the final surface to such an extent that other experimental data must be compiled. Such data must show the increase of diameter brought about by differences in flashing methods, and by the various percentage reductions of resistance between the flashed and unflashed states. When the filaments are gauged after flashing, these varying conditions need not be taken into account; but, generally speaking, the more usual and convenient way is to ascertain the diameter before the flashing operation.

At the same time as the surface per candle-power is being arranged, it is necessary to take into consideration the question of voltage. Each diameter of unflashed carbon of equal specific resistance has a fixed length for a required voltage, consequently to obtain a particular surface with a particular voltage, a diameter must be selected which will bring about these conditions. For this a diameter must be chosen which (when the length is cut to suit the required voltage) will correspond with the required surface. This diameter can be ascertained by photometric trials of filaments of various diameters and lengths.

Fig. 182. Loop Filament

After the gauging process the filaments are cut to the correct length on plates graduated to millimetres; and, to ensure greater accuracy in the case of loop or curl filaments, they are placed over metal knobs of the same diameter as the curls of the filament. Any sharp instrument, such as a knife, may be used for cutting the filaments to the length required. Allowance must be made for the extra length of filament necessary for the mount, and for this from 2 to 4 millimetres will generally be sufficient, but the amount depends on the kind and length of the final mount. The filaments, having now been gauged and cut, are ready for the subsequent process of jointing or mounting them to their conductors or "leading-in" wires.

"Leading-in" Wires and their Arrangement.—When the filaments have been assorted into various diameters, they are cut to certain lengths in accordance with the voltage, candle-power, and efficiency required, as described in the previous section. They are now ready for affixing to the conductors

or "leading-in" wires. Various devices have been tried to cheapen the materials used for this purpose. The substance used must have a high melting-point in order to withstand the heat of the glass-blower's flame. It must also be unoxidizable when in that flame; and, above all, it must have the same coefficient of expansion as the glass through which it passes. The only known material which fulfils all these conditions is platinum; and, to obviate the use of so expensive a metal, various alloys have been tried, but the result of their use has been excessive leakage due to the cracking of the lamps. One method which promised success was to take tubes of glass, painted with precipitated silver and Canada balsam, and heat them to burn-in the silver. Iron, nickel, or copper wires were then inserted in these glass tubes, and an air-tight joint of solder was made between the metal wire and the tubes coated with burnt-in silver. It was found that the expense of these operations was more than that of using short lengths of platinum, and as many improvements have been made in respect of economizing the lengths, the value of substitutes has been considerably discounted. For a 16-candle-power 100-volt lamp, platinum of 0.20 mm. diameter, and of only 3 to 5 mm. in length, can be used. To one end of this wire a length of copper wire is fixed, and to the other end a short length of nickel wire. The platinum section is finally arranged so that it will occupy the centre of that portion of the glass seal through which the conductors or "leading-in" wires convey the current in and out of the evacuated bulb. The platinum and other wires used are first cut up by an automatic machine, working at a high rate of speed, and which can be regulated to cut wires of any desired length or diameter. Machines are also in use which, in addition to cutting the wires, will at the same time tubulate their ends, ready for the reception and subsequent mounting of the filaments. Tubulating the ends of the wires is of great advantage, as it renders the final joint electrically and mechanically more perfect. In pinch-seal lamps the conductors, or "leading-in" wires, are generally made entirely of platinum, which is tubulated at the mount end. In making any form of joint between platinum and glass it is always best to have both substances at the highest possible temperature, and thus to ensure a more thorough flow of glass on to the platinum. By

this means it will be found that a complete junction is secured, capable of withstanding the varying strains subsequently set up by evacuation and the sudden changes of temperature to which the lamp will be subjected.

Jointing or Mounting Process.—There are many methods in use for joining the filament to the "leading-in" wires used to convey the current through the glass globe, and these methods vary according to the kind and size of the lamp to be made. With pinch-seal lamps the filaments are first inserted into the tubulated ends of the platinum wires, then hard carbon is deposited over the juncture of the wire and filament. This operation has been elaborated in great detail in order to ensure constancy of results, expedition, and the safety of the operators. The filament is placed in a machine which allows the current to pass from one wire across one junction, then through a "short-circuiting" piece to the other junction, and so to the other wire, as shown in fig. 183. When the filament is placed in the machine the whole is lowered into the iron benzine receptacle, as depicted in fig. 184. As soon as the joint is well below the surface of the benzine, the current can be sent through. This is done by hand, but an automatic arrangement prevents the circuit being closed until the joint is well below the surface of the benzine. The current generates heat at these joints, and as soon as they reach a white heat benzine is there chemically split up, and its carbon constituent is thrown down or deposited. The joint thus gradually increases in diameter, and to keep up the necessary temperature of decomposition the current has to be increased in the same ratio as the increase in the sectional area of the mount. Bubbles of white gas rise to the surface as long as decomposition is in progress, and when a certain definite quantity of current has flowed, and the bubbles have diminished, the operator knows by previous experience that the joint or mount is finished. It is possible to arrange for the circuit to open automatically when the current passing through the mount reaches the afore-mentioned definite quantity. Instead of using benzine, other hydrocarbon liquids can be employed for jointing, such as petroleum of high flash point (paraffin),

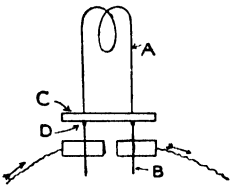


Fig. 183.—Detail of Jointing Apparatus

turpentine, or a mixture of the two. Cotton, rape, or linseed oil can also be used for the purpose, but, generally speaking, benzine is the cleanest, most satisfactory, and rapid method. This process of depositing a joint was invented by Lane-Fox, who used hydrocarbon in the gaseous form. Subsequently

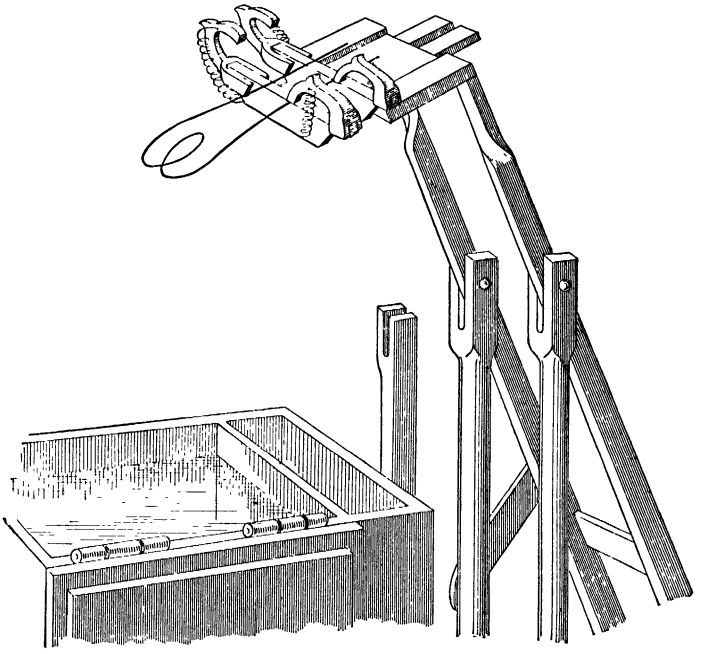


Fig. 184.—Jointing Apparatus, showing benzine receptacle

Swan used liquid hydrocarbon, and this is the process in general use at the present time.

"Flashing" or Depositing. — The process of "flashing" was originally introduced by Sawyer and Mann; but the first to apply it to the manufacture of high-resistance lamps was Lane-Fox, and his method of employing it is the foundation of the various methods in use at the present day. By means of "flashing", the filaments are reduced from their previous state of high and irregular resistance to one in which their resistance is lower but uniform. The process consists of heating the filaments in a hydrocarbon gas or liquid up to the point when their temperature is sufficiently high to chemically resolve the hydrocarbon into its constituents (hydrogen and carbon), the latter

being deposited in a solid form on the surface of the filament where the carbon is nascent. A very important result arising from this process is that, if there be any thin places in the filament, these places will be at the highest temperature; consequently those parts of the filament will decompose the hydrocarbon at a greater rate, and carbon will therefore be deposited on them more rapidly than on other parts. There is thus an automatic levelling up of previously uneven diameters,

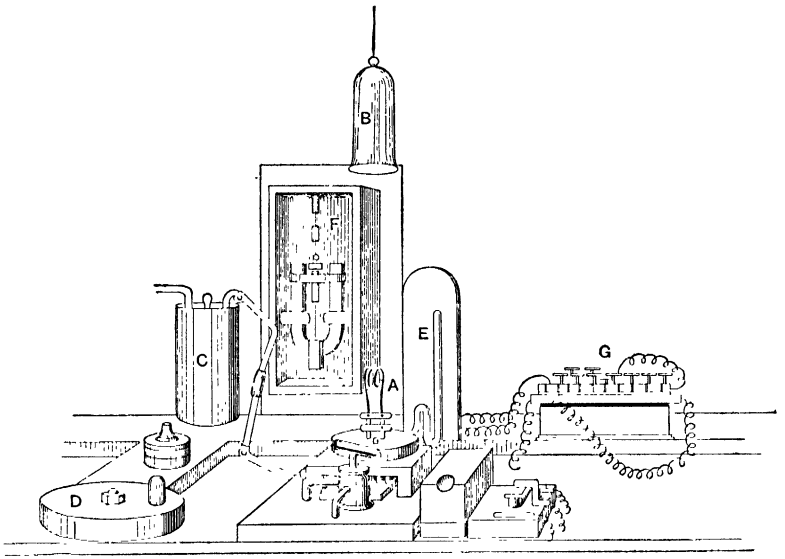


Fig. 185.- Flashing Apparatus

A, Filament in clip. B, Receiver to be placed over A. C, Hydrocarbon generator. D, Current regulator. E, Vacuum gauge. F, Galvanometer. G, Wheatstone bridge.

and finally the filament becomes throughout of even diameter and surface. At the same time the whole section of the filament is rendered more homogeneous by means of the great temperature to which it is raised when surrounded by the comparatively good vacuum used in modern methods of "flashing".

The diagram (fig. 185) illustrates the method of "flashing" generally employed. The filament A is placed in a holder under the receiver of an air pump B, which, after being exhausted of air, is filled with attenuated hydrocarbon vapour, usually obtained from benzine or its derivatives. These operations are controlled by a handle connected to a tap, as shown

in front of the sketch, so designed that a vacuum can first be produced; after which benzine vapour is admitted with or without a vacuum, and then air. These connections are so arranged that there is no possibility of explosion from the ignition of a mixture of benzine vapour and air. The gauge E is connected to the receiver B, and shows when the proper conditions are established in the latter. The filament A is then gradually heated up by the passing of an electric current through it. This current is increased by slow degrees until the filament is at a very high temperature, and capable of decomposing its surrounding atmosphere of hydrocarbon vapour. This decomposition splits the hydrocarbon vapour into its two constituents, carbon and hydrogen; the former is deposited on the white-hot filament and the latter removed by the vacuum pump. Various methods are employed for ascertaining the completion of the "flashing" operation. Some "flash" to a definite hot resistance; others to a definite cold resistance; while a few "flash" to a fixed current value. The method illustrated in fig. 185 is that of "flashing" to a cold resistance. For this a Wheatstone bridge is connected in the usual way to the filament, and when balance is established in a local battery circuit the galvanometer F gives a zero indication. In all the methods it is necessary to have a very steady potential, otherwise there will be variation in either the final ohmic or specific resistance. When the filament has been reduced to a fixed amount the receiver is filled with air and the filament removed. It is now examined and checked for hot or cold resistance, and at this stage it should have a shiny silvery and smooth surface, retaining all its original flexibility and hardness. "Flashing" can also be carried out with paraffin vapour, and with plain or enriched coal gas. On the completion of the "flashing" process the filaments are ready to be sealed into the glass bulbs or globes.

Methods of Glass Blowing.—The manufacture of the glass bulbs, tubes, &c., used in lamp making is in itself an industry. The greater number of glass bulbs used are of one or two standard sizes, and such bulbs are best when made out of pot glass blown in moulds. The method employed is similar to that for bottle blowing. The bulbs when blown are of the shape shown in fig. 186. The long neck is that portion which

was originally attached to the iron blowing rod, from which it is broken off at the end farthest from the bulb. The latter and the large portion of the neck are formed in a mould of carbon or similar heat-resisting material. Round-shaped mould-blown bulbs have advantages over those blown by hand, as naturally they are of more uniform size and shape, and the glass is better distributed over the whole bulb. On the end of such a bulb, shown in fig. 186, will be seen a small protuberance, and the first operation in the lamp works, after cleaning and washing the bulb, is to melt this protuberance in a small blowpipe flame in order to form a small opening into the bulb. This is generally done by heating the extremity of the protuberance and then blowing into the bulb, thus blowing the molten portion through. In the opening thus made is affixed a short length of glass quill tube (fig. 187). This tube is eventually used for sealing or joining the lamps to the vacuum pumps, and is also of great help in the handling of the bulb in the glass-blowing operations. The above operation is known as "stemming", and the next stage in the work is that of sealing-in the filament, which is performed in a variety of ways in different factories. The seals in most common use for this purpose are: (1) The pinch seal; (2) the American or tube seal; (3) the German or vacuum-tube seal.

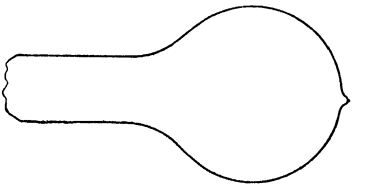


Fig. 186.—Blown Bulb

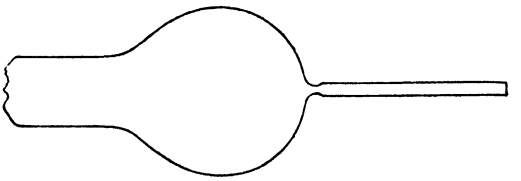


Fig. 187.—Bulb with Glass Quill Tube

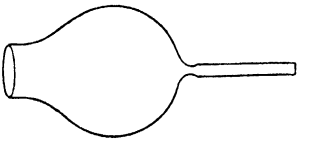


Fig. 188.—Bulb Prepared for Filament

Pinch-seal lamps as generally made have their "leading-in" wires entirely of platinum, but in some factories a compound wire of nickel and platinum is employed. The neck of the bulb is first melted and drawn off, the closed-up end is then reheated and blown away, in order to obtain an opening, which while hot is worked up into the shape shown

in fig. 188. Through this opening the mounted filaments are introduced, the platinum wires are melted on to the opening, and the opening itself is then well heated up and "pinched" together with a flat pair of tongs, in such a way as to close the molten glass well round the platinums. After this the latter are formed into loops and the filaments finally adjusted in the centre of the bulb. The ridge formed by the "pinching" tool enables the lamp to hold better in the cemented caps. The lamps at this stage are placed in annealers and allowed to cool down slowly, in order to avoid the liability to crack.

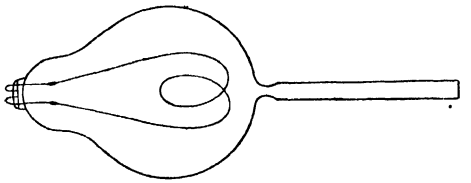


Fig. 189.—Lamp with Filament Mounted

Fig. 189 shows the lamp on the completion of the sealing-in process. The lamps thus "sealed-in", after being carefully examined for cracks and other defects, are ready for the next process, that of exhaustion or pumping.

Methods of Exhaustion.—The methods for exhausting the bulbs in use in different lamp factories are as varied as are the exhaust or vacuum pumps by means of which the process is carried out. There are, however, three principal methods, viz.—

(1st) By mercurial pumps of the Sprengel or Geissler type.
(2nd) By mechanical pumps.
(3rd) By chemical combination.

The actual exhaustion of the lamps is accomplished as follows: The glass stem of the lamp is attached to the glass fork of the pump, by both being melted together by means of a small hand blowpipe. While exhaustion is proceeding, wires are joined to each lamp to enable them to be supplied with a regulated current. At the same time the lamps are heated by air-gas flames, and a small amount of current is sent through the lamps. The additional heat thus produced enables the filament to throw out a large amount of its occluded gases, and as soon as these have been removed by the pump the current is increased, with a similar result as to the evolution of occluded gases; but this time they are evolved in diminished quantity,

this diminution continuing with each increase of current sent through, till, finally, the filaments are brought up to an incandescence far exceeding that at which the lamps will subsequently be used. Voltmeters are employed to control these operations, and when all indications of gas are removed the current is switched off, and the lamps are ready for removal from the pump. The wires are now detached from each lamp, and with a small blowpipe the operator heats the constricted portion of the lamp stem next to the bulb, which renders the glass soft, and, the pressure of the atmosphere immediately collapsing the fine tube at this point, it becomes solid, and the lamp is then lifted away from the fork piece, and any superfluous glass melted off, at which stage the lamp appears as in fig. 190. The next step is for the operator to test each lamp for vacuum by placing it in contact with a pole connected to an induction coil, or similar high-voltage transformer. This process will give a Geissler-tube effect if the vacuum is bad, but will give no indication of glow if the lamp has a good vacuum. No indication of glow might also be caused by the lamp being full of air. But most of these have been sorted out by the operators, who notice that the filaments of such lamps do not tremble when the lamps are shaken; they are finally eliminated by the subsequent running at full voltage, which destroys the filaments in the lamps containing air.

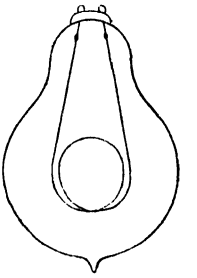


Fig. 190.—Sealed Lamp

Testing.—After exhaustion, the lamps, appearing as in fig. 190, are conveyed to the testing department, where they are first examined for the detection and elimination of defective and broken lamps. The good lamps are then hung on hooks, and are flashed up and down by a weak current to find out if the filaments, mounts, &c., are perfect. After being raised to a high voltage, much in excess of that they will be used for, they are tested for vacuum by the induction-coil test. The lamps then pass to the photometers, where each lamp is compared with a standard lamp, which can be either an incandescent lamp, run at constant voltage, or a standard gas burner, fitted with a Methven screen. These standards are kept in constant calibration with some form of standard lamp, such as the Vernon-

Harcourt Pentane lamp, the Hefner Von Altneck Amyl Acetate lamp, or the Dibdin Pentane Air-gas Burner, of 10 or 16 candle-power, which are all good and most reliable standards from which to keep the sub-standards in calibration.

Photometry.—A photometer is a piece of apparatus for estimating the intensity of the light emitted from a source in a given direction.

In general the estimation is made by comparing the illumination of adjacent surfaces (or adjacent parts of the same surface) receiving light from the unknown source and from a standard source respectively.

The difficulty of obtaining a quantitative determination of intensity of light is considerable, since no method of physical measurement is available. It might be suggested that the chemical action of light on a sensitive salt could be utilized in making the measurement, but photometry deals exclusively with light of wave lengths lying within the region of visibility, and all light-sensitive salts are also affected by radiations of wave lengths lying outside this region. A method depending on the effect of the intensity of the light falling on a crystal of selenium on the electrical resistance of the crystal has been suggested, but so far has not taken a practical form.

In all photometric measurements the result depends on a comparison made by the eye. The accuracy of the determination is therefore influenced by the experience of the operator. The different types of photometer have been designed with a view to making this comparison as simple and definite as possible.

In all cases the complete apparatus consists of the photometer proper (or photometer head as it is sometimes called), a graduated bar forming the axis of the apparatus, a support for holding the test lamp, and a standard lamp directing light of known intensity into the photometer.

Either the photometer head, or one or other of the lamps, is made to move backwards and forwards along the graduated bar as required.

The fundamental principle underlying all photometry is the Law of Inverse Squares, which may be stated as follows:—

The illumination of a given surface varies inversely as the square of its distance from the source of light, the surface being everywhere perpendicular to the rays falling upon it.

Rumford Photometer.—One of the early forms of photometer was that invented by Rumford in 1792. In its simplest form it consisted of a rod casting two shadows on a white screen. The standard and the test lamps were moved until the depth of the two shadows was judged to be the same. The candle-powers of the two lamps were then directly as the squares of their distances from the screen, and the candle-power of the test lamp could be determined.

The Rumford principle is a direct application of the Law of Inverse Squares.

The shadow cast by one lamp is illuminated by the other lamp, and consequently when the shadows are of equal density the illumination due to one lamp is equal to that due to the other lamp. The illuminations being equal, the candle-powers of the lamps must be directly as the squares of their distances from the shadows.

Let us suppose a standard spermaceti candle is being used as the standard, and that the distance of the candle from the screen is l_2. If the densities of the shadows appear to be equal when the test lamp is l_1 from the screen, then the candle-power of the test lamp is

$$\text{candle-power} = \frac{l_1^2}{l_2^2}.$$

Bunsen Photometer.—A reliable form of photometer for practical use is that due to Bunsen. In its modern form the photometer head consists of a blackened box divided into two compartments by a screen of white paper. This screen, with the exception of a small disk in the middle, is rendered translucent by being treated with wax. The photometer head is mounted on a graduated bar, and can be moved in either direction along the bar. In order to make the observation of the illumination of the two sides of the opaque disk simpler, two mirrors, inclined to one another at about 140 degrees, are placed at the back of the box, one on either side of the screen.

Looking into the box from the front, both sides of the spot are seen simultaneously. The position of the photometer head is adjusted until the two images appear to be of equal brilliancy relatively to the surrounding parts of the screen. The ratio of the candle-power of the test lamp to the candle-power of the standard lamp is equal to the ratio of the square of the distance

of the test lamp to the square of the distance of the standard lamp. Referring to fig. 191, S represents the standard lamp, which for the sake of example may be taken to be a 10-candle-power pentane lamp. If the figure represents the conditions for a balance, and the candle-power of the tested lamp is x, then

$$x = 10\frac{l_1^2}{l_2^2}.$$

The bar is usually graduated directly in candle-powers based on a unity standard. Consequently when 10 candle-power standard is being used the scale reading multiplied by 10 represents the actual candle-power.

The making of an efficient screen demands considerable care. Any inequality between the two sides of the disk will affect the

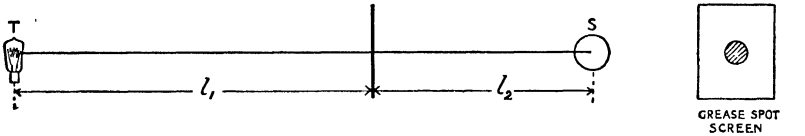


Fig. 191.—Bunsen Photometer

accuracy of the results unless some method of double balancing is employed.

Quite often only the central spot is made translucent, the rest of the screen being left opaque. Sometimes the spot is made star shaped instead of circular. A small spot is generally preferred to a large one. The inside of the photometer head, and also the wooden screens placed in front of the standard and test lamps are painted a dead black, in order to avoid errors due to reflection. The object of the wooden screens is to cut off all the rays from the lamps except those proceeding directly to the photometer head. The eyes of the operator are thus shielded from direct light.

There are many more complicated forms of photometer in use, but in the hands of an experienced tester a Bunsen with a well-made disk will give as accurate results as any.

The room in which a photometer is used should be perfectly light-tight, and the walls should be painted dull black. Furthermore, reflecting surfaces of all kinds must be carefully avoided.

Standards of Light.—The unit of intensity of light is the

"candle-power", and the official standard for the British Isles is the standard candle of spermaceti weighing $\frac{1}{6}$ lb. and burning 120 grains of sperm per hour.

The standard most commonly used in this country is the Vernon-Harcourt or pentane lamp.

10-Candle-power Pentane Standard.—This lamp, which burns a mixture of air and pentane vapour, is shown in outline in fig. 192. The liquid pentane (C_5H_{12}) is contained in a flat tank carried by a bracket at the top of the hollow pillar supporting the lamp.

This tank is fitted with two stopcocks, one of which is fitted with a metal siphon tube, which is connected by a length of rubber tubing to the base of the burner. The lamp is provided with a long metal chimney, the distance between the base of the chimney and the top of the burner being 47 mm. A second chimney encloses the first, and serves as a heater for the air supplied to the burner. The hot air passes up between the chimneys, down the hollow pillar, and thus to the metal casing enclosing the burner. The flame is protected from draughts by a light metal shield.

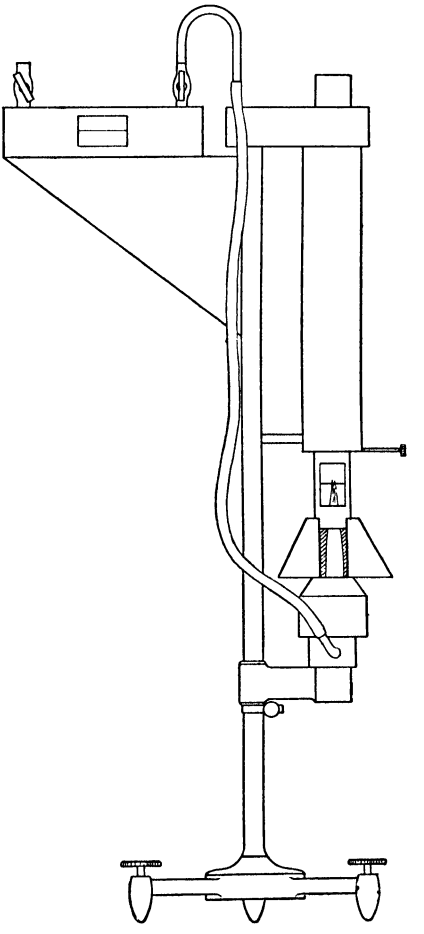


Fig. 192.—10-Candle-power Pentane Standard Lamp

There is a window in the lower part of the inner chimney, and the tip of the flame is adjusted to the cross wire on this window.

When the lamp is to be used, pentane is poured into the tank until it reaches the gauge mark on the inspection window.

The stopcocks are opened, and the heavy vapour falls down the tube and escapes from the ring of holes at the top of the burner. The vapour is ignited, and the height of the flame is regulated by opening and closing the air-inlet stopcock.

The lamp should be lighted a quarter of an hour or so before it is required, and the flame adjusted at intervals, the final adjustment being made immediately before a test is taken.

Methven Screen.—A simple standard for practical use is provided by the Methven screen. The screen consists of a flat plate of metal with an opening covered by a sliding metal plate with a rectangular hole.

Ordinary coal gas is supplied to an Argand burner placed behind the screen. The flame is enclosed by a glass chimney, and is adjusted to the higher of the two gauge wires mounted on the chimney. The light emitted from the slot is equivalent to that of two standard candles.

The small candle-power is a distinct disadvantage, and consequently the Methven screen is not nearly so often used as the 10-candle-power pentane standard.

In the photometry of electric lamps it is generally more convenient to use a high candle-power sub-standard. This sub-standard may be a carbon- or metallic-filament incandescent lamp, which has been carefully standardized for a definite potential difference between its terminals.

The lamp is run at this particular potential difference, and is used in place of a standard.

Dr. J. A. Fleming has introduced a sub-standard incandescent lamp which is said to be constant over fairly long periods. The filament is mounted in an ordinary bulb, and run for 100 hours at a potential difference 5 per cent above the normal. The filament is then removed and mounted in a bulb 6 to 8 in. in diameter. The sub-standard must be restandardized at frequent intervals, and the fixed potential difference between the terminals must be accurately maintained during a test.

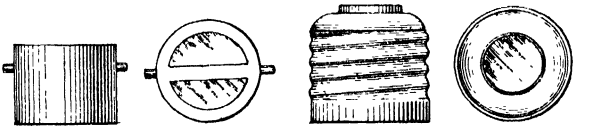
Other Standards.—The German standard is the Hefner amyl acetate lamp. The flame is orange in colour, and is very sensitive to draughts.

The Carcel lamp is the official standard in France.

According to a recent determination the relative values in candle-powers of the standards are as follows:—

Pentane.		Hefner.		Carcel.
10.0	.....	0.9	.....	9.62

Capping, Testing, and Final Operations.—Two types of lamp caps in use are shown in fig. 193. Quite 95 per cent of the caps used in this country are of the bayonet type. In order to fit bayonet caps to the kind of lamp illustrated in fig. 190 short lengths of tinned copper wire are needed. These are soldered at one end to the platinum loops, their other ends being subsequently soldered to the contact plates at the base



B.C. (Bayonet Cap) — No. 2, E.S. (Edison Screw)

Fig. 193.—Incandescent Lamp Caps

of the lamp cap. The ring of the cap having been fixed to the lamp by means of plaster of Paris or other cement, the two contact plates are then soldered to the ends of the copper wire, which have been previously cut off to a suitable length. These two contact plates are placed in position in the brass ring or cap, and the latter is now filled up with cement, which is so levelled off that it remains flush with the contact plates and the edge of the brass ring. With high-voltage lamps it is now becoming general to use "vitrite" caps. These have the brass contact plates and the pins for the bayonet catch embedded in a glass compound which is strong and has an extremely high resistance. As these vitrite caps are shaped to fit the lamp necks they only require a small quantity of cement to fix them to the lamps.

The lamps are then run up to their full brightness and compared together, by which means it is possible to detect any faulty lamps (such as those with uneven or spotty filaments, bad mounts, or unsatisfactory caps) that may have inadvertently been passed. Such faulty lamps must be thrown out, for it is found quite useless to attempt to repair them, as owing to the nature of the glass, &c., they would, if repaired, never be as

reliable as new ones; they are therefore immediately broken up. The lamps, having undergone this further test, are handed over to the cleaners, who clean up the bulbs and caps and remove any superfluous cement. This process being completed, they pass into the hands of the stampers, who place the lamps in heated boxes, and by means of rubber stamps and a fluoric acid compound brand them with the trade name, voltage, and candle-power. The efficiency in watts per candle-power is also generally given either as a number or with equivalent letters. During these operations some faults will have developed, consequently the lamps are again subjected to all the tests previously described, in order to eliminate any that may have become

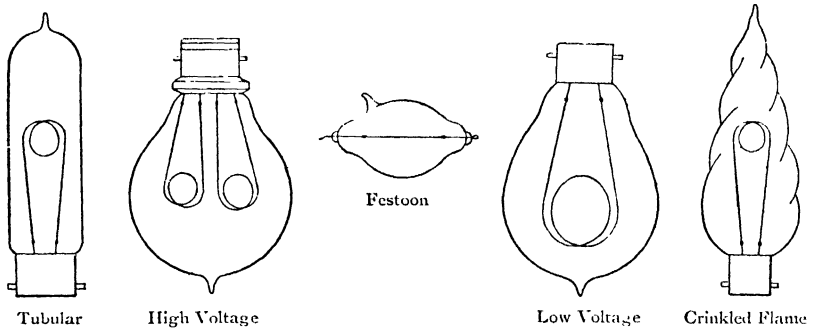


Fig. 194.—Some Incandescent Lamp Patterns

defective through the cleaning and stamping they have undergone. This final testing results in the "survival of the fittest", and the lamps that successfully stand it are ready for packing.

The various operations described above are those in most general use for the production of the usual standard lamps of from 50 to 250 volts and from 5 to 50 candle-power in bulbs of the ordinary standard shape. In addition to these there are hundreds of patterns of lamps differing in both size and shape made for special purposes, and each required at a different voltage, candle-power, and efficiency. A few common forms of these various patterns are to be seen in fig. 194. Other varieties are special lamps in different-coloured glass bulbs, reflector lamps of various shapes, opal and half-opal lamps, and lamps wholly or part obscured. In addition to these there are also made telephone indicator lamps, gun-sighting lamps, ship-lighting and signalling lamps, resistance lamps, radiator

or heating lamps, high candle-power lamps up to 1000 candle-power, and lamps as low as 1½ volts, representing only a fraction of a candle-power. For the manufacture of these many different types of lamps, special apparatus, machines, and tools are required.

The Practical Aspect of the Incandescent Lamp.—The incandescent lamp has many practical advantages over other forms of illumination. Its chief points of excellence are:—

1. It is the only practical illuminant which does not vitiate the atmosphere or alter its constitution in any way.

2. It removes all risk of fire and is the safest known illuminant.

3. It can be instantly lit and extinguished.

4. It provides a large amount of light for the comparatively small space it occupies.

5. It lends itself readily to artistic and decorative effects, and can be used at any inclination and in the most confined spaces.

With incandescent lamps the number of candle-power obtained for a given number of watts (or its "efficiency") may be anything chosen, according to the temperature at which the filament is run. This temperature is proportioned to the volts, and the higher the temperature or "efficiency" the shorter is the life of the lamp. Owing to the varying cost of electrical energy (Board of Trade Units), and the variations in the "useful" life and cost of different lamps, there is a best

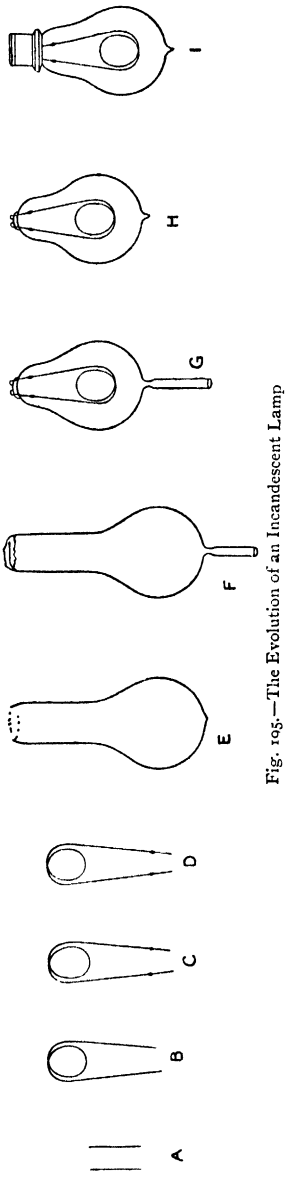


Fig. 195.—The Evolution of an Incandescent Lamp

A, Platinum wires. B, Filament. C, Filament mounted on platinum wires. D, Filament with platinum wires burnt in. E, Bulb as received from furnace. F, With stem attached for exhausting. G, With filament sealed in. H, Lamp exhausted of air. I, Finished lamp.

efficiency at which to run a lamp on any circuit in order to obtain the maximum of economy, i.e. the minimum of operating expenses. When the energy is cheap and the lamps expensive, the efficiency should be low; when the reverse is the case, the lamps should be run at a high efficiency. In practice it has been found that the total operating expenses are at the minimum when the cost of lamp renewals is about 15 per cent of the cost of the initial lamp plus the current consumed. If the renewals cost more than 15 per cent, the lamps are being used at too high an efficiency, and vice versa. The user will generally consider the "useful" life of a lamp as being limited to the period when its original candle-power has not diminished by more than 25 to 30 per cent. This is not a definite limit to the utility of the lamp, but its illumination will then become unsatisfactory for most purposes.

With a perfectly steady supply voltage the behaviour of lamps is quite different from that met with in ordinary practice, and very efficient lamps can be used. The great effect on the efficiency of lamps on unsteady circuits can be seen by the inspection of Table I, which shows the variations of candle-power and efficiency by over-running a 100-volt 16-candle-power 60-watt lamp. From this it will be seen that the efficiency is largely increased by a small increase in voltage.

TABLE I

Voltage.	Candle-power.	Efficiency in Watts per Candle-power.
100	16	3·75
101	16.8	3·62
102	17·7	3·5
103	18.5	3·4
104	19.2	3·3
105	20.1	3·2
107	21.8	3
109	24·3	2.8
112	29	2.5
116	35	2.25
120.5	43·5	2

The appended table shows the effect on the life of a 100-volt lamp by allowing the voltage to increase. The first column

gives the watts per candle-power at the normal voltage (100 volts), the next column gives the average life in hours when the lamps are run off accumulators with a steady voltage. The succeeding columns show the effects of increments of 2 volts on the initial efficiency, and on the life of the lamp at the higher voltages.

TABLE II

Watts per Candle-power.	Life in Hours at 100 Volts.	2 Volts up.		4 Volts up.		6 Volts up.	
(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
4·5	2200	4·1	1700	3·75	1300	3·45	950
4·0	1550	3·65	1150	3·35	850	3·1	660
3·5	1000	3·22	750	3·0	600	2·82	470
3·0	600	2·82	470	2·65	380	2·45	290
2·5	300	2·34	250	2·18	200	2·0	150
2·0	150	1·85	120	1·75	95	—	—

The chief causes of the falling off of candle-power in lamps are (1) alteration of the character of the surface of the filament, and (2) blackening of the bulb. Of these the greatest effect results from the first cause, for with a well-made lamp the deterioration in 1000 hours owing to the blackening of the bulb ought not to be more than 10 per cent of the total deterioration. The deterioration by change of the character of the surface of the filament is owing to the fact that particles of carbon are thrown off, leaving the surface of the filament an aggregation of fine particles of carbon with many elevations and depressions. There is, therefore, a larger radiating surface than was previously presented by the shiny grey surface of the new filament. This rough, blackened, and increased surface causes the filament to be more emissive than when new and smooth, consequently it radiates its heat more easily, and thus keeps cooler and duller although supplied with the same voltage and watts as when new.

Non-carbon Incandescent Electric Lamps.—Among this class those that have been worked out to the greatest extent are:—

1. The electrolytic lamp (of Nernst, &c.).
2. Metallic-filament lamp.
3. The mercury-vapour lamp.

The Electrolytic Lamp.—The main principles of the lamps in this class were first patented by Jablochkoff in 1877. He found that the oxides and nitrates of the earths, which are non-conductors when cold, are good conductors when hot. In his patent he mentions a method of first heating his filament or rod by covering it with a thin layer of carbon, through which a current was passed. This layer became hot, and lowered the resistance of the rod sufficiently to make the latter become a conductor and glow and give off light by incandescence. Professor Nernst, at the end of 1897, went over this ground again, and worked out methods of supplying preliminary heat to the filament rod (or "glower") in order to make it conductive. He and other inventors have introduced various electric devices which have been patented. Improvements have been made in the composition of the glower, the heating circuit, and the mounts and terminals of the lamps. The composition of the filaments or glowers of these electrolytic lamps varies with different makers, but is generally a mixture of one or more of the oxides of zirconium, yttrium, thorium, cerium, &c. These bodies are said to be capable of being raised to a greater temperature than carbon, and without so rapid a change. In any case this is so in air, as carbon would burn. Electrolytic lamps in air are rated at 1.8 to 2 watts per candle-power, the latter being taken with the usual frosted globes off. When these are on, as in practical use, the efficiency falls about 15 to 20 per cent. The diffusion in this class of lamp is not so good as in a carbon lamp, owing to the short, thick rod used as the glower. The actual efficiency in watts per mean spherical candle-power has been found to be about 3 watts per candle-power. The Nernst lamp has become obsolete, owing to the troubles experienced in the mounts, its slowness in lighting up, and general unreliability. There is also the fact that its efficiency is below that of the Tantalum, Osram, and other new metallic-filament lamps.

Metallic-filament Lamps.—The first incandescent lamps were made with metallic filaments: first of iron in air and in vacuo, then of platinum and of platinum iridium in air and in vacuo.

With these metals there was a considerable and a fairly quick change of their structure as the current passed through them, and they were very inconstant in their results.

In 1878 and 1879 Lane Fox and Edison were working with platinum, platinum iridium, and platinum alloys.

The greatest drawback with these metals was that, in order to get anything like economical results, they had to be run very close to their melting-points, and when metals are kept at such a high temperature they change rapidly in resistance and structure.

In 1880 and 1881 considerable advance was made in obtaining carbons of small diameter and even structure. Carbon possesses considerable advantages over platinum and its alloys, as it has a melting-point stated to be about 3600° C., whereas that of platinum is about half this value, and tungsten about 2950° C. Carbon also has a very important advantage, owing to its high specific resistance, which enables one to obtain small candle-powers in quite a small space, even with voltages of 250 volts. With metals, however, their low specific resistance makes it necessary to employ thinner and longer lengths than with carbon.

The advisability of finding bodies with higher melting-points than platinum or its alloys was fully recognized as far back as 1878 by Lane Fox and others, and even earlier than this; but the then impracticability of obtaining such metals in a ductile form prevented any further progress.

The methods now used for obtaining tungsten in a "wire-like" form were evolved slowly, but the chief lines of progress were those invented by Auer von Welsbach for his osmium filaments. It is on the almost identical lines he used for osmium that the present-day tungsten lamps are manufactured.

With tantalum, however, we have made a distinct progress in conquering the obstacles presented to previous inventors—that is, to obtain a metal of higher infusibility than platinum and its alloys, and yet sufficiently ductile to enable one to obtain wires thin enough for small units of candle-power.

The Osmium Lamp.—A lamp of this type, giving an efficiency of 1.5 watts per candle-power, was introduced by Auer von Welsbach.

These lamps were exceedingly fragile, and were not transportable in a practical way. The highest voltage made was 35 to 45 volts. Owing to the great fragility and cost of production of these filaments it was necessary to look round for

another body possessing the great infusibility of osmium together with increased mechanical strength. In this direction investigations were carried out by most of the larger firms, and one of the first lamps to appear that got over this question of fragility was the Tantalum lamp.

The Tantalum Lamp.—This has been brought to a great state of perfection in its manufacturing details chiefly by Von. Bolton (chemist to Messrs. Siemens & Halske). They are run at an efficiency of 1.7 watts per candle-power.

Tantalum, as an element, was discovered and investigated by Ekelberg in 1802, but it was extremely difficult to prepare, and the pure metal in a ductile form has only been known during the past few years. It has a specific gravity of from 16.8 to 17.0, and its melting-point is between 2250° C. and 2300° C. The chief sources from which it is prepared are the ores called "Tantalite", "Ferrotantalite", and "Manganotantalite". These mineral ores are very widely distributed, and are chiefly found in Australia, Norway and Sweden, North America, and Cornwall. "Manganotantalite" is insoluble in sulphuric, nitric, and hydrochloric acids, but is completely soluble in hydrofluoric acid in the cold. The mixed pentoxides of tantalum and niobium are then precipitated by ammonium chloride or carbon dioxide. This precipitate is treated with hot solution of acid potassium fluoride, which produces the mixed potassium fluotantalate and fluoniobate. The latter salt is seventeen times more soluble in water than the former, and by mixing with the proper proportions of water they can be separated by crystallization.

The pentoxide of tantalum is next prepared from the potassium fluotantalate residue by treatment with sulphuric acid, excess of which is subsequently removed with ammonium carbonate. This oxide is pressed into a small button, and placed in an electric furnace so arranged that a vacuum can be maintained by an air pump during the melting operation. By this means the oxygen is removed as fast as it is given off. On cooling, a button of metal remains, which is again melted in vacuo, and finally the pure metal, of specific gravity 16.8 to 17.0, is produced, the heat used being about 2300° C. This metal can be hammered out, and by means of diamond dies can be drawn out into wires of extreme thinness. For a 110-volt,

20 candle-power lamp the filament is 0.05 mm. diameter, and is 650 mm. long. Although a comparatively dear metal to prepare, yet as 1 lb. weight is about sufficient for 25,000 lamps of 25 candle-power, 100 volts, it is not so seriously dear as might at first be thought.

The Tungsten Lamp.—Dr. Just and R. Falk, in their endeavours to obtain a self-lighting Nernst burner, used rare earth oxides and tungsten, and in these investigations they seem to have come across the adaptability of tungsten by itself as a means of obtaining incandescence, and without associating it with any other bodies. Dr. Just, in alliance with Franz Hanamann, appears to be the first to describe a practical method of making pure tungsten filaments. The methods covered by them are among the most successful practised to-day, and, working very much on the same lines as Auer von Welsbach had done with his osmium filaments, they worked the process of depositing tungsten on thin carbon filaments by heating the latter in the presence of chloride or oxichloride of tungsten. They also describe the method of ridding the filament of its carbon by raising it to a high temperature in damp hydrogen. Filaments prepared in this way are successfully manufactured to-day, and appear to be excellent in every way. They have an efficiency of about 1 watt per candle-power.

Tungsten, owing to its great conductivity, has to be manufactured of very small cross section and of great length, in order to give sufficient resistance on present-day circuits of 100 volts and upwards. Filaments of about .04 mm. diameter are required in order to give lamps of 32 candle-power at 100 volts.

To Dr. Just and F. Hanamann must be ascribed the credit of being the first persons to make a distinct claim for manufacturing an incandescent body suitable for incandescent electric lamps, consisting of tungstenate molybden, or an alloy of the two metals, as mentioned in their first claim. In their second claim they cover the production of tungsten filaments, produced by mixing these bodies with organic binding media. The latter method has been brought to a successful manufacturing stage, and is carried on to-day by the Deutsche Gasgluhlicht Aktiengesellschaft, who take the oxide of the metal, suitably prepared and reduced to the metallic state by hydrogen, together with

a slight amount of organic binding material, and squirt this mixture by means of a press through dies or fine jets.

These filaments, after drying, are subjected to heat in the presence of hydrogen to render the filament slightly conductive. Then a current is passed through them whilst surrounded with hydrogen, and the filament then sinters up, as described by Just and Hanamann, and the carbon is thus got rid of, and only the metal remains. It is by various modifications in the process just described that the well-known Osram lamp is manufactured.

Another great advantage that tungsten possesses over carbon in everyday use is the fact that lamps can be manufactured to far closer limits of wattage; also the metal filament possesses the property of but slight change in resistance with large variation of current, consequently there is less danger of them being destroyed, as compared with carbon filaments, on circuits with large variations of voltage.

TABLE SHOWING PERCENTAGE EFFECT ON VOLTAGE AND CANDLE-POWER BY OVERRUNNING AND UNDERRUNNING AN "OSRAM" LAMP OF 100 VOLTS, 56 WATTS, 56 CANDLE-POWER, 1.0 WATT PER CANDLE-POWER.

Per cent Volts.	Per cent Candle-power.	Per cent Volts.	Per cent Candle-power.	Per cent Volts.	Per cent Candle-power.
−15	−43	± 0	± 0	+40	+234
−13½	−38	+ 5	+ 16	+50	+315
−10	−30	+10	+ 37	+60	+422
− 9	−27	+15	+ 68	+70	+547
− 5	−16	+20	+ 90	+80	+665
− 4½	−15½	+30	+156	+90	+728

The table shows the effect of increasing and decreasing the voltage when applied to a tungsten filament. It will be seen that these figures, compared with carbon, show a very different curve for candle-power and watts, with different percentages of overrunning and underrunning.

Beyond the manufacture of the tungsten filament there are many points of difficulty in the manufacture of lamps with such filaments. Chief among these is the question of a suitable joint between the filament and its conducting wires. In this respect the most successful method appears to be that employed by Auer von Welsbach with his osmium filaments, which he attached to the metal conductors by fusing up the latter into

small beads which embraced the filament, and so produced a strong and perfect electric joint. This fusing is done by temporarily attaching the filaments to the metallic wires at a point slightly distant from the ends of the latter. The whole arrangement is then put in an atmosphere of hydrogen, and a small arc is struck on to the ends of the wires, which causes them to fuse up into a bead which runs back until it meets the filament temporarily attached to same. The arc is stopped as soon as this bead surrounds the filament. Other methods of jointing are also employed by using pastes composed of carbon, also of tungsten and other bodies, but none of these appears to be so simple and perfect as the Auer process just described.

Further difficulties with these lamps have been met in relation to the supporting of the filaments. Some supports have a tendency to adhere to or to react on the filament, and, owing to the soft state of the latter when incandescent, the sagging produced has usually led to the destruction of the lamp. This trouble has been got over in various ways, among which the chief are the use of oxides of the rare earths, coated or mixed with carbon, by using carbon only, and also by depositing tungsten on a metallic wire support, or by using platinum-iridium wire supports.

In the manufacture of the metallic-filament lamp a great deal of labour and cost is expended in the preparation of the mounts which have to carry so many filaments as compared with the carbon lamp. The glass-blowing of these parts is done by mechanical means. The wires of these mounts are shaped by machines, and after the filaments are affixed thereto they are ready to be introduced, or sealed, into the bulbs in the same way as with carbon lamps. The fixing of the exhaust tube to the bulb is done mechanically, in the same way as in carbon lamps. The final treatment of the lamps for exhaustion only differs from the carbon lamp to a slight extent, but a still further difficulty has been in obtaining sufficiently good vacuum for these lamps. In this respect they appear to be much more sensitive than carbon, as a slight deterioration of vacuum in the latter class of lamp is not so detrimental to its life as in the case of tungsten filaments. There have been many methods of obtaining a vacuum which have been recently introduced, in order to obtain a sufficiently good vacuum together with a process

which was reasonably quick as compared to the well-known, but rather slow, method of using the Sprengel pump. A quicker method of exhausting is that known as chemical exhaustion. Another method that is at present in use, and gives fairly good results, is by means of the "Gaede" mercury pump, which is a mechanically driven pump of very rapid action, but rather liable to breakage, as some of its parts are very fragile.

It is questionable, as regards the exhaustion of metallic-filament lamps, whether any improvement will be made on the oldfashioned method with the Sprengel pump, but a good deal of work is being done in order to obtain a more rapid process, and nothing definite can be yet stated on this point. The chief additional process is that known as *running*. It is found that when the lamps leave the pumps there is a continual rise in candle-power, accompanied with a decreased resistance of the filament, and to compensate for this the lamps have to be run in order that their resistance shall be brought to a steady point. For this purpose lamps are run at varying periods from 5 to 30 hours, according to the percentage of overrunning and treatment they have been subjected to. It is not until the lamps are so overrun and steadied that they are classified in the photometers. It is found that the steadying process is very effective, as a lamp classified, for instance, as 100 volts 32 candle-power, at 1 to $1\frac{1}{4}$ watts per candle-power, will remain very constant, and at the end of 1000 hours will not have dropped more than 3 or 4 per cent from its original candle-power.

In general use the metallic-filament lamps have largely displaced the carbon-filament lamp for shop lighting, street lighting, and illumination of large areas of any kind, and there is but little doubt that they will in a short time displace all the smaller arc lamps at present so much in vogue. As there is every possibility of being able to make metallic-filament lamps of a wide range of candle-power, then, as well as displacing the smaller arc lamps, they will also displace the larger-sized incandescent carbon lamps, and eventually may even displace the largest sizes of ordinary arc lamps.

The metallic-filament lamp has been of great assistance in cheapening the cost of small isolated plants, such as in country houses, &c. It is found by using this type of lamp, of about 25 volts, that the number of accumulators can be cut down to

one-half, one-third, or even a fourth of the number at present used with carbon lamps, consequently the cost of such an installation is brought down to a figure which puts it into competition with various kinds of gas-lighting which (previously) largely filled this field.

Fig. 196 shows the variation of resistance, and fig. 197 the variation of watts per candle-power with variation of terminal

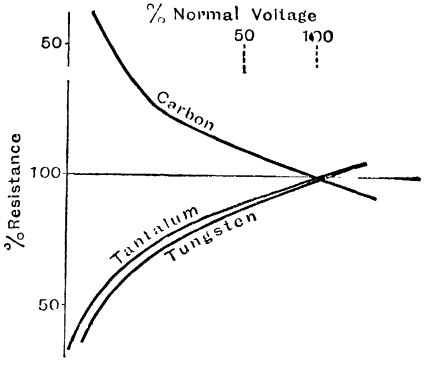


Fig. 196

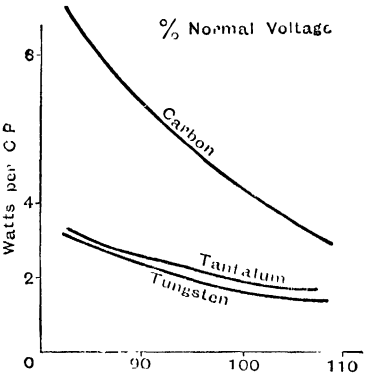


Fig. 197

voltage for incandescent lamps of carbon, tantalum, and tungsten respectively.

Tungsten-wire Lamps.—The problem rendering the brittle metal tungsten ductile is as old as the tungsten lamp itself, but it was not until early in 1911 that commercial tungsten lamps were produced having an incandescent body which was no longer composed of a certain number of separate filaments, but consisted of a single continuous piece of drawn tungsten *wire*.

The old tungsten-filament lamp had already almost superseded the majority of other incandescent lamps owing to its extraordinarily favourable economy in spite of the comparative brittleness and fragility of its filaments. The invention of the tungsten wire established the final superiority of this type of incandescent lamp, especially over the carbon-filament lamp, although the strength of the filament, combined with the low selling price of the carbon lamp, had deceived many a consumer as to the disadvantage of its large consumption of current, even long after the introduction of the tungsten lamp.

It was known that the tungsten filament, brittle at ordinary

temperature, became soft and flexible at the high temperature of operation in the lamp, but that it lost this quality again so soon as it had cooled down. Coolidge discovered that it had the property of softening even below red heat, and he was able to work tungsten at relatively low temperatures. He further discovered a process by which it is possible to render tungsten ductile by a mechanical treatment of the material in the heated state, and to also give it a permanent ductility whilst in the cold state. According to this process, pure tungsten powder is pressed into rods, then rendered coherent by heating, hammered whilst hot, and finally drawn out into wire through diamond dies.

Lamps manufactured from such wire show not only the great economy, the long life and constancy in candle-power, of the old tungsten-filament lamps, together with their greater steadiness under fluctuations of the voltage, but, moreover, are equal in strength to the carbon-filament lamps.

A further important success was attained after the discovery of this process, namely, the manufacturing of incandescent bodies of a considerably finer diameter than was possible from the old squirting process, so that tungsten lamps of 16 candle-power and even of 10 candle-power are now obtainable for pressures of 200 volts and upwards.

The mounting of the wire simply consists of it being continuously wound zigzag shape from a reel round a carrier of glass which has at the top and bottom a certain number of metal supports arranged in the shape of a star, and insulated by the central glass rod of the carrier. One star consists of strong and stiff material, whereas the second consists of very fine refractory wires, as, for instance, molybdenum used in the Osram lamps of the present time. These fine supports, acting as springs, keep the incandescent wire of the lamp securely in its original shape, in whatever position the lamp burns.

The completion of the wire lamp as to sealing-in and exhaustion is mainly the same as described for the squirted-filament lamp.

The tungsten lamp has been the means of introducing a vast extension of electric lighting and popularizing of electricity generally. It has thereby rendered the greatest service to the development of the whole electrical industry.

The Mercury-vapour Lamp.—This type of lamp consists of a vacuum tube with mercury as one of its poles. Such lamps were experimentally made many years ago, and the same results are occasionally obtained when testing mercurially pumped carbon lamps on a coil to ascertain the degree of evacuation. If mercury accidentally obtains access to the lamp bulb and the vacuum be poor, a fairly strong bluish light is obtained by passing the current from an induction coil through the evacuated space. The light given by them is of a most disagreeable colour, and gives an unexpected and peculiar colour to surrounding objects quite irrespective of their natural colour. Thus reds appear sometimes as greens, &c., and on this account such lamps can have only very restricted use, as for tunnel illumination, and drawing in black and white, &c.

Westinghouse Silica and Cooper-Hewitt Mercury-vapour Lamps.—The "Cooper-Hewitt" lamp as originally introduced, consists essentially of a high-vacuum glass tube having a mercury electrode at one end, and an iron electrode at the other. The current is conducted through the glass by means of platinum wires connected to the electrodes above referred to.

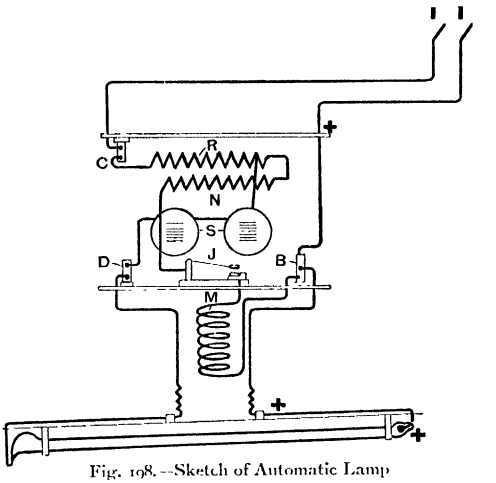


Fig. 198.—Sketch of Automatic Lamp

The light is due to the incandescence of mercury vapour between the electrodes in the form of a luminous column filling the tube. The light is emitted in uniform intensity from every part of the tube.

To light the lamp it is necessary to tilt the tube; this may be done by hand or automatically. Fig. 198 is a sketch of an automatic lamp. On switching on, the current flows through the tilting solenoid M, through the cut-out J, through the substitutional resistance N, and through the series resistance R back to line. The current flowing in solenoid M tilts the tube.

and a parallel circuit is provided through the mercury which flows in a continuous stream between the electrodes. This current flowing through the inductive resistance S in series with the tube operates the cut-out J, and the tube returns to its normal position by gravity.

The lamp is well adapted for buildings where well-diffused illumination is required. Where the characteristic colour of the light is a disadvantage, red rays may be added by means of a light-transforming reflector, which renders the colour of the light normal. The reflector takes advantage of the phenomenon of fluorescence, altering the wave length of certain light falling on it, a proportion of this being reflected as red. Certain invisible rays are reflected as visible, and these in a large degree compensate for reflector absorption.

Silica lamps differ from Cooper-Hewitt lamps in the respect that the tube is of transparent quartz which withstands the high temperature at which the lamp is operated. The tube is comparatively short, and is placed in an enclosed globe, giving the lamp the general appearance of an ordinary arc lamp. The globe is necessary for the proper operation of the lamp, and to cut off the ultra-violet rays in the mercury spectrum which the transparent quartz allows to pass. The lamp is lighted by the tilting of the tube in the manner already described.

The light is well diffused, of an agreeable white colour, and is admirably suitable for lighting workshops, docks, railway yards, and depots, &c. Being always connected in parallel, the lamps are quite independent. No attention is necessary during operation. The average life of the burner is from 2000 to 3000 hours. The lamps are made in various sizes, from 800 to 3000 candle-power, for continuous current from 100 to 500 volts.

Moore Light.—This is not really a "mercury"-vapour lamp, but a Geissler tube giving light by incandescence of gases in a partial vacuum. This is a class of lamp which appears to be slowly making headway, and is very promising where large spaces require illumination. By this method of lighting, efficiencies are obtained between 1.35 and 1.75 watt per candle-power, according to the gas with which the tube is partly filled.

By arranging the kind of gas used one can obtain various colour effects; for instance, white, yellow, blue, red, or any intermediate shade. This form of illumination is apparently

perfectly constant as to efficiency and candle-power after several thousand hours' burning.

It has the drawback, however, that it is only applicable for the illumination of large spaces, or places so situated that one can obtain a continuous run of the tube. The latter is generally made up in continuous lengths up to about 200 ft., but the two ends must be brought to near the same point.

This form of lighting amounts to a Geissler tube, with arrangements for keeping the gas pressure constant. Moore does this by means of an automatic regulating valve, which regulates the entry of the gas into the tube in accordance with the current strength, the current varying directly with the pressure of the gas in the tube. It was found that the pressure of the gas decreased during the passage of the current through it, and if this were not compensated for, as previously described, the light would be seriously diminished, and would eventually entirely disappear.

CHAPTER XX

ELECTRIC MAINS AND DISTRIBUTION

Dielectrics and Armourings.—The dielectric material used as an insulating sheathing in connection with electric conductors may be classified under two headings, viz.:

Material which is hygroscopic.
,, ,, non-hygroscopic.

The hygroscopic, or porous materials, chiefly used are jute and paper.

The non-hygroscopic, or waterproof materials, are rubber and vulcanized bitumen.

Cables insulated with materials which will absorb moisture necessarily must be covered with a sheathing of non-hygroscopic material. This is effected by passing the insulated cable through a press containing lead at a temperature just below melting-point, the lead being pressed through dies on to the surface of the cable insulation, and covering it with a solid

tubular lead sheath. As an alternative to lead the dielectric may be sheathed with rubber or bitumen, the rubber or bitumen being then put through a vulcanizing process.

Manila is generally considered the best class of paper for dielectric purposes. It is the custom to impregnate both this and jute, which is also used as a dielectric, with an oil or bituminous compound to enhance the insulating qualities, such a preparation tending to render the material more pliable, and to prevent it from absorbing moisture.

Considerable quantities of moisture are found in both paper and jute in their manufactured state, and when supposed to be suitable for insulating purposes, and some of the more prominent cable manufacturers have now erected plants for abstracting moisture from this class of dielectric when in position on the stranded conductor. The drums of insulated conductors are placed in exhaust chambers under a vacuum, and after a short time the moisture may be seen to drip in beads from the cable dielectric. The withdrawal of moisture is considered by some engineers to tend to render the dielectric less pliable and less capable of withstanding the disruptive strain when an alternating pressure test at high potential is applied to the cable. It, however, increases the insulation resistance of the dielectric.

Many engineers consider it more desirable that the dielectrics should be of such a nature that they will withstand high disruptive strains rather than give a high insulation resistance, and this is a matter which necessarily is greatly influenced by the potential at which the conductors are supposed to be worked.

Most of the bitumen used for insulating purposes is obtained from the Trinidad bitumen lakes, and goes through many refining processes until absolutely free from all impurities. It is then worked up into a plastic mass and sulphur is added, and it is applied at a warm temperature to the copper-stranded conductors (which are first served with a covering of yarn or paper). It is then forced through dies, through which the conductor is made to pass, the thickness of the bitumen covering being regulated by the gauge of the die, the sheathing being solidified by passing the conductor through a long trough of water before winding it on to a drum. When the conductor has been covered in the manner described the drum containing it is removed to one of the curing rooms, where the curing process

proceeds until the sheathing comes into a condition in which it is known as vulcanized bitumen.

The best class of rubber for insulating purposes is that obtained from Brazil, and is known as Para. This is mechanically treated, and cut into thin strip, which strip is wound next to the copper conductors, and forms the separator between the tinned copper and the outer covering of vulcanized rubber. This latter is made from a different quality of rubber, which is first refined and is then mixed with various compounds. With this mass a certain quantity of sulphur is mixed to form the vulcanizing agent. The copper conductor with its covering of Para rubber is then covered with this unvulcanized compound, which is put into curing chambers and subjected to a relatively high temperature for a certain length of time. The time and degree of this exposure depends somewhat upon the composition of the rubber compound.

Rubber of good quality, on being burned, should leave nothing but a small white residue of soft ash. Poor classes of rubber will not only leave an ash of practically the same dimensions in bulk as before burning, but the ash will be of a hard substance resembling plaster of Paris. For making a simple and effective rough-and-ready test of the insulating qualities of rubber the above process is one of the best, as it can be carried out anywhere and without special appliances.

In vulcanized-rubber insulations one weakness to be guarded against is the perforations that may in time become apparent, owing to the sulphur used in vulcanizing forming crystals in the vulcanizing process and dissolving out when the cable is exposed to moisture or chemicals when in use under the ground. These crystals, which are the effect of unequal mixing, or perhaps the presence of excess of sulphur in the manufacturing process, are formed during the vulcanizing operations, and as they may take time and certain conditions to cause them to dissolve, the cable may successfully stand the usual tests carried out at the manufacturer's works, and be passed out as in perfect condition. Modern appliances and experience have, however, reduced this risk to a minimum.

Where electric conductors are laid directly in the ground, and not drawn into ducts or laid in troughs, the armouring or mechanical protection usually provided consists of—

1. A double layer of steel tape, one layer overlapping the other, so that there is no space exposed between the joints (see fig. 199).

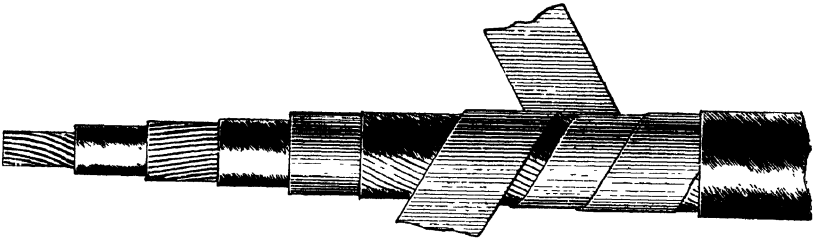


Fig. 199.—Steel-tape Armoured Cable

2. Round steel wires (see fig. 200).

3. A special segmental wire, shaped so that as the wires are wound round the cable each is locked into the side of the other,

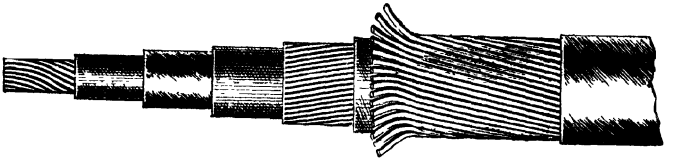


Fig. 200.—Steel-wire Armoured Cable

this style of steel sheathing being known as "lock-coil armouring" (fig. 201).

In cases of non-hygroscopic insulations either of the three above-mentioned forms of steel sheathings may be applied directly on the top of the insulation, which, however, is first served with a strong tape impregnated with insulating compound.

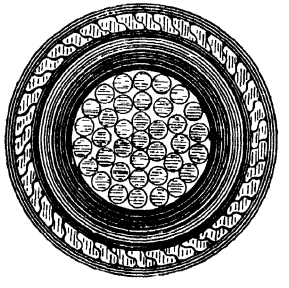


Fig. 201.—Lock-coil Steel Armoured Cable

But where the insulation consists of a hygroscopic material it will be either necessary to sheathe it with rubber or vulcanized bitumen before steel-armouring, or to lead-sheathe the insulation, and then apply the steel armouring on the top of the lead, which is first taped over or covered with yarn.

Systems of Underground Mains.—1. "Draw-in System."—Conductors with insulated sheathings drawn into ducts.

2. "Armoured Cable System."—Conductors with an insulated sheathing and steel armouring laid directly in the ground.

3. "Solid System."—Conductors with an insulated sheathing laid in troughs, and surrounded with a filling of pitch or bitumen compound.

Draw-in System.—The advantages claimed for draw-in systems are that when once the ducts have been laid, repairs or

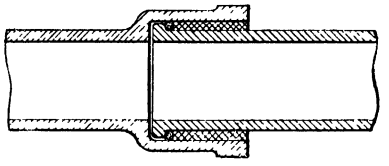


Fig. 202.—Caulked Lead Joint

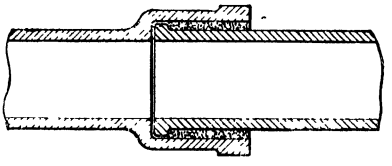


Fig. 203.—Caulked Yarn and Red-lead Joint

additions to the system may be made without the necessity for excavating the streets, excavations and reinstatements being a very heavy item in the cost of underground work. Cables, however, under such conditions are much more liable to the destructive effects produced by moisture, which may be found to have percolated through some defective joint in the ducts, or be produced by condensation.

Moreover, the ducts are liable to be charged with explosive gases obtained either from the subsoil, which in most towns is charged to a certain extent with coal gas through leaks in the gas-supply mains, or from the effects of leaks on the electric conductors themselves, the insulation of which may give off gas of an explosive nature. It is therefore necessary to consider the question of the careful levelling, draining, and ventilating of ducts used on a draw-in system, and this adds to the expense of the system, and in a measure counterbalances its economy in effecting a reduction of excavations and reinstatements.

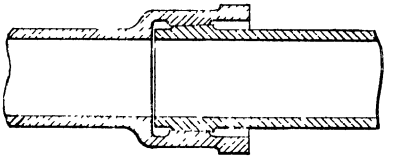


Fig. 204.—Turned and Bored Joint

The most common forms of duct joints in a cast-iron-pipe system are:—

1. Caulked lead joint (fig. 202).
2. Caulked yarn and red-lead joint (fig. 203).
3. Turned and bored joint (fig. 204).

Fig. 205 illustrates a section of cast-iron-pipe draw-in system with brick pit, and manhole frame and cover. Earthenware ducts can be manufactured so that several separate ways may be contained in one section of duct, as illustrated in fig. 206; and this is an advantage in comparison with cast iron, where,

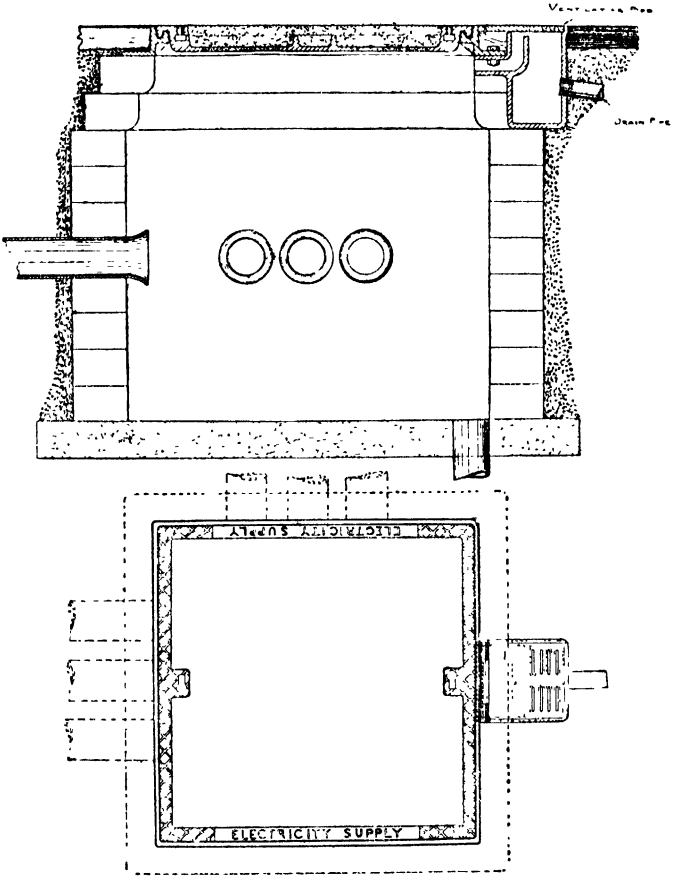


Fig. 205.—Brick Pit Manhole with Frame and Cover

owing to the difficulties in casting, it is not found convenient to manufacture material in this manner. Cast iron, however, is mechanically stronger than either of the other two forms of ducts.

The earthenware duct joint (fig. 207) is specially worthy of notice, as it is designed on a ball-and-socket principle, with a

view to allowing for the shifting of the ducts due to subsidence of the subsoil—a very common occurrence where streets are frequently disturbed and heavy traffic is predominant.

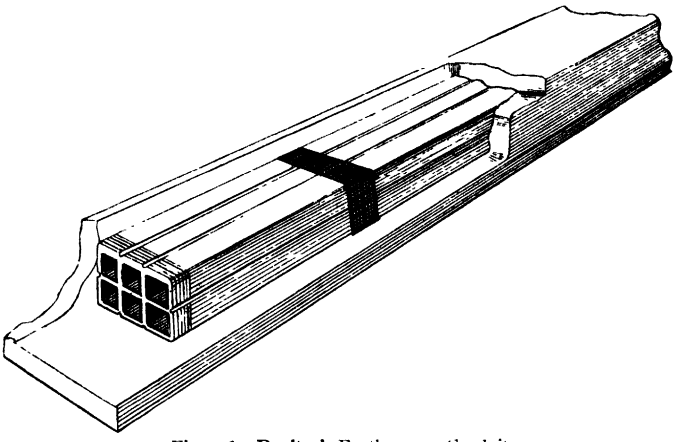


Fig. 206.—Doulton's Earthenware Conduit

These conduits are arranged with circular ducts, which admit of the cables being drawn in easily, and offer little resistance

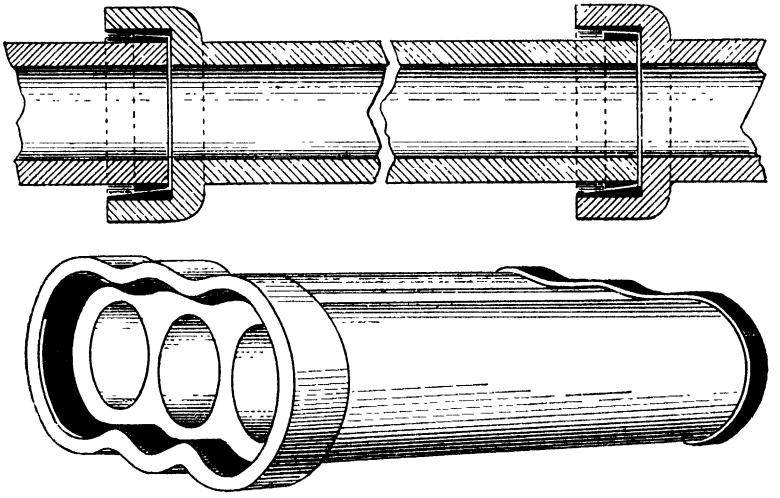


Fig. 207.—Doulton's Patent Flexible-jointed Duct

and consequently little risk of damage to the cables; they are made with spigot-and-socket ends, and fitted with self-centring joints, whereby a perfect joint is secured; the composition

linings are fitted to secure absolute alignment without the aid of mandrels; the patent joints admit of the conduits being laid rapidly; there is no cement used for jointing, so that the ground can be filled in at once. The conduits are made in "stoneware", which will resist all acids; it is non-porous and close in texture.

Armoured Cables Laid Direct in the Ground.—The easiest form of mains laying consists in the burying of armoured cables

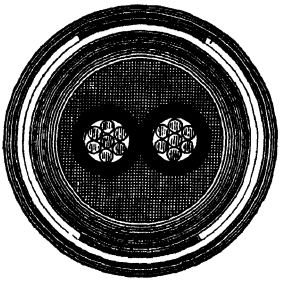


Fig. 208.—Twin or Two-core Cable

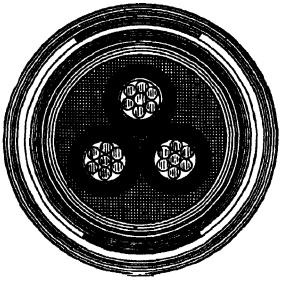


Fig. 209.—Three-core Cable

directly in the ground. The work can be carried out rapidly in fine or wet weather, and can be suspended at any time, and with the least amount of inconvenience in the public thoroughfares. This is not the case with other systems, in which all the work of armouring or providing mechanical protection for the cables is carried out in the trench, and necessitates longer time for operations, favourable conditions of weather, and greater lengths of street openings. In laying armoured cables, trenches may be opened for a few yards in length, the cable payed off from the drum, and the ground immediately filled in as the cable is laid, the drum following the trench opening. It is usual to allow a 15-in.-deep trench under footways and 30-in.-deep trench in roadways, and to cover the cable with either metallic blue bricks or lengths of wood steeped in a preservative free from acid or corrosives liable to damage the cable armouring. The thickness of the bricks or wood should not be less than $1\frac{1}{2}$ in., and wood is preferable as offering a greater resistance to a blow from a pick, shovel, or pinch than a tile.

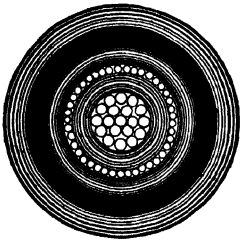


Fig. 210.—Concentric Cable

In the early days of two- and three-wire systems of distribution it was the usual custom to lay two or three separate conductors, but latterly armoured cables have been laid, consisting of two- or three-core or concentric and triple concentric cables, as illustrated in figs. 208, 209, 210, 211.

Solid System.—The solid system of laying mains, as originated by the Callender Cable and Construction Company, is a method which is daily becoming more popular.

The two chief reasons for this are that—

1. The armouring, when effected by laying cables on the solid system, forms a better and more mechanical protection.

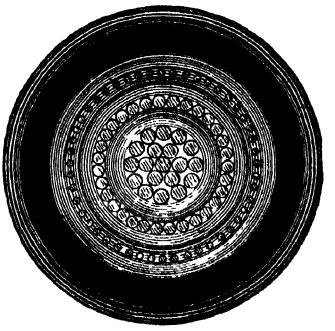


Fig. 211.—Triple Concentric Cable

2. The cable, being surrounded by a non-hygroscopic filling, is not only well protected from corrosive effects due to chemicals found in moist subsoils, but it is also insulated, and thereby safe from electrolytic action, which may be caused by currents flowing between different points of the earth.

Serious damage is known to have been caused to metal-sheathed cables from this source, the importance of which has led to the more prevalent use of the solid system, which is a sure safeguard against electrolytic effects caused by stray currents.

The troughs used consist of iron, earthenware, or wood, or a combination consisting of asphalt-lined iron troughing. Comparing the qualities of the three materials, iron is the most costly, even when of thin section (usually $\frac{3}{16}$ in. thick), but is not considered as durable as earthenware or wood.

Earthenware cannot be obtained in greater lengths than 3 ft., owing to the difficulties met with in manufacture, and therefore requires double the number of joints necessary with iron and five times the number required with wood.

Wood can be obtained in lengths of from 12 to 16 ft., and therefore claims an advantage over the other materials as requiring less joints.

Well-made earthenware is practically everlasting, but troughs

made of well-seasoned deal, and impregnated with preservative tar or oil, are very durable and easily manipulated, besides offering a great resistance to implements used for street excavating. Great care should be observed to ensure that the tar or oil used for preserving the wood is free from acid, as some of the creosoting processes entail the use of oil containing a high percentage of acid detrimental to cable sheathings, and such oils should on no account be used.

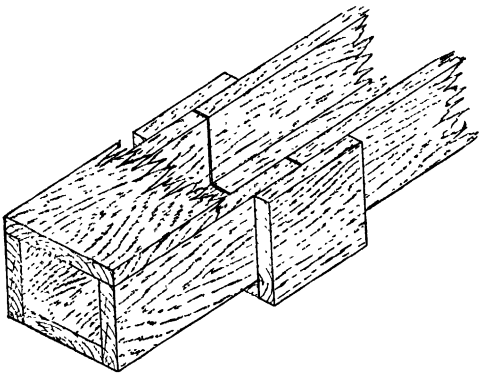


Fig. 212.—Wood Trough for Solid System

The filling of the troughs is usually carried out with refined bitumen or the best coal-tar pitch, rendered plastic by means of the introduction of a certain quantity of shale oil.

Trinidad bitumen is the most satisfactory of materials to use as a filling for the solid system, as in the semi-refined condition in which it is supplied for the purpose, it contains natural oils, which render it most adhesive and not liable to crack; whereas coal-tar pitch, unless very carefully mixed with shale oil, is

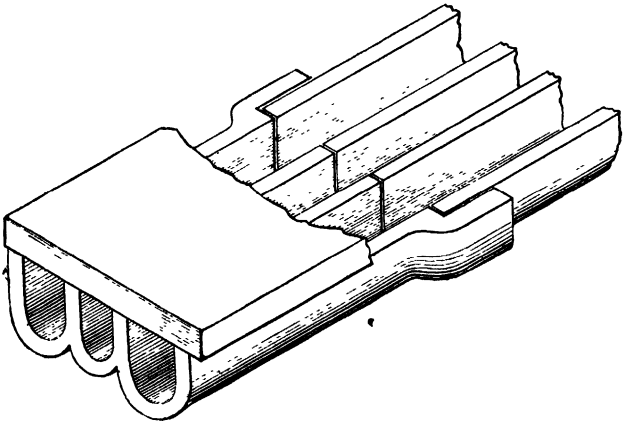


Fig. 213.—Earthenware Duct for Solid System

liable to crack, and allow the exposure of the cable sheathing to the subsoil.

Where cast-iron and earthenware troughs are used, the troughs are usually supplied with socket ends, whereby each length of trough is joined with the next. With wood troughs, however, it is necessary to use wood brackets, into which the ends of each trough are brought, and it is well to secure the ends by means of driving a nail in a downward direction through the trough ends and the bracket. On no account should ends

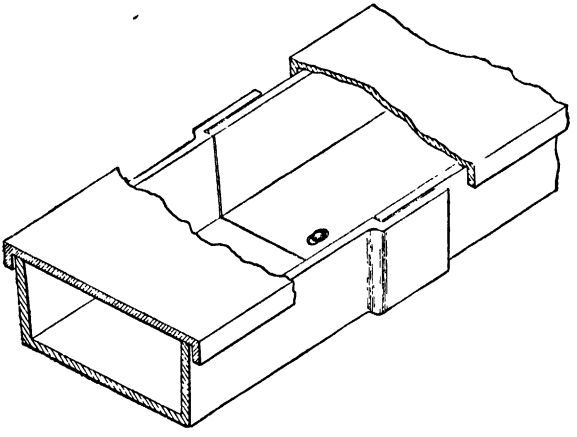


Fig. 214.—Cast-iron Trough for Solid System

be butted, as the subsoil at certain points will in time subside, and tend to break the joint where the troughing ends butt.

Figs. 212, 213, 214 show a section of wood, earthenware, and cast-iron trough and cover at the points where the joint is effected.

Fusing.—The insertion of fuses at certain points in a distribution system is the most common practice in use, as a safety device against causing unnecessary damage to a faulty main, or a main subject to heating through overload. Automatic cut-outs, that is to say switches which act automatically in opening a circuit under circumstances of overload, are also used, but there are two chief objections to this class of apparatus at present.

The first is that there is a difficulty in obtaining an inexpensive cut-out with a time limit, most of the ordinary types acting instantaneously. Instant action is not a necessity at times of

overload, and in fact is a source of much unnecessary inconvenience, as defects or accidents may happen on the supply system which, while causing a momentary flow of excessive current, are not of sufficient import to necessitate the instantaneous disconnection of the supply to the main affected by the fault. Fuses either of copper, tin, or lead have a time limit, as they do not act until the flow of current has had sufficient time in which to heat and melt the fuse.

Secondly, on a system of underground mains, the conditions of the atmosphere in a chamber in the subsoil are most detrimental to the efficient working of automatic mechanical appliances, and in consequence such appliances very often fail to act when most required to do so. Tin is the metal most commonly in use for fusing purposes, although many engineers abroad prefer silver, which metal does not maintain an arc to the same extent. Copper is also used, but is not considered so satisfactory as either tin or silver, as, the melting-point of copper being at so high a temperature, the ratio of the carrying capacity to the fusing-point (usually arranged in practice to be 1 to 3) necessitates the fuse under normal conditions working at a very high temperature. The nature of the metal changes under such conditions, and moreover the heating and cooling causes expansion and contraction, and the contacts at the terminals are liable to become loose. This trouble is also found with other metals in a minor degree, and for this reason all fuse terminals should not depend for security and conductivity upon the contact obtained by nuts or setscrews. Spring contacts obviate this trouble if of proper design.

Aluminium is used to a considerable extent for fuses, and, owing to its non-arcing properties and low melting-point as compared with copper, forms a very efficient fuse.

In moist climates a galvanic action sets up between aluminium and the copper where copper clips form the fuse terminals. This may be obviated in some cases by the use of a mixture of graphite and vaseline rubbed on the points of contact between the aluminium and copper, which, while excluding the moisture, does not appreciably spoil the conductivity at the contacts.

Notes on Balance and Uniformity of Pressure.—Alternating single-phase and continuous-current systems have hitherto been

the usual means by which a town supply has been achieved, the continuous-current system being the more popular of the two as lending itself in many more ways to meeting the requirements of the domestic supply. The hum produced in an alternating electric arc lamp, the inconvenience in starting motors on an alternating circuit, and the inadaptability of alternating currents to so many of the medical appliances in everyday use have all tended to the conclusion that a continuous-current supply is more suited to meet the many requirements of the consumer. The general tendency, therefore, at present is to put down a system in which a low-tension continuous-current network is provided. With the growing demand for electricity, however, the difficulties of efficiently feeding the network with low-tension feeders direct from generating stations is becoming more apparent. There are many reasons for this.

In working the feeders at their highest carrying capacity, which in practice is calculated to be not higher than at 1000 amperes per square inch section of copper, a drop of 2.5 volts per 100 yd. in length will be obtained in each feeder.

It will thus be seen that the efficiency of feeders of considerable length soon becomes low as the load increases, and it is therefore essential that the site for the generating station must be in as close proximity as possible to that portion of the distribution in which the demand is the greatest.

Under these circumstances a station may have to be erected on a site which, being in the centre of a very busy and important part of the town, is excessively costly to purchase. Also the facilities for fuel conveyance and water for condensing purposes will probably be very poor. With such hindrances an economical supply on the direct-current system may be impossible, and the alternative is to choose a site in a more convenient district on the outskirts of the city, or perhaps some miles away, where better facilities are available and the site is less costly.

The question then to be considered is the best means by which to supply the network from such a position, and the scheme resolves itself into one in which current is generated at high pressure and conveyed through high-tension feeders to sub-stations situated at suitable points of the network, where it is transformed down and converted through static

transformers and rotary converters to continuous current. Low-tension feeders will then be laid from the sub-stations to suitable points of distribution into the network.

Assuming that this will probably be the system most commonly adopted in future, the question of low-tension continuous-current may next be considered. This is generally effected by the three-wire system.

Diagram 215 illustrates the method of connection for such a system working at a pressure of 400 volts across the outer mains AC. Two generators, D and E, are connected in series, the positive terminal of one being connected to the main A, and the negative of the other to the main C, the point at which the

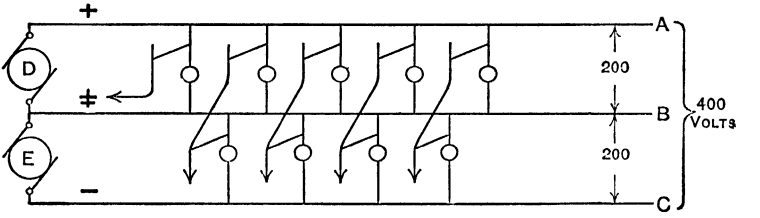


Fig. 215.—Three-wire Distribution

two generators are connected together being also connected to main B.

Each of these generators is working at a pressure of 200 volts, and therefore with both in series the resultant voltage will be 400. There is therefore a difference of potential of 400 volts between mains A and C, and 200 volts between A and B, and B and C. By this means two distinct voltages are obtainable, viz. 200 and 400, from a combination of machines working each at a pressure of 200 volts. A system so arranged is more efficient than with each machine connected to a distinct circuit, the middle wire being merely a connecting link by means of which the batches of lamps on the two circuits are connected together to run in series. With an unequal number of lamps on the two circuits, the extra energy required for the side which contains the most lamps is supplied from the generator on that side through the middle wire connected to the terminal of that machine. The arrows on the diagram indicate the direction taken by the current in effecting the circuit. It will thus be seen that the requisite carrying capacity of the middle wire need

be very small in comparison with the capacity of the other and outer conductors, and the outer conductors are of half the size that would be required if the same number of lamps were connected to them in parallel at the 200-volt pressure. It is generally the custom in practice to calculate the size of the middle wire to be one-third the size of the outer conductors. In the use of such a system it is obviously necessary to ensure as perfect a balance as possible between the lamps connected to the two sides, and in a perfectly balanced system it might be assumed that the load would balance across the outer conductors, and there would be no necessity to bring the middle wire back to the generators at all. There are many reasons why this is necessary. However well the balance may be effected on a three-wire system, the consumers suit their convenience in switching on and off their various circuits, and thus tend to vary the amount of current passing on the individual sides periodically.

It is found in practice that on an interconnected system, however little current may be observed to be passing through the middle wire between the generating station and the network, the conditions of balance are continually changing to a large degree in local sections owing to the changing demands of the consumers, and for this and other reasons fuses should never on any account be inserted in the run of any of the middle-wire connections. The necessity for ensuring a direct connection between the middle wire and the generators, as previously shown, requires that the respective lengths of middle-wire conductors should be solidly connected together by links or connectors of the full copper section and carrying capacity as the conductor itself. Even where the middle wire is efficiently continuous throughout, if it is of smaller section in comparison with the outers, the lamps on one side of the system may be burst due to excessive pressure on the occurrence of a fault, as illustrated by the accompanying diagram, fig. 216. Assume that the lamps connected are well balanced on either side of the system, and at a point considerably distant from the generating station a fault occurs which short-circuits the positive and neutral main. Owing to the high resistance of the middle wire in proportion to the resistance of either of the outers, the effect of the short-circuit is that there is a circuit at a pressure of

considerably in excess of 200 volts across the lamps on the negative side of the system momentarily until the generator on the short-circuited side of the system begins to lose its field. Owing to this, great care should be used to allow a larger capacity in the neutral wire of systems in which the network is fed by two wire feeders across the outers, the out-of-balance and difference-of-potential effects being largely accentuated when a fault occurs on such a network, owing to the absence of the heavier section of copper which would be afforded by a middle wire if run in connection with each feeder.

The middle- or neutral-wire connection to the point between the two generators ensures the balance being maintained, each

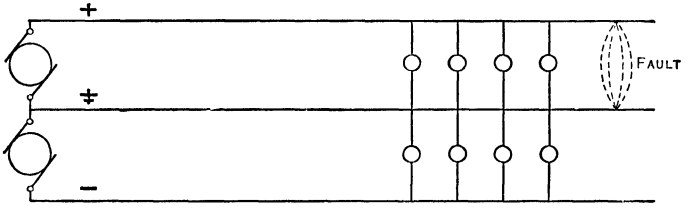


Fig. 216.—Fault on Three-wire Distribution

generator having a proper control of its own circuit. Moreover, if a fault occurs on one of the outer mains, it will not materially affect the conditions of supply on the other, whereas without the return of the middle wire to the generators the lamps connected to both circuits would be cut off.

Fault Localizing.—On an interconnected three-wire system in which the mains consist of triple concentric distributors and feeders with the outer conductors of the cables forming a neutral wire earthed at the generating station, apart from defects in the manufacture of the dielectric, the mains may be more effectually kept in order than any system of single conductors, for the reason that if the outer conductor is kept efficiently insulated from earth, the inner conductors are protected from harm, the outer conductor forming an insulating sheathing for the inners throughout the whole length of the cable. This is one great advantage of the use of a triple-concentric system, as it necessitates the insulation of one conductor alone being watched to ensure the insulation efficiency of the whole of the distribution.

Loop Test.—A milli-voltmeter of the lowest possible resist-

ance should be obtained containing a central zero and about 15 degrees marked off on each side of the scale. This takes the place of the galvanometer as generally used.

A porcelain insulated board containing the resistance or bridge can be constructed with insulators upon which to wind the resistance or bridge wire. The board should be about 15 in. square, and consist of thoroughly well-seasoned teak or some suitable wood which will not be liable to shrink or alter in shape. Twenty feet of No. 18 gauge tinned copper wire (commonly known as 18-gauge binding wire) can then be wound round the insulators provided for it, as illustrated, and if the insulators are measured out at accurate distances, and the wire

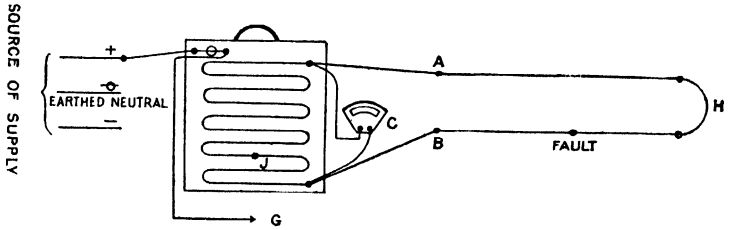


Fig. 217.—Loop Test

properly stretched upon it, a scale could be drawn across the board at right angles to the wire, dividing it into $\frac{1}{4}$ in., so that the exact length of the wire at any point between its terminals could be easily calculated. The terminals should be of very large dimensions, so that good metallic contacts are provided between the ends of the loop to be tested and the bridge wire. A bayonet lamp socket is attached to one end of the board with suitable terminals, one for connecting to the source of supply and the other to the test wire.

In fig. 217, B represents conductor, with fault as shown. A represents another, but sound, conductor of the three-wire main, of the same sectional area as B. H represents a connecting piece of conductor of the same section as A and B. H is joined as securely as possible to A and B, care being taken that the joint is a good metallic one and offers no high electrical resistance to the passage of the fault-localizing current.

With the connections made as shown, first examine the pointer on milli-voltmeter dial to ensure that it is directly over

the zero point. Then, taking the wire G, which is connected with the source of supply, make a contact with any part of the bridge wire and watch the milli-voltmeter, sliding the wire G along the bridge in one direction or the other until a point, say J, is reached at which the milli-voltmeter needle stands exactly at zero, and in the position in which it rested before test wire G was brought into action. When the needle is in its normal position (this can be proved by making intermittent contact on the bridge and noting that the needle does not shift) take notice of the exact point at which contact is made with the bridge, and measure the lengths between this point and each end of the bridge respectively.

The position of the fault on the loop will be in a direct ratio with the different lengths obtained between the test contact and the two ends of the bridge, the fault on the loop AB being nearest to that end of the bridge which gives the shortest length between its end and the test contact. Supposing the test contact as marked on the illustration came out at 5 in. from B and 15 in. from A. Measure up the total length of the loop AB, and if the total length is 200 yd. the fault will be $\frac{5}{20} \times 200$ yd. from one end, and $\frac{15}{20} \times 200$ yd. from the other end, 20 being of course the full length of the test wire, i.e. the fault will be 50 yd. from one end and 150 from the other.

Following the rule that the fault will be nearest to that end of the loop which is connected to the shortest length of bridge, which is BH, we find that the fault is 50 yd. from B.

Where it is not possible to obtain another and equal section of underground main to form the loop with a defective main, a length of good cable may be unwound from a drum and run over the ground, being connected at the far end in the usual manner, but it must be of the same section as the conductor containing the fault, otherwise the test will be useless. The test could be taken with a loop consisting of two equal lengths of cable of different section, provided that the ohmic resistance of each length of different section were first ascertained by actual test, and the ratio of the one to the other taken into account when working out the fault calculation.

If there is a short circuit and no earth between two conductors only, on a three-wire main, and the remaining good

conductor is the same size as one of the faulty conductors, the same test may be applied as follows:—

A is .1 section faulty.
B is .03 section faulty.
C is .1 section.

Loop faulty A with C, and connect to bridge in the usual manner. Earth B at testing end, and test as before described, calculating fault on length of A + C.

Where all three are short-circuited and not earthed the same test will apply, with a length of good cable to complete loop with one of the faulty mains, and another of the faulty mains earthed at testing end.

This loop test is the best practical method known for fault localization, especially if carried out with a small current, as this

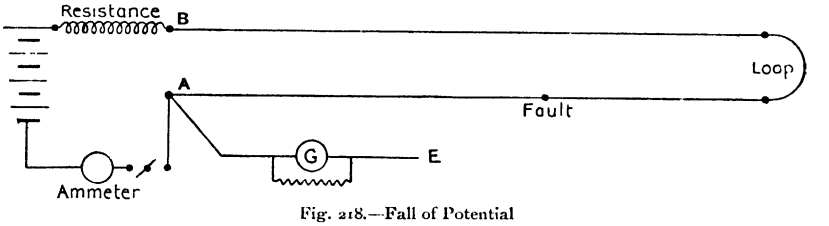


Fig. 218.—Fall of Potential

is less likely to affect the resistance of the fault than a heavy current, and, moreover, any fluctuation caused in the flow of current due to variation of pressure from the source of supply, or variation in the resistance in the earth portion of the circuit, alters both sides of the loop simultaneously, and does not affect the reading of the voltmeter. The test is also an extremely simple one, which necessitates the least degree of mathematical calculation, and so may easily be taught to a jointer or mains employee not skilled in calculations.

Where a supply of energy is not available from the mains the test may be made as follows by the fall-of-potential method (fig. 218): Connect a battery of four storage cells, for obtaining from 15 to 20 amperes at 8 volts, to the terminals of the looped cables. Put an ammeter and switch and a suitable resistance (about 1 ohm) in circuit, and connected to one of the two cables forming the loop, using in place of the milli-voltmeter before mentioned a galvanometer of 50,000 ohms resistance shunted

with $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{300}$ shunts. (This is a standard galvanometer supplied with the portable Kelvin testing sets as used so commonly on mains work.) One lead of the galvanometer is connected to the loop contact at A, which belongs to the *faulty* one of the pair of cables forming the loop, and the other end to earth (E). When the current is switched on to the loop, note deflection D_1 of galvanometer, using the shunts if necessary. When a steady reading has been obtained, shift the galvanometer lead, which was attached to A, over to B contact of the

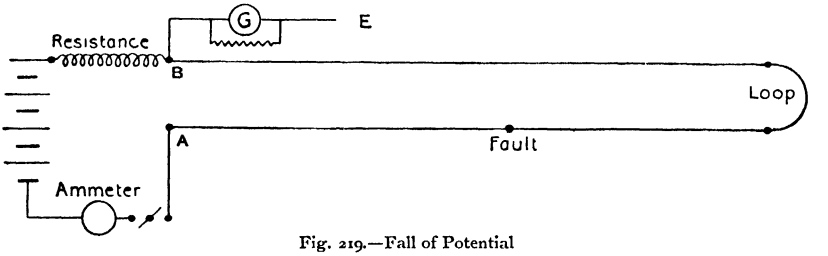


Fig. 219.—Fall of Potential

loop (fig. 219), and again note deflection D_2. The formula for calculation is then $\frac{D_1}{D_1 + D_2} \times$ total length of loop = position of fault from end first tested, which is A end of loop.

Suppose loop = 200 yd.

D_1 = 4° galvanometer deflection when connected to A,
D_2 = 16° ,, ,, ,, B;

then $\frac{4}{4+16} \times 200 = \frac{4}{20} \times 200 = 40$ yd. from A.

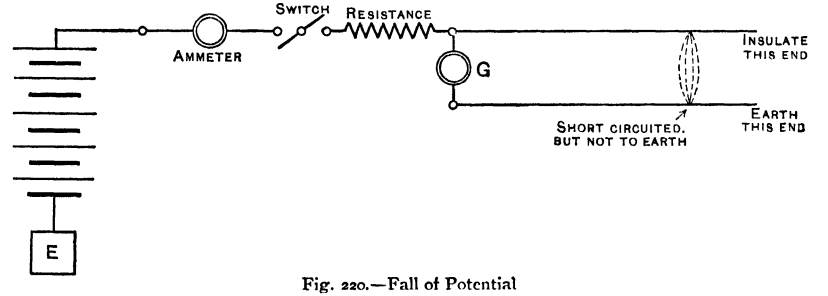


Fig. 220.—Fall of Potential

In localizing a short-circuit on a concentric or between two mains which are not earthed, the following fall-of-potential test may be used:—

For the first test connect the battery, resistance, and galvanometer, as shown in fig. 220, earthing the far end of the one conductor not connected to the battery, and making certain at the same time that the ends of the conductor connected to the positive side of the battery are well insulated. After obtaining galvanometer deflection convey apparatus to the other end of the main, connecting positive side of battery to the same conductor as before, removing the earth on the other conductor to the far end of this conductor (fig. 221). Obtain galvanometer

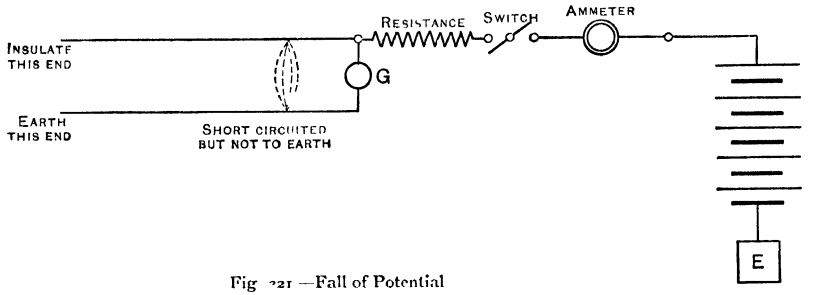


Fig. 221.—Fall of Potential

readings in the usual manner, and the formula for the test becomes $\frac{D_1}{D_1 + D_2} \times$ length in yards of one main only = distance in yards of fault from end first tested.

Joints and Jointing Appliances.—With the exception of feeders, the mains on a town distributing system become in course of time so many joints connected together by short pieces of undisturbed cable. Assuming that every shop or business premises in a street requires a supply, it is the usual method to lay a separate branch into each distinct tenement, and considering that frontages in a busy neighbourhood are on an average not more than 15 ft. in length, a joint is necessitated at every 15-ft. length in the run of the distributing main in the street.

This illustrates the importance that should be attached to the obtaining of joint boxes, which, while being economical in space, permit of the jointing of the conductors in such a manner that a thoroughly good electrical conductivity will be obtained between the fittings and the cable conductors.

In the early days, when rubber was the only dielectric available for distributing mains, all joints used to be sealed with

rubber, the rubber being vulcanized in position and forming a continuous vulcanized rubber sheathing throughout the length of the cable, including all joints.

Fig. 222 illustrates a neat form of joint for triple-concentric high-tension work, the different strands of the two outer conductors being brought on to a base of strip copper wrapped

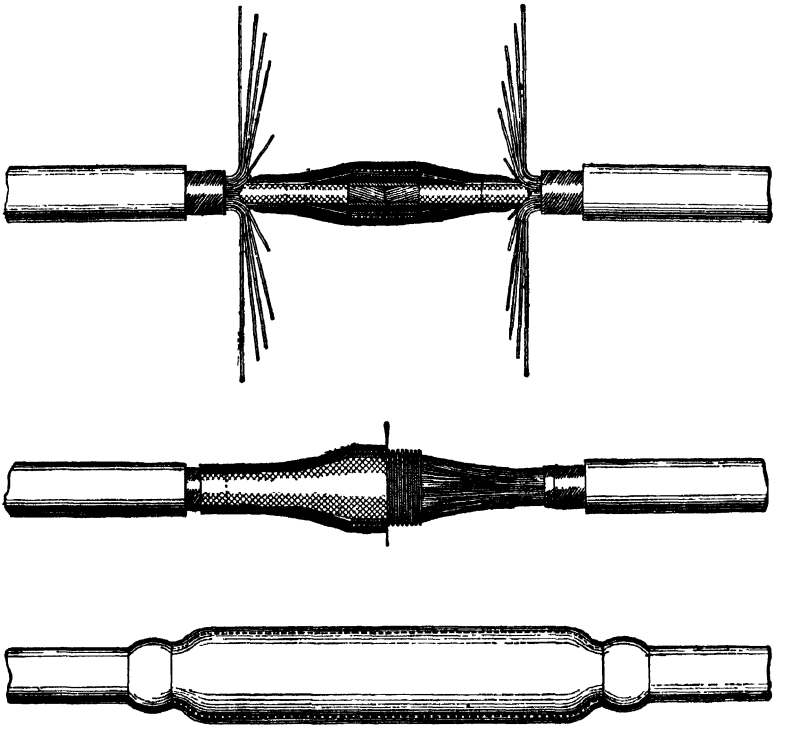


Fig. 222.—Triple Concentric Straight-through Lead-sleeve Joint Box

round the insulation of the underneath conductor; the strands overlap in length and are bound in position with copper binding wire, the whole being sweated.

The insulating material generally used is Indian cotton tape which has been boiled in resin oil, and between the layers strips of mica are bound, there being never less than three layers used, but the number of layers depends upon the size of the conductor and its dielectric. As a general rule the joint, when completed, measured over the whole of the made-up insulation, should not exceed 1.5 times the size of the original

cable measured over the insulation. The mica may be thinned down as much as possible when used in conjunction with the tape, and a properly made joint of this description will stand up to 60,000 volts. This type may be either enclosed in a lead sleeve with wiped joints, or put in a cast-iron box and filled in with jointing compound.

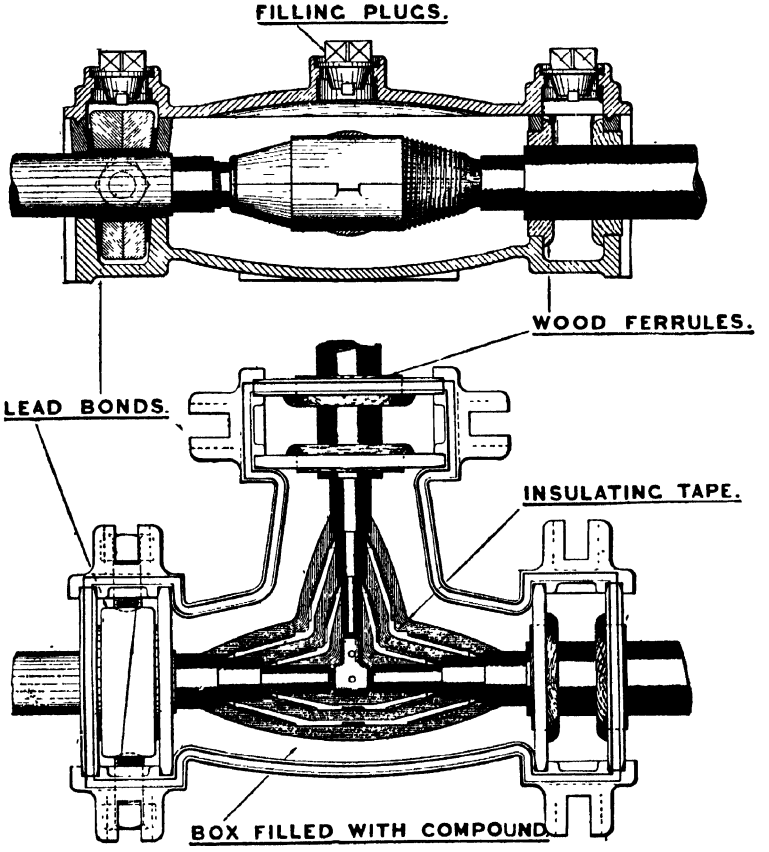


Fig. 223.—Triple Concentric Branch or Service Box

Tee or branch joints may be made with the cylindrical fittings as illustrated in fig. 223.

Fig. 224 illustrates tee joints for three-core cables. In spreading the cables for jointing purposes it is usual to allow a distance of not less than $\frac{3}{4}$ in. between the conductor joints, and between a conductor joint and the side of the cast-iron box.

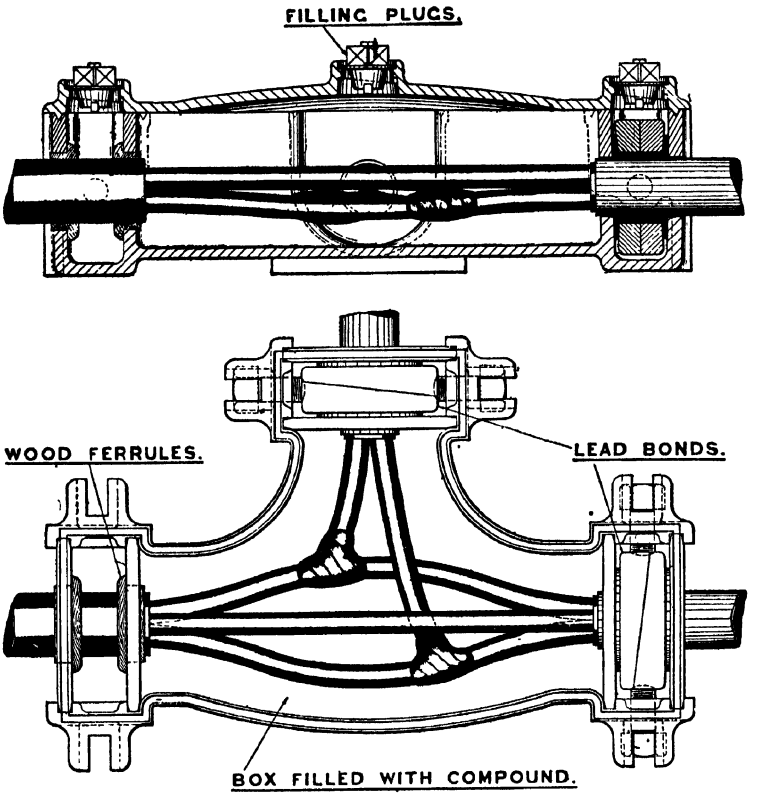


Fig. 224.—Three-core Branch or Service Box

Expenditure of Energy in Conductors. — The resistance offered by conductors demands expenditure of energy to send the current along them. The energy thus spent appears in the form of heat, raising the temperature of the conductor until such time as the rate of radiation and the rate of generation are the same, when the temperature will reach a maximum and remain steady. In systems of electric lighting and power distribution this energy is waste, and must be kept as low as possible, consistent with economy in other matters connected with the distribution. Consideration of the chief items of cost in electrical distribution led Lord Kelvin in 1881 to give the following as a solution of the problem: "What is the most economical size of conductor to use in a given case of distribution?" Solution: "If capital outlay in conductors varied in strict proportion to

weight of metal in the conductor, then the most economical size of conductor was that for which annual cost, interest, and depreciation were equal to annual cost of energy wasted". Applying this solution, Lord Kelvin gave the following formula:—

$$R = \sqrt{\frac{67.84\,\frac{ias}{b}}{I}}$$

where R = most economical resistance per mile in ohms,
I = current in amperes to be transmitted,
i = rate of interest on money value of conductor,
a = cost in pounds sterling of a ton of copper,
s = resistance of 1 mile of copper conductor 1 sq. in. in section,
b = annual cost in pounds sterling of an electrical horse-power for the number of hours the power is used.

The problem of the most economical conductor is really rather complex. For example—

1. The cost of cables does not vary directly with the weight of copper.

2. The cost of labour in laying conductors underground, or erecting them overhead, does not vary much with size of cable.

3. The number of hours during which power transmitted by the cable will be in use is difficult to estimate.

4. The temperature to which the cable may be raised without danger or injury to its insulation may in some cases of cheaply obtained power affect the question.

Loss of Energy due to Heating of Conductors.—The rise of temperature allowable in a conductor varies for different cases, depending upon position and object of conductor. In the case of covered wires the temperature must never be so high as to endanger the insulating covering. To calculate the heat generated in a wire of resistance R ohms when a current I amperes flows through it—

$$\text{Watts lost} = I^2R;$$
$$\text{but 1 watt} = 10^7 \text{ ergs per second},$$
$$\text{1 calorie} = 4.2 \times 10^7 \text{ ergs},$$
$$\text{therefore 1 watt} = \frac{1}{4.2} \text{ calories per second},$$
$$\text{and calories per second} = \frac{I^2R}{4.2}.$$

In electric lighting and transmission of power the object, of course, is the transmission of the energy with as little loss as possible. This can be attained by making the conductors of low resistance, or, in other words, by making them of large cross section.

INTERIOR WIRING

Limits of Current and Temperature.—For electric cables for interiors of buildings where the conductor is enclosed in a wooden casing Mr. Kennelly has given the following formulas:—

$$I = 560 \times d^{\frac{3}{2}}, \text{ } d \text{ being in inches,}$$
$$\text{or } I = 138 \times d^{\frac{3}{2}}, \text{ } d \text{ being in centimetres,}$$

I being the current in amperes which will raise the temperature of the conductor 10° C. from 24° C, d being the diameter of the copper core of conductor. The usual limit of current density is 1000 amperes per square inch for house wiring.

For bare overhead wires Professor G. Forbes has given the current which can be carried without overheating as

$$I = D^3 t \frac{\pi^2 \times H}{4R \times .24} = \frac{D^3 t}{320 R},$$

where I = current in amperes,
D = diameter of wire in centimetres,
t = excess of temperature of wire over air in degrees Centigrade,
H = coefficient of radiation and convection .0003.
R = specific resistance of material at limiting temperature.
.24 = calories in a joule.

The limiting temperature to which a conductor may be raised will be decided by one of several conditions.

1. Increase of resistance of conductor due to heat.
2. Decrease of resistance of insulation due to heat.
3. Softening of some insulators with heat.
4. Danger of fire due to overheating.

The rise in temperature of a conductor will depend upon: (1) the electrical conductivity; (2) the nature and extent of the radiating surface; (3) the nature and condition of the surrounding medium; and (4) the specific heat of the metal.

The radiating surface depends on the form of the conductor. The following gives the relative cooling surface of conductors of the same cross-sectional area:—

Shape.	Relative Cooling Surface.
Flat strip	2.75
Circular rod	1
Cables, 7 strands of 7 wires	1.71
Strands, 37 conductors	1.48

A rough surface radiates better than a polished one, and a black and rough surface is practically twice as good as a polished one. The addition of black insulating material over a conductor will actually promote radiation.

Fig. 225.—Three-stranded Cable

Conductors and Size.—Conductors for electrical purposes

Fig. 226.—1-, 7-, 19-, and 37-stranded Cable

should be of high-conductivity copper, not less than 100 per cent conductivity, and where sulphur or other substance liable to attack bare copper is contained in the insulation they should be tinned.

It is the regular practice now to use stranded cables only, owing to there being less chance of a stranded cable breaking, and also owing to the fact that such a cable is much more flexible than a solid wire of the same area.

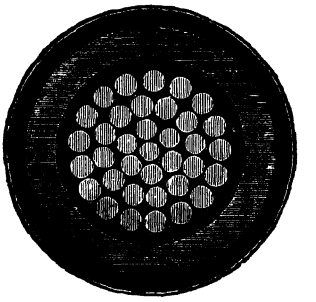


Fig. 227.—Section of 37-stranded Cable

Fig. 225 shows the arrangement of the strands in a three-core cable, and fig. 226 in a 7, 19, and 37 cable.

Fig. 227 is a section of a 37-stranded cable.

In any installation requiring more than 100 amperes the wiring should be split up into two or more circuits each carry-

ing 100 amperes, and no smaller conductor than a cable of three 22's should be used. Should a larger main conductor be required than one capable of carrying 100 amperes, the current density ought to be lower than at the rate of 1000 amperes per square inch, and should, when the current reaches 500 amperes, have only a density of 750 amperes per square inch.

Insulation.—The insulating material used for covering the conductor is of as great importance as the conductor itself. The properties for a good insulating substance are: (1) It must prevent leakage and consequent appreciable waste of energy, as well as allow the free handling of the conductor without any resultant shock. (2) It must be mechanically strong and tough, so that it cannot be easily torn or split. (3) It must be flexible, so that it can be bent without cracking. (4) It must be capable of standing fairly high temperatures without softening or permanent injury. (5) It should be unaffected by gases or acids with which it may come in contact. (6) It should be perfectly homogeneous and of a lasting nature.

No metal is known whose conductivity is sufficiently low or bad to be of use for this purpose. The insulating material most commonly used for "wiring", as distinct from underground mains, is pure and vulcanized indiarubber. When this is used it becomes necessary to tin the copper forming the cable, or at least the outer layer, to prevent the sulphur in the vulcanized rubber attacking the copper. It is customary first of all to serve the cable with a coating of pure rubber; on this the unvulcanized rubber is placed with a covering of tape, and the whole is vulcanized. The thickness of this insulating cover should be, according to the general rules for wiring for the supply of electrical energy issued by the Institution of Electrical Engineers, not less in thickness, measured radially, than 30 mm. plus one-tenth of the diameter of the conductor, and it should not soften at a lower temperature than 170° F.

The rubber covering of the cables should be protected from slight mechanical injury by being covered with a stout braid prepared so as to resist moisture; and, if the cables are to be drawn into pipes or conduits, they should have a further protection of stout tape over the braiding.

Various tests have been suggested for detecting inferior qualities of vulcanized rubber. The following test generally

brings to light any inferiority that may exist, though some inferior rubber mixtures will pass most of the test apparently satisfactorily.

Let, say, 14 in. of the cable be cut from a coil, and the tape and braiding removed. If possible, withdraw the conductors without cutting the rubber covering. If the rubber has to be cut, this should be done with a sharp knife, and a perfectly clean cut made. One end of the rubber covering should then be fixed in a wooden clamp or vice and stretched to double its length, i.e. to say 24 in., allowing 1 in. at each end for fixing. It should be left in this state for 24 hours, and should remain so without breaking. At the end of 24 hours it should be released and allowed 24 hours to recover. The permanent elongation of 12 in. after this time should certainly not exceed 15 per cent. A rubber under-vulcanized or under-cured, but quite good, may either break or have a greater elongation than this. The different layers of pure and vulcanized rubber should show no signs of separating, and it should be difficult to separate one layer from another. Three inches of the rubber covering should next be carefully weighed, and afterwards thoroughly consumed. The amount of ash or solid matter left in the crucible should not exceed 40 per cent. Bad rubber will show as high as 80 or even 90 per cent. The ash of a bad rubber with a large amount of foreign or earthy matter in it will retain the form of a tube, while the ash from a good rubber will take the form of a powder.

The braiding and taping should be examined for fineness of texture, mechanical strength, and thorough impregnation of waterproof compound. Rubber-covered cable has given a lot of trouble, but in the majority of cases this has been due to the cable being subjected to alternations of moisture and great dryness, or to considerable variations in temperature. To get over the former, the cable, if it is to be used in damp places, may be covered with a lead covering, which will also very often lessen the evil effects arising from variations in temperature. Rubber has remarkably good insulating properties, and cable has been made showing a minimum insulation resistance after one minute's electrification of anything between 300 and 5000 megohms per mile. Substitutes for rubber have been tried, but with little success. Gutta percha, which has high insulating properties,

becomes brittle, and has a low melting-point; bitumen has been used, but only on large-sized cables; a mixture of a waxy nature called "ozokerite" was tried a few years ago, but made little progress where used alone. Ozokerite tapes, and paper saturated in ozokerite, are still used for covering cables, but the latter requires to have a covering of some non-hygroscopic material such as lead or bitumen.

Another class of insulating materials for covering cables consists of fibrous material, such as yarn, jute, or paper, saturated with oil or waxes. This class must be provided with a water-tight envelope, usually lead, to keep out moisture. Recently cable manufacturers have saturated paper with a bituminous oil, and used vulcanized bitumen as the waterproof envelope. This makes the covering rather larger, and so far cables covered in this way have only been used on underground mains. Great care and the utmost precaution must be taken with this class of cable to protect exposed ends of the conductors, where they enter the terminals of switches, fuses, and other appliances, from the possible access of moisture, which might creep along the insulating material within the waterproof covering. This is usually done by means of prepared ends consisting of a lead coupling soldered at one end on to the lead covering and screwed at the other end, to receive a porcelain or earthenware corrugated insulator, through which a solid conductor, sweated to the stranded conductor, passes, or the lead covering is sweated on to a lead or brass back block filled up solid with pitch or bitumen, to which the fitting is screwed.

The insulation resistance per mile of ordinary rubber-covered cable for wiring purposes should not be less than 2400 megohms at 60° F. after one minute's electrification, if the wire to be used in any situation shows the least sign of dampness. In perfectly dry situations cables showing an insulation of 600 megohms per mile may safely be used.

Twin conductor cables consist of two conductors, each insulated separately with indiarubber, paper, or jute, saturated with insulating oil or wax, laid up and braided and taped together, or, in the case of the paper- or jute-covered cables, covered with one lead covering. This would form a flat conductor, and in order to make it circular, strands of yarn are used to make up the space, or (as is done by one cable manu-

facturing company) each conductor is made semicircular in shape so that the two covered conductors take a fairly circular shape.

Fig. 228 shows a twin lead-covered conductor of the flat shape.

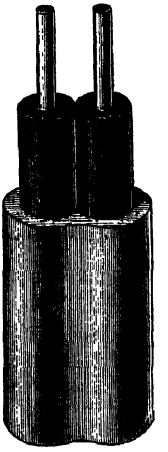


Fig. 228.—Twin Lead-covered Wires

Concentric conductors consist of an inner insulated conductor, and on the outside of the insulation is laid the second conductor concentric with the inner. This outer conductor may or may not be insulated, depending upon the system of wiring to be adopted. To form a triple concentric main this outer conductor is insulated, and a third conductor is placed around the insulation of the outer conductor. This third conductor, again, may or may not be insulated. As the diameter over which the outer conductor of a concentric cable is to be laid is, in small cables, of such a size that to use the same gauge of wire would mean considerable space between the strands, it is customary to use a greater number of wires of a smaller gauge to cover the diameter, or (as is the case in one system of wiring) the outer conductor is made up of iron wire of sufficient size to make the conductivity of the outer equal, if not more than, the conductivity of the inner conductor. In lead-covered concentric cables the lead is frequently used as a conductor, and if the lead is also armoured with iron wire or ribbon, the two—the lead and the armouring—are made of sufficient area to carry the current; or the lead, if not armoured, has copper wires laid in below it so as to make the conductivity of lead and copper equal to that of the core.

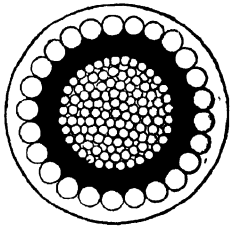


Fig. 229.—Section of Unarmoured Lead-sheathed Conductor

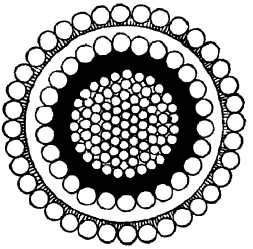


Fig. 230.—Section of Armoured Lead-sheathed Conductor

Another class of conductors are those known as flexible conductors, i.e. those made up of a number of wires not larger

than No. 40 S.W.G., insulated with pure or with pure and vulcanized indiarubber, and then served with a covering of glacé, silk, or heavy braiding. Two such conductors are usually twisted together so as to form a double or twin conductor. The glacé- or silk-covered flexibles are used for attachment to portable fittings or appliances, or for the wiring of fittings, and should never be used as part of the fixed wiring behind skirting-boards, floors, linings, or put in any inaccessible position. The heavy-braided twin flexible is for use where the conductors are liable to be roughly handled, such as in workshops or in connection with flexible connections to motors or heavy machinery. If pure rubber is used for the insulation it should be laid on in two layers, care being taken that these lap joint. The radial

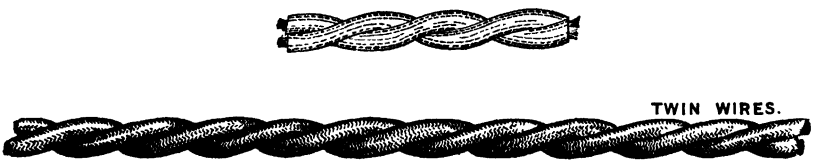


Fig. 231.—Twin Flexible Conductors

thickness of the dielectric should never be less than 3/4 mm. The rubber, either pure or vulcanized, must be quite equal to that used on the main wiring, but the following further test is given in the rules of the Institution of Electrical Engineers. A piece 1 yd. in length is to be cut from each coil and hung not more than 3 ft. over a pan of boiling water from which steam is rising, and an alternating pressure of 1000 volts, at a 40 to 100 periods per second frequency, is to be applied between the conductors. The insulation of the twin flexible conductors should be subjected to the test for five minutes, and show no signs of breaking down.

No smaller flexible than 34/40's, or the equivalent thereof, should be used. As flexible conductors are very often subjected to rough handling, and in counterweight fittings are continually under strain and made to run over two and sometimes more pulleys, only flexible made by reliable firms should be used. In fact, it is a good plan to renew flexible conductors periodically in places where there is considerable heat, or where they are used for supporting a cluster of incandescent lamps,

as the rubber, gradually hardened by the heat, in course of time cracks, and permits the conductors to get in contact with one another, causing a short-circuit. This has frequently happened, with the result that not only is the cluster of lights dropped, but an arc plays between the conductors and burns up the flexible for some length before the fuse goes.

Joints on Conductors.—The joints and ends of a cable are undoubtedly the chief points to which one must look for leakage and trouble. All joints should be mechanically and electrically perfect, i.e. two or more lengths of a conductor, after being jointed, should hold together without soldering, and at the same time the conductivity of the piece of cable containing the joint should be at least equal to, if not more than, an equal length of the conductor without a joint in it. If this is not the case heating may take place at the joint.

Fig. 232.—Straight-through Joint, ends prepared

To make a straight-through joint, i.e. one cable to another, pare off the insulation in separate layers as shown in fig. 232, so as to leave a length of cable exposed equal to about ten times the diameter of the cable. Spray out the wires of the outer layer of the cable in open umbrella form from about the middle of the exposed conductor. Cut out all the conductors inside this at the point where the outer layer sprays out. Do the same with both ends of the cable, and carefully clean, and if necessary tin all the separated-out wires as well as the exposed cable. Put the two ends of the cable together and interlace the separate wires, twisting up tightly the wires of No. 1 cable end round the exposed part of No. 2 cable, and vice versa, as shown in fig. 233. Solder the whole joint by pouring solder over it, or in small joints by holding a heated soldering-bolt against the joint until it is sufficiently warm to run a stick of solder held against the joint. Rosin only should be used as a soldering flux, and fluids containing acid or other corrosive substances should never be used.

Fig. 233.—Straight-through Joint, conductors twisted up

A single solid conductor can be jointed to a stranded cable by putting the solid conductor into the core of the stranded one and then twisting up the strands of the cable around the solid wire. To make a T joint, the cable that is to be teed on is prepared as before, but only about five diameters of both cables need be bared. The end of the cable requires to be opened up for the whole length of the bared part and the central conductors cut off. Half the sprayed-out conductors are laid flat together and separate from the other half similarly placed so as to form a Y on the end of the conductor. This Y is put into the centre of the bared part of the straight-through cable, as shown in fig. 234, one leg of the Y being twisted round

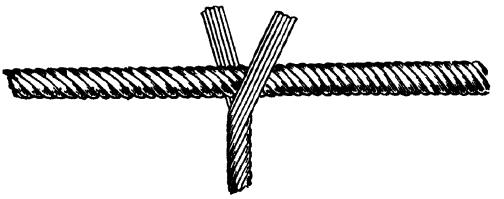


Fig. 234.—T Joint before Conductors are twisted up

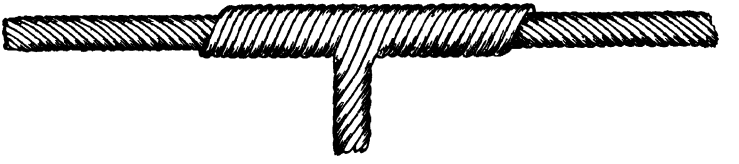


Fig. 235.—T Joint, conductors twisted up

to the right and the other to the left of the whole conductor. the completed joint having an appearance as shown in fig. 235. Recently jointing pieces or thimbles, straight-through, T, and four-way, have been devised, into which the bared ends of the cables are laid, and hammered up and then soldered in. Fig.

Fig. 236.—A Second Method of making a Straight-through Joint

236 shows a method of making a straight-through joint by means of a ferrule, and fig. 237 a joint which is used for temporary work or where the joint is to be in an insulating box. Fig. 238 shows an end connection of the same design as fig. 237, and fig. 239 is a combination of figs. 237 and 238.

The insulating of the joint has now to be done, and this

should be made as near as may be equal, so far as insulation is concerned, to the rest of the cable. This can only be done if the joint is vulcanized.

Joints are, as a rule, however, not vulcanized in ordinary

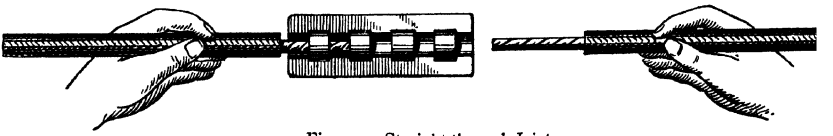


Fig. 237.—Straight-through Joint

wiring work, although recently a firm of cable makers has brought out a tape which is said to be self-curing or self-vulcanizing, or is already vulcanized. The usual practice is to simply cover the conductor with a pure rubber joint. The conductor should be cleaned with a dry rag and any soft or burnt rubber cut off. Whenever possible the end of the rubber should next be cut to a long bevel or cone, and the pure rubber strip wound tightly round the conductor, stretching it almost to the breaking point. For the first layer the rubber should be wound on the top of the cone, and each succeeding layer shortened so as to make the joint parallel, the two surfaces being brought together. The pure rubber should be applied in layers in this manner until the diameter is equal to that of the original insulation, care being taken that the end of the pure rubber is tacked down by means of solution. After leaving the joint for a few minutes to allow the spirit in the solution to evaporate, a prepared waterproof tape should be applied for one or more layers over the whole joint, finishing about 2 in. over the braiding at each end, and the whole should be coated with shellac varnish.

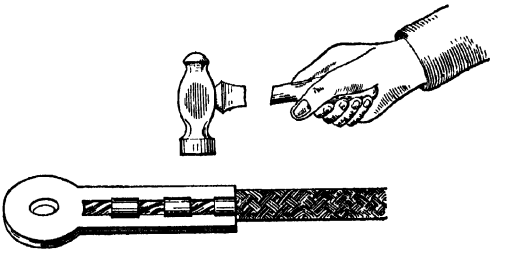


Fig. 238.—End Connection

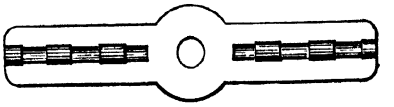


Fig. 239

It should always be remembered that joints constitute a source of weakness, and should therefore be avoided as far as possible. These can be reduced considerably by the looping system of wiring referred to later.

The General Arrangements of Wiring.—Wiring for electric lighting may be installed on what is known as the "Tree" principle or on the "Centre of Distribution" principle, or a combination of the two.

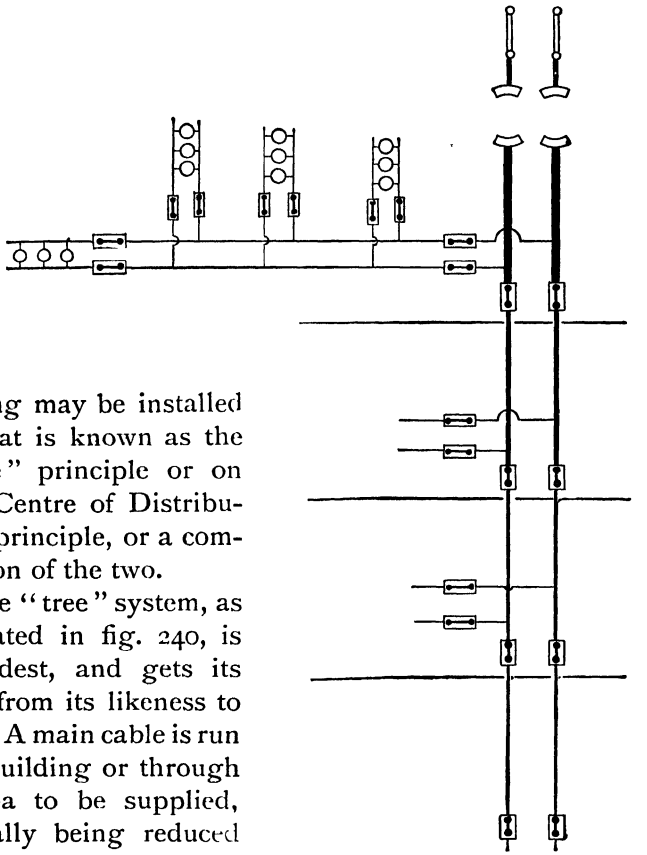


Fig. 240.—Tree System of Wiring

The "tree" system, as illustrated in fig. 240, is the oldest, and gets its name from its likeness to a tree. A main cable is run up a building or through an area to be supplied, gradually being reduced in size as sub-mains or tappings are taken off it, these sub-mains being reduced in a similar way in size as branches are taken off them. This system probably requires less cable than the other methods, but it necessitates a great many joints and fuses scattered over the building in odd places, as a fuse is necessary wherever a reduction in the size of cable takes place. Two or more "tree systems" started throughout a building would reduce the amount of current going through

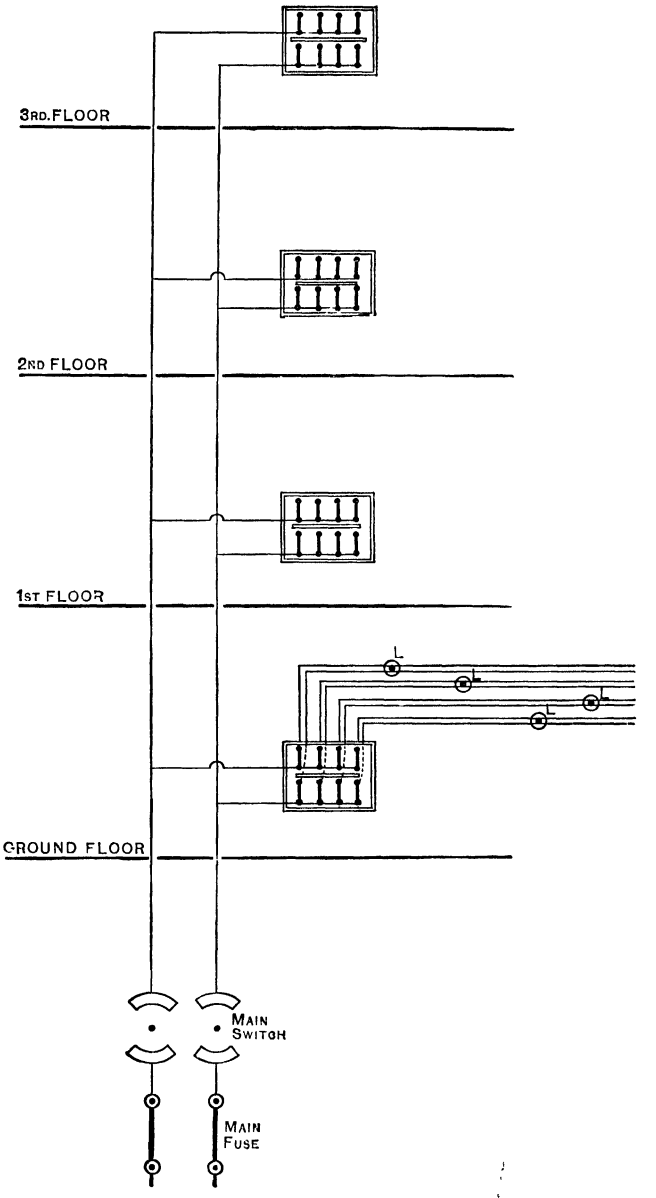


Fig. 241.—Distribution-board System of Wiring

one pair of mains. It is not advisable, however, to have the whole current going through one circuit, and the second or

"centre of distribution" system, as shown in figs. 241, 242, is now admitted to be the best, and is the one most in use.

This system consists of running the mains to a distribution board from which a number of sub-mains, after passing through fuses, run to branch distribution boards. From these branch distribution boards the wiring is carried out to the lights or points, after passing through fuses of a proper size, without any further reduction in size and therefore without further fuses.

This system possesses the advantages that all the fuses are collected together at centres, and one knows at once where to go to replace or draw a fuse; and, further, an installation can be carried out on this system with practically no joints on the mains or sub-mains and with a minimum of joints on the branch wiring.

Two combinations of the two systems are obvious: the mains to each part on a distribution-board system, and the sub-mains and branch wiring on the centre-of-distribution system.

In the Institution of Electrical Engineers' rules, and in those of most public supply companies, it is now recommended that the wiring be carried out from these centres of distribution, on small circuits carrying not more than 3 or 4 amperes. If this course is pursued the fuses in the distribution boards will be ample to protect every conductor even to one flexible conductor carrying current to a single lamp. In the wiring of a large hotel or block of offices the mains running up the building—a pair to each flat—should be on the distribution system, but with one sub-main run round past all the large rooms or different offices, tappings being taken off this through a fuse and into a distribution board. If this is not done a large number of sub-main fuses all in one box are necessary, and there is no means of indicating which fuse is for any particular office or room.

The ideal system from the branch distribution board would be to carry a pair of conductors from the board to each point of use, without joint or tapping. This can also frequently be managed for more than one point by means of looping the wiring down to switches and at the points. As joints are the greatest source of trouble in wiring, and the point at which bad work is likely to take place, looping to a considerable extent will pay. It means more wire, but the saving in labour, in

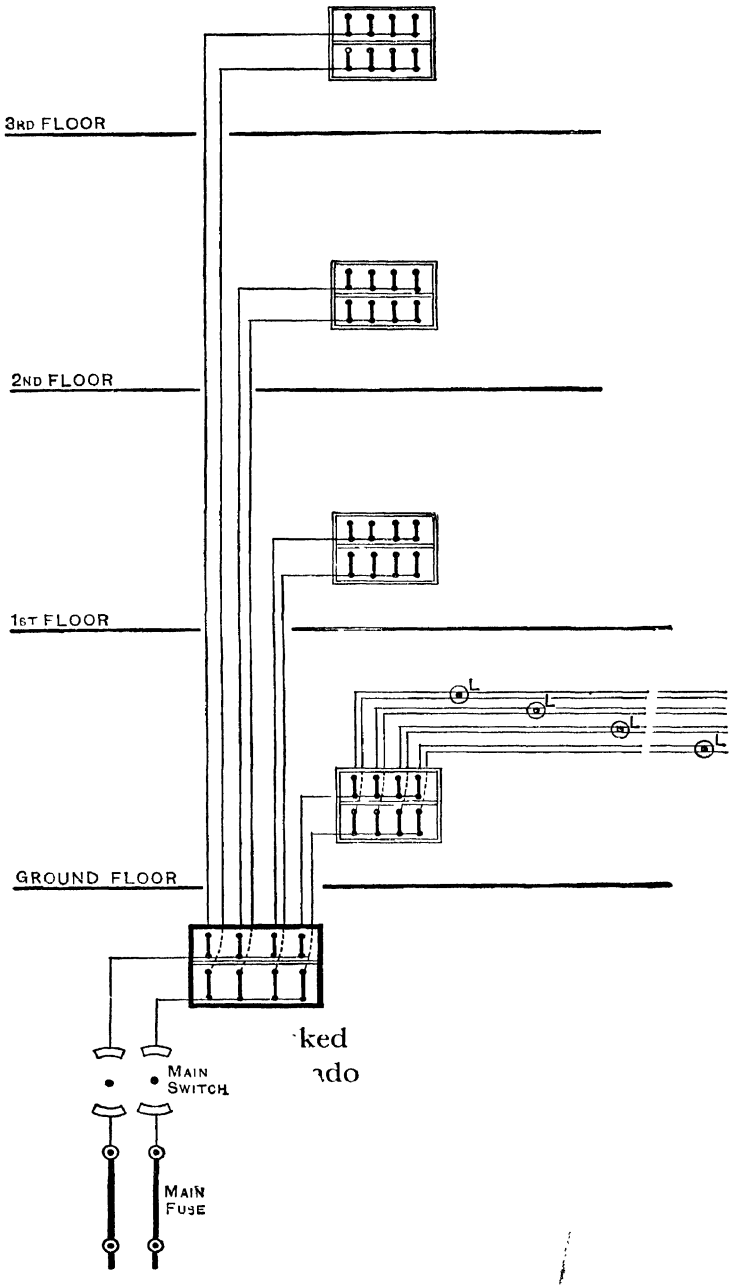


Fig. 242.—Distribution System of Wiring

jointing and jointing material, and the entire absence of joints, more than compensates for the extra wire required.

Looping consists of taking, say, the positive conductor, after leaving the branch distribution board, to the first switch in the circuit, and, without cutting it, carrying it to the point where the light is to be, and then on to a second switch. A separate length of wire is run from the switch to the light it is to control. The negative conductor is treated in the same way, but it is only necessary to loop it in at the lighting point. This means, of course, three wires in many places from switch to light, as shown in fig. 243. Not only can looping be done in branch wiring, but, in order to save joints, mains or sub-mains are now

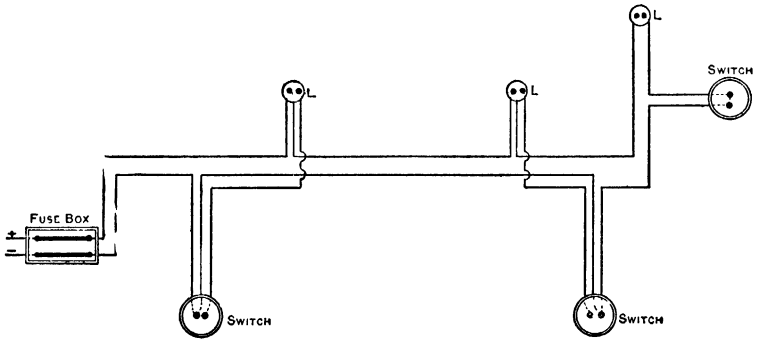


Fig. 243.—Circuit of Three Lighting Points, showing looping in at switch and light

frequently looped into distribution boards. Looping is also resorted to in the wiring of fittings, thus doing away entirely with T joints.

With the pressures now in use, 200 to 250 volts (practically double what they were before 1896), there is nothing like the trouble in, or necessity th[illegible]e is to be for, equalizing the pressure at all the lamps th[illegible] a building, or in limiting the fall to 2 or 3 per cent. [illegible] a cable carrying current at the rate of 1000 amperes per square inch, the loss in pressure is roughly $2\frac{1}{2}$ volts per 100 yd., i.e. $2\frac{1}{2}$ volts per 50 yd. lead and return. On 250 volts this only equals 1 per cent, and very few circuits will be 50 yd. in length. If by running a cable at the rate of 1000 amperes per square inch the fall is to exceed 3 per cent, either a larger conductor should be used or fewer lights put on the circuit, necessitating, of course, more circuits.

As by far the greatest amount of wiring is for connection to public supply mains at a fixed or declared pressure, no attempt is now ever made in the wiring to reduce the pressure at all the lamps to the same voltage or to have different voltage lamps on the same installation.

Methods of Protecting Conductors.—The method in which the insulated conductors are to be fixed in position and protected throughout their course in a building is a subject to which much thought has been given, resulting in the introduction of many ingenious devices. A perfect system for the protection of conductors has to fulfil a very wide range of requirements. The conductors must be mechanically protected, and the mechanical protection should be such as to resist vermin of any kind. Further, it should be water-tight, rendering the access of water to the conductors impossible; it should reduce to a minimum the pulling about of the conductors during erection; if metallic, it should be absolutely electrically continuous, and such that the conductors will be accessible at all times even if concealed, either for the detection of faults in the conductors themselves and bad workmanship in their erection, or for the purpose of tapping the conductors with a view to supplying additional lights or switches; and of course cheapness here, as everywhere else, is of importance. No protection yet used is adaptable to all places and situations, but the outside appearance should always be such as not to be unsightly when exposed nor so cumbersome as to necessitate damaging a building to any great extent in order to conceal the work.

Casing.—The first idea of wiring was simply to support or fix the cables about every 2 ft. by means of wooden saddles or staples, but this primitive method was discarded, as the cables very soon got knocked about and sagged between the supports. This led to the adoption of what is now no doubt the most extensively used system and the one which has been most experimented with, viz. the running of the cables protected throughout their entire length in a wooden casing. This casing usually consists of continuous lengths of wood of about 12 ft., from $1\frac{1}{2}$ to 6 in. wide, depending upon the size of cable it is to hold, and from $\frac{1}{2}$ to $1\frac{1}{2}$ in. deep, with two U-shaped grooves cut in the strip of wood in such a position that there is a continuous fillet of wood between the grooves of not less than $\frac{1}{2}$ in. in the

smallest size of casing and 1 in. in the larger sizes. A cover varying in thickness according to the size of casing is placed over this, and secured to the casing by means of screws after the cables have been placed in the grooves. This cover should have some moulding or cutting on it to indicate the position of the grooves in which the conductors are run, leaving plain the space where it is safe to place a screw. Very often wooden casing and cover are run so as to form a picture mould or top of a dado in cases where it is an old building that is to be wired. The casing should in every case be securely fixed in position, and fully qualified joiners should be engaged to do this part of the work. All joints, tees, and bends should be carefully mitred, and casing fixed throughout the entire length of the run of the conductors.

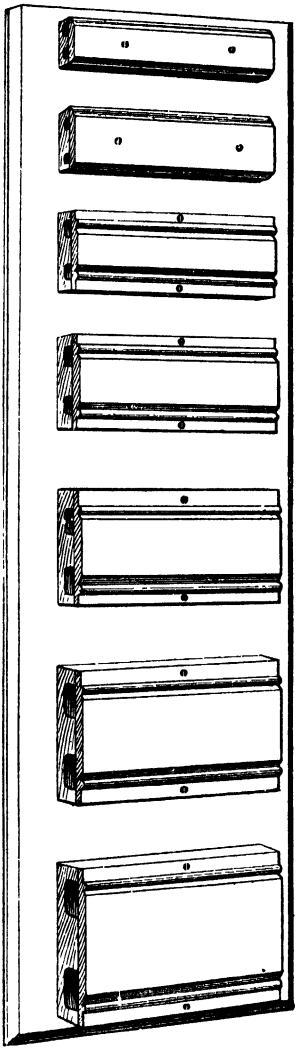


Fig. 244.—Wooden Casing

If wooden casing is to be used in a building in course of erection it is customary either to sink it into the plaster deep enough to make the face of the casing flush with the rest of the wall (in which case the covering will project beyond the wall), or to such a depth that the cover will be flush with the walls. The casing should never be entirely buried in the plaster unless it is of such a size that the conductors can be threaded into it, and inspection and draw-in boxes should be left at all bends and at frequent intervals.

In old buildings the casing as a rule has to be run on the surface, as already stated. This can be done in many cases without causing great unsightliness. The casing with the cover screwed on can occasionally be slipped behind the lath and plaster of the walls and secured at

the top and bottom of the run, the wires being threaded through afterwards. This necessitates running a larger size of casing than would otherwise be used if the wires were placed in the grooves.

Fig. 244 illustrates the six different sizes of casing most commonly used.

Wooden casing was very severely condemned a few years ago by enthusiasts of iron-pipe work and advocates of lead-covered wire, but recent experience has shown that even these other and newer systems have serious drawbacks. Wooden casing should not be used in excessively damp places, and where there is a chance of even slight dampness lead-covered wires should preferably be adopted.

The whole object of casing is, of course, to give some kind

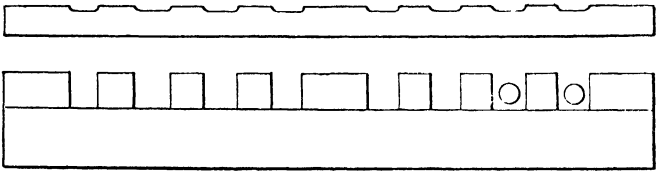


Fig. 245.—Built-up Casing

of mechanical protection to the conductors, and to hold them apart throughout their run. "Bunching" of conductors in one groove should not be encouraged. It is often said that if the conductors are all of the same polarity no danger will ensue from "bunching", but where a group of switch wires are run in one groove of casing it is of course only when all the switches are "on", or when they are all "off", that the conductors are at the same potential. Trouble has arisen due to conductors of the same polarity getting in contact with one another, and causing one fuse in a distribution board to supply current to all the other conductors, with the result that the fuses melt one by one as they are switched into circuit. Casings for holding conductors are occasionally built up in position. A flat board is securely fixed along the course which the wires are to run, and then a number of strips of wood about $\frac{1}{2}$ in. square are fixed on to this board, leaving spaces for the conductors between. The whole is then covered by a suitable covering board, as shown in fig. 245.

Conduit Systems.—A number of years ago a system of protecting and running conductors, consisting of a plain bitumened-paper tubing, was introduced. It was intended that this tubing should either be buried in the walls or fixed on the surface, and that at suitable joint or inspection boxes the cables or wires should be pulled into the tubes. The tubing is supplied in standard 10-ft. lengths, and the different lengths coupled together by means of a thin brass corrugated ferrule, as shown in figure 246. Bends and elbows are also made for going round corners and curves, as shown in fig. 247. This tubing does not present any greater mechanical protection to the cables than wooden casing, while the trouble of threading through the conductors and the jointing of these in small joint boxes presents difficulties. The insulating tube no doubt improves the insulation of the installation, but it is questionable if this is permanent.

Fig. 246.—Bitumened-paper Tubing

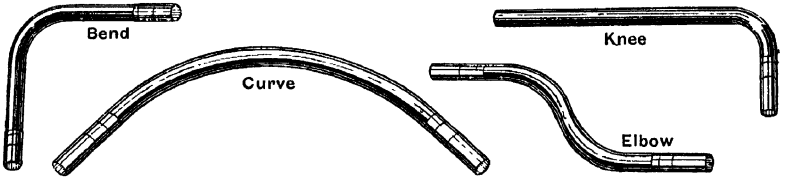


Fig. 247.—Bent Tubing

Fig 248.—Steel-armoured Insulated Conduit

A further development of this tubing is the steel insulated interior conduit. This consists of a plain paper tubing placed within a standard iron pipe, as shown in fig. 248. The insulating tube is firmly attached and cemented to the interior, and is so firmly connected to the metal tube that it practically forms part of it. Flexibility of the tubing is maintained by the

use of various junction boxes, elbows, &c., also covered inside with an insulating paper lining, as shown in figs. 249, 250, and 251. All sizes are supplied in 10-ft. lengths with screwed ends, and each length has a coupling on one end. Special pipe cutters and hack-saws are used for cutting the pipes if shorter lengths are required. Care must, however, be taken that cut ends of the pipe are finally trimmed off clean and square,

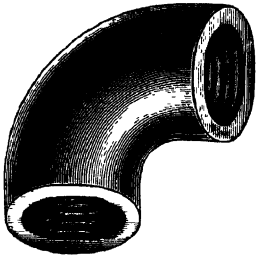


Fig. 249

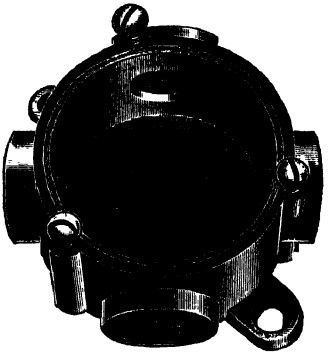


Fig. 250.—Four-way 2½-inch Insulated Iron Box

and a butt reamer is supplied for this purpose. It is preferable, when bends and elbows are required, to use standard elbows; but with careful handling the tube can be set or bent cold. The friction of drawing in the conductors is reduced to a minimum by the use of soapstone powder both in the tubes and on the wires. This tubing is, of course, expensive, but offers the greatest possible mechanical protection. Tubing is unsightly if exposed in a good building, and it is impossible to tap off lights from it.

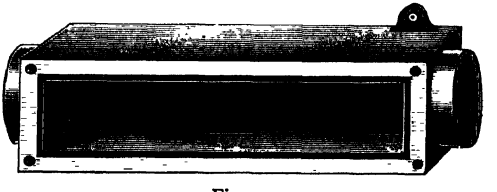


Fig. 251

In all tube systems the wires are subjected to considerable wear and tear while being pulled into place, and consequently it is advisable to have the insulation of the conductors further protected by an extra taping. In all draw-in systems and systems where the insulated conductors are in contact with one another it is customary not to run more than one circuit, i.e. one positive and one negative wire, in one pipe. Two switch wires controlling a group of

lights or an electrolier may be allowed as one, but, beyond this, bunching of conductors in pipes or casing should be avoided. One conductor going wrong will very probably put the others wrong, and in all likelihood it will not be till after all the conductors have gone wrong that anything will be found out. It will then be found exceedingly difficult to withdraw any one or all of them. As mentioned before, the cables frequently get damaged in the pulling in, and a point in favour of casing is the fact that the conductors are placed in position and secured with very little handling. Further, with wood casing in dry places the two conductors are held apart throughout their entire length by means of a fair insulator; whereas, in a draw-in system they are pressed together and surrounded with the very thing it is most desirable to avoid, viz. a metal connection to "earth". A perfect system of protecting the conductors would be one consisting of a fireproof, dampproof casing with a water-tight cover, the material being also a fair insulator.

Lead-covered Wires.—In place of protecting the conductors with wooden casing or iron pipes a great deal of wiring has been carried out with insulated conductors sheathed with lead. In many cases each conductor has had its own sheathing, but the most common arrangement is to have two insulated conductors under one lead sheathing, known as "twin lead-covered cable". The lead offers no more mechanical protection than wooden casing, but it is certainly more water-tight. The lead-covered wires are simply cleated to the walls of the building, or laid below flooring, lining, or plasterwork in the same way as block-tin or compo gas tubing is laid.

Concentric Wires.—In the system of concentric wiring the lead covering of the conductors is used as one conductor. This lead, if not large enough in section to equal or exceed the conductivity of the copper or inner conductor, is supplemented by copper strands laid in below the lead, or the lead is frequently armoured. All switches, fuses, and fitting bases are enclosed in cast-iron or brass boxes, to which the lead sheathing of the cables is hermetically sealed. The continuous metallic sheathing thus provided in conjunction with concentric conductors also ensures a mechanically strong conductor which dispenses with the need of wood casing. The general plan of wiring is to lead cables from a main switch and fuse to dis-

tributing centres. These distributing centres have cables of a uniform size radiating from them, and are arranged to supply from six to ten lights each. No smaller conductors are used; the same section of conductor is carried into every switch and every lamp holder, thus avoiding the necessity for any further fuses than those in the distributing centres or boards.

Wiring for Power Purposes.—If the current is also to be used for power purposes it is necessary to carry out the wiring for the power circuits on entirely separate conductors, and as current to be used for power purposes is generally supplied at a rate per unit lower than that charged for lighting, meters are,

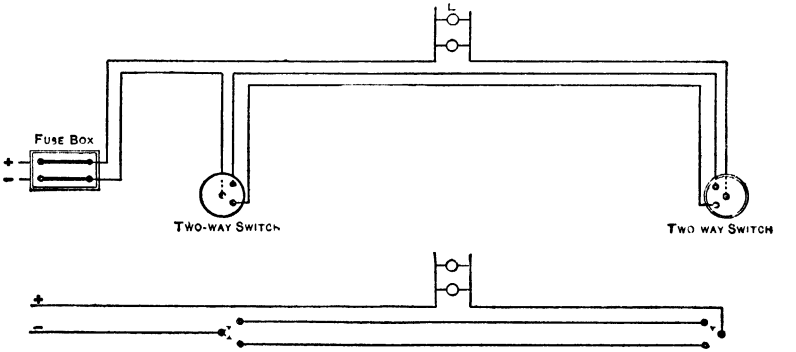


Fig. 252.—Two-way Stair Lighting

as a rule, supplied to record the consumpt for power separately. For power purposes, also, the branch circuits are generally of varying sizes and of more than 3 amperes, and this leads to the running of conductors each calculated to suit a particular motor or motors. It is therefore seldom possible or necessary to tap off these branch circuits for extra motors, and hence it follows that a system of wiring for power purposes need not be such that it will be easy to tap off. Further, motors are not readily moved from one place to another, and consequently, when once the position is fixed, it need not be considered whether it would be easy to alter the wiring or not. Supply companies do not care for mains carrying current to be used for power purposes, and supplied at a lower rate, being run or protected in such a way that they can be easily tapped by consumers, either technical or lay, for a supply for lighting. For these reasons the wiring for power supplies is carried out in

strong iron tubing, the different lengths being screwed together, or in steel-taped armoured conductors. The frequent switching on of large currents is liable to cause fluctuations in the voltage quite noticeable in the lighting where the supplies are carried out on the same circuits.

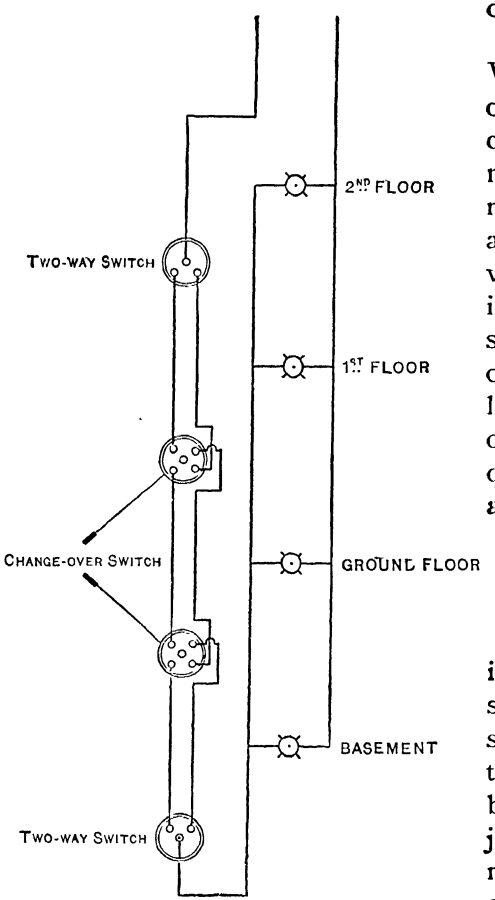


Fig. 253.—Two-way Corridor Lighting

Special Wiring.—Wiring to allow of lights or groups of lights being controlled from two or more points has led to a multiplicity of two-, three-, and four-way switches. A very common requirement is that the lights on a staircase can be switched off or on from any flat or landing of a house or office, or it may be required that the lights in a corridor be controllable from either end. Figs. 252 and 253 show how this can be done.

Earthing and Bonding.—It is absolutely essential to have all metal-sheathed cables and metal tubes or conduits perfectly bonded together at all joints and effectively connected together, and at one point connected to "earth".

In wiring for connection to a public supply company's mains the best practice is to run the tubing or lead-covered conductors insulated by wooden boards or tiles throughout their entire length, and then make one good earth connection at the main distributing board or boards, and by means of an insulated wire also. The reason for this is that, should the insulation of one of the conductors break down and come in con-

tact with the metal tubing or sheathing, a considerable current may momentarily flow along the metal sheathing to "earth", owing to the middle wire of the three-wire system of the supply company being "earthed". This current, no doubt, will go principally by means of the metal covering, but if the metal sheathing is in contact with "earth" at several points, the current will assuredly divide up and part of it go by these other ways. In its course it may come across gas and water pipes, the former in many cases being soft compo or block-tin tubes, and fuse these, causing a spark which melts the pipes. In the case of gas pipes, fires have frequently been caused in this way, the spark melting the pipe and firing the gas in it. The gas flame so produced melts the pipe and is apt to spread fire with great rapidity.

The "earth" connection should not be a soil pipe, rhone, or gas pipe, but a water-main supply pipe if possible, or a length of metal pipe sunk into the ground.

When conductors are carried from one building to another through main walls, or from one fire risk to another, additional protection is required, such as porcelain pipes or other tubes, which can be filled up with sand or other chemically inert incombustible substance, so as to prevent the spread of fire through the openings made for the conductors passing through such walls. Where conductors cannot be run in sight they should be made as accessible as possible. Conductors should never be put into casing till any dampness which may exist has dried out. In new buildings it may be easily arranged to have built chambers or ways left in the walls into which the conductors can afterwards be drawn.

Arrangement of Fuses.—Every conductor should be protected by means of a fuse, and each size of conductor has a corresponding size of fuse, which should be in circuit with it to prevent overheating of the conductor. The fuse should melt or "go" before the current exceeds twice what the conductor can safely bear or what it is meant to carry, i.e. before the current exceeds 2000 amperes per square inch, as a rule. This further means that wherever a conductor is reduced in size, or a branch is taken off a conductor of smaller area than the principal conductor, a fuse must be inserted to protect this smaller size of conductor. It may, however, happen that in some cases

a larger conductor is run for the sake of giving a less drop in potential to a distant point, in which case the conductor may be reduced, or tappings of a smaller size of conductor may be taken off without a fuse, provided, of course, that the fuse on the main conductor is of a size corresponding to the smaller size of branch or reduced conductor. With the system of distribution boards the reduction in size of the conductors takes place at the board, and no other fuses should be necessary. It is advisable, if at all possible, to arrange all the circuits leaving one distribution board so that they will be of the same size or carrying capacity. This

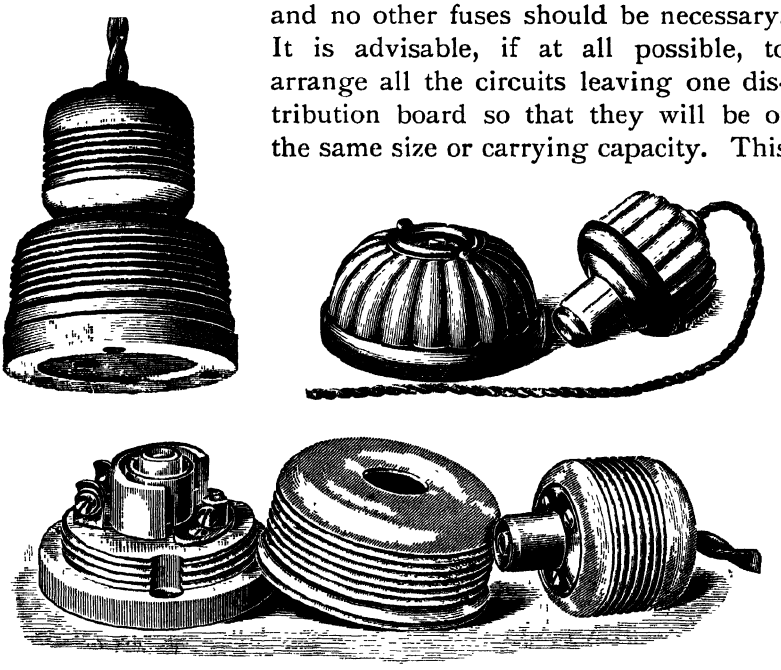


Fig. 254.—Concentric Wall Plugs

makes all the fuses of one size and prevents any accident through putting the wrong size of fuse in.

For ordinary incandescent wiring, 3-ampere circuits with 3/20 cable on 200 or 250 volts, and 5-ampere circuits with 7/21½ cable on 100 and 125 volts is the usual practice.

Circuits for arc lamps and motors or other special purposes should be kept on separate boards, and should be as small as possible, so as to avoid the necessity for any fuses other than those on the distributing board. With this arrangement no fuses in ceiling roses or other inaccessible places are necessary. With the "tree" system of distribution, fuses are required on

each pole wherever a branch leaves the main, and also at points where conductors are forked or reduced in size. In wiring a building for connection to a public supply main it is advisable that all the single-pole switches should be on the same pole of the installation, so that the supply company or corporation can

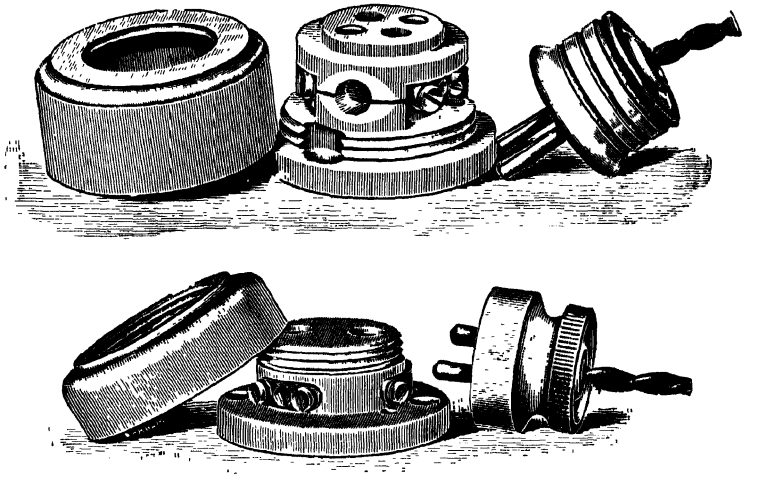
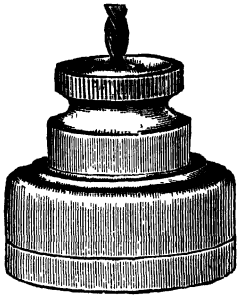


Fig. 255.—Wall Socket for Two-pin Socket



connect that pole to the pole of their system which is not connected to "earth" or at earth's potential. This will ensure that when the current is turned "off" at any fitting or piece of apparatus there is no difference of potential between the wiring of the fitting and earth.

Wall Sockets, &c.—There are different types of concentric sockets, but only those constructed so that they cannot be readily short-circuited by a piece of metal, such as a pin or a pencil case, should be used. This feature is secured by having an insulating column raised from the base of the socket to a little beyond the face of it, so as to prevent the formation of an arc should the connector be pulled out when current is flowing.

Figs. 254 and 255 are samples of wall plugs. The best form is undoubtedly the socket suitable for a two-pin plug as illustrated in fig. 255.

DIRECT-CURRENT GENERATORS

CHAPTER XXI

ARMATURES AND ARMATURE WINDINGS

Introduction.—The construction of a dynamo depends on the discovery made by Faraday in 1831 that when a conductor moves across a magnetic field an E.M.F. is induced in it, which E.M.F. is proportional to the rate of cutting of magnetic lines of force.

A dynamo is a machine, therefore, in which a certain arrangement of conductors is moved rapidly across a strong magnetic field, the various E.M.F.'s induced in the separate conductors being summed up, giving the resultant E.M.F. of the dynamo. When a current flows from the dynamo to an external circuit the mechanical energy given out by the prime mover is converted into electrical energy.

A dynamo consists of two main portions: (*a*) The field magnet system, whose function is to produce and serve as a path for the magnetic lines of force constituting the magnetic field. It consists usually of three portions, namely, the poles on which the exciting coils are wound, the yoke, and the pole shoes. (*b*) The armature, which consists of a number of copper bars, insulated from each other. These are arranged and connected in a definite order. Since the armature bars or conductors must be supported, they are, in practice, supported by a mass of iron, which forms part of the magnetic circuit, and so strengthens the flux in the gap between the pole and the armature.

Homopolar or Unipolar Dynamo.—The simplest method, which was that adopted by Faraday, is to provide a uniform magnetic field and move the conductor with a constant velocity across this field. In practice, however, this method is difficult to carry out, since the speed must be high, and the E.M.F. obtained is small.

Fig. 256 shows a dynamo built on this principle. The steel castings AA form an annular electromagnet excited by coil B. The flux passes radially from the outer to the inner pole. A copper cylinder D rotates in the gap between the poles, acting as a conductor of large cross section, and moving always in the same direction across the field. An E.M.F. is therefore induced between the ends of the cylinder. The circuit is completed through the external resistance R by means of the two brushes EE. The cylinder is supported and driven by the pulley F.

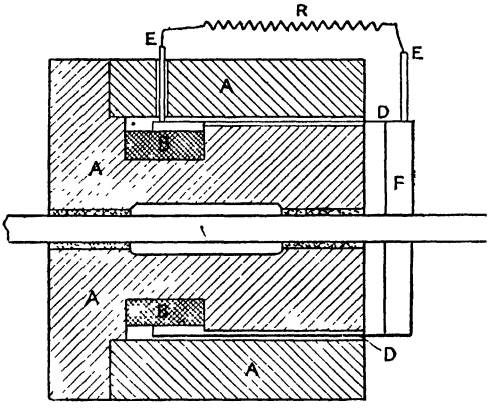


Fig. 256.—Homopolar Dynamo

The E.M.F. induced can be easily calculated from the fundamental E.M.F. formula, as follows:—

D = diameter of cylinder = 60 cm.
l = length of cylinder = 50 cm.
H = density in the air-gap = 10000 lines per square cm.
n = revolutions per second = 16.

From the fundamental equation

$$E = Hlv \times 10^{-8} \text{ volts} = Hl \times \pi Dn \times 10^{-8} \text{ volts}$$
$$= 10000 \times 50 \times \pi \times 60 \times 16 \times 10^{-8} \text{ volts}$$
$$= 15.05 \text{ volts.}$$

It will be seen from this calculation that the E.M.F. induced is low and the speed must be high. In addition, this design is very heavy, and the brushes are troublesome to keep in order, so that, although a few machines have been built, the homopolar dynamo has not come much into use.

E.M.F. Induced in a Coil Rotating in a Uniform Field.—Instead of the above, the following arrangement, or rather a modification of it, is used in practice. A simple loop is rotated in the uniform field between the north and south poles of a magnet. The ends of the loop are connected to two insulated metal slip rings, as shown in fig. 257. Two metal brushes, B_1B_2, press on the slip rings, and when the loop is revolving an E.M.F. may be obtained between them. The direction of the E.M.F. induced at any instant can be found by applying the *right-hand* rule.

In fig. 257 the direction of rotation is shown by the arrow, then the direction of the E.M.F. induced in A is from back to front, and the E.M.F. induced in B is from front to back for the position of the loop as shown in the diagram. These two E.M.F.'s add, and the E.M.F. generated by the loop is thus twice that generated by one conductor. The direction of the current through the external resistance is shown by the full arrow. When the loop has revolved through 90 degrees from the horizontal position, the E.M.F. induced in the loop is zero, since in this position the conductors are moving parallel to the lines of force. Rotating the loop farther, we find that the E.M.F. in A has reversed, and is now from front to back, so that the current in the external circuit is also reversed in direction, as shown by the dotted arrows in the figure. Continuing the rotation till the loop has moved through 180 degrees from the first position, the E.M.F. induced increases till the loop has revolved 180 degrees. The E.M.F. induced has reached its maximum value, since the conductors are now moving practically at right angles to the lines of force. Again revolving the loop, the E.M.F. falls to zero, when the loop has rotated through 90 degrees, and the conductors are moving parallel to the lines of force. During each revolution the current in the external circuit reverses, rises to a maximum, and falls to zero twice.

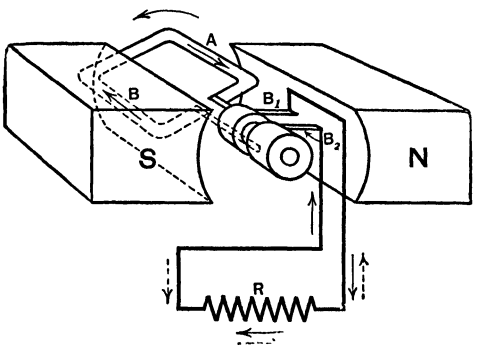


Fig. 257.—Simple Loop Dynamo

The E.M.F. induced at any instant, e.g. when the loop has revolved through an angle θ, can be calculated as follows: OO′ is the initial position of the loop, and LL′ is the position after the loop has rotated through an angle θ. The diameter of the loop is d cm.—that is, the distance between A and B in fig. 257; the length of the loop is l cm.—that is, the length of A or B; the strength of the magnetic field is H lines per square centimetre, and the speed of rotation is n revolutions per second. The formula for the E.M.F. induced in one conductor is, as before,

$$E = H \times l \times v \times 10^{-8} \text{ volts},$$

where v is the velocity of the conductor at right angles to the

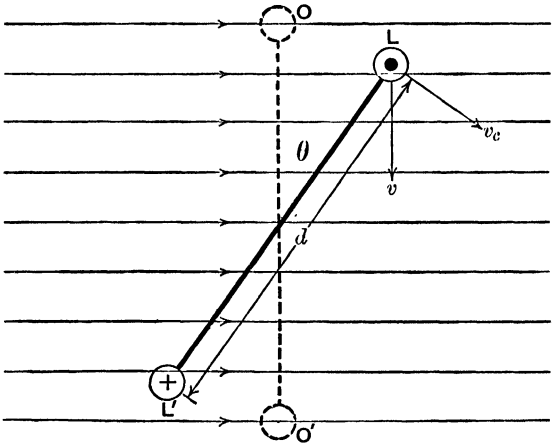


Fig. 258.—Coil in Uniform Field

lines of force. In fig. 258 v is evidently the vertical component of the speed $v_c = \pi dn$ of the conductor,

$$v = \pi dn \sin \theta.$$

Now if N is the maximum flux through the loop, $N = dl \times H$.

$$\begin{aligned} \therefore e &= H \times l \times v \times 10^{-8} \text{ volts} \\ &= H \times l \times \pi dn \sin \theta \times 10^{-8} \text{ volts} \\ &= \pi N n \sin \theta \times 10^{-8} \text{ volts}. \end{aligned}$$

For a loop of S turns there are 2S conductors in series, and the E.M.F. equation becomes

$$e = 2\pi N n S \sin \theta \times 10^{-8} \text{ volts}.$$

In this equation, when $\sin \theta = 1$, the value of e is a maximum, and is equal to $2\pi NnS$, so that calling this E_m we have the instantaneous value of the E.M.F.'s given by $e = E_m \sin \theta$. The curve connecting E.M.F. and angular displacement is therefore a simple sine curve (fig. 259). Such a current is called an *alternating current*, and the simple loop forms an *alternating-current* generator.

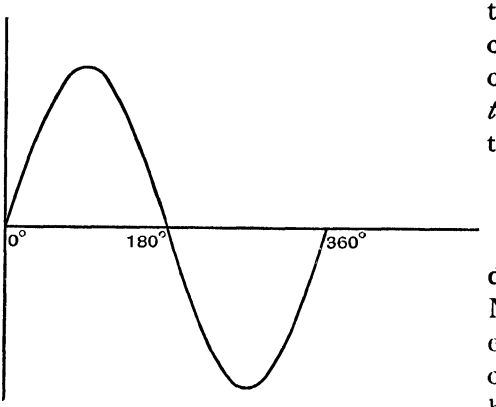


Fig. 259.—Sine Curve

Production of a Unidirectional Current. — Now suppose that the two ends of the loop are not connected to slip rings, but to a split metal ring, the two halves of which are insulated from each other. Such a device forms a *commutator* with two segments, S_1S_2.

Two brushes, B_1B_2 (fig. 260) are placed so as to press on opposite portions of the commutator. Their position is so arranged that when the loop is vertical, and the E.M.F. induced

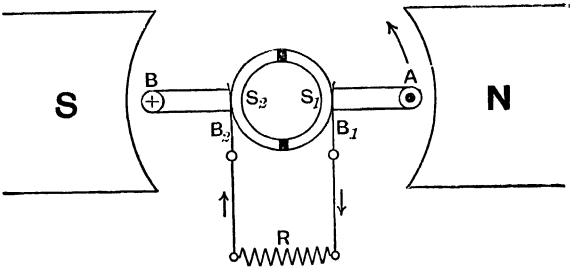


Fig. 260. - Simple Loop with Commutator

is therefore zero, the brushes rest over the insulation between the rings. In the figure the current is flowing from wire A to segment S_1, and then through brush B_1, the external circuit, brush B_2, segment S_2, to wire B, and so returns to A. The direction of current in the external circuit is as shown by the arrow. When the loop has revolved through 180 degrees A

has moved into the place of B_1 and B into that of A. The current now flows from B_1 to segment S_2, but S_2 is now connected to brush B, and the current again flows in the external circuit in the same direction, though the direction of current in the loop has reversed. The current in the external circuit is therefore unidirectional, though it varies from zero to a maximum twice in each revolution. This variation is shown in fig. 261. The loop with a commutator forms a method of generating a *unidirectional* or *direct current*, and is therefore known as a direct-current generator or simply a dynamo.

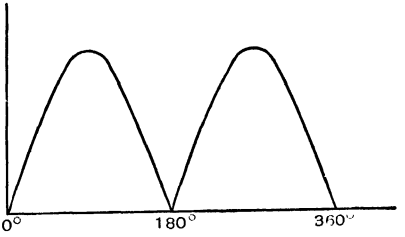


Fig. 261.—Commutated E.M.F. Curve

Drum and Ring Windings.—To increase the E.M.F. produced, and also to eliminate the fluctuations in the E.M.F., the number of inductors in series must be increased and also displaced relatively to each other, so that the time of zero E.M.F. is different for each inductor.

This can be accomplished by two methods. Fig. 262 represents a number of conductors arranged round a core. Now

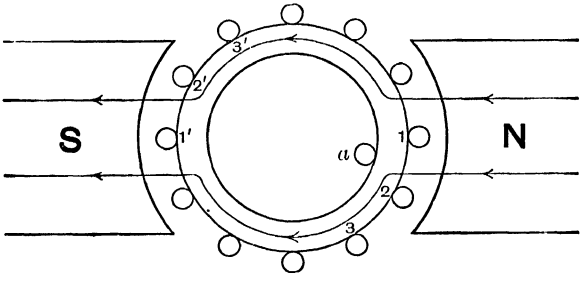


Fig. 262.—Simple Armature

inductors 1 and 2 can be connected in series, if the back end of 1 is connected to the front of 2 by means of a wire coming along inside the core, as shown at *a*. This method gives the winding known as *gramme* or *ring* winding. The lines of force emanating from the north pole of the field pass through the core and enter the south pole. Therefore, as the armature rotates only the conductors on the surface of the core

cut lines of force, so that the conductors inside the core have no E.M.F. induced in them, and only serve as connectors. In fact, if an E.M.F. is induced in the conductors inside the core, this E.M.F. acts in opposition to that induced in the main conductors.

The other method is to connect two inductors in series which lie under poles of opposite sign. This can be done if inductor 1 is connected to inductor 1′ at the back, and then inductor 1′ is connected to inductor 2 at the front; and so on. This gives a drum winding. In the drum winding all the inductors on the armature surface are active, while the cross connectors at the back and front are inactive.

4-coil Ring Winding.—Fig. 263 shows a 4-coil ring wind-

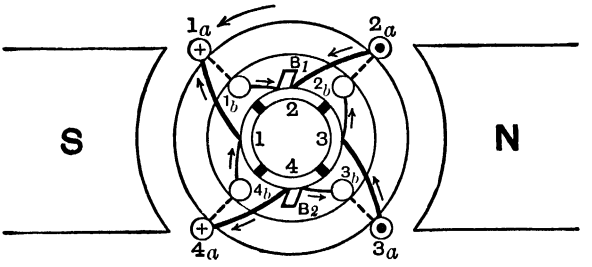


Fig. 263.—4-coil Ring Winding

ing. The active conductors are 1_a, 2_a, 3_a, 4_a, while 1, 2_b, 3_b, 4_b are the return conductors. 1_a is connected to segment 1 of the commutator at the front, and at the back it is connected to 1_b. 1_b is connected to commutator segment 2, and also to 2_a. Repeating this for all the four coils, the winding as drawn in the figure will be obtained.

It is evident that the armature winding forms a closed circuit, i.e. if the winding is traced out from one inductor, each inductor is only passed through once, before returning to the original starting inductor. Such a winding is said to be *singly re-entrant.*

Starting from brush B_1, it is found that there are two paths in parallel through the armature to brush B_2, viz.—

$$B_1 \text{ (+ve)} \begin{cases} \text{seg. } 2 - 2_a - 2_b - 3_a - 3_b - \text{seg. } 4 \\ \text{seg. } 2 - 1_b - 1_a - 4_b - 4_a - \text{seg. } 4 \end{cases} B_2 \text{ (−ve)}$$

It will be found that the E.M.F.'s in these two parallel paths will be equal in magnitude but opposite in direction, so that no

current circulates round the armature. If, however, brushes B_1 and B_2 are joined externally by a resistance R, a current will flow from B_1 to B_2 in the external circuit, while a current of half this value will flow in each half of the armature circuit. It will be found that all closed armature windings behave similarly to this 4-coil ring winding, in that they have at least two parallel paths between the positive and negative brushes, and that there is no resultant E.M.F. acting round the closed circuit.

The instantaneous E.M.F. at the terminals of the external circuit is evidently the sum of the commutated E.M.F.'s in inductors 1_a and 3_a added to the sum of the E.M.F.'s in inductors 2_a and 4_a. In fig. 264 curve I represents the commutated E.M.F. curve for inductors 1_a and 3_a. Now since 2_a and 4_a are displaced 90 degrees in space from 1_a and 3_a, the commutated E.M.F., curve II, lagging 90 degrees behind curve I, will represent the E.M.F. of inductors 2_a and 4_a. Their sum is represented by curve III. From this curve it is evident that the fluctuation has been greatly reduced since the E.M.F. no longer falls to zero.

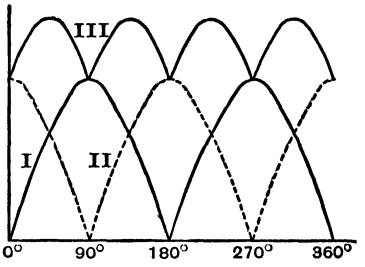


Fig. 264.—E.M.F. Curve Four-part Commutator

The fluctuation has been reduced to about 30 per cent by the use of two coils in series. It can be still further reduced by using more coils in series, each coil connected to a commutator segment. In practice any number of segments per pole greater than 30 may be regarded as giving a practically constant E.M.F.

4-coil Drum Winding.—The next winding to be considered is a 4-coil drum winding. There are eight surface inductors, as shown in fig. 265. These are numbered 1, 2, 3, 4, 1′, 2′, 3′, 4′, and are connected as follows: Segment 1 is connected to inductor 1 at the front. At the back 1 is connected to 1′, lying under a pole of opposite sign, and 1′ at the front is connected to segment 2. The remainder of the winding is finished in the same way. Again the winding forms a closed circuit, and there are two parallel paths between the brushes, namely—

$$B_1 \text{ (+ve)} \begin{cases} \text{seg. } 3 - 3 - 3' - 4 - 4' - \text{seg. } 1 \\ \text{seg. } 3 - 2' - 2 - 1' - 1 - \text{seg. } 1 \end{cases} B_2 \text{ (−ve)}$$

There is no resultant E.M.F. round the closed circuit, exactly as in the case of the ring winding. The fluctuation of the E.M.F. between the brushes is exactly the same as for the

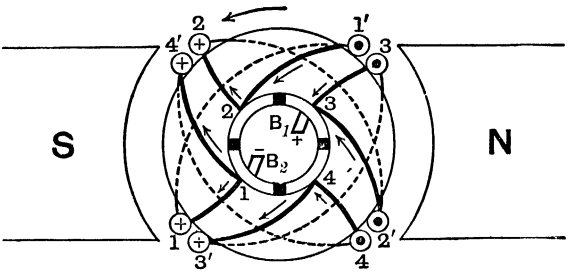


Fig. 265.—4-coil Drum Winding

ring winding, and the fluctuation may be reduced in the same manner.

Multipolar Drum Windings.—Ring winding for direct-current armatures, though it was the earliest, has practically become obsolete. Its only advantage over drum windings is, that the difference of potential between adjacent conductors is small and equal to the E.M.F. induced in one turn, while in a drum winding the difference of potential is equal to the full voltage given by the dynamo. Ring winding is therefore advantageous in the case of high-voltage dynamos since less insulation is required. On the other hand, a ring winding is costly to manufacture, and requires more copper than a drum winding. For these reasons, therefore, the ring winding has become disused.

Modern dynamos are usually made multipolar, i.e. they have more than one pair of poles, and have drum windings. Before proceeding to discuss these windings, however, the following definitions must be understood: The name *inductor* is given to that part of a conductor which cuts the magnetic flux, and therefore has an E.M.F. induced in it. In a drum winding these inductors are formed into loops or coils, each of which has two active inducing sides. Each of these sides constitutes one winding *element*. A winding element may consist of one bar or a group of wires forming the coil side. The group is treated as if it were a single bar, which amounts to replacing the multi-turn coil by a loop of one turn. This is exactly the same as in the practical case, since a multi-turn coil is made up on a

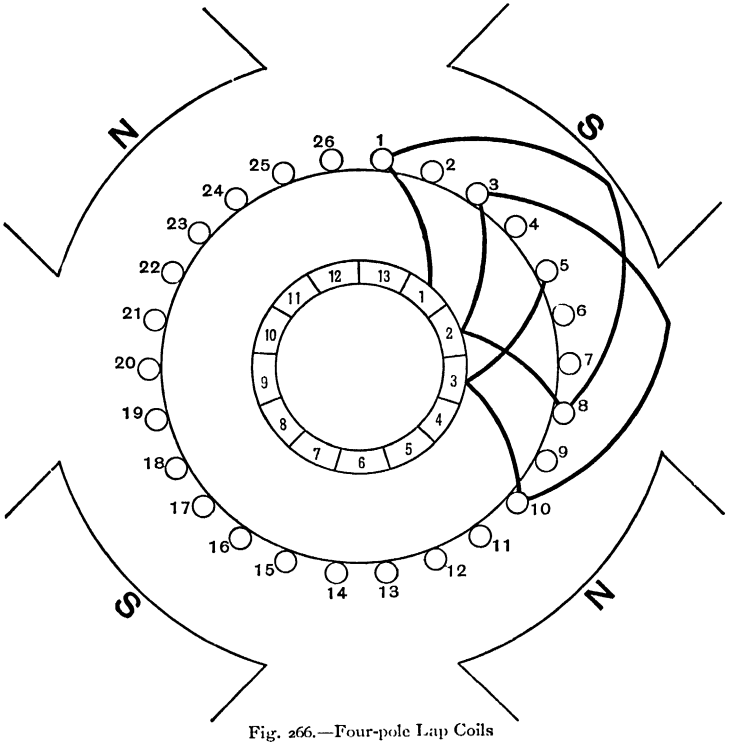


Fig. 266.—Four-pole Lap Coils

former, and formed to the proper shape, two ends only being left for connection to the commutator segment, thus replacing the multi-turn coil by one with a single turn. Between each two adjacent commutator segments there are two winding elements. In a drum winding, elements are connected in series which lie under poles of opposite polarity, and the span between elements connected in series is approximately a pole pitch. It may be shorter in order to save copper, but must be greater than the polar arc, otherwise it is possible for the two elements in series to lie under one pole and have E.M.F.'s which are in opposition induced in them, thus reducing the E.M.F. of the dynamo. As explained before, a drum winding must be closed or re-entrant, i.e. if the winding is traced through from any point, each element must be passed once through and no more before the starting-point is again reached. Care must therefore be taken, when choosing the pitches of the coils, that this condition is fulfilled.

Take the case of a 4-pole surface-wound armature (fig. 266) with 26 surface conductors. To start the winding, segment 1 on the commutator is connected to element 1 of the winding at the front. To form a loop element, 1 must be connected to an element distant one pole pitch, i.e. separated by 90 degrees, from element No. 1. There are two elements which could be chosen, element 8 or element 9. Element 9 must not be chosen, since

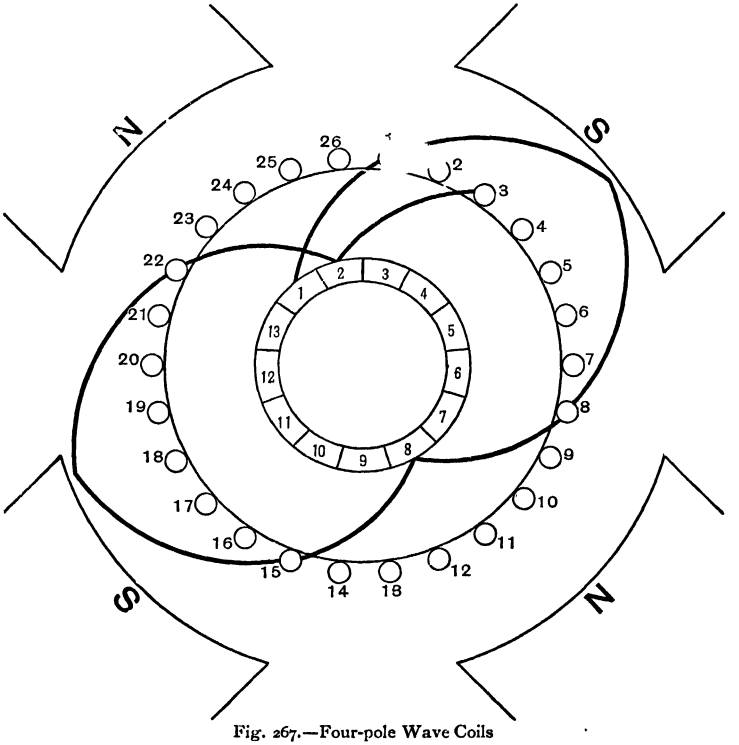


Fig. 267.—Four-pole Wave Coils

it will be required later on for the start of another loop. The first loop can now be formed by joining element 1 to element 8 at the back. The winding pitch at the back, or shortly the back pitch y_B, is the space between two elements connected in series, measured in elements passed over. In the example $y_B = 8 - 1 = 7$. Now the back pitch should be approximately a pole pitch, therefore if z_1 be the number of winding elements and p the number of pole pairs, the back pitch y_B should be nearly equal to $\frac{z_1}{2p}$, and the nearest odd number has been chosen, viz.

$\frac{s_1 + 2}{2p} = \frac{26 + 2}{4} = 7$. The loop spans slightly more than a pole pitch. An odd number must be chosen for the back pitch, so that the winding may be re-entrant.

The connection to the next loop may be made in two ways: (*a*) by joining the front of 8 to commutator segment 2 and to the front of element 3 (fig. 266), (*b*) by joining the front of 8 to

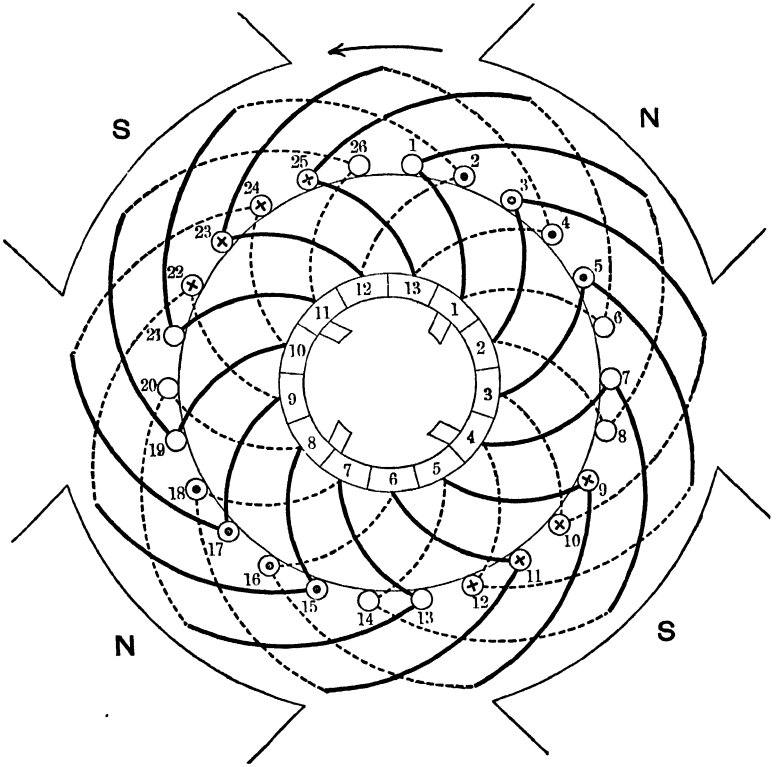


Fig. 268.—Four-pole Lap Winding

segment 8 and to the front of 15 (fig. 267). The first method gives a lap winding, the second a wave winding.

Lap Winding.—In lap winding the loop is completed by connecting to the next element but one to the element from which the loop started. The front pitch y_F is therefore measured in the opposite direction to the back pitch, and if the back pitch is called the positive direction the front pitch is negative, and smaller than the back pitch.

The winding is completed by forming loops similar to the original loop. Fig. 266 shows the completion of two loops, the second loop being formed as follows: Element 3 is connected at the back to element 10 (this element is found by adding the back pitch y_B to the number of the element from which the

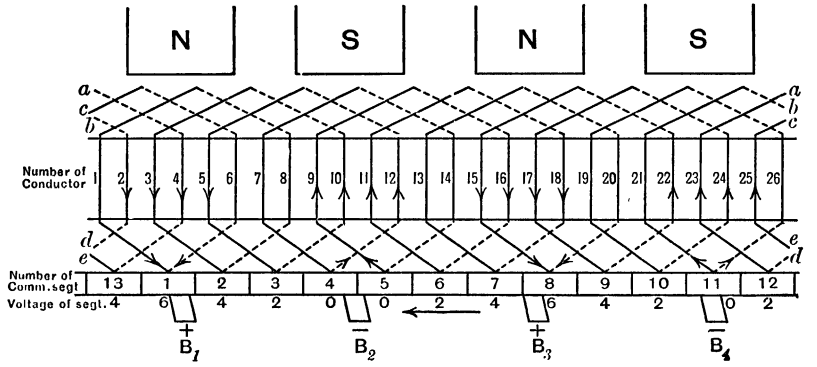
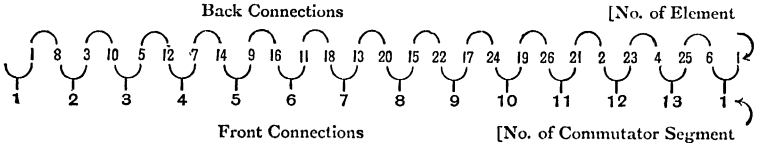


Fig. 269.—Development of a Four-pole Lap Winding

loop starts), element 10 is connected at the front to commutator segment 3 and element 5. Proceeding in a similar manner to form the other loops the winding can be completed as shown in figs. 268 and 269. Fig. 269 is merely the winding shown in fig. 268 developed. The connections between the elements are given in the following table:—



From this table it is evident that although the pitches are alternately measured backwards and forwards, the winding as a whole progresses forward till it closes at segment 1.

In the developed winding, fig. 269, the arrows show the direction of the E.M.F. induced in the windings. Assuming that 1 volt is induced in each element under a pole, and that the potential of segment 11 is zero, then the potential of each segment of the commutator is given by the numbers below the segment. It is evident that there are two points of maximum potential and two points of zero potential on the commutator.

Brushes can now be placed at the segments of maximum potential and of zero potential. The brushes of maximum potential are positive, and those of zero potential negative, and corresponding brushes are connected together to form the terminals of the machine. On tracing through the winding it will be found that there are four circuits, and that the paths through the armature are as follows:—

$$+ \begin{cases} B_1 \begin{cases} 1 - 8 - 3 - 10 - 5 - 12 - B_2 \\ 6 - 25 - 4 - 23 - 2 - 21 - B_4 \end{cases} - \\ B_3 \begin{cases} 15 - 22 - 17 - 24 - 19 - 26 - B_4 \\ 20 - 13 - 18 - 11 - 16 - 9 - B_2 \end{cases} \end{cases}$$

The elements 7 and 14 are short-circuited by brush B_2 for the position of the winding as shown in the diagram. The short-circuited coil changes from brush to brush as the armature revolves.

The distinguishing feature of lap winding, as illustrated above, is that there are as many paths through the armature, from positive to negative brushes, as there are poles.

The formulas and results obtained may now be summarized.

A lap winding is characterized by the formation of loops which overlap each other, hence the name *lap winding*. The number of paths through the armature is always equal to the number of poles. The back and front pitches must both be odd and differ by 2. The back pitch is approximately equal to $\frac{z_1}{2p}$. The front pitch is negative, and is obtained from the formula

$$y_B + y_F = \pm 2.$$

The number z_1 of elements may have any even value, but is usually arranged to be divisible by the number of poles, so that the winding may be symmetrical. It will be noted that the winding given above does not fulfil this condition, and is therefore not symmetrical.

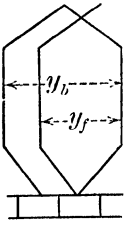


Fig. 270

Fig. 270 shows a developed single-turn coil with two elements, and indicates the pitches. In high-voltage slow-speed dynamos it is found impossible to insert a sufficient number of segments in the commutator to enable single-turn coils to be used, so that coils having 2, 3, or 4 turns in series are

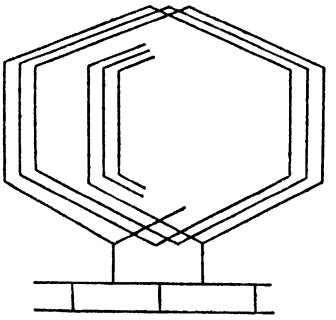


Fig. 271.—Multiturn Lap Coil

required. Fig. 271 shows a development of such a 3-turn coil. As explained above, each side of the coil forms a winding element, and is numbered as a whole instead of assigning separate numbers to each inductor in the coil.

Wave Winding.—In wave winding, or as it is sometimes called two-circuit winding, the first loop is connected to a similar loop similarly situated under the next pole pair. In the example (fig. 267) this next loop is formed from elements 15 and 22, the back pitch y_B for this loop being as before equal to 7. In a wave winding, therefore, both the front

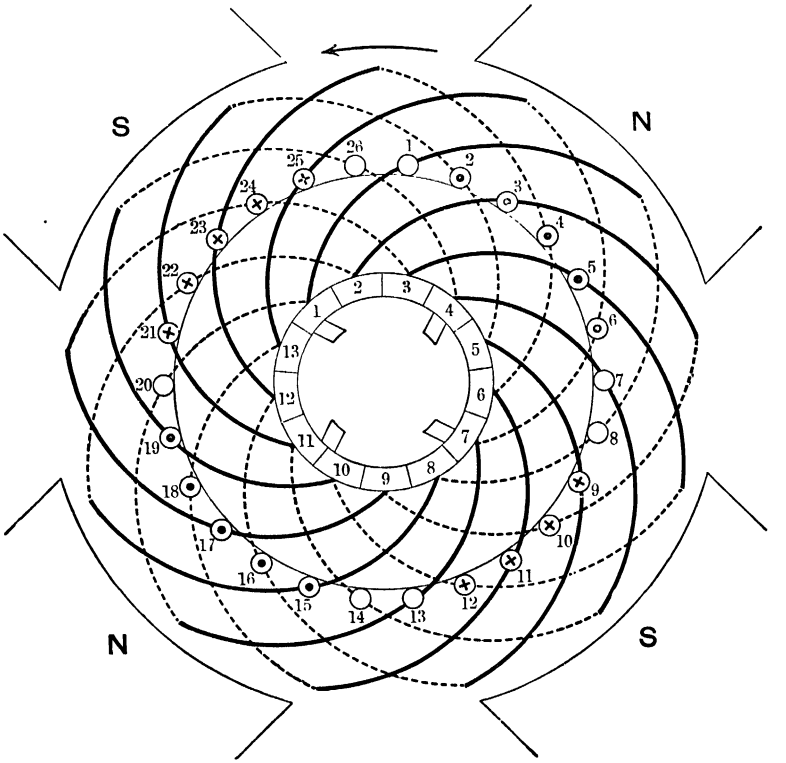


Fig. 272.—Four-pole Wave Winding

and back pitches are measured in the same direction, and must be odd if the winding is to be closed. The element next but one to the element from which the winding started is only reached after a complete tour round the armature has been made. To complete the winding, similar tours are made till the winding closes.

In the example, the winding is started by connecting commutator segment 1 to element 1 at the front. Add on the back

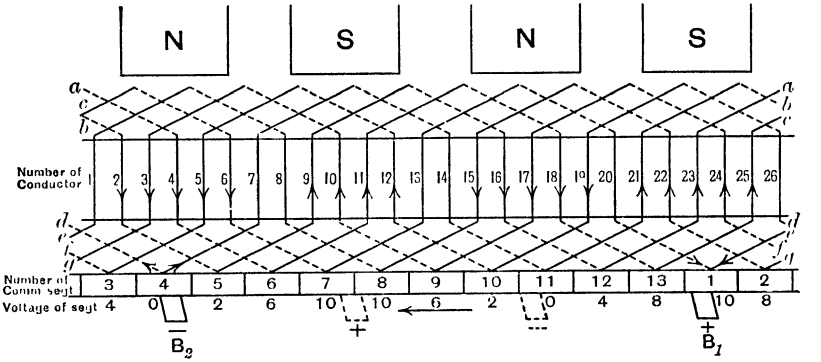
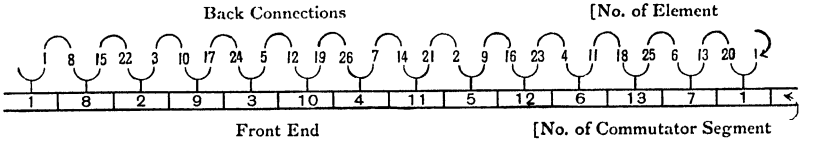


Fig. 273.—Development of a Four-pole Wave Winding

pitch $y_B = 7$ to find the element 1 is connected to at the back. This gives element 8. Element 1 is now connected to element 8 at the back, then element 8 is connected at the front to segment 8 and element 15 (this element is found by adding on the front pitch $y_F = 7$). Element 15 is next connected to element 22 at the back. The tour round the armature is then completed by connecting element 22 at the front to segment 2 and element 3. Similar tours are made, till the winding is closed at segment 1 (figs. 272 and 273). The complete winding is given in the following table:—



The average winding pitch $= \frac{y_B + y_F}{2} = 7$.

Now, in the above example it will be seen that the distance apart, measured in segments, of the points where the winding

touches the commutator is 7. So that in a wave winding the winding touches the commutator segments whose pitch is equal to the average of the front and back pitches of the winding. The number of tours round the armature before the winding closes is also equal to the average of the winding pitches.

Since the winding after one tour round the armature reaches the element next but one to the start, the number of elements and the average winding pitch are not independent of each other, but are connected by the following equation:—

$$z_1 = 2p\,y_{av} \pm 2.$$

$$\text{or} \quad y_{av} = \frac{z_1 \pm 2}{2p}.$$

Now y_{av} must be a whole number, therefore only certain numbers of elements are permissible for a wave winding.

Assuming, as before, that an E.M.F. of 1 volt is induced in each element under a pole, the numbers below the commutator segments (fig. 273) give the potentials of the segments, assuming that segment 4 is at zero potential. From the diagram it is found that there are two points of zero potential and two points of maximum potential, but the two points of zero potential belong to the same coil, and the two points of maximum potential also belong to the same coil, so that only two brushes are required for a wave winding. On tracing through the winding from the positive to the negative brush it will be seen that there are only two circuits through the armature, as follows:—

$$\begin{array}{l} B_1 \\ +ve \end{array} \begin{array}{l} \swarrow 1 - 8 - 15 - 22 - 3 - 10 - 17 - 24 - 5 - 12 - 19 - 26_1 \longrightarrow B_2 \\ \searrow 20 - 13 - 6 - 25 - 18 - 11 - 4 - 23 - 16 - 9 - 2 - 21 - 14 - 7 \nearrow -ve. \end{array}$$

In wave windings the number of elements in each half of the winding is usually not the same. This is unavoidable from the nature of the winding. Since no E.M.F. is induced in the extra conductors their presence does not interfere with the balance of the winding, and in large armatures the effect on the resistance of each path is very small. As the armature rotates the extra conductors shift from one circuit to the other and back again.

When two sets of brushes are used in a wave winding, they may be placed at any angle equal to the angle between a north and south pole, e.g. in a 6-pole dynamo the angle between the brushes may be either 60 degrees or 180 degrees.

In order to keep down the length of the commutator, as many sets of brushes as there are poles are used. In fig. 273 brushes may be placed in the positions shown dotted. These brushes bear on the same coils as the brushes B_1 and B_2, so that they assist B_1 and B_2 to collect the current. The size of the brushes B_1 and B_2 may be reduced, but this reduction is counterbalanced by the increased tendency to spark due to unequal distribution of current between the brushes. In short, the wave winding is characterized by the following rules:—

Both back and front pitches are measured in the same direction, and are odd.

The back pitch may be equal to, less, or greater than the front pitch.

The average pitch $= \frac{y_B + y_F}{2} = \frac{z_1 \pm 2}{2p}$.

The number of elements and the average pitch are not independent of each other.

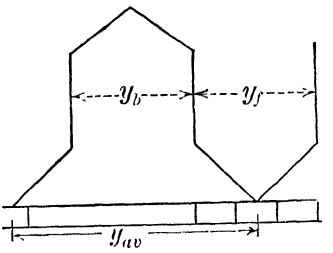


Fig. 274.—Single-turn Wave Coil

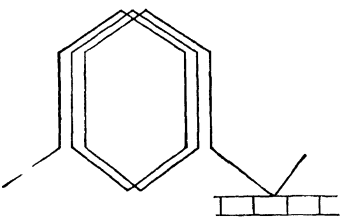


Fig. 275.—Multiturn Wave Coil

There are only two paths through the armature from a positive to a negative brush, and half the armature conductors are in series.

Fig. 274 illustrates a single-turn wave loop, and fig. 275 illustrates a multi-turn wave loop.

Examples of Lap and Wave Windings.—Fig. 276 shows a diagram of a lap winding for a 6-pole machine with 48 elements and 24 commutator segments The back pitch is therefore from the formula—

$$y_B = \frac{z_1}{2p} = \frac{48}{6} = 8.$$

y_B is therefore chosen equal to 7.

$$y_F = y_B \pm 2 = 7 + 2 = 9.$$

The back pitch is therefore 7 and the front pitch 9.

The radial lines represent elements, the connecting lines inside the front connections, and those outside the back connections. The thick black line traces out one coil or section of the armature between two commutator segments. Fig. 277 shows a diagram of wave winding for a 4-pole dynamo with 46 elements and 23 commutator segments.

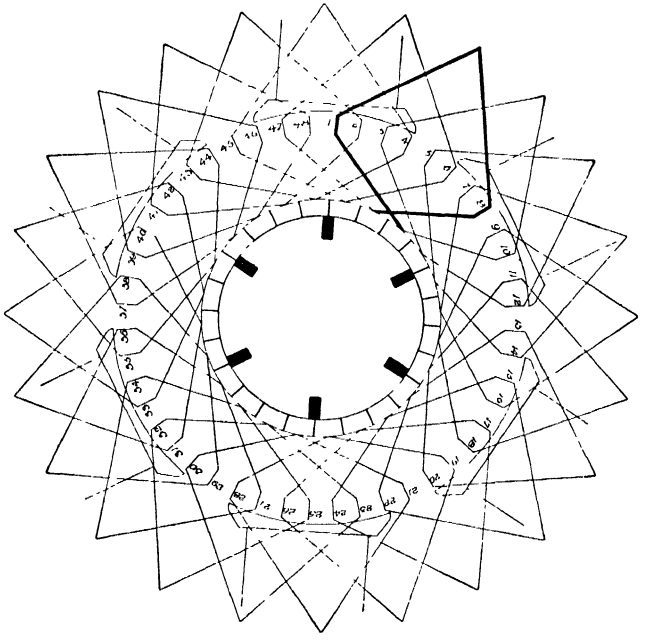


Fig. 276.—Diagram of Lap Winding

The average pitch is $= \frac{z_1 \pm 2}{2p} = \frac{46 - 2}{4} = 11$. The back and front pitches are chosen equal to 11. The winding touches the commutator at every eleventh segment. The thick black line, as before, shows a section or coil between two commutator segments.

The elements in a winding are usually arranged two or more in a slot in two layers, as shown in fig. 278. It is usual to number them so that odd numbers are at the top and even at the bottom of the slot. For mechanical reasons it is advantageous to connect an element in the top layer to one in the bottom layer, and vice versa, so that the pitches must be odd.

In using wave windings it will often be found convenient

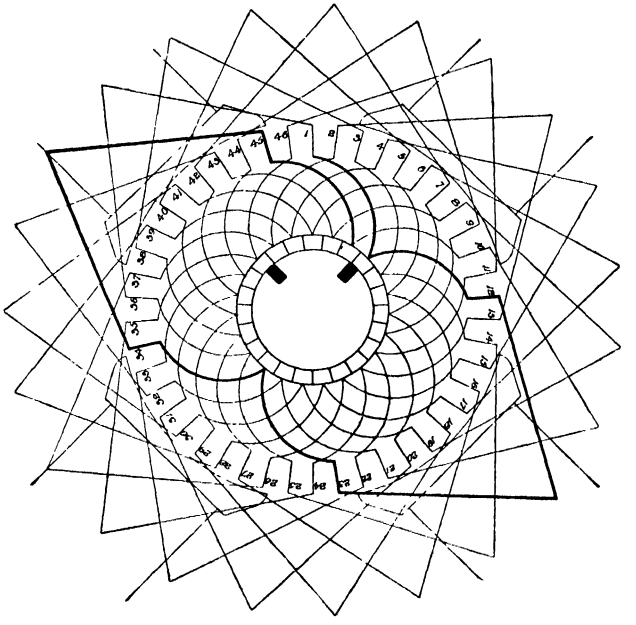


Fig. 277.—Diagram of Wave Winding

to use a number of slots which is not an even multiple of the number of commutator bars, as, for example, in an armature wound with 198 inductors connected to 99 commutator bars, and having 50 slots.

In this case a dead section or 2 dead inductors may be used to fill up. Thus in the above armature there would be four inductors per slot or 200 total, i.e. 198 + 2 dead inductors. In numbering the inductors these dead bars are omitted. They help to preserve the mechanical balance of the armature.

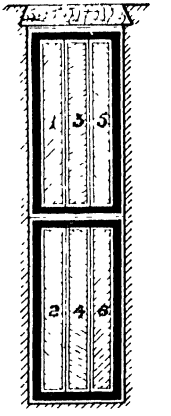


Fig. 278.—Diagram of Slot

With machines of any size, with lap-wound armatures, it is practically impossible to avoid small pressure differences between the circuits of the armature, due to unequal strength of the magnetic circuits. These inequalities may be due to eccentric boring of poles, unequal quality of iron in each pole, brackets and projections on yoke, or to various other causes, and may cause serious

sparking trouble at the commutator, due to the unequal loads on individual lines of brushes.

In order to counteract this, it will be found necessary, with lap winding, to use equalizing rings or cross connectors. These are used to couple up points in the armature circuits, which should be at the same potential, whereby any internal currents generated by the differences of pressure mentioned may be prevented from flowing through the brushes and causing sparking. It will generally be found sufficient if 4 or 5 points in each circuit of an armature between the brushes are coupled up thus.

If 4 points are used there will be 8 rings necessary, and if 5 points 10 rings necessary, and so on. If p be the number of pairs of poles in the machine, each ring will be coupled to the armature at p equidistant points.

E.M.F. Induced in Drum Windings.—The E.M.F. induced in a wire is equal to the rate of cutting lines of force, and if 10^8 lines of force are cut per second the E.M.F. induced is 1 volt. Consider the case of a 2-pole dynamo whose armature rotates at a speed of n revolutions per second. If the flux issuing from the north pole is N lines of force, then an inductor cuts 2 N lines of force in one revolution of the armature. The time of one revolution is $\frac{1}{n}$ sec. The average E.M.F. induced is equal to the flux cut divided by the time taken to cut it, so that the average E.M.F. induced in the inductor is equal to $2\,N \Big/ \frac{1}{n} \times 10^{-8}$ volts, i.e. $2\,Nn \times 10^{-8}$ volts. Now in a 2-pole armature with Z surface conductors there are Z/2 inductors in series between the brushes, so that the average E.M.F. between brushes is Z/2 times the E.M.F. induced in one wire.

$$\therefore E = NZn \times 10^{-8} \text{ volts (2-pole).}$$

In the case of a multipolar dynamo (with $2p$ poles) the above formula is modified slightly. The flux cut in one revolution is now $2pN$, and the average E.M.F. induced in one wire is $2pNn \times 10^{-8}$ volts. In the case of a lap winding there are $2p$ paths through the armature, so that the total number of inductors in series is $Z/2p$. The average E.M.F. induced between brushes is

$$\begin{aligned} E &= Z/2p \times 2pNn \times 10^{-8} \text{ volts} \\ &= NZn \times 10^{-8} \text{ volts (lap winding).} \end{aligned}$$

A wave winding has only two circuits through the armature, and has therefore Z/2 conductors in series. The average E.M.F. induced between brushes is now

$$\begin{aligned} E &= Z/2 \times 2pNn \times 10^{-8} \text{ volts} \\ &= pNZn \times 10^{-8} \text{ volts (wave winding).} \end{aligned}$$

These two formulas for the E.M.F. induced in wave and lap windings can be combined as follows. If $2q$ is the number of parallel paths through the armature, then the number of conductors in series is $Z/2q$. The average E.M.F. induced between brushes therefore

$$E = Z/2q \times 2pNn \times 10^{-8} \text{ volts} = p/q \times NZn \times 10^{-8} \text{ volts.}$$

Since the E.M.F. induced in a wave winding is greater in the ratio $p/1$ than that induced in a lap winding for the same number of inductors, wave windings are employed for the armatures of small machines designed to give high voltages. It is also suitable for machines, up to 200 or 300 kilowatts output, running at slow speeds. Lap winding, on the other hand, has as many paths through the armature as there are poles, and is therefore used in small dynamos which have to give large currents, since the current per path is smaller and the size of the armature conductor is smaller. It is also used on large machines for all outputs.

Calculation of E.M.F.—A 4-pole 7.5-kilowatt dynamo has 990 surface conductors arranged in a wave winding. On open circuit the E.M.F. induced when running at 800 r.p.m. is 250 volts. Find the flux per pole.

The formula for the E.M.F. induced in a wave winding is

$$\begin{aligned} E &= pNZn \times 10^{-8} \text{ volts.} \\ \therefore N &= \frac{E \times 10^8}{pZn} \\ &= \frac{250 \times 10^8}{2 \times 990 \times \frac{800}{60}} = 0.95 \times 10^6 \text{ lines.} \end{aligned}$$

The flux per pole is therefore 0.95×10^6 lines.

A 6-pole, 150-kilowatt generator with a lap-wound armature is to give 250 volts at 400 r.p.m. If the flux per pole is 6.06×10^6 lines, calculate the number of surface conductors.

For a lap winding

$$E = NZn \times 10^{-8} \text{ volts.}$$

$$\therefore Z = \frac{E \times 10^8}{Nn}$$

$$= \frac{250 \times 10^8}{6.06 \times 10^6 \times \frac{400}{60}} = 618.$$

The armature will have 618 surface conductors.

Construction of Armature.—When a solid block of iron is rotated in a magnetic field the direction of the flux through it is reversed, so that hysteresis loss occurs. In addition to this effect, however, since iron is a conductor, E.M.F.'s are induced in the core, which set up currents (known as eddy currents) in directions as shown in fig. 279. Though these E.M.F.'s are small, the resistance of the circuits through which they flow is small, so that the currents and consequent energy loss may be very great. To reduce the loss, the resistance of the circuit is increased, and the E.M.F. induced is decreased by building up the core from thin plates. The armature built up of these thin plates is said to be laminated, and the plane of lamination is at right angles to the shaft and to the active conductors. The laminations are insulated from each other by a thin sheet of paper or by a layer of varnish. The energy lost in eddy currents is therefore reduced, but cannot be eliminated by decreasing the thickness of the plates.

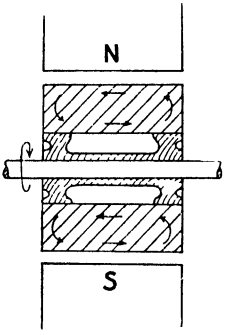


Fig. 279.—Eddy Currents in Armature Core

In practice, armature cores are built of laminations of soft annealed iron or steel, or of special alloys, such as stalloy, which have low hysteresis and low eddy-current losses. These laminations are from 0.35 mm. to 0.5 mm. thick, usually the latter thickness, and when built up under pressure the actual iron length of the core is from 85 to 90 per cent of the gross length, the remainder being taken up by the insulation. The slots are punched in the core before assembly. When the core has been assembled the slots must not be filed or machined. Filing the slots burrs over the edges of the laminations and so increases the eddy-current losses. These laminations are supplied in plates either 1 m. square or 2 m. by 1 m., so that in armatures up to

90 cm. diameter the stampings are in the form of a continuous ring, while above this diameter the core is built up of segments.

In small armatures the stampings are threaded directly on the shaft, a parallel key being fitted in the shaft to keep them in position, as shown in fig. 280. The end plate C_1 is placed in position on the shaft, and rests against the shoulder D. The laminations are now slipped on, being kept in alignment by drifts placed in a few of the slots and by the key K. The end plate C_2 is put in position, and the laminations are pressed together. When the core is sufficiently compressed, the end plate C_2 is locked by the circular key S, which springs into a square groove cut in the shaft. Instead of this key a nut is

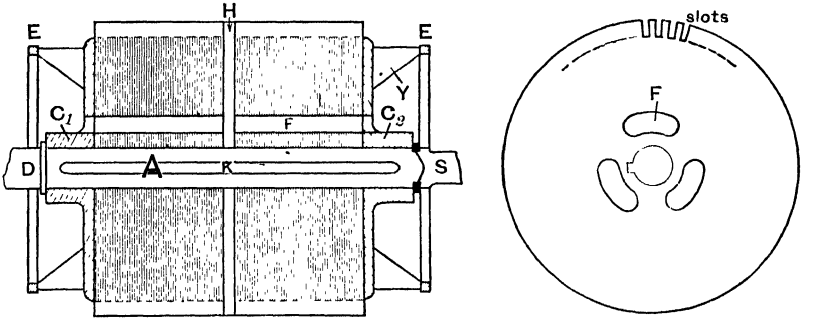


Fig. 280.—Armature Core for Small-diameter Armatures

often used, the shaft being screwed. The end plates C_1 and C_2 carry end rings E, which support the windings, the end connections being held down on them by a layer of binding wire. These end rings are supported by vanes which blow air through the windings to cool them. The diagram on the right of fig. 280 shows one stamping. When there is sufficient cross section of iron without utilizing the whole radial depth of the core, part of the core is cut away as at F to provide an air duct. The air passes in at the ends of the core and out at H, thus cooling the armature iron.

In armatures of larger diameter, where the segment is in the form of a complete ring, and where a small radial depth of stamping is sufficient to give the desired flux density, the stampings are mounted on a cast-iron spider B, which is pressed and keyed on to the shaft A (fig. 281). Keyways are punched in the interior of the ring, which fit on the arms D of the spider.

One end plate C_1 is either cast solid with the spider or bolted to it. The laminations, after being assembled, are compressed by the second end plate C_2, which is held in position by the spring key S. Sometimes bolts are used which pass through the core and so compress the plates. The armature core is thus self-

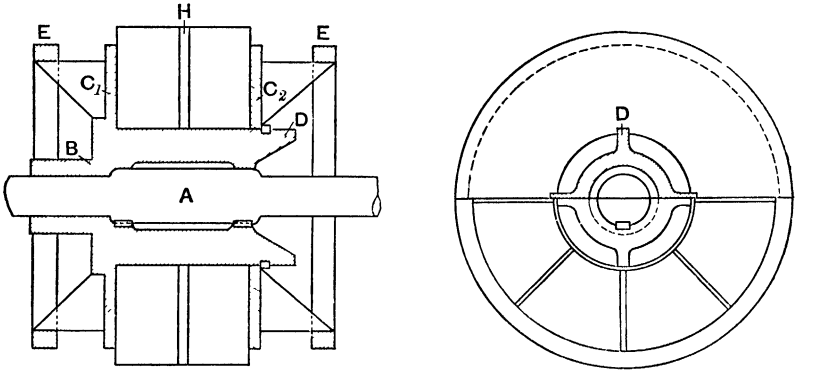


Fig. 281.—Armature Core for Medium-diameter Armature

contained, and a new shaft may be fitted without rebuilding the core. The end rings E carry the windings.

In armatures of still larger diameter, which from considerations of cost are built up of segments, the construction shown in fig. 282 is adopted. The laminations are built up on a cast-iron spider, which is pressed and keyed on to the shaft. The

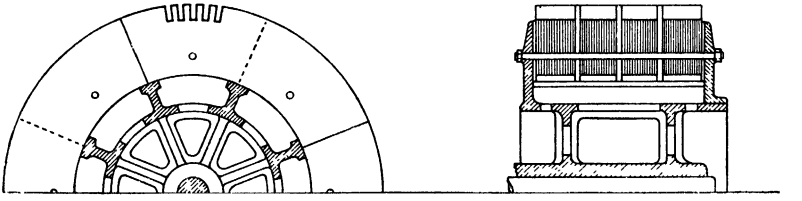


Fig. 282.—Armature Core for Large-diameter Armature

laminations are built up with butt joints, and are so arranged that the joints in successive layers do not overlap. In the drawing the laminations rest on the ends of the spider arms which are turned to the correct diameter. To prevent the segments from moving radially under the action of the centrifugal force due to rotation, they are held by bolts passing through the core and end plates. These bolts also compress

the laminations between the end plates. Sometimes the segments have wedge-shaped lugs, which dovetail into recesses cut in the spider arms, and so retain the segments against the action of centrifugal force. As before, one end plate is cast solid with the spider, while the other is held in position by bolts passing through the core. In the figure a ventilating spacer is placed between each end plate and the core.

In order that the heat generated in the armature may be dissipated, the armature core must be provided with channels

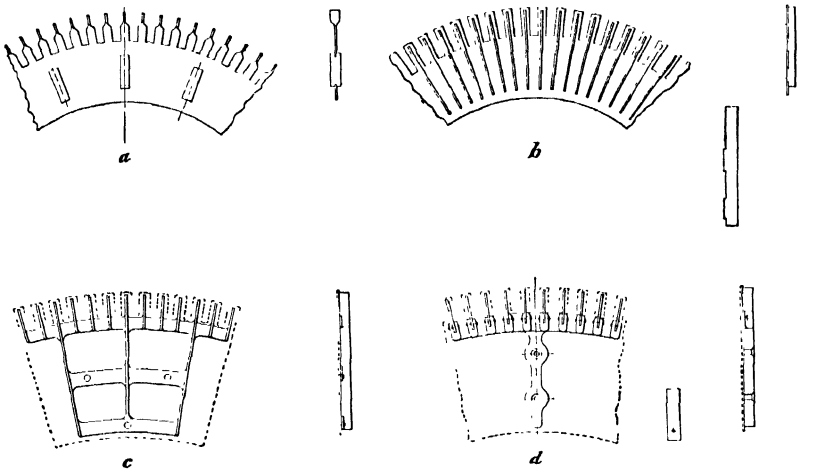


Fig. 283.—Types of Ventilators

or ducts to allow air free passage into and through the core. For this purpose the laminations are separated at intervals by distance pieces, so as to form radial air ducts, as shown at H in figs. 280, 281, and 282. These must be made of as light conconstruction as is consistent with mechanical strength, so that they offer no obstruction to the passage of air. If the ventilators are properly designed, they act as fans to drive air through the core.

Ventilating ducts should be provided at least every 5 cm., and these are generally from 1 cm. to 1.25 cm. wide. Various types of ventilators are employed, and fig. 283 shows drawings of some of these. The types shown in *a* and *b* are very simple and effective, being made from 0.15-cm.-thick plates, stamped with teeth of the same dimensions and number as the

rest of the disks. In the case of fig. 283, *a*, the tops of these teeth are turned round at right angles to the plate. Pieces of bar iron 1 cm. square are fitted in slots punched in the plates below the teeth as shown, in order to keep the body of the armature disks apart. In fig. 283, *b*, the teeth are not turned, but strips of 0.15-cm.-thick iron are fitted to the ventilator plate at each tooth as shown. In fig. 283, *c* and *d*, the distance pieces are brass castings which are riveted on to 0.15-cm. plates.

In reckoning up the net effective length of armature iron, the space occupied by these ventilators must be deducted from the total length of armature over the disks.

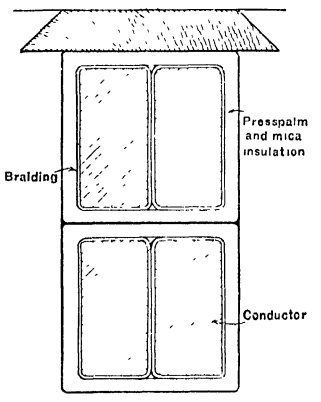


Fig. 284.—Section of Slot

All end flanges, winding supports, &c., made of unlaminated iron should be kept away from the magnetic field, or eddy currents will be induced in them. Any bolts passing through the core must either be placed where they cut few lines of force or be insulated by fibre tubes, so as to decrease the eddy-current loss.

The copper conductors are wound into coils on formers which give the coil the proper shape. The conductors are insulated with either a double cotton covering or braiding whose thickness varies from 0.3 to 0.6 mm. The portion of the coil which lies in the slot is then insulated with presspahn, or micanite. Since presspahn, or micanite, when used in thick sheets, is liable to crack at sharp bends, the insulation is built of a number of turns of thin material. Very often both materials are used. The usual thickness of insulation is about 1.5 mm. for 500-volt dynamos. The whole coil is then taped with linen tape to a thickness of 0.3 mm. The completed coil is now dried under vacuum and dipped into an insulating varnish, which prevents moisture entering the coil and so destroying its insulation. Before placing the coil in the slot a thin lining of presspahn is inserted to prevent the rough edges of the slot cutting the insulation. Fig. 284 is a drawing of a slot showing the conductors and insulation. Coils lying in the top layer must be well insulated from those in the bottom layer, since

the full potential difference of the machine may exist between them. The coils are held in position against the action of centrifugal force in the following manner. In large machines the portion of the coils inside the slots is kept in place by beech wedges, which fit into recesses in the teeth. The end connections, and in the case of small machines the portions lying in the slots, are held in position by external bands of wire. The binding wire is phosphor bronze or steel, and has a diameter of from 0.5 to 1.8 mm. The wire is wound on under tension on the top of a band of micanite which prevents the cutting of the coil insulation by the wire and insulates it from the conductors. The width of the band should not exceed 2 cm., so as to prevent excessive eddy-current loss. Copper clips are placed at intervals round the circumference. These clips are soldered to the binding wire, and serve both to hold it in place and secure the ends.

Commutator Construction and Brush Gear. — The commutator is usually built up from hard drawn, or rolled copper segments of tapered cross section, insulated from each other by mica insulation.

The copper must be of uniform hardness, so that it will wear uniformly. Mica is used as the insulator, since it wears at approximately the same rate as copper. In addition it is not hygroscopic, and does not carbonize when sparking takes place at the commutator. Mica also cleaves easily into laminæ of uniform thickness. Green-coloured or Canadian amber mica is the best, but, since large sheets of it are very expensive, built-up mica, which is made by sticking flakes of mica together with shellac, and then compressing the sheet while heated, is generally used.

The segments and mica insulators are built up into a ring of approximately the correct diameter. The ring is then fixed in a clamping ring, which can be tightened so as to exert a radial pressure on the segments. The commutator is heated, and as it cools the clamping ring is tightened. When cool the V's are turned in the ends of the segments, and the end clamping rings are fitted into position and bolted together. The outer surface of the commutator is turned up, and the commutator is ready for connecting up to the winding. The clamping rings are insulated from the segments by mica cones 2 to 3 mm.

thick. The angle of the cone is about 35 degrees, 5 degrees above and 30 degrees below the horizontal. The top part of the cone serves to produce a good seating for the bars, whilst the clamping down and securing against centrifugal force is done by the under part of the cone.

The commutator diameter is usually about 0.7 to 0.8 of the

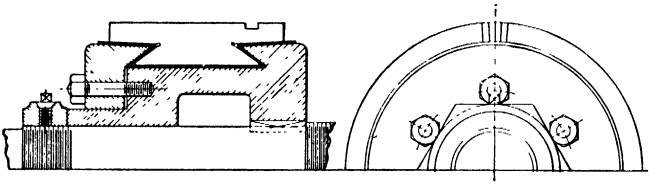


Fig. 285.--Commutator for Small Armature

armature diameter. The thickness of the segment at the top should not be less than 4 mm. and the thickness of the mica is usually about 0.8 mm.

Fig. 285 shows a construction suitable for a small armature. One clamping ring forms a spider which is keyed on to the shaft, and is kept in position against a shoulder by a nut. The loose ring, which is a sliding fit on the spider, is held in position by studs.

Fig. 286 is a drawing of a commutator for a large generator.

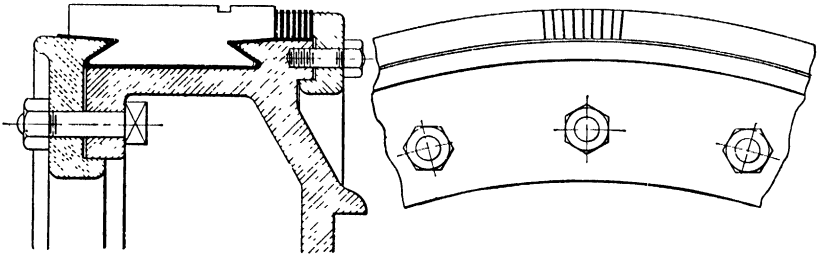


Fig. 286.--Commutator for Large Armature

The commutator spider in this case is bolted to a continuation of the armature spider.

Carbon brushes are now used exclusively on commutators. At one time copper gauze brushes were used almost exclusively, but the increased liability to sparking and excessive wear on the commutator have caused their disuse. The carbon brushes are arranged to cover between 1 and 3 segments on the commutator.

Several small brushes are used on the same brush spindle instead of one large one, since a small brush is more likely to give good contact with the commutator over all its surface than a large brush.

In fig. 287 A shows a box-type brush holder. In this type

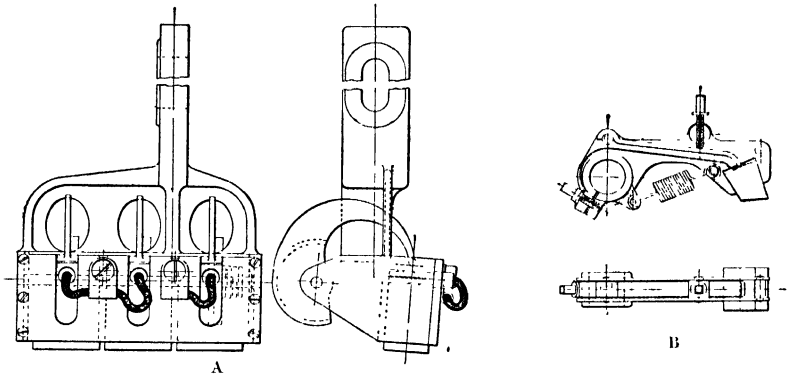


Fig. 287.—Box and Hammer Type Brush Holders

the brush slides up and down in a box forming part of the holder, the current being led into the brush by a flexible copper "pigtail". Fig. 288 shows a brush suitable for use in this type of holder. A small brass tube is let into the carbon block; into this tube a split pin slides, and from this split pin the flexible connection is taken to the holder. The brush is pressed against the commutator by the bent arm which rests on it. The pressure is applied to this arm by altering the tension of a spring, one end of which is attached to the arm.

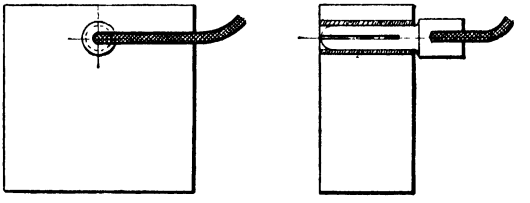


Fig. 288.—Sketch of Carbon Blocks

In fig. 287, B shows a hammer-type brush. In this type the brush is held rigidly in an arm which rotates round a bush fixed rigidly to the spindle. The pressure on the brush is given by a spring which is stretched between one end of the arm and the bush on the spindle. The disadvantages of this type are that: (1) the position of the brush alters slightly as the brush wears;

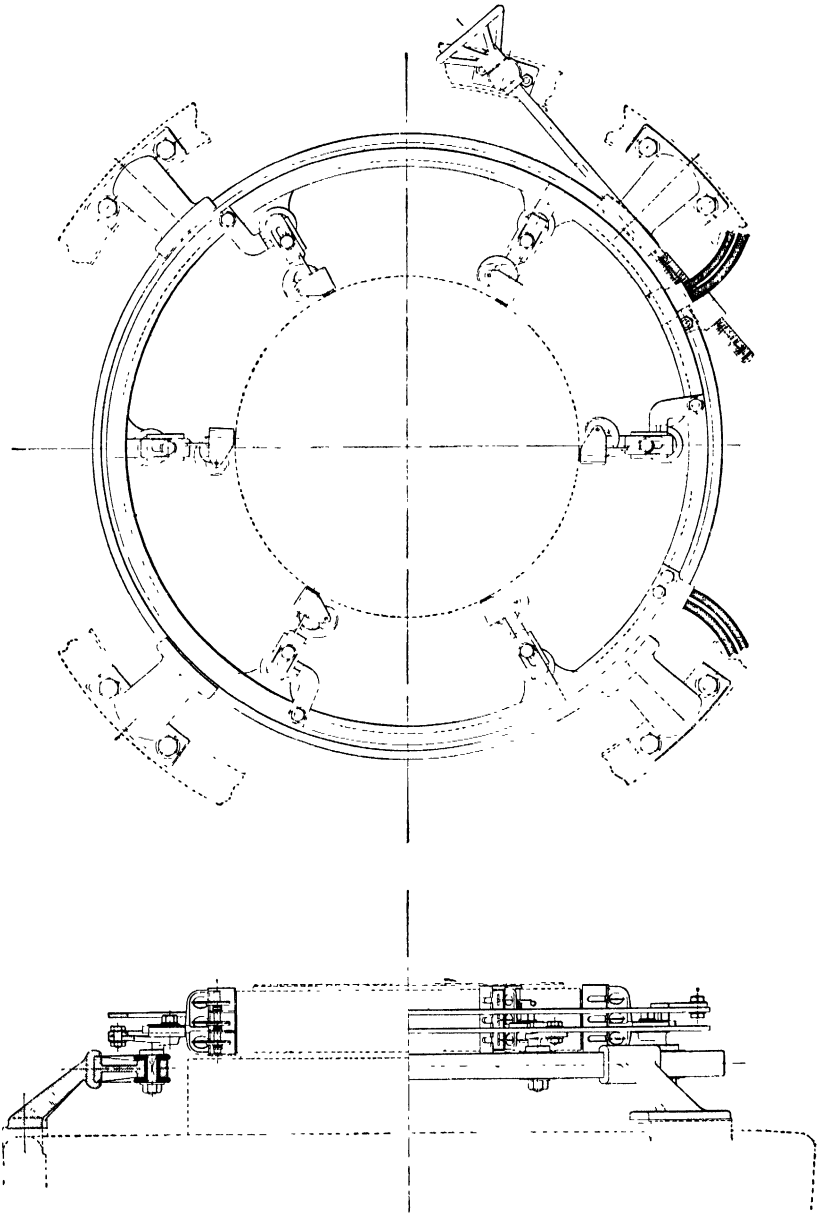


Fig. 280.—Brush Collecting Gear

(2) there is a danger of the arm touching the commutator when the brush gets worn down, and so causing sparking.

In small machines the brushes are mounted on an insulated spindle, which is held in arms projecting from a ring which is capable of rotation round the bearing at the commutator end. This rotation provides for adjustment of the brush position.

In large machines the brush spindles are supported by forks which are supported by and insulated from a ring. This ring is carried on brackets bolted to the field yoke (fig. 289). Provision is made for altering the brush position by means of the handle wheel. The spindle of this wheel gears with a worm bolted on to the ring.

CHAPTER XXII

THE FIELD MAGNET

Types of Field Magnets.—Originally dynamos were built with two pole fields, as shown in fig. 290, A. Both limbs were

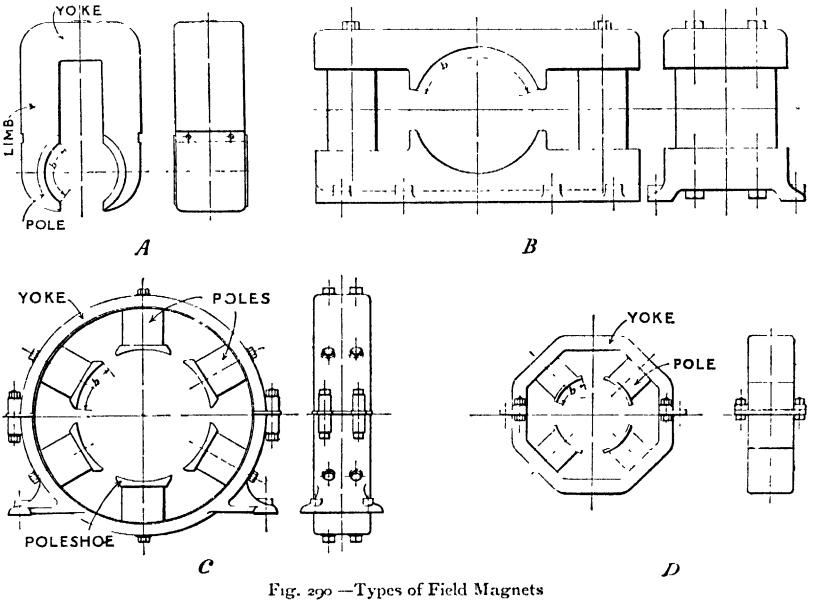


Fig. 290.—Types of Field Magnets

wound with exciting coils. This type of field had to be carried on a bedplate made of non-magnetic material, or supported

from a cast-iron bedplate by means of gunmetal supports, to reduce the leakage of flux from pole to pole through the bedplate. In small machines the poles were often inverted, so that the yoke became part of the bedplate. A great objection to this construction was the large leakage field produced. The Manchester type (fig. 290, B) was next adopted. The advantage of this type is that the magnetic circuit is completely ironclad, and the leakage flux is therefore reduced. It was found, however, that it was very difficult and uneconomical to build these machines for large outputs (above 300 kilowatts), so that multipolar designs (fig. 290, C and D) are now used. These designs are much more economical in material, and are used for all sizes from 1 kilowatt upwards. In fig. 290, D the poles are cast solid with the yoke, and in fig. 290, C they are bolted on to the yoke. The poles end in projections or pole shoes, which serve to distribute the flux over a larger portion of the armature surface. The edges of these pole shoes should be chamfered or rounded off to prevent too sudden changes of flux in the teeth as they enter or leave the pole face. These changes in the flux cause eddy-current losses in the shoe, which is laminated to decrease the loss.

Construction.—In a modern multipolar dynamo the yoke is made of cast iron or cast steel. The cross section of a cast-steel yoke only requires to be half that of cast iron, on account of the higher permeability and mechanical strength of the cast steel. The dynamo is therefore lighter although the relative cost is the same. The yoke is usually cast in the form of a ring, of oval or rectangular cross section. For ease in handling, the ring is often split, as in fig. 290, C.

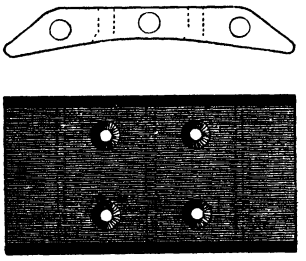


Fig. 291.—Laminated Pole Shoe

The poles are made of cast or rolled steel or of steel laminations similar to those used for the armature. If made of cast steel they may be either of rectangular or round cross section; if of laminations they are usually rectangular in cross section. When solid steel poles are used the pole shoe is built up of laminations riveted together, as shown in fig. 291, and held on to the poles by screws. The

shoe also serves to keep the exciting coils in place. Laminated poles are built up of stampings riveted together, and having the pole shoe as part of the stamping.

The poles are fixed to the yoke in many different ways. The chief difficulty is to get a good magnetic joint between the poles and the yoke, as the presence of a small air-gap at this portion of the magnetic circuit causes a large increase in the excitation required for the field.

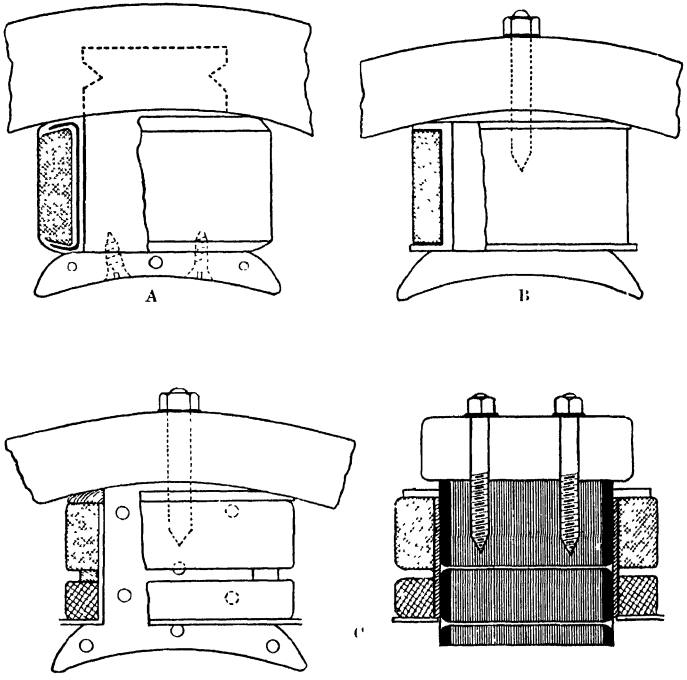


Fig. 292.—Types of Pole and Field-coil Construction

When cast-iron yokes are used the best method is to make the poles from steel forgings, and then cast them into the yoke (fig. 292, A). A good magnetic joint is obtained between the yoke and the pole. Bolted joints do not give satisfaction with cast-iron yokes, except when the poles are recessed into the yoke, and even in this case only with good workmanship. Cast-steel poles are often cast solid with the steel yoke, the pole shoe being held on by countersunk screws as explained above.

Laminated poles (fig. 292, C) and solid steel poles (fig. 292, B)

are usually bolted to the steel yoke. The yoke is bored out internally to the proper diameter, while the poles are turned to the same diameter, and a good magnetic joint is obtained when the poles are bolted to the yoke. Another method is often used, in which seatings are cast on the interior of the yoke. The seatings are machined flat, and have the poles bolted to them.

Field Coils.—Field coils may be divided into two classes, namely, those wound on bobbins and those wound on formers. Fig. 292, B shows a coil wound on a bobbin. The bobbin is made of sheet iron with brass flanges, or of moulded insulating material. The metal bobbin is usually insulated with press-pahn to a thickness of 2 or 3 mm. before commencing winding. When shunt and series coils are wound on the same bobbin they must be well insulated from each other. Fig. 292, A shows a taped former-wound coil. During winding, strips of tape are interlaced with the coil and tied together, to keep the coil in the proper shape, which is given by the flanged wooden former on which the coils are wound. When the coil is finished, the former is taken apart. The coil is next dried and dipped into insulating varnish, and covered with strips of presspahn or mica at the places where they are in contact with the poles, and taped all over with varnished tape.

Fig. 292, C shows another type of former-wound coil which is not taped over, but kept from touching the pole and pole shoe by strips of insulating fibre. In this example the series coil is separated from the shunt in the same manner. The advantage of this type of construction is that the air has a free passage behind the coil as well as over its surface, and therefore is better ventilated. It is not very suitable for damp situations, as it cannot be made completely water and moisture proof.

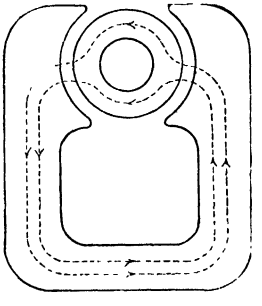


Fig. 293.—Flux Path in a 2-Pole Dynamo

The Magnetic Circuit. — Fig. 293 shows the path of the flux in a 2-pole dynamo. The flux passes through the yoke poles, two air-gaps, and the armature in series. In the ring armature shown the cross section of iron in the armature is twice the cross section of the ring, since the flux divides into two parts on entering the armature.

The flux paths (the lines drawn show the mean paths) for a multipolar dynamo are shown in fig. 294. The flux, leaving a north pole, passes through the air-gap and teeth, and then divides into two, each half completing the magnetic circuit through the armature core, an adjacent south pole, and the yoke. The flux in the armature core is therefore half the flux crossing the air-gap, while the flux in the yoke is half that in the pole. Owing to magnetic leakage the flux in the pole and the air-gap are not equal.

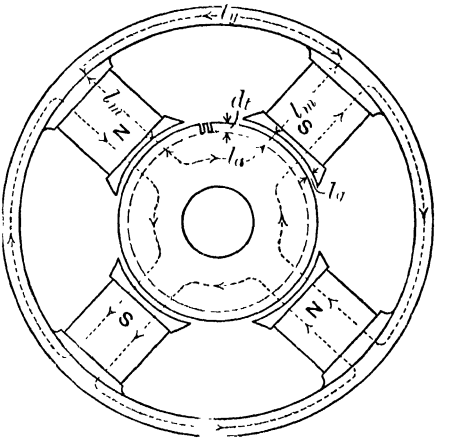


Fig. 294.—Flux Paths for a 4-Pole Dynamo

The ampere turns required for the magnetic circuit are calculated from the equations for the magnetic circuit, namely:—

$$\text{M.M.F.} = \text{Flux} \times \text{Reluctance},$$

which may be written as—

$$\text{Total ampere turns} = 0{\cdot}8 \times \text{Flux} \times \text{Reluctance},$$

and

$$\text{Reluctance} = \Sigma \frac{l}{A\mu};$$

where l, A, and μ refer to the separate portions of the circuit, which are as follows:—

1. Armature core.
2. Armature teeth.
3. Air-gaps.
4. Magnet limbs and pole shoes.
5. Yoke.

1. The armature core is built of laminations, as described previously. The area of the core is generally arranged so that the flux density in it is from 10000 to 12000 lines per square centimetre, in order to reduce the iron losses. If l_i is the net iron length of the core, that is the length between end plates minus the total breadth of the air ducts, the useful iron length

is about $0{\cdot}9 \times l_i$, the remaining 10 per cent being the insulation between the laminations. The area of the core is therefore $0.9 \times l_i \times h_a$, where h_a is the radial depth of the core below the teeth.

The flux density in the core is $B_a = \frac{N}{2 \times 0{\cdot}9 \times l_i h_a}$, where N is the total flux crossing the air-gap. From the drawing of the armature the length of the mean flux path through the core is estimated as l_a. From a table the value of the magnetizing force H_a is obtained corresponding to B_a, and the ampere turns for the core are—

$$X_a = 0.8\, H_a\, l_a.$$

2. The tooth density varies from 19000 to 23000 lines per square centimetre, so that the teeth are fairly well saturated. By so doing the space available for copper in the slots is increased, and at the same time the armature cross flux (referred to later) is reduced. Since the pole shoe does not cover the whole of the pole pitch, the whole of the teeth in one pole pitch do not carry flux at one time. It would not be correct, however, to assume that only the teeth under the polar arc carry flux, since the flux does not pass radially from the pole to the armature, but spreads out (fig. 295), so that an allowance must be made for the teeth outside the polar arc which carry flux. As a first approximation, this can be done by increasing the polar arc by about 8 to 12 per cent. If t_1 is the tooth pitch at the armature circumference, the number of teeth carrying flux is $\frac{b_p + 10 \text{ per cent}}{t_1}$. Since the sides of the slot are parallel, the tooth is tapered, and the cross section and flux density vary from point to point along the tooth.

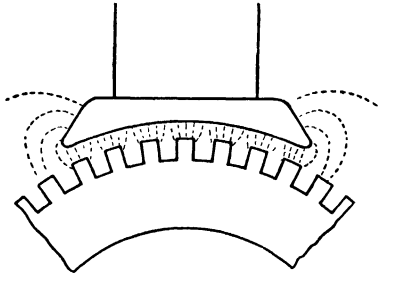


Fig. 295.—Flux Distribution in the Air-gap

In approximate calculations the value of B is found for a point halfway down the tooth, and the value of H corresponding to this value of B is taken as the mean value for the whole tooth. The breadth of the tooth at this point is equal to $(t_2 - s)$ where t_2 is the tooth pitch at this point and s the slot width. As in

the case of the core, the iron length is $0.9 \times l_i$, so that the area A_t of the teeth carrying flux is $\left(\frac{b_p + 10 \text{ per cent}}{t_1}\right)(t_2 - s)(0.9)\, l_i$. The mean flux density B_t in teeth is therefore $\frac{N}{A_t}$, and the magnetizing force H_t corresponding to this flux density is found from tables. The ampere-turns for the teeth are

$$X_t = 0.8\, H_t \times 2d_t,$$

where d_t is the depth of the slot.

3. The air-gap and the length of path in the air-gap are both difficult to determine accurately. The flux leaving the pole spreads over the whole of the pole pitch on the armature surface. The flux density in the gap is a maximum under the pole shoe, and falls off rapidly for points on the armature surface beyond the shoe, as shown in fig. 296. The area of the gap giving a density equal to the maximum density in the gap is therefore greater than the area of the pole shoe. On the other hand, the presence of the teeth reduce the effective area of the gap, so that these two effects tend to cancel out. It is often sufficiently accurate to assume that they do cancel out, and therefore that the area of the air-gap is equal to the area of the pole shoe, while the length of the gap is equal to radial distance from the armature surface to the pole-shoe surface. This distance in ordinary machines varies from 0.2 to 0.6 cm. If b_p is the breadth and l_s the length of the pole shoe, the area of the gap is $b_p \times l_s$ and $B_g = \frac{N}{b_p \times l_s}$. The ampere-turns for the gap are therefore

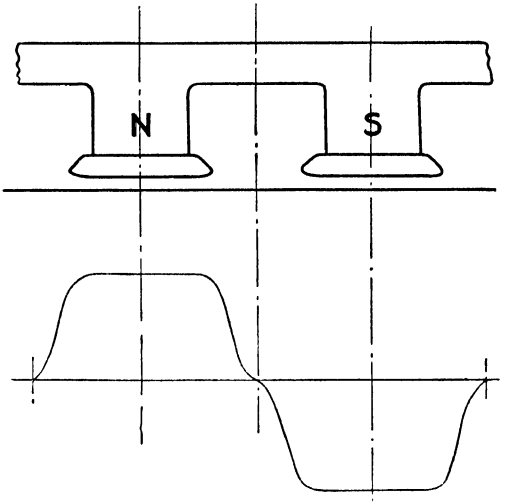


Fig. 296.—Field Distribution on No Load

$$X_g = 0.8\, B_g \times 2 \times l_g$$

where l_g is the length of the gap and $H_g = B_g$, since the path is in air.

4. Before commencing the calculation of the ampere-turns required for the magnet limb and yoke, the leakage flux from the magnet and yoke must be ascertained. Since air is not a good magnetic insulator, part of the flux produced by the magnetizing coils on the limbs therefore leaks across the inter-polar gap from pole shoe to pole shoe, and from pole to pole. In addition there are flux leakages from the poles to the yoke. These leakage paths are shown by the dotted lines in fig. 297. The leakage flux between any two points is proportional to the M.M.F. existing between them, and to the area at right angles

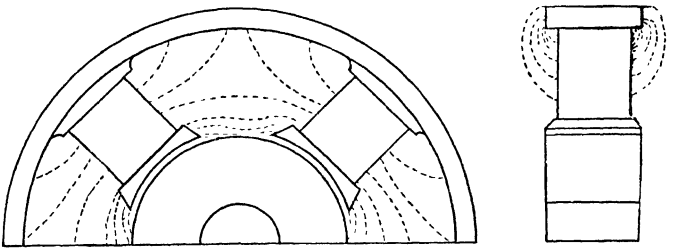


Fig. 297.—Leakage Flux Distribution

to the leakage path, while it is inversely proportional to the distance between the points. The leakage flux therefore depends on the design of the magnet frame and on the M.M.F. produced by the field coils. The flux therefore increases from the air-gap to the yoke, being a maximum at the junction of the yoke with the poles. In order to simplify calculations, the flux is usually taken as constant for the pole and yoke, this flux being found by multiplying the air-gap flux by a constant λ, known as the leakage coefficient. This coefficient has been determined for various machines by measuring ballistically the air-gap flux and the magnet-limb flux when the leakage coefficient

$$\lambda = \frac{\text{Magnet-limb flux}}{\text{Air-gap flux}}.$$

The value of λ varies from 1.6 for small 2-pole machines to 1.12 for large multipolar dynamos. For multipolar dynamos with slotted armatures it can be taken as 1.15 for machines above 100 kilowatts, and varying up to 1.2 for machines of

10–15 kilowatts output. The pole span b_p, or length of the polar arc on the curve, is usually from 0.60 to 0.70 of the pole pitch: 0.65 is an average value. This fixes the air-gap flux for a given armature length, the flux density in the gap being from 8000 to 10000 lines per square centimetre. The flux density in the poles (including leakage flux) is usually from 16000 to 17000 lines per square centimetre, so that the area of the poles is easily determined. The length of pole (radially) is fixed by the space required for the copper of the field coils. In the magnetic-circuit calculation the length of the path l_m in the pole includes the pole shoe. So that finally $B_m = \frac{\lambda N}{A_m}$ and

$$X_m = 2 \times l_m \times 0.8 H_m,$$

where H_m corresponds to B_m.

5. The flux through the yoke is half that through the poles. The area of the yoke is therefore arranged to give a flux density of from 11000 to 13000 lines per square centimetre for cast steel and from 5000 to 7000 lines per square centimetre for cast iron. The length of path l_y can be obtained from a drawing, and $B_y = \frac{\lambda N}{2A_y}$. If H_y is the magnetizing force corresponding to B_y the ampere-turns for the yoke are

$$X_y = 0.8 H_y l_y.$$

Adding together the ampere-turns for each part of the magnetic circuit, the total ampere-turns for the magnetic circuit are

$$X = 0.8 (H_a l + 2 H_t d_t + 2 B_g l_g + 2 H_m l_m + H_y l_y).$$

Since in the usual design each pole has an exciting coil, the magnetic circuit is interlinked with two exciting coils.

The ampere-turns in each field coil are therefore $\frac{X}{2}$ ampere-turns.

To illustrate the above calculation, an example will be taken of the magnetic circuit for a 4-pole, $7\frac{1}{2}$-kilowatt generator. The terminal voltage on full load is 250 volts when running at a speed of 800 r.p.m.

The armature has 990 surface conductors, and is wave wound. A sketch of the magnetic circuit is shown in fig. 298, where the dotted line represents the mean flux path.

The air-gap flux per pole on no load is

$$N = \frac{E \times 10^8}{pnZ} = \frac{250 \times 10^8}{2 \times \frac{800}{60} \times 990} = 0.95 \times 10^6 \text{ lines.}$$

The ampere-turns required for the various portions of the magnetic circuit can now be found.

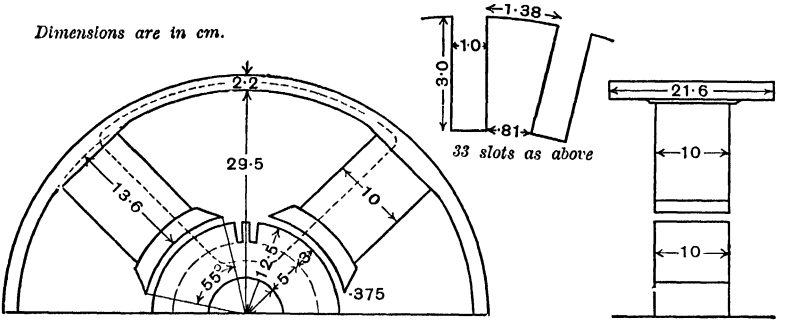


Fig. 298.—Magnetic Circuit for 4-pole Dynamo

1. **Armature Core.**—The gross length of the core is 10 cm. Since there are no ventilating ducts, the gross iron length is 10 cm. and the net iron length is $0.9 \times 10 = 9$ cm., allowing 10 per cent of the gross length for insulation. The depth of the core below the teeth is 5 cm., so that the area of iron in the core $= 5 \times 9 = 45$ sq. cm. The flux density is therefore $= \frac{\cdot 95 \times 10^6}{2} / 45 = 10500$ lines per square centimetre approximately. The armature is built of soft-iron laminations, and from the B-H curve for this material (see Table III) the value of H is 8.75. The length of path is measured on the drawing as 8 cm. approximately. The ampere-turns for the armature are therefore

$$X_a = 0.8 \times H \; l_a = 0.8 \times 8.75 \times 8 = 56.$$

2. **Teeth.**—There are 33 teeth in the armature, so that the tooth pitch at the top is $\frac{2\pi \times 12.5}{33} = 2.38$ cm. The breadth of a tooth at the top is therefore $(2.38 - 1.0)$ cm. $= 1.38$ cm., the slot being 1 cm. broad. In the same way the breadth of a tooth at the bottom is found to be 0.81 cm. The mean breadth is therefore 1.095 cm. and the mean area of the tooth is $1.095 \times 9 = 9.85$ sq. cm., the length of iron in the tooth parallel to the shaft being 9 cm. as found for the armature. The polar arc

subtends an angle of 55 degrees, so that the breadth of the pole shoe $= \frac{55}{360} \times \pi \times (25 + 0.75) = 12.35$ cm. Allowing 10 per cent for fringing, the number of teeth carrying flux is $= \frac{12.35 \times 1.1}{2.38} = 5.7$. The total area of the teeth is therefore $5.7 \times 9.85 = 56.2$ sq. cm. The mean tooth density is therefore $= \frac{.95 \times 10^6}{56.2} = 16900$ lines per square centimetre. The value of H corresponding to B = 16900 is 53, and since the length of path for the teeth is twice the length of one tooth, i.e. 6 cm, the total ampere-turns are

$$X_t = 0.8 \times 53 \times 6 = 255.$$

3. **Air-gaps.**—The area of the gap is taken equal to the area of the pole-shoe face. The breadth of the shoe has been found to be 12.35 cm., while the length is 10 cm., so that the area is 123.5 sq. cm. The flux density in the gap is therefore $\frac{.95 \times 10^6}{123.5}$ = 7700 lines per square centimetre. Since for the gap H is equal to B, and the length of the two gaps in series is 0.75 cm., the ampere-turns are

$$X_g = 0.8\, B_g \times 2\, l_g = 0.8 \times 7700 \times 0.75 = 4620.$$

4. **Magnets.**—The magnet limb is of cast steel, and is circular in section, so that the area is $= \frac{\pi \times 10^2}{4} = 78.54$ sq. cm. Assuming a leakage coefficient for the poles of 1.2, the flux through the poles is $0.95 \times 10^6 \times 1.2 = 1.14 \times 10^6$ lines. The flux density is therefore $\frac{1.14 \times 10^6}{78.54} = 14500$ lines per square centimetre. The value of H from the table for cast steel is 16.3. The length of path including the pole shoes is 27.2 cm. The ampere-turns are

$$X_m = 0.8\, H_m\, l_m = 0.8 \times 16.3 \times 27.2 = 354.$$

5. The flux in the yoke is half that in the magnet, i.e. 0.57×10^6 lines. The yoke is of cast steel, and its area is $= 21.6 \times 2.2 = 47.5$ sq. cm. The flux density $= \frac{.57 \times 10^6}{47.5} = 12000$. The corresponding value of H is 12.5. The length of path is, from the drawing, 38 cm., so that

$$X_y = 0.8 \times 12.5 \times 38 = 380.$$

The total ampere-turns required for the field circuit is therefore

$$X = (56 + 255 + 4620 + 354 + 380) = 5665,$$

and the ampere-turns on one field coil are approximately 2833 or, say, 2840. On full load it is found that to produce the same voltage an increased number of ampere-turns are required. On load, due to the ohmic drop in the armature caused by the armature, the induced E.M.F. must be greater than the no-load induced E.M.F. if the terminal voltage is to remain constant. In addition, the current flowing in the armature windings produces a magnetic effect (known as armature reaction) on the magnetic circuit, which reduces the flux from the pole. To overcome this, the field ampere-turns must be increased.

Armature Reaction.—Suppose, first, that the field coils of a 2-pole dynamo are excited, and that there is no current flowing in the armature, then the flux distribution is as shown in fig. 299. If a line is drawn joining the two points $b\,b'$, where no flux enters or leaves the armature, this line is called the neutral axis, and is at right angles to the axis of the poles. If the brushes are placed on the segments of the commutator, which are connected to the conductors on the neutral axis, the brushes will be connected to conductors in which no E.M.F. is induced, since they cut no flux, and therefore in which the E.M.F. and current is about to reverse in direction. The brushes are then said to be in the neutral position, and reversal of current in the coil will take place sparklessly.

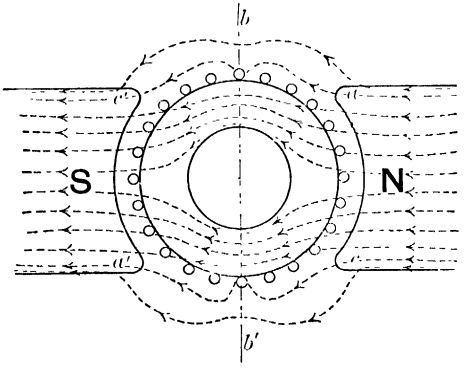


Fig. 299.—Main Flux on No Load

Next, suppose that the field is unexcited, and that a current is sent through the armature from the negative brush to the positive brush, i.e. in the same direction as it would flow if the field was excited and the external circuit closed. Then the direction of the current is given by the dots and crosses in

fig. 300. A dot represents a current from back to front, and a cross represents a current from front to back.

The current flowing in the armature converts the armature core into an electromagnet with its north pole at the bottom and south pole at the top, the axis of the magnet being the brush axis *bb'*, at right angles to the main field. The lines of force set up by the armature M.M.F. follow the paths shown in fig. 300. The flux (known as the cross flux) across the gap changes sign at the centre of the pole face.

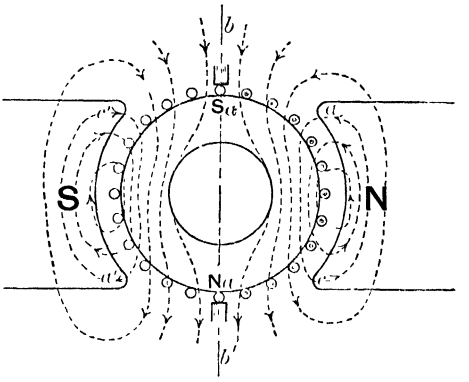


Fig. 300.—Armature Flux

This flux distribution is not altered by the rotation of the armature, since if the armature has a large number of surface conductors, the current distribution on the armature surface remains fixed in space though the current flows in succession through different conductors as the armature rotates.

To obtain the resultant flux on load, the main flux and the cross flux must be added together. Where the fluxes flow in opposite directions, they cancel out, and where they coincide, they add. This has been done in fig. 301. At the pole tips *a* and *a'* the fluxes coincide in direction, and the flux density in the air-gap is increased. On the other hand, at the pole tips *c* and *c'* the fluxes are in opposition, so that the flux density in the gap is decreased. Since the armature rotates towards pole tips *c* and *c'*, these pole tips are called the *leading pole tips* or *horns*, while *a* and *a'* are called the *trailing pole tips* or *horns*.

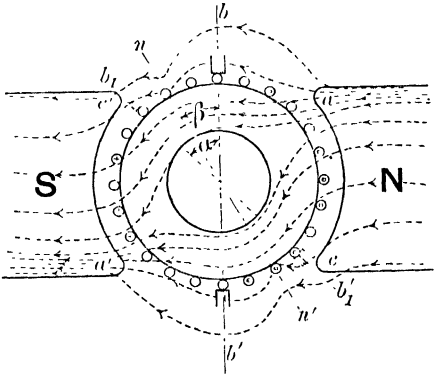


Fig. 301.—Resultant Flux in Armature

The air-gap flux density is reduced under the leading horns and strengthened under the trailing horns. Since the gap flux distribution has been altered, and is no longer symmetrical about the field axis, the neutral axis must have altered. From the diagram it will be seen that it has moved forwards through an angle β to the position nn'. To distinguish between the axis bb' and the axis nn', the former is called the geometrical neutral, while the latter is the actual neutral axis. Now if the brushes remain on the axis bb', an E.M.F. will be induced in the coils undergoing short-circuit by the brush, and when the coil moves from under the brush the circuit through the brush and the short-circuited coil is broken, causing sparking. In order to assist the reversal or commutation of the current the coil must be moved, in the direction of rotation, into a field of the opposite sign, so that an E.M.F. tending to reverse the current will be induced in the coil. The brushes are therefore moved through an angle a from the geometrical neutral position greater than β, to the position $b_1b'_1$. a is known as the *angle of lead.*

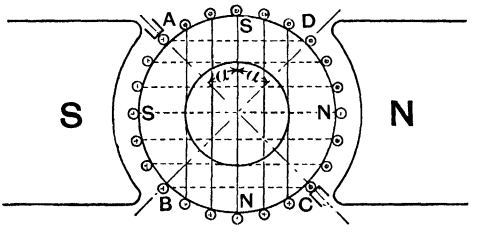


Fig. 302.—Cross and Back Ampere-turns

Back and Cross Ampere-turns.—The armature reaction is easily divided into two components whose ratio is given by the angle of brush shift a. When this angle is known, the conductors can be joined together in two groups, as shown in fig. 302, without affecting the magnetic effect of the armature.

The conductors in arc AD together with the conductors in the arc BC form an electromagnet whose axis is the axis of the main field. The M.M.F. of this solenoid, however, is opposed to the M.M.F. of the main field, and therefore tends to decrease the main flux. The ampere-turns produced by this solenoid are known as the armature *back ampere-turns.* The ampere-turns acting on the magnetic circuit are evidently equal to the number of bars in the arc AD multiplied by the current per bar. Since $\angle AOD = 2a$, the number of bars in arc AD $= \frac{2a}{360} \times Z$ where Z is the total number of surface conductors,

and if i_a is the current per bar, the armature back ampere-turns are

$$X_B = \frac{2a}{360} \times Zi_a.$$

In order to compensate for these demagnetizing ampere-turns the field ampere-turns per magnetic circuit must be increased by an amount equal to X_B, or the ampere-turns in one field coil must be increased by $\frac{X_B}{2}$.

If the brushes are moved in the direction opposite to that of rotation through an angle a from the geometrical neutral position, the armature ampere-turns in arc AD are magnetizing, and tend to strengthen the field. This, however, is not practicable on load unless special aids to commutation are used.

The conductors in the arc AB, together with those in arc DC (fig. 302), form an electromagnet whose axis is at right angles to the axis of the main field, and which produces the cross flux. The ampere-turns producing this cross flux are equal to the number of conductors in arc DC multiplied by the current per bar, i.e. the cross ampere-turns

$$X_C = \frac{(180^\circ - 2a)}{360} \times Z \times i_a.$$

The magnetic circuit for the cross flux is the pole shoe, air-gaps, teeth, and core, as shown in fig. 300. The air-gap flux distribution is given by Curve II (fig. 303), where the ordinates represent flux density in the gap and the abscissæ distance along the armature surface Since most of the reluctance is in the air-gap, the flux density in the gap due to the cross ampere-turns is proportional to the dis-

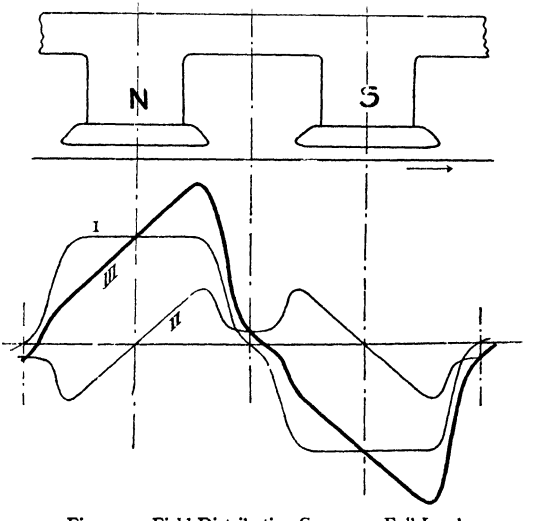


Fig. 303.—Field Distribution Curves on Full Load

tance from the centre of the pole arc. In fig. 303 the original field distribution is given by Curve I, and the resultant field distribution by Curve III (the sum of I and II).

The area of Curve III represents the total flux, since the ordinates are flux density and the abscissæ distances. It will be found that the areas of Curve I and Curve III are equal, so that the cross ampere-turns do not alter the total flux entering the armature, but merely alter the distribution of the flux. This statement, however, is only true so long as the teeth are not saturated. If the teeth are saturated, the effect of the cross ampere-turns is to reduce the total flux slightly.[1]

In multipolar armatures the armature reaction ampere-turns can be divided up into cross and demagnetizing ampere-turns for each magnetic circuit in a similar manner. For an angle of lead a the armature back ampere-turns per magnetic circuit will be

$$X_B = \frac{2a}{360} \times Z \times i_a,$$

so that the back ampere-turns to be compensated for on each field coil are $\frac{X_B}{2}$. If the dynamo has p pairs of poles, one pole pitch spans an angle of $\frac{360^\circ}{2p}$, so that the arc carrying the cross ampere-turns spans an angle of $\left(\frac{360^\circ}{2p} - 2a\right)$, so that the cross ampere-turns per magnetic circuit are

$$X_C = \frac{\left(\frac{360}{2p} - 2a\right)}{360} Z i_a,$$

and the cross ampere-turns per pole are $\frac{X_C}{2}$.

Calculation of Exciting Ampere-turns on Full Load.—The exciting ampere-turns required to give normal voltage at the terminals of the 7½-kilowatts dynamo described above will now be calculated.

The full-load armature current is 32 amperes (including the current in the shunt field coil), and the armature resistance is 0.53 ohm, so that the IR drop in the armature is 17 volts. Allowing 2 volts for the drop in the brushes, the total induced voltage on full load is (250 + 17 + 2) = 269 volts.

[1] For the method of calculating this reduction the reader is referred to *The Dynamo*, Vol. II, by Hawkins and Wallis.

The flux corresponding to this voltage is

$$N = \frac{269 \times 10^8}{2 \times \frac{800}{60} \times 990} = 1.02 \times 10^6 \text{ lines.}$$

The calculation of the ampere-turns required for the magnetic circuit to produce a flux of 1.02×10^6 lines is given in the following tabular statement:—

Part.	Flux.	Area.	B.	H.	*l*cm.	0.8 H*l*.
		Sq. cm.	Lines/sq. cm.			
Armature core	$\frac{1.02 \times 10^6}{2}$	45	11350	8.1	8	52
Teeth ...	1.02×10^6	56.2	18200	115	6	552
Air-gap ...	1.02×10^6	123.5	8270	8270	0.75	4960
Poles ...	$1.2 \times 1.02 \times 10^6$	78.54	15600	27.5	27.2	600
Yoke	$\frac{1.2 \times 1.02 \times 10^6}{2}$	47.5	12900	10	38	380
				Total		6544

The brushes are given a lead of 7.3 degrees, so that the armature back ampere-turns are

$$X_B = \frac{2\alpha}{360} \times Z \times i_a = \frac{7.3 \times 2}{360} \times 990 \times 16 = 636.$$

The current per bar is half the total armature current, 32 amperes, since the armature is wave wound with 2 circuits in parallel between brushes.

The total ampere-turns required for the magnetic circuit is the sum of the ampere-turns required to produce the flux and the ampere-turns to compensate for the armature back ampere-turns.

The total ampere-turns per magnetic circuit is

$$X = 6544 + 636 = 7180.$$

The ampere-turns wound on each pole are therefore 3590 ampere-turns.

Each of the four field coils must therefore be designed to give about 3600 ampere-turns.

Commutation.—By the action of the commutator and brushes the alternating E.M.F.'s induced in the individual armature conductors are made unidirectional across the external circuit.

Since the brush has a thickness greater than the thickness of the insulation between the two segments, it must periodically bridge across at least two segments, and frequently across more than two segments, depending on the width of the brush. During the time the brush is bridging over the two segments the local circuit through the coil and the brush is closed, and the coil is said to be short-circuited. During the time of short-circuit the current in the coil must fall to zero, reverse, and rise to a maximum in the opposite direction. This process is known as *commutation*. In order that there may be no sparking as the brush leaves the segment, the current in the coil must just have reached its correct value at that instant. The resistance of the short-circuited coil circuit is = resistance of the coil + resistance of the connectors to the commutator + resistance of the brush contacts + resistance of the brush itself.

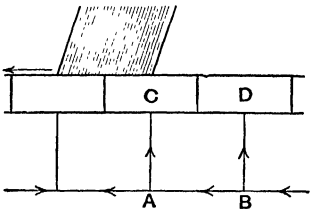


Fig. 304

In order to understand what occurs during commutation, the simplest case will be considered, and the deductions obtained from it will be modified to suit the actual circumstances.

The simplest case of commutation is obtained when it is assumed that all the resistances except the brush-contact resistances in the short-circuited coil circuit are negligible, and that no E.M.F.'s are induced in the short-circuited coil. All the segments of the commutator under the brush are therefore at the same potential, so that the current flowing in any segment will be inversely proportional to the contact resistance between that segment and the brush. Now the contact resistance is inversely proportional to the area of the brush in contact with the segment, so that the current is proportional to the area of the brush in contact with the segment.

Fig. 304 represents a portion of an armature lap winding with commutator and brushes. The armature winding is represented by straight lines joining the commutator connectors.

Immediately before short-circuit, the current in coil AB is equal to half the armature current, but when the brush touches segment D a new path is opened through connector BD to segment D and the brush. The current flowing in this path will be

proportional to the area of brush to the right of E (fig. 305). Since the commutator is moving at a constant speed, this area increases at a constant rate, and the current through BD increases at a constant rate. An increase in the current through BD means, however, an equal decrease in the current through BA. The current through the coil BA decreases uniformly, and reaches zero when E is at the middle of the brush. Since current in BD goes on increasing, the current in BA must reverse and increase negatively at a uniform rate till at the instant the brush leaves segment C the current in AB is again equal to the armature current and the current in AC is zero, so that there can be no sparking as the brush leaves segment C.

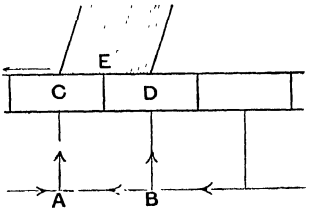


Fig. 305

Fig. 306 shows the current change graphically. The line AB is the axis of time, and the ordinates represent the current in the coil AB. *a* is the beginning and *b* the end of short-circuit.

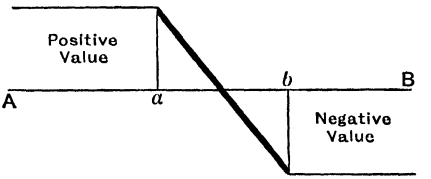


Fig. 306.—Ideal Commutation Curve

The above assumptions are difficult to realize in practice, however, the main error lying in the fact that there always is an E.M.F. induced in the short-circuited coil. The current flowing in the coil produces a flux linked with it. When the current changes, the change in this flux (by Lenz's Law) induces an E.M.F. (called the E.M.F. of self-induction) in the coil, which opposes the change of current. The rate of current change is no longer constant, and the line showing the current change is no longer straight, but will probably be curved, as shown in fig. 307. Where the self-induction of the coil is small the E.M.F. induced may not be great, and it may be possible for the current to rise to its

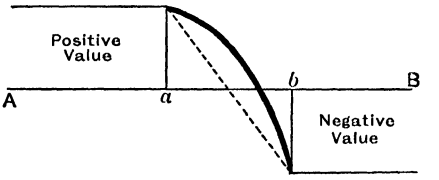


Fig. 307.—Actual Commutation Curve

proper value before the short-circuit is ended, as in fig. 307, thus causing no sparking. If, however, the current does not rise to its proper value by the end of short-circuit there will be a sudden change of current in the coil as the segment leaves the brush. This sudden change of current induces a large E.M.F. between the brush tip and the receding segment, which causes a spark to jump across from the segment to the brush as the former leaves the brush. This sparking roughens the commutator, which makes the evil worse.

Prevention of Sparking.—In order to prevent sparking, various methods are employed. Since the sparking is due primarily to the E.M.F. of self-induction in the armature coil, this E.M.F. should be reduced as much as possible. This can be done by reducing the number of turns in the armature coil, the limit being when the coil has only one turn.

If brushes having a high contact resistance are used, the effect of the induced E.M.F. in altering the rate of current change is diminished, so that the sparking is decreased. For this reason high-resistance carbon brushes have taken the place of the low-resistance copper gauze brushes.

Since the sparking is due to the E.M.F. of self-induction, it can be neutralized by inducing in the short-circuited coils an E.M.F. which is equal and opposite to it. This can be done in two ways, namely, by moving the brushes, or by the use of auxiliary or commutating poles.

The E.M.F. of self-induction opposes the change in the current, hence the E.M.F. induced in the short-circuited coil must be in such a direction as to reverse the current in the coil. In a generator such an E.M.F. can be obtained by arranging for the short-circuit to take place while the coil is moving across the fringe of flux from a pole of opposite polarity to the one it has just left. The flux from this pole cutting the coil induces a reversing E.M.F. in it. The brushes are therefore moved in the direction of rotation (fig. 301) to a position such as $b_1 b'_1$, where the coils connected to them are in a reversing field. The E.M.F. induced must be sufficient to first reduce the current to zero, and then to induce a current in the opposite direction which at the end of short-circuit will be equal to the normal current in the armature bars.

At no load there is no current to reverse, so that the brushes

are in the geometrical neutral position, and as the armature current increases the brushes are moved forward so that the coil is in a field of the proper strength to induce a sufficient reversing E.M.F. The reversing E.M.F. required increases with the armature current, and therefore the brush lead must also increase, since to increase the reversing E.M.F. the coil must cut a stronger field. The lead is increased until at a certain load it is impossible to obtain sparkless commutation, owing to the absence of a strong enough reversing field. This fixes the maximum load on the dynamo.

In modern dynamos the brush position is fixed at a constant angle of lead for all loads, so that perfect commutation can only be obtained for one load, but with high-resistance brushes the commutation is quite good at all loads between no load and full load.

Commutating Poles.—To obtain perfect commutation it is necessary for the short-circuited coil to move in a reversing field of the proper strength. Now, if a small pole is placed in the centre of the interpolar gap, and excited so as to be of the same polarity as the next pole in the direction of rotation, then this pole will give a field of the proper direction for producing a reversing E.M.F.

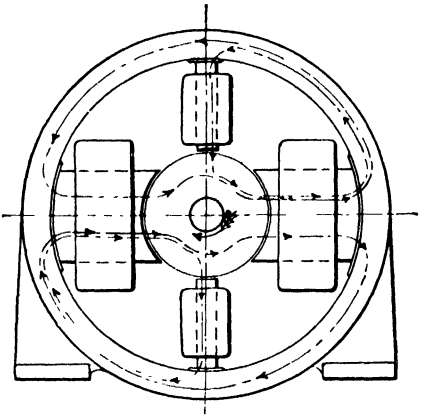


Fig. 308.—Flux Paths for Commutating Pole Machine

These small poles are called *commutating* poles, and are excited by a coil which carries the armature current. If the maximum density at full load is kept below 13000 lines/square centimetres, the flux through the pole will be proportional to the armature current. Since the reversing E.M.F. and the E.M.F. of self-induction both depend on the current, it is possible to make them neutralize each other, not only during the whole of short-circuit, but also for any value of the armature current, and in this way perfect commutation is obtained.

Fig. 308 shows a diagrammatic sketch of the flux paths for the main and commutating poles, while fig. 309 shows the flux distribution in the air-gap. It will be noticed that the flux density under the commutating pole is uniform. This ensures that the two E.M.F.'s neutralize each other during the whole of short-circuit.

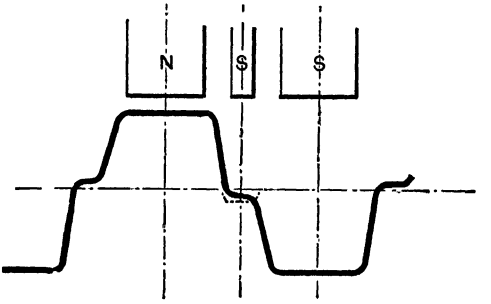


Fig. 309

The ampere-turns wound on the commutating poles must be greater than, and opposite to, those produced by the armature. Since the brush remains in the neutral position the whole of the armature ampere-turns are cross ampere-turns, and there are no demagnetizing ampere-turns.

Description of a Complete Machine.—Fig 310 shows a direct-current engine-type generator for direct coupling to high-speed engine. This machine has an output of 650 amperes at

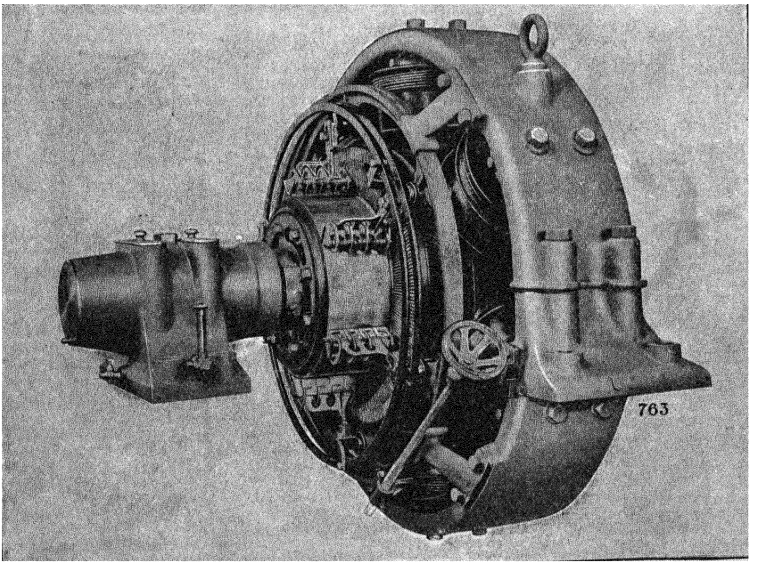


Fig. 310.—Direct-current Engine-type Generator

230 volts, or 150 kilowatts at 380 revolutions per minute, and the principal dimensions are as follows:—

Number of poles	6	
Armature, diameter over all	91.5	cm.
,, length over conductors	62.8	,,
,, diameter at bottom of slots	83.36	,,
,, internal diameter	54	,,
,, length over iron	30.5	,,
,, effective length, magnetic iron	25.1	,,
,, depth of slot	4.07	,,
,, width of slot	1.14	,,
,, number of slots	124	
,, width of tooth at root	.975	,,
,, depth of conductor	1.78	,,
,, width of conductor	.318	,,
,, space factor	.486	,,
,, number of ventilating ducts	2	
,, width of each duct	1.27	,,
,, core	mild steel	
,, spider	cast-iron	
,, yoke	cast-steel	
Magnet poles	,,	
Brushes	carbon	

Magnetic Core—

Length of pole face	30.5	cm.
Length of pole arc	32.8	,,
Pole arc ÷ pitch	.7	
Radial length	3.1	,,
Diameter	26.7	,,
Bore of field	92.08	,,
Depth of air-gap	.79	,,
Outside diameter of yoke	169	,,

Commutator—

Diameter	71.2	cm.
Rubbing length	25.4	,,
Number of segments	248	
Width of segment at top	.8	,,
Thickness of mica insulation	0.0762	,,

Brushes—

Number of sets	6
Number in one set	5
Contact size of each brush	12.9 sq. cm.
Number of inductors	496
Inductors per slot	4
Style of winding	lap or six-circuit single
Mean length of inductor	91.5 cm.
No-load voltage	230
Resistance between brushes	.00402 ohm
Current density in inductors	191 amps. per sq. cm.
Average voltage between commutator segments	5.6

Armature ampere-turns per pole	4480	
Ampere-turns to overcome armature reaction	1710	
Reactance voltage per segment	3.28	
Armature flux (full load)	8.1	megalines
Leakage coefficient = λ	1.15	
Flux in magnet cores	9.3	megalines
Section of armature	735	sq. cm.
Magnetic length through armature ...	27.5	cm.
Area of teeth under one pole	412.5	sq. cm.
Magnetic length of teeth	8.14	cm.
Area of gap	1111.5	sq. cm.
Length of gap	1.58	cm.
Section of magnet core	572	sq. cm.
Magnetic length of core	59.75	cm.
Area of yoke	625	sq. cm.
Magnetic length through yoke	84	cm.

Bobbin Windings—

Ohms per bobbin (shunt)	10	
Size of wire	.203	cm.
Number of shunt turns per bobbin	1630	
Cross section of series wire	5.16	sq. cm.
Number of series turns per bobbin	$3\frac{1}{2}$	
Ampere-turns (shunt) per circuit	12533	
Ampere-turns (series)	4550	
Total ampere-turns at full load ...	17083	

Curves of performance of this machine are given in fig. 311.

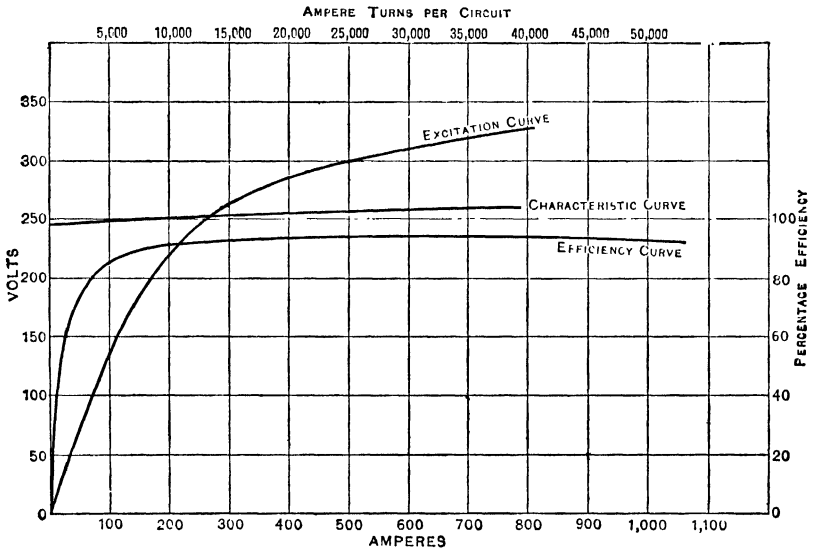


Fig. 311

It will be noticed from the efficiency curve that the commercial efficiency of the machine is 94 per cent at full load, and is very

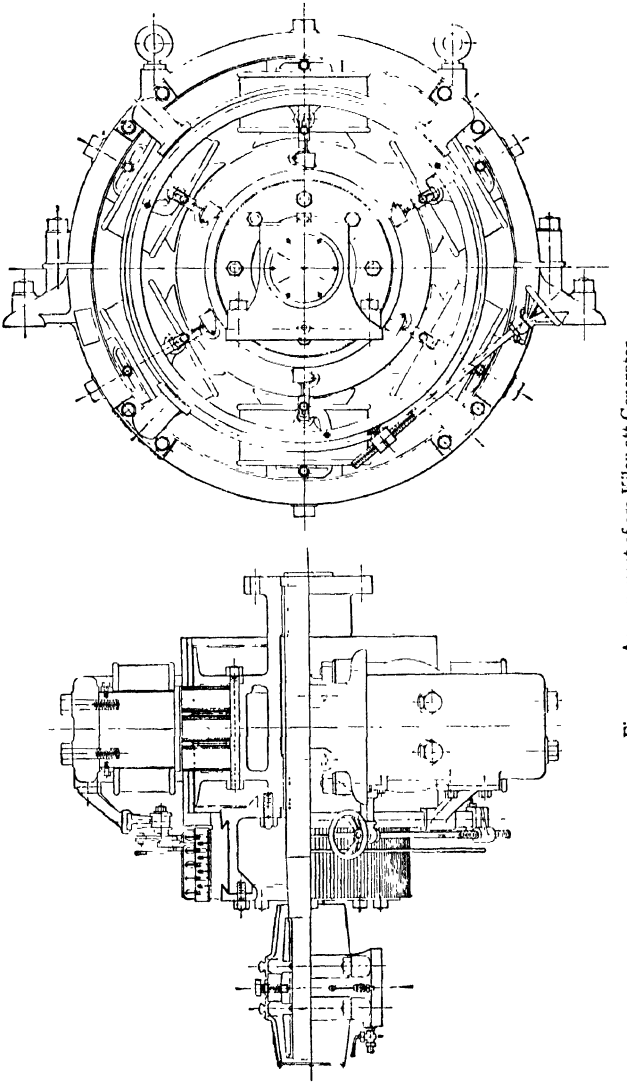


Fig. 312.—Arrangement of 150-Kilowatt Generator

nearly constant down to below half-load, a property which is of great value in the case of machines working on fluctuating loads, such as are met with in traction power stations.

Fig. 312 gives a section through the centre of the machine,

showing the details of construction of the armature and commutator. The spider, which is of cast iron, is of very solid construction, keyed to the shaft by two keys driven in from both ends. One of the armature end plates clamping the core disks, and also supporting the windings, is cast solid with the spider; the other is drawn by turned steel bolts, which also form keys for driving the core disks. The half-coupling for bolting to the engine flywheel is cast solid with the spider, thus the torsional stresses between the flywheel and armature are taken directly by the spider and not by the clamping ring. The commutator centre is bolted to a flange cast on the spider, making a very solid and lasting construction.

The brush gear is supported from the yoke by cast-iron brackets. The rocker ring is adjustable by means of a hand wheel and screw. The brushes, however, can be set so that they do not require adjustment from no load to 25 per cent overload. The hand wheel and screw, therefore, could very well be dispensed with, but are fitted simply in accordance with the usual practice.

The outer bearing is of the self-aligning type, with white-metal bearing surfaces. Continuous lubrication is provided by means of oiling rings. This type of bearing is typical of modern standard practice, and is of the type almost universally adopted on this class of machine.

CHAPTER XXIII

TYPES OF DYNAMOS AND THEIR CHARACTERISTICS

Types of Dynamos. — Dynamos can be divided into five classes, depending on the manner in which the field is excited, as follows:—

(*a*) *Magnetos or Permanent-magnet Dynamos.*—The field flux in these dynamos is produced by permanent magnets. This was the earliest type of dynamo, and is still used for machines of very small output, such as generators for insulation testing, magnetos for ignition purposes, and for telephone use. This type has the disadvantages of being heavy and

bulky for the output. The maximum flux density in the air-gap is low, and the armature must therefore have more copper than the ordinary types.

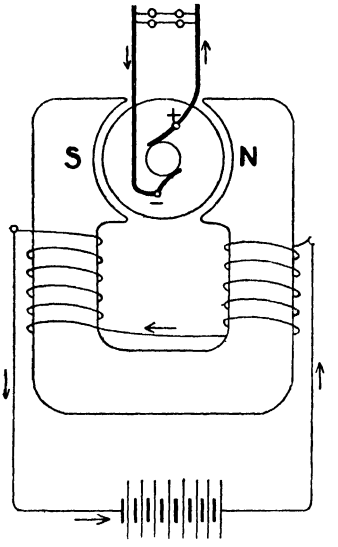


Fig. 313.—Separately Excited

(*b*) *Separately Excited Dynamos.*—The field magnets have exciting coils which are supplied with current from a separate dynamo or from a battery (fig. 313). This type of dynamo is used for low-voltage dynamos for electroplating purposes, and for boosters.

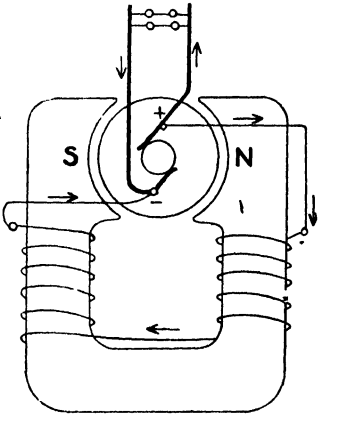


Fig. 314.—Shunt

(*c*) *Shunt Dynamo.*—The field coils are connected in parallel with the external circuit across the brushes (fig. 314). From the diagram it is evident that the coils are connected as a shunt to the external circuit, hence the name *shunt dynamo.* The field coils are wound with many turns of fine wire, so that they have a high resistance, and the energy required for excitation varies from 1 to 6 per cent of the output, depending on the size of the dynamo.

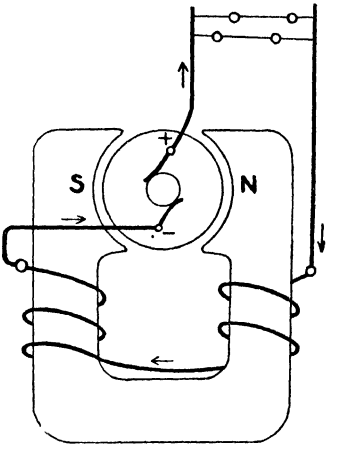


Fig. 315.—Series

(*d*) *Series Dynamo.*—In this type the field coils are wound

with a few turns of heavy wire, and are connected in series with the external circuit, hence the name *series dynamo* (fig. 315).

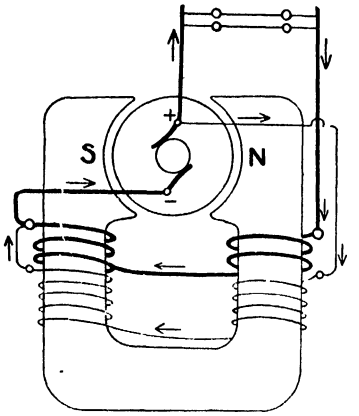


Fig. 316.—Compound Short Shunt

(*e*) *Compound-wound Dynamo.*—The fields have both series and shunt winding (fig. 316), so that the characteristics of this dynamo are a combination of the characteristics of the above two types.

Self-excitation.—The last three types of dynamos are self-exciting, i.e. if run at their normal speed they will gradually build up a voltage depending on the resistances of the field and external circuits, and on the design of the armature.

All dynamos, when unexcited, have a certain amount of remanent magnetism. When the machine is running at its normal speed this remanent magnetism induces a small E.M.F. in the armature. If the field circuit is closed, this small E.M.F. sends a small current through the field coils, which increases the flux through the armature, and again increases the induced E.M.F. This action and reaction continues until, due to field saturation, the induced E.M.F. reaches a stable value. In a series dynamo the external circuit must be closed, and have a resistance less than a certain maximum, before the generator is self-exciting, while in a shunt dynamo the external circuit need not be closed, and if it is closed the external resistance must be greater than a certain minimum value before the machine is self-exciting.

The field connections must be arranged so that the current sent through the field coils by the E.M.F., induced by the remanent magnetism, tends to increase the flux from the field, not to decrease it.

Characteristics.—The *magnetic characteristic*, also called the open-circuit characteristic, of a dynamo gives the relation between the flux entering the armature and the ampere-turns on the field when there is no current flowing in the armature coils. It can be calculated when the dimensions of the machine are given. In the previous chapter one point on such a magnetic

characteristic has been calculated. By repeating the calculation of the ampere-turns required on open circuit to produce various fluxes, the characteristic can be plotted, as in fig. 317. It can also be determined experimentally. The dynamo is run at normal speed, while the field coils are disconnected from the armature, and separately excited from an independent source of current. A resistance is inserted in series with the field coils, so that the field current may be varied. Readings are then taken of the P.D. between brushes and field current for various values of the field current, and the results are plotted, as shown in fig. 317. When the details of the armature and field windings are known, the experimental results can be converted and plotted in terms of flux and ampere-turns.

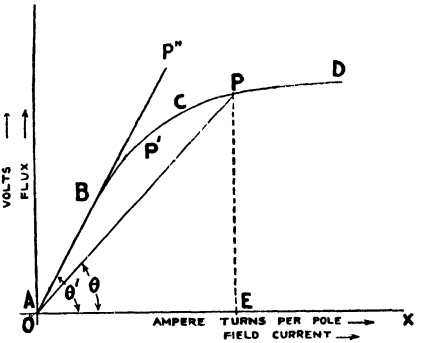


Fig. 317.—Magnetic Characteristic

The magnetic characteristic does not start from zero, but at a point A a little above zero. The distance OA, measured on the flux scale gives the flux produced by the remanent magnetism of the poles.

The curve can be divided into three portions: AB, BC, and CD. In the first portion, namely, A to B, the curve is nearly a straight line. The M.M.F. of the field coils is practically all required to overcome the reluctance of the air-gap, since the flux densities in the iron portions of the magnetic circuit are low and the permeability is therefore high.

In the part BC of the curve (called the *knee* of the magnetic characteristic) the ampere-turns increase more rapidly than the flux, showing that the permeability of the iron is decreasing. Above C the curve again becomes nearly straight, showing that the iron is saturated.

In shunt and compound generators the point representing normal voltage should be well above C, say at point P, in order that the machine may be stable in operation. A small decrease in engine speed produces a small decrease in the voltage across the field coils, and a corresponding decrease in the shunt

current. If the normal voltage corresponds to point P′, however, a small decrease in shunt field current produces a much larger change of flux than would take place for the same percentage decrease in field current at the point P. The effect of armature reaction is also reduced for the same reason.

Again, PE represents the armature voltage and OE the field current. The resistance of the field circuit is equal to armature voltage divided by field current, i.e. $(R_s) = \frac{PE}{EO} = \tan \angle POE$. The tangent of $\angle POE$ $(= \theta)$ represents the resistance of the field circuit. Now, as the resistance of the field is increased $\angle POE$ must increase, and P moves along the curve towards A. P reaches A when P″O is parallel to the tangent to the curve at A, so that tan $\angle P''OE$ represents the maximum resistance of the shunt circuit for which the machine is self-exciting. This resistance is called the *critical* resistance of the shunt circuit. For a given resistance of the shunt circuit a point can be obtained on the characteristic, which gives the maximum voltage obtainable from the dynamo.

The *external* characteristic of a dynamo gives the relation between the current flowing in the load circuit and the P.D. across the load circuit for a constant speed and constant resistance of the field circuit.

The *total* characteristic shows the relation between the induced E.M.F. and the armature current under the same conditions.

The external characteristic can be obtained experimentally by running the dynamo at constant speed. The external resistance is varied in steps from infinity to a minimum value (fixed by the maximum permissible armature current), while the corresponding values of external current and P.D. across the external circuit are noted. In the case of shunt or compound machines the shunt circuit resistance is kept constant.

The total characteristic is usually deduced graphically from the external characteristic.

Separately Excited Generator.—The external characteristic for a separately excited generator with constant excitation is shown in fig. 318. The external current is plotted as abscissæ, and terminal voltage as ordinates.

From the curve it will be evident that the voltage falls as the

external current increases. This fall is caused by (1) the voltage drop in the armature, and (2) by the decrease in the flux caused by the armature reaction.

The voltage induced in the armature is equal to the terminal voltage plus the voltage drop in the armature.

$$E_a = E_e + I_a R_a \text{ and } I_a = I_e.$$

The armature current for a separately excited dynamo is equal to the external current. In fig. 318 for the external current I_e = OD, the armature drop $I_a R_a$ is equal to CD (measured on the volt scale). OC is joined and produced to A. Then the intercept on any ordinate of the curve represents the armature drop for the current corresponding to that ordinate. Since the external E.M.F. corresponding to current I_e = OD is represented by DB, the induced E.M.F. is represented by DG, where BG = CD. G is therefore a point on the total characteristic. This process is repeated for other points on the external characteristic, and the total characteristic HG is obtained. If a horizontal line HF is drawn through H, FD represents the E.M.F. induced on no load, so that FG represents the effect of the armature reaction in decreasing the flux on load.

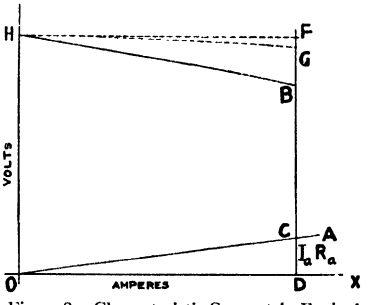


Fig. 318.—Characteristic Separately Excited

Shunt Dynamo.—Fig. 319 is a diagram of connections for a shunt dynamo. The voltage given by the machine can be varied by altering the resistance connected in series with the shunt field.

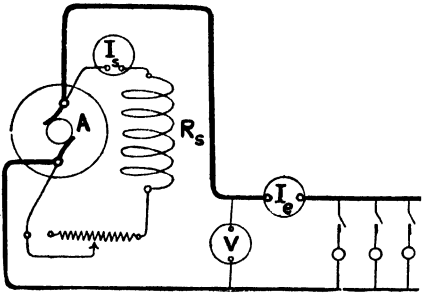


Fig. 319.—Shunt Connections

The external characteristic is shown in fig. 320. The maximum terminal voltage occurs at no load, and as the load current increases the voltage falls steadily. This drop is due primarily to (1) the armature resistance, and (2) the armature reaction.

Since the field coils are connected in parallel with the external circuit, the drop in P.D. across them decreases the field current and so causes a further drop in the terminal voltage. This decrease in voltage and increase in armature current goes on till a point is reached at which a further decrease in external resistance causes a decrease in both external current and P.D., and a further small increase causes both current and P.D. to fall practically to zero. At a point on the characteristic beyond the maximum the dynamo is working on the straight portion of the magnetic characteristic, and is therefore unstable. As in the case of the magnetic characteristic, the tangent of the angle COR gives the critical resistance of the external circuit. If the external circuit has a resistance smaller than this critical value the dynamo will not be self-exciting. This is of great practical importance, since if a shunt machine is accidentally short-circuited while running, the armature winding is not damaged by the continued passage of an abnormally large current, but the machine is demagnetized, and gives only a small current.

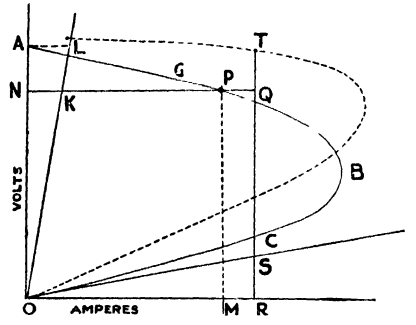


Fig. 320.—Shunt Characteristic

Usually it is impossible to obtain the maximum current point B on account of the heating of the armature. The maximum current is from two to three times the normal current, and the working portion of the external characteristic is the part AG.

The armature current I_a in a shunt dynamo is the sum of the shunt current I_s and the external current I_e. The induced E.M.F. is $E_a = E_e + I_a R_a = E_e + R_a (I_e + I_s)$.

The following method can be used for obtaining graphically the total characteristic from the external characteristic.

From A draw a horizontal line AL to represent the shunt current with voltage OA across the field circuit. Then join OL, and the intercept on any horizontal line represents the shunt current for that voltage. From R draw a vertical line RS to represent the armature drop corresponding to current OR. Join OS and produce it. Take any point P on the ex-

ternal characteristic. Draw PN horizontally and PM vertically. On PN produced set off PQ = NK = shunt current for the voltage PM. Then draw QR parallel to OA. Then OR is the armature current corresponding to the external current OM, and RS is the armature drop for this current. Set off QT = RS. Then T is a point on the total characteristic, since TR = RQ + RS, and is therefore the induced E.M.F. for the armature current OR. Repeating this for other points on the external characteristic the total characteristic (shown dotted) is obtained.

Another characteristic which is sometimes of great service is the regulation characteristic connecting excitation and external current when the dynamo field is varied so as to maintain constant voltage at all loads from zero to full load (fig. 321). If OC in fig. 321 is the external current at full load, then OB represents the exciting current required to give this current OC at normal voltage. The exciting current at no load is represented by OA, and the shunt field resistance must be designed to reduce the current from the value OB to the value OA. Usually the shunt regulator has more resistance than is necessary for this purpose, in order to take account of the changes in resistance of the field coils due to heating.

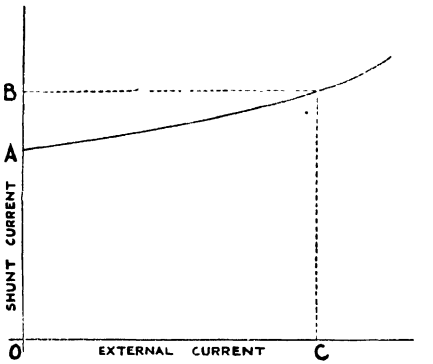


Fig. 321.—Regulation Characteristic

The regulation curve of a shunt dynamo is not a straight line but a curve. In order to keep the terminal voltage constant, the induced E.M.F. and the flux must be increased in proportion to the load. Since the magnetic characteristic is curved, equal increments of flux do not correspond to equal increments of excitation, but the greater the flux the greater is the increment of excitation required to produce a given change in the flux. The regulation characteristic is therefore concave upwards.

Shunt winding is more generally used than any other form of field winding for generators. In the case of large generators the decrease in voltage with increase in load is not excessive

within normal limits of working, so that shunt-wound generators may be regarded as giving a roughly constant terminal voltage. The bulk of the large generators in central stations, supplying current to lighting loads, are shunt wound. The load only changes slowly, so that the voltage is easily maintained constant by altering the resistance in the shunt field, while the switch gear is much simpler than that for compound dynamos.

Series Dynamo.—The connections for a series dynamo are shown diagrammatically in fig. 322. The voltage given by the dynamo can be regulated either by altering the number of turns in the field coils or by altering the resistance

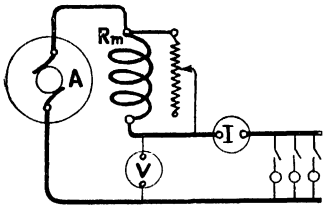


Fig. 322.—Series Connections

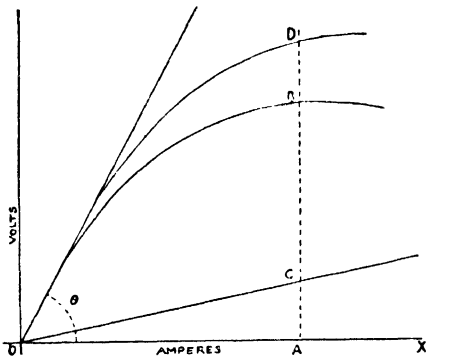


Fig. 323.—Series Characteristic

connected in parallel with the series coils. This resistance is usually known as a "diverter". By decreasing this resistance more of the external current passes through it and less through the series coil. The external characteristic is shown in fig. 323. When the external circuit is open the terminal E.M.F. is that due to residual magnetism. As the external resistance is reduced the external P.D. and current increase together to a certain point B on the curve. Beyond this point the external P.D. decreases with an increase in external current. The field coils of a series dynamo are excited by the external current so that the external characteristic will be similar to the magnetic characteristic. This is so up to the point B on the characteristic. At this point, however, the iron portions of the magnetic circuit are becoming saturated, with the result that a large increase in the external current only produces a small increase in the flux and induced E.M.F. The effect of this flux is wholly neutralized by the increased IR drop in the armature and field and the increased armature reaction, so that the external characteristic

droops (fig. 323). The external current and the armature current are the same, while the induced E.M.F. is equal to the sum of the external P.D. and the armature IR drop. The total characteristic is obtained graphically by adding the armature IR drop. In fig. 323 AC represents the drop in the armature for the current OA, and BD is set off equal to AC, so that AD = AB + AC = induced E.M.F. for the armature current OA. D is therefore a point on the total characteristic. The slope of the straight part of the external characteristic gives the critical resistance of the external circuit. (The reasoning is the same as for that of the shunt machine.) If the external resistance is greater than this "critical" value the dynamo will fail to excite.

The series dynamo is not much used at the present time. The principal objection to it is that the voltage varies with the load, and since practically all distribution systems are fed from a constant-voltage supply, the series dynamo is useless. Series-wound dynamos are sometimes used as boosters, but this application will be referred to later. They are also adopted in a Thury constant-current system of high-tension transmission.

Compound Dynamos.—Compound-wound dynamos can be divided into two classes, depending on the manner in which the field coils are connected: (*a*) short shunt, (*b*) long shunt. In the short-shunt compound dynamo (fig. 324*a*) the field coils are shown connected across the armature, so that the shunt field is excited from the brush voltage and the external current alone passes through the series coils. In the long-shunt compound dynamo (fig. 324*b*), however, the shunt field coils are connected across the terminals of the dynamo, and the whole armature current passes through the series coils. In the case of large machines there is very little difference in the external characteristic of the dynamo when it is connected either way, but in small machines a change in the method of connection may make a large difference in the characteristic.

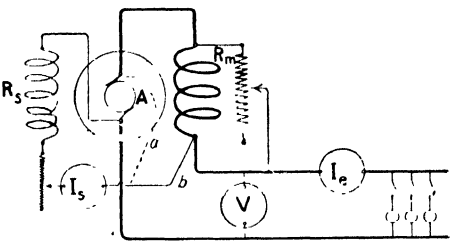


Fig. 324.—Short-shunt Compound Connections (*a*)
Long-shunt Compound Connections (*b*)

Since a shunt dynamo gives a decreasing voltage with an increase in load, and a series dynamo gives an increasing voltage with an increase in load, it is possible by combining a shunt and series dynamo in one machine, as in a compound dynamo, to obtain a machine which gives a constant or even an increasing voltage with increase in load. The shape of the external characteristic depends on the amount of series winding on the field. In the case of a long-shunt dynamo, the number of turns on the series field to give constant voltage can be easily determined from the regulation characteristic. Let i_s be the increase in shunt current required to maintain constant voltage, then the increase in field ampere-turns per pole is i_s multiplied by the number of turns in the shunt field coil S_s per pole. Then the number of turns on the series field coil per pole is

$$S_m = \frac{i_s \times S_s}{I_m},$$

and the external characteristic up to the full-load external current will be very nearly a straight line. If the dynamo has fewer turns in the series coil, the characteristic will droop as the load increases, and the dynamo is said to be "undercompounded"; and if there are more turns in the series coil the characteristic will rise as the load increases, and the dynamo is overcompounded. Fig. 325 gives the external characteristics for a level-compound dynamo, an undercompound dynamo, and an overcompound dynamo.

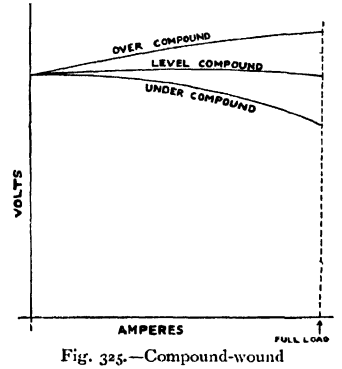


Fig. 325.—Compound-wound Characteristic

The regulation curve of a shunt dynamo is not a straight line, but bends upwards, so that a dynamo whose series field is arranged to give the same voltage at no load and full load will give a slightly higher voltage at loads between these values. The ampere-turns on the field increase proportionately to the load, and therefore at loads between no load and full load the field ampere-turns will be more than are necessary to maintain constant voltage. A level compound dynamo only compounds truly at one particular

speed of rotation. At higher speeds it will be overcompound, and at lower speeds it will be undercompound.

The voltage on open circuit can be adjusted by the shunt-regulating resistance, while the amount of compounding can be adjusted by altering the "diverter" resistance which is shunted across the field coils. The diverter resistance is useless when the dynamo is running in parallel with other compound dynamos, and the compounding is then adjusted by a small resistance in series with the series coil.

In a short-shunt dynamo the P.D. across the shunt coils is equal to the external P.D. plus the IR drop in the series coils, so that the current in the shunt circuit is $I_s = \frac{E_e + I_e R_m}{R_s}$, where E_e and I_e are the external P.D. and current respectively, and R_m and R_s the resistances of the series and shunt field coils. The armature current is $I_s + I_e$, so that the induced E.M.F. is

$$E_a = E_e + I_e R_m + (I_s + I_e)R_a$$
$$= E_e + I_e(R_m + R_a) + \frac{E_e + I_e R_m}{R_s} \times R_a.$$

This equation can be worked out graphically as in fig. 326. A is a point on the external characteristic corresponding to the external current OB. BC represents the IR drop in the series field coils, so that if AD is set off equal to BC, BD represents the P.D. at the armature terminals. LS (set off as in the shunt dynamo) represents the shunt current for a voltage OL = BD and DK = LS. OF therefore represents the armature current, and FG the IR drop in the armature. KH is set off = FG, and HF is the internal induced E.M.F. for the armature current OF. H is therefore a point on the total characteristic.

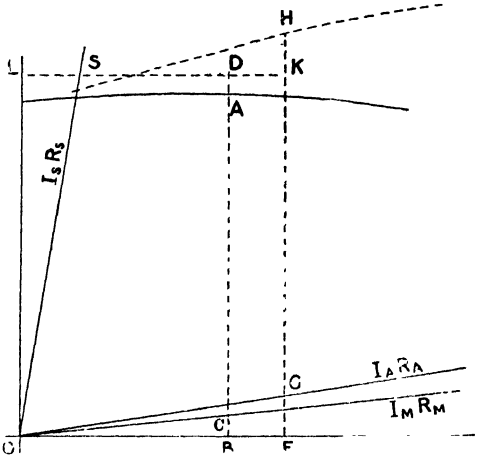


Fig. 326.—Short-shunt Compound Characteristic

In the long-shunt dynamo the shunt current is $I_s = \frac{E_e}{R_s}$. The armature and series field currents are the same, and are equal to the external current plus the shunt current. The induced E.M.F. is therefore

$$E_a = (I_e + I_s)(R_a + R_m).$$

The total characteristic can be deduced in a similar manner to that of the shunt dynamo.

The compound dynamo is used for supplying lighting and power (mainly in small stations) and for traction work. Generators used for supplying a traction load are overcompounded in order to compensate for the drop in the feeders.

Rosenberg Constant-current Dynamo.—In the majority of cases dynamos are required to give constant voltage when driven at a constant speed, but for certain purposes, such as running arc lamps in series, or for supplying single arc lamps, or for train lighting, a dynamo is required which is capable of giving a constant current in spite of variations in the external resistance or in the speed at which the dynamo is driven. A dynamo for train lighting is usually driven by belt from the axle of the carriage, while a battery of storage cells is installed to take the load when the train is moving very slowly or is at rest. When the train has reached a certain low speed, the dynamo must come automatically into action, and supply current sufficient to charge the battery and supply the lamps. This current must remain approximately constant over a wide speed range. In Rosenberg's constant-current dynamo these characteristics are obtained in a very simple manner. Fig. 327 represents a 2-pole dynamo of this type. The yoke and pole limbs have a very small cross-sectional area, and are "saturated" under

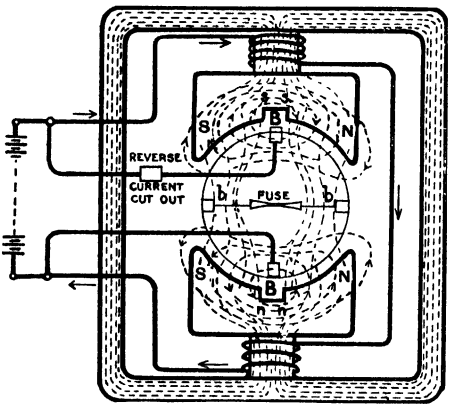


Fig. 327.—Rosenberg Constant-current Dynamo

normal conditions of working. The flux in the poles and yoke is produced by a small magnetizing current obtained either from the dynamo or the battery, the top pole being south and the bottom pole north. In the position *bb*, where the brushes in an ordinary dynamo would be placed, are a pair of brushes which are short-circuited through a fuse. The field magnets do not provide the useful flux which generates the working E.M.F. in the armature, but an auxiliary flux which produces a current through the short-circuited armature. This current produces a large cross flux which constitutes the main flux for the armature, and is at right angles to the auxiliary flux. The massive pole shoes serve as the return path for it. In the armature this cross flux is from right to left. The E.M.F. induced by this cross flux is at maximum between conductors whose position is that of B.B. A pair of brushes, which are the main brushes, are placed in this position, and are joined to the battery terminals. Since a small E.M.F. induced in the armature by the auxiliary flux will produce normal current in the short-circuited circuit, only a small auxiliary flux is required even at low speeds, and the section of the pole limbs and yoke is therefore small. The main or cross flux on the other hand is large so that the pole shoes must have a large cross section.

Since the direction of the short-circuit current, and hence of the cross flux, depends on the rotation, so that a reversal of the direction of rotation reverses the cross flux at the same time, the polarity of the main brushes BB will therefore be unaltered if the direction of rotation is reversed.

With increasing speed the E.M.F. due to the cross flux increases at first rapidly and then more slowly. When it rises above that of the battery it will charge the battery and supply current to the lamps.

The brushes BB are set so that the armature ampere-turns due to the current flowing in the external circuit tend to produce a flux in the opposite direction to the auxiliary flux, and so an increase in the external current decreases the auxiliary flux. This decrease in the auxiliary flux decreases the short-circuit current and the main cross flux, thus reducing the E.M.F. across the main brushes BB. In this way the external current is kept approximately constant for great changes in speed. With an increase in speed there is a decrease in the auxiliary

flux. The external current increases slowly with speed, the limit being reached when the armature demagnetizing ampere-turns are nearly equal to the field ampere-turns, which only occurs at an infinite speed.

The speed external-current characteristic is shown in fig. 328. The current flowing in the short-circuited brushes is shown as decreasing almost inversely in proportion to the speed. Thus the product of cross flux (which is proportional to the short-circuit current) and speed, that is, the induced E.M.F., remains nearly constant.

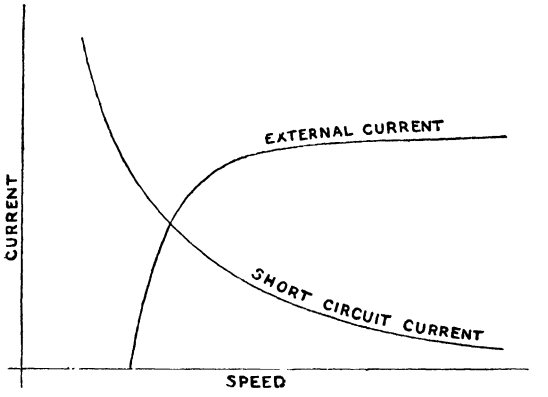


Fig. 328.—Rosenberg External-current Characteristic

No damage to the dynamo can be caused by a short-circuit in the external circuit, since the current cannot increase above a certain maximum value. On the other hand, if the external circuit is opened and the dynamo is running at a high speed, the demagnetizing effect of the armature is removed, and the auxiliary flux will increase to its maximum value, causing a large current to flow in the short-circuited armature, which may damage it. To prevent this a fuse is inserted in the connection between the short-circuited brushes.

To prevent the battery from being short-circuited when the dynamo is at rest a reverse-current cut-out is inserted in one of the leads to the battery. It consists of a cell containing a solution of ammonium borate and two plates—one of aluminium, the other of iron—dipping into the solution. So long as the aluminium plate is the cathode the current passes freely through the cell; but if the current reverses, a layer of non-conducting oxide of aluminium is formed on the aluminium plate, which insulates it and stops the current.

When a dynamo of this type is used to supply a single large arc lamp it is driven at constant speed, and the field has a series winding. The connections are shown in fig. 329. The

dynamo now gives a varying voltage and an almost constant current over a wide range in voltage.

The exact shape of the external characteristic depends on the relative value of the field and armature ampere-turns. The larger the field turns are compared with the armature ampere-turns the greater will be the auxiliary flux and therefore the main cross flux. The greater the main flux is, the greater will be the current through a given external resistance. The dynamo can be adjusted to give any definite current by shunting the field coils by a resistance. Fig. 330 shows two characteristics — the full curve corresponds to the case where the full current passes through the field coils, while the dotted curve shows the effect of shunting the field coils. One great advantage of this type of dynamo is that steadying resistances are unnecessary, since a change in length of the arc reduces the current slightly, but increases the voltage across the arc by a large amount. The machine cannot be overloaded by leaving the carbons short-circuited.

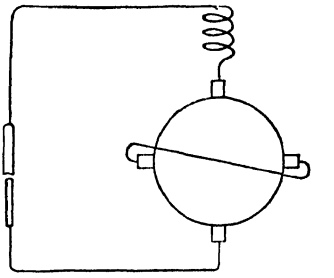


Fig. 329.—Connections with Series Field

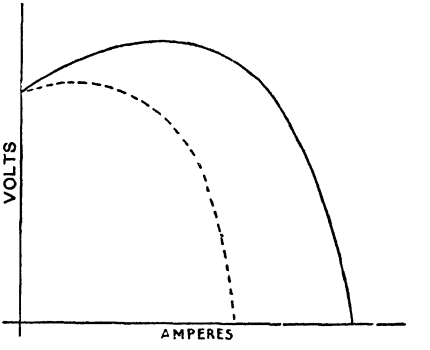


Fig. 330.—Characteristic of Rosenberg Series Dynamo

Paralleling of Shunt Dynamos.—The load on an electricity-supply station is not constant, but varies from time to time during the day. The time of greatest load is usually in the evening, when there is a large amount of power required for lighting. The load also varies with the seasons, being greater in winter than in summer. Now the efficiency of an engine and dynamo falls off very quickly as the load on the set is reduced, so that it would be uneconomical to run a large generating set at much below full load. For this reason the total output of the station is divided into several units, which are so arranged that at times of small load it is always possible

to have the dynamo supplying the load running at its best efficiency, while at times of large load several machines are running in parallel, and the efficiency is again the same.

The connections for running two shunt dynamos in parallel are given in fig. 331. The positive and negative terminals of each dynamo are connected through switches to the common conductors B+ and B−. These are known as bus-bars, and from them feeders run to the network of street mains. In series with the positive lead from each dynamo are connected ammeters A_1 and A_2, while the voltmeters V_1 and V_2 indicate the P.D. of each dynamo. The shunt coils have regulating resistances R_1 and R_2 in series with them. Dynamo D_1 is supplying the load and dynamo D_2 is to be paralleled with it. Before two dynamos can run in parallel the following conditions must be fulfilled, namely: (*a*) their polarity must be the same — that is, the positive terminals of each must be connected to the same bus-bar, and similarly for the negative terminals; (*b*) they must be excited to give exactly the same terminal voltage.

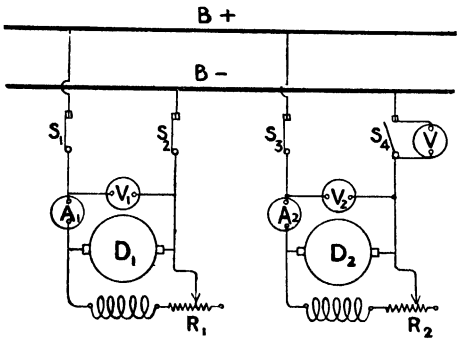


Fig 331.—Paralleling Shunt Dynamos

The method adopted is as follows: The second dynamo D_2 is run up to its proper speed, and its excitation is altered till the terminal E.M.F. of D_2 is approximately the same as that of D_1. Switch S_3 is now closed, and a voltmeter, known as a paralleling voltmeter, is connected across S_4, as in fig. 331. If the polarity of D_2 is right, this voltmeter will read approximately zero. The voltmeter reading is now reduced to zero by altering the excitation of D_2 by means of the field resistance, and the switch S_4 is closed. D_2 will then be running in parallel with D_1, but will not take any load. The dynamo is made to take its proper share of the load by increasing the field excitation, which increases the induced E.M.F. above the bus-bar P.D., thus causing a current to flow from the dynamo to the bus-bars and load the dynamo D_2. At the same time the load on

D_1 will be decreased. To shut down a dynamo the field excitation is reduced till the dynamo is delivering no current, when the switches may be opened.

Shunt dynamos running in parallel are quite stable. If, for example, one dynamo should increase its speed, the induced E.M.F. in the armature will be increased, and the dynamo will deliver an increased current to the bus-bars. Owing to the increased load, the engine driving the dynamo will tend to slow down, which stops the increase in load. At the same time the load on the other dynamos will be slightly reduced, tending to make them run slightly faster, and so check the decrease in load on them. The result, therefore, of a change in load of one dynamo is to call into play electrical actions and reactions in the other machines running in parallel with the first, which tend to oppose the change of the load on it.

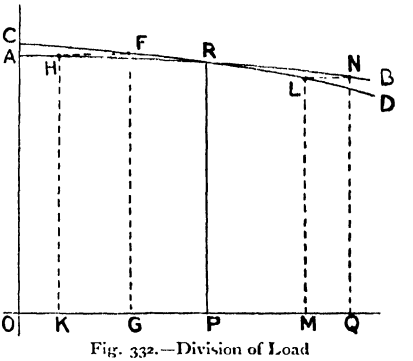


Fig. 332.—Division of Load

The division of an increase or decrease in load among machines running in parallel depends on the shape of the external characteristic curves. To illustrate this the case of two machines of equal size running in parallel will be taken, each giving an output equal to half full load. The first machine has a flat external characteristic AB (fig. 332), while the other has a sloping characteristic CD. The two characteristics intersect at the point R, which corresponds to an output from each machine equal to half full load. Now, the terminal voltage of each dynamo must be equal for any given total load. Assume that the external load decreases, and that the voltage of the bus-bars is now represented by HK. Then the line HF is drawn parallel to the current axis, cutting the external characteristic AB in H and CD in F. The currents corresponding to this new value of the terminal voltage are OK and OG respectively. The machine with the flat characteristic therefore delivers a smaller current than the one with the sloping characteristic. The reverse occurs for an increase in load, the first dynamo delivering a greater current than the other. In short, the dynamo

with the steeper characteristic tends to give a constant output, while the other tends to take all the variations in load.

Dynamos which are to run in parallel must therefore have similar characteristics if the load is to be divided between them proportionally to their individual output.

Paralleling of Compound Dynamos.—In the parallel running of compound dynamos there are other conditions to be fulfilled besides those mentioned in the case of shunt dynamos.

To investigate these conditions it will be assumed that two compound dynamos are running in parallel, with connections as shown in full lines in fig. 333. In addition, suppose that the speed of D_1 is increased. This increase in speed will increase the induced E.M.F. in the armature, and will therefore increase the armature current. This increase in current flowing in the series coils will increase the flux entering the armature, which in turn increases the induced E.M.F., causing a still larger armature current. At the same time the armature current in D_2 is reduced, causing a reduction in the induced E.M.F. by weakening the series field. This reduction in the induced E.M.F. causes a further reduction in the armature current. Both these actions increase, till finally the current from D_1 is reduced to zero, and D_2 reverses and runs as a motor. The running of compound-wound dynamos, connected as shown in fig. 333, is therefore unstable. If the ends of the series field coils connected to the brushes are connected together (shown dotted in fig. 333) by a conductor whose resistance is negligible compared with that of either series coil, the machines will be stable. They are in reality running as shunt dynamos connected to the negative bus-bar and the low-resistance bar, which is known as the equalizing bus-bar. The current in the series coils will be independent of the load on each dynamo, and the division of the total current between them will only depend on their relative resistances, since all points on the negative bus-

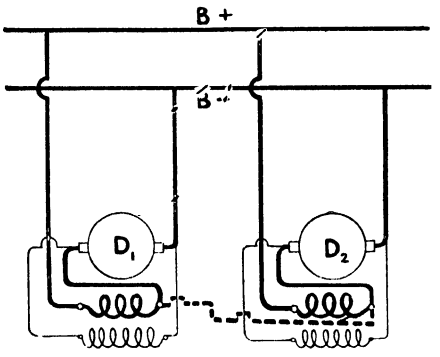


Fig. 333.—Paralleling Compound Dynamos

bar are at practically the same potential, and all points on the equalizer bar are at practically the same potential.

A compound dynamo is paralleled with others running on load in the following manner. The dynamo D_2 (fig. 334) is run up to its proper speed, and its field is excited. The equalizing switch S_5 and switch S_6 are next closed, thus exciting the series field coil. The shunt excitation is now altered by means of the shunt resistance R_2 till the paralleling voltmeter V connected across the switch S_4 reads zero. S_4 is then closed, and the dynamo is running in parallel with the other machines, and may be loaded by increasing the excitation of the shunt field coils. To shut down a compound dynamo the load is reduced to zero by altering the shunt field, and then the switches are opened in the order S_4, S_5, and finally S_6. In the diagram of connections the ammeters A_1 and A_2 give the output of each machine. An ammeter placed in the lead to the negative bar only gives the current flowing in the series field coil, which is not necessarily the current delivered by the armature.

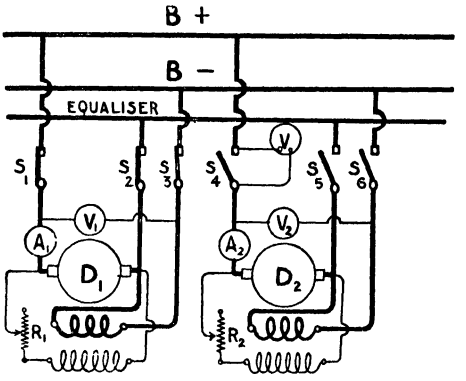


Fig. 334.—Paralleling Compound Dynamos

As in the case of shunt dynamos, compound dynamos which are to run in parallel must have similar external characteristics. In order to adjust the compounding of a single dynamo, independently of the others in parallel with it, a resistance must be inserted in series with the series coils. A resistance connected, in parallel with the field coil, between the negative bus-bar and the equalizer bus-bar alters the compounding of all the machines which are running in parallel.

Boosters.—A booster is a dynamo which is used to alter the voltage of a circuit in which an E.M.F. already exists. A booster may be driven by an electric motor or by an engine. Usually, however, it is driven at constant speed by a shunt motor. The booster armature is connected in series with the

circuit whose voltage has to be altered, while the field may be separately or self excited.

The P.D. between the positive and negative leads at all points in a distributing network should be approximately constant. If the feeders to the various parts of the network are connected to the same bus-bar it is not possible to fulfil this condition, owing to the varying drops of potential in the feeders, which are not all of the same length. By connecting a series-wound booster in series with each feeder it is possible to compensate for the drop of potential in the feeder, and so maintain a P.D. equal to the bus-bar P.D. at the far end of the feeder. In fig. 335 B+ and B− are the station bus-bars and F+ and F− the positive and negative leads of one feeder. In series with the positive lead are connected the armature A and field coils R_m of the booster. The booster is driven at constant speed by the shunt-motor M. Now, since the field current of the booster is the current flowing in the feeder, the E.M.F. added by the booster will vary with this current. Now, the fall of potential in the feeder is proportional to the feeder current, so that if the booster is to compensate for this fall of potential the E.M.F. given by it must be proportional to the current. The booster is therefore worked on the straight portion of the magnetic characteristic, i.e. the part AB (fig. 317) where the flux and the induced E.M.F. are proportional to the field excitation. In this manner the drop in the feeder is compensated, and the P.D. at the far end of the feeder will be equal to that of the bus-bar for all loads if the booster is designed to give a terminal E.M.F. equal to the drop in the feeder when full-load current is passing through them both.

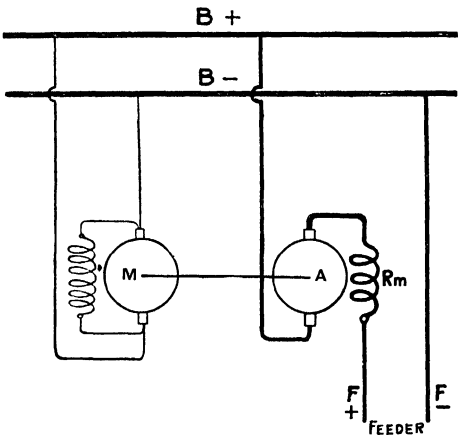


Fig. 335.—Feeder Booster

Another application of boosters is that of charging accumulators. During the charge of a battery the voltage across one

cell varies from 1.8 volts to 2.6 volts, so that if the cells are charged directly from the dynamo the dynamo must be capable of giving this variation in voltage. By using a booster in series with the dynamo the dynamo can be run at a constant voltage, and the booster gives the additional voltage required to charge the battery. Fig. 336 is a simple diagram of connections for such a booster. The armature A of the booster is driven by a shunt-wound motor M, and the field coils F are excited from the main bus-bars. The voltage given by the booster can be varied by the field resistance R. The booster armature is connected

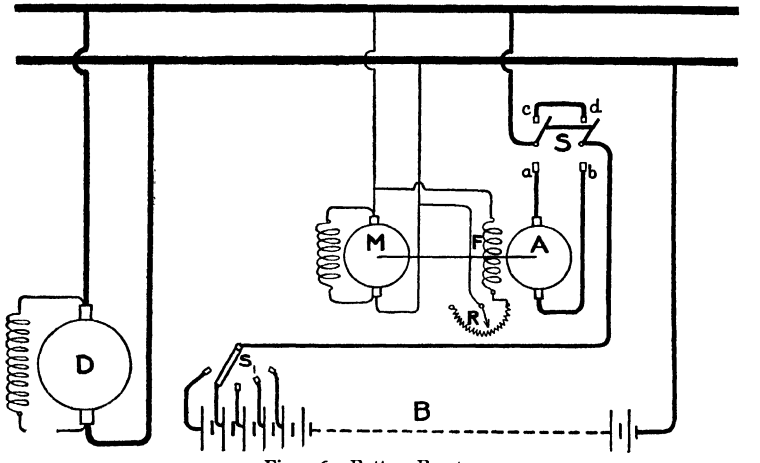


Fig. 336.—Battery Booster

to the terminals *ab* of the double-pole change-over switch S. When the battery is to be charged the switch is changed over to position *ab*, thus connecting the booster armature in series with the battery, and the booster voltage is added to that of the bus-bar. The booster field current is regulated by the resistance R till the proper charging current flows in the battery. As the charge proceeds the charging voltage is increased by cutting out resistance in the field circuit. When the end cells of the battery are fully charged they can be cut out of circuit by means of the regulating switch S_1. During discharge the double-pole switch S is thrown over into the upper position *cd*, thus cutting the booster armature out of circuit. The battery voltage during discharge can be regulated by altering the number of cells in circuit by means of the regulating switch S_1.

Efficiency and Losses.—During the transformation of mechanical energy into electrical energy in a dynamo a certain amount of it is always lost. The ratio of the electrical energy given out to the energy supplied is called the *efficiency*, which therefore must always be less than unity. There are various sources of losses in a dynamo, namely:

(*a*) Losses due to the heating of the field and armature conductors by the currents flowing through them. These losses are usually known as the "copper losses".

(*b*) Hysteresis and eddy-current losses in the iron of the armature and pole faces.

(*c*) Mechanical losses, namely, friction of the bearings and of the brushes on the commutator, and energy loss in churning the surrounding air (known as "windage").

(*a*) The losses in the armature and field coils are easily calculated. If R_a is the armature resistance and I_a the armature current, then the copper loss in the armature is $I_a^2R_a$. Similarly the shunt field copper loss is $I_s^2R_s$, and the series field loss (if there are series field coils) is $I_m^2R_m$. In addition to the purely copper losses there is a loss in the contact resistance of the brushes on the commutator. Now the contact resistance of carbon brushes on a copper commutator is almost inversely proportional to the current flowing, so that the loss is practically proportional to the current. For ordinary hard-carbon brushes the brush contact drop is from 2 to 2.5 volts for the two brushes in series. The total copper losses are therefore

$$W_{cu} = I_a^2R_a + I_s^2R_s + I_m^2R_m + 2\,I_a.$$

(*b*) The hysteresis and eddy-current losses in the armature can only be calculated with difficulty. A formula has been given before for the hysteresis loss, namely—

$$W_h = \eta V B^{1.6} f \times 10^{-7} \text{ watts},$$

where η = hysteresis constant, V = volume of iron in cubic centimetres, B = maximum induction, and f = magnetic cycles per second. For armature laminations η has a value varying from 0.0015 to 0.003. The values of the hysteresis loss for the teeth and core should be calculated separately, since the density in the teeth is much greater than that in the core. For high

densities the $B^{1.6}$ law does not hold for hysteresis, and the calculation is therefore not very accurate.

In addition to the hysteresis loss there is a loss due to the eddy currents produced in the plates by rotation in a magnetic field. From theoretical considerations a formula has been deduced for the eddy-current losses.

$$W_e = \beta V B^2 f^2 t^2,$$

where β is the eddy-current coefficient, V, B, and f are the same as for the hysteresis loss, and t is the thickness of the plate in centimetres. β is approximately 7.5×10^{-11} for ordinary iron stampings. Experience has shown, however, that although this formula is accurate enough for calculating the losses due to eddy currents in a single stamping, the actual losses in the built-up core are from two to three times the calculated value. This discrepancy is due to the fact that (1) the plates are not completely insulated from each other, and (2) there are eddy currents induced in neighbouring masses of iron which are not taken account of in the formula. The eddy-current loss in the pole shoes is also very difficult to calculate, but this loss can be reduced by laminating them.

It is evident, therefore, that the calculation of the hysteresis and eddy-current losses from the theoretical formulas is not accurate, since there are many factors which cannot be taken account of in the calculation. In practice, empirical formulas are used which are based on experiments on built-up cores. These formulas will give sufficiently accurate figures for dynamos of similar design, and which are assembled in the same manner, but which do not necessarily hold good for other designs. The following empirical formula, given by Esterline and Reid, may be used for calculating the approximate iron loss (hysteresis and eddies):—

$$\text{Watts per kilogram of core} = 16.4\,x + 22\,x^2$$

where $x = Bf10^{-6}$. The losses in teeth and core are calculated separately and added.

When the armature is loaded, the air-gap flux distribution is distorted by armature reaction from that on no load, so that the iron losses on load are greater than those on no load. This increase cannot be calculated very accurately.

(*c*) The mechanical losses—friction and windage—are always determined from empirical formulas derived from experiment. This loss varies from 0.5 per cent of the output in the case of 1000-kilowatt machines to 2 per cent in the case of 50-kilowatt machines.

The efficiency is equal to the output divided by the input. Since the difference is equal to the losses in the dynamo the efficiency $= \frac{\text{Output}}{\text{Output} + \text{losses}}$. We can therefore write

$$\eta = \frac{W_o}{W_1} = \frac{W_o}{W_o + \text{losses}} = \frac{W_o}{W_o + W_{cu} + W_i + W_f},$$

where W_{cu} = copper loss, W_i = iron loss, W_f = mechanical losses. The efficiency varies with the load, and is a maximum when $W_{cu} = (W_i + W_f)$.

Heating.—The energy lost in a dynamo is all turned into heat. The generation of heat is continuous, and is accompanied by a rise in temperature of the machine above that of the surrounding air. The temperature rises until the rate of generation is equal to the rate of loss of heat by radiation, convection, and conduction. The steady temperature reached by any part of the machine depends on the cooling surface provided, so that in a design sufficient cooling surface must be arranged for in order that the temperature rise may not be excessive.

If cotton or other fibrous insulation is kept at a temperature of about 80° C. it begins to char, and if there is any vibration it will crumble away, leaving the conductors uninsulated, thus increasing the chances of breakdown. The hysteresis loss in iron kept at a temperature of more than 80° C. increases continuously with time, and in the case of annealed iron this increase may be very great. This increase is known as "ageing". For these reasons the maximum temperature of a dynamo is limited to about 60° C., which gives a temperature rise of about 40° C., measured from the average summer temperature in this country. In specifications, the temperature rise above the surrounding air is therefore specified to be less than 40° C.

Considering the case of a field coil whose external diameter is d cm. and length l cm., and in which W watts are dissipated, the temperature rise of the outside layer may be estimated from the formula

$$\theta = \frac{360W}{\pi dl}.$$

The temperature of the centre of the coil is greater (10 to 50 per cent) than this, depending on the depth of the winding.

The temperature rise of a dynamo armature is much more difficult to determine. For large well-ventilated armatures the temperature rise may be taken as

$$\theta = \frac{350W}{A(1 + .24v)},$$

where W = the sum of the armature copper and iron loss, A = the circumferential area in sq. cm. of the armature (including the winding), and v = the peripheral speed in metres per second. The formulas given above have constants which have been determined experimentally, and the constants given must be checked experimentally on actual machines.

From the preceding it is evident that the output of a dynamo is limited by the maximum temperature rise allowable. In the case of direct-current generators and motors there is another condition which limits the output, namely, sparking. Generally speaking, however, the output is limited by thermal considerations, since commutation troubles can be overcome by using commutating poles to improve commutation.

Testing.—In the preceding paragraphs the method of calculating the losses and the efficiency from constructional data has been indicated. The next point to be explained is the experimental determination of these quantities.

The method to be adopted depends largely on the size of the machine and the facilities for testing.

In the case of small motors the efficiency may be determined by the direct measurement of input and output. The output is absorbed and measured by a rope brake, while the input is measured by means of an ammeter and voltmeter. The brake consists of two ropes, each led once round the pulley. These ropes are held in place by wooden blocks attached to them. One end of the ropes is attached to a large weight W_1 in such a manner that the motor tends to lift the weight while the other end is attached to a spring balance whose reading is W_2. The total pull on the rope is $(W_1 - W_2)$, and this pull acts at a distance r—equal to radius of rope + radius of pulley—from the motor shaft. The torque is therefore $(W_1 - W_2)r$, and the angular velocity is $2\pi n$ radians per second. The work done

per second is equal to torque multiplied by angular velocity, and the brake horse-power developed is the work done per second divided by 550, since 1 horse-power is equivalent to 550 ft.-lb. per second, so that finally

$$\text{B.H.P.} = \frac{(W_1 - W_2)\, r \times 2\pi n}{550}.$$

The output in watts is B.H.P. × 746. The input current is I amperes, and the P.D. applied to the motor terminals is E volts, so that the input is EI watts, and the efficiency

$$\eta = \frac{\text{B.H.P.} \times 746}{EI}.$$

All the power measured by the brake is wasted in heating the motor pulley. In the case of large motors the brake must be water-cooled, or the heat generated is liable to burn the brake ropes, and in addition makes the readings unsteady.

Small generators may be tested for efficiency by running them as motors at a slightly higher terminal voltage, so that the induced E.M.F. is the same as for the normal case of running as a generator. The losses are then the same whether the machine is running as a generator or motor, and the efficiency can be calculated independently from the output as a generator, and the losses measured in this manner for the same induced voltage and armature current.

Another convenient method, which is adopted when large numbers of approximately similar machines have to be tested, is to drive the generator directly by a motor whose efficiency is known accurately for all inputs. The output of the motor is the input to the generator, and if w_i is the input of the motor, whose efficiency at this input is η_1, the generator input is $\eta_1 w_i$. The generator efficiency is therefore $\frac{w_o}{\eta_1 w_i}$, where w_o is the output of the generator.

The methods given above are inconvenient in the case of large machines, since the power required to drive them at full load is all wasted. The most satisfactory test for machines whose output is greater than 20 kilowatts is that generally known as the Hopkinson test. This test requires two machines, which must be exactly similar in all respects, and which are mechanically coupled. They are also electrically connected together, as shown in fig. 337. The field of one of the machines

is slightly weakened, while that of the other is strengthened, so that the induced E.M.F.'s in the two machines are no longer equal, and a circulating current flows in both armature windings. One machine is loaded as a motor, the other as a dynamo. Power must be supplied to the combination in order to keep the machines running, and this power is equal to the sum of all the losses. In the original Hopkinson test this power was supplied mechanically, and was difficult to measure. It was modified by Kapp, and the modified form is that shown in fig. 337. The losses are now supplied electrically by an auxiliary generator.

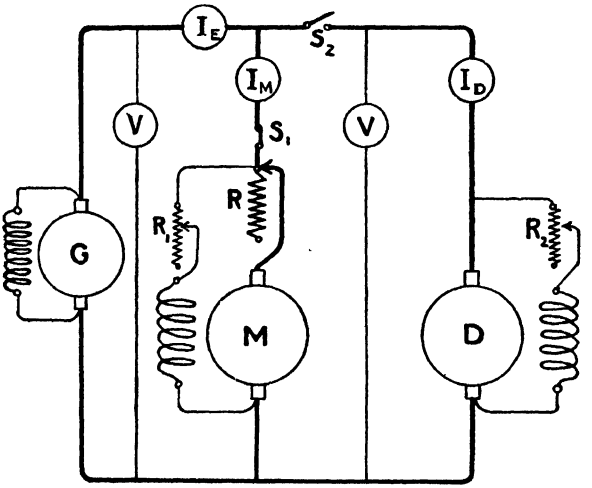


Fig. 337.—Hopkinson Test

The two machines to be tested are M and D, of which M is supposed to run as a motor and D as a generator. The motor is connected to the terminals of the auxiliary generator, whose voltage is the same as that of M, through a starting resistance R, ammeter, and switch S_1, while the generator has a switch S_2 in series with it. Care must be taken that terminals having the same polarity are connected together. The motor is started by means of the resistance R, and its speed is adjusted to the proper value by means of the field resistance R_1. Switch S_2 is closed when the voltage of the dynamo D is equal to that of the mains. By adjusting the field resistances full-load current can be made to flow in the armature of the generator. Now, if I_D is the current delivered by the generator, I_M the current taken by

the motor, the current supplied by the auxiliary generator I_E, then $I_M = I_D + I_E$. The two machines are nearly equally loaded, and their efficiencies may be assumed to be equal. This is not strictly correct, since their induced E.M.F.'s are not equal, and the armature currents are not exactly the same. Let η be the efficiency of each machine, then the output of the motor is ηVI_M or $\eta V(I_D + I_E)$. But this is the power supplied to the dynamo, so that the output of the dynamo is $\eta \times \eta V(I_D + I_E)$. The dynamo output is also VI_D, therefore $\eta^2 V(I_D + I_E) = VI_D$, and the efficiency of either is $\eta = \sqrt{\frac{VI_D}{V(I_D + I_E)}} = \sqrt{\frac{I_D}{I_D + I_E}}$.

The current developed in the generator is not wasted, but helps to drive the motor, so that full-load tests on large machines may be carried out with a small expenditure of power.

It will be noticed that the motor armature is more heavily loaded than the generator armature, while its field is weaker, so that the efficiencies cannot be exactly equal, and the above formula gives the geometrical mean of the two efficiencies. In the case of large machines the difference in load will practically not affect the efficiency, and the results will be accurate. In small machines, where the efficiency is low, the difference in load may be so great that the assumption of equal efficiency is no longer valid, and the method breaks down.

When the Hopkinson test cannot be adopted, some other method of testing must be adopted, such as that due to Swinburne, which consists in measuring these stray losses and calculating the efficiency from them and the known output of the machine. The machine is run light as a motor, with a voltage applied to the brushes equal to the full-load induced E.M.F., and the excitation is then adjusted to give the normal speed.

The power input to the armature is measured, and the armature copper loss for the no-load current is subtracted from it. The remainder W_{io} gives the iron and frictional losses for this induced voltage and speed. The input to the shunt field gives the excitation loss in the shunt coils. The resistance of the armature and series field coils is carefully measured by a suitable method, and the loss in them calculated for the full-load current I_e. The total copper losses (calculated as explained before) are W_{cu}, and the iron and friction losses are W_{io} for the

full-load conditions, so that the efficiency $\eta = \frac{W_o}{W_o + W_{io} + W_{cu}}$, where W_o is the full-load output of the machine. The efficiency for other loads can be found in a similar manner. This method, however, is not quite accurate, since the iron losses on load are higher than those measured with the same excitation but with the machine running light, owing to the field distortion caused by armature reaction.

CHAPTER XXIV

DIRECT-CURRENT MOTORS

Introduction.—When a current is sent through a conductor placed at right angles to a magnetic field, the conductor is acted on by a force. This force tends to move it in a direction given by the Left-hand Rule, at right angles to both the field and the conductor. In fig. 338 is shown a conductor C, placed on the surface of an iron core A, which is free to rotate in the gap between the poles NS of an electromagnet. When a current flows through the conductor in a direction away from the observer, the force acting tends to move it upwards as indicated by the arrow, and so long as the wire is under the north pole the force will be in the same direction. When the wire is moved into a position under the south pole, however, the direction of the force acting will be reversed, if the direction of the current remains the same. In order that the pull on the conductor may always have the same direction, the current must reverse as a conductor passes from one pole to the next one. A commutator is therefore necessary to reverse the current. An arrangement of conductors mounted on an iron core, and provided with means by which a unidirectional current is converted into an alternating current, has

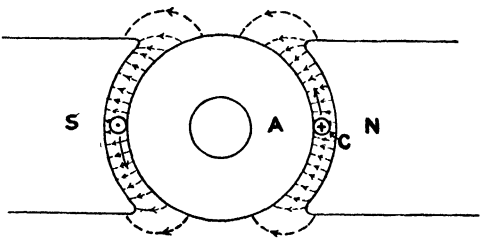


Fig. 338.—Simple Motor

been described before as the armature of a direct-current dynamo. Hence any dynamo will run as a motor if supplied with direct current.

The electrical and magnetic design of a motor is exactly the same as that of a dynamo, but the mechanical details are often quite different. A dynamo usually runs in a dry, clean situation, so that a very open design is an advantage. A motor, however, may be installed anywhere, and is usually placed where there is dust and dirt, so that the windings are covered over to prevent the entry of dust. To make the motor as self-contained as possible, the bearings are carried in castings bolted on to the yoke of the field magnets. These castings are also arranged to shield the windings and the commutator. Motors which run in damp situations, as, for example, the motors used for driving electric cars, are usually completely enclosed.

Direction of Rotation.—Since in a generator a torque must be applied to the armature to rotate it against the opposing torque due to the pull on the armature conductors when carrying a current, the direction of rotation of a motor must be the opposite of that of a generator with the same polarity of the field and direction of armature current. To reverse the direction of rotation either the armature current or the polarity of the field must be reversed. If both are reversed the direction of rotation will remain unaltered.

The rotation of a shunt dynamo run as a motor with the same polarity of its terminals will be unaltered, since the polarity of the field remains unchanged, but the direction of the armature current will be reversed. It now flows through the armature from the positive brush to the negative instead of from the negative brush to the positive, as in a shunt dynamo.

In a series dynamo run as a motor the rotation is reversed, since in this case both the field polarity and the armature current are reversed. Hence to make a series dynamo run as a motor with the same direction of rotation the connections of either the field coils or the armature must be interchanged.

Torque.—In a previous chapter it has been shown that the pull on a wire of length l, carrying a current of I amperes, and placed in a magnetic field of strength H is

$$f = \mathrm{H}l\frac{\mathrm{I}}{10} \text{ dynes.}$$

To find the torque exerted on the armature of a motor we can proceed as follows.

The pull exerted on one conductor is

$$f = B_g \times l \times \frac{I_c}{10} \text{ dynes},$$

where B_g = flux density in the air-gap, l = the active length of one conductor, and I_c = the current flowing in the conductor in amperes.

If the motor has p pairs of poles and Z conductors on the armature surface, the number of conductors lying under one pole is $\frac{Z}{2p} \times \beta$, where $\beta = \frac{\text{polar arc}}{\text{pole pitch}}$. Since there are $2p$ poles the total number of conductors lying in a magnetic field is therefore βZ.

The total pull exerted on the armature conductors is therefore

$$F = \beta Z \times B_g \times l \times \frac{I_c}{10} \text{ dynes}.$$

Now if the armature diameter is d, the polar arc is equal to $\frac{\pi d}{2p} \beta$. The total flux N issuing from one pole is the air-gap flux density × the polar arc × the axial length of the pole shoe, that is $N = B_g \times \frac{\pi d \beta}{2p} \times l$, since the active length of the conductor is equal to the length of the pole shoe.

The torque on the motor armature is

$$\tau = F \frac{d}{2} = \beta Z \frac{d}{2} \times B_g \times l \times \frac{I_c}{10} \text{ dyne-cm.},$$

substituting for $\beta d B_g l$, its value $\frac{N \times 2p}{\pi}$.

$$\text{The torque } \tau = 2p \times \frac{N}{2\pi} \times Z \times \frac{I_c}{10} \text{ dyne-cm.}$$

$$= 2p \times \frac{N}{2\pi} \times Z \times \frac{I_c}{9.81} \times 10^{-8} \text{ Kg.-m.}$$

Since 1 Kg.-m. = 9.81×10^7 dyne-cm.

If q = half the number of parallel paths in a winding $I_c = \frac{I_a}{2q}$, where I_a is the total current entering the armature, the final expression for the torque is

$$\tau = \frac{p}{q} \times \frac{N}{2\pi} \times Z \times \frac{I_a}{9.81} \times 10^{-8} \text{ Kg.-m.}$$

From the above equation it is evident the torque is proportional to the product of the flux per pole and the armature current, for a given winding.

The torque calculated in this manner is the total torque applied to the armature, and includes the torque necessary to overcome the iron and frictional losses of the motor armature. The torque, equivalent to these losses, must be deducted from the calculated value to find the useful torque available for overcoming the load resistance.

Speed.—Immediately the armature begins to rotate, the conductors on its surface cut the lines of force in the air-gap, and an E.M.F. is induced in them. By Lenz's Law this E.M.F. opposes the flow of the current in the armature, and is therefore known as the back E.M.F. The current I_a in the motor armature is produced by the resultant of the applied E.M.F. and the back E.M.F., so that

$$I_a = \frac{E_e - E_b}{R_a}.$$

Again, from the equation for the E.M.F. induced in a winding

$$E_b = \frac{p}{q} NZn \times 10^{-8} \text{ volts},$$

also

$$E_b = E_e - I_a R_a.$$

So that the speed of rotation $n = \dfrac{(E_e - I_a R_a) \times 10^8}{\frac{p}{q} NZ}$ r.p.s.

In a motor the $I_a R_a$ drop is small compared with the applied voltage E_e, so that $I_a R_a$ may be neglected. Then, if the field flux N is constant, the speed will be approximately proportional to the applied E.M.F. The speed can therefore be varied by varying the applied E.M.F. If the applied voltage E_e is maintained constant, while the flux per pole is varied, the speed will vary. From the equation for E_b it is evident that the product Nn must be constant if E_b, that is $E_e - I_a R_a$, is constant. The speed of the motor will therefore vary inversely as the flux per pole.

Output.—The output of a motor is the product of the torque on its armature and the angular velocity, so that the output is therefore

$$\text{Output} = \left(\frac{p}{q} \times \frac{N}{2\pi} \times Z \times \frac{I_a}{9.81}\right) \times (2\pi n) \times 10^{-8} \text{ Kg.-m. per second.}$$

$$= E_b I_a \text{ watts,}$$

$$\text{for } \frac{p}{q} \times NZn \times 10^{-8} = E_b \text{ and 1 watt} = 9.81 \text{ Kg.-m. per second.}$$

The useful output is, of course, less than this by the power wasted in the iron and friction losses.

The output equation can also be written in the form

$$\text{Output} = (E_e - I_a R_a) I_a = E_e I_a - I_a^2 R_a,$$

and it can be shown by differentiating this equation that the maximum output is obtained when $E_b = \frac{1}{2} E_e$. The efficiency is then only 50 per cent, and half the energy supplied is wasted in heating the armature conductors. When a motor is giving its maximum output, however, the current flowing in the armature will be so great that the armature will probably be burnt out.

Armature Reaction.—Since in a motor the armature current flows in the reverse direction to the current in a dynamo for the same direction of rotation, the cross flux produced by the armature ampere-turns will also be reversed. The main flux is distorted in a direction opposite to that of rotation, as shown in fig. 339, with the result that the flux density in the air-gap under the trailing tips is decreased, while that under the leading tips is increased. The true neutral position is therefore moved backwards from the geometrical neutral bb_1 through an angle β to the position nn_1, and the brushes must be moved backwards through an angle α greater than β to position BB_1, if perfect commutation is to be obtained. Since the armature current is reversed from that of a dynamo, the

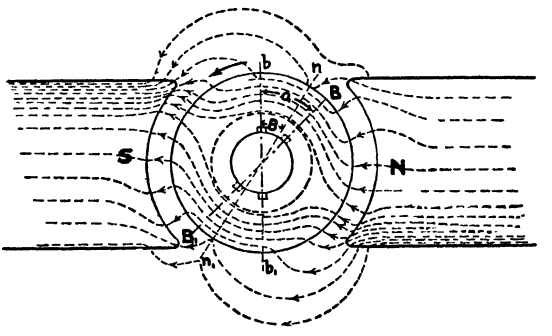


Fig. 339.—Armature Reaction in a Motor

ampere-turns set up by the conductors in the arc subtended by 2α are demagnetizing, and can be calculated in the same manner as those of a dynamo. Motors which have to run in either direction have their brushes fixed in the geometrical neutral position, and commutation occurs when the coil is cutting lines of force from the next pole. Commutation takes place therefore under unfavourable conditions, since the E.M.F. induced in the coil by this flux tends to maintain the current flowing in the coil. The field magnets and teeth should be nearly saturated, so as to reduce the distortion on load as much as possible, and reduce the flux cutting the coil during commutation.

In motors with a fixed direction of rotation the commutation can be improved by moving the brushes backward. This brings the coil undergoing short-circuit into the fringe of flux from the receding pole. The E.M.F. induced in the coil by this flux tends to reverse the current, since the E.M.F. induced in a motor armature always opposes the flow of the current. If commutating poles are used, their polarity must be the same as that of the pole preceding them in the direction of rotation.

Shunt Motor.—In a shunt motor the field is connected as a shunt to the armature, and both are supplied from constant-potential mains. Neglecting the heating of the field coils, and the effect of armature reaction, the flux through the armature will be constant, because the field coils have a constant exciting current flowing through them. The torque, which is proportional to the product of flux and armature current, will be proportional, therefore, to the armature current only. This relation is shown in fig. 340, where OA is the line giving the relation between torque and current. By setting off OB to represent the armature current required to overcome the friction and iron losses on no load, and then drawing BC parallel to OA, the line BC shows the relation between the armature current and the useful torque available for overcoming a given load.

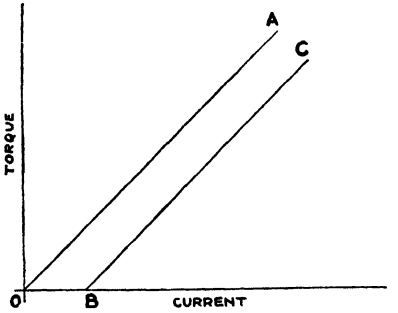


Fig. 340.—Shunt-motor Torque Curves

Again, the speed is given by the equation

$$n = \frac{E_e - I_a R_a}{\frac{p}{q} \times N \times Z} \times 10^8 \text{ r.p.s.}$$

The only variable in this equation is I_a, the armature current, so that the speed will drop slightly as the armature current I_a increases (fig. 341). As $I_a R_a$ at full-load current is usually not greater than 4 per cent of E_e, the speed variation from no load to full load is also not greater than 4 per cent. For most purposes this variation in speed is negligible, so that a shunt motor can be regarded as a constant-speed motor. The effect of armature reaction is to weaken the main field, so that if the brushes are displaced backwards the speed variation will be less than that caused by the armature resistance, for both N and $(E_e - I_a R_a)$ will decrease with load. The speed can be maintained exactly constant by giving the brushes sufficient backward lead.

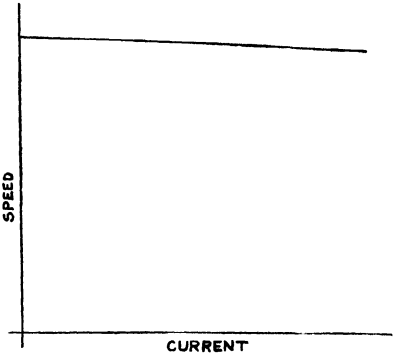


Fig. 341.—Shunt-motor Speed Curves

When a motor is started after a period of rest the field coils will be cold, so that the current through them will be greater than the normal value attained when the motor has been running for some time. The speed will rise as the coils warm up, since the flux decreases.

Most stationary motors, such as those used for driving shafting in factories, machine tools, &c., are shunt wound.

Starting of Shunt Motors.—When the armature is at rest there is no back E.M.F., and the only resistance in circuit is that of the armature, so that if the armature is switched directly on to the main supply the current flowing through the armature would be excessive, and there would be a risk of burning out the armature. Consider the case of a 5 horse-power 250-volt motor which has a full-load current of approximately 18 amperes and an armature resistance of 0.4 ohm. If the full P.D. of 250 volts is applied to the armature, the current flowing will

be $= \frac{250}{0.4} = 625$ amperes. In order to reduce the current taken at starting, a resistance is inserted in series with the armature, the field circuit being excited by the full voltage, and as the motor speed increases the resistance is cut out gradually, and is finally short-circuited. The value of this resistance is such that the current through the armature is about one and one-half times the normal current when the motor is starting against full-load torque.

In the previous example the full-load current is 18 amperes, so that a resistance of $\frac{250}{18 \times 1.5} = 9.25$ ohms will be required.

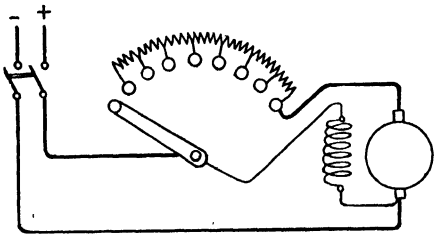


Fig. 342.—Starting Resistance of a Shunt Motor

The simplest method of arranging this starting resistance is that shown in fig. 342. The disadvantage of this simple arrangement is that it is possible to open the armature circuit and leave the field coils connected to the supply mains. When the double-pole switch is opened the highly inductive field-coil circuit is broken, which causes a great rise in the voltage across the field. This rise in voltage may be sufficient to pierce the insulation of the field coils and so damage them.

To prevent this the connections are arranged as shown in fig. 343. The negative brush on the armature and the negative terminal of the field coils are connected to the negative main through a fuse and one pole of the double-pole switch T. The positive main is connected through the other pole of the switch T and a fuse to the terminal L of the starter, while the positive brush and positive field terminal are connected to the terminals A and F respectively. The starter consists of a switch arm BD, which can be moved through an angle about the point B, so that the end D moves over and makes contact with the studs on the slate base of the starter. The starting resistance R is divided into sections, a connection being taken from the end of each section to a stud, as shown in the figure. When the double-pole switch is closed and the starting switch BD moved, so as to make contact with stud 1, the field coils are connected

directly across the mains, while the armature is connected through the whole of the starting resistance. In this manner the field coils are fully excited, while the armature current is reduced to an amount that gives the necessary torque to start the motor. As the speed increases, the switch arm BD is moved to the right over studs 1 to 10, reducing the resistance in series with the armature until it reaches the last stud, as shown by the dotted lines, when the starting resistance is short-circuited and the motor armature is connected directly across the mains.

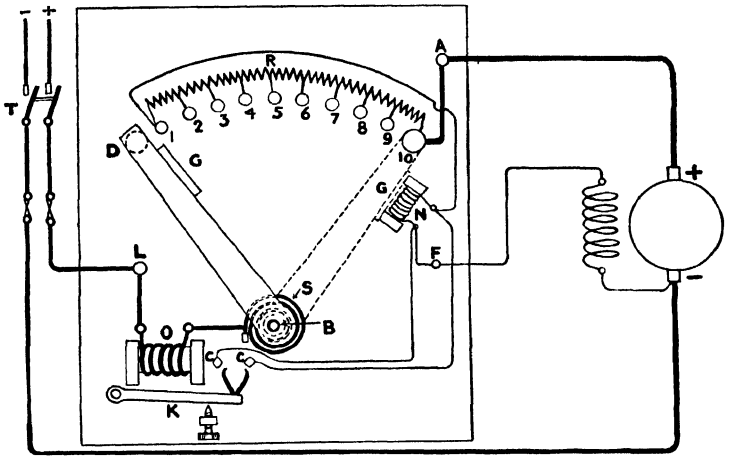


Fig. 343.—Shunt-motor Starter

When the supply to the motor is interrupted the motor will stop, and if the switch is left in the short-circuited position a large current will flow through the armature and damage it when the supply is restored. A device called "a no-voltage release" is therefore fitted, which automatically returns the switch to the off position when the supply or the field circuit is interrupted. This release consists of an electromagnet N, whose exciting coil is connected in series with the field coils. When the starting resistance is short-circuited, this electromagnet attracts an armature G loosely fitted to the switch arm, and holds it against the recoil of a spring S, which tends to move the switch to the off position. If the supply is interrupted, or the motor field circuit broken, the electromagnet is de-energized and the starting switch is pulled back into the off position by the spring.

Again, the motor may be overloaded, with the result that a large current flows through the armature, which may overheat and damage it. To prevent this overloading, a device, known as the "overload" release, is often fitted. This release consists of an electromagnet O, whose exciting coil carries the main current to the motor. When this current exceeds a certain maximum value the armature K is attracted, and the copper fork on it short-circuits the two contacts CC, each of which is connected to one end of the no-voltage coil. The no-voltage electromagnet N is thus de-energized, the switch arm returns to the "off" position, and the motor is stopped. The current at which the overload release acts is adjusted by altering the distance of the armature K from the electromagnet O.

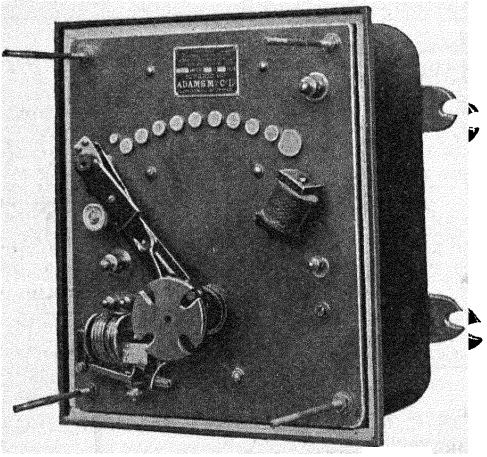


Fig. 344.—Simple Wire-resistance Starter, with Overload and No-volt Release; Iron Cover, with operating handle removed

It is evident from the diagram that the shunt circuit is always connected to the armature circuit, whether the motor is at rest or running. If the double-pole switch is opened when the motor is running, the field coils are left connected across the armature, and the magnetic flux through the field begins to disappear; but, since the armature is still running, it maintains the excitation, so that the flux dies away gradually, and no rise in the P.D. across the field coils takes place. Since the field is fully excited when the starting switch is on the first contact, the motor gives the greatest torque per ampere of the armature current when starting.

Fig. 344 shows the appearance of a starter similar to the one just described. The starting switch and the safety devices are mounted on a slate base. The studs are connected at the back of the base to the starting resistance, which is contained in a cast-iron box. The starter has an enclosing iron cover, which

has been removed in the view shown. The switch arm is moved from the front of the case by a handle which engages with the coupling disk, which may be seen attached to the spindle.

Speed Control of Shunt Motors.—The speed of a shunt motor can be varied by altering either the flux or the E.M.F. applied to the brushes. It has already been shown that the speed varies inversely as the flux, so that by decreasing the flux the speed may be increased. The motor is designed to run at the lowest speed required, with the normal applied voltage and full field current. To obtain the higher speeds a resistance is inserted in series with the field, which decreases the field current, and as a result the flux. Fig. 345 shows the relation between the speed and field current.

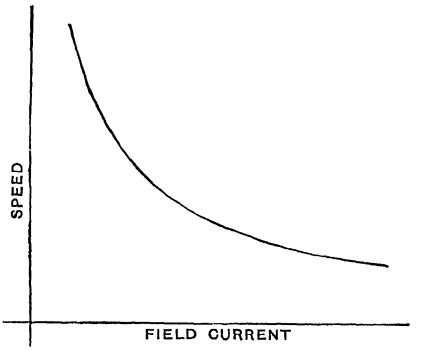


Fig. 345.—Speed Control by Field Variation

When a motor is running with a weaker field than normal, good commutation is difficult to obtain, as a result both of the increased speed and of the greater distortion caused by the armature reaction. For this reason the speed variation obtainable in machines of ordinary design is about 50 per cent. If, however, special devices for obtaining good commutation are used, a speed variation of from normal to six times normal is quite practicable.

The E.M.F. applied to the brushes can be varied (*a*) by inserting a resistance in series with the armature, (*b*) by using a multiple voltage supply, (*c*) by using a special motor generator.

(*a*) In this method of speed control a resistance, connected in series with the armature, is used to reduce the voltage of the supply to the value corresponding to the required speed. The method is uneconomical, since if, for example, half normal speed is required, the voltage applied to the brushes is approximately half the normal value, and hence the drop of volts in the resistance is half the normal value. The power wasted in the resistance is therefore half the power taken from the mains, and

the efficiency of the system is less than 50 per cent. Another objection is that the speed varies with every change in the resisting or load torque. If the torque increases, the armature current and the drop in the series resistance also increase, so that the voltage applied to the brushes decreases and the speed falls. This effect is shown in fig. 346, where the torque speed curves for a motor with different external resistances are given. The speed rises to the normal value as the load and current are diminished.

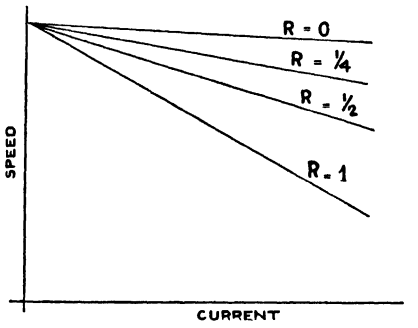


Fig. 346.—Speed Control by Resistance

(*b*) In the multiple-voltage method the field is excited from constant-potential mains, while the voltage applied to the armature is varied by connecting it to mains supplied at different voltages. For example, suppose that the supply is a four-wire one, with voltages between wires, as shown in fig. 347. The field is permanently connected across the 500-volt mains AD. The lowest speed is obtained when the armature is connected to the mains AB when the applied voltage is 120 volts. The next speed is obtained by connecting the armature to the mains BC when the applied voltage is 160 volts, giving a speed approximately 30 per cent greater. By changing the connections in this manner a speed range of approximately 1 to 4 times the lowest speed, in increments of about 30 per cent, can be obtained. To effect these changes a controller is used, which changes over the connections from main to main, and at the same time inserts a resistance in series with the armature, which is afterwards cut out in steps when the motor has risen to the speed corresponding to the new value of the applied voltage.

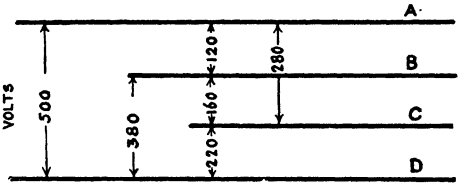


Fig. 347.—Multiple-voltage System of Speed Control

As the difference in speed between the steps is only 30 per cent, intermediate speeds can be obtained by weakening the

field. In this manner, by a combination of field weakening and varying armature-applied E.M.F.'s, an unlimited number of speeds can be obtained between the lowest and highest speeds. Since the E.M.F. applied to the brushes remains constant for all loads, the motor speed also remains approximately constant for all loads on the motor.

The motors used in this system of speed control are ordinary standard motors, and therefore cheaper than the special types used when the speed control is obtained by field weakening, but, on the other hand, the supply for this system is a 4-wire one with complicated arrangements for maintaining constant voltage

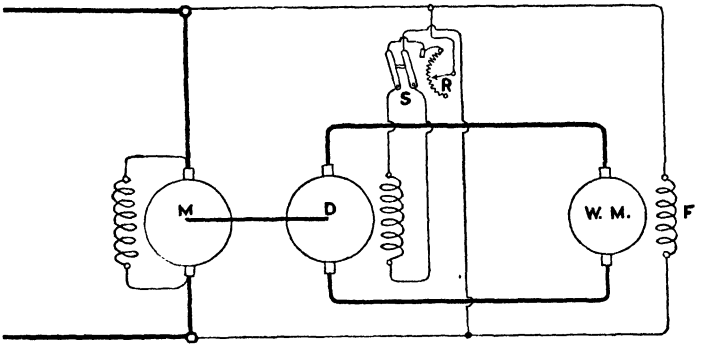


Fig. 348.—Ward Leonard System

between the various wires, while when field weakening is used only a 2-wire supply and a much simpler generating plant is required, so that unless a large number of variable-speed motors are required, the system of speed control by field weakening is usually the cheapest.

(*c*) In the motor-generator or Ward Leonard system the motor is separately excited, and its armature is supplied from the generator of a motor-generator set. In fig. 348 WM is the main working motor, whose field is separately excited from the supply mains. The armature of the working motor is connected directly to the armature of the generator D of the motor-generator set. The field of the generator is separately excited from the supply mains through a variable resistance R and switch S, which serves to reverse the generator field. The motor M of the motor generator is an ordinary constant-speed shunt motor.

When the working motor is at rest the generator field

circuit is open, and no E.M.F. is induced in the generator armature circuit. To start the working motor the generator field is excited till the current required to give the starting torque flows in the motor armature and the motor starts. As it speeds up, the generator field is increased till the proper motor speed is obtained. To stop the motor the generator field is decreased, thus decreasing the generator-induced E.M.F. below the motor-back E.M.F. The motor now supplies power back to the supply through the motor generator, and is therefore braked electrically. The motor stops when the generator field circuit is broken. Reversal of the rotation of the working motor is effected by the reversing switch S in the generator field, which reverses the generator flux and induced E.M.F.

While this system of speed control is very flexible, it is also very costly. The whole power supplied to the working motor passes through the motor-generator set, so that three machines are required, each of capacity equal to or greater than that needed for the actual work. The motor generator, however, can run at a high speed so that its cost can be made less than that of the main motor. This system is advantageous when the working motor is started and stopped frequently, since at starting there is no loss in starting resistances, and where power is expensive the decreased cost of running may more than cover the increased initial cost of the equipment. It is mainly used for driving winding engines for colliery work and reversing rolling mills in steel works, where exact control and a wide speed range are required.

When a heavy flywheel is fitted to the motor-generator set, and the driving motor is arranged to drop in speed when the load increases, this system is that usually known as the "Ilgner" system. If the load variations are rapid, the flywheel supplies energy when the load is great, and stores it when the load is small, and therefore tends to make the driving motor take a constant input from the mains.

Series Motor.—In a series motor the field and armature windings are in series with each other, so that the field excitation and flux from the poles depend on the armature current.

It has already been shown that the torque of any motor varies with the product of the armature current and field flux. Since at low flux densities in the magnetic circuit the flux is

proportional to the exciting current, the torque will be proportional to the current squared, but as the iron approaches saturation the torque becomes more nearly proportional to the first power of the current. Fig. 349 shows the relation between current and torque. As both flux and armature current are independent of the applied voltage, the torque of a series motor is independent of the applied voltage. On the other hand, in a shunt motor the torque depends on the applied voltage, since the flux depends on the field excitation, which varies with the applied voltage. For traction work, where there is often a large drop in voltage in the overhead feeders both when starting and running, a series motor will give a larger torque for the same armature current than a shunt motor will, although they may both give the same torque when supplied at the normal voltage. This is often a great advantage.

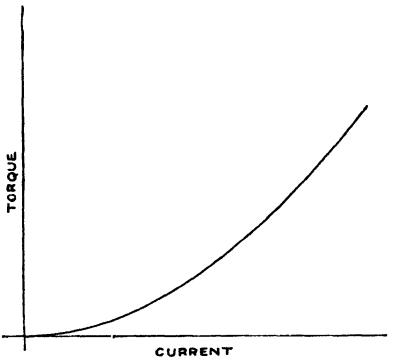


Fig. 349.—Series-motor Torque Curve

The speed of a motor varies inversely as the flux, so that in a series motor the speed will fall as the armature current increases, the exact amount depending on the saturation of the magnetic circuit. The general formula for the speed of a motor is

$$n = \frac{(E_e - I_a R_a) \times 10^8}{\frac{p}{q} NZ} \text{ r.p.s.}$$

The flux N is practically the only variable, for $I_a R_a$ is small compared with E_e, and the flux depends on the field current.

At low densities the flux increases almost directly with the current, so that the speed-current curve will be a hyperbola, since speed × current = speed × flux = constant. As the iron parts become saturated the flux does not increase so rapidly with the current, and the speed therefore does not fall so quickly, with the result that the speed-current curve becomes flatter for large currents. The speed tends to a constant value when magnetic circuit is saturated, which usually occurs for a current at or about the full-load value. The curve is shown in fig. 350.

As the load diminishes, the speed increases rapidly. At no load the torque is small since the no-load losses are small, and the current necessary to give this torque will also be small. The flux will therefore be small, so that the speed to give a back E.M.F. approximately equal to E_e applied will therefore be high. This tendency to very high speeds when the load is reduced is a bad feature of the series motor. The great centrifugal force acting on the armature conductors and the commutator segments at high speed may damage the armature. Series motors are therefore only used where there is a sufficient load due to friction of gearing, &c., to keep the speed down to a safe value, or where the load cannot be removed, as in traction motors. Series motors, when used for driving pumps or cranes, must always be geared or direct-coupled to prevent a breakage of the connection to the load which might occur if a belt was used.

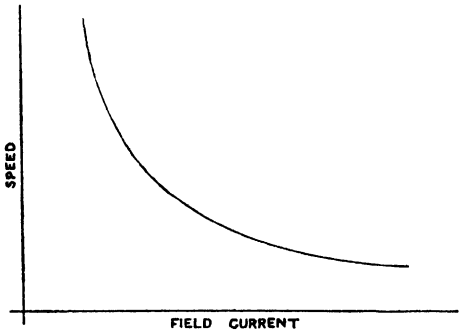


Fig. 350.—Series Motor Speed Curve

From a comparison of the torque-current and speed-current curves it is evident that the maximum torque occurs at the lowest speed. A series motor therefore causes a smaller maximum power demand on the supply mains than does a shunt motor for the same overload in torque. The shunt motor runs at an approximately constant speed for all loads, while a series motor runs slower for a heavier load, and thus takes a smaller power input for an overload.

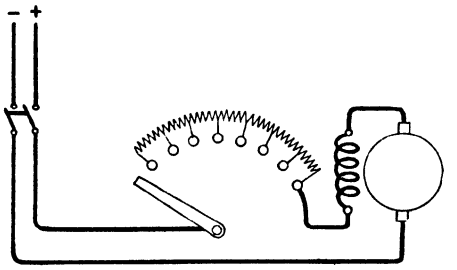


Fig. 351.—Simple Starter for Series Motor

Starting of Series Motors.—The connections for starting a series motor are shown in fig. 351. As the field is in series with the armature the starter can be a simple resistance con-

nected in series with both. The starter can be arranged in a similar manner to that of the shunt motor (fig. 343) with the following alterations: The connection from the first step of the starter to the field terminal F is omitted, and the no-voltage coil is wound with a few turns in series with the main circuit. The number of turns in the coil is so arranged that so long as the current is greater than the value corresponding to the maximum safe speed the starting switch is held in the "on" position against the pull of the spring. If, however, the current is reduced below this value the pull of the spring overcomes the pull of the electromagnet, and the starting switch returns to the "off" position.

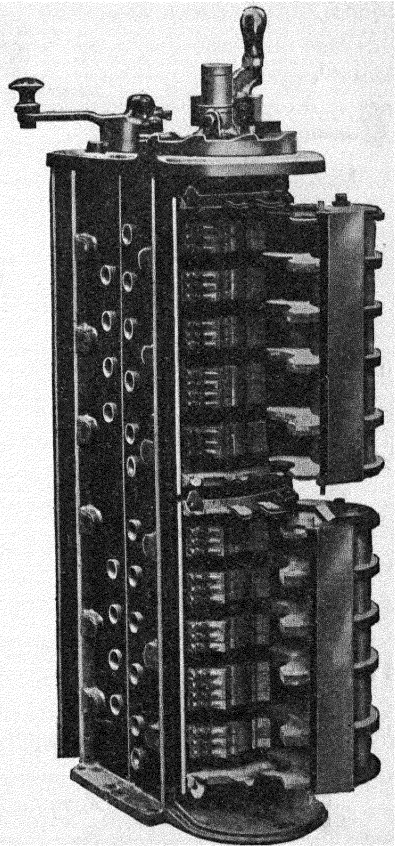


Fig. 352.—Crane Controller, showing Metallic Shield Blow-out Coils opened to give access to Main contacts

Since the field is in series with the armatures, opening the main circuit causes arcing, due to the self-induction of the field, so that where the motor is to be frequently started and stopped, heavy and substantial contacts must be used. For large motors a controller type of starter is used.

Fig. 352 shows a crane controller of the drum type with reversing and notching-up barrels separate, there being an operating handle for each. The drum is insulated from the main spindle, and the insulated portion carries copper segments, which turn under fixed contacts and serve to bridge them, thereby completing certain portions of the circuit controlled. The fixed contacts have a special hinged finger, controlled by a setscrew against the tension of a spring. This design is found necessary to deal with the heavy currents controlled.

A powerful magnetic blow-out is provided for each main contact, as the circuit is broken at certain steps.

The magnetic field is created by a solenoid, in series with the circuit controlled, and the final break of the circuit is placed within the lines cut by this field. The arc, forming as it does one pole of the circuit, is repelled from this field, and is thereby forced away from the two contacts which it would otherwise leap horizontally across. The rupture of the circuit is very sharp with these blow-outs, the magnet taking immediate effect on the arc, which breaks with a spiteful crack, or a noise resembling a bellow, depending on the inductance of the circuit broken.

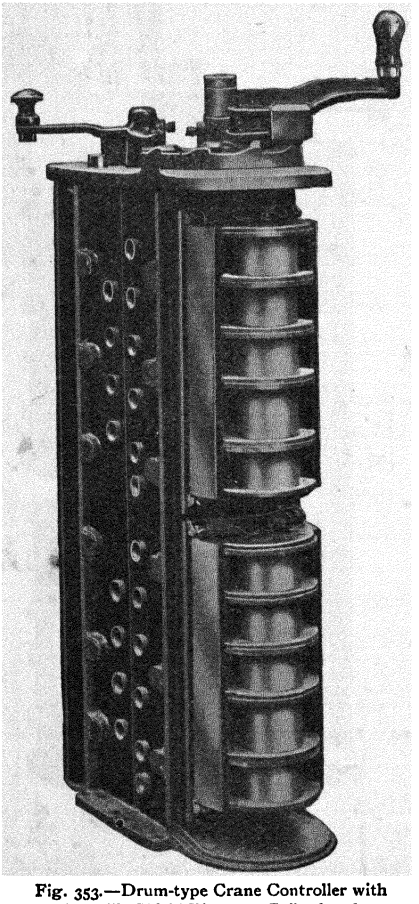


Fig. 353.—Drum-type Crane Controller with Metallic Shield Blow-out Coils closed

Fig. 353 shows the blow-out coils closed and the catches holding them in position.

The speed of series motors is usually controlled by means of a resistance in the armature circuit. The method is practically the same as for shunt motors, except that only the armature is affected in the latter. The same objections apply, however, in both cases. There are great changes in speed for small changes in torque and low efficiency.

Where two motors are to be controlled simultaneously a series-parallel control can be used. At the start the two motors are connected in series with each other and with a starting resistance, which is gradually reduced till each motor receives one-half of the line voltage. The motors are then connected in parallel with each other and in series with a resistance, which is again cut out in steps till

each motor receives full voltage. This method gives two efficient speeds with no external resistance in circuit.

Since the voltage applied to the motor when it is operated in series is half the voltage applied in parallel operation the respective speeds are approximately in the ratio of one to two.

As the torque of a series motor increases more rapidly than the armature current, it will have for the same starting current a greater starting torque than a shunt motor. For this reason it is suitable for traction and crane work where the starting torque is usually greater than the running torque.

Compound Motors.—There are two types of compound motors. In the one the series ampere-turns assist the shunt ampere-turns, and the flux therefore increases with load; and in the other the series ampere-turns oppose the shunt ampere-turns, so that the flux decreases with an increase in load. The first type is known as the cumulative-compound, or often as the compound motor, while the second type is known as the differential-compound. In the cumulative-compound motor the speed will decrease with increased load, since the field flux increases with load. The extent of this decrease depends on the ratio between series and shunt ampere-turns at full load. If, for example, the shunt ampere-turns at full load are twice the series ampere-turns, then, neglecting the effect of magnetic saturation, the field flux at no load is two-thirds of the flux at full load, so that the speed of the motor at full load will be two-thirds of the speed at no load. If the magnetic circuit is saturated, the speed variation will be smaller between no load and full load. Fig. 354 shows the effect of different ratios of shunt to series ampere-turns. Curve A is the speed-current curve for a motor in which the shunt ampere-turns are approximately three times the series ampere-turns at full load, while curve B, on the other hand, is the speed-current

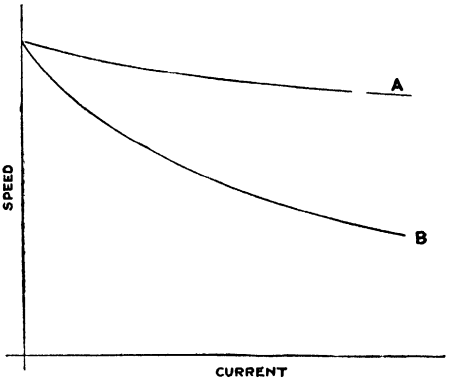


Fig. 354.—Compound Motor Speed Curves

curve for the same motor, but with the series ampere-turns three times the shunt ampere-turns.

The torque of a compound motor will increase more rapidly than the armature current, due to the fact that the flux increases with an increase in the armature current due to the effect of the ampere-turns in the series coil. Hence the compound motor combines the characteristics of the shunt and series types, having a speed not extremely variable under load changes but developing a large starting torque.

The stronger the shunt-field flux (or no-load flux) the more nearly will the speed-current and torque-current characteristics resemble those of a shunt motor; the weaker the shunt field the more closely will the motor resemble a series motor.

A compound motor is started in exactly the same manner as a shunt motor, and the same type of starter is employed. The speed control employed to vary the speed of a compound motor may be any of the methods described in connection with shunt motors.

Compound motors with characteristics resembling those of a shunt motor are used for driving machinery where the motor may be required to start under a heavy load, but must maintain an approximately constant speed when running even if the load is completely removed. If the motor has to deal with intermittent and severe overloads, a flywheel can be fitted which tends to equalize the demand on the supply mains.

A heavily compounded motor is often used in preference to the series motor in cases where a variable load is to be dealt with, for the shunt field limits the maximum speed obtainable, even if the load is entirely removed.

In a differential compound motor the series coil opposes the shunt coil, and thus weakens the field as the load increases. The object is to compensate for the fall in speed due to the I_aR_a drop, and thus maintain the speed constant at all loads. These motors are satisfactory so long as they are not overloaded, but if an overload occurs, the field flux is so much reduced that the torque is not sufficient to maintain the rotation, and the motor stops. Another objection is the difficulty of obtaining a sufficient starting torque. The series field has a lower inductance than the shunt field, and the current in the series field and the armature will, therefore, rise to the normal value before

the current in the shunt coil reaches its steady value, so that if the motor is started quickly the series field will overcome the shunt field, and the motor will rotate in the opposite direction. The series winding must, therefore, be short-circuited during the starting of the motor. Since a constant speed at all loads can be obtained by moving the brushes of a shunt motor backwards, the differential compound motor is seldom used.

ALTERNATING CURRENTS

CHAPTER XXV

ELEMENTARY THEORY OF ALTERNATING CURRENTS

Introductory.—An alternating current is one which, instead of flowing continuously in one direction round a circuit, periodically reverses its direction.

The time which elapses between successive reachings of zero value in the *same* direction is defined to be the *period* of the

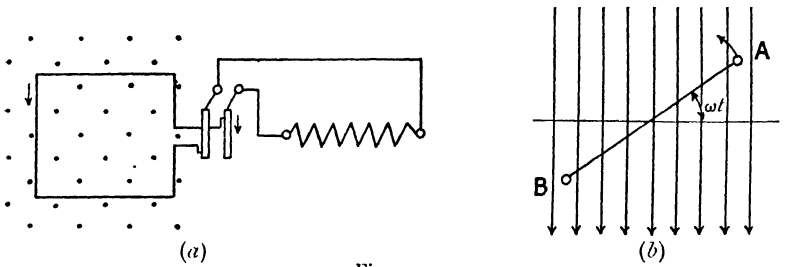


Fig. 355

alternating current, and the current is said to have completed one *cycle* during this time. The number of complete cycles performed per second is called the *frequency*. These statements apply equally well to alternating electromotive forces, potential differences, and magnetic fluxes.

Production of an Alternating E.M.F.—A simple alternating E.M.F may be produced by rotating a loop of wire with a uniform angular velocity in a uniform magnetic field, as shown in fig. 355 *a*. Let the loop be rotated in a counter-clockwise direction (fig. 355 *b*), and let time be measured from an instant when the loop is horizontal with the conductor A to the right.

Let A = area enclosed by the loop.
H = strength of the magnetic field.
ϕ = flux embraced by the loop at any time t.
$\Phi_{max.}$ = maximum flux embraced by the loop.
e = generated E.M.F. in the loop at instant t.
ω = angular velocity of the loop.

At any instant t the angle through which the loop has rotated will be ωt, and therefore

$$\phi = HA \cos \omega t.$$

Since the E.M.F. generated in the loop is at any instant

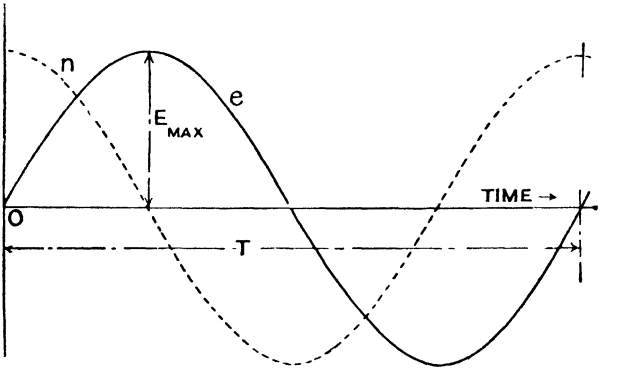


Fig. 356

equal to the time rate of change of the magnetic flux linked with the loop, and is in a direction tending to oppose the change,

$$e = -\frac{d}{dt}(HA \cos \omega t)$$
$$= HA\omega \sin \omega t.$$
But $HA = \Phi_{max.}$,
$$\therefore e = \Phi_{max.}\, \omega \sin \omega t, \text{ in C.G.S. units.}$$

This equation is represented graphically in fig. 356, where $E_{max.}$ = max. value of e, T = period in seconds, and n stands for $\omega\phi$.

Since, during the time that the loop rotates through an angle of 2π radians, the E.M.F. goes through one complete cycle, the period of the E.M.F. is equal to $\frac{2\pi}{\omega}$. Or, if f = frequency,

$$f = \frac{\omega}{2\pi}.$$

The E.M.F. equation may now be re-written

$$e = 2\pi f \Phi_{max.} \sin \omega t, \text{ in C.G.S. units.}$$

It is immaterial how the variation of the interlinkages of the loop with the magnetic flux is brought about, and the equation still holds if a magnetic flux varying according to the equation $\phi = \Phi_{max.} \cos \omega t$ is interlinked with a stationary loop.

The latter corresponds to the generation of an E.M.F. in a static transformer, and the former to the generation of an E.M.F. in a rotational machine.

In either case, if the loop consists of T turns so placed that

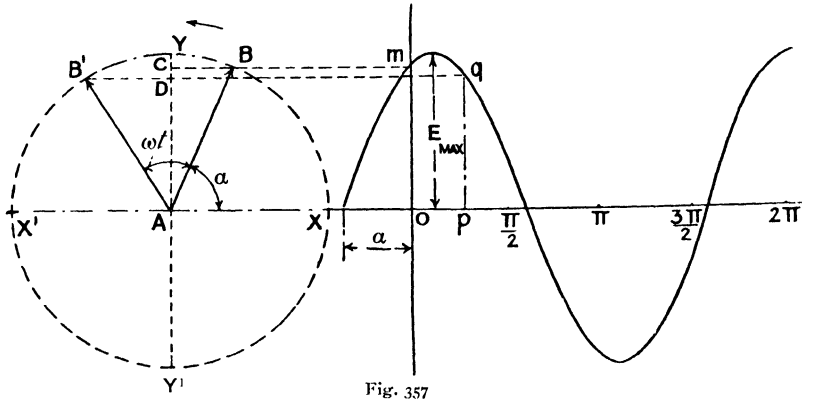


Fig. 357

the magnetic flux interlinked is the same for every turn at any instant, the equation becomes

$$e = 2\pi f T \Phi_{max.} \sin \omega t, \text{ in C.G.S. units.}$$

Vector Representation.—An alternating E.M.F., current, or magnetic flux, when of simple sine form, may very conveniently be represented by a vector, whose length represents to some scale the maximum value of the function, and which rotates with a constant angular velocity.

The direction of rotation is immaterial, but for the sake of uniformity it has been agreed that the direction of rotation shall invariably be taken as counter-clockwise. Consider, for example, an E.M.F. varying according to the equation

$$e = E_{max.} \sin (\omega t + \alpha).$$

Let AB in fig. 357 represent the maximum value $E_{max.}$ of this

E.M.F., and let AB rotate about the centre A with a constant angular velocity ω in a counter-clockwise direction. At any instant the projection of AB on the vertical axis Y'Y represents the value of the E.M.F. e at that instant, to the same scale as AB represents $E_{max.}$ At the instant t = zero, AC is the projection on the vertical axis,

$$\text{and } Om = AC = AB \sin \alpha = E_{max.} \sin \alpha.$$

At a time t seconds later, let the vector AB occupy the position AB'. AD is now the projection on the vertical axis,

$$\text{and } pq = AD = AB \sin (\omega t + \alpha) = E_{max.} \sin (\omega t + \alpha).$$

The ordinates of the curve drawn through such points as m, q, &c., will thus represent graphically the equation

$$e = E_{max.} \sin (\omega t + \alpha),$$

$$\text{or } e = E_{max.} \sin (2\pi f t + \alpha).$$

The value of e is zero when ωt is equal to $-\alpha$, $(\pi - \alpha)$, $(2\pi - \alpha)$, &c. The curve of e is, in effect, a sine curve displaced to the left along the axis of abscissæ by an amount α from zero. This angle α is known as the phase angle. The value of a vector diagram becomes apparent when a circuit having several E.M.F.'s or currents is considered. For example, let two E.M.F.'s, of which $E_{1max.}$ and $E_{2max.}$ are the maximum values, exist in a circuit, and let their instantaneous values be given by the equations

$$e_1 = E_{1max.} \sin \omega t \quad \text{.....(a)}$$

$$e_2 = E_{2max.} \sin \left(\omega t + \frac{\pi}{2}\right) \quad \text{.....(b)}$$

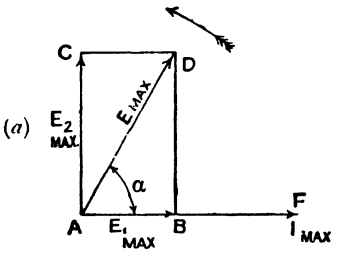
Let also the instantaneous value of i, the current flowing in the circuit, be given by

$$i = I_{max.} \sin \omega t \quad \text{.....(c)}$$

The magnitude of the maximum value of the resultant E.M.F. in the circuit, and the angle of phase difference between the current and the resultant E.M.F. can be readily found by means of the vector diagram. In fig. 358, let the vector AB represent $E_{1max.}$ From equation (b) it will be seen that $E_{2max.}$ is $\frac{\pi}{2}$ ahead of $E_{1max.}$ Therefore AC drawn at right angles to AB

and of length representing the magnitude of $E_{2max.}$ will represent $E_{2max.}$ in magnitude and phase.

$E_{max.}$, the maximum value of the resultant E.M.F., will be represented by the vector AD obtained by completing the parallelogram ABDC, for the projection of AD on the vertical axis is at every instant equal to the sum of the projections of AB and AC; i.e. $e = e_1 + e_2$, where e is the instantaneous value of the resultant E.M.F.



The vector AF of length representing the magnitude of $I_{max.}$ and drawn coincident with AB [see equations (c) and (a)], will represent $I_{max.}$; FAD = α is the angle of phase difference

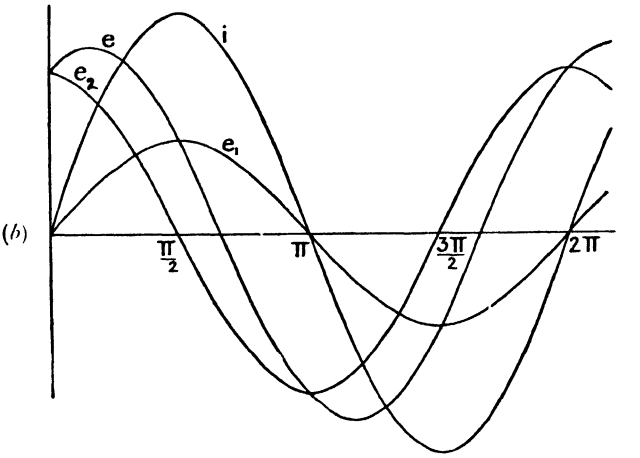


Fig. 358

between the current and the resultant E.M.F. The sine curves illustrated in fig. 358 *a* are plotted from the rotating vectors in fig. 358 *b*, and show the instantaneous values of the various E.M.F.'s and the current.

The vector diagram may be very rapidly drawn, and gives simply and effectively the information required.

The vector diagram is a very valuable adjunct where an analytical method is used in solving an alternating-current problem. In this case there is no need to draw the vector

diagram to scale. All that is required is a rough sketch which will indicate the general phase relationships of quantities involved in the problem. This sketch then serves as a guide in making the calculation.

Unless the vector diagram is to be used for projecting the actual instantaneous values, there is no need to let the vectors represent maximum values of the quantities. In practice, vector diagrams are almost invariably drawn to represent effective values.

It should be especially noted that *effective* values are *vector* quantities, and must be added vectorially. It is only when

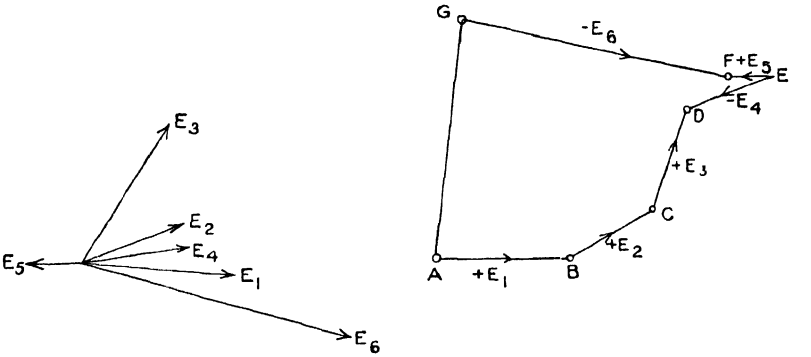


Fig. 359

instantaneous values are being used that *algebraic* addition may be employed.

Vector Equations and Vector Addition.—In many alternating-current problems it is convenient to write down a set of vector equations for the various quantities involved. In the following, vector quantities will be indicated by a dot placed below the symbol. Thus the vector equation for the E.M.F.'s in a given case may be

$$\dot{E}_1 + \dot{E}_2 + \dot{E}_3 - \dot{E}_4 + \dot{E}_5 - \dot{E}_6 = \dot{E}_R$$

Now draw these vectors at their correct phase angles, as shown to the left in fig. 359. Starting from any convenient point A, draw the vector polygon ABCDEFG. Indicate by an arrow-head the direction of each vector, noting carefully that where the sign of the vector in the vector equation is positive, the direction of the arrow-head on that particular vector must be in the direction of progress round the polygon from A to G,

whereas when the sign is negative the direction of the arrow-head must be opposite to the direction of progress round the polygon. The resultant vector, representing E_R is obtained by joining the *starting point* A to the *finishing point* G. By attending strictly to this procedure, the resultant of any system of vectors, however complicated, may be obtained quickly and with certainty. The procedure may be summarized as follows: (*a*) Draw each vector in its correct phase position; (*b*) where a *negative* sign occurs draw the vector in such a position that, in passing round the polygon from start to finish, the progress is

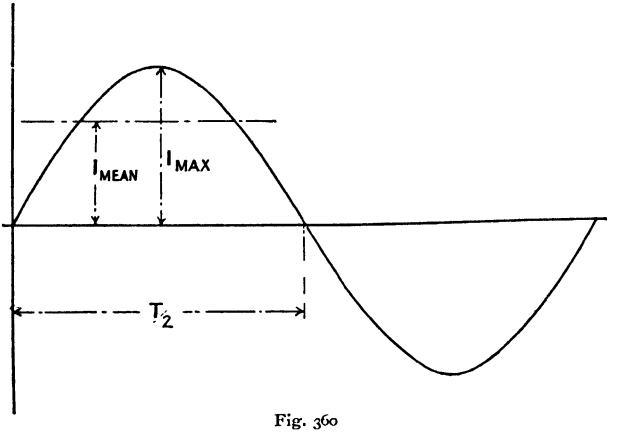


Fig. 360

against the direction of the arrow-head in this particular vector; (*c*) the resultant vector is represented in magnitude by the length of the straight line joining the *starting point* to the *finishing point*, and the positive direction of the resultant is from the *starting point* to the *finishing point.*

Mean and Effective Values.—The expression mean value as applied to an alternating current denotes only that value which is obtained if the mean of the instantaneous values of the current be taken over one half-period between successive zero values of the current; i.e. it is the greatest mean value obtainable over any half-period.

Thus let an alternating current as shown in fig. 360 be represented by the equation $i = I_{max.} \sin \omega t$.

Then the mean value is

$$I_{mean} = \int_0^{\frac{T}{2}} I_{max.} \sin \omega t dt \div \frac{T}{2},$$

where T is the period in seconds,

$$= \frac{2}{T} I_{max.} \frac{1}{\omega} \left[- \cos \omega t \right]_0^{\frac{T}{2}}.$$

Or, since $\omega = \frac{2\pi}{T}$,

$$I_{mean} = \frac{2}{T} I_{max.} \frac{T}{2\pi} \left[- \cos \frac{2\pi}{T} t \right]_0^{\frac{T}{2}}$$

$$= \frac{2}{\pi} I_{max.} = 0{\cdot}636\ I_{max.}$$

The mean value is not of itself important, but, as will be seen later, is involved in the form factor. The effective value is of direct practical importance, since all power measurements involve the effective values of current and pressure.

The effective value of an alternating current or E.M.F. may be defined as equal to the value of that direct current or E.M.F. which will produce the same heating effect when acting in any given circuit.

Let i = instantaneous value of the alternating current,
I = effective value of the alternating current,
r = resistance of the circuit.

Then from Joule's law the mean rate of heat production is

$$I^2 r = \int_0^T i^2 r dt \div T$$

$$= \frac{1}{T} \int_0^T I^2_{max.}\, r \left(\sin^2 \frac{2\pi}{T} t \right) dt$$

$$= \frac{I^2_{max.}\, r}{T} \int_0^T \frac{1}{2} \left(1 - \cos \frac{4\pi}{T} t \right) dt.$$

$$\text{Or } I^2 = \frac{I^2_{max.}}{2},$$

$$\text{and } I = \frac{I_{max.}}{\sqrt{2}} = 0{\cdot}707\ I_{max.}$$

In a similar way,

$$E = \frac{E_{max.}}{\sqrt{2}}.$$

All alternating-current ammeters and voltmeters read the effective value of the current or pressure, as the case may be.

Instead of the expression *effective value* the expressions *virtual value* or *root-mean-square value* (R.M.S. value) are

frequently used—the latter owing to the fact that the effective value is obtained by taking the root of the mean of the squares of the instantaneous values.

Form Factor.—The form factor is defined to be the ratio of the effective to the mean value of the wave. Thus if the form factor be denoted by γ,

$$\text{then } \gamma = \frac{\text{effective value}}{\text{mean value}}.$$

Thus for the case of a simple sine function

$$\gamma = \frac{\frac{I_{max.}}{\sqrt{2}}}{\frac{2}{\pi} I_{max.}} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11.$$

Referring to the simple example of a loop rotated in a magnetic field (see fig. 356) it will be seen that during one half-cycle of the E.M.F. between successive zeros, the flux linked with the loop changes from a positive maximum through zero to a negative maximum. Since the loop consists of a single turn, the total change in flux interlinkages is $2\Phi_{max}$. The mean rate of change of flux interlinkages is the total change divided by the time in seconds taken to complete the change, and by definition this is the mean value of the generated E.M.F.

$$\text{Therefore } E_{mean} = 2\Phi_{max.} \div \frac{1}{2f}, \text{ in C.G.S. units.}$$

$$\text{Or } E_{mean} = 4f\Phi_{max.}10^{-8} \text{ volts.}$$

Since the above involves no assumptions as to wave-form, it is evident that the mean value of the generated E.M.F. is independent of the wave-form. On the other hand, the effective value does depend on the wave-form, and may be obtained by multiplying the mean value by the form factor.

$$\text{Thus } E = 4\gamma f\Phi_{max.}10^{-8} \text{ volts.}$$

If the flux is linked with T turns instead of with a single turn, then $E = 4\gamma f T\Phi_{max.}10^{-8}$ volts.

For a simple sine function, since $\gamma = 1{\cdot}11$,

$$E = 4{\cdot}44 f T\Phi_{max.}10^{-8} \text{ volts.}$$

Referring to the equation previously deduced, i.e.

$$E_{max.} = 2\pi f T \Phi_{max.} 10^{-8},$$

since $$E = \frac{E_{max.}}{\sqrt{2}},$$

$$E = 4{\cdot}44 f T \Phi_{max.} 10^{-8} \text{ volts.}$$

INDUCTANCE

When dealing with the flow of a direct current through a circuit it is sufficient to know what is the resistance of the circuit, in order to determine the current that will flow when a given potential difference is applied to the circuit. If the applied potential difference is a varying one, a number of other factors have to be considered in calculating the current. At present we will deal with one only of these, namely the inductance of the circuit. When a current flows in a circuit, lines of magnetic force are set up. The shape, size, and relative position of the various parts of the circuit will determine the number of interlinkages between the lines of force and the circuit. If an alternating current is flowing in the circuit, the interlinkages are alternating also. It therefore follows that an additional E.M.F. is introduced into the circuit, this E.M.F. being equal to the time-rate of change of the interlinkages. The current now depends not merely on the applied P.D. and the resistance of the circuit, but also upon this additional E.M.F.—usually spoken of as the self-induced E.M.F.

Consider a circular coil consisting of T turns so concentrated that all the lines of magnetic force link with all the turns of the coil, and suppose that a current of i absolute units is flowing in the coil. In accordance with the law of the magnetic circuit, the instantaneous value of the flux produced by this current is given by $\phi = \frac{4\pi T i}{\rho}$ where ρ is the reluctance of the magnetic circuit in which the flux exists. Also the instantaneous value e of the E.M.F. induced by the varying interlinkages is given by

$$e = -\frac{d}{dt}(T\phi).^1$$

[1] The magnitude of the induced E.M.F. is equal to the time-rate of change of the flux interlinkages, and in accordance with Lenz's law, is in a direction opposite to the cause producing it—hence the negative sign.

Inserting the value of ϕ,

$$e = -\frac{d}{dt}\left(\frac{4\pi T^2 i}{\rho}\right).$$

We may replace $\frac{4\pi T^2}{\rho}$ by a coefficient L, giving

$$e = -\frac{d(Li)}{dt}.$$

Coefficient of Self-Induction.—The coefficient L is known as the coefficient of self-induction, or simply as the inductance, and may be defined as follows:—

"The inductance of a circuit is the number of interlinkages of the circuit with the lines of magnetic force produced by unit current flowing in the circuit."

In accordance with this definition, $L = \frac{T\phi}{i}$ absolute units.

From this we obtain, by inserting the value of ϕ,

$$L = \frac{T}{i}\left(\frac{4\pi T i}{\rho}\right) = \frac{4\pi T^2}{\rho} \text{ absolute units,}$$

which is the expression for L which we have already made use of above.

Mutual Inductance.—Mutual inductance is similar in its nature, and may be defined as follows:—

"The mutual inductance of two circuits is the number of interlinkages of *the one circuit* with the lines of magnetic force produced by unit current flowing in *the other circuit.*"

Inductance, whether "self" or "mutual", depends upon the reluctance of the path of the flux, and is therefore not necessarily constant. For example, when the circuit includes magnetic material, the inductance has a different value for every value of the current.

Units of Inductance.—The absolute unit of inductance may be defined as the inductance of a circuit in which the number of interlinkages of the circuit with the lines of magnetic force produced by one absolute unit of current flowing in the circuit, is unity.

Since a current varying uniformly at the rate of one absolute unit per second will give rise to a steady induced E.M.F. numerically equal to the inductance of the circuit, the absolute unit of inductance may be alternatively defined as the inductance

of a circuit in which one absolute unit of E.M.F. is induced by a current which varies uniformly at the rate of one absolute unit per second. It is on this basis that the practical unit of inductance—*the henry*—is defined. Thus a circuit in which an E.M.F. of one volt is induced by a current varying uniformly at the rate of one ampere per second is said to have an inductance of one henry. The henry has the value of 10^9 in terms of the absolute C.G.S. unit.

Energy stored in a Magnetic Field.—If a steady E.M.F. be applied to an inductive circuit, the current will rise until it attains a final value I. During the rise of the current, energy is being stored in the magnetic field produced by the current and linked with the circuit. A component e_L of the applied E.M.F. is devoted, during the time the current is changing, to balancing the self-induced E.M.F. e_{si}.

The remaining component of the applied E.M.F. causes the current to flow against the resistance of the circuit. This component multiplied by the current gives the rate at which energy is being absorbed by the resistance of the circuit. The rate at which energy is being stored in the magnetic field is given by $e_L i$, and the total energy stored when the current has reached its final value I is

$$W = \int_{i=0}^{i=I} e\, i dt.$$

$$\text{But } e_L = -e_{si} = +\frac{d(Li)}{dt}.$$

$$W = \int_{i=0}^{i=I} i\left(L\frac{di}{dt}\right)dt$$

$$= \int_0^I L i di$$

$$= \tfrac{1}{2}LI^2.$$

If L is in henrys and I in amperes, then the stored energy in joules is, $W = \frac{1}{2} LI^2$.

In highly inductive circuits carrying considerable currents, such as the field windings of large machines, the stored energy may be very great, and disastrous results may follow the sudden opening of such a circuit. The stored energy is suddenly released by the collapse of the magnetic field, and the excessive voltage thus induced may wreck the insulation of the field coils.

Inductive Reactance.—Referring again to the circular loop of T turns, let the current flowing in the loop be

$$i = I_{max.} \sin \omega t.$$

The self-induced E.M.F. will then be

$$e_{si} = -\frac{d(Li)}{dt}$$
$$= -L\frac{d}{dt}(I_{max.} \sin \omega t),$$

since there is no magnetic material in the circuit and therefore

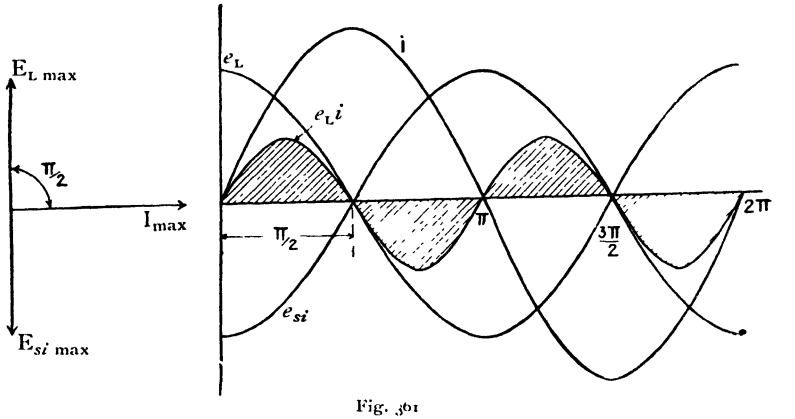


Fig. 301

L is constant. Assuming that the loop has negligible resistance, the applied E.M.F. has simply to overcome[1] the self-induced E.M.F. in order to cause the current i to flow,

$$\text{i.e. applied E.M.F. } e_L = -e_{si} = L\frac{d}{dt}(I_{max.} \sin \omega t)$$
$$= \omega L I_{max.} \cos \omega t.$$

From these equations we see that e_L leads the current i by $\frac{\pi}{2}$, while e_{si} lags behind i by the same amount. It is more usual to think of the phase position of the current relative to the E.M.F., and it will be noted that in a purely inductive circuit the current *lags by an angle* $\frac{\pi}{2}$ behind the applied E.M.F. As will be seen later, if the circuit has appreciable resistance, the current lags by an angle *less than* $\frac{\pi}{2}$ behind the E.M.F.

[1] The difference between the self-induced E.M.F. and the E.M.F. required to overcome it must be kept clearly in mind.

In fig. 361, in addition to the curves of e_{si} and e_L, the curve of the product $e_L i$ has been plotted. The area included between this curve and the horizontal axis represents the energy alternately stored in and restored by the magnetic field produced by the current and linked with the circuit. It should be noted that the positive and negative areas are equal, so that the net amount of energy supplied to the circuit is zero.

From the equation $e_L = \omega L I_{max.} \cos \omega t$, we at once obtain the maximum and effective values of the applied E.M.F.,

$$E_{L\,max.} = \omega L I_{max.}, \quad \text{and} \quad E_L = \omega L I.$$

The factor ωL may be replaced by a single coefficient x_L. This coefficient is called the *inductive reactance*, and may be defined as the ratio of the component E_L of the applied E.M.F. which overcomes the self-induced E.M.F., to the current I flowing in the circuit

$$x_L = \frac{E_L}{I} = \omega L = 2\pi f L.$$

The reactance of an inductive circuit varies directly as the frequency. A reactance has the dimensions of a resistance, and the practical unit of reactance is the ohm. The expression for the effective value of the component of the E.M.F. to overcome the self-induced E.M.F. in an inductive circuit therefore becomes

$$E_L = I x_L \text{ volts},$$

where I is the effective value of the current in amperes, and x_L is the inductive reactance in ohms.

Capacity.—When a condenser, say for example a pair of metallic plates separated by an air space, is connected to a source supplying a steady E.M.F., a current will flow momentarily until a quantity Q of electricity has been transferred to the condenser. Q is directly proportional to the potential difference between the plates, and is given by

$$Q = CV$$

where C is the capacity of the condenser, and V is the P.D. between the plates of the condenser.

In the case of a steady applied P.D., the flow of current ceases as soon as the condenser has acquired its charge Q. If, however, an alternating P.D. is maintained at the terminals of

the condenser, an alternating current will flow in the circuit. This current is a displacement current. As the P.D. changes from a negative maximum to a positive maximum, electrons are moving away from one of the plates, leaving that plate with a resultant positive charge. At the same time, electrons are accumulating on the other plate leaving it with a resultant negative charge. During the remaining half-cycle, the opposite process is taking place. The electron movement towards and away from the plates constitutes the current. It should be especially noted that there is no transfer of electricity from one side of the circuit to the other, except in so far as electrons leak across the dielectric of the condenser due to the fact that the dielectric is not a perfect insulator. The action of a condenser may be illustrated by an hydraulic analogy. Thus in fig. 362 let AB be a closed pipe filled with water and divided into two sections A and B by the piston P and a rubber diaphragm D.

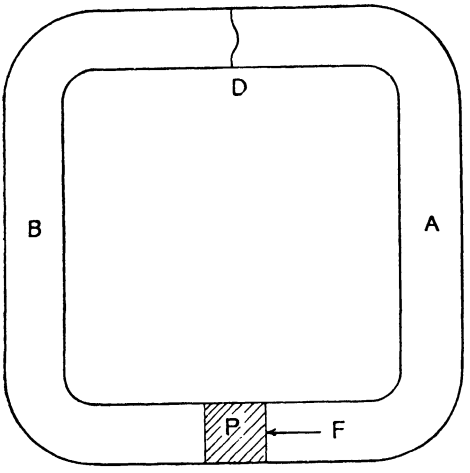


Fig. 362

If a steady force F be applied to the piston from right to left, the diaphragm D will be distorted as shown in the figure. A flow of water will take place up B and down A, but this will cease as soon as D has reached the distortion corresponding to the force F. If now a force acting alternately in opposite directions be applied to P, the diaphragm will be distorted first in one direction and then in the other, an alternating flow of water taking place in A and B. The alternating flow of water produced by the alternating force applied to P corresponds to the alternating current of electricity produced by the alternating E.M.F. in the electric circuit, and the distortion of the diaphragm corresponds to the strain set up in the dielectric of the condenser.

The amount of distortion of the diaphragm for a given force

applied to P is dependent on the dimensions of the diaphragm and its material, just as the charge given to the condenser by a given P.D. between its plates is dependent on the dimensions and material of the dielectric of the condenser. If the force applied to P is increased until the diaphragm breaks down, a transfer of water will take place from the one side to the other. Similarly, if the P.D. between the plates of the condenser is increased until the dielectric breaks down, a spark will pass from one plate of the condenser to the other and a transference of electricity from the one section of the circuit to the other will thus take place.

Practical Unit of Capacity.—The practical unit of capacity is the farad, and is defined as the capacity of a condenser to which a charge of one coulomb is given by a difference of potential of one volt between its terminals. This unit is inconveniently large for ordinary purposes, and the microfarad ($= 10^{-6}$ farad) is usually employed.

Capacity Reactance.—Let us consider a circuit having zero resistance and inductance, and containing a condenser of capacity C.

Let an E.M.F. given by the equation $e = E_{max.} \sin \omega t$ be applied to this circuit. The charge q of the condenser at any instant is $q = Cv$, where v is the P.D. between the terminals of the condenser. Since a current is equal to the rate at which electricity is flowing, the current i at any instant is given by

$$i = \frac{dq}{dt}$$

Inserting the value of q,

$$i = \frac{d}{dt}(Cv) = \frac{d}{dt}(Ce),$$

since the circuit is without resistance and inductance.

$$\therefore i = C\frac{de}{dt}$$

$$= C\frac{d}{dt}(E_{max.} \sin \omega t)$$

$$= \omega C E_{max.} \cos \omega t.$$

$$\therefore I_{max.} = \omega C E_{max.}$$

It will be seen that the current *leads* the E.M.F. by an angle of $\frac{\pi}{2}$, and has a maximum value of $\omega C E_{max.}$

In fig. 363 curves of e, i, and the product ei are shown. The area included between the last-mentioned and the horizontal axis represents the energy alternately given to and restored by the condenser. As in the case of the purely inductive circuit, the net energy given to the circuit is zero. In this case the energy is stored in the electrostatic field produced between the plates of the condenser. In practical condensers there is a certain amount of energy absorbed in the dielectric and the amount of energy supplied exceeds the amount restored by an

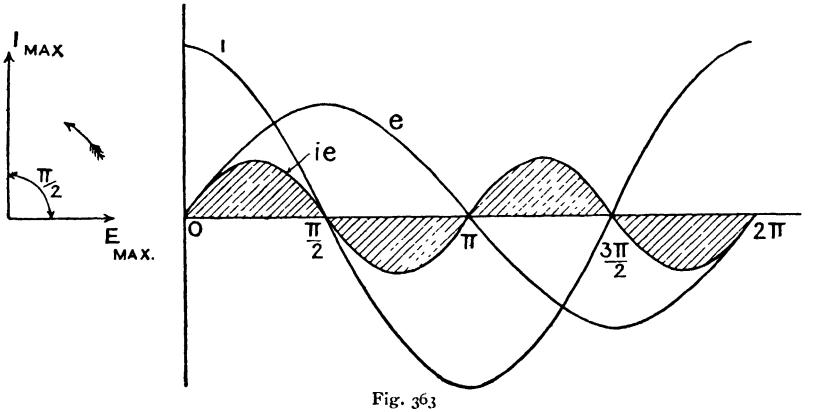


Fig. 363

amount equal to this loss. This means also that the current leads the E.M.F. by an angle *slightly less than* $\frac{\pi}{2}$.

From the last equation,

$$I = \omega CE,$$

$$\text{or } E = \frac{I}{\omega C}.$$

As in the case of the inductive circuit, this equation may be written in the form

$$E = Ix_c,$$

where x_c is called the *capacity reactance* of the circuit, and is equal to $\frac{I}{\omega C}$.

The capacity reactance of a circuit may be defined as the ratio of that component E_c of the applied E.M.F. which is devoted to charging the capacity, to the current flowing in the circuit.

If C is expressed in farads, then

$$x_c = \frac{1}{2\pi f C} \text{ ohms,}$$

$$\text{and } E_c = Ix_c \text{ volts.}$$

Capacity in Electric Circuits.—It may be pointed out that, apart from the presence of condensers as such, all electric circuits have a certain amount of capacity, although this may be so small as to be quite negligible at the frequencies used in the generation and transmission of electric power. In large alternating-current installations, especially where underground cables are used, the capacity is an item of considerable impor-

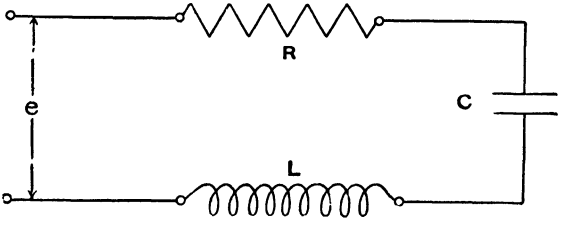


Fig. 364

tance, and the charging current, apart altogether from the useful load current, may be considerable. Where the frequency is very high, as in radiotelegraphy and radiotelephony circuits, extremely small capacities due to the proximity of different parts of the circuits may have a very great effect on the working of the circuits. The reason for this will be seen at once from the fact that the capacity current is directly proportional to the frequency. Where very short wave-lengths are employed, special precautions have to be taken to keep the self-capacity of the circuits sufficiently low.

Impedance.—In practical electric circuits any combination of resistance, inductance, and capacity may occur. We shall now consider a general case in which all three of these quantities have to be considered.

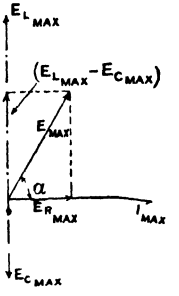
Resistance, Inductance, and Capacity in Series.—In the circuit shown in fig. 364 let R be the resistance in ohms, L the inductance in henrys, and C the capacity in farads, and let the current resulting from the sine wave applied E.M.F. e be given by the equation, $i = I_{max.} \sin \omega t$. In this case the applied E.M.F. e must supply three components, viz.

e_L to overcome the self-induced E.M.F.,
e_C to charge the condenser,
e_R to overcome the resistance.

Therefore $e = e_R + e_L + e_C$.

It has already been shown that e_L has a maximum value $\omega L I_{max}$ and leads e_R by $\frac{\pi}{2}$;

$$\therefore\ e_L = \omega L I_{max} \sin\left(\omega t + \frac{\pi}{2}\right).$$



Also e_C has a maximum value $\frac{I_{max}}{\omega C}$ and lags $\frac{\pi}{2}$ behind e_R,

$$e_C = \frac{I_{max}}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right),$$

and $e_R = I_{max} R \sin \omega t$.

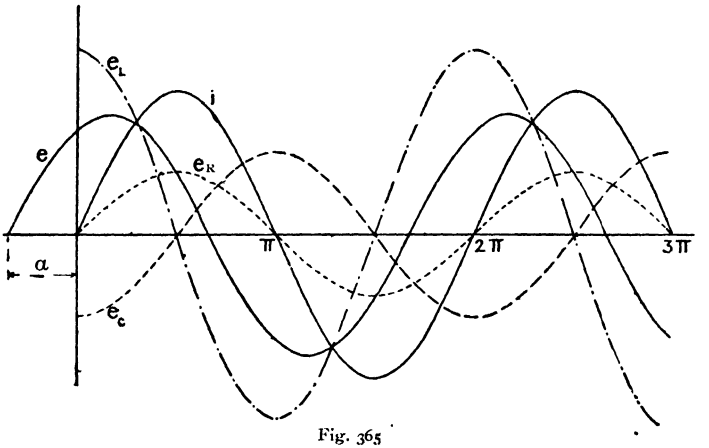


Fig. 365

The curves of e, e_R, e_L, e_C, and i have been plotted in fig. 365, and the corresponding vector diagram is shown to the left. From the vector diagram it will be seen that

$$\therefore\ E = \sqrt{E_R^2 + (E_L - E_C)^2}.$$

Inserting the values of E_R, E_L, and E_C,

$$E = \sqrt{I^2R^2 + \left(\omega LI - \frac{I}{\omega C}\right)^2}$$

$$= I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

This may be written $E = I\sqrt{R^2 + x^2}$, where x is the *total* reactance of the circuit, and is equal to $\left(\omega L - \frac{1}{\omega C}\right)$. The equation may be reduced to the still simpler form, $E = Iz$, where z is the total impedance of the circuit, and is equal to $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$.

The impedance of a circuit may be defined as the ratio of the applied E.M.F. to the current produced.

The above expression for the impedance is a complete one, since, by making the terms R, L, or C zero, an expression for the impedance in a circuit in which any of these quantities is absent may be obtained.

From the vector diagram

$$\tan \alpha = \frac{E_{L\,max} - E_{C\,max}}{E_{R\,max}},$$

$$\therefore \alpha = \tan^{-1} \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}.$$

This, again, is a complete expression, and by making the appropriate terms zero, the phase angle in any simpler case may be obtained.

Resonance.—When the factors L and C and the frequency f happen to have such values that the phase angle is zero, a state of resonance is said to exist in the circuit. In a series circuit this occurs when $\omega L = \frac{1}{\omega C}$, i.e. when the inductive reactance is numerically equal to the capacity reactance. The expression $E = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ then reduces to $E = IR$, and the circuit behaves as if both inductance and capacity were absent. Such a condition is capable of producing very serious results if it occurs accidentally. As an example, let the resistance, inductance, and capacity in fig. 364 be as follows: R = 0·2 ohm, C = 50 microfarads, and L = 0·2025 henry.

Let a sine wave of effective value 100 volts at 50 cycles per second be applied to the circuit,

$$x = x_L - x_C = (2\pi \times 50 \times 0{\cdot}2025) - \left(\frac{10_0}{2\pi \times 50 \times 50}\right)$$
$$= 63{\cdot}7 - 63{\cdot}7 = 0.$$

and $z = \sqrt{R^2 + x^2} = 0{\cdot}2$ ohm,

$$\therefore I = \frac{100}{0{\cdot}2} = 500 \text{ amperes.}$$

Also the P.D. across the inductance $= LI = 500 \times 63{\cdot}7 = 31{,}850$ volts, and the P.D. across the capacity $= \frac{I}{\omega C} = 500 \times 63{\cdot}7 = 31{,}850$ volts. It will be seen from this example that quite a low applied E.M.F. may give rise to very large P.D.'s across the inductance and capacity.

We will now consider the effect of changing the frequency to 25 cycles per second in the above example.

$$x_L = 2\pi \times 25 \times 0{\cdot}2025 = 31{\cdot}85 \text{ ohms.}$$

$$x_c = \frac{10^6}{2\pi \times 25 \times 50} = 127{\cdot}4 \text{ ohms.}$$

$$x = -95{\cdot}55 \text{ ohms.}$$

$$\therefore z = \sqrt{0{\cdot}04 + 9130} = 95{\cdot}55 \text{ ohms.}$$

$$I = \frac{E}{z} = \frac{100}{95{\cdot}55} = 1{\cdot}048 \text{ amperes.}$$

Let V_L = PD across the inductance

$= \omega LI = 31{\cdot}85 \times 1{\cdot}948 = 33{\cdot}4$ volts,

V_c = PD across the capacity

$= \frac{I}{\omega C} = 127{\cdot}4 \times 1{\cdot}948 = 133{\cdot}6$ volts.

$$a = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \tan^{-1}\left(\frac{-95{\cdot}55}{0{\cdot}2}\right)$$

$$= \tan^{-1}(-477{\cdot}75) = -89^\circ\ 50'.$$

Effect of Variation of Voltage and Frequency.—The effects of the variation of the applied E.M.F. and the frequency can be readily seen by plotting curves of impedance, reactance, phase angle, and current, on a base of (*a*) applied E.M.F., (*b*) frequency.

(1) *Resistance and Inductance.*—

$$x = \omega L.$$

x is therefore independent of the applied E.M.F., and varies directly as the frequency.

$$z = \sqrt{R^2 + x^2}.$$

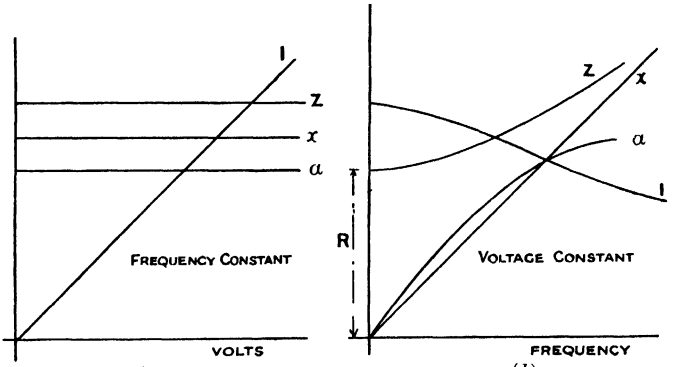
z is also independent of the applied E.M.F., and is a function of the frequency.

$$\alpha = \tan^{-1}\left(\frac{\omega L}{R}\right).$$

α is also independent of the applied E.M.F., and its tangent varies directly as the frequency.

$$I = \frac{E}{z}.$$

I varies directly as the applied E.M.F., and inversely as the impedance in all cases.



(*a*) Fig. 366 (*b*)

The curves are shown in figs. 366 *a* and *b*.

It will be noticed that the curve of z (fig. 366 *b*) cuts the axis of ordinates at a value equal to R, and is asymptotic to the line of x, i.e. when $f = 0$, $z = R$, and as f increases z becomes more and more nearly equal to x. α is zero when f is zero, and approaches $\frac{\pi}{2}$ as f approaches infinity.

(2) *Resistance and Capacity.*—

$$x = \frac{1}{\omega C}.$$

$$z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}.$$

$$\alpha = \tan^{-1}\left(\frac{1}{\omega C R}\right).$$

As before, x, z, and α are independent of the applied E.M.F.

x varies inversely as the frequency, z is an inverse function of the frequency, and the tangent of α varies inversely as the frequency.

From fig. 367 (b) it will be seen that the curve of z is asymptotic to the axis of ordinates, and to the straight line representing R, i.e. z approaches infinity as f approaches zero, and approaches the value of R as f approaches infinity.

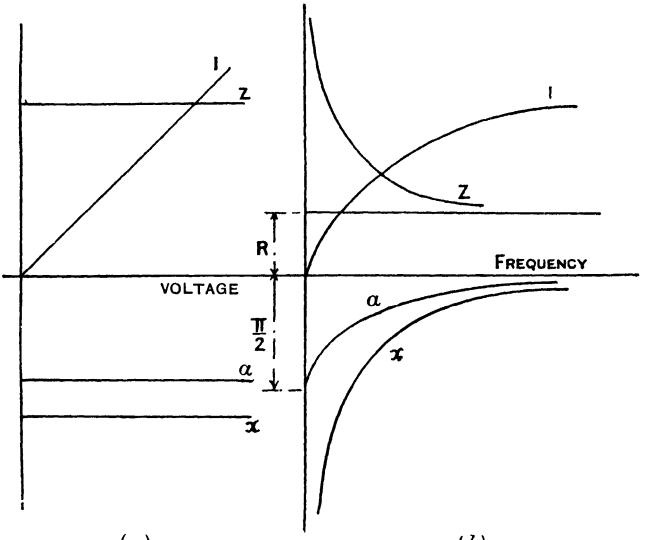


Fig. 367.

α is negative (i.e. the current leads the E.M.F.), and approaches $\frac{\pi}{2}$ as f approaches zero, and approaches zero as f approaches infinity.

(3) *Resistance, Inductance, and Capacity.*—

$$x = x_L - x_C = \omega L - \frac{1}{\omega C}.$$

$$z = \sqrt{R^2 + x^2}.$$

$$\alpha = \tan^{-1}\left(\frac{x}{R}\right).$$

x, z, and α are therefore all independent of the applied E.M.F., and are functions of the frequency.

As will be seen from fig. 368 (b), z decreases as f is increased until it reaches a minimum value equal to R. The inductive and

capacity reactances are then equal, and α is zero (resonance condition). As f increases to infinity, z approaches the value of x_L, and α the value $+\frac{\pi}{2}$. For the sake of clearness, the

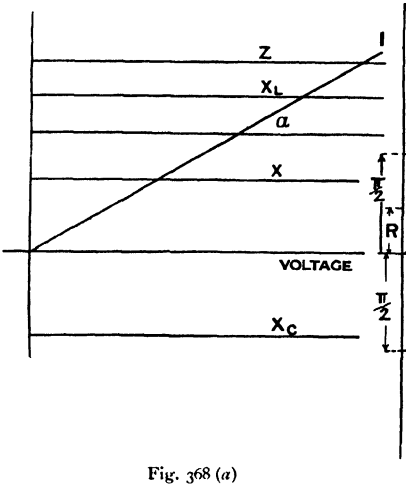


Fig. 368 (*a*)

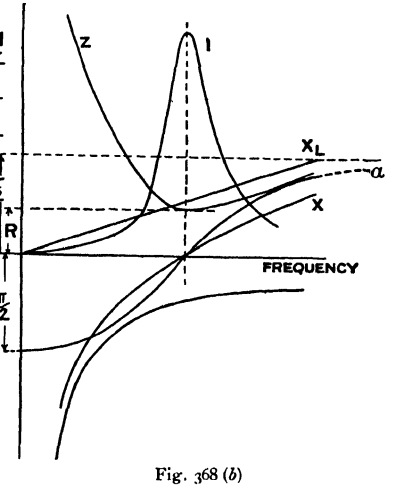


Fig. 368 (*b*)

curves of V_C and V_L (P.D.'s across the capacity and inductance respectively) are shown separately in fig. 368 (*c*). When f is zero, V_L is obviously zero, since I is then zero. The whole applied E.M.F. is therefore concentrated at the condenser, and $V_C = E$. The P.D. at the condenser, V_C (equal to $\frac{I}{\omega C}$), will come to a maximum before I, since before its maximum I is increasing much less rapidly than ωC. Also V_L (equal to ωLI) will come to a maximum after I, since immediately after its maximum I is decreasing much less rapidly than ωL is increasing.

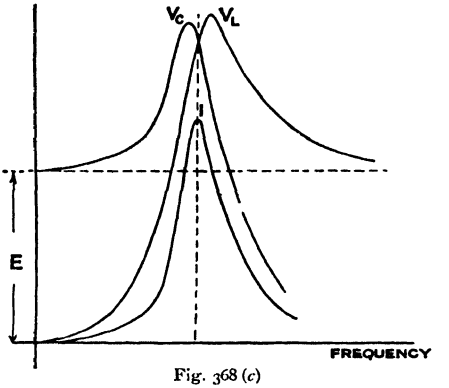


Fig. 368 (*c*)

Impedances in Series.—It should be specially noted that the impedance is a vector quantity, and that therefore, when impedances have to be added, they cannot be added alge-

braically. For example, let fig. 369 represent two impedances, z_1 and z_2, connected in series, and let fig. 370 represent the vector diagram for the complete circuit. From the diagram,

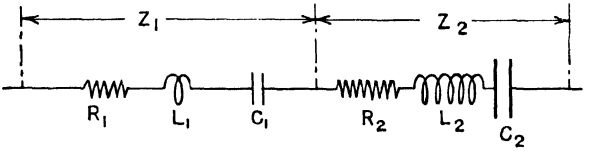


Fig. 369

OB (representing the applied E.M.F.—E) is equal to the square root of $(OA)^2 + (AB)^2$. Also $OA = IR_1 + IR_2$, and $AB = Ix_1 + Ix_2$.

Therefore $E = Iz = I\sqrt{(R_1 + R_2)^2 + (x_1 + x_2)^2}$.

Or $z = \sqrt{(R_1 + R_2)^2 + (x_1 + x_2)^2}$.

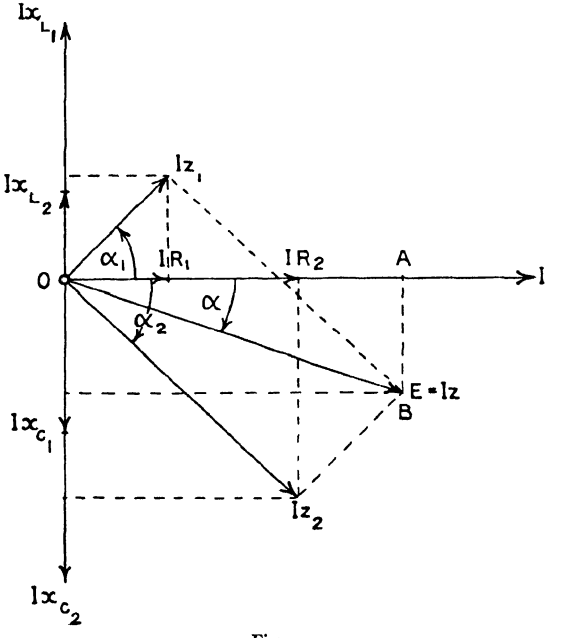


Fig. 370

In general, the resultant impedance of a number of impedances in series is given by

$$\sqrt{(\text{sum of resistances})^2 + (\text{sum of reactances})^2}.$$

Impedances in Parallel.—Fig. 371 represents two impedances, z_1 and z_2, connected in parallel, and the vector diagram is shown in fig. 372. The current, I, taken from the supply is the vector sum of the currents I_1 and I_2, in the two parallel branches of the circuit. Resolve each current into two components, one in phase with E and one at right angles to E $\left(\frac{\pi}{2} \text{ out of phase}\right)$. Components in phase with E (watt components of the currents),

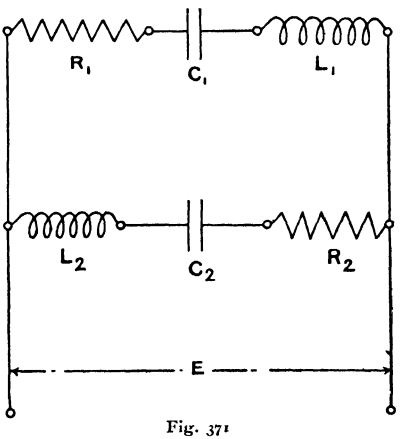


Fig. 371

$$I \cos \alpha = I_1 \cos \alpha_1 + I_2 \cos \alpha_2.$$

Components $\frac{\pi}{2}$ out of phase with E (wattless components of the currents),

$$I \sin \alpha = I_1 \sin \alpha_1 + I_2 \sin \alpha_2.$$

In the triangle Oab, the angle Oab is equal to α,

$$\cos \alpha_1 = \cos Oab = \frac{ab}{Oa} = \frac{I_1 R_1}{E}.$$

Also $\cos \alpha_1 = \frac{I_1 R_1}{I_1 z_1} = \frac{R_1}{z_1}$.

Similarly, $\cos \alpha_2 = \frac{R_2}{z_2}$, and

$$\cos \alpha = \frac{R}{z},$$

where z_1 and z_2 are the impedances of branches 1 and 2 respectively, z is the resultant impedance, and R is the resultant resistance of the whole circuit.

In a similar way it may be shown that

$$\sin \alpha_1 = \frac{x_1}{z_1}, \quad \sin \alpha_2 = \frac{x_2}{z_2},$$

$$\text{and } \sin \alpha = \frac{x}{z},$$

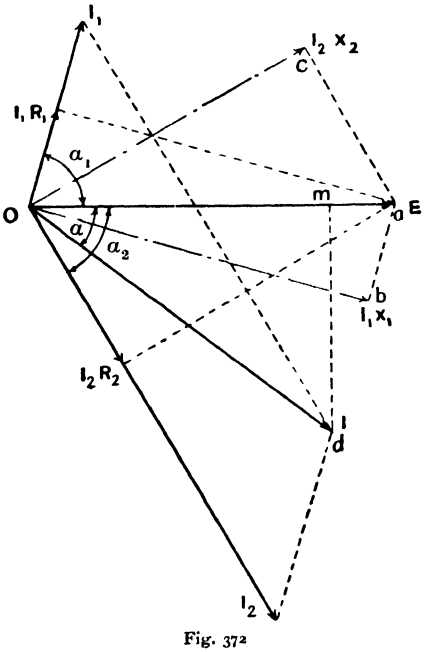


Fig. 372

where x_1, x_2, and x are respectively the reactances of branches 1, 2, and of the whole circuit (resultant reactance).

From the equations for the current components

$$I\frac{R}{z} = \left[I_1\frac{R_1}{z_1} + I_2\frac{R_2}{z_2}\right], \text{ and } I\frac{x}{z} = \left[I_1\frac{x_1}{z_1} + I_2\frac{x_2}{z_2}\right].$$

But from the definition of impedance

$$I = \frac{E}{z}, \quad I_1 = \frac{E}{z_1}, \text{ and } I_2 = \frac{E}{z_2}.$$

Therefore $E\frac{R}{(z)^2} = E\frac{R_1}{(z_1)^2} + E\frac{R_2}{(z_2)^2}$; or $R = \left[\frac{R_1}{(z_1)^2} + \frac{R_2}{(z_2)^2}\right]z^2$.

Similarly, $x = \left[\frac{x_1}{(z_1)^2} + \frac{x_2}{(z_2)^2}\right]z^2$.

Referring to the right-angled triangle Omd,

$$(Od)^2 = (Om)^2 + (md)^2,$$

i.e. $$I^2 = (I\cos\alpha)^2 + (I\sin\alpha)^2$$

$$= (I_1\cos\alpha_1 + I_2\cos\alpha_2)^2 + (I_1\sin\alpha_1 + I_2\sin\alpha_2)^2.$$

$$= E^2\left[\frac{R_1}{(z_1)^2} + \frac{R_2}{(z_2)^2}\right]^2 + E^2\left[\frac{x_1}{(z_1)^2} + \frac{x_2}{(z_2)^2}\right]^2.$$

Or $$I = E\sqrt{\left[\frac{R_1}{(z_1)^2} + \frac{R_2}{(z_2)^2}\right]^2 + \left[\frac{x_1}{(z_1)^2} + \frac{x_2}{(z_2)^2}\right]^2}.$$

From the definition of impedance, the resultant impedance z is given by

$$z = \frac{E}{I} = \frac{1}{\sqrt{\left[\frac{R_1}{(z_1)^2} + \frac{R_2}{(z_2)^2}\right]^2 + \left[\frac{x_1}{(z_1)^2} + \frac{x_2}{(z_2)^2}\right]^2}}.$$

The phase angles in the main and branch circuits are obtained as follows:—

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{x}{z}}{\frac{R}{z}} = \frac{\frac{x}{z^2}}{\frac{R}{z^2}},$$

$$\tan\alpha_1 = \frac{\frac{x_1}{z_1^2}}{\frac{R_1}{z_1^2}}, \text{ and } \tan\alpha_2 = \frac{\frac{x_2}{z_2^2}}{\frac{R_2}{z_2^2}}.$$

Or $$\alpha = \tan^{-1}\left(\frac{\frac{x_1}{z^2}}{\frac{R}{z^2}}\right), \quad \alpha_1 = \tan^{-1}\left(\frac{\frac{x_1}{z_1^2}}{\frac{R_1}{z_1^2}}\right), \text{ and } \alpha_2 = \tan^{-1}\left(\frac{\frac{x_2}{z_2^2}}{\frac{R_2}{z_2^2}}\right).$$

The procedure is similar for any number of impedances in parallel.

Where the impedances in ... x are the resultant impedance complicated circuit, the first ste... the whole circuit. resultant impedance z, ...ngements the expressions for having the components ...en expressed in terms of the R (resultant resistance), and x (resultant reactance). This single impedance may then be combined with the other impedances until a final single resultant impedance for the whole circuit is obtained.

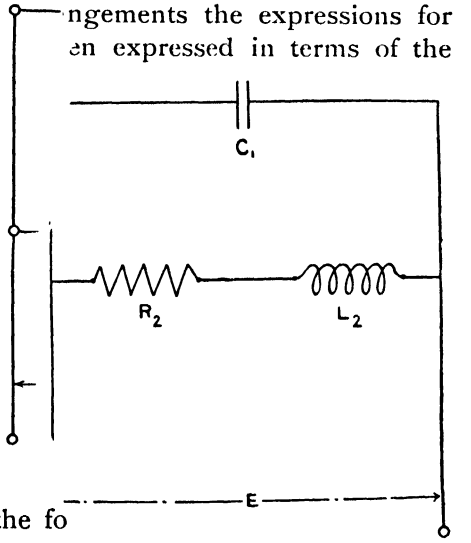


Fig. 374

As a practical example of a calculation involving two impedances in parallel, let us find the currents and phase angles in the circuit shown in fig. 373.

Let the constants have the fo...

$R_1 = 5$ ohms, $C_1 = 50$ n...

$L_2 = 0{\cdot}2$ henry, and E = ...

...tively the resistance ...

Then $z_1 = \sqrt{(R_1)^2 + (x_{L_1} - x_{C_1})^2}$

$= \sqrt{25 + \left(0 - \frac{10^{-6}}{2\pi \times 50 \times 50}\right)^2}$

$= \sqrt{25 + 4058} = \sqrt{4083}.$

Also $z_2 = \sqrt{(R_2)^2 + (x_{L_2} - x_{C_2})^2}$

$= \sqrt{400 + (2\pi \times 50 \times 0{\cdot}2 - 0)^2}$

$= \sqrt{4350}.$

$$z = \frac{1}{\sqrt{\left(\frac{5}{4083} + \frac{20}{4350}\right)^2 + \left(\frac{-63{\cdot}7}{4083} + \frac{62{\cdot}8}{4350}\right)^2}}$$

$$= \frac{1}{\sqrt{(12{\cdot}2 \times 10^{-4} + 46 \times 10^{-4})^2 + (-155{\cdot}8 \times 10^{-4} + 144{\cdot}5 \times 10^{-4})^2}}$$

$= \frac{10^4}{68{\cdot}2} = 146{\cdot}6$ ohms.

$I = 100 \times \frac{68{\cdot}2}{10^4} = 0{\cdot}682$ ampere.

$I_1 = \frac{100}{4083} = 1{\cdot}563$ amperes.

where x_1, x_2, and x are respectiv
2, and of the whole circuit (resul res.

From the equations for the

$$I\frac{R}{z} = \left[I_1\frac{R_1}{z_1} + I_2\frac{R_2}{z_2}\right], \quad \left[\frac{x_1}{(z_1)^2} + \frac{x_2}{(z_2)^2}\right].$$

But from the definition of

$$I = \frac{E}{z}, \quad I_1 =$$

Therefore $E\frac{R}{(z)^2} = E\frac{R_1}{(z_1)^2} + E$ (6·2 × 10⁻⁴.

Similarly, $x = \left[\frac{x_1}{(z_1)^2} + \frac{x_2}{(z_2)^2}\right.$

Referring to the right-angle 10° 59′.

$$(Od)^2 = (Om)^2 + (md)^2,$$

i.e. $I^2 = (I\cos\alpha)^2 + (I\sin$ 63·7 ÷ 5 4083 4083

$$= (I_1\cos\alpha_1 + I_2\cos$$

$$= E^2\left[\frac{R_1}{(z_1)^2} + \frac{R_2}{(z_2)^2}\right]^2 + \frac{-155\cdot8\times10^{-4}}{12\cdot2\times10^{-4}}$$

Or $I = E\sqrt{\left[\frac{R_1}{(z_1)^2} + \frac{R_2}{(z_2)^2}\right]}$ (−12·75) ÷ −85° 31′.

From the definition 4350 ÷ 20 4350 $= \tan^{-1}\left[\frac{144\cdot5\times10^{-4}}{46\times10^{-4}}\right]$

is given by z_2^2 ÷ $\tan^{-1}(3\cdot14)$ ÷ +72° 20′.

A negative value of α indicates, as before, that the current is leading the E.M.F. Some confusion may arise on referring this to the vector diagram, since α is, at a first glance, apparently positive when the current is leading.

This difficulty will be removed at once if it is remembered that in all cases α is measured *from* the current vector *to* the E.M.F. vector.

Resonance in Parallel Arrangements.—The condition for resonance, viz. the phase angle between the current and E.M.F. vectors for the main circuit equal to zero, applies also to parallel arrangements, but the effects of resonance are very different from those in the series case.

At resonance the current is, in this case, a minimum, and the impedance a maximum.

For resonance $\alpha = 0$, therefore $\tan\alpha = \frac{x}{R} = 0$, and

therefore $\frac{x}{z^2} = 0$,[1] where z and x are the resultant impedance and reactance, respectively, of the whole circuit.

In all but the simplest arrangements the expressions for z and x are very cumbrous when expressed in terms of the constants of the circuit.

Usually in practical applications the circuit is a simple one in which a condenser is connected in parallel with a circuit which is in itself inductive. In power circuits the object usually is to secure zero phase angle between the main current and the applied E.M.F., and in radio circuits to secure maximum resultant impedance. In each case the condition is the same, viz. resonance condition. As an example, take the circuit shown in fig. 374. R_2 and L_2 are respectively the resistance and inductance of the circuit in which the effect of the inductance is to be neutralized as far as the main circuit is concerned. This is to be done by connecting a condenser C_1 in parallel with the inductive circuit.

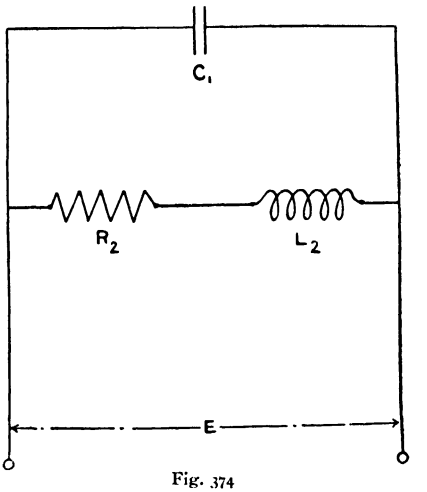


Fig. 374

$$\frac{x}{z^2} = -2\pi f C_1 + \frac{2\pi f L_2}{(R_2)^2 + (2\pi f L_2)^2} = 0.$$

This gives

$$C_1 = \frac{L_2}{R_2^2 + (2\pi f L_2)^2}.$$

Special attention may be called to the fact that the resonance condition is here *dependent on the value of the resistance*, whereas in *series* circuits it is *always independent of the resistance*.

Let $L_2 = 0{\cdot}2$ henry; $R_2 = 5$ ohms; and $f = 50$ cycles per second.

$$C_1 = \frac{0{\cdot}2}{25 + (100\pi \times 0{\cdot}2)^2}$$
$$= 50{\cdot}4 \text{ microfarads.}$$

[1] Provided $\frac{R}{z^2}$ is not also zero.

Until recently this method of neutralizing the effect of inductance in power circuits has not been employed owing to the difficulty in securing the necessary condensers capable of dealing with large currents. Now, however, suitable condensers are on the market, and the method is more commonly made use of.

APPENDIX

UNITS

In physical science it is essential to have certain definite units in terms of which results of observation, and physical quantities dealt with generally, can be expressed. All physical units may be divided into fundamental units and derived units. That is, a few units are chosen and fixed arbitrarily, so that all other units in physics can be expressed in terms of them; then the former are said to be fundamental units, and the latter to be derived units. Theoretically, any three independent units are necessary and sufficient, so that the selection of the three fundamental units is one of convenience rather than of necessity. The three fundamental units so chosen are: the unit of length, the unit of mass, and the unit of time.

There are in general use two systems of units—the British system and the metric or French system. The latter is so much simpler that experimental results are generally expressed in metric units (centimetre-gramme-second units, or C.G.S. units). But there are many results which are so well known that they are more conveniently expressed in British units (foot-pound-second units).

The British unit of length is the Imperial yard. It was enacted by Parliament (18 and 19 Vict., c. 72, July 30, 1855) "that the straight line between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the office of the Exchequer shall be the genuine standard yard at 62° F., and if lost it shall be replaced by means of its copies", which are preserved at the Royal Mint, the Royal Society of London, the Royal Observatory at Greenwich, and the new palace at Westminster. The unit generally used is a foot, which is a third of a yard.

The French unit of length is a metre, and is defined as the

distance between the ends of a rod of platinum made by Borda, the rod being at the temperature of melting ice. This length was originally intended, according to the measurements of Delambre and Mechain, to be the ten-millionth part of the earth's meridian passing through Paris. But the measurements of Delambre and Mechain were subsequently found not to be quite accurate, so that the actual definition of the metre is that above stated. The unit generally used is a centimetre, which is the one-hundredth part of a metre.

The British unit of mass is the Imperial pound. It is the quantity of matter in a piece of platinum marked "P.S., 1844, 1 lb." deposited in the office of the Exchequer, and the Act quoted above states that it "shall be the legal and genuine standard measure of weight, and shall be and be denominated the Imperial standard pound avoirdupois, and shall be deemed to be the only standard measure of weight from which all other weights and other measures having reference to weight shall be derived, computed, and ascertained, and one equal seven-thousandth part of such pound avoirdupois shall be a grain, and five thousand seven hundred and sixty such grains shall be and be deemed to be a pound troy. If at any time hereafter the said Imperial standard pound avoirdupois be lost or in any manner destroyed, defaced, or otherwise injured, the Commissioners of Her Majesty's Treasury may cause the same to be restored by reference to or adoption of any of the copies aforesaid, or such of them as may remain available for that purpose."

The French unit of mass is the kilogramme des Archives, and though originally intended to be the mass of a cubic decimetre of distilled water at its maximum density (approximately at the temperature of 4° C.), it is really defined as the mass of a piece of platinum constructed by Borda in accordance with a decree of the French Republic in 1795.

The metric system is decimal, Greek prefixes denoting multiples, and Latin prefixes denoting sub-multiples.

1 kilometre	= 1000 metres	1 kilogramme	= 1000 grammes
1 hektometre	= 100 ,,	1 hektogramme	= 100 ,,
1 dekametre	= 10 ,,	1 dekagramme	= 10 ,,
1 metre	= 1 metre	1 gramme	= 1 gramme
1 decimetre	= $\frac{1}{10}$,,	1 decigramme	= $\frac{1}{10}$,,
1 centimetre	= $\frac{1}{100}$,,	1 centigramme	= $\frac{1}{100}$,,
1 millimetre	= $\frac{1}{1000}$,,	1 milligramme	= $\frac{1}{1000}$,,

The relations between the British and metric units of length and mass are:

1 yard	=	0.91439179 metre.
1 metre	=	1.09362311 yards = 39.370432 inches.
1 pound	=	0.453593 kilogramme.
1 kilogramme	=	2.2046212 pounds.

The following are, however, near enough approximations:

1 foot	=	30.48 centimetres.
1 pound	=	453.6 grammes.

The unit of time universally adopted is a mean solar second. This unit of time differs from a solar second and from a sidereal second. A sidereal day is the time that elapses between two successive transits of any fixed star at any chosen meridian, or it is the time of the earth's rotation on its own axis. A sidereal second is, therefore, the time that the earth takes to rotate through an angle of one two hundred and fortieth of a degree.

A solar day is defined as the time that elapses between two successive transits of the sun at any given place on the earth's surface. For two reasons this time is of variable length. Firstly, the path of the earth round the sun is an ellipse, and hence its motion is sometimes quicker and sometimes slower. Secondly, the motion of the earth is in the plane of the ecliptic and not in the plane of the equator.

All electrical quantities can be expressed in terms of the mechanical, magnetic, chemical, or thermal effects produced by electricity. Magnetic, chemical, and thermal effects are, however, measured in terms of some mechanical unit, and hence it is only necessary to state the relations between the electrical quantities and the mechanical units. We may start from the experimental results of Coulomb, by defining the force between two quantities of static electricity, or the force between two quantities of magnetism. We thus arrive at what is commonly called the *electrostatic* units and the *electromagnetic* units.

Electrostatic System of Units.—Coulomb, experimenting with his torsion balance, found that the force of repulsion between similar quantities of electricity varied inversely as the square of the distance between them, and directly as the product of the two quantities of electricity. In symbols, if q, q^1 be the

two quantities of electricity, and r the distance between them, then $F \propto \frac{qq^1}{r^2} = \text{constant} \times \frac{qq^1}{r^2}$.

Faraday proved that different substances have different powers of transmitting electric disturbances, so that the force of repulsion between two similar quantities of electricity depends not only on the quantities and on their distance apart, but also on the intervening medium. Let us denote this power of transmitting electric disturbances, or the specific inductive capacity of the medium, by k, then $F = \text{constant} \times \frac{qq^1}{kr}$.

It is thus necessary to fix on some standard medium for defining the electrostatic quantity of electricity. In the ordinary electrostatic system the standard medium is air, at temperature o° C., and at standard atmospheric pressure. The constant in the equation will be unity if we define unit quantity of electricity as that quantity which, concentrated on a particle 1 cm. from an equal and similar quantity also concentrated on a particle, would repel it with a force of 1 dyne, when the medium across which the electric action is transmitted is air. The dimensional formula for unit *quantity* of electrostatic electricity is therefore

$$\begin{aligned}[q] &= [F \cdot L^2]^{\frac{1}{2}}; \\ &= [LMT^{-2} \cdot L^2]^{\frac{1}{2}}; \\ &= [L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}].\end{aligned}$$

Electric *surface density*, σ, of a conductor at any point is measured by the number of units of electricity per unit of area. Hence its dimensional formula is

$$[\sigma] = [qL^{-2}] = [L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].$$

The unit of *current*, i, in a conducting wire is the current in which the unit quantity passes in the unit of time. Hence its dimensional formula is

$$[i] = [qT^{-1}] = [L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}].$$

The *intensity of an electric field* at a point, f, or the electric force at any point in an electric field, is the force with which a unit of electricity would be acted on if placed at that point. The force, F, on a quantity of electricity, q, in a field of electric force

of intensity, f, is $F = fq$. Therefore the dimensional formula for the electric force at a point is

$$\begin{aligned}[f] &= [Fq^{-1}];\\ &= [LMT^{-2} \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T];\\ &= [L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].\end{aligned}$$

Unit *difference of electric potential*, v, exists between two points in an electric field when unit work is done in carrying unit quantity of electricity from the one point to the other against the electric forces. Hence if q units of electricity is carried through a difference of electric potential expressed by v, the work done $W = qv$. Therefore the dimensional expression for difference of electric potential is

$$\begin{aligned}[v] &= [W \cdot q^{-1}];\\ &= [L^2MT^{-2} \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T];\\ &= [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].\end{aligned}$$

Electric potential at a point due to a quantity, q, of electricity is the quotient obtained by dividing the quantity of electricity by the distance.

Thus $$[v] = [qL^{-1}] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].$$

The *capacity* of a conductor, c, is the quotient obtained by dividing the quantity of electricity with which it is charged by the potential which this charge produces in it. Hence its dimensional formula is

$$[c] = [qv^{-1}] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1} \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T] = [L].$$

Hence the dimensions of capacity will be that of length, and is correctly expressed as so many centimetres.

The *resistance* of a conductor, r, is by Ohm's law measured by the difference of potential at its extremities divided by the current produced in it thereby. Therefore its dimensional formula is

$$[r] = [vi^{-1}] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1} \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T^2] = [L^{-1}T].$$

Conductance is the reciprocal of resistance, and its dimensional expression is therefore $[LT^{-1}]$.

These electrostatic quantities may now be put into a tabulated form for convenience of reference.

Name.	Symbol.	Relation to other Quantities.	Dimensions in LMT.
Quantity	q	$\sqrt{\text{force} \times (\text{distance})^2}$	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$
Surface density ... ... / Electric displacement ...	σ	quantity ÷ area	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Current	i	quantity ÷ time	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$
Intensity of electric field ...	f	force ÷ quantity	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Potential	v	work ÷ quantity / quantity ÷ distance	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Capacity	c	quantity ÷ potential	L
Resistance	r	potential ÷ current	$L^{-1}T$
Conductance	$1/r$	current ÷ potential	LT^{-1}

Magnetic Units.—The unit *magnetic pole*, m, is defined in exactly the same manner as the unit quantity of electrostatic electricity. Hence the dimensional formula for strength of magnetic pole is

$$[m] = [F \cdot L^2]^{\frac{1}{2}};$$
$$= [L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}].$$

Magnetic *surface density*, σ^1, *intensity of magnetic field*, H, and *magnetic potential*, V, are defined similar to the corresponding quantities in electrostatics respectively, and hence their dimensional formulas are:

$$[\sigma^1] = [L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}];$$
$$[H] = [L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}];$$
$$[V] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].$$

The *moment* of a magnet, **M**, is the strength of either pole into the distance between the poles. Hence its dimensional formula is

$$[\mathbf{M}] = [m \times L];$$
$$= [L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1}].$$

If a bar magnet is placed in a uniform field of force, it experiences a couple tending to set it with the lines joining its poles in the direction of the force of the field. When the magnet is so placed that the line joining its poles is at right angles to the direction of the lines of force of the field, the couple is equal to the product of the magnetic moment into the strength of the field. Hence

$$[\mathbf{M}] = [\tau \cdot H^{-1}];$$
$$= [L^2MT^{-2} \times L^{\frac{1}{2}}M^{-\frac{1}{2}}T];$$
$$= [L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1}].$$

Intensity of magnetization, I, of a uniformly magnetized magnet is measured by its magnetic moment divided by its volume. Hence its dimensional formula is

$$[I] = [L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1} \times L^{-3}];$$
$$= [L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].$$

This is equal to the surface distribution of the magnetism over the ends of the bar, and its dimensional formula is therefore the same as that of magnetic surface density.

Magnetic induction, B, is proved to be equal to the resultant of H and $4\pi I$. If the substance be isotropic and has no magnetism except what exists on account of the magnetizing force, H, then

$$B = H + 4\pi I;$$
$$= H(1 + 4\pi\kappa);$$
$$= H\mu.$$

The dimensional formula for magnetic induction is therefore

$$[B] = [L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}],$$

the same as that of intensity of magnetization.

These magnetic quantities may now be put into a tabulated form for convenience of reference.

Name.	Symbol.	Relation to other Quantities.	Dimensions in LMT.
Quantity of magnetism Magnetic pole... ...	m	$\sqrt{\text{force} \times (\text{distance})^2}$	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$
Magnetic surface density	σ^1	magnetic pole ÷ area	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Intensity of magnetic field	H	force ÷ strength of pole	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Magnetic force ...			
Magnetic potential ...	V	work ÷ strength of pole	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Magnetic moment ...	M	strength of pole × distance	$L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1}$
Intensity of magnetization	I	moment ÷ volume	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Magnetic induction ...	B	magnetic force × permeability	

Electromagnetic System of Units.—In the electromagnetic system all the units are derived from the definition of magnetic pole. From the experiments of Oersted and of Ampère we are

led to regard a linear conductor carrying a current as producing a magnetic field. If the linear conductor be bent into a circle of unit radius, then unit magnetic pole placed at the centre is acted on by unit force per unit length of the conductor when unit current is flowing in it. The total force acting on the unit pole is therefore 2π units. If I be the symbol used to represent current, then

$$\begin{aligned}[I] &= [FLm^{-1}];\\ &= [LMT^{-2} \times L \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T];\\ &= [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].\end{aligned}$$

If we have, as in a practical experiment we would have, a magnet of length l and strength of pole m suspended at the centre of a coil of mean radius r, large in comparison with l, and if n be the number of turns in the coil, and θ the angle between the plane of the coil and the axis of the magnet, then neglecting the depth and breadth of the coil, the moment τ tending to turn the magnet when a current I is in the coil, is

$$\begin{aligned}\tau &= I \times m \times 2\pi nr \times \tfrac{1}{r^2} \times l \cos\theta;\\ \therefore [\tau] &= [Im];\\ [I] &= [\tau m^{-1}] = [FLm^{-1}],\end{aligned}$$

which is the same as above.

Let us suppose the coil to be fixed so that its plane is vertical and in the magnetic meridian, that the intensity of the magnetic field at the magnet is H; then when the deflection of the magnet due to the current I in the coil is θ, the moment τ tending to bring the magnet back to its original position is

$$\tau = mHl \sin\theta.$$

Equating these two expressions for τ we get

$$I = \frac{r}{2\pi n} H \tan\theta.$$

$$\therefore [I] = [LH] = [L \times L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}].$$

The quantity of electricity, Q, is equal to the product of the current into the time during which it lasts.

$$[Q] = [IT] = [L^{\frac{1}{2}}M^{\frac{1}{2}}].$$

The work done in urging a quantity, Q, of electricity through a difference of potential, V, is QV.

$$\therefore [V] = [WQ^{-1}] = [L^2MT^{-2} \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}]$$
$$= [L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}].$$

The capacity of a conductor is the quotient obtained by dividing the quantity of electricity with which it is charged by the potential which this charge produces in it. Hence

$$[C] = [QV^{-1}];$$
$$= [L^{\frac{1}{2}}M^{\frac{1}{2}} \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T^2];$$
$$= [L^{-1}T^2].$$

The dimensions of a resistance are

$$[R] = [VI^{-1}];$$
$$= [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2} \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T];$$
$$= [LT^{-1}].$$

The coefficient of self-induction of a circuit, in a medium of constant permeability, is the ratio of the counter electromotive force of the circuit to the rate of variation of the current producing it. In symbols

$$L_1 = e \div \frac{dI}{dt},$$

where L_1 is the coefficient of self-induction,

$$\therefore [L_1] = [L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2} \div L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1} \times T^{-1}]$$
$$= [L].$$

The simplest way of defining the coefficient of self-induction is to say that it is the ratio of the total induction in the circuit to the current producing it. If N be the total induction or total flux,

$$L_1 = N \div I = BA \div I.$$
$$\therefore [L_1] = [L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1} \times L^2 \times L^{-\frac{1}{2}}M^{-\frac{1}{2}}T]$$
$$= [L].$$

If M_1 be the coefficient of mutual induction between two circuits, then the counter electromotive force e^1, in one circuit, due to a current I in the other circuit, is $e^1 = M_1 \frac{dI}{dt}$,

$$\therefore [M_1] = [L].$$

These electromagnetic quantities may now be put into a tabulated form for convenience of reference:

Name.	Symbol.	Relation to other Quantities.	Dimensions in LMT.
Current	I	Length × magnetic force	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Quantity	Q	current × time	$L^{\frac{1}{2}}M^{\frac{1}{2}}$
Potential	V	work ÷ quantity	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}$
Electromotive force ...	E		
Capacity	C	quantity ÷ potential	$L^{-1}T^{2}$
Resistance	R	Electromotive force ÷ current	LT^{-1}
Self-induction ...	L_1	$\left(\frac{\text{Electromotive force}}{\text{rate of variation of current}}\right)$	L
Mutual induction ...	M_1		

Name.	Symbol.	Dimensions in Electrostatic System.	Symbol.	Dimensions in Electro-magnetic System.	Ratio.
Quantity ...	q	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$	Q	$L^{\frac{1}{2}}M^{\frac{1}{2}}$	$\frac{q}{Q} = LT^{-1} = v$
Current ...	i	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$	I	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	$\frac{i}{I} = LT^{-1} = v$
Potential ...	v	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	V	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$	$\frac{v}{V} = L^{-1}T = \frac{1}{v}$
Capacity ...	c	L	C	$L^{-1}T^{2}$	$\frac{c}{C} = L^{2}T^{-2} = v^{2}$
Resistance ...	r	$L^{-1}T$	R	LT^{-1}	$\frac{r}{R} = L^{-2}T^{2} = \frac{1}{v^{2}}$

1 electromagnetic unit of quantity = v electrostatic units;
,, ,, current = v ,, ,,
,, ,, potential = $\frac{1}{v}$,, ,,
,, ,, capacity = v^2 ,, ,,
,, ,, resistance = $\frac{1}{v^2}$,, ,,

where v is a velocity of 3×10^{10} cm. per second, or 300000 Km. per second.

Practical Electric Units.—Let a conducting circuit be placed in a magnetic field, and let the current in this circuit be I under the action of an electromotive force E. Let the circuit be so moved that in time t the number of lines of force passing through it is increased by N, then the work done against the electro-

magnetic forces is measured by the product IN. But the work done on the current is EIt.

$$\therefore EIt + IN = 0;$$
$$\therefore E = -\frac{N}{t}.$$

Hence, if the number of lines of force enclosed by a circuit be increased, an electromotive force in the negative direction is set up, and the measure of this electromotive force is the number of lines of force added per unit of time.

Unit electromotive force is produced in any body which moves so that it cuts one line of magnetic force per second.

This unit is found to be too small for practical purposes, and a unit one hundred million times greater, called the *volt*, is taken as the practical unit.

An electromotive force of 1 volt = 10^8 C.G.S. units.

The dimension of resistance is a velocity.

In the centimetre-gramme-second system of units, unit velocity is a centimetre per second, hence unit resistance on this system is a centimetre per second. But for practical purposes this unit is too small, and a unit one thousand million times greater is taken as the unit of resistance, and is called an *ohm*.

A resistance of 1 ohm = 10^9 C.G.S. units.

The practical unit of current derived by Ohm's law from these practical units of electromotive force and resistance is called an *ampere*.

A current of 1 ampere = $10^8 \div 10^9$ C.G.S. units;
= 10^{-1} C.G.S. unit.

The quantity of electricity conveyed by an ampere in one second is called a *coulomb*. An ampere is a coulomb per second.

A quantity of 1 coulomb = 10^{-1} C.G.S. unit.

The practical unit of capacity is called the "farad", and a condenser has unit capacity when a charge of 1 coulomb raises it to an electromotive force of 1 volt.

A capacity of 1 farad = $10^{-1} \div 10^8$ C.G.S. units;
= 10^{-9} C.G.S. unit.

The work done in raising a quantity of electricity Q, through a difference of potential E, is equal to the product of these two quantities. Hence

$$W = EQ = EIt = I^2Rt = E^2t/R.$$

If these quantities are expressed in the absolute electromagnetic centimetre-gramme-second system of units, the work done in any time t is expressed in centimetre-dynes or ergs. If they are expressed in terms of the practical units (for example, E in volts and Q in coulombs)

$$W = E \times 10^8 \times Q \times 10^{-1} = EQ \times 10^7 \text{ ergs.}$$

The work done by raising a coulomb of electricity through a difference of potential of 1 volt, is, on the suggestion of Sir William Siemens, called a *joule*.

$$1 \text{ joule of work} = 10^7 \text{ ergs.}$$

Thus, if the difference of potential be expressed in volts, the current in amperes, and the resistance in ohms, the work done in any time t seconds in an electric circuit is

$$\begin{aligned} W &= EIt \text{ joules;} \\ &= I^2Rt \quad ,, \\ &= E^2t/R \quad ,, \end{aligned}$$

If a joule of work be done in the circuit during each successive second of time, and if the work be done uniformly, then the rate of doing work is called a watt.

$$\begin{aligned} 1 \text{ watt} &= 1 \text{ joule per second;} \\ &= 10^7 \text{ ergs} \quad ,, \end{aligned}$$

Hence, expressing E in volts, I in amperes, and R in ohms, the activity in an electric circuit is

$$\begin{aligned} A &= EI \text{ watts;} \\ &= I^2R \quad ,, \\ &= E^2/R \quad ,, \end{aligned}$$

To reduce this activity to horse-power, we have to divide by 746, for 1 horse-power = 550 ft.-lb. per second

$$\begin{aligned} &= 746 \times 10^7 \text{ ergs per second;} \\ &= 746 \text{ watts.} \end{aligned}$$

$$1 \text{ kilowatt} = 1000 \text{ watts} \doteqdot 1\tfrac{1}{3} \text{ horse-power.}$$

The Board of Trade supply unit is 1 kilowatt for one hour, or a kilowatt-hour. It is, therefore, equal to 3600×1000 joules.

$$= 3{\cdot}6 \times 10^6 \text{ joules.}$$

The following table gives the ratio of these practical units to the units both in the electrostatic and electromagnetic systems:—

	Name.	Symbol.	Value in Electrostatic Units.	Value in Electromagnetic Units.
Quantity of electricity ...	Coulomb	$q\mathrm{Q}$	3×10^9	10^{-1}
Current	Ampere	$i\mathrm{I}$	3×10^9	10^{-1}
Potential difference ...	Volt	$e\mathrm{E}$	$1/300$	10^8
Resistance	Ohm	$r\mathrm{R}$	$1/9 \times 10^{11}$	10^9
Capacity	Farad	$c\mathrm{C}$	9×10^{11}	10^{-9}
	Microfarad		9×10^5	10^{-15}

As these ratios are of great importance in the reduction of experimental results, they are given directly in terms of one another as follows:—

1 ampere $= 3 \times 10^9$ electrostatic units of current;

$= \frac{1}{10}$ electromagnetic unit of current.

1 volt $= \frac{1}{300}$ electrostatic unit of potential;

$= 10^8$ electromagnetic units of potential;

1 ohm $= \frac{1}{9 \times 10^{11}}$ electrostatic unit of resistance;

$= 10^9$ electromagnetic units of resistance.

1 microfarad $= 9 \times 10^5$ electrostatic units of capacity;

$= \frac{1}{10^{15}}$ electromagnetic unit of capacity.

Quantity can be measured practically by a ballistic galvanometer.

Let wk^2 = moment of inertia of needle in ballistic galvanometer;

M = magnetic moment of needle in ballistic galvanometer;

G = galvanometer constant,

$= \frac{2\pi n}{r}$, if n = number of turns, and r = mean radius;

Q = quantity of electricity that flows in time τ.

Then strength of field at centre of coil $= Gi$, where i = the current.

Moment of the force $= MGi$.

Impulse $= MGi\tau = MGQ = wk^2\omega$, if ω is the angular velocity with which the needle begins to move.

$$\therefore \tfrac{1}{2}wk^2\omega^2 \text{ (the kinetic energy)} = \tfrac{1}{2}wk^2\left(\frac{MGQ}{wk^2}\right)^2 = \frac{M^2G^2Q^2}{2wk^2}.$$

The work done on the needle $= 2Hml(1 - \cos\alpha)$, if α is the angle of deflection, m the strength of the pole, and l half the length of the needle; $= MH\,2\sin^2\frac{\alpha}{2}$.

$$\therefore \frac{M^2G^2Q^2}{2wk^2} = MH\,2\sin^2\frac{\alpha}{2}.$$

Hence
$$Q = \frac{2}{G}\sin\frac{\alpha}{2}\sqrt{\frac{wk^2H}{M}}.$$

If the deflection is small and the motion simple harmonic, the period of the needle vibrating in the magnetic field, H, is

$$T = 2\pi\sqrt{\frac{wk^2}{MH}}.$$

$$\therefore Q = \frac{H}{G}\cdot\frac{T}{\pi}\cdot\sin\frac{\alpha}{2}.$$

(In any simple harmonic motion the period is equal to $2\pi\sqrt{\frac{\text{Mass}}{\text{Force per unit displacement}}}$. Now, in rotational motion, moment of inertia and torque are analogues to mass and force in translational motion.

Hence, by analogy, $T = 2\pi\sqrt{\frac{\text{Moment of inertia}}{\text{Torque per radian}}}$.)

If the galvanometer be slightly damped, and if λ be the logarithm of the ratio of each half amplitude to the next, then

$$Q = \frac{H}{G}\cdot\frac{T}{\pi}\left(1 + \frac{\lambda}{2}\right)\sin\frac{\alpha}{2}.$$

$\frac{H}{G}$ can be found by sending a steady current i through the galvanometer, giving a deflection θ.

Then
$$i = \frac{H}{G}\tan\theta.$$

$$\therefore Q = \frac{iT}{\pi}\frac{\sin\frac{\alpha}{2}}{\tan\theta}\left(1 + \frac{\lambda}{2}\right) \text{ coulombs, if } i \text{ is in amperes.}$$

TABLES

TABLE I.—Specific Inductive Capacity

Substance.	Specific Inductive Capacity.
Air	1.000
Carbon dioxide	1.0008
Hydrogen	.9998
Olefiant gas	1.0007
Sulphur dioxide	1.0037
Benzene	2.2
Carbon disulphide	1.0023
Castor oil	4.78
Petroleum (common)	2.10
Sperm oil	3.02
Turpentine (commercial)	2.23
Water at 25° C.	75.7
Chatterton's compound	2.55
Ebonite	2.28
Glass, flint (density 3.66)	7.38
,, crown (density 2.485)	6.96
Gutta percha	3.3–4.9
Indiarubber (pure)	2.34
Mica	6.64
Paper	1.5
Paraffin wax	1.98
Resin	2.50
Shellac	3.10
Sulphur	2.58

TABLE II.—Dielectric Strength of Insulating Materials

Material.	Approximate Dielectric Strength in Volts per Millimetre (V_{MAX}).
Asbestos	5000
Ebonite	40000
Fibre (red vulcanized)	8000
Glass (2 mm. thick)	16000
Gutta percha	11000
Leatheroid	7000
Megohmit	36000
Mica	58000
Paraffin	20000
Paper	9000
Paper (oiled)	24000
Porcelain	14000
Presspahn	6000

The above figures apply to thicknesses of 1 mm. or under, except where otherwise stated.

TABLE III.—B.H. Curve for Sheet Iron

B. Lines/Sq. Cm.	H. C.G.S. Units.	B. Lines/Sq. Cm.	H. C.G.S. Units.
2000	0.6	14000	19.1
3500	1.3	15000	25.4
5000	2.0	16000	35.5
6000	2.8	17000	56.5
7000	3.8	18000	103.7
8000	4.9	19000	163.2
9000	6.2	20000	257.5
10000	7.8	21000	399.5
11000	9.9	22000	710.0
12000	12.3	23000	1330.0
13000	15.2	24000	2475.0

TABLE IV.—B.H. Curve for Pole-core Steel

B. Lines/Sq. Cm.	H. C.G.S. Units.	B. Lines/Sq. Cm.	H. C.G.S. Units.
13000	10.3	16500	44.3
13500	11.9	17000	65.5
14000	13.8	17500	88.5
14500	16.3	18000	118.0
15000	20.4	18500	149.4
15500	26.1	19000	184.7
16000	35.0	19500	226.0

TABLE V.—B.H. Curve for Cast Iron

B. Lines per Square Centimetre.	H. C.G.S. Units.
1000	4.6
2000	9.4
3000	14.6
4000	20.4
4500	23.7
5000	27.4
5500	32.1
6000	38.0
6500	45.5
7000	55.0
7500	67.5
8000	83.6
8500	108.0

TABLE VI

Specific Resistance (approximate) of Liquids, Carbons, and Insulators

It decreases with increase of temperature

Substance.	Specific Resistance in Ohms per Cubic Centimetre.	Temperature, C.
Bichromate of potash (saturated solution)	29.6	—
Carbon, Edison-Swan	4.0×10^{-3}	—
Carbon arc	7.0×10^{-3}	—
Common salt, saturated	4.7	18
Nitric acid (density 1.36)	1.4	—
Sulphate of copper, saturated	29.3	10
Sulphate of zinc, saturated	21.5	14
Sulphuric acid (density 1.2)	8.0	12
Water	3.4×10^{5}	11
Water distilled	37.6×10^{5}	—
Ebonite	28.0×10^{15}	46
Glass (below 40° C., very large)	6.2×10^{15}	46
Gutta percha	450.0×10^{12}	24
Ice	284.0×10^{0}	−0.2
Mica	84.0×10^{12}	20
Paraffin	34.0×10^{15}	46
Shellac	9.0×10^{15}	28

TABLE VII

Resistance of Electrolytes in Ohms per Centimetre Cube at 0° C.

Composition Per Cent.	Sulphuric Acid, H_2SO_4.		Copper Sulphate, $CuSO_4$.		Sodium Chloride, NaCl.		Salammoniac, NH_4Cl.	
	Ohms.	Temp. Coefft.	Ohms.	Temp. Coefft.	Ohms.	Temp. Coefft.	Ohms.	Temp. Coefft.
5	4.82	1.2 %	52.3	2.2 %	14.9	2.2 %	10.9	2.0 %
10	2.57	1.3 %	31.4	2.3 %	8.33	2.1 %	5.67	1.0 %
15	1.85	1.4 %	24.1	2.3 %	6.15	2.1 %	3.89	1.7 %
20	1.54	1.5 %	—	—	5.14	2.2 %	2.98	1.6 %
25	1.40	1.5 %	—	—	4.70	2.3 %	2.50	1.5 %
30	1.36	1.6 %	—	—	—	—	—	—

In the above table the first column shows the amount of Salt, &c., dissolved in water, expressed as a percentage.

Under temperature coefficient is given the percentage decrease in resistance per 1° C. rise of temperature.

TABLE VIII

ELECTRO-CHEMICAL EQUIVALENTS, CALCULATED ON THE BASIS OF LORD RAYLEIGH'S ELECTRO-CHEMICAL EQUIVALENT OF SILVER

Elements.	Valency.	Atomic Weight.	Chemical Equivalent.	Electro-Chemical Equivalent (Milligrammes per Coulomb).	Coulombs per Gramme.	Grammes per Ampere-hour.
Aluminium	3	27.3	9.1	0.0925	10583.0	0.3402
Copper (cupric) ...	2	63.3	31.65	0.3287	3042.0	1.1833
Copper (cuprous) ...	1	63.3	63.3	0.6574	1521.0	2.3666
Gold	3	196.2	65.0	0.6791	1473.5	2.4448
Hydrogen	1	1.0	1.0	0.0104	96293.0	0.0374
Iron (ferric)	3	55.9	18.64	0.1936	5166.4	0.6968
Iron (ferrous) ...	2	55.9	27.95	0.2904	3445.5	1.0448
Lead	2	206.4	103.2	1.0716	933.2	3.8578
Magnesium	2	23.94	11.97	0.1243	8045.0	0.4475
Mercury (mercuric)	2	199.8	99.9	1.0374	964.0	3.7345
Mercury (mercurous)	1	199.8	199.8	2.0747	482.0	7.4690
Nickel	2	58.6	29.3	0.3043	3286.8	1.0953
Potassium	1	39.04	39.04	0.4054	2467.5	1.4595
Silver	1	107.66	107.66	1.1180	894.4	4.0250
Sodium	1	22.99	22.99	0.2387	4188.9	0.8594
Tin (stannic) ...	4	117.8	29.45	0.3058	3270.0	1.1009
Tin (stannous) ...	2	117.8	58.9	0.6116	1635.0	2.2018
Zinc	2	64.9	32.45	0.3370	2967.1	1.2133
Electro-negative:						
Bromine	1	79.75	79.75	0.8281	1207.6	2.9812
Chlorine	1	35.37	35.37	0.3673	2722.5	1.3223
Iodine	1	126.53	126.53	1.3139	761.1	4.7300
Nitrogen	3	14.01	4.67	0.0485	20618.6	0.1746
Oxygen	2	15.96	7.98	0.0829	12062.7	0.2984

TABLE IX

COPPER VOLTAMETER CONSTANT

Area of Cathode in Square Centimetres per Ampere.	Values of Apparent Electro-chemical Equivalent of Copper.				
	2° C.	12° C.	23° C.	28° C.	35° C.
50	.0003288	.0003287	.0003286	.0003286	.0003282
100	.0003288	.0003284	.0003283	.0003281	.0003274
150	.0003287	.0003281	.0003280	.0003278	.0003267
200	.0003285	.0003279	.0003277	.0003274	.0003259
250	.0003283	.0003278	.0003275	.0003268	.0003252
300	.0003282	.0003278	.0003272	.0003262	.0003245

TABLE X

Resistances of Metals and Alloys at 0° C.

Name of Metal or Alloy.	Resistance in Microhms per Centimetre Cube.	Approximate Resistance relatively to Copper.	Approximate per Cent Increase in Resistance per Degree C.
Aluminium (annealed)	2.886	1.825	0.390
Antimony (pressed)	35.13	2.22	0.389
Bismuth (pressed)	129.8	82.0	0.354
Copper (annealed)	1.582	1.0	0.388
Copper (hard-drawn)	1.620	1.02	0.4
Gold (annealed)	2.039	1.29	0.365
Gold (hard-drawn)	2.075	1.31	—
Iron (annealed)	9.614	6.08	0.463
Lead (pressed)	19.42	12.6	0.387
Mercury	94.10	59.5	0.072
Nickel (annealed)	12.35	7.81	0.365
Platinum (annealed)	8.971	5.67	0.247
Silver (annealed)	1.491	0.945	0.379
Silver (hard-drawn)	1.618	1.02	—
Tantalum (hard-drawn)	*16.5	10.42	—
Tin (pressed)	13.07	8.25	0.365
Tungsten (hard-drawn)	5.50	3.92	†0.51
Tungsten (annealed)	4.44	3.16	—
Zinc (pressed)	5.568	3.52	0.365
Constantan (copper-nickel alloy) ...	44.3	28.0	practically zero.
German silver (copper-nickel-zinc alloy)	20.78	13.12	0.044
Gold silver (2 Au. + 1 Ag.)	10.77	6.8	0.065
Manganese steel	68.0	43.0	0.122
Manganin (84 Cu. + 12 Mn. + 4 Ni.)	42.0	26.0	practically zero.
Platinoid (copper-nickel-zinc alloy + 1 % W)	32.7	20.7	0.021
Platinum iridium (9 Pt. + 1 Ir.) ...	21.58	13.62	0.021
Platinum silver (2 Pt. + 1 Ag.) ...	24.21	15.3	0.031
Reostene (steel alloy)	76.7	44.5	0.104
Telegraphic siliceous bronze	1.62	1.02	—
Telephonic siliceous bronze	4.53	2.86	—
Telephonic phosphor bronze	5.95	3.78	—

* Specific resistance at 15° C. The specific resistance at the temperature of the glowing filament in a tantalum lamp is about 85 microhms per cubic centimetre.

† This value only holds for temperatures between 0° C. and 170° C. The specific resistance of tungsten at the temperature of the glowing filament is about 12.5 times that at 15° C.

I.E.E. WIRING RULES.—TABLE XI

The figures in this table apply to situations where the maximum temperature of the air does not exceed 100° F. (37.7° C.). A margin in the maximum possible temperature of the cables has been allowed to provide for contingencies. The figures in columns 3, 3*a*, 4, and 4*a* have been supplied by the National Physical Laboratory to comply with Rule 36.

Gauge.	Section.	Rubber Insulated Cables.		Paper or Fibre Insulated Cables.		Minimum Insulation Resistance of One Mile in Megohms at 60° F.			Resistance.	Minimum Radial Thickness.				Gauge.
						Vulcanized Rubber.				Vulcanized Rubber.				
Number of Wires and Gauge in S.W.G. or Inches.	Nominal Sectional Area of Conductors.	Current. Maximum Current Permissible.	Volts Drop. Approximate Total Length in Circuit (Lead and Return) for 1 Volt Drop (col. 3).	Current. Maximum Current Permissible.	Volts Drop Approximate Total Length in Circuit (Lead and Return) for 1 Volt Drop (col. 4).	Up to 250 Volts.	Up to 650 Volts.	Paper or Fibre (Class B).	Conductor Resistance in Standard Ohms per 1000 Yards.	Up to 250 Volts.	Up to 650 Volts.	Paper or Fibre (Class B).	Lead Sheath.	Number of Wires and Gauge in S.W.G. or Inches
1	2	3*	3*a*	4*	4*a*	5	6	7	8	9	10	11	12	13
	sq in.	amperes.	yards.	amperes.	yards.	megohms	megohms.	megohms.	ohms.	inches	inches.	inches.	inches.	
Fittings 3/25 Wires	.0009	3.7	10	3.7	10	1250	4500	—	26.01	.034	.062	—	—	3/25 Fittings Wires
3/24	.0011	4.5	10	4.5	10	1250	4500	—	21.50	.034	.062	—	—	3/24
3/23	.0013	5.3	10	5 3	10	1250	4500	—	18.07	.035	.062	—	—	3/23
1/18	.0018	7.2	10	7.2	10	2000	5000	—	13.29	.035	.062	—	—	1/18
3/22	.0018	7.2	10	7.2	10	1250	4500	—	13.27	.036	.062	—	—	3/22
7/25	.0022	8.6	10	7.2	10	1250	4500	—	11.12	.036	.062	—	—	7/25
3/21	.0024	9.5	10	9.5	10	1250	4500	—	10.16	.038	.062	—	—	3/21
1/17	.0025	9.8	10	9.8	10	2000	5000	—	9.761	.036	.062	—	—	1/17
7/24	.0026	10.4	10	10.4	10	1250	4500	—	9.19	.037	.062	—	—	7/24
3/20	.0030	12.0	10	12.0	10	1250	4500	—	8.0	.038	.062	—	—	3/20
7/23	.0031	12.4	10	12.4	10	1250	4500	—	7.72	.037	.062	—	—	7/23
1/16	.0032	12.9	10	12.9	10	2000	5000	—	7.47	.036	.062	—	—	1/16
3/19	.0037	14.8	10	14.8	10	1250	4500	—	6.504	.039	.062	—	—	3/19
1/15	.0041	16.3	10	16.3	10	1250	4500	—	5.905	.037	.062	—	—	1/15
7/22	.0042	17.0	10	17.0	10	1250	4500	—	5.672	.038	.062	—	—	7/22
1/14	.0050	19.0	10	20.1	10	1250	4500	—	4.783	.038	.062	—	—	1/14
3/18	.0053	20 0	11	21.2	10	1250	4500	—	4.516	.040	.062	—	—	3/18
7/21	.0055	21.0	11	22.1	10	1250	4500	—	4.343	.040	.062	—	—	7/21
7/20	.0070	24	12	28	10	900	4000	—	3.431	.041	.062	—	—	7/20
7/19	.0086	28	12	34.6	10	900	4000	—	2.779	.042	.062	—	—	7/19
7/18	.0125	34	14	50	10	900	4000	140	1.930	.044	.062	.080	.060	7/18

7/17	.017	40	17	65	10	900	4000	140	1.418	.047	.062	.080	.060	7/17
19/20	.019	43	18	69	11	750	3500	140	1.266	.048	.062	.080	.060	19/20
7/16	.022	46	19	75	11	900	3500	140	1.086	.049	.062	.080	.060	7/16
19/19	.023	47	19	76	12	750	3500	140	1.0260	.050	.062	.080	.060	19/19
7/.068"	.025	50	20	81	12	750	3500	140	.6618	.050	.062	.080	.060	7/.068"
7/15	.028	53	21	86	12	750	3500	140	.8578	.052	.062	.080	.060	7/15
19/18	.034	59	23	96	13	750	3000	120	.7125	.054	.062	.080	.060	19/18
7/14	.035	60	23	97	13	750	3500	120	.6949	.054	.062	.080	.060	7/14
19/17	.046	70	26	114	15	750	3000	120	.5234	.058	.062	.080	.060	19/17
7/.097"	.050	74	27	120	16	750	3500	120	.4727	.059	.062	.080	.060	7/.097"
19/.058"	.050	74	27	120	16	750	3000	120	.4880	.059	.062	.080	.060	19/.058"
19/16	.060	83	29	135	17	750	3000	110	.4007	.062	.066	.080	.070	19/16
19/.072"	.075	97	31	157	18	600	3000	110	.3167	.066	.066	.080	.070	19/.072"
19/14	.094	113	33	183	19	600	3000	100	.2565	.070	.071	.090	.070	19/14
19/.083"	.100	118	34	191	20	600	3000	100	.2383	.071	.071	.090	.070	19/.083"
37/16	.117	130	36	210	21	600	3000	90	.2059	.075	.076	.090	.070	37/16
19/.092"	.125	134	37	219	21	600	3000	90	.1940	.076	.076	.090	.070	19/.092"
37/.072"	.150	152	39	246	23	600	3000	90	.1627	.080	.080	.090	.080	37/.072"
19/.101"	.150	152	39	246	23	600	3000	90	.1610	.081	.080	.090	.080	19/.101"
37/14	.182	172	42	275	24	600	2500	90	.1318	.086	.087	.090	.080	37/14
37/.083"	.200	184	43	296	25	600	2500	90	.1224	.087	.087	.090	.080	37/.083"
37/.092"	.250	214	47	343	27	600	2500	80	.0997	.094	.094	.100	.090	37/.092"
37/.104"	.300	240	50	385	29	600	2500	80	.0780	.103	.101	.100	.090	37/.104"
37/.112"	.350	264	53	425	31	600	2500	80	.0672	.107	.107	.100	.090	37/.112"
61/.092"	.400	288	55	464	32	600	2500	80	.0605	.113	.113	.100	.100	61/.092"
61/.097"	.450	310	58	502	34	600	2500	80	.0544	.121	.118	.100	.100	61/.097"
61/.104"	.500	332	60	540	35	600	2500	80	.0473	.121	.121	.100	.100	61/.104"
61/.108"	.550	357	61	583	36	600	2500	80	.0439	.125	.125	.110	.110	61/.108"
61/.112"	.600	384	62	624	36	600	2500	80	.0408	.125	.125	.110	.110	61/.112"
61/.118"	.650	410	63	662	37	600	2500	80	.0368	.129	.129	.110	.110	61/.118"
91/.098"	.700	434	64	700	38	600	2500	70	.0357	.129	.129	.110	.110	91/.098"
91/.101"	.750	461	65	738	38	600	2500	70	.0336	.131	.131	.110	.110	91/.101"
91/.108"	.800	488	65	776	39	600	2500	70	.0294	.133	.133	.120	.120	91/.108"
91/.112"	.900	540	66	855	39	600	2500	70	.0273	.137	.137	.120	.120	91/.112"
91/.118"	1.000	595	67	932	40	600	2500	70	.0246	.141	.141	.130	.120	91/.118"
127/.101"	1.000	595	67	932	40	600	2500	70	.0241	.141	.141	.130	.120	127/.101"

* N.B.—It must not be assumed that this current is always permissible, especially for lighting circuits where the determining factor is the drop in volts.

TABLE XII

Resistance, Conductivity, and Weight of Hard-drawn Pure Copper Wires at 15° C. Calculated by taking the Specific Resistance of Copper at 0° C. to be 1620×10^{-9} Board of Trade Ohm, and the Conductivity at Temperature $t°$ C. $= 1 - 0.0038701\,t + 0.000009009\,t^2$.

No of Wire Gauge.	Diameter.		Area of Cross Section.		Weight. Density = 8 95.		Conductivity.		Resistance.	
	Inches.	Centimetres.	Square Inches.	Square Centimetres.	Lb. per Yard.	Grammes per Metre.	Yards per B.T. Ohm.	Metres per B.T. Ohm.	B.T. Ohms per Yard.	B.T. Ohms per Metre.
0000000	0.500	1.270	0.1964	1.2668	2.285	1134	8073	7382	0.000124	0.000135
000000	0.464	1.179	0.1691	1.0909	1.970	976.3	6952	6357	0.000144	0.000157
00000	0.432	1.097	0.1466	0.9456	1.706	846.3	6026	5510	0.000166	0.000181
0000	0.400	1.016	0.1257	0.8107	1.463	725.6	5166	4724	0.000194	0.000212
000	0.372	0.945	0.1087	0.7012	1.265	627.6	4468	4086	0.000224	0.000245
00	0.348	0.884	0.0951	0.6136	1.107	549.6	3910	3576	0.000256	0.000280
0	0.324	0.823	0.0824	0.5319	0.960	476.1	3390	3099	0.000295	0.000323
1	0.300	0.762	0.0707	0.4560	0.823	408.1	2906	2657	0.000344	0.000376
2	0.276	0.701	0.0598	0.3860	0.696	345.4	2460	2249	0.000407	0.000445
3	0.252	0.640	0.0499	0.3218	0.581	288.0	2051	1875	0.000488	0.000533
4	0.232	0.589	0.0423	0.2727	0.492	244.0	1738	1589	0.000575	0.000629
5	0.212	0.538	0.0353	0.2277	0.411	203.8	1452	1327	0.000689	0.000754
6	0.192	0.488	0.0290	0.1868	0.337	166.8	1190	1089	0.000840	0.000919
7	0.176	0.447	0.0243	0.1570	0.283	140.5	1000	914	0.00100	0.00109
8	0.160	0.406	0.0201	0.1297	0.234	116.1	827	756	0.00121	0.00132
9	0.144	0.366	0.0163	0.1051	0.190	94.0	670	612	0.00149	0.00163
10	0.128	0.335	0.0129	0.0830	0.150	74.3	529	484	0.00189	0.00207
11	0.116	0.295	0.0106	0.0682	0.123	61.0	435	397	0.00230	0.00252
12	0.104	0.264	0.00849	0.0548	0.0989	49.0	349	319	0.00286	0.00313
13	0.092	0.234	0.00665	0.0429	0.0774	38.4	273	250	0.00366	0.00400
14	0.080	0.203	0.00503	0.0324	0.0585	29.0	207	189	0.00484	0.00529
15	0.072	0.183	0.00407	0.0263	0.0474	23.5	167	153	0.00597	0.00653
16	0.064	0.163	0.00322	0.0208	0.0374	18.6	132	121	0.00756	0.00827
17	0.056	0.142	0.00246	0.0159	0.0287	14.2	101	92.6	0.00988	0.0108

18	0.048	0.122	0.00181	0.0117	0.0211	10.4	74.4	68.0	0.0134	0.0147
19	0.040	0.102	0.00126	0.00811	0.0146	7.26	51.7	47.2	0.0194	0.0212
20	0.036	0.0914	0.00102	0.00657	0.0118	5.88	41.9	38.3	0.0239	0.0261
21	0.032	0.0813	0.000804	0.00519	0.00936	4.64	33.1	30.2	0.0302	0.0331
22	0.028	0.0711	0.000616	0.00397	0.00717	3.56	25.3	23.1	0.0395	0.0432
23	0.024	0.0610	0.000452	0.00292	0.00526	2.61	18.6	17.0	0.0538	0.0588
24	0.022	0.0559	0.000380	0.00245	0.00443	2.19	15.6	14.3	0.0640	0.0700
25	0.020	0.0508	0.000314	0.00203	0.00366	1.80	12.9	11.8	0.0774	0.0847
26	0.018	0.0457	0.000254	0.00164	0.00296	1.47	10.5	9.59	0.0956	0.105
27	0.0164	0.0417	0.000211	0.00136	0.00246	1.22	8.68	7.94	0.115	0.126
28	0.0148	0.0376	0.000172	0.00111	0.00200	0.893	7.07	6.47	0.141	0.155
29	0.0136	0.0345	0.000145	0.000937	0.00169	0.839	5.97	5.46	0.167	0.183
30	0.0124	0.0315	0.000121	0.000779	0.00141	0.697	4.96	4.54	0.201	0.220
31	0.0116	0.0295	0.000106	0.000682	0.00123	0.610	4.35	3.97	0.230	0.252
32	0.0108	0.0274	0.0000916	0.000591	0.00107	0.529	3.77	3.44	0.266	0.290
33	0.0100	0.0254	0.0000785	0.000507	0.000914	0.453	3.23	2.95	0.310	0.339
34	0.0092	0.0234	0.0000665	0.000429	0.000774	0.384	2.73	2.50	0.366	0.400
35	0.0084	0.0213	0.0000554	0.000358	0.000645	0.320	2.28	2.08	0.439	0.480
36	0.0076	0.0193	0.0000454	0.000293	0.000548	0.262	1.87	1.71	0.536	0.586
37	0.0068	0.0173	0.0000363	0.000234	0.000423	0.210	1.49	1.37	0.670	0.732
38	0.0060	0.0152	0.0000283	0.000182	0.000329	0.163	1.16	1.06	0.860	0.941
39	0.0052	0.0132	0.0000212	0.000137	0.000247	0.123	0.873	0.798	1.15	1.25
40	0.0048	0.0122	0.0000181	0.000117	0.000211	0.104	0.744	0.680	1.34	1.47
41	0.0044	0.0112	0.0000152	0.0000981	0.000177	0.0878	0.625	0.572	1.60	1.75
42	0.0040	0.0102	0.0000126	0.0000811	0.000146	0.0726	0.517	0.473	1.94	2.12
43	0.0036	0.00914	0.0000102	0.0000657	0.000118	0.0588	0.419	0.383	2.39	2.61
44	0.0032	0.00813	0.00000804	0.0000519	0.0000936	0.0464	0.331	0.302	3.03	3.31
45	0.0028	0.00712	0.00000616	0.0000397	0.0000717	0.0356	0.253	0.232	3.95	4.32
46	0.0024	0.00610	0.00000452	0.0000292	0.0000527	0.0261	0.186	0.170	5.38	5.88
47	0.0020	0.00508	0.00000314	0.0000203	0.0000366	0.0181	0.129	0.118	7.74	8.47
48	0.0016	0.00406	0.00000201	0.0000130	0.0000234	0.0116	0.0827	0.0756	12.1	13.2
49	0.0012	0.00305	0.00000113	0.00000730	0.0000132	0.00653	0.0465	0.0425	21.5	23.5
50	0.0010	0.00254	0.000000785	0.00000507	0.00000914	0.00453	0.0323	0.0295	31.0	33.9

TABLE XIII

Aluminium Conductors

Size S.W.G.	Diameter.		Area of Cross Section.		Weight. Lb. per Mile.	Resistance. Ohms per Mile.
	Inches.	Centimetres.	Square Inches.	Square Centimetres.		
0000000	0.500	1.270	0.196	1.2668	1194	.353
000000	0.464	1.179	0.169	1.0909	1018	.410
00000	0.432	1.097	0.147	0.9456	891.5	.476
0000	0.400	1.016	0.126	0.8107	766.4	.554
000	0.372	0.945	0.109	0.7012	661.1	.642
00	0.348	0.884	0.0951	0.6136	578.5	.731
0	0.324	0.823	0.0824	0.5319	507.8	.840
1	0.300	0.762	0.0707	0.4560	429.9	.980
2	0.276	0.701	0.0598	0.3860	363.7	1.16
3	0.252	0.640	0.0499	0.3218	304.1	1.40
4	0.232	0.589	0.0423	0.2727	253.3	1.64
5	0.212	0.538	0.0353	0.2277	214.7	1.97
6	0.192	0.488	0.0290	0.1868	176.1	2.40
7	0.176	0.447	0.0243	0.1570	148.0	2.86
8	0.160	0.406	0.0201	0.1297	126.3	3.45
9	0.144	0.366	0.0163	0.1051	99.1	4.28
10	0.128	0.335	0.0129	0.0830	79.6	5.38
11	0.116	0.295	0.0106	0.0682	63.0	6.56
12	0.104	0.264	0.00849	0.0548	48.7	8.20
13	0.092	0.234	0.00665	0.0429	39.6	10.5
14	0.080	0.203	0.00503	0.0324	31.4	13.9
15	0.072	0.183	0.00407	0.0263	24.8	18.1
16	0.064	0.163	0.00322	0.0208	19.8	21.7
17	0.056	0.142	0.00246	0.0159	15.7	28.2
18	0.048	0.122	0.00181	0.0117	10.9	38.4
19	0.040	0.102	0.00126	0.00811	7.81	55.2
20	0.036	0.0914	0.00102	0.00657	6.20	68.1
21	0.032	0.0813	0.000804	0.00519	4.92	86.3
22	0.028	0.0711	0.000616	0.00397	3.73	113
23	0.024	0.0610	0.000452	0.00292	2.74	154
24	0.022	0.0559	0.000380	0.00245	2.45	182
25	0.020	0.0508	0.000314	0.00203	1.94	222
26	0.018	0.0457	0.000254	0.00164	1.54	272
27	0.0164	0.0417	0.000211	0.00136	1.22	328
28	0.0148	0.0376	0.000172	0.00111	1.03	403
29	0.0136	0.0345	0.000145	0.000937	0.884	478
30	0.0124	0.0315	0.000121	0.000779	0.73	576

TABLE XIV.—Flexible Conductors

Size.		Equivalent in Solid Conductor.		Current at 1000 Amperes per Square Inch.	Maximum Current Permissible (I.E.E. Rules).
		S.W.G.	Cross Section Square Inches.		
34/40	22/38	22	.0006	0.616	2.5
56/40	36/38	20	.0010	1.018	4.1
100/40	64/38	18	.0018	1.81	7.2
178/40	114/38	16	.0032	3.2	12.9
278/40	178/38	14	.0050	5.0	19.0
367/40	235/38	13	.0066	6.65	23.5
301/38	187/36	12	.0085	8.5	27.5
	220/36	—	.010	10.0	31.0
	330/36	—	.015	15	37.0
	441/36	—	.020	20	44.0
	551/36	—	.025	25	50.0

TABLE XV.—Approximate Fusing Currents (Preece)

Current in Amperes	Size of Copper Cable which Fuse will Protect.	Tin Wire.		Lead Wire.		Copper Wire.		Iron Wire.	
		Diameter, Inches.	Approximate S.W.G.	Diameter, Inches.	Approximate S.W.G.	Diameter, Inches.	Approximate S.W.G.	Diameter, Inches.	Approximate S.W.G.
1	1/22	.0072	36	.0081	35	.0021	47	.0047	40
2	3/22	.0013	31	.0128	30	.0034	43	.0074	36
3	3/21½	.0149	28	.0168	27	.0044	41	.0097	33
4	3/21	.0181	26	.0203	25	.0053	39	.0117	31
5	3/21	.0210	25	.0236	23	.0062	38	.0136	29
10	7/21⅓	.0334	21	.0375	20	.0098	33	.0216	24
15	7/20	.0437	19	.0491	18	.0129	30	.0283	32
20	7/19	.0529	17	.0595	17	.0156	28	.0343	20.5
25	7/18	.0614	16	.0690	15	.0181	26	.0398	19
30	19/21	.0694	15	.0779	14	.0205	25	.0450	18.5
35	7/17	.0769	14.5	.0864	13.5	.0227	24	.0498	18
40	7/16	.0840	13.5	.0944	13	.0248	23	.0545	17
45	19/19	.0909	13	.1021	12	.0268	22	.0589	16.5
50	19/18	.0975	12.5	.1095	11.5	.0288	22	.0632	16
60	19/18	.1101	11	.1237	10	.0325	21	.0714	15
70	19/18	.1220	10	.1371	9.5	.0360	20	.0741	14
80	19/18	.1334	9½	.1499	8.5	.0394	19	.0864	13.5
90	19/17	.1443	9	.1621	8	.0426	18.5	.0935	13
100	19/17	.1548	8½	.1739	7	.0457	18	.1003	12
120	19/16	.1748	7	.1964	6	.0516	17.5	.1133	11
140	19/16	.1937	6	.2176	5	.0572	17	.1255	10
160	19/15	.2118	5	.2379	4	.0625	16	.1372	9.5
180	19/14	.2291	4	.2573	3	.0676	15	.1484	9
200	19/14	.2457	3.5	.2760	2	.0725	15	.1592	8
250	19/13	.2851	1.5	.3203	0	.0841	13.5	.1848	6.5

The formula used is $I = \text{constant} \times d^{3/2}$ where d is the diameter of the wire in centimetres, and the constant is 405.5, 340.6, 2530, and 777.4 for tin, lead, copper, and iron respectively. When d is in inches the constants are 1642, 1379, 10244, and 3148.

TABLE XVI

Table showing the Thermo-electric E.M.F. in Microvolts for a Circuit of Lead and any Substance, one Junction being at 0° C. and the other at 100° C.

Metal.	Thermo-electric E.M.F. in Microvolts.
Bismuth	+ 9700
Nickel	+ 2460
Cobalt	+ 2200
German silver	+ 1310
Palladium	+ 800
Platinum (soft)	+ 115
Aluminium	+ 52
Mercury	+ 42
Tin	+ 15
Lead	0
Copper	− 184
Magnesium	− 186
Platinum (hard)	− 222
Silver	− 289
Gold	− 334
Zinc	− 350
Cadmium	− 480
Platinum (95 per cent), Irid. (5 per cent) ...	− 595
Antimony (wire pressed)	− 600
Steel	− 975
Iron	− 1491
Telurium	− 50200
Selenium	− 80100

NOTE.—The thermo-electric E.M.F. for a circuit of any two substances in the tables for the given temperatures can be obtained by taking the algebraic difference of the two numbers.

QUESTIONS AND ANSWERS

1. Give the Board of Trade definition of the following units:—

1. The ohm.
2. The ampere.
3. The volt.

What is the relation between these units and the corresponding C.G.S. units?

2. A conductor of length "l" cm. moves with a constant velocity "v" cm. per second in a direction perpendicular both to the field of strength "H" and to its own length. What is the E.M.F. induced in the conductor?

Hence deduce the absolute unit of E.M.F.

A conductor 1 ft. in length moves with a constant velocity of 30 ft. per second in a field of strength H = 10000 lines per square centimetre. The motion is at right angles both to the length of the conductor and to the field. What is the E.M.F. in volts induced in the conductor?

3. Define the absolute units (electromagnetic) of current, E.M.F., and resistance. Give also the Board of Trade definitions of the ampere, the volt, and the ohm, and state the relations between these units and the corresponding absolute units.

4. Define the practical unit of activity, i.e. the "watt". What relation does it bear to the absolute C.G.S. unit?

Deduce the relation between the "horse-power" and the "kilowatt".

5. A copper conductor carrying a current I is bent into the form of a circle of radius r.

Find an expression for the force along the axis on unit magnet pole placed at any point on the axis which passes through the centre of the circle and is perpendicular to the plane of the circle. Hence find the force at the centre of the circle, and define the absolute unit of current.

6. What effect has a rise of temperature on the resistance of a conductor?

Define "temperature coefficient of resistance".

What is the physical meaning of a negative temperature coefficient?

A field-magnet coil has a resistance R_{t_1} ohms at $t_1°$ C. Deduce the equation, giving the resistance R_t at $t°$ C. in terms of "R_{t_1}", "t_1", "t", and "α", where "α" is the temperature coefficient of the wire of the coil.

7. Describe a method of measuring the resistance of the armature of a D.C. machine. What precautions must be taken to ensure an accurate result?

8. What do you understand by "specific resistance"? Describe how you would obtain experimentally the specific resistance of a metal given you in the form of a length of wire.

Give details of the calculations you would have to make.

9. Describe a method of determining experimentally the temperature coefficient for a metal given you in the form of a coil of wire. Give details of the method of calculating the coefficient from your results, and deduce any equations you may use.

10. Four resistances, AB, BC, CD, and DA are connected up in the form of a square.

The positive pole of a 4-volt accumulator is connected to the point A, and the negative pole to the point C. If the potential at B is the same as that at D, find what current is taken from the cell.

The resistance of AB is 2 ohms, of CD 3 ohms, and of DA 5 ohms.

Find also the currents passing through AB and DA respectively.

11. How would you proceed to locate a fault on an underground cable? Explain fully the principle of the method you would use.

Two cables of the same cross section are laid side by side from a sub-station to a feeding-point half a mile distant.

One of these cables went to earth at some point between the sub-station and the feeding-point. The other cable was used to complete the circuit, and a Murray loop test was performed at the sub-station end. It was found that a balance was obtained when the tapping contact was 68 cm. from the end of the potentiometer wire connected to the faulty cable. Find the position of the fault if the whole length of the potentiometer wire is 300 cm.

12. State Kirchoff's Laws, and express them algebraically.

13. A primary cell is connected in series with a 5-ohm resistance, the leads having a total resistance of 0.1 ohm. The cell has an internal resistance of 2 ohms and an E.M.F. on open circuit of 1.3 volts.

Find the current flowing in the circuit and the terminal E.M.F. of the cell.

14. The field winding of a shunt motor has a resistance of 150 ohms at 15° C. The motor is started, and after running for some hours the field current is found to be 1.4 amperes. The motor is connected to a 250-volt supply.

What is the average temperature of the field coils if the temperature coefficient is .004 per degree C.?

15. What do you understand by the insulation resistance of a cable?

If the cable is uniformly insulated, how does the insulation resistance vary with the length of the cable?

Describe the Kelvin testing set, make a sketch of the apparatus, and show how you would use it to measure the insulation resistance of a cable.

16. Two resistances, r_1 and r_2, are connected in series, and a P.D. of 10 volts is maintained continuously between the ends.

It is found that the heating of the resistance r_2 is three times that of r_1.

If the resistance of r_1 is 2 ohms, find the current flowing in the circuit.

The two resistances are now connected in parallel, and the same P.D. is applied across them. Find the current through each resistance. What is now the ratio of the heat developed in r_2 to that developed in r_1?

17. When a galvanometer is being used with a shunt, and it is necessary that the total resistance of the apparatus should be the same with and without the shunt, a compensating resistance is put into the main circuit when the galvanometer is shunted. Explain fully the reason for this. A battery of

internal resistance "b"; E.M.F. "E" is connected in series with a resistance "R" to a galvanometer of resistance "g". What is the current in the main circuit? If the galvanometer is now shunted by a resistance "S", find the value of the compensating resistance which must be added to the main circuit so that the current taken from the battery is the same as before.

18. Two resistances, r_1 and r_2, are connected in parallel, and an E.M.F. E_a is applied to the combination. The resistances are then connected in series, and an E.M.F. E_b is applied. If the resistance r_1 is 2 ohms and the E.M.F. E_b has to be four and a half times as great as E_a in order that the total current may be the same in both cases, find the resistance of r_2.

19. Describe fully how you would measure—

(*a*) A high resistance.
(*b*) A very low resistance.

In each case explain carefully the principle of the method you employ, and deduce any equations you would make use of in your calculations.

20. Four resistances, AB, BC, CD, and DA are arranged to form the sides of a square.

A battery is connected to the points A and C. The resistance of AB is 2 ohms, of BC 8 ohms, and of DA 5 ohms. The battery maintains a constant P.D. between A and C of 10 volts, and the current flowing in the battery is $2\frac{1}{4}$ amperes. What is the resistance of CD?

21. Describe the construction of a mirror galvanometer. Why may the deflection on the scale be taken as directly proportional to the current passing through the galvanometer?

22. Describe carefully the Crompton potentiometer, and make simple diagrammatic sketches of the apparatus. Explain fully how the instrument is used.

23. Explain fully how you would calibrate an ammeter by means of the Crompton potentiometer.

24. How is a shunt used to alter the range of a continuous-current ammeter?

A certain ammeter has a resistance of 1.0 ohm, and the full scale deflection is obtained when a current of 0.05 ampere flows through the ammeter. Find what resistance the shunt must have so that the ammeter will read up to—

(*a*) 10 amperes.
(*b*) 100 amperes.

What do you understand by the "multiplying power" of a shunt?

25. Describe briefly the principle and construction of any three types of continuous-current ammeters in common use.

Give the relative advantages and disadvantages of each type.

26. A moving-coil millivoltmeter has a resistance of 1 ohm, and its maximum reading direct is 150 millivolts. How could you measure voltages of—

(*a*) 15 volts,
(*b*) 150 volts,

with this instrument? Find the value of the external resistance required in each instance.

27. How may the Crompton potentiometer be used to calibrate a voltmeter?

Give a complete diagram of connections, and state the precautions necessary in performing the calibration.

28. Describe briefly any ampere-hour meter in common use. Give a simple diagram of connections. Can the meter be used to measure energy instead of quantity? If so, state under what conditions it could be so used.

29. Describe briefly the various damping devices used in electrical instruments. What is their function? Explain fully the action of each type you describe.

30. Describe briefly the principle and construction of a hot-wire ammeter.

To what errors is an instrument of this type liable, and from what errors would you expect it to be free?

31. Describe briefly an instrument of the moving-coil type. What is the deflecting force if the coil is "h" cm. high, has "n" turns, moves in a uniform field of strength H, and carries a current of I amperes?

Thus show that if H is constant in the air gap, "θ", the angle of deflection of the coil, is proportional to the current passing through the coil.

32. Describe briefly the Siemens electrodynamometer.

How is it used to measure (1) current? (2) energy? Illustrate your answer with simple diagrammatic sketches.

Show that when the instrument is used as an ammeter $C \propto \sqrt{\theta}$, and that when it is used as a wattmeter $EC \propto \theta$.

33. Deduce an expression for the force at the centre of a circular loop of radius "r" carrying a current "I".

A tangent galvanometer has a coil consisting of 50 turns, the mean diameter of the coil being 20 cm. Find the current in the coil when a steady deflection of 60 degrees is obtained.

The value of H is 0.18 C.G.S. units.

34. What are the essential differences, in construction and use, between an ammeter and a voltmeter? Is it possible to use one and the same instrument to measure amperes and volts?

If so, describe the method you would employ to adapt, say, a milliammeter to measure volts.

35. Describe briefly the Kelvin balance. What is the principle of the instrument?

Show that the displacement of the weight is proportional to the square of the current.

How would you use the balance to calibrate an ammeter?

36. Describe briefly the principle and action of an electrostatic voltmeter.

What advantages and disadvantages has an instrument of this type?

37. What are the usual sources of error in "moving-iron" instruments?

Explain fully how each of these errors is minimized in practice.

38. Give a simple diagrammatic sketch of the Wheatstone Bridge, and deduce the equation connecting "unknown" resistance with the "known" and "ratio arm" resistances for zero galvanometer deflection.

Describe the essential modification in any two pieces of apparatus in common use for resistance measurement, working on the Wheatstone Bridge principle.

39. Describe the post-office box, and draw a simple diagram of the connections.

For what range of resistance measurement may it be used?

40. Describe briefly the principle of the tangent galvanometer. Why must the magnetic needle be very short? State and prove the law of the instrument $I = \frac{H}{G} \tan \theta$.

41. Describe carefully any instrument with which you are familiar which indicates directly the insulation resistance of, say, a cable under test. What is the principle upon which the instrument works? Give a simple diagrammatic sketch of the apparatus.

42. In calibrating an ammeter by means of a copper voltameter it was found that the gain plate increased in weight by 5.9 g. in 1 hr. The average reading of the ammeter under test was 4.5 amperes. What is the percentage error of the ammeter?

The electrochemical equivalent of copper (cupric) is 0.328 mg. per coulomb.

Draw a diagram of the connections you would use in performing such a calibration. What precautions are necessary to ensure an accurate result?

43. Describe the fall-of-potential method of comparing very low resistances. Why is this method better than a Wheatstone-Bridge method? How is the effect of the resistance of the connections got rid of in the Thomson Bridge?

Give a diagrammatic sketch of the Thomson Bridge.

44. Describe briefly the commercial production of a metal by a process depending solely on—

(*a*) Electrolysis.

(*b*) Chemical action brought about by electrical heating.

If 96540 coulombs of electricity liberate 1 g. of hydrogen, and if the chemical equivalent of copper (cupric) is 31.65, find the number of hours for which a current of 4000 amperes will have to flow through a solution of copper sulphate in order to deposit 1 ton of copper.

45. What quantity of water will be brought to the boiling-point in 15 min. in an electric kettle taking 4 amperes on a 250-volt circuit, if the efficiency of the kettle is 85 per cent and the initial temperature of the water is 12° C.?

Joules equivalent = 42×10^6 ergs per gramme degree Centigrade.

46. State briefly your conception of the passage of a current of electricity through an electrolyte.

Describe the chemical changes which occur in the electrolysis of copper-sulphate solution.

Give a brief description of the practical applications of the electrolytic deposition of copper.

47. State Lenz's Law. In what cases would it be simpler to employ Lenz's Law than the right-hand rule in determining the direction of the E.M.F. induced in an electric circuit? A coil consisting of a large number of turns is connected to a galvanometer. A bar magnet is brought along the axis of the coil with the N. pole towards the coil. Trace the changes in the deflection on the galvanometer as the magnet passes through the coil and moves away.

Draw a curve of deflections on a base of magnet positions.

48. What do you understand by "polarization" in a primary cell? What is polarization due to? and what means are taken to overcome it? Explain why a Leclanché cell is particularly suitable for intermittent work such as operating electric bells.

49. In what form is the energy dissipated when a current flows through a resistance?

Define Joule's equivalent. Describe a method of determining Joule's equivalent electrically, and deduce any equations you would make use of in your calculations.

50. Describe carefully any standard cell with which you are familiar. What are the advantages and disadvantages of the particular type you describe?

51. Define "electrochemical equivalent", and state Faraday's Laws of Electrolysis.

A current of 5 amperes is passed through a solution of copper sulphate for 4 hr. What weight of copper will be deposited at the cathode?

The electrochemical equivalent of copper (cupric) may be taken as 0.328 mg. per coulomb.

52. What do you understand by "lines of force"? Sketch the form and direction of the lines of magnetic force surrounding a long straight conductor carrying a current.

If a second straight conductor carrying a current be brought near to the first conductor and placed parallel to it, the form of the lines of force is changed. Make two simple sketches showing the form and direction of the lines of force when—

(*a*) The current is in the same direction in the conductors.

(*b*) The current is in the opposite direction in the conductors.

In each case state whether the conductors will attract or repel one another.

53. Describe a type of secondary cell with which you are familiar, and give the chemical reactions which take place during charging and discharging.

Draw typical E.M.F. characteristic curves for charge and discharge.

54. What external voltage must a shunt-wound D.C. generator give to charge a battery of 64 cells, each cell having a resistance of 5×10^{-3} ohms and a back E.M.F. of 2.2 volts?

The resistance of the leads is 0.01 ohm, and the charging current is 60 amperes.

55. Draw a simple diagram of connections for an automatic switch which connects a storage-cell battery to a dynamo when the E.M.F. of the dynamo is high enough to charge the cells, and disconnects them before the E.M.F. falls sufficiently to allow the cells to discharge back through the dynamo.

56. What is the effect of an excessive rate of discharge on a cell?

How does the capacity vary with discharging current? Give typical curves for normal discharge current, and twice normal discharge current, plotting terminal volts on a time base.

57. What precautions as to charging, discharging, density of electrolyte, &c., have to be taken if a battery of storage cells is to be kept in good con-

dition? What troubles are storage cells liable to? How would you recognize that a cell is not in good order, and what means would you take to get rid of the trouble?

58. Describe briefly the method of employing a storage-cell battery in parallel with a D.C. generator to maintain a constant load on the generator, when the external load is constantly varying. What difficulties would you expect to meet with in practice? Suggest methods of overcoming them.

59. A battery has a terminal E.M.F. of 2.0 volts per cell, which falls to 1.9 volts at the end of the discharge. The load consists of 500 metallic filament lamps, each of 32 candle-power, taking 1.2 watts per candle-power. The terminal voltage is kept constant at 250 volts. If the internal resistance of each cell is 5×10^{-3} ohms, find the number of cells in circuit—

(*a*) At the beginning of the discharge.
(*b*) At the end of the discharge.

60. Define the "watt-hour" and "ampere-hour" efficiencies of an accumulator.

Is the ampere-hour efficiency greater than the watt-hour efficiency? Give full reasons for your answer.

61. Describe briefly the manufacture of the plates for a storage cell. Of what duration should the first charge of a new cell be? What chemical changes take place during charging, and how does the density of the electrolyte vary?

62. Describe briefly with illustrative sketches an "open" type arc lamp. Why is a steadying resistance used in series with the lamp? What are the advantages and disadvantages of the lamp you describe?

63. The efficiency of a 110-volt tantalum lamp is 1.7 watts per candle-power. The filament is 65 cm. long, and has a diameter of 0.04 mm. The specific resistance of the filament when cold is 16.5 microhms per cubic centimetre, and the ratio $\frac{\text{resistance hot}}{\text{resistance cold}} = 5.0$.

Calculate the candle-power of the lamp.

64. How does the candle-power of an incandescent lamp vary with the applied voltage? Are carbon-filament and metallic-filament lamps for the same voltage and of the same candle-power affected to the same extent by a change in applied voltage?

Write down the equation connecting candle-power and applied voltage. Give normal values for the index for carbon-filament and metallic-filament lamps.

65. Describe briefly the principle of the enclosed arc lamp. What are its advantages and disadvantages compared with the open arc?

66. Describe briefly the manufacture of a tantalum lamp.

How does the energy taken by the lamp per candle-power vary with the life of the lamp? Plot a typical curve of watts per candle-power on a time base for a metallic-filament lamp.

67. A carbon-filament lamp costs 9*d.* and has a useful life of 800 hr. A metallic-filament lamp costs 2*s.* 9*d.* Both lamps are of 16 candle-power, the carbon taking 3.5 watts per candle-power and the metallic-filament 1.2 watts per candle-power.

Find what the useful life of the metallic-filament lamp must be, so that the total cost for each may be the same.

The cost of energy is 3*d.* per Board of Trade unit. If both lamps are used for 800 hr., what is the saving effected by the metallic-filament lamp?

68. An Osram lamp costs 2*s.* 9*d.* and a carbon lamp costs 9*d.*, the candle-power of each being 30.

Assuming that the watts per candle-power taken by each lamp remains constant, and is 1.2 watts per candle-power in the case of the Osram and 3.5 watts per candle-power in the case of the carbon lamp, find the number of hours the lamps must burn to make the total cost, light for light, the same. Energy costs 3*d.* per Board of Trade unit.

69. Describe a form of photometer for use in testing lamps giving light of a colour widely different from that of the standard lamp used. How is the colour difficulty eliminated in the apparatus you describe?

70. What do you understand by the efficiency of an electric lamp? Discuss the advantages and disadvantages of metallic-filament and carbon-filament lamps.

How does a change in terminal voltage, above or below normal, affect the candle-power of these lamps?

71. Describe the action and construction of a mercury-vapour lamp. Give illustrative sketches and describe how the lamp is started.

What advantages and disadvantages has the mercury-vapour lamp?

72. Describe a modern flame arc lamp, and make a diagrammatic sketch of the lamp.

How does the distribution of light differ from that of an ordinary open arc lamp?

Illustrate your answer by typical polar diagrams showing the candle-power at all angles.

73. What voltage is necessary to maintain the arc in an ordinary open-type arc lamp? Draw a diagram showing how this voltage is absorbed in the different parts of the lamp. What is the relative rate of consumption of the two carbons?

74. Describe the "Grease Spot" photometer, and explain how you would make a complete photometric test of a metallic-filament lamp, using a 10 candle-power Pentane standard.

Draw typical curves for a good 32-candle-power 250-volt metallic-filament lamp, showing candle-power, watts per candle-power, and current on a voltage base.

75. Calculate the size of leads necessary to transmit 50 horse-power to a 500-volt motor 1 mile away, the loss in transmission to be 5 per cent.

76. A dynamo has to light a building by 150 32-candle-power lamps, each taking $1\frac{1}{2}$ watts per candle.

If the resistance of the leads is .015 ohm, and the total current taken is 72 amperes, find the terminal potential difference at the lamps and at the dynamo.

77. Describe briefly the "three-wire" system of distribution. Why is it necessary to use a balancer on such a system?

Draw a simple diagram showing the arrangement of generator, balancer, and load for a simple three-wire system.

78. A cable delivers 10 kilowatts to a point 100 yd. from the dynamo. The cross section of each core is .01 sq. in.

If the loss in transmission is 2 per cent, what is the pressure at the receiving end?

The resistance of an inch cube of copper is 0.66 microhm.

79. The cross section of each cable of a two-wire distribution is 0.4 sq. in.

The generating station supplies 110 kilowatts. The cable is 1 ml. long. What is the power delivered at the receiving end? and what is the pressure across the terminals there if the pressure at the station end is 550 volts?

80. A house is illuminated by 50 16-candle-power lamps. The supply voltage is 110, and each lamp takes 1.7 watts per candle-power. The leads from the generating plant to the house are 100 yd. long.

What must the cross section of the leads be if the drop is not to exceed 2 volts?

Take the specific resistance of copper as 0.66 microhm per inch cube.

81. The leads to a motor are 500 yd. long, and have a cross section of 0.08 sq. in.

The voltage at the feeding-point is 250 volts. At the moment of switching on the motor the reading of a voltmeter placed across the leads near the motor falls to 226 volts, and gradually rises to 235 volts as the motor comes up to speed. Find the current taken by the motor—

(*a*) When starting.
(*b*) When running at full speed.

Take the specific resistance of copper as 0.66 microhm per inch cube.

82. If the resistance of an inch cube of copper is 0.66 microhm, and the diameter of each of a pair of wires be 0.064 in., what total length of circuit, go and return, can be used so that the drop shall be 2 per cent when the pressure between the wires at one end is 200 volts and the current flowing through lamps connected between the wires at the other end is 2 amperes?

83. A 40-horse-power motor is fed from a point 100 yd. away. The P.D. at the motor terminals is 500 volts, and the motor has an efficiency of 90 per cent. If the leads from the feeding-point to the motor have a uniform cross section of 0.066 sq. in., find the P.D. at the feeding-point and the loss in watts in the leads.

What per cent of the power taken from the mains at the feeding-point is converted into useful work at the motor?

The specific resistance of copper may be taken as 0.66 microhm per inch cube.

84. In a water-power plant the dynamo which produces a fixed P.D. between the terminals of 120 volts is 300 yd. from the house. The usual load consists of 200 100-volt 35-watt glow lamps in parallel. What size of leads should be employed if the resistance of a cubic inch of copper be 0.66 microhm?

85. Two circuits, A and B, are connected in parallel to the mains at a point C 1000 yd. distant from the generator.

The length of circuit A is 250 yd.; it carries a current of 50 amperes, and has a cross section of 0.05 sq. in. Circuit B is 100 yd. long; it carries a current of 100 amperes, and has a cross section of 0.1 sq. in.

The P.D. at the generator terminals is 500 volts, and the cross section of the mains is 0.15 sq. in.

Assuming the specific resistance of the copper of the cables to be 0.66 microhm per inch cube, find the P.D. between the leads at the far end of—

(*a*) Circuit A.
(*b*) Circuit B.

86. A continuous-current generator is connected to mains $\frac{1}{2}$ ml. long. The cross section of the mains is 0.092 sq. in., and is uniform for the whole length. A lighting circuit A is connected to the extreme end of the mains. This circuit is 100 yd. long, and the load consists of 200 240-volt 56-watt lamps. Another lighting circuit B is connected to the mains at a point 400 yd. from the generator terminals. The length of this circuit is 100 yd., and the load consists of 100 250-volt 56-watt lamps.

Find the cross section of the leads in each circuit so that the voltage across the lamps may be of the correct amount at full load.

The voltage at the generator terminals is 270. Take the specific resistance of the copper in the cables as 0.7 microhms per inch cube.

87. The ampere-turns per pole in a 4-pole dynamo are 1820. If the field coils are in series with one another, and are connected as a shunt across the dynamo terminals, find the size of wire for the coils. The length of a mean turn is 63 cm., and the terminal E.M.F. of the dynamo is 200 volts. The specific resistance of copper is 1.7 microhm per centimetre cube.

88. An iron ring has a mean diameter of 40 cm. The cross section is circular throughout, and has an area of 8 sq. cm. for one-half of the ring and an area of 4 sq. cm. for the other half. Calculate the number of turns with which the ring must be wound in order that a flux of 32000 lines may be produced in the core when a current of 5 amperes flows in the winding. The material of the ring is the same throughout, and has a permeability of 2600 at a flux density of 4000 lines per square centimetre, and 1630 at a flux density of 8000 lines per square centimetre. Calculate also the number of turns required when a gap 1 mm. in width has been made in the ring at a part which has a cross section of 8 sq. cm.

89. Deduce an expression for the force exerted on a unit pole placed at the centre of a long solenoid when a current I is flowing in the solenoid.

Explain why a plunger of soft iron is drawn into such a solenoid when the current is switched on.

90. Define the term "hysteresis".

What are the iron losses of a dynamo? and how are they produced? How may these losses be reduced?

A mass of iron is made to revolve for 30 min. in a field so that the periodicity is 50 times per second.

The strength of the field is 8500 lines per square centimetre, and at this flux density the loss in ergs per cubic centimetre per cycle is 9000. Specific heat of iron, 0.11; specific gravity, 7.8; heat lost by radiation, 25 per cent of that generated. Find rise of temperature of the iron.

91. What is a "B.H." curve? Describe how you would determine experimentally the B.H. curve for a sample of iron given you in the form of a long rod. Give full details of the calculations you would have to make.

92. Draw typical B.H. curves for—

(*a*) Wrought iron.
(*b*) Hard steel.

State how you would determine from your curves the permeability of the material at different flux densities. What is meant by "coercive force" and "remanent magnetism"?

Illustrate your answer from your curves.

93. A ring 10 cm. internal diameter and 15 cm. external diameter is built up of sheet-iron stampings. The ring is wound with 500 turns of wire. It is found that when a current of 0.8 ampere is flowing in the coil, the flux density in the core is 12,200 lines per square centimetre. What is the permeability of the iron? If the cross section of the ring is rectangular, what is the depth of the ring, assuming that 90 per cent of the total area is iron area? The total flux is 82,300 lines.

94. Calculate the ampere turns required per magnetic circuit to produce a flux of 7.8 megalines per pole for the following 6-pole machine:—

Armature.—External diameter of core = 117 cm.; internal diameter of core = 77 cm. Gross length of core = 30 cm. There are four ducts each 1.25 cm. broad. Length of magnetic path in core = 68 cm. There are 150 slots each 30 mm. × 14.2 mm.

Air gap.—Length of single gap = 6 mm.

Poles.—Cast steel; pole arc ÷ pole pitch = 0.7; diameter = 30 cm.; length of magnetic path = 32.5 cm.; leakage coefficient = 1.18.

Yoke.—Cast steel; area = 400 sq. cm.; length of path = 100 cm.

95. A 250-kilowatt 6-pole D.C. generator runs at 375 R.P.M. and gives a P.D. of 440 volts on O.C. The armature is lap wound, and there are 918 armature conductors wound in 115 slots. The slots are radial and 4.5 cm. deep, and are 8.7 mm. wide at the bottom.

The armature is 31 cm. in gross axial length, with four air ducts each 1 cm. wide. The external diameter of the armature is 86 cm., and its internal diameter is 46 cm. It is built up of sheet-iron disks. Assume that 90 per cent of the total area is iron area.

The poles are of steel, and have a square section 24.25 cm. by 24.25 cm.

The pole shoe arc = 0.68 τ where τ = pole pitch. The axial length of the pole shoe is the same as that of the armature.

The air gap is 5 mm. in width.

The yoke is of cast steel, and has a cross section of 340 sq. cm.

The mean lengths of the paths in the various parts are as follows:—

Armature core ...	25 cm.	for one complete magnetic circuit.
Yoke	81 ,,	
Pole core ...	68 ,,	

Assume that 87 per cent of the pole flux crosses the air gap.

Calculate the number of ampere-turns per pole required to give 440 volts on O.C., and tabulate your results.

96. Define magneto-motive force, and state the law of the magnetic circuit.

Show how to calculate the ampere-turns per pole required to produce

a given flux in the armature of a dynamo. You are given the cross section and permeability of each part and the length of the path of the magnetic lines.

97. Explain carefully how you would find graphically the current with which a given shunt dynamo will charge a battery of storage cells, the characteristic curve of the dynamo, the E.M.F. of the battery, the resistance of the battery and the resistance of the leads being given you.

98. What do you understand by the magnetization curve of a dynamo?

Describe a method of determining the curve experimentally, explaining fully any calculations used in reducing your results to the required form.

99. Discuss the reasons for shifting the brushes on a D.C. generator as the load increases. How is the necessity for shifting the brushes obviated in modern machines?

100. Give a concise description of two methods of measuring the change of potential from segment to segment round the commutator of a 2-pole dynamo.

Draw a typical curve for a well-designed machine.

101. What are the relative advantages and disadvantages of ring and drum windings? Give a simple diagrammatic sketch of each winding.

102. Describe with the aid of sketches the differences between lap and wave winding.

Give the relations holding between the number of surface conductors, the number of pairs of poles, and the back and front pitches for each winding.

103. What do you understand by the characteristic curve of a dynamo?

Sketch typical characteristic curves for—

(*a*) a shunt dynamo,
(*b*) a series dynamo,
(*c*) a compound dynamo,

and discuss briefly the reasons for the difference in shape of the various curves.

104. Enumerate the losses in a D.C. generator. Describe how you would determine the various losses experimentally, and state how each loss varies with the load on the machine.

105. Describe the various methods of exciting the field of a D.C. generator.

Give sketches of the field connections for—

(*a*) A shunt machine.
(*b*) A series machine.
(*c*) A long-shunt compound machine.
(*d*) A short-shunt compound machine.

106. What conditions must be fulfilled before a D.C. generator can be switched into parallel with generators already on the bus-bars? What relation has the share of the load each machine will take, as the load varies, to the shapes of the characteristic curves of the various machines?

107. What conditions determine whether a dynamo will excite or not?

How are these affected by speed and external resistance in the case of—

(*a*) A shunt dynamo?
(*b*) A series dynamo?

108. What is the function of the commutator in D.C. machinery?

With the aid of a simple diagrammatic sketch, explain fully how commutation is effected. Give a brief description of the construction of a modern commutator.

109. Describe fully how you would draw the "total characteristic" of a compound-wound dynamo, assuming that you have already obtained the "external characteristic" experimentally. What other measurements would you require to make?

Indicate briefly the methods you would employ.

110. Explain fully why series and compound dynamos are unsuitable for charging accumulators.

111. What are the principal causes of sparking at the brushes in D.C. machines?

Indicate briefly the various methods employed in practice (including the use of commutating poles) to prevent sparking.

112. How many brush sets are necessary for a machine having a simple wave-wound armature? Could more sets be used?

Draw a development of a wave winding for a 4-pole machine, having 34 armature conductors. Show the positions of the poles and brushes, and indicate the direction of the current in the conductors.

113. Describe a method of running two compound-wound D.C. generators in parallel. Draw a diagram of the connections, and state what precautions are necessary in switching the one machine into parallel with the other. In what order must the various switches be closed?

114. What are the essential features of a simple lap winding? Is a lap winding more suitable for a machine of large output or for one of small output?

Draw a development of a simple lap winding for a 4-pole machine, having 36 armature conductors. Show the positions of the poles and brushes, and indicate the direction of the current in each conductor. What is the function of equalizing rings? How are they connected? Could equalizing rings be used with the winding you have drawn?

115. What effect has the current in the armature of a D.C. generator on the distribution of the magnetic flux in the air gap? What are the cross ampere-turns and back ampere-turns of the armature?

Give diagrammatic sketches of—

(*a*) the flux distribution due to the ampere-turns on the magnet poles alone,

(*b*) the flux distribution due to the ampere-turns of the armature alone,

(*c*) the resultant flux distribution for the loaded machine,

for a 2-pole dynamo.

116. Establish the equation $E = NZn\ 10^{-8}$ for a 2-pole dynamo.

What modification has to be made in this formula in the case of a multipolar dynamo—

(*a*) Lap wound?

(*b*) Wave wound?

A D.C. generator has a lap-wound armature. There are 252 slots with 6 conductors per slot.

If the generator has 6 poles, and the flux per pole linked with the armature conductors is 4.0×10^6 lines, what is the E.M.F. induced in the armature when it is running at 500 R.P.M.?

117. What are "eddy currents"? How may the loss due to eddy currents in the armature core of a machine be minimized?

Describe carefully how the eddy currents in a disk of metal moving between the poles of a magnet may be utilized to provide the necessary damping effect in a measuring instrument.

118. Upon what does the magnitude of the E.M.F. induced in a coil of wire by a change in the number of magnetic lines of force interlinked with the coil depend?

A 2-pole dynamo has a ring armature. A coil of wire is wound on to the armature midway between the poles. The ends of this coil are connected to a sensitive mirror galvanometer. If the armature be suddenly turned through a small angle, would you expect any perceptible deflection on the galvanometer? If the current in the field coils be suddenly reversed (the coil on the armature being midway between the poles), would you expect the galvanometer deflection to be greater or less than in the former case?

Give reasons for your answer.

119. Describe briefly the "Ward Leonard" system of control as applied to, say, a large motor which has to be frequently started and stopped. What are the advantages of the system as regards—

(*a*) Starting?
(*b*) Speed control?

120. Describe a starter for a shunt motor, and give a clear diagrammatic sketch of the connections, including those of the overload and no-voltage releases.

121. The speed of a motor is observed gradually to increase for some time after the motor is started. Show that this effect could be produced by the gradual heating of the field-magnet coils. The current flowing in the field coils of a motor running on a 250-volt supply is 2 amperes when the field coils are at a temperature of 15° C.

What will be the current flowing in the field coils when they are at a temperature of 30° C.?

The temperature coefficient of copper may be taken as .00428 ohm per degree Centigrade.

122. What changes take place in the speed, as the load is increased, in the case of—

(*a*) A series motor?
(*b*) A shunt motor?
(*c*) A compound motor (cumulative)?
(*d*) A compound motor (differential)?

Why is it dangerous to throw the load off a series motor, and to load a differential compound motor beyond a certain maximum?

123. A motor is required to drive a train of 80 tons weight along a level

track at 40 ml. per hour. The tractive force is 15 lb. per ton of dead-weight.

If the supply voltage is 550 volts, and the efficiency of the motor and gearing is 70 per cent, find the current taken by the motor.

Find also what current will be taken by the motor when the train is running up an incline of 1 in 120 at a speed of 20 ml. an hour.

124. What type of D.C. motor is most suitable for tramway work? State clearly the characteristics required in a motor for traction purposes, and state to what extent they are realized in the type of motor you recommend.

125. A D.C. machine is used as a generator, and when viewed from the commutator end the armature is seen to be rotating clockwise. If the connections are not altered in any way, and current is now taken from the bus-bars to run the machine as a motor, what will be its direction of rotation if the machine is—

(*a*) Shunt wound?
(*b*) Series wound?
(*c*) Compound wound?

Make simple diagrammatic sketches showing the direction of the currents in armature and field windings, and the direction of rotation for each case.

126. A 10-horse-power motor takes a current of 17 amperes when running at full load on a 500-volt circuit. The resistance of the field coils is 811 ohms, and of the armature 1.53 ohms. Calculate the sum of the iron, friction, and windage losses of the motor.

127. A constant P.D. of 500 volts is maintained at the terminals of a 40-horse-power motor.

The generator driving the motor is 100 yd. away, and the leads to the motor have a cross section of 0.07 sq. in.

The efficiency of the motor is 90 per cent.

What is the output of the generator when the motor is running on full load?

The specific resistance of the material of the leads is 0.66 microhm per inch cube.

128. Upon what does the torque of a D.C. motor depend? Explain carefully why the starting torque of a series motor is greater than that of a shunt motor. Also explain in detail why the series turns in a differential compound motor are cut out during starting.

129. What easily variable factors determine the speed at which a continuous-current motor will run? Describe two methods of speed control in practical use, in both of which the speed of the motor is approximately the same at full load and at no load.

130. How may the direction of rotation of a motor be changed? Describe, with the aid of sketches, how you would change the direction of rotation of—

(*a*) A shunt motor.
(*b*) A series motor.
(*c*) A compound motor.

Indicate the direction of the current in the armature and field windings in each case.

131. A 10-horse-power motor runs at 900 R.P.M., and takes a current of 17.5 amperes when running on a 500-volt supply.

What is the efficiency and what is the pull in pounds weight at the rim of the pulley if the pulley is 9 in. in diameter?

132. What do you understand by "the R.M.S. value of an alternating current", and "the phase difference between two alternating currents"? Evolve the relationship between the R.M.S. value and the maximum value of a sine wave alternating current.

133. Plot sine wave values of current and voltage showing the phase relationship in a single-phase circuit in which the voltmeter reading is 120, the ammeter reading is 65, and the power factor is 0.85.

134. Assuming that an alternating E.M.F. follows the simple harmonic law, obtain expressions for the average value and the effective or root-mean-square value in terms of the maximum value. Hence define the term "form factor" and obtain its value in the above case.

135. Plot the waves—

$$i = 14.14 \sin (2\pi\ 50t)$$

$$e = 7.07 \sin \left(2\pi\ 50t + \frac{\pi}{4}\right)$$

and draw the vector diagram for the current and E.M.F.

136. Show that the total energy of the magnetic field in which there are two circuits carrying currents I_1 and I_2 may be put in the form

$$\tfrac{1}{2}L_1I_1^2 + M.I_1I_2 + \tfrac{1}{2}L_2I_2^2.$$

Explain the meaning of the terms L_1, L_2, and M.

137. Define "inductance of a circuit", and "mutual inductance of two circuits".

The inductance of a circuit is found to be 8 millihenrys. Find the energy in foot-pounds stored up in the magnetic field when the current in the circuit is 5 amperes.

138. What voltage at a frequency of 25 cycles per second must be applied to a circuit of negligible resistance and inductance 0.5 henry to produce a current of 7 amperes?

139. In a certain circuit having an inductance of $\frac{1}{6}$ henry the voltmeter reading is 220 and the ammeter reading is 3.5. What is the frequency?

140. An E.M.F. of R.M.S. value 150 volts is applied to an inductance of 0.3 henry. What will be the R.M.S. value of the current if the frequency is 100 cycles per second?

141. Find the inductance of a coil of negligible resistance if an alternating E.M.F. of 200 volts at a frequency of 50 cycles per second produces through it a current of 1.5 amperes.

142. A P.D. $v = 141.4 \sin(2\pi ft)$ is maintained at the terminals of a coil having an inductance 0.0318 henry and a resistance 10 ohms. The frequency is varied from 0 to 100, the maximum value of v being kept constant at 141.4. Plot on a frequency base

(*a*) Reactance of the coil.
(*b*) Impedance of the coil.
(*c*) Effective value of the current.
(*d*) Phase angle between current and P.D.

143. A circuit has a resistance of 300 ohms. Calculate the inductance if the current lags 35° behind the E.M.F. at a frequency of 50 cycles per second.

144. A non-inductive circuit has a resistance of 10 ohms. How much inductance must be added to the circuit in order that the R.M.S. value of the current may not exceed 2 amperes when a simple harmonic E.M.F. of R.M.S. value 80 volts with a frequency of 50 cycles per second is applied to the circuit?

145. What is meant by "impedance"? If the self-induction of a circuit is 0.1 henry and its resistance 10 ohms, at what frequency will an increase of 1 per cent on the self-induction be more important than a diminution of 50 per cent in the resistance? Prove that when very high-frequency alternating currents are employed, the self-induction of even a straight piece of wire is more important than its resistance.

146. An alternator is supplying energy to an inductive circuit. Write down in your own words the fundamental principles from which you are able to state an equation, the mathematical solution of which gives you the current.

A circuit contains in series an unknown resistance and an inductance of 0.5 henry. An E.M.F. of R.M.S. value 100 volts at a frequency of 70 cycles per second is applied to the terminals and a current of 0.4 ampere is found to flow. What is the value of the resistance?

147. An alternator consisting of an armature having 10 coils and a field magnet with 10 pairs of poles (a N. pole on one side of the field facing a S. pole on the other), is running at 1000 revolutions per minute. If the potential difference between the armature terminals is 100 volts, and the outside circuit consists of a solenoid having a resistance of 5 ohms and an inductance of 0.01 henry, what is the current?

148. A certain coil of wire has an inductance of 0.3 henry. If a steady current of 3 amperes is flowing in the coil, and is brought to zero at a uniform rate in 0.02 second, what will be the induced E.M.F.?

149. A circuit of resistance 8 ohms, and inductance 0.01 henry, has applied to it a simple harmonic E.M.F. at a frequency of 60 cycles per second, and a current of 16.97 amperes is found to pass through the mains. Calculate the value of the applied E.M.F. and the phase angle of the current.

150. An inductive circuit of 4 ohms resistance and inductance 0.02 henry has applied to it a virtual alternating voltage of 100 at a frequency of 50 cycles per second. Calculate the virtual current and its angle of lag. Calculate also the activity of the circuit: (*a*) in watts, (*b*) in horse-power.

151. An inductance of 0.02 henry is connected in series with a non-inductive resistance of 20 ohms, and the two are connected to an A.C. supply at 150 volts and 50 cycles per second. Calculate the voltage across each portion of the circuit (assuming the resistance of the inductance to be negligible), the current, and the power factor of the circuit.

If the frequency is 100 what will be the values of the above quantities?

152. A choking coil having a negligible resistance passes a current of 6 amperes when connected to a supply voltage of 120 at 25 cycles per second, and a non-inductive resistance connected independently to the same supply passes 12 amperes. Find the current and power factor when the two are connected in series to a 200-volt supply at 50 cycles per second.

153. For what frequency will a current of 12 amperes be taken from a 250-volt supply by a circuit having a resistance of 13.7 ohms and an inductance of 0.05 henry?

154. A choking coil has a resistance of 15 ohms, and when it is connected to a 100-volt, 50-cycle circuit the current taken is 2.5 amperes. If the coil has 2000 turns of wire, what flux is produced by a steady current of 3 amperes?

155. The back E.M.F. of a choking coil is 30 volts when carrying 10 amperes at 50 cycles per second. Given that the maximum flux density in the core is 5000 lines per square centimetre, and the area is 15 sq. cm., find the number of turns on the coil and the length of the air-gap. Neglect the ampere-turns required for the iron portion of the circuit, and take the area of the air-gap as equal to the area of the core.

156. Calculate the capacity current taken by an unloaded concentric cable 10 miles long, subjected to a sinusoidal alternating E.M.F. of 10,000 volts at 50 cycles per second.

The capacity of the cable is 0.2 microfarad per mile.

157. A circuit having a capacity of 0.25 microfarad is connected to supply mains having a P.D. of 150 volts at 50 cycles per second. Calculate the current taken from the mains.

158. What is the capacity per mile of a concentric cable which when connected to a 20,000-volt supply at 25 cycles per second takes a current of 15 amperes? The length of the cable is 22 miles.

159. At what supply voltage will a current of 0.55 ampere flow in a circuit having a condenser of capacity $3\frac{1}{8}$ microfarads, if the frequency is 80 cycles per second?

160. A condenser having a capacity of 5 microfarads was connected across 200-volt A.C. mains, and a current of 0.44 ampere was indicated by an ammeter in the circuit. What was the frequency of the supply?

161. A 120-volt, 50-watt lamp is to be used on mains having a P.D. of 250 volts at frequency 100. What condenser capacity will have to be used, and what will be the power factor when the lamp is lighted?

162. A cable having a capacity of 15 microfarads has applied to it for testing purposes an alternating E.M.F. of 6500 volts at frequency 50 through a resistance of 150 ohms. What current will be taken from the mains and what will be the angle of phase difference between current and E.M.F.?

163. What must be the frequency of a 5000-volt supply if the current taken from the mains by a 1.33-microfarad condenser connected in series with a non-inductive resistance of 3000 ohms is 1 ampere?

164. A condenser of 14 microfarads capacity is to be connected in series with a resistance to 500-volt, single-phase A.C. mains in which the frequency is 50 cycles per second. If the current is to be limited to 2 amperes, what must be the value of the resistance?

165. A circuit consists of a condenser of capacity 8 microfarads connected in series with a non-inductive resistance of 18 ohms. What voltage at a frequency of 100 cycles per second must be applied to the circuit in order that an ammeter in the circuit may read 1.13 amperes?

166. Describe in general terms the effect of putting in an alternating-current circuit

(*a*) an inductance.
(*b*) a condenser.
(*c*) an inductance and a condenser in series with one another.

167. Find the resistance, reactance, impedance, and power-factor of a single-phase circuit in which the voltmeter reading is 1520, the ammeter reading is 150, and the wattmeter reading is 156 kilowatts.

168. Define "impedance", and "reactance" as applied to A.C. circuits, and draw a vector diagram showing the phase relation between E.M.F. and current in a circuit of which the impedance is 5 ohms, and reactance 4 ohms. What is the resistance of the circuit?

169. A circuit supplied from an alternator takes a current of 10 amperes at a pressure of 100 volts, its resistance being 8 ohms. Find the lag of the current; and the capacity in microfarads of the condenser which must be used in series with the circuit in order that the current may be 12.5 amperes and not lag at all. The frequency is 50 cycles per second.

170. An alternating E.M.F. of 500 volts and frequency 50 is applied to a circuit of resistance 20 ohms, inductance 0.05 henry, and capacity 200 microfarads. Calculate (*a*) the current in the circuit, (*b*) the angle of phase difference between current and E.M.F. Deduce the formulæ you use with these calculations.

171. A choking coil of resistance 6 ohms and inductance 0.2 henry is connected in series with a condenser of capacity 4 microfarads. Find the voltage that must be applied to the circuit at frequency 70 if the current is to be 7 amperes. Find also the angle of phase difference between current and voltage.

172. A choking coil has an inductance of 1.4 henrys and is connected in series with a 7-microfarad condenser to 440-volt, 50-cycle mains from which the current taken is 18 amperes. What is the resistance of the coil and the power factor of the circuit?

173. Calculate the inductance of a choking coil of resistance 10 ohms if when it is connected in series with a condenser of capacity of 7 microfarads to a 500-volt supply at 50 cycles, the current taken is 16.05 amperes.

174. Find the frequency for which a current of 0.95 ampere will be taken from a 250-volt supply by a circuit consisting of an inductance of 1.4 henrys, a non-inductive resistance of 20 ohms, and a condenser of capacity 10 microfarads, all in series. Find also the voltage across each portion of the circuit.

175. An inductance of 0.042 henry is connected in series with a resistance of 0.5 ohm, and a capacity of 2 microfarads to an A.C. supply at 220 volts and 550 cycles. Calculate the voltage across each portion of the circuit.

176. A set of 120-volt lamps taking 6 amperes is supplied from 250-volt A.C. mains. What must be the reactance and impedance of a choking coil of which the resistance is 8 ohms if such is to be used in series with the lamps? The frequency is 100 cycles per second.

What is the power-factor of the circuit, and how does the power wasted compare with that of a purely non-inductive circuit employed in a similar manner?

177. An alternator is supplying an inductive circuit. Describe what alteration takes place in the current in the main circuit if the inductive circuit

is shunted with a condenser of which the capacity is increasing. Discuss also the effect on the lag and lead of the current.

178. A circuit contains an inductance of 0.25 henry and a resistance of 3 ohms. Find the current flowing when an E.M.F. of 400 volts at 50 cycles per second is applied. What capacity placed in parallel will relieve the main of the wattless component of the current?

179. In the above question calculate the capacity when the resistance has values 10, 100, 500, and 1000 ohms, and draw a curve showing how the capacity must vary with resistance to maintain the resonance condition.

180. You are given the values of certain impedances and their respective phase angles. Describe how you would obtain graphically the resultant impedance and the resultant phase angle when they are connected (*a*) in series, (*b*) in parallel.

181. A circuit consists of two impedances connected in series, and the values are as follows: -

$R_1 = 4$ ohms,	$R_2 = 12$ ohms,
$L_1 = 0.03$ henry,	$L_2 = 0.2$ henry,
$C_1 = 2$ microfarads,	$C_2 = 0.7$ microfarad,

these components also being connected in series. Find the total impedance of the circuit and the current that will flow when an E.M.F. of 440 volts at 50 cycles per second is applied.

ANSWERS

2. 2.79 volts.
10. $1\frac{3}{4}$ amperes, $1\frac{1}{4}$ amperes, $\frac{1}{2}$ ampere.
11. 1197 ft. from sub-station.
13. 0.183 ampere, 0.934 volt.
14. 65.5° C.
16. 1.25, 5, $\frac{5}{3}$, $\frac{1}{3}$.
17. $I = \frac{E}{R+b+g}$, $g - \frac{gS}{S+g}$.
18. 1 ohm, 4 ohms.
20. 3 ohms.
24. (*a*) .00503 ohm, (*b*) .00051 ohm.
26. (*a*) 99 ohms, (*b*) 999 ohms.
33. .0995 ampere.
42. 9.95 per cent.
44. 215 hr.
45. 2.07 kg.
51. 23.6 g.
54. 160.7 volts approximately.
59. (*a*) 155 cells; (*b*) 165 cells.
63. 16 candle-power.
67. 270 hr., 5*s*. $4\frac{1}{2}d$.
68. 116 hr.
75. 0.25 sq. in.
76. 100 and 101.08.
78. 483 volts approximately.
79. 101.64 kilowatts, 508.2 volts.
80. .03 sq. in.
81. (*a*) 80.7 amperes; (*b*) 50.5 amperes.
82. 270 yd.
83. 504.77 volts, 316 watts, 89.2 per cent.
84. .0499 sq. in.
85. A, 440.6 volts. B, 447.8 volts.
86. A, 0.0225 sq. in. B, 0.0908 sq. in.
87. 0.003897 sq. cm.
88. 66 turns, 130 turns.
90. 16.9° C.
93. 953, 3 cm.
94. 9032 ampere-turns.
95.

Part.	Flux.	Area	Flux Density.	*l*.	Ampere-turns per Centimetre.	Ampere-turns per Pole Pair.
		Sq. cm.				
Arm core ...	7.68×10^6	376	10200	25	6.5	163
Teeth top ...	7.68×10^6	435	18600	9	110.0	990
Teeth middle		413				
Teeth bottom		390				
Air gap ...	7.68×10^6	930	8270	1.0	6616	6616
Pole core ...	8.84×10^6	589	15000	68.0	16.2	1005
Yoke ...	8.84×10^6	340	13000	81.0	8.0	648
						9422

116. 504 volts.
121. 132.5 ohms.
123. 248 amperes, 278 amperes.
126. 321 watts.
127. 33.4 kilowatts.
131. 85.25 per cent, 155.8 lb. weight.

137. 0.0738 ft.-lb.
138. 550 volts.
139. 60 cycles per second.
140. 0.795 ampere.
141. 0.424 henry.
143. 0.668 henry.
144. 1.23 henrys.
145. 97.5 cycles per second, and over.
146. 118.74 ohms.
147. 8.62 amperes.
148. 45 volts.
149. 150 volts; 25° 11′ lagging.
150. 13.42 amperes; 57° 32′; (*a*) 720.52 watts; (*b*) 0.966 horse-power.
151. E_R = 143 volts; E_L = 44.97 volts; 7.15 amperes; 0.9537 lagging; E_R = 127 volts; E_L = 79.82 volts; 6.35 amperes; 0.846 lagging.
152. 4.47 amperes; 0.447 lagging.
153. 50 cycles per second.
154. 19091 lines.
155. 127 turns; 0.32 cm.
156. 6.286 amperes.
157. 11.79 milliamperes.
158. 0.217 microfarad.
159. 350 volts.
160. 70 cycles per second.
161. 3.22 microfarads; 0.503 leading.
162. 25.02 amperes; 54° 44′ leading.
163. 120 cycles per second.
164. 104.15 ohms.
165. 226 volts.
167. 7.6 ohms; 6.7 ohms; 10.13 ohms; 0.75.
168. 3 ohms.
169. 36° 52′; 533.3 microfarads.
170. (*a*) 25 amperes; (*b*) 0° 33′ leading.
171. 3361.54 volts; 85° 42′ leading.
172. 19.64 ohms; 0.8035 leading.
173. 0.735 henry.
174. 60 or 30 cycles per second; E_R = 19 volts; E_L = 501.8 or 250.9 volts; E_C = 251.9 or 503.8 volts.
175. E_R = 144.735 volts; E_L = 42031.04 volts; E_C = 41865.5 volts.
176. x = 13.83 ohms; z = 15.78 ohms; 0.897 lagging; for inductive circuit 288 watts; for non-inductive circuit 780 watts.
178. 5.087 amperes; 40.43 microfarads.
179. 39.83 microfarads; 15.42 microfarads; 0.954 microfarad; 0.248 microfarad.
181. 6125.79 ohms; 0.072 amperes.

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