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PLANE TABLE MAPPING

HARPER'S GEOSCIENCE SERIES

CAREY CRONEIS, EDITOR

PLANE TABLE MAPPING

by

JULIAN W. LOW

RESEARCH GEOLOGIST

THE CALIFORNIA COMPANY

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To My Parents

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EDITOR'S INTRODUCTION

Low's *Plane Table Mapping* has been prepared to fill a rather unusual gap in the literature of the Earth Sciences. The author, therefore, has consciously limited the scope of this field manual to the hitherto somewhat neglected subject at hand. Professional engineers and geologists thus will possibly note the deliberate exclusion of closely related topics commonly treated in the standard texts and manuals on surveying, aerial photography, and field geology.

All such works, including the United States Geological Survey's well-known guide, *Topographic Instructions,* mention plane table mapping and, in varying detail, describe its applications and its advantages. But in the opinion of many "teachers"—whether they be instructing students in college or directing young engineers and petroleum and mining geologists in their early professional work—there has previously been no single book which describes in detail the plane table methods by which a survey is carried out from its inception to its logical end-product, a finished map.

Mr. Low originally started to remedy this deficiency under some compulsion. In 1940 the California Company, by which he was employed, asked him to prepare a manual for use by its many scientific trainees. The latter, although educated in various outstanding colleges and universities were, with few exceptions, inadequately trained in and lacked experience with common field procedures. As a consequence, most of them needed to be drilled in plane table mapping methods. Mr. Low's "company manual" proved to be very useful for this special type of post-graduate instruction and it was reprinted several times. In a sense, then, this volume in Harper's Geoscience Series is based as much on tested professional practice as it is on typical instructional procedure. It has, of course, had a pre-publication review by a number of college teachers, especially by members of the staff of the University of Colorado and of the Colorado School of Mines; and they have

not only stated their criticisms but voiced their enthusiastic approval.

It is not surprising that such approval should be forthcoming. Teachers in many of the interrelated engineering subjects have long realized the importance of plane table mapping and have wished to send students into the industrial fields adequately trained and experienced in its methods. Moreover, although simple surface types of mapping and exploration are fast giving way to complex geophysical and geochemical procedures, none of the latter will ever remove the need for an accurate framework of ground control —indeed, as the author points out in his Preface, each seems likely to increase that need.

The decision to publish Low's *Plane Table Mapping* as a "pocket manual" was reached after careful and thorough deliberation. The resultant format is, we believe, not only attractive but functional, and it should increase the usefulness of the volume in the field.

Both as a student of geology and later as one who conducted geological field courses I commonly wished for such a guide as Low's *Plane Table Mapping*. It seems rather certain, therefore, that present students and instructors in the various subdivisions of the Earth Sciences will also warmly welcome the publication of this very helpful manual. But it is probable that Low's book will be most appreciated by experienced economic geologists, and by the host of professional engineers, both civilian and military, whose role in our modern economy is still growing in importance.

CAREY CRONEIS

Beloit College

PREFACE

It is strange that a method of mapping so universally employed as the plane table should be so neglected in technical literature. Although many short articles have been written on special applications and techniques and a single chapter is devoted to the subject in several handbooks on general surveying, no single textbook brings together the numerous methods that are peculiar to this type of mapping. If the plane table method of mapping were becoming obsolete with the development of applications of aerial photographs, precise range finders, phototransits, shoran and other adaptations of radar, the absence of such a book might be understood; but the use of these newer techniques has only created a still greater need for the plane table, either as an adjunct to the method being employed or as a means of setting up a reliable graphic framework of control on the ground whereby the indirect methods can be checked and held within the limits of error demanded of most modern mapping.

The mapping of the United States has gone through a number of phases, and the plane table has played an important role in each for the last three-quarters of a century. It is not surprising that this was the primary instrument during the early period of topographic mapping by the United States Geological Survey, for the plane table is especially suited to this kind of work. The fact that more plane tables are in use today than at any other period in our history is revealing of the flexibility and adaptability of the method.

Commensurate with the expanding use of the plane table is the need for a text and handbook which facilitates teaching of the subject and at the same time provides a reference for the practicing engineer or geologist. In most beginning courses only fundamentals are taught. This is necessary because of the small amount of time that can be given to the subject; but a great deal more is required of the "trained" man who goes into the field with the responsibility of carrying out an extensive mapping project. It is

hoped that this book will be found useful in such circumstances.

Two objectives have been foremost in assembling and arranging the text materials. The first was to place in the hands of teachers and students an elementary text suited to beginning courses in field mapping. Because of the varying emphasis placed on plane table surveying in different colleges, it was not possible to segregate the subject matter in a way that would be ideal for all. Therefore, the principles and elementary methods are confined to the first three chapters. Since each of the numbered topics is somewhat independent of the others, many may be omitted without seriously impairing the continuity of the text. For example, in Chapter III only one of the several methods for solving the three-point problem need be learned in a first course.

The second objective of the writer was to assemble a large number of methods adaptable to certain types of mapping in order to make the book more convenient for the man in the field. These methods are presented in Chapters IV, V, and VI. The methods given therein are referred to the map scale ranges to which they are applicable. Since all the methods contained in this book have been tested in the field by the writer over the past twenty-five years, it was possible to classify them according to their relative merits as to speed and accuracy. Generally speaking, there is a close relationship between the scale of the map and the precision that can be attained by the plane table method. For this reason much care has been exercised in placing methods into categories defined by scale ranges. In other words, the scale classification as used here is also a classification of the accuracy and speed of the methods. Only precise methods are found in the chapter dealing with large scales, while less accurate but faster methods are given in the chapter on reconnaissance mapping. This arrangement should be convenient for the field engineer in that he will find all, or nearly all, the methods applicable to a given project in one of the chapters.

The last two chapters are included in order to aid the novice mapper in presenting his work more effectively. It is not uncommon to see a fine plane table sheet ruined in the final drafting phase of the work. If the subject of map drafting were treated in detail, a full-sized book would be required. Obviously, this could not be done in a handbook on plane table surveying; but this brief

discussion will enable one to follow accepted practices in the construction and presentation of maps.

Aerial photographs are likewise treated very briefly. Discussions are confined to methods which require only ordinary drafting instruments. There are no less than ten good books on the general subject of aerial photographs; hence, the methods presented in this book are those which would be used where the plane table constitutes an important phase of the project.

It is suggested that the beginner read the two chapters dealing with the care and adjustment of instruments. While he should not attempt to make the adjustments described until considerable experience has been gained, he will learn much of value about the construction of the instruments and how to care for them.

Contributions to the subject matter of this book by engineers and geologists have extended over many years. None has been direct, but without the special methods used by these professional men the text would be far less complete. To name each one who has given of his time and knowledge would require a long list, and one that would be difficult to compile. The writer is grateful to all.

To a few the writer is especially indebted. Most credit is due the late Ross L. Heaton, who gave the writer his first instruction in plane table mapping and supervised the field procedures. His example of professional excellence has long been the writer's guide to better work. To Max Gleissner and William McLaughlin of the United States Geological Survey much credit is due for methods dealing with triangulation and large-scale mapping. For methods of sketching on intermediate scales most credit is due Fred Cottrell, of the United States Geological Survey.

The manuscript was read critically by Warren O. Thompson and Larry Warner, of the University of Colorado, and L. W. Le Roy and others of the Colorado School of Mines. Significant changes were made in the arrangement of the book according to their suggestions, and there is no doubt that it is better for their having devoted many hours to the task.

Photographs used as illustrations of the Brunton pocket transit and the plane table instruments were made by W. McLaughlin and D. H. Pfeiffer.

J. W. L.

December, 1951

CHAPTER 1

Concepts and Definitions

1. Introductory Remarks

Surveying may be defined as the process of measuring the spatial relationships of features on or near the earth's surface. Surveys are classified in a great variety of ways, according to their purposes, relative precisions, the instruments employed, and so on. Thus, a transit survey utilizes the engineers' transit as the principal instrument; a plane table survey uses the alidade and plane table; a geologic survey is so designated because its purpose is to determine the relationships of geologic features, though such a survey may employ the transit, plane table, and many other instruments.

Plane table surveying is probably the easiest of all types to understand, but the most difficult in actual practice to master thoroughly. The statement has been made by competent teachers of engineering that months are required on the plane table to weeks on the transit or level in order to reach a comparable degree of proficiency with each. If a high degree of skill is attained the proportionate time on the plane table should probably be increased.

The principal difference between the plane table and other methods is that the solutions of many surveying problems are graphic, whereas with other instruments they are mathematical. Because the plane table method is largely graphic, it is positively essential that good habits in drafting be developed concurrently with the gain in knowledge of the theoretical and mathematical aspects of the subject. Many errors in plane table surveying are

attributable to poor drafting rather than poor instrumental work. In other words, there is an art as well as a science to plane table surveying, and neither can be slighted if excellence is to be attained.

The plane table is less stable than either the transit or level, and for this reason greater care and dexterity must be exercised in its operation. Because of this characteristic instability together with the fact that the plane table is frequently used in surveys of low precision, the method is considered by many to be suitable for only approximate measurements. This reputation, however, is not justified, especially when the accuracy and completeness of the finished map are taken into account in the comparison with other methods. What the plane table method does reveal is the natural aptitude or ability of the engineer. Careless habits soon come to light through the errors introduced in the map.

The elements and practice of surveying can be learned according to two different systems: The first is sometimes called the "rule of thumb" method. It consists of memorizing the instrumental operations and a long list of "practical" rules for making and applying the necessary mathematical computations. The steps in the process are performed without regard to, or knowledge of, the underlying principles or theory of surveying. The obvious weakness in this approach is that when a step in the field procedure or a rule governing the computations is temporarily forgotten, the surveyor cannot resort to the fundamental principles of surveying to overcome the immediate problem.

The second system emphasizes the concepts of earth relationships, the theory of surveying, sound map constructions and map projections, the causes of errors, the limitations of instruments and methods, etc. With such a background of training the plane table surveyor need have no fear of the temporary loss of a simple rule, for his general knowledge of the subject will enable him to analyze the problem and formulate a solution.

It is highly desirable for the plane table surveyor to develop the ability to think visually. Many of his problems involve three-

dimensional features, and, as stated earlier, the plane table method, itself, is largely a graphic method. When all the facts of a problem are clearly seen in the mind's eye, it is usually not difficult to arrive at a solution.

2. The Earth

Although the earth is a spheroid, somewhat flattened in the polar regions, the requirements of even moderately precise surveys are satisfied if it is treated as a perfect sphere. The actual dimensions of the earth as used by the United States government surveys are:

Polar axis	41,710,242 feet	(7,899.6 miles)
Equatorial axis	41,852,124 feet	(7,926.5 miles)

The departure from a true spherical shape affects measurements and computations most in the high latitudes. In the high latitudes approaching the poles the problems of maintaining correct directional relationships far surpass those resulting from the flattening mentioned above.

In plane table work the spherical shape of the earth is entirely neglected in the measurement of distance; however, as will be shown later, this factor must be considered in the determination of elevation even in approximate surveys.

For reasons of convenience the *mean surface* of the earth is generally accepted as *mean sea level,* or the elevation midway between high and low tides of the oceans.

3. Level Surfaces and Level Lines

A *level surface* is one which is everywhere parallel to the mean surface of the earth. A level line is an element of a level surface. It is clear, then, that a level surface is essentially spherical, and a level line is the arc of a circle whose radius varies according to the elevation above or below sea level. In surveys of small areas, or when all sight lines are short, level surfaces are assumed to be planes, and level lines are assumed to be straight lines.

4. Plumb Line

A plumb line is a line that is normal to a level surface; hence, all plumb lines may be considered as radii of the earth and as converging to a point at the center of the earth.

5. Vertical Plane

A vertical plane is a plane which bisects the earth. Obviously, a plumb line is an element of a vertical plane.

6. Elevation Datum

An elevation datum, sometimes called a "datum plane," is a selected level surface to which the elevations of a specific survey are referred. The datum is called a *real* datum if it coincides with a known level surface or established elevation. If the starting point of a survey is arbitrarily chosen, without regard to its actual elevation, the datum is *assumed.* The fact that the datum is assumed in no way affects the accuracy of the survey. It does mean, however, that the elevations of the survey bear an unknown relationship to sea level or any other established altitude.

7. Elevation and Difference in Elevation

The *elevation* of a point is the vertical distance above or below the datum. The elevation is referred to as plus if situated above the datum, and minus, if below. The *difference* in *elevation* between two points is the vertical distance between two imaginary level surfaces, each containing one of the points (Fig. 1). If the elevation of one of the points is known, that of the other can be determined simply by adding or subtracting, as the case may be. the difference in elevation.

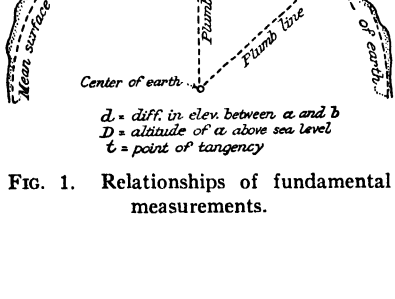


FIG. 1. Relationships of fundamental measurements.

8. Horizontal Plane and Horizontal Distance

A *horizontal plane* is a plane tangent to a level surface (Fig. 1). It is a level plane only at the point of tangency and diverges rapidly from the level surface outwardly from this point. A *horizontal line* is an element of a horizontal plane. Just as a circle arc may be approximated by a series of connected short chords, a sphere is approximated by many small intersecting planes each of which is tangent to the sphere at the central point. All surveys make use of these principles.

Horizontal distance is the distance projected vertically on to a horizontal plane. In surveying, all distances measured in the field are reduced to the horizontal equivalents by mathematical means.

9. Great Circles, Latitudes, and Longitudes

Figure 2 shows a sphere in the dashed outline cut through the center point by two vertical planes. The traces of the lines of intersection of the sphere and the planes are shown in heavy solid lines. In surveying, these lines are called *great circles*. A great circle is the trace of a level line on a vertical plane. Certain field methods in plane table surveying involve the principle of great circles.

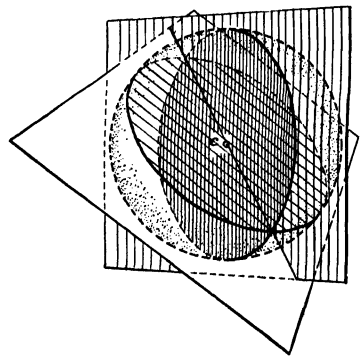


FIG. 2. The earth, vertical planes, and great circles.

Geographic *meridians* and *parallels* are a system of spherical coordinates by means of which it is possible to locate points or describe precisely the locations of points on the surface of the earth. *Geographic meridians* are great circles (Fig. 3) since the imaginary planes bisect the earth. In this special case, these planes also bisect the polar axis. The *equatorial plane* bisects

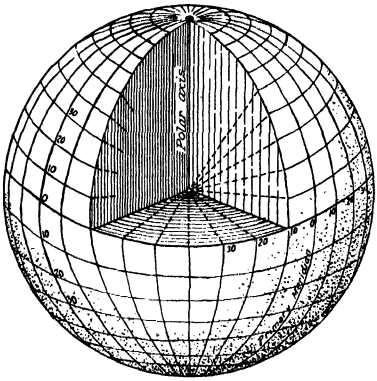


FIG. 3. The angular designation of latitudes and longitudes.

the earth normal to the polar axis; hence, the equator is a great circle. As can be seen in Fig. 3, parallels other than the equator are not great circles, but may be considered as a series of equally spaced planes parallel to the equatorial plane.

Figure 4 is a geometric construction illustrating the method of designating latitudes. The center of the earth is the point of origin for the angular designations given to both latitudes and longitudes.

Longitudes are described as east or west of any selected meridian, which is called a *primary meridian*. They never exceed 180 degrees east or 180 degrees west of the primary meridian. Longitude lines, or meridians, are true north-south lines; hence, no two north-south lines are exactly parallel. The primary meridian most commonly used in this country is the one passing through the observatory at Greenwich, England.

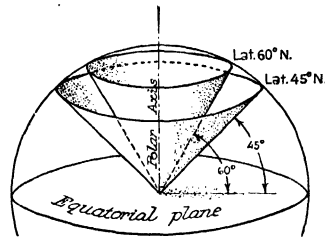


FIG. 4. Geometric development of parallels.

10. Bearings and Azimuths

In plane table mapping the directions of lines are obtained graphically on the plane table sheet, but in order to use certain types of surveying data to supplement the plane table work, it is necessary to have a "working" knowledge of common practices in expressing precisely the directions of lines obtained by non-graphic methods. These methods fall into two main groups: bearings and azimuths. Both are measured in terms of degrees, minutes, and seconds of

arc, from a selected line of reference, or meridian. Azimuths are measured clockwise through 360 degrees, starting from the reference meridian. They may be referred to either the north or south point, but never both on the same survey or in the same map area.

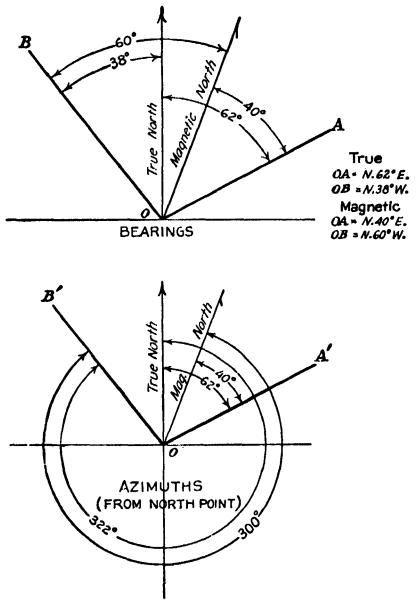


FIG. 5. The relations of magnetic and true bearings and azimuths.

Bearings are measured either clockwise or counterclockwise in quadrants, and never exceed 90 degrees.

Bearings are *true, magnetic,* or *assumed,* depending on the status of the reference line used. If the reference is a geographic meridian (Art. 9), then the bearings or azimuths are true; if the reference line is arbitrarily selected for the immediate purpose of the survey, the bearings or azimuths are assumed; and if the bearings or azimuths are referred to a magnetic north (compass) line they are magnetic. These relationships are shown in Figure 5.

11. Magnetic Meridians and Magnetic Declination

The magnetic poles of the earth do not coincide with the geographic poles, and for this reason the compass needle does not point to true north, except in certain localities, as explained later. The line along which the compass needle points is called a *magnetic meridian,* and the angular deviation of the magnetic meridian from a true, or geographic, meridian (Art. 9) is called *magnetic declination.* Magnetic declinations are east if the compass needle points east of true north; and west, if the needle points west of true north. The amount and direction of declination depend upon the geographic position of the observer and the year, day, etc., in which the observations are made. The time has to do with shifting of the earth's magnetic poles and may be summarized as follows.

The *annual variation* is an annual swing of the compass needle, the period of which is approximately 1 year. This variation is small (about 01′ of arc), and need not be considered in surveying.

The *solar diurnal variation* is a daily periodic swing of the needle, which amounts to about 06′, an amount too small to be of much concern to the surveyor.

The *secular variation* is a gradual change, whose period may be as long as 300 years. The amount of change (yearly) varies according to locality and in certain places in the United States is as large as 05′. Inasmuch as the period of the secular variation is extremely long, the total amount of change in declination for a given locality may be several degrees over a period of years.

Irregular variations are not well understood, but among other possible causes the most important is thought to be magnetic disturbances, which, in turn, are thought to be associated with Aurora Borealis. These disturbances are quite unpredictable and locally may amount to many degrees of deflection.

Local attractions are caused by magnetic substances, such as iron, nickel, and metalliferous rocks. The effect of local attraction depends upon the proximity of the material to the compass and the mass of the material. Small metallic objects carried on the

person of the observer may deflect the needle as much as several degrees.

12. The Isogonic Chart

The isogonic chart (Fig. 149) consists of a small scale map upon which are shown a set of roughly parallel solid lines, each representing a line of equal magnetic declination, and another set of dash lines (cutting across the solid lines) each of which indicates the annual rate of change in the amount of declination. All solid lines east of the agonic (zero) line show west declinations; those west of the agonic line, east declinations. Along the agonic line the compass needle points to true north, provided the date of the chart is such that no correction for annual change is necessary. The double dash line is the line of no annual change, i.e., anywhere along this line the compass needle points the same direction year after year. All dash lines north of the double line represent a westward swing of the compass needle in the amount per year as indicated by the numbers on the lines, and all dash lines south of the zero line show the eastward swing of the needle.

Isogonic charts are dated so that correct declinations can be determined for any year (later than the date on the chart). The original charts are prepared by the United States Coast and Geodetic Survey and are revised and brought up to date periodically.

13. Subdivision of Public Lands

It is to the interest of anyone engaged in mapping work to have some knowledge of the more common forms of land subdivision. While the plane table man will not be required to make new land surveys for the purpose of establishing boundaries, he is frequently called upon to "recover" old boundaries and section and township corners. In order to accomplish this with a minimum expenditure of field time, he must be familiar with the more prevalent, and inherent, irregularities in the township system.

Various schemes and units have been used in the United States at different times, due in part to the fact that certain parts of the

country have in the past been under the domination, or influence, of other governments. During the early periods in the settlement of North America, large grants of land were made to individuals. The boundaries of these grants were in many cases poorly defined and poorly marked on the ground. The common practice was to define the boundaries of properties by means of convenient natural (and artificial) marks, such as stream courses, ridges, and the margins of wooded areas, with the result that original boundaries often changed or became obliterated with the passing of time.

Much of the southwestern part of the United States, formerly under the domination of Spain (and Mexico) was subdivided on the basis of a Spanish unit of length, the *vara*. Varas of three different lengths have been used in the southwest regions, as follows:

1 vara (in Texas)	33⅓ inches
1 vara (in California)	33 inches
1 vara (in Mexico)	32.993 inches

In 1785 a law was enacted for the purpose of subdividing public lands into square townships and sections, the townships to be 6 miles square, and the sections 1 mile square. The sections were to be numbered from 1 to 36, beginning with 1 in the southeast corner of the township, and ending with 36 in the northwest corner, the numbers for each south-north row of sections starting at the south boundary. In 1796 the law was amended so that sections were to be numbered in east-west rows, alternately west and east, starting with 1 in the northeast corner and ending with 36 in the southeast (Fig. 7). This is the system in use today for all new surveys.

Because of the convergence of all true north-south lines (meridians), it is not possible to lay out a truly rectangular system of coordinates over large areas and maintain true east-west and north-south boundaries. For this reason all townships are not 6 miles square, nor are all sections 1 mile square, even though all the field measurements have been carried out without error. An attempt is therefore made to lay out the townships and sections

in such a manner as to place the necessary corrections for convergence in predetermined rows of sections within the townships.

The starting point for the township numbers (the *initial point*) is a selected intersection of a geographic meridian, called a *principal meridian,* and a geographic parallel, called a *base line.* At the present time it is the practice to establish secondary meridians east and west of the principal meridian at intervals calculated to contain a number of full-sized townships, usually 4 townships, or 24 miles. Such secondary meridians are designated 1st, 2d, etc., *guide meridians east* or *west,* depending on whether they lie east or west, respectively, of the principal meridian. At intervals of 24 miles, or other multiples of 6 miles, secondary parallels of latitude are established north and south of the base line. These lines are designated 1st, 2d, etc., *standard parallels north* or *south,* as the case may be.

The principal meridian and all base lines, standard parallels, and north and south boundaries of townships are continuous true north-south and east-west lines. Guide meridians are true north-south lines, but are continuous only for the distance between two standard parallels (Fig. 6). Along the base lines and standard parallels, guide meridians (and range lines) are offset in order to adjust for convergence.

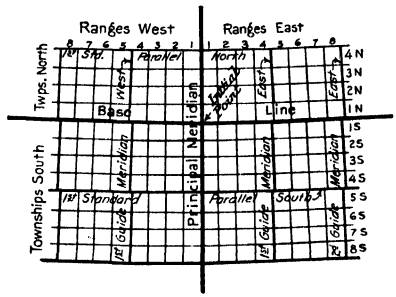


FIG. 6. Scheme of land subdivision and nomenclature.

Theoretically, the east and west sides of townships are exactly 6 miles long, any discrepancies being entirely due to errors in measurement. Along the north and south boundaries, only the south sides of those townships bordering on, and immediately north of, the base line and standard parallels are full 6 miles wide, all others being 6 miles, less the amount of convergence. The east

and west boundary lines of townships are called *range lines,* or *ranges;* the north and south boundaries are *tiers,* or more commonly *township lines.*

In subdividing townships into sections, the work progresses from the east range line to the west, all sections being as nearly as possible parallel to the east range line. Since the range lines converge to the north (in northern latitudes), it is obvious that the sections in the westernmost row will not be rectangular, nor will they be a full square mile in area. If all field measurements were without error, the east and west sides of sections would be exactly 1 mile in length; but since errors in varying magnitudes are usually present it is common practice, wherever feasible, to throw these errors in the northern half of the northernmost row of sections. Therefore, those sections most likely to be irregular in shape and size are: 1, 2, 3, 4, 5, 6, 7, 18, 19, 30, and 31.

Because of the necessity of making new surveys in isolated regions opened for settlement, particularly in the west, it was not always possible or expedient to conduct the work in a systematic manner from a few well-chosen initial points. Large errors in geographic position and in the bearings of lines were thus allowed to develop so that when the various isolated surveys were finally joined to more reliable work the errors were found to be too large to adjust by approved means. Therefore, it is not unusual to find offsets along north-south lines, or two or three marks representing the same land corner, or the two sides of a supposedly regular section or township diverging by 10 to 15 degrees. There are, likewise, a number of areas where small groups of townships do not fit into the regional system of numbering, and it is difficult, or perhaps impossible, to determine from maps the base lines and meridians used in the original surveys.

In Canada the scheme of land subdivision is very similar to that used in the United States, except that there are fewer irregularities in the Canadian surveys. The townships are subdivided into sections 1 mile square, but the numbering starts in the southeast corner and ends in the northeast. Sections are subdivided into 16

parts, called *legal subdivisions*, which are numbered from 1 to 16 starting in the southeast corner of the section and ending in the northeast. Each legal subdivision in a regular section contains 40 acres (Fig. 7). Townships are numbered north from the International boundary in western Canada, and ranges east and west from the principal meridian.

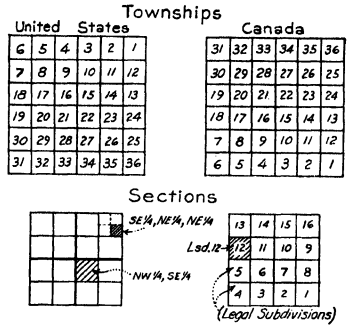


FIG. 7. Numbering of United States and Canadian sections and the principle of describing subdivisions of the section.

14. Map Projections

A map is a representation of a part, or all, of the earth's surface, showing the various terrestrial features in approximately their correct relationships.

Inasmuch as the spheroidal surface of the earth is represented on the plane surface of the map, it is not possible to maintain exact interrelationships of terrestrial points on maps covering large areas. In order to develop terrestrial positions on the map plane, various geometrical constructions are used, and these constructions are called *map projections*. Many different kinds of projections are in use, each being distinguished by its own peculiar type of distortion. Thus, one projection might faithfully produce directional relationships by sacrificing scaling properties, while another would show the shapes of features with considerable fidelity, but would be lacking in other important map qualities. Projections for "all-purpose" maps are those which tend to distribute the inherent distortions equally; that is, the distortions of scale, angles between lines, areas, and shapes. It is this type of conventional projection that is most often used in plane table and other kinds of intermediate and large-scale work.

A brief description of a few of the more common projections is given below:

Orthographic Projection

This projection is seldom used in the construction of maps, except for illustrative purposes, because the edges of orthographic projections are greatly distorted. It is mentioned here for the reason that it is the simplest of all the constructions, consisting merely of projecting by straight and parallel lines the points on the earth to a plane that is tangent at the equator (usually) and parallel to the polar axis.

Mercator's Projection

Mercator's projection is widely used for the construction of navigation maps for the reason that the bearing to any point, as shown on the map, is the course which a ship would steer to reach the point by the shortest route. Briefly, the theory of the projection is as follows.

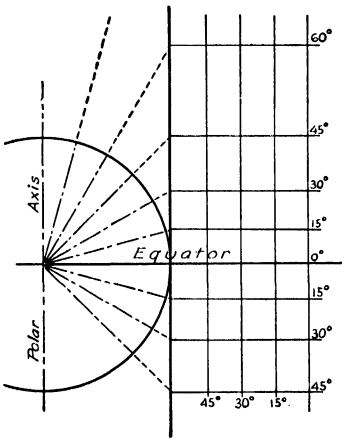


Fig. 8. The principle of the Mercator map projection.

Imagine a cylinder parallel to the polar axis, completely encircling the globe, but touching it only along the equator, where the sphere and cylinder are tangent. Straight radiating lines, originating at the center of the earth, are projected through points on the sphere to the surface of the cylinder. Meridians so projected are equally spaced and parallel on the cylinder; parallels are parallel on the cylinder, but the spacing between adjacent parallels is increasingly greater toward the poles (Fig. 8). The poles cannot be shown on a Mercator projection for the reason that the projection lines of the poles are parallel to the cylinder, and, hence, never intersect it. Now if the theoretical cylinder is cut along one of the

parallel meridian lines and unrolled, it constitutes a map in a plane.

A Mercator projection has no value where distances are to be obtained by scaling on the map. The scale rapidly increases from

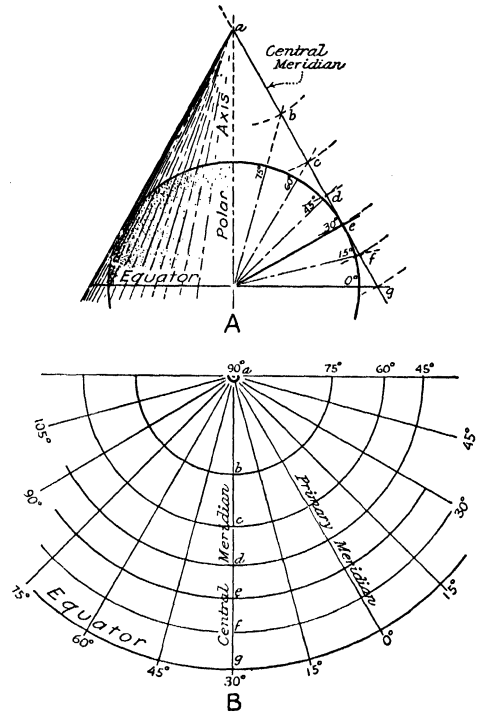


FIG. 9. A. Development of the conic projection. B. Character of the conic (polar) projection.

the equator toward the poles with the result that areas in the higher latitudes are vastly out of proportion to those in the intermediate and low latitudes; for example, on a Mercator projection, South America is only half as large as North America, and Greenland is nearly as large as North America.

Conic Projection

The conic projection is commonly used in the construction of small-scale maps of fairly large areas. If the area included is not too large the projection is moderately free from large distortions. The theory of the projection assumes a cone placed so that the apex lies on the extension of the polar axis, the cone being tangent to the earth at some predetermined geographic parallel (Fig. 9) usually the middle parallel of the map. Geographic parallels are projected on the cone from the center of the earth, as shown in Fig. 9A. Using point *a* as center, parallels are drawn as circle arcs (Fig. 9B), whose radii are determined from the first construction (Fig. 9A).

Meridians are radial lines drawn from point *a* to degree marks laid off by protractor or other means along one of the outer concentric circle arcs.

Polyconic Projection

The polyconic is the most widely used projection for maps of all scale classes. It is the most satisfactory thus far devised for

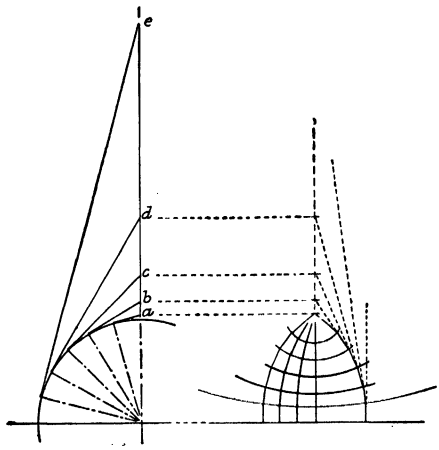


Fig. 10. Theory of the polyconic projection.

all-purpose maps, and is used almost exclusively by the different government surveys. The theory of the polyconic projection is similar to that of the conic: Instead of one cone, as in Fig. 9, consider a cone for each degree of latitude, the apex of each being determined by the intersections of the tangent lines with the prolongation of the polar axis. Figure 10 shows the construction for three such cones, though there may be any number, depending on the scale of the map and the precision required.

Tables have been prepared by the U.S. Coast and Geodetic Survey and the U.S. Geological Survey, by means of which the polyconic projection can be easily drawn in a wide range of map scales. (See Chapter XII.)

CHAPTER II

Elementary Plane Table Surveying

15. General Remarks

Plane table surveying consists of many individual manual operations, each of which must be executed in a manner that will tend to reduce the possibilities for error. Some of the most frequently occurring errors, and often the most difficult to trace back to their sources, are those made in the simpler operations. Much time may be lost as a result of failure to appreciate fully the importance of performing properly each of the many details of operation.

As a means of avoiding errors or inaccuracies in the work and increasing efficiency in general, the beginner should attempt to systematize his work. When the various steps are performed in orderly sequence, always in the same manner, the procedure soon becomes habitual, and the chances for errors and omissions in the minor operations are minimized. The sequence in which a given group of operations should be performed is, within certain limitations, a matter of personal choice. It cannot be presumed that any one procedure will be entirely efficient under all the variable conditions encountered in actual field practice. Conversely, the routine should be altered to fit the different field conditions or to meet the requirements of the specific type of survey. But some sort of routine which seems to fit the immediate problems should be set up as soon as possible so that the utmost efficiency can be attained.

The following articles deal with the elementary principles and operations of the plane table. Some of these may at first appear too elementary—too obvious; but, as stated above, experience has definitely shown that it is in these very simple and apparently minor steps that most errors occur.

The beginner should strive for accuracy before attempting to develop speed. The two—speed and accuracy—can hardly be developed simultaneously; and the logical order is *accuracy first, then speed.*

16. Plane Table Instruments

The essential pieces of the plane table outfit are the alidade, tripod, table (or board), stadia rod, and accessories, such as scales, plotting needles, notebooks, and various carrying cases (Fig. 11).

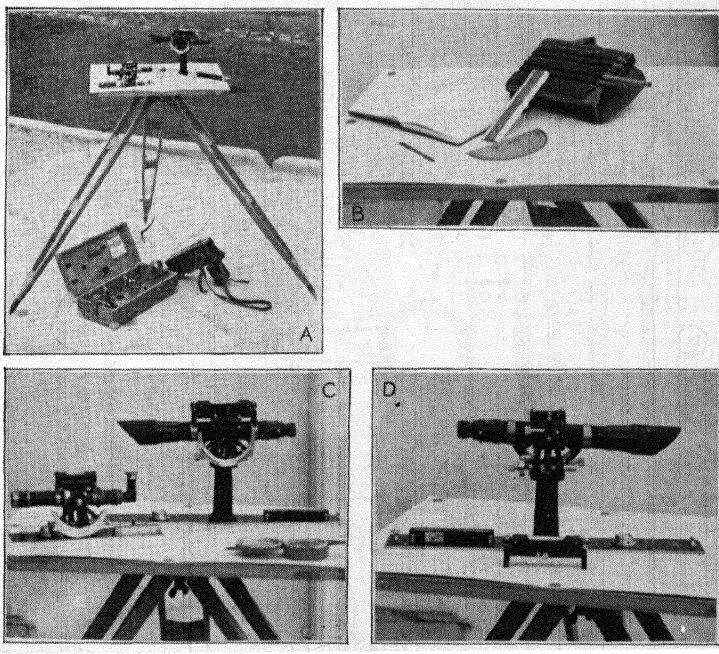
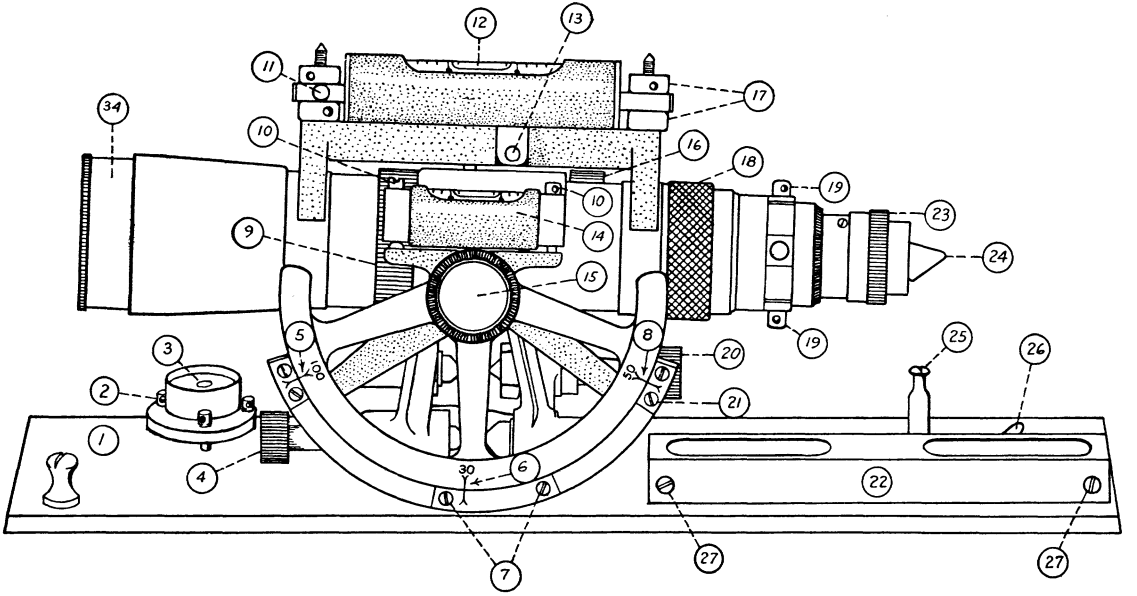


FIG. 11. Photograph showing the complete plane table outfit.

A. The Johnson tripod, plane table, explorers' and topographic alidades, and their carrying cases.
B. Accessory equipment: plotting scale, protractor, field notebook, and notebook and pencil carrying case.
C. Comparison of explorers' and topographic alidades.
D. Right side of the topographic alidade with the striding level dismounted.

FIG. 12. Explorers' (expedition) alidade, showing the principal external parts.



Principal external parts of the alidade as shown in Figs. 12 and 13.

(1) Blade or base. (2) Adjustment screw for universal level. (3) Universal level center mark. (4) Vertical angle and Beaman arc index tangent screw. (5) Horizontal distance arc index. (6) Vertical angle arc index. (7) Vertical angle arc index adjustment plate. (8) Beaman arc index. (9) Knurled telescope holding collar. (10) Control level adjustment screw. (11) Lateral adjustment screw for striding level. (12) Striding level bubble vial. (13) Striding level spring lock. (14) Control level sleeve shield. (15) Telescope axis bearing. (16) Axis clamp (on right side of instrument). (17) Vertical adjustment capstan nuts of striding level. (18) Knurled focusing ring. (19) Capstan adjustment screws for adjusting colli-

mation. (20) Telescope tangent screw (with or without gradienter attachment). (21) Beaman arc index adjustment screws. (22) Magnetic needle box or trough. (23) Cross wire or eyepiece focusing ring. (24) Prismatic eyepiece on Fig. 12, inverting eyepiece on Fig 13. (25) Keeper stud for striding level. Used when instrument is laid in case. (26) Needle lifting lever. (27) Compass box assembly screws. (28) Striding level stud. (29) Ring foot bearing for striding level. (30) Plunger opposing Beaman tangent screw. (31) Pedestal or standard of topographic type alidade. (32) Striding level spring lock releasing screw. (33) Finder sights on striding, not on all instruments. (34) Dust protector cap for objective lens.

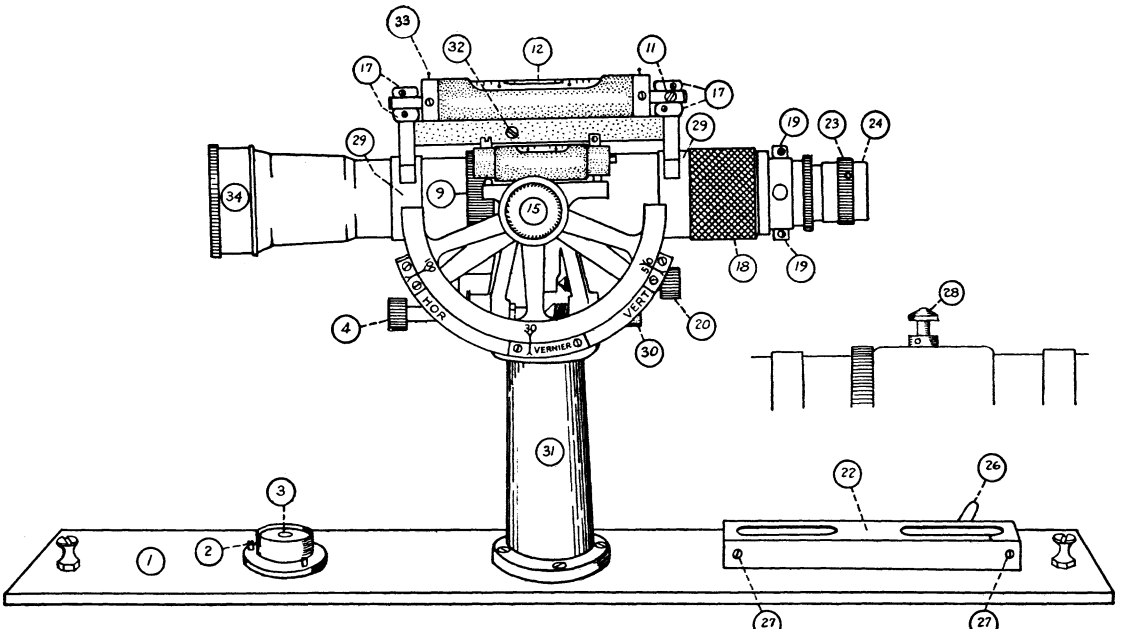


FIG. 13. The topographic (high standard) alidade, showing principal parts.

Alidades

There is a wide variety of alidades, each being adapted to certain types of work. The simplest alidade is a flat ruler a few inches long with two folding sights aligned parallel to the edge of the ruler. This is called a peep sight alidade and is used only for approximate work or as an aid in sketching details on a map whose main features are determined by other means.

Telescopic alidades are of three general types: explorers' (or expedition), topographic, and triangulation. All telescopic alidades have the following common features: a base or blade which is a straightedge beveled on one or both sides, a telescope containing a cross-hair reticule, one or more spirit levels, a compass needle, and graduated arcs for measuring the inclination of the telescope.

The explorers', or expedition alidade, is shown in Fig. 12. It differs from other types in that the telescope is mounted close to the base in order to make the instrument compact and easily carried. The length ranges from 10 to 14 inches, and the telescope powers from 10 to 14.

The topographic (high standard) alidade, shown in Fig. 13 is a somewhat larger and more cumbersome instrument to carry than the explorers' model; however, it is faster and more convenient to use on the plane table. The length of the base ranges from 16 to 20 inches, and the powers of the telescope from 14 to 20.

The triangulation or high precision alidade is similar to the topographic type in overall design. The blade may be as long as 30 inches with a slot in the central part so that a beveled straightedge parallel to the telescope lies directly beneath the center of the telescope. The powers of the telescope range from 18 to 32.

Tripods

The tripods on which the drawing board, or plane table, proper is mounted are of two general designs: the rigid leg and the extension, or telescopic. The rigid leg type shown in Fig. 11 is best for

most work because it is stronger and provides a more solid support for the plane table. The extension leg type is best where transportation facilities are strictly limited, as on boat or pack trips. In precipitous terrain the legs may be shortened or lengthened to provide the most stable setup.

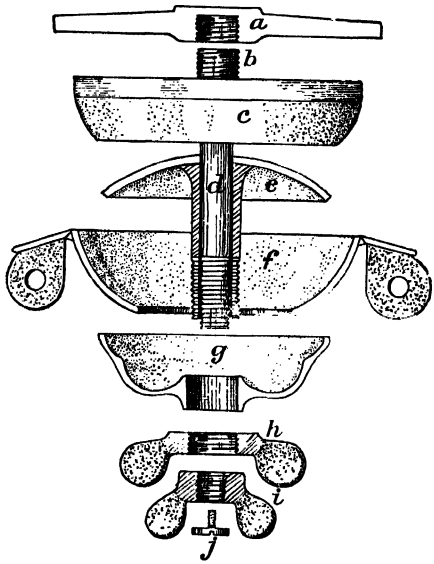


FIG. 14. Partially sectioned view of the Johnson tripod head assembly.

The traverse table has no leveling device for the table. It is leveled approximately by shifting the legs. The Johnson tripod head is shown schematically in Fig. 14. When the wing nuts *h* and *i* are loosened the table can be rotated or tilted in any direction. When the table is leveled, the nut *h* is tightened. This prevents the table from tilting, but allows it to be rotated in a horizontal plane. Wing nut *i* locks the rotational movement. The screw *j* forms a cap on *d*, which prevents the wing nuts from falling off when they are loosened.

Precision plane table tripods are equipped with a leveling device similar to the foot screws on a transit. This type is used by the U.S. Coast and Geodetic Survey.

Plane Tables

Plane tables are simply drawing boards with various fixtures for holding the paper and holding the board on the tripod. The standard sizes are, in inches, 15 by 15, 18 by 24, and 24 by 31. They may be constructed to any dimensions so long as they remain stable on the tripod.

17. Preparation of the Plane Table Sheet

Many kinds of plane table sheets are employed in actual mapping work, but only the cloth-mounted paper ones are considered here. The *single-mounted* sheet consists of strong cloth, usually linen, to which is tightly glued a good grade of drawing paper. The *double-mounted* sheet has the drawing paper on both sides of the cloth.

Plane table sheets should be "seasoned" for a few days (or weeks) fully exposed to the air so that the shrinkage or expansion adjustments to local atmospheric conditions will have taken place before any map work has been started.

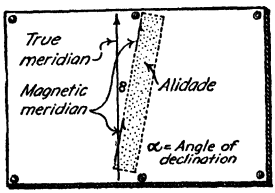


Fig. 15. View of the plane table and alidade base illustrating orientation by magnetic needle.

In preparing the sheets for the board, punch the thumbscrew holes considerably larger than the screw, but not quite so large as the screw head. The large holes allow for adjustment of the sheet according to temporary variations in humidity.

With a steel straightedge (or the fiducial edge of the alidade) and a 7H to 9H pencil with a wedge-shaped point, draw a fine north-south line across, and near the center of, the plane table sheet. From this true meridian lay off the correct magnetic declination (Art. 11). Show the magnetic merid-

ian by two fine lines, each 2 to 3 inches long, near the ends of the alidade blade (Fig. 15).

18. Setting Up the Tripod and Plane Table

Screw the plane table on the tripod so that the table-bearing plate is firmly seated on the Johnson head. Tighten the wing nuts on the bolts which fasten the legs to the tripod head. Spread the legs widely and, while holding the tripod so the legs are clear of the ground, loosen each wing nut only enough to allow each leg to fall into a vertical position by its own weight. The plane table and tripod are now ready to set up at the starting point.

In setting up the tripod on level ground, place two legs firmly in the ground and approximately level the table (by inspection) by moving the third leg from side to side. Be sure each leg is solidly planted so that it cannot easily be pressed any deeper. On sloping ground, always put one leg on the up-hill side and the other two on the down-hill (Fig. 16). This provides the greatest stability for the table.

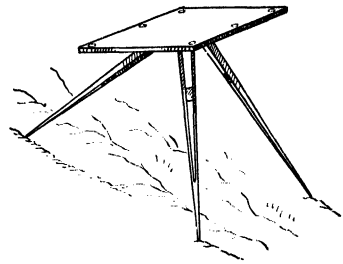


FIG. 16. Setting up the tripod on sloping ground.

19. Leveling and Orienting the Plane Table

Loosen both clamps of the Johnson head by turning the wing nuts underneath. Place the alidade *in the center* of the plane table so that the center of gravity of the instrument is directly above the tripod head. Watch the instrument closely, for the table is likely to tip and allow the alidade to fall to the ground. Bring the bubble of the universal level ("3" in Figs. 12 and 13) to center by tapping on the higher side of the table. When the bubble is centered, tighten the *upper* wing nut of the Johnson movement. The table can now be rotated while remaining level.

To orient the table so that the true meridian drawn on the sheet is parallel to a true meridian on the ground (Art. 9), place the fiducial (beveled) edge of the alidade blade exactly on the *magnetic meridian* (Fig. 15). Release the needle lock ("26" in Figs. 12 and 13) and rotate the table until the freely swinging needle comes to rest at the zero, or index, mark. Now tighten the *lower* wing nut of the Johnson head. Check the needle again after tightening the wing nut, as the table might have been moved slightly.

If the needle tends to cling to the glass cover, moisten the finger and rub it across the glass to discharge static electricity which may have accumulated. When the needle is sluggish, lightly tap the table or alidade blade with the fingers to aid in overcoming the friction between the needle bearing and the privot.

Great care must be taken that the table is not oriented by compass near large masses of steel or iron, such as bridges, railroads, conduits, and hydrants, and that no small magnetic objects are on the table or the person operating the plane table, as the needle will be deflected by the attraction of such objects.

When the amount of declination is not known, the table can be oriented, and a magnetic meridian drawn on the sheet for future use by the following method.

Draw a true north-south meridian on the sheet. Set up the tripod on some visible north-south line on the ground, preferably several miles long, such as a section or township line road or fence line. Level the table and place the alidade blade on the true meridian previously drawn on the sheet. Take care that the alidade is not disturbed from this position hereafter. Now direct the line of sight by rotating the table until the vertical cross wire exactly bisects the road or fence at the most distant visible point. Tighten the lower wing nut of the Johnson head and sight through the telescope again to ascertain that the vertical wire is still on the distant point. The plane table is now correctly oriented; the north line on the sheet has been made parallel to a north line on the ground.

Turn the alidade so that the blade approximates the direction of the magnetic meridian. Release the compass needle lock and

rotate the alidade (not the table) to the right or left until the needle comes to rest at the index mark. With a hard pencil, draw the magnetic meridian along the fiducial edge. This line can now be used for future compass orientation.

20. Directions of Sight Lines

When the plane table is properly oriented, the direction in which the telescope is pointed is represented on the sheet by the edge of the blade. It follows, then, that if the edge of the blade is pivoted about a point on the sheet representing the instrument station, the directions of successive sights to rod stations or other ground points are accurately shown on the sheet by drawing lines along the fiducial edge from the plotted instrument point toward the points sighted (Fig. 17). The lines radiating out from an instrument station are called *rays,* or *sight rays.*

21. Map Scales

The scale of a map is a comparison of the distance between two points, as measured on the map, with the distance between corresponding points, as measured on the ground, it being understood that only horizontal distances are considered. The scale of a map may be expressed in a variety of ways: 1 inch = 1 mile; 1 inch = 5280 feet; 1:63,360 or 1/63,360. The latter expression of scale is called a *fractional scale* or, less commonly, *scale ratio,* because it expresses the relationship between map and ground distances in simple ratios, rather than specific units. Thus, 1 foot on the map = 63,360 feet on the ground, or 1 centimeter on the map = 63,360 centimeters on the ground.

In field practice the first two designations are usually employed, the fractional scale being too cumbersome.

22. Scaling and Plotting Distances

New points are located on the sheet by drawing sight rays from the map position of the instrument station toward the points and plotting their positions on the lines according to the respective

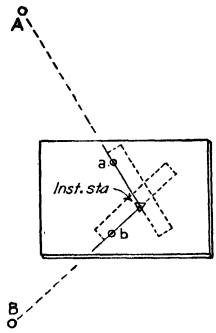


FIG. 17. View of the plane table and alidade base showing the principle of locating points on the plane table sheet.

horizontal distances (Points *a* and *b*, Fig. 17). The horizontal distances are obtained by stadia measurements, explained in Art. 29.

It is good practice, when plotting distances, to lay the plotting scale along the ray so that the correct distance graduation is on the old point (the instrument station) and the zero of the scale is at the new point (Fig. 18).

Points should be plotted by pricking small holes in the sheet with a plotting needle, which may easily be prepared as follows.

Taper the end of a light, round wooden stick, about the diameter of a pencil, and wind the tapered end with strong thread. Now drive a fine sewing needle in the end of the stick, leaving about ¾ inch of the pointed end protruding (Fig. 19a). Heat a small amount of sealing wax and mould a symmetrical point well down over the needle (Fig. 19b).

The holes in the sheet made with the plotting needle should be only large enough to be seen clearly without the aid of a lens.

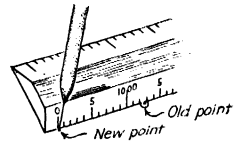


FIG. 18. Correct method for plotting a distance on the sheet.

23. Focusing the Telescope

The telescope must always be in perfect focus, or the observations on the rod will be inaccurate. To focus the telescope properly, proceed as follows.

Point the telescope toward the sky or some white well-illuminated surface, such as the wall of a building. Turn the knurled ring* ("23" in Figs. 12 and 13) until the cross and stadia wires are sharp, black, and clear. This focuses the microscope of the eyepiece on the wires.

* Some instruments are provided with knurled knobs, or disks, on the right side or top of the telescope. This is true for most external focusing instruments.

Now point the telescope toward some distant, clearly defined object, a stadia rod being best for the purpose. Bring the object into sharp focus by turning the forward knurled ring "18." The telescope is now apparently focused, but must be tested further, as follows.

24. Parallax

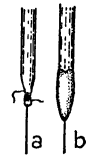


FIG. 19. The plotting needle.

When the instrument is apparently focused, set the cross wires on a clearly defined small point, such as a graduation on the rod. Move the eye back and forth across the eyepiece while observing the cross wires on the point. If the wires appear to move with respect to the point, the telescope is imperfectly focused. This error is called parallax.

Refocus the object lens until the wires remain stationary on the point when the position of the eye is shifted. It may be necessary to change the focus of the eyepiece microscope slightly.

When the eyepiece has once been properly focused, it should need little or no adjustment thereafter; however, the object lens must be refocused for each new point, if there is any appreciable difference in the distance.

Always check the parallax before taking a rod reading or adjusting the alidade. The telescope may appear to be out of adjustment when actually it is only out of focus.

25. Determination of Distance by Stadia

Toward the back of the telescope, but in front of the eyepiece, is a ring, or reticule, on which are strung either 3 or 4 horizontal wires and 1 vertical wire (Fig. 96). The vertical and middle horizontal wires are called *cross wires,* and the remaining 2 or 3 horizontal ones, *stadia wires.* The spacing between the upper and lower stadia wires is such that when a graduated rod is viewed through the telescope, the portion of the rod apparently intercepted by the two wires is proportional to the distance on the ground from the

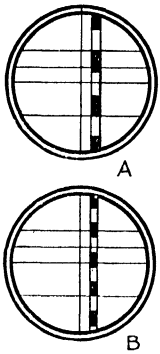


FIG. 20. View of the stadia rod through the telescope showing (A) a distance of 300 feet and (B) a distance of 450 feet.

instrument to the rod. Usually this relationship is 1:100; i.e., for every foot, or fraction of a foot, intercepted between the upper and lower stadia wires, the rod is 100 feet, or fraction of 100 feet, distant from the instrument.

EXAMPLE:

In Fig. 20A the apparent rod intercept is 3.0 feet between the upper and lower stadia wires, and the distance from the instrument to the rod is $3 \times 100 = 300$ feet.

In Fig. 20B the apparent intercept is 4.5 feet. and the ground distance is, therefore, $4.5 \times 100 = 450$ feet.

26. The Stadia Interval

The rod intercept between the upper and lower stadia wires is called the *stadia interval.* Obviously the value of the stadia interval varies according to the distance from the instrument to the rod. At 1000 feet distance, the stadia interval is 10.0 feet, at 1550 feet it is 15.5 feet. It is evident that as the rod is moved away from the instrument, a point will finally be reached where the stadia interval exceeds the length of the rod. In this case, the *half-interval* is used (Fig. 21A), this being the intercept between the horizontal cross wire and either the upper or the lower stadia wire. Similarly, if the distance is too great for a half-interval intercept, the *quarter-interval* is used (Fig. 21B).

To obtain the full interval value from half- or quarter-interval readings, multiply the intercept by 2 or 4, respectively.

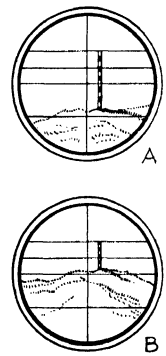


FIG. 21. View through the telescope showing the half-interval and quarter-interval intercepts.

EXAMPLE:

The quarter-interval intercept is 9.3 feet. The full interval is $9.3 \times 4 = 37.2$ feet; and the distance from the instrument to the rod is $37.2 \times 100 = 3720$ feet.

27. Stadia Interval Factor

The stadia interval factor is the number which, when multiplied by the full stadia interval, will give the correct distance to the rod. In the explanation above it is assumed to be 100. While this factor is ordinarily so near 100 that no great error will result if 100 is used, the actual value of the factor may be 98,101, etc., depending on the spacing of the stadia wires. The determination of the stadia interval factor is described in Chapter IV.

28. Reading Stadia Distances

Except in very special cases, the rod should be held as nearly vertical as possible, regardless of how steeply the telescope is inclined. Stadia conversion tables are computed on the assumption that the rod is vertical, and it is necessary to maintain this condition when the tables are used for certain conversions.

The rod is much more easily read when in direct sunlight, and for this reason the rodman should attempt to turn the rod so that it "catches" the direct light and still presents a sufficiently wide target for the instrumentman.

Always read a full interval if a sufficient length of rod is visible. Any errors made in reading the rod are multiplied by 2 or 4 if the half- or quarter-interval is used.

29. Reduction to Horizontal Distance

The distances represented on the plane table sheet are horizontal distances (Art. 8); that is, they are the bases of right triangles whose hypotenuses are the stadia, or slope, distances (Fig. 22). For inclinations of the telescope less than about 4 degrees, corrections are not ordinarily made, because there is no

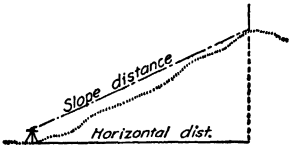


FIG. 22. Relations of slope (stadia) and horizontal distances.

appreciable difference in the lengths of the base and hypotenuse. If the inclination (vertical angle) is greater than 4 degrees, the stadia distance is reduced to the horizontal equivalent before plotting on the sheet.

On the vertical arc of the alidade is a scale marked "H" or "Hor," which is designed for making corrections to horizontal distance (Fig. 23). The graduations are in percentages, and the corrections are made as follows.

If the arc reads 100 at the index when the telescope is level, the graduations represent the percentage of stadia distance equal to the horizontal distance.

EXAMPLE:

Vertical angle = 15°; stadia distance = 1200 feet; correction indicated on arc (for 15°) = 93.3%.

Horizontal distance = 1200 × 0.933 = 1119.6 feet

If the arc reads 0 at the index when the telescope is level, the graduations represent the percentage *reduction* in stadia distance.

EXAMPLE:

Vertical angle = 15°; stadia distance = 1200 feet; correction indicated on arc = 6.7%.

Correction = 1200 × 0.067 = 80.4 feet.
Horizontal distance = 1200 − 80.4 = 1119.6 feet.

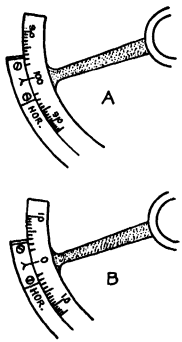


FIG. 23. Vertical arcs showing the horizontal distance correction scales by (A) percentage of stadia distance and (B) percentage correction to be subtracted from the stadia distance.

On some instruments the horizontal correction scale is placed directly above the Beaman stadia arc at the right end of the vertical arc. Others may not have a reduction scale.

A table for the reduction of stadia distances to horizontal distances is given in the Appendix.

30. Traversing

It has been shown how to measure distance by means of stadia; how to reduce sloping ground measurements to their horizontal equivalents, and then to plot the map positions of ground points along rays drawn from a point on the sheet representing the instrument station.

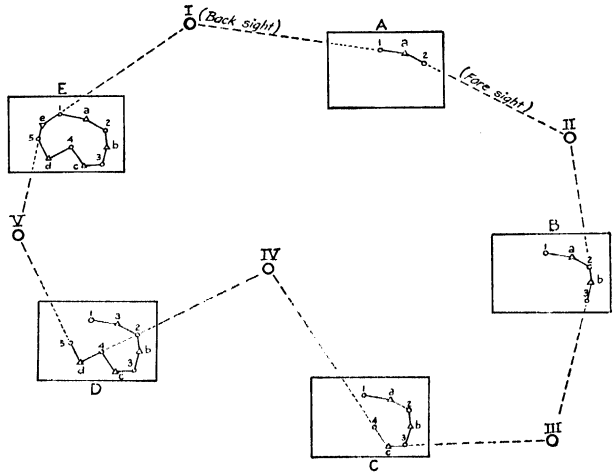


FIG. 24. The development of a plane table stadia traverse

Now if the map position of the *instrument station* is not known, it is determined by a process similar to that outlined above for rod stations.

In Fig. 24 the rectangles, *A*, *B*, *C*, *D*, and *E* are the ground positions of successive instrument stations, and *a*, *b*, *c*, *d*, and *e*, are the corresponding map positions of the stations plotted on the plane table sheet. Likewise, I, II, III, IV, and V are successive rod stations on the ground, and 1, 2, 3, 4, and 5 are the corresponding points on the sheet. Rod stations such as these, used for the purpose of establishing the locations of instrument stations, are called *turning points,* or "T.P.'s."

Turning again to Fig. 24, assume that *A* is the first instrument station and that turning point 1 is the only position plotted on the sheet. To locate instrument station *A* on the sheet, proceed as follows.

Level the plane table and orient it by means of the compass needle. Using the turning point 1 as a pivot for the alidade blade, rotate the instrument until the rod, held on I is brought into the line of sight (the vertical cross wire should appear to bisect the rod). Draw the ray from point 1 toward the instrument station. Read the stadia distance, and, if necessary, compute the horizontal distance, and plot the instrument station *a* on the ray.

The rod is now held at II, and, using *a* as the pivot point, a ray is drawn from *a* toward II, the horizontal distance is determined from a stadia reading, and the map point *2* is plotted.

The instrument is next taken to *B*, the table is oriented as before, and the map position *b* is determined by sighting back on II and using *2* as the pivot point. This procedure is repeated for each new instrument station and turning point. Such a succession of interrelated rod and instrument stations is called a *traverse*, and the procedure outlined above is termed *traversing*.

The observations on new turning points, as from *A* to II, are called *foresights;* while those establishing the positions of new instrument stations, as from I to *A*, are *backsights*.

Evidently, the map location of any rod or instrument station is dependent on all previously established points; so an error in one of the stations affects all succeeding ones. If the traverse has been run without error, the polygon from I to *E* will be exactly represented on the sheet on a reduced scale by the figure from 1 to *e*.

31. Horizontal Closure

A traverse line which eventually is run back to the starting point is called a *closed traverse*, or *loop*. Traverses should always be tied back either into the starting point or some other point that has been accurately plotted on the sheet independently of the traverse.

In Fig. 24, all locations have been made when instrument sta-

tion e has been established on the sheet. However, it is not yet known what total error may exist in the work at this point. This is determined as follows. From instrument station E sight on the rod held on the point I, the starting point. Treat this sight as if it were a new turning point—a foresight. Draw the ray from e and plot the distance obtained from a rod reading. If there are no errors in the traverse in orientations, in the determinations of distances, or in the plotting of the horizontal distances on the sheet, the position of 1, as determined from E, will coincide with the original point. On the other hand, if the position of e is incorrect, as a result of errors somewhere in the traverse, or if the table is misoriented at E, then the closing sight will fall at some distance from 1, as shown by 1′, in Fig. 25. This discrepancy is termed the *closing error.*

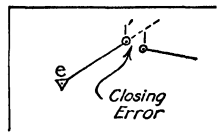


FIG. 25. Horizontal closure error in a stadia traverse.

32. The Starting (Initial) Point

The starting point, such as 1, in Fig. 24, may be plotted arbitrarily at any convenient place on the sheet, unless projection, grid, or section and township lines have been drawn previously. The map position of the initial point should be chosen so that all the contemplated traverse will fall on the sheet. This may be done best by sketching the area covered by the sheet on a small scale index map of the region, and from this determine approximately the position of the starting point relative to the boundaries of the sheet.

When projection or section lines are drafted on the sheet first, it is then necessary to plot the initial point correctly with reference to these lines. More will be said of this in a later chapter.

33. Elevations

The previous articles have dealt with establishing the horizontal positions of ground points on the map. It is ordinarily required that the vertical positions, or elevations, of the stations also be determined. The two, horizontal and vertical, are usually run

concurrently. The starting elevation, as in the case of the initial horizontal position, may be assumed; but the common practice is to start the elevations from some point whose elevation above sea level has been established from a previous survey. A point that is permanently marked on the ground with a bronze tablet, concrete post, or other enduring materials, and whose elevation above sea level has been definitely established, is called a *bench mark,* or B.M. A point whose elevation is established, but which is less permanently marked on the ground, is called a *temporary bench mark,* or T.B.M.

34. Difference in Elevation and H.I.

The difference in elevation (Art. 7) between the instrument and a rod station is spoken of as *plus* or *minus,* depending on whether the point whose elevation is being determined lies above or below, respectively, the point of known elevation. For example, assume that a bench mark has an elevation of 4000 feet. The difference in elevation between the bench mark and a near-by instrument station is found by measurement to be 50 feet, and the bench mark is known to be higher than the instrument station. The elevation of the instrument station is, therefore, 4000 − 50 = 3950 feet, or if the bench mark is lower than the instrument, the elevation of the latter is 4050 feet.

When the elevation of an instrument station is determined with the alidade, it is the elevation of the *center of the instrument* that is obtained, *not* that of the ground under the table. This elevation is called the *height of instrument,* or more commonly, H.I. It is often a matter of some confusion to the beginner that it is only the H.I. elevation, and not the ground elevation of an instrument station, with which the surveyor is concerned. It will be shown later why this is true.

35. Leveling the Telescope

Set up the tripod and plane table, and level the table by means of the circular level on the alidade base. Place the striding level

("12" in Figs. 12 and 13) on the telescope, being careful to press the spring lock pin on the side of the level so that it can be set down easily over the stud on the telescope. Sight the telescope on a stadia rod; and by means of the axis clamp ("16" in Fig. 12) and the right tangent screw "20" bring the bubble exactly to the center of the level tube. Without disturbing the telescope, lift the striding level, turn it end for end, and replace it on the telescope. If the bubble returns to center, it is in adjustment. The line of sight is now level; that is, any point cut by the horizontal cross wire is at the same elevation as the center of the telescope. Evidently, points above the middle wire are higher than the instrument, and those below are lower; so the horizontal cross wire reading on the rod is a direct determination of the *difference* in elevation between the rod station and the center of the instrument. For example, if the level cross wire reads 10.5 feet on the rod, the rod station is 10.5 feet lower elevation than the instrument.

In order to obtain a cross-wire reading with a level telescope, it is required that the rod station be lower than the center of the telescope.

36. Differential Leveling

Differential leveling consists of establishing the elevations of alternating rod and instrument stations by means of level sight lines. This is the most direct and the most accurate way of determining elevations, and should be used wherever possible on traverses. As in the case of traverses, the observations are called backsights and foresights, and the rod stations are called turning points. The procedure is as follows.

In Fig. 26, the plane table is set up at A, and a backsight is taken on the rod held on BM_1, whose elevation is 2500 feet. The level wire reading on the rod is 10.0 feet; therefore, the H.I. at A is $2500 + 10 = 2510$ feet. The rod is next taken to T.P.2. The level wire reading here is 1.5 feet, and since the rod point is lower than the instrument at A, the elevation is $2510.0 - 1.5 = 2508.5$ feet. Now the instrument is moved to station B and a level back-

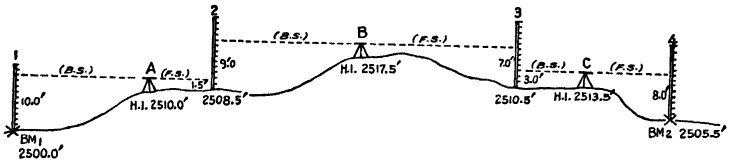


FIG. 26. Differential level line with alternating rod and instrument stations.

sight is taken to T.P.2. This level reading is 9.0 feet; and, as the H.I. is higher than T.P.2, the elevation is 2508.5 + 9.0 = 2517.5 feet.

It is evident from the foregoing that when the elevation of the instrument is determined from a middle wire reading on the rod, the difference in elevation is always added to the elevation of the turning point in order to obtain the elevation of the instrument. Conversely, if the elevation of the turning point is being determined by a level foresight, the difference in elevation is subtracted from the H.I.

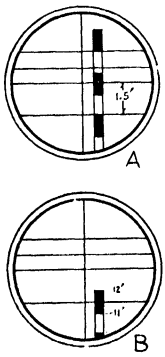


FIG. 27. View through the telescope showing (A) the stadia intercept and (B) the lower wire near the top of the rod with the telescope level.

LOWER WIRE READINGS

When the instrument is set up more than the length of the rod above the rod station, the horizontal cross wire will be above the top of the rod when the telescope is level. However, if the lower wire intersects the rod when the telescope is level, the difference in elevation can be determined.

It has already been shown (Art. 26) that the apparent stadia interval is a function of the distance from the instrument to the rod. Therefore, the intercept on the rod is, in effect, a vertical measurement of the distance between the wires, and this varies only according to the length of the sight. In Fig. 27A, the stadia interval is 3 feet; and the distance between the middle and lower wires is 1.5 feet. When the telescope is leveled, the lower wire cuts the rod at 11.5 feet,

as shown in Fig. 27B. Obviously, the middle wire is 1.5 feet above the lower wire (at this distance); so the height of the level sight line above the rod station, or the difference in elevation, is 11.5 + 1.5 = 13.0 feet. In all lower wire determinations the difference in elevation is the lower wire reading (with the telescope level) plus the stadia half-interval; and the rod station is always below the instrument.

Upper Wire Readings

When the rod is too high for a level middle wire reading, but low enough so that the upper stadia wire is on the rod, the difference in elevation may be computed from the upper wire reading and the half-interval intercept.

In Fig. 28A, the half-interval is shown to be 2.5 feet. After leveling the telescope, the upper wire cuts the rod at 0.7 feet (Fig. 28B). In this case the ground at the rod station is higher than the level (middle) wire; therefore, the rod station is above the H.I. The difference in elevation is 2.5 (the half-interval) − 0.7 = 1.8 feet.

If the elevation of the rod station is not known (as in a foresight), the difference in elevation is *plus,* i.e., it is added to the elevation of the H.I. If the H.I. is not known, the difference in elevation is *minus* and is subtracted from the elevation of the turning point to obtain the H.I.

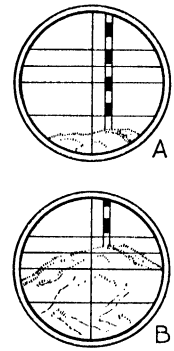


Fig. 28. View through the telescope showing (A) the stadia intercept and (B) the upper wire near the base of the rod with the telescope level.

To recapitulate, Fig. 29 represents the lines of sight projected from the horizontal cross wire and the upper and lower stadia wires, *H, U,* and *L,* respectively. It is evident here that the difference in elevation for sights from each of the three H.I.'s, *A, B,* and *C,* is as follows:

From *A:* Middle wire reading; backsight, plus; foresight, minus.

From *B:* Lower wire reading plus half-interval; backsight, plus; foresight, minus.

From *C:* Half-interval minus upper wire reading (telescope level); backsight, minus; foresight, plus.

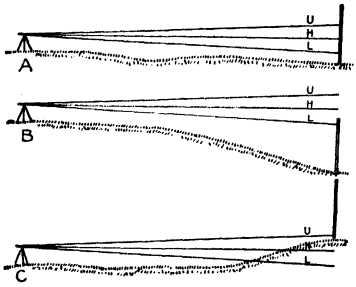


FIG. 29. "Level" wire readings: (A) middle wire, (B) lower wire, and (C) upper wire.

When running levels, the lengths of foresights and backsights should be equalized in order to eliminate the effects of certain instrumental errors.

37. Closure of Level Lines

Level lines are closed back to the starting point or to some other point of known elevation, much the same as are traverses. Figure 26 shows how elevations are carried from station to station as the work progresses. In the figure, the starting point is BM_1, and the elevation of H.I. *A* is determined by the backsight to this point. Elevations for succeeding rod and instrument stations are obtained by alternate foresights and backsights. When the H.I. *C* is established (2513.5 feet) the rod is held at BM_2, whose elevation is known to be 2505.5 feet. The difference in elevation between this point and H.I. *C* is determined by foresight to be 8.0 feet; and the elevation of BM_2, as computed from *C*, is 2505.5 feet. The closure error is 0.0 feet. If, on the other hand, this difference in elevation had been 7.0 feet, the computed elevation of foresight 4 would be 2506.5 feet, and the closure error would be +1.0 foot; or had the difference in elevation been 9.0 feet, the closure error would be −1.0 feet.

38. Indirect (Trigonometric) Leveling

In indirect leveling the difference in elevation is computed from the measured stadia interval and a measurement of the amount of inclination of the telescope, the latter being with reference to a

level line at the elevation of the H.I. The four principal methods for determining the difference in elevation indirectly are by vertical angles, stepping, the Beaman stadia arc, and the gradienter or Stebinger screw.

39. Vertical Angles

The vertical angle arc and arc vernier ("6" in Fig. 12) measure the inclination of the telescope in degrees and minutes of arc. The upper scale (Fig. 30A) is graduated in degrees and half-degrees,

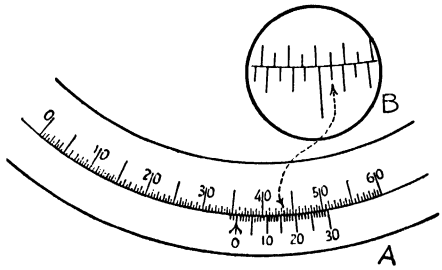


FIG. 30. Vertical angle arc and vernier. (B) magnified vernier.

and the index is the zero line of the lower or vernier scale. The minutes of arc are read on the vernier, graduated from 0′ to 30′. The spacing of the graduations on the upper and lower scales is such that only one pair of the opposing graduations is coincident at one time. In reading the vernier, the coincident graduation is the number of minutes that must be added to the degree (plus) or half-degree (plus) reading of the upper scale. For example, in Fig. 30A, the upper scale reads 35°30′, plus. The coincident graduation on the vernier (Fig. 30B) is 16′. The correct arc reading is, therefore, 35°46′.

When the telescope and the plane table are level, the vertical arc should read 30°00′; or if the index (control) level is used, the effect on the vertical angle is the same as with a level table, even though the table is inclined. The actual vertical angle, then, is the algebraic difference between the arc reading and 30°00′. If the

arc reading is greater than 30°, the sign of the vertical angle is plus, and the telescope is elevated; if the arc reading is smaller than 30°, the sign of the vertical angle is *minus,* and the telescope is depressed.

When only the striding level is used, it cannot be assumed that the *level* reading of the arc will be exactly 30°00′, for this will depend on how nearly level the table happens to be. For this reason, when the striding level only is used, the arc must be read with the telescope trained on the target and again with the telescope in the level position. The actual vertical angle in this case is the algebraic difference between the two arc readings.

Conversion Tables

Vertical angle conversion tables (see Appendix) are used in the computation of difference in elevation from vertical angle observations. These tables are computed on the basis of a stadia interval as determined from a *rod held in a vertical position.* It is therefore essential that the rod be vertical when making observations, no matter how large the angle is. The tables cannot be used in conjunction with horizontal distances or true slope distances, if the vertical angles are large.

Procedure and Computations

In reading the vertical angle it is common practice to set the horizontal cross wire always on the same foot mark on the rod, such as the 5-foot graduation. Some prefer to set the wire on the 5-foot mark for plus angles, and the 10-foot mark for minus angles. It is not important what procedure is used so long as it is followed consistently. The cross-wire reading on the rod is called the *rod correction* and is added to the number computed from the vertical angle when the latter is minus and subtracted from the vertical angle computation when the angle is plus.

The general field procedure is as follows. Read the stadia distance on the rod held in a vertical position. Record the reading in the notebook (Art. 48). Set the horizontal cross wire on the 5-

foot mark on the rod. Read the arc vernier with a hand lens of about 3 diameters magnification. Record the reading. Level the telescope with the striding level, read the vernier, and record. This reading should be very near 30°00′.

EXAMPLE 1 (Plus vertical angle):
Stadia interval 12.7 feet
First arc reading 36°45′
Level arc reading 30°11′
Rod correction −5.0 feet
Vertical angle 36°45′ − 30°11′ = +6°34′

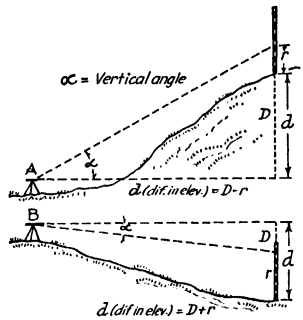


FIG. 31. Vertical angles. (A) plus angle, and (B) minus angle.

Under 6°34′ in the conversion tables (Appendix) is found the factor 11.36. The difference in elevation from the H.I. to the 5-foot mark on the rod is: 12.7 × 11.36 = 144.3 feet. Since the telescope was elevated to a point on the rod through, and above, the ground point at the rod station, the actual difference in elevation between the H.I. and the rod station is: 144.3 − 5.0 = 139.3 feet. (Fig. 31A).

EXAMPLE 2 (Minus vertical angle):
Stadia interval 13.2 feet
First arc reading 24°58′
Level arc reading 30°10′
Rod correction +10.0 feet
Vertical angle 24°58′ − 30°10′ = −5°12′

Conversion factor for 5°12′ is 9.03. The difference in elevation from the H.I. to the 10-foot mark on the rod is 13.2 × 9.03 = 119.2 feet. In Fig. 31B, it is obvious that the difference in elevation has been measured only down to the 10-foot mark on the rod; so the difference in elevation between the H.I. and the ground at the rod station is 119.2 + 10.0 = 129.2 feet.

For backsights, plus vertical angle sights give minus differences in elevation, and minus angles give plus differences in elevation. These conditions are reversed for foresights.

When the arc index or control level is used, the procedure is somewhat different from the above. This level is adjusted so that the vernier always reads 30°00′ when the control bubble is centered, irrespective of the inclination of the table. As a rule, the control level is not quite so precise as the striding level, and so should not be used where great precision is required in the elevations. However, it is much faster than the striding level and is accurate enough for the greater part of plane table work. It is used as follows.

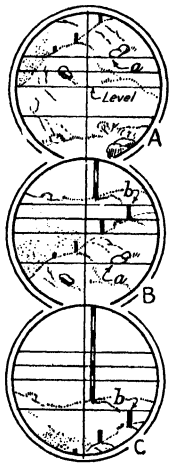


FIG. 32. View through the telescope while employing the step method of determining difference of elevation. A, B, and C represent three successively elevated interval steps.

Read the stadia interval, and set the cross wire on the 5- or 10-foot mark on the rod. Now, by means of the left (arc) tangent screw ("4" in Figs. 12 and 13), bring the control level bubble to the center. Read the arc vernier. The vertical angle is the arc reading minus 30°00′.

40. Stepping Method

This method of measuring the difference in elevation is both fast and accurate, if the relationship between distance to the rod and difference in elevation is such that the line of sight is not too steeply inclined. The measuring is done by counting the number of stadia intervals through which the telescope is depressed or elevated in order to bring the horizontal cross wire from the level position to some point on the rod.

Figure 32 represents a situation where the rod station is above the H.I. The stadia interval is assumed to be 5.0 feet. Level the telescope, and by inspection of the landscape through the telescope, select a small, clearly defined object that is exactly cut by the upper stadia wire, such as the base of the stone *a* in Fig. 32A. Using the right tangent screw, elevate the telescope until the lower wire is brought up to point

a, and select the next point *b* on the ground that is cut by the upper wire (Fig. 32B). Now bring the lower wire up to point *b,* as in Fig. 32C. The horizontal cross wire is now on the rod at the 1.5-foot mark.

The telescope has been elevated through 2 full stadia intervals; but, as the middle wire is 1.5 feet above the ground at the rod station, this amount must be subtracted from the computed difference in elevation. The actual difference in elevation between the H.I. and the ground at the rod station is, therefore, 5.0 (the stadia interval) $\times$ 2 (the number of steps) $-$ 1.5 feet $=$ 8.5 feet.

When the rod is below the H.I., the ground markers are selected along the lower wire, and the upper wire is stepped down successively to these points. The rod correction in this case, as for minus vertical angles, is added to the product of the stadia interval and the number of steps.

In stepping, it makes no difference whether the ground markers are nearer or more distant than the rod station, so long as they are within the range of clear focus of the telescope. Where the sights are very short, the markers must be more nearly the same distance from the instrument as the rod station, because on short sights the range of clear focus, free from parallax (Art. 24), is much shorter than for long sights.

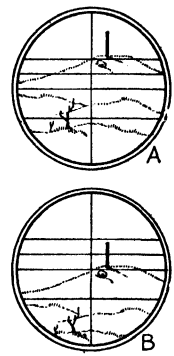


FIG. 33. The half-interval step.

HALF-INTERVAL STEP

Occasionally, it is necessary to use a step of a half-interval, when the distance to the rod is so far that the half-interval exceeds the length of the rod. In Fig. 33A, the final full interval step places the upper wire just below the base of the rod. One more full interval step will put the middle wire above the top of the rod and the lower wire below the base. This difficulty is overcome by stepping the *middle* wire up into the position previously occupied by the upper

wire. The quarter-interval wire then falls on the rod, as shown in Fig. 33B.

EXAMPLE:
Length of rod 12 feet
Full stadia interval 26.8 feet (From quarter-interval stadia reading)
Number of steps 2½
Quarter wire reading on rod 5.2 feet

Since the middle wire is below the base of the rod (Fig. 33B), the rod correction is 6.7 feet (the quarter-interval) minus 5.2, or 1.5 feet.

The difference in elevation $= 2.5 \times 26.8 + 1.5 = 68.5$ feet.

41. Beaman Stadia Arc

The Beaman stadia arc is graduated so that the scale reads 50 when the vernier reads 30°00′ and the telescope is level. Plus angles are indicated by readings above 50, and minus angles by those below 50. Thus, the number of arc divisions through which the telescope is depressed or elevated is obtained by subtracting 50 from the arc setting made with the cross wire on the rod. The graduations of the Beaman arc are shown in Fig. 34.

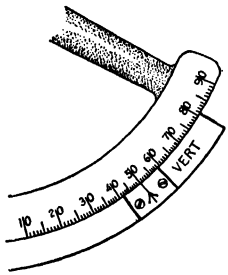


FIG. 34. The Beaman stadia arc.

When only the striding level is used, the procedure is as follows. Read the stadia interval, and record the interval in the notebook. Level the telescope with the axis clamp and the right tangent screw. While viewing the arc with a hand lens (if not provided on the instrument), set the arc index exactly on 50 by means of the index tangent screw on the left side of the alidade. Check the striding level bubble to ascertain that the telescope has not been disturbed while setting the index. Loosen the axis clamp and elevate or depress the telescope until the horizontal cross wire is approximately in the middle of the rod, and tighten the clamp. Using the *right* tangent screw, set the arc index exactly on the nearest arc graduation. Read the arc and record the reading.

EXAMPLE 1 (Rod station above the instrument):

Observed stadia interval	12.3 feet
Beaman arc 61 − 50 =	+11 divisions
Rod correction	6.7 feet (cross-wire reading)

Difference in elevation = 12.3 × 11 − 6.7 = 128.6 feet

EXAMPLE 2 (Rod station below the instrument):

Observed stadia interval	14.6 feet
Beaman arc 35 − 50 =	−15 divisions
Rod correction	7.8 feet (cross-wire reading)

Difference in elevation = 14.6 × 15 + 7.8 = 226.8.

If the instrument is provided with an index control level, the Beaman method becomes much simpler and faster:

Read the stadia distance and record in the notebook. Set the cross wire in the middle region of the rod. With the *left* (index) tangent screw center the control bubble. Now, using the *right* tangent screw, set the arc index on the nearest arc graduation. Read the cross wire on the rod.

The computations of difference in elevation are the same as given in the examples above.

Inasmuch as the striding level is usually of somewhat higher precision than the control level, it is better to use the striding level method on traverses where very accurate elevations are required.

42. The Gradienter (Stebinger) Screw

The gradienter screw replaces, or is a part of, the right tangent screw, as shown in Figure 35. Referring to this figure, the drum *b* is graduated on the periphery from 1 to 100, each division being $\frac{1}{100}$ of a revolution of the drum. The scale *e* measures the number of revolutions of the drum, the spaces between the graduations being exactly equal to the distances between the threads of the tangent screw. When the gradienter screw is properly adjusted one

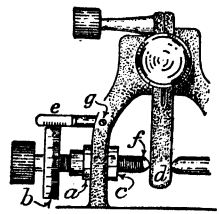


FIG. 35. The gradienter or Stebinger screw.

revolution of the drum elevates or depresses the telescope one stadia interval; therefore, each division on the periphery of the drum represents 0.01 interval.

Theoretically, the gradienter is a more precise measurement than the vertical angle, because the smallest division on the vernier (01 minute of arc) is approximately 0.03 interval, whereas, the gradienter reads directly to 0.01 interval. However, the actual precision obtained with the gradienter depends largely on the amount of mechanical wear in the instrument, exact adjustment of the screw, the precision with which the threads of the tangent screw are cut, and extreme care in operation. The measurements of difference in elevation become less accurate as the elevation or depression of the telescope is increased. For this reason, the gradienter should not be used for measuring differences in elevation where the inclination of the telescope exceeds 3 or 4 intervals or revolutions of the drum.

Even though the gradienter has been adjusted so that one revolution of the drum equals one interval of elevation, this condition will obtain only so long as the axis arm *d* (Fig. 35) is not moved far from the vertical position. As the arm moves from the vertical, the gradienter factor increases, and may be as large as 1.03 when the arm is in an extreme position.

When using the gradienter screw, it is highly important always to turn the screw to the right, i.e., with a continuous thrust motion, so that there will be no back-lash in the instrument during observations. If the rod is above the H.I., bring the striding level bubble to center with a right-hand motion, read the drum, and then proceed with a right motion of the screw until the cross wire is set on the desired graduation on the rod. When the rod is below the H.I., first set the cross wire on an even foot mark on the rod, for example, the 10-foot mark, with a thrust motion of the screw, read the drum, and then carefully bring the bubble to center with a continuous thrust motion of the screw. This procedure reduces the possibility for back-lash.

As a rule, it is easier and faster to use the step method than the

gradienter; and if ground markers are numerous and clearly defined, the step method is probably more accurate, as the measurements do not depend on precise mechanical operation of the instrument.

For determining difference in elevation, the gradienter screw is most useful when the stadia wires are but a fraction of an interval from the top or base of the rod with the telescope level, particularly if there are no convenient markers for stepping. The following examples will serve to illustrate two commonly used methods.

EXAMPLE 1 (Rod station above the H.I.):

Observed stadia interval 8.4 ft.
Gradienter drum (telescope level) .26
Gradienter drum (cross wire on 3-foot mark) .93
Difference in readings $.93 - .26 = .67$
Difference in elevation $= 8.4 \times .67 - 3.0 = 2.6$ feet.

This method assumes that one revolution of the drum moves the telescope through exactly one interval.

EXAMPLE 2 (Rod station above the H.I.):

Observed stadia interval 5.6 feet.

Level the telescope and read the gradienter drum. Using the right tangent screw, elevate the telescope until the upper wire is on the 1-foot

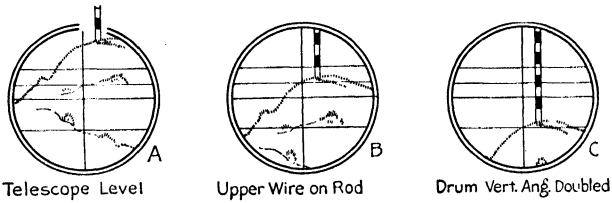


FIG. 36. Successive steps in the operation of the gradienter screw as discussed in the text in method 2.

graduation of the rod. (If the base is clearly defined, the rod correction can be eliminated by setting the wire on the base.) Read the drum again with the telescope in this position (Fig. 36B). Assume the difference in the two drum readings is 62 drum divisions. Now turn the drum

through 62 more divisions, and read the rod again. Assume the difference between the two rod readings is 3.7 feet (Fig. 36B and 36C). This is a measure of the vertical distance through which the upper wire has been moved from the original position with the telescope level up to the 1-foot mark on the rod. It is apparent, then, that the upper wire was 3.7 — 1.0 ft., or 2.7 feet, below the point.

The difference in elevation = 2.8 (the half-interval)
+3.7 — 1.0 = 5.5 feet.

This method differs from Example 1 in that it is in no way dependent on the adjustment of the drum. The distance of the upper wire (for plus angles) below the point is first obtained in terms of drum graduations and is then translated into feet difference in elevation by turning off the same number of divisions where the movement of the wire can be observed on the rod. Since these measurements are made with reference to the upper wire, it is necessary to convert the results thus obtained to the middle wire equivalent by adding the half-interval.

If the upper wire is more than the length of the rod below the point, obviously the gradienter equivalent cannot be measured directly on the rod. In this case, measure the equivalent for half the number of divisions turned, and multiply the result by 2.

For minus angles the procedure is the same as given above, except that measurements are made from the lower wire, and the rod correction (read near, or at, the top of the rod) is added in the computations for difference in elevation.

43. Control for Sketching

The foregoing articles have described various methods for determining the map positions and elevations for instrument and rod stations on traverses. It is evident that from the plotted position of an instrument station whose H.I. has been established, the map position and elevation of any point within range of the telescope can be determined by foresighting on a rod held on the point. Rod stations which are established for the purpose of locating map features and are not an integral part of the traverse are called *side*

shots. Side shots, together with traverse stations useful in sketching the map, are called, collectively, the *sketching control* of the map.

In plane table work it is customary to sketch map features by means of side shots at the time the traverses are being run. When a new instrument station is established, side shots are taken on the features whose locations or elevations are required. In order to avoid needless expenditure of time and effort, the area in the immediate vicinity should be studied critically so that sufficient control for accurate sketching can be obtained with a minimum number of located points. Wherever convenient, instrument stations and turning points should be located so that they furnish additional control for sketching. It is sometimes desirable to run a short traverse off the main (closed) traverse in order to get the instrument within range of necessary map features or to gain better vantage points for sketching. Such lines, consisting of two or three instrument stations, not tied back into the main traverse, are called *spur traverses*.

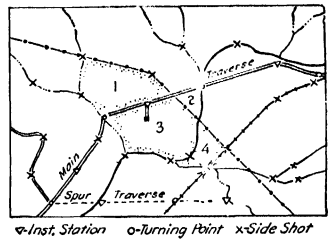


FIG. 37. Map showing traverse along a highway with side shots used as control for sketching other map details.

It is not the number of located points within a given area, but rather the distribution of the points, that determines the accuracy of the sketching. In Fig. 37, the main traverse is run along a principal highway. The traverse stations not only accurately locate the highway, but also all the important features nearby or crossing it, such as streams and junctions. Side shots are taken on the confluences of streams and the main bends in linear features. The stations on the spur traverse are chosen so that they afford control for two streams and the railroad. The importance of learning to place traverse as well as side locations so that they serve multiple purposes can hardly be overemphasized.

In field sketching, the best procedure is to first locate the linear

features accurately on the sheet by means of well-placed side shots and traverse points. This will break down the larger area into a number of segments, such as 1, 2, 3, and 4 in the figure. After the map area has thus been subdivided, the sketching of further map detail by inspection and comparison of relative positions becomes a simple operation.

44. Intersected Points

It has been shown (Art. 20) that the map position of a ground point sighted through the telescope lies somewhere on the ray drawn from the plotted position of the instrument station. Therefore, if rays are drawn to the same ground point from the respective plotted positions of two different instrument stations, the two rays will intersect at a point, which is the map location of the point sighted.

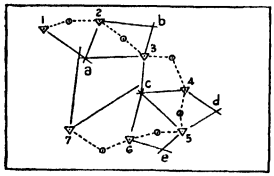


FIG. 38. Points located by the intersections of rays drawn from traverse stations.

In Fig. 38, a ray is drawn from station 1 toward a point on the ground, which is visible also from other locations along the traverse. The traverse is run to station 2, from which the second ray is drawn toward the point. This line intersects the first at *a*. From station 2 a ray is drawn to point *b*. The traverse is now continued to station 3, from where it is possible to draw rays to both *a* and *b*, and to a new point *c*. The map location of *a* is verified by the third ray. This procedure is repeated from each successive station. After a two-ray intersection is obtained, all additional rays drawn to the same point serve to check not only the map position of the point sighted, but also the accuracy of the traverse. Thus, when station 7 is located on the map, rays are drawn to *a* and *c*. Since these points have been checked by three- and four-ray intersections, station 7 is apparently in error, either in the orientation of the table at station 7, or in the plotted position of the station. When such an error is discovered in a traverse, it should be cor-

rected before the traverse is continued. In the case illustrated, first check the orientation of the table. If this fails to reveal any error, the traverse must be rerun from the last station checked, which, here, would be instrument station 6.

The difference in elevation between the H.I. and the intersected point is measured by vertical angle (Art. 39). The distance to the point is obtained by measuring the distance on the sheet with the plotting scale. Since the measurement thus obtained is the horizontal distance, trigonometric tables (not stadia tables) are used in the computations.

EXAMPLE:

Scaled horizontal distance	2650 feet
Vertical angle	+5°20′

In the table of natural tangents (Appendix) opposite the angle 5°20′ is found the factor .0934.

The difference in elevation = 2650 × .0934 = 247.4 feet.

The elevation of the intersected point should be computed from at least two different instrument stations, and preferably three. As in the case of horizontal positions, the elevations of the traverse stations are checked by repeated observations on the same (intersected) point by the method outlined above.

If any horizontal distance used in the computation of difference in elevation exceeds 1 mile, a correction for earth curvature and refraction should be made.

45. Marking and Reoccupation of Stations

Instrument stations and turning points should be marked on the ground so they may be found again at later dates. After a traverse or level line has been closed and, if necessary, adjusted, the stations may be used as starting points for new traverses or points into which new traverses can be tied. When reoccupying an old station, it is advisable to take a backsight to one of the old turning points to establish the new H.I.; for unless the tripod is set

up exactly as before, the new H.I. will differ from the old. It is good practice to measure the height of the instrument above the ground with the stadia rod at all new instrument stations and record the height in the notebook for future reference in case of reoccupation of the point.

46. Standard Map Symbols

As a result of many years of use by our larger government surveys, the symbols for all the features commonly shown on maps have become standardized and have been almost universally adopted by other organizations engaged in the construction of maps. For this reason, it is urged that a symbol never be invented for use on the plane table sheet if there is a standard symbol which will adequately represent the feature. Standard symbols for all common features, such as bridges, railroads, bench marks, section corners and boundary marks, should be memorized so that they can be used on the plane table sheet without the necessity of referring to printed examples, as shown in this book in the chapter on map drafting. In preparing for field work, the beginner should practice drawing the symbols freehand.

47. Station Numbers

Some types of surveys require that the instrument stations and all rod stations be numbered on the sheets and in the notebook. Various systems have been devised for showing on the sheet from which instrument station a given group of rod stations has been established. While in some instances such a system might be advantageous, it is usually more cumbersome to use and more complicated than simple, consecutive numbering, without index letters or subscripts. The U.S. Geological Survey uses no station numbers on the plane table sheet in topographic work. Other organizations number only the traverse points. Still others number all stations in consecutive order, without regard to the status of the station—instrument, T.P., side shot, etc. This is probably the best method because of the simplicity in both using it and in later checking.

When consecutive numbers are used, the instrument turning points, and side shots can be differentiated on the sheet by symbols, such as triangles, circles, and small crosses (Fig. 37). Intersected points may be identified by lower case letters of the alphabet and triangulation points (Chapter III) by capital letters or geographic names.

48. Field Notes

Field notes should be made with a sharp pencil of hardness 4H to 6H; for notes made with softer ones are liable to blur and become illegible after the book has been used in the field for a few weeks. Figures should be fairly small so that enough space is left between the lines for corrections. Various notes and remarks should be made in Reinhardt slant letters rather than longhand script, as there is less chance of misinterpretation if the notes are later used by someone else. The book must be protected against loss or damage, for it contains the information necessary for compilation of the final map. The loss of one page of notes might require several days of extra work in the field rerunning level lines or regaining other data.

The following instructions are not based on the preferences of the author, but rather, on principles well established by the engineering professions through many years of experience. Much more could be written on the keeping of good field notes, but if the ten items here discussed are put into practice, no difficulties should be encountered.

(1) Record instrumental observations exactly as made, without even the simplest mental calculations. For example, record a quarter-interval reading as 4×8.3, the actual rod reading (not 3320 feet, the computed distance); or a half-interval reading as 2×9.1 (not 18.2). The figures recorded for vertical angles and Beaman arc should be the actual figures read, not the differences in readings. For Beaman arc record 46, 73, etc. (not -4, $+23$).

(2) Record the date of each day worked in the field and general weather or other conditions affecting the accuracy of the

observations and the working efficiency of the party. It often happens that several days, or even weeks, elapse before a traverse or level line can be closed, and if a large error is discovered when the line is closed, a daily record of working conditions may be of considerable value in selecting portions of the work where mistakes were most likely to have been made.

(3) Record each observation immediately after it is made. A small amount of time might be saved by making several observations, such as stadia interval and vertical angle, and then recording all at one time; but the possibility for transposition of figures and other mistakes more than justifies the slightly slower but safer procedure.

(4) Record observations in such a way that the field method used is evident from the notes. For example, a difference in elevation measured by stepping, with the gradienter screw, or with the Beaman stadia arc, all use the stadia interval as the basis of measurement and can be recorded in identical ways; but it is better to record each method in a different manner, somewhat as follows:

Beaman Arc	*Gradienter*	*Steps*
51	G + 38	U 1
48	G − 12	D 2
47	G − 18	D 3

(5) Record as supplementary notes any changes in the routine or interruptions of the field work, such as "Striding level adjusted," "Rod elevated 3 ft.," "Rod in shadow, indistinct," or similar statements which may aid later in evaluation of certain portions of the work if discrepancies appear. Small freehand diagrams with dimensions written in are helpful in showing local ground conditions.

(6) The lines on the pages of most vertical angle and Beaman record books are numbered. By numbering the pages also, cross references are easily made. Thus, if current work is being recorded on page 40, but a backsight is taken to an old T.P. re-

corded on line 6 of page 15, the reference on page 40 is written "B.S. on P15 L6." This simple reference accurately identifies the T.P. and shows exactly where it may be found in the book.

(7) Descriptions of bench marks and other permanent marks should appear in the notes adjacent to the instrumental notes relating to them. Such descriptions should be made on the following general plan, always starting with the broad geographic position and ending with an intimate description of the mark itself.

State, County, or Parish
Mileage and direction from a near-by town or other well-known landmark.
Township and Range (section and quarter-section when known).
Footage and directions from two or three easily recognized local landmarks, not over a few hundred feet distant.
Complete description of the mark, such as "Granite boulder, 3 ft. diam., half buried, chiseled cross on top," "2-inch iron pipe, 1 foot above the ground," etc.
Finally, the actual part of the mark on which the observations were made: "Elev. top of pipe," etc.

(8) If several books are used in the course of a survey, or if the notes in one book pertain to more than one plane table sheet, number both the books and the sheets and indicate on both where the pertinent information can be found on the other. For references of this kind, together with a general index, leave a few blank pages in the front of the book when the field notes are begun. Below is an example:

Notebook No. 5

Plane table sheet No. 3—pp. 6–21, 28–42
" " " " 2 " 22–27
" " " " 4 " 43–50

(9) In vertical angle record books, the backs of the ruled pages are subdivided into small rectangles, which are numbered to correspond to the numbers on the lines. These rectangles are provided

P. 12 (2)

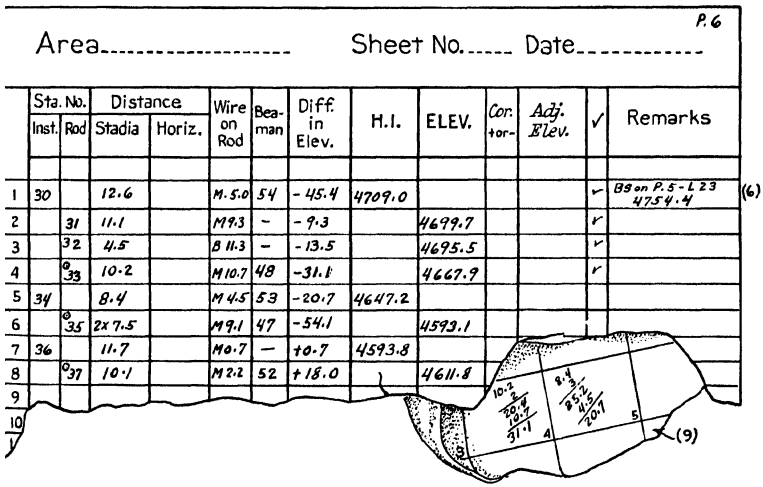
Map Area *Pine River, Colo.* Sheet No. *3* Date *10/22-42 Clear, cold.*

	Sta. No. Inst	Sta. No. Rod	Distance Stadia	Distance Horiz.	Wire on Rod	Vertical Angle Level	Vertical Angle + or −	Vertical Angle Diff.	Diff. in Elev.	H.I.	ELEV.	Cor. + or −	Adjusted Elev.	✓	Remarks
						Inst. – J.L. Condor, Recorder – T.N. Williams, Rod – R.M. Brown									
1	1		10.2		M 11.6				+ 11.6	5654.4				✓	B.S. on U.S.G.S. B.M. SE 1/4
2		2	15.6		B 10.1				− 17.9		5636.5			✓	Sec. 14-T10N-R.35W
(1) 3		3	2×10.1		T 1.5				+ 8.6		5643.2			✓	B.M. Elev. 5642.8
4		⊙4	10.5		M 3.4				− 3.4		5651.0			✓	
5	5		12.6		M 11.3				+ 11.3	5662.3				✓	
6		⊙6	9.3		M 4.5				− 4.5		5657.8			✓	
7	7		12.4		M 10.2				+ 10.2	5668.0				✓	
8		A		1530		30° 10′	37° 35′	+ 7° 25′	+ 199.2		5867.2	-.2	5867.0	✓	El. top church steeple
9		⊙8	9.7		M 8.3				− 8.3		5659.7			✓	[cor. Sec 31-12-4
(7) 10		×9	5.3		M 6.1				− 6.1		5661.9			✓	T. B. M. top pipe, NE
(6) 11	10		11.5		M 8.7				+ 8.7	~~5668.4~~ ~~5660.6~~				✓	B.S. on P12-L9
12		A		1910		30° 05′	36° 01′	+ 5° 56′	+ 198.4		5866.8	+.2	5867.0	✓	Top church steeple
13		⊙11	4.2		M 12.1				− 12.1		5656.3			✓	
(4) 14		12	10.6		M 5.0	Beaman 56		+ 6	+ 58.6					✓	
(4) 15		13	9.4		M 9.1		Step	D 2	− 28.9					✓	Intercept posib. 4x9.3 (5)
(4) 16		14	4x9.2		M 1.0		G	+ .71	+ 25.1					✓	Wrong elev.
(10) 17	~~15~~		~~11.7~~		~~M 7.1~~				~~+ 7.1~~	~~5619.6~~					BS on P2-L3 ~~5610.5~~
(10) 18		~~16~~	~~12.3~~		~~M 2.1~~				~~− 2.1~~		~~5615.5~~				5618.9
19	15		11.7		M. 7.1				+ 7.1	5626.0				✓	
20		16	12.3		M. 2.1				− 2.1		5623.9			✓	Checked to here by R.M.B.

FIG. 39. Page from a vertical angle record book.

for computing difference in elevations, curvature and refraction, etc. Use the rectangle having the same number as the line on which the instrumental data are recorded, as this greatly facilitates the checking of the notes.

(10) Field notes should be checked at least every day, and if opportunities arise during the day, they should be checked then.



P. 6

Area_____ Sheet No._____ Date_____

	Sta. No.		Distance		Wire on Rod	Beaman	Diff. in Elev.	H.I.	ELEV.	Cor. +or-	Adj. Elev.	✓	Remarks
	Inst.	Rod	Stadia	Horiz.									
1	30		12.6		M. 5.0	54	- 45.4	4709.0				✓	BS on P. 5 - L 23 4754.4 (6)
2		31	11.1		M 9.3	–	- 9.3		4699.7			✓	
3		32	4.5		B 11.3	–	- 13.5		4695.5			✓	
4		33	10.2		M 10.7	48	-31.1		4667.9			✓	
5	34		8.4		M 4.5	53	-20.7	4647.2					
6		35	2x 7.5		M 9.1	47	-54.1		4593.1				
7	36		11.7		M 0.7	–	+0.7	4593.8					
8		37	10.1		M 2.2	52	+18.0		4611.8				
9													
10													

(9)

FIG. 40. Page from a Beaman arc record book illustrating the use of numbered rectangles for computations.

It must be remembered that once an error is introduced into the work, it remains and affects all points established thereafter; therefore, it is to the advantage of the responsible members of the party to discover the errors as early as possible. Whenever possible, the checking should be done by one of the party who has had no part in computing the original notes. In any case, the original computations should not be referred to except where apparent errors have been discovered. The checker should never erase or otherwise alter the figures made by the instrumentman in the original computations.

When errors (or apparent errors) in recorded data have been found, *do not erase the original recordings.* Draw a thin line through the erroneous notes, without destroying their legibility, and write the corrected notes immediately above or below. This is a policy almost universally practiced by engineers for two principal reasons: First, it is not always a certainty that the original notes are in error, even though they appear to be. Later work may show that the supposed error in data was actually an error in computations, or that the error was actually in an unsuspected part of the work. Second, if the original notes are entirely free from alterations of any kind, there are poor grounds for suspecting that the notes have been falsely altered or "faked" to show satisfactory ties.

Figures 39 and 40 are examples of field notes for vertical angles and Beaman arc, respectively. The numbers in parentheses refer specifically to the principles discussed in the foregoing numbered paragraphs.

49. Suggestions for Rodmen

The accuracy and progress of the plane table party depend to a very large degree on the manner in which the rodman carries out his part of the work. In topographic, hydrographic, and other types of mapping, it has been shown statistically that the productivity of the field party can be increased as much as 20 or 30 per cent by simply improving the effectiveness of the rodmen. It is to the advantage of both the instrumentman and the rodman that the latter learn as quickly and thoroughly as possible the general plan of procedure in the survey and the details of his portion of the work. When two rodmen are used, it is a good plan to allow one to remain at the table while the other is sent out to give a series of shots. In this way the rodman at the table can quickly learn what is expected of him while rodding, and can better understand the delays that result from errors in judgment on the part of the rodman.

The following suggestions are applicable in practically all types of surveys.

(1) Hold the rod in a vertical position. This may be done by "balancing" the rod between the hands until there is little tendency for it to lean or fall (Fig. 41).

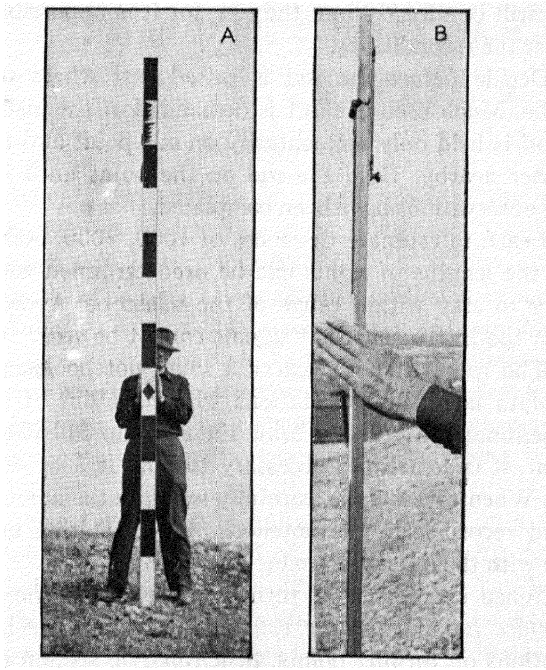


FIG. 41. A. Correct stance for holding the rod vertical. B. Balancing the rod between the hands.

(2) Be sure that *all* of the rod is visible to the instrumentman In brushy or hilly country the lower four or five feet of the roc may be hidden from view of the instrumentman, yet the latter b entirely in sight from the rodman's point of view. The rodman should check questionable locations by sighting the instrumentman from a position close to the ground.

(3) Whenever possible keep the face of the rod in full sunlight. This will sometimes necessitate turning the rod so that the face is not normal to the line of sight; but an oblique view in sunlight is better than a full view in shadow. An effort should be made to avoid setting up the rod where the instrumentman's line of sight will be directly into the sun, for it is impossible to read a rod under these conditions.

(4) Decide *before* the rod is raised just where the location should be. Much useless effort is demanded of the instrumentman if the rod is held only momentarily on one point and then shifted to another nearby. Hold the rod on the point until it is certain that the observations have been completed.

(5) Learn to estimate distances of 1000, 2000, 3000 feet, etc., so that the lengths of sights can be predetermined with sufficient accuracy to stay within range of the telescope. Avoid getting so far from the instrument that signals cannot be seen.

(6) The rodman should carry a small notebook and pencil to record data near the rod stations which are out of sight of the instrumentman. In order to bring the rod into full view of the instrument, it is sometimes necessary to hold it 3 or 4 feet off the ground. When this is done, carefully measure the amount of elevation and record it in the notebook. Check station numbers frequently with the instrumentman.

(7) When establishing a turning point, signal the instrumentman *before* holding up the rod on the point. The instrumental observations on turning points, bench marks, section corners, and all ties are usually made with considerably more refinement than for ordinary side shots; so it is important that the instrumentman be shown by signal when a shot such as the above is being given.

(8) Keep in the notebook a copy of the signals agreed upon with the instrumentman. Some signals will be used rarely, but when unforeseen situations arise in the field, it is convenient to have at hand a means of signaling the instrumentman.

CHAPTER III

Plane Table Triangulation

50. Remarks

The triangulation method of plane table surveying is best adapted in regions where long sight lines from many different points are possible. Where topographic relief is very low, it is essential that there be little tall vegetation or that it be sparsely distributed. In hilly country, points can usually be found high enough above the tops of trees, brush, and other obstructions. Occasionally, it is necessary to construct wooden towers a few feet high so that the instrument can be elevated above the near-by obstructions. But this is not done in plane table surveying if it can be avoided.

Where the country is at all suitable, triangulation is the fastest and the most accurate method of establishing horizontal control over large map areas. It is the only method that can be satisfactorily carried on by the instrumentman alone, after a small amount of initial work, requiring two or three men, has been completed. The actual operations and principles of plane table triangulation are simpler than traversing. There are fewer sources of error, though somewhat greater care and skill may be required. Errors in triangulation tend to be cumulative, while those in compass traversing are more likely to be compensating. Because of the longer sight lines, instrumental measurements must be made with considerably more refinement than for ordinary needle traverses. This is particularly true for vertical arc and gradienter measurements.

51. Reconnaissance Work

Before any instrumental work is begun, the map area should be examined in some detail to determine the best locations for the prospective stations, and to select the best site for the base line (Art. 54). Station signals are erected at this time, and, if necessary, brush, trees, etc., are cleared away from the area immediately around the points so that clear intervisibility between the points is assured. Except in very large-scale work it is not necessary to set up the tripod directly over the triangulation point; therefore, the signals should be so placed that sufficient space remains to set the tripod closely adjacent to any side.

If small-scale maps of the region are available, they should be used during the reconnaissance work for the purpose of plotting the approximate locations of stations, the best routes to them, and other information which will be helpful to the instrumentman when the surveying starts. By using a sketch map for this part of the work, a better distribution of stations can be attained.

52. Construction of Station Signals

Triangulation stations are always located in open places or on high points, and for this reason the signals are subjected to strong winds for periods of several weeks or even months. It is, therefore, necessary to construct the signals so they can withstand the elements for long periods without being blown down or otherwise disturbed. In planning the construction of the signals, some consideration should be given to the materials conveniently near the stations, the lengths of sights anticipated, the length of time that they will be needed, and the general coloration of the landscape that will be the background for the target. When all these factors are taken into account, it may be found that several different kinds of signals should be used in various parts of the area. As a matter of fact, it is good practice to vary the signals or targets enough that the different ones can be identified by some distinguishing character; for when two stations lie nearly in the same line of sight

from the instrument, it may be difficult for the instrumentman to determine just which station he is sighting.

In wooded country, towers or tripods can easily be constructed from poles which will be near at hand. These should not be any higher than necessary to place the target in full view of the other stations. Such a tripod is shown in Fig. 42A. If a single pole is erected, it should be firmly planted in the ground, and supported by three wire guys, attached near the top of the pole and to "deadmen" or stakes at some distance from the base (Fig. 42B). Where the ground is so hard or rocky that the pole cannot be set deeply, particular attention should be given to the guy wires, which will tend to slacken and finally break as a result of wind action. The guys will remain taut if a stone weighing several pounds is suspended from one or more of them, as shown in the figure. The wires should be tied in links at the pole, "deadman," and where the weight is suspended, so that constant swaying of the pole in the wind will not tend to twist the wires off.

In barren, rocky country, it is customary to build stone cairns on the stations. These range from 3 feet to 8 feet in height, depending on the local requirements and the availability of materials. In building the cairn lay the stones so that the spaces in each layer offset those in the layer below. If a target, such as a flag, is to be placed on the top, the stones in the upper 2 or 3 feet should be laid in tightly around the pole (Fig. 42C).

As will be shown later, where the distances between triangulation stations are relatively short, it is necessary to set up the instrument directly under the station signal; otherwise, serious errors in orientation will be introduced in the work. In extensive and detailed surveys, such as those conducted by the Topographic Branch of the U.S. Geological Survey, strong towers capable of supporting the instruments and observer are constructed on the stations. The tripod is set up on a platform a few feet above the ground. The station signal itself is sometimes built large enough that the plane table may be set up between the supporting legs. Towers of these types are costly in both time and materials and

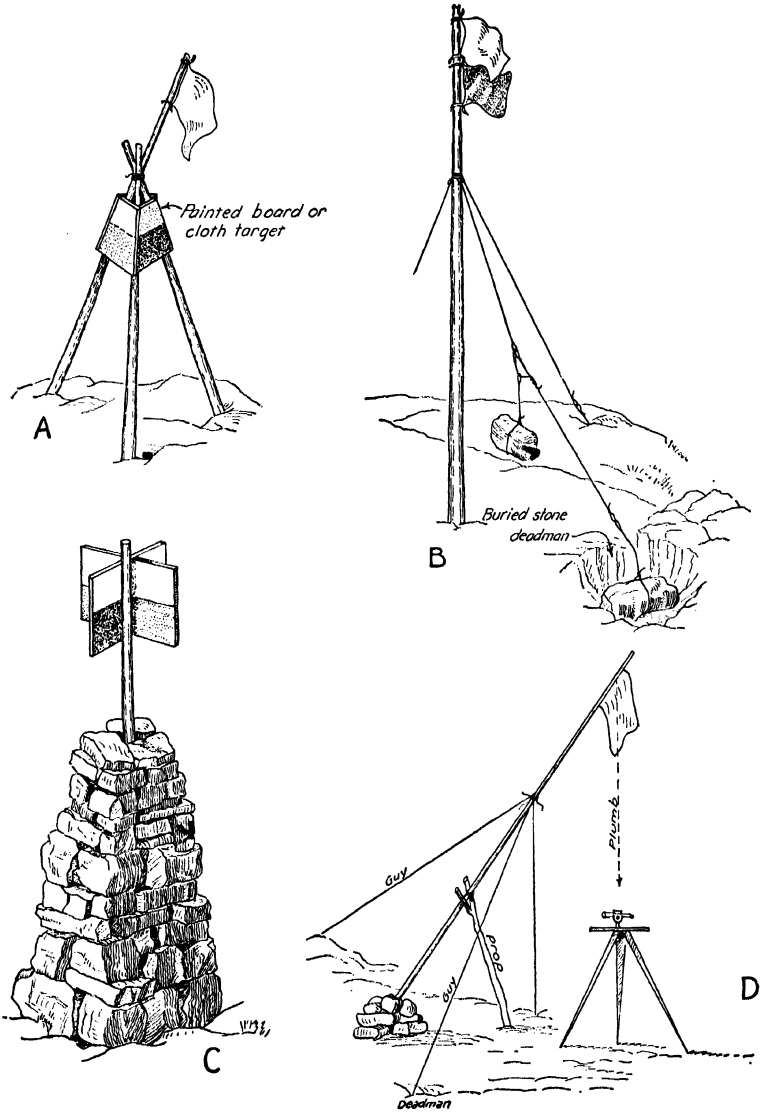


FIG. 42. Various types of triangulation signals constructed from materials near at hand.

in some cases are difficult to construct because of the problems of transporting the necessary materials. The simpler structures can be torn down for the purpose of occupying the station and can then be rebuilt when the observations are completed.

In large classes of surveying it is sometimes desirable for different groups to occupy several stations of the net simultaneously. This procedure makes it necessary for the different parties to make observations on stations being occupied by others. Obviously, the signals cannot be removed without interfering with these observations. Professor Warner, of the University of Colorado, has devised a signal which meets the necessary requirements of the work, yet is easy and economical to construct. It is shown in Fig. 42D. The leaning pole with the signal flag tied at the end permits setting the tripod on the correct ground point without disturbing observations being made from other stations. The pole must be firmly set in the ground and strongly guyed to deadmen. A forked branch from a tree or similar prop, as shown in the figure, adds much to the stability of the structure.

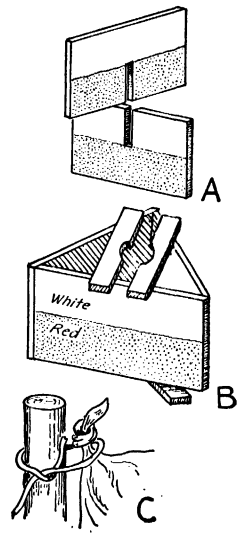


FIG. 43. Details of signal construction.

There are several types of simple targets that are satisfactory for plane table work where sight lines will not be more than a very few miles in length. Figure 43A shows one commonly used, which consists of two boards, painted red and white, and nailed to the pole at right angles to each other. Another similar board target is shown in Fig. 43B.

Cloth flags are the most convenient to carry and use, but have the disadvantage of presenting too small a mark to the instrument if there is no wind or if the wind happens to be blowing parallel to the line of sight when observations are being made. The usual practice is to use two flags of contrasting

colors, attached to the pole one above the other. Bright red, orange-yellow, and white are probably the best colors for the purpose. Inexpensive bunting or muslin is durable enough for most work. Unless the cloth is properly tied to the pole it will soon wear out and be blown away. Tie knots in the two corners adjacent to the pole and bind the flag to the pole with strips of cloth or very soft cotton cord, as shown in Fig. 43C. The dimensions of the flags have to be gaged according to the distances to the other stations from which observations will be made. Other factors, such as the clarity of the atmosphere, coloration of the background, and magnification of the alidade telescope affect the visibility. Using an alidade of 16- to 18-power under ordinary atmospheric conditions, a flag 1½ feet by 2 feet should be clearly visible up to about 4 miles; and one 3 feet by 4 feet, up to about 15 miles. Flags larger than this are not very satisfactory under most field conditions because they tend to wrap around the pole and thus become ineffective as targets.

53. Principle of Triangulation

The principle of triangulation is much the same as for intersected points, described in Art. 44, where it has been shown that the map locations of ground points are determined by the intersection of two or more rays drawn toward the point from two different instrument stations. In plane table triangulation only two stations (at the beginning) are located by direct measurement, all others thereafter being located on the sheet by the intersections of rays. The line of measurement between the first two stations is called the *base line.* The principal functions of the base line are to predetermine the scale of the map and the orientation of the sheet. As will be shown shortly, all future locations, orientations, and elevations are dependent on the base line. Errors made in measuring the base line on the ground or in plotting the scaled distance on the sheet result in a corresponding error in the scale of the map.

The *relative horizontal positions* of triangulation stations, called collectively triangulation *net* or *system,* are unaffected by errors

made in base line measurement or plotting. Such an error affects only the *scale* of the net as a whole. Thus, any length of line could be laid off on the sheet to represent the distance between the two ends of the base line on the ground, and the resulting horizontal interrelationships of stations in the system would be correct, though the *scale* of the system would not be known. However, since the elevations of the stations are computed from vertical angles and distances scaled on the sheet, it is necessary that the scale of the map be precisely determined; and for this reason, the measurement and plotting of the base line should be done with extreme care.

54. Measurement of the Base Line

Wherever possible, the base line should be measured on open, flat ground with the two ends so situated that they are intervisible from each other and from other points in the vicinity that will be used as triangulation stations. The location of the base line within the map area will be somewhat dependent on suitable sites; but if the area is large, the base line should be located somewhere in the central part so that the net will not have to be extended too far in any one direction. If the map area is greatly elongated, the base line should be oriented as nearly parallel as possible to the longer dimension of the area. The reason for this is explained later.

In certain types of very rough work the base line may be measured by means of stadia; or if the terrain is extremely rough, the stadia may be the best means of measurement. Under ordinary conditions, however, it is better to make these very important measurements with a steel tape or chain.

55. Chaining

Methods of chaining commonly used in connection with plane table work are described in the following paragraphs.

Level Ground

Set up the plane table on one end of the selected base line and sight the alidade on a flag or other signal set on the other end. The

instrumentman, by the use of hand signals, keeps the two chainmen constantly on this line of sight while they are taping the distances. The head (leading) chainman carries a counted number of small stakes or chaining pins and a notebook for recording the distances as measured.

While the rear chainman holds the zero end of the steel tape on the ground mark at the plane table, the head chainman pulls the tape taut and sets a stake on the line of sight at the even 100-, 200-, or 300-foot mark, according to the length of the tape being used. The distance is then recorded in the notebook.

Now, the rear chainman moves forward and holds the zero mark at the stake just set, and the head chainman sets another stake on line as before. After this measurement is made the rear chainman pulls up the first stake and moves on to the second. This process is continued until the rear chainman has collected all the stakes except the last one set by the head chainman. The number of stakes now held by the rear chainman, together with the last one still in the ground, equals the number of full chain lengths measured on the course, and should likewise equal the number of entries in the notebook.

The pins or stakes are easier for the rear chainman to find if small trips of red flagging material are tied to the tops. As in the case of traversing, the measuring is finally checked by measuring back to the starting point. If the two measurements agree within reasonable limits, the mean of the two is taken as the length of the base line. As a further check against the possibility of having dropped a full chain length in recording, the length of the line may be approximately measured by two or three stadia determinations.

Sloping Ground

There are three commonly used methods of obtaining horizontal distance by taping on sloping ground. The chainmen are kept on the line as described in the previous article.

(1) The *"broken" chain* method requires a hand level and a plumb bob having a cord about 5 feet long, and the procedure is as follows.

If the chaining is progressing down-hill, the head chainman carries the zero end of the tape, the plumb bob, and the hand level. The rear chainman records the distances measured. When going up-hill the conditions are reversed.

Figure 44 illustrates chaining down-hill from *A*, at the instrument, to *B*. The head chainman, carrying the zero end of the tape, moves down the slope to point *a*, where, by means of the hand level, he has determined that his shoulders are approximately level with the stake at the plane table. The rear chainman holds the tape on the stake to a convenient 1-, 5-, or 10-foot mark. The head chainman draws the tape taut at shoulder height (level), with the plumb bob hanging a few inches above the ground from the cord held at the zero mark on the tape. When the bob has ceased swinging, it is suddenly allowed to drop to the ground. A stake or pin is now driven in the ground at the mark made by the plumb bob. The rear chainman moves to *a*, and the head chainman moves on down the slope to *b*, at which point his shoulders are level with the stake at *a*, and the procedure outlined above is repeated in measuring the distance from *a* to *b*. Obviously, the distance measured each time depends on the steepness of the slope. Where the grade is low, it may be convenient to use a full tape length at a time; while in other places, perhaps, only a few feet can be measured with the tape held in a level position.

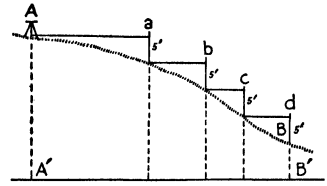


FIG. 44. Chaining on a slope by breaking chains.

The total *horizontal* distance *A'B'* is the sum of the horizontal distances *Aa*, *ab*, *bc*, etc.

Much care must be taken in correctly reading the tape and recording the distances. The number of pins collected by the rear chainman provides a check for the number of entries in the notebook; but, since partial chains, varying in length, have been used, it is probable that any error discovered in this way cannot be corrected except by rechaining back along the route.

(2) The *trigonometric* method consists simply in measuring the slope distance parallel to the ground, and then reducing this to the horizontal equivalent by computation.

In Fig. 45, the plane table is set up at A on one end of the base line, and the alidade is sighted to a signal on the opposite end. The vertical angle α is determined to B, the first change in the average slope angle. The slope distance AB is measured with the tape parallel to the ground. The instrument is next set up on the point B, sighted again on the signal at the end of the base line, and the vertical angle β determined to the point C at the next change in the slope angle. The distance BC is taped as above.

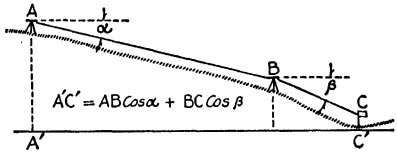


FIG. 45. Chaining along the slope and correcting to horizontal distance by means of the vertical angles.

The horizontal distance $A'C'$ is $AB \cos \alpha + BC \cos \beta$, etc.

(3) The *ratio method* is sometimes more convenient to use than the trigonometric in that it does not require moving the instrument to obtain direct vertical angles or the determination of difference in elevation to accomplish the same purpose indirectly. The alidade need not be moved so long as the chainman can be kept on line from the first setup.

The method consists in determining by means of the tape the ratio between the horizontal and slope distances, and then proceeding with slope measurements to the end of the course or leg of the course.

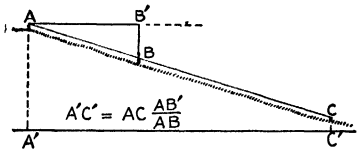


FIG. 46. Chaining along the slope and correction by means of ratios.

In Fig. 46, the slope distance AB is carefully measured with the tape. The tape is then held in a level position, and with a plumb bob centered on the stake at B, the horizontal distance AB' is measured, as in broken chaining. After the relationship between the slope and horizontal distance has thus been established, the

chainmen proceed along the course, measuring slope distances by full chain lengths.

EXAMPLE:

$$AB = 25 \text{ feet}, AB' = 24.5 \text{ feet}, AC = 505.5 \text{ feet}.$$

$$\text{Horizontal distance, } A'C' = 505.5 \times \frac{24.5}{25.0} = 495.3 \text{ feet}.$$

When this method is used, a new ratio must be determined for each change in the grade of the slope. The notes should indicate clearly the correct factor to be used in computing the horizontal distance for each leg of the course.

56. Base Net

After the base line is measured, and the signals are constructed on the prospective triangulation stations, instrumental work can be started on the triangulation system. The first step is to establish elevations on the base line stations. Where sea level elevations are required, a level line (Art. 36) is run to one or both stations from some near-by bench mark. It is not necessary to plot the instrument stations and turning points on this line, but a complete record of the work should be kept in the field notebook.

The next operation consists in accurately plotting the base line on the sheet to the desired scale of the map. This is done as follows.

Set up the tripod on one end of the base line, and level and orient the table by means of a magnetic meridian previously drawn on the sheet. The initial point a (Fig. 47A), in this case being the occupied end of the base line, is plotted on the sheet in such a position that the whole map area is assured of falling within the margins of the sheet. The fiducial edge of the alidade is now pivoted about this point until the vertical cross wire bisects the signal on the opposite end of the base line, and a ray is drawn toward the signal. Be sure that the line is long enough to include the full length of the measured base on the scale of the map. Before the alidade is moved, draw short, fine lines near each *end* of the blade,

as shown in Figure 47A. These lines, called *scores,* will be used for future orientation of the table, to be explained shortly. From point *a,* scale the measured (horizontal) distance of the base line and plot point *b.* Use extreme care in plotting this distance exactly.

Pivot the alidade about station *a* and draw rays to signals *c* and *d,* situated at the two sides of the base line, in each instance drawing also the scores near the ends of the blade. Read and record vertical angles to *b, c,* and *d.* Now, set up the tripod on *b,* at the

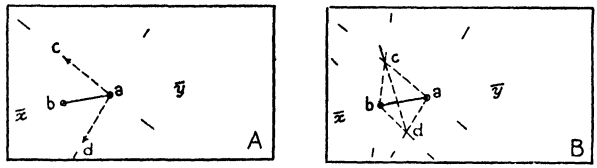


FIG. 47. Constructing the triangulation base net.

other end of the base line, and level the table. Place the fiducial edge on the scores *x* and *y* and rotate the table until the signal on *a* is bisected by the vertical wire. Clamp the Johnson head and check the position of the vertical wire on the signal. The table is now correctly oriented with respect to the orientation at station *a.* This is called *backsight orientation* and is the method used almost exclusively in triangulation. Rays are now drawn to the two signals at the sides of the base line, and points *c* and *d* are thus located on the sheet by the intersections. Draw the scores as before (Fig. 47B) and record vertical angles to *a, c,* and *d.*

The plane table is next set up at station *d* and oriented by placing the fiducial edge on the scores drawn from station *b* and rotating the table until the signal at *b* is cut by the vertical wire. Check the orientation by backsighting as above on station *a.*

Using *d* as the pivot, sight on the signal at *c* and draw the ray toward this point. If there are no errors in the work, this ray will exactly cut the two-ray intersection previously made. Check the results of these operations by occupying the station at *c* and sighting back on the other three stations. From each station occupied,

read and record the vertical angles to the other stations. Carefully prick the locations of *c* and *d* at the three-ray intersections.

The line *cd* is called the *secondary base line,* and the figure formed by the base line and all the rays is called the *base net.*

During the process of setting up the base net on the sheet, rays are drawn and vertical angles are read to all other visible signals. These rays may be temporarily identified by lightly lettering the station numbers on the respective lines.

57. Function of the Secondary Base Line

Obviously, the accuracy in length and orientation of the secondary base line is entirely dependent on the measured base line. The remainder of the triangulation system is likewise dependent to a very large degree on the length and orientation of the secondary base line. Within the scaling possibilities of the map, the secondary base is nearly as accurate as the plotted base line, though the latter is measured directly. Since it is often difficult and slow to tape a long base line, the advantages of a long base are obtained by establishing the secondary base, as described in the preceding paragraphs.

The accuracy of intersected locations depends on the width of the angles between the intersecting rays. The location is strongest when this angle is near 90 degrees. The location is weak if the angle between rays is less than 30 degrees (or greater than 150 degrees). For example, in Fig. 48, the angles between rays at *c* and *d,* drawn from the base line stations *a* and *b* should not be smaller than about 45 degrees, if the required accuracy in the secondary base is to be obtained. In the figure, the secondary base is more than twice as long as the measured base.

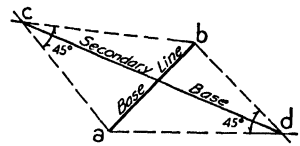


FIG. 48. Relations of the primary and secondary base lines.

It was stated earlier that if ground conditions are favorable, it is desirable to locate the base line approximately parallel with the

longest dimension of the map area. In Fig. 49, in which the base line is oriented in this way, it will be seen that the long line *FCA* is established by three-ray intersections directly from base net stations. The strong figure *cFAd*, extending to the side limits of

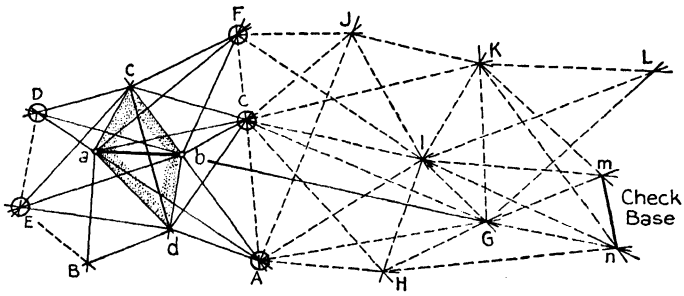


FIG. 49. Development of the triangulation system.

the system, is the result of the longer *secondary* base being oriented perpendicular to the direction of greatest expansion.

58. Expanding the Triangulation Net

The triangulation is extended over the map area in precisely the same manner in which the base net was established. From stations in the base net, rays are drawn toward the signals on all visible points. Some of these points will actually be located on the sheet at this time by three-ray intersections, as shown in Fig. 49; others, such as station *B*, will be located by only two rays, while a few may have only one ray drawn from the base stations, such as *G* in the figure. In expanding the system, set up the instrument first on those stations which are already the most strongly located from the base net, such as *C*, *E*, *D*, *F*, and *A*, in the order named. The relative strength of these locations is determined by comparing the angles of the intersecting rays and the number of rays drawn to the points. Referring to Fig. 49 as an example, the procedure is as follows.

Set up and level the plane table as close as practicable to the signal on station *C* and orient the table to base net stations from

which the rays were drawn. Pivot the alidade about the point *C* on the sheet and draw lines to *F* and *A*, which, like *C*, are already located by three lines. Next, draw rays to all other signals that are clearly visible, such as *J* and *I*. Read and record vertical angles to all stations sighted and record the height of the target above the H.I. at *C*.

The instrument is now set up at *F* and *A*, and the process outlined above is repeated. As stations ahead are located by two or three rays, they are occupied by the plane table, and the system is extended farther from these new vantage points.

Assume that the distant station *G* is visible from only one of the base net stations. The ray *bG* is drawn nevertheless. Later, when *G* has been located by lines from other stations, and is finally occupied, the long line back to the base net station *b* is used for the backsight orientation. This provides an excellent check for the orientation of the triangulation net as a whole.

Figure 50 shows four examples of triangulation systems, the shaded area in each case representing the base net. The chain of

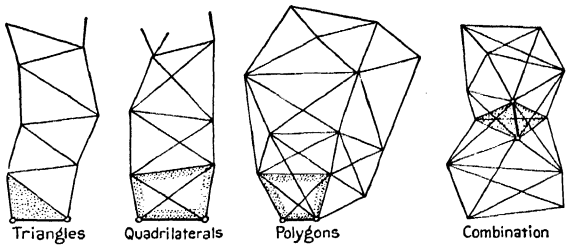


FIG. 50. Four types of triangulation systems.

triangles is considered the weakest construction of the four. The base net consists of the stations at the ends of the base line and only one other, which, in turn, is a two-ray location. The base nets in the quadrilateral and polygon types are of strong construction, the former being the stronger in the figure only because the base line is longer. The fourth system is probably the strongest of the group, for these reasons: (1) The full length of the base line is

utilized more effectively in producing the widest possible angles between the rays which locate the ends of the secondary base. (2) Lines to the extreme limits of the map area from the base stations tend to maintain rigidity in the orientation of the entire network of stations. (3) Although the length of the map area is comparable to the other three, expansion from the base net in any one direction is only about half as far.

59. Check Base Line

When the triangulation system has been greatly expanded from the base net, or if there are other reasons to suspect that the system has gotten "out of scale" or orientation, it is advisable to measure a check base with a steel tape at some convenient location near the extremities of the work.

Signals are constructed at each end of the measured line, and then located on the sheet by intersection from three or more of the previously established triangulation stations. In Fig. 49, the check base stations *m* and *n* are located by intersections from *H*, *I*, *K*, and *G*. Since the table at *G* is oriented by a backsight to *b*, a station in the base net, the lines *Gm* and *Gn* are used for orientation when the plane table is set up on the two ends of the check base line. The orientations at *m* and *n* are checked by sighting on other stations. If the work has been carried out with no appreciable error, the taped length of the line *mn* should be the same as that determined by scaling the distance between the intersections of the corresponding points on the sheet.

60. Resection

In Fig. 51A, the plane table is set up on triangulation station *C* and oriented by a backsight to station *D*. The ray *cI* is drawn toward the point *I*, whose location as an instrument station is required. The plane table is next set up on *I* and oriented by a backsight to *C* (Fig. 51B). Using *d* as a pivot, the alidade is rotated until the signal at *D* is on the line of sight, and a ray from

the map point *d* is drawn to intersect the line *cI*. The intersection is the map location of the instrument station, but this is checked by sighting on triangulation station *E*, and drawing the ray *eI*. This method of determining the map position of the instrument station is called *resection*.

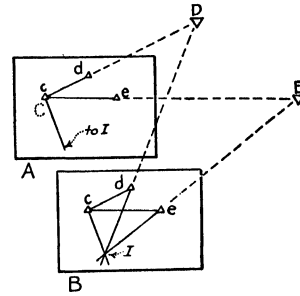


FIG. 51. Location by resection.

By way of comparison, the intersection method locates the point on the sheet by intersection before the corresponding ground point is occupied; the resection method provides an orientation line, but the ground point must be occupied in order to obtain the location.

61. The Three-Point Method

When the plane table is correctly oriented by means of the compass needle, the map position of the instrument station is determined by the intersection of rays drawn from the plotted positions of three triangulation stations successively sighted through the telescope, as shown in Fig. 52A. This is called the *three*-point method, and the process is called *three-pointing*. However, if the plane table is not correctly oriented, as shown in Fig. 52B, the three rays will not intersect at a point, but instead will form a small triangle in the general region of the instrument location. This is called the *triangle of error*.

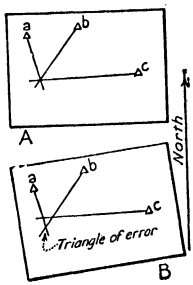


FIG. 52. A. Three-ray intersection on a correctly oriented plane table. B. Triangle of error formed with the table incorrectly oriented.

After a triangle of error has been obtained, the true map position of the instrument station can be determined by several different graphic methods, which will be explained later.

Consider Fig. 53, where the three triangulation stations are represented by *A*, *B*, and *C*. The triangle *ABC* is called the *great tri-*

angle, and the imaginary circle passing through these points is called the *great circle* (not to be confused with the great circles discussed in Art. 9).

In Fig. 53A, the plane table is set up on the great circle, and point *a,* the instrument station, is located on the sheet by the usual procedure of laying the fiducial edge on the plotted positions of the triangulation stations *A, B,* and *C* and sighting the signals of

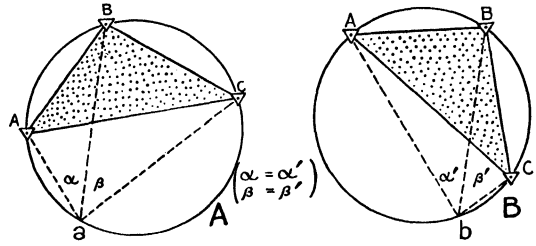


FIG. 53. The "great circle" and great triangle, showing indeterminate positions of the plane table on the "great circle."

the respective stations. The intersecting rays are shown by dashed lines, and the horizontal angles between the lines are indicated by α and β. In Fig. 53B, the orientation has been changed by rotating the table. The actual directions of the lines of sight, and consequently the horizontal angles, are unaffected by rotation of the table; but in this special case, the sight lines passing through the plotted positions of the stations still intersect at a point (point *b* in the figure), even though the table is incorrectly oriented. Compare this result of misorientation with that in Fig. 52B, where rotation of the table introduces a triangle of error.

It is evident, then, that *when the plane table is set up on the great circle, the map position of the instrument station cannot be determined by the three-point method.*

The map position of the point sought, relative to the triangle of error, depends on where the plane table is stationed with reference to the great triangle and the great circle. These relationships are shown in Fig. 54, to which the numbers of the following rules refer.

When the table is misoriented, the *correct map location of the instrument station lies always on the same side of each of the three rays drawn from the triangulation points.* That is, if the point sought is on the right side of the ray from station *A*, it will likewise be on the right side of the ray from *B* and from *C*, right or left being determined while facing the respective stations.

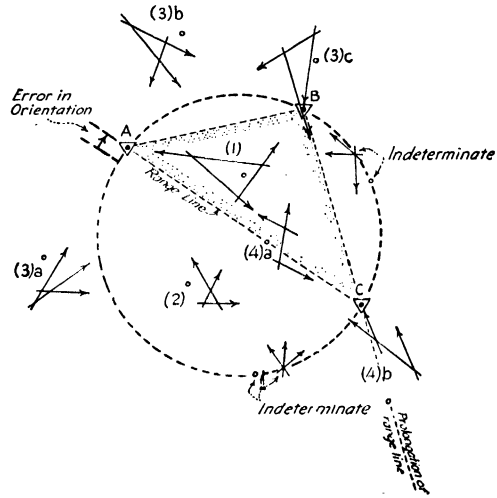


FIG. 54. The effect on three-ray intersections depending on the location of the plane table.

(1) When the plane table is inside the great triangle, the point sought is inside the triangle of error.

(2) When the plane table is in a segment between the great circle and the great triangle, the ray to the middle triangulation station lies between the point sought and the intersection of the other two rays.

(3) When the plane table is outside the great circle, the point sought lies on the same side of the ray from the most distant station as the intersection of the other two rays.

(4) When the plane table is on a range line formed by two of the triangulation stations, or a prolongation of such a range line,

two of the rays will be parallel and these will be intersected by the third. In this instance a triangle of error is never formed.

In Fig. 54, all parts of the drawing are in correct relationship to each other. The error in orientation, as indicated at station *A*, is the same for all plane table locations shown. It is, therefore, evident that the size of the triangle of error and the distance of the point sought from the respective triangle depends not only on the amount of misorientation, but to an even greater extent upon the position of the plane table relative to the three triangulation stations.

The strength of the three-point location can be judged by the shape and size of the triangle of error and the distance from the point sought to this triangle. The strongest location is inside the great triangle, because the point sought is always within the triangle of error, and the latter is comparatively large in area for a given degree of misorientation. Locations on a range line between two of the triangulation stations are strong. A comparison of (3)*a* and (3)*c* shows that locations outside the great circle are weak if the middle station is farther away than either of the other two, and strong if the middle station is the nearest. A location on the prolongation of a range line (4)*b* is strong if the angle made by the parallel rays and the ray to the third station is not less than 30 degrees.

In field practice it is often possible to select three out of a larger number of visible triangulation stations. Under such circumstances, the resection rays should be drawn from the three stations which will produce the strongest location for the plane table.

62. Methods for Solving the Three-Point Problem

The methods described in the following paragraphs are used at various times in actual field practice. Each method has certain advantages, as well as disadvantages, over the other methods. These considerations will be discussed later; but for the time being, suffice it to state that a working knowledge of several methods is highly desirable in ordinary plane table mapping, for certain situ-

ations arise which may make one of the methods temporarily more adaptable than the others.

63. Lehmann's (Coast Survey) Method

This method is a development of the four rules given in Art. 61. Since it is essentially a trial-and-error method, it is best adapted for those who have had much experience in plane table triangulation. The principal advantages are that it is rapid when employed by an experienced man, and that no construction lines or additional points need be plotted on the sheet.

At location (1) in Fig. 54, let the perpendicular distance from the point sought to each of the three rays be designated *pa, pb,* and *pc,* to correspond with the triangulation stations *A, B,* and *C.* Similarly, let the distance from the point to each of the stations be *pA, pB,* and *pC.* By measuring these distances it can be shown that $pa/pA = pb/pB = pc/pC$; or that the distance from the point sought to each of the rays is proportional to the distance from the instrument station to the respective triangulation stations. This relationship holds for any location of the plane table. It can be shown further that for a given amount of error in orientation, the ratio *pa/pA,* etc., is the same for all triangles made by sights on the same three triangulation stations.

In practice the method is applied as follows.

By inspection it is ascertained whether the triangle of error falls within the great triangle, between the great circle and the great triangle, or outside the great circle. A trial point is then plotted in a position estimated to be in the correct relationship to the three rays, as outlined in the preceding paragraph. The fiducial edge is now laid on this point and the plotted position of the *farthest* triangulation station, and the table is rotated until the station signal is brought into the line of sight. The Johnson head is locked, and a new set of rays is drawn from the three stations. If the trial point was correctly located, the new orientation will be correct, and the three rays will intersect at the point. If a new similar but smaller triangle of error is formed, the table should be rotated still farther

in the same direction; but if the triangle is reversed, the table has been rotated too far in the one direction, and, therefore, must be rotated a part of the way back. When a three-ray intersection is finally obtained, prick the location with a needle and erase all but the correct set of rays.

An experienced instrumentman will usually obtain a three-ray intersection on the second trial, less frequently on the first, and will seldom have to draw more than two sets of rays.

64. Tracing Paper Method

This method is quite accurate when the angles between rays are wide, provided the location is not too near the great circle. If more than three station signals are visible, rays should be drawn on the tracing paper to all of them. The procedure is as follows.

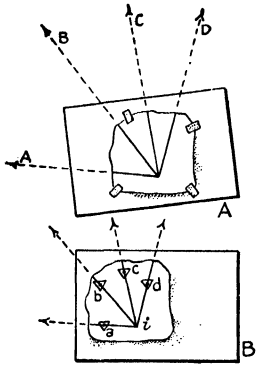


FIG. 55. Tracing paper solution of the three-point problem.

Tighten the Johnson head so the table cannot move. Fasten a piece of tracing paper on the board with adhesive tape. Plot a small point on the paper at any convenient place; and, using this point as a pivot for the alidade blade, draw rays to three or more triangulation stations, as shown in Fig. 55A. In this operation, pay no attention to any points plotted on the plane table sheet.

Now remove the tape and shift the tracing until the three rays exactly cut the corresponding plotted positions of the triangulation stations on the sheet, and then prick the point *i* through the tracing onto the plane table sheet, as in Fig. 55B.

To orient the table, place the fiducial edge on point *i* and the farthest triangulation point (either *b* or *c* in the figure), and rotate the table until the station signal is in the line of sight. Lock the Johnson head and check by backsighting on the other stations.

65. Intersecting Circle Method

The chief advantage in this method is that the position of the instrument station can be determined without the necessity of re-orienting the table or resighting on the triangulation points. Occasionally, this may be important, when the time available for field operations is too limted to permit completion of the solutions at the stations. After a triangle of error has been obtained, the process of locating the instrument station on the sheet can be completed in the drafting room; though it is better to check the results in the field.

The disadvantages are (1) that a number of construction lines have to be drawn on the plane table sheet, and (2) in some instances the necessary graphic constructions may fall off the sheet. When the solution is completed on a large table, additional drawing paper can be fastened to the sheet to provide space for the constructions.

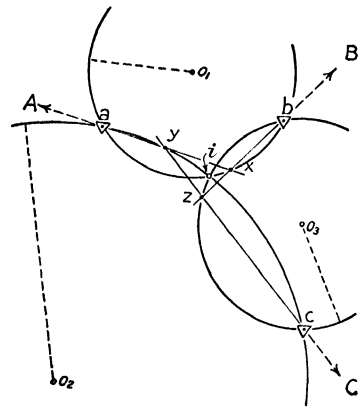


FIG. 56. Intersecting circle solution of the three-point problem.

In Fig. 56, a triangle of error has been obtained by resection from points *A*, *B*, and *C*. Of the points plotted on the sheet, consider only *a* and *b*, and the intersection *x* of the two rays drawn from *a* and *b*. Erect perpendicular bisectors from two of the chords *ab*, *ax*, or *bx*, and, by using the intersection of the bisectors as center, strike the circle arc through the three points. A drafting compass is needed for this purpose. In the same manner, strike another circle arc through the three points *a*, *c*, and *y*, the latter being the intersection of the rays from *a* and *c*. This arc will inter-

sect the first one drawn at a point on the sheet which is the correct position of the instrument station. Using the points *b* and *c* and the intersection *z*, strike the third arc passing through these points. If all these constructions are carefully executed, the three arcs will intersect at the point sought. When the plane table is on the great circle, obviously the construction arcs and the great circle coincide; and since there can be no points of intersection, the position of the table is indeterminate. Similarly, when stations are too near the great circle, the three construction arcs intersect at small angles, and the location becomes weak.

66. "Similar" Triangle Method

This method gives only the approximate map position of the plane table and, consequently, should not be used where very precise locations are required. It is a modification of the circle arc method designed for giving a rough location of the table quickly and without the necessity of drawing complicated or large constructions on the sheet.

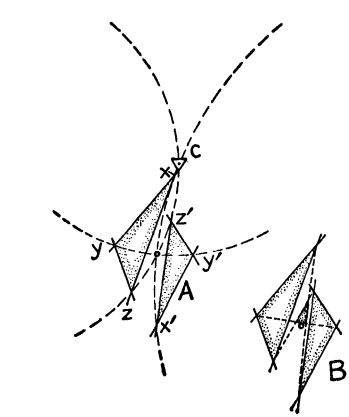


FIG. 57. "Similar" triangle solution of the three-point problem.

In Fig. 57A, the triangle of error *xyz* is obtained by resecting on stations *A*, *B*, and *C*. The table is now misoriented in the opposite direction, and three new rays are drawn from the stations to produce the reversed triangle *x'y'z'*. In locating the instrument by the arc method, as illustrated, it will be seen that each arc passes through similar vertices of the two triangles of error. In Fig. 57B, straight lines, instead of circle arcs, are drawn through the similar vertices. These lines intersect each other to make a very small triangle; and the point

sought is always very near, or within, this triangle. These lines are chords of the construction arcs shown in Fig. 57A. By estimating the radii of the construction circles, the arcs are sketched freehand to intersect at a point.

The triangle method is fast, but the location of the instrument station must be checked by reorienting and sighting on the three triangulation stations.

67. Bessel's (Italian) Method

Bessel's method (sometimes called the Italian method) is quite accurate, but, like the arc method, requires graphic constructions which occasionally fall off the sheet. It differs from the other methods in that no triangle of error or preliminary rays from the three triangulation stations are needed for establishing the map location and orientation of the table. The field procedure is as follows.

The plane table is set up and leveled, but the horizontal movement is left free so that the table may be rotated. In Fig. 58A, the fiducial edge is placed on the plotted triangulation points *a* and *c*, and the table is rotated until the signal on *c* is in the line of sight. This operation is the same as would be made if the table were set up on *A*, instead of the unknown point, and oriented by a backsight on *C*. The table is clamped in this position, and, using *a* as the pivot for the fiducial edge, station *B* is sighted and a ray toward this station is drawn on the sheet.

In Fig. 58B, the table is again loosened so it can be rotated. The alidade is laid on *c* and *a* (with the front toward *a*), and the table is rotated until the signal on *A* is in the line of sight. After clamping the table in this orientation, using *c* as the pivot point, *B* is again sighted with the telescope, and a ray is drawn from *c* toward *B*. This line intersects the one drawn from *a*, at point *x* in the figure. The point *x* is simply a part of the construction and does not represent any particular location on the ground. With a straightedge, a line is drawn passing through *x* and *b*, the plotted position of the triangulation station *B*. The point sought always lies somewhere on this line.

In Fig. 58C, the alidade is placed on the line bx, and the table is rotated until the signal on B is in the line of sight, and then clamped in this position. The table is now correctly oriented, and the position of i, the point sought, is obtained by resecting from stations A and C, and drawing the respective rays from a and c. After i is located and pricked on the sheet, the construction lines may be erased.

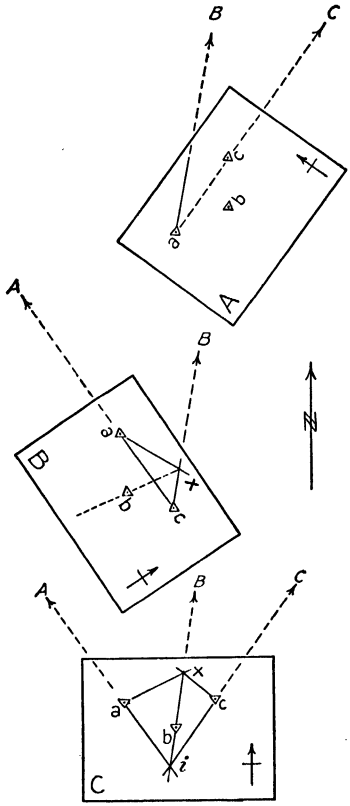


FIG. 58. The Italian method of solving the three-point problem.

68. The Two-Point Method

Occasionally, it is necessary to set up the plane table on a point from which only two triangulation stations are visible. However, the table can be correctly oriented, and the position of the instrument station accurately plotted, by the methods described below.

When the plane table can be set up on a prolongation of the range line through the two triangulation stations, the procedure is as follows.

In Fig. 59, the table is first set up at C, exactly on range with the triangulation stations A and B, and is oriented by laying the fiducial edge on the plotted positions a and b and rotating the table until the signals on the stations are cut by the vertical wire. The alidade is then sighted on a temporary signal previously set on the desired instrument station at D, and short orientation scores are drawn near the ends of the

fiducial edge. A temporary target, such as a small flag, is now set on C, and the plane table is set up on D and is oriented by placing the fiducial edge on the scores and backsighting to the signal on C. The table is now correctly oriented with respect to the two triangulation stations. The map position of D is determined by resecting from A and B. It should be clearly understood that the setup on C is for the sole purpose of obtaining a correct orientation

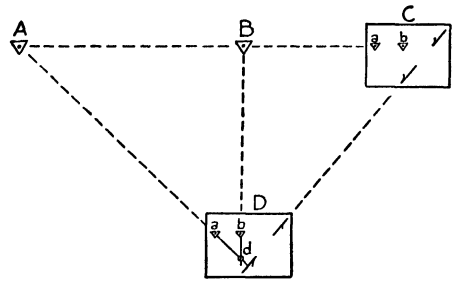


FIG. 59. Range line orientation.

at D, and there is no necessity for determining the map position of C.

It may be that points on the prolongation of the range line, such as C, are inaccessible, but the plane table can be set up on range between the two stations. In this case, several trial setups may be required before the plane table is on the range line. To ascertain that the table is on line, orient by laying the fiducial edge on the plotted range and sighting on one of the points. Lock the table in this position and reverse the alidade on the plotted line and sight the other station. If the plane table is on the range line, the signals on the stations will be bisected by the vertical wire in either position of the alidade. The orientation scores are then drawn toward the desired location at D.

When it is inconvenient to set up on the range line, the procedure is somewhat different.

In Fig. 60A, a temporary signal is set on the desired instrument site D. The plane table is then set up and oriented by compass at

any convenient location C roughly parallel to the line connecting the two control stations and at a considerable distance from D. The temporary point c on the sheet is obtained by resecting from A and B; but this is not the true map position of C because of the error in orientation. Using c as a pivot, a ray is drawn toward the

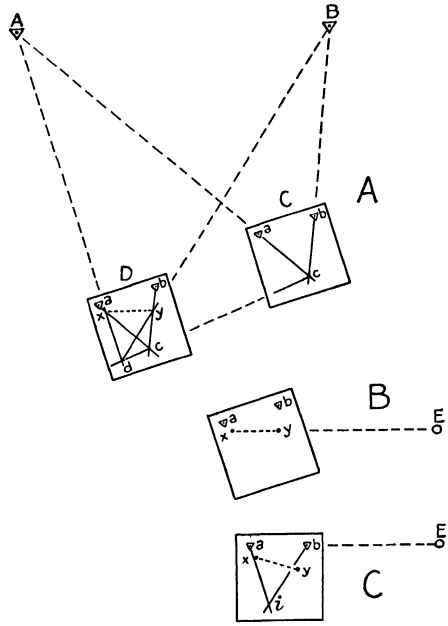


FIG. 60. Solution of the two-point problem.

signal on D, and d is plotted on this line according to the *estimated* distance from C to D.

After a temporary signal is erected on C, the plane table is moved to D and oriented by means of the line cd and a sight to the signal on C. The table is now aligned exactly as at C. Sights are now taken to A and B, and the respective rays are drawn from d. These rays intersect those drawn from C to x and y. The quadrilateral $cdxy$ is exactly the same as $CDAB$, but the figure on the

sheet is rotated with respect to the one on the ground by an amount equal to the error in orientation. This error in the orientation of the table is corrected as follows.

Lay the fiducial edge of the alidade on the line xy and select some small, clear object on the landscape that is cut by the vertical cross wire, such as E in Fig. 60B. Next loosen the Johnson head clamp, place the fiducial edge on the line ab, and rotate the table until the same object is again cut by the vertical wire, as in Fig. 60C. The table is now correctly oriented, and the position of the plane table i is obtained by resecting from A and B.

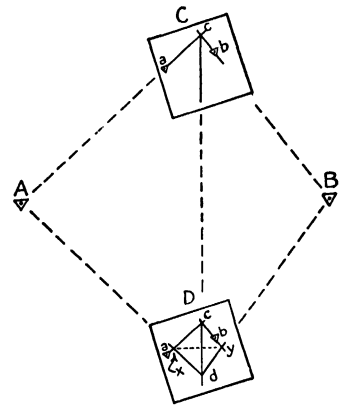


FIG. 61. Solution of the two-point problem.

Figure 61 illustrates a slightly different application of the method described above. The procedure in this case is the same as that shown in Fig. 60, but this arrangement of points C and D permits laying out the trial base CD normal to the range of the two triangulation stations.

69. Eccentric Instrument Stations

The importance of setting the plane table directly over the triangulation station mark depends on the scale of the map, the distance to the station used for orientation, and the distance to the farthest point to be located. Usually it is inconvenient to set up exactly over the point, for this involves tearing down and rebuilding the signal. Furthermore, it frequently happens that the visibility from a station is restricted to only the higher portions of the area, and for this reason much advantage is gained if the table can be set up at some distance from the station. The best means of gaging the allowable eccentricity of the plane table with refer-

ence to the triangulation station is by plottable distances and angles on the sheet. The smallest distance that can be plotted consistently is about 0.02 inch, and the smallest distance that can be measured between two plotted points with the scale is approximately 0.01 inch. On a scale of 2 inches to 1 mile, 0.01 inch represents a distance of 26.4 feet on the ground; so in so far as the position of the table is concerned, it may be set up as far as 15 to 20 feet from the station signal without seriously affecting the work. If the scale is larger, then the permissible eccentricity must be proportionately smaller. However, the orientation of the table is also affected by setting up to one side of the station. When the instrument is on range between the two stations, such as *i* in Fig. 62, orientation of the table is nearly as precise as if the table were at *a;* but when the table is in a position such as *i'* and the alidade is placed on the plotted points *a* and *b* for orienting, an obvious error in orientation results. Therefore, the limitations of eccentricity must be defined also in terms of angular discrepancy. For accurate work this angular error should not exceed 05 minutes of arc when the map distance of the orientation line and rays to new points are about equal and are no longer than 5 inches.

FIG. 62. Eccentric instrument locations.

To return to the example above, the scale is 2 inches to 1 mile, and the distance from the table to the triangulation station signal, approximately normal to the line of sight, is 20 feet. If the distance from the table to the station sighted for orientation is 2 miles (4 inches on the sheet), the resulting error in orientation is about 7 minutes of arc; if the distance to the station is 1 mile, the angular error is approximately 14 minutes. It is evident that the allowable amount of eccentricity in the setups must take into account the effects on orientation as well as the plottable error in position.

When it is necessary to set up at a considerable distance from the station signal, the following procedure is recommended. First,

set up the plane table on, or very near, the station, and orient by sighting on the distant signal. Next, draw a short ray toward the desired instrument point, measure the distance from the station to this point by stadia, tape, or pacing, and plot the point on the sheet. Now, move the table to this point, and orient by laying the fiducial edge on the newly plotted point and the plotted position of any distant triangulation station, and then rotate the table until the corresponding signal is on the line of sight. The new instrument station, instead of the near-by triangulation station, is now used for the origin of rays drawn toward new locations. Of course, a correction for H.I. elevation must also be made.

70. Elevations

When elevations are determined by triangulation methods, it is essential that the greatest care be taken in all instrumental adjustments and observations, because of the longer lines of sight with consequent increase in the magnitude of common errors. For example, an error in the parallelism of the line of sight and the axis of the level tube amounting to 0.02 foot for a sight 100 feet in length can be neglected in most small-scale traverse work; but in a sight 8 miles long, frequently encountered in triangulation, this discrepancy in parallelism introduces an error of 4.0 feet in elevation. Similarly, in most stadia work vertical angles are read only to the nearest minute of arc, which permits errors up to ½ minute; but an error of ½ minute in the vertical angle for a sight of 5 miles results in an error of 3.7 feet in elevation.

In triangulation only two methods of determining difference in elevation are commonly used, and these are the vertical angle and the gradienter screw. It will be seen that these methods are applied in a manner somewhat different from the procedures previously described.

71. Earth Curvature and Atmospheric Refraction

The effects of curvature and refraction in the determination of elevations are so small in sights less than 3000 feet in length that

they may be neglected in most plane table work. Furthermore, in traversing, the errors due to these causes tend to compensate one another. However, in longer sight lines, discrepancies due to curvature and refraction increase disproportionately with increases in the lengths of the lines of sight, and for this reason it is necessary to make certain corrections in computing the difference in elevation.

In Fig. 63, the initial position of the instrument is at *i*. At this point the level line of sight is horizontal, but since the level line is parallel to the mean surface of the earth, the two lines diverge at an increasing rate out from the point of tangency. At the left side of the figure the errors due to curvature in a series of short sights are indicated by e_1, e_2, etc., and the total error to *A* is the sum of all these small errors. Now, consider the sight from *i* to *B*, which is the same distance as from *i* to *A*. In this case, however, the curvature discrepancy *e* is much greater because of the longer sight line. The effect of curvature alone is approximately $0.66D^2$ feet, where *D* is the distance in miles from the instrument to the point sighted.

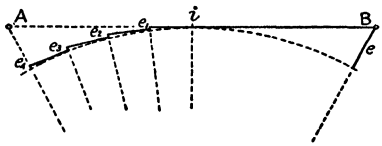


FIG. 63. Relations of long sights to the curvature correction.

For the short sights used in traverse work the line of sight may be considered as mathematically straight. This does not hold for the long sights encountered in triangulation. Because of the difference in density of the atmosphere at high and low levels, the line of sight is refracted downward from the point of tangency. The form of this line is circular, but the radius of curvature is a great deal longer than that of the earth.

In Fig. 64, the line of sight is shown as a solid line. At a distance *iA* the amount of curvature is $c + r$. However, since the line of sight is depressed below the horizontal at *A* by an amount equal to *r*, the correction which must be applied in computing the elevation is *c*. In the same figure, assume that the instrument is set up

at i and the telescope is leveled. The line of sight cuts the top of the mountain at B. Thus, the top of the mountain would appear to be at the same elevation as i; but because of the combined effects of curvature and refraction, the correction d must be added to the H.I. at i to obtain the elevation of B. In this case, the difference in elevation is exactly equal to the curvature correction.

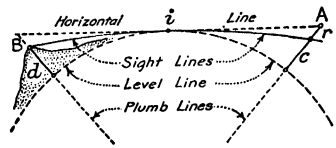


FIG. 64. Relations of the curvature and refraction correction.

In so far as plane table work is concerned, the value for the curvature correction as given by the formula $0.66D^2$ may be considered applicable for all regions. The value for the refraction correction, however, is subject to some modification according to the altitude of the region above sea level. This variation is small, in most cases amounting to less than errors from other sources. The expression most commonly used for refraction alone is $0.09D^2$ feet, where D is the distance in miles.

Since the correction for refraction is opposite to that for curvature, the formula used in the computation of elevations is the difference between $0.66D^2$ and $0.09D^2$, or $0.57D^2$. If the term D is in thousands of feet, the formula is $0.02D^2$.

72. Application of the Curvature Correction

When the elevation at the instrument is known and the *difference* in elevation to a foresight point is being determined, the algebraic sign of the curvature correction is plus. In Fig. 65, assume the H.I. elevation e to be 1500 feet at the station i. Since the foresight point A is cut by the level line of sight from i, the *apparent* difference in elevation is zero. Assume the curvature correction c to be 500 feet. As the sign of the correction is plus, the

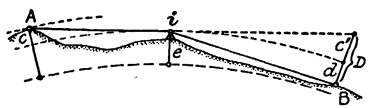


FIG. 65. Application of the curvature and refraction correction.

actual difference in elevation is $0 + 500 = +500$ feet; and the elevation of A is $1500 + 500 = 2000$ feet.

Now consider a sight from i to B. The apparent difference in elevation is $-D$, which for the purpose of the example will be given a value of -2300 feet. The curvature correction c' is $+1400$ feet. The actual difference in elevation is $-2300 + 1400 = -900$ feet; and the elevation of B is $1500 - 900 = 600$ feet.

When the elevation of the instrument is being determined (a backsight), the algebraic sign of the curvature correction is minus in the computation of *difference* in elevation. For example, in the illustration above, the elevation of B is 600 feet. The apparent difference in elevation is $+2300$ feet, and the curvature correction is -1400 feet. The actual difference in elevation is $+2300 - 1400 = +900$; and the elevation of i is $600 + 900 = 1500$ feet.

It sometimes happens that the apparent difference in elevation is minus, although the actual difference is plus, due to the effects of curvature. Imagine a foresight on a station which lies just below the level line of sight. Assume this distance from the middle wire down to the point is computed to be 10 feet. The apparent difference in elevation would be -10 feet (for a foresight). But if the distance to the point is great enough that the numerical value of the curvature correction exceeds the apparent difference in elevation, say 25 feet, then the actual difference in elevation would be $-10 + 25 = +15$ feet.

Convenient tables of curvature and refraction corrections will be found in the Appendix.

73. Reciprocal Sights

The correction for curvature and refraction can be avoided by the reciprocal sight method. Some instrumental errors, such as poor striding level adjustment, are also eliminated. The procedure is as follows.

In Fig. 66, the instrument is first set up at A, a sight is taken to B and the vertical angle α is carefully determined. The plane table is then moved to B, a sight is taken on A, and the vertical

angle β is determined. In each case the station target is set at the height of the instrument above the ground.

For the purposes of surveying, the angles ϕ and ϕ' may be considered as 90 degrees; and the arc distance H on the datum equal to the horizontal distances as shown in the diagram.

The *apparent* difference in elevation observed from A is d', and the curvature and refraction correction is c'. The *actual* difference

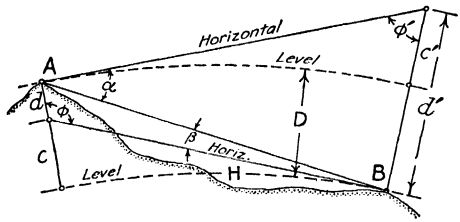


FIG. 66. Illustrating the theory of reciprocal sights.

in elevation D is $d' - c'$. Similarly, the apparent difference in elevation from B is d, and the actual difference in elevation D is $d + c$. Also, $D = (H \tan \alpha) - c'$, and $D = (H \tan \beta) + c$; therefore, $2D = (H \tan \alpha) - c' + (H \tan \beta) + c$. Since c' and c, the curvature and refraction corrections, are of practically equal value, these terms cancel out, leaving

$$2D = H \tan \alpha + H \tan \beta$$

or

$$D = \frac{H}{2}(\tan \alpha + \tan \beta)$$

Thus, the difference in elevation between two points is one-half the horizontal distance multiplied by the sum of the tangents of the reciprocal vertical angles, and no correction for curvature and refraction is made.

74. Vertical Angles

As stated before, vertical angles must be determined with extreme care where the lines of sight are long. The striding level, because of its higher precision, should be used instead of the con-

trol level. On long sights an effort should be made to read the vernier somewhat closer than even minutes. Fairly accurate estimates of half- and quarter-minutes can be made in the following way.

Place the alidade near the center of the table when determining difference in elevation. Center the bubble of the striding level and, with the left (index) tangent screw, set the vernier at exactly 30°00′. Check the bubble again to ascertain that it has not been disturbed while setting the arcs. Loosen the telescope axis clamp and elevate (or depress) the telescope until the horizontal cross wire is nearly on the station signal, then tighten the clamp, and complete the setting on the target with the right tangent screw. With a magnifying glass read the vernier to the nearest minute.

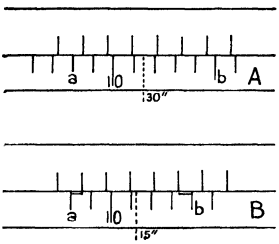


FIG. 67. Estimation of seconds on an arc that reads directly only to minutes.

If two of the graduations on the lower scale are about equally coincident with the opposing graduations on the upper scale, the actual vertical angle includes a fraction of a minute, which is near one-half minute, or 30 seconds. In Fig. 67A, such a condition is illustrated. The portion of the vertical angle stated in the smaller units is 11′30″. It will be seen in the figure that the amount of offset is approximately the same for a comparable number of units on the two sides of the eleventh and twelfth graduations, as shown at *a* and *b*.

In Fig. 67B, it is evident that the eleventh minute is the most nearly coincident, although there is some visible offset in the opposing lines. A comparison of graduations *a* and *b* shows that the angle is greater than 11 minutes, but less than 11′30″; so it is concluded that the most probable estimate is 11′15″.

Another method for estimating a fractional part of a minute consists of the following procedure. As in the method described above, the vernier is carefully set on 30°00′ when the telescope is level. After the middle wire has been set on the station target,

instead of estimating the fraction on the vertical arc, move the telescope up or down enough to bring the graduation most nearly coincident into exact coincidence. Through the telescope observe the position of the middle wire with respect to the station target. Now move the wire across the target, above or below, as the case may be, to the next exact minute graduation on the arc. The wire has now been moved through one minute of arc, and the position of the target between the two observed positions of the cross wire provides a means of estimating the fraction of the one-minute interval to be applied to the vertical angle.

75. The Gradienter Screw in Triangulation

In flat country where vertical angles of less than one-half degree are frequently encountered, the difference in elevation between triangulation stations can be conveniently and accurately measured with the gradienter screw. Although the method is subject to certain instrumental errors, these can be overcome by repetition, as explained later.

By means of the right tangent screw adjust the axis arm until it is approximately vertical. For a *plus* vertical angle sight the procedure is as follows.

With a continuous right-hand motion of the tangent screw, bring the bubble of the striding level to center. Read the gradienter drum and through the telescope select some sharply defined object on the landscape bisected by the upper wire, as in the step method. Continuing with a thrust motion of the screw, set the middle wire on the station target and read the drum again. Complete the interval step by raising the lower wire to the object formerly cut by the upper wire and read the gradienter drum.

EXAMPLE OF COMPUTATIONS:

Horizontal distance (scaled on the sheet)	10,260 feet
First drum reading (telescope level)	36
Second drum reading (wire on target)	84
Third drum reading (interval step)	38
Difference between first and second readings	48
Difference between first and third readings	102

$$\text{Difference in elevation (foresight)} = + \frac{48}{102} \times \frac{10{,}260}{100^*} + 0.021D^2$$

The value of D is here expressed in thousands of feet, so,

$$\text{Difference in elevation} = + \frac{48 \times 102.6}{102} + 0.021 \ (10.26)^2$$

$$= 48.2 + 1.2 = + 49.4 \text{ feet}$$

* Or the exact stadia interval factor if appreciably different from 100.

In order to maintain the necessary thrust motion of the tangent screw, the procedure for *minus* vertical angle sights is somewhat different;

Level the telescope and select an object exactly bisected by the lower wire. With the tangent screw, depress the telescope so that the upper wire is somewhat below the object, and then bring the upper wire up to the point with a thrust motion of the screw and read the drum. Elevate the telescope farther, until the middle wire cuts the target, and read the drum again. Complete the interval step by raising the lower wire until it cuts the object formerly bisected by the upper wire and read the drum. When this operation is completed the bubble of the striding level should come to rest in the center and, therefore, constitutes a check on the interval step.

EXAMPLE OF COMPUTATIONS:

Horizontal distance (scaled on the sheet)	20,440 feet	
First drum reading (upper wire on object)	83	
Second drum reading (middle wire on target)	46	
Third drum reading (telescope level)	82	
Difference between second and third readings		36
Difference between first and third readings		99

$$\text{Difference in elevation} = -\frac{36}{99} \times \frac{20{,}440}{100} + 0.021D^2$$

$$= -\frac{36 \times 204.4}{99} + 0.021(20.44)^2$$

$$= -74.3 + 8.8 = -65.5 \text{ feet}$$

To check the observations and computations, loosen the axis clamp and turn the tangent screw two or three revolutions and then proceed as outlined above. This brings a different part of the screw into play, and although numerical readings will be different, the final computed difference in elevation should be the same. If the second set of readings and computations does not agree within reasonable limits with the first group, turn the tangent screw again and make a third set of observations and take the mean of all the computations for the difference in elevation.

76. Inverting the Telescope

Unless the striding level is perfectly adjusted by the two-peg method, there may be a lack of parallelism between the line of sight and the longitudinal axis of the level. The effects of such a discrepancy are practically eliminated by the following procedure.

Level the telescope and determine the vertical angle, or the gradienter equivalent, according to previous instructions. Remove the striding level, loosen the knurled ring in front of the axis bearing so that the telescope turns freely in its sleeve, rotate the telescope 180 degrees, and then tighten the knurled ring again. Replace the striding level and repeat the determination of the vertical angle with the telescope in the inverted position. The mean of the two vertical angles thus measured is used in the computation of difference in elevation.

When the ring bearings upon which the legs of the striding level sit are badly worn, an average of four readings may be necessary for very accurate measurements. This method differs from that described above in that the angle is determined also with the striding level turned end for end on the telescope in both the upright and inverted positions.

77. Transit or Theodolite Control

The primary control for many plane table surveys is established by transit or theodolite triangulation. This is true for most departments of the federal government engaged in field mapping. If the

areas to be surveyed are large, it is advisable to utilize any such established positions as may be available from earlier and more precise work. In some instances this may require drawing a polyconic projection for the sole purpose of accurately plotting the transit stations on the plane table sheet; but the time required to do this is usually small as compared to the time saved in the field in setting up independent control.

When three or more transit triangulation points are carefully plotted on the sheet, the plane table triangulation consists of filling in with secondary stations, which are located by intersection from the primary stations. No measured base line is necessary, although check bases may be used to advantage in areas remote from the primary stations.

Methods for plotting transit stations on the plane table sheet are described in Chapter XII. After the stations are plotted, the sheet should be taken into the field and the plotting checked by setting up the plane table on each of the stations and by sighting on the others. At each station setup the table should be oriented by sighting on the most distant clearly visible signal whose position is plotted on the sheet. Signals should be previously erected on proposed new points so that they may be sighted in, the necessary rays drawn toward them, and the vertical angles determined when the primary stations are first occupied. This is particularly important where the primary points are difficultly accessible.

78. Supplementary Vertical Control

The discussion in earlier paragraphs has considered the determination of station elevations by vertical angles and scaled distances. Where very precise elevations are required, closed and adjusted level lines should be run from reliable bench marks. While it is not necessary to plot the positions of instrument stations and turning points on level lines, due consideration should be given to the possible later use of such plotted traverses. Very little additional time is required to determine the stadia distances and plot

the points on the sheet, when the alidade and plane table are used for running the levels; and the plotted traverse, with stations also marked on the ground, may be of great help in delineating map features. The traverses should be tied into the triangulation directly, or by three-pointing, and horizontal positions should be adjusted according to the ties.

All elevations previously established by government surveys should be utilized. When the elevations of more than one government survey are used, it is advisable to ascertain that the same datum was employed, making any correction necessary to reduce all to the same datum.

79. Precautions

Much useless effort and loss of field time may be avoided if reasonable precautions are observed. Suggestions made in the following paragraphs apply to plane table work other than triangulation, but they are considered of particular significance in triangulation because in this department of surveying errors tend to accumulate and are carried through the whole system. Furthermore, it is usually more difficult to reoccupy triangulation stations than the stations on a traverse.

Local differential atmospheric refraction, not to be confused with atmospheric refraction discussed earlier, may be caused by unequal temperatures of the air along the line of sight. The effects are unpredictable, but it is known that the line of sight may be seriously deflected either vertically or horizontally if it passes through atmospheric regions varying appreciably in temperature. This hazard is reduced by avoiding sights that pass close to the ground, cliffs, shrubbery, trees, etc., where the temperature is likely to be somewhat higher than at places where the line of sight passes well above such obstructions. A small bush in the sunlight and near the line of sight may introduce an error of several feet in elevation or horizontal position. Experience has shown that, as a rule, it is better to determine elevations in the morning and

evening and avoid the mid-day hours, whereas horizontal determinations are usually more accurate at mid-day, provided there is little wind.

Local refraction is often very apparent as "heat" waves. Usually this type of refraction does not cause large errors, but is troublesome in that visibility is reduced and it is difficult to set the wires accurately on the target when the latter is apparently moving violently. The "heat" wave phenomenon is most bothersome when the wind is blowing at a moderate rate for then large masses of air having different temperatures are moved across the line of sight. When the air is very quiet a far more serious condition of refraction is liable to exist for the reason that there is little tendency for the relatively hot and cool masses to mix. Local refraction under these conditions, regardless of the magnitude, may have no discernible manifestations, such as "heat" waves, nor will the visibility necessarily be affected. It should be clearly understood that local refraction is caused by *differences* in temperature (and occasionally humidity) of the air, and is not necessarily affected by the average temperature of the area. Deviations of large magnitude have been observed in winter with average temperatures well below zero.

On sights where the vertical angles are large, especial care must be taken in leveling the plane table; for if the table is tilted, the telescope does not move in a vertical plane, and this results in errors in both elevation determinations and the directions of rays drawn on the sheet.

Make all gradienter settings with a thrust or clockwise motion of the tangent screw so that the axis arm is thereby operating against the opposing spring. When settings are made with a left or counterclockwise motion, there is danger of back-lash or a belated movement of the telescope, which may result in erroneous readings.

If a station is occupied for a long period of time, check the orientation at rather frequent intervals to make certain that the table has not been disturbed by working around the tripod. At all

setups check the orientation after the last observation on other points has been made.

Draw rays with a hard pencil (7H to 9H) with a polished, sharp wedge- or chisel-shaped point. Hold the pencil in a vertical position with the flat side of the point pressed firmly against the fiducial edge when drawing the rays.

On long sights, where objects appear small in the telescopic field, a common error is that of confusing the quarter wire with the horizontal cross wire. Before setting the cross wire on a target, glance over the field to make certain that all the wires are accounted for and that the proper one is set on the target.

When a station is occupied, draw rays to all visible new stations. Some of these rays will not be necessary for locating the new point by intersection, but may be of great future value for orientation, or in analyzing and tracing errors that may occur.

Use the full length of the fiducial edge for orienting the table, irrespective of the distance from the instrument to the far station signal. Whenever a ray is drawn to a new station, draw also the short scores near the ends of the base and lightly label them with the numbers of the two stations lying on the same range line. In triangulation, misorientation outweighs all other sources of error in horizontal position, and, therefore, every possible precaution should be taken to insure accurate orientation. The accuracy of orientation is affected by both the length of the line on the sheet and the distance to the station sighted. The first condition is disposed of by using the full length of the blade on every backsight, and the second, by sighting on the most distant point that is clearly visible.

Always use the same end of the needle when orienting the table by compass. If the two ends of the needle do not simultaneously coincide with their respective index marks, do not attempt to use a mean position, which would be difficult to duplicate exactly in other setups. By using only one end (either the north or the south) exactly at the index mark, a higher degree of consistency can be expected.

When locating the instrument by three-pointing, be sure that the location is not too near the great circle, for in this position a three-line intersection will be obtained even though the table is incorrectly oriented. Where four or more stations are visible, select the three that will give the strongest location as, for example, within the great triangle.

Protect the plane table sheet from moisture. When the sheet is not in use, loosen all the thumbscrews so that it can expand and contract with changes in atmospheric humidity. Season the sheet as long as possible before it is used in the field by exposing it to the air in the locality where it is to be used. Since cloth-mounted sheets expand more in one direction than another, shrinkage causes distortion as well as scale change.

CHAPTER IV

Small and Intermediate Scale Mapping

80. Definition and Remarks

The terms small scale and intermediate scale have been defined in various ways, but for the purposes of the discussions that follow they will be considered as ranging from 1:96,000 (1 inch = 8000 feet) to 1:6000 (1 inch = 500 feet). Scales smaller than 1:96,000 are here considered suited for only a reconnaissance type of survey and are, therefore, treated under that general heading. Similarly, those larger than 1:6000 demand a refinement in field methods not consistent with other intermediate scales and so are included with the larger scales.

By far the greater part of plane table mapping done by government surveys and commercial organizations falls in the categories of small and intermediate scales as here defined. On a scale of 1:96,000 the smallest distance that can be plotted under field conditions is about 100 feet; while on a scale of 1:6000 the smallest plottable distance is about 5 feet. It is obvious that mapping requirements will be satisfied within so broad a range.

The methods described in Chapters II and III are applicable to small and intermediate scales. Some of the methods are used in connection with all these scales, others are more adaptable to the smaller scales of the group, and still others to the larger. The application of the various methods for different scales will necessarily have to be judged by field conditions, the abilities of the instrumentman, and many other factors which have a bearing on the accuracy of the map. Experience has shown quite clearly that

in many instances where poor results have been attributed to "inaccurate" methods, the actual difficulty lay in the fact that a method used was designed for a different kind of work. Any method which will permit the determination of a distance with an error not exceeding 25 feet is applicable for scales in which the expected plotting error is 100 feet. Conversely, the same method would be wrongly applied if used with a map scale which permitted plotting to within 5 or 10 feet.

81. Planning a Survey

The great variety of factors which must be considered in planning a large mapping project makes it impracticable to formulate rules that will aid materially in the problem. However, if the interdependencies of the various factors are fully appreciated and the main objectives are clearly defined, estimates of cost and time will be more accurate and the resulting map will be more satisfactory.

82. Purpose of the Survey

The question "What is a good map?" can be correctly answered "Any map which fully satisfies the purpose for which it was made." It follows, then, that in order to make a "good" map it is first necessary to define very clearly what purpose the map is to serve. This is the first step in planning a survey and probably the most important, for upon it will depend most of the other decisions that must be made. Although the primary purpose of the map should be kept uppermost in mind during the planning stages, and later during the field work, the principal advantage of the plane table method, namely, that of accurately and rapidly obtaining details of the terrain, should not be neglected. For example, the main object may be to obtain geological data; but if a more useful map can be made by sketching hydrographic, topographic, or cultural features, this should be done, so long as it does not seriously affect the general progress of the work.

83. Scale of the Map

In selecting the proper scale for any specific project a number of pertinent factors must be taken into account: the required horizontal limits of error in plotted positions, and vertical limits where the elevations are determined by intersection methods; density of details to be shown; distribution of primary control points; slope angles and contour interval; and field methods to be used.

Limits of Plotting Errors

The limit of error in plotted positions has a direct bearing on the size of the scale that should be adopted. It has been concluded, after examining plotting by many experienced topographic engineers, that the highest precision that can be expected is about 0.01 inch, and the average about 0.015 inch. To plot distances consistently within these limits of error demands considerable care in placing the plotting scale and pricking the new point. However, since such standards can be maintained, the figure 0.015 inch can be used as a basis for predetermining the scale which will permit plotting to any required accuracy. For example, if it is desired to keep plotting errors within 20 feet, the smallest scale that should be considered is $20/0.015 = 1333$ feet per inch. The nearest standard scale that will insure this precision is 1:12,000 (1 inch = 1000 feet). If the permissible plotting error is 50 feet, the corresponding scale would be $50/0.015 = 3333$ feet per inch; the nearest practical scales being 1:31,680 (2 inches = 1 mile, or 1 inch = 2640 feet) and 1:48000 (1 inch = 4000 feet). The plotting errors of these two scales, computed on the basis of 0.015 inch, are 36 feet and 60 feet, respectively.

When distances are obtained by scaling on the sheet for the purpose of computing difference in elevation by intersection methods (Art. 44), any error in distance results in an error in the computed difference in elevation. For a given error in distance, irrespective of the length of line scaled, the resulting error in

difference in elevation varies according to the size of the vertical angle, as shown in the table below. This table is based on a scaling error of 0.01 inch.

Scale	Probable Scaling Error	Maximum Vertical Angle	Probable Error in Difference of Elevation
1:96,000 (1″ = 8000′)	80 feet	1°	1.4 feet
		5	7.0
		10	14.1
		20	29.1
1:31,680 (2″ = 1 mile)	25 feet	1°	0.4
		5	2.2
		10	4.4
		20	9.1
1:12,000 (1″ = 1000 feet)	10 feet	1°	0.2
		5	0.9
		10	1.8
		20	3.6

Density of Details

In some kinds of plane table mapping the size of the scale will be determined not so much by precision requirements as by the amount of sketching detail that can be drawn to scale on the sheet. A good example would be the complicated pattern of topographic contours or outcropping geological formations in typical badlands terrain. Others would be complicated irrigation systems, land classification, oil fields, etc.

Distribution of Primary Control Points

Where the primary control has been previously established by transit or theodolite triangulation, it might be necessary to use a small scale in order to plot the required number of points on each

plane table sheet. Unless at least two or three such control points can be plotted on the sheet, they have little value in so far as expansion of the primary control by plane table methods is concerned.

Slope Angles and Contour Interval

If a topographic contour map is to be drawn with a specified contour interval, the grade of the slopes in the map area in some instances will be the determining factor in selection of a suitable scale.

The ability to draw closely spaced, yet well-formed contours, with a sharp sketching pencil is highly developed in most professional topographers, and for this reason, it is not uncommon to see penciled contour lines so closely spaced on field sheets that they cannot be inked. In such cases, the selection of a suitable scale might depend largely upon the ability of the draftsman who will ink the field sheets or draw the final map.

The ground spacing of contours is easily obtained by multiplying the contour interval by the natural cotangent of the angle of slope. For example, assume the slope angle is 20 degrees and the contour interval is 20 feet. The horizontal distance between contours (on the ground) is 20×2.75 ($= \cot 20°$) $= 55$ feet. On a scale of 1 inch $= 2000$ feet, the distance between contours on the sheet is $55/2000 = 0.027$ inch. Since contours spaced 0.02 inch apart can be inked without difficulty, the relationships cited above would present no particular problem from this viewpoint.

Field Methods to Be Used

It has been intimated earlier that the scale of the map should be consistent with the proposed field methods. Generally speaking, either the field methods or the map scale may be adjusted to bring about the correct relationship; but when the field methods are definitely specified in order to maintain a certain desired degree of precision in the work, the scale chosen should be of such size that plotting accuracy will be in proper balance with instrumental

measurements. When stadia measurements are to be determined to the nearest one-tenth of a foot on the rod, then the scale of the map should be large enough to plot a distance of 10 feet. On the other hand, if an error of 100 feet in the plotted position of any point is permissible, it is of little practical value to require field measurements of distance within the nearest 10 feet, except when stadia determinations are made also for the purpose of computing difference in elevation by means of vertical angles.

From the preceding remarks it is evident that the specifications for a plane table survey should exhibit a nice balance between scale and field methods. There are few examples where one can logically be considered without regard for the other. Even during the planning stages of a mapping project, it may be necessary to alter proposed methods or instrumental limits of error in order to bring about a better adjustment to a given scale.

84. Primary and Secondary Control

The primary horizontal control for the survey may be transit traverse, transit or theodolite triangulation, plane table traverse or triangulation, or a combination of these methods. When transit or theodolite control is used, it is necessary to provide for plotting the points accurately on the sheet. (Methods for plotting transit stations are given in Chapter XII.) If the scale is large and the area small, rectangular coordinates may be drawn on the sheet and the points plotted by latitudes and departures. If the scale is small and the area to be mapped is very extensive, it is better to construct a polyconic projection on the sheet and plot the transit or theodolite stations by geographic coordinates. Either of these methods of plotting is better than protractor or tangents, used with bearings and distance between stations, because the plotting of each station is entirely independent of the plotted positions of other stations.

For the plane table triangulation it is often advantageous to use a plane table somewhat larger than usual. The reason for doing this is that a larger number of the triangulation stations can

be located on a single sheet, thus permitting a greater number of observations in the field for the purpose of checking the positions of plotted points. The large sheets have the further advantage of reducing errors due to transferring points from one sheet to another in the control series. The disadvantages in the large boards are that they are awkward to use in the field and are less stable in the wind or when the alidade is placed near the edge.

A practice sometimes employed, and one that is not without merit, is that of using a smaller scale for the triangulation than for the detailing sheets. Obviously, a certain degree of scaling accuracy must be sacrificed in reducing the scale; but this effect may be more than neutralized by the relatively larger number of triangulation points that will fall on the sheet. The best method for transferring stations from the triangulation sheets to the detailing sheets is by rectangular coordinates drawn to the scale of the respective sheets.

Specifications for the survey should include provisions for secondary control, of what it shall consist, the allowable limits of error, and the field procedures that are to be employed. Generally speaking, the precision of secondary triangulation and plane table traverse should be maintained about midway between the primary control and the more important detailing though, of course, this will vary greatly in different projects. The secondary control of many surveys is simply the traverse or three-point locations made for the purpose of sketching map details.

Because of the manner in which systematic errors or, periodically, compensating errors, can quickly get out of hand, the overall primary control is ordinarily established by much more precise methods throughout than the scale of the map or the requirements of the sketched details would suggest. In preceding discussions a few of the many sources of error have been mentioned. Any one of these errors for a single observation, if well within the prescribed limits for the most important map features, would suggest no hazard of consequence. But it must be kept in mind that so-called compensating errors may operate in only one direction for a

number of times and that a number of different types of errors may occasionally operate together so that the combined effect is cumulative rather than compensating. Therefore, the primary control should be established by methods that will insure a relatively high degree of precision so that it will serve not only as a means of maintaining greater accuracy in the general map structure, but also as a yardstick for determining the amount of error at any given point. Unless the primary control is established by the safest and most accurate methods, it cannot logically serve as a standard for comparison or a means for adjusting the control of lower rank. For these reasons, it is good practice to run the primary control by approximately the same methods, whether the map scale is 1:96,000 or 1:6000.

85. Funds Available

The controlling factor in planning and conducting the project may be the amount of money available for the work. If the financial margin of safety is small, with consequent danger of exhausting the funds before the planned work is completed, steps should be taken to insure either a less detailed map of the whole area or completion of the more important parts of the area according to the original specifications.

Organizations which have engaged in extensive mapping operations for many years, such as some of our government surveys, have at their disposal the cost records of various kinds of mapping in a number of dissimilar regions. These statistics either are, or readily can be, broken down so that the cost per square mile may be determined for any phase of the work. The cost estimates of a proposed survey are made by referring to the records of a similar project and making minor adjustments according to differences in topography, scale, personnel, etc. Unfortunately, such statistics are not available to other organizations whose mapping activities and experience have been limited to comparatively restricted areas, less diversified terrains, fewer kinds of maps, or less experienced personnel; and the task of estimating costs is, therefore, more difficult.

The ordinary procedure is that of breaking down the project into the various component parts, such as triangulation, levels, detailing, transportation, etc., and if need be, analyzing each of these in terms of individual operations. In this way it is possible to arrive at fairly accurate estimates of cost per square mile, and thence to the cost of the whole project. Although a detailed example would be of doubtful value, the following may serve a useful purpose in illustrating a method of approach to the problem.

Primary Plane Table Triangulation

Area to be surveyed	300 square miles
Number of stations to be established	50
Average time for occupying each station, including time to move from one station to another	3 hours
Average number of stations completed per day	2½
Required number of full work days	21
Estimated loss of time due to unfavorable weather, moving, reoccupation of stations, etc.	10 days
Total estimated time for one field party	31 days
Daily cost of maintaining one party in the field (on basis of one instrumentman and two assistants)	$ 24.00
Total personnel costs	744.00
Transportation. 80 miles per day at $0.05 per mile	125.00
Incidentals: signal towers, flagging, repairs, etc.	100.00
Total	$969.00

Thus, the estimated cost of triangulation for this particular project is $19.00 per station, or $3.20 per square mile. Each main division of the survey is treated in a similar manner. Obviously, some knowledge of the topography and accessibility of the area is necessary in order to make a reasonably accurate estimate of the cost of mapping it.

86. Time Limitation

As in the case of costs, the time available for the field work may be the dominant factor in planning. In many of the regions where plane table mapping will be carried on extensively, climatic conditions are favorable for field work only 3 to 5 months out of

the year. If a project is not completed in a single season, the work should be suspended in a condition that will permit continuance at a later date with a minimum of duplication of effort. Traverse and level ties should be completed, important stations well marked on the ground, sheets properly matched, and field notebooks brought up to date, checked, and indexed. These and many other operations are very important when work must be suspended for a period of several months. The loss of a few stations may result in costly delays when the work is resumed.

87. Preparation of Plane Table Sheets

The best sheet for triangulation is the double-mounted zinc board, which consists of a thin sheet of zinc on both sides of which a fine grade of drawing paper is firmly glued. Atmospheric humidity and temperature have no appreciable effect on the scale, and the drawing surfaces are hard and smooth. The main objections to this type of sheet are that they are relatively expensive, heavy to carry in the field, and because of their extreme weight are difficult to ship or file.

Double-mounted sheets on cloth, mentioned in Art. 17, are the most widely used and generally satisfactory. The paper used in the better grades of double-mounted sheets is made so that differential shrinkage is reduced to a minimum. Despite this precaution, even the highest grade sheet usually has a tendency to contract and expand more in one direction than another, and for this reason it is wise to prepare them for field work as follows.

As stated before, the sheets should be seasoned, fully exposed to the air, for as long as practicable before the projection is drawn or any control points are plotted. It is very bad practice to "sweat" or stretch a sheet on the plane table, for one mounted on the board in this manner is bound to exhibit irregular scale changes later. In seasoning the sheets, hang them out in the open, away from walls, preferably out-of-doors where they are exposed to direct sunlight. Sheets piled one on top of another are likely to adjust to local climatic conditions only around the edges, and

consequently, the scale changes which occur later will be erratic and difficult to adjust.

At the time the first plotting is done, draw a sharp line about ¾ inch from the edge along one end and one side, or both ends and both sides. With the plotting needle and scale (or beam compass and standard bar scale) divide the lines into intervals of 1 inch, pricking a small hole at each of the intervals. As the field work progresses and the sheet is subjected to varying weather conditions, some shrinkage or expansion will probably take place, and the measured lines along the margins provide a means of determining the exact amount at any time by remeasuring the intervals.

88. Recording Scale Change

When the coordinate lines are remeasured, the lengths of the intervals and the dates of measurement should be recorded so that proper adjustments can be made in the positions of points plotted during different stages of the field work. These measurements and dates should be recorded on the sheet within the interval to which they apply, as shown in Fig. 68. It is fully as important to record the dates when no scale changes are observed as when large changes are noted. The length of time between measurements of the coordinate lines must be judged by conditions under which the field work is done and the kind of sheet being used. Under equable weather conditions, with a sheet that has been well seasoned, scale changes may be so small as to require no adjustment, even though the work may extend over a period of many weeks. On the other hand, if a poorly made, unseasoned sheet is used, and weather conditions are variable, large changes in scale may take place within the period of two or three days. The

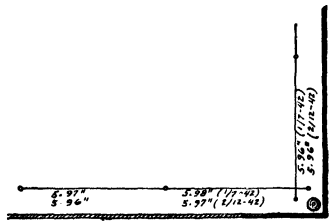


FIG. 68. Method of determining and recording change of scale on the plane table sheet.

safest procedure is to measure the lines every few days, and make permanent recordings of the measurements on the sheet at least every week or two.

89. Adjustment of Sheet Distortion

When the measured intervals at both ends and both sides of the sheet show the same amount of scale change, there will be no change in the relative positions of points plotted on the sheet, but only a change in the *scale* of the map as a whole. In this case, the plotting of new positions must take into account the change in scale that has affected earlier work, i.e., the new positions must be plotted on the current scale of the earlier work in order to maintain correct map relationships. For example, assume that the original 6-inch intervals are found to measure 5.92 inches. It is desired to plot a stadia distance of 2850 feet. Thus, the new scale is $592/600 = 0.99$ of the original scale used, requiring a reduction of 1 per cent in new distances plotted on the sheet. Using the regular plotting scale, the distance actually laid off on the sheet is $2850 - 28 = 2822$ feet.

Now in the case cited above if the distance to an intersected point is obtained by scaling for the purpose of computing the difference in elevation, the scaled distance is multiplied by the factor $600/592 = 1.13$, or an increase of approximately 1 per cent.

When the scale change along one coordinate is greater than along the other, the interrelationships of plotted positions are distorted, that is, the map is elongated in one direction. If the difference in scale along the two coordinates is large, the situation is difficult to cope with, because the amount of correction varies according to the direction of the line under consideration. A greatly exaggerated situation of this kind is shown in Fig. 69. For the purposes of discussion, the north-south ordinate is supposed to have undergone no change in scale, while the east-west ordinate has contracted. The correct positions of plotted points are i, a, b, c, and d, and the positions after shrinking of the sheet are i', a', b', c', and d'. It will be seen that the directions of lines parallel to the

coordinates are unaffected by shrinkage. The scale along north-south lines remains unchanged, while the maximum scale change occurs along east-west lines. The maximum error in azimuth occurs in lines approximately 45 degrees to the coordinates. Correctly plotted points lying on the same straight line, such as *a*, *i*, and *c*, will still lie on a straight line after distortion, but the orientation of the line will be rotated with reference to other points on the sheet, as in the case of the line *a'*, *i*, *c'*. Therefore, if it is desired to plot new locations on a sheet whose earlier work is distorted, attention must be given to the orientation of the ray drawn from the plotted position of the instrument toward the new point in order to maintain correct relationships between the new and old work. In the figure, if point *e* is to be located from instrument station *i*, the table is reoriented by backsighting on *b'*, which is approximately on the same range line. The point *e* is plotted on the ray thus drawn, but the scale reduction must be made nevertheless.

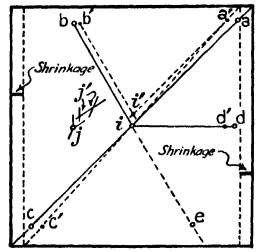


FIG. 69. Distortions resulting from sheet shrinkage.

When locations are made by intersection or resection, the results are unpredictable. Assume that the correct location of the instrument is at *i*. The resection rays are drawn from the distorted map positions *a'* and *b'*, and the resulting intersection occurs at *i'*. Now if the instrument is set up at a point whose true map position is *j* and rays are drawn from the distorted positions *c'*, *b'*, and *a'*, the resulting intersections occur as shown at *j'*.

No simple set of rules can be set down for handling this situation satisfactorily. The best advice is to use every possible means to prevent distortion of the sheet, for no matter what is done after distortion occurs, adjustments will be only approximations. If the sheet contains but a small amount of plotted work when the shrinkage (or expansion) takes place, the work may be transferred to a new sheet, the original scale being restored in the process. The

following coordinate method, which is used also for compilation from a distorted sheet to the final base, is probably the most accurate and convenient to use.

In Fig. 70, rectangle *B* is the distorted field sheet, and *A* is the base upon which the corrected positions of the points are plotted. Any point located somewhere in the central portion of the distorted sheet and whose position can be accurately plotted on the correct

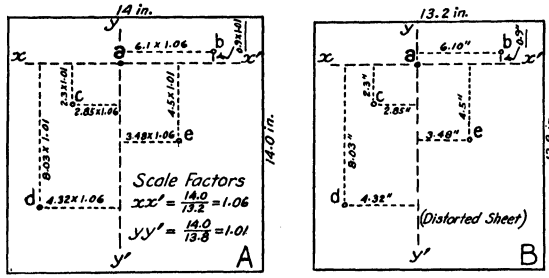


FIG. 70. Adjustment of locations distorted by sheet shrinkage.

base, such as point *a* in the figure, may be used as the starting point for measurements in making the adjustments. Through this point coordinates are drawn parallel to the edges of the plane table sheet. The scale factors are determined as described at the beginning of this Article, the factors here being 1.06 and 1.01. Using a scale graduated in decimals of an inch, latitudes and longitudes (departures) are measured from the axes xx' and yy' for all the points on the distorted sheet. These distances are then multiplied by the appropriate factors, and the results plotted from corresponding xx' and yy' axes on the compilation sheet. Points *b*, *c*, *d*, and *e* are thus transferred and corrected in the figure cited.

90. Stadia Rods

The subject of stadia rods is probably the most controversial one among plane table surveyors, and, for that matter, a great deal of latitude in design is permissible within the broad limits of practicality, for much depends on the type of work being done,

the character of the country, the kinds of transportation used, etc. However, there are four features which should be considered in evaluating a rod for general use in small and intermediate scale work: *legibility, durability, portability,* and *weight.*

Legibility

In mapping on scales ranging from 8000 feet per inch to 500 feet per inch, the distances determined by direct or indirect stadia intercept may vary from a few feet to 6000 or 8000 feet. The permissible error in stadia readings may be as little as 0.05 foot, or as great as 2.0 feet (0.5 foot error in the quarter-interval reading). When the stadia rod is used as a level rod, the survey specifications may require that the level readings be made to the nearest 0.01 foot. After many years of experimentation under a wide variety of field conditions, certain government surveys have adopted an all-purpose design which satisfactorily meets the requirements stated above. This rod is shown in Fig. 71.

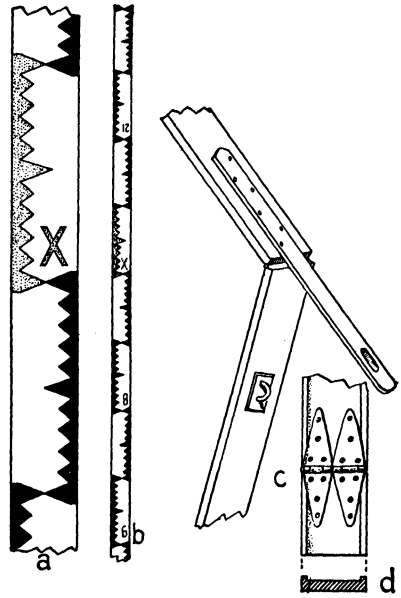


Fig. 71. Construction of the all-purpose stadia rod.

The smallest graduations are 0.05 foot, but by interpolation the rod is easily read to 0.01 foot. Using a 16-power telescope under favorable atmospheric conditions, the smallest graduations are distinct at distances up to 1200 or 1400 feet, while the half-foot marks can be seen at 2500 to 3000 feet. Because of the fact that the graduations for each foot are

staggered in alternate feet from one side to the other, the foot marks can be distinguished for nearly as far as the rod itself is visible. For convenience in close work, alternate feet are numbered; and as an aid in very long sights, the graduations at intervals of 5 feet are painted in Chinese red. Thus, in distant sights the rod appears as shown in Fig. 71b; the graduations are not visible, but the tenths of a foot are estimated by interpolation within the foot marks.

Another type of stadia rod widely used in small-scale work consists of alternate solid black and white feet, with the fifth, tenth, and, in some instances, the fifteenth graduated into one-tenth foot triangles. The graduated feet are sometimes painted Chinese red.

In addition to the type of graduation used, the legibility is affected also by the width of the board, the distinctness of the graduation lines, and the reflective properties of the enamel. The rod should be from 3 inches to 4½ inches wide and should be painted with weather-resisting, glossy enamel. Graduations are more clear-cut if embossed, as stated in Art. 174.

Durability

Because of the large dimensions of stadia rods used in small-scale plane table work, they are usually made of soft, light woods. Hinged or bolted joints should, therefore, be reinforced with thin iron plates, and the ends should likewise be bound with iron to prevent splitting. On folding types, the hinges should be as wide as the board, or two hinges used at each joint (Fig. 71c). During construction of the rod, if the central portion of the graduated face is planed more deeply than the edges, as shown in Fig. 71d, the graduations will be protected from rubbing when the rod is folded.

Portability

When the rodman must do a great deal of strenuous walking or climbing, the weight of the rod is an important consideration.

Rods of extreme widths, though perhaps easier to read at great distances, are very burdensome for the rodman to carry, and it is doubtful that their use improves the efficiency of the party as a whole. If the rod is to be carried on pack animals, inside automobiles, or other restricted spaces, it must be so constructed that it may be knocked down or folded. For most work, the folded length should be no more than 7 or 8 feet, and preferably less.

91. Stadia Constants

Figure 72 illustrates the principle of the stadia in relation to the optics of the telescope. In the figure, L is the center of the objective

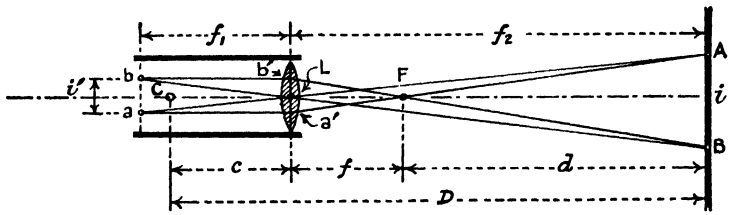


FIG. 72. Optical constants of the alidade.

lens, a and b the stadia wires, and A and B are the points on the stadia rod apparently cut by the stadia wires.

If a point-source of light were at a, one ray parallel to the optical axis of the telescope would be deviated by the lens so that it would intersect the optical axis at F. Likewise, a ray originating at b, also parallel to the axis, would pass through F. This point F is called the *principal focus* and for a given lens is at a fixed distance in front of the lens. This distance is called the *focal length* of the lens. Rays originating at a and b which pass through the center of the lens at L emerge without deviation. Thus, the two rays from a intersect at A, and the two from b intersect at B. The pairs of points a and A, and b and B, are called *conjugate foci;* and the distance f_1 and f_2 are called the *conjugate focal lengths.*

Now if light rays originate at A and B, the above conditions will be reversed, and the rays will be brought to focus at a and b.

That is, rays starting at one conjugate focus and traveling parallel to the optical axis and through the center of the lens will intersect each other at the opposite conjugate focus.

In the figure, the following relations are evident:

By similar triangles

$$\frac{f}{i'} = \frac{d}{i}, \quad \text{or} \quad d = \frac{f}{i'i}$$

The horizontal distance d is from the principal focus F to the rod. Therefore, the horizontal distance from the center of the instrument to the rod is

$$D = \frac{f}{i'} i + (f + c)$$

For all practical purposes the conjugate focal length f_1 may be considered equal to c, the distance from the center of the instrument to the lens when the latter is focused on a sight of average length.

The optical or $(f + c)$ constant is usually determined by the manufacturer of the instrument and recorded on the inside of the instrument case. For most large-scale stadia work the $(f + c)$ constant is assumed to be 1 foot, and it is entirely neglected in small and intermediate scale work, except when a very precise determination of difference in elevation is being made by means of vertical angle.

Most of the newer types of alidades are internally focusing, and in this optical arrangment the $(f + c)$ constant is very near zero; so no correction is necessary.

92. Determination of Stadia Interval Factor

In previous discussions it has been assumed that when the intercept between the upper and lower wires is 1 foot, the rod is 100 feet from the center of the instrument. It has just been shown that the $(f + c)$ constant must be added to the observed stadia distance. Now, in addition to the $(f + c)$ constant, is another con-

sideration which often is far more important. It is the number by which the stadia intercept is multiplied to give the stadia distance from the principal focus to the rod and is called the *stadia interval factor.*

In Fig. 72, assume that the stadia interval factor is 100 and that the horizontal distance d from the principal focus to the rod is 100 feet. The intercept on the rod i should, therefore, be 1 foot. However, this intercept is entirely dependent on the spacing of the stadia wires a and b. If the stadia wires are too closely spaced, then the factor by which all observed intercepts must be multiplied in order to give the correct stadia distances will have to be correspondingly larger. Thus, if $d = 100$ feet and $i = 0.97$ foot, the stadia interval factor for the instrument is $100/0.97 = 103$. If an observed stadia intercept with this instrument is 15.3 feet, the stadia distance from the center of the instrument is

$$15.3 \times 103 + (f + c) = 1577 \text{ feet}$$
$$(1 \text{ ft.})$$

The stadia interval factor is determined by observations as follows.

Set up the plane table on a stretch of level ground. Starting at a point directly beneath the principal focus (1 foot in front of the center of the instrument), measure a line 500 feet long with a steel tape and set a stake at each 100-foot interval. Now carefully determine the stadia intercepts with the rod held on the stakes. Great care must be taken that the telescope is perfectly focused for each distance so that no parallax is present. The rod used for this purpose should be graduated so that readings can be made to the nearest 0.01 foot.

EXAMPLE:

Observed stadia intervals: 0.98, 1.96, 2.94, 3.92, 4.89
Taped Distances : 100, 200, 300, 400, 500
Sum of observed stadia intervals = 14.69 feet
Sum of taped distances = 1500 feet

$$\text{Stadia interval factor} = \frac{1500}{14.69} = 102$$

93. Cover Sheets or Sheet Masks

Plane table sheets are usually exposed to a great deal of dust, which, together with perspiration from the hands, tends to soil and discolor the paper. This can be avoided by the use of cover sheets or so-called sheet masks. The best material for this use is heavy brown wrapping paper, cut to the size of the plane table.

Fasten the cover sheet to the plane table by the thumbscrews in the same manner as the regular plane table sheet. The portion of the plane table sheet where work is being done is exposed by tearing out a hole in the mask, as shown in Fig. 73. As the work

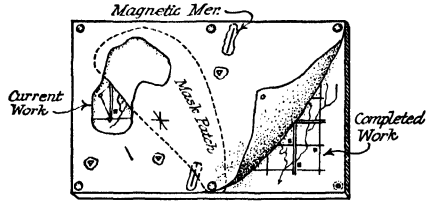


FIG. 73. Use of the cover sheet or mask and the movable patch.

progresses, the hole is enlarged. After a considerable area of the plane table sheet is thus exposed, a flap or patch of the brown paper is inserted beneath the mask. If this patch is fastened by only one of the thumbscrews, as shown in the figure, it may be rotated so that new map work is covered as soon as it is completed. Small holes are cut near the ends of the magnetic meridian and at triangulation points being currently used.

Intersections of side shot value may be made by drawing the rays on the cover sheet, thereby avoiding unnecessary lines on the map. When the intersected point is obtained on the cover sheet, the location is pricked through on the plane table sheet. Descriptive notes may be written on the cover sheet to aid in identifying natural objects selected for intersection.

94. Overlay Sheets

In order to avoid congestion of lines, points, numbers, etc., on the plane table sheet, much of the map data may be plotted on

overlay sheets, which consist of vellum or tracing paper cut in sheets the size of the plane table. Small crosses are drawn at the four corners of the plane table sheet and then carefully traced on the oversheet so that the latter may be repeatedly registered exactly over the plane table sheet. In addition to these register marks, it is desirable to trace the magnetic meridian.

After a portion of the area is mapped, the overlay is correctly placed on the plane table, and, using the underlying map as a guide, supplemental map data, such as wooded areas, areal geology, and irrigation systems, are sketched on the tracing. If it is inconvenient to carry the large sheets into the field, small pieces of tracing paper, which can be carried in the notebook, are used for the overlay material for each day's work, and the data on the small sheets are traced on the large overlays. Instrument and rod stations may be used for registering the various sheets.

It has long been the practice of the U.S. Geological Survey and other mapping organizations to require a separate oversheet for each main class of map data. Thus, on one sheet will be shown only the geographic names of the area, another will show only the elevations which are to be inked on the final map, while a third may show telegraph, telephone, and power transmission lines, and railroads with their respective names. Though it is usually not necessary to break down the material to this extent, the overlays should never be crowded. Perhaps two or three classes of subjects can be clearly drawn on one sheet.

In addition to the regular classified oversheets, there should be one on which may be plotted a variety of supplementary data and explanatory notes. During inking of the plane table sheet, it is often advisable to trace areas where lines, numbers, and map symbols are congested so that the inking of certain features will not destroy the record of others. On the "General Information" sheet a graphic record of horizontal adjustments in traverses or other positions may be made for future reference.

Through the use of overlays, a vast amount of map data can be compiled with but little extra time spent in the field. Since the overlays are not in themselves finished maps, it is usually not

necessary, or even advisable, to ink them. They should be clearly legible, but little time should be spent in drafting solely for the sake of appearance. Colored pencils can be used to much advantage in blocking out areas, such as woodlands, fields, and pastures. Likewise, different types of fences, canals, and other linear features are distinctly classified by different colored lines.

Each overlay should bear the title of the map area, the number of the plane table sheet which it supplements, and a subtitle descriptive of the information which it contains.

95. Preliminary Preparations

Before field work is begun, all information on the map area should be examined. Such information would include other maps, township plats and measurements of the U.S. General Land Office, records and maps from county or city engineers, U.S. Coast and Geodetic and Geological Survey descriptions of bench marks, triangulation and traverse stations, and boundary marks, various data from state surveys, aerial photographs, and any other material that might be of use in planning the survey or supplementing the work to be done in the field. Much of the easily available data may be compiled on vellum to the scale of the proposed new work so that usable portions can be readily transferred to the plane table sheets for field checking.

The compilation of earlier work to the scale of the plane table sheet should be made on vellum so that the compilation sheet, in turn, can be registered over the plane table sheet for transferring the data. Unless extreme care is exercised, "carbon" paper is not satisfactory for transferring the compiled data for the reason that it is difficult to erase and may smudge the plane table sheet. The following procedure is recommended.

By means of pantograph, photostats or photographs, proportional dividers, etc., reduce the scale of earlier work to that of the proposed plane table survey and compile the desired data on a sheet of vellum or thin tracing paper, using a sharp pencil of about 6H hardness. Turn the tracing back side up and retrace the

lines showing through with a comparatively soft, pointed pencil of hardness H or 2H. Now place the tracing over the plane table sheet, register it correctly by locations common to both sheets, and fasten it securely in this position with thumbtacks or drafting tape. Retrace the lines again with a sharp pencil of hardness 6H to 8H, when the softer graphite on the back side will be transferred from the tracing to the plane table sheet. In the last step care should be taken not to cut or impress the lines too deeply, for later adjustments would then be difficult to make.

A method of transfer used by the U.S. Geological Survey is very convenient where the necessary facilities are at hand. The compilation tracing is carefully inked with black drawing ink. The plane table sheet is sensitized in a dark room with a photographic emulsion. When the sheet is dry, the tracing is placed over it and is then exposed to the light. The plane table sheet is developed, and the resulting pale blue lines serve as "copy" to be checked in the field. The portions to be retained in the final map are penciled along with other field work. The blue lines not retraced with pencil are omitted when the sheet is inked.

96. Sheet Arrangement and Index Map

When the map area is so large as to require a number of plane table sheets, an index map showing the arrangement of the sheets should be prepared. Any existing small scale map of the region is usually satisfactory for this purpose. Control points, such as triangulation or traverse stations, should be plotted on the index map as accurately as is consistent with the scale or precision of the map so that the best possible arrangement of sheets, with respect to available control, can be attained. A convenient method for making the index is to cut out of stiff paper or cardboard rectangles representing the exact area of the plane table sheet on the scale of the index map. These rectangles may then be moved about until complete coverage of the area, sufficient overedges, inclusion of control stations, etc., are assured. For example, assume that the scale of the index map is 1 inch = 4 miles, and the pro-

posed mapping scale is 2 inches = 1 mile, and the size of the plane table sheet is 18 inches by 24 inches. Leaving a 1-inch margin, each sheet represents an area whose dimensions are 8 miles by 11 miles. On the scale of the index map this area would be represented by a rectangle 2 inches by 2¾ inches.

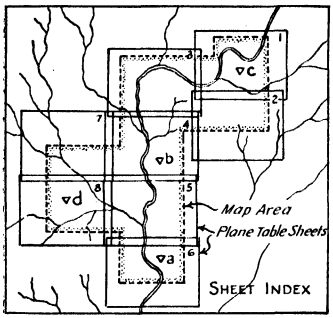


FIG. 74. Plane table sheet layout on a small scale map.

A sheet index map is shown in Fig. 74. The map area is shown by the dash lines, and the plane table sheets by solid lines. Points *a*, *b*, *c*, and *d* are control stations. Sheets should be numbered according to a definite geographical plan, rather than by the order in which each sheet is begun. Thus, it might be more convenient to begin the survey on sheet number 6, followed by 5, 8, etc. Unless an index map is made before the actual field mapping is begun, it is difficult to predetermine the designations of the various sheets, maintain consistent amounts of overlap, or insure complete coverage without gaps between the sheets.

97. Accessory Materials

Before starting into the field it should be ascertained that all essential materials have been procured. Every effort must be made to anticipate the needs of the survey parties so that unnecessary delays will be avoided. The following list is by no means exhaustive, but there are few projects that could not be carried on efficiently if all these materials were available; and, likewise, most projects will require the great majority of articles tabulated.

Scales—celluloid faced boxwood, flat type graduated for convenient use on the map scale, and one long engineers scale in inches.

Pencils—Hardness ranging from 2H to 9H, for use on the plane table sheet and field notebook.

Plotting needles—fine embroidery or beading needles.
Colored pencils—several each of four or five colors, wax base, waterproof.
India (waterproof) inks—black, blue, brown, red, and green.
Pens—fine and medium points and appropriate holders.
Protractor and celluloid or steel triangles.
Straightedge—steel, preferably beveled, at least 2 feet long.
Hand lens—magnification of 1½ to 3 diameters.
Drafting tape, thumb tacks, rubber bands, erasers, and other miscellaneous materials.
Vellum and light-weight tracing paper for overlays and transfer sheets.
Crayon—black, blue, or red, for marking stations on the ground.
Cotton flag material—white, bright red, orange, etc.
Field notebooks, plane table sheets, Manila wrapping paper for cover sheets, stadia reduction tables and trigonometric tables.
Stadia slide rule or computer, if available.
Iron wire—several hundred feet, medium gage, for signal guys.
Small instrument screwdriver, pointed pincers, steel adjusting pins.
Repair materials for alidades and rods—striding level tube, plaster of Paris, glue, shellac, spider web, watch oil, etc.; black, white, and red enamel, assorted wood screws, fine copper wire, and other items that might be useful in making emergency repairs on instruments and equipment.

The variety and amount of materials taken into the field should be gaged by the remoteness of the map area. If the area is far removed from a source of supplies the preparations must be more thorough.

98. Sketching Pencils

Pencils of 8H and 9H hardness are adapted for use on hard surfaced paper in regions where humidity is very low, as in much of our western and mountainous country. However, since the paper of plane table sheets tends to become soft when the humidity is high, it may be found that the 8H or 9H pencil cuts the surface of the sheet. Therefore, even in regions where hard pencils can usually be used satisfactorily, there may be damp days when it is advisable to use softer grades. In selecting the proper hardness, be

guided by the following. A moderately sharp point should make a clearly legible line that is gray in color but not impressed into the surface of the sheet. If the line is black, the pencil is too soft, and the lines are liable to smear as work progresses. If the point tends to make grooves in the paper, the pencil is too hard, and erasures and later inking will be more difficult. After sketching has been done with a soft pencil, it is advisable to remove the cover sheet from time to time to ascertain that the lines remain legible and, if necessary, to retrace them with a harder pencil.

Freehand sketching should be done with a sharp conical point that has been polished on a block of wood, leather, or the Manila paper of the cover sheet. A pencil freshly sharpened on sandpaper will make lines uneven in both width and tone. Straight lines made by means of a ruler or straightedge should be drawn with a chisel- or wedge-shaped point with the flat side of the point held firmly against the straightedge.

Most sandpaper pencil pointers are too coarse for sharpening the pencils used on the plane table sheet. Emery cloth of grade 600 or 700 is more satisfactory. Rustproof metal pencil files or nail files are convenient to carry in the pencil case and produce a good drawing point.

99. Special Plane Table Sheets

In rainy climates it is sometimes necessary to proceed with the field work while rain is actually falling. Under such climatic conditions paper sheets are entirely unsatisfactory. Celluloid and other plastic sheets having a specially prepared drawing surface are available for this type of work. Because of the fine grain of the celluloid sheet, pencils of medium hardness (4H to 6H) are most adaptable. These sheets are subject to rapid and drastic changes in scale, but the variations are ordinarily uniform, so adjustments are not difficult. One of the main disadvantages of celluloid sheets is that the pencil lines are easily rubbed off or washed off when the sheets are wet. A common practice is to transfer each day's work to a compilation sheet, which is kept indoors.

Cellulose acetate about 0.008 inch thick, having one surface frosted so that it will readily take ink, is probably the best material for the compilation sheet.

100. Practical Field Methods

As stated earlier, the methods described in previous chapters are adaptable to small and intermediate scale mapping. The main purpose in following articles is to present practical field methods that have not heretofore been described. It will be observed that certain methods are to be used only occasionally, according as field circumstances or the scale of the map demand. Some of these are unnecessarily precise and, consequently, too slow for practical use with small scales, being designed primarily for intermediate scales; the reverse is true for other methods. All the procedures discussed have been thoroughly tested under average field conditions and have been found entirely adequate when properly employed. As space does not permit outlining the many diverse conditions that are encountered from time to time in the field, much is left to the judgment and resourcefulness of the instrumentman in the proper application of the methods. Like the "good map," a "good" method may be considered as one which accomplishes the purpose *for which it was devised.*

101. Orientation by Backsight

This method of orienting the table was described in Art. 56 in connection with triangulation. The same principle is employed in backsight orientation for traversing. The procedure is as follows.

The plane table is set up on the initial point and oriented by the compass* needle. The fiducial edge is rotated about this point until the rod, held on the first turning point, is exactly bisected by the vertical wire. The ray to the turning point is drawn in the

* This is done when the directions on the map are not definitely fixed by previously plotted points or lines. If a projection or geographic meridian is already plotted, the first orientation should be obtained by sighting from one station to another, both of which are accurately plotted on the sheet.

usual manner, and in addition to this line, a short score is drawn along the fiducial edge near each end of the blade. After observations for distance and difference in elevation are completed, the tripod is next set up over the turning point, and the rod is held on the first instrument station. The table is leveled and then oriented by aligning the alidade on the two scores (reversed from the first position), and rotating the table until the rod is cut by the vertical wire. The table is clamped by means of the lower Johnson head wing nut. After clamping the table, always check the orientation by ascertaining that the vertical wire is still on the rod. This procedure is repeated at each turning point, every one of which must be occupied by the plane table.

The elevations of the turning points are obtained by direct instrumental observations; but the H.I.'s are determined by measuring with the rod the height of the center of the telescope above the turning point hub and adding this amount to the elevation of the turning point.

102. Discussion

In both theory and practice, the backsight method is the most accurate means of orienting the plane table. But unless every operation is performed with extreme care, errors may develop which far surpass in magnitude those which are liable to occur in compass work. Once an error is introduced in the work, it affects all succeeding orientations and, hence, the positions of new points as well. Errors are essentially cumulative, though in the course of a long traverse there may be sufficient compensation to obscure the presence or magnitude of errors that exist in different sections of the traverse. For example, assume that an error of 10 minutes is made at the second instrument station. From this point on, the error in orientation will remain 10 minutes, and the resulting error in position will accumulate at the rate of approximately 15 feet per mile for as many miles as the work extends from the station where the error was made. Thus, at a point 7 miles away, the error in position due to this cause alone is about 100 feet.

Suppose that another error of the same magnitude, but in the opposite direction, is now made. Succeeding orientations will then be correct, but the 100-foot error in position will remain. If still a third error is made, in the same direction as the second, the error in position may gradually be eliminated so that none is indicated when the traverse is finally closed.

The following precautions should be observed when the backsight method is used. Avoid steeply inclined sights and, where they must be used, be sure that the table is level. Always utilize the full length of the blade in drawing the orientation scores and again when the alidade is reversed along the scores. Take care that the table is well centered over the turning point hub, and that the rod is held on the original hub when the rodman returns to a former instrument station for the backsight. Avoid extremely short foresights and backsights. For a sight 200 feet in length an error of approximately 15 minutes in orientation results if either the table or the rod is 1 foot off the correct ground point. When a very short sight is necessary, pay particular attention to the alignment of the instruments with reference to the points on the ground.

103. Practical Use of the Backsight Method

The most extensive use of this method is in very large scale or very precise work (Chapter V), but it is also invaluable as a means of orientation where local attraction seriously affects the magnetic needle. In many regions the most convenient route for the traverse is along a railroad, but the compass cannot be used satisfactorily because of the magnetic effect of the iron rails. This effect is greatest where the tracks run in approximately a north-south direction and is at a minimum where the tracks run east and west, due to the fact that the earth's magnetic "lines" are more strongly attracted by linear magnetic features which are nearly parallel to them. Buried iron ore deposits or other magnetic rocks strongly affect the compass needle, and where such geologic conditions prevail in or near the map area, there is no recourse but to use the backsight or similar method. In regard to magnetic deposits, it

should be emphasized that the effect on the traverse is often cumulative. For example, assume that such deposits lie roughly parallel to, and near, only one side of the traverse loop. If the magnetic effect were approximately equable along this portion, the errors in position would not be detected by checking back to the previously established points, for they, also, would be out of position a corresponding amount. This type of error might become apparent only after the traverse had progressed beyond the point where the magnetic material influenced the compass needle.

The backsight method is often used for only short portions of a needle traverse where local attraction, such as steel bridges, pipelines, or various large masses of iron are likely to disturb the compass needle, or when the needle has been damaged in the course of the work.

104. Orientation by Foresight—Range Line Method

This method has very limited application but under special conditions is useful. It is most adaptable in areas of flat topography with few obstructions to visibility. The field procedure is as follows.

The plane table is set up and oriented as precisely as possible by one of the methods previously described. While pivoting the fiducial edge about the plotted position of the instrument station, the telescope is pointed in the general direction in which the traverse is to be run. A far distant point that can be seen from any position on the proposed traverse route, such as a windmill, smoke stack, or sharp crag, is brought exactly into the line of sight. Now a long line is drawn along the fiducial edge from the instrument point toward the distant point. If this line extends to the edge of the sheet and is still shorter than the alidade blade, draw a score near the end of the blade on the opposite side of the plotted instrument station. All stations of the traverse will be plotted on the long line.

With the alidade thus oriented on the sheet, the first turning point is established by signaling the rodman exactly on the line of

sight at the desired distance toward the distant point. The instrument is next set up over the turning point, and the table is oriented by placing the fiducial edge along the long line and rotating the table until the distant point is again brought into the line of sight. This procedure is repeated for all succeeding stations. Since all stations lie on the same straight line, accumulated plotting errors can be corrected by adding several observed distances together and plotting the total from the starting point.

It is a good practice to set a flag on the first station and at several places along the traverse so that the line can be checked by reversing the alidade along the plotted line and backsighting.

If it is desired to change the direction of the traverse, the alidade is pivoted about the last established instrument station, a new line is drawn toward another distant point, and the work is carried on from here in the same manner as described above.

105. Application and Discussion

This method is useful in recovering old, or establishing new, boundary lines and marks, township lines and corners, etc., or for establishing a reliable reference line across the map area. Occasionally it may be used for staking location for telephone and power transmission lines.

There is one inherent source of error which is cumulative in character. Although it is too small to be of much consequence in most work, it is reasonable to assume that occasions will arise when it would have to be taken into account. It will be recalled from Art. 9 that a great circle is the trace on the surface of the earth made by an imaginary plane passing through the center of the earth. By definition, such a plane is vertical, and the sight line through the telescope is an element of a vertical plane. Now assume that the site of the following example is at latitude 40° north (in the central part of the United States). The plane table is correctly set up and leveled, and the telescope is directed along a line that is due west at the table. Because of the fact that the line of sight lies in a vertical plane, a series of points along this line

would delineate a great circle on the ground. If such a line were prolonged far enough, it would pass through latitude 40° south at a point on the earth diametrically opposite the starting point. Therefore, it is evident that a due west (or east) sight line at the instrument deviates from a true west (or east) line on the ground at an increasing rate away from the point of observation. Where distances of several miles are involved, the resulting gradual rotation in the orientation of the table may become serious, depending in part on the direction of the line of sight. At latitude 50° north (north of the northern boundary of the U.S.) the deviation in the sight line from a true east-west line on the ground is approximately 7 feet in the first 3 miles, 28 feet in the first 6 miles, and so on, increasing about as the square of the distance.

Sight lines in a north-south direction can be prolonged indefinitely without affecting the orientation, because geographic meridians are also great circles, whereas geographic parallels are not.

106. The Baldwin Solar Chart

This is a graphic device for determining the orientation of the plane table by means of a shadow cast across the sheet by a plumb line, folding carpenters' scale, or any other vertical object with a straight, smooth edge. The solar chart is ordinarily used where field conditions make other methods impracticable and is not recommended for continued use where the compass needle or direct sighting methods are suitable. The chart and instructions for its use are contained in Bulletin 788-E of the U.S. Geological Survey, which can be purchased from the Superintendent of Documents, Washington, D.C.

When only one copy of the chart is readily available, it should be prepared for use in the field as follows.

Mount the chart on a piece of fine-textured, preshrunk linen with rubber cement. A good grade of tracing linen that has been thoroughly washed and dried is ideal for the purpose. Apply an even coating of the cement to both the cloth and the back of the chart and allow this to dry before pressing the two together. After

the chart is thus mounted, trim the edges, and apply a thin coat of transparent varnish on both sides. This will reduce distortion in the chart due to changes in humidity, and will protect the fine printed lines from rubbing off with continued use. The chart should be kept between two sheets of heavy cardboard when not in use so that it will lie flat on the plane table.

When using a plumb line for casting the shadow, care should be taken that the line is not moved off the vertical by wind.

107. Limitations

The Baldwin solar chart is adaptable to only small-scale or other low precision work. Under certain circumstances it is moderately accurate, but does not compare favorably with other methods of orientation previously discussed and, therefore, should be considered as an emergency method.

One source of error lies in the difficulty of precisely determining the edge of a shadow cast by a plumb line, straightedge, or other device. This might easily amount to more than half a degree. It is positively essential to know the time within two or three minutes, and under certain conditions, within one minute. The errors in azimuth due to an error of three minutes in time will range from one-half degree to six degrees. Obviously, such errors are too large to meet the requirements of most projects. Errors in azimuth due to incorrect time are greatest in the middle portion of the day. Such errors are smallest at low latitudes and greatest at high latitudes; but the rate of increase in error during the mid-day hours is greatest in the low latitudes.

108. General Remarks on Orientation

Under most circumstances the accuracy of the map is not affected by any discrepancy in the parallelism between the line of sight of the telescope and the fiducial edge of the blade. The result of this condition is simply that all the lines on the sheet are rotated as a unit with respect to corresponding lines on the ground, and it

is corrected by rotating the north point an equal amount. The cases in which this lack of parallelism does affect the work are: (1) when the true north line, grid, polyconic projection, land net, and other definite map points are plotted on the sheet previous to the plane table work, and then the plane table is oriented by means of a similarly plotted magnetic meridian; (2) when two alidades having different relationships between the fiducial edge and the line of sight are used on the same sheet with the same magnetic meridian; (3) when the true north point is determined by the Baldwin solar chart.

When orienting by magnetic needle, guard against small magnetic objects carelessly left on the table or in the clothing of the instrumentman. Plotted distances should always be less than half the length of the needle used to orient the table. To check a compass orientation, after the needle has once come to rest at the index mark, set it in motion again by passing a pocketknife or other magnetic object near either end of the needle, and note if the needle comes to rest at the same place the second time. As mentioned earlier, a charge of static electricity may accumulate on the compass box cover with the result that the needle may behave in an erratic manner, or even be held tightly to the box cover. As a precaution, always pass a slightly moistened finger over the cover to remove any charge that may exist.

109. Determination of Distance and Position

The principles of the stadia and intersection methods of determining distance and position on the map are described in Chapter II; therefore, only the special adaptations of these methods will be considered in following articles. In addition to the methods already described, are several that are used frequently in small and intermediate scale work. Certain of these require considerable skill, patience, and judgment, for they are subject to gross errors if not properly performed. Despite this fact, they are invaluable as a means of obtaining distance where unusual field conditions prevent the use of more orthodox methods.

110. Distance by Gradienter Screw

The gradienter drum provides a convenient and moderately accurate means of determining distance where a direct stadia intercept cannot be obtained. It is most useful when the rod is so far from the instrument that the quarter-interval space is greater than the length of the rod, or where a part of the rod is hidden from view so that a quarter-interval intercept cannot be obtained. The accuracy of the method depends on the amount of mechanical wear in the bearings and tangent screws of the alidade, the methods of observation and computation employed, and the care with which the operations are executed. When using the gradienter drum for any measurement, always make the settings of the drum with a thrust motion of the tangent screw so that the opposing spring is being constantly compressed.

METHOD 1

(1) Set the upper stadia wire on the lowest clearly visible graduation of the rod, and record the indicated reading of the drum scale. (2) Now, by turning the gradienter tangent screw, elevate the upper wire to the top of the rod and record the drum reading. (3) Move the lower wire up to the position on the rod first occupied by the upper wire and record the drum reading. The two points on the rod used in these operations are recorded in the notebook.

EXAMPLE:

From (1): upper wire reading 2.0′; drum reading 0.02
From (2): upper wire reading 13.0′; drum reading 0.19
From (3): lower wire reading 2.0′; drum reading 0.01
Difference between rod readings (1) and (2) 11.0 feet
Difference between drum readings (1) and (2) 0.17
Difference between drum readings (1) and (3) 0.99 (one interval)

$$\text{Stadia interval} = \frac{11.0 \times 0.99}{0.17} = 64.06 \text{ feet}$$

Assuming that the stadia interval factor is 102, then

$$\text{Stadia distance} = \frac{11.0 \times 0.99 \times 102}{0.17} = 6534 \text{ feet}$$

This is the *slope* stadia distance. If the vertical angle is large, the usual correction for horizontal distance is made.

METHOD 2

(1) Set the upper wire on the top of the rod and read the drum. (2) Elevate the middle wire* to a graduation near the base of the rod, and read the drum. (3) Determine the difference between readings (1) and (2), and then turn the drum an amount exactly equal to this difference, and record the new position of the middle wire on the rod.

EXAMPLE:
From (1): upper wire reading 14.0; drum reading 0.14
From (2): middle wire reading 1.0; drum reading 0.33
Difference between (1) and (2): rod 13.0 ft., drum 0.19
From (3): drum turned 0.19 revolution farther; middle wire on rod 4.0 feet

Difference between rod readings (2) and (3): 3.0 feet.

Stadia half-interval = 13.0′ + 3.0′ = 16.0 feet

Slope stadia distance = 16.0 × 2 × (stadia interval factor)

METHOD 3

This method should be used only for a fairly rapid, though approximate, determination of distance.

(1) Set the middle wire on the lowest rod graduation, and record the rod and drum readings. (2) Next, set the middle wire on the uppermost rod graduation, or the top of the rod, and record the rod and drum readings.

EXAMPLE:
From (1): middle wire reading 1.0′; drum reading 0.03
From (2): middle wire reading 14.0′; drum reading 0.44
Difference between (1) and (2): rod 13.0 feet, drum 0.41

$$\text{Slope stadia distance} = \frac{13.0 \times 100}{0.41} = 3171 \text{ feet}$$

* This example assumes that the telescope is not equipped with a quarter-interval wire. When the instrument has a quarter wire, the operations outlined above are carried out within the quarter-interval rather than the half-interval, and the final multiplication is by 4 instead of 2.

DISCUSSION

Method 2 is the most rapid and probably the most accurate; Method 1 is quite accurate, but somewhat slower to use, as both observations and computations are more complex. Methods 1 and 2 take into account the fact that one full-interval step may or may not register as 100 divisions on the gradienter drum. In Method 1, the drum is calibrated by completing the interval step. In Method 2, the distance which the rod lacks to complete a half-interval is first measured in terms of drum units, then a similar portion of the drum is calibrated by turning off the same number of units where the stadia values can be read directly on the rod. Method 3 assumes that one full-interval step equals exactly one revolution of the drum; hence, one drum unit equals 0.01 interval. This is seldom exactly true, and frequently the drum interval will vary as much as 4 or 5 units, depending on the wear in the instrument and the position of the axis arm at the time the observations are made.

With telescopes of average power (12 to 16 diameters magnification), the error due to mechanical causes should be no greater than the usual errors in reading the rod, if the observations are carefully made according to methods 1 and 2. The drum of the gradienter is easily read to 0.005 interval by interpolating for one-half unit on the drum scale; and, unless the alidade is badly worn, readings can be made with this degree of refinement without exceeding the mechanical precision of the instrument. As stated earlier, with good visibility and a 16-power telescope, a 0.05-foot graduation can be seen when the rod is 1200 to 1400 feet distant. At 1200 feet, 0.005 interval is 0.06 foot; so it is doubtful that any advantage is gained by attempting to read the gradienter drum more closely than one-half unit.

The principal hazard in the gradienter method lies in the fact that an error in mechanical operation or reading of the drum may be added to an error in reading the rod, and then the whole amount is multiplied by a factor which is, in turn, the full stadia interval divided by the length of rod used. For example, assume that a

13-foot portion of the rod is utilized in determining a distance of 6500 feet. If the error in reading the rod is 0.4 foot (0.2 foot at each setting of the stadia wire), and the error in reading and operating the drum is one-half drum unit in the same direction, the resultant errors in distance would be, respectively,

$$0.005 \times \frac{65}{5} \times 6500 = 162 \text{ feet}$$

and

$$0.4 \times \frac{65}{5} = 200 \text{ feet}$$

or a total error of approximately 360 feet. Ordinarily, errors will not accumulate to this extent; and the example given above is to show what might be expected to happen occasionally, even though each operation is performed with reasonable accuracy. The errors in distance increase disproportionately with an increase in the distance sighted. In the example given, the error in distance is approximately 5½ per cent; but, using the same bases for computation and a rod of the same length, at 7800 feet the error would be nearly 7 per cent.

Check Readings

A distance determined by gradienter is best checked by using a different part of the tangent screw and repeating the readings. Turn the drum through two or three revolutions, then loosen the axis clamp and bring the wires back to the rod by moving the telescope with the hand. Be sure to tighten the clamp again. Make a completely new set of observations and computations by the same method formerly used. If the second computed distance does not agree with the first within the desired limits, make a third set and use the average of the three.

111. Special Methods

A number of methods, usually used in reconnaissance or other approximate work, are sometimes adaptable to portions of surveys of generally higher precision. Descriptions of these methods are given in Chapter VI.

The *micrometer eyepiece,* a useful device for determining long distances, but one not widely used because it is relatively expensive, is subject to getting out of adjustment unless very carefully handled, with consequent gross cumulative errors; and much experience in instrumental work, particularly with the eyepiece, is necessary before it can be properly calibrated and operated with consistent accuracy. The eyepiece is simply a means of measuring accurately a very small deflection angle between two successive lines of sight to the two ends of a previously measured base line or points on a stadia rod. With this angle and the measured base, the distance from the instrument to the base is computed.

In areas where thick timber or brush makes it difficult to take stadia readings for sketching detail in the vicinity of the table, the distances may be more easily obtained with a measuring line. Steel or cloth tapes are not very satisfactory under these conditions for the reason that they are too easily snarled in the brush. Cotton woven rope of ¼ to ⅜ inch diameter thoroughly soaked in hot paraffin is best for the purpose. Tight knots are tied at intervals of 50 feet, and those at 100-foot intervals are painted different colors so that the chainman can readily determine the distance, which, in turn, is called back to the instrumentman. This method is also useful for measuring distances to points covered with water, as in river, lake, reservoir, and coast sounding surveys. The plane table is set up as close as possible to the water line. One end of the line is firmly anchored near the tripod, but between the latter and the water. The other end is carried by the chainman in the boat. When a sounding is made, the line is drawn up taut, and the distance is noted and called back to the man at the plane table. The instrumentman sights the direction with the alidade, and plots the distance furnished by the chainman.

112. Adjustment of Horizontal Closure Errors

Many methods have been devised for adjusting the horizontal errors of closure in traverses. Most of these methods are mathematical and are more applicable to transit traverses than to those made with the alidade and plane table. All methods for adjusting

errors are based on certain assumptions which may or may not be true. In regard to horizontal closures, it is assumed that the errors in direction or distance, or both, are mainly systematic in character and, therefore, accumulate throughout the length of the traverse. On this assumption, the error in closure is distributed back through the traverse, each station of which is shifted in position by an amount proportional to its distance from the starting point. When all field operations have been carried out in a manner which greatly reduces accidental errors, such a procedure of adjustment results in fixing the locations of stations very near their true positions, for the closure error actually represents the cumulative effect of inherent instrumental errors. Unfortunately, the closure errors in plane table traverses are usually of a different nature. This method, more than any other, is subject to a great many accidental errors. When the backsight method is used, the closing error might well be principally one of direction (rather than distances) which accumulates at a constant rate from the station where the table was misoriented. In this case a uniform adjustment results in nearly correct positions of the traverse stations. The adjustments also apply where the instrumentman tends to read or plot distances always too short or too long, provided the tie is made on an established position far from the starting point. Where the tie is made on the starting point, such errors tend to cancel out when the tie point is reached.

The accidental errors so common in plane table work are not actually corrected by most of the methods employing a uniform shift of the stations, whether by simple proportion or according to the square of the distance from the starting point. The "adjustment" simply distributes the error among the stations so that it is presumably within the permissible limits at all points. This procedure often results in shifting a point which is correctly located on the sheet. In other words, an "adjustment" is a process of partial correction by balancing the errors.

Opinions and practices vary greatly on adjustments of plotted traverses. Graphic constructions which result in bringing tie points

together look authentic on the sheet, but it is often better to adjust the traverse on the basis of what is known about working conditions at the time observations were made at various stations. The instrumentman can sometimes recall certain stations where visibility was poor or where an approximate method of measurement was used. On critical examination of the area, features which might deflect the magnetic needle may be discovered.

Before any attempt is made to adjust a traverse the following steps should be taken.

(1) Check the computations of distance to ascertain that no errors have been made.

(2) Rescale all the distances between instrument stations and turning points to make sure that no error has been made in the original plotting.

(3) If intermediate ties in elevation were made and vertical angles were used in the traverse, a poor elevation check might indicate a gross error in the stadia distance, and would thus suggest the portion of the traverse where the horizontal error was made.

(4) Give particular attention to those sights which are roughly parallel to the direction from the starting point to the plotted (erroneous) position of the closing point (the last observation on the starting point).

(5) Whenever practicable, run a line across the traverse loop from a station on one side to one on the other. This procedure will show (1) in what part of the traverse the principal error occurs, or (2) if the error is essentially cumulative. If the first condition is found, either rerun the erroneous portion, or cut this part again by a short bisecting traverse. In the second case, adjust by a graphic method.

Figure 75A, shows a traverse loop with instrument and rod stations numbered from 1 to 8, the tie shot on the starting point being 1′. In this case the obvious places to look for gross accidental errors are between 1 and 2 and 5 and 6, for these lines are nearly parallel to 11′. If various checks indicate systematic errors, proceed with the adjustment as follows.

On a piece of tracing paper draw a straight, thin line, *ab* in Fig. 75B. Plot the distances between the traverse stations 1 to 1′ as if the traverse lay along this line and erect a perpendicular at each station. On the end perpendicular *ac* plot the distance of the closing error (11′ Fig. 75A), and draw a line from this point to 1, making triangle *acb*. The lengths of these perpendiculars are the distance corrections to be applied to the corresponding stations.

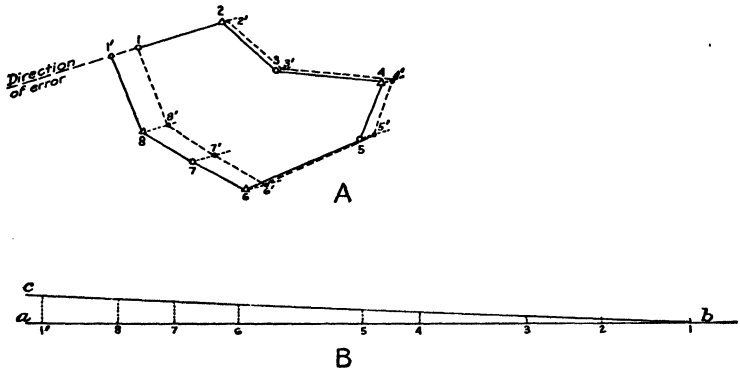


FIG. 75. Adjustment of a traverse loop.

The direction, or bearing, of the error is shown by the extension of the line 11′ in the upper figure. Draw lines through each of the traverse stations parallel to the direction of error. Now lay the tracing paper *B* so that the perpendicular distance 8 coincides with the line 88′ of the traverse. With a plotting needle prick the point 8′. Proceed in a like manner for each of the stations, being careful that the distance plotted on the parallel lines from the traverse stations is the right one for the station. The corrections should always be plotted in a direction from each of the stations corresponding to the direction from the closing point (1′) to the starting point. For this reason 2′ lies outside the loop, while 8′ lies inside. The adjusted loop is shown by the dashed line through the "prime" stations to 1.

When a traverse starting from one fixed point and ending at

another is to be adjusted, the procedure is somewhat different. In Fig. 76 a traverse is run from boundary marks *A* to *B* with a closing error *BB'*. The distance corrections for the traverse stations is determined from triangle *acb*, as in the case described above. Like-

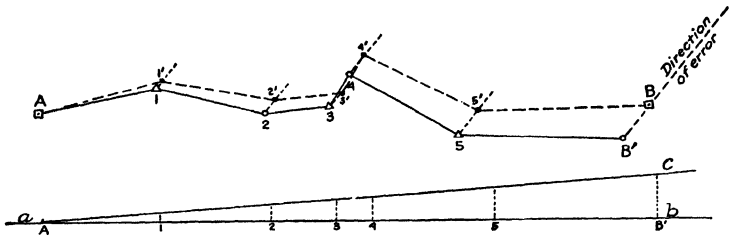


FIG. 76. Adjustment of a traverse between two known points.

wise, the positions of the stations are adjusted according to these distances along lines parallel to the direction of the closing error. The adjusted traverse is shown as a dashed line.

When a portion of a traverse is found to be in error it is adjusted as follows.

Figure 77 shows a portion of a traverse from stations 6 to 11. This portion is known to be too long. Draw a straight line from 6 to 11, shown as long dashes in the figure. From 6 measure the true length of the straight line between these points and plot a point *a*. Draw a line from 6 to *C*, which is located at any convenient place at some distance from 6. Draw a line from *C* to 11. Now draw a line from *a* parallel to 6*C*, cutting line C11 at 11'. This intersection is the corrected position of 11. From 11' draw a line parallel to 6–11, cutting 6*C* at 6', the corrected position of 6. Now draw radial lines from *C* to the remaining

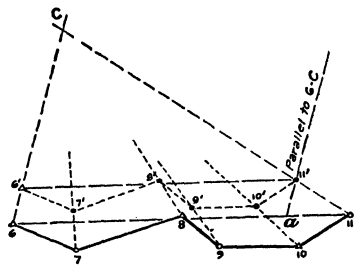


FIG. 77. Adjustment of a portion of a traverse.

traverse stations, shown as thin dashed lines in the figure. Between 6′ and 11′ draw each leg of the traverse parallel to the unadjusted legs. Where these lines intersect the radials are the adjusted positions of the stations 7′, 8′, 9′, and 10′.

113. Elevations

The determination of elevations by differential leveling, stepping, the Beaman stadia arc, gradienter screw, vertical angles with stadia, and vertical angles with intersections is discussed in Chapters II and III. The barometric method is discussed in Chapter VI. All these methods are used in small and intermediate scale plane table work. The purpose in following articles will be to outline more definitely the practical applications, various field procedures, sources of error, instrumental limitations, and other factors affecting elevation work.

114. Level Precision

The striding and control level tubes of the alidade, as well as levels on all other surveying instruments, are ground on a circle arc. The sensitivity or precision of the level tube is largely dependent on three factors; the radius of curvature on which the tube is ground, the precision of the grinding, and the accuracy of mounting the tube in the striding level (in certain types of tubes).

The arc grinding in some tubes is on only one side. In such types it is necessary to mount the tube so that the arc will lie exactly in the vertical plane of movement of the telescope; otherwise the bubble will tend to be sluggish, or will move erratically along the tube. A better type of tube is one in which the ground interior is similar to the inside of a barrel, i.e., all the interior of the tube is ground to the same curvature and, consequently, any portion will present an accurate curve along which the bubble can move. This kind of tube (sometimes called a "barrel tube") can be mounted in the striding level only approximately aligned and can then be adjusted into the correct vertical and horizontal planes by methods described in Chapter VII.

The degree of precision or sensitiveness of a bubble tube may be stated in terms of the radius of curvature on which the tube is ground, as 50-foot or 75-foot length of radius; or in seconds of arc, such as 30″ or 20″. The sensitiveness of the tube is directly proportional to the length of the radius of curvature; therefore, a tube arc of 100-foot radius is about twice as sensitive as one of 50-foot radius. The expression of tube precision in terms of seconds of arc refers to the central angle whose arc length is one division of the tube graduations. Thus, the level sensitiveness is inversely proportional to the number of seconds for a tube graduation of given length. Assuming that the spacing of tube graduations is $\frac{1}{10}$ inch in each case, a 30″ tube is approximately twice as precise as a 60″ tube. Since manufacturers do not adhere to a uniform practice in the matter of graduation spaces, two 30″ tubes may, or may not, be of the same precision.

The precision of alidade striding levels ranges from about 20″ (about 85-foot radius) to 60″ (about 28-foot radius). Most topographic alidade levels are 30″ to 45″ precision. The precision of the alidade striding level may be approximately determined in the field as follows. (1) Set up the tripod on solid ground, and place the alidade in the center of the table, directly over the Johnson head. (2) Hold the rod (graduated to 0.01 foot) on a firm foundation about 200 feet from the instrument. Determine this distance accurately. (3) With a thrust motion of the tangent screw bring the front end of the bubble to a graduation mark near the front end of the tube; then read the middle wire on the rod to the nearest 0.005 foot, or if possible to the nearest 0.001 foot. With a very soft pencil, mark on the level tube the position of the back end of the bubble. (4) Now depress the telescope until the back end of the bubble is at a graduation near the back end of the tube; mark the position of the front end of the bubble with the soft pencil and read the middle wire on the rod. (5) Scale on the level tube the distances traveled by the two ends of the bubble and determine the average. (6) Repeat the whole procedure a number of times and then compute the average of the movement of the bubble and the

movement of the middle wire on the rod. The rod can be read more closely if the rodman will move a pointed pencil up and down the face of the rod according to signals from the instrumentman. When such a target is used, the rodman determines the exact position of the wire and calls this back to the instrumentman.

If d is the mean distance traversed by the middle wire, b is the mean of the bubble movements in decimals of a foot, and D is the distance from the instrument to the rod in feet, then the

$$\text{Radius of curvature of the level tube} = \frac{b}{d} \cdot D$$

From the foregoing it should be clear that the importance of exactly centering the bubble while running elevations will depend to a great extent upon the precision of the level being used. A simple and practical way to determine the magnitude of error that may result in not centering the bubble is as follows. Set up the rod 100 feet from the instrument, level the alidade, and read the position of the middle wire on the rod to the nearest 0.005 foot. Make a series of similar readings with the forward end of the bubble set at successive graduations on the front end of the tube. Repeat the procedure for the back end of the tube. The difference between any two successive rod readings corresponding to one tube graduation is the amount of error in elevation for each 100 feet of the total length of the sight line. The average of all the readings should be used as the most probable error for a sight 100 feet in length.

115. Excess of Backsight or Foresight Distance

There are two sources of error that operate cumulatively in level lines when the lengths of the foresights exceed those of the backsights, or vice versa. These cumulative errors are due to earth curvature and small maladjustments in the level.

In Art. 156 (the two-peg adjustment), it is shown that even with a level that is out of adjustment, if the H.I. is determined by a "level" backsight and then a foresight elevation is established at a distance equal to the length of the backsight, the computed ele-

vation of the foresight will be correct. For example, assume that the error in the level adjustment amounts to 0.03 foot per 100 feet of the sight distance. If the length of the backsight is 1500 feet, then the error in the H.I. elevation due to this cause is

$$\pm \frac{1500}{100} \times 0.03 = \pm 0.45 \text{ foot}$$

Now if a turning point is established at a distance of 1500 feet, the error will again be ±0.45 foot; but, since the signs of foresights and backsights are reversed, the error is cancelled out, and the elevation of the turning point will be correct. On the other hand, if the distance of the turning point is only 500 feet, the resulting error in elevation due to the excess in the length of the backsight is

$$\pm \frac{1500 - 500}{100} \times 0.03 = \pm 0.30 \text{ foot}$$

The sign of the level correction depends on whether the front of the striding level is too high or too low.

It is evident that the elevations of alternate points in the traverse will be correct, regardless of any error in level adjustment, so long as the lengths of foresights and backsights are balanced. If the first instrument station is on the bench mark, and therefore has the correct elevation, all succeeding H.I.'s will also be correct, but the intervening turning points will each be in error according to the lengths of the respective foresights. If the first rod point is on the bench mark and the elevation of the instrument is determined by sighting on the rod, the conditions are reversed; and the H.I.'s will be in error according to the lengths of the backsights. This holds for elevation lines run by either direct methods or vertical angles.

When a balance of foresights and backsights is maintained, the correction for earth curvature cancels out, as in reciprocal sights (Art. 73). If an excess in either develops, an error due to curvature accumulates. The magnitude of the error depends on (1) the total amount of the excess and (2) the lengths of the individual sights.

Where sights are comparatively short, it is usually not necessary to make corrections for curvature, though the accumulated excess may be large. The two following examples will show why this is true:

EXAMPLE:

1. Average length of sight in which excess developed 1,500 feet
   Total excess (backsight or foresight) in level line 18,000 feet

Curvature correction for 1500-foot sight is

$$\pm\frac{(1500)^2}{(1000)} \times 0.02 = \pm 0.04 \text{ feet}$$

Accumulated error due to curvature is

$$\pm\frac{18{,}000}{1500} \times 0.04 = \pm 0.48 \text{ foot}$$

EXAMPLE:

2. Average length of sight in which excess developed 2,000 feet
   Total excess in the level line 18,000 feet

Curvature correction for 2000-foot sight is

$$\frac{(2000)^2}{(1000)} \times 0.02 = \pm 0.08 \text{ foot}$$

Accumulated error due to curvature is

$$\frac{(18{,}000)}{(2{,}000)} \times 0.08 = \pm 0.72 \text{ foot}$$

In most plane table level lines the curvature correction is not applied. However, when the elevations for permanent bench marks are established with the alidade and plane table used as an engineers' level and the average distances of either the backsights or the foresights greatly exceed the other, it is customary to consider this factor for each leg of the traverse in order to obtain the best possible adjustment of the bench mark elevations.

116. Field Procedures

The general procedure for differential leveling is described in Art. 36. Various methods have been devised and are being used in some government surveys to reduce the accidental or so-called personal errors. When the field party is made up of one or two rodmen, a recorder and computer, and the instrumentman, a commonly used method is as follows.

(1) The instrumentman reads the stadia distance to the recorder, who immediately computes the half-interval. (2) The telescope is leveled, and the middle wire position on the rod is given to the recorder, who, by adding or subtracting the half-interval, obtains the calculated positions of the upper and lower wires, respectively. (3) The instrumentman now observes on the rod the positions of the upper and lower wires. (4) The observed readings are then checked against the calculated positions, and if there is an error of consequence, the entire procedure is repeated.

This method is valuable where an important traverse is to be run without closing back to the starting point or to any other point of known position and elevation. As the above procedure does not check the actual leveling of the telescope, it is sometimes modified in the following way.

(1) The instrument is carefully leveled. (2) The positions of the upper and lower wires are observed and given to the recorder, who computes the position of the middle wire. (3) The telescope is now thrown out of the level position and releveled. (4) The position of the middle wire is observed, and checked with the calculated position.

EXAMPLE:

(1) Telescope leveled.
(2) Observed: upper wire 13.75′; lower wire 3.22′.
Computed: half-interval $= (13.75 - 3.22)/2 = 5.26'$;
middle wire position $= 3.22 + 5.26 = 8.48'$.
(3) Telescope releveled.
(4) Observed: middle wire position 8.49′;
Check: $8.49 - 8.48 = 0.01$ foot.

In some hydrographic and other special kinds of mapping, it is desirable to determine very accurate elevations, even though the scale of the map is small, and, consequently, considerable tolerance is permitted in the plotted positions of the stations. Elevations are usually determined by direct leveling with the alidade and plane table. In order to avoid plotting the points too close together, it is common practice to plot only every third or fourth station, depending on the distances between stations and the density of points required on the sheet. To illustrate the application of the method, assume that such a traverse is run down a highway whose average grade is 5 per cent (5 feet elevation per 100 feet distance). The length of the rod is 15 feet, and the average H.I. above the ground is 4 feet. The length of the rod limits the length of foresights to about 300 feet, and the H.I. limits backsights to about 80 feet. Plotted stations are desired at intervals of 1000 to 2000 feet. The procedure is as follows.

Starting from T.P. 1, the bench mark, which is already plotted on the sheet, the elevations of alternating H.I.'s and T.P.'s are run in the usual way, except that the sheet positions are not plotted—only the elevations are determined, and stadia distances are recorded in the notebook. When the level traverse has progressed to a point where a plotted position is required, the rod is held on T.P. 1, the starting point, the table is oriented and the stadia distance determined, and the instrument station is plotted on the sheet accordingly. This instrument station is clearly marked on the ground, and when the level traverse reaches a point ahead where another location is needed, the rodman returns to the former instrument station for the backsight, which locates the second plotted point.

By this method all plotted points are instrument stations, as all sights made for the purpose of establishing points on the sheet are backsights. The table is leveled for the intervening level sights, but is oriented only where a location is to be plotted on the sheet. When making a backsight for the purpose of locating the station, it is good practice to read the vertical angle and compute the difference in elevation so that any gross error in the level line will be

brought to light and can be corrected before the work progresses farther.

117. Vertical Angles in Level Lines

When it is necessary to use a vertical angle for determining the elevation of a station in a carefully run level line, every precaution should be observed to avoid the common errors of this method. A method in which the telescope is inverted to eliminate instrumental errors is described in Art. 76. Another checking method is performed as follows.

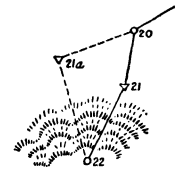


FIG. 78. Vertical angle closure of a level line.

In Fig. 78, stations 20 and 21 are on the main level line, and it is desired to extend the traverse to station 22, which is situated on a hill considerably above the general elevation of the main traverse. The elevation of 22 is first determined by vertical angle from station 21. Then a temporary instrument station is established at 21a by a level backsight to T.P. 20. From station 21a the elevation to 22 is again determined by means of another vertical angle. Since the H.I. elevation, the distance of 22, and the vertical angle are independent of station 21, an excellent check is provided for the elevation of 22.

118. General Precautions

Generally speaking, leveling is one of the most exacting of all surveying operations, and this is particularly true when the leveling is done with the alidade and plane table, largely because of the instability of the table itself. Great care must be taken that the table is not disturbed while making the observations. The alidade should always be placed in the center of the table directly over the Johnson head of the tripod. The tripod should be set low enough that no part of the levelman's body or clothing touches the table when he bends over the instrument in sighting. The weight of a coat sleeve resting on the table may cause a serious error on a long sight.

After the bubble is centered, the levelman should wait for a

short period of time before reading the rod to make certain that all movement of the bubble has ceased. Always check the bubble again after making a reading of the rod. When the tripod is set up on frozen ground or ice there is often a marked tendency for the legs to "creep" as the ice thaws under the pointed ends of the legs. Avoid such setups where possible.

The effect of local differential refraction is an ever present hazard, regardless of the temperature of the air; therefore, instrument stations and turning points should be selected on elevated points so that the line of sight is well above the ground. This effect is often very pronounced where the line of sight passes over broad stretches of snow, ice, or water.

One of the most common errors is that of reading the wrong wire on the rod, or recording a reading as a level wire, when the reading was made with one of the stadia wires. This mistake cannot be made when the methods described in Art. 116 are used.

When the rod is waving back and forth, as in a high wind, the lowest point on the rod cut by the middle wire is accepted as the correct reading. Any movement of the rod off the vertical results in a reading that is too high.

Turning points, as well as instrument stations, should be on solid ground, and on the highest point in the immediate vicinity. Be sure that the rod is correctly graduated by carefully measuring it with a standardized steel or invar tape. Cloth or so-called flexible rods are unsatisfactory for most level work.

Check the bubble adjustment frequently. Always check the adjustment if the instrument has been subjected to a severe shock or radical changes in temperature.

119. Precision of Plane Table Leveling

Remarkably accurate levels can be run with a properly adjusted alidade if sound methods are employed and reasonable care is exercised in performing routine field operations. There are many instances in hydrographic surveys where the results of plane table leveling were nearly or quite as good as those obtained in the same

area with engineers' levels. This calls for very strict observance of the best field practices and complete familiarity with the plane table method of leveling. While the quality of the instruments used plays an important part in the precision that might be expected, far more depends on the ability of the levelman. When using a telescope of 16- to 18-power, a self-reading rod graduated so that readings are made to the nearest 0.01 foot, limiting the lengths of sights to about 600 feet, and balancing the lengths of foresights and backsights, the maximum error in feet should not exceed

$$0.20\sqrt{\text{Distance in miles}}$$

Thus, the error in elevation in a traverse 4 miles long would be about 0.40 foot, and in one 9 miles long, about 0.60 foot.

If the average length of sight is increased to 1000 feet, the rod is read to the nearest 0.05 foot, and the other conditions are as outlined above, the error should be in the order of

$$0.30\sqrt{\text{Distance in miles}}$$

According to this formula the error in a traverse 4 miles long would be 0.60 foot, and in one 9 miles long, 0.90 foot. It is not especially difficult to maintain this order of precision on plane table leveling when ordinary precautions are observed.

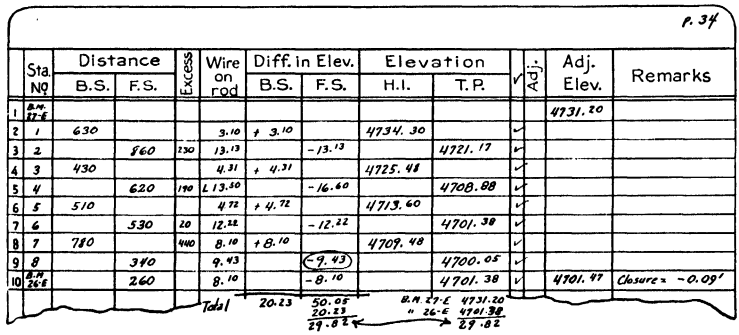
120. Plane Table Level Notes

In most plane table mapping a large portion of field data are recorded directly on the plane table sheet; so the loss of field notes, though perhaps serious, does not necessarily require that traverses be rerun. This does not hold for leveling notes, which often are the sole record of the field work. For this reason, it is particularly important to safeguard the level book and to make certain that records are legible, accurate, and complete. The page form of the notebook should be such that ordinary corrections and adjustments can be made without obscuring the record of field observations and at the same time provide a simple procedure of recording, computation, and checking. Figure 79 is a note form that embodies these qualities.

The column "excess" is for recording the difference in the lengths

of foresights and backsights, so that the total can be readily determined at any point in the traverse. This is convenient in keeping the foresights and backsights properly balanced, or in adjusting the elevations where the sights cannot be balanced.

The column "Wire on Rod" is for recording the level wire readings. Usually the initial letters "L" and "U" are used to designate



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	Sta. No	Distance B.S.	Distance F.S.	Excess	Wire on rod	Diff. in Elev. B.S.	Diff. in Elev. F.S.	Elevation H.I.	Elevation T.P.	✓	Adj.	Adj. Elev.	Remarks
1	B.M. 27-E											4731.20	
2	1	630			3.10	+ 3.10		4734.30		✓			
3	2		860	230	13.13		- 13.13		4721.17	✓			
4	3	430			4.31	+ 4.31		4725.48		✓			
5	4		620	190	L 13.50		- 16.60		4708.88	✓			
6	5	510			4.72	+ 4.72		4713.60		✓			
7	6		530	20	12.22		- 12.22		4701.38	✓			
8	7	780		440	8.10	+ 8.10		4709.48		✓			
9	8		340		9.43		(9.43)		4700.05	✓			
10	B.M. 26-E		260		8.10		- 8.10		4701.38	✓		4701.47	Closure = -0.09'
					Total	20.23	50.05	B.M. 27-E 4731.20					
							20.23	" 26-E 4701.38					
							29.82		29.82				

FIG. 79. Field notes of a level line.

a lower or upper wire reading, it being understood that all others are middle wire observations.

When a level line is adjusted for systematic errors, the amount of the adjustment with the appropriate sign should be recorded in the column "Adj." An explanation of the adjustment should be recorded under "Remarks." It is also good practice to indicate precisely which group of stations is affected.

Every computation should be checked by someone who has had no part in the original computations. A column is provided for marking the notes which have been checked. A fast method of checking the computations between any two widely separated stations in the traverse is as follows. Add all the differences in elevation of foresights and backsights that have been used in the computation of the elevation of the last station considered. The difference between these two sums will equal the computed difference in elevation between the two stations, if no errors have been

made in the notes. In the figure, T.P.8 is omitted because it is not in the line tying into B. M. 26-E.

121. Adjustment of Vertical Closure Errors

Closing errors in elevation may be adjusted in ways similar to the adjustments of horizontal errors. As in the case of horizontal errors, the plane table is more susceptible to errors in elevation than the transit or engineers' level. In consideration of this fact it is prudent to review some of the more common causes of errors before applying any method of rigid adjustment. By studying the field notes and recalling conditions under which each observation was made it may be deduced that the closing error is principally the result of one or two gross accidental errors, rather than a systematic accumulation of small individual errors. In such a case the logical procedure is to rerun the suspected portion of the level line. When the traverse has had to be adjusted for horizontal position, or if the horizontal closure is poor, pay especial attention to sights which employed the Beaman arc or vertical angle; for an error in distance would result in an error in elevation. Following is a list of common sources of error.

SYSTEMATIC ERRORS

(1) Striding level poorly adjusted in vertical plane. This may result in cumulative errors if the two-peg adjustment is used, but the tube is reversed on a number of instrument stations.

(2) Striding level not properly adjusted in horizontal plane (lateral adjustment). The line of sight will not be parallel to the axis of the level tube.

(3) Sum of backsight distances greatly exceeding those of foresights, or vice versa.

(4) Stadia rod too short or too long, resulting in both distance and elevation errors.

(5) Failure to correct for curvature when there is a large excess of foresights or backsights, especially if most of the sight lines are long.

(6) "Centering" the bubble while standing back of the alidade. Always stand to the side to avoid faulty centering.

ACCIDENTAL ERRORS

(1) Moving table while setting or reading the arcs.

(2) Making elevation observations with the alidade too near the edge of the table.

(3) "Creep" of the tripod when set up on ice or frozen ground.

(4) Weight of hands, notebooks, coat sleeves, etc., depressing the table while reading the rod.

(5) Sand grains lodged under a leg of the striding level. Always check the striding level legs and the ring bearings for dust.

(6) Failure of rodman to hold on the same point for foresight and backsight.

(7) Effect of wind lifting one side of the table.

(8) Differential refraction from sight line passing close to ground, rocks, bushes, etc.

(9) Misreading the rod by 1 or 5 feet, or the alidade by one-half or one-quarter interval.

(10) Errors in computations.

(11) Telescope or cross wires out of focus at time observations are made.

(12) Rod leaning toward or away from the instrument. This results in a level reading on the rod that is too large. If the elevation is determined by vertical angle (or Beaman arc) the difference in elevation will be too large because of the error in stadia distance.

It is evident that there are many causes of accidental errors in elevation. These possible causes should be checked in an effort to limit the known error to a portion of the loop, which can then be checked in the field with a few observations.

When an entire loop is to be adjusted, the following method may be used.

Let E represent the closing error, L the length of the circuit, and d_1, d_2, d_3, etc., the respective distances from the starting point to stations along the level line.

The correction C to be applied to the elevation of the first station is

$$C = \frac{d_1}{L} E$$

The correction to be applied to the second station is

$$C = \frac{d_2}{L} E$$

and so on for each station in the circuit.

If the closing elevation is higher than the starting elevation, subtract the amount of the correction from the elevation of each station; if the closing elevation is too low, add the corrections to station elevations.

EXAMPLE:
Length of circuit (L) is 12 miles
Closing error (E) is + 2.4 feet

Distances from starting point to stations G and H are 3.6 miles and 4.1 miles, respectively; and the *observed* elevations of these stations are 4006.3 and 4142.5 feet.
The adjusted elevation of G is

$$4{,}006.3 - \frac{3.6}{12} \times 2.4 = 4005.6$$

The adjusted elevation of H is

$$4142.5 - \frac{4.1}{12} \times 2.4 = 4141.7$$

The computations are made in this way for each rod and instrument station in the loop. The correction on the final tie point is

$$\frac{12 \text{ (distance from starting point)}}{12 \text{ (length of loop)}} \times 2.4, \text{ or the entire amount of the closing error}$$

No method of adjustment can correct a poorly executed traverse or level line. The additional time required in the field in taking all reasonable precautions against systematic and accidental errors is time well spent; for there is no rewarding satisfaction in "patching up" a poor piece of work.

122. Use of Intersected Points and Linear Features

When the first instrument station of the traverse is established, rays should be drawn to several prominent points that will be visible from future stations along the traverse. Vertical angles are determined and recorded in the note book. At the next instrument station, the points are again sighted, rays are drawn, and vertical angles recorded. In many instances the intersections made by this set of rays and the first set will be too acute to define clearly the positions of the points; they should be drawn, nevertheless, for the intersection, poor as it may be, will aid in identifying the points until better intersections can be obtained. Likewise, the computations of elevations will have to be deferred until the points are accurately located, but once the locations are established by cuts from later stations, the elevations can be computed from the first stations. From the second instrument station, new points ahead are selected for intersection, and so on for every station of the traverse. This is the best possible procedure for maintaining a constant check on the positions and elevations of the traverse stations and should be employed wherever field conditions permit.

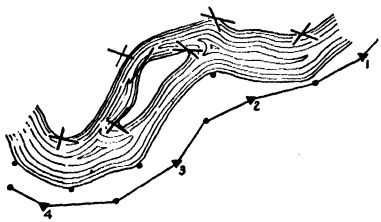


FIG. 80. Mapping by a parallel system of traverse stations and intersections.

In addition to this valuable function, intersected points furnish very good control for mapping details, often supplying control for sketching in places inaccessible to the rodman. A convenient adaptation of the method is shown in Fig. 80, where one bank of a river is mapped by side shots from the traverse stations, while the opposite bank is accurately located solely by intersections. In the figure the north side of the island is visible from only station 4; so from this position single rays are used to define the north limits of the shore line.

Figure 81 represents a plotted plane table traverse from which much use has been made of intersections and linear planimetric features. In this example, it will be noticed that instrument stations have been carefully selected with reference to the straight linear features that were to be mapped.

In the figure, dash lines are rays drawn on the sheet for the purpose of locating map features, and solid lines are the map

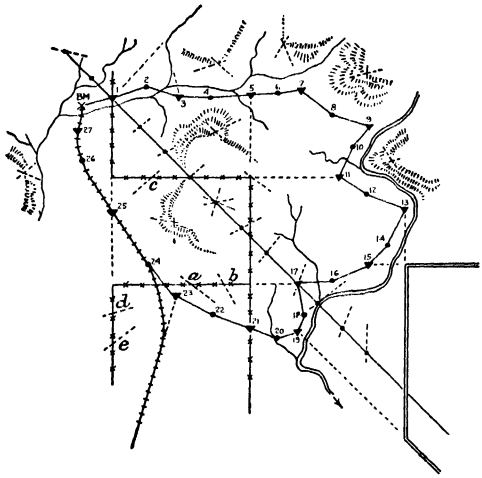


FIG. 81. Portion of a map showing maximum utilization of cultural features.

features. The first setup is located at the intersection of a fence and an electric transmission line so that their alignments on the map can be ascertained by sighting with the alidade. The direction of the upper portion of the small creek in the northern part of the map area is also established on the sheet by a direct sight from this station. Other rays are drawn to definite objects on the landscape, such as trees and boulders, for later intersection.

By carefully selecting the position on the ground (by inspection only), station 5 is so placed that it is exactly on the range of a north-south fence in the middle of the area. When the table is oriented, a long ray is drawn along this line of sight.

When the traverse has been run to the east side of the map area, station 11 is located on range with the east-west fence. The long line drawn along this sight defines the fence corner as well as the direction of the fence.

The highway in the south-east, though remote from any established map positions, is accurately drawn on the map by range line sights from stations 13, 15, and 19. Stations 17 and 21 are carefully located on the long lines previously drawn from stations 1 and 5, respectively. When these stations are located by means of the backsights to turning points, their positions and the orientation of the table are checked and, if necessary, corrected. From 17, a long sight establishes another fence corner and fixes the direction of the east-west fence. At station 23 the two tangents of the railroad are drawn, as indicated.

Cuts are made on the suspension towers of the transmission line from a number of stations along the traverse, and elevations of the towers are determined by vertical angles. A high tower approximately in the middle of the line and visible from many points on the traverse is sighted on repeatedly for the purpose of checking the positions and elevations of the traverse.

It is noteworthy that when a straight-line feature is located on the sheet by one long direct sight, a single cut on this line from any instrument station establishes a definite map point, and when a vertical angle is also determined at a station from which a cut is made, the elevation of the point can be computed. A visible object on the straight line is not necessary for making the cut, though it is somewhat easier to sight on a post or similar object. For example, points *a, b, c, d,* and *e* in the figure are cuts on the fence lines previously established from range line stations 1, 11, and 17. To determine the elevation of any of these points, the cross wires are set at a place on the ground immediately beneath the fence, and the vertical angle is observed.

Figure 81 illustrates how a limited number of rod and instrument stations are used advantageously as control for accurately sketching a relatively large amount of map detail. The plane table

mapper should be alert to the possibilities afforded for utilizing to the greatest extent every station located by conventional methods. Particular attention should be given to straight linear features, such as fences, power, telephone and telegraph lines, straight highways and railroads, and pipelines. Even though these features are not desired on the finished map, it is well to appraise their value carefully as control for other sketching before passing them by with the traverse.

123. Relationship of Scale and Density of Control

Consider a map area of 1 square mile, which is to be mapped topographically on a scale of 1 inch = 1 mile. It is determined that to produce a map of a certain quality as to accuracy, detail, etc., there will have to be approximately 30 rod stations for sketching control. Now if the scale is increased to 2 inches = 1 mile, and a commensurate fidelity in sketching is maintained, approximately 120 stations will be needed; if the scale is increased to 3 inches = 1 mile, 270 points will be required. Naturally, this relationship does not apply in many cases, especially where there are unusual conditions of topography, planimetry, etc.; but where field conditions are average and no unusual factors must be considered, it is a good working principle that the number of sketching control points will vary according to the square of the scale of the map. This may be stated in terms of the density of plotted points on the sheet. To maintain in the maps a quality that is commensurate with the scale, the number of sketching control points within a unit of sheet area must remain approximately constant, irrespective of the scale of the map.

CHAPTER V

Large-Scale Mapping

124. Definition and Remarks

Large scales, as herein defined, range from 1:6000 (1 inch = 500 feet) to about 1:120 (1 inch = 10 feet). Plotting limits range accordingly from approximately 8 feet to less than 6 inches. When scales larger than 1:120 are employed in areas of any appreciable extent, it is doubtful that the plane table is the best method, and for this reason larger scales are not considered.

In general, large-scale work is less difficult than small- and intermediate-scale mapping discussed in preceding chapters. Although a relatively high degree of precision is required in all the field measurements and drafting, the methods suited to this kind of work are more limited in number, and, therefore, the field procedure is correspondingly simpler. Small-scale mapping of large areas is complicated by problems such as earth curvature, long sight lines, poor visibility, and so on, that need not be considered in large-scale mapping of comparatively small areas. The former type of work demands much in general surveying experience, the ability to sketch accurately at long range, and resourcefulness in coping with variable field conditions; the latter requires less aptitude or experience in sketching, because all features mapped are closely controlled by actual measurements, and as a rule the more rigid specifications of the map will not permit carrying on the field work under too unfavorable working conditions. On the other hand, large-scale work requires more attention and care in routine instrumental operations and general field procedure. All instrumental observations are conducted in a more exacting manner,

actual mapping to scale of small details replaces the symbolization used in small scales, and the exact ground positions of the plane table and rod have greater significance.

Large scales are usually employed in mapping towns and cities, archeological sites, bridge, dam, and reservoir sites, and with detailed topographic contours for the purpose of computing volumes of irregular masses, such as earthen fills and excavations, and reservoir capacities. A number of the larger cities have been mapped by various organizations on a scale of 1:2400 and a contour interval of 2 feet. This scale permits plotting to about 3 feet and scaling to less than 2 feet.

125. Adaptation of the Plane Table for Large Scales

In order to reduce the number of sheets whose edges must be carefully matched, it is common practice in large-scale work to use tables and sheets somewhat larger than ordinary. Since the areas covered in a day's work are comparatively small, the large boards do not present the inconveniences of transportation that restrict their use in small-scale mapping.

The U.S. Coast and Geodetic Survey plane table is built so that a long strip of drawing paper can be used. The paper is held down tightly on the table by flat spring clamps along the edges, and the portion of paper not in current use is kept rolled at the two ends of, and beneath, the table. As the work progresses, the completed part is rolled at one end while fresh paper is unrolled at the other. The disadvantages of this arrangement are that (1) only medium weight, unmounted drawing paper can be used satisfactorily, (2) there is danger of tearing the completed map where it is brought over the edge of the table into the roll, and (3) having been rolled, the sheets tend to retain this shape and are consequently more difficult to ink.

Most large-scale work is done on a standard-sized table 24 inches by 31 inches, with either a Johnson or Coast and Geodetic type tripod head. The Geodetic type head is somewhat more stable than the Johnson head, and since it is equipped with leveling

screws, it can be leveled with greater precision. Tables larger than 31 inches by 31 inches are not recommended for general use.

Special bases for the pedestal type alidade are available for use on the large plane tables. These bases or blades are 25 inches to 30 inches long, some being slotted to provide a fiducial edge directly below the axis of the telescope. The alidade standard is attached near the rear end of the blade so that the instrumentman can stand back of the table when using a plotted point on the side of the sheet nearest the ground point sighted. Alidades with long blades are awkward to carry from one station to another, but the advantages in large-scale work are obvious. When a blade of ordinary length is being used and situations arise where a long blade is required, as in triangulation or long backsights, a light steel straightedge can be held against the fiducial edge so that the two plotted positions are properly aligned with reference to the line of sight.

126. Setting Plotted Position over Ground Point

It will be seen shortly that it is often necessary to set up the plane table so that the plotted position of a station on the sheet is vertically above the corresponding station or hub on the ground. When the map position of the point is near one edge of the sheet, the tripod will not be centered over the ground mark, but instead will have to be placed so that the plotted station occupies this position. Since no provision is made to shift the table on the tripod head, the only way that the desired relationship can be obtained is through shifting the tripod and table together.

The point can be brought near enough to the plumb position for most work by leveling and orienting the board by inspection (without using the alidade) and then by moving it bodily the small distance necessary to bring the plotted point over the station hub. A small pebble dropped from a point directly beneath the plotted station will indicate the plumb point on the ground. Where a more exact location is required, a plane table plumbing bar is used. This device is shown in Fig. 82. The end of the upper limb

is placed on the plotted point, and the tripod is shifted until the suspended plump bob coincides with the corresponding mark on the ground.

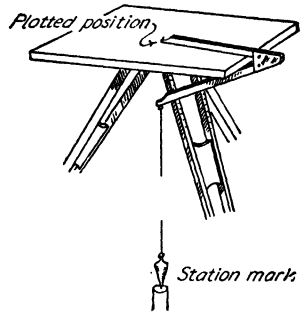


FIG. 82. Use of the plumbing arm and plumb bob.

127. Primary Control

The primary control for practically all city surveys is transit-tape traverse and transit triangulation. The transit traverse lines are established by precision methods. Horizontal angles are determined to within a few seconds of arc, and taping is done with a steel or invar tape that has been calibrated under standard conditions. Uniform tension of the tape when making measurements of distance is maintained by means of tension handles, which "weigh" the amount of pull. Adjustments are made in the observed distances according to the temperature of the air at the time the measurements were made and the coefficients of expansion of the tape used. It is customary in this type of work to use two or more so-called pocket thermometers at different places along the tape when it is drawn taut for the measurement. All tape measurements are made between chisel marks on the heads of metal rivets or spikes that have been driven into the solid ground or pavement.

While many city surveys are conducted under less stringent requirements, it is the policy of the U.S. Geological Survey and other organizations to employ the methods outlined above.

The transit triangulation used in city surveys is principally for the purpose of strengthening the transit traverse control, as most of the stations are situated so that they are of little direct use in the mapping operations. Both traverse and triangulation stations are plotted on the plane table sheets by accurate drafting room methods, usually by the coordinate method. The positions of the plotted control should be checked by every possible means before the sheets are sent into the field, and then should be checked again

in the field by instrumental methods before detailing is begun.

In city surveys it is common practice to lay out the primary transit traverses in grid form, the distance between lines ranging from 1000 feet to 2500 feet. In addition to these lines, secondary transit traverses, run with considerably lower precision, are sometimes used to further subdivide the map area into convenient-sized blocks for detailing. However, this part of the control is ordinarily run with the plane table. In this case, the plotted traverses are run from a transit station on one line to another transit point on another. Closure errors are adjusted before the detailed work is started. When this method is used, of course it is essential to mark the stations on the ground so they can easily be reoccupied.

The primary control for many kinds of large-scale work is run with the alidade and plane table. Triangulation should be used wherever field conditions are favorable. When plane table traverse is used temporary signals should be erected at various stations on the traverse so that the accuracy of the work can later be checked by backsighting from other points.

Just as primary control for small-scale mapping should be of a generally higher order of precision than detailing traverses, so should the primary control for precise large-scale mapping be of a relatively higher order. This may necessitate measuring all the distances on the primary traverse with a steel tape, rather than with stadia; and centering the plotted positions of the instrument stations over the corresponding ground points by means of a plumb bob, as stated in Art. 126. Stakes are driven in the ground for turning points before the foresight is given in order to insure using exactly the same point on the backsight.

128. Orientation

The magnetic needle may be used for orientation when the distances plotted do not exceed one-half the length of the compass needle, or when a large scale is used solely for the purpose of showing more detail and not for reasons of attaining a higher degree of precision.

The backsight method is best for this type of work. The field procedure differs from that previously described only in the care with which each step of the process must be performed. The importance of setting the plotted position of the instrument station vertically over the ground mark and holding the rod on exactly the same point for backsight and foresight can be readily appreciated in the following table for orientation errors that result from eccentric setups. Since a distance of 1 foot can be plotted on all scales larger than 1 inch = 50 feet (1:600), the table applies to these scales.

The column "Effective Eccentricity" refers to the combined effect of both rod and instrument being off their respective ground marks. If both are eccentric by the same amount, and in the same direction, the orientation of the table will not be affected; but if they are off the ground marks in opposite directions and normal to the line of sight, the effective eccentricity is the sum of the two errors. It will be observed in the table that the error in orientation depends not only on the amount of eccentricity but also on the length of the backsight.

Error in Orientation Resulting from Eccentricity of Stations

Effective Eccentricity (feet)	Length of Backsight (feet)	Error in Orientation	
		Angular	Feet per 100 feet of distance
1.0	100	34′	1.0
	500	07′	0.2
1.5	100	52′	1.5
	500	10′	0.3
2.0	100	1°09′	2.0
	500	14′	0.4

129. Measurement of Distance

When scales large enough to plot a distance of 1 foot are used, the field measurements on traverses and other important distances should be made with a steel tape, though the stadia method is also adequate, provided instrument constants have been accurately determined and stadia intercepts are carefully observed.

The $(f + c)$ constant should be added to all stadia distances if the alidade is externally focusing. As mentioned earlier, stadia distances with internally focusing telescopes are referred to the center of the instrument; therefore, no correction is necessary. The $(f + c)$ constant is determined by the manufacturer and recorded on the inside of the instrument case. However, this record may become lost, or instruments may be put in the wrong cases so that the correct constant is uncertain. As the value of $(f + c)$ is always within a few tenths of 1 foot, it is ordinarily well within the requirements of plotting accuracy simply to add 1 foot to all stadia determinations. If it is desired to determine the constant more closely, the following procedure may be used. The rod must be held perfectly vertical, the telescope must be level, and the rodman should use a sharp-pointed target, such as a lead pencil, so that readings can be made to the nearest 0.001 foot. The determinations should be conducted indoors, or where there will be no interference by wind. When this method is used, the $(f + c)$ constant and the stadia interval factor are determined simultaneously.

In Fig. 83, the lower figures are distances carefully measured in 10-foot intervals with a steel tape, and the upper ones are stadia intercepts read at the measured intervals. The unknown $(f + c)$

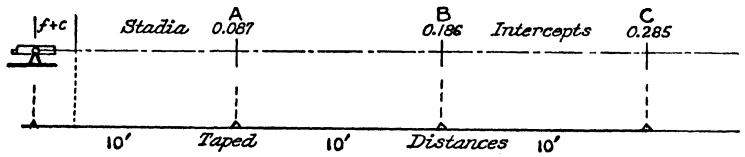


FIG. 83. Simultaneous determination of the $(f + c)$ constant and the stadia interval factor.

constant affects all the observed stadia intercepts approximately the same amount. From point A, the first rod reading, the difference between any two successive readings, such as A and B or B and C, should be the same. In the example this difference is 0.099 foot; therefore, the *stadia interval factor* is $10.00/0.099 = 101$. This factor should be checked by several readings. At A the stadia reading is 0.087. The $(f + c)$ constant, in terms of the stadia interval, is $0.099 - 0.087 = 0.012$, and in feet is $0.012 \times 101 = 1.21$. Since the value of c varies slightly according to the distance for which the telescope is focused, the distance from the axis to the objective lens is measured first when the telescope is focused at the mean distance of the above determinations, and then again when focused on a distant point. One-half the difference between these measurements is added to the figure 1.21 to give the mean value of $(f + c)$.

When using a large plane table and an alidade with a long base, it is sometimes difficult to keep the center of the instrument at the plotted point while reading the rod. The distance may be read with the alidade at a convenient position back of the plotted point, and then this amount is subtracted from the observed distance.

130. Stadia Rods for Precise Work

An all-purpose rod graduated in 0.05-foot intervals and capable of being read by interpolation to 0.01 foot is described in Art. 90. This rod is satisfactory for most large-scale surveying. If distances are to be determined within 1 foot, a rod of the Philadelphia type should be used. The Philadelphia rod, shown in Fig. 84, is graduated in 0.01-foot intervals and by interpolation can be read to a few thousandths of a foot. Numbers of feet are in red and those of

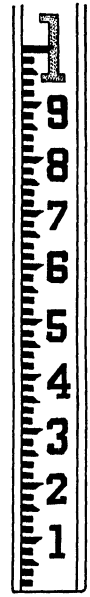


FIG. 84. Portion of a level rod used for precise stadia and level surveying.

tenths of feet are in black. Although this type of rod is designed primarily for levels, it is very adaptable for large-scale stadia work where the length of sights does not greatly exceed 600 feet; beyond this distance the small graduations appear blurred when viewed through a 16-power telescope.

131. Inclined Sights

Steeply inclined sights should be avoided in large-scale work wherever possible. When it is necessary to make a stadia reading with a high vertical angle, particular pains are to be taken that the plane table is level and the rod vertical. The conversion to horizontal distance is made as previously described.

The distance computed from a stadia reading on a vertical rod with an inclined line of sight is not the true slope distance along that line. When it is desired to obtain a slope distance by stadia, the following method is used.

The rodman carries a small T-square or any other right-angle straightedge with which he can obtain a rough line of sight normal to the rod.

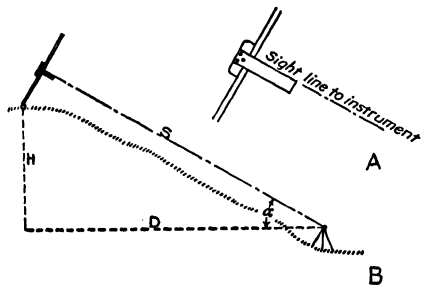


FIG. 85. Determination of a true slope distance.

Figure 85A shows how the T-square is held against the rod and the rod tilted so that the instrument is sighted along the straightedge. From Fig. 85B, the following relations are apparent:

a = the vertical angle

The slope distance, S = rod intercept $\times$ stadia interval factor $+ (f + c)$

Horizontal distance, $D = S \cos\alpha$

Difference in elevation, $H = S \sin\alpha$, or $H = D \tan\alpha \pm$ (rod correction)

It should be clearly understood that when the rod is tilted for a reading, conversions for horizontal distance or computations of difference in elevation cannot be made with stadia conversion tables or the stadia slide rule. All computations are made with trigonometric tables of natural functions or with a slide rule having a natural functions scale.

132. Lengths of Traverses and Checks

Traverses should be closed and errors properly adjusted at frequent intervals. The distance between ties may be governed largely by the number and distribution of plotted control stations, character of the terrain, visibility, and other factors over which the instrumentman has little control. Where such restrictions are of little consequence, the distance between ties should properly depend on (1) the scale of the map, (2) the permissible limits of error in horizontal position, (3) the field methods employed and the opportunities afforded for indirect checking of position.

(1) It was stated in Art. 123 that the density of points used in sketching topography to a degree of accuracy commensurate with the scale is approximately the same in terms of sheet area, irrespective of the map scale. Under ordinary conditions the principle of this rule may be applied to the distances between ties on traverses run for secondary control. For example, assume that for a given type of work on a scale of 1 inch = 1 mile the horizontal closure errors can be held consistently within the allowable limits if the traverses do not exceed 20 miles between ties into points whose positions are known. This distance is represented on the sheet by a line 20 inches long. Now if the same *relative* quality of work is to be maintained on a scale of 1 inch = 500 feet, the closed traverses should not exceed 2 miles (approximately 20 inches on the sheet) in length. It is assumed that in each case appropriate methods are used in the field work and that the char-

acter of the country does not especially favor either scale. It is evident that this rule stresses two steps in the procedure of running plane table traverses: orienting the table and plotting; but as these are the most frequent sources of error in position, the rule still holds in most cases.

Obviously, in very precise work all control stations must be held within comparatively narrow limits of error. The amount of error at closing stations is not in itself a positive criterion of the maximum error that might exist at various places along the traverse. However, this is the best means of gaging the total amount of error in the traverse at the point where the closure is made, and it is necessary to make adjustments on the assumption that errors have been systematic or cumulative, rather than accidental or compensating. On long traverses the compensating type of error is more likely to cancel out than in a short traverse. This is not true of systematic errors, which increase in magnitude according to the length of the traverse; hence, this type of error, in most instances, will be largest at the end of the traverse where the closure is made. It is principally the systematic errors that must be considered in fixing the limit for the length of traverses. Errors in position due to inaccurate plotting are usually accidental in character; those caused by misorientation, where the backsight method is used, are cumulative.

(2) The limit of error for horizontal positions of stations in the secondary traverses may be defined in two ways: the total error in position permissible at any point, or the amount of error per mile of traverse. The first definition, often used in connection with precise surveys, is more rigid and requires ties into the primary points at more frequent intervals. The second definition is adaptable to less precise work and, therefore, is more widely employed. It permits larger total errors and places smaller limitations on the lengths of the traverses, provided closing errors of appreciable size are adjusted. In either case, the distance between ties will depend largely on how far a traverse can be extended without adjustment and remain within the prescribed limits.

(3) It is obvious that in order to maintain a specified degree of accuracy in the work, selected field methods must be used. No method should be considered if it may introduce an error exceeding the prescribed limit. The instrumentman should take advantage of every opportunity to check his position by sighting back to previously established points. This method of checking by resection is used in all kinds of plane table work, but it is particularly adaptable to large-scale work where sight lines are short and stations are therefore easily marked for later checking purposes. Where intersection and resection are used continuously, traverses may safely be run much greater distances without tying into the primary control, for errors of appreciable magnitude should be disclosed by the resections. In precise mapping, resection from plane table traverse stations should be considered only as a means of roughly checking the position of the station, not as a final closure. Traverse closures should always be made on known points by direct measurement.

133. Elevations

The plane table methods suitable for determining elevations in large-scale work are essentially the same as for intermediate scales, except that only direct leveling methods should be used. The lengths of sights should be limited to 400 to 600 feet, and the lengths of foresights and backsights should be kept in proper balance in order to avoid large cumulative errors. As in the case of plotted traverses, level lines should be tied into bench marks or adjusted level stations at frequent intervals, and closing errors in the lines should be carefully adjusted.

134. Detail Mapping

The process of mapping planimetric and cultural features on large scales differs from small-scale work in that many of the details are represented true to scale and shape, rather than by a uniform-sized symbol. The degree to which this practice is carried out depends largely on the plotting limits of the scale used, though

other factors, such as the general map specifications and the kinds of data required, may play an important part. So-called sketching in this kind of work is more strictly mechanical drawing. Map features are closely controlled by rod stations and direct sight lines, and much of the drawing is done by means of straightedge, irregular curve, compass, and other drafting instruments.

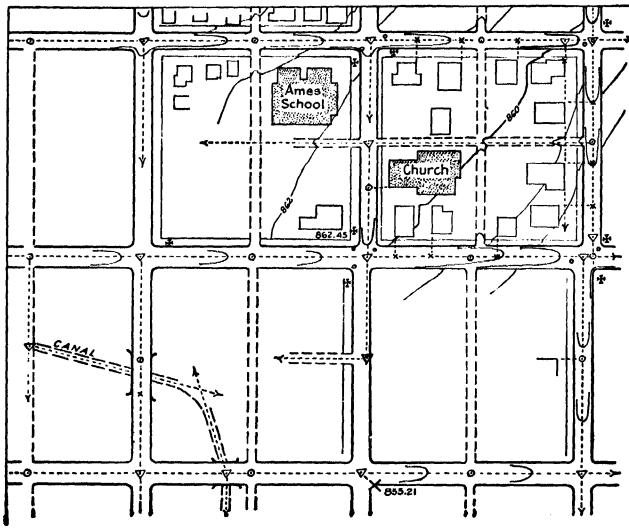


FIG. 86. Portion of a city map in various stages of completion.

Figure 86 is a portion of a city topographic map in various stages of construction. Plane table traverses are represented by dotted lines with conventional symbols for instrument stations and turning points. Rays drawn on the sheet by means of sighting with the alidade are shown as dotted lines with arrows indicating the direction of the sight. Side shots are shown as x's. The planimetry and topography are completed in the northeast block and partly completed in the adjacent block to the west. The remaining blocks are only the skeleton of the map, consisting of street and alley lines. The field procedure in mapping an area of this kind is about as follows.

Plotted traverses are first run from one primary control station to another at intervals of several blocks. These serve to break down the large areas lying between transit lines into smaller quadrangles. Only those features situated near the traverse stations are mapped at this time, though some effort should be made to locate the instrument stations at street intersections so that the center lines of cross streets can be drawn by sighting; and turning points should likewise be selected at positions in line with features to be mapped, such as buildings and alleys, as shown in the figure. Contours crossing the traverse line are indicated by lightly sketched lines. When these secondary traverses have been closed on primary points, all necessary adjustments are made before the work proceeds farther. The widths of streets are measured, and the curb lines are drawn accordingly.

The next step is the delineation of all streets by means of tertiary traverses. These traverses start at one adjusted secondary line and tie into another. The detailing of planimetry and topography is done concurrently with the running of the tertiary traverses. Much time and effort will be saved if some thought is given to advantageous placing of side shots used for sketching planimetric and topographic features. In the figure, rod points in the streets determine the positions of contours; but, since they are situated on line with building walls, they serve to reduce the number of points necessary to plot accurately the buildings on the sheet. There are boundless opportunities in city mapping to utilize alignments of features for rapid and accurate plotting.

The example of city mapping is given here because it illustrates, perhaps better than any other, the advantages of the plane table method as applied to precise mapping. The principle of subdividing the map area into smaller and more easily mappable units by means of adjusted traverses along linear ground features is an obvious procedure in the case of city mapping. While it may be less obvious in other kinds of work, it is, nevertheless, the safest method for all types of precise mapping, for when errors in position or elevation occur during the detailing, relatively small portions of the map area are affected.

CHAPTER VI

Reconnaissance Mapping

135. Definitions and Remarks

A plane table reconnaissance map is one that is constructed for the purpose of fulfilling a temporary and immediate need. Although reconnaissance maps may be drawn to any scale, all field maps on scales smaller than 1:96,000 (1 inch = 8000 feet) should properly be placed in this category, because the scaling limits are so large that approximate methods of measurement can safely be used in the field.

The U.S. Geological Survey considers as reconnaissance maps all topographic sheets that are constructed in the field on scales smaller than 1:96,000. Except for somewhat broader generalization in the forms of contours, these maps are very similar in appearance to the standard quadrangle sheets on a scale of 1:62,500, but close examination of the reconnaissance maps in the field often discloses omissions of important map features that are quite large enough to have been mapped to scale. In most instances these omissions, as well as other large discrepancies, are the result of sketching from great distances and with a minimum of control, both being necessary expedients in this type of work where speed in covering the map area is given precedence over accuracy or completeness. It should not be inferred that these maps are made without restricting limitations on accuracy, but rather that the limits are sufficiently broad to permit very rapid progress, yet at the same time narrow enough to insure localization of any large errors that might occur.

A characteristic of many reconnaissance maps is a marked lack

of uniformity in the relative amount of detail shown in different parts of the area, or disproportionate emphasis in depicting a few classes of map features. More will be said of this later.

To those not familiar with the problems of plane table reconnaissance mapping, it might appear that this is the simplest kind of plane table work because of the low precision requirements. In the majority of instances this point of view is erroneous. Reconnaissance mapping with the plane table requires more extensive knowledge of field procedures, broader experience in surveying, and more resourcefulness in coping with adverse working conditions than any other kind of mapping. The fact that comparatively large local errors are allowed is misleading in that it suggests a permissible laxity in the manner in which field procedures may be conducted. This type of work may utilize the methods employed with large, small, and intermediate scales, and in addition to these, a number that are suited for only approximate work. Obviously, the range of precision in these various methods is very wide, and it must devolve upon the instrumentman to select the appropriate method for any circumstance that may arise. Strangely, the wide limits of error allowed in reconnaissance work are exceeded more frequently than are the comparatively narrow limits of large and intermediate scale work; and the cause can usually be attributed to a poor choice of methods at a critical point in the structure of the map. This is likely to occur when an approximate method is used in establishing the principal control, particularly where any error made will have a cumulative and far-reaching effect on the map as a whole. It is a good practice to weigh carefully the effect that a large error might have on the map and on future work before using a method that is subject to large errors.

136. Purpose of Reconnaissance Surveys

The purpose in making plane table reconnaissance surveys is to obtain certain kinds of field information in a minimum length of time and at low cost. Topographic reconnaissance attempts to provide a much generalized map of all the principal topographic

features of the region, together with such other information as will add to the general usefulness of the map. No effort is made to define topographic details or to include features that are of doubtful value on a regional map. Other kinds of reconnaissance maps are made for the purpose of assembling information of a specific nature, such as areal or structural geology, hydrography, land and soil classification, and road or canal location. In order to maintain rapid progress and low mapping costs, it is essential that the field man keep the main purposes constantly in mind and, likewise, direct his efforts toward accomplishing these purposes. It is almost as important to avoid mapping those features that do not add materially to the value of the map as it is to obtain all the essential ones; for to crowd the sheet with relatively useless detail serves only to obscure the important features.

137. Office Compilation

The small scales and low precision of most reconnaissance maps allow extensive use of data which are not suitable for more accurate maps. Such material should be prepared so that it is easily incorporated in the map or compiled in pencil directly on the plane table sheets. The scales of earlier maps can be enlarged or reduced to that of the new map by means of pantograph, plotting and resketching by grid, or photostating. Drainage patterns and other useful data may be traced from aerial photographs on tracing paper and then be reduced to the desired scale by any of the above means. It should be understood that aerial photographs are not maps and are subject to large distortions and variation in scale (Chapter IX); hence, any compilation from them should be segregated or otherwise differentiated from material obtained from other sources. It is a good practice to indicate the sources of all the compiled information, and, when possible, to evaluate the relative accuracies so that discrepancies can be adjusted accordingly.

Although it is usually easier and more economical to compile existing map data to the scale of the reconnaissance map in the office, when this cannot be done the materials should be made

available to the plane table parties so that compilation can be carried on concurrently with the field mapping. Much time and effort in the field may be saved by utilizing these data to the fullest extent. Where the accuracy of the compilation is doubtful, it may be checked by instrumental methods.

138. Control for Reconnaissance Mapping

The horizontal control for many reconnaissance maps of areas that have been surveyed by the U.S. General Land Office consists of township and section corners accurately plotted on the plane table sheets before the latter are taken into the field. Because of the irregularities in land surveys, it is usually not sufficiently accurate to lay out a land grid to the proper scale on the assumption that the sections are 1 mile square and the townships 6 miles square (Art. 13). The corners and lines should be plotted according to the distances and bearings shown on General Land Office Township plats by means of vernier protractor, drafting machine, or by the tangent method of laying off horizontal angles. The lines and corners on the plats are sometimes inaccurately plotted, and this, together with the distortions of photostatic reproductions, makes them unfit for use as control for moderately accurate maps.

After the land nets are plotted, the sheets are taken into the field for checking. The problem of locating the corners on the ground is not difficult where roads run along section lines. Plats obtainable from county agents or engineers may be of considerable help in locating the corners. Where the map area lies in or near a national forest reserve, small-scale maps showing the main drainage and topographic features, trails and roads, "found" land corners, and other general data, may be available from the U.S. Forest Service.

When a mark (stone, iron pipe, bronze tablet, etc.) is found, the descriptive numbers and markings should be carefully examined and recorded in the notebook for future reference. Check the descriptions against the supposed position on the sheet to ascertain that the location has been correctly identified on the latter. Marks

bearing the letters "WC" are reference or "witness corners" used where obstacles prevent setting a mark at the true corner. These marks may be from a few tens of feet to several hundred feet from the true corner. In some instances the distance and bearing to the correct corner will be indicated on the mark, while in others the information will have to be obtained from the township plats.

By erecting signal towers or large flags at corners situated in open country or on prominent topographic features, triangulation methods may be conveniently employed both for checking the plotting of the land net and locating the position of the instrument by resection or three-pointing. In these operations the following precaution should be observed.

The earlier discussions of the three-point method have assumed that the three control stations are correctly plotted on the sheet and that the triangle of error is caused solely by incorrect orientation of the plane table. Therefore, to eliminate the triangle of error it is only necessary to reorient the table until the rays drawn from the control stations intersect at a point. When the plotted positions of the stations are in error, as might be the case in using a land corner net, this principle does not necessarily hold. A triangle of error may be formed when the plane table is correctly oriented, and a three-ray intersection may be obtained when the table is misoriented. For this reason, the safest procedure is to draw rays from at least four of the flagged corners wherever possible. The orientation of the table should then be adjusted until the intersections of the four or more rays are brought as near as possible to a common point. If a four-ray intersection is unattainable, use a mean point between the intersections as the position of the instrument.

Plane table triangulation and stadia traverses as described in previous chapters constitute the best control for reconnaissance mapping. Even though the sketching limits of the map are broad, experience has shown that it is poor practice to allow too wide limits in the primary control, and for this reason it is recommended that the principal network of stations be run by standards

suitable for small-scale detailed maps. In most triangulation the length of time spent in making instrumental observations is small in comparison with that used in traveling from station to station, setting up the instrument, and in constructing station signals. These operations are the same for any type of work; so the additional time required to make careful and complete instrumental observations, once the station is occupied, is small in relation to the total.

A special method combining the principles of both stadia and triangulation has been used to advantage mainly in Canada, but to some extent in the United States. It is adaptable in regions where timber is present but not so dense or continuous as to preclude long sights. As will be seen, the advantage in this method is that it provides a wider choice of instrument stations than might be possible where three-pointing alone is used.

Instead of building a conventional signal on a point selected for triangulation, a long pole is first painted in alternating bands of black and white similar to a stadia rod and then firmly planted in the ground and guyed with wires. The width of the black and white bands is usually 3 to 5 feet, the lowermost being white. Sometimes a red band is painted somewhere near the top of the pole. The length of the poles will depend upon the material available and the length of sights anticipated, but in actual practice poles up to about 25 feet in height have been used.

After the poles have been set on the selected sites and obstructions to visibility have been removed, a base line is measured and the stations are triangulated in the usual manner. When the triangulation net is thus set up, the instrument may be located anywhere in the map area by three-pointing where three stations are visible or by determining the stadia interval by means of the painted bands where only one pole is visible. The gradienter drum, micrometer eyepiece, or vernier method (Art. 141) can be used for sights that are too distant for a direct stadia intercept.

If only two poles are visible, a two-ray intersection is obtained on the sheet, and this position can then be checked by determining

the stadia distance to each of the poles. The location obtained by the resection is the true horizontal position of the instrument station; if the vertical angle is large, the stadia determinations must be converted to horizontal equivalents in order to check the resection.

The computation of difference in elevation from vertical angle readings is made with stadia tables or stadia slide rule if the distance is determined by stadia methods; and by trigonometric functions tables or engineers' slide rule if the distance is scaled off the sheet, as in an intersected position.

139. Approximate Determination of Distance

In addition to the ordinary surveying methods of determining distance described previously are a number of approximate methods suitable for only reconnaissance mapping. The following descriptions of these methods are taken roughly in the order of relative accuracy, the most accurate being the first considered.

140. Horizontal (Ground) Rod Method

This is a practical method for determining exceptionally long distances with a moderate degree of accuracy. As will be seen, it is necessary for the instrumentman and rodman to have arranged a system of signals for distant communications before the work starts. No accessory equipment is needed. The field procedure follows.

(1) At a signal from the instrumentman, the rodman drives a stake in the ground and holds the rod in a horizontal position normal to the line of sight with one end resting at a mark on the top of the stake (Fig. 87A).

(2) The knurled ring on the telescope is loosened, and the telescope is rotated 90 degrees in its sleeve so that the stadia wires are vertical, and the ring is tightened again. The alidade is moved horizontally until the upper (or lower) wire apparently cuts the end of the rod not resting on the stake. The instrument must not

be moved from this position until all observations are completed (Fig. 87B).

(3) The rodman is again signaled, and the rod is moved to the opposite side of the stake with the 1-foot graduation nearest the stake. The rod must be oriented perpendicular to the line of sight (Fig. 87B).

(4) The instrumentman now reads the point on the rod cut by the middle wire (Fig. 86B).

(5) The half-interval intercept is one full rod length plus the middle wire reading, and the slope distance is therefore

2 × (1 rod + Middle wire reading) × Stadia interval factor

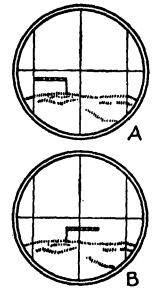


FIG. 87. The ground rod method of obtaining long distances.

In most work this method will not be employed where the vertical angle is large, but in this instance a true slope distance is obtained, and the correction to horizontal distance and the computation of difference in elevation are to be made as outlined in Art. 131.

141. The Vernier Method

This method is subject to large errors unless the vernier readings are made with extreme care. The instrumental procedure follows.

(1) Set the middle wire at a graduation near the base of the rod, and by means of the *left* (index) tangent screw set the vernier exactly on an even minute graduation.

(2) Move the middle wire to a position near the top of the rod, and with the *right* tangent screw set the vernier to the nearest minute graduation. The difference in the two vernier readings is the angle subtended by the rod.

(3) Read the position of the middle wire on the rod.

Record all vernier and rod readings. Repeat the readings several times in order to obtain reliable averages.

$$\text{Stadia interval} = \frac{\text{Difference in rod readings} \times 34.38}{\text{Difference in vernier readings}}$$

EXAMPLE:
From (1): middle wire 2.0 feet; vernier 30°02′
From (2): middle wire; vernier 30°13′
From (3): middle wire 12.3 feet

$$\text{Interval} = \frac{(12.3 - 2.0) \times 34.38}{30°13' - 30°02'} = \frac{10.3 \times 34.38}{11} = 32.19 \text{ feet}$$

$$\text{Stadia distance} = 32.19 \times 100 = 3219 \text{ feet}$$

142. Automobile Speedometer

The distance recorded on an automobile speedometer, or odometer, is sufficiently accurate for sketching purposes in some surveys. Since the speedometer measures distance along the surface of the ground, the accuracy of the distances depends largely on the character of the surface. In open flat country where courses can be run in straight lines, moderately accurate results are obtained by this method. The speedometer used for this purpose should first be tested by running the automobile along a level measured stretch of road several miles long. A correction factor can then be determined and later applied to distances measured. For example, if the speedometer records a measured distance of 5.00 miles as 4.85 miles, then all distances are obtained by multiplying the speedometer reading by the factor 500/485 = 1.03, or an increase of approximately 3 per cent of recorded distances.

143. The Measuring Wheel

Distances along the surface of the ground are sometimes measured by counting the revolutions of a wheel whose circumference has been carefully measured. As in the case of the automobile speedometer, the accuracy of this method depends to a large extent upon the flatness of the ground and obstacles that might cause the course to deviate from a straight line. A measuring wheel is easily made by mounting a bicycle wheel in a light fork with a push-bar, which in turn are made from light tubular metal. A piece of light spring steel, such as piano wire, may be attached

on the rim of the wheel so that it will strike against the fork with a sharp clicking noise at each revolution, and thus enable the operator to count the revolutions without actually watching the wheel continuously.

This method is often used to measure the surface distances between points on winding forest trails where customary surveying methods would be difficult to use or to determine quickly the length of canals, ditches, and other meandering lines.

144. Pacing

Perhaps the oldest known method of measuring distance is that of counting the steps or paces while walking, and it is likewise one of the most useful in reconnaissance mapping. A large number of very useful and moderately accurate maps have been constructed with distances measured solely by pacing and bearings determined with a pocket compass. Despite the fact that many instrumental methods are available in plane table work, pacing is widely used in connection with reconnaissance mapping, and in certain instances it is the best method for obtaining an approximate distance. In broken or wooded country the rodmen can often supply a great deal of important map data by pacing from rod stations to points not visible to the instrumentman; or when the rodmen are far afield, the instrumentman can map features near the table by sighting the directions to the points with the alidade and pacing the distances.

To pace distances with consistent accuracy requires practice. It is far better to use a "natural" step than to attempt to use one of predetermined length, such as 30 inches or 3 feet. The natural step is less tiring and is more likely to be of uniform length. The average pace length is determined by counting the steps while walking over a course whose distance has been measured by stadia or some other accurate means. The average length of pace is the distance divided by the number of steps. This should be done on level ground, up-hill, and down-hill, the pace length being ascertained for each type of surface likely to be encountered during the

survey. There is a tendency for some individuals gradually to increase or to decrease the length of step during the course of the field work, and for this reason it is advisable to redetermine the pace length from time to time.

Various schemes have been devised for keeping an accurate count of the number of paces while measuring long distances. An error of a few paces is ordinarily not important in work that employs such approximate methods; but the error most frequently made is that of miscounting tens or hundreds of steps, and obviously this is important. One of the oldest, simplest, and most reliable practices is to carry a handful of pebbles, and at each tenth or hundredth count to drop a pebble in the pocket. At the end of a course the pebbles in the pocket are counted, and if necessary a permanent record is made in a notebook.

The *pedometer* is a device that is attached to the belt or strapped to the leg in such a way that the motion of walking activates a simple mechanism, which, in turn, causes a recording dial to rotate. The pedometer is fairly reliable where the course is relatively free from obstructions, but since it is operated by any jarring motion of the wearer's body, large errors may occur in country where walking is difficult.

The *tally register* is a watch-sized device which registers on the metal dials every time a small button is pressed with the thumb. When used in connection with pacing, the button is pressed for every step or every fourth step. This is the most convenient of all methods in that the record of paces is always complete. The traverse can be temporarily terminated at any point and later resumed without danger of confusing the record.

Much reconnaissance is done in areas where there are few or no roads and horseback is the only means of transportation in the field. Experience has shown that the pace length of a horse is usually more uniform than that of a man; therefore, the length of the animal's step can be determined over a measured line, as described earlier, and this method of measuring distance made available in the construction of the map.

145. Gradienter—Ground Base Method

The principle of this method is similar to that employing the micrometer eyepiece. It is suitable for only reconnaissance work.

It often happens that two points already plotted on the sheet can be seen from the proposed instrument station, but the angle between two rays drawn to the points is too narrow to give a satisfactory intersection for locating the instrument by a two-point resection. The procedure is as follows.

In Fig. 88A, rays have been drawn from points *a* and *b* toward *i*, the instrument station. The long intersection prevents plotting

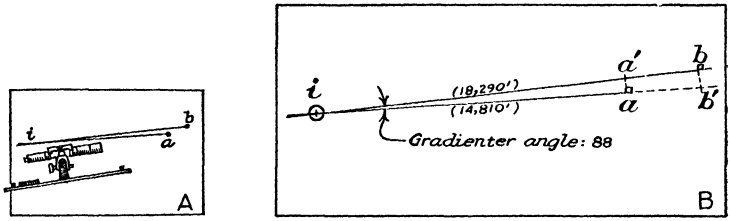


FIG. 88. The gradienter—ground base method of obtaining long distances.

an accurate location. Remove the striding level and loosen the holding collar of the telescope. Rotate the telescope 90 degrees so that the stadia wires are horizontal. Lay the alidade on its left side and turn the prismatic eyepiece with the aperture up. Align the middle wire (now vertical) on the right-hand point (*a* in the figure). Read the gradienter drum. Turn the gradienter until the point *b* is cut by the middle wire and read the drum again. The difference in drum readings is the horizontal angle in terms of drum units. Repeat the process several times in order to check the drum interval. The drum calibration should be determined by stepping the lower wire into the position of the upper wire and determining the number of drum units, as described in Art. 110.

Figure 88B is an enlargement of the rays of A, cited above. Draw a line perpendicular to *a* to cut the ray to *b*. Draw a per-

pendicular from b to cut the extension of the ray to a. Carefully scale distances a–a' and b–b'.

Assume the full interval step to be 101 and the gradienter angle to be 88 drum units. The scaled distance a–a' is 170 feet, and b–b' is 210 feet.

The distance a–i is

$$\frac{88}{101} \times 170 \times 100 \text{ (or stadia interval factor)} = 14{,}810 \text{ feet}$$

The distance b–i is

$$\frac{88}{101} \times 210 \times 100 = 18{,}290 \text{ feet}$$

When these distances are plotted from a and b along their respective rays, they should fall on the same point at i.

This method is subject to a number of errors, and the following precautions should be observed.

(1) The table must be correctly oriented.
(2) The angle should not be larger than one degree when the gradienter is used.
(3) The perpendicular base distances must be scaled with extreme care.
(4) The points a and b must be located accurately on the sheet.
(5) Great care must be taken in all instrumental manipulations.

146. Elevations

All the methods for determining difference in elevation described in previous chapters may be used in reconnaissance mapping. Where moderately accurate elevations are required, a few trigonometric or differential level lines should be run across the map area by methods suitable for standard small-scale work. Such lines will constitute the primary vertical control, and like the primary horizontal control, should be of somewhat higher order of precision than that required in the secondary or tertiary stations. The elevations computed from vertical angles in conjunction with triangulation are usually accurate enough for the primary control in reconnaissance mapping.

Full advantage should be taken of level lines established by earlier surveys. Much valuable data can often be obtained from railroad, utilities, and irrigation companies, river surveys, government barometric stations, the U.S. Forest and Park services, and published maps.

147. Barometric Leveling

Barometric leveling consists of determining elevations or differences in elevation by measuring the pressure of the atmosphere.

It is commonly known that air pressures are greater at low altitudes than at high; but there are also several important causes of variations in atmospheric pressure that are entirely unrelated to elevation, and these variations must always be taken into account when determining elevations with any kind of barometer (mercurial, altimeter, aneroid, etc.). The most important factors affecting the atmospheric pressure at any given point are general storms, high winds, temperature of the air, composition of the air, and a normal daily fluctuation called the *diurnal range* or *diurnal variation.*

Before the surveying aneroid can be used with assurance, it is necessary to know something about the magnitude of these variations relative to those caused by difference in elevation, when and how corrections are made, and under what conditions surveying by this method should not be attempted.

The atmosphere, being a mixture of a number of gases and vapors, does not behave in exactly the same way as a true gas, and for this reason, even though all other variations were eliminated, it would be almost impossible to calibrate the instrument so that exact elevations could be determined by air pressures. Manufacturers have largely overcome this difficulty, however, so that errors arising from this source are small as compared to those from other causes.

The effect of storms, especially severe local ones, is frequently so great that determinations of elevation by aneroid are quite worthless. Within a short period of time severe storms accompanied

by high winds may cause variations in pressure amounting to many hundreds of feet in terms of elevation. There is no way to cope with this situation, except to wait for more favorable weather.

Gusty winds sometimes cause very erratic fluctuations in pressure with consequent effects on elevation that are quite unpredictable. Broad regional storms have profound effects on the average pressures over large areas. Although disturbances of this nature are likely to affect the barometric elevations of the whole map area, they are usually less troublesome than the local storms in determining the difference in elevation between two points situated fairly close together.

Certain restricted localities have been noted where the atmospheric pressures are consistently lower or higher than the surrounding district. These local abnormalities appear to be related to "pockets" of warm or cold air, which in turn may be influenced by topographic conditions or peculiar air currents; but the causes are not fully understood.

As mentioned before, the atmosphere is composed of a mixture of gases, but the relative proportions of the various gases are not everywhere the same. Because of differences in density or weight, there are localities where an unusual concentration of one or more of the components will produce an abnormal effect on the pressures. Usually these conditions are local and temporary, but occasionally they will measurably affect the difference in elevation determined by barometer. There is little the surveyor can do about this situation, except to determine experimentally a calibration factor by repeated readings on points of known elevation.

148. Effect of Atmospheric Temperature

Inscribed on most modern aneroids is the notation "Compensated for temperature." This refers exclusively to the mechanical operation of the instrument, meaning simply that changes in temperature will not distort the mechanism so as to impair its precision in recording atmospheric pressures. Changes in temperature of the atmosphere always affect the pressure, and no matter

what type of barometer is used, corrections must be made for temperature variations.

The barometer is essentially an instrument for "weighing" the column of air pressing down on it, and since the weight, or density, of this column depends on the temperature, surveying aneroids are calibrated for specific temperatures. Most instruments are calibrated for 50° Fahrenheit.

Figure 89 is an exaggerated representation of three columns of air at different temperatures and a scale of altitudes. The horizontal lines of *a*, *b*, and *c* represent units of pressure. Column *b* shows the density of the air at the temperature of calibration of the barometer (50° F.). At an altitude of 200 the pressure is approximately 21 units at standard temperature (column *b*). Now consider column *a*, which represents the pressures at a temperature above the calibration standard. At elevation 200 the pressure is only 17.5 units, because of the smaller "weight" of air pressing down on the instrument. On the other hand, column *c*, at a temperature below the calibration standard, is heavier; and the pressure at elevation 200 is 25.6 units.

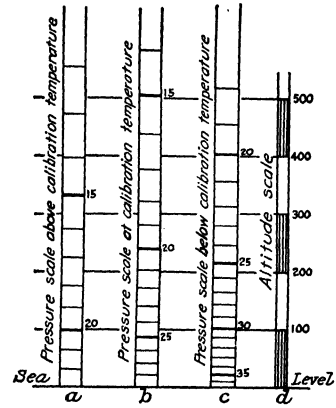


FIG. 89. Comparison of barometric calibration temperatures and altitudes.

Now assume that a barometric reading for altitude is made at temperature *a* (above the standard for the instrument), and the pressure is 15 units. The indicated elevation on the dial of the barometer is 500 (scales *b* and *d*). But this indicated elevation is obviously too high, for the actual elevation is about 330, as shown by scales *b* and *d*. Thus the difference between 500 and 330 is the *temperature correction.*

Since the pressure is (roughly) inversely proportional to the

altitude, it is clear that when the temperature is *above* the calibration temperature of the instrument, readings of elevation on the dial will be too high; hence, the correction is *minus*. When the temperature is *below* the instrument standard, the correction is *plus*.

The value of the temperature correction is approximately: 0.002 × Number of degrees Fahrenheit above or below standard × Indicated elevation on the dial.

EXAMPLE:

(1) Temperature 85° F.; apparent elevation 4000 feet
Elevation = 4000 − (35 × 0.002 × 4000) = 3720 feet

(2) Temperature 30° F.; apparent elevation 4000 feet
Elevation = 4000 + (20 × 0.002 × 4000) = 4160 feet

In ordinary surveying, barometric elevations referred to sea level will usually be obtained by determining the difference in elevation between the unknown point and one whose elevation is known. The "known" elevation may or may not have been established by barometric means. When the barometer is used for determining the difference in elevation, the temperature correction is applied in a different manner, for the temperatures at both stations must be considered. The commonest procedure is to treat the two simultaneously.

If the temperatures at the stations are 75° F. and 85° F. and the difference in barometric readings is 200 feet, the real difference in elevation is:

$$200 + \left[\frac{(85 - 50) + (75 - 50)}{2} \times 0.002 \times 200\right] = 212 \text{ feet}$$

The setup is simplified by using the factor 0.001, instead of 0.002, as follows:

$$200 + (85 + 75) - (2 \times 50) \times 0.001 \times 200 = 212 \text{ feet}$$

or the plus or minus correction is 0.1 per cent of the difference in the sum of the two temperatures and the doubled calibration temperature, multiplied by the difference in barometer readings.

If the instrument in the example above were calibrated for 60° F., the difference in elevation would be:

$$200 + (160 - 120) \times 0.001 \times 200 = 208 \text{ feet}$$

When the sum of the two temperatures is less than the doubled calibration temperature, the correction is subtracted from the apparent difference in elevation.

149. Diurnal Variation in Pressure

This variation in pressure takes place even though the weather, temperature, and other disturbing factors are stable. It follows a qualitatively characteristic rhythm, but it is by no means identical from day to day. Apparently the magnitude of the diurnal varia-

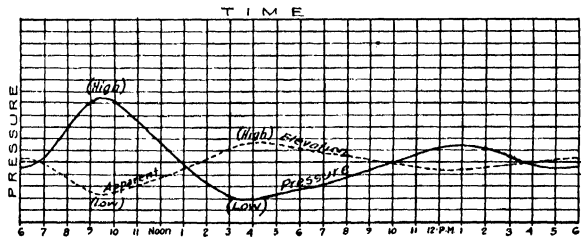


FIG. 90. Comparison of barometric pressures and elevations.

tion decreases from the equator toward the poles, but since regional storms influence the range of the variation as well as general pressures, it has been difficult to ascertain precisely just how much of the total fluctuation is attributable to this cause. According to the U.S. Weather Bureau, at latitude 30° the average range is approximately 100 feet in terms of elevation. At latitudes north of 50° the writer has observed daily ranges of over 200 feet for periods of several weeks duration.

Figure 90 is a schematic diagram of an "average" diurnal curve showing pressures and elevations. Absolute values are not known, but the following features may be accepted as generally characteristic of all normal diurnal curves.

The highest pressures (lowest relative elevations) usually occur three or four hours after sunrise, the lowest in late afternoon, a secondary high near midnight, and a secondary low about sunrise or somewhat earlier. The most rapid rate of change in pressure occurs during the hours from mid-morning to mid-afternoon.

150. Field Procedure Using Several Barometers

When several barometers are available for use in the same map area, a common practice is to keep from one to three instruments on widely distributed fixed stations within the area. Readings of pressure or apparent elevation are made at regular intervals of 5 to 15 minutes throughout the day. With other barometers observations of apparent elevation are made on the required map points. At the end of the day the diurnal correction curves are drawn from the data obtained at the base stations, and from these curves the apparent elevations obtained by the mappers are corrected to true elevations. In making the corrections for map points situated between two base stations, and where there is an appreciable difference in the curves of the base stations, adjustments are made for the relative positions of the three points concerned.

This method has proved satisfactory and has been widely employed in reconnaissance topographic mapping, especially in Canada. The principal objection is that several instruments, together with observers, are needed except when recording barometers are used, and only a part of the number used can be actively engaged in acquiring map data. It is therefore not suitable for work where only one or two instruments are available.

151. Field Procedure with One Barometer

There are two methods that are adaptable for work employing only one instrument. The first is used in areas where a number of known elevations already exist and are distributed in such a way that the mapping is carried on within a comparatively short traveling time to one of the known points. The second is designed

primarily for use where there are few known elevations or where the known elevations are difficult to reach at frequent intervals during the day.

METHOD 1

This method consists in reading the barometer on a station of known elevation, proceeding to the desired station where a reading is made, and then repeating the reading at the first station or any other near-by station whose elevation is known. As in the case of all barometric work, the time to the nearest minute is recorded whenever a reading is made. At the close of the day's work, a diurnal correction curve is drawn from the data acquired at the stations whose elevations are already known, and the new apparent elevations are adjusted according to the correction curve. Since any point of known elevation can be used for checking, it is desirable to have these stations plotted on a map for use in the field so that traverses can be planned to close on the nearest checking station.

METHOD 2

The field procedure for this method is the same as in method 1, except that the "repeat" readings are made on any of the stations recently determined by barometer, regardless of the fact that their actual elevations may not yet be known. The main object is to have at the end of the day a series of repeated readings made at frequent intervals of time. The difference in any two barometric readings on the same station represents the *change* in atmospheric pressure for the elapsed time between the readings. If the elevations are to be referred to sea level, it is necessary that at least one of the stations be one of known elevation. If only one such station is available, the first reading in the morning should be on this station: and it is still better that one or two others be made on it in the middle of the day and in the evening, though this is not positively essential. As will be shown shortly, a very satisfactory correction curve can be constructed from the data obtained by rereading the barometer on stations whose elevations are not known.

152. Barometric Field Notes

Various types of record books for barometric work are available. A convenient form for use with one instrument, as in the methods described above, is shown in Fig. 91. The back of each sheet should be cross-ruled for plotting the correction graphs.

Map Area ..... Page 4
Observer ..... Date July 13, 1945

	Sta	Time	Barometric Apparent Elevation	Cor. + or −	Elevations Corrected	Elevations Adjusted	Remarks
1	1	A.M. 8:40	2522.0	+41.0		2563.8	U.S.G.S. B.M. SE cor. Sec. 6, T. 21 N. R. 5 W. Temp. 73°F.
2	2	8:55	2521.0	+44.0	2565.0		Clear, wind 5 m.p.h.
3	3	9:02	2607.0	+46.0	2653.0		
4	(1)	9:09	2515.0	+48.0	2563.0		Temp. 77°
5	4	9:18	2610.0	+52.0	2662.0	2662.0	
6	5	9:32	2620.0	+57.0	2677.0		
7	6	9:52	2591.0	+64.0	2655.0		
8	(4)	10:20	2588.0	+74.0	2662.0		
9	7	10:41	2710.0	+70.0	2780.0		
10	(4)	11:10	2598.0	+65.0	2663.0		
11	8	11:45	2684.0	+55.0	2739.0		
12	(4)	12:05	2610.0	+52.0	2662.0		Temp. 91°
13	9	12:30	2603.0	+44.0	2647.0	2647.5	
14	(9)	12:55	2612.0	+35.0	2647.0		
15	10	1:30	2540.0	+22.0	2562.0		
16	(9)	1:58	2635.0	+13.0	2648.0		
17	(9)	2:15	2647.0	+1.0	2648.0		
18	11	2:35	2581.0	−9.0	2572.0	2572.5	Temp. 91°
19	12	3:05	2703.0	−16	2687.0		
20	(9)	3:40	2662.0	−15.0	2647.0		
21	(11)	4:10	2580.0	−7.0	2573.0		Temp. 87°. Wind 15 m.p.h.
22	13	4:15	2	−6.0			

FIG. 91. Field notes of a barometric survey.

In the figure, repeated stations are indicated by enclosing the station numbers in parentheses. The barometric or apparent elevations are recorded at the time the observations are made. The plus or minus correction values are obtained during the day for only those stations on which two or more readings have been made; the remainder of the correction values cannot be recorded until the correction curve has been drawn. When a number of slightly varying elevations have been computed for a station, the mean is taken as the elevation of the point, and this is recorded in

the column "Adjusted Elevations," as are all other known and accepted elevations, such as station 1, which is a U.S. Geological Survey bench mark. Barometric elevations like stations 4, 9, and 11 may be used as the starting points for other traverses.

153. Construction of the Correction Curve

The correction curve shown in Fig. 92 is constructed from the field notes shown in Fig. 91. The upper graph shows only the

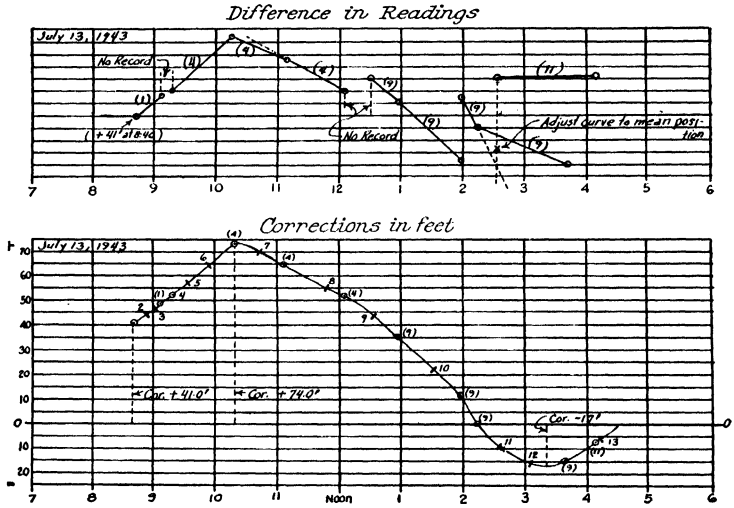


FIG. 92. Construction of the barometric diurnal curve from segments obtained by field observations.

difference in barometric readings without regard for the actual amount of variation from true elevation. The points are plotted at any convenient vertical position, each pair of points representing the time of the readings and the total *change* in apparent elevation. The stations used in plotting these segments are indicated by the numbers in parentheses, which in turn refer to the station numbers in the field notes. Two gaps will be noted in the record of pressure changes; one from 9:09 to 9:18, and the other from 12:05 to

12:30. This condition should be avoided, for such a gap in the pressure record may be the cause of raising or lowering all succeeding elevations in the traverse. When such omissions do occur the downward or upward rate of change in the adjacent segments is used to reconstruct the missing portion, as shown in the lower drawing.

During the hours between 2:35 and 4:10 the readings on station 11 overlap those on station 9. Also during this period the rate of change in pressure is markedly different. This discrepancy is adjusted by selecting a mean point upon which to construct the curve.

The lower drawing is the correction curve used in the adjustment of the barometric elevations. In Fig. 91 the barometric elevation of station 1 is 2522 and the actual elevation is 2563. Since the barometric elevation is 41 feet too low, the correction is plus 41. This starting point is plotted in the correct position relative to the zero (no correction) line on the graph. From this point on, the lower curve is constructed from the partial curves in the upper graph. After the correction curve is thus drawn, the corrections for all the stations are obtained simply by reading the plus or minus values from the curve for the particular time of day that the barometric elevation was obtained.

154. General Remarks

The methods described in the previous articles are used principally where the barometric leveling is to extend over large areas while maintaining a moderately high degree of accuracy in the elevations. For less accurate work, or where only a few stations are to be established, it is usually not essential to draw a correction curve. A simple method of determining the difference in elevation between two points follows.

Read the barometer on one station, and record the time; now go as quickly as possible to the second station and read the barometer and record the time; then return to the first station and repeat the readings. The difference in elevation is the apparent

(barometric) difference in the first two readings plus or minus one-half the difference in the first and third readings on the first station. If the third reading is larger than the first, the correction is minus and is subtracted from the apparent difference in elevation; if the third reading is smaller than the first, the correction is added. This method assumes that the period of time between the first and second observations is approximately the same as between the second and third, and that the rate of change in atmospheric pressure is constant. Both assumptions are more than likely to be somewhat in error, but if the time interval between readings is not too great, fairly good results will be obtained. When there is a very large difference in the two intervals, the correction should be roughly proportional to the time.

The rate of change in pressure cannot be predicted with assurance from one day to the next, or even from one hour to the next. As stated before, the most rapid changes ordinarily take place in the middle part of the day, but approaching storms which may not be apparent can cause serious fluctuations. In certain instances, the change in pressure may amount to 30 or 40 feet per hour, and the rate of change may vary from 3 feet at the beginning of the hour to 30 feet at the end of the hour.

The correction curve takes into account all changes in atmospheric pressure not related to elevation. It is, therefore, not necessary to make any correction for temperature when this method is used for determining the relative altitude of stations. The temperature correction is applied when it is desired to obtain an approximate elevation above the sea level datum and there is no established bench mark in the region.

In working over broad areas it is not uncommon to obtain an irregular correction curve. The irregularities usually reflect local differences in pressure; and since the new stations as well as the base stations are affected by any local conditions, it is better to retain the irregularities in the curve than to "smooth" it out by approximation.

The following precautions should be observed in all barometric

work: The surveying barometer is an extremely delicate instrument and can easily be ruined by rough handling. It should never be subjected to any jarring or violent vibration. When it can be avoided, the barometer should not be taken into, or even shipped through, regions where the elevations exceed the altitude range of the dials on the instrument; this may seriously impair the sensitiveness of the vacuum box for recording small differences in pressure.

Never leave a barometer where one side will be at a much higher temperature than the other, such as by laying it on snow, ice, or frozen ground while the sun shines directly on the upper side. Although this should not cause permanent injury to the mechanism, it is likely to result in erroneous readings for a considerable length of time.

Only specially trained persons with proper facilities to work with are capable of repairing or correctly adjusting a barometer. If a barometer becomes damaged or badly out of adjustment, send it to the manufacturer for the necessary work.

When the instrument is kept indoors at night, it should be taken out in the morning some time before the first observation is to be made so that complete adjustment to the outside temperature will have taken place.

The instrument should be held somewhat above the ground when making a reading. If it is placed on the ground or any object, differential stresses may be set up in the mechanism, owing to certain parts being at different temperatures from others.

CHAPTER VII

Adjustments of Instruments

155. Remarks

Surveying instruments, even though well cared for, are subject to getting out of adjustment. This may be caused by the wearing of different parts, or by differential stresses within the instrument, which, in turn, may be caused by certain screws being tighter than others, or slightly different coefficients of expansion in the various parts. Instruments sometimes get out of adjustment when subjected to widely varying temperatures, even though they are not in use. The unavoidable jarring and manipulations during field work normally cause some change in the adjustments. It is important, therefore, to check them frequently. Checks of certain parts, especially the levels, should be made at least once a day, and under some circumstances, two or more times a day. Other parts may need checking only every week or at periods of several weeks, depending on how roughly the instrument has been used.

The alidade (and other instruments) may be temporarily out of adjustment for the sole reason that one part is warmer than another. In this case it will "come back" into adjustment when an equable state is again reached. Level tubes are very susceptible to small differences in temperature, and for this reason, great care must be taken that heat from the hands does not distort the tube either while doing actual mapping or while making adjustments.

Before starting the adjustment of the alidade, examine it carefully for loose parts and for parts that might be screwed down too tight. Screws should be just tight enough to withstand ordinary use without working loose, but not so tight as to put unnecessary

strains on any part of the instrument or the threads of the screws. Clean all dust and gummy grease from tangent screws and spring plungers opposing them.

If possible keep the instrument entirely in the shade while making adjustments. When this procedure is contemplated, allow the instrument to stand for about half an hour where the adjusting is to be done. Where it is desirable or necessary to do the adjusting in direct sunshine, leave the alidade in the sunshine for a while prior to beginning the work. When adjusting an instrument in sunshine, avoid casting shadows over it.

In order to reduce errors caused by refraction and differential temperatures, it is desirable to adjust instruments on days when the sun is hidden by clouds, but not to the extent that light is poor. Choose flat, level ground, and set up the table where the footing is firm. When making those adjustments requiring the use of the telescope, extreme care must be taken that the cross wires and telescope are in perfect focus (Art. 23).

156. The Striding Level

LATERAL ADJUSTMENT

The longitudinal axis of the level tube should be in the same vertical plane as the longitudinal axis of the telescope. This may be tested as follows (refer to Figs. 12 and 13).

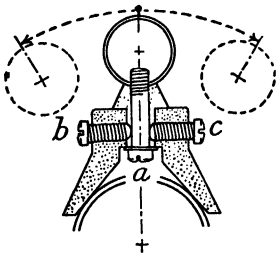


FIG. 93. Lateral adjustment of the striding level.

(1) Unscrew the center stud "28," place the striding level on the telescope, and center the bubble. (2) Move the striding level from side to side around the telescope, about 30 degrees on each side (Fig. 93). If the bubble does not move toward the ends of the tube, the lateral adjustment is correct. If the bubble tends to "crawl," the axis of the tube is not parallel to the telescope and, therefore, must be adjusted. (3) Loosen screw *a* (Fig. 93), turn screw *b* slightly to the

left, and turn screw *c* to the right until firm, but not tight, against *a*. (4) Replace the striding level on the telescope and test as in (2). If the movement of the bubble is in the same direction, but farther from the center, the adjustments of screws *b* and *c* were in the wrong directions, and are corrected by turning *b* to the left and *c* to the right. Repeat this process until the bubble does not move when the level is rotated about the telescope. When the lateral adjustment is complete, tighten screw *a*.

Vertical Adjustment

On the opposite end of the tube from the lateral adjustment screws is a screw for the vertical adjustment. On some instruments the screw operates against, and inside, a coil spring; but most new instruments are provided with a pair of capstan nuts instead of a spring.

Reversing Method

In order to avoid disturbing the telescope while making adjustments, the spring catch and boss should be removed from the striding level by turning screw "32" (Fig. 13). (1) Place the striding level on the telescope and center the bubble by means of the axis clamp and the right tangent screw. (2) Lift the level off the telescope, turn it end for end, and carefully replace it, *without disturbing the telescope*. If the level is in adjustment, the bubble will return to the center. If the bubble comes to rest some distance from the center, elevate (or depress) the telescope by means of the tangent screw until the bubble is midway between this point and the center. (3) Now, by means of the capstan nuts *a* and *b* (Fig. 94) adjust the level tube until the bubble comes to the center when the striding level is placed on the telescope in the same position as in (2). Reverse the level again to check the adjustment.

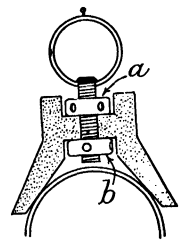


Fig. 94. Vertical adjustment of the striding level.

Note: If it is desired to raise this adjustment end of the tube, first turn nut *b* to the left, and then turn nut *a* also to the left until firmly opposing *b*. If this end is to be lowered, first turn *a* to the right, and then tighten the adjustment by turning *b* to the right.

Two-Peg Method

When the two-peg method of adjustment is used, it is imperative that the striding level be used thereafter always in the same position; i.e., with the same end forward. This is a more exact adjustment than the reversal method, in that the axial line of the level is made parallel with the line of sight, whereas the reversal method adjusts to the alignment of the ring bearings on the outside of the telescope. The procedure follows.

(1) With tape or stadia measure a line 200 or 400 feet long on level ground. Set up the plane table exactly in the middle of the line and place stadia rods or stationary poles at the two ends. (2) Level the telescope and sight on each of the two poles and mark the positions cut by the horizontal cross wire (points *a* and *b*, Fig. 95A). These two points have the same elevation, regardless

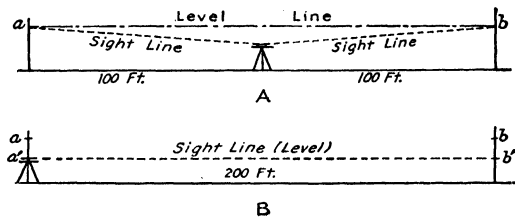


Fig. 95. Establishing a level line with a bubble out of adjustment, and then (in B) using this line to adjust the level by the two-peg method.

of the amount of error in the level. (3) Move the table to one end of the line, level the telescope, and sight on the near pole (point *a*, Fig. 95B). With scale or tape, measure the distance between the two points *a* and *a'* on the near pole, and then use this distance to establish point *b'* on the far pole. (4) Now sight

the horizontal wire exactly on point b' on the far pole. If the operations have been carried out without error, the sight line is now level. (5) Without disturbing the telescope, adjust the level tube until the bubble comes to rest in the center. (6) Check the adjustment by setting up at the second pole, and, proceeding as above, sight back on the first.

If the lateral adjustment is made first, check it again after making the vertical; or if the vertical adjustment is made first, check it again after making the lateral.

157. Auxiliary (Control) Level

The auxiliary level "14" (Figs. 12 and 13) is an arc index level, operating independently of the telescope. When properly adjusted, the auxiliary bubble is centered when the telescope is level and the vertical arc reads 30°00′. (1) After adjustment of the striding level, carefully level the telescope. (2) Using the arc index tangent screw "4" (Fig. 13), set the arc indices at their respective zero points. (3) Now adjust the auxiliary bubble by turning adjustment screws "10."

158. Universal (Circular) Level

To adjust the universal level "3" it is first necessary to level the plane table by means of the striding level. (1) Loosen the Johnson tripod head and place the alidade in the middle of the table. (2) Set the vertical arc on 30°00′ and approximately level the table by centering the auxiliary level. (3) By means of the right tangent screw, center the bubble of the striding level, then carefully lift the alidade and turn it end for end. (4) If the bubble is off center, bring it halfway back with the right tangent screw, and the remainder of the way by tilting the table.

This levels the table in one direction. (5) Now turn the alidade 90 degrees, being careful not to move the table, and repeat the process as outlined above. Clamp the Johnson head and check the leveling of the table with the alidade oriented in several directions.

(6) Adjust the universal level until centered by means of the screws "2" (Figs. 12 and 13).

In adjusting the universal level it is important to keep the alidade directly above the Johnson tripod head, as the weight of the instrument will tend to tilt the table if too near the edge.

159. Finder Sights

The finder sights "33" on the striding level should be in both lateral and vertical alignment with the telescope. To align the sights, move the telescope until the cross wires are centered on some clearly defined small object a few hundred feet distant from the table. Sight along the finder sights with the eye about 2 feet back of the rear sight. With pincers bend the sights into proper lateral alignment. If the alignment of the tops of the sights is too high or too low, file the top off one of the posts until the correct elevation is obtained.

160. Cross-Wire Alignment

When the telescope is elevated or depressed, it should move in the same vertical plane as that containing the vertical cross wire. To determine this relationship, proceed as follows.

Set one end of the vertical cross wire on a small, distinct point a few hundred feet from the instrument. With the right tangent screw, elevate (or depress) the telescope. The vertical wire should trace the point throughout its length. Now loosen the knurled collar "9" (Figs. 12 and 13), rotate the telescope 180 degrees until it strikes the stop, tighten the collar again, and repeat the above test with the telescope in the inverted position. If the vertical wire moves off the point as the telescope is elevated or depressed, an adjustment is necessary.

(1) Loosen the cross-wire reticule by turning the four capstan screws, *a*, *b*, *c*, and *d* (Fig. 96). (2) Lightly tap the collets so the reticule is rotated in the barrel of the telescope. (3) Check by testing as in the paragraph above. (4) Tighten the capstan screws and

check again, then tighten the knurled collar with the telescope in the correct position.

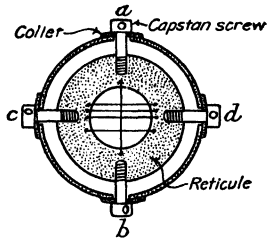


FIG. 96. Construction of the cross-wire reticule.

161. Collimation

An instrument is in correct collimation when the two cross wires and the optical axis of the telescope all intersect at a common point. To test the collimation, loosen the knurled collar "9" and sight the cross wires on a small, clear, distant point. Now, carefully rotate the telescope in its sleeve. The cross wires should remain on the point while the telescope is being rotated. If they move off the point during this operation, but return to the point when the telescope is brought back to the original position, an adjustment of the cross wires is required. The vertical and horizontal wires are adjusted independently as follows.

Set up a stadia rod from 100 to 200 feet in front of the instrument. Loosen the collar "9" so the telescope can easily be rotated. (1) Level the telescope with the striding level and read the *horizontal* cross-wire position on the rod, point *a* (Fig. 97A). (2) Turn

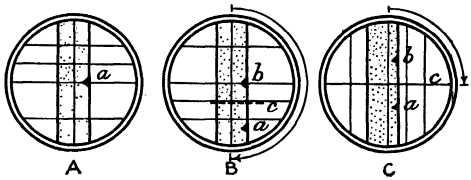


FIG. 97. Steps in the adjustment of collimation.

the telescope through 180 degrees until it strikes the stop, check the bubble, or relevel the telescope, and read the rod again, point *b* (Fig. 97B). (3) If the second reading is higher than the first, as shown in the figure, loosen capstan screw *a* and tighten *b* (Fig. 95) until the cross wire is midway between the two previous readings, point *c* (Fig. 97B). (4) The horizontal wire should now be in ad-

justment, but check it by righting and inverting two or three times, making slight adjustments as above until the wire reads exactly the same in either position.

To adjust the *vertical* cross wire, turn the telescope through 90 degrees, so the (normally) vertical wire is in a horizontal position. Level the telescope and read the rod. When in adjustment, the vertical wire will read exactly the same as the horizontal, point *c* (Fig. 97C). If the vertical wire is above or below point *c*, adjust it to this position by means of screws *c* and *d* (Fig. 96).

Be sure the telescope and cross wires are in perfect focus when making these adjustments.

162. Beaman Stadia Arc Index

If the instrument is in perfect adjustment, the Beaman stadia arc index "8" (Fig. 12) will read 50 when the arc vernier reads 30°00′. To adjust the Beaman index, loosen the holding screws "21," and lightly tap the index plate into the correct position, then tighten the holding screws again. When the control level has been adjusted to the Beaman index, adjust the vernier by loosening the holding screws "7."

When the vernier is set at 30°00′, the horizontal distance correction index "5" should read either 0 or 100, depending on whether the graduations on the particular instrument indicate percentage correction or percentage distance. Adjust by loosening the holding screws on the plate as for the other indices.

163. The Magnetic (Compass) Needle

Because of the vertical component of the earth's magnetic force, the north end of the compass needle tends to dip downward, and the amount of this dip depends on the locality. It is, therefore, necessary to "level" the needle when the instrument is taken from one place to another if there is much difference in the vertical intensity of the magnetic attraction.

The dip of the compass needle is compensated for by a small

counter balance which consists of a small coil of fine, non-magnetic wire, wound around the shaft of the needle near the south* end. If the north end of the needle is too low, move the wire farther toward the south end of the needle. The needle should be perfectly balanced when the base of the alidade is level.

When working with the compass needle, be sure that the hands are dry. It is good practice to rub a little oil on the hands before touching the needle.

164. Alignment of Base, or Blade

Under most surveying conditions it is not necessary that the *fiducial* (beveled) edge of the blade be exactly parallel with the line of sight, though this relationship is desirable. Some manufacturers put slots and bosses on the blade and pedestal so that alignment is effected when the two parts are put together. Others are held in alignment only by the ring clamp at the base of the pedestal, the clamp being fastened by holding screws through the blade. The following method of alignment should be undertaken only where there is no wind, preferably indoors.

(1) Set up and level the plane table at the same height as, and near the end of, a bench or flat plank 15 to 20 feet long. (2) Tightly stretch a fine, strong thread along the top of the bench and across the plane table, being careful that it is not pulled out of a straight line by dragging on the surface of the bench. (3) At the far end of the bench set vertically a fine sewing needle to one side of the thread at a distance equal to one-half the width of the alidade blade (Fig. 98). (4) On the plane table set two needles on the line of the thread, the distance between the needles being slightly less than the length of the blade. (5) Loosen the holding screws on the bottom of the blade so that the pedestal and telescope can easily be rotated with reference to the blade. (6) Now place the fiducial (beveled) edge of the blade against the two needles (Fig. 98A) and adjust the telescope until the far needle is

* In northern latitudes.

exactly cut by the vertical cross wire of the telescope. Tighten the holding screws and then check the alignment by sighting on the needle again.

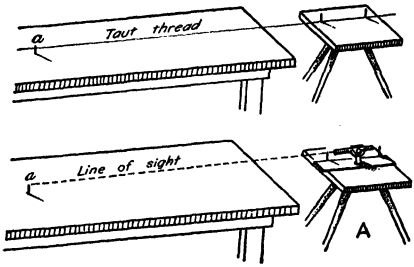


FIG. 98. Adjustment in the alignment of the alidade base.

165. The Gradienter (Stebinger) Screw

If the gradienter screw "20" (Fig. 12) is in perfect adjustment and the instrument is relatively free from wear, one revolution of the drum *b* (Fig. 35) will elevate or depress the telescope exactly one interval (when the telescope is operating near the level position). The tests and adjustments are as follows.

(1) With steel tape measure a distance 100 feet from the center of the instrument and set up a rod on this point. (*Note:* If the instrument is external focusing, add the amount of the $(f + c)$ constant to the 100 feet.) (2) Level the alidade and mark the position of the horizontal cross wire on the rod. Scale and clearly mark on the rod two lines exactly 1 foot apart, situated 6 inches above and below the first mark. (2) Set the horizontal cross wire on the lower mark, using a thrust motion of the tangent screw, and read the gradienter drum *b* (Fig. 35). (3) Now set the cross wire on the upper mark and read the drum again. If the second drum reading is the same as the first, the gradienter is in adjustment.

If the drum has turned through more than one revolution, loosen the nut *c* and turn the eccentric bushing *a* so that the point of the screw *f* is brought to bear at a higher position on the axis arm *d*. If the drum has been turned through less than one revolution for

1 foot on the rod, turn the eccentric bushing so that the point of the tangent screw is lowered on the axis arm. Repeat the adjustments and tests until the proper relationships are attained.

The index scale *e* (Fig. 35) should be close to, but not quite touching, the gradienter drum. To adjust, loosen the set screw *g*, turn the scale shaft to the desired position, and then tighten the set screw.

The bearing surface of the axis arm *d* and the tapered end of the tangent screw *f* should be smooth and highly polished. If they have become worn or pitted, they may be dressed with a very fine grade oil stone and polished with a razor hone. Lubricate these surfaces with a soft lead pencil.

166. Brunton Pocket Transit

The adjustments of the Brunton pocket transit (Fig. 99) are most easily made with the aid of the alidade and plane table. Set up and orient the plane table, and, by means of the alidade, select a small object a few hundred feet distant that is the same elevation

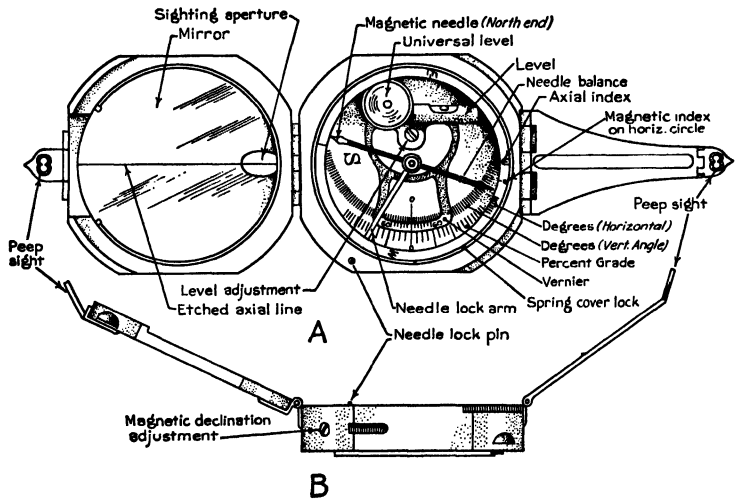


FIG. 99. Parts of the Brunton pocket transit.

as the table and clearly visible to the unaided eye. Now place the Brunton transit on the table in an upright position, as shown in Fig. 99A, so that the line of sight through the peep sights cuts the object. While the Brunton is in this position, center the bubble of the tubular level by means of the lever on the back of the case. Read the vernier. If the tubular level is in adjustment, the vernier will read 0°00′. If an adjustment is necessary, proceed as follows. (1) With a small knife blade, pry out the spring wire which holds the glass cover in place. (2) Remove the cover and carefully lift the compass needle off its pivot, together with the needle lock arm and the circular bushing on the pivot. (3) Loosen the level holding screw just enough to allow the level to be moved with reference to the vernier frame on which it bears. (4) Shift the position of the level a small amount, replace the transit on the table, and test again as in the paragraph above. Repeat the adjustments and tests until the vernier reads zero when the bubble is centered and the sight line exactly cuts the object.

If the plane table is not available, adjust the Brunton transit level between any two points of equal elevation or by the two-peg method.

To adjust the circular level, place the transit *in the center* of a carefully leveled plane table in the horizontal position shown in Fig. 99B. Hold the transit down firmly against the table with one hand while adjusting the bubble to center by bending the soft metal frame which attaches the level to the axis screw. Balance the magnetic needle by sliding the coil of wire along the south end of the needle shaft.

The needle locking arm should lift the needle just high enough to touch the glass cover when the lock pin is pressed down flush with the surface of the case.

To set off the correct declination, orient the sights on a true north-south line and turn the magnetic adjustment screw until the north end of the needle reads zero on the graduated horizontal circle. If the angular value of the declination is known, turn the adjustment screw until a corresponding angle is indicated at the

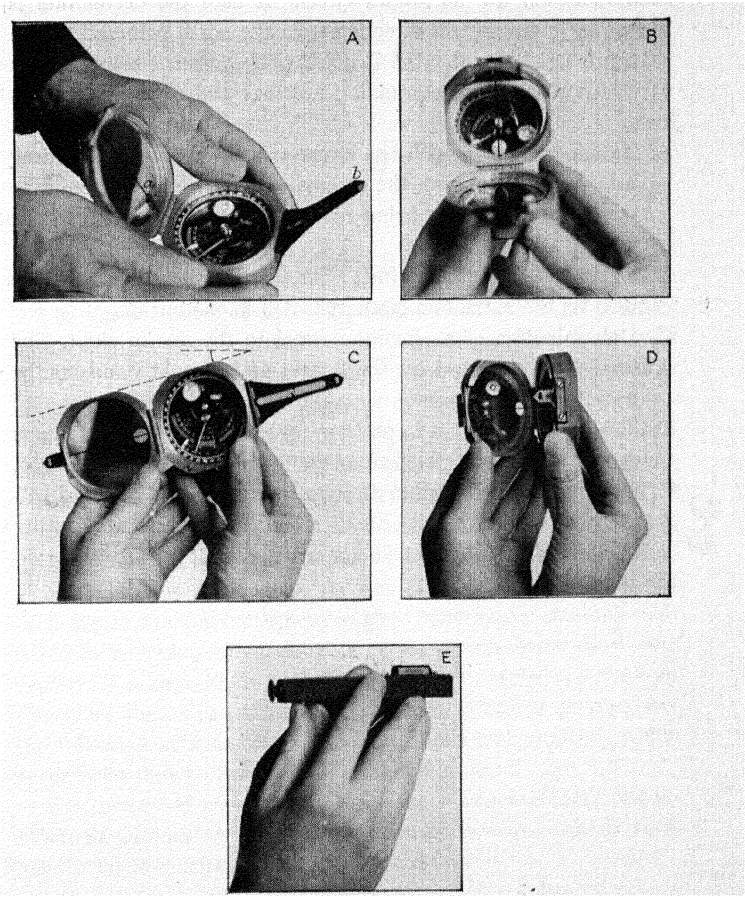


FIG. 100. Illustrating the various uses of the Brunton pocket transit.

axial index on the horizontal circle. Be sure the declination is turned off in the correct direction from the north point.

Figure 100 illustrates the Brunton pocket transit being held in various positions for determining bearings and elevations as follows:

A. Taking a bearing with the Brunton held so that the sighting arm *b* is aligned with the collimation line *a* and the point to which the bearing is being read. This point is viewed in the mirror.
B. Taking a bearing by a direct sight on the point. The bearing is read on the horizontal circle reflected in the mirror.
C. Determination of an incline normal to the line of sight. The level is manipulated by the fingers of the right hand on the back of the instrument.
D. Holding the Brunton in position for use as a hand level or for determining the angle of an incline parallel to the line of sight.
E. Holding the hand level in position for sighting. Note that in all the positions shown the hands are braced against each other to provide a steady rest for the instrument.

CHAPTER VIII

Care and Repair of Instruments

167. General Remarks

The various parts of surveying instruments are fitted together with extremely small tolerances so that no discernible play is present in any of the moving parts. Optical and mechanical alignments and fine adjustments of the different measuring devices are essential features of the alidade and other surveying instruments. If any one of these essential qualities is impaired, the result is an inaccurate instrument or one that is awkward to use. Consequently, it is to the advantage of anyone using surveying equipment to keep it always in good repair, and whenever possible to take precautions against undue wear or minor damage. The amount of time spent each day in keeping equipment in good condition is small as compared to field time lost through equipment failure resulting from neglect. It is beyond the scope of this book to include all the contingencies that arise during the course of extensive surveys, but the following articles cover those conditions and problems which occur from time to time during ordinary projects.

Novices should never attempt to clean or repair a surveying instrument without the close supervision of someone who is thoroughly familiar with its construction and use.

168. Transportation of Instruments

In packing instruments for shipment it should be borne in mind that the package may be subjected to much rough handling before reaching its destination. Therefore, it is necessary to provide a strong box which can withstand rough treatment and to pack the

instrument in such a way that any shocks that the box may receive will not be transferred to the instrument.

Alidades, telescopic stadia hand levels, compasses, and similar instruments should first be wrapped in paper to prevent dust and fine silt getting into the vital parts. They should then be packed firmly in the carrying cases with wadded or shredded paper, just tight enough so that they will not rattle when the case is roughly shaken, but not so tight as to put any appreciable strain on the instrument. If the distance is great or the mode of transportation rough, the carrying case in turn should be packed in a strong box of such size that a layer of excelsior 2 inches to 6 inches thick can be tightly packed completely around the case.

Barometers are extremely delicate and must be handled with great care if they are to function properly. If it is possible to avoid doing so, they should not be carried or shipped through regions where the altitudes greatly exceed the altitude range indicated on the instrument dial, as the low (or high) atmospheric pressures cause deleterious strains on the vacuum chamber. They are particularly sensitive to shocks, so in packing for shipment, thick layers of excelsior or other resilient material must entirely surround the instrument case.

Tripod legs should be loosened by means of the wing nuts, and the wing nuts should be tied to the tripod with strong cord to prevent their being lost. The tripod head can be protected by padding with several layers of burlap bound around with cord.

Stadia rods are best prepared for shipment by first wrapping each section in heavy paper, then binding the several sections together in burlap. As a further safeguard against the rubbing of the graduated face, small pads of soft cotton cloth may be placed between the sections at intervals of 2 or 3 feet before the sections are bound in burlap.

Plane table sheets must never be rolled. They should be packed flat with strong cardboards or boards on both sides of the sheets so that they can be neither bent nor punctured. If the sheets are likely to be exposed to rain or any excessive moisture, they should

be enclosed in an envelope of oilcloth or other waterproof material.

Steel tapes, chaining pins, and other steel or iron equipment must be thoroughly dried and preferably oiled to prevent rusting during shipment.

169. Cleaning and Lubrication of Instruments

In regions where there is a minimum of dust in the air, the working parts of the alidade may be lubricated with a very light grade of machine oil, such as typewriter oil. If, however, there is much dust where the instrument is being used, it is bad practice to use oil, as it tends to collect sand and silt and carry them into the bearings, where they act as abrasive agents. A good dry lubricant is made by pulverizing the "lead" from a pencil of HB to 2H hardness. This (graphite) powder is rubbed on the clean, dry surfaces of the moving parts of the instrument.

Before lubricating any part of an instrument clean it thoroughly with gasoline or alcohol. Never use acid solvents, such as gun cleaners, on surveying instruments. The points (as well as threads) of tangent screws and the hardened plates against which they bear will work more smoothly if they are covered with a thin coating of graphite from a soft pencil.

Compass needles with jewel bearings should not be lubricated.

170. Carrying Cases and Brackets

Leather carrying cases for alidades, hand levels, compasses, etc., must be kept in good repair if they are to protect the instruments. The instruments should fit snugly in their respective cases when the covers are fastened, but after much use leather cases tend to become stretched so that the instruments no longer fit exactly. This can be remedied by relining the case with thick felt, or by padding beneath the old lining with layers of blotting paper. The handles and carrying straps ought to be inspected frequently to ascertain that they are strongly fastened to the case. If the leather becomes hard or checked, it can be restored by washing with saddle soap, and when fully dry, dressing with neatsfoot oil.

Brackets on automobiles, used for carrying stadia rods and tripods, should be well padded with cloth or rubber.

171. Cleaning Lenses and Arcs

Never put hot water on a lens, as it may cause minute fractures over the surface or may actually break it. Dusty lenses must be wiped straight across *only once with the same part* of the cloth or paper tissue. Use a clean part of the cloth each time that the lens is wiped to prevent scratching it. If a lens is greasy, wipe it with a soft cotton cloth moistened with alcohol. It sometimes happens that the lenses of surveying instruments get smeared with pitch from trees or shrubs. This may be safely removed with pure turpentine; however, the turpentine leaves an objectionable oily film on the lenses, and this can be washed off with alcohol. It is a good practice to keep in the instrument case a pack of soft cotton cloth or absorbent tissue cut in 3-inch squares and enclosed in a dustproof envelope. Coated lenses should be "dusted" with a camel's-hair brush.

The silver vertical arcs on the alidade sometimes become so blackened that the graduations are difficult to read. This tarnish is readily removed by any good silver "polish." After the tarnish has been removed, the graduations are made more legible as follows. With a soft brush apply a thin coat of black waterproof drawing ink, working it down into the engraved graduations with the brush. Using a soft cloth slightly moistened with water, wipe off the ink, when it is dry, with a straight motion across the engraved lines. This will leave ink only in the engraved portions of the arc. Be careful that ink does not get down between the arcs.

172. Grinding Johnson Head Bearings

The Johnson tripod head contains two bearings, one allowing the plane table to rotate in only the horizontal plane and the other permitting a universal movement (Fig. 14). These spherical bearings sometimes become deeply scratched and grooved by sharp sand which gets between the plates, and the effect of this is to

make the movement jerky so that it is difficult to level or orient the plane table exactly. The bearings can be satisfactorily ground in the following manner.

First, remove the bearings and, to remove all dirt and grease, wash all the parts, including the plate attached to the tripod, with gasoline or alcohol. Now, on all of the bearing surfaces, spread thinly and evenly a coat of fine grade automobile valve grinding compound. Place the plates back in their proper positions on the tripod and grind the surfaces with an undulating, circular motion so that the *entire* surfaces are ground at the same time. Frequently lift the plates slightly and spin them around to be sure that the compound is evenly distributed. After a few minutes of grinding, remove the plates, clean them, and put on a fresh supply of the compound. When the surfaces are quite smooth, wash all the parts thoroughly, dry them, and then lubricate by rubbing with a soft lead pencil.

173. Loose Plane Table Bearing Plate

On the under side of the plane table is a metal plate with a threaded socket which screws on the upright threaded lug of the Johnson head. This plate is fastened to the plane table with short wood screws which often become loose through shrinking or warping of the plane table. Remove the metal plate and plug the screw holes with soft pine pegs covered with glue. Replace the plate so that the screws can be driven in new holes drilled between the old ones. Plane tables may be refinished with white shellac.

174. Stadia Rods

Stadia rods (sometimes called stadia boards) are made of lightweight soft wood, usually white pine, spruce, or poplar. While the light weight of the soft woods is advantageous, they are easily broken and are subject to warping if not properly cared for. For the latter reason, it is good practice to store the rods in a dry place, preferably flat on the floor. If the rods are of the hinge or section type, the metal fixtures have to be checked frequently, and if

found to be loose, they must be repaired immediately, for if rivets or screws are not kept tight the soft wood quickly tears out around the fixtures so that repair is difficult or impossible.

The paint on rods will gradually deteriorate because of constant exposure to the sun's rays, rain, mud, etc. When the paint becomes dull, it does not reflect the light properly, and, consequently, the rod is difficult to read. It is therefore necessary to repaint the rod from time to time.

Before repainting a rod, thoroughly wash and dry it, and lightly sandpaper the surface. Paint the white background first and allow it to dry before the black (or red) graduations are painted. Use a small, soft, pointed brush for painting the graduations so that sharp, exact outlines are obtained. The paint used should be enamel with a fairly glossy finish.

Newly constructed rods should be painted first with a hard wood filler coat, and then two thin coats of white enamel, each coat being allowed to dry before the next is applied. The graduations are then carefully measured and drawn on the rod with a soft lead pencil. Considerable improvement in legibility is gained if the rod is "embossed" by cutting V-shaped grooves along the pencil lines with a small sharp knife. These grooves should be about 0.1 inch deep. The graduations are now painted with two coats of gloss-finish enamel or paint.

The paint on rods is made more resistant to weather and the sun's rays by applying a thin coat of clear, white boat varnish.

175. Worn Axis Bearings

If the axis bearings of the alidade are badly worn, the equipment of an instrument shop may be required to restore them to proper working condition. However, it is occasionally necessary to make repairs in the field. To tighten the telescope axis, first remove the screws and bearing caps. If there are shims (paper-thin brass sheets) under the caps, remove them and replace the caps and screws. If the axis is still loose, or if no shims are present, cut a very small amount of metal off the flat part of the bearing cap

with a fine flat file. Be careful that all the surface is being cut simultaneously. Test the cap on the bearing frequently in order to avoid cutting away too much metal from the cap. Before putting the instrument together, wash all filings off with gasoline and lubricate the surfaces of the bearings with a soft lead pencil.

176. Stadia and Cross Wires

The stadia wires in most modern instruments are finely drawn platinum. In all the older types, and some of the new, very fine spider web is used. Regardless of the kind of wire originally used in the instrument, emergency replacements can be made with spider web. Any web used should first be examined under a magnifying glass to be sure that it is free from adhering dust particles, and is of small, constant diameter. It may be necessary to search in shrubbery or other dark places to find (preferably small) spiders from which clean, fine strands can be obtained. The spider can usually be made to "spin" a web by shaking him off the end of the stick. A fine web will ordinarily be found attached to the end of the stick. After a satisfactory strand has been obtained proceed as follows.

Remove the cross-wire reticule by loosening the capstan screws, being careful not to break any of the undamaged wires remaining in the reticule. Place the reticule on a dust-free table with the cross wires on the upper side. Examination with a magnifying glass will reveal minute scratches on the rim of the reticule, which indicate the correct locations of the cross and stadia wires. With a sliver of wood, such as a toothpick, place very small drops of glue or shellac on two—opposite—scratches. Now on a light forked branch cut from a tree or bush, pick up a section of the spider web, as shown in Fig. 101A. A small twig may be suspended at the end of the web so that it will be under slight tension. Turn the fork over and, while using the magnifying glass,

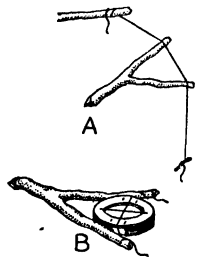


FIG. 101. Replacement of stadia wires.

press the taut strand down into the glue exactly on the register scratches, where the moist glue will hold the strand in place (Fig. 101B). Repeat this process for each wire.

After new cross wires or stadia wires have been put into the telescope, the instrument must be checked and (usually) adjusted for collimation, alignment, and stadia interval factor.

In order to safeguard the stadia wires never remove the eyepiece in a dusty or windy place. Never attempt to blow dust particles off the stadia or cross wires, as they are likely to be stretched or broken.

177. Refinishing Alidade Base

Alidades are now made with bases plated with stainless metal. The bases of earlier types are coated with white porcelaneous enamel, which gradually chips and wears off so that the plane table sheets are blackened by the exposed bronze of the blade. Ordinary paints are not satisfactory for this use because they wear off on the sheets and make the later inking of the sheets difficult. A temporary protective surface may be made as follows.

In 2 or 3 ounces of commercial acetone dissolve sufficient celluloid to make a solution having about the viscosity of thin syrup. The best celluloid for this purpose is old camera negatives from which the silver emulsion has been washed with hot water. The celluloid is cut into small pieces and dropped into the acetone, where it will dissolve in a few hours.

Clean the surface of the alidade blade with alcohol, and if necessary sandpaper lightly with a very fine grade of sandpaper or emery cloth. Apply the acetone solution as rapidly as possible with a broad, soft brush. The acetone dries so fast that the alidade can be used within a few minutes after the solution has been applied. To remove the film from the blade for repainting, simply wipe the surface with a cotton cloth saturated with acetone.

178. Steel and Cloth Tapes

Steel tapes (or chains) should be wiped with an oily cloth before they are taken into the field, if they are likely to get wet during

the day's work. If the tape has gotten wet or muddy, it should be washed and thoroughly dried, and then oiled, as soon as possible after the work is finished. Steel tapes have a great deal of tensile strength so long as they contain no kinks or loops, but are very easily broken if allowed to become twisted; therefore, particular care must be taken in coiling the tape, or using it in brushy country.

Cloth tapes are treated with a waterproofing solution to prevent their becoming wet and consequently shrinking. After the tape has been in use for some time, however, this waterproofing wears off. To waterproof a cloth tape, first check the length with a steel tape. If it is found to be too short (from shrinkage) soak it for a few minutes in water, after which it may be stretched to the correct length. Fasten it (stretched to the correct length) so that it will dry evenly, and when it is completely dry treat it with white shellac which has been thinned with alcohol.

If the tape is too long, soak it in water, and hang it loosely by the two ends with two or three supports between. Check it again with the steel tape after it is dry, before treating it with shellac.

CHAPTER IX

Characteristics of Vertical Aerial Photographs

179. Introductory Remarks

A handbook on plane table mapping would be incomplete if it did not also contain certain methods of mapping by vertical aerial photographs. In some respects the plane table and photographic methods are similar, and for this reason it is not difficult for a person familiar with one to grasp readily the principles of the other. There are now several very good manuals and textbooks on the general subject of aerial photographs, and for this reason the following treatment of the subject is in no sense exhaustive. It is the intention in following pages to define the advantages and limitations of aerial photomapping, to describe those photogrammetric methods which require little or no special equipment, and to outline the uses of photographs in connection with plane table methods.

It has been demonstrated on a number of large mapping projects that a combination of vertical aerial photographs and the plane table comprises a method that is usually superior to either one alone. The advantages of one complement those of the other: the photographs supply a great wealth of map detail, often in areas almost or quite inaccessible to the plane table; the plane table provides the most convenient and rapid means of obtaining accurate quantitative relationships of the features shown on the photographs. The advantages of vertical aerial photographs are sometimes not fully exploited in plane table mapping. Apparently,

this is due to several causes: a reluctance to alter the older "ground" methods to permit full use of photographs; lack of training and experience in aerial photogrammetry, or skepticism resulting from previous misuse of photographic methods with unsatisfactory results; and the erroneous impression that photographs and special equipment are too costly for the advantages that may be gained through their use. It has already been intimated that very little special equipment is necessary for using the photographs effectively. The costs of photographic prints vary according to the remoteness of the map area from suitable flying bases, the number of prints required, and general weather conditions. If the area has been flown by government agencies or any of the large commercial companies engaged in this kind of work, the prints can be obtained at very reasonable cost.

The relative value of aerial photographs in plane table surveys depends on a number of conditions. One of the most important considerations is the type of map that is to be made—geologic, topographic, hydrographic, small scale, large scale, etc. In some areas the photographs will be of little use to the plane table man because of the paucity of well-defined map features. The picture of grassy plains with little topographic relief would be of questionable value, except possibly in geologic or soil mapping. The photographs of precipitous mountain country of high relief can often be used to great advantage, but in this case photographic distortions are bound to be large, and a clear understanding of these distortions and the principles of aerial photogrammetry is an essential requisite to the proper use of the pictures. As a rule, the photographs of greatest use to the plane table man are those which exhibit an abundance of map detail; for, as will be shown later, photographic images that can also be identified on the ground are necessary in the mapping processes.

180. Kinds of Aerial Photographs

There are three general classes of aerial photographs used in the construction of maps: the oblique, which is a perspective view

taken with the camera inclined considerably from the vertical; the composite, which is taken with a multilens camera, producing (usually) one central vertical and a number of peripheral obliques that are later transformed into vertical equivalents; and the vertical, which is taken with the optical axis of the camera in as nearly a vertical position as possible. It is the vertical photograph that will be considered in the following pages.

Photographic prints (positives) may be obtained on papers having a wide variety of textures and lusters. The glossy finish produces sharp definition of the picture images, but is unsuited for pencil work. India ink can be used satisfactorily on the glossy finish. If much pencil work is to be done directly on the surface of the picture, such as topographic contouring, a soft matte finish should be used. This finish also provides an excellent surface for inking. Picture images are not quite so clear on the matte as on the gloss finish because of the texture itself, and also because the range of tone is narrower.

Scales of prints larger than the negative are obtained by projection, as for any other photographic enlargement; scales smaller than the negative, by rephotographing the contact prints made from the original negative. Some photographic definition is lost in these processes, but usually not to the extent that their value is seriously impaired. However, in addition to the loss of definition, certain distortions may be introduced that will cause trouble in using the prints for precise photogrammetric work. Enlargements used for ordinary mapping work should not exceed about three times the negative scale.

181. Flight Lines, Overlap, Crab, and Drift

A flight map is any existing intermediate or small-scale map on which are drawn the lines of flight the photographing airplane must follow to insure complete coverage of the map area with the required amount of overlap in the pictures along the edges of adjacent flight strips. When the pilot is able to fly precisely according to the predetermined flight lines, the strip of photographs will

appear as shown in Fig. 102A. The flight line is straight and is parallel to the edges of the pictures.

If the wind is blowing against the side of the plane, the latter will tend to drift off the course. In order to overcome the effects of drift, it is necessary to crab the plane slightly into the wind. It frequently happens that the photographer fails to turn the camera accordingly, with the result that the photographs in that strip are *crabbed,* as in Fig. 102B.

When the plane is held parallel to the predetermined flight course while a side or quartering wind is blowing, the resultant strip will appear as shown in Fig. 102C.

The photographs within a single flight strip should overlap one another by about 65 per cent. This is called the *end lap.* Because of the two-thirds end lap, all ground points within a strip will appear on three different photographs, such as point *a* in Fig. 102D. The different flight strips should overlap each other from 25 to 35 per cent (*side lap*). Within the limits of the side lap of two strips, all ground points will appear on from four to six different photographs, as point *b* in Fig. 102D.

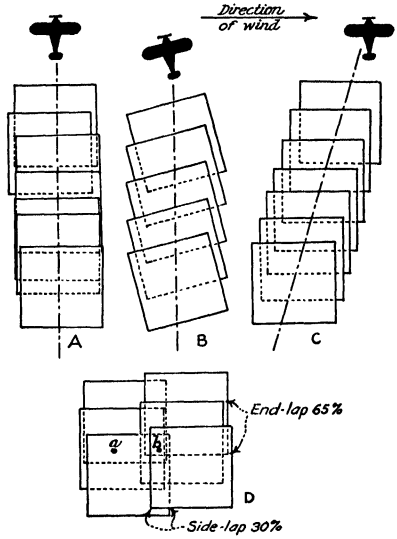


FIG. 102. Effects of side winds on the alignment of aerial photos in a strip.

182. Definitions

The definitions of flight line, side lap, end lap, crab, drift, and flight map are given in the preceding article and, therefore, are not included in the brief list of definitions that follow. Refer to Fig. 103 which illustrates most of the definitions.

The *principal point* is the point where the optical axis of the camera strikes the photograph and is at or very near the center of the picture.

Collimation marks are small marks either at the four sides or the corners of the photograph. The principal point is located on some types of photographs by the intersection of straight lines drawn through diametrically opposed collimation marks (Fig. 101).

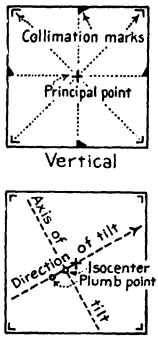


FIG. 103. Comparison of tilted and vertical aerial photographs.

The *plumb point* or *nadir point* is the point vertically beneath the lens of the camera at the instant of exposure. The plumb point may be referred to either as the point on the ground or the picture position of the ground point. In a perfectly vertical photograph the principal point and the plumb are coincident.

A *tilted* photograph is one made while the optical axis of the camera is inclined from a truly vertical position.

The *angle of tilt* is the amount of deviation of the optical axis from the plumb or vertical.

The *plane of tilt* is the vertical plane which contains both the optical axis and the plumb line in a tilted photograph.

The *direction of tilt* is the imaginary trace line where the plane of tilt intersects the plane of the photograph.

The *isocenter* is the point where the bisector of the angle of tilt strikes the photograph. If the angle of tilt is small, the isocenter is situated on the line of tilt nearly midway between the principal point and the plumb point.

Rectified prints are photographic prints in which the effects of tilt have been partially corrected. Where much ground relief is present, rectification does not eliminate all the effects of tilt.

A *vertical equivalent* is a projection representing a vertical photograph at the position of the projected tilted (or oblique) photograph.

Flight altitude (in aerial photography) is the height of the plane above the mean surface of the ground—not the altitude of the plane above sea level.

Aerial photogrammetry is the process of measuring and computing distances, elevations, and directions from aerial photographs.

A *stereo-pair* is any two photos overlapping in area, but taken from different camera positions. The area of overlap in two successive photos is normally about 60 per cent of the area of one photo.

A *stereo-triplet* is a set of three successive photos. Usually the center photo is entirely overlapped by the other two.

A *stereogram* is a pair of overlapping photos, separated, oriented, and mounted for viewing with a stereoscope.

A *mosaic* is a composite picture made by matching common picture images on a number of overlapping exposures and then gluing them to some sort of backing, such as cardboard.

A *controlled mosaic* is a mosaic made from rectified prints and assembled so that identified picture points are placed on the corresponding true map positions of the points. The images along the edges of adjoining prints may not match as exactly as on an uncontrolled mosaic, but distance relationships are more accurate.

A *radial line* is a line whose origin is the principal point or isocenter. Displacements due to relief are along radial lines from the plumb point or picture center; displacements caused by tilt of the camera are radial from the isocenter.

Restitution is the process of determining the true map positions of features whose positions on the photo are distorted. There are several methods by which this can be done.

Pseudoscopic effect is the reversal of topographic relief occurring under the stereoscope when the "right" and "left" prints of the stereoscopic pair are reversed. Valleys appear as ridges, and vice versa (Fig. 116B).

A *ground control point* is any point whose map position has been determined by surveying methods and has been used to adjust photographs in making a map.

The *focal length* of the camera is the distance from the lens to the film, and the *focal plane* is the plane of the film, which is normal to the optical axis of the lens.

183. Computation of Photograph Scale

In Fig. 104, A and B are two points on the ground, a and b are the corresponding points on the picture, f is the focal length of the camera, and H is the altitude of the plane above the ground; L is the camera lens. Since ALB and aLb are similar triangles, the scale fraction of the photograph is $AB/ab = H/f$. As H and f are usually expressed in feet and in inches, respectively, the scale fraction of the photograph will likewise be in these units; for example, let $H = 10{,}000$ feet and $f = 8$ inches; 10,000 feet/8 inches = 1250, or, 1 inch = 1250 feet. The scale of the photograph *varies directly with the focal length of the lens:* 10,000 feet/10 inches = 1 inch = 1000 feet; and the scale of the photograph *varies inversely with the altitude of the plane:* 15,000 feet/10 inches = 1 inch = 1500 feet.

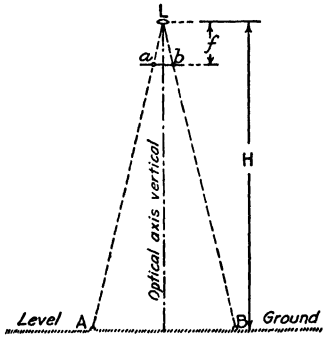


FIG. 104. Determination of the scale of a vertical photograph.

It is evident that for a lens of known focal length, the height above the ground at which the plane must fly to obtain photographs of a certain scale is determined by the equation $H = f(AB/ab)$; for example, if the focal length is 10 inches and the scale desired is 1 inch = 2000 feet, then the plane must fly at an altitude of $10 \times (2000/1) = 20{,}000$ feet.

184. Effects of Ground Relief

A strictly vertical photograph of perfectly level ground is a true map in that the relative distances and directions between points on the picture are the same as those on the ground. These

are the only conditions under which correct relationships are maintained. If either ground relief or tilt of the photograph is present, both the scale and the directional relationships are distorted, and the magnitude of the distortion varies according to the amount of tilt or relief. The effects of relief will be considered first.

It was shown in Art. 183 that the smaller the value of H in the scale fraction H/f, the larger will be the scale of the photograph. Since H is the height of the plane above the surface, and the photographic plane flies as nearly level as possible, it follows that the scale of the photograph varies according to the ground relief, i.e., the scale will be largest over portions of the area that have the highest elevations. In Fig. 105, A and B have the same ground dimensions, but it is apparent that A is appreciably larger than B on the photograph because A is at the higher elevation. Such a condition is not difficult to detect in pictures showing regular planimetric features whose dimensions are known; but it should be kept in mind when using photographs as maps that scale changes are present where there is surface relief even though they may not be discernible by casual inspection.

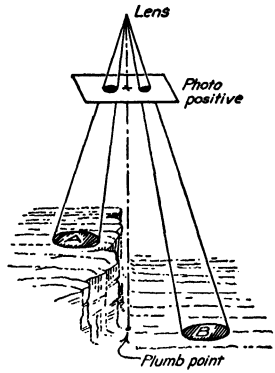


FIG. 105. Difference in scale of portions of a photo caused by ground relief.

In addition to causing a change in scale, ground relief produces a condition in the photograph called *lean*. Vertical photographs may be considered as conic projections of the irregular ground surface onto a horizontal plane, with the origin of the radial lines of projection being at the lens of the camera. In this type of projection, relief on the projected surface causes certain displacements on the projection plane, and these displacements are always along the radial projection planes. In aerial photographs displacements due to ground relief are along radial lines that originate at the plumb point; and in a vertical photograph, where the

plumb point and the principal point are coincident, these displacements are radial from the picture center, or principal point.

Figure 106 is a profile along a radial line passing through the principal point of a vertical photograph. *A*, *B*, *C*, and *D* are points on the ground, and *A'*, *B*, *C*, and *D'* are the vertical projections of

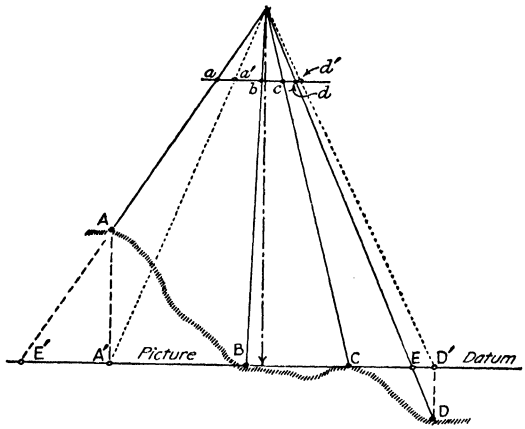


FIG. 106. Displacements of images above and below the selected datum.

these points on a horizontal plane, here designated the picture datum; *a*, *b*, *c*, and *d* are the picture images of the corresponding ground points. It will be observed that the image *a* is displaced *outward* from the center and the true map position at *a'*. This distance *aa'* is called the *parallactic displacement*. The horizontal position of *D* is at *D'*; the picture image *d* is displaced *inward* toward the center from the true map position at *d'*. Points *B* and *C* lie in the datum plane, and for this reason there is no relative displacement on the photograph, that is *a'*, *b*, *c*, and *d'* are in the correct relative positions on the horizontal plane of the picture. It will be observed further that images *a* and *d* appear on the photograph as if the corresponding ground points were at *E'* and *E*, respectively,

Moving the datum plane up or down has no effect on the above

relationships; it only changes the scale of the projection; therefore, the following important principles are evident from the figure.

Distortions due to relief are radial from the plumb point (the center of a vertical photograph) and are proportionate to the elevation of the point above or below the datum plane and the horizontal distance from the plumb point.

Points lying above the datum plane are displaced outward from the plumb point; those below the datum are displaced inward toward the plumb point. Points situated in the plane selected for the datum are not displaced on the photograph. All points lying in the same horizontal plane (at the same elevation) will appear on the *vertical* photograph in their correct relative positions, but will be out of adjustment with all points above or below that plane, because of the difference in scale and the lean from or toward the center.

Some of the effects of ground relief discussed above are shown in plan in Fig. 107. The left drawing is a topographic contour

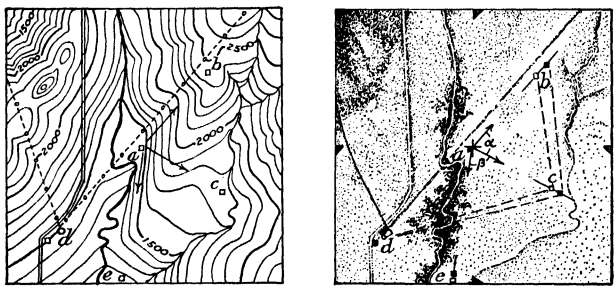


FIG. 107. Comparison of a map and a photograph of the same area showing parallactic displacements of features on the photo.

map; the right is an idealized drawing representing a vertical photograph of the same area. The distortions caused by topographic relief are exaggerated for the purpose of illustration.

On the map the rectangles *a*, *b*, *c*, and *e* are buildings, *a* being at a point corresponding to the principal point of the photograph,

and *d* is an electric power plant. The datum plane of the photograph is selected at the elevation 1500 feet, that is, at an elevation of 1500 feet the scale of the photo is the same as that of the map. On the picture, the black rectangles are the picture images of the buildings, and the open rectangles are the true map positions. Point *d,* which is at the datum elevation and the same elevation as the picture center, is also at the correct map position. Points *b* and *c* lie above the datum and are therefore displaced away from the center, while *e,* at a lower elevation, is displaced toward the center. Although the house at *d* is in the correct position, the top of the smokestack, because of its elevation, appears to lean radially from the center. The true position of the stack is the center of the base, not the top.

The transmission line extending across the area in a northeasterly direction is the same on the photograph as on the map for the reason that it passes near enough to the principal point that it is essentially a radial line. Due to the combined effects of increasing elevation and distance from the picture center, the other transmission line appears as a curved line whose mean bearing is considerably farther to the west on the photograph than on the map. After passing over the crest of the ridge, the line tends to curve back toward the true position, because the progressively lower elevations are approaching the datum elevation.

The bearing of the north-south road is somewhat west of true north on the picture. It remains a straight line, however, because at the south end, where the grade is steeper, the distance from the center is less than at the north end, where the grade is more gentle. In this case the character of the linear feature is not distorted, but the alignment is in error.

The error in bearing of the dash lines from *d* to *c,* and from *c* to *b* is evident. The correct bearings are the lines joining the open rectangles.

From the principal point of a vertical photograph, the horizontal angles between radial lines to definite picture points are the same as angles on the ground or on an accurate map. This holds regard-

less of the amount of relief in the area. In the figure, the angles α and β on the photograph are the same as the corresponding angles on the topographic map. As will be seen later, this fact makes it possible to construct very accurate planimetric maps from photographs having a great deal of relief distortion.

185. Effects of Tilt

Although most so-called vertical photographs are tilted less than 1 degree, very few are strictly vertical. Occasionally only a few pictures in a flight strip will be badly tilted, the ones on both

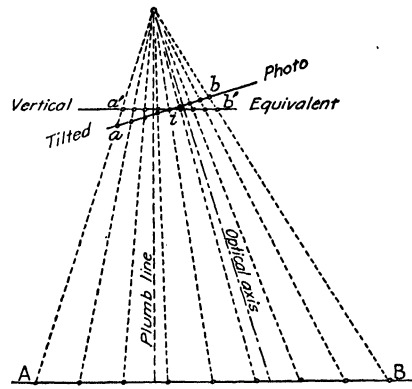


FIG. 108. Relation of the tilted photo to the vertical equivalent.

sides being well within the prescribed limits of 2 to 4 degrees. Because of the occasional tilted exposures, which must be used in the construction of the map, it is important to know the character of the resulting distortion. In order to illustrate better the principles involved, the degree of tilt used in the figures is much greater than that ordinarily encountered in vertical photographs.

In Fig. 108, it is assumed that the ground is level, and that distortion of the photograph is due entirely to tilt of the camera. Such distortions follow the laws of perspective. On the ground the points between *A* and *B* are equally spaced. On the tilted photo-

graph the spacing between corresponding points from *a* to *b* is progressively smaller toward *b*. From the diagram it is evident that the scale of a tilted photograph of level terrain becomes gradually smaller in the direction of the inclination of the optical axis, or opposite to the direction of tilt of the camera at the time the exposure was made. The line *a'b'* represents a vertical equivalent, that is, a vertical photograph taken from the same camera station as the tilted photograph. It will be observed that the scale on the vertical equivalent is constant. Point *i* is the end view of the axis of tilt, which lies in a plane bisecting the angle of tilt, here represented by the angle between the optical axis and the plumb line. The axis of tilt on the photograph may be considered as the intersection of the plane of the tilted photograph and that of the vertical equivalent, the point of intersection being on the bisector of the tilt angle. The point of intersection of the axis of tilt and the plane of tilt is called the *isocenter*. Since the isocenter is on the bisector of the tilt angle, and the bisector does not cut the plane of the picture at 90 degrees, the isocenter is always somewhat closer to the principal point than the plumb point, but nearly midway on a straight line between the two. Distortions due to tilt are radial from the isocenter; those due to relief, as stated earlier, are radial from the plumb point.

Figure 109 illustrates the effects of tilt in ground plan; *A* is a portion of a vertical photograph (or map), and *B* is a tilted photograph taken from the same camera station.

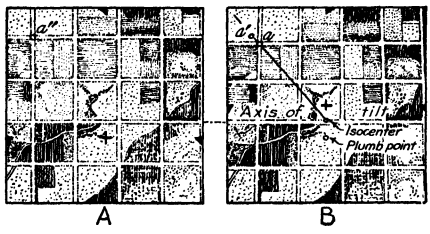


FIG. 109. A. Vertical photo. B. Tilted photo. Displacements due to tilt are radial from the isocenter.

Along the axis of tilt the scales of A and B are the same. This is the only line on the tilted photograph where the scale is the same as the vertical equivalent. However, all lines parallel to the axis of tilt exhibit true, but progressively larger, scales in the direction of tilt (opposite to the direction in which the optical axis is inclined from the vertical). In the figure, note that the *relative* distances between roads are correct along all east-west lines, but the scale becomes gradually smaller from south to north. In other words, the north-south roads converge to the northward. No two parts of any of the north-south lines have the same scale, as can be seen by comparing directly with the vertical.

The road intersection a, as well as all other points, is displaced from its true map position along a radial line from the isocenter, the correct position being at a'. The position of a' on the tilted photograph is the same as a'' on the vertical.

186. Effects of Tilt and Relief Combined

The effects of tilt and relief would not be particularly difficult to deal with if they could be treated separately; but, unfortu-

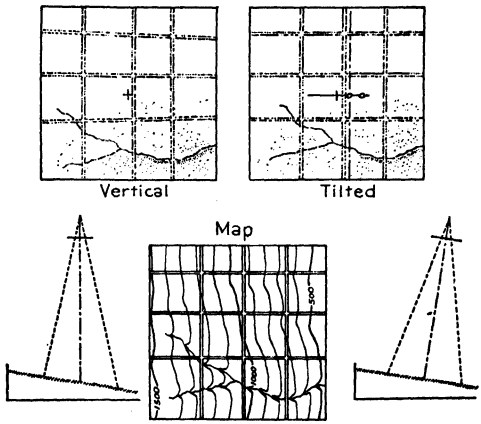


FIG. 110. A case where the tilted photo appears to be vertical and the vertical appears to be tilted. The topography of the area is shown below.

nately, this is usually not possible. When moderate to high relief and tilt are both present the types of distortion become very complicated. In some places they may tend to compensate each other, while in others they will be cumulative. Figure 110 shows an idealized condition where a vertical photograph exhibits all the characteristics of tilt, while the tilted photograph lacks these ordinarily diagnostic characters. The evident reason for this is that the plane of the tilted picture is parallel to the sloping ground surface. Although such a photograph is not exactly true to scale and free from distortions, it so closely simulates conditions on the ground that the existence of an appreciable amount of tilt may not be suspected. This is clearly a case of compensation. Now if the tilt were in the same amount but in the opposite direction, the results would be far different: In this case the tilted picture would show a rate of convergence and scale change more than twice as great as that illustrated in the vertical picture. The complexity of scale changes that result from relief and tilt are indicated in Fig. 111, which depicts a tilted photograph of a series of symmetrical hills and valleys.

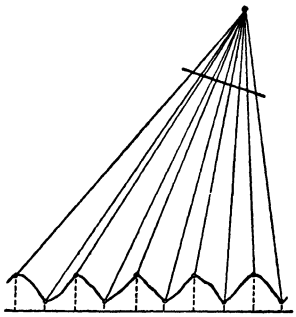


FIG. 111. Character of distortions caused by a combination of ground relief and tilt.

187. Distortions from Other Causes

A small amount of distortion in photographic prints is caused by differential shrinkage in both the film and the print paper during the drying process after development. Usually all reasonable precautions are taken that no stretching occurs while the film is drying and that it dries evenly and slowly; but despite these preventive measures, a few negatives may be somewhat distorted. Most print papers have a tendency to shrink more in one direction

than in another with the result that the picture may be elongated. Little can be done to rectify this condition, though the error is frequently of sufficient size to be of considerable annoyance in large-scale or precise work.

Although the lenses in aerial cameras are carefully ground and mounted to reduce aberration, it has not been entirely eliminated. Distortions due to this cause are usually small and are present only near the edges of the photographs; hence, they can be avoided by using only the central portions of the pictures.

During the processes of enlarging and reducing, distortions not present in the original negatives may be introduced. The most frequent cause of the errors is lack of parallelism between the aerial photographic negative or positive and the projection enlarger or the reducing camera. These distortions simulate those of tilted photographs, but are not exactly the same, especially in projection enlargements. When distortions in enlargements or reductions are suspected, the source may be ascertained by examining the original negatives or contact prints.

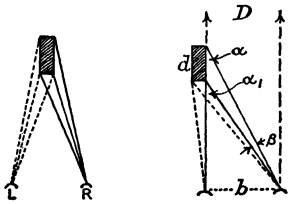
188. Stereoscopic Properties

When two overlapping aerial photographs are properly oriented with reference to each other and viewed through a stereoscope, a realistic illusion of the third dimension, or depth, is obtained. This is a very important property of aerial photographs in that features of the terrain may be studied in their proper spatial relationships, and with special equipment differences in elevation may be measured with a small percentage of error. The relative amount of ground relief is greatly exaggerated in the stereoscopic view, a condition which is often of much advantage in photogrammetric work. The uses of various expensive photomapping machines are not discussed in this book, but the use of the stereoscope in any extensive use of aerial photographs is considered essential to their correct interpretation and therefore is described in some detail in following pages.

189. Principle of Stereoscopic Vision

Three factors are operative in producing the perception of depth or distance: the apparent relative sizes of familiar objects, or those whose approximate dimensions are known; the atmospheric effects associated with distance; and binocular, or stereoscopic, vision. It is the last with which the study of aerial photographs is concerned and which is briefly explained below.

If a small object is viewed from a distance of a few inches and the eyes are alternately opened and closed, it will be observed that the left eye sees only the front and left side, while the right sees only the front and right side. These two different images are impressed on the retinas of the left and right eyes, respectively, and in some manner, not fully understood, are fused in the brain to form a single composite image having the property of the third dimension, or depth. This principle is shown in Fig. 112.



FIGS. 112 AND 113. Stereoscopic vision.

The perception of depth likewise results if one eye sees only the front of the object, while the other sees both the front and the side, as shown in Fig. 113. In this figure, if an object D is viewed from a great distance, the rays indicated by the dash lines are nearly parallel, with the result that the object appears at an indefinite distance away. On the other hand, if the rays come from the front and back of an object d, as shown by the solid lines, the object will appear at a definite distance, and the back will appear farther away than the front. Considering the three solid rays further: the angles α and α_1, made by the intersections of the rays to the right eye and the one to the left, are called the *angles of parallax*. It is evident that $\alpha_1 - \alpha = \beta$, and this angle is called the *differential parallax* of the object.

The interpupillary distance b is called the eye base, and averages

about 2½ inches, seldom varying as much as ¼ inch in different individuals.

The smallest parallactic angle that the eye can sense is about 20 seconds of arc. Using the average eye base of 2½ inches, it will be seen that the limiting angle β is reached when the distance to the object is approximately 2100 feet; hence, it may be concluded that stereoscopic vision ceases to function at this distance.

The stereoscopic effect diminishes as the angles of parallax decrease; therefore, to heighten the stereoscopic effect, the angles

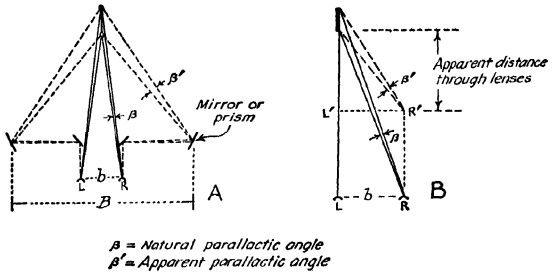


FIG. 114. Principle of the stereoscope. A. Mirror type. B. Lens type.

of parallax are enlarged. This may be done by lengthening the eye base or by magnifying the object. In Fig. 114A, the eye base is increased by a system of prisms. If b is the normal eye base, and B is the stereoscopic base, then the stereoscopic vision is increased according to the ratio B/b. The effect of magnifying the field is to reduce the distance to the object, as shown in Fig. 114B. If the magnification is two diameters, the stereoscopic effect is approximately doubled. The stereoscopic vision is greatly increased by a combination of prisms and lenses. Thus, if the magnification is 3 diameters and the eye base is increased 3 times, the stereoscopic effect is increased $3 \times 3 = 9$ times.

190. Stereoscopic Examination of Photographs

In Art. 184 it was shown that relief on the ground causes parallactic displacements on the photographs. The direction and amount

of the displacement for a given feature depend on the position of the camera station. As stated before, the displacements due to relief are radial from the plumb point. In Fig. 115, the two vertical photographs *A* and *B*, are made from the camera stations *A′* and *B′*.

If *A′* and *B′* were considered as the two ends of the observer's eye base, it is apparent that the stereoscopic effect would be enor-

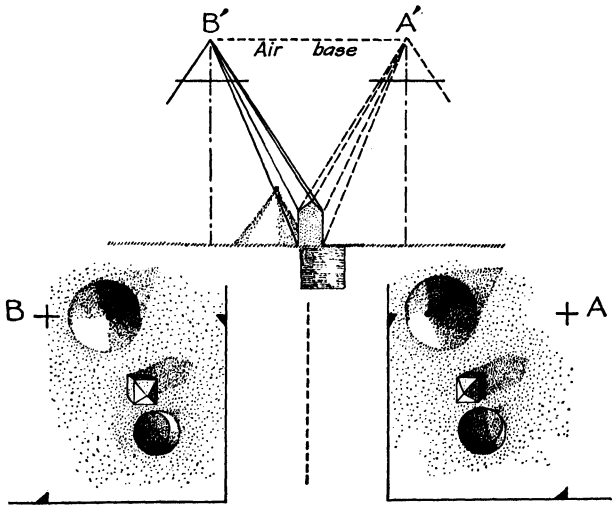


FIG. 115. A stereogram of geometric figures. The upper figure shows the relative positions of the objects on two overlapping aerial photos.

mously magnified because of the consequent widened parallactic angles. Since the parallactic displacements of objects are correctly registered on a vertical photograph, a similar relative magnification is obtained in the stereoscopic model when the two photos are viewed under the stereoscope.

The "photographs" *A* and *B* are correctly oriented with reference to each other, and then, without any rotation, are separated so that images common to both are about 2 inches apart. Photographs or stereoscopic drawings mounted in this fixed position are

called *stereograms*. To bring up the third dimension of the stereogram shown in Fig. 115, proceed as follows.

Place a piece of cardboard about 12 inches high along the medial dash line. Lower the head so that the upper edge of the cardboard is between the eyes and along the bridge of the nose. It is now possible for the right eye to see only *A*, and the left to see only *B*. With both eyes wide open try to look "through" the plane of the stereogram as if viewing a distant object. The reason for this is to make the lines of sight of the two eyes as nearly parallel as possible. When this condition is attained, the three pairs of geometric figures should move toward each other and finally fuse into only three figures, which will be seen as solid three-dimensional objects. Some practice is usually necessary in order easily to obtain the stereoscopic model without the use of instruments. If the desired fusion is not attained, the following exercise will help.

On a small piece of transparent celluloid or cellophane draw with black ink two dots about $\frac{1}{10}$ inch in diameter and $1\frac{1}{2}$ inches apart. Now hold this flat against a window pane with the line of the two dots parallel to the eye-base line. Keeping the line of sight near and just above or below the dots, focus the eyes on a distant object seen through the window. The dots should begin to move toward each other, and with some effort can be made to fuse. After the dots are apparently fused, try to direct the gaze on them without allowing the lines of sight to converge toward either one. A cardboard held between the eyes may be of some aid in this exercise.

When stereoscopic fusion is obtained, the three-dimensional image or model will appear clear and vivid. In addition to this will be faint images seen from the tails of the eyes; these are called *phantom images*. The phantom images may at first be bothersome, but after a little experience in stereoscopic work they will scarcely be noticed.

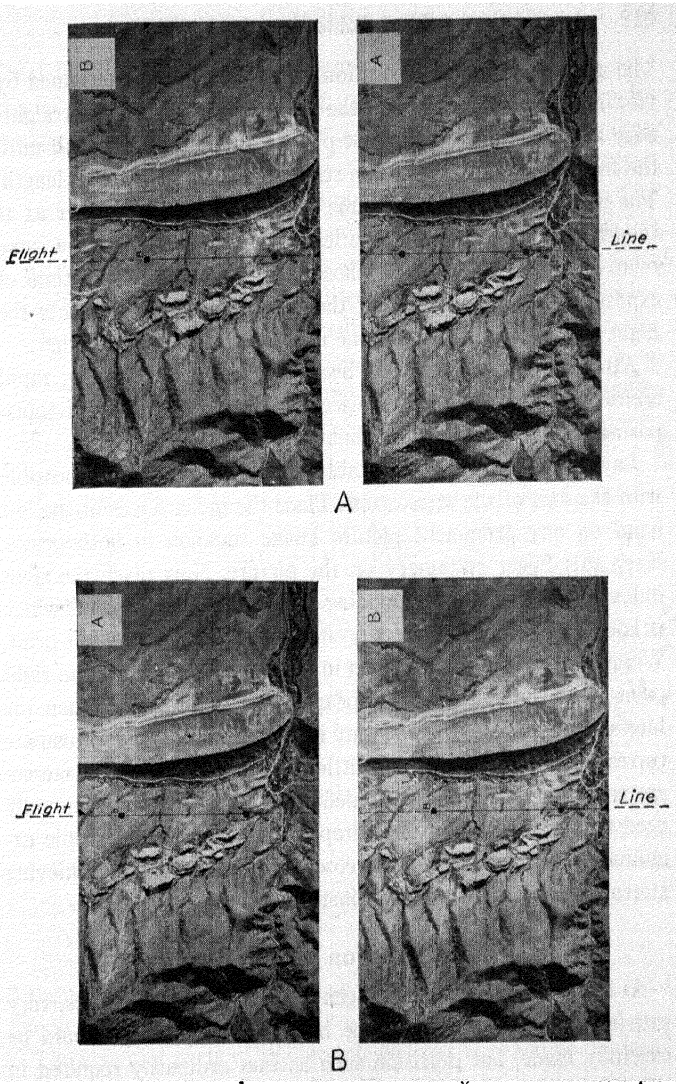
If a stereo-pair is placed under a stereoscope with the "left" picture on the right side and the "right" picture on the left, the resulting model will be reversed; that is the ridges will appear

as valleys, and canyons will appear as sharp ridges. Such a reversed image is called a *pseudoscopic model.* Figure 116A is a stereo-pair of the Red Rocks park area of Colorado. The lower pair of prints have been reversed (Fig. 116B) to give the pseudoscopic effect. Occasionally a pseudoscopic effect is obtained when the actual source of light falling on the work is opposite to that shown in the pictures. This condition is rectified by changing the source of light or rotating the two pictures as a unit through 180 degrees.

Correct orientation of the photographs with respect to each other and with reference to the axis of the stereoscope is highly important. Errors in orientation introduce "false parallax," which not only results in erroneous interpretations of relief features, but also puts unnecessary strain on the eyes. Until such time as considerable familiarity with stereoscopic work is gained, the following procedure for accurately orienting the pictures is advised. This method should *always* be used in orienting the pictures for stereoplotting instruments.

(1) By means of the collimation marks, carefully locate the principal points (centers) of the two photographs and prick the points with a fine plotting needle. (2) By comparison of these points with near-by images, transpose the principal point of photo *A* to the proper relative position on photo *B,* and vice versa. (3) With a very soft pencil (HB) or a fine pen and India ink draw a thin line on each picture, joining the center and transposed center and extending beyond the edge of the print. Now when the two photographs are laid so the two centers and two transposed centers are on the same straight line, the photographs are properly oriented with reference to each other. To orient the pair correctly under the stereoscope, proceed as follows.

(1) Place one picture under the stereoscope oriented so that the fine pencil line is parallel to the axis of the lenses or mirrors, and fasten it in this position with drafting tape. (2) Lay the second picture on the first, but with a horizontal separation of identical images approximately correct for the instrument being used.



(A) the photos are placed in the same relationship as the ground features; in (B) the photos are reversed, left to right, though still correctly oriented along the flight line. This inverts the ground relief, a condition called pseudoscopic effect.

The amount of lateral separation necessary can be determined by viewing the pictures through the stereoscope. (3) Lay a straightedge along the line on the first picture and shift the second until the fine line coincides with the straightedge throughout its length. The pictures now bear the same relationship to each other as at the time of exposure, and the lenses or mirrors are in the same relative spatial positions as those of the camera at the time of exposure. The line drawn on the print, likewise, represents the flight line of the plane from one exposure station to the other.

After some experience has been acquired, the following rapid approximate method may be used without the necessity of transposing centers or drawing the flight lines.

Lay the left print on the table oriented as closely as possible with the axis of the stereoscope. Place the index finger of the left hand on any prominent picture image common to both prints. Keep this finger stationary on the picture. Now place the right index finger on the corresponding image of the right print, which is laid roughly in the correct position to the side of the left print. While viewing the photographs in the stereoscope, shift the right print until the two fingers appear to merge into one. When the fingers are now raised, the picture images should be in approximate correspondence, and with but little further adjustment the stereoscopic model should be clearly defined. This is the most commonly used method of placing photographs for cursory stereoscopic examination, either under a stereoscope or as an aid in achieving stereoscopic vision without an instrument.

191. Construction of Stereoscopes

At costs ranging from a few cents to three or four dollars, very satisfactory stereoscopes can be built at home. They should be carefully made, but precision such as that ordinarily required in other kinds of optical instruments is not necessary.

A simple lens type is shown in Fig. 117. The lenses for this instrument need not be optically perfect, those available at ten-cent stores being adequate for most work. They should have a

focal length of 6 to 8 inches, and a magnification of 1½ to 2 diameters. Greater magnification restricts the field of vision too greatly. It is important that the focal lengths and the magnification of the two lenses be about equal. The focal length of the lens may be determined as follows. Lay a sheet of white paper on a table directly beneath a lighted electric bulb. Hold the lens between the paper and the bulb, moving it up and down until a sharp image of the bulb is cast on the paper.

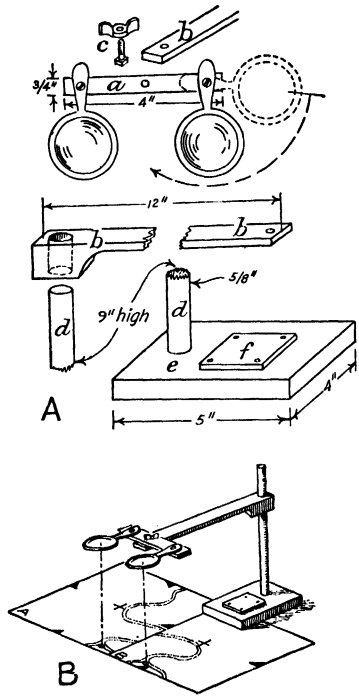


FIG. 117. Construction of a simple lens stereoscope.

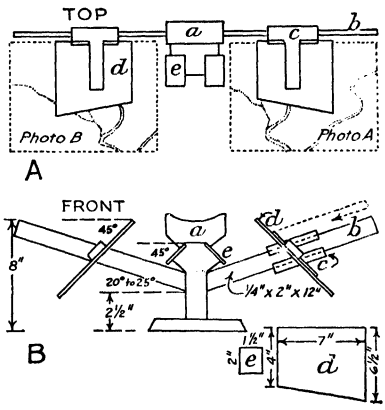


FIG. 118. Construction of an adjustable mirror stereoscope.

Measure the height of the lens above the paper. This is the focal length.

The essential parts of the lens stereoscope are the two "paired" lenses and any kind of mount by which they are held at the approximate focal length above the photographs. The one shown in Fig. 117 may be built from any tough wood, such as birch or cherry, any of several plastic materials, or a combination of wood

and plastics. The lens mount *a* and the swinging arm *b* should be of light construction. The bolt and wing nut *c*, likewise, should be small. The dowel pin *d* should fit closely in the hole in the end of *b*, but not so tight as to prevent sliding the arm up and down or rotating it sidewise. The pin *d* fits tightly into the socket of the base *e*. If the latter is too light and tends to tip when the stereoscope is assembled, a metal weight, such as *f*, may be added.

Figure 118 is a dimension drawing of a wooden mirror type stereoscope. The essential features of this type are the two pairs of mirrors and a frame that holds the two mirrors of each side parallel to each other. This frame may be constructed in a number of different ways, depending on the materials available, but the type shown has some advantageous features not common to all wooden stereoscopes of the mirror type.

The standard *a* is cut from heavy stock about 2 inches by 4 inches. The arms *b* are inserted into slots cut into *a*. The end of *b* is slightly tapered where it fits into the slot in *a* and is held rigidly in position by friction. The arms are thus readily removed for transporting. The mirror mount *c* must be rigid enough to hold the large mirrors *d* parallel to the eyepiece mirrors *e*, yet free enough to be moved along the arm, as indicated in the lower drawing. Both sets of mirrors should be inclined 45 degrees, and the arms supporting the large mirrors are inclined from 20 to 25 degrees from the horizontal. The top of the standard *a* is curved to form a support for the forehead while the stereoscope is being used. This may be padded with felt or leather for further comfort.

The mirrors should be moderately free from optical distortions, or serious false parallax and poor definition of the "plastic model" may result. The best mirrors for this purpose are silvered on the front and upper surfaces of plane ground plate glass.

192. Comparison of Lens and Mirror Stereoscopes

It is evident in Figs. 117B and 118A that the mirror type has a much broader field of view; the two photos do not touch each other, and, therefore, the entire overlapping area can be viewed

stereoscopically with but little adjustment of the pictures or the mirrors. The mirror type is somewhat easier on the eyes with continued use of several hours' duration. On the other hand, the images appear much larger in the lens type, and for this reason lenses are better in studying small details or for examining terrain having very low topographic relief. It has the further advantage of being more compact when set up for use and when knocked down for packing into carrying or shipping cases.

When the scales of the two photographs are different, the prints must be shifted from time to time in order to bring images into correspondence if the lens type instrument is used. The apparent scales of the photographs may be changed with the mirror type by sliding the large mirrors along the supporting arm: the scale becomes smaller as the mirror is moved outward. The scale adjustment for the whole picture is thus made, and later shifting is unnecessary.

CHAPTER X

Plane Table Applications of Vertical Photographs

193. Contact Prints

Contact prints are used extensively in the field as an adjunct to conventional plane table methods. They serve a number of purposes, depending on the type of survey and the character of the terrain. In country of rugged topography the photos are very helpful in laying out the routes of traverses, selecting triangulation points, locating trails and stream crossings, and as an aid in estimating distances. The last use, alone, may save the instrumentman a great deal of time in that the approximate distances to planned rod stations can be predetermined so that the rod will always be held within the range of the alidade.

Contact prints of terrain of high relief *always* contain certain distortions, even though such distortions may not be apparent. It should be kept in mind that the displacement of points in rolling topography is quite as great as that in very precipitous topography if the total difference in elevation within the areas of the pictures is the same and the high and low points are similarly located on the two prints. The distortions of rugged topography are only more apparent.

Since distortions due to relief and tilt are emphasized toward the margins of the photos, only the central portions should be used when the scaling properties are important. For this reason it is advantageous to use all the photos in an overlapping series, rather than only alternate prints, although the alternate prints provide full ground coverage.

194. Rectified Prints

Most of the effects of tilt are removed in rectified prints. The relationships of picture points are almost as accurate as those on vertical photos. The effects of relief are not reduced in the process of rectification; actually, there is some residual distortion caused by the combined effects of relief and tilt. Where the country is flat and nearly level, either vertical or rectified prints have scaling properties that compare favorably with good maps on the same scale. If it is desired to make the photos to an exact scale fraction it is necessary to provide ground control that has been tied into the picture images of the control points. When the photo scale is smaller than that desired on the map, it is enlarged by projection from the negative on to an enlarging table on which has been taped a plat of the area with a few ground stations plotted to scale. The enlarging table is adjusted until the projected photo images coincide with the corresponding plotted points. The enlargement is then made with the projector and table in this position. When the scale of the map is smaller than that of the photo, the latter must be rephotographed down to the correct scale. In this case the ground control is plotted on the ground glass in the back of the camera, and the camera and aerial photo are adjusted until corresponding points are brought into coincidence.

195. Photo-Plane Table Sheets

Aerial photos are often used on the plane table much as any other plane table sheet. In order to do this effectively for most work, it is necessary to rectify the prints, if there is any appreciable amount of tilt, and reduce or enlarge them to the desired scale. The prints may be mounted on heavy cardboard the size of the table or fastened directly to the table with drafting tape.

196. Sketching by Photo Images

Vertical or rectified prints of moderately flat country are usually sufficiently free from distortions to permit the plotting of distances

determined by ground methods. The photo may be oriented in the following way. Select a point on the picture which can also be identified on the ground. The point should be small and of such nature that the tripod can be set up either directly on it or immediately alongside. A small patch of bare earth, a low bush, the intersection of two trails, or similar features are excellent for the purpose. Set up the plane table on this point and level it. Using the backsight method described in Art. 101, align the alidade blade on the picture image of the point and the most distant point that can be seen on both the photo and the ground. Rotate the table until the ground object is brought exactly into the line of sight, and then clamp the table. As in backsighting in triangulation, check the orientation by sighting on another distant object.

In using this method of orientation it is important that either the instrument station or the backsight station be very near the center of the picture, for the sight line will then be on or very near a radial line, and will, therefore, be unaffected by ground relief. If a flag is planted on the ground position of the picture center the photo-plane table sheet may be oriented from any other position that can be occupied by the plane table and identified on the photo and the ground. When this method is used the backsight is used always on the flag.

The scaling accuracy of the photograph should be determined at a number of localities by reading a stadia distance to some well-defined object and then scaling the same line on the photo. It may be found necessary to use slightly varying scale factors for different parts of the photo or for lines running in different directions; for example, it may be found that the scale is moderately true in a generally east and west direction, but 5 per cent too small in a north and south measurement. In this case, all stadia distances in northerly or southerly directions should be reduced by 5 per cent before they are plotted on the photo if it is desired to maintain a constant relationship to other picture points.

One of the advantages of aerial photos is the abundance of detail available in "map" form. With a small amount of adjust-

ment for map position, a great deal of this detail, such as the courses of streams, geologic boundaries, and roads, may be incorporated into the plane table map. In order to take full advantage of the photos the rodman should be cognizant of the value of holding the rod wherever possible on an object that can be identified on the picture. This provides the necessary ground control for the adjustment of photographic distortions.

197. Transparent Overlay Sheet

Instead of sketching directly on the photo, a sheet of cellulose acetate of medium to heavy weight and with one frosted surface may be fastened to the table over the photo. Before any mapping is done the corners of the photo should be marked with fine inked crosses or right angles and these marks should then be carefully traced on the acetate. It is then a simple matter to register the acetate over the photo at any future time. The registration should be checked frequently during the course of the field work. The photo should be fastened to the under side of the acetate with drafting tape—not to the plane table. Any movement of the acetate sheet will then cause a corresponding movement of the photo.

The hard pencils ordinarily used on plane table sheets will not mark on cellulose acetate, even though the surface is heavily frosted. Hardnesses ranging from 2H to 5H are satisfactory. Since the penciled lines are easily rubbed off the acetate, completed portions of the map should be covered with a smooth paper. If the acetate becomes oily from handling, it will not take or hold pencil lines. This can be corrected by rubbing with drafting pounce or a soft rubber eraser.

198. Mosaic Sheets

Full-sized photo-plane table sheets may be made by matching and joining together a number of rectified prints. The principal point should be marked in waterproof ink on each of the prints before they are cut or torn for assembling into the mosaic. When used in the field the plane table is oriented according to the proce-

dure outlined previously, using only the principal point and one other point on the same partial print. The table should not be oriented by backsighting from the center of one print to an image point on another adjoining one, or vice versa; nor should instrumental determinations of distances be plotted across the junction of two partial matched prints, except for the purpose of checking the accuracy of the mosaic.

Unless the mosaic is exceptionally well constructed the orientations of the table will vary slightly from one print to another. Likewise, there will be slight discrepancies in the locations of plotted positions from stadia or other ground measurements. There should be no appreciable errors attributable to the photos in the relative positions of rod stations on any one print if they were determined from an instrument station on the same print. The discrepancies will likely occur between two such groups on different prints. These errors can be adjusted later if a common rod station is determined from instrument stations on two or more prints. The error in the plotted position of the rod station as established from the two instrument stations is a measure of the error in the mosaic. Such points should be determined at convenient places over the entire map area. While the mosaic sheet cannot be corrected, the compilation tracing can.

199. Transfer of Detail

Much time and effort in the field can be avoided if details are transferred from the photograph to the plane table sheet. In order to do this it is first necessary to provide a certain amount of plotted ground control. The ground control may be plane table triangulation, traverse, or both. The essential factor is that the ground points whose positions are determined in the field by instrumental means be identifiable on the photos. This is made simple if the photos are taken into the field and used by the field party at the time the preliminary control is set up. These points should be distributed so that at least three will fall on each print, though it is better if there are more.

If the photo is to the scale of the map and relatively free from

distortions, the transfer of map details is simply a tracing process; however, some variations are normally present, and for this reason it is necessary to "stretch" or constrict the scale of the photo, as the need may be, in order to place the features correctly on the plane table sheet. Several methods have been devised for this purpose, including projection of the photo on to the sheet and then outlining the projected image; but only those methods which need no special equipment are described here.

200. Triangular Division

This method consists of dividing the total area of the photo into a number of small units corresponding to a similar, but not identical, division of the map.

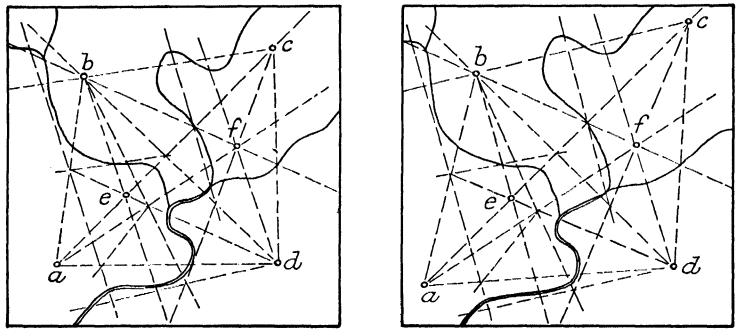


FIG. 119. Adjustment of photograhic detail by triangular division. A. Photo. B. Map.

In Fig. 119B, points *a* to *f* are accurately located on the plane table sheet. These points are located on a sheet of tracing paper, which will be used for compiling and adjusting the photo detail. The corresponding points are located on the photo by inspection and are marked so that they can be seen through the tracing. The problem is to correct the distorted position and course of the stream system shown on the photograph (Fig. 119A).

All the control points are connected by straight lines in every

possible manner. This subdivides the quadrilateral *abcd* into a number of small triangles and quadrilaterals. The intersections of these straight lines now serve as additional points through which other straight lines can be drawn. By utilizing the points and intersections on both the tracing and photo, the photo details can be sketched accurately on the tracing according to the relative positions within the figures thus obtained. When the compilation is finished, the tracing is registered exactly on the plane table sheet, and the map features are transferred by means of hard carbon paper.

201. Radial Line Method

This method is used to reduce an offscale photo to the scale of the map, the assumption being that there is no appreciable amount

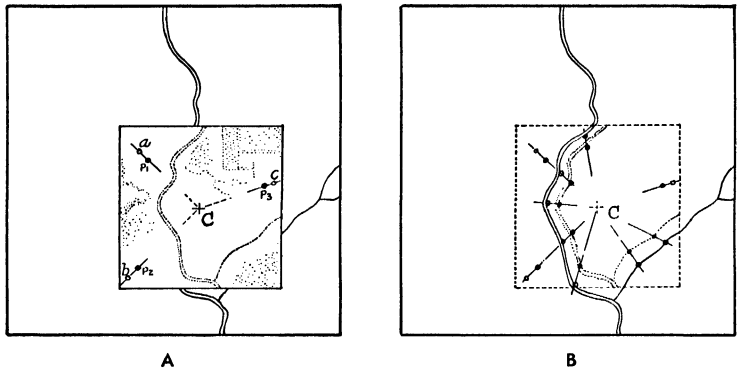


FIG. 120. Adjustment of photo detail by radial lines and scale ratio. A. Oriented Photo. B. Restored Detail.

of tilt and what distortions are present are due to topographic relief.

In Fig. 120A, the large rectangle represents a transparent compilation sheet, on which are plotted control points *a*, *b*, and *c*. On the underlying photo the corresponding picture points p_1, p_2, and p_3 are marked, and radial lines are drawn toward the principal

point of the photograph. Now, according to the tracing paper method of solving the three-point problem in triangulation described in Art. 64, the compilation tracing is shifted until the control points *a*, *b* and *c* lie on the radial lines on the photograph. The photo is now correctly oriented with reference to the sheet, and its center is correctly located on the map. Using a scale corresponding to the scale of the map, the distances from the picture center *C* to *b* and to p_2 are measured. From these measurements the scale ratio of the photograph and map are determined. The ratios are also determined *C* to *a* and p_1, and *c* and p_3. Applying the ratios determined for each segment of the photo, the computed distances are plotted on the radial lines as shown by the small open circles on the stream. In Fig. 120A, the stream at the edges of the photo offsets that correctly located on the map because of the differences in scale. However, in Fig. 120B, the portion of the stream appearing on the photo has been brought up to the scale of the map and coincides with the portion located by ground methods.

202. Line Division by Inspection

This method is similar to *triangular division* and is likewise subject to the inaccuracies of any method whose construction lines are not drawn toward the center of the photo. Nevertheless, it is widely used and is moderately accurate in areas of low topographic relief where scale differences between the map and the photograph are likely to be fairly uniform.

In Fig. 121A, a photo is oriented under a tracing on which are plotted the control points *a*, *b*, and *c*, corresponding to the image points p_1, p_2, and p_3, respectively. The tracing is shifted until *a*, p_1, *b* and p_2 all lie on the same straight line. The road intersection, *i*, is about midway between *a* and *b*; therefore, the tracing is again shifted along the line until the distance a–p_1 is equal to b–p_2. Now the intersection is correctly placed and can be drawn on the map. The same procedure is repeated in locating the road intersection *j*, (Fig. 121B) except that the points *a* and *c* are aligned and the spacing is adjusted along this line. After the north-south road is

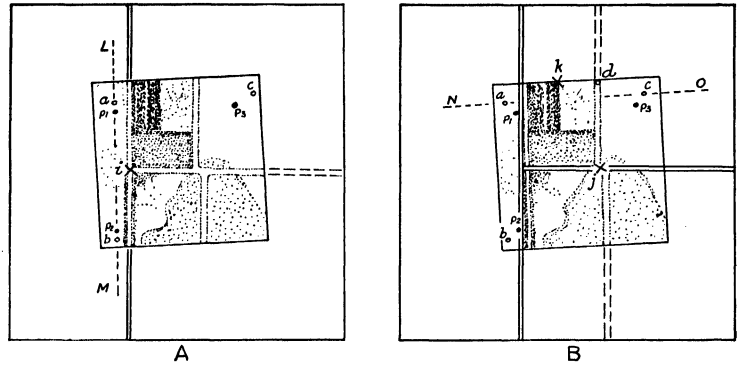


FIG. 121. Adjustment of photo scale to map scale by line division.

located on the sheet, it also may be used for further adjustments and sketching. Thus, if point *k*, situated midway between *a* and *d*, is to be located on the map, the tracing is shifted to the west so that the error is equally divided, and *k* is traced on the sheet.

203. Uncontrolled Mosaics

A mosaic is a composite picture made from a number of separate prints, properly matched and joined together. An *uncontrolled* mosaic is one made without ground control. With good photos of very flat terrain an uncontrolled mosaic can be constructed which compares favorably with a map of the same scale, provided the area does not include too many prints and the matching has been carried out with extreme care. The several factors which determine the accuracy of the mosaic are uniformity of scale in the prints, lack of tilt, low topographic relief, *method* of construction, and *care* in construction. Only the last two factors can be controlled by the person laying the mosaic.

204. Preparation and Materials

Much time will be saved and a more satisfactory mosaic obtained if the preparatory work is well done. It is well worth while,

and costs little more, to obtain the complete set of overlapping prints rather than a bare area coverage, for then only the central portions need be used. Get prints on thin paper with semigloss finish. Examine all the photos by pairs under the stereoscope and by inspection sketch an estimated contour line in the approximate areas where the prints are to be joined. Refer to topographic sheets when available. Use a soft pencil. If the relief is quite low this step is omitted; and in this case the irregular boundaries of dark areas are used for the matching lines. Note those prints which appear to have an appreciable amount of tilt. These may be discarded later, or larger portions of the adjoining prints may be used so that only the small central part of the tilted one need be utilized in the mosaic.

When the matching lines have been marked with a soft pencil, roughly trace the line on thin paper. Now lay the print face down on a table and register the thin paper, *penciled side down,* on the back of the print. With a hard pencil, retrace the line showing through. This will transfer the pencil lead from the first line drawn to the back of the print.

The next step should be practiced with heavy paper or discarded prints before it is tried on the prints to be used on the mosaic. Lay the left hand flat on the back side of the portion of print to be used with the finger tips near the transferred match line. With the right hand, *tear* the print as closely as possible to this line. If the portion in the right hand is bent back sharply during the tearing, a thin feather edge will be formed. This is what is desired. It is expected that the line can be followed only in a general way. Unless there is a specific reason for doing so, do not attempt to go back over the torn edge to "refine" it. The results are usually disappointing. The prints should be torn as the mosaic is laid—not beforehand.

The base for the mosaic should be a good grade of cardboard, very compact, but not too thick. Treat both sides with one coat of shellac. Tape or thumbtack this to a table, near the edges of the cardboard, so that the tacks will not interfere with the mosaic.

Have on hand a good supply of water-soluble mucilage, such as gum arabic.

205. Laying the Mosaic

Glue the central print of the mosaic in a corresponding central position on the cardboard. Next, tear any adjoining print as described above. Cover the entire back with mucilage, and that portion of the cardboard base, also. Lay the torn print on the first so that all images along the torn line are matched as perfectly as possible. Pat down the second print and wipe off excess mucilage around the edges with a damp cloth. Proceed to the next print and repeat the process just outlined. Work radially out from the central print and *work rapidly*.

When most of the prints have been laid, it may be that some difficulty will be encountered in further matching of picture images. This is due to progressive errors in the alignment of photos already laid. Make approximate adjustments at the time, but go on to the remaining prints until all are laid.

When the gum arabic is used (in considerable quantity) the prints can be shifted by steady directed pressure with the hands laid flat on the surface. Using this method, called "floating," shift prints slightly here and there until the entire mosaic is brought into the best adjustment.

Wipe the surface with a damp cloth, being careful not to remove mucilage from under the edges of matched prints. Examine the mosaic from time to time while it is drying to be sure that edges do not curl. When the mosaic is nearly dry, burnish all the picture edges with a bone burnisher or a polished rounded metal object, such as a teaspoon.

206. Controlled Mosaics

As stated earlier, a controlled mosaic is one which is tied into ground control at definite picture points. The density of such points may vary from one point to every 4 or 5 prints to 3 or more points per print. A controlled mosaic differs from an uncon-

trolled one in that its overall scale is usually more constant, though it may exhibit greater local discrepancies. The process of constructing a controlled mosaic is as follows.

The control points, obtained from existing maps, plane table surveys, or any other source, are plotted on the mounting board on the average scale of the photos. Unless the scale and quality of the photos are especially good, rectified prints should be obtained. Images on the photos corresponding to the plotted points are marked with pencil or ink. The picture centers are also marked and located on the base by radial line or other methods; these centers are connected by straight lines.

The laying of the controlled mosaic is much the same as laying an uncontrolled one, except that the centers of individual prints are laid exactly over the plotted locations of the centers, and the photos are oriented by aligning flight lines on the pictures with those connecting centers on the base.

207. Photo Triangulation Methods

There are several methods of photo-mapping that employ the principles of graphic triangulation described in Chapter III. All these methods depend on the four generalizations listed below:

1. Parallactic displacements caused by differences in elevation are radial from the nadir, or plumb point.

2. Displacements caused by tilt in the photograph are radial from the isocenter.

3. It is assumed that the photographs are essentially vertical and, therefore, that the plumb point and isocenter are coincident with the principal point, or picture center.

4. When photographs have little, or no, tilt, horizontal angles between radial lines are identical to angles determined on the ground from a point corresponding to the picture center.

Generally speaking, these four conditions are true; therefore, the principal points of the photos are used as the origins of radial lines, just as the instrument station is used as the origin of rays in plane table intersection work. If, in the discussions that follow,

the centers of photos are thought of as plane table stations and the picture flight lines as backsights and foresights, the theory of photo triangulation becomes quite simple. The beginner should acquire this concept as soon as possible.

Occasionally, a badly tilted print, or group of prints, will cause considerable trouble, because the angles between radial lines drawn from the principal point will be distorted, that is, they will be too large on one side of the center and too small on the other. Sometimes the photos are slightly distorted from unequal shrinkage of the paper on which they are made. If this is suspected, the troublesome prints should be replaced with new ones.

As mentioned earlier, there are several methods of photo triangulation, or radial line intersection. Some require special equipment, such as the slotted template method; others, like the acetate template method, need a larger quantity of expensive materials; and some are only moderately accurate. Only one method will be described here, because it is relatively free from the objections cited, and it is particularly adapted to plane table surveys of fairly large scope.

208. Control Sheet Method of Photo Triangulation

The accuracy of the method depends largely on the number of ground control points per photo and the distribution of these points. For this reason the best procedure is to take a set of the photos into the field when the ground control is being established and select the picture points that give the strongest photo control. These points should be well distributed over the picture, somewhat nearer the margins than the center. There must be not less than three such points for each print, and it is much better if there are four or five, all being identified and marked on the photos by inspection.

A great deal of care and discrimination must be exercised in the selection of picture images and in marking them for later use. The best way to mark the images exactly is with a hand lens and a fine plotting needle. In addition to the pricked exact location, the

point should be encircled with a bow pen and colored ink. It may also be numbered to correspond to the station number of the ground survey. These numbers are placed in open places so that photographic detail is not unnecessarily obliterated.

209. Transposing Centers

The picture centers are pricked and marked with a circle or cross. After the picture centers are marked on all the photos, the centers of adjacent overlapping prints are transposed to each print. This should be done with extreme care, and by inspection of photographic detail only. It often happens that the picture center does not fall on any identifiable detail of the photograph. In this case select two or three images as near to the center as possible and situated in such a way that the center falls on the line or within the triangle formed when these points are joined by straight lines. The same figure is made at the location of the transposed center, and the exact point is plotted in the same relative location on the line or within the triangle. By using proportionate distances, the center may be transposed to another print accurately even though there is considerable difference in scale or some marginal distortion. Two examples of this method are shown in Fig. 122. The center of photo number 3 falls on the straight line between the road corner and another photo detail to the northwest. The scaled distance between these points is 1.63 inch. The distance from c to the road corner is 0.92 inch; and the proportion of the total distance is $0.92/1.63 = 0.56$. The total distance from the corner to the same photo detail on photo number 2 is 1.74 inch; therefore, the distance from the corner to the position of the transposed center is $1.74 \times 0.56 =$

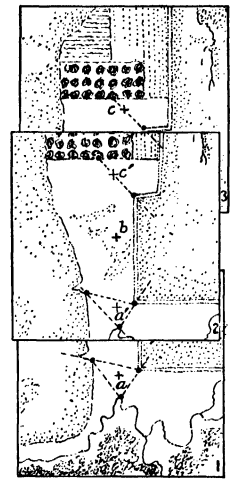


FIG. 122. Transfer of principal points by picture image control.

0.974 inch. This distance is scaled from the road corner, and the transposed center of photo number 3 is plotted at c'.

The principle in the transposition of the center of photo number 1 is essentially the same. In this instance a falls within a triangle constructed on picture images. If the triangles differ in size or shape, the transposed center may be located within it by computing proportionate distances, as described in the preceding paragraph.

When the photos are nearly to the same scale, details around the center and the center point itself may be traced on a scrap of tracing paper. The tracing paper is superposed on the second print and the picture points carefully registered. The center is then transposed by pricking through the paper with a plotting needle.

210. Marking Secondary Points

The main reason for using aerial photos is that much field time and expense can be saved by compiling map detail directly from the photos. Since the desired details on the photo may be distorted in shape and displaced in position, it is necessary to control all such features by secondary control points. These points are located entirely by the process of photo triangulation instead of field methods.

The secondary control points are selected wherever it is desired to locate a map feature. These points should be marked on the photos in a color readily distinguishable from that used in marking transposed centers and ground control.

The selection of points for later intersection should be guided by two considerations: (1) the proximity of the feature to be located and (2) the exactness with which it can be identified and marked on all the photos where the area occurs. In the side-lap zone, points occur on 6 prints; in the end lap, on 3 prints.

211. Compilation of Control Sheet

The best material for the control and compilation sheet is cellulose acetate with one side "frosted" so that pencils of medium

hardness can be used on it for sketching. The sheet should be larger than the map area on all sides by about half the width of a photo; i.e., so that the center points of marginal photos outside the map area will fall on the sheet.

All points located by surveying methods are carefully plotted on the sheet to the desired scale. It is preferable if the scale of the map is near, but somewhat smaller than the average scale of the photos. Linear features, such as roads, telephone lines, fences, and railroads, that have been located in the field should be plotted on the control sheet. These ground control points are then numbered to agree with corresponding numbers on the photographs. If there is a difference in the probable accuracy of the ground points, such as between triangulation and stadia traverse, the difference should be indicated by distinguishing symbols, such as squares or triangles.

When the points are marked on the photos, short radial lines from the principal point are drawn through each. These lines should be made with a fine pen or pencil and a beveled steel straightedge. The radial lines should be about three-fourths of an inch long and must coincide exactly with the point. Figure 123 shows a photograph completely prepared for intersection.

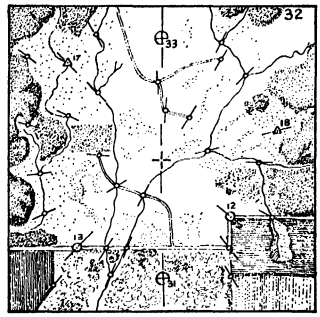
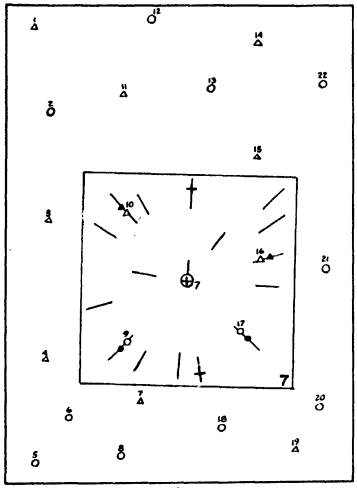
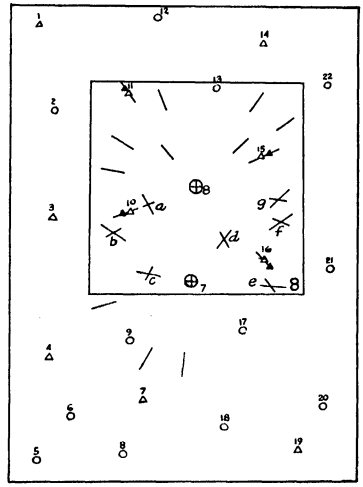


FIG. 123. Photograph marked for radial line triangulation.

212. Orientation and Center Location

It has been shown in Art. 201 that the photo is correctly oriented when the control points on the control tracing lie on the radial lines drawn through corresponding points on the photograph. When these conditions are fulfilled, the principal point of the photograph is at the correct map position of the point, regardless of the relative scales of the photo and map. In Fig. 124A,





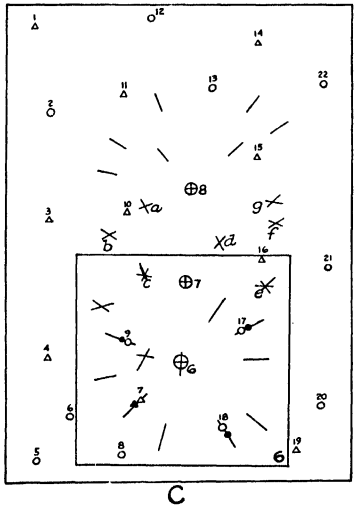


FIG. 124. Showing successive steps in the process of radial line triangulation. The large rectangle is the control sheet with plotted ground control, the smaller squares are three overlapping photos numbered 6, 7, and 8.

photo 7 is oriented by means of points 9, 10, 16, and 17, which are plotted on the control sheet. The center point at 7 is now correctly located and is traced onto the control sheet and numbered for future reference. Being very careful not to shift the tracing, short radial lines are drawn through all secondary control points. These radial lines are shown in the figure cited.

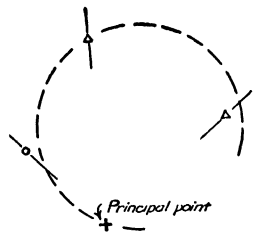


FIG. 125. Arrangement of the principal point and the two or three control points along the "danger" circle; orientation is indeterminate.

It can be seen that the process of orienting the picture and locating the principal point is identical to the tracing paper solution of the three-point problem. Like the three-point problem, the accuracy of the orientation and the locating of the principal point are dependent on the position of the principal point in relation to the ground control points. When the principal point and the control points lie near the arc of a circle, as in Fig. 125, neither the orientation nor the location is reliable.

213. Location by Intersection

In Fig. 124B, photograph 8 has been oriented by points 10, 11, 15, and 16. Picture center 8 is located on the sheet, and radial lines are drawn to points selected for secondary control. In this step in the process certain rays drawn from picture 7 are intersected by those drawn from the center of 8 to the same points, thus locating correctly on the map secondary control points *a*, *b*, *c*, *d*, *e*, *f*, and *g*. As in plane table mapping, the two-ray intersections should be checked by a third set of radial lines. This check is made in the case of points *c* and *e* in Fig. 124C, when photo 6 is placed under the tracing.

The process outlined above is continued until all the photos in a strip (along one flight line) have been used. It is then necessary to construct another chain of intersections along the adjacent flight line.

214. Side-Lap Intersections

While each photo in the process described is worked more or less independently of the others, it is sometimes necessary to make adjustments which are almost wholly dependent upon the judgment of the cartographer. The discrepancies which require adjustment come to light when two adjacent strips are tied together in the zone of side lap, such as at points *b*, *g*, and *f* in Fig. 124B. When the three radial lines do not intersect at a point, the photos are shifted slightly, either in position or orientation, until the best adjustment is attained. It may be impossible to obtain perfect three-line intersections at all points if some of the prints are tilted; but the errors can be distributed so that they are not excessively large at any one locality.

215. Sketching Detail

Several methods for adjusting photo detail to the scale of the map were described in Art. 199. These methods are applied when transferring details from the photos to the compilation sheet. In this case the intersected secondary control points together with the plotted picture centers and ground control provide the basis for adjustment of the desired features on the photos. In practice, scale ratios are not computed. Important features are located by radial line intersections and other points of known location. These features are drawn first. The adjustments of other features are made according to the principles given in Art. 202.

216. Discussion of Photo Triangulation

In contrast to other methods of photo triangulation, the control sheet method just described requires a relatively large number of ground control prints. This method was selected for two reasons: first, it is assumed here that photos are to be used as a supplement to conventional plane table methods; therefore, the ground control may be established rather easily by plane table triangulation and traverse. Second, because of the closely knit ground control.

this is the most accurate photo method, and such errors as do occur are usually quite apparent; so the necessary steps can be taken to correct them. The distribution of the ground control prevents serious progressive errors, such as might occur in radial line intersection or slotted template methods. Since the position and orientation of each photo is determined independently, the photo mapping need not be carried over the entire area.

There is no advantage in using photos in conjunction with the plane table in areas where the desired map details are poorly defined in the pictures. Their greatest advantage is where details are abundant and in areas difficult to reach or survey on the ground, such as craggy mountains, heavily wooded areas, and swamps. The photos are simply a means of bringing field data into the office, and when used judiciously, they are of great assistance to the plane table surveyor.

217. Supplemental Photo-Ray Intersections

It sometimes happens during the course of plane table surveys that one ray and a vertical angle can be determined to a natural point, but for one reason or another the necessary ray from another instrument station cannot be obtained, or can be obtained only by reoccupying an old station. The second ray, which results in an intersection and location, can be obtained with an aerial photograph if the point sighted with the instrument can be identified on the photo.

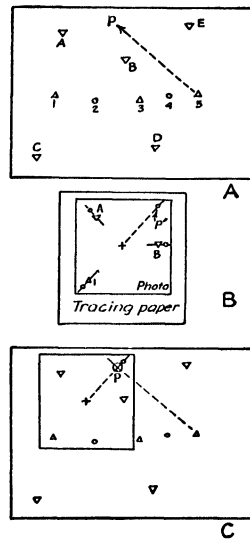


FIG. 126. Illustrating the use of the plane table for one ray and an oriented photo for the second ray to obtain a map location by intersection.

Figure 126A shows a plane table sheet with a few triangulation stations and a traverse. At station 5 a sight is made on a natural object p and a ray is drawn, but it is

difficult to get the intersecting ray without reoccupying stations 1 or 3. The photograph is used instead.

A piece of tracing paper somewhat larger than the photo is laid on the plane table sheet and triangulation stations A and B and instrument station 1 are traced. The tracing is then laid on the photo with the points on the corresponding rays, as shown in B. The tracing is replaced on the plane table sheet, correctly registered on the control stations, and the radial line is transferred to the plane table sheet. This line gives the desired intersection at p. The elevation of p can be computed from the vertical angle read at station 5 and the distance scaled on the sheet from that point.

CHAPTER XI

Drawing Instruments and Their Uses

218. General Remarks

The drafting of maps is a specialized field requiring some knowledge of elementary engineering, mechanical drawing, freehand drawing, and lettering, the construction of map projections, and other technical and manual skills peculiar to the profession. Years of continuous practice are necessary for complete mastery of the subject; and practice is the only means to mastery, for it is a manual art, and like all arts, perfection comes only through practice. No amount of knowledge of map drafting can substitute for actual practice, for it is the hand and eye that must be trained.

The map draftsman must be familiar with the uses and maintenance of drawing instruments, their applications and limitations, and the principles of certain graphic constructions made with them. Limited space does not permit an exhaustive treatment of drawing instruments and mechanical constructions. The following pages are devoted only to those operations and problems encountered in ordinary map work and to the instruments required for carrying out the proper drafting of maps.

219. Straightedges and Triangles

For most work the best straightedge is of stainless steel, beveled on only one edge. It is desirable to have several sizes, but if only one can be had, it should be about 2 inches wide and 36 inches long.

The steel straightedge must be handled carefully, for it is ruined for certain types of work if the edges become even slightly

nicked or dented. It should be kept in a drawer or hanging flat against a wall. If the edges are rough, they may be polished with a very fine oil hone. When honing, run the stone the full length of the edge, even though only a portion needs smoothing.

Steel straightedges tend to slip when used on sloping drafting tables. This tendency is reduced by sticking a strip of crepe drafting tape on the bottom side.

Triangles are made in a wide variety of sizes and shapes. The size is designated by the length along the longest edge from the 90-degree corner; the shape by the angles at the corners, such as 45-degree and 30–60 degree. The minimum equipment should include a 45-degree 4 inches long and one 10 to 14 inches long; a 30–60 degree 8 inches and one about 14 inches. In addition to these, two or three of various lengths, but beveled on one leg, are necessary for certain operations.

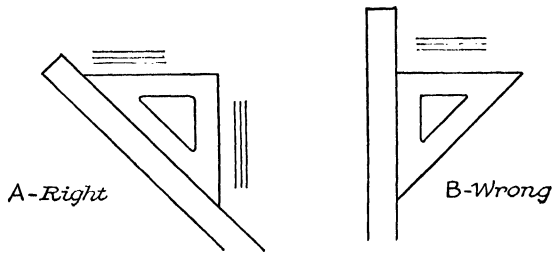


FIG. 127. Use of the triangle and straightedge.

Figure 127A shows how to place the triangle against the straightedge to draw parallel lines. When placed in these positions, the pressure of the pencil or pen against the edge tends to press the triangle firmly against the straightedge. When used as in Fig. 127B, the pressure is likely to move the triangle away from the straightedge.

220. Checking the Triangle

The accuracy of the right angle of a triangle may be checked as follows. First, fasten the straightedge down on a piece of white

paper with drafting tape. Place the triangle with the base against the straightedge and draw a fine perpendicular line along the other (altitude) edge. Now turn the triangle over as the page of a book with the same edge against the straightedge, but with the triangle reversed. If the right angle is true, the perpendicular leg will coincide with the line.

The angles and edges of non-metallic triangles can be made true by cutting the edges on a large flat file. The file should be somewhat longer than the triangle and should be taped flat on a table. The triangle is held upright as nearly vertical as possible and drawn evenly along the top of the file. Keep the entire length of the triangle in contact with the file, but apply the greatest pressure near the end where most of the material is to be removed. This procedure insures a straight edge when the correct angle is attained. Check the angle as outlined in the preceding paragraph.

221. Irregular Curves

Three types of irregular curves are shown in Fig. 128. These three types in sizes from 3 inches to 10 inches in length will suffice for most map work. Irregular curves are used for drawing and inking features with smooth curves, such as railroads, highways, and other works which have been laid out on the ground by engineering methods. Although no exact method can be stated for the use of irregular curves, there are a few general rules which are helpful. When a smooth curve is to be drawn through a series of points by means of irregular curves, the curve is placed so that its edge is in contact with not fewer than three points. At each new position of the curve, the edge should be placed against the last point and a portion of the curve previously drawn. The

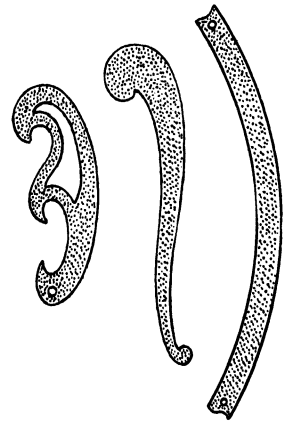


FIG. 128. Irregular (French) curves.

general alignment of the points can be determined by casual inspection. It should be determined in which direction the rate of curvature is increasing, and the irregular drafting curve is then placed with the end of increasing curvature in the same direction.

Figure 129 shows the application of one form of curve and the straightedge to draw the series of curves and tangents of a rail-

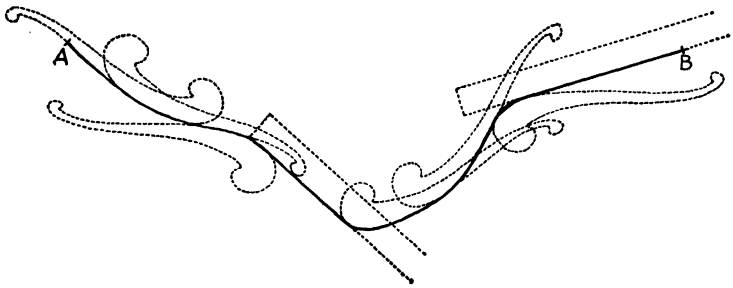


FIG. 129. Application of the straightedge and one irregular curve in drafting the curves and tangents of a railroad.

road from *A* to *B*. The dotted outlines show successive positions of the instruments.

222. Plotting Scales

For accurate plotting of distances, scales should be beveled to thin edges so that the graduations are brought close to the surface of the paper. The bevels should be faced with white celluloid or similar material. The "engineers'" scale is graduated in tenths, twentieths, thirtieths, and on to sixtieths of an inch. The smallest graduations (sixtieths) are difficult to read and are exceedingly tiring to the eyes. It has been shown in numerous tests where much plotting is done that the average accuracy is higher when a scale with larger graduations is used and the smaller units of distance are interpolated. The "architects'" scale is graduated in fourths, eighths, sixteenths, and up to sixty-fourths of an inch. The "fractional" scales are based on the proportionate length of a scale unit to the distance on the ground which it represents. For example, a

scale of 1:24,000 (or 1/24,000) means that one unit on the scale represents 24,000 units of the same length on the ground. This is a convenient system because it is applicable to any system of measurement: inches, feet, centimeters, meters, etc. The inch-mile scales are graduated so that a given number of inches represents 1 mile on the ground. On a 2-inches-equals-one-mile scale the graduations within each 2-inch interval may represent 20-, 40-, or 100-foot distances, or, on some types, tenths, fiftieths, or one-hundredths of a mile. The draftsman should make certain of the kind of scale before using it to plot distances on the map.

When plotting distances, the scale should be used as shown in Fig. 130. In the case illustrated the distance to be plotted is 2.96 inches. It is necessary to interpolate between 2.9 and 3.0. Since it is easier to lay the scale with the interpolated distance at a point already plotted, the scale is placed in this position, and the new point is plotted at the zero end of the scale. In other words, it is easier and faster to place the plotting needle exactly on a scale graduation than at an interpolated distance between graduations. All distances should be plotted in this manner, for there is less danger of making errors.

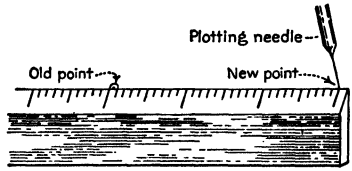


FIG. 130. Correct use of the scale and plotting needle (or pencil).

223. Protractors

The simplest type of protractor is a half-circle or complete circle graduated in degrees or half-degrees along the periphery. Protractors are made in stainless steel, brass, and transparent plastics. For most uses the transparent protractors are best. Protractors are used mostly for laying out bearings and azimuths or deflection angles of traverses on maps.

The angles are measured from some reference line already on the map—usually a north-south or an east-west line. If no such line is present near the location where the bearing is to be drawn,

then one must be drawn for temporary use as a reference. This line should be lightly penciled so that it can be cleanly erased when it has served its purpose.

There are two ways in which an angle can be laid off with a simple protractor: (1) The protractor is placed with the center of the circle (or semicircle) on the plotted point and oriented with the zero (or 90-degree) graduation on the reference line. A point is plotted at the correct angle graduation on the edge, and then a line is drawn from the old point to the new. (2) Another method, which is not so widely used, though more convenient, is illustrated in Fig. 131. A bearing of north 30 degrees east is to be laid off from point *c*. The center of the protractor at *o* is placed on the reference line, and the protractor is rotated about this point until the 30-degree graduation also falls on the line. Now the protractor is moved along the reference line until the base touches the point *c*, and the bearing is drawn through the point along the base of the protractor.

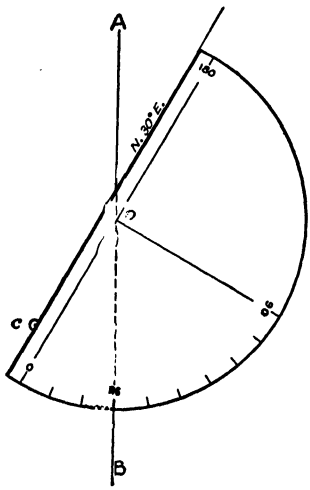


FIG. 131. Laying off a bearing with a simple protractor.

Some protractors are provided with a straightedge aligned with the center of the protractor. The straightedge rotates about this axis, and angles are read on the periphery of the circle. In some types, a vernier is attached to, or is a part of, the straightedge. The vernier is graduated to 10 minutes of arc.

224. The Drafting Machine

This is a device which has become standard equipment in most drafting rooms. The several types on the market, while differing in details, operate on the same principle. The base of the instru-

ment is clamped on the edge of the drafting table. An arm extending over the table is constructed on the principle of a mechanical parallelogram, so that a scale or straightedge attached at the end maintains a constant bearing while being moved over the drawing. A graduated circle and vernier arc provide the means for setting the straightedge to any angle. The verniers of some drafting machines read to one minute of arc. The drafting machine should be checked from time to time to ascertain that all moving parts fit snugly. The machine can be checked quickly as follows. Tape a large piece of cross-section paper to the drafting table. Set the machine straightedge exactly on a horizontal line at the base of the sheet, then move it to the top and check on another line on the paper. Now turn the straightedge 90 degrees and check on vertical lines across the table.

225. The Pantograph

The pantograph is a mechanical parallelogram attached to a stationary base at one corner and equipped with a tracing stylus and pencil at two others. When the stylus is moved along a line on the map, the pencil draws the same figure on another sheet. By setting the scales in different positions the pantograph will either enlarge or reduce the map features to any desired scale.

For precise work with the larger machines it is essential for the table to be level and to have a hard, smooth top. The suspension wires should be adjusted so that most of the weight of the arms is carried on the staff.

226. The Planimeter

The polar planimeter is a simply constructed device for measuring the areas of irregularly bounded map figures, such as lakes, fields, and so forth. The simplest and least expensive type has no adjustment for changing the scale. This has no effect on the accuracy of the measurements, but does make it necessary to compute the final results in terms of the desired units of area. Like

the pantograph the planimeter should be used on smooth, level surfaces.

In making measurements of area, a steel stylus mounted at the end of a horizontal tracing arm is moved along the boundary of the area in a clockwise direction. A small, graduated drum records the area encompassed. In some types the arm is adjustable, so that the drum readings can be made to correspond to a convenient number of areal units, such as square inches or square feet.

When using the instruments without adjustable arms, a calibration factor must be determined. This can be done in two ways. (1) Some planimeter sets are provided with a metal calibration bar with a needle pivot at one end and a small hole at the other. The distance between the point and the hole is the radius of a circle whose area is exactly 10 square inches. Set the needle point in the paper and the stylus of the tracing arm in the hole. Mark the starting point of the stylus and record the drum reading. Now, move the stylus through the complete circle back to the starting point, and read the drum again. The difference in the drum readings is the number of instrument units equal to 10 square inches. Thus, if the difference in drum readings is 10.125, the factor is $10/10.125 = 0.9875$. In measuring areas with this instrument, the indicated area on the drum is multiplied by the factor to obtain the correct area. (2) If the set does not contain a testing bar, a rectangle of exact dimensions may be drawn on the sheet and traced with the stylus. In this case, a straightedge of some kind should be placed on the lines to serve as a guide for the stylus so that exact tracing is assured. Although any size of rectangle may be used, it is convenient to draw one 2 inches by 5 inches, or 10 square inches. Determination of the factor is the same as given above.

The movement of the graduated drum is dependent on the small disks which roll on the surface of the map as the stylus is moved. Therefore, the readings may be erroneous if the paper has a high gloss or if the drawing is on untreated acetate. Determinations of area under these circumstances should be repeated at

least three times. Unless one of the three is much different from the others, use the average.

227. Beam Compass and Bar Scale

The beam compass consists of a hardwood or metal rod from 1 to 4 feet long with precision divider points which can be moved along the rod to any desired spacing. The dividers are equipped with tangent screws, which permit settings to a very fine degree.

The steel bar scale is shown in Fig. 132. All lines on the scale are finely engraved so that the points of the beam compass can be set exactly on the graduations. Horizontal lines are spaced one-

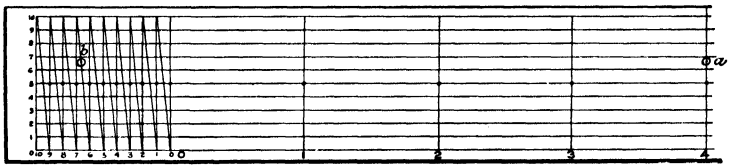


FIG. 132. Standard bar scale.

tenth inch apart. Vertical lines are 1 inch apart except the last inch of the scale, which is divided into one-tenth inch intervals. Diagonal lines traverse the one-inch by one-tenth inch rectangles at the end of the scale. A row of small holes extends down the center of the scale, each hole being exactly centered on one of the vertical lines. These holes hold the points of the compass for making approximate settings. The following example will illustrate the use of the beam compass and bar scale.

The points of the compass are to be set at a span of 4.667 inches. The right point is set in a hole on the 4-inch line, and the left is moved along the beam until the point drops in the hole on the 6-tenths graduation, and the compass points are tightened on the beam. Now the beam is lifted, and the right point is carefully set in the groove at position *a*, approximately 0.66 inch above the base line. Using the tangent screw to adjust the point spacing, the left point is adjusted until it rests in the diagonal between the

6-tenths and 7-tenths vertical lines, but 67-hundredths of an inch above the base, at point *b* in the figure. After making the final adjustment, the beam is lifted and then is carefully replaced in the same position to be sure that no strains occurred in the instrument during the setting.

It is clear that exact settings can be made to one-hundredth inch, for each hundredth is at the intersection of a horizontal and a diagonal line. Thousandth parts are interpolated along the diagonal between these intersections.

These instruments have many uses in the drafting room. The steel bar scale is particularly useful in laying out precise map projections and grids and for determining exact distances between triangulation stations from the plotted positions. Pencil and pen tips may be substituted for the divider points of the beam compass for drawing arcs of long radii.

228. Pens and Pencils

The variety of both pens and pencils is so great that only a small percentage can be discussed here. However, it should be apparent that the reason for such variety is that each has qualities that are desirable for specific jobs or for use on different types of drawing papers, linens, and other materials. Likewise, much depends on the natural tendencies of the individual who uses the equipment. The best advice is to experiment with different pens and pencils until the "right" ones are discovered. The following suggestions will serve as a general guide.

In selecting steel nib pen points for non-mechanical drawing, three features should be sought: width of line, flexibility, and size of holder. Some pens have extremely fine points but are also very flexible. Only highly skilled drafstmen can maintain an even width of line with this kind of pen. Unless one's hand is exceptionally steady and the "touch" evenly light, very flexible points should be avoided. Points ranging from very fine to coarse can be obtained in both flexible and stiff grades.

A pen point should not be strained to make lines wider, for this

will soon ruin it for making the weight of line for which it was intended. Widen the line by retracing or, better, use a heavier point. Keep the point clean by frequent wiping with a soft cotton cloth. Ink will not flow properly from a pitted or encrusted point. Furthermore, frequent wiping polishes the point so that ink flows from it better the longer it is used. For this reason, experienced draftsmen dislike "breaking in" a new point and take extremely good care of one that has been used for some time. Points are often ruined by rolling off the table. This may be avoided by flattening one or two sides of the holder, so that it will not roll easily, or by reversing the point in the holder when not in use. It is good practice to turn the point in the holder when it is laid away in a box or drawer.

Steel pen points may be sharpened on a fine oil hone, such as Arkansas novaculite. A hand magnifying lens may be used to check the point from time to time during the honing. The point should be put in a holder and honed with a back and forth motion parallel to the holder so that the nibs are not sprung apart. Professional topographic draftsmen often hone new points to exactly the desired sharpness, and then touch them up from time to time as they become worn. After honing a point with oil, wash it with soap and hot water, otherwise it will not hold the ink.

Pencils used for drawing lines with a straightedge should be sharpened with a wedge-shaped point. A conical point wears down so fast that one end of a long line will be considerably wider than the other.

After sharpening a pencil, "polish" the point on a piece of paper before using it on the map. A polished point lasts longer and does not strew graphite dust on the drawing. The point can be kept sharp by touching it up occasionally on a scrap of heavy Manila paper.

229. Ruling, Swivel, and Railroad Pens

The ruling pen consists of two tapering blades which are almost or quite identical in profile and sharpened to thin edges at the tips.

The spacing between the blades is adjustable by means of a small thumbscrew, and this determines the width of the line. There are many sizes of ruling pens, but only two general classes: the narrow and the broad, or "border" pen. The map draftsman should have at least two of the narrow types, one small and one medium sized, and one large "border" pen.

The ink is placed between the blades of the pen with a quill or dropper and is held there by the surface tension of the ink. When the pen is too heavily charged with ink it is likely to break the surface tension and cause a damaging blot on the map. Likewise, if the blades are set for a line too wide for the size of the pen, a large flow will occur when the pen touches the paper.

The broad blade pen should be used for drawing wide lines. It has a large ink capacity and safely holds larger quantities than the narrow ones. For fine lines the ink flows better from a narrow pen.

Ruling pens of all kinds are used primarily for drawing lines along straightedges or irregular drafting curves. They are not intended for freehand work and are unsatisfactory for this purpose.

The tips of ruling pens must be kept sharp in order to function properly. Sometimes one blade wears more than the other so that the two do not have the same contour. A pen in this condition cannot be expected to produce an even width of line. Figure 133

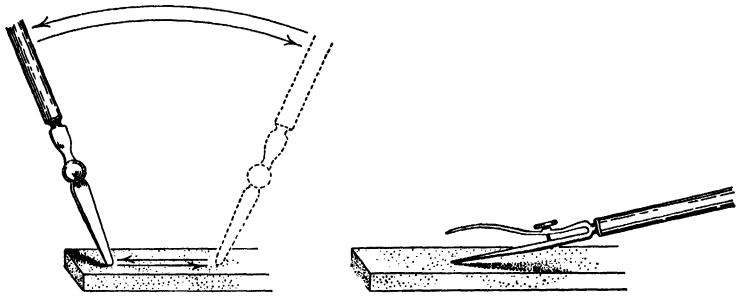


FIG. 133. Shaping and sharpening a ruling pen.

illustrates the correct way to reshape and sharpen a ruling pen. First set the blades so that they just touch each other at the tips. Holding the pen as in Fig. 133A, draw it back and forth along the full length of the hone, reversing the angle of the stock gradually along each stroke. This is to produce identically rounded profiles on the two blades. When the shape of the pen is satisfactory, open the blades and grind each seperately, as shown in Fig. 133B. Use a circular motion on the stone and constantly rotate the stock slightly so that a rounded surface is obtained. The tips should be sharp, but not so sharp that they tend to cut the drawing paper. After sharpening, the high polish may be restored by rubbing on a piece of oiled leather with a little polishing rouge.

The railroad pen is essentially a double ruling pen in which the width of lines and spacing between lines are independently adjustable. It is used for drawing double-line roads, railroads, ditches, etc., along irregular curves or straightedges. Railroad pens are sharpened as outlined in the preceding paragraph.

The swivel pen, sometimes called a "contouring" pen, is a recurved ruling pen which rotates on the stock (Fig. 134). It is designed for drawing irregular curved lines freehand. The swivel pen is quite difficult to use. The stock is held in a vertical position so that the swivel will operate freely. Since the tip of the pen trails somewhat behind the stock, allowance for this must be made in inking curved penciled lines. The eyes must follow the tip of the pen—not the stock. The swivel pen is best used for broad, sweeping curves and is likely to make heavy, blotchy lines where curves are sharp.

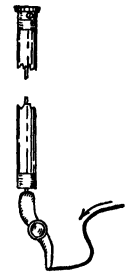


FIG. 134. The swivel or contour pen.

230. Lettering Guides and Transfers

The appearance and legibility of a map depend to a large degree upon the lettering. A finely drafted map loses much of its effectiveness if the lettering is poorly done. While expert freehand lettering

is superior to printing or various "mechanical" methods, the freehand lettering of the average engineer does not compare favorably with the guide lettering.

There are several types and brands of lettering guides on the market, all of which are capable of producing neat, legible letters in a wide variety of sizes and line weights in upper and lower case and in vertical or slanting letters. The two principal brands of lettering guides are the Wrico and LeRoy. The Wrico consists of a strip of transparent plastic with perforated letters, and a tubular tank pen which is guided by the slotted letters. The guide is butted against a straightedge and the letters are drawn directly beneath the guide. The LeRoy uses a similar tubular pen, which is inserted in a special holder. The holder operates in a slotted guide, placed against a straightedge, and is adjustable for making either slant or vertical letters with the same template. The LeRoy makes the letters in front of the guide, which is advantageous in spacing the letters correctly.

Alphabets of different sizes and styles printed on transparent transfer paper may be purchased at small cost. Small squares each containing one letter are cut from the sheet, placed on light pencil guide lines on the map, and burnished with a bone burnisher. A wax-like coating causes the transfer to stick to the tracing.

231. Perforated Line Spacers

Much time can be saved in drawing guide lines for freehand lettering if a spacing guide is used. The most inexpensive type is an ordinary triangle with rows of conical perforations. The triangle is placed against a straightedge, a sharp pencil is inserted in one of the holes, and the triangle is slid along the straightedge. The pencil draws a fine line parallel to the straightedge. Another perforation is selected and the process is repeated, and another parallel line is drawn. A large variety of spacing can be obtained either by turning the triangle with different sides against the straightedge or by using a hole in one row for one line and one in another row for the second line. The Ames lettering guide is a

perforated disk set in a frame in such a way that it can be rotated to provide any desired spacing of lines. It is used similarly to a lettering triangle, with the frame being placed against the straightedge.

232. Tracing Materials

The three principal classes of tracing materials are papers, linens, and plastics. Each has general characteristics which should be considered when selecting a material for a map or drawing.

The papers, of which there is a large variety, are the least expensive. They are more fragile than linen or acetate and, therefore, should not be used for maps which are subject to much revision or rough handling. Good grades of vellum tracing paper have a low shrinkage factor and are desirable for maps which must hold their scales under variable conditions of humidity. They take ink very well and are superior to all other materials for pencil work which must be reproduced directly from pencil copy by photoreproduction processes. The light-transmission properties vary according to the kind of material and the thickness. The heavier material is easier to work on and withstands considerable erasing; but it does not yield as good prints from pencil copy as the thinner grades. Inked drawings produce good prints from the heaviest grades of vellum if lines are not exceedingly fine. Some papers become very brittle with age.

Tracing linen is strong, flexible, and long-lived. It takes ink moderately well, but only soft to medium pencils can be used. Pencil copy does not reproduce well. While scale changes due to atmospheric variations are not large, they may vary in direction and amount in different parts of the sheet, thus causing distortions in the relative locations of map features. Under ordinary conditions these distortions are not large enough to cause any appreciable inconvenience.

Both linen and tracing paper have a tendency to shrink or expand more along one coordinate than the other. The tendency is more pronounced in linens. For this reason, when a map is laid out

in a number of sheets which must match along their edges, the sheets should be cut so that the linen (or paper), as it comes off the roll is oriented in the same direction.

Cellulose acetate is superior to paper or linen in several respects. It has much higher light transmission for photo-copy reproduction. Common drawing ink can be removed with a cloth moistened with dilute ammonia without effect on the surface, or it may be scraped off with a sharp erasing knife. The material is unaffected by complete immersion in water. Cellulose acetate requires soft pencils, for it is difficult to obtain pencil lines dark enough to give good photo reproductions with hard ones. It is more responsive to scale changes due to humidity than are paper and linen, but the scale changes are uniform and can be controlled if the humidity of the air can be controlled. Ink tends to flake off if the acetate is rolled; therefore, drawings should be stored on a flat surface.

CHAPTER XII

Methods and Practices in Map Construction

233. Remarks

Because of the consistently high quality of drafting in the published maps of various branches of the U.S. Geological Survey, these maps are generally accepted as standard by other organizations both within and outside the government. The beginner in map drafting can gain much by studying the modes of representation on the Geological Survey maps. Other maps of high quality are available for study in libraries, stationers' stores, or distribution offices of the federal government. These maps include those of the Coast and Geodetic Survey, Army Engineers, Forest Service, National Park Service, and Soil Conservation. In many of these examples it will be seen that a great variety and density of ground detail are shown without seriously affecting the legibility. This is the result of careful planning and fine discrimination in the selection of symbols, weights of lines, classification of data, and placement and choice of lettering.

234. Order of Procedure

Maps which are intended to present a large variety of detail must be carefully planned or they are likely to become a hopeless confusion of lines, symbols, and lettering. It may be that only a portion of the data can be shown on one sheet; therefore, it is of great importance to predetermine the classes of data and set them down in the order of their importance to the project. For example, if the map is to be primarily a geologic map, then all pertinent geologic features should take precedence over, say, hydrographic fea-

tures. When the map is to be drawn on a specified scale, this selection of material may become a major problem; for it may be necessary to choose one of two classes of data, both of which are considered of some importance. On the other hand, when a scale most suited to the volume of data can be selected by the draftsman, the grading of map data is not so difficult.

The first step in drawing the map is the construction of the base. This may be a system of rectangular coordinates, geographic coordinates, township and section lines, or county and legal subdivisions. Usually, these lines are not broken for others that cross them; therefore, they may be inked before other data are drawn. This avoids their being erased or confused during compilation of other data. After the base is inked another class of data, such as all water features, may be compiled, checked, and inked on the base.

It is often desirable to letter all the names on the body of the map before lines, boundaries, and other features are compiled. This permits breaking such lines as contours and streams at the lettered names. When acetate is used space can be cleared for lettering by scraping off inked map detail with an erasing knife. There is some advantage in this procedure in that the names can be placed where there is least interference with details already drawn on the map.

235. Boundaries, Grids, and Land Lines

Before inking any lines on the base, a trial sheet showing all the orders of subdivisions should be made. As an example, the following boundaries are given in order from the heaviest to the lightest lines: international boundary, state line, county line, principal or guide meridian and standard parallel or base line, township and range, section, and grid or geographic coordinates. The system given above would require eight different weights of lines. In a range of weights as large as this, two classes of boundaries are sometimes drawn with the same weight of line, but are distinguished from each other by the kind of line.

It is customary to show international boundaries with one long

dash and two (or three) short ones, all in heavy lines. State lines are one long and two short dashes, in moderately heavy lines; and county lines are one short dash in lines of lighter weights than state lines.

Land lines and coordinates are solid lines. Township and range lines are lighter than county lines, but heavier than section lines. Grids and geodetic coordinates are usually the lightest straight lines on the map.

While the *relative* weights of lines are more or less standard, *actual* weights should be adjusted to the scale of the map, i.e., the lines on a small-scale map should be lighter than those on a large-scale one.

236. Inking Straight Lines

Heavy lines, such as state boundaries, should be inked with a border pen. If the lines are exceptionally heavy, the inking must be done by drawing two medium weight lines that touch or overlap. When this is done, wait until the first line is dry before inking the second. There is a definite limit to the width of line that even the largest pen will make without danger of discharging too much ink. Even though the line is satisfactory as to evenness and width, an excessive amount of ink on linen or tracing paper will cause puckering. Therefore, wide lines should always be drawn in two stages.

An overcharged pen will make a wider line where the pen first touches the paper. This is particularly noticeable where the pen has been refilled at a middle portion of the line. Always refill the pen before all the ink has been discharged. When the pen is nearly empty the line becomes too light, and the small amount left in the pen may dry into a hard crust before it can be filled again. A certain amount of ink dries along the edges of the blades even while the pen is in constant use. Wash and dry it occasionally in order to avoid poor lines.

Medium and fine lines can be drawn either with ruling pens of appropriate sizes or with ordinary steel nib drawing points. The

latter are preferred by some organizations for two reasons: (1) there is no tendency to impress the ink into the paper; hence, erasures are made more easily; (2) the points do not have to be reset for a given line weight, and a point of different, but standard, size is used for each different weight of line. These points, however, must be used with care and in a specific manner, or they will blot under the triangle or straightedge. Use only a straightedge with a bevel. Run the thumb nail along the edge to be sure that it is perfectly smooth. If any nicks can be felt, hone them out as explained in Art. 219. Lay the straightedge with the bevel *down,* so that the edge is above the surface of the paper. This edge is aligned parallel to, but slightly to the side of, the line to be drawn. Place the pen against the upturned beveled edge as shown in Fig. 135. If there are any nicks in the straightedge, the sharp edge of the steel point will catch in them and cause the nibs to open, which, in turn, will flick ink over the map or form a blot. A small amount of ink will collect along the edge of the straightedge. This must be wiped off as each line is completed, for when it dries to a hard crust it affects the pen the same as nicks in the straightedge. Ink will not readily collect on the straightedge if a finger is first slightly moistened with oil and then wiped along the beveled edge, care being taken that no oil gets on any part of the straightedge that touches the drawing.

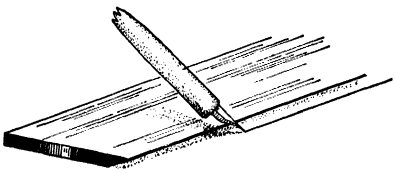


FIG. 135. Use of the nib pen with a beveled straightedge.

The nib pen and beveled straightedge method is superior to the ruling pen in drawing broken, dashed, or dotted lines, or a number of short straight lines. It is also best for long, very fine lines, such as those of the grid or projection. Some very expert topographic draftsmen use no ruling pens whatever, all straight line work being done with different sizes of steel nib pens.

Triangles are beveled for this type of work by holding them flat

on a table with the edge extending about ½ inch beyond the edge of the table and then filing the bevel with a large flat file. The newly cut bevel can be smoothed by scraping with a sharp knife or sanding with fine emery cloth. The final polish is gotten by rubbing with a block of hardwood. Only the hypotenuse edge should be beveled, the other two edges being left square for use in combination with a straightedge.

237. Kinds and Treatment of Inks

Most map drafting is done with waterproof drawing inks; however, water-soluble colored inks are more opaque than the waterproof kinds, and for this reason are somewhat better for photo reproductions. Black waterproof ink is a carbon suspension which thickens as the liquid medium evaporates. Obviously, the bottle should be kept closed except for the momentary reinking of pens. The hard crust which forms around the rim of the bottle should be wiped away with a damp cloth to prevent the hard chips from falling into the bottle and later causing trouble with pens.

It is sometimes necessary to thin carbon ink for use in very fine pens. Use distilled water to which is added a little ammonia. In humid climates the drying of ink on the map is hastened if a little ethyl alcohol is added to the ink. Electric driers which blow a stream of hot air are also used for this purpose.

Special inks are made for use on cellulose acetate or film. As mentioned before, ordinary inks do not adhere strongly to the impervious material and tend to flake off when dry. If these inks are not available, a satisfactory substitute can be made as follows.

Dissolve a few small pieces of cellulose acetate in pure acetone, which may be obtained at any drug store. Use a piece the size of the thumb nail to about 1 ounce of acetone. Allow ten to twenty minutes for complete solution. Shake the bottle occasionally. Add about half a teaspoonful of ammonia, a few drops at a time, mixing thoroughly by shaking the bottle during the process. If the mixture becomes fibrous, strain it through a fine cloth. Now introduce this solution, a drop at a time, into the waterproof drawing

ink, mixing it thoroughly as it is added. About one teaspoonful to a bottle of ink is sufficient to make the ink "bite" into the acetate. If the ink tends to flocculate, either too much acetone has been added or it has been poured in too rapidly. When ink has been treated in this way, it must be kept tightly stoppered, for the acetone evaporates very rapidly. When the ink is in constant use, add a few drops of the acetone solution each day. Pens used with this ink should be cleaned with acetone.

238. Freehand Inking

Much of the inking on maps must be done freehand, without the support of mechanical instruments or guides. Skill in freehand work is acquired only by much practice. Various exercises have been devised for developing a steady hand and a firm but light pressure with the pen. This is particularly important in the use of the finest pens. Generally speaking, it is easier to draw curved lines which arc away from the hand, rather than toward it. Also, it is better to confine movement to the wrist and fingers when inking irregular lines, because it is difficult to control the pen pressure when the pen is directed by arm movement. A line should not be prolonged to a point where the position of the hand is cramped. Work by progressive stages, a few inches for each position. This procedure is much less tiring, and results in smoother curves and more even weights of lines.

Figure 136 shows a number of groups of curved lines which have been found very helpful in developing pen skill. Draw upper and lower guide lines in pencil, then sketch the curved figures lightly in pencil. Ink the figures with a very fine pen, keeping the lines of even weight and the curves smooth. Do not turn the sheet while inking, for the objective is to practice inking the curves in all directions.

239. Standard Symbols

A great many map symbols have become standardized through years of usage. For example, a small triangle representing a tri-

angulation station is used by practically all organizations concerned with plotting triangulation stations on maps. The U.S. Geological Survey has made available at a cost of a few cents, a sheet showing all the map symbols used by that organization. Everyone engaged in the construction of maps should have one of these sheets for reference, and the standard symbols should be used

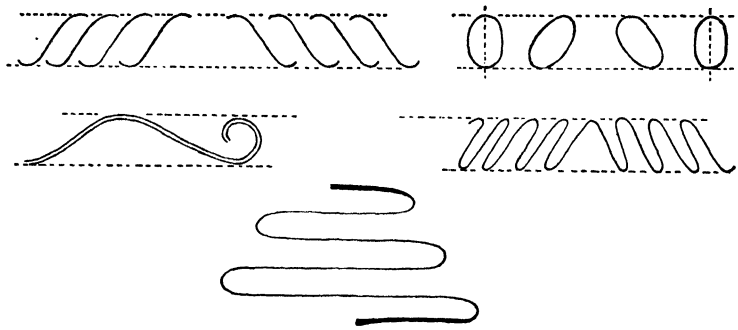


FIG. 136. Figures for practice in inking freehand.

wherever practicable. Figures 137, 138, 139, and 140 include many of the Geological Survey symbols together with some from other sources.

It is sometimes necessary to invent a new symbol to show some special map feature. The sheet of standard symbols should be checked to ascertain that a like figure has not been used to represent an entirely different feature.

240. Legends, Notes, and Explanations

Occasionally, a map is drafted, or even published, with insufficient notes or points and lines of reference to determine the geographic positions of the principal features shown in the body of the map. When a map is supposedly finished, it should be edited with extreme care to make certain that state and county names, township and range numbers, geographic coordinates (when used), and any other similar location references are fully labeled on the map.

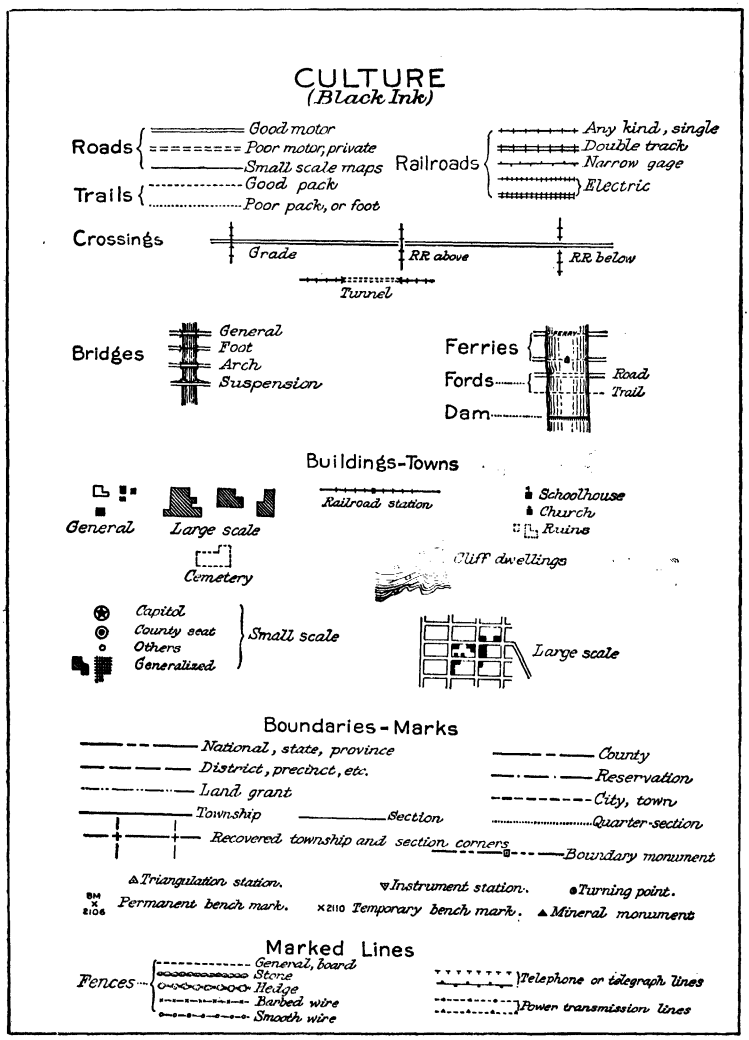


FIG. 137. Standard symbols shown in black ink.

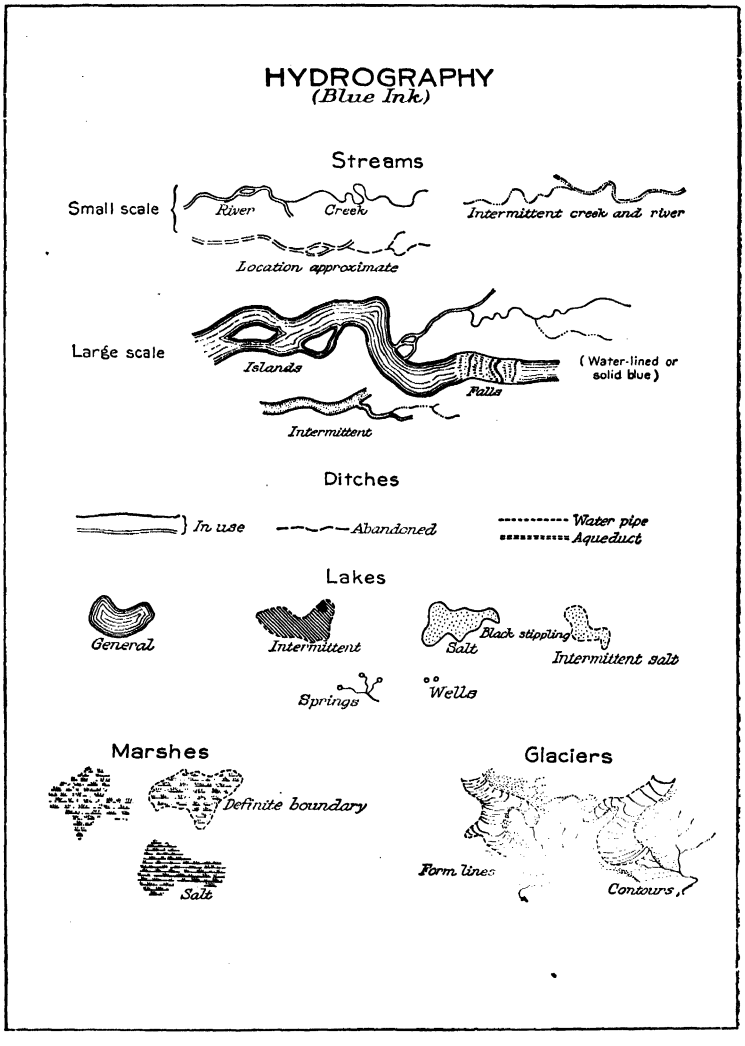


FIG. 138. Standard symbols shown in blue ink.

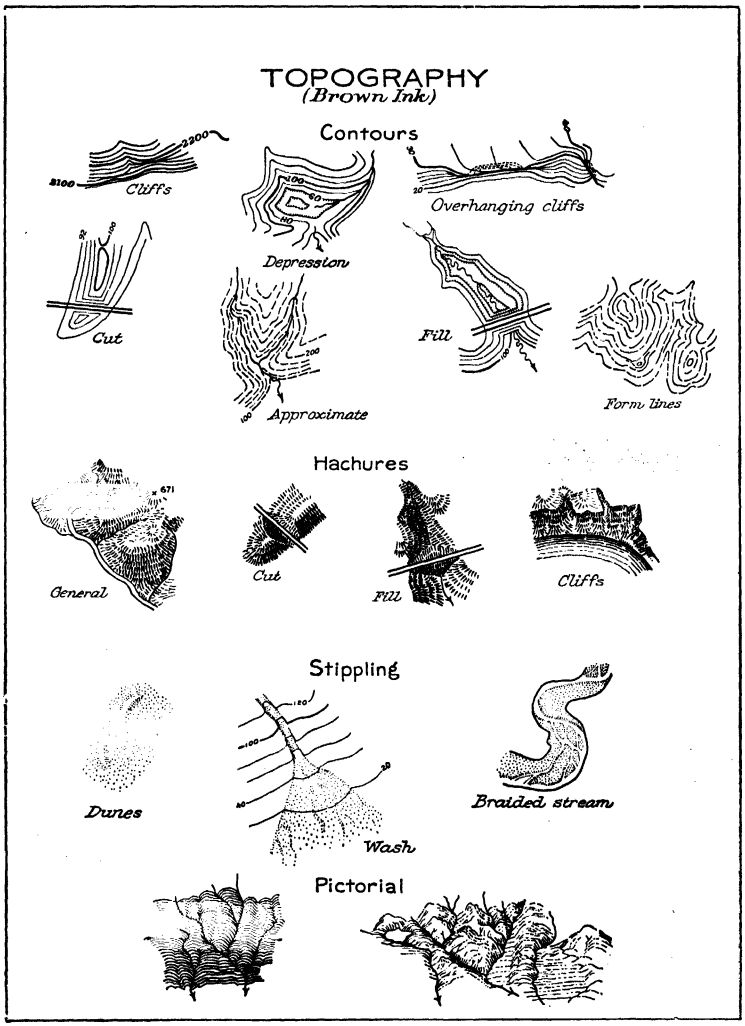


FIG. 139. Standard symbols shown in brown ink.

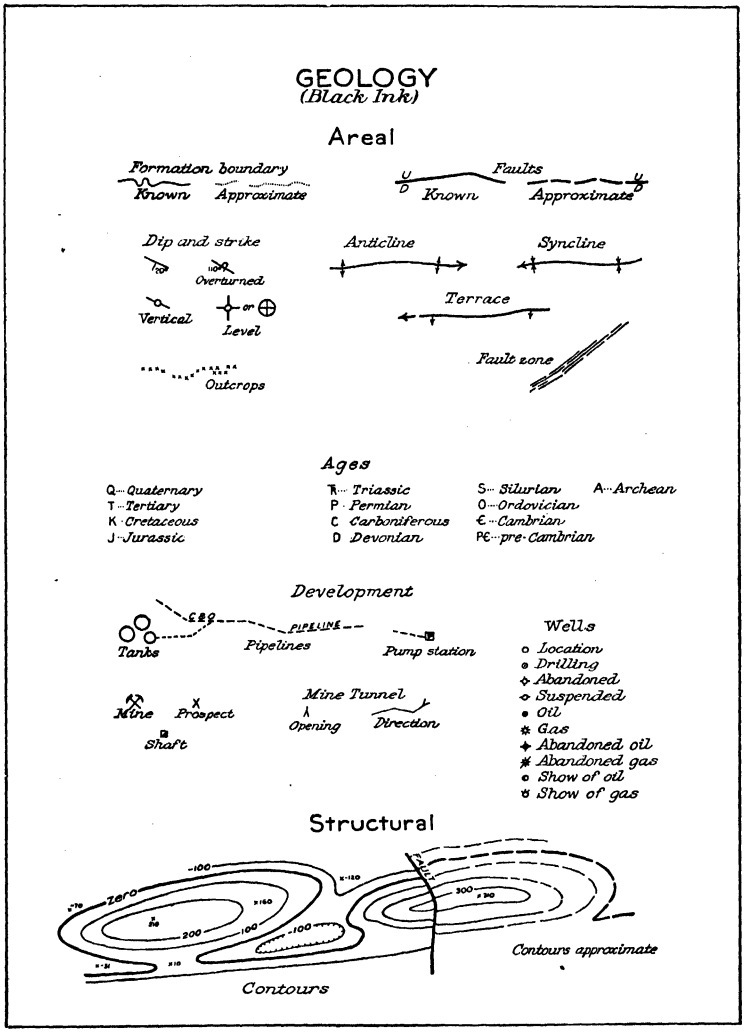


FIG. 140. Standard symbols shown in black or red ink.

The legend should illustrate and explain every symbol used on the map. If colored lines or areas are used they should be represented in the legend exactly as they appear on the map, for even minor variations in this respect are likely to be confusing to those not already familiar with the subject matter.

Although it is necessary on many maps to include a complete legend, notes on the map itself at the proper locations are even better, provided that lettering does not obscure important details. Marginal notes are often overlooked, but the map user cannot fail to see a label on or adjacent to the feature to which it applies. All explanatory notes should be in the same kind of lettering so that they will stand apart from near-by place names.

Whenever practicable, notes, explanations, and legends should be grouped in one part of the margin, preferably near the title. A map whose explanatory matter is scattered around the four sides is inconvenient to use.

241. Titles

The principal title of a map should indicate the *entire* subject matter. Thus, a map showing topographic contours and areal geology should be titled "Geologic and Topographic Map," not just "Geologic Map." One seeking a topographic map of the area would not be likely to suspect that the map bearing the simpler title would also show the topography. A map embracing all of Wayne County and a part of Jackson County should reveal in the title that a portion of Jackson County is included. The potential user of the map might be interested in only the portion in Jackson County.

Subtitles, shown in smaller letters than the principal title, should give additional information about the area, subject, or the survey. Within the title layout other important and comprehensive information may be included, for example, the contour interval, scale, dates of the survey, always the names of the authors, and similar data which apply to the map as a whole.

Although nicely centered titles add to the appearance of the

map, it is better to place them in one of the four corners. The common practice is to put the title in the lower right-hand corner. The reason is that when the map is folded the title will be on the outside of the folded sheet.

242. Lettering

The mechanical methods of lettering, mentioned in Art. 230, are used far more extensively than freehand lettering. However, it is generally conceded that hand lettering done by experts is superior to that done by means of any of the mechanical devices. Despite the fact that most draftsmen have neither the time nor the skill to equal the performance of these lettering guides, it is to their advantage to practice the freehand method sufficiently to do a professional-like job, for lettering sets may not always be available. All lettering devices are limited to letters of constant line weight, i.e., without shading; and none has provisions for making *serifs*, the cross lines at the tops and bottoms of the letters. Letters drawn freehand can be made to follow irregular curved linear features somewhat better than can the mechanical ones. The weights of freehand lines can be made only a fraction of the lightest possible line in the sets of letter guides. In short, there are many reasons that the map draftsman should learn and practice lettering.

243. Characteristics of Letters

If the letter *S*, for example, is drawn so that the upper half is exactly the same size as the lower, the upper half will appear to be much larger. Similarly, if a capital *H*, especially in a shaded style, is divided exactly in half by the cross line, it will look top-heavy. This can be verified by selecting these capital letters from any printed matter, examining them for symmetry, then viewing the same letters with the printing inverted. The top portions will now appear far too massive. The illusion holds for certain other letters, such as *B*, which have a "middle" line. This apparent mid-line is called the visual center of gravity.

Shaded (Roman) letters with rounded tops or bottoms, like *O*

and *C*, and those with angle tops or bottoms, like *A* and *V*, must be slightly taller than letters such as *E* or *L* in order to appear to be the same size. This is due to the relative masses. The top of an *O* is a thin line, and the shaded sides become gradually thinner toward the top and bottom. Likewise, the lower right side of the letter *N* and the bottom of the *V* have less mass than the bottom of the *I* or *F;* therefore these points extend slightly below the lower guide line.

Refer to the lettering chart in Fig. 141. Group (1) is the alphabet and numerals in upper case (capitals) modified Gothic. Group (2) shows a few letters and the numerals in Gothic slant, or Italic. Note that the axes of these letters are parallel, i.e., they are inclined the same amount. The inclined axes of the *A* and *V* cut the vertices and a point on the guide line midway between the "feet" of the letter. In order to visualize the forms of slanting letters, imagine the normal vertical *O* in a right-angled box. Now in group (2) the box is pressed to the right to form a rhomb, and the *O* still touches the sides of the distorted box in the same manner as the vertical *O* in the rectangular box. Slanting, or italic, letters always bear this relationship to the vertical letters of the same alphabet.

Group (3) shows the effect of compressing or extending letters of the same alphabet. It should be observed that letters of the same height look much larger in the extended form and smaller in the compressed. Much use is made of this illusion in the lettering of maps where uniform appearance is desired but the available space necessitates a variation in actual heights. It is in adjustments such as this that freehand lettering is better than the fixed forms of mechanical lettering sets.

Group (5) shows the direction the pen should travel and the order in which lines should be drawn in making the letters. Only key letters are shown, but similar lines in all letters are drawn in the same manner.

Figure 142, group (1) is the lower case Gothic alphabet corresponding to the upper case letters discussed above.

FIG. 141. Lettering chart showing upper-case letters.

Group (4) of Fig. 141 shows the upper case letters of a modified Roman alphabet commonly used on maps because of their high degree of legibility. As mentioned previously, certain letters must extend above and below the guide lines.

Group (6) is the related italic form, and group (2) of Fig. 142 shows corresponding lower case forms.

Group (7) of Fig. 141 and (3) of 142 are an embellished form of the Gothic hair-line letter. This alphabet has serifs, while the true Gothic does not.

The forms of different letters in any given alphabet are closely related to each other. The basic structure must be maintained in order to produce a harmonious effect when they are combined into words. New alphabets are composed from certain "key" letters. These "key" letters are *O, E, R, N,* and *V*. In group (4) of Fig. 142, thirteen letters are constructed on the basic form of the letter *O*. If the *O* were extended to a flattened ellipse, then the basic form of all these other letters would likewise be a flattened ellipse. The *f* and *j* are the same basic figure. Whatever is done to embellish the *V* of an alphabet must be done also to the *Y, W,* and *X;* or, if the shape of the lower case *n* is altered, the *m* and *h* must be changed to correspond. A little study of the letters of any standard alphabet will reveal many interrelationships of form or decoration.

As in the basic form of the letter, the serif, or base line, has a definite form, and this, in turn, has much to do with the legibility, neatness, and beauty of the letter. The simplest form of serif is a straight line of constant thickness. This serif should be used with alphabets whose letters consist of simple, even-weight lines, such as group (7) of the figure cited. Shaped and block serifs are used on shaded and block letters, respectively.

In group (2) of Fig. 142, four kinds of serifs are shown on the bases of boldface letters. Only the first and fourth are good forms. These are based on circle arcs tangent to both the staff of the letter and the base line. The second and third forms are sometimes used, but they add little to the character of the letter.

Correct spacing of letters in a word can be taught only by exam-

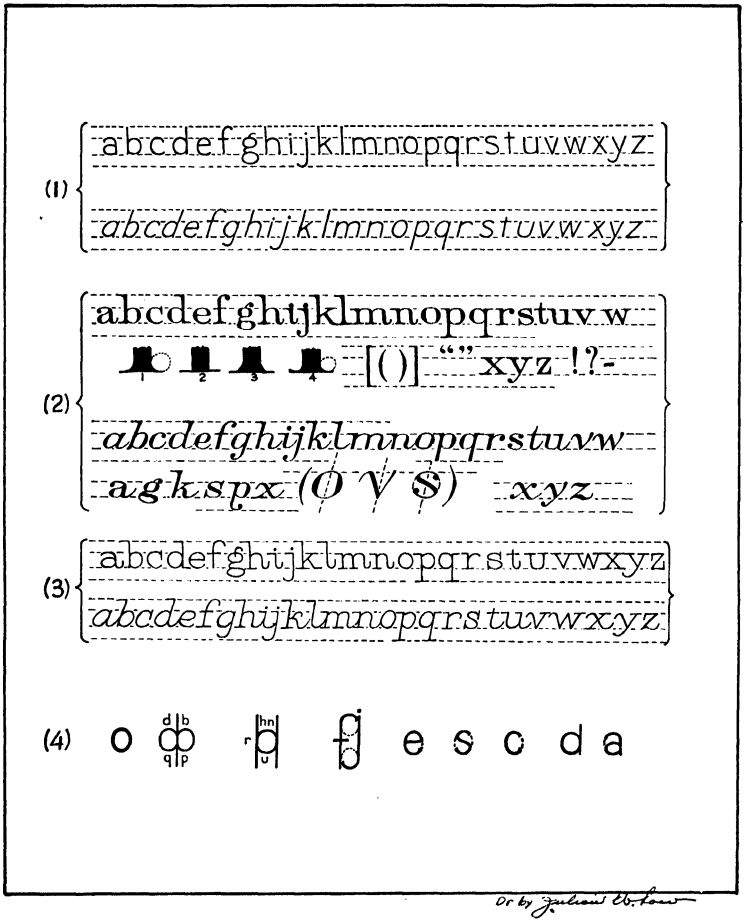


FIG. 142. Lettering chart showing lower-case letters.

ple, for there is no method of measurement that can be used, except in the case of very heavy block letters.

Referring again to group (4) of Fig. 141, it will be seen that the largest letters in the figure make three distinct groups: OIE, AV, and *B*. The spacing between *A* and *V* is wider than between *V* and *B*, but *A* and *V* appear to be more closely spaced because the heavy masses of the shaded sides are adjacent. In order to make OIE appear evenly spaced it is necessary to join the serifs of *I* and *E*. In group (7) the distance between the serifs of *A* and *B* is the same as that between *H* and *I* of the same line, but the letter *A* stands apart because the right side slopes away from the *B*.

From the preceding examples it is clear that the spacing of letters in a word must be judged solely on the visual distribution of the masses. It is a trial-and-error method, which can be perfected only through practice and experience.

244. Lettering of Maps

The standards and practices of map lettering vary greatly, not only among the different organizations, but for different classes of maps within one organization. For this reason, it is not practicable to attempt more than a general guide in the following discussion, which will point out generally accepted practices.

A map must be considered primarily as a functional drawing, not a work of art. The lettering (and other freehand drawing) can do much toward achieving both; but the functional worth should never be sacrificed for the sake of artistic effect. The lettering on a map serves the purpose of labeling and explaining map features; therefore, it should be done in a manner which most effectively accomplishes these purposes.

Lettering should always be right-side up. Names on vertical lines read from the bottom up, as do those extending in a northeasterly direction; those on northwesterly lines read from the top down. This rule applies to all map features: streams, railroads, boundaries, etc.

Although different sizes of letters may be used, only one style

should be used for a given subject. For example, use Roman italic letters for hydrographic features, vertical Gothic for land, Roman for towns and cities, Gothic italic for man-made features. When the style of letter is associated with a specific category of features, there need be no confusion in names where two dissimilar features lie close together on the map.

Study the map carefully and plan the lettering so that it will be as uniformly distributed as possible. Adjust the relative sizes of letters according to the relative importance or sizes of the features,

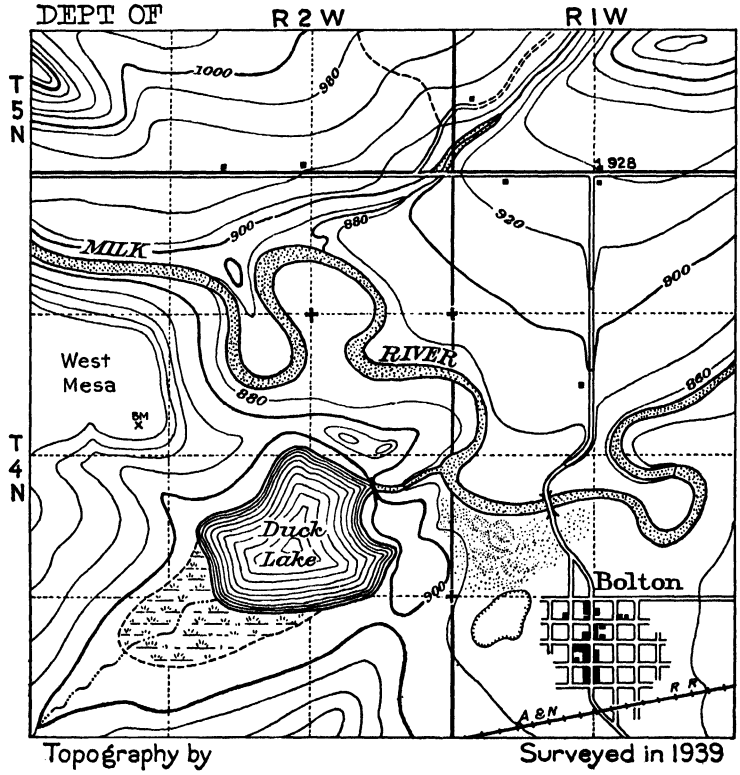


FIG. 143. Portion of a topographic map showing the uses of different alphabets and standard symbols.

that is, letter the names of cities in large Roman upper case, large towns in small upper case, and villages in lower case.

Long linear features, such as highways, need widely spaced extended letters so that some of the letters of the name will be seen no matter what portion of the line is caught by the eye.

Figure 143 is a portion of a topographic map illustrating the uses of a number of standard symbols and lettering styles. Observe that the water features are labeled in slant letters, and land features, such as "West Mesa" in vertical thin line Gothic.

245. Plotting Locations

Several methods are used for the accurate plotting of locations on a map from measurements made in the field. The map draftsman should be familiar with a number of procedures for the reason that some are quite precise, while others are faster though less accurate. He should be able to choose a method which best meets his needs.

246. Protractor and Scale

This method is moderately accurate with a simple protractor, and considerably more so with a vernier type. In order to use the protractor method it is necessary for one point to be located on the map by some other means and then to know the bearing and distance to the next point. A north line is drawn through the first point, the bearing is laid off toward the second, and the distance between the two is plotted along the line. When several points in a series such as transit traverse are plotted by bearings and distances, each new bearing may be laid off as just described by drawing a north line through each new point plotted, or the last bearing plotted may be used as a reference line. The latter method is illustrated in Fig. 144.

The bearing from station 1 to station 2 is N 40° E. This is laid off from the north line through station 1, and station 2 is plotted according to the distance. The bearing line is made longer than necessary for the plot. The bearing from station 2 to station 3 is N 48° E. The difference is plus 8°; so that angle is laid off from

the first bearing in a clockwise direction, and station 3 is plotted. From station 3 to station 4 the bearing is N 38° E. The difference between this and the last bearing is minus 10°. This angle is laid off in a counterclockwise direction from the last bearing plotted.

This method is subject to errors which may be cumulative, because each new bearing is dependent on all preceding ones.

When a drafting machine is used the attached scale is laid on a north-south line, and the vernier is set at zero. Henceforth, the scale is turned until the vernier reads the same as the bearing. Check the instrument occasionally by resetting the vernier on zero and laying the scale along the north line.

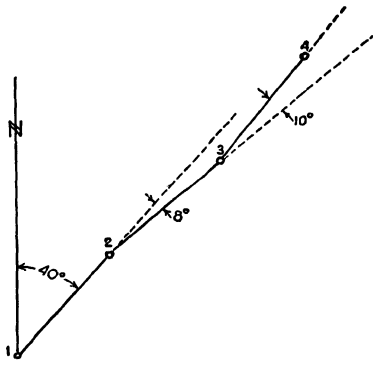


FIG. 144. Plotting locations with scale and protractor.

247. Locations by Intersecting Arcs

When the distances from two located points to the unplotted point are known, the new point is quickly located by the intersection of two circle arcs. A drafting, or beam, compass is set on the distance from one of the plotted points to the unknown location, and with the center on the plotted station, an arc is drawn in the vicinity of the new location. When this operation is repeated for the second plotted point, this arc intersects the first one at the location of the new point. This method is also used for transferring points from one map to another.

248. Tangent Offset Method

Precise bearings may be laid off by means of a triangle, engineers' scale, straightedge, and a table of natural tangents. It is

fundamentally the same as the protractor method. The procedure is as follows.

In Fig. 145A, the bearing north 30 degrees east is to be laid off from point 1 toward point 2. Along the north line from point 1, measure a distance of 10 to 100 units on any convenient scale (in this case, 10 inches). Erect a perpendicular, *ab*, at this point. In the tables of natural tangents the value for 30 degrees is .577.

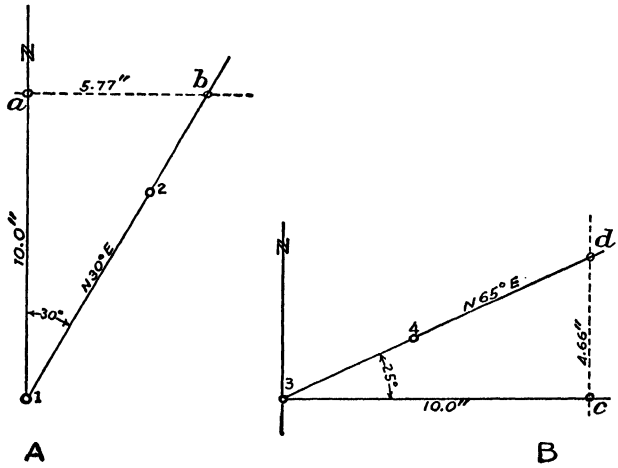


FIG. 145. Plotting locations by the tangent offset method.

Since 10 units is used as the base, the tangent factor is multiplied by 10, and this distance (5.77 inches) is plotted at *b*. Now the line 1*b* is drawn. This is the bearing north 30 degrees east; so station 2 can be plotted according to its distance from station 1. If the base is 100 scale units, multiply the tangent factor by 100, instead of 10.

When the bearing exceeds 45 degrees from the north point, the base is laid out on an east-west line, as shown in Fig. 145B. In this case the bearing from station 3 to station 4 is north 65 degrees east. The value .466 is the cotangent of 65 degrees, or the tangent of 25 degrees, the supplementary of the angle 65 degrees.

The tangent method is not only more precise than the simple protractor, but is a good method to employ on large-scale work where the distance to be plotted exceeds the length of the protractor.

249. Rectangular Coordinates

This method consists in first expressing the geographic position of a point in terms of its distance north (or south) and east (or west) of the plotted position of the station to which the distance and bearing are referred, and then plotting the point according to these distances. It is customary to call the north and south components *latitudes,* and the east and west ones, *departures.*

In Fig. 146 long north-south and east-west coordinates are drawn for reference lines, or cross-section paper may be used. Sta-

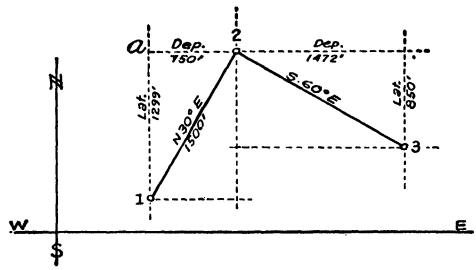


FIG. 146. Plotting locations by latitudes and departures.

tion 1 is plotted, and it is required to plot station 2, on a bearing of north 30 degrees east and 1500 feet distant from station 1. The latitude, or north component is 1500 cos 30 degrees = 1299 feet. This distance is plotted at *a* along a north line from station 1. The departure, or east component, is 1500 sin 30 degrees = 750 feet. This distance is plotted due east of the point *a,* and thus establishes the map position of station 2.

Station 3 is located 1700 feet south 60 degrees east of station 2. The latitude is 1700 cos 60 degrees = 850 feet; the departure is 1700 sin 60 degrees = 1472 feet. Using these values station 3 is located as shown in the figure.

According to the examples given it is clear that the following relationships are constant:

Latitude = Distance × Cosine of the bearing
Departure = Distance × Sine of the bearing

250. Plotting a Polyconic Projection

The limited space in this book does not permit a detailed discussion of the methods for drawing polyconic projections. The reader is referred to Bulletin 809 of the U.S. Geological Survey and Special Publication 5 of the U.S. Coast and Geodetic Survey for complete details on the subject. These publications contain the tables necessary for the construction of the projection. The tables are also available in Bulletin 650 of the U.S. Geological Survey.

The accuracy of a polyconic projection depends largely on the care with which the coordinate axes are drawn. It is recommended that these axes be constructed in the following manner.

Refer to Fig. 147. With a long steel straightedge and a sharp

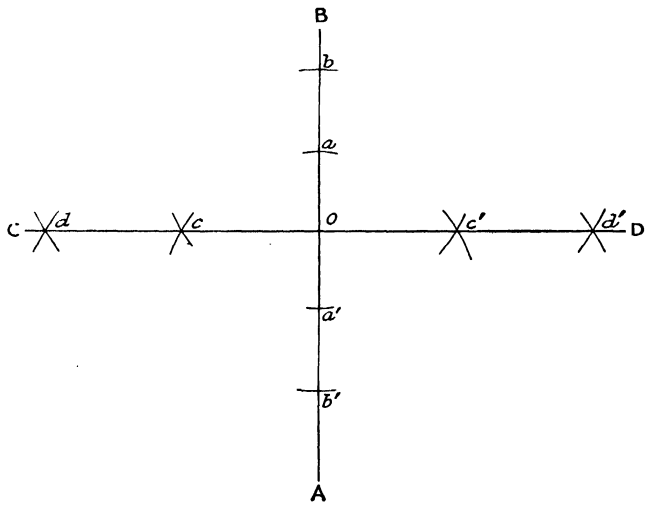


FIG. 147. Erecting a perpendicular in the construction of a polyconic projection.

chisel-pointed pencil draw line AB down the middle of the sheet. Select a point o at about the middle of this line, and using this point as center strike circle arcs a, b, a', and b'. Now, using these intersections as centers, strike arcs c, d, c', and d'. Draw the horizontal line CD through this set of intersections. When a perpendicular is constructed in this way there is little chance for error. If the perpendicular does not cut all the intersections exactly, erase the construction and repeat the process.

251. Character of the Polyconic Projection

The plotted meridians of the polyconic projections converge from south to north (in north latitudes). The rate of convergence depends on the latitude of the area, being almost negligible at the equator and very high in Arctic regions. Strictly speaking the meridians are curved lines, arcing outward from the central meridian of the projection; but in quadrangles of moderate geographic scope they are drawn as straight lines.

The parallels of latitude (in north latitudes) curve upward to the north toward the edges of the sheet. In practice, they are drawn as a series of short straight lines approximating a curved line. In maps of moderate geographic scope the parallels are drawn as straight lines normal to the central meridian of the projection.

While it is necessary to exercise reasonable care in the mechanics of constructing a polyconic projection, the principles and processes are neither complex nor difficult. Once the steps in the procedure are learned, the construction becomes quite simple. The publications cited earlier give detailed instructions and examples for plotting the projections on different scales. One example of a simple and widely used projection is given here.

252. Method of Construction

In the example illustrated the specifications of the quadrangle are:

Area: Latitude 33° 52′ 30″ North to 34° 00′ 00″ North
Longitude 99° 45′ 00″ West to 99° 52′ 30″ West

Coordinate intervals between meridians and parallels:
0° 2′ 30″
Scale: 1:24,000

Use U.S. Geological Survey "Tables for the Construction of Polyconic Projections," Table 4.

First, draw the coordinate axes shown in Fig. 147 by the method outlined in Art. 250. The following steps in the actual construction are illustrated in Fig. 148.

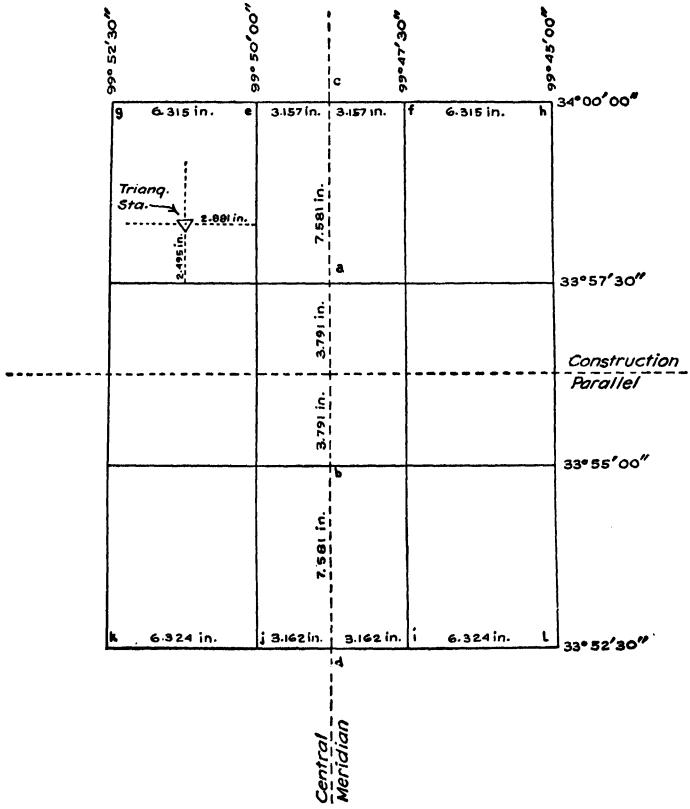


FIG. 148. Laying out a polyconic projection.

In the tables, under the heading "Meridional Distance," latitude 34°, line 2½ minutes, find the value 7.581 inches. This is the spacing of the parallels along the central meridian. Plot one-half this value, 3.791 inches, on each side of the central construction parallel, at *a* and *b*. Now scale 7.581 inches on either side of *a* and *b* and *c* and *d*. The geographic parallels of the projection will cross the central meridian at these points. For a projection of this scope the parallels may be drawn as straight lines, for the ordinate offset (curvature) is only 0.001 inch for 2½ minutes of latitude. Draw horizontal lines, the geographic parallels, through points *a*, *b*, *c*, and *d*.

In the tables under the heading "Longitude Interval," column 2½ minutes, line 34° latitude, find the value 6.315 inches. This is the spacing of 2½-minute meridians along the 34-degree parallel, at the north side of the projection. Plot one-half this distance, 3.157 inches, on each side of the central meridian at points *e* and *f*. Now plot points *g* and *h* 6.315 inches along the parallel outside the points *e* and *f*.

In the table at latitude 33° 52½′ find the value 6.324 inches. Plot one-half this distance on each side of the central meridian along the south parallel of the projection at points *i* and *j*, then plot *k* and *l* 6.324 to the east and west. The meridian can now be drawn as indicated in the figure.

If the scale of the map is small and the area covered is very large, the distance between each of the meridians should be determined along every parallel and the meridians plotted accordingly. Likewise the ordinates of the parallels must be gotten from the tables and plotted so that the approximate curvature of the parallels is developed in the construction.

253. Plotting Control

When the geographic coordinates of control points, such as triangulation stations, are given, these points are plotted on the sheet according to their longitude and latitude positions. This is done in much the same manner in which points are located for controlling

the construction of the polyconic projection itself. It consists in measuring the correct distance east or west, as the case might be, from a given meridian, and north or south of the designated parallel. Scales may be purchased which are graduated in degrees and minutes of longitude and latitude on the basis of most standard map scales. Lacking such a scale, the position of the point may be reduced to footage equivalents, which, in turn, are reduced to the scale of the map in inches and then plotted by coordinates with the proper map scale.

254. Coloring of Maps

Many kinds of maps must be hand-colored in the drafting room, and it is, therefore, essential for the draftsman to know different methods and mediums and the results that may be expected from each. The following articles outline some of the more common methods.

255. Wax-Base Crayons or Pencils

An even, moderately clear color can be obtained on paper maps with wax-base colored crayons or pencils. The method is fairly rapid, but the range of color values is quite limited. They are applied as follows.

The materials needed are a set of the colored pencils, a few sizes from 3 to 8 of *paper* charcoal drawing blenders, or stumps, paper facial cleansing tissues, or clean blotting paper, and a small bottle of white gasoline, benzene, or mineral spirits.

Cover only those areas of the same color at each step. Use only a small amount of brilliant or dark colors, and spread the color as evenly as possible. Now dip a charcoal stump in the gasoline (or other solvent), and, with a circular motion, lightly rub the colored areas until the color tone is even. Keep the stump moist, but not wet, by recharging it with solvent. Work first around the borders of the colored area. If the area is large, use gasoline-moistened blotting paper or cleansing tissue folded into a pad.

After the colors have been treated with the solvent, they become

somewhat waterproof. They are not easily erased or rubbed off, and pen and ink can be used on the colored surface without difficulty.

256. Water-Soluble Indelible Pencils

The colors in these pencils are soluble in water or alcohol, but relatively insoluble in gasoline and similar solvents. They are applied in the same manner as wax-base colors, but are smoothed with dry charcoal stumps or blotting paper. These pencils are easier and faster to use than the wax base, but they tend to rub off on the hands or clothing and turn to brilliant hues on contact with water. Because of the fact that they can be erased with a soft gum eraser, they are convenient for coloring work sheets which are subject to alteration during the course of the work. The colors are moderately opaque and, consequently, tend to obscure lettering and other map details.

Very brilliant and transparent colors are obtained by applying a water wash with a large sable artists' brush after the dry color has been spread with a stump. This method, except in special cases, has two disadvantages: (1) In order to control the wash, one must be trained in the techniques used by artists; and (2) wetting the map is likely to cause serious wrinkling and consequent scale distortion.

257. The Air Brush

The air brush is standard equipment of commercial artists and is sometimes used for coloring maps. It is a spray, or atomizer, operated by compressed air. The tips, or nozzles, are interchangeable and adjustable so that either very fine lines or broad areas can be colored. Waterbase colors similar to poster inks are used in them. Considerable practice with the air brush is necessary before consistently good work can be done; however, the air brush is capable of producing very subtle gradations of tone when properly handled. The colors used in the air brush must be perfectly free from lumps or grains. They should be thoroughly mixed and

strained through a cloth, such as muslin, immediately before using in the air brush. Tips must be kept free from dried color, rust, and corrosion.

When using the air brush all the map except the portion to be colored must be masked to prevent the spray from reaching those areas that are to be done in another color. Ordinary heavy wrapping paper may be used for this purpose. It is cut to conform with the boundaries and fastened to the map with drafting tape. Perforated or slotted stencils can be used for pattern effects, such as those used on various types of geologic maps. The stencils are cut from heavy, hard paper or thin acetate. All areas except the one to be colored are masked, and then the stencil is laid over the exposed area, care being taken that it is everywhere in close contact with the map so that the color spray will not creep under the stencil.

In using the air brush, first turn the spray on a blank paper to the side while it is being adjusted. Hang the map vertically against a wall, and stand several feet distant, depending on the size of the tip. Use a back and forth motion, starting at the top and gradually working downward. Keep the spray moving constantly, never holding it in one place long enough for the liquid to collect in droplets.

258. Printers' Inks

When correctly employed, colored printers' inks provide a very satisfactory method for hand-coloring maps. The process is as follows. (1) Thoroughly mix the thick printers' ink with mineral spirits until it has the consistency of thin syrup (about 3 parts mineral spirits to 1 part of ink).

(2) Apply this to the map with an artists' red sable round brush (sizes 4 to 10). There is *no* need for laying the color on smoothly, but be sure it covers all the surface and is spread exactly to the boundaries.

(3) Carefully lay a sheet of facial cleansing tissue flat on the moist surface and pat it down until it absorbs the "free" ink. Do

not use the same tissue on another place, unless it lies wholly within the area where the color has been spread.

(4) Make a pad, or wad, of clean tissue and wipe or rub all excess color from the surface of the map. Wipe from the border toward the center of the colored area, and use a clean portion of the tissue each time. The color will have a perfectly even tone, with no overlaps, no matter how large the area. The color penetrates deeply into absorbent paper, such as that in ozalid prints or drawing paper; so it should be wiped until there is no stickiness.

(5) Apply the darkest colors first, working up the scale toward the lightest shades. When a dark color has been applied and is given 5 to 10 minutes to "dry," the paper becomes repellent to further applications. Therefore, boundaries need be traced with only one color, the adjacent lighter one being brushed to, and over, the boundary of the darker color. When the lighter shade is wiped, all overlaps will disappear.

The viscous inks may be mixed (before adding mineral spirits) to obtain an infinite range of colors. Obviously, they cannot be combined by separate applications on the map for the reasons given in (5).

This method is generally the most satisfactory of all hand processes. It is positively waterproof and does not alter the scale of the map, therefore the map can be wet-mounted to cloth without danger of disturbing the colors. Ink, pencil, or colored pencil can be used on the colored surface. There is only one important disadvantage: since the color penetrates so deeply, it cannot be erased, washed out, or otherwise removed.

259. Maps for Reproduction

In the great majority of cases, the maps produced in the drafting room must be reproduced by one means or another. The original map may be perfectly drawn, but not suitable for reproduction. The draftsman should determine beforehand if the map is to be reproduced, by what method, and then do the drafting accord-

ingly. Otherwise, much patient work on the original drawing may be lost in the reproduction.

260. Photo-Copy Reproductions

This term is used here in a broad sense to include all processes of reproduction in which the copies are made on sensitized paper that is exposed to light and then "developed."

It is essential that all lines be quite opaque. When inking is done on acetate or other hard-surface materials, the ink may be spread so thin that light will penetrate it during exposure in the printing machine. Such lines will "burn out." View the tracing toward a strong light from the back, and mark the lines which appear to transmit light so that they may be reinked.

Soft pencil smutches or other dark spots should be removed, for they may be much stronger in the prints.

Extremely thin lines can be printed from acetate originals, but may disappear in prints made from vellum or linen originals.

Maps made for photo reproduction should not be colored. Even though the colors appear to be of an even tone on the tracing, they will generally be splotchy in the black and white reproductions. Only black patterns should be used to differentiate areas. Lines in colored inks, likewise, are likely to be ragged in the prints.

It should be ascertained before the map is started that its outside dimensions do not exceed those of the printing machines available.

261. Plate Reproductions

As used here, these processes include lithograph, metal lithograph, zinc offset, halftone, and related methods.

When maps are made to be reproduced by these methods, it is better if the original is drawn to a scale considerably larger than that required in the reproduction. The reason for this is that the irregularities of lines and letters are reduced in the same ratio as the reduction in scale.

Assuming that a reduction to one-half the original scale is

planned, the map must be drawn so that all features, symbols, lettering, and so on will be legible and correctly proportioned in the one-half reduction. Here, again, it is important to gage fine lines carefully. If the lines are too fine in the original, they will be lost in the etching process of making the plates. Likewise, very sharp, thin angles or small closed figures may "fill in" either in the plate itself, or in the prints made from the plate.

When a reduction of two or three diameters is to be made, more attention should be given to the overall proportions of figures than to the smoothness of lines with which they are drawn. The lines will become smooth, but the gross proportions will remain relatively unchanged.

For black-and-white reproductions, the original should be inked in black. Red and yellow reproduce in black moderately well, but blue on the original should be avoided. If the map is to be reproduced in, say, three colors (black, blue, and brown), then three color-separation drawings should be made, each showing all lines, symbols, and lettering that will appear in one color.

These three sheets should be made on transparent material, such as acetate, that is not subject to appreciable scale distortion. The sheets must bear matching marks so that they can be registered perfectly. All are drawn in black ink, even though each will be printed in a different color.

Appendix

TABLE 1. Contour Spacing from Slope Angles

Contour Interval Ft. =	1	2	5	10	20	25	50	100
Slope Angle in Degrees	Contour Spacing in Feet on the Ground							
1°	57.3	104.6	286.4	573	1046	1432	2864	5729
2	28.6	56.3	143.2	286	563	716	1432	2864
3	19.1	38.2	95.4	191	382	477	954	1908
4	14.3	28.6	71.5	143	286	357	715	1430
5	11.4	22.9	57.1	114	229	285	571	1143
6	9.5	19.0	47.5	95	190	237	475	951
7	8.1	16.3	40.7	81	163	203	407	814
8	7.1	14.2	35.5	71	142	177	355	711
9	6.3	12.6	31.5	63	126	157	315	631
10	5.6	11.3	28.3	57	113	141	283	567
11	5.1	10.3	25.7	51	103	128	257	514
12	4.7	9.4	23.5	47	94	117	235	470
13	4.3	8.7	21.6	43	87	108	216	433
14	4.0	8.0	20.0	40	80	100	200	401
15	3.7	7.5	18.6	37	75	93	186	373
16	3.5	7.0	17.4	35	70	87	174	349
17	3.3	6.5	16.3	33	66	81	163	327
18	3.1	6.2	15.4	31	62	77	154	308
19	2.9	5.8	14.5	29	58	72	145	290
20	2.7	5.5	13.7	27	55	68	137	270
21	2.6	5.2	13.0	26	52	65	130	260
22	2.5	4.9	12.3	25	49	62	123	247
23	2.3	4.7	11.7	23	47	59	117	235
24	2.2	4.5	11.2	22	45	56	112	225
25	2.1	4.3	10.7	21	43	53	107	214
26	2.0	4.1	10.2	20	41	51	102	204
27	2.0	3.9	9.8	20	39	49	98	196
28	1.9	3.8	9.4	19	37	47	94	188
29	1.8	3.6	9.0	18	36	45	90	180
30	1.7	3.5	8.6	17	35	43	86	173
31	1.7	3.3	8.3	16	33	41	83	166
32	1.6	3.2	8.0	16	32	40	80	160
33	1.5	3.1	7.7	15	31	38	77	154
34	1.5	3.0	7.4	15	30	37	74	148
35	1.4	2.9	7.1	14	29	35	71	143
36	1.4	2.8	6.9	14	28	34	69	138
37	1.3	2.7	6.6	13	27	33	66	133
38	1.3	2.6	6.4	13	26	32	64	128
39	1.2	2.5	6.1	12	25	31	61	123
40	1.2	2.4	5.9	12	24	30	59	119
41	1.1	2.3	5.7	11	23	29	57	115
42	1.1	2.2	5.5	11	22	28	55	111
43	1.1	2.1	5.3	11	21	27	53	107
44	1.0	2.0	5.1	10	20	26	51	103
45	1.0	2.0	5.0	10	20	25	50	100

EXAMPLE 1. Find the contour spacing for slope or dip of 19 degrees, contour interval 25 feet. In left-hand column find line 19; on top line column 25. At intersection is contour spacing 72 feet. *EXAMPLE 2.* Find slope or dip angle with contour interval of 50 feet and scaled spacing of 135 feet.

In the table, 135 falls between 130 and 137; in the degrees column, between 20° and 21°. Interpolating between 130 and 137, spacing of 135 feet shows a slope angle of 20¼ degrees.

TABLE 2. Conversion from Per Cent Grade to Vertical Angle

Per Cent Grade	Vertical Angle	Per Cent Grade	Vertical Angle	Vert. Angle	Per Cent Grade
1%	00°35′	51%	27°01′	1°	1.7%
2	01 09	52	27 28	2	3.5
3	01 44	53	27 56	3	5.2
4	02 18	54	28 22	4	7.0
5	02 52	55	28 49	5	8.7
6	03 26	56	29 15	6	10.5
7	04 00	57	29 41	7	12.3
8	04 35	58	30 07	8	14.0
9	05 09	59	30 33	9	15.8
10	05 43	60	30 58	10	17.6
11	06 17	61	31 23	11	19.4
12	06 51	62	31 48	12	21.2
13	07 24	63	32 13	13	23.1
14	07 58	64	32 37	14	24.9
15	08 32	65	33 02	15	26.8
16	09 05	66	33 26	16	28.7
17	09 39	67	33 49	17	30.6
18	10 12	68	34 13	18	32.5
19	10 45	69	34 36	19	34.4
20	11 19	70	34 59	20	36.4
21	11 51	71	35 22	21	38.4
22	12 24	72	35 45	22	40.4
23	12 57	73	36 08	23	42.4
24	13 30	74	36 30	24	44.5
25	14 02	75	36 52	25	46.6
26	14 34	76	37 14	26	48.8
27	15 06	77	37 36	27	50.9
28	15 39	78	37 57	28	53.2
29	16 10	79	38 18	29	55.4
30	16 42	80	38 39	30	57.7
31	17 13	81	39 00	31	60.1
32	17 44	82	39 21	32	62.5
33	18 16	83	39 41	33	64.9
34	18 46	84	40 01	34	67.4
35	19 17	85	40 22	35	70.0
36	19 48	86	40 42	36	72.6
37	20 18	87	41 02	37	75.3
38	20 48	88	41 21	38	78.1
39	21 18	89	41 40	39	81.0
40	21 48	90	41 59	40	83.9
41	22 18	91	42 19	41	86.9
42	22 47	92	42 37	42	90.0
43	23 16	93	42 55	43	93.2
44	23 45	94	43 14	44	96.6
45	24 13	95	43 32	45	100.0
46	24 42	96	43 50	46	103.5
47	25 11	97	44 07	47	107.2
48	25 39	98	44 25	48	111.1
49	26 06	99	44 43	49	115.5
50%	26°34′	100%	45°00′	50°	119.2%

Conversion from per cent grade to vertical angles in degrees and minutes of arc.

Conversion from degrees to per cent of grade.

TABLE 3. Corrections for Curvature and Refraction

Correction in feet $= 0.0206 \left(\frac{d}{1000}\right)^2$ (When d is in feet)

Dist. in Feet	Cor. in Feet	Dist.	Cor.	Dist.	Cor.
2,000	00.1	18,000	06.6	34,000	23.7
3,000	00.2	19,000	07.4	35,000	25.2
4,000	00.3	20,000	08.2	36,000	26.7
5,000	00.4	21,000	09.0	37,000	28.2
6,000	00.6	22,000	09.9	38,000	29.7
7,000	00.8	23,000	10.8	39,000	31.3
8,000	01.2	24,000	11.8	40,000	32.9
9,000	01.6	25,000	12.8	41,000	34.6
10,000	02.1	26,000	13.9	42,000	36.3
11,000	02.5	27,000	14.9	43,000	38.1
12,000	03.0	28,000	16.0	44,000	39.9
13,000	03.5	29,000	17.2	45,000	41.7
14,000	04.0	30,000	18.4	46,000	43.6
15,000	04.6	31,000	19.7	47,000	45.5
16,000	05.2	32,000	20.9	48,000	47.5
17,000	05.9	33,000	22.2	49,000	49.5
				50,000	51.5

Correction in feet $= 0.574D^2$ (When D is in miles)

Distance in Miles	Cor. in Feet	Distance in Miles	Cor. in Feet
1	00.6	13	97.0
2	02.3	14	112.5
3	05.2	15	129.1
4	09.2	16	146.9
5	14.4	17	165.8
6	20.6	18	185.9
7	28.1	19	207.2
8	36.7	20	229.5
9	46.4	21	253.1
10	57.4	22	277.7
11	69.4	23	303.6
12	82.7	24	330.5
		25	358.6

AGONIC LINE
FOR YEAR 1935
NO CHANGE
0
500 miles
(AFTER U.S.C. & G.S.)

TABLE 4. The Isogonic Chart

Lines of equal magnetic declination are shown as solid lines. The values of these lines apply without correction only to the year 1935, the date of the chart. East of the AGONIC, or zero, line declinations are west; west of this line declinations are east; along the agonic line the compass needle points to true north (on the date of the chart).

The dashed lines indicate the annual rate of change in declination. Along the double dashed line there is no annual change in the amount of declination. North and east of this line the needle shifts to the west at a yearly rate indicated by the one-minute lines; and south and west of the double line the needle shifts to the east. At points *A* and *B* the needle points to true north year after year.

Example:

Year: 1950 (15 years after date of chart)
Location: At *C* in Wyoming
Indicated declination is 16°00′ East
Rate of annual change is 01′
Corrected declination is
$16°00' - (15 \times 01') = 15°45'$ East

TABLE 5. Natural Sines, Cosines, Tangents, and Cotangents

′	0°		1°		2°		3°		4°		′
	N. sine	N. cos.	N. sine	N. cos.	N.sine	N.cos.	N.sine	N.cos.	N.sine	N.cos.	
0	.00000	1.00000	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	029	000	774	984	519	938	263	861	.07005	754	59
2	058	000	803	984	548	937	292	860	034	752	58
3	087	000	832	983	577	936	321	858	063	750	57
4	116	000	862	983	606	935	350	857	092	748	56
5	145	000	891	982	635	934	379	855	121	746	55
6	175	000	920	982	664	933	408	854	150	744	54
7	204	000	949	981	693	932	437	852	179	742	53
8	233	000	978	980	723	931	466	851	208	740	52
9	262	000	.02007	980	752	930	495	849	237	738	51
10	.00291	1.00000	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	320	.99999	065	979	810	927	553	846	295	734	49
12	349	999	094	978	839	926	582	844	324	731	48
13	378	999	123	977	868	925	611	842	353	729	47
14	407	999	152	977	897	924	640	841	382	727	46
15	436	999	181	976	926	923	669	839	411	725	45
16	465	999	211	976	955	922	698	838	440	723	44
17	495	999	240	975	984	921	727	836	469	721	43
18	524	999	269	974	.04013	919	756	834	498	719	42
19	553	998	298	974	042	918	785	833	527	716	41
20	00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	611	998	356	972	100	916	844	829	585	712	39
22	640	998	385	972	129	915	873	827	614	710	38
23	669	998	414	971	159	913	902	826	643	708	37
24	698	998	443	970	188	912	931	824	672	705	36
25	727	997	472	969	217	911	960	822	701	703	35
26	756	997	501	969	246	910	989	821	730	701	34
27	785	997	530	968	275	909	.06018	819	759	699	33
28	814	997	560	967	304	907	047	817	788	696	32
29	844	996	589	966	333	906	076	815	817	694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	902	996	647	965	391	904	134	812	875	689	29
32	931	996	676	964	420	902	163	810	904	687	28
33	960	995	705	963	449	901	192	808	933	685	27
34	989	995	734	963	478	900	221	806	962	683	26
35	.01018	995	763	962	507	898	250	804	991	680	25
36	047	995	792	961	536	897	279	803	.08020	678	24
37	076	994	821	960	565	896	308	801	049	676	23
38	105	994	850	959	594	894	337	799	078	673	22
39	134	994	879	959	623	893	366	797	107	671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	193	993	938	957	682	890	424	793	165	666	19
42	222	993	967	956	711	889	453	792	194	664	18
43	251	992	996	955	740	888	482	790	223	661	17
44	280	992	.03025	954	769	886	511	788	252	659	16
45	309	991	054	953	798	885	540	786	281	657	15
46	338	991	083	952	827	883	569	784	310	654	14
47	367	991	112	952	856	882	598	782	339	652	13
48	396	990	141	951	885	881	627	780	368	649	12
49	425	990	170	950	914	879	656	778	397	647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	483	989	228	948	972	876	714	774	455	642	9
52	513	989	257	947	.05001	875	743	772	484	639	8
53	542	988	286	946	030	873	773	770	513	637	7
54	571	988	316	945	059	872	802	768	542	635	6
55	600	987	345	944	088	870	831	766	571	632	5
56	629	987	374	943	117	869	860	764	600	630	4
57	658	986	403	942	146	867	889	762	629	627	3
58	687	986	432	941	175	866	918	760	658	625	2
59	716	985	461	940	205	864	947	758	687	622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
	N. cos.	N. sine	N. cos.	N. sine	N.cos.	N.sine	N.cos.	N.sine	N.cos.	N.sine	
′	89°		88°		87°		86°		85°		′

TABLE 5. (*Continued*)

′	0°		1°		2°		3°		4°		′
	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	
0	.0000	Infinite	.0175	57.2900	.0349	28.6363	.0524	19.0811	.0699	14.3007	60
1	03	3437.75	77	56.3506	52	3994	27	18.9755	.0702	2411	59
2	06	1718.87	80	55.4415	55	1664	30	8711	05	1821	58
3	09	1145.92	83	54.5613	58	27.9372	33	7678	08	1235	57
4	12	859.436	86	53.7086	61	7117	36	6656	11	0655	56
5	15	687.549	89	52.8821	64	4899	39	5645	14	0079	55
6	17	572.957	92	0807	67	2715	42	4645	17	13.9507	54
7	20	491.106	95	51.3032	70	0566	44	3655	20	8940	53
8	23	429.718	98	50.5485	73	26.8450	47	2677	23	8378	52
9	26	381.971	.0201	49.8157	75	6367	50	1708	26	7821	51
10	.0029	343.774	.0204	49.1039	.0378	26.4316	.0553	18.0750	.0729	13.7267	50
11	32	312.521	07	48.4121	81	2296	56	17.9802	31	6719	49
12	35	286.478	09	47.7395	84	0307	59	8863	34	6174	48
13	38	264.441	12	0853	87	25.8348	62	7934	37	5634	47
14	41	245.552	15	46.4489	90	6418	65	7015	40	5098	46
15	44	229.182	18	45.8294	93	4517	68	6106	43	4566	45
16	47	214.858	21	2261	96	2644	71	5205	46	4039	44
17	49	202.219	24	44.6386	99	0798	74	4314	49	3515	43
18	52	190.984	27	0661	.0402	24.8978	77	3432	52	2996	42
19	55	180.932	30	43.5081	05	7185	80	2558	55	2480	41
20	.0058	171.885	.0233	42.9641	.0407	24.5418	.0582	17.1693	.0758	13.1969	40
21	61	163.700	36	4335	10	3675	85	0837	61	1461	39
22	64	156.259	39	41.9158	13	1957	88	16.9990	64	0958	38
23	67	149.465	41	4106	16	0263	91	9150	67	0458	37
24	70	143.237	44	40.9174	19	23.8593	94	8319	69	12.9962	36
25	73	137.507	47	4358	22	6945	97	7496	72	9469	35
26	76	132.219	50	39.9655	25	5321	.0600	6681	75	8981	34
27	79	127.321	53	5059	28	3718	03	5874	78	8496	33
28	81	122.774	56	0568	31	2137	06	5075	81	8014	32
29	84	118.540	59	38.6177	34	0577	09	4283	84	7536	31
30	.0087	114.589	.0262	38.1885	.0437	22.9038	.0612	16.3499	.0787	12.7062	30
31	90	110.892	65	37.7686	40	7519	15	2722	90	6591	29
32	93	107.426	68	3579	42	6020	17	1952	93	6124	28
33	96	104.171	71	36.9560	45	4541	20	1190	96	5660	27
34	99	101.107	74	5627	48	3081	23	0435	99	5199	26
35	.0102	98.2179	76	1776	51	1640	26	15.9687	.0802	4742	25
36	05	95.4895	79	35.8006	54	0217	29	8945	05	4288	24
37	08	92.9085	82	4313	57	21.8813	32	8211	08	3838	23
38	11	90.4633	85	0695	60	7426	35	7483	10	3390	22
39	13	88.1436	88	34.7151	63	6056	38	6762	13	2946	21
40	.0116	85.9398	.0291	34.3678	.0466	21.4704	.0641	15.6048	.0816	12.2505	20
41	19	83.8435	94	0273	69	3369	44	5340	19	2067	19
42	22	81.8470	97	33.6935	72	2049	47	4638	22	1632	18
43	25	79.9434	.0300	3662	75	0747	50	3943	25	1201	17
44	28	78.1263	03	0452	77	20.9460	53	3254	28	0772	16
45	31	76.3900	06	32.7303	80	8188	55	2571	31	0346	15
46	34	74.7292	08	4213	83	6932	58	1893	34	11.9923	14
47	37	73.1390	11	1181	86	5691	61	1222	37	9504	13
48	40	71.6151	14	31.8205	89	4465	64	0557	40	9087	12
49	43	70.1533	17	5284	92	3253	67	14.9898	43	8673	11
50	.0145	68.7501	.0320	31.2416	.0495	20.2056	.0670	14.9244	.0846	11.8262	10
51	48	67.4019	23	30.9599	98	0872	73	8596	49	7853	9
52	51	66.1055	26	6833	.0501	19.9702	76	7954	51	7448	8
53	54	64.8580	29	4116	04	8546	79	7317	54	7045	7
54	57	63.6567	32	1446	07	7403	82	6685	57	6645	6
55	60	62.4992	35	29.8823	09	6273	85	6059	60	6248	5
56	63	61.3829	38	6245	12	5156	88	5438	63	5853	4
57	66	60.3058	40	3711	15	4051	90	4823	66	5461	3
58	69	59.2659	43	1220	18	2959	93	4212	69	5072	2
59	72	58.2612	46	28.8771	21	1879	96	3607	72	4685	1
60	.0175	57.2900	.0349	28.6363	.0524	19.0811	.0699	14.3007	.0875	11.4301	0
	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	Cotg	Tang	
	89°		88°		87°		86°		85°		′

TABLE 5. (*Continued*)

′	5°		6°		7°		8°		9°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	**60**
1	745	617	482	449	216	251	946	023	672	764	59
2	774	614	511	446	245	248	975	019	701	760	58
3	803	612	540	443	274	244	.14004	015	730	755	57
4	831	609	569	440	302	240	033	011	758	751	56
5	860	607	597	437	331	237	061	006	787	746	55
6	889	604	626	434	360	233	090	002	816	741	54
7	918	602	655	431	389	230	119	.98998	845	737	53
8	947	599	684	428	418	226	148	994	873	732	52
9	976	596	713	424	447	222	177	990	902	728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	**50**
11	034	591	771	418	504	215	234	982	959	718	49
12	063	588	800	415	533	211	263	978	988	714	48
13	092	586	829	412	562	208	292	973	.16017	709	47
14	121	583	858	409	591	204	320	969	046	704	46
15	150	580	887	406	620	200	349	965	074	700	45
16	179	578	916	402	649	197	378	961	103	695	44
17	208	575	945	399	678	193	407	957	132	690	43
18	237	572	973	396	706	189	436	953	160	686	42
19	266	570	.11002	393	735	186	464	948	189	681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	**40**
21	324	564	060	386	793	178	522	940	246	671	39
22	353	562	089	383	822	175	551	936	275	667	38
23	382	559	118	380	851	171	580	931	304	662	37
24	411	556	147	377	880	167	608	927	333	657	36
25	440	553	176	374	908	163	637	923	361	652	35
26	469	551	205	370	937	160	666	919	390	648	34
27	498	548	234	367	966	156	695	914	419	643	33
28	527	545	263	364	995	152	723	910	447	638	32
29	556	542	291	360	.13024	148	752	906	476	633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	**30**
31	614	537	349	354	081	141	810	897	533	624	29
32	642	534	378	351	110	137	838	893	562	619	28
33	671	531	407	347	139	133	867	889	591	614	27
34	700	528	436	344	168	129	896	884	620	609	26
35	729	526	465	341	197	125	925	880	648	604	25
36	758	523	494	337	226	122	954	876	677	600	24
37	787	520	523	334	254	118	982	871	706	595	23
38	816	517	552	331	283	114	.15011	867	734	590	22
39	845	514	580	327	312	110	040	863	763	585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	**20**
41	903	508	638	320	370	102	097	854	820	575	19
42	932	506	667	317	399	098	126	849	849	570	18
43	961	503	696	314	427	094	155	845	878	565	17
44	990	500	725	310	456	091	184	841	906	561	16
45	.10019	497	754	307	485	087	212	836	935	556	15
46	048	494	783	303	514	083	241	832	964	551	14
47	077	491	812	300	543	079	270	827	992	546	13
48	106	488	840	297	572	075	299	823	.17021	541	12
49	135	485	869	293	600	071	327	818	050	536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	**10**
51	192	479	927	286	658	063	385	809	107	526	9
52	221	476	956	283	687	059	414	805	136	521	8
53	250	473	985	279	716	055	442	800	164	516	7
54	279	470	.12014	276	744	051	471	796	193	511	6
55	308	467	043	272	773	047	500	791	222	506	5
56	337	464	071	269	802	043	529	787	250	501	4
57	366	461	100	265	831	039	557	782	279	496	3
58	395	458	129	262	860	035	586	778	308	491	2
59	424	455	158	258	889	031	615	773	336	486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	**0**
	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	
′	84°		83°		82°		81°		80°		′

TABLE 5. (*Continued*)

′	5°		6°		7°		8°		9°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.0875	11.4301	.1051	9.5144	.1228	8.1443	.1405	7.1154	.1584	6.3138	**60**
1	78	11.3919	54	9.4878	31	248	08	004	87	019	59
2	81	540	57	614	34	054	11	7.0855	90	6.2901	58
3	84	163	60	352	37	8.0860	14	706	93	783	57
4	87	11.2789	63	090	40	667	17	558	96	666	56
5	90	417	66	9.3831	43	476	20	410	99	549	55
6	92	048	69	572	46	285	23	264	.1602	432	54
7	95	11.1681	72	315	49	095	26	117	05	316	53
8	98	316	75	060	51	7.9906	29	6.9972	08	200	52
9	.0901	11.0954	78	9.2806	54	718	32	827	11	085	51
10	.0904	11.0594	.1080	9.2553	.1257	7.9530	.1435	6.9682	.1614	6.1970	**50**
11	07	237	83	302	60	344	38	538	17	856	49
12	10	10.9882	86	052	63	158	41	395	20	742	48
13	13	529	89	9.1803	66	7.8973	44	252	23	628	47
14	16	178	92	555	69	789	47	110	26	515	46
15	19	10.8829	95	309	72	606	50	6.8969	29	402	45
16	22	483	98	065	75	424	53	828	32	290	44
17	25	139	.1101	9.0821	78	243	56	687	35	178	43
18	28	10.7797	04	579	81	062	59	548	38	066	42
19	31	457	07	338	84	7.7882	62	408	41	6.0955	41
20	.0934	10.7119	.1110	9.0098	.1287	7.7704	.1465	6.8269	.1644	6.0844	**40**
21	36	10.6783	13	8.9860	90	525	68	131	47	734	39
22	39	450	16	623	93	348	71	6.7994	50	624	38
23	42	118	19	387	96	171	74	856	53	514	37
24	45	10.5789	22	152	99	7.6996	77	720	55	405	36
25	48	462	25	8.8919	1302	821	80	584	58	296	35
26	51	136	28	686	05	647	83	448	61	188	34
27	54	10.4813	31	455	08	473	86	313	64	080	33
28	57	491	33	225	11	301	89	179	67	5.9972	32
29	60	172	36	8.7996	14	129	92	045	70	865	31
30	.0963	10.3854	.1139	8.7769	.1317	7.5958	.1495	6.6912	.1673	5.9758	**30**
31	66	538	42	542	19	787	97	779	76	651	29
32	69	224	45	317	22	618	1500	646	79	545	28
33	72	10.2913	48	093	25	449	03	514	82	439	27
34	75	602	51	8.6870	28	281	06	383	85	333	26
35	78	294	54	648	31	113	09	252	88	228	25
36	81	10.1988	57	427	34	7.4947	12	122	91	124	24
37	83	683	60	208	37	781	15	6.5992	94	019	23
38	86	381	63	8.5989	40	615	18	863	97	5.8915	22
39	89	080	66	772	43	451	21	734	.1700	811	21
40	.0992	10.0780	.1169	8.5555	.1346	7.4287	.1524	6.5606	.1703	5.8708	**20**
41	95	483	72	340	49	124	27	478	06	605	19
42	98	187	75	126	52	7.3962	30	350	09	502	18
43	.1001	9.9893	78	8.4913	55	800	33	223	12	400	17
44	04	601	81	701	58	639	36	097	15	298	16
45	07	310	84	490	61	479	39	6.4971	18	197	15
46	10	021	87	280	64	319	42	846	21	095	14
47	13	9.8734	89	071	67	160	45	721	24	5.7994	13
48	16	448	92	8.3863	70	002	48	596	27	894	12
49	19	164	95	656	73	7.2844	51	472	30	794	11
50	.1022	9.7882	.1198	8.3450	.1376	7.2687	.1554	6.4348	.1733	5.7694	**10**
51	25	601	.1201	245	79	531	57	225	36	594	9
52	28	322	04	041	82	375	60	103	39	495	8
53	30	044	07	8.2838	85	220	63	6.3980	42	396	7
54	33	9.6768	10	636	88	066	66	859	45	297	6
55	36	493	13	434	91	7.1912	69	737	48	199	5
56	39	220	16	234	94	759	72	617	51	101	4
57	42	9.5949	19	035	97	607	75	496	54	004	3
58	45	679	22	8.1837	99	455	78	376	57	5.6906	2
59	48	411	25	640	.1402	304	81	257	60	809	1
60	.1051	9.5144	.1228	8.1443	.1405	7.1154	.1584	6.3138	.1763	5.6713	**0**
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	84°		83°		82°		81°		80°		′

TABLE 5. (*Continued*)

′	10°		11°		12°		13°		14°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	**60**
1	393	476	109	157	820	809	523	430	220	023	59
2	422	471	138	152	848	803	552	424	249	015	58
3	451	466	167	146	877	797	580	417	277	008	57
4	479	461	195	140	905	791	608	411	305	001	56
5	508	455	224	135	933	784	637	404	333	.96994	55
6	537	450	252	129	962	778	665	398	362	987	54
7	565	445	281	124	990	772	693	391	390	980	53
8	594	440	309	118	.21019	766	722	384	418	973	52
9	623	435	338	112	047	760	750	378	446	966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	**50**
11	680	425	395	101	104	748	807	365	503	952	49
12	708	420	423	096	132	742	835	358	531	945	48
13	737	414	452	090	161	735	863	351	559	937	47
14	766	409	481	084	189	729	892	345	587	930	46
15	794	404	509	079	218	723	920	338	615	923	45
16	823	399	538	073	246	717	948	331	644	916	44
17	852	394	566	067	275	711	977	325	672	909	43
18	880	389	595	061	303	705	.23005	318	700	902	42
19	909	383	623	056	331	698	033	311	728	894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	**40**
21	966	373	680	044	388	686	090	298	784	880	39
22	995	368	709	039	417	680	118	291	813	873	38
23	.18023	362	737	033	445	673	146	284	841	866	37
24	052	357	766	027	474	667	175	278	869	858	36
25	081	352	794	021	502	661	203	271	897	851	35
26	109	347	823	016	530	655	231	264	925	844	34
27	138	341	851	010	559	648	260	257	954	837	33
28	166	336	880	004	587	642	288	251	982	829	32
29	195	331	908	.97998	616	636	316	244	.25010	822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	**30**
31	252	320	965	987	672	623	373	230	066	807	29
32	281	315	994	981	701	617	401	223	094	800	28
33	309	310	.20022	975	729	611	429	217	122	793	27
34	338	304	051	969	758	604	458	210	151	786	26
35	367	299	079	963	786	598	486	203	179	778	25
36	395	294	108	958	814	592	514	196	207	771	24
37	424	288	136	952	843	585	542	189	235	764	23
38	452	283	165	946	871	579	571	182	263	756	22
39	481	277	193	940	899	573	599	176	291	749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	**20**
41	538	267	250	928	956	560	656	162	348	734	19
42	567	261	279	922	985	553	684	155	376	727	18
43	595	256	307	916	.22013	547	712	148	404	719	17
44	624	250	336	910	041	541	740	141	432	712	16
45	652	245	364	905	070	534	769	134	460	705	15
46	681	240	393	899	098	528	797	127	488	697	14
47	710	234	421	893	126	521	825	120	516	690	13
48	738	229	450	887	155	515	853	113	545	682	12
49	767	223	478	881	183	508	882	106	573	675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	**10**
51	824	212	535	869	240	496	938	093	629	660	9
52	852	207	563	863	268	489	966	086	657	653	8
53	881	201	592	857	297	483	995	079	685	645	7
54	910	196	620	851	325	476	.24023	072	713	638	6
55	938	190	649	845	353	470	051	065	741	630	5
56	967	185	677	839	382	463	079	058	769	623	4
57	995	179	706	833	410	457	108	051	798	615	3
58	.19024	174	734	827	438	450	136	044	826	608	2
59	052	168	763	821	467	444	164	037	854	600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	**0**
	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	
′	79°		78°		77°		76°		75°		′

TABLE 5. (*Continued*)

′	10°		11°		12°		13°		14°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.1763	5.6713	.1944	5.1446	.2126	4.7046	.2309	4.3315	.2493	4.0108	**60**
1	66	617	47	366	29	4.6979	12	257	96	058	59
2	69	521	50	286	32	912	15	200	99	009	58
3	72	425	53	207	35	845	18	143	.2503	3.9959	57
4	75	329	56	128	38	779	21	086	06	910	56
5	78	234	59	049	41	712	24	029	09	861	55
6	81	140	62	5.0970	44	646	27	4.2972	12	812	54
7	84	045	65	892	47	580	30	916	15	763	53
8	87	5.5951	68	814	50	514	33	859	18	714	52
9	90	857	71	736	53	448	36	803	21	665	51
10	.1793	5.5764	.1974	5.0658	.2156	4.6382	.2339	4.2747	.2524	3.9617	**50**
11	96	671	77	581	59	317	42	691	27	568	49
12	99	578	80	504	62	252	45	635	30	520	48
13	.1802	485	83	427	65	187	49	580	33	471	47
14	05	393	86	350	68	122	52	524	37	423	46
15	08	301	89	273	71	057	55	468	40	375	45
16	11	209	92	197	74	4.5993	58	413	43	327	44
17	14	118	95	121	77	928	61	358	46	279	43
18	17	026	98	045	80	864	64	303	49	232	42
19	20	5.4936	.2001	4.9969	83	800	67	248	52	184	41
20	.1823	5.4845	.2004	4.9894	.2186	4.5736	.2370	4.2193	.2555	3.9136	**40**
21	26	755	07	819	89	673	73	139	58	089	39
22	29	665	10	744	93	609	76	084	61	042	38
23	32	575	13	669	96	546	79	030	64	3.8995	37
24	35	486	16	594	99	483	82	4.1976	68	947	36
25	38	397	19	520	.2202	420	85	922	71	900	35
26	41	308	22	446	05	357	88	868	74	854	34
27	44	219	25	372	08	294	92	814	77	807	33
28	47	131	28	298	11	232	95	760	80	760	32
29	50	043	31	225	14	169	98	706	83	714	31
30	.1853	5.3955	.2035	4.9152	.2217	4.5107	.2401	4.1653	.2586	3.8667	**30**
31	56	868	38	078	20	045	04	600	89	621	29
32	59	781	41	006	23	4.4983	07	547	92	575	28
33	62	694	44	4.8933	26	922	10	493	95	528	27
34	65	607	47	860	29	860	13	441	99	482	26
35	68	521	50	788	32	799	16	388	2602	436	25
36	71	435	53	716	35	737	19	335	05	391	24
37	74	349	56	644	38	676	22	282	08	345	23
38	77	263	59	573	41	615	25	230	11	299	22
39	80	178	62	501	44	555	28	178	14	254	21
40	.1883	5.3093	.2065	4.8430	.2247	4.4494	.2432	4.1126	.2617	3.8208	**20**
41	87	008	68	359	51	434	35	074	20	163	19
42	90	5.2924	71	288	54	374	38	022	23	118	18
43	93	839	74	218	57	313	41	4.0970	27	073	17
44	96	755	77	147	60	253	44	918	30	028	16
45	99	672	80	077	63	194	47	867	33	3.7983	15
46	.1902	588	83	007	66	134	50	815	36	938	14
47	05	505	86	4.7937	69	075	53	764	39	893	13
48	08	422	89	867	72	015	56	713	42	848	12
49	11	339	92	798	75	4.3956	59	662	45	804	11
50	.1914	5.2257	.2095	4.7729	.2278	4.3897	.2462	4.0611	.2648	3.7760	**10**
51	17	174	98	659	81	838	65	560	51	715	9
52	20	092	.2101	591	84	779	69	509	55	671	8
53	23	011	04	522	87	721	72	459	58	627	7
54	26	5.1929	07	453	90	662	75	408	61	583	6
55	29	848	10	385	93	604	78	358	64	539	5
56	32	767	13	317	96	546	81	308	67	495	4
57	35	686	16	249	99	488	84	257	70	451	3
58	38	606	19	181	.2303	430	87	207	73	408	2
59	41	526	23	114	06	372	90	158	76	364	1
60	.1944	5.1446	.2126	4.7046	.2309	4.3315	.2493	4.0108	.2679	3.7321	**0**
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	79°		78°		77°		76°		75°		′

TABLE 5. (*Continued*)

′	15°		16°		17°		18°		19°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	910	585	592	118	265	622	929	097	584	542	59
2	938	578	620	110	293	613	957	088	612	533	58
3	966	570	648	102	321	605	985	079	639	523	57
4	994	562	676	094	348	596	.31012	070	667	514	56
5	.26022	555	704	086	376	588	040	061	694	504	55
6	050	547	731	078	404	579	068	052	722	495	54
7	079	540	759	070	432	571	095	043	749	485	53
8	107	532	787	062	460	562	123	033	777	476	52
9	135	524	815	054	487	554	151	024	804	466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	191	509	871	037	543	536	206	006	859	447	49
12	219	502	899	029	571	528	233	.94997	887	438	48
13	247	494	927	021	599	519	261	988	914	428	47
14	275	486	955	013	626	511	289	979	942	418	46
15	303	479	983	005	654	502	316	970	969	409	45
16	331	471	.28011	.95997	682	493	344	961	997	399	44
17	359	463	039	989	710	485	372	952	.33024	390	43
18	387	456	067	981	737	476	399	943	051	380	42
19	415	448	095	972	765	467	427	933	079	370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	471	433	150	956	821	450	482	915	134	351	39
22	500	425	178	948	849	441	510	906	161	342	38
23	528	417	206	940	876	433	537	897	189	332	37
24	556	410	234	931	904	424	565	888	216	322	36
25	584	402	262	923	932	415	593	878	244	313	35
26	612	394	290	915	960	407	620	869	271	303	34
27	640	386	318	907	987	398	648	860	298	293	33
28	668	379	346	898	.30015	389	675	851	326	284	32
29	696	371	374	890	043	380	703	842	353	274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	752	355	429	874	098	363	758	823	408	254	29
32	780	347	457	865	126	354	786	814	436	245	28
33	808	340	485	857	154	345	813	805	463	235	27
34	836	332	513	849	182	337	841	795	490	225	26
35	864	324	541	841	209	328	868	786	518	215	25
36	892	316	569	832	237	319	896	777	545	206	24
37	920	308	597	824	265	310	923	768	573	196	23
38	948	301	625	816	292	301	951	758	600	186	22
39	976	293	652	807	320	293	979	749	627	176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	032	277	708	791	376	275	034	730	682	157	19
42	060	269	736	782	403	266	061	721	710	147	18
43	088	261	764	774	431	257	089	712	737	137	17
44	116	253	792	766	459	248	116	702	764	127	16
45	144	246	820	757	486	240	144	693	792	118	15
46	172	238	847	749	514	231	171	684	819	108	14
47	200	230	875	740	542	222	199	674	846	098	13
48	228	222	903	732	570	213	227	665	874	088	12
49	256	214	931	724	597	204	254	656	901	078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	312	198	987	707	653	186	309	637	956	058	9
52	340	190	.29015	698	680	177	337	627	983	049	8
53	368	182	042	690	708	168	364	618	.34011	039	7
54	396	174	070	681	736	159	392	609	038	029	6
55	424	166	098	673	763	150	419	599	065	019	5
56	452	158	126	664	791	142	447	590	093	009	4
57	480	150	154	656	819	133	474	580	120	.93999	3
58	508	142	182	647	846	124	502	571	147	989	2
59	536	134	209	639	874	115	529	561	175	979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	
′	74°		73°		72°		71°		70°		′

TABLE 5. (*Continued*)

′	15°		16°		17°		18°		19°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.2679	3.7321	.2867	3.4874	.3057	3.2709	.3249	3.0777	.3443	2.9042	**60**
1	83	277	71	836	60	675	52	746	47	015	59
2	86	234	74	798	64	641	56	716	50	2.8987	58
3	89	191	77	760	67	607	59	686	53	960	57
4	92	148	80	722	70	573	62	655	56	933	56
5	95	105	83	684	73	539	65	625	60	905	55
6	98	062	86	646	76	506	69	595	63	878	54
7	.2701	019	90	608	80	472	72	565	66	851	53
8	04	3.6976	93	570	83	438	75	535	69	824	52
9	08	933	96	533	86	405	78	505	73	797	51
10	.2711	3.6891	.2899	3.4495	.3089	3.2371	.3281	3.0475	.3476	2.8770	**50**
11	14	848	.2902	458	92	338	85	445	79	743	49
12	17	806	05	420	96	305	88	415	82	716	48
13	20	764	08	383	99	272	91	385	86	689	47
14	23	722	12	346	.3102	238	94	356	89	662	46
15	26	680	15	308	05	205	98	326	92	636	45
16	29	638	18	271	08	172	.3301	296	95	609	44
17	33	596	21	234	11	139	04	267	99	582	43
18	36	554	24	197	15	106	07	237	.3502	556	42
19	39	512	27	160	18	073	10	208	05	529	41
20	.2742	3.6470	.2931	3.4124	.3121	3.2041	.3314	3.0178	.3508	2.8502	**40**
21	45	429	34	087	24	008	17	149	12	476	39
22	48	387	37	050	27	3.1975	20	120	15	449	38
23	51	346	40	014	31	943	23	090	18	423	37
24	54	305	43	3.3977	34	910	27	061	22	397	36
25	58	264	46	941	37	878	30	032	25	370	35
26	61	222	49	904	40	845	33	003	28	344	34
27	64	181	53	868	43	813	36	2.9974	31	318	33
28	67	140	56	832	47	780	39	945	35	291	32
29	70	100	59	796	50	748	43	916	38	265	31
30	.2773	3.6059	.2962	3.3759	.3153	3.1716	.3346	2.9887	.3541	2.8239	**30**
31	76	018	65	723	56	684	49	858	44	213	29
32	80	3.5978	68	687	59	652	52	829	48	187	28
33	83	937	72	652	63	620	56	800	51	161	27
34	86	897	75	616	66	588	59	772	54	135	26
35	89	856	78	580	69	556	62	743	58	109	25
36	92	816	81	544	72	524	65	714	61	083	24
37	95	776	84	509	75	492	69	686	64	057	23
38	98	736	87	473	79	460	72	657	67	032	22
39	.2801	696	91	438	82	429	75	629	71	006	21
40	.2805	3.5656	.2994	3.3402	.3185	3.1397	.3378	2.9600	.3574	2.7980	**20**
41	08	616	97	367	88	366	82	572	77	955	19
42	11	576	.3000	332	91	334	85	544	81	929	18
43	14	536	03	297	95	303	88	515	84	903	17
44	17	497	06	261	98	271	91	487	87	878	16
45	20	457	10	226	.3201	240	95	459	90	852	15
46	23	418	13	191	04	209	98	431	94	827	14
47	27	379	16	156	07	178	.3401	403	97	801	13
48	30	339	19	122	11	146	04	375	.3600	776	12
49	33	300	22	087	14	115	08	347	04	751	11
50	.2836	3.5261	.3026	3.3052	.3217	3.1084	.3411	2.9319	.3607	2.7725	**10**
51	39	222	29	017	20	053	14	291	10	700	9
52	42	183	32	3.2983	23	022	17	263	13	675	8
53	45	144	35	948	27	3.0991	21	235	17	650	7
54	49	105	38	914	30	961	24	208	20	625	6
55	52	067	41	879	33	930	27	180	23	600	5
56	55	028	45	845	36	899	30	152	27	575	4
57	58	3.4989	48	811	40	868	34	125	30	550	3
58	61	951	51	777	43	838	37	097	33	525	2
59	64	912	54	743	46	807	40	070	36	500	1
60	.2867	3.4874	.3057	3.2709	.3249	3.0777	.3443	2.9042	.3640	2.7475	**0**
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	74°		73°		72°		71°		70°		′

TABLE 5. (*Continued*)

′	20°		21°		22°		23°		24°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	**60**
1	229	959	864	348	488	707	100	039	700	343	59
2	257	949	891	337	515	697	127	028	727	331	58
3	284	939	918	327	542	686	153	016	753	319	57
4	311	929	945	316	569	675	180	005	780	307	56
5	339	919	973	306	595	664	207	.91994	806	295	55
6	366	909	.36000	295	622	653	234	982	833	283	54
7	393	899	027	285	649	642	260	971	860	272	53
8	421	889	054	274	676	631	287	959	886	260	52
9	448	879	081	264	703	620	314	948	913	248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	**50**
11	503	859	135	243	757	598	367	925	966	224	49
12	530	849	162	232	784	587	394	914	992	212	48
13	557	839	190	222	811	576	421	902	.41019	200	47
14	584	829	217	211	838	565	448	891	045	188	46
15	612	819	244	201	865	554	474	879	072	176	45
16	639	809	271	190	892	543	501	868	098	164	44
17	666	799	298	180	919	532	528	856	125	152	43
18	694	789	325	169	946	521	555	845	151	140	42
19	721	779	352	159	973	510	581	833	178	128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	**40**
21	775	759	406	137	.38026	488	635	810	231	104	39
22	803	748	434	127	053	477	661	799	257	092	38
23	830	738	461	116	080	466	688	787	284	080	37
24	857	728	488	106	107	455	715	775	310	068	36
25	884	718	515	095	134	444	741	764	337	056	35
26	912	708	542	084	161	432	768	752	363	044	34
27	939	698	569	074	188	421	795	741	390	032	33
28	966	688	596	063	215	410	822	729	416	020	32
29	993	677	623	052	241	399	848	718	443	008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	**30**
31	048	657	677	031	295	377	902	694	496	984	29
32	075	647	704	020	322	366	928	683	522	972	28
33	102	637	731	010	349	355	955	671	549	960	27
34	130	626	758	.92999	376	343	982	660	575	948	26
35	157	616	785	988	403	332	.40008	648	602	936	25
36	184	606	812	978	430	321	035	636	628	924	24
37	211	596	839	967	456	310	062	625	655	911	23
38	239	585	867	956	483	299	088	613	681	899	22
39	266	575	894	945	510	287	115	601	707	887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	**20**
41	320	555	948	924	564	265	168	578	760	863	19
42	347	544	975	913	591	254	195	566	787	851	18
43	375	534	.37002	902	617	243	221	555	813	839	17
44	402	524	029	892	644	231	248	543	840	826	16
45	429	514	056	881	671	220	275	531	866	814	15
46	456	503	083	870	698	209	301	519	892	802	14
47	484	493	110	859	725	198	328	508	919	790	13
48	511	483	137	849	752	186	355	496	945	778	12
49	538	472	164	838	778	175	381	484	972	766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	**10**
51	592	452	218	816	832	152	434	461	.42024	741	9
52	619	441	245	805	859	141	461	449	051	729	8
53	647	431	272	794	886	130	488	437	077	717	7
54	674	420	299	784	912	119	514	425	104	704	6
55	701	410	326	773	939	107	541	414	130	692	5
56	728	400	353	762	966	096	567	402	156	680	4
57	755	389	380	751	993	085	594	390	183	668	3
58	782	379	407	740	.39020	073	621	378	209	655	2
59	810	368	434	729	046	062	647	366	235	643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	**0**
	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	
′	69°		68°		67°		66°		65°		′

TABLE 5. (*Continued*)

′	20°		21°		22°		23°		24°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.3640	2.7475	.3839	2.6051	.4040	2.4751	.4245	2.3559	.4452	2.2460	**60**
1	43	50	42	28	44	30	48	39	56	43	59
2	46	25	45	06	47	09	52	20	59	25	58
3	50	00	49	2.5983	50	2.4689	55	01	63	08	57
4	53	2.7376	52	61	54	68	58	2.3483	66	2.2390	56
5	56	51	55	38	57	48	62	64	70	73	55
6	59	26	59	16	61	27	65	45	73	55	54
7	63	02	62	2.5893	64	06	69	26	77	38	53
8	66	2.7277	65	71	67	2.4586	72	07	80	20	52
9	69	53	69	48	71	66	76	2.3388	84	03	51
10	.3673	2.7228	.3872	2.5826	.4074	2.4545	.4279	2.3369	.4487	2.2286	**50**
11	76	04	75	04	78	25	83	51	91	68	49
12	79	2.7179	79	2.5782	81	04	86	32	94	51	48
13	83	55	82	59	84	2.4484	89	13	98	34	47
14	86	30	85	37	88	64	93	2.3294	.4501	16	46
15	89	06	89	15	91	43	96	76	05	2.2199	45
16	92	2.7082	92	2.5693	95	23	.4300	57	08	82	44
17	96	58	95	71	98	03	03	38	12	65	43
18	99	34	99	49	.4101	2.4383	07	20	15	48	42
19	.3702	09	.3902	27	05	62	10	01	19	30	41
20	.3706	2.6985	.3906	2.5605	.4108	2.4342	.4314	2.3183	.4522	2.2113	**40**
21	09	61	09	2.5583	11	22	17	64	26	2.2096	39
22	12	37	12	61	15	02	20	46	29	79	38
23	16	13	16	39	18	2.4282	24	27	33	62	37
24	19	2.6889	19	17	22	62	27	09	36	45	36
25	22	65	22	2.5495	25	42	31	2.3090	40	28	35
26	26	41	26	73	29	22	34	72	43	11	34
27	29	18	29	52	32	02	38	53	47	2.1994	33
28	32	2.6794	32	30	35	2.4182	41	35	50	77	32
29	36	70	36	08	39	62	45	17	54	60	31
30	.3739	2.6746	.3939	2.5386	.4142	2.4142	.4348	2.2998	.4557	2.1943	**30**
31	42	23	42	65	46	22	52	80	61	26	29
32	45	2.6699	46	43	49	02	55	62	64	09	28
33	49	75	49	22	52	2.4083	59	44	68	2.1892	27
34	52	52	53	00	56	63	62	25	71	76	26
35	55	28	56	2.5279	59	43	65	07	75	59	25
36	59	05	59	57	63	23	69	2.2889	78	42	24
37	62	2.6581	63	36	66	04	72	71	82	25	23
38	65	58	66	14	69	2.3984	76	53	85	08	22
39	69	34	69	2.5193	73	64	79	35	89	2.1792	21
40	.3772	2.6511	.3973	2.5172	.4176	2.3945	.4383	2.2817	.4592	2.1775	**20**
41	75	2.6488	76	50	80	25	86	2.2799	96	58	19
42	79	64	79	29	83	06	90	81	99	42	18
43	82	41	83	08	87	2.3886	93	63	.4603	25	17
44	85	18	86	2.5086	90	67	97	45	07	08	16
45	89	2.6395	90	65	93	47	.4400	27	10	2.1692	15
46	92	71	93	44	97	28	04	09	14	75	14
47	95	48	96	23	.4200	08	07	2.2691	17	59	13
48	99	25	.4000	02	04	2.3789	11	73	21	42	12
49	.3802	02	03	2.4981	07	70	14	55	24	25	11
50	.3805	2.6279	.4006	2.4960	.4210	2.3750	.4417	2.2637	.4628	2.1609	**10**
51	09	56	10	39	14	31	21	20	31	2.1592	9
52	12	33	13	18	17	12	24	02	35	76	8
53	15	10	17	2.4897	21	2.3693	28	2.2584	38	60	7
54	19	2.6187	20	76	24	73	31	66	42	43	6
55	22	65	23	55	28	54	35	49	45	27	5
56	25	42	27	34	31	35	38	31	49	10	4
57	29	19	30	13	34	16	42	13	52	2.1494	3
58	32	2.6096	33	2.4792	38	2.3597	45	2.2496	56	78	2
59	35	74	37	72	41	78	49	78	60	61	1
60	.3839	2.6051	.4040	2.4751	.4245	2.3559	.4452	2.2460	.4663	2.1445	**0**
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	69°		68°		67°		66°		65°		′

TABLE 5. (*Continued*)

′	25°		26°		27°		28°		29°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	**60**
1	288	618	863	867	425	087	973	281	506	448	59
2	315	606	889	854	451	074	999	267	532	434	58
3	341	594	916	841	477	061	.47024	254	557	420	57
4	367	582	942	828	503	048	050	240	583	406	56
5	394	569	968	816	529	035	076	226	608	391	55
6	420	557	994	803	554	021	101	213	634	377	54
7	446	545	.44020	790	580	008	127	199	659	363	53
8	473	532	046	777	606	.88995	153	185	684	349	52
9	499	520	072	764	632	981	178	172	710	335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	**50**
11	552	495	124	739	684	955	229	144	761	306	49
12	578	483	151	726	710	942	255	130	786	292	48
13	604	470	177	713	736	928	281	117	811	278	47
14	631	458	203	700	762	915	306	103	837	264	46
15	657	446	229	687	787	902	332	089	862	250	45
16	683	433	255	674	813	888	358	075	888	235	44
17	709	421	281	662	839	875	383	062	913	221	43
18	736	408	307	649	865	862	409	048	938	207	42
19	762	396	333	636	891	848	434	034	964	193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	**40**
21	815	371	385	610	942	822	486	006	.49014	164	39
22	841	358	411	597	968	808	511	.87993	040	150	38
23	867	346	437	584	994	795	537	979	065	136	37
24	894	334	464	571	.46020	782	562	965	090	121	36
25	920	321	490	558	046	768	588	951	116	107	35
26	946	309	516	545	072	755	614	937	141	093	34
27	972	296	542	532	097	741	639	923	166	079	33
28	999	284	568	519	123	728	665	909	192	064	32
29	.43025	271	594	506	149	715	690	896	217	050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	**30**
31	077	246	646	480	201	688	741	868	268	021	29
32	104	233	672	467	226	674	767	854	293	007	28
33	130	221	698	454	252	661	793	840	318	.86993	27
34	156	208	724	441	278	647	818	826	344	978	26
35	182	196	750	428	304	634	844	812	369	964	25
36	209	183	776	415	330	620	869	798	394	949	24
37	235	171	802	402	355	607	895	784	419	935	23
38	261	158	828	389	381	593	920	770	445	921	22
39	287	146	854	376	407	580	946	756	470	906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	**20**
41	340	120	906	350	458	553	997	729	521	878	19
42	366	108	932	337	484	539	.48022	715	546	863	18
43	392	095	958	324	510	526	048	701	571	849	17
44	418	082	984	311	536	512	073	687	596	834	16
45	445	070	.45010	298	561	499	099	673	622	820	15
46	471	057	036	285	587	485	124	659	647	805	14
47	497	045	062	272	613	472	150	645	672	791	13
48	523	032	088	259	639	458	175	631	697	777	12
49	549	019	114	245	664	445	201	617	723	762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	**10**
51	602	.89994	166	219	716	417	252	589	773	733	9
52	628	981	192	206	742	404	277	575	798	719	8
53	654	968	218	193	767	390	303	561	824	704	7
54	680	956	243	180	793	377	328	546	849	690	6
55	706	943	269	167	819	363	354	532	874	675	5
56	733	930	295	153	844	349	379	518	899	661	4
57	759	918	321	140	870	336	405	504	924	646	3
58	785	905	347	127	896	322	430	490	950	632	2
59	811	892	373	114	921	308	456	476	975	617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	**0**
	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	
′	64°		63°		62°		61°		60°		′

TABLE 5. (*Continued*)

′	25°		26°		27°		28°		29°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.4663	2.1445	.4877	2.0503	.5095	1.9626	.5317	1.8807	.5543	1.8040	**60**
1	67	29	81	2.0488	99	12	21	1.8794	47	28	59
2	70	13	85	73	.5103	1.9598	25	81	51	16	58
3	74	2.1396	88	58	06	84	28	68	55	03	57
4	77	80	92	43	10	70	32	55	58	1.7991	56
5	81	64	95	28	14	56	36	41	62	79	55
6	84	48	99	13	17	42	40	28	66	66	54
7	88	32	.4903	2.0398	21	28	43	15	70	54	53
8	91	15	06	83	25	14	47	02	74	42	52
9	95	2.1299	10	68	28	00	51	1.8689	77	30	51
10	.4699	2.1283	.4913	2.0353	.5132	1.9486	.5354	1.8676	.5581	1.7917	**50**
11	.4702	67	17	38	36	72	58	63	85	05	49
12	06	51	21	23	39	58	62	50	89	1.7893	48
13	09	35	24	08	43	44	66	37	93	81	47
14	13	19	28	2.0293	47	30	69	24	96	68	46
15	16	03	31	78	50	16	73	11	.5600	56	45
16	20	2.1187	35	63	54	02	77	1.8598	04	44	44
17	23	71	39	48	58	1.9388	81	85	08	32	43
18	27	55	42	33	61	75	84	72	12	20	42
19	31	39	46	19	65	61	88	59	16	08	41
20	.4734	2.1123	.4950	2.0204	.5169	1.9347	.5392	1.8546	.5619	1.7796	**40**
21	38	07	53	2.0189	72	33	96	33	23	83	39
22	41	2.1092	57	74	76	19	99	20	27	71	38
23	45	76	60	60	80	06	.5403	07	31	59	37
24	48	60	64	45	84	1.9292	07	1.8495	35	47	36
25	52	44	68	30	87	78	11	82	39	35	35
26	55	28	71	15	91	65	15	69	42	23	34
27	59	13	75	01	95	51	18	56	46	11	33
28	63	2.0997	79	2.0086	98	37	22	43	50	1.7699	32
29	66	81	82	72	.5202	23	26	30	54	87	31
30	.4770	2.0965	.4986	2.0057	.5206	1.9210	.5430	1.8418	.5658	1.7675	**30**
31	73	50	89	42	09	1.9196	33	05	62	63	29
32	77	34	93	28	13	83	37	1.8392	65	51	28
33	80	18	97	13	17	69	41	79	69	39	27
34	84	03	.5000	1.9999	20	55	45	67	73	27	26
35	88	2.0887	04	84	24	42	48	54	77	15	25
36	91	72	08	70	28	28	52	41	81	03	24
37	95	56	11	55	32	15	56	29	85	1.7591	23
38	98	40	15	41	35	01	60	16	88	79	22
39	.4802	25	19	26	39	1.9088	64	03	92	67	21
40	.4806	2.0809	.5022	1.9912	.5243	1.9074	.5467	1.8291	.5696	1.7556	**20**
41	09	2.0794	26	1.9897	46	61	71	78	.5700	44	19
42	13	78	29	83	50	47	75	65	04	32	18
43	16	63	33	68	54	34	79	53	08	20	17
44	20	48	37	54	58	20	82	40	12	08	16
45	23	32	40	40	61	07	86	28	15	1.7496	15
46	27	17	44	25	65	1.8993	90	15	19	85	14
47	31	01	48	11	69	80	94	02	23	73	13
48	34	2.0686	51	1.9797	72	67	98	1.8190	27	61	12
49	38	71	55	82	76	53	.5501	77	31	49	11
50	.4841	2.0655	.5059	1.9768	.5280	1.8940	.5505	1.8165	.5735	1.7437	**10**
51	45	40	62	54	84	27	09	52	39	26	9
52	49	25	66	40	87	13	13	40	43	14	8
53	52	09	70	25	91	00	17	27	46	02	7
54	56	2.0594	73	11	95	1.8887	20	15	50	1.7391	6
55	59	79	77	1.9697	98	73	24	03	54	79	5
56	63	64	81	83	.5302	60	28	1.8090	58	67	4
57	67	49	84	69	06	47	32	78	62	55	3
58	70	33	88	54	10	34	35	65	66	44	2
59	74	18	92	40	13	20	39	53	70	32	1
60	.4877	2.0503	.5095	1.9626	.5317	1.8807	.5543	1.8040	.5774	1.7321	**0**
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	64°		63°		62°		61°		60°		′

TABLE 5. (*Continued*)

′	30°		31°		32°		33°		34°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	025	588	529	702	.53017	789	488	851	943	887	59
2	050	573	554	687	041	774	513	835	968	871	58
3	076	559	579	672	066	759	537	819	992	855	57
4	101	544	604	657	091	743	561	804	.56016	839	56
5	126	530	628	642	115	728	586	788	040	822	55
6	151	515	653	627	140	712	610	772	064	806	54
7	176	501	678	612	164	697	635	756	088	790	53
8	201	486	703	597	189	681	659	740	112	773	52
9	227	471	728	582	214	666	683	724	136	757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	277	442	778	551	263	635	732	692	184	724	49
12	302	427	803	536	288	619	756	676	208	708	48
13	327	413	828	521	312	604	781	660	232	692	47
14	352	398	852	506	337	588	805	645	256	675	46
15	377	384	877	491	361	573	829	629	280	659	45
16	403	369	902	476	386	557	854	613	305	643	44
17	428	354	927	461	411	542	878	597	329	626	43
18	453	340	952	446	435	526	902	581	353	610	42
19	478	325	977	431	460	511	927	565	377	593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	528	295	026	401	509	480	975	533	425	561	39
22	553	281	051	385	534	464	999	517	449	544	38
23	578	266	076	370	558	448	.55024	501	473	528	37
24	603	251	101	355	583	433	048	485	497	511	36
25	628	237	126	340	607	417	072	469	521	495	35
26	654	222	151	325	632	402	097	453	545	478	34
27	679	207	175	310	656	386	121	437	569	462	33
28	704	192	200	294	681	370	145	421	593	446	32
29	729	178	225	279	705	355	169	405	617	429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	779	148	275	249	754	324	218	373	665	396	29
32	804	133	299	234	779	308	242	356	689	380	28
33	829	119	324	218	804	292	266	340	713	363	27
34	854	104	349	203	828	277	291	324	736	347	26
35	879	089	374	188	853	261	315	308	760	330	25
36	904	074	399	173	877	245	339	292	784	314	24
37	929	059	423	157	902	230	363	276	808	297	23
38	954	045	448	142	926	214	388	260	832	281	22
39	979	030	473	127	951	198	412	244	856	264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	029	000	522	096	.54000	167	460	212	904	231	19
42	054	.85985	547	081	024	151	484	195	928	214	18
43	079	970	572	066	049	135	509	179	952	198	17
44	104	956	597	051	073	120	533	163	976	181	16
45	129	941	621	035	097	104	557	147	57000	165	15
46	154	926	646	020	122	088	581	131	024	148	14
47	179	911	671	005	146	072	605	115	047	132	13
48	204	896	696	.84989	171	057	630	098	071	115	12
49	229	881	720	974	195	041	654	082	095	098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	279	851	770	943	244	009	702	050	143	065	9
52	304	836	794	928	269	.83994	726	034	167	048	8
53	329	821	819	913	293	978	750	017	191	032	7
54	354	806	844	897	317	962	775	001	215	015	6
55	379	792	869	882	342	946	799	.82985	238	.81999	5
56	404	777	893	866	366	930	823	969	262	982	4
57	429	762	918	851	391	915	847	953	286	965	3
58	454	747	943	836	415	899	871	936	310	949	2
59	479	732	967	820	440	883	895	920	334	932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	
′	59°		58°		57°		56°		55°		′

TABLE 5. (*Continued*)

′	30°		31°		32°		33°		34°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.5774	1.7321	.6009	1.6643	.6249	1.6003	.6494	1.5399	.6745	1.4826	60
1	77	09	13	32	53	1.5993	98	89	49	16	59
2	81	1.7297	17	21	57	83	.6502	79	54	07	58
3	85	86	20	10	61	72	06	69	58	1.4798	57
4	89	74	24	1.6599	65	62	11	59	62	88	56
5	93	62	28	88	69	52	15	50	66	79	55
6	97	51	32	77	73	41	19	40	71	70	54
7	.5801	39	36	66	77	31	23	30	75	61	53
8	05	28	40	55	81	21	27	20	79	51	52
9	08	16	44	45	85	11	31	11	83	42	51
10	.5812	1.7205	.6048	1.6534	.6289	1.5900	.6536	1.5301	.6787	1.4733	50
11	16	1.7193	52	23	93	1.5890	40	1.5291	92	24	49
12	20	82	56	12	97	80	44	82	96	15	48
13	24	70	60	01	.6301	69	48	72	.6800	05	47
14	28	59	64	1.6490	05	59	52	62	05	1.4696	46
15	32	47	68	79	10	49	56	53	09	87	45
16	36	36	72	69	14	39	60	43	13	78	44
17	40	24	76	58	18	29	65	33	17	69	43
18	44	13	80	47	22	18	69	24	22	1.4659	42
19	47	02	84	36	26	08	73	14	26	50	41
20	.5851	1.7090	.6088	1.6426	.6330	1.5798	.6577	1.5204	.6830	1.4641	40
21	55	79	92	15	34	88	81	1.5195	34	32	39
22	59	67	96	04	38	78	85	85	39	23	38
23	63	56	.6100	1.6393	42	68	90	75	43	14	37
24	67	45	04	83	46	57	94	66	47	05	36
25	71	33	08	72	50	47	98	56	51	1.4596	35
26	75	22	12	61	54	37	.6602	47	56	86	34
27	79	11	16	51	58	27	06	37	60	77	33
28	83	1.6999	20	40	63	17	10	27	64	68	32
29	87	88	24	29	67	07	15	18	69	59	31
30	.5890	1.6977	.6128	1.6319	.6371	1.5697	.6619	1.5108	.6873	1.4550	30
31	94	65	32	08	75	87	23	1.5099	77	41	29
32	98	54	36	1.6297	79	77	27	89	81	32	28
33	.5902	43	40	87	83	67	31	80	86	23	27
34	06	32	44	76	87	57	36	70	90	14	26
35	10	20	48	65	91	47	40	61	94	05	25
36	14	09	52	55	95	37	44	51	99	1.4496	24
37	18	1.6898	56	44	99	27	48	42	.6903	87	23
38	22	87	60	34	.6403	17	52	32	07	78	22
39	26	75	64	23	08	07	57	23	11	69	21
40	.5930	1.6864	.6168	1.6212	.6412	1.5597	.6661	1.5013	.6916	1.4460	20
41	34	53	72	02	16	87	65	04	20	51	19
42	38	42	76	1.6191	20	77	69	1.4994	24	42	18
43	42	31	80	81	24	67	73	85	29	33	17
44	45	20	84	70	28	57	78	75	33	24	16
45	49	08	88	60	32	47	82	66	37	15	15
46	53	1.6797	92	49	36	37	86	57	42	06	14
47	57	86	96	39	40	27	90	47	46	1.4397	13
48	61	75	.6200	28	45	17	94	38	50	88	12
49	65	64	04	18	49	07	99	28	54	79	11
50	.5969	1.6753	.6208	1.6107	.6453	1.5497	.6703	1.4919	.6959	1.4370	10
51	73	42	12	1.6097	57	87	07	10	63	61	9
52	77	31	16	87	61	77	11	00	67	52	8
53	81	20	20	76	65	68	15	1.4891	72	44	7
54	85	09	24	66	69	58	20	82	76	35	6
55	89	1.6698	28	55	73	48	24	72	80	26	5
56	93	87	33	45	78	38	28	63	85	17	4
57	97	76	37	34	82	28	32	54	89	08	3
58	.6001	65	41	24	86	18	37	44	93	1.4299	2
59	05	54	45	14	90	08	41	35	98	90	1
60	.6009	1.6643	.6249	1.6003	.6494	1.5399	.6745	1.4826	.7002	1.4281	0
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	59°		58°		57°		56°		55°		′

TABLE 5. (*Continued*)

′	35°		36°		37°		38°		39°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	381	899	802	885	205	846	589	783	955	696	59
2	405	882	826	867	228	829	612	765	977	678	58
3	429	865	849	850	251	811	635	747	.63000	660	57
4	453	848	873	833	274	793	658	729	022	641	56
5	477	832	896	816	298	776	681	711	045	623	55
6	501	815	920	799	321	758	704	694	068	605	54
7	524	798	943	782	344	741	726	676	090	586	53
8	548	782	967	765	367	723	749	658	113	568	52
9	572	765	990	748	390	706	772	640	135	550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	619	731	037	713	437	671	818	604	180	513	49
12	643	714	061	696	460	653	841	586	203	494	48
13	667	698	084	679	483	635	864	568	225	476	47
14	691	681	108	662	506	618	887	550	248	458	46
15	715	664	131	644	529	600	909	532	271	439	45
16	738	647	154	627	553	583	932	514	293	421	44
17	762	631	178	610	576	565	955	496	316	402	43
18	786	614	201	593	599	547	978	478	338	384	42
19	810	597	225	576	622	530	.62001	460	361	366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	857	563	272	541	668	494	046	424	406	329	39
22	881	546	295	524	691	477	069	405	428	310	38
23	904	530	318	507	714	459	092	387	451	292	37
24	928	513	342	489	738	441	115	369	473	273	36
25	952	496	365	472	761	424	138	351	496	255	35
26	976	479	389	455	784	406	160	333	518	236	34
27	999	462	412	438	807	388	183	315	540	218	33
28	.58023	445	436	420	830	371	206	297	563	199	32
29	047	428	459	403	853	353	229	279	585	181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	094	395	506	368	899	318	274	243	630	144	29
32	118	378	529	351	922	300	297	225	653	125	28
33	141	361	552	334	945	282	320	206	675	107	27
34	165	344	576	316	968	264	342	188	698	088	26
35	189	327	599	299	991	247	365	170	720	070	25
36	212	310	622	282	.61015	229	388	152	742	051	24
37	236	293	646	264	038	211	411	134	765	033	23
38	260	276	669	247	061	193	433	116	787	014	22
39	283	259	693	230	084	176	456	098	810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	330	225	739	195	130	140	502	061	854	959	19
42	354	208	763	178	153	122	524	043	877	940	18
43	378	191	786	160	176	105	547	025	899	921	17
44	401	174	809	143	199	087	570	007	922	903	16
45	425	157	832	125	222	069	592	.77988	944	884	15
46	449	140	856	108	245	051	615	970	966	866	14
47	472	123	879	091	268	033	638	952	989	847	13
48	496	106	902	073	291	016	660	934	.64011	828	12
49	519	089	926	056	314	.78998	683	916	033	810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	567	055	972	021	360	962	728	879	078	772	9
52	590	038	995	003	383	944	751	861	100	754	8
53	614	021	.60019	.79986	406	926	774	843	123	735	7
54	637	004	042	968	429	908	796	824	145	717	6
55	661	.80987	065	951	451	891	819	806	167	698	5
56	684	970	089	934	474	873	842	788	190	679	4
57	708	953	112	916	497	855	864	769	212	661	3
58	731	936	135	899	520	837	887	751	234	642	2
59	755	919	158	881	543	819	909	733	256	623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	N.cos.	N. sine	
′	54°		53°		52°		51°		50°		′

TABLE 5. (*Continued*)

′	35°		36°		37°		38°		39°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.7002	1.4281	.7265	1.3764	.7536	1.3270	.7813	1.2799	.8098	1.2349	60
1	06	73	70	55	40	62	18	92	.8103	42	59
2	11	64	74	47	45	54	22	84	07	34	58
3	15	55	79	39	49	46	27	76	12	27	57
4	19	46	83	30	54	38	32	69	17	20	56
5	24	37	88	22	58	30	36	61	22	12	55
6	28	29	92	13	63	22	41	53	27	05	54
7	32	20	97	05	68	14	46	46	32	1.2298	53
8	37	11	.7301	1.3697	72	06	50	38	36	90	52
9	41	02	06	88	77	1.3198	55	31	41	83	51
10	.7046	1.4193	.7310	1.3680	.7581	1.3190	.7860	1.2723	.8146	1.2276	50
11	50	85	14	72	86	82	65	15	51	68	49
12	54	76	19	63	90	75	69	08	56	61	48
13	59	67	23	55	95	67	74	00	61	54	47
14	63	58	28	47	.7600	59	79	1.2693	65	47	46
15	67	50	32	38	04	51	83	85	70	39	45
16	72	41	37	30	09	43	88	77	75	32	44
17	76	32	41	22	13	35	93	70	80	25	43
18	80	24	46	13	18	27	98	62	85	18	42
19	85	15	50	05	23	19	.7902	55	90	10	41
20	.7089	1.4106	.7355	1.3597	.7627	1.3111	.7907	1.2647	.8195	1.2203	40
21	94	1.4097	59	88	32	03	12	40	99	1.2196	39
22	98	89	64	80	36	1.3095	16	32	.8204	89	38
23	.7102	80	68	72	41	87	21	24	09	81	37
24	07	71	73	64	46	79	26	17	14	74	36
25	11	63	77	55	50	72	31	09	19	67	35
26	15	54	82	47	55	64	35	02	24	60	34
27	20	45	86	39	59	56	40	1.2594	29	53	33
28	24	37	91	31	64	48	45	87	34	45	32
29	29	28	95	22	69	40	50	79	38	38	31
30	.7133	1.4019	.7400	1.3514	.7673	1.3032	.7954	1.2572	.8243	1.2131	30
31	37	11	04	06	78	24	59	64	48	24	29
32	42	02	09	1.3498	83	17	64	57	53	17	28
33	46	1.3994	13	90	87	09	69	49	58	09	27
34	51	85	18	81	92	01	73	42	63	02	26
35	55	76	22	73	96	1.2993	78	34	68	1.2095	25
36	59	68	27	65	.7701	85	83	27	73	88	24
37	64	59	31	57	06	77	88	19	78	81	23
38	68	51	36	49	10	70	92	12	83	74	22
39	73	42	40	40	15	62	97	04	87	66	21
40	.7177	1.3934	.7445	1.3432	.7720	1.2954	.8002	1.2497	.8292	1.2059	20
41	81	25	49	24	24	46	07	89	97	52	19
42	86	16	54	16	29	38	12	82	.8302	45	18
43	90	08	58	08	34	31	16	75	07	38	17
44	95	1.3899	63	00	38	23	21	67	12	31	16
45	99	91	67	1.3392	43	15	26	60	17	24	15
46	.7203	82	72	84	47	07	31	52	22	17	14
47	08	74	76	75	52	00	35	45	27	09	13
48	12	65	81	67	57	1.2892	40	37	32	02	12
49	17	57	85	59	61	84	45	30	37	1.1995	11
50	.7221	1.3848	.7490	1.3351	.7766	1.2876	.8050	1.2423	.8342	1.1988	10
51	26	40	95	43	71	69	55	15	46	81	9
52	30	31	99	35	75	61	59	08	51	74	8
53	34	23	.7504	27	80	53	64	01	56	67	7
54	39	14	08	19	85	46	69	1.2393	61	60	6
55	43	06	13	11	89	38	74	86	66	53	5
56	48	1.3798	17	03	94	30	79	78	71	46	4
57	52	89	22	1.3295	99	22	83	71	76	39	3
58	57	81	26	87	.7803	15	88	64	81	32	2
59	61	72	31	78	08	07	93	56	86	25	1
60	.7265	1.3764	.7536	1.3270	.7813	1.2799	.8098	1.2349	.8391	1.1918	0
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	54°		53°		52°		51°		50°		′

TABLE 5. (*Continued*)

′	40°		41°		42°		43°		44°		′
	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	N.sine	N. cos.	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	**60**
1	301	586	628	452	935	295	221	116	487	914	59
2	323	567	650	433	956	276	242	096	508	894	58
3	346	548	672	414	978	256	264	076	529	873	57
4	368	530	694	395	999	237	285	056	549	853	56
5	390	511	716	375	.67021	217	306	036	570	833	55
6	412	492	738	356	043	198	327	016	591	813	54
7	435	473	759	337	064	178	349	.72996	612	792	53
8	457	455	781	318	086	159	370	976	633	772	52
9	479	436	803	299	107	139	391	957	654	752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	**50**
11	524	398	847	261	151	100	434	917	696	711	49
12	546	380	869	241	172	080	455	897	717	691	48
13	568	361	891	222	194	061	476	877	737	671	47
14	590	342	913	203	215	041	497	857	758	650	46
15	612	323	935	184	237	022	518	837	779	630	45
16	635	304	956	165	258	002	539	817	800	610	44
17	657	286	978	146	280	.73983	561	797	821	590	43
18	679	267	.66000	126	301	963	582	777	842	569	42
19	701	248	022	107	323	944	603	757	862	549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	**40**
21	746	210	066	069	366	904	645	717	904	508	39
22	768	192	088	050	387	885	666	697	925	488	38
23	790	173	109	030	409	865	688	677	946	468	37
24	812	154	131	011	430	846	709	657	966	447	36
25	834	135	153	.74992	452	826	730	637	987	427	35
26	856	116	175	973	473	806	751	617	.70008	407	34
27	878	097	197	953	495	787	772	597	029	386	33
28	901	078	218	934	516	767	793	577	049	366	32
29	923	059	240	915	538	747	814	557	070	345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	**30**
31	967	022	284	876	580	708	857	517	112	305	29
32	989	003	306	857	602	688	878	497	132	284	28
33	.65011	.75984	327	838	623	669	899	477	153	264	27
34	033	965	349	818	645	649	920	457	174	243	26
35	055	946	371	799	666	629	941	437	195	223	25
36	077	927	393	780	688	610	962	417	215	203	24
37	100	908	414	760	709	590	983	397	236	182	23
38	122	889	436	741	730	570	.69004	377	257	162	22
39	144	870	458	722	752	551	025	357	277	141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	**20**
41	188	832	501	683	795	511	067	317	319	100	19
42	210	813	523	664	816	491	088	297	339	080	18
43	232	794	545	644	837	472	109	277	360	059	17
44	254	775	566	625	859	452	130	257	381	039	16
45	276	756	588	606	880	432	151	236	401	019	15
46	298	738	610	586	901	413	172	216	422	.70998	14
47	320	719	632	567	923	393	193	196	443	978	13
48	342	700	653	548	944	373	214	176	463	957	12
49	364	680	675	528	965	353	235	156	484	937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	**10**
51	408	642	718	489	.68008	314	277	116	525	896	9
52	430	623	740	470	029	294	298	095	546	875	8
53	452	604	762	451	051	274	319	075	567	855	7
54	474	585	783	431	072	254	340	055	587	834	6
55	496	566	805	412	093	234	361	035	608	813	5
56	518	547	827	392	115	215	382	015	628	793	4
57	540	528	848	373	136	195	403	.71995	649	772	3
58	562	509	870	353	157	175	424	974	670	752	2
59	584	490	891	334	179	155	445	955	690	731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	**0**
	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	N. cos.	N. sine	
′	49°		48°		47°		46°		45°		′

TABLE 5. (*Continued*)

′	40°		41°		42°		43°		44°		′
	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	
0	.8391	1.1918	.8693	1.1504	.9004	1.1106	.9325	1.0724	.9657	1.0355	**60**
1	96	10	98	1.1497	09	00	31	17	63	49	59
2	.8401	03	.8703	90	15	1.1093	36	11	68	43	58
3	06	1.1896	08	83	20	87	41	05	74	37	57
4	11	89	13	77	25	80	47	1.0699	79	31	56
5	16	82	18	70	30	74	52	92	85	25	55
6	21	75	24	63	36	67	58	86	91	19	54
7	26	68	29	56	41	61	63	80	96	13	53
8	31	61	34	50	46	54	69	74	.9702	07	52
9	36	54	39	43	52	48	74	68	08	01	51
10	.8441	1.1847	.8744	1.1436	.9057	1.1041	.9380	1.0661	.9713	1.0295	**50**
11	46	40	49	30	62	35	85	55	19	89	49
12	51	33	54	23	67	28	91	49	25	83	48
13	56	26	59	16	73	22	96	43	30	77	47
14	61	19	65	10	78	16	.9402	37	36	71	46
15	66	12	70	03	83	09	07	30	42	65	45
16	71	06	75	1.1396	89	03	13	24	47	59	44
17	76	1.1799	80	89	94	1.0996	18	18	53	53	43
18	81	92	85	83	99	90	24	12	59	47	42
19	86	85	90	76	.9105	83	29	06	64	41	41
20	.8491	1.1778	.8796	1.1369	.9110	1.0977	.9435	1.0599	.9770	1.0235	**40**
21	96	71	.8801	63	15	71	40	93	76	30	39
22	.8501	64	06	56	21	64	46	87	81	24	38
23	06	57	11	49	26	58	51	81	87	18	37
24	11	50	16	43	31	51	57	75	93	12	36
25	16	43	21	36	37	45	62	69	98	06	35
26	21	36	27	29	42	39	68	62	.9804	00	34
27	26	29	32	23	47	32	73	56	10	1.0194	33
28	31	22	37	16	53	26	79	50	16	88	32
29	36	15	42	10	58	19	84	44	21	82	31
30	.8541	1.1708	.8847	1.1303	.9163	1.0913	.9490	1.0538	.9827	1.0176	**30**
31	46	02	52	1.1296	69	07	95	32	33	70	29
32	51	1.1695	58	90	74	00	.9501	26	38	64	28
33	56	88	63	83	79	1.0894	06	19	44	58	27
34	61	81	68	76	85	88	12	13	50	52	26
35	66	74	73	70	90	81	17	07	56	47	25
36	71	67	78	63	95	75	23	01	61	41	24
37	76	60	84	57	.9201	69	28	1.0495	67	35	23
38	81	53	89	50	06	62	34	89	73	29	22
39	86	47	94	43	12	56	40	83	79	23	21
40	.8591	1.1640	.8899	1.1237	.9217	1.0850	.9545	1.0477	.9884	1.0117	**20**
41	96	33	.8904	30	22	43	51	70	90	11	19
42	.8601	26	10	24	28	37	56	64	96	05	18
43	06	19	15	17	33	31	62	58	.9902	1.0099	17
44	11	12	20	11	39	24	67	52	07	94	16
45	17	06	25	04	44	18	73	46	13	88	15
46	22	1.1599	31	1.1197	49	12	78	40	19	82	14
47	27	92	36	91	55	05	84	34	25	76	13
48	32	85	41	84	60	1.0799	90	28	30	70	12
49	37	78	46	78	66	93	95	22	36	64	11
50	.8642	1.1571	.8952	1.1171	.9271	1.0786	.9601	1.0416	.9942	1.0058	**10**
51	47	65	57	65	76	80	06	10	48	52	9
52	52	58	62	58	82	74	12	04	54	47	8
53	57	51	67	52	87	68	18	1.0398	59	41	7
54	62	44	72	45	93	61	23	92	65	35	6
55	67	38	78	39	98	55	29	85	71	29	5
56	72	31	83	32	.9303	49	34	79	77	23	4
57	78	24	88	26	09	42	40	73	83	17	3
58	83	17	94	19	14	36	46	67	88	12	2
59	88	10	99	13	20	30	51	61	94	06	1
60	.8693	1.1504	.9004	1.1106	.9325	1.0724	.9657	1.0355	1.0000	1.0000	**0**
	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	Cotg.	Tang.	
′	49°		48°		47°		46°		45°		′

TABLE 6. Measurements and Areas

1 Mile = 80 chains
320 rods
1760 yards
5,280 feet
63,360 inches

1 Chain = 4 rods
22 yards
66.0 feet
100 links
792 inches

1 Rod = 5.5 yards
16.5 feet
1,980 inches

1 Link = 7.92 inches
0.66 foot

1 Sq. Mile = 640 acres

1 Acre = 10 sq. chains
160 sq. rods
4,840 sq. yards
43,560 sq. feet

1 Kilometer = 1,000 meters
0.62137 mile

1 Meter = 100 centimeters
1.0936 yard
3.2808 feet
39.37 inches

1 Centimeter = 10 millimeters
0.3937 inch

1 Millimeter = 0.03937 inch

1 Vara = 32.993 inches (Mexico)
33.000 inches (California)
33.333 inches (Texas)

1 Sq. Kilometer = 0.386101 sq. mile (approx.)

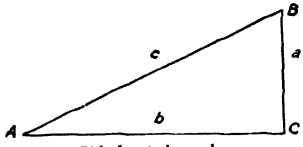
1 Sq. Kilometer = 247.1 acres (approx.)

$$\frac{\text{Area in square feet}}{43{,}560} = \text{Acres}$$

Area in square feet $\times$ 0.0000229 = Acres (approximate)

TABLE 6 (*Continued*)

Trigonometric Formulas



Right triangle

$$\sin A = \frac{a}{c} = \cos B$$

$$\cos A = \frac{b}{c} = \sin B$$

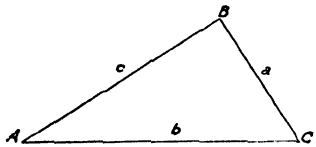
$$\tan A = \frac{a}{b} = \cot B$$

$$\cot A = \frac{b}{a} = \tan B$$

$$a = c \sin A = c \cos B = b \tan A = b \cot B$$

$$b = c \cos A = c \sin B = a \cot A = a \tan B$$

$$c = \frac{a}{\sin A} = \frac{a}{\cos B} = \frac{b}{\sin B} = \frac{b}{\cos A}$$



Oblique triangle

Known a,B,A

$$b = \frac{a}{\sin A} \cdot \sin B$$

$$c = \frac{a}{\sin A} \cdot \sin (A + B)$$

Known a,b,A

$$\sin B = \frac{\sin A}{a} \cdot b$$

$$c = \frac{a}{\sin A} \cdot \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a = \frac{b}{\sin B} \cdot \sin A = \frac{c}{\sin C} \cdot \sin A$$

$$b = \frac{a}{\sin A} \cdot \sin B = \frac{c}{\sin C} \cdot \sin B$$

$$c = \frac{a}{\sin A} \cdot \sin C = \frac{b}{\sin B} \cdot \sin C$$

TABLE 7. Stadia Conversion Tables

For computing difference of elevation and horizontal distance.

Anderson's: U.S. Geological Survey

0°		1°		2°		3°		4°	
′	100	′	100	′	100	′	100	′	100
1	0.03	0	1.74	0	3.49	0	5.23	0	6.96
2	0.06	1	1.77	1	3.52	1	5.26	1	6.99
3	0.09	2	1.80	2	3.55	2	5.29	2	7.02
4	0.12	3	1.83	3	3.58	3	5.31	3	7.05
5	0.15	4	1.86	4	3.60	4	5.34	4	7.08
6	0.18	5	1.89	5	3.63	5	5.37	5	7.10
7	0.20	6	1.92	6	3.66	6	5.40	6	7.13
8	0.23	7	1.95	7	3.69	7	5.43	7	7.16
9	0.26	8	1.98	8	3.72	8	5.46	8	7.19
		9	2.01	9	3.75	9	5.49	9	7.22
10	0.29	10	2.04	10	3.78	10	5.52	10	7.25
11	0.32	11	2.06	11	3.81	11	5.54	11	7.28
12	0.38	12	2.09	12	3.84	12	5.58	12	7.30
13	0.38	13	2.12	13	3.86	13	5.60	13	7.33
14	0.41	14	2.15	14	3.89	14	5.63	14	7.36
15	0.44	15	2.18	15	3.92	15	5.66	15	7.39
16	0.47	16	2.21	16	3.95	16	5.69	16	7.42
17	0.49	17	2.24	17	3.98	17	5.72	17	7.45
18	0.52	18	2.27	18	4.01	18	5.75	18	7.48
19	0.55	19	2.30	19	4.04	19	5.77	19	7.51
20	0.58	20	2.33	20	4.07	20	5.80	20	7.53
21	0.61	21	2.36	21	4.10	21	5.83	21	7.56
22	0.64	22	2.38	22	4.13	22	5.86	22	7.59
23	0.67	23	2.41	23	4.16	23	5.89	23	7.62
24	0.70	24	2.44	24	4.18	24	5.92	24	7.65
25	0.73	25	2.47	25	4.21	25	5.95	25	7.68
26	0.76	26	2.50	26	4.24	26	5.98	26	7.71
27	0.79	27	2.53	27	4.27	27	6.01	27	7.74
28	0.81	28	2.56	28	4.30	28	6.04	28	7.76
29	0.84	29	2.59	29	4.33	29	6.06	29	7.79
30	0.87	30	2.62	30	4.36	30	6.09	30	7.82
31	0.90	31	2.65	31	4.39	31	6.12	31	7.85
32	0.93	32	2.68	32	4.42	32	6.15	32	7.88
33	0.96	33	2.70	33	4.44	33	6.18	33	7.91
34	0.99	34	2.73	34	4.47	34	6.21	34	7.94
35	1.02	35	2.76	35	4.50	35	6.24	35	7.97
36	1.05	36	2.79	36	4.53	36	6.27	36	7.99
37	1.08	37	2.82	37	4.56	37	6.30	37	8.02
38	1.11	38	2.85	38	4.59	38	6.32	38	8.05
39	1.13	39	2.88	39	4.62	39	6.35	39	8.08
40	1.16	40	2.91	40	4.65	40	6.38	40	8.11
41	1.19	41	2.94	41	4.68	41	6.41	41	8.14
42	1.22	42	2.97	42	4.71	42	6.44	42	8.17
43	1.25	43	3.00	43	4.73	43	6.47	43	8.20
44	1.28	44	3.02	44	4.76	44	6.50	44	8.22
45	1.31	45	3.05	45	4.79	45	6.53	45	8.25
46	1.34	46	3.08	46	4.82	46	6.56	46	8.28
47	1.37	47	3.11	47	4.85	47	6.59	47	8.31
48	1.40	48	3.14	48	4.88	48	6.61	48	8.34
49	1.42	49	3.17	49	4.91	49	6.64	49	8.37
50	1.45	50	3.20	50	4.94	50	6.67	50	8.40
51	1.48	51	3.23	51	4.97	51	6.70	51	8.43
52	1.51	52	3.26	52	5.00	52	6.73	52	8.45
53	1.54	53	3.29	53	5.02	53	6.76	53	8.48
54	1.57	54	3.31	54	5.05	54	6.79	54	8.51
55	1.60	55	3.34	55	5.08	55	6.81	55	8.54
56	1.63	56	3.37	56	5.11	56	6.85	56	8.57
57	1.66	57	3.40	57	5.14	57	6.88	57	8.60
58	1.69	58	3.43	58	5.17	58	6.90	58	8.63
59	1.72	59	3.46	59	5.20	59	6.93	59	8.65
Horz. Dist.	99.99	Horz. Dist.	99.93	Horz. Dist.	99.81	Horz. Dist.	99.63	Horz. Dist.	99.38

TABLE 7 (*Continued*)

5°		6°		7°		8°		9°	
′	100	′	100	′	100	′	100	′	100
0	8.68	0	10.40	0	12.10	0	13.78	0	15.45
1	8.71	1	10.42	1	12.12	1	13.81	1	15.48
2	8.74	2	10.45	2	12.15	2	13.84	2	15.51
3	8.77	3	10.48	3	12.18	3	13.87	3	15.53
4	8.80	4	10.51	4	12.21	4	13.89	4	15.56
5	8.82	5	10.54	5	12.24	5	13.92	5	15.59
6	8.85	6	10.57	6	12.27	6	13.95	6	15.62
7	8.88	7	10.59	7	12.29	7	13.98	7	15.64
8	8.91	8	10.62	8	12.32	8	14.01	8	15.67
9	8.94	9	10.65	9	12.35	9	14.03	9	15.70
10	8.97	10	10.68	10	12.38	10	14.06	10	15.73
11	9.00	11	10.71	11	12.41	11	14.09	11	15.76
12	9.03	12	10.74	12	12.43	12	14.12	12	15.78
13	9.06	13	10.76	13	12.46	13	14.14	13	15.81
14	9.08	14	10.79	14	12.49	14	14.17	14	15.84
15	9.11	15	10.82	15	12.52	15	14.20	15	15.86
16	9.14	16	10.85	16	12.55	16	14.23	16	15.89
17	9.17	17	10.88	17	12.58	17	14.26	17	15.92
18	9.20	18	10.91	18	12.60	18	14.28	18	15.95
19	9.23	19	10.94	19	12.63	19	14.31	19	15.98
20	9.25	20	10.96	20	12.66	20	14.34	20	16.00
21	9.28	21	10.99	21	12.69	21	14.37	21	16.03
22	9.31	22	11.02	22	12.72	22	14.40	22	16.06
23	9.34	23	11.05	23	12.74	23	14.42	23	16.09
24	9.37	24	11.08	24	12.77	24	14.45	24	16.11
25	9.40	25	11.11	25	12.80	25	14.48	25	16.14
26	9.43	26	11.13	26	12.83	26	14.51	26	16.17
27	9.46	27	11.16	27	12.86	27	14.54	27	16.20
28	9.48	28	11.19	28	12.88	28	14.56	28	16.22
29	9.51	29	11.22	29	12.91	29	14.59	29	16.25
30	9.54	30	11.25	30	12.94	30	14.62	30	16.28
31	9.57	31	11.28	31	12.97	31	14.65	31	16.31
32	9.60	32	11.30	32	13.00	32	14.67	32	16.33
33	9.63	33	11.32	33	13.02	33	14.70	33	16.36
34	9.65	34	11.36	34	13.05	34	14.73	34	16.39
35	9.68	35	11.39	35	13.08	35	14.76	35	16.42
36	9.71	36	11.42	36	13.11	36	14.79	36	16.44
37	9.74	37	11.45	37	13.14	37	14.81	37	16.47
38	9.77	38	11.47	38	13.17	38	14.84	38	16.50
39	9.80	39	11.50	39	13.20	39	14.87	39	16.53
40	9.83	40	11.53	40	13.22	40	14.90	40	16.55
41	9.86	41	11.56	41	13.25	41	14.92	41	16.58
42	9.88	42	11.59	42	13.28	42	14.95	42	16.61
43	9.91	43	11.62	43	13.31	43	14.98	43	16.64
44	9.94	44	11.64	44	13.33	44	15.01	44	16.66
45	9.97	45	11.67	45	13.36	45	15.04	45	16.69
46	10.00	46	11.70	46	13.39	46	15.06	46	16.72
47	10.02	47	11.73	47	13.42	47	15.09	47	16.74
48	10.05	48	11.76	48	13.45	48	15.12	48	16.77
49	10.08	49	11.79	49	13.47	49	15.15	49	16.80
50	10.11	50	11.81	50	13.50	50	15.17	50	16.83
51	10.14	51	11.84	51	13.53	51	15.20	51	16.86
52	10.17	52	11.87	52	13.56	52	15.23	52	16.88
53	10.20	53	11.90	53	13.59	53	15.26	53	16.91
54	10.22	54	11.93	54	13.61	54	15.28	54	16.94
55	10.25	55	11.96	55	13.64	55	15.31	55	16.96
56	10.28	56	11.98	56	13.67	56	15.34	56	16.99
57	10.31	57	12.01	57	13.70	57	15.37	57	17.02
58	10.34	58	12.04	58	13.73	58	15.40	58	17.05
59	10.37	59	12.07	59	13.75	59	15.42	59	17.08
Horz. Dist.	99.08	Horz. Dist.	98.72	Horz. Dist.	98.29	Horz. Dist.	97.82	Horz. Dist.	97.28

TABLE 7 (*Continued*)

10°		11°		12°		13°		14°	
′	100	′	100	′	100	′	100	′	100
0	17.10	0	18.73	0	20.34	0	21.92	0	23.47
1	17.13	1	18.76	1	20.36	1	21.94	1	23.50
2	17.16	2	18.78	2	20.39	2	21.97	2	23.52
3	17.18	3	18.81	3	20.42	3	22.00	3	23.55
4	17.21	4	18.84	4	20.44	4	22.02	4	23.58
5	17.24	5	18.86	5	20.47	5	22.05	5	23.60
6	17.26	6	18.89	6	20.50	6	22.08	6	23.63
7	17.29	7	18.92	7	20.52	7	22.10	7	23.65
8	17.32	8	18.95	8	20.55	8	22.13	8	23.68
9	17.35	9	18.97	9	20.58	9	22.15	9	23.70
10	17.37	10	19.00	10	20.60	10	22.18	10	23.73
11	17.40	11	19.03	11	20.63	11	22.21	11	23.76
12	17.43	12	19.05	12	20.66	12	22.23	12	23.78
13	17.46	13	19.08	13	20.68	13	22.26	13	23.81
14	17.48	14	19.11	14	20.71	14	22.28	14	23.83
15	17.51	15	19.13	15	20.73	15	22.31	15	23.86
16	17.54	16	19.16	16	20.76	16	22.34	16	23.88
17	17.56	17	19.19	17	20.79	17	22.36	17	23.91
18	17.59	18	19.22	18	20.81	18	22.39	18	23.94
19	17.62	19	19.24	19	20.84	19	22.41	19	23.96
20	17.65	20	19.27	20	20.87	20	22.44	20	23.98
21	17.67	21	19.30	21	20.89	21	22.47	21	24.01
22	17.70	22	19.32	22	20.92	22	22.49	22	24.04
23	17.73	23	19.35	23	20.95	23	22.52	23	24.06
24	17.76	24	19.38	24	20.97	24	22.54	24	24.09
25	17.78	25	19.40	25	21.00	25	22.57	25	24.11
26	17.81	26	19.43	26	21.03	26	22.60	26	24.14
27	17.84	27	19.46	27	21.05	27	22.62	27	24.16
28	17.86	28	19.48	28	21.08	28	22.65	28	24.19
29	17.89	29	19.51	29	21.10	29	22.67	29	24.22
30	17.92	30	19.54	30	21.13	30	22.70	30	24.24
31	17.95	31	19.56	31	21.16	31	22.73	31	24.27
32	17.97	32	19.59	32	21.18	32	22.75	32	24.29
33	18.00	33	19.62	33	21.21	33	22.78	33	24.32
34	18.03	34	19.64	34	21.24	34	22.80	34	24.34
35	18.05	35	19.67	35	21.26	35	22.83	35	24.37
36	18.08	36	19.70	36	21.29	36	22.86	36	24.39
37	18.11	37	19.72	37	21.32	37	22.88	37	24.42
38	18.14	38	19.75	38	21.34	38	22.91	38	24.44
39	18.16	39	19.78	39	21.37	39	22.94	39	24.47
40	18.19	40	19.80	40	21.39	40	22.96	40	24.50
41	18.22	41	19.83	41	21.42	41	22.98	41	24.52
42	18.24	42	19.86	42	21.45	42	23.01	42	24.54
43	18.27	43	19.88	43	21.47	43	23.04	43	24.57
44	18.30	44	19.91	44	21.50	44	23.06	44	24.60
45	18.32	45	19.94	45	21.53	45	23.09	45	24.62
46	18.35	46	19.96	46	21.55	46	23.11	46	24.65
47	18.38	47	19.99	47	21.58	47	23.14	47	24.67
48	18.41	48	20.02	48	21.60	48	23.17	48	24.70
49	18.43	49	20.04	49	21.63	49	23.19	49	24.72
50	18.46	50	20.07	50	21.66	50	23.22	50	24.75
51	18.49	51	20.10	51	21.68	51	23.24	51	24.77
52	18.51	52	20.12	52	21.71	52	23.27	52	24.80
53	18.54	53	20.15	53	21.74	53	23.29	53	24.82
54	18.57	54	20.18	54	21.76	54	23.32	54	24.85
55	18.60	55	20.20	55	21.79	55	23.34	55	24.87
56	18.62	56	20.23	56	21.81	56	23.37	56	24.90
57	18.65	57	20.26	57	21.84	57	23.40	57	24.92
58	18.68	58	20.28	58	21.87	58	23.42	58	24.95
59	18.70	59	20.31	59	21.89	59	23.45	59	24.98
Horz. Dist.	96.68	Horz. Dist.	96.03	Horz. Dist.	95.32	Horz. Dist.	94.55	Horz. Dist.	93.73

TABLE 7 (*Continued*)

15°		16°		17°		18°		19°	
′	100	′	100	′	100	′	100	′	100
0	25.00	0	26.50	0	27.96	0	29.39	0	30.78
1	25.02	1	26.52	1	27.99	1	29.42	1	30.81
2	25.05	2	26.55	2	28.01	2	29.44	2	30.83
3	25.08	3	26.57	3	28.04	3	29.47	3	30.85
4	25.10	4	26.60	4	28.06	4	29.49	4	30.87
5	25.13	5	26.62	5	28.08	5	29.51	5	30.90
6	25.15	6	26.64	6	28.10	6	29.53	6	30.92
7	25.18	7	26.67	7	28.13	7	29.56	7	30.94
8	25.20	8	26.69	8	28.15	8	29.58	8	30.97
9	25.23	9	26.72	9	28.18	9	29.60	9	30.99
10	25.25	10	26.74	10	28.20	10	29.62	10	31.01
11	25.28	11	26.77	11	28.22	11	29.65	11	31.04
12	25.30	12	26.79	12	28.25	12	29.67	12	31.06
13	25.33	13	26.82	13	28.27	13	29.69	13	31.08
14	25.35	14	26.84	14	28.30	14	29.72	14	31.10
15	25.38	15	26.86	15	28.32	15	29.74	15	31.13
16	25.40	16	26.89	16	28.34	16	29.76	16	31.15
17	25.43	17	26.91	17	28.37	17	29.79	17	31.17
18	25.45	18	26.94	18	28.39	18	29.81	18	31.19
19	25.48	19	26.96	19	28.42	19	29.83	19	31.22
20	25.50	20	26.99	20	28.44	20	29.86	20	31.24
21	25.53	21	27.01	21	28.47	21	29.88	21	31.26
22	25.55	22	27.04	22	28.49	22	29.90	22	31.28
23	25.58	23	27.06	23	28.51	23	29.93	23	31.30
24	25.60	24	27.09	24	28.54	24	29.95	24	31.33
25	25.63	25	27.11	25	28.56	25	29.97	25	31.35
26	25.65	26	27.13	26	28.58	26	30.00	26	31.38
27	25.68	27	27.16	27	28.61	27	30.02	27	31.40
28	25.70	28	27.18	28	28.63	28	30.04	28	31.42
29	25.73	29	27.21	29	28.66	29	30.07	29	31.44
30	25.75	30	27.23	30	28.68	30	30.09	30	31.47
31	25.78	31	27.26	31	28.70	31	30.11	31	31.49
32	25.80	32	27.28	32	28.73	32	30.14	32	31.51
33	25.83	33	27.30	33	28.75	33	30.16	33	31.54
34	25.85	34	27.33	34	28.77	34	30.19	34	31.56
35	25.88	35	27.35	35	28.80	35	30.21	35	31.58
36	25.90	36	27.38	36	28.82	36	30.23	36	31.60
37	25.93	37	27.40	37	28.85	37	30.26	37	31.63
38	25.95	38	27.43	38	28.87	38	30.28	38	31.65
39	25.98	39	27.45	39	28.89	39	30.30	39	31.67
40	26.00	40	27.48	40	28.92	40	30.32	40	31.69
41	26.02	41	27.50	41	28.94	41	30.36	41	31.72
42	26.05	42	27.52	42	28.96	42	30.37	42	31.74
43	26.08	43	27.55	43	28.99	43	30.39	43	31.76
44	26.10	44	27.57	44	29.01	44	30.41	44	31.78
45	26.12	45	27.60	45	29.04	45	30.44	45	31.81
46	26.15	46	27.62	46	29.06	46	30.46	46	31.83
47	26.18	47	27.65	47	29.08	47	30.49	47	31.85
48	26.20	48	27.67	48	29.11	48	30.51	48	31.87
49	26.22	49	27.69	49	29.13	49	30.53	49	31.90
50	26.25	50	27.72	50	29.15	50	30.55	50	31.92
51	26.27	51	27.74	51	29.18	51	30.58	51	31.94
52	26.30	52	27.77	52	29.20	52	30.60	52	31.96
53	26.32	53	27.79	53	29.23	53	30.62	53	31.99
54	26.35	54	27.82	54	29.25	54	30.65	54	32.01
55	26.37	55	27.84	55	29.27	55	30.67	55	32.03
56	26.40	56	27.86	56	29.30	56	30.69	56	32.05
57	26.42	57	27.89	57	29.32	57	30.72	57	32.07
58	26.45	58	27.91	58	29.34	58	30.74	58	32.09
59	26.47	59	27.94	59	29.37	59	30.76	59	32.12
Horz. Dist.	92.86	Horz. Dist.	91.93	Horz. Dist.	90.96	Horz. Dist.	89.93	Horz. Dist.	88.86

TABLE 7 (*Continued*)

20°		21°		22°		23°		24°	
′	100	′	100	′	100	′	100	′	100
0	32.14	0	33.46	0	34.73	0	35.97	0	37.16
1	32.16	1	33.48	1	34.75	1	35.99	1	37.18
2	32.18	2	33.50	2	34.77	2	36.01	2	37.20
3	32.21	3	33.52	3	34.80	3	36.03	3	37.22
4	32.23	4	33.54	4	34.82	4	36.05	4	37.23
5	32.25	5	33.57	5	34.84	5	36.07	5	37.25
6	32.27	6	33.59	6	34.86	6	36.09	6	37.27
7	32.30	7	33.61	7	34.88	7	36.11	7	37.29
8	32.32	8	33.63	8	34.90	8	36.13	8	37.31
9	32.34	9	33.65	9	34.92	9	36.15	9	37.33
10	32.36	10	33.67	10	34.94	10	36.17	10	37.35
11	32.39	11	33.70	11	34.96	11	36.19	11	37.37
12	32.41	12	33.72	12	34.98	12	36.21	12	37.39
13	32.43	13	33.74	13	35.00	13	36.23	13	37.41
14	32.45	14	33.76	14	35.02	14	36.25	14	37.43
15	32.47	15	33.78	15	35.05	15	36.27	15	37.45
16	32.49	16	33.80	16	35.07	16	36.29	16	37.47
17	32.51	17	33.82	17	35.09	17	36.31	17	37.49
18	32.54	18	33.84	18	35.11	18	36.33	18	37.51
19	32.56	19	33.87	19	35.13	19	36.35	19	37.53
20	32.58	20	33.89	20	35.15	20	36.37	20	37.54
21	32.61	21	33.91	21	35.17	21	36.39	21	37.56
22	32.63	22	33.93	22	35.19	22	36.41	22	37.58
23	32.65	23	33.95	23	35.21	23	36.43	23	37.60
24	32.67	24	33.97	24	35.23	24	36.45	24	37.62
25	32.70	25	33.99	25	35.25	25	36.47	25	37.64
26	32.72	26	34.01	26	35.27	26	36.49	26	37.66
27	32.74	27	34.04	27	35.29	27	36.51	27	37.68
28	32.76	28	34.06	28	35.31	28	36.53	28	37.70
29	32.78	29	34.08	29	35.34	29	36.55	29	37.72
30	32.80	30	34.10	30	35.36	30	36.57	30	37.74
31	32.83	31	34.12	31	35.38	31	36.59	31	37.76
32	32.85	32	34.14	32	35.40	32	36.61	32	37.77
33	32.87	33	34.16	33	35.42	33	36.63	33	37.79
34	32.89	34	34.18	34	35.44	34	36.65	34	37.81
35	32.91	35	34.21	35	35.46	35	36.67	35	37.83
36	32.93	36	34.23	36	35.48	36	36.69	36	37.85
37	32.96	37	34.25	37	35.50	37	36.71	37	37.87
38	32.98	38	34.27	38	35.52	38	36.73	38	37.89
39	33.00	39	34.29	39	35.54	39	36.75	39	37.91
40	33.02	40	34.31	40	35.56	40	36.77	40	37.93
41	33.05	41	34.33	41	35.58	41	36.79	41	37.95
42	33.07	42	34.35	42	35.60	42	36.80	42	37.96
43	33.09	43	34.38	43	35.62	43	36.82	43	37.98
44	33.11	44	34.40	44	35.64	44	36.84	44	38.00
45	33.13	45	34.42	45	35.66	45	36.86	45	38.02
46	33.15	46	34.44	46	35.68	46	36.88	46	38.04
47	33.18	47	34.46	47	35.70	47	36.90	47	38.06
48	33.20	48	34.48	48	35.72	48	36.92	48	38.08
49	33.22	49	34.50	49	35.74	49	36.94	49	38.10
50	33.24	50	34.52	50	35.76	50	36.96	50	38.11
51	33.26	51	34.54	51	35.78	51	36.98	51	38.13
52	33.28	52	34.57	52	35.80	52	37.00	52	38.15
53	33.31	53	34.59	53	35.83	53	37.02	53	38.17
54	33.33	54	34.61	54	35.85	54	37.04	54	38.19
55	33.35	55	34.63	55	35.87	55	37.06	55	38.21
56	33.37	56	34.65	56	35.89	56	37.08	56	38.23
57	33.39	57	34.67	57	35.91	57	37.10	57	38.25
58	33.41	58	34.69	58	35.93	58	37.12	58	38.26
59	33.44	59	34.71	59	35.95	59	37.14	59	38.28
Horz. Dist.	87.74	Horz. Dist.	86.57	Horz. Dist.	85.36	Horz. Dist.	84.10	Horz. Dist.	82.80

TABLE 7 (*Continued*)

25°		26°		27°		28°		29°	
′	100	′	100	′	100	′	100	′	100
0	38.30	0	39.40	0	40.45	0	41.45	0	42.40
1	38.32	1	39.42	1	40.47	1	41.47	1	42.42
2	38.34	2	39.44	2	40.49	2	41.48	2	42.43
3	38.36	3	39.46	3	40.51	3	41.50	3	42.45
4	38.38	4	39.47	4	40.52	4	41.52	4	42.46
5	38.40	5	39.49	5	40.54	5	41.54	5	42.48
6	38.41	6	39.51	6	40.55	6	41.55	6	42.49
7	38.43	7	39.53	7	40.57	7	41.57	7	42.51
8	38.45	8	39.55	8	40.59	8	41.58	8	42.53
9	38.47	9	39.56	9	40.61	9	41.60	9	42.54
10	38.49	10	39.58	10	40.62	10	41.61	10	42.56
11	38.51	11	39.60	11	40.64	11	41.63	11	42.58
12	38.53	12	39.61	12	40.66	12	41.65	12	42.59
13	38.55	13	39.63	13	40.68	13	41.67	13	42.60
14	38.56	14	39.65	14	40.69	14	41.68	14	42.62
15	38.58	15	39.67	15	40.71	15	41.70	15	42.64
16	38.60	16	39.69	16	40.72	16	41.71	16	42.65
17	38.62	17	39.71	17	40.74	17	41.73	17	42.66
18	38.64	18	39.72	18	40.76	18	41.74	18	42.68
19	38.66	19	39.74	19	40.78	19	41.76	19	42.70
20	38.67	20	39.76	20	40.79	20	41.77	20	42.71
21	38.69	21	39.78	21	40.81	21	41.79	21	42.72
22	38.71	22	39.79	22	40.82	22	41.81	22	42.74
23	38.73	23	39.81	23	40.84	23	41.83	23	42.76
24	38.75	24	39.83	24	40.86	24	41.84	24	42.77
25	38.76	25	39.85	25	40.88	25	41.86	25	42.78
26	38.78	26	39.86	26	40.89	26	41.87	26	42.80
27	38.80	27	39.88	27	40.91	27	41.89	27	42.82
28	38.82	28	39.90	28	40.92	28	41.90	28	42.83
29	38.84	29	39.92	29	40.94	29	41.92	29	42.85
30	38.86	30	39.93	30	40.96	30	41.93	30	42.86
31	38.88	31	39.95	31	40.98	31	41.95	31	42.88
32	38.89	32	39.97	32	40.99	32	41.97	32	42.89
33	38.91	33	39.99	33	41.01	33	41.99	33	42.91
34	38.93	34	40.00	34	41.02	34	42.00	34	42.92
35	38.95	35	40.02	35	41.04	35	42.02	35	42.94
36	38.97	36	40.04	36	41.06	36	42.03	36	42.95
37	38.99	37	40.06	37	41.08	37	42.05	37	42.97
38	39.00	38	40.07	38	41.09	38	42.06	38	42.98
39	39.02	39	40.09	39	41.11	39	42.08	39	43.00
40	39.04	40	40.11	40	41.12	40	42.09	40	43.01
41	39.06	41	40.13	41	41.14	41	42.11	41	43.03
42	39.08	42	40.14	42	41.16	42	42.12	42	43.04
43	39.10	43	40.16	43	41.18	43	42.14	43	43.06
44	39.11	44	40.18	44	41.19	44	42.15	44	43.07
45	39.13	45	40.20	45	41.21	45	42.17	45	43.09
46	39.15	46	40.21	46	41.22	46	42.19	46	43.10
47	39.17	47	40.23	47	41.24	47	42.21	47	43.12
48	39.18	48	40.24	48	41.26	48	42.22	48	43.13
49	39.20	49	40.26	49	41.28	49	42.24	49	43.15
50	39.22	50	40.28	50	41.29	50	42.25	50	44.16
51	39.24	51	40.30	51	41.31	51	42.26	51	43.17
52	39.26	52	40.31	52	41.32	52	42.28	52	43.18
53	39.27	53	40.33	53	41.34	53	42.30	53	43.20
54	39.29	54	40.35	54	41.35	54	42.31	54	43.21
55	39.31	55	40.37	55	41.37	55	42.33	55	43.23
56	39.33	56	40.38	56	41.39	56	42.34	56	43.24
57	39.35	57	40.40	57	41.41	57	42.36	57	43.26
58	39.36	58	40.42	58	41.42	58	42.37	58	43.27
59	39.38	59	40.44	59	41.43	59	42.39	59	43.29
Horz. Dist.	81.47	Horz. Dist.	80.09	Horz. Dist.	78.68	Horz. Dist.	77.23	Horz. Dist.	75.75

TABLE 8. Scale Conversions

Fraction or Ratio	Inches to Miles	Inches to Feet
1:120	1 in. = 0.0019 mi.	(1 in. = 10 ft.)
1:600	1 in. = 0.009 mi.	(1 in. = 50 ft.)
(1:1,200)	1 in. = 0.0189 mi.	(1 in. = 100 ft.)
(1:2,400)	1 in. = 0.038 mi.	(1 in. = 200 ft.)
1:6,000	1 in. = 0.094 mi.	(1 in. = 500 ft.)
(1:12,000)	1 in. = 0.189 mi.	(1 in. = 1,000 ft.)
1:12,672	(1 in. = 0.20 mi.)	1 in. = 1,056 ft.
1:15,840	(1 in. = 0.25 mi.)	1 in. = 1,320 ft.
1:21,120	(1 in. = 0.333 mi.)	1 in. = 1,760 ft.
(1:24,000)	1 in. = 0.378 mi.	1 in. = 2,000 ft.
1:31,680	(1 in. = 0.500 mi.)	1 in. = 2,640 ft.
(1:48,000)	1 in. = 0.757 mi.	1 in. = 4,000 ft.
(1:62,500)	1 in. = 0.986 mi.	1 in. = 5,208 ft.
1:63,360	(1 in. = 1.000 mi.)	1 in. = 5,280 ft.
(1:96,000)	1 in. = 1.515 mi.	1 in. = 8,000 ft.
(1:125,000)	1 in. = 1.972 mi.	1 in. = 10,416 ft.
(1:250,000)	1 in. = 3.945 mi.	1 in. = 20,833 ft.
(1:500,000)	1 in. = 7.891 mi.	1 in. = 41,666 ft.
(1:1,000,000)	1 in. = 15.782 mi.	1 in. = 83,333 ft.
(1:2,000,000)	1 in. = 31.565 mi.	1 in. = 166,666 ft.
(1:2,500,000)	1 in. = 39.456 mi.	1 in. = 208,333 ft.
(1:3,000,000)	1 in. = 47.348 mi.	1 in. = 250,000 ft.

( ) Indicates the form most commonly used.

Compiled by J.W. Low

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