MATHEMATICS

OF

Air and Marine

NAVIGATION

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PREFACE

The purpose of this text is twofold — to provide the future navigator with a substantial foundation of mathematical theory, and to acquaint the interested layman with some of the problems encountered in navigating aircraft and surface vessels. It may be successfully studied by anyone who has completed the usual high school course in plane trigonometry.

The treatment is unusually concrete and direct. Numerous diagrams make the explanations easy to follow. Although the essential parts of spherical trigonometry have been carefully developed, the student has not been burdened with unnecessary formulas nor with all the cases of triangle solution. Attention is called to the use of haversines, which simplify both formulas and computations.

Five-place tables have been provided for the solution of numerical problems. Since the data are generally given to four significant figures and the nearest minute of angle, no interpolation is required except with the haversine table. Many of the problems are solved by graphical construction; with these not more than three-place accuracy can be expected. A few problems require special charts or reference to the *Nautical Almanac* for their solution.

References to other books and to maps and charts are given at the end of each chapter. The following books are mentioned in the bibliographies by author's name only:

Dutton, Benjamin, *Navigation and Nautical Astronomy*, U. S. Naval Institute, Annapolis, Md., 1942.

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The author wishes to thank Professor Clifford B. Upton of Teachers College, Columbia University, for valuable advice and criticism. He has also received numerous helpful suggestions from members of his classes. And finally he wishes to acknowledge the assistance of Gladys M. Bradley in the preparation of the manuscript.

TABLE OF CONTENTS

CHAPTER I

GEOMETRY OF THE EARTH

The navigator of an airplane or a ship must solve two problems: One is to determine the direction and distance of one point from another so that he may "chart his course"; the second is to determine the position of his plane or ship at frequent intervals so that he may know whether he is actually following the desired route. Chapter I will explain some of the principles of geometry which enter into these two problems.

Shape and Size of the Earth. It has long been known that the earth is nearly spherical in form. This conclusion was reached by different lines of reasoning. As a ship at sea approaches land, an observer on the ship sees the highest points on the land first. Similarly, when a ship sails away, the lower part is seen to sink from sight first and then finally the top part of the ship disappears. Ships and planes have traveled around the earth and returned to their starting point. When an observer looks across the earth's surface, the earth and sky appear to meet in a circle called the horizon. A perfect horizon may be seen from a ship or plane or possibly from a tall building. The first photograph showing the curve of the horizon was made in 1935 by Stevens and Anderson from a record height of 13.71 miles.

Reasons like the ones given above do not prove that the earth is spherical but show merely that it is a curved surface. Experiments and measurements have shown that the earth curves nearly the same in all directions. Scientists have devoted much time and effort to finding the exact size and shape of the earth. Careful measurements have shown that the distance from the earth's center to the poles is approximately 3950 miles and that the radius at the equator is 3963 miles. This difference of 13 miles is relatively very small in comparison with the distances of nearly four thousand miles from the center; therefore for all purposes of navigation the earth is considered to be a sphere.

For ordinary purposes, distances are measured in the usual mile of 5280 feet. The mile of 5280 feet is really a *land mile* or *statute* *mile*, and must be distinguished from a different unit called the *nautical mile* which is used by the navigator. The length of our common mile was not derived from measuring the earth but probably has come down to us from the thousand paces (*millia passuum*) of the Roman soldier. The nautical mile used by navigators is the length of one minute of the earth's circumference. Since the earth's circumference contains 360° or 21,600′, it also contains 21,600 nautical miles. The length of one minute of the earth's circumference depends on whether it is measured along the equator or on a circumference passing through the poles. This variation is quite small and the standard nautical mile adopted by the United States Government is 6080.27 feet.

Problems

1. Show that 1 nautical mile = 1.152 statute miles, and that 1 statute mile = .8684 nautical mile.

2. Using the formula that the circumference of a circle equals 2π times the radius ($C = 2\pi r$), find the radius of the earth in nautical and statute miles.

3. Find the earth's circumference in statute miles.

4. The area of a sphere is $4\pi r^2$. Find the area of the earth in square statute miles and square nautical miles.

5. The volume of a sphere is $4/3\,\pi r^3$. Find the volume of the earth in cubic statute miles and cubic nautical miles.

Earth Co-ordinates. There are many ways of describing the location of an aircraft or a ship or any point on the earth. The position of a plane may be given as ten miles northeast of LaGuardia Field, or a ship may be located by saying it is two miles southeast of the Ambrose Channel Lightship. Statements such as these cannot be used to locate a plane flying over the ocean or to give the position of a ship which is beyond the sight of land. In many situations, therefore, the navigator must state his position in terms of earth co-ordinates which are called latitude and longitude.

Latitude and longitude correspond rather closely to the x, y co-ordinates which are used in algebra. The earth co-ordinates, however, are measured in degrees and minutes of angle instead of some measure of length such as miles or feet.

It is known from geography that the earth rotates or spins once each day about a diameter which joins the north and south poles. The circle around the earth midway between the poles is called the equator. In other words, the equator is a circle which lies in a plane perpendicular to the polar diameter. Figure 1 is a cross section of the earth through the north and south poles P and P′; EE′ is a diameter of the equator; and AA′ is a line parallel to the equator with the arc EA equal to 40°. If Figure 1 is rotated about the line PP′, then the line EE′ will trace out the equator and AA′ a smaller circle parallel to the equator as shown in Figure 2.

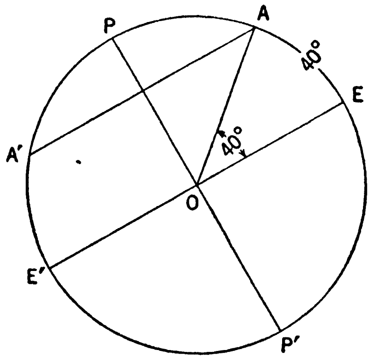

FIGURE 1.

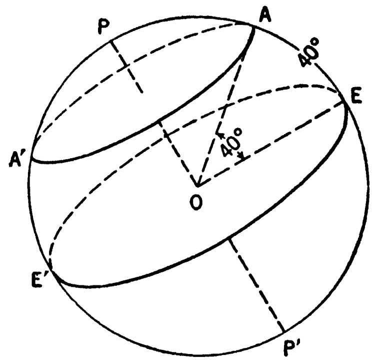

FIGURE 2.

The circle AA′ of Figure 2 is called a parallel of latitude and all the points on it are said to have a latitude of 40° north (40° N.). The latitude of 40° N. is measured either by the central angle AOE or by the corresponding arc on the circumference of the earth, AE.

All points on the equator have a latitude of 0°. Latitude varies from 90° S. at the south pole to 90° N. at the north pole. South latitudes are sometimes called negative. The equator corresponds to the X axis or $y = 0$, and the parallels of latitude to the lines parallel to the X axis or $y = 1$, $y = 2$, etc. Latitude alone does not fix the position of a point, except in the case of the north and south poles, any more than $y = 2$ fixes a point.

In Figure 3, PEP′E′ and PBP′B′ are two circles formed by taking cross sections of the earth through the polar diameter. The semicircles like PBP′ and PEP′ are called *meridians.* Let PEP′ be the meridian of Greenwich, England (latitude 51° 29′ N.), and let EB be an arc of the equator

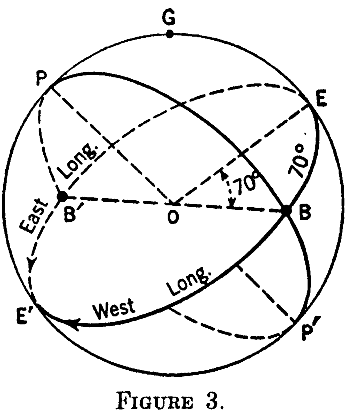

FIGURE 3.

equal to 70°. All points on PEP′, the meridian of Greenwich, have a longitude equal to 0°, and all points on the meridian PBP′ have a longitude of 70° west (70° W.). Longitude is measured east or west to 180° from the meridian of Greenwich. Since the arc EB and the angle EOB equal 70°, the arc EB′ and the angle EOB′ equal 110°; hence all points on PB′P′ are in longitude 110° E. The meridian PE′P′ contains all points of longitude 180°.

The meridian of 0° corresponds to the Y axis and the other meridian to the lines $x = 1$, $x = 2$, etc., although the meridians meet at the poles. In Figure 3, longitude was seen to be measured either by an arc of the equator or an angle at the earth's center. Longitude may also be measured along any parallel.

> Latitude is the angular distance north or south of the equator measured along the arc of a meridian.
>
> Longitude is the angular distance east or west of the meridian of Greenwich measured along the equator or along a parallel.

It is quite natural to measure latitude north or south from the equator, since the equator is the largest parallel of latitude. The meridians are all semicircles of the same size, and any meridian might have been chosen as the prime meridian. The meridian of Greenwich was adopted by the English after the establishment of the Observatory there and has since been adopted by most countries. It may be interesting to note that the meridian of Washington has sometimes been used as a reference meridian by our Government. In 1864 Congress passed an act stating that the boundaries of Colorado should be the parallels of latitude of 37° N. and 41° N. and the meridians of 25° and 32° west from Washington.

Problems

1. Find the latitude and longitude of your school by reference to the best map available.

2. Take as many maps of the world as are available and find the interval in degrees between successive meridians and parallels.

3. The latitudes and longitudes given below locate six important cities. Identify the cities from a map or globe.

Latitude:	38° 55′ N.	33° 52′ S.	51° 29′ N.	34° 40′ N.	55° 45′ N.	22° 54′ S.
Longitude:	77° 04′ W.	151° 12′ E.	0° 0′	135° 11′ E.	37° 34′ E.	43° 10′ W.

4. Find from a globe or map the approximate latitude and longitude of the following places: Cairo, Valparaiso, Singapore, Melbourne, Cologne.

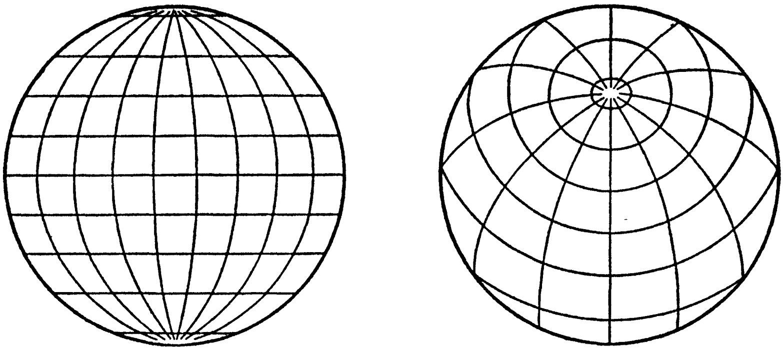

FIGURE 4.

Great Circles on the Earth. The parallels of latitude and the meridians form a network of intersecting circles on the surface of the earth. The parallels decrease in size toward the poles, while the meridians are all the same size. The equator and the meridians are examples of *great circles.*

> A great circle on the earth or any sphere is formed by the intersection of the sphere and a plane which passes through its center.

There is one great circle through any two points on a sphere, since three points determine a plane. Great circles are important in navigation because of the following theorem:

> The shortest distance between two points on the surface of a sphere is the shorter arc of the great circle which joins them.

The great circle between two points on the earth may be easily indicated on a terrestrial globe. Take a square of stiff cardboard and at its center construct a circle equal to the equator of the globe, cut out the circle and mark off degrees around its circumference as in Figure 5. This cardboard may be used as a "great circle protractor" to find the approximate number of degrees in the great circle arc between two points. It will also indicate the

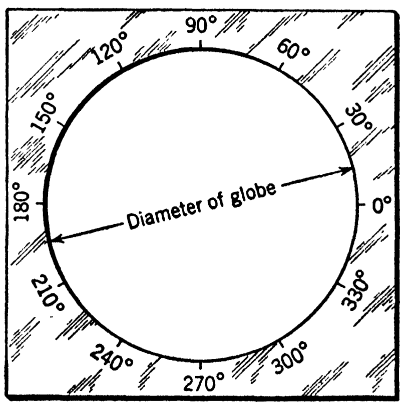

FIGURE 5.

two points where the great circle intersects the equator and the two points nearest the poles. For example, the great circle arc between San Francisco and Tokyo has a length of approximately 74° or 4440 nautical miles and the point nearest the north pole is about 49° N., 170° W. If this great circle arc is extended, it will cross the equator at longitude 80° W. and 100° E.

The accuracy of the answers obtained by this method will depend upon the size of the globe which is available and the accuracy of the cardboard protractor. In a later chapter a method of finding great circle distances by trigonometry will be developed.

The cardboard protractor and the equator of the globe illustrate another theorem.

> Two great circles intersect in two points which are the ends of a diameter of the sphere.

The meridians also illustrate this theorem.

Problems

Find the approximate great circle distances in degrees and nautical miles between the following points.

1. Natal (5° 47′ S., 35° 13′ W.) to Dakar (14° 40′ N., 17° 25′ W.)
2. New York (40° 43′ N., 74° 00′ W.) to Cape of Good Hope (34° 21′ S., 18° 29′ E.)
3. Sydney (33° 52′ S., 151° 12′ E.) to Valparaiso (33° 01′ S., 71° 39′ W.)
4. Auckland (36° 50′ S., 174° 46′ E.) to Sitka (57° 03′ N., 135° 20′ W.)
5. Harbour Grace (47° 43′ N., 53° 08′ W.) to Moscow (55° 45′ N., 37° 34′ E.)
6. Curtiss Field (40° 45′ N., 73° 37′ W.) to Paris (48° 50′ N., 2° 20′ E.)

Small Circles on the Earth. In later problems it will be necessary to consider *small circles* on the earth. The parallels of latitude are examples of small circles. In general every plane will intersect a sphere in a circle. If the plane passes through the center of the sphere, the intersection of the plane and the sphere is a great circle. If the plane does not pass through the center of the sphere, the intersection is called a small circle. Figure 6 shows a plane intersecting a sphere in a small circle.

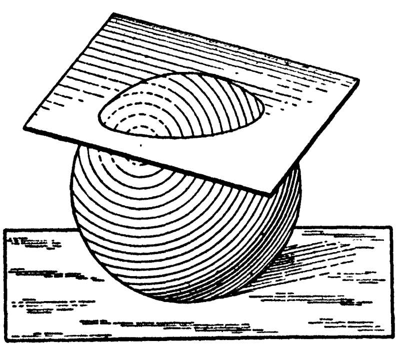

FIGURE 6.

All the points on the parallel of 80° N. are 10° or 600 nautical miles from the north pole. If the parallels were not marked on a globe, they could easily be constructed in much the same way that circles are drawn with the compasses in a plane. To construct the parallel of 80° N. one would place one point of the compasses at 0° longitude on the equator, and the other point at 10° W. on the equator. Then using this "radius" the parallel could be drawn on the globe with the north pole as "center." In a similar way any parallel might be drawn.

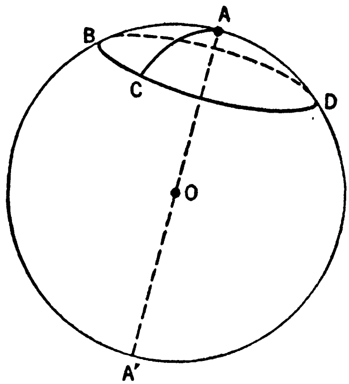

FIGURE 7.

Suppose that a small circle equal to the parallel of 80° N. is drawn with Washington, D. C., as "center"; this circle will contain all the points on the earth which are 600 nautical miles or 10° distant from Washington. Washington is said to be one of the *poles* of this small circle and the 10° distance is called the polar distance of the circle. The definitions of poles and polar distances of a small circle are:

> The poles of a small circle are the ends of the diameter of the sphere which is perpendicular to the plane of the small circle.
>
> The polar distance of a circle of a sphere is the arc of a great circle from the nearer pole to any point on the small circle.

In Figure 7, A and A′ are the poles of the small circle BCD; and the polar distance of the small circle is measured by any of the great circle arcs AB, AC, etc. The polar distance of any parallel of latitude is the complement of the latitude.

Problems

1. Construct on a globe small circles of polar distances of 10°, 20°, and 30° which have Chicago (41° 50′ N., 87° 37′ W.) as a pole.

2. Construct the small circles which have Halifax (44° 40′ N., 63° 35′ W.) as a pole and which have polar distances of 315, 430, 635 nautical miles.

3. Construct a small circle with Lima (12° 03′ S., 77° 00′ W.) for a pole and a polar distance of 30°. Find the intersections of this circle with a small circle which has New York (40° 43′ N., 74° 00′ W.) for a pole and a polar distance of 40°.

4. Find the longitudes at which the small circle with New York for pole and polar distance of 40° crosses the parallels of 10° N., 20° N., 30° N., 40° N.

5. Find the latitudes at which the small circle with Lima for pole and polar distance of 30° crosses the meridians of 60° W., 75° W., 90° W., and 105° W.

Triangles on the Earth. In plane trigonometry one studies various rules for solving plane triangles, that is, for finding certain sides and angles when certain other sides and angles are given. Navigation, like plane surveying, makes use of plane trigonometry. Plane trigonometry, however, does not make any allowance for the curvature of the earth and, when the distances involved are large, it is necessary to work with triangles on the surface of a sphere. A spherical triangle is a part of the surface of a sphere which is bounded by three great circles.

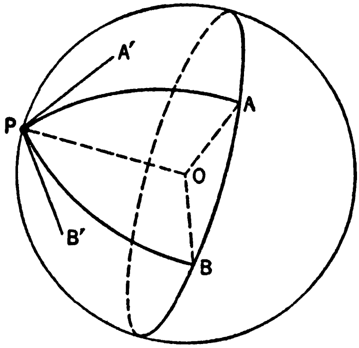

FIGURE 8.

Consider a spherical triangle PAB in Figure 8. It is formed by the arcs of three great circles, the half meridians PA and PB, and the arc of the equator AB. Since the sides of the triangle are arcs of great circles, they can be measured in degrees. PA and PB are each equal to 90°. AB can be found from the longitudes of points A and B. If PA is the meridian of 75° W. and PB the meridian of 120° W., then the arc AB = 45°. PAB is a special case of a spherical triangle since two of its sides are quadrants.

The angle between two great circles, commonly called a *spherical angle,* is defined as the angle between the tangents to the circles at their point of intersection. In Figure 8, angle P, or the angle between PA and PB, is, by definition, the angle between the tangents to PA and PB or angle A′PB′. Notice that the angles A′PB′ and AOB are equal because their sides are parallel; therefore angle P in the triangle is equal to the arc AB or 45°. The angles A and B in the triangle are each equal to 90°. Hence the sum of the angles of PAB is 225°.

Suppose that PB moves westward, then angle P increases and the sum of the three angles increases. If B moves eastward toward A, the sum of the three angles decreases. This illustrates an important theorem.

The sum of the angles of a triangle formed by the arcs of three great circles is greater than 180° and less than 540°.

In the previous example two vertices of the spherical triangle were on the equator. Consider now a triangle formed on the earth by the meridian arcs from the north pole to San Francisco (37° 47′ N., 122° 26′ W.) and to Tahiti (17° 30′ S., 149° 30′ W.) and the great circle arc which joins these two places (see Figure 9).

The side PS = 90° − 37° 47′ = 52° 13′.
The side PT = 90° + 17° 30′ = 107° 30′.
Angle P = 149° 30′ − 122° 26′ = 27° 04′.

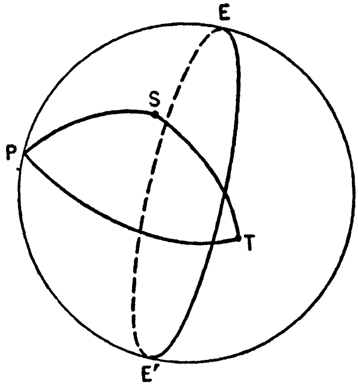

FIGURE 9.

Construct this triangle on a globe and determine approximately the length of the arc ST. Estimate the angles, and verify that their sum is greater than 180°.

In conclusion, the sides and angles of spherical triangles are both measured in degrees and minutes, and the sum of the angles is always greater than 180°. Spherical triangles like the triangle PST occur in many of the problems of navigation.

Problems

For each of the problems on page 6, construct the triangle formed by the north pole and the two given points. Find the exact values of the angle at the pole and its two including sides. Estimate the other two angles and verify that the sum of the angles is greater than 180°.

Mapping the Earth. The problem of constructing accurate maps of part or all of the earth's surface has engaged man's attention from the earliest times. The Babylonians drew maps on clay tablets, and it is said that Rameses II had a map of the estates along the Nile in 1300 B.C. The early Polynesian navigators made maps of the Pacific to show the location of islands and the directions of winds and currents. Maps made during medieval times were naturally incomplete, and unexplored parts of the world were filled in with fantastic pictures of strange men and beasts.

There is some interest in comparing early maps of the New World with our present-day maps. A "Map of New York I. with adjacent Rocks and other remarkable Parts of Hell Gate" drawn in 1778 by the British Hydrographer has an error of 1′ in the latitude and 14′ in the longitude.

Several departments of the United States Government are directly concerned with map making. The United States Geological Survey is engaged in preparing a large-scale topographic map of the entire country. The United States Hydrographic Office supplies the charts necessary in navigation and together with the British Admiralty has issued most of the navigation charts in general use. For 120 years the United States Coast and Geodetic Survey has been issuing charts of harbors and coasts, which show buoys, lights, shoals, channel depths, and prominent landmarks. The United States Department of Agriculture publishes large-scale maps which indicate the types of soil to be found in a given locality. No maps approach the detail of those of the British Ordnance Survey, which has prepared a map of the United Kingdom on a scale of twenty-five inches to the mile.

There are two problems in map making. One is to construct a network of lines to represent the meridians and parallels. The second is to locate coast lines, islands, rivers and all the other details at their proper positions on the network.

In a sense the problem of mapping the earth is the same as making a baseball cover. Examine a baseball closely and notice that at certain places the cover is slightly wrinkled and at others it is slightly stretched. The stretching and wrinkling occur because the surface of a sphere cannot be flattened out in a plane without distorting it in some way. This difficulty does not occur with the cone and cylinder. The surface of a right cone may be formed from the sector of a circle and the surface of a right cylinder from a rectangle (see Figure 10).

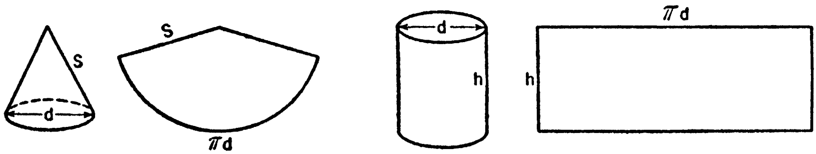

FIGURE 10.

Many different types of world maps have been developed for special purposes. In some maps the meridians are parallel lines and "the top of the map is north." Other maps are centered on the north pole and show the meridians as straight lines passing through this point; in such maps north is toward the center of the map. On some maps both the meridians and parallels are straight lines, and on others either the meridians or parallels or both may be curves.

Problems

1. Take the world maps in an atlas or geography and compare the meridians and parallels. Notice whether the meridians pass through a common point and whether they are straight lines or curves. Notice whether the parallels have the same length or whether they become smaller toward the pole.

2. Make a similar comparison for maps of the Polar Regions.

Mercator's Map of the World. One of the most common world maps is called *Mercator's Projection;* in fact, most maps of the world

FIGURE 11.

are drawn on Mercator's plan. In the Mercator map the meridians are parallel straight lines, equal distances apart; the parallels of latitude are also parallel lines, but the distance between the parallels increases toward the poles.

It is worth while to compare the Mercator map with the globe. All the parallels on the map have the same length; on the globe of course the parallels become shorter toward the poles. At latitude 60° on the map, distances in both the east-west and north-south directions are doubled in length. Hence an island at latitude 60° is four times as large as it should be. Figure 11 shows how areas are enlarged by the Mercator map. Egypt, Anglo-Egyptian Sudan, and the Belgian Congo have been placed on the map of Greenland. Greenland has an area of only 827,300 square miles, while the combined area of the other three territories is 2,276,500 square miles. This shows that the Mercator map should not be used for comparison of areas.

Mercator's map has one important property which makes it important for navigation. Any two points on the earth may be connected by a curve which does not change direction; such a curve is called a *rhumb line.* A rhumb line is not to be confused with a great circle. The great circle between two points is the shortest distance between

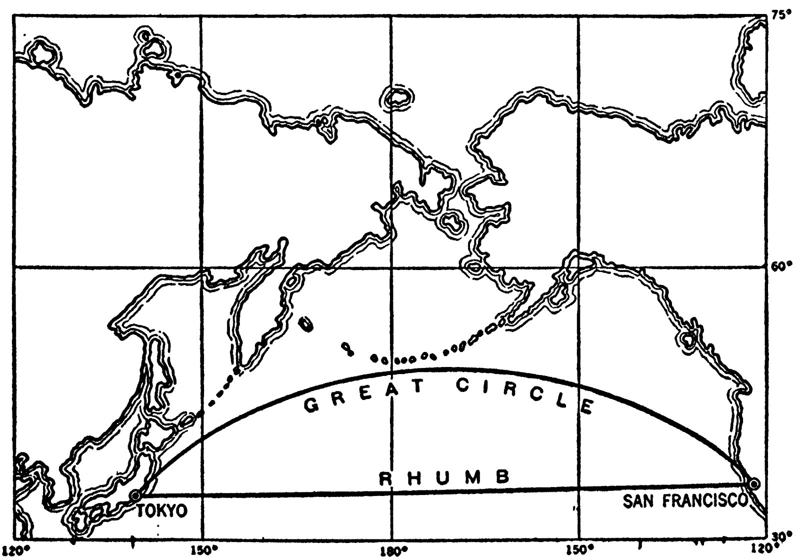

FIGURE 12. RHUMB LINE AND GREAT CIRCLE FROM SAN FRANCISCO TO TOKYO.

them, but as one follows a great circle, his direction will change from point to point as he moves along the arc. In place of the great circle one might follow the rhumb line between two points; in this case the direction would remain the same, but the total distance traveled would be longer. Only when two places are on the equator or on the same meridian will the rhumb line and the great circle between them coincide.

Figure 12 shows the rhumb line and the great circle between San Francisco and Tokyo plotted on the Mercator map. Notice that the rhumb line is a straight line on the map and the great circle is a curve, and hence *appears* to be longer than the rhumb line. Actually the rhumb line is 266 nautical miles longer than the great circle. Notice also that the great circle changes direction since it goes as far north as latitude 49°.

The importance of maps cannot be overemphasized. However, a true conception of areas and distances on the earth can be gained only from a globe.

Problems

In each of these problems a series of points on a great circle is given. Plot the great circles on a Mercator map and draw the corresponding rhumb lines.

1. Curtis Field (40° 45′ N., 73° 37′ W.) to Paris (48° 50′ N., 2° 20′ E.)

Latitude:	46° 50′ N.	51° 30′ N.	52° 10′ N.	49° 30′ N.
Longitude:	60° W.	40° W.	20° W.	0° W.

2. Cape Race (46° 39′ N., 53° 04′ W.) to Cape of Good Hope (34° 21′ S., 18° 29′ E.)

Latitude:	40° N.	20° N.	0°	20° S.
Longitude:	42° 15′ W.	22° 25′ W.	8° 27′ W.	5° 31′ E.

3. Honolulu (21° 18′ N., 157° 52′ W.) to Vancouver (48° 34′ N., 124° 08′ W.)

Latitude:	30° 24′ N.	35° 12′ N.	39° 19′ N.	42° 48′ N.
Longitude:	150° W.	145° W.	140° W.	135° W.

4. Cape Farewell (59° 47′ N., 43° 40′ W.) to Bouvet Island (54° 28′ S., 3° 30′ E.)

Latitude:

50° N.	30° N.	10° N.	10° S.	30° S.	40° S.

Longitude:

35° 20′ W.	26° 04′ W.	20° 08′ W.	14° 58′ W.	9° 02′ W.	0° 14′ E.

The Types of Navigation. The problems of determining the direction and distance of one point from another and of fixing the position of an airplane or a ship are solved in a variety of ways. Position may be determined by the aid of maps and visible landmarks; this is called *piloting.* Position is also found by *dead reckoning;* that is, calculating the new position from the distance and direction flown or sailed from some given position. Radio has become increasingly important in navigation. The *radio beam* and the *radio direction finder* tell the direction of the airplane from the radio station and enable the navigator to plot these direction lines on a map. Finding latitude and longitude from observations of the sun and stars is called *celestial navigation.* Whenever possible these four methods of navigation are used together as is shown in this description of the training at the Kelly Field Navigation School.

Actually the aerial navigator has to learn four types of navigation and practice all of them in the one hundred hours in the air he gets here besides the classroom work. In one of AT-7 Beechcraft navigation-training planes used here you will find three student navigators all working at once, but separately, while the ship is speeding on one of the training missions. These average one thousand miles apiece, and take the boys to every part of this country within a 500-mile radius.

One man will be piloting, with the aid of maps, landmarks, beacons and some of the plane's instruments. Another will be working on dead reckoning, using compass, drift indicator, clock, and other instruments. The third will be navigating wholly by radio, correlating radio beacons, beams, and other sources of information obtainable by radio.

Or in place of one of these three, there being room for only three students in an AT-7, there will be one man practicing celestial navigation, taking sights with a bubble sextant through the plane's top hatch or windows of the sun, moon or stars, and working them out with his own tables and instruments. If all three men come out with the same answer, and the plane is where they think it is when they say it should be, the mission has been a success.

— WILLIAM H. TAYLOR in the *New York Herald Tribune,* July 26, 1942

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Deetz and Adams: Part I.
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CHAPTER II

THE SAILINGS AND DEAD RECKONING

Dead reckoning is the process of finding the new position of a ship or plane from the distance and direction it has traveled from some given position. The various methods used in dead reckoning and in the reverse problem of finding the direction and distance between two points are called *sailings*.

Measurement of Directions or Bearings. In problems of navigation it is necessary to have an exact method of indicating directions. The direction of point A from point B is called the *bearing* of A from B and is expressed as an angle. Bearings are indicated from 0° to 360°; north is 0°, east 90°, south 180°, and west 270°, as shown in Figure 13.

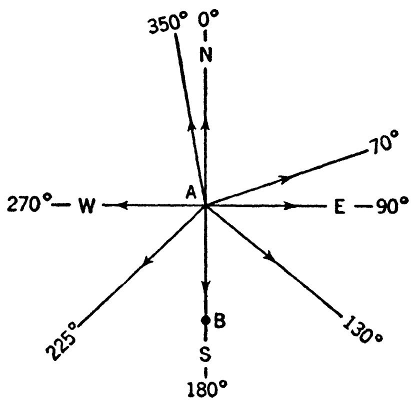

FIGURE 13.

If point B is due south of point A, then the bearing of B from A is 180°, and a plane flying from A to B would be on a *course* of 180°. If a plane follows a course of 130°, then it is flying in a direction which is 40° toward the south from east. A northeast course or bearing is 45°, southeast 135°, southwest 225°, and northwest 315°. This method of indicating bearings and courses has largely replaced the older method of dividing the 360° into the 32 points of the compass.

The most common instrument for measuring bearings is the magnetic compass. The compass needle does not usually point to the true north and hence a correction must be applied for this difference. The difference between true north and magnetic north is called *magnetic variation,* or declination, and may be east or west. The values of the magnetic variation for a few places in the United States for 1942 are given on page 16.

Topeka	9° 13′ E.	Bismarck	13° 47′ E.
Louisville	0° 40′ E.	Houston	9° 07′ E.
Annapolis	7° 42′ W.	Norfolk	6° 19′ W.
Buffalo	7° 34′ W.	Cheyenne	14° 44′ E.

The *magnetic bearing* of a line is its direction measured from magnetic north and is not to be confused with the bearing measured from true north. Suppose that the magnetic bearing of a line is 120° at a place where the magnetic variation is 15° E. In Figure 14 let ON represent true north and OM magnetic north, then angle MOA represents the magnetic bearing of 120°. The true bearing is angle NOA or $15° + 120° = 135°$. The correction of magnetic bearings may be reduced to a rule:

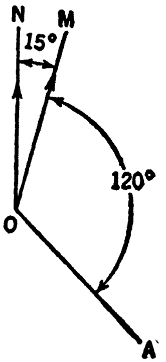

FIGURE 14.

> To correct magnetic bearing to true bearing, add east variation, subtract west variation.

This rule should not be applied arbitrarily until it has been verified by many examples.

There are other errors of the compass needle caused by local magnetism and by the magnetism of an aircraft. Such errors must be determined and applied in the same way as variation.

Problems

1. Construct a figure like Figure 13 to show bearings or courses of 320°, 190°, 10°, 97°, 210°, 175°, 290°.

2. Find your local magnetic variation by reference to a table or large-scale map.

3. Complete the following table. In each case construct a figure like Figure 14. (Do not write in your text.)

Magnetic Bearing	*Variation*	*True Bearing*
73°	5° E.	......
310°	10° E.	......
230°	15° W.	......
90°	4° E.	......
175°	9° W.	......
260°	10° W.	......
20°	8° E.	......
130°	6° W.	......

4. Complete the following table.

Magnetic Bearing	*Variation*	*True Bearing*
------	6° W.	30°
------	11° W.	63°
------	14° E.	330°
------	5° W.	250°
------	10° E.	185°
------	15° W.	80°
------	8° E.	240°

Plane Sailing. When it is required to find merely the relation between the course, distance sailed, and the corresponding north-south and east-west distances, a method known as plane sailing is used. Plane sailing is concerned with a triangle like ABC, in Figure 15, where angle C is the course from C to A, CB is the meridian through C, and BA the parallel through A. Plane trigonometry may be used to determine the various parts of triangle ABC.

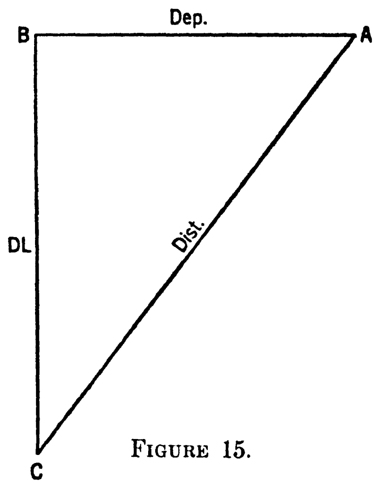

FIGURE 15.

CA is called the distance sailed (Dist.).
CB the difference of latitude (DL).
BA is called the departure (Dep.).

Notice that the length of CB in nautical miles may be found directly from the difference of the two latitudes; if A is in latitude 47° and C in latitude 43°, then CB = 4° or 240 nautical miles. BA, however, is an arc of a parallel and its length in miles would *not* be equal to the difference of longitude. One minute on the equator is one nautical mile, but one minute on the other parallels will vary according to the latitude. Hence difference of latitude (DL) may refer either to nautical miles or difference of minutes of latitude, but departure refers only to a distance expressed in nautical miles. From this it follows that plane sailing may not be used to find the longitude of a new position.

The two problems of plane sailing are:

(1) Given the course and distance, to find the difference of latitude and departure.

(2) Given the difference of latitude and departure, to find the course and distance.

The formulas for plane sailing are simply the formulas for solving a plane right triangle.

$$\text{DL} = \text{Dist. } \cos C$$
$$\text{Dep.} = \text{Dist. } \sin C$$
$$\tan C = \frac{\text{Dep.}}{\text{DL}}$$

When the third formula is used to find angle C, an acute angle will be found in the tables; this acute angle will be the angle from the meridian to the distance. The relationship between angle C and the course, expressed in the usual 0° to 360° manner, is shown in Figure 16. In plane sailing as in most problems of trigonometry a figure should always accompany the solution of a problem. Draw figures for examples 1 and 2.

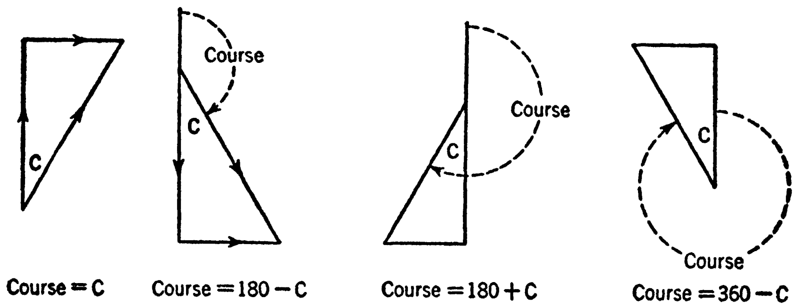

FIGURE 16.

Plane sailing may not be used to find the difference of longitude between C and A, and may not be used when the distances involved become too long, because of the neglect of curvature.

Example 1. A plane flies a course of 335° from Mitchel Field (40° 45′ N., 73° 58′ W.) for 200 nautical miles. Find the difference of latitude and departure.

log 200 =	2.30103	log 200 =	2.30103
log cos 25° =	9.95728	log sin 25° =	9.62595
log DL =	2.25831	log Dep. =	1.92698
DL =	181.3 nautical miles	Dep. =	84.52 nautical miles

Example 2. Find the course and distance from C to A if A is 215 nautical miles west and 183 nautical miles south of C.

log 215 =	2.33244	log 215 =	2.33244
log 183 =	2.26245	log sin 49° 36′ =	9.88169
log tan C =	.06999	log Dist. =	2.45075
C =	49° 36′	Dist. =	282.3 nautical miles
Course =	229° 36′		

Problems

1. A ship sails a course of 140° for a distance of 283 miles. Find the difference of latitude and departure.

2. A ship changes her latitude 207 miles to the north and makes a departure of 118 miles to the east. What course has she followed, and what is the distance made good?

3. A plane follows a course of 310° for $2\frac{1}{2}$ hours at 100 miles per hour. Find the difference of latitude and departure.

4. Find the course and distance in the following:

	DL	*Dep.*
(*a*)	250 miles north	300 miles east
(*b*)	298 miles south	267 miles west
(*c*)	315 miles south	245 miles east
(*d*)	135 miles north	210 miles west

5. Find the difference of latitude and departure in the following:

	Course	*Distance*
(*a*)	37° 34′	384 miles
(*b*)	307° 10′	242 miles
(*c*)	145° 00′	317 miles
(*d*)	210° 40′	289 miles
(*e*)	298° 30′	117 miles

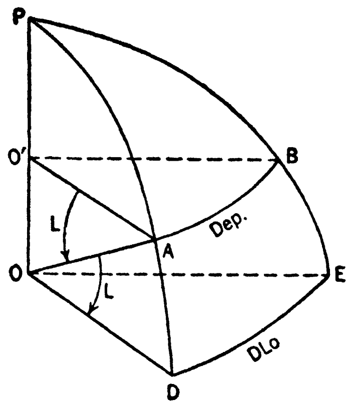

FIGURE 17.

Parallel Sailing. If the course is due east or west, it is easy to convert the nautical miles of departure into degrees and minutes of longitude, or to convert a given difference of longitude into nautical miles. In Figure 17, AB is an arc of the parallel of L°, DE the corresponding arc of the equator, O the center of the earth, and O′A the radius of the parallel. In right triangle OAO′,

$$O'A = OA \cos L = OD \cos L$$

$$\frac{AB}{DE} = \frac{O'A}{OD} = \frac{OD \cos L}{OD}$$

Hence $$AB = DE \cos L$$

AB is the departure or the distance along the parallel, and DE is the corresponding distance along the equator. The number of nautical miles in DE equals the difference of longitude, since on the equator one minute of longitude equals one nautical mile. Difference of longitude is abbreviated DLo. For parallel sailing the formulas are:

$$\text{Dep.} = \text{DLo} \cos \text{L, and DLo} = \frac{\text{Dep.}}{\cos \text{L}}$$

Notice that the difference of longitude, DLo, is the number of degrees and minutes of longitude between the two meridians and is not always the differences between the two longitudes. For the three examples below DLo = 10° = 600′.

60° W. to 70° W.	70° − 60° = 10°
5° E. to 5° W.	5° + 5° = 10°
175° W. to 175° E.	360 − (175° + 175°) = 10°

Example 1. Find the distance along the parallel of 37° 30′ S. from White Island, New Zealand (177° 11′ E.), to Cape Howe, Victoria (149° 59′ E.).

DLo = 177° 11′ − 149° 59′ = 27° 12′ = 1632′
Dep. = 1632 cos 37° 30′ = 1295 nautical miles

Example 2. A ship sails east from (32° 00′ S., 2° 10′ W.) for 300 nautical miles. Find the longitude of the new position.

DLo = 300/cos 32° = 354′ = 5° 54′
New longitude = 2° 10′ W. − 5° 54′ = 3° 44′ E.

Problems

1. Complete the following table. Find the DLo in degrees and minutes.

43° 10′ W.	54° 48′ W.
178° 13′ W.	176° 19′ E.
6° 17′ E.	5° 46′ W.
74° 00′ W.	81° 19′ W.
120° 47′ E.	122° 27′ E.
174° 39′ E.	169° 43′ W.

2. Find the number of nautical miles in 10° of longitude on each of the parallels of 15°, 30°, 45°, 60°, 75°, 90°.

3. Find the number of minutes of longitude in 600 nautical miles on each of the parallels of 15°, 30°, 45°, 60°, 75°.

4. Find the distance along the parallel of 35° 39′ N. from Tokyo (139° 45′ E.) to Wang-kia-tia Bay, China (119° 52′ E.).

5. A ship in latitude 58° 00′ N. and longitude 23° 45′ W. sails west 195 nautical miles. Find the longitude of the new position.

6. Find the distance along the parallel of 41° 09′ N. from Bridgeport Harbor (73° 11′ E.) to the north point of Gardiners Island (72° 09′ W.).

7. A ship in latitude 32° 22′ N. and longitude 52° 20′ W. sails west 365 miles. Required, her distance from the Island of Bermuda in the same latitude and longitude 64° 43′ W. (from Norie's *Navigation*).

8. Find the distance along the parallel of 41° 09′ N. from Oporto Light (8° 41′ W.) to Block Island Light (71° 33′ W.).

Middle Latitude Sailing. When the course between two points does not follow a parallel of latitude, some method must be found to obtain the longitude of the new position. In parallel sailing, difference of longitude was found by the formula DLo = Dep./cos L. In middle latitude sailing, nautical miles of departure are converted to minutes of longitude by dividing by the cosine of the average of the two latitudes, that is, the middle latitude. Middle latitude is abbreviated L_m.

Middle latitude sailing is equivalent to solving a plane right triangle ABC, Figure 18, where the sides are DL, DLo cos L_m, and the distance. Angle C is related to the course as was shown in Figure 16, page 18. The formulas for finding course and distance are:

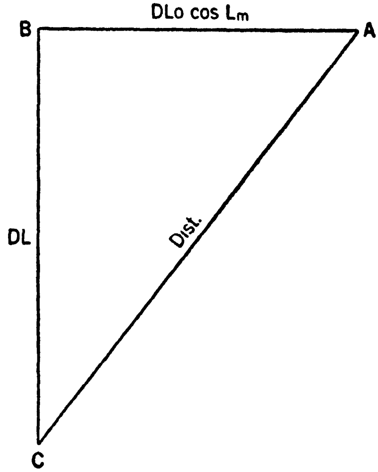

FIGURE 18.

$$\tan C = \frac{\text{Dep.}}{\text{DL}} = \frac{\text{DLo} \cos L_m}{\text{DL}}, \text{ and Dist.} = \frac{\text{Dep.}}{\sin C}$$

Example 1. Find the course and distance by middle latitude sailing from (35° 00′ N., 61° 31′ W.) to (30° 00′ N., 53° 22′ W.).

$$\text{DL} = 35° 00' - 30° 00' = 5° 00' = 300' = 300 \text{ nautical miles}$$
$$\text{DLo} = 61° 31' - 53° 22' = 8° 09' = 489'$$
$$L_m = \frac{35° 00' + 30° 00'}{2} = 32° 30'$$

log DLo(489) =	2.68931	log Dep. =	2.61534
log cos L_m(32° 30′) =	9.92603	log sin C(53° 58′) =	9.90777
log Dep. =	2.61534	log Dist. =	2.70757
log DL(300) =	2.47712	Dist. =	510.0 nautical miles
log tan C =	0.13822		
C =	53° 58′		
Course =	126° 02′		

When the course and distance are given, the formulas for finding the latitude and longitude of the new position are as follows:

$$DL = \text{Dist. cos. } C \qquad DLo = \frac{\text{Dist. sin } C}{\cos L_m}$$

The difference of latitude and longitude are then applied to the given latitude and longitude to find the new position.

Example 2. A plane leaves Dayton Airport (39° 53′ N., 84° 12′ W.) and flies a course of 225° for 400 miles. Find the latitude and longitude of the new position.

$$\begin{aligned} \log \text{Dist.}(400) &= 2.60206 \\ \log \cos C(45°) &= \underline{9.84949} \\ \log DL &= 2.45155 \\ DL \text{ (nautical miles)} &= 282.8 \\ DL &= 4° 42'.8 \\ L_m &= 39° 53' - \frac{4° 42'.8}{2} \\ L_m &= 37° 32' \end{aligned}$$

$$\begin{aligned} \log \text{Dist.}(400) &= 2.60206 \\ \log \sin C(45°) &= \underline{9.84949} \\ \log \text{Dep.} &= 2.45155 \\ \log \cos L_m(37° 32') &= \underline{9.98927} \\ \log DLo &= 2.55228 \\ DLo &= 356'.6 \\ DLo &= 5° 57' \end{aligned}$$

New position: 39° 53′ N. − 4° 43′ = 35° 10′ N.
84° 12′ W. + 5° 57′ = 90° 09′ W.

Middle latitude sailing is an approximate method and cannot be used when the distance is too great. The error is considered to be "immaterial for distances ranging up to 200 nautical miles, but for distances of 600 nautical miles the default from accuracy would be of the order of one per cent when the middle latitude is as great as 60°" (Bowditch). Middle latitude cannot be used for courses crossing the equator.

Problems

1. Find the course and distance between the following points on the great circle from New York to the Cape of Good Hope.

A. (25° 00′ N., 46° 36′ W.)
B. (20° 00′ N., 40° 41′ W.)
C. (15° 00′ N., 35° 20′ W.)
D. (10° 00′ N., 30° 13′ W.)

2. Find the new position from the following initial positions and courses and distances.

A. (49° 42′ N., 50° 00′ W.), course 49° 20′, distance 248.6 nautical miles
B. (52° 24′ N., 45° 00′ W.), course 53° 18′, distance 222.5 nautical miles
C. (54° 37′ N., 40° 00′ W.), course 56° 42′, distance 210.6 nautical miles

3. A ship sails 268 nautical miles on a course of 223° from latitude 39° 37′ S. and longitude 3° 42′ E. Find the new position.

4. Find the course and distance by middle latitude sailing from Vladivostok (43° 05′ N., 131° 53′ E.) to Niigati (37° 55′ N., 139° 03′ E.).

5. Find the position reached by a ship sailing a course of 315° for 300 nautical miles from (48° 46′ N., 178° 15′ W.).

Bibliography

World Almanac, 1942: "Table of Magnetic Variation," 175.

Magnetic variation is indicated on the charts published by the U. S. Hydrographic Office. These charts cover nearly every part of the world; inquiries to the Hydrographic Office should state whether a general chart of the entire region is desired or whether the writer wishes the more detailed, larger-scale charts covering it. Magnetic variation is also indicated on the "Sectional and Regional Aeronautical Charts of the United States" sold by the U. S. Coast and Geodetic Survey.

The Magnetic Compass: Dutton, Chapter III; H. O. 216, 39–56; Bowditch, Chapter III.

Plane, Parallel and Middle Latitude Sailing: Dutton, 83–91; Bowditch, 98–103.

Mercator Sailing: Dutton, 91–93; Bowditch, 103–105.

Air Navigation by Dead Reckoning: Lyon, Chapter V.

CHAPTER III

PILOTING

Navigation beyond the sight of land, where observations of the sun and stars are necessary for the determination of latitude and longitude, is called *celestial navigation.* Celestial navigation is to be distinguished from navigation along coasts and in harbors which is termed *piloting.* "Piloting, in the sense given to the word by modern and popular usage, is the art of conducting a vessel in channels and harbors and along coasts, where landmarks and aids to navigation are available for finding the position, and where the depth of water and dangers to navigation are such as to require a constant watch to be kept upon the vessel's course and frequent changes to be made therein" (Bowditch). The greatest dangers to ships usually occur when the ship is near land or is entering or leaving a harbor; and while errors in finding position on the high seas may be corrected by later observations, an error in piloting is frequently disastrous.

In modern piloting the navigator is provided with detailed charts of the coasts, lists of landmarks such as lights and buoys, and precise information concerning the tides, currents, and depth of the water. Soundings are of the utmost importance in piloting. Modern instruments for taking soundings measure the time interval required for a sound wave transmitted from the bottom of the vessel to be echoed back to the vessel. Since sound travels at a constant speed in water, the depth of the water can be automatically determined and plotted by the sounding machine. When the position of an aircraft, or rather the point on the earth directly beneath it, is determined from visible landmarks, methods corresponding to those of piloting may be used. Air navigation, however, is complicated by the rapid change of position and by frequent poor visibility. The aeronautical charts of the United States issued by the U. S. Coast and Geodetic Survey provide the navigator of a plane with detail similar to that of the coast and harbor charts.

Position by Cross Bearings. When it is possible to observe the bearings of two, or more, objects which are indicated on the navigational or aeronautical chart, position is easily found by plotting these bearing lines. The navigator observes the bearing of a lighthouse from his ship, but must plot on the map the bearing of the ship from the lighthouse. In Figure 19 the bearing of point B measured from A is 110°, but the bearing of A from B is 110° + 180° or 290°; similarly the bearing of D measured from C is 240°, but the bearing of C from D is 60°, or 240° − 180°.

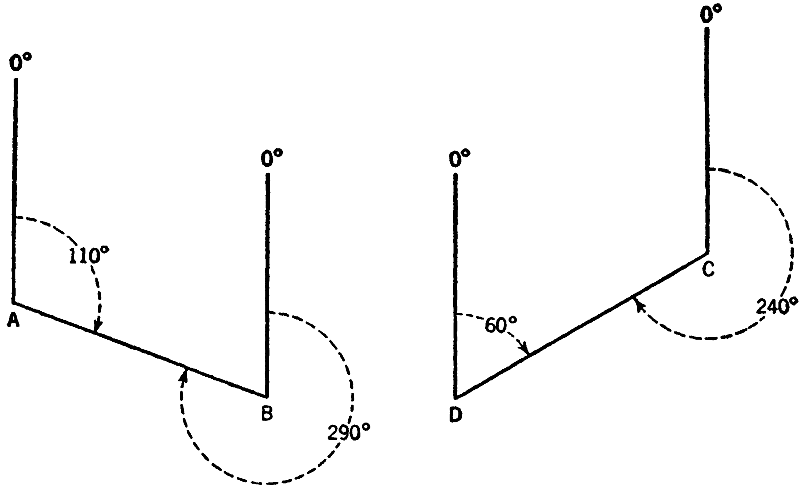

FIGURE 19.

In Figure 19 if A represents a ship and B a lighthouse, then the navigator measures the bearing of 110°, but must plot the *reciprocal* bearing of 290° on his chart. Reciprocal bearings differ by 180°. In problems of this kind the meridians through A and B are considered to be parallel straight lines. Two bearing lines from different objects will intersect in a point, but give no check on the accuracy of the intersection. When possible it is desirable to obtain three bearings and have a check on the accuracy of the intersection, or *fix*.

Example. A plane flying over Long Island Sound observes the bearings of three lighthouses.

Plum Island, 275°
Gardiners Island, 265°
Montauk Point, 140°

Find the position of the plane on the chart.

The plane bears

95° from Plum Island
85° from Gardiners Island
320° from Montauk Point

The three bearings do not determine a point but a small triangle, the so-called "cocked hat," or triangle of error. The accuracy of the fix depends of course on the size of this triangle.

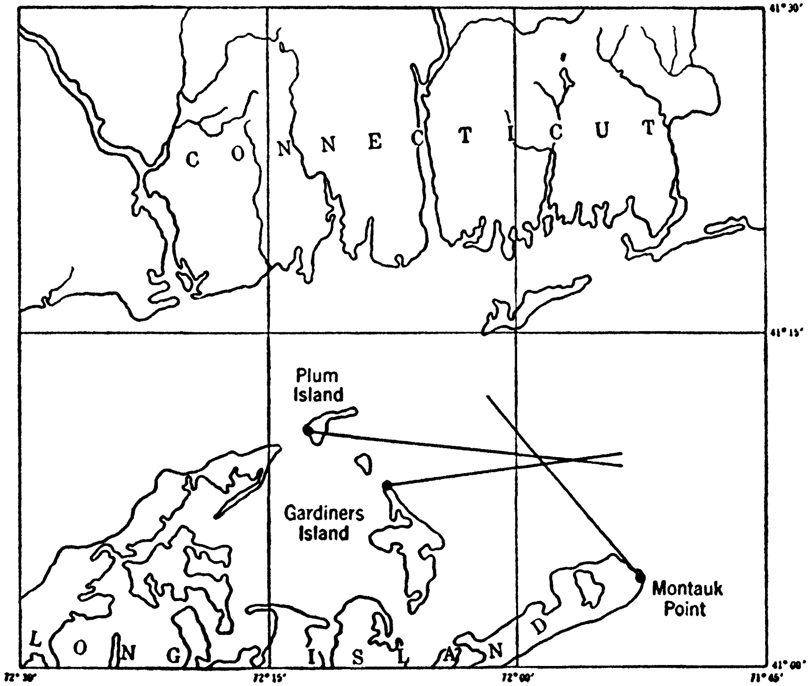

FIGURE 20.

Problems

1. Find the reciprocal bearings for the following bearings. Construct in each case a figure like those of Figure 19. (Do not write in your text.)

Bearing of A from B	*Bearing of B from A*
350°	------
175°	------
87°	------
247°	------
190°	------
10°	------
298°	------
90°	------
37°	------

2. Construct a triangle ABC with AB = 3.5 miles, BC = 4.0 miles, and CA = 5.4 miles. Make the direction of AB, 220°. AC is east of B. A navigator observes these bearings from his ship; A, 295°; B, 270°; C, 250°. Find the position of his ship on your figure.

3. AB is 3.8 miles long and has a direction of 130°, BC is 2.8 miles, and CA 5.6 miles. AC is west of B. A navigator observes the bearing of A to be 35°, B 60°, and C 80°. Find the position of his ship.

Radio Navigation. Various radio devices have been developed which are of the greatest importance in modern air and marine navigation. There are three means of finding position by radio: the radio beam, bearings of the plane or ship from the radio station, and bearings of the radio stations from the plane or ship. These methods are used to determine position in much the same way as cross bearings.

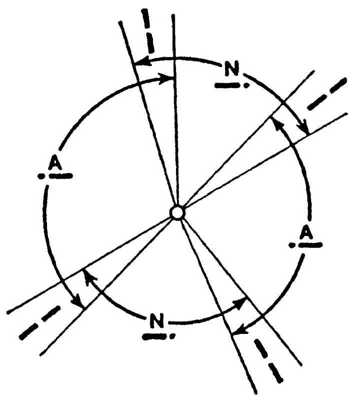

FIGURE 21.

Along the main airlines in the United States there are stations at frequent intervals which broadcast signals for the guidance of pilots. These signals consist simply of the letters A (• –) and N (– •) in Morse code sent out in opposite directions, so that each letter is audible in two opposite quadrants or sectors. Along the boundaries of adjacent sectors the two signals merge in a continuous signal (– –); these equisignal zones are the *radio beams.* A pilot approaching the transmitting station will receive one letter more pronounced if he is to the left of the beam and the other more pronounced if he is to the right of the beam. The letters A and N are interrupted frequently for other signals which identify the direction of the beam.

The *radio direction finder* is an instrument which determines the direction from which a radio message is received. By means of the radio direction finder a station may determine the bearing of a plane from the station and then transmit this bearing back to the plane. Two stations can give the plane intersecting bearings which determine the position of the plane. In preparation for their trans-Pacific flights the engineers of Pan American Airways developed radio direction finders with a range of two thousand miles. Radio direction finders have also been developed for the plane to carry, so that it may find its own bearing from a ship or land station. Radio direction finders of this type have been developed with a range of one thousand miles.

Radio bearing lines may be plotted on a chart and used to find a *fix* in much the same way as the bearings obtained from visible objects. Radio waves follow a great circle, and for long distances the great circle will not have the same direction as the rhumb line (see Figure 12). Before a radio bearing is plotted as a straight line on a Mercator chart, the bearing must be corrected by special tables.

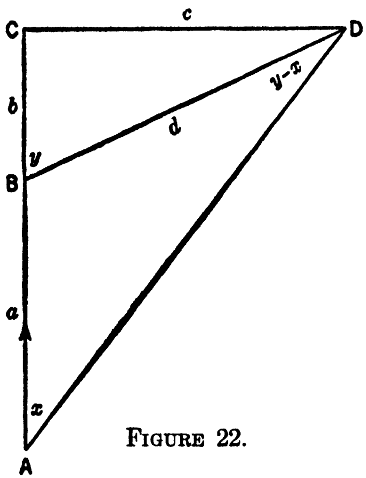

FIGURE 22.

Position by Two Bearings of a Single Object. The position of a ship may be determined by two bearings of a single object obtained at different times. In Figure 22, A and B are two positions of a ship following the course AB, and D is the object sighted. The angles x and y, called *angles on the bow*, are obtained from the bearings of D, measured at A and B. AB is known, since it is the distance run between the times of the two observations. Figure 22 is simply a triangle ABD with altitude c, and plane trigonometry may be used to find d, the distance from the object at the time the second bearing is taken, or to find c the perpendicular. The distance CD (c) is called the *distance at which the object is passed abeam.* In practice the use of trigonometry is avoided by special tables.

There are numerous special cases of Figure 22. If angle $y = 2\,x$, then BD = AB, or the distance run equals the distance to the object at the second observation. This is called *doubling the angle on the bow.* In this case any convenient value of x may be taken and the angle observed until it has doubled.

There is a special method of finding DC, the distance abeam. This consists of selecting the two angles x and y so that $\cot x - \cot y = 1$. For example, if $x = 30°$, then $\cot x = 1.7321$ and $\cot y = .7321$, using a table $y = 53° 48'$. The navigator is supplied with special tables which give corresponding values of x and y for this special case.

Problems

1. Prove that $d = \dfrac{a \sin x}{\sin (y - x)}$ and that $c = \dfrac{a \sin x \sin y}{\sin (y - x)}$.
2. Prove that BD = AB if $y = 2x$.
3. Prove that if $\cot x - \cot y = 1$, then $a = c$.
4. Prove that $\dfrac{\sin x \sin y}{\sin (y - x)} = \dfrac{1}{\cot x - \cot y}$.

5. Find values of x and y, such that $\cot x - \cot y = 1$ by completing the following table. Find the other angles to the nearest degree only. Hint: Unless you have access to a table of natural trigonometric functions, you will have to use both the logarithms of numbers (pages ii–xix) and the log cotangents (pages xx–xlii) to solve this problem.

Cot x	*x*	*y*	*Cot y*
2.475	22°	----	----
----	----	41°	1.150
1.963	27°	----	----
----	----	51°	0.810
1.600	32°	----	----
----	----	79°	0.194

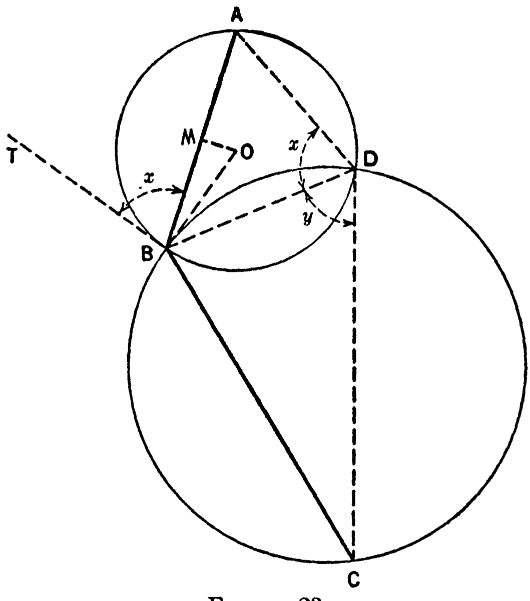

FIGURE 23.

The Three-Point Problem. An important method of fixing position is to measure the angles which the ship makes with three known points. In Figure 23, let A, B, and C be three visible points and D the ship. D may be determined on the chart by the angles x and y. Since the solution depends only on x and y and not on bearings, the angles may be measured with a sextant and compass errors eliminated. Errors caused by estimated distances are also eliminated.

The simplest solution consists merely in laying out the angles on a sheet of tracing paper and moving the paper over the chart until the sides of the angles pass through A, B, and C. D may be located in a similar manner by using a three-arm protractor. Since D lies on the intersection of two circles which have AB and BC as chords, the problem may also be solved by a construction of geometry: "To construct, on a line segment [AB] as a chord, a segment of a circle in which a given angle [x] may be inscribed." If this construction is repeated with the line BC and angle y, D is found by the intersection of the two circles.

To determine O, the center of the circle through A and B, construct:

angle ABT = angle x
BO perpendicular to BT
MO the perpendicular bisector of AB

The position of D is indeterminate from the given data when the two circles coincide; and for an accurate fix it is necessary to avoid the case where the centers are close together. In general, the position of D is determined more accurately when the angles x and y are large and the distances from D to A, B, and C are short.

The three-point problem was first used by the Dutch mathematician, Willebrord Snell, who published a trigonometrical solution in 1617. It has many applications in marine surveying and military engineering as well as in navigation. Many solutions other than the geometrical one have been devised.

Problems

1. Use the construction of the three-point problem to locate a ship D from three lights, A, B, and C. AB = 2.1 miles and bears due south, BC = 3.2 miles and bears 164°. Angle ADB = 33°, angle BDC = 68°. The ship is on the east side of AB.

2. Give the proof for the geometric construction in Figure 23.

3. Construct the circle through AB in Figure 23 by making an isosceles triangle on AB as a base. Make each base angle of the isosceles triangle equal to $90° - x/2$.

4. Construct the circle through AB in Figure 23 by constructing a triangle on AB as a base with the sum of the base angles equal to $180° - x$.

The Danger Angle. Another application of elementary geometry is used in sailing along a coast where there are hidden reefs or other dangers. In Figure 24, A and B are two lighthouses and C a hidden danger to be avoided. The navigator constructs a circle on the chart which passes through A and B and safely includes C. He then measures the angle ADB inscribed in the larger arc AB. In order to avoid C, the ship must be steered so that the angle ASB, which it makes with A and B, is less than angle ADB.

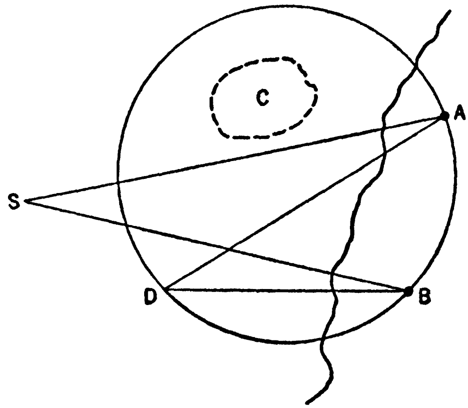

FIGURE 24.

Problems

1. Prove that (*a*) the ship is inside the circle when angle S is greater than angle D; (*b*) the ship is on the circle when angle S equals angle D; (*c*) the ship is outside the circle when S is less than angle D.

2. Construct a figure to show how a double danger angle might be used to steer a safe course between two dangers.

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CHAPTER IV

SPECIAL PROBLEMS OF AIR NAVIGATION

The same basic problems occur in both air and sea navigation. Courses and distances must be calculated for both planes and ships, and frequent determination of position must be made to see if the plane or vessel is on course. The aerial navigator must work more rapidly and frequently with more uncertain data. Ocean currents have been charted so that their effect on the course of a vessel can be accurately determined. Although it is impossible to chart the winds in the same way as the ocean currents, the navigator of a plane must make due allowance for the speed and direction of the wind in determining his course. The actual direction in which a plane goes, called its *track*, is the result of the wind direction and the direction in which the plane is aimed or pointed. Similarly the speed of a plane is the result of the wind velocity and the speed which the plane has in calm air.

The high speed of modern aircraft makes it imperative that computations of course and distance be made rapidly. If the position of a ship is in doubt, the vessel may "heave to" until an accurate fix has been obtained. The aerial navigator must make the best determination possible while traveling at a relatively high speed. Many of the special problems of air navigation are solved by graphic construction, since the solutions must be obtained quickly, and since the given data are frequently approximate. This chapter will present solutions of certain problems peculiar to air navigation.

The Triangle of Velocities. An important problem in air navigation is to determine the effect of a wind of known velocity and direction on the course and speed of a plane. Consider a northwest wind of forty miles an hour and a plane capable of making 150 miles an hour in calm air. Suppose the plane starts from O, Figure 25, in a northeast direction — that is, on a course of 45°. C in Figure 25 represents the position of the plane after one hour if there is no wind; OC has a bearing of 45° and a length representing 150 miles to scale. The effect of the wind alone would be to move the plane in a southeast direction. OA with a bearing of 135° and a length of 40 miles to the same scale as AC represents the effect of the wind, that is, the wind itself in one hour would move the plane from O to A. The plane of course does not follow either OA or OC, but its actual direction lies between OA and OC.

To determine the actual course and speed of the plane, construct the parallelogram OABC and draw the diagonal OB. The plane actually moves along the line OB, with a speed represented by the length of OB, and a course represented by the direction of OB. For this example, the effective speed of the plane is 155 miles an hour and its actual course is 60°.

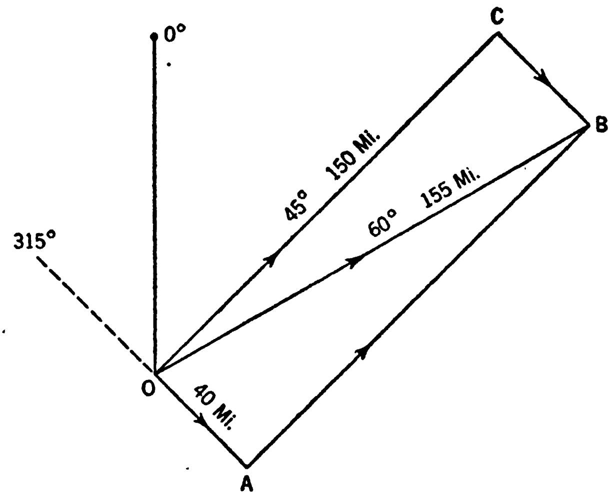

FIGURE 25.

The above example shows that it is necessary to distinguish carefully between the direction in which the plane is pointed and the direction in which it actually goes. *Heading* is the direction in which a plane is pointed; in Figure 25 the heading is 45°. *Track* is the direction of travel over the ground, 60° in Figure 25. The *air speed* or the speed which the plane has in calm air must also be carefully distinguished from the speed over the ground or *ground speed*. In Figure 25 the air speed is 150 miles, measured on OC, and the ground speed is 155 miles, measured along OB.

The *drift angle* is the angle between the heading and the track. In Figure 25 the drift is angle COB, 15° to the right. Drift angle, track, and ground speed may be determined by drawing only triangle OAB instead of the entire parallelogram.

In Figure 25 a heading was assumed and the corresponding track was found. In general the navigator has to find the heading which corresponds to a given track. This problem is also solved by constructing a triangle or parallelogram of velocities. Suppose it is required to find the heading for a track of 80° with the same wind and air speed as before. In Figure 26, construct OA with a bearing of 135° and a length of 40 miles to represent the wind. Draw the indefinite line OB with a bearing of 80° to represent the given track; then with center A, and a radius of 150 miles, draw an arc intersecting OB at B. Complete the parallelogram OABC. As before, head-

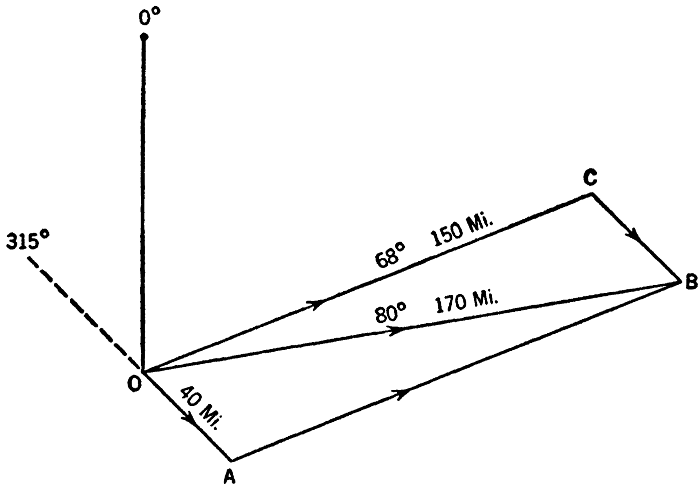

FIGURE 26.

ing and air speed are represented by OC, and track and ground speed by OB.

OC represents the given air speed, 150 mi./hr., and the required heading, 68°.

OB represents the required ground speed, 170 mi./hr., and the given track, 80°.

The drift angle is 12° to the right, angle COB.

In Figure 26, the pilot must steer the plane 12° to the left of the desired track, in order to offset the force of the wind driving him off his course. That is, the pilot steers in the direction OC, in order for the plane to go in the direction OB.

In aerial navigation, heading is determined from the compass, and air speed from the air-speed indicator. Track and ground speed must be found by some method corresponding to the above. The wind direction is indicated in the usual manner, from 0° to 360°; however, the wind is said to be *from some given direction;* a north wind is a *wind from 0°;* a southeast wind, a *wind from 135°*, etc.

Problems

1. Taking a convenient scale, indicate graphically the following wind velocities and directions: (*a*) 35 miles from 70°; (*b*) 15 miles from 160°; (*c*) 40 miles from 340°; (*d*) 10 miles from 250°.

2. A plane has an air speed of 150 mi./hr.; there is a 40-mile wind from 315°. Find the ground speed, track, and drift for the following headings, by constructing a figure like that of Figure 25. (Do not write in your text.)

Heading	*Ground Speed*	*Track*	*Drift*
315°	------	------	------
0°	------	------	------
45°	------	------	------
90°	------	------	------
135°	------	------	------

3. A plane has an air speed of 150 mi./hr.; there is a 40-mile wind from 315°. Find the heading, ground speed, and drift to fly the following tracks, by constructing figures like that of Figure 26.

Heading	*Ground Speed*	*Track*	*Drift*
------	------	315°	------
------	------	340°	------
------	------	40°	------
------	------	80°	------
------	------	115°	------

4. A plane has an air speed of 200 mi./hr.; there is a 30-mile wind from 210°. Complete the following table by finding heading or track, ground speed, and drift.

Heading	*Ground Speed*	*Track*	*Drift*
------	------	0°	------
0°	------	------	------
------	------	30°	------
30°	------	------	------
60°	------	------	------
------	------	60°	------
90°	------	------	------
------	------	90°	------
120°	------	------	------

5. What heading shall a plane take in order to track 225°, if there is a 30-mile wind from 170°? The air speed is 130 mi./hr.

6. There is a wind of 25 miles per hour from a direction of 320°. Find the track, drift angle, and ground speed of a plane heading 60° with an air speed of 120 mi./hr.

7. Find the heading for a plane to track 310°, if the wind is 20 mi./hr. from 40° and the air speed 90 knots.

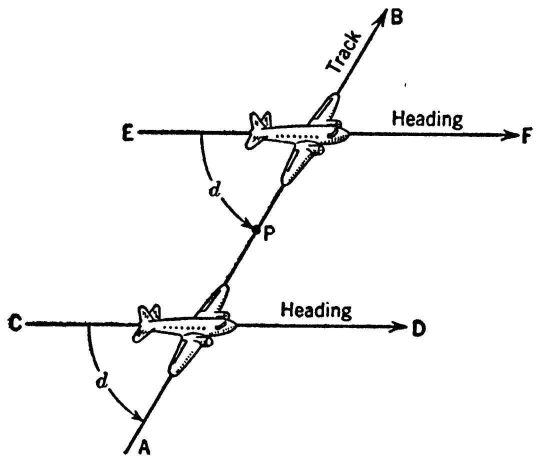

FIGURE 27.

The Wind Star. In the problems of the previous section, it was assumed that the wind direction and velocity were known to the navigator. In many cases it is necessary to determine the heading for a given track without a knowledge of the wind's velocity and direction. This problem may be solved by measuring the drift on two headings.

In order to measure the drift it is necessary to sight back on some fixed point on the surface, over which the plane has passed. Figure 27 shows two positions of a plane following a track AB, with a heading represented by the parallel lines CD and EF. The drift angle is the angle between the fore-and-aft axis of the plane and the track, angle *d* in Figure 27. Angle *d* may be measured by sighting back on some fixed point like P over which the plane has passed. In order to measure drift on ocean flights, flares have been developed which will burn for some time after they have struck the water. For daylight flying, Pan American Airways developed a special "bomb" filled with aluminum powder which forms a visible mark after it strikes the water.

The *wind star* is a graphic method of finding the heading for a required track from measurements of drift on two headings.

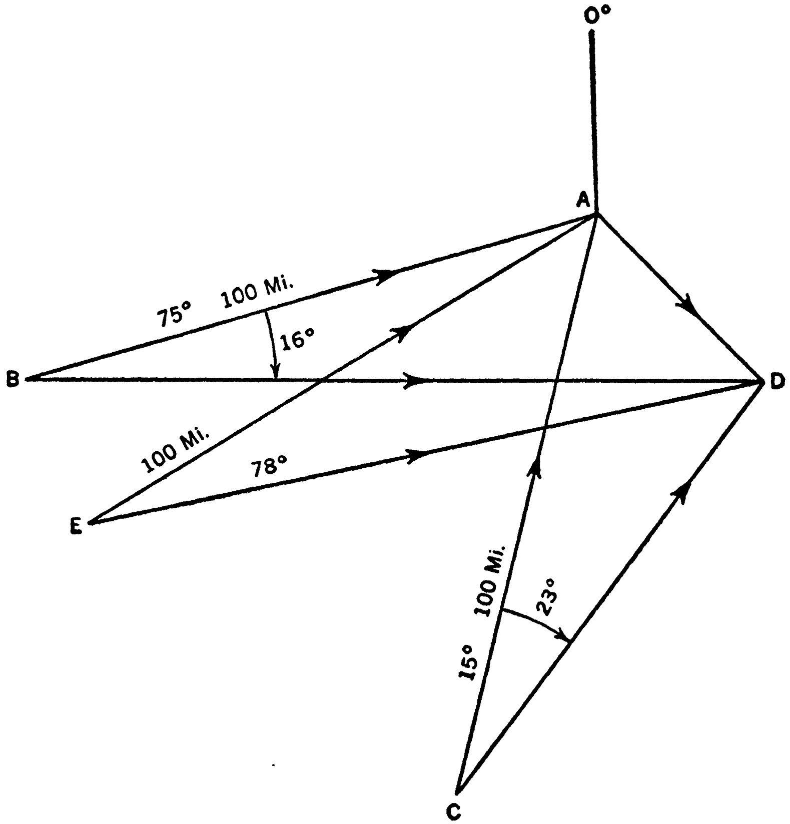

FIGURE 28. THE WIND STAR.

Example. A plane with an air speed of 100 mi./hr. is to be flown on a track of 78°. By trial it has been found that the drift is 23° to the right, on a heading of 15°, and 16° to the right on a heading of 75°.

This problem may be solved graphically by plotting the reciprocal bearings (see page 25). In Figure 28 draw AB with a bearing of 255° and a length of 100 miles. Since the bearing of AB is 255°, then the bearing of BA = 255° − 180° = 75°. Draw the indefinite line BD, making angle ABD = 16°. Next construct AC with a length of 100 miles, and a bearing of 195° (15° + 180°); lay off angle ACD = 23°. Extend BD and CD to intersect at point D.

If the plane started from B, in calm air, it would arrive at A in one hour; but since it has a 16° right drift on this heading, it follows the track BD. Similarly, if the plane started from C, with no wind, it would reach A in one hour; actually starting from C it will follow the track CD due to the wind. Hence the wind direction and velocity are represented by the bearing and length of AD.

To determine the heading which the plane must take to follow the required track of 78°, construct DE with a bearing of 258°, so that ED represents the track of 78°. Then, with A as a center, and a radius of 100 miles, the air speed, draw an arc intersecting DE at E.

The required heading is the bearing of EA, 58°.
The drift angle is angle AED, 20°.
The ground speed is the length of ED, 113 miles.

This method may be summarized by noting that the bearings of all lines drawn to A represent headings, and that the bearings of all lines drawn to D represent tracks. The lengths of all lines drawn to A represent air speed; and the lengths of all lines drawn to D represent ground speeds. There are various means for solving the wind star mechanically.

Problems

1. An airplane making 120 mi./hr. air speed drifts 15° to the left on a heading of 140°, and 8° to the right on a heading of 40°. Find the heading, ground speed, and angle of drift for a track of 95°.

2. It is desired to fly due west with a plane having an air speed of 80 mi./hr. On a heading of 300° the drift is 9° to the right and on a heading of 240°, 21° to the right. Find the heading, ground speed, and drift angle.

3. Find the heading to track 38°, with a plane making 200 mi./hr. air speed, if the drift is 15° left on a heading of 88°, and 17° left on a heading of 68°.

4. A plane which has an air speed of 175 knots drifts 6° right on a heading of 265°, and 10° right on a heading of 306°. What heading should the plane take in order to track 302°? What ground speed will it have on this heading?

5. It is desired to track 116° with a plane making 200 mi./hr. air speed. The drift is 12° right on a 90° heading, and 15° right on a 60° heading. What heading should the plane take and what ground speed will it have?

6. The air speed of a plane is 150 knots. It drifts 10° to the right on a heading of 290°, and 6° to the right on a heading of 260°. Find the heading, ground speed, and drift for the plane to track 253°.

7. A plane with an air speed of 200 knots heads 53° and drifts 6° to the left; on a heading of 81° it drifts 8° to the left. Find the heading, drift, and ground speed for a track of 102°.

Interception. There is an old rule in navigation which says that "constant bearing means collision." This means that if the bearing of one ship from another is constant, the ships will collide. In Figure 29, let A, A′, A″, etc. represent different positions of a ship following the course AC; and B, B′, B″, etc. the corresponding positions of a second ship following the course BC. If the bearing of the second ship from the first remains constant, then the line connecting the two ships is always parallel to the base AB, and the ships must reach C at the same time.

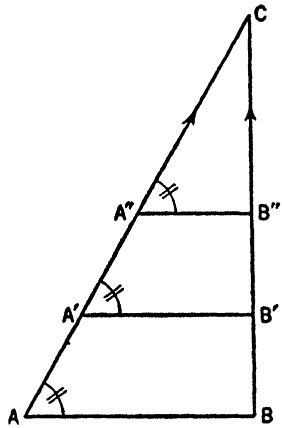

FIGURE 29.

In air navigation the rule about constant bearing may be changed to "constant bearing means interception." The problem of interception has many obvious uses; planes are required to meet for patrol, to intercept hostile aircraft or ships, or to return to a carrier after a flight. The problem of interception is solved graphically.

Example 1. Interception without wind and in less than one hour.

Point A (Figure 30) is 60 miles south and 70 miles east of point B. A plane leaves A on a track of 280° with a speed of 150 mi./hr. At the same time, a second plane with a speed of 130 mi./hr. leaves B. What is the track of the second plane; when and where will the interception occur?

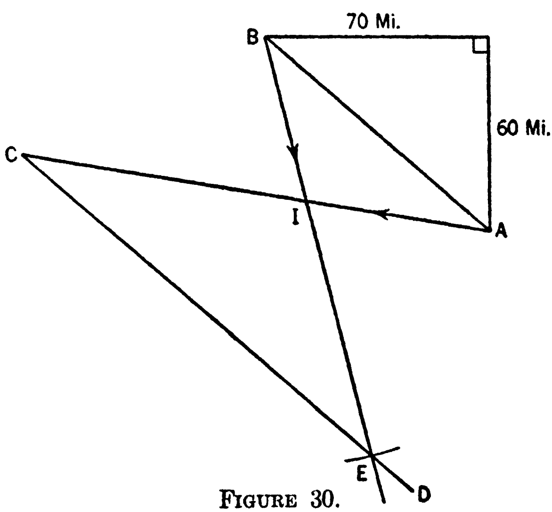

FIGURE 30.

Since there is no wind, track and heading are the same; and air speed and ground speed are the same. Construct the line AC with a bearing of 280° and a length of 150 miles. Through C draw CD parallel to BA. With B as a center and a radius of 130 miles, draw an arc intersecting CD at E. BE represents the track and speed of the plane from B. The interception occurs at point I. The problem may be checked by finding the number of hours required for each plane to reach point I.

Time for plane from B: BI/130 = 52/130 = .40 hour.
Time for plane from A: AI/150 = 60/150 = .40 hour.

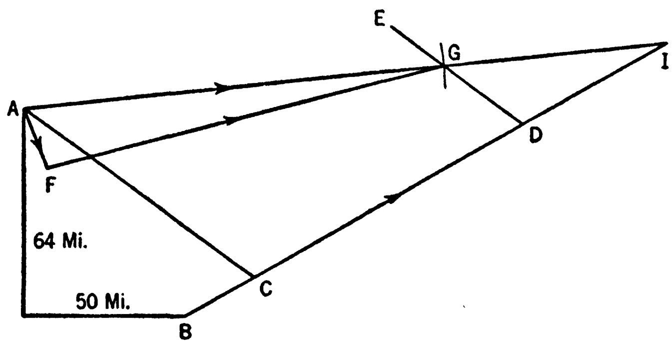

FIGURE 31.

Example 2. Interception with wind and after one hour.

Point B (Figure 31) is 50 miles east and 64 miles south of point A. A plane is observed passing over point B on a track of 60° with a ground speed of 100 mi./hr. There is a 20-mile wind from 340°. Fifteen minutes later a plane is dispatched from A, which has an air speed of 130 mi./hr. Find the heading, track, and ground speed required for the plane from A to intercept the plane from B.

Construct the line BI with a bearing of 60° to represent the track of the plane from B. Make BC = 25 miles, the distance covered by the first plane in the 15-minute interval; then lay off CD = 100 miles, the distance covered by the first plane in one hour. Draw DE parallel to CA. Construct AF to represent the wind in one hour, 20 miles from 340°. With F as a center and a radius equal to 130 miles, draw an arc intersecting DE at G. AFG is the usual wind triangle for the plane from A; FG representing heading and air speed; AG the track and ground speed. Extend AG to intersect BI at point I. I is the point of interception.

The heading of the pursuing plane is the bearing of FG, 75°.

The track of the pursuing plane is the bearing of AG, 84°.

Its ground speed is the length of AG, 134 mi./hr.

Time for plane from A to reach I = AI/AG = 203/134 = 1.51 hours.

Time for plane from B to reach I = BI/100 = 176/100 = 1.76 hours.

The difference of .25 hours between the two times is the fifteen-minute start of the plane from B.

The problem of interception depends on the intersection of a line and a circle. In example 2, the circle, with center F and radius 130, intersects the line DE. In some cases the circle will not intersect the line, and then no interception is possible. In other cases there may be two intersections; the two intersections, however, do not always give two solutions for the problem of interception.

Problems

1. In example 1, change the heading of the plane from A to 252°. Show that interception takes place in one hour.

2. Show that interception is impossible in example 1 if the heading of the plane from A is changed to 240°.

3. Point A is 59 miles east and 70 miles north of B. A plane leaves A on a heading of 250° at 140 mi./hr. air speed. At the same time a plane leaves B at 110 mi./hr. There is no wind. Find the heading of the plane from B and the time of interception.

4. B is 60 miles east and 90 miles south of A. There is a 40-mile wind from 315°. A plane from B heads 60° at 120 knots air speed. Fifteen minutes later a plane leaves A at 175 knots air speed. Find the tracks and ground speeds of the two planes and the time required for interception.

5. A plane is sighted over Montauk Point on Long Island, heading 240° and making 150 miles per hour. Fifteen minutes later a plane is to be dispatched from Mitchel Field to intercept the first plane. The pursuing plane is also capable of making 150 miles per hour. Mitchel Field is 80.9 miles east and 19 miles south of Montauk Point. Find the heading of the pursuing plane and how far from Mitchel Field the interception takes place.

6. A is 95 miles north and 45 miles east of B. A plane is to start from B on a track of 300° and has a ground speed of 90 miles an hour. At the same time a plane is to take off from A to join the plane from B. The plane from A has an air speed of 130 miles an hour. There is a 26-mile wind from the direction of 40°. What heading should the plane from A take, and where will the interception occur?

7. B is 74 miles north and 51 miles west of A. A plane leaves A and flies due east at 90 miles an hour. At the same time a second plane making 120 miles per hour leaves B to overtake the first plane. There is no wind. Find the heading the pursuing plane should take, and determine where the interception occurs.

8. Solve problem 7 if there is a 30-mile wind from 225°.

Radius of Action Returning to a Fixed Base. It is frequently necessary to determine how far a plane can fly from a given point, and return within a fixed time. The total time which is allowed for flight depends upon the fuel supply. If there is no wind, the distance the plane can fly is determined at once by the air speed, and the "time out" will be equal to the "time in." When there is a wind, the ground speed on the outbound trip will be different from the ground speed of the return flight. In this case it is necessary to determine these two ground speeds, and use them to calculate the distance, called *radius of action,* which the plane may safely fly.

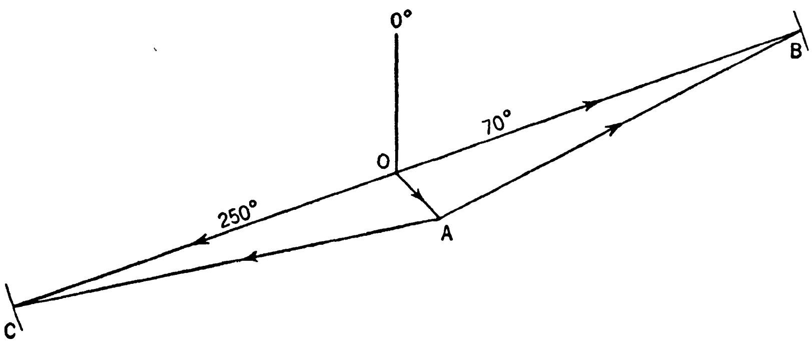

FIGURE 32. RADIUS OF ACTION — FIXED BASE.

Example. A plane is ordered to leave point O (Figure 32) and scout in a direction 70°. The plane has an air speed of 225 mi./hr., and the total time of flight is to be three hours. There is a 30-mile wind from 315°. Find the radius of action.

The first step is to find the ground speeds for the track out (70°) and the track in (250°), under the given wind conditions. These two triangles may be constructed together as is shown in Figure 32. Draw OA to represent the wind, and the line COB to represent the tracks of 70° and 250°. From A as a center, and with a radius equal to the air speed of 225 mi./hr., draw arcs intersecting CB at C and B.

The ground speed out or $g_1 = 240$ mi./hr.
The ground speed in or $g_2 = 214$ mi./hr.

Denote the time out by t_1, the time in by t_2, the total time by T, and the radius of action by R, then:

$$t_1 = \frac{R}{g_1}, \quad t_2 = \frac{R}{g_2}$$

$$T = t_1 + t_2 = \frac{R}{g_1} + \frac{R}{g_2}$$

$$T = \frac{R(g_1 + g_2)}{g_1 g_2}$$

$$R = \frac{T g_1 g_2}{g_1 + g_2}$$

For this problem, $T = 3$ hours, $g_1 = 240$ mi./hr., $g_2 = 213$ mi./hr., and

$$R = \frac{3(240)(213)}{453} = 339 \text{ miles}$$

$$t_1 = \frac{T g_2}{g_1 + g_2} = \frac{3(213)}{453} = 1.41 \text{ hours}$$

$$t_2 = \frac{T g_1}{g_1 + g_2} = \frac{3(240)}{453} = 1.59 \text{ hours}$$

Hence the plane follows the track 70° for 1.41 hours with a ground speed of 240 mi./hr.; it turns and follows the return track of 250° for 1.59 hours with a ground speed of 213 mi./hr.

In all of the problems which involve wind velocity, the answer obtained is valid for the given wind conditions, and any change in the wind requires that the problem be reworked with the new data.

Problems

Find the radius of action for each of the following:

	Total Time	*Track Out*	*Air Speed*	*Wind*
1.	4 hours	30°	160 knots	40 miles from 90°
2.	4 hours	30°	160 knots	40 miles from 270°
3.	4 hours	210°	160 knots	40 miles from 45°
4.	3 hours	210°	160 knots	40 miles from 45°
5.	3 hours	330°	200 mi./hr.	20 miles from 220°
6.	2.5 hours	195°	175 mi./hr.	15 miles from 70°
7.	3 hours	130°	140 mi./hr.	20 miles from 100°
8.	2 hours	75°	200 knots	15 miles from 0°
9.	2 hours	255°	200 knots	15 miles from 180°

Radius of Action Returning to a Moving Base. When a plane leaves a carrier, the method of the last section does not apply, since the carrier will not remain in a fixed position. The problem of returning to a moving base is a combination of the simple wind triangle and the problem of interception.

Example. A carrier is following a course of 0° with a speed of 20 knots. There is a 25-knot wind from 310°. A plane with 90-knot air speed and fuel for 4 hours is ordered to scout on a track of 60°. How long may the plane scout in this direction, before changing course to return to the carrier?

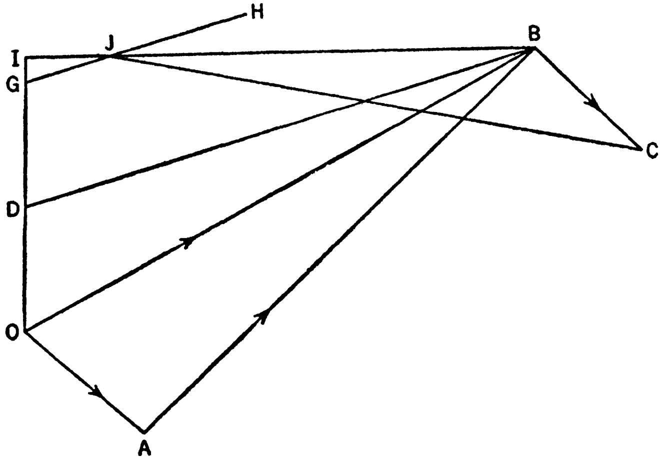

FIGURE 33. RADIUS OF ACTION — MOVING BASE.

The problem is first solved in terms of a one-hour flight from the carrier. In Figure 33, triangle OAB is constructed to determine the heading and ground speed for the 60° track from point O. The ground speed out is 95 knots (OB) and the heading 45° (AB). At the end of one hour the plane is at point B and the carrier at D; D being 20 miles north of O. It is required to find the time for the plane to intercept the carrier. Construct DG = 20 miles and GH parallel to DB. BCJ is the usual wind triangle constructed to determine the heading, track, and ground speed for the plane from point B. The time required for the plane from B to intercept the carrier is

$$BI/BJ = 82/69 = 1.19 \text{ hours.}$$

Hence, if the plane were to return to the carrier after following a track of 60° for only one hour, it would take 1.19 hours for it to return. The times out and in for the flight of four hours are easily obtained from the times for the flight of 2.19 hours.

$$\frac{\text{Total time}}{2.19} = \frac{4}{2.19} = \frac{\text{time out}}{1.00} = \frac{\text{time in}}{1.19}.$$

$$\text{Time out} = 1.83 \text{ hours.}$$
$$\text{Time in} = 2.17 \text{ hours.}$$

The ground speed out is the length of OB, or 95 knots; the ground speed in is the length of BJ, 69 knots.

The distance out is $95 \times 1.83 = 174$ miles.
The distance in is $69 \times 2.17 = 150$ miles.

The problem of returning to a moving base is essentially the same as that of starting from one fixed base and returning to a second fixed base.

Problems

1. A carrier is following a course of 130° at 25 knots. A plane with an air speed of 160 knots is ordered to patrol on a track of 30°, and return in four hours. There is a 30-knot wind from 90°. Find the time out, and the ground speeds out and in.

2. Answer problem 1 if the track out is 60°.

3. Answer problem 1 if the track out is 90°.

4. Answer problem 1 if the track out is 30° and the wind is 20 knots from 120°.

5. A carrier is following a course of 75° at 22 knots. A plane is to scout 3 hours on a track of 350°. The plane has an air speed of 120 knots and the wind is 15 knots from 270°. Find the time out, and the ground speeds out and in.

6. Answer problem 5 if the track is 10°.

7. Answer problem 5 if the wind is 15 knots from 225°.

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CHAPTER V

SPHERICAL TRIGONOMETRY OF THE EARTH

In Chapter I, the lengths of great circle arcs were found by using a "protractor" on a globe. This method was approximate only; even on a very large globe, it would be impossible to obtain an answer correct to the nearest minute. Finding an accurate answer for a great circle arc is a problem of spherical trigonometry. In problems of piloting, such as doubling the angle on the bow or finding the distance at which an object is passed abeam (page 28), the navigator is using a plane triangle to determine a line which he cannot measure. Spherical triangles on the earth, that is, triangles formed by great circles, are used in a similar manner to find great circle arcs.

Four methods of finding position have been mentioned; they are piloting, dead reckoning, radio navigation, and celestial navigation. Celestial navigation, which is finding latitude and longitude from observations of the sun and stars, also depends on the solution of spherical triangles.

In this chapter two fundamental formulas of spherical trigonometry will be developed and applied to earth problems. These two formulas are all that are necessary for finding great circle arcs, and for working the main problems of celestial navigation. There are many other formulas of spherical trigonometry which are not considered here. The computations of spherical trigonometry are made much shorter if a function called the *haversine* is used. The haversine function will be studied briefly before beginning the study of spherical trigonometry.

The Haversine Function. The sine, cosine, and other functions of an angle may be defined both as ratios and lines. For either of the angles marked A in Figure 34

$$\sin A = y/r \quad \text{and} \quad \cos A = x/r.$$

If $r = 1$, then

$$\sin A = y \quad \text{and} \quad \cos A = x.$$

The distance BD in Figure 34 is $r - x$, or if $r = 1$,

$$BD = 1 - x = 1 - \cos A.$$

BD, like the sine and cosine, is a function of angle A. It was formerly

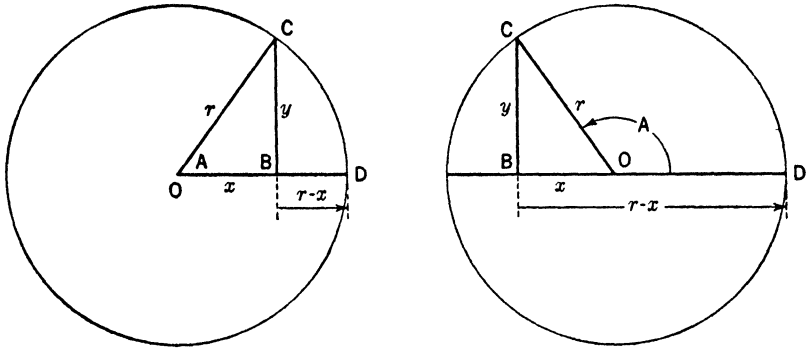

FIGURE 34.

called the sagitta, or arrow, since if CB is extended to intersect the circle, the figure resembles a bow and arrow. Later BD was called the versed sine.

It is convenient to take one half of BD. BD itself will vary from 0 to 2; and hence BD/2 varies from 0 to 1. The function BD/2 is called the *haversine*, half of the versed sine, and is abbreviated *hav*. Hence

$$\text{haversine } A = \text{hav } A = \frac{1 - \cos A}{2}$$

Both logarithmic and natural haversines are used in solving spherical triangles. Haversines may be found from the known values of the natural cosines.

$$\text{hav } 0° = \frac{1 - \cos 0°}{2} = \frac{1 - 1}{2} = 0$$

$$\text{hav } 60° = \frac{1 - \cos 60°}{2} = \frac{1 - .50}{2} = .25$$

$$\text{hav } 90° = \frac{1 - \cos 90°}{2} = \frac{1 - 0}{2} = .50$$

$$\text{hav } 120° = \frac{1 - \cos 120°}{2} = \frac{1 + .50}{2} = .75$$

$$\text{hav } 180° = \frac{1 - \cos 180°}{2} = \frac{1 + 1}{2} = 1$$

$$\log \text{hav } 60° = \log .25 = 9.39794$$
$$\log \text{hav } 90° = \log .50 = 9.69897$$
$$\log \text{hav } 120° = \log .75 = 9.87506$$

The haversine increases from 0 to 1 as the angle increases from 0° to 180°; the haversines of all angles are positive and never greater than 1.

The haversine is a means of simplifying certain formulas of plane trigonometry. For example, for $\sin\frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}$, one may write $\sin\frac{x}{2} = \sqrt{\text{hav } x}$. The formula $\sin\frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}}$ is used to find angle A in triangle ABC when the three sides are given. The square root may be avoided by writing, $\text{hav A} = \frac{(s - b)(s - c)}{bc}$.

Problems

Solve the following problems without using the haversine tables. Find the natural haversines from the given natural cosines. (Do not write in your text.)

1. Prove that hav $(-x)$ = hav x and that hav $(360° - x)$ = hav x.

2. Complete the following table:

x	*Cos x*	*Hav x*
0°	1.000	------
30°	.866	------
45°	.707	------
60°	.500	------
90°	.000	------
120°	− .500	------
135°	− .707	------
150°	− .866	------
180°	− 1.000	------

3. From the results of problem 2 find the values of the haversines of 210°, 225°, 240°, 270°, 300°, 315°, 330° without further computation.

4. Use the results of problems 2 and 3 to plot the graph of y = hav x from $x = 0$ to $x = 360°$.

5. Complete the following:

x	*Cos x*	*Hav x*	*Log Hav x*
3° 37′	.99801	------	------
3° 38′	.99799	------	------
11° 28′	.98004	------	------
11° 29′	.97998	------	------
36° 52′	.80003	------	------
36° 53′	.79986	------	------

6. From the results of problem 5 complete the following.

(*a*) Log haversines of angle between 36° 53′ and 180° have a characteristic of ------.

(*b*) Log haversines of angles between ------ and ------ have a characteristic of 8 − 10.

(*c*) Log haversines of angles between ...... and ...... have a characteristic of 7 − 10.

7. Complete the following table.

x	*Cos x*	*Log Hav x*	*Hav x*
53° 00′	.60182	......	......
53° 10′	.59949	......	......
53° 20′	.59716	......	......

The Haversine Table. Pages xliv–xlviii contain a table of haversines for angles from 0° to 180°, at ten-minute intervals. The angle, its log haversine, and the natural haversine, are printed in adjacent columns. On page xlv the first angle is 40° 00′, printed 40°. Hence from page xlv:

log hav 40° 00′ = 9.06810
nat hav 40° 00′ = .11698
log hav 40° 10′ = 9.07157
nat hav 40° 10′ = .11791

On page xliv the characteristic of the log haversine varies from 4 − 10 to 9 − 10; the characteristic is printed for each angle on this page. On pages xlv–xlviii all the characteristics (except for the three last entries) are 9 − 10 and are printed only in the headings of the columns. In using the haversine table, it is necessary to interpolate to the nearest minute.

Example 1. Find log hav 36° 54′ and hav 36° 54′.

To find the log haversine 36° 54′, find the difference of log hav 36° 50′ and log hav 37° 00′, multiply this difference by .4, and add to log hav 36° 50′. To find nat hav 36° 54′, subtract the natural haversines of 36° 50′ and 37° 00′, multiply the difference by .4, and add to the natural haversine 36° 50′. This process of interpolation may be represented as follows.

Angle		*Log Hav*		*Nat Hav*	
37° 00′		9.00295		.10068	
[36° 54′		[9.00068		[.10016	
4′	10′	x	378	y	87
[36° 50′		[8.99917		[.09981	

$$\frac{4}{10} = \frac{x}{378} \qquad \frac{4}{10} = \frac{y}{87}$$

$$x = 151 \qquad y = 35$$

This diagram to represent interpolation is intended only as a means of making the process clear. Frequently the arithmetic involved in interpolation can be performed mentally; in any case the written work of interpolation should be reduced to a minimum.

Example 2. Find angle x if log hav x = 9.04270.

From page xliv 9.04270 is between the log haversines of 38° 50′ and 38° 40′.

$$\log \text{hav } 38^\circ 50' - \log \text{hav } 38^\circ 40' = .00359$$
$$\log \text{hav } x - \log \text{hav } 38^\circ 40' = .00288$$
$$\tfrac{288}{359} \times 10' = 8'$$
$$x = 38^\circ 40' + 8' = 38^\circ 48'$$

To clarify the process, the example may be represented in a manner similar to example 1.

	Angle			*Log Hav*	
	38° 50′			9.04341	
	x			9.04270	
y		10′	288		359
	38° 40′			9.03982	

$$\frac{y}{10} = \frac{288}{359}$$
$$y = 8'$$

Problems

Use the haversine table to solve the following problems.

1. Find the log and natural haversines of the following angles: (*a*) 110° 32′, (*b*) 56° 47′, (*c*) 11° 25′, (*d*) 15° 18′, (*e*) 97° 45′, (*f*) 3° 34′, (*g*) 27° 33′, (*h*) 36° 57′, (*i*) 119° 43′, (*j*) 179° 57′.

2. Find the angle corresponding to the following log haversines: (*a*) 9.18998, (*b*) 9.90621, (*c*) 8.23331, (*d*) 9.91067, (*e*) 6.93166, (*f*) 9.99806, (*g*) 7.14506, (*h*) 8.10216, (*i*) 9.11227, (*j*) 9.45725.

3. Find the angle corresponding to the following natural haversines: (*a*) .80594, (*b*) .97994, (*c*) .00186, (*d*) .76495, (*e*) .63472, (*f*) .01579, (*g*) .64693, (*h*) .84231, (*i*) .00942, (*j*) .79832.

The Haversine Formula for Great Circle Arcs. In plane trigonometry the Law of Cosines, $c^2 = a^2 + b^2 - 2\,ab \cos C$, is used to find the third side of a triangle when the other two sides and their included angle are given. The purpose of this section is to develop a formula for finding the third side of a spherical triangle when two sides and the included angle are given.

Spherical triangles are formed on the surface of a sphere by great circle arcs. The sides and angles of spherical triangles are both measured in degrees and minutes (see Triangles on the Earth, pages 8–9).

Plane triangles are used in proving relationships between the sides and angles of spherical triangles.

Figure 35 represents a spherical triangle drawn on a sphere with center O and radius r. The sides BC, CA, and AB are denoted by a, b, and c. The plane triangle CDE is formed by drawing the tangents to AC and BC at C; these tangents meet OA and OB extended at D and E. The four plane triangles in Figure 35 are drawn separately in Figure 36. Angle DOC equals the arc AC or b; angle OCD is a right angle; hence in triangle COD,

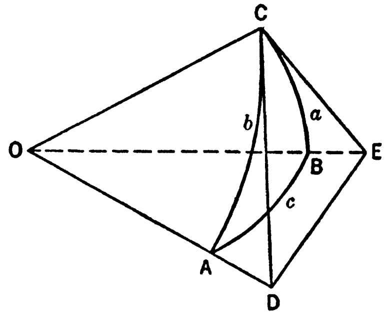

FIGURE 35.

$$CD = r \tan b \quad \text{and} \quad OD = r \sec b$$

Similarly in triangle COE,

$$CE = r \tan a \quad \text{and} \quad OE = r \sec a$$

Angle C, in triangle CDE, is the angle between the two tangents and is equal to angle C in the spherical triangle. Angle EOD equals the arc AB or c.

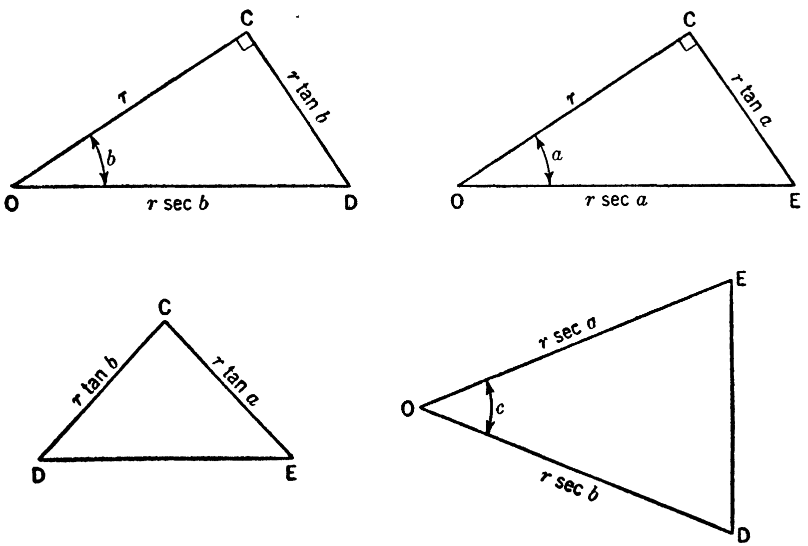

FIGURE 36.

Two sides of both triangles CDE and EOD are known from the right triangles OCD and COE. Applying the Law of Cosines to EOD,

$$\overline{DE}^2 = r^2 \sec^2 a + r^2 \sec^2 b - 2\, r^2 \sec a \sec b \cos c. \qquad (1)$$

Applying the Law of Cosines to CDE,

$$\overline{DE}^2 = r^2 \tan^2 a + r^2 \tan^2 b - 2\, r^2 \tan a \tan b \cos C. \qquad (2)$$

Subtracting (2) from (1) and dividing by r^2,

$$0 = \sec^2 a - \tan^2 a + \sec^2 b - \tan^2 b - 2 \sec a \sec b \cos c + 2 \tan a \tan b \cos C.$$

Applying the formulas $\sec^2 a - \tan^2 a = 1$ and $\sec^2 b - \tan^2 b = 1$,

$$0 = 2 - 2 \sec a \sec b \cos c + 2 \tan a \tan b \cos C.$$

Transposing and dividing by 2,

$$\sec a \sec b \cos c = 1 + \tan a \tan b \cos C.$$

Multiplying both sides by $\cos a \cos b$,

$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \qquad (3)$$

Formula (3) is called the *sine-cosine formula* and may be used to find side c from a, b, and C; or to find angle C, if the three sides are given. In the sine-cosine formula, $\cos c$ is the sum of two products. Either product may be positive or negative; $\cos a \cos b$ is negative when either a or b is obtuse; $\sin a \sin b \cos C$ is negative when C is obtuse.

The sine-cosine formula is not well adapted for computation by logarithms. Since $\operatorname{hav} c = \dfrac{1 - \cos c}{2}$, $\cos c$ may be replaced by $1 - 2 \operatorname{hav} c$, and $\cos C$ by $1 - 2 \operatorname{hav} C$. A shorter and more convenient form is easily obtained by making these substitutions for $\cos c$ and $\cos C$.

$$1 - 2 \operatorname{hav} c = \cos a \cos b + \sin a \sin b\,(1 - 2 \operatorname{hav} C).$$
$$1 - 2 \operatorname{hav} c = \cos a \cos b + \sin a \sin b - 2 \sin a \sin b \operatorname{hav} C.$$

But $\cos a \cos b + \sin a \sin b = \cos (a - b) = 1 - 2 \operatorname{hav} (a - b)$.

Hence, $1 - 2 \operatorname{hav} c = - 2 \sin a \sin b \operatorname{hav} C + 1 - 2 \operatorname{hav} (a - b)$.

This reduces to

$$\operatorname{hav} c = \sin a \sin b \operatorname{hav} C + \operatorname{hav} (a - b)$$

the *haversine formula* for spherical triangles.

Problems

1. Prove that $\cos b = \cos a \cos c + \sin a \sin c \cos B$ and that $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

2. Prove that $\operatorname{hav} b = \sin a \sin c \operatorname{hav} B + \operatorname{hav} (a - c)$ and that $\operatorname{hav} a = \sin b \sin c \operatorname{hav} A + \operatorname{hav} (b - c)$.

3. Find formulas for cos C, cos B, cos A from the sine-cosine formula.

4. Find formulas for hav C, hav B, and hav A from the haversine formula.

Great Circle Distances on the Earth. When the latitudes and longitudes of two places are given, the haversine formula may be used to find the great circle arc which joins them. If the two latitudes are denoted by L_1 and L_2, the difference of longitude by DLo, and the great circle arc by D; then the haversine formula becomes

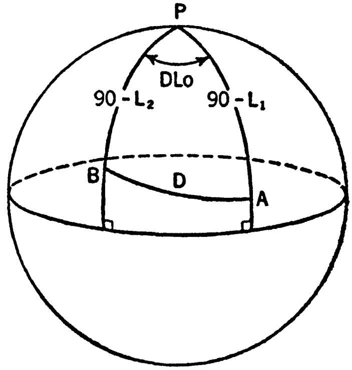

FIGURE 37

$$\text{hav } D = \cos L_1 \cos L_2 \text{ hav DLo} + \text{hav } (L_1 - L_2)$$

This formula is easily obtained from the haversine formula of the last section. In Figure 37 triangle PAB is formed by two points and the pole.

$$PB = 90° - \text{latitude of B} = 90° - L_2$$
$$PA = 90° - \text{latitude of A} = 90° - L_1$$
$$P = \text{difference of longitude} = \text{DLo}$$
$$AB = \text{required arc} = D$$
$$\sin (PA) = \sin (90° - L_1) = \cos L_1$$
$$\sin (PB) = \sin (90° - L_2) = \cos L_2$$
$$\text{hav } (PB - PA) = \text{hav } (90° - L_2 - 90° + L_1) = \text{hav } (L_1 - L_2)$$

Applying the formula $\text{hav } c = \sin a \sin b \text{ hav } C + \text{hav } (a - b)$ to triangle PAB,

$$\text{hav } D = \cos L_1 \cos L_2 \text{ hav DLo} + \text{hav } (L_1 - L_2)$$

Example 1. Find the great circle distance between San Francisco (37° 50′ N., 122° 26′ W.) and Tokyo (35° 39′ N., 139° 45′ E.).

$$L_1 = 37° 50', L_2 = 35° 39', L_1 - L_2 = 2° 11'$$
$$\text{DLo} = 360° - (122° 26' + 139° 45') = 97° 49'$$
$$\text{hav } D = \cos 37° 50' \cos 35° 39' \text{ hav } 97° 49' + \text{hav } 2° 11'$$

log cos L_1	(37° 50′)	=	9.89752
+ log cos L_2	(35° 39′)	=	9.90987
log hav DLo	(97° 49′)	=	9.75435
			9.56174
antilog 9.56174		=	.36453
+ nat hav ($L_1 - L_2$)	(2° 11′)	=	.00036
	nat. hav D	=	.36489
	D	=	74° 20′
	D	=	4460 nautical miles

In using the haversine formula, *south latitudes are negative.* DLo is either the sum, difference, or 360° minus the sum of the longitudes (see DLo under Parallel Sailing, pages 19–20). All the functions in the haversine formula are positive, even when one or both of the latitudes are negative.

Example 2. Find the great circle distance between San Francisco and Tahiti (17° 30′ S., 149° 30′ W.).

$$L_1 = 37° 50', \quad L_2 = -17° 30'$$
$$L_1 - L_2 = 37° 50' - (-17° 30') = 55° 17'$$
$$\text{DLo} = 149° 30' - 122° 26' = 27° 04'$$
$$\cos L_2 = \cos(-17° 30') = \cos 17° 30'$$

log cos L_1	(37° 50′)	= 9.89781
+ log cos L_2	(17° 30′)	= 9.97942
log hav DLo	(27° 04′)	= 8.73847
		8.61570
antilog 8.61570		= .04128
nat hav ($L_1 - L_2$)	(55° 17′)	= .21524
	nat hav D	= .25652
	D	= 60° 52′
	D	= 3652 nautical miles

In some cases $L_1 - L_2$ is negative, but since hav $(-x)$ = hav x, hav $(L_1 - L_2)$ is positive.

Problems

1. Show that the computation is unchanged in examples 1 and 2 if L_1 and L_2 are interchanged.

Find the great circle distance between the two given places in each of the following problems.

2. Harbour Grace (47° 43′ N., 53° 08′ W.) to Moscow (55° 45′ N., 37° 34′ E.).

3. New York (40° 43′ N., 74° 00′ W.) to Cape of Good Hope (34° 21′ S., 18° 30′ E.).

4. From Manila (14° 36′ N., 120° 57′ E.) to Hong Kong (22° 16′ N., 114° 10′ E.).

5. From Oporto Light (41° 09′ N., 8° 41′ W.) to Block Island Light (41° 09′ N., 71° 33′ W.). Compare this answer with the distance along the parallel of 41° 09′ N.

6. From Port Elizabeth (33° 58′ S., 25° 37′ E.) to Freemantle (32° 03′ S., 115° 44′ E.).

7. Auckland (36° 50′ S., 174° 46′ E.) to Sitka (57° 03′ N., 135° 20′ W.).

8. Pitcairn Island (25° 04′ S., 130° 08′ W.) to Easter Island (27° 10′ S., 109° 26′ W.).

9. Lakehurst (40° 01′ N., 74° 20′ W.) to Frankfurt-am-Main (50° 07′ N., 8° 39′ E.).

10. White Island, New Zealand (37° 30′ S., 177° 11′ E.) to Cape Howe, Victoria (37° 30′ S., 149° 59′ E.). Compare with example 1, page 20.

11. Choshi, Japan (35° 42′ N., 140° 52′ E.) to Cape Horn (55° 59′ S., 67° 16′ W.).

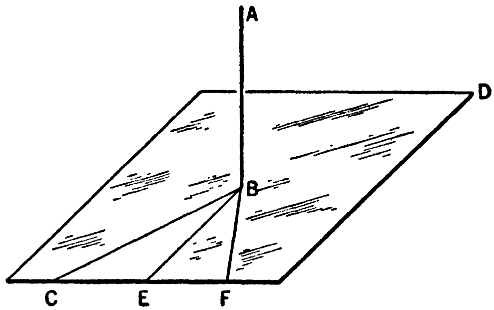

FIGURE 38.

12. If the great circle arc of problem 11 is continued, it will intersect the coast of Africa at (8° 10′ N., 13° 06′ W.) between Cape St. Ann and Cape Sierra Leone, making one of the longest possible great circle courses entirely over water. Find the distance from Choshi to the point on the African coast.

The Sine Formula for Spherical Triangles. Angles in a spherical triangle may be found by a formula which is similar to the Law of Sines for plane triangles. Plane triangles are used in proving this sine formula in much the same way as they were used in proving the haversine formula.

The proof of the sine formula depends upon a theorem of solid geometry which is illustrated in Figure 38. If a given line is perpendicular to a plane, it is perpendicular to every line in the plane which passes through the foot of the given line. In Figure 38, AB is perpendicular to the plane CD; the theorem states that AB is also perpendicular to BC, BE, BF, etc.

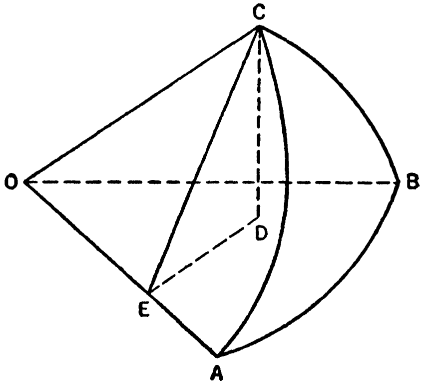

FIGURE 39.

In Figure 39, ABC is any spherical triangle drawn on a sphere with center O. Two plane right triangles are formed by the following constructions. Draw CD perpendicular to the plane OAB; construct the plane CDE perpendicular to OA. From the

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theorem stated above, CDE and OCE are right triangles. Angle A in the spherical triangle is the angle between the tangents to CA and BA at A; the lines CE and DE are parallel to these tangents. Hence CED equals angle A.

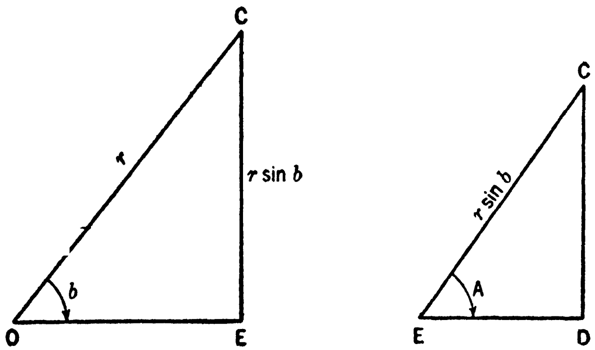

FIGURE 40.

In Figure 40 the right triangles COE and CED are drawn separately.

From COE, $\quad CE = r \sin b.$

From CED, $\quad CD = \overline{CE} \sin A = r \sin b \sin A.$

If a plane is constructed through CD perpendicular to OB, then it could be proved that

$$CD = r \sin a \sin B.$$

Hence, $\quad CD = r \sin b \sin A = r \sin a \sin B.$

Then, $\quad \dfrac{\sin A}{\sin a} = \dfrac{\sin B}{\sin b}$

Since A and B are any two angles of ABC,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

This is the *sine formula* for spherical triangles. The sine formula may be used to find the angles A and B in Figure 37, after the arc D has been found.

$$\frac{\sin A}{\cos L_2} = \frac{\sin B}{\cos L_1} = \frac{\sin DLo}{\sin D}$$

$$\sin A = \frac{\sin DLo \cos L_2}{\sin D} \qquad \sin B = \frac{\sin DLo \cos L_1}{\sin D}$$

Example 1. Find the angles in the spherical triangle formed by the pole, San Francisco, and Tokyo.

Let A be the angle at San Francisco, and B the angle at Tokyo. From example 1, page 53, $L_1 = 37° 50'$, $L_2 = 35° 39'$, $DLo = 97° 49'$, $D = 74° 20'$.

$$\sin A = \frac{\sin DLo \cos L_2}{\sin D} = \frac{\sin 97° 49' \cos 35° 39'}{\sin 74° 20'} \qquad \sin B = \frac{\sin DLo \cos L_1}{\sin D} = \frac{\sin 97° 49' \cos 37° 50'}{\sin 74° 20'}$$

$$\begin{aligned}
&\log \sin DLo \ (97° 49') &= 9.99595\\
-&\log \sin D \ (74° 20') &= \underline{9.98356}\\
&\log \text{quotient} &= 0.01239\\
+&\log \cos L_2 \ (35° 39') &= \underline{9.90987}\\
&\log \sin A &= 9.92226\\
&A &= 56° 44'\\
&\log \text{quotient} &= 0.01237\\
&\log \cos L_1 \ (37° 50') &= \underline{9.89752}\\
&\log \sin B &= 9.90989\\
&B &= 54° 21'
\end{aligned}$$

Since $\sin A = \sin (180° - A)$, the sine function cannot distinguish between acute angles and their supplements. In the above example $\log \sin A = 9.92226$, hence A equals either $56° 44'$ or $180° - 56° 44'$. This ambiguity may be avoided by using other rules of spherical trigonometry. The haversine formula may be written

$$\text{hav } C = \frac{\text{hav } c - \text{hav } (a - b)}{\sin a \sin b}$$

and used to find the angles. However, it is seldom necessary to use other rules. If the triangle is marked out on a globe with the "great circle protractor" used in Chapter I, it is easy to tell whether the angles are acute or obtuse. In example 1, it is obvious from the globe that both of the angles are acute.

The angles found by the sine formula give the *bearing* of the great circle at A and B. However, the direction or bearing of the great circle changes continually. Only the *initial course* can be obtained from the values of A and B.

Example 2. Find the angles in the triangle formed by San Francisco (A), Tahiti (B), and the pole.

From example 2, page 54, $L_1 = 37° 50'$, $L_2 = -17° 30'$, $DLo = 27° 04'$, $D = 60° 52'$.

Here it is obvious that angle A is obtuse and B acute.

$$\sin A = \frac{\sin 27^\circ 04' \cos 17^\circ 30'}{\sin 60^\circ 52'} \qquad \sin B = \frac{\sin 27^\circ 04' \cos 37^\circ 50'}{\sin 60^\circ 52'}$$

$$\begin{aligned}
&\log \sin \text{DLo} \quad (27^\circ 04') &= 9.65804 \\
-\ &\log \sin \text{D} \quad (60^\circ 52') &= \underline{9.94126} \\
&\log \text{quotient} &= 9.71678 \\
+\ &\log \cos L_2 \quad (17^\circ 30') &= \underline{9.97942} \\
&\log \sin A &= 9.69620 \\
&\qquad\qquad\qquad A &= 180^\circ - 29^\circ 47' = 150^\circ 13' \\
&\log \text{quotient} &= 9.71678 \\
+\ &\log \cos L_1 \quad (37^\circ 50') &= \underline{9.89752} \\
&\log \sin B &= 9.61430 \\
&\qquad\qquad\qquad B &= 24^\circ 18'
\end{aligned}$$

Problems

Use the results of the problems on page 54 and find the angles of the spherical triangles formed by the places named and the north pole. In each case verify that the sum of the three angles is between 180° and 540°.

1. The triangle of problem 2.
2. The triangle of problem 3.
3. The triangle of problem 4.
4. The triangle of problem 5.
5. The triangle of problem 6.
6. The triangle of problem 7.
7. The triangle of problem 8.
8. The triangle of problem 9.

Bibliography

Great Circle Sailing: Dutton, Chapter VI; Bowditch, 105–108; Bradley, A. D., *Proceedings of the U. S. Naval Institute*, Feb. 1941, 175–178.

Great Circle Charts: Great circle charts (the gnomonic projection) show great circles as straight lines. Great circle charts of the North Atlantic, South Atlantic, North Pacific, South Pacific, and Indian Oceans are published by the U. S. Hydrographic Office. The U. S. Coast and Geodetic Survey publishes a *Great Circle Chart of the United States.*

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CHAPTER VI

NAUTICAL ASTRONOMY

For centuries, people living in the northern latitudes have known that the North Star appears to have a fixed position in the heavens. From very early times, observations of the North Star have been used to determine latitude and to fix directions on the earth. It is common knowledge that the stars appear to move across the sky from east to west, and that each star appears to describe a circle around the North Star. Certain stars are seen to rise and set each day, and others are always observed to be above the horizon. In reality this daily motion of the stars is caused by the turning of the earth on its axis. To an observer, however, it seems that the earth is fixed and that the rising and setting of the stars is caused by a rotation of the sky.

It was long believed that the earth was stationary and that the stars were fixed points on an immense sphere, having the same center as the earth. While it is now known that this sphere is purely imaginary, it is convenient to use it for many problems of astronomy and celestial navigation. The daily motion of the stars may be explained by a daily rotation of this imaginary sphere; the stars and planets may be located on its surface by co-ordinates, just as points on the earth are located by latitude and longitude.

The Celestial Sphere. The immense sphere on which the stars appear to be located is called the *celestial sphere.* If the polar axis of the earth, *pp′* in Figure 41 (page 60), is extended, it intersects the celestial sphere in two points P and P′. P and P′ are the *north* and *south celestial poles.* The North Star, or Polaris, is very near the north celestial pole. The *celestial equator* is a great circle of the celestial sphere midway between the celestial poles. The celestial equator is marked EE′ in Figure 41.

The point on the celestial sphere which is directly overhead is called the *zenith;* the point diametrically opposite the zenith is called the *nadir.* In Figure 41, ZA is the diameter of the celestial sphere which joins zenith and nadir. Observers at different positions on the earth will not have the same zenith. To fix the position of the zenith for a given point *a* on the earth, draw the radius of the earth, *Oa*, and extend it to intersect the celestial sphere in point Z. Midway between the zenith and nadir is a great circle called the *celestial horizon*, NS in Figure 41.

If one were to move northward, the pole and zenith would become nearer together, that is, the arc PZ in Figure 41 would decrease. The angle between the equator and horizon, or arc ES, would also decrease. At the north pole, P would coincide with Z, and equator and horizon would also coincide. On the earth's equator, P and N would coincide, and the celestial equator would be perpendicular to the horizon. For southern latitudes the north pole, P, would be below the horizon and the south pole, P′, would be elevated above it.

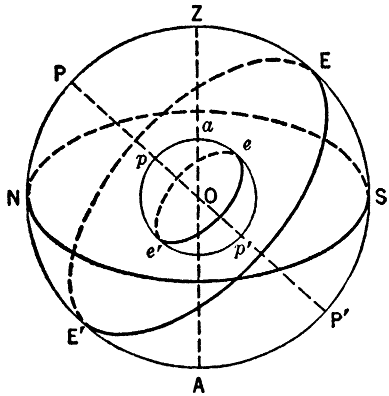

FIGURE 41.

The arc *ea* on the earth is the observer's latitude. The arc EZ, measured from the celestial equator to the zenith, is equal to the observer's latitude.

Problems

1. Show that the angle of elevation of the pole, arc NP in Figure 41, is the observer's latitude.

2. Draw figures like Figure 41 for latitudes of 15° N. and 75° N. Indicate the poles, celestial equator, zenith, and celestial horizon.

3. Repeat problem 2 for latitudes of 15° S. and 75° S.

4. Make a drawing of the celestial sphere in which the zenith and north pole coincide, that is, for latitude 90° N.

5. Draw a figure showing how the celestial sphere appears to an observer on the equator.

Co-ordinates on the Celestial Sphere; Right Ascension and Declination. The stars appear to have fixed positions on the celestial sphere, just as cities on the earth have a fixed latitude and longitude. The planets, on the other hand, appear to move among the stars or to change their co-ordinates, just as a moving ship on the earth changes its latitude and longitude. In fact, the name *planet* means wanderer. The moon may be seen to change its position among the stars. Observations of the moon on successive nights will show that its angular distances from the stars along its path change rapidly. If the sun and stars could be observed at the same time, the sun would also appear to change its position relative to the stars.

If the changing position of the sun were plotted on the celestial sphere, its course would be found to be a great circle. This great circle is called the *ecliptic,* and makes an angle of 23° 27′ with the equator. The sun moves eastward along the ecliptic and makes a complete journey around it, in one solar year or 365.2422 days. This annual movement of the sun around the ecliptic is, of course, distinct from its daily rising and setting. The sun has two motions; each day it moves from east to west across the sky as the celestial sphere makes a rotation, each year it completes one circuit of the ecliptic. The equator and the ecliptic intersect in two points, A and B, in Figure 42. The sun crosses the equator at point A, in a north-bound direction, about March 21; it crosses the equator again at B, in a south-bound direction, about September 21. Point A is called the *first point of Aries* or the *vernal equinox.* Point B is the autumnal equinox.

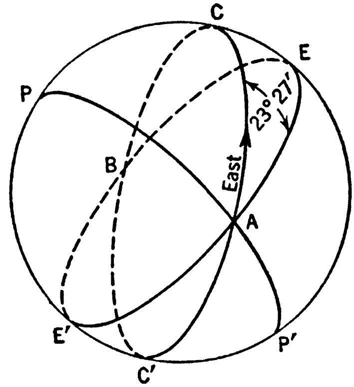

FIGURE 42.

Stars have a co-ordinate system which corresponds closely to latitude and longitude on the earth. The celestial co-ordinate corresponding to latitude is called *declination.* Like latitude, declination is measured north or south from the equator. The declination of all points on the equator is 0°; the north pole has a declination of 90° N., or + 90°; the declination of the south pole is 90° S., or − 90°. All points of a given declination lie on a small circle parallel to the equator.

The celestial co-ordinate corresponding to longitude is called *right ascension.* All the points on the half great circle or meridian PAP′, which is drawn through the vernal equinox, have a right ascension of 0° (Figure 42). Right ascension, unlike longitude, is measured only eastward along the equator, from 0° to 360°. It is also expressed in hours and minutes.

24 hours, or $24^h = 360°$ 4 minutes, or $4^m = 1°$
1 hour, or $1^h = 15°$ 1 minute, or $1^m = 15'$
1 second, or $1^s = 15''$

In Figure 43 the small circle BB′ contains all the points of 60° N. declination. The meridian PCP′ is drawn so that the arc AC = 15°, or 1^h. The meridian PDP′ is drawn so that the arc ACED = 330°, or 22^h. The star S_1 has a declination 60° N. and a right ascension of 0^h. The co-ordinates of the star S_2 are 60° N. and 1^h. The star S_3 has the co-ordinates 60° N. and 22^h.

For some purposes a co-ordinate called *sidereal hour angle* is used in place of right ascension. Sidereal hour angle is measured *westward* along the equator, and is usually expressed in degrees and minutes, not in hours and minutes.

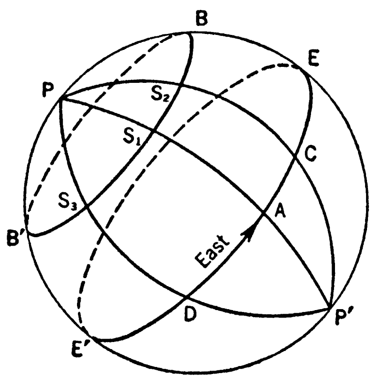

FIGURE 43.

One advantage of the co-ordinates right ascension and declination is that these co-ordinates are nearly constant for the stars. The right ascension and declination of the sun, moon, and planets change. Right ascension and declination are tabulated for the use of the navigator in the *American Nautical Almanac*, published yearly by the United States Naval Observatory.

Problems

1. The star Vega has a right ascension of $18^h\ 35^m$ and a declination of + 38° 44′. Draw a figure similar to Figure 43 and locate Vega by the intersection of its meridian and declination circle.

2. Repeat problem 1 for the following stars:

Star	*Right Ascension* h	m	*Declination* °	′
Mizar	13	22	+ 55	13
Capella	5	12	+ 45	56
Pollux	7	42	+ 28	10
Regulus	10	05	+ 12	15
Altair	19	48	+ 8	43
Antares	16	26	− 26	18
Sirius	6	43	− 16	38
Rigel	5	12	− 8	16

3. Plot the positions of the stars given in problem 2 on a sheet of rectangular graph paper. Let the X axis represent the equator, and mark off the hours of right ascension at equal intervals along it; number the hours, 0 to 24, from *right to left.* Mark the right side of the map *west,* and the left side *east.* Draw the meridians perpendicular to the equator, and use a scale so that 15° of declination has the same length as 1^h of right ascension.

4. Plot the stars of problem 2 on a map centered on the north pole. From the north pole as a center, draw equally spaced circles to represent the circles of declination. Draw radii from the pole, at intervals of 15°, to represent the meridians. Number the meridians counterclockwise from 0^h to 23^h.

Star Maps. Maps of the celestial sphere may be made in the same way as maps of the earth. In problems 3 and 4 of the last section, two kinds of star maps were made. One had the north pole as center; on the other the meridians and circles of declination were represented by straight lines. The same difficulties occur in making star maps as in making maps of the earth. It is inevitable that distances and directions are distorted. In the map of problem 3, for example, the lines representing the circles of declination are too long. In the map of problem 4, the distances along the meridians are correct, but the distances along the circles of declination are not.

In star maps of the type constructed in problem 3, the *right* side of the map is the west side. This map may be placed in its proper position by facing south and holding it over the head; this makes the right ascension lines point to the poles, and brings the east and west sides of the map in their proper position. The map of problem 4 should also be held over the head.

In orienting a star map, it is necessary to determine which one of the right ascension lines coincides with the observer's meridian. At some time during the twenty-four hours of the day each one of the right ascension lines coincides with the celestial meridian of any given place. The easiest method of finding the right ascension of the meridian is by using a small star map known as a *planisphere.* The planisphere is similar to the polar map of problem 4, but it is arranged so that the time of day may be set opposite the date. This determines the right ascension of the meridian mechanically.

An approximate method for finding the right ascension of the observer's meridian is as follows:

> Multiply the number of months since September 21 by two and add to the number of hours since midnight.

Example 1. Find the right ascension of the meridian for 10 : 00 P.M. local time on August 11.

10 : 00 P.M. = 22 hours since midnight.
Sept. 21 to Aug. 11 = 10.7 months.
$10.7 \times 2 + 22 = 43.4$ hours.

Since 43.4 is more than 24, the right ascension of the meridian is $43.4 - 24 = 19.4$ hours approximately.

Example 2. Find the right ascension of the meridian for 4 : 00 A.M. local time on December 31.

4 : 00 A.M. = 4 hours since midnight.
Sept. 21 to Dec. 31 = 3.3 months.
$3.3 \times 2 + 4 = 10.6$ hours approximately.

This method is approximate only but is sufficiently accurate for orienting a star map. In using this rule, standard time should be used and not daylight saving or war time.

Problems

1. Find the approximate right ascension of the meridian for 9 : 00 P.M. local time for the first of each month.

2. Find the approximate right ascension of the meridian for the fifteenth of each month at 3 : 00 A.M. local time.

3. Plot the positions of the moon for January, 1943, on a star map. Use the moon's right ascension and declination for zero hours (midnight) Greenwich time for each day of the month. This data is given in the 1943 *Nautical Almanac* on pages 40–47. Connect the points with a smooth curve to show the moon's path among the stars.

4. Plot the position of the planet Venus for 1943. Use the right ascension and declination for the first and fifteenth of each month. (*Nautical Almanac*, pages 138–141.)

5. Repeat problem 5 for the planet Mars. (*Nautical Almanac*, pages 142–145.)

Declination and Hour Angle; the Second Co-ordinate System. The co-ordinates right ascension and declination have two advantages. They are independent of the place of observation, and they are nearly constant for the stars. On the other hand, the hours of right ascension are not directly connected with hours of time. For some problems it is convenient to use a co-ordinate called *hour angle* in place of right ascension.

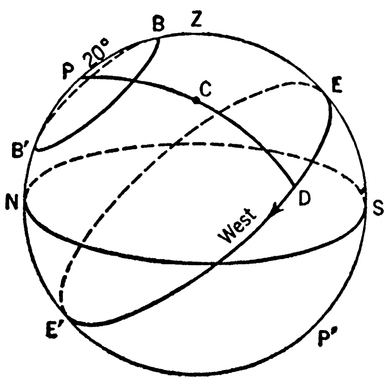

FIGURE 44.

Figure 44 represents the celestial sphere; P and P′ are the poles, EE′ the equator, NS the celestial horizon, and Z the zenith. At some time each day any given star crosses the local meridian PZP′. During the day the star will be carried around the pole in a circle. For example, if the star has a declination of 70° N., it will describe a circle, BB′ in Figure 44, which is 20° from the pole.

When the star crosses the arc of the meridian which contains the zenith, it is said to be making an *upper transit.* B, in Figure 44, shows the star at upper transit. When the star crosses PNP′, it is said to be making a *lower transit.* B′ shows the star at lower transit. For all localities in the United States, both the upper and lower transits of a star of declination 70° N. are above the horizon NS. For certain stars only the upper transit is above the horizon. For other stars, both upper and lower transits are below the horizon. The time between two successive upper transits of a star is about four minutes less than twenty-four hours.

The angle between the meridian of the place of observation and the meridian through the star is called the *star's hour angle.* The hour angle of a star at upper transit is 0°, and at lower transit its hour angle is 180°. When the hour angle of the sun is 0°, it is noon by the sun or *local apparent noon.* Unlike right ascension, the hour angle is measured westward. Suppose a star is at point C, west of the meridian, in Figure 44; its hour angle is angle EPD, or the arc ED along the equator. When a star is 15° east of the meridian, its hour angle is 345°. Stars may be located on the surface of the celestial sphere by hour angle and declination. The hour angle, of course, varies with time; hence this co-ordinate system fixes the position of a star for a given time only.

Problems

In each of the following problems draw a figure to represent the celestial sphere. Locate the star from the given co-ordinates. Mark off the hour angle on the equator and the declination on a meridian.

	Hour Angle	*Declination*
1.	0°	+ 10°
2.	330°	− 20°
3.	180°	0°
4.	135°	+ 30°
5.	270°	+ 60°
6.	210°	+ 20°
7.	45°	− 15°

Hour Angle and Time. The *Greenwich hour angle* of a star is its hour angle measured from the meridian of Greenwich. The *American Nautical Almanac* and the *American Air Almanac* give the Greenwich hour angle (G. H. A.) of the sun, the moon, Venus, Mars, Jupiter, Saturn, and 55 of the brighter stars. The G. H. A. is tabulated for frequent intervals of time, and special tables are given to aid in interpolation.

The sun's hour angle is a natural measure of time, one hour of time corresponding to a change of 15° in the hour angle. Noon by the sun, or local apparent noon, is the time when the sun reaches its greatest angle of elevation and has an hour angle of 0°. Suppose one observed the sun on the meridian of Greenwich and noted the Greenwich standard time at which the sun reached its greatest altitude above the horizon. The sun would not reach its highest point at exactly twelve noon by the Greenwich clock and the interval between two successive noons by the sun would not be exactly 24 clock hours. For example, on April 1, 1943, the sun at Greenwich reaches its highest altitude at 12 : 04 : 07.5 P.M.; on April 2, 1943, the highest altitude occurs at 12 : 03 : 49.5. The irregularity of the sun's motion is due to two causes; the sun moves along the ecliptic instead of on the equator; and its daily motion eastward along the ecliptic is not the same for all times of the year.

To explain the standard time in ordinary use, it is necessary to invent a "fictitious sun," which is called the *mean sun*. The mean sun is assumed to travel around the equator at a constant rate equal to the average rate of the real sun. The standard time of any meridian corresponds to the hour angle of the mean sun measured from that meridian. Thus:

Hour Angle	*Time*	
180°	0^h	Midnight
270°	6^h	6 : 00 A.M.
360° or 0°	12^h	Noon
90°	18^h	6 : 00 P.M.
180°	24^h	Midnight

Standard time may be based on any meridian. The standard times used in the United States and their meridians are: Eastern, 75° W.; Central, 90° W.; Mountain, 105° W.; and Pacific, 120° W. The War Times now in use and their meridians are Eastern, 60° W.; Central, 75° W.; Mountain, 90° W.; Pacific, 105° W. Data in the *Nautical Almanac* and *Air Almanac* are tabulated for *Greenwich Civil Time* (G. C. T.), which is the standard time of the 0° meridian. In using one of these almanacs, it is necessary to find the G. C. T. which corresponds to the standard time of some other meridian.

The system of time keeping used in the United States Navy extends the principle of standard time to the entire globe. The Navy uses twenty-four time zones as shown in Figure 45. The ship time in any zone is the standard time of the mid-meridian of the zone. The numbers in each zone show the number of hours to be added to zone time to obtain G. C. T. In zone − 7, between longitude 97° 30′ E. and 112° 30′ E., a ship keeps the standard time of the meridian of 105° E. To change ship time in this zone to G. C. T. subtract 7 hours.

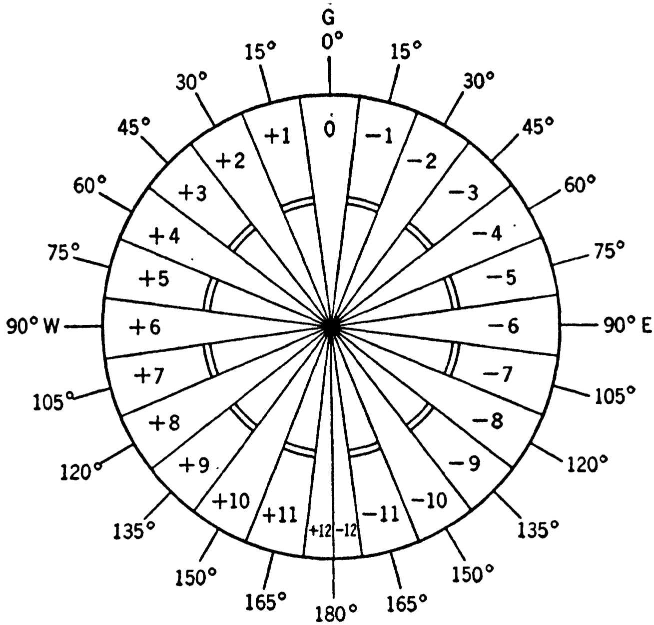

FIGURE 45. (After Dutton.)

Example 1. Find the G. C. T. and date corresponding to 9 : 30 P.M. E. W. T., January 27, 1943.

E. W. T.	9 : 30 P.M.	
Zone Time	$21^h\ 30^m$	January 27
Zone	$+ 4^h$	
	$25^h\ 30^m$	
G. C. T.	$1^h\ 30^m$	January 28

Notice in this example that the Greenwich date is later than the E. W. T. date.

Example 2. Find the G. C. T. and date corresponding to $8^h\,00^m$ Feb. 3, 1943, Zone − 10.

Zone Time	$8^h\,00^m$	February 3
Zone	$-\,10^h\,00^m$	
G. C. T.	$22^h\,00^m$	February 2

In this example, the Greenwich date is earlier than the local date, since $8^h\,00^m - 10^h\,00^m = -\,2^h\,00^m$ or $22^h\,00^m$ of the previous day. In other examples the local date and the Greenwich date may be the same.

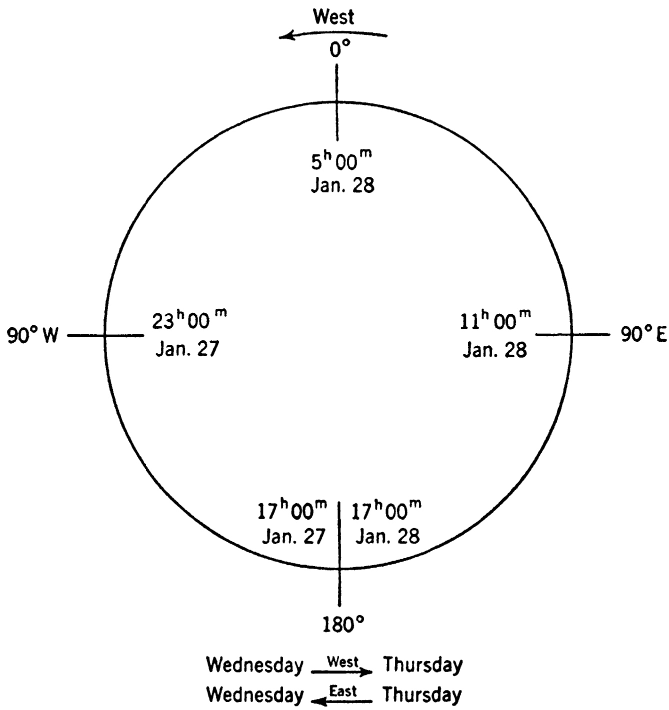

FIGURE 46.

The 12-hour zone in Figure 45 is divided into two parts, + 12 and − 12. Between 172° 30′ W. and 180°, the local time is 12 hours slower than G. C. T.; between 172° 30′ E. and 180° the local time is twelve hours faster than G. C. T. The date must be changed when the meridian of 180° is crossed. The reason for this change of date is shown in Figure 46. Suppose it is $5^h\,00^m$ on January 28 at Greenwich; west of Greenwich the local time is earlier, and hence at 180° the local time would be $17^h\,00^m$, January 27. East of

Greenwich the local time is ahead of G. C. T., and hence at 180° it would be $17^h\ 00^m$ January 28.

Problems

1. Find the G. C. T. and date corresponding to the following local times and dates.

(*a*) 11 : 00 P.M. E. W. T., Jan. 27
(*b*) 10 : 00 A.M. P. S. T., Jan. 29
(*c*) 12 : 00 M. C. S. T., Feb. 14
(*d*) 4 : 00 A.M. E. S. T., Mar. 22
(*e*) 7 : 30 P.M. C. W. T., Mar. 25
(*f*) 5 : 30 A.M. P. W. T., April 3
(*g*) 9 : 30 P.M. M. S. T., April 17
(*h*) 11 : 30 A.M. M. W. T., April 21

2. Find the G. C. T. and date corresponding to the following:

(*a*) $10^h\ 00^m$, Jan. 3, Zone + 4
(*b*) $2^h\ 00^m$, Feb. 19, Zone − 6
(*c*) $2^h\ 00^m$, Mar. 3, Zone + 12
(*d*) $3^h\ 00^m$, April 16, Zone − 10
(*e*) $17^h\ 00^m$, May 23, Zone − 12
(*f*) $17^h\ 00^m$, May 29, Zone + 8
(*g*) $15^h\ 00^m$, June 1, Zone − 9

Altitude and Azimuth; the Third Co-ordinate System. The first co-ordinate system considered was right ascension and declination; the second was hour angle and declination. The system of right ascension and declination is independent of the place of observation, so that these co-ordinates may be used in making star maps. The hour angle was found to be a means of measuring time. The co-ordinates, right ascension, declination, and hour angle cannot be measured directly by an observer.

The co-ordinates which an observer might determine for himself are the angle of elevation of a star and its bearing or direction. In astronomy and navigation the angle of elevation of a star is called its *altitude*. The direction or bearing of the star is called its *azimuth*. Azimuth, like bearing on the earth, is measured from 0° to 360° — north, 0°; east, 90°; south, 180°; west, 270°.

Figure 47 represents the celestial sphere; PZS, the meridian; Z, the zenith; and N, E, S, and W, the north, east, south, and west points of the horizon. Altitude is measured from 0° on the horizon to 90° at the zenith. A star below the horizon has a negative altitude. Stars of equal altitude lie on a small circle, which is parallel to the horizon. AA′ in Figure 47 contains all points which have an altitude of 60°. The points on the arc ZPN are due north or have an azimuth of 0°. If great circle arcs are drawn from Z to E, S, and W, then all points on ZE have an azimuth of 90°, all points on ZS an azimuth of 180°, etc. The azimuth may be regarded either as an angle at Z or as an arc of the horizon, measured from N in an eastward direction.

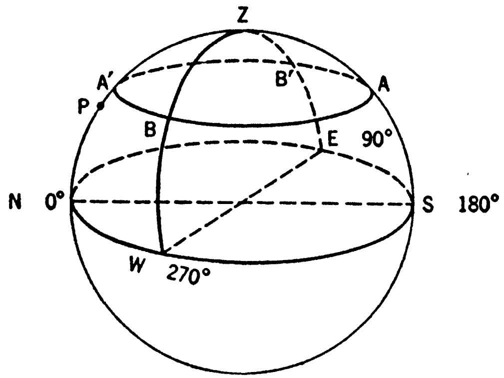

FIGURE 47.

In Figure 47 the altitude and azimuth of the four points A′, B′, A, and B are as follows:

Point	*Altitude*	*Azimuth*
A′	60°, arc NA′	0°
B′	60°, arc EB′	90°, arc NE
A	60°, arc SA	180°, arc NES
B	60°, arc WB	270°, arc NESW

Problems

On a separate sheet of paper complete the following statements:

1. The distance of a star from the equator is ------; from the celestial horizon is ------.

2. Declination is 0° on the ------; + 90° at the ------; − 90° at the ------. Altitude is 0° on the ------; + 90° at the ------; − 90° at the ------.

3. Stars of equal altitude lie on a ------ parallel to the ------. Stars of equal declination lie on a ------ parallel to the ------.

4. A star is at upper transit when it crosses the great circle arc containing the poles and the ------. A star is at lower transit when it crosses the great circle arc containing the poles and the ------.

5. The local hour angle of a star is the angle between the local meridian and the ------. At upper transit the hour angle = ------. At lower transit the hour angle = ------.

6. Stars that have hour angles between 0° and 180° are ------ of the local meridian. Stars that have hour angles between 180° and 360° are ------ of the local meridian.

7. Right ascension is the angle between the meridian through the star, and the meridian through the ...... It is measured ...... from 0° to 360°, or from ...... to ......

8. Azimuth is the angle between the great circle through the zenith and north pole, and the great circle through the ...... Azimuth is measured from 0° to ...... through ......

9. The right ascension of the sun changes 360° or 24^h in one ...... The right ascension of the moon changes 360° or 24^h in about ...... days.

10. Tell what co-ordinates of a star vary with time? with the place of observation?

11. What co-ordinates are used for star maps? Why are these co-ordinates used?

12. Why do star maps not show the sun, moon, and planets?

The Astronomic Triangle. Most of the problems in celestial navigation are concerned with solving a spherical triangle. This is the triangle formed on the celestial sphere by the pole, the observer's zenith, and a given star. In Figure 48 the triangle PZS_1 is formed by the north pole, P, the observer's zenith, Z, and a star, S_1. The great circle EE′ is the equator, and NS is the horizon. The sides of this triangle are easily related to various co-ordinates.

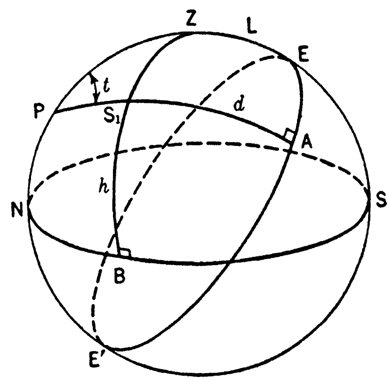

FIGURE 48.

$$EZ = \text{observer's latitude, } L \qquad PZ = 90° - L$$
$$AS_1 = \text{star's declination, } d \qquad PS_1 = 90° - d$$
$$BS_1 = \text{star's altitude, } h \qquad S_1Z = 90° - h$$

South latitudes and declinations are considered negative, so the expressions for sides of the triangle are true for all latitudes and declinations.

In Figure 48 the star is west of the meridian, and the angle ZPS_1 = the star's local hour angle. When a star is east of the meridian, the angle ZPS_1 would be 360 minus the local hour angle. If angle P is denoted by t and the local hour angle by L. H. A., then,

$$\text{angle } P = t = \text{L. H. A., or } 360 - \text{L. H. A.}$$

In this figure the azimuth is the arc NSB of the horizon or the corresponding angle at Z; and angle PZS_1, or angle Z in this triangle, is

360° minus the azimuth. When the star is east of the meridian, then angle Z in the triangle equals the azimuth. Hence, if the azimuth is denoted by Z_n, then

$$\text{angle } Z = Z_n, \quad \text{or} \quad 360 - Z_n$$

The relations between the sides and angles of the astronomic triangle and the celestial co-ordinates may be summarized:

$PZ = 90° -$ latitude $= 90° - L$, south latitudes negative
$ZS_1 = 90° -$ altitude $= 90° - h$
$PS_1 = 90° -$ declination $= 90° - d$, south declinations negative

For a star west of the meridian:

angle $ZPS_1 = t =$ local hour angle = L. H. A.
angle $PZS_1 = 360 -$ azimuth $= 360° - Z_n$

For a star east of the meridian:

angle $ZPS_1 = t = 360 -$ local hour angle $= 360° -$ L. H. A.
angle $PZS_1 =$ azimuth $= Z_n$

Angle ZS_1P is not involved in the problems of celestial navigation.

The astronomic triangle corresponds exactly to a triangle on the earth, formed by the pole of the earth, the point of observation, and the point directly underneath the star. There are many problems in which it is required to solve the astronomic triangle. We shall learn in the next chapter that the main problem in celestial navigation is to compute the altitude and azimuth from the latitude, declination, and hour angle corresponding to a given time and place. This problem may be solved by the haversine and sine formulas of Chapter V.

Declination and G. H. A. are given in the *Nautical Almanac* and *Air Almanac* at frequent intervals of G. C. T. From the G. H. A. one may find either the L. H. A. or 360 − L. H. A., that is, angle t in the triangle.

Example 1. Using the *Nautical Almanac*, find the values of angle t and the declinations for the stars Aldebaran, Capella, and Regulus at 40° 37′ N., 92° 36′ W. for February 25, 1943, at 10 : 32 : 45 P.M. C. S. T.

C. S. T.	10 : 32 : 45 P.M.
Zone time	$22^h\ 32^m\ 45^s$
Zone	$+ 6^h$
G. C. T.	$4^h\ 32^m\ 45^s$, February 26

The G. H. A. and declinations are found in the 1943 *Nautical Almanac* under "Stars, February, 1943," pages 170–173. The stars are listed in the order of right ascension. Under the name of each star is found its right ascension, its declination, and the time it transits the meridian of Greenwich on the first of the month. For each star the G. H. A. is given for 0^h G. C. T. for each day of the month. The numbers in parentheses below refer to pages in the *Nautical Almanac.*

	Aldebaran	*Capella*	*Regulus*
Declination	16° 24′ (170)	+ 45° 56′ (170)	+ 12° 15′ (171)
G. H. A. 0^h Feb. 26	86° 48′.4 (170)	76° 50′.9 (171)	3° 37′.8 (171)
Corr. for 4^h 32^m	68° 11′.2 (214)	68° 11′.2 (214)	68° 11′.2 (214)
Corr. for 45^s	11′.3 (214)	11′.3 (214)	11′.3 (214)
G. H. A. for 4^h 32^m 45^s	155° 11′	145° 13′	72° 00′
Longitude	92° 36′	92° 36′	92° 36′
	62° 35′ west	52° 37′ west	20° 36′ east

The stars Aldebaran and Capella are west of the meridian of 92° 36′. The answer of 20° 36′ for Regulus is of course 360° − L. H. A., since the star is east of the meridian. In order to avoid errors in finding the angle t, diagrams as shown in Figure 49 should always be drawn.

The three diagrams in Figure 49 show how the longitude and the G. H. A. are combined to find angle t. These diagrams represent the angles plotted on the equator as they would appear if viewed from the south. In each case draw the vertical diameter Mpm; Mp being the local meridian of 92° 36′ W. Draw the line Gpg making angle MpG = 92° 36′, so that pG represents the meridian of Greenwich. The star's G. H. A. is laid off to the westward of pG. Angle t in each case is the angle ★ pM. It is obvious from the figures when angle t is west and when it is east of pM.

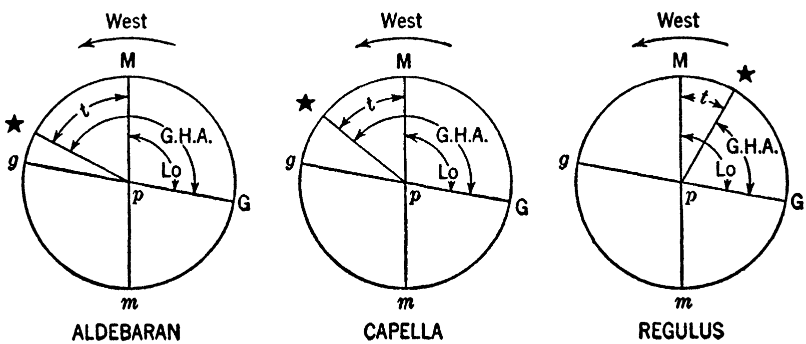

FIGURE 49.

Example 2. Find angle t and the declination of the sun for 15° 23′ S., 44° 48′ E. at $15^h 45^m 37^s$ zone time on March 11, 1943 (see Figure 50).

Zone time	$15^h 45^m 37^s$
Zone	-3^h
G. C. T.	$12^h 45^m 37^s$, March 11

"The Sun for March 11" is page 10 in the *Nautical Almanac*, and the "Correction Table for the G. H. A." is page 12. The sun's declination is − 3° 56′; found by inspection on page 10 between the tabulated values for 12^h and 14^h.

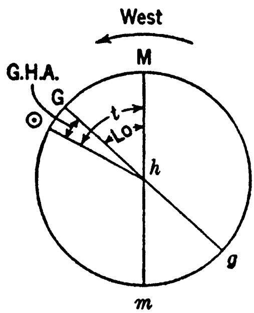

FIGURE 50.

G. H. A. for 12^h	357° 25.5	(10)
Corr. for 45^m	11° 15.0	(12)
Corr. for 37^s	09.3	(12)
	368° 50′	
G. H. A.	8° 50′	
Longitude	44° 48′	
	53° 38′ west	

Examples of finding G. H. A.'s for the moon and the planets are given in the 1943 *Nautical Almanac* on pages 304–305. In the *Air Almanac* the G. H. A. and declination are tabulated at ten-minute intervals for the sun, moon, and planets. The G. H. A. of a star is found from the star's sidereal hour angle and the tabulated value of the G. H. A. of the vernal equinox, or first point of Aries. The G. H. A. of the first point of Aries is indicated by the symbol G. H. A. ♈.

Example 3. Find the G. H. A. of the Sun, Mars, and Moon for $4^h 27^m 32^s$ G. C. T. November 13, 1942, using the *Air Almanac* for September–December, 1942.

	Sun	*Mars*	*Moon*
G. H. A. for $4^h 20^m$ (633)	248° 56′	261° 18′	182° 51′
Corr. for $7^m 32^s$ (inside front cover)	1° 53′	1° 53′	1° 49′
Additional corr. for Moon (633)			− 1′
G. H. A.	250° 49′	263° 11′	184° 39′
Declination (633)	− 17° 47′	− 13° 41′	− 18° 30′

Example 4. Find the G. H. A. and declination of the stars Antares and Canopus for $4^h 27^m 32^s$ G. C. T., November 13, 1942 (see Figure 51).

	Antares	*Canopus*
G. H. A. ♈ for $4^h 20^m$ (633)	116° 39′	116° 39′
Corr. for $7^m 32^s$ (inside front cover)	1° 53′	1° 53′
Sidereal H. A. of star (inside back cover)	113° 32′	264° 20′
G. H. A. of star	232° 04′	22° 52′
Declination (inside back cover)	− 26° 18′	− 52° 40′

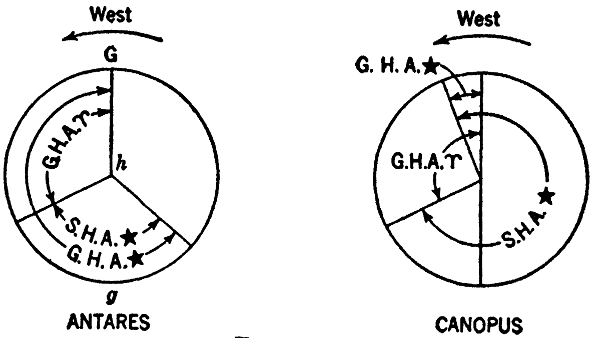

FIGURE 51.

Problems

1. Find the values of angle *t*, which correspond to G. H. A.'s of 10° 15′, 275° 49′, 110° 27′, 235° 29′, 342° 25′, for the following longitudes:

(*a*) 15° 18′ W.; (*b*) 135° 42′ E.; (*c*) 175° 43′ W.; (*d*) 96° 18′ E.

2. Find the values of angle *t*, which correspond to G. H. A.'s of 317° 42′, 43° 25′, 286° 24′, 135° 49′, 93° 23′, for longitudes:

(*a*) 74° 00′ W.; (*b*) 74° 00′ E.; (*c*) 145° 33′ E.; (*d*) 145° 33′ W.

3. Using the *Nautical Almanac* for 1943, find the G. H. A., angle *t*, and declination of the stars Dubhe, Regulus, and Spica:

(*a*) For a point in longitude 110° 23′ E. at $3^h\ 40^m\ 27^s$ zone time, January 15 (pages 166–169).

(*b*) For a point in longitude 127° 49′ W. at $21^h\ 20^m\ 33^s$ zone time, April 14 (pages 178–181).

4. Using the *Nautical Almanac* for 1943, find the G. H. A., angle *t*, and declination of the stars Vega, Deneb, and Fomalhaut:

(*a*) For a point in longitude 96° 43′ W. at $4^h\ 15^m\ 55^s$ zone time, July 5.

(*b*) For a point in longitude 89° 33′ E. at $22^h\ 04^m\ 19^s$ zone time, October 6.

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Nautical Astronomy:

Dutton, Chapter VIII; Bowditch, Chapter VII; Lyon, Chapter X.

Time:

The *Nautical Almanac;* the *Air Almanac;* Dutton, Chapter XII; Bowditch, Chapter IX; Lyon, Chapter X.

(The *American Nautical Almanac* is published for each year in advance by the United States Naval Observatory. The *American Air Almanac* also is published by the United States Naval Observatory; each volume covers a period of four months.)

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CHAPTER VII

CELESTIAL NAVIGATION; THE SUMNER LINE

From time immemorial observations of the stars have been used to determine directions and to aid in finding position on the earth. History records many early navigators who used celestial navigation. The methods and instruments used in modern celestial navigation are, however, of comparatively recent development. Columbus relied principally on dead reckoning but used the North Star to tell the time at night, to check the variation of the compass, and to find his latitude. The early Polynesian navigators sailed for thousands of miles across the Pacific and fixed their courses by the stars.

> Kupé, the first Polynesian discoverer of New Zealand, who is said to have lived about the thirteenth or fourteenth century, gave to succeeding expeditions from Tahiti very particular sailing directions, that they should steer always a little to the right of the place where the Sun, Moon, and Venus all set at the beginning of the hot season.
>
> T. R. St. Johnston in *Islanders of the Pacific*

The early Hawaiians sailed to Tahiti, a distance of more than 2300 nautical miles. The return voyage was directed to the eastward of Hawaii, and the proper time to turn west was determined by observing the Pole Star until it had an altitude equal to the latitude of Hawaii.

The development of modern precise navigation is due in no small part to the work of the self-taught American, Nathaniel Bowditch. In 1795, Bowditch went to sea, as supercargo on a vessel engaged in the China trade. He devoted the leisure which this position gave him to the study of navigation and the correction of the existing manual of navigation. *The American Practical Navigator*, published at intervals by the United States Hydrographic Office, bears on its title page the statement, "originally by Nathaniel Bowditch, LL.D," and is universally known simply as *Bowditch*.

Before 1761 the determination of longitude at sea was very difficult because time pieces were not sufficiently accurate to be reliable on a long voyage. The British Government offered large rewards for an accurate clock, or *chronometer*, suited to the purposes of navigation.

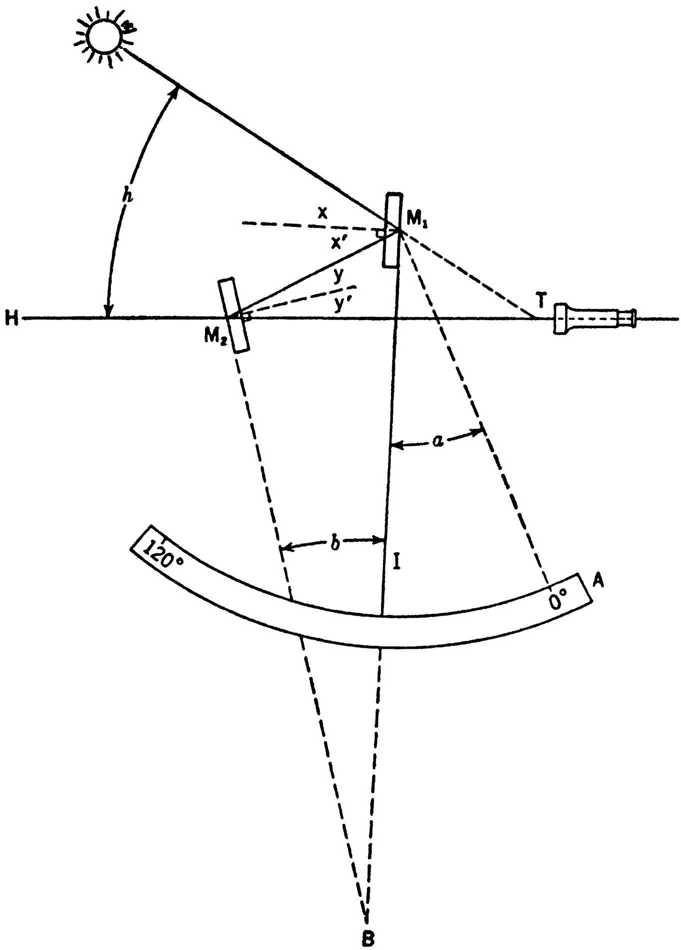


FIGURE 52.

The first chronometer suitable for finding the longitude at sea was invented by John Harrison in 1761. Harrison received a reward of £20,000 from the British Government.

The altitudes of the sun and stars are measured by an instrument called the *sextant*. The sextant measures the angle subtended at the observer's eye by distant objects. The name sextant is derived from the length of the arc on the instrument, 60°. The *octant* is similar to the sextant except that it has an arc of 45°. The principle of measuring angles by reflection, which is used in the sextant, was first applied by the Englishman, Hadley, and an American, Godfrey, about 1730. The sextant or octant replaced an earlier instrument, the *astrolabe*, which was used for measuring altitudes. The name astrolabe means "to take a star." Finding the altitude of the sun or a star is called "shooting" the sun or star.

Measurement and Correction of Observed Altitudes. The essential parts of the *marine sextant* are shown in Figure 52:

A is the arc, or limb, graduated from 0° to 120°.

I is the movable index arm, which is mounted to swing about the center of the arc, A.

M_1 is a mirror, called the index glass; it is mounted on the index arm and is perpendicular to the plane of the arc.

M_2 is the horizon glass. Its plane is also perpendicular to the arc, A. The right half of the horizon glass is a mirror, the left half is clear glass.

T is a telescope directed to the horizon glass. The axis of the telescope is parallel to the plane of the arc.

Sextants are provided with dark glasses for use in observing the sun. To measure the sun's altitude, the sextant is held vertically, and the horizon, H, is viewed through the telescope in the line TH. The index arm is turned until the reflected image of the sun appears tangent to the horizon as shown in Figure 53. The sun's altitude above the horizon is angle h in Figure 52. The angle through which the index mirror has been turned is angle a. The arc is 60° in length, but the scale is doubled so that the angle read is $2a$. The principle of the sextant is that

$$\text{angle } h = 2 \text{ angle } a.$$

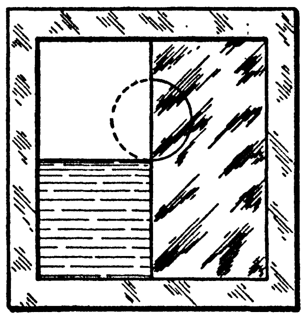


FIGURE 53.

When a sextant is in perfect adjustment, the line from M_1 to 0° on the arc, is parallel to the horizon glass; that is, the line

$$M_1 0° \text{ is parallel to } M_2B$$

and hence

$$\text{angle } a = \text{angle } b.$$

When light strikes a mirror and is reflected, the ray that strikes the mirror and the reflected ray make equal angles with the perpendicular to the mirror (*i.e.*, the angle of incidence equals the angle of reflection). In Figure 52

$$\text{angle } x = \text{angle } x' \qquad \text{angle } y = \text{angle } y'.$$

It follows from geometry, that h, the required altitude, equals $2\,a$, which is the angle read from the arc, for

in triangle TM_1M_2 $\quad h = (x + x') - (y + y')$

$$= 2\,x' - 2\,y$$

in triangle M_1M_2B $\quad b = 180° - (90° + y) - (90° - x')$

$$= x' - y$$

hence $\quad h = 2\,b$

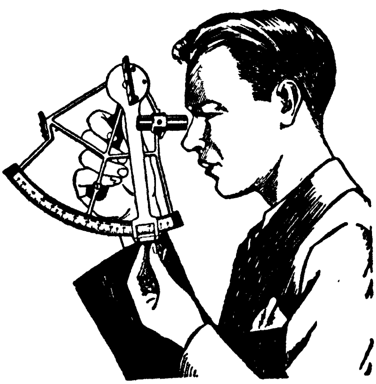
Since $\quad b = a,\ h = 2\,a$

Sextant altitudes must be corrected for an *index error.* Index error is most accurately determined by making the direct and reflected images of a star coincide. If the angle reads zero, when the two images coincide, there is no index error; if the reading is 1° to the *left* of zero on the arc, then one degree must be *subtracted* from all altitudes. If the reading is 1° to the right, then one degree must be added to all altitudes. The navigator uses the following rule to tell when to add and subtract index error:

If it's on, it's off; if it's off, it's on.

Index error may also be determined by making the direct and reflected images of the horizon coincide.

On land, altitudes of the sun and stars may be observed with a surveying transit, if the altitudes are not greater than 45°. Instrument companies supply sun glasses for use with a transit. When a transit is used to observe star altitudes, the cross wires must be illuminated. This may be done by directing the light of a small flashlight down the telescope, in such a way as not to obstruct the vision. The transit should first be focused on a distant object.



Courtesy Yoder Instruments

USING THE MARINE SEXTANT.

The marine sextant is used when a perfect sea horizon is available. *Bubble sextants* have been developed which eliminate the necessity for sighting the sea horizon. They are particularly suited for aerial navigation.

A number of corrections must be applied to observed altitudes. These corrections are tabulated in the *Nautical Almanac* and the *Air Almanac* and are easy to apply in practice.

(1) *Index Error.* The index error of a sextant, which has just been discussed, must be determined frequently and applied to all altitudes measured with the sextant.

(2) *Dip or Height of Eye.* This is a correction applied only to altitudes measured from the visible sea horizon; it does not apply to altitudes measured with the bubble sextant or transit. The line of sight toward the horizon is below the horizontal, hence the angle observed is too large. Corrections for dip are always subtracted. Corrections for various heights of eye are given in Table C, page 295, in the *Nautical Almanac*, and on the back cover of the *Air Almanac*.

(3) *Semidiameter.* In Figure 52, the altitude of the sun's lower edge was observed. This altitude must be corrected to obtain the altitude of the sun's center. Corrections for semidiameter are combined with corrections for refraction and parallax. When a bubble sextant is used, the center of the sun or moon is brought into coincidence with the bubble, and there is no semidiameter correction. The lower edge of the sun or moon is called the *lower limb.*

(4) *Refraction and Parallax.* The "bending" or refraction of light by the earth's atmosphere causes the stars to appear higher than

TABLE 1

CORRECTIONS TO OBSERVED ALTITUDES FROM THE "NAUTICAL ALMANAC"

	Marine Sextant	*Bubble Sextant*	*Transit*
	Always correct for index error. Correct for Dip, Table C, page 295.	Always correct for index error. No Dip Correction.	No Index Error. No Dip.
Sun's Lower Limb	Table A, page 295.	------	Table A
Sun's Center	------	Table E, page 298.	------
Star or Planet	Table A or Table E	Table A or Table E	Table A or Table E
Moon's Lower Limb [1]	Table D, pages 296–297.	------	Table D
Moon's Upper Limb [1]	Table D	------	Table D
Moon's Center [1]	------	Table F, page 299.	------

[1] In correcting altitudes of the moon, always find the horizontal parallax for the date of observation. Horizontal parallax is given at the bottom of each page for the "Moon 1943," pages 40–131. Use the horizontal parallax to find the correction in Table D, E, or F.

they are. Theoretically, an altitude should be measured from the center of the earth, instead of on its surface. Parallax is the difference between an altitude measured on the earth's surface and the same altitude measured from the earth's center. Parallax is small except in the case of the moon. Corrections for refraction and parallax are combined in a single table. For the sun and moon, corrections for semidiameter, refraction, and parallax are combined.

Table 1 (page 81) shows what tables from the *Nautical Almanac* should be used for correcting the various altitudes measured with sextants or with a transit.

Corrections for refraction and dip are given on the back cover of the *Air Almanac*. The semidiameters of the sun and moon are given on the A.M. side of each daily sheet. Corrections for the moon's parallax are also given on the A.M. side of each daily sheet.

Example 1. The sun's lower limb and a star are observed with a marine sextant having an index error of − 02′. The height of eye is 37 feet. The altitude of the sun's lower limb is 42° 36′, and the star's altitude is 37° 42′. Find the corrected altitudes.

Nautical Almanac

Sun's Lower Limb		Star	
Index	− 02′	Index	− 02′
Dip 37 feet, Table C	− 06′	Dip	− 06′
Table A	+ 15′	Table A	− 01′
Correction	+ 7′	Correction	− 09′
Observed Altitude	42° 36′	Observed Altitude	37° 42′
Obs. Alt. Corr.	42° 43′	Obs. Alt. Corr.	37° 33′

Air Almanac

Sun's Lower Limb		Star	
Index	− 02′	Index	− 02′
Dip, Back Cover	− 06′	Dip, Back Cover	− 06′
Refraction, Back Cover	− 01′	Refraction, Back Cover	− 01′
Semidiameter, Daily Sheet	+ 16′	Correction	− 09′
Correction	+ 7′		

Example 2. The altitude of the moon's center, observed on Nov. 27, 1942, with a bubble sextant, is 66° 18′. The sextant has an index error of + 05′. Find the corrected altitude.

In the *Nautical Almanac* find the horizontal parallax for Nov. 27 at bottom of page 99. Use this value to obtain the correction from table F. In the *Air Almanac* find the correction on the right side of page 661.

Nautical Almanac		*Air Almanac*	
Horizontal Parallax	54′ (page 122)	Index	+ 05′
Index Error	+ 05′	Refraction, Back Cover	00′
Table F	+ 21′	Parallax (page 661)	+ 21′
Correction	+ 26′	Correction	26′
Observed Altitude	66° 18′	Obs. Altitude	66° 18′
Obs. Alt. Corr.	66° 44′	Obs. Alt. Corr.	66° 44′

Problems

Correct the following altitudes.

	Instrument	*Index Error*	*Obs. Alt.*
1. Star	Bubble Sextant	− 05′	29° 32′
2. Sun's Lower Limb	Transit	------	35° 47′
3. Mars	Bubble Sextant	+ 03′	28° 13′
4. Jupiter	Bubble Sextant	− 08′	45° 53′
5. Star	Transit	------	44° 18′
6. Sun's Lower Limb	Bubble Sextant	+ 07′	27° 19′

7. Star altitudes of 32° 37′, 65° 19′, and 29° 18′ have been found with a marine sextant having an index error of − 05′. The height of eye is 60 feet.

8. Correct the following altitudes of the Moon.

	Altitude	*Date*	*Index Error*
Upper Limb	32° 19′	Jan. 13, 1943	0
Center	45° 15′	Feb. 27, 1943	+ 04′
Lower Limb	63° 47′	Mar. 11, 1943	− 03′
Center	23° 56′	June 17, 1943	+ 05′

Latitude by Meridian Altitude. The latitude of a place may be determined by measuring the altitude of a star or other celestial body when it crosses the meridian. North of latitude 23° 27′ N., the sun is always south at local apparent noon (see local apparent noon, page 66). Between the tropics (that is, between 23° 27′ N. and 23° 27′ S.) the sun may be north or south, depending on the time of the year. In the southern hemisphere, south of latitude 23° 27′ S., the sun will always be north at noon.

Example 1. A navigator finds the sun's corrected altitude to be 33° 44′ when the sun is crossing the meridian to the south. The sun's declination is 25° 33′ S. What is the latitude?

The latitude may be determined by constructing Figure 54. Draw the semicircle NZS, to represent the meridian; N is the north point, Z the zenith, and S the south point. Plot the sun on this figure by making angle h equal to the altitude, 33° 44′. The intersection E of the meridian and the equator is found by making angle d equal to the declination 25° 33′ S. Since the sun is south of the equator, the equator is north of the sun. The angle EOZ is the observer's latitude, L. For this figure:

$$L + d + h = 90°$$
$$L = 90° - (d + h)$$
$$L = 30° 43' \text{ N.}$$

In Figure 54, the latitude is north, since the zenith is north of the equator.

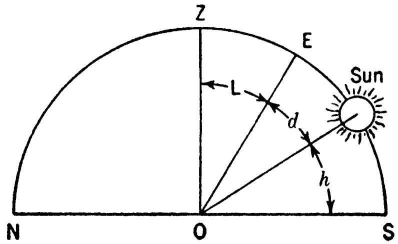


FIGURE 54.

The sun, in Figure 54, is said to be making an *upper transit to the south* (see upper and lower transit, page 65). An *upper transit to the north,* means that the sun or a star crosses the meridian between the zenith and the north pole.

Example 2. The sun's altitude is 75° 38′ at upper transit to the south. Its declination is 25° 33′ S. Find the latitude. (See Figure 55.)

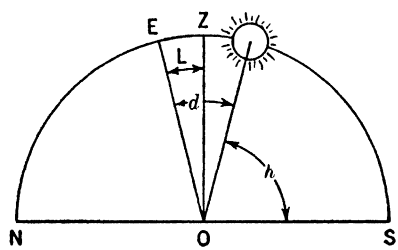


FIGURE 55.

When the latitude and declination are laid off on Figure 55, the zenith is south of the equator, and hence the latitude is south. For this figure:

$$h + d - L = 90°$$
$$L = h + d - 90°$$
$$L = 11° 11' \text{ S.}$$

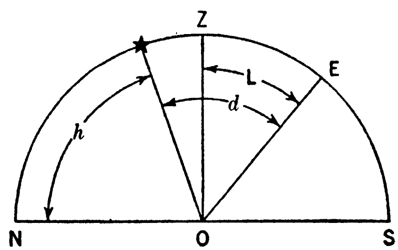


FIGURE 56.

Formulas may be derived for the various cases of north and south transits, and north and south declinations. In general, it is better to construct a diagram like Figure 54 or Figure 55 and not to rely on formulas. Latitude may be determined in a similar manner from the meridian altitudes of the stars at upper or lower transit. Figure 56 represents a star of north declination making an upper transit to the north. In this figure,

$$h + d - L = 90°$$

Problems

Find the latitude from the following meridian altitudes of the sun.

	1943 Date G. C. T.	*Upper Transit*	*Declination*	*Altitude*
1.	July 18, $20^h 00^m$	North	21° 05′ N.	58° 42′
2.	Oct. 9, $4^h 00^m$	South	5° 55′ S.	42° 48′
3.	Dec. 21	South	23° 27′ S.	71° 27′
4.	Nov. 8, $8^h 00^m$	North	16° 21′ S.	48° 56′
5.	Aug. 2, $2^h 00^m$	South	18° 02′ N.	67° 16′
6.	July 27, $10^h 00^m$	South	19° 23′ N.	20° 18′
7.	Sept. 16, $18^h 00^m$	North	2° 47′ N.	80° 15′

Find the latitude from the meridian altitudes of the following stars.

	Star	*Upper Transit*	*Declination*	*Altitude*
8.	Vega	North	38° 44′ N.	60° 21′
9.	Fomalhaut	South	29° 56′ S.	20° 13′
10.	Capella	North	45° 56′ N.	44° 04′
11.	Antares	South	26° 18′ S.	18° 29′
12.	Arcturus	North	19° 29′ N.	76° 24′
13.	Sirius	South	16° 38′ S.	45° 17′

Latitude from Altitudes of Polaris. The *Nautical Almanac* and the *Air Almanac* provide a short and easy method of finding the latitude from the North Star. The altitude is observed and corrected. The approximate local hour angle is found from the G. H. A. and the approximate longitude. Table III in the *Nautical Almanac*, page 284, gives a correction to apply to the altitude to obtain the latitude. This correction may be positive or negative, and is given for all values of the local hour angle.

Example. At 10 : 00 P.M. E. W. T. on January 16, 1943, the altitude of Polaris is found to be 38° 17′. There is no index error. The approximate longitude is 76°.

E. W. T.	10 : 00 P.M.	January 16
Zone Time	$22^h 00^m$	January 16
Zone	+ 4	
G. C. T.	$2^h 00^m$	January 17

Nautical Almanac

G. H. A. 0^h	89° 20.5	(page 280)
Corr. 2^h	30° 04.9	(page 214)
G. H. A.	119° 25′	
Longitude	76°	(approximate)
L. H. A.	43° 25′	(approximate)
Observed Altitude	38° 17′	
Table A	− 1	
Corr. Alt.	38° 16′	
Table III (page 284)	− 43′	
Latitude	37° 33′	

The correction − 43′, obtained from Table III, is found from the local hour angle.

Notice that the longitude does not need to be exact, since the corrections in Table II change very slowly, as the local hour angle changes. Hence the navigator may find his correct latitude, from his dead reckoning longitude.

The *Air Almanac* provides a similar method. In using the *Air Almanac,* find the L. H. A. ♈ (*i.e.*, the local hour angle of the first point of Aries) and then use the correction table on the back of the star chart.

The Circle of Position. One observation of the altitude of a celestial body does not fix position. This is true for the examples of the two previous sections. When a navigator has found his latitude, he has merely located his plane or ship on a circle on the earth. Any single observation will always locate the navigator on a circle on the earth, although not always on a parallel of latitude.

Suppose a navigator finds that the altitude of the star Vega is 66° 00′ at $2^h\,00^m$ G. C. T., May 14, 1943. From the *Nautical Almanac* he finds that Vega has a declination of 38° 44′ and a G. H. A. of 342° 11′. Using the altitude, the G. H. A., and the declination, it would be easy for him to construct a *circle of position* on a globe. Vega is the zenith of a certain point on the earth; this point is called the *substellar* point of Vega. The latitude of the substellar point is 38° 44′ N., the declination of Vega. The longitude of the substellar point is 342° 11′ west from Greenwich or 17° 49′ E. (360° − 342° 11′). The navigator's distance from the substellar point is the arc SZ of the astronomic triangle, or 90° − 66° 00′ = 24° 00′ (see the Astronomic Triangle, page 71). The position of point Z in the astronomic triangle is not known, and hence the navigator's position is somewhere on a small circle which has (38° 44′ N., 17° 49′ E.) for a pole and a polar distance of 24° 00′. This small circle is shown in Figure 57. V is the substellar point of Vega; PP′ the meridian of 17° 49′ E., and EE′ the equator.

Observations of two stars at the same time would determine two circles of position. The position of the plane or ship would be one of the intersections of these two circles (see problem 3, page 7).

Suppose that navigators in different ships find the altitude of Vega at the same time. The first obtains an altitude of 66° 00′ and is somewhere on the circle with a polar distance of 90° − 66° 00′, shown in Figure 57. The second finds an altitude of 60° 00′, and his ship will be on the circle of polar distance 90° − 60° 00′, in Figure 57. Notice that for all the points on the *larger* circle the altitude of Vega is 60° 00′, and for all the points on the *smaller* circle Vega has an altitude of 66° 00′. *The places at which the altitude is greater are nearer the substellar point.*

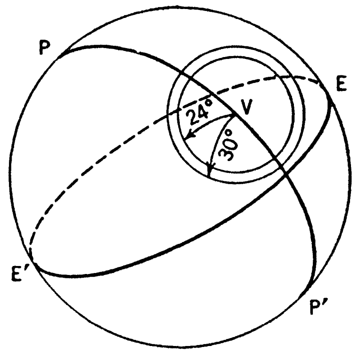


FIGURE 57.

Problems

Draw on a globe the circles of position defined as follows.

	Declination	*G. H. A.*	*Altitude*
1.	+ 30°	240°	40°
2.	+ 30°	240°	45°
3.	+ 30°	240°	35°
4.	− 20°	80°	60°
5.	− 20°	80°	65°
6.	− 20°	80°	70°

The Sumner Line. If a navigator observed the altitude of two stars at the same time, he might draw two circles of position. His ship would be at one of the points of intersection of the two circles. Figure 64, page 98, shows two position circles plotted on a Mercator map. In this case, at least, there is no doubt about which intersection is the position. It is not possible, of course, to determine with any accuracy the latitude and longitude of the intersection of two circles drawn on a globe.

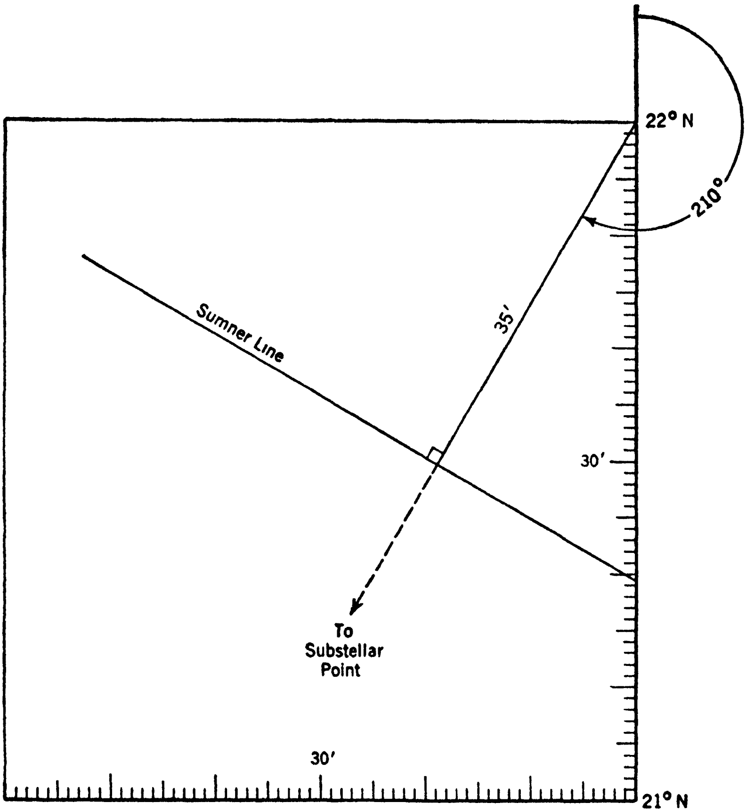


FIGURE 58.

In practice, small arcs of the position circles are drawn on a chart. Unless the observed altitude is greater than 80°, the arc of the position circle is drawn as a straight line. These *lines of position* are called *Sumner lines*, after T. H. Sumner, who discovered this method of finding position in 1837.

The Sumner line may be plotted by calculating the latitude and longitude of two points; this was the method used by Capt. Sumner. The method explained here, which is now in general use, is due to Marcq St. Hilaire, an admiral in the French Navy. The St. Hilaire method uses an assumed position; the assumed position may be that obtained by dead reckoning, or any convenient point which is near the dead-reckoning position.

In the St. Hilaire method the altitude of a star is observed. The altitude and azimuth of the star are calculated for the time of observation and for the assumed position. Sumner lines are usually plotted on Mercator charts, like the "position plotting sheets" published by the U. S. Hydrographic Office. Figure 58 shows a Sumner line plotted on a section of the position-plotting sheet for latitudes 21° to 24°.

Example 1. A navigator near (22° N., 45° W.) finds the corrected observed altitude of a star to be 45° 51′. The altitude and azimuth calculated for (22° N., 45° W.) are 45° 16′ and 210° 00′. Plot the Sumner line (see Fig. 58).

observed altitude corrected	h_o =	45° 51′
calculated altitude	h_c =	45° 16′
	difference =	35′ toward the substellar point.

Any meridian on the plotting sheet may be taken as 45° W. The line bearing 210° from (22° N., 45° W.) points to the substellar point. As one moves along the azimuth line toward the substellar point, the altitude of the star increases. The observed altitude is 35′ greater than the calculated altitude; hence the point of observation lies on

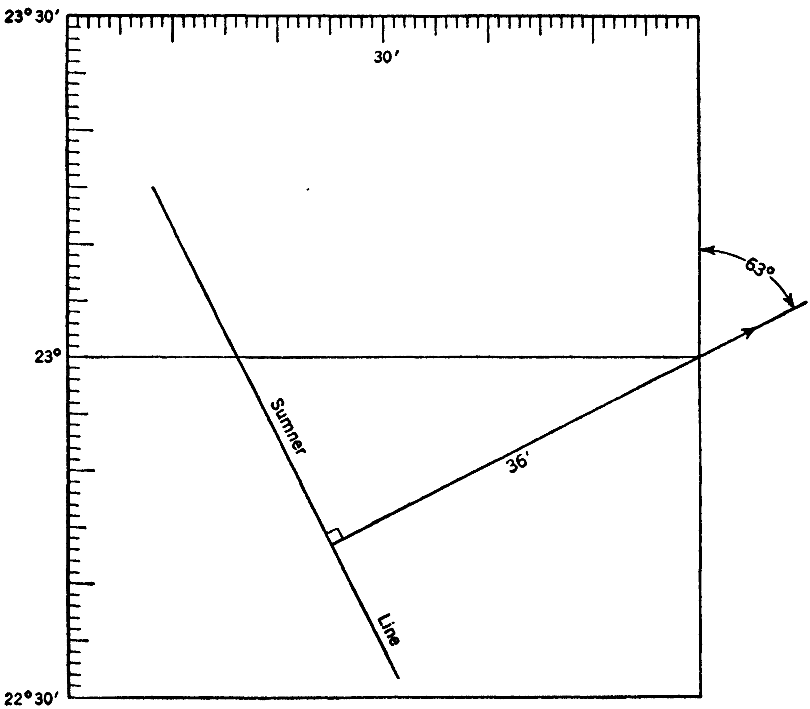


FIGURE 59.

a circle of position, which is 35′ or 35 nautical miles nearer the substellar point. The distance of 35 miles is laid off on the azimuth line and the Sumner line is drawn perpendicular to the azimuth line. This distance of 35′ or 35 nautical miles is made equal to 35′ of latitude (1 nautical mile = 1′ of latitude, not longitude).

In Figure 57, the Sumner line represents a small arc of a circle of position. The azimuth line represents a small arc of the great circle joining the assumed position to the substellar point. Within the limits of the chart no appreciable error is introduced by drawing these arcs as straight lines.

Example 2. Assumed position is (23° N., 46° W.). Calculated altitude, or $h_c = 27°\,38'$. Calculated azimuth, or $Z_n = 63°$. Corrected observed altitude, or $h_o = 27°\,02'$. (See Figure 59.)

$$\begin{aligned} h_o &= 27°\,02' \\ h_c &= \underline{27°\,38'} \\ \text{difference} &= \quad 36' \text{ away from substellar point.} \end{aligned}$$

Since the observed altitude is less than the calculated altitude, the Sumner line is 36′ or 36 nautical miles farther away from the substellar point than the assumed position.

Observed altitude *greater* than calculated altitude, Sumner line is *toward* the substellar point.
Observed altitude *less* than calculated altitude, Sumner line is *away* from substellar point.

Problems

Plot Sumner lines in the following problems. Draw two perpendicular lines for the assumed meridian and parallel. Use any convenient scale for the difference of altitude.

	Calculated Azimuth Z_n	*Observed Alt. Corr.* h_o	*Calculated Altitude* h_c
1.	135°	45° 58′	45° 48′
2.	310°	63° 27′	63° 40′
3.	70°	20° 40′	20° 30′
4.	240°	27° 08′	27° 18′
5.	195°	75° 42′	76° 00′
6.	233°	69° 47′	69° 30′
7.	150°	33° 29′	34° 00′
8.	280°	56° 18′	56° 04′

Altitude on the Prime Vertical. Plotting a Sumner line involves calculating altitude and azimuth. There is a somewhat general misconception that the sun in north latitudes never has an azimuth between north and east or between west and north. It is the purpose of this section to develop a formula for finding the latitudes at which certain stars, or the sun, at certain times of the year, have an azimuth of 90° or 270°.

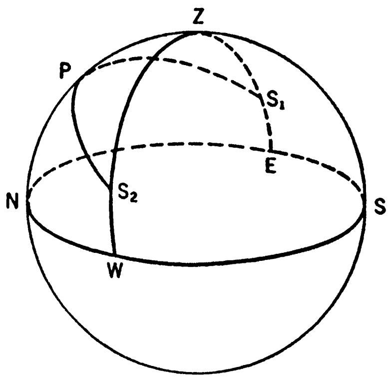


FIGURE 60.

Figure 60 shows the celestial sphere, and the line WZE passes through the zenith and the east and west points of the horizon. WZE is called the *prime vertical.* S_1 is a star which is rising and has an azimuth of 90° or due east. S_2 is a star which is setting and has an azimuth of 270° or due west. In triangle PZS_1:

$$PZ = 90° - L$$
$$PS_1 = 90° - d$$
$$PZS_1 = 90°$$

and $ZS_1 = 90° - h$, where h is the altitude on the prime vertical.

On page 52, the *sine-cosine* formula,

$$\cos c = \cos a \cos b + \sin a \sin b \cos C,$$

was proved for spherical triangles. If in a spherical triangle $C = 90°$, $\cos C = 0°$, and the sine-cosine formula becomes

$$\cos c = \cos a \cos b$$

If the formula $\cos c = \cos a \cos b$ is applied to triangle PZS_1,

$$\cos (90° - d) = \cos (90° - L) \cos (90° - h),$$

or

$$\sin h = \frac{\sin d}{\sin L}.$$

Example 1. At what altitude is Arcturus on the prime vertical in latitude 40° 43′ N.? The declination of Arcturus is 19° 29′ N.

$$\sin h = \frac{\sin 19° 29'}{\sin 40° 43'}$$

$$\begin{aligned} \log \sin d \ (19° 29') &= 9.52314 \\ - \log \sin L \ (40° 43') &= \underline{9.81446} \\ \log \sin h \qquad &= 9.70868 \\ h &= 30° 45' \end{aligned}$$

Hence when Arcturus is *rising in this latitude,* for altitudes —

(*a*) less than 30° 45′, its azimuth is between 0° and 90°.

(*b*) greater than 30° 45′, its azimuth is between 90° and 180°.

When Arcturus is *setting in this latitude,* for altitudes —

(*a*) greater than 30° 45′, its azimuth is between 180° and 270°.

(*b*) less than 30° 45′, its azimuth is between 270° and 360°.

Example 2. Find the altitude at which the sun is on the prime vertical for latitude 30° 20′ N. on July 27, 1942. The approximate declination is 19° 23′ N.

$$\begin{aligned} \log \sin d \ (19^\circ 23') &= 9.52099 \\ -\log \sin L \ (30^\circ 20') &= \underline{9.70332} \\ \log \sin h \qquad &= 9.81767 \\ h &= 41^\circ 05' \end{aligned}$$

For this latitude and date, the sun will be east of north in the morning until it reaches an altitude of 41° 05′. In the afternoon it will be north of west, when its altitude is less than 41° 05′.

Problems

1. Find the altitude on the prime vertical for latitude 35° N. for the stars, Altair (+ 8° 43′), Regulus (+ 12° 15′), and Aldebaran (+ 16° 24′).

2. Show that stars of south declination are never observed on the prime vertical in northern latitudes. (Show that sin h is negative.)

3. Prove that the sun rises north of east and sets north of west for latitudes 23° 27′ N. to 66° 33′ N., between March 21 and September 21.

4. Restate problem 3 for south latitudes.

5. Show that a star does not cross the prime vertical in north latitudes if the star's declination is greater than the latitude.

Calculation of Altitude and Azimuth. The problem of calculating altitude is similar to the problem of finding great circle arcs. Altitude is found by the *haversine formula* and azimuth by the *sine formula.* The sides of the astronomic triangle are

$$\begin{aligned} PZ &= 90^\circ - L, \text{ south latitudes negative} \\ PS &= 90^\circ - d, \text{ south declinations negative} \\ ZS &= 90^\circ - h_c \end{aligned}$$

Angle P = t and is found from the G. H. A. and the longitude. The haversine formula is

$$\text{hav } c = \sin a \sin b \text{ hav } C + \text{hav } (a - b)$$

For triangle PZS, the haversine formula becomes

$$\text{hav } (90^\circ - h_c) = \cos L \cos d \text{ hav } t + \text{hav } (L - d).$$

Angle PZS in the astronomic triangle is either the azimuth Z_n, or 360 − Z_n. The *sine formula* for spherical triangles is

$$\sin B = \frac{\sin C \sin b}{\sin c}$$

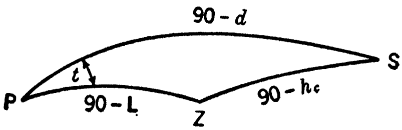


FIGURE 61.

For triangle PZS the sine formula may be written

$$\sin PZS = \frac{\sin t \cos d}{\cos h_c}$$

In using the sine formula for angle PZS, an acute angle is found from the tables; this acute angle is the acute angle from the meridian to the arc ZS. Denote this acute angle by Z; then

$$\sin Z = \frac{\sin t \cos d}{\cos h_c}.$$

The relation between the acute angle Z and the azimuth, Z_n, is as follows:

Star is north and east, then $Z_n = Z$.
Star is east and south, then $Z_n = 180° - Z$.
Star is south and west, then $Z_n = 180° + Z$.
Star is west and north, then $Z_n = 360° - Z$.

The relation between Z and Z_n is the same as that between angle C and the course, Figure 16, page 18.

An approximate determination of the azimuth may be made with the compass. In all but a few cases this rough observation will determine the quadrant of the azimuth. In the few doubtful cases, the altitude on the prime vertical may be computed and compared with the observed altitude (see examples 1 and 2 above); this will determine whether the star is north of east, or south of east, etc. The azimuth could be found by the haversine formula. When the haversine formula is used for finding azimuth, the angle PZS of the triangle is found and there is no ambiguity.

Example 1. Find the altitude and azimuth of the star Spica for (40° N., 50° W.) at $1^h 00^m$ G. C. T., May 3, 1943. The azimuth is between 90° and 180°.

$$\begin{aligned}
\text{G. H. A. } 0^h &= 19°\ 28.1 \\
\text{Corr. } 1^h &= \underline{15°\ 02.5} \\
\text{G. H. A.} &= 34°\ 31' \\
\text{Long.} &= \underline{50°\ \text{W.}} \\
t &= 15°\ 29'\ \text{east} \qquad d = -\ 10°\ 52'
\end{aligned}$$

$$\text{hav}\,(90° - h_c) = \cos 40° \cos 10°\ 52'\ \text{hav}\ 15°\ 29' + \text{hav}\ 50°\ 52'$$

$$\begin{aligned}
&\log \cos L && (40°\ 00') = 9.88425 \\
+\ &\log \cos d && (10°\ 52') = 9.99214 \\
&\log \text{hav}\ t && (15°\ 29') = \underline{8.25878} \\
& && \ 8.13517 \\
&\text{antilog } 8.13517 && = .01365 \\
+\ &\text{nat hav } (L-d)\ (50°\ 52') && = \underline{.18444} \\
&\text{nat hav } (90° - h_c) && = .19809 \\
& && 90° - h_c = 52°\ 51' \\
& && h_c = 37°\ 09'
\end{aligned}$$

$$\sin Z = \frac{\sin 15°\ 29' \cos 10°\ 52'}{\cos 37°\ 09'}$$

$$\begin{aligned}
&\log \sin t && (15°\ 29') = 9.42644 \\
+\ &\log \cos d && (10°\ 52') = \underline{9.99214} \\
& && \ 9.41858 \\
-\ &\log \cos h_c && (37°\ 09') = \underline{9.90149} \\
&\log \sin Z && = 9.51709 \\
& && Z = 19°\ 12' \\
& && Z_n = 180° - Z = 160°\ 48'
\end{aligned}$$

In this example, $L - d = 40°\ 00' - (-\ 10°\ 52') = 50°\ 52'$. Since Spica was observed to the south and east, the acute angle Z is subtracted from 180° to obtain the azimuth.

Example 2. Find the altitude and azimuth of Regulus for the same time and position as example 1. The azimuth is between 180° and 270°.

$$\begin{aligned}
\text{G. H. A. } 0^h &= 68°\ 41.0 \\
\text{Corr. } 1^h &= \underline{15°\ 02.5} \\
\text{G. H. A.} &= 83°\ 44' \\
\text{Long.} &= \underline{50°\ \text{W.}} \\
t &= 33°\ 44'\ \text{west} \qquad d = +\ 12°\ 15'
\end{aligned}$$

$$\log \text{hav}\,(90° - h_c) = \cos 40° \cos 12°\ 15'\ \text{hav}\ 33°\ 44' + \text{hav}\ 27°\ 45'$$

$$\begin{aligned}
&\log \cos L && (40°\ 00') = 9.88425 \\
+\ &\log \cos d && (12°\ 15') = 9.99000 \\
&\log \text{hav}\ t && (33°\ 44') = \underline{8.92523} \\
& && \ 8.79948
\end{aligned}$$

$$
\begin{array}{lll}
\text{antilog } 8.79948 & & = .06303 \\
+ \text{ nat hav } (L-d)\ (27^\circ 45') & & = \underline{.05751} \\
\text{nat hav } h_c & & = .12054 \\
 & 90^\circ - h_c & = 40^\circ 38' \\
 & h_c & = 49^\circ 22'
\end{array}
$$

$$\sin Z = \frac{\sin 33^\circ 44' \cos 12^\circ 15'}{\cos 49^\circ 22'}$$

$$
\begin{array}{lll}
\log \sin t & (33^\circ 44') & = 9.74455 \\
+ \log \cos d & (12^\circ 15') & = \underline{9.99000} \\
 & & 9.73455 \\
- \log \cos h_c\ (49^\circ 22') & & = \underline{9.81372} \\
\log \sin Z & & = 9.92083 \\
 & Z & = 56^\circ 27' \\
 & Z_n & = 180^\circ + 56^\circ 27' = 236^\circ 27'
\end{array}
$$

Problems

Find the altitude and azimuth in problems 1–6.

	Position	*1943 Date G. C. T.*	*Declination*	*G. H. A.*	*Azimuth between*
1. Arcturus	40° 49′ N. 73° 57′ W.	July 21 $4^h 00^m$	+ 19° 29′	144° 47′	270° & 360°
2. The Sun	22° 54′ S. 43° 10′ W.	Aug. 3 $18^h 00^m$	+ 17° 36′	88° 28′	270° & 360°
3. Fomalhaut	31° 16′ N. 32° 19′ E.	Aug. 15 $1^h 00^m$	− 29° 55′	353° 56′	180° & 270°
4. Altair	17° 30′ S. 149° 30′ W.	Sept. 20 $8^h 00^m$	+ 8° 43′	181° 20′	270° & 360°
5. Aldebaran	60° 10′ N. 24° 57′ E.	Nov. 24 $2^h 00^m$	+ 16° 24′	23° 59′	180° & 270°
6. Canopus	28° 16′ S. 64° 44′ W.	Dec. 1 $2^h 00^m$	− 52° 40′	3° 22′	90° & 180°

In problems 7–10 find first the zone and then the G. C. T. from the zone time. Find the G. H. A. and the declination from the *Nautical Almanac* or the *Air Almanac.* Calculate the altitude and azimuth for the given position.

	Position by Dead Reckoning	*1943 Date Zone Time*
7. Antares	22° 16′ S. 166° 27′ E.	Mar. 23 $1^h 30^m$
8. Deneb	45° 00′ N. 50° 00′ W.	June 4 $23^h 00^m$
9. Arcturus	9° 25′ S. 160° 05′ E.	May 18 $22^h 00^m$
10. Sirius	20° 10′ N. 55° 06′ W.	Dec. 16 $2^h 30^m$

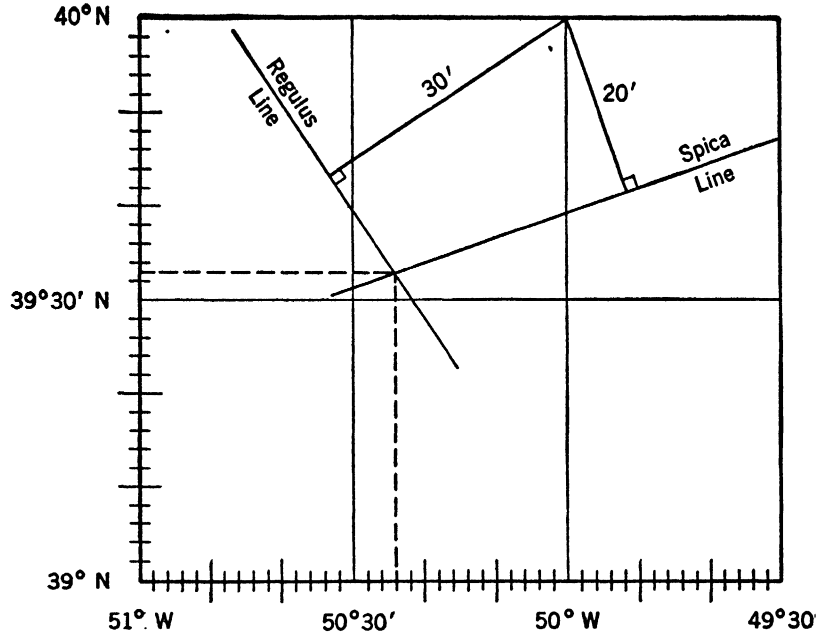


FIGURE 62.

Position by Sumner Lines. Position may be determined by the two Sumner lines resulting from two simultaneous altitudes of different stars. For the best *fix*, two stars should be selected which differ by about 90° in azimuth.

Example. A navigator in the North Atlantic finds the corrected altitude of Spica to be 37° 29′ and the corrected altitude of Regulus to be 49° 52′, at $1^h\,00^m$ G. C. T., May 3, 1943. Find his position, using (40° N., 50° W.) for an assumed position.

The calculated altitudes and azimuths were found in the examples of the last section.

	Spica		*Regulus*
h_o =	37° 29′	h_o =	49° 52′
h_c =	37° 09′	h_c =	49° 22′
	20′ toward		30 toward
Z_n =	160° 48′	Z_n =	236° 27′

The Sumner lines from these two observations are shown on a plotting sheet in Figure 62. The ship's position determined by their intersection is (39° 33′ N., 50° 24′ W.).

In this example the two altitudes were found at the same time. In air navigation there will generally be a time interval between the observations. In Figure 63, the navigator of a plane has plotted line A, and then line B, but the plane has gone 45 nautical miles on a course of 225° between the times of the observations. The navigator allows for this change of position by constructing EF with a bearing of 225° and a length of 45 miles. Line C is drawn parallel to line A, and P is the position at the time of the second observation.

The two observations which were used to plot the Sumner lines of Figure 62 actually determine two circles of position. Figure 64 shows these two circles of position and the substellar points of Regulus and Spica. The circles appear distorted because they are plotted on a map. The two circles of Figure 64 have two intersections, but the intersection in the South Pacific is easily eliminated. If the difference in azimuth of two stars is near 0° or near 180°, they should not be used for a fix by Sumner lines. The best fix is obtained when the difference of azimuth is near 90°.

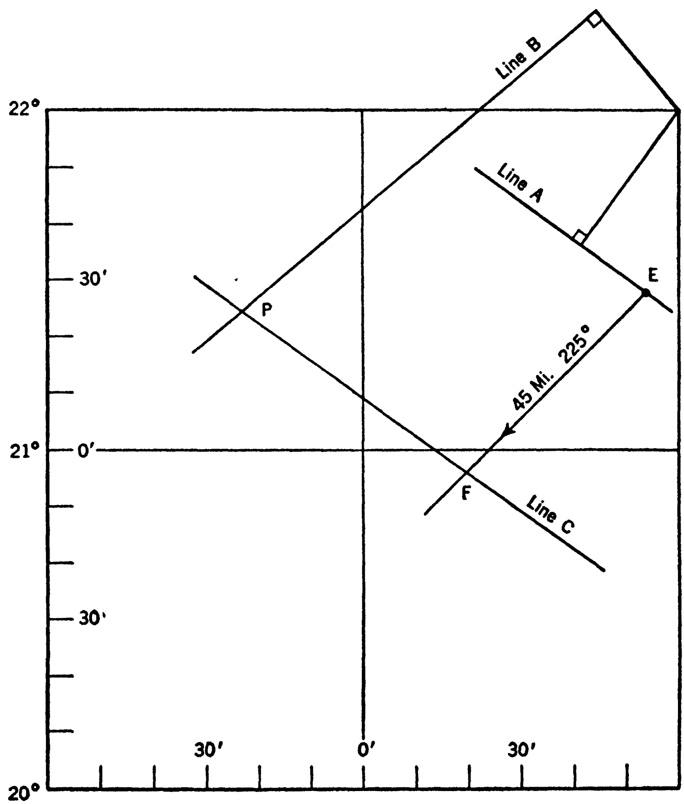


FIGURE 63.

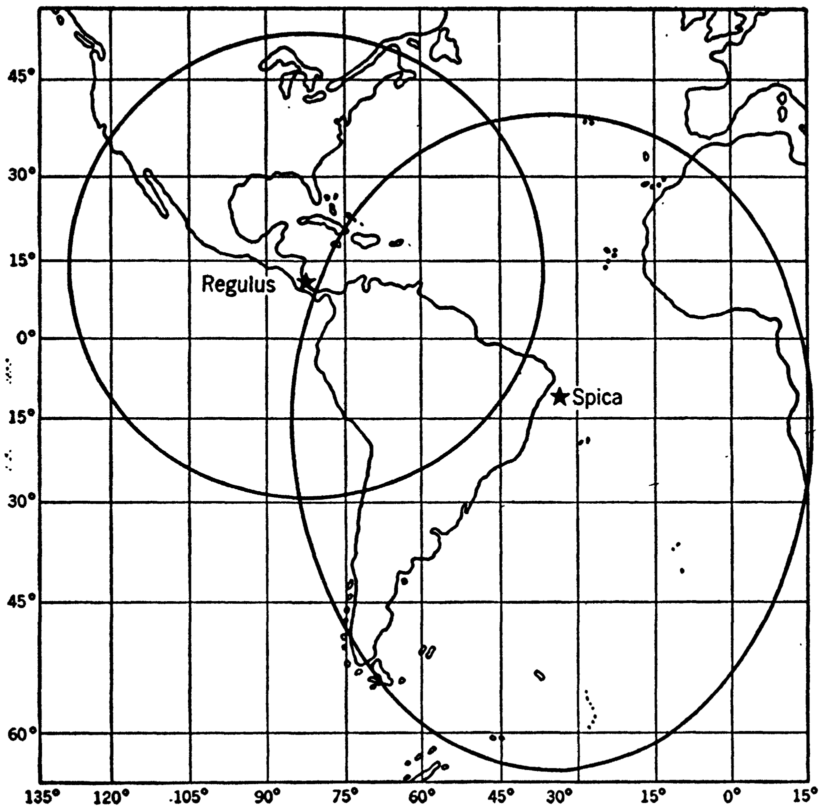


FIGURE 64.

One Sumner line is frequently valuable even though it does not fix position. If the Sumner line runs north and south, the longitude is determined. If it runs east and west, the latitude is determined. The incident which led to the discovery of the Sumner line is a striking illustration of the value of a single position line. In 1837 Captain Sumner sailed from Charleston for Greenock. As he neared his destination, his position became doubtful because he had been unable to take celestial observations. On November 27 he was able to observe a single altitude of the sun. The line of position which he plotted from this observation ran through Small's Light, and consequently the original Sumner line was the means of Sumner's ship reaching port safely.

Many recent developments have increased both the accuracy of modern navigation and the rapidity with which calculations can be made. Numerous special tables have been prepared for calculating altitude and azimuth. The Greenwich hour angle was introduced to avoid the more cumbersome use of right ascension. The bubble sextant and the second setting watch have been developed to aid the aerial navigator. The *Air Almanac* has been published to provide all the essential astronomical data in as compact a form as possible.

Problems

Find the latitude and longitude of the point of observation in the following problems. In problems 3 and 4, find the G. H. A. and the declination from the *Nautical Almanac.* If plotting sheets are not available, they may be constructed by taking 12.00 cm. to equal 1° of longitude in each problem; then, in No. 1, 1° of latitude = 14.24 cm.; in No. 2, 1° of latitude = 13.78 cm.; in No. 3, 1° of latitude = 12.10 cm.; and in No. 4, 1° of latitude = 15.84 cm.

	Assumed Position	*1943 Date G. C. T.*	*Star*	*Dec.*	*G. H. A.*	*Obs. Alt.*	*Azimuth between*
1.	32° 00′ N. 136° 00′ W.	Aug. 10 $10^h 00^m$	Vega	+ 38° 44′	189° 25′	46° 44′	270° & 360°
			Fomalhaut	− 29° 55′	124° 22′	26° 47′	90° & 180°
2.	30° 00′ S. 11° 00′ E.	Sept. 10 $21^h 00^m$	Peacock	− 56° 55′	358° 43′	62° 30′	180° & 270°
			Nunki	− 26° 22′	21° 04′	61° 05′	180° & 270°

3. 10° 00′ N. Oct. 14 Observed altitude of Aldebaran = 74° 04′
   60° 00′ E. $22^h 00^m$ Observed altitude of Hamal = 65° 42′

4. 41° 00′ N. Mar. 18 Observed altitude of Regulus = 38° 58′
   74° 00′ W. $00^h 08^m$ Observed altitude of Mizar = 28° 42′

Bibliography

Historical References:

"Columbus and Polaris" by Samuel E. Morison, *The American Neptune,* 1941, 6–25, 123–127.

"The Ocean Navigation of Columbus" by J. W. McElroy, *The American Neptune,* 1941, 209–240.

"South Sea Sailors Steer by the Stars" by M. W. Makemson, *The Sky,* Jan., 1939.

"The Sacred Calabash" by Hugh S. Rodman, *Proceedings of the U. S. Naval Institute*, August, 1927.

Discovery of the Sumner Line, *Bowditch*, 177.

Nathaniel Bowditch, *Bowditch*, 3.

"Historic Flights" by Bradley Jones, *Avigation*, Chapter XIV.

The Sextant:

Bowditch, 120–128; Dutton, Chapter X; *Field Mathematics* by Shuster and Bedford, Chapter XII.

Sextants and other instruments suitable for school use are sold by Lafayette Instruments, Inc., 252 Lafayette St., New York.

Plotting Sheets:

Position plotting sheets are published by the United States Hydrographic Office. The Coast and Geodetic Survey Aeronautical Charts may also be used for plotting.

Position by Sumner Lines:

Dutton, Chapters XIV, XV; Bowditch, Chapters XIV, XV; Lyon, 191–194, 203.

Special Tables, U. S. Hydrographic Office Publications:

No. 208, *Navigation Tables for Mariners and Aviators*, Dreisonstok.

No. 211, *Dead Reckoning, Altitude and Azimuth Tables*, Ageton.

No. 214, *Tables of Computed Altitude and Azimuth*, nine vols., each covering 10° of latitude.

Other Special Tables:

Star Altitude Curves, separate volumes for each 10° band of latitude, Weems System of Navigation, Annapolis.

Weems' Line of Position Book, Weems System of Navigation, Annapolis.

Manual of Celestial Navigation. Ageton, Van Nostrand, New York.

INDEX

FIVE PLACE

LOGARITHMS OF NUMBERS

Pages ii-xix

AND

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

FOR EACH MINUTE OF ARC

Pages xx-xlii

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
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121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
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127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
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133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
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136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
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142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
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152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
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171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
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183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
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201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
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219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
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242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
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252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
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256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
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264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
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271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
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275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
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283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
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285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
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291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
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295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
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303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
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311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
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337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
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356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
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366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
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383	58 320	58 331	58 343	58 354	58 365	58 377	58 388	58 399	58 410	58 422
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387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
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392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
393	59 439	59 450	59 461	59 472	59 483	59 494	59 506	59 517	59 528	59 539
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402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
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405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
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407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
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409	61 172	61 183	61 194	61 204	61 215	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 395	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
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414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
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419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
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423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
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438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
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442	64 542	64 552	64 562	64 572	64 582	64 591	64 601	64 611	64 621	64 631
443	64 640	64 650	64 660	64 670	64 680	64 689	64 699	64 709	64 719	64 729
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445	64 836	64 846	64 856	64 865	64 875	64 885	64 895	64 904	64 914	64 924
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448	65 128	65 137	65 147	65 157	65 167	65 176	65 186	65 196	65 205	65 215
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454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
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456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
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487	68 753	68 762	68 771	68 780	68 789	68 797	68 806	68 815	68 824	68 833
488	68 842	68 851	68 860	68 869	68 878	68 886	68 895	68 904	68 913	68 922
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497	69 636	69 644	69 653	69 662	69 671	69 679	69 688	69 697	69 705	69 714
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No.	0	1	2	3	4	5	6	7	8	9

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521	71 684	71 692	71 700	71 709	71 717	71 725	71 734	71 742	71 750	71 759
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523	71 850	71 858	71 867	71 875	71 883	71 892	71 900	71 908	71 917	71 925
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526	72 099	72 107	72 115	72 123	72 132	72 140	72 148	72 156	72 165	72 173
527	72 181	72 189	72 198	72 206	72 214	72 222	72 230	72 239	72 247	72 255
528	72 263	72 272	72 280	72 288	72 296	72 304	72 313	72 321	72 329	72 337
529	72 346	72 354	72 362	72 370	72 378	72 387	72 395	72 403	72 411	72 419
530	72 428	72 436	72 444	72 452	72 460	72 469	72 477	72 485	72 493	72 501
531	72 509	72 518	72 526	72 534	72 542	72 550	72 558	72 567	72 575	72 583
532	72 591	72 599	72 607	72 616	72 624	72 632	72 640	72 648	72 656	72 665
533	72 673	72 681	72 689	72 697	72 705	72 713	72 722	72 730	72 738	72 746
534	72 754	72 762	72 770	72 779	72 787	72 795	72 803	72 811	72 819	72 827
535	72 835	72 843	72 852	72 860	72 868	72 876	72 884	72 892	72 900	72 908
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539	73 159	73 167	73 175	73 183	73 191	73 199	73 207	73 215	73 223	73 231
540	73 239	73 247	73 255	73 263	73 272	73 280	73 288	73 296	73 304	73 312
541	73 320	73 328	73 336	73 344	73 352	73 360	73 368	73 376	73 384	73 392
542	73 400	73 408	73 416	73 424	73 432	73 440	73 448	73 456	73 464	73 472
543	73 480	73 488	73 496	73 504	73 512	73 520	73 528	73 536	73 544	73 552
544	73 560	73 568	73 576	73 584	73 592	73 600	73 608	73 616	73 624	73 632
545	73 640	73 648	73 656	73 664	73 672	73 679	73 687	73 695	73 703	73 711
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547	73 799	73 807	73 815	73 823	73 830	73 838	73 846	73 854	73 862	73 870
548	73 878	73 886	73 894	73 902	73 910	73 918	73 926	73 933	73 941	73 949
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No. 0 1 2 3 4 5 6 7 8 9

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551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
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558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
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564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
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567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
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571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
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575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
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586	76 790	76 797	76 805	76 812	76 819	76 827	76 834	76 842	76 849	76 856
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591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
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594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444
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596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
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602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
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605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
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611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
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615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
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619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
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621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	79 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
629	79 865	79 872	79 879	79 886	79 893	79 900	79 906	79 913	79 920	79 927
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631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
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633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
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644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
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648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
No.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

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652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
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660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
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665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
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685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
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687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
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691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
No.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

No.	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
No.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

No.	0	1	2	3	4	5	6	7	8	9
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

No.	0	1	2	3	4	5	6	7	8	9
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
No.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

No.	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
No.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

0°

′	Log sin	Log tan	Log cot	Log cos	′
0	—	—	—	10.00 000	**60**
1	6.46 373	6.46 373	13.53 627	10.00 000	59
2	6.76 476	6.76 476	13.23 524	10.00 000	58
3	6.94 085	6.94 085	13.05 915	10.00 000	57
4	7.06 579	7.06 579	12.93 421	10.00 000	56
5	7.16 270	7.16 270	12.83 730	10.00 000	55
6	7.24 188	7.24 188	12.75 812	10.00 000	54
7	7.30 882	7.30 882	12.69 118	10.00 000	53
8	7.36 682	7.36 682	12.63 318	10.00 000	52
9	7.41 797	7.41 797	12.58 203	10.00 000	51
10	7.46 373	7.46 373	12.53 627	10.00 000	**50**
11	7.50 512	7.50 512	12.49 488	10.00 000	49
12	7.54 291	7.54 291	12.45 709	10.00 000	48
13	7.57 767	7.57 767	12.42 233	10.00 000	47
14	7.60 985	7.60 986	12.39 014	10.00 000	46
15	7.63 982	7.63 982	12.36 018	10.00 000	45
16	7.66 784	7.66 785	12.33 215	10.00 000	44
17	7.69 417	7.69 418	12.30 582	9.99 999	43
18	7.71 900	7.71 900	12.28 100	9.99 999	42
19	7.74 248	7.74 248	12.25 752	9.99 999	41
20	7.76 475	7.76 476	12.23 524	9.99 999	**40**
21	7.78 594	7.78 595	12.21 405	9.99 999	39
22	7.80 615	7.80 615	12.19 385	9.99 999	38
23	7.82 545	7.82 546	12.17 454	9.99 999	37
24	7.84 393	7.84 394	12.15 606	9.99 999	36
25	7.86 166	7.86 167	12.13 833	9.99 999	35
26	7.87 870	7.87 871	12.12 129	9.99 999	34
27	7.89 509	7.89 510	12.10 490	9.99 999	33
28	7.91 088	7.91 089	12.08 911	9.99 999	32
29	7.92 612	7.92 613	12.07 387	9.99 998	31
30	7.94 084	7.94 086	12.05 914	9.99 998	**30**
31	7.95 508	7.95 510	12.04 490	9.99 998	29
32	7.96 887	7.96 889	12.03 111	9.99 998	28
33	7.98 223	7.98 225	12.01 775	9.99 998	27
34	7.99 520	7.99 522	12.00 478	9.99 998	26
35	8.00 779	8.00 781	11.99 219	9.99 998	25
36	8.02 002	8.02 004	11.97 996	9.99 998	24
37	8.03 192	8.03 194	11.96 806	9.99 997	23
38	8.04 350	8.04 353	11.95 647	9.99 997	22
39	8.05 478	8.05 481	11.94 519	9.99 997	21
40	8.06 578	8.06 581	11.93 419	9.99 997	**20**
41	8.07 650	8.07 653	11.92 347	9.99 997	19
42	8.08 696	8.08 700	11.91 300	9.99 997	18
43	8.09 718	8.09 722	11.90 278	9.99 997	17
44	8.10 717	8.10 720	11.89 280	9.99 996	16
45	8.11 693	8.11 696	11.88 304	9.99 996	15
46	8.12 647	8.12 651	11.87 349	9.99 996	14
47	8.13 581	8.13 585	11.86 415	9.99 996	13
48	8.14 495	8.14 500	11.85 500	9.99 996	12
49	8.15 391	8.15 395	11.84 605	9.99 996	11
50	8.16 268	8.16 273	11.83 727	9.99 995	**10**
51	8.17 128	8.17 133	11.82 867	9.99 995	9
52	8.17 971	8.17 976	11.82 024	9.99 995	8
53	8.18 798	8.18 804	11.81 196	9.99 995	7
54	8.19 610	8.19 616	11.80 384	9.99 995	6
55	8.20 407	8.20 413	11.79 587	9.99 994	5
56	8.21 189	8.21 195	11.78 805	9.99 994	4
57	8.21 958	8.21 964	11.78 036	9.99 994	3
58	8.22 713	8.22 720	11.77 280	9.99 994	2
59	8.23 456	8.23 462	11.76 538	9.99 994	1
60	8.24 186	8.24 192	11.75 808	9.99 993	**0**
′	Log cos	Log cot	Log tan	Log sin	′

89°

1°

′	8 L sin	8 L tan	11 L cot	9 L cos	′
0	.24 186	.24 192	.75 808	.99 993	60
1	.24 903	.24 910	.75 090	.99 993	59
2	.25 609	.25 616	.74 384	.99 993	58
3	.26 304	.26 312	.73 688	.99 993	57
4	.26 988	.26 996	.73 004	.99 992	56
5	.27 661	.27 669	.72 331	.99 992	55
6	.28 324	.28 332	.71 668	.99 992	54
7	.28 977	.28 986	.71 014	.99 992	53
8	.29 621	.29 629	.70 371	.99 992	52
9	.30 255	.30 263	.69 737	.99 991	51
10	.30 879	.30 888	.69 112	.99 991	50
11	.31 495	.31 505	.68 495	.99 991	49
12	.32 103	.32 112	.67 888	.99 990	48
13	.32 702	.32 711	.67 289	.99 990	47
14	.33 292	.33 302	.66 698	.99 990	46
15	.33 875	.33 886	.66 114	.99 990	45
16	.34 450	.34 461	.65 539	.99 989	44
17	.35 018	.35 029	.64 971	.99 989	43
18	.35 578	.35 590	.64 410	.99 989	42
19	.36 131	.36 143	.63 857	.99 989	41
20	.36 678	.36 689	.63 311	.99 988	40
21	.37 217	.37 229	.62 771	.99 988	39
22	.37 750	.37 762	.62 238	.99 988	38
23	.38 276	.38 289	.61 711	.99 987	37
24	.38 796	.38 809	.61 191	.99 987	36
25	.39 310	.39 323	.60 677	.99 987	35
26	.39 818	.39 832	.60 168	.99 986	34
27	.40 320	.40 334	.59 666	.99 986	33
28	.40 816	.40 830	.59 170	.99 986	32
29	.41 307	.41 321	.58 679	.99 985	31
30	.41 792	.41 807	.58 193	.99 985	30
31	.42 272	.42 287	.57 713	.99 985	29
32	.42 746	.42 762	.57 238	.99 984	28
33	.43 216	.43 232	.56 768	.99 984	27
34	.43 680	.43 696	.56 304	.99 984	26
35	.44 139	.44 156	.55 844	.99 983	25
36	.44 594	.44 611	.55 389	.99 983	24
37	.45 044	.45 061	.54 939	.99 983	23
38	.45 489	.45 507	.54 493	.99 982	22
39	.45 930	.45 948	.54 052	.99 982	21
40	.46 366	.46 385	.53 615	.99 982	20
41	.46 799	.46 817	.53 183	.99 981	19
42	.47 226	.47 245	.52 755	.99 981	18
43	.47 650	.47 669	.52 331	.99 981	17
44	.48 069	.48 089	.51 911	.99 980	16
45	.48 485	.48 505	.51 495	.99 980	15
46	.48 896	.48 917	.51 083	.99 979	14
47	.49 304	.49 325	.50 675	.99 979	13
48	.49 708	.49 729	.50 271	.99 979	12
49	.50 108	.50 130	.49 870	.99 978	11
50	.50 504	.50 527	.49 473	.99 978	10
51	.50 897	.50 920	.49 080	.99 977	9
52	.51 287	.51 310	.48 690	.99 977	8
53	.51 673	.51 696	.48 304	.99 977	7
54	.52 055	.52 079	.47 921	.99 976	6
55	.52 434	.52 459	.47 541	.99 976	5
56	.52 810	.52 835	.47 165	.99 975	4
57	.53 183	.53 208	.46 792	.99 975	3
58	.53 552	.53 578	.46 422	.99 974	2
59	.53 919	.53 945	.46 055	.99 974	1
60	.54 282	.54 308	.45 692	.99 974	0
′	8 L cos	8 L cot	11 L tan	9 L sin	′

88°

2°

′	8 L sin	8 L tan	11 L cot	9 L cos	′
0	.54 282	.54 308	.45 692	.99 974	60
1	.54 642	.54 669	.45 331	.99 973	59
2	.54 999	.55 027	.44 973	.99 973	58
3	.55 354	.55 382	.44 618	.99 972	57
4	.55 705	.55 734	.44 266	.99 972	56
5	.56 054	.56 083	.43 917	.99 971	55
6	.56 400	.56 429	.43 571	.99 971	54
7	.56 743	.56 773	.43 227	.99 970	53
8	.57 084	.57 114	.42 886	.99 970	52
9	.57 421	.57 452	.42 548	.99 969	51
10	.57 757	.57 788	.42 212	.99 969	50
11	.58 089	.58 121	.41 879	.99 968	49
12	.58 419	.58 451	.41 549	.99 968	48
13	.58 747	.58 779	.41 221	.99 967	47
14	.59 072	.59 105	.40 895	.99 967	46
15	.59 395	.59 428	.40 572	.99 967	45
16	.59 715	.59 749	.40 251	.99 966	44
17	.60 033	.60 068	.39 932	.99 966	43
18	.60 349	.60 384	.39 616	.99 965	42
19	.60 662	.60 698	.39 302	.99 964	41
20	.60 973	.61 009	.38 991	.99 964	40
21	.61 282	.61 319	.38 681	.99 963	39
22	.61 589	.61 626	.38 374	.99 963	38
23	.61 894	.61 931	.38 069	.99 962	37
24	.62 196	.62 234	.37 766	.99 962	36
25	.62 497	.62 535	.37 465	.99 961	35
26	.62 795	.62 834	.37 166	.99 961	34
27	.63 091	.63 131	.36 869	.99 960	33
28	.63 385	.63 426	.36 574	.99 960	32
29	.63 678	.63 718	.36 282	.99 959	31
30	.63 968	.64 009	.35 991	.99 959	30
31	.64 256	.64 298	.35 702	.99 958	29
32	.64 543	.64 585	.35 415	.99 958	28
33	.64 827	.64 870	.35 130	.99 957	27
34	.65 110	.65 154	.34 846	.99 956	26
35	.65 391	.65 435	.34 565	.99 956	25
36	.65 670	.65 715	.34 285	.99 955	24
37	.65 947	.65 993	.34 007	.99 955	23
38	.66 223	.66 269	.33 731	.99 954	22
39	.66 497	.66 543	.33 457	.99 954	21
40	.66 769	.66 816	.33 184	.99 953	20
41	.67 039	.67 087	.32 913	.99 952	19
42	.67 308	.67 356	.32 644	.99 952	18
43	.67 575	.67 624	.32 376	.99 951	17
44	.67 841	.67 890	.32 110	.99 951	16
45	.68 104	.68 154	.31 846	.99 950	15
46	.68 367	.68 417	.31 583	.99 949	14
47	.68 627	.68 678	.31 322	.99 949	13
48	.68 886	.68 938	.31 062	.99 948	12
49	.69 144	.69 196	.30 804	.99 948	11
50	.69 400	.69 453	.30 547	.99 947	10
51	.69 654	.69 708	.30 292	.99 946	9
52	.69 907	.69 962	.30 038	.99 946	8
53	.70 159	.70 214	.29 786	.99 945	7
54	.70 409	.70 465	.29 535	.99 944	6
55	.70 658	.70 714	.29 286	.99 944	5
56	.70 905	.70 962	.29 038	.99 943	4
57	.71 151	.71 208	.28 792	.99 942	3
58	.71 395	.71 453	.28 547	.99 942	2
59	.71 638	.71 697	.28 303	.99 941	1
60	.71 880	.71 940	.28 060	.99 940	0
′	8 L cos	8 L cot	11 L tan	9 L sin	′

87°

3°

′	8 L sin	8 L tan	11 L cot	9 L cos	′
0	.71 880	.71 940	.28 060	.99 940	**60**
1	.72 120	.72 181	.27 819	.99 940	59
2	.72 359	.72 420	.27 580	.99 939	58
3	.72 597	.72 659	.27 341	.99 938	57
4	.72 834	.72 896	.27 104	.99 938	56
5	.73 069	.73 132	.26 868	.99 937	55
6	.73 303	.73 366	.26 634	.99 936	54
7	.73 535	.73 600	.26 400	.99 936	53
8	.73 767	.73 832	.26 168	.99 935	52
9	.73 997	.74 063	.25 937	.99 934	51
10	.74 226	.74 292	.25 708	.99 934	**50**
11	.74 454	.74 521	.25 479	.99 933	49
12	.74 680	.74 748	.25 252	.99 932	48
13	.74 906	.74 974	.25 026	.99 932	47
14	.75 130	.75 199	.24 801	.99 931	46
15	.75 353	.75 423	.24 577	.99 930	45
16	.75 575	.75 645	.24 355	.99 929	44
17	.75 795	.75 867	.24 133	.99 929	43
18	.76 015	.76 087	.23 913	.99 928	42
19	.76 234	.76 306	.23 694	.99 927	41
20	.76 451	.76 525	.23 475	.99 926	**40**
21	.76 667	.76 742	.23 258	.99 926	39
22	.76 883	.76 958	.23 042	.99 925	38
23	.77 097	.77 173	.22 827	.99 924	37
24	.77 310	.77 387	.22 613	.99 923	36
25	.77 522	.77 600	.22 400	.99 923	35
26	.77 733	.77 811	.22 189	.99 922	34
27	.77 943	.78 022	.21 978	.99 921	33
28	.78 152	.78 232	.21 768	.99 920	32
29	.78 360	.78 441	.21 559	.99 920	31
30	.78 568	.78 649	.21 351	.99 919	**30**
31	.78 774	.78 855	.21 145	.99 918	29
32	.78 979	.79 061	.20 939	.99 917	28
33	.79 183	.79 266	.20 734	.99 917	27
34	.79 386	.79 470	.20 530	.99 916	26
35	.79 588	.79 673	.20 327	.99 915	25
36	.79 789	.79 875	.20 125	.99 914	24
37	.79 990	.80 076	.19 924	.99 913	23
38	.80 189	.80 277	.19 723	.99 913	22
39	.80 388	.80 476	.19 524	.99 912	21
40	.80 585	.80 674	.19 326	.99 911	**20**
41	.80 782	.80 872	.19 128	.99 910	19
42	.80 978	.81 068	.18 932	.99 909	18
43	.81 173	.81 264	.18 736	.99 909	17
44	.81 367	.81 459	.18 541	.99 908	16
45	.81 560	.81 653	.18 347	.99 907	15
46	.81 752	.81 846	.18 154	.99 906	14
47	.81 944	.82 038	.17 962	.99 905	13
48	.82 134	.82 230	.17 770	.99 904	12
49	.82 324	.82 420	.17 580	.99 904	11
50	.82 513	.82 610	.17 390	.99 903	**10**
51	.82 701	.82 799	.17 201	.99 902	9
52	.82 888	.82 987	.17 013	.99 901	8
53	.83 075	.83 175	.16 825	.99 900	7
54	.83 261	.83 361	.16 639	.99 899	6
55	.83 446	.83 547	.16 453	.99 898	5
56	.83 630	.83 732	.16 268	.99 898	4
57	.83 813	.83 916	.16 084	.99 897	3
58	.83 996	.84 100	.15 900	.99 896	2
59	.84 177	.84 282	.15 718	.99 895	1
60	.84 358	.84 464	.15 536	.99 894	**0**
′	8 L cos	8 L cot	11 L tan	9 L sin	′

86°

4°

′	8 L sin	8 L tan	11 L cot	9 L cos	′
0	.84 358	.84 464	.15 536	.99 894	**60**
1	.84 539	.84 646	.15 354	.99 893	59
2	.84 718	.84 826	.15 174	.99 892	58
3	.84 897	.85 006	.14 994	.99 891	57
4	.85 075	.85 185	.14 815	.99 891	56
5	.85 252	.85 363	.14 637	.99 890	55
6	.85 429	.85 540	.14 460	.99 889	54
7	.85 605	.85 717	.14 283	.99 888	53
8	.85 780	.85 893	.14 107	.99 887	52
9	.85 955	.86 069	.13 931	.99 886	51
10	.86 128	.86 243	.13 757	.99 885	**50**
11	.86 301	.86 417	.13 583	.99 884	49
12	.86 474	.86 591	.13 409	.99 883	48
13	.86 645	.86 763	.13 237	.99 882	47
14	.86 816	.86 935	.13 065	.99 881	46
15	.86 987	.87 106	.12 894	.99 880	45
16	.87 156	.87 277	.12 723	.99 879	44
17	.87 325	.87 447	.12 553	.99 879	43
18	.87 494	.87 616	.12 384	.99 878	42
19	.87 661	.87 785	.12 215	.99 877	41
20	.87 829	.87 953	.12 047	.99 876	**40**
21	.87 995	.88 120	.11 880	.99 875	39
22	.88 161	.88 287	.11 713	.99 874	38
23	.88 326	.88 453	.11 547	.99 873	37
24	.88 490	.88 618	.11 382	.99 872	36
25	.88 654	.88 783	.11 217	.99 871	35
26	.88 817	.88 948	.11 052	.99 870	34
27	.88 980	.89 111	.10 889	.99 869	33
28	.89 142	.89 274	.10 726	.99 868	32
29	.89 304	.89 437	.10 563	.99 867	31
30	.89 464	.89 598	.10 402	.99 866	**30**
31	.89 625	.89 760	.10 240	.99 865	29
32	.89 784	.89 920	.10 080	.99 864	28
33	.89 943	.90 080	.09 920	.99 863	27
34	.90 102	.90 240	.09 760	.99 862	26
35	.90 260	.90 399	.09 601	.99 861	25
36	.90 417	.90 557	.09 443	.99 860	24
37	.90 574	.90 715	.09 285	.99 859	23
38	.90 730	.90 872	.09 128	.99 858	22
39	.90 885	.91 029	.08 971	.99 857	21
40	.91 040	.91 185	.08 815	.99 856	**20**
41	.91 195	.91 340	.08 660	.99 855	19
42	.91 349	.91 495	.08 505	.99 854	18
43	.91 502	.91 650	.08 350	.99 853	17
44	.91 655	.91 803	.08 197	.99 852	16
45	.91 807	.91 957	.08 043	.99 851	15
46	.91 959	.92 110	.07 890	.99 850	14
47	.92 110	.92 262	.07 738	.99 848	13
48	.92 261	.92 414	.07 586	.99 847	12
49	.92 411	.92 565	.07 435	.99 846	11
50	.92 561	.92 716	.07 284	.99 845	**10**
51	.92 710	.92 866	.07 134	.99 844	9
52	.92 859	.93 016	.06 984	.99 843	8
53	.93 007	.93 165	.06 835	.99 842	7
54	.93 154	.93 313	.06 687	.99 841	6
55	.93 301	.93 462	.06 538	.99 840	5
56	.93 448	.93 609	.06 391	.99 839	4
57	.93 594	.93 756	.06 244	.99 838	3
58	.93 740	.93 903	.06 097	.99 837	2
59	.93 885	.94 049	.05 951	.99 836	1
60	.94 030	.94 195	.05 805	.99 834	**0**
′	8 L cos	8 L cot	11 L tan	9 L sin	′

85°

5°

′	8 L sin	8 L tan	11 L cot	9 L cos	′
0	.94 030	.94 195	.05 805	.99 834	60
1	.94 174	.94 340	.05 660	.99 833	59
2	.94 317	.94 485	.05 515	.99 832	58
3	.94 461	.94 630	.05 370	.99 831	57
4	.94 603	.94 773	.05 227	.99 830	56
5	.94 746	.94 917	.05 083	.99 829	55
6	.94 887	.95 060	.04 940	.99 828	54
7	.95 029	.95 202	.04 798	.99 827	53
8	.95 170	.95 344	.04 656	.99 825	52
9	.95 310	.95 486	.04 514	.99 824	51
10	.95 450	.95 627	.04 373	.99 823	50
11	.95 589	.95 767	.04 233	.99 822	49
12	.95 728	.95 908	.04 092	.99 821	48
13	.95 867	.96 047	.03 953	.99 820	47
14	.96 005	.96 187	.03 813	.99 819	46
15	.96 143	.96 325	.03 675	.99 817	45
16	.96 280	.96 464	.03 536	.99 816	44
17	.96 417	.96 602	.03 398	.99 815	43
18	.96 553	.96 739	.03 261	.99 814	42
19	.96 689	.96 877	.03 123	.99 813	41
20	.96 825	.97 013	.02 987	.99 812	40
21	.96 960	.97 150	.02 850	.99 810	39
22	.97 095	.97 285	.02 715	.99 809	38
23	.97 229	.97 421	.02 579	.99 808	37
24	.97 363	.97 556	.02 444	.99 807	36
25	.97 496	.97 691	.02 309	.99 806	35
26	.97 629	.97 825	.02 175	.99 804	34
27	.97 762	.97 959	.02 041	.99 803	33
28	.97 894	.98 092	.01 908	.99 802	32
29	.98 026	.98 225	.01 775	.99 801	31
30	.98 157	.98 358	.01 642	.99 800	30
31	.98 288	.98 490	.01 510	.99 798	29
32	.98 419	.98 622	.01 378	.99 797	28
33	.98 549	.98 753	.01 247	.99 796	27
34	.98 679	.98 884	.01 116	.99 795	26
35	.98 808	.99 015	.00 985	.99 793	25
36	.98 937	.99 145	.00 855	.99 792	24
37	.99 066	.99 275	.00 725	.99 791	23
38	.99 194	.99 405	.00 595	.99 790	22
39	.99 322	.99 534	.00 466	.99 788	21
40	.99 450	.99 662	.00 338	.99 787	20
41	.99 577	.99 791	.00 209	.99 786	19
42	.99 704	.99 919	.00 081	.99 785	18
43	.99 830	.00 046	.99 954	.99 783	17
44	.99 956	.00 174	.99 826	.99 782	16
45	.00 082	.00 301	.99 699	.99 781	15
46	.00 207	.00 427	.99 573	.99 780	14
47	.00 332	.00 553	.99 447	.99 778	13
48	.00 456	.00 679	.99 321	.99 777	12
49	.00 581	.00 805	.99 195	.99 776	11
50	.00 704	.00 930	.99 070	.99 775	10
51	.00 828	.01 055	.98 945	.99 773	9
52	.00 951	.01 179	.98 821	.99 772	8
53	.01 074	.01 303	.98 697	.99 771	7
54	.01 196	.01 427	.98 573	.99 769	6
55	.01 318	.01 550	.98 450	.99 768	5
56	.01 440	.01 673	.98 327	.99 767	4
57	.01 561	.01 796	.98 204	.99 765	3
58	.01 682	.01 918	.98 082	.99 764	2
59	.01 803	.02 040	.97 960	.99 763	1
60	.01 923	.02 162	.97 838	.99 761	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

84°

6°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.01 923	.02 162	.97 838	.99 761	60
1	.02 043	.02 283	.97 717	.99 760	59
2	.02 163	.02 404	.97 596	.99 759	58
3	.02 283	.02 525	.97 475	.99 757	57
4	.02 402	.02 645	.97 355	.99 756	56
5	.02 520	.02 766	.97 234	.99 755	55
6	.02 639	.02 885	.97 115	.99 753	54
7	.02 757	.03 005	.96 995	.99 752	53
8	.02 874	.03 124	.96 876	.99 751	52
9	.02 992	.03 242	.96 758	.99 749	51
10	.03 109	.03 361	.96 639	.99 748	50
11	.03 226	.03 479	.96 521	.99 747	49
12	.03 342	.03 597	.96 403	.99 745	48
13	.03 458	.03 714	.96 286	.99 744	47
14	.03 574	.03 832	.96 168	.99 742	46
15	.03 690	.03 948	.96 052	.99 741	45
16	.03 805	.04 065	.95 935	.99 740	44
17	.03 920	.04 181	.95 819	.99 738	43
18	.04 034	.04 297	.95 703	.99 737	42
19	.04 149	.04 413	.95 587	.99 736	41
20	.04 262	.04 528	.95 472	.99 734	40
21	.04 376	.04 643	.95 357	.99 733	39
22	.04 490	.04 758	.95 242	.99 731	38
23	.04 603	.04 873	.95 127	.99 730	37
24	.04 715	.04 987	.95 013	.99 728	36
25	.04 828	.05 101	.94 899	.99 727	35
26	.04 940	.05 214	.94 786	.99 726	34
27	.05 052	.05 328	.94 672	.99 724	33
28	.05 164	.05 441	.94 559	.99 723	32
29	.05 275	.05 553	.94 447	.99 721	31
30	.05 386	.05 666	.94 334	.99 720	30
31	.05 497	.05 778	.94 222	.99 718	29
32	.05 607	.05 890	.94 110	.99 717	28
33	.05 717	.06 002	.93 998	.99 716	27
34	.05 827	.06 113	.93 887	.99 714	26
35	.05 937	.06 224	.93 776	.99 713	25
36	.06 046	.06 335	.93 665	.99 711	24
37	.06 155	.06 445	.93 555	.99 710	23
38	.06 264	.06 556	.93 444	.99 708	22
39	.06 372	.06 666	.93 334	.99 707	21
40	.06 481	.06 775	.93 225	.99 705	20
41	.06 589	.06 885	.93 115	.99 704	19
42	.06 696	.06 994	.93 006	.99 702	18
43	.06 804	.07 103	.92 897	.99 701	17
44	.06 911	.07 211	.92 789	.99 699	16
45	.07 018	.07 320	.92 680	.99 698	15
46	.07 124	.07 428	.92 572	.99 696	14
47	.07 231	.07 536	.92 464	.99 695	13
48	.07 337	.07 643	.92 357	.99 693	12
49	.07 442	.07 751	.92 249	.99 692	11
50	.07 548	.07 858	.92 142	.99 690	10
51	.07 653	.07 964	.92 036	.99 689	9
52	.07 758	.08 071	.91 929	.99 687	8
53	.07 863	.08 177	.91 823	.99 686	7
54	.07 968	.08 283	.91 717	.99 684	6
55	.08 072	.08 389	.91 611	.99 683	5
56	.08 176	.08 495	.91 505	.99 681	4
57	.08 280	.08 600	.91 400	.99 680	3
58	.08 383	.08 705	.91 295	.99 678	2
59	.08 486	.08 810	.91 190	.99 677	1
60	.08 589	.08 914	.91 086	.99 675	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

83°

7°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.08 589	.08 914	.91 086	.99 675	**60**
1	.08 692	.09 019	.90 981	.99 674	59
2	.08 795	.09 123	.90 877	.99 672	58
3	.08 897	.09 227	.90 773	.99 670	57
4	.08 999	.09 330	.90 670	.99 669	56
5	.09 101	.09 434	.90 566	.99 667	55
6	.09 202	.09 537	.90 463	.99 666	54
7	.09 304	.09 640	.90 360	.99 664	53
8	.09 405	.09 742	.90 258	.99 663	52
9	.09 506	.09 845	.90 155	.99 661	51
10	.09 606	.09 947	.90 053	.99 659	**50**
11	.09 707	.10 049	.89 951	.99 658	49
12	.09 807	.10 150	.89 850	.99 656	48
13	.09 907	.10 252	.89 748	.99 655	47
14	.10 006	.10 353	.89 647	.99 653	46
15	.10 106	.10 454	.89 546	.99 651	45
16	.10 205	.10 555	.89 445	.99 650	44
17	.10 304	.10 656	.89 344	.99 648	43
18	.10 402	.10 756	.89 244	.99 647	42
19	.10 501	.10 856	.89 144	.99 645	41
20	.10 599	.10 956	.89 044	.99 643	**40**
21	.10 697	.11 056	.88 944	.99 642	39
22	.10 795	.11 155	.88 845	.99 640	38
23	.10 893	.11 254	.88 746	.99 638	37
24	.10 990	.11 353	.88 647	.99 637	36
25	.11 087	.11 452	.88 548	.99 635	35
26	.11 184	.11 551	.88 449	.99 633	34
27	.11 281	.11 649	.88 351	.99 632	33
28	.11 377	.11 747	.88 253	.99 630	32
29	.11 474	.11 845	.88 155	.99 629	31
30	.11 570	.11 943	.88 057	.99 627	**30**
31	.11 666	.12 040	.87 960	.99 625	29
32	.11 761	.12 138	.87 862	.99 624	28
33	.11 857	.12 235	.87 765	.99 622	27
34	.11 952	.12 332	.87 668	.99 620	26
35	.12 047	.12 428	.87 572	.99 618	25
36	.12 142	.12 525	.87 475	.99 617	24
37	.12 236	.12 621	.87 379	.99 615	23
38	.12 331	.12 717	.87 283	.99 613	22
39	.12 425	.12 813	.87 187	.99 612	21
40	.12 519	.12 909	.87 091	.99 610	**20**
41	.12 612	.13 004	.86 996	.99 608	19
42	.12 706	.13 099	.86 901	.99 607	18
43	.12 799	.13 194	.86 806	.99 605	17
44	.12 892	.13 289	.86 711	.99 603	16
45	.12 985	.13 384	.86 616	.99 601	15
46	.13 078	.13 478	.86 522	.99 600	14
47	.13 171	.13 573	.86 427	.99 598	13
48	.13 263	.13 667	.86 333	.99 596	12
49	.13 355	.13 761	.86 239	.99 595	11
50	.13 447	.13 854	.86 146	.99 593	**10**
51	.13 539	.13 948	.86 052	.99 591	9
52	.13 630	.14 041	.85 959	.99 589	8
53	.13 722	.14 134	.85 866	.99 588	7
54	.13 813	.14 227	.85 773	.99 586	6
55	.13 904	.14 320	.85 680	.99 584	5
56	.13 994	.14 412	.85 588	.99 582	4
57	.14 085	.14 504	.85 496	.99 581	3
58	.14 175	.14 597	.85 403	.99 579	2
59	.14 266	.14 688	.85 312	.99 577	1
60	.14 356	.14 780	.85 220	.99 575	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

82°

8°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.14 356	.14 780	.85 220	.99 575	**60**
1	.14 445	.14 872	.85 128	.99 574	59
2	.14 535	.14 963	.85 037	.99 572	58
3	.14 624	.15 054	.84 946	.99 570	57
4	.14 714	.15 145	.84 855	.99 568	56
5	.14 803	.15 236	.84 764	.99 566	55
6	.14 891	.15 327	.84 673	.99 565	54
7	.14 980	.15 417	.84 583	.99 563	53
8	.15 069	.15 508	.84 492	.99 561	52
9	.15 157	.15 598	.84 402	.99 559	51
10	.15 245	.15 688	.84 312	.99 557	**50**
11	.15 333	.15 777	.84 223	.99 556	49
12	.15 421	.15 867	.84 133	.99 554	48
13	.15 508	.15 956	.84 044	.99 552	47
14	.15 596	.16 046	.83 954	.99 550	46
15	.15 683	.16 135	.83 865	.99 548	45
16	.15 770	.16 224	.83 776	.99 546	44
17	.15 857	.16 312	.83 688	.99 545	43
18	.15 944	.16 401	.83 599	.99 543	42
19	.16 030	.16 489	.83 511	.99 541	41
20	.16 116	.16 577	.83 423	.99 539	**40**
21	.16 203	.16 665	.83 335	.99 537	39
22	.16 289	.16 753	.83 247	.99 535	38
23	.16 374	.16 841	.83 159	.99 533	37
24	.16 460	.16 928	.83 072	.99 532	36
25	.16 545	.17 016	.82 984	.99 530	35
26	.16 631	.17 103	.82 897	.99 528	34
27	.16 716	.17 190	.82 810	.99 526	33
28	.16 801	.17 277	.82 723	.99 524	32
29	.16 886	.17 363	.82 637	.99 522	31
30	.16 970	.17 450	.82 550	.99 520	**30**
31	.17 055	.17 536	.82 464	.99 518	29
32	.17 139	.17 622	.82 378	.99 517	28
33	.17 223	.17 708	.82 292	.99 515	27
34	.17 307	.17 794	.82 206	.99 513	26
35	.17 391	.17 880	.82 120	.99 511	25
36	.17 474	.17 965	.82 035	.99 509	24
37	.17 558	.18 051	.81 949	.99 507	23
38	.17 641	.18 136	.81 864	.99 505	22
39	.17 724	.18 221	.81 779	.99 503	21
40	.17 807	.18 306	.81 694	.99 501	**20**
41	.17 890	.18 391	.81 609	.99 499	19
42	.17 973	.18 475	.81 525	.99 497	18
43	.18 055	.18 560	.81 440	.99 495	17
44	.18 137	.18 644	.81 356	.99 494	16
45	.18 220	.18 728	.81 272	.99 492	15
46	.18 302	.18 812	.81 188	.99 490	14
47	.18 383	.18 896	.81 104	.99 488	13
48	.18 465	.18 979	.81 021	.99 486	12
49	.18 547	.19 063	.80 937	.99 484	11
50	.18 628	.19 146	.80 854	.99 482	**10**
51	.18 709	.19 229	.80 771	.99 480	9
52	.18 790	.19 312	.80 688	.99 478	8
53	.18 871	.19 395	.80 605	.99 476	7
54	.18 952	.19 478	.80 522	.99 474	6
55	.19 033	.19 561	.80 439	.99 472	5
56	.19 113	.19 643	.80 357	.99 470	4
57	.19 193	.19 725	.80 275	.99 468	3
58	.19 273	.19 807	.80 193	.99 466	2
59	.19 353	.19 889	.80 111	.99 464	1
60	.19 433	.19 971	.80 029	.99 462	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

81°

9°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.19 433	.19 971	.80 029	.99 462	**60**
1	.19 513	.20 053	.79 947	.99 460	59
2	.19 592	.20 134	.79 866	.99 458	58
3	.19 672	20 216	.79 784	.99 456	57
4	.19 751	20 297	.79 703	.99 454	56
5	.19 830	.20 378	.79 622	.99 452	55
6	.19 909	.20 459	.79 541	.99 450	54
7	.19 988	.20 540	.79 460	.99 448	53
8	.20 067	.20 621	.79 379	.99 446	52
9	.20 145	.20 701	.79 299	.99 444	51
10	.20 223	.20 782	.79 218	.99 442	**50**
11	.20 302	.20 862	.79 138	.99 440	49
12	.20 380	.20 942	.79 058	.99 438	48
13	.20 458	.21 022	.78 978	.99 436	47
14	.20 535	.21 102	.78 898	.99 434	46
15	.20 613	.21 182	.78 818	.99 432	45
16	.20 691	.21 261	.78 739	.99 429	44
17	.20 768	.21 341	.78 659	.99 427	43
18	.20 845	.21 420	.78 580	.99 425	42
19	.20 922	.21 499	.78 501	.99 423	41
20	.20 999	.21 578	.78 422	.99 421	**40**
21	.21 076	.21 657	.78 343	.99 419	39
22	.21 153	.21 736	.78 264	.99 417	38
23	.21 229	.21 814	.78 186	.99 415	37
24	.21 306	.21 893	.78 107	.99 413	36
25	.21 382	.21 971	.78 029	.99 411	35
26	.21 458	.22 049	.77 951	.99 409	34
27	.21 534	.22 127	.77 873	.99 407	33
28	.21 610	.22 205	.77 795	.99 404	32
29	.21 685	.22 283	.77 717	.99 402	31
30	.21 761	.22 361	.77 639	.99 400	**30**
31	.21 836	.22 438	.77 562	.99 398	29
32	.21 912	.22 516	.77 484	.99 396	28
33	.21 987	.22 593	.77 407	.99 394	27
34	.22 062	.22 670	.77 330	.99 392	26
35	.22 137	.22 747	.77 253	.99 390	25
36	.22 211	.22 824	.77 176	.99 388	24
37	.22 286	.22 901	.77 099	.99 385	23
38	.22 361	.22 977	.77 023	.99 383	22
39	.22 435	.23 054	.76 946	.99 381	21
40	.22 509	.23 130	.76 870	.99 379	**20**
41	.22 583	.23 206	.76 794	.99 377	19
42	.22 657	.23 283	.76 717	.99 375	18
43	.22 731	.23 359	.76 641	.99 372	17
44	.22 805	.23 435	.76 565	.99 370	16
45	.22 878	.23 510	.76 490	.99 368	15
46	.22 952	.23 586	.76 414	.99 366	14
47	.23 025	.23 661	.76 339	.99 364	13
48	.23 098	.23 737	.76 263	.99 362	12
49	.23 171	.23 812	.76 188	.99 359	11
50	.23 244	.23 887	.76 113	.99 357	**10**
51	.23 317	.23 962	.76 038	.99 355	9
52	.23 390	.24 037	.75 963	.99 353	8
53	.23 462	.24 112	.75 888	.99 351	7
54	.23 535	.24 186	.75 814	.99 348	6
55	.23 607	.24 261	.75 739	.99 346	5
56	.23 679	.24 335	.75 665	.99 344	4
57	.23 752	.24 410	.75 590	.99 342	3
58	.23 823	.24 484	.75 516	.99 340	2
59	.23 895	.24 558	.75 442	.99 337	1
60	.23 967	.24 632	.75 368	.99 335	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

80°

10°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.23 967	.24 632	.75 368	.99 335	**60**
1	.24 039	.24 706	.75 294	.99 333	59
2	.24 110	.24 779	.75 221	.99 331	58
3	.24 181	.24 853	.75 147	.99 328	57
4	.24 253	.24 926	.75 074	.99 326	56
5	.24 324	.25 000	.75 000	.99 324	55
6	.24 395	.25 073	.74 927	.99 322	54
7	.24 466	.25 146	.74 854	.99 319	53
8	.24 536	.25 219	.74 781	.99 317	52
9	.24 607	.25 292	.74 708	.99 315	51
10	.24 677	.25 365	.74 635	.99 313	**50**
11	.24 748	.25 437	.74 563	.99 310	49
12	.24 818	.25 510	.74 490	.99 308	48
13	.24 888	.25 582	.74 418	.99 306	47
14	.24 958	.25 655	.74 345	.99 304	46
15	.25 028	.25 727	.74 273	.99 301	45
16	.25 098	.25 799	.74 201	.99 299	44
17	.25 168	.25 871	.74 129	.99 297	43
18	.25 237	.25 943	.74 057	.99 294	42
19	.25 307	.26 015	.73 985	.99 292	41
20	.25 376	.26 086	.73 914	.99 290	**40**
21	.25 445	.26 158	.73 842	.99 288	39
22	.25 514	.26 229	.73 771	.99 285	38
23	.25 583	.26 301	.73 699	.99 283	37
24	.25 652	.26 372	.73 628	.99 281	36
25	.25 721	.26 443	.73 557	.99 278	35
26	.25 790	.26 514	.73 486	.99 276	34
27	.25 858	.26 585	.73 415	.99 274	33
28	.25 927	.26 655	.73 345	.99 271	32
29	.25 995	.26 726	.73 274	.99 269	31
30	.26 063	.26 797	.73 203	.99 267	**30**
31	.26 131	.26 867	.73 133	.99 264	29
32	.26 199	.26 937	.73 063	.99 262	28
33	.26 267	.27 008	.72 992	.99 260	27
34	.26 335	.27 078	.72 922	.99 257	26
35	.26 403	.27 148	.72 852	.99 255	25
36	.26 470	.27 218	.72 782	.99 252	24
37	.26 538	.27 288	.72 712	.99 250	23
38	.26 605	.27 357	.72 643	.99 248	22
39	.26 672	.27 427	.72 573	.99 245	21
40	.26 739	.27 496	.72 504	.99 243	**20**
41	.26 806	.27 566	.72 434	.99 241	19
42	.26 873	.27 635	.72 365	.99 238	18
43	.26 940	.27 704	.72 296	.99 236	17
44	.27 007	.27 773	.72 227	.99 233	16
45	.27 073	.27 842	.72 158	.99 231	15
46	.27 140	.27 911	.72 089	.99 229	14
47	.27 206	.27 980	.72 020	.99 226	13
48	.27 273	.28 049	.71 951	.99 224	12
49	.27 339	.28 117	.71 883	.99 221	11
50	.27 405	.28 186	.71 814	.99 219	**10**
51	.27 471	.28 254	.71 746	.99 217	9
52	.27 537	.28 323	.71 677	.99 214	8
53	.27 602	.28 391	.71 609	.99 212	7
54	.27 668	.28 459	.71 541	.99 209	6
55	.27 734	.28 527	.71 473	.99 207	5
56	.27 799	.28 595	.71 405	.99 204	4
57	.27 864	.28 662	.71 338	.99 202	3
58	.27 930	.28 730	.71 270	.99 200	2
59	.27 995	.28 798	.71 202	.99 197	1
60	.28 060	.28 865	.71 135	.99 195	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

79°

11°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.28 060	.28 865	.71 135	.99 195	60
1	.28 125	.28 933	.71 067	.99 192	59
2	.28 190	.29 000	.71 000	.99 190	58
3	.28 254	.29 067	.70 933	.99 187	57
4	.28 319	.29 134	.70 866	.99 185	56
5	.28 384	.29 201	.70 799	.99 182	55
6	.28 448	.29 268	.70 732	.99 180	54
7	.28 512	.29 335	.70 665	.99 177	53
8	.28 577	.29 402	.70 598	.99 175	52
9	.28 641	.29 468	.70 532	.99 172	51
10	.28 705	.29 535	.70 465	.99 170	50
11	.28 769	.29 601	.70 399	.99 167	49
12	.28 833	.29 668	.70 332	.99 165	48
13	.28 896	.29 734	.70 266	.99 162	47
14	.28 960	.29 800	.70 200	.99 160	46
15	.29 024	.29 866	.70 134	.99 157	45
16	.29 087	.29 932	.70 068	.99 155	44
17	.29 150	.29 998	.70 002	.99 152	43
18	.29 214	.30 064	.69 936	.99 150	42
19	.29 277	.30 130	.69 870	.99 147	41
20	.29 340	.30 195	.69 805	.99 145	40
21	.29 403	.30 261	.69 739	.99 142	39
22	.29 466	.30 326	.69 674	.99 140	38
23	.29 529	.30 391	.69 609	.99 137	37
24	.29 591	.30 457	.69 543	.99 135	36
25	.29 654	.30 522	.69 478	.99 132	35
26	.29 716	.30 587	.69 413	.99 130	34
27	.29 779	.30 652	.69 348	.99 127	33
28	.29 841	.30 717	.69 283	.99 124	32
29	.29 903	.30 782	.69 218	.99 122	31
30	.29 966	.30 846	.69 154	.99 119	30
31	.30 028	.30 911	.69 089	.99 117	29
32	.30 090	.30 975	.69 025	.99 114	28
33	.30 151	.31 040	.68 960	.99 112	27
34	.30 213	.31 104	.68 896	.99 109	26
35	.30 275	.31 168	.68 832	.99 106	25
36	.30 336	.31 233	.68 767	.99 104	24
37	.30 398	.31 297	.68 703	.99 101	23
38	.30 459	.31 361	.68 639	.99 099	22
39	.30 521	.31 425	.68 575	.99 096	21
40	.30 582	.31 489	.68 511	.99 093	20
41	.30 643	.31 552	.68 448	.99 091	19
42	.30 704	.31 616	.68 384	.99 088	18
43	.30 765	.31 679	.68 321	.99 086	17
44	.30 826	.31 743	.68 257	.99 083	16
45	.30 887	.31 806	.68 194	.99 080	15
46	.30 947	.31 870	.68 130	.99 078	14
47	.31 008	.31 933	.68 067	.99 075	13
48	.31 068	.31 996	.68 004	.99 072	12
49	.31 129	.32 059	.67 941	.99 070	11
50	.31 189	.32 122	.67 878	.99 067	10
51	.31 250	.32 185	.67 815	.99 064	9
52	.31 310	.32 248	.67 752	.99 062	8
53	.31 370	.32 311	.67 689	.99 059	7
54	.31 430	.32 373	.67 627	.99 056	6
55	.31 490	.32 436	.67 564	.99 054	5
56	.31 549	.32 498	.67 502	.99 051	4
57	.31 609	.32 561	.67 439	.99 048	3
58	.31 669	.32 623	.67 377	.99 046	2
59	.31 728	.32 685	.67 315	.99 043	1
60	.31 788	.32 747	.67 253	.99 040	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

78°

12°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.31 788	.32 747	.67 253	.99 040	60
1	.31 847	.32 810	.67 190	.99 038	59
2	.31 907	.32 872	.67 128	.99 035	58
3	.31 966	.32 933	.67 067	.99 032	57
4	.32 025	.32 995	.67 005	.99 030	56
5	.32 084	.33 057	.66 943	.99 027	55
6	.32 143	.33 119	.66 881	.99 024	54
7	.32 202	.33 180	.66 820	.99 022	53
8	.32 261	.33 242	.66 758	.99 019	52
9	.32 319	.33 303	.66 697	.99 016	51
10	.32 378	.33 365	.66 635	.99 013	50
11	.32 437	.33 426	.66 574	.99 011	49
12	.32 495	.33 487	.66 513	.99 008	48
13	.32 553	.33 548	.66 452	.99 005	47
14	.32 612	.33 609	.66 391	.99 002	46
15	.32 670	.33 670	.66 330	.99 000	45
16	.32 728	.33 731	.66 269	.98 997	44
17	.32 786	.33 792	.66 208	.98 994	43
18	.32 844	.33 853	.66 147	.98 991	42
19	.32 902	.33 913	.66 087	.98 989	41
20	.32 960	.33 974	.66 026	.98 986	40
21	.33 018	.34 034	.65 966	.98 983	39
22	.33 075	.34 095	.65 905	.98 980	38
23	.33 133	.34 155	.65 845	.98 978	37
24	.33 190	.34 215	.65 785	.98 975	36
25	.33 248	.34 276	.65 724	.98 972	35
26	.33 305	.34 336	.65 664	.98 969	34
27	.33 362	.34 396	.65 604	.98 967	33
28	.33 420	.34 456	.65 544	.98 964	32
29	.33 477	.34 516	.65 484	.98 961	31
30	.33 534	.34 576	.65 424	.98 958	30
31	.33 591	.34 635	.65 365	.98 955	29
32	.33 647	.34 695	.65 305	.98 953	28
33	.33 704	.34 755	.65 245	.98 950	27
34	.33 761	.34 814	.65 186	.98 947	26
35	.33 818	.34 874	.65 126	.98 944	25
36	.33 874	.34 933	.65 067	.98 941	24
37	.33 931	.34 992	.65 008	.98 938	23
38	.33 987	.35 051	.64 949	.98 936	22
39	.34 043	.35 111	.64 889	.98 933	21
40	.34 100	.35 170	.64 830	.98 930	20
41	.34 156	.35 229	.64 771	.98 927	19
42	.34 212	.35 288	.64 712	.98 924	18
43	.34 268	.35 347	.64 653	.98 921	17
44	.34 324	.35 405	.64 595	.98 919	16
45	.34 380	.35 464	.64 536	.98 916	15
46	.34 436	.35 523	.64 477	.98 913	14
47	.34 491	.35 581	.64 419	.98 910	13
48	.34 547	.35 640	.64 360	.98 907	12
49	.34 602	.35 698	.64 302	.98 904	11
50	.34 658	.35 757	.64 243	.98 901	10
51	.34 713	.35 815	.64 185	.98 898	9
52	.34 769	.35 873	.64 127	.98 896	8
53	.34 824	.35 931	.64 069	.98 893	7
54	.34 879	.35 989	.64 011	.98 890	6
55	.34 934	.36 047	.63 953	.98 887	5
56	.34 989	.36 105	.63 895	.98 884	4
57	.35 044	.36 163	.63 837	.98 881	3
58	.35 099	.36 221	.63 779	.98 878	2
59	.35 154	.36 279	.63 721	.98 875	1
60	.35 209	.36 336	.63 664	.98 872	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

77°

13°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.35 209	.36 336	.63 664	.98 872	60
1	.35 263	.36 394	.63 606	.98 869	59
2	.35 318	.36 452	.63 548	.98 867	58
3	.35 373	.36 509	.63 491	.98 864	57
4	.35 427	.36 566	.63 434	.98 861	56
5	.35 481	.36 624	.63 376	.98 858	55
6	.35 536	.36 681	.63 319	.98 855	54
7	.35 590	.36 738	.63 262	.98 852	53
8	.35 644	.36 795	.63 205	.98 849	52
9	.35 698	.36 852	.63 148	.98 846	51
10	.35 752	.36 909	.63 091	.98 843	50
11	.35 806	.36 966	.63 034	.98 840	49
12	.35 860	.37 023	.62 977	.98 837	48
13	.35 914	.37 080	.62 920	.98 834	47
14	.35 968	.37 137	.62 863	.98 831	46
15	.36 022	.37 193	.62 807	.98 828	45
16	.36 075	.37 250	.62 750	.98 825	44
17	.36 129	.37 306	.62 694	.98 822	43
18	.36 182	.37 363	.62 637	.98 819	42
19	.36 236	.37 419	.62 581	.98 816	41
20	.36 289	.37 476	.62 524	.98 813	40
21	.36 342	.37 532	.62 468	.98 810	39
22	.36 395	.37 588	.62 412	.98 807	38
23	.36 449	.37 644	.62 356	.98 804	37
24	.36 502	.37 700	.62 300	.98 801	36
25	.36 555	.37 756	.62 244	.98 798	35
26	.36 608	.37 812	.62 188	.98 795	34
27	.36 660	.37 868	.62 132	.98 792	33
28	.36 713	.37 924	.62 076	.98 789	32
29	.36 766	.37 980	.62 020	.98 786	31
30	.36 819	.38 035	.61 965	.98 783	30
31	.36 871	.38 091	.61 909	.98 780	29
32	.36 924	.38 147	.61 853	.98 777	28
33	.36 976	.38 202	.61 798	.98 774	27
34	.37 028	.38 257	.61 743	.98 771	26
35	.37 081	.38 313	.61 687	.98 768	25
36	.37 133	.38 368	.61 632	.98 765	24
37	.37 185	.38 423	.61 577	.98 762	23
38	.37 237	.38 479	.61 521	.98 759	22
39	.37 289	.38 534	.61 466	.98 756	21
40	.37 341	.38 589	.61 411	.98 753	20
41	.37 393	.38 644	.61 356	.98 750	19
42	.37 445	.38 699	.61 301	.98 746	18
43	.37 497	.38 754	.61 246	.98 743	17
44	.37 549	.38 808	.61 192	.98 740	16
45	.37 600	.38 863	.61 137	.98 737	15
46	.37 652	.38 918	.61 082	.98 734	14
47	.37 703	.38 972	.61 028	.98 731	13
48	.37 755	.39 027	.60 973	.98 728	12
49	.37 806	.39 082	.60 918	.98 725	11
50	.37 858	.39 136	.60 864	.98 722	10
51	.37 909	.39 190	.60 810	.98 719	9
52	.37 960	.39 245	.60 755	.98 715	8
53	.38 011	.39 299	.60 701	.98 712	7
54	.38 062	.39 353	.60 647	.98 709	6
55	.38 113	.39 407	.60 593	.98 706	5
56	.38 164	.39 461	.60 539	.98 703	4
57	.38 215	.39 515	.60 485	.98 700	3
58	.38 266	.39 569	.60 431	.98 697	2
59	.38 317	.39 623	.60 377	.98 694	1
60	.38 368	.39 677	.60 323	.98 690	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

76°

14°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.38 368	.39 677	.60 323	.98 690	60
1	.38 418	.39 731	.60 269	.98 687	59
2	.38 469	.39 785	.60 215	.98 684	58
3	.38 519	.39 838	.60 162	.98 681	57
4	.38 570	.39 892	.60 108	.98 678	56
5	.38 620	.39 945	.60 055	.98 675	55
6	.38 670	.39 999	.60 001	.98 671	54
7	.38 721	.40 052	.59 948	.98 668	53
8	.38 771	.40 106	.59 894	.98 665	52
9	.38 821	.40 159	.59 841	.98 662	51
10	.38 871	.40 212	.59 788	.98 659	50
11	.38 921	.40 266	.59 734	.98 656	49
12	.38 971	.40 319	.59 681	.98 652	48
13	.39 021	.40 372	.59 628	.98 649	47
14	.39 071	.40 425	.59 575	.98 646	46
15	.39 121	.40 478	.59 522	.98 643	45
16	.39 170	.40 531	.59 469	.98 640	44
17	.39 220	.40 584	.59 416	.98 636	43
18	.39 270	.40 636	.59 364	.98 633	42
19	.39 319	.40 689	.59 311	.98 630	41
20	.39 369	.40 742	.59 258	.98 627	40
21	.39 418	.40 795	.59 205	.98 623	39
22	.39 467	.40 847	.59 153	.98 620	38
23	.39 517	.40 900	.59 100	.98 617	37
24	.39 566	.40 952	.59 048	.98 614	36
25	.39 615	.41 005	.58 995	.98 610	35
26	.39 664	.41 057	.58 943	.98 607	34
27	.39 713	.41 109	.58 891	.98 604	33
28	.39 762	.41 161	.58 839	.98 601	32
29	.39 811	.41 214	.58 786	.98 597	31
30	.39 860	.41 266	.58 734	.98 594	30
31	.39 909	.41 318	.58 682	.98 591	29
32	.39 958	.41 370	.58 630	.98 588	28
33	.40 006	.41 422	.58 578	.98 584	27
34	.40 055	.41 474	.58 526	.98 581	26
35	.40 103	.41 526	.58 474	.98 578	25
36	.40 152	.41 578	.58 422	.98 574	24
37	.40 200	.41 629	.58 371	.98 571	23
38	.40 249	.41 681	.58 319	.98 568	22
39	.40 297	.41 733	.58 267	.98 565	21
40	.40 346	.41 784	.58 216	.98 561	20
41	.40 394	.41 836	.58 164	.98 558	19
42	.40 442	.41 887	.58 113	.98 555	18
43	.40 490	.41 939	.58 061	.98 551	17
44	.40 538	.41 990	.58 010	.98 548	16
45	.40 586	.42 041	.57 959	.98 545	15
46	.40 634	.42 093	.57 907	.98 541	14
47	.40 682	.42 144	.57 856	.98 538	13
48	.40 730	.42 195	.57 805	.98 535	12
49	.40 778	.42 246	.57 754	.98 531	11
50	.40 825	.42 297	.57 703	.98 528	10
51	.40 873	.42 348	.57 652	.98 525	9
52	.40 921	.42 399	.57 601	.98 521	8
53	.40 968	.42 450	.57 550	.98 518	7
54	.41 016	.42 501	.57 499	.98 515	6
55	.41 063	.42 552	.57 448	.98 511	5
56	.41 111	.42 603	.57 397	.98 508	4
57	.41 158	.42 653	.57 347	.98 505	3
58	.41 205	.42 704	.57 296	.98 501	2
59	.41 252	.42 755	.57 245	.98 498	1
60	.41 300	.42 805	.57 195	.98 494	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

75°

15°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.41 300	.42 805	.57 195	.98 494	**60**
1	.41 347	.42 856	.57 144	.98 491	59
2	.41 394	.42 906	.57 094	.98 488	58
3	.41 441	.42 957	.57 043	.98 484	57
4	.41 488	.43 007	.56 993	.98 481	56
5	.41 535	.43 057	.56 943	.98 477	55
6	.41 582	.43 108	.56 892	.98 474	54
7	.41 628	.43 158	.56 842	.98 471	53
8	.41 675	.43 208	.56 792	.98 467	52
9	.41 722	.43 258	.56 742	.98 464	51
10	.41 768	.43 308	.56 692	.98 460	**50**
11	.41 815	.43 358	.56 642	.98 457	49
12	.41 861	.43 408	.56 592	.98 453	48
13	.41 908	.43 458	.56 542	.98 450	47
14	.41 954	.43 508	.56 492	.98 447	46
15	.42 001	.43 558	.56 442	.98 443	45
16	.42 047	.43 607	.56 393	.98 440	44
17	.42 093	.43 657	.56 343	.98 436	43
18	.42 140	.43 707	.56 293	.98 433	42
19	.42 186	.43 756	.56 244	.98 429	41
20	.42 232	.43 806	.56 194	.98 426	**40**
21	.42 278	.43 855	.56 145	.98 422	39
22	.42 324	.43 905	.56 095	.98 419	38
23	.42 370	.43 954	.56 046	.98 415	37
24	.42 416	.44 004	.55 996	.98 412	36
25	.42 461	.44 053	.55 947	.98 409	35
26	.42 507	.44 102	.55 898	.98 405	34
27	.42 553	.44 151	.55 849	.98 402	33
28	.42 599	.44 201	.55 799	.98 398	32
29	.42 644	.44 250	.55 750	.98 395	31
30	.42 690	.44 299	.55 701	.98 391	**30**
31	.42 735	.44 348	.55 652	.98 388	29
32	.42 781	.44 397	.55 603	.98 384	28
33	.42 826	.44 446	.55 554	.98 381	27
34	.42 872	.44 495	.55 505	.98 377	26
35	.42 917	.44 544	.55 456	.98 373	25
36	.42 962	.44 592	.55 408	.98 370	24
37	.43 008	.44 641	.55 359	.98 366	23
38	.43 053	.44 690	.55 310	.98 363	22
39	.43 098	.44 738	.55 262	.98 359	21
40	.43 143	.44 787	.55 213	.98 356	**20**
41	.43 188	.44 836	.55 164	.98 352	19
42	.43 233	.44 884	.55 116	.98 349	18
43	.43 278	.44 933	.55 067	.98 345	17
44	.43 323	.44 981	.55 019	.98 342	16
45	.43 367	.45 029	.54 971	.98 338	15
46	.43 412	.45 078	.54 922	.98 334	14
47	.43 457	.45 126	.54 874	.98 331	13
48	.43 502	.45 174	.54 826	.98 327	12
49	.43 546	.45 222	.54 778	.98 324	11
50	.43 591	.45 271	.54 729	.98 320	**10**
51	.43 635	.45 319	.54 681	.98 317	9
52	.43 680	.45 367	.54 633	.98 313	8
53	.43 724	.45 415	.54 585	.98 309	7
54	.43 769	.45 463	.54 537	.98 306	6
55	.43 813	.45 511	.54 489	.98 302	5
56	.43 857	.45 559	.54 441	.98 299	4
57	.43 901	.45 606	.54 394	.98 295	3
58	.43 946	.45 654	.54 346	.98 291	2
59	.43 990	.45 702	.54 298	.98 288	1
60	.44 034	.45 750	.54 250	.98 284	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

74°

16°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.44 034	.45 750	.54 250	.98 284	**60**
1	.44 078	.45 797	.54 203	.98 281	59
2	.44 122	.45 845	.54 155	.98 277	58
3	.44 166	.45 892	.54 108	.98 273	57
4	.44 210	.45 940	.54 060	.98 270	56
5	.44 253	.45 987	.54 013	.98 266	55
6	.44 297	.46 035	.53 965	.98 262	54
7	.44 341	.46 082	.53 918	.98 259	53
8	.44 385	.46 130	.53 870	.98 255	52
9	.44 428	.46 177	.53 823	.98 251	51
10	.44 472	.46 224	.53 776	.98 248	**50**
11	.44 516	.46 271	.53 729	.98 244	49
12	.44 559	.46 319	.53 681	.98 240	48
13	.44 602	.46 366	.53 634	.98 237	47
14	.44 646	.46 413	.53 587	.98 233	46
15	.44 689	.46 460	.53 540	.98 229	45
16	.44 733	.46 507	.53 493	.98 226	44
17	.44 776	.46 554	.53 446	.98 222	43
18	.44 819	.46 601	.53 399	.98 218	42
19	.44 862	.46 648	.53 352	.98 215	41
20	.44 905	.46 694	.53 306	.98 211	**40**
21	.44 948	.46 741	.53 259	.98 207	39
22	.44 992	.46 788	.53 212	.98 204	38
23	.45 035	.46 835	.53 165	.98 200	37
24	.45 077	.46 881	.53 119	.98 196	36
25	.45 120	.46 928	.53 072	.98 192	35
26	.45 163	.46 975	.53 025	.98 189	34
27	.45 206	.47 021	.52 979	.98 185	33
28	.45 249	.47 068	.52 932	.98 181	32
29	.45 292	.47 114	.52 886	.98 177	31
30	.45 334	.47 160	.52 840	.98 174	**30**
31	.45 377	.47 207	.52 793	.98 170	29
32	.45 419	.47 253	.52 747	.98 166	28
33	.45 462	.47 299	.52 701	.98 162	27
34	.45 504	.47 346	.52 654	.98 159	26
35	.45 547	.47 392	.52 608	.98 155	25
36	.45 589	.47 438	.52 562	.98 151	24
37	.45 632	.47 484	.52 516	.98 147	23
38	.45 674	.47 530	.52 470	.98 144	22
39	.45 716	.47 576	.52 424	.98 140	21
40	.45 758	.47 622	.52 378	.98 136	**20**
41	.45 801	.47 668	.52 332	.98 132	19
42	.45 843	.47 714	.52 286	.98 129	18
43	.45 885	.47 760	.52 240	.98 125	17
44	.45 927	.47 806	.52 194	.98 121	16
45	.45 969	.47 852	.52 148	.98 117	15
46	.46 011	.47 897	.52 103	.98 113	14
47	.46 053	.47 943	.52 057	.98 110	13
48	.46 095	.47 989	.52 011	.98 106	12
49	.46 136	.48 035	.51 965	.98 102	11
50	.46 178	.48 080	.51 920	.98 098	**10**
51	.46 220	.48 126	.51 874	.98 094	9
52	.46 262	.48 171	.51 829	.98 090	8
53	.46 303	.48 217	.51 783	.98 087	7
54	.46 345	.48 262	.51 738	.98 083	6
55	.46 386	.48 307	.51 693	.98 079	5
56	.46 428	.48 353	.51 647	.98 075	4
57	.46 469	.48 398	.51 602	.98 071	3
58	.46 511	.48 443	.51 557	.98 067	2
59	.46 552	.48 489	.51 511	.98 063	1
60	.46 594	.48 534	.51 466	.98 060	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

73°

17°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.46 594	.48 534	.51 466	.98 060	**60**
1	.46 635	.48 579	.51 421	.98 056	59
2	.46 676	.48 624	.51 376	.98 052	58
3	.46 717	.48 669	.51 331	.98 048	57
4	.46 758	.48 714	.51 286	.98 044	56
5	.46 800	.48 759	.51 241	.98 040	55
6	.46 841	.48 804	.51 196	.98 036	54
7	.46 882	.48 849	.51 151	.98 032	53
8	.46 923	.48 894	.51 106	.98 029	52
9	.46 964	.48 939	.51 061	.98 025	51
10	.47 005	.48 984	.51 016	.98 021	**50**
11	.47 045	.49 029	.50 971	.98 017	49
12	.47 086	.49 073	.50 927	.98 013	48
13	.47 127	.49 118	.50 882	.98 009	47
14	.47 168	.49 163	.50 837	.98 005	46
15	.47 209	.49 207	.50 793	.98 001	45
16	.47 249	.49 252	.50 748	.97 997	44
17	.47 290	.49 296	.50 704	.97 993	43
18	.47 330	.49 341	.50 659	.97 989	42
19	.47 371	.49 385	.50 615	.97 986	41
20	.47 411	.49 430	.50 570	.97 982	**40**
21	.47 452	.49 474	.50 526	.97 978	39
22	.47 492	.49 519	.50 481	.97 974	38
23	.47 533	.49 563	.50 437	.97 970	37
24	.47 573	.49 607	.50 393	.97 966	36
25	.47 613	.49 652	.50 348	.97 962	35
26	.47 654	.49 696	50 304	.97 958	34
27	.47 694	.49 740	.50 260	.97 954	33
28	.47 734	.49 784	.50 216	.97 950	32
29	.47 774	.49 828	.50 172	.97 946	31
30	.47 814	.49 872	.50 128	.97 942	**30**
31	.47 854	.49 916	.50 084	.97 938	29
32	.47 894	.49 960	.50 040	.97 934	28
33	.47 934	.50 004	.49 996	.97 930	27
34	.47 974	.50 048	.49 952	.97 926	26
35	.48 014	.50 092	.49 908	.97 922	25
36	.48 054	.50 136	.49 864	.97 918	24
37	.48 094	.50 180	.49 820	.97 914	23
38	.48 133	.50 223	.49 777	.97 910	22
39	.48 173	.50 267	.49 733	.97 906	21
40	.48 213	.50 311	.49 689	.97 902	**20**
41	.48 252	.50 355	.49 645	.97 898	19
42	.48 292	.50 398	.49 602	.97 894	18
43	.48 332	.50 442	.49 558	.97 890	17
44	.48 371	.50 485	.49 515	.97 886	16
45	.48 411	.50 529	.49 471	.97 882	15
46	.48 450	.50 572	.49 428	.97 878	14
47	.48 490	.50 616	.49 384	.97 874	13
48	.48 529	.50 659	.49 341	.97 870	12
49	.48 568	.50 703	.49 297	.97 866	11
50	.48 607	.50 746	.49 254	.97 861	**10**
51	.48 647	.50 789	.49 211	.97 857	9
52	.48 686	.50 833	.49 167	.97 853	8
53	.48 725	.50 876	.49 124	.97 849	7
54	.48 764	.50 919	.49 081	.97 845	6
55	.48 803	.50 962	.49 038	.97 841	5
56	.48 842	.51 005	.48 995	.97 837	4
57	.48 881	.51 048	.48 952	.97 833	3
58	.48 920	.51 092	.48 908	.97 829	2
59	.48 959	.51 135	.48 865	.97 825	1
60	.48 998	.51 178	.48 822	.97 821	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

72°

18°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.48 998	.51 178	.48 822	.97 821	**60**
1	.49 037	.51 221	.48 779	.97 817	59
2	.49 076	.51 264	.48 736	.97 812	58
3	.49 115	.51 306	.48 694	.97 808	57
4	.49 153	.51 349	.48 651	.97 804	56
5	.49 192	.51 392	.48 608	.97 800	55
6	.49 231	.51 435	.48 565	.97 796	54
7	.49 269	.51 478	.48 522	.97 792	53
8	.49 308	.51 520	.48 480	.97 788	52
9	.49 347	.51 563	.48 437	.97 784	51
10	.49 385	.51 606	.48 394	.97 779	**50**
11	.49 424	.51 648	.48 352	.97 775	49
12	.49 462	.51 691	.48 309	.97 771	48
13	.49 500	.51 734	.48 266	.97 767	47
14	.49 539	.51 776	.48 224	.97 763	46
15	.49 577	.51 819	.48 181	.97 759	45
16	.49 615	.51 861	.48 139	.97 754	44
17	.49 654	.51 903	.48 097	.97 750	43
18	.49 692	.51 946	.48 054	.97 746	42
19	.49 730	.51 988	.48 012	.97 742	41
20	.49 768	.52 031	.47 969	.97 738	**40**
21	.49 806	.52 073	.47 927	.97 734	39
22	.49 844	.52 115	.47 885	.97 729	38
23	.49 882	.52 157	.47 843	.97 725	37
24	.49 920	.52 200	.47 800	.97 721	36
25	.49 958	.52 242	.47 758	.97 717	35
26	.49 996	.52 284	.47 716	.97 713	34
27	.50 034	.52 326	.47 674	.97 708	33
28	.50 072	.52 368	.47 632	.97 704	32
29	.50 110	.52 410	.47 590	.97 700	31
30	.50 148	.52 452	.47 548	.97 696	**30**
31	.50 185	.52 494	.47 506	.97 691	29
32	.50 223	.52 536	.47 464	.97 687	28
33	.50 261	.52 578	.47 422	.97 683	27
34	.50 298	.52 620	.47 380	.97 679	26
35	.50 336	.52 661	.47 339	.97 674	25
36	.50 374	.52 703	.47 297	.97 670	24
37	.50 411	.52 745	.47 255	.97 666	23
38	.50 449	.52 787	.47 213	.97 662	22
39	.50 486	.52 829	.47 171	.97 657	21
40	.50 523	.52 870	.47 130	.97 653	**20**
41	.50 561	.52 912	.47 088	.97 649	19
42	.50 598	.52 953	.47 047	.97 645	18
43	.50 635	.52 995	.47 005	.97 540	17
44	.50 673	.53 037	.46 963	.97 636	16
45	.50 710	.53 078	.46 922	.97 632	15
46	.50 747	.53 120	.46 880	.97 628	14
47	.50 784	.53 161	.46 839	.97 623	13
48	.50 821	.53 202	.46 798	.97 619	12
49	.50 858	.53 244	.46 756	.97 615	11
50	.50 896	.53 285	.46 715	.97 610	**10**
51	.50 933	.53 327	.46 673	.97 606	9
52	.50 970	.53 368	.46 632	.97 602	8
53	.51 007	.53 409	.46 591	.97 597	7
54	.51 043	.53 450	.46 550	.97 593	6
55	.51 080	.53 492	.46 508	.97 589	5
56	.51 117	.53 533	.46 467	.97 584	4
57	.51 154	.53 574	.46 426	.97 580	3
58	.51 191	.53 615	.46 385	.97 576	2
59	.51 227	.53 656	.46 344	.97 571	1
60	.51 264	.53 697	.46 303	.97 567	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

71°

19°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.51 264	.53 697	.46 303	.97 567	**60**
1	.51 301	.53 738	.46 262	.97 563	59
2	.51 338	.53 779	.46 221	.97 558	58
3	.51 374	.53 820	.46 180	.97 554	57
4	.51 411	.53 861	.46 139	.97 550	56
5	.51 447	.53 902	.46 098	.97 545	55
6	.51 484	.53 943	.46 057	.97 541	54
7	.51 520	.53 984	.46 016	.97 536	53
8	.51 557	.54 025	.45 975	.97 532	52
9	.51 593	.54 065	.45 935	.97 528	51
10	.51 629	.54 106	.45 894	.97 523	**50**
11	.51 666	.54 147	.45 853	.97 519	49
12	.51 702	.54 187	.45 813	.97 515	48
13	.51 738	.54 228	.45 772	.97 510	47
14	.51 774	.54 269	.45 731	.97 506	46
15	.51 811	.54 309	.45 691	.97 501	45
16	.51 847	.54 350	.45 650	.97 497	44
17	.51 883	.54 390	.45 610	.97 492	43
18	.51 919	.54 431	.45 569	.97 488	42
19	.51 955	.54 471	.45 529	.97 484	41
20	.51 991	.54 512	.45 488	.97 479	**40**
21	.52 027	.54 552	.45 448	.97 475	39
22	.52 063	.54 593	.45 407	.97 470	38
23	.52 099	.54 633	.45 367	.97 466	37
24	.52 135	.54 673	.45 327	.97 461	36
25	.52 171	.54 714	.45 286	.97 457	35
26	.52 207	.54 754	.45 246	.97 453	34
27	.52 242	.54 794	.45 206	.97 448	33
28	.52 278	.54 835	.45 165	.97 444	32
29	.52 314	.54 875	.45 125	.97 439	31
30	.52 350	.54 915	.45 085	.97 435	**30**
31	.52 385	.54 955	.45 045	.97 430	29
32	.52 421	.54 995	.45 005	.97 426	28
33	.52 456	.55 035	.44 965	.97 421	27
34	.52 492	.55 075	.44 925	.97 417	26
35	.52 527	.55 115	.44 885	.97 412	25
36	.52 563	.55 155	.44 845	.97 408	24
37	.52 598	.55 195	.44 805	.97 403	23
38	.52 634	.55 235	.44 765	.97 399	22
39	.52 669	.55 275	.44 725	.97 394	21
40	.52 705	.55 315	.44 685	.97 390	**20**
41	.52 740	.55 355	.44 645	.97 385	19
42	.52 775	.55 395	.44 605	.97 381	18
43	.52 811	.55 434	.44 566	.97 376	17
44	.52 846	.55 474	.44 526	.97 372	16
45	.52 881	.55 514	.44 486	.97 367	15
46	.52 916	.55 554	.44 446	.97 363	14
47	.52 951	.55 593	.44 407	.97 358	13
48	.52 986	.55 633	.44 367	.97 353	12
49	.53 021	.55 673	.44 327	.97 349	11
50	.53 056	.55 712	.44 288	.97 344	**10**
51	.53 092	.55 752	.44 248	.97 340	9
52	.53 126	.55 791	.44 209	.97 335	8
53	.53 161	.55 831	.44 169	.97 331	7
54	.53 196	.55 870	.44 130	.97 326	6
55	.53 231	.55 910	.44 090	.97 322	5
56	.53 266	.55 949	.44 051	.97 317	4
57	.53 301	.55 989	.44 011	.97 312	3
58	.53 336	.56 028	.43 972	.97 308	2
59	.53 370	.56 067	.43 933	.97 303	1
60	.53 405	.56 107	.43 893	.97 299	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

70°

20°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.53 405	.56 107	.43 893	.97 299	**60**
1	.53 440	.56 146	.43 854	.97 294	59
2	.53 475	.56 185	.43 815	.97 289	58
3	.53 509	.56 224	.43 776	.97 285	57
4	.53 544	.56 264	.43 736	.97 280	56
5	.53 578	.56 303	.43 697	.97 276	55
6	.53 613	.56 342	.43 658	.97 271	54
7	.53 647	.56 381	.43 619	.97 266	53
8	.53 682	.56 420	.43 580	.97 262	52
9	.53 716	.56 459	.43 541	.97 257	51
10	.53 751	.56 498	.43 502	.97 252	**50**
11	.53 785	.56 537	.43 463	.97 248	49
12	.53 819	.56 576	.43 424	.97 243	48
13	.53 854	.56 615	.43 385	.97 238	47
14	.53 888	.56 654	.43 346	.97 234	46
15	.53 922	.56 693	.43 307	.97 229	45
16	.53 957	.56 732	.43 268	.97 224	44
17	.53 991	.56 771	.43 229	.97 220	43
18	.54 025	.56 810	.43 190	.97 215	42
19	.54 059	.56 849	.43 151	.97 210	41
20	.54 093	.56 887	.43 113	.97 206	**40**
21	.54 127	.56 926	.43 074	.97 201	39
22	.54 161	.56 965	.43 035	.97 196	38
23	.54 195	.57 004	.42 996	.97 192	37
24	.54 229	.57 042	.42 958	.97 187	36
25	.54 263	.57 081	.42 919	.97 182	35
26	.54 297	.57 120	.42 880	.97 178	34
27	.54 331	.57 158	.42 842	.97 173	33
28	.54 365	.57 197	.42 803	.97 168	32
29	.54 399	.57 235	.42 765	.97 163	31
30	.54 433	.57 274	.42 726	.97 159	**30**
31	.54 466	.57 312	.42 688	.97 154	29
32	.54 500	.57 351	.42 649	.97 149	28
33	.54 534	.57 389	.42 611	.97 145	27
34	.54 567	.57 428	.42 572	.97 140	26
35	.54 601	.57 466	.42 534	.97 135	25
36	.54 635	.57 504	.42 496	.97 130	24
37	.54 668	.57 543	.42 457	.97 126	23
38	.54 702	.57 581	.42 419	.97 121	22
39	.54 735	.57 619	.42 381	.97 116	21
40	.54 769	.57 658	.42 342	.97 111	**20**
41	.54 802	.57 696	.42 304	.97 107	19
42	.54 836	.57 734	.42 266	.97 102	18
43	.54 869	.57 772	.42 228	.97 097	17
44	.54 903	.57 810	.42 190	.97 092	16
45	.54 936	.57 849	.42 151	.97 087	15
46	.54 969	.57 887	.42 113	.97 083	14
47	.55 003	.57 925	.42 075	.97 078	13
48	.55 036	.57 963	.42 037	.97 073	12
49	.55 069	.58 001	.41 999	.97 068	11
50	.55 102	.58 039	.41 961	.97 063	**10**
51	.55 136	.58 077	.41 923	.97 059	9
52	.55 169	.58 115	.41 885	.97 054	8
53	.55 202	.58 153	.41 847	.97 049	7
54	.55 235	.58 191	.41 809	.97 044	6
55	.55 268	.58 229	.41 771	.97 039	5
56	.55 301	.58 267	.41 733	.97 035	4
57	.55 334	.58 304	.41 696	.97 030	3
58	.55 367	.58 342	.41 658	.97 025	2
59	.55 400	.58 380	.41 620	.97 020	1
60	.55 433	.58 418	.41 582	.97 015	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

69°

21°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.55 433	.58 418	.41 582	.97 015	**60**
1	.55 466	.58 455	.41 545	.97 010	59
2	.55 499	.58 493	.41 507	.97 005	58
3	.55 532	.58 531	.41 469	.97 001	57
4	.55 564	.58 569	.41 431	.96 996	56
5	.55 597	.58 606	.41 394	.96 991	55
6	.55 630	.58 644	.41 356	.96 986	54
7	.55 663	.58 681	.41 319	.96 981	53
8	.55 695	.58 719	.41 281	.96 976	52
9	.55 728	.58 757	.41 243	.96 971	51
10	.55 761	.58 794	.41 206	.96 966	**50**
11	.55 793	.58 832	.41 168	.96 962	49
12	.55 826	.58 869	.41 131	.96 957	48
13	.55 858	.58 907	.41 093	.96 952	47
14	.55 891	.58 944	.41 056	.96 947	46
15	.55 923	.58 981	.41 019	.96 942	45
16	.55 956	.59 019	.40 981	.96 937	44
17	.55 988	.59 056	.40 944	.96 932	43
18	.56 021	.59 094	.40 906	.96 927	42
19	.56 053	.59 131	.40 869	.96 922	41
20	.56 085	.59 168	.40 832	.96 917	**40**
21	.56 118	.59 205	.40 795	.96 912	39
22	.56 150	.59 243	.40 757	.96 907	38
23	.56 182	.59 280	.40 720	.96 903	37
24	.56 215	.59 317	.40 683	.96 898	36
25	.56 247	.59 354	.40 646	.96 893	35
26	.56 279	.59 391	.40 609	.96 888	34
27	.56 311	.59 429	.40 571	.96 883	33
28	.56 343	.59 466	.40 534	.96 878	32
29	.56 375	.59 503	.40 497	.96 873	31
30	.56 408	.59 540	.40 460	.96 868	**30**
31	.56 440	.59 577	.40 423	.96 863	29
32	.56 472	.59 614	.40 386	.96 858	28
33	.56 504	.59 651	.40 349	.96 853	27
34	.56 536	.59 688	.40 312	.96 848	26
35	.56 568	.59 725	.40 275	.96 843	25
36	.56 599	.59 762	.40 238	.96 838	24
37	.56 631	.59 799	.40 201	.96 833	23
38	.56 663	.59 835	.40 165	.96 828	22
39	.56 695	.59 872	.40 128	.96 823	21
40	.56 727	.59 909	.40 091	.96 818	**20**
41	.56 759	.59 946	.40 054	.96 813	19
42	.56 790	.59 983	.40 017	.96 808	18
43	.56 822	.60 019	.39 981	.96 803	17
44	.56 854	.60 056	.39 944	.96 798	16
45	.56 886	.60 093	.39 907	.96 793	15
46	.56 917	.60 130	.39 870	.96 788	14
47	.56 949	.60 166	.39 834	.96 783	13
48	.56 980	.60 203	.39 797	.96 778	12
49	.57 012	.60 240	.39 760	.96 772	11
50	.57 044	.60 276	.39 724	.96 767	**10**
51	.57 075	.60 313	.39 687	.96 762	9
52	.57 107	.60 349	.39 651	.96 757	8
53	.57 138	.60 386	.39 614	.96 752	7
54	.57 169	.60 422	.39 578	.96 747	6
55	.57 201	.60 459	.39 541	.96 742	5
56	.57 232	.60 495	.39 505	.96 737	4
57	.57 264	.60 532	.39 468	.96 732	3
58	.57 295	.60 568	.39 432	.96 727	2
59	.57 326	.60 605	.39 395	.96 722	1
60	.57 358	.60 641	.39 359	.96 717	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

68°

22°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.57 358	.60 641	.39 359	.96 717	**60**
1	.57 389	.60 677	.39 323	.96 711	59
2	.57 420	.60 714	.39 286	.96 706	58
3	.57 451	.60 750	.39 250	.96 701	57
4	.57 482	.60 786	.39 214	.96 696	56
5	.57 514	.60 823	.39 177	.96 691	55
6	.57 545	.60 859	.39 141	.96 686	54
7	.57 576	.60 895	.39 105	.96 681	53
8	.57 607	.60 931	.39 069	.96 676	52
9	.57 638	.60 967	.39 033	.96 670	51
10	.57 669	.61 004	.38 996	.96 665	**50**
11	.57 700	.61 040	.38 960	.96 660	49
12	.57 731	.61 076	.38 924	.96 655	48
13	.57 762	.61 112	.38 888	.96 650	47
14	.57 793	.61 148	.38 852	.96 645	46
15	.57 824	.61 184	.38 816	.96 640	45
16	.57 855	.61 220	.38 780	.96 634	44
17	.57 885	.61 256	.38 744	.96 629	43
18	.57 916	.61 292	.38 708	.96 624	42
19	.57 947	.61 328	.38 672	.96 619	41
20	.57 978	.61 364	.38 636	.96 614	**40**
21	.58 008	.61 400	.38 600	.96 608	39
22	.58 039	.61 436	.38 564	.96 603	38
23	.58 070	.61 472	.38 528	.96 598	37
24	.58 101	.61 508	.38 492	.96 593	36
25	.58 131	.61 544	.38 456	.96 588	35
26	.58 162	.61 579	.38 421	.96 582	34
27	.58 192	.61 615	.38 385	.96 577	33
28	.58 223	.61 651	.38 349	.96 572	32
29	.58 253	.61 687	.38 313	.96 567	31
30	.58 284	.61 722	.38 278	.96 562	**30**
31	.58 314	.61 758	.38 242	.96 556	29
32	.58 345	.61 794	.38 206	.96 551	28
33	.58 375	.61 830	.38 170	.96 546	27
34	.58 406	.61 865	.38 135	.96 541	26
35	.58 436	.61 901	.38 099	.96 535	25
36	.58 467	.61 936	.38 064	.96 530	24
37	.58 497	.61 972	.38 028	.96 525	23
38	.58 527	.62 008	.37 992	.96 520	22
39	.58 557	.62 043	.37 957	.96 514	21
40	.58 588	.62 079	.37 921	.96 509	**20**
41	.58 618	.62 114	.37 886	.96 504	19
42	.58 648	.62 150	.37 850	.96 498	18
43	.58 678	.62 185	.37 815	.96 493	17
44	.58 709	.62 221	.37 779	.96 488	16
45	.58 739	.62 256	.37 744	.96 483	15
46	.58 769	.62 292	.37 708	.96 477	14
47	.58 799	.62 327	.37 673	.96 472	13
48	.58 829	.62 362	.37 638	.96 467	12
49	.58 859	.62 398	.37 602	.96 461	11
50	.58 889	.62 433	.37 567	.96 456	**10**
51	.58 919	.62 468	.37 532	.96 451	9
52	.58 949	.62 504	.37 496	.96 445	8
53	.58 979	.62 539	.37 461	.96 440	7
54	.59 009	.62 574	.37 426	.96 435	6
55	.59 039	.62 609	.37 391	.96 429	5
56	.59 069	.62 645	.37 355	.96 424	4
57	.59 098	.62 680	.37 320	.96 419	3
58	.59 128	.62 715	.37 285	.96 413	2
59	.59 158	.62 750	.37 250	.96 408	1
60	.59 188	.62 785	.37 215	.96 403	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

67°

23°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.59 188	.62 785	.37 215	.96 403	60
1	.59 218	.62 820	.37 180	.96 397	59
2	.59 247	.62 855	.37 145	.96 392	58
3	.59 277	.62 890	.37 110	.96 387	57
4	.59 307	.62 926	.37 074	.96 381	56
5	.59 336	.62 961	.37 039	.96 376	55
6	.59 366	.62 996	.37 004	.96 370	54
7	.59 396	.63 031	.36 969	.96 365	53
8	.59 425	.63 066	.36 934	.96 360	52
9	.59 455	.63 101	.36 899	.96 354	51
10	.59 484	.63 135	.36 865	.96 349	50
11	.59 514	.63 170	.36 830	.96 343	49
12	.59 543	.63 205	.36 795	.96 338	48
13	.59 573	.63 240	.36 760	.96 333	47
14	.59 602	.63 275	.36 725	.96 327	46
15	.59 632	.63 310	.36 690	.96 322	45
16	.59 661	.63 345	.36 655	.96 316	44
17	.59 690	.63 379	.36 621	.96 311	43
18	.59 720	.63 414	.36 586	.96 305	42
19	.59 749	.63 449	.36 551	.96 300	41
20	.59 778	.63 484	.36 516	.96 294	40
21	.59 808	.63 519	.36 481	.96 289	39
22	.59 837	.63 553	.36 447	.96 284	38
23	.59 866	.63 588	.36 412	.96 278	37
24	.59 895	.63 623	.36 377	.96 273	36
25	.59 924	.63 657	.36 343	.96 267	35
26	.59 954	.63 692	.36 308	.96 262	34
27	.59 983	.63 726	.36 274	.96 256	33
28	.60 012	.63 761	.36 239	.96 251	32
29	.60 041	.63 796	.36 204	.96 245	31
30	.60 070	.63 830	.36 170	.96 240	30
31	.60 099	.63 865	.36 135	.96 234	29
32	.60 128	.63 899	.36 101	.96 229	28
33	.60 157	.63 934	.36 066	.96 223	27
34	.60 186	.63 968	.36 032	.96 218	26
35	.60 215	.64 003	.35 997	.96 212	25
36	.60 244	.64 037	.35 963	.96 207	24
37	.60 273	.64 072	.35 928	.96 201	23
38	.60 302	.64 106	.35 894	.96 196	22
39	.60 331	.64 140	.35 860	.96 190	21
40	.60 359	.64 175	.35 825	.96 185	20
41	.60 388	.64 209	.35 791	.96 179	19
42	.60 417	.64 243	.35 757	.96 174	18
43	.60 446	.64 278	.35 722	.96 168	17
44	.60 474	.64 312	.35 688	.96 162	16
45	.60 503	.64 346	.35 654	.96 157	15
46	.60 532	.64 381	.35 619	.96 151	14
47	.60 561	.64 415	.35 585	.96 146	13
48	.60 589	.64 449	.35 551	.96 140	12
49	.60 618	.64 483	.35 517	.96 135	11
50	.60 646	.64 517	.35 483	.96 129	10
51	.60 675	.64 552	.35 448	.96 123	9
52	.60 704	.64 586	.35 414	.96 118	8
53	.60 732	.64 620	.35 380	.96 112	7
54	.60 761	.64 654	.35 346	.96 107	6
55	.60 789	.64 688	.35 312	.96 101	5
56	.60 818	.64 722	.35 278	.96 095	4
57	.60 846	.64 756	.35 244	.96 090	3
58	.60 875	.64 790	.35 210	.96 084	2
59	.60 903	.64 824	.35 176	.96 079	1
60	.60 931	.64 858	.35 142	.96 073	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

66°

24°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.60 931	.64 858	.35 142	.96 073	60
1	.60 960	.64 892	.35 108	.96 067	59
2	.60 988	.64 926	.35 074	.96 062	58
3	.61 016	.64 960	.35 040	.96 056	57
4	.61 045	.64 994	.35 006	.96 050	56
5	.61 073	.65 028	.34 972	.96 045	55
6	.61 101	.65 062	.34 938	.96 039	54
7	.61 129	.65 096	.34 904	.96 034	53
8	.61 158	.65 130	.34 870	.96 028	52
9	.61 186	.65 164	.34 836	.96 022	51
10	.61 214	.65 197	.34 803	.96 017	50
11	.61 242	.65 231	.34 769	.96 011	49
12	.61 270	.65 265	.34 735	.96 005	48
13	.61 298	.65 299	.34 701	.96 000	47
14	.61 326	.65 333	.34 667	.95 994	46
15	.61 354	.65 366	.34 634	.95 988	45
16	.61 382	.65 400	.34 600	.95 982	44
17	.61 411	.65 434	.34 566	.95 977	43
18	.61 438	.65 467	.34 533	.95 971	42
19	.61 466	.65 501	.34 499	.95 965	41
20	.61 494	.65 535	.34 465	.95 960	40
21	.61 522	.65 568	.34 432	.95 954	39
22	.61 550	.65 602	.34 398	.95 948	38
23	.61 578	.65 636	.34 364	.95 942	37
24	.61 606	.65 669	.34 331	.95 937	36
25	.61 634	.65 703	.34 297	.95 931	35
26	.61 662	.65 736	.34 264	.95 925	34
27	.61 689	.65 770	.34 230	.95 920	33
28	.61 717	.65 803	.34 197	.95 914	32
29	.61 745	.65 837	.34 163	.95 908	31
30	.61 773	.65 870	.34 130	.95 902	30
31	.61 800	.65 904	.34 096	.95 897	29
32	.61 828	.65 937	.34 063	.95 891	28
33	.61 856	.65 971	.34 029	.95 885	27
34	.61 883	.66 004	.33 996	.95 879	26
35	.61 911	.66 038	.33 962	.95 873	25
36	.61 939	.66 071	.33 929	.95 868	24
37	.61 966	.66 104	.33 896	.95 862	23
38	.61 994	.66 138	.33 862	.95 856	22
39	.62 021	.66 171	.33 829	.95 850	21
40	.62 049	.66 204	.33 796	.95 844	20
41	.62 076	.66 238	.33 762	.95 839	19
42	.62 104	.66 271	.33 729	.95 833	18
43	.62 131	.66 304	.33 696	.95 827	17
44	.62 159	.66 337	.33 663	.95 821	16
45	.62 186	.66 371	.33 629	.95 815	15
46	.62 214	.66 404	.33 596	.95 810	14
47	.62 241	.66 437	.33 563	.95 804	13
48	.62 268	.66 470	.33 530	.95 798	12
49	.62 296	.66 503	.33 497	.95 792	11
50	.62 323	.66 537	.33 463	.95 786	10
51	.62 350	.66 570	.33 430	.95 780	9
52	.62 377	.66 603	.33 397	.95 775	8
53	.62 405	.66 636	.33 364	.95 769	7
54	.62 432	.66 669	.33 331	.95 763	6
55	.62 459	.66 702	.33 298	.95 757	5
56	.62 486	.66 735	.33 265	.95 751	4
57	.62 513	.66 768	.33 232	.95 745	3
58	.62 541	.66 801	.33 199	.95 739	2
59	.62 568	.66 834	.33 166	.95 733	1
60	.62 595	.66 867	.33 133	.95 728	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

65°

25°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.62 595	.66 867	.33 133	.95 728	**60**
1	.62 622	.66 900	.33 100	.95 722	59
2	.62 649	.66 933	.33 067	.95 716	58
3	.62 676	.66 966	.33 034	.95 710	57
4	.62 703	.66 999	.33 001	.95 704	56
5	.62 730	.67 032	.32 968	.95 698	55
6	.62 757	.67 065	.32 935	.95 692	54
7	.62 784	.67 098	.32 902	.95 686	53
8	.62 811	.67 131	.32 869	.95 680	52
9	.62 838	.67 163	.32 837	.95 674	51
10	.62 865	.67 196	.32 804	.95 668	**50**
11	.62 892	.67 229	.32 771	.95 663	49
12	.62 918	.67 262	.32 738	.95 657	48
13	.62 945	.67 295	.32 705	.95 651	47
14	.62 972	.67 327	.32 673	.95 645	46
15	.62 999	.67 360	.32 640	.95 639	45
16	.63 026	.67 393	.32 607	.95 633	44
17	.63 052	.67 426	.32 574	.95 627	43
18	.63 079	.67 458	.32 542	.95 621	42
19	.63 106	.67 491	.32 509	.95 615	41
20	.63 133	.67 524	.32 476	.95 609	**40**
21	.63 159	.67 556	.32 444	.95 603	39
22	.63 186	.67 589	.32 411	.95 597	38
23	.63 213	.67 622	.32 378	.95 591	37
24	.63 239	.67 654	.32 346	.95 585	36
25	.63 266	.67 687	.32 313	.95 579	35
26	.63 292	.67 719	.32 281	.95 573	34
27	.63 319	.67 752	.32 248	.95 567	33
28	.63 345	.67 785	.32 215	.95 561	32
29	.63 372	.67 817	.32 183	.95 555	31
30	.63 398	.67 850	.32 150	.95 549	**30**
31	.63 425	.67 882	.32 118	.95 543	29
32	.63 451	.67 915	.32 085	.95 537	28
33	.63 478	.67 947	.32 053	.95 531	27
34	.63 504	.67 980	.32 020	.95 525	26
35	.63 531	.68 012	.31 988	.95 519	25
36	.63 557	.68 044	.31 956	.95 513	24
37	.63 583	.68 077	.31 923	.95 507	23
38	.63 610	.68 109	.31 891	.95 500	22
39	.63 636	.68 142	.31 858	.95 494	21
40	.63 662	.68 174	.31 826	.95 488	**20**
41	.63 689	.68 206	.31 794	.95 482	19
42	.63 715	.68 239	.31 761	.95 476	18
43	.63 741	.68 271	.31 729	.95 470	17
44	.63 767	.68 303	.31 697	.95 464	16
45	.63 794	.68 336	.31 664	.95 458	15
46	.63 820	.68 368	.31 632	.95 452	14
47	.63 846	.68 400	.31 600	.95 446	13
48	.63 872	.68 432	.31 568	.95 440	12
49	.63 898	.68 465	.31 535	.95 434	11
50	.63 924	.68 497	.31 503	.95 427	**10**
51	.63 950	.68 529	.31 471	.95 421	9
52	.63 976	.68 561	.31 439	.95 415	8
53	.64 002	.68 593	.31 407	.95 409	7
54	.64 028	.68 626	.31 374	.95 403	6
55	.64 054	.68 658	.31 342	.95 397	5
56	.64 080	.68 690	.31 310	.95 391	4
57	.64 106	.68 722	.31 278	.95 384	3
58	.64 132	.68 754	.31 246	.95 378	2
59	.64 158	.68 786	.31 214	.95 372	1
60	.64 184	.68 818	.31 182	.95 366	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

64°

26°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.64 184	.68 818	.31 182	.95 366	**60**
1	.64 210	.68 850	.31 150	.95 360	59
2	.64 236	.68 882	.31 118	.95 354	58
3	.64 262	.68 914	.31 086	.95 348	57
4	.64 288	.68 946	.31 054	.95 341	56
5	.64 313	.68 978	.31 022	.95 335	55
6	.64 339	.69 010	.30 990	.95 329	54
7	.64 365	.69 042	.30 958	.95 323	53
8	.64 391	.69 074	.30 926	.95 317	52
9	.64 417	.69 106	.30 894	.95 310	51
10	.64 442	.69 138	.30 862	.95 304	**50**
11	.64 468	.69 170	.30 830	.95 298	49
12	.64 494	.69 202	.30 798	.95 292	48
13	.64 519	.69 234	.30 766	.95 286	47
14	.64 545	.69 266	.30 734	.95 279	46
15	.64 571	.69 298	.30 702	.95 273	45
16	.64 596	.69 329	.30 671	.95 267	44
17	.64 622	.69 361	.30 639	.95 261	43
18	.64 647	.69 393	.30 607	.95 254	42
19	.64 673	.69 425	.30 575	.95 248	41
20	.64 698	.69 457	.30 543	.95 242	**40**
21	.64 724	.69 488	.30 512	.95 236	39
22	.64 749	.69 520	.30 480	.95 229	38
23	.64 775	.69 552	.30 448	.95 223	37
24	.64 800	.69 584	.30 416	.95 217	36
25	.64 826	.69 615	.30 385	.95 211	35
26	.64 851	.69 647	.30 353	.95 204	34
27	.64 877	.69 679	.30 321	.95 198	33
28	.64 902	.69 710	.30 290	.95 192	32
29	.64 927	.69 742	.30 258	.95 185	31
30	.64 953	.69 774	.30 226	.95 179	**30**
31	.64 978	.69 805	.30 195	.95 173	29
32	.65 003	.69 837	.30 163	.95 167	28
33	.65 029	.69 868	.30 132	.95 160	27
34	.65 054	.69 900	.30 100	.95 154	26
35	.65 079	.69 932	.30 068	.95 148	25
36	.65 104	.69 963	.30 037	.95 141	24
37	.65 130	.69 995	.30 005	.95 135	23
38	.65 155	.70 026	.29 974	.95 129	22
39	.65 180	.70 058	.29 942	.95 122	21
40	.65 205	.70 089	.29 911	.95 116	**20**
41	.65 230	.70 121	.29 879	.95 110	19
42	.65 255	.70 152	.29 848	.95 103	18
43	.65 281	.70 184	.29 816	.95 097	17
44	.65 306	.70 215	.29 785	.95 090	16
45	.65 331	.70 247	.29 753	.95 084	15
46	.65 356	.70 278	.29 722	.95 078	14
47	.65 381	.70 309	.29 691	.95 071	13
48	.65 406	.70 341	.29 659	.95 065	12
49	.65 431	.70 372	.29 628	.95 059	11
50	.65 456	.70 404	.29 596	.95 052	**10**
51	.65 481	.70 435	.29 565	.95 046	9
52	.65 506	.70 466	.29 534	.95 039	8
53	.65 531	.70 498	.29 502	.95 033	7
54	.65 556	.70 529	.29 471	.95 027	6
55	.65 580	.70 560	.29 440	.95 020	5
56	.65 605	.70 592	.29 408	.95 014	4
57	.65 630	.70 623	.29 377	.95 007	3
58	.65 655	.70 654	.29 346	.95 001	2
59	.65 680	.70 685	.29 315	.94 995	1
60	.65 705	.70 717	.29 283	.94 988	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

63°

27°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.65 705	.70 717	.29 283	.94 988	**60**
1	.65 729	.70 748	.29 252	.94 982	59
2	.65 754	.70 779	.29 221	.94 975	58
3	.65 779	.70 810	.29 190	.94 969	57
4	.65 804	.70 841	.29 159	.94 962	56
5	.65 828	.70 873	.29 127	.94 956	55
6	.65 853	.70 904	.29 096	.94 949	54
7	.65 878	.70 935	.29 065	.94 943	53
8	.65 902	.70 966	.29 034	.94 936	52
9	.65 927	.70 997	.29 003	.94 930	51
10	.65 952	.71 028	.28 972	.94 923	**50**
11	.65 976	.71 059	.28 941	.94 917	49
12	.66 001	.71 090	.28 910	.94 911	48
13	.66 025	.71 121	.28 879	.94 904	47
14	.66 050	.71 153	.28 847	.94 898	46
15	.66 075	.71 184	.28 816	.94 891	45
16	.66 099	.71 215	.28 785	.94 885	44
17	.66 124	.71 246	.28 754	.94 878	43
18	.66 148	.71 277	.28 723	.94 871	42
19	.66 173	.71 308	.28 692	.94 865	41
20	.66 197	.71 339	.28 661	.94 858	**40**
21	.66 221	.71 370	.28 630	.94 852	39
22	.66 246	.71 401	.28 599	.94 845	38
23	.66 270	.71 431	.28 569	.94 839	37
24	.66 295	.71 462	.28 538	.94 832	36
25	.66 319	.71 493	.28 507	.94 826	35
26	.66 343	.71 524	.28 476	.94 819	34
27	.66 368	.71 555	.28 445	.94 813	33
28	.66 392	.71 586	.28 414	.94 806	32
29	.66 416	.71 617	.28 383	.94 799	31
30	.66 441	.71 648	.28 352	.94 793	**30**
31	.66 465	.71 679	.28 321	.94 786	29
32	.66 489	.71 709	.28 291	.94 780	28
33	.66 513	.71 740	.28 260	.94 773	27
34	.66 537	.71 771	.28 229	.94 767	26
35	.66 562	.71 802	.28 198	.94 760	25
36	.66 586	.71 833	.28 167	.94 753	24
37	.66 610	.71 863	.28 137	.94 747	23
38	.66 634	.71 894	.28 106	.94 740	22
39	.66 658	.71 925	.28 075	.94 734	21
40	.66 682	.71 955	.28 045	.94 727	**20**
41	.66 706	.71 986	.28 014	.94 720	19
42	.66 731	.72 017	.27 983	.94 714	18
43	.66 755	.72 048	.27 952	.94 707	17
44	.66 779	.72 078	.27 922	.94 700	16
45	.66 803	.72 109	.27 891	.94 694	15
46	.66 827	.72 140	.27 860	.94 687	14
47	.66 851	.72 170	.27 830	.94 680	13
48	.66 875	.72 201	.27 799	.94 674	12
49	.66 899	.72 231	.27 769	.94 667	11
50	.66 922	.72 262	.27 738	.94 660	**10**
51	.66 946	.72 293	.27 707	.94 654	9
52	.66 970	.72 323	.27 677	.94 647	8
53	.66 994	.72 354	.27 646	.94 640	7
54	.67 018	.72 384	.27 616	.94 634	6
55	.67 042	.72 415	.27 585	.94 627	5
56	.67 066	.72 445	.27 555	.94 620	4
57	.67 090	.72 476	.27 524	.94 614	3
58	.67 113	.72 506	.27 494	.94 607	2
59	.67 137	.72 537	.27 463	.94 600	1
60	.67 161	.72 567	.27 433	.94 593	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

62°

28°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.67 161	.72 567	.27 433	.94 593	**60**
1	.67 185	.72 598	.27 402	.94 587	59
2	.67 208	.72 628	.27 372	.94 580	58
3	.67 232	.72 659	.27 341	.94 573	57
4	.67 256	.72 689	.27 311	.94 567	56
5	.67 280	.72 720	.27 280	.94 560	55
6	.67 303	.72 750	.27 250	.94 553	54
7	.67 327	.72 780	.27 220	.94 546	53
8	.67 350	.72 811	.27 189	.94 540	52
9	.67 374	.72 841	.27 159	.94 533	51
10	.67 398	.72 872	.27 128	.94 526	**50**
11	.67 421	.72 902	.27 098	.94 519	49
12	.67 445	.72 932	.27 068	.94 513	48
13	.67 468	.72 963	.27 037	.94 506	47
14	.67 492	.72 993	.27 007	.94 499	46
15	.67 515	.73 023	.26 977	.94 492	45
16	67 539	.73 054	.26 946	.94 485	44
17	.67 562	.73 084	.26 916	.94 479	43
18	.67 586	.73 114	.26 886	.94 472	42
19	.67 609	.73 144	.26 856	.94 465	41
20	.67 633	.73 175	.26 825	.94 458	**40**
21	.67 656	.73 205	.26 795	.94 451	39
22	.67 680	.73 235	.26 765	.94 445	38
23	.67 703	.73 265	.26 735	.94 438	37
24	.67 726	.73 295	.26 705	.94 431	36
25	.67 750	.73 326	.26 674	.94 424	35
26	.67 773	.73 356	.26 644	.94 417	34
27	.67 796	.73 386	.26 614	.94 410	33
28	.67 820	.73 416	.26 584	.94 404	32
29	.67 843	.73 446	.26 554	.94 397	31
30	.67 866	.73 476	.26 524	.94 390	**30**
31	.67 890	.73 507	.26 493	.94 383	29
32	.67 913	.73 537	.26 463	.94 376	28
33	.67 936	.73 567	.26 433	.94 369	27
34	.67 959	.73 597	.26 403	.94 362	26
35	.67 982	.73 627	.26 373	.94 355	25
36	.68 006	.73 657	.26 343	.94 349	24
37	.68 029	.73 687	.26 313	.94 342	23
38	.68 052	.73 717	.26 283	.94 335	22
39	.68 075	.73 747	.26 253	.94 328	21
40	.68 098	.73 777	.26 223	.94 321	**20**
41	.68 121	.73 807	.26 193	.94 314	19
42	.68 144	.73 837	.26 163	.94 307	18
43	.68 167	.73 867	.26 133	.94 300	17
44	.68 190	.73 897	.26 103	.94 293	16
45	.68 213	.73 927	.26 073	.94 286	15
46	.68 237	.73 957	.26 043	.94 279	14
47	.68 260	.73 987	.26 013	.94 273	13
48	.68 283	.74 017	.25 983	.94 266	12
49	.68 305	.74 047	.25 953	.94 259	11
50	.68 328	.74 077	.25 923	.94 252	**10**
51	.68 351	.74 107	.25 893	.94 245	9
52	.68 374	.74 137	.25 863	.94 238	8
53	.68 397	.74 166	.25 834	.94 231	7
54	.68 420	.74 196	.25 804	.94 224	6
55	.68 443	.74 226	.25 774	.94 217	5
56	.68 466	.74 256	.25 744	.94 210	4
57	.68 489	.74 286	.25 714	.94 203	3
58	.68 512	.74 316	.25 684	.94 196	2
59	.68 534	.74 345	.25 655	.94 189	1
60	.68 557	.74 375	.25 625	.94 182	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

61°

29°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.68 557	.74 375	.25 625	.94 182	**60**
1	.68 580	.74 405	.25 595	.94 175	59
2	.68 603	.74 435	.25 565	.94 168	58
3	.68 625	.74 465	.25 535	.94 161	57
4	.68 648	.74 494	.25 506	.94 154	56
5	.68 671	.74 524	.25 476	.94 147	55
6	.68 694	.74 554	.25 446	.94 140	54
7	.68 716	.74 583	.25 417	.94 133	53
8	.68 739	.74 613	.25 387	.94 126	52
9	.68 762	.74 643	.25 357	.94 119	51
10	.68 784	.74 673	.25 327	.94 112	**50**
11	.68 807	.74 702	.25 298	.94 105	49
12	.68 829	.74 732	.25 268	.94 098	48
13	.68 852	.74 762	.25 238	.94 090	47
14	.68 875	.74 791	.25 209	.94 083	46
15	.68 897	.74 821	.25 179	.94 076	45
16	.68 920	.74 851	.25 149	.94 069	44
17	.68 942	.74 880	.25 120	.94 062	43
18	.68 965	.74 910	.25 090	.94 055	42
19	.68 987	.74 939	.25 061	.94 048	41
20	.69 010	.74 969	.25 031	.94 041	**40**
21	.69 032	.74 998	.25 002	.94 034	39
22	.69 055	.75 028	.24 972	.94 027	38
23	.69 077	.75 058	.24 942	.94 020	37
24	.69 100	.75 087	.24 913	.94 012	36
25	.69 122	.75 117	.24 883	.94 005	35
26	.69 144	.75 146	.24 854	.93 998	34
27	.69 167	.75 176	.24 824	.93 991	33
28	.69 189	.75 205	.24 795	.93 984	32
29	.69 212	.75 235	.24 765	.93 977	31
30	.69 234	.75 264	.24 736	.93 970	**30**
31	.69 256	.75 294	.24 706	.93 963	29
32	.69 279	.75 323	.24 677	.93 955	28
33	.69 301	.75 353	.24 647	.93 948	27
34	.69 323	.75 382	.24 618	.93 941	26
35	.69 345	.75 411	.24 589	.93 934	25
36	.69 368	.75 441	.24 559	.93 927	24
37	.69 390	.75 470	.24 530	.93 920	23
38	.69 412	.75 500	.24 500	.93 912	22
39	.69 434	.75 529	.24 471	.93 905	21
40	.69 456	.75 558	.24 442	.93 898	**20**
41	.69 479	.75 588	.24 412	.93 891	19
42	.69 501	.75 617	.24 383	.93 884	18
43	.69 523	.75 647	.24 353	.93 876	17
44	.69 545	.75 676	.24 324	.93 869	16
45	.69 567	.75 705	.24 295	.93 862	15
46	.69 589	.75 735	.24 265	.93 855	14
47	.69 611	.75 764	.24 236	.93 847	13
48	.69 633	.75 793	.24 207	.93 840	12
49	.69 655	.75 822	.24 178	.93 833	11
50	.69 677	.75 852	.24 148	.93 826	**10**
51	.69 699	.75 881	.24 119	.93 819	9
52	.69 721	.75 910	.24 090	.93 811	8
53	.69 743	.75 939	.24 061	.93 804	7
54	.69 765	.75 969	.24 031	.93 797	6
55	.69 787	.75 998	.24 002	.93 789	5
56	.69 809	.76 027	.23 973	.93 782	4
57	.69 831	.76 056	.23 944	.93 775	3
58	.69 853	.76 086	.23 914	.93 768	2
59	.69 875	.76 115	.23 885	.93 760	1
60	.69 897	.76 144	.23 856	.93 753	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

60°

30°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.69 897	.76 144	.23 856	.93 753	**60**
1	.69 919	.76 173	.23 827	.93 746	59
2	.69 941	.76 202	.23 798	.93 738	58
3	.69 963	.76 231	.23 769	.93 731	57
4	.69 984	.76 261	.23 739	.93 724	56
5	.70 006	.76 290	.23 710	.93 717	55
6	.70 028	.76 319	.23 681	.93 709	54
7	.70 050	.76 348	.23 652	.93 702	53
8	.70 072	.76 377	.23 623	.93 695	52
9	.70 093	.76 406	.23 594	.93 687	51
10	.70 115	.76 435	.23 565	.93 680	**50**
11	.70 137	.76 464	.23 536	.93 673	49
12	.70 159	.76 493	.23 507	.93 665	48
13	.70 180	.76 522	.23 478	.93 658	47
14	.70 202	.76 551	.23 449	.93 650	46
15	.70 224	.76 580	.23 420	.93 643	45
16	.70 245	.76 609	.23 391	.93 636	44
17	.70 267	.76 639	.23 361	.93 628	43
18	.70 288	.76 668	.23 332	.93 621	42
19	.70 310	.76 697	.23 303	.93 614	41
20	.70 332	.76 725	.23 275	.93 606	**40**
21	.70 353	.76 754	.23 246	.93 599	39
22	.70 375	.76 783	.23 217	.93 591	38
23	.70 396	.76 812	.23 188	.93 584	37
24	.70 418	.76 841	.23 159	.93 577	36
25	.70 439	.76 870	.23 130	.93 569	35
26	.70 461	.76 899	.23 101	.93 562	34
27	.70 482	.76 928	.23 072	.93 554	33
28	.70 504	.76 957	.23 043	.93 547	32
29	.70 525	.76 986	.23 014	.93 539	31
30	.70 547	.77 015	.22 985	.93 532	**30**
31	.70 568	.77 044	.22 956	.93 525	29
32	.70 590	.77 073	.22 927	.93 517	28
33	.70 611	.77 101	.22 899	.93 510	27
34	.70 633	.77 130	.22 870	.93 502	26
35	.70 654	.77 159	.22 841	.93 495	25
36	.70 675	.77 188	.22 812	.93 487	24
37	.70 697	.77 217	.22 783	.93 480	23
38	.70 718	.77 246	.22 754	.93 472	22
39	.70 739	.77 274	.22 726	.93 465	21
40	.70 761	.77 303	.22 697	.93 457	**20**
41	.70 782	.77 332	.22 668	.93 450	19
42	.70 803	.77 361	.22 639	.93 442	18
43	.70 824	.77 390	.22 610	.93 435	17
44	.70 846	.77 418	.22 582	.93 427	16
45	.70 867	.77 447	.22 553	.93 420	15
46	.70 888	.77 476	.22 524	.93 412	14
47	.70 909	.77 505	.22 495	.93 405	13
48	.70 931	.77 533	.22 467	.93 397	12
49	.70 952	.77 562	.22 438	.93 390	11
50	.70 973	.77 591	.22 409	.93 382	**10**
51	.70 994	.77 619	.22 381	.93 375	9
52	.71 015	.77 648	.22 352	.93 367	8
53	.71 036	.77 677	.22 323	.93 360	7
54	.71 058	.77 706	.22 294	.93 352	6
55	.71 079	.77 734	.22 266	.93 344	5
56	.71 100	.77 763	.22 237	.93 337	4
57	.71 121	.77 791	.22 209	.93 329	3
58	.71 142	.77 820	.22 180	.93 322	2
59	.71 163	.77 849	.22 151	.93 314	1
60	.71 184	.77 877	.22 123	.93 307	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

59°

31°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.71 184	.77 877	.22 123	.93 307	**60**
1	.71 205	.77 906	.22 094	.93 299	59
2	.71 226	.77 935	.22 065	.93 291	58
3	.71 247	.77 963	.22 037	.93 284	57
4	.71 268	.77 992	.22 008	.93 276	56
5	.71 289	.78 020	.21 980	.93 269	55
6	.71 310	.78 049	.21 951	.93 261	54
7	.71 331	.78 077	.21 923	.93 253	53
8	.71 352	.78 106	.21 894	.93 246	52
9	.71 373	.78 135	.21 865	.93 238	51
10	.71 393	.78 163	.21 837	.93 230	**50**
11	.71 414	.78 192	.21 808	.93 223	49
12	.71 435	.78 220	.21 780	.93 215	48
13	.71 456	.78 249	.21 751	.93 207	47
14	.71 477	.78 277	.21 723	.93 200	46
15	.71 498	.78 306	.21 694	.93 192	45
16	.71 519	.78 334	.21 666	.93 184	44
17	.71 539	.78 363	.21 637	.93 177	43
18	.71 560	.78 391	.21 609	.93 169	42
19	.71 581	.78 419	.21 581	.93 161	41
20	.71 602	.78 448	.21 552	.93 154	**40**
21	.71 622	.78 476	.21 524	.93 146	39
22	.71 643	.78 505	.21 495	.93 138	38
23	.71 664	.78 533	.21 467	.93 131	37
24	.71 685	.78 562	.21 438	.93 123	36
25	.71 705	.78 590	.21 410	.93 115	35
26	.71 726	.78 618	.21 382	.93 108	34
27	.71 747	.78 647	.21 353	.93 100	33
28	.71 767	.78 675	.21 325	.93 092	32
29	.71 788	.78 704	.21 296	.93 084	31
30	.71 809	.78 732	.21 268	.93 077	**30**
31	.71 829	.78 760	.21 240	.93 069	29
32	.71 850	.78 789	.21 211	.93 061	28
33	.71 870	.78 817	.21 183	.93 053	27
34	.71 891	.78 845	.21 155	.93 046	26
35	.71 911	.78 874	.21 126	.93 038	25
36	.71 932	.78 902	.21 098	.93 030	24
37	.71 952	.78 930	.21 070	.93 022	23
38	.71 973	.78 959	.21 041	.93 014	22
39	.71 994	.78 987	.21 013	.93 007	21
40	.72 014	.79 015	.20 985	.92 999	**20**
41	.72 034	.79 043	.20 957	.92 991	19
42	.72 055	.79 072	.20 928	.92 983	18
43	.72 075	.79 100	.20 900	.92 976	17
44	.72 096	.79 128	.20 872	.92 968	16
45	.72 116	.79 156	.20 844	.92 960	15
46	.72 137	.79 185	.20 815	.92 952	14
47	.72 157	.79 213	.20 787	.92 944	13
48	.72 177	.79 241	.20 759	.92 936	12
49	.72 198	.79 269	.20 731	.92 929	11
50	.72 218	.79 297	.20 703	.92 921	**10**
51	.72 238	.79 326	.20 674	.92 913	9
52	.72 259	.79 354	.20 646	.92 905	8
53	.72 279	.79 382	.20 618	.92 897	7
54	.72 299	.79 410	.20 590	.92 889	6
55	.72 320	.79 438	.20 562	.92 881	5
56	.72 340	.79 466	.20 534	.92 874	4
57	.72 360	.79 495	.20 505	.92 866	3
58	.72 381	.79 523	.20 477	.92 858	2
59	.72 401	.79 551	.20 449	.92 850	1
60	.72 421	.79 579	.20 421	.92 842	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

58°

32°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.72 421	.79 579	.20 421	.92 842	**60**
1	.72 441	.79 607	.20 393	.92 834	59
2	.72 461	.79 635	.20 365	.92 826	58
3	.72 482	.79 663	.20 337	.92 818	57
4	.72 502	.79 691	.20 309	.92 810	56
5	.72 522	.79 719	.20 281	.92 803	55
6	.72 542	.79 747	.20 253	.92 795	54
7	.72 562	.79 776	.20 224	.92 787	53
8	.72 582	.79 804	.20 196	.92 779	52
9	.72 602	.79 832	.20 168	.92 771	51
10	.72 622	.79 860	.20 140	.92 763	**50**
11	.72 643	.79 888	.20 112	.92 755	49
12	.72 663	.79 916	.20 084	.92 747	48
13	.72 683	.79 944	.20 056	.92 739	47
14	.72 703	.79 972	.20 028	.92 731	46
15	.72 723	.80 000	.20 000	.92 723	45
16	.72 743	.80 028	.19 972	.92 715	44
17	.72 763	.80 056	.19 944	.92 707	43
18	.72 783	.80 084	.19 916	.92 699	42
19	.72 803	.80 112	.19 888	.92 691	41
20	.72 823	.80 140	.19 860	.92 683	**40**
21	.72 843	.80 168	.19 832	.92 675	39
22	.72 863	.80 195	.19 805	.92 667	38
23	.72 883	.80 223	.19 777	.92 659	37
24	.72 902	.80 251	.19 749	.92 651	36
25	.72 922	.80 279	.19 721	.92 643	35
26	.72 942	.80 307	.19 693	.92 635	34
27	.72 962	.80 335	.19 665	.92 627	33
28	.72 982	.80 363	.19 637	.92 619	32
29	.73 002	.80 391	.19 609	.92 611	31
30	.73 022	.80 419	.19 581	.92 603	**30**
31	.73 041	.80 447	.19 553	.92 595	29
32	.73 061	.80 474	.19 526	.92 587	28
33	.73 081	.80 502	.19 498	.92 579	27
34	.73 101	.80 530	.19 470	.92 571	26
35	.73 121	.80 558	.19 442	.92 563	25
36	.73 140	.80 586	.19 414	.92 555	24
37	.73 160	.80 614	.19 386	.92 546	23
38	.73 180	.80 642	.19 358	.92 538	22
39	.73 200	.80 669	.19 331	.92 530	21
40	.73 219	.80 697	.19 303	.92 522	**20**
41	.73 239	.80 725	.19 275	.92 514	19
42	.73 259	.80 753	.19 247	.92 506	18
43	.73 278	.80 781	.19 219	.92 498	17
44	.73 298	.80 808	.19 192	.92 490	16
45	.73 318	.80 836	.19 164	.92 482	15
46	.73 337	.80 864	.19 136	.92 473	14
47	.73 357	.80 892	.19 108	.92 465	13
48	.73 377	.80 919	.19 081	.92 457	12
49	.73 396	.80 947	.19 053	.92 449	11
50	.73 416	.80 975	.19 025	.92 441	**10**
51	.73 435	.81 003	.18 997	.92 433	9
52	.73 455	.81 030	.18 970	.92 425	8
53	.73 474	.81 058	.18 942	.92 416	7
54	.73 494	.81 086	.18 914	.92 408	6
55	.73 513	.81 113	.18 887	.92 400	5
56	.73 533	.81 141	.18 859	.92 392	4
57	.73 552	.81 169	.18 831	.92 384	3
58	.73 572	.81 196	.18 804	.92 376	2
59	.73 591	.81 224	.18 776	.92 367	1
60	.73 611	.81 252	.18 748	.92 359	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

57°

33°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.73 611	.81 252	.18 748	.92 359	60
1	.73 630	.81 279	.18 721	.92 351	59
2	.73 650	.81 307	.18 693	.92 343	58
3	.73 669	.81 335	.18 665	.92 335	57
4	.73 689	.81 362	.18 638	.92 326	56
5	.73 708	.81 390	.18 610	.92 318	55
6	.73 727	.81 418	.18 582	.92 310	54
7	.73 747	.81 445	.18 555	.92 302	53
8	.73 766	.81 473	.18 527	.92 293	52
9	.73 785	.81 500	.18 500	.92 285	51
10	.73 805	.81 528	.18 472	.92 277	50
11	.73 824	.81 556	.18 444	.92 269	49
12	.73 843	.81 583	.18 417	.92 260	48
13	.73 863	.81 611	.18 389	.92 252	47
14	.73 882	.81 638	.18 362	.92 244	46
15	.73 901	.81 666	.18 334	.92 235	45
16	.73 921	.81 693	.18 307	.92 227	44
17	.73 940	.81 721	.18 279	.92 219	43
18	.73 959	.81 748	.18 252	.92 211	42
19	.73 978	.81 776	.18 224	.92 202	41
20	.73 997	.81 803	.18 197	.92 194	40
21	.74 017	.81 831	.18 169	.92 186	39
22	.74 036	.81 858	.18 142	.92 177	38
23	.74 055	.81 886	.18 114	.92 169	37
24	.74 074	.81 913	.18 087	.92 161	36
25	.74 093	.81 941	.18 059	.92 152	35
26	.74 113	.81 968	.18 032	.92 144	34
27	.74 132	.81 996	.18 004	.92 136	33
28	.74 151	.82 023	.17 977	.92 127	32
29	.74 170	.82 051	.17 949	.92 119	31
30	.74 189	.82 078	.17 922	.92 111	30
31	.74 208	.82 106	.17 894	.92 102	29
32	.74 227	.82 133	.17 867	.92 094	28
33	.74 246	.82 161	.17 839	.92 086	27
34	.74 265	.82 188	.17 812	.92 077	26
35	.74 284	.82 215	.17 785	.92 069	25
36	.74 303	.82 243	.17 757	.92 060	24
37	.74 322	.82 270	.17 730	.92 052	23
38	.74 341	.82 298	.17 702	.92 044	22
39	.74 360	.82 325	.17 675	.92 035	21
40	.74 379	.82 352	.17 648	.92 027	20
41	.74 398	.82 380	.17 620	.92 018	19
42	.74 417	.82 407	.17 593	.92 010	18
43	.74 436	.82 435	.17 565	.92 002	17
44	.74 455	.82 462	.17 538	.91 993	16
45	.74 474	.82 489	.17 511	.91 985	15
46	.74 493	.82 517	.17 483	.91 976	14
47	.74 512	.82 544	.17 456	.91 968	13
48	.74 531	.82 571	.17 429	.91 959	12
49	.74 549	.82 599	.17 401	.91 951	11
50	.74 568	.82 626	.17 374	.91 942	10
51	.74 587	.82 653	.17 347	.91 934	9
52	.74 606	.82 681	.17 319	.91 925	8
53	.74 625	.82 708	.17 292	.91 917	7
54	.74 644	.82 735	.17 265	.91 908	6
55	.74 662	.82 762	.17 238	.91 900	5
56	.74 681	.82 790	.17 210	.91 891	4
57	.74 700	.82 817	.17 183	.91 883	3
58	.74 719	.82 844	.17 156	.91 874	2
59	.74 737	.82 871	.17 129	.91 866	1
60	.74 756	.82 899	.17 101	.91 857	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

56°

34°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.74 756	.82 899	.17 101	.91 857	60
1	.74 775	.82 926	.17 074	.91 849	59
2	.74 794	.82 953	.17 047	.91 840	58
3	.74 812	.82 980	.17 020	.91 832	57
4	.74 831	.83 008	.16 992	.91 823	56
5	.74 850	.83 035	.16 965	.91 815	55
6	.74 868	.83 062	.16 938	.91 806	54
7	.74 887	.83 089	.16 911	.91 798	53
8	.74 906	.83 117	.16 883	.91 789	52
9	.74 924	.83 144	.16 856	.91 781	51
10	.74 943	.83 171	.16 829	.91 772	50
11	.74 961	.83 198	.16 802	.91 763	49
12	.74 980	.83 225	.16 775	.91 755	48
13	.74 999	.83 252	.16 748	.91 746	47
14	.75 017	.83 280	.16 720	.91 738	46
15	.75 036	.83 307	.16 693	.91 729	45
16	.75 054	.83 334	.16 666	.91 720	44
17	.75 073	.83 361	.16 639	.91 712	43
18	.75 091	.83 388	.16 612	.91 703	42
19	.75 110	.83 415	.16 585	.91 695	41
20	.75 128	.83 442	.16 558	.91 686	40
21	.75 147	.83 470	.16 530	.91 677	39
22	.75 165	.83 497	.16 503	.91 669	38
23	.75 184	.83 524	.16 476	.91 660	37
24	.75 202	.83 551	.16 449	.91 651	36
25	.75 221	.83 578	.16 422	.91 643	35
26	.75 239	.83 605	.16 395	.91 634	34
27	.75 258	.83 632	.16 368	.91 625	33
28	.75 276	.83 659	.16 341	.91 617	32
29	.75 294	.83 686	.16 314	.91 608	31
30	.75 313	.83 713	.16 287	.91 599	30
31	.75 331	.83 740	.16 260	.91 591	29
32	.75 350	.83 768	.16 232	.91 582	28
33	.75 368	.83 795	.16 205	.91 573	27
34	.75 386	.83 822	.16 178	.91 565	26
35	.75 405	.83 849	.16 151	.91 556	25
36	.75 423	.83 876	.16 124	.91 547	24
37	.75 441	.83 903	.16 097	.91 538	23
38	.75 459	.83 930	.16 070	.91 530	22
39	.75 478	.83 957	.16 043	.91 521	21
40	.75 496	.83 984	.16 016	.91 512	20
41	.75 514	.84 011	.15 989	.91 504	19
42	.75 533	.84 038	.15 962	.91 495	18
43	.75 551	.84 065	.15 935	.91 486	17
44	.75 569	.84 092	.15 908	.91 477	16
45	.75 587	.84 119	.15 881	.91 469	15
46	.75 605	.84 146	.15 854	.91 460	14
47	.75 624	.84 173	.15 827	.91 451	13
48	.75 642	.84 200	.15 800	.91 442	12
49	.75 660	.84 227	.15 773	.91 433	11
50	.75 678	.84 254	.15 746	.91 425	10
51	.75 696	.84 280	.15 720	.91 416	9
52	.75 714	.84 307	.15 693	.91 407	8
53	.75 733	.84 334	.15 666	.91 398	7
54	.75 751	.84 361	.15 639	.91 389	6
55	.75 769	.84 388	.15 612	.91 381	5
56	.75 787	.84 415	.15 585	.91 372	4
57	.75 805	.84 442	.15 558	.91 363	3
58	.75 823	.84 469	.15 531	.91 354	2
59	.75 841	.84 496	.15 504	.91 345	1
60	.75 859	.84 523	.15 477	.91 336	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

55°

35°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.75 859	.84 523	.15 477	.91 336	**60**
1	.75 877	.84 550	.15 450	.91 328	59
2	.75 895	.84 576	.15 424	.91 319	58
3	.75 913	.84 603	.15 397	.91 310	57
4	.75 931	.84 630	.15 370	.91 301	56
5	.75 949	.84 657	.15 343	.91 292	55
6	.75 967	.84 684	.15 316	.91 283	54
7	.75 985	.84 711	.15 289	.91 274	53
8	.76 003	.84 738	.15 262	.91 266	52
9	.76 021	.84 764	.15 236	.91 257	51
10	.76 039	.84 791	.15 209	.91 248	**50**
11	.76 057	.84 818	.15 182	.91 239	49
12	.76 075	.84 845	.15 155	.91 230	48
13	.76 093	.84 872	.15 128	.91 221	47
14	.76 111	.84 899	.15 101	.91 212	46
15	.76 129	.84 925	.15 075	.91 203	45
16	.76 146	.84 952	.15 048	.91 194	44
17	.76 164	.84 979	.15 021	.91 185	43
18	.76 182	.85 006	.14 994	.91 176	42
19	.76 200	.85 033	.14 967	.91 167	41
20	.76 218	.85 059	.14 941	.91 158	**40**
21	.76 236	.85 086	.14 914	.91 149	39
22	.76 253	.85 113	.14 887	.91 141	38
23	.76 271	.85 140	.14 860	.91 132	37
24	.76 289	.85 166	.14 834	.91 123	36
25	.76 307	.85 193	.14 807	.91 114	35
26	.76 324	.85 220	.14 780	.91 105	34
27	.76 342	.85 247	.14 753	.91 096	33
28	.76 360	.85 273	.14 727	.91 087	32
29	.76 378	.85 300	.14 700	.91 078	31
30	.76 395	.85 327	.14 673	.91 069	**30**
31	.76 413	.85 354	.14 646	.91 060	29
32	.76 431	.85 380	.14 620	.91 051	28
33	.76 448	.85 407	.14 593	.91 042	27
34	.76 466	.85 434	.14 566	.91 033	26
35	.76 484	.85 460	.14 540	.91 023	25
36	.76 501	.85 487	.14 513	.91 014	24
37	.76 519	.85 514	.14 486	.91 005	23
38	.76 537	.85 540	.14 460	.90 996	22
39	.76 554	.85 567	.14 433	.90 987	21
40	.76 572	.85 594	.14 406	.90 978	**20**
41	.76 590	.85 620	.14 380	.90 969	19
42	.76 607	.85 647	.14 353	.90 960	18
43	.76 625	.85 674	.14 326	.90 951	17
44	.76 642	.85 700	.14 300	.90 942	16
45	.76 660	.85 727	.14 273	.90 933	15
46	.76 677	.85 754	.14 246	.90 924	14
47	.76 695	.85 780	.14 220	.90 915	13
48	.76 712	.85 807	.14 193	.90 906	12
49	.76 730	.85 834	.14 166	.90 896	11
50	.76 747	.85 860	.14 140	.90 887	**10**
51	.76 765	.85 887	.14 113	.90 878	9
52	.76 782	.85 913	.14 087	.90 869	8
53	.76 800	.85 940	.14 060	.90 860	7
54	.76 817	.85 967	.14 033	.90 851	6
55	.76 835	.85 993	.14 007	.90 842	5
56	.76 852	.86 020	.13 980	.90 832	4
57	.76 870	.86 046	.13 954	.90 823	3
58	.76 887	.86 073	.13 927	.90 814	2
59	.76 904	.86 100	.13 900	.90 805	1
60	.76 922	.86 126	.13 874	.90 796	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

54°

36°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.76 922	.86 126	.13 874	.90 796	**60**
1	.76 939	.86 153	.13 847	.90 787	59
2	.76 957	.86 179	.13 821	.90 777	58
3	.76 974	.86 206	.13 794	.90 768	57
4	.76 991	.86 232	.13 768	.90 759	56
5	.77 009	.86 259	.13 741	.90 750	55
6	.77 026	.86 285	.13 715	.90 741	54
7	.77 043	.86 312	.13 688	.90 731	53
8	.77 061	.86 338	.13 662	.90 722	52
9	.77 078	.86 365	.13 635	.90 713	51
10	.77 095	.86 392	.13 608	.90 704	**50**
11	.77 112	.86 418	.13 582	.90 694	49
12	.77 130	.86 445	.13 555	.90 685	48
13	.77 147	.86 471	.13 529	.90 676	47
14	.77 164	.86 498	.13 502	.90 667	46
15	.77 181	.86 524	.13 476	.90 657	45
16	.77 199	.86 551	.13 449	.90 648	44
17	.77 216	.86 577	.13 423	.90 639	43
18	.77 233	.86 603	.13 397	.90 630	42
19	.77 250	.86 630	.13 370	.90 620	41
20	.77 268	.86 656	.13 344	.90 611	**40**
21	.77 285	.86 683	.13 317	.90 602	39
22	.77 302	.86 709	.13 291	.90 592	38
23	.77 319	.86 736	.13 264	.90 583	37
24	.77 336	.86 762	.13 238	.90 574	36
25	.77 353	.86 789	.13 211	.90 565	35
26	.77 370	.86 815	.13 185	.90 555	34
27	.77 387	.86 842	.13 158	.90 546	33
28	.77 405	.86 868	.13 132	.90 537	32
29	.77 422	.86 894	.13 106	.90 527	31
30	.77 439	.86 921	.13 079	.90 518	**30**
31	.77 456	.86 947	.13 053	.90 509	29
32	.77 473	.86 974	.13 026	.90 499	28
33	.77 490	.87 000	.13 000	.90 490	27
34	.77 507	.87 027	.12 973	.90 480	26
35	.77 524	.87 053	.12 947	.90 471	25
36	.77 541	.87 079	.12 921	.90 462	24
37	.77 558	.87 106	.12 894	.90 452	23
38	.77 575	.87 132	.12 868	.90 443	22
39	.77 592	.87 158	.12 842	.90 434	21
40	.77 609	.87 185	.12 815	.90 424	**20**
41	.77 626	.87 211	.12 789	.90 415	19
42	.77 643	.87 238	.12 762	.90 405	18
43	.77 660	.87 264	.12 736	.90 396	17
44	.77 677	.87 290	.12 710	.90 386	16
45	.77 694	.87 317	.12 683	.90 377	15
46	.77 711	.87 343	.12 657	.90 368	14
47	.77 728	.87 369	.12 631	.90 358	13
48	.77 744	.87 396	.12 604	.90 349	12
49	.77 761	.87 422	.12 578	.90 339	11
50	.77 778	.87 448	.12 552	.90 330	**10**
51	.77 795	.87 475	.12 525	.90 320	9
52	.77 812	.87 501	.12 499	.90 311	8
53	.77 829	.87 527	.12 473	.90 301	7
54	.77 846	.87 554	.12 446	.90 292	6
55	.77 862	.87 580	.12 420	.90 282	5
56	.77 879	.87 606	.12 394	.90 273	4
57	.77 896	.87 633	.12 367	.90 263	3
58	.77 913	.87 659	.12 341	.90 254	2
59	.77 930	.87 685	.12 315	.90 244	1
60	.77 946	.87 711	.12 289	.90 235	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

53°

37°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.77946	.87711	.12289	.90235	60
1	.77963	.87738	.12262	.90225	59
2	.77980	.87764	.12236	.90216	58
3	.77997	.87790	.12210	.90206	57
4	.78013	.87817	.12183	.90197	56
5	.78030	.87843	.12157	.90187	55
6	.78047	.87869	.12131	.90178	54
7	.78063	.87895	.12105	.90168	53
8	.78080	.87922	.12078	.90159	52
9	.78097	.87948	.12052	.90149	51
10	.78113	.87974	.12026	.90139	50
11	.78130	.88000	.12000	.90130	49
12	.78147	.88027	.11973	.90120	48
13	.78163	.88053	.11947	.90111	47
14	.78180	.88079	.11921	.90101	46
15	.78197	.88105	.11895	.90091	45
16	.78213	.88131	.11869	.90082	44
17	.78230	.88158	.11842	.90072	43
18	.78246	.88184	.11816	.90063	42
19	.78263	.88210	.11790	.90053	41
20	.78280	.88236	.11764	.90043	40
21	.78296	.88262	.11738	.90034	39
22	.78313	.88289	.11711	.90024	38
23	.78329	.88315	.11685	.90014	37
24	.78346	.88341	.11659	.90005	36
25	.78362	.88367	.11633	.89995	35
26	.78379	.88393	.11607	.89985	34
27	.78395	.88420	.11580	.89976	33
28	.78412	.88446	.11554	.89966	32
29	.78428	.88472	.11528	.89956	31
30	.78445	.88498	.11502	.89947	30
31	.78461	.88524	.11476	.89937	29
32	.78478	.88550	.11450	.89927	28
33	.78494	.88577	.11423	.89918	27
34	.78510	.88603	.11397	.89908	26
35	.78527	.88629	.11371	.89898	25
36	.78543	.88655	.11345	.89888	24
37	.78560	.88681	.11319	.89879	23
38	.78576	.88707	.11293	.89869	22
39	.78592	.88733	.11267	.89859	21
40	.78609	.88759	.11241	.89849	20
41	.78625	.88786	.11214	.89840	19
42	.78642	.88812	.11188	.89830	18
43	.78658	.88838	.11162	.89820	17
44	.78674	.88864	.11136	.89810	16
45	.78691	.88890	.11110	.89801	15
46	.78707	.88916	.11084	.89791	14
47	.78723	.88942	.11058	.89781	13
48	.78739	.88968	.11032	.89771	12
49	.78756	.88994	.11006	.89761	11
50	.78772	.89020	.10980	.89752	10
51	.78788	.89046	.10954	.89742	9
52	.78805	.89073	.10927	.89732	8
53	.78821	.89099	.10901	.89722	7
54	.78837	.89125	.10875	.89712	6
55	.78853	.89151	.10849	.89702	5
56	.78869	.89177	.10823	.88693	4
57	.78886	.89203	.10797	.89683	3
58	.78902	.89229	.10771	.89673	2
59	.78918	.89255	.10745	.89663	1
60	.78934	.89281	.10719	.89653	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

52°

38°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.78934	.89281	.10719	.89653	60
1	.78950	.89307	.10693	.89643	59
2	.78967	.89333	.10667	.89633	58
3	.78983	.89359	.10641	.89624	57
4	.78999	.89385	.10615	.89614	56
5	.79015	.89411	.10589	.89604	55
6	.79031	.89437	.10563	.89594	54
7	.79047	.89463	.10537	.89584	53
8	.79063	.89489	.10511	.89574	52
9	.79079	.89515	.10485	.89564	51
10	.79095	.89541	.10459	.89554	50
11	.79111	.89567	.10433	.89544	49
12	.79128	.89593	.10407	.89534	48
13	.79144	.89619	.10381	.89524	47
14	.79160	.89645	.10355	.89514	46
15	.79176	.89671	.10329	.89504	45
16	.79192	.89697	.10303	.89495	44
17	.79208	.89723	.10277	.89485	43
18	.79224	.89749	.10251	.89475	42
19	.79240	.89775	.10225	.89465	41
20	.79256	.89801	.10199	.89455	40
21	.79272	.89827	.10173	.89445	39
22	.79288	.89853	.10147	.89435	38
23	.79304	.89879	.10121	.89425	37
24	.79319	.89905	.10095	.89415	36
25	.79335	.89931	.10069	.89405	35
26	.79351	.89957	.10043	.89395	34
27	.79367	.89983	.10017	.89385	33
28	.79383	.90009	.09991	.89375	32
29	.79399	.90035	.09965	.89364	31
30	.79415	.90061	.09939	.89354	30
31	.79431	.90086	.09914	.89344	29
32	.79447	.90112	.09888	.89334	28
33	.79463	.90138	.09862	.89324	27
34	.79478	.90164	.09836	.89314	26
35	.79494	.90190	.09810	.89304	25
36	.79510	.90216	.09784	.89294	24
37	.79526	.90242	.09758	.89284	23
38	.79542	.90268	.09732	.89274	22
39	.79558	.90294	.09706	.89264	21
40	.79573	.90320	.09680	.89254	20
41	.79589	.90346	.09654	.89244	19
42	.79605	.90371	.09629	.89233	18
43	.79621	.90397	.09603	.89223	17
44	.79636	.90423	.09577	.89213	16
45	.79652	.90449	.09551	.89203	15
46	.79668	.90475	.09525	.89193	14
47	.79684	.90501	.09499	.89183	13
48	.79699	.90527	.09473	.89173	12
49	.79715	.90553	.09447	.89162	11
50	.79731	.90578	.09422	.89152	10
51	.79746	.90604	.09396	.89142	9
52	.79762	.90630	.09370	.89132	8
53	.79778	.90656	.09344	.89122	7
54	.79793	.90682	.09318	.89112	6
55	.79809	.90708	.09292	.89101	5
56	.79825	.90734	.09266	.89091	4
57	.79840	.90759	.09241	.89081	3
58	.79856	.90785	.09215	.89071	2
59	.79872	.90811	.09189	.89060	1
60	.79887	.90837	.09163	.89050	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

51°

39°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.79887	.90837	.09163	.89050	**60**
1	.79903	.90863	.09137	.89040	59
2	.79918	.90889	.09111	.89030	58
3	.79934	.90914	.09086	.89020	57
4	.79950	.90940	.09060	.89009	56
5	.79965	.90966	.09034	.88999	55
6	.79981	.90992	.09008	.88989	54
7	.79996	.91018	.08982	.88978	53
8	.80012	.91043	.08957	.88968	52
9	.80027	.91069	.08931	.88958	51
10	.80043	.91095	.08905	.88948	**50**
11	.80058	.91121	.08879	.88937	49
12	.80074	.91147	.08853	.88927	48
13	.80089	.91172	.08828	.88917	47
14	.80105	.91198	.08802	.88906	46
15	.80120	.91224	.08776	.88896	45
16	.80136	.91250	.08750	.88886	44
17	.80151	.91276	.08724	.88875	43
18	.80166	.91301	.08699	.88865	42
19	.80182	.91327	.08673	.88855	41
20	.80197	.91353	.08647	.88844	**40**
21	.80213	.91379	.08621	.88834	39
22	.80228	.91404	.08596	.88824	38
23	.80244	.91430	.08570	.88813	37
24	.80259	.91456	.08544	.88803	36
25	.80274	.91482	.08518	.88793	35
26	.80290	.91507	.08493	.88782	34
27	.80305	.91533	.08467	.88772	33
28	.80320	.91559	.08441	.88761	32
29	.80336	.91585	.08415	.88751	31
30	.80351	.91610	.08390	.88741	**30**
31	.80366	.91636	.08364	.88730	29
32	.80382	.91662	.08338	.88720	28
33	.80397	.91688	.08312	.88709	27
34	.80412	.91713	.08287	.88699	26
35	.80428	.91739	.08261	.88688	25
36	.80443	.91765	.08235	.88678	24
37	.80458	.91791	.08209	.88668	23
38	.80473	.91816	.08184	.88657	22
39	.80489	.91842	.08158	.88647	21
40	.80504	.91868	.08132	.88636	**20**
41	.80519	.91893	.08107	.88626	19
42	.80534	.91919	.08081	.88615	18
43	.80550	.91945	.08055	.88605	17
44	.80565	.91971	.08029	.88594	16
45	.80580	.91996	.08004	.88584	15
46	.80595	.92022	.07978	.88573	14
47	.80610	.92048	.07952	.88563	13
48	.80625	.92073	.07927	.88552	12
49	.80641	.92099	.07901	.88542	11
50	.80656	.92125	.07875	.88531	**10**
51	.80671	.92150	.07850	.88521	9
52	.80686	.92176	.07824	.88510	8
53	.80701	.92202	.07798	.88499	7
54	.80716	.92227	.07773	.88489	6
55	.80731	.92253	.07747	.88478	5
56	.80746	.92279	.07721	.88468	4
57	.80762	.92304	.07696	.88457	3
58	.80777	.92330	.07670	.88447	2
59	.80792	.92356	.07644	.88436	1
60	.80807	.92381	.07619	.88425	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

50°

40°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.80807	.92381	.07619	.88425	**60**
1	.80822	.92407	.07593	.88415	59
2	.80837	.92433	.07567	.88404	58
3	.80852	.92458	.07542	.88394	57
4	.80867	.92484	.07516	.88383	56
5	.80882	.92510	.07490	.88372	55
6	.80897	.92535	.07465	.88362	54
7	.80912	.92561	.07439	.88351	53
8	.80927	.92587	.07413	.88340	52
9	.80942	.92612	.07388	.88330	51
10	.80957	.92638	.07362	.88319	**50**
11	.80972	.92663	.07337	.88308	49
12	.80987	.92689	.07311	.88298	48
13	.81002	.92715	.07285	.88287	47
14	.81017	.92740	.07260	.88276	46
15	.81032	.92766	.07234	.88266	45
16	.81047	.92792	.07208	.88255	44
17	.81061	.92817	.07183	.88244	43
18	.81076	.92843	.07157	.88234	42
19	.81091	.92868	.07132	.88223	41
20	.81106	.92894	.07106	.88212	**40**
21	.81121	.92920	.07080	.88201	39
22	.81136	.92945	.07055	.88191	38
23	.81151	.92971	.07029	.88180	37
24	.81166	.92996	.07004	.88169	36
25	.81180	.93022	.06978	.88158	35
26	.81195	.93048	.06952	.88148	34
27	.81210	.93073	.06927	.88137	33
28	.81225	.93099	.06901	.88126	32
29	.81240	.93124	.06876	.88115	31
30	.81254	.93150	.06850	.88105	**30**
31	.81269	.93175	.06825	.88094	29
32	.81284	.93201	.06799	.88083	28
33	.81299	.93227	.06773	.88072	27
34	.81314	.93252	.06748	.88061	26
35	.81328	.93278	.06722	.88051	25
36	.81343	.93303	.06697	.88040	24
37	.81358	.93329	.06671	.88029	23
38	.81372	.93354	.06646	.88018	22
39	.81387	.93380	.06620	.88007	21
40	.81402	.93406	.06594	.87996	**20**
41	.81417	.93431	.06569	.87985	19
42	.81431	.93457	.06543	.87975	18
43	.81446	.93482	.06518	.87964	17
44	.81461	.93508	.06492	.87953	16
45	.81475	.93533	.06467	.87942	15
46	.81490	.93559	.06441	.87931	14
47	.81505	.93584	.06416	.87920	13
48	.81519	.93610	.06390	.87909	12
49	.81534	.93636	.06364	.87898	11
50	.81549	.93661	.06339	.87887	**10**
51	.81563	.93687	.06313	.87877	9
52	.81578	.93712	.06288	.87866	8
53	.81592	.93738	.06262	.87855	7
54	.81607	.93763	.06237	.87844	6
55	.81622	.93789	.06211	.87833	5
56	.81636	.93814	.06186	.87822	4
57	.81651	.93840	.06160	.87811	3
58	.81665	.93865	.06135	.87800	2
59	.81680	.93891	.06109	.87789	1
60	.81694	.93916	.06084	.87778	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

49°

41°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.81 694	.93 916	.06 084	.87 778	**60**
1	.81 709	.93 942	.06 058	.87 767	59
2	.81 723	.93 967	.06 033	.87 756	58
3	.81 738	.93 993	.06 007	.87 745	57
4	.81 752	.94 018	.05 982	.87 734	56
5	.81 767	.94 044	.05 956	.87 723	55
6	.81 781	.94 069	.05 931	.87 712	54
7	.81 796	.94 095	.05 905	.87 701	53
8	.81 810	.94 120	.05 880	.87 690	52
9	.81 825	.94 146	.05 854	.87 679	51
10	.81 839	.94 171	.05 829	.87 668	**50**
11	.81 854	.94 197	.05 803	.87 657	49
12	.81 868	.94 222	.05 778	.87 646	48
13	.81 882	.94 248	.05 752	.87 635	47
14	.81 897	.94 273	.05 727	.87 624	46
15	.81 911	.94 299	.05 701	.87 613	45
16	.81 926	.94 324	.05 676	.87 601	44
17	.81 940	.94 350	.05 650	.87 590	43
18	.81 955	.94 375	.05 625	.87 579	42
19	.81 969	.94 401	.05 599	.87 568	41
20	.81 983	.94 426	.05 574	.87 557	**40**
21	.81 998	.94 452	.05 548	.87 546	39
22	.82 012	.94 477	.05 523	.87 535	38
23	.82 026	.94 503	.05 497	.87 524	37
24	.82 041	.94 528	.05 472	.87 513	36
25	.82 055	.94 554	.05 446	.87 501	35
26	.82 069	.94 579	.05 421	.87 490	34
27	.82 084	.94 604	.05 396	.87 479	33
28	.82 098	.94 630	.05 370	.87 468	32
29	.82 112	.94 655	.05 345	.87 457	31
30	.82 126	.94 681	.05 319	.87 446	**30**
31	.82 141	.94 706	.05 294	.87 434	29
32	.82 155	.94 732	.05 268	.87 423	28
33	.82 169	.94 757	.05 243	.87 412	27
34	.82 184	.94 783	.05 217	.87 401	26
35	.82 198	.94 808	.05 192	.87 390	25
36	.82 212	.94 834	.05 166	.87 378	24
37	.82 226	.94 859	.05 141	.87 367	23
38	.82 240	.94 884	.05 116	.87 356	22
39	.82 255	.94 910	.05 090	.87 345	21
40	.82 269	.94 935	.05 065	.87 334	**20**
41	.82 283	.94 961	.05 039	.87 322	19
42	.82 297	.94 986	.05 014	.87 311	18
43	.82 311	.95 012	.04 988	.87 300	17
44	.82 326	.95 037	.04 963	.87 288	16
45	.82 340	.95 062	.04 938	.87 277	15
46	.82 354	.95 088	.04 912	.87 266	14
47	.82 368	.95 113	.04 887	.87 255	13
48	.82 382	.95 139	.04 861	.87 243	12
49	.82 396	.95 164	.04 836	.87 232	11
50	.82 410	.95 190	.04 810	.87 221	**10**
51	.82 424	.95 215	.04 785	.87 209	9
52	.82 439	.95 240	.04 760	.87 198	8
53	.82 453	.95 266	.04 734	.87 187	7
54	.82 467	.95 291	.04 709	.87 175	6
55	.82 481	.95 317	.04 683	.87 164	5
56	.82 495	.95 342	.04 658	.87 153	4
57	.82 509	.95 368	.04 632	.87 141	3
58	.82 523	.95 393	.04 607	.87 130	2
59	.82 537	.95 418	.04 582	.87 119	1
60	.82 551	.95 444	.04 556	.87 107	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

48°

42°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.82 551	.95 444	.04 556	.87 107	**60**
1	.82 565	.95 469	.04 531	.87 096	59
2	.82 579	.95 495	.04 505	.87 085	58
3	.82 593	.95 520	.04 480	.87 073	57
4	.82 607	.95 545	.04 455	.87 062	56
5	.82 621	.95 571	.04 429	.87 050	55
6	.82 635	.95 596	.04 404	.87 039	54
7	.82 649	.95 622	.04 378	.87 028	53
8	.82 663	.95 647	.04 353	.87 016	52
9	.82 677	.95 672	.04 328	.87 005	51
10	.82 691	.95 698	.04 302	.86 993	**50**
11	.82 705	.95 723	.04 277	.86 982	49
12	.82 719	.95 748	.04 252	.86 970	48
13	.82 733	.95 774	.04 226	.86 959	47
14	.82 747	.95 799	.04 201	.86 947	46
15	.82 761	.95 825	.04 175	.86 936	45
16	.82 775	.95 850	.04 150	.86 924	44
17	.82 788	.95 875	.04 125	.86 913	43
18	.82 802	.95 901	.04 099	.86 902	42
19	.82 816	.95 926	.04 074	.86 890	41
20	.82 830	.95 952	.04 048	.86 879	**40**
21	.82 844	.95 977	.04 023	.86 867	39
22	.82 858	.96 002	.03 998	.86 855	38
23	.82 872	.96 028	.03 972	.86 844	37
24	.82 885	.96 053	.03 947	.86 832	36
25	.82 899	.96 078	.03 922	.86 821	35
26	.82 913	.96 104	.03 896	.86 809	34
27	.82 927	.96 129	.03 871	.86 798	33
28	.82 941	.96 155	.03 845	.86 786	32
29	.82 955	.96 180	.03 820	.86 775	31
30	.82 968	.96 205	.03 795	.86 763	**30**
31	.82 982	.96 231	.03 769	.86 752	29
32	.82 996	.96 256	.03 744	.86 740	28
33	.83 010	.96 281	.03 719	.86 728	27
34	.83 023	.96 307	.03 693	.86 717	26
35	.83 037	.96 332	.03 668	.86 705	25
36	.83 051	.96 357	.03 643	.86 694	24
37	.83 065	.96 383	.03 617	.86 682	23
38	.83 078	.96 408	.03 592	.86 670	22
39	.83 092	.96 433	.03 567	.86 659	21
40	.83 106	.96 459	.03 541	.86 647	**20**
41	.83 120	.96 484	.03 516	.86 635	19
42	.83 133	.96 510	.03 490	.86 624	18
43	.83 147	.96 535	.03 465	.86 612	17
44	.83 161	.96 560	.03 440	.86 600	16
45	.83 174	.96 586	.03 414	.86 589	15
46	.83 188	.96 611	.03 389	.86 577	14
47	.83 202	.96 636	.03 364	.86 565	13
48	.83 215	.96 662	.03 338	.86 554	12
49	.83 229	.96 687	.03 313	.86 542	11
50	.83 242	.96 712	.03 288	.86 530	**10**
51	.83 256	.96 738	.03 262	.86 518	9
52	.83 270	.96 763	.03 237	.86 507	8
53	.83 283	.96 788	.03 212	.86 495	7
54	.83 297	.96 814	.03 186	.86 483	6
55	.83 310	.96 839	.03 161	.86 472	5
56	.83 324	.96 864	.03 136	.86 460	4
57	.83 338	.96 890	.03 110	.86 448	3
58	.83 351	.96 915	.03 085	.86 436	2
59	.83 365	.96 940	.03 060	.86 425	1
60	.83 378	.96 966	.03 034	.86 413	**0**
′	9 L cos	9 L cot	10 L tan	9 L sin	′

47°

43°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.83 378	.96 966	.03 034	.86 413	60
1	.83 392	.96 991	.03 009	.86 401	59
2	.83 405	.97 016	.02 984	.86 389	58
3	.83 419	.97 042	.02 958	.86 377	57
4	.83 432	.97 067	.02 933	.86 366	56
5	.83 446	.97 092	.02 908	.86 354	55
6	.83 459	.97 118	.02 882	.86 342	54
7	.83 473	.97 143	.02 857	.86 330	53
8	.83 486	.97 168	.02 832	.86 318	52
9	.83 500	.97 193	.02 807	.86 306	51
10	.83 513	.97 219	.02 781	.86 295	50
11	.83 527	.97 244	.02 756	.86 283	49
12	.83 540	.97 269	.02 731	.86 271	48
13	.83 554	.97 295	.02 705	.86 259	47
14	.83 567	.97 320	.02 680	.86 247	46
15	.83 581	.97 345	.02 655	.86 235	45
16	.83 594	.97 371	.02 629	.86 223	44
17	.83 608	.97 396	.02 604	.86 211	43
18	.83 621	.97 421	.02 579	.86 200	42
19	.83 634	.97 447	.02 553	.86 188	41
20	.83 648	.97 472	.02 528	.86 176	40
21	.83 661	.97 497	.02 503	.86 164	39
22	.83 674	.97 523	.02 477	.86 152	38
23	.83 688	.97 548	.02 452	.86 140	37
24	.83 701	.97 573	.02 427	.86 128	36
25	.83 715	.97 598	.02 402	.86 116	35
26	.83 728	.97 624	.02 376	.86 104	34
27	.83 741	.97 649	.02 351	.86 092	33
28	.83 755	.97 674	.02 326	.86 080	32
29	.83 768	.97 700	.02 300	.86 068	31
30	.83 781	.97 725	.02 275	.86 056	30
31	.83 795	.97 750	.02 250	.86 044	29
32	.83 808	.97 776	.02 224	.86 032	28
33	.83 821	.97 801	.02 199	.86 020	27
34	.83 834	.97 826	.02 174	.86 008	26
35	.83 848	.97 851	.02 149	.85 996	25
36	.83 861	.97 877	.02 123	.85 984	24
37	.83 874	.97 902	.02 098	.85 972	23
38	.83 887	.97 927	.02 073	.85 960	22
39	.83 901	.97 953	.02 047	.85 948	21
40	.83 914	.97 978	.02 022	.85 936	20
41	.83 927	.98 003	.01 997	.85 924	19
42	.83 940	.98 029	.01 971	.85 912	18
43	.83 954	.98 054	.01 946	.85 900	17
44	.83 967	.98 079	.01 921	.85 888	16
45	.83 980	.98 104	.01 896	.85 876	15
46	.83 993	.98 130	.01 870	.85 864	14
47	.84 006	.98 155	.01 845	.85 851	13
48	.84 020	.98 180	.01 820	.85 839	12
49	.84 033	.98 206	.01 794	.85 827	11
50	.84 046	.98 231	.01 769	.85 815	10
51	.84 059	.98 256	.01 744	.85 803	9
52	.84 072	.98 281	.01 719	.85 791	8
53	.84 085	.98 307	.01 693	.85 779	7
54	.84 098	.98 332	.01 668	.85 766	6
55	.84 112	.98 357	.01 643	.85 754	5
56	.84 125	.98 383	.01 617	.85 742	4
57	.84 138	.98 408	.01 592	.85 730	3
58	.84 151	.98 433	.01 567	.85 718	2
59	.84 164	.98 458	.01 542	.85 706	1
60	.84 177	.98 484	.01 516	.85 693	0
′	9 L cos	9 L cot	10 L tan	9 L sin	′

46°

44°

′	9 L sin	9 L tan	10 L cot	9 L cos	′
0	.84 177	.98 484	.01 516	.85 693	60
1	.84 190	.98 509	.01 491	.85 681	59
2	.84 203	.98 534	.01 466	.85 669	58
3	.84 216	.98 560	.01 440	.85 657	57
4	.84 229	.98 585	.01 415	.85 645	56
5	.84 242	.98 610	.01 390	.85 632	55
6	.84 255	.98 635	.01 365	.85 620	54
7	.84 269	.98 661	.01 339	.85 608	53
8	.84 282	.98 686	.01 314	.85 596	52
9	.84 295	.98 711	.01 289	.85 583	51
10	.84 308	.98 737	.01 263	.85 571	50
11	.84 321	.98 762	.01 238	.85 559	49
12	.84 334	.98 787	.01 213	.85 547	48
13	.84 347	.98 812	.01 188	.85 534	47
14	.84 360	.98 838	.01 162	.85 522	46
15	.84 373	.98 863	.01 137	.85 510	45
16	.84 385	.98 888	.01 112	.85 497	44
17	.84 398	.98 913	.01 087	.85 485	43
18	.84 411	.98 939	.01 061	.85 473	42
19	.84 424	.98 964	.01 036	.85 460	41
20	.84 437	.98 989	.01 011	.85 448	40
21	.84 450	.99 015	.00 985	.85 436	39
22	.84 463	.99 040	.00 960	.85 423	38
23	.84 476	.99 065	.00 935	.85 411	37
24	.84 489	.99 090	.00 910	.85 399	36
25	.84 502	.99 116	.00 884	.85 386	35
26	.84 515	.99 141	.00 859	.85 374	34
27	.84 528	.99 166	.00 834	.85 361	33
28	.84 540	.99 191	.00 809	.85 349	32
29	.84 553	.99 217	.00 783	.85 337	31
30	.84 566	.99 242	.00 758	.85 324	30
31	.84 579	.99 267	.00 733	.85 312	29
32	.84 592	.99 293	.00 707	.85 299	28
33	.84 605	.99 318	.00 682	.85 287	27
34	.84 618	.99 343	.00 657	.85 274	26
35	.84 630	.99 368	.00 632	.85 262	25
36	.84 643	.99 394	.00 606	.85 250	24
37	.84 656	.99 419	.00 581	.85 237	23
38	.84 669	.99 444	.00 556	.85 225	22
39	.84 682	.99 469	.00 531	.85 212	21
40	.84 694	.99 495	.00 505	.85 200	20
41	.84 707	.99 520	.00 480	.85 187	19
42	.84 720	.99 545	.00 455	.85 175	18
43	.84 733	.99 570	.00 430	.85 162	17
44	.84 745	.99 596	.00 404	.85 150	16
45	.84 758	.99 621	.00 379	.85 137	15
46	.84 771	.99 646	.00 354	.85 125	14
47	.84 784	.99 672	.00 328	.85 112	13
48	.84 796	.99 697	.00 303	.85 100	12
49	.84 809	.99 722	.00 278	.85 087	11
50	.84 822	.99 747	.00 253	.85 074	10
51	.84 835	.99 773	.00 227	.85 062	9
52	.84 847	.99 798	.00 202	.85 049	8
53	.84 860	.99 823	.00 177	.85 037	7
54	.84 873	.99 848	.00 152	.85 024	6
55	.84 885	.99 874	.00 126	.85 012	5
56	.84 898	.99 899	.00 101	.84 999	4
57	.84 911	.99 924	.00 076	.84 986	3
58	.84 923	.99 949	.00 051	.84 974	2
59	.84 936	.99 975	.00 025	.84 961	1
60	.84 949	.00 000	.00 000	.84 949	0
′	9 L cos	10 L cot	10 L tan	9 L sin	′

45°

FIVE PLACE

LOGARITHMIC AND NATURAL HAVERSINES

FOR EACH TEN MINUTES OF ARC

Pages xliv-xlviii

0°–40°

° ′	Log hav	Nat hav	° ′	Log hav	Nat hav	° ′	Log hav	Nat hav	° ′	Log hav	Nat hav
0	——	.00 000	**10**	7.88 059	.00 760	**20**	8.47 934	.03 015	**30**	8.82 599	.06 699
10	4.32 539	.00 000	10	7.89 491	.00 785	10	8.48 648	.03 065	10	8.83 069	.06 772
20	4.92 745	.00 001	20	7.90 900	.00 811	20	8.49 355	.03 116	20	8.83 537	.06 845
30	5.27 963	.00 002	30	7.92 286	.00 837	30	8.50 056	.03 166	30	8.84 002	.06 919
40	5.52 951	.00 003	40	7.93 650	.00 864	40	8.50 752	.03 218	40	8.84 464	.06 993
50	5.72 332	.00 005	50	7.94 992	.00 891	50	8.51 442	.03 269	50	8.84 923	.07 067
1	5.88 168	.00 008	**11**	7.96 315	.00 919	**21**	8.52 127	.03 321	**31**	8.85 380	.07 142
10	6.01 557	.00 010	10	7.97 617	.00 947	10	8.52 806	.03 373	10	8.85 834	.07 217
20	6.13 155	.00 014	20	7.98 899	.00 975	20	8.53 479	.03 426	20	8.86 286	.07 292
30	6.23 385	.00 017	30	8.00 163	.01 004	30	8.54 147	.03 479	30	8.86 735	.07 368
40	6.32 536	.00 021	40	8.01 409	.01 033	40	8.54 810	.03 533	40	8.87 182	.07 444
50	6.40 814	.00 026	50	8.02 637	.01 063	50	8.55 467	.03 587	50	8.87 626	.07 521
2	6.48 371	.00 030	**12**	8.03 847	.01 093	**22**	8.56 120	.03 641	**32**	8.88 068	.07 598
10	6.55 323	.00 036	10	8.05 041	.01 123	10	8.56 767	.03 695	10	8.88 507	.07 675
20	6.61 759	.00 041	20	8.06 218	.01 154	20	8.57 410	.03 751	20	8.88 944	.07 752
30	6.67 751	.00 048	30	8.07 379	.01 185	30	8.58 047	.03 806	30	8.89 379	.07 830
40	6.73 355	.00 054	40	8.08 525	.01 217	40	8.58 680	.03 862	40	8.89 811	.07 909
50	6.78 620	.00 061	50	8.09 656	.01 249	50	8.59 308	.03 918	50	8.90 241	.07 987
3	6.83 584	.00 069	**13**	8.10 772	.01 282	**23**	8.59 931	.03 975	**33**	8.90 668	.08 066
10	6.88 279	.00 076	10	8.11 873	.01 314	10	8.60 550	.04 032	10	8.91 094	.08 146
20	6.92 733	.00 085	20	8.12 961	.01 348	20	8.61 164	.04 089	20	8.91 517	.08 226
30	6.96 970	.00 093	30	8.14 035	.01 382	30	8.61 773	.04 147	30	8.91 938	.08 306
40	7.01 009	.00 102	40	8.15 096	.01 416	40	8.62 379	.04 205	40	8.92 356	.08 386
50	7.04 869	.00 112	50	8.16 144	.01 450	50	8.62 979	.04 264	50	8.92 773	.08 467
4	7.08 564	.00 122	**14**	8.17 179	.01 485	**24**	8.63 576	.04 323	**34**	8.93 187	.08 548
10	7.12 108	.00 132	10	8.18 202	.01 521	10	8.64 168	.04 382	10	8.93 599	.08 630
20	7.15 513	.00 143	20	8.19 212	.01 556	20	8.64 756	.04 442	20	8.94 009	.08 711
30	7.18 790	.00 154	30	8.20 211	.01 593	30	8.65 340	.04 502	30	8.94 417	.08 794
40	7.21 947	.00 166	40	8.21 199	.01 629	40	8.659 20	.04 562	40	8.94 823	.08 876
50	7.24 993	.00 178	50	8.22 175	.01 666	50	8.66 496	.04 623	50	8.95 227	.08 959
5	7.27 936	.00 190	**15**	8.23 140	.01 704	**25**	8.67 067	.04 685	**35**	8.95 628	.09 042
10	7.30 782	.00 203	10	8.24 094	.01 742	10	8.67 635	.04 746	10	8.96 028	.09 126
20	7.33 538	.00 216	20	8.25 037	.01 780	20	8.68 199	.04 808	20	8.96 426	.09 210
30	7.36 209	.00 230	30	8.25 971	.01 818	30	8.68 760	.04 871	30	8.96 821	.09 294
40	7.38 800	.00 244	40	8.26 894	.01 858	40	8.69 316	.04 934	40	8.97 215	.09 379
50	7.41 315	.00 259	50	8.27 807	.01 897	50	8.69 869	.04 997	50	8.97 607	.09 464
6	7.43 760	.00 274	**16**	8.28 711	.01 937	**26**	8.70 418	.05 060	**36**	8.97 997	.09 549
10	7.46 138	.00 289	10	8.29 605	.01 977	10	8.70 963	.05 124	10	8.98 384	.09 635
20	7.48 452	.00 305	20	8.30 490	.02 018	20	8.71 505	.05 189	20	8.98 770	.09 721
30	7.50 706	.00 321	30	8.31 366	.02 059	30	8.72 043	.05 253	30	8.99 154	.09 807
40	7.52 902	.00 338	40	8.32 233	.02 101	40	8.72 578	.05 318	40	8.99 536	.09 894
50	7.55 045	.00 355	50	8.33 091	.02 142	50	8.73 109	.05 384	50	8.99 917	.09 981
7	7.57 135	.00 373	**17**	8.33 940	.02 185	**27**	8.73 637	.05 450	**37**	9.00 295	.10 068
10	7.59 176	.00 391	10	8.34 782	.02 227	10	8.74 162	.05 516	10	9.00 672	.10 156
20	7.61 170	.00 409	20	8.35 614	.02 271	20	8.74 683	.05 582	20	9.01 047	.10 244
30	7.63 120	.00 428	30	8.36 439	.02 314	30	8.75 201	.05 649	30	9.01 420	.10 332
40	7.65 026	.00 447	40	8.37 256	.02 358	40	8.75 715	.05 717	40	9.01 791	.10 421
50	7.66 891	.00 467	50	8.38 065	.02 402	50	8.76 227	.05 785	50	9.02 161	.10 510
8	7.68 717	.00 487	**18**	8.38 867	.02 447	**28**	8.76 735	.05 853	**38**	9.02 528	.10 599
10	7.70 505	.00 507	10	8.39 660	.02 492	10	8.77 240	.05 921	10	9.02 894	.10 689
20	7.72 257	.00 528	20	8.40 447	.02 538	20	8.77 742	.05 990	20	9.03 259	.10 779
30	7.73 974	.00 549	30	8.41 226	.02 584	30	8.78 241	.06 059	30	9.03 621	.10 870
40	7.75 657	.00 571	40	8.41 998	.02 630	40	8.78 737	.06 129	40	9.03 982	.10 960
50	7.77 308	.00 593	50	8.42 764	.02 677	50	8.79 230	.06 199	50	9.04 341	.11 051
9	7.78 929	.00 616	**19**	8.43 522	.02 724	**29**	8.79 720	.06 269	**39**	9.04 699	.11 143
10	7.80 519	.00 639	10	8.44 273	.02 772	10	8.80 207	.06 340	10	9.05 055	.11 234
20	7.82 081	.00 662	20	8.45 018	.02 820	20	8.80 691	.06 411	20	9.05 409	.11 326
30	7.83 615	.00 686	30	8.45 757	.02 868	30	8.81 172	.06 482	30	9.057 62	.11 419
40	7.85 122	.00 710	40	8.46 489	.02 917	40	8.81 651	.06 554	40	9.06 113	.11 511
50	7.86 603	.00 735	50	8.47 215	.02 966	50	8.821 26	.06 626	50	9.06 462	.11 604
10	7.88 059	.00 760	**20**	8.47 934	.03 015	**30**	8.82 599	.06 699	**40**	9.06 810	.11 698

° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav
40	.06 810	.11 698	**50**	.25 190	.17 861	**60**	.39 794	.25 000	**70**	.51 718	.32 899
10	.07 157	.11 791	10	.25 460	.17 972	10	.40 012	.25 126	10	.51 898	.33 036
20	.07 501	.11 885	20	.25 729	.18 084	20	.40 230	.25 252	20	.52 078	.33 173
30	.07 845	.11 980	30	.25 998	.18 196	30	.40 447	.25 379	30	.52 257	.33 310
40	.08 186	.12 074	40	.26 265	.18 308	40	.40 663	.25 506	40	.52 436	.33 447
50	.08 526	.12 169	50	.26 532	.18 421	50	.40 879	.25 632	50	.52 613	.33 584
41	.08 865	.12 265	**51**	.26 797	.18 534	**61**	.41 094	.25 760	**71**	.52 791	.33 722
10	.09 202	.12 360	10	.27 061	.18 647	10	.41 308	.25 887	10	.52 968	.33 859
20	.09 538	.12 456	20	.27 325	.18 761	20	.41 521	.26 014	20	.53 144	.33 997
30	.09 872	.12 552	30	.27 587	.18 874	30	.41 734	.26 142	30	.53 320	.34 135
40	.10 205	.12 649	40	.27 848	.18 988	40	.41 946	.26 270	40	.53 495	.34 273
50	.10 536	.12 746	50	.28 109	.19 102	50	.42 157	.26 398	50	.53 670	.34 411
42	.10 866	.12 843	**52**	.28 368	.19 217	**62**	.42 368	.26 526	**72**	.53 844	.34 549
10	.11 194	.12 940	10	.28 627	.19 332	10	.42 578	.26 655	10	.54 017	.34 688
20	.11 521	.13 038	20	.28 885	.19 447	20	.42 787	.26 784	20	.54 190	.34 826
30	.11 847	.13 136	30	.29 141	.19 562	30	.42 996	.26 913	30	.54 363	.34 965
40	.12 171	.13 235	40	.29 397	.19 677	40	.43 203	.27 042	40	.54 535	.35 103
50	.12 494	.13 333	50	.29 652	.19 793	50	.43 411	.27 171	50	.54 707	.35 242
43	.12 815	.13 432	**53**	.29 906	.19 909	**63**	.43 617	.27 300	**73**	.54 878	.35 381
10	.13 135	.13 532	10	.30 158	.20 026	10	.43 823	.27 430	10	.55 048	.35 521
20	.13 454	.13 631	20	.30 410	.20 142	20	.44 028	.27 560	20	.55 218	.35 660
30	.13 771	.13 731	30	.30 662	.20 259	30	.44 232	.27 690	30	.55 387	.35 799
40	.14 087	.13 832	40	.30 912	.20 376	40	.44 436	.27 820	40	.55 556	.35 939
50	.14 402	.13 932	50	.31 161	.20 493	50	.44 639	.27 951	50	.55 725	.36 078
44	.14 715	.14 033	**54**	.31 409	.20 611	**64**	.44 842	.28 081	**74**	.55 893	.36 218
10	.15 027	.14 134	10	.31 657	.20 729	10	.45 044	.28 212	10	.56 060	.36 358
20	.15 338	.14 236	20	.31 903	.20 847	20	.45 245	.28 343	20	.56 227	.36 498
30	.15 647	.14 337	30	.32 149	.20 965	30	.45 446	.28 474	30	.56 393	.36 638
40	.15 955	.14 440	40	.32 394	.21 083	40	.45 645	.28 606	40	.56 559	.36 778
50	.16 262	.14 542	50	.32 638	.21 202	50	.45 845	.28 737	50	.56 725	.36 919
45	.16 568	.14 645	**55**	.32 881	.21 321	**65**	.46 043	.28 869	**75**	.56 889	.37 059
10	.16 872	.14 748	10	.33 123	.21 440	10	.46 241	.29 001	10	.57 054	.37 200
20	.17 175	.14 851	20	.33 365	.21 560	20	.46 439	.29 133	20	.57 218	.37 340
30	.17 477	.14 955	30	.33 605	.21 680	30	.46 635	.29 265	30	.57 381	.37 481
40	.17 778	.15 058	40	.33 845	.21 800	40	.46 831	.29 398	40	.57 544	.37 622
50	.10 877	.15 163	50	.34 084	.21 920	50	.47 027	.29 530	50	.57 706	.37 763
46	.18 376	.15 267	**56**	.34 322	.22 040	**66**	.47 222	.29 663	**76**	.57 868	.37 904
10	.18 673	.15 372	10	.34 559	.22 161	10	.47 416	.29 796	10	.58 030	.38 045
20	.18 968	.15 477	20	.34 795	.22 282	20	.47 610	.29 929	20	.58 191	.38 186
30	.19 263	.15 582	30	.35 031	.22 403	30	.47 803	.30 063	30	.58 351	.38 328
40	.19 557	.15 688	40	.35 266	.22 525	40	.47 995	.30 196	40	.58 511	.38 469
50	.19 849	.15 794	50	.35 500	.22 646	50	.48 187	.30 330	50	.58 671	.38 611
47	.20 140	.15 900	**57**	.35 733	.22 768	**67**	.48 378	.30 463	**77**	.58 830	.38 752
10	.20 430	.16 007	10	.35 965	.22 890	10	.48 568	.30 597	10	.58 989	.38 894
20	.20 719	.16 113	20	.36 196	.23 012	20	.48 758	.30 732	20	.59 147	.39 036
30	.21 006	.16 220	30	.36 427	.23 135	30	.48 948	.30 866	30	.59 304	.39 178
40	.21 293	.16 328	40	.36 657	.23 258	40	.49 137	.31 000	40	.59 461	.39 320
50	.21 578	.16 436	50	.36 886	.23 381	50	.49 325	.31 135	50	.59 618	.39 462
48	.21 863	.16 543	**58**	.37 114	.23 504	**68**	.49 512	.31 270	**78**	.59 774	.39 604
10	.22 146	.16 652	10	.37 342	.23 627	10	.49 699	.31 405	10	.59 930	.39 746
20	.22 428	.16 760	20	.37 569	.23 751	20	.49 886	.31 540	20	.60 085	.39 889
30	.22 709	.16 869	30	.37 794	.23 875	30	.50 072	.31 675	30	.60 240	.40 032
40	.22 989	.16 978	40	.38 020	.23 999	40	.50 257	.31 810	40	.60 395	.40 174
50	.23 268	.17 087	50	.38 244	.24 124	50	.50 442	.31 946	50	.60 549	.40 317
49	.23 545	.17 197	**59**	.38 468	.24 248	**69**	.50 626	.32 082	**79**	.60 702	.40 460
10	.23 822	.17 307	10	.38 691	.24 373	10	.50 809	.32 217	10	.60 855	.40 602
20	.24 098	.17 417	20	.38 913	.24 498	20	.50 992	.32 353	20	.61 008	.40 745
30	.24 372	.17 528	30	.39 134	.24 623	30	.51 174	.32 490	30	.61 160	.40 888
40	.24 646	.17 638	40	.39 355	.24 749	40	.51 356	.32 626	40	.61 312	.41 031
50	.24 918	.17 749	50	.39 575	.24 874	50	.51 538	.32 762	50	.61 463	.41 174
50	.25 190	.17 861	**60**	.39 794	.25 000	**70**	.51 718	.32 899	**80**	.61 614	.41 318

80°–120°

° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav
80	.61 614	.41 318	**90**	.69 897	.50 000	**100**	.76 851	.58 682	**110**	.82 673	.67 101
10	.61 764	.41 461	10	.70 023	.50 145	10	.76 957	.58 826	10	.82 761	.67 238
20	.61 914	.41 604	20	.70 149	.50 291	20	.77 062	.58 969	20	.82 849	.67 374
30	.62 063	.41 748	30	.70 274	.50 436	30	.77 167	.59 112	30	.82 937	.67 510
40	.62 212	.41 891	40	.70 399	.50 582	40	.77 272	.59 255	40	.83 025	.67 647
50	.62 361	.42 035	50	.70 524	.50 727	50	.77 377	.59 398	50	.83 112	.67 783
81	.62 509	.42 178	**91**	.70 648	.50 873	**101**	.77 481	.59 540	**111**	.83 199	.67 918
10	.62 657	.42 322	10	.70 772	.51 018	10	.77 585	.59 683	10	.83 285	.68 054
20	.62 804	.42 466	20	.70 896	.51 163	20	.77 689	.59 826	20	.83 372	.68 190
30	.62 951	.42 610	30	.71 019	.51 309	30	.77 792	.59 968	30	.83 458	.68 325
40	.63 097	.42 753	40	.71 142	.51 454	40	.77 895	.60 111	40	.83 544	.68 460
50	.63 243	.42 897	50	.71 265	.51 600	50	.77 998	.60 253	50	.83 630	.68 595
82	.63 389	.43 041	**92**	.71 387	.51 745	**102**	.78 101	.60 396	**112**	.83 715	.68 730
10	.63 534	.43 185	10	.71 509	.51 890	10	.78 203	.60 538	10	.83 800	.68 865
20	.63 678	.43 330	20	.71 630	.52 036	20	.78 305	.60 680	20	.83 885	.69 000
30	.63 823	.43 474	30	.71 751	.52 181	30	.78 406	.60 822	30	.83 969	.69 134
40	.63 966	.43 618	40	.71 872	.52 326	40	.78 507	.60 964	40	.84 054	.69 268
50	.64 110	.43 762	50	.71 992	.52 472	50	.78 608	.61 106	50	.84 138	.69 403
83	.64 253	.43 907	**93**	.72 112	.52 617	**103**	.78 709	.61 248	**113**	.84 221	.69 537
10	.64 396	.44 051	10	.72 232	.52 762	10	.78 809	.61 389	10	.84 305	.69 670
20	.64 538	.44 195	20	.72 352	.52 907	20	.78 909	.61 531	20	.84 388	.69 804
30	.64 679	.44 340	30	.72 471	.53 052	30	.79 009	.61 672	30	.84 471	.69 937
40	.64 821	.44 484	40	.72 589	.53 198	40	.79 108	.61 814	40	.84 554	.70 071
50	.64 962	.44 629	50	.72 708	.53 343	50	.79 208	.61 955	50	.84 636	.70 204
84	.65 102	.44 774	**94**	.72 825	.53 488	**104**	.79 306	.62 096	**114**	.84 718	.70 337
10	.65 242	.44 918	10	.72 943	.53 633	10	.79 405	.62 237	10	.84 800	.70 470
20	.65 382	.45 063	20	.73 060	.53 778	20	.79 503	.62 378	20	.84 882	.70 602
30	.65 521	.45 208	30	.73 177	.53 923	30	.79 601	.62 519	30	.84 963	.70 735
40	.65 660	.45 353	40	.73 294	.54 068	40	.79 699	.62 660	40	.85 044	.70 867
50	.65 799	.45 497	50	.73 410	.54 213	50	.79 796	.62 800	50	.85 125	.70 999
85	.65 937	.45 642	**95**	.73 526	.54 358	**105**	.79 893	.62 941	**115**	.85 206	.71 131
10	.66 074	.45 787	10	.73 642	.54 503	10	.79 990	.63 081	10	.85 286	.71 263
20	.66 212	.45 932	20	.73 757	.54 647	20	.80 087	.63 222	20	.85 366	.71 394
30	.66 348	.46 077	30	.73 872	.54 792	30	.80 183	.63 362	30	.85 446	.71 526
40	.66 485	.46 222	40	.73 987	.54 937	40	.80 279	.63 502	40	.85 526	.71 657
50	.66 621	.46 367	50	.74 101	.55 082	50	.80 374	.63 642	50	.85 605	.71 788
86	.66 757	.46 512	**96**	.74 215	.55 226	**106**	.80 470	.63 782	**116**	.85 684	.71 919
10	.66 892	.46 657	10	.74 328	.55 371	10	.80 565	.63 922	10	.85 763	.72 049
20	.67 027	.46 802	20	.74 442	.55 516	20	.80 660	.64 061	20	.85 841	.72 180
30	.67 161	.46 948	30	.74 554	.55 660	30	.80 754	.64 201	30	.85 920	.72 310
40	.67 295	.47 093	40	.74 667	.55 805	40	.80 848	.64 340	40	.85 998	.72 440
50	.67 429	.47 238	50	.74 779	.55 949	50	.80 942	.64 479	50	.86 076	.72 570
87	.67 562	.47 383	**97**	.74 891	.56 093	**107**	.81 036	.64 619	**117**	.86 153	.72 700
10	.67 695	.47 528	10	.75 003	.56 238	10	.81 129	.64 758	10	.86 230	.72 829
20	.67 828	.47 674	20	.75 114	.56 382	20	.81 222	.64 897	20	.86 307	.72 958
30	.67 960	.47 819	30	.75 225	.56 526	30	.81 315	.65 035	30	.86 384	.73 087
40	.68 092	.47 964	40	.75 336	.56 670	40	.81 407	.65 174	40	.86 461	.73 216
50	.68 223	.48 110	50	.75 446	.56 815	50	.81 500	.65 312	50	.86 537	.73 345
88	.68 354	.48 255	**98**	.75 556	.56 959	**108**	.81 592	.65 451	**118**	.86 613	.73 474
10	.68 485	.48 400	10	.75 666	.57 103	10	.81 683	.65 589	10	.86 689	.73 602
20	.68 615	.48 546	20	.75 775	.57 247	20	.81 775	.65 727	20	.86 764	.73 730
30	.68 745	.48 691	30	.75 884	.57 390	30	.81 866	.65 865	30	.86 840	.73 858
40	.68 875	.48 837	40	.75 993	.57 534	40	.81 956	.66 003	40	.86 915	.73 986
50	.69 004	.48 982	50	.76 101	.57 678	50	.82 047	.66 141	50	.86 990	.74 113
89	.69 132	.49 127	**99**	.76 209	.57 822	**109**	.82 137	.66 278	**119**	.87 064	.74 240
10	.69 261	.49 273	10	.76 317	.57 965	10	.82 227	.66 416	10	.87 138	.74 368
20	.69 389	.49 418	20	.76 424	.58 109	20	.82 317	.66 553	20	.87 212	.74 494
30	.69 516	.49 564	30	.76 531	.58 252	30	.82 406	.66 690	30	.87 286	.74 621
40	.69 644	.49 709	40	.76 638	.58 396	40	.82 495	.66 827	40	.87 360	.74 748
50	.69 770	.49 855	50	.76 745	.58 539	50	.82 584	.66 964	50	.87 433	.74 874
90	.69 897	.50 000	**100**	.76 851	.58 682	**110**	.82 673	.67 101	**120**	.87 506	.75 000

° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav
120	.87 506	.75 000	**130**	.91 455	.82 139	**140**	.94 597	.88 302	**150**	.96 989	.93 301
10	.87 579	.75 126	10	.91 514	.82 251	10	.94 643	.88 396	10	.97 022	.93 374
20	.87 652	.75 251	20	.91 573	.82 362	20	.94 689	.88 489	20	.97 056	.93 446
30	.87 724	.75 377	30	.91 631	.82 472	30	.94 734	.88 581	30	.97 089	.93 518
40	.87 796	.75 502	40	.91 689	.82 583	40	.94 779	.88 674	40	.97 123	.93 589
50	.87 868	.75 627	50	.91 747	.82 693	50	.94 824	.88 766	50	.97 156	.93 660
121	.87 939	.75 752	**131**	.91 805	.82 803	**141**	.94 869	.88 857	**151**	.97 188	.93 731
10	.88 011	.75 876	10	.91 862	.82 913	10	.94 914	.88 949	10	.97 221	.93 801
20	.88 082	.76 001	20	.91 919	.83 022	20	.94 958	.89 040	20	.97 253	.93 871
30	.88 153	.76 125	30	.91 976	.83 131	30	.95 003	.89 130	30	.97 285	.93 941
40	.88 223	.76 249	40	.92 033	.83 240	40	.95 047	.89 221	40	.97 317	.94 010
50	.88 294	.76 373	50	.92 090	.83 348	50	.95 090	.89 311	50	.97 349	.94 079
122	.88 364	.76 496	**132**	.92 146	.83 457	**142**	.95 134	.89 401	**152**	.97 381	.94 147
10	.88 434	.76 619	10	.92 202	.83 564	10	.95 177	.89 490	10	.97 412	.94 215
20	.88 503	.76 742	20	.92 258	.83 672	20	.95 221	.89 579	20	.97 443	.94 283
30	.88 573	.76 865	30	.92 314	.83 780	30	.95 264	.89 668	30	.97 474	.94 351
40	.88 642	.76 988	40	.92 369	.83 887	40	.95 306	.89 756	40	.97 505	.94 418
50	.88 711	.77 110	50	.92 425	.83 993	50	.95 349	.89 844	50	.97 536	.94 484
123	.88 780	.77 232	**133**	.92 480	.84 100	**143**	.95 391	.89 932	**153**	.97 566	.94 550
10	.88 848	.77 354	10	.92 534	.84 206	10	.95 433	.90 019	10	.97 597	.94 616
20	.88 916	.77 475	20	.92 589	.84 312	20	.95 475	.90 106	20	.97 627	.94 682
30	.88 984	.77 597	30	.92 643	.84 418	30	.95 517	.90 193	30	.97 656	.94 747
40	.89 052	.77 718	40	.92 698	.84 523	40	.95 559	.90 279	40	.97 686	.94 811
50	.89 120	.77 839	50	.92 751	.84 628	50	.95 600	.90 365	50	.97 716	.94 876
124	.89 187	.77 960	**134**	.92 805	.84 733	**144**	.95 641	.90 451	**154**	.97 745	.94 940
10	.89 254	.78 080	10	.92 859	.84 837	10	.95 682	.90 537	10	.97 774	.95 003
20	.89 321	.78 200	20	.92 912	.84 942	20	.95 723	.90 621	20	.97 803	.95 066
30	.89 387	.78 320	30	.92 965	.85 045	30	.95 763	.90 706	30	.97 831	.95 129
40	.89 454	.78 440	40	.93 018	.85 149	40	.95 804	.90 790	40	.97 860	.95 192
50	.89 520	.78 560	50	.93 071	.85 252	50	.95 844	.90 874	50	.97 888	.95 254
125	.89 586	.78 679	**135**	.93 123	.85 355	**145**	.95 884	.90 958	**155**	.97 916	.95 315
10	.89 651	.78 798	10	.93 175	.85 458	10	.95 924	.91 041	10	.97 944	.95 377
20	.89 717	.78 917	20	.93 227	.85 560	20	.95 963	.91 124	20	.97 972	.95 438
30	.89 782	.79 035	30	.93 279	.85 663	30	.96 002	.91 206	30	.97 999	.95 498
40	.89 847	.79 153	40	.93 331	.85 764	40	.96 042	.91 289	40	.98 027	.95 558
50	.89 912	.79 271	50	.93 382	.85 866	50	.96 081	.91 370	50	.98 054	.95 618
126	.89 976	.79 389	**136**	.93 433	.85 967	**146**	.96 119	.91 452	**156**	.98 081	.95 677
10	.90 040	.79 507	10	.93 484	.86 068	10	.96 158	.91 533	10	.98 108	.95 736
20	.90 104	.79 624	20	.93 535	.86 168	20	.96 196	.91 614	20	.98 134	.95 795
30	.90 168	.79 741	30	.93 585	.86 269	30	.96 234	.91 694	30	.98 161	.95 853
40	.90 232	.79 858	40	.93 636	.86 369	40	.96 272	.91 774	40	.98 187	.95 911
50	.90 295	.79 974	50	.93 686	.86 468	50	.96 310	.91 854	50	.98 213	.95 968
127	.90 358	.80 091	**137**	.93 736	.86 568	**147**	.96 347	.91 934	**157**	.98 239	.96 025
10	.90 421	.80 207	10	.93 785	.86 667	10	.96 385	.92 013	10	.98 264	.96 082
20	.90 484	.80 323	20	.93 835	.86 765	20	.96 422	.92 091	20	.98 290	.96 138
30	.90 546	.80 438	30	.93 884	.86 864	30	.96 459	.92 170	30	.98 315	.96 194
40	.90 608	.80 553	40	.93 933	.86 962	40	.96 495	.92 248	40	.98 340	.96 249
50	.90 670	.80 668	50	.93 982	.87 060	50	.96 532	.92 325	50	.98 365	.96 305
128	.90 732	.80 783	**138**	.94 030	.87 157	**148**	.96 568	.92 402	**158**	.98 389	.96 359
10	.90 794	.80 898	10	.94 079	.87 254	10	.96 604	.92 479	10	.98 414	.96 413
20	.90 855	.81 012	20	.94 127	.87 351	20	.96 640	.92 556	20	.98 438	.96 467
30	.90 916	.81 126	30	.94 175	.87 448	30	.96 676	.92 632	30	.98 462	.96 521
40	.90 977	.81 239	40	.94 223	.87 544	40	.96 712	.92 708	40	.98 486	.96 574
50	.91 037	.81 353	50	.94 270	.87 640	50	.96 747	.92 783	50	.98 510	.96 627
129	.91 098	.81 466	**139**	.94 318	.87 735	**149**	.96 782	.92 858	**159**	.98 533	.96 679
10	.91 158	.81 579	10	.94 365	.87 831	10	.96 817	.92 933	10	.98 557	.96 731
20	.91 218	.81 692	20	.94 412	.87 926	20	.96 852	.93 007	20	.98 580	.96 782
30	.91 277	.81 804	30	.94 458	.88 020	30	.96 886	.93 081	30	.98 603	.96 834
40	.91 337	.81 916	40	.94 505	.88 115	40	.96 921	.93 155	40	.98 625	.96 884
50	.91 396	.82 028	50	.94 551	.88 209	50	.96 955	.93 228	50	.98 648	.96 935
130	.91 455	.82 139	**140**	.94 597	.88 302	**150**	.96 989	.93 301	**160**	.98 670	.96 985

160°–180°

° ′	9 L hav	Nat hav	° ′	9 L hav	Nat hav
160	.98 670	.96 985	**170**	.99 669	.99 240
10	.98 692	.97 034	10	.99 680	.99 265
20	.98 714	.97 083	20	.99 691	.99 290
30	.98 736	.97 132	30	.99 701	.99 314
40	.98 758	.97 180	40	.99 712	.99 338
50	.98 779	.97 228	50	.99 722	.99 361
161	.98 801	.97 276	**171**	.99 732	.99 384
10	.98 822	.97 323	10	.99 742	.99 407
20	.98 842	.97 370	20	.99 751	.99 429
30	.98 863	.97 416	30	.99 761	.99 451
40	.98 884	.97 462	40	.99 770	.99 472
50	.98 904	.97 508	50	.99 779	.99 493
162	.98 924	.97 553	**172**	.99 788	.99 513
10	.98 944	.97 598	10	.99 797	.99 533
20	.98 964	.97 642	20	.99 805	.99 553
30	.98 983	.97 686	30	.99 814	.99 572
40	.99 003	.97 729	40	.99 822	.99 591
50	.99 022	.97 773	50	.99 830	.99 609
163	.99 041	.97 815	**173**	.99 838	.99 627
10	.99 059	.97 858	10	.99 845	.99 645
20	.99 078	.97 899	20	.99 853	.99 662
30	.99 096	.97 941	30	.99 860	.99 679
40	.99 115	.97 982	40	.99 867	.99 695
50	.99 133	.98 023	50	.99 874	.99 711
164	.99 151	.98 063	**174**	.99 881	.99 726
10	.99 168	.98 103	10	.99 887	.99 741
20	.99 186	.98 142	20	.99 894	.99 756
30	.99 203	.98 182	30	.99 900	.99 770
40	.99 220	.98 220	40	.99 906	.99 784
50	.99 237	.98 258	50	.99 912	.99 797
165	.99 254	.98 296	**175**	.99 917	.99 810
10	.99 270	.98 334	10	.99 923	.99 822
20	.99 287	.98 371	20	.99 928	.99 834
30	.99 303	.98 407	30	.99 933	.99 846
40	.99 319	.98 444	40	.99 938	.99 857
50	.99 335	.98 479	50	.99 943	.99 868
166	.99 350	.98 515	**176**	.99 947	.99 878
10	.99 366	.98 550	10	.99 951	.99 888
20	.99 381	.98 584	20	.99 955	.99 898
30	.99 396	.98 619	30	.99 959	.99 907
40	.99 411	.98 652	40	.99 963	.99 915
50	.99 425	.98 686	50	.99 967	.99 924
167	.99 440	.98 719	**177**	.99 970	.99 931
10	.99 454	.98 751	10	.99 973	.99 939
20	.99 468	.98 783	20	.99 976	.99 946
30	.99 482	.98 815	30	.99 979	.99 952
40	.99 496	.98 846	40	.99 982	.99 959
50	.99 510	.98 877	50	.99 984	.99 964
168	.99 523	.98 907	**178**	.99 987	.99 970
10	.99 536	.98 937	10	.99 989	.99 974
20	.99 549	.98 967	20	.99 991	.99 979
30	.99 562	.98 996	30	.99 993	.99 983
40	.99 575	.99 025	40	.99 994	.99 986
50	.99 587	.99 053	50	.99 996	.99 990
169	.99 599	.99 081	**179**	.99 997	.99 992
10	.99 611	.99 109	10	.99 998	.99 995
20	.99 623	.99 136	20	.99 999	.99 997
30	.99 635	.99 163	30	.99 999	.99 998
40	.99 646	.99 189	40	.00 000	.99 999
50	.99 658	.99 215	50	.00 000	1.00 000
170	.99 669	.99 240	**180**	.00 000	1.00 000